A WATER BALANGCE APPROACH TO THE ESTIMATION OF ANNUAL RUNOFF : A CASE STUDY OF THE EDEN CATCHMENT, FIFE, SCOTLAND

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## DECLARATION

I certify that this thesis is my own work and has not been submitted for any degree other than that of Master of Philosophy in the University of Edinburgh.
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## ABSTRACT

The determination of catchment runoff from rainfall data is a long-standing problem in water resource management. Traditional methods of annual runoff derivation are largely empirical, and designed for use in upland areas, where they are reasonably successful. However, in lowland catchments where evapotranspiration is a much larger part of the moisture budget, these methods are very inaccurate. A recent paper by Thom and Ledger (1976) suggested that an alternative measure of annual runoff - excess winter rain - may generate more realistic estimates of runoff in lowland areas. The purpose of this thesis is to examine the validity of the Thom and Ledger approach by applying it to an area in south-east Scotland.

The basic Thom and Ledger method is applied to the Eden catchment and the nearby Leuchars rainfall record. Over an 8-year period it is shown that 92 per cent of the variability in annual runoff from the Eden catchment can be explained by variation in potential excess winter rain at Leuchars. A significant modification to the original Thom and Ledger method is devised which gives improved estimates of runoff during the years 1968-76. However, inaccuracies still exist: the main causes are identified as errors in rainfall measurement and the inability of the water balance model to account for changes in groundwater storage.

Analysis of excess winter rain data at Leuchars and Blackford Hill, Edinburgh, reveals that the recent drought in the Eden catchment was the worst for approximately 160 years ; the one in a 100year drought event occurred during the runoff year 1972-73. It is suggested that the severity of this recent occurrence be taken into account when appraising the water resource potential of the Fife area.

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INTRODUCTION
Drought is not a phenomenon usually associated with Scotland, as is apparent from the title of the 1973 Scottish Development. Department publication concerning national water resources: "A Measure of Plenty". However, between 1969 and 1976 large areas of Scotland experienced a period of severe rainfall deficiency, as indicated in Figure 1.1. It is evident from the figure that the eastern side of the country was the most seriously affected, in particular the south-east: over the 5 -year period 1970-1974 less than 80 per cent of the (1916-1950) mean precipitation was received (Figure 1.1a); 40 out of 60 months during the same time span were drier than the (1916-1950) monthly average (Figure l.1b). This rainfall deficiency was sufficiently serious to cause acute water supply problems, as reflected by press reports at the time. For example: "Fife and Kinross water Board report reservoirs in their area 54 per cent full, the most satisfactory position since 1971" (Scotsman 1974a); "Standpipes being erected in Fife towns, consumers warned of their imminent use" (Scotsman 1974b).

The severity of this recent event, and the embarrassment it caused those responsible for providing water supplies, has aroused considerable interest in the water resources in the south-east region of Scotland. Indeed, the drought provided the impetus for the present study: how often can drought conditions of this magnitude be expected to occur in the future ? The prediction of the probability of occurrence of future hydrological events relies upon the existence


Figure 1.1a. Precipitation over period 1970-74 expressed as a percentage of the (1916-50) 5-year mean.

Figure 1.16. No of months of deficient precipitation 1970-74 based on ( $1916=50$ ) mean momthnly values.
of long-term hydrological records of which there are few in southeast Scotland. Moreover, the substantial records are of precipitation, while a more meaningful measure of available water is precipitation minus rainfall, i.e. runoff. Unfortunately, if rainfall long-term records in the region are rare, then long-term rainft records are virtually non-existent.

Various methods for the calculation of runoff from rainfall data have been proposed and employed over the years, with varying degrees of success. For example, Hawksley's Rules, the Deacon diagram and the Lapworth chart were formulated to provide guaranteed minimum yields from reservoirs during extreme drought situations. All of these approaches involve the deduction from annual rainfall of a constant figure to account for evaporation to give an annual runoff value: over a calendar year this evaporative loss is taken to be $355 \mathrm{~mm}-405 \mathrm{~mm}$, depending on the location in the British Isles (Manual of British Water Supply Practice 1954). The work of Thom and Ledger (1976) appeared to offer the possibility of establishing more accurate runoff estimates than those obtained from the empirical methods of Hawksley and his counterparts. Simply, Thom and Ledger calculated monthly balances of rainfall, $p$, and potential evaporation, $e_{p}$, (the latter estimated via the Penman formula), these monthly balances being summed to give an annual value of the difference "precipitation minus potential evaporation", $\left(P-E_{p}\right)$. Thom and Ledger applied this technique to the long-term Blackford Hill, Edinburgh, rainfall record, and hence generated a synthetic runoff series for this site for the period 1785-1976.

This thesis set out to examine the validity of the Thom and Ledger (1976) approach by applying it to another relatively lengthy rainfall record and nearby catchment. The rainfall station and catchment chosen were Leuchars, Fife, and the River Eden respectively. THE EDEN CATCHMENT

The Eden catchment area, comprising $307 \mathrm{~km}^{2}$, is situated approximately 30 km north of Edinburgh, and lies centrally in the Fife peninsula which is bounded by the Firths of Tay and Forth to the north and south respectively. (Figure 1.2). The Ochil Hills form the northern limit of the catchment, while the Lomond Hills constitute the southern boundary. Rising in the Ochils, the River Eden flows some 40 km in a north-easterly direction to enter the sea in St. Andrews Bay. The basin is relatively low-lying, having a mean height of 102 m , with some 40 per cent of the catchment below 50 m , as shown in Figure 1.3. Land-use in the catchment is predominantly agricultural.

The Eden catchment was chosen for this study because few (if any) water balance studies have been carried out in Scotland in areas which normally have limited water resources, such as this. (In fact few water balance studies have been undertaken anywhere in Scotland!) The lack of water in the Fife region is directly related to climatic factors, the available water being simplistically the difference between rainfall and evaporation. Average annual rainfall over the catchment is fairly low - 839 mm - with spring and early summer being generally dry, as is evident from Table 1.la. Table l.lc illustrates the rainfall pattern over the basin: precipitation decreases from


FIGURE 1.2 : Location of the Eden catchment
west to east, Gateside having a mean annual rainfall of 920 mm , while Leuchars receives only 661 mm on average. The location of the raingauge sites is given in Figure 1.5 (Data for Tables 1.la and 1.1 c were supplied by the Meteorological Office). In contrast to the low rainfall, evaporation in the Eden catchment is relatively high, annual potential evaporation being 471 mm on average; potential evaporation during the period April to August generally exceeds 50 mm , with June and July values being more than 80 mm on average. Table 1.1b, calculated from the Ministry of Agriculture, Fisheries and Food Technical Bulletin No. 16 (1967), gives further details. TABLE 1.1

EDEN CATCHMENT : CLIMATIC FACTORS

| Table | 1a | 1941-70 MEAN RAINFALL OVER CATCHMENT |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Month | J. |  | M. | A. | M. | J. | J1 | A. | S. | 0. | $N$. |  | Year |
| Rainfall | 85 | 57 | 54 | 49 | 66 | 55 | 82 | 82 | 73 | 86 | 80 |  | 839 |
| Table | 1b | 1950-64 MEAN POTENTIAL EVAPORATION AT MEAN CATCHMENT HEIGHT |  |  |  |  |  |  |  |  |  |  |  |


| Month | J. | F. | M. | A. | M. | J. | J1. | A. | S. | 0. | N. | D. | Year |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P.E. | 0.4 | 8 | 29 | 53 | 76 | 89 | 86 | 67 | 40 | 20 | 3 | 0.1 | 471 |

Table $\quad$ lc $\frac{\text { ANNUAL RAINFALL DISTRIBUTION OVER CATCHMENT }}{(1941-70 \text { MEAN VALUES })}$

Station Gateside Letham Pitlair Pitlessie West Hall Clatto Leuchars

| Rainfall | 920 | 767 | 750 | 800 | 740 | 840 | 661 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

(All values in mm)


In terms of moisture balance, therefore, it is evident, that in an average year the water surplus in the catchment is marginal - some 370 mm ; in the mean year evaporation exceeds rainfall from April to July resulting in a soil moisture deficit.

However, this unfavourable water balance is offset to some extent by the groundwater resources within the catchment, which the Scottish Development Department consider substantial enough to be treated as a regional resource (Scottish Development Department 1975). Figure 1.4 shows the solid geology of the basin, the 01d Red Sandstone strata - known locally as Knox Pulpit formation - constituting the underground reservoir, the rock having specific yields of up to 15 per cent. The sandstone is overlain largely by fluvoglacial deposits of sand, gravel and boulder clay. In view of the possible exploitation of the underlying 01d Red Sandstone aquifer, officials of the Fife and Kinross Water Board expressed considerable interest in a study which would give further information on the "usual" surface water conditions in the Eden catchment, since the runoff of the River Eden has been gauged only since October, 1967: This was another important factor in the selection of the Eden catchment for this study.

A further factor in the choice of the Eden basin was its excellent instrumentation network,shown in Figure 1.5. The proximity of the Leuchars climatological station to the catchment, shown in Figure 1.5, was also very significant, Leuchars having extensive meteorological data, and in particular a rainfall record continuous since 1922. A final determinant was one of convenience: the

catchment is not far from Edinburgh - more or less 30 km ; most sources of information are also relatively close at hand, the Fife and Kinross Water Board Offices, for example, being situated in Glenrothes, the Tay River Purification Board headquarters being in Perth.

ORGANIZATION OF THE THESIS
Chapter 2 reviews the methods of deriving runoff from rainfall data, including a detailed analysis of Penman's revolutionary formula for the calculation of potential evapotranspiration; the work of Thom and Ledger (1976) is also briefly outlined. In Chapter 3 the Thom and Ledger approach is applied to the Eden catchment and the Leuchars rainfall record: the methodology involved is scrutinized critically. The sources of error in the Thom and Ledger (1976) method are identified at the beginning of Chapter 4. This chapter is subsequently divided into five sections, the first of which proposes a significant modification to the Thom and Ledger approach to annual runoff estimation; each of the remaining sections examines further possible sources of error.

Chapter 5 investigates the accuracy of the basic data employed in the Thom and Ledger procedure. These data are potential evapotranspiration as deduced by the Penman formula; precipitation; and annual runoff, the vector which is used to check the accuracy of the annual runoff index suggested by Thom and Ledger. In Chapter 6 synthetic annual runoff distributions for the Eden catchment are analysed, and the problems of frequency analysis applied to hydrological data are briefly discussed. Finally, some concluding comments are made in Chapter 7.


Key to Gauging Stations
and Post Gauge Sites
O A KEMBACK (G.S.)
© в PITLESSIE (P.G.S.)
© c GATESIDE (P.G.S.)

Key to Rain Gauge Sites $/$

| $1 \Delta$ | LEUCHARS | $5 \Delta$ | PITLESSIE |
| :--- | :--- | :--- | :--- |
| $2 \Delta$ | WEST HALL | $6 \Delta$ | CLATTO |
| $3 \Delta$ | LETHAM | $7 \Delta$ | GATESIDE |

$3 \Delta$ LETHAM
4 A PITLAIR

## METHODS OF DERIVING RUNOFF FROM RAINFALL : A REVIEW

Since the time in the mid-nineteenth century when reservoir construction began on a large scale to meet industrial and population needs, the estimation of annual water yield has been of vital importance to engineers. In engineering terms the yield of an impounding reservoir is the quantity of water which can be drawn off for use, and depends upon runoff from the catchment and the storage capacity of the scheme. Whilst this thesis is not concerned with the storage aspect, it is most certainly concerned with the runoff situation, since Thom and Ledger (1976) suggested a possible improved way in which this could be calculated from available longterm rainfall records. It is hoped that by examining the problems of yield estimation historically, the Thom and Ledger process, which is by no means a novel idea, may be put into perspective.

## EARLY DEVELOPMENT AND THE RELIABLE YIELD CONCEPT

The decade 1841-50 was probably the most prolific in the development of reservoir schemes in upland areas, but due to their remote location the projects commissioned at this time had very few years runoff data on which their yield estimation could be based. Then, as now, the yields were based largely on the derivation of runoff values from the more extensive rainfall data available from the catchment and surrounding region. Thus, Hawksley (1847) - cited in Lloyd (1947) illustrates the procedure used at the time: "If evaporation be deduced from rainfall upon a district, the remainder is the quantity which may be safely calculated upon being received by the reservoir".

It has been customary in estimating runoff from rainfall in upland surface catchments in Britain to make a deduction for this evaporative loss of between 355 mm and 405 mm , the lower value being used in Northern England and Scotland (Manual of British Water Supply Practice 1954).

In an effort to standardize an absolute minimum yield value for any new reservoir scheme, Hawksley (1868) - cited in the Manual of British Water Supply Practice (1954) - introduced the Reliable or Guaranteed Yield concept. This he defined as the yield obtained numerically on the basis of runoff estimated from rainfall in the three consecutive driest years in the rainfall record. Hence, Hawksley published his " 1868 Rule":

Reliable number of days supply $=\frac{500}{\sqrt[3]{\mathrm{R}_{3 \mathrm{~d}^{-E}}}}$ - 2.1.

Where $R_{3 d}=$ rainfall in the three consecutive driest years E . = evaporation loss, taken as $355 \mathrm{~mm}-405 \mathrm{~mm}$ depending on geographical location

The rather arbitrary nature of the formula is apparent.
In 1892, in the aftermath of the serious drought five years previously, the eminent water engineer A.R. Binnie formalized the reliable yield idea. He generalized the three consecutive driest years as being equal to 240 per cent of "normal" rainfall, this value being equivalent to the runoff value in a hypothetical 80 per cent runoff year. From this was subtracted the accepted 380 mm for evaporation, giving the runoff basis for reliable yield calculations. This'"Reliable Yield" rule of 1892 forms the present legal basis for

British water supply projects. It is relevant to note that the very doubtful assumptions concerning the steady nature of annual evaporation and the generalization of the rainfall in the three driest years are inherent in this "Reliable Yield" rule.

Hawksley's (1868) rule envisaged a reservoir of one stated size for a particular catchment. At the turn of the century G.F. Deacon produced his well-known "Deacon Diagram" to allow for more flexibility in storage capacities and thus design of impounding schemes. The nature and form of the diagram are not relevant to this work, but it is pertinent to point out that he persisted with the assumed 380 mm annual evaporation loss to transform rainfall to streamflow for reliable yield calculations.

Despite the empiricism involved, the above methods are regularly used when calculating yields for new reservoirs, even to the present day. For example, Lewis (1957) cited the Deacon method being used for the reliable yield calculations for the Alwen reservoir in North Wales, with the customary 380 mm (approximately) evaporation deduction to derive annual runoff values. Similarly, when considering the Rivington scheme in Lancashire, Lloyd (1947) used Deacon's and Hawksley's procedures to estimate yields. Risbridger and Godfrey (1954) used both the above methods in giving yields for the Elan and Claerwen catchments in Wales.

However, the publications quoted above are largely investigative, and the authors showed considerable dissatisfaction with these "rule-of-thumb" methods. Apart from theoretical considerations, the evidence of rainfall and accompanying streamflow data - accumulated
since the time the reservoirs were constructed - have shown the rainfall/runoff assumptions inherent in the methods to be excessively simple. Thus, Lewis (1957) stated: "no definite relationship exists between rainfall and runoff", and suggested that over a period of years annual runoff varied with rainfall in an oscillatory manner. Risbridger and Godfrey (1954) found that wide divergences in runoff occurred between years of sensibly equal precipitation. Morton - quoted in Lloyd (1947) - was of the opinion that streamflow gaugings are the only reliable basis for estimating yield, since losses, the chief one being evaporative, are extremely uncertain.

In the preceding discussion it has been noted that the rules and formulas derived have been for application to upland areas, since reservoirs by their nature are usually located in these regions due to the high precipitation incident on their contributory areas. Considerable doubt has been expressed about the application of these speculative methods to the rainfall/runoff relationships in these upland locations. However, these relationships may be described as excellent when compared with the association of the two variables in lowland catchments. This is illustrated in Figure 2.1., where annual runoff and precipitation data for the years 1958-1976 - based on the hydrological year October-September - are plotted for lowland and upland catchments in Scotland. Allt Uaine is located in a wet upland area of Argyllshire in western Scotland, and the River Tyne catchment is situated in the relatively dry low-lying eastern part of the country.

The correlation coefficient $-r=0.95$ - for the Allt Uaine

catchment indicates a close relationship between rainfall and runoff. A more helpful parameter, the coefficient of determination, $r^{2}$, indicates that 90 per cent of the variability in annual runoff can be accounted for by variation in annual rainfall. Clearly, this close association must vindicate to some extent the traditional methods of reliable yield estimation, in particular the assumption that annual evaporation is approximately constant. Indeed, Law (1953) published a very close correlation between annual rainfall and runoff for the Derwent catchment in Yorkshire and, hence, considered that runoff could be satisfactorily deduced directly from rainfall in this catchment. However, for the low-lying Tyne catchment in East Lothian, only 44 per cent of the variation in annual runoff is attributable to variability in annual rainfall. In this case evaporation can not be taken as some arbitrary constant, and methods of measuring or estimating it appear very necessary.

At the turn of the century the prospects of accurate estimation seemed gloomy. Thus, Abbe (1905) mirrored the views held by many at that time: "The loss of water evaporation in nature is so mixed up with leakage and consumption by plants that our meteorological data are of comparatively little importance".

Nevertheless, over the years, various equations have been published to quantify this evaporation loss factor. The Institution of Water Engineers (1936) came to the conclusion that the deduction of 355 mm to 405 mm annually for evaporation loss in the three driest years was quite insufficient, and from examination of their records developed the expression:

$$
L=C 4.8 \sqrt[3]{\mathrm{R}}
$$

where $L=$ total average loss (in inches) per annum
$R=$ long-term average rainfall in inches
$C=$ coefficient depending on the hydrological characteristics of the individual drainage area (commonly assumed to be unity)

Another equally theoretically doubtful formula was stated by Vermeule - cited in Lewis (1957) - as below:

$$
E=15.5+0.16 R
$$

where $E=$ evaporation (in inches)
$R=$ annual runoff (in inches)
The equation is designed for use at $50^{\circ} \mathrm{F}$; a correction factor of ( $0.05 \mathrm{~T}-1.48$ ) is applicable, where $T$ is the temperature in degrees Fahrenheit.

On a slightly more secure theoretical basis is the formula derived for any catchment area by Lloyd (1942), for annual evaporation loss, $E$, in inches, based on rainfall, $R$, at a mean air temperature, $T$, (in degrees Fahrenheit), and duration of sunshine hours, $S$, as:
$E=0.57 \mathrm{R}^{0.97}+1.10(\mathrm{~T}-48)+0.006(\mathrm{~S}-1450)-2.4$.
While Lloyd's (1942) formula takes account of some of the factors determining evaporation - sunshine and air temperature - the equation is still rather hypothetical, employing an abundance of empirically derived constants, e.g. (S-1450).

Equations 2.1. - 2.4. were designed for use in the British Isles and, in fact, produce very different estimates of evaporative loss.

These discrepancies further emphasize the empiricism of the formulae and thus must cast serious doubt on their use.

The major breakthrough in evaporation estimation came with the publication of the work of H.L. Penman and C.W. Thornthwaite. In Britain and America respectively, the two men and their research groups had independently developed the concept of evapotranspiration over a period of years, and produced their now well-known formulas. Since both formulas are widely used in estimating evaporation in a water balance approach to runoff estimation, each will now be examined. THE THORNTHWAITE METHOD

After earlier work with Holzman using an aerodynamic approach (Thornthwaite and Holzman 1939), Thornthwaite presented his formula in 1948, and later modified it in co-operation with Mather (1954, 1955). The formula is largely empirical, based only on mean air temperature, $T$, with adjustments for length of daylight hours. A monthly heat index, $i$, is evaluated from the mean monthly temperature, $T$, in degrees Celsius as below:

$$
\left.\mathbf{i}=\left\lvert\, \frac{1}{5}\right.\right)^{1.514}
$$

Twelve consecutive monthly indices are obtained and summed to give the annual heat index, $I$, of the site. The potential evapotranspiration, PE, defined by Thornthwaite (1954) as: "the amount of water which will be lost from a surface completely covered with vegetation if there is sufficient water in the soil at all times for the use of the yegetation" is calculated as:

$$
P E=\frac{1.6 \mathrm{~b}(10 \mathrm{~T})^{a}}{\mathrm{I}} \mathrm{~mm} / \text { day } \quad-2.6
$$

where $T=$ mean air temperature (in ${ }^{\circ} \mathrm{C}$ ) over the whole period $a=$ cubic function of annual heat index b $=$ correction factor for season and latitude Thornthwaite's formula has been widely criticised, mainly because of its inherent empiricism. Over the years Thornthwaite has attempted to answer his critics, although his arguments have been less than convincing. For example, Thornthwaite (1954) justified his selection of mean air temperature as the main parameter influencing $P E$ on the grounds that there is a fixed relationship between that part of net radiation which is used for heating, and that part which is used for evaporation, when the soil is continuously moist. Thus, Thornthwaite claimed his method estimates PE by an indirect reference to the radiation balance at the earth's surface. However, there is no such thing as a fixed relationship between sensible heating and evaporation, even when water is non-limiting, i.e. the relationship between the two variables depends on radiation and ventilation in a very complex manner; it is in fact a function of climate (Thom. pers. comm). Thornthwaite's (1954) claim that mean air temperature is a conservative parameter which integrates humidity factors and air movement is somewhat more realistic. An extremely serious defect in the Thornthwaite formula is in locations where climatic conditions are seriously affected by the advection of moist or dry air, since this results in frequent rapid changes in mean air temperature, the very parameter upon which the formula is based. This is the case in

Britain where advection effects are substantial due to its maritime position and the continual interchange of polar and maritime air masses: hence the formula is relatively unsuccessful in Britain (Ward 1975).

Despite the criticism, the formula is frequently employed, although mainly in areas where meagre climatic records exist. It is probably for this reason that Ward (1975) considered the application of the Thornthwaite formula to be reasonable in many climatic areas of the world. In Britain, however, there is little justification for its uses since, apart from theoretical considerations, adequate meteorological records are generally available.

THE PENMAN METHOD
The Penman (1948) formula combined the turbulent transfer and energy balance approaches to evaporation estimation. In its original form (1948) the method was essentially a two-stage process, involving the calculation of evaporation from a hypothetical open water surface, $E_{0}$, and then the conversion of $E_{0}$ to the potential evapotranspiration, $P E$, from short green vegetation completely covering the ground, and adequately supplied with water. Penman expressed the relationship between $E_{0}$ and $P E$ as:

$$
P E=f E_{0}
$$

where the empirical factor, $f$, was found to be 0.8
in the summer, and 0.6 in the winter.
Although these values of $f$ were determined for south-east England, they are probably valid to within about 15 per cent in all temperate climates (Monteith 1973).

The turbulent transfer or aerodynamic element in the Penman equation, $E_{0}$, is firmly based on the Daltonian expression:

$$
E=f(u)\left(e_{s}-e_{a}\right)
$$

where $E=$ rate of evaporation
$f(u)=$ function of horizontal windspeed
$e_{s}=$ saturated vapour pressure at the surface
$e_{a}=$ atmospheric vapour pressure

Thus, the removal of water vapour from the evaporating surface is governed by the gradient of humidity and by windspeed. Penman (1948) expressed this "drying power" of the air as:

$$
E_{a}=0.35\left(1+9.8 \times 10^{-3} u_{2}\right)\left(e_{a}-e_{d}\right) \mathrm{mm} / \mathrm{day}
$$

where $E_{a}=$ rate of evaporation
$e_{a}=$ saturated vapour pressure over water at the mean air temperature, $\mathrm{T}^{\circ} \mathrm{F}$, in mm Hg
$e_{d}=$ mean vapour pressure of the atmosphere in mm Hg
$u_{2}=$ windspeed in miles per day measured at 2 m
In equation 2.9. the vapour pressure gradient, as a difference, is implicit in the product $0.35\left(e_{a}-e_{d}\right)$, and the windspeed is accounted for in $u_{2}$, which can be equated to ( $\left.u_{2}-0\right)$, since the surface windspeed is zero. Following experimental work at Lake Hefner in the U.S.A., Penman (1956a), modified his aerodynamic term to:

$$
E_{a}=0.35\left(0.5+10^{-2} u_{2}\right)\left(e_{a}-e_{d}\right) \text { mm/day } \quad-2.10
$$

All the variables are as previously defined.
Clearly, Penman considered that this was the proper wind function to apply to open water.

The radiation term in the Penman equation, $H$, known as the surface net radiation (or radiation balance at the surface), is the difference between absorbed incoming short-wave radiation, $R_{I}$, and net terrestrial radiation, $R_{B}$, as shown in equation 2.11:

$$
H=R_{I}(1-r)-R_{B}
$$

where $r=$ the reflectivity or albedo of the surface
Penman (1948) quantified this equation using the theoretical radiation intensity at the ground surface in the absence of an atmosphere, $R_{a}$, the ratio $n / N$ of actual ( $n$ ) to possible (N) hours of sunshine and the theoretical black body terrestrial radiation leaving the ground in the absence of atmosphere, $\sigma T_{a}^{4}$. Thus, equation 2.11 became:

$$
\begin{aligned}
H= & (1-r) R_{a}\left(0.18+0.55^{n} / N\right)-\sigma T_{a}^{4} \\
& \cdot\left(0.56-0.09 \sqrt{e_{d}}\right)\left(0.10+0.90^{n} / N\right) m m / \text { day }
\end{aligned}
$$

where $\sigma=$ Stefan's constant

$$
T_{a}=\text { mean air temperature in degrees Kelvin }\left({ }^{0} K\right)
$$

This net input of radiative energy, $H$, is expended as shown in equation 2.13.
$H=E+K+(S+C+M) \quad-2.13$.
where $E=$ energy used in evaporation (latent heat)
$K=$ energy used in heating the air (sensible heat)
$S=$ heat transfer to the surface (sensible heat)
$C=$ heat conducted in or out of the surface
(sensible heat)
$M=$ energy used in melting snow (latent heat)
Since, for practical purposes, S, C, and M, are relatively small,
Penman apportioned $H$ between sensible heat transfer to the air, $K$, and
evaporation, E. Thus, equation 2.12. was simplified to:

$$
H=E+K
$$

Penman then estimated the proportion of net radiation, $H$, used in evaporation by combining equations 2.10 and 2.11 to give the wellknown Penman formula:

$$
E_{0}=\frac{Y H+E_{a}}{\underline{\Delta}+T} \mathrm{~mm} / \text { day }
$$

$\gamma$
where $\Delta=$ slope of the saturation vapour pressure curve against temperature at $T_{a}$ in $m m$ of mercury ( mmHg ) per degree Fahrenheit ( ${ }^{\circ} \mathrm{F}$ ) $Y=$ psychometric constant -0.27 mm Hg per ${ }^{\mathrm{O}} \mathrm{F}$ at sea-level

Subsequent work on albedo measurements by Monteith (1959) led Penman (1962) to the conclusion that the two stage process to estimate PE, expressed in equation 2.7, was unnecessary. The magnitude of the factor, $f$, and its seasonal behaviour were found to be consistent with the respective albedos of water and short green vegetation (about 0.06 and 0.25 ), so that the net radiative energy input was determined by:

$$
\begin{aligned}
H= & 0.75 R_{a}\left(0.18+0.55 \frac{n}{N}\right)-\sigma T_{a}^{4} \\
& \left(0.56-0.09 \sqrt{e_{d}}\right)(0.10+0.90: n / N)-m m / \text { day }
\end{aligned}
$$

In order to account for the extra "roughness" of a crop compared with open water, the aerodynamic term became:

$$
E_{a}=0.35\left(1+10^{-2} u_{2}\right)\left(e_{a}-e_{d}\right) \mathrm{mm} / \text { day } \quad-2.17
$$

Thus, the resulting evapotranspiration from a vegetated surface, PE, became:
$\Delta$


The Penman method of estimating PE requires only the measurement of mean temperature, mean vapour pressure, mean daily wind-speed, and duration of bright sunshine, all these variables being readily measured at meteorological stations. Therefore, as Penman intended, only standard meteorological data are needed as input for the formula, rather than measurements of surface temperature, and its associated parameter, surface vapour pressure, which are rarely made.

According to Ward (1975), the Penman formula is probably the most comprehensive approach to the estimation of PE which has so far been devised and takes into account almost all of the factors which are known to influence it. Thom and 0liver (1977) considered that Penman gave the first physically sound treatment of the difficult problem of estimating evaporation from a natural surface, and further that Penman's method is simple and realistic. Even Thornthwaite stated that the Penman method must be regarded as one of the most widely known and respected systems. (Thornthwaite and Hare 1965). Nevertheless, criticism of the method is frequent. This is not really surprising since Penman's (1956b) definition of PE: - "the amount of water to be transpired in unit time by an extended surface of short green crop of uniform height, completely shading the ground, and never short of water" - was framed to exclude the following
important factors:
(a) advective effects since the surface is extended ;
(b) physical characteristics of the vegetation, i.e. height, shape, and surface "roughness", as the crop is "uniform" ;
(c) moisture movement and moisture control in the plant and soil since the crop is "never short of water" ;
and (d) albedo effects since the crop is uniformly green.
From his definition of PE, it is clear that Penman considered evaporative loss from vegetated surfaces to be controlled entirely by meteorological conditions, i.e. Penman considered $P E$ to be a function of climate. Unfortunately, subsequent research has raised severe doubts concerning Penman's suppositions. For example, Chang (1965), Van Hylckama (1963), Rijtema (1966), Van Bavel et al (1963), have published data which strongly suggest plant control over water loss. De Vries and Van Duin (1953), and others, have presented well documented evidence which directly opposes Penman's premise of insignificant aerodynamic roughness. With reference to supposition (a) concerning advective effects, it is not clear whether by "extended" Penman meant bigger than -
( i) a plant pot ; or
(ii) a bowling green ; or
(iii) a field of crops ; or
(iv) a county ; or
$\because$ (v) a country.

- (Thom. pers. comm.). This is important because it has bearing on the meaning of the words "advective effects" ; these occur on all
scales from (i) to (v) and even bigger, indeed up to the global scales. In (i) and $(i \mathbf{i})$ however, they may properly be referred to as edge effects, and it is only these which Penman's premise excludes. Effects of advection on larger scales are truly part of climate and, as such, are dealt with (or at least covered) by the Penman approach.

The Penman formula is also criticised because of the empiricism it contains. (This appears to be an occupational hazard for the initiators of evapotranspiration formulae). The constants in the equation have been empirically derived and therefore are not universal. In particular, the constants used in the energy budget term are especially weak, since the amount of sunshine and theoretical radiation intensity are equated to total incoming radiation. However, this depends upon a statistically random distribution of bright sunshine throughout the day, which Baier and Robertson (1965) considered limits the formula's usefulness to periods of at least 10 days.

- Over the years since its initial publication, the Penman formula has been modified, by Penman himself and others, to overcome some of its comparatively minor inadequacies. The majority of the suggested modifications concern the radiation terms, given in equation 2.12. For example, an important addition to the Penman (1948) equation for outgoing radiation, $R_{B}$, is the multiplication of this term by 0.95 , thus allowing for the vegetated surface not acting as a perfect "black body" (Budyko 1958). An alteration of the incoming short-wave radiation, $R_{I}$, term was proposed by Glover and McCulloch (1958):

$$
R_{I}=(1-r) R_{a}(0.29 \cos (1 a t)+0.52 n / N) m m / \text { day }-2.19
$$

This proposal was based upon regression analysis of actual values of radiation and hours of sunshine, and gives a better relationship over the range of latitude from $0^{\circ}$ to $60^{\circ}$ (Ward 1975). Later modifications of the radiation terms were put forward by L. P. Smith of the Meteorological Office. Smith's modifications have been put into practice in Ministry of Agriculuture, Fisheries and Food Technical Bulletin No. 16 (1967), where tables of the average potential evapotranspiration of each county in Britain are given. Smith (1967) proposed that:

$$
\begin{align*}
& H= 0.75 \mathrm{R}_{1}-\sigma \mathrm{T}_{\mathrm{a}}{ }^{4}\left(0.47-0.075 \Gamma_{\mathrm{e}_{\mathrm{d}}}\right)(0.17+0.83 \mathrm{n} / \mathrm{N}) . \\
& \mathrm{mm} / \mathrm{day}
\end{align*}
$$

where $R_{1}=R_{a} f_{a}(n / N)$, where $f_{a}$ takes the following forms $R_{1}=R_{a}(0.135+0.68 \mathrm{n} / N)$ for smokey areas when $n / \mathrm{N} \quad 0.40$ $R_{T}=R_{a}(0.16+0.62 \mathrm{n} / \mathrm{N})$ for latitude south of $54 \frac{1}{2} \mathrm{O}_{\mathrm{N}}$ $R_{1}=R_{a}(0.155+0.69 \mathrm{n} / \mathrm{N})$ for latitudes $54 \frac{1}{2}{ }^{\mathrm{o}} \mathrm{N}$ to $56^{\circ} \mathrm{N}$ (i.e. the British Isles)

Smith's (1967) amended radiation terms are in fact those used in this work to derive potential evapotranspiration for the Eden catchment.

Modifications to the ventilation term, $E_{a}$, in the Penman equation to allow for "plant factors" have also been suggested. Monteith (1965) introduced factors concerned with the nature of the crop, namely its aerodynamic resistance ( $r_{a}$ ) and surface (stomatal) resistance $\left(r_{s}\right)$ to estimate the latent heat flux ( $\lambda E$ ) from an unsaturated surface:

$$
\begin{aligned}
& \lambda E=\frac{\Delta H+\varnothing c\left(e_{s}(T)-e_{d}\right) / r_{a}}{\Delta+Y\left(1+r_{s} / r_{a}\right)} \\
& \text { where } \varnothing=\text { density of air } \\
& c=\text { specific heat of the air } \\
& e_{s}(T)=e_{a}
\end{aligned}
$$

Other parameters are as previously defined.
Monteith's (1965) form of the equation has the aim of supplementing the largely meteorological approach with physiological concepts of the control of the passage of water vapour from the leaf to the atmosphere ; one control is the size and distribution of the stomata, the other the aerodynamic properties of the plant surface. Unfortunately, the main problem with this variant of the Penman approach is the difficulty of obtaining adequate measurements of the crop factor. However, several studies have been made in which values of $r_{s}$ and $r_{a}$ have been determined, the results of Szeicz et al (1969) being given in Table 2.1. The considerable variations of $r_{a}$ and $r_{s}$ from month to month, and surface to surface, are clear.

Thom and Oliver (1977) considered that the form of the Penman equation most extensively applied in hydrometeorology, given in equation 2.18., underemphasises the importance of ventilation relative to radiation in maintaining regional evaporation. Therefore, Thom and Oliver (1977) proposed a generalized ventilation term, $\mathrm{E}_{\mathrm{a}}$, to account for the aerodynamic roughness of the vegetated surface. Their amended $E_{a}$ term was expressed as:

TABLE 2.1.
VALUES OF AERODYNAMIC RESISTANCE, $r_{a}$, AND SURFACE (STOMATAL) RESISTANCE, $r_{s}$, AND EVAPORATION, E, FOR VARIOUS SURFACES IN SOUTHERN ENGLAND. (All values in mm/day) (from Szeicz et al (1969))

$E_{a}=13.8\left(e_{a}-e_{d}\right)\left(1-10^{-2} u_{2}\right) /\left(\operatorname{Ln}\left({ }^{z} / z_{0}\right)\right)^{2} \mathrm{~mm} /$ day -2.22.
where $z=2 \mathrm{~m}$ above the effective surface of the vegetation $z_{0}=$ aerodynamic roughness parameter of the vegetation (of the order $h / 10$, where $h$ is vegetation height)
The overall effect of this amendment to the $\mathrm{E}_{\mathrm{a}}$ term is the modification of the Penman equation 2.18 to:

$$
\begin{aligned}
& P E=\frac{\Delta(H-G)+m Y E_{a}}{\Delta+Y(1+n)} \\
& \text { where } n= r_{S / d a y}=1.4 \text { (2) for a typical unforested } \\
& \text { lowland English county } \\
& m= 2.5=f\left(z / z_{0}\right) \\
& G= \text { heat flux to the soil (proposed separately } \\
& \text { by Monteith but included by Thom and 01iver } \\
& \text { (1977) }
\end{aligned}
$$

Equation 2.23 predicts 40 mm more evaporation in the winter half-year, and 40 mm less in the summer, than does equation 2.18. (Thom and Oliver 1977).

Despite the criticism and subsequent modifications, the frequently used form of the Penman equation, given in equation 2.18, must still be regarded as an extremely attractive method of estimating PE. It has sound basis in theoretical considerations, it employs easily obtainable data and significantly: "the formula has worked satisfactorily in a wide range of climates" (Penman 1963). Moreover, it is clear that the Penman approach is in a different class from the Thornthwaite method. As Prus-Chacinski (1977) stated: "the Penman formula still reigns supreme worldwide". A more detailed examination
of the accuracy of the Penman formula applied to British conditions is undertaken in Chapter 5.

THE APPLICATION OF THE PENMAN FORMULA TO A LOWLAND CATCHMENT
Penman's (1950) work on the Stour catchment, Essex, is a good example of the application of his equation in water balance calculations. An accurate catchment water balance for any time period can be expressed as:

```
P-Q - E - \DeltaS - \DeltaG = 0,
where P = precipitation
    E = actual evapotranspiration
    Q = runoff
    \DeltaS = change in soil moisture
    \DeltaG= change in ground water storage
```

- 2.24.

All values in mm.
Penman's purpose in this particular work was to discover how well his evapotranspiration estimate, E, compared with evaporation estimated from measured catchment data $-P, Q, \Delta S$ and $\Delta G$ in equation 2.24.

The procedure adopted by Penman (1950) initially involved the calculation of potential evapotranspiration, PE, via his equation. Since the requirements of PE - a continuous water supply and complete vegetation cover - are seldom fulfilled, these PE values are generally greater than the corresponding actual evapotranspiration, $E$, values. Therefore, to allow for this difference between PE and E, Penman proposed the following model of soil moisture movement: Field capacity was defined as the maximum amount of undrainable water held by the soil against gravity. When transpiration occurs, the moisture content
of the soil is reduced to an amount less than this field capacity. The difference between field capacity and the reduced amount Penman referred to as the Soil Moisture Deficit. As the drying of the soil increases through the summer, Penman proposed that a stage would be reached when potential transpiration, PE , would fall below its maximum rate, since the moisture present in the soil would be outwith the range of the vegetation roots. To allow for this, Penman (1949, 1950) introduced the concept of a Root Constant, C, which could vary with root development and, therefore, crop type. This root constant, C, was defined as: "the amount of water within the vegetational rooting zone plus 25 mm which is available for transpiration at the potential rate". After this amount has been transpired, the rate was expected to fall towards about 10 per cent of the potential rate (Penman 1949).

Penman then applied the root constant idea to the Stour catchment. The catchment was divided into three: the riparian area, those parts of the catchment remote from the river, and the intermediate areas. It was assumed that within the riparian areas there was never any check to transpiration, but that root constants of 200 mm and 130 mm applied to the intermediate and remote areas respectively. Employing different proportions of the three areas, potential evaporation values were amended to actual values for the basin. To complete water balance data, well levels in the catchment were monitored on a monthly basis to give groundwater storage changes, $\Delta G$; precipitation, $P$, was measured daily at four gauges within the area ; runoff, $Q$, was recorded at the Stratford gauging station ; and soil
moisture changes, $\Delta S$, were estimated.
Penman achieved what he considered to be highly satisfactory results for data covering the years 1933-1948. In fact, the basin evaporation estimated using 25 per ceńt riparian, 25 per cent intermediate, and 50 per cent remote, turned out to be exactly the same as the 1933-1948 mean water balance estimate.

THE THOM AND LEDGER (1976) METHOD
The Thom and Ledger (1976) method was designed to deduce annual runoff, Q, and utilized a simplified water balance approach. Thom and Ledger introduced the parameter Excess Winter Rain, R, which was equated to the difference between annual rainfall, $P$, and annual evaporation, E :

$$
R=P-E \quad m m
$$

Changes in water storage $-\Delta G, \Delta S$ - were assumed to be negligible, so that $R$ was deemed to be equivalent in an uncomplicated fashion to annual runoff, Q :

$$
\mathrm{R} \equiv \mathrm{Q} \quad \mathrm{~mm}
$$

Actually, Thom and Ledger (1976) utilized (a) mean annual potential evaporation values, $\bar{E}_{p}$, derived using L.P. Smith's (1967) tables of mean (1950-1964) county evaporation and, (b) annual rainfall values, $P$, for the Blackford Hill station, Edinburgh, to generate potential values of excess winter rain, $R_{p}$, as:

$$
R_{p}=P-\bar{E}_{p} m m
$$

$R_{p}$ values were derived in this fashion for each year in the long-term - 1785-1976 - Blackford Hill precipitation record. Since actual evaporation tends to lag behind potential evaporation, in general
these $R_{p}$ values would be smaller than the corresponding $R$ values, and, therefore, $R_{p}$ would be expected to. underestimate $Q$ if employed directly, i.e. in equation 2.27. Nevertheless, $R_{p}$ can be expected to be unambiguously related to annual runoff, $Q$. Thus, in effect, Thom and Ledger (1976) established a long-term runoff record for this particular Edinburgh site.

However; to generate confidence in this artificial runoff record Thom and Ledger (1976) decided that it was necessary to compare their deduced $R_{p}$ values with available measured annual runoff, $Q$, values of rivers in the proximity of Edinburgh. Employing a regression analysis technique, good agreement between $Q$ and $R_{p}$ was found for nearby Lothian catchments - the Tyne, Esk, and Peffer/Pilmuir. Unfortunately, the longest runoff record for these rivers was only 13 years, so that it was considered unwise to calibrate a 200 -year synthetic runoff series using such limited data. Therefore, the hydrological characteristics of the several catchments were used to deduce the annual runoff from each as a function of potential excess winter rain, $R_{p}$, at Blackford Hill.

When the measure of annual runoff-potential catchment excess winter rain, $R_{p c}$ - derived via the hydrological characteristics of each catchment, was compared with $R_{p}$ at Blackford Hill, satisfactory agreement was evident. However, $R_{p c}$ underestimated $Q_{C}$ for all three catchments. Even when these $R_{p c}$ values were converted to actual catchment excess winter rain, $R_{c}$, values via Penman's (1950) root constant distribution concept, underestimation of measured runoff, $Q_{C}$, was still apparent. Nevertheless, with some justification, Thom and

Ledger (1976) considered excess winter rain - either potential, $R_{p}$, or actual, $R_{C}$, to be a reasonably reliable index of annual runoff, $Q$. Moreover, it was suggested that the synthetic runoff series derived from this index could be of possible use to those concerned with water supply schemes.

However, as Thom and Ledger (1976) used a single rainfall record and catchments within a limited area, it is possible that their encouraging results were mere coincidence, particularly since the available runoff records were relatively short. Furthermore, the Thom and Ledger (1976) water balance model was based on monthly data, whereas Grindley (1967) suggested daily balances were very necessary for reasonable accuracy. The realism of Thom and Ledger's assumption of negligible storage in their water balance must also be questioned. Not surprisingly,Law (1977) considered that the Thom and Ledger (1976) method should be tested thoroughly in any new situation before being used. Therefore, to check the validity and accuracy of the Thom and Ledger method, their approach was applied to a different rainfall record and associated catchment - that of the Leuchars meteorological station, Fife, and the nearby Eden catchment.

THE THOM/LEDGER METHOD APPLIED TO THE LEUCHARS
RAINFALL RECORD AND THE EDEN CATCHMENT .
THE LEUCHARS RAINFALL SERIES
Precipitation has been recorded on a daily basis at Leuchars since 1922, and during this time the gauge site has been moved once, in 1959, when it was re-located 1.1 km to the north-west of its original position. Leuchars is listed as one of the Meteorological Office "Key Stations" in the British rainfall network, which indicates a reasonable degree of accuracy. Further, the new site is considered by a Meteorological Office Inspector to be "very good", particularly as regards exposure. (Wilson, Meteorological Office, pers. comm.). The gauge movement obviously renders the rainfall record non-homogeneous, although for the purposes of this work the move was not considered very significant since the gauge was moved only a short distance over level ground, and there was no reason to expect a great difference in "catch". Thus the Leuchars precipitation record was assumed to be a reasonably accurate single data set. INDEX OF ANNUAL RUNOFF

The index of annual runoff used by Thom and Ledger (1976) - excess winter rain, $R$, was defined as: "the amount by which precipitation exceeds (actual) evaporation between the (autumnal) date of return to field capacity and the date of the initiation of the following year's soil moisture deficit". They stated their idea of soil moisture deficit, D, as: "the net amount by which (actual) evaporation exceeds effective precipitation from the time when the soil is at
field capacity", and effective precipitation as: "the (usually large) part of the precipitation falling in a period of soil moisture deficit which serves to reduce that deficit". This effective rainfall clause was designed to ignore the direct runoff component resulting from intense rainfall. However, in hydrological terms "effective rainfall" is usually interpreted as meaning precipitation that causes runoff. Hence, in accordance with Rodda's (1977) criticism "effective precipitation" is now amended by the author to "accepted precipitation".

Since potential evaporation values are readily available from the Penman formula, and the actual values are far more complex to deduce, it is not surprising that Thom and Ledger decided to use potential values of excess winter rain and soil moisture deficit. Therefore potential excess winter rain, $R_{p}$, was formally defined as: "the amount by which precipitation exceeds potential evaporation between the potential date of return to field capacity and the date of the initiation of the following year's (actual or potential) soil moisture deficit". Derivations of $R_{p}$ and $D_{p}$ - maximum potential soil moisture deficit - for an average year at Leuchars is shown in Table 3.1 and interpreted diagramatically in Figure 3.1. Potential soil moisture deficit was calculated by summing the monthly differences between estimated potential evaporation, $\bar{e}_{p}$, and measured rainfall, $p$, starting in the month when $e_{p}$ exceeded $p$. These accumulated values give the maximum potential soil moisture deficit, $D_{p}$, which occurred during 1970 in July. After the maximum deficit has been achieved rainfall gradually decreases the deficit,


FIGURE 3.1 : Sample derivation of potential excess winter rain, $R_{r}$, and maximum potential soil
runoff year, 978 isture deficit, $D p$, at. Leuchars durínc a normal
until the soil is once more at field capacity. Once field capacity has been reached, the excess of rainfall, $p$, over potential evaporation, $\bar{e}_{p}$, is accumulated to give the potential excess winter rain value, $\mathrm{R}_{\mathrm{p}}$.
TABLE 3.1.
DERIVATION OF $R_{p}, D_{p}$, FOR RUNOFF YEARS AT LEUCHARS, FIFE:
Table 3.1. A NORMAL RUNOFF YEAR 1970-71
Year mm J. F. M. A. M. J. J1. A. S. $\quad$. $\quad$. N. D.

|  | $e_{p}$ | 0.0 | 10.2 | 30.5 | 54.6 | 80.0 | 91.4 | 88.9 | 69.9 | 41.9 | 21.6 | 5.1 | 0.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1970 | P | 91.4 | 41.2 | 12.7 | 24.1 | 56.9 | 49.8 | 62.7 | 81.5 | 61.5 | 35.3 | 102.1 | 32.3 |
|  | $d_{p}$ | $0.0$ | 0.0 | 17.8 | 48.3 | 71.4 | 113.0 | 139.2 | 127.6 | 108.0 | 94.3 | 0.0 | 0.0 |
|  | p |  | - | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 . | 0.0 | 2.7 | 35.0 |
| 1971 | P | 43.9 | 17.3 | 30.5 | 59.2 | 57.5 | 34.4 | 77.2 | 60.6 | 9.0 | 37.7 | 57.1 | 15.9 |
|  |  | 0.0 | 0.0 | 0.0 | 0.0 | 22.5 | 79.5 | 91.2 | 100.5 | 133.4 | 117.3 | 65.3 | 49.4 |
|  | $r_{\text {p }}$ | 78.9 | 86.0 | 86.0 | 90.6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

From the table above or Figure 3.1., it can be seen that the potential excess winter rain, $\mathrm{R}_{\mathrm{p}}$, for $1970-71$ was 91 mm , that the maximum potential soil moisture deficits, $D_{p}$, during 1970 and 1971 were 139 mm and 133 mm respectively, and that the potential date of return to field capacity in 1970 was approximately November 29 th.

The mean monthly potential evaporation values, $\overline{\mathrm{e}}_{\mathrm{p}}$, were taken from the Ministry of Agriculture, Fisheries and Food Technical Bulletin No. 16 (1967). They were deduced for a location 10 m above mean sea-level in the county of West Fife, i.e. the Leuchars site. These values are based on the years 1950-64, which happen to be
reasonably representative of the period 1785-1975 for the B1ackford Hill (Edinburgh) station (Thom and Ledger 1976). Thom and Ledger pointed out that potential evaporation itself is a remarkably conservative parameter, the standard deviation of calculated monthly values being approximatley 10 per cent of their long-term mean. This is to be compared with the corresponding value for monthly rainfall which is perhaps $\pm 40$ per cent (Thom and Ledger 1977).

Referring to Leuchars data, the standard deviation of monthly values of potential evaporation are given in Table 3.2. TABLE 3.2.

MONTHLY STANDARD DEVIATIONS OF. POTENTIAL
EVAPOTRANSPIRATION AT LEUCHARS, 1957-76

| MONTH | MEAN P.E. <br> $(\mathrm{mm})$ | STANDARD <br> DEVIATION <br> $(\mathrm{mm})$ | STANDARD DEVIATION AS A <br> PERCENTAGE OF THE MEAN |
| :--- | :---: | :---: | :---: |
| APRIL | 52.3 | 6.0 | 11.4 |
| MAY | 73.2 | 9.9 | 13.5 |
| JUNE | 88.9 | 11.6 | 13.1 |
| JULY | 87.9 | 11.1 | 12.6 |
| AUGUST | 66.8 | 7.4 | 11.1 |
| SEPTEMBER | 41.9 | 5.3 | 12.7 |

The potential evaporation data used were based on meteorological variables recorded during the period 1957-76, and were calculated using L. P. Smith's (1967) version of the Penman formula. It is evident from the table that May had the greatest standard deviation over this 19 year period, some 13.5 per cent, whilst August had the least, 11 per cent. When averaged over the summer months the
standard deviation was 12.4 per cent of the mean, confirming the suspected steady nature of potential evapotranspiration. Moreover, similar values to these have been published for the Kew Observatory and Camden Square sites in the London area : Wales-Smith (1973) gave the standard deviation of calculated monthly values of potential evapotranspiration as more or less 10 per cent of their longterm mean. Altogether this evidence lends support to the Thom and Ledger use of mean values of potential evapotranspiration. Using the procedure outlined in Table 3.1. the excess winter rain, Rp, series was obtained for Leuchars for the years 1922-76: these $R_{p}$ values are given in Table 3.3. The corresponding $D_{p}$ values are given in Appendix I. Clearly, the runoff years 1955-56 and 1972-73 have negative $R p$ values. These do not follow directly from the formal definition of $R_{p}$ given previously. TABLE 3.3.

POTENTIAL EXCESS WINTER RAIN, $\mathrm{R}_{\mathrm{p}}$, AT LEUICHARS, FOR THE PERIOD 1922-76
(All values in mm)

| WINTER <br> STARTING | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1920 |  |  | 67 | 92 | 384 | 175 | 211 | 394 | 147 | 146 |
| 1930 | 336 | 234 | 191 | 53 | 129 | 303 | 152 | 184 | 329 | 136 |
| 1940 | 297 | 119 | 37 | 34 | 205 | 242 | 291 | 108 | 197 | 118 |
| 1950 | 267 | 155 | 5 | 92 | 266 | -38 | 230 | 161 | 117 | 222 |
| 1960 | 272 | 138 | 166 | 253 | 261 | 298 | 256 | 100 | 283 | 105 |
| 1970 | 91 | 136 | -82 | 17 | 93 | 18 |  |  |  |  |

For the runoff year 1955-56 for example, the formal definition would have given $R_{p}=0$, and hence for the runoff years 1955-56 and 1956-57 taken together an $R_{p}$ value of 192 mm would have been obtained for the latter year. However, since during the winter of 1955-56 the potential soil moisture deficit failed to return to zero, by 38 mm , it is helpful to equate $R_{p}$ to -38 mm for that year, and to add an identical positive quantity to $R_{p}$ for the following year to give $R_{p}(1956-57)=230 \mathrm{~mm}$. Such a procedure enhances the value of $R p$ as an index of annual runoff: it is clearly realistic, since any potential moisture deficit - here 38 mm - carried over from a previous runoff year (1955-56) must be satisfied in the following year before any runoff can be generated.

CORRELATION WITH MEASURED RUNOFF
It was initially intended by Thom and Ledger to terminate their work with the derivation of the $R_{p}$ series for Blackford Hill. However, on the suggestion of M. Beran of the Institute of Hydrology, it was decided to compare derived $R_{p}$ values with measured runoff from local catchment areas, and hence test the use of $R_{p}$ as an index of annual runoff. Thom and Ledger used a straightforward regression analysis; of the form shown in equation 3.1. below:

$$
Q_{c}=a R_{p}+b
$$

where $Q_{C}$ is measured catchment annual runoff
The main problem here lay in the choice of an appropriate runoff year. Thom and Ledger decided that a July - June runoff year is the most applicable to conditions in south-east Scotland, since it involves least carryover from one year to the next of runoff caused by excess winter rain. The hydrograph based on mean monthly
data for the River Eden for the period 1968-76 - shown in Figure
3.2- tends to support their use of this particular runoff year. However, it is interesting to note that inspection of Figure 3.2. suggests that August to July would have been a marginally better choice than July to June for these particular years in the Eden catchment. Nevertheless, for all years the July - June period enclosed the hydrograph peaks, and it seems that generally the effect of any excess winter rain is negligible by June, when baseflow from storage appears to predominate.

## RESULTS OF REGRESSION ANALYSIS APPLIED TO THE RIVER

EDEN CATCHMENT AND LEUCHARS STATION
Annual runoff values, $Q_{C}$, for the River Eden at Kemback were plotted against the derived $R_{p}$ values for Leuchars for the period 1968-76, i.e. the length of the runoff record, as shown in Figure 3.3. The $Q_{c}$ and $R_{p}$ data used are listed in Table 3.4. The results of the regression analysis are given in Table 3.5.

TABLE 3.4.
MEASURED AND ESTIMATED RUNOFF VALUES FOR
THE EDEN CATCHMENT, 1968-76

| Year. | Measured <br> Runoff <br> Qc $(m m)$ | Potential Excess <br> Winter Rain, <br> Leuchars $R_{p}(\mathrm{~mm})$ | Potential Catchment <br> Excess Winter Rain <br> $R_{p c}(\mathrm{~mm})$ | Actual Catchment <br> Excess Winter <br> Rain $R_{c}(\mathrm{~mm})$ |
| :---: | :---: | :---: | :---: | :---: |
| $1968-69$ | 457 | 283 | 565 | 565 |
| $1969-70$ | 305 | 105 | 196 | 196 |
| $1970-71$ | 358 | 91 | 313 | 327 |
| $1971-72$ | 341 | 136 | 336 | 342 |
| $1972-73$ | 170 | -82 | -31 | -27 |
| $1973-74$ | 206 | 17 | 243 | 309 |
| $1974-75$ | 311 | 93 | 229 | 239 |
| $1975-76$ | 226 | 18 | 233 | 244 |



TABLE 3.5.
REGRESSION DATA FOR RIVER EDEN MEASURED RUNOFF,
$\underline{Q_{c},}$ AGAINST $R_{p}$. FOR LEUCHARS, $R_{p c}$ AND $R_{c}$

| Parameter | a | b | Correlation Coefficient <br> $r$ | Coefficient of <br> Determination, $r^{2}$ |
| :--- | :---: | :---: | :---: | :---: |
| Rp Leuchars $^{0.84}$ | 227 | 0.96 | 0.92 |  |
| P $_{\text {pc }}$ | 0.49 | 168 | 0.88 | 0.77 |
| R $_{C}$ | 0.46 | 170 | 0.82 | 0.67 |

It is apparent that a highly satisfactory correlation has been achieved, the correlation coefficient - r - being very high, and significant at 0.01 probability level. Further, the coefficient of determination, $r^{2}$, indicates that 92 per cent of the variation in annual runoff, $Q_{c}$, can be accounted for by variation in $R_{p}$. There is good reason to expect a close correlation between these two closely linked variables, especially since the rainfall data site is only 5 km to the east of the Eden catchment and should, therefore, experience reasonably representative weather conditions. However, reference to the Qc: $R_{p}$ 1:1 line in Figure 3.3. reveals that generally Leuchars receives approximately 200 mm less rain annually than the Eden catchment.

Examining Figure 3.3., it can be seen that the regression was based on 8 years runoff data only. It would have increased confidence in the relationship if data for 1976-77 were available, but these have yet to be processed (Beale, Tay River Purification Board, pers. comm.). Thom and Ledger were also working with relatively short runoff records for their catchments, the River Tyne 13-year


FIGURE 3.3 : Relationship between annual runoff of the River Eden, $Q_{C}$, and :
(i) Leuchars potential excess winter rain, $R_{p}$; (ii) Eden catchment potential excess winter rain, $R_{p c}$; (iii) Eden catchment actual excess winter rain, $R_{C}$, for the years 1968-76
record being the longest, and they deemed it essential that the physical reality of the regression lines be established. Thus the next stage in the Thom and Ledger method involved the transposition of excess winter rain, $R_{p}$, from the long-term rainfall station to the catchment, using the hydrological characteristics of the catchment to achieve this. This entailed the use of long-term average values of precipitation, $\overline{\mathrm{P}}$, and mean potential evapotranspiration, $E_{p}$, in the derivation of potential catchment excess winter rain, $R_{p c}$, as detailed below.

ANNUAL RUNOFF BY DEDUCTION
The Derivation of Potential Catchment
Excess Winter Rain
The following derivation is reproduced largely from Thom and Ledger (1976), and since it forms an integral part of this work, the reasoning is examined where considered appropriate.

For a particular catchment (subscript c refers to catchment), and assuming no change in groundwater or soil moisture storage, then equation 2.24. can be written as:

$$
R_{c}=Q_{c}
$$

It is reasonable to propose that the actual excess winter rain for a catchment, $R_{c}$, is greater than its potential value, $R_{p c}$, by the amount the actual evapotranspiration, $E_{C}$, falls short of $i t s$ potential value, $E_{p c}$, as given below:

$$
R_{c}=R_{p c}+\left(E_{p c}-E_{c}\right)
$$

However, at this juncture, the potential catchment value is required, therefore, $E_{C}$ is approximated by $E_{p c}$, and a first estimate of $Q_{c}$,
catchment annual runoff, is obtained, namely its potential value:

$$
Q_{p c}=R_{p c}
$$

The parameter $R_{p c}$, in relation to $R_{p}$ for any long-term station, may be accurately stated as:

$$
R_{p c}=R_{p}+\left(P_{c}-P\right)-\left(E_{p c}-E_{p}\right)
$$

where the subscript $c$ refers to the catchment and no subscript refers to the, long-term station
Using long-term mean values of $P_{C}$ and $P$ - that is $P_{C}$ and $P-a$ first estimate of the component ( $\mathrm{Pc}-\mathrm{P}$ ) is:

$$
P_{c}-P=\left(\bar{P}_{c}-\bar{P}\right) \cdot P
$$

$$
=\left(\rho_{c}-1\right) P,
$$

where $\rho_{c}$ is the ratio of $\bar{P}_{c}$ to $\bar{P}$
Inherent in equation 3.6. is the assumption that there is a fixed relationship between the rain incident on the catchment, $P_{C}$, and long-term rainfall station, $P$, and further that the ratio of long-term mean precipitation on the catchment and rainfall station, - $\rho_{c}$ - can be used to transform accurately $P$ values into the corresponding $P_{C}$ values. $P_{C}$ and $P$ values for the period 1957-76 for the Eden catchment and Leuchars are given in Table 3.6. The $P_{C}$ values were calculated using the standard Meteorological Office isohyetal method, as detailed in Chapter 4. The ratio $\rho_{c}$ used to deduce the catchment rainfall, $\rho_{c} P$,values was 1.27. This was based on the Leuchars (1922-76) mean precipitation $-\bar{P}=661 \mathrm{~mm}$ - and a $\bar{P}_{c}$ (191650) value of 839 mm for the Eden catchment, as estimated and supplied by the Meteorological Office. It is evident from Table 3.6. that

TABLE 3.6.
EDEN CATCHMENT ESTIMATED ANNUAL RAINFALL AND LEUCHARS
RECORDED ANNUAL RAINFALL VALUES FOR THE PERIOD 1957-76


| 1957-58 | 957 | 780 | 1.23 | 991 | 34 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1958-59 | 771 | 564 | 1.37 | 716 | -55 |
| 1959-60 | 886 | 742 | 1.19 | 942 | 56 |
| 1960-61 | 881 | 730 | 1.21 | 927 | 46 |
| 1961-62 | 765 | 617 | 1.24 | 784 | 19 |
| 1962-63 | 916 | 729 | 1.26 | 926 | 10 |
| 1963-64 | 912 | 754 | 1.25 | 958 | 46 |
| 1964-65 | 684 | 533 | 1.28 | 677 | -7 |
| 1965-66 | 1022 | 828 | 1.23 | 1051 | 29 |
| 1966-67 | 808 | 688 | 1.17 | 874 | 66 |
| 1967-68 | 761 | 734 | 1.04 | 932 | 171 |
| 1968-69 | 931 | 813 | 1.14 | 1033 | 102 |
| 1969-70 | 642 | 523 | 1.23 | 664 | 22 |
| 1970-71 | 774 | 615 | 1.26 | 781 | 7 |
| 1971-72 | 772 | 633 | 1.22 | 804 | 32 |
| 1972-73 | 447 | 344 | 1.30 | 437 | -10 |
| 1973-74 | 610 | 560 | 1.09 | 711 | 101 |
| 1974-75. | 701 | 549 | 1.28 | 697 | -4 |
| 1975-76 | 651 | 553 | 1.17 | 702 | 51 |

the $\rho_{c}$ ratio in fact varies widely from year to year: in the years 1968-69 and 1973-74, $\rho_{C} P$ overestimated $P_{C}$ by over 100 mm , and in 1958-59 the $\rho_{C}$ expression underestimated $P_{C}$ by 55 mm . On average, $\rho_{C} P(1957-76)$ overestimated $P_{C}(1957-76)$ by 38 mm ; $\rho_{c}$ (1957-76) was in fact 1.21. Regression analysis of Eden catchment annual rainfall, $P_{C}$, and Leuchars annual precipitation, $P$, over the period 1957-76 was undertaken to give further information on the association between the two parameters. Figure 3.4. illustrates graphically the relationship between $P_{C}$ and $P$ : it is apparent that the relation between the two parameters can be defined by a straight line of the form:

$$
P_{c}=a P+b
$$

Table 3.7. details the results of the regression analysis. Clearly, the two parameters are quite closely associated as indicated by the correlation coefficient of 0.86 ; the coefficient of determination, $r^{2}$, suggests that some 74 per cent of the variation in $P_{C}$ can be explained by the variability of $P$. However, both the figure and the statistical analysis provide further evidence that although $\mathrm{P}_{\mathrm{C}}$ and $P$ are reasonably closely related, there is no constant relation between the two: Indeed, it is extremely optimistic to expect any TABLE 3.7.
REGRESSION DATA FOR EDEN CATCHMENT ANNUAL RAINFALL, $\mathrm{P}_{\mathrm{C}}$, PLOTTED AGAINST LEUCHARS ANNUAL PRECIPITATION, P, FOR THE PERIOD 1957-76

| $a$ | $b$ | correlation coefficient, $r$ | coefficient of determination <br> $r^{2}$ |
| :---: | :---: | :---: | :---: |
| 1.09 | 81 | 0.86 | 0.74 |


two hydrological parameters to exhibit such a restrictively constant association. Thus the use of the $\rho_{c}$ ratio to derive catchment.rainfall appears unsatisfactory, and can be expected to introduce considerable inaccuracy into the $R_{p c}$ estimate. An alternative method of estimating catchment rainfall from rainfall recorded at a long-term station is via a regression equation of the form shown in equation 3.7., but substantial errors would also be involved in this method of estimation. Unfortunately, it seems that there is an insurmountable problem in translating rainfall totals from one location to another.

Turning to evapotranspiration considerations, values of $E_{p}$ were expected to have a negative correlation with rainfall, but Figure 3.5. illustrates the insignificant dependence between the two variables for leuchars meteorological station. The analysis was carried out for the period April 1971 - September 1975, excluding the periods October - March inclusively, since well over 90 per cent of potential evaporation occurs during the summer months. The evapotranspiration values used were those calculated from L. P. Smith's (1967) version of the Penman formula, based on the available meteorological data recorded at Leuchars. No relationship at all was evident and thus it was considered pointless either drawing some arbitrary regression line or carrying out any statistical analysis for dependence between rainfall and potential evapotranspiration. These results can be compared with those given in Thom and Ledger (1976) where a similar lack of dependence between the two variables was illustrated for Turnhouse, Tiree and Eskdalemuir. Acknowledging this lack of dependence of $E_{p}$ on $P, E_{p}$


FIGURE 3.5 : Relationship between potential evapotranspiration and precipitation during summer months at Leuchars, Fife
can be equated to $\bar{E}_{p}$. The difference in evaporation between the catchment and rainfall station $-E_{p c}-E_{p}$ - may then be expressed in terms of height difference alone, so that:

$$
E_{p c}-E_{p}=\bar{E}_{p c}-\bar{E}_{p}=0.29\left(H_{c}-H\right)
$$

where $H_{C}$ and $H$ are mean catchment height and height of the longterm rainfall station respectively.

The assumption Thom and Ledger made in the above is that potential transpiration decreases in a linear fashion with the height of the ground above sea-level. However, the change in evapotranspiration with height is not as straightforward as this, since temperature and sunshine usually decrease, and windspeed and relative humidity increase. Nevertheless, integrating all these contributory factors, it is simple and not too unrealistic to accept a linear height correction. Therefore, on an annual basis, for Scotland, a reduction of 0.29 mm per metre above sea-level was used (Ministry of Agriculture, Fisheries and Food, Technical Bulletin No. 16,1967 ), as incorporated into equation 3.8.

As a height correction alone was used to allow for differences in evaporation, it was essential that the mean catchment height be estimated with a fairly high degree of accuracy. This was attempted using a standard dot-grid method on a 1:250,000 Ordnance Survey map of the area, each dot representing $0.466 \mathrm{~km}^{2}$ on the ground. The height distribution calculated is given in Table 3.8 below. The areal distribution can be seen in Figure 1.3. It is noted that the figures in Table 3.8. were the mean of three estimations, and the resulting
mean catchment height, $H_{C}$, is 102 m . This estimate appears reasonable since the Ministry of Agriculture, Fisheries and Food Technical Bulletin No. 16 (1967) estimated the average county height of East Fife as 76 m , and a corresponding value of 116 m for the more mountainous western region. It is noted that the derived catchment area shown in Table 3.8 . is $295 \mathrm{~km}^{2}$, some $12 \mathrm{~km}^{2}$ less than the Tay River Purification Board's estimate of the basin area above Kemback gauging station. Nevertheless, this is TABLE 3.8.

HEIGHT DISTRIBUTION IN THE EDEN CATCHMENT

| Height $(\mathrm{m})$ | Area $\left(\mathrm{km}^{2}\right)$ |
| ---: | :---: |
| $0-50$ | 88 |
| $50-100$ | 97 |
| $100-150$ | 79 |
| $150-200$ | 16 |
| $200-250$ | 9 |
| $>250$ | 6 |

still a good result, considering the inherent errors of the dot-grid method, and individual errors in its use. Thus,referring to equation 3.8., the difference $\left(H_{c}-H\right)$ is 92 m , giving an annual adjustment of 26 mm .

From the above reasoning it follows that for the July - June runoff year, $R_{p}=\bar{R}_{p}+(P-\bar{P})=P-\bar{E}_{p}$
which when combined with equations 3.5., 3.6., and 3.8., gives for any catchment:

$$
R_{p c}=\rho_{c} P-\bar{E}_{p}+0.29\left(H_{c}-H\right)
$$

## RESULTS FOR THE EDEN CATCHMENT

- Applying the data for the Eden catchment and Leuchars rainfall station - shown in Table 3.9. - to equation 3.10., and inserting annual (July-June) rainfall values, $P$, for the Leuchars station for the period 1968-76, eight years of catchment potential excess winter rain, $R_{p c}$, values were deduced: these $R_{p c}$ values are given in Table 3.4.

TABLE 3.9.
CATCHMENT AND RAINFALL STATION PARAMETERS

| $\bar{P}_{C}(1916-50)$ | $=839 \mathrm{~mm}$ | $H_{C}=102 \mathrm{~m}$ |
| :--- | :--- | :--- |
| $\bar{P}(1922-76)$ | $=661 \mathrm{~mm}$ | $H=10 \mathrm{~m}$ |
| $\rho_{c}\left(\frac{\bar{P}_{C}}{\bar{P}}\right)$ | $=1.27$ | $\bar{E}_{p}(1950-64)=495 \ldots \mathrm{~mm}$ |
| $\bar{R}_{p}(1922-76)$ | $=170 \mathrm{~mm}$ |  |

The derived $R_{p c}$ values were plotted against the River Eden measured catchment runoff, $Q_{C}$, as shown in Figure 3.3., giving a line of the form:

$$
Q_{c}=a R_{p c}+b
$$

The results of the regression analysis carried out on the variables are given in Table 3.5. The correlation coefficient $r=$ 0.88 indicates a close relationship between $Q_{c}$ and $R_{p c}$, endorsed by the coefficient of determination $r^{2}=0.77$. These results are encouraging and generate some confidence in equation 3.10.

When the $R_{p c}$ and $Q_{c}$ values - shown in Table 3.4. - were compared, it was apparent that $Q_{c}$ was generally underestimated by $R_{p c}$. This is illustrated diagramatically in Figure 3.6., where $R_{p c}$ and $Q_{c}$ values are plotted against $R_{p}$ values for Leuchars. Inspection of the $R_{p c}$ and $Q_{c}$ lines in the figure reveals that in the mean year when $R_{p}=R_{p}=170 \mathrm{~mm}, Q_{C}$ was underestimated by more or less 85 mm . It is also evident that as the years became "drier", i.e. $Q_{C}$ and $R_{p}$ decreased, the underestimation increased. Thus in the driest year on record, 1972-73, when $Q_{c}$ was $170 \mathrm{~mm}, R_{p c}$ underestimated $Q_{C}$ by 201 mm , while in a comparatively wet year - 1970-71 - when $Q_{C}$ was 358 mm , the underestimate fell to 45 mm . However, Table 3.4 . also reveals that there were three years when $R_{p c}$ gave higher values of annual runoff than the measured $Q_{C}$ values, these years being 1968-69, 1973-74, and 1975-76. Reference to Table 3.6. gives the probable reason for this, the use of the constant $\rho_{c}$ overestimating precipitation input into the catchment by $102 \mathrm{~mm}, 101 \mathrm{~mm}$, and 51 mm respectively. Nevertheless, the $R_{p c}$ values deduced purely from hydrological characteristics of the catchment and rainfall station did give a fairly reasonable first estimate of annual runoff, and could be said to establish the validity of the $Q_{c}: R_{p}$ relationship. Rpc values deduced for the period 1922-76 are given in Appendix II.

Thom and Ledger (1976) produced similar results for their. three selected catchments, and concluded that the obvious way to reduce the general underestimate was to obtain actual rather than potential evapotranspiration estimates, since actual values are rarely as high as the potential values, especially in "dry" summers.


FIGURE 3.6 : Potential excess winter rain, $R_{p}$, at Leuchars plotted against (i) River Eden measured runoff, ${ }^{Q_{C}}$, and (ii) estimated runoff, $R_{p c}$ and $R_{C}$, for the years 1968-76

The procedure they used to allow for this difference is now dealt with.

ACTUAL EXCESS WINTER RAIN, $R_{C}$, ESTIMATION
Thom and Ledger (1976) initially equated the amount by which annual actual evapotranspiration, $E_{C}$, fell behind annual potential evapotranspiration, $E_{p c}$, to the difference between the maximum potential soil moisture deficit, $D_{p c}$, and the lesser actual soil moisture deficit value, $D_{C}$, as below:

$$
E_{p c}-E_{c}=D_{p c}-D_{c}
$$

From the previously established equation 3.3., it follows that:

$$
R_{c}=R_{p c}+\left(D_{p c}-D_{c}\right)
$$

All the parameters are as previously defined.
Examining equation 3.13., it can be seen that as $R_{p c}$ has already been estimated, the problem hinges on the actual and potential soil moisture deficits for the catchment. Potential values of soil moisture deficit were derived in a similar fashion to potential excess winter rain, i.e. utilizing long-term average values, as shown below:

$$
D_{p}=\bar{D}_{p}-1 / 3(P-\bar{P})+2 / 3 \cdot\left(E_{p}-\bar{E}_{p}\right)
$$

In the equation, $D_{p}$ is the potential maximum soil moisture deficit for Leuchars and $D_{p}$ is the series mean value for Leuchars. All the parameter values are given in Table 3.9. Equation 3.14. assumes that $\begin{gathered}a n \\ \alpha\end{gathered}$ annual potential evapotranspiration occur during the period when $D_{p}$ is set-up, this time being April - July inclusive. Referring to Table 1.1., it can be seen that over this time period the Eden
catchment receives 30 per cent of the annual rainfall on average, and loses 64 per cent of the average annual potential evaporation. These figures illustrate that for the Eden catchment, at least, the approximations in equation 3.14. are valid.

Using these now vindicated approximations, maximum potential soil moisture deficit for any catchment, $D_{p C}$, may be expressed as:

$$
D_{p c}=D_{p}-1 / 3\left(P_{c}-P\right)+2 / 3\left(E_{p c}-E_{p}\right) \quad-3.15
$$

Applying the logic and assumptions given in equation 3.6., and 3.8., it follows that:

$$
D_{p c}=\bar{D}_{p}-1 / 3\left(p_{c} P-\bar{P}\right)-0.20\left(H_{C}-H\right)
$$

Thus, using the data given in Table 3.9., $\mathrm{D}_{\mathrm{p},}$ for the Eden catchment can be calculated for any year. $D_{p c}$ values for the years 1922-76 are given in Appendix II. In order to get actual excess winter rain however, the actual soil moisture deficit, $D_{C}$, must be calculated, and hence ( $D_{p c}-D_{C}$ ). To allow for evapotranspiration at less than the potential rate, the Penman "Root Constant" hypothesis was utilized. In this hypothesis evapotranspiration is assumed to continue at a constant high rate almost throughout the range from field capacity to permanent wilting point, the latter being the lowest soil moisture content at which the plant can extract water from the soil. Numerous experiments conducted by Veihmeyer and associates (Veihmeyer and Hendrickson 1955) form the basis for the above assumption. It should be stated that Veihmeyer's work assumed that stomatal openings are not affected by soil dryness, which Penman (1963) confirmed for corn plants, but which is doubtful for many plants in various environmental conditions (Rijtema 1966). Nevertheless, it can be proposed that a limiting condition
will be reached when the depth of soil "dryness" reaches a short distance below the root limit, and then evapotranspiration will fall dramatically below the maximum rate. Penman's (1949) introduction of a root constant was designed to define this distance and determine how much moisture is readily available to the vegetation, and hence the quantity of water that can be extracted in total. Thus,for a root crop, e.g. sugar beet with a root constant of $100 \mathrm{~mm}, 100 \mathrm{~mm}$ of moisture is easily removed from the soil, and a further 25 mm can be extracted almost as easily by capilliary movement. Any further amounts will be transpired only with great difficulty, so that little increase in soil moisture deficit beyond approximately 140 mm is possible.

This root constant idea is translated into the family of curves shown in Figure 3.7., derived by Thom and Ledger (1976), from which actual soil moisture deficit, $D_{c}$, can be read off for any potential soil moisture deficit, $D_{p c}$, for any given root constant. Hydrological Memorandum No. 38 (Grindley 1969) presented the same data in tabulated form. Hence, for pasture land with a root constant of 75 mm , a potential moisture deficit of say 150 mm is reduced to an actual value of 120 mm . Therefore, the actual evaporation, $E$, falls short of the potential amount, $E_{p}$, by some 30 mm , and the actual excess winter rain is increased by this amount. Thus, if a representative estimate of the vegetational root constant distribution within a catchment can be established, then the difference between actual and potential evaporation can be calculated for the catchment as a whole, and the runoff estimate subsequently adjusted.


FIGURE 3.7 : Relationship between actual and potential soil moisture deficit for selected root constant values

## APPLICATION OF THE PENMAN ROOT CONSTANT THEORY

## TO THE EDEN CATCHMENT

In his original paper on the River Stour catchment, Penman (1950) used an extremely arbitrary root constant distribution, employing only three types of land-use: (a) an area drawing directly on the water-table, constituting 20 per cent of the catchment area; (b) an area for which the root constant is 200 mm , encompassing 30 per cent of the catchment area; and (c) an area for which the root constant is 125 mm , comprising 50 per cent of the catchment.

Category (a) is commonly referred to as the riparian zone, where the land is close to the watercourse, and within this area the optimal supply of water to plant roots is considered available throughout the year. Area (b) is tree-covered or having other deep rooted vegetation, and with a root constant of 200 mm it is only in rare summers, e.g. 1972, 1973, that a deficit of this magnitude will be built up, resulting in the evaporation rate falling below the maximum. Classification (c) is for short-rooted vegetation and pasture land. The model outlined above is that used in the United Kingdom Meteorological Office for the preparation of its bi-monthly soil moisture deficit maps (Grindley 1970).

Penman (1950) experimented with various percentage areas of these three root constant classifications for the River Stour catchment, and concluded;"the important point that emerges is that any intelligent guess at the distribution of vegetation in the catchment will be adequate, and calculations based on changes in the guess will do little more than introduce a second order correction
into the estimate of annual evaporation based on meteorological data". Thom and Ledger (1976) accepted this and persisted with this approach, although they additionally divided up each catchment into three selected height intervals and then designated four root constant classifications to assumed percentage land-use areas within each height interval. The root constant values employed are those commonly accepted for the appropriate land-use types, published for example in Grindley (1970), and shown in Table 3.10. These root constant values were then weighted by the percentage areas of each catchment within each height range to give the overall root constant distribution for each catchment.

A similar procedure was used for the River Eden catchment. A preliminary survey of catchment land-use was undertaken, and 1:50,000 Ordnance Survey maps of the area were consulted. The overall impression this work conveyed of land-use in the area is given in Table 3.10., where the assumed root constant distribution is shown for the chosen height intervals. The previously calculated areas within each height interval (see Table 3.8.) are also included in Table 3.10. The weighting procedure was then carried out, and the root constant distribution was deduced for the catchment as a whole, as shown in Table 3.11.

Utilizing the values given in Table 3.11 and Figure 3.7., potential maximum soil moisture deficit values, $D_{p c}$, were modified to give actual maximum soil moisture deficits, $D_{C}$, for each year during the period 1922-76. The differences, $\left(D_{p c}-D_{c}\right)$ for the Eden catchment were plotted in Figure 3.8., and show the increasing gap


FIGURE 3.8 : Relationship between actual and potential soil moisture deficit in the Eden catchment
between potential and actual maximum soil moisture deficit values as the moisture deficit increases. For example, in the extremely TABLE 3.10.

THE PERCENTAGE ASSUMED DISTRIBUTION OF ROOT CONSTANTS,
c, FOR SELECTED HEIGHT INTERVALS:RIVER EDEN CATCHMENT

| Area $\left(\mathrm{km}^{2}\right)$ | Height <br> Interval <br> $(\mathrm{m})$ | $c=0 \mathrm{~mm}$ | $c=25 \mathrm{~mm}$ | $c=100 \mathrm{~mm}, c=200 \mathrm{~mm}$ |  |
| :--- | ---: | :--- | :---: | :---: | :---: | :---: |
| 185 | 200 | 5 | 10 | 70 | 15 |
| 95 | $100-200$ | 10 | 30 | 50 | 10 |
| 15 | 200 | 10 | 80 | - | -5 |
| Land-use <br> type |  | Bare rock, <br> urban | Rough <br> pasture | Root crops, <br> field crops | Trees, <br> Riparian 1and |

dry summer of 1973 a potential maximum deficit of 197 mm was reduced to an actual deficit of 131 mm , while in the relatively wet year, 1971, a potential deficit of 82 mm was reduced to an actual value of 76 mm . $A l l D_{c}$ and ( $D_{p c}-D_{c}$ ) values are given in Appendix II. TABLE 3.11.

THE PERCENTAGE DEDUCED ROOT CONSTANT DISTRIBUTION FOR THE
RIVER EDEN CATCHMENT

| $c=0 \mathrm{~mm} \quad c=25 \mathrm{~mm}$ | $c=100 \mathrm{~mm}$ | $c=200 \mathrm{~mm}$ |
| :---: | :---: | :---: |
| 4 | 20 | 63 |

The differences $\left(D_{p e}-D_{c}\right)$ for each of the runoff years during the period 1968-76 were substituted into equation 3.13., and the actual excess winter rain, $R_{c}$, values calculated. These values are shown in Table 3.4., and the $R_{c}: Q_{c}$ regression line of form $Q_{C}=a R_{C}+b$ is plotted in Figure 3.3. The regression data given in Table 3.5., show a less close relationship between $Q_{C}$ and $R_{C}$ than $Q_{C}$ and $R_{p C}, \quad r=0.82$ compared with $r=0.88$. This, however, was not surprising since the èrror in the $R_{p c}$ estimates due to the overestimation of precipitation caused by the use of the $\rho_{c}$ ratio will be augmented by the allowance for the difference between actual and potential soil moisture deficit, $\left(D_{p c}-D_{c}\right)$, in the derivation of $R_{C}$ for some runoff years. For example, ( $D_{p c}-D_{C}$ ) in 1973 was 66 mm , and the $R_{p c}$ estimate for the runoff year 1973-74 was 243 mm , the measured runoff being 206 mm . From Table 3.6., it is apparent that this $R_{p c}$ estimate is based on a catchment rainfall over 100 mm in excess of the actual rainfall, and the addition of a further 66 mm only serves to make the $R_{c}$ estimate for 1973-74-309 mm - even more inaccurate.

When $R_{C}$ was plotted against $R_{p}$ - see Figure 3.6. - slightly better agreement with measured runoff values, $Q_{c}$, was evident. However, from Figure 3.6. (or alternatively Figure 3.3. by comparison of the $R_{C}$ against $Q_{C}$ regression line and the $R_{C}: Q_{C} 1: 1$ line), it is evident that in the (1922-76) mean year, when $R_{p}=170 \mathrm{~mm}, R_{c}$ still underestimated $Q_{C}$ by approximately 75 mm , although in the years 1968 $-69,1973-74$, and $1975-76, R_{C}$ values gave overestimates of $Q_{C}$ since the $R_{p c}$ values in these years were already in excess of the $Q_{C}$
measured values. The derived $R_{C}$ values for the period 1922-76 Which were hypothesized to be equivalent to $Q_{c}$ - are given in Appendix II.

The results obtained have illustrated the basic. soundness of the Thom and Ledger (1976) method, the parameter excess winter rain having generated reasonably satisfactory estimates of annual runoff. However, in the light of the general underestimate of $Q_{C}$ by $R_{C}$, and in particular due to the occasional overestimate, it was considered vital that the sources of these discrepancies were identified, examined, and, where possible, accounted for. The origins of the discrepancies were classified under two general headings: (a) deficiencies in the Thom and Ledger method and its assumptions, and (b) inaccuracies of the actual data used in the procedure. Categories (a) and (b) are dealt with in Chapters 4 and 5 respectively.

## CHAPTER 4

THE THOM AND LEDGER METHOD - AN ASSESSMENT OF ERRORS
The likely causes of inaccuracy in the estimation of annual runoff in the River Eden by actual excess winter rain, $R_{C}$, were identified as:- (i) The use of 1950-64 average values of potential evaporation to derive excess winter rain values. (ii) The transposition of Leuchars rainfall data to the River Eden catchment using the ratio of catchment and rainfall station long-term average values, $f_{c}$. (iii) The employment of an assumed distribution of land-use types and hence root constants to transform potential catchment excess winter rain, $R_{p c}$, to actual values. (iv) An assumption that all rain falling during the time of an existing soil moisture deficit was accepted by the soil, and resulted in no direct runoff component. (v) The supposition that changes in groundwater, $\Delta G$, from year to year were negligible. (vi) The conjecture that monthly time steps would adequately sum daily moisture balances. (vii) A presumption that input of water from other areas would have no significant effect on annual water balances.

It was realized that points (i) - (iii) inclusive could be conveniently accounted for by modifying the basic Thom and Ledger (1976) procedure, as detailed below.

A MODIFIED THOM AND LEDGER APPROACH TO ANNUAL RUNOFF ESTIMATION
(A) THE DIRECT CALCULATION OF POTENTIAL

CATCHMENT EXCESS WINTER RAIN, Rpc
If monthly catchment precipitation, $p_{C}$, estimates are available
and monthly values of potential evapotranspiration, $e_{p c}$, are calculated, then potential catchment excess winter rain, $R_{p c}$, can be deduced directly as:
$R_{p c}=\sum p_{c}-e_{p c}$
from the time. $p_{c} \geqslant e_{p c}$
Similarly, $\mathrm{D}_{\mathrm{pc}}$, potential catchment maximum soil moisture deficit can be calculated by:
$D_{p c}=\sum e_{p c}-p_{c}$
from the time $e_{p c} \geqslant p_{c}$
The prime advantage of equations 4.1. and 4.2. is the exclusion of the unsatisfactory $\rho_{c}$ : ratio which was used by Thom and Ledger (1976) to generate "catchment" rainfall from the rainfall record of a single station. Sample derivations of $R_{p c}$ and $D_{p c}$ for 1973-1974 are given in Table 4.1., the procedure described in Chapter 3 being employed.

From Table 4.1 it is evident that potential catchment excess winter rain, Rpc, for the runoff year, 1973-74 was 107 mm , and that the maximum potential soil moisture deficits reached in September, 1973, and August, 1974, were 172 mm and 170 mm respectively. Derived values of $R_{p c}$ and $D_{p c}$ for the period 1957-76 are given in Appendix III, the meteorological elements required to calculate $e_{p c}$ being available for this time period only. Full details of the methods employed to estimate $e_{p c}$ and $p_{c}$ are given below.

TABLE 4.1.
DERIVATION OF $R_{p C}, D_{p C}$ FOR THE RUNOFF YEAR 1973-74 (All values in mm)

| 1973 | J | F | M | A | M | J | 11 | A | S | 0 | $N$ | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| epc | 1.2 | 10.6 | 40.5 | 61.4 | 62.0 | 97.1 | 80.7 | 70.0 | 43.7 | 14.9 | 7.4 | 4.5 |
| Pc | 60.0 | 13.0 | 16.0 | 32.0 | 69.0 | 28.0 | 71.0 | 37.0 | 30.0 | 38.0 | 26.0 | 67.0 |
| difference (epc - pc) | - | - | 24.5 | 29.4 | -7.0 | 69.1 | 9.7 | 33.0 | 13.7 | -23.1 | -18.6 | -62.5 |
| ${ }^{\text {dpc }}$ | - | - | 24.5 | 53.9 | 46.9 | 116.0 | 125.7 | 158.7 | 172.4 | 149.3 | 130.7 | 68.2 |
| $r_{p c}$ | - | - | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

1974
$\begin{array}{llllllllllllll}e_{p c} & 13.2 & 11.4 & 23.5 & 41.2 & 80.8 & 87.5 & 98.2 & 73.0 & 41.3 & 24.6 & 6.8 & 15.5\end{array}$
$\begin{array}{lllllllllllllll} \\ \mathrm{P}_{\mathrm{C}} & & 89.0 & 80.0 & 84.0 & 19.0 & 50.0 & 45.0 & 50.0 & 47.0 & 85.0 & 40.0 & 106.0 & 88.0\end{array}$
difference $-75.8-38.6-60.5 \quad 22.2 \quad 30.8 \quad 42.5 \quad 48.2 \quad 26.0$ $\left(e_{p c}-p_{c}\right)$

| $d_{p c}$ | 0.0 | 0.0 | 0.0 | 22.2 | 53.0 | 95.5 | 143.7 | 169.7 | 121.0 | 105.6 | 6.4 | 0.0 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $r_{p c}$ | 7.6 | 46.2 | 106.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 66.1 |

FULL "PENMAN FORMULATION" CALCULATION OF POTENTIAL EVAPOTRANSPIRATION FOR THE RIVER EDEN CATCHMENT

The full Penman monthly potential evaporation values for Leuchars airfield were calculated using L. P. Smith's (1967) version of the Penman formula, stated in equations 2.17. and 2.20. From these equations it is apparent that the formula requires the following meteorological elements as input - (a) the actual sunshine hours, n. This was measured at Leuchars using a standard Campbe11-Stokes recorder.
(b) The saturation deficit of the air, $\left(e_{a}-e_{d}\right)$, air temperature, $T_{a}$, and vapour pressure, $e_{d}$. Saturation deficits were obtained from the
average daily Stevenson screen temperature at $03,09,15$, and 21 hours GMT, and the average of the vapour pressures at the same times; temperatures and vapour pressures were available from the same observations. Wales-Smith (1971) published data which suggest that the use of air temperature measured four times a day to approximate twenty-four individual hourly values to provide monthly mean temperature results in mean errors in potential evaporation of approximately 0.04 mm per day. Similarly, when vapour pressures were measured at the same times errors in potential evaporation estimation exceeded $0.06 \mathrm{~mm} /$ day on only 1 per cent of occasions. These "errors" were deduced by comparison with lysimeter measurements, and could hardly be considered very significant in view of the approximate nature of the Penman formula itself.

The third required input was (c) the run-of-wind at 2 m above the ground ( $u_{2}$ ). Effective windspeed was measured at Leuchars at 10 m ( u 10 ) , and the normal Penman correction factor of 0.78 applied to adjust $u_{10}$ to the equivalent speed at 2 m , i.e. $u_{2}$. This will probably have resulted in some error, although this should not be very great since the wind is by no means a dominant factor in the Penman equation, particularly during the summer months. The last data input was (d) values for the theoretical radiation at the top of the atmosphere, $R_{a}$, and mean possible sunshine hours, $N$. Values of $R_{a}$ and $N$ for the relevant latitude and time of year are published in the Smithsonian Meteorological Tables.

Monthly mean values of these meteorological variables were calculated, and used to derive monthly potential evapotranspiration at

Leuchars for the period 1957-76. This process was carried out by the Meteorological Office using a standard computer programme. The potential evaporation estimates for Leuchars were then transposed to the Eden catchment using a height correction alone, the simple linear relationship between height and potential evaporation discussed in Chapter 3 being assumed. An allowance of 0.29 mm per metre altitude over the 12 month period was applicable, and since mean catchment height, $H_{c}=102 \mathrm{~m}$, and Leuchars is situated 10 m above mean sea-level, a reduction in potential evaporation of 26 mm was used, distributed as shown in Table 4.2. The emphasis is on the TABLE 4.2.

ALLOWANCE FOR THE REDUCTION IN POTENTIAL EVAPORATION ON A MONTHLY BASIS BETWEEN LEUCHARS AND THE RIVER EDEN CATCHMENT

```
Month J F M A M J J Jl A Slllllll
Reduction 1.0 1.0
        (mm)
```

summer months as the majority of evaporation occurs during this time. When the Leuchars monthly estimates had been corrected accordingly, monthly "full Penman" estimates of potential evaporation, $e_{p c}$, for the River Eden catchment were available for the period 1957-76. The other element required for catchment excess winter rain - $R_{p c}-$ generation was monthly catchment rainfall, $\mathrm{P}_{\mathrm{C}}$. THE CALCULATION OF CATCHMENT RAINFALL

Monthly catchment precipitation values for the duration of the River Eden runoff record, i.e. from October, 1967 onwards, had
already been calculated by the Meteorological Office, for use by the Tay River Purification Board. These areal values were estimated using the standard Meteorological Office procedure outlined below.

The Eden catchment boundary above Kemback gauging station was traced out on a $1: 63,360$ Ordfnänce Survey map, and this tracing subsequently reduced to a $1: 250,000$ scale. Average annuai rainfall (AAR) isohyets for the period 1941-70 were marked on the tracing, and the areas between isohyets measured using planimetry. The mean average annual rainfall between each pair of isohyets was estimated, i.e. the rainfall "depth", taking into account catchment topography, aspect and prevailing wind direction, as well as the individual station average annual rainfalls available. Calculated areas were then multiplied by rainfall "depth" to give volumes, and the sum of the volumes divided by the sum of the areas gave the average annual general rainfall (AAGR) for the catchment, i.e. 839 mm.

A network of rainfall gauges across the catchment and around its boundaries was then chosen, and from the individual gauge totals areal rainfall for any period of time was calculated from:

$$
\begin{aligned}
& P_{c}=\frac{A A G R}{N} \sum_{n=1}^{n=N} \frac{T_{n}}{A A R_{n}} m m \\
& \text { where } T_{n}=\text { rainfall total for a given period (mm) } \\
& N=\text { number of stations used }
\end{aligned}
$$

(Prior, Meteorological Office, pers. comm.).

To obtain the required monthly values of areal precipitation from 1957 onwards, this Meteorological Office method was followed, the network of rainfall stations used being shown in Figure 4.1. The individual stations employed in the network were those which were reputedly reliable, and were selected on the recommendation of Shaw (Meteorological Office, pérs. comm.). This particular network of stations, given in Table 4.3., was also designed to be representative of the "wetter" and "drier" climatic areas in the Eden catchment, as inspection of the average annual rainfalls for each station given in Table 4.3. reveals.

TABLE 4.3.
THE NETWORK OF RAINFALL STATIONS EMPLOYED TO DETERMINE
MONTHLY EDEN CATCHMENT RAINFALL, 1957-76
STATION MEAN ANNUAL PRECIPITATION (mm) (1941-70)
Leuchars661
St. Andrews ..... 682
Carriston ..... 833
Clatto ..... 840
pittait Pitlessie ..... 750
Loch Leven Sluices ..... 935

[^0]However, as the raingauge network used by the Meteorological Office to calculate areal precipitation values for 1967-76 was unknown, it was considered wise to check the agreement of catchment precipitation values estimated using the different raingauge networks. The


Loch Leven Sluices
years 1968 and 1969 were chosen for comparison, the results being given in Table 4.4. From the table it is apparent that both networks give similar estimates of areal precipitation, indicating that catchment rainfall for the years 1957-76 was at least consistently calculated.

TABLE 4.4.
EDEN CATCHMENT RAINFALL AMOUNTS FOR THE YEARS 1968 AND 1969
CALCULATED USING METEOROLOGICAL OFFICE AND PERSONALLY
SELECTED RAINGAUGE NETWORKS (All values in mm)
Month J F M A M J Jl A S 0 N D Year 1968
Meteoro-
$\begin{array}{lllllllllllll}\text { logical } & 55 & 53 & 46 & 70 & 125 & 22 & 93 & 35 & 79 & 133 & 66 & 52\end{array} \quad 829$
Office
Estimate
Personal 55514769126209540821356451835
Estimate

| Month <br> 1969 | J | F | M | A | M | J | J A | A | S | O | N | D | Year |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Meteoro- <br> logical <br> Office | 96 | 49 | 46 | 60 | 150 | 72 | 37 | 53 | 50 | 33 | 85 | 60 | 791 |
| Estimate |  |  |  |  |  |  |  |  |  |  |  |  |  |

Clearly, the Meteorological Office process is based on the isohyetal method for areal precipitation estimation, this method being generally accepted as most accurate. . For example, Ward (1975) stated:"If used with skill, the isohyetal method should provide the most accurate determination of average precipitation, since it
enables a large number of factors such as relief, aspect and direction of storm movement to be taken into account, albeit rather subjectively". The subjectivity of the method caused Rodda (1969). to indicate a preference for the more objective Thiessen polygon approach, although he admitted the deficiencies of the latter method when applied to hilly terrain. Therefore, it seems reasonable to suggest that the isohyetal procedure carried out by experienced Meteorological Office personnel should prove to be the most accurate method for the calculation of catchment precipitation. However, when evaluating the merits of methods by which areal precipitation may be estimated, the errors inherent in each procedure must be weighed against the inaccuracies of the individual point measurements of precipitation upon which all the methods are dependent.

Despite being one of the earliest measured meteorological parameters, there are still many problems concerning the measurement of rainfall by a raingauge. It is normally assumed that a standard raingauge measures the amount of water reaching the earth's surface. However, investigations by many researchers have shown that this is not the case. For example, (Rodda 1970a) published data which indicate that the standard Meteorological Office Mark II gauge, standing 305 mm above the ground, catches some 6 per cent less rain than is incident at ground level.Nevertheless, these inaccurate individual measurements (taken over an area of only 1.267 $\times 10^{-6}$ of a hectare by the Meteorological Office Mark II gauge), are then extrapolated to give areal values for the catchment. As

Rodda (1969) stated, "It is highly probable that the error in determining the mean rainfall for an area will be appreciable, even when the most satisfactory instruments are combined with the best techniques of network design and computation of the mean". RESULTS OF THE DIRECT ESTIMATION OF POTENTIAL CATCHMENT EXCESS WINTER RAIN, Rpc

The directly calculated catchment potential excess winter rain values, Rpc, were plotted against measured annual runoffs, $Q_{c}$, for the period 1968-76, generating a correlation line of the form $Q_{c}=$ ${ }^{\mathrm{a}} \mathrm{R}_{p c}+\mathrm{b}$, as shown in Figure 4.2. The results of the regression analysis carried out on the two variables are given in Table 4.5. It is clear that $Q_{c}$ and $R_{p c}$ are very closely related: The correlation coefficient, $r=0.98$, is extremely high, and is an improvement on the correlation coefficient of 0.96 obtained when $Q_{c}$ was plotted against $R_{p}$ at Le uchars. Inspection of the coefficient of determination, $r^{2}$, shows that 95 per cent of the variability in $Q_{c}$ can be accounted for by variation in $R_{p c}$.

TABLE 4.5.
REGRESSION DATA FOR THE RIVER EDEN MEASURED RUNOFF, Qc,
PLOTTED AGAINST CATCHMENT EXCESS WINTER RAIN

| ParameterCorrelation <br> Coefficient $r$ | Regression <br> Eoefficient | Coefficient of <br> Determination | $r^{2}$ | Base <br> Constant |
| :--- | :---: | :---: | :---: | :---: |
| $R_{p c}$ | 0.977 | 0.69 | 0.95 | 158 |
| $R_{c}$ | 0.976 | 0.76 | 0.95 | 128 |

From Table 4.6., below, it is also apparent that the derived $R_{p c}$ values underestimated $Q_{c}$ in all the runoff years on record.

These data are shown diagramatically in Figure 4.2., where comparison of the $R_{p c}: Q_{c}$ correlation line and the $R_{p c}: Q_{c} 1: 1$ line reveals that the underestimation consistently decreased as the years became wetter. For example, in relatively dry years, i.e. when $Q_{c}<200 \mathrm{~mm}, \mathrm{R}_{\mathrm{p}}$ underestimated $\mathrm{Q}_{\mathrm{c}}$ by more than 100 mm , whereas in wet years, i.e. $Q_{C}>450 \mathrm{~mm}$, the underestimate was less than 20 mm . Thus it is evident that $R_{p c}$ derived from catchment precipitation and potential evaporation data is a more reliable index of $Q_{c}$ than $R_{p c}$ values generated using Leuchars rainfall and the $\rho_{c}$ ratio and average potential evapotranspiration values. The basis for this statement lies in the fact that: (a) a higher correlation coefficient has been achieved - 0.98 - compared with 0.88 (see

Table 3.5.), and, (b) the difference between $Q_{c}$ and $R_{p c}$ exhibited a consistent trend, i.e. $\left(Q_{c}-R_{p c}\right)$ increased as the years became drier; in all runoff years $Q_{c}$ exceeded $R_{p c}$.

TABLE 4.6.
MEASURED AND ESTIMATED ANNUAL RUNOFF FOR THE EDEN
CATCHMENT, 1968-76

| Year | Measured Runoff <br> $Q_{c}(\mathrm{~mm})$ | Catchment Potential <br> Excess Winter Rain <br> Rpc $\left.^{(m m}\right)$ | Catchment Actual <br> Excess Winter Rain <br> $R_{C}(\mathrm{~mm})$ |
| :--- | :---: | :---: | :---: |
| $1968-69$ | 457 | 451 | 452 |
| $1969-70$ | 305 | 215 | 220 |
| $1970-71$ | 358 | 286 | 292 |
| $1971-72$ | 340 | 243 | 245 |
| $1972-73$ | 170 | 55 | 70 |
| $1973-74$ | 206 | 107 | 145 |
| $1974-75$ | 311 | 209 | 246 |
| $1975-76$ | 226 | 53 | 110 |



FIGURE 4.2 : Measured annual runoff of the Eden, $Q_{C}$, plotted against catchment potential excess winter rain, $R_{p c}$, and catchment actual excess winter rain, $R_{C}$, for the years 1968-76

The next stage was the deduction of actual catchment excess winter rain values, $R_{c}$, from the potential, $R_{p c}$, values. This was achieved using the relationship $R_{C}=R_{p c}+\left(D_{p c}-D_{c}\right)$ discussed and established in Chapter 3. Dpc values for each runoff year were available directly as exemplified in Table 4.1. for the years 1973 and 1974. To transform these potential values to the requisite actual maximum soil moisture deficits, $D_{C}$, a land-use survey of the Eden catchment was undertaken; by this means an actual root constant distribution for the catchment was generated. It was hoped that: (i) a more accurate estimate of the difference ( $\mathrm{D}_{\mathrm{pc}}{ }^{-}$. $\mathrm{D}_{\mathrm{C}}$ ) would result, and further that: (ii) this work would check Penman's (1950) supposition that the root constant distribution employed to derive $D_{C}$ is not especially important, provided that the chosen distribution is sensible.
(B) THE DERIVATION OF ACTUAL CATCHMENT EXCESS

WINTER RAIN, RC, VIA A LAND-USE SURVEY OF
THE EDEN CATCHMENT
During the years 1965-67 the Second Land-use Survey of Great Britain, directed by Miss A. Coleman, was carried out. In Fife, school children from Dunfermline and Kirkcaldy High Schools surveyed the region field by field, and covered most of the Eden catchment. Their results, presumably checked by responsible persons, were entered on 1:10,560 maps of the area. The land-use types they employed have been reduced to six basic categories, as shown in Table 4.7. below.

TABLE 4.7.
LAND-USE CLASSIFICATIONS EMPLOYED IN THE SURVEY OF FIFE

\left.| General Category | Inclusive Land-use Types |
| :--- | :--- |
| Forest | Forestry plantations, shelterbelts, "Natural" |
| woodland. |  |$\right\}$| Dairy and beef cattle grazing, sheep pasture, |  |
| :--- | :--- |
| Grassland | rotational grass. |
| Rough Pasture | Heather, bracken, gorse, "scrub" grass, bilberry. |
| Field Crops | Wheat, barley, oats. |
| Root Crops | Potatoes, sugarbeet, turnips, kale. |
| Urban | Water-proofed surfaces. |

A visual method was used to estimate percentage forest, root crop etc., on each of the original 1:10,560 maps of the Eden catchment and the estimates summed to give percentage values for the catchment as a whole. This procedure was executed by two individuals, and the results averaged. It is evident that the land-use estimates of the two individuals show reasonable agreement, which indicates a fairly accurate or, at least consistent, procedure. The land-use distribution in the Eden catchment is illustrated in Figure 4.3., and percentages of the selected land-use categories are given in Table 4.8. The riparian area estimate of 5 per cent was considered realistic for a low-lying catchment which is relatively densely drained, as shown in Figure 1.3. Overall, the percentage land-use values in Table 4.8. seemed reasonable, since the Eden catchment is by and large agricultural, with very little urbanization. Comparison


TABLE 4.8.

## LAND UTILIZATION IN THE EDEN CATCHMENT

| Land-use CategoryPercentage of Catchment <br> Area | Assigned Root <br> Constant (mm) |
| :---: | :---: |

Riparian Land 5

| Woodland | 11 |  | 200 |  |
| :--- | :--- | :--- | :--- | :--- |
| Field Crops | 26 |  |  | 120 |

Root Crops 16 . 100

Grassland 31 75
Rough Pasture 1025
Urban 1 . 0
of the values in Table 4.8. with the typical distribution of landuse for a predominantly farming area in East Anglia, published in Grindley (1970), revealed that the percentage land-uses were at worst of the same order of magnitude and in some cases reasonably similar. Grindley's (1970) distribution is reprơduced in Table 4.9. below.

It was realized that land-use in the catchment could well have changed in the years since the original survey was completed. However, since the agriculture is operated on a rotational basis, the values for 1965-67 were expected to be reasonably representative of the longer period under consideration. The catchment was also extensively surveyed, and any obvious changes in land-use, e.g. forestry plantations, were allowed for when the percentage land-uses, given in Table 4.8., were estimated.

THE DISTRIBUTION OF LAND-USE FOR A PREDOMINANTLY
FARMING AREA IN EAST ANGLIA

| Land-use Type | Percentage of the Total | Area |
| :--- | :---: | :---: |
| Cereals, etc. | 60.8 | Root Constant (mm) |
| Root Crops, etc. | 10.7 | 140 |
| Permanent Grass, etc. | 10.2 | 97 |
| Temporary Grass, etc. | 9.8 | 75 |
| Rough Grazing | 1.0 | 56 |
| Woodland | 2.6 | 13 |
| Fallow | 0.9 | 000 |
| Urban | 0.1 | 0 |
| Riparian | 3.9 |  |

Although most of the root constant values assigned to land-uses in the Eden catchment were those employed by the United Kingdom Meteorological Office, reference to Grindley's (1970) root constant values in Table 4.9. reveals that the root constant for field crops, i.e. cereals, applied to the Eden catchment is some 20 mm less than the Meteorological Office value. It was hoped that this lower root constant value would allow for the incomplete ground cover in spring, and also for the reduction in evapotranspiration after harvesting, which usually occurs in south-east Scotland during late August. However, it must be stated that the root constant values used are not especially accurate, or indeed constant. Specific root constants for an individual crop will vary with soil type and
depth. Thus, for example, for permanent grassland - here assigned a root constant of 75 mm - values between 70 mm and 120 mm have been calculated and used (Ward 1975). Moreover, research has indicated that the range $70-120 \mathrm{~mm}$ may well be too high (New Scientist 1961). With this inemind a study of the distribution of soil type in the Eden catchment - summarized in Figure 4.4. - was performed, and allowance made for apparently poor soils. For example, heather and gorse have fairly deep, well developed rooting systems, but are usually found on shallow soils: therefore, they were assigned a root constant of 25 mm . However, as reference to Figure 4.4. indicates, the soils in the catchment are largely deep and fertile, as reflected by the crops they support, and vegetation type alone is generally more than adequate as andication of the appropriate root constant value.

THE RESULTS OF (A) THE TRANSFORMATION OF POTENTIAL CATCHMENT MAXIMUM SOIL MOISTURE DEFICIT, Dpc, TO ITS ACTUAL VALUE, $D_{C}$, AND (B) THE SUBSEQUENT GENERATION OF ACTUAL CATCHMENT EXCESS WINTER RAIN, R $\mathrm{R}_{\mathrm{C}}$, USING THE ESTIMATED ROOT CONSTANT DISTRIBUTION IN THE EDEN CATCHMENT

The $D_{p c}$ values given in Appendix III were converted to actual values, $D_{C}$, employing the Eden çatchment root constant distribution shown in Table 4.8., and the set of curves given in Figure 3.7. These actual maximum soil moisture deficit, $D_{C}$, values have been plotted in Figure 3.8., where the annual ( $\mathrm{D}_{\mathrm{pc}}-\mathrm{D}_{\mathrm{c}}$ ) differences produced by employing an actual rather than an assumed root constant distribution are easily discernible. Presuming that the land-use


Filluvium
survey (and hence the root constant distribution) was accurate, it is apparent from Figure 3.8 that the assumed root constant distribution made excessive allowance for the amount by which actual evapotranspiration fell below the estimated potential value. For example, catchment potential soil moisture deficit values of 100 mm and 150 mm produced actual values of 94 mm and 126 mm respectively for the actual root constant distribution: the corresponding values for the assumed root constant distribution were 90 mm and 120 mm respectively.

However, even in extremely dry summers - when $D_{p c}>200 \mathrm{~mm}$ - the difference in ( $D_{p c}-D_{c}$ ) generated using an assumed as opposed to an actual root constant distribution would only be 6 mm . Hence, Penman's (1950) statement concerning the insensitivity of ( $D_{p c}-D_{c}$ ) to selected root constant values seems well justified. In consequence, in terms of the precision of determination of actual excess winter rain, $R_{c}$, it seems that the time and effort expended on the land-use survey of the Eden catchment was largely wasted.

The $R_{c}$ values generated for the runoff years 1968-76 - given in Table 4.6. - were plotted against $Q_{C}$, as displayed in Figure 4.2., the regression parameters being shown in Table 4.5. Clearly, $R_{C}$ and $Q_{c}$ are closely related, as indicated by the correlation coefficient of 0.98 . Each of the individual $R_{c}$ estimates obtained is an improvement on the corresponding $R_{p c}$ estimate of measured runoff, $Q_{C}$, as the rotation of the $R_{c}$ line towards the $R_{p C}: Q_{c} 1: 1$ line in Figure 4.2 demonstrates. It is interesting to note that
the use of the assumed root constant distribution for the Eden catchment to calculate ( $D_{p c}-D_{c}$ ) and hence $R_{c}$, would have placed all the $R_{C}$ values closer to the $Q_{C}$ points.

Nevertheless, $R_{C}$ still underestimated $Q_{C}$ for all runoff years, although this underestimate only amounted to 5 mm in the wet runoff year 1968-69, when $R_{C}=452 \mathrm{~mm}$ and the catchment runoff, $Q_{C}$, was recorded as 457 mm . However, in dry years the underestimate was considerable. For example, in the runoff years 1972-73 and 1975-76, $R_{C}$ underestimated $Q_{C}$ by 100 mm and 116 mm respectively: this constitutes 59 and 51 per cent of the measured runoff in those years.

At this juncture it is relevant to consider the representativeness of the time-span of the measured runoff record, i.e. 1968-76. Reference to the mean catchment hydrological parameters for the periods 1957-76 and 1968-76 given in Table 4.10 illustrate the extreme "dryness" of the latter period.

TABLE 4.10.
MEAN VALUES OF VARIOUS HYDROLOGICAL PARAMETERS FOR THE
EDEN CATCHMENT FOR THE PERIODS(1957-75) AND (1968-76)

| Period | $\bar{Q}_{c}$ | $\bar{D}_{p c}$ | $\bar{D}_{c}$ | $\bar{P}_{c}$ | $\bar{R}_{p c}$ | $\bar{R}_{c}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1957-76$ | - | 102 | 88 | 784 | 311 | 323 |
| $1968-76$ | 297 | 138 | 114 | 691 | 202 | 223 |

(All the values are in mm).
Mean catchment rainfall, $\overline{\mathrm{P}}_{\mathrm{C}}$, was some 90 mm less over the years 1968-76 compared with 1957-76, and some 150 mm less than the

Meteorological Office computed 1941-70 catchment precipitation estimate of 839 mm . The mean actual catchment maximum soil moisture deficit, $\bar{D}_{C}$, for the period 1968-76 exceeded the corresponding 1957-76 deficit by 26 mm . Furthermore, the mean actual catchment excess winter rain, $\bar{R}_{C},(1968-76)$ was exactly 100 mm less than the $\bar{R}_{C}(1957-76)$ value of 323 mm , and consequently $R_{C}(1968-76)$ underestimated measured runoff $Q_{c}$ by 74 mm : it is evident from Figure 4.2. that $R_{C}(1957-76)$ on $1 y$ underestimated $Q_{C}$ by 45 mm . Thus it is apparent that the discrepancy between measured annual runoff, $Q_{C}$, and $R_{C}$ during the period 1968-76 was unusually high, undoubtedly due to the excessively dry nature of this period.

However, it was obvious that there remained some basic cause or causes for the systematic underestimation of measured annual runoff, $Q_{c}$, by the rigorously calculated actual catchment excess winter rain parameter, $R_{C}$. An obvious weakness in the water balance model proposed by Thom and Ledger (1976) to derive $R_{C}$ was that no proviso was made for direct surface runoff, hence this deficiency is now examined.

## II. AN ESTIMATION OF DIRECT RUNOFF IN THE EDEN CATCHMENT

AND ITS SIGNIFICANCE IN ACTUAL CATCHMENT EXCESS WINTER
RAIN, $\mathrm{R}_{\mathrm{C}}$, GENERATION
In order to estimate the amount of rain falling on the Eden catchment which rapidly travelled by various routes to drainage channels as "direct runoff" rather than reducing existing soil moisture deficits, a hydrograph analysis was carried out on River Eden daily flow data. Numerous techniques have been suggested and employed to separate hydrographs into various arbitrary components, i.e. channel precipitation, overland flow, interflow, and groundwater discharge. However, according to Ward (1975), the simplest and probably the most logical approximation for the separation of direct runoff is that a horizontal line drawn from the sharp break of slope where the discharge begins to increase (point $x$ ) to its intersection with the recession link (point $y$ ) as illustrated in the inset in Figure 4.5. This was the method employed in this investigation. Hydrographs of the River Eden at Kemback were prepared for the years 1968-75, daily mean discharge (in cubic metres per second) being plotted semilogarithmically with time, as exemplified in Figure 4.5. for the year 1972, the shaded area in the figure being the estimated direct runoff component. The period analysed in each year was from April until the month of occurrence of maximum potential soil moisture deficit - normally August or September in the Eden catchment. Deduced monthly and yearly direct runoff estimates, converted to millimetres (mm), are given in Table 4.11. for the years 1968-75.


OIRECT RUNOFF

FIGURE 4.5 : Daily average flows for the Eden recorded at Kemback gauging station, April - October, 1972

TABLE 4.11.
DIRECT RUNOFF ESTIMATES (mm) FOR THE RIVER EDEN, 1968-75,
FOR THE PERIOD APRIL UNTIL MONTH OF OCCURRENCE OF MAXIMUM
POTENTIAL SOIL MOISTURE DEFICIT

|  | April | May | June | July | Aug. | Sept. | Oct. | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1968 | 4.0 | 13.3 | 0.0 | 0.8 | 0.1 |  |  | 18.2 |
| 1969 | 2.8 | 13.8 | 0.7 | 0.5 | 0.6 |  |  | 18.4 |
| 1970 | 0.3 | 0.5 | 0.2 | 0.3 | 3.8 |  |  | 5.1 |
| 1971 | 0.3 | 0.3 | 0.1 | 0.9 | 2.5 | 0.1 |  | 5.1 |
| 1972 | 1.4 | 1.3 | 1.8 | 0.1 | 0.2 | 0.0 | 0.0 | 4.9 |
| 1973 | 0.1 | 0.9 | 0.2 | 0.5 | 0.2 | 0.1 |  | 2.0 |
| 1974 | 0.1 | 0.3 | 0.2 | 0.2 | 0.1 | 0.4 | 1.3 |  |
| 1975 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.8 |  | 1.3 |
|  |  |  |  |  |  |  | Mean | 7.0 |

From Table 4.11. it is evident that the calculated direct runoff values were generally low, the exceptions being the month of May in 1968 and 1969, when intense rainfall produced some 13 mm and 14 mm respectively of storm runoff, this amounting to 10 and 9 per cent of the corresponding catchment rainfall. These events apart, the greatest monthly and yearly values were only 4 mm and 5 mm in April 1968 and 1970 respectively. These estimates amount to barely 6 and 2 per cent of the precipitation incident on the catchment during these times. In some months, for example, June, 1968, and September, 1972, (see Figure 4.5.), hydrograph analysis indicated that no direct runoff at all was generated, the low rainfall amounts
of 22 mm and 8 mm respectively falling on the catchment area in these months being "accepted" by the soil. For the period 1968-75, assuming that one-third of the average annual rain fell during the time when the maximum soil moisture deficit was built up, i.e. 1/3 $P_{C}(1968-75)=230 \mathrm{~mm}$ (from Table 4.10.), and since the mean direct runoff value for the same period was 7 mm (from Table 4.11.), it follows that on average only 3 per cent direct runoff was generated. This estimate for the Eden catchment is consistent with the 5 per cent direct runoff deduced for the topographically and climatically similar Peffer/Pilmuir catchment in East Lothian by Thom and Ledger (1976).

The generally low values for the Eden catchment are far from surprising in view of the low, gently undulating nature of the area. Also the soil types found in the catchment - shown in Figure 4.4. are largely deep and fertile, and thus have relatively high infiltration rates. Reference to Painter (1971) indicated that the Brown Forest soils which cover the majority of the catchment have minimum infiltration rates between 5 mm and 8 mm per hour, the corresponding rate for the sandy alluvial soil present being as high as 9 mm per hour. According to Horton's (1945) classical model, these infiltration rates must be exceeded by rainfall intensity to produce surface runoff, and as Table 4.11. reflects, this is a rare occurrence in south-east Scotland, especially during the period April-September. Furthermore, the vegetation cover in the catchment during this time can be expected to effectively increase the infiltration rates by (a) promoting a more open pore structure and higher permeability in
the soils, and, (b) breaking the impact of raindrops on the soil surface which can result in fine material being thrown into suspension on impact, and then deposited as an almost impermeable surface skin, a process which can lower infiltration rates by as much as ten times (Kirkby 1969). Thus, as Kirkby (1969) pointed out, Hortonian overland flow is very unusual in locations where a dense vegetation cover has been established. Consequently, the bulk of the direct runoff quantities deduced from hydrograph analysis and presented in Table 4.11 will probably have resulted from interflow, i.e. water which has infiltrated the soil surface and moved laterally without penetrating to the underlying zone of saturation. Support for this view was found in Kirkby and Chorley (1967), where it was suggested that overland flow is rare in Britain, since rain intensity is usually less than infiltration capacity, and that interflow (or throughflow) is the more important process.

It is relevant to point out that the direct runoff estimates are very approximate, since the traditional methods of hydrograph analysis, which are based on the oversimple Hortonian infiltration theory, have been largely discredited in recent years. Thus, Freeze (1972) stated that hydrograph separation appears to be "little more than a convenient fiction", and Nash and Sutcliffe (1970) called for a rejection of "the a priori division of hydrographs into $i 11$ defined components", arguing that there is "a continuum of different paths by which runoff reaches the streams".

Nevertheless, while acknowledging all these misgivings, hydrograph separation still gives some useful indication of the amount
of rain which falls during the times of existing soil moisture deficit and does not contribute to the reduction of that deficit. Thus, unaccepted rain, i.e. direct runoff, effectively increased the maximum potential catchment soil moisture deficit, $\mathrm{D}_{\mathrm{pc}}$, in the years under investigation. For example, from Table 4.11 it is evident that in 19705 mm of rain ran off directly, and hence a maximum potential deficit of 100 mm was amended to a value of 105 mm , as shown in Table 4.12. These amended catchment maximum potential soil moisture deficit values, $D_{p c}$, for each year were then employed, together with Figure 3.8., to deduce directly the modified difference between actual and potential moisture deficits, ( $D_{p c}-D_{c}$ ). Values for the years 1968-75 are given in Table 4.12., where, for example, it can be seen that in 1969 ( $\mathrm{D}_{\mathrm{pc}}-\mathrm{D}_{\mathrm{c}}$ ) was altered from 5 mm to 7 mm , and the actual catchment excess winter rain, $R_{C}$, for the subsequent runoff year 1969-70 was increased by this amount - 2 mm .

From Table 4.13., however, it is clear that the adjustments made in $R_{C}$ due to the allowance for direct runoff are insignificant in terms of the underestimation of measured runoff, $Q_{c}$, by the actual excess winter rain parameter, $R_{C}$. The greatest adjustment was a mere 2 mm in the runoff year 1969-70, and this still left a discrepancy of 83 mm between $Q_{C}$ and $R_{C}$ to be accounted for. In the runoff years 1974-75 and 1975-76 the allowances made for unaccepted rainfall made no difference at all to the $R_{C}$ estimates, the latter values falling short of $Q_{C}$ by 65 mm and 116 mm respectively.

TABLE 4.12.
ADJUSTMENTS MADE IN MAXIMUM POTENTIAL CATCHMENT SOIL MOISTURE DEFICITS, $\mathrm{D}_{\mathrm{pc}}$, AND IN THE DIFFERENCES BETWEEN ACTUAL AND POTENTIAL DEFICITS ( $\mathrm{D}_{\mathrm{Pc}}-\mathrm{D}_{\mathrm{C}}$ ) TO ALLOW FOR DIRECT RUNOFF
ESTIMATION (All values in mm)

| Year | Calculated <br> $D_{p c}$ | Amended <br> $D_{p c}$ | Calculated <br> $D_{p c}-D_{c}$ | Amended <br> $D_{p c}-D_{c}$ | Increase in <br> subsequent runoff <br> years $R_{c}$ estimate |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1968 | 62 | 80 | 1 | 2 | 1 |
| 1969 | 94 | 112 | 5 | 7 | 2 |
| 1970 | 100 | 105 | 6 | 7 | 1 |
| 1971 | 74 | 79 | 2 | 2 | 0 |
| 1972 | 130 | 135 | 15 | 16 | 1 |
| 1973 | 172 | 174 | 38 | 39 | 1 |
| 1974 | 170 | 171 | 37 | 37 | 0 |
| 1975 | 193 | 194 | 57 | 57 | 0 |

It is pertinent to point out that even if a different hydrograph separation technique had been employed which doubled or trebled the calculated direct runoff estimates, the increase in generated $R_{c}$ values would still have been slight. These findings provide support for Grindley's (1970) statement that the assumption of no surface runoff for much of lowland Britain gives rise to no serious error in the estimate of maximum soil moisture deficits, (and hence excess winter rain, $R_{c}$, values).

TABLE 4.13.
ADJUSTMENT OF ACTUAL CATCHMENT EXCESS WINTER RAIN, R $R_{C}$, VALUES
ALLOWING FOR DIRECT RUNOFF ESTIMATION, 1968-76
(Al. 1 values in mm)

|  | Calculated <br> $R_{C}$ | Amended <br> $R_{C}$ | Measured Runoff <br> $Q_{C}$ |
| :--- | :---: | :---: | :---: |
| $1968-69$ | 452 | 453 | 457 |
| $1969-70$ | 220 | 222 | 305 |
| $1970-71$ | 292 | 293 | 358 |
| $1971-72$ | 245 | 245 | 341 |
| $1972-73$ | 70 | 71 | 170 |
| $1973-74$ | 145 | 146 | 206 |
| $1974-75$ | 246 | 246 | 311 |
| $1975-76$ | 110 | 110 | 226 |

## III AN APPRAISAL OF THE ACCURACY OF EMPLOYING MONTHLY

## TIME STEPS IN THE ESTIMATION OF ACTUAL CATCHMENT

## EXCESS WINTER RAIN, RC

In determining maximum potential catchment soil moisture deficit, $D_{p c}$, monthly differences between potential evapotranspiration, epc, and catchment rainfall, Pc, were accumulated, as is evident from equation 4.2. However, Grindley (1967) was of the opinion that it is necessary to employ daily time steps if systematic underestimation of soil moisture deficit is to be avoided. Grindley (1967) reasoned in the following fashion.

Essentially,a soil moisture deficit is established when evaporation exceeds rainfall, and vegetation has to draw on moisture reserves in the soil to satisfy the requirements of transpiration. Clearly,a slight moisture deficit will arise (even in winter) on any day when precipitation is nil or daily evaporation exceeds daily rainfall. Such deficits may be temporary and quickly eliminated by a succeeding day or days in which precipitation exceeds moisture deficit plus evaporation on the succeeding day or days, or they may persist and accumulate. If only monthly totals are dealt with, the fact that such an accumulation is not taken into account may lead to serious error in the calculated total soil moisture deficit over a season. For example, if a dry period, possibly of up to two weeks duration, occurs in the second half of a month in which the precipitation in the first half of the month has already exceeded evaporation for the whole month, then the excess precipitation in the first part of the month must be assumed to have run off
or percolated to permanent groundwater (the water retained in the zone of permanent saturation below the water table). It is then necessary to estimate evaporation over the period of dryness, and to make allowance as required for different rates of evaporation in different parts of the month, this being particularly important in spring months when evaporation rates normally increase rapidly. Grindley (1967) cited the example shown in Table 4.14., in which the last 11 days of May were assumed to be dry, and the excess precipitation in the first part of the month was counted as lost to permanent groundwater or runoff. Evaporation over the last 11 days then represented the excess of evaporation over rainfall, and a moisture deficit was considered to have been set up from May 20th. Grindley (1967) estimated that 39 per cent of the May evaporation normally occurs in the last 11 days of the month, and from Table 4.14., 39 per cent of the total evaporation - 83.8 mm - for this particular May was 32.8 mm . Therefore, the soil moisture deficit at the end of May would have been 32.8 mm instead of zero, and the deficit at the end of June would have been 80.3 mm and not 47.5 mm as calculated using monthly data.

TABLE 4.14.
GRINDLEY'S (1967) SIMPLE EXAAMPLE OF SOIL MOISTURE DEFICIT
CALCULATION (A11 values in mm)

| Month | Rainfall | Evaporation | Soil Moisture Deficit |
| :--- | :---: | :---: | :---: |
| April | 53.1 | 47.2 | 0.0 |
| May | 99.8 | 83.8 | 0.0 |
| June | 42.7 | 90.2 | 47.5 |

The values given in Table 4.14 are in fact reasonably similar to those found in the Fife area, and thus offer a convenient method of demonstrating the way in which the use of monthly time steps could affect actual catchment excess winter rain, $R_{C}$, estimates for the Eden catchment. If it is assumed that Grindley's preceding . conjecture regarding the rainfall/evaporation distribution in May was actual fact, and further that no similar "dry spells" occurred during any of the other summer months, then maximum potential soil moisture deficit, $\mathrm{Dpc}_{\mathrm{p}}$, for the Eden catchment would be underestimated by some 33 mm . This underestimate would then have bearing'on the amendment of the difference between potential and actual catchment soil moisture deficit - ( $D_{p c}-D_{C}$ ) - since the extent of the amendment depends on the absolute value of $D_{p c}$ for the year in question. For example, if this 33 mm underestimate occurred in the summer of 1971 when $D_{p c}$ was 74 mm (see Appendix III) then the amended $D_{p c}$ value would be $(74+33) \mathrm{mm}$, i.e. 107 mm . The resulting amended $\left(D_{p c}-D_{c}\right)$ value - deduced directly from Figure 3.8. - would be 7 mm , as opposed to the ( $D_{p c}-D_{c}$ ) value of 2 mm for the original $D_{p c}$ value of 74 mm . Since from the equation 3.13., $R_{c}=R_{p c}+\left(D_{p c}-D_{c}\right)$, the actual catchment excess winter rain, $\mathrm{R}_{\mathrm{C}}$, estimate for the runoff year 1971-72 would be revised upwards by some 5 mm . If this 33 mm underestimate of $\mathrm{D}_{\mathrm{pc}}$ caused by the employment of monthily time steps occurred in the very dry runoff year 1972-73, when $D_{p c}=130$ mm , then by the same reasoning $R_{c}$ would be corrected upwards by 17 mm .

It is clear that occasional significant underestimation of actual catchment excess winter rain, $\mathrm{R}_{\mathrm{C}}$, must arise in the fashion
outlined above. If, for example, the conjectured 17 mm increase in the $R_{C}$ estimate in 1972-1973 is actually correct, then this would account for some 22 per cent of the 78 mm underestimation of annual measured runoff, $Q_{C}$, by $R_{C}$ in that year. However, perusal of the available daily rainfall records for Leuchars station revealed that precipitation in the Fife area appears to be reasonably randomly distributed in time. Moreover, it has been demonstrated in the previous section that generally little or no runoff occurs in the Eden catchment, particularly during the April-August period when soil moisture deficits are usually set up. Thus any rainfall incident on the catchment following a few days of zero precipitation would be expected to contribute quickly and efficiently to the reduction of the moisture deficit. Also,since the monthly evapotranspiration values used in this work were the sum of daily estimates calculated from recorded meteorological variables at Leuchars, then the "allowance for different rates of evaporation in different parts of the month" recommended by Grindley (1967) was effectively made.

Obviously, as in any moisture balance calculation, it would have been preferable to employ daily rather than monthly time steps in the calculation of actual catchment excess winter rain, $R_{C}$. However, this would involve a vast increase in workload, and generally this does not appear to be worthwhile in terms of the increase in precision of the $R_{c}$ estimate achieved. Indeed, if $R_{C}$ is to be employed as an index of annual runoff, then part of its attraction ${ }^{-}$ must surely lie in its ease of calculation.

## ITS CONTRIBUTION TO ANNUAL STREAMFLOW

Inherent in the Thom and Ledger (1976) definition of excess winter rain is the premise that the net change in groundwater storage, $\Delta G$, during the July - June runoff year is negligible, i.e.

$$
\Delta G=G_{r}-G_{d}=0
$$

where $G_{r}$ is the recharge to groundwater and $G_{d}$ is discharge from groundwater. This assumption is probably reasonably accurate in catchments with little groundwater storage, but in catchments with substantial groundwater reservoirs such as the Eden, it seems unlikely, particularly in years of sub-normal precipitation. Throughout the 5-year period 1970-1974 less than 80 per cent of the (1916-50) average rain fell on the Eden catchment - see Figure 1.1. ; in 1973 analysis of meteorological data indicated that only 60 per cent of the long-term average precipitation was received. Clearly, groundwater recharge, $G_{r}$, would fall below its "normal" level since in the final analysis virtually all groundwater owes its existence directly or indirectly to precipitation (Ward 1975). Furthermore, it would be expected that groundwater discharge, $G_{d}$, would augment streamflow during this dry period, with the attendant decrease in the volume of water in storage. Thus, there appeared to be a strong possibility that the underestimation of River Eden measured annual runoff, $Q_{C}$, by actual excess winter rain, $R_{C}$, could be at least partially explained by changes in groundwater storage.

Further support for this view was apparent from the results of Thom and Ledger (1976) for the Esk catchment in the Lothians, which
was known to have considerable groundwater resources (Wright 1968). The Thom and Ledger (1976) graph of Blackford Hill potential excess winter rain $R_{p}$, against $Q_{C}$ and $R_{C}$ for the Esk catchment, together with the data for Leuchars and the River Eden plotted in an identical fashion, is shown in Figure 4.6. It is suggested that the remarkable similarity of the Rc curve in relation to the $Q_{c}$ regression line for the Esk and Eden is more than coincidence. Indeed, the Thom and Ledger (1976) $R_{C}$ and $Q_{C}$ lines for the Peffer/Pilmuir and Tyne catchments, both of which have negligible groundwater resources (Wright 1968), serve by reason of their contrasting shape to highlight the similarity of the Esk and Eden plots. It seems feasible to hypothesize that the increasing difference between measured and estimated runoff - $\left(Q_{c}-R_{c}\right)$ - for both the Esk and the Eden as the years become drier is due to the increasing contribution of groundwater discharge to streamflow. In order to substantiate the above hypothesis, the groundwater characteristics of the Eden catchment were examined, and available evidence of groundwater change was analysed.

## THE GROUNDWATER RESERVOIR

Due to the extreme drought conditions during the summer of 1972, the Fife and Kinross Water Board decided to explore the groundwater resources of the Loch Leven and Stratheden areas with a view to developing them for public supply purposes. Following successful preliminary drilling in the Devonian Upper 01d Red Sandstone aquifer in early 1973, the Instiute of Geological Sciences was invited by the Scottish Development Department to undertake a hydrogeological



IGURE 4.6 : Measured and estimated runoff for the Esk and Eden plotted against potential excess winter rain, $R_{p}$, at Blackford Hill, Edinburgh, and Leuchars respectively
reconnaissance of the area to determine whether the groundwater of the Upper 01d Red Sandstone could be regarded as a regional resource. The results of this investigation were recorded in Groundwater Resources in Scotland (Scottish Development Department 1975) and Groundwater Storage in Fife and Kinross (Foster, Stirling and Paterson 1976) : Most of the following information has been derived from these sources. THE EXTENT AND STRUCTURE OF THE GROUNDWATER RESERVOIR

Reference to the work of Chisholm and Dean (1974) indicated that the Upper 01d Red Sandstone aquifer comprises the following subdivisions:
(4) The Kinneswood Formation
(3) The Knox Pulpit Formation
(2) The Glenvale Formation (including the Glen Burn Member at the top)
(1) The Burnside Formation

During the course of the aforementioned hydrogeological survey it became apparent from investigations of the hydraulic properties of the cored borehole specimens that the upper part of the Upper 01d Red Sandstone, with the exception of the uppermost 50-60 m, i.e. the Kinneswood Formation, forms the main aquifer in the region, and is of much greater water supply potential than the lower part. The precise lower boundary of this principal aquifer is somewhat uncertain, but it was tentatively placed at the base of the Glen Burn Member by Foster, Stirling and Paterson (1976). Hence, the principal aquifer is believed to comprise the Glen Burn Member and the Knox Pulpit Formation.


The distribution and extent of this aquifer is shown in Figure 4.7., where it is evident that the aquifer has a narrow outcrop on the Lomond Hills, and then expands considerably and extends along the south side of Stratheden, narrowing again and disappearing east of Cupar. The area of this principal aquifer within the Eden catchment was estimated as $126 \mathrm{~km}^{2}$ by Foster, Stirling and Paterson (1976). A sample geological section through the Upper 01d Red Sandstone - K - K' in Figure 4.7. - is presented in Figure 4.8., where the relative position and thickness of the water yielding strata is clarified, the dip of the 01d Red Sandstone to the southeast being evident. The borehole logs used to construct the crosssection are also included in the figure. Lastly,it is relevant to note that the Loch Leven and Stratheden region is highly faulted, as is evident from Figures 4.7. and 4.8. HYDRAULIC PROPERTIES OF THE PRINCIPAL AQUIFER

As Ward (1975) stated: "groundwater is mainly rock", and consequently, even in high-yielding aquifers, water can only occur in the voids or interstices between solid rock particles and fragments. Thus the fundamental geological factor affecting the occurrence of groundwater is the nature of these interstices, and particularly their size, shape and distribution through the zone of saturation. The actual amount of groundwater stored in saturated rock depends upon its porosity. Meinzer (1923) defined the porosity of a rock as its property of containing interstices, porosity normally being expressed in terms of the total volume of material which is represented by its interstices. However, while the porosity of a
material determines how much water a saturated rock can hold, by no means all of this water will be readily available for movement in the context of the hydrological cycle. For example, the porosity range of clay, a material which transmits only very small quantities of water, was given by Dixey (1950) as 50-60 per cent, while known good aquifers, e.g. sandstones, have only low to medium porosities - 10-20 per cent (Todd 1959). Therefore, a more useful index of the "mobile" water which flows readily from a rock to constitute the groundwater component of streamflow is given by the specific yield of the material. The specific yield represents the "effective" porosity of the water-bearing material, being defined as the amount of water which will drain from the material under the influence of gravity (Lindenbergh 1958). Specific yield is usually expressed as a percentage by volume of the drained material.

With reference to the Eden catchment, detailed laboratory investigations of core samples from the principal aquifer suggested that the Knox Pulpit sandstones had porosities in excess of 20 per cent, a specific yield of over 15 per cent being expected for these horizons. The Glen Burn Member sandstones were found to be more variable with porosities between 10 and 20 per cent, and associated specific yields under gravity of 5 to 10 per cent (Foster, Stirling and Paterson 1976). After discussion with W. G. N. Stirling (Fife and Kinross Water Board, pers. comm.), it was decided that an assumption of a specific yield of 15 per cent for the whole of the principal aquifer would not be too inaccurate. Hence this was the value used in subsequent calculations.

Foster, Stirling and Paterson (1976) considered that a reasonably confident estimate of the major groundwater component of River Eden annual flow, $Q_{C}$, could be made by employing a hydrograph separation technique, as exemplified in Figure 4.9., for the calendar years 1972 and 1973. The Foster, Stirling and Paterson (1976) derived estimates of groundwater contribution to measured streamflow at Kemback gauging station, for the period 1968-1973, are given in Table 4.15., these values being for the standard October - September runoff year. From Table 4.15 it is apparent that even in relatively "normal" rainfall years there is a substantial groundwater component in River Eden annual runoff. For example, in the 1969-1970 runoff year, when calculated catchment precipitation differed from the (1941 - 1970) estimated average catchment rainfall by only 8 per cent, 193 mm - or 59 per cent - of the annual runoff was derived from rainfall infiltration to the groundwater reservoir, and subsequent throughflow in the Upper 01d Red Sandstone to the river. Significantly in the very dry 1972-1973 hydrological year when catchment rainfall was some 60 per cent of the longterm mean, hydrograph separation indicated that some 65 per cent of the recorded streamflow was due to groundwater discharge. Moreover, Foster, Stirling and Paterson (1976) reported hearsay evidence which suggested that in 1973 the water table over extensive areas of the Upper 01d Red Sandstone was considerably below average, possibly by as much as 2 m . It was inferred from this that approximately 54 mm of River Eden baseflow could have been derived from groundwater storage, this comprising 35 per cent of the October - September annual runoff.

TABLE 4.15.
THE GROUNDWATER COMPONENT OF RIVER EDEN ANNUAL STREAMFLOW
AND CORRESPONDING CATCHMENT RAINFALL DATA, SEPTEMBER -
OCTOBER RUNOFF YEARS, 1968-1973

| Runoff Year | Groundwater | Groundwater | Catchment | Catchment Rain |
| :---: | :--- | :--- | :--- | :--- |
| (October - | Component of | Component As | Rainfall | As A Percentage |
| September) | Riverflow $(\mathrm{mm})$ | A Percentage | (mm) | Of The (1941- |
|  |  | Of Total |  |  |
|  |  | Riverflow |  |  |
|  |  |  |  |  |


| $1968-1969$ | 182 | 40 | 864 | 103 |
| :--- | :--- | :--- | :--- | :--- |
| $1969-1970$ | 193 | 59 | 775 | 92 |
| $1970-1971$ | 172 | 50 | 677 | 81 |
| $1971-1972$ | 143 | 44 | 677 | 81 |
| $1972-1973$ | 105 | 65 | 506 | 60 |

Clearly, if this was indeed true, this groundwater storage change would be a very good reason why $R_{C}$ underestimated measured runoff, $Q_{C}$, for this particular year, and thus further investigation was undertaken. Extensive research into the Fife and Kinross Water Board borehole level records revealed that during the years 1971-1974 fluctuations in the order of $1-4 \mathrm{~m}$ had occurred in the water table. However, these records were unrealiable since: (a) they were incomplete, and, (b) substantial volumes of water had been extracted from most of the boreholes, with an attendant fall in borehole water level. Significantly, there was one fortunate exception: complete data for the period July - June, 1972-1973, had been recorded for the Pitlessie Maltings observational borehole (shown in Figure 4.10.), this 12 -month period coinciding exactly with the runoff year for which $R_{C}$ was derived. W. G. N. Stirling
(Fife and Kinross Water Board, pers. comm.), stated that the water level in the Pitlessie borehole was virtually unaffected by pumping from nearby production boreholes, and further that changes in the water table at the Pitlessie site could be assumed to be reasonably representative of groundwater changes occurring throughout the entire Upper 01d Red Sandstone aquifer. It was apparent from the borehole data that the water level had fallen more or less 1.5 m - from 5.72 m on July 19th, 1972 to 7.17 m on June $20 \mathrm{th}, 1973$. Thus, an estimate of 1.5 m drop in groundwater level was used in subsequent calculations, rather than the 2 m estimate suggested by Foster, Stirling and Paterson (1976).

THE SIGNIFICANCE OF THE CHANGE IN GROUNDWATER STORAGE
IN THE YEAR 1972 - 1973, IN RELATION TO ACTUAL CATCH-
MENT EXCESS WINTER RAIN, $R_{C}$
The groundwater component of annual runoff resulting from change in groundwater storage can be calculated from the equation below, published in Rodda et al (1976):
specific yield $x$ change in aquifer storage $=$
volume of groundwater discharge
Thus, assuming a specific yield of 15 per cent for the principal aquifer, with the water table falling 1.5 m over an area of $126 \mathrm{~km}^{2}$, 38 mm of the 170 mm measured annual runoff, $\mathrm{Q}_{\mathrm{C}}$, for the July - June runoff year (1972-1973), was due to change in groundwater storage. From Table 4.6., it is evident that the $R_{C}$ value of 92 mm underestimated $Q_{C}$ by 78 mm in this particular year, and hence 49 per cent of this difference can be explained by groundwater contribution to streamflow, leaving a residual underestimate of 40 mm .

Since the sample 1972-1973 July - June runoff year was the. driest on record, with $Q_{C}=170 \mathrm{~mm}$ and $P_{C}=447 \mathrm{~mm}$, it seems reasonable to suggest that the decrease in groundwater storage $-\Delta G$ - during this year was probably unusually large. However, it is apparent that in some of the other drier runoff years, e.g. 19731974 and 1975 - 1976; when $Q_{c}$ was 206 mm and 226 mm respectively, $\Delta G$ would also be considerable. Unfortunately, it was impossible to quantify $\Delta G$ for these runoff years due to the lack of basic data for the Upper 01d Red Sandstone aquifer.

Conversely, in the "wetter". runoff years groundwater recharge, $G_{r}$, would also be significant, resulting in an increase in the volume of water in storage. For example in the July - June runoff year 1968-1969, with $Q_{c}=457 \mathrm{~mm}$, it is plausible to postulate that some of the 931 mm of precipitation incident of the Eden catchment contributed to groundwater recharge, $G_{r}$, at the expense of measured streamflow, $Q_{c}$. Thus the actual catchment excess winter rain, $R_{c}$, estimate of 452 mm for this runoff year is probably misleadingly close to the measured runoff value of 457 mm , since from precipitation and evaporation considerations alone, i.e. ignoring groundwater recharge, the runoff value would be expected to exceed the actual measured value, $Q_{C}$. The overall effect of the estimated groundwater discharge, $G_{d}$, in 1972 - 1973 and the postulated groundwater recharge, $G_{r}$, in 1968-1969, would be the rotation of the $R_{C} / Q_{c}$ line shown in Figure 4.2., about some mid-point where $G_{r}=G_{d}$, i.e. when $\Delta G=0$.

However, the above discussion is largely conjectual, lacking any real supporting evidence, even the groundwater discharge, $G_{d}$,
value of 38 mm for 1972-1973 being rather approximate. Unfortunately, all that it is realistic to state is that groundwater changes will almost certainly have a significant effect on measured runoff, $Q_{C}$, values for most years in the Eden catchment, in particular during the relatively dry 8-year period, 1968-1976, under examination.

## IV B A CONSIDERATION OF THE POSSIBLE EFFECTS OF THE

## GROUNDWATER CONTRIBUTION FROM THE ADJACENT LEVEN

## AQUIFER ON RIVER EDEN ANNUAL STREAMFLOW, $Q_{c}$

In addition to changes in storage, any study of the groundwater balance of a catchment must necessarily include:: (a) the examination of the precise definition of the groundwater catchment, which as Ineson and Downing (1965) stated:"may or may not have the same limits as the surface catchment" and, (b) the associated consideration of groundwater leakage and inflow from adjacent catchments. The only catchment which could have any significant groundwater effect on the Eden catchment is the adjacent Loch Leven/River Leven system, which is now investigated. It is relevant to note that for the Eden and Leven catchments, (a) and (b) are intrinsically interrelated as is evident from Figure 4.7., where it is apparent that the catchments "share" the same underlying Upper 01d Red Sandstone aquifer.

However, with reference to item (a), Foster, Stirling and Paterson (1976) considered that the groundwater divide between the Eden valley and the Leven basin could be adequately defined on the basis of groundwater level contours, and concluded that the subsurface boundary differed only slightly from the surface water divide. Foster, Stirling and Paterson's (1976) estimated groundwater level contours in metres above Ordnance Datum on the main water table for the 1971-1974 drought years are shown in Figure 4.10., where the surface catchment of the Eden above Kemback gauging station is also marked. Examination of Figure 4.10 tends to substantiate Foster,


Stirling and Paterson's conclusion, the groundwater contours for the Leven catchment overlapping only marginally into the estimated Eden surface catchment.

Significantly, though, the groundwater contours in Figure 4.10 suggest that groundwater inflow into the Eden basin from the Leven aquifer is a distinct possibility, since there is a considerable height difference between the two groundwater reservoirs. Ward (1975) identified this possible subsurface water movement, stating that:"aquifer recharge may also result from groundwater inflow from higher level groundwater basins within a complete system". With its extensive faulting the Eden/Leven area is certainly "complex" in hydrogeological terms, the faults allowing considerable scope for groundwater movement. Indeed, Stirling (Fife and Kinross Water Board, pers. comm.), stated that regional groundwater flow from the Leven catchment into the Eden system was "extremely likely", for the reasons outlined above.

However, it is only possible to identify this groundwater inflow from the Leven catchment as a probable contributory factor in the underestimation of $Q_{C}$, Eden measured annual runoff, by actual catchment excess winter rain, $R_{C}$. No quantification. of this component is possible since, as Ward (1975) stated: "At the present stage of our knowledge and techniques, it is almost impossible to quantify the amount of water involved in groundwater leakage and inflow, and the estimates which are normally made of this component of the groundwater balance equation may often be considerably in error". Significantly, Foster, Stirling and Paterson (1976) rejected any attempt at a groundwater balance in the Eden catchment for these very reasons.

It is clear from the preceding section that actual excess winter rain, $R_{C}$, estimates, derived from precipitation and evapotranspiration considerations alone, cannot account for any complications caused by the groundwater regime of a catchment. Therefore, in terms of accurate runoff calculation, the parameter $R_{C}$ will only have limited success in catchments with substantial groundwater reservoirs, such as the River Eden, particularly in "dry" years.

## ITS EFFECT ON ANNUAL STREAMFLOW, $Q_{C}$

A further possible reason for the discrepancy between Eden measured annual runoff, $Q_{C}$, and actual catchment excess rain, $R_{C}$, was identified as the introduction of water into the Eden catchment for public and industrial use, its subsequent discharge into the River Eden artificially augmenting annual streamflow. A diagramatic summary of inflow and outflow in the Eden catchment is given in Figure 4.11. A 508 mm regional main introduces water into the catchment at New Inn, this inflow being metered. Of this water entering the catchment, only a proportion is used, the bulk leaving the area by the same main at Tarvit service reservoir. Additionally, approximately 0.31 Ml of water goes daily from this main to supply the needs of Balmerino, Gauldry, Rathillet, etc., these towns being outwith the catchment area. Into this self-contained system, water is also introduced, from boreholes within the Eden basin, most of the water being extracted from the Kettlebridge and Balmalicolm bores.

Data for the 1974 - 1975 July - June runoff year, supplied by the Fife Regional Council Water Division, are given in Table 4.16. For the purposes of this investigation, all borehole water was treated as additional water in the Eden scheme, although it is clear that a percentage of this extracted groundwater would have contributed to streamflow anyway. Indeed, Foster, Stirling and Paterson (1976) concluded that the perennial exploitation of the Eden groundwater resources was undesirable due to the adverse interference with the surface water system. Nevertheless, with reference to the notation


IGURE 4.11 : Regional inflow/outflow in the Eden catchment: a simplified representation

TABLE 4.16.
MONTHLY INFLOW AND OUTFLOW WITHIN THE EDEN BASIN IN THE
SAMPLE RUNOFF YEAR, 1974-1975 (All values in millions of litres (M1))

| 1974 | Inflow Through <br> Regional Main <br> New Inn | Inflow from <br> Boreholes | Outflow Through <br> Tarvit Service <br> Reservoir |
| :--- | :---: | :---: | :---: |
| Month | 89.68 | 98.99 | 137.92 |
| July | 19.58 | 188.30 | 130.41 |
| August | 20.09 | 136.02 | 66.54 |
| September | 61.70 | 123.56 | 84.03 |
| October | 27.37 | 128.65 | 68.86 |
| November | 86.39 | 27.97 | 66.13 |

1975

| January | 103.33 | - | 68.74 |
| :--- | :---: | :---: | :---: |
| February | 104.95 | - | 68.77 |
| March | 100.38 | - | 82.65 |
| April | 72.87 | - | 68.19 |
| May | 47.12 | 79.12 | 92.18 |
| June | 47.23 | 136.55 | 87.88 |
| Total | 780.69 (A) | $919.16(B)$ | $1023.30(C)$ |

(D) 112.83 M1 supplied annually to Balmerino
and Gauldry, etc.
in Table 4.16, water introduced into the Eden catchment was calculated by:
. Introduced water $=(A+B)-(C+D)$
The resulting estimate of the volume of introduced water was 564.7 million litres, (M1), this constituting 1.8 mm of River Eden annual runoff for the year 1974-1975. Clearly, not all of the 565 MI . introduced into the catchment would have reachied the Eden drainage system, some actually being "used". However, even this overestimated value of additional runoff is insignificant in terms of the ( 1968 - 1976) 74 mm mean underestimate of measured runoff, $Q_{C}$, by actual excess winter rain, $R_{C}$.
A SUMMARY OF THE ERRORS INHERENT IN THE THOM AND

## LEDGER (1976) ME THOD

The amendment of the Thom and Ledger method, i.e. the use of the full Penman formula evaporation values and catchment precipitation estimates resulted in a consistent trend in the underestimation of measured annual runoff, $Q_{C}$, by actual catchment excess winter rain, $R_{C}$. In terms of accounting for the discrepancy between $Q_{C}$ and $R_{C}$, it has been shown that: (a) direct runoff in the Eden catchment and, (b) the input of water into the area are both insignificant. Further, it has been suggested that generally, (c) the use of monthly time steps in moisture balance calculations is similarly inadequate in explaining the ( $1968-76$ ) 74 mm mean underestimate of $Q_{C}$ by $R_{C}$. However, the effects of changes in groundwater storage on annual river flows in the Eden have been identified as highly significant. In the sample year 1972-1973, approximately 50 per cent of the underestimate could be explained by change in the groundwater regime.

Thus, although groundwater changes accounted for a significant proportion of the discrepancy between $Q_{C}$ and $R_{C}$, it was clear that some supplementary factors must also have contributed to the underestimate. Hence Chapter 5 now deals with deficiencies in data input which could have accounted for the recorded differences between measured and estimated runoff values.

There are two elements in the calculation of actual catchment excess winter rain, $R_{C}$ : evapotranspiration and precipitation, these being subject to error of estimation and measurement respectively. Measured annual catchment runoff, $Q_{C}$, has been employed to appraise the accuracy of $R_{C}$, although $Q_{C}$ itself is still subject to errors of measurement. Therefore, all three factors are examined in an attempt to explain the observed discrepancies between $R_{C}$ and $Q_{C}$. THE ACCURACY OF PENMAN EVAPOTRANSPIRATION ESTIMATES

It has been seen in Chapter 4 that actual excess winter rain, $R_{C}$, values for the Eden catchment were derived from full Penman formulation estimates of potential evapotranspiration, PE, utilizing the root constant distribution concept recommended by Penman (1950). Thus it is essentially the precision of the basic Penman formula which is under discussion in this section, since this forms the foundation upon which the amendment to produce the required actual evapotranspiration values is based.

According to Thom and Ledger (1976), little can be said concerning the accuracy of Penman evapotranspiration estimates, possibly because, as Penman himself has commented, his method has been used rather than tested (Rodda, Downing and Law 1976). Nevertheless, some investigations into the accuracy of the Penman formula have been carried out, adopting comparisons with either: (a) evapotranspiration calculated by the catchment water balance
approach or, (b) direct instrumental measurement of evaporation or evapotranspiration.

THE CATCHMENT WATER BALANCE APPROACH TO THE ASSESSMENT OF THE ACCURACY OF THE PENMAN METHOD

In this approach independent measurements of evaporative loss from catchments, obtained from the difference between rainfall and runoff, are compared with actual and/or potential Penman evapotranspiration estimates. In fact, as Pegg (1970) stated, the majority of the investigative work has been undertaken into the accuracy of potential rather than actual evapotranspiration, since the derivation of the former is far more straightforward. For instance Pegg himself (1970) carried out a rigorous water balance study of the small ( $15.5 \mathrm{~km}^{2}$ ) experimental clay Catchwater Drain catchment in east Yorkshire. Apart from rainfall and runoff measurements, changes in surface retention, soil moisture storage and groundwater storage were closely monitored. However, considering precipitation, P, and runoff, Q, alone, Pegg (1970) found excellent agreement between Penman estimated potential evapotranspiration, PE, values and measurements of "evaporative loss", $E$, for the calendar years 1966 and 1967, as shown in Table 5.1.

TABLE 5.1.
A COMPARISON OF EVAPOTRANSPIRATION ESTIMATES CALCULATED USING
(A) PENMAN'S FORMULA AND (B) CATCHMENT DATA (from Pegg (1970))
Year Rainfall Runoff Evapotranspiration Penman PE Values

| $P-Q$ | $=$ | $E$ | $P E$ |
| ---: | :---: | :---: | :---: |
| $1966781.9-300.4=$ | 481.5 | 470.0 |  |
| $1967651.0-186.7=$ | 464.3 | 455.0 |  |

[^1]It was noted that the $(P-Q)$ values exceed $P E$ in both years, the possible reasons including changes in storage and surface retention, as well as inaccuracies in the measurement of $P$ an $Q$ and $a$ non-watertight catchment (Pegg 1970). Nevertheless, Penman PE estimates accurate to within 3 per cent of the annual "rainfall minus runoff" values seem to be very good approximations.

Similar types of study have also been carried out elsewhere in the British Isles. In the classical investigation on the Stour catchment, Penman (1950) matched annual rainfall minus runoff differences with "root constant" derived actual evapotranspiration for the period 1933-1948, taking changes in groundwater and soil moisture storage into account. Penman recognised the possibilities of error in the data input, errors of observation, and errors in the method itself. However,it was considered that these errors might compensate for each other to a large extent, so that the final actual evapotranspiration estimate was likely to be within 5 per cent of the values deduced from water balance calculations (Penman 1950).

Penman (1955) carried out a similar investigation into the water balance of the Thames basin above Teddington Weir. Seasonal changes in groundwater storage, deduced from measurements of rainfall, $P$, and runoff, $Q$, and estimates of actual evaporation, $E$, (calculated using the standard Penman formula and the Penman (1950) root constant concept), were compared with changes in the level of three wells within the catchment. The results were encouraging ; in fact Penman (1955) stated: "The general agreement is sufficiently good to induce a feeling of complacency", although he added: "this
agreement could well be the result of happy guesses and fortunate assumptions needed to cover up ignorance". Overall, Penman (1955) suggested that his PE estimates were approximately 5 per cent too high, and had random errors imposed. Nevertheless, as Penman himself stated: "It still remains a considerable achievment for formulae to predict annual evaporation out of doors to within 5 per cent of the true value".

In Scotland few investigations of the hydrological cycle within catchment areas have been carried out. Apart from the outdated work of McClean on the rivers Garry (1927) and Dee (1935), Reynold's (1969) study of the small ( $3.1 \mathrm{~km}^{2}$ ) Allt Uaine catchment, Argyllsinire, is the only publication of which this author is aware. Reynolds (1969) reported less than satisfactory agreement between Penman PE estimates and evapotranspiration - calculated from the difference between rainfall and runoff - in this particularly wet upland catchment. However, Reynolds himself (1969) stated that this investigation was not a fair test of Penman's method, due mainly to the considerable significance of the errors in precipitation and streamflow measurement in this particular catchment. To elucidate, (1950 1961) mean annual catchment rainfall and runoff were 3455 mm and 3200 mm respectively, so that even Reynold's (1969) optimistic estimates of 3 and 5 per cent accuracy in these measurements still represent errors of 104 mm and 160 mm respectively. Thus, matching the corresponding mean Penman PE estimates of 320 mm against errors of this magnitude is clearly profitless.

The work of Smith (1965) further illustrates the problems of assessing the accuracy of evapotranspiration formulae using
comparisons with measured catchment data. Smith (1965) carried out a water balance study of the Hunsingore and Gouthwaite catchments within the Nidd basin. From available long-term rainfall and runoff records, average annual evaporative losses for the two catchments were calculated as 378 mm and 353 mm respectively. However, the PE value of 412 mm derived by Smith (1965) from the Penman formula was appreciably higher than these calculated values. It was tentatively suggested that the differences were associated with rainfall measurements. However, Smith himself (1965) emphasized that the discrepancies must remain largely unexplained, since none of the data dealt with in the water balance study could be assessed with sufficient accuracy for a definite conclusion to be reached.

Nevertheless, other catchment water balance studies have produced extremely close agreement between Penman evapotranspiration estimates and measurements of evaporative losses. For instance, Edwards and Rodda (1972) compared Penman potential transpiration amounts with "losses" from a small clay catchment in Oxfordshire. Differences between the two sets of values were found month by month over the period 1965-1968, the Penman PE figures exceeding values determined from the water balance in the summer, while the reverse was true during the winter. Clearly, some of these differences represent the margin between actual and potential evapotranspiration. Edwards and Rodda (1972) further identified heat storage in the soil as a major source of the observed differences. Nevertheless, these discrepancies served to compensate for each other to a large extent, resulting in very close agreement between Penman PE
estimates and evaporative losses on an annual basis. Indeed, the general confidence in the Penman method was reflected by PrusChacinski (1963) who suggested that when Penman evapotranspiration estimates and catchment evaporative losses do not agree to within 10 per cent, then this may indicate a non-watertight catchment, differences between surface and groundwater watersheds, and errors in the relevant measurements.

THE USE OF INSTRUMENTS TO ESTIMATE THE PRECISION OF
PENMAN FORMULATED EVAPORATION, $E_{0}$, AND POTENTIAL
EVAPOTRANSPIRATION, PE, VALUES
Two of the more commonly used instruments to measure (i) evaporation and, (ii) potential evapotranspiration, are evaporation pans (or tanks) and lysimeters respectively. The former are simply vessels containing water, e.g. a water filled bucket, while the latter have been defined by Harrold (1966) as small units of vegetated soil on which water balance values can be obtained. As stated in Chapter 2, Penman's (1948) original formula was derived to estimate evaporation, $E_{0}$, from a hypothetical open water surface, and was subsequently adapted to estimate potential evapotranspiration, PE, from a vegetated surface. Therefore, to test the Penman formula accurately, evaporation determined by pan measurements should strictly be matched against Penman, $E_{0}$, estimates for open water, while lysimeter measurements should be compared with Penman, PE, values for vegetated surfaces. With evaporation pan data, however, this logical comparison is frequently not made, pan "losses" often being matched against Penman PE estimates.

However, before proceding with comparisons, it is relevant to question the representativeness of the aforementioned instruments. What, precisely, do their measurements actually mean ? How does the pan measured evaporation relate to the loss from an imaginary lake or reservoir at that site ? Can the results of a lysimeter planted with wheat be extrapolated to the wheat fields surrounding it ? The answers to these questions become apparent when the use of each instrument is examined in more detail.
(i) EVAPORATION PANS (OR TANKS)

The basis of evaporation determination by a pan, or tank, is the establishment of a simple water balance of the form:

$$
\begin{aligned}
& E_{0}=P-W m m \\
& \text { where } P=\text { precipitation } \\
& W=\text { change in the water level } \\
& E_{0}=\text { evaporation from the pan, or tank }
\end{aligned}
$$

$$
-5.1
$$

These $E_{0}$ measurements supposedly reflect accurately the evaporation from a free extended water surface which is exposed to the same meteorological conditions. However, a brief examination of the British Standard (Meteorological Office) tank reveals why this expectation is not realized.

The British tank is 610 mm deep and has sides 1.83 m long. It is installed with its rim protruding 76 mm above ground level, and is filled so that the water surface is approximately level with the ground (Bilham 1938). Unavoidably the air flow across the water surface in the tank will be affected by the upstanding rim, and inevitably there will be some screening effect from surrounding
vegetation. Both these factors will distort the "natural" ventilation of the tank. Turning to energy considerations, Nordenson and Baker (1962) found that in sunken pans there was considerable heat exchange between the pan and the surrounding soil. A further factor in the discrepancy between open water evaporation and pan "loss" is due to the "oasis" effect caused by the advection of warm dry air over the limited water surface, which maintains an artificially high rate of evaporation during the day (Ward 1975). Additional factors that can distort pan measurement are splash-in and splash-out, undetected leaks, interference by plants and animals and algal growth (Rodda, Downing and Law 1976). A concluding illustration of these inaccuracies was found in Wales-Smith (1971) where it was stated: "It is known that there are daily differences between the readings of identical tanks thought to have identical exposures at the same meteorological station".

Against this background of error and uncertainty in evaporation pan measurements, comparison of Penman $E_{0}$ estimates have to be made. Surprisingly, Rijtema (1965) reported data for the Netherlands which indicate a close agreement between measured water loss from sunken pans and the $E_{0}$ values calculated with the Penman equation. Stanhill (1958a) reported exceptionally good agreement between weekly Penman $E_{0}$ estimates and evaporation tank measurements. With data covering 52 weeks for the Wellesbourne station, Warwickshire, Stanhill (1958a) produced the regression equation:
$y=1.05 x-0.05$
where $y=7$-day evaporation tank estimate
$x=7$-day Penman $E_{0}$ estimate

The gradient 7.05 , and the intercept on the $y$ - axis -0.05 , indicate the closeness of the two variables to a $1: 1$ relationship. Wales-Smith (1971) also found that 5-day means of measured tank evaporation were good approximations of Penman 5 -day mean $E_{0}$ values. However, Ward (1975) summed up the results of other experiments with the statement: "Very rarely are pan measurements able to provide a value which closely approximates potential water loss".

Although it seems unrealistic to expect a water surface to behave like a normal vegetated area, comparison of Penman PE values and evaporation pan "losses" are frequently made, often with surprisingly good results. For example, Wales-Smith (1971) compared British Standard tank evaporation with Penman PE estimates for the Kew Observatory site in London. The Penman estimates were calculated using three measures of radiation - (i) net radiation, (ii) incoming radiation or, (iii) duration of bright sunshine. Daily, 5day and 10-day mean Penman PE estimates and evaporation tank measurements were compared, as shown in Figure 5.1. Regression analysis was carried out on the variables, the results for the specified lengths of period being given in Table 5.2. (PE values in the table were calculated using "duration of bright sunshine" as a radiation index).

It is clear that Penman PE estimates give results approximating more and more closely to tank evaporation with increasing length of period, correlation coefficients of 0.96 and 0.971 being calculated for 5- and 10-day means respectively. Indeed reference to the correlation and "1:7" lines in Figure 5.1. further reveals the


FIGURE 5.1 : Wales-Smith's (1971) plots of Penman PE estimates (PE) against standard tank evaporation measurements ( $T$ ) for various time periods
extreme closeness of the relationship between Penman PE estimates and evaporation tank measurements: For the 10-day mean values the correlation line and "l:1" lines are virtually indistinguishable. TABLE 5.2.

WALES-SMITH'S (1971) RESULTS OF THE REGRESSION ANALYSIS
OF PENMAN PE ESTIMATES (PE) AGAINST BRITISH STANDARD
TANK EVAPORATION MEASUREMENTS (T) FOR VARIOUS MEAN TIME PERIODS

| Mean Period | Regression Equation | Correlation Coefficient |
| :--- | :--- | :---: |
| 1-day | $P E=0.604 \mathrm{~T}+1$ | 0.799 |
| 5-day | $\mathrm{PE}=0.954 \mathrm{~T}+0.102$ | 0.96 |
| 10-day | $\mathrm{PE}=0.999 \mathrm{~T}+0.039$ | 0.971 |

Similar results were found for the other two measures of radiation and, thus, it is not surprising that Wales-Smith (1971) stated: "Penman PE estimates made for 5 days, and 5 -day mean values of British Standard tank evaporation are meaningful and comparable measures of evaporation". Support for Wales-Smith's (1971) view was found in Rijtema (1965), where it was stated that under "normal" climatological conditions in the Netherlands, maximum evapotranspiration from a grass cover of $100-500 \mathrm{~mm}$ height closely approximates the evaporation rate from sunken pans.

## (ii) LYSIMETERS

Most lysimeters are installed so that their rims are level with the ground. In optimal circumstances the lysimeter is filled with an undisturbed block of soil, although it is more normal for soil to
be introduced into the lysimeter in the correct sequence and compacted by some means. The soil is then covered with turf, the turf being irrigated regularly to maintain the soil at field capacity. The potential evapotranspiration, PE, is calculated from a water balance of the form:

$$
\begin{aligned}
P E=P & +I-D m m \\
\text { where } P & =\text { precipitation amount } \\
I & =\text { amount of irrigation } \\
D & =\text { amount of drainage }
\end{aligned}
$$

$$
-5.2
$$

The drainage, $D$, is conducted to a suitable vessel where it is measured ; precipitation, $P$, is usually measured by a nearby raingauge ; and the irrigation amount, $I$, is metered. An experienced operator is required if the lysimeter is to function satisfactorily (Green F. 1959).

Lysimeters and evaporation pans suffer from some common disadvantages: foreign matter, growth of algae, interference by birds and animals, undetected leaks, "splash-in" and "splash-out" are all factors that can distort records. Air flow disturbance and heat exchange in surrounding soil are also problems common to both instruments. Lysimeters suffer from a number of additional disadvantages. For example, a lysimeter with uncompacted soil has infiltration characteristics that differ from those of its surroundings ; water may drain down the inside walls and at the bottom the soil will become saturated. Clearly, the siting of a lysimeter is particularly important ; the thermal and moisture regimes of the lysimeter should match those of its surroundings as closely as possible. The
centre of a large area of uniform vegetation is ideal, since the air passing over the lysimeter has travelled across a large fetch. In this case an."oasis" effect should be avoided, although it is not clear how large the fetch should be (Rodda, Downing and Law 1976). With these shortcomings in mind it seems unrealistic to expect lysimeters to be very representative of "natural" conditions.

Ward (1963) compared Penman PE estimates and lysimeter data for an experimental site on the Thames floodplain. Daily observations of temperature, windspeed, humidity, bright sunshine and rainfall were made, these climatological data being used as the basis for the estimation of PE by the Penman formula. All these instruments together with two identical lysimeters were situated in a large field of permanent grass, so that uniform vegetation conditions existed on and around the lysimeter tanks. Furthermore, the water table in this part of the floodplain was seldom more than 1.2 m below the ground surface. For the greater part of the period of observations, therefore, it is probable that evapotranspiration occurred at the potential rate from the area surrounding the instruments and thus site conditions for both the climatological observations and direct measurement of PE were very satisfactory (Ward 1963).

Ward's (1963) results for the long-period (15-month) PE and summer (April - September) PE are reproduced in Table 5.3. It is evident that there is a very good agreement between total measured PE and that estimated by the Penman formula ; the lysimeter measurements are in fact less than 3 per cent in excess of the calculated Penman value. The summer values showed even closer agreement,

Penman estimated PE being a mere 0.01 per cent greater than measured PE.

TABLE 5.3.
THE RELATIONSHIP BETWEEN SUMMER PE; LONG PERIOD PE
(15 MONTHS), AND LYSIMETER MEASUREMENTS (After Ward 1963)

| Method | Long Period PE | Summer PE |
| :---: | :---: | :---: |
|  | (April 1959 - July 1960) <br> exciuding June 1959 | (April - Sept.) |


|  | $(\mathrm{mm})$ | $(\mathrm{mm})$ |
| :--- | :---: | :---: |
| Penman | 826 | 750 |
| Lysimeter | 843 | 743. |

Short-term - 10-day - totals of measured and estimated PE also showed close agreement. Ward (1963) carried out a regression analysis on these 10 -day totals over a 15 -month period, producing a regression equation:

$$
\begin{array}{rl}
Y=0.99 & x+0.22 \\
\text { where } \quad Y & =P E \text { estimated using Penman's formula } \\
& x=\text { lysimeter measured } P E
\end{array}
$$

The gradient and intercept on the $Y$-axis- 0.99 and 0.22 respectively, indicate how closely the measured and estimated figures approximate the line of the unit slope. Ward (1963) emphasized that the optimal siting of his lysimeters reduced advective effects over the soil tanks to a minimum, thus explaining to some extent the apparent "accuracy" of the lysimeter data. It was also suggested that the lysimeter data indicate that the Penman formula overestimated PE during the summer half-year, and underestimated PE during the winter half-year. The same type of study was made by

Stanhill (1958a) with similar results. Close agreement between 7-day measured and estimated PE values was reported, and the same seasonal discrepancies between Penman estimated PE and lysimeter measurements were apparent.

The seasonal discrepancies between measured PE and estimated PE using the standard Penman formula are consistent with theoretical considerations. The Penman formula tends to overestimate PE in the spring time since: (a) no direct allowance is made for the heating of the soil ; and (b) no account is taken of stomatal resistance, $r_{s}$, and aerodynamic resistance, $r_{a}$. Indeed, the amended Penman formula proposed by Thom and 01 iver (1977) takes account of these factors, (see Chapter 2). However, on an annual basis, it is reasonable to suggest that the seasonal overestimates and underestimates generated by the standard Penman formula will cancel to a large extent.

THE ACCURACY OF THE PENMAN FORMULA : A CONCLUSION
There is insurmountable difficulty in assessing the accuracy of the standard Penman formula: it is impossible to obtain indisputably precise "control" PE values against which Penman formulated values can be compared. Seldom, if ever, do instruments truly represent the natural evaporation situation ; the components in catchment water balance studies are subject to significant errors of measurement thus rendering deduced evapotranspiration amounts fairly uncertain. However, evaporation pans, lysimeters and catchment studies all indicate that Penman estimates of potential evapotranspiration, PE, are at worst of the right order of magnitude. Furthermore, it seems reasonable to suggest that Penman's (1950)
assessment of the accuracy of his formula - $\pm 5$ per cent of the true annual evaporation value - is in fact realistic, particularly since the seasonal inaccuracies of the formula will cancel over the 12 -month period. Moreover, this 5 per cent is clearly not a systematic error. In some years true evapotranspiration will be overestimated by the Penman formula, in other years it will be underestimated. Overall, therefore, the 25 per cent discrepancy between 1968-1976 mean measured and estimated runoff, $Q_{C}$ and $R_{C}$ respectively, is considered unlikely to be attributable to any great extent to inaccuracies of Penman formulated evapotranspiration values. THE ACCURACY OF PRECIPITATION MEASUREMENT

In Chapter 2 the problem of extrapolating point rainfall measurements to catchment areas was examined and the uncertainty of the individual point measurements was also discussed, albeit briefly. As Rodda (1969) stated, the true rainfall at a point is not known because there is no absolute standard of rainfall measurement as, for example, there is in the case of river discharge. Hence, all measurements of rainfall made in the conventional manner are relative, and in spite of numerous experiments with different gauges, there is still no method of measuring the rain falling at a particular point on the earth's surface to a known degree of accuracy. Nevertheless, it is possible to draw some conclusions regarding the relative accuracy of standard - Meteorological Office Mark II raingauge measurements, and this is attempted in this section.

The literature concerning raingauge studies is immense: Kurtyka
(1953) published a monumental work which provided over 1000 references on the subject ! Bleasdale (1959) suggested that this comprehensive publication was compulsory reading for anyone who wished to claim a serious interest in the problems of rainfall measurement. However, for the purposes of the present work, a detailed literature review is considered superfluous. The sole intention is to assess the probable magnitude of error in standard raingauge measurements. Nevertheless, for reference purposes the main sources of error in raingauges are summarized in Figure 5.2., reproduced from Rodda (1969). Of these sources of error the wind is by far the most important, as was convincingly demonstrated by the wind-tunnel experiments of Rodda and Robinson (1969) at Southampton University.

The sum total of the exhaustive research that has been carried out on the design and accuracy of standard raingauges is that precipitation is systematically underestimated by these instruments. This is by no means a recent revelation:as Dines (1958) stated: "the fact that a raingauge with its rim level with the ground will record more than one at the standard height of 305 mm (i.e. the Meteorological Office Mark II gauge) .... must not be allowed to pass as a new discovery". For example, Heberden (1769), Bache (1838), Stevenson (1842), Jevons (1861) and Stow (1871) amongst others, expressed a knowledge of this fact.

Recently, however, quantification of this underestimation has been attempted, mainly by comparison of measurements made with standard raingauges and raingauges installed at ground level. At Wellesbourne, Warwickshire, Stanhill (1958b) compared the precipitation collected in a Mark II Meteorological Office raingauge buried


CONCEPTUAL MODEL OF THE PROCESSES INVOLVED IN DETERMINING RAINFALL WITH A CONVENTIONAL RAINGAUGE.
with its rim 20 mm above ground level with an identical gauge some 18 m away whose rim was at the usual height - 305 mm - above short grass. The ground level gauge was surrounded by a coarse doormat with the object of minimizing in-splashing. Over a 12-month period the total rainfall collected in the ground level gauge was 5.0 per cent more than that collected by the raingauge at standard height. Green M. (1970) matched the rainfall catch in a standard Mark II gauge with the mean catch of a nine-hole ground level raingauge at the Turville Hill site in the Chilterns. Over a 5 -year period the Mark II raingauge with its rim at 305 mm caught 3.6 per cent less than the groundlevel raingauge. At the Wallingford experimental station, Rodda (1970a) reported that one groundlevel raingauge caught 6.4 per cent more rain over a period of 8 years than a 305 mm high Meteorological Office Mark II standard raingauge some 5 m away from it. The ground level gauge was installed in a pit and surrounded by a metal grid to avoid splash-in. Over the period August, 1968 December, 1969 (excluding months with snowfall) the same groundlevel gauge recorded 4.8 per cent more rainfall than the standard Mark II gauge. Rodda (1970b) published results of groundlevel/standard Mark II raingauge comparisons for sites throughout the British Isles. These are reproduced in Table 5.4. All groundlevel gauges were surrounded by non-splash devices. With the exception of the exposed gauges located in extremely wet mountainous areas in Wales, the underestimates recorded by the standard gauges were relatively consistent, varying between 2.7 and 7.9 per cent. Ignoring the Welsh data, the mean discrepancy between the groundlevel and standard

TABLE 5.4.
THE CATCHES OF STANDARD METEOROLOGICAL OFFICE MARK II RAINGAUGES
(AT 305 mm ) AND GROUNDLEVEL RAINGAUGES FOR VARIOUS SITES IN THE
BRITISH ISLES (FROM RODDA 1970b)

|  |  |  |  | Total Rainfall |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Site |  |  |  |  |  |  |
|  | Period of | Altitude | Mean | Standard | Ground | Increase |
|  | Record | $(\mathrm{m})$ | Annual | Gauge | Level at Ground |  |
|  | (Months) |  | Rain- | $(\mathrm{mm})$ | $(\mathrm{mm})$ | Level |
|  |  |  | fall |  |  | (per cent) |


| Carregwen, Plynlimon | 20 | 579 | 2743 | 2906 | 3520 | 21.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cwm Stradllyn, Moel Hebog | 16 | 494 | 1778+ | 328 | 389 | 18.6 |
| Coalburn, Cumberland | 15 | 274 | 1067 | 2670 | 2873 | 7.6 |
| Slaidburn, Stocks Reservoir, Yorks | 96 | 201 | 1499 | 11953 | 12591 | 5.3 |
| Grendon Underwood, Bucks. | 80 | 67 | 660 | 4394 | 4544 | 3.4 |
| Wallingford Berks. | 96 | 48 | 635 | 4582 | 4877 | 7.1 |
| Benson, 0xon | 27 | 68 | 660 | 1461 | 1506 | 3.1 |
| Harmondsworth, Middlesex | 4 | 24 | 635 | . 368 | 386 | 4.8 |
| Kew <br> Observatory | 17 | 5 | 610 | 925 | 978 | 7.9 |
| Farnham, Surrey | 5 | 104 | 795 | 360 | 370 | 2.7 |
| Tarbet, Dumbartonshire | 7 | - | 2921 | 1783 | 1885 | 6.0 |

gauges was 5.3 per cent.
While potential evapotranspiration, PE, is rightly considered to be more difficult to measure directly than rainfall, Green $F$. (1958) cited lysimeter data which he considered throws light on the accuracy of measurement of the latter. Over a period of four years the discepancies between measured and computed PE values could be accounted for by the addition of 5 per cent to the observed rainfall in an equation of the form:

$$
\begin{aligned}
P E= & (I+P)-D m m \\
\text { where } P E= & P E \text { measured by the lysimeter } \\
P= & \text { observed precipitation in a } \\
& \text { nearby standard gauge } \\
I= & \text { amount of irrigation } \\
D= & \text { amount of drainage }
\end{aligned}
$$

$$
-5.3
$$

Law (1959) reported a similar lysimeter experiment concluding that rainfall is underestimated by the standard Meteorological Office gauge by 4-6 per cent.

Rodda (1970b) stated: "The underestimation of rainfall may not be particularly important to the meteorologist, but in other studies the significance of a systematic error in rainfall measurements may be considerable. In hydrology, for example, it is particularly serious where catchment water balances are being studied". Actual catchment excess winter rain, $R_{c}$, is deduced via a water balance approach $\left(R_{c}=\sum p_{C}-e_{C}\right)$, and it is very probable that the systematic underestimation of rainfall contributed significantly to the discrepancy between $R_{C}$ and measured annual catchment runoff, $Q_{C}$.

If a 5 per cent underestimation of precipitarion is accepted then the calculated $R_{c}$ values for the Eden catchment for the runoff years during the period 1968-1976 are amended to the values shown in Table 5.5. It is apparent that with the exception of the wet. TABLE 5.5.

THE AMENDMENT OF THE EDEN CATCHMENT ACTUAL EXCESS
WINTER RAIN, $R_{c}$, VALUES ALLOWING FOR A 5 PER CENT
UNDERESTIMATION OF RAINFALL (All values in mm)

| Runoff | Calculated |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Catchment <br> Crecipitation | Increment | Rainfallated <br> $R_{C}$ | Amended <br> $R_{C}$ | Measured <br> Annual |
|  | $\mathrm{P}_{\mathrm{C}}$ |  |  |  |  |
|  |  |  |  |  |  |


| $1968-69$ | 931 | 47 | 452 | 499 | 457 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $1969-70$ | 642 | 32 | 220 | 252 | 305 |
| $1970-71$ | 774 | 39 | 292 | 331 | 358 |
| $1971-72$ | 772 | 39 | 245 | 284 | 340 |
| $1972-73$ | 447 | 22 | 70 | 92 | 170 |
| $1973-74$ | 610 | 31 | 145 | 176 | 206 |
| $1974-75$ | 701 | 35 | 246 | 279 | 311 |
| $1975-76$ | 651 | 33 | 110 | 143 | 226 |
| Mean | 691 | 34 | 223 | 257 | 297 |

$\begin{array}{lllll}\text { Relative values expressed as } \% \text { of } Q_{C} & 75 & 86.5 & 100\end{array}$
year 1968-1969, the $R_{c}$ values are closer approximations to the corresponding measured runoff, $Q_{C}$, values ; the (1968-1976) mean
discrepancy between $R_{C}$ and $Q_{C}-74 \mathrm{~mm}$ - is reduced by 34 mm , this representing 46 per cent of the difference.

It appears from the evidence presented in this section that a 5 per cent allowance for rainfall underestimation is rather conservative. However, as the Eden catchment is relatively low-lying, and the raingauges in the area are reasonably well sited (Wilson, Meteorological Office, pers. comm.), this relatively cautious estimate appears reasonable. No account has been taken of the additional inaccuracies in extrapolating point rainfall measurements to the catchment, although even if it was possible to quantify the inaccuracy, it would be impossible to say whether the extrapolation process augments or reduces the proposed 5 per cent underestimation of rainfall.

Thom and Ledger (1976) found that the differences between the mean values of actual catchment excess winter rain and measured annual runoff $\left(\bar{Q}_{c}-\bar{R}_{c}\right)$, for the Tyne, Esk and Peffer/Pilmuir catchments. could be accounted for by the addition of 6.7 per cent, 4.5 per cent, and 5.2 per cent respectively to the catchment rainfall totals. They suggested that it was more than mere chance that these values were of much the same order of magnitude as the underestimation of rainfall by the standard Meteorological Office Mark II gauge. Therefore, it was tentatively suggested that perhaps these results constitute valid supportive evidence of the systematic underestimation of precipitation in Great Britain by approximately 5.5 per cent on average (Thom and Ledger 1976).

In the study of the Eden catchment, the (1968-1976)
discrepancy between $\bar{Q}_{C}$ and $\bar{R}_{C}$ was found to be 25 per cent, as shown in Table 5.5. The addition of a 5 per cent increment to catchment rainfall resulted in an increase of 11.5 per cent in $\bar{R}_{C}$, although a substantial discrepancy between $\bar{Q}_{C}$ and $\bar{R}_{C}$ still existed - some 13.5 per cent. Overall, there can be absolutely no doubt that the underestimation of precipitation contributes largely to the discrepancy between $Q_{C}$ and $R_{C}$. Indeed, the results for the Eden catchment can be interpreted as providing further evidence of the systematic underestimation of rainfall by the standard raingauge. However, the deficiencies in the Thom and Ledger (1976) derivation of actual catchment excess winter rain, $R_{C}$, have been highlighted in the present rigorous study of the Eden catchment. For example, the estimation of catchment precipitation via the $o_{c}$ ratio has been identified as a source of considerable error ; the premise of zerochange in groundwater storage, $\Delta G$, from year to year is very doubtful ; and it has been surmised that evaporation cannot be ascertained within 5 per cent of the "true" value, the very figure that Thom and Ledger (1976) suggest is atțributable to rainfall underestimation. Therefore, on balance, it seems likely that the convenient "fit" of the discrepancy between mean measured and estimated runoff, $\left(\bar{Q}_{C}-\bar{R}_{C}\right)$, recorded by Thom and Ledger (1976) was, in fact, largely coincidental. AN APPRAISAL OF THE ACCURACY OF THE RIVER EDEN FLOW RECORD

The water level of the River Eden is measured at the Kemback gauging station by a continuous flow recorder. This instrument provides a graphical record of the changing water level, the level being
recorded on a chart with an appropriate time scale and level range. The recorded levels are converted into discharges via a stagedischarge relationship. This is a curve or table which expresses the relation between the stage - the elevation of the surface of the stream relative to a datum - and the discharge in an open channel at a given cross-section for a given condition of flow, i.e. steady, rising or falling (British Standards Institution 1964).

The discharge is determined by the precise measurement of (a) the cross-sectional area and, (b) the water velocity, the latter usually being recorded with a sensitive reference current meter at two depths in an adequate number of "verticals" across the open channel. The chosen cross-section is thus divided into specific "velocity-areas", and the discharge, $Q$, is calculated via an equation of the form:

$$
Q=\left(A_{1} V_{1}+A_{2} V_{2}+A_{3} \quad V_{3}+A_{x} V_{x}\right) m^{3} \bar{s} 1 \quad-5.4
$$

where $A_{x}=a$ component area
$V_{X}=$ velocity of component area
An adequate number of observations suitably distributed throughout the whole range of stage is taken, each preferably made at a steady stage since there is generally a tend $e^{e} n c y$ for measurement points obtained during rising stages to indicate higher discharges than those determined by measurements at comparable falling stages (British Standards Institution 1964).

The determined discharges are then matched against the corresponding water level or stage measured by the flow recorder, and the stage-discharge relation established. Usually this relationship can
be expressed by an equation of the form $Q=\mathrm{Ch}^{\mathrm{X}}$ (where $Q$ is the discharge in $m^{3-1}$, $h$ is the gauge height in $m$, and $C$ and $x$ are coefficients) over the whole range of discharges, or more often by two or more similar equations each relating to a portion of the range. However, if the zero of the gauge does not coincide with the zero flow, a correction, $a$, must be applied to $h$, making the equation:

$$
Q={ }^{-} C(h+a)^{x}
$$

An equation of this type plots as a straight line on logarithmically divided graph paper. Consequently, if the points are plotted, Q as abcissae against $h$ as ordinates on paper of this type, it is possible to draw one or more straight lines through the points. The quantity a is usually determined in advance from consideration of the levels on the control cross-section in relation to the zero of the gauge (British Standards Institution 1964).

The stage-discharge curve for the River Eden at Kemback gauging station is shown in Figure 5:3. This particular stage-discharge relationship plots as two straight lines, one covering the lower end of the range, one the higher end ; the correction factor, $a$, has a value of 0.35 m , as indicated on the diagram. This graph is now used to read off the appropriate discharges for the recorded stages. (Figure 5.3. was drawn and supplied by the Tay River Purification Board).

The British Standards Institution (1964) laid down stringent requirements for: (a) the installation, operation and maintenance of the flow recorder ; (b) the selection of the cross-section and its

measurement ; (c) the procedure to be followed, and the precautions to be taken, when measuring velocities. It was also specified that the stage-discharge relation should be carefully and frequently checked by one or more prescribed methods. There is no reason to suppose that these requirements were not fulfilled. Indeed, Beale (Tay River Purification Board, pers. comm.) stated that he was very satisfied with the Kemback station, pointing out that a regular weekly gauging measurement is taken at the station every Monday, which he considered must make it one of the most regularly gauged stations in the United Kingdom. Thus, Beale was confident of the stage-discharge relationship.

However, there is one problem: weed growth in the river at Kemback during the summer months. This is illustrated in Plates 5.1 and 5.2 where the abundant weed is clearly visible. (The plates were taken during August, 1976). This weed effectively raises the water level in the river, which results in an overestimation of discharge. This is interpreted in Figure 5.3., the shaded area representing the scatter of stage-discharge points due to the summer weed growth. It is evident from the figure that the largest possible variation in river flow caused by weed growth was approximately $1 \mathrm{~m}^{3-1}$, which over a period of 100 days - a reasonable value for the duration of the summer weed effect - is equivalent to 0.09 mm runoff. The (1968-1976) mean annual runoff in the River Eden was 297 mm , and clearly this 0.09 mm overestimation is negligible in comparison. Thus, with Beale's statements regarding the Kemback gauging station in mind, the margin of error in runoff measurement can reasonably be taken to


Plate 5.1. Weed growth in the Eden at Kemback
gauging station, August, 1976.


Plate 5.2. Weed growth in the Eden upstream of
the Kemback gauging station, August, 1976.
lie within the limits given in the British Standards Institution (1964) publication: "When the stage-discharge curve is kept under frequent observation and is stable, and when the record is prepared from a flow-measurement station operated and maintained in accordance with this British Standard, the monthly and annual runoff will be more accurate than any individual measurement and may reasonably be taken to be of the order of $\pm 3$ per cent".

There is no reason to expect this 3 per cent error to constitute a consistent overestimate, or indeed a consistent underestimate of annual runoff in the years under consideration. Clearly, in some years the error in streamflow measurement could make a small contribution to the discrepancy between measured runoff, $Q_{C}$, and the index of annual runoff, $R_{c}$. For example, in the runoff years 1970-1971, when $Q_{C}=358 \mathrm{~mm}$, a 3 per cent overestimate of $Q_{C}$ would be equivalent to approximately 11 mm runoff - the discrepancy between $Q_{c}$ and $R_{c}$ for this particular year was 66 mm . However, it is equally likely that in this runoff year $Q_{C}$ could be underestimated by the same amount ! Overall it seems probable that the (1968-1976) mean runoff value $Q_{c}$ - 297 mm - was more or less unaffected by inaccuracies in streamflow measurement, since any overestimates and underestimates would be expected to cancel to a large extent. For the purposes of this work, it must be concluded that the degree of accuracy of the Kemback gauging station flow record is more than satisfactory.

SYNTHETIC RUNOFF SERIES FOR THE RIVER EDEN, FIFE
SOURCES OF THE RUNOFF SERIES AND THEIR RELATIVE ACCURACY
Although the major part of this work is concerned with the methodology of the Thom and Ledger (1976) approach, in particular the assessment of its deficiencies and overall accuracy,'various potentially useful annual runoff series have also been generated. To summarize, they have originated from the following sources:
(a) Potential excess winter rain, $R_{p}$, data at Leuchars for the years 1922 - 1976 inclusive. Annual runoff amounts in the Eden catchment, $Q_{C}$, for each of these years can be calculated via the regression equation $Q_{c}=0.84 R_{p}+227$ (see Table 3.5.). $R_{p}$ values for the period 1922 - 1976 are given in Table 3.3.
(b) Actual catchment excess winter rain, $R_{c}$, derived from 1950-64 mean potential evapotranspiration estimates and the long-term ratio between Leuchars and Eden catchment precipitation - $\rho_{c}$; an assumed root constant distribution was used to convert catchment potential excess winter rain, $R_{p c}$, to actual, $R_{c}$, values. The detailed reasoning has been presented in the series of equations 3.2. - 3.14., and discussed in Chapter 3. The parameter $R_{c}$ was designed to be equivalent to annual measured runoff, $Q_{C}$ : values of $R_{c}$ for the period 1922-76 are shown in Appendix II.
(c) Actual catchment excess winter rain, $R_{c}$, calculated directly from monthly catchment rainfall estimates and full Penman formulated values of monthly catchment potential evaporation values, as expressed in equation 4.1. An actual root constant distribution based
on a land-use survey was employed to transform $R_{p c}$ to the corresponding $R_{C}$ values. These $R_{c}$ values for the period 1957-1976 are presented in Appendix III.

In addition to these sources, there was one very interesting possibility: one of the longest rainfall records in the British Isles exists in Edinburgh, some 30 km from the Eden catchment. Could this record be used to produce a substantially longer runoff record for the Eden than the approximately 50 -year record derived from Leuchars data ? Rainfall has been measured continuously in Edinburgh since 1785, although the record is only homogeneous since 1895 when the raingauge site was fixed at the Royal Observatory, Blackford Hill. (In fact this is not strictly true: the raingauge has been moved in the environs of Blackford Hill at least twice since 1895 (Shaw, Meteorological Office, pers. comm.)). Prior to this date the gauge resided at various sites: Charlotte Square ; Duddingston ; "within one mile of the castle" ; Blackford Hill itself ; and "other places in Edinburgh" (Mossman 1896). However, as variations in mean rainfall from site to site probably lie within $\pm 5$ per cent of the overall mean (Thom and Ledger 1976), and since it has been seen that rainfall measurements are to say the least rather approximate, the errors involved in assuming the "Blackford Hill series" - 1785-1976 - to be homogeneous can be safely assumed to be relatively significant.

Thom and Ledger (1976) derived potential excess winter rain, $R_{p}$, values for Blackford Hill for the period 1785-1976: the complete data set is reproduced in Table 6.1. It was found that $R_{p}$ (Blackford

TABLE 6.1.
POTENTIAL EXCESS WINTER RAIN, $R_{p}$, AT BLACKFORD HILL, EDINBURGH
$\begin{array}{lllllllllll}\text { Winter Starting } & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\end{array}$

| 1730 | - | - | - | 52. | 218 | - | - | - | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1770 | 244 | 164 | 410 | 232 | 370 | 342 |  |  |  |  |
| 1780 | 137 | - | - | - | - | 307 | 135 | 351 | 76 | 229 |
| 1790 | 267 | 257 | 460 | 3 | 370 | 376 | -86 | 247 | 76 | 261 |
| 1800 | 57 | 31 | 64. | 21. | 63 | -18 | 16 | 155 | 331 | 263 |
| 1810 | 205 | 408 | 117 | 38 | 87 | 58 | 181 | 296 | 150 | 95 |
| 1820 | 176 | 163 | 192 | 217 | 149 | 116 | 91 | 216 | 240 | 252 |
| 1830 | 439 | 75 | 151 | 92 | 66 | 245 | 266 | 255 | 238 | 159 |
| 1840 | 149 | 237 | -78 | 146 | 48 | 261 | 236 | 282 | 196 | 111 |
| 1850 | 82 | 97 | 281 | 183 | 19 | 113 | 206 | 170 | 234 | 184 |
| 1860 | 307 | 380 | 300 | 199 | 208 | 200 | 272 | 312 | 197 | 142 |
| 1870 | 60 | 253 | 426 | 228 | 225 | 362 | 460 | 333 | 143 | 223 |
| 1880 | 167 | 271 | 249 | 140 | 105 | 44 | 111 | 113 | 72 | 181 |
| 1890 | 156 | 156 | 103 | 218 | 118 | 157 | 185 | 81 | 314 | 148 |
| 1900 | 287 | 83 | 190 | 194 | 94 | 63 | 185 | 351 | 131 | 244 |
| 1910 | 113 | 84 | 265 | 59 | 47 | 350 | 366 | 124 | 159 | 192 |
| 1920 | 217 | 114 | 71 | 106 | 420 | 254 | 240 | 498 | 213 | 224 |
| 1930 | 357 | 349 | 230 | 143 | 128 | 306 | 154 | 222 | 307 | 17 |
| 1940 | 330 | 138 | 70 | 134 | 364 | 187 | 260 | 193 | 353 | 262 |
| 1950 | 320 | 249 | 100 | 204 | 396 | -21 | 304 | 136 | 225 | 65 |
| 1960 | 145 | 147 | 202 | 262 | 85 | 339 | 316 | 238 | 245 | 115 |
| 1970 | 105 | 129 | -87 | 65 | 153 | 114 |  |  |  |  |

(All values in mm)

Hill) and the annual runoff of the Eden catchment, $Q_{c}$, for the 8year period 1968 - 1976 plots as a straight line of the familiar form $Q_{C}=a R_{p}+b$. Regression analysis revealed that the Blackford Hill $R_{p}$ values and River Eden $Q_{C}$ values are indeed closely related. The relevant quantitative details are given in Table 6.2. TABLE 6.2. REGRESSION DATA FOR RIVER EDEN MEASURED ANNUAL.

RUNOFF, $Q_{c}$, AND $R_{p}$ FOR BLACKFORD HILL, EDINBURGH

| Parameter | a | Correlation Coefficient, <br> $r$ | Coefficient of <br> Determination, $r^{2}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $R_{p}$ | 0.86 | 218 | 0.90 | 0.81 |

The correlation coefficient - $\mathrm{r}=0.90$ - is significant at 0.01 probability level; the coefficient of determination $-r^{2}=0.81-$ indicates that 81 per cent of the variability in River Eden annual streamflow can be accounted for by variation in $R_{p}$ at Blackford Hill, Edinburgh. The application of the Edinburgh $R_{p}$ data to runoff from the Eden catchment, Fife, is clearly valid.

The relative accuracy of the individual runoff series is reflected by regression analysis of the independent variables - $R_{p}$ (Leuchars), $R_{p}$ (Blackford Hill), $R_{c}$ (Leuchars data), $R_{c}$ (catchment data) - and the dependent variable, $Q_{C}$, River Eden annual runoff. The results of the regression analysis have been presented earlier in the thesis, but are brought together in Table 6.3 for convenient reference. In all cases close correlations are apparent: the variables have an obvious cause-and-effect relationship to one another, the results of the
analysis being far from coincidental. Not surprisingly, it is apparent from Table 6.3 that the best index of annual runoff is $R_{C}$ deduced from catchment precipitation estimates and full Penman evaporation values $-r=0.98$. However, the synthetic runoff record generated using this parameter is of only 19 years duration, a rather brief period for worthwhile frequency analysis. Therefore,it was considered more useful to examine the lengthier runoff frequency distributions generated using the index potential excess winter rain, $R_{p}$, for Leuchars and Blackford Hill.
TABLE 6.3.
A SUMMARY OF THE RESULTS OF REGRESSION ANALYSIS OF VARIOUS INDICES OF ANNUAL RUNOFF AND MEASURED RUNOFF IN THE RIVER EDEN

Runoff Index Correlation Coefficient, $\quad$| Coefficient of |
| :--- |
| Determination, $r^{2}$ |

| $R_{p}$ (Leuchars) | 0.96 | 0.92 |
| :--- | :--- | :--- |
| $R_{c}$ (Leuchars data) | 0.82 | 0.77 |
| $R_{c}$ (catchment data) | 0.98 |  |
| $R_{p}$ (Blackford Hill) | 0.90 | $\ddots$ |

A RUNOFF SERIES FOR THE RIVER EDEN USING LEUCHARS
POTENTİAL EXCESS WINTER RAIN, $R_{p}$, DATA (1922 - 1976)
The cumulative frequency distribution of $R_{p}$ at Leuchars, plotted from the data given in Table 3.3, is presented in Figure 6.1. From the straight line drawn through the data points in this figure it can be deduced that 60 per cent of all values of $R_{p}$ lie within 80 mm or 47 per

cent, of the series mean value of 170 mm . However, of much greater interest to those concerned with water supply schemes are the extremes of the distribution. Only 5 per cent of $R_{p}$ values lie below 0 mm , and some 10 per cent of $R_{p}$ values lie above 300 mm : in terms of the return period, these represent the 20 -year and 10 -year events respectively. 10 per cent of the time $R_{p}$ would be expected to fall below 40 mm , but for three out of the four years in the period 1972 - $1976 R_{p}$ was less than this, i.e. - $82 \mathrm{~mm}, 17 \mathrm{~mm}, 93 \mathrm{~mm}$ and 18 mm respectively. The runoff year 1972 - 1973 was in fact the driest on record $-R_{p}=-82 \mathrm{~mm}$ - and this is the 100-year event. It is possible to transform linearly the cumulative frequency distribution established for $R_{p}$ in Figure 6.1 by way of the correlation line $Q_{C}=0.84 R_{p}+227$ plotted in Figure 3.5, to obtain the corresponding distribution in amount of annual runoff from the Eden catchment. This is accomplished in Figure 6.1 itself by the inclusion of the additional ordinate scale in $Q$, annual runoff, from which it can be deduced, for example, that 68 per cent of all values of annual runoff from the Eden catchment can be expected to lie within 25 per cent of the long-term mean value of 400 mm ; 40 per cent of all runoff values lie within 50 mm or 12.5 per cent of the long-term mean. In terms of extreme values, only once in fifty years or so is the annual runoff from the Eden catchment likely to fall below 200 mm - 50 per cent of the long-term mean runoff value. Conversely, only once in approximately thirty-three years is it likely that the Eden annual runoff will exceed 600 mm - 150 per cent of the mean value.

In Figure 6.2 the $R_{p}$ data sequence is presented as a 3 -year

running mean $-\bar{R}_{p 3}$ - the "driest three consecutive years" being a relevant and frequently quoted parameter in reservoir design and operation (see Chapter 2). The $\overline{\mathrm{R}}_{\mathrm{p} 3}$ values are given in Table 6.4. From the figure and the table it is apparent that the wettest 3-year period was 1964-1967- $\bar{R}_{\mathrm{p} 3}=272 \mathrm{~mm}$; 1925-1928 and 1938-1941 were also relatively wet, with $\bar{R}_{p 3}$ values of 260 mm and 254 mm respectively. Dry periods occurred during the years 1941-1944 and 19511954 when $\bar{R}_{\mathrm{p} 3}$ was 63 mm and 84 mm respectively. However, by far the worst dry spel1 during the 54-year Leuchars record was 1972-1975 when $\bar{R}_{\mathrm{p} 3}$ was a mere 28 mm . A RUNOFF SERIES FOR THE RIVER EDEN EMPLOYING BLACKFORD HILL,

EDINBURGH, POTENTIAL EXCESS WINTER RAIN, $R_{p}$, DATA (1785-1976)
The cumulative frequency distribution of $R_{p}$ at Blackford Hill is shown in Figure 6.3 (from Thom and Ledger 1976). The regression equation, $Q_{c}=0.86 R_{p}+218$ was used to generate the corresponding distribution in annual runoff values for the Eden catchment, the ordinate scale in $Q$ being included in Figure 6.3. It is interesting to note that the frequency distribution of Eden annual runoff, $Q_{C}$, derived from Blackford Hill and Leuchars data are remarkably similar. Table 6.5. - deduced from Figures 6.1 and 6.3 - shows this similarity more clearly. For example, from the table it can be seen that the Blackford Hilll series mean (1785-1976) $\bar{Q}_{C}$ value is 415 mm , compared with a value of 400 mm deduced from the 54 -year Leuchars record ; the 90 per cent probability level is in fact the same for both the Blackford Hill and Leuchars distribution - 270 mm . The Blackford Hill frequency distribution shown in Figure 6.3 indicates that annual runoff


TABLE 6.4.
3-YEAR RUNNING MEAN, $\bar{R}_{p 3}$, OF POTENTIAL EXCESS WINTER
RAIN AT LEUCHARS, FIFE

| Period | $\bar{R}_{\mathrm{p} 3}(\mathrm{~mm})$ | Period | $\bar{R}_{\mathrm{p} 3}(\mathrm{~mm})$ |
| :---: | :---: | :---: | :---: |
| 1922-25 | 181 | 1948-51 | 157 |
| 1923-26 | 217 | 1949-52 | 180 |
| 1924-27 | 257 | 1950-53 | 142 |
| 1925-28 | 260 | - 1951-54 | 84 |
| 1926-29 | 251 | 1952-55 | 121 |
| 1927-30 | 229 | 1953-56 | 107 |
| 1928-31 | 210 | 1954-57 | 153 |
| 1929-32 | 239 | 1955-58 | 118 |
| 1930-33 | 254 | 1956-59 | 169 |
| 1931-34 | 159 | 1957-60 | 167 |
| 1932-35 | 124 | 1958-61 | 204 |
| 1933-36 | 162 | 1959-62 | 211 |
| 1934-37 | 257 | 1960-63 | 192 |
| 1935-38 | 213 | 1961-64 | 186 |
| 1936-39 | 222 | 1962-65 | 227 |
| 1937-40 | 217 | 1963-66 | 271 |
| 1938-41 | 254 | 1964-67 | 272 |
| 1939-42 | 184 | 1965-68 | 218 |
| 1940-43 | 151 | 1966-69 | 213 |
| 1941-44 | 63 | 1967-70 | 163 |
| 1942-45 | 92 | 1968-71 | 160 |
| 1943-46 | 160 | 1969-72 | 111 |
| 1944-47 | 246 | 1970-73 | 48 |
| 1945-48 | 214 | 1971-74 | 71 |
| - 1946-49 | 199 | 1972-75 | 28 |
| .1947-50 | 141 | 1973-76 | 43 |

from the Eden catchment can be expected to fall below 200 mm once every 33 years or so ; a $Q_{c}$ value in excess of 600 mm is likely to occur once in 20 years. The corresponding return periods derived from Leuchars data are 50 years and 33 years respectively. This likeness of the Blackford Hill and Leuchars distributions suggests that the Leuchars 54-year sample period is in fact reasonably representative of the parent "annual runoff" population. TABLE 6.5.

RELATIVE VALUES OF ANNUAL RUNOFF FOR THE RIVER EDEN
CALCULATED FROM (A) BLACKFORD HILL $R_{p}$ DATA AND (B)
LEUCHARS $R_{p}$ DATA FOR SELECTED PROBABILITIES OF OCCURRENCE
Percentage Probability Estimated Runoff (mm) Estimated Runoff (mm) of Runoff being Equalled (Leuchars data) (Blackford Hill data) or Exceeded

| 1 | 635 | 685 |
| :--- | :--- | :--- |
| 5 | 570 | 605 |
| 20 | 480 | 510 |
| 50 | 400 | 415 |
| 80 | 270 | 320 |
| 90 | 235 | 270 |
| 95 | 170 | 225 |
| 99 |  | 145 |

The 3 -year running mean, $R_{p 3}$, of potential excess winter rain at Blackford Hill is shown in Figure 6.4. A very significant fact is immediately apparent: a drought occurred at the turn of the nineteenth

century which was far worse in terms of both severity and persistence than the dry period in the 1970's $-\bar{R}_{p}$ for the 7-year period $1800-$ 1807 was only 33 mm . In comparison, the 6-year period 1970-1976 with $\bar{R}_{p}=80 \mathrm{~mm}$ appears as a relatively minor feature. $\bar{R}_{p 3}$ for the years 1804-1807 was $20 \mathrm{~mm} / a n n u m$, considerably less than the value of 36 mm recorded for 1971-1974. However, this should not obscure the important point that the recent dry spell was the worst for 163 years. In the interim period there has been only one comparable drought: this occurred during the years 1842-1845 when $\bar{R}_{p 3}$ was a mere 40 mm .
THE USE OF STATISTICAL ANALYSIS IN HYDROME TEOROLOGY :
A BRIEF CRITICISM
In the preceding paragraphs it has been assumed that the hydrological data under discussion are perfectly suitable for statistical? analysis. However, this is far from the case: as Chow (1964) stated: "Hydrological phenomena seldom completely satisfy the requirements of statistical theory". It is important to recognise some of the defects inherent in the excess winter rain, $R_{p}$, data.

The 54-year record of $R_{p}$ at Leuchars, and the approximately 200year record of $R_{p}$ at Blackford Hill, are only samples of the "excess winter rain" history, i.e. the $R_{p}$ parent population at these sites. Statistical theory is based on the concept that any sample is assumed to be representative of the population and therefore deductions can be made from the sample concerning the nature of the population. In order to draw inferences about any population, the data in the sample must be (a) random, (b) independent, and (c) homogeneous.

When every item in the population has an independent chance of being chosen, the sample is then said to be random. In most hydrological studies there can be very little control over the selection of a sample: as Bruce and Clark (1966) stated: "the information nature can provide over a period must be used". Thus, for example, the whole length of the Leuchars rainfall record - 1922-1976 - has been employed to give the Leuchars $R_{p}$ "sample". With regard to supposition (b), successive values of any meteorological variable (annual or otherwise) are never totally independent. This is admirably illustrated in Figures 6.2 and 6.4 (more conclusively in the latter since a longer period is shown), where it is evident that wet and dry years tend to occur in groups rather than in random sequences. For example, the plot of the 3 -year running mean $\bar{R}_{p 3}$ at Blackford Hill (Figure 6.4) reveals that particularly wet spells occurred during the years 1871-1880, 1924-1931, and 1946-1951 ; dry periods occurred during the years 1800-1807, and 1969-1975, among others. Moreover, many more sequences of particularly dry years are apparent in Figure 6.4 than would be likely if $R_{p}$ were a truly random variable: just how many could be obtained from more detailed statistical analysis.

Excess winter rain is the result of the processes of precipitation and evaporation. Undoubtedly, precipitation is the dominant factor in the determination of $R_{p}$ : with reference to requirement (c) it has been seen that neither the Blackford Hill nor the Leuchars rainfall data are homogeneous, the Leuchars raingauge site having been moved only once, while the Blackford Hill data are a conglomeration of the rainfall records from various sites in Edinburgh. These
already statistically dubious $R_{p}$ data are then subjected to the frequency analysis technique.

Frequency analysis assumes that the $R_{p}$ and hence the deduced annual runoff, $Q_{C}$, values follow, or tend toward, a normal or Gaussian distribution. This assumption is true for the $R_{p}$ data, as indicated by the fact that the $R_{p}$ values plot as a straight line on normal probability paper - see Figures 6.1 and 6.3. However, a difficult problem encountered in frequency analysis is that of sampling errors. As an illustration, consider that a very iong riverflow record - 10,000 years - is available for frequency analysis. In such a record, the magnitude of, for example, the 100-year flood will be well established. The question is: How often will there be no flood as big as the 100-year flood in a period of 100 consecutive years, how often one, and how often more than one 100-year flood in 100 years ? Linsley, Kohler and Paulus (1958) published details of the theoretical return periods of floods of various magnitudes, as shown in Table 6.6. (It is assumed that the floods are distributed fortuitously ; this may not be exactly correct, but it does offer a reasonable means of estimating sampling errors).

From the table it can be seen, for example, that 25 per cent of the intervals between floods equal to or greater than the 100-year flood will be less than 29 years, while an equal number will be in excess of 139 years.

According to various authors, frequency analysis applied to drought studies involves even larger sampling errors! The technique was used in the Missouri basin above Sioux City, Iowa, to
determine the need for design after the severe 1930's drought in the American "Mid-West". The estimated return periods of the minimum observed 1, 2, 3, and 5-year precipitation during the period 1930 1940 are $560,170,2000$, and 1100 years respectively. However,there is a 5 per cent chance that the true average return period is as short as 30 years for each event (Linsley et al 1958). TABLE 6.6.

THEORETICAL DISTRIBUTION OF FLOOD RETURN PERIODS
Average Actual Return Period $\mathrm{T}_{\mathrm{r}}$ (years) exceeded various Return

| Period | $1 \%$ | $5 \%$ | $25 \%$ | $50 \%$ | $75 \%$ | $95 \%$ | $99 \%$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Ir (years) | $1 \%$ | 5 | 3 | 1 | 0 | 0 | 0 |
| 2 | 2 | 22 | 14 | 7 | 3 | 1 | 0 |
| 10 | 45 | 28 | 14 | 7 | 3 | 0 | 0 |
| 10 | 137 | 89 | 42 | 21 | 8 | 2 | 0 |
| 10 | 459 | 300 | 139 | 69 | 29 | 5 | 1 |
| 100 | 4620 | 3000 | 1400 | 693 | 288 | 51 | 10 |

Another factor that should be considered in interpreting the past record of hydrological events as a measure of future probabilities of occurrence is the question of the effect of climatic change. It seems reasonably safe to assume that chance is the major influence in changes in rainfall over periods of several years, e.g. decades ; however, there are undoubtedly changes in climatic conditions over several centuries. H.H. Lamb in particular has made a substantial contribution to the study of climatic variability. A brief indication of the variations in rainfall during the present interglacial are given in Table 6.7. (from Lamb 1972).

AVERAGE RAINFALL ESTIMATES FOR ENGLAND AND WALES
DURING THE PRESENT INTERGLACIAL

| NameEpoch Date | Percentage of the 1916-50 <br> Average Rainfall |  |
| :--- | :---: | :---: |
| Boreal | C. 6500 BC | $92-95$ |
| Atlantic | C. 4000 BC | $110-115$ |
| Sub-Boreal | C .2000 BC | $100-105$ |
| Sub-Atlantic | $900-450 \mathrm{BC}$ | $103-105$ |
| Little Optimum | $1150-1300$ | 103 |
| Little Ice Age | $1300-1700$ | 93 |
| Warm Decades | $1900-1950$ | 99 |

Because of these climatic fluctuations, it seems doubtful that any long hydrological record, e.g. the Blackford Hill $R_{p}$ data should be regarded as a sample from one population - the data are probably not homogeneous. Thus the $R_{p}$ record at Leuchars would appear to be better suited to frequency analysis than that of Blackford Hill , particularly since the longer Blackford Hill record is also a very poor second to Leuchars as regards "raingauge site" homogeneity. Conversely, it must be realized that the shorter-term Leuchars $R_{p}$ record could cover a period of "irregular" hydrological conditions, although it seems that this latter argument does not appear to be significant in this case. Generally, however, it is true that the shorter the hydrological record, the greater will be the sampling error. It is emphasized that length of record is a factor that must be considered very carefully indeed.

## ANNUAL RUNOFF SERIES

The long-term excess winter rain, $R_{p}$, records at Blackford Hill and Leuchars have been correlated with the short-term - 8-year - runoff record of the River Eden to generate synthetic annual runoff records of 191 years and 54 years duration respectively. Clearly, use of runoff estimates derived from correlation are in some ways an unsatisfactory substitute for actual discharge records. Like any hydrological record, a streamflow record of any length, represents only a sample of the flow of the stream, and as such contains an inherent sampling error. This sampling error, which cannot be reduced by measuring the same samples more accurately, is often large in comparison to the errors that would be introduced by correlation with a longer record. This is certainly true in the case of the 8 year Eden record, the 1968 - 1976 period encompassing the driest conditions to occur for over a century! Thus correlation with longer records can serve to reduce the sampling error and to provide a more reliable base from which to estimate the characteristics of future flow.

Langbein and Hardison (1955) demonstrated that extending runoff data by the correlation technique is profitable wherever the correlation coefficient, $r$, exceeds 0.44 and one record is at least 25 per cent longer than the short record. These conditions are met for all the indices of annual runoff given at the beginning of the chapter and in Table 6.3. Furthermore, Langbein and Hardison (1955) have shown that the effective period of record, $N$, of a combined shortterm and extended record is approximately:

where $N_{r}=$ number of years of short-term record
$N_{e}=$ number of years of extension
$r=$ correlation coefficient
From the formula, it is apparent that the effective period of the runoff record derived via Leuchars $R_{p}$ data is approximately 34 years ; the corresponding effective record derived from Blackford Hill data is circa 28 years.

The conclusions to be drawn from this work can be conveniently divided into two categories: those relating to the methodology of the Thom and Ledger (1976) approach and its modified version; and those concerning the hydrological conditions in the Eden catchment over the years 1785-1976.

## CONCLUSIONS REGARDING METHODOLOGY

The analysis in this thesis has shown that the parameter potential excess winter rain, $R_{p}$, is a useful and accurate index of annual runoff. For example, regression analysis indicates that 92 per cent of the variablility in annual runoff in the Eden catchment can be explained by variation in $R_{p}$ at Leuchars. Thus $R_{p}$ can be confidently used to give a meaningful ranking of annual runoff events in the Eden catchment. Moreover, $R_{p}$ derived for Blackford Hill, Edinburgh - some 30 km distant from the the Eden catchment is also a relatively reliable measure of Eden annual runoff. This indicates that Edinburgh and the Fife peninsula are relatively homogeneous in terms of climate. The runoff year selected by Thom and Ledger (1976) to test the derived values of $R_{p}$ was taken to run from July to June: this appears to be very suitable for the Eden catchment, and indeed for south-east Scotland in general.

The parameters catchment potential and actual excess winter rain $-R_{p c}$ and $R_{c}$ respectively - determined by consideration of the hydrological characteristics of the Eden catchment and Leuchars longterm rainfall station are reasonable indices of Eden annual runoff:
correlation coefficients between measured annual runoff, $Q_{C}$, and $R_{p c}$ and $R_{c}$ were found to be 0.88 and 0.82 respectively. Clearly, these two parameters are less reliable than $R_{p}$ derived for Leuchars as measures of annual runoff magnitudes in the Eden catchment. Nevertheless, the series of equations 3.2 - 3.14 proposed by Thom and Ledger to determine $R_{p c}$ and $R_{c}$ are more than "interesting conceptually" as suggested by Law (1977). The equations - which were applied by Thom and Ledger (1976) to the Blackford Hill rainfall record and nearby Lothian catchments - have been tested thoroughly in a different situation - the Leuchars site and the Eden catchment - and have proved to be satisfactory. For example, in the runoff years 1970-71 and 1971-72 when measured runoff was recorded as 358 mm and $341 \mathrm{~mm}, R_{p c}$ gave estimates of 313 mm and 336 mm respectively; $R_{c}$ gave corresponding values of 327 mm and 342 mm respectively. However, a major flaw in the equations is the use of the $\rho_{c}$ ratio to translate rainfall from the long-term station to the catchments. Considerable inaccuracies in runoff estimation result in some years due to this: in the runoff year 1973-74 the $\rho_{c}$ term overestimated Eden catchment precipitation by more or less 100 mm , and hence $R_{p c}$ and $R_{c}$ gave substantial overestimates of measured runoff - 37 mm and 103 mm respectively. Unfortunately, the accurate translation of rainfall amounts from one location to another appears to be an insurmountable problem.

The modification of the Thom and Ledger method enables catchment potential excess winter rain, $R_{p c}$, and catchment maximum potential soil moisture deficit, $D_{p c}$, to be calculated directly as:

$$
\begin{aligned}
R_{p c} & =\sum p_{c}-e_{p c} \\
D_{p c} & =\sum e_{p c}-p_{c}
\end{aligned}
$$

The data input - monthly catchment precipitation, $p_{c}$, calculated via the standard Meteorological Office isohyetal procedure, and monthly catchment potential evapotranspiration, $e_{p c}$, calculated from measured meteorological variables via the full Penman formula is more accurate than that employed in the basic Thom and Ledger approach. Thus it is not surprising that $R_{p c}$ calculated directly gives improved estimates of measured runoff, $Q_{C} .95$ per cent of the variation in $Q_{c}$ can be attributed to variation in $R_{p c}$.

The potential values of catchment excess winter rain $-R_{p c}$ were changed to actual values $-R_{c}$ - via the equation $R_{c}=R_{p c}+$ $\left(D_{p c}-D_{c}\right)$. An actual root constant distribution - determined by land-use survey of the Eden catchment - was employed to derive actual soil moisture deficit, $D_{c}$, from the corresponding $D_{p c}$ values. It is clear that the use of an actual rather than an assumed root constant distribution to deduce $D_{c}$ (the latter method employed by Thom and Ledger (1976)) makes little difference to the adjustment of $R_{p c}$ to give the corresponding. $R_{c}$ value. Thus Penman's (1950) supposition regarding the insensitivity of ( $D_{p c}-D_{c}$ ) to the selection of root constant distribution appears to be well-founded, and the time and effort expended on the land-use survey seems unnecessary.

The calculated $R_{c}$ values were extremely closely related to measured annual runoff in the Eden catchment: a correlation coefficient of 0.98 was achieved for the period 1968-76. However $\bar{R}_{c}$ (196876) underestimated $\bar{Q}_{C}(1968-76)$ by some 74 mm - or 25 per cent of $\bar{Q}_{c}$.

Table 7.1 gives a summary of the possible contributory factors to this underestimate and their magnitudes. The notation and information in the table is itemized in the following paragraphs.

TABLE 7.1.
SUMMARY OF THE CONTRIBUTORY FACTORS IN THE UNDERESTIMATION
OF MEASURED ANNUAL RUNOFF, $Q_{C}$, BY ACTUAL CATCHMENT EXCESS
WINTER RAIN, $\mathrm{R}_{\mathrm{c}}$

| Source | a | b | c | d | e | f | g |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Estimated contribution to the discrepancy $\left(Q_{c}-R_{c}\right)(m m)$ | 3 | 1.8 | 11 (hypothe value | $\begin{aligned} & \text { tica1 }{ }^{38} \\ & \text { e) } \end{aligned}$ | $\pm 5 \%$ | 34 | $\pm 3 \%$ |
| Period | $\begin{aligned} & \text { 1968-76 } \\ & \text { mean } \\ & \text { value } \end{aligned}$ | 1974-75 | 1973-74 | 1972-73 | 12 months | 1968-76 mean value | 12 months |
| Contribution expressed as a percentage of 1968-76 ( $\mathrm{Q}_{\mathrm{c}}-\mathrm{R}_{\mathrm{c}}$ ) value ( 74 mm ) | 4.0 | 2.5 | 15.0 | 51.5 | - | 46.0 | - |
| Contribution expressed as a percentage of 1968-76 ̄̄c value (297 mm) | 1.0 | 0.5 | 4.0 | 13.0 | - | 11.5 | - |

It is concluded that the following make a negligible contribution to the discrepancy $\left(\overline{Q_{C}-R_{C}}\right)$ : (a) The assumption that all rain falling during times of existing soil moisture deficit is accepted by the soil and results in no direct runoff : this accounts for a mere 4 per cent of the 1968-76 discrepancy between $\overline{\hat{Q}}_{c}$ and $\overline{\mathrm{R}}_{c}$. (b) The
import of water from outwith the Eden catchment for industrial and domestic purposes, its subsequent discharge into the river. system augmenting artificially annual runoff. The maximum addition to annual runoff in the sample year $1974-75$ was less than 2 mm : this represents less than one per cent of the $1968-76 Q_{C}$ value. Of slightly more significance is (c) the use of monthly time steps to reflect accurately daily moisture balances. For example in the runoff year 1973-74 it is hypothesised that perhaps some 15 per cent of the 1968-76 ( $Q_{C}-R_{c}$ ) value of 74 mm can be attributed to the use of monthly time steps. However, it is emphasised that this procedure will result in discrepancies of this magnitude only occasionally in the Eden catchment.

However,a major deficiency in the Thom and Ledger model is (d) the supposition that changes in groundwater storage - $\Delta G$ - from year to year are negligible. This deficiency is especially serious in catchments with substantial groundwater reservoirs - such as the Eden catchment - particularly in dry years. Thus in the very dry runoff year 1972-73, some 38 mm - or 52 per cent - of the 1968-76 discrepancy between mean estimated and measured runoff could be attributed to changes in the groundwater regime; this 38 mm constitutes some 22 per cent of the measured runoff during this 12 -month period. Further, it is also probable that in all years the River Eden annual streamflow was augmented by groundwater seepage into the catchment from the adjacent Loch Leven basin. Thus, in its inability to. account for groundwater complications, the Thom and Ledger model suffers the same disadvantage as many water balance approaches.

With regard to the data used in the modified Thom and Ledger approach, the accuracy of (e) Penman formulated potential evapotranspiration, PE, values seems to be satisfactory. A review of the relevant literature revealed that Penman PE values for lowland, agricultural catchments can be expected to be accurate to within approximately 5 per cent of the true values. It is deemed that the inaccuracy of PE estimation made little contribution to the 1968-76 mean discrepancy between $Q_{C}$ and $R_{c}$, since in some years the true $P E$ amount will be overestimated while in other years it will be underestimated. There can be little or no doubt that the Penman formula is the best available for evaporation estimation in lowland locations in the British Isles.

However, (f) the systematic underestimation of precipitation by the Meteorological Office Mark II raingauge has been identified as a highly significant contributory factor in the inaccuracy of the parameter catchment excess winter rain. The addition of a 5 per cent increment to the annual catchment rainfall figures accounts for some 46 per cent of the 1968-76 mean discrepancy between measured and estimated runoff $-\left(\overline{Q_{c}}-R_{c}\right)$. Thom and Ledger (1976) suggested that the fact that the recorded discrepancies between $\bar{Q}_{c}$ and $\bar{R}_{c}$ for the Esk, Tyne and Peffer/Pilmuir catchments could be explained by the addition to annual rainfall of an increment of circa 5 per cent provided further evidence of the systematic underestimation of precipitation in Great Britain - by around 5 per cent on average. However, the present detailed analysis of water balance components in the Eden catchment strongly suggests that the recorded discrepancies for

Thom and Ledger's selected catchments were, in fact, largely coincidental. In order to make a suggestion of this kind, it is necessary that the procedure involved in the estimation of $R_{c}$ is very precise, and further that the data employed are of equal or better accuracy than $\pm 5$ per cent. The use of the $P_{c}$ ratio and the assumption that groundwater changes are negligible are contrary to the former supposition; the fact that potential evapotranspiration can be determined at best to within $\pm 5$ per cent contradicts the latter.

Finally, the measurement of River Eden runoff at the Kemback gauging station - item (g) in Table 7.1. - is concluded to be very satisfactory: over the requisite 12 -month runoff period the recorded value is probably accurate to within $\pm 3$ per cent of the true amount. To all intents and purposes, therefore, errors in runoff measurement are of little or no significance in the discrepancy $\left(Q_{c}-R_{c}\right)$. HYDROLOGICAL CONDITIONS IN THE EDEN CATCHMENT, 1785-1976

The synthetic runoff series generated for the Eden from the indices potential excess winter rain, $R_{p}$, at Blackford Hill, Edinburgh, and Leuchars, Fife, are remarkably similar. Thus the series mean annual runoff values for the Eden derived from Blackford Hill (1785-1976) and Leuchars (1922-1976) data are 415 mm and 400 mm respectively. The Blackford Hill frequency distribution suggests that annual runoff from the Eden catchment can be expected to fall below 200 mm once every 33 years or so; a $Q_{C}$ value in excess of 600 mm is likely to occur once in 20 years. The corresponding return periods derived from Leuchars data are 50 and 33 years
respectively. Values derived from Leuchars and Blackford Hill data for the one in a 100 -year drought event are 170 mm and 145 mm respectively.

The driest year during the period 1922-76 was 1972-73, when $R_{p}$ at Leuchars was -82 mm : this is equivalent to a runoff value of 170 mm . (The measured runoff value for this year was in fact 170 mm ). Only in one year in ten would $R_{p}$ be expected to fall below 40 mm , but this occurred three times in four years during the period 197276 ; for the years 1972-75, the mean potential excess winter rain value, $\bar{R}_{p 3}$, was $28 \mathrm{~mm} /$ annum. Other dry spells occurred during the years 1941-44 and 1951-54 when $\bar{R}_{p 3}$ was 63 mm and 84 mm respectively.

Turning to the runoff series derived from Blackford Hill data for the longer period 1785-1976, it can be seen that 1972-73 was, in fact, the driest single runoff year during this 191-year period, with $R_{p}=-87 \mathrm{~mm}$. It is followed closely by the year 1806-07, when $R_{p}$ was -86 mm ; another very dry runoff year was 1842-43 when $R_{p}$ was -78 mm . However, in terms of "dry spells", it is clear that a drought occurred at the beginning of the nineteenth century which was even more severe than that of the 1970's: $\bar{R}_{p}$ for the 7-year period 1800-07 was a mere 33 mm . $\bar{R}_{p 3}$ for the years 1804-07 was 20 $\mathrm{mm} /$ annum, considerably less than the value of $36 . \mathrm{mm}$ recorded for 1971-74. Nevertheless, to put the recent drought into the correct perspective, it was still the worst experienced by south-east Scotland in circa 160 years !

It is noted that the absolute magnitudes of potential excess winter rain, $R_{p}$, values, and hence deduced annual runoff, $Q_{c}$, values
are by no means precise. Nevertheless, it must be argued that the estimates produced using the Thom and Ledger (1976) method are more exact, and indeed reliable, than those very approximate values devised via the largely empirical methods of Hawksley, Binnie and others. The index excess winter rain integrates the best estimates of precipitation and evaporation to produce a realistic estimate of available water: it is as good an index of annual runoff as any other, and surely better than most. Thus the $R_{p}$ series for Blackford Hill and Leuchars are proposed as useful and valid sources of annual runoff information for those concerned with water resources design and appraisal in Fife and nearby areas.

APPENDIX I

Maximum potential soil moisture deficit (mm) at
Leuchars, Fife, for the period 1922-76.

| Summer <br> of | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | ---: | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1920 |  |  | 150 | 169 | 67 | 161 | 125 | 118 | 147 | 148 |
| 1930 | 152 | 20 | 145 | 204 | 83 | 162 | 141 | 70 | 92 | 165 |
| 1940 | 164 | 164 | 139 | 131 | 132 | 64 | 142 | 133 | 100 | 155 |
| 1950 | 127 | 82 | 150 | 170 | 108 | 256 | 124 | 127 | 72 | 204 |
| 1960 | 103 | 150 | 159 | 132 | 148 | 90 | 76 | 158 | 102 | 126 |
| 1970 | 128 | 133 | 223 | 307 | 250 | 185 |  |  |  |  |

## APPENDIX II

Values of excess winter rain (mm) and soil moisture deficit (mm)
for the Eden catchment for the period 1922-76 calculated via
equations $3.2-3.10$ and $3.12-3.16$. (See Chapter 3 )
$\underset{\text { Year }}{\text { Runoff }} \quad R_{p c} \quad R_{c} \quad D_{p c} \quad D_{c} \quad\left(D_{p c}-D_{c}\right) \cdot \underset{\text { Year }}{\text { Runoff }} \quad R_{p c} \quad R_{c} \quad D_{p c} \quad D_{c} \quad\left(D_{p c}-D_{c}\right)$

| $1922-23$ | 250 |  | 103 | 94 | 9 | $1951-52$ | 345 | 345 | 72 | 69 | 3 |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $1923-24$ | 344 | 353 | 72 | 68 | 4 | $1952-53$ | 139 | 142 | 140 | 115 | 25 |  |
| $1924-25$ | 669 | 673 | -35 | -35 | 0 | $1953-54$ | 364 | 389 | 66 | 63 | 3 |  |
| $1925-26$ | 363 | 363 | 66 | 63 | 3 | $1954-55$ | 410 | 413 | 50 | 49 | 1 |  |
| $1926-27$ | 410 | 413 | 50 | 49 | 1 | $1955-56$ | 125 | 126 | 144 | 113 | 26 |  |
| $1927-28$ | 697 | 698 | -44 | -44 | 0 | $1956-57$ | 451 | 477 | 37 | 37 | 0 |  |
| $1928-29$ | 313 | 313 | 82 | 76 | 6 | $1957-58$ | 526 | 546 | 12 | 12 | 0 |  |
| $1929-30$ | 332 | 338 | 76 | 72 | 4 | $1958-59$ | 248 | 248 | 104 | 94 | 10 |  |
| $1930-31$ | 742 | 746 | -59 | -59 | 0 | $1959-60$ | 474 | 484 | 29 | 29 | 0 |  |
| $1931-32$ | 361 | 361 | 67 | 64 | 3 | $1960-61$ | 459 | 459 | 33 | 33 | 0 |  |
| $1932-33$ | 316 | 319 | 78 | 73 | 5 | $1961-62$ | 316 | 316 | 81 | 75 | 6 |  |
| $1933-34$ | 303 | 308 | 86 | 79 | 7 | $1962-63$ | 458 | 464 | 34 | 34 | 0 |  |
| $1934-35$ | 335 | 342 | 75 | 71 | 4 | $1963-64$ | 490 | 490 | 23 | 23 | 0 |  |
| $1935-36$ | 448 | 452 | 38 | 38 | 0 | $1964-65$ | 209 | 209 | 117 | 103 | 14 |  |
| $1936-37$ | 441 | 441 | 47 | 46 | 1 | $1965-66$ | 604 | 618 | -8 | -8 | 0 |  |
| $1937-38$ | 393 | 394 | 56 | 54 | 2 | $1966-67$ | 406 | 406 | 51 | 50 | 1 |  |
| $1938-39$ | 493 | 495 | 32 | 32 | 0 | $1967-68$ | 464 | 465 | 32 | 32 | 0 |  |
| $1939-40$ | 297 | 297 | 88 | 81 | 7 | $1968-69$ | 565 | 565 | -2 | -2 | 0 |  |
| $1940-41$ | 603 | 610 | -13 | -13 | 0 | $1969-70$ | 196 | 196 | 121 | 107 | 14 |  |
| $1941-42$ | 351 | 351 | 70 | 67 | 3 | $1970-71$ | 313 | 327 | 82 | 76 | 6 |  |
| $1942-43$ | 261 | 264 | 100 | 90 | 10 | $1971-72$ | 336 | 342 | 74 | 70 | 4 |  |
| $1943-44$ | 194 | 204 | 122 | 108 | 14 | $1972-73$ | -31 | -27 | 197 | 131 | 66 |  |
| $1944-45$ | 464 | 478 | 33 | 33 | 0 | $1973-74$ | 243 | 309 | 105 | 95 | 10 |  |
| $1945-46$ | 393 | 393 | 56 | 54 | 2 | $1974-75$ | 229 | 239 | 110 | 99 | 11 |  |
| $1946-47$ | 651 | 653 | 17 | 17 | 0 | $1975-76$ | 233 | 244 | 109 | 98 | 11 |  |
| $1947-48$ | 267 | 267 | 98 | 89 | 9 |  | . |  |  |  |  |  |
| $1948-49$ | 293 | 302 | 85 | 82 | 3 |  |  |  |  |  |  |  |
| $1949-50$ | 342 | 345 | 73 | 69 | 4 |  | 6 |  |  |  |  |  |
| $1950-51$ | 531 | 535 | 10 | 10 | 0 |  |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |  |  |  |  |

```
where R }\mp@subsup{R}{pc}{}=\mathrm{ catchment potential excess winter rain (mm)
    R
    D
        deficit (mm)
    D
        deficit (mm)
```

Interpretation of the data given in Appendix II :
the sample runoff year 1970-71
$R_{p c}$ and $R_{c}$ values $=R_{p c}$ and $R_{c}$ values for the winter starting 1970.
$D_{p c}$ and $D_{c}$ values $=D_{p c}$ and $D_{c}$ values for the summer of 1971.
Calculated $\left(D_{p c}-D_{C}\right)$ value -6 mm - is added to the $R_{p c}$ value for 1971-72 to give the $R_{c}$ value for the latter year.

## APPENDIX III

Excess winter rain and maximum soil moisture deficit values for the
Eden catchment for the period 1957-76 calculated via the modified
Thom and Ledger approach. (See Chapter 4)

| Runoff Year | $\mathrm{R}_{\mathrm{pc}}$ | $\mathrm{R}_{\mathrm{c}}$ | $\mathrm{D}_{\mathrm{pc}}$ | ${ }^{\text {c }}$ | $\left(D_{p c}-D_{c}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1956-57 | - | - | 46 | 46 | 0 |
| 1957-58 | 382 | 382 | 16 | 16 | 0 |
| 1958-59 | 413 | 413 | 154 | 127 | 27 |
| 1959-60 | 394 | 421 | 69 | 68. | 1 |
| 1960-61 | 478 | 479 | 121 | 111 | 10 |
| 1961-62 | 277 | 287 | 131 | 116 | 15 |
| 1962-63 | 347 | 362 | 27. | 27 | 0 |
| 1963-64 | 501 | 501 | 96 | 91 | 5 |
| 1964-65 | 222 | 227 | 8 | 8 | 0 |
| 1965-66 | 574 | 574 | 41 | 41 | 0 |
| 1966-67 | 379 | 379 | 115 | 106 | 9 |
| 1967-68 | 314 | 323 | 62 | 61 | 1 |
| 1968-69 | 451 | 452 | 94 | 89 | 5 |
| 1969-70 | 215 | 220 | 100 | 94 | 6 |
| 1970-71 | 286 | 292 | 74 | 72 | 2 |
| 1971-72 | 243 | 245 | 130 | 115 | 15 |
| 1972-73 | 55 | 70 | 172 | 134 | 38 |
| 1973-74 | 107 | 145 | 170 | 133 | 37 |
| 1974-75 | 209 | 246 | 193 | 136 | 57 |
| 1975-76 | 53 | 110 | 174 | 135 | 39 |

(All values in mm)
The notation and interpretation of data are as given in Appendix II.

ABBE, C., The Piche evaporimeter, Monthly Weather Rev., 33, 253-255 (1905)

BACHE, A.D., Note on the effect of deflected amounts of air on the quantity of rain collected in a raingauge, Trans. Ass. Adv. Sci:, p25 (1838)

BAIER, W. and ROBERTSON, G. W., Estimation of latent evaporation from simple weather observations, Can. J. Plant Sci:, 45, 275-284 (1965)

BAVEL, C. H. M. van, FRITSCHEN, L. J. and REEVES, W. E., Transpiration of sudangrass as an externally controlled process, Science, 141, 269-270 (1963)

BEALE, G., Tay River Purification Board, Perth, personal communication

BILHAM, E. G., The Climate of the British Isles, Macmillan and Co., London (1938)

BLEASDALE, A., The measurement of rainfal1, Weather, 14, 12-18 (1959)

BRITISH STANDARDS INSTITUTION, Methods of measurement of iqquid flow in open channels, Part 3 : Velocity area methods, British Standard 3680 (1964)

BRUCE, J. P. and CLARK, R.H., Introduction to Hydrometeorology, Pergamon Press, 0xford (1966)

BUDYKO, M. I., The heat balance of the earth's surface, Transl. by N. A. Stepanova, Office of Technical Services, U.S. Dept. of Commerce, Washington D.C. (1958)

CHANG, J. H., On the study of evapotranspiration and water balance, Erdkunde, 19, 141-150.(1965)

CHISHOLM, J. I. and DEAN, J. M., The upper old red sandstone of Fife and Kinross, Scot J. Geol., 10, 1-30 (1974)

CHOW, V. T., Handbook of Applied Hydrology, McGraw-Hill, New York (1964)

DINES, J. S., Rainfall measurements at ground level, Weather, 13, p211 (1958)

DIXEY, F., A Practical Handbook of Water Supply, 2nd edn., Murby, London (1950)

EDWARDS, K. A. and RODDA; J. C., A preliminary study of the water balance of a small clay catchment, Proc. IASH Symp. on results of research in representative and experimental basins, Wellington, IASH Pub. No. 97, 187-199 (1972)

FOSTER, S. S. D., STIRLING, W. G. N. and PATERSON, I.B., Groundwater storage in Fife and Kinross : Its potential as a regional resource, IGS Report Series (1976)

FREEZE, R. A., Role of subsurface flow in generating surface runoff 1: Baseflow contributions to channel flow, WRR, 8, 609-623 (1972)

GLOVER, J. and MCCULLOCH, J. S. G., The empirical relation between solar radiation and hours of sunshine, Q.J.R. Met. Soc.., 84, 172-175 (1958)

GREEN, M. J., Some factors affecting the catch of raingauges, Met. Mag., 99, 10-20 (1970)

GREEN, F. H. W., Rainfall at ground leve1, Weather, 13, 275-277 (1958)

GREEN, F. H. W., Some observations of potential evapotranspiration, 1955-57; Q.J.R.Met. Soc., 35, 152-155. (1959)

GRINDLEY, J., The estimation of soil moisture deficits, Met. Mag., 102, 97-108 (1967)

GRINDLEY, J., The calculation of actual evaporation and soil moisture deficit over specified catchment areas, Met. Off. Hyd. Mem. No. 38 (1969)

GRINDLEY, J., Estimation and mapping of evaporation, in Symp. on World Water Balance, IASH. Publ. 92, 1:200-213 (1970)

HARROLD, L. L., Measuring evapotranspiration by lysimetry, in Evapotranspiration and its role in water resources management, Conf. Proc. Amer. Soc. Agric. Eng., 28-33 (1966)

HEBERDEN, W., of the different quantities of rain, which appear to fall at different heights, over the same spot of ground, Phil. Trans., 59, p359 (1769)

HORTON, R. E., Erosional development of streams and their drainage basins : hydrophysical approach to quantitative morphology, Geol. Soc. Amer. Bull., 56, 275-370 (1945)
hYLCKAMA, T. E. A. van., Growth, development and water use by saltcedar under different conditions of weather and access to water, IASH Gen. Ass. Berkeley, Cttee.for Evaporation, Publ. 62, 75-86 (1965)

INESON, J. and DOWNING, R. A., Some hydrogeological factors in permeable catchment studies, J. Instn Wat. Engnrs., 19, 59-80 (1965)

INSTITUTION OF WATER ENGINEERS, Interim Report of the Hydrological Committee (1936)

INSTITUTION OF WATER ENGINEERS, Manual of British Water Supply Practice, 2nd edn., Cambrịdge (1954)

JEVONS, W. S., On the deficiency of rạin in an elevated raingauge as caused by wind, Phil. Mag., Series 4, 22, p421 (1861)

KIRKBY, M. J., Infiltration, throughflow, and overland flow, in Chorley, R. J., (Ed.) Introduction to Physical Hydrology, Methuen, London (1969)

KIRKBY, M. J. and CHORLEY, R. J., Throughflow, overland flow and erosion, Bull. IASH, 12, 5-21 (1967)

KURTYKA, J. C., Precipitation measurements study, Report of Invest. No. 20, State Water Survey Division, Illinois (1953)

LAMB, H. H., Climate - present past and future. Vol. 1. Fundamentals and climate now. Methuen, London (1972)

LANGBEIN, W. B. and HARDISON, C. H., Extending streamflow data, Am. Soc. Civ. Engnrs. Proc., Paper 826 (1955)

LAW, F., The estimation of the reliable yield of a catchment by correlation of rainfall and runoff, Journ: Instn. Wat. Engnrs., 7, 273-293 (1953)

LAN, F., The measurement of rainfall, Weather, 14, pl87 (1959)

LAW, F. M: , Discussion of "Rainfall, runoff and climatic change" (A. S. Thom and D. C. Ledger) Proc. Instn. Civ. Engnrs., 63, 721-731 (1977)

LEDGER, D. C. and THOM, A. S., 200 years of potential soil moisture deficit in south-east Scotland, Weather, 32, 342349 (1977)

LEWIS, W. K., Investigation of rainfall, runoff, and yield on the Alwen and Brenig catchments, Proc. Instn. Civ. Engnrs., 8, 17-51 (1957)

LINDENBERGH, P. C., Movement of underground water below and above phreatic level, IASH Gen. Ass. Toronto, 2, 193-198 (1958)

LINSLEY, R. K., KOHLER, M. A. and PAULUS, J. L. H., Hydrology for Engineers, McGraw-Hi11, New York (1958)

LLOYD, D., Evaporation loss from land areas, Trans. Instn. Wat. Engnrs., 47, 59-73 (1942)

LLOYD, D., The reliable yield of an impounding reservoir, Journ. Instn. Wat. Engnrs., 1, 213-221 (1947)

McCLEAN, W. N., Rainfall and flow-off in the River Garry, Invernessshire, Trans. Instn. Wat. Engnrs., 32, 110-146 (1927)

MCCLEAN, W. N., River flow records : River Dee, Aberdeenshire, Q.J.R. Met. Soc., 61, 433-435 (1935)

MEINZER, O. E., Outline of groundwater hydrology, USGS Wat. Supp. Pap. 494 (1923)

MONTEITH, J. L., The reflection of short-wave radiation by vegetation, Q.J.R. Met. Soc., 85, 386-392 (1959)

MONTEITH, J. L., Evaporation and environment,.Proc. Symp. Expt1. Biol., 19, 205-234 (1965)

MONTEITH, J. L., Principles of Environmental Physics, Arnold, London (1973)

MOSSMAN, R. C., The meteorology of Edinburgh, Trans. Roy. Soc. Edin., 38, 681-775 (1896)

NASH, J. E. and SUTCLIFFE, J. V., River flow forecastings through conceptual models, Part I - A discussion of principles, J. Hydrol., 10, 282-290 (1970)

NEW SCIENTIST, Making irrigation in Britain a more exact science, New Scientist, 11, pll (1961)

NORDENSON, T. J. and BAKER, D. R., Comparative evaluation of evaporation instruments, JGR, 67, 671-679 (1962)

PAINTER, R. B., A hydrological classification of the soils of England and Wales, Proc. Instn. Civ. Engnrs., 48, 8 93-95 (1971)
PEGG, R. K., Evapotranspiration and the water balance in a small clay catchment, in Taylor J. A. (Ed.) The Role of Water in Agriculture, Pergamon, 0xford (1970)

PENMAN, H. L., Natural evaporation from open water, bare soil and grass, Proc. R. Soc., Series A, 193, 120145 (1948)
PENMAN, H. L., The dependence of transpiration on weather and soil conditions, J. Soil Sci., 1, 74-89 (1949)

PENMAN, H. L., The water balance of the Stour catchment area, Journ. Instn. Wat. Engnrs., 4, 457-469 (1950)

PENMAN, H. L., Components in the water balance of a catchment area, Q.J.R. Met. Soc., 81, 280-283 (1955)

PENMAN, H. L., Estimating evaporation, Trans. Am. Geophys. Union, 37, 43-50 (1956a)
PENMAN, H. L. and othërs. Discussions of evaporation, etc., Neth. J. Agric.. Sci., 4, 87-97 (1956b)

PENMAN, H. L., Woburn irrigation, 1951-1959, J. Agric. Sci., 58, 343-379 (1962)

PENMAN, H. L., Vegetation and Hydrology, Commonwealth Agric. Bur., Farnham Royal (1963)

PRIOR, M. J., Meteorological Office, Bracknell, personal communication.

PRUS-CHACINSKI, T. M., Discussion of "Some aspects of the hydrology of the Thames basin", (F. M. Ändrews), Proc. Instn. Civ. Engnrs., 24, 247-287 (1963)

PRUS-CHACINSKI, T. M., 'Discussion of "Rainfal1, runoff and climatic change", (A. S. Thom and D. C. Ledger), Proc. Instn. Civ. Engnrs., 63, 721-731 (1977)

REYNOLDS, G., Rainfall, runoff and evaporation on a catchment in west Scotland, Weather, 24, 90-98 (1969)

RIJTEMA, P. E., An analysis of actual evapotranspiration, Agr: Res. Dept., 659, Pudoc, Wageningen (1965)

RIJTEMA, P. E., Transpiration and production of crops in relation to climate and irrigation, Inst. for Land and Water Management Res. Tech. Bull., 44 (1966)

RISBRIDGER, C. A. and GODFREY, W. H., Rainfall, runoff and storage : Elan and Claerwen gathering grounds, Proc. Instn. Civ. Engnrs., Part III, 3, 345-388 (1954)

RODDA, J. C., The assessment of precipitation, in Water, Earth and Man, Ed. R. J. Chorley, Methuen, London, 130134 (1969)

RODDA, J. C., On the questions of rainfall measurement and representativeness, Symp. on World Water Balance, IASH Publ. 92, 1: 173-186 (1970a)

RODDA, J. C., Definitive rainfall measurements and their significance for agriculture, in The Role of Water in Agriculture, Ed. J. A. Taylor, Pergamon, Oxford, (1970b)

RODDA, J. C., Discussion of "Rainfall, runoff and climatic change", (A. S. Thom and D. C. Ledger), Proc. Instn. Civ. Engnrs., 63, 721-731 (1977)

RODDA, J. C., DOWNING, R. A. and LAW, F. M., Systematic Hydrology, Butterworth, London (1976)

RODDA, J. C. and ROBINSON, A. C., Rain, wind and the aerodynamic characteristics of raingauges, Met. Mag., 98, 113-120 (1969)

SCOTTISH DEVELOPMENT DEPARTMENT, A measure of plenty : Water resources in Scotland, a general survey, HMSO, Edinburgh (1973)

SCOTTISH DEVELOPMENT, DEPARTMENT, Groundwater resources in Scotland with particular reference to Fife and Kinross, Applied Res. and Deve1.Report No. 2, Edinburgh (1975)

SCOTSMAN, Board lift bans on water use, Scotsman, 15th Oct. (1974a)

SCOTSMAN, Water stand-pipes go up in Fife, Scotsman, 22nd Aug. (1974b)

SHAW, J. B., Meteorological Office, Edinburgh, personal communication

SMITH, K., Rainfall, runoff and water balance in the Nidd Valley, Yorkshire, Water and Water Engng., 69, 510 (1965)

SMITH, L. P., Potential Transpiration. Min. Ag. Fish. and Food, Tech. Bull. No. 16, London (1967)

STANHILL, G., . The accuracy of meteorological estimates of evapotranspiration, J. Instn Wat. Engnrs., 12, 377-383 (1958a)

STANHILL, G., Rainfall measurement at ground leve1, Weather, 13, 33-34 (1958b)

STEVENSON, T., On the defects of raingauges with description of an improved form, Edinburgh New Phil. Journ., 33, 12-21 (1842)

STIRLING, W. G. N., Fife and Kinross Water Board, personal communication.

STOW., F. W., Raingauge experiments at Hawsker, near Whitby, Yorkshire, Brit. Rainfall, p16.(1871)

SZEICZ, G., ENDRODI, G. and TAJCHMAN, S., Aerodynamic and surface factors in evaporation, Water Resources Research, 5, 380-394 (1969)

THOM, A. S. Department of Meteorology, University of Edinburgh, personal communication.

THOM, A. S. and LEDGER, D. C., Rainfall, runoff and climatic change, Proc. Instn. Civ. Engnrs., Part 2, 61, 633-652 (1976).

THOM, A. S. and OLIVER, H. R., On Penman's equation for estimating regional evaporation, Q.J.R. Met. Soc., 103, 345357 (1977)

THORNTHWAITE, C. W., An approach towards a rational classification of climate, Geog. Rev., 38, 55-94 (1948)

THORNTHWAITE, C. W., A re-examination of the concept and measurement of potential evapotranspiration, Publ. Climatol., 7, 12-29 (1954)

THORNTHWAITE, C. W. and HARE, F. K., The loss of water to the air, Meteorological Monographs, 6, 163-180 (1965)

THORNTHWAITE, C. W. and HOLZMAN, B.; The determination of evaporation from land and water surfaces, Mon. Wea. Rev., 67, 4-11 (1939)

THORNTHWAITE, C. W. and MATHER, J. R., The computation of soil moisture, Pub1. Climatol., 7, 390-402 (1954)

THORNTHWAITE, C. W. and MATHER, J. R., The water balance, Publ. Climatol., 8, 1-86 (1955)

VEIHMEYER, F. J. and HENDRICKSON, A. H., Does transpiration decrease as the soil moisture decreases ? Trans. AGU, 36, 425-428 (1955)

VRIES, D. A. de and DUIN, R. H. A. van, Some considerations on the diurnal variation of transpiration, Neth. J. Agric. Sc., 1, 27-39 (1953)

WALES-SMITH, B. G., The use of the Penman formula in hydrology, Met: Off. Hyd., Mem. No. 39 (1971)

WALES-SMITH, B. G., Evaporation in the London area from 1698 to 1970, Met. Mag., 102, 281-291 (1973)

WARD, R. C., Observations of potential evapotranspiration (PE) on the Thames floodplain, 1959 - 1960, J. Hydrology, $1,183-194$ (1963)

WARD, R. C., Principles of Hydrology, 2nd Edn. McGraw-Hill, London (1975)

WILSON, W., Meteorological Office, Edinburgh, personal communication.

WRIGHT, C. E., The rainfall/runoff relationship as affected by geological characteristics with particular reference to the Lothians, private communication (1968)


[^0]:    AVERAGE ANNUAL GENERAL RAINFALL, EDEN CATCHMENT $=839 \mathrm{~mm}$

[^1]:    (All values in mm)

