Logical Ambiguity

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Declaration

I declare that this thesis has been composed by myself and that the research reported therein has been conducted by myself.

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I still don't understand anything despite this excellent start and the blame for this must be placed with all those I have talked with since, especially Ian Lewin, Lex Holt, Mark Hepple, Guy Barry, Glyn Morrill, Jaap van der Does and Henk Zeevat. Finally, it is to be doubted that I would be here were it not for the efforts of an incredibly large number of ocean bottom dwelling fish, whom I would like to thank, but upon whom, needless to say, no blame can rest for mistakes in what follows.

Abstract

The thesis presents research in the field of model theoretic semantics on the problem of ambiguity, especially as it arises for sentences that contain junctions (and,or) and quantifiers (every man, a woman). A number of techniques that have been proposed are surveyed, and I conclude that these ought to be rejected because they do not make ambiguity 'emergent': they all have the feature that subtheories would be able to explain all syntactic facts yet would predict no ambiguity. In other words these accounts have a special purpose mechanism for generating ambiguities.

It is argued that *categorial* grammars show promise for giving an 'emergent' account. This is because the only way to take a subtheory of a particular categorial grammar is by changing one of the small number of clauses by which the categorial grammar axiomatises an infinite set of syntactic rules, and such a change is likely to have a wider range of effects on the coverage of the grammar than simply the subtraction of ambiguity.

Of categorial grammars proposed to date the most powerful is *Lambek Categorial Grammar*, which defines the set of syntactic rules by a notational variant of Gentzen's sequent calculus for implicational propositional logic, and which defines meaning assignment by using the Curry-Howard isomorphism between Natural Deduction proofs in implicational propositional logic and terms of typed lambda calculus. It is shown that no satisfactory account of the junctions and quantifiers is possible in Lambek categorial grammar.

I introduce then a framework that I call *Polymorphic Lambek Categorial Grammar*, which adds variables and their universal quantification, to the language of categorisation. The set of syntactic rules is specified by a notational variant of Gentzen's sequent calculus for quantified propositional logic, and which defines meaning assignment by using Girard's Extended Curry-Howard isomorphism between Natural Deduction proofs in quantified implicational propositional logic and terms of 2nd order polymorphic lambda calculus. It is shown that this allows an account of the junctions and quantifiers, and one which is 'emergent'.

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Chapter 1

Introduction

Succinctly put the aim of the thesis is to provide a solution to the problem of logical ambiguity. We explain below what 'logical ambiguity' is, and why it is a problem.

There are infinitely many sequences of English words which count as grammatical sentences. This means that a theory aiming to predict which sequences of English words are grammatical sentences cannot take the form of a finite listing of the right sequences. Instead a theory must include some inductive/recursive rules, of which the following is an example:

(1) *if* X *is a sentence and* Y *is a sentence, then* X and Y *is a sentence.*

Each of the infinitely many sequences of English words which count as grammatical sentences also has a meaning. Therefore, one might try to construct a theory predicting what meaning was assigned to what sentence. A semantic theory of this kind seems vexed by conceptual difficulties that do not occur in a syntactic theory. A syntactic theory has only ultimately to assign a yes/no answer to a string, and there is no conceptual difficulty in describing that task. However, supposing that a semantic theory has ultimately to assign a meaning to a string, involves one immediately in deciding what 'meanings' are. Because of the conceptual difficulties here, semantic theorising has often taken for itself a different aim. Examples of such substitute aims have been (i) the specification of the situations in which the sentence would be true or (ii) specification of which sentences logically follow from which sentences.

Whatever one might take the aim of semantic theories to be, one aspect remains in common with syntactic theories: that the number of facts to be explained is infinite. Therefore the theory must include some inductive/recursive rules, such as:

(2) *if* X *is true in the situation and* Y *is true in the situation, then* X and Y *is true in the situation.*

Now the semantic theory could either exploit or ignore the inductive syntactic theory. For example, if the syntactic theory has inductive clauses defining 'new' sentences from 'old', the semantic theory might add to this rule a part specifying the semantic properties of the new sentence from the semantic properties of the old. (2) is such an exploitation of (1). On the other hand, ignoring the syntactic theory, the semantic theory might start afresh on its own inductive definition.

Of these two options, it has seemed overwhelmingly more likely that the semantic theory should exploit the syntactic theory than that it should not. Perhaps one argument for this is that the ability that humans have to distinguish grammatical utterances from ungrammatical is a rather pointless ability in isolation. Yet we have it, and plausibly, the reason is that it is a subcapacity of our capacity for comprehending these utterances: the semantic theory exploits the syntactic theory.

However, the phenomenon of ambiguity poses a problem for this plausible view that a semantic theory should exploit a syntactic theory. Both of the sentences below are ambiguous:

(3) a. I like old men and women

b. Every man loves a woman

The simplest kind of inductive definition of the grammaticality of (3a) (and related sentences such as I like men, I like old men, I like men and women) is given in (4).

(4) *1.* men *is a CN and* women *is a CN*

2. if X *is a CN, then* I like X *is an S*

3. if X *is a CN then* old X *is a CN*

4. if X *is a CN and* Y *is a CN, then* X and Y *is a CN*

(4) uses the labels 'CN' and 'S' to classify sequences of words. It is not necessary to try to attribute a significance to these classifications. (4) simply presents a system for assigning labels to sequences of words which has the effect that all the sequences labelled 'S' are sequences speakers of English would recognise as grammatical. On the basis of (4), there are *two* different inductive justifications of the fact that (3a) is a sentence, which is to say, that according to the theory in (4), (3a) is syntactically ambiguous:

(i) *1,4* ⇒ men and women *is a CN,*
∴*3* ⇒ old men and women *is a CN,*
∴ ⇒ I like old men and women *is an S*

(ii) 1,3 ⇒ old men is a CN,
∴1,4 ⇒ old men and women is a CN,
∴2 ⇒ I like old men and women is an S.

Therefore (3a) corroborates the view that a semantic theory should exploit a syntactic theory, for if one could add to the clauses of the theory, specifications of semantic properties of the strings involved, then parallel to the above two different inductive justifications of the sentencehood of (3a) would be two inductive definitions of a semantic property of (3a), and probably these two definitions would not come to the same conclusion.

The simplest kind of inductive definition of the grammaticality of (3b) (and of related sentences like a woman loves every man, John loves a woman, every man loves John) is given in (5)

(5) *1.* every man *is an NP and* a woman *is an NP*

2. if X *is an NP, then* loves X *is a VP*

3. if X *is an NP and* Y *is a VP, then* XY *is an S*

(5) gives only one inductive justification of the fact that (3b) is grammatical, that is according to (5), (3b) is not syntactically ambiguous:

1,2 ⇒ loves a woman *is a VP,*
∴*1,3* ⇒ every man loves a woman *is an S*

(3b) was an example of semantic ambiguity unaccompanied by syntactic ambiguity [1] and therefore gives counterevidence to the view that a semantic theory should exploit a syntactic theory. The single inductive justification of its grammaticality should imply that it has only one meaning (or 'semantic property'). There are many other examples of semantic ambiguity unaccompanied by syntactic ambiguity, such as:

(6) Most stupid and temperamental students cry all day long
Every student did not stay awake all night typing

We will call words like most, every and a, *determiners* , words like and and or we will call *junctions* and words like not we will call *negations* . Determiners, junctions and negations together form a group of words often called the *logical constants.* The presence in a sentence of one of these expressions is a fairly reliable sign that the sentence will exhibit semantic ambiguity unaccompanied by syntactic ambiguity. We will dub this phenomenon *logical ambiguity.* The objective of the thesis is to give a solution to the problem of logically ambiguous sentences, in some way that does not discard the principle that the semantic theory should exploit the syntactic theory.

There are two kinds of solution that have been proposed. The first kind of solution moves to a 'non-deterministic' kind of exploitation of the syntactic theory by the semantic theory: associated with a rule defining new syntactic objects from old could be a semantic part defining a *range* of possible semantic properties of the new syntactic object given the possible meanings of the old. The second kind of solution is to revise the syntactic theory so that there are as many inductive justifications of a sentence as there are possible meanings of it. For example one might add to the mini-grammar of (5) the following inductive clause:

(7) *if* X *is an NP and* ...he ... *is an S, then* ...X ... *is an S*

According to the expanded version of (5) there will then be *three* different inductive justifications of the sentence every man loves a woman, that is, it will be threeways syntactically ambiguous.

The solution to the problem of logical ambiguity that will be put forward in this thesis is an instance of this second kind of solution. So a syntactic approach will be advocated according to which there are (at least) as many inductive justifications of a logically ambiguous sentence as there possible meanings of it. I will argue that my particular syntactic solution is an improvement both over other syntactic solutions and over solutions of the other above mentioned kind, that use a non-deterministic exploitation of syntax. The ground on which it will be argued that the solution is an improvement over others is that logical ambiguity should not be a *modular* feature of a theory. This is to say, it should not be possible a take a theory that *does* accounts for logical

[1] In describing things this way one has to bear in mind that 'syntactic ambiguity' is not a property that a sentence has in its own right, it is a property that it has only in relation to some hypothesised syntactic theory.

ambiguity, among other things, and by simplifying it into a subtheory, to subtract from the coverage *solely* the explanation of logical ambiguity (in other words the subtheory would be able to explain all the same syntactic facts as the larger theory, but to logically ambiguous sentences it would assign only one reading). If a theory was resistant to such modular simplification, one would expect that the reason for this would be that its explanation of ambiguity was not due to some special purpose device but arises from the interaction of several different devices, each of which has its part to play in the explanation of other phenomena than ambiguity. Therefore, altering these devices will not only affect the coverage of ambiguity phenomena but the coverage of other phenomena also. Of such a theory, we will say that it meets the *emergence* criterion.

Existing accounts seemingly do make ambiguity a modular feature. The option of using a non-deterministic exploitation of syntax could be said to associate with a syntactic rule a disjunction of semantic rules. Throwing away all but one of the disjuncts will lose simply the explanation of ambiguity. The second kind of account, most famously instanced in Montague 73, has the feature that is possible to cover all syntactic facts with a subset of the syntactic rules proposed. What is lost if not all the rules are used is the explanation of ambiguity. The justification of the claim that the solution that I propose meets the emergence criterion will have to wait until it has been described.

There is nothing I can cite in evidence for the intuition that ambiguity should be an emergent feature. Nonetheless, it is this feature that makes the theory I propose most unlike other accounts, and it this feature that makes it more falsifiable than others. For example, if a stroke could cause a person to lose to the ability to perceive logical ambiguity, but to experience no other deficit to their language capacity, that would seem to be strong evidence for the modularity of logical ambiguity, and correspondingly strong falsification of the non-modular theory that I will propose. A number of other accounts could accommodate such data.

So much by way of introduction to the general argument that will be made in the thesis. It is difficult without delving into technical terms to give much in the way of introduction to, or summary of, the finer detail of this argument and the structure of the thesis. There follows a brief sketch, highlighting what may be found in what follows that is novel.

Chapter 2: 'Universal Grammar' gives an introduction to a general form that a syntactic and semantic proposal concerning a particular language might take, a general form that was first proposed in Montague 70 (though almost all the individual parts had been seen before). One of its most important features is that it delineates a precise sense in which a semantic theory can exploit a syntactic theory. It also gives a certain form to the non-novel idea that by associating with expressions certain set-theoretical objects (sets, sets of sets), it is possible to given an inductive definition of what sentences entail what sentences - that is the valid entailments. It is by use of just such set-theoretical machinery that the semantic theories to be proposed here operate.

Chapter 3, 'Semantic facts about English' more or less identifies the facts about entailment in English that one would want to account for with a semantic theory couched in the UG framework described in Chapter 2. There is also some discussion of whether there are any other kinds of semantic fact, besides entailment, that one could hope to capture with a theory of the Montagovian kind. This relates to the point made above that there are conceptual difficulties in stating the aims of a semantic theory simply as a correct pairing of sentences with meanings. We said above that substitute aims might be (i) to predict the situations in which a sentence is true or (ii) to predict the valid entailments. One of the things that we do in this chapter is to argue that (i) is not an aim of a Montagovian theory, though it has often been construed that way. It is also argued that a corollorary of this is that there is no place in the description of the data for the terminology of 'scope-ambiguity'.

Chapter 4, 'Introduction to Categorial Grammar', describes the framework of *Lambek categorial grammar*, briefly LCG (Lambek 58, van Benthem 86, Moortgat 88). The first essential feature of categorial grammar is that has an infinite, inductively defined *language* of categories, defined from some *atomic* categories and the *categorial connectives*, / and \. For example it is the case that if x and y are categories so is x/y. The second essential feature of *Lambek* categorial grammar (though not of other kinds of categorial grammar) is that there is an inductive definition of syntactic rule. This is an important distinction as compared with the illustrations of syntactic theories that were given above in (4) and (5). (4) and (5), by a finite enumeration of syntactic rules, gives an inductive definition of grammatical strings. LCG, on the other hand, by means of a finite device known as a *sequent calculus*, gives an inductive definition of a set of syntactic rules, keyed on patterns of connectives. The so defined rules then participate in the inductive definition of a set of syntactic strings in the usual way.

It is the structure of the inductive justification of a syntactic rule that a semantic theory in LCG exploits. It is shown in a preliminary fashion in this chapter that a theory cast in the LCG framework has some chance of accounting for logical ambiguity, and in a way likely to meet the emergence criterion. One can get a hint of why ambiguity will not be modularly removable by reflecting on the fact that the LCG rule set is inductively defined. This inductive definition has a small set of clauses (in fact 5), and any descriptive success that an LCG has stems from them. Discarding any one of them can be expected to have far-reaching consequences on the coverage of the account, more far-reaching than simply eliminating ambiguity. Hence the difficulty of modular simplification to eliminate ambiguity. Chapter 6 will consider in a more systematic fashion whether an account of logical ambiguity can be formulated within the LCG framework.

Chapter 5, 'Arguments for Locality and Minimality' is primarily a chapter in which a number of the accounts that have been given of junctions and quantifiers are described and the descriptive adequacy and emergence criteria are applied. The accounts considered do not meet the criteria. The chapter also serves to introduce some 'basic semantic technology', something one would have to do at some point in any case.

Chapter 6, 'The failure of LCG to account for the Logical Constants' argues that LCG, despite the preliminary indications noted in Chapter 4, does not allow a solution to logical ambiguity.

Chapter 7, 'Polymorphic Lambek Categorial Grammar' first introduces an extension of the LCG framework. This first part of the chapter is general and not concerned particularly with the problems posed by junctions and determiners - afterwards we formulate a particular account within the framework and investigate whether it constitutes a solution to logical ambiguity. It was mentioned above that categorial grammars have as a definitive feature, an inductively defined set of categories. *Polymorphic Lambek Categorial Grammar* (PLCG) extends LCG's inductive definition of the categories, introducing *category-variables* and their *universal quantification*, (that is allowing for categories such as $\forall$X.X/(X\np)). It was also mentioned above that LCG had the distinctive feature of inductively defining the syntactic rules. This inductive definition is also widened in the case of PLCG, including clauses to govern the $\forall$ connective. As with LCG, it is the structure of the inductive justification of a syntactic rule that the semantic theory exploits. The effect of PLCG's category notation is that if there were a number of LCG categorisations one would like to have assigned to an expression, all of which were instances of a category *schema*, such as $x/(x\backslash \mathrm{np})$, one can gain the same effect in PLCG by assigning the categorisation $\forall$X.X/(X\np).

After the general description of the PLCG framework, a particular example is formulated, aimed at accounting for the junctions and determiners. Junctions and determiners are given categorisations that exploit the increased categorial language of PLCG. We then proceed to show that this PLCG account captures the distributional behaviour of junctions and determiners, and that it gives a descriptively adequate account of logical ambiguity. We claim that this account also meets the criterion of emergence: simplifications would not simply subtract from the coverage of logical ambiguity, but from the coverage of syntactic phenomena also.

Chapter 8: 'Comparison with Hendriks' Type Flexibility proposal' discusses one further account of the logical ambiguity problem, the *type-flexibility* proposal of Hendriks 88. This is an account that takes the evidence that logically ambiguous sentences are not syntactically ambiguous at face value, and solves the problem by defining a certain way in which the semantic theory non-deterministically can exploit the syntactic theory. The first point made about this is that it makes of ambiguity a modular feature. However, the real reason for considering Hendriks' is not simply to apply the criterion of emergence to it. The main reason for making this comparison is that there is a strong theoretical affinity between the two proposals, and to try to illuminate this, a translation is outlined which will transform an analysis within the type-flexibility account into an analysis within the PLCG account. This translation also brings to light two further ways that one could build an account of logical ambiguity on top of LCG, accounts that have been briefly considered in Moortgat 88 and Moortgat 90. I will give some examples that show that these possibilities have to be ruled out.

Figure 1.1 gives an indication of the main logical dependencies between the chapters. The reader may also find it useful to look from time to time to the end of the final chapter, where there is a certain amount of indexing to the various definitions given in the thesis. There are indices for the main definitions, the hypotheses, the languages, the models, and the theories. There is also a general index to help locate 'in-text' definitions.

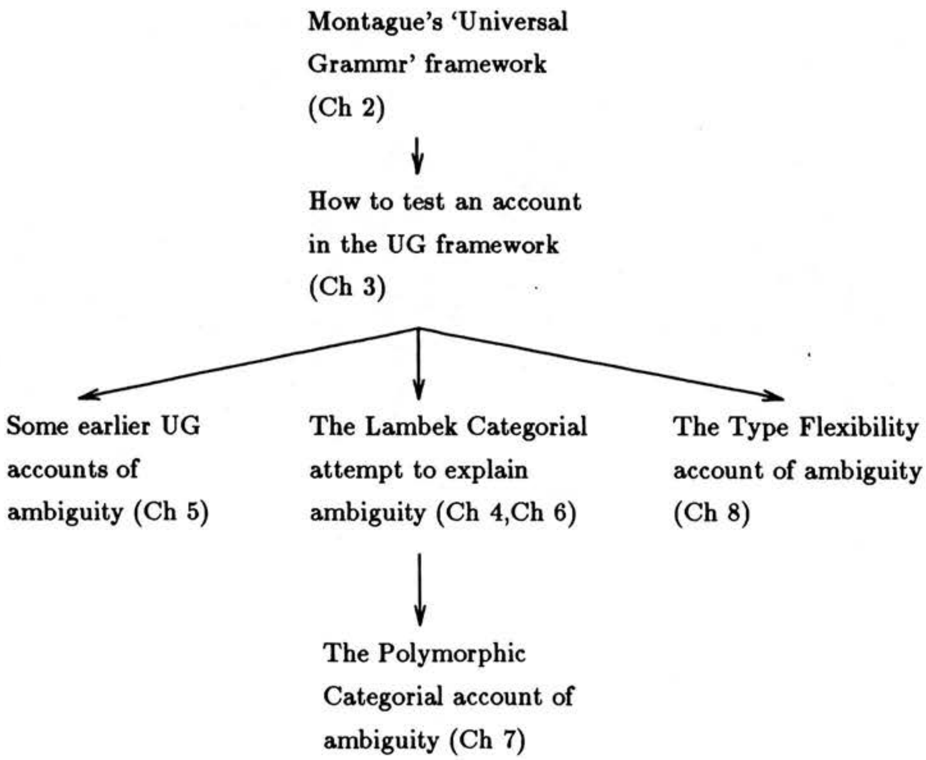


Figure 1.1: Some logical dependencies amongst the chapters

Chapter 2

Universal Grammar

1 Introduction

In the 1970 paper *Universal Grammar* (henceforth referred to as UG) Richard Montague proposed a general framework in which to describe the syntax of languages and alongside this a general specification of how to develop a formal semantics for a language whose syntax is characterised in the way he describes.[1] This chapter is nothing more than a recapitulation of the proposals made in UG. We have gone through this exercise for a number of reasons.

The main aim of the research described in this thesis was to formulate a *compositional* account of certain natural language *ambiguities* and in UG, Montague gave a general form for such accounts. By making our proposals within this general framework we will be adopting Montague's interpretation of the terms *compositionality* and *ambiguity*. One of the reasons for using Montague's definitions rather than relying on the intuitive understanding of these terms is that it resolves what at least are *prima-facie* conflicts between the intuitive meanings of the terms. The intuition behind compositionality is that once the interpretations of the parts of an expression are fixed, so is the interpretation of the whole. Yet this is just what is not true of an ambiguous expression - fixing the interpretation of the parts still leaves two possibilities at least for the interpretation of the whole.

A second reason is that it is only from the perspective of Montague's definition of compositionality that it is possible to relate the kind of compositionality enjoyed by categorial grammars to the kind enjoyed by other kinds of grammar. In particular, if one had only the 'Rule to Rule' notion of compositionality, then it would be impossible to say in what sense categorial grammar, of the Lambek calculus variety, is compositional.

Thirdly, the UG framework gives a kind of level ground on which to compare accounts not themselves formulated overtly in the framework, by comparing their representatives within the framework.

In section 2, an overview of the UG framework will be given briefly. This hopefully provides orientating background for the more detailed material in section 3, concerning the syntactic aspects of the framework and section 4, concerning the semantic aspects. Section 5 refers very briefly to a topic which is discussed in the UG paper but will be here largely ignored, namely the use of *intermediate translation languages*. Finally in section 6, the notion of *polymorphism* is introduced and it is noted the extent to which polymorphism is allowed by the UG framework.

[1] The semantical ideas all had more or less long histories: denotational semantics (Frege 1892), possible-worlds semantics (Carnap 47, Kripke 63), type-theory (Church 41)

2 Overview of the UG framework

The Montagovian picture involves three kinds of objects and two connections chaining them together. The three objects are LANGUAGES[2], DISAMBIGUATED LANGUAGES and SEMANTIC ALGEBRAS and the two connections are the DISAMBIGUATION RELATION and the MEANING ASSIGNMENT FUNCTION.

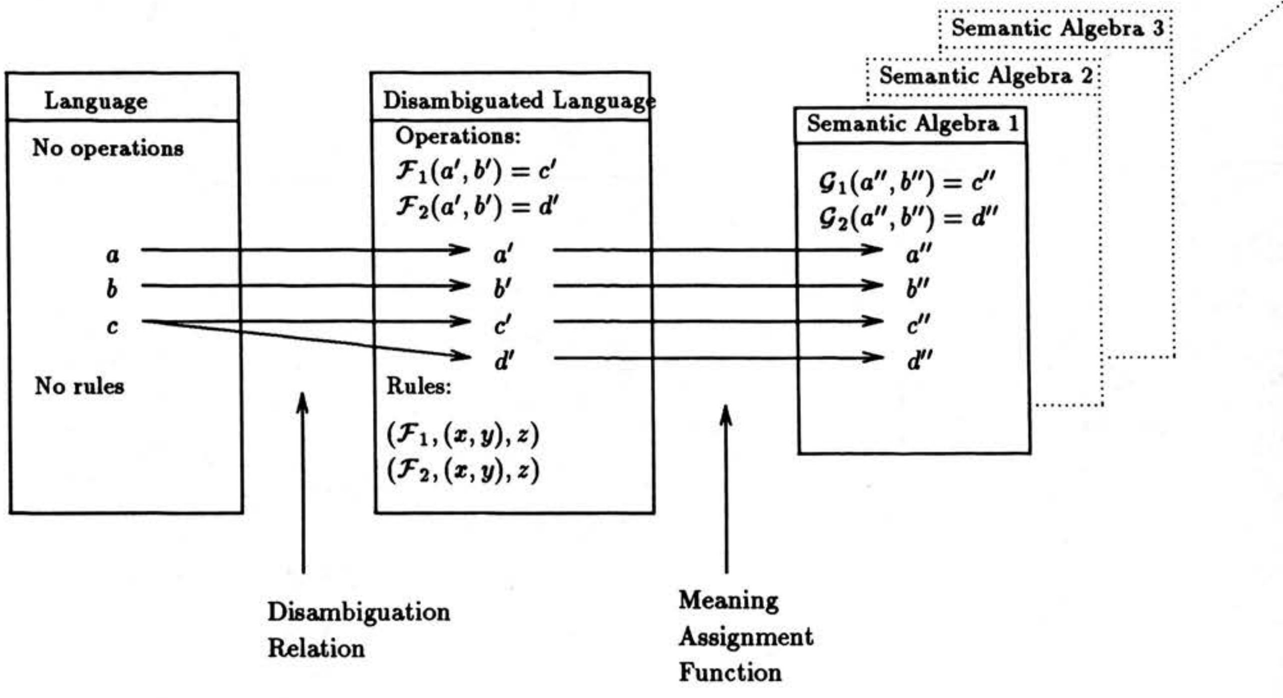


Figure 2.1: Syntax and Semantics in the UG framework

In the following sections these notions will be defined. Here, by way of introduction, approximate descriptions will be given, and we will indicate which parts are carrying what burden.

LANGUAGE Roughly speaking a LANGUAGE can be thought of as a set of expressions. Importantly, these expressions are *not* the primary members of syntactic categories, nor are they the primary bearers of meaning. The expressions of a LANGUAGE have categories and meanings only derivatively, once the LANGUAGE has been linked to a DISAMBIGUATED LANGUAGE, by a DISAMBIGUATION RELATION.

DISAMBIGUATED LANGUAGE Roughly speaking, this can be thought of as a SYNTACTIC ALGEBRA plus a GRAMMAR. A SYNTACTIC ALGEBRA is an expression set closed under the application of certain syntactic operations. Because a DISAMBIGUATED LANGUAGE includes a grammar, it is intrinsic to it that it define categorisation facts concerning its

[2] Terms are given in small caps when it is important to distinguish Montague's technical definition of the term from either every day meaning or definitions by other people.

expressions. A LANGUAGE (that is a set of expressions), $\mathcal{L}$, and a DISAMBIGUATED language, $\mathcal{L}'$, are said to be connected by a DISAMBIGUATION RELATION [3], $\mathcal{R}$, if the domain of $\mathcal{R}$ is $\mathcal{L}$ and the range is the *carrier set* of the algebra of $\mathcal{L}'$. Via this relation, $\mathcal{L}$ is understood to inherit categorisation facts from $\mathcal{L}'$. For example:

If ζ', an expression of $\mathcal{L}'$, is of category δ, and ζ is an expression of $\mathcal{L}$ such that $\zeta\mathcal{R}\zeta'$, then ζ is of category δ.

SEMANTIC ALGEBRA This is a realm of meaning objects, closed under the application of some semantic operations. The carrier set of the SYNTACTIC ALGEBRA of a DISAMBIGUATED LANGUAGE and the carrier set of a SEMANTIC ALGEBRA can be linked by an INTERPRETATION FUNCTION. This function is actually defined only on the *lexical* expressions. The MEANING ASSIGNMENT FUNCTION g is the unique homomorphic extension of f, assigning meanings to all expressions of the DISAMBIGUATED LANGUAGE.

If $\mathcal{L}'$ is a DISAMBIGUATED LANGUAGE, whose expressions are assigned meanings by some g, then as with categorisation, expressions of a LANGUAGE $\mathcal{L}$ can inherit these meanings via a DISAMBIGUATION RELATION from $\mathcal{L}$ to $\mathcal{L}'$.

If ζ', an expression of $\mathcal{L}'$, has meaning m, and ζ is an expression of $\mathcal{L}$ such that $\zeta\mathcal{R}\zeta'$ then ζ has meaning m.

By chaining the *relational link*, $\mathcal{R}$, between $\mathcal{L}$ and $\mathcal{L}'$ with the *functional link*, g, between $\mathcal{L}'$ and the semantic algebra, one has a *relational link*, $\mathcal{R} \circ g$, between language and meaning. One would expect there to be this kind of relational link between language and meaning because of ambiguity. However, within Montague's UG framework this relational link is not a 'primitive' meaning-connection. The 'primitive' relation is the *functional* meaning assignment function. The essential feature in the Montagovian architecture that allows a representation of *ambiguity* is the relational link between LANGUAGES and DISAMBIGUATED LANGUAGES.

3 Syntax in the UG framework

3.1 DISAMBIGUATED LANGUAGE

It was said above, that a DISAMBIGUATED LANGUAGE was some kind of combination of a syntactic algebra with a grammar, and on this comment we will now expand.

A syntactic algebra is a pair $\langle \mathcal{A}, (\mathcal{F}_\gamma)_{\gamma\in\Gamma}\rangle$, where $(\mathcal{F}_\gamma)_{\gamma\in\Gamma}$ represents a family of operations, indexed by Γ under the application of which $\mathcal{A}$ is closed. Roughly speaking, the objects of $\mathcal{A}$ are conceived of as the expressions of the language, with the operations performing the work of

[3] This is in fact a slight departure from Montague's formulation, in which he spoke of an 'ambiguation' relation, for which the carrier set of the syntactic algebra is the *domain* and not the *range*.

deriving large, complex expressions from smaller simpler expressions. For example, if the algebra contains an operation to perform concatenation and contains the objects *John* and *walks*, then the $\mathcal{A}$ must also contain *John walks*, *John John* and *walks walks*. Because of this closure property of $\mathcal{A}$, to call the members of $\mathcal{A}$ 'expressions of the language' is something of an extension of the ordinary usage.

The grammar part of a disambiguated language defines the *categorisation* facts concerning objects of the algebra. There are two components: (i) the categorisation facts concerning *basic expressions* and (ii) *syntactic rules.*

The categorisation facts concerning basic expressions are given by defining an indexed family of sets, $(\mathcal{X}_\delta)_{\delta\in\Delta}$, each $\mathcal{X}_\delta$ being the set of objects of $\mathcal{A}$ that may be called *lexical δ phrases.* Δ will be referred to as the set of *phrase-set indices.*

The syntactic rules are of the form $\langle\mathcal{F}_\gamma, \langle\delta_1, \ldots \delta_n\rangle, \delta_{n+1}\rangle$, which may be interpreted as stating that whenever the operation $\mathcal{F}_\gamma$ is applied to a n-tuple of representatives of the categories $\delta_1, \ldots, \delta_n$, the resulting object is of category δ_{n+1} (this will be made more precise in a moment).

The syntactic algebra and grammar are combined together as a 5-tuple:$\langle\mathcal{A}, (\mathcal{F}_\gamma)_{\gamma\in\Gamma}, (\mathcal{X}_\delta)_{\delta\in\Delta}, \mathcal{S}, \delta_0\rangle$, where $\mathcal{S}$ is simply the set of syntactic rules and δ_0, is a distinguished phrase set index, the significance of whose status can only be explained when semantics is turned to.

Not all such 5-tuples qualify as DISAMBIGUATED LANGUAGES: there are some additional characteristics that reflect the presence of the word *disambiguated.* Before the definition of these, an example of what is at least the *kind* of thing that a disambiguated language is:

Let $\mathcal{L}^1$ be the five tuple $\langle\mathcal{A}^1, (\mathcal{F}^1_\gamma)_{\gamma\in\Gamma^1}, (\mathcal{X}^1_\delta)_{\delta\in\Delta^1}, \mathcal{S}^1, \delta_0\rangle$, where the elements in this tuple are defined below :

The ambiguous language $\mathcal{L}^1$

1. the set of phrase-set indices $\Delta^1 = \{$ NP, S, VP, PV, ADV $\}$
2. distinguished phrase-set index $\delta^1_0 = \mathrm{S}$
3. the family of sets of basic δ-phrases called $(\mathcal{X}^1_\delta)_{\delta\in\Delta}$
   $\mathcal{X}^1_{\mathrm{S}} = \emptyset$, $\mathcal{X}^1_{\mathrm{NP}} = \{\mathsf{John}, \mathsf{Mary}\}$, $\mathcal{X}^1_{\mathrm{VP}} = \{\mathsf{died}\}$, $\mathcal{X}^1_{\mathrm{PV}} = \{\mathsf{said}\}$, $\mathcal{X}^1_{\mathrm{ADV}} = \{\mathsf{yesterday}\}$
4. a family of syntactic operations, $(\mathcal{F}^1_\gamma)_{\gamma\in\Gamma^1}$, where $\Gamma^1 = \{0\}$
   Whatever α, $\beta\in \mathcal{A}^1$, $\mathcal{F}^1_0(\alpha, \beta) = \alpha\beta$
5. the set, $\mathcal{A}^1$, which is the closure of $\cup(\mathcal{X}^1_\delta)_{\delta\in\Delta^1}$ under the operations $(\mathcal{F}^1_\gamma)_{\gamma\in\Gamma^1}$
6. the syntactic rules $\mathcal{S}^1 =$
   $\{\langle\mathcal{F}^1_0, \langle\mathrm{NP}, \mathrm{VP}\rangle, \mathrm{S}\rangle, \langle\mathcal{F}^1_0, \langle\mathrm{PV}, \mathrm{S}\rangle, \mathrm{VP}\rangle, \langle\mathcal{F}^1_0, \langle\mathrm{VP}, \mathrm{ADV}\rangle, \mathrm{VP}\rangle\}$

The following three points cover general facts that cannot be conveyed by this specific example.

First it should be noted that although the objects that feature in Δ^1, the phrase-set-indices, are familiar syntactic labels, they need not be. Also the set phrase-set-indices, Δ^1, the set of operation indices Γ^1, and the set of syntactic rules $\mathcal{S}^1$, though finite in the above, need not be. In Chapter 4, categorial grammars are represented by DISAMBIGUATED LANGUAGES, and the phrase-set indices, the operation indices and the syntactic rules will all be infinite sets.

Second, we should make clear exactly how $(\mathcal{X}^1_\delta)_{\delta\in\Delta}$ and $\mathcal{S}^1$ combine to define categorisation. There is a 'once and for all' definition of how this is done for any DISAMBIGUATED LANGUAGE. The set Δ was used to index the family of sets of lexical δ-phrases. Δ is also used to index a family of larger sets of expressions, $(\mathcal{C}_\delta)_{\delta\in\Delta}$. This family is the family of *phrase-sets* [4] and is defined:

Definition 1 (The family of phrase-sets)

A DISAMBIGUATED LANGUAGE *generates the family of phrase-set* $(\mathcal{C}_\delta)_{\delta\in\Delta}$ *if and only if (1)* $(\mathcal{C}_\delta)_{\delta\in\Delta}$ *is a family of subsets of* $\mathcal{A}$*, indexed by* Δ*; (2)* $\mathcal{X}_\delta \subseteq \mathcal{C}_\delta$ *for all* $\delta \in \Delta$*; (3) whenever* $\langle \mathcal{F}, \langle \delta_1, \ldots, \delta_n \rangle, \delta_{n+1} \rangle \in \mathcal{S}$ *and* $\alpha_n \in \mathcal{C}_{\delta_n}$*,* $\mathcal{F}(\alpha_1, \ldots, \alpha_n) \in \mathcal{C}_{\delta_{n+1}}$*; (4) whenever* $(\mathcal{C}'_\delta)$ *satisfies (1)-(3),* $\mathcal{C}_\delta \subseteq \mathcal{C}'_\delta$

Forbidding though this appears, what is states is in fact very intuitive. (2) says that lexical δ-phrases are in the phrase-set indexed by δ. (3) simply says that phrase-sets are closed under application of the syntactic operations in accordance with the syntactic rules. A rule such as the first appearing rule in $\mathcal{S}^1$ may thus by seen as shorthand for

if $\alpha \in \mathcal{C}_{\mathrm{NP}}$, $\beta \in \mathcal{C}_{\mathrm{VP}}$ *then,* $\mathcal{F}^1_0(\alpha, \beta) \in \mathcal{C}_{\mathrm{S}}$

In the 1973 paper, *The Proper Treatment of Quantification* (henceforth PTQ), Montague gave syntactic rules in this longhand form.

A third point that needs to be emphasised is that Montague construed the syntactic operations as extensional objects, which is to say that they are one and the same as certain sets of sequences of objects. One cannot therefore propose two *different* syntactic operations which have the *same* effect on all sequences of expressions, as the operations would then be identical to the same set of sequences of expressions. This extensional construal has consequences because Montague goes on to map the syntactic operations onto semantic ones, with the requirement therefore that syntactic operations be discriminated at least as finely as semantic ones. Two different semantic operations must correspond to two 'visibly' different syntactic operations.

We turn now to those characteristics a 5-tuple like $\mathcal{L}^1$ must have if it is to qualify as DISAM-

[4] We are using 'phrase-set' where Montague used the term 'category'. This is because we wish to reserve the term 'category' for later use in the discussion of 'categorial grammar'.

BIGUATED. Consider the following two true identities in the algebra of $\mathcal{L}^1$:

(1) John said Mary died yesterday

$= \mathcal{F}_0^1(\text{John}, \mathcal{F}_0^1(\text{said}, \mathcal{F}_0^1(\text{Mary}, \mathcal{F}_0^1(\text{died}, \text{yesterday}))))$

John said Mary died yesterday

$= \mathcal{F}_0^1(\text{John}, \mathcal{F}_0^1(\mathcal{F}_0^1(\text{said}, \mathcal{F}_0^1(\text{Mary}, \text{died})), \text{yesterday}))$

These identities say that the object John said Mary died yesterday can be derived in two ways. The essence of the disambiguation conditions for a 5-tuple like $\mathcal{L}^1$ is that there should be one or less derivations of any object of the algebra. Suppose α is an object of the algebra and one wants to know whether this object may be derived. In other words one wants to know whether there are $\mathcal{F}_\gamma, \beta_1, \ldots, \beta_n$ such that the following identity holds:

$\alpha = \mathcal{F}_\gamma(\beta_1, \ldots, \beta_n)$

In a DISAMBIGUATED LANGUAGE the following facts about search for a solutions to this equation are required :

1. If for some δ, $\alpha \in \mathcal{X}_\delta$, (that is α is a basic expression), there is no solution to the above equation.
2. If α is not a basic expression, there is at most one solution to the above equation

3.2 LANGUAGES and the DISAMBIGUATION RELATION

We turn now to LANGUAGES and their connection with DISAMBIGUATED LANGUAGES via a DISAMBIGUATION RELATION. We said above that a LANGUAGE was simply to be taken as an expression set, not associated directly with any grammatical rules. This is essentially true, but is not quite how things are defined. The formal definition actually has a LANGUAGE as a *pair*, $\langle \mathcal{L}, \mathcal{R} \rangle$ where $\mathcal{L}$ is any DISAMBIGUATED LANGUAGE, $\langle \mathcal{A}, \mathcal{F}_\gamma, \mathcal{X}_\delta, \mathcal{S}, \delta_0 \rangle$ and $\mathcal{R}$ is any binary relation, with $Ran(\mathcal{R}) \subseteq \mathcal{A}$, (referred to as the DISAMBIGUATION RELATION).

Henceforth LANGUAGE will be taken in this sense. Its expressions are those objects which stand in the DISAMBIGUATION RELATION to expressions of $\mathcal{A}$.

There is a definition of the way in which a LANGUAGE $\langle \mathcal{L}, \mathcal{R} \rangle$ inherits categorisation from $\mathcal{L}$ (where $\mathcal{L}$ is a DISAMBIGUATED LANGUAGE, and $\mathcal{R}$ a DISAMBIGUATION RELATION):

Definition 2 (Phrase-Set of $\langle \mathcal{L}, \mathcal{R} \rangle$) *$\langle \mathcal{L}, \mathcal{R} \rangle$ generates a family of phrase-set $(\mathcal{CAT}_{\delta,\langle \mathcal{L},\mathcal{R}\rangle})_{\delta\in\Delta}$, where $\mathcal{CAT}_{\delta,\langle \mathcal{L},\mathcal{R}\rangle}$ is the set of objects, s, such that $s\mathcal{R}\alpha$ for some $\alpha \in \mathcal{C}_\delta$, one of the phrase-sets generated by $\mathcal{L}$*

What one finds in Montague's PTQ paper is not the definition of a DISAMBIGUATED LANGUAGE together with a DISAMBIGUATION RELATION. Instead one finds the definition of a 5-tuple that is very much like $\mathcal{L}^1$, in that it meets all the requirements of a DISAMBIGUATED LANGUAGE except the disambiguation conditions. However, it is possible to *extract* from the definition of the ambiguous 5-tuple the definition of a DISAMBIGUATED LANGUAGE and a DISAMBIGUATION RELATION. First the DISAMBIGUATED LANGUAGE will be defined and then it will be explained how it may be seen as extracted from the ambiguous 5-tuple $\mathcal{L}^1$.

$\mathcal{L}^2$: a disambiguated version $\mathcal{L}^1$

1. $\Delta^2 = \Delta^1$
2. distinguished phrase-set index = S
3. $(\mathcal{X}^2_\delta)_{\delta\in\Delta}$: if $s_1 \in \mathcal{X}^1_\delta$ then $\langle s_1, \langle\rangle, \delta\rangle \in \mathcal{X}^2_\delta$
4. $(\mathcal{F}^2_\gamma)_{\gamma\in\Gamma^2}$, where $\Gamma^2 = \Gamma^1$
   for all $\gamma \in \Gamma^2$, for all $\langle\alpha_\zeta\rangle, \langle\beta_\eta\rangle \in \mathcal{A}^2$
   $\mathcal{F}^2_\gamma(\langle\alpha_\zeta\rangle, \langle\beta_\eta\rangle) = \langle\mathcal{F}^1_\gamma(\alpha_1, \beta_1), \langle\langle\alpha_\zeta\rangle, \langle\beta_\eta\rangle\rangle, \gamma\rangle$
5. the set $\mathcal{A}^2$ which is the closure of $\cup(\mathcal{X}^2_\delta)_{\delta\in\Delta^2}$ under the operations $(\mathcal{F}^2_\gamma)_{\gamma\in\Gamma^2}$
6. the syntactic rules $\mathcal{S}^2$ are: exactly the same as those of $\mathcal{L}^1$, except for the replacement of $\mathcal{F}^1_\gamma$ by $\mathcal{F}^2_\gamma$

$\mathcal{L}^2$ satisfies the disambiguation conditions. The operations are defined so as to record in the output of the operation which operation it is and what the inputs were. Nor can basic expressions be derived as the third coordinate of a basic expression is always a phrase-set index and not an operation index. For example, concerning $\mathcal{L}^1$ there was the identity,

(2) $\mathcal{F}^1_0(\mathsf{John}, \mathcal{F}^1_0(\mathsf{said}, \mathcal{F}^1_0(\mathsf{Mary}, \mathcal{F}^1_0(\mathsf{died}, \mathsf{yesterday}))))$
$= \mathcal{F}^1_0(\mathsf{John}, \mathcal{F}^1_0(\mathcal{F}^1_0(\mathsf{said}, \mathcal{F}^1_0(\mathsf{Mary}, \mathsf{died})), \mathsf{yesterday}))$

Whereas if $\mathcal{F}^1_0$ is changed to $\mathcal{F}^2_0$, the corresponding identity does not hold of the algebra in $\mathcal{L}^2$. One can see this clearly if the *triples* in $\mathcal{A}^2$ are depicted as *trees*, taking first and third coordinates are as a mother node and the second coordinate as the sequence of daughters of that node. See Figure 2.2 and Figure 2.3.

What these pictures also make clear is that the *objects* of $\mathcal{L}^2$ are *analysis trees* of the objects of $\mathcal{L}^1$. [5] In general, the way that an ambiguous 5-tuple like $\mathcal{L}^1$ can effectively define a DISAMBIGUATED LANGUAGE, is by taking the objects of the DISAMBIGUATED LANGUAGE to be analysis trees of objects of the ambiguous 5-tuple.

The ambiguous 5-tuple, $\mathcal{L}^1$, also effectively defines a LANGUAGE, because one can take as the DISAMBIGUATION RELATION the relationship which holds between the terminal yield of an analysis tree and an analysis tree.

[5] These 'analysis trees' do not depict process of *categorisation* in $\mathcal{L}^1$.

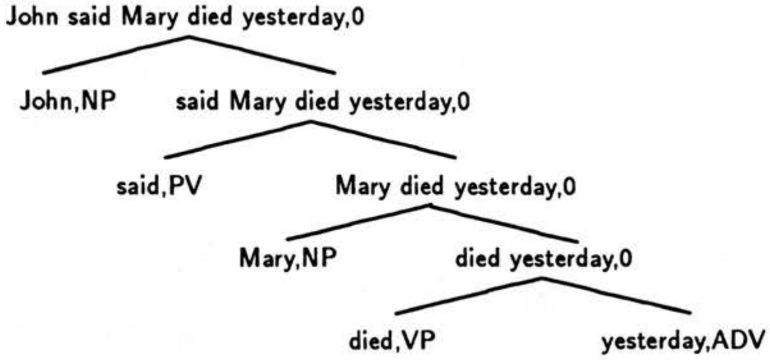


Figure 2.2: First $\mathcal{L}^2$ disambiguation of John said Mary died yesterday

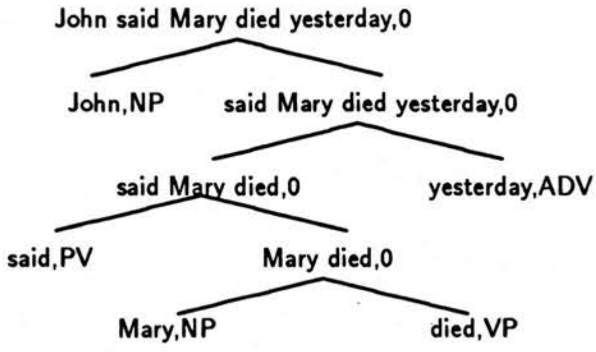


Figure 2.3: Second $\mathcal{L}^2$ disambiguation of John said Mary died yesterday

Whilst some pairings of DISAMBIGUATED LANGUAGE and DISAMBIGUATION RELATION can be seen as effectively defined by an ambiguous 5-tuple, some cannot. The DISAMBIGUATION RELATION defined by the ambiguous 5-tuple, $\mathcal{L}^1$ is a *one-to-many* relation but the UG framework allows also for DISAMBIGUATION RELATIONS to be *many-to-one* or *many-to-many*, and such DISAMBIGUATION RELATIONS cannot be regarded as pairings of terminal yield and analysis tree.

Whether or not is appropriate of the UG framework to allow the DISAMBIGUATION RELATION to be other than *one-to-many* is a question that one might ask. There are some natural language phenomena that lend some support to the unrestricted nature of the DISAMBIGUATION RELATION. The cases where one might exploit this lack of restriction occur when there are, so to speak, several ways of saying the same thing. In English, one could argue that the active and passive form of a sentence are just two ways of saying the same thing, and one could reflect this by having them share the same disambiguation:

$\mathcal{R}$(John likes Mary,$\langle$John likes Mary$, d_1, \epsilon_1\rangle$)

$\mathcal{R}$(Mary was liked by John, $\langle$John likes Mary$, d_1, \epsilon_1\rangle$)

Another example is provided by the relatively free-word order of the np arguments of a verb in German. Arguably all the orders are different ways of saying the same thing and one might explain this by postulating that they share the same disambiguation:

$\mathcal{R}$(er liebte ihn,⟨er liebte ihn, d_1, ϵ_1⟩)

$\mathcal{R}$(ihn liebte er,⟨er liebte ihn, d_1, ϵ_1⟩)

There are not the only approaches to passivisation and relatively free-word order, and in pointing them out I am not claiming that these phenomena can *only* be accounted for by exploiting the unrestricted nature of the disambiguation relation. The point is simply that if the disambiguation relation were to be restricted to be a *one-to-many* relation, the above approaches could not be carried out, and this would not be consistent with the aimed for generality of the UG framework.

4 Semantics in the UG framework

Having described Montague's conception of syntax in the UG framework, we will now consider Montague's conception of meaning and the way in which an expression may be linked with its meaning.

The algebraic theme is prominent again here: Montague proposed that the materials of the semanticist should form part of an algebra, a closed semantic world of semantic objects subject to semantic operations. Objects in *syntactic* algebras are mapped onto objects in *semantic* algebras by a *meaning assignment function*. The pairing of a meaning assignment function with a semantic algebra is called an INTERPRETATION.

The main reason for bringing the algebraic perspective to language and meaning is that the idea of a compositionality can be given a very exact and general formulation: given a syntactic and a semantic algebra which are *similar*, a mapping from the syntactic objects to the semantic objects is a compositional meaning assignment if it is the homomorphic extension of a mapping from the basic syntactic objects to the semantic objects.

In section 4.1, 'Theories of Reference', Montague's notions of *semantic algebra*, *interpretation* and *compositionality* are defined. About the *objects* inhabiting semantic algebras no assumptions are made, and as a result there is no notion of *truth* of a sentence on a given INTERPRETATION. It is in section 4.2, 'Theories of Reference', that semantic algebras are described in which feature more Fregean and familiar semantic values. These are the algebras of *type-theoretical, possible-worlds* semantics, and emergent is a notion of *truth* of a sentence, *relative* to a given *Fregean* INTERPRETATION and possible-world, (or MODEL as such a pairing of a INTERPRETATION and possible world is called).

At this point of section 4.2, one could be forgiven for thinking that to make a syntactic/semantic proposal concerning a natural language within the UG framework would be to define two things: a LANGUAGE and a MODEL. However, the final characterisation of syntactic/semantic proposals is that they should pair a language not with *a* MODEL but with a *set* of possible MODELS. This is because, it is only with respect to such a pairing that a notion of *entailment* can be defined.

4.1 Theories of Meaning

A semantic algebra is a pair $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma} \rangle$. $\mathcal{B}$ is a set of objects, the $\mathcal{G}_\gamma$ are operations which are total on $\mathcal{B}$ and under which $\mathcal{B}$ is closed. The $\mathcal{G}_\gamma$ are extensionally conceived. In these respects semantic algebras are exactly like the syntactic algebras which feature as parts of DISAMBIGUATED LANGUAGES. A difference is that there is no correspondent of the disambiguation conditions for the semantic algebras.

Now on Montague's conception the fundamental language-meaning connection is between the syntactic algebra of a DISAMBIGUATED LANGUAGE and a semantic algebra. One makes such a connection by specifying an INTERPRETATION. An INTERPRETATION for a DISAMBIGUATED LANGUAGE $\mathcal{L}$, where $\mathcal{L} = \langle \mathcal{A}, (\mathcal{F}_\gamma)_{\gamma \in \Gamma}, (\mathcal{X}_\delta)_{\delta \in \Delta}, \mathcal{S}, \delta_0 \rangle$, is defined thus:

Definition 3 (INTERPRETATION for $\mathcal{L}$)
Is a triple $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f \rangle$, *where*
(i) $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma} \rangle$ *and* $\langle \mathcal{A}, (\mathcal{F}_\gamma)_{\gamma \in \Gamma} \rangle$ *are* similar *algebras and*
(ii) f *is a function from* $\cup(\mathcal{X}_\delta)_{\delta \in \Delta}$ *into* $\mathcal{B}$

Here the *similarity* of the algebras $\langle \mathcal{A}, (\mathcal{F}_\gamma)_{\gamma \in \Gamma} \rangle$ and $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma} \rangle$ requires that for all γ, $\mathcal{F}_\gamma$ and $\mathcal{G}_\gamma$ have the same number of argument places.

The notion of INTERPRETATION concerns only the basic expressions. The term MEANING ASSIGNMENT is used to describe the extension of the mapping to include also the compound expressions of the DISAMBIGUATED LANGUAGE:

Definition 4 (MEANING ASSIGNMENT for $\mathcal{L}$)
The unique homomorphism, $[\![]\!]$, *from* $\langle \mathcal{A}, (\mathcal{F}_\gamma)_{\gamma \in \Gamma} \rangle$ *to* $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma} \rangle$ *such that* $f \subset [\![]\!]$, *where* f *is the interpretation function defined above.*

The $f \subset [\![]\!]$ part simply says that $[\![]\!]$ coincides with f on the basic expressions, and the fact that $[\![]\!]$ is a homomorphism simply says that for all compound expressions, $\mathcal{F}_\gamma(\alpha_1, \ldots, \alpha_n)$,

$$[\![\mathcal{F}_\gamma(\alpha_1, \ldots, \alpha_n)]\!] = \mathcal{G}_\gamma([\![\alpha_1]\!], \ldots, [\![\alpha_n]\!])$$

One is justified in speaking of the *unique* homomorphism as it can be quickly proved that there is only one mapping $[\![]\!]$ extending f and satisfying the homomorphism property. Though the term 'compositionality' is not mentioned here, the above definition of MEANING assignment embodies Montague's conception of 'compositionality'. On the question of whether this corresponds with the intuitive definition of what is required for a meaning association to be 'compositional', I offer only the following paragraph.

The compositionality slogan: 'the meaning associated with a complex whole is a function of the meaning associated with its parts'. This has can be read as trivial, for given the meanings of the parts of a sentence and the meaning of the whole there are indefinitely many functions which may be regarded as relating these as arguments to value. A more significant reading of the compositionality slogan is that the meanings of sentences S and S' are always the same, if S' is derived from S by replacing some word with another of the *same meaning* and *same grammatical category*. However, because S and S' may well be ambiguous, one cannot speak of *the* meaning of S being preserved. Possibly at this point one should claim that there is an intuitive notion of the 'mode of composition' of S, and that one may speak of S and S' 'sharing' a mode of composition. This view is reflected by modifying the compositionality slogan to include the phrase, 'a function of the meaning of the parts and their *mode of composition*'. Such a reading, the UG framework reflects very well, for a *mode of composition* is a DISAMBIGUATION, and substitutability holds without qualification for objects of the DISAMBIGUATED LANGUAGE.

Just as a LANGUAGE may inherit categorisation facts from a DISAMBIGUATED LANGUAGE that it bears a DISAMBIGUATION RELATION to, so a LANGUAGE may also inherit MEANING ASSIGNMENT facts. For this inherited MEANING ASSIGNMENT the terminology s_1 *means* m_1 *in* $\langle \mathcal{L}, \mathcal{R} \rangle$ is used which is defined :

Definition 5 (s_1 means m_1 in $\langle \mathcal{L}, \mathcal{R} \rangle$)
s_1 *means* m_1 *in* $\langle \mathcal{L}, \mathcal{R} \rangle$ *iff there exists* $\alpha \in \cup_{\delta \in \Delta} C_\delta$ *such that* $s_1 \mathcal{R} \alpha$ *and* $g(\alpha) = m_1$

The MEANING ASSIGNMENT is defined on DISAMBIGUATED LANGUAGES. Why is this ?

Given that meanings are assigned by a function, it is clear that a semantically disambiguated language must supply the arguments to this map, the expressions of which should be related by an ambiguation relationship to expressions of a LANGUAGE. This argues for the fact that the intervening language must be semantically disambiguated, but does not argue for it being *syntactically disambiguated.*

It is the formulation of compositionality as homomorphism that is at the root of the requirement that syntactic algebras meet the disambiguation conditions. The relevance is that if the syntactic algebra meets the disambiguation conditions and one has a *similar* semantic algebra then *any* map from the basic expressions to semantic objects has a homomorphic extension. This is to be contrasted with when the syntactic algebra fails to meet the disambiguation conditions - in other words some derivational histories converge to the syntactic same object. Then unless the semantic algebra reflects these identities there can be no homomorphic connection.[6]

The definition of 'means in $\langle \mathcal{L}, \mathcal{R} \rangle$' is the last truly 'universal' definition of the UG paper, where

[6] The relevance of the homomorphism requirement to use of disambiguated algebras should not be overplayed: an ambiguous syntactic algebra can be homomorphically linked to a semantic algebra, but not to just any *similar* algebra.

no assumptions are made about the nature of the objects in the syntactic and semantic algebras. In the next section of the UG paper and of this chapter, semantic algebras of a particular kind are described, the algebras of type-theoretical, possible worlds semantics.

4.2 Theories of Reference

In constructing the sets of objects $\mathcal{B}$ of such algebras, there are three degrees of freedom: the choice of a set, $\mathcal{E}$, of entities, the choice of a set, $\mathcal{I}$, of possible worlds and a set, $\mathcal{J}$, of contexts of use. From the entities $\mathcal{E}$, the worlds $\mathcal{I}$ and the fixed set of truth values, there is fixed construction of set of possible DENOTATIONS, and from this and and the cartesian product $\mathcal{I} \times \mathcal{J}$, there is a fixed construction of the set of possible MEANINGS.

The set of possible DENOTATIONS is a set of sets, each indexed by a *type*. Montague called the set of types, T. I shall define a slight enlargement of this set of types and call it $\text{TJ}^{\rightarrow}$, the reason for which name will be given later.

Definition 6 ($\text{TJ}^{\rightarrow}$)

a. e, t and s are $\in \text{TJ}^{\rightarrow}$

b. if a and b are $\in \text{TJ}^{\rightarrow}$ *then* $(a \rightarrow b)$ *is* $\in \text{TJ}^{\rightarrow}$

Relative to the sets $\mathcal{E}$ and $\mathcal{I}$, the set of DENOTATIONS which has index a, D_a is given by [7]:

Definition 7 (Denotation set, D_a) $D_e = \mathcal{E},\ D_s = \mathcal{I},\ D_t = \{0, 1\},\ D_{(a \rightarrow b)} = D_b^{D_a}$

The exponent notation A^B refers to the set of total functions from B into A. From the family of denotation sets $(D_a)_{a \in T}$, the family of meaning sets $(\mathcal{M}_a)_{a \in T}$ is defined as

Definition 8 (Meaning set, $\mathcal{M}_a$) $\mathcal{M}_a = D_a^{\mathcal{I} \times \mathcal{J}}$

Therefore the types index both the MEANING sets and DENOTATION sets, with a particular MEANING of type a being a set of pairs, whose first coordinate is member of $\mathcal{I} \times \mathcal{J}$ and whose second coordinate is a DENOTATION of type a. This set of pairs is moreover a function. It is the objects in $\cup(\mathcal{M}_a)_{a \in T}$ that serve as objects in a semantic algebra $\langle \mathcal{B}, \mathcal{G}_\gamma \rangle$, which we shall a *Fregean* algebra . A FREGEAN INTERPRETATION is more or less the application of the notion of INTERPRETATION in the context of above described kind of algebra. What is additional is that a FREGEAN INTERPRETATION must make reference to a type mapping, ν, relating the phrase-set indices of Δ to the types in $\text{TJ}^{\rightarrow}$. Such mappings are arbitrary save for the requirement that $\nu(\delta_0) = t$. Given this, a FREGEAN INTERPRETATION is defined thus (for a DISAMBIGUATED LANGUAGE $\mathcal{L}$, where $\mathcal{L} = \langle \mathcal{A}, (\mathcal{F}_\gamma)_{\gamma \in \Gamma}, (\mathcal{X}_\delta)_{\delta \in \Delta}, \mathcal{S}, \delta_0 \rangle$):

[7] Another term used for the same thing is *type domain*.

Definition 9 (**FREGEAN INTERPRETATION of $\mathcal{L}$, associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$**)

Is a triple $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f \rangle$, *where*

(i) $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f \rangle$ *is an* INTERPRETATION, *and* $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma} \rangle$ *is a Fregean algebra*

(ii) whenever $\delta \in \Delta$ *and* $\alpha \in \mathcal{X}_\delta, f(\alpha) \in \mathcal{M}_{\nu(\delta)}$

(iii) whenever $\langle \mathcal{F}_\gamma, \langle \delta_\eta \rangle, \epsilon \rangle \in \mathrm{S}$ *and* $m_\eta \in \mathcal{M}_{\nu(\delta_\eta)}$ *then* $\mathcal{G}_\gamma(\langle m_\eta \rangle) \in \mathcal{M}_{\nu(\delta)}$

Because of the nature of the objects in FREGEAN INTERPRETATIONS, it is possible to define the notion of TRUTH of a sentence, *relative* to a particular INTERPRETATION and a particular $\langle i, j \rangle \in \mathcal{I} \times \mathcal{J}$. In fact the notion defined is that of TRUTH in a MODEL, where MODELS are taken to be a pair of a FREGEAN INTERPRETATION and some $\langle i, j \rangle$.

Definition 10 (**MODEL for $\mathcal{L}$**)

Is a pair $\langle \Im, \langle i, j \rangle \rangle$, *where* $\Im$ *is a* FREGEAN INTERPRETATION *of* $\mathcal{L}$ *associated with some* $E, \mathcal{I}, \mathcal{J}, \nu$ *and* $\langle i, j \rangle \in \mathcal{I} \times \mathcal{J}$

Definition 11 (**TRUTH**)

α *is a* TRUE *sentence of* $\mathcal{L}$ *with respect to* $\langle \Im, \langle i, j \rangle \rangle$ *iff* $[\![\alpha]\!](\langle i, j \rangle) = 1$

In a related fashion there is an obvious definition of the *denotation* of any category of expression with respect to a MODEL.

Finally a notion of *entailment* is defined. This is defined not relative to a single MODEL, but relative to a class $\mathcal{K}$ of MODELS:

Definition 12 (**α $\mathcal{K}$-ENTAILS β**)

α $\mathcal{K}$-ENTAILS β *iff for every* MODEL $\langle \Im, \langle i, j \rangle \rangle$ *in* $\mathcal{K}$, *if* α *is* TRUE *with respect to* $\langle \Im, \langle i, j \rangle \rangle$ *then* β *is* TRUE *with respect to* $\langle \Im, \langle i, j \rangle \rangle$.

It will be convenient in what follows to use the terminology of a THEORY OF REFERENCE for the joint specification of (i) a LANGUAGE, $\langle \mathcal{L}, \mathcal{R} \rangle$ and (ii) a class $\mathcal{K}$ of MODELS of $\mathcal{L}$. Note that a THEORY OF REFERENCE is the Montagovian characterisation of a *particular* language, and not a general framework for such characterisations.[8]

The notions of TRUTH in a MODEL and of $\mathcal{K}$-ENTAILMENT are the cornerstones of the empirical dimension of a THEORY OF REFERENCE, what enables one to say whether a THEORY OF REFERENCE is right or wrong. This is discussed in Chapter 3.

The definitions of MODEL, TRUTH and $\mathcal{K}$-ENTAILS are rather similar to ones given in textbooks in Predicate Logic, with one significant difference. The above notion of MODEL has as a component a semantic algebra, which is a pair $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma} \rangle$. The usual notion of model would have

[8] When Tarski speaks of a 'truth-definition' (Tarski 32), and when Davidson speaks of a 'theory of truth' (Davidson 67), it also is in this language particular rather than general sense.

something corresponding to the $\mathcal{B}$, but not have anything corresponding to the semantic operations, $(\mathcal{G}_\gamma)_{\gamma\in\Gamma}$. Now a didactic point often made is that in varying *models* one should not vary one's assumptions about how the semantic values of parts combine to give the semantic value of a complex whole. This point is not intrinsically true on Montague's definition of MODEL, for varying the MODELS means varying the semantic algebra, and that can mean varying the semantic operations. However, one can reintroduce the didactic point by suitably restricting the class $\mathcal{K}$ of MODELS. In fact securing the desired performance from a THEORY OF REFERENCE is largely a matter of giving the right restrictions of the class of MODELS.

5 Translation languages

A subject treated in the UG paper that will not be treated here is that of indirect interpretation by means of *translation* languages. The idea is that if one defines a *translation*, t from one DISAMBIGUATED LANGUAGE, $\mathcal{L}$, to another DISAMBIGUATED LANGUAGE, $\mathcal{L}^T$, for which one has defined a MEANING ASSIGNMENT, g, then a 'meaning assignment' for $\mathcal{L}$ can be defined as $g \circ t$. The reason for putting 'meaning assignment' in quotes is that generally $g \circ t$ will not be regardable as the homomorphic extension of the interpretation function f, of some INTERPRETATION. However, provided the translation function, t, is a homomorphism then the 'meaning assignment' function, $g \circ t$, will actually be a MEANING ASSIGNMENT.

6 Polymorphism in Universal Grammar

A one place function will be said to be *polymorphic* if two different denotations sets, D_a and D_b are subsets of the domain of the function. Correspondingly for n-place functions.

The algebra of a FREGEAN INTERPRETATION has functions occurring at two levels, the operations of the algebra and the objects of the algebra, and concerning both one can investigate for polymorphism.

Concerning the operations: the answer is that *every* operation of the semantic algebra is polymorphic. This property follows from their totality: every operation has for its domain the entire set of meanings $\mathcal{B}$ and every denotation set is a subset of $\mathcal{B}$.

Concerning the objects: the answer is that *no* object of the algebra is polymorphic.

Chapter 3

Semantic facts about English

1 Introduction

It is clear how to at least begin a list of pretheoretical facts by which to assess the syntactic part of a Montagovian THEORY OF REFERENCE; the list could be begun by grammaticality judgements. To be sure there will be problematic borderline cases for which there do not seem to be hard and fast pretheoretical judgements of grammaticality, but there is still a large group of uncontroversial judgements the capturing of which may be taken as the first obligation of the syntactic part of a Montagovian THEORY OF REFERENCE. It is less obvious with what pretheoretical facts one can even begin to assess the semantic part. Section 2 attempts the task of identifying the appropriate *kind* of fact. 'Translations into formal languages' are considered, as well as 'truth conditions'. Both are rejected and it is claimed that the appropriate semantic facts are 'truth intuitions'. These, as will be explained, are statements *interrelating the truth in a situation of several sentences*, and a special case is a statement of entailment. All this may seem not at all controversial, but a perhaps surprising consequence of adhering to a view of semantic data that does not encompass 'translational' facts is that there can be no mention of the vocabulary of 'scope-ambiguity' in the description of facts about English.

We then proceed in section 3 to set out some particular 'truth intuitions' that we will be making it our concern in later chapters to capture. In section 3.1 are noted intuitions of the special kind customarily called *transparency* and *opacity* intuitions. In sections 3.2, 3.3 and 3.4 'truth intuitions' concerning junctions and (determiner + common noun) combinations (= *quantifiers*) are given. It is noted that although valid entailments are a potential kind of semantic data, one cannot make any generalisations over several logical constants if entailments are the *only* kind of data. This is simply because the valid inferences for a group of logical constants do not differ from each other merely by the substitution of one constant for another but are of radically different *forms*. However, if one gives data in the more general form of 'truth intuitions', one can make generalisations over logical constants. In section 3.5, 'truth intuitions' are given intended to characterise the semantic contribution of *embedded* sentences, VP's and quantifiers. It is claimed that to do so a truth predicate must be used and 'higher-order' quantification.

2 Kinds of semantic data

Three kinds of semantic data are considered in this section: 'truth-conditional', 'translational', and 'truth-intuitional', and only the third accepted as proper for a THEORY OF REFERENCE. The grounds for rejecting 'truth-conditional' and 'translational' facts are given only briefly. However, it will not matter too much if the reader is not convinced by these arguments, so long as they *are* convinced that *one* possible form of data are 'truth-intuitions'.

2.1 Truth conditions

Is the data for semantics, facts about truth conditions ? To answer that question two conceptions of 'truth condition' will be described below.

The first conception of 'truth condition' is the Davidsonian one, and an example of a Davidsonian 'truth condition' is (1) [1] :

(1) snow is white is true iff snow is white.

Note this is an English sentence which refers to another English sentence. It is a biconditional. A necessary condition for a biconditional to count as the truth condition of snow is white is that in the righthand side there occur no *references to* expressions.

Such truth conditions do *not* form the basic data for a THEORY OF REFERENCE. They *are* the basic data for another kind of semantic theory, namely a Davidsonian truth theory. This approach to semantics was proposed first in the 1967 paper, *Truth and Meainng* and the 1984 book, *Inquiries into Truth and Interpretation*, collects together essays by him developing and defending the approach. Just to distinguish such a theory from Montague's conception of a THEORY OF REFERENCE, the following is offered by way of brief description of Davidsonian truth theories.

In such a theory a definition of an *artificial* sentence predicate TRUE is given. The aim is that from the definition one should be able to deduce 'TRUTH conditions', that is biconditionals like (1), except that TRUE appears in the place of true. Truth conditions may be used as data for such a TRUTH-theory by requiring that when one can derive a 'TRUTH condition', it should be the case that when the defined word TRUE is replaced by the real English word true, the biconditional is correct. There are some further requirements suggested by Davidson on allowable forms that such a TRUTH-theory might take, but for present purposes these further details are not important.

What *is* important to note is that although in the UG framework there is a technically defined notion of TRUTH, it is not the case that from a THEORY OF REFERENCE, TRUTH-conditions will be derivable. Instead what follows from a THEORY OF REFERENCE are statements such as:

For any MODEL, $\langle \Im, \langle i,j \rangle \rangle$,
$\mathcal{F}(\alpha_1, \ldots, \alpha_n)$ is TRUE with respect to $\langle \Im, \langle i,j \rangle \rangle$ iff $\mathcal{G}([\![\alpha_1]\!], \ldots, [\![\alpha_n]\!])(i,j) = 1$

Such a statement is not a TRUTH-condition for at least two reasons. First, the righthand side contains many *references* to expressions, and secondly a *relational* notion, TRUTH *wrt.* $\langle i,j \rangle$ appears, rather than a *monadic* one.

[1] From amongst the many conventions one might adopt for *referring* to sentences, we will adopt the convention that a sentence in sans-serif is a sentence name.

On the second conception of 'truth condition' a 'truth condition' is not an English sentence. It is a 'graph of truth against situation', that is record of which situations the sentence is true in. We have straightaway used an informal term here: 'situation', and it will be used many times in the sequel. Many questions could be asked seeking clarification of the term, for example whether counterfactual situations should be considered, or impossible situations, whether situations are synonymous with possible worlds, whether they are total and so on. However, the points we will make and the use to which the term will be put in what follows, do not require that such questions be settled in any particular way.

So to continue: the second conception of 'truth condition' is that of graph of truth against situation. Under this second conception it also *not* the case that truth conditions form the basic data for a THEORY OF REFERENCE, though it is often suggested that they are (see for example DWP 81, p4-7) This is because a THEORY OF REFERENCE does not specify for a sentence a graph of truth against situation, but specifies a graph of truth against MODEL. MODELS were defined in the previous chapter, and they are primarily a mapping of expressions onto set-theoretical objects. Whatever a situation is, it is not, *prima-facie* at least, a MODEL.

The only way that 'truth conditions', under the second conception, could be data for a THEORY OF REFERENCE would be if one gave a way to *decode a* MODEL *into a specification of a certain situation*. Then one could ask whether, *under the decoding*, the THEORY OF REFERENCE entailed for a sentence the appropriate graph of situation against truth. However, we shall not be putting forward 'decoding' schemes for MODELS.

2.2 Translations

A consequence of the fact that I shall not be decoding MODELS into situations is that I shall not attempt to make statments concerning the *translation* of English into FOL or some other formal language, statements such as:

(2) every man loves a woman may be translated into First Order Logic (FOL) thus: $\forall x(man'(y) \rightarrow \exists y(woman'(y) \wedge love'(x,y)))$

The reason that I will not make such statements is that unless a decoding is given from MODELS of FOL to situations, it is impossible to say what makes (2) true, for compare (2) with another kind of translation statement, this time concering translation into French:

(3) every man loves a woman may be translated into French thus: chaque homme adore une femme

What makes (3) a fact, is that the graph of situations against truth for every man loves a woman is the same as that for chaque homme adore une femme. *Prima-facie* at least, the same cannot be said of (2). There is a graph of situation against truth for the English sentence, but what there is for the FOL formula is a graph of MODEL against truth. Without a decoding of MODELS of FOL into situations, (2) is simply comparing incommensurable things.

The main price to be paid for resolving not to make translation statements such as (2) is that there is no place in what follows for the vocabulary of *scope-ambiguity*. It is a standard practice to use the notion of *scope* that applies to the language of FOL to define a relation of *scope-alternative* holding between expressions of FOL, and then use this *scope-alternative* relation to frame hypotheses about the extent of natural language ambiguity. Such hypotheses are called 'scope ambiguity' hypotheses and a typical such hypothesis would be: *if the sentence α has the translation Φ, then it also has the translation Φ', where Φ' is a* scope-alternative *of Φ*. A typical notion of *scope-alternative* would have it that the only scope alternative of the formula mentioned in (2) is:

(4) $\exists y(woman'(y) \wedge \forall x(man'(x) \rightarrow love'(x, y)))$

In view of (2) and (4), one might say that every man loves a woman was *completely scope ambiguous*, meaning that the set of its 'translations' into FOL was closed under the scope-alternative relation. However, tidy though this description of the extent of ambiguity in English is, unless there *are* 'translation facts' to start with, it cannot be used [2].

2.3 Truth intuitions

We shall give the name 'truth intuitions' to statements such as the following:

(5) For any situation, s, snow is white and grass is green is true in s if and only if Snow is white is true in s and grass is green is true in s.

(6) For any situation s, a man died is true in s if John is a man is true in s and John died is true in s

(5) is close to the first conception of truth condition discussed above. However, (5), is *not* a truth condition . This because a *relational* version of true occurs rather than a *monadic* one, and the righthand side of the biconditional is full of references to expressions. (6) is not even a biconditional. Essentially a truth intuition *univerally quantifies over situations and interrelates several instances of the predicate 'true in s', each concerning different sentences.*

Truth intuitions can be used as semantic data for a THEORY OF REFERENCE in a similar fashion to the way in which truth conditions may be used as semantic data for a Davidsonian truth

[2] There seems in any case a problem in characterising natural language ambiguity by 'scope-ambiguity' hypotheses, because logically equivalent FOL formulae are presumably all equally 'right' translations, but the scope alternatives of logically equivalent FOL expressions are not always logically equivalent.

theory. A THEORY OF REFERENCE will entail statements in which figure *correlates* of some of the terms in the truth intuition. For example a THEORY OF REFERENCE might entail:

(7) For any MODEL, $\langle \Im, \langle i,j\rangle\rangle \in \mathcal{K}$, **snow is white and grass is green** is TRUE with respect to $\langle \Im, \langle i,j\rangle\rangle$ if and only if **snow is white** is TRUE with respect to $\langle \Im, \langle i,j\rangle\rangle$ and **grass is green** is TRUE with respect to $\langle \Im, \langle i,j\rangle\rangle$.

(8) For any MODEL, $\langle \Im, \langle i,j\rangle\rangle \in \mathcal{K}$, if **John is male** is TRUE with respect to $\langle \Im, \langle i,j\rangle\rangle$ and **John died** is TRUE with respect to $\langle \Im, \langle i,j\rangle\rangle$ then **a man died** is TRUE with respect to $\langle \Im, \langle i,j\rangle\rangle$.

Truth-intuitions (5) and (6) can function as data for a THEORY OF REFERENCE that entails (7) and (8) by requiring that (7) and (8) become the correct truth intuitions (5) and (6) under the substitutions [3] :

'MODEL $\langle \Im, \langle i,j\rangle\rangle$' $\Rightarrow$ 'situation s'

'TRUE' $\Rightarrow$ '**true**'

What we are doing here is not the same thing as providing a decoding scheme from MODELS into situations, at least not on a MODEL by MODEL basis. It is simply being proposed that an entailment of a THEORY OF REFERENCE that universally quantifies over MODELS can be read as quantifying over situations.

Judgments about the *validity of arguments* can also be seen as truth intuitions in other guise: if when one says 'p validly entails q' one means 'for all situations, if p is true in s, then q is true in s'. So an equivalent to the intuition about 'truth in s' given in (6) is the following intuition about validity:

(9) **John is a man**
John died
$\therefore$**a man died** is a valid inference

From a THEORY OF REFERENCE it either will or will not be the case that (see the definition of $\mathcal{K}$-ENTAILS given in Chapter 2):

(10) **John is man, John died** $\mathcal{K}$-ENTAILS **a man died**

It will be required that if (10) follows from a THEORY OF REFERENCE, then (10) should be transformed into the correct validity intuition (9) under the substitution:

'$\alpha_1, \ldots, \alpha_n$ $\mathcal{K}$-ENTAILS β' $\Rightarrow$ '$\alpha_1, \ldots, \alpha_n \therefore \beta$ is a valid inference'

[3] This is not quite true because the objects of which TRUE is predicated are expressions of a DISAMBIGUATED LANGUAGE and typically therefore will not be expressions of English; the relevance of the disambiguation relation to the data will be looked at in a moment.

Truth intuitions can therefore be adopted as a kind of semantic fact by which a THEORY OF REFERENCE can be assessed, and we shall do so. In fact it will be usually be the case that the truth intuitions will be more complex then those illustrated above. This arises because of *ambiguity* and *context-sensitivity*.

Ambiguity

If a truth intuition refers to an ambiguous sentence, it may be a compelling intuition when we focus on one interpretation of the sentence, but uncompelling on a different interpretation. Consider for example the following validity intuition:

(11) John did not sleep well
Mary did sleep well
∴.everyone did not sleep well is a valid inference

Assuming that only the conclusion of the inference is ambiguous, one can say that on one interpretation of every one did not sleep well the observation is true, but on another interpretation it is false. For this reason, truth and validity intuitions will often be proposed that mention *readings*:

(12) There is a reading of every one did not sleep well on which
John did not sleep well
Mary did sleep well
∴.everyone did not sleep well is a valid inference

The equivalent statement not in inference form is:

(13) There is a reading r of every one did not sleep such that whatever situation s,
if John did not sleep well is true in s
and Mary did sleep well is true in s,
then everyone did not sleep well is true in s on reading r

It will be a convention that if a truth-intuition is given which omits to relativise truth to a reading, then the sentence of which truth is predicated is an unambiguous sentence. Such truth-intuitions will become data by taking universal and existential quantifications over *disambiguations* in entailments of a THEORY OF REFERENCE and respectively dropping them and turning them into existential quantifications over readings. So a THEORY OF REFERENCE will be held to have accounted for (13) if it entails:

(14) there is a disambiguation β_1 of everyone did not sleep well such that whatever model, $\langle \Im, \langle i,j\rangle\rangle$, whatever disambiguation β_2 of John did not sleep, whatever disambiguation β_3 of Mary did sleep well, if $[\![\beta_2]\!](i,j) = 1$ and $[\![\beta_3]\!](i,j) = 1$, then $[\![\beta_1]\!](i,j) = 1$

Context-sensitivity

Natural language contains context-sensitive referring expressions, such as that in that is a man. To give truth-intuitions concerning such expressions, we make appeal to a notion of situations which is such that a situation is understood to fix a reference of a context-sensitive referring expression. Using the notation s_x^{that} for the situation that differs from s solely in that it fixes the referent of that a certain way we can formulate truth intuitions such as the following:

(15) Whatever situation s, this is a man is true in s_x^{this} iff that is a man is true in s_x^{that}

There is a problem posed by the formulation of truth intuitions for sentences which have two occurrences of the same context sensitive expression, such as this goes with this. At one moment this refers to one thing and at another it refers to another. A crude response to this is to pretend that English does not have the demonstrative pronoun this but has instead a number of subscripted pronouns this_1, this_2 and so on.[4]

We turn in the next section from the description of the format of truth intuitions, to the identification of the crucial truth intuitions that will be used as semantic data.

3 Facts about English

3.1 Transparency and Opacity intuitions

Consider the truth intuitions:

(16) for any situation s, if Cicero is Tully is true is s then Cicero is handsome is true in s iff Tully is handsome is true in s

(17) there is a situation s such that Cicero is Tully is true is s, Cicero is handsome is true in s and Tully is handsome is false in s

Using the terminology of Quine (Quine 1963, essay 8), one could say that 'Cicero is Tully is true in s' is a way of saying that Cicero is *co-extensive* with Tully in s and that (16) (resp (17)) claims that Cicero occurs *transparently* (resp. *opaquely*) in Cicero is handsome.

Truth intuitions that amount to claims about *transparency* and *opacity* constitute a fundamental core of semantic facts to be accounted for by a THEORY OF REFERENCE. We will first define the technical terms *co-extension*, *transparency* and *opacity* in terms of truth intuitions, and then enumerate some of the most important data concerning *transparency* and *opacity*.

[4] A better response seems to be to regard linguistic objects a real physical events, extended in time. Then all words are effectively 'subscripted' by their time of utterance.

First we define the notion: *co-extensive in s*. There are two cases, the 'referring' expressions, where these encompass proper names and pronouns, and the 'predicating' expressions, where these encompass sentences (S), common-nouns (CN), verb-phrases (VP), transitive verbs (TV) and ditransitive verbs (TTV). In order to be able to give one definition that embraces all 'predicating' expressions, it is useful to define the following function $PRED$, which maps a sequence of referring expressions and a predicating expression into a sentence :

$$PRED(\langle\rangle, \beta^S) = \beta^S$$
$$PRED(\langle\gamma_1^{NP}\rangle, \beta^{VP}) = \gamma_1^{NP}\beta^{VP}$$
$$PRED(\langle\gamma_1^{NP}\rangle, \beta^{CN}) = \gamma_1^{NP}\text{is a } \beta^{CN}$$
$$PRED(\langle\gamma_1^{NP}, \gamma_2^{NP}\rangle, \beta^{TV}) = \gamma_1^{NP}\beta^{TV}\gamma_2^{NP}$$
$$PRED(\langle\gamma_1^{NP}, \gamma_2^{NP}, \gamma_3^{NP}\rangle, \beta^{TTV}) = \gamma_1^{NP}\beta^{TTV}\gamma_2^{NP}\gamma_3^{NP}$$

Definition 13 (Co-extension)

Referring terms: *For any situation s, α and β are* co-extensive *in s iff α* is *β is true in s.*

Predicating Expressions: *For any situation s, α and β are* co-extensive *in s iff for all proper names and pronoun sequences $\langle\gamma\rangle_\zeta$ of appropriate length, $PRED(\langle\gamma\rangle_\zeta, \alpha)$ is true in s iff $PRED(\langle\gamma\rangle_\zeta, \beta)$ is true in s, where the appropriate length of sequence is 0 if α and β are S's, 1 if they are VP's or CN's, 2 if they are TV's and 3 if they are TTV's.*

As it stands this definition does not make sense if applied to *ambiguous* expressions. For example if α is gave everyone a piece of advice and α' is is stupid, then amongst the things that the definition indicates as crucial to whether α is co-extensive with α' is whether:

John gave everyone a piece of advice is true in s iff John is stupid is true in s

However, owing to the ambiguity of John gave everyone a piece of advice this biconditional cannot be evaluated. Rather than try to complicate the definition to embrace ambiguous expressions we will leave matters as they are and will only invoke the concept of co-extension for unambiguous expressions.

Transparency [5] can now be defined as 'preservation of truth value under co-extensive substitution':

Definition 14 (Transparent and Opaque occurrences)

α occurs transparently *in* S[α] *if for all situations s, if α and α' are co-extensive in s then* S[α] *is true in s iff* S[α'/α] *is true in s. Otherwise α occurs* opaquely.

(18) a (John)$_e$ walks - John occurs *transparently*

b John wants (to go)$_i$ - to go occurs *opaquely*

[5] Sometimes 'transparency of occurence' is defined so as to be relative to another expression in the sentence, which expression is then referred to as a 'creator of a transparent' context. The notion we define is not so relativised.

As with the definition of co-extension, the above definition of transparency has glossed over the issue of ambiguity. This time we will adjust the definition to account for ambiguity. We must do this because a certain kind of reading distinction uses a notion of *reading-relative* transparency. For example, it is often said that there are two readings of John believes the murderer of Smith used a knife, one according to which murderer of Smith occurs *opaquely* and one according to which it occurs *transparently*.

Formulating a more precise definition is a tricky matter: under what reading of $S[\alpha'/\alpha]$ should its truth be compared to the truth of $S[\alpha]$ on a given reading r ? The answer cannot be the reading r, because presumably r is not a reading of $S[\alpha'/\alpha]$ at all. One would like to appeal to a notion of *sub*-readings and substitution of sub-readings, though it is a little strange to bring such technical vocabulary into what after all are supposed to be the description of intuitions about sentences. In certain cases, however, the notions of sub-reading and substitution do seem quite intuitive. Consider for example the conjunction of two ambiguous sentences, S_1 and S_2, where these have m and n distinct readings respectively. Then it seems intuitive to think of a reading, R, of S_1 and S_2 as no more than a *pairing* of readings of the conjuncts, $\langle r_1, r_2\rangle$. Then reading r_1 of is a subreading of R, and $R' = \langle r'_1, r_2\rangle$ is obtained from R by the substitution of subreading r'_1 for subreading r_1. At all events the reading-relativised definition is:

Definition 15 (Transparent and Opaque occurrences)

According to reading R of $S[\alpha]$, *α occurs* transparently *if for all situations s, for all α',*
if α is co-extensive in s with α'
then $S[\alpha]$ *on R is true in s iff* $S[\alpha'/\alpha]$ *on R' is true in s, where $R' = R[r'/r]$, where r' and r are readings of α' and α. Otherwise, according to reading R of* $S[\alpha]$, *α occurs* opaquely.

In the data below concerning transparency or opacity of an occurrence, we have marked with a subscript e those expressions which occur transparently on all readings, with a subscript i those expressions which occur opaquely on all readings and with a subscript i/e those expressions which occur opaquely on some readings and transparently on some other readings.

(19) a $\text{John}_e\ \text{walks}_e$

b $\text{John}_e\ (\text{loves}_e\ \text{Dave}_e)_e$

c $\text{DET}\ (\text{CN})_e\ (\text{VP})_e$

d $\text{John}_e\ \text{loves}_e\ \text{DET}\ (\text{CN})_e$

e $\text{DET}\ (\text{CN})_e\ (\text{loves})_e\ \text{DET}\ (\text{CN})_e$

f $\text{John}_e\ \text{believes}\ (\text{S})_i$

g $\text{John}_e\ \text{wants}\ (\text{VP})_i$

h $\text{John}\ (\text{seeks})_{i/e}\ \text{a}\ (\text{man})_{i/e}$

i $(\text{S}^1)_e\ \text{and}\ (\text{S}^2)_e$

j $\text{John}_e\ (\text{VP}^1)_e\ \text{and}\ (\text{VP}^2)_e$

k John$_e$ (TV^1)$_e$ and (TV^2)$_e$ (Mary)$_e$

Besides these assorted observations concerning *opacity* and *transparency*, there seems to be at least one general principal, depicted approximately below and then defined in Hypothesis 1

opacity: $(\ldots[\ldots\alpha\ldots]_i\ldots) \Rightarrow (\ldots[\ldots\alpha_i\ldots]_i\ldots)$

Hypothesis 1 (Downward heritability of opacity) *if there is a reading r of* $(\ldots[\ldots\alpha\ldots]\ldots)$ *according to which* $[\ldots\alpha\ldots]$ *occurs* opaquely *then there is a reading r′ of* $(\ldots[\ldots\alpha\ldots]\ldots)$ *according to which* α *occurs* opaquely

So for example, (19f) and (19g) together with Hypothesis 1 imply (20f′) and (20g′) below (where this time the *i* subscript indicates opacity on *a* reading):

(20) f′ John believes (a man$_i$ walks$_i$)$_i$

g′ John wants (to marry a (blond)$_i$)$_i$

A sister property to that of the *downward heritability of opacity* might be the property below of *upward heritability of transparency*, but this property actually holds by definition of transparency:

$(\ldots[\ldots\alpha\ldots]_e\ldots)$ and $[\ldots\alpha_e\ldots] \Rightarrow (\ldots[\ldots\alpha_e\ldots]_e\ldots)$

As a point of terminology, we will say that a quantified noun phrase, DET CN, has a *de-dicto* (resp. *de-re*) occurrence on a reading if and only if the common noun part CN has an *opaque* (resp. *transparent* occurrence) on that reading. In this terminology, part of the facts observed in (20) which are the consequences of the downward heritability of opacity might equally be stated thus: for quantifiers in embedded sentences and embedded VP's, there is a reading according to which the quantifier occurs *de-dicto*.

For most expressions, if they have an opaque occurrence on *one* reading they have an opaque occurrence on *all* readings. The main exception to this are the quantifiers: many would assent to the hypothesis that whatever sentence in which DET CN occurs, there is a reading upon which the CN occurs *transparently*:

Hypothesis 2 (Common noun transparency) *if* α *is a sentence in which* DET CN *occurs then there is a reading of* α *according to which* CN *occurs transparently.*

3.2 Preliminary Truth Intuitions concerning junctions

In this section we will present discuss some of the truth intuitions that are going to form the data which theories in later chapters will aspire to capture. We begin by giving some specific intuitions concerning particular sentences. Discussion moves then to whether one can state any generalisations which would entail the specific observations. We note some difficulties in finding a vocabulary in which to state generalisations. It is because of these difficulties that the section is entitled 'Preliminary'. It will be in section 3.4 that I state with more finality what I take the semantic facts concerning junctions to be.

For and and or the obvious validity intuitions to point out are that the following are valid inferences:

S_1 and S_2	S_1
$\therefore S_1$	$\therefore S_1$ or S_2

Factual though these observations are they do not invite generalising across junctions: the inference do not differ merely be the substitution of one lexical item for another but are actually of different forms. Better prospects for generalisation are offered by the following pair of truth intuitions:

(21) There is a reading r of John walks and Mary talks such that for any situation s,
John walks and Mary talks is true on r in s iff
John walks is true in s *and* Mary talks is true in s

(22) There is a reading r of John walks or Mary talks such that for any situation s,
John walks or Mary talks is true on r in s iff
John walks is true in s *or* Mary talks is true in s

We can generalise over both of these if we exploit the obvious correspondence between mentions and uses of a junction:

(23) For any junction, JUNCT, there is a reading r of John walks JUNCT Mary talks such that, where J corresponds to JUNCT, for any situation s,
John walks JUNCT Mary talks is true on r in s iff
John walks is true in s J Mary talks is true in s,

Having generalised for *lexical* material of a junction, we might attempt to generalise for *syntactic* context. Consider for example the following instances of *sub-sentential* junctions.

(24) a John walks_{VP_1} JUNCT talks_{VP_2}
b John loves_{TV_1} JUNCT hates_{TV_2} Mary

For each of these a separate generalisation could be given, along the lines of that just given for the sentential junctions. There would, however, be a substantial similarity between the generalisations for subsentential junctions and the sentential junctions so one could try to formulate a single generalisation embracing at once all kinds of occurrence of junctions. Here is a first pass:

(25) **Junction generalisation: first attempt**

Any instantiation of $S[X_1$ JUNCT $X_2]$ has a reading r such that, where J corresponds to JUNCT, for any situation s,

$S[X_1$ JUNCT $X_2]$ is true on r in s iff

$S[X_1/X_1$ JUNCT $X_2]$ is true in s J $S[X_2/X_1$ JUNCT $X_2]$ is true in s

This generalisation appears to be true of all the junction containing sentences that we have considered so far, and also of many of the possible instantiations of $S[X_1$ JUNCT $X_2]$ that we have not yet considered. There will also be some rather controversial instances, as for example in the case of a man came in and sat down. In a little while I discuss further some of these controversial cases. To give advance warning of the position that I will adopt, however, it will be that this generalisation (or something rather like it) should be adopted as one of the entailments one will be hoping to obtain from a THEORY OF REFERENCE.

However, there is fundamental problem with this generalisation as it stands, a problem which means it is not at all apt to be adopted as an empirical yardstick, a problem which I shall call the Recursive Ambiguity problem. It arises because for many instantiations of $S[X_1$ JUNCT $X_2]$, the instantiations of $S[X_1/X_1$ JUNCT $X_2]$ and $S[X_2/X_1$ JUNCT $X_2]$ are *ambiguous* and therefore it does not make sense to predicate the property 'true in s' of them. Such a sentence is: every man loves a woman and every marriage has a good day. The application of the generalisation (25, p39) to this involves statements like 'every man loves a woman is true in s' which does not make sense because the sentence in question is ambiguous. So (25, p39) will not do as a generalisation embracing *all* kinds of occurrence of junctions.

Let us call the strings $S[X_1/X_1$ JUNCT $X_2]$ and $S[X_2/X_1$ JUNCT $X_2]$ that are mentioned in (25) the *reducts* of $S[X_1$ JUNCT $X_2]$. (25) was aiming to make a true generalisation concering the triple consisting of $S[X_1$ JUNCT $X_2]$ and its two reducts. (25) fails as a generalisation as it does not anticipate the possibility that the reducts may be ambiguous. If there is a generalisation to be made, it must a statement connecting together this triple of potentially ambiguous strings, in some way quantifying over the readings of all three members of the triple.

Now by and large it appears to be the case that *for any* supposed pair of readings of the reducts, there is a corresponding reading of $S[X_1$ JUNCT $X_2]$. Therefore one can canvas a generalisation that has a $\forall\forall\exists$ form:

(26) **Junction generalisation: second attempt**

Whatever instantiation of $S[X_1$ JUNCT $X_2]$, whatever reading r_2 of $S[X_1/X_1$ JUNCT $X_2]$, whatever reading r_3 of $S[X_2/X_1$ JUNCT $X_2]$, there is a reading r_1 of $S[X_1$ JUNCT $X_2]$, such that, where J corresponds to JUNCT, for any situation s,

S_1 JUNCT S_2 is true on r_1 in s iff S_1 is true on r_2 in s J S_2 is true on r_3 in s

(26) succeeds then in being a generalisation which can at least meaningfully be applied to all junction-containing sentences, and so is an improvement over the first attempt, (25,p39). Still leaving to one side the question of whether this generalisation is empirically over-generous, it is from the point of view of testing THEORIES OF REFERENCE still not ideal on grounds of form alone. Because of the universal quantification over readings of the reducts, one will have a case to consider for *all* the disambiguations that the THEORY OF REFERENCE provides for the reducts. It is easier to test a THEORY OF REFERENCE against the kind of truth intuition which mentions only one ambiguous string, all others that are mentioned having only one reading each. The instances of (26) are not of this pattern. I will return to this problem after a preliminary discussion of the truth intuitions for sentences containing quantifiers.

3.3 Preliminary Truth Intuitions concerning quantifiers

As with the junctions, if the determiners are taken one by one then there are obvious validity intuitions that present themselves as data:

every man is mortal	Socrates is mortal	no man is mortal	most Greeks are men
Socrates is a man	Socrates is a man	Socrates is a man	most Greeks are mortal
∴Socrates is mortal	∴a man is mortal	∴Socrates is not mortal	∴a man is mortal

There are many more such inferences but as with the junctions the problem with presenting the semantic data in this form is that it offers no way to make generalisations across determiners: the argument forms are specific to the determiners in them. There is a better chance of making generalisations if we start out with truth intuitions such as the following two:

(27) There is a reading r_1 of every man walks such that for any situation s,
every man walks is true in s on r_1 iff { x: he_1 is a man is true in $s_x^{\text{he}_1}$ }
is a subset of
{ x: he_1 walks is true in $s_x^{\text{he}_1}$ }

(28) There is a reading r_1 of a man walks such that for any situation s,
a man walks is true in s on r_1 iff { x: he_1 is a man is true in $s_x^{\text{he}_1}$ }
has a non-null intersection with
{ x: he_1 walks is true in $s_x^{\text{he}_1}$ }

As there was with the junctions there is the possibility of generalising over these two by taking careful note of the correspondence between *mention* of a determiner and *use* of a term describing a binary relation between sets. To assist with this we define some notation for compactly describing relations between sets:

'$EVERY(S_1, S_2)$' means 'S_1 is a subset of S_2'

'$A(S_1, S_2)$' means 'S_1 has a non-null intersection with S_2'

'$NO(S_1, S_2)$' means 'S_1 has a null intersection with S_2'

'$MOST(S_1, S_2)$' means '$\|S_1 \cap S_2\| > \|S_1\|$'

Now we can make the following generalisation concerning any subject occurrences of a quantifier:

(29) Whatever DET CN, there is a reading r_1 of DET CN walks such that, where D corresponds to DET, for any situation s,

DET CN walks is true in s on r_1 iff $D($ { x: he_1 is a CN is true in $s_x^{\text{he}_1}$ }
{ x: he_1 walks is true in $s_x^{\text{he}_1}$ }
)

For exactness here is a specification of the correspondence between DETs and D's:

If DET is instantiated to every, all or each, D should be instantiated to $EVERY$

If DET is instantiated to a, one or some, D should be instantiated to A

If DET is instantiated to no, D should be instantiated to NO.

If DET is instantiated to most, D should be instantiated to $MOST$.

(29, p41) quantifies over *all* DET CN. Really the quantification should be over just those DET that have had a corresponding set relation defined for them.

Moving from generalising over lexical items to generalising over syntactic locations, we might formulate independent generalisations concerning the two kinds of non-subject occurrence of quantifiers shown below:

(30) a John loves DET CN

b John told DET CN to go

However these two additional generalisations would be very similar to what we have in (29, p41) . Rather as with the junctions, we can try to generalise over occurrence of DET's in number of different syntactic contexts:

(31) **Determiner generalisation: first attempt**

For any instantiation of S[DET CN], there is a reading r_1 such that, where D corresponds to DET, for any situation s,

S[DET CN] is true in s on r_1 iff $D($ { x: he_1 is a CN is true in $s_x^{\text{he}_1}$ }
{ x: S[he_1/DET CN] is true in $s_x^{\text{he}_1}$ }
)

This is true for all the sentences considered so far, and for many other sentences besides. I will discuss later some potential counter-examples to this generalisation. However, as there was with out first attempt at a generalisation for junctions, (25, p39), there is a funamental problem with the generalisation (31) as it stands. For some instantiations of S[DET CN], the sentences mentioned in the right-hand side of the biconditional are ambiguous. An example is: every man gave a policeman 2 flowers. If a policeman is chosen as the instantiation of DET CN then generalisation (31) will make mention of 'every man gave he_i 2 flowers is true in s', which is non-sensical:

(32) there is a reading r_1 of every man gave a policeman 2 flowers such that whatever situation s, every man gave a policeman 2 flowers is true in s on r_1 iff

$A($ { x: he_1 is a policeman is true in $s_x^{he_1}$ }

{ x: every man gave he_1 a flower is true in $s_x^{he_1}$ }

)

I will for the moment call the strings he_1 is a CN and S[he_i/DET CN] that are mentioned in (31), the *reducts* of S[DET CN]. (31) was aiming to make a true generalisation concering the triple consisting of S[DET CN] and its two reducts. (31) fails as a generalisation as it does not anticipate the possibility that the reducts may be ambiguous. If there is a generalisation to be made, it must be a statement connecting together this triple of potentially ambiguous strings, in some way quantifying over the readings of all three members of the triple.

Now, as was the case with the junctions, it appears to be the case that *for any* supposed pair of readings of the reducts, there is a corresponding reading of S[DET CN]. Therefore one can canvas the following hypothesis:

(33) **Determiner generalisation: second attempt**

For any instantiation of S[DET CN], whatever reading r_2 of he_1 is a CN, whatever reading r_3 of S[he_1/DET CN], there is a reading r_1 of S[DET CN] such that, where D corresponds to DET, for any situation s,

S[DET CN] is true in s on r_1 iff $D($ { x: he_1 is a CN is true in $s_x^{he_1}$ on r_2 }

{ x: S[he_1/DET CN] is true in $s_x^{he_1}$ on r_3 }

)

The generalisation (33) is an improvement on (31, p41) through being at least meaningful. Still leaving to one side the question of whether the generalisation is empirically over-generous, it is from the point of view of testing THEORIES OF REFERENCE, still not ideal. As with (26, p39), one will have a case to consider for *all* the disambiguations that the THEORY OF REFERENCE provides for the reducts. Matters are far easier if a truth intuition mentions only one ambiguous string.

To summarise what I have said so far concerning junctions and quantifier. For both, I first suggested simple generalisations, which had many intuitively true instances, but which had unfortunately a number of non-sensical instances. For the prototype junction generalisation, (25, p39), the problem arose with a sentence such as every man loves a woman and every marriage

has a good day. For the prototype determiner generalisation, (31, p41), the problem arose with a sentence such as **every man gave a policeman 2 flowers**. I then gave two further generalisations, (26, p39) and (33, p42), whose instances would be truth intuitions with a ∀∀∃ format. These generalisations had many true instances, and did not have the non-sensical instances of the prototype generalisations, (25, p39) and (31, p41). In other words they had something at least meaningful to say about **every man loves a woman and every marriage has a good day** and **every man gave a policeman 2 flowers**. However, I observed also that the ∀∀∃ format of the instances of generalisations (26, p39) and (33, p42) made them not ideal from the point of view of testing a THEORY OF REFERENCE. If possible it would be preferrable to have generalisations whose instances would mentions only *one* ambiguous string. In the following section I will see whether there are correct truth intuitions, of the desired form, concerning sentences such as **every man loves a woman and every marriage has a good day** and **every man gave a policeman 2 flowers**.

3.4 Recursive Ambiguity Intuitions for junctions and quantifiers

Although the *instances* of generalisations (26, p39) and (33, p42) are not truth-intuitions of the desired form, there are *entailments* of these generalisations which are *are* truth intuitions of the desired form.

For example, the quantifier generalisation, (33, p42), has an instance for **every man gave a policeman 2 flowers**, and this will be a truth intuition that mentions the following pair of sentences:

he$_1$ is a policeman

every man gave he$_1$ 2 flowers

There is no ambiguity of the first. There is a further instance of the quantifier generalisation for the second, and this will be truth intuition that mentions the following pair of sentences:

he$_2$ is a flower

every man gave he$_1$ he$_2$

Again, there is no ambiguity of the first, and there is a further instance of the quantifier generalisation concering the second, and this will be a truth intuition that mentions the following pair of sentences:

he$_3$ is a man

he$_3$ gave he$_1$ he$_2$

Neither of these is ambiguous. A truth intuition of the desired form will follow from these three instance of the quantifier generalisation, namely the following one:

(34) There is a reading r_1 of every man gave a policeman 2 flowers such that whatever situation, s, every man gave a policeman 2 flowers is true in s on iff

A

{ x: he_1 is a policeman is true in $s_x^{\text{he}_1}$ }

{ x: TWO { y: he_2 is a flower is true in $s_{x,y}^{\text{he}_1,\text{he}_2}$ } }

{ y: $EVERY$ { z: he_3 is a man is true in $s_{x,y,z}^{\text{he}_1,\text{he}_2,\text{he}_3}$ } }

{ z: he_3 gave he_1 he_2 is true in $s_{x,y,z}^{\text{he}_1,\text{he}_2,\text{he}_3}$ }

Something similar applies in the case of every man loves a woman and every marriage has a good day. One first notes the truth intuition concerning this that arises via the junction generalisation, (26, p39). This truth intuition will make mention also of the following two sentences:
every man loves a woman
every marriage has a good day

One can then note the truth intuitions concering these that arise via the quantifier generalisation, (33, p42). These will mention still further sentences which are either unambiguous or concerning which there are truth intuitions arising from the quantifier hypothesis. Then from all these noted truth intuitions one will able to infer a single truth intuition concering every man loves a woman and every marriage has a good day.

Therefore, with sufficient application, one can infer from the generalisations given in (26, p39) and (33, p42), truth intuitions of the desired from. In other words, the two generalisations given together constitute a kind of declarative specification of truth intuitions of the desired form. I will be concerned now with a rather more direct procedural specification of these truth-intuitions.

Casting one's mind back to the first attempt at a determiner generalisation, (31, p41), one could say that there was a non-deterministic rewrite rule implicit in it, a rewrite which may take one from 'S[DET CN] is true in s' to $D(\{x : \text{he}_1$ is a CN is true in $s_x^{he_1}\}, \{x :$ S[he_1/DET CN] is true in $s_x^{he_1}\})$.

Also casting one's mind back to the first attempt at a junction generalisation, (25, p39), one can say that there was another implicit non-deterministic rewrite rule, a rewrite which may take one from 'S[X_1 JUNCT X_2] is true in s' to 'S[X_1/X_1 JUNCT X_2] is true in s J S[X_2/X_1 JUNCT X_2] is true in s'

Once one has reflected a while on the way in which the desired truth intuitions are *inferred* from the junction and determiner generalisations, (26, p39) and (33, p42), it should be clear that the same truth intuitions could instead be *constructed* via these non-deterministic rewrite rules, as follows. One starts with a statement T, which is 'S is true in s'. One recursively applies the non-deterministic rewrites to T, obtaining some eventual result T_n, and one then proposes the truth intuition 'there is a reading r of S such that whatever situation, s, S is true in s under r

iff T_n'.

Therefore, if we can get more precise about this rewrite procedure, the way is opened to formulate generalisations to the effect that there are concerning junction and determiner containing sentences all the truth intuitions that one would expect under the rewrite procedure.

To this end I define a relation, $\mathcal{RA}$, which will have 5 arguments: a sentence, an occurrence of an expression in the sentence, a situation, a statement about the truth of sentences and finally a number, there to help with subscripting pronouns:

Definition 16 (Recursive Ambiguity Relation)

(i) where p is a sentence containing no junctions or determiners, then whatever string β, *whatever situation s, whatever i,*
$\mathcal{RA}(p, \beta, s, i, p$ *is true on its only reading in s).*

(ii) $\mathcal{RA}$(S[X_1 JUNCT X_2], X_1 JUNCT X_2, s, i, J (T_1, T_2)), *if there exist* β_1, β_2 *such that* $\mathcal{RA}$(S[X_1/X_1 JUNCT X_2], β_1, s, i, T_1) *and* $\mathcal{RA}$(S[X_2/X_1 JUNCT X_2], β_2, s, i, T_2)
where J corresponds to JUNCT

(iii) $\mathcal{RA}$(S[DET CN],DET CN, $s, i, D(\{ x: T_1 \},\{ x: T_2 \})$), *if there exists* β_1, β_2 *such that* $\mathcal{RA}$(he_i is a CN, $\beta_1, s_x^{he_i}, i+1, T_1$) *and* $\mathcal{RA}$(S[he_i/DET CN], $\beta_2, s_x^{he_i}, i+1, T_2$)
where D corresponds to DET.

Some illustration:

(35) $\mathcal{RA}$(every man loves a woman, every man, s, 1

EVERY { x: he_1 is a man is true in $s_x^{he_1}$ }
{ x: *SOME* { y: he_2 is a woman is true in $s_{x,y}^{he_1,he_2}$ }
{ y: he_1 loves he_2 is true in $s_{x,y}^{he_1,he_2}$ } }

)

(36) $\mathcal{RA}$(every man loves a woman, a woman, s, 1

SOME { x: he_1 is a woman is true in $s_x^{he_1}$ }
{ x: *EVERY* { y: he_2 is a man is true in $s_{x,y}^{he_1,he_2}$ }
{ y: he_2 loves he_1 is true in $s_{x,y}^{he_1,he_2}$ } }

)

(37) There are no other T or β such that $\mathcal{RA}$(every man loves a woman, β, s, 1, T)

In terms of the $\mathcal{RA}$ relation, I will now define a notion: 'α is recursively ambiguous wrt. the subexpression β', by means of which it will become possible to refer to certain sets of truth intuitions that one could have inferred from the junction and determiner generalisations, (26, p39) and (33, p42).

Definition 17 (α is recursively ambiguous wrt. β) *α is recursively ambiguous wrt. β iff whatever T such that $\mathcal{RA}(\alpha,\beta, T)$, there is a reading r of α such that α is true on r iff T.*

Truth intuitions (of the desired form) concerning a determiner containing sentence that one could *infer* at some length from the earlier junction and determiner generalisations (26, p39) and (33, p42), one can now infer rather directly from the following hypothesis.

Hypothesis 3 (Recursive ambiguity of Quantifiers) *for any sentence β that contain a quantifier,* DET CN, *β is recursively ambiguous wrt.* DET CN

As I mentioned earlier, I will be adopting the above hypothesis. In other words, Hypothesis 3 will be taken to be one of the desired entailments of a THEORY OF REFERENCE.

As to whether Hypothesis 3 is over-generous, there is at least one widely agreed upon exception to it, first noted by Rodman (Rodman 76), and it occurs when the DET CN occurs in a syntactic island: Guinevere has a bone which is in every room is *not* recursively ambiguous wrt. every room. This comes down to the fact the following truth intuition is not compelling:

(38) There is a reading r of Guinevere has a bone which is in every room such that for any situation s,
Guinevere has a bone which is in every room is true on r in s iff
EVERY
{ y: he$_2$ is a room is true in $s^{y}_{he_1}$ }
{ y: *SOME* { x: he$_1$ is a bone which is in he$_2$ is true in $s^{x,y}_{he_1,he_2}$ } }
{ x: Guinevere has he$_1$ is true in $s^{x,y}_{he_1,he_2}$ }

For the main part of the thesis, this exception to Hypothesis 3 will be ignored. It will, however, receive some consideration in the final chapter.

It should be noted that there is a connection between the recursive ambiguity data and the transparency data: in claiming that S[DET CN] is recursively ambiguous wrt. DET CN we are claiming that there is a reading on which CN occurs *transparently* in S[DET CN]. Therefore Hypothesis 3(p46) entails Hypothesis 2(p37).

In a similar vein to Hypothesis 3, an hypothesis concerning junctions and recursive ambiguity will now be framed. From it one will be to infer rather directly those truth intuitions of the desired form that one could have inferred rather indirectly from the earlier determiner and junction generalisations, (26, p39) and (33, p42):

Hypothesis 4 (Recursive ambiguity of Junctions) *For any β that contains an* X_1 JUNCT X_2, *β is recursively ambiguous wrt.* X_1 JUNCT X_2.

As I mentioned earlier, I will be adopting this as one of the desired entailments of a THEORY OF REFERENCE. I consider now the question of whether this empirically over-generous.

I have so far considered only sentences such that outwith the joined elements there is no logical complexity, in other words sentences with forms like:

(39) a S_1 JUNCT S_2

b John VP_1 JUNCT VP_2

c John TV_1 JUNCT TV_2 Mary

For such sentences the claim of recursive ambiguity with respect to the joined elements seems uncontroversial. However, Hypothesis 4 also has the following more controversial entailment for a sentence that is not of the above mentioned form:

(40) a man came in and sat down is recursively ambiguous wrt. came in and sat down

Whether (40) is reasonable comes down to whether the following truth intuition is compelling:

(41) There is a reading such that a man came in and sat down is true iff

$$
\begin{array}{lll}
\text{AND(} & \text{SOME} & \{ x\text{: he}_1 \text{ is a man is true in } s_x^{\text{he}_1} \} \qquad) \\
 & & \{ x\text{: he}_1 \text{ came in is true in } s_x^{\text{he}_1} \} \\
 & \text{SOME} & \{ x\text{: he}_1 \text{ is a man is true in } s_x^{\text{he}_1} \} \\
 & & \{ x\text{: he}_1 \text{ is a woman is true in } s_x^{\text{he}_1} \}
\end{array}
$$

Undeniably this not the preferred reading of the sentence. The question is whether it is a possible reading. If it were not a possibility, then one might be inclined to revise Hypothesis 4 (p46) to apply to only those sentences lacking logical complexity outwith the joined elements. On this score, I have two observations to make. One is that substitution of different lexical material seems to change the intuitions for sentences such as (40). The other is that there are quite a number of other sentences exhibiiting logical complexity outwith the joined elements that do seem to have the reading predicted by Hypothesis 4 (p46).

First, substituting different lexical material into the same syntactic framework. On the subject of government responses to student demonstrations, one might say:

(42) A brutal police baton charge disrupted considerably the 1963 Paris student demonstations and dispersed completely the 1990 Tianenmen Square demonstration.

The reading predicted by Hypothesis 4 (p46) seems possible, even preferred. My intuitions are the same for the following sentence, said on the subject of Napoleonic successes throughout Europe:

(43) That year, a French flag flew above a captured British fleet in Portugal, fluttered from the roof of the Austrian parliament building, and draped the gates to St. Petersburg.

There follow some examples of sentences exhibitting logical complexity outwith the joined items, and possessing, it seems to me, the readings predicted by Hypothesis 4 (p46):

(44) a. every man and woman died

b. every tall and handsome man died

c. every house in London and Paris has an escalating price

d. the judge will say either that they are guilty or that they are not guilty

e. he wants not to praise Caesar but to bury him

Therefore, as I said above, I will proceed on the assumption that Hypothesis 4 (p46) is something one should expect as an entailment of a THEORY OF REFERENCE. In the concluding chapter, I will discuss a little further the question of exceptions to the hypothesis.

3.5 Further Truth Intuitions for creators of Opaque Contexts

3.5.1 Truth Intuitions for Sentence Embedders

The truth intuitions for a sentence p that we have given so far have been of the form:

(45) For all situations s, p_1 is true in s iff $\Phi(q_1$ is true in $s, \ldots, q_n$ is true in $s)$

Here $\Phi(q_1$ is true, $\ldots$, q_n is true$)$ has been a logical complex of statements about the truth of other sentences. Typically if 'q_i is true in s' were replaced by a logical equivalent, then the truth intuition would remain correct. Now no truth intuition for John believes that p of the form 'John believes that p is true iff Φ(p is true in $s, \ldots$)' can be correct *if 'p is true in s' is replaceable in it by a logical equivalent.* This is because if such a truth intuition were correct it would imply that p occurs *transparently* in John believes p, whereas, of course, it occurs opaquely.

However, if it is the case that in the truth intuition for John believes that p, 'p is true in s' *cannot* be replaced by a logical equivalent, then the truth intuition for John believes that p cannot be related to truth intuitions for p. Not only does this seem unnatural, it also leaves theories surprisingly underconstrained: with no link in the data between embedded and unembedded occurrences of a sentence, there need not be any in a theory that aims to account for the data.

Therefore it would be desirable to give truth intuitions for John believes p containing p is true in s, and such that p is true in s is replaceable by a logical equivalent. I believe this is possible. For example, there is a reading of John believes a man died, and a reading of a man died such that the following inference is valid:

(46) John believes a man died

a man died

∴ A proposition that John believes is true

The relevant reading of John believes a man died is one according to which man occurs opaquely. It should be noted that in the truth-intuition above there is an object-language occurrence of true and quantification over *propositions*. Since Tarski it has been known that there are profound difficulties in providing a THEORY OF REFERENCE for a language that contains a truth predicate. Because of this it may seem a waste of effort to put forward truth intuitions concerning sentences featuring object language occurrences of true. This issue will receive attention in Chapter 5. Here we give the generalisation for the embedding verb believes:

Hypothesis 5 (The sentence embedding verb, believe) *Whatever reading r_{embed} of* p, *there is a reading r of* John believes p *such that whatever situation s, if* John believes that p *is true on r on s and* p *is true on r_{embed}, then* a proposition that John believes is true *is true in s*

Hypothesis 5 concerns only believes. For many other sentence-embedding verbs something similar could be formulated.

By using the kind of inferences referred to by Hypothesis 5 one can distinguish readings which we have hitherto given no means for distinguishing. For example there is an intuition that there are *two* distinct readings of John believes every man loves a woman such that the quantifiers are interpreted *de-dicto*. These two readings can be distinguished by considering the validity of the inference:

John believes every man loves a woman

every man loves a woman

∴ a proposition that John believes is true

There are two readings of Every man loves a woman, one logically stronger than the other. Suppose r_{embed} is a the logically weaker reading of. Then assuming the reading r_{embed} of the second premise, one of the *de-dicto* readings of John believes every man loves a woman is such that the above inference is valid and on the other de-dicto reading the inference is invalid.[6]

3.5.2 Truth Intuitions concerning VP embedders

As with the sentence embedders, the opacity of VP occurrence following a verb like wants means that it is impossible to formulate truth intuitions that relate an opaque occurrence of a VP biconditionally to a transparent occurrence. That is to say, one can be sure that there will not be correct truth intuitions of the form

For all s, John want to VP is true in s iff Φ(S[VP] is true in $s, \ldots$)

where Φ supports substitution of logical equivalents and S[VP] is a sentence that has VP occurring transparently. Such truth intuitions cannot be correct, because they entail that VP occurs

[6] If r_{embed} were the logically stronger of the readings of the embedded sentence then on either of the *de-dicto* readings of John believes every man loves a woman the inference would be invalid.

transparently in John wants to VP.

Nonetheless there are truth intuitions that relate a transparent occurrence of the VP to the opaque occurrence. There is reading of John wanted to marry a blond such that the following inferences are valid:

John wanted to marry a blond	John wanted to marry a blond
John married a blond	Dave married a blond
∴.an act that John wanted to do, was done by John	∴.an act that John wanted to do, was done by Dave

The relevant reading of John wanted to marry a blond is one according to which a blond has a *de-dicto* interpretation. The generalisation is:

Hypothesis 6 (The VP embedding verb, want) *Whatever reading* r_{embed} *of* a VP's, *there is a reading r of* b wanted to VP *such that whatever situation s, if* b wanted to VP *is true in s on r and* a VP's *is true in s on* r_{embed}, *then* an act that b wanted to do was done by a *is true in s.*

Hypothesis 6 concerns the specific verb want. However, there are many other VP embedding verbs for which something similar could be formulated.

3.5.3 A truth intuition concerning seek

John seeks a unicorn is recursively ambiguous wrt. a unicorn, a fact which it requires it to have one reading characterised by following through the $\mathcal{RA}$ relation. There is also reading on which a unicorn is interpreted *de-dicto*, concerning which, one can say that it is reading upon which the following inference is valid[7]:

John sought a unicorn

Dave found a unicorn

∴.an act that John tried to do was done by Dave

This inference is lexically specific to the verb seek.

3.6 Junction and Quantifier phenomena that have been ignored

The data recorded in this chapter will be the data that the subsequently developed THEORIES OF REFERENCE will be assessed by. It ought to be said at this point, however, that there are other junction and quantifier phenomena that have been entirely overlooked.

For quantifiers, what is perhaps the most drastic omission has been the '*same per/different per*'

[7] this assumes seeks is synonymous with try to find.

contrast. Consider the following sentence:

(47) most people who graduate from the school can do two things

The 'same-per' reading is one on which requires there to be a *pair* of accomplishments and a *majority* of the school body such that every one in that majority can do *both* of the things in the pair.

(48) there is a reading r of (47) such that whatever situation s, (47) is true in s on r iff
there is a pair $\langle x, y \rangle$ such that
he$_1$ is an accomplishment is true in $s^{x}_{he_1}$
he$_2$ is an accomplishment is true in $s^{y}_{he_2}$
$MOST$ ({z : he$_3$ is a student is true in $s^{x,y,z}_{he_1,he_2,he_3}$})
{z : he$_3$ does he$_1$ and he$_3$ does he$_2$ is true in $s^{x,y,z}_{he_1,he_2,he_3}$}

This reading seems a natural one and is probably more natural than the reading required by recursive ambiguity wrt. two things - which it is not the same as.

For junctions, what is perhaps the most drastic omission has been the 'non-boolean' interpretation of and, that is possible in:

(49) John and Mary met in town

The possibility of these 'non-boolean' coordinations leads to certain kinds of ambiguity, as for example in:

(50) John and Mary carried a piano upstairs

There is an ambiguity here as to whether John and Mary acted collectively, jointly shouldering a single piano, or each separately carried a piano. This kind of ambiguity will not be dealt with. For an approach to them see Link (1983).

Chapter 4

Introduction to Categorial Grammar

To be introduced in this chapter are the frameworks of *Ajdukiewicz/Bar-Hillel categorial grammar* (ABG) and *Lambek categorial grammar* (LG). These are not overtly simply specialisations of the UG framework. The UG framework specifies (i) a certain way to define a meaning relation between strings and objects residing in semantic algebras and (ii) a certain way to define a categorisation relation between strings and categories. Both relations are mediated by a DISAMBIGUATED LANGUAGE. The ABG and LG frameworks specify these relations in *different* ways to the UG framework, making no (overt) appeal to a DISAMBIGUATED LANGUAGE. The emphasis of the presentation will be on the meaning relation, as it is in this respect that the LG framework is most unorthodox. The meaning relation for ABG's is looked at primarily to point up the contrast with the meaning relation for LG's. The 'working definition' of the meaning relation will first be put forward, as propounded by Moortgat and Hendriks. This will then be subjected to a conceptual analysis in order to highlight the role played in this definition by a fact drawn from an ostensibly different discipline. The discipline is proof-theory and the fact is the *Curry-Howard isomorphism*. This isomorphism comes to the statement that proofs of Natural deduction and typed λ-calculus terms are notational variants.

Once the LG framework has been described 'from the inside' as it were, we will re-examine the claim that it is not a specialisation of the UG framework. What one finds is that any LG definition of a categorisation and meaning relation can be duplicated by a UG proposal that defines the same relations. This means that LG is, by UG standards, *compositional*, despite the apparently unorthodox manner of defining the meaning relation. This is a point as much in favour of the generality of the UG framework as in favour of the LG framework, for it would be agreed on all sides that the LG framework is 'intuitively compositional', yet is only the UG formalisation of compositionality that can recognise the LG framework as technically compositional. In particular, on the 'Rule-to-Rule' formulation of compositionality, it is not possible to recognise the LG framework as compositional.

1 Basic Concepts of Categorial Grammar

We can begin by introducing what is most distinctive of categorial grammars, the *categories*. We will also say a little about the categorial grammarian's use of term *syntactic rule*, because it differs from the UG use of the term.

The categories

A distinctive feature of categorial grammars is the language used to categorise expressions. This language shall be referred to as the set of *categories* and is inductively defined from a finite set of *basic categories*, using *categorial connectives*. For the following definition of the set of *bidirectional* categories, some set of basic categories, BASCAT must be assumed. Then, where x and y are metavariables over $\text{CAT}^{(/,\backslash)}$:

Definition 18 (The categorial language $\text{CAT}^{(/,\backslash)}$)

(i) $\text{BASCAT} \subset \text{CAT}^{(/,\backslash)}$

(ii) if x and y are $\in \text{CAT}^{(/,\backslash)}$, then x/y and $x\backslash y$ are $\in \text{CAT}^{(/,\backslash)}$.

Thus categorial grammars, assume an *infinite* number of different syntactic kinds and assume *relationships* between the names of these kinds. This might be contrasted with what would be usual for a simple [1] phrase-structure grammar, where at most *finitely* many different syntactic kinds are assumed to exist and *no* relationships hold between the names of the kinds.

The set of categories, $\text{CAT}^{(/,\backslash)}$, just defined is in fact just one among a family of others that we might have given, and represents a certain choice as to what to count as the categorial connectives. One might instead use only one of the connectives, / or \, defining thereby $\text{CAT}^{/}$ or $\text{CAT}^{\backslash}$. $\text{CAT}^{/}$ is the oldest set of categories having been proposed by Ajdukiewicz in 1935. One aspect of the development of categorial grammars since then has been the extension of the set of categories. $\text{CAT}^{(/,\backslash)}$ was first proposed in 57 by Bar-Hillel. By 58, in a somewhat independent development, Lambek had suggested the set $\text{CAT}^{(/,\backslash,\cdot)}$, and in 61, $\text{CAT}^{(/,\backslash,\cdot,\wedge)}$. Recent years have seen something of an acceleration in this process, with a number of further binary and unary connectives being proposed in response to various kinds of syntactic phenomena. In Morril, Leslie, Hepple and Barry (1990), for example, one finds $\text{CAT}^{(/,\backslash,\cdot,\wedge,\vee,\Box,\Diamond,\lhd,\rhd,\ldots)}$ In time, we also propose a set of categories that is an extension of $\text{CAT}^{(/,\backslash)}$, namely $\text{CAT}^{(/,\backslash,\forall)}$, which adds variables and quantifiers. However, for the purposes of this chapter, the set of categories shall be $\text{CAT}^{(/,\backslash)}$.

A point of terminology regarding the bidirectional set of categories. We shall say y is the *argument* and x is the *value* in both x/y and $x\backslash y$. [2]

The categorisation relation

In contrast to UG, the inhabitants of categories are not expressions of a DISAMBIGUATED LANGUAGE but are simply *strings*. Akin to the specification of the *basic phrase sets* is the specification of a *categorial lexicon*. This lexicon constitutes the definition of the categorisation relation as it concerns words.

This relation is extended by the set of syntactic rules, where a syntactic rule in categorial grammar is of the form (where x_i and $y \in \text{CAT}^{(/,\backslash)}$) :

[1] By simple we are assuming no attachment to X-bar theory.

[2] A rival convention governing the use of this notation,(one used by Lambek), says that whilst y and x are argument and value in x/y, they are value and argument in $x\backslash y$.

$x_1, \ldots, x_n \Rightarrow y$

Note that this differs from the UG definition of a syntactic rule by lacking a syntactic operation. UG syntactic rules were read as stating categorisation facts concerning non-basic expressions of a DISAMBIGUATED LANGUAGE. The categorial syntactic rule above is read as simply stating a categorisation facts concerning *strings*:

if α_1 *is of category* x_1*, and* α_2 *is of category* x_2*, ..., and* α_n *is of category* x_n *then the* concatenation *of* $\alpha_1, \ldots, \alpha_n$ *is of category* y*.*

This reading of a syntactic rule as making a specifically *concatenative* commitment is definitive of much recent work in recent categorial grammar work (see for example, Moortgat 88, Steedman 85, Szabolsci 87), though there are exceptions (Bach 79, 80).

The AB grammarian is free to choose the set of basic categories, BASCAT, but once that set is chosen, the set of syntactic rules is *fixed*. The same goes for the L grammarian, for whom a set of syntactic rules is fixed once BASCAT is chosen, though it is a different set from that of the AB grammarian. In both cases the set of syntactic rules is *infinite* but specified in a finite way: *rule-schemata* for ABG and *induction* for LG. This implies a certain closure property of the rule set: if $x_1 \ldots, x_n \Rightarrow y$ is a syntactic rule then so also is $x'_1, \ldots, x'_n \Rightarrow y'$, where the primed categories come from the unprimed by replacing the basic categories with some other set of categories. In other words, rule-hood is contingent not so much upon the particular categories present but on the *pattern of connectives*. As an aside it might be noted that on this count the grammar part of Montague 73 is not a categorial grammar, though often described as such. For example, the category part of the 'Quantifying-in' rule is $t/IV, t \Rightarrow t$, but there is no rule $IV/t, IV \Rightarrow IV$: rulehood is contingent not simply on the pattern of connectives.

The next section concerns the ABG framework: its particular rule-set and fashion of defining the meaning relation. For subsequent chapters it is the LG framework that is important but the ABG framework is first described because it is widely known yet significantly different. To avoid confusions then, we have presented the ABG framework the better to be able to distinguish the LG framework from it.

2 Ajdukiewicz/Bar-Hillel categorial grammar

2.1 Categorising

We have already indicated how a categorial syntactic rule is to be read as specifying a categorisation fact about strings. It just remains to say what the rules are. The AB rule set is (based on a set of basic categories BASCAT) :

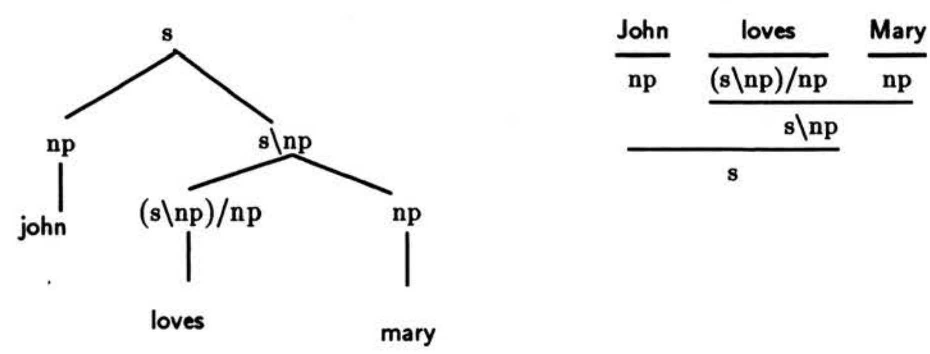
$\{x/y, y \Rightarrow x : x, y \in \text{CAT}^{(/,\backslash)}\} \cup \{y, x\backslash y \Rightarrow x : x, y \in \text{CAT}^{(/,\backslash)}\}$

Thus the rule-set is defined by *rule-schemata*. These have the respective names: Forward Function Application (>) and Backwards Function Application (<). To illustrate the definition of a categorisation relation by an ABG we will now consider the categorisation relation defined by the conjunction of the above rule set with the lexicon (supposing BASCAT= $\{s, np\}$):

(1) John :: np
loves :: (s\np)/np
Mary :: np

One can infer from this that s is a categorisation of **John loves Mary**. It is the presence in the AB rule set of the two rules: $(s\backslash np)/np, np \Rightarrow s\backslash np$ and $np, s\backslash np \Rightarrow s$ that entails this categorisation and in the leftmost of the illustrations below we have used a conventional phrase-structure tree to depict graphically the inferences made in coming to the conclusion that s is a categorisation of **John loves Mary**. On the one hand such a tree simply presents categorisation facts concerning **John loves Mary** and substrings thereof: the tree has the property that for any node x, the lexical material dominated by x is of category x. On the other hand, such a tree presents a record of inferences that have to be made in order to conclude that **John loves Mary** has category s. It is more common-place in the categorial literature to present such trees upside down, as illustrated in the rightmost picture.[3]

(2)



2.2 Assigning a meaning

We will assume that the destination for meaning assignment will be as it is for any THEORY OF REFERENCE, namely FREGEAN semantic algebras. A THEORY OF REFERENCE specifies *indirectly* a meaning relation between strings and objects in a FREGEAN semantic algebra by defining two things. First a disambiguation relation between the set of strings and a DISAMBIGUATED LANGUAGE. Second a FREGEAN INTERPRETATION, $\langle \mathcal{B}, \mathcal{G}_\gamma, f \rangle$. The meaning relation is defined *directly* in the ABG framework. The relation is defined in the case of words by something akin to an INTERPRETATION, which we shall call an AB-interpretation. This will be $\langle \mathcal{B}, \mathcal{G}_\gamma, f^{AB} \rangle$, where f^{AB} is a function on *categorised lexical strings*. This is in contrast to f, which was a

[3] originated by Mark Steedman.

function on basic expressions of a DISAMBIGUATED LANGUAGE. As was the case for FREGEAN INTERPRETATIONS, an AB-interpretation will be associated with a category-to-type map, which f^{AB} must conform to.

All AB-interpretations which are associated with the same category-to-type map in fact involve same semantic algebra.[4] This is in contrast with the situation for INTERPRETATIONS, where the design of the semantic algebra is in the hands of the semantic theorist. The algebras associated with the following two kinds of category-to-type maps will be described below:

Definition 19 (Extensional and intensional category-to-type maps)
ν is an extensional category-to-type map if $\nu(x/y) = \nu(x\backslash y) = (\nu(y) \rightarrow \nu(x))$
ν is an intensional category-to-type map if $\nu(x/y) = \nu(x\backslash y) = ((s \rightarrow \nu(y)) \rightarrow \nu(x))$

Superscripting with 'e' will indicated that category-to-type map is extensional, and superscripting with 'i' will indicate that it is intensional. The operations of the semantic algebra that features in every AB-interpretation associated with a category-to-type map ν^e are given by applying a certain *rule-to-operation map*, h^e_{AB}, to the AB rule set, and therefore the algebra may be described as $\langle \mathcal{B}, \{h^e_{AB}(s) : s \text{ is an AB rule}\}\rangle$. Similarly each ν^i is associated with another rule-to-operation map, h^i_{AB}, and therefore with algebras, $\langle \mathcal{B}, \{h^i_{AB}(s) : s \text{ is an AB rule}\}\rangle$. The rule-to-operation maps are defined below:

Definition 20 (The extensional AB Rule-to-Operation map, h^e_{AB})
$h^e_{AB}(x/y, y \Rightarrow x) = m_1^{\nu^e(x/y)} \mapsto m_2^{\nu^e(y)} \mapsto (w,j) \mapsto m_1(w,j)(m_2(w,j))$
$h^e_{AB}(y, x\backslash y \Rightarrow x) = m_1^{\nu^e(y)} \mapsto m_2^{\nu^e(x\backslash y)} \mapsto (w,j) \mapsto m_2(w,j)(m_1(w,j))$

Definition 21 (The intensional AB Rule-to-Operation map, h^i_{AB})
$h^i_{AB}(x/y, y \Rightarrow x) = m_1^{\nu^i(x/y)} \mapsto m_2^{\nu^i(y)} \mapsto (w,j) \mapsto m_1(w,j)(w' \mapsto m_2(w',j))$
$h^i_{AB}(y, x\backslash y \Rightarrow x) = m_1^{\nu^i(y)} \mapsto m_2^{\nu^i(x\backslash y)} \mapsto (w,j) \mapsto m_2(w,j)(w' \mapsto m_1(w',j))$

An AB-interpretation concerns only words, and some means must be given for extending the meaning relation to compound strings. Recall that given a INTERPRETATION, $\langle \mathcal{B}, \mathcal{G}_\gamma, f\rangle$, a way was defined in Chapter 2 of extending f to a function $[\![]\!]$ embracing non-basic expressions of the disambiguated language: $[\![]\!]$ was the homomorphic extension of f. The f^{AB} of an AB-interpretation is not turned into a meaning relation by extension in this way. It cannot be done in this way, because the strings do not have any algebraic structure. Moreover, f^{AB} must be extended to a meaning *relation*. The extension of f^{AB} will still be referred to as $[\![]\!]$, but instead of writing $[\![[\alpha]_z]\!] = m$ we will write $[\alpha]_z [\![]\!] m$, to emphasise the relational nature. The definition of how an AB-interpretation leads to a meaning relation given below covers the cases of extensional and intensional category-to-type maps, according to whether δ is read as e or i.

[4] More exactly, one should speak of a family of algebras, indexed by choices of $\mathcal{E}$, $\mathcal{I}$, $\mathcal{J}$.

Definition 22 (AB meaning relation) *Relative to an AB-interpretation,* $\langle \mathcal{B}, \{h^{\delta}_{AB}(s) : s \text{ is an } AB \text{ rule}\}, f^{AB}\rangle$ *associated with category to type map* ν^{δ}*, and relative to a categorial lexicon, the AB meaning relation,* $\blacksquare$ *is defined:*

$[\alpha]_z \blacksquare f^{AB}([\alpha]_z)$ *if* α *is a basic expression of category* z*.*

$[\alpha\beta]_z \blacksquare h^{\delta}_{AB}(x, y \Rightarrow z)(m_2, m_3)$*, if there exist meanings* m_2, m_3 *and categories* x, y *such that* α *may be categorised as* x*,* β *may be categorised as* y*,* $[\alpha]_x \blacksquare m_2$*,* $[\beta]_y \blacksquare m_3$*, and* $x, y \Rightarrow z$ *is present in the rule set.*

To illustrate the definition, recall the lexicon given in (1) and suppose an AB-interpretation, $\langle\langle \mathcal{B}, \{h^{e}_{AB}(s) : s \text{ is an AB rule}\}\rangle, f^{AB}\rangle$. The definitions given so far entail the following concerning loves, Mary and loves Mary:

$[\text{loves}]_{(s\backslash np)/np} \blacksquare f^{AB}([\text{loves}]_{(s\backslash np)/np})$

$[\text{Mary}]_{np} \blacksquare f^{AB}([\text{Mary}]_{np})$

$\therefore [\text{loves Mary}]_{s\backslash np} \blacksquare h^{e}_{AB}((s\backslash np)/np, np \Rightarrow s\backslash np)(f^{AB}([\text{loves}]_{(s\backslash np)/np}), (f^{AB}([\text{Mary}]_{np}))$

$\therefore [\text{loves Mary}]_{s\backslash np} \blacksquare (w, j) \mapsto f^{AB}([\text{loves}]_{(s\backslash np)/np})(w, j)(f^{AB}([\text{Mary}]_{np})(w, j))$

Different though it is to the UG definition of meaning assignment, the definition of the AB meaning relation shares with the UG definition the feature that it accords quite well with the intuitive notion of compositionality: 'the meaning of a whole is a function of the meaning of the parts and its mode of composition'. Each possible meaning of a whole is a function of some possible meanings of its parts and the determinant of which function to apply to the parts is the *categorisation* tree for the expression. AB grammars are far from alone in adopting such a definition of the meaning relation. Meaning relations are so often defined in the above fashion, particularly by the invocation of a rule-to-operation map [5] , that the format is sometimes regarded as definitional of 'compositional', as opposed to Montague's definition. This theme of notions of compositionality will be picked up again when Lambek categorial grammars are considered in due course.

By an AB-theory we will mean the specification of a categorial lexicon and a set of possible AB-models, where these are pairings of AB-interpretations and *world:context of use* pairs.

Without going into details, it may be clear that natural facts about well-formedness or meaning may outrun the resources of AB-theories, and there have been extrapolations of the AB framework to respond to this, by the addition of new rule-schemata, and the definition of cor-

[5]The format is often referred to as the 'Rule-to-Rule' format. The reason for the second occurrence of the word 'Rule' in place of the word 'Operation' is that by and large semantic algebras are not explicitly used, and instead of semantic operations there are rules.

respondingly more inclusive rule-to-operation maps. For example:

(3) Rule $x \Rightarrow y/(y\backslash x)$

Value of h^e $m_1 \mapsto (w,j) \mapsto d_2 \mapsto d_2(m_1(w,j))$

Value of h^i $m_1 \mapsto (w,j) \mapsto d_2 \mapsto d_2(w)(w' \mapsto m_1(w',j))$

Rule $x/y \Rightarrow (x/z)/(y/z)$

Value of h^e $m_1 \mapsto (w,j) \mapsto d_2 \mapsto d_3 \mapsto m_1(w,j)(d_2(d_3))$

Value of h^i $m_1 \mapsto (w,j) \mapsto d_2 \mapsto d_3 \mapsto m_1(w,j)(w' \mapsto d_2(w')(d_3))$

Rule $x/y,\ y/z \Rightarrow x/z$

Value of h^e $m_1 \mapsto m_2 \mapsto (w,j) \mapsto d_3 \mapsto m_1(w,j)(m_2(w,j)(d_3))$

Value of h^i $m_1 \mapsto m_2 \mapsto (w,j) \mapsto d_3 \mapsto m_1(w,j)(w' \mapsto m_2(w',j)(d_3))$

The rule-schemata above have the respective names: Type Raising, Geach and Composition, and papers arguing for the use of some or all of these are: Geach (1972), Ades and Steedman (1982) and Dowty (1988). These proposals have a kinship with the basic ABG framework via the use of schemata to define the rule-set and the use of a rule-to-operation maps to define the meaning relation. No more will be said of these extensions of AB grammar because our goal is that of explaining Lambek categorial grammar and that destination will not be arrived at by further extrapolation along the present dimensions. For the LG framework the rule set is *not* defined by schemata, but instead by a *sequent calculus*, and the definition of the meaning relation does *not* invoke a rule-to-operation map but invokes instead a *proof*-to-operation map.

3 Lambek Categorial Grammar

3.1 Categorising

Lambek categorial grammar is based on an analogy between syntactic rules and *propositional sequents*. These were introduced in Gentzen 34 and are objects $U \Rightarrow a$, where U, the *antecedent* is a possibly empty sequence of *formulae* and a, the *succedent* is a single formula.[6]

What a propositional sequent, $U \Rightarrow a$, means, is that a follows from the formulae in U. One of the two proof systems introduced by Gentzen 34 is *sequent calculus*, which is a proof system the principal objects of which are such sequents, inductively defining the set of sequents which make valid statements about entailment. Lambek 58 introduced a sequent calculus, $\mathrm{L}^{(/,\backslash)}$, for *categorial* sequents, $U \Rightarrow x$, where U is a sequence of *categories* and x is a single category.

[6] One can also have multiply conclusioned sequents and it is in terms of these that the most elegantly symmetrical formulations of propositional logic take place.

Unlike the propositional case U is not allowed to be empty. $L^{(/,\backslash)}$, essentially gives an inductive definition of '$U \Rightarrow x$ is a syntactic-rule'. The axiom sequents, of form $x \Rightarrow x$, are the base-cases of the induction, whilst the inference rules are the inductive cases. Here is the calculus[7]:

Definition 23 (The Lambek Calculus, $L^{(/,\backslash)}$)

(Ax) $\quad x \Rightarrow x$

(Cut) $$\frac{U, x, V \Rightarrow w \quad T \Rightarrow x}{U, T, V \Rightarrow w}\ Cut$$

(/L) $$\frac{U, y, V \Rightarrow w \quad T \Rightarrow x}{U, y/x, T, V \Rightarrow w}\ /L$$

(/R) $$\frac{T, x \Rightarrow y}{T \Rightarrow y/x}\ /R$$

(\L) $$\frac{T \Rightarrow x \quad U, y, V \Rightarrow w}{U, T, y\backslash x, V \Rightarrow w}\ \backslash L$$

(\R) $$\frac{x, T \Rightarrow y}{T \Rightarrow y\backslash x}\ \backslash R$$

The notation conventions are that U, T, V stand for sequences of categories, with U and V possibly empty. w, x, y stand for single categories. We will define here a piece of terminology concerning the *two* premise rules, though it will not be used for some time: the $T \Rightarrow x$ premise will be called the *minor* premise, and the other premise the *major* premise.

Although the direction of implication is down the page, one invariably unfolds a proof from conclusion towards premises. Thus one speaks of 'applying' a rule to a sequent where this means finding the premises from which the sequent could have been inferred by the rule. With the exception of the Cut rule, one is only in a position to apply an inductive case (= inference rule) concerning the potential rule-hood of a sequent, $T \Rightarrow x$, if there is a non-basic category in the antecedents or succedent. If there is a non-basic category in the antecedent T then one of the rules (/L) or (\L) may be applied. These are known as 'left' rules, because there must be a connective on the left-hand side of a sequent for them to apply. If there is a non-basic category in the succedent x, one of the rules (/R) or (\R) applies. These are known as 'right' rules, because there must be a connective on the right-hand side of a sequent for them to apply. One is always in a position to use the Cut rule.

In this way, given a sequent r whose rule-hood is unknown, a tree of sequents can be built up above it, with r the *root* of the tree. At each stage of construction of such a tree we will refer to the *uppermost* sequent on any branch as a *leaf*. If all the leaves of such a tree are axioms, then the tree is a *proof* and the root, r, is said to be proved.

Some examples of how this inductive definition of rule-hood works follow:

[7] In fact this lacks the left and right rules for product which were given in the original calculus. We have not included product as one of the categorial connectives.

(4) Function Application:
$$\frac{\mathrm{np} \Rightarrow \mathrm{np} \quad \mathrm{s} \Rightarrow \mathrm{s}}{\mathrm{np}, \mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}}\backslash L$$

Type Raise:
$$\frac{\dfrac{\mathrm{np} \Rightarrow \mathrm{np} \quad \mathrm{s} \Rightarrow \mathrm{s}}{\mathrm{np}, \mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}}\backslash L}{\mathrm{np} \Rightarrow \mathrm{s}/(\mathrm{s}\backslash\mathrm{np})}/R$$

(5) Composition:
$$\frac{\dfrac{\dfrac{\mathrm{s} \Rightarrow \mathrm{s} \quad \mathrm{cn} \Rightarrow \mathrm{cn}}{\mathrm{s}/\mathrm{cn}, \mathrm{cn} \Rightarrow \mathrm{s}}/L \quad \mathrm{np} \Rightarrow \mathrm{np}}{\mathrm{s}/\mathrm{cn}, \mathrm{cn}/\mathrm{np}, \mathrm{np} \Rightarrow \mathrm{s}}/L}{\mathrm{s}/\mathrm{cn}, \mathrm{cn}/\mathrm{np} \Rightarrow \mathrm{s}/\mathrm{np}}/R$$

Geach:
$$\frac{\dfrac{\dfrac{\dfrac{\mathrm{s} \Rightarrow \mathrm{s} \quad \mathrm{cn} \Rightarrow \mathrm{cn}}{\mathrm{s}/\mathrm{cn}, \mathrm{cn} \Rightarrow \mathrm{s}}/L \quad \mathrm{np} \Rightarrow \mathrm{np}}{\mathrm{s}/\mathrm{cn}, \mathrm{cn}/\mathrm{np}, \mathrm{np} \Rightarrow \mathrm{s}}/L}{\mathrm{s}/\mathrm{cn}, \mathrm{cn}/\mathrm{np} \Rightarrow \mathrm{s}/\mathrm{np}}/R}{\mathrm{s}/\mathrm{cn} \Rightarrow (\mathrm{s}/\mathrm{np})/(\mathrm{cn}/\mathrm{np})}/R$$

The following observations have a bearing on the process of search (proved by Lambek in his 1958 paper. See also Moortgat 88 for exposition):

- The derivability of a sequent in $L^{(/,\backslash)}$ − Cut is decidable.
- $L^{(/,\backslash)}$ − Cut and $L^{(/,\backslash)}$ allow the derivation of exactly the same set of sequents, that is, Cut may be eliminated.

Together they entail that the categorisation question for an L-grammar is decidable. As the connection of the above facts with the decidability of categorisation is not immediately obvious, we will pause awhile to see just why this is.

Take for example the question of whether loves John Mary may be categorised as s, on the lexicon given in (1).

The first strategy might be called the *flat* strategy. One looks for a 'special purpose' rule for the expression, relating the categories of the words in the expression to s. This may be called a *categorising sequent*. In the present case it must be checked whether (s\np)/np, np, np ⇒ s is derivable in $L^{(/,\backslash)}$. Because of Cut elimination, it is sufficient to check for derivability in $L^{(/,\backslash)}$ − Cut. The decidability property of $L^{(/,\backslash)}$ − Cut guarantees an effective procedure for answering this question and using it we find the answer is 'no'. Hence the categorising sequent is not derivable.

That would be that as far as categorisation went were it not for the existence of a *non-flat* strategy, which does not rely on the derivability of the categorising sequent. For example, because of the concatenative interpretation of syntactic rules, loves John Mary will have category s if there are categories x, y such that

(i) loves John has category x, Mary has category y

(ii) the sequent $x, y \Rightarrow$ s is derivable in $L^{(/,\backslash)}$.

The inquiry gone though in the *flat* strategy does not *prima-facie* settle the answers to the above questions. The general *non-flat* strategy is:[8]

[8] The flat strategy is the degenerate case where U is segmented into itself only and the succedent category is

(1) make the 'current' sequence of categories the sequence of lexical categories.

(2) segment the current sequence of categories U into adjacent subsequences $U_1, \ldots, U_n$

(3) find n succedent categories $y_1, \ldots, y_n$ such that each $U_i \Rightarrow y_i$ is derivable in $\text{L}^{(/,\backslash)}$, then make the current sequence $y_1, \ldots, y_n$. If the current sequence is s, stop and if not return to (2).

Now as long as at least two circuits of this loop are made, the ***non-flat*** strategy will discover ***grounds*** for the categorisation of an expression that cannot be found by the ***flat*** strategy and so, ***prima facie*** at least, the failure of the ***flat*** strategy to categorise the expression as s does not prove that it cannot be so categorised.

However, there is a problem in adopting the more comprehensive ***non-flat*** strategy: it is unterminating. The problem is at step (2), finding a mother category for a given sequence of daughters. This will be problematic because if for some x one can derive that $T \Rightarrow x$ then there are infinitely many other x' such that one can derive $T \Rightarrow x'$.

Given these observations it is puzzling how it can be that the categorisation question for Lambek categorial grammar is decidable. The solution to the puzzle lies in in the fact that each of the following sentences implies its successor:

(1) The expression is successfully categorised by the ***non-flat*** strategy.

(2) There is a Cut based proof of the categorising sequent.

(3) There is a Cut free proof of the categorising sequent.

(4) The expression is successfully categorised by the ***flat*** strategy.

It follows that the ***flat*** strategy is a complete categorisation procedure: failure to categorise by the ***flat*** strategy implies failure to categorise by the ***non-flat*** strategy. The implication from (1) to (2) requires inspection of the related structure of analyses using the ***non-flat*** strategy and Cut based sequent proofs. There is an illustration of this below.

Suppose the following were a categorisation tree for loves John Mary produced by pursuing the ***non-flat*** strategy.

$$\cfrac{\cfrac{\cfrac{\text{loves}}{(\text{s}\backslash\text{np})/\text{np}} \quad \cfrac{\text{John}}{\text{np}}}{\text{x}} \qquad \cfrac{\cfrac{\text{Mary}}{\text{np}}}{\text{y}}}{\text{s}}$$

This presupposes that there is a proof P_1 of $(\text{s}\backslash\text{np})/\text{np}, \text{np} \Rightarrow x$, a proof P_2 of $\text{np} \Rightarrow y$ and a proof P_3 of $x, y \Rightarrow \text{s}$. One can therefore produce the following *Cut*-based proof of the categorising sequent, $(\text{s}\backslash\text{np})/\text{np}, \text{np}, \text{np} \Rightarrow \text{s}$:

---

chosen to be s,

$$\dfrac{\dfrac{P_1}{(\mathrm{s}\backslash \mathrm{np})/\mathrm{np}, \mathrm{np} \Rightarrow x} \qquad \dfrac{\dfrac{P_2}{\mathrm{np} \Rightarrow y} \quad \dfrac{P_3}{x, y \Rightarrow \mathrm{s}}}{\mathrm{x}, \mathrm{np} \Rightarrow \mathrm{s}}\,\mathrm{Cut}}{(\mathrm{s}\backslash \mathrm{np})/\mathrm{np}, \mathrm{np}, \mathrm{np} \Rightarrow \mathrm{s}}\,\mathrm{Cut}$$

The implication from (2) to (3) is the Cut elimination theorem and the implication from (3) to (4) is given simply by the definition of the *flat* strategy.

String Semantics

For a propositional sequent one would expect to use a semantics for the propositional language to relate the sequent arrow to a semantically defined entailment relation. With respect to the semantically defined entailment relation, one could ask whether a given calculus was sound and complete. One of the most remarkable features of the categorial framework is that there is a semantics for the categorial language, $\text{CAT}^{(/,\backslash)}$, which allows the definition of a semantic entailment relationship. One can then compare this semantic entailment relationship with syntactic definitions of a set of sequents, definitions such as the AB rule-schemata or the Lambek calculus. The semantics for $\text{CAT}^{(/,\backslash)}$ will be presented below and the important result of Buskowski 86 concerning the Lambek calculus.

In presenting this so-called 'string semantics', we will revert to the usual conventions observed in giving an interpretation to a formal language, and will not reformulate the language of categories, $\text{CAT}^{(/,\backslash)}$, as an instance of a DISAMBIGUATED LANGUAGE.

The possible interpretations of the language of categories, $\text{CAT}^{(/,\backslash)}$, are based on algebras $\langle A, \cdot \rangle$, where $\cdot$ is a binary associative operation - the most natural example is choosing A to be be strings and $\cdot$ to be *concatenation*. The interpretation function, I, may assign to the members of BASCAT *any* subset of A, and this function is extended to an interpretation function for the whole language $\text{CAT}^{(/,\backslash)}$ thus:

Definition 24 (String semantics for $\text{CAT}^{(/,\backslash)}$) *Relative to an interpretation function, I, which assigns to the members of* BASCAT *a subset of A,*

$[\![x]\!] = I(x)$, *if* $x \in \text{BASCAT}$

$[\![x/y]\!] = \{a : \forall b \in [\![y]\!], a \cdot b \in [\![x]\!]\}$ *(equivalently* $\{a : a \cdot [\![y]\!] \subseteq [\![x]\!]\}$*)*

$[\![x\backslash y]\!] = \{a : \forall b \in [\![y]\!], b \cdot a \in [\![x]\!]\}$

With respect to this notion of interpretation, one can define a semantic notion of categorial derivability:

Definition 25 (String semantic entailment for $\text{CAT}^{(/,\backslash)}$)

$x_1, \ldots, x_n$ *string semantically entails* y *iff for all interpretations,* $[\![x_1]\!] \cdot \ldots \cdot [\![x_n]\!] \subseteq [\![y]\!]$.

In the usual way, one sometimes says '$x_1, \ldots, x_n \Rightarrow y$ is string-semantically valid' instead of '$x_1, \ldots, x_n$ string-semantically entails y'.

Every rule in the AB rule set is string-semantically valid. However, many string-semantically valid sequents are not in the AB rule set, for example all instances of the schemata Type-raising, Geach and Composition, given earlier.

Every $\text{L}^{(/,\backslash)}$ derivable sequent is string-semantically valid, which is to say that $\text{L}^{(/,\backslash)}$ is *sound*. This follows from the fact that axiom sequents are string-semantically valid, and that the rules of $\text{L}^{(/,\backslash)}$ preserve validity.

Even more significantly, it has been shown by Buskowski that the other direction of implication also holds: every string-semantically valid sequent is derivable in $\text{L}^{(/,\backslash)}$, which is to say that $\text{L}^{(/,\backslash)}$ is *complete.*

Therefore, if one adheres to the string-semantic interpretation of the categorial language $\text{CAT}^{(/,\backslash)}$, $\text{L}^{(/,\backslash)}$ represents a ceiling to the sequents one can help oneself to.[9] The significance I attribute to this is that to postulate additional rules or axioms would be to postulate entailments that *do not follow from the meaning of the categorial language,* $\text{CAT}^{(/,\backslash)}$. However, for balance it must be said that amongst the practitioners of categorial grammar, the relevance of the string semantics is not settled. Further discussion of this will be postponed until section 3.2.7, by which time the question of how meanings are assigned in the Lambek categorial grammar framework will have been discussed.

3.2 Assigning a meaning

Analogously to the AB grammars, we will suppose that semantic proposals will be made in the form of the specification of a class of $\text{L}^{(/,\backslash)}$-interpretation [10] , where this is $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f^L \rangle$, with f^L a function on categorised strings. $\text{L}^{(/,\backslash)}$-interpretations will be associated with a category-to-type map, and as with the AB-interpretations the only cases that will be considered are extensional and intensional category-to-type maps. All $\text{L}^{(/,\backslash)}$-interpretations associated with a particular ν^e involve the same semantic algebra and all $\text{L}^{(/,\backslash)}$-interpretations associated with a particular ν^i involve the same semantic algebra. Where the algebras associated with particular category-to-type maps were defined in the case of AB grammars with the assistance of extensional and intensional *rule*-to-operation maps, h^e_{AB} and h^i_{AB}, the algebras for $\text{L}^{(/,\backslash)}$ grammars will be defined by extensional and intensional *proof*-to-operation maps, H^e_L and H^i_L. The definition of the $\text{L}^{(/,\backslash)}$-meaning relation given below assumes that these maps have been defined.

[9] It should be noted that this statement is relative to a particular categorial language. For a more inclusive language there will additional string-semantically valid sequents.

[10] These are not to be confused with *string-semantic* interpretations of the language of categories itself, $\text{CAT}^{(/,\backslash)}$

Definition 26 ($L^{(/,\backslash)}$ meaning relation)

Relative to an $L^{(/,\backslash)}$-interpretation, $\langle \mathcal{B}, \{H^{\delta}_{L}(P) : P \text{ is a proof of } L^{(/,\backslash)}\}, f^{L}\rangle$, associated with the category-to-type map ν^{δ}, and relative to a categorial lexicon, the $L^{(/,\backslash)}$-meaning relation, $[\!]$ is defined:

$[\alpha]_z [\!] (f^L([\alpha]_z))$ *if α is basic*

$[\alpha\beta]_z [\!] H^{\delta}_{L}(P)(m_2, m_3)$ *if there exist meanings m_2, m_3, a proof P and categories x, y such that α may be categorised as x, β may be categorised as y, $[\alpha]_x [\!] m_2$, $[\beta]_y [\!] m_3$, and P is a proof of $x, y \Rightarrow z$.*

As with the AB meaning relation, one can ask whether this accords with the intuitive notion of compositionality. Each possible meaning of a whole is a function of possible meanings of its parts, but where is the 'mode of composition' that intuitively one expects to be the determinant of which function is applied to the parts ? Unlike the AB meaning relation, this determinant is *not* the categorisation tree. The determinant is in fact the *proof* of the sequent used to categorise the expression. To pick up the thread again of notions of compositionality, this means that the definition of the Lambek meaning relation does *not* conform to the 'Rule-to-Rule' notion of compositionality. In fact it appears that the Lambek meaning relation enjoys its own unique brand of compositionality. In section 4 this compositionality will be considered further, and it will be claimed that the kind of compositionality enjoyed by the Lambek calculus can be easily fitted to Montague's definition. But before looking on Lambek categorial grammar from the outside we had will continue the description from the inside.

We take up the deferred matter of the proof-to-operation maps, H^{e}_{L} and H^{i}_{L}. First we will encode operations by terms of a typed λ-calculus language. The task is then transformed into one of defining *proof-to-term* maps [11]. So, delaying once more, before explaining how to define a proof-to-term map, the term language will be defined and it will be explained how a term defines an operation.

To define the set of *typed terms*, $\mathcal{L}^{\lambda}$, a set of typed variables, VAR^{λ}, is assumed.

Definition 27 (Typed Term of $\mathcal{L}^{\lambda}$)

a. if $\Phi^a \in \text{VAR}^{\lambda}$, Φ^a is $\in \mathcal{L}^{\lambda}$

b. if $\Phi^{(a\rightarrow b)}$ and Ψ^a are $\in \mathcal{L}^{\lambda}$, then $(\Phi^{(a\rightarrow b)}\Psi^a)^b \in \mathcal{L}^{\lambda}$

c. if $\Phi^a \in \text{VAR}^{\lambda}$ and $\Psi^b \in \mathcal{L}^{\lambda}$ then $(\lambda\Phi^a\Psi^b)^{(a\rightarrow b)} \in \mathcal{L}^{\lambda}$

In specifying the language $\mathcal{L}^{\lambda}$ we have not adhered to the algebraic UG format, but have reverted to the usual practice for specifying a formal language. For a term of $\mathcal{L}^{\lambda}$ to define an operation, the language must have a semantics. As with the syntax, we will revert to usual practices in specifying this semantics.

[11] The Rule-to-Operation maps for AB grammars could have been specified by going via a Rule-to-Term map also.

We have a choice whether to associate with terms meanings which are functions from a world-assignment pair to a denotation of type a, or meanings which are a function from an assignment to a denotation of type $(s \rightarrow a)$. For present purposes it seems the latter choice is more convenient. It must be borne in mind, however, that it is the intention to use the terms to define operations on *Fregean meanings*, which themselves are functions on *world-context of use* pairs.

The assignments of an interpretation associated with $\mathcal{E}$ and $\mathcal{I}$ are all the functions g with domain VAR^λ such that $g(\Phi^a) \in D_a$. Let f^λ be the interpretation function, assigning to variables of type a a function from assignments to denotations of type a such that: $f^\lambda(\Phi^a)(g) = g(\Phi^a)$. The extension of f^λ to embrace complex expressions is defined:

Definition 28 (Meaning of Typed Terms of $\mathcal{L}^\lambda$) *Relative to an interpretation f^λ associated with $\mathcal{E}$ and $\mathcal{I}$, $[\![]\!]^\lambda$ is defined:*
$[\![\Phi^a]\!]^\lambda(g) = f^\lambda(\Phi^a)(g)$ *if* $\Phi^a \in \text{VAR}^\lambda$
$[\![\lambda\Phi^a.\Psi^b]\!]^\lambda(g)$ *is that function* $d_1 \in D_{(a\rightarrow b)}$ *such that for any member* $d_2 \in D_a$, $d_1(d_2) = [\![\Psi^b]\!]^\lambda(g^{d_2}_{\Phi^a})$
$[\![\Phi^{(a\rightarrow b)}(\Psi^b)]\!]^\lambda(g) = [\![\Phi^{(a\rightarrow b)}]\!]^\lambda(g)([\![\Psi^b]\!]^\lambda(g))$

Now to consider how a typed term may define an operation of a *Fregean algebra*.[12] Suppose Φ^b is a typed term, the free variables of which are $x_1^{a_1}, \ldots, x_n^{a_n}$. First we substitute for free variables $x_j^{a_j}$, the *terms*, $y_j^{(s\rightarrow a_j)}(i)$, where the $y_j^{(s\rightarrow a_j)}$ are variables. Then prefix λi to the front and call the result Φ^*.

Definition 29 (Operation defined by Φ^*) *Let h be an arbitrary $\mathcal{L}^\lambda$ assignment, the operation defined by Φ^* is:* $m_1, \ldots, m_n \mapsto (w,j) \mapsto [\![\Phi^*]\!](h[w' \mapsto m_1(w,j)/y_1, \ldots, w' \mapsto m_n(w',j)/y_n])(w)$

Some example of operations defined by terms (some space saving abbreviations of types are used here, so for example $(s\rightarrow(e\rightarrow t))$ may be abbreviated to (s,e,t) or set):

(6) $\Phi = x_1^{et}(x_2^e)$
$\Phi^* = \lambda i[y_1^{(s,et)}i(y_2^{(s,e)}i)]$,
Operation: $m_1^{(e\rightarrow t)}, m_2^e \mapsto (w,j) \mapsto m_1(w,j)(m_2(w,j))$

(7) $\Phi = x_1^{(se,t)}(\lambda i.x_2^e)$
$\Phi^* = \lambda i[y_1^{(s,se,t)}i(\lambda i[y_2^{(s,e)}i])]$
Operation: $m_1^{(se,t)}, m_2^e \mapsto (w,j) \mapsto m_1(w,j)(w' \mapsto m_2(w',j))$

[12] or more exactly a family of operations across a family of algebras, indexed by choices of $\mathcal{E}$ and $\mathcal{I}$.

(8) $\Phi = \lambda u_1^{(e,t)}.u_1(x_1^e)$
$\Phi^* = \lambda i[\lambda u_1^{(e,t)}.u_1(y_1^{(s,e)}i)]$
Operation: $m_1^e \mapsto (w, j) \mapsto d_1^{(e,t)} \mapsto d_1(m_1(w, j))$

(9) $\Phi = \lambda u_1^{(s,se,t)}[u_1(i)(\lambda i[x_1^e])]$
$\Phi^* = \lambda i \lambda u_1^{(s,se,t)}[u_1(i)(\lambda i[y_1^{(s,e)}i])]$
Operation $= m_1^e \mapsto (w, j) \mapsto d_1^{(s,se,t)} \mapsto d_1(w)(w' \mapsto m_1(w', j))$

We now finally we come to the heart of the process by which meaning is assigned in a Lambek categorial grammar: the *proof-to-term* maps. These maps will share the names H_L^e and H_L^i with the proof-to-operation maps, because the proof-to-term maps are essentially all there is to the proof-to-operations maps.

First we will describe the method by which in practice one calculates the values of H_L^e and H_L^i on a given $\text{L}^{(/,\backslash)}$ proof. This involves the 'term-associated' calculi that have been proposed by Hendriks and Moortgat. Subsequently, we will describe what lies in the background of these practical definitions.

3.2.1 Working definitions of H_L^e and H_L^i

To calculate the term to be associated with a given proof according to H_L^e and H_L^i one uses 'term-associated' calculi. The term-associated calculus for H_L^e will be referred to as $\text{L}_e^{(/,\backslash)}$ and that for H_L^i will be referred to as $\text{L}_i^{(/,\backslash)}$, with $\text{L}_\delta^{(/,\backslash)}$ generic between the two. Both $\text{L}_e^{(/,\backslash)}$ and $\text{L}_i^{(/,\backslash)}$ are a complication of $\text{L}^{(/,\backslash)}$ and for them sequents are not built out of categories but out of *category:term* pairs. $\text{L}_e^{(/,\backslash)}$ and $\text{L}_i^{(/,\backslash)}$ follow below (see immediately after the definitions for the notation conventions):

Definition 30 (Extensionally Term Associated Lambek Calculus, $\text{L}_e^{(/,\backslash)}$)

(Ax) $x : \Delta \Rightarrow x : \Delta$

(Cut) $$\frac{U,\ x : \Delta,\ V \Rightarrow w \qquad T \Rightarrow x : \Delta}{U,\ T,\ V \Rightarrow w}\ Cut$$

(/L) $$\frac{U,\ y : \Delta\Gamma,\ V \Rightarrow w \qquad T \Rightarrow x : \Gamma}{U,\ y/x : \Delta,\ T,\ V \Rightarrow w}\ /L$$

(/R) $$\frac{T,\ x : \zeta \Rightarrow y : \Gamma}{T \Rightarrow y/x : \lambda\zeta\Gamma}\ /R$$

(\L) $$\frac{T \Rightarrow x : \Gamma \qquad U,\ y : \Delta\Gamma,\ V \Rightarrow w}{U,\ T,\ y\backslash x : \Delta,\ V \Rightarrow w}\ \backslash L$$

(\R) $$\frac{x : \zeta,\ T \Rightarrow y : \Gamma}{T \Rightarrow y\backslash x : \lambda\zeta\Gamma}\ \backslash R$$

Definition 31 (Intensionally Term Association Lambek Calculus, $L_i^{(/,\backslash)}$)

(Ax) $x : \Delta \Rightarrow x : \Delta$

(Cut) $$\frac{U,\ x : \Delta,\ V \Rightarrow w \quad T \Rightarrow x : \Delta}{U,\ T,\ V \Rightarrow w}\ Cut$$

(/L) $$\frac{U,\ y : \Delta(\lambda i.\Gamma),\ V \Rightarrow w \quad T \Rightarrow x : \Gamma}{U,\ y/x : \Delta,\ T,\ V \Rightarrow w}\ /L$$

(/R) $$\frac{T,\ x : \zeta(i) \Rightarrow y : \Gamma}{T \Rightarrow y/x : \lambda\zeta\Gamma}\ /R$$

(\L) $$\frac{T \Rightarrow x : \Gamma \quad U,\ y : \Delta(\lambda i.\Gamma),\ V \Rightarrow w}{U,\ T,\ y\backslash x : \Delta,\ V \Rightarrow w}\ \backslash L$$

(\R) $$\frac{x : \zeta(i),\ T \Rightarrow y : \Gamma}{T \Rightarrow y\backslash x : \lambda\zeta\Gamma}\ \backslash R$$

The notation conventions now differ from those of Definition 23: U, T, V abbreviate sequences of *category:term* pairs, U and V possibly empty. w is a single category:term pair. x and y are single categories. Δ, Γ and ζ are single λ-calculus terms, with ζ a variable.

The value of H_L^δ on some proof P of $x_1, \ldots, x_n \Rightarrow y$ is $\Phi^{\nu^\delta(y)}$ if (i) there is a proof P^δ in $L_\delta^{(/,\backslash)}$ of $x_1 : \zeta_1^{\nu^\delta(x_1)}, \ldots, x_n : \zeta_n^{\nu^\delta(x_n)} \Rightarrow y : \Phi^{\nu^\delta(y)}$ and (ii) P^δ is of the *same form* as P.

van Benthem (1986) proposes a term association for a variant of $L^{(/,\backslash)}$ which is unidirectional Lambek calculus and has a rule of permutation. Moortgat (1988) proposed the extensional term association for $L^{(/,\backslash)}$, that is the calculus $L_e^{(/,\backslash)}$, whilst the intensionally term associated calculus $L_i^{(/,\backslash)}$ was first proposed by Hendriks (1990).

To illustrate the definitions, we will consider the meaning [Mary walks]$_s$ would be entailed to stand in the $[\!]$ relation to by the lexicon given in (1) and an $L^{(/,\backslash)}$-interpretation, $\langle\langle \mathcal{B}, \{H_L^e(P) : P \text{ is an } L^{(/,\backslash)} \text{ proof}\}\rangle, f^L\rangle$, associated with the *extensional* category-to-type map that maps np to e and s to t. One meaning is obtained from a proof in $L^{(/,\backslash)}$ of the categorising sequent, np, s\np $\Rightarrow$ s:

(10) $$\frac{\text{np} \Rightarrow \text{np} \quad \text{s} \Rightarrow \text{s}}{\text{np}, \text{s}\backslash\text{np} \Rightarrow \text{s}}\backslash\text{L}$$

Then from the definition of the $L^{(/,\backslash)}$-meaning relations, assuming P_1 is the above proof, one has the following fact about $[\!]$:

$$[\text{Mary walks}]_s [\!] H_L^e(P_1)(f^L([\text{Mary}]_{np}, f^L([\text{walks}]_{s\backslash np}))$$

To determine $H_L^e(P_1)$ one must find Φ^t such that (i) the the sequent np : x_1^e, s\np : $x_2^{(e\rightarrow t)} \Rightarrow$ s : Φ^t is provable in $L_e^{(/,\backslash)}$ and (ii) the proof in $L_e^{(/,\backslash)}$ is of the same form as P_1. The Φ^t that satisfies these requirements is $x_2^{(e\rightarrow t)}(x_1^e)$ and the requisite proof of $L_e^{(/,\backslash)}$ follows:

(11) $$\frac{\mathrm{np} : x_1^e \Rightarrow \mathrm{np} : x_1^e \quad \mathrm{s} : x_2^{(e,t)}(x_1^e) \Rightarrow \mathrm{s} : x_2^{(e,t)}(x_1^e)}{\mathrm{np} : x_1^e,\ \mathrm{s}\backslash\mathrm{np} : x_2^{(e,t)} \Rightarrow \mathrm{s} : x_2^{(e,t)}(x_1^e)}\backslash L$$

Referring back to (6) for the operation defined by $x_2^{(e\longrightarrow t)}(x_1^e)$, we have the following fact about $[\!]$:

$$[\text{Mary walks}]_{\mathrm{s}}[\!](w,j) \mapsto f^L([\text{walks}]_{\mathrm{s}\backslash\mathrm{np}})(w,j)(f^L([\text{Mary}]_{\mathrm{np}})(w,j))$$

If instead of an *extensional* $\mathrm{L}^{(/,\backslash)}$-interpretation, one took an *intensional* one, $\langle\langle\mathcal{B}, \{H_L^i(P) : P \text{ is an } \mathrm{L}^{(/,\backslash)} \text{ proof}\}\rangle, f^L\rangle$, associated with the *intensional* category-to-type map that still maps np to e and s to t, then the entailed fact about $[\!]$ and [Mary walks]$_{\mathrm{s}}$ would be:

$$[\text{Mary walks}]_{\mathrm{s}}[\!]H_L^i(P_1)(f^L([\text{Mary}]_{\mathrm{np}}), f^L([\text{walks}]_{\mathrm{s}\backslash\mathrm{np}}))$$

To determine $H_L^i(P_1)$ one must find Φ^t such that (i) the sequent $\mathrm{np} : x_1^e, \mathrm{s}\backslash\mathrm{np} : x_2^{(se,t)} \Rightarrow \mathrm{s} : \Phi^t$ is provable in $\mathrm{L}_i^{(/,\backslash)}$ and (ii) the proof in $\mathrm{L}_i^{(/,\backslash)}$ is of the same form as P_1. This Φ^t is $x_2^{(se,t)}(\lambda i x_1^e)$, the requisite proof of $\mathrm{L}_i^{(/,\backslash)}$ being:

(12) $$\frac{\mathrm{np} : x_1^e \Rightarrow \mathrm{np} : x_1^e \quad \mathrm{s} : x_2^{(se,t)}(\lambda i[x_1^e]) \Rightarrow \mathrm{s} : x_2^{(se,t)}(\lambda i[x_1^e])}{\mathrm{np} : x_1^e,\ \mathrm{s}\backslash\mathrm{np} : x_2^{(se,t)}, \Rightarrow \mathrm{s} : x_2^{(se,t)}(\lambda i[x_1^e])}\backslash L$$

Then consulting (7) for the operations defined by $x_2^{(se,t)}(\lambda i[x_1^e])$, the entailed fact about $[\!]$ would be:

$$[\text{Mary walks}]_{\mathrm{s}}[\!](w,j) \mapsto f^L([\text{walks}]_{\mathrm{s}\backslash\mathrm{np}})(w,j)(w' \mapsto f^L([\text{Mary}]_{\mathrm{np}})(w',j))$$

In use one tends not to have a completed proof of $\mathrm{L}^{(/,\backslash)}$ for which one wishes to know the appropriately associated semantic operation. It is a more likely that one has an incompletely specified term-associated sequent of the form $x_1 : \zeta_1, \ldots, x_n : \zeta_n \Rightarrow y : \Phi_n$ where Φ_n is an unknown; one is interested in answer substitutions for Φ_n that will make the sequent derivable in $\mathrm{L}_e^{(/,\backslash)}$ or $\mathrm{L}_i^{(/,\backslash)}$. In this situation one uses the calculi to replace the problem with a set of simpler problems, in unknowns $\Phi_{n+1}, \ldots, \Phi_{n+m}$, and some identities relating Φ_n to $\Phi_{n+1}, \ldots, \Phi_{n+m}$. One arrives in time at axiom problems which do not allow the generation of further problems, they simply generate an identity. One then can pool all the identities generated and simultaneous solution of these will give a solution for original unknown Φ_n. There follows an example of this taking 1.1 below as a problem for $\mathrm{L}_e^{(/,\backslash)}$(for compactness the type information on terms has been omitted):

1.1: $\mathrm{s/s} : x_1 \Rightarrow (\mathrm{s/np})/(\mathrm{s/np}) : \Phi_{1.1}$

The (/R) rule allows us to replace this with another problem and an understood identity:

2.1: $s/s : x_1,\ s/np : u_1 \Rightarrow s/np : \Phi_{2.1} \quad Id : \Phi_{1.1} = \lambda u_1 \Phi_{2.1}$

Applying the (/R) rule again produces another problem and another identity:

3.1 $s/s : x_1,\ s/np : u_1,\ np : u_2 \Rightarrow s : \Phi_{3.1} \quad Id : \Phi_{2.1} = \lambda u_2 \Phi_{3.1}$

Use of the (/L) rule will now produce two problems, which share an unknown:

4.1 $s/np : u_1,\ np : u_2 \Rightarrow s : \Phi_{4.1}$

4.2 $s : x_1(\Phi_{4.1}) \Rightarrow s : \Phi_{4.2} \quad Id : \Phi_{3.1} = \Phi_{4.2}$

A strategic question now facing one is which of the problems to work on. Problem 4.1 involves one unknown and 4.2 involves two. 4.1 can be tackled first and solved, with the result that 4.2 actually then involves only one unknown.

4.1 is tackled by the (/L) rule, generating:

5.1 $np : u_2 \Rightarrow np : \Phi_{5.1}$

5.2 $s : u_1(\Phi_{5.1}) \Rightarrow s : \Phi_{5.2} \quad Id : \Phi_{4.1} = \Phi_{5.2}$

The same strategy should be pursued here, and problem 5.1 considered first. It is in fact a problem that may be solved using the axiom case, giving our first solution, $\Phi_{5.1} = u_2$. This information can be borne in mind in solving problem 5.2, because where it mentions $\Phi_{5.1}$ as an unknown we in fact know the solution. 5.2 is also an axiom case, and gives the next solution, $\Phi_{5.2} = u_1(u_2)$, and together with the identity, $\Phi_{4.1} = \Phi_{5.2}$, this gives a solution for $\Phi_{4.1}$, $\Phi_{4.1} = u_1(u_2)$. We can now turn to the 4.2 problem. This is another axiom problem, and so the solution for $\Phi_{4.2}$ is generated, $\Phi_{4.2} = x_1(u_1(u_2))$.

All that now remains to be done is to pool the identities to arrive at the solution:$\Phi_{1.1} = \lambda u_1 \lambda u_2 [x_1(u_1(u_2))]$

By going through this process one has essentially worked out the following proof of $L_e^{(/,\backslash)}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{s : u_1(u_2) \Rightarrow s : u_1(u_2) \qquad np : u_2 \Rightarrow np : u_2}{s/np : u_1,\ np : u_2 \Rightarrow s : u_1(u_2)}/L \qquad s : x_1(u_1(u_2)) \Rightarrow s : x_1(u_1(u_2))}{s/s : x_1,\ s/np : u_1,\ np : u_2 \Rightarrow s : x_1(u_1(u_2))}/L}{s/s : x_1,\ s/np : u_1 \Rightarrow s/np : \lambda u_2 x_1(u_1(u_2))}/R}{s/s : x_1 \Rightarrow (s/np)/(s/np) : \lambda u_1 \lambda u_2 x_1(u_1(u_2))}/R$$

In section 3.1 it was observed that to settle categorisation questions in LCG it is sufficient to apply what was called the 'flat' strategy - that is simply to look for an $L^{(/,\backslash)}$ proof fo the categorising sequent of a string. There is an analogous issue whether the flat strategy is sufficient to settly meaning assignment questions. The flat strategy is again sufficient, and showing that this is so is essentially the task of showing that for every Cut-based proof, P, of a sequent s, there is a an equivalent Cut-free proof, P', of s, such that $H_L^{\delta}(P)$ is the same operation as

$H^\delta_L(P')$. This has been called the semantic Cut Elimination theorem for $\textsc{l}^{(/,\backslash)}$, and it has been shown by Moortgat (1989) and Hendriks (1989).

3.2.2 Decomposition of the Proof-to-Term maps

There is a logical ancestry to the maps H^e_L and H^i_L: they constitutes the rediscovery by categorial grammarians of something discovered by proof-theorists: the *Curry-Howard* isomorphism. It will be the concern of sections 3.2.3, 3.2.4 and 3.2.5 to lay bare this logical ancestry. The term-associated calculi were presented as *fait-accompli* by their originators, with little comment on why they are the way they are. It is said, for example, that 'Elimination rules ([/L],[\L]) correspond to functional application...Introduction rules ([/R],[\R]) correspond to lambda abstraction' (Moortat 88,p37). But what kind of correspondence is this ? On the face of it nothing could be simpler: there is the class of $\textsc{l}^{(/,\backslash)}$ proofs, inductively defined by two kinds of clause, Left and Right, and on the other hand there is the class of $\mathcal{L}^\lambda$ terms, inductively defined by two kinds of clause, for application and abstraction terms. Yet the inductions cross-cut in a curious way. Looking at the the Left rules of the term associated calculi, there is a sense in which a term part of the minor premise and a term part of the *conclusion* are put together in the major premise. One of the virtues of laying bare the logical ancestry is that it gives one a better view of what is going on here. It also casts light on why the semantic Cut-Elimination theorem holds for $\textsc{l}^{(/,\backslash)}$.

The essential point is that both H^e_L and H^i_L may be defined as the composition of *three* other maps, as illustrated in Figure 4.1, and defined in Definition 32.

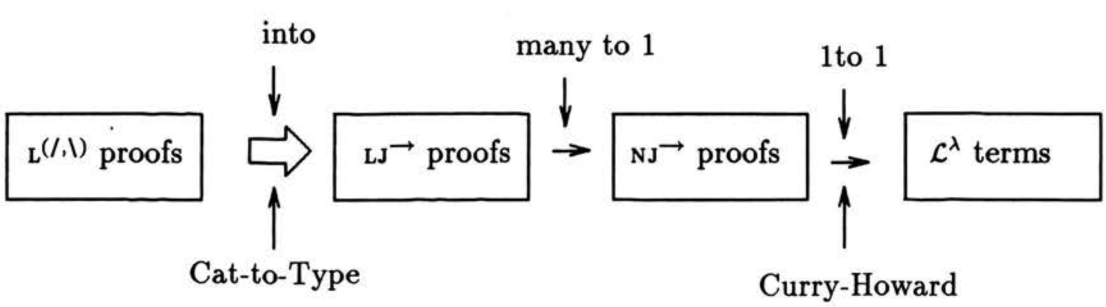


Figure 4.1: The Proof-to-Operation mapping

Definition 32 (The proof-to-term map, H^δ_L)
$H^\delta_L = \ulcorner\ \urcorner \circ * \circ \overline{\nu^\delta}$, where [13]

1. $\overline{\nu^\delta}$ is a map from $\textsc{l}^{(/,\backslash)}$ *proofs of sequents over* $\textsc{cat}^{(/,\backslash)}$ *to proofs of sequents over* $\textsc{tj}^{\rightarrow}$ *. The calculus in which the proofs of* $\textsc{tj}^{\rightarrow}$ *sequents are constructed will be referred to as* $\textsc{lj}^{\rightarrow}$*. 2. * is a map from* $\textsc{lj}^{\rightarrow}$ *proofs to* Natural Deduction *proofs (from assumptions) of* $\textsc{tj}^{\rightarrow}$ *propositions.*

[13] $(\ulcorner\ \urcorner \circ * \circ \overline{\nu^\delta})(P) = \ulcorner *(\overline{\nu^\delta}(P)) \urcorner$

The Natural Deduction proof system will be referred to as NJ$^{\rightarrow}$.

3. $\ulcorner\ \lrcorner$ *is a map from* NJ$^{\rightarrow}$ *proofs to* terms of $\mathcal{L}^{\lambda}$

The first destination in the sequence of steps from L$^{(/,\backslash)}$ to $\mathcal{L}^{\lambda}$ is LJ$^{\rightarrow}$. This is a sequent calculus for sequents over TJ$^{\rightarrow}$. Moreover, LJ$^{\rightarrow}$is not just *a* sequent calculus: it is the implicational part of Gentzen's sequent calculus for propositional logic (Gentzen 34). One can apply to Gentzen's sequent calculus rules to sequents over TJ$^{\rightarrow}$, the set of types, because *types* of TJ$^{\rightarrow}$are formally identical to *well-formed formulae of implicational propositional logic*. To turn a proof of L$^{(/,\backslash)}$ into a proof of LJ$^{\rightarrow}$ turns out to require nothing more than the application of an extensional or intensional *category-to-type* map. It is the choice of extensional versus intensional category-to-type map that results in the possibility of the two proof-to-term maps, H_L^e and H_L^i. The other two links in the above three part definition of H_L^δ are constant.

The second link is possible essentially because when Gentzen proposed the sequent calculus method of proof, it was as a *meta-language* for the description of Natural Deduction proofs. We call the map *.

The third link is the Curry-Howard isomorphism: a remarkable discovery (Howard 1980) that there is a one-to-one correspondence between the terms of $\mathcal{L}^{\lambda}$ and the proofs of NJ$^{\rightarrow}$. It is part of the correspondence that one identifies an occurrence of *a* as a *formula* in a proof, with an occurrence of *a* as the *type* of a $\mathcal{L}^{\lambda}$ term. We call this map $\ulcorner\ \lrcorner$.

What the decomposition shows is that to propose a proof-to-operation map is essentially to re-observe the Curry-Howard isomorphism, modulo the addition of the category-to-type maps.

These mappings will be described in more detail below, the logical progression being from right to left relative to Figure 4.1. We therefore start by defining NJ$^{\rightarrow}$ and the map from proofs of NJ$^{\rightarrow}$ to terms of $\mathcal{L}^{\lambda}$.

3.2.3 The Natural Deduction system NJ$^{\rightarrow}$, and the Curry-Howard isomorphism

A subpart of the Gentzen's rules for constructing *Natural Deduction* proofs for propositional logic (Gentzen 34, Sundholm 83) is presented in the first three rows of Table 4.1. It is the part concerning only the implication connective $\rightarrow$, and will be referred to as NJ$^{\rightarrow}$. This language of implicational propositional logic is formally identical to the set of types, TJ$^{\rightarrow}$, and the *a*'s and *b*'s in the table range over types of TJ$^{\rightarrow}$. Capital 'I' in a rule name is an abbreviation for 'Introduction', whilst capital 'E' in a rule name is an abbreviation for 'Elimination'.

The rules given in the first three rows of Table 4.1 define a certain set of trees, the leaves of which are called the *assumptions*. These assumptions fall into two sets: the discharged, $[a]_i$ and the undischarged, a. The set of trees in turn defines an entailment relationship between a *set* of premise propositions $\{a_1, \ldots, a_n\}$, and a conclusion b: $\{a_1, \ldots, a_n\} \vdash b$ iff there is an NJ$^{\rightarrow}$

a	there is a proof of a from the assumption a
$U, [a]_i$ ⋮ (→I): $\dfrac{b}{a\to b}\,i$	if you have a proof of b from assumptions U, a, you can make a proof of $a \to b$, ***discharging*** the assumption a
$U \quad V$ ⋮ ⋮ (→E): $\dfrac{a\to b \quad a}{b}$	if you have a proof of $a \to b$ with assumptions U, and a proof of a from assumptions V, you can make a proof of b with assumptions U, V
$\dfrac{\dfrac{\overset{U,[a]}{\vdots}\; b}{a\to b} \quad \overset{V}{\vdots}\; a}{b} \;\rhd\; \overset{V}{\vdots}\; U, a \;\vdots\; b$	An Introduction followed by an Elimination is a detour

Table 4.1: Construction and Normalisation of proofs of $\mathrm{NJ}^{\to}$

proof P of b such that (i) each a_i occurs 0 or more times as an undischarged assumption, and (ii) P has no undischarged assumptions that are not members of $\{a_1, \ldots, a_n\}$. The proof may, and typically will, have many additional ***discharged assumptions***. The relationship $\vdash$ extracted from the proofs of $\mathrm{NJ}^{\to}$ coincides with the semantic entailment relationship as defined by the standard semantics for implicational propositional logic.

We will have more to say about the bearing of semantics on the language $\mathrm{TJ}^{\to}$ in a little while. The subject that we wish to turn to now is the relationship between $\mathrm{NJ}^{\to}$ proofs and terms of $\mathcal{L}^{\lambda}$. Before this, attention should be drawn to the fourth row of Table 4.1. This does not define a further rule for constructing proofs. Instead this turns some complex proof with assumptions U, V and conclusion a into a simpler proof with the same assumptions and the same conclusion. Here for example is a rather indirect proof of b from the assumptions a and $(a \to b)$:

$$\dfrac{\dfrac{\dfrac{a \quad [a\to b]_i}{b}\,(\to\mathrm{E})}{(a\to b)\to b}\,i\;(\to\mathrm{I}) \qquad a\to b}{b}\,(\to\mathrm{E})$$

Here is a rather more direct proof of b from the same assumptions:

$$(\to\mathrm{E}):\ \dfrac{a \quad a\to b}{b}$$

$\rhd$ will convert the less direct proof into the more direct proof, a process referred to as ***normalisation***.

Turning now to the relationship between NJ$^{\rightarrow}$ proofs and $\mathcal{L}^\lambda$ terms, the essential point to be made is *that* NJ$^{\rightarrow}$ *proofs and* $\mathcal{L}^\lambda$ *terms are notational variants of each other.* Also the process of *normalisation* in one system corresponds exactly to the notion of *normalisation* in the other. This is the content of the *Curry-Howard isomorphism.*

Part of what it takes to demonstrate this is to define a map, $\ulcorner\ \urcorner$, which given a proof, $\vdots\, a$, of the proposition a (with assumptions b_1 to b_n), returns a $\mathcal{L}^\lambda$ term, Φ^a (the types of its free variables being b_1 to b_n). The essence of its action is depicted below:

$$\left\ulcorner \begin{array}{c} \vdots \\ a \end{array} \right\urcorner = \Phi^{a}$$

At one end of the map one has a proof structure in which an item a figures as a *formula*. At the other end of the map one has a term structure in which the item a figures as a *type*. The 'formulae as types' slogan refers to the fact that this identification is crucial in the argument that NJ$^{\rightarrow}$ proofs and $\mathcal{L}^\lambda$ terms are notational variants.

Definition 33 (One direction of the Curry-Howard isomorphism, $\ulcorner\ \urcorner$)

Assumptions *The assumptions of the proof (both discharged an undischarged), are associated with typed variables, the variable having as its type the assumption with which it is associated.*

Steps of ($\rightarrow$I) *If a proof* $P^{(a\rightarrow b)}$ *is obtained from a proof* P^b *by an ($\rightarrow$I) step which discharges 0 or more occurrences of assumptions of* a *in* P^b *then,* $(\lambda x^a[\Phi^b])^{(a\rightarrow b)}$ *is the term associated with* $P^{(a\rightarrow b)}$ *if* Φ^b *would be associated with* P^b *when* x^a *is associated with each of the occurrences of* a *that are discharged by the ($\rightarrow$ I) step.*

Steps of ($\rightarrow$E) *If* P^b *is obtained by a step of ($\rightarrow$E) from* $P^{(a\rightarrow b)}$ *and* P^a*, then* $(\Phi^{(a\rightarrow b)}\Psi^a)^b$ *is the term associated with* P^b *if* $\Phi^{(a\rightarrow b)}$ *is the term associated with* $P^{(a\rightarrow b)}$ *and* Ψ^a *is the term associated with* P^a*.*

In figure 4.2 there is an illustration.

'Classical' and 'Constructive' semantics

There are two kinds of semantics that might bring to bear on the proof-system NJ$^{\rightarrow}$, the 'classical' and the 'constructive'.

The 'classical' semantics would be one that interpreted the propositions of TJ$^{\rightarrow}$as *sets* and gave the $\rightarrow$ connective its usual definition: $[\![a\rightarrow b]\!] = [\![a]\!]^C \cup [\![b]\!]$. The purpose of this classical semantics is to define valid *entailment* and so to allow the proof-system to be assessed for *soundness* and *completeness.*

The 'constructive' semantics would be one that interpreted a proposition a as what was defined in Chapter 2 as D_a, and which also interpreted the *proofs* of NJ$^{\rightarrow}$, assigning them the same interpretation as that of the isomorphic $\mathcal{L}^\lambda$ terms. The purpose of this constructive semantics

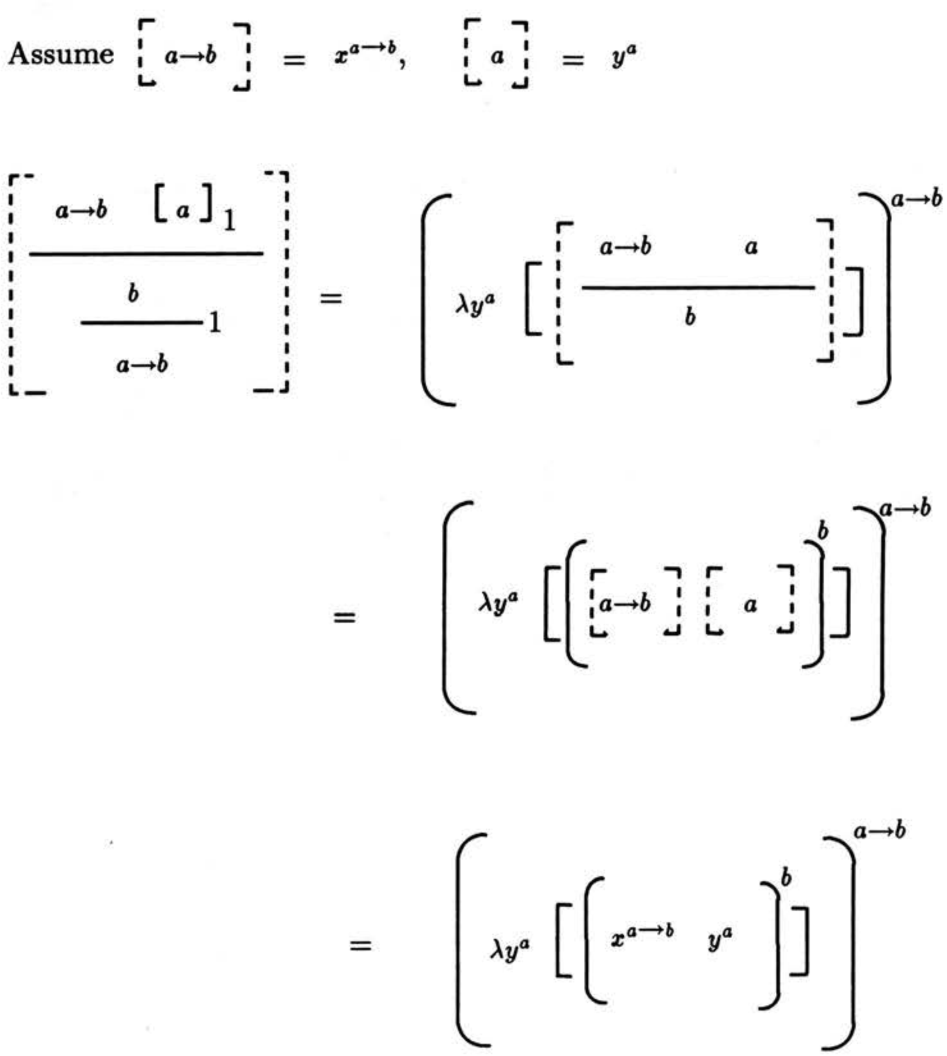


Figure 4.2: Illustration of the Curry-Howard isomorphism

is not to define *entailment*, but is to use the *proof*-interpretations to reflect the process of *normalisation*.

One way to emphasise the difference between the 'classical' and the 'constructive' semantics is to see what happens if the 'constructive' interpretation of propositions of TJ$^{\rightarrow}$ is used for the same purpose as the 'classical' semantics, that it is to define *entailment*. The first thing that is different to the 'classical' semantics is that there is no uniform operation that will combine the constructive interpretations of the premises. For the 'classical' semantics the interpretations of the individual premises are combined by *intersection* and the result is then checked for inclusion in the interpretation of the conclusion. With the 'constructive' interpretation of the propositions, there are lots of different combining operations that one might try, depending what type-domains one is trying to combine. If the domains are $D_{(a\rightarrow b)}$ and D_a say, the natural operation is pointwise function application. This will yield a subset of the D_b. On this basis one could say the NJ$^{\rightarrow}$ defined entailment '$\{(a\rightarrow b), a\} \vdash b$' is constructively valid. However, to similarly be able to say that the NJ$^{\rightarrow}$ defined entailment '$\{(a\rightarrow b), (b\rightarrow c)\} \vdash (a\rightarrow c)$' is semantically valid, the operation combining the domains must be *composition*. Different premise-sets will invite different operations, with in fact the space of possible operations that one might apply to the domains $D_{a_1}, \ldots, D_{a_n}$ being defined by the $\mathcal{L}^{\lambda}$ terms with free variables of type $a_1, \ldots, a_n$. The definition of entailment on the 'constructive' semantics then wouid have to be:

'$\{a_1, \ldots, a_n\} \vdash b$' is constructively valid iff *there is a* $\mathcal{L}^{\lambda}$ *definable operation which combines* $D_{a_1}, \ldots, D_{a_n}$ *together to make a subset of* D_b.

But because of the Curry-Howard isomorphism, this purportedly *semantic entailment* relation is just the *syntactic entailment* in a different guise: there will be a $\mathcal{L}^{\lambda}$ definable operation *exactly* when there is an NJ$^{\rightarrow}$ proof with assumptions $a_1, \ldots, a_n$ and conclusion b. Therefore it is going to be very unrewarding to ask questions of soundness and completeness with respect to the definition of 'constructive entailment' - NJ$^{\rightarrow}$ *is trivially sound and complete on this definition of entailment.*

Of the two styles of semantics, it is the 'constructive' semantics that is of relevance to the process we are engaged on, that of associating semantic operations with L$^{(/,\backslash)}$ proofs. We have depicted this process as taking an L$^{(/,\backslash)}$ proof, transforming it successively into a LJ$^{\rightarrow}$ proof, a NJ$^{\rightarrow}$proof, a $\mathcal{L}^{\lambda}$ term and then essentially taking the interpretation of this term as the operation. It amounts to the same thing to transform the L$^{(/,\backslash)}$ proof into the NJ$^{\rightarrow}$ proof and then to take the *constructive* interpretation of that NJ$^{\rightarrow}$ proof.

This completes the discussion of the relationship between NJ$^{\rightarrow}$ and $\mathcal{L}^{\lambda}$. In the next two sections we retrace back further to the left in Figure 4.1.

3.2.4 A map from LJ$^{\rightarrow}$ to NJ$^{\rightarrow}$

We saw in the above that from the possible proofs of NJ$^{\rightarrow}$ an entailment relation between a set of types and single type can be defined. This same relation can be defined inductively with a sequent calculus, the system, LJ$^{\rightarrow}$, presented below. More precisely it should be said that the sequent calculus defines inductively a relationship, ' $\Rightarrow$ ', between a *sequence* of types and a single type and from this can be defined an entailment relationship between a *set* of types and a single type, the set being precisely the propositions having at least one occurrence in the sequence.

Definition 34 (LJ$^{\rightarrow}$, a sequent calculus for TJ$^{\rightarrow}$)

(Ax) $a \Rightarrow a$

(Cut) $$\frac{U, a \Rightarrow c \quad V \Rightarrow a}{U, V \Rightarrow c}\,Cut$$

($\rightarrow$L) $$\frac{T \Rightarrow a \quad U, b, V \Rightarrow c}{U, T, a\rightarrow b, V \Rightarrow c}\rightarrow L$$

($\rightarrow$R) $$\frac{a, U \Rightarrow b}{U \Rightarrow a\rightarrow b}\rightarrow R$$

$$\frac{U \Rightarrow c}{U, a \Rightarrow c}Weak \qquad \frac{U, a, a \Rightarrow c}{U, a \Rightarrow c}Cont \qquad \frac{U \Rightarrow c}{\pi(U) \Rightarrow c}Perm$$

The last three rules are usually called the 'structural rules'. Similar observations were made by Gentzen about LJ$^{\rightarrow}$ in 1934 as were made by Lambek about L$^{(/,\backslash)}$ in 1957:

- leaving Cut to one side, the derivability of a sequent is decidable (there are some complications concerning the deployment of the structural rules)
- the Cut rule is superfluous: any proof of a sequent that uses the Cut rule can be replaced by one that does not use it.

There are several ways to show that the entailment relation defined by LJ$^{\rightarrow}$ and NJ$^{\rightarrow}$ are the same. One is to show that they are both sound and complete with respect to the 'classical' semantics. Another is to show that proofs of NJ$^{\rightarrow}$ and proofs of LJ$^{\rightarrow}$ may be interconverted [14]. One of the directions of this conversion is the second of the links involved in defining the proof-to-operation map, H^{δ}_{L}. This is the conversion from LJ$^{\rightarrow}$ to NJ$^{\rightarrow}$ and it will be referred to as $*$.

The first step in the direction of defining $*$ is made by reading LJ$^{\rightarrow}$ sequents as making statements about the existence of NJ$^{\rightarrow}$ proofs:

[14] Gentzen showed the equivalence by introducing a third corner, a *Hilbert-style* axiom-system, and showing the possibility of traversing from NJ$^{\rightarrow}$ to LJ$^{\rightarrow}$ to a Hilbert-style system to NJ$^{\rightarrow}$.

Definition 35 (Proof-theoretic interpretation of sequents) *'$U \Rightarrow a$' is true iff there is a* $\text{NJ}^{\to}$ *proof that has U as its undischarged assumptions and a as its conclusion.*

All the sequents derivable in $\text{LJ}^{\to}$ are true on this interpretation. One can see this because the axioms are true on this interpretation and the inference rules of $\text{LJ}^{\to}$ are sound on it. (13), (14), (15) and (16) display the facts about $\text{NJ}^{\to}$ proofs that make $\text{LJ}^{\to}$ axioms true, and the rules ($\to$R), ($\to$L) and Cut sound. The same can be done for the structural rules of Weakening, Contraction and Permutation.

(13) *a is proof*

(14) *if* $\begin{array}{c} U, a \\ \vdots \\ b \end{array}$ *is a proof, then* $\dfrac{\begin{array}{c} U, [a]_i \\ \vdots \\ b \end{array}}{a \to b}\,i$ *is a proof.*

(15) *If* $\begin{array}{c} T \\ \vdots \\ a \end{array}$ *and* $\begin{array}{c} U, b, V \\ \vdots \\ c \end{array}$ *are proofs, then* $\begin{array}{c} \dfrac{\begin{array}{cc} T & \\ \vdots & \\ a & a \to b \end{array}}{U, b, V} \\ \vdots \\ c \end{array}$ *is a proof*

(16) *If* $\begin{array}{c} T \\ \vdots \\ a \end{array}$ *and* $\begin{array}{c} U, a, V \\ \vdots \\ c \end{array}$ *are proofs, then* $\begin{array}{c} T \\ \vdots \\ U, a, V \\ \vdots \\ c \end{array}$ *is a proof*

Note especially (15). The two *premises* of the ($\to$L) inference describe a *top* and a *bottom* part of an $\text{NJ}^{\to}$ proof. This 'corkscrew' in the relationship of $\text{LJ}^{\to}$ tp $\text{NJ}^{\to}$ is the cause of the impression that one gets from working with the term associated calculi, $\text{L}_{\delta}^{(/,\backslash)}$, that terms 'flow' from the minor premise into the major premise.

Now making the observation that the derivable sequents of $\text{LJ}^{\to}$ are all true on the 'proof-theoretic' interpretation is enough the demonstrate that the entailment relation defined by $\text{LJ}^{\to}$ is a sub-relation of the entailment relation defined by $\text{NJ}^{\to}$. However, we persevere and show that any $\text{LJ}^{\to}$ *proof* can be converted to an $\text{NJ}^{\to}$ proof.

Let the proof-context of a sequent $U \Rightarrow a$ be understood to be the subtraction of $U \Rightarrow a$ from a proof of $U \Rightarrow a$. Since $U \Rightarrow a$ plus its proof context is a proof, we can take the map to be defined upon a sequent and proof-context.

The leaves of a proof have no context. $*$ is therefore defined with reference to the sequent alone. The leaves must be axiom sequents, $a \Rightarrow a$, and $*(a \Rightarrow a)$ is simply the $\text{NJ}^{\to}$ proof a. All non-leaf sequents have as their context the one or more proofs terminating in the premises that lead to the non-leaf sequent. If one supposes that $*$ has been defined for these premise sequents (given their context), it is clear to see how to define $*$ for the non-leaf sequent in its context: one simply refers to the facts about $\text{NJ}^{\rightarrow}$ proofs that make the sequent calculus rules sound, as noted in (14), (15) and (16). This terse description should be made clearer by the example in Figure 4.3, which illustrates the map $*$ on a certain $\text{LJ}^{\rightarrow}$ proof of $a \Rightarrow (a \rightarrow b) \rightarrow b$.

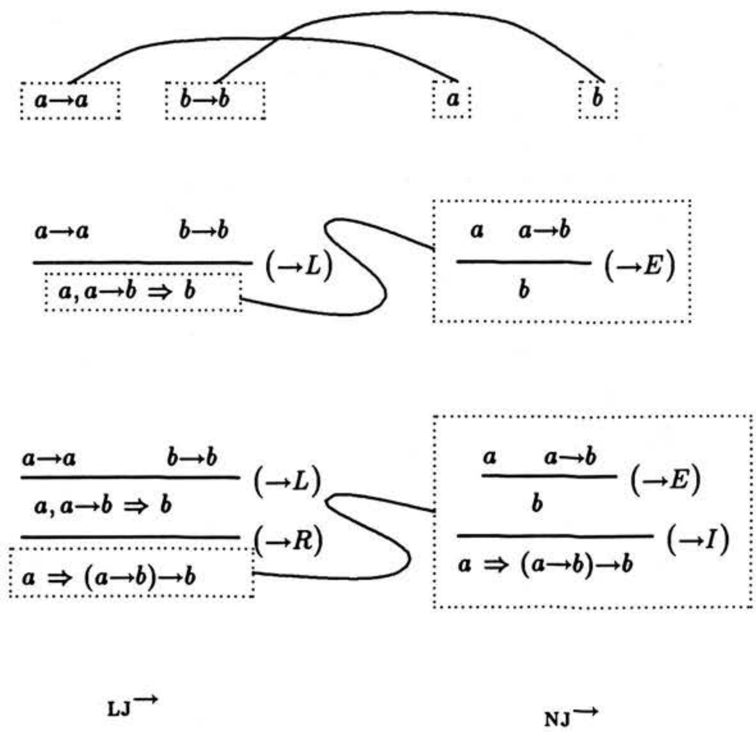


Figure 4.3: From $\text{LJ}^{\rightarrow}$ to $\text{NJ}^{\rightarrow}$

3.2.5 From $\text{L}^{(/,\backslash)}$ to $\text{LJ}^{\rightarrow}$

The last link in the chain of mappings defining H^{δ}_{L} is the link from $\text{L}^{(/,\backslash)}$ proofs to $\text{LJ}^{\rightarrow}$ proofs. H^{e}_{L} and H^{i}_{L} differ by the different ways this link is accomplished.

$\overline{\nu^e}$: the extensional link

Imagine applying a ν^e throughout an $\text{L}^{(/,\backslash)}$ proof, and deleting the labelling of the steps. The result is a tree of $\text{TJ}^{\rightarrow}$ sequents. One can wonder whether any such tree represents an inference obtainable in $\text{LJ}^{\rightarrow}$. The answer is yes. That this is so is obvious for the leaf-trees which will originate from axiom $\text{L}^{(/,\backslash)}$ proofs. One can also see that the images of (\L) and (\R) steps are (→L) and (→R) steps. Finally, although the images of (\L) and (\R) steps are not simply (→L) and (→R) steps, they are transitions derivable by (→L) and (→R) in combination with the Perm rule. In this way one can see how, by making reference to an extensional type-map,

ν^e, a particular $\mathrm{LJ}^{\rightarrow}$ proof may be associated with a $\mathrm{L}^{(/,\backslash)}$ proof. The details of association are given in Table 4.2, with $\overline{\nu^e}$ notating the conversion.

$\mathrm{L}^{(/,\backslash)}$ proof	image in $\mathrm{LJ}^{\rightarrow}$ under $\overline{\nu^e}$
$x \Rightarrow x$	$\nu^e(x) \Rightarrow \nu^e(x)$
$\dfrac{P_1 \quad P_2}{U, T, y\backslash x, V \Rightarrow z}\backslash\mathrm{L}$	$\dfrac{\overline{\nu^e}(P_1) \quad \overline{\nu^e}(P_2)}{\nu^e(U), \nu^e(T), \nu^e(x)\rightarrow\nu^e(y), \nu^e(V) \Rightarrow \nu^e(z)}\rightarrow\mathrm{L}$
$\dfrac{P_1 \quad P_2}{U, y/x, T, V \Rightarrow z}/\,\mathrm{L}$	$\dfrac{\dfrac{\overline{\nu^e}(P_2) \quad \overline{\nu^e}(P_1)}{\nu^e(U), \nu^e(T), (\nu^e(x)\rightarrow\nu^e(y)), \nu^e(V) \Rightarrow \nu^e(z)}\rightarrow\mathrm{L}}{\nu^e(U), (\nu^e(x)\rightarrow\nu^e(y)), \nu^e(T), \nu^e(V) \Rightarrow \nu^e(z)}\mathrm{Perm}$
$\dfrac{P_1}{U \Rightarrow y\backslash x}\backslash\mathrm{R}$	$\dfrac{\overline{\nu^e}(P_1)}{\nu^e(U) \Rightarrow (\nu^e(x)\rightarrow\nu^e(y))}\rightarrow\mathrm{R}$
$\dfrac{P_1}{U \Rightarrow y/x}/\,\mathrm{R}$	$\dfrac{\dfrac{\overline{\nu^e}(P_1)}{\nu^e(x), \nu^e(U) \Rightarrow \nu^e(y)}\mathrm{Perm}}{\nu^e(U) \Rightarrow (\nu^e(x)\rightarrow\nu^e(y))}\rightarrow\mathrm{R}$

Table 4.2: $\overline{\nu^e}$: $\mathrm{L}^{(/,\backslash)} \mapsto \mathrm{LJ}^{\rightarrow}$

<u>$\overline{\nu^i}$: the intensional link</u>

With reference to an *intensional* category-to-type map, it is also possible to link $\mathrm{L}^{(/,\backslash)}$ proofs to $\mathrm{LJ}^{\rightarrow}$ proofs. However, matters do not run so smoothly as for the extensional maps: this time, when a map ν^i is applied to an $\mathrm{L}^{(/,\backslash)}$ proof, the resulting tree of sequents does not represent derivable transitions within $\mathrm{LJ}^{\rightarrow}$. For example the image of a (/R) step is unsound on a classical semantics, and so does not represent a possible transition within $\mathrm{LJ}^{\rightarrow}$.

That the image of a (/R) step is unsound

The premise sequent will be $\nu^i(U)$, $\nu^i(y) \Rightarrow \nu^i(x)$ and the conclusion sequent $\nu^i(U) \Rightarrow ((s\rightarrow\nu^i(y))\rightarrow\nu^i(x))$.

To suppose the premise is valid is to suppose that:

(1) Whatever interpretation, $[\![\nu^i(U)]\!] \cap [\![\nu^i(y)]\!] \subseteq [\![\nu^i(x)]\!]$

For the conclusion to be valid then we require:

(2) Whatever interpretation, $[\![\nu^i(U)]\!] \subseteq ([\![s]\!] \cap [\![\nu^i(y)]\!]^c) \cup [\![\nu^i(x)]\!]$

For a counterexample, suppose on an particular interpretation, $[\![s]\!] = [\![\nu^i(y)]\!] = [\![\nu^i(x)]\!] = \{a\}$

and $[\![\nu^i(U)]\!] = \{a, b\}$. This refutes (2) but is consistent with (1) □

Although the tree of sequents obtained by applying the ν^i map does not represent derivable transitions in $\text{LJ}^{\rightarrow}$, a simple modification of it will make it do so: s should be added to the antecedents of all sequents. In Table 4.3 the details of the intensional conversion from $\text{L}^{(/,\backslash)}$ to $\text{LJ}^{\rightarrow}$ are given, using this device of adding s to the antecedents. The table makes reference to the derived rules ($\overset{s}{\rightarrow}$L) and ($\overset{s}{\rightarrow}$R) and here are the sequences of steps in $\text{LJ}^{\rightarrow}$ that I will be using these derived rule to abbreviate:

Abbreviated rule	Unabbreviated rule
$\dfrac{s, T' \Rightarrow a \quad s, U', b, V' \Rightarrow c}{s, U', T', (s \rightarrow a) \rightarrow b, V' \Rightarrow c}\overset{s}{\rightarrow}\text{L}$	$\dfrac{\dfrac{s, T' \Rightarrow a}{T' \Rightarrow s \rightarrow a}\rightarrow\text{R} \quad s, U', b, V' \Rightarrow c}{s, U', T', (s \rightarrow a) \rightarrow b, V' \Rightarrow c}\rightarrow\text{L}$
$\dfrac{s, U', a \Rightarrow b}{s, U' \Rightarrow ((s \rightarrow a) \rightarrow b)}\overset{s}{\rightarrow}\text{R}$	$\dfrac{\dfrac{\dfrac{s \Rightarrow s \quad s, U', a \Rightarrow b}{s, U', s, (s \rightarrow a) \Rightarrow b}\rightarrow\text{L}}{s, U', (s \rightarrow a) \Rightarrow b}\text{Cont}}{s, U' \Rightarrow ((s \rightarrow a) \rightarrow b)}\rightarrow\text{R}$

$\text{L}^{(/,\backslash)}$ proof	image in $\text{LJ}^{\rightarrow}$ under $\overline{\nu^i}$
$x \Rightarrow x$	$\dfrac{\nu^i(x) \Rightarrow \nu^i(x)}{s, \nu^i(x) \Rightarrow \nu^i(x)}\text{Weak}$
$\dfrac{P_1 \quad P_2}{U, T, y\backslash x, V \Rightarrow z}\backslash\text{L}$	$\dfrac{\overline{\nu^i}(P_1) \qquad \overline{\nu^i}(P_2)}{s, \nu^i(U), \nu^i(T), ((s \rightarrow \nu^i(x)) \rightarrow \nu^i(y)), \nu^i(V) \Rightarrow \nu^i(z)}\overset{s}{\rightarrow}\text{L}$
$\dfrac{P_1}{U \Rightarrow y\backslash x}\backslash\text{R}$	$\dfrac{\overline{\nu^i}(P_1)}{s, \nu^i(U) \Rightarrow (s \rightarrow \nu^i(x)) \rightarrow \nu^i(y)}\overset{s}{\rightarrow}\text{R}$

Table 4.3: $\overline{\nu^i}$: $\text{L}^{(/,\backslash)} \mapsto \text{LJ}^{\rightarrow}$

3.2.6 Multiplicity of proofs as a source of ambiguity

The Lambek categorial framework seems to hold some promise for explaining the recursive ambiguity facts that were described in Chapter 3. For example, to account for the categorisation of **every man loves a woman**, although in LCG only one sequent is needed, the framework cannot help but provide more than the sequent: it must provide a proof or *proofs* of the sequent, and since the $L^{(/,\backslash)}$-meaning relation is 'driven off' proofs, two proofs of a single sequent may lead to two meaning assignments. We say 'may' rather than 'will lead to two meaning assignments', because it is often the case that different proofs are associated with semantically equivalent terms. Examples of significantly different proofs are given in (17) and (18), where two proofs are given of $\mathrm{s/(s\backslash np), (s\backslash np)/np, s\backslash(s/np)} \Rightarrow s$. This is a categorising sequent for **every man loves a woman** on a not unreasonable choice of categorisation for **every man**, **loves** and **a woman**. On both proof-to-operation maps, these two proofs will be associated with significantly different operations.

(17)

$$\dfrac{\mathrm{s \Rightarrow s} \qquad \dfrac{\dfrac{\dfrac{\mathrm{np, (s\backslash np)/np, np \Rightarrow s}}{\mathrm{np, (s\backslash np)/np \Rightarrow s/np}}\,/\mathrm{R} \qquad \mathrm{s \Rightarrow s}}{\mathrm{np, (s\backslash np)/np, s\backslash(s/np) \Rightarrow s}}\,\backslash\mathrm{L}}{\mathrm{(s\backslash np)/np, s\backslash(s/np) \Rightarrow s\backslash np}}\,\backslash\mathrm{R}}{\mathrm{s/(s\backslash np), (s\backslash np)/np, s\backslash(s/np) \Rightarrow s}}\,/\mathrm{L}$$

The value of H^e_L on the above proof is $x^{(et,t)}(\lambda u^e_2[z^{(et,t)}(\lambda u^e_5[y^{(eet)}u_5u_2])])$, assuming $x^{(et,t)}$, $y^{(eet)}$ and $z^{(et,t)}$ are associated with the antecedents. Therefore, the operation defined is:

$m_1, m_2, m_3, (w, j) \mapsto [\![\lambda i[y_1^{(s\rightarrow(et,t))}(i)(\lambda u^e_2[y_3^{(s\rightarrow(et,t))}(i)(\lambda u^e_5[y_2^{(s\rightarrow eet)}(i)(u_5)(u_2)])])]]\!](h)(w)$
where $h(y_1) = w' \mapsto m_1(w', j)$, $h(y_2) = w' \mapsto m_2(w', j)$, $h(y_3) = w' \mapsto m_3(w', j)$

$= m_1, m_2, m_3, (w, j) \mapsto m_1(w, j)(y^e \mapsto m_3(w, j)(x^e \mapsto m_2(w, j)(x)(y)))$

(18)

$$\dfrac{\dfrac{\dfrac{\mathrm{s \Rightarrow s} \qquad \dfrac{\mathrm{np, (s\backslash np)/np, np \Rightarrow s}}{\mathrm{(s\backslash np)/np, np \Rightarrow s\backslash np}}\,\backslash\mathrm{R}}{\mathrm{s/\,(s\backslash np), (s\backslash np)/np, np \Rightarrow s}}\,/\,\mathrm{L}}{\mathrm{s/(s\backslash np)\ (s\backslash np)/np, \Rightarrow s/np}}\,/\,\mathrm{R} \qquad \mathrm{s \Rightarrow s}}{\mathrm{s/(s\backslash np), (s\backslash np)/np, s\backslash(s/np) \Rightarrow s}}\,\backslash\mathrm{L}$$

The value of H^e_L on the above proof is $z^{(et,t)}(\lambda u_2.x^{(et,t)}(\lambda u_5.y^{eet}u_2u_5))$

Therefore the operation defined is (where h is the assignment mentioned above):
$m_1, m_2, m_3, (w, j) \mapsto [\![\lambda i[y_3^{(s\rightarrow(et,t))}(i)(\lambda u^e_2[y_3^{(s\rightarrow(e,et))}(i)(\lambda u^e_5[y_2^{(s\rightarrow eet)}(i)(u_2)(u_5)])])]]\!](h)(w) =$

$m_1, m_2, m_3 \mapsto (w, j) \mapsto m_3(w, j)(x \mapsto m_1(w, j)(y \mapsto m_2(w, j)(x)(y)))$

This then is an example of the potential of LCG to account for ambiguity. Such potential has also been noted by others (Moortgat 88, Hendriks 90). It has also been concluded that these early promising signs are more or less misleading. The question is looked at in some detail in Chapter 6. What we will do here is indicate why such an account, should it be forthcoming, would be especially interesting. There are, after all, other kinds of account of ambiguity - some will be considered in Chapter 5. The criterion on which a categorial grammar account might have an advantage over other accounts is the criterion of *emergence*. This was the criterion mentioned in the introductory chapter, and it requires that ambiguity should not be a modularly subtractable part of the coverage of an account. That is, it should not be possible, by a simplification to make the theory into one that accounts for the very same syntactic facts as before, but which finds all sentences unambiguous. The reason that a descriptively adequate LCG account would meet the emergence criterion is that it would owe its success in explaining ambiguity to the structure of $L^{(/,\backslash)}$ proofs, and these are defined by just 5 clauses. Removing any of the clauses is likely to not only lessen the extent to which ambiguity is explained, but also to affect the theory's ability to account for the syntactic facts also. It is not worth laboring this point until one has on one's hands a descriptively adequate LCG account. Suffice it to say that LCG arguably holds out the promise of particularly theoretically interesting account of ambiguity.

3.2.7 The strangeness and potential significance of the string-semantics

It was mentioned in section 3.1 that there is no consensus amongst categorial grammar researchers on the significance of the string-semantics. This is probably because there seems something 'inverted' (see below) about the priorities of a string-semantic interpretation. In this section there are no arguments settling once and for all the significance of the string-semantics. What there is first is an attempt to articulate what the 'strangeness' of the string-semantics is. After that there is an acknowledgement of the fact that this 'strangeness' can be see to lesson the force of the statement made earlier: that to use sequents other than those present in the Lambek calculus is to use sequents that depart from the meaning of the slash-connective. Finally, I will try out an argument to the effect that an explanation of a phenomenon in LCG is especially interesting because of the existence of the string-semantics.

One way to regard a string semantic interpretation is as defining in an unorthodox way a *categorisation relation* between strings and categories. As such, it bears comparison with the orthodox way of defining a categorisation relation, which is by specifying a grammar. The following vocabulary will be useful in the comparison: a categorisation fact is *string-simple* if the string is a *word* and it is *category-simple* if the category is *basic*.

Now a string-semantic interpretation consists in the direct specification of all *category-simple* categorisation facts. All the other facts are defined inductively from these. A grammar on the other hand consists in the direct specification of all *string-simple* categorisation facts, and then other facts are inductively defined from these. From the perspective of a grammar based definition of a categorisation relation then, the string semantic based definition is bizarre, and in this resides the difficulty of settling the significance of the string-semantics.

One thing this strangeness does is allow one to be less than persuaded that $L^{(/,\backslash)}$ represents a genuine ceiling on possible sequents (involving the slash-connective). Consider the significance of the fact that a particular sequent is not derivable in $L^{(/,\backslash)}$. This means that the sequent describes categorisation facts which do not hold in all 'possible' categorisation relations, where the possible categorisation relations are given by the string-semantics. If one finds the string-semantic definition of a categorisation relation bizarre, then the associated notion of possible categorisation relation will seem bizarre and therefore the fact that the sequent is underivable in the Lambek calculus need not seem very insignificant.

One final point will be considered in this section. One could argue that once the category-simple facts have been settled, the issue of which proofs the sentence should be associated with is an *empirical matter*, akin to the issue of whether the sentence contains three occurrences of the letter 'A'. The empirically testable nature of association between a sentence and a proof, may affect the significance of the discovery that a given phenomenon can be accounted for by an 'induced' categorial grammar. Consider how significant it would seem if one could predict the number of different readings of a sentence from the empirical matter of the number of occurrences of the letter 'A' in it. Such a correspondence would be hard to put down to coincidence, and one would have to conclude that occurrences of the letter 'A' *really* are a causal factor in determining the number of readings of a sentence. Equally if the number of readings of a sentence could be predicted from the number of proofs associated with it, could one put the correspondence down to coincidence ?

It is crucial to this line of reasoning that that one be able to claim that the association between sentence and proof is an *empirical* matter. If instead one simply defines the association by proposing a grammar, and then should one obtain a correspondence between the proofs and the readings, the correspondence could be put down not to coincidence, nor causation, but design. So its crucial whether string-semantically induced associations of a sentence with a proof are as empirical a matter as the number of occurrences of the letter 'A' in a sentence. This status is vitiated by the fact that the string-semantically induced category-complex facts depend on the specification by a grammar of category-simple facts, and these facts can only be instrumentally justified. However, even given this, if it should turn out that ambiguity was explicable by a string-semantically induced categorial grammar, could one put it down to coincidence and maintain that none the less the induced categorial account had nothing to do with what is *actually* at the bottom of ambiguity ? I have to simply leave this as a question.

4 Universal Grammar

Until now in this chapter, we have ignored the UG definition of the link between an ambiguous language and a semantic algebra and substituted other definitions. Lambek categorial grammar has been projected as having a definition special to it of the language-meaning relation: the link is made at a lexical level by proposing an $\mathrm{L}^{(/,\backslash)}$-interpretation, and the link at the non-lexical level is made via the $\mathrm{L}^{(/,\backslash)}$ meaning relation. Let us call the combination of a $\mathrm{L}^{(/,\backslash)}$-interpretation with a (w, j) an $\mathrm{L}^{(/,\backslash)}$-*model*, and the combination of a $\mathrm{L}^{(/,\backslash)}$-grammar and a class of possible $\mathrm{L}^{(/,\backslash)}$-models an $\mathrm{L}^{(/,\backslash)}$-*theory*. Therefore to work in the LCG framework is to construct $\mathrm{L}^{(/,\backslash)}$-theories. What we will attempt to do in this section is identify a region within the space of possible THEORIES OF REFERENCE that may be thought of as *representing* every possible $\mathrm{L}^{(/,\backslash)}$-theory. We have to say 'representing' as the THEORIES OF REFERENCE will not actually be $\mathrm{L}^{(/,\backslash)}$-theories. They will represent $\mathrm{L}^{(/,\backslash)}$-theories by replicating their syntactic and semantic entailments.

We begin by noting the kind of entailments that an $\mathrm{L}^{(/,\backslash)}$-theory has. They are of two forms, defining the categorisation relation and the meaning relation:

a *categorisation* of s is x

a *meaning* of $[s]_x$ is m, relative to $\mathrm{L}^{(/,\backslash)}$-interpretation, $\langle \mathcal{B}, \{H^{\delta}_{L}(P) : P \text{ is an } \mathrm{L}^{(/,\backslash)} \text{ proof}\}, f^{L}\rangle$

Here s is a *string*, and is not understood to have any algebraic structure. A THEORY OF REFERENCE also entails facts concerning the categorisation and meaning of *strings*, though the chain of implication is more extended than for an $\mathrm{L}^{(/,\backslash)}$-theory; the most immediate entailments of a THEORY OF REFERENCE concern not *strings* but *expressions of a* DISAMBIGUATED LANGUAGE:

a *categorisation* of α is x

the *meaning* of α is m, relative to an INTERPRETATION, $\langle \mathcal{B}, (\mathcal{G}_{\gamma})_{\gamma\in\Gamma}, f\rangle$

As explained in Chapter 2, from these central entailments concerning expressions of a disambiguated language, entailments concerning simply *strings* follow when the DISAMBIGUATION RELATION is taken account of. Therefore a THEORY OF REFERENCE has the same kind of entailments as an $\mathrm{L}^{(/,\backslash)}$-theory and what we need to do is show given any $\mathrm{L}^{(/,\backslash)}$-theory, θ, a simulating THEORY OF REFERENCE, $\mathcal{T}^{\theta}$ can be found.

The DISAMBIGUATION RELATION of $\mathcal{T}^{\theta}$ will be as usual the first argument projection function. To replicate the syntactic entailments of θ, $\mathcal{T}^{\theta}$ will be taken to involve the following DISAMBIGUATED LANGUAGE:

$\mathcal{L}^{\theta}$: part of $\mathcal{T}^{\theta}$, which simulates the $\mathrm{L}^{(/,\backslash)}$-theory, θ

1. *The phrase-set indices*, $\Delta = \mathrm{CAT}^{(/,\backslash)}$ (relative to BASCAT),
2. *Set of basic phrase-sets*, $\mathcal{X}_{\delta}$: $\langle \alpha, \langle\rangle, x\rangle \in \mathcal{X}_x$ iff x is a categorisation of the lexical item

α according to θ.

3. *The operation indices*, Γ = the set of possible proofs in $L^{(/,\backslash)}$.
4. *The operations*, $\mathcal{F}_\gamma$ operate on triples in the usual way, amounting to concatenation on strings. That is, $\forall\gamma \in \Gamma$, $\mathcal{F}_\gamma$ is an n-place operation, where n is the number of antecedent categories of the conclusion sequent of γ, and $\forall\langle\alpha^1_{\zeta_1}\rangle, \langle\alpha^2_{\zeta_2}\rangle, \ldots, \langle\alpha^n_{\zeta_n}\rangle$,
$\mathcal{F}_\gamma(\langle\alpha^1_{\zeta_1}\rangle, \langle\alpha^2_{\zeta_2}\rangle, \ldots, \langle\alpha^n_{\zeta_n}\rangle) = \langle\alpha^1_1\alpha^2_1 \ldots \alpha^n_1, \langle\langle\alpha^1_{\zeta_1}\rangle, \langle\alpha^2_{\zeta_2}\rangle, \ldots, \langle\alpha^n_{\zeta_n}\rangle\rangle, \gamma\rangle$
5. *The set of rules* $\mathcal{S} = \{\langle\mathcal{F}_\gamma, U, x\rangle : \gamma \in \Gamma, U \Rightarrow x \text{ is the conclusion of } \gamma\}$

The DISAMBIGUATED LANGUAGES that correspond to $L^{(/,\backslash)}$-grammars we will call DISAMBIGUATED LAMBEK LANGUAGES.

Now θ will specify a certain set of possible $L^{(/,\backslash)}$-models, each model being $\langle\langle\mathcal{B}, \{H^\delta_L(P) : P \text{ is an } L^{(/,\backslash)} \text{ proof}\}, f^L\rangle, \langle w, j\rangle\rangle$. To turn these $L^{(/,\backslash)}$-models specified by θ into MODELS, specified by $\mathcal{T}^\theta$, it is simply necessary to replace f^L with a function f defined on basic expressions of $\mathcal{L}^\theta$. The relationship between f^L and f is:

$$f(\langle\alpha, \langle\rangle, x\rangle) = f^L([\alpha]_x)$$

Before we endeavour to show that $\mathcal{T}^\theta$ defined above really does reproduce the categorisation and meaning relation facts of θ, we will illustrate the above construction by applying it to a particular $L^{(/,\backslash)}$-theory. The categorial lexicon of the $L^{(/,\backslash)}$-theory will be that given in (1), and for the models of the $L^{(/,\backslash)}$-theory we will allow any $\langle\langle\mathcal{B}, \{H^e_L(P) : P \text{ is an } L^{(/,\backslash)} \text{ proof}\}, f^L\rangle, \langle w, j\rangle\rangle$, associated with the extensional category-to-type map that maps np to e and s to t.

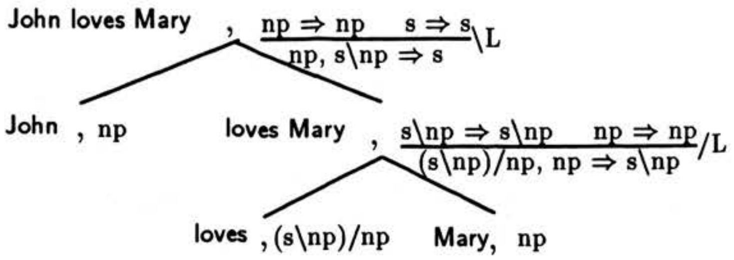
Now a categorisation entailment of such an $L^{(/,\backslash)}$-theory will be that a categorisation of John loves Mary is s. To show that the same fact is entailed by the corresponding THEORY OF REFERENCE, an object of the disambiguated language must be found which is in C_s and which stand in the ambiguation relationship to John loves Mary . The required object is described in (19):

(19) $\mathcal{F}_{PROOF_2}(\langle\text{john}, \langle\rangle, \text{np}\rangle, \mathcal{F}_{PROOF_1}(\langle\text{loves}, \langle\rangle, \text{np}\rangle, \langle\text{mary}, \langle\rangle, \text{np}\rangle))$

where the operation indices $PROOF_1$ and $PROOF_2$ are the following proofs:

(20) PROOF1: $\dfrac{\text{s}\backslash\text{np} \Rightarrow \text{s}\backslash\text{np} \quad \text{np} \Rightarrow \text{np}}{(\text{s}\backslash\text{np})/\text{np}, \text{np} \Rightarrow \text{s}\backslash\text{np}}/L$ PROOF2: $\dfrac{\text{np} \Rightarrow \text{np} \quad \text{s} \Rightarrow \text{s}}{\text{np}, \text{s}\backslash\text{np} \Rightarrow \text{s}}\backslash L$

The object described in (19) is depicted in tree form below:



(21)

The tree in (21) should not be confused with the *categorisation trees* shown earlier in (2).

With respect to an arbitrary $L^{(/,\backslash)}$-interpretation allowed by the $L^{(/,\backslash)}$-theory, a possible meaning of John loves Mary is

(22) $\mathcal{G}_{PROOF_2}(f^L([\text{John}]_{np}, \mathcal{G}_{PROOF_1}(f^L([\text{loves}]_{(s\backslash np)/np}), f^L([\text{Mary}]_{np})))$

For a THEORY OF REFERENCE, a possible meaning of John loves Mary is the meaning assigned to any disambiguation of it. Therefore with respect to the corresponding INTERPRETATION, a possible meaning of John loves Mary is

(23) $[\![\mathcal{F}_{PROOF_2}(\langle \text{john}, \langle\rangle, np\rangle, \mathcal{F}_{PROOF_1}(\langle \text{loves}, \langle\rangle, np\rangle, \langle \text{mary}, \langle\rangle, np\rangle))]\!]$
$= \mathcal{G}_{PROOF_2}([\![\langle \text{john}, \langle\rangle, np\rangle]\!], \mathcal{G}_{PROOF_1}([\![\langle \text{loves}, \langle\rangle, np\rangle]\!], [\![\langle \text{mary}, \langle\rangle, np\rangle]\!]))$
$= \mathcal{G}_{PROOF_2}(f(\langle \text{john}, \langle\rangle, np\rangle), \mathcal{G}_{PROOF_1}(f(\langle \text{loves}, \langle\rangle, np\rangle), f(\langle \text{mary}, \langle\rangle, np\rangle)))$

By the definition of what is for an INTERPRETATION to correspond to an $L^{(/,\backslash)}$-interpretation, the above describes the same meaning as that in (22).

Now to compare more generally *any* $L^{(/,\backslash)}$-theory, θ, and what has been defined above as the corresponding THEORY OF REFERENCE. Firstly, it is clear that the THEORY OF REFERENCE $\mathcal{T}^\theta$ will agree with θ on lexical facts concerning categorisation and meaning.

Now suppose that according to θ:

$\beta_1 \dots \beta_n$ may be categorised as y

$[\beta_1 \dots \beta_n]_y$ has possible meaning m according to the L $L^{(/,\backslash)}$-interpretation $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f^L \rangle$

If θ entails the categorisation fact, it must be the case that (i) there are $x_1, \dots, x_n$ such that the β_i may be categorised as x_i, and (ii) the sequent $x_1, \dots, x_n \Rightarrow y$ is provable in $L^{(/,\backslash)}$. One can infer from (i) that in $\mathcal{L}^\theta$ there must be the basic expressions: $\langle \beta_i, \langle\rangle, x_i \rangle$. From (ii) one can infer that there must be a rule $\langle \mathcal{F}_P, \langle x_1, \dots, x_n \rangle, y \rangle$, where P is a possible proof of $x_1, \dots, x_n \Rightarrow y$. These two facts about $\mathcal{L}^\theta$ entail that there is a disambiguation of $\beta_1 \dots \beta_n$ that is of category y, namely $\mathcal{F}_P(\langle \beta_1, \langle\rangle, x_1 \rangle, \dots, \langle \beta_n, \langle\rangle, x_n \rangle)$. Therefore, $\mathcal{T}^\theta$ entails the same categorisation fact concerning $\beta_1 \dots \beta_n$ as does θ.

For θ also to entail the meaning relation fact, it must be the case that there are $m_1, \dots, m_n$ which are the meanings of the categorised expressions $[\beta_1]_{x_1}, \dots, [\beta_n]_{x_n}$ according to $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f^L \rangle$

and that $m = H^{\delta}_{L}(P)(m_1, \ldots m_n) = \mathcal{G}_P(m_1, \ldots, m_n)$. By definition of the corresponding INTERPRETATION, $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f \rangle$, for each of the $\langle \beta_i, \langle\rangle, x_i \rangle$, $f(\langle \beta_i, \langle\rangle, x_i \rangle) = m_i$. Therefore by the definition of the homomorphic extension $[\![]\!]$ of f,

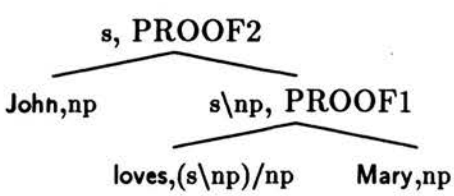
$[\![\mathcal{F}_P(\langle \beta_1, \langle\rangle, x_1 \rangle, \ldots, \langle \beta_n, \langle\rangle, x_n \rangle)]\!] = \mathcal{G}_P(m_1, \ldots, m_n)$

Therefore $\mathcal{T}^\theta$ entails the same meaning relation fact as θ.

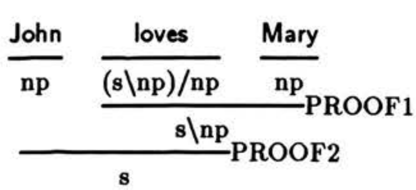
From now on we will not present $\mathrm{L}^{(/,\backslash)}$-theories but instead equivalent THEORIES OF REFERENCE, to be called $\mathrm{L}^{(/,\backslash)}$-THEORIES OF REFERENCE.

The discussion of categorisation and Cut elimination can be transferred to the arena of DISAMBIGUATED LAMBEK LANGUAGES. Where we spoke of 'pursuing the *flat* categorisation strategy' now we should speak of 'search for *flat* members of the carrier set'. The *flat* members are the basic expressions and those triples $< s_1, < \alpha_i >, \gamma >$ where $< \alpha_i >$ is a sequence basic expressions. Where we observed that any categorisation obtained by the non-flat strategy could be obtained using the flat strategy and Cut, now we should observe that the ambiguation of any non-flat member must be the same as the ambiguation of a flat member in whose operation index the Cut rule is repeatedly used.

To display an expression of a DISAMBIGUATED LAMBEK LANGUAGE in full is a rather strenuous task, as witnessed by the picture in (21). Because all syntactic operations of a DISAMBIGUATED LAMBEK LANGUAGE amount to string-concatenation, it is possible to streamline the representation. The non-leaf nodes of the tree in (21) are of the form *string:proof*, where the proof is an $\mathrm{L}^{(/,\backslash)}$ proof of some sequent $U \Rightarrow x$. In the streamlined representation such nodes are replaced by *x:proof*, giving the streamlined version of (21):

(24)


In the usual fashion of categorial grammar, we will usually present such a tree upside down:

(25)


Chapter 5

Arguments for locality and minimality

1 Introduction

In this chapter I will look at some strategies that one may adopt towards accounting for the junctions and quantifiers. Section 2 considers a certain swathe of the possible accounts and identifies a criterion according to which one should prefer over these the Polymorphic categorial account that will be proposed in Chapter 7. The criterion put forward will be called *emergence*, where this roughly is whether or not the account has features whose purpose is simply the explanation of ambiguity. The reason for calling this aspect *emergence* is the fashion in which we will test for it: by seeing whether it is impossible, by some easy simplification, to turn the account from one that explains both the facts concerning semantic ambiguity and the facts concerning extensive privileges of occurrence, into one that explains only the extensive privileges of occurrence. An account for which such a simplification is not possible will be called *emergent*.

The accounts of junctions considered in section 2 are those based on the 'Conjunction Reduction'(CR) transformation, and those based on Gazdar's Cross-categorial Coordination proposal. It is convenient to refer to these as the *non-local* and the *local* approaches, where *locality* is roughly the requirement that the analysis of a sentence builds the sentence from substrings of itself. The accounts of quantification considered are those based on the 'Quantifier Lowering' transformation, and another somewhat nameless, though quite familiar account which we will refer to as Cross-Categorial Quantification. These also will be referred to as the non-local and the local approaches.

These accounts are considered first from the point of view of descriptive adequacy: a descriptively adequate account of junctions and quantifiers would be one that captured both the extensive privileges of occurrence of these expressions and the semantic ambiguities that they engender. None of these non-local and local approaches to junctions and quantifiers are descriptively adequate, at least not if given in their simplest form. It is then considered whether elaborations of the non-local and local approaches can achieve descriptive adequacy. One elaboration is to combine the non-local and local approaches, and such a combination is probably the most often given kind of account. This combined account is descriptively adequate. This means that descriptive adequacy alone cannot be the reason for special interest in the Polymorphic categorial grammar account and that is why attention is then given to the criterion of *emergence*. The Polymorphic categorial grammar account is emergent, though the demonstration of this will have to wait until Chapter 7. What is seen at the end of section 2 is that the descriptively adequate *combined* non-local/local account is not emergent. The possibility of elaborating one of the non-local approaches in some other way than simply adding the local approaches is also considered. Such other elaborations are seen to lead semantic overgeneration, however. It is important to consider such elaborations because the non-local approach though descriptively inadequate, *is* an emergent account.

Therefore, by the end of section 2 a number of accounts of junctions and quantifiers will have been considered and all seen either to be not descriptively adequate or not emergent. This discussion will not have encompassed *all* hitherto proposed accounts, and at the end of the section we will note the accounts that have been missed out and indicate where in the thesis these accounts receive consideration.

Section 2 also has two ancillary purposes to the main one of considering whether accounts are emergent. The first of these is that some of the ground covered should make more readily understandable the Polymorphic categorial grammar account in Chapter 7. This is because the *local* analyses of junctions and quantifiers described in Section 2 involve certain *polymorphic operations*, and if these operations are understood then it may help to regard the proposal of Chapter 7 as a *lexicalisation* of these operations. The second additional purpose of section 2 is to introduce certain pieces of basic 'semantic technology'. Amongst these are (i) the relation between typing and transparency/opacity, (ii) the fundamental denotations for junctions and (iii) the fundamental denotations for determiners.

The way that a course is charted in section 2 through various accounts of junctions and quantifiers is by means of first specifying a core-account and then growing it in various directions. This core-account makes certain choices concerning types. Section 3, entitled 'Arguments for minimality', considers an account that makes a different choice of typing, a so-called 'non-minimal' typing of verbal terms. This is therefore one of the accounts referred to above as being missed by the considerations of section 2. Besides pointing out that this account a further *non-emergent* account, we make in section 3 some objections to this non-minimal typing. The first objection is to the explanation of 'intensional' transitive verbs that the typing has been argued to provide. The second objection is that certain semantic undergenerations can only be got around by using the non-local account of quantifiers and the result is the correlation of the most *natural* interpretation with the most *complex* of analyses. The relevance to the rest of the thesis of these criticisms of the non-minimal typing lies in Chapter 6, the project of which is to show that no account of the junctions and quantifiers can be found using Lambek categorial grammar; the elimination of the option of using non-minimal typing simplifies this project somewhat.

2 Arguments for Locality

In this section we will consider some of the accounts of the junctions and quantifiers that have been proposed and identify a criterion for preferring over these the Polymorphic categorial account that will be put forward in Chapter 7. We will do this by first giving a core-account which explains a number of phenomena but which does not allow for the extensive privileges of occurrence of junctions and quantifiers. A number of different ways of expanding on this core-account will then be considered. The core-account given is intended to be the simplest possible one. Sections 2.1, 2.2 and 2.3 specify this core-account. Section 2.1 covers conventional techniques for handling the syntax and semantics of verbal terms, proper names and sentence and VP embedding constructions. Section 2.2 gives a preliminary account of *junctions* and *quantifiers*, one accounting only for *sentential* occurrences of junctions and *subject* occurrences of quantifiers. This gives an opportunity to show that what one might call the orthodox denotations for junctions and quantifiers may be seen as more or less emergent from the recursive ambiguity data recorded in Chapter 3. There is one final development of the core-account in section 2.3. In order to bring to bear the data in section 3.5, Chapter 3 (the truth intuitions for sentence and VP embedding verbs), one must allow for truth-predicates and higher-order quantification. Section 2.3.1 discusses the problem that was postponed from section 3.5, Chapter 3 - the problem of truth paradoxes.

In section 2.4 we present two broad approaches to expanding the core-account into one encompassing the non-sentential junctions, approaches that may be divided by a *locality* criterion. This was mentioned in the above Introduction, and now it will be defined: a disambiguation $\overline{\alpha}$ is *local* if the *subparts* are disambiguations of *substrings* of α (the 'over-line' notation will be used to refer to a member of the syntactic algebra by mentioning only its *string* part). The non-local approach to junctions is that associated with the transformation of Conjunction Reduction, and this is described in section 2.4.1. Section 2.4.2 describes a local approach to junctions, and it is the UG embodiment of Gazdar's Cross-categorial Coordination proposal.

Section 2.5 is a sister section to section 2.4, this time concerning quantifiers. The non-local approach is the UG version of the transformational idea of Chomsky-adjunction and Quantifier lowering, and this is described in section 2.5.1. The other approach is local and somewhat nameless, though a familiar enough part of semantic 'folk-wisdom'. It will be referred to as Cross-categorial Quantification, and is described in section 2.5.2.

Each of the sections 2.4.1, 2.4.2, 2.5.1 and 2.5.2 not only describes an approach but also semantically assesses it. None of the approaches is descriptively adequate. In both the case of junctions and quantifiers, although the non-local approach is a prolific generator of readings, there remain some problems of semantic undergeneration. The local approaches are less prolific generators of readings, but the undergenerations of the local approaches do not intersect with the undergenerations of the non-local approaches.

Section 2.6 considers more fully the accounts that are possible with devices described in section 2.4 and section 2.5, and in particular whether it is possible to obtain a descriptively adequate, emergent account. A *combined* account is considered, which remedies the undergenerations of the non-local accounts by adding the local mechanisms. This gives a descriptively adequate but not emergent account. Another account is considered which remedies the under-generations of the non-local approaches in some other way than simply incorporating the local analyses. This involves introducing several other kinds of non-locality. Objections to these additional instances of non-locality are (i) they cannot be given the syntactic motivation that can be given for the non-local junction and quantifier analyses, and (ii) they introduce certain kinds of semantic *over*-generation of their own.

This completes the task of section 2, which was to show that a number of possible accounts are not emergent. At the very end of section 2.6 there is an indication of which accounts are left to be considered and a moral is drawn concerning the direction in which one must go to obtain an emergent account.

2.1 Basic Montagovian semantics

2.1.1 Verbal terms and Proper names

In this section a simple THEORY OF REFERENCE will be defined, accounting for the syntax and semantics of English sentences consisting of verbal terms and proper names. First the DISAMBIGUATED LANGUAGE part :

$\mathcal{L}^4$: verbal terms and proper names

1. set of phrase set indices: $\Delta = \{\mathrm{NP}, \mathrm{VP}, \mathrm{TV}, \mathrm{TTV}, \mathrm{S}\}$
2. the family of sets of basic δ-phrases: $\mathcal{X}_{\mathrm{NP}} = \{\langle \mathsf{John}, \langle\rangle, \mathrm{NP}\rangle, \langle \mathsf{Mary}, \langle\rangle, \mathrm{NP}\rangle\}$, $\mathcal{X}_{\mathrm{VP}} = \{\langle \mathsf{walks}, \langle\rangle, \mathrm{VP}\rangle\}$, $\mathcal{X}_{\mathrm{TV}} = \{\langle \mathsf{loves}, \langle\rangle, \mathrm{TV}\rangle, \langle \mathsf{is}, \langle\rangle, \mathrm{TV}\rangle\}$, $\mathcal{X}_{\mathrm{TTV}} = \{\langle \mathsf{gave}, \langle\rangle, \mathrm{TTV}\rangle\}$
3. the syntactic operations: $\Gamma = \{<, >\}$ The string part of both $\mathcal{F}_<$ and $\mathcal{F}_>$ is *concatenation*
4. the syntactic rules: $\{\ \langle \mathcal{F}_<, \langle \mathrm{NP}, \mathrm{VP}\rangle, \mathrm{S}\rangle,\ \langle \mathcal{F}_>, \langle \mathrm{TV}, \mathrm{NP}\rangle, \mathrm{VP}\rangle,\ \langle \mathcal{F}_>, \langle \mathrm{TTV}, \mathrm{NP}\rangle, \mathrm{TV}\rangle\ \}$

As was noted in Chapter 2, the least restriction one can expect on a set of possible models is a restriction relating the operations of the algebras of the models to each other. The easiest way to do this seems to be define 'algebra-spanning' functions, which take any possible choice of $\mathcal{E}$, $\mathcal{I}$ and $\mathcal{J}$, into some particular operation defined over the meanings sets generated by that choice of $\mathcal{E}$, $\mathcal{I}$ and $\mathcal{J}$. The operations of an allowable algebra associated with a particular $\mathcal{E}$, $\mathcal{I}$ and $\mathcal{J}$ are then the values of such 'algebra-spanning' functions at $\mathcal{E}$, $\mathcal{I}$ and $\mathcal{J}$. Before giving the set of possible models to be associated with $\mathcal{L}^4$ we will first define four such 'algebra spanning' functions: $\mathcal{H}^E_<$, $\mathcal{H}^E_>$, $\mathcal{H}_<$ and $\mathcal{H}_>$. The superscript 'E' indicates that $\mathcal{H}^E_<$ and $\mathcal{H}^E_>$ are functions which when applied to a choice of $\mathcal{E}$, $\mathcal{I}$ and $\mathcal{J}$ return an operation on *denotations*, unlike $\mathcal{H}_<$ and $\mathcal{H}_>$, which return operations on *meanings*.

Definition 36 ($\mathcal{H}^E_<$,$\mathcal{H}^E_>$,$\mathcal{H}_>$, and $\mathcal{H}_<$) *For any sets $\mathcal{E}$, $\mathcal{I}$ and $\mathcal{J}$, for any types a,b $\in$ TJ$^{\rightarrow}$, for any $d_1 \in D_a$, any $d_2 \in D_{(a\rightarrow b)}$,*
$\mathcal{H}^E_<(\mathcal{E},\mathcal{I},\mathcal{J})(d_1,d_2) = d_2(d_1)$,
$\mathcal{H}^E_>(\mathcal{E},\mathcal{I},\mathcal{J})(d_2,d_1) = d_2(d_1)$,
and for any $m_1 \in M_a$, any $m_2 \in M_{(a\rightarrow b)}$, any $(w,j) \in \mathcal{I} \times \mathcal{J}$
$\mathcal{H}_<(\mathcal{E},\mathcal{I},\mathcal{J})(m_1,m_2)(w,j) = \mathcal{H}^E_<(\mathcal{E},\mathcal{I},\mathcal{J})(m_1(w,j),m_2(w,j))$,
$\mathcal{H}_>(\mathcal{E},\mathcal{I},\mathcal{J})(m_2,m_1)(w,j) = \mathcal{H}^E_>(\mathcal{E},\mathcal{I},\mathcal{J})(m_2(w,j),m_1(w,j))$

These operations, and others that will be subsequently defined, will be assumed to return the undefined object at arguments of types other than those mentioned in the operation definition. $\mathcal{H}_>(\mathcal{E},\mathcal{I},\mathcal{J})$ (resp. $\mathcal{H}_<(\mathcal{E},\mathcal{I},\mathcal{J})$) will be referred to as the *extensional forward (resp. backward) function application operation.* We will now define the set of possible models for $\mathcal{L}^4$:

> **The set $\mathcal{K}^1$ of possible models for $\mathcal{L}^4$:** $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,j\rangle\rangle$ associated with $\mathcal{E},\mathcal{I},\mathcal{J},\nu$ is $\in \mathcal{K}^1$ iff
>
> 1. Type mapping: $\nu^1(\text{NP}) = e$, $\nu^1(\text{S}) = t$, $\nu^1(\text{VP}) = (e\rightarrow t)$, $\nu^1(\text{TV}) = (e\rightarrow(e\rightarrow t))$, $\nu^1(\text{TTV}) = (e\rightarrow(e\rightarrow(e\rightarrow t)))$
> 2. Constraints on f: only the Copula Postulate, defined below. The expressions with respect to which f is unconstrained are said to be *freely* interpreted in their type.
> 3. Algebraic constraints: $\Gamma = \{<,>\}$, $\mathcal{G}_< = \mathcal{H}_<(\mathcal{E},\mathcal{I},\mathcal{J})$, $\mathcal{G}_> = \mathcal{H}_>(\mathcal{E},\mathcal{I},\mathcal{J})$.

To give the Copula Postulate it is convenient to first define an 'algebra spanning' function $\mathcal{IS}$:

Definition 37 ($\mathcal{IS}$) *For any $\mathcal{E},\mathcal{I},\mathcal{J}$, for any $\langle w,j\rangle \in \mathcal{I} \times \mathcal{J}$, for any $d_1,d_2 \in D_e$,*
$\mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})(\langle w,j\rangle)(d_1)(d_2) = 1$ *iff* $d_1 = d_2$

Definition 38 (Copula Postulate) *For any model $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,j\rangle\rangle \in \mathcal{K}^1$, $f(\langle\text{is},\langle\rangle,\text{TV}\rangle) = \mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})$*

$\mathcal{T}^1$ will be understood to be the THEORY OF REFERENCE defined by the combination of $\mathcal{L}^4$ and $\mathcal{K}^1$. Because of the limited syntactic coverage, $\mathcal{T}^1$ can only be assessed by the following data on *transparent occurrences* (see (19a,b, p36) in Chapter 3, and recall that subscript e means transparency on all readings):

(1) a John$_e$ walks$_e$

b John$_e$ (loves$_e$ Dave$_e$)$_e$

$\mathcal{T}^1$ accounts for both of these. Consider (1a). Taking into account the fact that $\mathcal{T}^1$ provides only one disambiguation of an expression if it provides any, the condition for $\mathcal{T}^1$ to account for the unambiguous transparency of the occurrence of John in (1a) is :

(2) there is no $\overline{\alpha}$, where α is a referring expression, and no model, $\langle \Im, \langle w, j\rangle\rangle \in \mathcal{K}^1$, such that (i) $[\![\overline{\textsf{John is } \alpha}]\!](w,j) = 1$ and (ii) $[\![\overline{\textsf{John walks}}]\!](w,j) \neq [\![\overline{\alpha \textsf{ walks}}]\!](w,j)$.

One can show that the co-extension clause (i) entails identity of denotation, whilst (ii) entails difference of denotation. In other words, (i) and (ii) contradict each other:

(3) Entailments of (i):

$[\![\overline{\textsf{John is } \alpha}]\!](w,j) = 1$

$\leftrightarrow [\![\mathcal{F}_<(\overline{\textsf{John}}, \mathcal{F}_>(\overline{\textsf{is}}, \overline{\alpha}))]\!](w,j) = 1$

$\leftrightarrow [\![\overline{\textsf{is}}]\!](w,j)([\![\overline{\alpha}]\!](w,j))([\![\overline{\textsf{John}}]\!](w,j)) = 1$ (by definition of $\mathcal{G}_>$, $\mathcal{G}_<$)

$\leftrightarrow [\![\overline{\textsf{John}}]\!](w,j) = [\![\overline{\alpha}]\!](w,j)$ (by the meaning postulate for is)

(4) Entailments of (ii):

$[\![\overline{\textsf{John walks}}]\!](w,j) \neq [\![\overline{\alpha \textsf{ walks}}]\!](w,j)$

$\leftrightarrow \mathcal{G}_<([\![\overline{\textsf{John}}]\!], [\![\overline{\textsf{walks}}]\!])(w,j) \neq \mathcal{G}_<([\![\overline{\alpha}]\!], [\![\overline{\textsf{walks}}]\!])(w,j)$

$\leftrightarrow \mathcal{G}^E_<([\![\overline{\textsf{John}}]\!](w,j), [\![\overline{\textsf{walks}}]\!](w,j)) \neq \mathcal{G}^E_<([\![\overline{\alpha}]\!](w,j), [\![\overline{\textsf{walks}}]\!](w,j))$ (by definition of $\mathcal{G}_<$, and using the notation $\mathcal{G}^E_<$ for $\mathcal{H}^E_<(\mathcal{E}, \mathcal{I}, \mathcal{J})$)

$\leftrightarrow [\![\overline{\textsf{John}}]\!](w,j) \neq [\![\overline{\alpha}]\!](w,j)$

This explanation of the transparency of the occurrence John is often described with the following phrase: 'the denotation of $\overline{\textsf{John walks}}$ is a function of the denotation of $\overline{\textsf{John}}$'. This is a rather poor shorthand for 'whatever (w,j), there is function, f, such that whatever referring expression α, $[\![\overline{\alpha \textsf{ walks}}]\!](w,j) = f([\![\overline{\alpha}]\!](w,j))$'.

The transparency of the object NP position of (1b) is explained in like fashion. Finally the unambiguous transparency of the occurrences of walks in (1a) and loves in (1b) follow simply from the definition of transparency.

$\mathcal{T}^1$ is an example of *free* interpretation in the *right* types accounting for *transparency*. The type mapping of $\mathcal{T}^1$, ν^1, embodies what I wish to refer to as the hypothesis of *Minimal Types*. This comment will make more sense after the next section has been considered, where other typings will be considered under which *free* interpretation would not account for transparency, and instead careful constraint of interpretation by *Meaning Postulates* is required to account for transparency.

2.1.2 Embedding Verbs

With just the two syntactic operations that have so far been introduced, one can widen the syntactic coverage to encompass *sentence* embedding constructions simply (if crudely, for the complementiser that will be ignored) :

$\mathcal{L}^5$: incorporating sentence embedding constructions

1. Phrase set indices: as for $\mathcal{L}^4$, with the addition of PV
2. Sets of basic δ-phrases: as for $\mathcal{L}^4$ with the addition of $\mathcal{X}_{\mathrm{PV}} = \{\langle \mathsf{believe}, \langle\rangle, \mathrm{PV}\rangle\}$
3. Syntactic Operations: as for $\mathcal{L}^4$
4. Syntactic rules: as for $\mathcal{L}^4$ with the addition of: $\langle \mathcal{F}_>, \langle \mathrm{PV}, \mathrm{S}\rangle, \mathrm{VP}\rangle$

To extend syntactic coverage to the VP embedding constructions presents more difficulty, because of the obligatory presence of the infinitising to. However, there are semantic problems associated with the above syntactic analysis. The use of $\mathcal{F}_>$ in the rule to combine a sentence embedding verb with a sentence dictates that the semantic value of the combination be obtained by using the $\mathcal{G}_>$ operation, which in turn dictates that the type associated with such verbs is $(t \rightarrow (e \rightarrow t))$ (otherwise the value returned by $\mathcal{G}_>$ will be the undefined object). This will fail to respect the fact that embedded sentences occur unambiguously *opaquely*, as noted in (19f), Chapter 3. For example, to account for the fact that Mary walks occurs *opaquely* in John believes Mary walks, we require:

(5) there is an $\overline{\alpha}$, where α is a sentence, and there is a model, $\langle \Im, \langle w, g\rangle\rangle$, such that (i) $[\![\overline{\text{Mary walks}}]\!](w,j) = [\![\overline{\alpha}]\!](w,j)$ and (ii) $[\![\overline{\text{John believes Mary walks}}]\!](w,j) \neq [\![\overline{\text{John believes } \alpha}]\!](w,j)$

(ii) contradicts (i), as shown below (using the abbreviation m_1 for $[\![\overline{\text{Mary walks}}]\!]$, m_2 for $[\![\overline{\alpha}]\!]$, J for $[\![\overline{\text{John}}]\!]$, B for $[\![\overline{\text{believes}}]\!]$):

(6) Entailment of (ii):

$$[\![\overline{\text{John believes Mary walks}}]\!](w,j) \neq [\![\overline{\text{John believes } \alpha}]\!](w,j)$$
$$\leftrightarrow \mathcal{G}_<(J, \mathcal{G}_>(B, m_1))(w,j) \neq \mathcal{G}_<(J, \mathcal{G}_>(B, m_2))(w,j)$$
$$\leftrightarrow \mathcal{G}^E_>(J(w,j), \mathcal{G}_>(B, m_1)(w,j)) \neq \mathcal{G}^E_>(J(w,j), \mathcal{G}_>(B, m_2)(w,j))$$
$$\leftrightarrow \mathcal{G}^E_>(J(w,j), \mathcal{G}^E_>(B(w,j), m_1(w,j))) \neq \mathcal{G}^E_>(J(w,j), \mathcal{G}^E_>(B(w,j), m_2(w,j)))$$
$$\leftrightarrow m_1(w,j) \neq m_2(w,j)$$
$$\leftrightarrow [\![\overline{\text{Mary walks}}]\!](w,j) \neq [\![\overline{\alpha}]\!](w,j)$$

Because (ii) contradicts (i), (5) is false, and the opacity fact is not accounted for. The standard [1] response to this problem is to change the typing assumption for sentence embedding verbs so that they have type $((s \rightarrow t) \rightarrow (e \rightarrow t))$ and change the semantic operation that is implicated in the derivation of a disambiguation of believes α to the operation of *Intensional Forward Function Application*, the 'algebra-spanning' definition of which is:

[1] Montague 68 treats belief this way.

Definition 39 ($\mathcal{H}_{<_i}$**and** $\mathcal{H}_{>_i}$) *For any sets* $\mathcal{E},\mathcal{I},\mathcal{J}$*, for any types* $a,b \in$ TJ$^{\rightarrow}$*, for any* $m_1 \in M_a$*,* $m_2 \in M_{((s\rightarrow a)\rightarrow b)}$*, any* $(w,j) \in \mathcal{I} \times \mathcal{J}$*,*
$\mathcal{H}_{<_i}(\mathcal{E},\mathcal{I},\mathcal{J})(m_1,m_2)(w,j) = m_2(w,j)(w' \mapsto m_1(w',j))$,
$\mathcal{H}_{>_i}(\mathcal{E},\mathcal{I},\mathcal{J})(m_2,m_1)(w,j) = m_2(w,j)(w' \mapsto m_1(w',j))$

It is common to regard $\mathcal{G}_{>_i}$ as the combination of $\mathcal{G}_>$ with a 'taking the intension' operation, which we shall call $\mathcal{G}\uparrow$. This operation turns a meaning of type t into a meaning of type $(s\rightarrow t)$, the new meaning effectively denoting at every (w,j) the old meaning (note the definition makes it the case that $\mathcal{G}_{>_i}(m_1,m_2) = \mathcal{G}_>(m_1,\mathcal{G}_\uparrow(m_2))$):

Definition 40 ($\mathcal{H}_\uparrow$) *For any sets* $\mathcal{E},\mathcal{I},\mathcal{J}$*, for any type* $a \in$ TJ$^{\rightarrow}$*, for any* $m_1 \in M_a$*, any* $(w,j) \in \mathcal{I} \times \mathcal{J}$*,* $\mathcal{H}_\uparrow(\mathcal{E},\mathcal{I},\mathcal{J})(m_1) = (w,j) \mapsto w' \mapsto m_1(w',j)$

If the class of models for $\mathcal{L}^5$ were defined to associate $\mathcal{G}_{>_i}$ with $\mathcal{F}_>$, and $\mathcal{G}_<$ with $\mathcal{F}_<$, the opacity of **Mary walks** in **John believes Mary walks** would be accounted for. Referring back to (5), the condition for opacity, this time from (ii) cannot be deduced the negation of (i):

(7) Entailment of (ii):

$$[\![\overline{\text{John believes Mary walks}}]\!](w,j) \neq [\![\overline{\text{John believes } \alpha}]\!](w,j)$$
$$\leftrightarrow \mathcal{G}_<(J,\mathcal{G}_{>_i}(B,m_1))(w,j) \neq \mathcal{G}_{<_i}(J,\mathcal{G}_>(B,m_2))(w,j) =$$
$$\leftrightarrow \mathcal{G}^E_<(J(w,j),\mathcal{G}_{>_i}(B,m_1)(w,j)) \neq \mathcal{G}^E_<(J(w,j),\mathcal{G}_{>_i}(B,m_2)(w,j))$$
$$\leftrightarrow \mathcal{G}^E_>(J(w,j),B(w,j)(w' \mapsto m_1(w',j))) \neq \mathcal{G}^E_>(J(w,j),B(w,j)(w' \mapsto m_2(w',j)))$$
$$\leftrightarrow (w' \mapsto m_1(w',j)) \neq (w' \mapsto m_2(w',j))$$
$$\not\leftrightarrow [\![\overline{\text{Mary walks}}]\!](w,j) \neq [\![\overline{\alpha}]\!](w,j)$$

This explanation of opacity is sometimes summarised as: 'the denotation of $\overline{\text{John believes Mary walks}}$ is not a function of the denotation of $\overline{\text{Mary walks}}$'. This is rather poor shorthand for: 'it is not the case that whatever (w,j), there is function, f, such that whatever sentence α, $[\![\overline{\text{John believes } \alpha}]\!](w,j) = f([\![\overline{\alpha}]\!](w,j))$'. It should also be noted that Hypothesis 1 concerning the *downward* heritability of opacity, would be respected if the class of models for $\mathcal{L}^5$ was as is now being considered.

This explains how to achieve opaque effects for the combination of a sentence embedding verb with a sentence. There now arises the problem of how to fit this into a system like the pairing of $\mathcal{L}^4$ and $\mathcal{K}^1$, a system that succeeds in explaining some of the transparency data. The problem is that syntactically the facts suggest that we use the same syntactic operations to combine a TV with an object NP as we use to combine a sentence embedding verb with a sentence, the operation $\mathcal{F}_>$. However, to render occurrence of NP complements of TV's *transparent* it was necessary to associate $\mathcal{F}_>$ with the *extensional* function application operation $\mathcal{G}_>$, whereas to render the occurrence of sentential complements of sentence embedding verbs *opaque* it would be necessary to associate $\mathcal{F}_{>}$ with the *intensional* function application operation, $\mathcal{G}_{>_i}$. There are two strategies for responding to this problem.

The first strategy does not alter at all the syntactic choices that together constitute $\mathcal{L}^5$. Some of the semantic choices of $\mathcal{K}^1$ are revised, taking the lead from the devices used to achieve opacity for sentence embedding verbs: the category-to-type mapping is 'intensionalised' and $\mathcal{F}_{>}$ and $\mathcal{F}_{<}$ are associated not with $\mathcal{G}_{<}$ and $\mathcal{G}_{>}$ but with $\mathcal{G}_{<_i}$ and $\mathcal{G}_{>_i}$. This on the face of it undermines the account of the *transparency* data, but these are explained by placing additional constraints on interpretation functions, constraints that we will call Extensionality meaning postulates.

> **The set $\mathcal{K}^2$ of possible models for $\mathcal{L}^5$:** $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, j\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^2$ iff
>
> 1. Type mapping: if $\nu^1(\delta) = (a{\rightarrow}b)$ then $\nu^2(\delta) = ((s{\rightarrow}a){\rightarrow}b)$. Also $\nu^2(\mathrm{NP}) = e$, $\nu^2(\mathrm{S}) = t$, $\nu^2(\mathrm{PV}) = ((s{\rightarrow}t){\rightarrow}((s{\rightarrow}e){\rightarrow}t))$.
> 2. Constraints on f^2: expressions of type e are freely interpreted, but with respect to all expressions of functional type, f^2 is subject to the meaning postulates in Definition 42, below.
> 3. Algebraic constraints: $\Gamma = \{<_i, >_i\}$. In the usual way $\mathcal{G}_\gamma = \mathcal{H}_\gamma(\mathcal{E}, \mathcal{I}, \mathcal{J})$.

Before defining the meaning postulates for $\mathcal{K}^2$, it is of some convenience to first define a map, $\mathcal{I}^n$ which converts a meaning which in its nth argument place takes denotations of type b to a meaning which in its n^th^ argument place takes denotations of type $(s{\rightarrow}b)$. [2]

Definition 41 ($\mathcal{I}^n$) *Whatever types $\vec{a}$, b, $c \in \mathrm{TJ}^{\rightarrow}$ whatever meaning $m_1{}^{(\vec{a}\rightarrow(b\rightarrow c))}$, whatever sequence of denotations $\vec{x}^{\vec{a}}$, whatever denotation $y^{(s\rightarrow b)}$ and whatever $(w, j) \in \mathcal{I} \times \mathcal{J}$,*
$\mathcal{I}^n(m_1{}^{(\vec{a}\rightarrow(b\rightarrow c))})(w, j)(\vec{x})(y^{(s\rightarrow b)}) = m_1{}^{(\vec{a}\rightarrow(b\rightarrow c))}(w, j)(\vec{x})(yw)$

Definition 42 (Meaning Postulates for $\mathcal{K}^2$) *Whatever model, $\langle \Im, \langle w, j\rangle\rangle$, whatever $\alpha \in \mathcal{A}^5$*
if $\alpha = \langle \textsf{is}, \langle\rangle, \mathrm{TV}\rangle$, then $f^2(\alpha) = \mathcal{I}^1\mathcal{I}^2(\mathcal{IS}(\mathcal{E}, \mathcal{I}, \mathcal{J}))$ (Revised Copula Postulate)

$$\left.\begin{array}{l}
\text{if } \alpha \in \mathcal{X}^5_{\mathrm{VP}}, \text{ then there exists } m_1{}^{(e\rightarrow t)} \text{ such that } f^2(\alpha) = \mathcal{I}^1(m_1) \\
\text{if } \alpha \in \mathcal{X}^5_{\mathrm{TV}}, \text{ then there exists } m_1{}^{(e\rightarrow(e\rightarrow t))} \text{ such that } f^2(\alpha) = \mathcal{I}^1\mathcal{I}^2(m_1) \\
\text{if } \alpha \in \mathcal{X}^5_{\mathrm{TTV}}, \text{ then there exists } m_1{}^{(e\rightarrow(e\rightarrow(e\rightarrow t)))} \text{ such that } f^2(\alpha) = \mathcal{I}^1\mathcal{I}^2\mathcal{I}^3(m_1) \\
\text{if } \alpha \in \mathcal{X}^5_{\mathrm{PV}}, \text{ there exists } m_1{}^{((s\rightarrow t)\rightarrow(e\rightarrow t))} \text{ such that } f^2(\alpha) = \mathcal{I}^2(m_1)
\end{array}\right\} \textit{Extension-ality Postulates}$$

Let $\mathcal{T}^2$ be the combination of $\mathcal{L}^5$ with $\mathcal{K}^2$. Note that the *Revised Copula Postulate*, like the previous version, rules out all but one of the meanings possible for is given its type, whereas although the *Extensionality Postulates* rule out many of the meanings possible for the expressions

[2] The definition uses the notation of overlining with an arrow several times. This is an attempt to schematize what properly should be a recursive definition namely: $\mathcal{I}^n(m^{(a,b)})(w, j) = i^n(m(w, j), w)$; $i^n(d^{(a,c)}, w)(x^a) = i^{n-1}(d(x), w)$; $i^1(d^{(b,c)}, w)(y^{(s,b)}) = d(yw)$

concerned given their type, many possibilities are left remaining. The *Extensionality Postulates* basically say of certain expressions that their meaning in a MODEL in $\mathcal{K}^2$ must be derivable by some applications of the $\mathcal{I}^n$ operation from what *would* have been their meaning in a MODEL in $\mathcal{K}^1$. Below is an illustration of how these meaning postulates preserve the coverage of the transparency data.

That $\mathcal{T}^2$ accounts for the transparency of John *in* John walks

We can show that what is required for the opacity of the occurrence of John in walks is impossible. For opacity, the condition in (2) is required, repeated below:

there must be an α and a model $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,j\rangle\rangle\in\mathcal{K}^2$ such that (i) $[\![\text{John is }\alpha]\!](w,j)=1$ (ii) $[\![\text{John walks}]\!](w,j)\neq[\![\alpha\text{ walks}]\!](w,j)$

As before we wish to show that (i) and (ii) contradict each other, only this time the reasoning is a little different.

Entailments of (i):

$\rightarrow [\![\text{is}]\!](w,j)(w'\mapsto[\![\alpha]\!](w',j))(w'\mapsto[\![\text{John}]\!](w',j))=1$

$\rightarrow [\![\text{John}]\!](w,j)=[\![\alpha]\!](w,j)$ (because of the *Revised Copula Postulate*)

Entailments of (ii):

$\rightarrow [\![\text{walks}]\!](w,j)(w'\mapsto[\![\text{John}]\!](w',j'))\neq[\![\text{walks}]\!](w,j)(w'\mapsto[\![\alpha]\!](w',j))$

$\rightarrow$ there is an $m^{(e\rightarrow t)}$ such that $\mathcal{I}^1(m)(w,j)(w'\mapsto[\![\text{John}]\!](w',j))\neq\mathcal{I}^1(m)(w,j)(w'\mapsto[\![\alpha]\!](w',j))$ (because of the *Extensionality Postulate*)

$\rightarrow$ there is an $m^{(e\rightarrow t)}$ such that $m(w,j)([\![\text{John}]\!](w,j))\neq m(w,j)([\![\alpha]\!](w,j))$

$\rightarrow [\![\text{John}]\!](w,j)\neq[\![\alpha]\!](w,j)$

So (i) and (ii) contradict each other, and the condition for opacity is impossible □

The second strategy for achieving coverage of both the transparency and opacity data is to import the distinction between transparent and opaque occurrence into the syntactic domain in some form or another: though one *can* use the same syntactic operations to combine a TV with an object NP as is used to combine a sentence embedding verb with a sentence, one does not *have* to use the same operation. For the sentence embedding case one can define another kind of concatenation, $\mathcal{F}_{>_i}$, one made invisibly different in its effects from other kinds by means of the disambiguation relation. $\mathcal{F}_{>}$ can then be associated with $\mathcal{G}_{>}$, whilst $\mathcal{F}_{>_i}$ is associated with $\mathcal{G}_{>_i}$. Alternatively, one could view the embedding verb as not being combined with *sentence* but with a derivative of a sentence, in the derivation of which the $\mathcal{G}_{\uparrow}$ operation is implicated. This is what is done in the DISAMBIGUATED LANGUAGE defined below.

$\mathcal{L}^6$: a revision of $\mathcal{L}^5$ which recognises 'intensionalisation' as a syntactic operation

1. Phrase-set indices: as for $\mathcal{L}^5$, with the addition of SC, (for sentential complement)
2. Basic Phrase sets: as for $\mathcal{L}^5$

3. Syntactic Operations: $\Gamma^6 = \{<, >, \uparrow, \}$. $\mathcal{F}_<$ and $\mathcal{F}_>$ are as before. $\mathcal{F}_\uparrow$ as an operation on strings simply prefaces a string with that
4. Syntactic Rules: as for $\mathcal{L}^5$ except that the PV rule now features the category SC: $\langle \mathcal{F}_>, \langle \text{PV}, \text{SC}\rangle, \text{VP}\rangle$, and there is the additional rule: $\langle \mathcal{F}_\uparrow, \langle \text{S}\rangle, \text{SC}\rangle$

The set $\mathcal{K}^3$ of possible models for $\mathcal{L}^6$: $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, j\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^3$ iff

1. Type Mapping: extension of the *Minimal Types* mapping given for $\mathcal{K}^1$ as follows: $\nu^3(\text{PV}) = ((s{\rightarrow}t){\rightarrow}(e{\rightarrow}t))$, $\nu^3(\text{SC}) = (s{\rightarrow}t)$
2. Constraints on f^3: only that f^3 is subject to the Copula Postulate, Definition 38. Other than this all expressions are freely interpreted in their type.
3. Algebraic Constraints: $\Gamma = \{<, >, \uparrow\}$. In the usual way $\mathcal{G}_\gamma = \mathcal{H}_\gamma(\mathcal{E}, \mathcal{I}, \mathcal{J})$

Let $\mathcal{T}^3$ be the combination of $\mathcal{L}^6$ and $\mathcal{K}^3$. One could proceed to the consideration of the VP-embedding verbs on the basis of either $\mathcal{T}^2$ or $\mathcal{T}^3$. The latter will be adopted, the reason being the greater simplicity of the definition of $\mathcal{K}^3$ than the definition of $\mathcal{K}^2$. In particular, the interpretation function for models in $\mathcal{K}^3$ is constrained only with respect to is, whereas that for models in $\mathcal{K}^2$ is constrained with respect to almost every expression.

To increase the linguistic coverage to cover VP embedding constructions it should be recalled that the VP complement position was an opaque one (see section 3.1 of Chapter 3, example (19g), p36). Therefore the VP embedding verbs will have as an argument type $(s{\rightarrow}(e{\rightarrow}t))$. Mirroring the syntactic analysis of the sentence embedding verbs we will suppose that the embedding verb is not combined with a VP directly but with a descendant of the VP. The descendant is the infinitised form, and the associated semantic operation is $\mathcal{G}_\uparrow$.

$\mathcal{L}^7$: incorporating VP embedding constructions

1. Phrase-set indices: as for $\mathcal{L}^6$, with the addition of VVP,TVVP and VPC.
2. Basic phrase-sets: as for $\mathcal{L}^6$ with the addition of: $\mathcal{X}^7_{\text{VVP}} = \{\langle \text{wants}, \langle\rangle, \text{VVP}\rangle\}$, $\mathcal{X}^7_{\text{TVVP}} = \{\langle \text{told}, \langle\rangle, \text{TVVP}\rangle, \langle \text{asked}, \langle\rangle, \text{TVVP}\rangle\}$
3. Syntactic Operations: as for $\mathcal{L}^6$ with the addition of $\mathcal{F}_{(vp,\uparrow)}$: $\mathcal{F}_{(vp,\uparrow)}$ as an operation on strings simply prefaces the string with to.
4. Syntactic Rules: as for $\mathcal{L}^6$ with the addition of $\langle \mathcal{F}_>, \langle \text{TVVP}, \text{NP}\rangle, \text{VVP}\rangle$, $\langle \mathcal{F}_>, \langle \text{VVP}, \text{VPC}\rangle, \text{VP}\rangle$, $\langle \mathcal{F}_{(vp,\uparrow)}, \langle \text{VP}\rangle, \text{VPC}\rangle$

The set $\mathcal{K}^4$ of possible models for $\mathcal{L}^7$: $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, j\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^4$ iff

1. Type mapping: the extension of that for $\mathcal{K}^3$ as follows: $\nu^4(\text{VVP}) = ((s \rightarrow (e \rightarrow t)) \rightarrow (e \rightarrow t))$, $\nu^4(\text{TVVP}) = (e \rightarrow ((s \rightarrow (e \rightarrow t)) \rightarrow (e \rightarrow t)))$, $\nu^4(\text{VPC}) = (s \rightarrow (e \rightarrow t))$
2. Constraints on f^4: only that f^4 is subject to the Copula Postulate, Definition 38.
3. Algebraic constraints: as for $\mathcal{K}^3$. Additionally $\mathcal{G}_{\uparrow}$ also bears the index $(vp, \uparrow)$.

Let $\mathcal{T}^4$ be the the combination of $\mathcal{L}^7$ and $\mathcal{K}^4$. $\mathcal{T}^4$ is a THEORY OF REFERENCE that accounts for the syntactic and semantic properties of a fragment of English. In fact, the only [3] semantic facts brought to bear so far have concerned transparency and opacity, and on a particular typing assumption, (ν^4), this data has been accounted for invoking only a meaning postulate to fix the denotation of is. This is, however, is only the first of three phases of the development of the core-account from which we will go on to consider the extensive priveleges of occurrence of junctions and quantifiers. What we shall do now is turn to more extensive fragments of English that include junctions and quantifiers, but only in limited contexts, in particular, junctions between sentences, and quantifiers in subject position. More of the data set forth in Chapter 3 may then be brought to bear, including additional transparency data and recursive ambiguity data for junctions and quantifiers. We shall see once again that a certain typing assumption allows the explanation of the *transparency data* without the invocation of meaning postulates. We shall also see that to explain a small part of the recursive ambiguity data, the interpretation function with respect to junctions and quantifiers must be subject to the same kind of strict constraint as already been seen in the Copula Postulate, in effect requiring the constants to have the same denotation in every model.

2.2 Sentential Junctions and Subject Quantifiers

2.2.1 Syntax and Semantics for Sentential Junctions

What denotations should be chosen for sentential junctions ? Historically the answer comes from the truth-functional semantics for propositional logic. What we will do in this section is to see how the denotations are basically determined by a small part of the recursive ambiguity data that was set forth in Chapter 3. To do this, however, we must make some assumptions about the syntax of the structures using junctions. [4] Firstly junctions will be assigned their own category JUNCT. For rules we have two choices. On the one hand we could continue to have just binary rules and add the following:

$\langle \mathcal{F}_{>}, \langle \text{JUNCT}, \text{S} \rangle, \text{JUNCTS} \rangle$, $\langle \mathcal{F}_{<}, \langle \text{S}, \text{JUNCTS} \rangle, \text{S} \rangle$

[3] The further data concerning sentence embedding verbs that is expressed in Hypothesis 5 cannot be brought to bear because the language is too impoverished to express the conclusions of the inferences involved.

[4] one can alternatively assume the denotations and let the data determine the semantic operations.

On the other hand, we can use a ternary rule which implicates a syntactic operation that has not been encountered before:

$\langle \mathcal{F}_{\mathcal{J}}, \langle \text{S}, \text{JUNCT}, \text{S} \rangle, \text{S} \rangle$

On grounds of theoretical economy there is nothing to chose between them: using the binary approach requires no extension of the pool of syntactic operations but does require an increase in the pool of phrase-set indices, (to include JUNCTS), whereas the ternary approach requires an extension of the syntactic operations, (to include $\mathcal{F}_{\mathcal{J}}$), but preserves the phrase-set indices. We will adopt the ternary approach. $\mathcal{F}_{\mathcal{J}}$ will be associated with a semantic operation $\mathcal{G}_{\mathcal{J}}$. As was the case with $\mathcal{G}_{>}$ and $\mathcal{G}_{<}$, this operation will be required to be definable in terms of a certain operation on denotations, $\mathcal{G}^{E}_{\mathcal{J}}$. The following definition of $\mathcal{G}_{\mathcal{J}}$ defines it only at triples of arguments of type $t, (t \rightarrow (t \rightarrow t)), t$, leaving it to return the undefined object at arguments of other types. Later the range of types of arguments at which $\mathcal{G}_{\mathcal{J}}$ returns a significant value will be increased.

Definition 43 (**$\mathcal{H}^{E}_{\mathcal{J}}$ and $\mathcal{H}_{\mathcal{J}}$**) *For any $\mathcal{E}$, $\mathcal{I}$ and $\mathcal{J}$,*
whatever $d_2 \in D^{(t \rightarrow (t \rightarrow t))}$, $d_1 \in D^{t}$, $d_3 \in D^{t}$, $\mathcal{H}^{E}_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(d_1, d_2, d_3) = d_2(d_3)(d_1)$
whatever $m_2 \in M^{(t \rightarrow (t \rightarrow t))}$, $m_1 \in M^{t}$, $m_3 \in M^{t}$, $(w, j) \in \mathcal{I} \times \mathcal{J}$
$\mathcal{H}_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_1, m_2, m_3)(w, j) = \mathcal{H}^{E}_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_1(w, j), m_2(w, j), m_3(w, j))$

This dictates that the type correspondence be $(t \rightarrow (t \rightarrow t))$ for JUNCT. This type correspondence does not determine which of the 16 possible objects of type $(t \rightarrow (t \rightarrow t))$ should be given as denotations in any particular model to a junction. The set of possible models about to be defined will make reference to the Junction Meaning Postulate, but this postulate will not be given for a little while. As the recursive ambiguity data is considered, it will become clear what constraint on interpretation functions of possible models must be specified by the Junction Meaning Postulate.

$\mathcal{L}^{8}$: including sentential junctions

1. The set of phrase-set indices: as for $\mathcal{L}^{7}$, with the addition of JUNCT.
2. Basic phrase-sets: as for $\mathcal{L}^{7}$, with the addition of:
   $\mathcal{X}_{\text{JUNCT}} = \{\langle \text{and}, \langle\rangle, \text{JUNCT} \rangle, \langle \text{or}, \langle\rangle, \text{JUNCT} \rangle\}$
3. Syntactic operations: as for $\mathcal{L}^{7}$, with the addition of $\mathcal{F}_{\mathcal{J}}$, which as an operation on strings is a 3 place concatenation operation.
4. Syntactic rules: as for $\mathcal{L}^{7}$, with the addition of, $\langle \mathcal{F}_{\mathcal{J}}, \langle \text{S}, \text{JUNCT}, \text{S} \rangle, \text{JUNCT} \rangle$

The set $\mathcal{K}^{5}$ of possible models of $\mathcal{L}^{8}$: $\langle \langle \mathcal{B}, (\mathcal{G}_{\gamma})_{\gamma \in \Gamma}, f \rangle, \langle w, j \rangle \rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^{5}$ iff

1. Type mapping: the extension of the mapping given for $\mathcal{K}^4$, as follows: $\nu^5(\text{JUNCT}) = (t \rightarrow (t \rightarrow t))$.
2. Constraints on f^5: as for $\mathcal{K}^4$, with the addition of restrictions wrt. $\langle \mathsf{and}, \langle\rangle, \text{JUNCT}\rangle$ and $\langle \mathsf{or}, \langle\rangle, \text{JUNCT}\rangle$ as specified in the Junction Meaning Postulate, forthcoming in Definition 45.
3. Algebraic constraints: $\Gamma = \{<, >, \uparrow, (\uparrow, vp), \mathcal{J}\}$. In the usual way, $\mathcal{G}_\gamma = \mathcal{H}_\gamma(\mathcal{E}, \mathcal{I}, \mathcal{J})$.

The THEORY OF REFERENCE $\mathcal{T}^5$ will be understood to be the combination of $\mathcal{L}^8$ and $\mathcal{K}^5$. Data concerning transparency and recursive ambiguity can be brought to bear on $\mathcal{T}^5$. Firstly the transparency behaviour of sentential junctions noted in (19f, p36) of Chapter 3, section 3.1, an instance of which is (8i) below. Secondly, there is the recursive ambiguity data that is a consequence of Hypothesis 4 (p46), an instance of which is (8ii) below:

(8) i (John walks)$_e$ JUNCT (Mary talks)$_e$
ii John walks JUNCT Mary talks is recursively ambiguous wrt. John walks JUNCT Mary talks.

The transparency fact (8i) is accounted for in the way familiar from section 2.1: (a) co-extension of sentences implies identity of denotation and (b) denotation of a junction-containing sentence is given by the application of the semantic operation $\mathcal{G}^E_{\mathcal{J}}$ to the denotations of the junction, and the joined sentences. Because the explanation of the transparency of the sentential arguments of junctions is of a piece with the explanation of the transparency of the subject and object argument positions of verbs, the explanation holds with the junctions *freely* interpreted in their type, just as it holds for verbs, which are *freely* interpreted.

The recursive ambiguity claim of (8ii) above is equivalent to the claim that:

(9) There is a reading of John walks JUNCT Mary talks such that, where J corresponds to JUNCT, whatever situation s,
John walks JUNCT Mary talks is true in s iff J (John walks is true in s,
Mary talks is true in s)

In the way indicated in section 2.3 of Chapter 3, to the above there corresponds a condition on $\mathcal{T}^5$ (where $\overline{\text{John walks}}$ and $\overline{\text{Mary talks}}$ refer to the only possible disambiguations):

(10) there is a β such that $\mathcal{R}$(John walks JUNCT Mary talks, β) and such that, where J corresponds to JUNCT, whatever model $\langle\langle \mathcal{B}, \mathcal{G}_\gamma, f\rangle, \langle w, j\rangle\rangle \in \mathcal{K}^5$,
$[\![\beta]\!](\langle w, j\rangle) = 1$ iff
$\mathsf{J}([\![\overline{(\text{John walks}}]\!](w, j) = 1,$
$[\![\overline{\text{Mary talks})}]\!](w, j) = 1)$

Now does $\mathcal{T}^5$ entails (10) ? There is only one β such that $\mathcal{R}(\text{John walks JUNCT Mary talks},\beta)$, and $\mathcal{T}^5$ ensures that its denotation according to an arbitrary model $\langle \Im, \langle w, j \rangle \rangle \in \mathcal{K}^5$ will be:

(11) $[\![\overline{\text{JUNCT}}]\!](w,j)([\![\overline{\text{John walks}}]\!](w,j))([\![\overline{\text{Mary talks}}]\!](w,j))$

Therefore $\mathcal{T}^5$ entails the following:

(12) there is a β such that $\mathcal{R}(\text{John walks JUNCT Mary talks}, \beta)$ and such that whatever model $\langle\langle \mathcal{B}, \mathcal{G}_\gamma, f\rangle, \langle w, j\rangle\rangle$,
$[\![\beta]\!](\langle w,j\rangle) = 1$ iff $[\![\overline{\text{JUNCT}}]\!](w,j)([\![\overline{\text{John walks}}]\!](w,j))([\![\overline{\text{Mary talks}}]\!](w,j)) = 1$

Clearly the way to bring (12), what $\mathcal{T}^5$ *does* entail, into line with (10), what $\mathcal{T}^5$ *must* entail, is to make adjustments so that the following holds:

(13) where J corresponds to JUNCT, whatever model $\langle\langle \mathcal{B}, \mathcal{G}_\gamma, f\rangle, \langle w, j\rangle\rangle \in \mathcal{K}^5$,

$$\begin{aligned} [\![\overline{\text{JUNCT}}]\!](w,j) \quad & ([\![\overline{\text{John walks}}]\!](w,j)) = 1 \text{ iff J } ([\![\overline{\text{John walks}}]\!](w,j) = 1 \\ & ([\![\overline{\text{Mary talks}}]\!](w,j)) \qquad\qquad [\![\overline{\text{Mary talks}}]\!](w,j) = 1) \end{aligned}$$

This splits into 4 cases according to the 4 possible combinations of value of $\overline{\text{John walks}}$ and $\overline{\text{Mary talks}}$ at (w,j):

(14) $[\![\overline{\text{Junct}}]\!](w,j)(0)(0) = 1$ iff $\text{J}(0 = 1, 0 = 1)$
$[\![\overline{\text{Junct}}]\!](w,j)(0)(1) = 1$ iff $\text{J}(0 = 1, 1 = 1)$
$[\![\overline{\text{Junct}}]\!](w,j)(1)(0) = 1$ iff $\text{J}(1 = 1, 0 = 1)$
$[\![\overline{\text{Junct}}]\!](w,j)(1)(1) = 1$ iff $\text{J}(1 = 1, 1 = 1)$

Substituting and and AND into these 4 schematic biconditionals dictates that the value of and at (w,j) be the familiar Boolean truth function. Likewise for substituting or and OR. This is the content of the next two definitions.

Definition 44 ($\mathcal{AND}, \mathcal{OR}$) *For any $\mathcal{E}, \mathcal{I}$ and $\mathcal{J}$, whatever $(w,j) \in \mathcal{I} \times \mathcal{J}$*

$\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,j)$	$(0)(0) = 0$	$\mathcal{OR}(\mathcal{E},\mathcal{I},\mathcal{J})(w,j)$	$(0)(0) = 0$
	$(0)(1) = 0$		$(0)(1) = 1$
	$(1)(0) = 0$		$(1)(0) = 1$
	$(1)(1) = 1$		$(1)(1) = 1$

Definition 45 (Junction Meaning Postulate) *Whatever model $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^5$, $f(\overline{\text{and}}) = \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})$, $f(\overline{\text{or}}) = \mathcal{OR}(\mathcal{E},\mathcal{I},\mathcal{J})$*

2.2.2 Syntax and Semantics for Subject position quantifiers

What denotations should be chosen for quantifiers ? Up until the innovation of Generalised Quantifier theory [5] there was no satisfactory answer to this: the prevailing view was that quantifiers had in fact *no* denotation, but instead made a syncategorematic contribution to sentences in which they find themselves. However, using similar methods to those just used in the case of junctions, it can be seen how a particular denotational hypothesis is forced upon one, and that is the Generalised Quantifier denotational hypothesis.

One will be forced to this hypothesis if the following simple syntactic assumptions are made. Quantifier phrases will not be assigned the category NP. Quantifiers instead shall have their own category, QNP, which a determiner DET and a common noun phrase CN may combine to form. DET and CN are further phrase-set indices that we will use and we will assume that the type correspondence for CN is $(e \rightarrow t)$. We will also assume that the rule deriving QNP's from DET's and CN's implicates the syntactic operation $\mathcal{F}_{>}$. Because this operation is associated with the semantic operation $\mathcal{G}_{>}$, this means that the type for DET must be $((e \rightarrow t) \rightarrow a)$, for some a. Not identifying QNP with NP means that none of the rules for combining NP's with verbs can be seen as saying anything about the quantifiers. Therefore in order to account for the distribution of QNP's it appears to be necessary to define versions of all the rules that make mention of NP's, mentioning QNP's instead. For the moment, we shall only make provision for QNP's in subject position, and will do so by adapting the rule that generates NP's in subject position.

The revised rule for QNP's differs not only by replacing mention of NP by mention of QNP but also by replacing mention of the syntactic operation $\mathcal{F}_{<}$ by mention of the syntactic operation $\mathcal{F}_{>}$. This dictates that the type of QNP's be $((e \rightarrow t) \rightarrow t)$ and therefore $((e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t))$ for DET. This type association is a general tenet of Generalised Quantifier theory.

Having a type correspondence still does not determine *which* objects of type $((e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t))$ should be the values of the determiners, and there are very many to chose from (if the set $\mathcal{E}$ has just 2 individuals, there are 2^{16} different such functions). The definition of the class of MODELS will make reference to the Determiner Meaning Postulate, and the specification of this postulate will be held back for a little while. As was the case with junctions, when we consider the recursive ambiguity data, the necessary constraints on the interpretation function become clear.

To bring to bear the recursive ambiguity data concerning subject occurrences of quantifiers, there must be disambiguations of sentences such as he_1 is a man. It will be assumed for the moment that added to the set of basic expressions of category NP is a set of numbered nominal

[5] see Barwise and Cooper 81, Keenan and Stavi 86, both anticipated somewhat by Montague 73.

proforms, called NPPRO. Where the number of these nominal proforms is N, the set is defined:

$$\text{NPPRO} = \{\langle \mathsf{he}_n, \langle\rangle, \text{NP}\rangle : n \leq N\}$$

Dealing with semantics of these involves placing some constraints of the contexts of use, $\mathcal{J}$, until now unconstrained. Henceforth the contexts of use associated with a possible model will be required to be the set of functions from NPPRO to $\mathcal{E}$: $\mathcal{E}^{\text{NPPRO}}$. The is a construction will be treated syncategorematically.

$\mathcal{L}^9$: including pronouns and subject position quantifiers

1. The set of phrase-set indices: as for $\mathcal{L}^8$, with the addition of { DET,CN,QNP}
2. Basic phrase-sets: as for $\mathcal{L}^8$, with the addition of
   $\mathcal{X}^9_{\text{DET}} = \{\langle \mathsf{a}, \langle\rangle, \text{DET}\rangle, \langle \mathsf{every}, \langle\rangle, \text{DET}\rangle, \langle \mathsf{most}, \langle\rangle, \text{DET}\rangle, \langle \mathsf{no}, \langle\rangle, \text{DET}\rangle\}$,
   $\mathcal{X}^9_{\text{CN}} = \{\langle \mathsf{man}, \langle\rangle, \text{CN}\rangle, \langle \mathsf{woman}, \langle\rangle, \text{CN}\rangle, \langle \mathsf{beer}, \langle\rangle, \text{CN}\rangle, \langle \mathsf{pie}, \langle\rangle, \text{CN}\rangle\}$,
   $\mathcal{X}^9_{\text{QNP}} = \emptyset$
3. Syntactic operations: as for $\mathcal{L}^8$, with the addition of $\mathcal{F}_{isa}$, which as an operation on two strings s_1, s_2, simply *inserts* is a between s_1 and s_2.
4. The set of syntactic rules: as for $\mathcal{L}^8$ with the addition of
   $\{\langle \mathcal{F}_>, \langle \text{DET}, \text{CN}\rangle, \text{QNP}\rangle, \langle \mathcal{F}_>, \langle \text{QNP}, \text{VP}\rangle, \text{S}\rangle, \langle \mathcal{F}_{isa}, \langle \text{NP}, \text{CN}\rangle, \text{S}\rangle\}$

The set $\mathcal{K}^6$ of possible models for $\mathcal{L}^9$ $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^6$ iff

1. Type Mapping: the extension of the mapping given for $\mathcal{K}^5$, as follows: $\nu^6(\text{DET}) = ((e{\rightarrow}t){\rightarrow}((e{\rightarrow}t){\rightarrow}t))$, $\nu^6(\text{CN}) = (e{\rightarrow}t)$, $\nu^6(\text{QNP}) = ((e{\rightarrow}t){\rightarrow}t)$
2. Constraints on f^6: f^6 is subject to the Copula Postulate, the Junction Postulate, the Pronoun Postulates (Definition 46) and the Determiner Postulate (Definition 48.
3. Algebraic constraints: $\Gamma = \{<, >, \uparrow, (\uparrow, vp), \mathcal{J}, isa\}$. In the usual way $\mathcal{G}_\gamma = \mathcal{H}(\mathcal{E}, \mathcal{I}, \mathcal{J})$. $\mathcal{G}_<$ also bears the index *isa*.

Definition 46 (Pronoun Postulates) *For any model,* $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$*, associated with* $\mathcal{E}, \mathcal{I}, \mathcal{J}$*, whatever* $(w, g), (w, g') \in \mathcal{I} \times \mathcal{J}$*, whatever* $n \leq N$

(i) If $\alpha \notin$ NPPRO *then* $f(\alpha)(w, g) = f(\alpha)(w, g')$

(ii) If α *is the pronoun* $\langle \mathsf{he}_n, \langle\rangle, \text{NP}\rangle$, $f(\mathsf{he}_n)(w, g) = g(\mathsf{he}_n)$

By $\mathcal{T}^6$ we will intend to refer to the combination of $\mathcal{L}^9$ and $\mathcal{K}^6$. Given the syntactic coverage of $\mathcal{L}^9$, semantic data can be brought to bear of two kinds. Firstly there are the unambiguous transparency facts concerning unembedded subject position quantifiers that were noted as (19c, p36) in Chapter 3, and an example of which is given below in (15i). Secondly, there are the recursive ambiguity facts concerning subject position quantifiers that are entailed by Hypothesis 3 (p46), an example of which is (15ii) [6]

(15) i DET $(\mathsf{man})_e$ $(\mathsf{walks})_e$

ii DET man walks is recursively ambiguous wrt. DET man

(15i) is accounted for. To account for (15ii) the Determiner Meaning Postulate is required.

Unambiguous transparency facts for unembedded quantifiers

$\mathcal{T}^6$ is such that there is at most one disambiguation of any expression. Therefore, the condition for $\mathcal{T}^6$ to render the occurrence of man opaque in DET man walks is:

(16) there is an $\overline{\alpha}$, where α is a CN, and there is a model, $\langle \Im, \langle w, g\rangle\rangle$, such that (i) whatever proper name or pronoun β, $\llbracket\overline{\beta \text{ is a man}}\rrbracket(w,g) = \llbracket\overline{\beta \text{ is a } \alpha}\rrbracket(w,g)$ and (ii) $\llbracket\overline{\text{DET man walks}}\rrbracket(w,g) \neq \llbracket\overline{\text{DET } \alpha \text{ walks}}\rrbracket(w,g)$.

(i) entails that $\llbracket\overline{\text{man}}\rrbracket(w,g) = \llbracket\overline{\alpha}\rrbracket(w,g)$, whilst (ii) entail the negation of this. Therefore the condition for opacity is impossible and man is predicted to occur transparently in DET man walks. In a similar way the transparency of the occurrence of walks will be accounted for.

Recursive ambiguity data

(15ii) amounts to the claim that:

(17) there is a reading of DET man walks such that where D corresponds to DET, whatever situation s, DET man walks is true on the reading in s iff

D { x: he_1 is a man is true in $s_x^{he_1}$ }

{ x: he_1 walks is true in $s_x^{he_1}$ }

Therefore, the required entailment of $\mathcal{T}^6$ is (where $\overline{\mathsf{he}_1 \text{ is a man}}$ and $\overline{\mathsf{he}_1 \text{ walks}}$ refer to the only possible disambiguations):

(18) there is a β such that $\mathcal{R}(\text{DET man walks}, \beta)$ and such that, where D corresponds to DET, whatever model $\langle\langle \mathcal{B}, \mathcal{G}_\gamma, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^6$,

$\llbracket\beta\rrbracket(\langle w,g\rangle) = 1$ iff D { x: $\llbracket\overline{\mathsf{he}_1 \text{ is a man}}\rrbracket(\langle w, g_x^{\mathsf{he}_1}\rangle)$ }

{ x: $\llbracket\overline{\mathsf{he}_1 \text{ walks}}\rrbracket(\langle w, g_x^{\mathsf{he}_1}\rangle)$ }

[6] The presence of determiners is a step on the way to being able to express the sentences referred to in Hypothesis 5 (p49), but they remain inexpressible still.

Now according to $\mathcal{T}^6$, there is only one β such that $\mathcal{R}$(DET man walks, β). The denotation assigned to β at any model $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,g\rangle\rangle \in \mathcal{K}^6$ is:

(19) $[\![\overline{\text{DET}}]\!](w,g)([\![\overline{\text{man}}]\!](w,g))([\![\overline{\text{walks}}]\!](w,g))$

Therefore $\mathcal{T}^6$ entails:

(20) there is a β such that $\mathcal{R}$(DET man walks, β) and such that whatever model $\langle\langle\mathcal{B},\mathcal{G}_\gamma,f\rangle,\langle w,g\rangle\rangle \in \mathcal{K}^6$,
$[\![\beta]\!](\langle w,g\rangle) = 1$ iff $[\![\overline{\text{DET}}]\!](w,g)([\![\overline{\text{man}}]\!](w,g))([\![\overline{\text{walks}}]\!](w,g)) = 1$

Clearly unless the following holds, then (20), what $\mathcal{T}^6$ *does* entail, will not amount to (18), what $\mathcal{T}^6$ *must* entail:

(21) where D corresponds to DET, whatever $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,g\rangle\rangle \in \mathcal{K}^6$,
$[\![\overline{\text{DET}}]\!](w,g)\ ([\![\overline{\text{man}}]\!](w,g))\ ([\![\overline{\text{walks}}]\!](w,g)) = 1$ iff D $\{\, x\text{: } [\![\overline{\text{he}_1 \text{ is a man}}]\!](\langle w,g_x^{\text{he}_1}\rangle)\,\}$ $\{\, x\text{: } [\![\overline{\text{he}_1 \text{ walks}}]\!](\langle w,g_x^{\text{he}_1}\rangle)\,\}$

To simplify this one should note the following identities:

(22) $\{\, x\text{: } [\![\overline{\text{he}_1 \text{ is a man}}]\!](\langle w,g_x^{\text{he}_1}\rangle)\,\} = [\![\overline{\text{man}}]\!](w,g)$
$\{\, x\text{: } [\![\overline{\text{he}_1 \text{ walks}}]\!](\langle w,g_x^{\text{he}_1}\rangle)\,\} = [\![\overline{\text{walks}}]\!](w,g)$

Then because $[\![\overline{\text{man}}]\!](w,g)$ and $[\![\overline{\text{walks}}]\!](w,g)$ range over (characteristic functions of) arbitrary sets of individuals in the model, what we require can be rewritten:

(23) where D corresponds to DET, whatever $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,g\rangle\rangle \in \mathcal{K}^6$, whatever sets S_1, S_2, of individuals in the model,
$[\![\overline{\text{DET}}]\!](w,g)(cf(S_1))(cf(S_2)) = 1$ iff D (S_1,S_2)

Therefore, we require for all determiners denotations that the result of applying the function to two sets should be 1 exactly when the corresponding relation D holds between the sets. This is not going to the case with the determiners freely interpreted, and is exactly what the following definitions dictate.

Definition 47 (EVERY,A,NO,MOST) *For any $\mathcal{E},\mathcal{I}$ and $\mathcal{J}$, for all $(w,g) \in \mathcal{I}\times\mathcal{J}$, for all subsets S_1,S_2 of $\mathcal{E}$*
$\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(cf(S_1))(cf(S_2)) = 1$ *iff S_1 is a subset of S_2*
$\mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(cf(S_1))(cf(S_2)) = 1$ *iff S_1 has a non-null intersection with S_2*
$\mathcal{NO}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(cf(S_1))(cf(S_2)) = 1$ *iff S_1 has a null intersection with S_2*
$\mathcal{MOST}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(cf(S_1))(cf(S_2)) = 1$ *iff* $\|S_1 \cap S_2\| > \|S_1\|$

Definition 48 (Determiner Meaning Postulate) *For all models* $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,g\rangle\rangle \in \mathcal{K}^6$,

$f^6(\overline{\text{every}}) = f^6(\overline{\text{all}}) = f^6(\overline{\text{each}}) = \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})$

$f^6(\overline{\text{a}}) = f^6(\overline{\text{some}}) = f^6(\overline{\text{one}}) = \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})$

$f^6(\overline{\text{no}}) = \mathcal{NO}(\mathcal{E},\mathcal{I},\mathcal{J})$

$f^6(\overline{\text{most}}) = \mathcal{MOST}(\mathcal{E},\mathcal{I},\mathcal{J})$

This postulate essentially embodies the determiner denotations that have been put forward in the Generalised Quantifier literature.

Taking stock, we have now completed the second phase of the development of the core-account, and have now an account that syntactically encompasses sentential junctions and quantifiers in subject position. We are in fact now more or less in a position to go on to the consideration in Sections 2.4 and 2.5 of ways of extending coverage to non-sentential junctions and non-subject position quantifiers. However, the next section, section 2.3, contains a last phase of development of the core-account that will allow the truth intuitions for embedding constructions to be brought to bear.

2.3 Some further extensions

The reader may prefer to skip forward to section 2.4 and only refer back to this section when the sentences under consideration exhibit the syntactic forms that are the concern of this section, forms exemplified by:

a talented candidate applied a dog near John died a man that John likes died	} *Common noun modification*
a proposition that John believes is true an act that John wanted to do was done by Dave	} *Higher-order quantification and truth predicates*

By 'higher-order quantification and truth predicates' I am referring to the syntactic devices that occur in the conclusions of the inferences mentioned in Hypothesis 5 (p49) and Hypothesis 6 (p49). The new THEORY OF REFERENCE is simply presented below and then a few words of commentary are given. Following that, the issue of truth predicates is considered.

In the following definitions p and vp abbreviate $(s\rightarrow t)$ and $(s\rightarrow(e\rightarrow t))$.

$\mathcal{L}^{10}$: CN modifiers, higher order quantification and truth predicates

1. set of phrase set indices: as for $\mathcal{L}^9$, with the addition of

{ ADJ,PP,P,RC, RC^p, RC^{vp}, CN^p, CN^{vp}, DET^p, DET^{vp}, QNP^p, QNP^{vp}, NP^p, NP^{vp}, VP^p, VP^{vp}, TV^{vp} }

2. basic phrase sets: as for $\mathcal{L}^9$ with addition of:

   $\mathcal{X}_{\mathrm{ADJ}} = \{\langle \mathsf{tall}, \langle\rangle, \mathrm{ADJ}\rangle, \ldots\}$

   $\mathcal{X}_{\mathrm{PP}} = \emptyset$

   $\mathcal{X}_{\mathrm{P}} = \{\langle \mathsf{near}, \langle\rangle, \mathrm{P}\rangle, \ldots\}$

   $\mathcal{X}_{\mathrm{RC}} = \mathcal{X}_{\mathrm{RC}^p} = \mathcal{X}_{\mathrm{RC}^{vp}} = \emptyset$

   $\mathcal{X}_{\mathrm{CN}^p} = \{\langle \mathsf{proposition}, \langle\rangle, \mathrm{CN}^p\rangle\}$, $\mathcal{X}_{\mathrm{CN}^{vp}} = \{\langle \mathsf{act}, \langle\rangle, \mathrm{CN}^{vp}\rangle\}$

   $\mathcal{X}_{\mathrm{DET}^p} = \{\langle s_1, \langle\rangle, \mathrm{DET}^p\rangle : \langle s_1, \langle\rangle, \mathrm{DET}\rangle \in \mathcal{X}^9_{\mathrm{DET}}\}$

   $\mathcal{X}_{\mathrm{DET}^{vp}} = \{\langle s_1, \langle\rangle, \mathrm{DET}^{vp}\rangle : \langle s_1, \langle\rangle, \mathrm{DET}\rangle \in \mathcal{X}^9_{\mathrm{DET}}\}$

   $\mathcal{X}_{\mathrm{QNP}^p} = \mathcal{X}_{\mathrm{QNP}^{vp}} = \emptyset$

   $\mathcal{X}_{\mathrm{VP}^p} = \{\langle \mathsf{is\ true}, \langle\rangle, \mathrm{VP}^p\rangle\}$, $\mathcal{X}_{\mathrm{VP}^{vp}} = \{\langle \mathsf{was\ done\ by\ John}, \langle\rangle, \mathrm{VP}^{vp}\rangle\}$

   $\mathcal{X}_{\mathrm{TV}^{vp}} = \{\langle \mathsf{do}, \langle\rangle, \mathrm{TV}^{vp}\rangle\}$

   $\mathcal{X}_{\mathrm{NP}^p} = \{\mathsf{he}^p_i : i \leq N_p\}$

   $\mathcal{X}_{\mathrm{NP}^{vp}} = \{\mathsf{he}^{vp}_i : i \leq N_{vp}\}$

3. the syntactic operations: as for $\mathcal{L}^9$ with the addition of $\mathcal{F}_\cap$, and the family of operations $\mathcal{F}_{\lambda \mathsf{he}_i}$, $\mathcal{F}_{\lambda \mathsf{he}^p_i}$ and $\mathcal{F}_{\lambda \mathsf{he}^{vp}_i}$.

   $\mathcal{F}_\cap$ concatenates.

   $\mathcal{F}_{\lambda \mathsf{he}_i}$ deletes occurrences of he_i, and prefaces with **that**.

   $\mathcal{F}_{\lambda \mathsf{he}^p_i}$ deletes occurrences of he^p_i, and prefaces with **that**.

   $\mathcal{F}_{\lambda \mathsf{he}^{vp}_i}$ deletes occurrences of he^{vp}_i, and prefaces with **that**

4. the syntactic rules: as for $\mathcal{L}^9$ with the addition of:

   $\{\langle \mathcal{F}_\cap, \langle \mathrm{ADJ}, \mathrm{CN}\rangle, \mathrm{CN}\rangle, \langle \mathcal{F}_\cap, \langle \mathrm{CN}, \mathrm{PP}\rangle, \mathrm{CN}\rangle, \langle \mathcal{F}_\cap, \langle \mathrm{CN}, \mathrm{RC}\rangle, \mathrm{CN}\rangle,$

   $\langle \mathcal{F}_>, \langle \mathrm{P}, \mathrm{NP}\rangle, \mathrm{PP}\rangle, \langle \mathcal{F}_>, \langle \mathrm{PV}, \mathrm{NP}^p\rangle, \mathrm{VP}\rangle, \langle \mathcal{F}_>, \langle \mathrm{TV}^{vp}, \mathrm{NP}^{vp}\rangle, \mathrm{VP}\rangle,$

   $\langle \mathcal{F}_{\lambda \mathsf{he}_i}, \langle \mathrm{S}\rangle, \mathrm{RC}\rangle\}$ and for $\delta = p$ and $\delta = vp$, the rules:

   $\langle \mathcal{F}_>, \langle \mathrm{DET}^\delta, \mathrm{CN}^\delta\rangle, \mathrm{QNP}^\delta\rangle$

   $\langle \mathcal{F}_>, \langle \mathrm{QNP}^\delta, \mathrm{VP}^\delta\rangle, \mathrm{S}\rangle$

   $\langle \mathcal{F}_\cap, \langle \mathrm{CN}^\delta, \mathrm{RC}^\delta\rangle, \mathrm{CN}^\delta\rangle$ $\langle \mathcal{F}_{\lambda \mathsf{he}^\delta_i}, \langle \mathrm{S}\rangle, \mathrm{RC}^\delta\rangle$

Definition 49 ($\mathcal{H}_\cap$) *Whatever $\mathcal{E},\mathcal{I},\mathcal{J}$, whatever type $a \in \mathrm{TJ}^{\rightarrow}$, whatever m_1, m_2 of type $(a \rightarrow t)$, whatever $(w,g) \in \mathcal{I} \times \mathcal{J}$,*

$$\mathcal{H}_\cap(\mathcal{E},\mathcal{I},\mathcal{J})(m_1{}^{(a\rightarrow t)}, m_2{}^{(a\rightarrow t)})(w,g) = x^a \mapsto \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(m_1(w,g)(x))(m_2(w,g)(x))$$

Definition 50 ($\mathcal{H}_{\lambda \mathsf{he}_i}$,$\mathcal{H}_{\lambda \mathsf{he}^p_i}$,$\mathcal{H}_{\lambda \mathsf{he}^{vp}_i}$) *Whatever $\mathcal{E},\mathcal{I},\mathcal{J}$, whatever m_1 of type a,*

$$\mathcal{H}_{\lambda \mathsf{he}_i}(\mathcal{E},\mathcal{I},\mathcal{J})(m_1{}^a)(w,g) = x^e \mapsto m_1(w, g^x_{\mathsf{he}_i})$$
$$\mathcal{H}_{\lambda \mathsf{he}^p_i}(\mathcal{E},\mathcal{I},\mathcal{J})(m_1{}^a)(w,g) = x^p \mapsto m_1(w, g^x_{\mathsf{he}^p_i})$$
$$\mathcal{H}_{\lambda \mathsf{he}^{vp}_i}(\mathcal{E},\mathcal{I},\mathcal{J})(m_1{}^a)(w,g) = x^{vp} \mapsto m_1(w, g^x_{\mathsf{he}^{vp}_i})$$

The set $\mathcal{K}^7$ of possible models of $\mathcal{L}^{10}$ $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,g\rangle\rangle$ associated with $\mathcal{E},\mathcal{I},\mathcal{J},\nu$ is $\in\mathcal{K}^7$ iff

1. Type mapping: extension of the type mapping for $\mathcal{K}^6$ as follows: $\nu^7(\text{ADJ}) = \nu^7(\text{PP}) = \nu^7(\text{RC}) = (e\rightarrow t)$, $\nu^7(\text{P}) = (e\rightarrow(e\rightarrow t))$, $\nu^7(\text{RC}^p) = (p\rightarrow t)$, $\nu^7(\text{RC}^{vp}) = (vp\rightarrow t)$, $\nu^7(\text{CN}^p) = (p\rightarrow t)$, $\nu^7(\text{CN}^{vp}) = (vp\rightarrow t)$, $\nu^7(\text{DET}^p) = ((p\rightarrow t)\rightarrow((p\rightarrow t)\rightarrow t))$, $\nu^7(\text{DET}^{vp}) = ((vp\rightarrow t)\rightarrow((vp\rightarrow t)\rightarrow t))$, $\nu^7(\text{QNP}^p) = ((p\rightarrow t)\rightarrow t)$, $\nu^7(\text{QNP}^{vp}) = ((vp\rightarrow t)\rightarrow t)$, $\nu^7(\text{NP}^p) = p$, $\nu^7(\text{NP}^{vp}) = vp$, $\nu^7(\text{VP}^p) = (p\rightarrow t)$, $\nu^7(\text{VP}^{vp}) = (vp\rightarrow t)$, $\nu^7(\text{TV}^{vp}) = (vp\rightarrow(e\rightarrow t))$
2. Constraints on the interpretation function f^7: the same constraints as for $\mathcal{K}^6$, plus additional constraints concerning the members of $\mathcal{X}_{\text{DET}^p}$, $\mathcal{X}_{\text{DET}^{vp}}$, $\mathcal{X}_{\text{VP}^p}$, $\mathcal{X}_{\text{VP}^{vp}}$, $\mathcal{X}_{\text{TV}^{vp}}$
3. Algebraic constraints: $\Gamma = \{<,>,\uparrow,(vp,\uparrow),\mathcal{J},\lambda\text{he}_i,\lambda\text{he}_i^p,\lambda\text{he}_i^{vp},\cap\}$. In the usual way $\mathcal{G}_\gamma = \mathcal{H}_\gamma(\mathcal{E},\mathcal{I},\mathcal{J})$.

Definition 51 (Additional meaning postulates for $\mathcal{K}^7$)

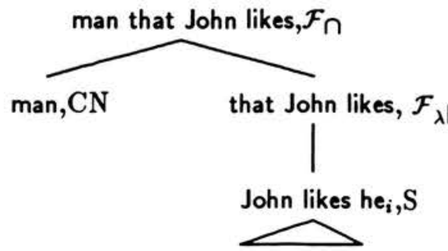
$\mathcal{X}_{\text{DET}^p},\mathcal{X}_{\text{DET}^{vp}}$: *$f^7$ is subject to a restriction that parallels the restriction concerning members of $\mathcal{X}_{\text{DET}}$.*

$\mathcal{X}_{\text{VP}^p}$: $f^7(\overline{\text{is true}}) = (w,g)\mapsto x^p\mapsto x(w)$.

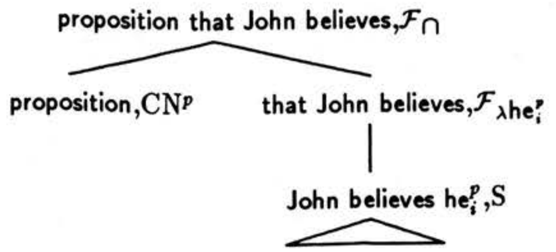
$\mathcal{X}_{\text{VP}^{vp}}$: $f^7(\overline{\text{was done by John}}) = (w,g)\mapsto P^{vp}\mapsto P(w)(f^7(\overline{\text{John}})(w,g))$

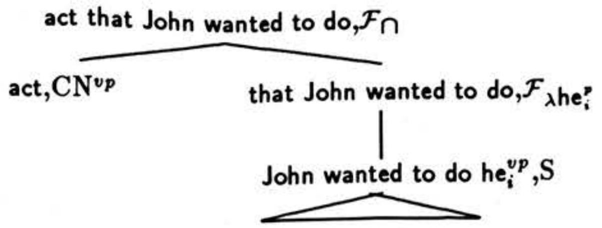
$\mathcal{X}_{\text{TV}^{vp}}$: $f^7(\overline{\text{do}}) = (w,g)\mapsto P^{vp}\mapsto x^e\mapsto P(w)(x)$

$\mathcal{T}^7$ is the combination of $\mathcal{L}^{10}$ and $\mathcal{K}^7$. $\mathcal{T}^7$ allows for three kinds of Common Noun modification: modification by *adjective*, by *preposition phrase* and by *relative clause*. In each case the common noun is combined with the common noun modifier by $\mathcal{F}_\cap$, and thereby the semantic operation $\mathcal{G}_\cap$ is implicated. This operation essentially takes the intersection of the denotations of the two parts contributing to the complex common noun. Of the Common Noun modifiers it is the relative clause which have the most complex derivation. The analysis suggested is along the same lines as that found in PTQ, whereby the relative clause is derived by a deletion operation from a pronoun-containing sentence. There is an illustration below of the disambiguation of man that John likes, which involves a relative clause:



$\mathcal{T}^7$ also provides disambiguations of proposition that John believes and act that John wants to do. According to $\mathcal{T}^7$ these are not categorised CN but as CN^p and CN^{vp} respectively. This is not the only instance of $\mathcal{T}^7$ adding two parallel categories to categories that are present in $\mathcal{L}^9$. The same happens for NP, DET, QNP, VP and RC. There follow illustrations of the disambiguation of proposition that John believes and act that John wanted to do:





2.3.1 Truth predicates

$\mathcal{L}^{10}$ includes a truth predicate is true, and something akin to one, was done by John. This is necessary if Hypothesis 5 (p49) and Hypothesis 6 (p50) are to be brought to bear because the inferences referred to by these hypotheses involved these predicates. Yet, as was noted when the inferences were introduced in section 3.5 of Chapter 3, there are certain dangers when one tries to provide a THEORY OF REFERENCE for a language containing a truth predicate. In this section I will try to isolate what exactly are the 'dangers' for a THEORY OF REFERENCE that includes a truth predicate and then argue that it is still worth considering the THEORY OF REFERENCE just presented.

The 'danger' for languages containing a truth predicate arises from the possibility of forming certain *paradoxical* sentences and discourses, the most familiar of which is the 'Liar sentence':

(24) this sentence is false

One can imagine a THEORY OF REFERENCE that treated this sentence as a context-sensitive referring expressions, receiving a reference through an assignment function, and that the references would be an expression of a formal language. is false would be a predicate that, given a (w, g), maps certain *expressions* to 1 and others to 0. The natural meaning postulate preventing is false from being *freely* interpreted would be:[7]

[7] This is a different postulate to that occurring in $\mathcal{T}^7$. This is because for the Liar, true is understood as a

Whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever sentence α,

$[\![\overline{\textsf{is false}}]\!](w,g)(\alpha) = 1$ iff $[\![\alpha]\!](w,g) = 0$

However, these assumptions actually entail that there is no model, $\langle \Im, \langle w, g\rangle\rangle$. If there was such a model, $[\![\overline{\textsf{is false}}]\!]$ would have to have impossible properties. The impossible behaviours occur when $[\![\overline{\textsf{is false}}]\!]$ is applied to any (w,g) such that $g(\overline{\textsf{this sentence}}) = \overline{\textsf{this sentence is false}}$. One can show that $[\![\overline{\textsf{is false}}]\!](w,g)(\overline{\textsf{this sentence is false}}) = 1$ iff $[\![\overline{\textsf{is false}}]\!](w,g)(\overline{\textsf{this sentence is false}}) = 0$:

$[\![\overline{\textsf{is false}}]\!](w,g)(\overline{\textsf{this sentence is false}}) = 1$
iff $[\![\overline{\textsf{this sentence is false}}]\!](w,g) = 0$ (by the meaning postulate for **is false**)
iff $[\![\overline{\textsf{is false}}]\!](w,g)([\![\overline{\textsf{this sentence}}]\!](w,g)) = 0$ (by breaking up the predicate argument structure)
iff $[\![\overline{\textsf{is false}}]\!](w,g)(\overline{\textsf{this sentence is false}}) = 0$ (because $g(\overline{\textsf{this sentence}}) = \overline{\textsf{this sentence is false}}$)

Because there are *no* possible models, every sentence is predicted to entail every other sentence, and the THEORY OF REFERENCE is devoid of any empirical force whatever.

Having observed the problems arising in a THEORY OF REFERENCE for the Liar sentence, the question to be considered is whether $\mathcal{T}^7$ is subject to a similar problems. $\mathcal{T}^7$ provides no disambiguation of the Liar sentence, so it cannot be subject to *exactly* the above problem. However, Kripke (in Kripke 75) has pointed out that more than simply avoiding the formation of overtly self-referential sentences is required if paradox is to be avoided: it is possible to formulate paradoxical discourses without these overtly self-referential devices. His examples are far more pressing as they are formulated within the syntax of $\mathcal{L}^{10}$. Here is a Kripkean discourse:

(25) **A believes exactly one proposition**
B believes exactly one proposition
A believes that every proposition that B believes is false
B believes that every proposition that A believes is true

The third and fourth sentences of the above discourse have disambiguations according $\mathcal{T}^7$. Strictly speaking, the first and second sentences do not have disambiguations according to $\mathcal{T}^7$. This is because they involve *non-subject* position quantifications. For the purposes of this section we will suppose that $\mathcal{T}^7$ is extended by the rule:

$\langle \mathcal{F}_{Kripke}, \langle \mathrm{PV}, \mathrm{QNP}^p\rangle, \mathrm{VP}\rangle$

and that $\mathcal{G}_{Kripke}$ has the definition:

$\mathcal{G}_{Kripke}(m_1^{(p\rightarrow(e\rightarrow t))})(m_2^{((p\rightarrow t)\rightarrow t)})(w,g) = x^e \mapsto m_2(w,g)(y^p \mapsto m_1(w,g)(y)(x))$.

There are two crucial intuitions about (25). First, that the sentences of (25) can all be simultaneously true. Second, that if the sentences of (25) are simultaneously true, there is no way to settle the issue of whether it is A or B who has the right beliefs. The second intuition is only

predicate applying to *sentences*, whereas the truth involving sentences of $\mathcal{T}^7$ predicate truth of propositions.

likely to emerge after some pondering:

Perhaps A is right. But then B must be wrong, and if B is wrong, then A must be wrong. So perhaps A is wrong. But then B must be right, and if B is right, then A is right.

If the first two sentences are true then *the* proposition that A believes concerns *the* proposition that B believes, which in turn concerns *the* proposition that A believes. In this roundabout way, the proposition that A believes concerns *itself*.

Now the effect of the meaning postulate invoked by $\mathcal{T}^7$ is that there are no models according to which all 4 of the sentences in the discourse are true.

That there is no model that makes the Kripkean discourse true

The conditions for the truth of the four sentences in an arbitrary model $\langle \Im, \langle w^*, g^* \rangle \rangle$ are given below, where a, b, bel are the denotations of 'A','B' and 'believes':

First sentence: There exists some p such that $bel(p)(a) = 1$ and whatever r, if $r \neq p$, then $bel(r)(a) = 0$

Second sentence: There exists some q such that $bel(q)(b) = 1$ and whatever r, if $r \neq q$, then $bel(r)(b) = 0$

Third sentence: $bel(w \mapsto [\text{every thing that B believes is false}](w, g^*))(a) = 1$

Fourth sentence: $bel(w \mapsto [\text{every thing that A believes is true}](w, g^*))(b) = 1$

Suppose $\langle \Im, \langle w^*, g^* \rangle \rangle$ meets the first two conditions. Let us call p^* and q^* those member of $D_{(s \to t)}$ that render the first two conditions true, and let P^* and Q^* refer to the following objects associated with $\langle \Im, \langle w^*, g^* \rangle \rangle$:

$P^* = w \mapsto [\text{every thing that B believes is false}](w, g^*)$

$Q^* = w \mapsto [\text{every thing that A believes is true}](w, g^*)$

Then for $\langle \Im, \langle w^*, g^* \rangle \rangle$ to additionally meet the third and fourth conditions, we must have the identities:

$p^* = P^*$

$q^* = Q^*$

However, these identities are impossible, for one can show that that either p^* and P^* will differ at w^* or q^* and Q^* will differ.

Case 1: if $p^*(w^*) = 0$ and $q^*(w^*) = 0$, then $P^*(w^*) = 1$ and $Q^*(w^*) = 0$
Case 2: if $p^*(w^*) = 0$ and $q^*(w^*) = 1$, then $P^*(w^*) = 0$ and $Q^*(w^*) = 0$
Case 3: if $p^*(w^*) = 1$ and $q^*(w^*) = 0$, then $P^*(w^*) = 1$ and $Q^*(w^*) = 1$
Case 4: if $p^*(w^*) = 1$ and $q^*(w^*) = 1$, then $P^*(w^*) = 0$ and $Q^*(w^*) = 1$

So there is no model that renders the Kripkean discourse true. □

So $\mathcal{T}^7$ fails to respect the intuition that all four of the sentence in the Kripkean discourse may be true: there are no models with that property. However, the class of models has not been shown to be empty, and therefore $\mathcal{T}^7$ does not allow that anything follows from anything. Therefore, unlike the THEORY OF REFERENCE for the Liar sentence, $\mathcal{T}^7$ retains empirical content. The closest thing to a collapse of entailment suffered by $\mathcal{T}^7$ is that there is predicted to be a valid inference from the Kripkean discourse to *any* Ψ. Now, even for a THEORY OF REFERENCE that avoided truth predicates, there will be this kind of mistaken prediction. For example in place of the Kripkean discourse one could have:

John believes that every woman died

John does not believes that it is not the case a woman did not die

Therefore $\mathcal{T}^7$ seems to suffer no more by way of mismatch with the data than the often pointed out defects accruing from the over-coarse identity conditions of propositions in possible worlds semantics.

$\mathcal{T}^7$ is the core-account on the basis of which we will now proceed to consider various accounts of junctions and quantifiers that may be proposed.

2.4 Non-sentential Junctions

A syntactic fact about junctions is that they have rather wide priveleges of occurrence, a reminder of which appear below:

(26) John walks JUNCT talks (VP case)
John loves JUNCT hates Mary (TV case)
John wants JUNCT needs to go (VVP case)
John told JUNCT asked Mary to go (TVVP case)

This is a fact that $\mathcal{L}^{10}$ does not allow for, catering only for sentential junctions. We will look now at two strategies for extending $\mathcal{T}^7$ to provide disambiguations of sentences such as these. The semantic properties of these sentences must also be accounted for. The significant *transparency* facts are that the verbs that occur in the above flanking the junction word occur transparently. Hypothesis 4 (p46) entails the following *recursive ambiguity* facts concerning the above sentences:

(27) John walks JUNCT talks is recursively ambiguous wrt. walks JUNCT talks
John loves JUNCT hates Mary is recursively ambiguous wrt. loves JUNCT hates
John wants JUNCT needs to go is recursively ambiguous wrt. wants JUNCT needs
John told JUNCT asked Mary to go is recursively ambiguous wrt. told JUNCT asked

Of the two strategies to be looked at, the first (section 2.4.1) is *non-local* and the second (section 2.4.2) is *local*. The *non-local* approach is based on the strategy adopted in (early) transformational grammar, the crucial ingredient of which is 'Conjunction Reduction'. The local approach is based on the proposal of Gazdar's in the early 80's that has come to be called 'Cross Categorial Coordination'.

The main empirical assessment of these two strategies that we will undertake is a semantic one. Concerning the non-local approach, we observe that, though a prolific generator of readings, it is prone to a particular kind of undergeneration, an undergeneration that were we using a First Order Logic translation language, we might call 'failing to give the junction ***narrow enough scope***'. Concerning the local approach we will observe that it is not prone to this particular semantic undergeneration problem, through prone to semantic undergeneration problems of its own. Neither approach is thus descriptively adequate.

2.4.1 The non-local approach: Conjunction Reduction

To begin with, the early Transformational Grammar analysis of non-sentential junctions will be described (Chomsky 57). After that, we will consider how to represent the same idea within a THEORY OF REFERENCE. The main idea is that the occurrence of junctions between ***non-sentential*** conjuncts is a 'surface' phenomena, the structures exhibiting it being derived from 'underlying' structures that exhibit only ***sentential*** junction. The name of the transformation that effects the mapping from underlying to surface structure is Conjunction Reduction and its action is depicted in Figure 5.1.

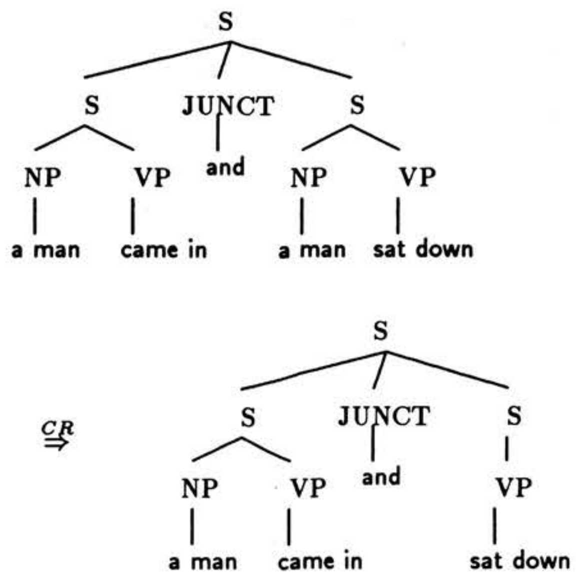


Figure 5.1: The Conjunction Reduction Transformation

To decide whether the CR transformation should apply one first looks at the terminal yield of the deep-structure tree, which in the case of Figure 5.1 is a man came in and a man sat down. Then one compares the two joined sentences in this, to see whether there is a substring α whose replacement by another string β will give the right conjunct. In other words one looks for X and Y such that the left conjunct is XαY and the right conjunct is XβY. If such X and Y can be found, then the transformation may apply and its output is a tree with terminal yield X α

and β Y. Some examples:

(28) [a man]$_X$ [came in]$_\alpha$ and [a man]$_X$ [sat down]$_\beta$
$\rightsquigarrow$ [a man]$_X$ [came in]$_\alpha$ and [sat down]$_\beta$

[John loves]$_\alpha$ [Peter]$_Y$ and [Mary hates]$_\beta$ [Peter]$_Y$
$\rightsquigarrow$ [John loves]$_\alpha$ and [Mary hates]$_\beta$ [Peter]$_Y$

[John]$_X$ [loves]$_\alpha$ [Peter]$_Y$ and [John]$_X$ [hates]$_\beta$ [Peter]$_Y$
$\rightsquigarrow$ [John]$_X$ [loves]$_\alpha$ and [hates]$_\beta$ [Peter]$_Y$

The meaning of the surface structure is inherited from the meaning of the deep-structure; the CR transformation is semantically an identity. The examples make clear that the idea is syntactically appealing at least: one manages to explain a very large number of instances of junctions by just 2 devices: the rule for sentence junction and the conjunction reduction operation. We will postpone consideration of its semantic appeal until we have seen how this idea can be fitted into the Universal Grammar framework.

There are two places that the work done by the CR transformation could be done in the UG framework. One place is in the transfer from disambiguated to ambiguous language, adjusting the disambiguation relation such that conjunction reduced sentences are the ambiguations of unreduced sentences. The other place is within the disambiguated language itself, adding a CR-like operation to the syntactic algebra. It is the latter that is implemented below.

$\mathcal{L}^{11}$:non-sentential junctions by adding CR as a syntactic operation

1. Phrase-set indices: as for $\mathcal{L}^{10}$
2. Basic phrase-sets: as for $\mathcal{L}^{10}$
3. Syntactic operations: as for $\mathcal{L}^{10}$ with the addition of the operation $\mathcal{F}_{\mathcal{CR}}$, the string part of which is such that if $s = Xs_1Y\text{and}Xs_2Y$, then the output is $Xs_1\text{and}s_2Y$, else the output is $Xs_1Y\text{and}Xs_2Y$.
4. Syntactic rules: as for $\mathcal{L}^{10}$ with the addition of $\langle\mathcal{F}_{\mathcal{CR}}, \text{S}, \text{S}\rangle$

The set of possible models, $\mathcal{K}^8$, for $\mathcal{L}^{11}$ has virtually the same definition as $\mathcal{K}^7$ (113). There should be the same type mapping and the same constraints on the interpretation function. The only difference will consist in the presence of as additional operation, $\mathcal{G}_{\mathcal{CR}}$, which is understood to be the identity operation. By $\mathcal{T}^8$ we will mean the combination of $\mathcal{L}^{11}$ and $\mathcal{K}^8$.

$\mathcal{L}^{11}$ clearly generates all the cases of non-sentential junctions and in the simple form it will also doubtless overgenerate, but instead of attending to the these syntactic imperfections we will press on with the semantic assessment.

Transparency

The transparency facts concerning the sentences in (26) are accounted for. For example for the case of junction of VP's we have:

(29) John $(\text{walks})_e$ JUNCT $(\text{talks})_e$

Noting that $\mathcal{T}^8$ provides at most one disambiguation of any expression, the condition for $\mathcal{T}^8$ to predict that walks occurs *opaquely* in (29) is

(30) there is an $\overline{\alpha}$, where α is a VP, and a model, $\langle \Im, \langle w, g \rangle \rangle$, such that (i) whatever proper name and pronoun β, $[\![\overline{\beta \text{ walks}}]\!](w,g) = [\![\overline{\beta\ \alpha}]\!](w,g)$, and (ii) $[\![\overline{\text{John walks JUNCT talks}}]\!](w,g) \neq [\![\overline{\text{John } \alpha \text{ JUNCT talks}}]\!](w,g)$

In the usual way, the co-extension clause (i) entails that $\overline{\text{walks}}$ and $\overline{\alpha}$ have the same denotation. This contradicts (ii):

(31) Entailments of (ii)

$[\![\overline{\text{John walks JUNCT talks}}]\!](w,g) \neq [\![\overline{\text{John } \alpha \text{ JUNCT talks}}]\!](w,g)$

$\leftrightarrow [\![\overline{\text{John walks JUNCT John talks}}]\!](w,g) \neq [\![\overline{\text{John } \alpha \text{ JUNCT John talks}}]\!](w,g)$

$\leftrightarrow [\![\overline{\text{John walks}}]\!](w,g) \neq [\![\overline{\text{John } \alpha}]\!](w,g)$

$\leftrightarrow [\![\overline{\text{walks}}]\!](w,g) \neq [\![\overline{\alpha}]\!](w,g)$

Recursive Ambiguity

The consequences of Hypothesis 4 (p46) for the sentences in (26) were listed in (27). These consequences are accounted for. As an example consider the TV case from (27):

(32) John loves JUNCT hates Mary is recursively ambiguous wrt. loves JUNCT hates

The account of this by $\mathcal{T}^8$ is shown by the equivalences below, the first of which is the condition on $\mathcal{T}^8$ to which the recursive ambiguity claim amounts:

(33) for all models $\langle \Im, \langle w, g \rangle \rangle$, $[\![\overline{\text{John loves JUNCT hates Mary}}]\!](w,g) = 1$
iff $J([\![\overline{\text{John loves Mary}}]\!](w,g) = 1, [\![\overline{\text{John hates Mary}}]\!](w,g) = 1)$

$\leftrightarrow$ for all models $\langle \Im, \langle w, g \rangle \rangle$, $[\![\overline{\text{John loves Mary JUNCT John hates Mary}}]\!](w,g) = 1$
iff $J([\![\overline{\text{John loves Mary}}]\!](w,g) = 1, [\![\overline{\text{John hates Mary}}]\!](w,g) = 1)$

$\leftrightarrow$ for all models $\langle \Im, \langle w, g \rangle \rangle$, $J([\![\overline{\text{John loves Mary}}]\!](w,g) = 1, [\![\overline{\text{John hates Mary}}]\!](w,g) = 1)$ iff $J([\![\overline{\text{John loves Mary}}]\!](w,g) = 1, [\![\overline{\text{John hates Mary}}]\!](w,g) = 1)$

Problems of semantic undergeneration

We will now consider three kinds of sentence with respect to which $\mathcal{T}^8$ semantically undergenerates:

(34) a a man came in and sat down

b a talented and hardworking candidate applied for the job

c John wanted to visit the cinema or read a book at home

The first two cases are sentences containing both a quantifier and a junction and therefore both Hypothesis 3 (p46) and Hypothesis 4 (p46) make predictions concerning readings. To say that a man came in and sat down is recursively ambiguous wrt. a man and wrt. came in and sat down is to say the following two things:

(35) a. there is a reading, r, of (34a) such that whatever situation s, a man came in and sat down is true in s on $r$iff

SOME { x: he$_1$ is a man is true in $s_x^{he_1}$ }

{ x: AND (he$_1$ came in is true in $s_x^{he_1}$) }

he$_1$ sat down is true in $s_x^{he_1}$

b. there is reading, r, of (34a) such that whatever situation s, a man came in and sat down is true in s on r iff

AND(SOME { x: he$_1$ is a man is true in $s_x^{he_1}$ })

{ x: he$_1$ came in is true in $s_x^{he_1}$ }

SOME { x: he$_1$ is a man is true in $s_x^{he_1}$ }

{ x: he$_1$ sat down is true in $s_x^{he_1}$ }

The reading described by (35a) seems a natural one, whilst the reading described by (35b) seems far less natural and is perhaps not possessed by the sentence at all. Now according to $\mathcal{T}^8$, (34a) is not ambiguous at all. This could be counted a minor defect if the reading corresponding to the one disambiguation was the natural one. However, as the reader may confirm, the reading accounted for by this disambiguation is the less natural one (described in (35b)). The more natural reading, (the one described in (35a)) is not accounted for. Therefore at best we have a problem of *undergeneration*. According to one's views on the unnaturalness of the reading described in (35b), we may also have a problem of *overgeneration*.

(34b) is a similar case. To say that a talented and hardworking candidate applied for the job is recursively ambiguous wrt. a talented and hardworking candidate and wrt. talented and hardworking is to say the following two things:

(36) a. there is a reading r such that whatever situations s, a talented and hardworking candidate applied for the job is true in s on r iff

SOME { x: AND (he_1 is a talented candidate is true in $s_x^{\text{he}_1}$) }

he_1 is a hardworking candidate is true in $s_x^{\text{he}_1}$

{ x: he_1 applied of the job is true in $s_x^{\text{he}_1}$ }

b. there is a reading r such that whatever situation s, a talented and hardworking candidate applied for the job is true in s iff

AND(a talented candidate applied for the job is true in s)

a hardworking candidate applied for the job is true in s

(34b) certainly has the reading described in (36a), whereas the reading described in (36b) is marginal. There is only one disambiguation of the sentence in $\mathcal{T}^8$, and one may confirm that this disambiguation is associated with the marginal reading. The natural reading goes unaccounted for.

The final example sentence (34c) presents a slightly different case. Hypothesis 4 (p46) predicts that John wanted to visit the cinema or read a book at home is recursively ambiguous wrt. visit the cinema or read a book at home, which is controversial. Hypothesis 6 (p50) predicts, uncontroversially that there is a reading of (34c) according to which the following argument is valid:

(37) John wanted to visit the cinema or read a book at home

John visited the cinema or read a book at home

∴ an act that John wanted to do, was done by John

According to $\mathcal{T}^8$ there is only one disambiguation of (34c). It has the properties that correspond to the marginal reading rather than to the preferred reading. Therefore $\mathcal{T}^8$ predicts that the above argument is unambiguously *invalid.*

To review these three problematic sentences then, everyone can agree then that $\mathcal{T}^8$ semantically *undergenerates* (Partee 70, and Lakoff 70 note similar undergeneration problems). It is also a debatable point whether there is *overgeneration* here, as the only readings predicted are at best marginal.

Sentences such as those in (34) pose a *semantic* problem for the viability of CR as a tactic for non-sentential conjunction. There are a number of ways one make adjustments to try get around the problem whilst retaining the CR analysis and until these are looked at, one cannot say that the semantic undergenerations are an insuperable problem. In section (2.6) we will give this matter fuller consideration. For the moment we will turn now to a wholly different strategy towards handling non-sentential junctions.

2.4.2 The local approach: Cross Categorial Coordination

In the local approach to be described, **a man came in and sat down** can only be generated if the substring **came in and sat down** is generated. In similar vein, the generation of **John loves and hates Peter** will depend on the generation of **loves and hates**, the generation of **a talented and hardworking candidate applied for the job** will depend on the generation of **talented and hardworking**. None of these statements are true of $\mathcal{T}^8$.

This is done by supposing a disambiguated language exactly like $\mathcal{L}^{10}$ with the addition, for every phrase-set index δ, of the rule:

$\langle \mathcal{F}_{\mathcal{J}}, \langle \delta, \text{JUNCT}, \delta \rangle, \delta \rangle$

Let this be $\mathcal{L}^{12}$. The class of models associated with $\mathcal{L}^{12}$ is defined by changing only very slightly the definition of $\mathcal{K}^7$ (p113), the class of models associated with $\mathcal{L}^{10}$. The only change is in fact in the 'algebra-spanning' function $\mathcal{H}_{\mathcal{J}}$. Previously this was defined to be significant only at a triple of arguments of types $t, (t \rightarrow (t \rightarrow t))$ and t. $\mathcal{H}_{\mathcal{J}}$ will be redefined to be significant at a number of other types. Therefore the class of models, $\mathcal{K}^9$, associated with $\mathcal{L}^{12}$, is exactly the same as $\mathcal{K}^7$ except that $\mathcal{H}_{\mathcal{J}}$ will be redefined. Just as it was possible to let some facts about recursive ambiguity dictate the denotations of the junctions (see section 2.2.1), it is possible to let further facts about recursive ambiguity dictate the definition of $\mathcal{H}_{\mathcal{J}}$ at the additional types. Therefore, the redefinition of $\mathcal{H}_{\mathcal{J}}$ will be postponed for a little while, until some relevant recursive ambiguity data has been considered. $\mathcal{T}^9$ will be understood to be the combination of $\mathcal{L}^{12}$ and $\mathcal{K}^9$. As will become clear, it is the UG embodiment of Gazdar's Cross-Categorial Coordination proposal (Gazdar 1980, see also Keenan and Faltz 1985). We will now semantically assess $\mathcal{T}^9$.

Transparency facts

As noted several times already, the generalisation concerning the sentences in (26) to be explained is that the constituents linked by the junction word occur transparently. To explain this, $\mathcal{T}^9$ needs simply to mimic the feature of $\mathcal{T}^5$ (p105) that allowed capture of the transparency facts for just sentential junctions. This feature was that the semantic operation on *meanings*, $\mathcal{G}_{\mathcal{J}}$, implicated by the derivation of the junction-containing sentence, had to be definable from a semantic operation, $\mathcal{G}^E_{\mathcal{J}}$, on *denotations*. If this feature is insisted on in the constraints on $\mathcal{G}_{\mathcal{J}}$ as an operation featuring in the models in $\mathcal{K}^9$, then the transparency data that was explained by the non-local approach, $\mathcal{T}^8$, will also be explained by the local approach $\mathcal{T}^9$.

Recursive Ambiguity

We pass on then to the recursive ambiguity intuitions that we gave in (27). To begin with consider just the TV case. The required entailment of $\mathcal{T}^9$ is given in (38a) whilst what $\mathcal{T}^9$

actually does entail is given in (38b):

(38) a. for all models $\langle \Im, \langle w, g\rangle\rangle$, $[\![\text{John loves JUNCT hates Mary}]\!](w,g) = 1$
iff $J([\![\text{John loves Mary}]\!](w,g) = 1, [\![\text{John hates Mary}]\!](w,g) = 1)$

b. there is a disambiguation β such that whatever model $\langle \Im, \langle w, g\rangle\rangle$, $[\![\beta]\!](w,g) = 1$ iff
$\mathcal{H}_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})([\![\text{loves}]\!], [\![\text{JUNCT}]\!], [\![\text{hates}]\!])(w,g)([\![\text{Mary}]\!](w,g))([\![\text{John}]\!](w,g)) = 1$

It can clearly only be the case that (38b) will amount to (38a) if $\mathcal{H}_{\mathcal{J}}$ is so constrained as to make (39a) hold:

(39) (a) whatever model, $\langle \Im, \langle w, g\rangle\rangle$,
$\mathcal{H}_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})([\![\text{loves}]\!], [\![\text{JUNCT}]\!], [\![\text{hates}]\!])(w,g)([\![\text{Mary}]\!](w,g))([\![\text{John}]\!](w,g)) = 1$ iff
$J([\![\text{loves}]\!](w,g)([\![\text{Mary}]\!](w,g))([\![\text{John}]\!](w,g)) = 1,$
$[\![\text{hates}]\!](w,g)([\![\text{Mary}]\!](w,g))([\![\text{John}]\!](w,g)) = 1)$

$\leftrightarrow$(b) whatever $\langle \Im, \langle w, g\rangle\rangle$, whatever d_1^e, d_2^e, P_3^{eet}, P_4^{eet}
$\mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(P_3, [\![\text{JUNCT}]\!](w,g), P_4)(d_1)(d_2) = [\![\text{JUNCT}]\!](w,g)(P_3 d_1 d_2)(P_4 d_1 d_2)$

$\leftarrow$ (c) whatever $\langle \Im, \langle w, g\rangle\rangle$, whatever d_1^e, d_2^e,P_3^{eet}, P_4^{eet}, $f^{(t\rightarrow(t\rightarrow t))}$
$\mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(P_3, f, P_4)(d_1)(d_2) = f(P_3 d_1 d_2)(P_4 d_1 d_2)$

(39b) is an equivalent to (39a). (39c) is a slightly stronger condition than (39b), concerning all members of $D^{(t\rightarrow(t\rightarrow t))}$ and not just the denotations of junctions. (39c) is a constraint on $\mathcal{H}_{\mathcal{J}}$, and what is presented below is the whole set of such constraints on $\mathcal{H}_{\mathcal{J}}$ generated by considering all of the sentences in (27):

(40) whatever $\langle \Im, \langle w, g\rangle\rangle$, whatever $f^{t(tt)}$
a. whatever P_1^{et}, P_2^{et},d_1^e,
$\mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(P_1, f, P_2)(d_1) = f(P_1 d_1)(P_2 d_1)$
b. whatever P_1^{eet}, P_2^{eet}, d_1^e, d_2^e,
$\mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(P_1, f, P_2)(d_1)(d_2) = f(P_1 d_1 d_2)(P_2 d_1 d_2)$
c. whatever $P_1^{(s,et)et}$, $P_2^{(s,et)et}$, d_1^{set}, d_2^e
$\mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(P_1, f, P_2)(d_1)(d_2) = f(P_1 d_1 d_2)(P_2 d_1 d_2)$
d. whatever $P_1^{e(s,et)et}$, $P_2^{e(s,et)et}$, d_1^e, d_2^{set}, d_3^e,
$\mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(P_1, f, P_2)(d_1)(d_2)(d_2) = f(P_1 d_1 d_2 d_3)(P_2 d_1 d_2 d_3)$

This then expresses some constraints that need to be placed on the function $\mathcal{H}^E_{\mathcal{J}}$ if this local approach to non-sentential junctions is going to conform to certain instances of the recursive ambiguity data. There is a finite set of types at which the $\mathcal{H}_{\mathcal{J}}$ operation is required to be significant by the above equations, a subset of the set of types that has come to be called the 'Conjoinable Types':

Definition 52 (Conjoinable Types) *Whatever type, $a \in \text{TJ}^{\rightarrow}$, (i) a is a conjoinable type if a is t, (ii) $(a \rightarrow b)$ is a conjoinable type if b is a conjoinable type.*

The redefinition of $\mathcal{H}_{\mathcal{J}}$ will render its value at $\mathcal{E}, \mathcal{I}$ and $\mathcal{J}$ an operation which is significant not just at the subset of the conjoinable types illustrated in (40), but at arguments of *all* conjoinable types:

Definition 53 (New $\mathcal{H}_{\mathcal{J}}$) *For any $\mathcal{E}, \mathcal{I}, \mathcal{J}$, for any $a \in \text{TJ}^{\rightarrow}$ where a is conjoinable, for any $P_1^a, P_2^a, f^{t(tt)}$*

if $a = t$, $\mathcal{H}^E_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(P_1, f, P_2) = f(P_1)(P_2)$,

if $a = (b \rightarrow c)$, $\mathcal{H}^E_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(P_1, f, P_2) = x^b \mapsto \mathcal{H}^E_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(P_1 x, f, P_2 x)$,

for any $m_1^a, m_2^{t(tt)}, m_3^a$,

$\mathcal{H}_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_1, m_2, m_3) = (w, g) \mapsto \mathcal{H}^E_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_1(w, g), m_2(w, g), m_2(w, g))$

The reader may confirm that with $\mathcal{H}_{\mathcal{J}}$ constrained as above, the conditions in (40) will be entailed by $\mathcal{T}^9$, and therefore the recursive ambiguity data given in (27) will be accounted for by $\mathcal{T}^9$. The explanation of the sentential junctions is also retained.

Problems of semantic undergeneration ?

Having now fully specified the local approach, $\mathcal{T}^9$, we will now consider the sentences that were semantically problematic to the proposal based on using Conjunction Reduction, $\mathcal{T}^8$. See (34, p121). Recall that $\mathcal{T}^8$ could not explain the most natural readings of sentences in (34). This problem is *not* faced by the local approach, $\mathcal{T}^9$. The disambiguations provided by $\mathcal{T}^9$ of the sentences in (34) are illustrated in the Figures 5.2, 5.3 and Figure 5.4 .

However, to say that $\mathcal{T}^9$ lacks the semantic undergenerations of $\mathcal{T}^8$ is not to say that $\mathcal{T}^9$ is descriptively adequate. $\mathcal{T}^9$ provides at most one disambiguation of any expression and therefore the full set of entailments of Hypothesis 4 (p46) will never be accounted for by $\mathcal{T}^9$.

2.4.3 Summary

What we have done in this section is describe two possible extensions of the core-account, $\mathcal{T}^7$ (p113), each aimed at accounting for extensive priveleges of occurrence of junctions. $\mathcal{T}^8$ (p119) embodied a non-local approach to junctions, whilst $\mathcal{T}^9$ (p123) embodied a local approach. Neither of the accounts was descriptively adequate. It was seen that certain semantic undergenerations that $\mathcal{T}^8$ is subject to, $\mathcal{T}^9$ is not.

This will not be our final word on the potential for a descriptively adequate account based on the mechanisms of $\mathcal{T}^8$ and $\mathcal{T}^9$. These two approaches will be considered further in section 2.6.

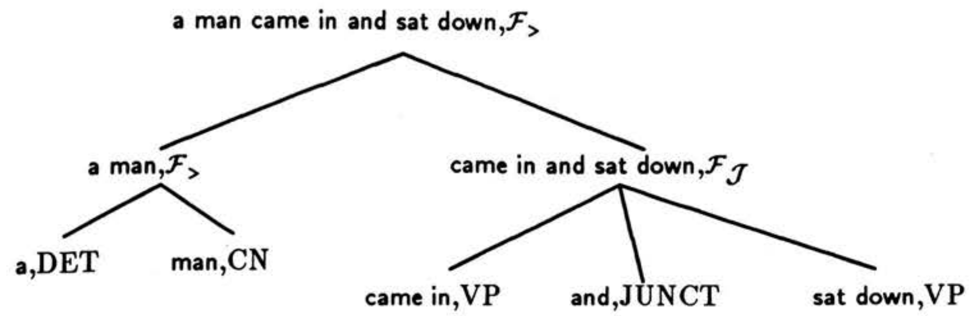


Figure 5.2:

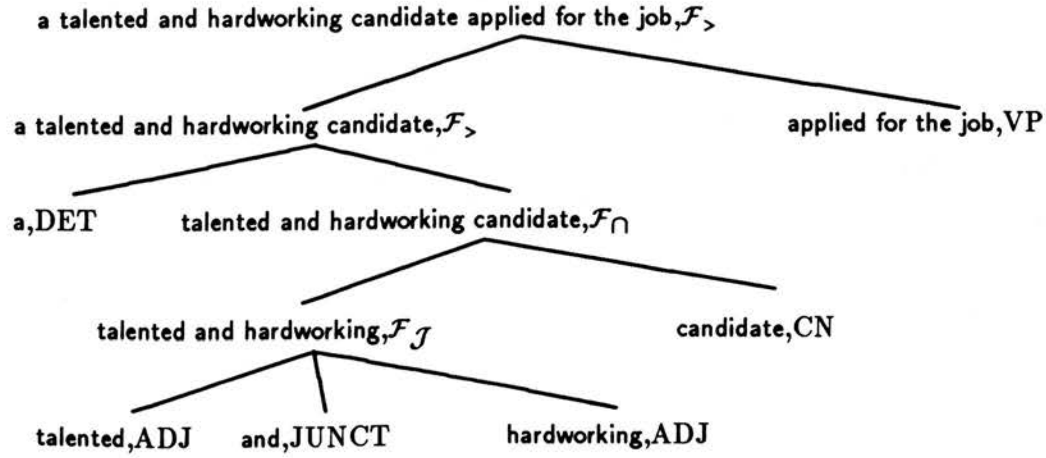


Figure 5.3:

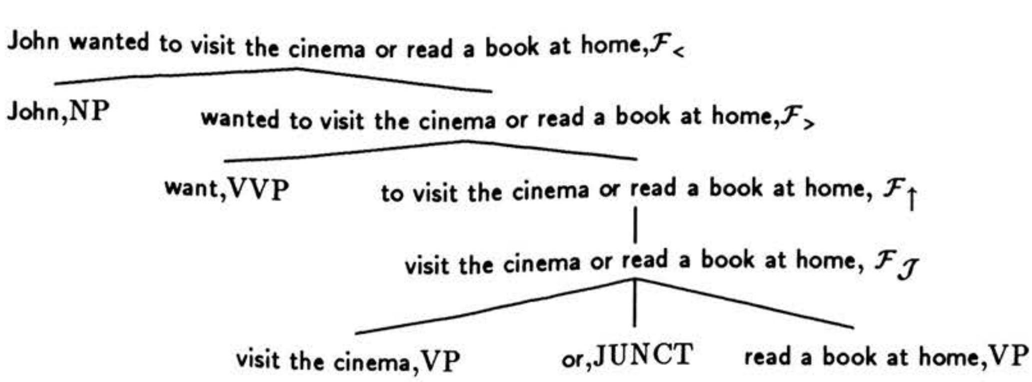


Figure 5.4:

For the moment, however, we will leave the non-sentential junctions and turn to the matter of non-subject quantifiers. As with junctions, it turns out for the quantifiers that theoreticians have proposed two kinds of account, one a non-local and the other local. These will be described in section 2.5. Section 2.6 will therefore not only consider further the potential of the non-local and local analyses of junctions, but will consider in parallel, the potential of the non-local and local analyses of quantifiers.

2.5 Non-subject position quantifications

Quantifiers have rather wide priveleges of occurrence:

(41) a. every man died
  b. Mary loves every man
  c. John gave every man Mary
  d. John told every man to go

As we did in the case of junction we will look at two strategies for accounting for this syntactic fact. The first approach that we will look at take its inspiration once again from transformational grammar, its crucial ingredient being the 'Quantifier Lowering' transformation. This is a *non-local* strategy. The second approach really has no name although it is very often used. We will call it Cross Categorial Quantification. It is a *local* strategy. Besides accounting for the syntactic fact of wide priveleges of occurrences, there are semantic facts to be accounted for, as ever falling into two kinds: transparency and recursive ambiguity. In the unambiguous sentences in (41), there are transparent occurrences of the common noun following the determiner, and also of the verbal term adjacent to the determiner. Concerning these sentences, Hypothesis 3 (p46) entails that the sentences are recursively ambiguous wrt. the QNP within them. This is the minimal semantic data that must be accounted for. Over and above there is also semantic data concerning genuine instances of ambiguity.

We will look at the approaches as different possible extensions to $\mathcal{T}^9$, which was the account given in section 2.4.2 embodying the *local* approach to junctions. This is convenient for expository purposes and nothing ultimately depends on it: it will be considered in section 2.6 whether the observations made here about mechanisms for quantification would be changed if the basis were other than the local account of junctions.

Each of the two approaches is semantically assessed. The result is that the non-local approach is subject to a particular kind of semantic undergeneration. The local approach is not subject to this kind of semantic undergeneration, though subject to many other instances of semantic undergeneration. Therefore neither approach is descriptively adequate.

2.5.1 The non-local approach: Quantifier Lowering

The approach to quantifiers that will be described is that which was current in the Transformational Grammar architecture of Chomsky 57 and Chomsky 65. The main idea here is that the location of quantifier phrases in NP positions is a 'surface' phenomenon only, and the structures that do exhibit QNP's in NP positions are derived from 'underlying' structures where the QNP's do not occupy NP positions but instead are akin to *sentence modifiers*, entering trees via 'Chomsky-adjunction'. The name of transformation that effects the mapping from deep-structure sentence-modifier quantifier to surface structure NP-located quantifiers is Quantifier Lowering. An example of its action is depicted in Figure 5.5.

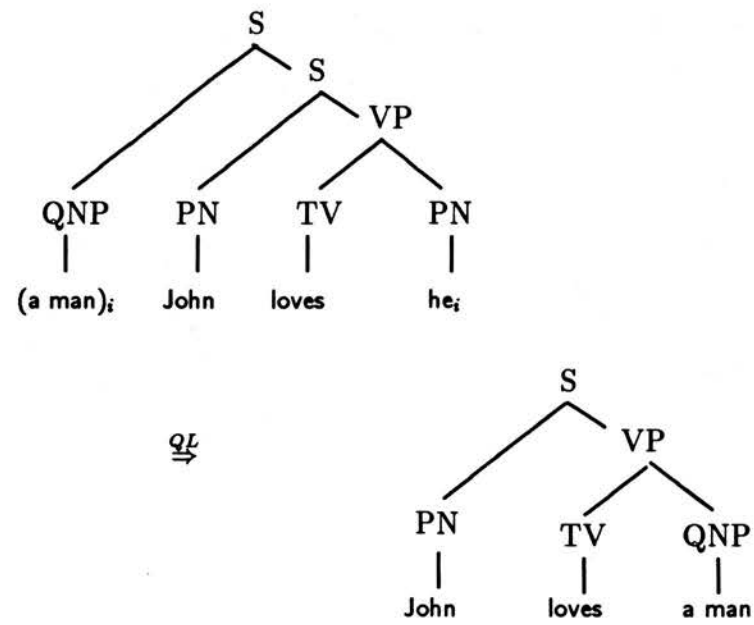


Figure 5.5: The Quantifier Lowering Transformation

As the input to the transformation one is looking for an S node whose left daughter is a subtree: $[(\beta)_i]_{QNP}$, and whose right daughter is another S. The daughter S node should itself dominate a subtree: $[\text{he}_i]_{NP}$. The output of the transformation is an amended version of the daughter S node, one which replaces the $[\text{he}_i]_{NP}$ subtree with $[\beta]_{QNP}$.

This is illustrated in (42).

(42) $[[(\text{a man})_i]_{QNP}\ [[\text{he}_i]_{NP}\ \text{walks}]_S]_S$

$\rightsquigarrow [[\text{a man}]_{QNP}\ \text{walks}]_S$

$[[(\text{a man})_i]_{QNP}\ [\text{Mary believes}\ [\text{he}_i]_{NP}\ \text{walks}]_S]_S$

$\rightsquigarrow [\text{Mary believes}\ [\text{a man}]_{QNP}\ \text{walks}]_S$

$[[(\text{a man})_i]_{QNP}$ [Mary loves $[\text{he}_i]_{NP}]_S]_S$

$\rightsquigarrow$ [Mary loves $[\text{a man}]_{QNP}]_S$

As with the CR transformation, the meaning of the surface structure is intended to be inherited from the deep structure; QL is semantically an identity. There are indices on the QNP phrases and the pronouns and these are essential. In the deep-structure, there is nothing configurational about the tree to indicate which argument slot of a relation a quantifier is concerned with. Yet it is the deep-structure that defines the meaning of the sentence. It is *co-indexing* that codes which argument role a quantifier is concerned with.

It is clear that this is in spirit a *non-local* approach to quantification, because the generation of the surface structure sentence depends on deep-structures whose terminal yields are not substrings of the sentence. There is a proposal akin to the above one within the more recent Transformational Grammar architecture, as in May 84. The difference is that besides Deep Structure there is another level with a relationship with Surface Structure, a level known as Logical Form. Logical Form is the level defining semantic interpretation is not involved in the inductive definition of the grammatical sentences, and is derived *from* Surface Structure. Figure 5.5 would also approximately represent the hypothesis concerning quantifiers in this architecture if the arrow were reversed and labelled 'Quantifier-Raising'. However, because I have not fully considered this latter-day TG architecture, I intend the comments below to concern only the earlier TG architecture.

One of the syntactic appeals of the (early) TG proposal is akin to that of CR: one manages to explain a very large number of instances of quantification by just three devices: the rules defining the distribution of pronouns, the rule of Chomsky-adjoining a QNP and sentence, and the Quantifier Lowering transformation.

As we did with Conjunction Reduction we will now address two questions. First the question of how to incorporate this transformational analysis into a UG-style fragment. Second, the semantic plausibility.

As for CR, there are two possible places in which QL could be represented in the Universal Grammar architecture: as a part of the disambiguation relation or as an operation in the syntactic algebra. Both are considered below.

To represent QL as part of the disambiguation relation the idea is to define the disambiguation relation so that a disambiguated expression $\langle\alpha\rangle_\zeta$ stands in the relation to any β such that β comes from α_1 by a series of quantifier lowerings. With such an alteration of the disambiguation relation, the remaining necessary syntactic change is to expand the set of rules and operations of $\mathcal{L}^{12}$ to include the rule of 'Chomsky Adjunction'.

$\mathcal{L}^{13}$: QL as part of the disambiguation relation

1. Phrase set indices: as for $\mathcal{L}^{12}$
2. Basic phrase-sets: as for $\mathcal{L}^{12}$
3. Syntactic operations: as for $\mathcal{L}^{12}$ with the addition with the family of operation $\mathcal{F}_{CHOM.i}$, whose operation on strings is:

   $\mathcal{F}_{CHOM.i}(\alpha, \beta) = (\alpha)_i\beta$
4. Syntactic rules: as for $\mathcal{L}^{12}$ with the addition of the family of syntactic rules:

   $\{\langle \mathcal{F}_{CHOM.i}, \langle \text{QNP}, \text{S}\rangle, \text{S}\rangle : i \leq N\}$
5. Disambiguation relation

   The new disambiguation relation will be defined with the assistance of a function $\mathcal{QL}_i$: For all $\langle s_1, d, \epsilon\rangle$, $\mathcal{QL}_i(\langle s_1, d, \epsilon\rangle) = \langle s_2, d, \epsilon\rangle$, where s_2 is the deletion from s_1 of the first occurrence of a bracketed string subscripted by i, $(\alpha)_i$ and the replacement of subsequent occurrences of he_i by α.

   $\mathcal{R}^{13}(\beta, \langle\alpha\rangle_\zeta)$ iff there is some sequence of $\mathcal{QL}_i$ such that β is the first argument projection of $\mathcal{QL}_1(\ldots \mathcal{QL}_n(\langle\alpha\rangle_\zeta))$, and β contains no bracketed and subscripted strings.

The definition of the set of possible models, $\mathcal{K}^{10}$, for $\mathcal{L}^{13}$ is almost exactly the same as that for $\mathcal{L}^{12}$. All that is additional is the definition of the semantic operation associated with the syntactic 'Chomsky-adjunction' operation, $\mathcal{F}_{MONT.i}$. This is defined from the following 'algebra-spanning' function $\mathcal{H}_{MONT.i}$.

Definition 54 ($\mathcal{H}_{MONT.i}$) *For any $\mathcal{E}, \mathcal{I}$ and $\mathcal{J}$, for any $m_1^{((e\rightarrow t)\rightarrow t)}$, for any $m_2{}^t$, for any $(w, g) \in I \times J$*

$\mathcal{H}_{MONT.i}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_1, m_2) = \mathcal{H}_{>}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_1, \mathcal{H}_{\lambda \mathsf{he}_i}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_2))$

The reason for naming the operation that corresponds to $\mathcal{F}_{CHOM.i}$, $\mathcal{G}_{MONT.i}$ is that we shall soon be using the very same semantic operation in association with a syntactic operation invented by Montague: the notorious 'Quantifying-in' operation.

Now to consider the option of having QL as a syntactic operation. The functions $\mathcal{QL}_i$ that were defined above to assist in the specification of the ambiguation relation, can be taken as syntactic operations in their own right. They should semantically be associated with identity operations. However, if we are to have such an operation in the algebra, it will have to be implicated by a syntactic rule and it is difficult to say what kind of rule that should be. One that maps from sentences to sentences will not do because that will entail that the effective input to the QL transformation is a grammatical sentence, which it is not.

One could try to add a new phrase-set index with the specific purpose of categorising the typical input to QL, giving the input a category thereby but not recognising it as a *sentence*. A more economical approach, however, is to *compose* together each of the 'Chomsky-adjunction' operations, $\mathcal{F}_{CHOM,i}$, with the corresponding Quantifier Lowering operation, $\mathcal{QL}_i$.

(43) $\langle$a man$, d_1, \epsilon_1\rangle$ $\langle$he$_i$ walks$, d_2, \epsilon_2\rangle$ $\Downarrow \mathcal{F}_{CHOM.i}$ $\langle$(a man)$_i$ he$_i$ walks$, d_3, \epsilon_3\rangle$ $\Downarrow \mathcal{QL}.i$ $\langle$a man walks$, d_3, \epsilon_3\rangle$

$\langle$a man$, d_1, \epsilon_1\rangle$ $\langle$he$_i$ walks$, d_2, \epsilon_2\rangle$ $\Downarrow \mathcal{F}_{MONT.i}$ $\langle$a man walks$, d_3, \epsilon_3\rangle$

Each $\mathcal{F}_{MONT.i}$ operation is therefore one which inserts a QNP phrase in the place of a pronoun he$_i$, and is implicated by a rule which maps from a QNP and S, to an S. These $\mathcal{F}_{MONT.i}$ operations are exactly Montague's 'Quantifying-in' operation, put forward in PTQ. The semantic operation that should be associated with each 'Quantifying-in' operation will be exactly the same as that which was associated with the 'Chomsky-adjunction' operation: $\mathcal{G}_{MONT.i}$.

$\mathcal{L}^{14}$: QL and Chomsky Adjunction as a single syntactic operation

1. Phrase-set indices: as for $\mathcal{L}^{12}$
2. Basic phrase-sets: as for $\mathcal{L}^{12}$
3. Syntactic operations: as for $\mathcal{L}^{12}$ with the addition of a family operations, $\mathcal{F}_{MONT,i}$, whose behaviour on strings is:

   $\forall n, \mathcal{F}_{MONT.n}(\alpha, \beta) =$ that string which results from replacing the first of any occurrences of he$_n$ with α.
4. Syntactic rules: as for $\mathcal{L}^{12}$ with the addition of the family of rules (recall that N is the number of members of of NPPRO):

   $\{\langle \mathcal{F}_{MONT.i}, \langle \text{QNP}, \text{S}\rangle, \text{S}\rangle : i \leq N\}$

The definition of the class of possible models, $\mathcal{K}^{11}$, for $\mathcal{L}^{14}$ is exactly the same as that for $\mathcal{L}^{13}$. $\mathcal{T}^{11}$ will be understood to be the combination of $\mathcal{L}^{14}$ and $\mathcal{K}^{11}$. In what follows we will be assuming the $\mathcal{T}^{11}$ method of incorporating in UG the 'Chomsky-Adjunction plus QL' transformational analysis. What follows is the semantic assessment of $\mathcal{T}^{11}$.

Transparency/Opacity

As a sample of the transparency data pertaining to the sentences listed in (41), consider whether $\mathcal{T}^{11}$ accounts for the fact that there is a reading of (41a) such that man occurs transparently.

(44) there is a disambiguation, $\overline{\text{every man walks}}$, that has a subpart a disambiguation, $\overline{\text{man}}$, such that whatever $\overline{\alpha}$, where α is a CN and whatever model, $\langle \Im, \langle w, g\rangle\rangle$, if (i) for all proper name and pronouns, β, $[\![\overline{\beta \text{ is a man}}]\!](w, g) = [\![\overline{\beta \text{ is a } \alpha}]\!](w, g)$ then (ii) $[\![\overline{\text{every man walks}}]\!](w, g) = [\![\overline{\text{every } \alpha \text{ walks}}]\!](w, g)$

A disambiguation of every man walks that has the property claimed in the above is:

$\mathcal{F}_{MONT.1}(\mathcal{F}_{>}(\overline{\text{every}}, \overline{\text{man}}), \mathcal{F}_{<}(\overline{\text{he}_1}, \overline{\text{walks}}))$

In the usual way one may conclude from the supposition of (i) that $[\![\overline{\text{man}}]\!](w,g) = [\![\overline{\alpha}]\!](w,g)$, and from this one may conclude the following identity:

$$[\![\mathcal{F}_{MONT.1}(\mathcal{F}_{>}(\overline{\text{every}}, \overline{\text{man}}), \mathcal{F}_{<}(\overline{\text{he}_1}, \overline{\text{walks}}))]\!](w,g) = [\![\mathcal{F}_{MONT.1}(\mathcal{F}_{>}(\overline{\text{every}}, \overline{\alpha}), \mathcal{F}_{<}(\overline{\text{he}_1}, \overline{\text{walks}}))]\!](w,g)$$

This is equivalent to (ii). The transparency of the occurrence of walks is not so easily seen. What has to be considered is whether it is possible to have $[\![\overline{\text{walks}}]\!](w,g) = [\![\overline{\alpha}]\!](w,g)$ yet have the non-identity shown as the first line of (45). (45) also shows the entailments of this non-identity and it leads to the contradiction of $[\![\overline{\text{walks}}]\!](w,g) = [\![\overline{\alpha}]\!](w,g)$. To do so it is necessary to use the Pronoun Postulate, Definition 46 (p108).

(45) $[\![\mathcal{F}_{MONT.1}(\mathcal{F}_{>}(\overline{\text{every}}, \overline{\text{man}}), \mathcal{F}_{<}(\overline{\text{he}_1}, \overline{\text{walks}}))]\!](w,g)$
$\neq [\![\mathcal{F}_{MONT.1}(\mathcal{F}_{>}(\overline{\text{every}}, \overline{\text{man}}), \mathcal{F}_{<}(\overline{\text{he}_1}, \overline{\alpha}))]\!](w,g)$
$\leftrightarrow [\![\overline{\text{every man}}]\!](w,g)(x \mapsto [\![\overline{\text{he walks}}]\!](w, g^x_{\text{he}_1}))$
$\neq [\![\overline{\text{every man}}]\!](w,g)(x \mapsto [\![\overline{\text{he }\alpha}]\!](w, g^x_{\text{he}_1}))$
$\leftrightarrow (x \mapsto [\![\overline{\text{he walks}}]\!](w, g^x_{\text{he}_1})) = (x \mapsto [\![\overline{\text{he }\alpha}]\!](w, g^x_{\text{he}_1}))$
$\leftrightarrow$ for some x, $[\![\overline{\text{he walks}}]\!](w, g^x_{\text{he}_1}) \neq [\![\overline{\text{he }\alpha}]\!](w, g^x_{\text{he}_1})$
$\leftrightarrow$ for some x, $[\![\overline{\text{walks}}]\!](w, g^{\text{he}_1}_x)(x) \neq [\![\overline{\alpha}]\!](w, g^{\text{he}_1}_x)(x)$
$\leftrightarrow$ for some x, $[\![\overline{\text{walks}}]\!](w, g^{\text{he}_1}_x) \neq [\![\overline{\alpha}]\!](w, g^{\text{he}_1}_x)$
$\leftrightarrow [\![\overline{\text{walks}}]\!](w,g) \neq [\![\overline{\alpha}]\!](w,g)$ (because of the Pronoun postulate)

The transparency facts for the other sentences in (41) are all accounted for in a similar fashion. For quantifiers in *embedded* sentences Hypothesis 2 (p37) and Hypothesis 1 (p37) are relevant. Hypothesis 2 predicts that embedded quantifiers have a *de-re* interpretation. The *downward* heritability of opacity described by Hypothesis 1 also predicts that an embedded the quantifier should have a *de-dicto* interpretation.

$\mathcal{T}^{11}$ conforms completely to the predictions of Hypothesis 2 but only partly to the predictions of Hypothesis 1: *de-dicto* interpretations of embedded quantifiers are only possible when the quantifier is a substring of an *sentence*. When the quantifier is a substring of an embedded VP, then there is no explanation of the possibility of a *de-dicto* interpretation. More will be said about this in a moment. Figures 5.6 and 5.7 depict the disambiguations that allow for *de-re* and the *de-dicto* interpretation of a man in John believes that a man came in.

Recursive Ambiguity

Hypothesis 3 (p46) entails that the sentences in (41) are recursively ambiguous wrt. the QNP within them, and this is accounted for by $\mathcal{T}^{11}$. Also in the case of a number of genuinely ambiguous sentences, $\mathcal{T}^{11}$ still accounts for the entailments of Hypothesis 3. For example, recursive ambiguity of every man loves a woman with respect to both the quantifiers, amounts

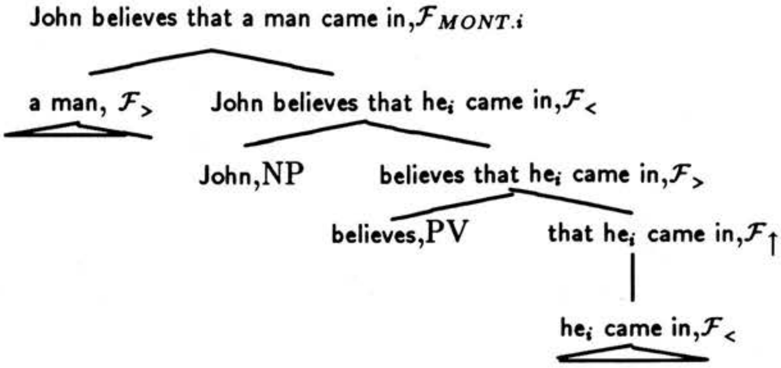


Figure 5.6: A *transparent* occurrence of man in John believes that a man walks

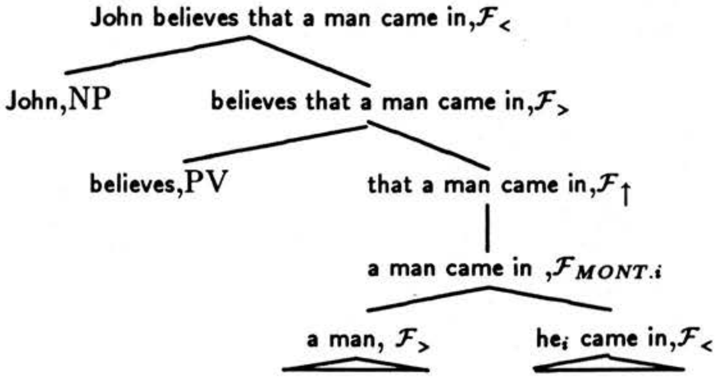


Figure 5.7: An *opaque* occurrence of man in John believes that a man walks

to requiring two readings (see p45) in Chapter 3). The disambiguations provided by $\mathcal{T}^{11}$ to account for the two readings are depicted in Figure 5.8.

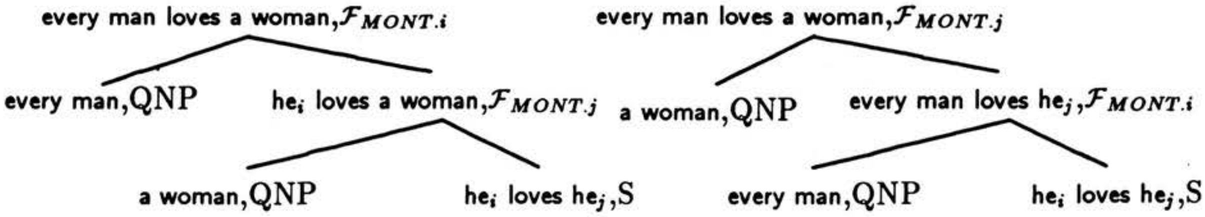


Figure 5.8: Two readings of every man loves a woman

Problems of semantic undergeneration

Below three kinds of sentence are shown with respect to which $\mathcal{T}^{11}$ semantically undergenerates:

(46) a. John consumed a pie or drank a beer

b. every dog near a door barked

c. John wanted to marry a blond

The first case involves a junction and a quantifier and therefore both Hypothesis 4 (p46) and Hypothesis 3 make predictions concerning it. $\mathcal{T}^{11}$ conforms to the predictions of Hypothesis 3 but not to the predictions of Hypothesis 4.

The prediction of Hypothesis 4 is that (46a) is recursively ambiguous wrt. consumed a pie or drank a beer, and this entails the existence of a reading with the property described below:

(47) there is a reading r of (46a) such that whatever situation s, (46a) is true in s on reading r iff OR (John consumed a pie is true in s,John drank a beer is true in s)

Bearing in mind that $\mathcal{T}^{11}$ contains a *local* analysis of junctions, the reader may confirm that $\mathcal{T}^{11}$ provides just the two disambiguations of (46a) shown in Figure 5.9. These are associated not with the reading required by Hypothesis 4, but with the readings required by Hypothesis 3.

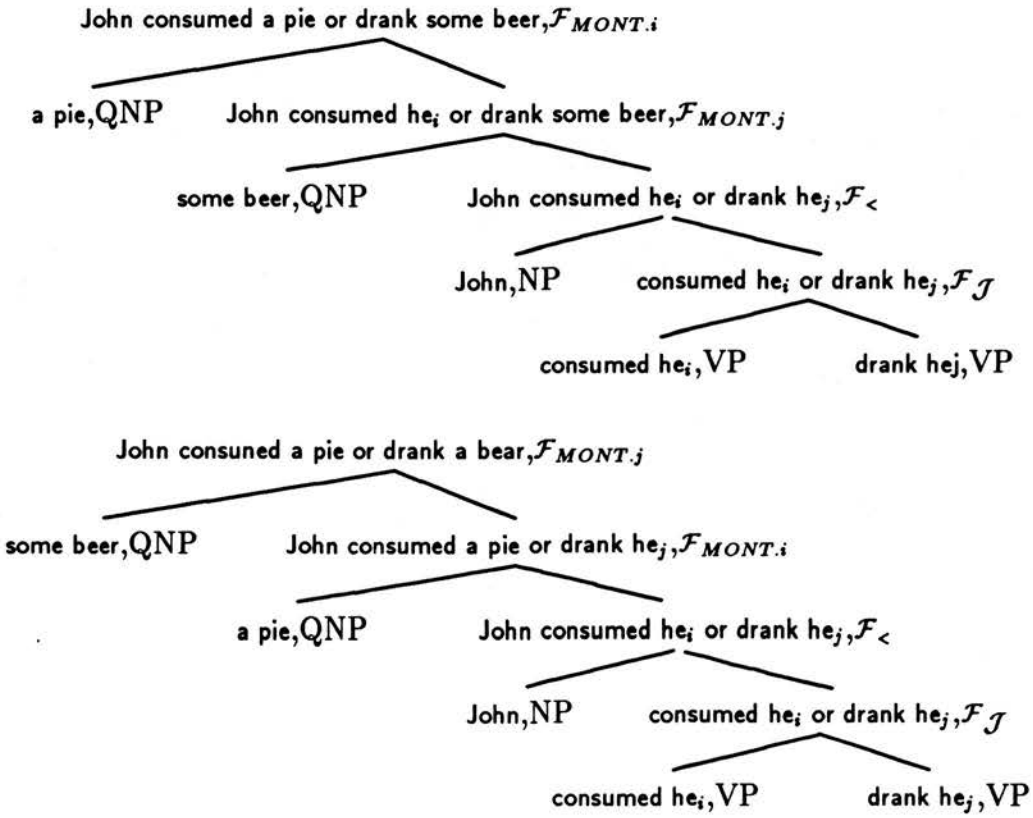


Figure 5.9: The two disambiguations of John consumed a pie or drank a beer

A way to underscore how serious a semantic defect this is is to observe that $\mathcal{T}^{11}$ predicts the following argument to be unambiguously valid.

(48) John consumed a pie or drank some beer.

$\therefore$. There were beer's

Turning to the next of the sentences from (46) concerning which $\mathcal{T}^{11}$ semantically undergenerates, (46b), one can see that Hypothesis 3 suggests that it is recursively ambiguous wrt. every dog near a door and wrt. a door. $\mathcal{T}^{11}$ allows only for recursive ambiguity wrt. a door. Recursive ambiguity wrt. every dog near a door entails the existence of a reading as described below:

(49) There is a reading r, such that whatever situation s, every dog near a door died is true on r in s iff EVERY { x: he$_i$ is a dog near a door is true in $s^x_{\mathsf{he}_i}$ }
{ x: he$_i$ died is true in $s^x_{\mathsf{he}_i}$ }

The only disambiguation of (46b) is depicted in Figure 5.10. It is not associated with the reading described in (49).

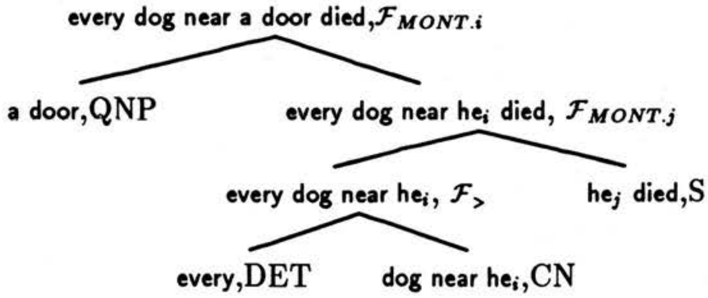


Figure 5.10: The only disambiguation of every dog near a door barked

Finally turning to the last of the problem sentences, (46c), one can see that there are predictions concerning it generated by Hypothesis 3 and by Hypothesis 6 (p50). The prediction of Hypothesis 3 is accounted for, but that of Hypothesis 6 is not. The reading predicted by Hypothesis 6 is that on which the CN, blond, occurs opaquely, and according to which the following inference is valid:

(50) John wanted to marry a blond
John married a blond
∴ an act that John wanted to do, was done by John

The only disambiguation of (46c) according to $\mathcal{T}^{11}$ is depicted in Figure 5.11. The semantic properties associated with this disambiguation are not such as render the occurrence of blond opaque, nor are they such as to render the argument in (50) valid. In fact the combination of $\mathcal{T}^{11}$ predicts the argument to be unambiguously invalid.

We have just seen that $\mathcal{T}^{11}$ undergenerates with respect to the sentences in (46), so that $\mathcal{T}^{11}$ cannot be counted as descriptively adequate. Semantic objections to the sufficiency of a 'Quantifying-in' type mechanism have been made before (Partee 70, for example), but they have usually involved difficulties with quantifier-pronoun interactions. The example used above, because not involving pronouns, are independent of any particular semantic assumptions concerning their treatment.

In section 2.6 we will consider how these problems of undergeneration may be overcome. Before that we will look at a *local* approach to quantification.

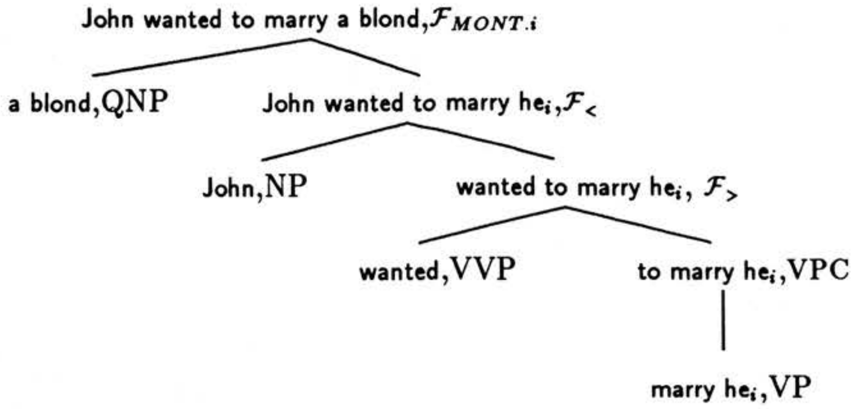


Figure 5.11: The only disambiguation of **John wanted to marry a blond**

2.5.2 The local approach: Cross Categorial Quantification

In $\mathcal{T}^{11}$ there were disambiguations of **loves John**, **gave John**, **told John** and **near John** but not of **loves every man**, **gave every man**, **told every man**, and **near every man**. In the THEORY OF REFERENCE to be considered in this section, there will be disambiguations also of these quantifier-containing subsentential expressions. We will have thereby a *local* approach to sentences containing quantifiers.

Recall that we are presenting the non-local and local approaches to quantification as different possible extensions of $\mathcal{T}^9$ (p123), the local approach to junctions. What we will now have is, corresponding to any rule of $\mathcal{L}^{12}$ (p123) that mentions NP's, a sister rule that mentions QNP's. In $\mathcal{L}^{12}$ we had a quantificational sister to the subject NP rule, and it made mention of $\mathcal{F}_>$. The rule for $\mathcal{L}^{15}$ will be:

$\langle \mathcal{F}_Q, \langle \mathrm{QNP}, \mathrm{VP} \rangle, \mathrm{S} \rangle$

This rules implicates the new syntactic operation $\mathcal{F}_Q$ which we have yet to define. This operation will also be implicated by the sister rules for the non-subject NP rules:

$\{\langle \mathcal{F}_Q, \langle \delta, \mathrm{QNP} \rangle, \sigma \rangle : \langle \delta, \sigma \rangle \in \{\langle \mathrm{TV}, \mathrm{VP} \rangle, \langle \mathrm{TTV}, \mathrm{TV} \rangle, \langle \mathrm{TVVP}, \mathrm{VVP} \rangle, \langle \mathrm{P}, \mathrm{PP} \rangle\}\}$

The typing and denotational assumptions of Generalised Quantifier theory will be carried forward, and will, in conjunction with the recursive ambiguity data, dictate the definition of the semantic operation $\mathcal{G}_Q$, that will be associated with $\mathcal{F}_Q$.

$\mathcal{L}^{15}$: incorporating a local approach to quantifiers

1. Phrase set indices: as for $\mathcal{L}^{12}$
2. Basic Phrase sets: as for $\mathcal{L}^{12}$
3. Syntactic Operation: as for $\mathcal{L}^{12}$ with the addition of $\mathcal{F}_Q$. $\mathcal{F}_Q$ as an operation on strings is concatenation.

4. Syntactic Rules: as for $\mathcal{L}^{12}$ except that $\langle \mathcal{F}_{>}, \langle \text{QNP}, \text{VP} \rangle, \text{S} \rangle$ is replaced by $\langle \mathcal{F}_{Q}, \langle \text{QNP}, \text{VP} \rangle, \text{S} \rangle$, and the following set of rules is added:
$\{\langle \mathcal{F}_{Q}, \langle \delta, \text{QNP} \rangle, \sigma \rangle : \langle \delta, \sigma \rangle \in \{\langle \text{TV}, \text{VP} \rangle, \langle \text{TTV}, \text{TV} \rangle, \langle \text{TVVP}, \text{VVP} \rangle, \langle \text{P}, \text{PP} \rangle\}\}$

The definition of the class of possible models, $\mathcal{K}^{12}$, for $\mathcal{L}^{15}$, is almost exactly the same as that for the models of $\mathcal{L}^{12}$. All that is additional is the specification of semantic operation corresponding to the new syntactic operation $\mathcal{F}_{Q}$. The 'algebra-spanning' definition of $\mathcal{H}_{Q}$ will be given below, in the course of the semantic assessment of $\mathcal{T}^{12}$, the combination of $\mathcal{L}^{15}$ and $\mathcal{K}^{12}$.

Transparency/opacity Facts

Starting with the transparency facts concerning the sentences in (41,p127), these will be accounted for simply if the operation $\mathcal{G}_{Q}$ is constrained to be one definable in terms of an operation on denotations. One might note that this explanation of transparency is noticeably more straightforward than that provided by $\mathcal{T}^{11}$, because the Pronoun postulates are not involved.

Concerning quantifiers occurring in embedded sentences and VP's, what is required by Hypothesis 1 (p37) is that there should be the possibility of a *de-dicto* interpretation and what is required by Hypothesis 2 (p37) is that there should be the possibility of a *de-re* interpretation.

With respect to Hypothesis 1, $\mathcal{T}^{12}$ is a success where $\mathcal{T}^{11}$ was a failure: it does explain availability of *de-dicto* interpretations for all kinds of embedded quantifier. It was the VP case that $\mathcal{T}^{11}$ did not allow, because according to $\mathcal{T}^{11}$ there was no disambiguation of to marry a blond. According to $\mathcal{T}^{12}$ there is a disambiguation of to marry a blond. This coverage by $\mathcal{T}^{12}$ of a semantic undergeneration suffered by $\mathcal{T}^{11}$ is part of a general story about which more will be said below.

However, with respect to Hypothesis 2, $\mathcal{T}^{12}$ is a failure where $\mathcal{T}^{11}$ was a success: it does not explain the availability of *de-re* interpretations of embedded quantifiers.

Recursive Ambiguity

The single quantifier sentences of (41) are recursively ambiguous wrt. the contained quantifier. Consider first how it is that $\mathcal{T}^{12}$ is to account for this fact in the case of (41b). The required entailment of $\mathcal{T}^{12}$ is given in (51a), whilst what $\mathcal{T}^{12}$ actually does entail is given in (51b).

(51) a. there is a β such that $\mathcal{R}((41b), \beta)$ and such that whatever model $\langle\langle \mathcal{B}, \mathcal{G}_{\gamma}, f \rangle, \langle w, g \rangle\rangle$,
$[\![\beta]\!](w,g) = 1$ iff D $\{ x\text{: } [\![\text{man}]\!](\langle w, g \rangle)(x) = 1 \}$
$\{ x\text{: } [\![\text{loves}]\!](w,g)(x)([\![\text{john}]\!](w,g)) = 1 \}$
where D corresponds to DET

b. there is a β such that $\mathcal{R}((41b), \beta)$ and whatever model $\langle \Im, \langle w, g \rangle\rangle$,
$[\![\beta]\!](w,g) = 1$ iff $\mathcal{H}_{Q}(\mathcal{E}, \mathcal{I}, \mathcal{J})([\![\text{DETman}]\!], [\![\text{love}]\!])([\![\text{John}]\!](w,g)) = 1$

It can clearly only be the case that (51b) that will amount to (51a) if $\mathcal{H}_{Q}$ is so contrained as to make (52a) hold.

(52) a. whatever model $\langle \Im, \langle w, g\rangle\rangle$,

$\mathcal{H}_Q(\mathcal{E},\mathcal{I},\mathcal{J})([\![\text{love}]\!],[\![\text{DETman}]\!],)([\![\text{John}]\!](w,g)) = 1$

iff D $\{ x: [\![\text{man}]\!](\langle w,g\rangle)(x) = 1 \}$

$\{ x: [\![\text{loves}]\!](w,g)(x)([\![\text{john}]\!](w,g)) = 1 \}$

$\leftrightarrow$ (b) whatever model $\langle \Im, \langle w, g\rangle\rangle$, whatever $P_2^{(e\to t)}, P_1^{(e\to(e\to t))}$, d_1^e,

$\mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(P_1,[\![\text{DET}]\!](w,g)(P_2))(d_1^e) = [\![\text{DET}]\!](w,g)(P_2)(x \mapsto P_1(x)(d_1))$

$\leftarrow$ (c) whatever model $\langle \Im, \langle w, g\rangle\rangle$, whatever $P_1^{(e\to(e\to t))}$, $f^{((e\to t)\to t)}$, d_1^e,

$\mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(P_1,f)(d_1) = f(x \mapsto P_1(x)(d_1))$

(52b) is an equivalent to (52a). (52c) is a slightly stronger condition, quantifying over all members of $D_{((e\to t)\to t)}$ and not just the denotations of QNP's. (52c) is a constraint on $\mathcal{H}_Q$ and what is presented below is the whole set of such constraints generated by considering all of the sentences in (41).

(53) a. whatever $f^{((e\to t)\to t)}$, $P_1^{(e\to t)}$,

$\mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(f,P_1) = f(x \mapsto P_1 x)$

b. whatever $f^{((e\to t)\to t)}$, $P_1^{(e\to(e\to t))}$, d_1^e,

$\mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(P_1,f)(d_1) = f(x \mapsto P_1 x d_1)$

c. whatever $f^{((e\to t)\to t)}$, $P_1^{(e\to(e\to(e\to t)))}$, d_1^e, d_2^e,

$\mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(P_1,f)(d_1)(d_2) = f(x \mapsto P_1 x d_1 d_2)$

d. whatever $f^{((e\to t)\to t)}$, $P_1^{(e\to((s\to(e\to t))\to(e\to t)))}$, d_1^{vp}, d_2^e,

$\mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(P_1,f)(d_1)(d_2) = f(x \mapsto P_1 x d_1 d_2)$

The equations concern the application of the operation, $\mathcal{H}_Q^E$ at denotations drawn from four sequences of types. We will actually define $\mathcal{H}_Q$ so that it returns an operation that is significant at infinitely many types. The postulate will concern the application of the operation to meanings of type $((e\to t)\to t)$ and of type a, where a is a Quantifiable Type, where these are defined:

Definition 55 (Quantifiable Types)
Whatever $a \in \text{TJ}^{\to}$, if a is Conjoinable, then $(e\to a)$ is Quantifiable.

Definition 56 ($\mathcal{H}_Q$) *For any $\mathcal{E},\mathcal{I},\mathcal{J}$, for any $a \in \text{TJ}^{\to}$ where a is quantifiable, whatever f^{ett}, P_1^a,*

if $a = (e\to t), \mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(P_1,f) = \mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(f,P_1) = f(P_1)$,

if $a = (e\to(b\to c)), \mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(P_1,f) = \mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(f,P_1)$,

$= (y^b \mapsto \mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(x^e \mapsto P_1 x y, f))$

whatever $m_2^{(et)t}, m_1^a$,

$$\mathcal{H}_Q(\mathcal{E},\mathcal{I},\mathcal{J})(m_1,m_2)(w,g) = \mathcal{H}_Q(\mathcal{E},\mathcal{I},\mathcal{J})(m_2,m_1)(w,g)$$
$$= \mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})(m_1(w,g),m_2(w,g))$$

To recap on where we are: we have just assessed $\mathcal{T}^{12}$ with respect to the requirement that some simple quantificational structures are recursively ambiguous wrt. the quantifiers within them, and used this data to define what the quantification operation, $\mathcal{G}_Q$, must do. These simple sentences were not in fact ambiguous at all and it is now time to consider how well $\mathcal{T}^{12}$ fares against the entailments of Hypothesis 3 (p46) for actually ambiguous structures, sentences such as the following (the number in brackets is the number of possible readings):

(54) a. every man loves a woman (2)

b. every man gave a woman two prizes (6)

c. John believes a man came in (2)

d. John believes every man loves a woman (6)

For each of these there is only *one* disambiguation according to $\mathcal{T}^{12}$, and therefore $\mathcal{T}^{12}$ falls very far short of the ambiguity requirements of Hypothesis 3 as regards these sentences. Thus there is a considerable problem of semantic undergeneration for $\mathcal{T}^{12}$. Next we will consider how $\mathcal{T}^{12}$ fares on the sentences that were problematic to the non-local account of quantification, $\mathcal{T}^{11}$.

Problems of semantic undergeneration ?

See (46,p133). Recall that for each of the sentences in (46) there was a natural reading which $\mathcal{T}^{11}$ could not account for.

For $\mathcal{T}^{12}$, matters are quite the reverse: the reading that previously could not be accounted for is the only reading that now *can* be accounted for. This means in the case of (46a), that $\mathcal{T}^{12}$ can explain where $\mathcal{T}^{11}$ could not, why there is a reading of (46a) according to which the following argument is *invalid*:

(55) John consumed a pie or drank some beer.

∴. there were beer's

In the case of (46b), $\mathcal{T}^{12}$ can explain where $\mathcal{T}^{11}$ could not, why (46b) is recursively ambiguous with respect to every dog near a door. Finally in the case of (46c), $\mathcal{T}^{12}$ can explain where $\mathcal{T}^{11}$ could not, why there is reading of (46c) according to which the following argument is valid:

(56) John wanted to marry a blond

John is married to a blond

∴.an act that John wanted to do, was done by John

There we will end the initial semantic comparison of $\mathcal{T}^{12}$ and $\mathcal{T}^{11}$.

2.5.3 Summary

In sections 2.5.1 and 2.5.2 two possible extensions of $\mathcal{T}^9$ (p123) have been described, each one a possible way of accounting for the extensive privileges of occurrence of quantifiers. $\mathcal{T}^{11}$ (p131) embodied a non-local approach to quantifiers, whilst $\mathcal{T}^{12}$ (p137) embodied a local approach. It has been noted that neither is descriptively adequate. Attention has also been drawn to a certain set of semantic undergenerations suffered by $\mathcal{T}^{11}$ and not by $\mathcal{T}^{12}$.

Putting together the observations of this section and section 2.4, we may say that none of the accounts described so far has been descriptively adequate. However, this is not intended to show that no descriptively adequate account can be built *based* on some of the mechanisms described in sections 2.4 and 2.5, and in the next section, we consider two ways in which one can try to obtain a descriptively adequate account from these mechanisms.

2.6 Local vs. Non local, a verdict

It was noted in section 2.4 that the undergenerations of the non-local approach to junctions, $\mathcal{T}^8$ (p119) were different to the undergenerations of the local approach to junctions, $\mathcal{T}^9$ (p123). It was also noted in section 2.5 that the undergenerations of the non-local approach to quantifiers, $\mathcal{T}^{11}$ (p131) were different to the undergenerations of the local approach to quantifiers, $\mathcal{T}^{12}$ (p137).

Therefore a way to obtain a descriptively adequate account is simply to combine the four accounts presented so far. Without giving the details of the specification of this combined account we will henceforth refer to it as $\mathcal{T}^{13}$. Such a combined account is descriptively adequate, but it is not *emergent*, which is to say by an easy simplification, it is possible to subtract from the coverage *just* the explanation of ambiguity: one can simply subtract from $\mathcal{T}^{13}$ what we have called the *non-local* approaches to junctions and quantifiers.

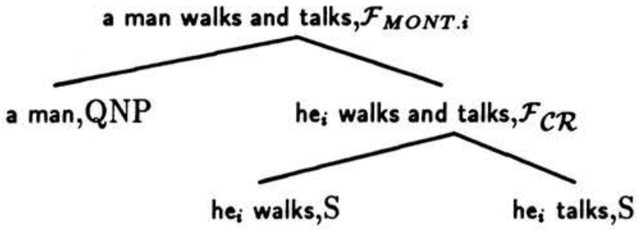
In saying that $\mathcal{T}^{13}$ is not emergent, I would claim to have done a large part of what is involved in establishing that the literature does not provide an account of junctions and quantifiers that meets the emergence criterion. Now, it might appear that $\mathcal{T}^{13}$ is my own strawman, and it is true that with no particular name or paper is the $\mathcal{T}^{13}$ account associated. However, I would argue that $\mathcal{T}^{13}$ does incorporate some of the most significant strategies for handling junctions and quantifiers that have been proposed in the literature, namely Conjunction Reduction, Cross Categorial Coordination, Quantifier Lowering and Cross Categorial Quantification. This should be clear from the references that I have given in this section. I have shown that the most straightforward way to obtain a descriptively adequate account on the basis of these resources is simply to combine them, and I have shown that such a combined account does not meet the emergence criterion.

There is, however, more to showing that the literature has not provided an emergent account than showing that $\mathcal{T}^{13}$ is not emergent. One thing to be acknowledged is that $\mathcal{T}^{13}$ assumes a simpler typing than that supposed by PTQ and in many proposals since. The next section considers whether the pictures changes with the change of typing assumption. This leaves Hendriks' 'type-flexibility' account (Hendriks 90) and Cooper's 'storage' account (Cooper 83) to be considered. The former is considered in Chapter 8 and the latter is considered in the final chapter.

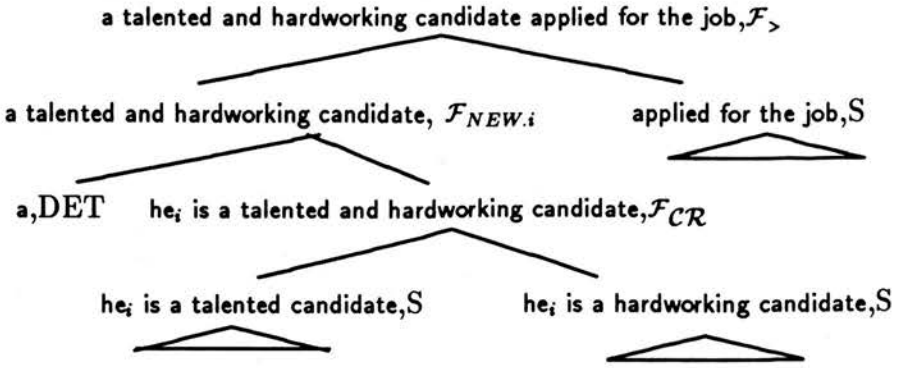
I will finish this section by considering one other possible though never proposed account based on some of the mechanisms described in sections 2.4 and 2.5. Each of the non-local accounts was very close to being a descriptively adequate, emergent account. The accounts were emergent because the relevant transformation (CR or QL) was the mechanism both behind the explanation of extensive priveleges of occurrence and ambiguity. The only way to subtract from the accounts the explanation of ambiguity, would be by making unavailable the relevant transformations, and this would subtract junctions and quantifiers almost completely from the language. For this reason, a promising strategy for finding a successful, emergent account of the junctions and quantifiers is by looking for ways to make up for the semantic undergenerations of the non-local approaches in a different way than simply adding the local approaches.

First we will consider the non-local approach to junctions as embodied by $\mathcal{T}^8$. (34a,p121) was the first of the sentences that it was noted as suffering undergeneration problems with respect to. The problem was to obtain the reading entailed by the fact that the sentence is recursively ambiguous wrt. a man. One could argue that the blame for this lies not with the non-local junction mechanism, but with the quantification mechanism: for subject quantifiers in $\mathcal{T}^8$ were treated *locally*. If we replace this local analysis of quantification with the non-local one, as described in $\mathcal{T}^{11}$, the missing reading could have been generated. It would be associated with the following disambiguation:

(57)

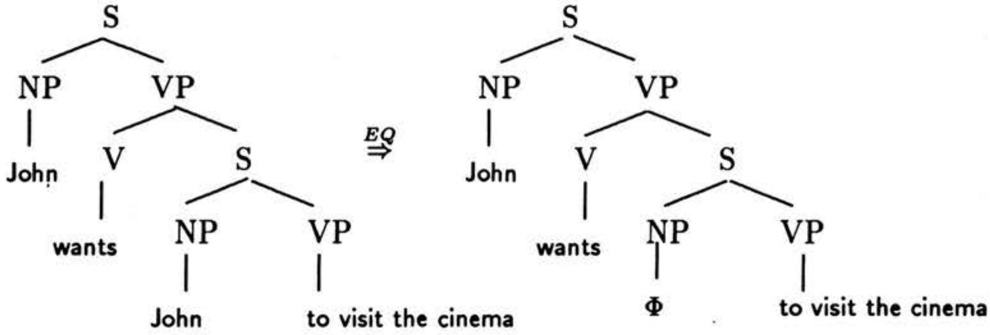


The second of the problematic sentences for the non-local approach to junctions was (34b,p121). The problem was that the reading entailed by the fact that the sentence is recursively ambiguous wrt. a talented and hard working candidate was not accounted for. This problem may be solved without adopting the local analysis of junctions in the following way: one adopts a non-local approach to the combination of a DET and a CN, viewing the combination of DET with CN as 'surface' only, deriving from a deep-structure combination of a DET with a *sentence*. Assuming such a view, here is the analysis which would secure the appropriate reading:



(58)

The third of the problematic sentences for the non-local approach to junctions was (34c,p121). The problem was to generate the reading predicted by Hypothesis 6 (p50). It is possible to get around this undergeneration problem without adopting the local approach to junctions in the following way: one adopts a non-local approach to the combination of a VVP with a VP. This non-local approach is again inspired by a certain transformational analysis, one in which the phenomenon of verbs taking VP complements is a 'surface' one only, structures exhibiting it in fact being derived from 'deep' structures in which the verb is combined with a sentence instead. The transformation effecting the map from deep to surface structure is the EQUI transformation, and the fashion in which one transformationally derive John wants to visit the cinema is illustrated below:



(59)

One can imagine adapting into a UG format this transformational analysis, in much the same way that the transformational analyses of conjunction and quantification were adapted in $\mathcal{T}^8$ and $\mathcal{T}^{11}$. The following illustrates a potential UG-style disambiguation of John wanted to visit the cinema or read a book at home, a disambiguation which would support the reading that is predicted by Hypothesis 6.

John wanted to visit the cinema or read a book at home,$\mathcal{F}_{EQUI}$

John wanted John visits the cinema or reads a book at home,$\mathcal{F}_{<}$

John,NP   wanted John visits the cinema or reads a book at home,$\mathcal{F}_{>}$

wanted,V   John visits the cinema or reads a book at home,$\mathcal{F}_{\mathcal{CR}}$

(60)   John visits the cinema or John reads a book at home,S

Therefore it seems that the undergeneration problems that were noted when the non-local approach to junctions, $\mathcal{T}^8$, was introduced *can* be overcome without adopting the local approach to junctions - though it involves adding a whole range of further transformation-like devices.

If we consider now the undergeneration problems that attended the non-local approach to quantification, we will find exactly the same picture emerges. Undergeneration can be overcome without adopting the local account of quantification but only by adopting a non-local approach to a number of other kinds of expression. The non-local approach to quantification was introduced in $\mathcal{T}^{11}$. The first of the examples with respect to which it suffered semantic undergeneration was (46a,p133). The problem was that one could not generate the reading required by the fact that the sentence is recursively ambiguous wrt. consumed a pie or drank a beer (the associated truth intuition is described in (47,p134)). $\mathcal{T}^{11}$, though it embodied a non-local approach to quantifiers, built on the local approach to junctions embodied in $\mathcal{T}^9$. It will perhaps be clear by now that there is an obvious potential solution to the undergeneration problem and this is to switch to the non-local approach to junctions.

The second problem case was:

(61) every dog near a door barked

The difficulty was to simply to explain the fact that the sentence is recursively ambiguous wrt. every dog near a door (the associated truth intuition is described in (49)). Adopting the non-local approach to DET CN combination that was described above will also solve this defect of the non-local approach to quantification.

The third problematic example was:

(62) John wanted to marry a blond

The difficulty was that one could not obtain the reading predicted by Hypothesis 6 (p50). Again it will perhaps be clear from what has gone before that an obvious way to solve this problem is by adding to $\mathcal{T}^{11}$, the UG version of the EQUI transformation that we just described.

We can summarise these findings on the issue of whether the semantic defects of the non-local approach to junctions and the non-local approach to quantifiers can be remedied by further additions as follows. Some of the undergenerations can be solved by simply combining the non-local approach to junctions with the non-local approach to quantifiers. This, however, does not solve all of the undergeneration problems. One must also adopt a non-local approach to VVP plus VP combinations *and* towards to DET plus CN combinations. Without giving its detailed specification we will refer to this account as $\mathcal{T}^{14}$. Is $\mathcal{T}^{14}$ a descriptively adequate and emergent account ? It is emergent insofar as no additional mechanisms have been introduced which duplicate the explanation of the distribution of the junctions and quantifiers. However, it is not descriptively adequate: the EQUI-like devices cause $\mathcal{T}^{14}$ to *overgenerates*. The examples are when the subject NP is quantified:

(63) Every girl wants to win the prize

There is an EQUI based derivation of this that has it interpreted roughly as:

(64) Every wants it to be the case that every girl wins the prize

I take it that there is no such reading. Though these observations are not conclusive, we have given evidence at least that the only way to make the non-local approaches descriptively semantically adequate is by adding the local mechanisms, and have therefore given evidence that there is no way to expand the non-local account into a descriptively adequate yet emergent account.

The very final thing that we will do in this section is try to extract a moral, a moral concerning the direction in which one must go to obtain an emergent account. For the non-local approaches certain sentences were noted whose most natural readings could not be accounted for. We have given evidence that the only way to account for the natural readings of these sentences is by adopting local analyses. In other words, we have given evidence that the local analyses will have to be present in any account. Because the local analyses account for the extensive privileges of occurrence of the junctions and quantifiers, the moral is that the only way one can hope to obtain an *emergent* account is by somehow turning the mechanisms used in the local analyses into mechanisms that can account for ambiguity. This, more or less, is what the Polymorphic categorial account to be proposed in Chapter 7 will do.

3 Arguments for Minimality

The typing of Verbal Terms that was adopted in $\mathcal{K}^1$ (p96) and maintained throughout section 2 has been called the 'minimal typing assumption'. The typing deserves the name because it appears to be the typing that allows for explanation of the semantic data with the minimum number of *meaning postulates*. An example of this has already been seen in the case of the combination of $\mathcal{L}^5$ (p97) and $\mathcal{K}^2$ (p100), which was suggested as a combination which could deal adequately with the opacity of sentential and VP complements. This adopted a non-minimal typing policy and in order for this combination to allow for the transparency data, it was necessary to define the 'Extensionality Meaning Postulates', given in Definition 42 (p100). This same transparency data was accounted for on the minimal typing assumption by the combination of $\mathcal{L}^6$ (p101) and $\mathcal{K}^3$ (p102), using only the Copula Postulate.

We in this section consider a 'non-minimal' set of typings for Verbal Terms deriving from Montague's influential PTQ paper.

The defininitive feature of Montague's typing of Verbal Terms is that in some of the argument positions one does not have the e type but one has a type related to that assigned to QNP's. We will discuss first an instance of this typing philosophy where in place of e one actually has the type of QNP's, that is, $((e \rightarrow t) \rightarrow t)$. We will call this the *Quantifiers as objects* typing, and it is considered in section 3.1. Second we will discuss an instance of the typing philosophy where in place of e one has $(s \rightarrow ((e \rightarrow t) \rightarrow t))$. We will call this the *Intensions of Quantifiers as objects* typing, and it is considered in section 3.2.

The first reason for looking at these accounts based on non-minimal typing is to see whether they achieve descriptive adequacy and emergence. We will quickly see that they do not.

The second reason for considering these accounts has to do with justification of starting points. In the next chapter we will be endeavouring to show that no account of junctions and quantifiers can be given using Lambek categorial grammar. Now showing this is easier if it is assumed that all accounts are based on the minimal typing assumption. Ignoring accounts based on the non-minimal typing assumption obviously needs some justification and it is this that this section goes some way towards supplying.

3.1 The 'Quantifiers as objects' PTQ typing

The following is the 'Quantifiers as objects' typing:

$\nu(\mathrm{TV}) = (e^t \rightarrow (e \rightarrow t))$
$\nu(\mathrm{TTV}) = (e^t \rightarrow (e^t \rightarrow (e \rightarrow t)))$
$\nu(\mathrm{TVVP}) = (e^t \rightarrow ((s \rightarrow (e \rightarrow t)) \rightarrow (e \rightarrow t)))$
$\nu(\mathrm{P}) = (e^t \rightarrow ((e \rightarrow t) \rightarrow (e \rightarrow t)))$

Around this typing a THEORY OF REFERENCE can be defined that eliminates the syntactic distinction between NP's and QNP's that was maintained throughout Section 2. The theory is presented below and can be thought of as deriving from $\mathcal{L}^{15}$ by getting rid of all the rules that mentioned QNP's and then changing all the rules that mentioned NP's to mention QNP's instead. Also the syntactic operation of the rule that generates subject NP's is changed from $\mathcal{F}_{<}$ to $\mathcal{F}_{>}$.[8]

A language that does not distinguish NP and QNP:$\mathcal{L}^{16}$

1. $\Delta^{16} = \{\mathrm{DET, CN, QNP, S, SC, VP, VPC, TV, TTV, PV, VVP, TVVP, P, PP, JUNCT}\}$
2. $\delta_0 = \mathrm{S}$
3. $\mathcal{X}_\delta^{16}$: Same as for $\mathcal{L}^{15}$ except
   (i) since NP $\notin \Delta^{16}$ there is no $\mathcal{X}_{\mathrm{NP}}^{16}$; instead $\mathcal{X}_{\mathrm{QNP}}^{16} = \{\langle \mathsf{John}, \langle\rangle, \mathrm{QNP}\rangle\} \cup \{\langle \mathsf{he_i}, \langle\rangle, \mathrm{QNP}\rangle : \mathsf{he}_i \in \mathcal{NPPRO}\}$
   (ii) $\mathcal{X}_{\mathrm{TV}}^{16}$ includes $\langle \mathsf{seek}, \langle\rangle, \mathrm{TV}\rangle$, $\mathcal{X}_{\mathrm{P}}^{16}$ includes $\langle \mathsf{about}, \langle\rangle, \mathrm{P}\rangle$
4. $\mathcal{F}_\gamma^{16}$. $\Gamma^{16} = \{<, >, \uparrow, (vp, \uparrow), \cap, \mathcal{J}\}$.
5. $\mathcal{S}^{16} =$
   $\langle \mathcal{F}_>, \langle \mathrm{DET, CN}\rangle, \mathrm{QNP}\rangle$, $\langle \mathcal{F}_\cap, \langle \mathrm{CN, PP}\rangle, \mathrm{CN}\rangle$, $\langle \mathcal{F}_>, \langle \mathrm{PV, SC}\rangle, \mathrm{VP}\rangle$,
   $\langle \mathcal{F}_>, \langle \mathrm{VVP, VPC}\rangle, \mathrm{VP}\rangle$, $\langle \mathcal{F}_\uparrow, \langle \mathrm{S}\rangle, \mathrm{SC}\rangle$, $\langle \mathcal{F}_\uparrow, \langle \mathrm{VP}\rangle, \mathrm{VPC}\rangle$
   $\langle \mathcal{F}_>, \langle \mathrm{QNP, VP}\rangle, \mathrm{S}\rangle$
   $\{\langle \mathcal{F}_>, \langle \delta, \mathrm{QNP}\rangle, \sigma\rangle : \langle \delta, \sigma\rangle \in \{\langle \mathrm{TV, VP}\rangle, \langle \mathrm{TTV, TV}\rangle, \langle \mathrm{TTVP, VVP}\rangle, \langle \mathrm{P, PP}\rangle\}\}$

The set $\mathcal{K}^{15}$ of possible models of $\mathcal{L}^{16}$ $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^{15}$ iff

1. Typing mapping: for DET,CN, QNP, VP, PV, VVP, PP and JUNCT, ν^{15} is as it was for $\mathcal{K}^{15}$. ν^{15} assigns no type to NP, because it is no longer a phrase-set index. The values of ν^{15} for TV, TTV, TVVP and P are different to those for $\mathcal{K}^{15}$:
   $\nu^{15}(\mathrm{TV}) = (e^t \rightarrow (e \rightarrow t))$, $\nu^{15}(\mathrm{TTV}) = (e^t \rightarrow (e^t \rightarrow (e \rightarrow t)))$, $\nu^{15}(\mathrm{TVVP}) = (e^t \rightarrow (vp \rightarrow (e \rightarrow t)))$, $\nu^{15}(\mathrm{P}) = (e^t \rightarrow (e \rightarrow t))$
2. Constraints on f^{15}: f^{15} is subject to restriction with respect to junctions and determiners (Definitions 45 and 48), with respect to proper names, pronouns, the copula, and with respect to 'extensional' verbal terms (see Definition 57).
3. Algebraic Constraints: $\Gamma = \{<, >, \uparrow, (vp, \uparrow), \cap\lambda\mathsf{he}_i, \lambda\mathsf{he}_i^{vp}, \lambda\mathsf{he}_i^{r}, \mathcal{J}\}$. In the usual way, $\mathcal{G}_\gamma = \mathcal{H}(\mathcal{E}, \mathcal{I}, \mathcal{J})$.

[8]Note also that those parts pertaining to higher order quantification and truth predicates have been discarded.

Before giving the postulates for $\mathcal{K}^{15}$, it is convenient to define first the map AR^n between type domains:

$$AR^n(m_1{}^{(\vec{a},e,\vec{c},t)})(w,g)(\vec{x_1}^{\vec{a}})(y^{e^t})(\vec{x_2}^{\vec{c}}) = y(z^e \mapsto m_1(w,g)(\vec{x_1})(z)(\vec{x_2}))$$

Definition 57 (Postulates for $\mathcal{K}^{15}$) *Whatever model $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,g\rangle\rangle$, whatever $(w,g) \in \mathcal{I} \times \mathcal{J}$, whatever $\alpha \in \mathcal{A}^{16}$,*
if α is a proper name, then $f(\alpha) = (w,g) \mapsto P \mapsto Pd$ for some $d^e \in \mathcal{D}_e$ (Proper name postulate)
if $\alpha = \langle \mathsf{he}_n, \langle\rangle, \mathrm{QNP}\rangle$, then $f(\alpha)(w,g) = P \mapsto P(g(\mathsf{he}_n))$ (Pronoun postulate)
if $\alpha = \langle \mathsf{is}, \langle\rangle, \mathrm{TV}\rangle$, then $f(\alpha) = AR^1(\mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J}))$ (Copula postulate)

$$\left.\begin{array}{l}
\textit{unless } \alpha \textit{ is } \overline{\mathsf{seeks}},\ \textit{if } \alpha \in \mathcal{X}^{15}_{\mathrm{TV}}, f^{15}(\alpha) = AR^1 m_1{}^{eet} \textit{ for some } m_1{}^{eet} \in \mathcal{M}_{eet} \\
\textit{if } \alpha \in \mathcal{X}^{15}_{\mathrm{TTV}}, f^{15}(\alpha) = AR^2 AR^1 m_1{}^{eeet} \textit{ for some } m_1{}^{eeet} \in \mathcal{M}_{eeet} \\
\textit{if } \alpha \in \mathcal{X}^{15}_{\mathrm{TVVP}}, f^{15}(\alpha) = AR^1 m_1{}^{e(set)et} \textit{ for some } m_1{}^{e(set)et} \in \mathcal{M}_{e(set)et} \\
\textit{unless } \alpha \textit{ is } \overline{\mathsf{about}},\ \textit{if } \alpha \in \mathcal{X}^{15}_{\mathrm{P}}, f^{15}(\alpha) = AR^1 m_1{}^{e(et)et} \textit{ for some } m_1{}^{e(et)et} \in \mathcal{M}_{e(et)et}
\end{array}\right\} \textit{Extensional VT plus Quantifier postulate}$$

Note that we have given the name 'Extensional $\mathcal{VT}$ plus Quantifier postulate' to the postulate that spell the restriction of f^{15} with respect to 'extensional' verbs. This is to distinguish it from the *Extensionality meaning postulates* that were encountered in section 2.1.2, Definition 42.

Let $\mathcal{T}^{15}$ be the combination of $\mathcal{L}^{16}$ and $\mathcal{K}^{15}$. About $\mathcal{T}^{15}$ I would like to make a number of observations. Firstly, $\mathcal{T}^{15}$ is extracted from Montague's account and this opportunity will be used to note how Montague's PTQ accounts fares by the criterion of emergence.

Secondly $\mathcal{T}^{15}$ offers a different *local* mechanism for accounting for quantifiers to that found in $\mathcal{T}^{12}$ (p137) and I would like to offer a number of reasons why I think the $\mathcal{T}^{12}$ mechanism is to be preferred. The non-minimal typing assumption of $\mathcal{T}^{15}$ could be argued to produce some kind of explanation of the behaviour of *intensional* verbs. I will question that. Also $\mathcal{T}^{15}$ should account for the properties of extensional verbs and I will question that.

Before beginning the reader should note that I am aware that the typing of $\mathcal{T}^{15}$ is a simplification of what is often called the non-minimal typing assumption - specifically the *s* type is absent - and in section 3.2 it will be considered whether the points made depend on this simplification of the typing. It should become apparent as we proceed, why I think it is useful to consider the simplified typing first.

The PTQ account and emergence

$\mathcal{T}^{15}$ as it stands is not descriptively adequate, for it provides at most *one* disambiguation of any sentence, and therefore will not account for ambiguity. To $\mathcal{T}^{15}$ could be added a variant of the non-local quantification mechanism seen in section 2.5.1. The result plainly would not be an emergent account. The expansion of $\mathcal{T}^{15}$ by the addition of the non-local quantification mechanism gives more or less an exact copy of the PTQ account. The main difference is the simplification of the typing. Therefore the assessment that the expanded version of $\mathcal{T}^{15}$ is not emergent can be taken as the assessment that the PTQ account is not emergent.

Intensional verbs

The table below shows three aspects of the behaviour of loves (an extensional transitive verb) and seeks (an intensional transitive verbs): (i) whether the occurrence of man in John TV a man is unambiguously transparent. (ii) whether the occurrence of TV in John TV a man is unambiguously transparent. (iii) whether the argument, John TV a man ∴ there are men, is unambiguously valid.

(65)

	loves	seeks
man is unambiguously transparent	+	-
TV is unambiguously transparent	+	-
Argument is unambiguously valid	+	-

Asking whether the argument referred to in the third row of the table is validated is a shorthand way to ask whether John TV a man is recursively ambiguous wrt. a man.

Here are the predicted properties for loves and seeks according to $\mathcal{T}^{15}$:

(66)

	loves	seeks
man is unambiguously transparent	+	+
TV is unambiguously transparent	+	-
Argument is unambiguously valid	+	-

We will trace the reasons why the entries for seeks in (66) are as they are. $\mathcal{T}^{15}$ predicts man to occur unambiguously *transparently*, as is easily seen by considering that the there is one disambiguation only and its denotation with respect to an arbitrary model, $\langle \Im, \langle w, g \rangle \rangle$, is:

(67) $[\![\overline{\text{John}}]\!](w,g)\ ([\![\overline{\text{seek}}]\!](w,g)([\![\overline{\text{a man}}]\!](w,g)))$

For any CN, α, (67) this could only differ from the denotation of the only disambiguation of John seeks a α, if the denotation of $\overline{\alpha}$ differed from the denotation of $\overline{\text{man}}$, and since for $\mathcal{T}^{15}$ co-extension of common nouns implies identity of denotation, the occurrence of man is predicted to be transparent.

Moving down the table, we come to the question of the transparency or otherwise of the occurrence of seeks. It is predicted to occur *opaquely*. This may seem a little surprising, because for any TV, α, (67) could only differ from the denotation of the only disambiguation of John α a man if the denotation of $\overline{\alpha}$ differed from the denotation of $\overline{\text{seeks}}$. However, coextension of seeks with α does *not* implies identity of denotation:

Co-extension of seeks *with some* α *does not guarantee identity of denotation*

For co-extension of seeks with α:

(1) For all proper names, x, y, $[\![\overline{\text{x seeks y}}]\!](w,g) = [\![\overline{\text{x } \alpha \text{ y}}]\!](w,g)$

Replacing the quantification over proper names in (1) with quantification over the possible denotations of proper names and ignoring for the moment the Proper Name meaning postulate:

(2) For all $d_1^{((e\rightarrow t)\rightarrow t)}, d_2^{((e\rightarrow t)\rightarrow t)}$, $d_1([\![\overline{\text{seeks}}]\!](w,g)(d_2)) = d_1([\![\overline{\alpha}]\!](w,g)(d_2))$

When we take into account the proper name meaning postulate, we see that in quantifying over all denotations of type $((e\rightarrow t)\rightarrow t)$ we have quantified over too large a domain, for proper names may have as denotations only those member of $\mathcal{D}_{et}$ of which the following is true:

$\exists x \in \mathcal{D}_e, d_1 = (P \mapsto Px)$

The set of all such 'individual sublimations' stands in one to one correspondence with $\mathcal{D}_e$. So instead of (2) we should have the logically weaker:

(3) For all a^e, b^e, $[\![\overline{\text{seeks}}]\!](w,g)(P \mapsto Pa)(b) = [\![\overline{\alpha}]\!](w,g)(P \mapsto Pa)(b)$

A model can respect (3) but not assign to seeks and α the same denotation: for a model to respect (3) all that is required is that the denotations of seeks and α are indistinguishable when applied to a pair of arguments consisting of an individual sublimation and an individual. The denotations may still be distinguishable when applied to a pair consisting of an *arbitrary* member of $D_{((e\rightarrow t)\rightarrow t)}$ and an individual, such as $[\![\overline{\text{a man}}]\!](w,g)$ and a^e. Therefore, co-extension of seeks with α does not imply identity of denotation. □

The final entry in the table concerning seeks is that concerning the validity of the inference John seeks a man ∴ there are men. According to $\mathcal{T}^{15}$, the inference is blocked.

That the inference is blocked

The inference will not validated if a model can be found with respect to which the premise is true and the conclusion false. Let M stand for $[\![\overline{\text{man}}]\!](w,g)$, AM stand for $[\![\overline{\text{a man}}]\!](w,g)$, S stand for $[\![\overline{\text{seeks}}]\!](w,g)$, and j stand for the individual of which $[\![\overline{\text{John}}]\!](w,g)$ is the individual sublimation.

For the conclusion to be false, M must the characteristic function of the empty set. Therefore AM must be that unique member of $D^{((e\rightarrow t)\rightarrow t)}$, which maps all $P^{(e\rightarrow t)}$ to 0: that is, the characteristic function of the empty set of sets of individuals.

Therefore to make the premise true, S most be such that $S(AM)(j) = 1$, where AM is the unique member of $D^{((e\rightarrow t)\rightarrow t)}$ just described. Since $\overline{\text{seeks}}$ is freely interpreted, there must be models such that $S(AM)(j) = 1$

As the table in (66) makes clear, the behaviour of a freely interpreted transitive verb, such as seeks, on the 'Quantifiers as arguments' typing is neither the typical behaviour of an extensional or an intensional verb.

It is more like an intensional verb than an extensional verb, and it is striking that the inference from John seeks a man to there are men is blocked, as one requires of intensional transitive verbs.

However, surprisingly, at the same time as the inference is blocked, the CN is predicted to occur unambiguously *transparently*, so that the verb lacks the most definitive feature of intensional transitive verb. In other words, according to the 'Quantifiers as objects' typing, seeks should share with loves the feature that the contribution of the common noun in John TV a man is its *denotation*. One might call verbs that have the properties arising from free interpretation under the 'Quantifiers as objects typing', *PTQ-verbs*, and it is this empty class of verbs, not the class of 'intensional' verbs that the 'Quantifiers as objects' typing provides an explanation of.

This is the main criticism that one can make of the $\mathcal{T}^{15}$ account of intensional verbs. A subsidiary point can also be made concerning certain valid inference that intensional transitive verbs participate in, such as the following:

(68) John wants a red hat

∴.John wants a hat

(69) John wants a red hat and a blue coat

∴.John wants a red hat

$\mathcal{T}^{15}$ will not predict these to be valid, as things stand (or rather the obvious extension of $\mathcal{T}^{15}$ that allows for adjectival modification amd junction of QNP's). This might be remedied by having a postulate for intensional verbs referring to an inclusion ordering on quantifier denotations. The postulate would require that the denotation of an intensional verb to be upward monotone in its first argument: if the relation holds between an individual and a quantifier Q, then it must also hold between the individual and any *larger* quantifier, Q'. Because, according to $\mathcal{T}^{15}$, $[\![\overline{\text{a red hat}}]\!](w,g) \subseteq [\![\overline{\text{a hat}}]\!](w,g)$ and $[\![\overline{\text{a red hat and a blue coat}}]\!](w,g) \subseteq [\![\overline{\text{a red hat}}]\!](w,g)$, such a postulate would predict, correctly, the validity of the above two arguments. However, it would also predict, incorrectly, the validity of the following:

(70) John seeks a woman

every woman is a vegetarian

∴.John seeks a vegetarian

When the 'Intensions of Quantifiers as objects' typing is considered in section 3.2, a way to distinguish the validity of these will be considered.

Extensional verbs

We will now consider the entries in (66) for loves, a verb that is subject the 'Extensional VT plus quantifier' meaning postulate. According to (66), $\mathcal{T}^{15}$ does capture the desired semantic properties of John loves a man. Below I have gone through the way in which the typing and the meaning postulate interact to bring about the desired effect. One of the reasons for doing this is to emphasise why the 'Extensional VT plus quantifier' postulate has the long-winded name, and is not simply called an 'Extensionality' postulate.

The first entry in (66) is obvious. In the explanation of the second and third entries the meaning postulate comes to the fore. Consider the transparency of the occurrence of loves in John loves a man. By analogy with what was observed for seeks, the crucial feature is whether co-extension of loves with α really does implies identity of denotation. It does.

That co-extension of loves *and* α *guarantees identity of denotation*

Co-extension of loves with α implies:

(1) for all a^e, b^e, $[\![\overline{\text{loves}}]\!](w,g)(P \mapsto Pa)(b) = [\![\overline{\alpha}]\!](w,g)(P \mapsto Pa)(b)$

To suppose that loves and α are subject to the extensionality postulate then:

(2) for some d_1^{eet}, $[\![\overline{\text{loves}}]\!](w,g) = Q^{(et,t)} \mapsto b^e \mapsto Q(a \mapsto d_1(a)(b))$
for some d_2^{eet}, $[\![\overline{\alpha}]\!](w,g) = Q^{(et,t)} \mapsto b^e \mapsto Q(a \mapsto d_2(a)(b))$

(2) and (1) may be combined as follows:

(3) there are d_1^{eet} and d_2^{eet} such that,
$[\![\overline{\text{loves}}]\!](w,g) = Q^{(et,t)} \mapsto b^e \mapsto Q(a \mapsto d_1(a)(b))$,
$[\![\overline{\alpha}]\!](w,g) = Q^{(et,t)} \mapsto b^e \mapsto Q(a \mapsto d_2(a)(b))$,
and for all a^e, b^e, $[\![\overline{\text{loves}}]\!](w,g)(P \mapsto Pa)(b) = [\![\overline{\alpha}]\!](w,g)(P \mapsto Pa)(b)$.

(3) entails:

(4) there are d_1^{eet} and d_2^{eet} such that,
$[\![\overline{\text{loves}}]\!](w,g) = Q \mapsto b \mapsto Q(a \mapsto d_1(a)(b))$,
$[\![\overline{\alpha}]\!](w,g) = Q \mapsto b \mapsto Q(a \mapsto d_2(a)(b))$,
and $d_1 = d_2$.

But this implies that $[\![\overline{\text{loves}}]\!](w,g) = [\![\overline{\alpha}]\!](w,g)$. □

Any constraint on f^{15} that had the effect that co-extension of extensional transitive verbs implies identity of denotation, would be sufficient to guarantee that $\mathcal{T}^{15}$ predicts the transparency of the occurrence of loves in John loves a man. Here are two alternatives to the 'Extensional VT plus quantifier' meaning postulate that would serve just as well for ensuring that the transparency data is accounted for:

(71) $[\![\overline{\text{loves}}]\!](w,g) = Q^{et,t} \mapsto b^e \mapsto [\![\overline{\text{loves}}]\!](w,g)(P \mapsto Pj)(b)$
$[\![\overline{\text{loves}}]\!](w,g) = Q^{et,t} \mapsto b^e \mapsto [\![\overline{\text{loves}}]\!](w,g)(P \mapsto Pb)(b)$

So the 'Extensional VT plus quantifier' postulate of Definition 57 (p147) does not *look* like the 'Extensionality' postulate seen in Definition 42 (p100), nor does it simply guarantee the transparency of the occurrence of the common noun. These are two good reasons to not simply call it the 'Extensionality' postulate.

The extra that the 'Extensional VT plus quantifier' postulate does that the constraints in (71) do not do is guarantee that John loves a man is recursively ambiguous wrt. a man:

That John loves a man *is recursively ambiguous wrt.* a man

$\mathcal{T}^{15}$ must entail:

(1) there is a disambiguation β of John loves a man such that whatever model $\langle \Im, \langle w, g \rangle \rangle \in \mathcal{K}^{15}$,
$[\![\beta]\!](w,g) = 1$ iff SOME $\{ x\colon [\![\overline{\text{he}_1 \text{ is a man}}]\!](w, g_x^{\text{he}_1}) = 1 \}$
$\{ x\colon [\![\overline{\text{John loves he}_1}]\!](w, g_x^{\text{he}_1}) = 1 \}$

The Copula postulate and the Pronoun postulate guarantee the following identities:
$\{ x\colon [\![\overline{\text{he}_1 \text{ is a man}}]\!](w, g_x^{\text{he}_1}) = 1 \} = \{ x\colon M(x) = 1 \}$
$\{ x\colon [\![\overline{\text{John loves he}_1}]\!](w, g_x^{\text{he}_1}) = 1 \} = \{ x\colon L(P \mapsto Px)(j) \}$

Using these identities, (1) is equivalent to:

(2) there is a disambiguation β of John loves a man such that whatever model $\langle \Im, \langle w, g \rangle \rangle \in \mathcal{K}^{15}$,
$[\![\beta]\!](w,g) = 1$ iff SOME $\{ x\colon M(x) = 1 \}$
$\{ x\colon L(P \mapsto Px)(j) = 1 \}$

Now, what $\mathcal{T}^{15}$ *does* entail is:

(3) there is a disambiguation β of John loves a man such that whatever model $\langle \Im, \langle w, g \rangle \rangle \in \mathcal{K}^{15}$,
$[\![\beta]\!](w,g) = 1$ iff $L(AM)(j) = 1$

For (3) to amount to (2) we require that:

(4) Whatever model $\langle \Im, \langle w, g \rangle \rangle \in \mathcal{K}^{15}$,
$L(AM)(j) = 1$ iff SOME $\{ x\colon M(x) = 1 \}$
$\{ x\colon L(P \mapsto Px)(j) = 1 \}$

One can show that (4) must be true given the 'Extensional VT plus quantifier' postulate, for to suppose (4) false is to suppose that:

(5) there is a model $\langle \Im, \langle w, g \rangle \rangle \in \mathcal{K}^{15}$, and there is d^{eet} such that
$AM(x \mapsto d(x)(j)) = 1 \not\leftrightarrow$ SOME $\{ x\colon M(x) = 1 \}$
$\{ x\colon d(x)(j) = 1 \}$

But (5) is possible given the Determiner postulate. Hence (4) is true, hence John loves a man is predicted to be recursively ambiguous wrt. a man.□

Note that a side effect of this is that the inference from John loves a man to there are men is validated.

One way to look at the 'Extensional VT plus quantifier' postulate is as relating the denotation of a verb plus quantifier at a model of $\mathcal{T}^{15}$ to what *would* have been the denotation of the combination at a model of $\mathcal{T}^{12}$. Roughly speaking, one could say that the postulate guarantees that the result of combining a verb with a quantifier at a model of $\mathcal{T}^{15}$ is exactly the same as what the result *would* have been at a model of $\mathcal{T}^{12}$. Such a way of looking at a meaning postulate was suggested in section 2.1.2.

However, it would be a mistake to think that the postulate succeeds completely in this internal simulation of $\mathcal{T}^{12}$. Consider (72)

(72) John loves JUNCT hates DET man

Concerning such sentences Hypothesis 4 (p46) predicts controversially that they be recursively ambiguous wrt. loves JUNCT hates, whilst the uncontroversial prediction of Hypothesis 3 (p46) is that they be recursively ambiguous wrt. DET man. We saw sentence like this when the non-local and local approaches to quantification were being considered in section 2. $\mathcal{T}^{12}$ (p137), the local approach to quantification based on *minimal types*, was able to account for the uncontroversial reading, namely recursive ambiguity wrt. DET man. $\mathcal{T}^{12}$ could not account for the controversial reading. It is exactly the other way round for $\mathcal{T}^{15}$, the local approach which is based on *non-minimal types*. This may be seen because the denotation of the only disambiguation of (72) in an arbitrary model $\langle \Im, \langle w, g\rangle\rangle$ is (where $L = [\![\text{loves}]\!](w,g)$, $H = [\![\text{loves}]\!](w,g)$, $J = [\![\text{JUNCT}]\!](w,g)$, $D = [\![\text{DET}]\!](w,g)$, $M = [\![\text{man}]\!](w,g)$, $j = [\![\text{John}]\!](w,g)$):

$$\mathcal{G}^{E}_{\mathcal{J}}(L,J,H)(DM)(j)$$
$$= J(L(DM)(j), H(DM)(j))$$
$$= [\![\text{JUNCT}]\!](w,g)([\![\text{John loves DET man}]\!](w,g), [\![\text{John hates DET man}]\!](w,g))$$

This semantic undergeneration is a detracting feature of the account of extensional verbs that is provided under the non-minimal typing of $\mathcal{T}^{15}$ (this has also been noted in Partee and Rooth 83). Because the undergenerations could be cured by certain additions to the account, one cannot say that they are an unanswerable objection to the non-minimal typing. However, if we are to insist on an account meeting the criterion of emergence, the possibility of *adding* further mechanism for simply semantic reasons is not there, and the undergeneration objection carries some weight (of ways to cure the undergenerations, one is to add the non-local approach to quantification, and the other is to add the *type-shifting* mechanisms that Partee and Rooth propose, of which more will be said in Chapter 8). There is also a criticism one could make of proposed extensions that is independent of the emergence criterion. It is that whatever the extensions, the result would be that the most *natural* reading of the sentence was not associated with the most *simple* analysis. Such a criticism would become a concrete objection if one insisted on a principle that canonicality of reading should be correlated with complexity of disambiguation. I am not going to argue for such a principle, and so not too much weight can be put on the second criticism.

Having made these observations of $\mathcal{T}^{15}$'s handling of intensional and extensional verbs we now leave the 'Quantifiers as objects', typing and consider the intensionalised version.

3.2 The 'Intensions of Quantifiers as objects' typing

The 'Intensions of Quantifiers as objects' typing is:

$\nu(\text{TV}) = ((s \rightarrow e^t) \rightarrow (e \rightarrow t))$.

$\nu(\text{TTV}) = ((s \rightarrow e^t) \rightarrow ((s \rightarrow e^t) \rightarrow (e \rightarrow t)))$.

$\nu(\text{TVVP}) = ((s \rightarrow e^t) \rightarrow (vp \rightarrow (e \rightarrow t)))$.

$\nu(\text{P}) = ((s \rightarrow e^t) \rightarrow (e \rightarrow t))$.

Around this typing one could build the theory below.

$\mathcal{L}^{17}$:another language that does not distinguish NP and QNP

1. Δ^{17}: as for $\mathcal{L}^{16}$, with the addition of QNPC
2. $\mathcal{X}^{17}_{\delta}$: Same as for $\mathcal{L}^{16}$
3. $\mathcal{F}^{17}_{\gamma}$: as for $\mathcal{L}^{16}$, with the addition of $\mathcal{F}_{q,\uparrow}$, which as an operation on strings is an identity.
4. $\mathcal{S}^{17} =$
   $\langle \mathcal{F}_{>}, \langle \mathrm{DET}, \mathrm{CN}\rangle, \mathrm{QNP}\rangle$, $\langle \mathcal{F}_{\cap}, \langle \mathrm{CN}, \mathrm{PP}\rangle, \mathrm{CN}\rangle$, $\langle \mathcal{F}_{>}, \langle \mathrm{PV}, \mathrm{SC}\rangle, \mathrm{VP}\rangle$,
   $\langle \mathcal{F}_{>}, \langle \mathrm{VVP}, \mathrm{VPC}\rangle, \mathrm{VP}\rangle$, $\langle \mathcal{F}_{\uparrow}, \langle \mathrm{S}\rangle, \mathrm{SC}\rangle$, $\langle \mathcal{F}_{(vp,\uparrow)}, \langle \mathrm{VP}\rangle, \mathrm{VPC}\rangle$
   $\langle \mathcal{F}_{q,\uparrow}, \langle \mathrm{QNP}\rangle, \mathrm{QNPC}\rangle$, $\langle \mathcal{F}_{>}, \langle \mathrm{QNP}, \mathrm{VP}\rangle, \mathrm{S}\rangle$
   $\{\langle \mathcal{F}_{>}, \langle \delta, \mathrm{QNPC}\rangle, \sigma\rangle : \langle \delta, \sigma\rangle \in \{\langle \mathrm{TV}, \mathrm{VP}\rangle, \langle \mathrm{TTV}, \mathrm{TV}\rangle, \langle \mathrm{TTVP}, \mathrm{VVP}\rangle, \langle \mathrm{P}, \mathrm{PP}\rangle\}\}$

The class of models $\mathcal{K}^{16}$ for $\mathcal{L}^{17}$ $\langle\langle \mathcal{B}, (\mathcal{G}_{\gamma})_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^{16}$ **iff**

1. Typing mapping: same as ν^{15}, except for the $\mathcal{VT}$'s and for QNPC
   $\nu^{16}(\mathrm{TV}) = ((s\rightarrow e^{t})\rightarrow(e\rightarrow t))$, $\nu^{16}(\mathrm{TTV}) = ((s\rightarrow e^{t})\rightarrow((s\rightarrow e^{t})\rightarrow(e\rightarrow t)))$, $\nu^{16}(\mathrm{TVVP}) = ((s\rightarrow e^{t})\rightarrow((s\rightarrow(e\rightarrow t))\rightarrow(e\rightarrow t)))$, $\nu^{16}(\mathrm{P}) = ((s\rightarrow e^{t})\rightarrow((e\rightarrow t)\rightarrow(e\rightarrow t)))$, $\nu^{16}(\mathrm{QNPC}) = (s\rightarrow((e\rightarrow t)\rightarrow t))$
2. Constraints on f^{16}: Postulates are as before, with the exception of the redefinition of the 'Copula' and 'Extensional VT plus quantifier' postulate (see below).
3. Algebraic Constraints: as for $\mathcal{K}^{15}$, with the addition the $\mathcal{G}_{\uparrow}$ additionally bears the index $(q, \uparrow)$.

Definition 58 (New postulates for $\mathcal{K}^{16}$) *Whatever model* $\langle\langle \mathcal{B}, (\mathcal{G}_{\gamma})_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^{16}$, *whatever* $\alpha \in \mathcal{A}^{17}$,
if $\alpha = \langle \text{is}, \langle\rangle, \mathrm{TV}$, *then* $f(\alpha) = \mathcal{I}^2\mathcal{I}^1 AR^1(\mathcal{IS}(\mathcal{E}, \mathcal{I}, \mathcal{J}))$ *(Copula postulate)*

unless α *is* seeks, *if* $\alpha \in \mathcal{X}^{16}_{\mathrm{TV}}, f^{16}(\alpha) = \mathcal{I}^1 AR^1 {m_1}^{eet}$ *for some* ${m_1}^{eet} \in \mathcal{M}_{eet}$
if $\alpha \in \mathcal{X}^{16}_{\mathrm{TTV}}, f^{16}(\alpha) = \mathcal{I}^2\mathcal{I}^1 AR^2 AR^1 {m_1}^{eeet}$ *for some* ${m_1}^{eeet} \in \mathcal{M}_{eeet}$
if $\alpha \in \mathcal{X}^{16}_{\mathrm{TVVP}}, f^{16}(\alpha) = \mathcal{I}^1 AR^1 {m_1}^{e(set)et}$ *for some* ${m_1}^{e(set)et} \in \mathcal{M}_{e(set)et}$
unless α *is* about, *if* $\alpha \in \mathcal{X}^{16}_{\mathrm{P}}, f^{16}(\alpha) = \mathcal{I}^1 AR^1 {m_1}^{e(et)et}$ *for some* ${m_1}^{e(et)et} \in \mathcal{M}_{e(et)et}$

Extensional VT plus Quantifier postulate

Let $\mathcal{T}^{16}$ be the combination of $\mathcal{L}^{17}$ and $\mathcal{K}^{16}$. The 'Intensions of quantifiers as arguments' typing can be argued for simply by asking oneself the question how it may be brought about that the common noun has an opaque occurrence. The other locations in which a CN may have an opaque occurrence have all arisen by the *downward heritability of opacity*, such as when the CN occurs in an embedded sentence or an embedded VP. The reason for the opaque contribution of the CN in this case is the opaque occurrence of the embedded sentence or embedded VP, and this is reflected by giving to sentence and VP embedders, types that have intensional types in argument position: $((s \rightarrow t) \rightarrow (e \rightarrow t))$ and $((s \rightarrow (e \rightarrow t)) \rightarrow (e \rightarrow t))$ for example. By analogy, it will be the case that if one gives to intensional transitives a type such that they apply to the intensions of quantifiers, the common noun will be predicted to occur opaquely. So the idea is to make the denotation of the only disambiguation of seeks a man be:

$[\![\overline{\text{seeks}}]\!](w, g)(w' \mapsto [\![\overline{\text{a}\ \overline{\text{man}}}]\!](w', g))$

It will then possible for this to be different to the denotation of the only disambiguation of seeks a α, even when the denotations of $\overline{\text{man}}$ and $\overline{\alpha}$ are identical, and thus man will be predicted to occur opaquely.

As as was the case with the introduction of the embedding verbs, once one has fixed on a certain idea for the analysis of the creator of the opaque contexts, there are a number choices about how to situate this in the wider setting of a whole language. As we observed then, one strategy is to carry out an across the board intensionalisation of types, use the *intensional* function application operation and recover the effect of transparency by certain 'extensionality' postulates. That was what Montague did in PTQ. The other strategy limits the extent of the intensionalisation of types to just those categories of expression in which one finds instances of creators of opaque contexts. Other expressions retain their extensional types and continue to be combined with other expressions by 'ordinary' function application. We pursued the latter strategy in $\mathcal{T}^{16}$. This is one difference between the PTQ account and $\mathcal{T}^{16}$. The other main difference is that, like $\mathcal{T}^{15}$, the 'Quantifying-in' rules have been left out.

What we will do now is consider whether the criticisms made in section 3.1 of the 'Quantifiers as objects' typing can also be made of the 'Intensions of Quantifiers as objects' typing (the discussion of emergence will not be repeated, as it seems clear that what was said of $\mathcal{T}^{15}$ carroes over to $\mathcal{T}^{16}$).

Intensional verbs

We observed that under the 'Quantifiers as objects' typing, not all the properties of an intensional transitive verb like seek were accounted for. The predicted behaviour of loves and seeks according to $\mathcal{T}^{16}$ is shown in (73), which should be compared with (66,p148), the corresponding table for $\mathcal{T}^{15}$.

(73)

	loves	seeks
man is unambiguously transparent	+	-
TV is unambiguously transparent	+	-
Argument is unambiguously valid	+	-

It was objected to the 'Quantifiers as objects' typing that it modeled not intensional verbs but

PTQ-verbs. One cannot make the same objection of the 'Intensions of quantifiers as objects' typing: intensional verbs and extensional verbs are given the right properties. However, a weaker form of the *PTQ-verb* objection will be made when the extensional verbs are considered below.

Another objection brought against the 'Quantifiers as objects' typing was that it rendered the verb plus quantifier combination too non-logical, ignoring that there were some valid inferences to be drawn (these were illustrated in (68,p150) and (69,p150)). A postulate was suggested for intensional verbs which validated these but which also validated incorrectly (70,p150).

Since $[\![\text{wants a red hat}]\!](w,g)$ is determined by the value of $w \mapsto [\![\text{a red hat}]\!](w,g)$, it would be natural to alter the postulate to be sensitive to the inclusion ordering in the type $(s\rightarrow((e\rightarrow t)\rightarrow t))$, similarly requiring upward monotonicity of $[\![\text{wants}]\!](w,g)$ with respect to this ordering. This seems to have the desired effect on the inferences (68), (69) and (70), validating the (68) and (69) and, moreover not validating the inference in (70).

Therefore neither of the criticisms made of the 'Quantifiers as objects' typing in respect of the treatment of intensional transitive verbs applies to the 'Intensions of Quantifiers as objects' typing.

Extensional verbs

The revised version of 'Extensional VT plus quantifier' postulate (Definition 58,p154) gives $\mathcal{T}^{16}$ the same coverage of semantic facts pertaining to combinations of extensional verbal terms and quantifiers as $\mathcal{T}^{15}$ had. In other words the properties of John loves a man will be accounted for, but the most natural reading of John loves and hates a man will not. Therefore in respect of the latter kind of sentence, the $\mathcal{T}^{16}$ is subject to the same criticism as $\mathcal{T}^{15}$.

The 'Extensional VT plus quantifier' postulate, Definition 58, is in fact a composition of Definition 42 (p100) and Definition 57 (p147). Definition 42 was the 'Extensionality' postulate put forward when the sentence embedders were introduced in section 2.1.2. Definition 57 was the previous version of the 'Extensional VT plus quantifier' postulate, that given for $\mathcal{T}^{15}$. The compound nature of Definition 58 leads one to make the following version of the *PTQ-verb* criticism.

Although $\mathcal{T}^{16}$ does not make verbs behave like *PTQ-verbs*, it certainly *allows* for the existence of such verbs. $\mathcal{T}^{16}$ could easily make extensional verbs behave like *PTQ-verb* if the postulate for extensional verbs were simplified into being a version of the 'Extensionality' postulate, Definition 42. That 'Extensionality' postulate makes items typed $((s\rightarrow\tau)\rightarrow b)$ behave as if they had been typed $(\tau\rightarrow b)$, and in the case of the 'Intensions of quantifiers as objects' typing, such a postulate would make 'extensional' verbs behave like *PTQ-verbs*.

This to be sure a rather weak criticism and to turn it into a very substantial objection, a certain tacit principle would have to be made explicit and argued for: the principle that that if a meaning postulate relates meanings of type a to meanings of type b in what seems a two step fashion, then one would expect to find expressions whose behaviour was predicted by going through only one of those two steps. Now this is a very informal principle and I am not going to attempt to make it more precise, or to argue for it. So $\mathcal{T}^{16}$'s allowance for the possibility of *PTQ-verbs* will be left a weak criticism only.

3.3 Summary

We have considered two accounts based on non-minimal types.

The first was the theory $\mathcal{T}^{15}$ (p147), based on the 'Quantifiers as objects' non-minimal typing. We claimed that if the mechanisms for non-local quantification were added, the result would be a near clone of Montague's PTQ account. We observed that the expanded version of $\mathcal{T}^{15}$ would not be an emergent account, and that observation is intended to hold for Montague's PTQ account also. We then made some observations concerning the local mechanism for quantification provided by $\mathcal{T}^{15}$. Firstly, $\mathcal{T}^{15}$ might be argued to account for intensional verbs. However, this appearance only arises if undue prominence is given to an inference invalidating property of such verbs, for this is indeed accounted for by $\mathcal{T}^{15}$. One may then overlook the canonical *opacity* producing behaviour of such verbs, a behaviour that is not accounted for by $\mathcal{T}^{15}$. We called the fictional, transparency inducing, inference invalidating class of verbs accounted for by $\mathcal{T}^{15}$, the *PTQ-verbs*. We also argued that certain valid arguments using intensional verbs are not validated by $\mathcal{T}^{15}$. We then considered the extensional verbs, and suggested that the role of the 'Extensional VT plus Quantifier' postulate is to allow $\mathcal{T}^{15}$ to simulate the result in $\mathcal{T}^{12}$ of combing a VT with a quantifier, gaining the effect of minimal types without actually having them. It was observed that such simulation is, however, imperfect.

The second theory was $\mathcal{T}^{16}$ (p154), based on the 'Intensions of Quantifiers as objects' non-minimal typing. The objections made concerning $\mathcal{T}^{15}$'s explanation of intensional verbs do not hold for $\mathcal{T}^{16}$. However, for $\mathcal{T}^{16}$ there are same undergenerations for sentence containing extensional verbs as there was for $\mathcal{T}^{15}$. It was also observed that the typing embodied $\mathcal{T}^{16}$ 'allows for' the existence of *PTQ-verbs*.

Therefore section 3 has achieved the following two things. First, a further account of junction and quantifiers that the literature provides has been considered and shown not to meet the emergence criterion. Second some objections have been made to the non-minimal typing used in PTQ, the main one being that it predicts a curious syndrome that no verb has.

4 Conclusions

The main conclusion of the chapter is that amongst the mechanisms for accounting for junctions and quantifiers that the literature provides are Conjunction Reduction, Cross-Categorial Coordination, Quantifier Lowering and Cross-Categorial Quantification, and on the basis of these one cannot construct a descriptively adequate, emergent account. This has been shown both for a minimal and for a non-minimal typing regime. Some further mechanisms that the literature provides will be considered in Chapters 8 and 9.

Also evidence has been given that one should expect a successful account to have certain features. One of these is locality and the other of these is minimality of typing. The evidence is not conclusive, but it is evidence nonetheless. For subsequent developments in the thesis it is not necessary that one be convinced of the necessity for locality and minimality. However, the Polymorphic categorial account has both of these features and it is attractive thought at least that it is the account logically to be expected if the features of locality and minimality were insisted on.

Chapter 6

The failure of LCG to account for the Logical Constants

Chapter 5 described various accounts that have been proposed to explain the following two properties of junctions and determiners:

- Junctions and determiners have extensive priveleges of occurrence.
- Junctions and determiners manifest recursive ambiguity phenomena as noted in Hypotheses 3 and 4.

Can these syntactic and semantic properties of junctions and determiners be accounted for by an LCG based account, that is by formulating a suitable $L^{(/,\backslash)}$-THEORY OF REFERENCE ? It is with this question that we will be concerned with in this chapter, and the answer given will be a negative one.

Now to make the claim that there is *no* $L^{(/,\backslash)}$-THEORY OF REFERENCE that accounts for junctions and determiners is to make a large claim indeed, and one very hard to prove: somehow the considerations must embrace all possible categorial lexicons. To not raise false expectations, it should be said at this early stage that we have no *proof* that a $L^{(/,\backslash)}$-THEORY OF REFERENCE cannot be found. Instead, recorded below are the results of a number of excursions into the space of possible $L^{(/,\backslash)}$-THEORIES OF REFERENCE in search of an account of junctions and determiners.

Moortgat and Hendriks have all also comes to the negative conclusion concerning accounting for the determiners in $L^{(/,\backslash)}$ (Moortgat 88 p223-225, Hendriks 90 p19-28, Bouma 86 also). Their conclusions, like those in this chapter, are inductively reached on the basis of 'trial and error' excursions into the space of possible categorisations (Hendriks more extensive than Moortgat). Something must then be said in explanation of why I think the investigation carried out in this chapter may be seen as improving upon the above mentioned discussions.

Firstly, here the junctions are considered alongside the determiners, and in the above discussions only the determiners were considered. Secondly, the above discussions did not emphasise the difficulty in even accounting for the basic syntactic facts, not just facts of ambiguity. Thirdly, there is need to give a rationale to the categorial lexicons considered, and this neither of the above mentioned discussions try to do.

One rationale that could be used but will not be, would be the principle that one would not give any meaning postulates for verbal terms, meaning postulates such as those seen in Chapter 5. In the absence of meaning postulates, the need to account for basic transparency facts more or less dictates then the minimal typing of verbal terms, and from that typing one could work backwards to a categorisation. However, to insist on using no verbal term meaning postulates, is to forgo the possibility of explaining opacity facts in LCG, as will be seen in section 1 below. I did not want to do that, and so I have to countenance at least some meaning postulates, and this immediately opens the way to many possible categorisations.

Section 1 and section 2 record attempts to construct a $L^{(/,\backslash)}$-THEORY OF REFERENCE that preserve the semantic analysis of junctions and determiners of Chapter 5. In particular these $L^{(/,\backslash)}$-THEORIES OF REFERENCE share with the accounts in Chapter 5 the feature that junctions and determiners are assigned just one *type*. Any such accounts we shall call a *monomorphic* $L^{(/,\backslash)}$-THEORY OF REFERENCE. Section 3 describes an attempt to account for junctions and determiners that jettisons the *monomorphism* assumption, and thereby differs in its analysis of junctions and determiner somewhat from any of the THEORIES OF REFERENCE in Chapter 5.

Note that the monomorphism assumption of sections 1 and 2 does *not* mean that the $L^{(/,\backslash)}$-THEORY OF REFERENCE explored must assign junctions and determiners just one *category*, only that if several categories are given, then the category-to-type map of the $L^{(/,\backslash)}$-THEORY OF REFERENCE must map them all to a single type. The difference between section 1 and section 2 consists in the assumptions made concerning the typing of verbal terms. Section 1 adheres as closely as possible to the *minimal typing assumption* of section 2 of Chapter 5, whilst section 2 drops the assumption that the verbal terms should have minimal types, and instead the Montagovian typing philosophy is adopted, as described in section 3 of Chapter 5. This is where the arguments for minimality that were made in Chapter 5 can have some relevance. If the reader was persuaded against the non-minimal typing then the further considerations of section 2 can be side-stepped. However, because the arguments against the typing in Chapter 5 were not conclusive, we have in any case considered what the possibilities are of finding a successful $L^{(/,\backslash)}$-THEORY OF REFERENCE based on the non-minimal typing philosophy.

None of the $L^{(/,\backslash)}$-THEORIES OF REFERENCE in section 1, 2 or 3 is able to account fully for the properties of junctions and determiners. Note that the criterion of emergence does not arise because there are no descriptively adequate accounts of the junctions and determiners.

1 A monomorphic $L^{(/,\backslash)}$-THEORY OF REFERENCE based on minimal types

To start with we will be concerned to formulate a DISAMBIGUATED LAMBEK LANGUAGE that covers verbal terms, proper names, VP and sentence embedders, but not junctions and determiners. We will in fact be attempting the same syntactic coverage with a DISAMBIGUATED LAMBEK LANGUAGE that was covered by section 2.1 of Chapter 5, termed there 'Basic Montagovian semantics'. It should be noted that for the moment, the *intensional* transitive verbs will not be considered. Note also that to specify a DISAMBIGUATED LAMBEK LANGUAGE it is only necessary to specify the phrase-set indices and the membership of the basic phrase-sets - the syntactic operations and rules are fixed by these.

A DISAMBIGUATED LAMBEK LANGUAGE for verbal terms, proper names and embedding verbs, $\mathcal{L}^{18}$

1. The phrase-set indices: the set $\text{CAT}^{(/,\backslash)}$ given as BASCAT, $\{\text{s}, \text{np}, \text{vpc}, \text{sc}\}$
2. The basic phrase sets: Whatever strings α, whatever categories $\delta \in \text{CAT}^{(/,\backslash)}$, $\langle \alpha, \langle\rangle, \delta\rangle \in \mathcal{X}_\delta$ iff α appears in the δ row of the tables below:

np	john, mary, he$_1$, he$_2$, ...	(s\np)/sc	believes
s\np	walks	sc/s	that
(s\np)/np	loves, is	(s\np)/vpc	wants
((s\np)/np)/np	gives	((s\np)/vpc)/np	told
		vpc/(s\np)	to

For models of $\mathcal{L}^{18}$ we will consider one class based on an extensional category-to-type map and one based on an intensional map (contexts of use, $\mathcal{J}$, will be considered to be *assignments*, as they were from section 2.2.2 on in the previous chapter).

The extensional class of possible models $\mathcal{K}^{17}$ for $\mathcal{L}^{18}$: $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^{17}$ iff

1. Type Mapping: $\nu^e(\text{np}) = e$, $\nu^e(\text{s}) = t$, $\nu^e(\text{vpc}) = \nu^e(\text{s}\backslash\text{np})$, $\nu^e(\text{sc}) = \nu^e(\text{s})$, $\nu^e(x/y) = \nu^e(x\backslash y) = (\nu^e(y) \rightarrow \nu^e(x))$
2. Constraints on f^{17}: all expressions are freely interpreted, except $\langle \text{is}, \langle\rangle, (\text{s}\backslash\text{np})/\text{np}\rangle$, for which $f(\langle \text{is}, \langle\rangle, (\text{s}\backslash\text{np})/\text{np}\rangle) = \mathcal{IS}(\mathcal{E}, \mathcal{I}, \mathcal{J})$.
3. Algebraic constraints: Γ is the set of possible $\text{L}^{(/,\backslash)}$ proofs, with $\mathcal{G}_\gamma = H^e_L(\gamma)$

$\mathcal{LT}^{17}$ will refer to the $\text{L}^{(/,\backslash)}$-THEORY OF REFERENCE that is constituted by the combination of $\mathcal{L}^{18}$ and $\mathcal{K}^{17}$. The category-to-type map for $\mathcal{LT}^{17}$ is chosen so as to render the typing of verbal terms as close as possible to what we have called the *minimal types*. In the case of the VP, TV, and TTV verbal terms, the types assigned are exactly the minimal types, and for the other verbal terms, the types assigned differ from the *minimal* types by the absence of the *s*-type.

Now $\mathcal{LT}^{17}$ will be assessed by some transparency/opacity data. There are readings of the following sentences according to which the expressions marked e(resp. i) occur transparently (resp. opaquely):

(1) a. (Mary)$_e$ walks

b. (John)$_e$ thinks that (Mary walks)$_i$

c. (John)$_e$ told Mary to (go)$_i$

$\mathcal{LT}^{17}$ can handle the transparency but not the opacity data. Because $\mathcal{LT}^{17}$ is a theory that assigns every expression indefinitely many disambiguations, if it assigns any at all, the condition that $\mathcal{LT}^{17}$ must fulfil if it is to account for the transparency data has to be formulated with a little care:

(2) there is a disambiguation β of Mary walks, involving a disambiguation $\overline{\text{Mary}}$ of category δ, such that whatever disambiguation $\overline{\alpha}$ of category δ, where α is a referring expression, whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever disambiguation $\overline{\text{Mary is } \alpha}$, if $[\![\overline{\text{Mary is } \alpha}]\!](w,g) = 1$, then $[\![\beta]\!](w,g) = [\![\beta[\overline{\alpha}/\overline{\text{Mary}}]]\!](w,g)$.

The quantifications over disambiguations may be replaced by quantifications over *flat* disambiguations (because of the semantic Cut Elimination theorem), and in each case there is only one possible *flat* disambiguation. The semantic operations defined by the proofs of the relevant categorising sequents are then such that (2) holds if the following implication does:

(3) for any model and for any $\overline{\alpha}$ of category np, if $[\![\overline{\text{is}}]\!](w,g)([\![\overline{\alpha}]\!](w,g))([\![\overline{\text{Mary}}]\!](w,g)) = 1$ then $[\![\overline{\text{walks}}]\!](w,g)([\![\overline{\text{Mary}}]\!](w,g)) = [\![\overline{\text{walks}}]\!](w,g)([\![\overline{\alpha}]\!](w,g))$

Due to the meaning postulate for is, this implication does hold and therefore $\mathcal{LT}^{17}$ accounts for (1a). To account for opaque occurrence of Mary walks in (1b), the following is required:

(4) there is a disambiguation β of John thinks that Mary walks, involving a disambiguation $\overline{\text{Mary walks}}$ of category δ, there is a disambiguation $\overline{\alpha}$ of category δ, where α is a sentence, there is a model, $\langle \Im, \langle w, g\rangle\rangle$, such that (i) $[\![\overline{\text{Mary walks}}]\!](w,g) = [\![\overline{\alpha}]\!](w,g)$ and (ii) $[\![\beta]\!](w,g) \neq [\![\beta[\overline{\alpha}/\overline{\text{Mary walks}}]]\!](w,g)$.

The first step cannot this time be to declare this equivalent to a claim about *flat* disambiguations, because there are *no* flat disambiguations of John thinks that Mary walks that involve a disambiguation of Mary walks. However, it suffices to consider only those disambiguations of John thinks that Mary walks that have, besides the disambiguation of Mary walks, only word disambiguations as parts. There as many of these as there are proofs of np, (s\np)/sc, sc/s, s $\Rightarrow$ s, and there are just two of these, both associated with the term $x_2(x_3(x_4))(x_1)$.

Given this it is clear to see that if the denotations of the disambiguations of Mary walks and α are identical as (i) requires, the denotation of any such disambiguation of John thinks that Mary walks cannot differ from the denotation of the corresponding disambiguation of John thinks that α, as (ii) requires. Therefore, $\mathcal{LT}^{17}$ does not account for the opaque occurrence of Mary walks in (1b).

In a similar way one can show that $\mathcal{LT}^{17}$ predicts go to occur transparently in (1c). Therefore, $\mathcal{LT}^{17}$ does not account for the opacity data. It might appear to be possible to solve this problem and yet retain an extensional category to type map by changing to $\nu(\text{sc}) = (s, t)$

and $\nu(\text{vpc}) = (s, e, t)$, leading to the types of **believes** and **wants** being (st, e, t) and (set, e, t). However, the types of **that** and **to** will have to be (t, st) and (et, set), and this will reintroduce transparency.

Now, what will be considered is the combination of $\mathcal{L}^{18}$ with a class of models based on an *intensional* category-to-type map that aims to adhere as closely as possible to the minimal types of verbal terms:

> **The intensional class of possible models $\mathcal{K}^{18}$ for $\mathcal{L}^{18}$:** $\langle\langle\mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^{18}$ iff
>
> 1. Type Mapping: $\nu^i(\text{np}) = e$, $\nu^i(\text{s}) = t$, $\nu^i(\text{vpc}) = \nu^i(\text{s}\backslash\text{np})$, $\nu^i(\text{sc}) = \nu^i(\text{s})$, $\nu^i(x/y) = \nu^i(x\backslash y) = ((s{\rightarrow}\nu^i(y)){\rightarrow}\nu^i(x))$
> 2. Constraints on f: with respect to certain verbal terms, f is subject to constraints as specified in the *Extensionality postulates*, defined below. f is also subject to constraint concerning **is**,**that** and **to**.
> 3. Algebraic constraints: Γ is the set of possible $\text{L}^{(/,\backslash)}$ proofs, with $\mathcal{G}_\gamma = H^i_L(\gamma)$

Definition 59 (Meaning postulates for $\mathcal{K}^{18}$) *Whatever model,* $\langle\langle\mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^{18}$,
if $\alpha = \langle \textsf{is}, \langle\rangle, (\text{s}\backslash\text{np})/\text{np}\rangle$, *then* $f(\alpha) = \mathcal{I}^1\mathcal{I}^2\mathcal{I}\mathcal{S}(\mathcal{E},\mathcal{I},\mathcal{J})$ *(Copula)*
if $\alpha = \langle \textsf{that}, \langle\rangle, \text{sc}/\text{s}, \rangle$, $f(\alpha) = (w, g) \mapsto p^{(s\rightarrow t)} \mapsto p(w)$ *(Complementiser)*
if $\alpha = \langle \textsf{to}, \langle\rangle, \text{vpc}/(\text{s}\backslash\text{np})\rangle$, *then* $f(\alpha) = (w, g) \mapsto P^{(s,se,t)} \mapsto P(w)$ *(Infinitiser)*
if $\alpha = \langle \textsf{he}_1, \langle\rangle, \text{np}\rangle$, *then* $f(\alpha) = (w, g) \mapsto g(\textsf{he}_1)$ *(Pronoun)*

$$\left.\begin{array}{l}
\textit{if } \alpha \in \mathcal{X}^{18}_{\text{s}\backslash\text{np}}, \textit{ there exists } m_1{}^{(e\rightarrow t)} \textit{ such that } f(\alpha) = \mathcal{I}^1(m_1) \\
\textit{if } \alpha \in \mathcal{X}^{18}_{(\text{s}\backslash\text{np})/\text{np}}, \textit{ there exists } m_1{}^{(e\rightarrow(e\rightarrow t))} \textit{ such that } f(\alpha) = \mathcal{I}^1\mathcal{I}^2(m_1) \\
\textit{if } \alpha \in \mathcal{X}^{18}_{((\text{s}\backslash\text{np})/\text{np})/\text{np}}, \textit{ there exists } m_1{}^{(e\rightarrow(e\rightarrow(e\rightarrow t)))} \textit{ such that } f(\alpha) = \mathcal{I}^1\mathcal{I}^2\mathcal{I}^3(m_1) \\
\textit{if } \alpha \in \mathcal{X}^{18}_{((\text{s}\backslash\text{np})/\text{sc})}, \textit{ there exists } m_1{}^{(st,et)} \textit{ such that } f(\alpha) = \mathcal{I}^2(m_1) \\
\textit{if } \alpha \in \mathcal{X}^{18}_{(\text{s}\backslash\text{np})/\text{vpc}}, \textit{ there exists } m_1{}^{((s,se,t),et)} \textit{ such that } f(\alpha) = \mathcal{I}^2(m_1) \\
\textit{if } \alpha \in \mathcal{X}^{18}_{((\text{s}\backslash\text{np})/\text{vpc})/\text{np}}, \textit{ there exists } m_1{}^{(e,(s,se,t),et)} \textit{ such that } f(\alpha) = \mathcal{I}^1\mathcal{I}^3(m_1)
\end{array}\right\} \begin{array}{l}\textit{Extension-}\\\textit{ality}\\\textit{postlates}\end{array}$$

The Extensionality Meaning Postulates are virtual carbon-copies of those in Definition 42 in section 2.1.2 of Chapter 5. See there also for the definition of $\mathcal{I}^n$, and illustrations there of how this saves transparency. The effect of the postulates for **that** and **to** is to ensure that the possible meanings of, for example **that Mary walks** and **to walk** are exactly the same as the possible meanings of **Mary walks** and **walks**. When later in this chapter, alternative typing regimes are considered, the associated postulates for **that** and **to** will not be explicitly stated but will be understood to be analogous to those above.

$\mathcal{LT}^{18}$ will refer to the $\mathrm{L}^{(/,\backslash)}$-THEORY OF REFERENCE that is the combination of $\mathcal{L}^{18}$ with $\mathcal{K}^{18}$. The types assigned to verbal terms by $\mathcal{LT}^{18}$ are the minimal types *plus* some additional occurrences of the s-type.

$\mathcal{LT}^{18}$ can account for the transparency and opacity data given in (1). There is the same range of disambiguations to be considered, only now the syntactic operations are associated with different semantic operation, being defined by the application of H^i_L rather than H^e_L to $\mathrm{L}^{(/,\backslash)}$ proofs.

The transparency of the occurrence of Mary in (1a) will be explained if the following implication holds:

(5) for any model and for any $\overline{\alpha}$ of category np, if $[\![\overline{\mathsf{is}}]\!](w,g)(w' \mapsto [\![\overline{\alpha}]\!](w',g))(w' \mapsto [\![\overline{\mathsf{Mary}}]\!](w',g)) = 1$ then $[\![\overline{\mathsf{walks}}]\!](w,g)(w' \mapsto [\![\overline{\mathsf{Mary}}]\!](w',g)) = [\![\overline{\mathsf{walks}}]\!](w,g)(w' \mapsto [\![\overline{\alpha}]\!](w',g))$

Now to suppose a counterexample to this to suppose a model and an $\overline{\alpha}$ such that

(6) $[\![\overline{\mathsf{is}}]\!](w,g)(w' \mapsto [\![\overline{\alpha}]\!](w',g))(w' \mapsto [\![\overline{\mathsf{Mary}}]\!](w',g)) = 1$
and $[\![\overline{\mathsf{walks}}]\!](w,g)(w' \mapsto [\![\overline{\mathsf{Mary}}]\!](w',g)) \neq [\![\overline{\mathsf{walks}}]\!](w,g)(w' \mapsto [\![\overline{\alpha}]\!](w',g))$

Taking into account the meaning postulates for is and for intransitive verbs, for some $m^{(e,t)}$, the above entails the following contradiction:

(7) $[\![\overline{\alpha}]\!](w,g) = [\![\overline{\mathsf{Mary}}]\!](w,g) = 1$ and $m(w,g)([\![\overline{\mathsf{Mary}}]\!](w,g)) \neq m(w,g)([\![\overline{\alpha}]\!](w,g))$

The condition for the opaque occurrence of Mary walks in (1b) now is:

(8) there exists a model and an α such that $[\![\overline{\mathsf{Mary\ walks}}]\!](w,g) = [\![\overline{\alpha}]\!](w,g)$ and
$[\![\overline{\mathsf{thinks}}]\!](w,g)(w' \mapsto [\![\overline{\mathsf{that}}]\!](w',g)(w'' \mapsto [\![\overline{\mathsf{Mary\ walks}}]\!](w'',g)))(w'''' \mapsto [\![\overline{\mathsf{John}}]\!](w'''',g))$
$\neq [\![\overline{\mathsf{thinks}}]\!](w,g)(w' \mapsto [\![\overline{\mathsf{that}}]\!](w',g)(w'' \mapsto [\![\overline{\alpha}]\!](w'',g)))(w'''' \mapsto [\![\overline{\mathsf{John}}]\!](w'''',g))$

Now taking into account the meaning postulates for thinks and that, this is equivalent to:

(9) there is some model, there is some α and there is there is some $m^{(st,et)}$ such that $[\![\overline{\mathsf{thinks}}]\!] = \mathcal{I}^2 m$ and $[\![\overline{\mathsf{Mary\ walks}}]\!](w,g) = [\![\overline{\alpha}]\!](w,g)$ and,
$m^{(st,et)}(w,g)(w'' \mapsto [\![\overline{\mathsf{Mary\ walks}}]\!](w'',g))([\![\overline{\mathsf{John}}]\!](w,g)) \neq m^{(st,et)}(w,g)(w'' \mapsto [\![\overline{\alpha}]\!](w'',g))([\![\overline{\mathsf{John}}]\!](w,g))$

There is no contradiction here, and hence the opacity is accounted for.

$\mathcal{LT}^{18}$ is the most minimally typed $\mathrm{L}^{(/,\backslash)}$-THEORY OF REFERENCE that can handle the opacity phenomena arising from sentence and VP embedding verbs, and $\mathcal{LT}^{18}$ will be the assumed core around which now a theory encompassing junctions and determiners will be sought.

Determiners cannot be introduced until common nouns have been. Nor will there be any possibility of using recursive ambiguity data unless there are disambiguations of sentences such as John is a man. The introduction of common nouns is most easily accomplished by the addition of a basic category: cn, for which the typing assumption is $\nu^i(\text{cn}) = (e \rightarrow t)$. In Chapter 5, section 2.2.2, the approach taken to the John is a man construction, was to assume a syntactic operation, $\mathcal{F}_{i\bullet\bullet}$, which combined a disambiguation of John with a disambiguation of man to give a disambiguation of John is a man. This is not a possibility for a DISAMBIGUATED LAMBEK LANGUAGE because there are only concatenative syntactic operations available. What we shall do is to count the John is a man construction amongst the distributional data concerning quantifiers to be accounted for . $\mathcal{LT}^{19}$ will be understood to the extension of $\mathcal{LT}^{18}$ to accommodate common nouns.

The search for categorisations of junctions and determiners by which to expand $\mathcal{LT}^{19}$ which will now take place has the following character: *types* for junctions and determiners will be assumed which, because of the already fixed category-to-type map ν^i, will fix the possible categorisations that may be considered.

Certain meanings [1] were assigned to junctions and determiners in Chapter 5, meanings which it is commonplace to assign them and on the basis of which it is known to be possible to build a successful THEORY OF REFERENCE. Therefore, in the present context of trying to construct a successful $\text{L}^{(/,\backslash)}$-THEORY OF REFERENCE it is a natural assumption that junctions and determiners should again have these familiar meanings. However, the assumption must be rejected, and it must be rejected because of the *types* of these familiar meanings:

(10) *Junctions*: $(t \rightarrow (t \rightarrow t))$
Determiners: $((e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t))$

$\mathcal{LT}^{19}$ is associated with an *intensional* category-to-type map and no categories of $\text{CAT}^{(/,\backslash)}$ are associated with the types in (10) by the *intensional* category-to-type map, ν^i.

Here then is an early cost to our choosing above an intensional category-to-type map in the face of opacity facts. One could at this point backtrack, revert to the extensional category-to-type map and abandon the attempt to explain opacity. Under the extensional category-to-type there is a non-empty pool of categories that are paired with the types in (10), and one could see whether any of these accounts for the vital syntactic and semantic phenomena concerning junctions and determiners. We will not do this. Instead we will adopt the strategy of giving the junctions and determiners unfamiliar denotations of types that are at least the image of some category and which may be related to the familiar denotations by meaning postulates.

The following seem the simplest types for junctions and determiners that meet the joint require-

[1] Or rather certain meaning postulates concerning junctions and determiners were defined which specified the meanings assigned in every possible model.

ments of being the images of categories and being relatable to the familiar types by meaning postulate:

(11) ***Junctions***: $((s\to t)\to((s\to t)\to t))$

Determiners: $((s\to(e\to t))\to((s\to((s\to e)\to t))\to t))$

The possible categorisations for junctions and determiners on the typing assumption in (11) are:

(12) ***Junctions***: (s/s)/s	***Determiners***: (s/(s/np))/cn
(s/s)\s	(s/(s/np))\cn
(s\s)/s	(s/(s\np))/cn
(s\s)\s	(s/(s\np))\cn
	(s\(s/np))/cn
	(s\(s/np))\cn
	(s\(s\np))/cn
	(s\(s\np))\cn

The following examples of a meaning postulates for junctions and determiners relate meanings of the orthodox types to meanings of the types in (11).

(13) Whatever MODEL, $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,g\rangle\rangle$, associated with $\mathcal{E},\mathcal{I},\mathcal{J}$,

$[\![\text{and}]\!](w,g)(x^{st})(y^{st}) = \mathcal{I}^1\mathcal{I}^2(\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J}))$

$[\![\text{every}]\!](w,g)(x^{set})(y^{s(se)t}) = \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(xw)(z^e\mapsto yw(w'\mapsto z))$,

What one can do now is see whether the syntactic and semantic facts about junctions and determiners can be accounted for by expanding $\mathcal{LT}^{19}$ with choices of categorisation from among those in (12). Before doing this, it should be said that there are probably other types than those in (11) that both are the images of at least some categories and which are relatable to the orthodox types. With each of these there is associated a further possible way to expand $\mathcal{LT}^{19}$. Nonetheless the further possibilities will not be considered.

Of the four junction categorisations in (12), those whose slashes all lean one way would lead to obvious overgenerations and can be discounted. The other two categorisations are instantiations of the two schemata $(x/y)\backslash z$ and $(x\backslash z)/y$. It is the case that for all x,y and z that $(x/y)\backslash z \iff (x\backslash z)/y$, and therefore only one of the pair need be considered. We shall choose (s\s)/s.

Four out of the eight determiner categorisations in (12) have as their argument, cn, introduced by a *backwards* slash. These may be disregarded on the grounds that the common noun *follows* rather than *precedes* the determiner. Of the remaining four possibilities, s\(s\np) may be discounted as leading to certain obvious overgenerations, such as walks every man. This leaves three possibilities, all instances of Q/cn, where Q is one the categories appearing below:

(14) s/(s/np), s/(s\np), s\(s/np)

By $\mathcal{LT}^{20}$ we shall refer to that extension of $\mathcal{LT}^{19}$ that categorises junctions as (s\s)/s, and categorises determiners as Q/cn, for all Q in (14). To be considered then is whether $\mathcal{LT}^{20}$ can account for the syntactic and semantic properties of junctions and determiners.

Some of the syntactic facts that need to be accounted for are given in (15) and (16): all the sentences in (15) and (16) must be categorisable as s.

(15) **John walks and Mary talks** (S case)
John walks or talks (VP case)
John loves and hates Mary (TV case)
John wants and needs to go (VVP case)
John told or asked Mary to go (TVVP case)

(16) **every man walks** (VP case)
John loves every man (TV case)
John gives every man Mary (TTV case)
John told every man to go (TVVP case)
John is a man (Copula case)

Now $\mathcal{LT}^{20}$ can account for the data in (15) only if the following categorising sequents are derivable in $\text{L}^{(/,\backslash)}$:

(17) np, s\np, (s\s)/s, np, s\np ⇒ s (S case)
np, s\np, (s\s)/s, s\np ⇒ s (VP case)
np, (s\np)/np, (s\s)/s, (s\np)/np, np ⇒ s (TV case)
np, (s\np)/vpc, (s\s)/s, (s\np)/vpc, vpc/(s\np), s\np ⇒ s (VVP case)
np, ((s\np)/vpc)/np, (s\s)/s, ((s\np)/vpc)/np, np, vpc/(s\np), s\np ⇒ s (TVVP case)

Because of the decidability property of $\text{L}^{(/,\backslash)}$ – Cut, the derivability of the above five sequents may be determined. The reader will quickly be able to find a proof for the S-case. By rather patient application, the reader should be able to convince him or herself that there is *no* proof for the other cases. Because of the property of ***string-semantic soundness and completeness***, one can also demonstrate these underivability results string semantically, by displaying a countermodel. This may prove less work and be more illuminating than the search for calculus proofs and an illustration of the method is given below in demonstrating the VP-case is underivable.

Proof that the VP-case for junctions is underivable in $\text{L}^{(/,\backslash)}$

We seek a countermodel to

(i) np, s\np, (s\s)/s, s\np ⇒ s

Suppose:

[np] = $\{\Box\}$, [s] is the smallest set such that $\Box\heartsuit \in$ [s] and if $a, b \in$ [s] then $a\Diamond b \in$ [s]

Then it follows that:

$\heartsuit \in$ [s\np], $\Diamond \in$ [(s\s)/s], $\Box\heartsuit\Diamond\heartsuit \notin$ [s],

$\therefore$.there exist a, b, c, d (namely $\Box, \heartsuit, \Diamond, \heartsuit$) such that $a \in$ [np], $b \in$ [s\np], $c \in$ [(s\s)/s], $d \in$ [s\np] and $a \cdot b \cdot c \cdot d \notin$ [s]

and this refutes (i). $\Box$

So one can show that $\mathcal{LT}^{20}$ does not fully account for *syntactic* properties of junctions in English, and *ipso facto*, does not fully account for their *semantic* properties. One can only bring semantic data to bear upon junction-containing sentences of which $\mathcal{LT}^{20}$ provides a disambiguation, and that is only for the sentential junctions. One can confirm that the above suggested meaning postulate for **and** does allow $\mathcal{LT}^{20}$ to account for the instances of Hypothesis 4 as applied to sentential junctions.

We will now consider whether $\mathcal{LT}^{20}$ can account for the syntactic properties of determiners,as set out in (16). This can only be so if the following categorising sequents are derivable in $L^{(/,\backslash)}$ − Cut:

(18) for some Q in (14),

Q/cn, cn, s\np $\Rightarrow$ s (VP case)

np, (s\np)/np, Q/cn, cn $\Rightarrow$ s (TV case and Copula case)

np, ((s\np)/np)/np, Q/cn, cn, np $\Rightarrow$ s (TTV case)

np, ((s\np)/vpc)/np, Q/cn, cn, vpc/(s\np), s\np $\Rightarrow$ s (TVVP case)

It should be noted that the sequents in (18) are not being put forward as *simultaneous* equations all of which some single value of Q/cn must satisfy; different values of Q/cn may be considered for each sequent, for we are not requiring that determiners have just one categorisation.

Choosing Q/cn to be (s/(s\np))/cn, the VP case has the proof given in (19a), and choosing Q/cn to be (s\(s/np))/cn, the TV case (which is also the Copula case) has the proof given in (19b).

(19) a.

$$\dfrac{\dfrac{s \Rightarrow s \quad s\backslash np \Rightarrow s\backslash np}{s/(s\backslash np),\ s\backslash np \Rightarrow s}/L \qquad cn \Rightarrow cn}{((s/(s\backslash np))/cn)/cn,\ cn,\ s\backslash np \Rightarrow s}/L$$

b.

$$\dfrac{\dfrac{\dfrac{np,\ (s\backslash np)/np,\ np \Rightarrow s}{np,\ (s\backslash np)/np \Rightarrow s/np}/R \qquad s \Rightarrow s}{np,\ (s\backslash np)/np,\ s\backslash(s/np) \Rightarrow s}\backslash L \qquad cn \Rightarrow cn}{np,\ (s\backslash np)/np,\ (s\backslash(s/np))/cn,\ cn \Rightarrow s}/\,L$$

So values of Q from the possibilities listed in (14) can be found that account for the first two cases in the list of distributional facts given in (16). In fact these will be the only success cases:

the TTV and TVVP cases are not accounted for under *any* of the possible values of Q. As with the failure cases for the junctions there are two ways to show this, one by search for possible $L^{(/,\backslash)}$ proofs and the other by reasoning string-semantically. Below, by basically string-semantic methods, the underivability of the TTV case is proved.

Part of the reasoning invokes a certain metamathematical property of the Lambek calculus. This is notion of *count-consistency* (van Benthem 86). The idea is to look at a sequent, whose a category and take a certain kind of census of the number of times it occurs in the sequent. Axiom sequents have an identical left and righthand side, so the census for each side will be identical. It turns out that there is a simple way to define how to take the census so that the tally for the lefthand side remains the same as the tally for the righthand side as new sequents are inferred from the axiom sequents: one simply counts an occurence as an argument negatively, and an occurence as a value positively. This way of counting is what is defined in the following:

Definition 60 (Count) *Whatever* $x \in$ BASCAT, *whatever* $y, z \in$ CAT$^{(/,\backslash)}$,
x-count of y is 1 if $y \in$ BASCAT and $x = y$
x-count of y is 0 if $y \in$ BASCAT and $x \neq y$
x-count of y/z or $y\backslash z$ is x-count of y minus *x-count of z*

It is easily proved (van Benthem 86) that for all derivable sequents of $L^{(/,\backslash)}$, for all basic category x, the sum of the x-counts of the antecedent categories equals the x-count of the succedent (van Benthem 86). I will refer to this as the *count-consistency* property. Now one can use this property whilst actively searching for a proof of a given sequent, by rejecting any subgoal that is a count-inconsistent sequent, and it is to this use that we put it in the following proof.

Proof that in $L^{(/,\backslash)}$ the TTV case is not derivable.

First we show that for whatever Q in (14), the categorising sequent,
(i) np, ((s\np)/np)/ np, Q/cn, cn, np ⇒ s
is derivable only if the following sequent is
(ii) np, ((s\np)/np)/np, Q, np ⇒ s
The form of (i) is such that it matches only the (/ L) rule. This it does in five different ways, and below are listed the five different possibilities for the minor premise:
(1) Q/cn ⇒ np
(2) Q/cn, cn ⇒ np
(3) Q/cn, cn, np ⇒ np
(4) cn ⇒ cn
(5) cn, np ⇒ cn

cn-count rules out (1). np-count rules out (3) and (5). (2) is not an axiom and matches against only the (/ L) rule, leading to the major premise Q ⇒ np. For none of the values of Q is this an axiom or matchable against a rule. Therefore (2) may be ruled out. (4) is the only remaining possibility and being an axiom it is derivable. The major premise of the (/L) inference for this match is np, ((s\np)/np)/np, Q, np ⇒ s, that is (ii) and so the proof of (i) reduces to (ii). We will give now a countermodel to all three of the sequents that (ii) represents under different instantiation of Q drawn from (14).

We suppose:

[np] = {□}, [s] = {□♡□□
◇♡□□
□♡□◇
◇♡□◇
◇□♡□
◇◇♡□
◇□♡◇
◇◇♡◇
◇◇□♡
◇◇◇♡}

It follows that,

[s\np] = {♡□□
♡□◇}

[s/np] = {□♡□
◇♡□
◇□♡
◇◇♡}

[s/(s\np)] = {□ ,
◇}

[s\(s/np)] = {□
◇}

[s/(s/np)] = {◇},

[(s\np)/np] = {♡□}

[((s\np)/np)/np] = {♡}

Therefore whether Q = s/(s\np) or s\(s/np) or s/(s/np), ◇∈ [Q], and so

there exists a,b,c,d (namely □, ♡, ◇, □) such that $a \in$ [np], $b \in$ [((s\np)/np)/np], $c \in$ [Q],

$d \in$ [np], $a \cdot b \cdot c \cdot d \notin$ [s]

and this refutes all the instantiations of (ii).□

Therefore, by failing to account for the TTV and TVVP cases, we can say that $\mathcal{LT}^{20}$ fails to account fully for the *syntactic* properties of determiners in English, and *ipso facto*, $\mathcal{LT}^{20}$ fails to account fully for their semantic properties. The fact that an LCG account, based on the above very natural categorisation assumptions, fails even to account for the *syntactic* properties of junctions and determiners is one of the most surprising aspects of $L^{(/,\backslash)}$. It is surprising because, as we saw in Chapter 4, LCG does produce a very attractive account of the syntactic *and* ambiguity facts for the structure QNP_1 TV QNP_2.

That is more or less that as far as showing that no *monomorphic*, $L^{(/,\backslash)}$-THEORY OF REFERENCE can be found adhering to minimal types. We turn in the next section to the possibility of non-minimal types. For the remainder of this section I will consider the semantic capabilities of $\mathcal{LT}^{20}$ for those sentences of which it does provide a disambiguation.

Examples of sentences for which $\mathcal{LT}^{20}$ provides disambiguations, are the VP-case and the TV-case of the sentences put forward as the syntactic data in (16). These are unambiguous and the single consequences of Hypothesis 3 for these sentences will be accounted for by $\mathcal{LT}^{20}$ if the meaning postulate for determiners given in (13) is adopted.

As was the case for the transparency data, the fact that $\mathcal{LT}^{20}$ provides indefinitely many disambiguations of an expression if it provides any at all, means that one has to formulate with care the condition to be entailed by $\mathcal{LT}^{20}$ if it is to be said to have accounted for a piece of recursive ambiguity data. For example we will say that to account for the fact that every man walks is recursively ambiguous wrt. every man, $\mathcal{LT}^{20}$ must entail:

(20) there is a disambiguation β of every man walks such that whatever $\overline{\text{he}_1\text{ is a man}}$, whatever $\overline{\text{he}_1\text{ walks}}$, whatever model $\langle \Im, \langle w, g\rangle\rangle$,

$$[\![\beta]\!](w,g) = 1 \text{ iff EVERY } \quad \{x : [\![\overline{\text{he}_1\text{ is a man}}]\!](w, g^x_{he_1}) = 1\}$$
$$\{x : [\![\overline{\text{he}_1\text{ walks}}]\!](w, g^x_{he_1}) = 1\}$$

The only possible *flat* disambiguation of every man walks has the property described in (20) as shown below.

That the flat disambiguation of every man walks *has the property described in (20)*

To suppose that the one possible flat disambiguation of every man walks does not have the property described in (20) is to suppose that

(1) There is a $\overline{\text{he}_1\text{ is a man}}$, there is a $\overline{\text{he}_1\text{ walks}}$, there is a model, $\langle \Im, \langle w, g\rangle\rangle$, such that $[\![\overline{\text{every man walks}}]\!](\langle w, g\rangle) = 1 \not\leftrightarrow$
EVERY $(\{\, x\text{: } [\![\, \overline{\text{he}_1\text{ is a man}}]\!](\langle w, g_x^{he_1}\rangle)\,\},\{\, x\text{: } [\![\, \overline{\text{he}_1\text{ walks}}]\!](\langle w, g_x^{he_1}\rangle)\,\})$

Now (1) is equivalent to a claim quantifying over *flat* disambiguations, and for each of the sentences involved, there is only only flat disambiguation, and henceforth $\overline{\text{he}_1\text{ is a man}}$ and $\overline{\text{he}_1\text{ walks}}$ shall be understood as referring to these. To derive entailments the denotations of $\overline{\text{every man walks}}$, $\overline{\text{he}_1\text{ is a man}}$ and $\overline{\text{he}_1\text{ walks}}$ must be determined:

$$[\![\overline{\text{every man walks}}]\!](\langle w, g\rangle) = [\![\overline{\text{every}}]\!](w,g)(w' \mapsto [\![\overline{\text{man}}]\!](w',g))(w' \mapsto [\![\overline{\text{walks}}]\!](w',g))$$

$$\{x : [\![\overline{\text{he}_1\text{ walks}}]\!](w, g^x_{he_1}) = 1\} = \{x : [\![\overline{\text{walks}}]\!](w,g)(w' \mapsto x) = 1\}$$

$$\{x : [\![\overline{\text{he}_1\text{ is a man}}]\!](w, g^x_{he_1}) = 1\} \quad \text{(i)}$$
$$= \{x : [\![\overline{\text{a}}]\!](w, g^x_{he_1})(w' \mapsto [\![\overline{\text{man}}]\!](w', g^x_{he_1}))(w' \mapsto d^{(s,e)} \mapsto [\![\overline{\text{is}}]\!](w', g^x_{he_1})(w' \mapsto dw')(w' \mapsto [\![\overline{\text{v}_1}]\!](w', g^x_{he_1}))) = 1\} \quad \text{(ii)}$$
$$= \{x : \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w',g))(y^e \mapsto [\![\overline{\text{is}}]\!](w,g)(w' \mapsto y)(w' \mapsto x)) = 1\} \quad \text{(iii)}$$
$$= \{x : \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w',g))(y^e \mapsto \mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(y)(x)) = 1\} \quad \text{(iv)}$$
$$= \{x : \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w',g))(cf(\{x\}))\} \quad \text{(v)}$$
$$= \{x : [\![\overline{\text{man}}]\!](w,g)(x) = 1\} \quad \text{(vi)}$$

From (ii) to (iii) requires the meaning postulates for a and pronouns, from (iii) to (iv) requires the meaning postulate for is, from (iv) to (v) relies on the definition of $\mathcal{IS}$ and from (v) to (vi) relies on the definition of $\mathcal{A}$.

Taking into account these identities (1) entails

(2) there is a model, $\langle \Im, \langle w, g\rangle\rangle$, such that
$[\![\overline{\text{every}}]\!](w,g)(w' \mapsto [\![\overline{\text{man}}]\!](w',g))(w' \mapsto [\![\overline{\text{walks}}]\!](w',g)) = 1$
$\not\leftrightarrow$ EVERY $(\{x : [\![\overline{\text{man}}]\!](w,g)(x) = 1\}, \{x : [\![\overline{\text{walks}}]\!](w,g)(w' \mapsto x)\})$

Taking into account the postulate for every, (2) entails:

(3) there is a model, $\langle \Im, \langle w, g\rangle\rangle$, such that
$\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w,g))(x \mapsto [\![\overline{\text{walks}}]\!](w,g)(w' \mapsto x)) = 1$
$\not\leftrightarrow$ EVERY $(\{x : [\![\overline{\text{man}}]\!](w,g)(x) = 1\}, \{x : [\![\overline{\text{walks}}]\!](w,g)(w' \mapsto x)\})$

Given the definition of $\mathcal{EVERY}$, (3) is a contradiction. Hence (1) is false. □

$\mathcal{LT}^{20}$ also provides disambiguations of genuinely ambiguous sentences, such as:

(21) a. every man loves a woman

b. John believes a man came in

Now although $\mathcal{LT}^{20}$ is rather a weak theory since it cannot account fully for the syntactic properties of determiners, $\mathcal{LT}^{20}$ has some strengths, for it is able to account for the ambiguity of (21a). The relevant two proofs have already been previewed in section 3.2.6 of Chapter 4 in illustration of the potential of $\mathrm{L}^{(/,\backslash)}$ for explaining ambiguities. See (17) and (18) of Chapter 4. On the strength of this one might entertain the hypothesis that for all ambiguous sentences for which $\mathcal{LT}^{20}$ provides at least *one* disambiguation, $\mathcal{LT}^{20}$ provides enough disambiguations to account for the ambiguity. (21b) is a counterexample to this hypothesis, as we will now show (the work done here will also be of use in Section 3) . To show this we will not consider the set of flat disambiguations but will simplify a little by considering the set of disambiguations that are *almost* flat, having as their one non-lexical subpart a disambiguation of a man. Therefore the disambiguations are of the form,

$$\dfrac{\dfrac{\text{John}}{\mathrm{np}}\quad \dfrac{\text{believes}}{(\mathrm{s}\backslash\mathrm{np})/\mathrm{sc}}\quad \dfrac{\text{that}}{\mathrm{sc}/\mathrm{s}}\quad \dfrac{\dfrac{\text{a}}{Q/\mathrm{cn}}\quad \dfrac{\text{man}}{\mathrm{cn}}}{Q}1\quad \dfrac{\text{came in}}{\mathrm{s}\backslash\mathrm{np}}}{\mathrm{s}}2$$

1 is a proof of Q/cn, $\mathrm{cn} \Rightarrow Q$,
2 is a proof of np, $(\mathrm{s}\backslash\mathrm{np})/\mathrm{sc}$, sc/s, Q, $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}$

np, $(\mathrm{s}\backslash\mathrm{np})/\mathrm{sc}$, sc/s, Q, $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}$ is provable only for $Q = \mathrm{s}/(\mathrm{s}\backslash\mathrm{np})$, as is shown below, followed by the 5 possible proofs for this value of Q (the proofs are not developed back as far as axioms but until sequents which either have exactly one proof or are semantically unequivocal).

That np, $(\mathrm{s}\backslash\mathrm{np})/\mathrm{sc}$, sc/s, Q, $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}$ *is provable only for* $Q = \mathrm{s}/(\mathrm{s}\backslash\mathrm{np})$

Consider in left-to-right order across the sequent the connectives to which one could apply a Slash-Left rule . For the principal connectives of $(\mathrm{s}\backslash\mathrm{np})/\mathrm{sc}$ and sc/s, the count-consistent combinations of major and minor premise are (1a,1b) and (2a,2b) below. Then there are the possibilities arising through the principal connective of Q. If $Q = \mathrm{s}/(\mathrm{s}\backslash\mathrm{np})$, then one is lead to the major and minor premises (3a,3b) below. If $Q = \mathrm{s}\backslash(\mathrm{s}/\mathrm{np})$, all possible minor premises are count inconsistent. If $Q = \mathrm{s}/(\mathrm{s}/\mathrm{np})$, the only possible minor premise is $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}/\mathrm{np}$, and this is not provable. Finally there is the principal connective of $\mathrm{s}\backslash\mathrm{np}$. There are two count-consistent possibilities for the minor premise, $Q \Rightarrow \mathrm{np}$, which one can see is provable for no value of Q, and np, $(\mathrm{s}\backslash\mathrm{np})/\mathrm{sc}$, sc/s, $Q \Rightarrow \mathrm{np}$, which is also provable for no value of Q, to see which one only has to note the count-inconsistency of all possible minor premises at the *next* Slash Left inference.

(1a) np, $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}$ (1b) sc/s, Q, $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{sc}$

(2a) np, $(\mathrm{s}\backslash\mathrm{np})/\mathrm{sc}$, $\mathrm{sc} \Rightarrow \mathrm{s}$ (2b) Q, $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}$

(3a) np, $(\mathrm{s}\backslash\mathrm{np})/\mathrm{sc}$, sc/s, $\mathrm{s} \Rightarrow \mathrm{s}$ (3b) $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s}\backslash\mathrm{np}$

(1a) has exactly one proof. The connectives in (1b) will be considered in left-to-right order. For the principal slash of sc/s, the only count-consistent pairing of major and minor premise is (1b.1a,1b.1b) below. Then there is the principal connective of Q. If Q = s/(s\np) then one is led to the pair (1b.2a,1b,2b). 1b is not provable for the other values of Q. The final (1b) possibility is the principal connective of s\np, for which the only count-consistent minor premise is the unprovable Q ⇒ np. (2a) has exactly one proof. (2b) is provable only for Q = s/(s\np), and then only one way. There are two count-consistent possible pairings of major and minor premise for (3a), shown below as (3a.1a,3a.1b) and (3a.2a,3a.2b). (3b) is semantically unambiguous.

(1b.1a) sc ⇒ sc (1b.1b) Q, s\np ⇒ s

(1b.2a) sc/s, s ⇒ sc (1b.2b) s\np ⇒ s\np

(3a.1a) np, s\np ⇒ s (3a.1b) sc/s, s ⇒ sc

(3a.2a) np, (s\np)/sc, sc ⇒ s (3a.2b) s ⇒ s

All of these except (1b.1b) have either exactly one proof or are semantically unambiguous. (1b.1b) is in fact provable only for Q = s/(s\np), then in only one way. Therefore we have shown that the only choice of Q for which np, (s\np)/sc, sc/s, Q, s\np ⇒ s is provable is Q = s/(s\np). □

$$\dfrac{\text{np, s\\np} \Rightarrow \text{s} \qquad \dfrac{\text{sc} \Rightarrow \text{sc} \qquad \text{s/(s\\np), s\\np} \Rightarrow \text{s}}{\text{sc/s, s/(s\\np), s\\np} \Rightarrow \text{sc}}/L}{\text{np, (s\\np)/sc, sc/s, s/(s\\np), s\\np} \Rightarrow \text{s}}/L \qquad \dfrac{\text{np, s\\np} \Rightarrow \text{s} \qquad \dfrac{\text{sc/s, s} \Rightarrow \text{sc} \qquad \text{s\\np} \Rightarrow \text{s\\np}}{\text{sc/s, s/(s\\np), s\\np} \Rightarrow \text{sc}}/L}{\text{np, (s\\np)/sc, sc/s, s/(s\\np), s\\np} \Rightarrow \text{s}}/L$$

$$\dfrac{\dfrac{\text{np, s\\np} \Rightarrow \text{s} \qquad \text{sc} \Rightarrow \text{sc}}{\text{np, (s\\np)/sc, sc} \Rightarrow \text{s}}/L \qquad \text{s/(s\\np), s\\np} \Rightarrow \text{s}}{\text{np, (s\\np)/sc, sc/s, s/(s\\np), s\\np} \Rightarrow \text{s}}/L$$

$$\dfrac{\dfrac{\text{np, s\\np} \Rightarrow \text{s} \qquad \text{sc/s, s} \Rightarrow \text{sc}}{\text{np, (s\\np)/sc, sc/s, s} \Rightarrow \text{s}}/L \qquad \text{s\\np} \Rightarrow \text{s\\np}}{\text{np, (s\\np)/sc, sc/s, s/(s\\np), s\\np} \Rightarrow \text{s}}/L \qquad \dfrac{\dfrac{\text{np, (s\\np)/sc, sc} \Rightarrow \text{s} \qquad \text{s} \Rightarrow \text{s}}{\text{np, (s\\np)/sc, sc/s, s} \Rightarrow \text{s}}/L \qquad \text{s\\np} \Rightarrow \text{s\\np}}{\text{np, (s\\np)/sc, sc/s, s/(s\\np), s\\np} \Rightarrow \text{s}}/L$$

Now despite the number of different proofs of np, (s\np)/sc, sc/s, s/(s\np), s\np ⇒ s there is no significant semantic diversity, for all five of these proofs are associated with the same term by H^i_L, assuming x^e_1,$x_2^{(st,se,t)}$,$x_3^{(st,t)}$, $x_4^{((s,se,t),t)}$ and $x_5^{(se,t)}$ are the terms associated with the antecedent categories:

$$x_2^{(st,se,t)}(\lambda i[x_3^{(st,t)}(\lambda i[x_4^{((s,se,t),t)}(\lambda i[x_5^{(se,t)}])])])(\lambda i[x_1^e])$$

Therefore all the disambiguations of John believes a man came in that we are considering will be assigned the same meaning, and we may say that $\mathcal{LT}^{20}$ fails to account for the ambiguity. With this observation about $\mathcal{LT}^{20}$, we conclude the assessment of its syntactic and semantic properties. To summarise the findings, one may say that $\mathcal{LT}^{20}$ does not account fully for the syntax of either junctions or determiners. For some of the determiner-containing sentences of which $\mathcal{LT}^{20}$ provides a disambiguation, $\mathcal{LT}^{20}$ accounts for their ambiguity and for some some it does not. This syntactic and semantic performance of $\mathcal{LT}^{20}$ is summarised in Table 6.1, along with that of some of the other $L^{(/,\backslash)}$-THEORIES OF REFERENCE that we shall be considering.

Because $\mathcal{LT}^{19}$ was the monomorphic $L^{(/,\backslash)}$-THEORY OF REFERENCE that adhered to the *minimal types* assumption, the fact that $\mathcal{LT}^{19}$ could not be extended to a satisfactory account of junctions and determiners shows that junctions and determiners cannot be accounted for by a monomorphic $L^{(/,\backslash)}$-THEORY OF REFERENCE that adheres to the *minimal types* assumption.

2 A monomorphic $L^{(/,\backslash)}$-THEORY OF REFERENCE not based on minimal types

In the previous section the $L^{(/,\backslash)}$-THEORY OF REFERENCE was so designed that the types associated with verbal terms were as close as possible to the minimal types. In this section, two alternative monomorphic $L^{(/,\backslash)}$-THEORIES OF REFERENCE will be looked which are so designed that the types associated with verbal terms are *non-minimal*, and in fact are very close to the Montagovian non-minimal typing of verbal terms that was discussed in section 3 of Chapter 5. The type-association aimed at is (where q stands for $((e,t),t)$):

(22) $\nu(\text{VP}) = ((s,q),t)$
$\nu(\text{TV}) = ((s,q),((s,q),t))$
$\nu(\text{TTV}) = ((s,q),((s,q),((s,q)),t)))$

There are two elements of choice involved in attempting to simulate this in a $L^{(/,\backslash)}$-THEORY OF REFERENCE. One is the categorisation of the verbal terms and the other is the values assigned to the atomic categories under the category-to-type map. There appear to be two pairings of these choices that succeed in simulating the verbal term to type association described in (22).

One pairing persists with the verbal term categorisations used in the previous section (introduced in $\mathcal{L}^{19}$) and changes the category-to-type mapping solely in the type associated with the atomic category np, from e to $((e,t),t)$. Thus the verbal term to type association is:

(23) $\nu(\text{s}\backslash\text{np}) = ((s,q),t)$
$\nu((\text{s}\backslash\text{np})/\text{np}) = ((s,q),((s,q),t))$
$\nu(((\text{s}\backslash\text{np})/\text{np})/\text{np}) = ((s,q),((s,q),((s,q),t)))$

The other pairing revises the categorisations of verbal terms used in the previous section, but retains the category-to-type map. We saw in the previous section several (in fact four) examples of categories assigned a q-like type under this mapping:

(24) s/(s/np), s/(s\np), s\(s/np), s\(s\np)

These are all assigned the q-like type

$$((s,((s,e),t)),t)$$

This q-like type shall be abbreviated as q^*. If the categorisation of verbal terms is changed in such a way that the np arguments are replaced by any of the categories in (24), the type associated with verbal terms will be resemble that in (22), with q^* in the place of q:

(25) where Q_i is a category in (24)
$\nu(\mathrm{s}\backslash Q_1) = ((s,q^*),t)$
$\nu((\mathrm{s}\backslash Q_1)/Q_2) = ((s,q^*),((s,q^*),t))$
$\nu(((\mathrm{s}\backslash Q_1)/Q_2)/Q_3) = ((s,q^*),((s,q^*),((s,q^*),t)))$

Both of these approaches are explored below to see whether explanation of junctions and determiners can be achieved, the first option under the title 'Simple categories, complex typing' in section 2.1 and the second option under the title 'Complex categories, simple typing' in section 2.2.

2.1 Simple categories, complex typing

In the previous section, we led up to junctions and determiners via a simple DISAMBIGUATED LAMBEK LANGUAGE not including junctions and determiners - the LANGUAGE featuring in $\mathcal{LT}^{19}$. In this section we start from the same DISAMBIGUATED LAMBEK LANGUAGE but in pairing it with a class of models, change the category-to-type map to:

(26) $\nu^i(\mathrm{np}) = ((e{\rightarrow}t){\rightarrow}t)$, $\nu^i(\mathrm{s}) = t$, $\nu^i(\mathrm{cn}) = (e{\rightarrow}t)$, $\nu^i(\mathrm{vpc}) = \nu^i(\mathrm{s}\backslash\mathrm{np})$, $\nu^i(\mathrm{sc}) = \nu^i(\mathrm{s})$,
$\nu^i(x/y) = \nu^i(x\backslash y) = ((s{\rightarrow}\nu^i(y)){\rightarrow}\nu^i(x))$

The change is that np is assigned not to e but to $((e{\rightarrow}t){\rightarrow}t)$. Under this typing the meaning postulates that are required for verbal terms will have to be different to those governing $\mathcal{LT}^{19}$. These will be left to one side for the moment as we now consider the types of the junctions and determiners.

As was the case for $\mathcal{LT}^{20}$, the types cannot be familiar types as shown in (10): they are not the images of any category under the new ν^i given in (26). Once more then we must assume that the junctions and determiners have types other than those in (10), choosing which other types on the basis of the joint requirements that the types be the images of at least some categories under ν^i and that they be relatable by meaning postulate to the orthodox types in (10).

The simplest typing meeting these requirements is:

(27) ***Junctions*** $((s \rightarrow t) \rightarrow ((s \rightarrow t) \rightarrow t))$

Determiners $((s \rightarrow (e \rightarrow t)) \rightarrow ((e \rightarrow t) \rightarrow t))$

This typing allows the following categorisations for junctions and determiners:

(28) *Junctions*: (s/s)/s *Determiners*: np/cn, np\cn

(s/s)\s

(s\s)/s

(s\s)\s

With respect to junctions the same postulate as was given in (13) may be used. For the determiners the postulate must be slightly different because the typing is different. Here in illustration of the postulates for determiners is the postulate concerning every:

(29) Whatever MODEL, $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f\rangle, \langle w, g\rangle\rangle$, associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}$,

$[\![\text{every}]\!](w,g)(x^{(s,et)})(y^{et}) = \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(xw)(y)$

We will consider now whether a $\mathrm{L}^{(/,\backslash)}$-THEORY OF REFERENCE exists whose LANGUAGE is an extension of the LANGUAGE part of $\mathcal{LT}^{19}$, by some or all of the categorisation of junctions and determiners in (28)

The categorisation possibilities for the junctions are here as they were in (12), and by the same arguments as given in section 1, it is necessary to consider only (s\s)/s. Also of the determiner categorisations that which introduces the cn argument with a backwards slash can be disregarded.

Therefore the $\mathrm{L}^{(/,\backslash)}$-THEORY OF REFERENCE whose performance we wish to assess has for its LANGUAGE the extension of the LANGUAGE part of $\mathcal{LT}^{19}$, by the categorisation of junctions as (s\s)/s, and the categorisation of determiners as np/cn. Its class of models will be based on the category-to-type map in (26). The resulting theory will be called $\mathcal{LT}^{21}$.

Besides the junction and determiner postulates $\mathcal{LT}^{21}$ will be governed by postulates concerning *proper name* NP's, pronouns, verbal terms, that, to and is. See Definition 61. The postulates concerning *proper name* NP's and verbal terms are virtual carbon copies of the postulates given in Definition 58 of section 3 of Chapter 5. As was pointed out in section 3 of Chapter 5, the postulate concerning verbal terms should not simply be entitled an 'extensionality' postulate, as that indicates that it shares with the other 'extensionality' postulates the purpose of securing otherwise lost transparency effects. A simpler postulate than the one given would be sufficient to secure transparency effects, a postulate which would give to verbal terms the behaviour of *PTQ-verbs* (see section 3 of Chapter 5 for the introduction of this term). What the postulate actually does is ensure that when verbal term are combined with a quantifier, the combination has 'the proper quantificational force'. It therefore has the rather clumsy title, 'Extensional $\mathcal{VT}$ plus Quantifier postulate'.

Definition 61 (Meaning postulates for $\mathcal{K}^{21}$) *Whatever model* $\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,\langle w,g\rangle\rangle \in \mathcal{K}^{21}$,
if $\alpha \in \mathcal{X}\text{np}$ *is a proper name, then* $f(\alpha) = (w,g) \mapsto P^{et} \mapsto Pd$ *for some* $d^e \in \mathcal{D}_e$ *(Proper Name)*
if $\alpha = \langle \mathsf{he}_i, \langle\rangle, \text{np}\rangle$, *the* $f(\alpha) = (w,g) \mapsto P^{et} \mapsto P(g(\mathsf{he}_i))$ *(Pronoun)*
if $\alpha = \langle \mathsf{is}, \langle\rangle, (\text{s}\backslash\text{np})/\text{np}\rangle$, $f(\alpha) = \mathcal{I}^2\mathcal{I}^1 AR^2 AR^1 \mathcal{I}\mathcal{S}(\mathcal{E},\mathcal{I},\mathcal{J})$ *(Copula)*

$$\left.\begin{array}{l}
\textit{if } \alpha \in \mathcal{X}^{21}_{\text{s}\backslash\text{np}}, f^{21}(\alpha) = \mathcal{I}^1 AR^1 m_1{}^{et} \textit{ for some } m_1{}^{et} \in \mathcal{M}_{et} \\
\textit{if } \alpha \in \mathcal{X}^{21}_{(\text{s}\backslash\text{np})/\text{np}}, f^{21}(\alpha) = \mathcal{I}^2\mathcal{I}^1 AR^2 AR^1 m_1{}^{eet} \textit{ for some } m_1{}^{eet} \in \mathcal{M}_{eet} \\
\textit{if } \alpha \in \mathcal{X}^{21}_{((\text{s}\backslash\text{np})/\text{np})/\text{np}}, f^{21}(\alpha) = \mathcal{I}^3\mathcal{I}^2\mathcal{I}^1 AR^3 AR^2 AR^1 m_1{}^{eeet} \textit{ for some } m_1{}^{eeet} \in \mathcal{M}_{eeet} \\
\textit{if } \alpha \in \mathcal{X}^{21}_{(\text{s}\backslash\text{np}/\text{vpc})/\text{np}}, f^{21}(\alpha) = \mathcal{I}^3\mathcal{I}^1 AR^3 AR^1 m_1{}^{e(set)et} \textit{ for some } m_1{}^{e(set)et} \in \\
\mathcal{M}_{e(set)et} \\
\textit{if } \alpha \in \mathcal{X}^{21}_{\text{s}\backslash\text{np}/\text{sc}}, f^{21}(\alpha) = \mathcal{I}^2 AR^2 m_1{}^{(st,et)} \textit{ for some } m_1{}^{(st,et)} \in \mathcal{M}_{(st,et)} \\
\textit{if } \alpha \in \mathcal{X}^{21}_{\text{s}\backslash\text{np}/\text{vpc}}, f^{21}(\alpha) = \mathcal{I}^2 AR^2 m_1{}^{(s(se)t,et)} \textit{ for some } m_1{}^{(s(se)t,et)} \in \mathcal{M}_{(s(se)t,et)}
\end{array}\right\} \textit{Extensional } \mathcal{VT} \textit{ plus Quantifier postulate}$$

Now to assess $\mathcal{LT}^{21}$'s performance in accounting for the properties of junctions and determiners.

Little needs to be said concerning the accounting of junctions by $\mathcal{LT}^{21}$ because the categorisations of junctions and verbal terms in $\mathcal{LT}^{21}$ are the same as those in $\mathcal{LT}^{20}$. Therefore $\mathcal{LT}^{21}$ captures exactly those syntactic properties that are captured by $\mathcal{LT}^{20}$, namely only the fact of sentential junctions.

Turning to determiners it is easy to see that the np/cn categorisation will allow an explanation of the distributional data (noted in 16). Therefore, in the shape of $\mathcal{LT}^{21}$, we have found a monomorphic L$^{(/,\backslash)}$-THEORY OF REFERENCE that accounts for the *syntactic* phenomena concerning determiners. We will assess now whether $\mathcal{LT}^{21}$ accounts for the following selection of *semantic* properties of sentences that contain determiners:

(30) a. every man walks is recursively ambiguous wrt. every man.

b. John loves every man is recursively ambiguous wrt. every man.

c. John gave every man Mary is recursively ambiguous wrt. every man.

d. John told every man to go is recursively ambiguous wrt. every man

e. every man loves a woman is recursively ambiguous wrt. every man and a woman

f. every man told a woman to go is recursively ambiguous wrt. every man and a woman.

g. John believes a $(\text{man})_{i/e}$ came in

(30a,b,c,d) constitute simple cases, recording the single consequences of Hypothesis 3 on the unambiguous sentences of (16). They therefore encapsulate what it is for the combination of verbal term and quantifier to have 'proper quantificational force'. (30e,f) are more demanding cases as they describe ambiguities. (30g) is a further instance of ambiguity, this time a transparency/opacity ambiguity.

Checking the unambiguous cases is largely a matter of checking that the meaning postulates specified for determiners and verbal terms have the desired effect. This is illustrated for (30a). The condition that the fact in (30a) equates to was given in (20), and it requires the existence of a disambiguation with certain properties. $\mathcal{LT}^{21}$ does furnishes a disambiguation with the required properties, the following one: [2]

(31)

$$\dfrac{\dfrac{\dfrac{\textsf{every}}{np/cn}\quad\dfrac{\textsf{man}}{cn}}{np}\,1\qquad\dfrac{\textsf{walks}}{s\backslash np}}{s}\,2$$

1. $\dfrac{np \Rightarrow np \qquad cn \Rightarrow cn}{np/cn,\ cn \Rightarrow np}/L$

2. $\dfrac{np \Rightarrow np \qquad s \Rightarrow s}{np,\ s\backslash np \Rightarrow s}\backslash L$

Now we will show that the disambiguation in (31) has the property described (20).

That the disambiguation in (31) accounts for (30a)

To suppose that the disambiguation in (31) does *not* account for (30a) is to suppose that the disambiguations does not have the property that was described in (20), and this is to suppose

(1) There is a $\overline{\textsf{he}_1\ \textsf{is a man}}$, there is a $\overline{\textsf{he}_1\ \textsf{walks}}$, there is a model, $\langle \Im, \langle w, g\rangle\rangle$, such that
$[31](\langle w, g\rangle) = 1 \not\leftrightarrow$ EVERY $(\{\, x\text{: } [\,\overline{\textsf{he}_1\ \textsf{is a man}}](\langle w, g_x^{he_1}\rangle)\,\},\{\, x\text{: } [\,\overline{\textsf{he}_1^e\ \textsf{walks}}](\langle w, g_x^{he_1}\rangle)\,\})$

Now (1) is equivalent to a claim quantifying over *flat* disambiguations, and for each of the sentences involved, there is only only flat disambiguation, and henceforth $\overline{\textsf{he}_1\ \textsf{is a man}}$ and $\overline{\textsf{he}_1\ \textsf{walks}}$ shall be understood as referring to these. To derive entailments of (1) we must determine the denotations of 31), $\overline{\textsf{he}_1\ \textsf{is a man}}$ and $\overline{\textsf{he}_1\ \textsf{walks}}$:

$$[31](\langle w, g\rangle) = [\overline{\textsf{walks}}](w,g)(w' \mapsto [\overline{\textsf{every}}](w',g)(w'' \mapsto [\overline{\textsf{man}}](w'',g)))$$

$$\{\, x\text{: } [\,\overline{\textsf{he}_1\ \textsf{walks}}](w, g_x^{he_1}) = 1\,\} = \{x : [\overline{\textsf{walks}}](w,g)(w' \mapsto P \mapsto Px)\}$$

$$\begin{aligned}
&\{\, x\text{: } [\,\overline{\textsf{he}_1\ \textsf{is a man}}](\langle w, g_x^{he_1}\rangle)\,\} && \text{(i)}\\
&= \{x : [\overline{\textsf{is}}](w, g^x_{he_1})(w' \mapsto [\overline{\textsf{a}}](w, g^x_{he_1})(w'' \mapsto [\overline{\textsf{man}}](w'', g^x_{he_1})))(w' \mapsto [\overline{\textsf{v}_1}](w', g^x_{he_1}))\} && \text{(ii)}\\
&= \{x : [\overline{\textsf{he}_1}](w, g^x_{he_1})(d_2 \mapsto [\overline{\textsf{a}}](w,g)(w'' \mapsto [\overline{\textsf{man}}](w'',g))(d_1 \mapsto \mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(d_1)(d_2))) = 1\} && \text{(iii)}\\
&= \{x : \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\overline{\textsf{man}}](w',g))(d_1 \mapsto \mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(d_1)(x)) = 1\} && \text{(iv)}\\
&= \{x : \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\overline{\textsf{man}}](w',g))(cf(\{x\})) = 1\} && \text{(v)}\\
&= \{x : [\overline{\textsf{man}}](w,g)(x) = 1\} && \text{(vi)}
\end{aligned}$$

From (ii) to (iii) uses the meaning postulate for is, from (iii) to (iv) uses the meaning postulates for a and pronouns, from (iv) to (v) relies on the definition of $\mathcal{IS}$ and from (v) to (vi) relies on the definition of $\mathcal{A}$.

Taking into account these identities (1) entails:

(2) there is a model, $\langle \Im, \langle w, g\rangle\rangle$, such that

[2] When one is trying to demonstrate the failure of an L$^{(/,\backslash)}$-THEORY OF REFERENCE to provide a disambiguation having certain properties, it pays to confine attention to the *flat* disambiguations. However, to demonstrate the success of an L$^{(/,\backslash)}$-THEORY OF REFERENCE, one might was well give a *non-flat* disambiguation.

$[\![\textsf{walks}]\!](w,g)(w' \mapsto [\![\textsf{every}]\!](w',g)(w'' \mapsto [\![\textsf{man}]\!](w'',g))) = 1$

$\not\leftrightarrow$ EVERY $(\{x : [\![\textsf{man}]\!](w,g)(x) = 1\}, \{x : [\![\textsf{walks}]\!](w,g)(w' \mapsto P \mapsto Px)\})$

Taking into account the meaning postulate for **walks**, (2) entails:

(3) there exists a model, $\langle \Im, \langle w,g\rangle\rangle$and there exist $m^{e,t}$ such that,

$\mathcal{I}^1 AR^1 m(w,g)(w' \mapsto [\![\textsf{every}]\!](w',g)(w'' \mapsto [\![\textsf{man}]\!](w'',g))) = 1$

$\not\leftrightarrow$ EVERY $(\{x : [\![\textsf{man}]\!](w,g)(x) = 1\}, \{x : \mathcal{I}^1 AR^1(m)(w,g)(w' \mapsto P \mapsto Px)\})$

By definition of $\mathcal{I}^1$and AR^1, (3) entails:

(4) there exists a model, $\langle \Im, \langle w,g\rangle\rangle$, there exists $m^{e,t}$ such that

$[\![\textsf{every}]\!](w,g)(w'' \mapsto [\![\textsf{man}]\!](w'',g))(x^e \mapsto m(w,g)(x)) = 1$

$\not\leftrightarrow$ EVERY $(\{x : [\![\textsf{man}]\!](w,g)(x) = 1\}, \{x : (m)(w,g)(x) = 1\})$

Given the postulate for **every**, (4) entails:

(5) there exists a model, $\langle \Im, \langle w,g\rangle\rangle$, there exists $m^{e,t}$ such that

$\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\textsf{man}]\!](w,g))(x^e \mapsto m(w,g)(x)) = 1$

$\not\leftrightarrow$ EVERY $(\{x : [\![\textsf{man}]\!](w,g)(x) = 1\}, \{x : (m)(w,g)(x) = 1\})$

Given the definition of $\mathcal{EVERY}$, (5) is a contradiction. Hence (1) is false □

One can demonstrate in similar fashion that the facts observed in (30b,c,d) are also accounted for by $\mathcal{LT}^{21}$.

Now we turn to (30e), which concerns genuine ambiguity. See (3.4) in Chapter 3 for the two separate existential claims concerning readings that (30e) amounts to. To account for these facts, $\mathcal{LT}^{21}$ must provide *two* significantly different disambiguations of **every man loves a woman**. It is necessary only to consider the *flat* disambiguations and there are as many of these are there are possible proofs of the categorising sequent. There are *seven* different possible proofs of the categorising sequent for **every man loves a woman**:

$$\dfrac{\dfrac{np,(s\backslash np)/np,np \Rightarrow s \quad cn \Rightarrow cn}{np,(s\backslash np)/np,np/cn,cn \Rightarrow s}/L \quad cn \Rightarrow cn}{np/cn,\ cn,\ (s\backslash np)/np,\ np/cn,\ cn \Rightarrow s}/L$$

$$\dfrac{\dfrac{np,s\backslash np \Rightarrow s \quad np/cn,\ cn \Rightarrow np}{np,(s\backslash np)/np,np/cn,cn \Rightarrow s}/L \quad cn \Rightarrow cn}{np/cn,\ cn,\ (s\backslash np)/np,\ np/cn,\ cn \Rightarrow s}/L$$

$$\dfrac{\dfrac{np/cn,\ cn \Rightarrow np \quad s \Rightarrow s}{np/cn,\ cn,\ s\backslash np \Rightarrow s}\backslash L \quad np/cn,cn \Rightarrow np}{np/cn,\ cn,\ (s\backslash np)/np,\ np/cn,\ cn \Rightarrow s}/L$$

$$\dfrac{\dfrac{np,\ s\backslash np \Rightarrow s \quad cn \Rightarrow cn}{np/cn,\ cn,\ s\backslash np \Rightarrow s}\backslash L \quad np/cn,cn \Rightarrow np}{np/cn,\ cn,\ (s\backslash np)/np,\ np/cn,\ cn \Rightarrow s}/L$$

$$\dfrac{\dfrac{\dfrac{np/cn,\ cn \Rightarrow np \quad s \Rightarrow s}{np/cn,\ cn,\ s\backslash np \Rightarrow s}\backslash L \quad np \Rightarrow np}{np/cn,\ cn,\ (s\backslash np)/np,\ np \Rightarrow s}/L \quad cn \Rightarrow cn}{np/cn,\ cn,\ (s\backslash np)/np,\ np/cn,\ cn \Rightarrow s}/L$$

$$\dfrac{\dfrac{\dfrac{np,\ s\backslash np \Rightarrow s \quad cn \Rightarrow cn}{np/cn,\ cn,\ s\backslash np \Rightarrow s}\backslash L \quad np \Rightarrow np}{np/cn,\ cn,\ (s\backslash np)/np,\ np \Rightarrow s}/L \quad cn \Rightarrow cn}{np/cn,\ cn,\ (s\backslash np)/np,\ np/cn,\ cn \Rightarrow s}/L$$

$$\dfrac{\dfrac{np,\ (s\backslash np)/np,\ np \Rightarrow np \Rightarrow s \quad cn \Rightarrow cn}{np/cn,\ cn,\ (s\backslash np)/np,\ np \Rightarrow s}/L \quad cn \Rightarrow cn}{np/cn,\ cn,\ (s\backslash np)/np,\ np/cn,\ cn \Rightarrow s}/L$$

However, these seven proofs are not significantly *semantically* different, being all associated with the same term, according to H^i_L, namely

$$x_3^{((s,e^t),(s,e^t),t)}(\lambda i[x_4^{((s,et),(s,e^t))}(\lambda i[x_5^{(et)}])])(\lambda i[x_1^{((s,et),(s,e^t))}(\lambda i[x_2^{(et)}])])$$

Therefore $\mathcal{LT}^{21}$ will not account for (30e). It is also the case that that (30f,g) will be unaccounted for.

In summary, one can say of $\mathcal{LT}^{21}$ that (i) neither syntax nor semantics of junctions are accounted for, and that (ii) although the syntax of determiners is accounted for, the semantics as regards ambiguity is not. The performance of $\mathcal{LT}^{21}$ is summarised in Table 6.1.

2.2 Complex categories, simple typing

As we said above, the other fashion in which non-minimal types may be associated with verbal terms begins by revising the categorisation of verbal terms that was used in section 1. By $\mathcal{LT}^{22}$ we will refer to the $\mathrm{L}^{(/,\backslash)}$-THEORY OF REFERENCE embodying the new categorisation of verbal terms.

At this point the reader should refer to the categorisation of basic-expressions given in the LANGUAGE part of $\mathcal{LT}^{19}$. Where np appears as an argument in the categorisation of an expression according to the LANGUAGE part of $\mathcal{LT}^{19}$, any one of the four categories in (24) can appear in the categorisation of the expression according to the LANGUAGE part of $\mathcal{LT}^{22}$. So for example,

(32) $\mathrm{s}\backslash Q$ is a categorisation of walks iff Q is a category in (24)
$(\mathrm{s}\backslash Q_1)/Q_2$ is a categorisation of loves iff Q_1, Q_2 are categories in (24)
$\mathrm{vpc}/(\mathrm{s}\backslash Q)$ is a categorisation of to iff Q is a category in (24).

Note the categorisation of John and the he_i is unchanged: it is still np. So much for the LANGUAGE part of $\mathcal{LT}^{22}$. For the category-to-type map of the class of models of $\mathcal{LT}^{22}$, one reverts to the category-to-type map of $\mathcal{LT}^{20}$, rather than category-to-type map of the previous section. In other words np once more is mapped to e.

In this way the association between verbal terms and types becomes that which was indicated in (25), an approximation of the association produced by $\mathcal{LT}^{21}$. The meaning postulates for $\mathcal{LT}^{22}$ are different again from the postulates for any of the $\mathrm{L}^{(/,\backslash)}$-THEORIES OF REFERENCE considered so far. Their further specification will be left until the categorisations of the junctions and determiners is considered.

The category-to-type map is exactly what it was in section 1, and therefore the same choices can be made here as there of the *types* for the junctions and determiners, their possible categorisations, and the meaning postulates governing them. See (11), (12) and (13).

By the same considerations as given in section 1, we need take only one of the junction categorisations from (12), namely (s\s)/s. Of the eight possible categorisations of determiners, those which introduce the argument cn by a backwards slash may be disregarded.

We will give the name $\mathcal{LT}^{23}$ to the extension of $\mathcal{LT}^{22}$ that gives to junctions the category (s\s)/s, and to determiners the categories Q/cn, where Q is a category in (24).

Concerning the meaning postulates governing $\mathcal{LT}^{23}$, we have said already that the postulates for the junctions and determiners are the same as those given for $\mathcal{LT}^{20}$.

It is not possible to give this time the postulate for verbal terms in terms of the function AR^n, which it will be recalled mapped from $m_1{}^{(\vec{a},e,\vec{c},t)}$ to $m_1{}^{(\vec{a},(et,t),\vec{c},t)}$. It has instead to be given in terms of a more complex function yet, *int-AR*^{n}, that maps from $m_1{}^{(\vec{a},e,\vec{c},t)}$ to $m_1{}^{(\vec{a},((s,se,t),t),\vec{c},t)}$. *int-AR*^{n} is defined below with AR^n repeated for comparison.

(33) $AR^n(m_1{}^{(\vec{a},e,\vec{c},t)})(w,g)(\vec{x_1}{}^{\vec{a}})(y^{e^t})(\vec{x_2}{}^{\vec{c}}) = y(z^e \mapsto m_1(w,g)(\vec{x_1})(z)(\vec{x_2}))$

int- $AR^n(m_1{}^{(\vec{a},e,\vec{c},t)})(w,g)(\vec{x_1}{}^{\vec{a}})(y^{((s,se,t),t)})(\vec{x_2}{}^{\vec{c}}) = y(w' \mapsto z^{(s,e)} \mapsto m_1(w',g)(\vec{x_1})$
$(z(w'))(\vec{x_2}))$

The 'Verbal term plus quantifier' postulate for $\mathcal{LT}^{23}$ is then exactly the same as that given in section 2.1, (61), save for the replacement of AR^n with *int-AR*^{n}.

We are now in a position to consider whether $\mathcal{LT}^{23}$ accounts for the syntactic and semantic properties of junctions and determiners.

First the syntactic properties of junctions will be considered. Taking into account $\mathcal{LT}^{23}$'s categorisation of verbal terms, one can say that the distributional data can only be accounted for if the following categorising sequents are derivable in $\text{L}^{(/,\backslash)}$:

(34) where Q_i is a category in (24)

np, s\\$Q_1$, (s\s)/s, np, s\\$Q_2$ ⇒ s (S case)

np, s\\$Q_1$, (s\s)/s, s\\$Q_2$ ⇒ s (VP case)

np, (s\\$Q_1$)/$Q_2$, (s\s)/s, (s\\$Q_3$)/$Q_4$, np ⇒ s (TV case)

np, (s\\$Q_1$)/vpc, (s\s)/s, (s\\$Q_2$)/vpc, vpc/(s\\$Q_3$), s\\$Q_5$ ⇒ s (VVP case)

np, ((s\\$Q_1$)/vpc)/$Q_2$, (s\s)/s, ((s\\$Q_3$)/vpc)/$Q_4$, np, vpc/(s\\$Q_5$), s\\$Q_6$ ⇒ s (TVVP case)

The S case may easily be proved for $Q_1 = Q_2 = $ s/(s\np). The VP case cannot be proved, as shown below.

That the VP case cannot be proved in $\text{L}^{(/,\backslash)}$

We wish to show that for no values of Q_1, Q_2 from the categories in (24) is there a proof of the following sequent

(i) np, s\\$Q_1$, (s\s)/s, s\\$Q_2$ ⇒ s

First note that whatever categories from (24) Q_1 and Q_2 are chosen to be, the np-count of Q_1 and Q_2 is 1. (i) matches only against Slash-Left rules, which it does in a number of different possible ways leading to the following pairings of major (marked a) and minor (marked b) premises to be proved:

(1a) s, (s\s)/s, s\\Q_2 ⇒ s (1b) np ⇒ Q_1

(2a) np, s\\$Q_1$, (s\s) ⇒ s (2b) s\\$Q_2$ ⇒ s

(3a) s ⇒ s (3b) np, s\\Q_1, (s\s)/s ⇒ Q_2

(4a) np, s ⇒ s (4b) s\\Q_1, (s\s)/s ⇒ Q_2

(5a) np, s\\Q_1, s ⇒ s (5b) (s\s)/s ⇒ Q_2

(1a),(2b),(3b),(4b),(5b) may all be ruled out by np-count. Because all avenues along which to develop a proof of (i) involves an unprovable subgoal whatever categories from (24) Q_1 and Q_2 are chosen to be, then is no proof of (i) for any possible choice of Q_1 and Q_2.

One can show in similar fashion that the TV, VVP and TVVP cases are also unprovable. Therefore, along with all the $L^{(/,\backslash)}$-THEORIES OF REFERENCE considered so far, $\mathcal{LT}^{23}$ cannot account for even the *syntactic* properties of junctions.

Now the determiners will be considered. Given the distributional data set out in (16) and $\mathcal{LT}^{23}$'s categorisation of verbal terms, one can say that the distributional data will only be accounted for if the following categorising sequents are derivable in $L^{(/,\backslash)}$:

(35) Where Q_i is a category in (24)

Q_1/cn, cn, s\\Q_2 ⇒ s (VP case)

np, (s\\Q_1)/Q_2, Q_3/cn, cn ⇒ s (TV case)

np, ((s\\Q_1)/Q_2)/Q_3, Q_4/cn, cn, np ⇒ s (TTV case)

np, ((s\\$Q_1$)/vpc)/$Q_2$, Q_3/cn, cn, vpc/(s\\$Q_4$), s\\$Q_5$ ⇒ s (TVVP case)

If all the Q_i in the above sequents chosen to be s/(s\np), they will be derivable. There are many other solutions. Therefore, we may say that $\mathcal{LT}^{23}$ accounts for the syntactic properties of determiners.

Having seen the success of $\mathcal{LT}^{23}$ in accounting for the syntactic properties of determiners, we wish now to see whether the semantic properties are accounted for.

Looking again at the semantic data set out in (30), then as was the case for $\mathcal{LT}^{21}$, checking whether the unambiguous cases, (30a,b,c,d), are accounted for amounts to little more than checking that the meaning postulates for verbal terms and determiners have been appropriately defined. We shall simply assume that this is so. On the ambiguous cases, $\mathcal{LT}^{23}$ has more success than $\mathcal{LT}^{21}$, but still is not wholly successful.

Consider (30e) first. Amongst the disambiguations of every man loves a woman provided by $\mathcal{LT}^{23}$ there are those of the form (here an abbreviation will be used:$np^{\wedge s}$ = s/(s\np), $np^{\vee s}$ = s\(s/np)):

$$\dfrac{\dfrac{\dfrac{\text{every}}{np^{\wedge s}/cn} \quad \dfrac{\text{man}}{cn}}{np^{\wedge s}}1 \quad \dfrac{\text{loves}}{(s\backslash np^{\wedge s})/np^{\vee s}} \quad \dfrac{\dfrac{\text{a}}{np^{\vee s}/cn} \quad \dfrac{\text{woman}}{cn}}{np^{\vee s}}3}{s}2$$

1 is a proof of $np^{\wedge s}/cn,\ cn \Rightarrow np^{\wedge s}$, 2 is a proof of $np^{\wedge s},\ (s\backslash np^{\wedge s})/np^{\vee s},\ np^{\vee s} \Rightarrow s$, 3 is a proof of $np^{\vee s}/cn,\ cn \Rightarrow np^{\vee s}$

There are two semantically significantly differing proofs of $np^{\wedge s}, (s\backslash np^{\wedge s})/np^{\vee s}, np^{\vee s} \Rightarrow s$ given in (36) and (38), with the terms associated by H^i_L given in (37) and (39):

(36)

$$\dfrac{\dfrac{\dfrac{s \Rightarrow s \qquad \dfrac{\dfrac{\dfrac{np \Rightarrow np^{\wedge s} \quad s \Rightarrow s}{np,\ s\backslash np^{\wedge s}}\backslash L \qquad np \Rightarrow np^{\vee s}}{np,\ (s\backslash np^{\wedge s})/np^{\vee s},\ np \Rightarrow s}/L}{(s\backslash np^{\wedge s})/np^{\vee s},\ np \Rightarrow s\backslash np}\backslash R}{np^{\wedge s},\ (s\backslash np^{\wedge s})/np^{\vee s},\ np \Rightarrow s}/L}{np^{\wedge s},\ (s\backslash np^{\wedge s})/np^{\vee s} \Rightarrow s/np}/R \qquad s \Rightarrow s}{np^{\wedge s},\ (s\backslash np^{\wedge s})/np^{\vee s},\ np^{\vee s} \Rightarrow s}\backslash L$$

(37) $x_3^{q^*}(\lambda i \lambda u_1^{se}[$
$x_1^{q^*}(\lambda i \lambda u_2^{se}[$
$x_2^{(sq^*,sq^*,t)}(\lambda u_3^{(s,se,t)}[u_3 i(\lambda i[u_1 i])])(\lambda u_4^{(s,se,t)}[u_4 i(\lambda i[u_2 i])])])])$

(38)

$$\dfrac{s \Rightarrow s \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{np \Rightarrow np^{\wedge s} \quad s \Rightarrow s}{np,\ s\backslash np^{\wedge s}}\backslash L \qquad np \Rightarrow np^{\vee s}}{np,\ (s\backslash np^{\wedge s})/np^{\vee s},\ np \Rightarrow s}/L}{np, (s\backslash np^{\wedge s})/np^{\vee s} \Rightarrow s/np}/R \qquad s \Rightarrow s}{np,\ (s\backslash np^{\wedge s})/np^{\vee s},\ np^{\vee s} \Rightarrow s}\backslash L}{(s\backslash np^{\wedge s})/np^{\vee s},\ np^{\vee s} \Rightarrow s\backslash np}\backslash R}{np^{\wedge s},\ (s\backslash np^{\wedge s})/np^{\vee s},\ np^{\vee s} \Rightarrow s}/L$$

(39) $x_1^{q^*}(\lambda i \lambda u_2^{se}[$
$x_3^{q^*}(\lambda i \lambda u_1^{se}[$
$x_2^{(sq^*,sq^*,t)}(\lambda u_3^{(s,se,t)}[u_3 i(\lambda i[u_1 i])])(\lambda u_4^{(s,se,t)}[u_4 i(\lambda i[u_2 i])])])])$

Without arguing it any further, we will take it that it is obvious from the terms in (37) and (39) that $\mathcal{LT}^{23}$ *will* account for (30e).

Now consider (30f). We wish to show that $\mathcal{LT}^{23}$ will not account for this piece of ambiguity data. To do this we should consider all the possible *flat* disambiguations of (30f), which means we should consider all possible proofs of:

(40) $Q_1/\text{cn}, \text{cn}, ((\text{s}\backslash Q_2)/\text{vpc})/Q_3, Q_4/\text{cn}, \text{cn}, \text{vpc}/(\text{s}\backslash Q_5), \text{s}\backslash Q_6 \Rightarrow \text{s}$

We will in fact make a simplifying assumption and consider instead a certain set of ***non-flat*** disambiguations, of the form:

(41)

every	man	told	a	woman	to	go
Q_1/cn	cn	$((\text{s}\backslash Q_2)/\text{vpc})/Q_3$	Q_4/cn	cn	$\text{vpc}/(\text{s}\backslash Q_5)$	$\text{s}\backslash Q_6$

$$\frac{\dfrac{\dfrac{\text{every}}{Q_1/\text{cn}} \quad \dfrac{\text{man}}{\text{cn}}}{Q_1}1 \quad \dfrac{\text{told}}{((\text{s}\backslash Q_2)/\text{vpc})/Q_3} \quad \dfrac{\dfrac{\text{a}}{Q_4/\text{cn}} \quad \dfrac{\text{woman}}{\text{cn}}}{Q_4}2 \quad \dfrac{\dfrac{\text{to}}{\text{vpc}/(\text{s}\backslash Q_5)} \quad \dfrac{\text{go}}{\text{s}\backslash Q_6}}{\text{vpc}}4}{\text{s}}3$$

There are as many semantically significantly different such disambiguations are there as significantly different proofs of the sequent below, to go into the above at 3.

(42) $Q_1, ((\text{s}\backslash Q_2)/\text{vpc})/Q_3, Q_4, \text{vpc} \Rightarrow \text{s}$

In investigating the possible proofs of (42) we will trace back avenues of possible proof until semantically unequivocal sequents are reached. (43) shows the only proof of (42) whose first development is through the principal connective of Q_1. That this is so is shown below

That (43) is the only proof of (42) developed first through Q_1.

First Q_1 must be so chosen that its principal connective is /, otherwise there is no proof development possible. For $Q_1 = \text{s}/(\text{s}/\text{np})$, there is just one count-consistent possible pairings of major and minor premise, and likewise for $Q_1 = \text{s}/(\text{s}\backslash\text{np})$:

(1a) $\text{s} \Rightarrow \text{s}$ (1b) $((\text{s}\backslash Q_2)/\text{vpc})/Q_3, Q_4, \text{vpc} \Rightarrow \text{s}/\text{np}$

(2a) $\text{s} \Rightarrow \text{s}$ (2b) $((\text{s}\backslash Q_2)/\text{vpc})/Q_3, Q_4, \text{vpc} \Rightarrow \text{s}\backslash\text{np}$

(1a) and (2a) are semantically unequivocal and (1b) actually has no proof, leaving just the developments of (2b) to be considered. Using the fact that Right rules may be ordered before Left rules, (2b) is developed into the single premise (2b.1):

(2b.1) $\text{np}, ((\text{s}\backslash Q_2)/\text{vpc})/Q_3, Q_4, \text{vpc} \Rightarrow \text{s}$

There would appear to be possible developments of (2b.1) through the principal connective of Q_4, but whatever instantiation is chosen of Q_4, the minor premise of the proof development is not count-consistent. The only other development of (2b.1) is through the principal connective of $((\text{s}\backslash Q_2)/\text{vpc})/Q_3$, and there is only one count-consistent pairing of major and minor premise:

(2b.1.1a) $\text{np}, (\text{s}\backslash Q_2)/\text{vpc}, \text{vpc} \Rightarrow \text{s}$ (2b.1.1b) $Q_4 \Rightarrow Q_3$

(2b.1.1a) and (2b.1.1b) are semantically unequivocal if provable at all, and thus need be developed no further. The proof thus traced out is (43).

In (44), (45) and (46) are shown the only proofs whose first development is through the principal slash of $((s\backslash Q_2)/vpc)/Q_3$. That this is so can be shown by similar reasoning to that above, applying count-consistency scrupulously. The principal connective of Q_4 also suggests proof-developments but all may be eliminated by count-consistency.

(43)
$$\frac{s \Rightarrow s \qquad \dfrac{\dfrac{np, (s\backslash Q_2)/vpc, vpc \Rightarrow s \qquad Q_4 \Rightarrow Q_3}{np, ((s\backslash Q_2)/vpc)/Q_3, Q_4, vpc \Rightarrow s}/L}{((s\backslash Q_2)/vpc)/Q_3, Q_4, vpc \Rightarrow s\backslash np}\backslash R}{s/(s\backslash np), ((s\backslash Q_2)/vpc)/Q_3, Q_4, vpc \Rightarrow s}/L$$

(44)
$$\frac{\dfrac{\dfrac{Q_1 \Rightarrow Q_2 \qquad s \Rightarrow s}{Q_1, s\backslash Q_2 \Rightarrow s}\backslash L \qquad vpc \Rightarrow vpc}{Q_1, (s\backslash Q_2)/vpc, vpc \Rightarrow s}/L \qquad Q_4 \Rightarrow Q_3}{Q_1, ((s\backslash Q_2)/vpc)/Q_3, Q_4, vpc \Rightarrow s}/L$$

(45)
$$\frac{\dfrac{\dfrac{s \Rightarrow s \qquad \dfrac{\dfrac{np \Rightarrow Q_2 \qquad s \Rightarrow s}{np, s\backslash Q_2 \Rightarrow s}\backslash L}{s\backslash Q_2 \Rightarrow s\backslash np}\backslash R}{Q_1, s\backslash Q_2 \Rightarrow s}/L \qquad vpc \Rightarrow vpc}{Q_1, (s\backslash Q_2)/vpc, vpc \Rightarrow s}/L \qquad Q_4 \Rightarrow Q_3}{Q_1, ((s\backslash Q_2)/vpc)/Q_3, Q_4, vpc \Rightarrow s}/L$$

(46)
$$\frac{\dfrac{s \Rightarrow s \qquad \dfrac{\dfrac{\dfrac{np \Rightarrow Q_2 \qquad s \Rightarrow s}{np, s\backslash Q_2 \Rightarrow s}\backslash L \qquad vpc \Rightarrow vpc}{np, (s\backslash Q_2)/vpc, vpc \Rightarrow s}/L}{(s\backslash Q_2)/vpc, vpc \Rightarrow s\backslash np}\backslash R}{Q_1, (s\backslash Q_2)/vpc, vpc \Rightarrow s}/L \qquad Q_4 \Rightarrow Q_3}{Q_1, ((s\backslash Q_2)/vpc)/Q_3, Q_4, vpc \Rightarrow s}/L$$

The terms associated with these four proofs are one or other of the following two terms (assuming that $T_1^{q^*}$, $V^{(sq^*,(s,sq^*,t),sq^*,t)}$,$T_2^{q^*}$,$\Phi^{(sq^*,t)}$ are the term associated with the antecedents of the sequent):

$T_1(\lambda i \lambda u_1[V(\lambda i[T_2])(\lambda i[\Phi])(\lambda i \lambda u_3[u_3(i)(\lambda i[u_1 i])])])$
$V(\lambda i T_2)(\lambda i \Phi)(\lambda i T_1)$

What we need to do now is to compare the denotations of the disambiguations of every man told a woman to go that results from making 3 in (41) a proof associated with the first or second of the above terms. Let us call these d_1 and d_2, and suppose that m_1,m_2, m_3 and m_4 are the meanings assigned to the disambiguation of every man, told, a woman and to go. Because of the meaning postulate concerning told, one can show that $d_1 = d_2$, and that therefore despite the multiplicity of proofs, $\mathcal{LT}^{23}$ does not account for (30f):

That $d_1 = d_2$

Suppose

(1) $d_1 \neq d_2$.

(1) implies:

(2) $m_1(w,g)(w_1 \mapsto x^{(s,e)} \mapsto m_2(w_1,g)(w_2 \mapsto m_3(w_2,g))(w_3 \mapsto m_4(w_3,g))(w_4 \mapsto Y^{(s,se,t)} \mapsto$
$Y(w_4)(w_5 \mapsto x(w_5))))$
$\neq m_2(w_1,g)(w_2 \mapsto m_3(w_2,g))(w_3 \mapsto m_4(w_3,g))(w_4 \mapsto m_1(w_4,g))$

Because of the 'VT plus Quantifier' meaning postulate concerning told, (2) implies:

(3) there exists m_* such that
$m_1(w,g)(w_1 \mapsto x^{(s,e)} \mapsto int{-}AR^3 int{-}AR^1(m_*)(w_1,g)(w_2 \mapsto m_3(w_2,g))(w_3 \mapsto m_4(w_3,g))(w_4 \mapsto$
$Y^{(s,se,t)} \mapsto Y(w_4)(w_5 \mapsto x(w_5))))$
$\neq int - AR^3 int - AR^1(m_*)(w_1,g)(w_2 \mapsto m_3(w_2,g))(w_3 \mapsto m_4(w_3,g))(w_4 \mapsto m_1(w_4,g))$

Using the definition of $int - AR^3$ (3) implies:

(4) there exists m_* such that
$m_1(w,g)(w_1 \mapsto x^{(s,e)} \mapsto int - AR^1(m_*)(w_1,g)(w_2 \mapsto m_3(w_2,g))(w_3 \mapsto m_4(w_3,g))x(w_1))$
$\neq (m_1(w_1,g))(w_4 \mapsto z^{(s,e)} \mapsto int{-}AR^1(m_*)(w_4,g)(w_2 \mapsto m_3(w_2,g))(w_3 \mapsto m_4(w_3,g))(z(w_4)))$

(4) is a contradiction. Therefore (1) is false. □

$\mathcal{LT}^{23}$ also fails to account for (30g). There are so many different proofs of the relevant categorising sequent that rather a lot of time would have to be spent to demonstrate $\mathcal{LT}^{23}$'s failing on (30g) in the same way as its failing on (30f) was demonstrated. However, by looking at the relatively small number of different ways such proofs might *begin*, it is possible to deduce enough about the terms that would be associated the completed proof to show that the ambiguity is not accounted for. Five ways in which a proof of the categorising sequent might be begun are given in (47), (48), (49), (50) and (51). The are other *apparently* possible ways to commence a proof, but all others can be quickly eliminated by the count-constraint. What one can show is that however these proofs are developed, the associated terms shall not have the term associated with Q_2 as the principal function: it must appear as an argument. Taking only the terms into consideration then a man will be predicted to occur opaquely. Of course it is possible that in concert with appropriate meaning postulates, a term predicting an opaque occurence can predict a transparent one. So to be sure that $\mathcal{LT}^{23}$ does not account for (30g) one should check out that there is not this kind of conspiracy of term and meaning postulate. I have checked this out, but the reader can avoid doing this if they believe that the grammar should in any case pass the the sterner test of providing a reading according to which an embedded quantifier occurs transparently, even when the verb with which the quantifier is combined itself usually generates an opaque context.

(47)
$$\dfrac{\dfrac{\vdots}{\text{np}:y_1^e,\ \text{s}\backslash Q_1:y_2(\lambda i[\Phi])\ \Rightarrow \text{s}:\Psi^t} \quad \dfrac{\vdots}{\text{sc/s}:y_3,\ Q_2:y_4,\ \text{s}\backslash Q_3:y_5\ \Rightarrow \text{sc}:\Phi^t}}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc/s}:y_3^{(st,t)},\ Q_2:y_4^{q^*},\ \text{s}\backslash Q_3:y_5^{(sq^*,t)}\ \Rightarrow \text{s}:\Psi^t}/\text{L}$$

$\Phi^t = (y_3, y_4, y_5)^t$, by which I mean that Φ is of type t and contains y_3, y_4 and y_5. $\Psi^t = (y_2(\lambda i[\Phi]), y_1)$. Therefore y_4 is not the principal function in Ψ^t.

(48)
$$\dfrac{\dfrac{\vdots}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc}:y_3(\lambda i[\Phi])\ \Rightarrow \text{s}:\Psi^t} \quad \dfrac{\vdots}{Q_2:y_4,\ \text{s}\backslash Q_3:y_5\ \Rightarrow \text{sc}:\Phi^t}}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc/s}:y_3^{(st,t)},\ Q_2:y_4^{q^*},\ \text{s}\backslash Q_3:y_5^{(sq^*,t)}\ \Rightarrow \text{s}:\Psi^t}/\text{L}$$

$\Phi^t = (y_4, y_5)^t$, $\Psi^t = (y_3(\lambda i[\Phi]), y_2, y_1)$. Therefore y_4 is not the principal function in Ψ^t.

(49)
$$\dfrac{\dfrac{\vdots}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc/s}:y_3^{(st,t)},\ \text{s}:y_4(\lambda i[\Phi])\ \Rightarrow \text{s}:\Psi^t} \quad \dfrac{\vdots}{\text{s}\backslash Q_3:y_5\ \Rightarrow \text{s}\backslash\text{np}:\Phi^{(se,t)}}}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc/s}:y_3^{(st,t)},\ Q_2:y_4^{q^*},\ \text{s}\backslash Q_3:y_5^{(sq^*,t)}\ \Rightarrow \text{s}:\Psi^t}/\text{L}$$

$\Phi^{(se,t)} = (y_5)$. Ψ^t must contain $(y_4(\lambda i[\Phi]))^t$. Since $(y_4(\lambda i[\Phi]))^t$ is of a non-functional type, it must be an argument. Therefore y_4 is not the principal function in Ψ^t.

(50)
$$\dfrac{\dfrac{\vdots}{Q_2:y_4^{q^*}\ \Rightarrow Q_3:\Phi^{q^*}} \quad \dfrac{\vdots}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc/s}:y_3^{(st,t)},\ \text{s}:y^5(\lambda i[\Phi])\ \Rightarrow \text{s}:\Psi^t}}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc/s}:y_3^{(st,t)},\ Q_2:y_4^{q^*},\ \text{s}\backslash Q_3:y_5^{(sq^*,t)}\ \Rightarrow \text{s}:\Psi^t}\text{L}$$

$\Phi^t = y_4^{q^*}$. Therefore Ψ^t must contain $(y^5(\lambda i[y^4]))^t$, which is of a non-functional type and so must must be an argument Ψ^t. Therefore y_4 is not the principal function in Ψ^t

(51)
$$\dfrac{\dfrac{\vdots}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc/s}:y_3^{(st,t)},\ Q_2:y_4^{q^*}\ \Rightarrow Q_3:\Phi^{q^*}} \quad \dfrac{\vdots}{\text{s}:y^5(\lambda i[\Phi])\ \Rightarrow \text{s}:\Psi^t}}{\text{np}:y_1^e,\ (\text{s}\backslash Q_1)/\text{sc}:y_2^{(st,sq^*,t)},\ \text{sc/s}:y_3^{(st,t)},\ Q_2:y_4^{q^*},\ \text{s}\backslash Q_3:y_5^{(sq^*,t)}\ \Rightarrow \text{s}:\Psi^t}\text{L}$$

$\Phi^{q^*} = (y_1, y_2, y_3, y_4)^{q^*}$. $\Psi = y^5(\Phi)$. Therefore y_4 is not the principal function in Ψ^t.

That completes the assessment of the abilities of $\mathcal{LT}^{23}$ to account for the semantic properties of determiners. We see that $\mathcal{LT}^{23}$, like all the L$^{(/,\backslash)}$-THEORIES OF REFERENCE that we have considered so far, is unable to account wholly for the semantic properties of the determiners. The performance of $\mathcal{LT}^{23}$ is summarised in Table 6.1.

3 A polymorphic L$^{(/,\backslash)}$-THEORY OF REFERENCE based on minimal types

We have seen that no satisfactory account of junctions and determiners seems possible in the form of a L$^{(/,\backslash)}$-THEORY OF REFERENCE that is *monomorphic* with respect to its typing of junctions and determiners. Now when the restriction to *monomorphic* L$^{(/,\backslash)}$-THEORIES OF REFERENCE is lifted and instead we permit ourselves to assign junctions and determiners several different types, one may quickly find a L$^{(/,\backslash)}$-THEORY OF REFERENCE that successfully accounts for at least the *syntax* of junctions and determiners.

We shall show this by returning to the L$^{(/,\backslash)}$-THEORY OF REFERENCE, $\mathcal{LT}^{19}$, which was the starting point in section 1. The categorisation of verbal term in $\mathcal{LT}^{19}$ one can say is the 'classic' categorisation, and the category-to-type map of $\mathcal{LT}^{19}$ was the one that gave to verbal terms the minimal types. We looked in section 1 at the questions whether there were possible extensions of $\mathcal{LT}^{19}$ that account for junctions and determiners, and here the same question will be looked at, only this time the requirement that the extension be *monomorphic vis-a-vis* the types assigned to junctions and determiners will be dropped.

In (52) are set out some conditions on junctions categories that if met by an extension of $\mathcal{LT}^{19}$ would mean that that extension accounts for the syntactic data concerning junctions.

(52) a. $s, C_1, s \Rightarrow s \equiv C_1 \Rightarrow (s\backslash s)/s$

b. $np, s\backslash np, C_2, s\backslash np \Rightarrow s \equiv C_2 \Rightarrow ((s\backslash np)\backslash(s\backslash np))/(s\backslash np)$

c. $np, (s\backslash np)/np, C_3, (s\backslash np)/np, np \Rightarrow s$

$\equiv C_3 \Rightarrow (((s\backslash np)/np)\backslash((s\backslash np)/np))/((s\backslash np)/np)$

d. $np, (s\backslash np)/vpc, C_4, (s\backslash np)/vpc, vpc \Rightarrow s$

$\equiv C_4 \Rightarrow (((s\backslash np)/vpc)\backslash((s\backslash np)/vpc))/((s\backslash np)/vpc)$

e. $np, ((s\backslash np)/vpc)/np, C_5, ((s\backslash np)/vpc)/np, vpc, np \Rightarrow s$

$\equiv C_5 \Rightarrow ((((s\backslash np)/vpc)/np)\backslash(((s\backslash np)/vpc)/np))/(((s\backslash np)/vpc)/np)$

The simplest set of solutions to these equations is one that could not be considered in section 1, leading as it must to the junctions being associated with several *different* types:

(53) $C_1 = (\mathrm{s}\backslash\mathrm{s})/\mathrm{s}, \quad \nu(C_1) = (st, st, t)$

$C_2 = ((\mathrm{s}\backslash\mathrm{np})\backslash(\mathrm{s}\backslash\mathrm{np}))/(\mathrm{s}\backslash\mathrm{np}), \quad \nu(C_2) = ((s, se, t), (s, se, t), (se, t))$

$C_3 = (((\mathrm{s}\backslash\mathrm{np})/\mathrm{np})\backslash((\mathrm{s}\backslash\mathrm{np})/\mathrm{np}))/((\mathrm{s}\backslash\mathrm{np})/\mathrm{np}),$

$\nu(C_3) = ((s, se, se, t), (s, se, se, t), (se, se, t))$

$C_4 = (((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})\backslash((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc}))/((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc}),$

$\nu(C_4) = ((s, (s, se, t), se, t), (s, (s, se, t), se, t), ((s, se, t), se, t))$

$C_5 = ((((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})/\mathrm{np})\backslash(((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})/\mathrm{np}))/(((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})/\mathrm{np}),$

$\nu(C_5) = ((s, se, (s, se, t), se, t), (s, se, (s, se, t), se, t), (se, (s, se, t), se, t))$

The simplicity of these solutions may perhaps be easier to see if some the categories and types involved are abbreviated a little. We will use the following abbreviations of categories $VP^C = \mathrm{s}\backslash\mathrm{np}$, $TV^C = VP^C/\mathrm{np}$, $TTV^C = TV^C/\mathrm{np}$, $VVP^C = VP^C/\mathrm{vpc}$, $TVVP^C = VVP^C/\mathrm{np}$, and the following abbreviations of types $vp = (se, t)$, $tv = (se, se, t)$.

(54) $C_1 = (\mathrm{s}\backslash\mathrm{s})/\mathrm{s}, \quad \nu(C_1) = (st, st, t)$

$C_2 = (VP^C\backslash VP^C)/VP^C, \quad \nu(C_2) = ((s, vp), (s, vp), vp)$

$C_3 = (TV^C\backslash TV^C)/TV^C, \quad \nu(C_3) = ((s, tv), (s, tv), tv)$

$C_4 = (VVP^C\backslash VVP^C)/VVP^C,$

$\nu(C_4) = ((s, (s, vp), vp), (s, (s, vp), vp), ((s, vp), vp))$

$C_5 = (TVVP^C\backslash TVVP^C)/TVVP^C,$

$\nu(C_5) = ((s, se, (s, vp), vp), (s, se, (s, vp), vp), (se, (s, vp), vp))$

The same exercise will be repeated for the determiners. In (58) are some conditions on determiner categorisations that if met by an extension of $\mathcal{LT}^{19}$ would guarantee that the syntactic properties of determiners were accounted for. One should note that (58e) would allow for the coverage of the as yet unconsidered case of occurrences of QNP's as the objects of prepositions, as in,

(55) every man near a woman died

John killed every man near a woman

(56) a. C_1, cn, $\mathrm{s}\backslash\mathrm{np} \Rightarrow \mathrm{s} \equiv C_1 \Rightarrow (\mathrm{s}/(\mathrm{s}\backslash\mathrm{np}))/\mathrm{cn}$

b. np, $(\mathrm{s}\backslash\mathrm{np})/\mathrm{np}$, C_2, cn $\Rightarrow \mathrm{s} \equiv C_2 \Rightarrow ((\mathrm{s}\backslash\mathrm{np})\backslash((\mathrm{s}\backslash\mathrm{np})/\mathrm{np}))/\mathrm{cn}$

c. np, $((\mathrm{s}\backslash\mathrm{np})/\mathrm{np})/\mathrm{np}$, C_3, cn, np $\Rightarrow \mathrm{s} \equiv C_3 \Rightarrow (((\mathrm{s}\backslash\mathrm{np})/\mathrm{np})\backslash(((\mathrm{s}\backslash\mathrm{np})/\mathrm{np})/\mathrm{np}))/\mathrm{cn}$

d. np, $((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})/\mathrm{np}$, C_4, cn, vpc $\Rightarrow \mathrm{s} \equiv$

$C_4 \Rightarrow (((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})\backslash(((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})/\mathrm{np}))/\mathrm{cn}$

e. $(\mathrm{cn}\backslash\mathrm{cn})/\mathrm{np}$, C_5, cn $\Rightarrow \mathrm{cn}\backslash\mathrm{cn} \equiv C_5 \Rightarrow ((\mathrm{cn}\backslash\mathrm{cn})\backslash((\mathrm{cn}\backslash\mathrm{cn})/\mathrm{np}))/\mathrm{cn}$

Again its is the case that the simplest solution set to these equations is one which could not be considered in section 1 because of the consequence that the determiners would be associated with several different types:

(57) $C_1 = \mathrm{s}/(\mathrm{s}\backslash\mathrm{np}) \quad \nu(C_1) = ((s, se, t), t)$

$C_2 = (\mathrm{s}\backslash\mathrm{np})\backslash((\mathrm{s}\backslash\mathrm{np})/\mathrm{np}) \quad \nu(C_2) = ((s, se, se, t), (se, t))$

$C_3 = ((\mathrm{s}\backslash\mathrm{np})/\mathrm{np})\backslash(((\mathrm{s}\backslash\mathrm{np})/\mathrm{np})/\mathrm{np}) \quad \nu(C_3) = ((s, se, se, se, t), (se, se, t))$

$C_4 = ((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})\backslash(((\mathrm{s}\backslash\mathrm{np})/\mathrm{vpc})/\mathrm{np})$

$\nu(C_4) = ((s, se, (s, se, t), se, t), ((s, se, t), se, t))$

$C_5 = (\mathrm{cn}\backslash\mathrm{cn})\backslash((\mathrm{cn}\backslash\mathrm{cn})/\mathrm{np}) \quad \nu(C_5) = ((s, se, (s, et), et), ((s, et), et))$

These solutions are repeated in abbreviated form in (58):

(58) $C_1 = \mathrm{s}/(\mathrm{s}\backslash\mathrm{np}) \quad \nu(C_1) = ((s, se, t), t)$

$C_2 = VP^C\backslash(VP^C/\mathrm{np}) \quad \nu(C_2) = ((s, se, vp), vp)$

$C_3 = TV^C\backslash(TV^C/\mathrm{np}) \quad \nu(C_3) = ((s, se, tv), tv)$

$C_4 = VVP^C\backslash(VVP^C/\mathrm{np}) \quad \nu(C_4) = ((s, se, (s, vp), vp), ((s, vp), vp))$

$C_5 = (\mathrm{cn}\backslash\mathrm{cn})\backslash((\mathrm{cn}\backslash\mathrm{cn})/\mathrm{np}) \quad \nu(C_5) = ((s, se, (s, et), et), ((s, et), et))$

Therefore clearly $\mathcal{LT}^{19}$ may be so extended as to account for the *syntax* of junctions and determiners.

By $\mathcal{LT}^{24}$ we shall refer to that extension of $\mathcal{LT}^{19}$ that categorises junction in all the ways indicated in (54), and determiners in all the ways indicated in (58). The meaning postulates by which $\mathcal{LT}^{24}$ is governed are those by which $\mathcal{LT}^{19}$ is governed together with junction and determiner postulates built on the pattern of the following postulates for disambiguations of and and every:

Definition 62 (Postulates for and) *Whatever model, $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$, associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}$ and v, all instances of the schematic condition (a) below hold, where the schematic category variable, x, and schematic type variable, a, may be jointly instantiated to any pair of values in:*

$\{\ \langle \mathrm{s}, t\rangle,\ \langle VP^C, vp\rangle,\ \langle TV^C, tv\rangle,\ \langle VVP^C, ((s, vp), vp)\rangle,\ \langle TVVP^C, (se, (s, vp), vp)\rangle\ \}$

(a) $f(\langle \mathsf{and}\rangle, \langle\rangle, (x\backslash x)/x\rangle) =$

$\mathcal{I}^2\mathcal{I}^1((w, g), d_1^a, d_2^a \mapsto \mathcal{H}^E_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(d_1^a, \mathcal{AND}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g), d_2^a))$

Definition 63 (Postulates for every) *Whatever model, $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$, associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}$ and v, (a) below holds and all instances of the schematic condition (b) below hold, where the schematic category variable, x, and schematic type variable, a, may be jointly instantiated to any pair of values in:*

$\{\ \langle VP^C, vp\rangle,\ \langle TV^C, tv\rangle,\ \langle VVP^C, ((s, vp), vp)\rangle,\ \langle (\mathrm{cn}\backslash\mathrm{cn}), ((s, et), et)\rangle\ \}$

(a) $f(\langle \textsf{every} \rangle, \langle\rangle, \mathrm{s}/(\mathrm{s}\backslash\mathrm{np}))) =$

$\mathcal{I}^2\mathcal{I}^1((w,g), d_1^{(et)}, d_2^{(se,t)} \mapsto$

$\quad \mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})((d_3^e \mapsto d_2^{(se,t)}(w \mapsto d_3^e)), \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(d_1^{(et)})))$

(b) $f(\langle \textsf{every} \rangle, \langle\rangle, x\backslash(x/\mathrm{np}))) =$

$\mathcal{I}^2\mathcal{I}^1((w,g), d_1^{(et)}, d_2^{(se,a)} \mapsto$

$\quad \mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})((d_3^e \mapsto d_2^{(se,a)}(w \mapsto d_3^e)), \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(d_1^{(et)})))$

The workings of these postulates will become clearer if some examples are given of how they lead to $\mathcal{LT}^{24}$ being able to account for the semantic data. Accordingly, we will assess whether $\mathcal{LT}^{24}$ can account for the semantic properties of junctions and determiners.

Beginning with junctions, $\mathcal{LT}^{24}$ can account for instances of Hypothesis 4 as applied to unambiguous sentences containing junctions, instances such as the following:

(59) a. **John walks and Mary talks** is recursively ambiguous wrt. **John walks and Mary talks**

b. **John walks and talks** is recursively ambiguous wrt. **walks and talks**

c. **John loves and hates Mary** is recursively ambiguous wrt. **loves and hates**

d. **John wants and needs to go** is recursively ambiguous wrt. **wants and needs**

The following disambiguations of the sentences in (15) account for the data in (59)

(60) John walks and Mary talks

np s\np C_1 np s\np

s s

a

s

$C_1 = (\mathrm{s}\backslash\mathrm{s})/\mathrm{s}$

(61) John walks and talks

np s\np C_2 s\np

b

s\np

s

$C_2 = (VP^C\backslash VP^C)/VP^C$

(62) John loves and hates Mary

np (s\np)/np C_3 (s\np)/np np

c

(s\np)/np

s

$C_3 = (TV^C\backslash TV^C)/TV^C,$

(63) John wants and needs to go

np (s\np)/vpc C_4 (s\np)/vpc vpc/(s\np) s\np

d

(s\np)/vpc

s

$C_4 = (VVP^C\backslash VVP^C)/VVP^C$

That $\mathcal{LT}^{24}$ accounts for the data in (59)

(S-case) there is a $\overline{\text{John walks and Mary talks}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{John walks}}$, whatever $\overline{\text{Mary talks}}$, $[\![\overline{\text{John walks and Mary talks}}]\!](w,g) = 1$ iff $AND([\![\overline{\text{John walks}}]\!](w,g) = 1, [\![\overline{\text{Mary talks}}]\!](w,g) = 1)$.

The disambiguation in (60) has this property. This is shown below where $\overline{\text{John walks and Mary talks}}$ refers to (60). Also $\overline{\text{John walks}}$ and $\overline{\text{Mary talks}}$ refer to the only possible flat disambiguations.

(i) $[\![\overline{\text{John walks and Mary talks}}]\!](w,g) = 1 \leftrightarrow$

(ii) $[\![\overline{\text{and}_{(s\backslash s)/s}}]\!](w,g)(w' \mapsto [\![\overline{\text{Mary talks}}]\!](w',g))(w' \mapsto [\![\overline{\text{John walks}}]\!](w',g)) = 1 \leftrightarrow$

(iii) $\mathcal{H}^{E}_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})([\![\overline{\text{Mary talks}}]\!](w,g), \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g), [\![\overline{\text{John walks}}]\!](w,g)) = 1 \leftrightarrow$

(iv) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{Mary talks}}]\!](w,g))([\![\overline{\text{John walks}}]\!](w,g)) = 1 \leftrightarrow$

(v) $AND([\![\overline{\text{John walks}}]\!](w,g) = 1, [\![\overline{\text{Mary talks}}]\!](w,g) = 1)$

(VP-case) there is a $\overline{\text{John walks and talks}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{John walks}}$, whatever $\overline{\text{John talks}}$, $[\![\overline{\text{John walks and talks}}]\!](w,g) = 1$ iff $AND([\![\overline{\text{John walks}}]\!](w,g) = 1, [\![\overline{\text{John talks}}]\!](w,g) = 1)$.

The disambiguation in (61) has this property. This is shown below where $\overline{\text{John walks and talks}}$ refers to (61). Also $\overline{\text{John walks}}$ and $\overline{\text{John talks}}$ refer to the only possible flat disambiguations. T, W and j are abbreviations of the meanings of talks, walks and John.

(i) $[\![\overline{\text{John walks and talks}}]\!](w,g) = 1 \leftrightarrow$

(ii) $[\![\overline{\text{and}_{(VPC\backslash VPC)/VPC}}]\!](w,g)(w' \mapsto \mathsf{T}(w',g))(w' \mapsto \mathsf{W}(w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$

(iii) $\mathcal{H}^{E}_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(\mathsf{T}(w,g), \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g), \mathsf{W}(w,g))$
$(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$

(iv) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{T}(w,g)(w' \mapsto \mathsf{j}(w',g)))(\mathsf{W}(w,g)(w' \mapsto \mathsf{j}(w',g))) = 1 \leftrightarrow$

(v) $AND([\![\overline{\text{John walks}}]\!](w,g) = 1, [\![\overline{\text{John talks}}]\!](w,g) = 1)$

(TV-case) there is a $\overline{\text{John loves and hates Dave}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{John loves Dave}}$, whatever $\overline{\text{John hates Dave}}$, $[\![\overline{\text{John loves and hates Dave}}]\!](w,g) = 1$ iff $AND([\![\overline{\text{John loves Dave}}]\!](w,g) = 1, [\![\overline{\text{John hates Dave}}]\!](w,g) = 1)$.

The disambiguation in (62) has this property. This is shown below where $\overline{\text{John loves and hates Dave}}$ refers to (62). Also $\overline{\text{John loves Dave}}$ and $\overline{\text{John hates Dave}}$ refer to the only possible flat disambiguations. H, L, d and m are the obvious abbreviations for meanings.

(i) $[\![\overline{\text{John loves and hates Dave}}]\!](w,g) = 1 \leftrightarrow$

(ii) $[\![\overline{\text{and}_{(TVC\backslash TVC)/TVC}}]\!](w,g)(w' \mapsto \mathsf{H}(w',g))(w' \mapsto \mathsf{L}(w',g))(w' \mapsto \mathsf{d}(w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$

(iii) $\mathcal{H}^{E}_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(\mathsf{H}(w,g), \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g), \mathsf{L}(w,g))$
$(w' \mapsto \mathsf{d}(w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$

(iv) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{H}(w,g)(w' \mapsto \mathsf{d}(w',g))(w' \mapsto \mathsf{j}(w',g)))(\mathsf{L}(w,g)(w' \mapsto \mathsf{d}(w',g))(w' \mapsto \mathsf{j}(w',g))) = 1 \leftrightarrow$

(v) $AND([\![\overline{\text{John loves Dave}}]\!](w,g) = 1, [\![\overline{\text{John hates Dave}}]\!](w,g) = 1)$

(VVP-case) there is a $\overline{\text{John needs and wants to go}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{John needs to go}}$, whatever $\overline{\text{John wants to go}}$, $[\![\overline{\text{John wants and needs to go}}]\!](w,g) = 1$ iff $AND([\![\overline{\text{John wants to go}}]\!](w,g) = 1, [\![\overline{\text{John needs to go}}]\!](w,g) = 1)$.

The disambiguation in (63) has this property. This is shown below where $\overline{\text{John wants and needs to go}}$ refers to (63). Also $\overline{\text{John wants to go}}$ and $\overline{\text{John needs to go}}$ refer to *any* possible flat disambiguation: there are several but all are semantically equivalent. N, W and j are obvious abbreviations for meanings.

(i) $[\![\overline{\text{John needs and wants to go}}]\!](w,g) = 1 \leftrightarrow$

(ii) $[\overline{\text{and}_{(VVP^C\backslash VVP^C)/VVP^C}}](w,g)(w' \mapsto \mathsf{N}(w',g))(w' \mapsto \mathsf{W}(w',g))$
$(w' \mapsto [\overline{\text{to go}_{\text{vpc}}}](w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$
(iii) $\mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(\mathsf{N}(w,g), \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g), \mathsf{W}(w,g))$
$(w' \mapsto [\overline{\text{to go}_{\text{vpc}}}](w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$
(iv) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{N}(w,g)(w' \mapsto [\overline{\text{to go}_{\text{vpc}}}](w',g))(w' \mapsto \mathsf{j}(w',g)))(\mathsf{W}(w,g)$
$(w' \mapsto [\overline{\text{to go}_{\text{vpc}}}](w',g))(w' \mapsto \mathsf{j}(w',g))) = 1 \leftrightarrow$
(v) $AND([\overline{\text{John wants to go}}](w,g) = 1, [\overline{\text{John needs to go}}](w,g) = 1)$

For all of the above the step from (ii) to (iii) is a matter of applying the meaning postulate for the relevant disambiguation of and, and using the definitions of $\mathcal{I}^1$ and $\mathcal{I}^2$. From (iii) to (iv) uses the definition of $\mathcal{H}^E_{\mathcal{J}}$. From (iv) to (v) uses the correspondence between $\mathcal{AND}$ and AND.

Now we turn to the determiners, considering in turn the unambiguous cases, (30a,b,c,d) and the ambiguous cases, (30e,f,g). The semantic observations (30a,b,c,d) are accounted for by the following disambiguations:

(64)
$$\dfrac{\dfrac{\dfrac{\text{every}}{C_1/\text{cn}} \quad \dfrac{\text{man}}{\text{cn}}}{C_1} \quad \dfrac{\text{walks}}{\text{s}\backslash\text{np}}}{\text{s}} \qquad C_1 = \text{s}/(\text{s}\backslash\text{np})$$

(65)
$$\dfrac{\dfrac{\text{John}}{\text{np}} \quad \dfrac{\text{loves}}{(\text{s}\backslash\text{np})/\text{np}} \quad \dfrac{\dfrac{\text{every}}{C_2/\text{cn}} \quad \dfrac{\text{man}}{\text{cn}}}{C_2}}{\text{s}} \qquad C_2 = VP^C\backslash(VP^C/\text{np})$$

(66)
$$\dfrac{\dfrac{\text{John}}{\text{np}} \quad \dfrac{\text{gave}}{((\text{s}\backslash\text{np})/\text{np})/\text{np}} \quad \dfrac{\dfrac{\text{every}}{C_3/\text{cn}} \quad \dfrac{\text{man}}{\text{cn}}}{C_3} \quad \dfrac{\text{Mary}}{\text{np}}}{\text{s}} \qquad C_3 = TV^C\backslash(TV^C/\text{np})$$

(67)
$$\dfrac{\dfrac{\text{John}}{\text{np}} \quad \dfrac{\text{told}}{((\text{s}\backslash\text{np})/\text{vpc})/\text{np}} \quad \dfrac{\dfrac{\text{every}}{C_4} \quad \dfrac{\text{man}}{\text{cn}}}{C_4} \quad \dfrac{\text{to}}{\text{vpc}/(\text{s}\backslash\text{np})} \quad \dfrac{\text{go}}{\text{s}\backslash\text{np}}}{\text{s}} \qquad C_4 = VVP^C\backslash(VVP^C/\text{np})$$

That $\mathcal{LT}^{24}$ can account for the data in (30a,b,c,d)

First we note the following fact about the only flat disambiguation of he_1 is a man

$(x \mapsto [\overline{\text{he}_1 \text{ is a man}}](w, g_x^{he_1}))$
$= (x \mapsto [\overline{\text{a}_{VP^C\backslash TV^C}}](w,g)(w' \mapsto [\overline{\text{man}}](w',g))(w' \mapsto [\overline{\text{is}}](w',g))(w' \mapsto [\overline{\text{v}_1}](w',g)))$
$= (x \mapsto \mathcal{H}^E_{\mathcal{Q}}(\mathcal{E},\mathcal{I},\mathcal{J})(d_3 \mapsto [\overline{\text{is}}](w,g)(w' \mapsto d_3), \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\overline{\text{man}}](w,g)))(w' \mapsto x))$
$= (x \mapsto \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\overline{\text{man}}](w,g))(d_3 \mapsto [\overline{\text{is}}](w,g)(w' \mapsto d_3)(w \mapsto x)))$
$= (x \mapsto \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\overline{\text{man}}](w,g))(d_3 \mapsto \mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(d_3)(x)))$

$= (x \mapsto [\![\text{man}]\!](w,g)(x))$

(VP-case) there is a $\overline{\text{every man walks}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{he}_1 \text{ is a man}}$, whatever $\overline{\text{he}_1 \text{ walks}}$, $[\![\text{every man walks}]\!](w,g) = 1 \leftrightarrow EVERY(\{x : [\![\text{he}_1 \text{ is a man}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\text{he}_1 \text{ walks}]\!](w, g^x_{he_1}) = 1\})$

(64) has this property, as shown below where $\overline{\text{every man walks}}$ stands for this disambiguation. Also $\overline{\text{v}_1 \text{ is a man}}$ and $\overline{\text{v}_1 \text{ walks}}$ stand for the only possible flat disambiguations. M and W are the obvious abbreviations for meanings.

(i) $[\![\text{every man walks}]\!](w,g) = 1 \leftrightarrow$
(ii) $[\![\text{every}_{s/(s\backslash np)}]\!](w,g)(w' \mapsto \mathsf{M}(w'g))(w' \mapsto \mathsf{W}(w',g)) = 1 \leftrightarrow$
(iii) $\mathcal{H}^E_Q(\mathcal{E},\mathcal{I},\mathcal{J})((d^e_3 \mapsto \mathsf{W}(w,g)(w \mapsto d^e_3)), \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w,g))) = 1 \leftrightarrow$
(iv) $\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w,g))(d^e_3 \mapsto \mathsf{W}(w,g)(w \mapsto d^e_3)) = 1 \leftrightarrow$
(v) $EVERY(\{x : [\![\text{he}_1 \text{ is a man}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\text{he}_1 \text{ walks}]\!](w, g^x_{he_1}) = 1\})$

(TV-case) there is a $\overline{\text{John loves every man}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{he}_1 \text{ is a man}}$, whatever $\overline{\text{he}_1 \text{ walks}}$, $[\![\text{John loves every man}]\!](w,g) = 1 \leftrightarrow EVERY(\{x : [\![\text{he}_1 \text{ is a man}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\text{John loves he}_1]\!](w, g^x_{he_1}) = 1\})$

(65) has this property, as shown below where $\overline{\text{John loves every man}}$ stands for this disambiguation. Also $\overline{\text{he}_1 \text{ is a man}}$ and $\overline{\text{John loves he}_1}$ stand for the only possible flat disambiguations. M, L and j are the obvious abbreviations for meanings.

(i) $[\![\text{John loves every man}]\!](w,g) = 1 \leftrightarrow$
(ii) $[\![\text{every}_{VPC\backslash(VPC/np)}]\!](w,g)(w' \mapsto \mathsf{M}(w'g))(w' \mapsto \mathsf{L}(w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$
(iii) $\mathcal{H}^E_Q(\mathcal{E},\mathcal{I},\mathcal{J})((d^e_3 \mapsto \mathsf{L}(w,g)(w \mapsto d^e_3)), \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w,g)))$
$(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$
(iv) $\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w,g))(d^e_3 \mapsto \mathsf{L}(w,g)(w \mapsto d^e_3)(w' \mapsto \mathsf{j}(w',g))) = 1 \leftrightarrow$
(v) $EVERY(\{x : [\![\text{he}_1 \text{ is a man}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\text{John loves he}_1]\!](w, g^x_{he_1}) = 1\})$

(TTV-case) there is a $\overline{\text{John gave every man Dave}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{he}_1 \text{ is a man}}$, whatever $\overline{\text{he}_1 \text{ walks}}$, $[\![\text{John gave every man Dave}]\!](w,g) = 1 \leftrightarrow EVERY(\{x : [\![\text{he}_1 \text{ is a man}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\text{John gave he}_1 \text{ Dave}]\!](w, g^x_{he_1}) = 1\})$

(66) has this property, as shown below where $\overline{\text{John gave every man Dave}}$ stands for this disambiguation. Also $\overline{\text{he}_1 \text{ is a man}}$ and $\overline{\text{John gave he}_1 \text{ Dave}}$ stand for the only possible flat disambiguations. M, G, d and j are the obvious abbreviations of meanings.

(i) $[\![\text{John gave every man Dave}]\!](w,g) = 1 \leftrightarrow$
(ii) $[\![\text{every}_{TVC\backslash(TVC/np)}]\!](w,g)(w' \mapsto \mathsf{M}(w'g))(w' \mapsto \mathsf{G}(w',g))(w' \mapsto \mathsf{d}(w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$
(iii) $\mathcal{H}^E_Q(\mathcal{E},\mathcal{I},\mathcal{J})((d^e_3 \mapsto \mathsf{G}(w,g)(w \mapsto d^e_3)), \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w,g)))$
$(w' \mapsto \mathsf{d}(w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$
(iv) $\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w,g))(d^e_3 \mapsto \mathsf{G}(w,g)(w \mapsto d^e_3)(w' \mapsto \mathsf{d}(w',g))(w' \mapsto \mathsf{j}(w',g))) = 1 \leftrightarrow$
(v) $EVERY(\{x : [\![\text{he}_1 \text{ is a man}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\text{John gave he}_1 \text{ Dave}]\!](w, g^x_{he_1}) = 1\})$

(TVVP-case) there is a $\overline{\text{John told every man to go}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{he}_1 \text{ is a man}}$, whatever $\overline{\text{John told he}_1 \text{ to go}}$, $[\![\text{John told every man to go}]\!](w,g) = 1 \leftrightarrow EVERY(\{x : [\![\text{he}_1 \text{ is a man}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\text{John told he}_1 \text{ to go}]\!](w, g^x_{he_1}) = 1\})$

(67) has this property, as shown below where $\overline{\text{John told every man to go}}$ stands for this disam-

biguation. $\overline{\text{he}_1 \text{ is a man}}$ stands for the only possible flat disambiguation and $\overline{\text{John told he}_1 \text{ to go}}$ stand for *any* possible flat disambiguation: there are several possibilities but all are semantically alike. M, T and j are the obvious abbreviations of meanings.

(i) $\llbracket\overline{\text{John told every man to go}}\rrbracket(w,g) = 1 \leftrightarrow$
(ii) $\llbracket\overline{\text{every}_{VVPC\backslash(VVPC/\text{np})}}\rrbracket(w,g)(w' \mapsto \mathsf{M}(w'g))(w' \mapsto \mathsf{T}(w',g))(w' \mapsto \llbracket\overline{\text{to go}_{\text{vpc}}}\rrbracket(w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$
(iii) $\mathcal{H}^E_Q(\mathcal{E},\mathcal{I},\mathcal{J})((d^e_3 \mapsto \mathsf{T}(w,g)(w \mapsto d^e_3)), \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w,g)))$
$(w' \mapsto \llbracket\overline{\text{to go}_{\text{vpc}}}\rrbracket(w',g))(w' \mapsto \mathsf{j}(w',g)) = 1 \leftrightarrow$
(iv) $\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w,g))(d^e_3 \mapsto \mathsf{T}(w,g)(w \mapsto d^e_3)(w' \mapsto \llbracket\overline{\text{to go}_{\text{vpc}}}\rrbracket(w',g))(w' \mapsto \mathsf{j}(w',g))) = 1 \leftrightarrow$
(v) $EVERY(\{x : \llbracket\overline{\text{he}_1 \text{ is a man}}\rrbracket(w, g^x_{he_1}) = 1\}, \{x : \llbracket\overline{\text{John told he}_1 \text{ to go}}\rrbracket(w, g^x_{he_1}) = 1\})$

From (ii) to (iii) is a matter of applying the meaning postulate for the relevant disambiguation of every, and using the definitions of $\mathcal{I}^1$ and $\mathcal{I}^2$. From (iii) to (iv) uses the definition of $\mathcal{H}^E_Q$. From (iv) to (v) uses the correspondence between $\mathcal{EVERY}$ and $EVERY$, as well as the meaning postulates for is and for pronouns.

So $\mathcal{LT}^{24}$ can account for (30a,b,c,d), the unambiguous cases. However, $\mathcal{LT}^{24}$ cannot account for the ambiguous cases, (30e,f,g). There will be no detailed demonstration of this, but for example consider (30g). We have already considered whether $\mathcal{LT}^{20}$ could account for (30g). $\mathcal{LT}^{20}$ and $\mathcal{LT}^{24}$ differ only on the categorisation of the junctions and determiners and so a great deal of what was said in the case of $\mathcal{LT}^{20}$ carries over to $\mathcal{LT}^{24}$. We showed that of the three possible quantifier categorisations allowed by $\mathcal{LT}^{20}$, the sequent np, (s\np)/sc, sc/s, Q, s\np $\Rightarrow$ s, was provable only for Q = s/(s\np). Now $\mathcal{LT}^{24}$ has more determiner categorisations, but with small adaptations to the proof one can show once again that it is still only the Q = s/(s\np) possibility for which np, (s\np)/sc, sc/s, Q, s\np $\Rightarrow$ s is provable. The fact that there is no significant semantic diversity amongst the possible proofs of np, (s\np)/sc, sc/s, Q, s\np $\Rightarrow$ s, for Q = s/(s\np), has already been shown, therefore, $\mathcal{LT}^{24}$, like $\mathcal{LT}^{20}$ is unable to account for (30g).

4 Conclusion

As was said at the beginning of the chapter, the concern has been to show that there is no accounting for junctions and determiners within the Lambek calculus framework. In the preceding sections we have varied along a number of theoretical coordinates, but have found no wholly successful account. The performance characteristics of the accounts considered are given in Table 6.1. What is perhaps most surprising is the acute shortcomings of the L$^{(/,\backslash)}$-THEORY OF REFERENCE based on minimal types: $\mathcal{LT}^{20}$. Not even the *syntactic* facts are accounted for. Perhaps surprising also is the fact that the syntactic properties of junctions could only be accounted for within LCG once it was assumed that the junctions belonged to several categories, as was the case in section 3.

	$\mathcal{LT}^{20}$		$\mathcal{LT}^{21}$		$\mathcal{LT}^{23}$		$\mathcal{LT}^{24}$	
	Syn	Sem	Syn	Sem	Syn	Sem	Syn	Sem
John walks and Mary talks	+	+	+	+	+	+	+	+
John walks or talks	-	-	-	-	-	-	+	+
John loves and hates Mary	-	-	-	-	-	-	+	+
John wants and needs to go	-	-	-	-	-	-	+	+
John told or asked Mary to go	-	-	-	-	-	-	+	+
Every man walks	+	+	+	+	+	+	+	+
John loves every man	+	+	+	+	+	+	+	+
John gives every man Mary	-	-	+	+	+	+	+	+
John told every man to go	-	-	+	+	+	+	+	+
every man loves a woman	+	+	+	-	+	+	+	-
John seeks a man	+	+	+	+	+	+	+	+
every man told a woman to go	-	-	+	-	+	-	+	-
John believes a man came in	+	-	+	-	+	-	+	-

Table 6.1: The syntactic and semantic performance of $\mathcal{LT}^{20}$, $\mathcal{LT}^{21}$, $\mathcal{LT}^{23}$ and $\mathcal{LT}^{24}$

We will end by making some observations on the last of the L$^{(/,\backslash)}$-THEORIES OF REFERENCE considered, $\mathcal{LT}^{24}$. Although in $\mathcal{LT}^{24}$ there were quite a large number of separate lexical entries for the junctions and determiners, these separate lexical entries were far from being unrelated. Rather, one could say there was a 'template' for a lexical entry of which several instantiations appeared. Consider the following as the 'template' for the lexical entries for and:

(68) A categorisation and a meaning for and are

$$(x\backslash x)/x + \mathcal{I}^2\mathcal{I}^1((w,g),d_1^a,d_2^a \mapsto \mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(d_1^a,\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g),d_2^a))$$

$\mathcal{LT}^{24}$ renders this statement true for $\langle x,a\rangle \in \{\ \langle \mathrm{s},t\rangle, \langle VP^C, vp\rangle, \langle TV^C, tv\rangle, \langle VVP^C, ((s,vp),vp)\rangle, \langle TVVP^C, (se,(s,vp),vp)\rangle\ \}$.

For the determiners there were two 'templates'

(69) A categorisation and a meaning of every is

$$x/(x\backslash \mathrm{np}) \quad +$$
$$\mathcal{I}^2\mathcal{I}^1((w,g),d_1^{(et)},d_2^{(se,a)} \mapsto$$
$$\mathcal{H}^E_{\mathcal{Q}}(\mathcal{E},\mathcal{I},\mathcal{J})((d_3^e \mapsto d_2^{(se,a)}(w \mapsto d_3^e)),\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(d_1^{(et)})))$$

(70) A categorisation and a meaning of **every** is

$x\backslash(x/\mathrm{np})$ +

$\mathcal{I}^2\mathcal{I}^1((w,g), d_1^{(et)}, d_2^{(se,a)} \mapsto$

$\mathcal{H}_Q^E(\mathcal{E},\mathcal{I},\mathcal{J})((d_3^e \mapsto d_2^{(se,a)}(w \mapsto d_3^e)), \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(d_1^{(et)})))$

$\mathcal{LT}^{24}$ renders the first statement true for $\langle x, a\rangle = \langle \mathrm{s}, t\rangle$ and the second statement true for $\langle x, a\rangle \in \{ \langle VP^C, vp\rangle, \langle TV^C, tv\rangle, \langle VVP^C, ((s, vp), vp)\rangle, \langle(\mathrm{cn}\backslash\mathrm{cn}), ((s, et), et)\rangle \}$.

The existence of such 'templates' signals the possibility of a kind of theory which could at least match the success of $\mathcal{LT}^{24}$.

The theory would have the general feature that a *schema* over categories would *be* a category, and the process of *instantiating* the variables would *be* a syntactic operation. If the theory were of this general kind, then in particular one could be sure of accounting for the syntactic properties of junctions and determiners within it.

The theory would also have the general feature that a *family* of meanings indexed by types would *be* a meaning, and the *selection* of one meaning from the family would *be* a semantic operation. If the theory were of this general kind then in particular one could be sure of accounting for the basic semantic properties of junctions and determiners - that is of unambiguous sentences containing them.

No $\mathrm{L}^{(/,\backslash)}$-THEORY OF REFERENCE has these features. The framework developed in the next chapter does.

Chapter 7

Polymorphic Categorial Grammar

We have just explored in Chapter 6 the difficulties of defining a $L^{(/,\backslash)}$-THEORY OF REFERENCE that would account for the syntactic and semantic properties of determiners and junctions. Here is a formulation of the problem, assuming that one insists on the 'classical' categorisation of verbal terms that was used in $\mathcal{LT}^{18}$:

(1) *There is a junction categorisation, $C_1 \in \text{CAT}^{(/,\backslash)}$, and two quantifier categorisations, $C_2, C_3 \in \text{CAT}^{(/,\backslash)}$, such that:*

for all x, $L^{(/,\backslash)}$ derives: $x, C_1, x \Rightarrow x$
for all x, $L^{(/,\backslash)}$ derives: $x/\text{np}, C_2 \Rightarrow x$
for all x, $L^{(/,\backslash)}$ derives: $C_3, x\backslash\text{np} \Rightarrow x$

} *and with sufficient diversity of* proofs *to account for ambiguity*

Admittedly this somewhat overstates what was being required of the LCG framework, for example by insisting that there be just two quantifier categorisations. However, if (1) had been true of the LCG framework, then it would have been the case that junctions and quantifiers could be accounted for. Expressions C_1, C_2 and C_3 of the classical categorial language, $\text{CAT}^{(/,\backslash)}$, could not be be found satisfying these requirements.

We will introduce in this chapter a generalisation of the notion of $L^{(/,\backslash)}$THEORY OF REFERENCE, to be referred to as a $L^{(/,\backslash,\forall)}$-THEORY OF REFERENCE.

The DISAMBIGUATED LANGUAGE part of a $L^{(/,\backslash,\forall)}$-THEORY OF REFERENCE will be based on an extension of the set of categories, $\text{CAT}^{(/,\backslash)}$ and its associated calculus $L^{(/,\backslash)}$. To $\text{CAT}^{(/,\backslash)}$ are added *category-variables* and their *universal quantification*, giving what will be refered to as the *Polymorphic categorial language*, $\text{CAT}^{(/,\backslash,\forall)}$. To $L^{(/,\backslash)}$ will be added adaptations of the ($\forall$R) and ($\forall$L) rules from sequent calculus presentations of predicate logic, giving what will be refered to as the *Polymorphic Lambek calculus*, $L^{(/,\backslash,\forall)}$. If (1) is read with $\text{CAT}^{(/,\backslash,\forall)}$ in the place of $\text{CAT}^{(/,\backslash)}$ and $L^{(/,\backslash,\forall)}$ in place of $L^{(/,\backslash)}$, (1) will turn out true, the categories C_1, C_2 and C_3 being:

(2) $C_1 = \forall X.(X\backslash X)/X$
$C_2 = \forall X.X\backslash(X/\text{np})$
$C_3 = \forall X.X/(X\backslash\text{np})$

The interpretations that figured in a $L^{(/,\backslash)}$-THEORY OF REFERENCE were of a particular kind. First, the semantic *objects* were typed by the set of types which was the language of *implicational propositional logic*, $\text{TJ}^{\rightarrow}$. Second, the semantic *operations* were effectively indexed by possible proofs of $L^{(/,\backslash)}$: the operations were encoded by $\mathcal{L}^{\lambda}$ terms and a map defined from $L^{(/,\backslash)}$ proofs to $\mathcal{L}^{\lambda}$ terms, being effectively the composition of category-to-type map with the Curry-Howard isomorphism. These features are extended by the interpretations that will figure in a $L^{(/,\backslash,\forall)}$-THEORY OF REFERENCE. The semantic *objects* will be typed by the set of types which is the language of *quantified implicational propositional logic*, $\text{TJ}^{(\rightarrow,\forall)}$, a set of types inclusive of

$\text{TJ}^{\rightarrow}$. The semantic operations will now effectively be indexed by possible proofs of $\text{L}^{(/,\backslash,\forall)}$: the operations will be encoded by terms of *2nd order polymorphic λ-calculus*, $\mathcal{L}^{(\lambda,\Delta)}$, a langauge inclusive of $\mathcal{L}^{\lambda}$, and a map defined from $\text{L}^{(/,\backslash,\forall)}$ proofs to $\mathcal{L}^{(\lambda,\Delta)}$ terms, being the composition of a category-to-type map with an extension of the Curry-Howard isomorphism.

In section 1 the framework of Polymorphic Lambek Categorial Grammar (PLCG) will be described. Section 2 gives the analysis within the PLCG framework of junctions and determiners. Section 3 is concerned to show that this polymorphic categorial proposal captures all of the syntactic and semantic phenomena concerning junctions and quantifiers which were set forth as the aim in Chapter 3. Section 4 considers whether there are any reasons to prefer the polymorphic categorial grammar account of the junctions and quantifiers to other accounts

1 Polymorphic Lambek Categorial Grammar

We will pursue the same overall scheme of description for PLCG as was used in Chapter 4 to describe LCG. Thus, there are two sections. Section 1.1, deals with matters of categorisation. Section 1.2, deals with matters of meaning assignment.

1.1 Polymorphic Lambek Calculus

As was said above, this section deals with matters of categorisation in the PLCG framework. To this end, first the *Polymorphic categorial language* is defined. Second, the question of the string-semantics for this language will be considered. Third, we will intrduce the new sequent rules that are associated with the expanded categorial language. Their soundness with respect to the string semantics will be demonstrated.

The polymorphic categorial language: $\text{L}^{(/,\backslash,\forall)}$

The first step is to extend the categorial language $\text{CAT}^{(/,\backslash)}$ to the categorial language $\text{CAT}^{(/,\backslash,\forall)}$, a language that has *category variables* and their *universal quantification*. For the definition of $\text{CAT}^{(/,\backslash,\forall)}$, a set of basic categories, BASCAT, must be assumed as for $\text{CAT}^{(/,\backslash)}$, and additionally a set of category variables CATVAR (including, we will assume X and Y). Then $\text{CAT}^{(/,\backslash,\forall)}$ has the following definition (where x and y are metavariables over $\text{CAT}^{(/,\backslash,\forall)}$ and Z is a metavariable over CATVAR):

Definition 64 (The Polymorphic Categorial Langauge, $\text{CAT}^{(/,\backslash,\forall)}$)
(i) BASCAT $\subset \text{CAT}^{(/,\backslash,\forall)}$
(ii) if x and y are $\in \text{CAT}^{(/,\backslash,\forall)}$, then x/y and $x\backslash y$ are $\in \text{CAT}^{(/,\backslash,\forall)}$.

(iii) CATVAR $\subset$ CAT$^{(/,\backslash,\forall)}$

(iv) if $Z \in$ CATVAR *and* $x \in$ CAT$^{(/,\backslash,\forall)}$, *then* $\forall Z.x \in$ CAT$^{(/,\backslash,\forall)}$.

The string semantic interpretation of CAT$^{(/,\backslash,\forall)}$

As a langauge containing variables and their universal quantification it seems natural that the category interpretations for CAT$^{(/,\backslash,\forall)}$ should not now be *string-sets*, as they were for CAT$^{(/,\backslash)}$, but *functions* from *assignments* to string sets, with assignments assigning *string-sets* to category variables. As with other such languages one can expect that the interpretation of variable-free categories and of closed categories to be a *constant* function on assigments, whilst the interpretation of open categories will be a function returing different values for different assignments.

What shall be the set of possible assignments ? The simplest answer would be to choose the set of possible assignments to be $(2^A)^{\text{CATVAR}}$, that is *any* string-set can be assigned to a category variable. We will give a hypothetical semantics based on this notion of assignment and then try to explain why this appears to be the wrong way to define assignment.

Definition 65 (Hypothetical String Semantics for CAT$^{(/,\backslash,\forall)}$)

Assignments: $(2^A)^{\text{CATVAR}}$

Interpretation function: I applies to member of BASCAT, *and returns* constant functions *from assigments to subsets of A*

$[\![]\!]$, *the extension of I to all members of* CAT$^{(/,\backslash,\forall)}$:

$[\![x/y]\!](g) = \{a : \forall b \in [\![y]\!](g), a{\cdot}b \in [\![x]\!](g)\}$

$[\![x\backslash y]\!](g) = \{a : \forall b \in [\![y]\!](g), b{\cdot}a \in [\![x]\!](g)\}$

$[\![Z]\!](g) = g(Z)$

$[\![\forall Z.x]\!](g) = \{a : \textit{for all } B \in 2^A \ a \in [\![x]\!](g^B_Z)\}$

This simple semantics has some rather surprising consequences:

(3) for all interpretations,I, for all assignments g,

$[\![\forall X.X/X]\!](g) = \emptyset$, $[\![\forall X.(X\backslash X)/X]\!](g) = \emptyset$, $[\![\forall X.X/(X\backslash \text{np})]\!](g) = [\![\text{np}]\!](g)$

That for all interpretations,I, for all assignments g,$[\![\forall X.X/X]\!](g) = \emptyset$

One can show the following supposition to be false: there is an I, there is a g, there is an a, such that $a \in [\![\forall X.X/X]\!](g)$.

We note the following entailments:

$a \in [\![\forall X.X/X]\!](g)$

$\Rightarrow$ for all B, $a \in [\![X/X]\!](g^B_X)$

$\Rightarrow$ for all B, for all $b \in [\![X]\!](g^B_X)$, $a{\cdot}b \in [\![X]\!](g^B_X)$

$\Rightarrow$ for all B, for all $b \in B$, $a{\cdot}b \in B$

But this last condition is impossible. For example pick B to be $\{a\}$. Then $a \cdot a \notin B$. So the supposition is false. □

That for all interpretations, for all assignments g, $[\forall X.(X\backslash X)/X](g) = \emptyset$

One can show that the following supposition is false:

there is an I, there is a g, there is an a, such that $a \in [\forall X.(X\backslash X)/X](g)$.

The supposition entails: for all B, for all $b \in B$, for all $c \in B$, $c \cdot a \cdot b \in B$.

This is impossible. For example pick B to be $\{a\}$. Then $c \cdot a \cdot b \notin B$. Therefore the supposition is false. □

That for all interpretations, I, for all assignments g, $[\forall X.X/(X\backslash np)](g) = [\![np]\!](g)$

First we show that $[np](g) \subset [\forall X.X/(X\backslash np)](g)$, by supposing otherwise:
there is an n such that $n \in [np](g)$ and $n \notin [\forall X.X/(X\backslash np)](g)$.
Note $n \notin [\forall X.X/(X\backslash np)](g) \Rightarrow$ there is B and b such that
(i) $b \in [X\backslash np](g^B_X)$ and
(ii) $n \cdot b \notin [X](g^B_X)$

Now consider the implications of $n \in [np](g)$. $n \in [np](g) \Rightarrow n \in [np](g^B_X)$. Therefore from (ii),

(iii) $b \notin [X\backslash np](g^B_X)$

(iii) contradicts (i). Therefore the supposition was false.

Second we show that $[\forall X.X/(X\backslash np)](g) \subset [np](g)$, by supposing otherwise:
there is a q such that $q \in [\forall X.X/(X\backslash np)](g)$ and $q \notin [np](g)$.
Now $q \in [\forall X.X/(X\backslash np)](g) \Rightarrow$ for all B, $q \cdot [X\backslash np](g^B_X) \subseteq [X](g^B_X) \Rightarrow$
(i) for all B, $q \cdot [X\backslash np](g^B_X) \subseteq B$

Then consider the set B which is the product of $[np](g)$ with some arbitrary b, that is let B be $\{n_1 \cdot b, n_2 \cdot b, \ldots\}$, where n_1, n_2 etc are the members of $[np](g)$. For this B, $[X\backslash np](g^B_X) = \{b\}$.

Therefore, for this choice of B, (i) implies:

$\{q \cdot b\} \subseteq \{n_1 \cdot b, n_2 \cdot b, \ldots\}$

$\Rightarrow q \cdot b = n_i \cdot b$, for some $n_i \in [np](g)$

$\Rightarrow q = n_i$ for some $n_i \in [np](g)$

$\Rightarrow q \in [np](g)$ which contradicts part of the original supposition. □

Its clear that if the langauge $\text{CAT}^{(/,\backslash,\forall)}$ is interpreted as above, it will *not* reflect the uses to which we intend to put it. The second identity for example means that one could not assign and to the polymorphic categorisation $\forall X.(X\backslash X)/X$, which we are intending to do. The third identity would mean that if every man is assigned the category, $\forall X.X/(X\backslash np)$, then it ought to have a disambiguation of category np, and on the assumption that np is assigned to e, this would mean that every man has a possible meaning of type e, which is bizarre. A different typing assumption would make this less bizarre, but still the hypothetical string-semantics seems to be forcing one to make certain typing assumptions. A final comment to be made is that it is hard to see what sequent calculus rules would allow the derivation of the sequent $\forall X.X/(X\backslash np) \Rightarrow np$, which should be derivable if the calculus is to be complete with respect to the above semantics.

We shall therefore reject the hypothetical string-semantics of Definition 65. The defect of the hypothetical sematncis appears to be that the domain of quantification is too wide. Consider for example, $\forall X.(X\backslash X)/X$. On the above semantics a string a could only be a member of this of category if all sets B were closed under under the operation of *inserting* a between members of B, and clearly no string could have that property. If, however, the quantification were restricted to those B that *are the interpretation of some category*, then it seems that the string and meets the requirements: if B is a set of English strings interpreting a *category*, then it should be the case that its is closed under the operation of taking pairs and inserting and between them.

Roughly speaking, the difference between the notion of assignment used in the above hypthetical string-semantics and the notion to be adopted is that whereas the above definition allowed variables to be assigned to *any* subset of A, the actual definition will allow variables to be assigned only to those subsets of A, *which are themselves the interpretation of some category.*

Translating this rough idea into something precise is a rather delicate matter. To begin with, since the interpretation of categories should be a function from *assignments*, the restriction on assignments needs to be: the string-set B may be assigned to a category variable if there is a category x and an assignment g such that $[\![x]\!](g) = B$. But this is circular.

Therefore it seems that to define the *range* for assignments a subsidiary interpretation of categories is required *that does not involve assignments.* The subsidiary interpretation concerns only $\textsc{cat}^{(/,\backslash)}$, and it is the same as the original notion of string-semantic interpretation of $\textsc{cat}^{(/,\backslash)}$, that was seen in Chapter 4. It is based on an interpretation function, I^*, for members of BASCAT, which interprets them as members of 2^A. This is extended to a function $[\![]\!]^*$ in the familiar way. This is all defined in Definition 66. We will use $[\![(\textsc{cat}^{(/,\backslash)})]\!]^*$ to refer to that subset of 2^A that is the set of $[\![]\!]^*$ values of members of $\textsc{cat}^{(/,\backslash)}$. That is

$$[\![(\textsc{cat}^{(/,\backslash)})]\!]^* = \{B : B \in 2^A, \text{there is an } x \in \textsc{cat}^{(/,\backslash)} \text{ such that } B = [\![x]\!]^*)\}$$

It will be the set $[\![(\textsc{cat}^{(/,\backslash)})]\!]^*$ that defines the range for assignments. That is, the assignments invoked in the main interpretation of $\textsc{cat}^{(/,\backslash,\forall)}$, the interpretation that concerns all the categories of $\textsc{cat}^{(/,\backslash,\forall)}$, will be understood to assign to variables the strings-sets in $[\![(\textsc{cat}^{(/,\backslash)})]\!]^*$.

Definition 66 (String Semantics of $\textsc{cat}^{(/,\backslash,\forall)}$)

The subsidiary interpretation of $\textsc{cat}^{(/,\backslash,\forall)}$ *has*

(i) No assignments

(ii) Interpretation function: I^* applies to members of BASCAT, *and returns any member of* 2^A

(iii) $[\![]\!]^*$, the extension of I^* to cover all of $\textsc{cat}^{(/,\backslash)}$*:*

$[\![x/y]\!]^*$ *is* $\{a : \forall b \in [\![y]\!]^*, a{\cdot}b \in [\![x]\!]^*\}$, *for* $x, y \in$ CAT$^{(/,\backslash)}$

$[\![x\backslash y]\!]^*$ *is* $\{a : \forall b \in [\![y]\!]^*, b{\cdot}a \in [\![x]\!]^*\}$, *for* $x, y \in$ CAT$^{(/,\backslash)}$

The main interpretation of CAT$^{(/,\backslash,\forall)}$ *has*

(i) Assignments: $([\![(\text{CAT}^{(/,\backslash)})]\!]^*)^{\text{CATVAR}}$.

(ii) Interpretation function: I applies to members of BASCAT *and* $I(x)(g) = I^*(x)$

(iii) $[\![\,]\!]$, *the extension of I to cover all of* CAT$^{(/,\backslash,\forall)}$:

$[\![x/y]\!](g)$ *is* $\{a : \forall b \in [\![y]\!](g), a{\cdot}b \in [\![x]\!](g)\}$, *for* $x, y \in$ CAT$^{(/,\backslash,\forall)}$

$[\![x\backslash y]\!](g)$ *is* $\{a : \forall b \in [\![y]\!](g), b{\cdot}a \in [\![x]\!](g)\}$, *for* $x, y \in$ CAT$^{(/,\backslash,\forall)}$

$[\![Z]\!](g) = g(Z)$, *for Z in* CATVAR

$[\![\forall Z.x]\!](g) = \{a : \text{for all } B \in [\![(\text{CAT}^{(/,\backslash)})]\!]^* \; a \in [\![x]\!](g^B_Z)\}$, *for Z in* CATVAR, *x in* CAT$^{(/,\backslash,\forall)}$

String-semantic entailment should be redefined to take account of the assignment relativised notion of interpretation:

$x_1, \ldots, x_n$ string semantically entails y iff for all interpretations, for all assignments, $[\![x_1]\!](g)\cdot \ldots \cdot[\![x_n]\!](g) \subseteq [\![y]\!](g)$.

With this string semantics in mind, we can turn to the sequent calculus for CAT$^{(/,\backslash,\forall)}$. The Polymorphic Lambek Calculus, L$^{(/,\backslash,\forall)}$, will be understood to be the addition of the following rules to the rules of the Lambek calculus, L$^{(/,\backslash)}$.

Definition 67 ((∀L) and (∀R))

(∀L) $$\frac{U, x[y/Z], V \Rightarrow w}{U, \forall Z.x, V \Rightarrow w}\forall L$$ *[y contains no quantifiers]*

(∀R) $$\frac{T \Rightarrow x}{T \Rightarrow \forall Z.x}\forall R$$ *[Z is not free in T]*

We can show that the additional rules of L$^{(/,\backslash,\forall)}$ are *sound* with respect to the proposed semantics

That the (∀L) inference is sound

The soundness depends on the following implication, which we will first prove and then show how soundness follows from it:

(1) for any a, if $a \in [\![\forall Z.x]\!](g)$ then for any y not containing quantifiers,$a \in [\![x[y/Z]]\!](g)$

To show this suppose (i) $a \in [\![\forall Z.x]\!](g)$.

∴(ii) for all $S \in I^*(\text{CAT}^{(/,\backslash)})$, $a \in [\![x]\!](g^S_Z)$

Now for any y not containing quantifiers, $[\![y]\!](g) \in I^*(\text{CAT}^{(/,\backslash)})$. So (ii) entails:

(iii) for any y not containing quantifiers, $a \in [\![x]\!](g_Z^{[\![y]\!](g)})$

∴(iv) For any y not containing quantifiers, $a \in [\![x[y/Z]]\!](g)$

This proves (1). Now we will show how (1) guarantees the soundness of the (∀L) rule. We need to show that invalidity of the conclusion of a (∀L) inference entails the invalidity of the premise. So suppose the conclusion $U, \forall Z.x, V \Rightarrow w$ is invalid. $\therefore$.

(i) there is an I,g and an a such that $a \in [\![U]\!](g)\cdot[\![\forall Z.x]\!](g)\cdot[\![V]\!](g)$, $a \notin [\![w]\!](g)$

$\therefore$.(ii) there is an I,g, b_1, b_2, b_3 such that $b_1 \in [\![U]\!](g)$, $b_2 \in [\![\forall Z.x]\!](g)$, $b_3 \in [\![V]\!](g)$, $b_1 b_2 b_3 \notin [\![w]\!](g)$ Using (1) from above, this entails:

(iii) there is an I,g, b_1, b_2, b_3 such that $b_1 \in [\![U]\!](g)$, $b_2 \in [\![x[y/Z]]\!](g)$, $b_3 \in [\![V]\!](g)$, $b_1 b_2 b_3 \notin [\![w]\!](g)$

$\therefore$.(iv) there is an I,g, a such that $a \in [\![U]\!](g)\cdot[\![x[y/Z]]\!](g)\cdot[\![V]\!](g)$, $a \notin [\![w]\!](g)$

$\therefore$.$U, x[y/Z], V \Rightarrow w$ is invalid. Therefore we have shown that invalidity of the conclusion entails invalidity of the premise. □

That the (∀R) rule is sound

Suppose $x_1, \ldots, x_n \Rightarrow y$ is string semantically valid and X is not free in the x_i $\therefore$.

(i) for all I, for all g, $[\![x_1]\!](g)\cdot \ldots \cdot[\![x_n]\!](g) \subseteq [\![y]\!](g)$

(ii) for all I, for all g, for all a, if $a \in [\![x_1]\!](g)\cdot \ldots \cdot[\![x_n]\!](g)$ then, for all h, $h \overset{X}{\sim} g$, $a \in [\![x_1]\!](h)\cdot \ldots \cdot[\![x_n]\!](h)$

(i) and (ii) entail:

(iii) for all I, for all g, for all a, if $a \in [\![x_1]\!](g)\cdot \ldots \cdot[\![x_n]\!](g)$, then, for all h, $h \overset{X}{\sim} g$, $a \in [\![y]\!](h)$

For suppose that (iii) were not true. Then,

there is an I, there is a g, there is an a such that $a \in [\![x_1]\!](g)\cdot \ldots \cdot[\![x_n]\!](g)$ and there is an h, $h \overset{X}{\sim} g$ such that $a \notin [\![y]\!](h)$

Because of (ii) this would entail

there is an I, there is a g, there is an a such that $a \in [\![x_1]\!](g)\cdot \ldots \cdot[\![x_n]\!](g)$ and there is an h, $h \overset{X}{\sim} g$ such that $a \in [\![x_1]\!](h)\cdot \ldots \cdot[\![x_n]\!](h)$ and $a \notin [\![y]\!](h)$

$\therefore$.there is an I, there is an h, there is an a such that $a \in [\![x_1]\!](h)\cdot \ldots \cdot[\![x_n]\!](h)$ and $a \notin [\![y]\!](h)$

This contradicts (i). Hence (iii) must be true. (iii) entails

for all I, for all g, for all a, if $a \in [\![x_1]\!](g)\cdot \ldots \cdot[\![x_n]\!](g)$ then $a \in [\![\forall X y]\!](g)$

$\therefore$.$x_1, \ldots, x_n \Rightarrow \forall X y$ is string-semantically valid. Therefore we have shown the validity of the premise entials the validity of the conclusion. □

If one were to ignore the side-condition on the (∀R) rule one can easily produce proofs that lead from valid premises to invalid conclusions. For example,

$$(4) \quad \frac{X\backslash(X/np) \Rightarrow X\backslash(X/np)}{X\backslash(X/np) \Rightarrow \forall X.X\backslash(X/np)}(\forall R)$$

That (4) is an unsound inference

The premise of (4) is clearly valid. Suppose the conclusion is valid:
Whatever I, g, a, if $a \in [\![X\backslash(X/np)]\!](g)$ then $a \in [\![\forall X.X\backslash(X/np)]\!](g)$
This is equivalent to
Whatever I, g, a, whatever $h, h \overset{X}{\sim} g$, if $a \in [\![X\backslash(X/np)]\!](g)$ then $a \in [\![X\backslash(X/np)]\!](h)$
$\therefore$ Whatever I, a, whatever g such that $g(X) = [\![(s\backslash np)\backslash((s\backslash np)/np)]\!](g)$, whatever $h, h \overset{X}{\sim} g$ such that $h(X) = [\![((s\backslash np)/np)\backslash(((s\backslash np)/np)/np)]\!](g)$, if $a \in [\![X\backslash(X/np)]\!](g)$ then $a \in [\![X\backslash(X/np)]\!](h)$
$\therefore$ Whatever I, a, whatever g such that $g(X) = [\![(s\backslash np)\backslash((s\backslash np)/np)]\!](g)$, whatever $h, h \overset{X}{\sim} g$ such that $h(X) = [\![((s\backslash np)/np)\backslash(((s\backslash np)/np)/np)]\!](g)$, if $a \in [\![(s\backslash np)\backslash((s\backslash np)/np)]\!](g)$ then $a \in [\![((s\backslash np)/np)\backslash(((s\backslash np)/np)/np)]\!](g)$
$\therefore$ Whatever I, a, whatever g, if $a \in [\![(s\backslash np)\backslash((s\backslash np)/np)]\!](g)$ then $a \in [\![((s\backslash np)/np)\backslash(((s\backslash np)/np)/np)]\!](g)$

This is equivalent to the claim that $(s\backslash np)\backslash((s\backslash np)/np) \Rightarrow ((s\backslash np)/np)\backslash(((s\backslash np)/np)/np)$ is valid on the $L^{(/,\backslash)}$ notion of validity, which because of the completeness of $L^{(/,\backslash)}$, entails that $(s\backslash np)\backslash((s\backslash np)/np) \Rightarrow ((s\backslash np)/np)\backslash(((s\backslash np)/np)/np)$ is derivable in $L^{(/,\backslash)}$. But $(s\backslash np)\backslash((s\backslash np)/np) \Rightarrow ((s\backslash np)/np)\backslash(((s\backslash np)/np)/np)$ is not derivable in $L^{(/,\backslash)}$. Therefore conclusion of (4) must be invalid □

One can also see the necessity for the side condition to ($\forall$R) simply by considering some of the rather unexpected categorisations that would be possible without the side condition. For example, if the side condition were ignored anything with category s/s would also have to have the category $(s/(s\backslash np))/cn$. This is because ignoring the side condition allows one to prove the sequent $s/s \Rightarrow \forall X.(\forall X.X^{\wedge s})/X$ (see 5), whilst ($\forall$L) inferences suffice to show $\forall X.(\forall X.X^{\wedge s})/X \Rightarrow (s/(s\backslash np))/cn$

(5)
$$\dfrac{\dfrac{\dfrac{\dfrac{s/s, X, s\backslash X \Rightarrow s}{s/s, X \Rightarrow s/(s\backslash X)}/R}{s/s, X \Rightarrow \forall X.X^{\wedge s}}\forall R}{s/s \Rightarrow (\forall X.X^{\wedge s})/X}/R}{s/s \Rightarrow \forall X.(\forall X.X^{\wedge s})/X}\forall R$$

On this string semantics also we do not have the identity $[\![np]\!](g) = [\![\forall X.X/(X\backslash np)]\!](g)$. In the above proof of the identity it was essential that we able to assume the quantified variable was assigned to $[\![np]\!](g)\cdot\{b\}$ for some arbitrarily chosen b. But $[\![np]\!](g)\cdot\{b\}$ will not necessarily be in $I^*(\text{CAT}^{(/,\backslash)})$.

It is worth noting that the calculus allows one to prove $np \Rightarrow \forall X.X/(X\backslash np)$, but not to prove $\forall X.X/(X\backslash np) \Rightarrow np$. Therefore np and $\forall X.X/(X\backslash np)$ are kept distinct, and therefore it will be possible to consider the typing which assigns np to e, without having to worry about the fact every man must then have a disambiguation of category np, and therefore having a meaning of type e.

It seems a good question whether a semantics could be found that did not mean that the ($\forall$L) rule had to be restricted. In the above, the subsidiary interpretation function, $[\![]\!]^*$, concerned just the members of $\text{CAT}^{(/,\backslash)}$. To lift the restriction on the ($\forall$L) rule it would be necessary to have the subsidiary interpretation, $[\![]\!]^*$, give interpretations to ***quantified categories***. The ***closed*** categories are good candidates, and we might have for a closed category $\forall X.x$,

$a \in [\![\forall X.x]\!]^*$ iff for every B such that there is a closed y such that $[\![y]\!]^* = B$, $a \in [\![x]\!]^{*'}$, where $[\![]\!]^{*'}$ is the homomorphic extension of that $I^{*'}$ that differs from I^* by assigning B to X.

The only difficulty is that this seems to be a circular definition also: there is certainly no way to calculate $[\![\forall X.x]\!]^*$ simply by checking for all a, whether $a \in [\![\forall X.x]\!]^*$. For each case, one of the things to be checked is whether $a \in [\![x]\!]^{*'}$, where $[\![]\!]^{*'}$ is the homomorphic extension of that $I^{*'}$ that differs from I^* by assigning $[\![\forall X.x]\!]^*$ to X.

$\text{CAT}^{(/,\backslash,\forall)}$ and $\text{L}^{(/,\backslash,\forall)}$ may be used in the definition of a DISAMBIGUATED LANGUAGE in exactly the same way as $\text{CAT}^{(/,\backslash)}$ and $\text{L}^{(/,\backslash)}$ were. The syntactic operations will again be indexed by the possible proofs, they will again operate on objects which are themselves ***triples***, and amount to concatenation if only the first coordinate is considered; the disambiguation relation will once more be the first coordinate projection function.

1.2 Assigning Meaning

In this section, we are concerned with matters of meaning assignment within the PLCG framework.

First we must recall that the interpretations that are parts of the models that featured in $\text{L}^{(/,\backslash)}$-THEORIES OF REFERENCE are at two levels of specialisation from the general notion of INTERPRETATION. The general notion of INTERPRETATION is of a triple, $\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f \rangle$. The first specialisation is to the case where the carrier set, $\mathcal{B}$, of the algebra involved must be $\cup(\mathcal{M}_a)$, where the $\mathcal{M}_a$ are the MEANING sets, indexed types in $\text{TJ}^{\rightarrow}$, constructed relative to three sets, $\mathcal{E}$, $\mathcal{I}$ and $\mathcal{J}$. Such INTERPRETATIONS were refered to as FREGEAN INTERPRETATIONS. The second specialisation is that each $\mathcal{G}_\gamma$ is $H^\delta_L(P)$, where P is some $\text{L}^{(/,\backslash)}$ proof, and H^δ_L is the proof-to-operation map that was defined in Chapter 4, section 3.2.

The interpretations that will be parts of the kind of model that will feature in a $\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE are also at two levels of specialisation from the general notion of INTERPRETATION, and these specialisations are parallel to but not the same as the specialisations that are invoked by a $\text{L}^{(/,\backslash)}$-THEORY OF REFERENCE.

Firstly the notion of INTERPRETATION is specialised to the case where the objects of the carrier set, $\mathcal{B}$, are defined by MEANING sets indexed by types from the set $\text{TJ}^{(\rightarrow,\forall)}$. $\text{TJ}^{(\rightarrow,\forall)}$ is that extension of the language of implicational propositional logic, $\text{TJ}^{\rightarrow}$, that allows for ***propositional variables*** and their ***universal quantification***. Such INTERPRETATIONS will be refered to as a

POLYMORPHIC FREGEAN INTERPRETATIONS. POLYMORPHIC FREGEAN INTERPRETATIONS are defined in section 1.2.1 below.

The second specialisation is that each $\mathcal{G}_\gamma$ is $H^{\delta}_{L\forall}(P)$, where P is a proof of $\mathrm{L}^{(/,\backslash,\forall)}$ and $H^{\delta}_{L\forall}$ is a new proof-to-operation map. This proof-to-operation map will be considered in sections 1.2.2, 1.2.3, 1.2.4, 1.2.5, 1.2.6 and 1.2.7.

1.2.1 Polymorphic Fregean Algebras

To define $\mathrm{TJ}^{(\rightarrow,\forall)}$, a set of *propositional variables*, TVAR, must be assumed. It will be assumed that in TVAR are at least the two variables π, ω. θ is a metavariable over members of TVAR. a and b serve as meta-variables over any type of $\mathrm{TJ}^{(\rightarrow,\forall)}$, the definition of which follows:

Definition 68 ($\mathrm{TJ}^{(\rightarrow,\forall)}$)

a. e, t and s are $\in \mathrm{TJ}^{(\rightarrow,\forall)}$

b. if $\theta \in$ TVAR, *then* $\theta \in \mathrm{TJ}^{(\rightarrow,\forall)}$

c. if a and b are $\in \mathrm{TJ}^{(\rightarrow,\forall)}$ *then* $(a \rightarrow b)$ *is* $\in \mathrm{TJ}^{(\rightarrow,\forall)}$

d. if $\theta \in$ TVAR *and a is* $\in \mathrm{TJ}^{(\rightarrow,\forall)}$, *then* $\forall\theta.a$ *is* $\in \mathrm{TJ}^{(\rightarrow,\forall)}$

Examples of types of $\mathrm{TJ}^{(\rightarrow,\forall)}$:

(6) $(\pi \rightarrow \pi)$, $\forall\pi((e \rightarrow \pi) \rightarrow \pi)$ and $\forall\pi(\pi \rightarrow \pi)$

Now the types in $\mathrm{TJ}^{\rightarrow}$ were used to index both a set of MEANINGS and a set of DENOTATIONS, with in fact the set of DENOTATIONS being the most fundamental and the MEANINGS being simply $D_a^{\mathcal{I}\times\mathcal{J}}$. Something akin to this will hold also for the types in $\mathrm{TJ}^{(\rightarrow,\forall)}$. We will define, relative to the sets $\mathcal{E}$ and $\mathcal{I}$, POLYMORPHIC DENOTATION sets having index a, where a is a *closed formula* in $\mathrm{TJ}^{(\rightarrow,\forall)}$. These will be notated PD_a

To do so we must reemphasise a perspective on the previous DENOTATION sets, $(D_a)_{a \in \mathrm{TJ}^{\rightarrow}}$, a perspective that we first introduced in Chapter 4, section 3.2.3, namely that a DENOTATION set D_a might be regarded as the 'constructive' interpretation of the formula a. The definition of the POLYMORPHIC DENOTATION set indexed by a formula, a, of $\mathrm{TJ}^{(\rightarrow,\forall)}$ will actually depend on the definition of a 'constructive' interpretation of a. This will all be defined below, but first we will summarise the general idea. All formulae of $\mathrm{TJ}^{(\rightarrow,\forall)}$ will be given an *assignment* dependent 'constructive' interpretation. Those formulae, a, of $\mathrm{TJ}^{(\rightarrow,\forall)}$ that are *closed*, will come to have the same interpretation no matter what assignment, and this assignment independent 'constructive' interpretation of a will defined to be the POLYMORPHIC DENOTATION set indexed by a. Those formulae of a that are *open* shall not index any POLYMORPHIC DENOTATION set.

So the heart of the matter of defining the carrier sets of a POLYMORPHIC FREGEAN INTERPRETATION is the definition of the 'constructive' interpretation of expressions of $\mathrm{TJ}^{(\rightarrow,\forall)}$. There are structural similarities of this definition with the string-semantic interpretation of $\mathrm{CAT}^{(/,\backslash,\forall)}$.

Recall that in the string-semantics for $\text{CAT}^{(/,\backslash,\forall)}$ there were two interpretations I^* and I. I^* gave to members of BASCAT assignment independent interpretations, and was extended to $[\![]\!]^*$, which gave assignment independent interpretations to all members of $\text{CAT}^{(/,\backslash)}$. I gave to members of BASCAT an interpretation which was a constant functions from assignments to the I^* value, and this was extended to $[\![]\!]$, applying to all of $\text{CAT}^{(/,\backslash,\forall)}$ and giving assignment dependent interpretations. The *range* for the assignments invoked by the I was the $[\![]\!]^*$ values. In a similar way there will a two-level approach to the 'constructive' interpretation of $\text{TJ}^{(\rightarrow,\forall)}$. Recall that the definition of the DENOTATION set, D_a, was relative to a *choice* of $\mathcal{E}$ and $\mathcal{I}$. One can look at varying the choice of $\mathcal{E}$ and $\mathcal{I}$ as varying the choice of an interpretation function which operates on the atomic types of $\text{TJ}^{\rightarrow}$. The already defined DENOTATION sets, D_a, can be seen as the extension of such an interpretation function to cover all of $\text{TJ}^{\rightarrow}$. We will given the name 'Level One interpretation' to this alternative view of the previous definition of the DENOTATION sets, D_a, relative to $\mathcal{E}$ and $\mathcal{I}$. For a given Level One interpretation of $\text{TJ}^{(\rightarrow,\forall)}$, we will use $\mathcal{D}$ for the set comprising of all the interpretations of the members of $\text{TJ}^{\rightarrow}$. That is:

$$\mathcal{D} = \{D_a : a \in \text{TJ}^{\rightarrow}\}$$

The Level One interpretation concerns only the $\text{TJ}^{\rightarrow}$ part of $\text{TJ}^{(\rightarrow,\forall)}$. The Level Two interpretation concerns all of $\text{TJ}^{(\rightarrow,\forall)}$and gives assignment dependent interpretations. The *range* for these assignments will be the set of Level One interpretations, that is, $\mathcal{D}$.

Definition 69 ('Constructive' interpretation of $\text{TJ}^{(\rightarrow,\forall)}$)

Level One interpretation of $\text{TJ}^{(\rightarrow,\forall)}$ *has:*

(i) No assignments

(ii) Interpretation function: $I^*(e) = \mathcal{E}, I^*(s) = \mathcal{I}, I^*(t) = \{0,1\}$

(iii) $[\![]\!]^*$*, the extension of* I^* *to* $\text{TJ}^{\rightarrow}$*:* $[\![a]\!]^*$ *is the* DENOTATION *set,* D_a*, relative to* $\mathcal{E},\mathcal{I}$.

Level Two interpretation of $\text{TJ}^{(\rightarrow,\forall)}$ *has:*

(i) Assignments: $g \in \mathcal{D}^{\text{TVAR}}$

(ii) Interpretation function: $I(e) = g \mapsto I^*(e),\ I(t) = g \mapsto I^*(t),\ I(s) = g \mapsto I^*(s)$

(iii) $[\![]\!]$*, the extension of* I *to all of* $\text{TJ}^{(\rightarrow,\forall)}$ *as follows*

$[\![\pi]\!](g) = g(\pi)$

$[\![a \rightarrow b]\!](g) = [\![b]\!](g)^{[\![a]\!](g)}$

$[\![\forall\pi.a]\!](g)$ *is* $\{f : dom(f) = \mathcal{D}, \textit{whatever } \tau \in \mathcal{D},\ f(\tau) \in [\![a]\!](g^{\tau}_{\pi})\}$

An example of a member of $[\![\forall\pi.(\pi \rightarrow \pi)]\!](g)$, where g is arbitary, is the function f whose value at every $\tau \in \mathcal{D}$ is the identity function on τ. We have to check whether $f(\tau) \in [\![(\pi \rightarrow \pi)]\!](g^{\tau}_{\pi})$:

$f(\tau) \in [\![(\pi \rightarrow \pi)]\!](g^{\tau}_{\pi})$

iff $f(\tau) \in [\![\pi]\!](g^{\tau}_{\pi})^{[\![\pi]\!](g^{\tau}_{\pi})}$

iff $f(\tau) \in \tau^{\tau}$

Clearly if $f(\tau)$ is the identity function on τ, it meets this condition.

Definition 70 (Polymorphic Denotation set PD_a) *Relative to $\mathcal{E},\mathcal{I}$, for any* closed *formula, a, of* TJ$^{(\rightarrow,\forall)}$, $PD_a = [\![a]\!](g)$, *where* $[\![]\!]$ *is the Level Two interpretation associated with $\mathcal{E},\mathcal{I}$, and g is any* $g \in \mathcal{D}^{\mathrm{TVAR}}$

The notion of POLYMORPHIC MEANING set, PM_a, is simply $PD_a^{\mathcal{I}\times\mathcal{J}}$.

Definition 71 (Polymorphic Fregean Algebra) $\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma}\rangle$ *is a Polymorphic Fregean Algebra iff for some $\mathcal{E}$, $\mathcal{I}$, $\mathcal{B}=\cup(\mathcal{PM}_a)$, where the $\mathcal{PM}_a$ are the* POLYMORPHIC MEANING *sets indexed by closed formulae of* TJ$^{(\rightarrow,\forall)}$, *relative to $\mathcal{E}$ and $\mathcal{I}$.*

A POLYMORPHIC FREGEAN INTERPRETATION is more or less the special case of a INTERPRETATION, where the algebra involved is a POLYMORPHIC FREGEAN ALGEBRAS. Additionally, a POLYMORPHIC FREGEAN INTERPRETATION must make reference to a mapping relating phrase-set indices to *closed formulae* of TJ$^{(\rightarrow,\forall)}$. Therefore a POLYMORPHIC FREGEAN INTERPRETATION for a DISAMBIGUATED LANGUAGE, $\mathcal{L}$, is defined thus:

Definition 72 (POLYMORPHIC FREGEAN INTERPRETATION **of $\mathcal{L}$, associated with $\mathcal{E},\mathcal{I},\mathcal{J},\nu$)**

Is a triple $\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle$, *where*
(i) $\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle$ *is an* INTERPRETATION, *and* $\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma}\rangle$ *is a* POLYMORPHIC FREGEAN ALGEBRA
(ii) whenever $\delta\in\Delta$ *and* $\alpha\in\mathcal{X}_\delta, f(\alpha)\in\mathcal{PM}_{\nu(\delta)}$
(iii) whenever $\langle\mathcal{F}_\gamma,\langle\delta_\eta\rangle,\epsilon\rangle\in\mathrm{S}$ *and* $m_\eta\in\mathcal{PM}_{\nu(\delta_\eta)}$ *then* $\mathcal{G}_\gamma(\langle m_\eta\rangle)\in\mathcal{PM}_{\nu(\delta)}$

Having thus specialised of INTERPRETATIONS in respect of the nature of the carrier set, we must now specialise in respect of the nature of the operations. The specialisation is that the operations should be the image of a proof-to-operation map, $H^{\delta}_{L\forall}$. This specialisation is the concern of sections 1.2.2, 1.2.3, 1.2.4, 1.2.5, 1.2.6 and 1.2.7.

1.2.2 2nd order Polymorphic λ-calculus

As we did in the case of H^{δ}_{L}, we will encode operations by terms of a typed λ-calculus language, so that the task is then transformed into one of defining a proof-to-term map. In this section the term language will be defined and it will be explained how a term encodes an operation.

The term language to be defined is $\mathcal{L}^{(\lambda,\Delta)}$. It is more or less the language referred to in Girard et al (1989) as '2nd order Polymorphic λ-calculus', the credit for the invention of which is usually given to both Girard (1972) and Reynolds (1974).

Unlike $\mathcal{L}^{\lambda}$, $\mathcal{L}^{(\lambda,\Delta)}$ has two distinct kinds of variable, TVAR and VAR$^{\lambda}$. TVAR has already been defined and is the set of *type-variables* of TJ$^{(\rightarrow,\forall)}$. VAR$^{\lambda}$ is once again a set of typed variables, this time having types drawn from TJ$^{(\rightarrow,\forall)}$.

Definition 73 (Typed Terms of $\mathcal{L}^{(\lambda,\Delta)}$)

a. if $\Phi^a \in$ VAR$^{\lambda}$, $\Phi^a \in \mathcal{L}^{(\lambda,\Delta)}$

b. if $\Phi^a \in$ VAR$^{\lambda}$ *and* $\Psi^b \in \mathcal{L}^{(\lambda,\Delta)}$ *then* $(\lambda\Phi^a\Psi^b)^{(a\rightarrow b)} \in \mathcal{L}^{(\lambda,\Delta)}$

c. if $\Phi^{(a\rightarrow b)}$ *and* Ψ^a *are* $\in \mathcal{L}^{(\lambda,\Delta)}$, *then* $(\Phi^{(a\rightarrow b)}\Psi^a)^b \in \mathcal{L}^{(\lambda,\Delta)}$

d. if $\theta \in$ TVAR, *then* $\theta \in \mathcal{L}^{(\lambda,\Delta)}$

e. if $\theta \in$ TVAR *and* $\Phi^a \in \mathcal{L}^{(\lambda,\Delta)}$ and θ is not free in the type part of any free variable of Φ^a, *then* $(\Delta\theta.\Phi^a)^{\forall\theta.a} \in \mathcal{L}^{(\lambda,\Delta)}$

f. if $\Phi^{\forall\theta.a} \in \mathcal{L}^{(\lambda,\Delta)}$ *and* $b \in$ TJ$^{(\rightarrow,\forall)}$ and b does not contain quantifiers *then* $(\Phi^{\forall\theta.a}(b))^{a[b/\theta]} \in \mathcal{L}^{(\lambda,\Delta)}$

It should be noted that vitally θ, mentioned in clause e of the above as being abstracted over by Δ, is a variable of the language TJ$^{(\rightarrow,\forall)}$, that is a member of TVAR. θ does not refer to a member of VAR$^{\lambda}$. Or putting things another way, terms of $\mathcal{L}^{(\lambda,\Delta)}$ feature occurences of types in other than superscript positions. The kinds of terms of $\mathcal{L}^{(\lambda,\Delta)}$ that are governed by clauses e and f of the above are *abstractions over types* and *applications to types*. There are restrictions on the formation of these terms. One can abstract a *type-variable* θ out of Φ^a only if θ is not *free* in the type part of any *free variable* of Φ^a. One can apply to a type b only if that type contains no quantifiers (this restriction is not incorporated in the definition of $\mathcal{L}^{(\lambda,\Delta)}$ in Girard et al (1989)). Some explanation of these restrictions can be offered when the semantics of $\mathcal{L}^{(\lambda,\Delta)}$ is considered.

An example of a typed term of $\mathcal{L}^{(\lambda,\Delta)}$:

(7) $(\Delta\pi(\lambda x^{\pi}.x^{\pi})^{(\pi\rightarrow\pi)})^{\forall\pi(\pi\rightarrow\pi)}$

As usual, some of the *type* parts will be surpressed so the term above might be given as:

(8) $\Delta\pi.\lambda x^{\pi}.x^{\pi}$

Examples that are excluded by the side-conditions on term formation:

(9) $\Delta\pi.x^{\pi}$, $x^{\forall\pi.a}(\forall\pi.a)$, $\Phi^{\forall\pi.\pi\rightarrow\pi}(\forall\pi.\pi\rightarrow\pi)(\Phi^{\forall\pi.\pi\rightarrow\pi})$

The semantic values of terms of $\mathcal{L}^{(\lambda,\Delta)}$ will be functions whose first argument is an *assignment*, as was the case for terms of $\mathcal{L}^{\lambda}$. However, the assignments will now concern both the type-variables, TVAR, and the other variables, VAR$^{\lambda}$.

The range of assignments as far as members of TVAR is concerned is $\mathcal{D}$. The range of the assignments as far as members of VAR^λ is concerned is $\cup(PD_a)_a$ is a closed formula of $\text{TJ}^{(\rightarrow,\forall)}$. For those members of VAR^λ, x^a, where a is a *closed* formula of $\text{TJ}^{(\rightarrow,\forall)}$, we require that $g(x^a) \in PD_a$. For those members of VAR^λ, x^a, where a is an *open* formula, there can be no such requirement since PD_a is not defined for *open formulae*. However, if $g(x^a) \notin [\![a]\!](g)$, we shall call the assignment a *pseudo-assignment*. *Pseudo-assignments* are required only to 'oil the wheels' of the definition.

Definition 74 (Interpretation of Typed Terms of $\mathcal{L}^{(\lambda,\Delta)}$)
Assignments:$\{g : g = g_1 \cup g_2\ g_1 \in \mathcal{D}^{\text{TVAR}}$, *where* $a \in \text{TJ}^{\rightarrow}$
$g_2 \in (\cup PD_a)^{\text{VAR}^\lambda}$, *where* $a \in \text{TJ}^{(\rightarrow,\forall)}$ *and is closed*
$g_2(x^a) \in PD_a$, *if* a *is closed*
Interpretation: $I(e) = g \mapsto \mathcal{E}, I(t) = \{0,1\}, I(s) = g \mapsto \mathcal{I}$
$[\![]\!]$, *which extends* I *to all of* $\mathcal{L}^{(\lambda,\Delta)}$:
$[\![\pi]\!](g) = g(\pi)$
$[\![a{\rightarrow}b]\!](g) = [\![b]\!](g)^{[\![a]\!](g)}$
$[\![\forall\pi.a]\!](g)$ *is* $\{f : dom(f) = \mathcal{D}, whatever\ \tau \in \mathcal{D},\ f(\tau) \in [\![a]\!](g^\tau_\pi)\}$
$[\![\Phi^a]\!](g) = g(\Phi^a)$, *if* $\Phi^a \in \text{VAR}^\lambda$
$[\![\lambda\Phi^a.\Psi^b]\!](g)$ *is that function* $d_1^{(a,b)}$ *such that for any member* d_2^a *of* $[\![a]\!](g)$, $d_1(d-2) = [\![\Psi^b]\!](g^{d_2}_{\Phi^a})$
$[\![\Phi^{(a\rightarrow b)}(\Psi^b)]\!](g) = [\![\Phi^a]\!](g)([\![\Psi^b]\!](g))$
$[\![\Delta\pi.\Phi^a]\!](g)$ *is that function* $d^{\forall\pi.a}$ *such that for any* $\tau \in \mathcal{D}$, $d(\tau) = [\![\Phi^a]\!](g^\tau_\pi)$. *Note* g^τ_π *is probably a pseudo-assignment*
$[\![\Phi^{\forall\pi.a}(b)]\!](g) = [\![\Phi^{\forall\pi.a}]\!](g)([\![b]\!](g))$

This semantics is intended to reflect the intuition that an object of type $\forall\pi.a$ is a function from a type b to an object of type $a[b/\pi]$. As an illustration consider the interpretation of $\Delta\pi\lambda x^\pi.x^\pi$ (τ is a member of $\mathcal{D}$):

$[\![\Delta\pi\lambda x^\pi.x^\pi]\!](g) = \tau \mapsto [\![\lambda x^\pi.x^\pi]\!](g^\tau_\pi)$

g^τ_π is probably a pseudo-assignment. Continuing with the interpretative procedure, however, we have (where d is a member of τ):

$[\![\Delta\pi\lambda x^\pi.x^\pi]\!](g) = \tau \mapsto d \mapsto [\![x^\pi]\!](g^{\tau,d}_{\pi,x^\pi})$
$= \tau \mapsto d \mapsto g^{\tau,d}_{\pi,x^\pi}(x^\pi)$
$= \tau \mapsto d \mapsto d.$

So the interpretation of the non-basic expression $\Delta\pi\lambda x^\pi.x^\pi$ at an assignment depended on the interpretation of the non-basic $\lambda x^\pi.x^\pi$ at a number of *pseudo-assignments*. However, the interpretation of the non-basic expression $\lambda x^\pi.x^\pi$ at whatever *pseudo-assignment* was required, depended on the interpretation of the basic expression x^π at an *assignment* not a *pseudo-assignment*. It will be the case that for all terms that do not violate the side-condition on type abstraction terms that the interpretation of an expression at an assignments will depend on the interpretations of its consituent *basic*-expression at assignments, not *pseudo-assignments*. On the other hand, a type-abstraction term which violates the above-mentioned restriction, such as $\Delta\pi.x^{\pi}$, would have a denotation at an assignment that was determined by the denotations of its constituent basic expressions,(that is x^{π}) at *pseudo-assignments*:

$[\![\Delta\pi.x^{\pi}]\!](g) = \tau \mapsto [\![x^{\pi}]\!](g^{\tau}_{\pi})$
$= \tau \mapsto g^{\tau}_{\pi}(x^{\pi})$

For all τ but one, g^{τ}_{π} is a pseudo-assignment. Notice also that $[\![\Delta\pi.x^{\pi}]\!](g)$ is a constant function on types, and not a function from a type to an object in the type. Therefore the terms which violate the side condition on type-abstraction receive anomolous denotations.

There is also some explanation of the restriction on the formation of type application terms. An example of such a term is $(\Delta\pi.\Phi^{a})(\forall\omega.b)$. Its interpretation would invoke assignments which do not exist:

$[\![(\Delta\pi.\Phi^{a})(\forall\omega.b)]\!](g)$
$= [\![\Phi^{a}]\!](g')$, where $g'(\pi) = [\![\forall\omega.b]\!](g)$

The assignments were defined as taking *type*-variables to members of $\mathcal{D}$, but $g'(\pi)$ is not a member of $\mathcal{D}$.

As with the string semantics for $\text{L}^{(/,\backslash,\forall)}$, one can ask whether a semantics could be provided that allows the restriction on type-application to be relaxed. This would seem to require that quantified types have Level One interpretations (i.e. not involving assignments) that could then be assigned to type variables. If we continue to use the notation D_a for the Level One interpretation of a type, then what is required is the definition of $D_{\forall\pi.a}$. The intuitive definition of $D_{\forall\pi.a}$ is as a function *on type-domains*. However, now we face a circularity, for we require an answer to $D_{\forall\pi.a}(D_{\forall\pi.a})$. Therefore, lifting the restriction on type-application is non-trivial.

There could be different motivations for embarking on such a reworking of the theory. One could be mathematical elegance. Another could be the empirical unserviceability of the restricted theory. As matters stand, empirical shortcomings of the theory that are traceable to this restriction have not been found and therefore there is not motivation of the second kind for revision of the theory. If such shortcomings were found then one would have to follow in the footsteps of Girard (1986), who has provided a semantics for $\mathcal{L}^{(\lambda,\Delta)}$ which is such that type application is unrestricted.[1]

The definition of how it is that a term represents an operation will be very similar in the case of $\mathcal{L}^{(\lambda,\Delta)}$ to the definition in the case of $\mathcal{L}^{\lambda}$. Suppose Φ^{b} is a typed term, the free term variables of which are $x_1^{a_1},\ldots,x_n^{a_n}$. First we substitute for the free term variables $x_j^{a_j}$, the *terms*, $y_j^{(s\rightarrow a_j)}(i)$,

[1]This semantics is rather far removed what is is the usual interpretation of types and functions in natural language semantics.

where the $y_j^{(s\rightarrow a_j)}$ are term variables. Then prefix λi to the front and call the result Φ^*.

Definition 75 (Operation defined by Φ^*) *Where h is an arbitary $\mathcal{L}^{(\lambda,\Delta)}$ assignment, the operation defined by Φ^* is:*

$m_1, \ldots, m_n \mapsto (w,g) \mapsto [\![\Phi^*]\!](h[w' \mapsto m_1(w',g)/y_1, \ldots, w' \mapsto m_n(w',g)/y_n])(w)$

Some examples of operations defined by terms:

(10) $\Phi = \Delta\pi\lambda x_1^{(e\rightarrow\pi)}[x_1(x_2^e)]$

$\Phi^* = \lambda i\Delta\pi\lambda x_1^{(e\rightarrow\pi)}[x_1(y_2^{(s\rightarrow e)}(i))]$

Operation: $m_1^e \mapsto (w,g) \mapsto \tau \mapsto d \mapsto d(m_1(w,g))$ (where $\tau \in \mathcal{D}$, $d \in \tau^{D_e}$)

(11) $\Phi = x_2^{\forall\pi((e\rightarrow\pi)\rightarrow\pi)}(t)(x_1^{(e\rightarrow t)})$

$\Phi^* = \lambda i[y_2^{(s\rightarrow\forall\pi((e\rightarrow\pi)\rightarrow\pi))}(i)(t)(y_1^{(s\rightarrow(e\rightarrow t))}(i))$

Operation: $m_1^{(e\rightarrow t)} \mapsto m_2^{\forall\pi((e\rightarrow\pi)\rightarrow\pi)} \mapsto (w,g) \mapsto m_2(w,g)(D_t)(m_1(w,g))$

Now we may turn to the definition of the proof-to-operation map, $H^\delta_{L\forall}$. In Chapter 4, we gave two definitions of the proof-to-operation map H^δ_L. The first was a 'practical definition' and invoked the notion of a term-associated calculus. The second was by decomposing H^δ_L into three steps, the first from L$^{(/,\backslash)}$ proofs to LJ$^{\rightarrow}$ proofs, the second from LJ$^{\rightarrow}$ proofs to NJ$^{\rightarrow}$ proofs and the third from NJ$^{\rightarrow}$ proofs to $\mathcal{L}^\lambda$ terms. For $H^\delta_{L\forall}$ there will again be two definitions. This time the decompositional definition will be the first considered.

1.2.3 Decomposition of the Proof-to-Term maps

The maps $H^e_{L\forall}$ and $H^i_{L\forall}$ may both be defined as the composition of three other maps, as illustrated in Figure 7.1 and defined in Definition 76.

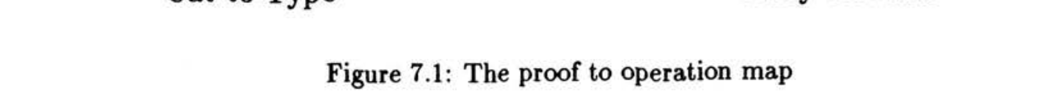


Figure 7.1: The proof to operation map

Definition 76 (The proof-to-term map)
$H^\delta_{L\forall} = \ulcorner\ \urcorner \circ * \circ \overline{\nu^\delta}$, *where*

1. ν is a map from L$^{(/,\backslash,\forall)}$ *proofs of sequents over* CAT$^{(/,\backslash,\forall)}$ *to proofs of sequents over* TJ$^{(\rightarrow,\forall)}$. *The calculus in which the proofs of* TJ$^{(\rightarrow,\forall)}$ *sequents are constructed will be refered to as* LJ$^{(\rightarrow,\forall)}$.

*2. * is a map from* $\text{LJ}^{(\rightarrow,\forall)}$ *proofs to* Natural Deduction *proofs (from assumptions) of* $\text{TJ}^{(\rightarrow,\forall)}$ *propositions. The Natural Deduction proof system will be refered to as* $\text{NJ}^{(\rightarrow,\forall)}$.
3. $\ulcorner\ \urcorner$ *is a map from* $\text{NJ}^{(\rightarrow,\forall)}$ *proofs to* terms of $\mathcal{L}^{(\lambda,\Delta)}$

The first destination is $\text{LJ}^{(\rightarrow,\forall)}$, a sequent calculus for sequents over $\text{TJ}^{(\rightarrow,\forall)}$. This is the calculus that results from the addition to $\text{LJ}^{\rightarrow}$ of Gentzen's sequent calculus rules for $\forall$. To turn a proof of $\text{L}^{(/,\backslash,\forall)}$ into a proof of $\text{LJ}^{(\rightarrow,\forall)}$ turns out to require nothing essentially more than the application of an extensional or intensional category-to-type map, mapping category-variables onto type-variables and mapping $\text{CAT}^{(/,\backslash,\forall)}$'s $\forall$ to $\text{TJ}^{(\rightarrow,\forall)}$'s $\forall$. The second leap is due to the fact that $\text{LJ}^{(\rightarrow,\forall)}$ can be viewed as a meta-language for a Natural Deduction system, this time $\text{NJ}^{(\rightarrow,\forall)}$. The third leap is the *extended* Curry-Howard isomorphism: an extension discovered by Girard linking the terms of $\mathcal{L}^{(\lambda,\Delta)}$ and the proofs of $\text{NJ}^{(\rightarrow,\forall)}$.

These stepping stones are described in more detail below, in sections 1.2.4, 1.2.5, and 1.2.6, progessing from right to left relative to Figure 7.1. Therefore we start with the Natural Deduction system $\text{NJ}^{(\rightarrow,\forall)}$ and the Extended Curry-Howard isomorphism.

1.2.4 The Natural Deduction system $\text{NJ}^{(\rightarrow,\forall)}$ and the extended Curry-Howard isomorphism

Table 7.1 should be looked at in conjunction with Table 4.1, Chapter 4, which defined the natural deduction system $\text{NJ}^{\rightarrow}$. Table 7.1 defines an introduction and an elimination rule for $\forall$. The tables together define $\text{NJ}^{(\rightarrow,\forall)}$.

For the system $\text{NJ}^{\rightarrow}$ there was a notion of ***normalisation*** that took one from a less direct proof to a more direct proof, keyed on the occurence in the proof of ($\rightarrow$I) followed by ($\rightarrow$E). There is an additional case of normalisation for the more inclusive system $\text{NJ}^{(\rightarrow,\forall)}$, keyed this time on proofs in which ($\forall$I) is followed by ($\forall$E). This is defined in the third row of Table 7.1.

The definitions given in Chapter 4, section 3.2.3 of how a Natural Deduction proofs defines an entailment relation again apply to the more inclusive $\text{NJ}^{(\rightarrow,\forall)}$ system. There follow some example proofs of $\text{NJ}^{(\rightarrow,\forall)}$:

(12) a.
$$\dfrac{\dfrac{[\pi]_1}{(\pi\rightarrow\pi)}(\rightarrow I_1)}{\forall\pi.(\pi\rightarrow\pi)}(\forall I)$$

b.
$$\dfrac{\dfrac{\dfrac{[(e\rightarrow\pi)]_1 \quad e}{\pi}(\rightarrow E)}{((e\rightarrow\pi)\rightarrow\pi)}(\rightarrow I_1)}{\forall\pi.((e\rightarrow\pi)\rightarrow\pi)}(\forall I)$$

$\dfrac{\begin{array}{c}U\\ \vdots\\ a\end{array}}{\forall\pi.a}(\forall\text{I})$	if you have a proof of a from assumptions U, you make a proof of $\forall\pi.a$. *π should not be free in any* undischarged *assumptions*
$\dfrac{\begin{array}{c}U\\ \vdots\\ \forall\pi.a\end{array}}{a[b/\pi]}(\forall\text{E})$	if you have a proof of $\forall\pi.a$, you can make a proof of $a[b/\pi]$ with assumptions U. *b should contain no quantifiers*
$\dfrac{\dfrac{\begin{array}{c}U\\ \vdots\\ a\end{array}}{\forall\pi.a}(\forall\text{I})}{a[b/\pi]}(\forall\text{E}) \quad \rhd \quad \begin{array}{c}U[b/\pi]\\ \vdots\\ a[b/\pi]\end{array}$	Intro followed by Elim is a detour

Table 7.1: Introduction and Elimination rules for $\forall$

(13) c.

$$\dfrac{e\to t \qquad \dfrac{\forall\pi.((e\to\pi)\to\pi)}{(e\to t)\to t}(\forall\text{E})}{t}(\to\text{E})$$

An example of proof normalisation:

(14)
$$\dfrac{\dfrac{\dfrac{[\pi]_1}{(\pi\to\pi)}(\to\text{I}_1)}{\forall\pi.(\pi\to\pi)}(\forall\text{I})}{e\to e}(\forall\text{E}) \quad \rhd \quad \dfrac{[e]_1}{(e\to e)}\to\text{I}_1$$

The extended Curry-Howard isomorphism is now defined. Additional to the clauses that defined $\ulcorner\ \lrcorner$ before are the following:

Definition 77 (Extended Curry-Howard isomorphism: $\ulcorner\ \lrcorner$)

($\forall$ I)-Step*: if Φ^a is the term associated with a proof that is input to a $\forall$-Introduction step, the term associated with the resulting proof is* $(\Delta\pi.\Phi^a)^{\forall\pi.a}$.

($\forall$ E)-Step*: If $\Phi^{\forall\pi.a}$ is the term associated with a proof that is input to a $\forall$-Elimination step, the term associated with the resulting proof is* $(\Phi^{\forall\pi.a}(b))^{a[b/\pi]}$.

On this basis the values under $\ulcorner\ \lrcorner$ of the proofs given in (12a), (12b) and (13) are given below:

(15)
$$\left\ulcorner \dfrac{\dfrac{[\pi]_1}{(\pi\to\pi)}(\to\text{I}_1)}{\forall\pi.(\pi\to\pi)}(\forall\text{I}) \right\lrcorner =$$

$(\Delta\pi.(\lambda x^{\pi}.x^{\pi})^{(\pi\to\pi)})^{\forall\pi.(\pi\to\pi)}$ which abbreviates to $\Delta\pi.\lambda x^{\pi}.x^{\pi}$.

$$(16)\ \left\ulcorner \dfrac{\dfrac{\dfrac{[(e{\rightarrow}\pi)]_1 \quad e}{\pi}(\rightarrow\mathrm{E})}{((e{\rightarrow}\pi){\rightarrow}\pi)}(\rightarrow\mathrm{I}_1)}{\forall\pi.((e{\rightarrow}\pi){\rightarrow}\pi)}(\forall\mathrm{I}) \right\lrcorner =$$

$$(\Delta\pi.(\lambda x^{(e\rightarrow\pi)}.(x^{(e\rightarrow\pi)}y^{e})^{\pi})^{((e\rightarrow\pi)\rightarrow\pi)})^{\forall\pi.((e\rightarrow\pi)\rightarrow\pi)}$$

which abbreviates to $\Delta\pi.\lambda x^{(e\rightarrow\pi)}.x(y^{e})$.

$$(17)\ \left\ulcorner \dfrac{e{\rightarrow}t \qquad \dfrac{\forall\pi.((e{\rightarrow}\pi){\rightarrow}\pi)}{(e{\rightarrow}t){\rightarrow}t}(\forall\mathrm{E})}{t}(\rightarrow\mathrm{E}) \right\lrcorner =$$

$((x^{\forall\pi.((e\rightarrow\pi)\rightarrow\pi)}(t))^{((e\rightarrow t)\rightarrow t)}(y^{(e\rightarrow t)}))^{t}$ which abbreviates to $x^{\forall\pi.((e\rightarrow\pi)\rightarrow\pi)}(t)(y^{(e\rightarrow t)})$

In figure 7.2 the step-wise process of calculating the value of $\ulcorner\ \lrcorner$ on the proof in 12a is gone through.

$$\left\ulcorner \dfrac{\dfrac{[\pi]\mathrm{i}}{\pi{\rightarrow}\pi}}{\forall\pi.(\pi{\rightarrow}\pi)} \right\lrcorner = \left(\Delta\pi.\left\ulcorner \dfrac{[\pi]\mathrm{i}}{\pi{\rightarrow}\pi} \right\lrcorner\right)^{\forall\pi.(\pi\rightarrow\pi)}$$

$$= \left(\Delta\pi\left(\lambda x^{\pi}\ulcorner [\pi]\mathrm{i} \lrcorner\right)^{\pi\rightarrow\pi}\right)^{\forall\pi.(\pi\rightarrow\pi)}$$

$$= \left(\Delta\pi\left(\lambda x^{\pi}.x^{\pi}\right)^{\pi\rightarrow\pi}\right)^{\forall\pi.(\pi\rightarrow\pi)}$$

Figure 7.2: Example of the extended Curry-Howard isomorphism

'Classical' and 'Constructive' semantics

As was the case with $\mathrm{NJ}^{\rightarrow}$, there are two kinds of semantic approach to $\mathrm{NJ}^{(\rightarrow,\forall)}$. There is a 'classical' semantics according to which the all members of $\mathrm{TJ}^{(\rightarrow,\forall)}$ are interpreted as sets (relative to an assignment). The purpose of this semantics is to define entailment and consider soundness and completeness. The 'constructive' semantics gives to the members of $\mathrm{TJ}^{(\rightarrow,\forall)}$ the functional interpretations that were encountered when the semantics of $\mathcal{L}^{(\lambda,\Delta)}$ was being considered above, and gives to the *proofs* of $\mathrm{NJ}^{(\rightarrow,\forall)}$ the same interpretations as is possessed by the corresponding term of $\mathcal{L}^{(\lambda,\Delta)}$.

One can try to justify the side-conditions that were imposed on the (∀I) and (∀E), from the point of view of either semantics. The 'constructive' semantics justification has already been considered.

On the 'classical' semantics one can explain the side-condition on (∀I) though not the side-condition on (∀E). The side-condition to (∀I) guarantees its soundness on the 'classical' semantics. The reasoning that this is so is almost exactly the same as that which showed the soundness of the (∀I) inference in $\text{L}^{(/,\backslash,\forall)}$, givens its side-condition.

That the (∀ I) rules is sound under the Classical Semantics

Suppose one has an $\text{NJ}^{(\rightarrow,\forall)}$proof, D, of a, from assumptions U, such that π does not occur free in any of the WFFs in U, and suupose that the entailment that this defines is valid.

Because of the validity of the entailment we may suppose:

(i) for all I, forall g, for all p if $p \in [\![U]\!](g)$ then $p \in [\![a]\!](g)$

Because π does not occur free in any of the WFFs in U:

(ii) for all I, forall g, if $p \in [\![U]\!](g)$ then for all h, $h \overset{\pi}{\sim} g$, $p \in [\![U]\!](h)$

from (i) and (ii) it follows that:

(iii) for all I, forall g, for all p if $p \in [\![U]\!](g)$ then for all h, $h \overset{\pi}{\sim} g$, $p \in [\![a]\!](h)$

For suppose (iii) was not true. Then,

there is an I, there is a g, there is a p such that $p \in [\![U]\!](g)$ and there is an h, $h \overset{\pi}{\sim} g$ such that $p \notin [\![a]\!](h)$

Because of (ii) this would entail:

there is an I, there is a g, there is a p such that $p \in [\![U]\!](g)$ and there is an h, $h \overset{\pi}{\sim} g$ such that $p \in [\![U]\!](h)$ and $p \notin [\![a]\!](h)$

$\therefore$.there is an I, there is an h, there is a p such that $p \in [\![U]\!](h)$ and $p \notin [\![a]\!](h)$

This contradicts (i). Hence (iii) must be true. (iii) entails:

for all I, forall g, for all p if $p \in [\![U]\!](g)$ then $p \in [\![\forall\pi.a]\!](g)$

Therefore if D is expanded by a (∀I) inference into D', the entialment that D' defines will be valid. □

It is easy to construct what are unsound inferences on the 'classical' semantics if the side-condition is ignored, for example:

(18) $$\frac{\pi}{\forall\pi.\pi}(\forall I)$$

It is not clear to me what the relationship between the 'classical' and 'contructive' explanations of the side-condition.

1.2.5 The sequent calculus, $\text{LJ}^{(\rightarrow,\forall)}$, and the map * to $\text{NJ}^{(\rightarrow,\forall)}$

The sequent calculus for the language $\text{TJ}^{(\rightarrow,\forall)}$ will be refered to as $\text{LJ}^{(\rightarrow,\forall)}$ and it consists of the rules of $\text{LJ}^{\rightarrow}$ together with the following ($\forall$R) and ($\forall$L) rules.

Definition 78 (Sequent Calculus Rules for $\forall$)

($\forall$L) $$\frac{U, a[b/\pi], V \Rightarrow w}{U, \forall\pi.a, V \Rightarrow w}\,\forall L$$ *[b is some chosen type, b is quantifier free]*

($\forall$R) $$\frac{U \Rightarrow a}{U \Rightarrow \forall\pi.a}\,\forall R$$ *[π is not free in U]*

Examples of proofs in $\text{LJ}^{(\rightarrow,\forall)}$:

(19) a. $$\dfrac{\dfrac{\pi \Rightarrow \pi}{\Rightarrow (\pi\rightarrow\pi)}\rightarrow\text{R}}{\Rightarrow \forall\pi.(\pi\rightarrow\pi)}\forall\text{R}$$

b. $$\dfrac{\dfrac{\dfrac{e \Rightarrow e \quad \pi \Rightarrow \pi}{(e\rightarrow\pi), e \Rightarrow \pi}\rightarrow\text{L}}{e \Rightarrow ((e\rightarrow\pi)\rightarrow\pi)}\rightarrow\text{R}}{e \Rightarrow \forall\pi((e\rightarrow\pi)\rightarrow\pi)}\forall\text{R}$$

c. $$\dfrac{\dfrac{(e\rightarrow t) \Rightarrow (e\rightarrow t) \quad t \Rightarrow t}{(e\rightarrow t), ((e\rightarrow t)\rightarrow t) \Rightarrow t}\rightarrow\text{L}}{(e\rightarrow t), \forall\pi((e\rightarrow\pi)\rightarrow\pi) \Rightarrow t}\forall\text{L}$$

As was the case with the sequent calculus $\text{LJ}^{\rightarrow}$, the sequents of $\text{LJ}^{(\rightarrow,\forall)}$ can be interpreted 'proof-theoretically' in terms of proofs of $\text{NJ}^{(\rightarrow,\forall)}$: an $\text{LJ}^{(\rightarrow,\forall)}$ sequent $U \Rightarrow a$ is 'true' if and only there is an $\text{NJ}^{(\rightarrow,\forall)}$ proof that has U as its undischarged assumptions and a as its conclusion. The rules ($\forall$R) and ($\forall$L) are sound on this interpretation, that facts making them so being indicated below. These facts effectively define the map $*$ from $\text{LJ}^{(\rightarrow,\forall)}$ proofs to proofs of $\text{NJ}^{(\rightarrow,\forall)}$:

(20) If $\begin{array}{c} U \\ \vdots \\ a \end{array}$ is a proof such that π does not occur free in U then $\begin{array}{c} U \\ \vdots \\ \dfrac{a}{\forall\pi.a}\forall\text{I} \end{array}$ is a proof

(21) If $\begin{array}{c} U, a[b/\pi] \\ \vdots \\ c \end{array}$ is a proof then $\begin{array}{cc} & \dfrac{\forall\pi.a}{a[b/\pi]}\forall\text{E} \\ U & \\ & \vdots \\ & c \end{array}$ is a proof.

Note that in (21), the premise of a (∀ L) inference describes the *bottom* part of a $\text{NJ}^{(\rightarrow,\forall)}$ proof (attention was drawn to something similar is the case of the relationship of →L in $\text{LJ}^{\rightarrow}$ to →E in $\text{NJ}^{\rightarrow}$). This should be made clearer by studying Figure 7.3, Figure 7.4 and Figure 7.5, which illustrate the operation of $*$ on proofs of $\text{LJ}^{(\rightarrow,\forall)}$.

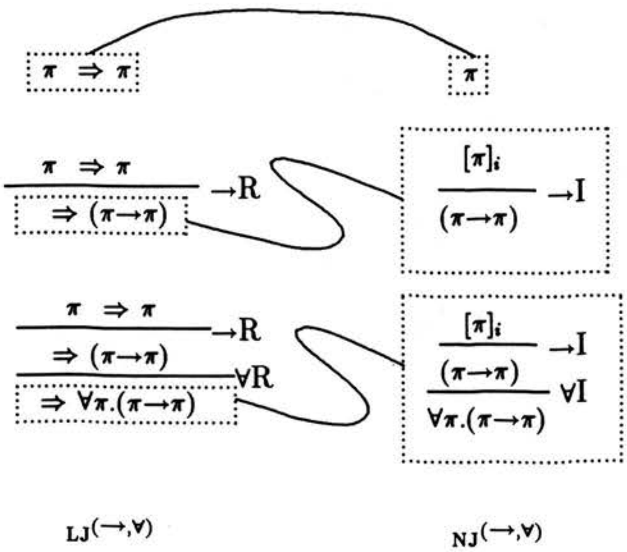


Figure 7.3: Example of $*$: from $\text{LJ}^{(\rightarrow,\forall)}$ to $\text{NJ}^{(\rightarrow,\forall)}$ proof of $\Rightarrow \forall\pi.(\pi\rightarrow\pi)$

1.2.6 From $\text{L}^{(/,\backslash,\forall)}$ to $\text{LJ}^{(\rightarrow,\forall)}$

Possible maps from Polymorphic Lambek calculus proofs to $\text{LJ}^{(\rightarrow,\forall)}$ are based on possible category to type maps. There were two classes of maps considered in the case of the Lambek calculus: the extensional, according to which $\nu^e(x/y) = (\nu^e(y)\rightarrow\nu^e(x))$ and the intensional, according to which $\nu^i(x/y) = ((s\rightarrow\nu^i(y))\rightarrow\nu^i(x))$. The same distinction will continue to be applied. Additionally we have to consider the value of the category-to-map when applied to those categories in $\text{CAT}^{(/,\backslash,\forall)}$ that are not in $\text{CAT}^{(/,\backslash)}$. There is no difference between ν^e and ν^i as far as this is concerned, and so the following uses ν^δ as indifferent between the two:

Whatever category-variable , Z, whatever category, x,

$\nu^\delta(Z) = \theta$, for some type-variable θ

$\nu^\delta(\forall Z.x) = \forall\nu^\delta(Z).\nu^\delta(x)$

On this basis, the maps, $\overline{\nu^e}$ and $\overline{\nu^i}$, introduced in Chapter 4 leading from $\text{L}^{(/,\backslash)}$ proofs to $\text{LJ}^{\rightarrow}$ proofs, may be extended to maps leading from $\text{L}^{(/,\backslash,\forall)}$ proofs to $\text{LJ}^{(\rightarrow,\forall)}$ proofs. Table 7.2 in

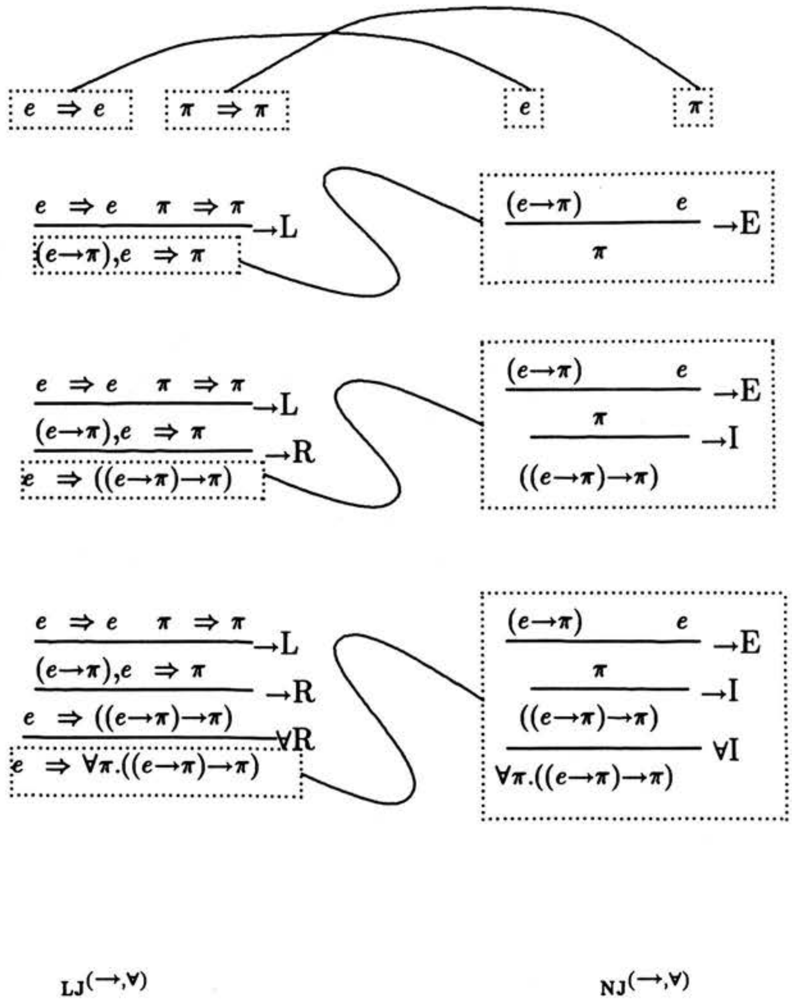


Figure 7.4: Example of $*$: from $LJ^{(\rightarrow,\forall)}$ to $NJ^{(\rightarrow,\forall)}$proof of $e \Rightarrow \forall\pi.((e\rightarrow\pi)\rightarrow\pi)$

conjunction with Table 4.2 defines $\overline{\nu^e}$ whilst Table 7.3 in conjunction with Table 4.3 defines $\overline{\nu^i}$.

1.2.7 Efficient Meaning Assignment: term associated calculus

In the case of the map from $L^{(/,\backslash)}$ proofs to typed terms of $\mathcal{L}^{\lambda}$, one could shortcut the intermediate stages between $L^{(/,\backslash)}$ and $\mathcal{L}^{\lambda}$ by defining a *term-associated* calculus. This applies also in the case of the map from $L^{(/,\backslash,\forall)}$ proofs to $\mathcal{L}^{(\lambda,\Delta)}$ terms. The term-associated versions of $L^{(/,\backslash,\forall)}$ build upon the *extensionally* and the *intensionally* term associated versions of $L^{(/,\backslash)}$. If one adds to the extensionally (resp. intensionally) term associated version of $L^{(/,\backslash)}$, the following *term-associated* versions of the (∀R) and (∀L), the result be refered to as the extensionally (resp. intensionally) term associated version of $L^{(/,\backslash,\forall)}$.

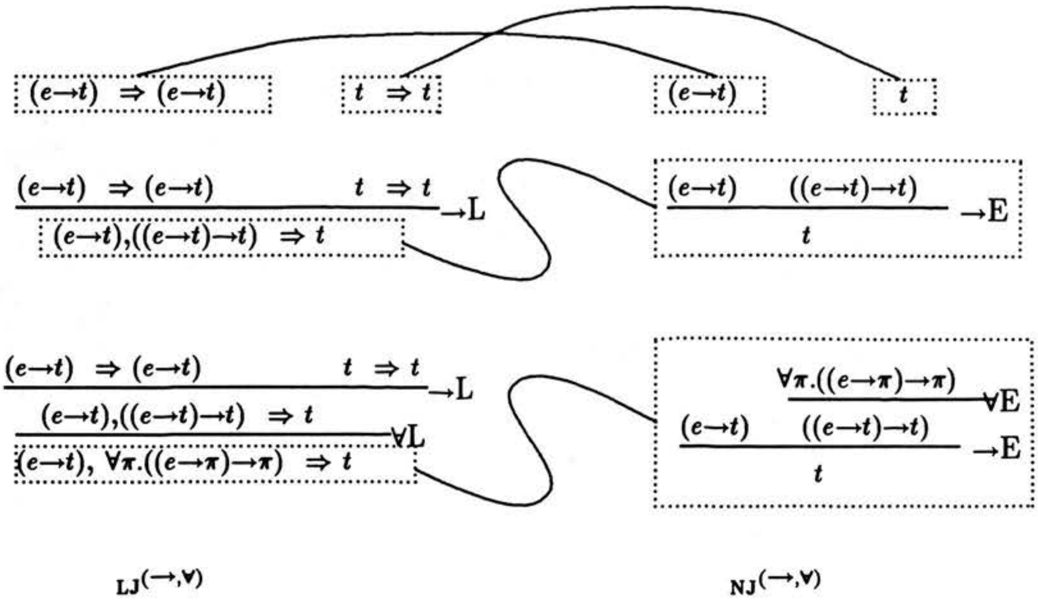


Figure 7.5: Example of $*$: from $\text{LJ}^{(\rightarrow,\forall)}$ to $\text{NJ}^{(\rightarrow,\forall)}$proof of $(e\rightarrow t), \forall pi.((e\rightarrow pi)\rightarrow\pi) \Rightarrow t$

$\text{L}^{(/,\backslash,\forall)}$ proof	image in $\text{LJ}^{(\rightarrow,\forall)}$ under $\overline{\nu^e}$
$\dfrac{P}{U \Rightarrow \forall Z.x}\forall R$	$\dfrac{\overline{\nu^e}(P)}{\nu^e(U) \Rightarrow \nu^e(\forall Z.x)}\forall R$
$\dfrac{P}{U, \forall Z.x, V \Rightarrow w}\forall L$	$\dfrac{\overline{\nu^e}(P)}{\nu^e(U), \nu^e(\forall Z.x), \nu^e(V) \Rightarrow \nu^e(w)}\forall L$

Table 7.2: $\overline{\nu^e}$: $\text{L}^{(/,\backslash,\forall)} \mapsto \text{LJ}^{(\rightarrow,\forall)}$

Definition 79 (Term Associated Polymorphic Lambek Calculus)

(∀L) $$\frac{U, x[y/Z] : \alpha(a), V \Rightarrow w}{U, \forall Z.x : \alpha, V \Rightarrow w}\forall L$$ *[where y is quantifier free and $\nu(y) = a$]*

(∀R) $$\frac{T \Rightarrow x : \alpha}{T \Rightarrow \forall Z.x : \Delta\theta.\alpha}\forall R$$ *[Z is not free in T, $\nu(Z) = \theta$]*

2 The junctions and determiners in PLCG

The previous section has defined the PLCG *framework*, that is to say, identified the class of $\text{L}^{(/,\backslash,\forall)}$-THEORIES OF REFERENCE. What we will do now is propose a particular $\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE. Section 2.1 gives the categorisation, the typing assumptions and most of the meaning postulates. The meaning postulates for junctions and determiners are considered in

$\text{L}^{(/,\backslash,\forall)}$ proof	image in $\text{LJ}^{(\rightarrow,\forall)}$ under $\overline{\nu^i}$
$\dfrac{P}{U \Rightarrow \forall Z.x}\forall\text{R}$	$\dfrac{\overline{\nu^i}(P)}{s,\nu^i(U) \Rightarrow \nu^i(\forall Z.x)}\forall\text{R}$
$\dfrac{P}{U,\forall Z.x, V \Rightarrow w}\forall\text{L}$	$\dfrac{\overline{\nu^i}(P)}{s,\nu^i(U),\nu^i(\forall Z.x),\nu^i(V) \Rightarrow \nu^i(w)}\forall\text{L}$

Table 7.3: $\overline{\nu^i}$: $\text{L}^{(/,\backslash,\forall)} \mapsto \text{LJ}^{(\rightarrow,\forall)}$

Section 2.2.

2.1 A $\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE for junctions and determiners

$\mathcal{L}^{19}$: a DISAMBIGUATED POLYMORPHIC LAMBEK LANGAUGE for verbal terms, proper names, embedding verbs, junctions and determiners

1. The phrase-set indices: the set $\text{CAT}^{(/,\backslash,\forall)}$, given as BASCAT, $\{\text{s}, \text{np}, \text{cn}, \text{vpc}, \text{sc}\}$, and CATVAR= $\{\text{X}, \text{Y}\}$
2. The basic phrase sets: whatever strings α, whatever categories $\delta \in \text{CAT}^{(/,\backslash,\forall)}$, $\langle\alpha, \langle\rangle, \delta\rangle \in \mathcal{X}_\delta$ iff α appears in the δ row of the tables below:

np	john, mary, he_1, he_2, ...	(s\np)/sc	believes
s\np	walks	sc/s	that
(s\np)/np	loves, is	(s\np)/vpc	wants
((s\np)/np)/np	gives	((s\np)/vpc)/np	told
		vpc/(s\np)	to

cn	man, woman
∀X.X/(X\np)/cn	every, a, no, most
∀X.X\(X/np)/cn	every, a, no, most
∀X.((X\X)/X)	and, or

The class of intensional possible models $\mathcal{K}^{25}$ for $\mathcal{L}^{19}$ $\langle\langle\mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^{25}$ iff

1. $\nu^i(\text{np}) = e$, $\nu^i(\text{s}) = t$, $\nu^i(\text{vpc}) = \nu^i(\text{s}\backslash\text{np})$, $\nu^i(\text{sc}) = \nu^i(\text{s}), \nu^i(\text{X}) = \pi, \nu^i(\text{Y}) = \omega, \nu^i(x/y) = \nu^i(x\backslash y) = ((s \rightarrow \nu^i(y)) \rightarrow \nu^i(x)), \nu^i(\forall Z.x) = \forall \nu^i(Z).\nu^i(x)$

2. Constraints on f^{25}: for the constraints on f with respect to is,that and to and certain verbal terms, see Definition 59, 1, Chapter 6. f is also subject to the Junction and Determiner meaning postulates defined below in Definition 80.
3. Algebraic constraints: all algebras contain the set of operations:
   $\{H^i_{L\forall}(\gamma) : \gamma \text{ is a proof of } \mathrm{L}^{(/,\backslash,\forall)}\}$

By $\mathcal{LT}^{25}$ we shall refer to the $\mathrm{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE which is the combination of the above DISAMBIGUATED LANGUAGE and class of MODELS.

As indicated above, we have yet to give a meaning postulate governing the values of the interpretation function when applied to the disambiguations of junctions and determiners. This is the concern of the next section.

2.2 Defining the polymorphic junctions and determiners

We defined in Chapter 5, certain 'algebra-spanning' functions associated with junctions and determiners, functions which for any choice of $\mathcal{E}, \mathcal{I}, \mathcal{J}$ return particular meanings of the types in (22), what one might call the 'basic' types:

(22) $(t \rightarrow (t \rightarrow t))$
$((e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t))$

When $\mathrm{L}^{(/,\backslash)}$-THEORIES OF REFERENCE were considered which did not assign junctions and determiners to these types, it was the policy to give meaning postulates for junctions and determiners that fixed a required relationship between the meaning assigned to the junction or determiner and the corresponding algebra spanning function. The same policy is to be pursued here.

Under the category-to-type map of $\mathcal{LT}^{25}$, the meanings assigned to junctions and determiners will be of meanings of types:

(23) $\forall \pi.((s \rightarrow \pi) \rightarrow ((s \rightarrow \pi) \rightarrow \pi))$.
$((s \rightarrow (e \rightarrow t)) \rightarrow \forall \pi.((s \rightarrow ((s \rightarrow e) \rightarrow \pi)) \rightarrow \pi))$

Therefore we wish to specify a relationship between meanings of the types in (22) and (23). Two strategies for defining this relation suggest themselves: (1) *by terms of* $\mathcal{L}^{(\lambda,\Delta)}$ or (2) *by recursion*, both explored below.

2.2.1 The impossibility of a $\mathcal{L}^{(\lambda,\Delta)}$ definition of the polymorphic junctions and determiners

We have already introduced the idea of defining a semantic operation by a terms of a formal language. Given the fact that many of the meaning postulates that we have defined so far involve the application of a semantic operation to a priveleged object of certain type, it will come as no surprise that meaning postulates could be specified with the assistance of terms of a formal language. Consider for example the meaning postulate governing the interpretation of is, in $\mathcal{LT}^{18}$ from definition 59:

Whatever model, $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^{18}$,
$f(\langle \mathsf{is}, \langle\rangle, (\mathrm{s}\backslash\mathrm{np})/\mathrm{np}\rangle) = \mathcal{I}^1\mathcal{I}^2\mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})$.

The very same constraint on the interpretation function can be specified in the following way:

Whatever model, $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^{18}$, *whatever* $\mathcal{L}^\lambda$ *assignment,* k,
$f(\langle \mathsf{is}, \langle\rangle, (\mathrm{s}\backslash\mathrm{np})/\mathrm{np}\rangle) = (w,g) \mapsto [\![\lambda i^s \lambda y^{(s,e)} \lambda z^{(s,e)} [x^{(e,e,t)}(yi)(zi)]]\!](k^{\mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)}_{x^{(e,e,t)}})(w)$

The crucial element is the $\mathcal{L}^\lambda$ term $\lambda i^s \lambda y^{(s,e)} \lambda z^{(s,e)} [x^{(e,e,t)}(yi)(zi)]$. This term has type $(s,(s,e),(s,e),t)$, which apart from the initial s, is the type assigned to is according to $\mathcal{K}^{18}$. The term also contains a single free variable, $x^{(e,e,t)}$, and the type of this is the 'basic' copula type (that is, the type of $\mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})$.

This is an instance of a general pattern for giving meaning postulates which relate the 'basic' meaning, in type a, of an expresssion to a 'derived' meaning in type b: one simply looks for a term of type b, whose single free variable is of type a.

On the same pattern, one could give a meaning postulate governing the polymorphic interpretation of junctions if there were some term, Φ_{JUNCT} of $\mathcal{L}^{(\lambda,\Delta)}$, of type $(s, \forall\pi.((s,\pi),((s,\pi),\pi)))$, having one free variable $x^{(t,(t,t))}$. A postulate for and could then be given: *Whatever model,* $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^{25}$, *whatever* $\mathcal{L}^{(\lambda,\Delta)}$ *assignment,* k
$f(\langle \mathsf{and}, \langle\rangle, \forall \mathrm{X}.(\mathrm{X}\backslash\mathrm{X})/\mathrm{X}\rangle) = (w,g) \mapsto [\![\Phi^{(s,\forall\pi.((s,\pi),((s,\pi),\pi)))}_{JUNCT}]\!](k^{\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)}_{x^{(t,(t,t))}})(w)$

Similarly, one could give a meaning postulate governing the polymorphic interpretation of determiners if there were some term Φ_{DET} of $\mathcal{L}^{(\lambda,\Delta)}$, of type $((s,et), \forall\pi.((s,se,\pi),\pi))$, whose sole free variable was $x^{((e\rightarrow t)\rightarrow((e\rightarrow t)\rightarrow t))}$. The postulate for every could then be given:

Whatever model, $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^{25}$, *where* k *is an arbitrary* $\mathcal{L}^{(\lambda,\Delta)}$ *assignment,*
$f(\langle \mathsf{every}, \langle\rangle, \forall \mathrm{X}.\mathrm{X}/(\mathrm{X}\backslash\mathrm{np})/\mathrm{cn}\rangle) = (w,g) \mapsto [\![\Phi^{(s,((s,et),\forall\pi.((s,se,\pi),\pi)))}_{DET}]\!](k^{\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)}_{x^{((et),((e,t),t))}})(w)$

To search for the terms $\Phi^{(s,\forall\pi.((s,\pi),((s,\pi),\pi)))}_{JUNCT}$ and $\Phi^{(s,((s,et),\forall\pi.((s,se,\pi),\pi)))}_{DET}$, one can exploit the Extended Curry-Howard isomorphism: there would be such terms if and only if :
(i) there is in $\mathrm{NJ}^{(\rightarrow,\forall)}$, a proof of $(s,\forall\pi.((s,\pi),((s,\pi),\pi)))$ having as its only undischarged assumption $(t,(t,t))$
(ii) there is in $\mathrm{NJ}^{(\rightarrow,\forall)}$, a proof of $(s,((s,e,t),\forall\pi.((s,(s,e),\pi),\pi)))$ having as its only undischarged assumption $((e\rightarrow t)\rightarrow((e\rightarrow t)\rightarrow t))$.

We wish to show that (i) and (ii) are false. Now to answer these two questions about $\mathrm{NJ}^{(\rightarrow,\forall)}$ one can reason either syntactically or semantically. That is, one can simply *search* for whether there are the proofs, or one can check whether the entailments defined by such proofs are valid:

because NJ$^{(\rightarrow,\forall)}$ is ***sound*** with respect to the 'classical' semantics it will not be possible to derive any entailments that are not valid on that classical semantics.

Considering (i), both $(s, \forall\pi.((s,\pi),((s,\pi),\pi)))$ and $(t,(t,t))$ are classical tautologies, which can be established either semantically or by observing the following two NJ$^{(\rightarrow,\forall)}$ proofs that have ***no*** undischarged assumptions:

(24) a.
$$\dfrac{\dfrac{[t]_1}{(t\rightarrow t)}\rightarrow I_1}{(t\rightarrow(t\rightarrow t))}\rightarrow I$$

b.
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[s]_2 \quad [(s\rightarrow\pi)]_1}{\pi}\rightarrow E}{((s\rightarrow\pi)\rightarrow\pi)}\rightarrow I_1}{((s\rightarrow\pi)\rightarrow((s\rightarrow\pi)\rightarrow\pi))}\rightarrow I}{\forall\pi((s\rightarrow\pi)\rightarrow((s\rightarrow\pi)\rightarrow\pi))}\forall I}{(s\rightarrow\forall\pi((s\rightarrow\pi)\rightarrow((s\rightarrow\pi)\rightarrow\pi)))}\rightarrow I_2$$

From the tautological status of $(t,(t,t))$ and $(s,\forall\pi.((s,\pi),((s,\pi),\pi)))$, and the soundness of NJ$^{(\rightarrow,\forall)}$ one can see that the entailment $(t,(t,t)) \Rightarrow (s,\forall\pi.((s,\pi),((s,\pi),\pi)))$ must be semantically valid. Therefore one cannot refute (i) semantically. (i) is still false, however. Here it is important to recall the definition of how it is that an Natural Deduction ***proof*** defines an entailment. See section 3.2.3, Chapter 4. For NJ$^{(\rightarrow,\forall)}$ to define the entailment $(t,(t,t)) \Rightarrow (s,\forall\pi.((s,\pi),((s,\pi),\pi)))$, it is sufficient that there be a proof from ***no assumptions*** of $(s,\forall\pi.((s,\pi),((s,\pi),\pi)))$, and such a proof was given above.

The same point must be borne in mind if we try to settle (i) by settling the related question about LJ$^{(\rightarrow,\forall)}$: is there an LJ$^{(\rightarrow,\forall)}$ proof $(t,(t,t)) \Rightarrow (s,\forall\pi.((s,\pi),((s,\pi),\pi)))$? Such an LJ$^{(\rightarrow,\forall)}$ proof will only imply that (i) is true, if $(t,(t,t))$ is ***not*** introduced into the antecedent by a Weakening inference. If $(t,(t,t))$ is introduced into the antecedent by a Weakening inference, then the LJ$^{(\rightarrow,\forall)}$ proof corresponds not to the NJ$^{(\rightarrow,\forall)}$ proof mentioned in (i) but instead to (24b). And indeed the only LJ$^{(\rightarrow,\forall)}$ proofs of $(t,(t,t)) \Rightarrow (s,\forall\pi.((s,\pi),((s,\pi),\pi)))$ that I can find, consist of a proof of $\Rightarrow (s,\forall\pi.((s,\pi),((s,\pi),\pi)))$ with an added step introducing $(t,(t,t))$ into the antecedent by a Weakening inference, such as:

(25)
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{s\rightarrow s \quad \pi\rightarrow\pi}{s,(s\rightarrow\pi)\Rightarrow\pi}\rightarrow L}{(s\rightarrow\pi),(s\rightarrow\pi),s\Rightarrow\pi}\text{Weak and Perm}}{s\Rightarrow((s\rightarrow\pi)\rightarrow((s\rightarrow\pi)\rightarrow\pi))}\rightarrow R\text{ and }\rightarrow R}{s\Rightarrow\forall\pi((s\rightarrow\pi)\rightarrow((s\rightarrow\pi)\rightarrow\pi))}\forall R}{\Rightarrow(s\rightarrow\forall\pi((s\rightarrow\pi)\rightarrow((s\rightarrow\pi)\rightarrow\pi)))}\rightarrow R}{(t\rightarrow(t\rightarrow t))\ \Rightarrow(s\rightarrow\forall\pi((s\rightarrow\pi)\rightarrow((s\rightarrow\pi)\rightarrow\pi)))}\text{Weak}$$

Now to consider (ii). In this case, matters are more straightforward because semantic methods may be used: one can show that the entailment defined by the supposed $\text{NJ}^{(\rightarrow,\forall)}$ proof is not valid

That $(et, et, t) \Rightarrow (s, (set), \forall\pi((s, se, \pi), \pi))$ *is not valid*

Suppose:

1. there is an interpretation I, an assignment g, and a p such that: $p \in [\![t]\!](g)$, $p \in [\![s]\!](g)$, $p \notin [\![e]\!](g)$

One can show that it follws from this supposition that $p \in [\![(et, et, t)]\!](g)$ and $p \notin [\![(s, (set), \forall\pi((s, se, \pi), \pi))]\!](g)$, and therefore that $(et, et, t) \Rightarrow (s, (set), \forall\pi((s, se, \pi), \pi))$ is not valid.

First we find a *sufficient* condition for $p \in [\![(et, et, t)]\!](g)$: $p \in [\![(et, et, t)]\!](g)$
if $p \notin [\![(et)]\!](g)$ or $p \in [\![(et, t)]\!](g)$
if $p \notin [\![(et)]\!](g)$ or $p \in [\![t]\!](g)$ (discarding first disjunct)
if $p \in [\![t]\!](g)$ (discarding first disjunct)

By assumption 1, the above is true, therefore $p \in [\![(et, et, t)]\!](g)$

Second we find a *sufficient* condition for $p \notin [\![(s, (set), \forall\pi((s, se, \pi), \pi))]\!](g)$:
$p \notin [\![(s, (set), \forall\pi((s, se, \pi), \pi))]\!](g)$
if $p \in [\![s]\!](g)$ and $p \in [\![(set)]\!](g)$ and $p \notin [\![\forall\pi((s, se, \pi), \pi)]\!](g)$
if $p \in [\![s]\!](g)$ and ($p \notin [\![s]\!](g)$ or $p \in [\![(et)]\!](g)$) and $p \notin [\![\forall\pi((s, se, \pi), \pi)]\!](g)$
if $p \in [\![s]\!](g)$ and ($p \in [\![(et)]\!](g)$) and $p \notin [\![\forall\pi((s, se, \pi), \pi)]\!](g)$
if $p \in [\![s]\!](g)$ and ($p \notin [\![e]\!](g)$ or $p \in [\![t]\!](g)$) and $p \notin [\![\forall\pi((s, se, \pi), \pi)]\!](g)$
if $p \in [\![s]\!](g)$ and ($p \in [\![t]\!](g)$) and $p \notin [\![\forall\pi((s, se, \pi), \pi)]\!](g)$
if $p \in [\![s]\!](g)$ and ($p \in [\![t]\!](g)$) and there is a B such that $p \notin [\![((s, se, \pi), \pi)]\!](g^B_\pi)$
if $p \in [\![s]\!](g)$ and ($p \in [\![t]\!](g)$) and there is a B such that ($p \in [\![(s, se, \pi)]\!](g^B_\pi)$ and $p \notin [\![\pi]\!](g^B_\pi)$)
if $p \in [\![s]\!](g)$ and ($p \in [\![t]\!](g)$) and there is a B such that (($p \notin [\![s]\!](g^B_\pi)$ or $p \in [\![(se, \pi)]\!](g^B_\pi)$) and $p \notin B$)
if $p \in [\![s]\!](g)$ and ($p \in [\![t]\!](g)$) and there is a B such that (($p \notin [\![(se)]\!](g^B_\pi)$ or $p \in [\![\pi]\!](g^B_\pi)$) and $p \notin B$)
if $p \in [\![s]\!](g)$ and ($p \in [\![t]\!](g)$) and there is a B such that (($p \in [\![s]\!](g^B_\pi)$ and $p \notin [\![e]\!](g^B_\pi)$) and $p \notin B$)

By taking B to be $[\![e]\!](g)$ one can see that supposition 1 entails that the above condition is true and therefore entails that $p \notin [\![(s, (set), \forall\pi((s, se, \pi), \pi))]\!](g)$ □

Therefore one cannot specify the meaning postulates for the polymorphic interpretation of junctions and determiners using terms of $\mathcal{L}^{(\lambda,\Delta)}$. In short one cannot define the polymorphic junctions and determiners in $\mathcal{L}^{(\lambda,\Delta)}$.

2.2.2 The possibility of a *recursive* definition of the polymorphic junctions and determiners

The domain of the function denoted by determiner or a junction according to the above $\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE is the set $\mathcal{D}$. Recall that the members of $\mathcal{D}$ are DENOTATION sets, indexed by types in $\text{TJ}^{\rightarrow}$. We will maintain that it is only with respect to those DENOTATION sets that are indexed by *conjoinable types* that the function need be constrained by a meaning postulate. At DENOTATIONS sets indexed by types other than these the determiner and junction functions are unconstrained.

Within the subset of $\mathcal{D}$ consisting of the DENOTATION sets that are indexed by *conjoinable types*, the polymorphic junction and determiner functions will be given a *recursive* definition, the value at a certain DENOTATION set indexed by conjoinable type a being defined in terms of the value at a DENOTATION set indexed by a *simpler* conjoinable type b. Grounding the recursion will be the value of the polymorphic function at the DENOTATION set indexed by the *simplest* conjoinable type, t, at which DENOTATION set the polymorphic determiners and junctions return the orthodox determiner and junction meanings.

This is embodied in the junction and determiner postulate below. $\mathcal{J}$ stands for one of the 'algebra-spanning' junction functions that specifies a 'basic' meaning given $\mathcal{E}, \mathcal{I}, \mathcal{J}$. $\mathcal{J}_\forall$ stands for the polymorphic meaning assigned to a corresponding lexical junction disambiguation in an arbitary model allowed by $\mathcal{LT}^{25}$. $\mathcal{Q}$ stands for any one of the 'algebra-spannning' determiner functions that specifies the 'basic' meaning given $\mathcal{E}, \mathcal{I}, \mathcal{J}$. $\mathcal{Q}_\forall$ stands for the polymorphic meaning assigned to a corresponding lexical determiner disambiguation in an arbitrary model allowed by $\mathcal{LT}^{25}$.

Definition 80 (Junction and Determiner Postulate)
For all models $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma \in \Gamma}, f\rangle, \langle w, g\rangle\rangle \in \mathcal{K}^{25}$,
whatever conjoinable types *type* a, b *defined relative to* $\mathcal{E}$, $\mathcal{I}$ *and* $\mathcal{J}$,

for any $P_1^{(s,a)}, P_2^{(s,a)}$,
if $a = t$, $\mathcal{J}_\forall(w,g)(D_a)(P_1)(P_2) = \mathcal{J}(w,g)(P_1(w))(P_2(w))$
if $a = (b,c)$, $\mathcal{J}_\forall(w,g)(D_a)(P_1)(P_2) = x^b \mapsto \mathcal{J}_\forall(w,g)(D_c)(w,g)(w' \mapsto P_1 w' x)(w' \mapsto P_2 w' x)$

for any $S^{(s,e,t)}, P^{(s,se,a)}$,
if $a = t$, $\mathcal{Q}_\forall(w,g)(S)(D_a)(P) = \mathcal{Q}(w,g)(S(w))(x^e \mapsto P(w)(w' \mapsto x))$
if $a = (b,c)$, $\mathcal{Q}_\forall(w,g)(S)(D_a)(P) = x^b \mapsto \mathcal{Q}_\forall(w,g)(S)(D_c)(w' \mapsto y^{s,e} \mapsto Pw'yx)$

This definition should be compared with the Definitions 53 and 56, from Chapter 5, sections 2.4.2 and 2.5.2, which were the recursive definitions of the cross-categorial junction operation, $\mathcal{H}_{\mathcal{J}}$, and the cross-categorial quantification operation, $\mathcal{H}_Q$. The similarity of these with Definition 80 is what lies behind the comment that was made in Chapter 5: that the polymorphic junction and quantifier denotations are the lexicalisations of the cross-categorial junction and quantifier operations. The link will perhaps be clearer with a couple of examples. To simplify, imagine for the moment that the above postulates had been made under an extensional rather than an intensional typing. Then the corresponding versions of the postulates would make it the case that $[\![\overline{\textsf{and}}]\!]$ could only be a possible meaning of **and** if:

$$[\![\overline{\textsf{and}}]\!](w,g)(D_a)(P_1)(P_2) = \mathcal{H}^E_{\mathcal{J}}(\mathcal{E},\mathcal{I},\mathcal{J})(P_1, \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g), P_2)$$

Similarly, $[\![\overline{\textsf{every man}}]\!]$ could only be a possible meaning of **every man** if:

$$[\![\overline{\textsf{every man}}]\!](w,g)(D_a)(P) = \mathcal{H}^E_Q(\mathcal{E},\mathcal{I},\mathcal{J})(\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\textsf{man}}]\!](w,g)), P)$$

Before proceeding to the empirical assessment of $\mathcal{LT}^{25}$, I will just note that in previous versions of the ideas reported here (Emms 89, Emms 90), I took it as an absolute necessity to express the meaning postulates for the polymorphic junctions and determiners via some kind of λ-calculus terms, in the fashion of section 2.2.1. To enable this I defined a further extension of $\mathcal{L}^{(\lambda,\Delta)}$, allowing *lists* of arguments to be terms, and the terms defining the polymorphic junctions and determiners featured abstractions over lists types. The resulting proposal has a greater 'pencil and paper' calculability than the current proposal, but at the cost of a lessening in simplicity of the relationship between the polymorphism in the syntax and that in the semantics.

3 The semantic assessment of the polymorphic proposal

In this section we will assess the semantic performance of $\mathcal{LT}^{25}$. Section 3.1 begins this with simple cases, unambiguous sentences with junctions and determiners. Then ambiguities are considered. Section 3.2 considers a number of determiner containing sentences that are ambiguous. Section 3.3 does the same for a number of ambiguous junction-containing sentences. Finally section 3.4 consider some ambiguities associated with embedding constructions.

3.1 Unambiguous sentences with Junctions and Determiners

The first kind of test to which $\mathcal{LT}^{25}$ will be put is that is should account for the instances of Hypotheses 4 and Hypothesis 3 when applied to unambiguous sentences containing a junction or a determiner. A sample of junction cases was given in (59a,b,c,d), Chapter 6 and of the determiner cases in (30a,b,c,d), Chapter 6.

$\mathcal{LT}^{25}$ accounts for (59a,b,c,d) and below there is some indication of how one might go about showing this in the cases of (59a) and (59d).

3.1.1 John walks and Mary talks

The required entailment of $\mathcal{LT}^{25}$ to account for (59a) is:

(26) There is a $\overline{\text{John walks and Mary talks}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{John walks}}$, whatever $\overline{\text{Mary talks}}$, $[\![\overline{\text{John walks and Mary talks}}]\!](w,g) = 1$ iff $AND([\![\overline{\text{John talks}}]\!](w,g) = 1, [\![\overline{\text{Mary talks}}]\!](w,g) = 1)$.

The disambiguation in (27) has the above described property.

(27)
$$\dfrac{\dfrac{\dfrac{\text{John}}{np}\quad\dfrac{\text{walks}}{s\backslash np}}{s}\quad \dfrac{\dfrac{\text{and}}{\forall X((X\backslash X)/X)}}{(s\backslash s)/s}a \quad \dfrac{\dfrac{\text{Mary}}{np}\quad \dfrac{\text{talks}}{s\backslash np}}{s}}{s}$$

PROOFa.
$$\dfrac{(s\backslash s)/s \Rightarrow (s\backslash s)/s}{\forall X((X\backslash X)/X) \Rightarrow (s\backslash s)/s}\forall L$$

Recall that all of the horizontal lines in the above should be understood to be anotated with certain proofs. Where this is not done, the simplest possible proof of the sequent linking what is above the line with what is below is understood to go in that place. The only line annotated in the above is labeled with 'a', and this is understood to refer to 'PROOFa', which is given alongside. This indeed also is the simplest possible proof of the sequent linking what is above the line with what is below. However, it is included to indicate the workings of the ($\forall$L) rule.

The chain of equivalences in (28) suffices to show that the disambiguation in (27) has the desired property, where $\overline{\text{John walks}}$ and $\overline{\text{Mary talks}}$ refer to the single possible flat disambiguations: we are assuming that there is no semantic diversity among the different possible disambiguations of John walks and Mary talks.

(28) (i) $[\![(27)]\!](w,g) = 1$

iff (ii) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{talks}}]\!](w,g)(w' \mapsto [\![\overline{\text{Mary}}]\!](w',g)))$ $= 1$

$([\![\overline{\text{walks}}]\!](w,g)(w' \mapsto [\![\overline{\text{John}}]\!](w',g)))$

iff (iii) $AND([\![\overline{\text{John walks}}]\!](w,g) = 1, [\![\overline{\text{Mary talks}}]\!](w,g) = 1)$

The equivalence of (28ii) and (28iii) is immediate oncee the denotations of the disambiguations of John walks and Mary talks are calculated, and account taken of the correspondence between $\mathcal{AND}$ and AND. For the equivalence of (28i) and (28ii) the denotation of (27) must be calculated, and this is shown in (29). There are two significant facts determining this denotation. First the operation associated with PROOFa by $H^i_{L\forall}$. This is $m^{\forall\pi((s,\pi),(s,\pi),\pi)} \mapsto (w,g) \mapsto m(w,g)(D_t)$, which is to say that the polymorphic junction denotation is applied to the t type. Second, there is the force of the junction postulate for the polymorphic junction function applied to the t type.

(29) $[\![(27)]\!](w,g)$

$$\begin{aligned}&= [\![\overline{\text{and}}]\!](w,g)(D_t)(w' \mapsto [\![\overline{\text{talks}}]\!](w',g)(w'' \mapsto [\![\overline{\text{Mary}}]\!](w'',g)))\\ &\qquad (w' \mapsto [\![\overline{\text{walks}}]\!](w',g)(w'' \mapsto [\![\overline{\text{John}}]\!](w'',g)))\\ &= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{talks}}]\!](w,g)(w' \mapsto [\![\overline{\text{Mary}}]\!](w',g)))\\ &\qquad ([\![\overline{\text{walks}}]\!](w,g)(w' \mapsto [\![\overline{\text{John}}]\!](w',g)))\end{aligned}$$

Given (29) the equivalence of (28i) and (28ii) is immediate.

3.1.2 John needs and wants to go

To account for (59d), the required entailment of $\mathcal{LT}^{25}$ is:

(30) There is a $\overline{\text{John needs and wants to go}}$ such that whatever model, $\langle \Im, \langle w,g\rangle\rangle$, whatever $\overline{\text{John needs to go}}$, whatever $\overline{\text{John wants to go}}$, $[\![\overline{\text{John wants and needs to go}}]\!](w,g) = 1$ iff $AND([\![\overline{\text{John wants to go}}]\!](w,g) = 1, [\![\overline{\text{John needs to go}}]\!](w,g) = 1)$.

The disambiguation in (31) has the required property.

(31)
$$\dfrac{\dfrac{\text{John}}{np}\quad \dfrac{\dfrac{\text{wants}}{(s\backslash np)/vpc}\quad \dfrac{\dfrac{\text{and}}{\forall X((X\backslash X)/X)}\,d}{C_4}\quad \dfrac{\text{needs}}{(s\backslash np)/vpc}}{(s\backslash np)/vpc}\quad \dfrac{\text{to}}{vpc/(s\backslash np)}\quad \dfrac{\text{go}}{s\backslash np}}{s}$$

$C_4 = (VVP\backslash VVP)/VVP$

This time none of proofs associated with the horizontal lines have been shown. d is put in as marker so that we may refer in a moment to the proof associated with this step. To save on space also, the inferred categorisation of **and** is given just as C_4, with alongside the information that $C_4 = (VVP\backslash VVP)/VVP$. Here we are using the same category abbreviations as were introduced in section 3 of Chapter 6, only without any longer the C superscript. They will be in heavy use throughout the rest of this chapter. Also, as we did in the above, we will often use the space immediate alongside the depiction of a disambiguation to 'decode' abbreviatory devices.

To show that the disambiguation in (31) has the required property, the chain of equivalences (32) suffices, where $\overline{\text{John wants to go}}$ and $\overline{\text{John needs to go}}$ refer to possible flat disambiguations and it is assumed that there is no semantic diversity among the different possible disambiguations :

(32) (i)$[\![31]\!](w,g) = 1$

iff (ii) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{needs}}]\!](w,g)(w' \mapsto [\![\overline{\text{to}}]\!](w',g)(w'' \mapsto [\![\overline{\text{go}}]\!](w'',g)))(w' \mapsto [\![\overline{\text{John}}]\!](w',g)))([\![\overline{\text{wants}}]\!](w,g)(w' \mapsto [\![\overline{\text{to}}]\!](w',g)(w'' \mapsto [\![\overline{\text{go}}]\!](w'',g)))(w' \mapsto [\![\overline{\text{John}}]\!](w',g))) = 1$

iff (iii) $AND([\![\overline{\text{John wants to go}}]\!](w,g) = 1, [\![\overline{\text{John needs to go}}]\!](w,g) = 1)$

The equivalence of (ii) and (iii) is immediate once the denotations of the disambiguations of **John wants to go** and **John needs to go** are calculated. For the equivalence of (i) and (ii) the denotation of (31) must be calculated and this is shown in (33). The operation associated with PROOFd by $H^i_{L\forall}$ is $m^{\forall\pi((s,\pi),(s,\pi),\pi)} \mapsto (w,g) \mapsto m(w,g)(D_{((s,vp),vp)})$, which is to say that the polymorphic junction denotation is applied to the $((s,vp),vp)$ type. (33) uses the these abbreviations: $\mathsf{TG} = (w' \mapsto [\![\overline{\mathsf{to}}]\!](w',g)(w'' \mapsto [\![\overline{\mathsf{go}}]\!](w'',g)))$, $\mathsf{j} = (w' \mapsto [\![\overline{\mathsf{John}}]\!](w',g))$ $\mathsf{N} = (w' \mapsto [\![\overline{\mathsf{needs}}]\!](w',g))$, $\mathsf{W} = (w' \mapsto [\![\overline{\mathsf{wants}}]\!](w',g))$.

(33) $[\![(31)]\!](w,g)$

$$\begin{aligned}
&= [\![\overline{\mathsf{and}}]\!](w,g)\ (D_{((s,vp),(se,t))})\ (w' \mapsto [\![\overline{\mathsf{needs}}]\!](w',g))\ (w' \mapsto [\![\overline{\mathsf{wants}}]\!](w',g))\ (w' \mapsto [\![\overline{\mathsf{to}}]\!](w',g)\\
&\quad (w'' \mapsto [\![\overline{\mathsf{go}}]\!](w'',g)))\ (w' \mapsto [\![\overline{\mathsf{John}}]\!](w',g))\\
&= [\![\overline{\mathsf{and}}]\!](w,g)(D_{((s,vp),(se,t))})(\mathsf{N})(\mathsf{W})(\mathsf{TG})^{(s,vp)}(\mathsf{j})\\
&= (x^{(s,vp)} \mapsto [\![\overline{\mathsf{and}}]\!](w,g)(D_{(se,t)})(w' \mapsto \mathsf{N}w'x)(w' \mapsto \mathsf{W}w'x))(\mathsf{TG})^{(s,vp)}(\mathsf{j})\\
&= [\![\overline{\mathsf{and}}]\!](w,g)(D_{(se,t)})(w' \mapsto \mathsf{N}w'(\mathsf{TG}))(w' \mapsto \mathsf{W}w'(\mathsf{TG}))(\mathsf{j})^{se}\\
&= (x^{(s,e)} \mapsto [\![\overline{\mathsf{and}}]\!](w,g)(D_t)(w' \mapsto \mathsf{N}w'(\mathsf{TG})x)(w' \mapsto \mathsf{W}w'(\mathsf{TG})x))(\mathsf{j})^{se}\\
&= [\![\overline{\mathsf{and}}]\!](w,g)(D_t)(w' \mapsto \mathsf{N}w'(\mathsf{TG})(\mathsf{j}))(w' \mapsto \mathsf{W}w'(\mathsf{TG})(\mathsf{j}))\\
&= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{N}w(\mathsf{TG})(\mathsf{j}))(\mathsf{W}w(\mathsf{TG})(\mathsf{j}))\\
&= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\mathsf{needs}}]\!](w,g)(w' \mapsto [\![\overline{\mathsf{to}}]\!](w',g)(w'' \mapsto [\![\overline{\mathsf{go}}]\!](w'',g)))(w' \mapsto [\![\overline{\mathsf{John}}]\!](w',g)))\\
&\quad ([\![\overline{\mathsf{wants}}]\!](w,g)(w' \mapsto [\![\overline{\mathsf{to}}]\!](w',g)(w'' \mapsto [\![\overline{\mathsf{go}}]\!](w'',g)))(w' \mapsto [\![\overline{\mathsf{John}}]\!](w',g)))
\end{aligned}$$

Givne (33) the equivalence of (i) and (ii) is immediate.

$\mathcal{LT}^{25}$ also accounts for the unambiguous determiner cases, (30a,b,c,d), and below there is some indication of how this might be shown in the cases of (30a) and (30d)

3.1.3 every man walks

The required entailment of $\mathcal{LT}^{25}$ to account for (30) is

(34) There is a $\overline{\text{every man walks}}$ such that whatever model, $\langle \Im, \langle w,g\rangle\rangle$, whatever $\overline{\text{he}_1 \text{ is a man}}$, whatever $\overline{\text{he}_1 \text{ walks}}$, $[\![\overline{\text{every man walks}}]\!](w,g) = 1 \leftrightarrow EVERY(\{x : [\![\overline{\text{he}_1 \text{ is a man}}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\overline{\text{he}_1 \text{ walks}}]\!](w, g^x_{he_1}) = 1\})$

The disambiguation in (35) has the property.

(35)

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{every}}{\forall X.X/(X\backslash np)/cn} \quad \dfrac{\text{man}}{cn}}{\forall X.X/(X\backslash np)}}{s/(s\backslash np)}\,a \quad \dfrac{\text{cried}}{s\backslash np}}{s}$$

The chain of equivalences in (36) suffices to show this:

(36) (i) $\llbracket 35 \rrbracket(w,g) = 1$

iff (ii) $\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\llbracket\overline{\mathsf{man}}\rrbracket(w,g))(x_i^e \rightarrow \llbracket\overline{\mathsf{walks}}\rrbracket(w,g)(w \mapsto x^e)) = 1$

iff (iii) $EVERY(\{x : \llbracket\overline{\mathsf{he_1\ is\ a\ man}}\rrbracket(w, g^x_{he_1}) = 1\}, \{x : \llbracket\overline{\mathsf{he_1\ walks}}\rrbracket(w, g^x_{he_1}) = 1\})$

$\overline{he_1\ \mathsf{walks}}$ refers to the only possible flat disambiguation, and $\overline{\mathsf{he_1\ is\ a\ man}}$ is the disambiguation in (37). We have assumed that there is no significant semantic diversity amongst the possible disambiguations of $\mathsf{he_1}$ is a man.

(37)

he$_1$	is	a	man
np	(s\np)/np	∀X.X\(X/np)/cn	cn
		∀X.X\(X/np)	
		VP\(VP/np)	
	s		

The equivalence of (ii) and (iii) requires one to consider the denotations of (37) and $\overline{\mathsf{he_1\ walks}}$, and the only non-trivial part is the demonstration that $\llbracket 37 \rrbracket(w,g) = \llbracket\overline{\mathsf{man}}\rrbracket(w,g)(g(he_1))$. As this involves a, we have have shown this as the first illustration of polymorphic determiner meanings. This time the polymorphic meaning is applied to the type (se, t).

(Key to abbreviations: $\mathsf{M} = (w' \mapsto \llbracket\overline{\mathsf{man}}\rrbracket(w',g))$, $\mathsf{I} = (w' \mapsto \llbracket\overline{\mathsf{is}}\rrbracket(w',g))$ $\nabla_3 = (w' \mapsto g(he_1)))$

(38) $\llbracket 37 \rrbracket(w,g)$

$= \llbracket\overline{\mathsf{a}}\rrbracket(w,g)(w' \mapsto \llbracket\overline{\mathsf{man}}\rrbracket(w',g))(D_{((s,e),t)})(w' \mapsto \llbracket\overline{\mathsf{is}}\rrbracket(w',g))(w' \mapsto g(he_1))$

$= \llbracket\overline{\mathsf{a}}\rrbracket(w,g)(\mathsf{M})(D_{((s,e),t)})(\mathsf{I})(\nabla_3)^{(s,e)}$

$= (\dot{x}^{(s,e)} \mapsto \llbracket\overline{\mathsf{a}}\rrbracket(w,g)(\mathsf{M})(D_t)(w' \mapsto y^{(s,e)} \mapsto \mathsf{I}w'yx))(\nabla_3)^{(s,e)}$

$= \llbracket\overline{\mathsf{a}}\rrbracket(w,g)(\mathsf{M})(D_t)(w' \mapsto y^{(s,e)} \mapsto \mathsf{I}w'y\nabla_3)$

$= \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w))(x^e \mapsto \mathsf{I}w(w' \mapsto x)\nabla_3)$

$= \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}(w))(x^e \mapsto \mathcal{IS}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)xg(he_1))$

$= \mathsf{M}(w)(g(he_1))$

$= \llbracket\overline{\mathsf{man}}\rrbracket(w,g)(g(he_1))$

The equivalence of (i) and (ii) requires only the calculation of the denotation of (35), shown below. The polymorphic determiner meaning is on this occasion applied to the type t (W =

$(w' \mapsto [\![\overline{\text{walks}}]\!](w', g)))$

(39) $[\![35]\!](w, g)$

$= [\![\overline{\text{every}}]\!](w, g)(w' \mapsto [\![\overline{\text{man}}]\!](w', g))(D_t)(w' \mapsto [\![\overline{\text{walks}}]\!](w', g))$

$= [\![\overline{\text{every}}]\!](w, g)(\mathsf{M})(D_t)(\mathsf{W})$

$= \mathcal{EVERY}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\mathsf{M}w)(x^e \mapsto \mathsf{W}w(w' \mapsto x))$

$= \mathcal{EVERY}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)([\![\overline{\text{man}}]\!](w, g))(x^e \mapsto [\![\overline{\text{walks}}]\!](w, g)(w' \mapsto x))$

3.1.4 John told every man to go

The required entailment of $\mathcal{LT}^{25}$ to account for (30d) is:

(40) there is a $\overline{\text{John told every man to go}}$ such that whatever model, $\langle \Im, \langle w, g \rangle \rangle$, whatever $\overline{\text{he}_1 \text{ is a man}}$, whatever $\overline{\text{John told he}_1 \text{ to go}}$, $[\![\overline{\text{John told every man to go}}]\!](w, g) = 1 \leftrightarrow EVERY(\{x : [\![\overline{\text{he}_1 \text{ is a man}}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\overline{\text{John told v to go}}]\!](w, g^x_{he_1}) = 1\})$

(41) has this property:

(41)
$$
\dfrac{\dfrac{\text{John}}{\text{np}} \quad \dfrac{\text{told}}{((\text{s}\backslash\text{np})/\text{vpc})/\text{np}} \quad \dfrac{\dfrac{\dfrac{\dfrac{\text{every}}{\forall X.X\backslash(X/\text{np})/\text{cn}} \quad \dfrac{\text{man}}{\text{cn}}}{\forall X.X\backslash(X/\text{np})}}{C_4}\text{d}}{} \quad \dfrac{\text{to}}{\text{vpc}/(\text{s}\backslash\text{np})} \quad \dfrac{\text{go}}{\text{s}\backslash\text{np}}}{\text{s}}
$$

$C_4 = VVP\backslash(VVP/\text{np})$

The chain of equivalences in (42) suffices to show this:

(42) (i) $[\![(41)]\!](w, g) = 1$

iff (ii) $\mathcal{EVERY}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)([\![\overline{\text{man}}]\!](w, g))(d^e_3 \mapsto [\![\overline{\text{told}}]\!](w, g)(w \mapsto d^e_3)(w' \mapsto [\![\overline{\text{to go}_{\text{vpc}}}]\!](w', g))$ $(w' \mapsto [\![\overline{\text{John}}]\!](w', g))) = 1$

iff (iii) $EVERY(\{x : [\![\overline{\text{he}_1 \text{ is a man}}]\!](w, g^x_{he_1}) = 1\}, \{x : [\![\overline{\text{John told he}_1 \text{ to go}}]\!](w, g^x_{he_1}) = 1\})$

$\overline{\text{John told he}_1 \text{ to go}}$ refers to one of the possible flat disambiguations and $\overline{\text{he}_1 \text{ is a man}}$ refers once again to (37). The equivalence of (ii) and (iii) is evident from the denotations of the disambiguations, taking note once again of the fact that $[\![37]\!](w, g) = [\![\overline{\text{man}}]\!](w, g)(g(he_1))$. The equivalence of (i) and (ii) is shown below ($\mathsf{G} = (w' \mapsto [\![\overline{\text{to}}]\!](w', g)(w'' \mapsto [\![\overline{\text{go}}]\!](w'', g)))$, $\mathsf{j} = (w' \mapsto$

$[\![\mathsf{John}]\!](w',g))$ $\mathsf{M} = (w' \mapsto [\![\mathsf{man}]\!](w',g))$, $\mathsf{T} = (w' \mapsto [\![\mathsf{told}]\!](w',g)))$

(43) $[\![(41)]\!](w,g)$

$$= [\![\mathsf{every}]\!](w,g)(w' \mapsto [\![\mathsf{man}]\!](w',g))(D_{((s,vp),(se,t))})(w' \mapsto [\![\mathsf{told}]\!](w',g))(w' \mapsto [\![\mathsf{to}]\!](w',g)$$
$$(w'' \mapsto [\![\mathsf{go}]\!](w'',g)))(w' \mapsto [\![\mathsf{John}]\!](w',g))$$
$$= [\![\mathsf{every}]\!](w,g)(\mathsf{M})(D_{((s,vp),(se,t))})(\mathsf{T})(\mathsf{G})^{(s,vp)}(\mathsf{j})$$
$$= (x^{(s,vp)} \mapsto [\![\mathsf{every}]\!](w,g)(\mathsf{M})(D_{(se,t)})(w' \mapsto y^{(s,e)} \mapsto \mathsf{T}w'yx))(\mathsf{G})^{(s,vp)}(\mathsf{j})$$
$$= [\![\mathsf{every}]\!](w,g)(\mathsf{M})(D_{(se,t)})(w' \mapsto y^{(s,e)} \mapsto \mathsf{T}w'y\mathsf{G})(\mathsf{j})^{se}$$
$$= (x^{(s,e)} \mapsto [\![\mathsf{every}]\!](w,g)(\mathsf{M})(D_t)(w' \mapsto y^{(s,e)} \mapsto \mathsf{T}w'y\mathsf{G}x))(\mathsf{j})^{(se)}$$
$$= [\![\mathsf{every}]\!](w,g)(\mathsf{M})(D_t)(w' \mapsto y^{(s,e)} \mapsto \mathsf{T}w'y\mathsf{Gj})$$
$$= \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}w)(x^e \mapsto \mathsf{T}w(w' \mapsto x)\mathsf{Gj})$$
$$= \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\mathsf{man}]\!](w,g))(x^e \mapsto [\![\mathsf{told}]\!](w,g)(w' \mapsto x)(w' \mapsto [\![\mathsf{to}]\!](w',g)$$
$$(w'' \mapsto [\![\mathsf{go}]\!](w'',g)))(w' \mapsto [\![\mathsf{John}]\!](w',g)))$$

3.2 Recursively ambiguous sentences with Determiners

Having confirmed that $\mathcal{LT}^{25}$ accounts for the semantic properties of unambiguous sentences we can now consider some ambiguous sentences. In (30e,f,g) of Chapter 6, section 2.1, were recorded three instances of the application of Hypothesis 3, and to begin with we will be concerned to show that $\mathcal{LT}^{25}$ accounts for these. Once these particular cases have been considered we will turn to whether $\mathcal{LT}^{25}$ accords more generally with the requirements of Hypothesis 3, concerning recursive ambiguity with respect to quantifiers.

3.2.1 a nun liked every boy

We consider first (30e), the fact that a nun liked every boy is recursively ambiguous wrt. *both* a nun and every boy. This is accounted for by the two disambiguations below.

(44)

a nun — liked — every boy

∀X.X/(X\np) — (s\np)/np — ∀Y.Y\(Y/np)

s/(s\np) — (s\np)\((s\np)/np)

s\np

s

(45)

a nun	liked	every boy
∀X.X/(X\np)	(s\np)/np	∀Y.Y\(Y/np)
s/(s\np)		s\(s/np)
s/np		
s		

The reading associated with the claim that a nun liked every boy is recursively ambiguous wrt. a nun is accounted for by (44), as should be plain from its denotation indicated in (46). Note that the immediate subparts of (44) are a disambiguation of a nun and a disambiguation of liked every boy.

The reading associated with the claim that a nun liked every boy is recursively ambiguous wrt. every boy is accounted for by (45), as should be plain from its denotation indicated in (46). Note that the immediate subparts of (45) are a disambiguation of every boy and a disambiguation of a nun liked.

(46) $N = (w' \mapsto [\![\langle \text{nun}, \langle\rangle, \text{CN}\rangle]\!](w', g))$

$B = (w' \mapsto [\![\langle \text{boy}, \langle\rangle, \text{CN}\rangle]\!](w', g))$

$L = (w' \mapsto [\![\langle \text{liked}, \langle\rangle, (\text{s}\backslash\text{np})/\text{np}\rangle]\!](w', g))$

$[\![(44)]\!](w, g)$

$= [\![\overline{\text{a}}]\!](w, g)(N)(D_t)(w' \mapsto [\![\overline{\text{every}}]\!](w', g)(B)(D_{(se,t)})(L))$

$= \mathcal{A}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(Nw)(y^e \mapsto \mathcal{EVERY}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(Bw)(x^e \mapsto Lw(w' \mapsto x)(w' \mapsto y)))$

$[\![(45)]\!](w, g)$

$= [\![\overline{\text{every}}]\!](w, g)(B)(D_t)(w' \mapsto d^{(s,e)} \mapsto [\![\overline{\text{a}}]\!](w', g)(N)(D_t)(w' \mapsto Lw'(d)))$

$= \mathcal{EVERY}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(Bw)(x^e \mapsto \mathcal{A}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(Nw)(y^e \mapsto Lw(w' \mapsto x)(w' \mapsto y)))$

Moving onto (30f,g), we will explain first how $\mathcal{LT}^{25}$ accounts for (30g) and then return to (30f). This is because the *de-re/de-dicto* contrast of (30g) is a little more easily explained than the two way recursive ambiguity of (30f).

3.2.2 John believes a man came in

Let us suppose $\mathcal{LT}^{25}$ would account for (30g) if it allowed two disambiguations of John believes that a man came in whose denotations were as follows:

(47) *de-dicto*: $\mathsf{B}w(w' \mapsto \mathcal{A}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w', g)(\mathsf{M}w')(x^e \mapsto \mathsf{C}w'(w'' \mapsto x)))(\mathsf{j})$

de-re: $\mathcal{A}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\mathsf{M}w)(x^e \mapsto \mathsf{B}w(w' \mapsto \mathsf{C}w'(w'' \mapsto x))(\mathsf{j}))$

To start off with, here is a disambiguation of John believes that a man came in that accounts for the *de-dicto* reading of a man.

(48)

John	believes	that	a man	came in
np	s\np/sc	sc/s	∀X.X/(X\np)	s\np
	(s\np)/s		s/(s\np)	
			s	
	s\np			
s				

This contains a disambiguation of a man came in, which has as immediate subparts a disambiguation of the quantified noun phrase and that disambiguation of came in which is the basic expression ⟨came in, ⟨⟩, s\np⟩. In any model allowed by $\mathcal{LT}^{25}$ this disambiguation of came in has a denotation of type $((s,e),t)$, which is a function of arity 1.

The key to accounting for the *de-re* interpretation of a man is that there are other disambiguations of came in that are associated with denotations of *larger arities.* In (49) there is a disambiguation of came in which would be associated with a meaning of arity 3.

(49)

$$\dfrac{\dfrac{\text{came in}}{\text{s\\np}}}{\text{(VP\\(VP/s))\\np}}\text{PROOF 1} \qquad \text{PROOF 1} \quad \dfrac{\dfrac{\text{VP/s, np, s\\np} \Rightarrow \text{VP}}{\text{np, s\\np} \Rightarrow \text{VP\\(VP/s)}}}{\text{s\\np} \Rightarrow \text{(VP\\(VP/s))\\np}}$$

$H^{i}_{L\forall}$(PROOF 1) is the operation defined by the term $\lambda x^{(s,e)}\lambda y^{(s,st,se,t)}[yi(\lambda i[u_1^{(se,t)}(\lambda i[xi])])]$, assuming that $u_1^{(se,t)}$ is associated with the antecedent. This operation is:

(50) $m^{(se,t)} \mapsto (w,g) \mapsto x^{(s,e)} \mapsto y^{(s,st,se,t)} \mapsto y(w)(w' \mapsto m(w',g)(x))$

Therefore the denotation of the non-basic disambiguation of came in in (49) is:

(51) $x^{(s,e)} \mapsto y^{(s,st,se,t)} \mapsto y(w)(w' \mapsto [\![\overline{\text{came in}}]\!](w',g)(x))$

This disambiguation of came in is of category (VP\(VP/s))\np, which is an instance of x\np. Therefore that disambiguation of a man which is of category (VP\(VP/s))/((VP\(VP/s))\np), could be combined with the disambiguation in (49) to obtain a disambiguation of a man came in of category (VP\(VP/s)):

(52)

$$\dfrac{\dfrac{\dfrac{\text{a man}}{\forall X.X/(X\backslash np)}}{(VP\backslash(VP/s))/((VP\backslash(VP/s))\backslash np)} \quad \dfrac{\dfrac{\text{came in}}{VP}}{(VP\backslash(VP/s))\backslash np}\text{PROOF1}}{(VP\backslash(VP/s))}$$

The denotation assigned to (52) is (using M as an abbreviation of $(w' \mapsto [\![\overline{\text{man}}]\!](w', g))$):

(53) $[\![\overline{\text{a man came in}}]\!]$

$= [\![\overline{\text{a}}]\!](w, g)(\mathsf{M})(D_{((s,st,se,t),se,t)})(w' \mapsto [\![(49)]\!](w', g))$

$= y^{(s,st,se,t)} \mapsto z^{se} \mapsto \mathcal{A}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\mathsf{M}w)(x^e \mapsto [\![(49)]\!](w, g)(w' \mapsto x)(y)(z))$

$= y^{(s,st,se,t)} \mapsto z^{se} \mapsto \mathcal{A}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\mathsf{M}w)(x^e \mapsto y(w)(w' \mapsto \mathsf{C}w'(w' \mapsto x))(z))$

Now if the above function is compared with what was indicated in (47) as the required denotation of the looked for disambiguation, one can see that if the function was to be applied to the 'right' arguments, the result would be the denotation in (47); the 'right' arguments are B and j. Therefore, if the (52) disambiguation of a man came in can be combined with disambiguations of believes and John by syntactic operations associated with *intensional function application*, the denotation of the resulting disambiguation will be what was sought. This *is* possible as indicated in the following:

(54)

$$\dfrac{\dfrac{\text{John}}{np} \quad \dfrac{\dfrac{\text{believes}}{(s\backslash np)/s} \quad \dfrac{\text{a man came in}}{(s\backslash np)\backslash((s\backslash np)/s)}}{s\backslash np}}{s}$$

Checking the denotation assigned to (54):

(55) $[\![(54)]\!](w, g)$

$= (y^{(s,st,se,t)} \mapsto z^{se} \mapsto \mathcal{A}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\mathsf{M}w)(x^e \mapsto y(w)(w' \mapsto \mathsf{C}w'(w' \mapsto x))(z)))(\mathsf{B})(\mathsf{j})$

$= \mathcal{A}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\mathsf{M}w)(x^e \mapsto \mathsf{B}(w)(w' \mapsto \mathsf{C}w'(w' \mapsto x))(\mathsf{j}))$

3.2.3 every man told a woman to go

Now we move onto (30f), and the task of accounting for the recursive ambiguity of every man told a woman to go wrt a woman. First we shall see whether we can replicate the strategy used for (30g).

The disambiguation of John believes a man came in for which (54) is a notation for a highly *non-flat* member of the carrier set of the syntactic algebra, reflecting in its structure the generally accepted constituent structure of the sentence. One can equally well arrive at the desired result based on a *flatter* member of the syntactic algebra, less reflective of the conventional constituent structure, namely

$$\dfrac{\dfrac{\text{John}}{np} \quad \dfrac{\text{believes that}}{(s\backslash np)/s} \quad \dfrac{\text{a man}}{\forall X.X/(X\backslash np)} \quad \dfrac{\text{came in}}{s\backslash np}}{s}\text{PROOF1}$$

where PROOF1 is the following proof (using PV for (s\np)/s):

(56)
$$\dfrac{\dfrac{np, PV, (s\backslash np)\backslash PV \Rightarrow s \quad \dfrac{np,\ PV,\ np,\ s\backslash np \Rightarrow s}{s\backslash np \Rightarrow ((s\backslash np)\backslash PV)\backslash np}\backslash R\backslash R\backslash R}{np, PV, ((s\backslash np)\backslash PV)/(((s\backslash np)\backslash PV)\backslash np),\ s\backslash np \Rightarrow s}/L}{np, PV, \quad \forall X.X/(X\backslash np) \quad ,\ s\backslash np \Rightarrow s}\forall L$$

Therefore PROOF1, the proof indexing the crucial operation may be seen as built up from a proof of np,PV, np, s\np ⇒ s. This sequent differs from the concluding sequent only by the presence of np instead of ∀X.X/(X\np).

Therefore we might seek a disambiguation of every man told a woman to go by starting from a proof of ∀X.X/(X\np), ((s\np)/vpc)/np, np, vpc ⇒ s, and building from it to a proof of ∀X.X/(X\np), ((s\np)/vpc)/np, ∀Y.Y\(Y/np), vpc ⇒ s, on the model of (56). Here then is a proof of ∀X.X/(X\np), ((s\np)/vpc)/np, np, vpc ⇒ s:

(57)
$$\dfrac{\dfrac{\dfrac{\dfrac{s \Rightarrow s \quad s\backslash np \Rightarrow s\backslash np}{s/(s\backslash np),\ s\backslash np \Rightarrow s}/L \quad vpc \Rightarrow vpc}{s/(s\backslash np),\ (s\backslash np)/vpc,\ vpc \Rightarrow s}/L \quad np \Rightarrow np}{s/(s\backslash np),\ ((s\backslash np)/vpc)/np,\ np,\ vpc \Rightarrow s}/L}{\forall X.X/(X\backslash np),\ ((s\backslash np)/vpc)/np,\ np,\ vpc \Rightarrow s}(\forall L)$$

Building on this in an analogous way to (56) would lead to the following:

(58)
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{s \Rightarrow s \quad s\backslash np \Rightarrow s\backslash np}{s/(s\backslash np),\ s\backslash np \Rightarrow s} \quad vpc \Rightarrow vpc}{s/(s\backslash np),\ (s\backslash np)/vpc,\ vpc \Rightarrow s} \quad np \Rightarrow np}{s/(s\backslash np),\ ((s\backslash np)/vpc)/np,\ np,\ vpc \Rightarrow s}}{\forall X.np^{\wedge X},\ ((s\backslash np)/vpc)/np,\ np,\ vpc \Rightarrow s}(\forall L)}{TVVP \Rightarrow ((s\backslash \forall X.np^{\wedge X})/vpc)/np} \quad \dfrac{\vdots}{\forall X.np^{\wedge X}, ((s\backslash \forall X.np^{\wedge X})/vpc), vpc \Rightarrow s}}{\dfrac{\forall X.np^{\wedge X}, TVVP, ((s\backslash \forall X.np^{\wedge X})/vpc)\backslash(((s\backslash \forall X.np^{\wedge X})/vpc)/np), vpc \Rightarrow s}{\forall X.np^{\wedge X}, TVVP, \quad \forall Y.Y\backslash(Y/np) \quad , vpc \Rightarrow s}(\forall L)}(/L)$$

However, this is not a possible proof, as in the last step, the (∀L) inference, as value for Y, the following is chosen, with abbreviations now done away with:

$$((s\backslash(\forall X.X/(X\backslash np)))/vpc)/np$$

This is a category in which a *polymorphic quantifier categorisation* figures and is therefore not a possible instantiation if we are to abide by the restriction on variable instantiation that we have decided upon in Definition 67. All is not lost, however.

(58) is built upon (57). In (57) there is there is a (∀L) inference involving $\forall X.X/(X\backslash np)$, at which point X is instantiated to s. This is in fact the last step of (57). Dropping this last step from (57) we have a a proof of $s/(s\backslash np), TVVP, np, vpc \Rightarrow s$. We will now build on this proof in somewhat the fashion that we built upon (57):

(59)
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{s \Rightarrow s \quad s\backslash np \Rightarrow s\backslash np}{s/(s\backslash np),\ s\backslash np \Rightarrow s} \quad vpc \Rightarrow vpc}{s/(s\backslash np),\ (s\backslash np)/vpc,\ vpc \Rightarrow s} \quad np \Rightarrow np}{s/(s\backslash np),\ ((s\backslash np)/vpc)/np,\ np,\ vpc \Rightarrow s}}{TVVP \Rightarrow ((s\backslash np^{\wedge s})/vpc)/np} \qquad \dfrac{\dfrac{\vdots}{np^{\wedge s}, ((s\backslash np^{\wedge s})/vpc), vpc \Rightarrow s}}{\forall X\ np^{\wedge X}, ((s\backslash np^{\wedge s})/vpc), vpc \Rightarrow s}\forall L}{\dfrac{\forall X\ np^{\wedge X}, TVVP, ((s\backslash np^{\wedge s})/vpc)\backslash(((s\backslash np^{\wedge s})/vpc)/np), vpc \Rightarrow s}{\forall X\ np^{\wedge X}, TVVP, \quad \forall Y.Y\backslash(Y/np) \quad , vpc \Rightarrow s}(\forall L)}(/L)$$

The disambiguation associated with this proof will account for the reading of every told a woman to go required by its recursive ambiguity wrt. a woman.

For, supposing that with the antecedents of the concluding sequent of (59) are associated the variables $x_{Q1}^{\forall\pi((s,se,\pi),\pi)}$, $x_2^{(se,(s,se,t),se,t)}$, $x_{Q2}^{\forall\pi((s,se,\pi),\pi)}$, $x_4^{(se,t)}$, then the term associated with the proof is:

(60) $x_{Q2}^{\forall\pi((s,se,\pi),\pi)}(TYPE)(\lambda i\beta)(\lambda i x_4)(\lambda i x_{Q1}^{\forall\pi((s,se,\pi),\pi)}(t))$
where $TYPE = ((s,se,t),(s,(s,se,t),t),t))$
$\beta = \lambda y_1^{se}\lambda y_2^{(s,se,t)}\lambda y_3^{(s,(s,se,t),t)}[y_3(i)(\lambda i x_2(\lambda i y_1 i)(\lambda i y_2 i))]$

Therefore the denotation of the disambiguation is:

(61) $[\![\overline{\text{a woman}}]\!](w,g)$
(D_{TYPE})
$(w', d_1^{se}, d_2^{(s,se,t)}, d_3^{(s,((s,se,t),t))} \mapsto$
$d_3(w')(w'' \mapsto [\![\overline{\text{told}}]\!](w'',g)(w'' \mapsto d_1(w''))(w'' \mapsto d_2(w''))))$
$(w' \mapsto [\![\overline{\text{to go}}]\!](w',g))\ (w' \mapsto [\![\overline{\text{every man}}]\!](w',g)(D_t))$

$= [\![\overline{\text{a woman}}]\!](w,g)(x^e \mapsto [\![\overline{\text{every man}}]\!](w,g)(y^e \mapsto [\![\overline{\text{told}}]\!](w,g)(w' \mapsto x)(w' \mapsto [\![\overline{\text{to go}}]\!](w',g))$
$(w' \mapsto y)))$

We have seen how that all the semantic data concerning determiners that was set out in (30), Chapter 6, is accounted for, both unambiguous and ambiguous cases. Next we consider more generally whether $\mathcal{LT}^{25}$ is in accord with Hypothesis 3.

3.2.4 General recursive ambiguity of S[DET CN] wrt. DET CN

We wish to consider now whether $\mathcal{LT}^{25}$ accords with Hypothesis 3, concerning recursive ambiguity with respect to quantifiers. To start with we will simplify and assume that DET CN is every man. Then if a THEORY OF REFERENCE is to accord with Hypothesis 3 it must be the case that:

(62) for *any* disambiguation of ... he_1 ..., there is *some* disambiguation of ... every man ... such that,

$$[\![\overline{\text{... every man ...}}]\!](w,g) =$$
$$\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w,g))(x \mapsto [\![\overline{\text{... he}_1 \text{ ...}}]\!](w,g^x_{\mathsf{he}_1}))$$

If the THEORY OF REFERENCE under consideration was for example $\mathcal{T}^{11}$, the non-local account of quantification that was described in section 2.5.1 of Chapter 5, it would be quite easy to demonstrate (62). This is because the obvious candidate for the required disambiguation of ... every man ... is that one which has the supposed disambiguation of ... he_1 ... as a *subpart*: such disambiguations of ... every man ... are provided for by the 'Quantifying-in' operation.

When $\mathcal{LT}^{25}$ is the THEORY OF REFERENCE under consideration, it is more difficult to to demonstrate (62). This is because no disambiguation of ... he_1 ... can be used as a *subpart* of a disambiguation of ... every man .... However, (62) can still be demonstrated, for it does not require a certain relationship between disambiguations, it requires a relationship between the denotations of those disambiguations. Essentially, we must look for a 'factorisation' of the ... every man ... denotation, revealing the presence of the ... he_1 ... denotation as a factor.

What we will do now is to present a simplified, outline form of the proof that according to $\mathcal{LT}^{25}$, (62) is true. After that, we will consider the effect of the simplifications made.

So, to begin with, we must consider an arbitrary sentence, ... he_1 ..., and an arbitary disambiguation of it. Considering an arbitrary sentence, ... he_1 ..., comes to considering any sequence of lexical categories that instantiate the pattern $U, T1, \mathrm{np}, T2, V$, where one of T_1 or $T2$ is not empty and both U and V may be empty. Then to suppose an arbitary disambiguation of ... he_1 ..., is to suppose that there is some proof, P_1, of $U, T1, \mathrm{np}, T2, V \Rightarrow \mathrm{s}$. Hence the supposition is:

there is a proof P_1 of U,T1, np, T2, V $\Rightarrow$ s:

$$\frac{\vdots}{U, T1, \mathrm{np}, T2, V \Rightarrow \mathrm{s}}$$

We will consider the case of this supposition such that it is $T2$ that is non-empty. It should be obvious how things would correspondingly go if $T1$ was taken to be non-empty. What we have to show on the basis of this supposition is (i) that there is disambiguation of ... every man ..., and (ii) furthermore that the relationship between the denotations of the disambiguations of ... he_1 ... and ... every man ... is as specified in (62).

Now the search for the disambiguation of ... every man ... may be equated to the search for a proof of $U, T1, Q, T2, V \Rightarrow \mathrm{s}$, where Q is one of the two polymorphic quantifier categorisations, $\forall$X.X/(X\np) or $\forall$X.X\(X/np). We will claim, more or less, that the proof whose existence is deducible from the supposition of the existence of the proof P_1 is the following one:

P_5, a proof of $U, T1, \forall \mathrm{X.X}/(\mathrm{X}\backslash\mathrm{np}), T2, , V \Rightarrow \mathrm{s}$:

$$\dfrac{\dfrac{\dfrac{\vdots}{U, T1, (\mathrm{s}/V)\backslash U.T1, V \Rightarrow \mathrm{s}} \qquad \dfrac{\dfrac{\vdots}{U, T1, \mathrm{np}, T2, V \Rightarrow \mathrm{s}}}{T2 \Rightarrow ((\mathrm{s}/V)\backslash U.T1)\backslash\mathrm{np}}\text{Slash R}}{U, T1, ((\mathrm{s}/V)\backslash U.T1)/(((\mathrm{s}/V)\backslash U.T1)\backslash\mathrm{np}), T2, V \Rightarrow \mathrm{s}}(/\mathrm{L})}{U, T1 \quad \forall \mathrm{X.X}/(\mathrm{X}\backslash\mathrm{np}) \quad , T2, V \Rightarrow \mathrm{s}}(\forall\mathrm{L})$$

A few words of explantion of P_5. The supposed proof P_1 is the 'top-right' part of P_5. 'Slash R' indicates that by a series of (\R) and (/ R) one deduces $T2 \Rightarrow ((\mathrm{s}/V)\backslash U.T1)\backslash\mathrm{np}$ $((\mathrm{s}/V)\backslash U.T1$ is an abbreviation for $(\mathrm{s}/V)\backslash\backslash U\backslash T_1)$. The sequent, $T2 \Rightarrow ((\mathrm{s}/V)\backslash U.T1)\backslash\mathrm{np}$ is then used as the minor premise in a (/L) inference, the major premise being $U, T1, ((\mathrm{s}/V)\backslash U.T1), V \Rightarrow \mathrm{s}$. This major premise can be given the obvious proof that proceeds by a series of Slash-Left inferences. The sequent resulting from the (/L) inference combining the major and minor premise is: $U, T1, ((\mathrm{s}/V)\backslash U.T1)/(((\mathrm{s}/V)\backslash U.T1)\backslash\mathrm{np}), T2, V \Rightarrow \mathrm{s}$. $((\mathrm{s}/V)\backslash U.T1)/(((\mathrm{s}/V)\backslash U.T1)\backslash\mathrm{np}$ is an instance of the schema $x/(x\backslash\mathrm{np})$, and so we have the premise for a ($\forall$L) inference, allowing the conclusion of $U, T1, \forall \mathrm{X.X}/(\mathrm{X}\backslash\mathrm{np}), T2, V \Rightarrow \mathrm{s}$, and this is the final step of P_5. We have here ignored the side-condition to the ($\forall$L) inference and not worried whether $((\mathrm{s}/V)\backslash U.T1)$ contains any *quantified* categories. This ignored complication will be considered in a while.

P_5 demonstrates the existence of disambiguation of ... every man ... on the basis of the supposed disambiguation of ... he_1 .... What it remains to do is to show that the relationship between the disambiguations associated with P_1 and P_5 is that required by (62). This requires us to show that:

(63) $\mathcal{G}_{P_5}(m_1^{\vec{U}}, m_2^{\vec{T}1}, [\![\overline{\textsf{every man}}]\!], m_4^{\vec{T}2}, m_5^{\vec{V}})(w,g)$
$= \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\textsf{man}}]\!](w,g))(x \mapsto \mathcal{G}_{P1}(m_1^{\vec{U}}, m_2^{\vec{T}1}, [\![\overline{\textsf{he}_1}]\!], m_4^{\vec{T}2}, m_5^{\vec{V}})(w, g^x_{\textsf{he}_1}))$

The notations used here should be more or less self-explanatory. For example, $m_1^{\vec{U}}$ refers to the sequence of meanings of the lexical items the sequence of whose cateogorisations U represented. $\vec{U}$ is intended to the the sequence of types associated with the sequence of categories U. Also $\overline{\textsf{every man}}$ is intended to refer to the polymorphic disambiguation of every man.

There is no elaborate reasoning involved in establishing (63). All that is required is consultation of the definitions of the proof-to-operation map, $H^i_{L\forall}$, and the meaning postulate for the polymorphic determiner, and a large amount of book-keeping.

That (63) is true

We consider first LHS of (63). Suppose on choosing $x_1^{\vec{U}}, x_2^{\vec{T}1}, x_3^e, x_4^{\vec{T}2}, x_5^{\vec{V}}$ are chosen for the antecedent categories of the concluding sequent of P_1 $H^i_{L\forall}(P_1) = \Psi$. Then if $u_1^{\vec{U}}, u_2^{\vec{T}1}, u_Q^{\forall\pi((s,se,\pi),\pi)}$, $u_4^{\vec{T}2}, u_5^{\vec{V}}$ are chosen for the antecedents of the concluding sequent of P_5, $H^i_{L\forall}(P_5)$ is:

(64) $u_Q^{\forall\pi((s,se,\pi),\pi)}(TYPE)(\lambda i\Sigma)(\lambda i u_2^{\vec{T}1})(\lambda i u_1^{\vec{U}})(\lambda i u_5^{\vec{V}})$
where $TYPE = ((s,\vec{T}1),(s,\vec{U}),(s,\vec{V}),t)$
$\Sigma = \lambda y_1^{(se)}\lambda y_2^{(s,\vec{T}1)}\lambda y_3^{(s,\vec{U})}\lambda y_4^{(s,\vec{V})}\Psi[y_3 i/x_1, y_2 i/x_2, y_1 i/x_3, u_4/x_4, y_4 i/x_5]$

The LHS of (63) is the value of the operation defined by the term above when applied to the meanings of the parts of ... every man ... and a (w,g), and this is calculated below. Note in the first step, each free variable, u^a, of the term above is replaced by the term $z^{(s,a)}i$, and the whole term prefaced with λi. Because u_4 is 'invisible' in $H^i_{L\forall}(P_5)$, the notation Σ_* is used for $\Sigma[z_4 i/u_4]$. The third step relies on the recursive determiner postulate. The foruth step is because Σ_* is $\lambda y_1\lambda y_2\lambda y_3\lambda y_4[\Psi[y_3 i/x_1, y_2 i/x_2, y_1 i/x_3, z_4 i/x_4, y_4 i/x_5]]$

(65) $[\![\lambda i z_Q^{\forall\pi((s,se,\pi),\pi)}(i)(TYPE)(\lambda i\Sigma_*)(\lambda i z_2^{(s,\vec{T}1)}i)(\lambda i z_1^{(s,\vec{U})}i)(\lambda i z_5^{(s,\vec{V})}i)]\!](k)(w)$
where k is a $\mathcal{L}^{(\lambda,\Delta)}$assignment such that: $k(z_1^{(s,\vec{U})}) = w' \mapsto m_1(w',g)$, $k(z_2^{(s,\vec{T}1)}) = w' \mapsto m_2(w',g)$, $k(z_Q^{(s,\forall\pi((s,se,\pi),\pi))}) = w' \mapsto [\![\overline{\textsf{every man}}]\!](w',g)$, $k(z_4^{(s,\vec{T}2)}) = w' \mapsto m_4(w',g)$, $k(z_5^{(s,\vec{V})}) = w' \mapsto m_5(w',g)$
$= [\![\overline{\textsf{every man}}]\!](w,g)(D_{TYPE})([\![\lambda i\Sigma_*]\!](k))([\![\lambda i z_2^{(s,\vec{T}1)}i]\!](k))([\![\lambda i z_1^{(s,\vec{U})}i]\!](k))([\![\lambda i z_5^{(s,\vec{V})}i]\!](k))$
$=\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\textsf{man}}]\!](w,g))$
$(x^e \mapsto [\![\lambda i\Sigma_*]\!](k)(w)(w' \mapsto x)([\![\lambda i z_2^{(s,\vec{T}1)}i]\!](k))([\![\lambda i z_1^{(s,\vec{U})}i]\!](k))([\![\lambda i z_5^{(s,\vec{V})}i]\!](k)))$
$=\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\textsf{man}}]\!](w,g))(x^e \mapsto [\![\Psi[y_3 i/x_1, y_2 i/x_2, y_1 i/x_3, z_4 i/x_4, y_4 i/x_5]]\!](k'))$,
where k' differs from k at most wrt. i, y_1, y_2, y_3, y_4, for which $k'(i) = w$, $k'(y_1) = (w' \mapsto x)$, $k'(y_2) = [\![\lambda i z_2^{(s,\vec{T}1)}i]\!](k) = w' \mapsto m_2(w',g)$, $k'(y_3) = [\![\lambda i z_1^{(s,\vec{U})}i]\!](k) = w' \mapsto m_1(w',g)$, $k'(y_4) = [\![\lambda i z_5^{(s,\vec{V})}i]\!](k) = w' \mapsto m_5(w',g)$

(65) is a form of the LHS of (63) and we will leave it for the moment in that form and turn to the RHS of (63), which is calculated below:

(66) $\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\textsf{man}}]\!](w,g))(x^e \mapsto [\![\lambda i\Psi[z_1 i/x_1, z_2 i/x_2, z_3 i/x_3, z_4 i/x_4, z_5 i/x_5]]\!](l)(w))$
where l is an $\mathcal{L}^{(\lambda,\Delta)}$ assignment such that:$l(z_1) = w' \mapsto m_1(w', g^x_{\textsf{he}_1})$,$l(z_2) = w' \mapsto m_2(w', g^x_{\textsf{he}_1})$, $l(z_3) = w' \mapsto [\![\overline{\textsf{he}_1}]\!](w', g^x_{\textsf{he}_1}) = w' \mapsto x$, $l(z_4) = w' \mapsto m_4(w', g^x_{\textsf{he}_1})$, $l(z_5) = w' \mapsto m_5(w', g^x_{\textsf{he}_1})$

Now comparing (65) and (66) it is clear that they could only differ if one of the following inequalities holds:

(67) $[\![y_3 i]\!](k') \neq [\![z_1 i]\!](l)$, $[\![y_2 i]\!](k') \neq [\![z_2 i]\!](l)$, $[\![y_1 i]\!](k') \neq [\![z_3 i]\!](l)$, $[\![z_4 i]\!](k') \neq [\![z_4 i]\!](l)$, $[\![y_4 i]\!](k') \neq [\![z_5 i]\!](l)$

But all of these inequalities are impossible. Therefore (63) is true □

That completes the simplified proof of the fact that $\mathcal{LT}^{25}$ is in accord with Hypothesis 3. Now to consider the simplifications that were made. There were two simplifications, the first and least important of which was that the CN part of DET CN was man. In fact it should be allowed that CN is a phrase, and the condition for accord with Hypothesis 3 should be revised to:

(68) for *any* disambiguation of ... he$_1$ ..., *any* disambiguation of v$_1$ is a CN, there is *some* disambiguation of ... every man ... such that,

$[\![\overline{\ldots\text{every man}\ldots}]\!](w,g) =$

$\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(x \mapsto [\![\overline{\text{he}_1 \text{ is a man}}]\!](w,g^{x}_{\text{he}_1}))(x \mapsto [\![\overline{\ldots\text{he}_1\ldots}]\!](w,g^{x}_{\text{he}_1}))$

Therefore the supposition of the proof should not only be the existence of a proof, P_1, of $U, T1, \text{np}, T2, V \Rightarrow \text{s}$, but also the existence of a proof, R, of np, (s\np)/np, (∀X.X/(X\np))/cn, W ⇒ s, where W is the sequence of lexical categories corresponding to CN. This also changes the aim of the proof to being one of demonstrating the existence of a proof of

U,$T1$,(∀X.X/(X\np))/cn,W,$T2$,V ⇒ s

The second and more important simplification that was made concerns the ignoring of the side-condition to the (∀L) inference. The simplified proof will not work if in $U, T1$ or V there are ***quantified*** categories. The first step of the revised proof concerns such occurences. If there are, it is assumed that in P_1 are (∀L) inferences, each instantiating some category variable Z to some category x. We infer the existence of a proof P_2, from which (∀L) inferences concerning ∀'s in $U, T1$ or V are absent, proving the sequent $U', T1', \text{np}, T2, V' \Rightarrow \text{s}$, where each of U',$T1'$, and V' differ from U, $T1$, and V by replacement of any category $\forall Z.y$ with $y[x/Z]$.

Then the required disambiguation of ... DET CN ... is associated with the following proof:

P_5: a proof of $U, T1, (\forall \text{X.X}/(\text{X}\backslash\text{np}))/\text{cn}, W, T2, V \Rightarrow \text{s}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{U \Rightarrow U' \quad T \Rightarrow T1' \quad V \Rightarrow V'}{U, T1, (\text{s}/V')\backslash U'.T1', V \Rightarrow \text{s}} \quad \dfrac{\overline{\quad\vdots\quad} \atop U', T1', \text{np}, T2, V' \Rightarrow \text{s}}{T2 \Rightarrow ((\text{s}/V')\backslash U'.T1')\backslash\text{np}}\,\text{Slash R}}{U, T1, ((\text{s}/V')\backslash U'.T1')/(((\text{s}/V')\backslash U'.T1')\backslash\text{np}), T2, V \Rightarrow \text{s}}\,(/\text{L})}{U, T1, \quad \forall\text{X.X}/(\text{X}\backslash\text{np}) \quad, T2, V \Rightarrow \text{s}}\,(\forall\text{L}) \quad \dfrac{\vdots}{\text{W} \Rightarrow \text{cn}}}{\text{U,T1,} \quad (\forall\text{X.X}/(\text{X}\backslash\text{np}))/\text{cn} \quad \text{W, T2,V} \Rightarrow \text{s}}\,/\text{L}$$

A few words of explantion of P_5. P_2 is the proof associated with the rightmost ellipsis, the proof of $U', T1', \text{np}, T2, V' \Rightarrow \text{s}$. Again 'Slash R' indicates a series of Slash Right inferences, this time leading to $T2 \Rightarrow ((\text{s}/V')\backslash U'.T1')\backslash\text{np}$. The sequent, $T2 \Rightarrow ((\text{s}/V')\backslash U'.T1')\backslash\text{np}$ is then used as the minor premise in a (/L) inference, the major premise being $U, T1, ((\mathrm{s}/V)\backslash U.T1), V \Rightarrow \mathrm{s}$. This major premise can be given the obvious proof that proceeds by a series of Slash-Left inferences, the minor premises for which are $U \Rightarrow U'$, $T1 \Rightarrow T1'$ and $V \Rightarrow V'$. These are understood to have proofs which include the (∀L) inferences that were absented from P_1 to give P_2. The sequent resulting from the (/L) inference combining the major and minor premise is: $U, T1, ((\mathrm{s}/V')\backslash U'.T1')/(((\mathrm{s}/V')\backslash U'.T1')\backslash \mathrm{np}), T2, V \Rightarrow \mathrm{s}$. $((\mathrm{s}/V')\backslash U'.T1')/(((\mathrm{s}/V')\backslash U'.T1')\backslash \mathrm{np}$ is an instance of the schema $x/(x\backslash \mathrm{np})$, and so we have the premise for a (∀L) inference, allowing the conclusion of $U, T1, \forall \mathrm{X}.\mathrm{X}/(\mathrm{X}\backslash \mathrm{np}), T2, V \Rightarrow \mathrm{s}$. This forms the major premise of a (/L) inference, whose minor premise is $W \Rightarrow \mathrm{cn}$. The proof of this minor premise is, R'. This is intended to be identical to a subproof of R. Recall R is a proof of np, (s\np)/np, $(\forall \mathrm{X}\ \mathrm{np}^{\wedge \mathrm{X}})/\mathrm{cn}$, $W \Rightarrow \mathrm{cn}$. Given the meaning postulates for he_1, is and a, the denotation of the disambiguation of he_i is a CN associated with the proof R, can be calculated from the proof R':

$$\mathcal{G}_R([\![\mathsf{he}_1]\!], [\![\mathsf{is}]\!], [\![\mathsf{a}]\!], m^{\vec{W}})(w, g^x_{he_1}) = \mathcal{G}_{R'}(m^{\vec{W}})(w, g)(x)$$

We now have to demonstrate that there the relationship between the disambiguations associated with P_1, R and P_5 that is required by (68), namely that:

$$\begin{aligned}(69)\ & \mathcal{G}_{P_5}(m_1^{\vec{U}}, m_2^{\vec{T1}}, [\![\mathsf{every}]\!], m_{CN}^{\vec{W}}, m_4^{\vec{T2}}, m_5^{\vec{V}})(w, g) \\ & = \mathcal{EVERY}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\mathcal{G}_R([\![\mathsf{he}_1]\!], [\![\mathsf{is}]\!], [\![\mathsf{a}]\!], m_{CN}^{\vec{W}})(w, g)) \\ & \qquad\qquad (x \mapsto \mathcal{G}_{P1}(m_1^{\vec{U}}, m_2^{\vec{T1}}, [\![\mathsf{he}_1]\!], m_4^{\vec{T2}}, m_5^{\vec{V}})(w, g^x_{\mathsf{he}_1}))\end{aligned}$$

Below, the fact, that (69) is true is demonstrated.

That (69) is true

Suppose on choosing $x_1^{\vec{U}'}, x_2^{\vec{T1}'}, x_3^e, x_4^{\vec{T2}'}, x_5^{\vec{V}'}$ for the antecedent categories of the concluding sequent of P_2, $H^{\mathsf{i}}_{L\forall}(P_2) = \Psi$. Suppose also that on choosing $c^{\vec{W}}$ for the antecedent category of R, $H^{\mathsf{i}}_{L\forall}(R) = \theta$. We will give $H^{\mathsf{i}}_{L\forall}(P_5)$ on the supposition that $u_1^{\vec{U}}$, $u_2^{\vec{T1}}$, $u_Q^{((s,et),\forall\pi((s,se,\pi),\pi))}$, $c^{\vec{W}}$, $u_4^{\vec{T2}}$, $u_5^{\vec{V}}$ are chosen for the antecedents. But first we must explain the notation used to register the effect of the ∀L inferences in the proofs of $U \Rightarrow U'$, $T1 \Rightarrow T1'$ and $V \Rightarrow V'$. If $u_1^{\vec{U}}, u_2^{\vec{T1}}$ and $u_5^{\vec{V}}$ were chosen for the antecedents of the these proofs, the terms for the proofs will be written $u_1^{\vec{U}}(t_1), u_2^{\vec{T1}}(t_2)$ and $u_5^{\vec{V}}(t_5)$, were t_1, t_2 and t_5 are types. Below is $H^{\mathsf{i}}_{L\forall}(P_5)$:

(70) $u_Q^{((s,et),\forall\pi((s,se,\pi),\pi))}(\lambda i\theta)(TYPE)(\lambda i\Sigma)(\lambda i u_2^{\vec{T1}}(t_2))(\lambda i u_1^{\vec{U}}(t_1))(\lambda i u_5^{\vec{V}}(t_5))$

where $TYPE = ((s, \vec{T1}'), (s, \vec{U}'), (s, \vec{V}'), t)$

$\Sigma = \lambda y_1^{(se)} \lambda y_2^{(s,\vec{T1}')} \lambda y_3^{(s,\vec{U}')} \lambda y_4^{(s,\vec{V}')} \Psi[y_3 i/x_1, y_2 i/x_2, y_1 i/x_3, u_4/x_4, y_4 i/x_5]$

The value of the operation defined by the term above when applied to the meanings of the parts of ...every CN ... and a (w, g) is calculated below. In the first step, free variables, u^a, are replaced by terms $z^{(s,a)}i$, the free variable $c^{\vec{W}}$ is replaced by the term $d^{(s,\vec{W})}i$ and the whole term prefaced with λi. u_4 and c are 'invisible' so $\Sigma_\bullet$ is used for $\Sigma[z_4 i/u_4]$ and $\theta_\bullet$ is used for $\theta[di/c]$. The third step relies on the recursive determiner postulate, and the fourth step on the fact that $\Sigma_\bullet$ is $\lambda y_1 \lambda y_2 \lambda y_3 \lambda y_4[\Psi[y_3 i/x_1, y_2 i/x_2, y_1 i/x_3, z_4 i/x_4, y_4 i/x_5]]$

(71) $[\![\lambda i z_Q^{(s,((s,et),\forall\pi((s,se,\pi),\pi)))}(i)(\lambda i\theta_*)(TYPE)(\lambda i\Sigma_*)(\lambda i z_2^{(s,\vec{T1})} i(t_2))(\lambda i z_1^{(s,\vec{U})} i(t_1))(\lambda i z_5^{(s,\vec{V})} i(t_5))]\!](k)(w)$

where k is a $\mathcal{L}^{(\lambda,\Delta)}$assignment such that: $k(z_1^{(s,\vec{U})}) = w' \mapsto m_1(w',g)$, $k(z_2^{(s,\vec{T1})}) = w' \mapsto m_2(w',g)$, $k(z_Q^{(s,(s,et),\forall\pi((s,se,\pi),\pi))}) = w' \mapsto [\![\overline{\text{every}}]\!](w',g)$, $k(z_4^{(s,\vec{T2})}) = w' \mapsto m_4(w',g)$, $k(z_5^{(s,\vec{V})}) = w' \mapsto m_5(w',g)$, $k(d^{(s,\vec{W})}) = w' \mapsto m_{CN}(w',g)$

$= [\![\overline{\text{every}}]\!](w,g)([\![\lambda i\theta_*]\!](k))(D_{type})([\![\lambda i\Sigma_*]\!](k))([\![\lambda i z_2^{(s,\vec{T1})} i(t_2)]\!](k))([\![\lambda i z_1^{(s,\vec{U})} i(t_1)]\!](k))([\![\lambda i z_5^{(s,\vec{V})} i(t_5)]\!](k))$

$= \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\lambda i\theta_*]\!](k)(w))$

$(x^e \mapsto [\![\lambda i\Sigma_*]\!](k)(w)(w' \mapsto x)([\![\lambda i z_2^{(s,\vec{T1})} i(t_2)]\!](k))([\![\lambda i z_1^{(s,\vec{U})} i(t_1)]\!](k))([\![\lambda i z_5^{(s,\vec{V})} i(t_5)]\!](k)))$

$= \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\lambda i\theta_*]\!](k)(w))(x \mapsto [\![\Psi[y_3 i/x_1, y_2 i/x_2, y_1 i/x_3, z_4 i/x_4, y_4 i/x_5]]\!](k'))$,

where k' differs from k at most wrt. i, y_1, y_2, y_3, y_4, for which $k'(i) = w$, $k'(y_1) = (w' \mapsto x)$, $k'(y_2) = [\![\lambda i z_2^{(s,\vec{T1})} i(t_2)]\!](k) = w' \mapsto m_2(w',g)(D_{t_2})$, $k'(y_3) = [\![\lambda i z_1^{(s,\vec{U})} i(t_1)]\!](k) = w' \mapsto m_1(w',g)(D_{t_1})$, $k'(y_4) = [\![\lambda i z_5^{(s,\vec{V})} i(t_5)]\!](k) = w' \mapsto m_5(w',g)(D_{t_5})$

This is a form of the LHS of (69), and leaving it in that form we now consider the RHS of (69). Choosing $v_1^{\vec{U}}, v_2^{\vec{T1}}, v_3^e, v_4^{\vec{T2}}, v_5^{\vec{V}}$ to be associated with the antecedents of the conclusion of P_1, the term associated with P_1 is:

$\Psi[v_1^{\vec{U}}(t_1)/x_1^{\vec{U}'}, v_2^{\vec{T1}}(t_2)/x_2^{\vec{T1}'}, v_3^e/x_3^e, v_4^{\vec{T2}}/x_4^{\vec{T2}}, v_5^{\vec{V}}(t_5)/x_5^{\vec{V}'}]$

Using this we can calculate the RHS of (69) to be:

(72) $\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\lambda i\theta_*]\!](k)(w))(x \mapsto [\![\Psi[z_1 i(t_1)/x_1, z_2 i(t_2)/x_2, z_3 i/x_3, z_4 i/x_4, z_5 i(t_5)/x_5]]\!](l))$

where l is an $\mathcal{L}^{(\lambda,\Delta)}$ assignment such that: $l(i) = w$, $l(z_1) = w' \mapsto m_1(w', g^x_{\text{he}_1})$, $l(z_2) = w' \mapsto m_2(w', g^x_{\text{he}_1})$, $l(z_3) = w' \mapsto [\![\overline{\text{he}_1}]\!](w', g^x_{\text{he}_1}) = w' \mapsto x$, $l(z_4) = w' \mapsto m_4(w', g^x_{\text{he}_1})$, $l(z_5) = w' \mapsto m_5(w', g^x_{\text{he}_1})$

Comparing (71) with the (72) it is clear that they could only differ if one of the following inequalities holds:

(73) $[\![y_3 i]\!](k') \neq [\![z_1 i(t_1)]\!](l)$, $[\![y_2 i]\!](k') \neq [\![z_2 i(t_2)]\!](l)$, $[\![y_1 i]\!](k') \neq [\![z_3 i]\!](l)$, $[\![z_4 i]\!](k') \neq [\![z_4 i]\!](l)$, $[\![y_4 i]\!](k') \neq [\![z_5 i(t_5)]\!](l)$

But all of these inequalities are impossible. Therefore (69) is true. □

3.2.5 A way of thinking about how the polymorphic proposal works

One may feel a little mystified about how the polymorphic proposal manages to account for ambiguity, in particular puzzled how it relates to the familiar 'Quantifying-In' account. Consider the *de-re* interpretation of a man in

(74) John believes that a man came in

Familiarity with the 'Quantifying-in' account gives one the impression that to obtain the reading one must 'deal' with John and believes before one deals with a man. This makes the polymorphic categorial analysis (repeated in (75)) puzzling, because one deals with John and believes *after* one deals with a man.

```
(75)  John    believes that      a man              came in
      ----    -------------   ------------          -------
       np       (s\np)/s      ∀X.X/(X\np)            s\np
                                              ---------------------------
                                              ((s\np)\((s\np)/s))\np
                              -------------------------------------------
                                        (s\np)\((s\np)/s)
              -------------------------------------------
                            s\np
      -------------------------------
                    s
```

The following comments may be of some help here. We will call the derived version of came in, of category ((s\np)\((s\np)/s))\np, (came in)^R, and make the simplifying assumption that we are using an extensional tying. The *denotation* of (came in)^R may be compared with the *assignment-meaning* of HE_{np} $\mathsf{HE}_{(s\backslash np)/s}$ he_1 came in, by which I mean that given either of (i) or (ii) below, one could calculate the other:

(i) $[\![(\text{came in})^R]\!](w^*, g^*)$

(ii) $(g \mapsto [\![\mathsf{HE}_{np}\ \mathsf{HE}_{(s\backslash np)/s}\ \mathsf{he}_1\ \text{came in}]\!](w^*, g))$

For example the result applying (i) to three arguments, x^e, $y^{(tet)}$ and z^e is exactly the same the result of applying (ii) to an assignment which assigns these values to he_1, $\mathrm{HE}_{(s\backslash np)/s}$, HE_{np}. Therefore one may look at (came in)^R as being a coded form HE_{np} $\mathsf{HE}_{(s\backslash np)/s}$ he_1 came in.

It is less obvious to see, but also true that the *denotation* of the result of combining the polymorphic a man with (came in)^R may be compared with the *assignment-meaning* of the result of 'quantifying-in' a man into HE_{np} $\mathsf{HE}_{(s\backslash np)/s}$ he_1 came in:

(i′) $[\![\text{a man came in}]\!](w^*, g^*)$

(ii′) $(g \mapsto [\![\mathsf{HE}_{np}\ \mathsf{HE}_{(s\backslash np)/s}\ \text{a man came in}]\!](w^*, g))$

For example applying (i′) to two arguments $y^{(tet)}$ and z^e is exactly the same as applying (ii′) to an assignment which assigns these values to $\mathrm{HE}_{(s\backslash np)/s}$, HE_{np}. Therefore, what one gets by using the polymorphic quantifier together with the derived version of (came in)^R is a coded version of one what one gets by combining the ordinary quantifier with HE_{np} $\mathsf{HE}_{(s\backslash np)/s}$ he_1 came in.

The final step of the categorial analysis is to apply (i′) to the pair of arguments 6believes that$]\!](w^*, g^*)$ and $[\![\text{John}]\!](w^*, g^*)$. Recalling what was just said above, this must give the same result as applying (ii′) to an assignment which assigns these values to $\mathrm{HE}_{(s\backslash np)/s}$ and HE_{np}, in other words: $[\![\mathsf{HE}_{np}\ \mathsf{HE}_{(s\backslash np)/s}\ \text{a man came in}]\!](w^*, h)$, where $h(\mathsf{HE}_{(s\backslash np)/s}) = [\![\text{believes that}]\!](w^*, g^*)$, and $h(\mathsf{HE}_{np}) = [\![\text{John}]\!](w^*, g^*)$

Therefore we have described a way of obtaining the desired reading, using the 'Quantifying-In' mechanism, and yet 'dealing' with a man *before* believes that and John. One can look upon the polymorphic categorial analysis as being essentially the same thing.

Having spent some time explaining the coverage of $\mathcal{LT}^{25}$ with respect to determiner-containing sentences, in the next section, we turn to ambiguous, junction-containing sentences.

3.3 Recursively ambiguous sentences with Junctions

3.3.1 a man came in and sat down

Hypothesis 3 and Hypothesis 4 each entail the existence of a reading of a man came in and sat down. Hypothesis 3 entails recursive ambiguity wrt. a man, and we can be sure that $\mathcal{LT}^{25}$ accounts for this reading; the above comments on the general explanation by $\mathcal{LT}^{25}$ of recursive ambiguity with respect to quantifiers, coupled with the fact that $\mathcal{LT}^{25}$ accounts for the recursive ambiguity of he_1 came in and sat down, guarantees this.

Hypothesis 4 entails recursive ambiguity wrt. came in and sat down, and this means that there should be a reading, r, of a man came in and sat down such that

(76) whatever situation s, a man came in and sat down is true in s on r iff a man came in is true in s and a man sat down is true in s

The corresponding condition for $\mathcal{LT}^{25}$ is

(77) there is a $\overline{\text{a man came in and sat down}}$ such that whatever model, $\langle \Im, \langle w, g\rangle\rangle$, whatever $\overline{\text{a man came in}}$, whatever $\overline{\text{a man sat down}}$,
$[\![\overline{\text{a man came in and sat down}}]\!](w,g) = 1$ iff
$[\![\overline{\text{a man came in}}]\!](w,g) = 1$ and $[\![\overline{\text{a man sat down}}]\!](w,g) = 1$

The disambiguation in (78) has this property:

(78)
$$\dfrac{\dfrac{\dfrac{\text{a}}{(\forall Y.Y/(Y\backslash np))/cn} \quad \dfrac{\text{man}}{cn}}{\forall Y.Y/(Y\backslash np)} \quad \dfrac{\text{came in}}{s\backslash np} \quad \dfrac{\text{and}}{\forall X.((X\backslash X)/X)} \quad \dfrac{\text{sat down}}{s\backslash np}}{s}\text{PROOFn}$$

(79) PROOFn:
$$\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{s\backslash np \Rightarrow s\backslash(np^{\wedge s})} \quad \dfrac{\dfrac{\vdots}{np^{\wedge s}, s\backslash(np^{\wedge s}) \Rightarrow s}}{\forall Y np^{\wedge Y}, s\backslash(np^{\wedge s}) \Rightarrow s}\forall L}{\forall Y np^{\wedge Y}, s\backslash np, (s\backslash(np^{\wedge s}))\backslash(s\backslash(np^{\wedge s})) \Rightarrow s}\backslash L \quad \dfrac{\vdots}{s\backslash np \Rightarrow s\backslash(np^{\wedge s})}}{\forall Y np^{\wedge Y}, s\backslash np, ((s\backslash(np^{\wedge s}))\backslash(s\backslash(np^{\wedge s})))/(s\backslash(np^{\wedge s})), s\backslash np \Rightarrow s}/L}{\forall Y np^{\wedge Y}, s\backslash np, \forall X.((X\backslash X)/X), s\backslash np \Rightarrow s}\forall L$$

The chain of equivalences in (80) suffices to show that (78) has the property described in (77).

(80) (i) $[\![(78)]\!](w,g) = 1$

iff

(ii) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w,g))(x^e \mapsto [\![\overline{\text{sat down}}]\!](w,g)(w' \mapsto x)))(\mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w,g))(x^e \mapsto [\![\overline{\text{came in}}]\!](w,g)(w' \mapsto x)))$

iff (iii) $[\![\overline{\text{a man came in}}]\!](w,g) = 1$ and $[\![\overline{\text{a man sat down}}]\!](w,g) = 1$

The equivalence of (ii) to and (iii) is obvious. For the equivalence of (i) and (ii) we need to calculate the denotation of (78). Firstly, assuming that the antecedents of the concluding sequent of (79) are associated with the variables $u_Q^{\forall\pi((s,se,\pi),\pi)}$, $u_2^{(se,t)}$, $u_J^{\forall\pi((s,\pi),(s,\pi),\pi)}$, $u_4^{(se,t)}$, then the term associated with (79) is:

(81) $u_J^{\forall\pi((s,\pi),(s,\pi),\pi)}(TYPE)$
$(\lambda i\lambda y_1^{(s,(s,se,t),t)}[y_1 i(\lambda i u_4^{(se,t)})])$
$(\lambda i\lambda y_2^{(s,(s,se,t),t)}[y_2 i(\lambda i u_2^{(se,t)})])$
$(\lambda i u_Q^{\forall\pi((s,se,\pi),\pi)}(t))$
where $TYPE = ((s,(s,se,t),t),t)$

Using this term, the denotation of (78) is calculated below (abbreviations $\mathsf{C} = w \mapsto [\![\overline{\text{came in}}]\!](w,g)$, $\mathsf{S} = w \mapsto [\![\overline{\text{sat down}}]\!](w,g)$, $\mathsf{M} = w \mapsto [\![\overline{\text{man}}]\!](w,g)$

(82) $[\![(78)]\!](w,g)$

$= [\![\overline{\text{and}}]\!](w,g)(D_{TYPE})(w' \mapsto d_1^{(s,(s,se,t),t)} \mapsto d_1(w')(w'' \mapsto \mathsf{C}w''))(w' \mapsto d_2^{(s,(s,se,t),t)} \mapsto d_2(w')(w'' \mapsto \mathsf{S}w''))(w' \mapsto [\![\overline{\text{a man}}]\!](w',g)(D_t))$

$= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{a man}}]\!](w,g)(D_t)(w'' \mapsto \mathsf{C}w''))([\![\overline{\text{a man}}]\!](w,g)(D_t)(w'' \mapsto \mathsf{S}w''))$

$= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w,g))(x^e \mapsto [\![\overline{\text{sat down}}]\!](w,g)(w' \mapsto x)))$
$(\mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\text{man}}]\!](w,g))(x^e \mapsto [\![\overline{\text{came in}}]\!](w,g)(w' \mapsto x)))$

Given (82) the equivalence of (i) and (ii) in (80) is immediate.

3.3.2 every man and woman died

Hypothesis 3 and Hypothesis 4 each entail the existence of a reading of every man and woman died. Hypothesis 3 entails recursive ambiguity wrt. every man and woman, and we can be sure that $\mathcal{LT}^{25}$ accounts for this reading if it accounts for the only reading of he_1 is a man and woman; $\mathcal{LT}^{25}$ clearly does.

Hypothesis 4 entails recursive ambiguity wrt. man and woman, and this means that there should be a reading, r, of every man and woman died such that

(83) whatever situation s, every man and woman died is true in s on r iff every man died is true in s and every woman died is true in s

The corresponding condition for $\mathcal{LT}^{25}$ is

(84) there is a $\overline{\text{every man and woman died}}$ such that whatever model, $\langle \Im, \langle w, g \rangle \rangle$, whatever $\overline{\text{every man died}}$, whatever $\overline{\text{every woman died}}$,
$\llbracket \overline{\text{every man and woman died}} \rrbracket (w, g) = 1$ iff
$\llbracket \overline{\text{every man died}} \rrbracket (w, g) = 1$ and $\llbracket \overline{\text{every woman died}} \rrbracket (w, g) = 1$

The disambiguation in (85) has this property:

(85)
$$\cfrac{\cfrac{\text{every}}{(\forall Y.Y/(Y\backslash np))/cn} \quad \cfrac{\text{man}}{cn} \quad \cfrac{\text{and}}{\forall X.((X\backslash X)/X)} \quad \cfrac{\text{woman}}{cn} \quad \cfrac{\text{died}}{s\backslash np}}{s}\,\text{PROOFn}$$

(86)
$$\cfrac{\cfrac{\cfrac{\cfrac{\vdots}{cn \Rightarrow cn^{\vee np^{\wedge s}}} \quad \cfrac{\cfrac{\vdots}{(\forall Y np^{\wedge Y})/cn \Rightarrow (np^{\wedge s})/cn} \quad \cfrac{\vdots}{np^{\wedge s}, s\backslash np \Rightarrow s}}{(\forall Y np^{\wedge Y})/cn, cn^{\vee np^{\wedge s}}, s\backslash np \Rightarrow s}\backslash L}{(\forall Y np^{\wedge Y})/cn, cn, cn^{\vee np^{\wedge s}}\backslash cn^{\vee np^{\wedge s}}, s\backslash np \Rightarrow s}\backslash L \quad \cfrac{\vdots}{cn \Rightarrow cn^{\vee np^{\wedge s}}}}{(\forall Y np^{\wedge Y})/cn, cn, (cn^{\vee np^{\wedge s}}\backslash cn^{\vee np^{\wedge s}})/cn^{\vee np^{\wedge s}}, cn, s\backslash np \Rightarrow s}/L}{(\forall Y np^{\wedge Y})/cn, cn, \forall X.((X\backslash X)/X), cn, s\backslash np \Rightarrow s}\forall L$$

The chain of equivalences in (87) suffices to show that (85) has the property described in (84).

(87) (i) $\llbracket (85) \rrbracket (w, g) = 1$ iff
(ii)
$\mathcal{AND}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\mathcal{EVERY}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\llbracket \overline{\text{man}} \rrbracket (w, g))(x^e \mapsto \llbracket \overline{\text{died}} \rrbracket (w, g)(w' \mapsto x)))(\mathcal{EVERY}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)(\llbracket \overline{\text{woman}} \rrbracket (w, g))(x^e \mapsto \llbracket \overline{\text{died}} \rrbracket (w, g)(w' \mapsto x)))$
iff (iii) $\llbracket \overline{\text{every man died}} \rrbracket (w, g) = 1$ and $\llbracket \overline{\text{every woman died}} \rrbracket (w, g) = 1$

The equivalence of (ii) to and (iii) is obvious. For the equivalence of (i) and (ii) we need to calculate the denotation of (85). Firstly, assuming that the antecedents of the concluding sequent of (86) are associated with the variables $u_Q^{((s,et),\forall\pi((s,se,\pi),\pi))}$, $u_2^{(e,t)}$, $u_J^{\forall\pi((s,\pi),(s,\pi),\pi)}$, $u_4^{(e,t)}$, $u_5^{(se,t)}$then the term associated with (86) is:

(88)
$$\begin{aligned} & u_J^{\forall\pi((s,\pi),(s,\pi),\pi)}(TYPE) \\ & \qquad (\lambda i \lambda y_1^{(s,(s,et),(s,se,t),t)}[yi(\lambda i u_4^{(e,t)})]) \\ & \qquad (\lambda i \lambda y_2^{(s,(s,et),(s,se,t),t)}[yi(\lambda i u_2^{(e,t)})]) \\ & \qquad (\lambda i \lambda y_3^{(s,et)}[u_Q^{((s,et),\forall\pi((s,se,\pi),\pi))}(\lambda i y i)(t)]) \\ & \qquad (\lambda i u_5) \end{aligned}$$

where $TYPE = ((s,(s,et),(s,se,t),t),(s,se,t),t)$

Using the above term, the denotation of (85) is calculated below (abbreviations, $\mathsf{M} = w \mapsto [\![\overline{\mathsf{man}}]\!](w,g)$, $\mathsf{W} = w \mapsto [\![\overline{\mathsf{woman}}]\!](w,g)$, $\mathsf{D} = w \mapsto [\![\overline{\mathsf{died}}]\!](w,g)$)

(89) $[\![(85)]\!](w,g)$

$= [\![\overline{\mathsf{and}}]\!](w,g)(D_{TYPE})(w' \mapsto d_1^{(s,(s,et),(s,se,t),t)} \mapsto d_1(w')(w'' \mapsto \mathsf{W}w''))(w' \mapsto d_2^{(s,(s,et),(s,se,t),t)} \mapsto d_2(w')(w'' \mapsto \mathsf{M}w''))(w' \mapsto d_3^{(s,et)} \mapsto [\![\overline{\mathsf{every}}]\!](w',g)(d_3)(D_t))(w' \mapsto [\![\overline{\mathsf{died}}]\!](w',g))$

$=$

$\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\mathsf{every}}]\!](w,g)(w' \mapsto \mathsf{W}w')(D_t)(w' \mapsto \mathsf{D}w'))([\![\overline{\mathsf{every}}]\!](w,g)(w' \mapsto \mathsf{M}w')(D_t)(w' \mapsto \mathsf{D}w'))$

$= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{W}w)(x \mapsto \mathsf{D}w(w' \mapsto x)))(\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathsf{M}w)(x \mapsto \mathsf{D}w(w' \mapsto x)))$

$= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\mathsf{woman}}]\!](w,g))(x^e \mapsto [\![\overline{\mathsf{died}}]\!](w,g)(w' \mapsto x)))(\mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\mathsf{man}}]\!](w,g))(x^e \mapsto [\![\overline{\mathsf{died}}]\!](w,g)(w' \mapsto x)))$

Given (89) the equivalence of (i) and (ii) in (87) is immediate.

Having seen that a couple of ambiguous junction-containing sentences are accounted for, we consider in the next section whether $\mathcal{LT}^{25}$ is in accord with Hypothesis 4.

3.3.3 General recursive ambiguity of S[X$_1$ JUNCT X$_2$] wrt. X$_1$ JUNCT X$_2$

Suppose α is ...X$_1$ JUNCT X$_2$ .... Suppose α_1 is α[X$_1$/X$_1$ JUNCT X$_2$] and α_2 is α[X$_2$/X$_1$ JUNCT X$_2$]. The condition for a THEORY OF REFERENCE to be in accord with Hypothesis 4 is:

(90) whatever disambiguation of α_1, whatever disambiguation of α_2, there is a disambiguation of ...X$_1$ JUNCT X$_2$ ... such that

$[\![\overline{\alpha}]\!](w,g) = 1$

iff $\mathcal{JUNCT}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)([\![\overline{\alpha_2}]\!](w,g))([\![\overline{\alpha_1}]\!](w,g))$

If the THEORY OF REFERENCE under consideration was for example $\mathcal{T}^8$, the non-local account of junctions that was described in section 2.4.1 of Chapter 5, it would be quite easy to demonstrate (90). This is because the obvious candidate for the required disambiguation of ...X$_1$ JUNCT X$_2$ ... is that one which has the supposed disambiguations of ...X$_1$ ... and ...X$_2$ ... as *subparts*: such disambiguations of ...X$_1$ JUNCT X$_2$ ... are provided for by the 'Conjunction Reduction' operation.

For $\mathcal{LT}^{25}$ matters are more difficult because no disambiguation of ...X$_1$ ... or ...X$_2$ ... can be used as a *subpart* of a disambiguation of ...X$_1$ JUNCT X$_2$ .... As was the case for the quantifiers, however, (90) can still be demonstrated, essentially by looking for a 'factorisation' of the ...X$_1$ JUNCT X$_2$ ... denotation featuring the denotations of ...X$_1$ ... and ...X$_2$ ..., which we will now do.

The sequence of lexical categories for α_1 may be supposed to be an instance of the pattern $U, T1, V$, where T_1 is not empty and both U and V may be empty. The sequence of lexical categories for α_2 may be supposed to be an instance of the pattern $U, T2, V$. The supposition is that there is a proof P_1 of $U, T1, V \Rightarrow$ s, and a proof R_1 of $U, T2, V \Rightarrow$ s:

there is a proof P_1 of U,T1, V $\Rightarrow$ s and a proof R_1 of U,T2,V $\Rightarrow$ s:

$$\frac{\vdots}{U, T1, V \Rightarrow \mathrm{s}} \qquad\qquad \frac{\vdots}{U, T2, V \Rightarrow \mathrm{s}}$$

We have to show that on the basis of this supposition there exists a proof P_6 of U,$T1$, $\forall$X(X\X)/X, $T2$,V $\Rightarrow$ s, and the disambiguations associated with P_1, R_1 and P_6 are semantically related in the way required by (90).

The required disambiguation of ... X_1 JUNCT X_2 ... is that associated with the following proof:

P_6, a proof of $U, T1, \forall X.(X\backslash X)/X, T2, V \Rightarrow$ s:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\vdots}{U', T1, V' \Rightarrow \mathrm{s}}}{T1 \Rightarrow (\mathrm{s}/V')\backslash U'}\text{Slash R} \qquad \cfrac{\vdots}{U, (\mathrm{s}/V')\backslash U', V \Rightarrow \mathrm{s}}}{U, T1, ((\mathrm{s}/V')\backslash U')\backslash((\mathrm{s}/V')\backslash U'), V \Rightarrow \mathrm{s}}(\backslash L) \qquad \cfrac{\cfrac{\vdots}{U', T2, V' \Rightarrow \mathrm{s}}}{T2 \Rightarrow (\mathrm{s}/V')\backslash U'}\text{Slash R}}{U, T1, (((\mathrm{s}/V')\backslash U')\backslash((\mathrm{s}/V')\backslash U'))/((\mathrm{s}/V')\backslash U'), T2, V \Rightarrow \mathrm{s}}/\mathrm{L}}{U, T1, \forall X.(X\backslash X)/X, T2, V \Rightarrow \mathrm{s}}\forall\mathrm{L}$$

Some comments on P_6. The first step towards inferring the existence of this proof concerns the presence of polymorphic categories in U, V in P_1 and R_1. If there are, it is assumed that in P_1 and R_1 are ($\forall$L) inferences, each instantiating some category variable Z to some category x. We infer the existence of proofs P_2, R_2, in which the only ($\forall$L) inference concern $\forall$'s that are in T_1 or T_2. P_2 and R_2 prove the sequents $U', T1, V' \Rightarrow$ s and $U', T2, V' \Rightarrow$ s, where in each of U' and V' differ from U and V by replacement of any category $\forall Z.y$ with $y[x/Z]$. P_2 is the proof associated with the leftmost ellipsis in P_6. R_2 is the proof associated with the rightmost ellipsis in P_6. By adding a series of (\R) and (/ R) steps to P_2 and R_2 one obtains $T1 \Rightarrow (\mathrm{s}/V')\backslash U'$ and $T2 \Rightarrow (\mathrm{s}/V')\backslash U'$. The sequent, $T1 \Rightarrow (\mathrm{s}/V')\backslash U'$, can be used as the minor premise in a (\L) inference, the major premise being $U, ((\mathrm{s}/V')\backslash U'), V \Rightarrow$ s. This major premise can be given the obvious proof that proceeds by a series of Slash-Left inferences, and this is what the middle ellipis in P_6 represents. Also in the course of P_6, the $\forall$L inferences are restored that were removed from P_1 and R_1 to make P_2 and R_2. The result of the (\L) inference is $U, T1, ((\mathrm{s}/V')\backslash U')\backslash((\mathrm{s}/V')\backslash U'), V \Rightarrow$ s. The sequent $T2 \Rightarrow (\mathrm{s}/V')\backslash U'$ can be used as the minor premise in (/L) inference, the major premise being $U, T1, ((\mathrm{s}/V')\backslash U')\backslash((\mathrm{s}/V')\backslash U'), V \Rightarrow$ s, just proved. In this way a proof of $U, T1, (((\mathrm{s}/V')\backslash U')\backslash((\mathrm{s}/V')\backslash U'))/((\mathrm{s}/V')\backslash U'), T2, V \Rightarrow$ s is obtained. $(((s/V')\backslash U')\backslash((s/V')\backslash U'))/((s/V')\backslash U')$ is an instance of the schema $(x\backslash x)/x$, and so we have the premise for a (∀L) inference, allowing the conclusion of $U, T1, \forall X.(X\backslash X)/X, T2, V \Rightarrow s$. The side-condition to the (∀L) inference will not be violated.

What it remains to show is the relationship between the disambiguations associated with P_1, R_1 and P_6 is as required by (90). This requires us to show that:

$$\text{(91) } \mathcal{G}_{P_5}(m_1^{\vec{U}}, m_2^{\vec{T1}}, [\![\overline{\text{and}}]\!], m_4^{\vec{T2}}, m_5^{\vec{V}})(w,g)$$
$$= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(\mathcal{G}_{R_1}(m_1^{\vec{U}}, m_2^{\vec{T2}}, m_4^{\vec{V}})(w,g))(\mathcal{G}_{P_1}(m_1^{\vec{U}}, m_2^{\vec{T1}}, m_4^{\vec{V}})(w,g))$$

That (91) is true

Consider first the LHS of (91) Suppose a particular P_2, and that with $x_1^{\vec{U}'}, x_2^{\vec{T1}}, x_5^{\vec{V}'}$ chosen for the antecedent categories, $H^i_{L\forall}(P_2) = \Psi^1$. Correspondingly for a particular R_2, with $x_1^{\vec{U}'}, x_4^{\vec{T2}}, x_5^{\vec{V}'}$ chosen for the antecedent categories, suppose $H^i_{L\forall}(R_2) = \Psi^2$ Then if $u_1^{\vec{U}}, u_2^{\vec{T1}}, u_J^{\forall\pi((s,\pi),(s,\pi),\pi)}, u_4^{\vec{T2}}, u_5^{\vec{V}}$ are chosen for the antecedents of the concluding sequent of P_6, $H^i_{L\forall}(P_6)$ is:

(92) $u_J^{\forall\pi((s,\pi),(s,\pi),\pi)}(TYPE)(\lambda i\beta_2)(\lambda i\beta_1)(\lambda i u_5^{\vec{V}}(t_5))(\lambda i u_1^{\vec{U}}(t_1))$

where $TYPE = ((s,\vec{V}'),(s,\vec{U}'),t)$

$\beta_2 = \lambda y_1^{(s,\vec{V}')}\lambda y_2^{(s,\vec{U}')}$

$\Psi^2[y_1^{(s,\vec{V}')}i/x_5^{\vec{V}'}, y_2^{(s,\vec{U}')}i/x_1^{\vec{U}'}, u_4^{\vec{T2}}/x_4^{\vec{T2}}]$

$\beta_1 = \lambda y_1^{(s,\vec{V}')}\lambda y_2^{(s,\vec{U}')}$

$\Psi^1[y_1^{(s,\vec{V}')}i/x_5^{\vec{V}'}, y_2^{(s,\vec{U}')}i/x_1^{\vec{U}'}, u_2^{\vec{T1}}/x_2^{\vec{T1}}]$

The LHS of (91) is the value of the operation that is defined by the term above applied to the meanings of the lexical parts of S[X₁ and X₂] and a (w,g). Note that in the first step each free variables, u^a, of the term above is replaced by the term $z^{(s,a)}i$. and the whole term prefaced with λi. Because u_2 and u_4 are 'invisible' β^2_* is used for $\beta^2[z_4^{(s,\vec{T2})}(i)/u_4^{\vec{T2}}]$, and β^1_* is used for $\beta^1[z_2^{(s,\vec{T1})}(i)/u_2^{\vec{T1}}]$.

(93) $[\![\lambda i z_J^{(s,\forall\pi((s,\pi),(s,\pi),\pi))}i(TYPE)(\lambda i\beta^2_*)(\lambda i\beta^1_*)(\lambda i z_5^{(s,\vec{V})}i(t_5))(\lambda i z_1^{(s,\vec{U})}i(t_1))]\!](k)(w)$

where k is a $\mathcal{L}^{(\lambda,\Delta)}$assignment such that: $k(z_1^{(s,\vec{U})}) = w' \mapsto m_1(w',g)$, $k(z_2^{(s,\vec{T1})}) = w' \mapsto m_2(w',g)$, $k(z_J^{(s,\forall\pi((s,\pi),(s,\pi),\pi))}) = w' \mapsto [\![\overline{\text{and}}]\!](w',g)$, $k(z_4^{(s,\vec{T2})}) = w' \mapsto m_4(w',g)$, $k(z_3^{(s,\vec{V})}) = w' \mapsto m_5(w',g)$

$= [\![\overline{\text{and}}]\!](w,g)(D_{TYPE})([\![\lambda i\beta^2_*]\!](k))([\![\lambda i\beta^1_*]\!](k))([\![\lambda i z_5^{(s,\vec{V})}i(t_5)]\!](k))([\![\lambda i z_1^{(s,\vec{U})}i(t_1)]\!](k))$

$= \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(([\![\lambda i\beta^2_*]\!](k)(w))([\![\lambda i z_5^{(s,\vec{V})}i(t_5)]\!](k))([\![\lambda i z_1^{(s,\vec{U})}i(t_1)]\!](k)))$

$(([\![\lambda i\beta^1_*]\!](k)(w))([\![\lambda i z_5^{(s,\vec{V})}i(t_5)]\!](k))([\![\lambda i z_1^{(s,\vec{U})}i(t_1)]\!](k)))$

Given that the abvoe is a form of the LHS of (91) is is clear that what is required for the truth of (91) are the following identities:

(94) (a) $[\![\lambda i\beta^2_*]\!](k)(w)([\![\lambda i z_5^{(s,\vec{V})}i(t_5)]\!](k))([\![\lambda i z_1^{(s,\vec{U})}i(t_1)]\!](k)) = \mathcal{G}_{R_1}(m_1^{\vec{U}}, m_2^{\vec{T2}}, m_4^{\vec{V}})(w,g)$

(b) $[\![\lambda i\beta^1_*]\!](k)(w)([\![\lambda i z_5^{(s,\vec{V})}i(t_5)]\!](k))([\![\lambda i z_1^{(s,\vec{U})}i(t_1)]\!](k)) = \mathcal{G}_{P_1}(m_1^{\vec{U}}, m_2^{\vec{T1}}, m_4^{\vec{V}})(w,g)$

Consider first the LHS of (94a):

(95) $[\![\lambda i\beta^2_*]\!](k)(w)([\![\lambda i z_5^{(s,\vec{V})}i(t_5)]\!](k))([\![\lambda i z_1^{(s,\vec{U})}i(t_1)]\!](k))$

$= [\![\Psi^2[y_2^{(s,\vec{U}')}i/x_1^{\vec{U}'}, y_1^{(s,\vec{V1}')}i/x_5^{\vec{V1}'}, z_4^{(s,\vec{T2})}i/x_4^{\vec{T2}}]]\!](k')$

where k' differs from k at most wrt. i, y_1, y_2, for which $k'(i) = w$, $k'(y_1^{(s,\vec{V1}')}) = [\![\lambda i z_5^{(s,\vec{V})}i(t_5)]\!](k) = w' \mapsto m_5(w',g)(D_{t_5})$ $k'(y_2^{(s,\vec{U}')}) = [\![\lambda i z_1^{(s,\vec{U})}i(t_1)]\!](k) = w' \mapsto m_1(w',g)(D_{t_1})$

For the RHS of (94a) we must consider the operation associated with R_1, the proof associated with the disambiguation of $\ldots X_2 \ldots$. Choosing $u_1^{\vec{U}}, u_4^{\vec{T2}}, u_5^{\vec{V}}$ to be associated with the antecedents of the conclusion of R_1, the term associated with R_1 is

$\Psi^2[u_1^{\vec{U}}(t_1)/x_1^{\vec{U}'}, u_4^{\vec{T}2}/x_4^{\vec{T}2}, u_5^{\vec{V}}(t_5)/x_5^{\vec{V}'}]$

The RHS of (94a) is the value of the operation applied the meanings of the lexical parts of S$[X_2]$ and to (w, g):

(96) $[\![\Psi^2[z_1^{(s,\vec{U})}i(t_1)/x_1^{\vec{U}'}, z_4^{(s,\vec{T}2)}i/x_4^{\vec{T}2}, z_5^{(s,\vec{V})}i(t_5)/x_5^{\vec{V}'}]]\!](l)$
where l is an $\mathcal{L}^{(\lambda,\Delta)}$ assignment such that: $l(i) = w$, $l(z_1^{(s,\vec{U})}) = w' \mapsto m_1(w', g)$, $l(z_4^{(s,\vec{T}2)})$ $= w' \mapsto m_4(w', g)$, $l(z_5^{(s,\vec{V})}) = w' \mapsto m_5(w', g)$

Comparing (95) with the (96) it is clear that they could only differ if one of the following inequalities holds:

(97) $[\![y_2 i]\!](k') \neq [\![z_1 i(t_1)]\!](l)$, $[\![z_4 i]\!](k') \neq [\![z_4 i]\!](l)$, $[\![y_1 i]\!](k') \neq [\![z_5 i(t_5)]\!](l)$

But all of these inequalities are impossible. Therefore (94a) is true. In an exactly similar fashion one can show that (94b) holds and therefore that (91) is true □

3.4 Ambiguities associated with embedding construction

Certain kinds of ambiguity fall outside the considerations of the previous two sections. These are ambiguities associated with embedding constructions:

(98) John believes every man loves a woman

Of the several readings of this, only two readings are associated with recursive ambiguity wrt. to either of the contained QNP's. Besides these there is one reading according to which every man and a woman are interpreted *de-re* and *de-dicto* respectively, then another reading according to which they are interpreted *de-dicto* and *de-re* respectively. Finally there are *two* readings according to which both every man and a woman are interpreted *de-dicto*. These then are further ambiguity phenomena that $\mathcal{LT}^{25}$ ought to capture. The reader will probably be able to imagine how $\mathcal{LT}^{25}$ accounts for these further ambiguities and it will not be spelt out here. What instead we will do is spell out how a more difficult case would be handled namely a certain reading of John believes that Mary thinks that every man loves a woman. It is a reading that might be described as having the following features:

(i) every man is interpreted *de-re with respect to* thinks and *de-re with respect to* believes
(ii) a woman is interpreted *de-re with respect to* thinks and *de-dicto with respect to* believes

However, this description uses vocabulary which we have not defined. In place of this we have to use the following considerably lengthier description:

(99) there is a reading r_1 of John believes Mary thinks every man loves a woman

there is a reading r_2 of John believes Mary thinks he_1 loves a woman

there is a reading r_3 of Mary thinks he_1 loves a woman such that r_1 and r_2 are related according to (i), r_2 and r_3 are related according to (ii) and r_3 has the property in (iii):

(i) whatever situation s, John believes Mary thinks every man loves a woman is true on r_1 in s iff

EVERY $\{x : \text{he}_1 \text{ is man is true in } s^x_{\text{he}_1}\}$

$\{x : \text{John believes Mary thinks he}_1 \text{ loves a woman is true on } r_2 \text{ in } s^x_{\text{he}_1}\}$

(ii) whatever situation s, if John believes Mary thinks he_1 loves a woman is true on r_2 and Mary thinks he_1 loves a woman is true on r_3 in s then a proposition that John believes is true is true in s

(iii) whatever situation s, Mary thinks he_1 loves a woman is true on r_3 in s iff

A $\{y : \text{he}_2 \text{ is a woman is true in } s^y_{\text{he}_2}\}$

$\{y : \text{Mary thinks he}_1 \text{ loves he}_2 \text{ is true in } s^y_{\text{he}_2}\}$

(i) gives a characteristic of the reading, r_1, according to which every man occurs *de-re* in John believes Mary thinks he_1 loves a woman. (iii) gives a characteristic of the reading, r_3, according to which a woman occurs *de-re* in Mary thinks he_1 loves a woman, and (ii) relates the reading r_3 of Mary thinks he_1 loves a woman to the reading r_1 of John believes Mary thinks he_1 loves a woman.

The truth intuition makes reference to a truth predicate and $\mathcal{LT}^{25}$ does not cover this construction. The details of the extension of $\mathcal{LT}^{25}$ will not be given - it may be inferred from the account given in Chapter 5, section 2.3.

So to begin the demonstration that $\mathcal{LT}^{25}$ does entail (99), we firstly note that there is a disambiguation, β_3, of Mary thinks he_1 loves a woman, that meets the condition corresponding to (99iii) - we can sure of this because $\mathcal{LT}^0$ accounts for recursive ambiguity facts:

$$[\![\beta_3]\!](w,g) =$$
$$\mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)$$
$$([\![\overline{\text{woman}}]\!](w,g))$$
$$(y^e \mapsto [\![\overline{\text{Mary thinks that}}]\!](w,g)(w' \mapsto [\![\overline{\text{loves}}]\!](w',g)(w' \mapsto y)(w' \mapsto g(\text{he}_1))))$$

Secondly, a disambiguation, β_2, of John believes Mary thinks he_1 loves a woman can be found such that the pair consisting of β_2 and β_3 meets the condition corresponding to (99ii) - β_2 is simply the natural disambiguation of John believes Mary thinks he_1 loves a woman that has β_3 as a part.

$[\![\beta_2]\!](w,g) =$

$[\![\overline{\text{John believes that}}]\!](w,g)$
$\quad (w' \mapsto \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w',g)$
$\qquad ([\![\overline{\text{woman}}]\!](w',g))$
$\qquad (y^e \mapsto [\![\overline{\text{Mary thinks that}}]\!](w',g)(w' \mapsto [\![\overline{\text{loves}}]\!](w',g)(w' \mapsto y)(w' \mapsto g(\text{he}_1)))$
$\qquad)$
$\quad)$

Given this, a disambiguation, β_1, of John believes Mary thinks every man loves a woman will be related to β_2 in the way required by (the correspondent of) (99i) if β_1 has the following denotation (using the abbreviations M for man, W for woman, JBT for John believes that, MTT for Mary thinks that) :

(100) $[\![\beta_1]\!](w,g) =$
$\quad \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)$
$\qquad ([\![\overline{\text{M}}]\!](w,g))$
$\qquad (x^e \mapsto [\![\overline{\text{JBT}}]\!](w,g)(w' \mapsto \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w',g)$
$\qquad\qquad ([\![\overline{\text{W}}]\!](w',g))$
$\qquad\qquad (y^e \mapsto [\![\overline{\text{MTT}}]\!](w',g)(w' \mapsto [\![\overline{\text{L}}]\!](w',g)(w' \mapsto y)(w' \mapsto x)))$
$\qquad\quad)$
$\qquad)$

The required disambiguation of John believes that Mary thinks that every man loves a woman, β_1, is associated with the following proof, P_1, shown below. The disambiguation has for subparts disambiguations of JBT, MTT, every man, loves and a woman.

(101)

$$\dfrac{\vdots}{a/a,\ \text{np},\ \text{TV},\ \text{np} \Rightarrow a}$$

$$\dfrac{\text{TV} \Rightarrow (a\backslash(a/a)\backslash\text{np})/\text{np} \qquad \dfrac{\vdots}{a/a,\ \text{np},\ (\text{s}\backslash(a/a))\backslash\text{np} \Rightarrow a}}{\dfrac{a/a,\ \text{np},\ \text{TV},\ (\text{s}\backslash(a/a)\backslash\text{np})\backslash((\text{s}\backslash(a/a)\backslash\text{np})/\text{np}) \Rightarrow a}{(a/a),\ \text{np},\ \text{TV},\ \text{np}^{\vee y} \Rightarrow a}\ \text{abbrev}}$$

$$\dfrac{\dfrac{\vdots}{b/b,\ a \Rightarrow b} \qquad (a/a),\ \text{np},\ \text{TV},\ \text{np}^{\vee y} \Rightarrow a}{\dfrac{b/b,\ a/a,\ \text{np},\ \text{TV},\ \text{np}^{\vee y} \Rightarrow b}{\text{TV} \Rightarrow (b\backslash(b/b)\backslash(a/a)/\text{np}^{\vee y})\backslash\text{np}}}\ /\text{L}$$

$$\dfrac{\dfrac{\vdots}{b/b,\ a/a,\ \text{s}\backslash(b/b)\backslash(a/a)/\text{np}^{\vee y},\ \forall\text{Y.np}^{\vee\text{Y}} \Rightarrow b} \qquad \text{TV} \Rightarrow (b\backslash(b/b)\backslash(a/a)/\text{np}^{\vee y})\backslash\text{np}}{\dfrac{\dfrac{b/b,\ a/a,(\text{s}\backslash(b/b)\backslash(a/a)/\text{np}^{\vee y})/(\text{s}\backslash(b/b)\backslash(a/a)/\text{np}^{\vee y})\backslash\text{np},\ \text{TV},\ \forall\text{Y.np}^{\vee\text{Y}} \Rightarrow b}{b/b,\ a/a,\ \text{np}^{\wedge x},\ \text{TV},\forall\text{Y.np}^{\vee\text{Y}} \Rightarrow b}\ \text{abbrev}}{b/b,\ a/a,\ \forall\text{X.np}^{\wedge\text{X}},\ \text{TV},\ \forall\text{Y.np}^{\vee\text{Y}} \Rightarrow b}\ \forall\text{L}}$$

Some words of guidance to the proof P_1 and its structure. First to distinguish the first occurrence of s/s in the sequent to be proved from the second, we have used b/b and a/a.

Looking to the topright of P_1 we find as a subproof a proof of a/a, np, TV, s\(a/a)\np\((s\(a/a)\np)/np) $\Rightarrow a$. This is proof that one could use to give the disambiguation, β_3, of MTT $\textsf{he}_1$ loves a woman. The next step of the proof, labelled 'abbrev', is not actually an inference at all, it just introduces the abbreviation y for s\(a/a)\np. The resulting sequent, $a/a, \mathrm{np}, TV, \mathrm{np}^{\vee y} \Rightarrow a$ is used as the minor premise of (/L) inference. giving $b/b, a/a, \mathrm{np}, TV, \mathrm{np}^{\vee y} \Rightarrow a$. The proof to this stage could be used to give the disambiguation, β_2 of JBT MTT $\textsf{he}_1$ loves a woman. Call the proof as developed up to this point, P_2. Now we have already seen a general strategy such that from the suppostion of a proof of ... $\textsf{he}_1$ ... one can infer a proof of ... every man .... The further development of P_2 into P_1 is simply the application of that strategy. All that needs commenting upon is the presence of another 'abbrev' step, this time introducing the abbreviation x for (s\(b/b)\(a/a)/np$^{\vee y}$).

Below it is proved that the denotation assigned to β_1 is as required by (100). Before that it may be useful to check a misconception that may have arisen from the above description of P_1. We observed that P_1, the proof allowing a disambiguation, β_1 of JBT MTT every man loves a woman, contained a *subproof*, P_2, one which could be used to give a disambiguation, β_2 of JBT MTT $\textsf{he}_1$ loves a woman. This *subproof* relationship between P_2 and P_1 should not be confused with a *subpart* relationship between β_2 and β_1: there is no *subpart* relation between β_2 and β_1.

The term associated with P_1, assuming $u_1^{(st,t)}$, $u_2^{(st,t)}$, $u_{D1}^{\forall\pi((s,se,\pi),\pi)}$, $u_5^{(se,se,t)}$, $u_{D2}^{\forall\pi((s,se,\pi),\pi)}$ are associated with the antecedents, is (where $TYPE2 = \nu(y) = (se, (s, st, t), t)$, and $TYPE1 = \nu(x) = ((s, (s, se, TYPE2), TYPE2), (s, st, t), (s, st, t), t))$:

(102) $u_{D1}(TYPE1)$
$(\Sigma)(\lambda i u_{D2}(TYPE2))$
$(\lambda i u_2)$
$(\lambda i u_1)$

where $\Sigma = \lambda i \lambda y_1^{se} \lambda y_2^{(s,(s,se,TYPE2),TYPE2)} \lambda y_3^{(s,st,t)} \lambda y_4^{(s,st,t)}$

$$
\begin{array}{l}
[y_4(i)(\lambda i[y_2(i) \\
\quad (\lambda i \lambda z_1^{se} \lambda z_2^{se} \lambda z_3^{(s,st,t)} \\
\quad\quad [z_3(i)(\lambda i[u_5(z_1)(z_2)])] \\
\quad) \\
\quad (y_1)(y_3) \\
]) \\
]
\end{array}
$$

On the basis of this term, the denotation of the disambiguation associated with the proof in (101) is calculated below:

$$
\begin{array}{l}
[\![\beta_1]\!](w,g) = [\![\overline{\text{every}}]\!](w,g) \\
\quad (w' \mapsto [\![M]\!](w,g)) \\
\quad (D_{TYPE1}) \\
\quad (\nabla_1) \\
\quad (w' \mapsto [\![\overline{\text{a}}]\!](w',g)(w' \mapsto [\![WOMAN]\!](w',g))(D_{TYPE2})) \\
\quad (w' \mapsto [\![MTT]\!](w',g)) \\
\quad (w' \mapsto [\![JBT]\!](w',g))
\end{array}
$$

$$
\begin{array}{l}
\text{where } \nabla_1 = w_1, d_1^{se}, d_2^{(s,(s,se,TYPE2),TYPE2)}, d_3^{(s,st,t)}, d_4^{(s,st,t)} \mapsto \\
\qquad d_4(w_1)(w_2 \mapsto d_2(w_2) \\
\qquad\qquad\qquad (w_3, f_1^{se}, f_2^{se}, f_3^{(s,st,t)} \mapsto \\
\qquad\qquad\qquad\quad f_3(w_3)(w_4 \mapsto [\![L]\!](w_4,g)(f_1)(f_2)) \\
\qquad\qquad\qquad) \\
\qquad\qquad\qquad (d_1)(d_3) \\
\qquad\quad)
\end{array}
$$

$$
\begin{array}{l}
\text{Therefore } [\![\beta_1]\!](w,g) = \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g) \\
\qquad\qquad\qquad\qquad\qquad ([\![M]\!](w,g)) \\
\qquad\qquad\qquad\qquad\qquad (x^e \mapsto \nabla_1(w) \\
\qquad\qquad\qquad\qquad\qquad\qquad (w' \mapsto x) \\
\qquad\qquad\qquad\qquad\qquad\qquad (w' \mapsto [\![A]\!](w',g)(w' \mapsto [\![W]\!](w',g))(D_{TYPE2})) \\
\qquad\qquad\qquad\qquad\qquad\qquad (w' \mapsto [\![MTT]\!](w',g)) \\
\qquad\qquad\qquad\qquad\qquad\qquad (w' \mapsto [\![JBT]\!](w',g)) \\
\qquad\qquad\qquad\qquad\qquad)
\end{array}
$$

$$
\begin{array}{l}
= \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g) \\
\qquad\quad ([\![M]\!](w,g)) \\
\qquad\quad (x^e \mapsto [\![JBT]\!](w,g)(w_2 \mapsto [\![A]\!](w_2,g) \\
\qquad\qquad\qquad\qquad\qquad\qquad (w' \mapsto [\![W]\!](w',g))(D_{TYPE2}) \\
\qquad\qquad\qquad\qquad\qquad\qquad (w_3, f_1, f_2, f_3 \mapsto f_3(w_3)(w_4 \mapsto [\![L]\!](w_4,g)(f_1)(f_2))) \\
\qquad\qquad\qquad\qquad\qquad\qquad (w' \mapsto x) \\
\qquad\qquad\qquad\qquad\qquad\qquad (w' \mapsto [\![MTT]\!](w',g)) \\
\qquad\qquad\qquad\qquad\quad) \\
\qquad\quad) \\
= \mathcal{EVERY}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g) \\
\qquad\quad ([\![M]\!](w,g)) \\
\qquad\quad (x^e \mapsto [\![JBT]\!](w,g)(w_2 \mapsto \mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g) \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad ([\![W]\!](w,g)) \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad (y^e \mapsto [\![MTT]\!](w_2,g)(w_4 \mapsto [\![L]\!] \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad (w_4,g)(w' \mapsto y)(w' \mapsto x))) \\
\qquad\qquad\qquad\qquad\quad) \\
\qquad\quad)
\end{array}
$$

Comparing the above with (100), one can see that the disambiguation associated with the proof in (101), β_1 is related to β_2 and β_3 in the way required by (99).

4 Conclusions

We have in the above both introduced the general framework of *Polymorphic Categorial Grammar* and made a particular proposal within that framework, namely $\mathcal{LT}^{25}$. We have shown that $\mathcal{LT}^{25}$ accounts for the extensive priveleges of occurrence of junctions and determiners and their tendency to engender ambiguity - that is to say $\mathcal{LT}^{25}$ accounts for the syntactic and semantic facts. However, $\mathcal{LT}^{25}$ is far from being the only account that has been of these facts - Chapter 5 drew attention to other accounts of this same range of facts. Is there anything to recommend $\mathcal{LT}^{25}$ over other accounts with the same coverage ?

One way to approach this would be to mount an argument for the great theoretical attractiveness of many of the features of the categorial grammar approach in general. If such an argument were given then that would effectively rule out the other accounts of the facts, and make the above described polymorphic categorial grammar account particularly important. I have not yet in this thesis campaigned for categorial grammar in this way and I will not start the campaign here. The reason is that although there are points in favour of the categorial grammar approach, these are not so overwhelming as to rule out any other kind of account.

Therefore we come again to the question of what there is to recommend $\mathcal{LT}^{25}$ over other accounts with the same coverage ? The question was anticipated in Chapter 5, in which I suggested that the polymorphic categorial account would be unique in meeting the criterion of *emergence*. The intuitive idea behind this criterion is to single out those acconts that do not have a special purpose mechanism for accounting for ambiguity. Intuitively, one can say this of $\mathcal{LT}^{25}$. We will try back up this intuition with the definition we gave to *emergence* in Chapter 5: an account would be *emergent* if it was resistant to a certain kind of modular simplification - namely a simplification that would take away the ambiguity but leave intact syntactic facts. We have argued already in Chapter 5 that a number of other accounts are not emergent. Is the polymorphic categorial account emergent ?

Two factors are crucial to $\mathcal{LT}^{25}$'s ability to account for ambiguity. The first is the ability to raise the arity of functions by use of the (Slash R) inferences. The second is the polymorphism of the logical constants that leaves them able to combine with an adjacent item when that item has undergone arity raising. Neither of these *seem* to be special purpose mechanisms for quantification. However, pursuing the formulation of emergence that we have given, let us see whether anything other than ambiguity would be lost if one either made unavailable the (Slash R) inferences or lessened the extent of polymorphism.

John told (Mary to go) and (Jack to stay) is an example of non-constituent coordination. For $\mathcal{LT}^{25}$ to derive this sentence, the sequent np, vpc $\Rightarrow x$ must be derivable in $\mathrm{L}^{(/,\backslash)}$ for some x. Without using (Slash R) inferences, there are no such x. With the (Slash R) inference, a value for x could be VP\((VP/vpc)/np). For more on non-constituent coordination see Moortgat 88 and relatedly Dowty 88. Therefore making the (Slash R) inferences unavailable would lose $\mathcal{LT}^{25}$ not only the account of ambiguity but also the account of non-constituent coordination.

$\mathcal{LT}^{25}$ as it stands covers a fragment that does not include extraction constructions. The standard categorial approach to extraction constructions would simply be to add relative pronouns to the lexicon, categorised as (cn\cn)/(s\np) and (cn\cn)/(s/np). However, as with non-constituent coordination, the success of this depends on having the (Slash R) inferences available (again see Moortgat 88). Therefore, once again, making the (Slash R) inferences unavailable would lose $\mathcal{LT}^{25}$ not only the account of ambiguity but also the account of extraction.[2]

[2]Note, however, that the standard categorial account of extraction is not wholly persuasive, being limited to 'peripheral' extraction. This point is returned to in the final chapter.

The second option for removing the explanation of ambiguity from $\mathcal{LT}^{25}$ is to somehow lessen the extent of the polymorphism of the logical constants. The problem with this is not so much that this is a 'simplification' that has unwanted side-effects, but that it is not a simplification at all. For example one could leave the constants with their quantified categorisations but alter the ($\forall$ L) rule to allow only particular instantiations of the categorisations. However, adding such side-conditions is obviously a complication of the theory, not a simplification of it. Instead of changing anything on the categorisation side, one could revise the meaning postulates to subtract from the number of types at which the functions are significant. Again this is more of a complication than anything else, and to do it in such a way that just the account of ambiguity is lost is difficult. For example, in 1 to 5 below, the skeletal indications are given of a number of disambiguations possible in $\mathcal{LT}^{25}$. 1 and 2 give the *de-re* interpretation of a man in John believes that a man came in, and 3, 4 and 5 account for certain simple readings. In order to subtract just ambiguity, 1 and 2 must be made semantically insignificant and yet 3,4, and 5 left significant. This requires making the determiner function not significant at the types for certain unary functions: $D_{((s,st,t),t)}$, and for certain binary functions: $D_{((s,st,se,t),se,t)}$, whilst leaving the determiner function significant at the types for other unary functions:$D_{(se,t)}$ and other binary functions, $D_{(se,se,t)}$ or $D_{(set,e,t)}$.

1. [John believes that]$_{s/s}$ [a man] [came in]$_{(s\backslash(s/s))\backslash np}$
2. [John]$_{np}$ [believes that]$_{s\backslash np/s}$ [a man] [came in]$_{(s\backslash np)\backslash(s\backslash np/s)\backslash np}$
3. [John]$_{np}$ [loves]$_{(s\backslash np)/np}$ [a man]
4. [John]$_{np}$ [gave]$_{((s\backslash np)/np)/np}$ [a man] [Bill]$_{np}$
5. [dog]$_{cn}$ [near]$_{(cn\backslash cn)/np}$ [a man]

To summarise the previous three paragraphs then, it appears that $\mathcal{LT}^{25}$ is an *emergent* account: the explanation of ambiguity emerges from features of the account whose purpose is to explain features of the language other than its ambiguity.

There are number of other features of the $\mathcal{LT}^{25}$ that would be lost if adjustments were made to lose the account of ambiguity, though these are features not pertaining directly to syntactic coverage. The account of ambiguity is in a sense emergent from these rather abstract features also. For example, $\text{L}^{(/,\backslash)}$ is sound and complete with respect to the string-semantics described in Chapter 4. Therefore if one made unavailable the (Slash R) inferences in order to lose the account of ambiguity, what would also be lost would be completeness with respect to the string-semantics. Taking the other route to losing the account of ambiguity, namely lessening the extent of the polymorphism of the quantifiers, lessens the extent to which, in theory, quantifiers can appear wherever referential np's can. For, suppose α is a quantifier and there is an x such that α does not have the category $x/(x\backslash\text{np})$ (or such that it is semantically insignificant for the corresponding type). This means there can be strings β, of category $x\backslash\text{np}$, such that $\alpha\beta$ is not grammatical (or not meaningful), but with a referential np in place of α, it is grammatical (and meaningful).

The significance of this latter kind of emergence is rather hard to say place, for where in the previous discussion of emergence one is referring to a familiar intuition that ambiguity ought to be emergent from syntactically motivated syntactic mechanisms, there is no corresponding intuition that ambiguity ought to be emergent from soundness and completeness wrt. a string semantics.

Looking back to earlier in the chapter now, to where the PLCG framework itself was introduced, two questions arise meriting some discussion.

The first concerns the way that certain circularity avoiding restrictions seems to be forced on one, both in providing a string-semantics for the polymorphic categorial language, $\text{CAT}^{(/,\backslash,\forall)}$, and in providing a function-based semantics for the term language $\mathcal{L}^{(\lambda,\Delta)}$. The expression being interpreted in each case are very different, and the semantics invoke different kinds of object, yet the restrictions correspond: the restrictions on the ($\forall$L) inference ensure that proofs are not produced which would be associated with terms that violate the restriction on the formation of type-application terms in $\mathcal{L}^{(\lambda,\Delta)}$. The question arising from this is why this should be. It is not a question that I have an answer to.

The second question concerns whether there are further uses that the PLCG framework could be put, other than accounting for the junctions and quantifiers. This appears to be the case, and tentative illustrations of this will be given in the concluding chapter.

Chapter 8

Comparisons with Hendriks' System

1 Introduction

In Chapter 7 I have outlined a categorial approach to junctions and quantification. In this chapter a number of comparisons of this will be made with the account found in 'Flexible Montague Grammar' (Hendriks 90, see also Hendriks 1987).

In section 2, the idea of *flexible* semantics is introduced. The particular case which is Hendriks' FMG account will be briefly described and its central feature presented, the 'Quantification calculus', $\mathrm{H}_e^{\rightarrow}$(we take for the purposes of comparison the *extensional* version of the type-change calculus).

There are two reasons for considering the FMG account. The first comes from the fact that I have argued that the PLCG account is special in meeting the criterion of emergence, and therefore I have to apply that criterion to the FMG account, and this is done in section 3. However, the main reason for considering the FMG account is that there is a strong theoretical affinity between the central mechanism of the FMG account and that of PLCG account. The FMG account makes vital use of a sequent calculus for inductively defining a set of *type-change* sequents, and it is the possibility of having several different proofs of one sequent that it is at the root of the explanation of ambiguity.

We will present in section 4 in outline a method of translating from the FMG anaysis of a sentence to a PLCG analysis. This shows that the coverage of the PLCG proposal is at least as great as that of the FMG proposal, and it illuminates the relationship between the two by giving an at least operational feeling for 'what part of the PLCG accout corresponds to what part of the FMG account'. There also emerges from this, some systems of interest at least to an LCG adherent, namely two potential ways to extend the LCG framework to gain an account of logical ambiguity, and both somewhat different from the extension by polymorphism that I have proposed. Problems of coverage for both of these systems are dicussed in section 5.1 and section 5.2.

It is not possible to indicate in further detail the topics that will be discussed in this chapter until the FMG proposal has been described.

2 Hendriks' Type Flexibility proposal

What is often called a 'semantic type shift' is essentially a *unary* semantic operation. Therefore $\mathcal{G}_{\uparrow}$, each of the $\mathcal{I}^n$, $\mathcal{G}_{\lambda \mathsf{he}_i}$, $\mathcal{G}_{\lambda \mathsf{he}_i^p}$, $\mathcal{G}_{\lambda \mathsf{he}_i^{vp}}$, and each of the AR^n are 'semantic type shifts'. Also the value of H_L^e on any $\mathrm{L}^{(/,\backslash)}$ proof of a sequent with *one* antecedent is a 'semantic type shift'.

Suppose the condition inherent to a THEORY OF REFERENCE that there be a *function* from phrase-set indices to types were relaxed to the requirement that there be a *relation*. Then for certain disambiguations the assigned meaning will be determined by the application of a semantic operation to objects which are not of the types such that the operation gives a significant result. However, there may be semantic type shifts which if applied to the arguments of the operation would yield arguments of types such that the operation gives a signficant result.

This is precisely what a *flexible semantics* envisages. So suppose T is a set of type shifts, that $\mathcal{T}$ is an example of the above described kind of *relationally typed* THEORY OF REFERENCE. Suppose also that $\mathcal{F}_\gamma(\alpha, \beta)$ is a disambiguation of s_1, and that *a* meaning of α is m_1 and *a* meaning of β is m_2. Then according to a *flexible semantics* based on T and $\mathcal{T}$, for any $t_i, t_j \in T$, *a* meaning of $\mathcal{F}_\gamma(\alpha, \beta)$ is $\mathcal{G}_\gamma(t_i(m_1), t_j(m_2))$. What we have here is an example of a system that allows semantics to 'non-deterministically' exploit syntax, because the use of the syntactic operation, $\mathcal{F}_\gamma$ is compatible with a *range* of semantic operations. Note that although $\mathcal{G}_\gamma$ is required to be involved in the calculation of the semantic result, $\mathcal{G}_\gamma$ is not required to be applied to m_1 and m_2, but is allowed to apply to *any* descendants of m_1 and m_2 under type-shifts. If there are two pairs of type-shifts such that $\mathcal{G}_\gamma$ may be applied significantly after the type-shifts, the effect is that $\mathcal{F}_\gamma$ is associated with at least two mappings from m_1 and m_2.

The earliest motivation that was put forward for extending the notion of THEORY OF REFERENCE in this way is that it allows the implication of semantic operations in the analysis of a sentence without the need for their reflection by possibly rather contrived syntactic operations (Benthem 86). For example, consider the case of John does not snore. One *could* propose a unary syntactic operation mapping a sentence modifying version of not to the VP modifying version, associating with the syntactic operation the following 'Geach' operation:

$$m^{(a \longrightarrow b)} \mapsto (w, g) \mapsto x^{(c \longrightarrow a)} \mapsto y^{(c \longrightarrow b)} \mapsto z^c \mapsto m(w, g)(x(z))(y(z))$$

However, one might complain that the syntactic operation was contrived. A *flexible* semantics whose set of type shifts included as t_{Geach} the above operation could explain the same facts without the contrived syntactic operation. A meaning of $\mathcal{F}_<$(John, $\mathcal{F}_>$(does, $\mathcal{F}_>$(not,snore))) would be:

$$\mathcal{G}_<([\![\text{John}]\!], \mathcal{G}_>([\![\text{does}]\!], \mathcal{G}_>(t_{Geach}([\![\text{not}]\!]), [\![\text{snores}]\!])))$$

Here, it is only if t_{Geach} is used that $\mathcal{G}_>$ can apply significantly, so the introduction of flexibility does not lead to there being *two* kinds of significance to the rule covering VP-negation. However, if there are more type-shifts than this, the effect may to give several semantic significances to a syntactic rule and this may lead to a resolution of the problem posed by logical ambiguity.

At the outset, the prospects for solving logical ambiguity in this way do not look good. One would not think that there was a way to explain the *de-re/de-dicto* ambiguity of John believes a man came in, by associating a large number of semantic operations with the rule that combines a man with came in. However, Hendriks 87, revealed that just this was possible.

The historical antecedent to this was Benthem 86, some comments in which can be read as suggesting that one make the very unrestrictive choice that T be the set of constructive interpretations of $\text{NJ}^{\rightarrow}$ proofs with a single undischarged assumption. Because of the Curry-Howard isomorphism, this is exactly the same as suggesting that for T one have all operations definable by a $\mathcal{L}^{\lambda}$ term with a single free variable. Benthem showed that ambiguity phenomena for QNP_1 TV QNP_2 sentences could be accounted for in this way. Even with such a large T, it was not obvious how the *de-re/de-dicto* ambiguities could be dealt with. What was obvious though, is that such a choice for T will lead to wild semantic overgeneration, for T will certainly include shifts like t_{flip}, below, and $n - ary$ analogues:

$m^{(a\rightarrow(b\rightarrow c))} \mapsto (w, g) \mapsto x^b \mapsto y^a \mapsto m(w, g)(y)(x)$.

t_{flip} will allow John loves Mary and Mary loves John to be synonymous. On the same topic of which shifts to use, there was also in Benthem 86 the suggestion that t_{Geach} should be included,not only for the above-mentioned VP-modifier case, but also to allow the combination of TV, type (e, e, t) with object-quantifier, type $((e, t), t)$ (the QNP is made the functor because $t_{Geach}(QNP)$ could give an object of type $((a, e, t), (a, t))$, for any a). However, this thematically misconstrues the TV QNP combination and this is an argument *against* having t_{Geach} in the type-shifts.

Here it is that we come Hendriks 87, which makes a particular proposal for the set of shifts, T, and shows how logical ambiguity can be accounted for on the basis of this choice. He specifies the set of type-shifts inductively, by way of a sequent calculus for the type-language $\text{TJ}^{\rightarrow}$. This is the 'Quantification calculus', which we shall refer to as $\text{H}_e^{\rightarrow}$. Here is $\text{H}_e^{\rightarrow}$:

The Quantification Calculus $\text{H}_e^{\rightarrow}$

AR $(\vec{u}\rightarrow a\rightarrow\vec{v}\rightarrow b) \Rightarrow (\vec{u}\rightarrow((a\rightarrow b)\rightarrow b)\rightarrow\vec{v}\rightarrow b)$

AL $((\vec{u}\rightarrow(a\rightarrow b)\rightarrow b)\rightarrow\vec{v}\rightarrow c) \Rightarrow (\vec{u}\rightarrow a\rightarrow\vec{v}\rightarrow c)$

VR $(\vec{u}\rightarrow a) \Rightarrow (\vec{u}\rightarrow((a\rightarrow b)\rightarrow b))$

Cut $$\frac{a \Rightarrow b \quad b \Rightarrow c}{a \Rightarrow c}$$

It should be noted that this is the calculus of Hendriks 87. It is therefore *extensional* rather than intensional as is the calculus of Hendriks 90, and it places does not place the restrictions that the types b in AR, AL and VR be t.

As we did for $\text{LJ}^{\rightarrow}$, one could define a mapping from proofs of $\text{H}_e^{\rightarrow}$ to proofs of $\text{NJ}^{\rightarrow}$, and using the Curry-Howard isomorphism, assign a term to that proof. Hendriks gives a *term-associated* version of $\text{H}_e^{\rightarrow}$, which follows, in the background of which one see this procedure of going through $\text{NJ}^{\rightarrow}$ if the axioms are associated with $\text{NJ}^{\rightarrow}$ proofs in the way indicated in (1), (2) and (3).

Term associated $\mathrm{H}_e^{\to}$

AR $(\vec{u}\to a\to\vec{v}\to b) : \Phi \Rightarrow (\vec{u}\to((a\to b)\to b)\to\vec{v}\to b) : \lambda \vec{x_1}^{\vec{u}} \lambda T^{((a\to b)\to b)} \lambda \vec{x_2}^{\vec{v}}[T(\lambda z^a[\Phi \vec{x_1}^{\vec{u}} z \vec{x_2}^{\vec{v}}])]$

AL $((\vec{u}\to(a\to b)\to b)\to\vec{v}\to c) : \Phi \Rightarrow (\vec{u}\to a\to\vec{v}\to c) : \lambda \vec{x_1}^{\vec{u}} \lambda y^a \lambda \vec{x_2}^{\vec{v}}[\Phi(\vec{x_1}^{\vec{u}})(\lambda z^{(a\to b)}[zy])(\vec{x_2}^{\vec{v}})]$

VR $(\vec{u}\to a) : \Phi \Rightarrow (\vec{u}\to((a\to b)\to b)) : \lambda \vec{x_1}^{\vec{u}} \lambda y^{(a\to b)}[y(\Phi \vec{x_1}^{\vec{u}})]$

Cut $\dfrac{a \Rightarrow b : \Phi \qquad b : \Phi \Rightarrow c}{a \Rightarrow c}$

(1) AR:

$$\dfrac{\dfrac{[((a\to b)\to b)]_3 \qquad \dfrac{\dfrac{(\vec{u}\to a\to\vec{v}\to b) \quad [\vec{u}]_4 \quad [a]_1 \quad [\vec{v}]_2}{b}(\to \mathrm{E})}{(a\to b)}(\to_1 \mathrm{I})}{b}(\to \mathrm{E})}{(\vec{u}\to((a\to b)\to b)\to\vec{v}\to b)}(\to_{2,3,4} \mathrm{L})$$

(2) AL:

$$\dfrac{\dfrac{(\vec{u}\to((a\to b)\to b)\to\vec{v}\to c) \quad [\vec{u}]_4 \quad \dfrac{\dfrac{[(a\to b)]_1 \quad [a]_3}{b}(\to \mathrm{E})}{((a\to b)\to b)}(\to_1 \mathrm{I}) \quad [\vec{v}]_2}{c}(\to \mathrm{E})}{(\vec{u}\to a\to\vec{v}\to c)}(\to_{2,3,4} \mathrm{I})$$

(3) VR:

$$\dfrac{\dfrac{[(a\to b)]_1 \qquad \dfrac{(\vec{u}\to a) \quad [\vec{u}]_2}{a}(\to \mathrm{E})}{b}(\to \mathrm{E})}{(\vec{u}\to(a\to b)\to b)}(\to_{1,2} \mathrm{I})$$

Flexible Montague Grammar, as defined in Hendriks 87, consists of the combination of the set of type shifts defined by $\mathrm{H}_e^{\to}$ with the following *relationally typed*, THEORY OF REFERENCE, $\mathcal{FT}^0$.

Extensional Flexible Montague Grammar

1. The phrase-set indices, $\Delta = \{NP, IV, S, TV, DET, CN, PV\}$
2. Whatever strings α, whatever categories $\delta \in \Delta$, $\langle \alpha, \langle\rangle, \delta \rangle \in \mathcal{X}_\delta$ iff α appears in the δ row of the table below:

NP	john, mary
DET	every, a, no, most
CN	man, woman
VP	walks
TV	loves, is
PV	believes

3. Syntactic operations: $\mathcal{F}_>, \mathcal{F}_<, \mathcal{F}_{AND}, \mathcal{F}_{OR}$. The string part of $\mathcal{F}_{AND}$ is $\mathcal{F}_{AND}(s_1, s_2) = s_1$ and s_2. Similarly for $\mathcal{F}_{OR}$.
4. Syntactic rules $\{\langle \mathcal{F}_<, \langle NP, VP\rangle, S\rangle, \langle \mathcal{F}_>, \langle TV, NP\rangle, VP\rangle, \langle \mathcal{F}_>, \langle DET, CN\rangle, NP\rangle,$ $\langle \mathcal{F}_>, \langle PV, S\rangle, VP\rangle,$
for $\delta = S, VP, NP, TV$, $\gamma = AND, OR$, the rule, $\langle \mathcal{F}_\gamma, \langle \delta, \delta\rangle, \delta\rangle\}$

The possible models $\langle\langle \mathcal{B}, (\mathcal{G}_\gamma)_{\gamma\in\Gamma}, f\rangle, \langle w, g\rangle\rangle$ associated with $\mathcal{E}, \mathcal{I}, \mathcal{J}, \nu$ is $\in \mathcal{K}^{26}$ iff

1. Type Relation: $r(\text{np}, e)$, $r(DET, ((e\rightarrow t)\rightarrow((e\rightarrow t)\rightarrow t)))$, $r(\text{CN}, (e\rightarrow t))$, $r(\text{VP}, (e\rightarrow t))$, $r(\text{TV}, (e\rightarrow(e\rightarrow t)))$,
$r(\text{PV}, (t\rightarrow(e\rightarrow t)))$, and if $r(\delta, a)$ and $a \Rightarrow b$ is derivable in $\text{H}_e^{\rightarrow}$, then $r(\delta, b)$.
2. Constraints on f: all expressions are freely interpreted in the most basic type assigned them by r, except for the determiners and is. In each model, each determiner is assigned $\mathcal{DET}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)$, and is is assigned $\mathcal{IS}(\mathcal{E}, \mathcal{I}, \mathcal{J})(w, g)$.
3. Algebraic constraints: Γ is $\{<, >, AND, OR\}$. $\mathcal{G}_>$ and $\mathcal{G}_<$ are familiar. $\mathcal{G}_{AND}(m_1, m_2) = \mathcal{H}_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_1, \mathcal{AND}(\mathcal{E}, \mathcal{I}, \mathcal{J}), m_2)$, and $\mathcal{G}_{OR}(m_1, m_2) = \mathcal{H}_{\mathcal{J}}(\mathcal{E}, \mathcal{I}, \mathcal{J})(m_1, \mathcal{OR}(\mathcal{E}, \mathcal{I}, \mathcal{J}), m_2)$.

The junctions and determiners are treated in rather different fashions to each other here. One difference is that the junctions are treated syncategorematically and the determiners categorematically. One could have categorematic treatments instead, using the polymorphic junction operation, $\mathcal{H}_{\mathcal{J}}$, from Chapter 5, or using the polymorphic junction denotation from the previous chapter. Not a great deal hangs on this, except a putative account of the Rodman observations p49-50 of Hendriks 90. Even if the junctions were treated categorematically in either of the ways indicated, there would remain a difference. One could quite legitimately say that the semantics of junction constructions is handled by ***polymorphism***, whereas the semantics of quantifier constructions is handled by ***type-flexibility***. The proposal is 'aimed at' quantifiers in a way that it is not 'aimed at' junctions. This is because although the type-shifts allows one to have the simplest type for quantifiers, $((e, t), t)$ and yet simply to use function application to combine with a quantifier with a verb, the type-shifts do not allow one to have the simplest type for junctions, $(t, (t, t))$ and yet simply to use function application to combine a junction with two verbs. This aspect should be borne in mind for the rest of this chapter, when we contrast FMG the 'type-flexibility' approach with PLCG, the 'polymorphism' approach: really FMG is a hybrid of type-flexibility and polymorphism.

Now that the FMG account has been described, it should be clear why a comparison of it with the PLCG is especially relevant. In the PLCG account, the key to ambiguity is in the space of possible proofs of $\text{L}^{(/,\backslash,\forall)}$. In the FMG account the key to ambiguity is the space of possible proofs of $\text{H}_e^{\rightarrow}$.

Section 3 is a competitive comparison of FMG and PLCG, trying to find arguments for favouring one over the other.

Section 4 is a less competitive comparison and in the spirit of trying to illuminate the connection between the two proposals, gives a translation 'from type-flexibility to quantifier polymorphism'. Also in the course of this translation a number of staging posts are passed through which are illuminating for anyone who is an adherent of LCG but is frustated at the impossibility of accounting for logical ambiguity within it and these will be discussed.

The first of the staging posts in the translation from 'from type-flexibility to quantifier polymorphism' is (roughly) what results from replacing the syntactic component of the FMG proposal, which was non-categorial, with an LCG component. This appears to offer an LCG adherent a solution to logical ambiguity by building on LCG, and in a different way than I have built on it to get the PLCG account. Just such a combination is proposed in Moortgat 88, and such a change of the syntactic component of the FMG is fully in the spirit in which the FMG account was proposed; such portability of the type-shifting component from one syntax to another has been stressed to be an attractive feature of the account. However, I will argue in section 5.2 that for an LCG adherent, bringing in the type-shifting axioms leads to semantic disasters. This observations should be interesting not only to an LCG adherent but to a type-flexibility advocate also: they force one to realise that type-shifts which are 'safe' in the context of one kind of syntax may not be in the context of another.

The second staging post in the 'translation from type-flexibility to quantifier polymorphism' is what one could call the categorial version of Hendriks' proposal: it adds category versions of Hendriks type-change axioms to the axiomatisation of $\mathrm{L}^{(/,\backslash)}$. All but one of the proposed additions turn out to be already derivable in $\mathrm{L}^{(/,\backslash)}$. This appears to offer the LCG adherent still another account of logical ambiguity that builds on LCG, again different to the PLCG account, and by taking the very small step of simply adding one axiom (this account has been touched upon in Moortgat 1990). However, I argue in section 5.1 that the additional axiom lead to syntactic and semantic overgenerations.

3 Type Flexibility or Categorial Polymorphism ?

Our main criterion in evaluating accounts of logical ambiguity has been the criterion of emergence. The FMG fares rather badly by this criterion. One can expect this from the rather modular design separating the syntactic rules and the type-flexibility component. This will mean that simplifications of the type-flexibility component will not effect the yield of the theory in terms of grammatical facts. Still, without the type-shifts, many sentences though parsed, would have no significant interpretation. Therefore, the question is whether it is possible to simplify the type-shifting component so that all sentences have a significant interpretation, and yet to lessen substantially the extent of ambiguity. To do this AR should be weakened so that only *first* arguments may be raised, and AL should be dropped. All of the following sentences would then become unambiguous, though interpretable:

(4) a. every man loves a woman
 b. every man told a woman to go
 c. John believes a man came in

Some sentences, though, would remain ambiguous simply by the presence of VR, such as **every man and woman died**. The only construction in the FMG fragment that actually depends on VR for interpretability is John and a man. In which case, specifying VR as specific to the type e, in tandem with other above mentioned simplifications of the type-shifting component, would reduce the extent of ambiguity to zero, and yet leave all sentences with an interpretation.

A further attractive feature of the PLCG account that is lost by the FMG account is the unity of the logical constants. In the PLCG account, the junctions and the determiners all had polymorphic denotations and had this denotation fixed by a meaning postulate. In the FMG account, the logical properties of junction containing expressions are fixed by a postulate determining a polymorphic operation, whereas the logical properties of determiner containing expressions are fixed by a postulate determining a monomorphic denotation.

Finally, although I have not tried to make an argument for the LCG framework, it seems rather a negative point about the flexibility approach that it is incompatible with an LCG grammar. That this is so will be seen in section 5.2.

4 From Type Flexibility to Quantifier Polymorphism

We want to show in this section that for every analysis of a reading provided by $\mathcal{FT}^0$ (the flexibility account), there is a corresponding analysis in $\mathcal{LT}^{25}$ (the PLCG account). The first step is to reach an account which is a certain kind of extension of an $\text{L}^{(/,\backslash)}$-THEORY OF REFERENCE, and extension based on the union of the Lambek calculus, $\text{L}^{(/,\backslash)}$, with a *categorial version* of $\text{H}_e^{\rightarrow}$, which we shall call $\text{H}^{(/,\backslash)}$. Any such extension of an LCG account we will call a $\text{H}^{(/,\backslash)},\text{L}^{(/,\backslash)}$-THEORY OF REFERENCE. $\text{H}^{(/,\backslash)}$ will be described further below, under the heading 'Category Flexibility', and the relevant facts about derivability in $\text{L}^{(/,\backslash)}$ noted. Then we will return to the matter of translating from the flexibility account, $\mathcal{FT}^0$ to a $\text{H}^{(/,\backslash)},\text{L}^{(/,\backslash)}$-THEORY OF REFERENCE.

4.1 Category Flexibility

The following will be counted as the categorial version of $\text{H}_e^{\rightarrow}$:

The Calculus $\mathrm{H}^{(/,\backslash)}$

/-AR $((b|Y)/a)|X : \Phi \rightarrow ((b|Y)/a^{\vee b})|X : \lambda \vec{x_1}^{\vec{u}} \lambda T^{((a\rightarrow b)\rightarrow b)} \lambda \vec{x_2}^{\vec{v}}[T(\lambda z^a[\Phi \vec{x_1}^{\vec{u}} z \vec{x_2}^{\vec{v}}])]$

\\-AR $((b|Y)\backslash a)|X : \Phi \rightarrow ((b|Y)\backslash a^{\wedge b})|X :$

/-AL $((c|Y)/a^{\vee b})|X : \Phi \rightarrow ((c|Y)/a)|X : \lambda \vec{x_1}^{\vec{u}} \lambda y^a \lambda \vec{x_2}^{\vec{v}}[\Phi(\vec{x_1}^{\vec{u}})(\lambda z^{(a\rightarrow b)}[zy])(\vec{x_2}^{\vec{v}})]$

\\- AL $((c|Y)\backslash a^{\wedge b})|X : \Phi \rightarrow ((c|Y)\backslash a)|X :$

/-VR $a|X : \Phi \rightarrow a^{\wedge b}|X : \lambda \vec{x_1}^{\vec{u}} \lambda y^{(a\rightarrow b)}[y(\Phi \vec{x_1}^{\vec{u}})]$

\\- VR $a|X : \Phi \rightarrow a^{\vee b}|X :$

Cut as before

The notation, $|X$, is shorthand for a list of syntactic arguments $x_1, x_2, \ldots, x_n$ introduced by a sequence of slashes of either directionality.

Some of the axioms of $\mathrm{H}^{(/,\backslash)}$ are derivable in $\mathrm{L}^{(/,\backslash)}$and some are not. The |-VR and |-AL axioms are derivable in $\mathrm{L}^{(/,\backslash)}$. |-AR is not. (This fact has also been noted by Hendriks 90 and Moortgat 88)

In (5), the /-VR axiom is derived and in (6), /-AL

(5) VR $(\vec{u}\rightarrow a) : \Phi \Rightarrow (\vec{u}\rightarrow a^b) \; : \; \lambda \vec{x_1} \lambda y^{(a\rightarrow b)}[y(\Phi(\vec{x_1}))]$

/-VR
$$\cfrac{\cfrac{a : \Phi(\vec{x_1}),\ b\backslash a : y \Rightarrow b : y(\Phi(\vec{x_1}))}{a : \Phi(\vec{x_1}) \Rightarrow a^{\wedge b} : \lambda y[y(\Phi(\vec{x_1}))]}}{a|X : \Phi \Rightarrow a^{\wedge b}|X : \lambda \vec{x_1} \lambda y y(\Phi(\vec{x_1}))}$$

(6) AL $(\vec{u}\rightarrow a^b\rightarrow\vec{v}\rightarrow c) : \Phi \Rightarrow (\vec{u}\rightarrow a\rightarrow\vec{v}\rightarrow c) : \lambda \vec{x_1} \lambda y[\Phi(\vec{x_1})(\lambda P[Py])]$

/-AL
$$\cfrac{\cfrac{c|Y \Rightarrow c|Y \qquad a \Rightarrow a^{\vee b}}{(c|Y)/a^{\vee b}, a \Rightarrow c|Y}}{((c|Y)/a^{\vee b})|X \Rightarrow ((c|Y)/b)|X}$$

As mentioned above it is Argument Raising that fails to have a valid categorial equivalent. AR will be valid only when the directionality of all other arguments is the reverse of that of the raised argument (the notation, $\backslash_n X$ is shorthand for some list of argument $x_1, x_2, \ldots, x_n$ introduced by n slashes all of which are leftward looking):

(7) AR $(\vec{u}\rightarrow a\rightarrow\vec{v}\rightarrow b) : \Phi \Rightarrow (\vec{u}\rightarrow a^b\rightarrow\vec{v}\rightarrow b) \; : \; \lambda \vec{x_1} \lambda T \lambda \vec{x_2}[T(\lambda z[\Phi \vec{x_1} z \vec{x_2}])]$

/-AR
$$\cfrac{\cfrac{\cfrac{\cfrac{Y, (b\backslash_n Y)/a \Rightarrow b/a \qquad b \Rightarrow b}{Y, (b\backslash_n Y)/a, a^{\vee b} \Rightarrow b}\backslash L}{(b\backslash_n Y)/a, a^{\vee b} \Rightarrow b\backslash_n Y}\backslash_n R}{(b\backslash_n Y)/a \Rightarrow (b\backslash_n Y)/a^{\vee b}}/R}{((b\backslash_n Y)/a)|X \Rightarrow ((b\backslash_n Y)/a^{\vee b})|X}$$

((s\np)/np)/np is an example of a category that does not meet the above stated conditions for the validity of the /-AR transition to $((s\backslash np)/np)/(s\backslash(s/np))$, and below a countermodel to the sequent is shown.

Proof that there is a countermodel to $((s\backslash np)/x)/np \Rightarrow ((s\backslash np)/x)/(s\backslash(s/np))$

Assume a model such that $[\![np]\!] = \{\Box\}$, and $[\![s]\!] = \{\Box\heartsuit\Box\Box, \Box\heartsuit\Box\Diamond\}$.

Clearly $\heartsuit \in [\![((s\backslash np)/np)/np]\!]$. Need to show that $\heartsuit \notin [\![((s\backslash np)/np)/(s\backslash(s/np))]\!]$. So we must consider $[\![s\backslash(s/np)]\!]$.

$[\![s/np]\!] = \{\Box\heartsuit\Box\}$

Therefore $\Diamond \in [\![s\backslash(s/np)]\!]$. Thus

$\exists a, b, c(a \in [\![s\backslash(s/np)]\!], b \in [\![x]\!], c \in [\![np]\!], c\heartsuit ab \notin [\![s]\!]$

Hence $\heartsuit \notin [\![((s\backslash np)/np)/(s\backslash(s/np))]\!]$, and the sequent is refuted.

A useful fact that we will note now for later use is that in certain cases the AR and VR axioms make the same statement:

(8) AR: $b/a \Rightarrow b/a^{\vee b}$ is the same as VR:$b/a \Rightarrow (b/a)^{\wedge b}$, because $b/a^{\vee b} = (b/a)^{\wedge b}$
AR: $b\backslash a \Rightarrow b\backslash a^{\wedge b}$ is the same as VR:$b\backslash a \Rightarrow (b\backslash a)^{\vee b}$, because $b\backslash a^{\wedge b} = (b\backslash a)^{\vee b}$

See section 1 of Chapter 7 for the definition of $\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE. A $\text{H}^{(/,\backslash)}, \text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE will be understood to be defined analogously.

4.2 From Type Flexibilty to Category Flexibilty

(9) is what one would write down in tracing through a particular meaning assignment that $\mathcal{FT}^0$ allows to the sentence John loves every man, and (10) shows a corresponding disambiguation in a $\text{H}^{(/,\backslash)}, \text{L}^{(/,\backslash)}$-THEORY OF REFERENCE.

(9)
$$\cfrac{\cfrac{\text{John}}{\mathrm{NP}{:}e} \qquad \cfrac{\cfrac{\cfrac{\text{loves}}{\mathrm{TV}{:}(e,e,t)}}{\mathrm{TV}{:}(e^t,e,t)}AR^1 \qquad \cfrac{\cfrac{\text{every}}{\mathrm{DET}{:}((e,t),e^t)} \quad \cfrac{\text{man}}{\mathrm{CN}{:}(e,t)}}{\mathrm{NP}{:}e^t}\mathcal{F}_>}{\mathrm{VP}{:}(e,t)}\mathcal{F}_>}{\mathrm{S}{:}t}\mathcal{F}_<$$

(10)
$$\cfrac{\cfrac{\text{John}}{np} \qquad \cfrac{\cfrac{\cfrac{\text{loves}}{(s\backslash np)/np}}{(s\backslash np)/np^{\vee s}}AR^1 \qquad \cfrac{\cfrac{\text{every}}{np^{\vee s}/cn} \quad \cfrac{\text{man}}{cn}}{np^{\vee s}}}{s\backslash np}}{s}$$

Seeing the pairing of (9) and (10) it probably does not seem surprising that for every analysis of a reading that $\mathcal{FT}^0$ allows, there is a corresponding disambiguation in a $\text{H}^{(/,\backslash)}, \text{L}^{(/,\backslash)}$-THEORY OF

REFERENCE. Note, however, that in (9) there is an invariant dimension that persists through type-change, and which determines what things combine and how. In (10), there is not this invariant dimension.

Accordingly, we will take the step from the flexibility analysis to the $H^{(/,\backslash)}$,$L^{(/,\backslash)}$ analysis a little more slowly. First note that (9) is not simply the description of a disambiguation that $\mathcal{FT}^0$ allows, it is a combination of a disambiguation with a certain pattern of type-shifting. (10), by contrast, is simply a disambiguation. This complicates what we mean when we compare an 'analysis of a reading' in the $\mathcal{FT}^0$ system and in the $H^{(/,\backslash)}$-$L^{(/,\backslash)}$ system. Now although one of the principal motivations for a flexible semantics is to be free of the need to have all aspects of the analysis of a reading present in the syntax, one can nonetheless recast a flexible semantics in the UG format, so that all aspects of the analysis are present in the syntax. This is the first thing that will be done with $\mathcal{FT}^0$. [1]

All we have to do is extend $\mathcal{FT}^0$'s syntax so that (9) can actually be seen as displaying a disambiguation. To do this the categorisations of $\mathcal{FT}^0$ are replaced by pairs $\langle x : a\rangle$, where x is a $\mathcal{FT}^0$ categorisation and a is a type. So for example John has the bipartite categorisation $\langle NP : e\rangle$ and every man has the bipartite categorisation $\langle NP : ((e,t),t)\rangle$.

Every type-shift, t, is now to be understood as an operation of the semantic algebra, sharing an index with a particular syntactic operation. If t leads from a to b, the index is the proof of $a \Rightarrow b$ in $H_e^{\rightarrow}$. To the syntactic rules of $\mathcal{FT}^0$ are added *unary* rules, $\langle \mathcal{F}_\gamma, \langle x : a\rangle, x : b\rangle$, where $\mathcal{F}_\gamma$ is a syntactic operation indexed by a $H_e^{\rightarrow}$ proof of $a \Rightarrow b$, which is intended to amount to an identity on strings.

The original syntactic rules of $\mathcal{FT}^0$ now become the source of a considerably expanded set. Where there was a rule featuring the category δ, now there will be a putative rule featuring $\langle \delta : a\rangle$ for any a such that $r(\delta, a)$. If the type parts of the putative rule 'agree' with the semantic operation associated with it, then the putative rule is a rule.

> *Step 1: convert $\mathcal{FT}^0$ into a* THEORY OF REFERENCE, *by including the* types *in the categorisation, and by turning the* type-shifts *into syntactic rules*

The THEORY OF REFERENCE thereby arrived at will be called a $H_e^{\rightarrow}$-THEORY OF REFERENCE. The $H_e^{\rightarrow}$-THEORY OF REFERENCE derived in this way from $\mathcal{FT}^0$ will be called $\mathcal{HT}^0$. We can now restate the aim of the section as being to show that there is a $H^{(/,\backslash)}$,$L^{(/,\backslash)}$-THEORY OF REFERENCE which provides an equivalent disambiguation to every disambiguation admitted by $\mathcal{HT}^0$.

First we transform the rules of $\mathcal{HT}^0$ by replacing the δ part of a $\langle \delta : a\rangle$ pair with a $CAT^{(/,\backslash)}$

[1] Hendriks 90 also recasts the FMG account within UG, by taking as the meaning of an expression the set of possibilities allowed to it by syntactic analysis and type-shifts.

category, according to the correspondence below.

(11)

Hendriks Lexicon	Categorial Lexicon	Examples
cn	cn	man, woman
IV	s\np	walk, be missing
NP	np	John, a unicorn
Det	np/cn	the, a
TV	(s\np)/np	find, seek
PV	s\np/ s	claim, believe

Looking at just the $\text{CAT}^{(/,\backslash)}$ parts of the rules obtained, in most cases the rule represents an $\text{L}^{(/,\backslash)}$-valid sequent, and the operation part of the derived rule will be understood to have as its index the $\text{L}^{(/,\backslash)}$ proof of the sequent. However, with only this much transformation of the rules of $\mathcal{HT}^0$, there arise a problem with the cross-categorial junction rules of $\mathcal{HT}^0$, all of the form $\langle \mathcal{F}_\gamma, \langle x:a, x:a\rangle, x:a\rangle$, where γ is AND or OR and x is S, VP, NP or TV. $x, x \Rightarrow x$ is just not $\text{L}^{(/,\backslash)}$ valid. To deal with this, we will presume that added to the lexicon are the polymorphic disambiguations of **and**, and of **or**, of category $\forall\text{X}((\text{X}\backslash\text{X})/\text{X})$ and that there are additional rules: $\langle \mathcal{F}_{PROOF}, \langle x:a, \forall\text{X}((\text{X}\backslash\text{X})/\text{X}) : \forall\pi(\pi,(\pi,\pi)), x:a\rangle, x:a\rangle$, where PROOF is an $\text{L}^{(/,\backslash,\forall)}$ proof of $x, \forall\text{X}((\text{X}\backslash\text{X})/\text{X}), x \Rightarrow x$.

> *Step 2: replace categories with* $\text{CAT}^{(/,\backslash)}$ *categories, and introduce polymorphic disambiguations of junctions, making the* category *parts of rules* $\text{L}^{(/,\backslash,\forall)}$ *sequents.*

Evidently, the derived theory will generate disambiguations which are equivalent to all the disambiguations that $\mathcal{HT}^0$ generates. We will call the resulting theory $\mathcal{HT}^1$. Where $\mathcal{HT}^0$ admitted (9), the derived theory admits:

(12)

$$\dfrac{\dfrac{\text{John}}{\text{np}:e} \quad \dfrac{\dfrac{\text{loves}}{(\text{s}\backslash\text{np})/\text{np}:(e,e,t)}}{(\text{s}\backslash\text{np})/\text{np}:(e^t,e,t)}AR^1 \quad \dfrac{\dfrac{\text{every}}{\text{np}/\text{cn}:((e,t),e^t)} \quad \dfrac{\text{man}}{\text{cn}:(e,t)}}{\text{np}:e^t}1}{\text{s}:t}2$$

The language of $\text{CAT}^{(/,\backslash)}$ and the language of $\text{TJ}^{\rightarrow}$ are structurally alike, and the next stage of the translation is to replace the *types*, a in a category:type pairs, $x:a$, with a categories y that stands in the category-to-type map to a. Such a change of type into category will be done to all the rules of $\mathcal{HT}^1$, and the result will be called $\mathcal{HT}^2$. Two examples of images of (12) in $\mathcal{HT}^2$ are given below.

(13)

$$\dfrac{\dfrac{\text{John}}{\mathrm{np}{:}np}\qquad \dfrac{\dfrac{\dfrac{\text{loves}}{(\mathrm{s}\backslash\mathrm{np})/\mathrm{np}{:}(s\backslash np)/np}}{(\mathrm{s}\backslash\mathrm{np})/\mathrm{np}{:}(s\backslash np)/np^{\vee s}}AR^1 \qquad \dfrac{\dfrac{\text{every}}{\mathrm{np}/\mathrm{cn}{:}np^{\vee s}/cn}\quad \dfrac{\text{man}}{\mathrm{cn}{:}cn}}{\mathrm{np}{:}np^{\vee s}}1}{\mathrm{s}\backslash\mathrm{np}{:}s\backslash np}}{\mathrm{s}{:}s}2$$

(14)

$$\dfrac{\dfrac{\text{John}}{\mathrm{np}{:}np}\qquad \dfrac{\dfrac{\dfrac{\text{loves}}{(\mathrm{s}\backslash\mathrm{np})/\mathrm{np}{:}(s/np)/np}}{(\mathrm{s}\backslash\mathrm{np})/\mathrm{np}{:}(s/np)/np^{\vee s}}AR^1 \qquad \dfrac{\dfrac{\text{every}}{\mathrm{np}/\mathrm{cn}{:}np^{\vee s}/cn}\quad \dfrac{\text{man}}{\mathrm{cn}{:}cn}}{\mathrm{np}{:}np^{\vee s}}1}{\mathrm{s}\backslash\mathrm{np}{:}s/np}2}{\mathrm{s}{:}s}3$$

Thus there are still bipartite categorisations, but in both parts members of $\text{CAT}^{(/,\backslash,\forall)}$ appear. We will call the categories that now appear in the *type* slot 'bidirectional logical types' (BLT's).

Step 3: Replace the types with corresponding categories

Now if only the BLT parts of (13) and (14) were retained, the remnant of (13) would be a disambiguation of a $\text{H}^{(/,\backslash)},\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE (in fact it would be (10)), but (14) would not (a slash direction is wrong at line 3).

What we wish to know now is whether from amongst the several different disambiguations of a sentence provided by $\mathcal{HT}^2$, there will always be one which when stripped down to its BLT's is a disambiguation of a $\text{H}^{(/,\backslash)},\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE. To see that there will be, consider again the disambiguations admitted by $\mathcal{HT}^1$. All steps of combination are steps of function application, where an arrow connective is eliminated from the type part of the functor word. This arrow that is eliminated may be traced back through all the unary type-shift steps, to the type.part of a lexical item: *the type-shifts never introduce an arrow that is eliminated at a combination step.* For disambiguations that $\mathcal{HT}^2$ generates, there is a corresponding feature: a slash connective is eliminated from the BLT part of the functor word, and that slash may be traced back through all unary BLT-shift steps, to the BLT part of a lexical item: *the BLT-shifts do not intoduce or change the directionality of a slash that is eliminated at a combination step.* Since there will be at least one version of a lexical item according to $\mathcal{HT}^2$ which gives the BLT part of the category:BLT pair the same directionality as the category part, there will be one disambiguation that $\mathcal{HT}^2$ provides which when reduced to just the BLT parts could count as an analysis in the $\text{H}^{(/,\backslash)},\text{L}^{(/,\backslash,\forall)}$ theory.

Step 4: throw away the category part and retain the BLT part

That completes the informal demonstration that there is a $\text{H}^{(/,\backslash)},\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE that provides a disambiguation corresponding to any disambiguation of $\mathcal{HT}^0$, the FMG analysis.

We will call this $\text{H}^{(/,\backslash)},\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE, $\mathcal{HT}^3$. Notice that if we had ignored the junction parts of $\mathcal{HT}^0$, a $\text{H}^{(/,\backslash)},\text{L}^{(/,\backslash)}$-THEORY OF REFERENCE would have sufficed to represent every disambiguation possible within $\mathcal{HT}^0$.

4.3 From Category Flexibility to Quantifier Polymorphism

We now come to the last and most important step, from $\mathcal{HT}^3$ to the PLCG account, $\mathcal{LT}^{25}$. We will not demonstrate that *every* disambiguation allowed by $\mathcal{HT}^3$ has a $\mathcal{LT}^{25}$ image, only that *those disambiguations in $\mathcal{HT}^3$ that are the images of $\mathcal{HT}^0$ disambiguations* have disambiguations in $\mathcal{LT}^{25}$. The distinction will be crucial in section 5.1.

One of the things that prevents the disambiguations that $\mathcal{HT}^3$ provides from counting as disambiguations of an $\text{L}^{(/,\backslash,\forall)}$-THEORY OF REFERENCE, such as $\mathcal{LT}^{25}$, are the occurences of steps of /AR and \AR. What will be described is a process of transformation that may be applied to a $\mathcal{HT}^3$ disambiguation that eliminates occurences of /AR and \AR. At the end of the transformation one has something which counts as a disambiguation of $\mathcal{LT}^{25}$.

Instances of AR in $\mathcal{HT}^3$ are either paired with an instance of VR or paired with an occurence of quantifier (here when we say AR and VR, this is intended to refer to the theorems of $\text{H}^{(/,\backslash)}$ and not to the theorems of $\text{H}_e^{\rightarrow}$). Basically, one by one these AR:VR or AR:Quant pairs are used to trigger transformations of the disambiguation: an AR:VR pair is replaced by a VR:VR pair, and an AR:Quant pair is replaced by a VR:PolyQuant pair.

First we co-index the members of AR:VR and AR:Quant pairs. One finds the pairs by first identifying for a syntactic functor what its arguments are. If VR has been applied to an argument, or if the argument is a quantifier, AR will have been applied to the functor. The VR or the quantifier should then be paired with the AR. We will illustrate this on a certain $\mathcal{HT}^3$ disambiguation (ttlg abbreviates 'Through the looking glass' and 1,2 and 3 are used to distinguish different 'levels' of s. Also only the unary steps are shown, the binary steps being left as understood):

$$
\frac{\text{[Fred}}{np}
\quad
\cfrac{\cfrac{\text{[claims}}{(1\backslash np/2)}}{1\backslash np/2^{\vee 1}}\,\text{AR}
\quad
\frac{\text{[every schoolboy}}{np^{\wedge 1}}
\quad
\cfrac{\cfrac{\cfrac{\cfrac{\text{[believes}}{2\backslash np/3}}{2^{\vee 1}\backslash np/3}\,\text{VR}}{2^{\vee 1}\backslash np^{\wedge 1}/3}\,\text{AR}}{2^{\vee 1}\backslash np^{\wedge 1}/3^{\vee 1}}\,\text{AR}
\quad
\frac{\text{[a mathematician}}{np^{\wedge 1}}
\quad
\cfrac{\cfrac{\cfrac{\text{[wrote}}{3\backslash np/np}}{3^{\vee 1}\backslash np/np}\,\text{VR}}{3^{\vee 1}\backslash np^{\wedge 1}/np}\,\text{AR}
\quad
\frac{\text{ttlg]]]]]}}{np}
$$

In (15) the co-indexing of AR:VR and AR:Quant pairs is done.

(15)
$$\frac{\text{[Fred}}{np}\quad \cfrac{\cfrac{\text{[claims}}{(1\backslash np/2)}}{1\backslash np/2^{\vee 1}}\text{AR}_i\quad \frac{\text{[every schoolboy}}{(np^{\wedge 1})_{ii}}\quad \cfrac{\cfrac{\cfrac{\cfrac{\text{[believes}}{2\backslash np/3}}{2^{\vee 1}\backslash np/3}\text{VR}_i}{2^{\vee 1}\backslash np^{\wedge 1}/3}\text{AR}_{ii}}{2^{\vee 1}\backslash np^{\wedge 1}/3^{\vee 1}}\text{AR}_{iii}\quad \frac{\text{[a mathematician}}{(np^{\wedge 1})_{iv}}\quad \cfrac{\cfrac{\cfrac{\text{[wrote}}{3\backslash np/np}}{3^{\vee 1}\backslash np/np}\text{VR}_{iii}}{3^{\vee 1}\backslash np^{\wedge 1}/np}\text{AR}_{iv}\quad \frac{\text{ttlg]]]]]}}{np}$$

In general if we have a coindexed AR:VR pair, then it will be of the following form (or a directional switch of it):

$$\frac{(b\ |Y)/a)\ |X}{((b\ |Y)/\ a^{\vee b})|X}\text{AR} \qquad\qquad \frac{a|Z}{a^{\vee b}|Z}\text{VR}$$

This is replaced by an 'ar':vr pair. We say 'ar' as it one of those instances of AR that is indistinguishable from an application of VR (see the end of section 4.1):

$$\frac{(b\ |Y)/a)\ |X}{((b\ |Y)/\ a^{\vee(b|Y)})\ |X}\text{`ar'} \qquad\qquad \frac{a|Z}{a^{\vee(b|Y)}|Z}\text{vr}$$

If we have a co-indexed AR:Quant pair, then it will be of the form:

$$\frac{(s\ |Y)/np)\ |X}{((s\ |Y)/\ np^{\vee s})|X}\text{AR} \qquad\qquad \frac{\alpha}{np^{\vee s}}$$

We replace this with a pair consisting of an 'ar' transition, and a quantifier disambiguation permitted by $\mathcal{LT}$[25]:

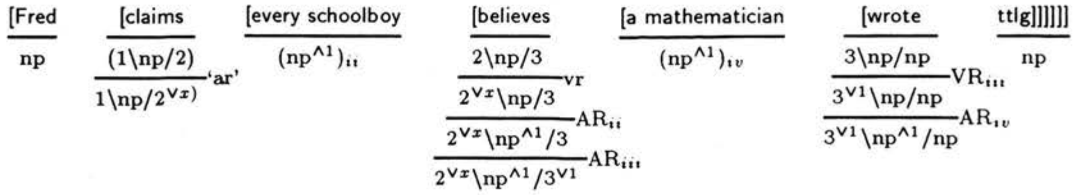
$$\frac{(s\ |Y)/np)\ |X}{((s\ |Y)/\ np^{\vee(s|Y)})\ |X}\text{`ar'} \qquad\qquad \cfrac{\cfrac{\alpha}{\forall X\ np^{\vee X}}}{np^{\vee(s|Y)}}$$

This is the essence of the transformation to be carried out.

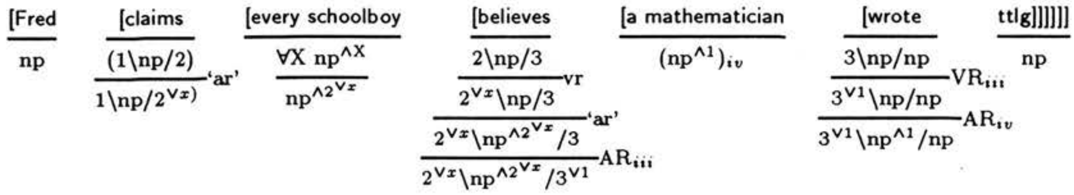
Step 5: Co-index the members of AR:VR and AR:Quant pairs.
Repace AR:VR with 'ar':vr and AR:Quant with 'ar':PolyQuant

A little more than this has to be said to handle cases where there are several AR:VR or AR:Quant pairs in one analysis. We need in such cases to transform the pairs in a certain order.

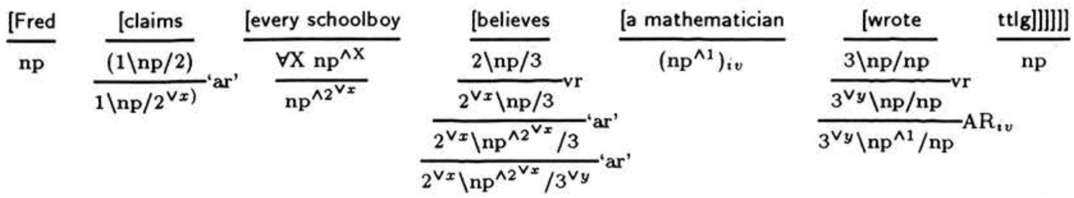
In such cases we envisage the transformation be effected in several passes over a derivation. In any pass a pair AR_i:VR_i or AR_i:Quant_i is chosen according to the nature of the branch upon which the AR_i occurs. If the branch is such that there are no *earlier* co-indexed transitions on the branch then the transformation is applied to the AR_i:VR_i or AR_i:Quant_i pair. This is illustrated below working on (15). In (16), we modify the pair AR_i:VR_i, because for all other pairs, the AR_n is preceded on its branch by a co-indexed type-shift ($x = \text{s}\backslash\text{np}$):

(16) 

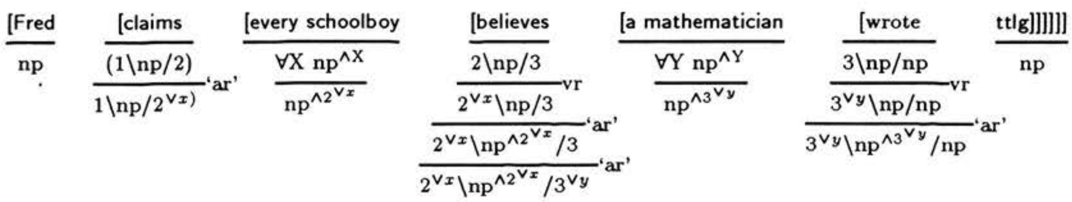
Next pair modified is AR_{ii} : $(\text{np}^{\wedge 1})_{ii}$ - again for other pairs the AR_n is preceded by a co-indexed transition:



The next pair modified is AR_{iii} : VR_{iii} ($y = (2^{\vee x}\backslash\text{np}^{\wedge 2^{\vee x}})$):



Finally the pair AR_{iv} : $(\text{np}^{\wedge 1})_{iv}$ is modified:



This is a disambiguation that exists in $\mathcal{LT}^{25}$.

Taking stock then, we have shown in this section and the previous section that a translation exists from disambiguations in the type-flexibility system $\mathcal{FT}^0$ to disambiguations in the polymorphic categorial system $\mathcal{LT}^{25}$. It ought now to be proved that the disambiguations that are made to correspond under this translation are semantically equivalent but this is not attempted here (there is some indication of what is required in Emms 90,p105-108).

5 Extending LCG to account for logical ambiguity

Having become convinced that pure LCG does not give an account of logical ambiguity, Moortgat (1988, p238-243) suggests an account of logical ambiguity that is *built* on pure LCG in a certain way. Firstly, it is a *flexible-semantics*, deploying the type-shifts, $\mathrm{H}_e^{\rightarrow}$, in combination with the syntax in just the fashion of FMG. Secondly, a polymorphic analysis of junctions is used. This is more or less exactly the same as the account $\mathcal{HT}^1$, which we used above as a stepping stone between the FMG account and the PLCG account.

The PLCG account is also an extension built upon the LCG framework, so if one was an adherent of the LCG framework and was looking for ways to extend it to secure an account of ambiguity, one would like to have reasons to choose between the two. I will make two observations on this score.

Firstly, in terms of ecomomy of the theoretical technology, the PLCG account, $\mathcal{LT}^{25}$, is preferable to the Moortgat 88 account, $\mathcal{HT}^1$. This is because $\mathcal{HT}^1$ adds to LCG both the technologies of polymorphism and type-flexibility, whereas $\mathcal{LT}^{25}$ adds to LCG just polymorphism.

Secondly, and more importantly, $\mathcal{HT}^1$ is prone to some rather dramatic semantic overgenerations. These will be shown in section 5.2 below. In anticipation of this we will consider $\mathcal{HT}^3$.

$\mathcal{HT}^3$ is another kind of extension of LCG that accounts for logical ambiguity. Firstly, it extends LCG by the addition of the /AR and \AR axioms, and secondly a polymorphic analysis of junctions is used. Again an LCG adherent would like to have reasons for choosing between this and either of $\mathcal{LT}^{25}$ and $\mathcal{HT}^1$.

Unlike $\mathcal{HT}^1$, $\mathcal{HT}^3$ never has been seriously considered as an account of logical ambiguity. Moortgat 90 (p76-78) considers 'interesting' a particular extension of LCG that allows the derivation of the /AR and \AR axioms, citing for example that it shows that it is not necessary to move to a category-type relation if one is to have minimal types in LCG. However, he moves quickly on to consider another account. Nonetheless, if you are looking for *extensions* of the LCG framework to enable account of ambiguity, adding /AR and \AR to the axiom set is a possibility one should consider: if it were entirely empirically adequate account, then $\mathcal{HT}^3$ would be an attractively economical extension of LCG.

Of course, about $\mathcal{HT}^3$, we can make the same observation was made about $\mathcal{HT}^1$, namely that $\mathcal{HT}^3$ uses polymorphism plus something, but $\mathcal{LT}^{25}$ shows that polymorphism alone suffices. However, ignoring that, in section 5.1 below, the descriptive adequecy of $\mathcal{HT}^3$ is considered. Syntactic and semantic overgenerations are found. Once one has seen these, one will immediately wonder whether there are correponding semantic overgenerations for $\mathcal{HT}^1$, and this is what is seen to be the case in section 5.2.

5.1 Using Category Flexibility instead of Type Flexibility

In section 5.1.1 we shall see that the addition of /AR and \AR axioms can lead to syntactic overgenerations (the use of $L^{(/,\backslash,\forall)}$ does not). Section 5.1.2 shows some wild semantic overgenerations that are induced by the additional axioms.

5.1.1 A Syntactic Comparison of $H^{(/,\backslash)}$ with Quantifier Polymorphism

Here we are comparing the syntactic performance of $\mathcal{HT}^3$ and $\mathcal{LT}^{25}$. The two theories agree almost entirely in the assignments of categories to words, the only difference being in the categorisation of determiners. $\mathcal{HT}^3$ has $(np^{\vee s})/cn$ and $(np^{\wedge s})/cn$ whilst $\mathcal{LT}^{25}$ has $(\forall X np^{\vee X})/cn$ and $(\forall X np^{\wedge X})/cn$.

We shall try to find strings that are categorised by $\mathcal{HT}^3$, but are not categorised by $\mathcal{LT}^{25}$, and then see if the strings are actually ungrammatical. We will probably have a case of a string categorised by $\mathcal{HT}^3$ but not by $\mathcal{LT}^{25}$ if we can find a pair of strings a, b, having the categories:

(17) $s/(s\backslash np)$, $s|X\backslash x\backslash np$

In such a case, $\mathcal{HT}^3$ will predict that ab has the category $s|X\backslash x$, but $\mathcal{LT}^{25}$ probably will not. Whether $\mathcal{LT}^{25}$ does so also depends on what other categorisations a has, other than $s/(s\backslash np)$. If a has the np categorisation, or has the polymorphic quantifier categorisation $\forall X.X/(X\backslash np)$, then $\mathcal{LT}^{25}$ will also predict that ab has the category $s|X\backslash x$. Therefore we are looking for a string a which is neither an np nor a quantifier, and a string b of category $s|X\backslash x\backslash np$.

There is a corresponding directionally reversed kind of case also, we seek strings b, a having the categories:

(18) $s|X/x/np$, $s\backslash(s/np)$

So long as a is not an np or a quantifier then $\mathcal{HT}^3$ will predict that ba has the category $s|X/np$, and $\mathcal{LT}^{25}$ will not.

Because the verbs in English that take two arguments in the same direction are *forward looking*, (such as gave, $((s\backslash np)/np)/np$, and told, $((s\backslash np)/vpc)/np$) our best chance is by looking for the second kind of case. However, finding a string a of category $s\backslash(s/np)$ that is *not* also of category np or $\forall X.X\backslash(X/np)$ is not very easy.

Suppose because in (19a) has category $(s\backslash s)/s$. Then Mary because it was broken has category $s\backslash(s/np)$, and is therefore an example of the a-string that we are looking for. Taking b to be told, of category $((s\backslash np)/vpc)/np)$, then $\mathcal{HT}^3$ predicts (19b) to be a sentence whilst $\mathcal{LT}^{25}$ does not.

(19) a. He hit [Mary because it was 3.00]

b. He told [Mary because it was 3.00] to leave(?)

The end result, (19b) is an unorthodox word order, but is not outright ungrammatical.

Suppose though in (20a) has category ((s/np)\(s/np))/(s/np). Then though Mary hates that man has the category s\(s/np), and so can be used as another example of an a string for a case of the (18) type. Taking b this time to be gave, of category ((s\np)/np)/np, then $\mathcal{HT}^3$ predicts (20b) to be a sentence whilst $\mathcal{LT}^{25}$ does not.

(20) a. John loves [though Mary hates that man]

b. Dave gave [though Mary hates that man] some money *

The end result, (20b) is ungrammatical. If only English had some verbs taking two arguments to the *left*, then we could find the required a strings for case for the (17) type quite easily: Josef thinks that Peter is a case in point. This suggests that for German, which is verb-final in subordinate clauses, the effects of adopting the /AR and \AR axioms might easier to show.

So suppose $\mathcal{HT}^3_g$ and $\mathcal{LT}^{25}_g$ are categorial accounts of German, based on the same categorisations of verbal terms (s\np:ging, (s\np)\np:liebte, s\np/sc:glaubt, s/sc:das), differing with respect to the categorisation of quantifiers, and with $\mathcal{HT}^3_g$ adding /AR and \AR to $L^{(/,\backslash)}$, whilst $\mathcal{LT}^{25}_g$ addes ∀L and ∀R.

The categorisation of liebte is that appropriate for subordinate clauses, where the verb is final. Josef glaubt das Peter may be categorised s/(s\np). Therefore $\mathcal{HT}^3_g$ predicts (21b) to be a grammatical subordinate clause, whereas $\mathcal{LT}^{25}_g$ does not.

(21) a. [Josef glaubt das Peter] ging (Josef thinks that Peter went)

b. das Hans [Josef glaubt das Peter] liebte * (that Hans (Josef thinks that Peter) likes)

(21b) is ungrammatical.

(20b) and (21b) demonstrate overgenerations that are brought about by adopting /AR and \AR. In sum, one can say that the adoption of /AR and \AR will lead to strings of category s/(s\np) or s\(s/np) being wrongly accorded some of the priveleges of occurence of NP's and quantifiers. [2] What we will see in the next section is that use of /AR and \AR accords strings simply of category s/(s\np) or s\(s/np) the *semantic* properties of quantifiers.

[2] There may well be more convincing counterexamples to AR, but I know of no-one else who has looked at the same question; despite the fact both Hendriks and Moortgat have considered the categorial version of AR, neither addressed the question of its linguistic plausibility. Instead they simply considered whether or not it was derivable in $L^{(/,\backslash)}$.

5.1.2 A semantic comparison of $\text{H}^{(/,\backslash)}$ with Quantifier Polymorphism

Consider the following four instantiations of Harry believes α came in:

$$\text{Harry believes that} \left\{ \begin{array}{l} \text{a man} \\ \text{yesterday a man} \\ \text{John shouted and} \\ \text{John thinks that Mary} \end{array} \right\} \text{came in}$$

$\mathcal{HT}^3$ provides disambiguations of the above which are such as to suggest that the above sentences may by paraphrased as below:

$$\left\{ \begin{array}{l} \text{There is a man who} \\ \text{Yesterday there was a man who} \\ \text{John shouted and is someone who} \\ \text{John thinks that Mary is someone who} \end{array} \right\} \text{Harry believes came in}$$

Only in the case of $\alpha =$ a man, does this appear to be correct. $\mathcal{LT}^{25}$ mimics this performance of $\mathcal{HT}^3$ only in respect of $\alpha =$ a man, and is therefore not subject to certain semantic overgenerations suffered by $\mathcal{HT}^3$. This is explained further below.

Below is a $\mathcal{HT}^3$ disambiguation of Harry believes that α came in, where α might be any of the expressions of category $\text{np}^{\wedge\text{s}}$ in (23):

(22)

$$\begin{array}{ccccc}
\underline{\text{Harry}} & \underline{\text{believes}} & \underline{\text{that}} & \underline{\alpha} & \underline{\text{came in}} \\
\text{np} & (\text{s}_1\backslash\text{np})/\text{sc} & \text{sc}/\text{s}_2 & \text{np}^{\wedge\text{s}_1} & \text{s}_2\backslash\text{np}
\end{array}$$

$$\text{np} \;\; (\text{s}_1\backslash\text{np})/\text{sc} \;\; \text{sc}/\text{s}_2 \Rightarrow \text{s}_1/\text{s}_2$$

$$\frac{\text{s}_2\backslash\text{np}}{((\text{s}_1\backslash(\text{s}_1/\text{s}_2))\backslash\text{np})}\text{VR}$$

$$\frac{((\text{s}_1\backslash(\text{s}_1/\text{s}_2))\backslash\text{np})}{((\text{s}_1\backslash(\text{s}_1/\text{s}_2))\backslash\text{np}^{\wedge\text{s}_1})}\text{AR}$$

$$\frac{\text{np}^{\wedge\text{s}_1} \quad ((\text{s}_1\backslash(\text{s}_1/\text{s}_2))\backslash\text{np}^{\wedge\text{s}_1})}{(\text{s}_1\backslash(\text{s}_1/\text{s}_2))}$$

$$\frac{\text{s}_1/\text{s}_2 \quad (\text{s}_1\backslash(\text{s}_1/\text{s}_2))}{\text{s}_1}$$

(23) a man

yesterday a man

John shouted and

John thinks that Mary

The denotation assigned to (22) is as follows:

(24) $[\![(22)]\!](w,g)$

$= (d_1^{e^t} \mapsto d_2^{(t,t)} \mapsto$

$d_1(x^e \mapsto d_2([\![\text{came in}]\!](w,g)(x))))([\![\overline{\alpha}]\!](w,g))([\![\overline{\text{Harry believes that}}]\!](w,g))$

$= [\![\overline{\alpha}]\!](w,g)(x^e \mapsto [\![\overline{\text{Harry believes that}}]\!](w,g)([\![\overline{\text{came in}}]\!](w,g)(x^e)))$

$= [\![\overline{\alpha}]\!](w,g)(x^e \mapsto [\![\overline{\text{believes}}]\!](w,g)([\![\overline{\text{came in}}]\!](w,g)(x))([\![\overline{\text{Harry}}]\!](w,g)))$

Only in the case that α is a man does this seem to correspond to a possible reading of the sentence.

Now we must see to what extent this semantic overgeneration repeated by $\mathcal{LT}^{25}$. I claim that there is in fact only a *semantically equivalent* $\mathcal{LT}^{25}$ representative of (22) in the case that α is a man. Here it is important to recall that we gave in section 4.3 only a *restricted* translation from disambiguations of $\mathcal{HT}^3$ into disambiguations of $\mathcal{LT}^{25}$, relying on the fact that instances of AR were either paired with an instance of VR or with a quantifier disambiguation. Should α be a man in (22), then AR is paired with a quantifier, and so to obtain an image in $\mathcal{LT}^{25}$, one would replace the $((s_1\backslash(s_1/s_2))\backslash np) \Rightarrow ((s_1\backslash(s_1/s_2))\backslash np^{\wedge s_1})$ instance of AR with the 'ar': $((s_1\backslash(s_1/s_2))\backslash np) \Rightarrow ((s_1\backslash(s_1/s_2))\backslash np^{\wedge(s_1\backslash(s_1/s_2))})$, and replace the $\mathcal{HT}^3$ disambiguation of a man with a $\mathcal{LT}^{25}$ disambiguation of category $np^{\wedge(s_1\backslash(s_1/s_2))}$.

However, if α is one of the other possibilities drawn from (23) then AR is paired neither with an instance of VR nor with a quantifier. It is paired simply with a disambiguation of α of category $np^{\wedge s}$. Now there is no guarantee that for a disambiguation of category $np^{\wedge s}$, there will necessarily be a $\mathcal{LT}^{25}$ disambiguation of category $np^{\wedge(s_1\backslash(s_1/s_2))}$, and if there is not, there is no way to copy the transformation used in the quantifier case.

In the case that α is yesterday a man, there is no $\mathcal{LT}^{25}$ disambiguation of category $np^{\wedge(s_1\backslash(s_1/s_2))}$.

Complicating the picture somewhat, when α is John shouted and or John thinks that Mary, there *are* $\mathcal{LT}^{25}$ disambiguations of category $np^{\wedge(s_1\backslash(s_1/s_2))}$. Nonetheless, if we mimic the transformmation of the $\mathcal{HT}^3$ disambiguation that is used in the quantifier case, using these $\mathcal{LT}^{25}$ disambiguations of α, the resulting $\mathcal{LT}^{25}$ disambiguations do not have the same denotations as the $\mathcal{HT}^3$ denotations.

For α = John shouted and, the denotations of the $\mathcal{HT}^3$ disambiguation and the 'corresponding' $\mathcal{LT}^{25}$ disambiguation are given in (25) and (26) below for comparison. Similarly (27) and (28) makes the comparison for α = John thinks that Mary.

(25) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)$

$$([\![\overline{\text{believes}}]\!](w,g)([\![\overline{\text{came in}}]\!](w,g)([\![\overline{\text{John}}]\!](w,g)))([\![\overline{\text{Harry}}]\!](w,g)))([\![\overline{\text{shouted}}]\!](w,g)([\![\overline{\text{John}}]\!](w,g)))$$

(26) $\mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)$

$$([\![\overline{\text{believes}}]\!](w,g)([\![\overline{\text{came in}}]\!](w,g)([\![\overline{\text{John}}]\!](w,g)))([\![\overline{\text{Harry}}]\!](w,g)))([\![\overline{\text{believes}}]\!](w,g)([\![\overline{\text{shouted}}]\!](w,g)([\![\overline{\text{John}}]\!](w,g)))([\![\overline{\text{Harry}}]\!](w,g)))$$

(27) $[\![\overline{\text{thinks}}]\!](w,g)([\![\overline{\text{believes}}]\!](w,g)([\![\overline{\text{came in}}]\!](w,g)([\![\overline{\text{Mary}}]\!](w,g)))$
$\qquad\qquad\qquad\qquad([\![\overline{\text{Harry}}]\!](w,g))$
$\qquad)$
$\qquad ([\![\overline{\text{John}}]\!](w,g))$

(28) $[\![\overline{\text{believes}}]\!](w,g)([\![\overline{\text{thinks}}]\!](w,g)([\![\overline{\text{came in}}]\!](w,g)([\![\overline{\text{Mary}}]\!](w,g)))$
$\qquad\qquad\qquad\qquad([\![\overline{\text{John}}]\!](w,g))$
$\qquad)$
$\qquad ([\![\overline{\text{Harry}}]\!](w,g))$

Summing up, one can say that adopting the /AR and \AR accords certain semantic properties to strings of category s/(s\np) and s\(s/np) which should be accorded only to quantifiers. $\mathcal{LT}^{25}$ does not do this. What this shows is that $\mathcal{HT}^3$ is not a good option for an extension of LCG that accounts for logical ambiguity. This is not in itself very important, because no-one has every seriously proposed $\mathcal{HT}^3$. What is more important whether the semantic overgenerations noted for $\mathcal{HT}^3$, also apply to $\mathcal{HT}^1$, which it will be recalled was simply the combination of LCG with type-flexibility (plus the polymorphism of junctions), and was the account put forward in Moortgat 88.

5.2 Combining Type Flexibility with Lambek Categorial Grammar

Recall that the categorisations of $\mathcal{HT}^1$ were bipartite, comprising a category and a type. John has for example bipartite categorisation np:e, and a man has bipartite categorisation np:$((e,t),t)$. Below is a $\mathcal{HT}^1$ disambiguation of Harry believes α came in, where α is any expression of category s/(s\np) : e^t:

(29)
$$\dfrac{\dfrac{\dfrac{\text{Harry believes that}}{\text{s/s}:(t,t)}}{\text{s/s}:((tt,t),t)}\text{AR} \quad \dfrac{\dfrac{\dfrac{\alpha}{\text{s/(s}\backslash\text{np)}:(et,t)}}{\text{s/(s}\backslash\text{np)}:(((et,t),(tt,t)),(tt,t))}\text{VR}\quad \dfrac{\dfrac{\dfrac{\text{came in}}{\text{s}\backslash\text{np}:(e,t)}}{\text{s}\backslash\text{np}:(e,(tt,t))}\text{VR}}{\text{s}\backslash\text{np}:(((et,t),(tt,t)))}\text{AR}}{\text{s}:(tt,t)}}{\text{s}:t}$$

Possible values for α are:

(30) John shouted and
John thinks that Mary

a man and yesterday a man are not on the list. In the case of a man this is because its categorisations is not s/(s\np) : (et,t), but np : (et,t). In the case of yesterday a man, although it has the categorisation s/(s\np) : (et,t), its denotation is the trivial undefined object (this is because of the term that is associated with the proof of the sequent s/s, np $\Rightarrow$ s/(s\np); it 'expects' np to have type e).

One can easily show that for both of the α in (30), the denotation of (29) is exactly the same as that of (22), given in (24). Therefore, for these α, $\mathcal{HT}^1$ produces the same semantic overgeneration on Harry believes that α came in as $\mathcal{HT}^3$.

It appears that for $\mathcal{HT}^3$, more or less all that is required of an expression for it to engender ambiguity is that it have type $((e,t),t)$. It is not so significant what the category part of its categorisation is. every man engenders ambiguity whilst categorised np, and John believes that Mary engenders ambiguity whilst categorised s/(s\np). Therefore, the problem for combining the type-flexibility proposal with LCG is that there will be many expressions (admittedly strange compounds) of type $((e,t),t)$, which do not engender ambiguity.

One might wonder whether this is a problem even when the type-flexibility is combined with a less powerful grammar than LCG. For again if there are expressions of type $((e,t),t)$, the flexibility approach is liable to see them as engendering ambiguity. For example, at a rudimentary level one might assign the type $((e,t),t)$ to an expression like is barbaric, used in sentences like to shout is barbaric, shouting is barbaric. If to shout and shouting had types (e,t), then the type-flexibility approach would allow one to interpret John thinks that to shout is barbaric as something like it is barbaric to be something that John thinks shouts.

6 Conclusions

We have compared the type-flexibility account with PLCG. About this we first noted that it is a slight misdescription to say type-flexibility offers an alternative account of logical ambiguity to that offered by PLCG - it is important to bear in mind that FMG itself adopts a *polymorphic* analysis of junctions. Second we argued that FMG does not fare well by the criterion of emergence. Third, we pointed out that FMG creates a disunity amongst the junctions and determiners that is not present in the PLCG account. A slightly different kind of comparison took the form of providing a translation from an FMG analysis into a PLCG analysis. Finally we considered two ways of building an account 'on top' of LCG that this translation suggests, one by adding categorial versions of the type-shift axioms to the categorial calculus and one by simply using LCG in tandem with a type-shifting component. Either option was seen to lead to semantic overgenerations.

Chapter 9

Conclusions

We have presented an extension to the LCG framework that allows an explanation of the extensive privileges of occurrence of junctions and quantifiers, and the ambiguities these words induce. I hope most would agree that there is some interest to the account put forward if only for the reason that it is surprising that an account whose basic ingredients are so elementary can achieve the coverage of ambiguity; ambiguity has often be seen as a quite sophisticated accomplishment of a grammar.

However, I have tried to argue that the PLCG account is interesting in more ways than its simplicity, and here I will draw together the various threads of argument that have been presented so far. The task of persuasion is different according to whether or not one can presume the audience is persuaded that LCG should be the core of any account. The discussion in section 5, Chapter 8 is addressed to such a person. Such a person is looking for reasons why the PLCG extension of LCG is better than other ambiguity-explaining extensions of LCG. Two such other extensions of LCG were considered, one the combination of LCG with the type-flexibility proposal and the other the addition to LCG of a categorial version of 'Argument-Raising' axiom. Semantic overgenerations that would arise for such accounts were pointed out that effectively rule them out as possible solutions.

I have also argued that the PLCG account should interest someone even if they are not persuaded that LCG should form the core of any account. Chapter 5 considered the classic transformational solutions to logical ambiguity that have been proposed (or Montagovian analogues of them) and it was argued that on the criterion of emergence, the PLCG account is to be preferred.

Moving the discussion towards accounts that allow a non-deterministic exploitation of syntax by semantics, Chapter 8 considered Hendriks' type-flexibility account and argued that on the grounds theoretical economy of theoretical technology and the criterion of emergence, the PLCG account is to be preferred.

There are further accounts that allow a non-deterministic exploitation of syntax by semantics that have not been considered so far, the most well known of which is 'Cooper Storage'. I shall offer a few comments on this proposal now, for the description of which the reader should consult Cooper 83. First, I take it that storage is a way to implement the effects of 'quantifying-in' without having it as a syntactic rule. Cooper presents a system that simulates the semantic effects of the non-minimally typed PTQ account. Storage is only an *option*, there being always another mechanism not using storage that will give one interpretation (but only one) to a quantifier-containing sentence. Such a two-route system does not fare well by the criterion of emergence: remove the option to 'store' quantifiers and one loses only the coverage of (quantifier-induced) ambiguity. In Chapter 5, we also considered the 'quantifying-in' strategy in the context of a minimally typed account, and for this also there is a parallel storage account. We concluded in Chapter 5, that one could not get by with just a rule that allowed quantifiers to be 'quantified-in' to a sentence, and that the most natural way to make up the shortfall is by adding what we called the 'local' account of quantification, Cross Categorial Quantification. The combined account inevitably fails by the emergence criterion. The same considerations apply to the account that has storage instead of 'quantifying-in'.

There is also the consideration that, as it stands, the storage account does not even address junction-induced ambiguities. One can see the lines on which a 'junction-storage' account could be formulated, but it would bear the same relation to the syntactic technique of Conjunction Reduction as quantifier-storage bears to the syntactic technique of 'quantifying-in'. Therefore the criterion of emeregence will apply again.

Having this summarised the comparisons that can be made between the PLCG account as so far developed and other accounts we will now consider potential developments of the PLCG account and highlight some particular areas needing further research.

Negation words were mentioned back in Chapter 1 as the third kind of 'logical constants' (the others being junctions and determiners). Like the others they engender ambiguity without obvious accompanying syntactic ambiguity. Like the others also, they have extensive privileges of occurrence:

(1) a. not every attendant had a good time
    b. every attendant did not have a good time
    c. I could not go
    d. I ought not to go
    e. every non-attendant had a good time
    f. I like Laurel but not Hardy

The privileges of occurrence could be accounted for by categorising not as $\forall X.X/X$. This could be associated with a polymorphic meaning, $\mathcal{N}_\forall$, of type $\forall\pi((s,\pi),\pi)$, based on a a propositional negation $\mathcal{N}$ of type (t,t), in the following way (see also Keenan and Faltz 85):

for any conjoinable type a, for any $P^{(s,a)}$,

if $a = t$, $\mathcal{N}_\forall(w,g)(D_a)(P) = \mathcal{N}(w,g)(Pw)$

if $a = (b,c)$, $\mathcal{N}_\forall(w,g)(D_a)(P) = x^b \mapsto \mathcal{N}_\forall(w,g)(D_c)(w' \mapsto Pw'x)$

As with the polymorphic analysis of junctions and determiners, PLCG will predict that not engenders ambiguity, and in some cases this is borne out by the facts, while in others this seems an overgeneration. For example, for (1b,c,d) there are will be readings obtainable by a pure applicative analysis and then one can investigate the possibility of readings that make them more or less synonymous with:

(2) b′. it is not the case that every attendant did have a good time

c′. it is not the case that I could go

d′. it is not the case that I ought to go

(2b′) and (2c′) seem genuine possibilities, whilst (2d′) does not. A skeletal indication of how the readings (2b′, c′) could be accounted for is given in (3). There is also an indication of how the only possible reading of (2f) is accounted for:

(3) $(\text{every attendant did})_{s/(s\backslash np)}\ (\text{not } (\text{have a good time})_{s\backslash(s/(s\backslash np))})_{s\backslash(s/(s\backslash np))}$

$(\text{I could})_{s/(s\backslash np)}\ (\text{not } (\text{go})_{s\backslash(s/(s\backslash np))})_{s\backslash(s/(s\backslash np))}$

$(\text{I like})_{s/np}\ ((\text{Laurel})_{s\backslash(s/np)}\ \text{but } (\text{not } (\text{Hardy})_{s\backslash(s/np)})_{s\backslash(s/np)})_{s\backslash(s/np)}$

Tense is another cause of syntactically unreflected ambiguity:

(4) everyone sitting in the room witnessed the crime

According to the context (4), can be construed as talking of a group of people presently gathered in a room, who in the past witnessed a crime (reading 1) or as talking of a past time at which a group of people were gathered and witnessed a crime (reading 2). Can this and other similar cases be explained within PLCG ? The problem presented is somewhat different to that found with the 'logical constants' because there may not be an ambiguity causing word, there may simply be an ambiguity causing feature on a word: in (4) there is the past-tense *form* of the verb witness, rather than a separate lexical item indicative of tense. If there *were* a separate lexical item indicative of tense, say PAST just before the verb witness, then the ambiguity could be explained in exactly the same manner as (1b), giving PAST the category $\forall X.X/X$, the type $\forall\pi((s,\pi),\pi)$, and relating this to a propositional tense operator, $\mathcal{PAST}$, as follows ($\mathcal{PAST}_\forall$ is the meaning to be associated with the polymorphic lexical item PAST):

whatever conjoinable type,a, whatever $P^{(s,a)}$,

$\mathcal{PAST}(w,g)(P^{(s,t)}) = 1$ *iff there exists* $w' < w$, *such that* $P(w') = 1$

if $a = t$, $\mathcal{PAST}_\forall(w,g)(D_t)(P) = \mathcal{PAST}(w,g)(P)$

if $a = (b,c)$,$\mathcal{PAST}_\forall(w,g)(D_a)(P) = x^b \mapsto \mathcal{PAST}_\forall(w,g)(D_c)(w' \mapsto Pw'x)$

reading 1: everyone sitting in the room $(\text{PAST } (\text{witness the crime})_{s\backslash np})_{s\backslash np}$

reading 2: everyone sitting in the room $(\text{PAST } (\text{witness the crime})_{s\backslash np^{\wedge s}})_{s\backslash np^{\wedge s}}$

Therefore what it takes to be able to use polymorphism to explain the ambiguity of (4) is the supposition of a certain *abstract* lexical item: the input to interpretation is not just the sequence of words that one is presented with, but a sequence that may include the result of decomposing certain lexical items into separate parts. In the present case it seems quite natural for morphological analysis to recover from witnessed the combination PAST walk. However, it must be conceded that in allowing for the possibility of such preprocessing, the falsifiability of the claim that a given phenomenon can be explained using polymorphism is considerably lessened.

There is some promise for the use of PLCG in a categorial adaptation of the semantic analysis of interrogatives that has been proposed by Groenendijk and Stokhof (84,87,90, henceforth G+S 90). They have proposed an analysis of both embedded (5a) and unembedded (5b) interrogatives:

(5) a. Mary knows who came in

b. who came in

(5b) is interpreted 'categorially' (as they put it), as a one-place function, roughly the set of people that came in: $[\![(5b)]\!](w,g) = x^e \mapsto [\![\text{came in}]\!](w,g)(x)$

With this semantic value for (5b), they can explain the question-answer relation between (5b) and an answer such as John. From the 'categorial' interpretation of a question, $[\![Q?]\!]_C$, a 'propositional' interpretation, $[\![Q?]\!]_P$, can be derived that is appropriate to embedded uses:

(6) $[\![Q?]\!]_P(w,g)(w') = 1$ iff $[\![Q?]\!]_C(w',g) = [\![Q?]\!]_C(w,g)$

$[\![(5a)]\!]_P(w,g) = [\![\text{know}]\!](w,g)(p)([\![\text{Mary}]\!](w,g))$, where $p(w') = 1$ iff $[\![\text{came in}]\!](w',g) = [\![\text{came in}]\!](w,g)$

Thus the 'categorial' interpretation is more basic, and it is proposed to be derived using the following syntactic and semantic mechanisms[1]. The syntactic origin of an interrogative containing n WH-phrases is a sentence containing n pronouns. Akin to the 'Quantifying-in' rules, there are 'WH-abstract' rules that act iteratively upon this, inserting a WH-phrase for a particular pronoun. The associated semantic operations are the abstraction operations which we have already encountered in Chapter 5, section 2.3, namely the operations $\mathcal{G}_{\lambda \text{he}_i}$. Thus the meaning of (5b), of type (e,t) is derived from a meaning of type t. If there were n WH-phrases, there would to begin with be a meaning of type t, then a meaning of type (e_1,t), and eventually a meaning of type $(e_n, \ldots (e_1,t) \ldots)$.

It is not obvious that to achieve this semantic coverage one needs to use such syntactic insertion operations and subscripted variables. The abstraction operations, $\mathcal{G}_{\lambda \text{he}_i}$, do not really change a meaning of type a into a meaning of type (e,a), they really just rearrange a function of type a. To begin with one has an n-place function in the form of a meaning of type t, depending on assignments, and discriminating between assignments on the basic of some difference in the values assigned to a certain set of n variables. After the abstraction operations, one has the same n-place function in the form of a meaning of type $(e_n, \ldots (e_1,t) \ldots)$, depending on assignments,

[1] This is actually a simplified version that allows only who to be a WH-phrase. It also only concerns 'in situ' WH-phrases.

but equating all assignments together. It seems one could arrive at the same end point in a different manner. To begin with one has an n-place function, and this is the meaning of the verb in the interrogative, of type $(e_1, \ldots (e_n, t) \ldots)$. Combinations with WH-phrases could then take place, each accompanied by a step of *argument permutation*, from a meaning of type $(e, \vec{a}, t)$ to a meaning of type $(\vec{a}, e, t)$. The end result will be a form of the verb function that one began with, a form of type $(e_n, \ldots (e_1, t) \ldots)$. This would use only concatenative syntactic operations and would give the same interpretations to who came in and who loves who as the G+S 90 proposal, and therefore suggest the possibility of a PLCG account, and such an account is outlined below.

For this, WH-phrases are given the same categorisation as quantifiers: ∀X.X/(X\np) and ∀X.X\(X/np). Their meaning shall not quite be the argument permutation operation, mentioned above. We will arrange things so that if α is an expression of category x/np, and contains n WH-phrases, its semantic type will be $(e, \vec{a}, \vec{b}, t)$, where $\nu(x) = (\vec{a}, t)$, and $\vec{b}$ is a sequence of types associated with the WH-phrases. Thus the argument sequence of the type is split into two parts, the first relating to yet to be encountered arguments, and the second relating to already encountered WH-phrases. On the permutation account described above, if α who were now built, the type of the result would be $(\vec{a}, \vec{b}, e, t)$, with e becoming added on to the end of $\vec{b}$. What we shall actually suppose is that e is added on to the *beginning* of $\vec{b}$, so that the type of α who is $(\vec{a}, e, \vec{b}, t)$. We can do this by defining the following *type-swallowing* functions as the meanings of WH-phrases:

$[\![\text{who}]\!](w, g)(D_{a,b})(d^{(e,a,c)}) = x^a \mapsto [\![\text{who}]\!](w, g)(D_b)(y^e \mapsto d(y)(x))$
$[\![\text{who}]\!](w, g)(D_t)(d^{(e,c)}) = d$

It has to be confessed that we have defined by recursion here a function which does not have a $\text{TJ}^{(\rightarrow,\forall)}$ type. Also the type of a consituent containing WH-phrases will not be the type corresponding to the category, though it will be a $\text{TJ}^{(\rightarrow,\forall)}$ type. There is a remnant of the category to type correspondence, however, in the each combination with the WH-phrase, lowers the 'linguistic-arity' of the function: there is one fewer argument place concerning which one can expect other expressions to contribute information.

Not all the argument places of a verb in a interrogative need be filled by WH phrases (though in English not just anything goes if the WH-phrase is 'in situ'). One could have who told who to go. The categorial syntax will suggest that told who, of category (s\np)/vpc, and to go could be combined by function application and if this is done the right result would be obtained. One ignores here the mismatch of the type of told who, $((e, t), e, e, t)$ with what the category-to-type map suggests. It is possible to do this because the operations dictated by the syntax are still compatible with the types. The same goes for interrogatives that include quantifiers together with WH phrases, as in who gave who a Xmas card. In this fashion, the interpretations that G+S 90 suggest for unembedded interrogatives may be derived. For embedded interrogatives it is necessary to suppose that the conversion operation from the 'categorial' interpretation to the 'propositional' interpretation could be included in the meaning of the embedding verb. The polymorphism of the quantifiers and WH phrases would allow one then to account for some ambiguities possible with embedded interrogatives:

(7) a. Bill knows who entered every shop

a1. (Bill knows)$_{s/s}$ ((who entered)$_{s/np}$ every shop)$_{s}$

a2. (Bill knows)$_{s/s}$ ((who entered)$_{(s\backslash(s/s))/np}$ every shop)$_{s\backslash(s/s)}$

b. John knows who said who was there

b1. John knows (who said)$_{s/s}$ (who (was there)$_{s\backslash np}$)$_{s}$

b2. John knows (who said)$_{s/s}$ (who (was there)$_{(s\backslash(s/s))\backslash np}$)$_{s\backslash(s/s)}$

To distinguish the readings of (7a), suppose some people entered at least one shop, while others entered *every* shop. One reading has it that Bill knows who the latter group are, and a skeletal disambiguation corresponding to this reading is given in (7a1). Another reading has it that for every shop, Bill knows who the group of people are who entered that shop, and a skeletal disambiguation corresponding to this reading is given in (7a2).[2] To distinguish the readings of (7b), suppose a crime was committed and a number of people were witnesses. Also two or three non-witnesses (they might be a doormen, or taxi-drivers) independently say who these witnesses are. Then one reading has it that John knows who these witness identifying people are, and (7b1) is the disambiguation for this reading. For the other reading, vary the situation to one in which each witness-identifier claims only to have seen one of the witnesses. This is shown in (7b2).

Negation, tense, and interrogatives then are three areas in which it may be argued some of the ambiguity data may be captured within PLCG, using much the same techniques that allowed explanation of the ambiguities engendered by junctions and quantifiers. We will next consider an application of polymorphism along another dimension to that considered so far.

Several times we have considered such quantified noun-phrases as a proposition which John believes and an act that John wanted to do, and referred to them as examples of 'higher-order' quantification. In Chapter 5, section 2.3, there were 'basic' determiners of type $((e,t),(e,t),t)$, and then additional determiners of types $((p,t),(p,t),t)$ and $((vp,t),(vp,t),t)$, where p and vp were abbreviations of (s,t) and (s,e,t) respectively. This suggests another opportunity to use polymorphism, by defining a determiner of type $\forall\omega((\omega,t),(\omega,t),t)$. Then reintroducing the kind of polymorphism we have already considered for the determiners, one could define a *doubly* polymorphic determiner of type $\forall\omega((\omega,t),\forall\pi((\omega,\pi),\pi))$. To illustrate this recall that whenever a

[2] Groenendijk and Stokhof distinguish also a further reading which they call the 'pair-list' reading, similar to the second reading we have described, except that every shop makes a *de-dicto* contribution. How this may be accounted for is a question for further research.

condition on models has been specified concerning determiners, this has involved making reference back to an 'algebra-spanning' function, such as $\mathcal{A}$, first defined in Definition 47. This was a type-specific function, but could easily be revised to be a type-unspecific function, as below:

for any $\mathcal{E},\mathcal{I}$ *and* $\mathcal{J}$, *for all* $(w,g) \in \mathcal{I} \times \mathcal{J}$, *whatever* D_a, *whatever* $P_1, P_2 \in D_{(a,t)}$
$\mathcal{A}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(P_1)(P_2) = 1$ *iff the set characterised by* P_1 *has a non-null intersection with the set characterised by* P_2

Then the condition on models giving a doubly polymorphic meaning to a could be:

whatever type $d \in \text{TJ}^{\rightarrow}$, *whatever* $P_1^{(d,t)}$, *whatever conjoinable type* a, *whatever* $P_2^{(d,a)}$,
if $a = t$, $[\![\mathsf{a}]\!](w,g)(D_d)(P_1)(D_a)(P_2) = \mathcal{A}(w,g)(P_1)(P_2)$
if $a = (b,c)$, $[\![\mathsf{a}]\!](w,g)(D_d)(P_1)(D_a)(P_2) = x^b \mapsto [\![\mathsf{a}]\!](w,g)(D_d)(P_1)(D_c)(y^d \mapsto P_2yx)$

How this polymorphic approach compares to other approaches remains to be considered. There follow a few comments on the 'individual-based' analysis of the phrases a proposition which John believes and an act that John wanted to do (Chierchia and Turner 88). This approach renders the common-noun parts (roughly) of type (e,t), which then allows one to suppose that the determiner has type $((e,t),(e,t),t)$. On this approach, to validate the inferences featuring the 'higher-order' noun phrases (shown below) it seems necessary to use in some fashion an 'individual-correlation' function, relating objects of type e to objects of type (s,t) and (s,e,t).

John believes a man died	John wanted to marry a blond
a man died	John married a blond
∴A proposition that John believes is true	∴An act that John wanted to do, was done by John

The polymorphic approach is perhaps more economical in not requiring such a function, and the model theory behind the polymorphic approach seems more simple than that required to support the idea of 'individual-correlates'. However, for any real comparison to take place, there would have to be a consideration of the other kinds of sentences at which the 'individual-based' semantics is aimed, especially 'self-predication' sentences, so the discussion of higher-order quantification will be left here.

There is also an application of the PLCG framework that is more syntactic than semantic. PLCG offers the possibility of extending the coverage of 'extraction' constructions over that possible within LCG. LCG allows only for those cases where the 'gap' for the 'extracted' NP is in sentence initial or sentence final position. Thus (8a,b) can be captured in LCG but not (8c) (the brackets mark a 'sentence but for missing NP' string, and e marks where the missing NP should go):

(8) a. the man who (e killed the dog)

b. the dog which (the man killed e)

c. the man who (John told e to kill the dog)

PLCG allows the coverage of cases of non-peripheral extraction like (8c) by categorising the relativiser as: ∀X.((cn\cn)/X)/((s/X)/np). The meaning of the relativiser that is appropriate to this categorisation would be (under an extensional typing regime):

whatever type a, whatever $P_1^{(e,a,t)}$, *whatever* P_2^a, *whatever* $P_3^{(e,t)}$:
$[\![\text{who}]\!](w,g)(D_a)(P_1)(P_2)(P_3) = x^e \mapsto \mathcal{AND}(\mathcal{E},\mathcal{I},\mathcal{J})(w,g)(P_1 x P_2)(P_3 x)$

For a full coverage of extraction, one would also have to retain the usual categorial categorisations of the relativiser, (cn\cn)/(s/np) and (cn\cn)/(s\np). Other categorial solutions to non-peripheral extraction have been proposed in Moortgat 88 (developed further in Moortgat 90 (Tilburg)) and in Morril, Leslie, Hepple and Barry (1990), both involving the increase of the categorial language by further connectives, governed by further inference mechanisms. Further research is required to establish criteria of comparison here, both amongst these categorial accounts and between these and accounts not centrally based on LCG (perhaps one could apply another criterion of emergence here, looking this time for whether coverage of extraction constructions was modularly removable).

We have now considered several areas where the syntactic and semantic coverage of the basic PLCG account that we have given may be enhanced by further uses of polymorphism. We turn now to a further area which must be covered but this time where mechanisms of polymorphism make no difference. If one considers Montague's PTQ, one can say that two jobs are performed by the 'Quantifying-in' rule in PTQ. One is the job of accounting for ambiguities caused by quantifiers. The other is the job of accounting for 'pronoun-binding', the semantic part of which is the explanation of the validity of inferences like:

a French man believes that he will one day play for England
no French man will one one day for England
∴ a proposition that a French man believes is false

We have ignored the phenomenon of pronoun-binding entirely so far, and the PLCG account that was given in Chapter 7 will not explain the validity of the above inference. It is also not obvious that an account can be formulated making especial use of polymorphism. Suppose the only way to account for pronoun binding phenomena was by means of the 'Quantifying-in' rule. Then even if one were to accept the PLCG account of ambiguity, one would have to suppose that on top of the PLCG account one also has the 'Quantifying-in' rule. However, such a composite account would be very unattractive, because the 'Quantifying-in' rule would make the entire PLCG account of (quantifier-induced) ambiguity redundant (well almost, recall the undergenerations of the 'Quantifying-in' account that were noted in Chapter 5). Therefore it had better not be that the 'Quantifying-in' account of pronoun binding phenomena is the only one.

In reaction to this I would say first that the 'Quantifying-in' account of pronoun binding phenomena, though capable of explaining quite a large part of the data, leaves substantial parts of it unexplained, especially intersentential anaphora and donkey sentences. Hendriks 90 makes more or less this same point in anticipation of a similar objection being levelled at his type-flexibility account. Secondly, speaking in the abstract, it seems that an equally or more satisfactory account of pronoun binding phenomena might well not share with the 'Quantifying-in' account the feature that it provides also an account of the ambiguity of sentences that themselves *contain no pronouns*. Therefore the combination of PLCG with a more satisfactory account of pronoun binding might well not render the PLCG account of ambiguity redundant. In fact, there is no need to speak in the abstract. Groenendijk and Stokhof's 'dynamic-semantics' account (Groenendijk and Stokhof 91, henceforth DMG. The account was actually put forward in 1989) captures the semantic force of pronoun-containing sentences (and discourses) in an essentially different way to the PTQ account, and it is not the case that the account itself explains quantifier induced ambiguities. Combination of PLCG with the DMG account of pronoun binding would not therefore render the PLCG account of (quantifier-induced) ambiguity redundant. For illustration of the fact that a dynamic semantics account is very compatible with a PLCG account, an indication is given below of a 'dynamic-semantics' model.

The contexts of use, $\mathcal{J}$, will not by assumed to be assignments, but will simply be a set of *states*, $\mathcal{S}$. There is now a *type* for these contexts of use, $*$, and $\mathcal{S} = D_*$. The typings of expressions are roughly changed from (a,t) to (a,T), where $T = ((*,t),t)$:

$\langle\langle\mathcal{B},(\mathcal{G}_\gamma)_{\gamma\in\Gamma},f\rangle,(w,s)\rangle$ *is a dynamic model, associated with* $\mathcal{E},\mathcal{I},\mathcal{S}$ *if*

Update: *in* $D_{(*,e)}$ *are* discourse markers, d_n, *and whatever* d_i, *whatever state* s, *whatever* x^e, *there exists a unique state* s' *such that* $d_i(s') = x$, *and whatever* $j, j \neq i$, $d_j(s) = d_j(s')$. *This* s' *will be notated as* $s_x^{d_n}$

CN: *there exists a* $d^{(s,e,t)}$, *call it man′, such that,* $[\![\mathsf{man}_n]\!](w)(s)(x^e)(p^{*t}) = 1$ *iff* $man'(w)(x) = 1$ *and* $p(s_x^{d_n}) = 1$

VP: *there exists a* $d^{(s,e,t)}$, *call it won′, such that,* $[\![\mathsf{won}]\!](w)(s)(x^e)(p^{*t}) = 1$ *iff* $won'(w)(x) = 1$ *and* $p(s) = 1$

DET: $[\![\mathsf{a}]\!](w)(s)(P^{(s,*,e,T)})(Q^{(s,*,e,T)})(p^{*t}) = 1$ *iff there is an* x^e *such that* $P(w)(s)(x)(s' \mapsto Q(w)(s')(x)(p)) = 1$

PV: *there exists a* $d^{(s,st,e,t)}$, *call it bel′, such that,* $[\![\mathsf{believes}]\!](w)(s)(\Phi^{(s,*,T)})(x^e)(p^{*t}) = 1$ *iff* $bel'(w)(w' \mapsto \Phi(w')(s)(p))(x) = 1$ *and* $p(s) = 1$

PRO: $[\![\mathsf{he}_n]\!](w)(s)(P^{(s,*,e,T)})(p^{*t}) = 1$ *iff* $P(w)(s)(d_n(s))(p)$ *and* $p(s) = 1$

$\Gamma = \{>_i\} : \mathcal{G}_{>},(m_1^{((s,*,a),b)}, m_2^a)(w,s) = m_1(w,s)((w',s') \mapsto m_2(w',s'))(x)$

One can assume a simple (not necessarily categorial) syntax, which operates on lexical items that bear indices. What is and what is not a possible indexing is not settled by this syntax. All syntactic combinations are associated with the $\mathcal{G}_{>,}$ operation. Below an example of how pronoun binding is accounted for is given (*true* is the characteristic function of $\mathcal{S}$)

(i) $[\![$a man$_n$ believes he$_n$ won$]\!](w)(s)(true) = 1$

$\leftrightarrow$ there is an x^e such that (ii) $[\![$man$_n]\!](w)(s)(x)(s' \mapsto [\![$believes he$_n$ won$]\!](w)(s')(x)(true)) = 1$

(ii) $\leftrightarrow man'(w)(x) = 1$ and (iii) $[\![$believes he$_n$ won$]\!](w)(s_x^{d_n})(x)(true) = 1$

(iii) $\leftrightarrow [\![$believes$]\!](w)(s_x^{d_n})(w' \mapsto s' \mapsto [\![he_n$ won$]\!](w)(s'))(x)(true) = 1$

$\leftrightarrow bel'(w)(w' \mapsto [\![he_n$ won$]\!](w)(s_x^{d_n})(true))(x) = 1$

$\leftrightarrow bel'(w)(w' \mapsto [\![he_n]\!](w')(s_x^{d_n})(w' \mapsto s' \mapsto [\![won]\!](w')(s'))(true))(x) = 1$

$\leftrightarrow bel'(w)(w' \mapsto [\![won]\!](w')(s_x^{d_n})(d_n(s_x^{d_n}))(true))(x) = 1$

$\leftrightarrow bel'(w)(w' \mapsto won'(w')(x))(x) = 1$

Combining (i), (ii) and (iii):

$[\![$a man$_n$ believes he$_n$ won$]\!](w)(s)(true) = 1$ iff there is an x such that $man'(w)(x) = 1$ and $bel'(w)(w' \mapsto won'(w)(x))$

One can say that DMG is a 'semantic' account of pronoun binding phenomena, because there is no syntactic rule that is at the heart of the explanation. The DMG account simply uses a standard syntax, changes the typing assumptions from what is normal, and uniformly uses a new kind of intensional function application to carry out semantic combinations. It is simply the meaning of the lexical items themselves that makes the account work. Because of the uniformity of the semantic operation, there is no obstacle to having an LCG syntax, so long as the proof-to-operation map is changed in the appropriate way. Therefore the essential ideas of DMG can be incorporated very directly into PLCG. However, further work needs to be done to see how well the combination performs: it might be that there will be cases where the PLCG analysis of a quite difficult reading somehow prevents the semantic machinery for pronoun binding from working in the intended way. Also there is an issue whether the dynamic semantics account is limited to the existential and universal quantifiers, or can be extended to cover all quantifiers (Chierchia 91).

From extending coverage, we turn now to the question of restraining it. A question for further research is whether the PLCG account can refined to give a more sensitive prediction of the data, both of reading impossibility and of reading preference. The basic yield of readings seems too great for certain sentences, and there is a question therefore of the possibilities within PLCG to make impossible readings underivable.

One kind of overgeneration that needs to be considered is that occurring with the Rodman sentences, noted in Chapter 3. The island-constraint violating extraction of (9a) should not be derivable, but it is under the PLCG account, nor relatedly should (9b) be interpretable as (9c), which it is under the PLCG account:

(9) a. (every room that Guinevere has a bone which is in)$_{\forall X.np^{\wedge X}}$

b. Guinevere has a bone which is in every room

c. every room is such that Guinevere has a bone which is in it

Though Rodman's observations have not emerged as a theorem of the basic PLCG account, the PLCG account can still be argued to show promise in relating the syntactic restriction to the semantic. This is because it seems that any steps taken to modify the PLCG account so as to account for the fact that (9a) is underivable seem likely to also rule out (9c) as a reading of (9b). There follows an example of this. (9a) could not be derived if (10a) could not be, and this requires that the sequent (10b) be underivable:

(10) a. (Guinevere has a bone which is in)$_{(s/np)}$

b. np, (s\np)/np, $(\forall Y.np^{\vee Y})$/cn, (cn\cn)/(s\np), (s\np)/np $\Rightarrow$ s/np

(10b) would be underivable if one had as a side condition to the (/R) inference that amongst the antecedents one could not have the relativiser categorisation, (cn\cn)/(s\np). (10b) is also a subgoal of the obvious way to derive reading (9c), so the side-condition to the (/R) inference would also rule out this route to the semantic overgeneration. There are several other routes to be considered, but all in fact are ruled out by the side-condition to the (/R) inference. Further work is required to explore such an explanation of Rodman's observation.

There are some further overgeneration problems to be considered with junctions. For all of the sentences below the PLCG account predicts that there is a reading according to which the sentence is equivalent to a particular junction of two sentences. Arguably this is an overgeneration in the case of (11a), though certainly not in the case of (11b,c,d,e).

(11) a. a man came in and sat down

b. every man and woman died

c. every house in London and Paris has an escalating price

d. the judge will say either that they are guilty or that they are not guilty

e. he wants not to praise Caesar but to bury him

It is difficult to see how the PLCG account could be adjusted to rule out just the overgeneration associated with (11a). This is because for all the sentences above, the PLCG analysis that makes the sentence equivalent to a junction of two sentences is very similar: the polymorphic junction is instantiated not to an immediately neighbouring category, but to something derivable by (Slash R) inferences from an immediately neighbouring category. The only way that I can see to proceed here would be to have the restriction that when the variable of a polymorphic junction is instantiated to a category, that category must derivable from some category in the set, {cn, np, sc, vpc}. In particular this set does not contain s\np. This is not a very elegant solution to the problem, and is made even less elegant by the fact that no such restriction is required for the polymorphic quantifier. However it should be borne in mind that both the Conjunction Reduction and FMG accounts of the ambiguities caused by junctions will suffer from the same overgenerations.

A question also for further research is the usefulness of instantiation restrictions to explain reading preferences. Consider the following sentence:

(12) every man loves a woman and every shop attracts a shopper.

This is a conjunction of two sentences, any one of which taken on its own is ambiguous. There is a strong tendency towards reading parallelism in such conjunctions, and nothing in the PLCG account leads one to expect this (nor in fact in any other account). Such reading parallelism could, however, be implemented by letting the instantiations of category variables of the quantifiers in the first conjunct dictate the instantiation possibilities of the corresponding quantifiers in the second conjunct.

I will comment finally now on two technical matters that deserve further investigation.

The problem of avoiding circularity was encountered both in giving a string-semantics for $\text{CAT}^{(/,\backslash,\forall)}$, and in giving a function-space semantics for $\mathcal{L}^{(\lambda,\Delta)}$. In the case of $\text{CAT}^{(/,\backslash,\forall)}$, it led to a restriction on the ($\forall$L) rule - the restriction that a quantified categorial variable cannot be instantiated to any category involving quantifiers. In the case of $\mathcal{L}^{(\lambda,\Delta)}$, it led to the restriction on the formation of type application terms that when a term of type $\forall\pi.a$ is applied to a type b, the result is only defined if b is a type which contains no type quantifiers. It remains an issue to be investigated whether the semantics for $\text{CAT}^{(/,\backslash,\forall)}$ and $\mathcal{L}^{(\lambda,\Delta)}$ could be such that such restrictions could be eliminated.

A less esoteric problem is that of decidability. The ($\forall$L) inferences invite one simply to *choose* a category, and if one cannot find a proof on a given choice one could always return to the ($\forall$L) inference and choose another category, and so on. One might think one could remain uncommitted in the choice of variable at the ($\forall$L) inference, and by unfolding the proof accumulate constraints on the unknown until such time as only one possibility remained for the unknown. This will not work because the typical next step after the ($\forall$L) inference is a (/L) inference, the minor premise of which is $T \Rightarrow x$, where x is the unknown and T is known. If there are any answers for x at this point, there are infinitely many. So decidability of the calculus remains a problem. In mitigation of this one can point out that several people have advocated the polymorphic approach to junctions, often together with mechanisms for type-shifting or category-shifting (Partee and Rooth 83, Moortgat 88, Hendriks 90), and these proposals would seem to suffer equally from the decidability problem. There are two kinds of response to this problem that invite further research. The first is suggested by section 3.2.4, chapter 7. There we *generated* the proof that explained recursive ambiguity of ... DET CN ..., *from* a proof for ... NP .... One might be able to design a procedure that take ones through the relevant set of possible proof solutions for a particular sentence by first finding one proof and then *generating* the others by a process of proof transformation. The second kind of solution to the decidability problem that could be investigated would be to have revisable bounds to the complexity of instantiation considered at a (∀L) inference.

Definitions

Hypotheses

Theories

Models

Languages

Page	Language	Indication of Content
15	$\mathcal{L}^1$	nearly a DISAMBIGUATED LANGUAGE
18	$\mathcal{L}^2$	DISAMBIGUATED LANGUAGE
86	$\mathcal{L}^\theta$	language part of $\mathcal{T}^\theta$, a THEORY OF REFERENCE which simulates the L-theory, θ
95	$\mathcal{L}^4$	verbal terms and proper names
97	$\mathcal{L}^5$	incorporating verbal terms and proper names
101	$\mathcal{L}^6$	a revision of $\mathcal{L}^5$ which recognises 'intensionalisation' as a syntactic operation
102	$\mathcal{L}^7$	incorporating embedding constructions
104	$\mathcal{L}^8$	including sentential junctions
108	$\mathcal{L}^9$	including pronouns and subject position quantifiers
111	$\mathcal{L}^{10}$	CN modifiers, higher order quantification and truth predicates
119	$\mathcal{L}^{11}$	non-sentential junctions by adding CR as a syntactic operation
123	$\mathcal{L}^{12}$	cross-categorial coordination
129	$\mathcal{L}^{13}$	QL as part of the disambiguation relation
131	$\mathcal{L}^{14}$	QL and Chomsky-Adjunction as a single syntactic operation
136	$\mathcal{L}^{15}$	incorporating a local approach to quantifiers
146	$\mathcal{L}^{16}$	a language that does distinguish NP and QNP
153	$\mathcal{L}^{17}$	another language that does not distinguish NP and QNP
164	$\mathcal{L}^{18}$	verbal terms, proper names and embedding verbs
228	$\mathcal{L}^{19}$	verbal terms, proper names, embedding verbs, junctions and determiners

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