Modeling the child's development of cardinality: from counting to conservation of number

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Dedicated to the memory of
my mother Luisa

Declaration

I declare that this thesis has been composed by myself and that the research reported therein has been conducted by myself unless otherwise stated.

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Abstract

This thesis proposes a domain-specific framework to deal with some of the problems of cognitive development raised by Post-Piagetian research, in particular early competence and heterogeneity in across-domains performance. The theoretical framework is explored in relation to the development of the domain of cardinal number: from counting to precocious number conservation to standard conservation. The results of three empirical studies support the interpretation that a same structure specialized in processing numerical information becomes operational on contents of increasing complexity:

1. on individual sets (as in set reproduction tasks);
2. on two or more sets (as in set comparison and modified conservation tasks);
3. on sets of sets (as in the Piagetian number conservation task).

The process by which the child discovers the import of the number-structure on new contents is modeled as a semantic process that transforms cardinal representations, entertained as irrelevant, into relevant representations. This transformation proceeds in a stage-like way which, at each stage, reveals new facts about number and brings about a restructuring and extension of the cardinal number concept.

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Chapter 1 General introduction

1.1 Early competence

Suppose we want to assess a 5-year-old child's understanding of cardinal number. We may begin by presenting the child with Piaget's well known number conservation task. We ask the child to take the same number of sweets as there are in a row before him. The child diligently places one sweet in front of each one of the row's items. Then we transform one of the rows into a longer array. We ask the child whether the two rows still have the same number of sweets or whether one of them has now more sweets than the other. The child replies that there are more sweets in the longer array and suggests that to have the same number again, the sweets which exceed the length of the shorter row have to be taken away.

The reply of our 5-year-old to the number conservation question and his attempt to re-establish the equinumerosity are indicative of an extremely limited understanding of what the cardinal number of a set is. His numerical inferences appear to be bound to the perceptual appearance of the set because the number is considered to change when the set's configuration is changed. It increases when the set looks bigger and decreases when it looks smaller. According to Piaget's characterization, this child's number concept is pre-operational, that is, perception-bound and irreversible.

Subsequently, we present the same child with another version of the conservation task, and, once the transformation has been performed, we require a separate count of the two rows before asking whether the number is the same or whether it has changed. The child now concludes that the rows have the same number of sweets and justifies this by pointing out that no sweets have been added nor taken away. At this point we may start suspecting that, in fact, the child does know something about cardinality and about what transformations affect or do not affect the numerosity of a set. And we may also start doubting of Piaget's clear-cut characterization of our 5-year-old child's number concept as perception bound and irreversible. This suspicion may be put under further test by presenting the child with another version of the task in which no counts are required. Here the spatial transformation of the row, instead of being performed by the experimenter, occurs as an accident. The array's configuration is changed by the intervention of a third disturbing agent, like a "naughty" teddy bear, who hits the row and mixes it up. With the two rows again looking different, we ask the child if they are still equinumerous or if one of them has more sweets. Here too the child confirms that the two rows are equinumerous and typically puts back the sweets into one-to-one correspondence. You may have recognized in these two modified conservation tasks the situations first used by Gréco (1962) and McGarrigle & Donaldson (1975). Results of this kind have been reported for a large number of Piaget's sensori-motor and concrete operational tasks and constitute evidence of what have been called *early competencies*: one of the major issues in cognitive development.

How are these early competencies to be accounted for? And more particularly what is that allows the child to solve the conservation task under certain conditions and what at the same time prevents him from confirming the equinumerosity in the standard Piagetian task? Is the understanding of conservation in the modified tasks itself the outcome of a developmental process? If this is so, what kind of change takes place between the phase in which the modified tasks are failed and the later phase in which they are solved? And between the period in which the modified tasks are understood and the later phase in which also the standard conservation is solved? These are the questions which I shall address in this thesis. But before introducing the specific aims and the plan of this work, I wish to draw the wider background against which these questions have arisen and to situate them with respect to the current debates in cognitive development research.

1.2 Background

The study of cognitive development has been deeply influenced by the ideas and the work of Jean Piaget. According to Piaget, the cognitive system develops through an ordered sequence of stages, each underlying a qualitatively distinct representational and reasoning structure. Piaget identified three overall stages which he characterized with three structures: the group of displacement, constructed during the sensori-motor period (between 0 and 2 years of age), the grouping of concrete operations, constructed during the representational (or semiotic) period (between 2 and 10 years of age) and the group of formal operations, constructed during the hypothetical-deductive period (from 10 years of age to adolescence).

The development of the diverse concepts reflects that of the general stages and is revealed by the increasing coherence, generality, and mobility of the concepts. Piaget studied the process by which these properties of concepts emerge in the solving of problem which have a same structure. The child is either guided to focus his attention on some component of the experimental situation (e.g. look at an object in the task about the permanence of the objects) or asked to perform an action (e.g. establish a one-to-one correspondence between two sets of objects in the task dealing with the conservation of the equinumerosity, or reproduce the water-line of a half-filled bottle in the task exploring the concept of horizontality). Then the element or the relation attended to undergoes some transformation. In the case of the permanence of the object task, for example, the object is hidden under a cover. More complex transformations involve hiding the object under a second cover, placed beside the first one; permuting the two covers or performing the different hiding operations out of the child's view. In the case of the conservation of number task, as we have seen, one of the two equinumerous rows is lengthened to destroy the spatial one-to-one correspondence between the rows' elements and the child has to recognize the equinumerosity beneath the length difference. In the water-line task, the bottle is tilted and the child is asked to draw the water-line in this non-canonical position.

From the evidence of the children's capacity to relate the initial, canonical states, the transformations and the new, modified states (demonstrated by the retrieval of the object in the object-permanence task; by the confirmation of equinumerosity in the conservation of number or by the drawing of a horizontal line to represent the water-line in the tilted bottle), the degree of generality, coherence and mobility of their concepts is inferred. Using tasks of this kind, Piaget analysed the genesis of most domains of knowledge, providing a very detailed description of the sensori-motor period (eight sub-stages) and a broader description of the operational periods. In the case of the genesis of most concrete operational concepts, in fact, Piaget identified three clearly distinct sub-stages:

1. the inability to establish the initial relation (for instance the one-to-one correspondence or the representation of the water-line);
2. the ability to establish that relation in the initial situation, but not in the modified one;
3. the ability to generalize the relation to the transformed situation.

The Piagetian theory has been submitted to an extensive and thorough empirical testing, which has occupied much of the research on cognitive development from the 50s well into the 80s. Two aspects of the theory have been examined with special attention:

1. The general stage hypothesis, that the different knowledge domains are organized by same structural principles and should hence develop with some degree of homogeneity;
2. The description of the development of individual concepts in a sequence of sub-stages and the account of the underlying change.

There is now a certain consensus among students of cognitive development that little evidence exists in support of (1) the hypothesis of major stages of cognitive development of the type described by Piaget and of (2) the account of the development of specific concepts (see Gelman & Baillargeon 1983, Carey 1984 for excellent and complete critical reviews of the literature).

Concerning the hypothesis of across-the-board stages, and the prediction extrapolated from it that cognitive abilities should emerge in a coordinate fashion, the research about correlations in the performance across knowledge domains found very low correspondences. These findings are particularly robust in the concrete operational stage, where the same children were often reported to behave according to Piaget's Stage 1 criteria for some concept and according to Stage 3 criteria for some other concept, or inversely. More problematically for the Piagetian hypothesis, similar results are also reported for the formal operational period, where the performance in logical reasoning tasks was found to be critically dependent on the familiarity and expertise of the subjects on the particular task content (Johnson-Laird 1983, Evans 1982).

Concerning the hypothesis of clear-cut changes in the development of individual concepts, the studies of replication of Piaget's results produce two apparently "contradictory" findings. On the one hand, the studies which employed tasks identical to the Piagetian original ones (also with different materials and modes of presentation), consistently replicated Piaget's findings. On the other hand, the studies which introduced more important changes to the task presentation (e.g. contextual and supplementary information are provided; the nature of the test display and of the response required are modified), while maintaining a very similar structure (i.e. the initial state, the transformation and the modified state), found that children could solve the tasks under these modified conditions before they could solve the original tasks. These findings, which exist for the great majority of Piagetian tasks, have been interpreted as evidence of early competence, i.e. under certain circumstances, young children behave in a way qualitatively equivalent to that of older children. The young children appear to be operating with concepts which do not differ in coherence and mobility, although they have a more limited domain of application.

In conclusion, the results of limited across-domains homogeneity of performance and of early competence levels beside the operational level described by Piaget call for a revision of the hypotheses of general stages of logical competence and of clear-cut differences between younger and older children's concepts.

To approach these anomalies of Piaget's theory, two theoretical tendencies have been emerging. On the one hand, the so-called neo-Piagetian theories reformulate the idea of general qualitative changes by defining a sequence of overall stages in terms of information-processing capacities, such as memory size, number of variables handled (see Case 1985, Halford 1982). On the other hand, the domain-specific paradigm abandons the ideas of across-the-board stages and of changes in reasoning and representational capacities in favour of the idea that cognition is organized in domains, or sub-systems which develop with some autonomy (see Carey 1985, Gelman & Baillargeon 1983, Karmiloff-Smith 1988a, 1988b, Keil 1986, Gopnick 1988). As this approach gives up the hypothesis of qualitatively different reasoning and representational systems, its basic assumption is that what develops are limited knowledge structures which become increasingly more explicit, abstract and general, as opposed to changing in kind.

The domain-specific approach seems to be a better candidate to account for the evidence of low correlations and early competencies and to exploit the extensive database on these phenomena. From a domain-specific perspective, the correlation data constitute evidence of the relative independence with which distinct conceptual fields develop. A same child can be in Stage 3 in the numerical domain and at the same time in Stage 1 in the spatial domain simply because he has elaborated a more sophisticated conception of number than of space. Another child can be doing just the opposite because his understanding of space is more articulate and complete than his understanding of number. The early competence data instead can be reinterpreted as prima facie evidence of the levels in the process of elaborating a knowledge domain, the equivalent of Piagetian stages within particular domains, and can be exploited to provide a much more detailed description of the developmental process in the diverse domains.

To specify the structure underlying domain specific-stages, two solutions have been proposed. Carey, Gopnick and Karmiloff-Smith draw an analogy between conceptual development and scientific progress. The domain-specific structures are attributed some of the properties that philosophers of science (see Kuhn 1970, Laudan 1977) have identified in scientific theories, like *incommensurability*. The developmental process is also interpreted in terms of some of the mechanisms underlying theory change in science, such as concept *differentiation* and *coalescence*. Gelman and Keil instead attribute basic, innate capacities in the forms of principles and rules structuring the different domains, and characterize the developmental process by two factors: the increasing *explicitness* of the rules and principles, and the improved ability to apply and use them.

1.3 Objectives

In this thesis, I study the development of cardinal number from counting to conservation of number with the aim of providing a domain-specific account of this genesis. By domain-specific I mean an account which explains the developmental process without invoking general changes in the cognitive system, that is changes outside the domain of cardinal number.

I argue that, from the first evidence of the child's ability to operate with cardinal number (e.g. numerical discrimination, counting, early forms of conservation) we can attribute to the child a basic structure for processing cardinal number information. This structure can be minimally defined as yielding representations of *serts* and *elements of sets* and of the relation of *(1-1)-correspondence* between elements of sets. Nevertheless, because of the very limited range of numerical situations in which the child is initially competent and because of the progressive widening of this range, I formulate the hypothesis that in the course of development the child learns the uses and imports of the cardinal number structure, that is, where cardinal representations are useful to solve problems, classify and predict events, etc.. The child gradually extends the domain of application of this structure by abstracting it over increasingly more complex contents, and, in the process, works out new aspects and properties of the number domain.

This developmental process goes through a sequence of stages which correspond to the contexts for which the child has worked out the applications of the number structure, that is, the class of "objects" upon which the structure is operational. The stage sequence is reflected in the child's increasing ability to handle numerical operations and to cope with situations and problems involving cardinal number.

To summarize, the building blocks of the domain-specific account of cognitive development I propose are: a) the distinction between a number structure and its domain of application, or the situations in which the child has worked out the import and the consequences of the applications of the structure, b) the principle that in the course of development the child generalizes this structure to a wider range of situations and in the process elaborates a more complete and abstract cardinal number concept.

1.4 Theoretical and empirical analysis of number development

The analysis of the developmental process is carried out on two levels: (1) the competence levels the child goes through, that is the *domain-specific knowledge states*; (2) the process by which the child moves from one competence level to the following, that is the *transition*. Regarding the knowledge states, I claim that, in the elaboration of the number domain, the child develops through a sequence of stages underlying qualitative different number concepts. These concepts have the same basic structure, but operate on different classes of objects. To illustrate how this account works, consider the case of Piaget's conservation of number task. In this task, the first expression of numerical understanding appears when the children (who are asked to take the same number of objects as there in a model set) stop copying the configuration of the set and begin to reproduce the set's number. In doing so, the children actively apply the 1-1-correspondence by putting in front of each object of the model set another object, thus reproducing successfully the model set's number. However, as the correspondence is destroyed when the experimenter lengthens one of the two rows, the child considers that the two sets now have a different number of elements. According to the account I propose, the first stage capacity to reproduce the number of objects in a set indicates that the child has a grasp of *the 1-1-correspondence* structure, but that the structure applies at that stage only to the elements of *individual sets*. This is sufficient to represent the cardinal property of a collection and thus to construct equinumerous sets. On the other hand, this is insufficient to deal with the conservation problem which also requires that the *1-1-correspondence* be established between the elements of *two sets*. Once the child has abstracted the structure over this more complex object, he can also operate on the relational aspects of cardinal number, like equinumerosity and order of sets, and work out the conditions of number conservation.

The nature of the interaction of numerical structure and content (or the objects that the structure can assimilate) can be studied experimentally using a variety of numerical problem-solving situations and determining the different competence levels in the solution of these problems. The analysis of the tasks solved at each level gives us the basis from which to infer the content of the numerical structure (the class of objects it applies to) and to characterize the kind of number concept elaborated.

Regarding the second aspect of the developmental process, i.e. the transition, I argue that the process by which the numerical structure comes to be applied to new situations and to express more complex objects (sets, relations between pairs of sets, etc.) is one of abstraction. Through the abstraction of the structure over new contexts, new properties and aspects of the concept are discovered. I specify this process in terms of a semantic model constructed by Richards (1985, 1987) to account for the development of the concept of object in the sensori-motor period. According to Richards, development is the process of "making information relevant", that is discovering the relevance of a concept in new contexts. In the transition process, the child moves from a state of irrelevance, where he does not see the bearing of the structure on some particular context, to an intermediate state of ambivalence and finally to a state of bivalence, which justifies the application of the structure to formulate hypotheses and verify or falsify them according to the circumstances. The mechanism of "making relevant" is defined by a logical algorithm which reduces the semantic space in which the child reasons from a four-valued to a two-valued reasoning framework.

From the synthesis of the account of knowledge states and transition, I propose the following picture of a portion of the development of cardinal number. I start from the assumption that children have a biologically determined structure specialized in processing numerical information. The earlier expression of cardinal competence which I shall consider, i.e. number reproduction, indicates that the foundational relation of *(1-1)-correspondence* is in place and is applied to the restricted domain of *elements of individual sets*. The process by which the child moves from this first expression to a subsequent, more advanced competence level (identified empirically as the capacity to make accurate number judgments), is one by which the relevance of the numerical structure is worked out for the new domain of relationships between two or more sets. The same process underlies the transition to conservation of number, where the relevance of the number structure is worked out for the more complex "object" set of sets. At each of these levels, the child's concept is thus defined by the numerical structure and by the objects it assimilates. Number is first a property of sets, then a relation between sets and then a relation between sets of sets.

This proposal suggests a synthesis of the two approaches of domain-specific knowledge structures (see section 1.2.): the more constructive "theory-based" approach of Carey, Karmiloff-Smith and Gopnick and the more innate "set of principles" account of Keil and Gelman. I characterize the knowledge states in terms of a structure which is present from very early on in development, a structure that however gives rise to different theories depending on the objects it applies to at the different stages. The formulation of the transition mechanism in terms of relevance specifies the process by which new meanings emerge from an innate structure. The concept becomes progressively more explicit (as Gelman suggests) and at the same time, by interacting with new contents, is reinterpreted and generalized (as Carey and Gopnick claim).

1.5 Predictions

The hypothesis that children have a basic understanding of number, i.e. a structure specialized for processing cardinal information, and that in the course of development this structure is relevant and operational over increasingly more complex objects entails that the children move through a fixed sequence of stages (without regressions) of greater numerical reasoning and problem solving ability. The hypothesis that the process of abstraction is three-phased, corresponding to the cognitive states of irrelevance, ambivalence and relevance predicts that the children confronted to a problem which is new for their stage, first do not see the relevance of the structure in that new context, produce an incorrect solution and do not recognize its inadequacy; secondly they start envisaging that the structure may apply and may lead to the solution of the problem, and oscillate between the correct and the incorrect solution; finally they observe the effect of applying the structure by formulating and testing the relative hypothesis.

1.6 Plan

Chapter 2 deals with the issue of representing knowledge states and transitions in conceptual development. I examine the structure of the Piagetian account: a) the sequence of stages of conceptual organization, each including, while extending, its predecessor, and b) the mechanism of reflective abstraction, by which a stage is transposed onto a higher level and organized in relation to the other elements present at that level. I then discuss some of the empirical and conceptual problems with Piaget's theory: the evidence of precocious successes in operational tasks, of low correlations in performance across different conceptual domains, of the contextual bias in adults' logical reasoning. As a way of solving some of these problems, I argue for a domain-specific approach to cognitive development.

In Chapter 3, I introduce the logical representations of stage structures and the algorithm of stage transition proposed by Richards for the development of the object concept. I suggest that Richards' model can be used to formally represent the structural differences between ordered stages and, above all, to simulate the process of transition from one stage to the following. I also argue that, because of its essentially logical nature, this model is far from providing an adequate account of the complexity of the psychological processes described. Richards' model should be simply taken as a possible example of how discontinuity and conceptual restructuring can be expressed in a domain-specific view.

In Chapter 4, I offer a reinterpretation of the early competence phenomenon in the terms of this theoretical framework. The case examined is that of the modified conservation task in which the spatial transformation occurs accidentally (McGarrigle & Donaldson 1975). The precocious success is explained by the fact that the modified task involves numerical relation on less complex objects (sets) than those involved in the standard conservation task (sets of sets).

In Chapter 5, I present some methodological considerations about the analysis of domain-specific development. I propose a method of hierarchical analysis to identify the order in which tasks, dealing with the domain, are solved. The hierarchy identifies different levels of problem-solving ability from which underlying conceptual organizations can be inferred. The advantages of the hierarchical analysis are that it permits us a) to integrate the data of early competencies, b) to evaluate the hypotheses that within domains the child develops through a sequence of ordered stages and c) subsequently to test precise hypothesis about the competence underlying the stages. In the last section, the chapter sets out the plan of the study of cardinal number development which is carried out in the remaining chapters.

In Chapter 6, I examine in some detail the existing literature on the development of the cardinal number concept. Three basic experimental paradigms have been used: the tasks of reproduction, comparison and conservation of sets, the latter being the most extensively studied. I emphasize two central findings: a) the same developmental shift from space-based to number-based estimations of sets' size is reported in all three experimental paradigms; b) the development of number conservation is at the same time a very robust result, as long as the standard test is used, and a very weak result when modified conservation formats are employed. Early forms of conservation have in fact been reported from age 4. These findings set the questions which are explored in the experimental component of the thesis (Chapters 7 and 8).

In Chapter 7, I introduce two experiments designed to determine whether the shift from space-based to number-based estimations of sets' size in set reproductions, comparisons and standard conservations occurs *all at once* or whether it appears first in a class of tasks, then in some other tasks, and so on. In the first experiment, I examine the acquisition order of the three basic tasks of number reproduction, comparison and conservation. In the second experiment, I replicate the previous study focusing on number reproduction and comparison, and introduce a new experimental condition of set comparison. The two experiments identify three stages of number competence. Stage 1 corresponds to the ability to *reproduce sets*; Stage 2 to *compare sets* and Stage 3 to *conserve number in the standard Piagetian task.* In the last section, I propose an account of the number concepts underlying this stage sequence. I argue that the Stage 1 number concept reflects the application of the number-domain structure to *individual sets* and expresses the property "cardinality" of sets of objects. The Stage 2 number concept results from the application of the number structure to *a pair of sets* and expresses the cardinal relations of "equinumerosity" or difference between sets. The Stage 3 number concepts reflects the abstraction of the number structure over *pairs of pairs of sets* and makes possible to derive the principle of equivalence conservation.

Chapter 8 presents a third experiment which provides a first test of this account of the development of cardinal representations. The experiment investigates the hypothesis that the modified conservation tasks, like set comparison, require the matching of two sets, an operation which is available from Stage 2. As predicted by the hypothesis, the early conservations (identified by the modified conservation tasks) emerge quasi concurrently with the solution of the set comparison tasks and precede conservation in the standard task. This finding corroborates the hypothesis that the child develops from simple counting to number conservation through three stages, each corresponding to a new, and more complex, content on which the number-domain structure is operational.

In Chapter 9, I discuss the account of cardinal number development proposed in relation to the alternative theories of number development of Piaget and Gelman. I then model the transition between the stages identified, by means of Richards' logical representations and algorithm. The model accomplishes only partially the descriptive work originally envisaged. In conclusion, I evaluate the domain-specific approach defended and suggest some new directions of enquiry.

Chapter 2 Domain-specific conceptual development

2.1 Introduction

This chapter examines some of the conceptual and empirical problems encountered by Piaget's theory of cognitive development and sets out an alternative, general theoretical framework. The argument advanced is that the fundamental problems with Piaget's theory resolve around the central claim that what changes in development are overall *reasoning and representational capacities*. What is not in question is Piaget's account of the developmental process as a sequence of stages, each including the preceding one, while reorganizing and extending it.

In order to deal with the problems with Piaget's theory while exploiting its strength, I propose a) to take the alternative perspective that what changes in development are conceptual contents *organized by domain-specific structures*, as opposed to overall logical systems, and b) to reformulate the Piagetian account of development within this domain-specific framework. From this perspective, cognitive development is envisaged as proceeding within knowledge domains (and in the local interactions between domains, an aspect which I shall leave for the time being) through a sequence of stages. The conceptual structure underlying each new stage includes the structure of the previous stage, while extending and reorganizing it.

The chapter is divided into three sections. In the first section, I introduce some elements of Piaget's theory and discuss in some detail his account of conceptual change. In the second section, I briefly examine the basis of the empirical and philosophical criticisms of the general stages theory. In the third part, I argue for a domain-specific account of cognitive development and propose a way of analyzing the domain-specific developmental process.

2.2 Piaget's theory of cognitive development

According to Piaget, cognitive development proceeds in a sequence of stages, each corresponding to a level of adaptation (or equilibrium) between the individual and the environment and each reflecting a general cognitive structure. Piaget discusses the notion of equilibrium by drawing an analogy between the stability of a cognitive state and the equilibrium of a mechanical system (1950). A mechanical system is in equilibrium when the set of the virtual works compatible with the relations of the systems gives a product (of its compositions) equivalent to 0, i.e. with exact compensation of + and -. Piaget writes[1]:

> Dire qu'un système réel est en équilibre revient ainsi à évoquer une composition entre des mouvements ou des travaux virtuels: parler d'équilibre c'est donc insérer le réel dans un ensemble de transformations, simplement possibles. Mais réciproquement, ces possibilités sont elles-mêmes déterminées par les 'liaisons' du système, c'est-à-dire le réel (1950, p.40).

Similarly, the cognitive system is in equilibrium when all the mental operations (corresponding to virtual transformations on the representational level) are compensated, that is, when for each possible mental operation there exists a corresponding operation of equal value, which can reverse the first. It is central to Piaget's theory that the system is reversible, that it contains the possibility of mentally reversing the operations executed. In practice, when the system is equilibrated, after an operation has been undertaken and a result obtained, the individual can represent a symmetrical operation which leads back to the initial state, without its having been permanently modified by the application of the first operation.

Number conservation, discussed in Chapter 1, constitutes one of the clearest illustrations of a reversible construct. Recall that in the conservation task, a spatial transformation is performed over two sets equivalent in number and distribution, so as to make them look different. A reversible concept allows the child to cancel mentally the spatial difference brought about by the transformation and to go back to the initial equinumerosity. This operation supports the conclusion that the two sets are still equinumerous, either because it is possible to go back to the initial one-to-one correspondence or because nothing has been added to or taken away from the sets.

2.2.1 The stages

According to Piaget's reconstruction of development, children move through three stages, underlying three cognitive structures in equilibrium[2]: the sensori-motor group of translations (around 2 years of age), the concrete operational grouping (between 6 and 9 years of age) and the formal operational group (between 12 and 14 years of age). Piaget represented the different structures using existing logico-mathematical systems,

[1]"To say that a system is in equilibrium is equivalent to attributing a law of composition between virtual movements and works: talking of equilibrium thus means inserting reality within a set of transformations, which are only possible. Inversely, these possibilities are themselves determined by the 'relationships' of the system, i.e. the real."

[2]The preoperational period is often referred to as a fourth stage, even though no general, stable structure is constructed.

such as Poincare's "groupe des déplacements" or Klein's INRC group, and formulating an ad hoc system to express concrete thinking, e.g. the grouping.

During the first stage, the sensori-motor schemas[3] are progressively differentiated and coordinated to form the *group of translations*. This geometrical structure reflects the organization of the child's knowledge of the movements and positions of the objects and of his own body. Take "A, B, C, D" to denote the starting and end points of a translation, "0" the null translation, "+" the composition of two translations and "=" the result of the composition. The structure in which the translations come to be related is the following:

a) AB + BC = AC

b) AB + BA = 0

c) AB + 0 = AB

d) AC + CD = AB + BD

Given the practical nature of the sensori-motor stage, the relations between translations require concrete, perceptual supports, i.e. the composition of translations relies on the recognition of the location and order of perceptual indices, such as reference points and marks in the environment. At the behavioural level, the group of translations is reflected in the action sequences that involve returning to an initial position (e.g. b: a movement in one direction can be canceled by a movement in the opposite direction), diverting an obstacle (e.g. d: a location can be reached by one among a choice of different routes), retracing the objects' displacements and locations (as in the permanence of the object tasks). The group expresses a primitive form of reversibility in action characteristic of the stage as the capacity to undo the result of the application of a sensori-motor scheme by invoking a second scheme which brings the child back to the initial situation.

During the pre-operational period, the structure of translations goes through a profound change as it is transposed onto the representational plane by the appearance of the symbolic function. In this period, the child begins to represent the actions and to reason about them, beside acting with them. These early representations have the form of pre-concepts and intuitions, as they conserve features of the sensori-motor schemes on the new plane of representation. In particular, the preoperational schemes, like actions, are irreversible in that they work exclusively in one direction. Piaget formally represents the preoperational thought in terms of a "semi-logic" corresponding to a

[3] The Piagetian *schèmes* which express the organization of actions as they are transferred or generalized by repetition in analogous circumstances).

system of one-way mappings: functions ($y = (f)x$) which represent an ordered couple or a unidirectional application (e.g. dependency relations and covariations)[4].

To become an operation, the internalized action has to be coordinated into a structured whole and, within this structure, to become reversible, such that it can be both carried out and canceled mentally. The equilibrium reached at the concrete stage corresponds to the construction of *groupings* of operations. Piaget's formalization of this structure attempts to capture the mobility and coherence characteristic of these operations. They have in fact the particularity of being concrete in that they reflect actual events occurring in the world and in that they proceed by means of contiguous overlappings (e.g. step by step). To express these properties, Piaget restricts the range of combinations allowed within the structure of the mathematical group and creates the groupings. Whereas in a group, the combination of two elements of the system produces a third element, without passing through intermediate steps; in a grouping the elements can only combine contiguously. In the grouping of classes, for instance, $((A + A) - A)$ is not equivalent to $(A + (A - A))$, as the former gives an empty class $(((A + A) = A) - A = 0)$ and the latter gives class A $(((A - A) = 0) + A = A)$. Piaget distinguishes eight such groupings divided into two types: the groupings of classes with *reversibility by inversion* (or negation), e.g. $(+ A - A = 0;)$ and the groupings of relations with *reversibility by reciprocity*, e.g. $(A > B; B > A)$ (permutation of the terms); $(A < B)$ (reversing of the relation).

The grouping structure is reflected in the reasoning patterns underlying the solution to tasks such as the conservation or the seriation tasks, to take just two examples. In the conservation of liquids, the child is asked to put the same amount of liquid in two identical containers A and B.

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[4]The attempt to define a structure underlying the pre-operational period plays a central role in Piaget's latest works, in particular *Epistemology and psychology of functions*, written with J.B. Grize & Vinh-Bang (Dordrecht, D.Reidel, 1977). This line of research has been recently taken up by P.M. Davidson in a paper called "Piaget's category-theoretic interpretation of cognitive development: a neglected contribution", published in Human Development 31, 1988 (p.225-244).

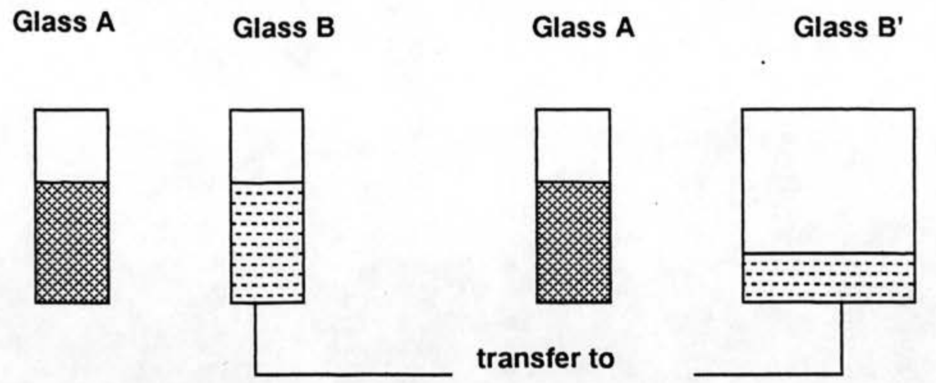

Fig. 2.1: the conservation of liquids task

Once the initial equivalence has been established, the content of one container (B) is poured into a second container of different shape (B'), that may be shorter and wider. After the transformation, the child is asked whether the amount of liquid has remained the same or whether there is more liquid in one of the two containers. Whereas preoperational children answer that the amount is different (in general they take the container where the level is higher as having more liquid), operational children conserve the quantity and justify their answers using either:

- a transitive argument (A = B' because A was equal to B and B is equal to B');
- inversion (A = B' because if we pour the content of B' back into B, the liquid will be level in the two containers);
- reciprocity of relations (A = B' because in B' the level is higher but the width narrower, while in B the level is lower and the width larger).

In the seriation task, where the child is asked to order a bunch of sticks according to their length, operational children use the relations "bigger than" and "smaller than" between any two sticks with equal ease, and succeed in ordering the sticks. They can also insert supplementary sticks in the right place along the series, when they are required to do so.

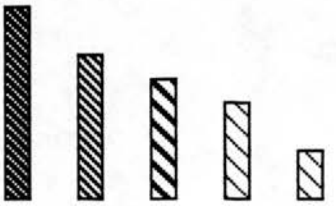

Fig. 2.2: Operational seriation

For the preoperational children instead, the application of one relation excludes the other (e.g. a stick cannot be at the same time longer than one stick and shorter than another stick), as these constitute unidirectional, unrelated schemes. They thus arrange the sticks in pairs or triplets with, for instance, a very long stick and two small ones. When the preoperational children succeed in building the series, they still cannot insert supplementary elements.

Fig. 2.3: Preoperational seriation

The last level of equilibrium is attained when the group of formal operations has been constructed. While the groupings of operations are applicable only to concrete objects and are related in a step by step fashion, the group of formal operations operate on non-concrete objects and engage in abstract, hypothetical reasoning about propositional objects. The group structure organizes propositions which contain the operations of classification, seriation, space and time of the previous level, i.e. they are "operations on operations" to use Piaget's terminology.

Piaget formalized this new combinatorial competence with the group INRC (identity, negation, reciprocal and correlative) which combines in one operation the reversibility by inversion and by reciprocity, that were separated in the grouping. The group expresses the whole set of transformations that may be performed on propositions in order to establish all the possible relations between them. Every operation, such as the implication $p > q$, has an inverse transformation N $p \cdot -q$ (read "p and not q"); a reciprocal R $q > p$; a correlated C $-p \cdot q$; an identity I which leaves the expression unchanged, as well as the whole set of combinations of the different transformations.

A formal operational system is required to deal with situations where two or more variables interact, such as problems that involve relative movements. The understanding of a system of relative movements (e.g. a moving object can go forward or backward (I and N) on a board, which itself can go forward or backward (R and C)) involves the combination of inversions and compensations. Thus to characterize the object's movements from the observer's viewpoint, the different combinations of the movement of the object and of the board have to be taken into account and related.

2.2.2 The properties of the stages

Although the stages define different levels of relational complexity, the three stages underlie cognitive structures in equilibrium which share the following properties:
1 - a structure is a whole, i.e. it is more than an aggregate of individual elements and has properties that none of its individual elements possess;
2 - a structure consists of its elements, their non-relational properties and the relations between the parts, what Piaget calls the composition laws, such as identity, reversibility, associativity, etc. The composition laws are the core of the structure, as they specify the class of transformations that the system can express at each stage, i.e. the relations and combinations between elements that may be established;
3 - a structure is a self-regulating system, with the corollary properties of self-maintenance and closure. Transformations from one set of elements to another never lead outside the system but always produce a result which belongs to the system. Because the structures are closed, they preserve their identity as a system.

2.2.3 The differences between stages

In the sequence of stages outlined by Piaget, each subsequent structure possesses greater reversibility and can handle a wider range of transformations. This progress is reflected in the increasing ability to take into account and combine dimensions, as in the solution of problems, in the classification, explanation and prediction of events, and in the more and more systematic exploration of the full range of possibilities. This greater internal articulation leads to a better equilibrium in the exchanges between the individual and the environment. I illustrate the qualitative differences between stages using two examples: 1) the shift from concrete operational to formal operational transitivity; 2) the shift from preoperational to operational classification.

According to Piaget, the grouping of concrete operations reflects a system of thought which operates on real events that have occurred or are occurring. For that reason, children at the concrete operational stage, who master seriation tasks bearing on concrete objects, fail equivalent problems of seriation which are presented in propositional form. For example, one of the problems studied by Piaget is the following (example 1.1):

- Edith is fairer than Susanne
- Edith is darker than Lili
- who is the darkest of the three?

Piaget (1967, p.159) reports this reply as paradigmatic of concrete operational thought:

- Edith and Susanne are fair
- Edith and Lili are dark

then Lili is the darkest, Susanne the fairest and Edith is in between.

When confronted with verbal transitive problems, the concrete operational children seem to make the same errors that preoperational children were doing in the seriation of sticks task. They compare the characters (corresponding to the sticks in the concrete seriation) two by two, instead of considering them at the same time darker (longer) than and fairer (shorter) than some other character (stick). The concrete operational thought cannot handle problems where the premises are given in the form of utterances and the conclusion is to be drawn logically from them, without concrete support.

Piaget represents the difference between the concrete groupings and the formal groups in terms of the combinatorial power of the two systems. This distinction emerges in particular in the systems' different types of reversibility. In the groupings there are two distinct types of reversibility:

1. inversion or negation (+A - A = 0) in the structures of classes;
2. reciprocity or reversal order (A = B; B = A) in the structures of relations.

In the group, the two reversibilities are synthesized in a single system, so that it is possible to move from one transformation to the other within the same system. The outcome is that for any complex proposition, four transformations can be simultaneously represented: Inversion (I), Negation (N), Reciprocal (R) and Correlative (C). This brings about new capacities of abstract reasoning, as children can operate a wider range of transformations on concrete operations, e.g. I, N, R and C; and also can combine formal operations between themselves, e.g. N + R = C.

The second illustration refers to the transition from preoperational schemes of classification to operations. In a classification task (example 1.2), the child is asked to put all the wooden beads (B) from a bunch (C) of wooden and plastic (B') beads in a container. The wooden beads (B) are either white (A) or brown (A'). Preoperational children handle the problem successfully and demonstrate understanding of the scheme A + A' = B. They also answer correctly the questions "if I take away all the wooden beads, will some beads be left in the container?": "*No, because they are all wooden*" and "If I take only the brown wooden beads away, will there be any bead left?": "*Yes, the white beads.*" Children also demonstrate understanding of the schemes B - A - A' = 0 and B - A' = A. When however the quantification of inclusion question is put to them "Are there more wooden beads or more brown wooden beads?" they answer that there are more brown wooden beads "*because there are only 2 white beads*".

Piaget explained this behaviour in terms of the unidirectionality characteristic of preoperational thinking. The child centres his attention on the whole B, or on the parts

A and A', alternatively. Once he centres on A, the whole B is destroyed. Thus the part A cannot be compared with the whole B (which does not exist any longer), and is compared with the remaining part A'. At the concrete operational stage, on the contrary, all the classification schemes have become operations. They are coordinated within a unique system and become reversible. The child can now invoke one operation and identify subclass A = B - A'; then reverse the scheme to identify B = A + A', compare B and A and conclude that B is greater than A. Within the structure, he can decompose the whole and later reconstruct it, without modifying it nor its parts permanently. In more theoretical terms, Piaget characterizes preoperational thought as the system where series of one-way relationships can be established, but where these functions are not coordinated into a single system. In the case of liquid conservation (fig.1.1) for instance, the preoperational child non-conservation reflects a one-way relationship of correlation between the height of the liquid in the container and its quantity. There is more liquid where the level is higher. The composition of these one-way relationships within the concrete operational system opens the way to compensations. The child recognizes that there is the same amount of liquid in the two glasses as the liquid in the tall glass reaches a higher level, but is narrower, while the liquid in the shorter glass is lower, but wider. In the case of classification, the compensation intervenes when A is considered at the same time a subclass of B and a class in itself, possibly superordinate with respect to other classes it may contain.

Let me stress that in the Piagetian analysis, the elements of the higher stage are already present at the lower level, even thought they are inserted in different relational systems. Once they are projected and coordinated on the higher level, they acquire new properties, and in particular greater flexibility, i.e. combinatorial and reversibility. Examining again the qualitative shift in classification, Piaget showed that the elements for classification were present at the pre-operational level, as unrelated schemas. Because their classification schemas are uncoordinated, pre-operational children do not understand a problem like the quantification of class-inclusion, which requires the simultaneous comparison of the total class and of one of its subclasses. Once the child has applied one of the schemes, i.e. A = B - A', the total class B is permanently modified and the classes that the child is left with are A and A', which he indeed compares. On the other hand the operational structure of classification allows the child to construct hierarchies of classes, that he can explore, decompose and recompose exhaustively, though step by step.

2.2.4 The transition between stages

The transition from a lower to a higher stage is characterized in terms of the mechanism of *reflective abstraction*, according to which the structure of the lower stage is projected onto a new plane of thought, integrated with the structure present there and extended. Piaget distinguishes two phases in the process:

> The reflecting abstraction includes two inseparable aspects: a 'reflecting' in the sense of projecting on a higher level what is happening on a lower level, and a 'reflection' in the sense of a cognitive reconstruction or reorganization (more or less conscious) of what has thus been transferred (1975, p.35).

The actions (and later operations) are projected onto the new plane, e.g. *réfléchissement* in a physical sense, and there reorganized, e.g. reflection in a psychological sense. The intuition behind this metacognitive principle is *based on a pun*. To become aware of what is going on in one's cognitive functioning, one has to look at one's own internal processes from some privileged, external point of view, i.e. an higher stage. The *reflection* has to occur on a higher plane than that of action (and later concrete operation) since it involves reasoning about the underlying processes, and reasoning about processes requires a representation (or conceptualization) of them. The inferior structure is thus reflected, projected on a superior plane, where it is understood in relation to the structure present on that plane.

The originality of the notion of reflective abstraction emerges when it is compared with empirical abstraction, the Piagetian term for induction. Piaget writes:

> As opposed to empirical abstraction which consists merely of deriving the common characteristics from a class of objects (by combination of abstraction and simple generalization), reflective abstraction consists in deriving from a system of actions or operations at a lower level, certain characteristics whose reflection (in the quasi-physical sense of the term) upon actions or operations of a higher level it guarantees; for it is only possible to be conscious of the processes of an earlier construction through a reconstruction on a new plan.....In short, reflective abstraction proceeds by reconstructions which transcend, whilst integrating, previous constructions (1966, p.189).

Empirical abstraction consists of abstracting a quality, such as colour or weight, from an object, and works by assimilating new contents to the existing structures. As such it can bring an increase in the extension of the structures, but no new structure. Reflective abstraction instead consists of abstracting properties not from an object, but from one's actions on the objects and in particular from the logical coordination of one's actions. Piaget provides the following example to illustrate abstraction from logical coordination. A child puts five pebbles in a row and discovers that the number remains 5 both when he counts from right to left and when he counts from left to right.

> This experience is of logico-mathematical nature because it does not relate the pebbles themselves, but to the relations between the activity of ordering and that of forming a sum. The linear order did not exist in the pebbles before the subject aligned them in a row. As for their sum, that too depends on the activity; that of addition which, on the one hand, ignores the other pebbles or objects placed on the table, and, on the other hand, constructs a totality by means of these few pebbles without omitting any of them or counting the same twice. What the child discovers is a property of his own activity; and not a property of the pebbles as such: it is the fact that the result of the operation of addition is independent of the order followed (1966, p.232).

The creative aspect of reflective abstraction is accomplished by the second component, or reflection. The first component, the réfléchissement, in fact consists of projecting structure which was already there. The second component instead reorganizes the projected structure by means of processes that Piaget called *constructive generalization* or *completive generalization*, since it involves generalization in both extension and comprehension. Reflective abstraction proceeds under two conditions, as Piaget writes:

> a) the new structure must first of all be a reconstruction of the preceding one if it is not to lack coherence and congruity; it will thus be the product of the preceding one on a plane chosen by it; b) it must also, however, widen the scope of the preceding one, making it general by combining it with the elements proper to the new place of thought; otherwise there will be nothing new about it (1967, p. 320).

2.2.5 The principle of Equilibration

Reflective abstraction is embedded within the more general principle of heightening (*majorante*) equilibration, the autoregulatory principle which governs the interaction between the cognitive system and the environment, and constitutes the essentially cognitive component of that principle. Underlying the functioning of reflective abstraction, and in particular the causes of its activation, is Piaget's theory of consciousness, a theory strongly influenced by Edouard Claparede's "loi de la prise de conscience". According to this law, consciousness of the self or of the internal functioning does not arise as long as we are successfully meeting our needs and adapting to the environment. Then we are only conscious of the results of our actions and not of the internal mechanisms organizing the action. We become aware of our selves and of our internal functioning only when some of our needs are frustrated: external obstacles appear, goals are not fulfilled, etc. In these cases, we have to analyze the reasons for the failure, and thus examine our own internal processes. Reflective abstraction captures the mechanism by which one takes some perspective over one's internal functioning and organization from a higher point of view (e.g. the mental processes themselves are the objects of cognitive activity). The level is higher in the sense of more abstract, as what previously organized our action now becomes the content of thought.

A few considerations about the explanatory nature of Piaget's Equilibration principle are necessary, as this issue has created many misunderstandings. Thanks to the reflections of Kitchener (1983, 1986), Overton and Reese (1973, 1981) and Overton (1984, 1985), the epistemological nature of Piaget's Equilibration theory has been much clarified. These authors argue that Piaget's explanation of conceptual change is of a retrodactive nature, by opposition to predictive, and mirrors the explanations given by the theories concerned with evolutionary and historical processes (e.g. theories for which change is the primitive), in biology as well as in psychology and philosophy. A retrodactive explanation specifies the laws and principles capable of reconstructing the evolutionary process as it has occurred. Hence Piaget's theory of equilibration is an attempt to reconstruct the rational progression of development by providing a description (interpretation) of the developmental process (that is, the sequence of stages) and by specifying the laws governing why (that is, the tendency toward the most advanced states of equilibrium possible) and how (that is, reflective abstraction) development proceeds.

2.2.6 The structure of Piaget's account of cognitive development: articulation of holism and cumulativity

To summarize, the equilibration theory offers a reconstruction of cognitive development which is based on the following principles:

1. Development proceeds through a sequence of cognitive stages. The order of succession of the stages is fixed;
2. Each stage has a holistic structure, with laws of composition, associativity, identity and reversibility, and is a self-regulating system. The structures constitute qualitatively different systems of thought, formalized as logical systems of different power;
3. In the sequence of stages, the structure of the preceding stage is an integral part of the structure of the subsequent stage. The new structure is at the same time based upon and more general than the earlier one. Thus, the earlier structure is at the same time included in the following structure, and radically changed by the new relations emerging with the new structure;

4. The transition from lower to higher stages, and the relative generalization, is accounted for by the mechanism of reflective abstraction. The structure is abstracted from the lower stage, projected on the higher stage and reconstructed into a new whole;

5. Each stage represents a particular degree of equilibrium between the individual and the environment; the sequence as a whole reflects the equilibration process, i.e. the progress towards greater problem-solving efficacy, coherence, increased objectivity.

This reconstruction rests on the articulation of holistic and cumulative aspects of conceptual growth: *holistic*, because each new stage constitutes a qualitatively different system of thought, when compared to the prior stage; *cumulative*, because each new stage incorporates the prior stage, and constitutes a generalization from it. As the new conceptual organizations include the prior ones, it follows that cognitive development proceeds unidirectionally towards progress (in the incremental sense), without loss of knowledge. The synthesis between the two components is made by means of the process of reflective abstraction. The projection of the lower structure onto the higher plane accounts for the cumulative aspect of inclusion; the reorganization and reconstruction on the new plane accounts for the holistic aspect of qualitatively different structure.

In order to appreciate the coherence and strength of Piaget's analysis of conceptual development, consider some of the problems that may arise from a simple, structural hypothesis. As we have seen, two stage structures constitute two holistic systems: their parts are in interaction with each other, such that each part derives its meaning from the whole. From this property, it follows that stage structures cannot be reduced to their more primitive elements, as a) these elements are defined within their respective whole structures and b) the structures have emergent properties that the elements do not possess. Borrowing from the works in the field of history of science (in particular Kuhn 1970), the two structures are said to be *incommensurable*, as their constitutive elements cannot be isolated, matched and then compared directly. Depending on the structural context of which they are part, they refer to different entities, they play different roles and they express different relations.

These considerations open a central problem for structural theories of cognitive development: if the stage-structures are incommensurable and cannot be compared, do later structural organizations constitute progress with respect to the prior ones? And if they do, how can one express and represent the progress that a later structure brings about over the preceding one?

In my view, the strength of Piaget's account lies in the fact that it does offer a way of approaching the fundamental issue of progress, by positing the inclusion (and conservation) of the previous structures into the subsequent ones, even though this structure comes to be radically changed in the process. This property of development is expressed by the fact that the included structure is still recognizable in the subsequent structure as a part of the network of schemes or operations. Nevertheless, in the higher stage the schemes and the operations are more highly interconnected to form more complex, flexible and coherent systems, giving a radical new interpretation of the concept.

This analysis is particularly clear concerning the different systems used to define preoperational and operational classifications, that I define above. The preoperational classification is represented by the classification schemes:

$A + A' = B$; $B - A - A' = 0$; $B - A' = A$;

These are unrelated and unidirectional. The operational classification emerges from the coordination in a single structure of these schemes and from the properties which derive:

composition: $A+A'=B$; $B+B'=C$; etc.

reversibility: $B-A'=A$; $C-B=B'$;

associativity: $(A+A`) +B'=A+ (A'+B')$;

identity: $A+0=A$

tautology: $A+A=A$

The projection and reorganization of the lower stage classification schemes bring about a qualitatively different classification system, and a radical redefinition of the class concept. Hence although the schemes are formally equivalent in the two stages, their interpretation, specified by their interconnections within the stage-structure, is radically different. In other words, the inclusion can be discerned on the basis of the individual schemes; while the restructuring and the emergence of new properties are identified by the new relations which arise between schemes. In the performance, the inclusion is reflected in the fact that the classification tasks understood by the children of the lower stage are also solved by the children of the higher stage. The progress instead is revealed by the new class of problems that the children can solve (e.g. class-inclusion).

Hence by specifying the structure of the different stages in terms of schemes and relations between schemes, Piaget succeeds in combining the two aspects of conceptual holism and cumulation, and at the same time avoids the relativism in which holistic views of knowledge seem to be inevitably trapped as well as the weakness of simply additive accounts of knowledge growth.

2.3 Problems with Piaget's theory of cognitive development

An extensive discussion of the philosophical, logical and empirical studies concerned with Piaget's genetic epistemology would by far exceed the scale of this dissertation. I shall thus limit myself to two types of problems and focus on the criticisms relative a) to the insufficient explicitness of the mechanisms invoked by the equilibration theory and b) to the empirical support for the general stages hypothesis.

2.3.1 Conceptual problems

The major criticism advanced against Piaget's equilibration theory is of being too descriptive and of giving of its central mechanism, reflective abstraction, a simply metaphorical characterization. Consider the quotations of Piaget, reported in Section 2.2.4. The two components of reflective abstraction, projection and reorganization, are described by analogy to two kinds of reflection. Projection, or the transfer of the structure of the lower stage onto the higher stage, is equated to optical reflection, while reorganization to reflection as a cognitive process. To my knowledge, nowhere does Piaget specify the functioning of the two components of reflective abstraction in greater detail. It thus remains unclear how the translation of knowledge from one level of representation to another may occur, and how the knowledge may be transformed in the course of the process. Similarly, if we take for granted that some structure is transferred to the new representational level, how this structure may interact with the structure already present on that level and how the global reorganization is achieved also remain unspecified. Hence, the equilibration theory fails to provide a sufficiently detailed explanation of the processes (of abstraction and reorganization) by which the cognitive structures develop in the direction of greater logical and representational power. These considerations are at the origin of Fodor's argument against Piaget's constructivism. Fodor in fact dismisses equilibration as a plausible alternative to induction to explain conceptual growth on the grounds that equilibration is

> entirely descriptive: there is simply no theory of the processes whereby equilibria are achieved (1975, p. 90).

On the basis of this judgment, Fodor claims that the only mechanism left capable of explaining conceptual acquisition is inductive generalization, i.e. the projection and confirmation of hypotheses. According to Fodor, the hypotheses are in the form of biconditionals. On the left-hand side, they specify the concept to be learnt and on the right-hand side the conditions under which the concept applies, i.e. the extension of the concept being learned. For instance, if C is the concept to be learnt and G specifies the conditions under which the concept is true, the hypothesis that is projected to learn C is the following:

(x) (x is C if and only if x is G)

The formulation of the hypothesis and its subsequent confirmation presuppose that one already knows G. But if G is required to understand C and C and G are coextensional, then C does not represent, in any non-trivial sense, a conceptual novelty. C is nothing more than a synonym of G. In other words, one cannot learn what C is (that C falls under G), unless one already has a language in which C and G are expressed. Fodor concludes that concepts cannot be learnt. Conceptual growth can only be the result of biological maturation or accident.

When this argument is directly applied to Piaget, the claim that children of different stages represent different concepts is disproved as it would be impossible for children of stage 1, for example, to learn a stage 2 concept C, since at stage 1 they could not express the concept G, of stage 2, necessary to formulate and verify the hypothesis about C. Fodor writes:

> Either the conditions on applying a stage 2 concept can be represented in terms of some stage 1 concept, in which case there is no obvious sense in which the stage 2 conceptual system is more powerful than the stage 1 conceptual system, or there are stage 2 concepts whose extension cannot be represented in the stage 1 vocabulary, in which case there is no way for the stage 1 child to learn them (p.90, 1975).

The conclusion about the impossibility of explaining cognitive development in terms of a change in the nature of the representational and reasoning system, reached by Fodor through philosophical argument, mirrors the conclusions that can be drawn from the results of the empirical research on Piaget's theory. In fact, no substantial evidence has been found to support Piaget's claim that development involves across-the-board changes in representations and reasoning capacities. Two findings are particularly critical: low correlations in performance levels across knowledge domains (i.e. heterogeneity) and the sensitivity of performance level to task presentation (i.e. early competencies).

2.3.2 Empirical problems

The discussion of the empirical evidence bearing on the general stage hypothesis will be extremely brief. As the amount of relevant research is simply enormous, I shall thus limit myself to defining the problems and illustrating them with some examples. Furthermore, the examples will be essentially drawn from studies of the preoperational-operational period, as it is in this phase that the general stage hypothesis has been most systematically investigated.

2.3.2.1 Heterogeneity

The general stage hypothesis claims that concepts are highly interconnected and homogeneous as they are organized according to common structural principles (e.g. the operational structures):

> each stage is characterized by an overall structure in terms of which the main behaviour patterns can be explained (Piaget & Inhelder 1969, p.153).

Concepts of equivalent operational complexity should thus appear in a related, quasi synchronous, fashion. For Piaget, support for this hypothesis came from the fact that children master, at approximately the same average age, various concepts of equivalent operational level. For instance, the concepts of number conservation, class-inclusion and transitive seriation, which in the Piagetian analysis underlie logico-mathematical operational structures of equal complexity (e.g. they are concrete and have one form of reversibility) all appear at around age 7 or 8. However, the empirical results provided by Piaget do not constitute conclusive evidence of the existence of homogeneous competence levels across conceptual domains, since Piaget studied the various notions using different groups of children. A firmer corroboration of the general stages hypothesis would require that the same children be examined across the different concepts for which synchronism is hypothesized and that these children be found to acquire these concepts in a close-to simultaneous order.

The studies which have compared the same children's ability to conserve (number, substance, length, weight, volume), classify, seriate, measure, predict physical phenomena, represent geometrical relations, etc., report low correlations between these different abilities (see the reviews by Brown & Desforges 1979, Carey 1984, Gelman & Baillargeon 1983). For instance, Tuddenham (1971) report that the median correlation in a range of across domains tasks that require isomorphic operations is only .30. Furthermore, there are widespread reports of children who solve some tasks in a fully operational way (e.g. corresponding to stage 3) and then perform in a typically preoperational way in other tasks (e.g. corresponding to stage 1).

The studies of Rieben, Ribaupierre and Lautrey (Ribaupierre et al. 1985, Lautrey et al. 1985, Rieben et al. 1986) constitute one of the most extensive and systematic investigation of the existence of concrete operational structures. The authors examined

154 children (from 6 to 12 years of age) in the solution of a battery of eight concrete operational tasks dealing with:

1. the logico-mathematical domain:
   classes, classification of objects in intersecting classes,
   probabilities, quantification of probabilities attached to the drawing of a red object from collections of different size;
2. the physical domain:
   conservation of substance, confirmation of equivalence for two balls of clay which go through a series of deformations,
   conservation of weight, confirmation of equivalence of weight for two balls of clay deformed and cut into pieces,
   conservation of volume, confirmation of equivalence of volume for liquid transferred in containers of different shape,
   construction of volumes, equivalent volumes (e.g. two houses) built on different bases (e.g. two islands);
3. the spatial domain:
   sectioning of volumes, anticipation and drawing of surfaces obtained by the sectioning of objects along different axes,
   unfolding of volumes, (anticipation and drawing of the development of plane or curvilinear surfaces of different shapes);
4. the mental imagery:
   folding of lines, anticipation and drawing of the figure obtained with geometrical figures folded in half,
   folds and holes, anticipation and drawing of the folding lines and the holes of a square sheet of paper first folded and then slightly cut (in a corner or in the middle of the fold).

Correlations between the performance level in pairs of the eight tasks are not very high, as the authors note, given the age span (from 6 to 12 years) examined. The values of Kendall's Tau Coefficient range from .23 to .50, with a median correlation of .39, and are statistically significant at a level of p less than .05. Considering the correlations higher than the median value, the authors identify two clusters of tasks. The first includes the logico-mathematical tasks and the conservation tasks. The second includes all the physical tasks and one task from each of the mental imagery and spatial domains. These results are indicative of the complexity and the fragmentation which seem to characterize the developmental process, when the performance of the same children is examined in different knowledge domains. This heterogeneity goes against the uniformity and unidirectionality hypothesized by Piaget.

The same kind of problems have been also pointed out regarding the notions that Piaget describes as emerging from the synthesis of specific concepts. For instance, Piaget claims that the concept of operational number emerges from the synthesis of the concepts of classes and relations. From this claim, it is legitimate to expect some form of quasi synchronism in the acquisition of the concept of operational number, transitivity and classification. However, when the performance levels of the same children in the tasks probing the operational understanding of the three concepts (i.e. conservation of number, quantification of class-inclusion, seriation) have been compared, no significant correlations have been found (Dodwell (1960, 1962), Brainerd (1978), Kofsky (1966), Little (1972)). The same problem has appeared in the domain of spatial relations. Lunzer (1960, 1965) does not find any synchronism between the solutions to the tasks of conservation and measure of volume or between conservation and seriation of length.

These phenomena were not unknown to Piaget himself who introduced the notion of "horizontal décalage" to account for the non-synchronic application of the same operations to different contents. His classical example of horizontal décalage is the conservation concept, where two years intervals separate the acquisition of the conservation of substance (around age 7-8 years), of weight (age 9-10 years) and of volume (11-12 years). Piaget explains[5]:

> La raison de ces décalages est naturellement à chercher dans les caractères intuitifs de la substance, du poids et du volume, qui facilitent ou retardent les compositions opératoires: une même forme logique n'est donc pas encore, avant 11-12 ans, indépendante de son contenu concret. (1967, p.157).

Nevertheless, contrary to what Piaget claims in this text, the same décalages, consequence of representing and reasoning upon different contents, are also present in the formal operational stage. It has been extensively demonstrated that the adult's performance in logical reasoning tasks is critically dependent on the particular content of the task (see the reviews of Wason & Johnson-Laird 1972, Evans 1982 and Johnson-Laird 1983).

As an illustration, consider the classic four- card selection task (Wason & Shapiro 1971, Johnson-Laird, Legrenzi & Sonino Legrenzi 1972, etc.). The task is designed to determine what evidence the subject takes to be relevant to establishing the truth or falsity of an assertion. The subject is presented with four cards showing a letter on one

[5] "The reason of these décalages lies in the intuitive aspects of substance, weight and volume. These characteristics can either facilitate or delay the operational compositions: a same logical form is not independent of its concrete content before 11-12 years of age" (my translation).

side and a number on the other side (e.g. A, D, 4, 7), and with the rule: "if a card has a vowel on one side, then it has an even number on the other side". The subject's task consists of choosing the cards that need to be turned over to find out whether the rule is true or false. In this abstract condition, the correct answer, i.e. turn A and 7, is given only by a very small number of subjects. When the task is instead presented with a more concrete, familiar material, the proportion of correct choices increases significantly. One of the tasks used (Johnson-Laird, Legrenzi & Sonino Legrenzi 1972) asks the subjects to choose the letters necessary to establish whether the following rule has been violated: "if a letter is sealed, then it has a 5d stamp on it". Four envelops are presented: the back sealed, the back unsealed, the front with a 5d stamp, the back with a 4d stamp. 22 out of 24 subjects solved the task correctly, lifting both the sealed envelop and that with the 4d stamp. Of the same 24 subjects, only 7 solved the abstract condition correctly.

In a short article, "Intellectual evolution from adolescence to adulthood", Piaget recognizes the role of factual knowledge (e.g. familiarity and expertise) in logical reasoning:

> We can retain the idea that formal operations are free from this concrete content, but we must add that this is true only on the condition that for the subjects the situations involve equal aptitudes or comparable vital interests (1977, p. 165).

However, although invoked by Piaget, neither "horizontal décalage" nor specific expertise can be accounted for in any explicit and systematic way within the operational stages theory. The low correlations in performance level across knowledge domains, both in the concrete operational stage (where this phenomenon may be expected given the role that the perceptual support plays on cognitive functioning at that stage) and in the formal operational stage (where this phenomenon is instead more difficult to account for), constitute an important anomaly for the general stages theory and identify a class of phenomena that alternative theories of cognitive development have to deal with.

2.3.2.2 Early competence

A second class of phenomena which constitute important anomalies for the the theory of general stages are the early competencies: under particular circumstances, children demonstrate precocious understanding of operational concepts. In Chapter 1, I introduced the case of early competencies in number conservation. Children appear to be failing the original Piagetian tasks while at the same time they can solve modified versions of the tasks, which still demand a mobile, articulate understanding of the concept examined (e.g. the lengthening of one of the rows is performed by a third, disturbing, agent, instead of the experimenter (McGarrigle & Donaldson 1975); the child is required to count the two rows after the spatial transformation has been performed (Gréco 1962)). These results go against the claim that clear-cut differences distinguish the concept at the concrete operational stage from its early forms, e.g. between the perception-bound pre-operational number concept and the reversible, mobile operational number concept.

More generally, from the studies which replicate most Piagetian experiments, a strong pattern emerges characterised by two apparently contradictory results. On the one hand, the Piagetian findings are confirmed when tasks conforming to those originally employed by Piaget are used. On the other hand, early competencies are exposed when modified versions of the original tasks are employed, which maintain the structure of the task while setting it into pragmatically, linguistically or physically different contexts. The data of early competencies are extensively discussed in the review chapters by Brown & Desforges (1978), Gelman & Baillargeon (1983) and Carey (1984). In Chapter 4, I shall give a detailed presentation of evidence of early competence in number conservation.

Here as illustration, consider the study by Bovet, Baranzini, Dami & Sinclair (1975) on conservation of volume and density, a paradigm case of the early competence issue. The original volume conservation task consists of two containers of equal size, half filled with water and of two objects, of same volume (and different weight in the density task). Once the child has confirmed that the water goes up the same height when the two objects are immersed in the containers, one of the objects goes through a series of transformation: it is made into different shapes or is divided into parts. The conservation question about whether, after the transformation, the two objects still take up the same space is then asked. A prediction that the level the water will go up is also required. The task is typically solved between the end of the stage of concrete operations and the beginning of the stage of formal operations, around age 12 - 14 years. Before that stage, children predict that the water will go up higher when the surface enveloping the objects looks larger (e.g. the object is divided into many parts) and when the object is heavier.

The modification of the task designed by Bovet et al. eliminates some of the complex physical characteristics. As index of space occupied, the water is replaced by bran flour; a move which eliminates physical aspects of the task linked to water, such as buoyancy, pressure, speed of fall, etc. Under these conditions, the authors find evidence of early understanding of volume and density, and introduce a pre-notion of "voluminosité"[6]:

> C'est an niveau de 7, 8 ans approximativement que se construisent les trois notions physiques élémentaires que sont le poids statique, la voluminosité et la conservation de la différence de lourdeur. Plus évolueées que la notion de substance, elles semblent constituer une première différenciation au sein de cette notion globale, en tant que contenu physique, et elles résultent d'une lente élaboration puisqu'avant cet âge on constate l'absence de conservation de la place occupée et de la différence de lourdeur.....Nous pensons pouvoir affirmer que chacune de ces notions représente un niveau préalable nécessaire en vue de la construction des notions achevées et complexes de l'adulte. Le poids statique pourrait ainsi préparer la notion de poids-force; la voluminosité, celle de volume physique et géométrique quantitativement mesurable; et la densité simple, celle de densité en tant que rapport du poids au volume (1975, p.78-79).

Similar results are reported for the task of seriation Gillieron (1976), class-inclusion (Markman 1978), horizontality (DeLisi, DeLisi & Youniss 1977), physical causality (Bullock & Gelman 1979) and many others.

The early competence data identify levels of conceptual elaboration of some complexity (e.g. the concept of "voluminosité"), where the Piagetian theory would only expect perceptually bound, global pre-concepts and intuitions. Together with the low correlations results, they indicate that developmental proceeds in a more articulated and fragmented fashion than originally predicted by the general stage theory, not only at the level of between domains performance (heterogeneity) but also at the level of single conceptual domains, where the child appears to go through several levels of competence.

2.4 Cognitive development from a domain-specific perspective

In the previous sections I have discussed some of the main problems encountered by Piaget's general stages hypothesis both on the conceptual and on the empirical levels.

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[6]"It is around the age of 7, 8 years that the three elementary physical notions of static weight, volume occupied and conservation of weight difference are acquired. These notions are more developed than the notion of substance. They seem to constitute a first differentiation within that global notion, as physical content, and emerge out of a slow elaboration since, before, before that age, there is no conservation of volume occupied nor of weight difference... We have grounds to claim that each of these notions represents a necessary prerequisite for the construction of the full-blown, complex, adult notions. Hence static weight could prepare the way for the notion of weight-force; the volume occupied that of (measurable) physical and geometrical volume; and the simple density, that of density as relationship between weight and volume" (my translation).

In recent years, an alternative perspective on cognitive development has been emerging which is designed to address these problems (among other issues). From their review of the literature on the development of operational concepts, Gelman & Baillargeon (1983) conclude that

> the experimental evidence available today no longer supports the hypothesis of a major qualitative shift from preoperational to concrete-operational thought. Instead we argue for domain-specific descriptions of the nature as well as the development of cognitive abilities (1983, p.167).

Similar conclusions are reached by Carey (1984), Feldman (1986), Turiel & Davidson (1986). The basis of the domain-specific view is the hypothesis that children have biologically determined structures specialized in processing different kinds of information pertaining to various knowledge domains. The cognitive system is hence envisaged as organized in subsystems, with limited interactions: a view which is the opposite of Piaget's general structures and which seems more adequate to account for the fact that conceptual development proceeds in a much more fragmented way than expected under Piaget's general stages hypothesis.

Because of the relative novelty of the domain-specific hypothesis and of the complexity of the issues that it raises, I have spread the introduction and the definition of the terms invoked and the discussion of its implications over the next two chapters. Here I wish to focus on the first-hand advantages that this view yields with respect to the problems I raised with Piaget's theory. I then move on to discuss the central issue of how to envisage conceptual development from biologically determined domain-specific structures.

First, consider the two developmental phenomena of heterogeneity and of early competencies. From the domain-specific perspective, heterogeneity is the rule, rather than the exception, as the domains are expected to develop autonomously (and not as a consequence of the development of overall operational systems, as Piaget claims). A child may in fact have elaborated a rich and articulated concept of physical causality (e.g. which would classify him at stage 3, for instance) and at the same time have elaborated a limited understanding of the domain of number (e.g. be able to reproduce sets, but not to conserve number, equivalent to stage 1). On the other hand, early competencies take the central place of initial evidence of the development of the different domains, as they identify levels of competence in the domain.

Consider then the issue of explaining conceptual change in development. The problem for Piaget's structural theory was that of explaining how new, more articulate and coherent relational systems emerged out of previous, simpler systems; an extremely abstract and complex process of which, I argued, Piaget provides an underspecified description. From a domain-specific perspective, the terms of the problem are radically changed as a) specialized structure is attributed to the child from the beginning and b) the changes are more local and concern aspects of the domains, rather than abstract and general relational systems. At the level of conceptual domains, the issue of explaining transition is thus formulated in terms that appear to be more tractable.

With a domain-specific hypothesis, the locus of the explanation of developmental change is transferred from general principles of conceptual organization to local conceptual structures. These structures have been described as theories by Carey (1985, 1988), Karmiloff-Smith (with Inhelder 1975, 1988b), Gopnick (1988) and as sets of principles or rules by Gelman (with Gallistel 1978, with Meck 1986), Keil (1986). I wish to advance and explore an alternative, though compatible, hypothesis which distinguishes between the ability to represent concepts and the ability to apply them correctly. Children have a basic structures specialized in processing domain-specific information (e.g. the conceptual expressability of Fodor), and, in the course of development, they work out the functions of these structures as a means to a) solve problems, b) actively organize, classify and explain events in the world and c) infer new possible states of affairs, like the prediction and anticipation of situations. The basic intuition is that as the child comes in contact with a wide range of contexts where these structures may apply, he has to work out where they to apply them and the consequences of their application. This proposal is consistent with Fodor's view of what may change in development:

> learning does not increase the *expressive power* of one's system of concepts (construed as the set of states of affairs that one can represent) though, of course, it can and often does increase one's information about which states of affairs in fact obtain (1975, p.93).

This dimension of development covers more fully the complexity of what understanding a concept means, since, as Fodor remarks, conceptual expressability alone cannot exhaust it:

> Consider the English predicate 'is a chair'. The present view is, roughly, that no one has mastered that predicate unless he has learned that it falls under some such generalization as 'y is a chair' is true iff Gx. But, of course, it does not follow that someone who knows what 'is a chair' means is therefore in command of a general procedure for sorting stimuli into chairs and and nonchairs. That would follow only on the added assumption that he has a general procedure for sorting stimuli into those which do, and those which do not, satisfy G. But that assumption is no part of the view that learning a language involves learning truth rules for its predicates. If, e.g., it is true that 'chair' means 'portable seat for one', then it is plausible that no one has mastered 'is a chair' unless he has learned that it falls under the truth rule 'y is a chair' is true iff x is a portable seat for one'. But someone might well

> know this about 'is a chair' and still not be able to tell about some given object (or, for that matter, about any object) whether or not it is a chair. He would be in this situation if, e.g., his way of telling whether a thing is a chair is to find out whether it satisfies the right-hand side of the truth rule, and if he is unable to tell about this (or any) thing whether it is a portable seat for one (1975, p.62).

There exists thus a clear dimension of conceptual development which concerns the discovery of the situations in the real world where some concept applies and of the (pragmatic and epistemic) consequences that follow from its application. These two aspects can be gradually worked out in the course of development by formulating and testing hypotheses relative to the concepts in different circumstances.

The hypothesis that cognition has a biologically determined conceptual expressability corresponding to the capacity to represent the structures founding a range of domains (e.g. cardinal number, physical causality, topology, etc.) and that cognitive development is the process by which the child works out the contexts of application for those structures, remains however too general to provide some insight into the complexity of the developmental processes or to motivate specific empirical research. The hypothesis does not in fact put any precise constraints on how the developmental process may occur: whether it proceeds in a simple cumulative way, e.g. the structure is applied to a larger and larger number of situations, as they present themselves to the child; or whether it proceeds in a gradual, step-by-step way, e.g. the structure is applied to a limited class of situations, then some generalization process goes on and the structure is applied to new situations and so on.

It is in addressing the issue of the constraints on development that the coherence and power of Piaget's account of development can be helpful. The process of domain-specific development can be viewed as consisting of a sequence of stages, each corresponding to the knowledge that the domain structure applies to a particular class of situations. Each new stage includes the knowledge formulated in the preceding stage (e.g. the domain structure, the applications of the domain structure characteristic of the stage and the consequences derived from these applications (properties, regularities, relations discovered)), and at the same time reorganizes and extends it. The application of the structure is worked out for a new set of situations, new facts are derived and a new stage organization emerges. The step-by-step elaboration reflects the increasing complexity of the objects to which the domain structure is applied and the role of prerequisite that each new applications plays for further acquisitions. In sum, this perspective mirrors, at the level of knowledge domains, the Piagetian account of development as it describes the elaboration of more abstract and articulate concepts from the assimilation of more and more complex contents to an innate structure, and the relative discovery of new aspects and properties of the domain.

This hypothesis, stated here in a very tentative form, will be examined from different perspectives. It will be reformulated in the terms of a formal model of transition according to which development is the process of "making information relevant": the specialized domain structure creates a representation which is not immediately relevant and whose import and implications have to be worked out. The hypothesis will be also examined with respect to a concrete case: the developmental phenomena of early competencies. The domain-specific perspective proposed provides in fact a new way of approaching these data.

2.5 Conclusions

According to Piaget, cognitive development proceeds in a fixed sequence of stages, underlying general representation and inference structures. Along the sequence, each new structure includes the previous one, and at the same time extends its generality and coherence. The mechanism responsible for the transition from the lower to the higher stage is reflective abstraction which consists of two processes: 1) the projection of the structure of the lower level onto a new plane of thought, and 2) the integration of it to the structure present at the higher level to form a new system. The originality and strength of this account lies in the synthesis of a holistic view of conceptual organization (structural stages) with a cumulative view of conceptual increase (inclusion of the structure of the lower stage into the higher stage). This synthesis makes it possible to avoid the problems in which holism (i.e. the incommensurability of concepts and the difficulty of characterizing the progress) and cumulativity (i.e. the inadequacy of an additive account of conceptual growth) taken alone run into.

However Piaget's characterization suffers from two fundamental problems. Empirically, the hypothesis of a sequence of clear-cut overall stages of reasoning and representational competence is undermined by the findings of heterogeneity and of early competencies. Conceptually, the process of reflective abstraction, as it is formulated, does not provide a sufficiently detailed psychological account of how the processes of structures' translation and reorganization work and succeed in achieving greater representational and logical capacities. The most vehement criticism of reflective abstraction comes from Fodor (1975), who qualifies this mechanism of "entirely descriptive".

Both empirical and conceptual problems point towards the same aspect of Piaget's theory, i.e. the view of cognition as a single system, organized by progressively stronger logical structures. On the one hand this view is not corroborated by the empirical evidence, which instead stresses the fragmentation (and the partiality) in the way cognition is organized; on the other hand, this view makes abstract sets of relations the locus of developmental change.

These problems appear to become more tractable if the perspective is shifted from the view of a unified cognitive system, organized by same underlying principles, to the alternative view that cognition is organized into partial, domain-specific structures, specialized in processing particular types of information. From this alternative perspective, development would proceed within individual domains and by means of local interactions between domains, and would result in the elaboration of progressively richer knowledge contents. Two major advantages follow from this view. We can reinterpret most of the empirical evidence of early competence as levels of elaboration in the development of the domains and heterogeneity as domains developing with some independence. We also approach the question of the transition processes on a level which appears to be more tractable, i.e. the changes concern partial structures of knowledge. In the next two chapters I try to make this view clearer, first through a formal model of developmental transition according to which development is the process of discovering the relevance of some structure in new contexts, then through a concrete example, the reinterpretation of the early competence phenomenon.

Chapter 3 Modeling domain-specific cognitive development

3.1 Introduction

In two articles (1985, 1987), Richards presents a formal model of the development of the object concept, from age 4 to 18 months. This model specifies logical representations for the sequence of substages originally described by Piaget and an algorithm for the processes of transition between substages.

Richards defines stage organizations in terms of propositional networks and captures the relations of inclusion and extension existing between successive stages in terms of the addition of new *relevant* propositions to the preceding network. The transition between stages is defined as the process of "making information relevant" and is brought about by the abstraction from weaker reasoning frameworks of stronger systems. This abstraction is governed by the semantic algorithm of *diagram pruning*. The semantic environment in which the child reasons is reduced and strengthened as certain propositions change from irrelevant to paradoxical to either true or false, and allow a greater number of inferences to be drawn.

Since Richards' model shifts the burden of the explanation of developmental change from the kind of representations that the child have to the interpretations that he entertains, this model is perfectly compatible with the view on cognitive development introduced at the end of the previous chapter. Development emerges from the tension between biologically determined cognitive structures specialized in processing specific kinds of information and learning the circumstances under which it is appropriate to use these structures to perform in the world. In other words, the child's cognitive system provides domain-specific representations of which the child works out the relevance in the course of development.

I make appeal to Richards' model as a way of explicitly formulating my view of development. I extend this model to provide a general purpose structural description of knowledge states and transitions in the genesis of conceptual domains. While offering a way of representing central aspects of the developmental process, the extended model provides only a purely formal and descriptive account. I therefore suggest that a complete analysis of developmental change has to include an account of the *content* of the concept at the different stages, beside its *structure*. This further layer of analysis is fundamental as far as generation of new research (and eventual educational applications) is concerned.

The chapter is divided in two sections. In the first section, I present in some detail the objectives and the general structure of Richards' model before discussing his account of transition from substage 3 to 5 of the object concept development. In the second section, I identify some problems with the explanatory value of the model and propose ways of dealing with them.

3.2 Richards' model of developmental transition

Richards' proposal is an attempt to reinterpret some aspects of the Piagetian account of conceptual development and at the same time deal with Fodor's compelling argument on innateness. The basic intuition behind Richards's model is that, in the course of development, the representational and inferential structure of the cognitive system remains unchanged, while its interpretation, or content, varies. If we assume that the bases of the cognitive system are propositions related by logical connectives, the developmental process may increase the number of true propositions as well as the number of inferences which can be drawn from them. The developmental changes are thus envisaged at the level of what is entertained (what is relevant for the child at a given stage), rather that at the level of conceptual expressability (e.g. Fodor's language of thought).

3.2.1 The objectives

Richards sets two central aims for the explanation of cognitive development:

1) to specify a psychological mechanism of conceptual development, which is explicit and capable of dealing with Fodor's nativist conclusions;
2) to represent the difference between lower and higher conceptual structures.

Richards deals with the first point by assuming a) that children can represent propositions and logical relations (e.g. they have an innate, though limited, logical competence) and b) that in the course of development they elaborate increasingly complex conceptual contents (e.g. at different stages they reason within different propositional networks). The change in the knowledge contents is expressed in semantic terms, as different truth-values are assigned to propositions. The process of reassigning values invokes a form of abstraction over the semantic environment in which the child reasons and brings about an internal restructuring of knowledge.

Regarding the difference between lower and higher conceptual structures, Richards assumes that the structures constitute holistic systems, i.e. the meaning of the concept at a stage is only specifiable as part of the system, of entities and relations, to which it belongs. From this property of concepts it follows that two (developmentally ordered) stage-structures are incommensurable, i.e. there is no univocal way of comparing their conceptual organization and of saying what has been added to the lower stage concept to form the higher stage concept. It is in principle impossible a) to characterize the concept that the child has at the various levels in a stage-independent way, b) to specify the differences between concepts of different levels and c) to identify the conceptual advance that later stages represent with respect to the preceding ones. In order to deal with the problem of incommensurability between structures, Richards recovers the Piagetian idea that concepts develop in a sequence of discrete stages, each including the preceding stage's structure and explores the possibility of characterizing the inclusion and the conceptual increment in a formal 'syntactic' way:

> If one takes a stage to be a network of connections among schemas, with schemas regarded as the analogues of sentences, Piaget's theory can be seen to inherit the coherence of Quine's account. From a 'syntactic' perspective one may discern a particular network as included in another, and yet semantically one may read the two networks in entirely different ways (p. 35, 1985).

The relations of inclusion and extension between two successive stages are captured by the fact that the structure of propositions of the more advanced stage contains the structure of the previous stage plus some new propositions which express the novelty and the conceptual advance of the new stage. Nevertheless on the semantic level, the two structures can receive two radically different interpretations and constitute two qualitatively different conceptual systems.

3.2.2 The structure of the model

Richards approaches the developmental question from a *formal semantics* perspective, undoubtedly an innovation in cognitive psychology. Richards' basic assumption is that, if the developmental process is of a psychological nature, then logical reasoning is to play a fundamental role in bringing about the conceptual changes; the conceptual advance is derived through problem solving and inference. Under this assumption, he notes that:

> Which logic we should envisage to be involved, however, is a matter for speculation. It would seem natural to consider classical logic, if only because Piaget regards it as determining the fourth stage. He does, nevertheless, entertain another possibility, viz., relevance logic. We shall explore the hypothesis that both logics are actually involved in development. Given that the mechanism need not be determined by a single logic, we shall suppose that it is in a certain sense unstable, sometimes using classical logic and other times employing relevance logic, i.e., the first-degree fragment (p.36, 1985).

Children have an innate internal language, where propositions and the logical connectives *and*, *or* and *not* are represented. However the interpretation of this language is not univocal. At different stages of development, and depending on the context, children can interpret propositions and relations within one of three logical systems: a fragment of *relevance logic* (first-degree entailment), *Kalman logic* or *classical logic.*

These logical systems are ordered according to their inferential power, that is the class of valid deductions that they allow. This difference in reasoning strength is central to capturing the difference between children of a lower stage and children of the higher stage and to modeling the transition between stages. The transition mechanism operates upon the weak logical structure of the lower stage (i.e. the four-valued first-degree entailment), and reduces its semantic space to three-valued Kalman logic, first, and then to two-valued classical logic. This semantic mechanism leads thus from the weaker to the stronger reasoning framework

3.2.2.1 The logical systems

The formal apparatus to characterize the logics consists of a propositional language which has the three basic connectives *and*, *or* and *not* (from which the other connectives can be introduced by definition if required) and a *nonempty set of atomic sentences*. The set of all possible sentences of the language can be defined inductively: where *A* and *B* are any sentences, so too are *(A and B)*, *(A or B)* and *not A*. Moreover, since we want to distinguish the conceptual stages in terms of the deductions that are produced, a relation of Entailment is defined at the metalinguistic level: =>.

The two principal logics are defined in semantic terms. In *Classical logic,* the propositions have one of two truth values: they can be either *true* or *false*. The meaning of the connectives *and*, *or* and *not* is defined as follows:

- a conjunction is true only when both conjuncts are true, otherwise it is false;
- a disjunction is true when at least one disjunct is true; it is false when both disjuncts are false;
- a negation is true when the proposition is false; false when the proposition is true;
- an entailment holds between A and B (A => B), when A and B are both true, both false, or when A is false and B is true. It does not hold if A is true and B is false.

In *First-degree Entailment*, a proposition can have one among four truth values: *true*, *false*, *both true and false* (btf) and *neither true nor false* (ntf). The connectives are redefined to include the case of propositions *btf* or *ntf*, which are incomparable between themselves.

The first-degree entailments for conjunction and disjunction are defined in the following diagrams:

A \ B	t	f	ntf	btf
t	t	f	ntf	btf
f	f	f	f	f
ntf	ntf	f	ntf	f
btf	btf	f	f	btf

Fig. 3.1: Truth-table for conjunction in first-degree entailment

A \ B	t	f	ntf	btf
t	t	t	t	t
f	t	f	ntf	btf
ntf	t	ntf	ntf	t
btf	t	btf	t	btf

Fig. 3.2: Truth-table for disjunction in first-degree entailment

The negation is defined as in classical logic for propositions which are true and false. For the other two cases, negation does not change the truth value of the proposition: if A is ntf, not A is ntf; and if A is btf, not A is btf. Concerning entailment, (A => B) holds when A and B are either both true or both false; and when A is false. (A => B) does not hold when A is true and B is false, ntf or btf, and also when A and B are incomparable.

The relationship between the theories of entailment defined by these two systems is central to this developmental model. The logic of first-degree entailment is wholly included in classical logic, as entailments which hold for the former holds also for the latter and constitutes a sub-theory of classical logic. This property reflects the fundamental inclusion relation existing between two ordered stages in development. *Kalman logic* occupies an intermediary position between the logic of first-degree entailment and classical logic, as Kalman entailment is strictly stronger than first-degree and strictly weaker than classical entailment. Kalman logic is a three valued theory, where the truth values are fixed as *true*, *false* and *both true and false* (btf).

In practice, when reasoning in the 4-value environment of the logic of first-degree entailment, children distinguish the class of entailments that are first-degree valid, i.e. what entails what in first-degree entailment. Similarly, when reasoning in the bivalued environment, children distinguish the more extended class of classical entailments. Disjunctive syllogism and transitive inference are instances of schemas that hold in classical logic, but that are not valid in the logics of Kalman and of first-degree entailment.

((A v B) & -A)=> B

ntf btf ntf btf

t

ntf

incomparable

Fig. 3.3: Proof that disjunctive syllogism is not valid in the logic of first degree entailment.

(A and not A) => (B or not B) is an instance of a schema that distinguishes Kalman logic, where it holds, from first-degree entailment, where it does not hold. Significantly, the strength of these logics is inversely related to the number of possible truth values: the class of entailments increases as the number of truth values decreases. It is around this property of the three logical systems that the transition algorithm is articulated.

3.2.2.2 The transition

The transition from the weak reasoning framework of the lower stage to the stronger reasoning framework of the higher stage is accounted for in terms of a strategy which operates upon the semantic structures of the different logics. The strategy consists of abstracting from the four-valued tree of first-degree entailment a three-valued tree first and a two-valued tree later. Technically, the abstraction is expressed by the pruning of the first-degree entailment semantic tree of the node for neither true nor false, and then by the pruning of the Kalman logic semantic tree of the node for both true and false.

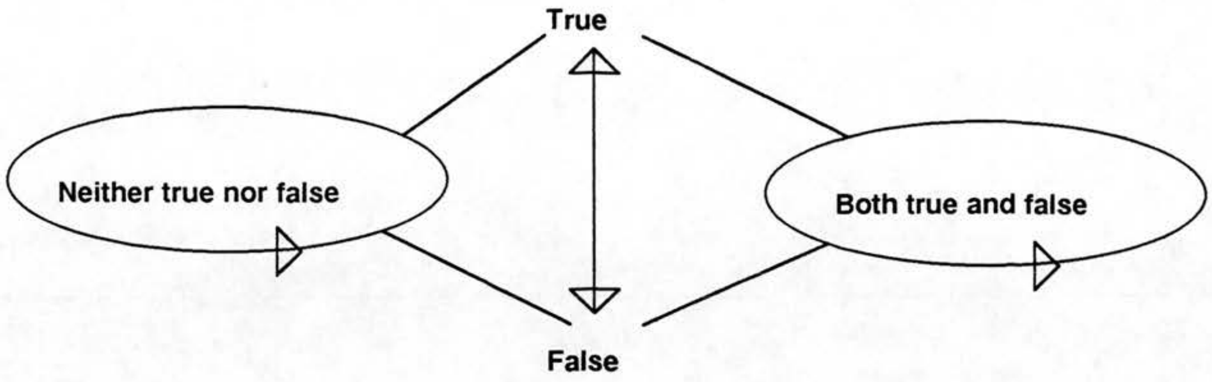

Fig. 3.4: The lattice of First-degree entailment

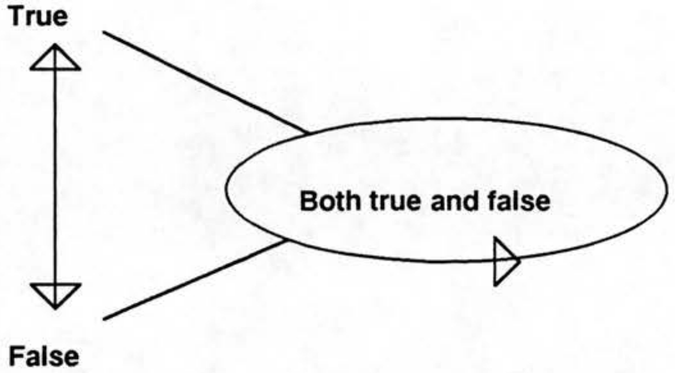

Fig. 3.5: The lattice of Kalman logic

Fig. 3.6: The lattice of classical logic

Each pruning forces a reassignment of truth values under the constraint of consistency, i.e. what is projected must be in accord with what is already known. The proposition that was initially entertained as ntf is reassigned the value btf: the initial belief that the proposition does not have any bearing on the situation at hand is partly conserved and combined with the new belief that the proposition might well be true in the situation. Similarly the proposition entertained as btf is reinterpreted within a bivalent semantic space and reassigned one of the values, true or false, after having been tested.

Defined in these terms, the mechanism gives an explicit account of the way in which abstraction may proceed and expresses the property of unidirectionality of development as change occurs from ntf to btf and from btf to true or false exclusively. It also accounts for the tri-partite nature of change in terms of the constraint of consistency. Development proceeds in a rational way as the initial abstraction corresponds to an attempt to combine the initial beliefs with new insights. The combination of these contrasting beliefs gives rise to an ambivalent state of mind, which itself motivates the search for a rational solution of the ambivalence by abstracting the propositions into a framework where they can actually be tested.

The transition algorithm is meta-conceptual in nature, since it is defined over the different logical systems, and is not strictly part of any of them.

3.2.3 The predictions

Richards' model captures conceptual transition at the level of change in problem-solving abilities. Schematically, a developmental transition is identified at the level of a *developmental task*, i.e. a task that enhances different patterns of solutions at different periods in the life-span and hence discriminates between competence levels (e.g. the task is failed at a lower stage and performed successfully at a later higher stage). The content of the propositions invoked by the model expresses the solution to developmental tasks. The interface between the model and the empirical data is within the developmental tasks. Richards writes:

> These problems constitute the determinants of the analysis; that is, they set the issues to be resolved and thereby fix the parameters necessary to specify the concept. The solutions provide an interpretation for these parameters which is given in the form of propositions which are held to be true (1985,p.61).

A proposition in the model can be either the solution of the tasks or a premise required to deduce the solution. When this proposition is true, it permits conclusive reasoning, that is to deduce a true proposition (the solution), given true premises (the problem). The transition corresponds to the process by which the proposition comes to be reinterpreted into a relevant proposition, i.e. a proposition which can operate as a premise in the inferential chain or as hypothesis, be eventually verified and added to the knowledge base. The model predicts that the transition between every pair of ordered stages is tripartite, uniform and unidirectional:

1) children who fail the task approach the problem within the logic of first-degree entailment and can only recognize the entailments valid in this logic. A proposition which is either one of the premises or the conclusion of the deduction that produces the solution is entertained as neither true nor false, that is irrelevant to the task in hand. As such the accurate solution cannot be deduced: it cannot contribute to its being realized, nor can it contribute to it being shown to be unrealizable;
2) the developmental strategy of abstracting a sublattice from the valuation space of first-degree entailment is applied. The proposition which was entertained as irrelevant is reinterpreted within the three-valued Kalman logic. In an attempt to combine the "intuition" that the proposition might be relevant (and even true) and the prior belief that it was not, children entertain the proposition as paradoxical: at the same time true and false. The model predicts failure in solving the task, since an ambivalent proposition, which plays the role of either premise or conclusion in a deduction, cannot yield conclusive reasoning. In this intermediate phase, the failure is accompanied by behavioural indices suggestive of internal conflict: any attempt to verify an ambivalent proposition being also attempts to falsify it, and vice versa;
3) the developmental strategy is again invoked to force a reinterpretation of the paradoxical proposition. The proposition is now entertained as either true or false and, depending on the state of the world, is falsified or verified. By reasoning from true premises to true conclusions, children can now increase their knowledge. The model predicts that the correct solution is derived through classical inference and that the testing of hypotheses is reflected, at the behavioural level, in verification procedures and eventual revisions or corrections.

Before approaching the second aspect of the model, that is, the representation of the conceptual increment between two ordered stages, let us examine the application of the model to the development of the object concept. A concrete example makes the way in which the model represents the stage structures clearer.

3.2.4 The model of the object concept development

Between 0 and 18 months of age, children learn a great deal about objects in the world, about their spatial and dynamic properties. In this section, we focus on the acquisition of object identity, or the permanence of the object in Piaget's terms. The understanding of object permanence is operationally defined as the capacity to look for objects that have disappeared from view in the location where they were last seen and to retrieve them. The structure of the object task, designed to investigate the development of this understanding, consists of attracting the infant's attention towards an object placed within reaching distance. The object is then hidden under a cover, with an action in full view of the child. Understanding of the permanence of the object is attributed, in case the child searches for the object and looks under the cover. In more complex tasks, the object is either hidden underneath a second cover, beside the one used in the previous task; or the object is hidden under two superposed covers; or else the cover under which the object lies, is exchanged location with a second cover.

According to Piaget's reconstruction, between 0 and 4 months infants develop the capacity to track moving objects first in their field of view and later also beyond it, with coordinated movements of the head and the eyes. By substage 3 (4 to 6 months), infants can reach out to pick up an object that they see. However if the object task is presented and the object is fully covered under a cloth or a cup, infants do not attempt to remove the cover and to retrieve the object. It is sufficient for the object to be only slightly uncovered and visible that the infants will not have any trouble in recovering it. In a sense, substage 3 infants act as if the object, once covered or disappeared, no longer existed. At substage 4 (6 to 12 months), infants can retrieve the hidden object. However, if in a second trial, the object is hidden under a second cover, always in full view of the child, another surprising response occurs. The infants go and look under the cover where they first found the object, instead of where the object had last disappeared, i.e. the second cover. Even after they fail to find the object, they do not look under the second cover. At substage 5, infants retrieve the object from the last hiding place and appear to be able to reconstruct the object's displacements. Nevertheless, they cannot retrieve the object when it is hidden under two covers. They lift the top cover and stop short of lifting the bottom cover. By 18 months, infants seem to possess a structured concept of object, as they can find objects after any visible displacement as well as after non visible ones, by verifying possible routes and hiding places.

Richards reanalyzes this developmental progression between the substages 3 to 6 (from the capacity to recover the object from under one cover to the ability to recover the object after double covering or swapping of location between two covers) in terms of his transition model.

3.2.4.1 The transition from Substage 3 to Substage 4

The object is hidden under one cover in full view of the infant. Since the infant seems interested in it, one would expect him to act to recover the object and hence to look under the cover. Instead the infant does not do anything of this sort; first he seems bewildered and then loses interest in the whole situation. The model describes this state of affairs as the infant representing the proposition:

(1) *The object is there*

(where the deictic 'there' refers to the location of disappearance) but projecting it as irrelevant. The proposition (1) has the truth value neither true nor false. Although the infant projects the appropriate proposition, he envisages the place of disappearance as irrelevant to the recovery of the object. The irrelevant cognitive state also explains why the reaction which accompanies the disappearance is one of bewilderment.

Later, the infant may modify his view and envisage (1) as more significant to the problem than he first thought. However the proposition (1) does not immediately emerge as an unambiguous hypothesis. The infant has reasons to suppose that (1) is true as to take it to be false, a state that in the model corresponds to projecting the proposition (1) as paradoxical, by invoking the truth value both true and false. The corresponding state of mind is one of distress and frustration, as often reported in the literature. In fact neither looking under the cover nor not looking would lead to a verification or a falsification of the hypothesis.

The way out is to project the proposition (1) as either true or false. The infant is now rationally motivated to act and check under the cover whether the object in fact is there or not, since what he now discovers he can understand. The infant can now retrieve the object from underneath the cover and have thus reached substage 4. The model defines the object concept elaborated at substage 4 as consisting of one atomic sentence (1) and a generalization of it in the atomic schema (2):

(1) *The object is there*

(2) *[The object is there]*$_{3.}$

The schema (identified by the square brackets) has object and place as parameters, and a subscript which indicates that the schema emerges from substage 3. The classical theory of the object elaborated at substage 4 consists of (2) and all the atomic sentences, like (1), that can be added under (2). The classical theory thus specifies the stage 4 conditions of admissibility for sentences to be instances of (2).

3.2.4.2 The transition from Substage 4 to Substage 5

The acquisition of the substage 3 theory does not require the application of any particular rule of inference. As the proposition (1) is seen as relevant and bivalent, the infant is motivated to act in order to determine whether the object is or is not under the cover. In the transition between substage 4 and substage 5 instead, inference plays a central role, as the hypothesis that is projected to solve the corresponding developmental task is to be deduced after proposition (1) has been falsified.

The critical task that differentiates substage 4 from substage 5 consists of a first trial, equivalent to the previous one (where the infant does not have any difficulty in retrieving the object) followed by a second trial. The experimenter, instead of putting the object again in the same location, puts it under a second cover placed just beside it. The surprising behaviour is that frequently the infants do not look immediately under this second cover, but rather lift the first one, where they had previously found the object. Even more surprising is the fact that, after not having found the object, they do not turn toward the second cover and look underneath it.

This new task demonstrates how the substage 4 concept of object identity is limited, as it is sufficient to move the object to a new location to observe behaviours similar to those characteristic of Substage 3. The model formulates this state of affairs as follows. The infant entertains two propositions[7]:

(3) *The object is* $there_i$

(4) *The object is* $there_i$ *or The object is* $there_{ii}$

The choice of propositions reflects the fact that the infants conceive the problem as one concerning '$there_i$' and not '$there_{ii}$'. In that respect, a disjunction is more appropriate than a conjunction for two reasons. From a conceptual point of view, the problems deals with one object moved between two locations, a situation that can be represented by a disjunction, but not by a conjunction. From a logical point of view, the complex proposition is relevant, even when one of the disjuncts is irrelevant (see the definition of disjunction in the logic of first-degree entailment), and this would not be the case with a conjunction.

The infant approaches the problem from the perspective of substage 4 theory. Hence he has good reasons to expect the vanished object to be where it had been found. Proposition (3) is an instance of the schema (2) constitutive of the Substage 4 theory, and as such it is considered relevant to the problem and true. The second disjunct of (4) is instead considered irrelevant to the task, as the infant is exclusively centred on the location '$there_i$'.

The infant takes (3) to be true, looks under the cover and finds no object there. If he had an adequate concept of object identity, he would consider that the object is under the second cover and would go and look there. If the infant were reasoning within a classical logic framework, he would go through a disjunctive argument:

[7]The subscritped deictics refer to the two locations: '*i*' is the first cover and '*ii*' the second. The three occurrences of '*object*' are taken to be co-referential.

(4) *the object is* $there_i$ *or the object is* $there_{ii}$

(3) *not the object is* $there_i$

---

(5) *the object is* $there_{ii}$.

Instead, substage 4 infants, after having failed to retrieve the object from under the original cover, do not attempt to reach for the second cover and appear to be bewildered. According to the model, they reason within the weak logic of first-degree entailment, where the disjunctive syllogism is not a valid entailment. In this framework, they do not seem to conceive of any alternative location of the object to the one in which it was previously found. The possibility that the object is '$there_{ii}$' is considered irrelevant to the solution of the problem.

In a subsequent phase, the child, while still failing to recover the object, shows reactions of frustration, rather than bewilderment. The change is interpreted as reflecting an emerging awareness that the object might be under the second cover. The object's move to this new location is now seen as being of some relevance to the task, even though the reasons for considering it unimportant are still present. The model represents this state of affairs in terms of the proposition (5) *the object is* $there_{ii}$ now being projected as paradoxical, i.e. both true and false. The task is now formulated within the environment of Kalman logic, where the disjunctive argument is still not valid. However even though the infants do not look under the second cover, they start suspecting that it might well be there. In these circumstances, they are not motivated to go and look under the second cover because both finding the object and not finding it would confirm the paradoxical hypothesis.

The model envisages the way out of the dilemma as the strengthening of the reasoning framework by abstraction of the bivalued classical logic from the three-valued Kalman logic. Now the disjunctive syllogism is valid and proposition (5) *the object is* $there_{ii}$. can be deduced as a true hypothesis, open to testing. The proposition which was paradoxical is thus reassigned the truth value: either true or false. The infant is now rationally motivated to look under the cover to check whether the object is there or not. The testing carries new knowledge as the infant can finally understand the outcome of his action. As he finds the object in the second location, he discovers new properties of object displacements and identity.

The ability to retrieve the object when it is moved between distinct locations corresponds to the attainment of substage 5. The model characterizes the conceptual advance as the extension of the theory of substage 4 by the addition of the schemas: (6)

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[*The object is there*$_{ii}$]$_4$ and (8) [*not the object is there*$_i$]$_4$ which reflects the fact that in the task with two covers, the proposition (3) is false: (7) *not the object is there*$_i$.

The substage 5 object concept is determined by the classical theory consisting of:

(2) *[The object is there]*$_3$

(6) [*The object is there*$_{ii}$]$_4$

(8) [*not the object is there*$_i$]$_4$

The theory specifies what instances are admissible, that is, which propositions the substage 5 infants hold as true.

3.3 Richards' model as an account of transition in domain-specific development

According to the domain-specific view proposed at the end of chapter 1, development emerges from the tension between biologically determined structures, specialized in processing specific kinds of information pertaining to the diverse knowledge domains, and discovery of the circumstances under which the application of these structures makes it possible to solve problems, identify properties, anticipate events, etc. Contrary to the general stage hypothesis, the development of a conceptual domain is not brought about by changes outside the domain. Furthermore, the development of the different knowledge domains is presented as proceeding in a discontinuous fashion, through a sequence of stages, each corresponding to a particular range of application of the domain structure. The stage sequence reflects the increasing generality of the concepts, as later stages have a more extended range of application and deal with more complex contents. The nature of the concept elaborated at the different stages is defined by the domain structure and by the contents that the structure assimilates at the stages. The model proposed by Richards is not only compatible with this view, but extremely useful in increasing its precision and explicitness. I now examine how the main components and articulations of the domain-specific framework presented can be specified by means of Richards' model.

3.3.1 The general principles

At the more general level, Richards starts from the assumption that children have an innate representational system, where propositions and logical relations are expressed and where complex reasoning can take place. At the same time, children cannot immediately access and exploit all the information represented, i.e. at the outset most representations are irrelevant and the range of inferences is extremely limited. The distinction between representing information and entertaining it as relevant corresponds to the distinction between having a structure to process domain information and knowing where to apply it and which consequences to draw from it. The opposite case involves representing a proposition, but entertaining it as irrelevant, i.e. neither true nor false. This is a logical formulation of the loose principle that, in presence of the appropriate information, the specialized domain structure is triggered and produces an internal representation, which need not be exploited to accomplish some action plan, to draw some inferences, etc.

The reason the cognitive system has irrelevant representations is that any input from the world is inherently ambiguous: it can be processed by many different specialized systems (like having different points of view on a same scene). For that reason, not all the representations produced can be entertained as relevant, at any one moment and for one situation. But how can a selection between the representations be made, and on which basis? Development can play the role of the process by which we work out the consequences of representing particular aspects of a situation and we fix, through hypothesis formation and testing, the representations that are more useful in the real world (e.g.for solving problems, organizing and explaining facts, anticipating events).

3.3.2 The transition process

Regarding the transition process, the tripartite, uniform and unidirectional model conforms to the idea that development proceeds in a similar, discontinuous manner in all the domains. More particularly, the model specifies an explicit mechanism capable of explaining how the process of gradually working out where the application of the domain structure is advisable and useful (e.g. to identify properties and relations, solve problems, anticipate facts, etc.). Moreover, as Richards defines it, the transition process conforms well to the behavioural patterns observed in the development of the object concept, as described by Piaget and later confirmed by numerous studies (see Bower 1974 for a review). I will now set out two examples: the transition between substage 4 and 5 of object concept development and the transition from substage 1 to 3 of the development of the conservation of liquids. In the examples I try to show how the generalization of domain-specific structures to new contexts integrates the account in terms of Richards' processes for making information relevant.

3.3.2.1 Transition in object concept development

The clearest illustration of the working of the developmental strategy described by Richards is found in the transition between substage 4 and 5 of the object concept. At substage 4, the infant has elaborated a concept of object, according to which an object can disappear at one location and can be found there. Thus, when the infant is faced with the A-B task (the object is transferred from under A, where the infant had found it before, to B) he formulates the hypothesis that the object is under the cover where he found it in previous experiences. The infant represents the transfer from A to B, but does not conceive that as relevant to identifying the location of the object. Therefore, even when he looks under A and does not find the object, he does not formulate the alternative hypotheses (e.g. the object is under B). In Richards' model this state of affairs is represented as the infant reasoning within the weaker, first-degree logic. Within this logic, the inference that if the object is either under A or under B; and the object is not under A, then it is under B, cannot be drawn.

In the terms of the domain-specific hypothesis, the child has a structure specialized to process information relative to objects and displacements. At substage 4, this structure applies (and is relevant) to the class of situations which involve one object and one location. It is because the substage 4 concept does not cover the displacements between locations that the transfer and the positioning of the object under B constitute irrelevant information for the child. Thus confronted with the A-B task, the substage 4 infant is bewildered at not finding the object under A, the only location where he conceives it to be.

The developmental strategy of making what is irrelevant paradoxical is then applied. The infant tries to square the evidence suggesting that the object might be under the cover B (e.g. resulting from the application of the domain-structure to the problem) with the original belief that the object is not under B and with the newly acquired knowledge of the fact that the object is not under A. The infant finds himself in an ambivalent state of mind with respect to the solution of the task, expressed by the proposition that it is both true and false that the object is under B. Cognitively, this does not motivate the action of verifying whether the object is indeed under B, as both finding and not finding the object would confirm the hypothesis. Although Kalman logic is stronger than the logic of first-degree, the inference which would lead from the falsification of the initial hypothesis to the correct solution is still not valid.

The developmental strategy of making what is paradoxical into a proposition which is either true or false further strengthens the reasoning. Now the disjunctive syllogism is valid and can take the infant from formulating the proposition as hypothesis to verify it, by searching for the object in the second location. The hypothesis testing corresponds to working out the consequences of applying the domain structure to this new class of situations, and to acquiring new knowledge about objects and displacements.

3.3.2.2 Transition in liquid conservation development

It also accounts well for the three phases of the developmental process that have been observed in most Piagetian experiments. Consider the case of the conservation tasks, and in particular the liquid conservation task (see section 2.2). In an initial phase, children do not conserve the amount of liquid after this has been poured from one of two identical containers into a container of different shape. One of the typical answers is that "there is more liquid to drink because the liquid comes up higher", a judgment based on the comparison between the levels of the liquid in the long, narrow glass and in the short, wide glass, which overlooks the fact that the amount of liquid itself is unchanged. The second phase is characterized by vacillations between non-conservation and conservation judgments. Either the children keep changing their mind from conservation to difference, or conserve after one transformation and do not conserve after a second, equivalent transformation. In the third phase, the child confirms the conservation of liquid quantity, regardless of the other changes, and justifies it by arguments based on logical identity, reversibility by cancellation of the change, and compensation between dimensions.

This developmental sequence can be characterized in terms of the three cognitive states postulated by the model as the process through which the child generalizes the specialized domain structure to a new set of situations. In the first period, the appropriate domain structure is not applied (e.g. it is taken to be irrelevant to the task at hand). It follows that the properties (e.g. the water is the same, nothing has been added nor taken away) and the relations (e.g. the level of the water in one container is higher, but the width is smaller) from which the conservation principle may be derived are not identified. The child represents the proposition that the amount of water is the same in A and B, since he has the structure appropriate to process this information. Yet, he entertains that proposition as irrelevant. The same information can in fact be processed by systems other than the appropriate one, systems that had worked in the past, e.g. global perceptual estimation of quantity, and of which the child knows the implications.

The vacillations typical of the intermediate period reflects an ambivalent state of mind, where the child starts envisaging that the application of the domain structure to the task can provide some perspective on the problem, as it suggests that the amount of water is the same. In an attempt to fit this conclusion with what he knows already, the child tries to conciliate that conclusion with the initial one, that the amount is different. Thus he entertains the proposition that the amount of water is the same in A and B as paradoxical, i.e. at the same time true and false.

Finally, the third period corresponds to the straightforward application of the domain structure to the task. The proposition becomes relevant and bivalent. The child formulates the hypothesis that the amount is the same, either justifies it by the argument of composition of width and height or tests it by pouring it back into the initial container, and derives the conservation principle.

3.3.3 The sequence of stages and their organizations

In Richards' model, the conceptual organization underlying the stages is defined by a classical theory consisting of a network of propositional schemas. The conceptual progress brought about with a new stage is specified in terms of the addition of new schemas to the classical theory of the previous stage. The classical theory of substage 4, for example, consists of the atomic schema

(2) $[\textit{The object is there}]_3$.

At the subsequent substage 5, two new atomic schemas are added:

(6) $[\textit{The object is there}_{ii}]_4$

(8) $[\textit{not the object is there}_i]_4$.

This same representation can be extended to define the stages in the development of conceptual domains. The network of propositions expresses the range of application of the domain structure at a stage, i.e. the class of situations where the relevance of the structure has been worked out. The inclusion of the lower stage into the subsequent stage and its reorganization are captured by the fact that a) the network corresponding to the lower stage is still recognizable in the higher stage and b) the network corresponding to the higher stage introduces supplementary propositions and relations between propositions, and thus constitutes a new network.

Under the assumption that the concepts in the network are holistic in nature and that the concept of different stages are incommensurable, the concept at the substage 4 ought to have a radically different interpretation from the concept of substage 5. This requirement is partially satisfied by the model. On the one hand, an understanding of objects based on the substage 4 classical theory, where only instances of (2) are true, is to be very different from that of substage 5, where instances of (6) and (8) are also admitted. On the other hand, schema (2) belongs to both classical theories and its content is constant within the two theories, i.e. the solution to object tasks with one disappearance location only. The fact that a schema constitutive of the classical theory of two stages has the same scope is not strictly compatible with the assumption that concepts of different stages have incommensurable interpretations, even though it reflects the inclusion relation at the basis of Piaget's analysis of stage structures.

Richards proposes to envisage the issue of incommensurable structures which share common components from two different perspectives, i.e. that of the cognitive scientist and that of the child. From the cognitive scientist's viewpoint each substage has a content, i.e the class of tasks in the scope of the substage's schemas. Development corresponds to an increase in competence (e.g. extension of the range of application of the concept) such that new substages include the preceding ones. Thus the structure of the predecessor (e.g. its schemas and their content) is included in the successor.

From the child's viewpoint, on the other hand, the meaning of the schemas changes radically from one substage to another. This meaning can thus be specified only in terms of different theories, characteristic of the different stages. The distinction between the child's and the cognitive theorist's perspectives leads Richards to conclude that:

> The difficulty is not merely that one cannot get inside the head of the child but that it would be useless even if one could. One would then have no perspective from which to articulate the difference between the theories. This can only be characterized in an extra-cognitive way. What one shares with the child is the sequence of classical theories, not their incommensurable interpretations (p.62, 1985).

This conclusion however is not without problems. If the logical structure of Richards' model is sufficiently plain, its status as a psychological account of conceptual development does present some shortcomings. The conclusion that it is impossible, as well useless, to attempt to give a characterization of conceptual contents entails that: a) one cannot specify the conditions under which the child entertains some concept as irrelevant, b) the developmental reconstruction is always *a posteriori*, on existing data. The most dramatic consequence is that if one refuses to make hypotheses about the content of the concepts the child has, one has no grounds upon which to generate new empirical questions and construct new experiments.

The same problem emerges concerning the status of the formal characterization of the stage-structures as a set of propositional schemas which correspond to the solutions to the class of tasks solved at the stage. On the one hand, Richards presents the propositional representation as a shorthand to express the fact that the solution of the tasks indicates the acquisition of a more general competence, at least that equivalent to solving a class of comparable problems. The use of schemas to express this generalization is presented as "a certain expedient (p.52, 1985)":

> to overcome the difficulty is to specify precisely what problems these are (....) What is needed is some kind of generalization which defines the range of appropriate instances. Unfortunately it is not easy to identify the relevant factors and even if it were, it would be cumbersome to formulate them into a suitable generalization (p.52, 1985).

On the other hand, Richards gives important theoretical motivation to the representation of stage knowledge as propositional schemas:

> It is natural to ask what the concept might be which is differentiated by the classical theory determined by schema (2). Here it is difficult to be explicit without resorting to metaphor or to the via negativa. Neither seems a satisfactory way to identify the conceptual content of this substage, or any other substage. Quine would seem to point to the only viable approach. Let us suppose that insofar as 'object-sentences' are concerned, the child accepts as true only those belonging to the adumbrated theory. Clearly he is going to have a very strange idea of objects, one that may actually be impossible to characterize other than by the theory itself. But what other characterization do we need? It seems sufficient to identify the range of schemas which the child takes to yield true sentences, in this case schema (2). This fixes, together with the logic, the concept he is entertaining and reveals it to be curious in the extreme (p.53, 1985).

Richards argues that, since there is no satisfying way of giving a direct characterization of the content of the concept, the only possibility left is to specify the concept in terms of the schemas which underlie the solutions of the tasks that the children solve at a stage. However this proposal risks becoming circular as we would say that a child is at a stage on the basis of the tasks that he solves and then would model the competence underlying the stage as a set of schemas corresponding to the solutions of the tasks.

Consider, for instance, the case of a stage$_n$ which is identified by the solution of tasks a and b. The model represents the concept at stage$_n$ by the schemas $[a]_n$ and $[b]_n$, which are the solutions to the class of tasks equivalent to a and b. Although formally irreproachable (e.g. this characterization satisfies the constraints of inclusion and extension as well as expressing the holistic properties of concepts), the characterisation of stage competence remains purely descriptive. In a sense it posits the same thing that has to be explained, i.e. the capacity to solve the tasks, as model of the competence, i.e. the schemas expressing the solutions of the tasks. This leads Richards to the quasi paradoxical conclusion:

> What is then the object concept? It is just the set of solutions to the characteristic problems (1985, p.52).

Beside circularity, two other important consequences follow from the choice of specifying the child's concepts solely on the basis of the tasks he can solve. Firstly, we could not generalize from the children's performance in the tasks to qualify the children's understanding of the concept, which strongly undermines the objectives of psychological research. Secondly, and more importantly, we could only offer an *a posteriori* characterization of the children's concepts on the basis of existing research, as we would not have the parameters to devise and study new tasks. These parameters can only be specified with respect to hypotheses about the capacities and the limitations defining the concept at a stage, hypotheses that can be tested by designing and administering new specific tasks.

In order to solve these problems, I suggest that the model be complemented with a characterization of the nature of the conceptual content at the diverse stages, underlying the classical theories described by the model. For instance, we can offer a tentative characterization of the nature of the object concept at stage 4, and from that create new tasks that, if our characterization is correct, should be easily solved by stage 4 children. Similarly, from our hypotheses, we can devise tasks that depend on aspects of objects which, according to our hypothesis, may be difficult to handle at stage 4 and predict failure in those tasks. Hence, by trying to capture the content of the concept, as well as its structure, we can motivate new empirical work and gradually refine our descriptions and interpretations of the developmental process.

Going back to the distinction that Richards introduces between the two perspectives for analyzing conceptual development, i.e. the child's and the cognitive scientist's, I am suggesting that both are necessary to account for the two related aspects of development: the stage organization and the transition process between stages. The external, formal perspective is indeed necessary to articulate the differences between the stages which otherwise would be unscrutable, and to capture the constructive process of stage transition. The sequence of stages can in fact only be articulated from the outside, as we observe the change in the behavioural patterns, reactions and solution strategies possibly of the same children at different moments in time or, more simply, of a group of children of different ages. The internal perspective is instead necessary to attempt a characterization of the children's competence at a particular stage. It is this analysis that leads to new research, as it demands that new situations be introduced to probe aspects of the children's understanding and to test the hypotheses about their competence.

The domain-specific perspective, introduced as a solution for some of the problems emerged with Piaget's theory, may provide the guide-lines for a detailed analysis of the conceptual structures at the different stages. According to the domain-specific hypothesis, the nature of the concept at the different stages can be characterized in terms of the interaction between the basic structure of the domain, that is specialized in processing information relative to the domain, and the contexts in which this structure is effectively applied to yield relevant information. The particular version of the domain-specific hypothesis I propose claims that along the sequence of stage concepts, the relevance of the domain structure is worked out for increasingly more complex contents. Each stage reflects the relevance of the domain-structure for a particular class of objects, and the simultaneous irrelevance of that structure over some other classes of more complex objects, and defines a specific concept. The meanings of contents, objects and order of complexity should become clearer when the domain of cardinal number will be analysed and a set of situations that are instances of the domain introduced.

3.4 Conclusions

According to the theoretical framework proposed at the end of chapter 2, conceptual development proceeds at the level of individual knowledge domains (and of local interactions between domains) and emerges from the tension between basic knowledge of the structure underlying the domains and gradual learning of the (pragmatic and epistemic) consequences of the application of the structure to real world situations. The model of Richards helps make this process more explicit.

In this model, the fact that children may represent but not use some structure, necessary to approach a particular problem, is expressed in terms of information which the child entertains as irrelevant (i.e. neither true nor false). Although the specialized structure applies to the test situation, the child does not appreciate its relevance to a full understanding of the situation, nor the consequences of its application, or the facts that he can derive from it. Subsequent development is characterized as the reinterpretation of this information first as paradoxical (at the same time true and false) and later as relevant. Only in this last stage, does the child proceed to testing whether the application of the structure in the context of the task yields useful information for the solution of the task. In the case that it does, the child has discovered a new situation that can be understood with that structure and has learnt new aspects and properties of the domain.

In conclusion, Richards' model makes it possible to capture the structure of the developing concept: represent the sequence of classical theories and the relations between them. This models also offers a way of envisaging the process that brings the child from less to more advanced conceptual organizations. In parallel with this analysis, I suggest that it is necessary to provide an account of stage concepts. The analysis of the kind of concepts that the children entertain at the different stages directs empirical research, addressing questions such as:

1. what is the nature of the understanding beneath the classical theory of the stage?
2. (and its complementary) why are some aspects of the concept irrelevant?
3. with respect to which situations is the child in an irrelevant cognitive state?

and more practical questions, such as:

4. which kind of experience can be more profitable to get the process of "making relevant" started?

Chapter 4 Early competences revisited

4.1 Introduction

In this chapter, I pursue the presentation of the domain-specific framework through the discussion of its application to a central issue in developmental psychology: the interpretation of precocious success in Piagetian tasks. I argue that the domain-specific perspective permits us to treat early success on modified tasks as evidence of a different level of understanding of a concept from the one which underpins later success on the standard task. The approach advocated provides a systematic treatment of intermediate competence levels.

Early competence, already introduced in Chapter 2, points up the limit of the analyses of cognitive development which take single tasks to be critical proof of the acquisition of a concept. Because early competence is highly sensitive to the mode of presentation of the task, it provides evidence for certain essential properties of concepts: their context sensitivity, limited application and specialization; and of concept development, gradual generalization and abstraction.

From the perspective of a domain-specific model, these properties of concepts in development are captured through the distinction between knowledge of the domain-structure and application of that knowledge. Each level of competence identified experimentally, be it with a modified task or with a traditional Piagetian task, is interpreted as evidence of the capacity to apply the domain-structure in the context set by the task. The application yields representations of properties and relations relevant for the solution of the task. The reason for the décalage from the modified task (i.e. the precocious success) to the standard task lies in the different complexity of the content invoked by the tasks. The structure of the modified and standard tasks is then reanalyzed in terms of the nature of the objects which are to be isolated and related to solve the task, i.e. the objects to which the domain-structure is applied and which are expressed in the relevant representation.

The chapter is divided into three sections. In the first section, I examine the general argument for attributing to the child early competencies on the basis of modified Piagetian tasks, and identify some problems with the argument. In the second section, I illustrate the argument with the case of the modified number conservation task of McGarrigle & Donaldson (1975). In the third section, I propose a reinterpretation of the precocious success data based on the domain-specific framework presented.

4.2 The argument for early competence

As we have seen in chapter 2, Piaget devised tasks which were critical for attributing a full-blown concept to the child, like the permanence of the object, the seriation, the class-inclusion, the conservation of matter, liquid, weight and volume. The conservation of number task, for instance, is the task which discriminates between children who have an operational concept of number: they can create a representation of the number of a collection and reason with it, and children who have a pre-operational concept of number: they have a pre-concept, or an intuition of number, based on the spatial extent and the configuration of a collection.

The conservation task is a critical test of number concept in that to maintain the equinumerosity between two collections after a spatial transformation is performed on one of them, the child has to go beyond the spatial difference and consider the spatial transformation as number-irrelevant. Different complex inferences could be involved in deriving the conservation principle:

a) the post-transformation configuration may be related back to the pre-transformation configuration, where the equinumerosity was first established (e.g. by reversibility, equinumerosity can be confirmed by going back to the one-to-one correspondence between the pre-transformation arrays);

b) the two arrays may be analysed in terms of density (the distance between elements) and length of array, dimensions which are then combined and matched (e.g. by composition, one is longer but more spaced, the other is shorter but more dense);

c) alternatively the operation performed on the array may be examined, to note that the number of objects in the modified array is always the same (e.g. by identity, nothing has been added nor taken away from it).

After Piaget, much research was aimed at devising tasks to control for other factors which may have been responsible for the preoperational type of response as an alternative to or along with logical competence. Larsen (1977) calls this approach the *simplification strategy:*

> The general strategy seems to be that since performance on any task is multiply determined, it is impossible to draw any conclusions about one aspect of the state of the child from his performance on the task until all other significant aspects of the child's state have been measured and controlled for. Specifically, one cannot conclude from a child's performance on, for instance, a conservation task that the child "has" or does not "have" conservation until other significant factors determining his performance, such as his attention or verbal ability, are also controlled for. In particular, a negative conclusion about a child's possession of conservation is unjustified (p. 1163).

Hence the basic schema behind the replication tasks is to maintain the structure of the original Piagetian tasks, while a) setting the task into different contexts (e.g. familiar situations), b) changing the material or the wording of the questions, and/or c) preceding the task with training sessions (e.g. for memory, attention, linguistic expressions used). For some of these tasks, precocious successes are reported, i.e. children could solve the modified task earlier than they could solve the original Piagetian task.

These results opened the way to the early competence argument that, since the Piagetian task is critical for attributing operational understanding of a concept, and children can solve tasks of same structure, but in different modes of presentation, before Piaget's task, then the children have the operational concept much earlier than Piaget claimed. In the standard task, the argument goes, their real competence is simply masked by performance factors. Larsen (1977) remarks that such an argument is in principle unfalsifiable:

> One could always argue that other factors prevent the expression of a given behaviour, and there are always other factors involved in any experiment (p. 1163).

Beside conceptual problems, there are also a number of empirical problems. First, the analysis of the behaviours associated with the correct response in the modified task suggests that they are qualitatively different from the behaviours characteristic of the correct response in the traditional task (e.g. non-operational justifications, response strategies based on spatial indices, on the order in which the transformations are performed or on the last word uttered. See the example which follows and Appendix 6.2 for more details) .

Second, given a Piagetian task, early competence has been exposed by replications that isolated a variety of factors, such as linguistic skills, pragmatic competence, memory load, attention, etc. The non-conservation responses have thus been interpreted as due to the difficulties with these various factors. In the case of number conservation, for instance, Mehler & Bever (1967) claim that conservation is one of the basic capacities already of 2-year-olds, and that only the maturity of memory and attention capacities stop them from expressing this competence in the standard task. Gelman (1969) explains non-conservation by the fact that the child does not attend to the connection between the initial pair of sets and the post-transformation pair. Later Gelman (1972) interprets non-conservation responses as indexing a lack of confidence in his own judgment due to the child's poor counting skills, as opposed to lack of cognitive ability. On the other hand, McGarrigle & Donaldson (1975) identify in pragmatic skills the reason for non-conserving responses.

Two problems arise from this accounts. The memory, attentional, counting or pragmatic skills are themselves cognitive processes that, like conservation, have to be explained by a response-independent factor, like the operational structures for Piaget. Moreover, the alternative accounts need to explain not only the reasons for failure in conservation, but also the development from non-conservation to conservation. Unfortunately, no precise account has been offered of how memory, attentional, counting or pragmatic skills develop to yield the solution of the traditional task.

Third, the solution of the modified task appears to be itself the outcome of a developmental process, as a substantial proportion of children is reported to fail even this task. A complete account of early competence should also explain the process by which the child arrives at the solution of the modified task.

To summarize, if, following the early competence argument, the concept is attributed to the child from his first success at the modified tasks, three main questions remain to be answered:

1. What is the conceptual basis of the qualitative differences in performance on modified and standard tasks, when the justifications are different, the strategies often heavily reliant on physical and spatial features of the modified task and the solutions do not easily generalize to equivalent problems;
2. What is the nature of the process which underlies both the solution of the modified task and the shift from solving the modified task only to solving both modified and standard tasks;
3. What is the cognitive organization in the period which precedes the solution of the modified task and the process by which the child acquires this ability.

In order to clarify and illustrate these points, I will discuss a case of precocious success in a Piagetian task: the studies of number conservation where the spatial transformation of the collections is introduced as an accident instead of being carried out directly by the experimenter.

4.3 A case of early competence in number conservation: conservation after accidental-incidental transformations

One of the most robust evidence of early competence comes from the modified task of McGarrigle & Donaldson (1975). McGarrigle & Donaldson analyzed the conservation task of number and of length from a pragmatic point of view and examined some of the factors which guide the child's interpretation of the conservation question ("is there more here or more here or are they both the same?"). They observed that a conflict may be brought in by that fact that the experimenter utters a sentence referring to the numerical relations between two collections, and precedes this utterance by an action (the lengthening of one of the arrays) which implicitly refers to the spatial extent of the collections. McGarrigle & Donaldson offered the hypothesis that it is the uncoupling between the adults' linguistic and non-linguistic behaviour (e.g. a state of affairs that goes against normal usage), which misleads the young child to interpret the conservation question as in fact bearing on the spatial size of the collections, and hence leads him to give a non-conservation response based on the comparison of spatial size. As McGarrigle and Donaldson write:

> It could be that the experimenter's simple direct action of changing the length of the row leads the child to infer an intention on the experimenter's part to talk about what he has just been doing. It is as if the experimenter refers behaviourally to length although he continues to talk about number (1975, p.343).

4.3.1 The accidental number conservation task

In order to test the pragmatics hypothesis, McGarrigle & Donaldson modified the way in which the transformation was carried out. They substituted the direct action of the experimenter on the display with an *accidental* transformation, brought about by a 'naughty teddy bear', whose own activity was explicitly directed towards the goal of spoiling the game. Under this accidental transformation condition, since the spatial transformation did not appear to have been produced with calculated intent by the experimenter, the child should not be led to make the inference that the experimenter was thinking of spatial size, while talking about number. McGarrigle & Donaldson's hypothesis was thus that children would have conserved from an earlier age in the accidental condition than in the standard conservation.

At the same time, the modification in the way the transformation was carried out did not seriously affect the structure of the conservation problem. Firstly, the child was introduced to the teddy bear, kept in a box, and was told that "teddy is very naughty and that he was liable to escape from his box from time to time and try to 'mess up the toys' and 'spoil the game' (McGarrigle & Donaldson, p.345, 1975)". The child was then presented either with the standard procedure first or with the modified one first[8].

[8]The 80 children (between 4,2 to 6,3 years of age) are divided into two groups where the order of presentation is counter-balanced. The children receive four tasks for each of the accidental and intentional conditions: conservation of number equality, conservation of number inequality,

He was asked whether two rows of counters of same number, same length and equally spaced, were equinumerous: "is there more here or more here or are they both the same?". In the intentional condition, after the initial judgment, the experimenter said "Now watch...", lengthened one of the rows and repeated the previous comparison question. In the accidental condition, after the initial judgment had been given, the experimenter expressed surprise and worry while he took the teddy bear out of the box and moved it towards the closest row of counters. He said: "it's naughty teddy!" or "Oh! look out, he's going to spoil the game", and moved the teddy over the row to disarrange the counters. At this point the comparison question was repeated. No justifications of the answer were required, because according to McGarrigle & Donaldson:

> the attempt to elicit justifications would have involved the child and E in further complex interaction, the characteristics of which could have influenced the child's subsequent behaviour in a number of ways (p.346, 1975).

4.3.2 The results

Significantly more children gave equivalence conservation responses in the accidental task (54 out of 80, 67.5%) than in the intentional task (33 also out of 80, 41.25%). When all the four tasks (e.g. conservation of equal number, unequal number, equal length, unequal length) were examined, the mean percentage of correct responses was of 71.9% in the accidental task, and of 33.7% in the intentional task. A better performance in the standard condition was found among the children who started with the accidental conservation than among those who started with the standard task (47.5% vs 35%). The number of conservation responses in the accidental condition was lower when children started with the intentional condition (55% vs 80%).

The results indicate a clear décalage between conservation in the accidental and in the intentional condition. More than 60% of the nursery children conserve number when the transformation is carried out accidentally, while only 30% of them conserve number when the transformation is carried out intentionally by the experimenter. These results corroborate McGarrigle & Donaldson's hypothesis that the intentional structure of the task misleads the children to interpret the conservation question as bearing on spatial size rather than number.

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conservation of length equality and conservation of length inequality. Here I shall limit my discussion to the number equality task only.

4.3.3 The replications of McGarrigle & Donaldson's study

4.3.3.1 Exact replications

The results of this experiment have been extensively replicated. Light, Buckingham and Robbins (1979, experiment 1) tested 60 children from 4.9 to 6.5 years of age on tasks of conservation of equal and unequal length in the two conditions, accidental and intentional. The procedure used matched exactly that of McGarrigle & Donaldson's original study. The results are presented in the following table:

		Accidental condition			
		Inequality		Equality	
		C	NC	C	NC
Standard Condition	C	11	0	8	2
	NC	10	39	9	41

Table 4.1: Number of conserving (C) and non-conserving (NC) judgments under the conditions accidental and standard for equal and unequal sets (from Light, Buckingham & Robbins 1979, p.306).

Even though less dramatic, the difference in the number of correct conservation responses in the accidental and intentional conditions is confirmed: around 30% correct judgments in the accidental condition (70% in McGarrigle & Donaldson) versus 17,5% in the standard condition (30% in McGarrigle & Donaldson). When Light et al.'s results are presented as cross-classification of responses, they reveal three main behavioural patterns in the solution of the two task conditions:

1. around 66% of the children do not conserve in either of the task conditions;
2. around 17% conserve in the accidental conservation, and do not conserve in the intentional conservation;
3. around 17% conserve in both accidental and intentional conditions of the conservation task.

Only two children do not conform to any of these behavioural patterns as, in the equal conservation task, they fail the accidental condition and succeed in the intentional condition.

Similar results are reported by Dockrell, Campbell & Neilson (1980), Hargreaves, Molloy & Pratt (1982)[9], S.A. Miller (1982, experiment 3), Parrat-Dayan & Bovet (1982), Neilson, Dockrell & McKechnie (1983), Moore & Frye (1986). The difference between rates of conservation response in the accidental and intentional conditions was confirmed also for the case of arrays of seven elements (Dockrell et al., Parrat-Dayan et al.), whereas McGarrigle & Donaldson had used collections of four elements. The only exception to the generalization of the phenomenon to larger collections is the study by Moore & Frye who report no statistically significant difference between modified and standard conservation for collections of seven items.

4.3.3.2 Modified procedures: incidental transformations

An even stronger facilitation effect was obtained by substituting the accidental transformation with an incidental one, that is, the change was made to look literally accidental, as opposed to being carried out by a teddy bear operated by the experimenter. This further modification was aimed at eliminating the role of the experimenter in the transformation. As Light, Buckingham & Robbins write:

> Since the child's interpretation of the tester's intentions is central to our concerns, it is unfortunate that the 'naughty teddy' device involves some ambiguities in this respect. While the children in the first experiment were willing to 'play the game' by attributing agency to the teddy bear, they clearly also knew that the tester was responsible for both introducing and manipulating it. The term 'accidental' is perhaps a misnomer because the teddy bear was supposedly trying to spoil the game. But the extent to which the child holds separate the intentions of the tester and those of the teddy must remain in doubt. As any parent knows, children at this age have an unnerving tendency to 'step outside' role-playing situations of this kind just when the adult has been drawn in most deeply! (p.307, 1979).

The authors devised a new version of the conservation task in which the task was embedded in a competitive game between two children. The game consisted of placing pasta shells on a grid; the winner was the first who placed all his pasta shells. The first step in the game was to establish two equivalent amounts of pasta shells kept in two containers, one of which had the particularity of being chipped. Once the equivalence was confirmed, the experimenter handed a container to each child, and, at that moment, 'remarked' that one of them had a very sharp chipped edge and that it could

[9]In this experiment the accidental transformation is carried out by a monkey manipulated by a second experimenter. This modification is introduced to eliminate any intervention of the experimenter who puts the question on the material (even through a puppet).

have been dangerous for the child. He thus searched for a new container and produced a larger one, where he poured the pasta shells from the chipped container. The conservation question was then put: whether the amount of pasta shells in the narrow and tall container was the same as the amount in the wide and short container (the level of pasta shells is different in the two containers). Moreover, the conservation question was pragmatically motivated by the fact that the game was fair only if both children had the same amount of pasta shells.

The incidental transformation paradigm led to even earlier conservations than the McGarrigle and Donaldson's procedure. Light et al. reported that 70% of the children conserved in the incidental condition versus 5% in the traditional condition[10]. Bovet, Parrat-Dayan & Deshusses-Addor (1981) followed the same procedure as Light et al. and, once the game had ended, put the candies back in the two containers and asked the conservation question again. Their results[11] confirmed the findings of the previous experiment and are summarized in the following table:

Standard \ Incidental	C	NC
C	8	0
NC	18	6

Table 4.2: Number of conserving (C) and non-conserving (NC) judgments under the two task conditions standard and incidental (from Bovet et al. 1981, p. 293).

Here, too, we observe the three behavioural patterns of a) nonconservation in both conditions (18.75%), b) conservation in the incidental condition and non-conservation in the standard condition (56.25%), and conservation in both task conditions (25%). No child conserves in the standard task and fails to conserve in the incidental task.

Two experiments (Hargreaves, Molloy & Pratt 1982 and S.A. Miller 1982) indicate that the incidental transformation format leads to significantly more conservations responses than the accidental format (e.g. 73% vs 93% in the latter experiment).

[10]The subjects of the experiment were 80 children between 5,7 and 6,7 years of age.
[11]The subjects were 32 children between 4,7 and 6,11 years of age.

4.3.3.3 Modified procedures: request of justifications

Parrat-Dayan & Bovet (1982) and Neilson, Dockrell & McKechnie (1983) addressed the question of whether accidental conservations constitute evidence of operational reasoning, as the *masking competence* hypothesis of McGarrigle & Donaldson suggests. They examined the justifications given by children to conservation in accidental transformation tasks to see if they conformed to the typical operational justifications which accompanied the solution of the classical task:

a) reversibility: the spatial transformation is irrelevant and could be undone, to go back to the initial configuration;

b) compensation: one row is longer, but its elements are more spaced, while the other row is shorter, but its elements are closer together;

c) identity: nothing has been added nor taken away, they are still the same collections as before.

Neilson et al. examined 128 children between age 4,2 and 6,9 years and reported 80% correct conservation responses in the accidental condition against 34% in the traditional condition, thus confirming the precocious conservation. When however, the responses were classified into conserving on the basis of the two criteria a) 'same number' response plus b) operational justifications, the percentage of conservation responses decreased to 25% in the accidental condition. Hence, if children are considered to conserve number only if they say that the number is still the same after the transformation and justify this using operational arguments, conservation performance in accidental and intentional conditions is equivalent. Furthermore the type of justification given tended to vary depending on the experimental condition: in the accidental task, 75% were of the reversibility type; in the intentional task 30% were of the reversibility type and 54% of the identity type.

Parrat-Dayan & Bovet observed the same phenomenon: of 39 children (between 4,8 to 6,5 years of age) 23 conserved number in the accidental condition (59%), 13 could justify their conservation response, using mainly arguments which made reference to the initial configuration (e.g. reversibility type). Often the justification took a practical form, in that the children put the elements disarranged by the doll back in one-to-one correspondence. Parrat-Dayan & Bovet considered that most of the justifications given corresponded in fact to a more primitive form of reversibility: the simple return to the point of origin, what Piaget called *empirical return* or *renversabilité* (translated as revertibility)[12]:

> il semblerait donc que le jouet ait le rôle d'activer le schème de correspondence terme à terme permettant l'affirmation de l'égalité par un raisonnement de renversibilité (p.243, 1982).

The authors concluded that the type of justification given by children to conservations in the accidental format did not support the claim that children's superior performance in the accidental condition reflects the same operational competence of the standard condition.

4.3.4 Discussion

This case of precocious success in number conservation provides an ideal context in which to evaluate the argument of early competence. Recall that precocious success in tasks which maintain the structure of the original Piagetian tasks was interpreted as conclusive evidence that a child had the basic numerical competence earlier than expected by Piaget. In Section 4.2, I pointed out three problems for the early competence argument: (1) qualitatively different functioning underlies the solutions of the modified task and of the standard task; (2) developmental theory still has to specify the processes underlying the solution of the modified task and underlying the shift to solving both tasks has to be specified; (3) the theory must also account for cognitive organization before the child solves a modified task itself as well as the process by which the child acquires the ability to solve it. Consider these three issues with respect to the accidental-incidental paradigm.

Few children justify accidental conservation by operational argument though many use such arguments on intentional tasks. The differences in the kind of justifications given are so pronounced that if the stricter scoring criterion of correct response plus operational justifications is adopted, the performance in the two tasks is not significantly different any more. The accidental task may do more than simply revealing the competence masked by the traditional task: it may show that such a task is, in effect, a different problem involving inferences of different nature.

Now let us turn to the suggestion that the capacity to solve the modified conservation task appears to be itself the outcome of a developmental process. In both the accidental

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[12] "It would hence appear that the toy plays the role of activating the one-to-one correspondence schema, making it possible to affirm the equivalence through reasoning by revertibility" (my translation).

condition and the incidental condition, a significant proportion of children did not conserve in either of the task conditions (68.3% in Light et al., 18.75% in Bovet et al.). Hence precocious success cannot be interpreted as the simple expression of a basic primitive competence. Either the modified task itself masks this primitive competence or it identifies a competence level which results from the development of prior incompetence.

Finally, let us consider the developmental change from the solution of the modified task alone to the solution of the standard task as well. The process by which the children would overcome their misinterpretation of the standard task remains unspecified. McGarrigle & Donaldson write:

> In the early stages of language acquisition, the child interprets the meaning of behaviour events to arrive at a notion of speaker's meaning and this knowledge is utilized to make sense of the language around him. Eventually the child acquires a semblance of linguistic meaning, in that he can respect certain properties of the language where the non-linguistic components of the speaker's activities do not conflict with utterance. During this phase the intentional nature of the speaker's activities, where this is at variance with the utterance, can govern what the child thinks is being talked about, so that his understanding of such concepts as number and length can be obscured (p.347, 1975).

And they conclude:

> It is possible that the achievements of the concrete operational stage are as much a reflection of the child's increasing independence from features of the interactional setting as they are evidence of the development of logical competence (p.349, 1975).

The processes through which independence from the interactional setting may be achieved remain unclear. Moreover, if we accept the view that the acquisition of general pragmatic rules underlies the solution of the traditional conservation task, how can we account for one of the most robust results in conservation studies, i.e. the décalage of two years separating the acquisition of the conservation of substance, of weight, of volume? If the solution of the first substance conservation task is explained by the acquired independence from the interactional setting, how could one explain the concurrent failure in the structurally equivalent weight conservation task? A first answer to this question is offered by Dockrell & al. who write:

> In conclusion, the existence of the phenomena of uncoupling is not being challenged. However, the assertion that the transition from pre-operational to operational thought is "as much a reflection of the child's increasing independence from the features of the interactional setting as they are evidence of a logical competence" is being questioned. It seems to us that behavioural interpretation of setting is not a variable that functions autonomously like counting or reading but rather pervades all communicative interactions and

> reflects the status of other aspects of the communicator's knowledge. It may be seen to override linguistic analysis of an utterance where the child has trouble understanding the linguistic terms involved or where he is unsure about a logical-cognitive judgment or about the precise demands of the task or even when he is unfamiliar with the material at hand. But likewise it itself can be dominated by other aspects of the child's knowledge, such as that of word meaning in alternative communicative contexts. What is required at this point is a clarification of the interaction between the child's use of behavioural strategies and his complete or partial understanding of the logical requirements of the conservation task (p.438-439, 1980).

In other words, faced with the conservation task, the child may be at loss in interpreting what the situation is about and thus finds the key to understanding the standard conservation task in the experimenter's action (e.g. if the experimenter changes configuration, than it is the comparison of spatial size that is at stake), but unfortunately for him the key is the wrong one. Later in development, he has the conceptual means to understand the conservation task autonomously and can ignore the length dimension highlighted by the experimenter's action. This same transition may occur within each of the tasks of conservation of number, substance, liquid, weight, volume, etc. at different moments of development, depending on the degree of elaboration of the concept involved. In other words, as long as the child has not elaborated a sufficiently articulated and general concept of substance, liquid, weight, he cannot make sense of the conservation question and resorts to the interactional setting to guide his interpretation of the problem. As these concepts are gradually acquired, the child has a clearer appreciation of the effects of transforming the objects' shapes and becomes more independent from the interactional cues provided by the experimenter's actions. Under this interpretation, although the pragmatic perspective sheds some light on the failure in the conservation task, it neither explains it nor says anything about the process leading to conservation. The pragmatic process appears to be an event associated with, but not the cause of failure to conserve. The burden of the explanation hence remains with conceptual organization and change.

The advantage of interpreting precocious success as reflecting a level of competence in its own right is that it makes possible to give a unified account of the three issues discussed here:

1. the qualitative differences between the solution of the modified and traditional conservation tasks express different underlying concepts and different ways in which the task is understood and solved;

2. the failure in the modified task indicates that there is a stage in which the child has not elaborated the concept sufficiently to deal even with the modified task, although he may well operate adequately with number in other circumstances;

3. the simultaneous success in the modified task and failure in the standard task are index of a stage of conceptual elaboration, sufficiently articulated to deal with the modified task, but not enough to deal with the standard conservation task.

If we posit the hypothesis that the child's cardinal number concept develops through a sequence of competence levels, corresponding to different forms of equinumerosity and conservation of increasing coherence and generality, both accidental and standard conservations can be seen as expressions of the understanding of properties of equinumerosity, although of different complexity. These levels of understanding can be spelt out by testing the same children on new tasks and by determining the extension, articulation and coherence of their number concepts beside accidental, incidental and intentional conservation. From this perspective, neither the complete understanding of the conservation principle à la Piaget nor the innate understanding of the conservation principle à la Mehler & Bever, Gelman and possibly McGarrigle & Donaldson would make much sense. The notion would always be understood relative to a problem space and to a stage in the development of the cardinal number concept.

To summarize, the Piagetian interpretation in terms of operational structures is too general and cannot discriminate between the accidental-incidental where undoubtedly the child goes beyond the difference in spatial extent between the two collections, and the traditional forms of conservation. The early competence interpretation is also too general and does not provide an adequate analysis of the child's initial failure on the modified task and of the evolution of success on the traditional conservation task. I have advanced the alternative view that precocious success reflect a level of numerical competence which is itself the outcome of a developmental process and the basis for the subsequent development of conservation in the traditional task. In the next section, I propose a way of envisaging the different levels of numerical competence.

4.3.5 The domain-specific account of early competence

The theoretical approach presented in the previous chapters provides a framework in which to specify the conceptual organization underlying the different competence levels and to capture the transition from less to more advanced organizations. According to this theoretical perspective, the child is endowed with biologically determined structures specialized in processing specific kinds of information. In the course of development, the child learns to discriminate the contexts in which the representations produced by the domain-specific structures are relevant, that is, they allow him to derive regularities and relations useful for pursuing his actions, for predicting events and for achieving goals. Development proceeds in a stage-like manner, with subsequent stages extending the range of application of the structure to more complex contents in the form of the class of objects assimilated by each new domain-structure.

When the early competence issue is examined from this angle, the modified and traditional tasks do indeed track the same concept, but operating on different objects. The child may thus work out the relevance of the number structure for the accidental-incidental situations, and derive from it the fact that the two collections are equinumerous, and at the same time may not see the relevance of the number structure in the traditional task, and so fail to discover the general conservation principle. The problem thus becomes that of defining in which sense the requirements of the accidental-incidental format and the traditional format are different, of determining why the child who can establish equinumerosity in the accidental-accidental format fails to apply the specialized number structure and discover equinumerosity also in the traditional task.

One interesting difference between the two tasks is that, when the conservation question is posed after the spatial transformation has occurred as an accident, the question can be seen as inviting a natural check for an accidental disruption of the two collections' numerosity. The child may thus answer the question by comparing the two post-transformation collections either via counting, matching or dimensional composition. The reports of children who put back the objects in one to one correspondence support this interpretation, as do the justifications by revertibility. Alternatively, the accidental transformation may be simply canceled, as a nuisance to the progress in the game, an interpretation much favoured by Bovet et al. In either case, the accidental-incidental tasks demand application of number structure to the pair of sets visible after the accident.

We have seen however that the same child is very likely not to confirm that the two collections are equinumerous when the transformation is performed by the experimenter. In this situation, he fails to see the relevance of the number structure application and cannot establish the equinumerosity. I wish to suggest that the transformation, because it is intentionally performed by the experimenter does not encourage the child to see whether equinumerosity has been upset. Rather than focusing the child's attention on the current pair of sets, the task invites the child to decide whether two collections are equinumerous on the basis of three facts: the initial equinumerosity of the two collections in one-to-one correspondence, the rearrangement of one of the collections gradually performed by the experimenter and the resulting pair of collections, very different in distribution. While in the accidental task, the child has to establish whether the two sets are still equinumerous after the accident, by applying the number structure to that pair of sets, in the standard task, the child has to establish whether a property of a pair of sets, i.e. their being equinumerous, is maintained after the transformation. This corresponds to a more complex application of the number structure: on a pair of pairs of sets (e.g. the initial pair and the post-transformation pair). The décalage between the two tasks can be thus explained in terms of the complexity of the objects over which the child has to abstract the number structure: pairs of sets in the accidental-incidental task and pairs of pairs of sets in the standard task.

To recapitulate, the décalage is due to the fact that, at the level of precocious conservations, the child has worked out the relevance of the number structure for pairs of sets and can determine whether two sets are equinumerous or not. This number concept however cannot assimilate situations, like the traditional task, which involve pairs of pairs of sets. The child does not see this task as bearing on number, and being at loss uses the experimenter's action in lengthening a row as index of what the task is about. He thus bases his judgment on spatial extent and abandons the initial equinumerosity.

And what about the period in which the children also fail the modified task? Here the existing studies tell us very little about such children's numerical abilities. Without this information, I can only extend this account and hypothesize that these children's number concept does not even apply to pairs of sets. For them, the modified conservation task involves a content which is too complex for a child who has not discovered the relevance of the number domain structure on pairs of sets. In the case of modified conservation tasks, as in the traditional, the child who fails bases his judgment on length.

We have been able, however tentatively, to characterize the competence underlying success first in the modified and later in the standard task. My hypotheses about competence levels can be tested by determining whether other tasks with equivalent requirements are solved concurrently either with the modified or the standard tasks.

4.4 Conclusions

The data identifying precocious success in Piagetian tasks pose a difficult problem of interpretation. If we attribute full competence to children who can solve the modified task, then we risk the positive error of attributing the concept to a child who in effect does not have it, as failure in the standard tasks would suggest. If on the other hand we attribute full competence only from the solution of the standard task, then we risk the negative error of not attributing the concept when the child has it, as the precocious success would suggest. A way out of this dilemma is to suppose that competence is relevant to the level of problem-solving ability. The child does not have the full concept either when he is capable of solving the modified task or when he solves the traditional task. He goes instead through different levels of conceptual elaboration of increasing complexity and generality. New theoretical and empirical questions arise, as regards experimentally describing the different competence levels and of characterising the underlying organizations. The next chapter is concerned with the methodology for a systematic investigation of competence levels and for evaluating accounts of conceptual organization and change.

Chapter 5 A research method for domain-specific cognitive development

5.1 Introduction

This chapter deals with the nature of the research and with the appropriate methodology for the study of within-domains development. The theoretical framework proposed in the preceding chapters addresses two basic empirical questions:

- whether the development of a conceptual domain proceeds according to a fixed sequence of steps corresponding to levels of increasing competence;
- whether the transition phases are characterized by typical behavioural patterns which reflect the sequence of cognitive states postulated by Richards' model.

The ideal strategy to explore these questions would require a longitudinal study in which the same children are examined on a set of tasks, tracking a conceptual domain, over a period of a few years at constant intervals. However this method presents both methodological (e.g. the effect of repeated testing on same problems) and above all practical shortcomings (e.g. the time scale, the children who drop out of the sample, etc.). The second best strategy (Wohlwill 1973) relies on cross-sectional studies in which children of different ages are examined on a same set of tasks. The cross-sectional studies bring out the response patterns characteristic of children in development.

The particular strategy that I suggest consists of administering a battery of tasks, each probing a different aspect of the concept investigated, to a same sample of children from different age groups. Response patterns across age and across tasks are identified by performing a hierarchical analysis of the order in which the tasks are solved: we need to know which tasks are solved *concurrently* (e.g. the children who fail one task fail also a second; children who succeed one task also succeed a second), which tasks are solved with a systematic, *collective décalage* (e.g. the children who solve one task still fail a second), and also which tasks are solved with an *individual décalage* (e.g. some children solve one task and fail the second, while other children fail the former task and solve the latter).

Hierarchical analysis captures the paths that children follow along these steps in the development of the domain. Concurrency successes represent a level of problem-solving ability, or stage, while collective décalage identifies differences in problem-solving ability and reveal the order between stages. The patterns of individual décalage are an index of the different paths children follow in reaching a stage.

The same analysis permits us to test hypotheses about the competence underlying the different stages and about the transition process. Hypotheses about stage competence are tested by devising new tasks which should require the conceptual organization postulated and by determining whether these tasks are solved concurrently with the other tasks typical of the stage. The study of transition, on the other hand, requires qualitative information about the procedures which children use to solve problems, to check and eventually correct their views; about the effect of countersuggestions, about the type of justifications, etc. All these characterize collective décalage. The adequacy of Richards' model (e.g. three ordered cognitive states) as an account of transition can be evaluated against these qualitative observations.

The chapter is divided into three parts. The first part is concerned with the argument for adopting batteries of tasks to assess competence and with the criteria for constructing batteries of tasks. The second section introduces the experimental designs and the statistical tools needed for the hierarchical analysis between task solutions. In the third section, I set out the plan for the analysis of the development of cardinal number, the domain which is investigated in the remaining chapters.

5.2 The study of conceptual development with batteries of tasks

In chapter 4, I argued for an account of precocious success in terms of levels of competence and have suggested that to investigate the nature of these conceptual organizations it is necessary to identify new situations where the same early competence may be expressed. In this section I discuss this research strategy in a more systematic way. First, I present the argument for assessing the children's competence using several tasks as opposed to one or two. I then deal with more practical issues such as the construction of a battery of related tasks and the selection of appropriate tasks to obtain information about the children's problem solving capacities and about their strategies and attitudes.

5.2.1 The argument for using battery of tasks

One of the central methodological issues in cognitive developmental studies concerns the criteria for attributing understanding of a concept to the child. In the Piagetian studies, and even more so in the replications, the assessment of the concepts is accomplished using critical tasks which discriminate between the partial, preoperational concepts and the fully operational ones (see Chapter 4, for a more complete discussion). Two significant and apparently unsolvable puzzles emerge from this strategy: the interpretation of precocious success in modified critical tasks, discussed in chapter 4, and the response criteria in conservation studies (Brainerd 1973, 1974, 1977; Reese & Schack 1974).

Consider the precocious success puzzle first. Two tasks T_1 and T_2 are taken to probe the same concept: they share the same structure, but differ in their mode of presentation (e.g. in the conservation task, for instance in T_1 the transformation occurs as an accident, in T_2 it is carried out intentionally). It is found that T_1 is solved before T_2. The solution of T_1 can be interpreted as evidence that the child has the concept, and that parasite, incidental performance factors stop him from expressing the same competence in T_2. Alternatively the earlier solution of T_1 can be interpreted as an artifact of the modifications introduced to the T_2 task, which alone is the proof that the child has the full concept. However, both these interpretations are prone to two kinds of errors: the false positive error of attributing the competence from the solution of T_1, when in fact the child does not have the concept; and the false negative error of doing the opposite, and of attributing the competence only if T_2 is solved, when in fact the child has the competence already when he solves T_1. In other words, the parasite performance factors, rather than competence, may guide the correct response in T_1 as much as they may lead to the incorrect response in T_2, and mask the underlying competence.

Similar problems have emerged when more qualitative observations, such as justifications, response to countersuggestions, etc. have been used to assess the child's concepts, alongside the success and failure in the task. In a conservation task, for instance, Piaget attributes understanding of conservation when the child confirms that the two quantities are still equivalent after the transformation and gives operational justifications for the equivalence. Brainerd (1973) criticizes this criterion on the ground that the ability to express the reasons of the conservation judgment verbally is not a requirement for understanding the concept and is therefore irrelevant to the issue of assessing conservation. The traditional Piagetian criterion for conservation on the basis of "same" answers plus operational justifications thus risks producing false negative errors. At the same time, however, Brainerd remarks that the criterion of attributing understanding of conservation on the basis of "same" answers alone introduces the risk of false positives, i.e. the child in fact does not understand conservation. Brainerd concludes that although one risks false positive errors in using only judgment, this error can be eliminated by proper procedural safeguards, which, however, he leaves largely unspecified. The false negative errors instead cannot be overcome.

I argue that neither puzzle can be solved by looking at the performance on single tasks, but that instead the assessment of the child's understanding has to be based on the child's performance on several related tasks, which embed the concept in diverse contexts, put different constraints, elicit familiar knowledge, etc. The risk of negative-positive errors in the attribution of a concept is substantially reduced if the success-failure in one task, here the accidental conservation, is then compared with the performance in some other equivalent tasks, here Gréco's counted conservation and set comparison tasks. In other words, the children's solution of specific tasks appears to be dependent on so many factors built into the task , that it is hopeless to think that one test can be critical in distinguishing children who have a concept from children who do not. The response pattern across tasks constitutes a more robust basis from which to draw inferences and formulate hypotheses about the child's competence.

This line of argument can be pushed even further to claim that the question of whether a child has a given concept or does not have it is quasi meaningless and certainly misleading. In a sense in fact, the child always has some understanding of a concept, as his ability to cope with a variety of situations in experimental settings and especially in real life indicates. This does not mean however that his understanding does not change and increase in the course of development. Rather than asking whether the child has a concept, the questions of what the nature of the concept is that he entertains, of how his concept is different from the adult's or from a younger child's should be addressed. In Piaget, this distinction is expressed in the qualification of a concept as either being pre-operational, concrete or formal operational. However, since Piaget defines pre-operational concepts essentially by via negativa, and not as achievements in themselves, Piaget also presents a dichotomy between no concept and full-blown, adult-like concept.

Moreover, the nature of the child's concept at different periods of his development and the difference between its earlier and later forms acquire new meaning in the light of such phenomena as early competence, heterogeneity, décalage, content-dependency of adults' logical reasoning and such models as domain-specific accounts which posit a sequence of levels of conceptual organization.

The advantage of testing children with a battery of tasks, all probing the same basic concept put in different contexts, is that we can discriminate between competence levels on the basis of the tasks the child solves, i.e. where he uses the concept correctly, and the tasks the child fails, i.e. where he does not apply the concept. In particular, in chapter 4 I have proposed a reinterpretation of the precocious success data in terms of expressions of levels of elaboration of the concept. The solution of T_1 is hence explained in relation to an underlying organization of the concept of some internal complexity and coherence. This concept however appears to be still too limited to solve T2. The subsequent solution of T2 is then explained in relation to a more advanced conceptual organization.

From this perspective, the main empirical objective becomes that of providing a detailed description of the different levels of conceptual elaboration and more particularly to identify what else the child knows when he solves the modified task in the lower stage and what else he knows when he solves the standard task in the higher stage. Using a battery of tasks tracking a same concept in different contexts, we can retrace the response patterns across tasks and determine which other tasks the children of the lower stage can solve, and do the same thing for the higher stage children.

To illustrate how this strategy may work, consider a fictional battery of eight tasks, all dealing with cardinal number in terms of set reproductions, comparison, and conservation. Let us represent the tasks as: T1, T2, T3, T4, T5, T6, T7, T8. T1 is the accidental conservation, T2 is the standard conservation. In order to assess whether the child who solves T1 has the full number concept (e.g. the early competence argument), whether the child who solves T2 has the full number concept (e.g. the Piagetian argument) or whether both the child who solves T1 and fails T2 and the child who succeeds both tasks have a coherent number concept, which however differs in extension and internal complexity (e.g. the interpretation I advance), we can see which other number tasks the children are capable of solving.

We may find out that T1, T6, T7 are solved concurrently, while the children who solve T2 also solve T3, T4. T5 and T8 instead are failed both by the children who succeed in T1 and by those who succeed in T2. These response patterns are represented schematically as levels of a developmental sequence in the following diagram:

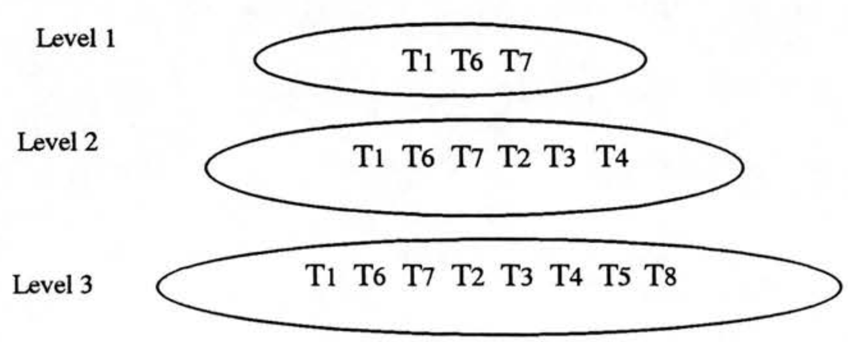

Fig 5.1: Three ordered levels of problem-solving ability

The analysis of the order in which the eight tasks are solved would reveal three ordered levels of problem-solving ability. Both the Piagetian and the early competence interpretations would have some difficulty in accounting for these data. The mature, operational concept postulated by Piaget fails to apply to the tasks T5 and T8, and some even more advanced concept (e.g. formal operational?) would have to be invoked. The primitive early competence instead reveals itself in T6 and T7, as well as T1, an outcome which would tend to corroborate the interpretation of precocious understanding. However, this interpretation should then account for the two layers of masked competence, that is what stops the primitive competence to be expressed in the solution of T3 and T4 first, and later in T5 and T8.

The most parsimonious account of this fictional developmental pattern is provided by the domain-specific interpretation. Throughout the three levels, the children demonstrate understanding of cardinal number, although this understanding becomes gradually more general as a wider range of tasks is solved. Between step 1 and 2, and step 2 step 3, the framework would claim that the child discovers the relevance of the representations produced by the structure specialized to process numerical information for new classes of situations. The reason for the décalage would then lie in the fact that the objects to which the specialized number structure applies are more complex at each successive step.

The experimental and statistical methods for investigating the response pattern across tasks will be dealt with in the next section. Before that, I shall briefly introduce some general principles regarding the construction and the selection of the tasks appropriate for carrying out this kind of analysis.

5.2.2 The construction of the battery of tasks

The construction of a battery of tasks sets three basic practical and theoretical questions: devising tasks which embed the concept investigated, providing a varied set of such tasks and choosing tasks which provide us with both information about the problem-solving skill of the child and about more qualitative aspects of the solution such as the strategies used, the capacity to correct the errors, to justify a solution, etc. The Piagetian research method provides us with some part of the answers.

5.2.2.1 The analysis of the conceptual domain

In the Piagetian methodology, the first step in approaching the development of a concept is the definition of the structure of the concept. The definition is articulated from the advanced scientific and epistemological analyses of the concept. This analysis brings out the entities and relations at the basis of the conceptual domain and to devise practical tasks which deal directly with these basic elements of the concept. More precisely, the solution of the tasks requires that the child establish these entities and relations in the context of a concrete situation and that he carry out operations and inferences on them to produce the correct solution. Consider, for instance, the case of the domain of cardinal number.

Piaget refers to Cantor's definition of cardinality: if A and B are two sets such that there exists a (1-1)-correspondence between the elements of A and the elements of B, than we say that A and B have the same cardinal number. Piaget's study of the development of cardinality will then focus on the children's capacity to establish the (1-1)-correspondence between the elements of two sets and to draw inferences about their equinumerosity or difference. To study these capacities, Piaget devises two related tasks. The first task is a simple set reproduction problem. The child is asked to take out of a group the same number of objects as there are in a array which is placed in front of him. The second task is the conservation of number, which I have already discussed in the previous chapters. One of the two equinumerous rows, constructed in the reproduction task, is changed into a longer row. The child is asked whether the two rows still have the same number of elements or whether one of them has more elements.

By means of these two tasks thus Piaget embeds the abstract relation of (1-1)-correspondence at the basis of cardinal number into two practical situations[13]. These tasks become the instrument for assessing the development of the understanding of cardinality in the child. Notice however that as Piaget considers the conservation task the ultimate test of the mature, operational number concept, this task has acquired the role of critical test, and we have been confronted with the puzzles associated with attributing competence on the basis of single tasks.

5.2.2.2 The variations of task format

The same Piagetian strategy can be extended to construct several tasks which deal with the entities and relations defined, but in varied situations, such as the practical

[13]By focusing directly on the conceptual basis of number, Piaget's studies constitute a radical innovation in the field of number development research which, before Piaget, was essentially concerned with dressing a catalogue of the numerical skills of the child (e.g. enumeration, counting, elementary arithmetical operations). The impact of the conservation paradigm is reexamined in the next chapter.

context in which they are embedded, the mode of response, (e.g. action, verbal response, drawing), the type of task (e.g. reproduction, comparison, anticipation).

Again Piagetian research, this time that concerned with the sensori-motor period, offers a model of how equivalent variations can be devised. Piaget has studied the genesis of the object concept using a number of variants of the basic object permanence task (see chapter 3). This task consists of hiding an object which the infant is interested in, under a cover within his reach and of observing whether the infant attempts to or succeeds in retrieving the object from under the cover. The diverse variations used by Piaget consist of:

a) hiding the object under a second cover, beside the one where the object disappeared in previous trials;
b) putting a second cover over the one where the object has disappeared;
c) switching round the two covers, one of which hides the object;
d) hiding the object under another cover in the proximal space, without the child seeing the object's transfer, to determine whether he can reconstruct possible displacements of objects in the space around him.

Using such a battery of tasks, Piaget identifies six substages in the development of the concept. The ordered substages reflect the infants' capacity to retrieve the object in situations involving displacements of increasing complexity. This same level of detail however is not achieved in Piaget's analysis of the genesis of concrete and formal operational concepts. Generally Piaget did not introduce systematic variations in the tasks' formats, and only modified the material used or the mode of response. With this reduced range of task types, Piaget typically identifies three substages in the operational period, with eventually some intermediate levels to classify particular behavioural patterns. The three substages correspond very schematically to the failure in the task, the oscillation between success and failure and the correct solution of the task, with appropriate justifications. These descriptions do not approach, either in precision or in detail, the one Piaget gave of object concept development[14]. The latter

[14]The reason of that appears to be historical. When Piaget studied the sensori-motor period (around 1937), his theory of the operational stages was not clearly spelt out yet. The first extended formulation of the operational theory is usually identified with the article "Le mécanisme du développement mental et les lois du groupement des operations. Esquisse d'une théorie opératoire de l'intelligence" published in 1941 in Archives de Psychologie 112, 215-285 (translation of the title "the mechanism of the development of the mind and the laws of the grouping of operations. Outline of an operational theory of intelligence"). The research on the sensori-motor stage thus respond to an essentially descriptive aim, while the studies of the operational period are focused on a specific property of

remains one of the best models of the accuracy with which an analysis of conceptual development might be carried out.

5.2.2.3 The type of tasks

The high replicability, the richness of responses enhanced and the variety in the behaviours described demonstrate the well-founded of Piaget's assumption that conceptual development is best captured in problem-solving situations that expose the children's active search of solutions to practical tasks, the strategies they adopt, the controls of the adequacy of the solution and the eventual corrections[15].

The main advantage of concrete tasks is that they reduce the degrees of freedom and set the framework for a rich interaction between the child, the specific problem space and the experimenter. From these interactions, the observers can obtain different sorts of information about:

1. the child's capacity to solve the problem and use the concept in the particular situation;
2. the physical and logical properties of these situation;
3. the procedures invoked to arrive at the solution;
4. the reasons offered for favouring this procedure;
5. the checks the child may carry out and the eventual corrections;
6. about the arguments the child can give against countersuggestions.

As the context set by the task remains fixed, and children from different age groups interact with that context, developmental change can be observed at each of levels listed above. The analysis of the particular requirements on concept application set by the tasks solved and failed at a stage permits to formulate hypotheses about the nature

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cognitive functioning, i.e. reversibility, and overlook the complexity of the pre-operational period.

[15]See the forward to Piaget & Szeminska's 'The child's conception of number':"In dealing with these new problems (development of operations which give rise to number and continuous quantities, to space, time and speed) appropriate methods must be used. We shall still keep with our original procedure of free conversation with the child, conversation which is governed by the questions put, but which is compelled to follow the direction indicated by the child's spontaneous answers. Our investigation of sensori-motor intelligence has, however, shown us the necessity for actual manipulation of objects. In The child's conception of physical causality, we saw, though it was not possible to take full advantage of the fact, that conversation with the child is much more reliable and more fruitful when it is related to experiments made with adequate material, and when the child instead of thinking in the void, is talking about actions he has just performed" (p.VII, 1952).

of the concept characteristic of that stage. The qualitative information about the checks that the children carry out on their solutions, the corrections they attempt and the justifications they give provide us with elements for characterizing their global cognitive attitude towards the task. Whether, for instance, the solution they propose is the only one they can envisage, and if inaccurate, cannot be corrected. Or whether, they are not very confident of the solution they propose, in which case the checks and corrections are most revealing. The first case would match the characterization of the lower stage, given by Richards' model in terms of irrelevant state of mind. The second case would match the characterization of the intermediate period in terms of a paradoxical state of mind.

5.3 The statistical methods and the experimental design

Once the battery of tasks has been constructed, a number of decisions have to be made regarding how to detect and analyse the change in the children's performance in the tasks. In general, the performance of children from different age groups has been compared. For instance, given two tasks T_1 and T_2, if the number of correct responses to T_2 is significantly greater among the older than among the younger children, while performance in T_1 does not differ significantly in the two age groups, developmental change has been reported. This change has been interpreted as evidence that the solution of T_1 is a temporal antecedent of the solution of T_2, and so is the developmental relationship between the acquisition of the concepts or skills (e.g. depending on the interpretation given) required to solve the two tasks.

Although the age dimension is important in the study of developmental processes, a more specific dimension of change can be identified by examining the performance of the same children in the series of tasks, within-subjects, rather than comparing groups of children, between-subjects. Cross-tasks comparisons reveal patterns of concurrency and order. Schematically the situation can be represented using a two-by-two contingency table, indicating the frequency of successes and failures for any pair of tasks.

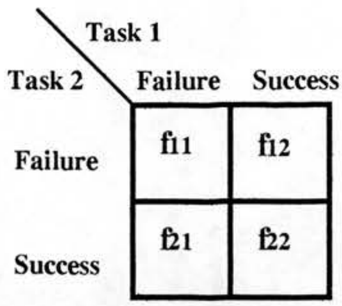

Fig. 5.2: Developmental contingency table

A pattern of concurrency emerges when the frequency in the two cells representing the combination of success in T1 and failure in T2 (e.g. f12), and success in T2 and failure in T1 (e.g. f21) approaches zero, within the limits of reliability of the response measures involved.

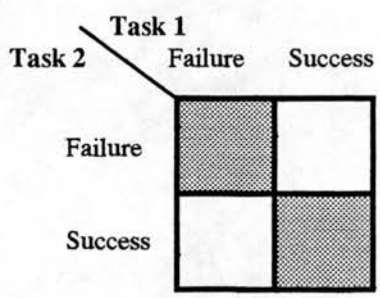

Fig. 5.3: The response pattern of concurrency (white cells are empty)

This response pattern is interpreted as evidence of two stages in the development of the concept: a lower stage in which the children fail both tasks and a higher stage in which children solve both tasks.

A pattern of collective décalage emerges when the frequency in one of the cells which represent the combination of success and failure approaches zero. The décalage would be in favour of T1 when the empty cell corresponds to success in T2 and failure in T1, i.e. only children child who succeed T1 also succeed T2, while no child who fails T1 succeed in T2. It would be in favour of T2, when the empty cell represents failure in T2 and success in T1.

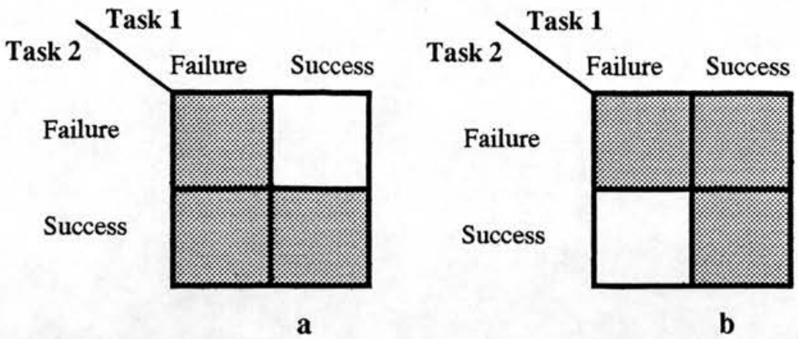

Fig. 5.4: Two examples of the response pattern of collective décalage: a) of T2 precedes T1, b) of T1 precedes T2

These response patterns are interpreted as evidence of three stages in the development of the concept. The first corresponds to the failure in both tasks, the intermediate to success in one task only, the third stage to success in both tasks.

Finally, a pattern of individual décalage emerges when no cell is empty. The children's responses distribute uniformly in the four cells, as some children solve both tasks, other children fail both tasks, and for some children T1 is solved and T2 failed while for the other children T1 is failed and T2 solved.

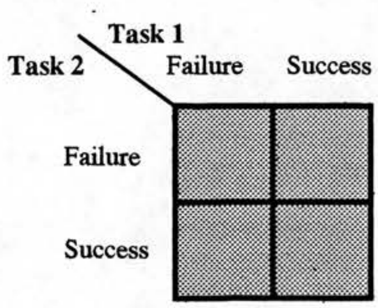

Fig. 5.5: The response pattern of individual décalage (no empty cells)

This last pattern is interpreted as indication of different paths that the children follow when developing from the lower stage, i.e. failure in both tasks, to the higher stage, i.e. success in both tasks. Some children elaborate the competence necessary to solve T1, but not adequate to solve T2, while other children do the opposite. This also suggests that the two tasks track forms of understanding which develop independently and which may come together when the higher stage is reached.

By systematically detecting these response patterns for each pair of tasks of the battery, we can obtain a detailed description of the sequence of levels of problem-solving ability and of the paths taken by children to move between levels. Consider for instance the case of six tasks dealing with different aspects of a conceptual domain. The analysis of the across tasks response patterns may result in the diagram below:

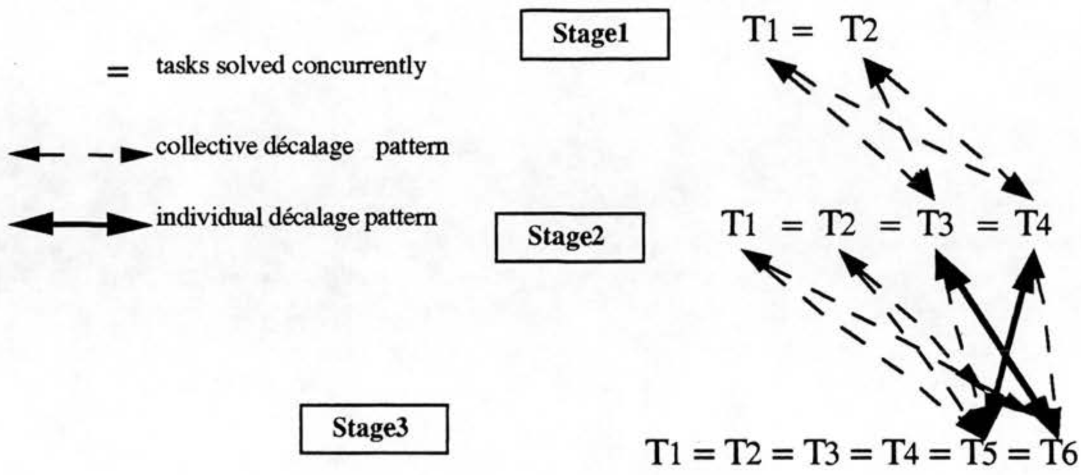

Fig 5.6: The hierarchical analysis of the solutions to tasks T1 to T6

The stages are identified by the tasks which are correctly solved at the same time. Stage 1 corresponds to the correct solution of T1 and T2; Stage 2 to the solution of the two previous tasks plus T3 and T4; Stage 4 to the solution of the four previous tasks plus T5 and T6. The order between stages is reflected in the patterns of collective décalage existing between the solution of particular tasks. Stage 1 children solve T1 and T2, but fail T3 and T4; Stage 2 children perform successfully in T1, T2, T3 and T4, but fail T5 and T6.

These patterns identify the situations which are critical for the children of a lower stage (e.g. the requirements of T3 and T4 are too complex for Stage 1 children) and which are solved at the higher stage to demonstrate the new conceptual advance. The transition from Stage 2 and Stage 3 instead goes through two different paths, as indicated by the individual décalage between T3 and T6, and between T4 and T5. At Stage 2, children fail tasks T5 and T6, which are instead solved at Stage 3. However some children reach Stage 3 by elaborating first the competence necessary to solve T5 and then generalize this competence to solve T6. Other children reach Stage 3 by the opposite path. First they acquire the competence to solve T6 and later generalize it to include the case of T5.

The stages are interpreted as expressions of competence levels in the conceptual domain. The décalages distinguish between less and more advanced competence levels, which are, according to the theoretical framework, abstractions over the preceding stage's organization. The stage sequence described would thus reflect a series of conceptual organizations, each corresponding to the application of the domain-structure to a particular class of objects and each being a prerequisite for the subsequent conceptual elaboration. In the next section I introduce a statistical method for performing the hierarchical analysis outlined.

5.3.1 The statistical method of prediction analysis of cross-classifications

The prediction analysis for cross-classifications of Hildebrand, Laing & Rosenthal (1977) provides a statistical index *Del* tailored to capture the phenomenon of discontinuity in developmental stage theory[16]. It consists of a statistical procedure for quantifying the extent to which a contingency table conforms to *triangular hypothesis* of concurrency, collective décalage or individual décalage. The Del is a proportionate reduction of error measure, which reflects the proportional improvement in the accuracy of an estimation based on a *prediction* (e.g. of concurrency or décalage) over the expected frequency. The measure Del is calculated for each logically distinct statement of the form: given that an observation has $X=x_1$ (e.g. success in T_1), we predict $Y=y_1$ (e.g. success in T_1 under the hypothesis of concurrence; failure in T_2 under the hypothesis of décalage).

The developmental data are presented in a two-by-two contingency table, where the rows correspond to success and failure in Task1 and the columns correspond to success and failure in Task2. The four cell entries are expressed as frequencies with regard to the total number of observations, N (see table 5.1). Over the contingency tables, four response patterns, or models, represent the order of acquisition between T_1 and T_2 (see tables 5.2 to 5.4).

The predictions are formulated as triangular hypotheses corresponding to each of the models and predicting the frequencies of success and failure in T_2 on the basis of the responses in T_1. The triangular hypothesis of concurrency states that failure in T_1 predicts failure in T_2, and that success in T_1 predicts success in T_2 and expects frequencies approaching zero to occur in all non-diagonal cells (e.g. $cell_{1.2}$ and $cell_{2.1}$ in table 5.1). Errors are those events that occur in the two cells that are predicted to be empty: If large enough, statistically, they falsify the hypothesis of concurrency. The triangular hypotheses of collective décalage in favour of T_1 states that failure in T_1 predicts failure in T_2, whereas success in T_1 predicts either success or failure in T_2. It

[16]The use of the statistical method in developmental studies is advocated by Hofmann (1983) and put into practice by Lautrey, Ribaupierre & Rieben (1985) and Rieben, Ribaupierre & Lautrey (1986).

expects frequencies approaching zero to be found in the bottom left-hand cell (e. g. cell2.1). Prediction errors are those events which occur in that cell. In the case of the opposite hypothesis of collective décalage in favour of T2, the prediction expects the top right-had cell to approach a zero frequency.

The Del measure reflects the proportionate reduction of error that is achieved by predicting certain cell frequencies of the table conditionally on the basis of the different triangular hypotheses instead of by chance. The Del is computed as the ratio of the observed frequency of the error cell(s) and the expected frequencies for the same cell(s). For instance, if the triangular hypothesis predicts cell2.1 to be empty,the expected error is calculated by multiplying the unconditional probabilities of the cell (e.g. the marginal total f2. and f1.) and dividing the product by the total number of observation N. The Del index corresponding to the prediction that cell2.1 is empty is calculated by substituting the observed error and expected error values in the following equation:

$$\mathrm{Del}_{2.1} = 1.0 - \frac{\text{observed error } (f_{2.1})}{\text{expected error } (f_{2.}\ f_{.1})}$$

The value of Del ranges between zero and one. A value of zero indicates that the triangular hypothesis makes absolutely no improvement over chance. An index of one indicates that the triangular hypothesis provides a considerable improvement over chance. Values between zero and one provide a measure of the proportionate extent to which the data support the triangular hypothesis.

The next step consists of determining whether an observed Del is significantly greater than a chance Del, always equivalent to zero. This is accomplished by calculating the standard error of Del first and then a simple one-tailed normal curve test to determine the significance of the Del with respect to chance:

$$z = \frac{\mathrm{Del}}{S}$$

A significant Del (the significance limit is set at $p<.05$) supports the prediction of the relative triangular hypothesis, that there are fewer subjects in the error cell(s) than the marginal frequencies would predict. In the same way the accuracy of different predictions (e.g. *a* and *b*) on a same contingency table can be compared[17]:

[17]As a general practice, the accuracy of a triangular hypothesis as the best predictor of order between tasks will be calculated on the basis of the significance of the associated Del and of the difference between this Del

$$z = \frac{Del_a - Del_b}{S_{b-c}}$$

5.3.2 The uses of the prediction analysis

The Del analysis can be employed to achieve both descriptive and predictive research aims. In a descriptive phase, the order of acquisition between tasks which have been studied in the literature can be more firmly established. When we identify the best model to fit the data. In a predictive phase, when hypotheses about stage organization are formulated, specific hypotheses can be tested about concurrency between tasks, if these tasks invoke the competence postulated, and about décalage, if these tasks set requirements that can not be satisfied by the concept postulated. In particular, if the child's solution of T_1 is explained in terms of him having elaborated a concept c_1, a new task T_2 can be constructed which sets requirements compatible with c_1. From that, children who solve T_1 are expected to solve T_2 as well, whereas children who fail T_1 should fail T_2 (e.g. a pattern of concurrency between the solution of T_1 and T_2). Alternatively, a task T_3 can be created which requires inferences that are too complex for c_1. In this case, only children who solve T_2, but not all of them, are expected to solve T_3 too, while no children who fail T_2 are expected to be able to solve T_3.

To test specific hypotheses, the level of significance of the Del which corresponds to the expected order is calculated and compared with the values of the two alternative orders. This serves to determine whether this Del is not only significant but also significantly greater than the alternatives. A specific prediction is thus verified when the corresponding Del both reaches the significance level against chance and its improvement is significantly greater than that corresponding to the two Dels yielded by the alternative models. The prediction is falsified when:

1. The corresponding Del does not reach the significance level, in which case the alternative models are tested post hoc to identify an eventual best model that fits the data;
2. No Del achieves the significance level. This result can be due to two different distributions: (a) the predictions are *under-determined* by the data, either because the

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and the Dels from the other possible ordering hypotheses. This further test is recommended by Lautrey, Ribaupierre & Rieben (1985) to neutralize the bias represented by the use of marginal frequencies. The second measure in fact balances the two (or three) Dels, which were all submitted to the same bias, one against the other, whereas the first measure tests a single Del against chance.

tasks are too easy, in which case the great majority of subjects are in the cell22 (success in both tasks), or because they are too difficult, so that subjects are in the cell11 (failure in both tasks). This situation identifies a problem of sampling, i.e. the sample examined is either too young or too old for the tasks at hand. (b) the responses are evenly distributed in the four cells. We conclude by default that an *individual décalage* exists in the acquisition of the two tasks;

3. The Del corresponding to the prediction is significant, but not significantly different from the Dels of the alternative models, which themselves offer a significant improvement over chance. Since all three triangular hypotheses which expect the cells 21 and 12 to be empty are simultaneously verified, and no model is significantly better than any other, we conclude that *weak concurrency* underlies the acquisition of the two tasks;

4. A special case of the previous situation is when the three Dels are equal to 1 (or close to). This indicates that the three hypotheses fit perfectly the distribution. From this result, I conclude that a *strict concurrency* holds in the acquisition of the two tasks, as the corner cells 21 and 12 are empty (or close to be empty);

5. The Del is significant, but not different from one of the alternative models. If one of the model is concurrency, then two models account for the order between the two tasks and coexist: *concurrency and collective décalage* in favour of one task. If instead the two models are of collective décalage, then the two tasks are acquired with *individual décalage.*

5.3.3 The between-groups comparisons

Situation (2a) above points out a important limitation of the prediction analysis method, that is it does not work when the responses are concentrated in one cell (e.g. the cell representing the success or the failure in both tasks). This problem can be tackled by carefully sampling children in the age range where developmental changes are known, or expected, to occur. The hierarchical analysis has thus to be complemented with the standard between age groups analysis of performance which identifies the periods in which the developmental change investigated occurs.

Since the problem-solving performance is measured by categorizing the subject's responses as success or failure, standard non-parametric statistical tests for nominal data (see Siegel 1956) are employed. The χ^2 statistic tests the difference between age groups (or orders) in the number of responses which fall into the two categories of success and failure. The Marascuilo & McSweeney (1967) instead tests the difference in response between tasks for each age group.

The Marascuilo & McSweeney test, as modified by Meddis (1984) to provide a test of specific experimental hypotheses, measures the effects associated with different conditions under which the same individuals are observed (e.g. in a repeated measures design). I employ the test to compare the cross-task performance differences within each age group and to obtain the complementary information to that provided by the χ^2 (e.g. across groups performance difference in single tasks). In the Marascuilo & McSweeney test, the null hypothesis is that if the different tasks are equivalent, then the children should have the same probability of success under all conditions. The statistics computes an index L, which reflects the change from correct solutions in one task to wrong solutions in subsequent tasks. L can be converted into a standard z score, from which the one-tailed significance level is calculated.

In my experiments, the two statistics χ^2 and L play a complementary role in that they permit us to capture the effect of the age variable on the performance in single tasks and the effect of task condition on the performance within each individual age group. These two tests are thus instrumental in identifying the age at which we are more likely to observe some particular developmental phenomenon with respect to a set of tasks, and thus to avoid making a task too easy or too difficult for the sample examined.

5.3.4 The experimental design

The experimental design which permits the two complementary analyses of cross-sectional response patterns and of between-groups comparisons of responses is a within-subjects design with subjects nested in age groups (or age x order groups, etc.). All the subjects are tested in all the tasks and are divided into groups constructed according to age and order of tasks.

5.4 The plan for the study of cardinal number development

The theoretical and methodological apparatus presented in the preceding chapters is applied to the study of cardinal number development, one of the most puzzling and well documented domains in cognitive developmental research. In this section I set out the plan for this study which is a direct implementation of the general research strategy introduced earlier. Before discussing the different parts of my study of cardinal number development, however it is necessary to delimit somewhat this vast domain. Psychological research has approached the question of cardinal number development from three different perspectives:

1. The acquisition of enumeration and counting skills, such as learning the number words sequence and the counting procedures;

2. The acquisition of the operations on sets of objects as in situations of reproduction, comparison and conservation of sets' number;

3. The acquisition of the arithmetic operations, both in their abstract form and embedded in practical situations like number word problems.

From these studies it emerges that in the course of development the three levels of number competence (e.g. enumeration-counting, cardinal representation and arithmetics) interact in a highly complex way. Thus in the concluding remarks of her extensive study of "Children's counting and concepts of number", Fuson writes that:

> In summary, over the age span from age 2 through 8, children come to understand increasingly complex relationships among the mathematically different situations in which number words are used. They gain a considerable amount of knowledge concerning different specific situations within each kind of situation. Larger and larger number words are learned. Important changes occur in children's conceptualizations of the sequence, counting and cardinal situations. Increasingly abstract and complex conceptual units are used in these situations. The relationships among sequence, counting and cardinal situations become closer and more automatic until finally these become integrated within the number-word sequence itself. At this level the number-word sequence is a seriated, embedded, unitized, cardinalized, truly numerical sequence (1988, p. 416-417).

The scope of a dissertation does not allow me to address the fundamental issue of the relationship between these different aspects of numerical knowledge in development. I shall thus focus on *the development of the capacity to represent the cardinal number of sets of objects* and *draw inferences from these cardinal representations*. In dealing with this question, I shall also be concerned with some aspects of the acquisition of the number-word sequence and of counting skills, as these constitute one of the privileged instruments for representing cardinality.

I have chosen to study representations of the cardinality of collections of objects for both theoretical and developmental reasons. On the theoretical level, this constitutes the most basic aspect of the cardinal number concept with the operations of addition and subtraction. As Wilder writes:

> For most mathematicians, numbers are concepts relating to the "size" of sets. The "size" of the set is the most basic aspect of its form - disregarding all other aspect such as colour, shape, substance and the like, if the set be a collection of physical objects; and disregarding order and other relations, operations and the like, if the set be a collection of mathematical entities - and corresponds to the "number" of its elements (p. 102-103, 1965).

The capacity to abstract the numerical "size" of a collection of objects from its other dimensions appears to be the outcome of a long and complex developmental process. Since the early studies on numerosity judgments (Binet 1890), it has clearly emerged that 3-, 4-year-old children tend to compare collections of objects on the basis of spatial features of the collections such as the size of the individual elements, the space occupied, the length, if the collection forms arrays, etc. From the age of 5-6 years, children base their comparisons on the number of objects in the collections, usually after having counted them. Since the pioneering experiment of Binet, the shift from initial space-based to number-based comparisons has been widely reported. Furthermore, Piaget's studies of the conservation of number find that under certain circumstances even 5-, 6-year-old children abandon an initial judgment of equinumerosity of two rows of objects, when one of the rows is spatially modified into a longer (or shorter) row. In the context of the conservation task then, the 5-, 6-year-old children make pre-numerical, space-based, judgments of numerosity equivalent to those observed with 3-, 4-year-olds by Binet.

The objective of the study of cardinal number development I undertake is to provide an account of the acquisition of the capacity to measure the size of a set of objects independently from other indices of extent, such as the space occupied by the set as a whole, the dimensions of its elements, etc. Firstly, I examine whether this capacity emerges through a sequence of distinguishable levels of increasing adequacy. Secondly, on the basis of the description of how development proceeds, I provide a characterization of the nature of the different number concepts elaborated. Thirdly, I model the developmental process using Richards' logical representations and algorithms.

To approach these questions, I have devised a battery of tasks which require that numerical representations of the set size be established and that inferences be drawn from them. The tasks are derived from the very extensive literature on this topic. The first phase of the study (Chapter 6) consists of a detailed review of the literature aimed at identifying the tasks which point out clear developmental changes and at isolating the periods of development in which these changes occur. The second phase (Chapter 7) consists of establishing the precise order in which these tasks are solved. The hierarchical analysis should reveal stages of competence, corresponding to the tasks which are solved concurrently, and transition phases, corresponding to the cases of collective décalage. On this first descriptive basis, I formulate hypotheses about the underlying number concepts. These hypotheses are tested with new experiments in the subsequent predictive phase (Chapter 8). The fourth phase consists of the modeling the overall developmental process using Richards' logical apparatus. With this transition model I formalize the competence levels as networks of propositions and the process leading from one level to the following as 1) the reinterpretation of propositions from irrelevant to relevant, 2) the testing of the relevant propositions and, in case they are verified, 3) their integration into a new propositional network.

Chapter 6 Review of the literature on cardinal number development

6.1 Introduction

Three main experimental paradigms have been employed to investigate the child's development of cardinal number representations as the capacity to abstract the numerical size of a set of objects from its other properties[18] and to draw inferences on these representations:

1) the Reproduction of sets;

2) the Comparison of sets;

3) the Conservation of number in various formats.

According to the literature, the acquisition of the solution to all the three tasks follows a similar developmental pattern: children move from *non-numerical representations* of the sets' size based on physical dimensions, such as space occupied, length, width or items size, to accurate *numerical representations,* based on cardinality alone. In solving number reproduction tasks, for instance, children move from constructing sets which reproduce the configuration of the collection (in particular its length), to accurate reproductions. These are carried out either by matching each element of the collection with one element or on counting out a same number of elements to form the new collection. In solving number comparison tasks, as well as conservation tasks, children move from judging the numerosity of the sets on the basis of their spatial dimensions (and again length in particular) to comparing their actual cardinality. The comparison is carried out either by determining whether a spatial (1-1)-correspondence holds between the elements of the sets, or by counting them and directly comparing the cardinal values obtained.

In this chapter, I examine in some detail the existing evidence of the shift from space- to number-based estimations of numerosity in the tasks of set reproduction, comparison and conservation. The analysis of the literature provides the initial information (a) to discriminate the circumstances under which the child can operate with number from the circumstances under which he cannot, (b) to follow the gradual

[18]These tasks have also provided a rich context in which to follow the genesis of *counting in use* (as opposed to disembedded counting or simple enumeration) as a means to represent the cardinality of collections of objects and to carry out comparisons, reproductions and conservation judgments.

extension of the domain of application of number and (c) to describe the developmental process of generalization in the transition from failure to success in specific tasks.

The chapter is divided into four sections. The first section is concerned with number conservation (Piaget & Szeminska (1941, English version 1952), achieved at around age 6-7. Piaget & Szeminska's results, where non-conservers abandoned their judgment of equinumerosity of two rows of objects when one row was spread out has been systematically replicated. First, I present the original Piagetian study and the replications which confirm the robustness of the number conservation phenomenon between age 5 and 7. I then examine the modifications to the original conservation task which find precocious forms of conservation among children younger than 5. Early forms of conservation have been found in the studies which introduced the following modifications to the original Piagetian procedure[19] :

a) the transformation occurs accidentally (McGarrigle & Donaldson, 1975) (see Chapter 4);

b) only one row is presented and transformed spatially; an identity conservation question is put: "do you think that the number of objects is the same as it was before?" (Elkind, 1967);

c) after the transformation, the child is requested first to count the two rows and then to answer the standard conservation question (Gréco, 1962).

In the second and third sections, I discuss the studies of the development of set reproduction and set comparison abilities. Finally in the fourth section, I examine the existing evidence about the order in which the three tasks are acquired.

6.2 The conservation task

6.2.1 The original Piagetian study

In the early years of experimental psychology, the study of number development was concerned with listing and describing *counting* and *enumeration abilities* (Descoeudres 1921, Douglass 1925, Grant 1938, McLaughlin 1935, Reiss 1943, Russell 1936, Woody 1931) and with the *perception of numerosity* (Binet 1890) in children. The field received new impulse from the works of Jean Piaget. Piaget in fact introduced an operational dimension to the study of number concepts and focused on the child's

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[19]In Appendix 3.1 and 3.2 I discuss two experimental paradigms which have identified very precocious forms of conservation: the magic conservation of Gelman (1972) and the additive and spatial transformation format of Mehler & Bever (1967).

numerical representations and inferences in problem-solving type situations. Gelman remarks this profound change of perspective:

> Piaget shifted the framework from one in which the child's responses to number were thought to indicate mastery of number facts to one in which the child's responses to number were thought to reveal the functioning of underlying cognitive operators. (1972, p.119)

To investigate the development of operational number, Piaget designed a battery of tasks which share a same basic structure. Firstly, the child is required to establish a particular relationship between two collections of objects. Secondly, the configuration of one of the two collections is changed, without changing its numerical dimensions. Fundamental tests of operational number are the conservation task, dealing with cardinal number, and the seriation task, dealing with ordinal number. The conservation task, already presented at various places in the preceding chapters, consists of asking the child[20]:

1. To construct a row with the same number of elements as a model row: "take just enough glasses off this tray for the bottles, one for each";
2. To confirm the equivalence verbally: "Is there the same number of bottles and glasses, or are there more glasses or more bottles?";

The arrangement of one of the sets is then modified: one row is either lengthened, shortened or made into a circle;

3. To confirm the equivalence between the two sets: "now, are there more bottles, more glasses or have we got the same number of bottles and glasses?";
4. To give a justification for the answer: "how do you know that the number is the same/different?" and reply to counter-suggestions like "you say that the number is the same, but you see this one is much longer and this one very short".

Piaget identifies an operational concept of cardinal number with responses where the child maintains the relation of equinumerosity, regardless of the change in shape, and

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[20]Piaget presented the problem using different materials: a) beads arranged in rows, b) collections of functionally-related objects, like eggs and egg-cups or vases and flowers, c) collections created by one for one exchange, d) bottles filled up with beads. As Piaget did not report significant differences across the range of presentations, I report the data as a whole and take the freedom of giving illustrations from the different task situations.

gives operational justifications, such as "nothing has been added nor taken away; this one is longer but more spaced, that one is shorter but more crowded; we can go back as it was before", etc..

In the seriation problem, the child is presented with a collection of objects, ordered according to one dimension, and is required to construct a second collection of objects of corresponding order. For example, the child is asked to find the sticks, from a bunch of sticks of different length, that go with a series of dolls of markedly different heights. Once the two rows have been arranged, one collection is modified in such a way that the corresponding elements are no longer opposite to one another. The child is asked the following questions:

a) for one object, which object from the corresponding collection goes with it;

b) the order of one series is reversed and the same question is put to the child;

c) the objects of each collections are mixed up, one object is picked up from a bunch; the child is required to take the objects from the second bunch that are either bigger or smaller than the selected objects.

An operational understanding of ordinal number is attributed to the child when he can establish the ordinal correspondence in the different contexts and solve the last task. For the rest of the discussion, I shall be concerned with the tasks on cardinal number, as it deals directly with cardinal concepts.

6.2.1.1 The rationale of the number conservation task

Piaget introduced the number conservation task as a test for discriminating between a primitive form of equivalence based on what Piaget qualifies as *intuitive correspondence* and the more advanced form of equivalence based on operational correspondence. Piaget argues that the child has a mature concept of number only when the correspondence between sets has become independent of perceptual features and in particular, when the child is capable of distinguishing between number-relevant (e.g. addition and subtraction) transformations and number irrelevant (e.g. permutation and partition) transformations, which only affect the configuration of the set. The conservation task constitutes a critical test of mature, operative cardinal number concept in that if the child's first equivalence relation is based on the optical correspondence between the elements and on the equivalence of shape between the two sets, and not on numerosity per se, then the child should abandon the equivalence when a change in shape is produced. The transformation destroys in fact both the original configuration and the optical correspondence, and introduces a marked difference of configuration.

Piaget explains the failure to conserve in terms of the pre-operational nature of the number concept. The pre-operational thinking lacks the mobility and the internal articulation to permit the child to go back mentally to the initial, pre-transformation, equinumerous configuration or to compose the two spatial dimensions: greater length with smaller density. For the pre-operational child thus, when the configuration changes, everything changes, and when one of the collections looks bigger or takes up more space, than it has a bigger number too. A further source of confirmation of the operational, logical nature of the child's reasoning in the number conservation task comes from the justifications that children give of their responses. Operational children give the following reasons:

1. Nothing has been added nor taken away;
2. It is possible to go back to the previous display;
3. One row is longer, but its elements are more spaced; and the second row is shorter, but its elements more crowded;
4. The enumeration of the two sets leads to the same number.

6.2.1.2 The development of number conservation

Piaget reports that between age 4 and 7, children move from non-conservation of number to consistent conservation. He identifies three basic types of solutions:

Substage 1: children do construct a set equivalent to the model set, but respond that one of the sets is more numerous after the transformation. In general, depending on the transformation, longer rows or taller piles are taken to be more numerous.
Substage 2: children maintain number until the sets are too markedly different, i.e. big differences in level or cross-section when the beads are in different bottles or marked difference in length or density when they are arranged in rows. Children also give inconsistent responses: conservation is immediately followed by non-conservation, or vice versa.
Substage 3: children confirm the equinumerosity irrespective of any change in the arrangement of their elements. Piaget illustrates the behavioural patterns characteristic of each substage with extracts from the experimental protocols.

6.2.1.2.1 Substage 1

Children are at the non-conservation substage 1 when they abandon the initial equinumerosity, estimating the numerical size of the sets after the transformation merely from single perceptual relationships: height or width in the case of beads in bottles; length or density in the case of rows. Enumeration itself appears to be subordinated to direct perceptual evaluation. A good illustration of the cognitive state characteristic of substage 1 children is provided by the protocol of Bab (age 4,6) (p.28, 1952)[21]: Bab put one bean on the table every time the experimenter did so. Have we got the same? - "Yes". Bab then put one bean into L each time the experimenter put one into P, and with each bean the child said spontaneously "It's the same". But when there were ten in each glass, and L was 1/2 full, Bab cried: "I've got a lot." - And what about me? - "Mine's quite full."- Are they the same? - "I've got a lot." - And what about me? - "Look! you've got only a few." - Why? - "Look there (pointing to the levels)". Bab then put a bead into E each time the experimenter put one in P: Make sure that we've both got the same. - "Me one and you one; me two and you two;...."(up to 6, when glass E was quite full). - Are they the same? ..... - If we made one necklace with your beads and one with mine, would they be the same? - "No, mine would be longer." - But if we took all your beads and all mine? - "No, yours won't be as long; we must fill your glass to have a necklace as long as mine." - Count them. - (Bab counted 6 in E and 6 in P) - Well? - "You'll have a little necklace." - But why have you got a lot? - "Look, they are low in your glass. It's me that's got a lot, mine's quite full."

Similarly Port (age 5,0) (p.26, 1952) confirms the equinumerosity of the beads contained in two identical glasses, and justifies it by saying: "Because there's the same height of green and red." When the content of one of the glasses is poured into a narrow and tall glass, the child claims that one of the glasses now has more and justifies that by saying "Because it's narrow and they go higher". In the situation of exchange of pennies for flowers, Gui (age 4,4) (p.57, 1952) exchanges 6 pennies for 6 flowers. The pennies are in a row while the flowers are bunched together. The experimenter asks: What have we done?- "We've exchanged them." And then is there the same number of pennies and flowers? - "No, it's more there", while pointing to the pennies.

Piaget concludes that children at substage 1 do not evaluate sets as made up of discrete units, but rather according to their global dimensions, such as length or width. Thus the same set can sometimes be more numerous, sometimes be less numerous

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[21]In the text the children's replies are presented between inverted commas and separated from the experimenter's questions and remarks by a dash. The children are referred to by the first three or four letters of their name.

depending on it being lengthened, shortened or poured into containers of different size and shape.

6.2.1.2.2 Substage 2

Children are classified in substage 2 when they do not consistently conserve number, in the sense that either their conservation depends on the degree of change in the configuration, or their conservation judgments are contradictory and unstable (e.g. the necklace made out of the set A is longer than that made out of set B as it is judged larger than B, but the set B is considered at the same time more numerous than set A, because the row it forms is denser than A). The equivalence of two sets is not immediately abandoned after the transformation, as it is typical of substage 1. Instead, the difference in configuration between the two rows makes the child oscillate between judgments of difference and equivalence. According to Piaget, substage 2 corresponds to a period of conflict between correspondence-based equivalence and configuration-based difference. Some examples from the protocols illustrate the behaviours which reflect this conflict. Marg (age 5,6) (p. 30, 1952) considers that pouring the content of glass A into a narrower and taller glass L will increase the number of beads ("There are more in the big one.... because it gets bigger here (pointing to the narrower column in L)"); however the necklaces made with the beads from A and L "they'll be the same length". Tis (5,1) (p.31, 1952) puts one bead into L every time the experimenter puts one into A. He also counts each bead as he puts it in and reaches the correct total of 12. L is then full, and Tis noticed: "I've got more." - Why? - "There are more in mine." - And if we make two necklaces? - "This one (L) will be longer." - Why? - "The glass is bigger, and that one (A) is smaller." - How did we put the beads in? - "We put two every time." - What will the necklaces be like? - "Yours will be long and mine will be the same length." - Why? - "Because this one (L) is big, and mine (A) is little. You've got a lot of beads in yours." - And what about you? - "Not as many, but a lot all the same".

According to Piaget, it is from the conflict between reasons to conserve and reasons to change that operational conservation will emerge. The conflict characterizing substage 2 leads in fact to the coordination of the different relations involved into a system susceptible of justifying conservation, while taking into account all the variations.

6.2.1.2.3 Substage 3

At substage 3, children consistently give conservation answers. Lau (6,2) (p.47, 1952) makes 6 glasses correspond to 6 bottles. The glasses are then grouped together and the experimenter asks whether they are still the same number; the child answers: "Yes, it's the same number of glasses. You've only put them closer together, but it's still the same number" Other children justify their conservation answers by saying: "That (transformation) hasn't changed anything" or "Because there are ten vases and here (flowers) there are ten" or else "Because they go like that (spontaneously putting one flower opposite each penny)".

6.2.1.3 Discussion of Piaget's conservation studies

Between the age of 4 and the age of 6/7 children shift from *non-conserving* responses, i.e. the spatial transformation of lengthening (or shortening) one of two equinumerous rows destroys the original equinumerosity and makes the modified row more numerous (or less numerous), to *conserving* answers, i.e. the numerosity of the two collections is unchanged by spatial modifications. Between non-conserving and conserving answers Piaget distinguishes an intermediate level of unstable responses, which is taken to be the expression of the conflict between number-based and space-based estimations of numerosity. The change from non-conservation to conservation reflects the overall change in cognitive organization, from irreversible, perception-bound preoperational schemas to reversible, mobile operational structures.

6.2.2 The replications confirming Piaget's results

6.2.2.1 The exact replications

In the early 1960s, the developmental change identified by the conservation task is put to severe test in a series of studies: Dodwell (1960, 1961), Gréco (1962), Hood (1962), Rothemberg (1969), Wohlwill & Lowe (1962) and Zimiles (1966). The objective of these studies is mainly to replicate Piaget's experiment using systematic procedures and precise statistical analyses. The flexible Piagetian "clinical procedure" was translated into a standardized experimental procedure where:

a) the initial equivalence is not established by the child himself. Instead two equinumerous parallel rows of equally spaced items are presented to the child; the child is only required to confirm the equinumerosity;

b) only one of two spatial transformations is effected: one row is either lengthened or shortened;

c) performance is generally scored on the basis of "same" answers: a child is a non-conserver when he says that one row has more elements; intermediate, when under one transformation he says that the rows are different and under the second transformation that they are equivalent; conserver, when he maintains that the rows have the same number, regardless of the difference in shape.

Regarding justifications, two positions have been adopted. Some researchers use a stricter criterion for classifying children: conservers give a "same" answer *and* justify it giving "operational" arguments. Other researchers instead consider that the request for justifications introduces a new conceptual dimension: a level of meta-conceptual understanding of conservation (see chapter 5 and the discussion of Brainerd's argument). Since no firm basis exists to decide whether requiring justifications does or does not implicate a different kind of conservation, it seems appropriate to adopt the salomonic position of scoring the subjects using both criteria and of testing for the significance of the eventual differences in correct response frequencies obtained with the two scoring procedures (a position also defended by S.A. Miller 1978).

The studies previously cited confirm the shift from non-conservation to conservation in the period between 4 and 6-7 years of age with both scoring procedures. The Piagetian findings have also been replicated in the context of scalogram studies (Wohlwill 1960, Smedslund 1966a, Wang, Resnick & Boozer 1971), learning studies (Wallach & Sprott 1964, Bearison 1969, Gelman 1969 and 1982, Beilin 1971, Winer 1968), numerical/arithmetic competence (Gelman 1972, Mpiangu & Gentile 1975, Russac 1978, Pennington & Wallach 1980, Fuson 1988), strategies analysis (Halford 1975, Cuneo 1982), relations to other conservation and logical tasks (Dodwell 1962, Smedslund 1966b, Winer 1974, Inhelder, Blanchet, Sinclair & Piaget 1975, Siegler 1981). Other studies introduced modifications to the display. For example, P.H.Miller, Heldmeyer & S.A.Miller (1975) add stripes to connect pairs of elements of the sets after the transformation and find that this more salient perceptual display elicits only a small anticipation.

To my knowledge, the only negative evidence is reported by Mehler & Bever (1967), who found a very precocious form of number conservation between age 2 and 3 years. These results have not however been replicated by any of the subsequent studies (see appendix 6.2).

From the vast amount of evidence confirming Piaget's original findings, it seems appropriate to conclude that during the preschool and early school years, between 4 and 7 years of age, children move from non-conservation. The phenomenon is well established for the specific context and procedure of the Piagetian problem and also generalizes to different modes of presentation of the task.

6.2.2.2 The replications with modified conservation tasks

Modifications of the original Piagetian conservation problem provided supplementary evidence confirming the developmental change in number conservation. S.A.Miller (1982) tests the conservation problem in familiar environments and with natural objects. Silverman & Schneider (1968), and S.A. Miller (1976a, b) devise non-verbal versions of the problems and Bryant (1972) studies the more general capacity to transfer numerical information between spatially modified collections. These experiments demonstrate the extent of the phenomena originally described by Piaget, as the same shift from non-conservation to conservation, and from perceptually-based to number based judgments is confirmed between the ages of 4 and 7 years.

6.2.2.2.1 Conservation tasks in ecological settings

S.A. Miller (1982, experiment 1) designs a conservation task in which the transformation occurs in an "ecologically natural manner", with minimal intervention of the experimenter. Whereas the objects of the original task (e.g. counters, candies, beads, egg-cups, etc.) can be moved only by the actions of the experimenter, Miller introduces elements that are either alive (e.g. crickets) or inanimate, but in a dynamic setting (e.g. cars on a slope; floating boats). These three *ecological* conditions are compared with three standard ones using the same items, in the case of the inanimate ones, or equivalent (plastic crickets instead of real ones), in the case of the alive ones. The three ecological conditions are:

1) Boats, the child is shown two rows of five boats each, floating in a tub of water; once the child has agreed to the equality, the experimenter pulls a hidden switch, releasing an underwater wire that was holding one row together. The boats of one row thus slowly float apart;

2) Cars, two rows of five cars each are presented on two inclined boards; a breaking pad on one board is removed, causing one row of cars to run down-hill and take up a more spread-out distribution;

3) Crickets, two groups of live crickets are presented in identical glass containers, divided in two compartments; once the child has agreed that they are the same number, one dividing wall is removed and the crickets spread over the entire container.

After each modification, children are asked whether the number of items in the two collections (e.g. bunches) is still the same or whether it has changed and one has more items in it. No significant differences are found between the conservation responses of 80 children (from 5 to 7 years of age) in the standard and ecological conditions. In the standard task, around 53% of the children conserve, while in the modified task around 57% do so[22].

6.2.2.2.2 Non-verbal conservation tasks

Experiments by Silverman & Schneider (1968), S.A. Miller (1976 a, b) compare the standard conservation task with a non-verbal version of it. In the non-verbal task, the children, instead of being asked whether the two rows have the same (or different) number of counters, are asked "to pick the candies that they wish to eat", from two rows of candies. This procedure involves a minimal use of quantitative terms both in the wording of the comparison question and in the practical mode of response (e.g. choice of one row). These non-verbal conservation problems produce minor anticipations of conservation. The difference between performance in the standard and in the non-verbal conditions reaches the significance level only in one of Miller's studies (1976b, Experiment 1). However Miller's own replication of that experiment (Experiment 2) has not confirmed the previous findings.

Another series of experiments by Miller (1976a) shed some light on the small advance in conservation responses in the non-verbal condition. It appears that, when the term "more" does not appear in the question, children's tendency to systematically choose the longer row of sweets decreases. This yields an increase in the number of correct responses which however does not exceed chance, that is, half of the time children choose the more numerous row correctly. The superiority of the non-verbal condition results thus from the fact that children are less likely to be consistently wrong on the non-verbal trials than on the standard trials, and not from the fact that they conserve systematically. For that to be the case, conservation responses should have been significantly better than chance.

From the evidence at our disposal then, non-verbal conservation problems constitute a further generalization of the number conservation phenomenon, which emerges also

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[22] Miller interprets these results as evidence against McGarrigle & Donaldson (1975)' s claim that the failure to conserve is essentially due to the nature of the action performed by the experimenter which misleads the child into interpreting the conservation question as referring to the dimension modified (e.g. length) and not to number. Here in fact the role of the experimenter is masked and the rearrangements are random.

when comparative terms are eliminated from the question and the response. Non-conservation answers then may not be simply attributed to poorly developed linguistic skills, like misunderstanding of the relational terms "more, less, same".

6.2.2.2.3 The inconsistency of numerical judgments

Some further qualification of the problems underlying non-conservation responses comes from the studies which examined the children's consistency in carrying a numerical judgment about two large collections (e.g. around 20 elements) through a series of number-irrelevant transformations of the collections. This alternative experimental paradigm was introduced by Bryant (1972). Bryant designs a modified conservation tasks where the post-transformation display consists of two rows of different number (difference = 1) but same length and irregular distribution. Prior pilot studies had revealed that these displays, unbiased with regard to the length cue, lead to chance-level choices, i.e. each of the two rows has the same probability of being judged to be the more numerous[23] .

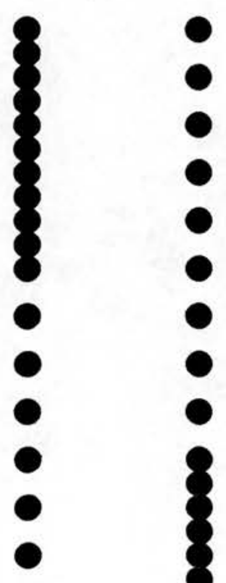

Figure 6.1 : Chance display, from Bryant (1972, p.81)

[23] Notice that the child is not given the possibility of answering that the two rows are equinumerous.

Bryant argues that the capacity to conserve a numerical relationship between two sets is more directly observed using the chance displays as post-transformation configurations since they do not have any misleading length cue which may distract the child. Bryant predicts that if children appreciate conservation, than their judgment of the chance display would be based on the previous judgment and this would lead to a percentage of correct judgments significantly above chance. Such an improvement in performance would indicate that the relation established in the pre-transformation display has been conserved and transferred to what is usually a chance-level display. Bryant's modified conservation task consists of:

a) two vertical arrays containing respectively 19 and 20 items, placed in spatial one-to-one correspondence, such that the more numerous array exceeds the length of the other array of one item;

b) both arrays are transformed, starting from the more numerous, to obtain two post-transformation arrays of equal length, i.e. the chance-level displays;

c) the child is asked to point to the more numerous array; no judgment of equality is allowed.

Bryant reports that from age 3, children respond consistently better to the modified task than to the standard task. Children correctly indicate the more numerous array in the chance display after the transformation between 80 and 90% of the time. Bryant concludes that children have a basic understanding of conservation and that this understanding is masked by the length difference cue between post-transformation collections in the standard conservation test. Hence when such a misleading cue is not present, as in his version of the task, children can express their understanding of conservation. This conclusion is however undermined by the results of subsequent studies which control for two factors which Bryant overlooked in the design of his experiment. The two factors are:

1. The order of manipulation, in Bryant's procedure the more numerous row is always manipulated first.
2. The questions used, Bryant does not allow 'same number' answers. Children are obliged to choose one of the rows as more numerous even when they may think that the two rows are equinumerous.

These two factors may influence the child's response in a significant way. Many studies of the preschool child in fact show that his response strategies are often based on indices of the experimental situation like the last word uttered, the order in which some manipulations are performed, the colour of the material used, etc. In the case of Bryant's experiment then, the child may answer by systematically choosing the modified collection as the more numerous, without going through a conservation inference. On the other hand, the studies of number judgment in preschool children indicate that children tend to compare collections on the basis of their spatial size. It is likely then that children would judge the two collections of the chance display as equinumerous on the basis of their equivalence in length. And indeed the experiments which controlled for the order of manipulation and introduced the "same" option do not confirm Bryant's findings.

Katz and Beilin (1976) modify Bryant's procedure by transforming only one row: either the more numerous is contracted or the less numerous lengthened. Furthermore, in a control condition, one row is modified so as to maintain the original length ratio between the two rows (e.g. the longer remains the longer). The results show that correct responses to the control condition significantly exceed the experimental conditions (original conservation and Bryant test) and that the two experimental conditions do not lead significantly different responses. The results suggest that instead of relying on the prior judgment, children base their responses on alternative strategies. They choose the array according to its colour (e.g. always the red row), according to its position (e.g. always the right row), or systematically choose the row that has been manipulated.

Starkey (1981) devises a study to examine the effect of the order of transformation variable: either the more numerous or the less numerous array is transformed first. The order of array transformation is found to be significant, with more correct judgments when the more numerous array is transformed first. Starkey thus confirms that Bryant's results may be due to the children's use of the order-of-array-transformation cue (correlated in Bryant's experiment with numerosity) rather than to the conservation of the initial numerosity judgment. Furthermore, the children's performance is radically different from that reported by Bryant when "same" judgments are allowed. Eighty-three per cent of the children respond that the post-transformation rows (e.g; the chance-level display) have the same number of items, when they are given the opportunity to do so. So, contrary to what claimed by Bryant, the numerosity judgment with the post-transformation, chance-level display is one of equinumerosity, even when the initial judgment identified one row as more numerous. Children seem thus to favour local, length-based comparisons, and to exploit other indices rather than transfer numerical judgements across number irrelevant, spatial transformations.

Halford and Boyle (1985, experiment 5) provide supplementary evidence of the radical difference in response when the 'same' answer is allowed. Their 1985 study provides the most systematic investigation of the consistency of children's numerical judgments through number irrelevant transformations. The study consists of a series of five experiments that used Bryant's chance-level displays as both pre-transformation and post-transformation displays. The authors argue that since chance-level displays do not seem to carry stable cues to the sets' numerosity, using them both before and after the transformation should show whether children's numerical judgments are independent, or whether children can maintain a numerosity judgment and can carry numerical information about a set across spatial transformations.

The experimental procedure consists of asking the children to choose which of two chance-level rows has more objects. Then the spatial transformations are performed, in such a way that the two rows remain equivalent to chance-level ones. The hypothesis is that if children understand invariance, than they should take the same row to be the more numerous, both before and after the transformation. The results indicate that, while 3-, 4-year-olds do not demonstrate any tendency to maintain the judgment across the transformations, their judgments being completely independent, 6-, 7-year-olds show a significant tendency to maintain their original judgments across transformations. Furthermore, an experimental condition where no transformation is performed, with the question simply repeated, examines whether the change of judgment is not simply due to the repetition of the question. When no transformation occurs and the comparison question is simply put again, all the children maintain their judgments between 84 and 88% of the time. The change in the children's numerosity judgments seems thus to be essentially due to the spatial transformation leading to a new configuration of collections.

In conclusion, and contrary to what initially claimed by Bryant, children between age 3 and 6 years appear to judge pairs of collections on an independent basis depending on the collections' configuration. There is no evidence that they understand conservation in the sense of carrying numerical information through from pre-transformation judgments to post-transformation ones.

6.2.2.3 Discussion of positive replications

From a wide range of conservation studies, varying in a) type of questioning, b) size of the collections, c) objects used and d) ecology of the situation, it emerges that between the age of 3/4 and the age of 6/7 the child's understanding of cardinal number goes through an important change. The child moves from considering that spatial transformations performed on a pair of collections, with the same number of elements, affect the equinumerosity of the two collections to maintaining their equinumerosity, regardless of modifications to their configuration. The younger children appear to overrely on the estimation of space occupied by the collections as a measure of numerosity. Thus the children can easily switch from judging one collection to be larger than another collection to the opposite judgment, once the latter collection is modified into, for instance, a longer row. Furthermore, the studies presented in section 6.2.3.3, suggest that young children tend to make independent and opposite judgments of numerosity about pairs of collections which are modified spatially.

These results confirm the Piagetian description and his account of the pre-operational number concept as perception-bound and undifferentiated from the spatial properties of the collections. However this picture of the development of cardinal number is challenged and greatly complicated by the results obtained using some other modified conservation tasks. These tasks have revealed that, under specific conditions, 4- and 5-year-old children can conserve number.

6.2.3 Precocious forms of conservations

Three tasks have produced the most robust evidence of early conservations:

1. Identity conservation;
2. Conservation of counted collections;
3. Conservation after accidental, incidental transformations.

There is strong evidence for collective décalage between conservation in the context set by these tasks and conservation in the standard Piagetian task. The third task has been discussed at some length in chapter 4. In this section, I shall present the first and second modified conservation tasks .

6.2.3.1 Identity vs. Equivalence conservation

Elkind (1967) argues that Piaget's conservation task does not distinguish between two types of conservation:

1. The conservation of the numerosity of the transformed set before and after the spatial transformation has occurred, what he names the *number identity*;
2. The conservation of the equinumerosity between the two sets before and after one of the sets has been transformed, or *number equivalence*.

According to Elkind's logical analysis of the task, the conservation concept corresponds to a conditional of the form, if $R_1 = R_2$ (read row1 is equivalent to row2) then $R_1 = R_2'$ (read row1 is equivalent to row2', the row2 modified spatially). On the assumption that the complex proposition $R_1 = R_2$ is entertained by the child as true, the conditional would hold under two conditions: if $R_2 = R_2'$ is also true and if the child can draw the transitive inference from $R_1 = R_2$ to $R_1 = R_2'$, via $R_2 = R_2'$. According to Piaget's theory, transitivity is an achievement of the operational stage, and that may partly explain the failure to conserve. But what about identity?

Elkind argues that knowledge of number identity (e.g. R2 = R2') is a necessary, but not sufficient condition for number equivalence, since knowing that R2=R2' does not automatically lead to the conclusion that R1=R2', unless the child combines this knowledge with the original equinumerosity of R1=R2, via a transitive inference. Elkind then formulates the hypothesis that identity conservation is a developmental antecedent and a conceptual prerequisite for equivalence conservation. The hypothesis is tested using a direct test of *identity conservation*, which consists of a simple modification of the original conservation. One row is presented to the child and is transformed into either a longer or shorter row.

Figure 6.2: the identity conservation format with a spreading out transformation

After the transformation, the conservation question is asked: "is there more, less or the same number of objects than there was before?". The developmental order between identity and equivalence conservation is examined by comparing the children's performance in the two tasks. Two experimental designs have been employed:

1. Between-subjects with age, task (identity vs. equivalence) and their interaction as independent variables, the dependent variable being the number of conservation responses;
2. Within-subject, with type of task as independent variable and frequencies in the four response categories: conservation in both tasks, non-conservation in both task or conservation in one task but not in the other, as dependent variable.

The results indicate a tendency for the identity conservation to be solved before the equivalence conservation task, although this evidence is somewhat contradictory. Among the studies using a between-subjects design, some articles report that 4 and 5 years-old are significantly better in the identity conservation test than in the equivalence conservation task: Hooper (1969), Elkind & Schoenfeld (1972), Brainerd & Hooper (1975), Rybash, Roodin & Sullivan (1975). Paradigmatic among the studies which report a décalage between identity and equivalence conservation is Elkind & Schoenfield's. Two groups of children, Nursery (mean-age: 4.5 years) and First-grade (mean-age: 6.3 years), are tested on both Elkind's and Piaget's tasks. The analysis of the distribution of correct conservation answers shows:

1. the age variable to be significant. Six-year-olds perform better than four-year-olds, with mean score respectively of 2.7 and 2.02 out of 3;

2. the task variable to be significant: overall children perform better in the Identity Task than in the Equivalence Task, with mean score 2.53 versus 2.18;

3. the interaction of age and task to be significant in the Nursery group, but not in the First-grade, for whom the two tasks are equally difficult.

Other studies, however, do not report significant differences between success in the identity and equivalence conservations (Northman & Grue 1970, Murray 1970, Papalia & Hooper 1971, Miller 1977). As illustration consider Northman & Grue's experiment. Sixty children from Second and Third Grade classes (age range between 6,11 and 9,8) took three identity conservation and three equivalence conservation tests. A correct judgment was scored 1 and an incorrect judgment 0. The results indicate that children conserve in an all or none fashion as most children conserved either in no trials (score between 0 and 1) or virtually in all trials (score between 5 and 6).

Also in studies which employed a within-subjects design, both décalage and concurrence have been reported. In the studies indicating concurrency in the solution of the two tasks, either children conserve in both tasks or they do not conserve in either task. Moynahan & Glick's study (1972) examined 96 children from a kindergarten (mean-age 5,11) and a first grade (mean-age 6,9) classes. The children's responses, scored as conservation when the child answers that the number is the same *and gives operational justifications*, have the following distribution[24]:

Equivalence \ Identity	C	NC
C	70	2
NC	1	23

Table 6.1: the relationship of conserving (C) and non-conserving (NC) judgments in the identity and equivalence tasks (from Moynahan & Glick 1972).

Koshinsky & Hall (1973) find that 86% of the children examined (72 children from three age groups: kindergarten around age 5, first grade age 6 and second grade age 7),

[24]In order to provide a uniform presentation of the data from the different experiments, I have adopted the format of two-by-two contingency tables, with in the rows the frequency of conservation and non-conservation responses to the standard conservation task and in the columns the frequency of conservation and non-conservation responses to the modified conservation task. This has been possible only for the studies which use a within-subjects design and specify the frequencies (or proportions and totals) of conservation/non-conservation responses.

either conserve in both identity and equivalence tasks or fail to conserve in the two tasks. The analysis of conservation and non-conservation responses in each age group shows that 80% of the kindergarten fail to conserve in both tasks against around 40% of the older children.The authors interpret this pattern as close to concurrency.

Equivalence \ Identity	C	NC
C	23	3
NC	7	39

Table 6.2: the relationship of conserving (C) and non-conserving (NC) judgments in the identity and equivalence tasks (from Koshinsky & Hall 1973).

The results of Hooper (1969) go in the opposite direction and indicate a clear décalage. Hooper (with Marshall 1968, cited in Hooper 1969) reports that 75% of the children fail both tasks, 13% succeed both tasks and 12% of the children conserve number in the identity task but not in the standard Piagetian task. Finally, the most recent study on the developmental relationship between identity and equivalence conservation, by Cowan (1979), provides strong evidence in support of a décalage with identity conservation acquired before equivalence conservation. The responses of 72 children (4.6- to 6.6-year-olds) have the following distribution:

Equivalence \ Identity	C	NC
C	8	3
NC	20	41

2 objects condition

a

	C	NC
C	19	2
NC	21	30

5 objects

b

	C	NC
C	23	0
NC	25	24

15 objects

c

Table 6.3: the relationship of conserving (C) and non-conserving (NC) judgments in the identity and equivalence tasks: a) for collections of 2 items, b) for collections of 5 items and c) for collections of 15 items (from Cowan 1979)

The discordant data about the order in which the tasks of identity and equivalence conservation are solved gave rise to an interesting debate in the Psychological Bulletin between Brainerd & Hooper (1975, 1978) on the one hand and S.A. Miller (1978) on the other hand. Brainerd & Hooper argue that Elkind's original prediction has been validated by all the studies that satisfy two conditions: 1) the sample examined includes children of around age 4, as with younger and older children ceiling effects may occur which mask the difference in performance between the two tasks; 2) the scoring of conservation responses takes into account the kind of justifications given. Miller challenges that conclusion on both factual and methodological grounds and claims that only a minor décalage effect is in fact present. The general agreement reached was that more research is needed to substantiate the claim that identity conservation is acquired before and is a prerequisite for the acquisition of equivalence conservation. The study by Cowan (1979) is, at my knowledge, the most recent attempt to resolve this issue. As I noted above, Cowan does find clear evidence in favour of the décalage with a sample of children from 4,6 to 6,6 years of age. The issue of the order of acquisition of identity and equivalence conservation is reexamined in my own experimental work presented in Chapter 8.

6.2.3.2 Conservation of counted collections

In a series of experiments on cardinal number development, Gréco (1962) introduces a variant of the Piagetian conservation of number task in which the child is required to *count* the two rows *after* the transformation has been carried out and *before* making his judgment about equinumerosity. With this situation, Gréco explores the distinction between "quotité" and quantity, a distinction introduced by Cournot (1861, quoted by Gréco, p.9) between number as measure (how many) and as quantity (how big)[25] :

> Cournot soutient que l'idée de quantité n'est pas une idée primitive, et que "l'esprit humain la construit au moyen de deux idées vraiment irréductibles et fondamentales, l'idée de nombre et l'idée de grandeur" (p.9, footnote 6).

Gréco argues that the original conservation task is a test of quantity, while his modified task, which involves counting, probes the child's understanding of "quotité". Following Cournot's analysis, he considers the conservation of counted sets to be a pre-requisite for standard conservation and expects it to be acquired prior to standard

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[25] Cournot maintains that the idea of quantity is not a primitive idea, and that "the human mind creates it from two absolutely irreducible and fundamental ideas:the idea of number and the idea of size (my translation).

conservation. Gréco employs a within-subject design and compares the performance of the same children in the classic test and in two modified formats of the counting task. The two tasks of 'quotité' are:

1. After the transformation, the child is asked to count one of the rows, while the second row is hidden with the hand by the experimenter. The child has to "guess" the number of objects in the hidden row. This is called the *inference task*;
2. After the transformation, the child counts both rows, the numbers counted are repeated and the conservation question is asked. This is called the *comparison task*.

The third problem is a classical conservation one. For each task, three different conditions are examined:

a) conservation of equinumerosity with a spreading-out transformation;

b) conservation of equinumerosity with a piling (of counters) transformation;

c) conservation of inequality.

Each child is tested on the three problems, presented in a fixed order using the same collections of objects. The inference task is first, followed by the enumeration task and by the conservation task. In the report, the order of presentation of the experimental conditions is not specified. The subjects are 85 children from age 5 to 8 years. The results indicate that the two counted conservation tasks are solved before the standard conservation task. This décalage appears clearly in the following contingency tables:

Standard \ Comparison	C	NC
C	25	0
NC	29	31

Spread-out condition

a

	C	NC
C	40	0
NC	26	19

Piling condition

b

	C	NC
C	34	0
NC	22	29

Inequality condition

c

Table 6.4: the relationship of conserving (C) and non-conserving (NC) judgments in the counted comparison and standard conservation tasks for the three conditions: a) spread-out transformation, b) piling transformation and c) unequal collections.

Inference / Standard

a) Spread-out condition

Standard \ Inference	C	NC
C	24	1
NC	13	47

b) Piling condition

Standard \ Inference	C	NC
C	40	0
NC	23	22

c) Inequality condition

Standard \ Inference	C	NC
C	34	0
NC	16	35

Tables 6.5: the relationship of conserving (C) and non-conserving (NC) judgments in the inference and standard conservation tasks for the three conditions: a) spread-out transformation, b) piling transformation and c) unequal collections.

These results constitute a perfect case of collective décalage. For both task formats and across all three conditions, the children either fail both the counted and standard tasks, or succeed both tasks or succeed the counted task, while failing the standard conservation task. Only one child (e.g. in the inference, spread-out condition) solves the more difficult standard task while failing the easier counted conservation task.

The failure in both tasks indicates that, in a first stage of development, the count of the two collections up to the same number is not sufficient for the children to confirm that the two collections have the same number nor to infer the cardinality of the hidden collection. The difference in configuration still plays a crucial role in judging numerosity. In a subsequent stage, the count information confirms the equinumerosity regardless of the difference in configuration, allowing them to infer the cardinality of the hidden collection. Nevertheless the same children do not solve the task when they are not explicitly required to count the two collections (e.g. the standard Piagetian conservation). Only at a later stage, the conservation task is correctly solved both with and without the count of the two collections after the transformation.

The comparison task appears to be solved before the inference tasks, even though the pattern of décalage is not as clear as it was for the previous pairs of tasks:

Comparison \ Inference	C	NC
C	34	20
NC	3	28

Spread-out condition

a

Comparison \ Inference	C	NC
C	57	9
NC	6	13

Piling condition

b

Comparison \ Inference	C	NC
C	46	10
NC	4	25

Inequality condition

c

Tables 6.6: the relationship of conserving (C) and non-conserving (NC) judgments in the two counted conservation tasks: comparison and inference, for the three conditions: a) spread-out transformation, b) piling transformation and c) unequal collections.

In all the three conditions, a tendency to solve the comparison task before the inference task emerges, and may be seen as further evidence that when the child has counted both collections then he can reason more adequately about numerosity.

Gréco reports that children give operational justifications for conservation in the counted tasks as they will later do in the standard task (e.g. "you did not add nor take any away", "you have just moved them"). Examples from the protocols cited by Gréco illustrate the case of children who at the same time solve the count task and fail the Piagetian task[26] :

Ves (4.9): Where is it more now? - "There (B', modified row), because it's very tall, it's a lot all these on the top of each other" - Count that (A) - "6" (wrong, A is 7) - And B', guess? - "Eight" (wrong, B' is 6) - Count B'! - "Six" - And how many in A'? - "Six" - And for B'? - "Six" - Then, where is it more? - "If we count, it's six in both, but if we do not count, it's more white (A), it's long, long..".

Tit (5.3): initially A is 8 and B is 7; B' is made into a longer row. Where is it more? - "There (B'), it's longer". - Yes longer, but the counters, are there more or less here? - "Less" - Where more? - "There" (B') - Count A - "Eight" - And B', guess how many without counting! - "Seven, I counted before" - Then is there more in A or more in B'..... - How many B' now? - "Seven" - And of A? - "Eight" - Then do we have more A or more B' - "Before we had more As, now more Bs". - How many Bs now? - "Seven" - And how many A? - "Eight" - More As or more Bs? - "More Bs, obviously!" - What is more, Eight or Seven? - "Eight, naturally". - But here, are the

[26]Instead of reporting the original french text with my English translation in the footnotes, as I have done up to now, I directly give the translation in the main text for reasons of space.

seven Bs more than the Eight As? - "No, eight is more than seven, but there, there are more reds (B')".

Cal (5.3): Is it the same now, here (A) and there (B', transformed)? - "It's not the same thing, but it's always the same number". - Why do you say that it's not the same thing? - "Because it's more down there" (the bottom row) - More of what? - "More buttons" - But is it the same number still? - "Yes, it's the same number". - Could you count the buttons here (upper row)? - "Eight" (wrong it's 7) - And there (bottom row)? - "Eight also". - But you say that there are more buttons here (bottom row)? - "Ah! Yes, sorry: then nine, I had forgot that one, pointing to the button that exceeds the length of the second row".

The décalage between the solution of counted conservation and of standard conservation has been confirmed in a study by Fuson, Secada & Hall (1983, experiment 1), who also controlled for order effects, a factor which was not taken into account in Gréco's study. In Fuson et al., the performance of a group of children from 4,4 to 5,6 years of age in a standard conservation task and in a conservation of counted sets equivalent to Gréco's is compared. The results indicate that:
a) significantly more children conserve in the count condition (11 out of 16) than in the standard condition (2 out of 14);
b) the number of conservation responses in the standard task is greater when the counting task has come first.
This second study confirms the main patterns of children who fail both tasks, solve the counted conservation and fail the standard one, and finally solve both tasks. Furthermore, it indicates that some transfer occurs between the counted procedure and the standard one, as children start using counting to compare the two post-transformation arrays also in the traditional task. The arguments given to justify the latter conservation are also based on the number counted.

In both experiments, one of the most surprising results is that a good proportion of 5-6-year-old children do not conserve number even when they have counted the two sets and have reached equivalent numbers. A study by Ginsburg (1975) examines the non-conservation strategies among children (between 2.6 and 5 years of age) who have counted the two sets both before and after the transformation. Ginsburg reports that regardless of the counts, the children still base their numerical judgments on the spatial extent of the sets, and in particular their length. Either they consider more numerous the longer row or the shorter and denser row, or alternatively, they do not seem to be using any consistent judgment strategy.

6.2.3.3 Discussion of precocious conservation

The studies on identity conservation and on conservation of counted collections identify two early forms of conservation. Together with the incidental-accidental conservation paradigm discussed in chapter 4, they indicate that under specific conditions, children younger than 5-6 years of age can maintain the numerosity of a collection, regardless of changes in the spatial size of the collection. Under these specific conditions, then, children that, according to Piaget, have a perception-bound, undifferentiated number concept, can overcome differences in shape and spatial extent and represent the equinumerosity of two collections. What are the critical conditions?

First, children can conserve number earlier when they are pushed to count the two collections and create an explicit numerical representation of the sets' number. This request, however, does not automatically entail conservation, as children at around age 4 may not conserve even after having counted the sets. The early acquisition of "quotité" sheds some light on the phase of development in which counting gradually becomes an effective, operational means of representing the cardinality of sets in practical, numerical tasks. Gréco describes this phase in terms of coordination of global set size, and "quotité", or the set's counted size[27]:

> L'extension est en effet l'indice d'une quantité qui n'est encore pas maitrisée par l'operation mentale. Assignée à l'espace, la quantité physique en est dépendente. La quotité n'est pas l'object d'une perception extensive: on perçoit la numerosité, non la quotité ou le nombre proprement dit. Et sept restent sept parce qu'il n'y a pas de raison pour qu'ils aient augmenté ou diminué (1962, p.68).

Second, children also appear to conserve the numerosity of a single set, which goes through spatial transformations, before they can conserve the equinumerosity of a pair of sets, one of which is transformed. They consider that the transformation carried out on one set is irrelevant with regards to the set's number, and elaborate, according to Elkind, one of the building blocks of the later (equivalence) conservation concept. Third, children not only conserve number identity and number equivalence after counting, they can also conserve number equivalence without any explicit request for a count, when the transformation of the collection occurs as an accident (see chapter 4). In the next sections, the analysis of the literature on number judgment and set

[27]Extension is the index of a concept of quantity which has not been mastered by mental operation. When attributed to space, physical quantity is dependent on it. "Quotité" is not the object of a perception of extension: we perceive numerosity, and not "quotité" or number as such. And seven remains seven because there is no reason for them to be more or less (my translation).

reproduction provides some new information about what the child knows in the period preceding the appreciation of standard number conservation.

6.3 Reproduction of Sets

The reproduction task deals with the child's ability to create a set equivalent to a given (or 'model') set. It provides a direct test of the child's understanding of the basis for equivalence, i.e. to draw a one-to-one correspondence between the elements of the model set and the elements of the new set. The correspondence can be achieved either by optical, spatial correspondence or by symbolic representations, i.e. number words in counting. When one-to-one correspondence is established through counting, the model set is counted and a same number of items is counted out (of a bunch) to create the equivalent set. When it is established through spatial correspondence, an item (from a bunch) is put in front of each element of the model set. The only requirement for successful reproduction is that the copy set contain the same number of items as the model set, regardless of equivalence or difference in configuration.

Two experimental paradigms are examined: 1) the studies of spontaneous reproductions; 2) the studies of reproduction using counting.

6.3.1 Piaget's study of spontaneous reproduction

Chapter IV of Piaget & Szeminska's "The Child's Conception of Number" (1952) focuses on the development of the child's spontaneous strategies to estimate the cardinal value of a set. The experimenter tells the child: "Here are some 'objects': pick out the same number", without suggesting any specific method. The objects forming the model set are laid on the table in different configurations. The objects to construct the copy are instead in a box. The task has been presented with a range of different materials and configurations:

a) the model collection is made up of matches, while the copy has to be made with counters, or vice versa;

b) the model collection is presented in parallel rows, closed shapes (like an oval or a house), closed shapes which require a specific number of counters (like a square), random arrangements;

c) the model collection is made of six beans in a row.

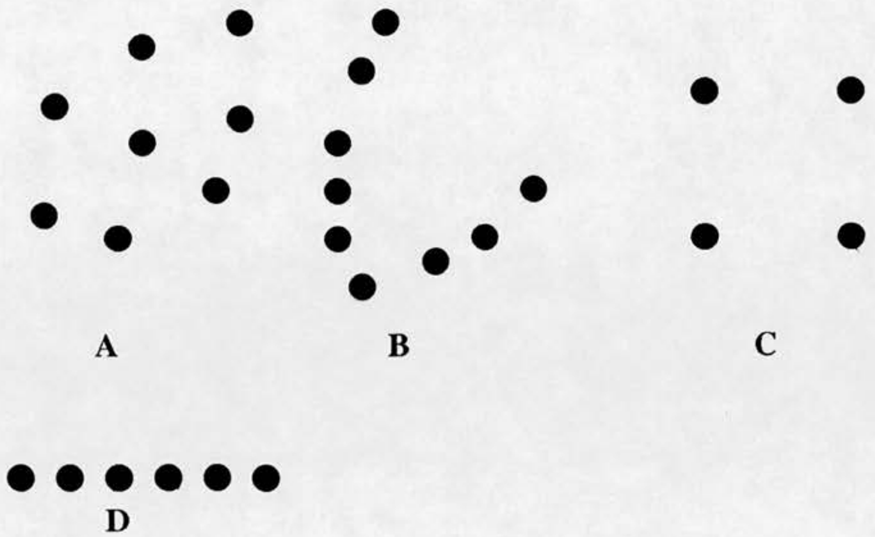


Fig. 6.3: Examples of sets' configurations used by Piaget & Szeminska (1952)

Piaget classifies the solutions offered and the corresponding strategies into two basic behavioural patterns, or substages.

6.3.1.1 Substage 1

The solution offered by Substage 1 children (around age 4) is based on global qualitative copies and not on numerical procedures. Some children, for example, take a small handful of objects from the box and try to arrange them so that they look like the model. Hug (5;0), once the copy is finished, answers the question of how he knows that the two collections have the same number of items, "I looked twice (once the model and once the copy). It's right." Most often children use more elements than necessary, especially by placing them very close together. As the overall shape is reproduced, they conclude that "there is the same number of counters" (Mul 4;1).

Reproducing collections arranged in rows, Piaget reports, children tend to reproduce the length of the row rather than to establish a correct correspondence. Piaget provides an illustration of this behaviour with an extract from the protocol of Boq (4;7): Put as many sweets here as there are there. Those (6) are for Roger. You have to take as many as he has. - (The child makes a compact row of about ten, which is shorter than the model). - Are there the same? - "Not yet" (adding two more sweets). - And now? - "Yes". - Why? - "Because they're like that" (indicating the length).

As a result the only configurations that Substage 1 children manage to reproduce are those where the number of objects determines the shape, like the four objects corresponding to the angles of a squares. It appears that for substage 1 children, equal counts do not guarantee equinumerosity if the collections are different in shape. Piaget cites the case of Min (5;0) who reproduces a row of 8 counters with 10 counters in a row of equivalent length. He correctly counts the two rows to 8 and 10, but still maintains that the two collections are equivalent.

6.3.1.2 Substage 2

Children are at Substage 2 (around age 5) when they establish an optical spatial correspondence between the items of the model set and those of the copy set. Whereas children at substage 1 usually begin by putting a pile of counters on the table and then arrange them to imitate the model, children here begin by taking the counters one by one and by reproducing the different parts of the model. Piaget exemplifies the new strategy: Ha (4;5) who first looks carefully at the pile of 15 counters, then puts down 16 elements one at a time, copying the configuration of the model one by one. The child spontaneously checks to see that the correspondence is accurate and counts the two collections: Are they the same? - "There (copy) is bigger. I'll take some away (removing the extra counter)". - Are they the same? - "Yes". Ba (4;9) justifies the equivalence by pointing to each pair of corresponding elements by saying at each pairing: "This one and that one, this one and that one,...".

At this substage, problems emerge only when copy and model are made up of objects which are markedly different in shape (e.g. counters and matches). This suggests that perceptual correspondence is still playing some basic role in the child's equinumerosity concept.

6.3.1.3 Other studies of spontaneous reproductions

Gréco (1962) replicates Piaget's study of reproduction and adds a supplementary numerical reasoning question. After the reproduction is achieved, the experimenter covers one of the two collections with his/her hand and asks the child to guess the number of objects in the hidden collection. Piaget's description of the development of reproduction in two substages is further decomposed into four substages:

Substage 0: the child does not seem to understand the problem. His actions appear to essentially playful and there is even no attempt to reproduce the figure;

Substage1: the child does a global copy of the collection which respects its spatial dimensions. When the child is asked to guess the number of objects of the hidden collection, he generally (85%) does not count the visible collection;

Substage 2: the child constructs the copy by systematic matching of the elements. Children make spontaneous use of counting to infer the number of elements in the hidden collection;
Substage 3: the child counts the model collection and takes an equivalent number of objects to construct the copy; the shape of the collection is rarely reproduced.

The distribution of children in the four behavioural categories is presented in the following table:

Age group (years)	**Substage**				**Total**
	0	1	2	3	
4 - 5	**6**	**12**	**2**	**0**	**20**
5 - 6	**1**	**4**	**13**	**7**	**25**
6 - 7	**0**	**5**	**4**	**11**	**20**
7 - 8	**0**	**0**	**4**	**16**	**20**

Table 6.7: Contingency table of the number of children in the four substages as a function of age (Gréco, 1962, p.42-43);

The increase in the number of children who reproduce the set correctly and infer the cardinality of the hidden set, by counting the number of the visible set, increases steadily with age. While 90% of the younger children fail the task (e.g. substages 0 and 1), reproducing the configuration rather than the number of the collection, only 15% of the children from age 5 onwards fail the tasks. The group of the 5-, 6-year-olds children tend to reproduce the collections using spatial matching, while the older groups more generally reproduce the set's number using counting.

Similar observations are reported by Comiti, Bessot & Pariselle (1980) in a series of experiments on the reproduction strategies that children employ spontaneously and on their efficacy. In their experiments, the reproduction task is followed by a verification question (e.g. "Are you sure?") and a comparison question (e.g. "Is the number the same or is it different?). The sample examined consists of children from the Cours Préparatoire (C.P., from age 6 to age 8 years) and Cours Elémentaire (C.E., from age

8 to age 10 years). The analysis of the spontaneous strategies indicate a clear difference between the two groups:

1. C.P. children use: counting (47%) , matching (38%) and perceptual estimation (15%);
2. C.E. children use counting (79%) and matching 21%.

A second experiment examines the change in the reproduction strategies as a function of the children's familiarity and expertise with the size of the collection to be reproduced. 68 children from Cours Préparatoire classes are tested on reproduction of *large collections* (37 items) in two sessions separated by three months. In the three month interval, the teaching program introduces the children to numbers bigger than 20. The following table presents the frequency with which the strategies of counting, spatial correspondence and perceptual copying are employed in the first test in March and in the second test in June. Their distribution is compared with that obtained in the Cours Elémentaire group of the previous experiment.

Strategy	March C.P.	June C.P.	C.E.
Counting	13	16	79
Matching	53	71	21
Perceptual Estimation	34	13	0

Table 6.8: Percentages of children for reproduction strategy categories in the first and second trial of the C.P. group and in the C.E. group of the first experiment (from Comiti et al. p.205, 1980)

The teaching program followed during the three month interval between trials does not lead to a significant increase in the use of the counting strategy . This strategy is rarely used and the 6-, 8-year-olds, especially when compared with the 8-, 10-year-olds . The overall improvement in the accuracy of the reproductions is essentially due to the more frequent use of spatial matching, which replaces reproductions based on perceptually estimated copies.

Comiti et al. report that in the first trial, when asked to verify their reproductions, the children who used perceptual estimation strategies in reproduction either confirm the equivalence or tend to modify the collections to make them look even more similar in shape. Among the children who used correspondence, the majority confirm the equinumerosity, often pointing to the one-to-one pairing of the elements of the two collections. The majority of children who succeed at reproduction, later confirm the equivalence. Only eight children do not seem confident and check the numerosity either by counting or by making the spatial pairing more precise. Among the 41 children who fail reproduction, 23 are able to correct using matching.

On the second trial, the children's strategies evolve as follows:

1. 9 remain perceptual estimators (against 24, three months before); 7 stay at the same level and 2 regress from matching to perceptual estimation;
2. 17 children move from perceptual estimation to matching (14) and counting (3);
3. more children (41 against 28) use matching spontaneously;
4. a similar number of children (12 vs. 14) use counting;

36 children out of 68 correctly reproduce the set. Among them 35 do so by matching. The following table presents the proportions of successful reproductions for the two strategies of counting and matching among the two C.P. groups in March and June and in the C.E. of the previous experiment.

Strategy	March C.P.	June C.P.	C.E.
Counting	.11	.10	.40
Matching	.75	.83	1

Table 6.9: Proportions of correct reproductions for the two strategies: counting and matching, in the first and second trial of the C.P. group and in the C.E. group of the first experiment (from Comiti et al. p.205, 1980).

In the whole sample tested, the matching strategy appears to be a much more reliable means to carry out accurate reproductions of large sets than counting. However, while this strategy is more common among younger children, older children favour the counting strategy. In the next section, I examine the studies on reproduction tasks which require the use of counting.

6.3.2 Studies of reproduction with counting

Along with spontaneous reproduction strategies, Fuson (1988) and Saxe (1977,1979) investigate children's responses when they are explicitly asked to count the model set. As in the studies on spontaneous reproduction, children here first reproduce the set configuration, and later make use of the count information.

6.3.2.1 The studies by Fuson

Fuson (1988) designed two variants of the reproduction task: one in which the model set is physically present, e.g. the "make an equivalent set" task, and one in which only the cardinal value of the model set is given, e.g. the "make a set of n" task.

In the "make an equivalent set" task, the children (between 4 and 6 years of age) are presented with a row of irregularly spaced (7, 9, 16 and 18) chips and are asked: "give me as many blue chips as there are red ones", from a pile of 20 blue chips. In the "make a set of n" task, a pile of chips is placed in front of the child, who is asked "Give me n poker chips" (sets of 7, 8, 17 and 19 chips). The results indicate that children's reproductions are:

a) Accurate less than 50% of the time before age 4,6;

b) Accurate above 75% of the time after age 5,6;

c) Dependent on set size, with sets bigger than 10 leading to a significantly worst performance;

d) More accurate in the "make a set of n" task than in the "make an equivalent set task".

In the "make an equivalent set" task:

a) 60% of the matching strategies[28] follow one of these patterns:

- Put-near match, the child moves a blue chip in front of each of the red chips, (more frequent);
- Look-match, the child looks at one red chip after the other and simultaneously puts a blue chip into a pile; eye fixation and pointing accompany this behaviour.

b) 20% simply count;

c) 20% either make a pile of chips (one-by-one or by handfuls) without looking at the model set, or make a row of chips (at the edge of the table or along the row).

In the "make a set of n" task:

a) 82% of the solutions rely on counting out a set of n;

b) 23% of the strategies are non-numerical: children simply push some chips on the table.

[28]The younger group (3.6 to 4.6 year-old) matchings were correct only 50% of the time. Often children put one chip too few or too many, or begin with correct matching and at some point started to put the chips down closer together for the rest of the row.

In her analysis of the errors, Fuson reports that most incorrect reproductions are due to the fact that children do not use the model set nor the cardinal value of the set as a limit to their activity and keep on adding blue chips until they have made a pile of all 20 blue chips. In the first condition, this behaviour is reported for 13 of the 24 younger children. In the second condition, the same behaviour is found among 12 young children.

6.3.2.2 The studies by Saxe

In a series of experiments, Saxe (1977, 1979) compares children's performance in two task conditions:

1.The model set is visible on the table in front of the child;

2. The model set is on the floor, in a position which does not permit the child to see the model set when he is constructing the copy set.

This second condition, by excluding the possibility of matching, requires the use of counting to transfer the numerical information from the model to the copy set. The reproduction has to be carried out either by putting the same number of objects on the table or by drawing them on a piece of paper. Throughout the testing session, and for both experimental conditions, the experimenter frequently suggests to the children that they should count the items (e.g. "Would counting help you?").

Saxe classifies children's strategies (between age 3 and 7) in pre-quantitative (level 1, subdivided in sub-level 1a and sub-level 1b) and quantitative (level 2, subdivided in 2a and 2b). The classification does not take counting accuracy into account.

6.3.2.2.1 Level 1

Level 1 strategies are those where counts are not used as a means to reproduce sets. At *sub-level 1a*: the child first makes an approximate copy of the model without counting. When explicitly asked to do so, he counts only the copy. Saxe reports examples like: the child makes a semicircle of 15 beads opposite the model. The experimenter suggests counting and the child counts to 14, gesturing back and forth over the model and the copy. Following the count, no corrections are made. Another child draws 11 circles, filling the page; after the suggestion he counts the copy to 10. In the situation where the model is on the floor, the child puts all the available objects on the table, counts them to 10 and, asked whether model and copy sets have the same number, answers: "yes, because I counted them".

At *Sub-level 1b*, the child makes an approximate copy of the model without counting. After direct suggestion, he counts both collections separately, but does not use this information to verify or correct his copy. Examples of this pattern are: a child puts six beads in a straight line opposite the model but not in one-to-one correspondence. After the suggestion, he counts: "you have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. I have 1, 2, 3, 4, 5, 6, 7." Even though he has reached different numbers, he does not modify the copy. In the drawing task, a subject draws 12 circles, filling the page; then counts the copy to 7 and the model to 11 and says "they're the same number". In the reproduction of the far collection, another subject places the entire available set on the table; after the suggestion he counts the objects on the table to 14 and the objects on the floor to 6 and confirms the equivalence justifying it: "yes, because I counted".

6.3.2.2.2 Level 2

Counting is used to produce numerical reproductions of arrays. In *sub-level 2a*: the child makes an approximate copy of the model without counting and then counts both collections separately. A trial-and-error method is used to attain the numerical equivalence between model and copy. Illustrations are: a child who makes a reproduction of 15 beads and matches the end points with the model. Asked whether they have the same number, he answers that they have not and makes a series of additions and subtractions interspersed with counts until the equality is achieved. Similarly a child draws 11 circles in a random order and, after the suggestion, counts both the model and the copy. He remarks the difference, draws the copy again and after a series of counts, erasures and recounts, produces a correct copy.

At *Sub-level 2b*, the child counts the model and then produces a numerically equivalent copy, or produces an approximate copy and then systematically adjust it through counting to attain numerical equivalence with the model. Typically children count the model collection and then count out the same number from the pile and place those opposite the model. Otherwise they count out the same number of circles while drawing them.

The same sequence of behavioural patterns from pre-quantitative to quantitative reproductions is observed across the task conditions. Ninety per cent of the 3-year-olds use strategy 1a; 4-year-olds are evenly distributed in the four sub-levels while 7-year-olds are consistently using strategy 2b. Overall, the situation with the model on the floor tends to be more difficult and a steady rise in accurate counting is found between sub-levels 1a and 2b.

In a subsequent, longitudinal study, Saxe (1977) follows the development of 9 of the 3-year-olds of the previous study. Whereas in the first test these children were all using pre-quantitative strategies (5 were at sub-level 1a and 3 at 1b), after 12 months all children use more advanced strategies: 5 are at sub-level 1b and 3 at sub-level 2a. Counting accuracy also improves. When the same children are tested after 6 more months, all children use quantitative counting strategies and all but one consistently count accurately.

6.3.3 Discussion of the development of reproduction

The results from the studies of the development of set reproduction identify three clearly distinct behavioural patterns:

1. Before age 5, children do not reproduce the cardinal number of a set of objects. They make global copies of the set's configuration without noting the number of elements used. In particular, they tend to put a handful of objects on the table and rearrange them to look like the model set, to use all the objects at their disposal, or to establish a precise correspondence but only for some parts of the configuration (e.g. the end points of a row or the angles of a closed figure). When the children are explicitly asked to count the model set before starting the reproduction, they do not make use of this numerical information in their reproduction. When instead they are asked to count the model and the copy set after the reproduction has been carried out, they count one of the sets only, or they count both sets separately but still conclude that they are equinumerous. As Saxe notes, in this first stage, counting information does not seem to have any quantitative meaning for the child. Thus, in Gréco's inference task, the children do not infer the cardinality of the hidden set, even when they are invited to count the visible set.
2. Between age 5 and 7, children reproduce the set correctly (e.g. 75% of the children older than 5,6 provide accurate reproductions in Fuson's study). They generally construct the copy by one-to-one correspondence (placing one object in front of each of the model set's elements until all the elements of the model are paired). The copy is also produced using counting, but more rarely. The cardinal number of the model is calculated and an equal number of objects is taken to build the copy. In Gréco's inference task, the children who have created the copy set by spatial matching solve the task by counting the visible set and saying that the hidden set has that same number. The shift taking place between stage 1 and 2 is reflected in the data of Comiti et al. on the reproduction strategies. In the group between 6 and 8 years of age, 15% of the responses are spatial copies, whereas none of the 8 to 10 years group uses this

strategy. Matching decreases slightly, while counting increases significantly as the favoured strategy. 79% of the children from the 8 to 10 years group use counting in their reproduction, versus 47% in the 6 to 8 group. In the older group also, the rate of corrections after inaccurate reproductions is high. Saxe notices that in this period, the task is solved by a trial and error method whereby the child puts down a bunch of objects, then counts the model and the copy, adds (or subtracts) one element from the copy, then counts the two sets again, adds (or subtracts) another object, counts again, and so on until the counts coincide.

3. After age 7, children solve the reproduction task using a systematic counting strategy. They count the model, and count the same number of objects to create a copy which can have a very different configuration. This pattern corresponds to Gréco's substage 3 and to Fuson's level 2b.

6.4 Relative Number Judgments

The number comparison task probes the understanding of equinumerosity in the context of determining whether two given sets of objects have the same number of elements or not. Whereas the reproduction task requires children to establish a match between elements of the model set and elements of the copy set, the comparison task requires the child to determine whether the matching between the elements of two given collections already holds or whether one is larger and has unmatched elements.

The standard comparison paradigm consists of showing children a series of stimuli (e.g. two rows of equal-size dots or counters) and of asking them to make a cardinal number judgments: "Is there the same number of 'objects' here and there (or the same number of reds and blues), or is it different?"; "I want you to look carefully and choose the row with the most marbles". Siegel (1971) introduces a variant of this paradigm with a matching to sample task, in which children are asked to choose among four possible cards, the card which has the same number of objects as the target collection. Non-verbal variants of the task consist of learning situations, in which judgments on the basis of number, as opposed to spatial extent, size of items, etc. are taught.

The development of comparison capacities was first studied by Binet (1890). Binet asked a group of 4-year-olds to judge whether two parallel rows of counters contained the same or a different number of elements. One row consisted of 15 large counters, the other of 18 small counters, so that the more numerous row was shorter than the less numerous one. The children consistently judged the longer row as having a greater number of counters. Even when Binet took some of the larger counters away, so as to reduce the length, the child maintained the same judgment, until the smaller row was also shorter. At the same time children were always correct when they had to judge rows of counters of equal size and equally spaced. From these results Binet concluded that 4-year-old children base their numerical comparison on a holistic estimation of the space occupied by the collection, instead of its cardinality.

These findings have been consistently confirmed: Wohlwill (1963), Siegel (1971, 1973, 1974, 1982), Pufall & Shaw (1972), Pufall, Shaw & Syrdal-Larsky (1973), Brainerd (1973,1977,1978), Lawson, Baron & Siegel (1974), Baron, Lawson & Siegel (1975), Smither, Smiley & Rees (1974), Estes & Combs (1966), Estes (1976), Saxe (1979), McLaughlin (1981), Fuson, Secada & Hall (1983), Michie (1984a, 1984b), Cowan (1984, 1987), Sophian (1987). The development of number comparison in these numerous studies can be summarized in the two behavioural patterns which follow:

1. Before age 6, children recognize that two rows are equinumerous when they have not only same number, but also same length and density. Similarly, children correctly judge a difference in number, when the two rows have different number as well as different length (or density), and the longer (or denser) row is also the more numerous. When instead number does not co-vary with the length or density dimensions of the row, the children do not make accurate judgments of numerosity. For instance, when the rows have:

a) the same number of elements, but different length and density, the longer row is judged as having more elements (less frequently the denser row is chosen);

b) different number if elements but equivalent length, the two rows are judged to be equinumerous;

c) different length and number, with the shorter row more numerous, the longer row is consider more numerous.

2. After age 6, children judge the two collections on the basis of number alone. So two collections are equinumerous if their respective counts produce the same cardinal number, regardless of differences in spatial extent.

As illustration, consider the results from the studies of Pufall & Shaw (1972):

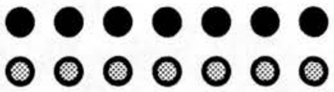
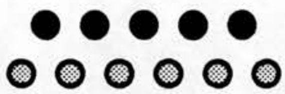
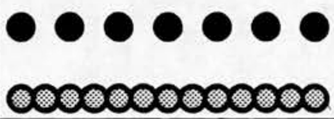
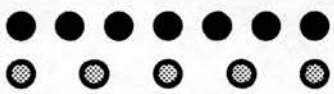
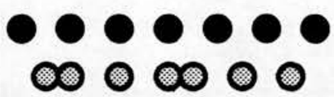
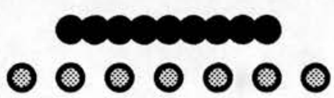
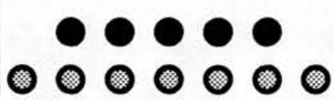
Configuration	Type of relation	Age: 3	4	5	6
	same density same length same number	.42	1.0	1.0	1.0
	same density different length different number	.9	.85	.95	.95
	same length different density different number	.58	.17	.29	.63
	as above	.84	.40	.63	.92
	same number different length different density	.11	.04	.02	.10
	different length different density different number (shorter row more numerous)	.53	.19	.46	.88
	as above (longer row more numerous)	.89	.90	.98	1.0

Table 6.10: Proportion of correct judgments as a function of age and arrays number and configuration (length and density) (from Pufall & Shaw, 1972, p.63-65)

Leaving beside the column corresponding to the 3-year-olds' responses for the moment[29], we observe very high accuracy across age groups in the judgments of arrays which have same length, density and number (e.g. top row); same density, different length and different number (e.g. second row from top; the longer array is also the larger); different density, length and number; the longer array is the most numerous (e.g. bottom row). We observe significantly higher performance between

29 The studies of conservation in 3 and 4 year-old children reviewed in Appendix 6.2, and in particular the replications of Mehler & Bever (1967) study, have shown that the judgments of children at that age are not reliable, as they are based on contingent strategies, such as repeating the last comparative term uttered, choosing the same array each time, or the array of same colour, etc. Their interpretation hence is very problematic and will not be addressed in this dissertation.

the 4- and the 5- and between the 5- and 6-year-olds groups when the arrays have same length but different density and number (e.g. third and fourth rows from top); and when the arrays have different length, density and number and the larger array is shorter than the less numerous one. The proportion of correct judgments is constantly high among the 6-year-olds (e.g. between .63 and 1.00, mean = .78), except for cases of two arrays of same number and different lengths (e.g. proportion of .10 correct responses, not different from the younger age group). This same developmental pattern (with only some variations on the size of the rows and the length, density differences are introduced) has also been generally confirmed in studies which set the numerical judgments in a learning context or in a matching-to-sample task.

I now turn to the question of the role of counting on numerical comparison accuracy. Do children spontaneously use counting to compare, and, when required to count, do they make use of this information regardless of differences in spatial size between the collections?

Cowan (1987) examines how children compare rows whose cardinality is within the range of children's counting competence and whose length provides a conflicting cue to number. In four experiments, children from 3 to 5 years of age, whose counting competence is assessed for consistency, are tested using two comparison procedures:

1. The child is asked to count each row and then to judge whether the two rows have the same number or whether one of them is bigger;
2. The experimenter does the counting, and the comparison question is put once the two collections have been counted.

This second condition is designed to control for the fact that children may not use counting to compare because they lack confidence in its accuracy and reliability, whereas the experimenter's count ensures the accuracy.

Cowan reports that very few competent counters aged under 6 years judge correctly on the basis of their counting when conflicting cues are present. Before that age, counting two rows of different length up to the same number does not convince children that the two sets are equinumerous. Instead the longer row is generally taken to have more elements.

In a first experiment (1984a), Michie reports that children 3- and 4-year-olds do not spontaneously use information from counting to make numerical judgments, unless they are explicitly asked to do so; instead they systematically base their judgments on the length cue. In a follow-up experiment (1984b), Michie investigates two factors that can play a role in the spontaneous use of counting:

a) the presence of the misleading length cues. To eliminate this factor, the elements of the two sets to be compared are counted and put into opaque boxes;

b) the role of memory. A visual display of numerals is used to indicate the last numeral counted.

The child is then provided with the numerals representing the cardinality of the sets, either with or without the set of objects present in view. Performance is significantly improved when the sets are hidden in the boxes and the numerals representing their cardinality are visible. The improvement is not significant when, both numerals and sets with conflicting arrangements (e.g. the longer row has fewer elements) can be seen. From these results, Michie draws a rather paradoxical conclusion:

> The result of their counting may be played down by children, not because they lack an understanding of number, but because they judge it to be less reliable than other sources of information relevant to number (p.356).

How can someone who understands number prefer the spatial size of two rows over the cardinal size to judge about numerosity? It seems rather that young children are capable of using the counting information only when this is the only information available about the two collections (e.g. they are into two opaque containers). When the younger child has information both about number and spatial extension, he will base his numerical judgments on the latter. In other words, Michie's findings indicate that young children can accurately judge the numeral "6" to be bigger than the numeral "5", while at the same time they conclude that a longer row that they have counted to "5" is more numerous than a shorter row that they have counted to "6". Observations of this kind are also reported in studies by Sophian (1987) and Saxe (1979).

6.5 Relationship between task acquisition

Very few studies have addressed the question of the order in which the tasks of set reproduction, comparison and conservation are solved. Gréco (1962) and Piaget & Szeminska (1952) examine the same children in both the reproduction and the conservation tasks and report that reproduction is correctly solved before the conservation, with a décalage of around two years. The substage 2 in Piaget's analysis, for instance, corresponds to the capacity to reproduce the collection while failing the conservation test when one of the collections is spread out.

Saxe (1977, 1979) examines all three tasks and finds that although the same transition from pre-quantitative to quantitative procedures is observed across each of the tasks, it occurs in comparison (his sublevel 2a) than in reproduction (Sublevel 2b), and in these two tasks than in conservation.

6.6 Conclusions and directions for new research

The extensive research on cardinal number development presented in this chapter indicates a radical change in the understanding and use of numerical relations between age 3 and age 7. Two periods can be distinguished. Before age 5-6, children systematically estimate the size of a collection of objects on the basis of its spatial extent. Hence, when they are asked to create a collection equivalent to a model one, they tend to reproduce the overall configuration rather then the cardinality of the model. Similarly when they are asked to judge the numerosity of two collections, they indicate as more numerous the collection which takes up more space, and particularly the one which is longer, taller or larger. When the collections have an equivalent spatial size (same length and similar density), they are also considered equinumerous. In this period children can count up to 20 with good consistency, but, they do not use counting spontaneously as a strategy for solving these quantitative problems. When they are explicitly required to do so, either they count one of the collections only, or both collections, but do not draw any conclusion about equinumerosity on this basis. Only the study by Michie obtains precocious success in set comparisons by making the children count the two collections into opaque containers. In this situation, the children do not have information about the spatial extent and indeed base their judgments on the counts. In this period then, children have some knowledge of number (e.g. they know a good portion of the number word sequence, they can count at least up to 20, they know that "5" is bigger than "4", etc.). However, they do not know that to reproduce a row of four they have to count out four elements, or match each element of the row with one object, and that to compare two rows they can either count them or put their elements into one-to-one correspondence to see whether one is bigger.

Between age 5-6 and age 7, children learn to reproduce and compare sets' cardinal numbers independently of their configurations. They can also confirm that a set's number is not changed when its configuration is modified (identity conservation task). They can confirm that two collections are equinumerous regardless of spatial modifications under two conditions: when the spatial transformation occurs as an accident and when they are required to count the two collections again before judging about their equinumerosity. However, only after age 7 can children conserve number in the standard Piagetian task, where no counting is asked and the transformation is performed by the experimenter.

The fascinating aspect of these results is that the same kind of developmental change is observed for each task at different periods. In each case, children evolve from evaluations of size based on the spatial dimensions to systematic numerical strategies that employ one-to-one correspondence and counting. The change takes place between ages 3 and 5-6 for reproduction, comparison, identity and counted conservation. It occurs between ages 5-6 and 7 for the standard Piagetian task. In other words, the children seem to go through a period in which they can construct cardinal representations, which are completely independent from spatial features of the set, for a particular class of situations. These same children however "regress" to pre-numerical representations when they are faced with the Piagetian conservation task. The range of tasks solved in the period preceding standard conservation are indicative of the numerical competence that children have before age 7, a competence which clearly exceeds the Piagetian account of the pre-operational number concept as irreversible and perception-bound. Children can in fact construct cardinal representations and draw inferences about equinumerosity, difference and conservation, even before succeeding the standard conservation task.

The reconstruction just given, however, can be only indicative of the kind of changes occurring in development, as it is based on the ages in which the various tasks are consistently solved, on different samples, in different countries with different teaching programs, etc. More precise data about the order in which the tasks are solved is needed. In the next two chapters I investigate this order using the hierarchical analysis method described in chapter 5. In particular I shall address the following questions:

1. The comparison and reproduction tasks appear to be acquired in the same period. Saxe (1977, 1979) however suggests that comparison is solved with a slight anticipation on reproduction. Are the reproduction and comparison tasks solved concurrently or does a collective décalage exist between the solution of comparison and reproduction, as suggested by Saxe?
2. The modified conservation task appears to be mastered in the same period. Does that mean that identity conservation, counted conservation and accidental conservation are solved concurrently?
3. Can the collective décalage between modified and standard conservation tasks be confirmed by the hierarchical analysis of performance across tasks?

Precise data about the order in which the diverse numerical tasks are solved is necessary to identify stages of number competence (the tasks solved concurrently), order among these stages (the increasing range of tasks solved concurrently) and to pin down the phases of transition between stages (the task failed at one stage, solved at the subsequent stage, and the response strategies, testing procedures, general attitudes observed). This information allows us to begin to articulate a characterization of the nature of the contents upon which the cardinal number structure operates and to capture the generalization of this structure to new contents. In the next two chapters, I shall try to answer the three questions above with three experiments. In the first two experiments, reported in the following Chapter 7, I explore the relationship between the acquisition of the three main tasks of set reproduction, comparison and conservation. On the basis of the developmental orders identified, I propose a characterization of the underlying number concepts. These hypotheses are tested in the third experiment which occupies Chapter 8 and which includes some of the modified versions of Piaget's conservation task.

Chapter 7 A study of cardinal number development

7.1 General introduction to the experimental component

Chapters 7 and 8 constitute the experimental component of the thesis and the application of the theoretical framework and of the methodology presented in the previous chapters. These chapters introduce three experiments which offer a systematic investigation of the development of the cardinal number concept from the acquisition of the ability to reproduce sets to the conservation of number in Piaget's standard task.

The two experiments presented in Chapter 7 have two descriptive objectives:
a) to determine whether the development from set reproduction to number conservation proceeds in a sequence of clearly distinguishable stages;
b) to gather observations about the phases of transition between stages.
The investigation of the developmental process arises from two questions which were left unanswered by the literature on cardinal number development:

1. Does the capacity to reproduce sets precede, follow or co-occur with the capacity to compare sets?
2. Does the capacity to compare sets precede, follow or co-occur with number conservation in the Piagetian task?

Answering these questions and confirming the reported décalage from the capacity to reproduce sets to number conservation will provide the basis from which hypotheses about the evolution of the child's number concepts will be advanced. The hypotheses will be articulated in the theoretical terms of the domain-specific framework proposed. Tasks solved concurrently identify a stage of competence and indicate the contexts in which the child has worked out the relevance of the number-domain structure. From the analysis of these tasks' requirements, the objects to which the number domain structure applies are specified as well as the nature of the number concept at that stage. Tasks solved with a systematic collective décalage reflect the steps in the developmental process and identify the transition phases when the relevance of the number structure is discovered for new contexts and the number concept redefined.

The third experiment, presented in Chapter 8, constitutes a first test of these hypotheses. It examines whether the solutions of the different modified conservation tasks emerge concurrently as well as concurrently with set comparison.

The experimental component of the thesis is hence divided in two chapters. In the first, more descriptive Chapter 7, I present two experiments which try to replicate and systematize some of the data about numerical development discussed in the literature review. This first part is composed of three sections corresponding to two experiments and to the general discussion and interpretation of their results. In the second, more predictive part (Chapter 8), I test the account of the development of cardinal number competence with a third experiment which introduces new tasks.

7.2 Experiment 1

7.2.1 Introduction

Experiment 1 initiates the hierarchical analysis of the development of cardinal number by comparing the performance of children age 4 to 7 years in the three tasks of:

1. *set reproduction*, the construction of a set with the same number of objects as a 'model' set;
2. *set comparison*, the judgment of numerosity of two sets of objects which are either equivalent in number and spatial extent (especially length) or equivalent in number and different in configuration;
3. *conservation of number*, the standard Piagetian test, where one of the two sets that the child has constructed in the reproduction task and has judged equinumerous is either lengthened or shortened by the experimenter. After the transformation, the child is asked whether the sets are still equinumerous or whether they are different in number.

In the literature, the three tasks have been studied independently and on different groups of children. In this experiment, the development of the ability to solve the three tasks and the order in which the tasks are acquired is examined on the same children who take the whole battery of tasks. The cross-sectional response patterns will constitute evidence of the steps in the development of the cardinal number concept from space dominant to number dominant quantifications of sets.

The literature summarized in Chapter 6 suggests that reproduction and comparison abilities are acquired by age 5-6 years and conservation by age 7. A collective décalage should therefore exist between the solution of reproduction and comparison, on the one hand, and of conservation on the other hand. Apart from Saxe's studies (1977, 1979) which indicate that numerical reasoning appears first in comparison tasks and later in reproduction tasks, the literature does not provide any clear evidence about the acquisition order between set reproduction and comparison. Experiment 1 examines this order by presenting the reproduction task in two conditions followed by a numerical judgment:

a) 'Visible': the set to be reproduced is physically present before the child;

b) 'Hidden': the set to be reproduced is out of the child's sight and is identified only by the numeral corresponding to its cardinality. After the reproduction, the screen hiding the model row is lifted.

The two conditions differ in that a correct visible reproduction yields a copy row which is generally equivalent in number and form[30] to the model row, whereas a correct hidden reproduction yields two rows equinumerous, but different in form[31]. When, after the reproduction, the children are required to compare the two sets, now both visible and one facing the other, they are confronted with coinciding information in the visible condition (both number and length are equivalent) and conflicting information in the hidden condition (same number and different length). This unusual format is designed to establish the order in which number reproduction and comparison abilities develop.

Furthermore, since over this same period the child's knowledge of the number word sequence and of counting develops rapidly, precautions have been taken to take into account the child's expertise of enumeration and counting. The children have been pre-tested on their counting and enumeration capacities and, on the basis of the results at the pre-test, set sizes have been chosen that the children could quantify reliably. Notice that if on the one hand this strategy introduces an element of variation, i.e. different children are exposed to sets of different cardinality, on the other hand all the children are faced with similar demands on their representation of numerosity. Small sets of three or four objects may in fact be just within the span of the children's counting skill at age 4, but may be very easy to handle, and may even function as units, for the 6 or 7 year-old child. For the older children, instead, the quantification of a set of eight or nine objects may present the same degree of difficulty as the small set for the younger child. In other words, the strategy of determining the size of the sets that the child is asked to reproduce, compare or conserve from a pre-test of counting skill should put all the children in a comparable situation. The choice of factoring out counting

[30]According to the literature, when the model set is present children spontaneously use spatial pairing to reproduce the set and, when correct, obtain sets which have both equivalent number and length (75% of children between 3 and 6 years according to Fuson 1988).

[31]Children do not have any information about the shape of the set and their reproduction is exclusively based on the numeral representing the cardinality of the set, without guarantee of similarity in shape.

expertise by using different sets, adapted to the children's capacity, also respond to the preoccupations expressed both by researchers, like Gelman (1972b), who distinguish between the capacity to create numerical representations and to reason with these representations, and by researchers, like Bovet et al.(1981), who consider that reasoning with small sets does not involve the same kind of processes (e.g. perceptual estimation or subitizing) as reasoning with medium-size and large sets.

7.2.2 Objectives

The objective of the study is to determine whether the solutions of the three tasks by the same children appear:

1. *concurrently* : either a child succeeds at both the tasks or fails them;
2. with *collective décalage*: a child solves some tasks while at the same time failing some other task;
3. with *individual décalage*, some children solve some tasks before others, while other children do the opposite.

Collective décalage between reproduction-comparison and conservation (e.g. a child may succeed in both reproduction and comparison tasks and then fail conservation, whereas no child who succeeds in conservation fails in reproduction or comparison) would indicate that the conservation task introduces aspects of the number concept which are *critical* for children who at the same time operate adequately with number in the context of the tasks of reproduction and comparison. The pattern of collective décalage would thus identify two stages of number concept development, corresponding to the competence underlying the ability to reproduce and compare sets, and to the competence underlying conservation; and a transition period (reflected by typical responses) when the understanding of conservation is elaborated.

Moreover children's number judgments after the two reproduction conditions (Hidden and Visible) can reveal whether the equinumerosity at the basis of accurate reproduction is systematically confirmed in a subsequent accurate comparison, regardless of the difference in the form of the sets, or whether accurate reproductions are followed by inaccurate comparisons when the spatial and numerical sizes do not coincide. Collective décalage between reproduction and comparison would identify two stages of numerical competence: a first stage in which the children can only reproduce sets and a second stage in which they can compare sets both when the numerical and spatial sizes coincide and when they do not. Notice that this sequence parallels stages in the acquisition of Piagetian conservation. In the conservation task, however, the conflict between number and spatial cues is a consequence of a transformation carried out by the experimenter, while in the comparison, the conflict is a by-product of the strategy used to construct the equivalent sets and no transformations are involved.

To summarize, the two conditions of the reproduction task followed by the comparison and conservation tasks confront the child with the problem of establishing and confirming the equinumerosity of two collections. The three tasks thus introduce similar contexts in which to probe the coherence and generality of the child's concept of cardinal number and in particular his ability to create a one-to-one correspondence between the elements of two collections and draw inferences about the collections' equinumerosity or difference[32], independently from the spatial cues to size.

7.2.3 Hypotheses

The literature on cardinal number development predicts that:

Hypothesis 1: there is a collective décalage between the solution of the reproduction and comparison tasks on the one hand and the later solution of the conservation task on the other hand.

Correct performance on the conservation task should be more strongly associated with correct performance on the reproduction and comparison tasks than with incorrect performance on these tasks, as in figures 7.1a, 7.1b.

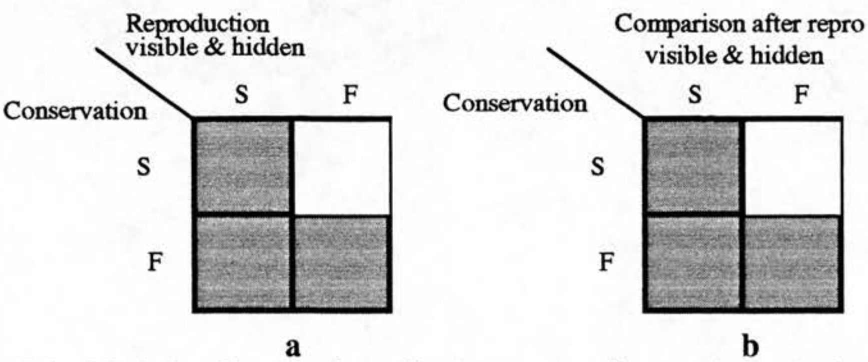


Fig. 7.1: Models of collective décalage according to hypothesis 1: a) between responses to the conservation and reproduction, b) between responses to conservation and comparison (white cells are the cells predicted to be empty).

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[32]The different tasks are presented in a related way so as to avoid that the comparison task be a mere perception of numerosity task (see chapter 6 section 6.4). By requiring the comparison after the set reproduction, the child has two sources of information on which to base his number judgment: 1) the original equinumerosity established in the reproduction task; 2) the perceptual estimation of size based on the distribution of the collections.

The developmental order between the tasks of reproduction and comparison will be investigated via the hidden condition of reproduction. After hidden reproduction task, the length difference between rows should create a conflict between numerical and spatial size and should introduce a new critical dimension (or conflict) in the comparison task that neither the reproduction task nor the comparison after visible reproduction (same length and same number) contain. Because of the conflict, the comparison of a pair of rows of different lengths constitutes a more complex operation than the reproduction of an individual row or the comparison of rows which coincide in number and length:

Hypothesis 2a: there is collective décalage between the solution of the reproduction tasks and the later solution of the comparison task after *hidden* reproduction task.

Correct performance on the comparison after hidden reproduction task should be associated with correct performance in the hidden reproduction task: no one should fail the former who succeeds at the latter, as in figure 7.2.

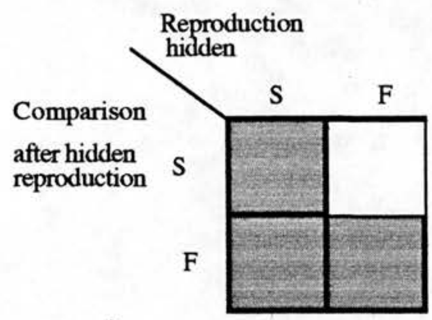

Fig. 7.2: Model of collective décalage between responses to hidden reproduction and to comparison after hidden reproduction according to hypothesis 2a.

Hypothesis 2b: the solution of the reproduction task in both visible and hidden conditions is concurrent with the solution of the comparison after visible reproduction task.

Correct performance on the comparison after *visible* reproduction should be strongly associated with correct performance in both reproduction tasks; incorrect performance on the comparison after visible reproduction is strongly associated with incorrect performance in the visible reproduction, as in figure 7.3.

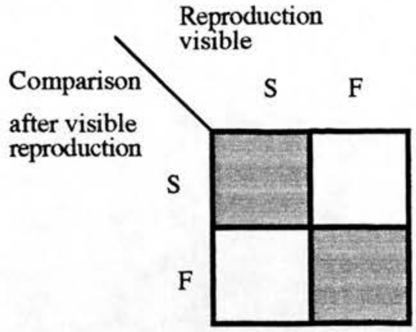

Fig. 7.3: Model of concurrency between responses to visible reproduction and to the following comparison according to hypothesis 2b.

7.2.4 Design

Each subject was presented the tasks in the following fixed order:

1. Number Reproduction: A Visible, B Hidden
2. Number Comparison
3. Number Conservation

Two variants of the hidden condition of reproduction (B) were used:

- 'Seen' condition (B1): the subject saw and counted the model set, before it was Hidden behind the screen;
- 'Unseen' condition (B2), the subject did not see the model set at all and was only informed of its cardinal number.

No specific hypotheses were attached to either of the two B conditions which were of an exploratory nature. The series was repeated three times for each child starting with the reproduction in the visible condition (A), where the set to be reproduced, or model set, was on the table before the child. In the next reproduction-comparison-conservation series, the child had one of the two hidden conditions (B), where the model set was behind a screen and was denoted by the numeral corresponding to its cardinality. In the final series, the other B condition was used. A always preceded B conditions. Half the subjects received the seen condition (B1) first and unseen (B2) second, while the other half did the opposite. I chose to start always with the visible condition A in order to let the children display spontaneous procedures of quantification. Starting with condition B, might have in fact strongly induced the use of counting also in the visible condition of reproduction. The orders of task presentations are summarized in the following schema:

Order 1

First trial	Second trial	Third trial
Repro visible (A)	Repro hid seen (B1)	Repro hid unseen (B2)

Comparison	Comparison	Comparison
Conservation	Conservation	Conservation

Order 2

First trial	Second trial	Third trial
Repro visible (A)	Repro hid unseen (B2)	Repro hid seen (B1)
Comparison	Comparison	Comparison
Conservation	Conservation	Conservation

The number and the type of objects were different at each trial to reduce repetition effects. The following *independent variables* were manipulated:

- tasks:
    - reproduction visible, hidden seen, hidden unseen;
    - comparison after reproductions visible, hidden seen, hidden unseen;
    - conservation after reproductions visible, hidden seen, hidden unseen.
- schooling/age group:
    - Nursery (between age 4 and 5)
    - Primary 1 (between age 5 and 6)
    - Primary 2 (between age 6 and 7)
- order of presentation of the hidden conditions (B): seen (B1) first or unseen (B2) first.

Dependent variables were the number of correct responses in each task and the contingencies between correct and incorrect responses in pairs of tasks.

In terms of statistical design, this experiment was within subjects (repeated measures), with subjects nested in age groups. In the within-subject dimension, I compared the same children across the whole set of tasks; while in the nested, between-subject dimension, I compared the performance of the age groups in each task and of single age groups across tasks.

7.2.5 Statistical analysis

The mixed design requires different statistical treatments. The order in which the tasks are solved can be analyzed at two levels:

1) at the level of age groups, by comparing the frequency of correct-incorrect responses to each task across the three age groups (e.g. significant difference in the number of success-failure responses between Nursery, Primary 1 and Primary 2 children) and by comparing the frequency of correct-incorrect responses within each

age group across task conditions (e.g. significant difference in the number of success-failure responses to reproduction, comparison and conservation tasks);
2) at the level of the whole sample, by looking for patterns of concurrency, collective décalage or individual décalage in the responses to each pair of tasks.

As I pointed out in section 5.3.2, both kinds of analysis are equally necessary since the hierarchical analysis, which tests more directly the hypotheses of order, necessitates a previous delimitation of the period in which developmental changes occur. This information guides the sampling and reduces the risk of obtaining very unbalanced contingency tables, in which most children find the two tasks either too simple or too difficult. Since the children's performance is measured at the nominal level (children are allocated in the two categories of success and failure), I use non-parametric statistics for both the age group and the task conditions analyses[33]:
a) χ^2 tests to evaluate the degree of relationship between the distribution of correct responses to each task condition and the age groups;
b) Marascuilo & McSweeney test (a variant of Cochran's Q test) to evaluate whether, within each age group, the performance follows the order of complexity assumed for the task conditions.

For the more specific question of the developmental ordering among problems (e.g. concurrency, collective décalage or individual décalage), I employ the Hildebrand, Laing & Rosenthal's Prediction analysis for cross-classifications. For each hypothesis listed above in 7.2.3, I devise a triangular hypothesis which identifies the cells of the two-by-two developmental contingency table predicted to be empty. The prediction is verified when the observed Del (which gives the improvement of fit over chance when one (or two) cell(s) of the table is predicted to be empty), is significantly greater than a chance Del[34] (at the level of $p<.05$, one-tail normal curve test, with the critical value of z to exceed 1.65) and is significantly different from the Del values of the alternative ordering patterns (see section 5.3.1 for a description).

7.2.6 Materials

Three sets of objects were used: red and blue wooden cylinders (1.7 cm high and 1.7 cm of diameter), small plastic animals (pigs and hippopotamuses of similar size), orange and yellow round sweets (2.5 cm of diameter). The set of objects given to the

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[33]The statistical methods are presented in some detail in sections 5.3.1,5.3.2.
[34]As a general policy, I shall give the precise significance level corresponding to the one-tail normal curve test z. The choice of a one-tail test is justified by the fact that the theory dictates only positive Del as being desirable (see Hofmann, 1983, p.35).

child to reproduce were kept in a box. In the hidden condition of reproduction, the screen was a red, rectangular cardboard. For each subject and each condition, a different set of objects was used, chosen on a random base.

7.2.7 Procedure

The children were seen individually and first were pre-tested on their counting abilities: "do you know how to count?" and "how far can you count?". If the child missed a number word from the sequence or stopped the recitation abruptly, the experimenter asked "what number comes after x (the problematic number word)?". On the basis of the portion of the sequence which the child could recite, the experimenter decided which set sizes to use in the following tasks. In a general way, Nursery children were tested with sets of between 4 and 6 objects; Primary 1 with sets between 7 and 10 and Primary 2 with sets between 7 and 12. The pre-test was followed by the reproduction, comparison and conservation tasks, repeated for each condition of reproduction according to the order defined in 7.2.4. The series were presented in two half-counterbalanced orders: of each age group half follows Order 1 and half Order 2.

7.2.7.1 Reproduction Task

The experimenter put down a row of objects and said "here I have a line (or row) of 'objects', I give you these" (the box containing the corresponding objects, between 12 and 15) and, pointing to the row, asked the child:

- in visible condition: "I would like you to take the same number of 'objects'[35] as there are here (or as I have) and make a row (or line) with them".
- in hidden seen condition: "Could you count this row of 'objects'?" After the count, the row was hidden behind the screen:"Now I cover it up, so that you cannot see them. I would like you to take the same number of 'objects' as there are here (pointing to the row behind the screen) and make a line (row) with them here (pointing to the child's side of the screen)".
- in hidden unseen condition: without ever showing the row, the experiment said "I'm making a line (or row) with 'n'[36] objects here. I would like you take the same number of 'objects' from the box and make a row with them on this side (or your side)".

[35]In the questioning the word "objects" was replaced by the word denoting the items that were used in the test, such as pigs, hippos, candies, rounds, etc.

[36]"n" was replaced by the chosen cardinal number

7.2.7.2 Comparison Task

After the reproduction task in the hidden condition, the screen was taken away. In all conditions, the child was asked: "Is there (pointing to one row) and there (pointing to the other row) the same number of objects, or does one of the rows have more objects?" In case the child responded that the two rows were different in number, the experimenter asked which of the rows had the most and then asked the child to correct the inequality: "What can you do to have the same number here and there (pointing to the two rows)?". If the child did not know how to achieve the equinumerosity, the experimenter suggested to establish a one-to-one correspondence by placing the objects from the two rows one in front of the other.

7.2.7.3 Conservation of Number Task

Once the child had reached the conclusion that the two rows had the same number of objects (by a correct judgment, by an incorrect judgment of equinumerosity for unequal sets, by the correction of an initially inaccurate judgment of difference, by the suggestion of the experiment to create a one-to-one correspondence), the experimenter presented him the conservation problem: "Look what I do now"; and transformed the arrangement of one of the two rows, making it longer (in half trials) or shorter (in the second half of the trials). The conservation question followed:"Is the number of objects the same here and there (pointing to the two rows), or is the number different?". The child was then asked to justify his answer:"why is it, or why do you think so?". The experimenter then modified the other row into a shorter one and asked the same series of question.

A particular procedure was followed in the case of children who, in the comparison task, judged as equinumerous sets which were in fact different. These children were not excluded from the third part of the experiment because some meaningful distinctions could still be carried out over their conservation responses. Two post-transformation responses are possible. Consider a response of difference first. A judgment of difference could either express a "correct" appreciation of the difference existing between the two sets or an "incorrect" estimation of set size based on spatial cues, i.g. the spatial difference introduced by the transformation. In order to discriminate between these two interpretations, the first transformation to be performed made the more numerous row into a shorter one. A choice of the longer row as the more numerous was evidence of the fact that the child was basing his judgments on the spatial extent of the sets. Instead, when the shorter row was chosen, we asked the child to make the two rows equinumerous (eventually suggesting the one-to-one matching) and we started the conservation task again. Consider then a response of equivalence. Since we knew from the literature that incorrect comparisons are generally based on an estimation of the spatial size of the sets rather than their numerical size, we did not expect that many children would have maintained their "incorrect" judgment of equinumerosity after the transformation that made the two rows look markedly different. In section 7.2.10.1.2.3, we report the number of responses who conform to these three behavioural patterns.

7.2.7.4 Interviews

For the Primary classes the interviews occurred in one of the rooms of the school, in a location familiar to the children. The Nursery children were interviewed within the Nursery itself, in a quiet corner. The experimenter was accompanied by a female colleague, who filled in an already prepared protocol-schema for the session (see Appendix 7.1) and took further notes about the children's manipulations and comments. The testing sessions lasted between 5 and 20 minutes.

7.2.8 Measure

Children's performance was measured by the number of correct reproductions, comparison and conservation responses. The scoring criteria were:

1. In the <u>reproduction task</u>, the number of items of the copy set had to be equivalent to the number of items of the model set;
2. In the <u>comparison task</u>, the judgment of equinumerosity had to conform to the actual situation; in the case of incorrect reproductions, comparisons were scored correct when the child recognized the difference *and* corrected it; children who admitted that the two sets were different but who did not know how to equalize them failed the task;
3. In the <u>conservation task</u>, children were scored correct when they answered that the two collections had the same number of elements after *both* spatial transformations[37].

[37]As I argued in section 6.2.2.1, two criteria for passing the conservation test have been employed in the literature: "same" answers and "same" answers plus operational justifications. I scored the children's responses according to the two criteria and have found a small difference in the number of conserving children (see 7.2.10.1.2.3). I have thus retained the "same"answer criterion alone because it provides the less conservative measure of conservation but at the same time the most conservative measure of across-task performance and décalage, e.g. children who succeed in the conservation task and fail in the reproduction or

A third transformation decided the status of particularly insecure and unstable responses.

7.2.9 Subjects

Sixty children from age 3,9 to age 7,2 were tested. They were divided into three class-age groups of 20 children each: Nursery (mean age = 4,4; SD = .39), Primary 1 (m = 5,6; SD = .24), Primary 2 (m = 6,7; SD = .28). The children were from the same school and came from a mixed social background, with a predominance of middle-class children. Three children were not native speakers of English, though they were very proficient.

7.2.10 Results

The analysis of the results was performed at three levels. The group performance analysis examined:

a) the effect of the variable order of presentation of the two reproduction conditions (independent variable) on the number of correct reproduction, comparison and conservation responses (dependent variables);
b) the effect of the variable schooling/age group (independent variables) on the performance in each of the tasks: reproduction, comparison and conservation (dependent variables).

The task analysis examined the effect of the tasks' complexity within each age group. The hierarchical analysis examined the order of acquisition of the tasks through the cross-sectional patterns of concurrency, collective décalage and individual décalage in the solution of each pair-wise combination of the three tasks.

7.2.10.1 Group analysis

7.2.10.1.1 Order of presentation

Tables 7.2.1, 7.2.2, 7.2.3 are contingency tables for the frequency of correct and incorrect reproductions (7.2.1), comparisons (7.2.2) and conservations (7.2.3) in the

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comparison task. The justifications are examined in the qualitative analysis of the results.

two hidden reproduction conditions, seen and unseen, when the condition seen is first (Order 1) and when the condition unseen is first (Order 2). Each table is followed by the statistics χ^2 (with Yates correction) computed on it.

Response \ Order	First	Second
C	22	28
I	8	2

(A) Hidden reproduction condition seen

Response \ Order	First	Second
C	22	27
I	8	3

(B) Hidden reproduction condition unseen

Table 7.2.1: Frequency of correct (C) and incorrect (I)*hidden reproduction* responses for condition seen (A) and unseen (B) as a function of order of presentation (A: χ^2 (1, N=60) = 4.32, $0.05 > p > 0.03$; B: χ^2 (1, N=60) = 2.7, $p = 0.1$).

Response \ Order	First	Second
C	20	23
I	10	7

(A) Comparison after visible Reproduction

Response \ Order	First	Second
C	18	21
I	12	9

(B) Comparison after hidden Reproduction

Table 7.2.2: Frequency of correct (C) and incorrect (I) *comparison* responses as a function of order of presentation of the initial visible reproduction (A: χ^2 (1, N=60) = 0.72, $0.5 > p > 0.3$) and hidden reproduction (B: χ^2 (1, N=60) = 0.6, $0.5 > p > 0.3$).

Response \ Order	First	Second
C	12	16
I	18	14

(A) Conservation after visible Reproduction

Response \ Order	First	Second
C	12	14
I	18	16

(B) Conservation after hidden Reproduction

Table 7.2.3: Frequency of correct (C) and incorrect (I) *conservation* responses as a function of order of presentation of the initial visible reproduction (A: χ^2(1, N=60) = 1.06, $0.5 > p > 0.3$) and hidden reproduction (B: χ^2(1, N=60) = 0.25, $0.9 > p > 0.8$).

The χ^2 analyses reveal that the distribution of correct responses in the comparison and conservation tasks for children starting with the seen condition is not significantly different from that of the children starting with the unseen condition. The distribution of correct reproductions in the hidden seen condition is instead affected by the order of presentation. When this task is presented after a trial which begins with the hidden unseen condition, the number of correct responses increases. A similar tendency appears also in the hidden unseen condition, without however reaching the level of significance set (p equal or smaller than .05). In the remaining discussion the variable order of presentation will be ignored, except when the performance differences in hidden reproduction tasks will be examined.

7.2.10.1.2 Age group comparisons

The age group analysis determines the age period in which some developmental change occurs on the basis of the difference in age groups' performance. Tables 7.2.4 to 7.2.12 present the contingency tables for the children's correct and wrong responses to each task condition as a function of age[38]. A χ^2 test is computed on each contingency table. When the value of more than two expected frequencies is less than five, as in Tables 7.2.5 and 7.2.6, I have collapsed the responses of Primary children and computed the χ^2 test on that distribution. Occasionally, a further breakdown of the table is required to identify the precise age groups in which the performance varies significantly. To carry out this specific analysis, goodness of fit tests on the number of correct and wrong responses are computed.

7.2.10.1.2.1 Reproduction Task

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	11	15	19
Failure	9	5	1

Table 7.2.4: Correct and failed reproduction responses by age group in the reproduction task, visible condition (χ^2 (2, N=60) = 8.53, p = 0.014).

[38]In general, in case of tables with two or more cells with expected frequencies of less than 5, I have collapsed the responses of children from Nursery-Primary 1 in conservation and Primary 1-2 in reproduction.

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	13	18	19
Failure	7	2	1

Table 7.2.5: Correct reproduction responses by age group in the reproduction task, hidden seen condition (χ^2 (1, N=60) = 7.04, 0.01 > p > 0.005).

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	12	18	19
Failure	8	2	1

Table 7.2.6: Correct reproduction responses by age group in the reproduction task, hidden unseen condition (χ^2 (1, N=60) = 9.59, 0.005 > p > 0.001).

The χ^2 test is significant for all three reproduction tasks. Between the Nursery and the Primary groups, the number of correct reproductions increases and the number of failed reproductions decreases. The younger children reproduce the sets with less accuracy, whereas no difference appears between the two Primary groups' responses (18 correct hidden reproductions in Primary 1 versus 19 in Primary 2; 15 correct visible reproductions in Primary 1 versus 19 in Primary 2).

7.2.10.1.2.2 Comparison task

The data for the comparison tasks which follow the three conditions of the reproduction task are summarized in Tables 7.2.7 to 7.2.9. Each contingency table of response category by age group is followed by the relative χ^2 statistics, and where necessary some further comparisons of parts of the table.

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	11	16	20
Failure	9	4	0

Table 7.2.7: Correct and failed comparison responses after visible reproductions by age group (χ^2 (2, N=60) = 11.98, p = 0.002).

The difference is homogeneously distributed across the three age groups and is more pronounced between Nursery and Primary 2 children, where the χ^2 test is highly significant (χ^2 (1, N=40) = 11.6, p < 001).

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	5	18	20
Failure	15	2	0

Table 7.2.8: Correct and failed comparison responses after hidden, seen reproductions by age group (χ^2 (2, N=60) = 32.67, p = 0.000001).

Nursery children usually failed the comparison task after hidden seen reproduction. Older children generally succeeded. Whereas the primary groups give an equivalent number of correct and wrong responses, the difference between Nursery and Primary 1 responses (and consequently also Primary 2) is highly significant (χ^2 (1, N=40) = 17.4, p < .001).

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	4	16	19
Failure	16	4	1

Table 7.2.9: Correct and failed comparison responses after hidden unseen reproductions by age group (χ^2 (2, N=60) = 27.69, p=0.00001).

Nursery children usually failed and primary children succeeded. The χ^2 test of the distribution of correct and wrong reproductions among Nursery and Primary 1 children is highly significant: χ^2 (1, N=40) = 14.4, $p < .001$.

The null hypothesis of homogeneity between age groups' responses is rejected for all comparison tasks. The Primary children make more correct comparisons than the nursery children. The difference in accuracy is particularly pronounced between Nursery and Primary children when the comparisons follow the hidden reproductions (see Tables 7.2.8 and 7.2.9)

7.2.10.1.2.3 Conservation task

The data for the conservation tasks which follow the three conditions of reproduction are summarized in Tables 7.2.10 to 7.2.12. Each contingency table of response category (correct-failed) by age group is followed by the relative χ^2 statistics.

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	1	7	18
Failure	19	13	2

Table 7.2.10: Conservation and non-conservation responses after visible reproduction by age group (χ^2 (2, N=60) = 30.271, p = 0.000001).

Both the difference between Nursery and Primary 1 correct responses (binomial test, p = .035) and between Primary 1 and Primary 2 correct and wrong conservations (χ^2 (1, N = 40) = 12.8, $p < .001$) are significant. Only in Primary 2 did most children conserve. Of the 26 children who confirmed that the two collections were equinumerous after the transformation, 22 justified the conservation with operational arguments (see 6.2.1.1). The remaining children either counted the two rows or could not explain the equinumerosity.

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	1	9	18
Failure	19	11	2

Table 7.2.11: Conservation and non-conservation responses after hidden seen reproduction by age group (χ^2 (2, N=60) = 29.062, p = 0.000001).

As in the previous distribution, the responses differ significantly both between the Nursery and Primary 1 groups (binomial test on conservation responses, p = .011) and between the Primary 1 and Primary 2 groups (χ^2 (1, N=40) = 9.2, .005 > p > .001). Again a majority conserved only in Primary 2. Of the 28 children who maintained the judgment of equinumerosity, 23 gave operational justifications.

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	1	7	18
Failure	19	13	2

Table 7.2.12: Conservation and non-conservation responses after hidden unseen reproduction by age group

The distribution is equivalent to that of Table 7.2.10 and so are the differences identified there. Here too, 23 of the 26 provided operational justifications for the conservation.

The χ^2 tests are significant for the conservation responses following all conditions of reproductions. The null hypothesis of homogeneity is thus rejected for all three conditions, as the children's responses have a distribution significantly different from chance. The origin of the difference lays principally in that fact that: Nursery children do not conserve with only one exception (always the same child); Primary 2 children systematically maintain the equinumerosity throughout the spatial transformations (18 of them out of 20) and Primary 1 children occupy an intermediate position, with around 30% of correct conservations. While in the case of the comparison task the difference was very noticeable between the Nursery and Primary groups, in the conservation the difference is more marked between Nursery and Primary 1 children, on the one hand, and Primary 2 children on the other hand. Notice that the distribution would not have been significantly different should the criteria for conservation have been "same" answer plus justifications. In fact more than 80% of the children classified as conservers on the basis of their answer do also provide operational justifications.

Let us briefly focus on the conservation responses of the children who fail the comparison task, and judge equinumerous sets which are in fact different. As outlined in 7.2.7.3, the row with fewer elements was systematically made into a longer row to evaluate the basis of judgments of difference. Thirty children are presented with a conservation task with unequal sets. Twenty-four children answer that the two rows are different, but indicate that the smaller and longer row is more numerous. The six remaining children remark the difference and correct it through spontaneous or suggested one-to-one correspondence. However only one of them succeeds the following conservation task. These observations indicate that the failure in the comparison task goes with the failure in the conservation task, a result which will clearly emerge from the hierarchical analysis which follows.

7.2.10.1.2.4 Discussion

Significant changes in performance with age are observed in all nine tasks. The developmental differences emerge more clearly between Nursery and Primary children in the case of the reproduction and comparison tasks, and between Nursery-Primary 1 and Primary 2 children in the case of the conservation task. The task analysis which follows focuses directly on the performance changes across tasks at each age level.

7.2.10.1.3 Task Analysis

The repeated testing of children in the reproduction, comparison and conservation tasks allows us to determine whether the difference in performance across tasks conforms to the expected degree of difficulty of the tasks, and above all whether these differences are specific to particular age groups. Since in my predictions I formulate specific alternative hypotheses about the order of difficulty, I favour Marascuilo & McSweeney test (1967) over Cochran Q test (1950), which is classically employed for repeated measure designs with dichotomous data (see Meddis 1984, p.230).

The hypotheses 1, 2a and 2b underlie specific predictions about the order of difficulty of the tasks. For each predicted order, there is an associated set of coefficients, adjusted to sum to zero so to simplify the computations, which reflect the complexity scale. Hypothesis 1 predicts that conservation will be the most difficult task of all. Hypotheses 2a and 2b predict:

1. In the visible condition, correct reproductions will produce two rows of equal length and number, facilitating the subsequent comparison by posing no conflict between number and space occupied. For all age-group, then, reproduction and comparison are expected to be equally difficult and both should be easier than conservation[39]:

A) Reproduction=Comparison > Conservation [-1 -1 +2]

2. In the hidden condition, correct reproductions will produce two rows matching in number but not in number and length. Because length and number cues conflict, comparison should now be more difficult than reproduction, but still easier than conservation.

B) Reproduction > Comparison > Conservation [-1 0 +1]

Tables 7.2.13 and 7.2.14 present the values of the Z statistics relative to the two predicted orders of task difficulty A and B for the three age groups. The values significant at $p < .05$ are marked by a star (*).

Order \ Age	Nursery	Primary 1	Primary 2
Ord A	4.47*	4.01*	1.22

Table 7.2.13: Marascuilo & McSweeney Z values associated with prediction A: reproduction and comparison are equally difficult and conservation is more difficult, for the age groups Nursery, Primary 1 and Primary 2

The predicted order of difficulty between the tasks of reproduction, comparison and conservation is confirmed for the two younger groups, Nursery ($z=4.47$, $p<0.00003$) and Primary 1 ($z=4.01$, $p<0.00003$). The result is not statistically significant for the Primary 2 group ($z=1.22$, $p=0.111$) and the null hypothesis cannot be rejected: the older children perform uniformly across the three tasks.

[39](=) stands for "the number of subjects correctly performing Task A is equivalent to the number of subjects correctly performing Task B"; (>) stands for "the number of subjects correctly performing Task A is greater that the number of subjects correctly performing Task B".

Order \ Age	Nursery	Primary 1	Primary 2
Ord B seen	4.24*	3.48*	0.7
Ord B unseen	4.06*	3.81*	0.7

Table 7.2.14: Marascuilo & McSweeney Z values associated with prediction B: reproduction easier than comparison easier than conservation in the three age groups

The result of the test is statistically significant in the two younger groups both for the hidden seen and for hidden unseen conditions of reproduction (Nursery: seen $z = 4.24$, $p < 0.00003$; unseen $z = 4.06$, $p < 0.00003$; Primary 1: seen $z = 3.48$, $p = 0.00034$; unseen $z = 3.81$, $p = 0.00007$). The Null hypothesis of homogeneity is rejected in favour of the alternative hypothesis of order for the two younger groups. The result is not statistically significant in the Primary 2 group (with both seen and unseen reproductions $z = 0.7$, $p = .242$) and the null hypothesis cannot be rejected. Also when the older children begin with the hidden reproductions, they perform equally well across the three tasks.

7.2.10.1.3.1 Discussion

The data support the predicted orders of complexity between tasks for the Nursery and Primary 1 children. As predicted by order A (e.g. visible reproduction = comparison > conservation) a significant number of children succeed in the reproduction and comparison tasks and fail in the conservation task. As predicted by order B (e.g. hidden reproduction > comparison > conservation), a significant number of children reproduce accurately, fewer children compare correctly and even fewer children conserve. The fact that the Primary 2 children's responses to the three tasks do not follow the predicted orders confirms the tendency emerged in the previous age group analysis, and indicates that the acquisition of the capacity to reproduce, compare and conserve number develops in the period between pre-school and the first year of school, and is fully acquired by the second year of school.

7.2.10.2 Prediction analysis of task solution orders

Children's performance across pairs of tasks provides the tests for specific developmental hypotheses of concurrency and décalage. The Hildebrand, Laing &

Rosenthal Prediction Analysis of Cross-Classifications (described in Section 3.5) tests order hypotheses directly. The procedure applies to two-by-two contingency tables, where the rows represent the frequency of correct or failed responses to the first task and the columns represent the frequency of correct or failed responses to the second task. In the language of prediction analysis, the two main hypotheses advanced by Experiment 1 are reformulated as follows:

- Hypothesis 1 (collective décalage from the solution of the reproduction and comparison tasks to the solution of the conservation task for all task conditions): failure in the reproduction and comparison tasks is predictor of failure in the conservation task;
- Hypothesis 2a (collective décalage from the solution of the tasks of reproduction and comparison after visible reproduction to the solution of the comparison after hidden reproduction): failure in the reproduction and comparison after visible reproduction is predictor of failure in the comparisons after hidden reproduction.
- Hypothesis 2b (concurrency between the solution of the reproduction task and the solution of the comparison after visible reproduction): failure in reproduction predicts failure in comparison; success in reproduction predicts success in comparison.

7.2.10.2.1 Hypothesis 1: collective décalage between reproduction-comparison and conservation

The triangular hypothesis associated with collective décalage in favour of reproduction and comparison solutions over conservation predicts that the cell corresponding to failure in reproduction, comparison plus success in conservation be empty. First I evaluate the hypothesis of collective décalage from reproduction to conservation. Then I examine the order of acquisition of comparison and conservation.

7.2.10.2.1.1 Collective décalage between reproduction and conservation

Tables 7.2.15 to 7.2.17 present the two-by-two contingency tables of the all of children's responses to the three conditions of the reproduction task and the following conservation task. After each table, I indicate the value of the Del index corresponding to triangular hypothesis of collective décalage between the ability to solve the reproduction task and the later ability to solve the conservation task. The cell predicted to be empty by the triangular hypothesis corresponds to the white cell (the other cells being shaded). For each table, I indicate the normal curve z test for Del[40] and its significance level. Another z test compares the difference between the Del corresponding to the main hypothesis and the Dels of the alternative hypotheses of concurrency and collective décalage in the opposite direction. This second comparison serves to determine whether the triangular hypothesis not only yields a significant improvement over chance but also provides a more accurate prediction than the other triangular hypotheses.

Cons \ Repro visible	S	F
S	25	1
F	20	14

Table 7.2.15: Contingency table for reproduction and conservation in the *visible* reproduction condition (Del = 0.83; z = 7.6, p < 0.00003)

Not only is the Del for Hypothesis 1 significant in the visible reproduction condition, but is a significantly better predictor of results than concurrency (z = 4.9, p < 0.00003) or décalage from conservation to reproduction (z = 4.6, p < 0.00003).

Cons \ Repro h seen	S	F
S	27	1
F	23	9

Table 7.2.16: Contingency table for reproduction and conservation in the *hidden seen* condition of reproduction (Del = 0.785 (z = 3.9, p<0.00005))

For the hidden seen condition, collective décalage in favour of reproduction predicts the significantly non-chance contingencies and is a significantly better predictor than concurrency (z = 3.1, p < 0.00097) or the reverse décalage (z = 3.6, p < 0.00016).

[40]The Del was calculated by means of statistical software designed by Lautrey, Ribaupierre & Rieben and kindly provided by them.

Cons \ Repro h uns	S	F
S	25	1
F	24	10

Table 7.2.17: Contingency table for reproduction and conservation in the *hidden unseen* condition of reproduction (Del = 0.615 ($z = 2.6$, $p < 0.004$))

Finally in the hidden unseen condition, collective décalage in favour of reproduction is significant and a significantly better predictor than concurrency ($z = 2.3$, $p < 0.01072$) and opposite décalage ($z = 2.5$, $p < 0.00621$).

In all three conditions, the children's performance in the reproduction task is a very reliable predictor of performance in the conservation task. When children solve the reproduction task correctly, they can either succeed or fail the more complex conservation task; but if they fail the simpler reproduction task, they systematically fail the conservation task. *As predicted by Hypothesis 1, the ability to reproduce number is a prerequisite for number conservation.*

7.2.10.2.1.2 Collective décalage between comparison and conservation

Tables 7.2.18 to 7.2.20 present the two-by-two contingency tables for correct and wrong responses to the comparison and the conservation tasks after each of the three conditions of the reproduction task. In each case Del indices comparisons are given.

Cons \ Comp visible	S	F
S	26	0
F	21	13

Table 7.2.18: Contingency table for comparison and conservation after *visible* reproduction (Del = 1 ($z = e$, $p < 0.00001$))

After visible reproduction, there is significant décalage from comparison to conservation. This model is a significantly better predictor than concurrency ($z = 7.4$, $p < 0.00003$) or conservation-to-comparison décalage ($z = 12.2$, $p < 0.00003$).

Cons \ Comp h seen	S	F
S	28	0
F	15	17

Table 7.2.19: Contingency table for comparison and conservation after *hidden seen* reproduction (Del = 1 ($z = e$, $p < 0.00001$))

After hidden seen reproduction, there is significant décalage from comparison to conservation. This model is a significantly better predictor than concurrency ($z = 5.1$, $p < 0.00003$) or conservation-to-comparison décalage ($z = 7.5$, $p < 0.00003$).

Cons \ Comp h uns	S	F
S	25	1
F	14	20

Table 7.2.20: Contingency table for comparison and conservation after *hidden unseen* reproduction (Del = 1 ($z = e$, $p < 0.00001$))

After hidden unseen reproduction, there is significant décalage from comparison to conservation. This model is a significantly better predictor than concurrency ($z = 3.4$, $p < 0.00034$) or conservation-to-comparison décalage ($z = 4.3$, $p < 0.00003$).

In all three conditions, children's performance in the comparison task is a very reliable predictor of performance in the conservation task. Only children who solve the simpler comparison task succeed the more advanced conservation task, though not all the children who succeed the comparison solve the conservation task correctly. *As Hypothesis 1 predicted, the ability to compare sets is a prerequisite for number conservation. This holds both for sets of same number and shape, after visible* *reproductions, and for sets of same number and different shape, after hidden reproductions.*

7.2.10.2.2 Hypothesis 2a: collective décalage between reproduction and comparison after hidden reproduction

Tables 7.2.21 and 7.2.22 present the two-by-two contingency tables of responses to the comparison tasks after the hidden reproduction tasks. Hypothesis 2a predicts décalage from hidden reproductions to comparison. The décalage is brought about by the fact that the hidden reproduction yields two rows of different length, and the length cue is expected to make the comparisons problematic. The triangular hypothesis associated to Hypothesis 2a predicts that the number of children failing the reproduction task and succeeding at the comparison task (white cells) will be close to zero.

Repro \ Comp h seen	S	F
S	41	9
F	2	8

Table 7.2.21: Contingency table for reproduction and comparison after *hidden seen* reproduction (Del = .72 ($z = 4.2$, $p < 0.00003$))

In the hidden unseen condition, collective décalage in favour of reproduction is significant and a significantly better predictor than concurrency ($z = 1.9$, $p < 0.0287$) and comparison-to-reproduction décalage ($z = 2.2$, $p < 0.0139$).

Repro \ Comp h uns	S	F
S	38	11
F	1	10

Table 7.2.22: Contingency table for reproduction and comparison after *hidden unseen* reproduction (Del = .86 ($z = 6.5$, $p < 0.00003$))

After hidden unseen reproduction, there is significant décalage from reproduction to comparison. This model is a significantly better predictor than concurrency ($z = 2.7$, $p < 0.00347$) or comparison-to-reproduction décalage ($z=3.5$, $p<0.00023$).

For both conditions, the children's performance in the reproduction task is a very reliable predictor of performance in the comparison task. When children fail the reproduction task, they systematically fail the comparison task too, while if they succeed at the reproduction task, they may either succeed at or fail the comparison task. *As Hypothesis 2a predicted, the ability to compare sets of different length and same number, emerges only when the ability to reproduce sets is in place.*

7.2.10.2.3 Hypothesis 2b: Concurrency between visible reproduction and comparison

Table 7.2.23 presents the two-by-two contingency table for the responses to the tasks of reproduction in the visible condition and the subsequent comparison of the two sets of same number and shape. The Hypothesis 2b predicts that the ability to reproduce the sets appears concurrently with the ability to compare them, when the sets have been constructed so as to have at the same time equivalent number and length. Children who fail reproduction are also expected to fail comparison and children who succeed at reproduction are expected to solve comparison too (see shaded cells on the contingency table).

Repro \ Comp visible	S	F
S	45	0
F	2	13

Table 7.2.23: Contingency table for comparison and reproduction in the *visible* condition (Del = .83 ($z = 7.6$, $p < 0.00003$))

The concurrency between visible reproduction and subsequent comparison is of a strict type because both alternative décalages are significant (in favour of reproduction Del =.83, $z = 7.6$, $p < 0.00003$; in favour of comparison Del = 1, $z = e$, $p < 0.00001$) while the concurrency pattern is significantly different from both ($z = 1.7$, $p < 0.0446$, for reproduction,; $z = 1.6$, $p < 0.0548$, for comparison). As it is expected in the case of strict synchronism, the two triangular décalage hypotheses, which predict that either the lower left cell or the higher right cell are empty, are verified (e.g. no child who fails one task succeeds at the other task). In other words, *success at visible reproduction perfectly matches success at comparison; and failure at visible reproduction corresponds perfectly to failure at comparison.*

To recapitulate, the analysis of acquisition orderings of the problems of reproduction and comparison provides support for the hypotheses. It shows that:

1. When set reproduction occurs with the set visible, the children who fail the reproduction task, also fail the comparison task, while children who succeed at the reproduction task constructing two sets equivalent in number and length, then succeed at comparison. The two tasks are acquired concurrently;
2. When instead the set is reproduced on the basis of its cardinal number alone, and no equivalence of length is created, children can succeed at reproduction and later fail at comparison. A significant number of children produce an accurate reproduction by counting out an equivalent number of objects but in the subsequent comparison task do not confirm the equinumerosity, basing their numerical judgment instead on the space occupied by the two sets. In general the longer row is taken to have a greater number of objects.

7.2.10.3 Some a posteriori comparisons

In introducing the two conditions of the reproduction task (visible and hidden), I assumed that when the row of objects is presented, the children construct the equivalent set by matching the objects one-to-one and obtain two rows which have same number and same length. When the row is instead hidden, the new row produced has the same number, and generally different length. The children examined in Experiment 1 conform to these behavioural patterns. They all use one-to-one matching in the visible reproduction condition and construct a set which is equivalent in distribution and number to the model. In the hidden reproduction condition, they count out and arrange the objects on one side of the screen. In this experiment, it never happened that two rows had equivalent length.

I have examined a posteriori the nature of the acquisition order between (a) the visible and hidden reproduction tasks (for both seen and unseen conditions), (b) visible reproduction and comparison after hidden reproduction and (c) comparison tasks which followed seen and unseen hidden reproductions. For each pair of tasks, I have calculated the Del indexes for the three hypotheses of concurrency, décalage in favour of one task and décalage in favour of the second task. The contingency tables for each pair of tasks are presented in the Appendix 7.2, together with the relative statistical tests. Here I briefly summarize the main findings.

Weak concurrency exists between visible and hidden seen reproductions and between the two conditions (seen and unseen) of the hidden reproductions. Individual décalage holds between visible and hidden unseen reproduction. Weak concurrency and individual décalage are found between visible reproduction and comparison after both conditions of hidden reproduction. These results are, however, only partially reliable as the underlying distributions are very unbalanced. The majority of children in fact consistently solve each of these pairs of tasks correctly (around 70% of the children). The failure to identify some clear-cut response patterns may hence be due to the fact that the reproduction task is too easy for this sample.

After hidden reproduction (seen and unseen), the response patterns indicate a concurrency accompanied by collective décalage in favour of the seen condition. The décalage component corresponds to 4 children out of 60 who have solved the comparison task after the reproduction in which they have counted the rows themselves and fail the second condition of comparison. The remaining 54 children's responses conform to a strict concurrency pattern.

7.2.10.4 Qualitative analysis of the responses

In this section, I present some general observations of the behaviours underlying the scoring of the responses as correct or wrong. I discuss the strategies used, the justification given and the procedures used to produce, explain and check the solutions offered. This more qualitative and descriptive analysis is performed separately on each task.

7.2.10.4.1 Reproduction Task

All 144 (out of 180) *correct* reproductions result from the same strategies:

1. In the visible reproductions, the children put the objects down one at a time, each one in front of, and very close to, one object of the model row. The strategy corresponds to establishing a spatial one-to-one correspondence between elements of the model set and elements of the new one. Among the older children, the matching is often accompanied by counting. Only one child employs a different strategy. Lan (6,10) from Primary 2 says "How many are there?", counts the model set and takes the same number of objects out of the box, in a bunch.

2. In the hidden reproductions, the children put the objects down one at a time and count; the action is stopped once the count has reached the number of the collection behind the screen.

The 36 *inaccurate* reproductions are produced by the following procedures:

1. In the visible reproductions (15 failures), children either use all the objects of the box (7 children of which 4 from Nursery, 2 Primary 1 and 1 Primary 2) mainly to reproduce the length of the row, or apply a non-systematic spatial matching (7 children: 4 Nursery, 3 Primary 1). In this case they end up with 1 or 2 objects too many or too few and reproduce the length of the model row. In the former case however, some children construct a series of rows parallel to and of same length as the model row until all the objects are used. One child from the Nursery group uses a completely different strategy and reproduces a six-object row with a four object row, starting from one end of the model and going down perpendicular to it. The resulting row is thus different in number and shape from the model set.

2. In the hidden reproductions (21 failures), children either use up all the objects in the box, without any overt quantification (7 children of which 5 from the Nursery, 2 from Primary 1), count incorrectly or employ various pre-quantitative strategies. The pre-quantitative strategies are more common in the Nursery group (8; 1 from Primary 1) and consist generally of putting down a random number of objects, without counting them in the process, or picking up all the objects in the box one by one.

Dan (5,1) constructs a row of eight objects to reproduce a set of five, without showing any form of explicit numerical quantification. Even when the experimenter repeats the question and the model set's number, the child does not recount the objects. Mich (4,0) has to reproduce a row of five objects. First she puts down three objects, then adds another object. At this point the experimenter repeats the question, clearly restating the cardinal number of the model set. Mich counts her row of four objects, adds one and immediately afterwards another object, to obtain a set of six objects. Inaccurate counts produce the remaining incorrect reproductions (2 children from Nursery, 1 from Primary 1 and 2 from Primary 2). Although they base their response on the cardinality of the model set, the children put down a number of objects which is different from what is required. In the hidden seen condition, Lu (5,6) counts the set of six objects accurately but then in the reproduction puts five objects down. The action of taking and placing the objects is not coordinated with the counting, and so she counts to six but puts down five. Kat (5,7) also counts the objects as she puts them down. The model row is made of seven elements. She puts down six objects, stops suddenly, counts the row to five and adds two more objects, thus creating a row of eight, instead of the required seven.

To recapitulate, accurate reproductions result from the sequential matching of each object of the model set with one object, i.e. the spatial correspondence characteristic of reproductions of visible sets, and via number words and counting in the case of reproductions of non-visible sets. Inaccurate reproductions result from:

a) indiscriminate use of all the objects given to construct the set;

b) reproduction of the length of the row, without attention to the number of objects used;

c) imprecise spatial one-to-one correspondence between elements of the two rows, with the exception of the two ends of the rows;

d) poor coordination between counting and putting the objects down;

e) general difficulty with understanding the task and using the numerical information about the model set, as in cases where the child puts down an arbitrary number of objects.

7.2.10.4.2 Comparison task

The strategies used in the comparisons after visible reproduction are analyzed separately from the comparisons after hidden reproduction (B). Particular attention is given to the situations where correct reproductions are followed by incorrect comparison or where inaccurate reproductions are corrected in the comparison task.

7.2.10.4.2.1 Comparison after Visible Reproduction

All the children who reproduce the set accurately also compare the sets correctly and confirm their equinumerosity. Only 2 of the 15 children who fail the reproduction recognize the difference between the sets and correct it. The two children belong to the Primary groups (1 Primary 1; 1 Primary 2). Pol (6,3) starts the reproduction using a spatial matching strategy but progressively shifts of strategy and puts the cylinders very close together, until they are all used. At the comparison she remarks the difference, asks for more objects and creates the correspondence by adding some objects to the model set and taking some away from her copy set. Nic (5,8) also uses all the objects. When she is asked the comparison question, she notices the big difference, counts them and takes away the correct difference from the copy set. Eight children, who had failed reproduction, judge the two sets to be different in number. Nevertheless they neither attempt to correct the difference nor succeed in equating the sets' numbers. Most of the tentative corrections consist of subtracting some objects from the row judged to be more numerous, without precise quantification. Mic (5,9) initially reproduces the length of the row, using more elements than necessary. At the comparison stage, he takes three objects away from the copy row, even though the difference is of four, and reconfirms the equinumerosity. Tho (5,6) uses all the objects at his disposal to reproduce the set, notices the difference and, without apparent quantification, takes away seven elements, where the difference is five. Five children, mainly from the Nursery group, confirm that the two rows are equinumerous, even though they are not. Their incorrect judgment seems to be based on the equivalent length of the rows or on the result of peculiar counts. Ad (4,8) counts the two rows as if they were one and concludes that they are the same number. Joe (5,8) puts five objects in a row with the ends matching those of a row of seven. At the comparison stage, he counts the model to five, recounts it to seven, counts the copy row to six and concludes that the two rows have the same number of objects.

7.2.10.4.2.2 Comparison after Hidden Reproduction

Around two thirds of the Nursery children abandon the equinumerosity established in by reproduction once the screen is taken away to reveal two rows of unequal length. This phenomenon is practically non-existent among older children. In all, out of 99 correct reproductions, 18 (16 in Nursery, 2 in Primary 1) pairs of rows are judged to be different in number at the subsequent comparison. After a judgment of difference, the children are required to re-balance the sets and correct the difference. Three characteristic behaviours are observed:

1. Children say that the rows have different number but cannot say what the difference is nor attempt any action to equate the sets;
2. Children reestablish equinumerosity by adding or subtracting some elements from one of the sets to obtain two rows of same length or, more rarely, change the form of the rows again to obtain two rows of equivalent length.
3. Children justify the difference by producing inaccurate counts that conform to their judgment. Ad (4,8) skips one of the elements of the shorter row and counts it as having one element less than the longer row. She (4,9) counts an element of the longer row twice and shows that the longer row has one element more than the shorter one. In both cases, the outcome is that the perceptual judgment is confirmed by the count and that the addition of one element to the shorter row, to produce matching end-points, gives two equivalent sets.

Among the children who have failed the initial reproduction task, I observe:

1. Children who confirm the 'equinumerosity';

2. Children who remark the difference in number but who either could not correct it or who produce an inadequate correction (e.g. modify the length, add or subtract elements to match the end points);
3. Children who succeed in reestablishing equinumerosity.
Nursery children tend to perform inaccurate correction. They modify the distribution of the elements to obtain an equivalent length of the rows. They add or subtract a wrong number of elements to achieve matching end-points. Most of the Primary children instead use the comparison task as a test of the accuracy of their previous reproduction. They count the number of objects in the two rows, and in case of difference, subtract or add the number of elements required.

To recapitulate, the conditions under which reproduction is performed produce different solutions and yield different procedures of comparison. In the older age groups, the comparison question is used to check the accuracy of the reproduction. The children count the two sets and, if required, correct inaccurate reproductions by adding or subtracting the appropriate number of elements. In the younger group, instead, the sets which are taken to be different in number and are corrected are those which have different length, regardless of number. To achieve equivalence of form, the children either lengthen one of the rows or add some objects to the shorter row to make the end-points match.

7.2.10.4.3 Conservation task

Classic conservation responses have been observed. Among the majority of Nursery and Primary 1 children, the lengthened row is systematically considered the more numerous. The justification given is that since it is longer, it has more elements. A few hesitations between conservation and difference are found among Primary 1 children, with conserving answers being immediately followed by non-conserving responses. Conservations are classically justified using the arguments that 1) nothing has been added nor taken away, 2) it is possible to go back to the original configuration and 3) one row is longer, but more spaced, while the second row is shorter and more crowded.

7.2.11 Discussion

The Age Group, Task Condition and Acquisition Order analyses provide substantial support for the three hypotheses of (1) collective décalage between reproduction-comparison and conservation, (2a) collective décalage between hidden reproduction and subsequent comparison, (2b) concurrency between visible reproduction and subsequent comparison. At all three levels of analysis, children appear to possess first the ability to solve the reproduction tasks and the comparison after visible reproduction task, then to solve the comparison of rows of different length (after hidden reproductions) and finally the understanding of conservation.

The comparison of performance across age groups indicates that the number of children who solve each of the tasks increases with age. More specifically, a clear performance gap emerges between Nursery and Primary children in the comparison tasks after hidden reproduction (25% of the Nursery children are correct versus 80% of the Primary 1 and 95% of the Primary 2) and between Nursery, Primary 1 children and Primary 2 children in the conservation task (5% Nursery and 35% Primary 1 children are correct versus 90% Primary 2 children). Along the age scale then, Nursery children appear to have some ability to reproduce sets (more than 50% correct) but very little competence in comparing and conserving number. Primary 1 children have the ability to solve the reproduction and comparison tasks (around 85% correct solutions), but have very little competence in conserving number (around 35% correct solutions). Primary 2 children instead master the whole battery of tasks, as their rate of success ranges between 90 and 100% correct solutions in all tasks.

This battery of tasks thus identifies two developmental changes in performance. Between the Nursery and the Primary 1 years, the children acquire the competence to reproduce and compare sets of objects. Between the Primary 1 and the Primary 2 years, the children acquire the competence to conserve number, across spatial transformations of the sets. The comparison of performance across tasks within each age group mirrors the preceding findings. While the performance of Nursery and Primary 1 children varies according to the predicted order of complexity of the tasks (e.g. reproduction > comparison > conservation), the Primary 2 children's performance does not change significantly across tasks. In particular, among Nursery and Primary 1 children when the set reproduced is visible and leads to two rows equivalent in number and length, reproduction is as difficult as comparison, and both tasks are easier than conservation. When the set reproduced is hidden, the rows may be equinumerous, but are different in length. The comparison task is here more complex than the reproduction task, as it introduces a conflict between number and length. Comparison however is still easier than conservation, where the conflict is produced by an active (dynamic and intentional) transformation.

Consider now the core of the developmental analysis, that is the hierarchical analysis of the order in which pairs of tasks are solved. Hypothesis 1 states that collective décalage exists between the solution of the tasks of reproduction and comparison and the later solution of the conservation Task. The triangular hypotheses which correspond to collective décalage between the solution of reproduction and conservation, on the one hand, and between the solution of comparison and conservation, on the other hand, describe the data with good accuracy. In the contingency tables relative to the different pairs of tasks, children distribute in the three cells which correspond to 1) failure in both tasks, 2) success in both tasks and 3) success in the simpler task (either reproduction or comparison) and failure in the more complex task (conservation). I have found only 5 responses out of 360 which correspond to the opposite pattern, i.e. failure in the less advanced task and success in the more advanced.

This finding conforms to the developmental progression from a level where both tasks are failed, to a second level where the simpler task is solved and at the same time the more advanced task is failed, to a third level where both tasks are succeeded. The solutions of the tasks of reproduction and comparison identify the intermediate level between complete failure and success, and track a stage of conceptual elaboration which appears to have a status and an organization of its own. Consider now the developmental relationship between reproduction and comparison: are these two tasks understood at this same intermediate stage in the elaboration of the number concept?

Hypothesis 2a states that collective décalage exists between the solution of the hidden reproduction task and the later solution of the comparison after hidden reproduction task. Hypothesis 2b, on the other hand, states that the solution of the visible reproduction task is concurrent with the solution of the comparison after visible reproduction task. The two triangular hypotheses that express the two relations of concurrency and collective décalage describe the data with good accuracy. Only two children out of 60 fail one task, while succeeding the second task. Only three responses out of 120 correspond to failures in the simpler task of visible reproduction and correct solutions to the more advanced comparison tasks. Also, these data are in agreement with the expected developmental progression, as at the first level children fail all the tasks; at the second level they succeed the reproduction and the comparison of rows equivalent in number and length, but fail the comparison of rows equinumerous but different inform; at the third level they solve all the reproduction and comparison tasks. The ability to solve the tasks of reproduction and of comparison of sets of same number and distribution thus identifies a competence level which precedes that of generalized comparisons independent of spatial cues.

This fairly linear picture of the development of cardinal number is, however, complicated when the developmental orders between (a) the solution of the two conditions (visible and hidden) of reproduction and (b) the solution of hidden reproduction and comparison after visible reproduction are examined. The findings of weak concurrency and individual décalage suggest that the competence levels may not be reached by the same path. At level 1 in particular, some children are able to solve visible reproductions using matching, but may fail hidden reproductions which requires the use of counting. Other children do the opposite. Similarly some children fail the hidden reproduction task and fail the comparison after visible reproduction. As I previously remarked, however, these results remain only indicative, because the contingency tables are very unbalanced by the high rate of success in both reproductions (70% of the trials). The developmental relationship between the different task conditions will be examined more thoroughly in a subsequent experiment with younger children, in order to reduce the proportion of children for which the two tasks are too easy and more clearly to identify eventual underlying response patterns.

In conclusion, Experiment 1 provides some robust evidence on the sequence of levels which lead to the understanding of cardinal relations required by the conservation task. The data confirm the order of acquisition between reproduction/comparison and conservation pointed out in the literature. They provide new evidence of a level of competence between the capacity to reproduce and conserve sets which corresponds to the ability to judge the numerosity of pairs of sets, independently from spatial indices. The experiment supports a finer decomposition of the developmental process:

Stage 0: children fail all the tasks. Failure in the visible reproduction task is generally due to the reproduction of the length of the row, rather than the number. Failure in hidden reproduction is due to the use of all the objects at the children's disposal or to counting inaccuracies;

Stage 1: children solve the reproduction tasks and the comparison task after visible reproduction, when the sets are equivalent in both length and number. They fail the comparison after hidden reproduction, where the previously established sets' equinumerosity conflicts with the sets' difference in shape. Children tend to judge the longer row as more numerous and seem to forget the equinumerosity that underlies the preceding reproduction;

Stage 2: children solve the reproduction and comparison tasks in all conditions, both when the reproduced sets have equivalent shape and when they are different. Often children use the comparison problem to check the accuracy of the previous reproduction. They fail the Piagetian conservation test and abandon the equinumerosity once the experimenter has lengthened one of the two rows;

Stage 3: children solve the three tasks of reproduction, comparison and conservation under all conditions.

The analyses of the children's solution strategies, justification and testing procedures give some insight into the nature of the conceptual problems characterizing the transition between stages. In Stage 1, when in the comparison after hidden reproduction task the screen is taken away and the children see that the two rows do not look identical (e.g. one is longer or shorter than the other), they judge the two sets to be different in number. The longer row is generally considered to have more elements. If they are asked to verify their answer, and eventually induced to count, they double count one of the elements of the longer row to reach a greater number. Alternatively they skip on one element of the shorter row to obtain a smaller cardinal number. Their counting is in a sense manipulated to fit the numerical judgment. In those cases, the children reestablish equivalence by adding one element to the shorter row and in the process matching an end-point of the two rows. Other children count both rows as one to conclude that the sets are indeed different. When they are asked to reestablish the equivalence, they either modify the distribution of the two rows, to make them of same length, or they add (or subtract) some elements to the shorter (longer) row to match the end-points. Behaviours which are even more suggestive of some conceptual conflict are observed among the children who judge the two equinumerous rows to be different on the basis of their length, and then add some elements to equate the rows' length, count them again and discover that the two rows have different number. They then add the elements necessary to cancel the difference. This action however introduces a new difference in length. Again the children say that the two rows are different and add some elements to compensate the difference in length. The subsequent count to verify the equivalence reveals the new difference, which calls for a new addition of elements, and so on until all the objects at the children's disposal have been used. This puzzling behavioural pattern has been observed in at least four cases.

The children reach Stage 2 when they have elaborated the knowledge necessary to solve reproductions and comparisons under all conditions. Comparison judgments based on length differences are verified and corrected using counting. Alternatively, counting or one-to-one correspondence is used from the start as the basis for the numerical judgment. Although these children cope with the length difference in the static context of the comparison task, however they fail to assimilate the length difference brought about by the spatial transformation in the conservation task. In some cases, the conservation task is presented starting from two rows which, although different in length, are correctly considered equinumerous. Even in these circumstances, after the transformation has occurred, the children claim that the two rows are different in number and that the row which is now longer is more numerous.

Counting, which these children used spontaneously in the comparison task, is surprisingly not invoked to solve the conservation.

Behaviours that can indicate periods intermediate between Stage 2 and Stage 3 are characterized by oscillations between conservation and non-conservation answers. Some children first say that the number has remained the same after the transformation, then move to a non-conservation response. Often the switch is triggered by the request to justify the answer or by the counter-suggestions, such as one row is much longer than the other.

7.3 Experiment 2

7.3.1 Introduction

Experiment 1 has found evidence of three stages in the elaboration of the concept of cardinal number. The stages correspond to the ability to reproduce sets and to compare sets equivalent in number and length (Stage 1), to compare also sets equivalent in number but different in length (Stage 2) and to conserve number across transformation on the sets' distribution (Stage 3). Experiment 2 focuses on the period encompassing Stage 1 and Stage 2 and is designed to replicate the new result of Experiment 1 that the ability to reproduce sets emerges prior to the ability to compare sets which are equinumerous, but different in configuration. In order to determine whether the difficulties that children experience with comparison are due only to the misleading length difference, Experiment 2 introduces a second condition of comparison in which the sets to be compared form two arrays of same length and differ cardinal number.

Experiment 2 presents children with the tasks of set reproduction in the two visible and hidden (unseen) forms and of set comparison under three conditions:

a) two rows of same number and length (after correct visible reproduction);
b) two rows of same number and different length (after correct hidden reproduction);
c) two rows of same length and different number (as the child is not required to construct the set himself but is presented with two already made rows, in a new condition, *direct comparison.*).

The two conditions of reproduction and comparison are exact replicas of the tasks used in Experiment 1. The third condition of comparison is instead a new situation and consists of placing before the child two sets forming two rows of equivalent length, matching end-points and a minor difference of spacing between items. To achieve this perceptual similarity the numerical difference between the two sets is of one element.

Since according to Experiment 1 the ability to reproduce and compare sets emerges between age 4 and 5 years, in Experiment 2 the battery of numerical tasks is presented to a sample of 40 Nursery and 20 Primary 1 children.

7.3.2 Objectives

Experiment 2 has three basic objectives. First, it should replicate the acquisition orders identified by Experiment 1. Second, it addresses the question of whether the correct judgments observed in Stage 1 (for rows equivalent in number and length) underlie some form of numerical competence or whether they are simply length based. To answer this question the new comparison task of sets of same length and different number is introduced. Collective décalage between comparison after visible reproduction and the new direct comparison task (children judge the two rows to be equinumerous) would indicate that Stage 1 children systematically base their numerosity judgments on the spatial size of the sets, although they are able to reproduce sets correctly. Concurrency between success in comparison after visible reproduction and the new task would indicate that in some conditions the Stage 1 children can work out the numerical difference beneath the length equivalence and can make accurate numerical judgments. If this is so, failure in the comparison after hidden reproduction (for rows equivalent in number, but different in length) would constitute a special case rather than the expression of a general conceptual deficit, i.e. the inability to compare the cardinality of sets of objects.

Third, Experiment 2 examines the order in which the solutions of visible and hidden reproductions emerge. Experiment 1 in fact identifies a pattern of weak décalage between the two conditions of reproduction, a finding which deserves further scrutiny. To consider this issue, a larger sample of Nursery children is examined.

7.3.3 Hypotheses

Experiment 2 investigates the following response patterns. First, I expect to replicate the findings of Experiment 1:

1. *Collective décalage* from hidden reproduction to comparison after hidden reproduction;
2. *Concurrency* between visible reproduction and comparison after visible reproduction;
3. *Tendency towards individual décalage* in the solution of reproduction in the two conditions visible and hidden.

Second, there are predictions about the relationship between the comparisons after hidden and visible reproductions and the new direct comparison task, in particular:

4. *Concurrency* between comparison after hidden reproduction and direct comparison, also equivalent to

5. *Collective décalage* between visible reproduction, subsequent comparison and direct comparison.

Three groups of hypotheses are formulated. The first group, consists of the hypotheses already investigated in Experiment 1, and invites replication with a larger sample of young children.

Hypothesis 1a: there is a collective décalage from the solution of the hidden reproduction task to the solution of the comparison after hidden reproduction task.

Correct performance on the comparison after the hidden reproduction task should be more strongly associated with correct performance on the hidden reproduction task than with incorrect performance on this task, as in figure 7.2 (p. 148).

Hypothesis 1b: The solution of the visible reproduction task is concurrent with the solution of the comparison after visible reproduction task.

Correct performance on the comparison after visible reproduction task should be strongly associated with correct performance in the visible reproduction task; incorrect performance on comparison after visible reproduction should be strongly associated with incorrect performance in visible reproduction, as in figure 7.3 (p.148).

The second group of hypotheses examines some generalizations from hypothesis 2b. The two comparison conditions: direct and after hidden reproduction introduce a conflict between the length and the cardinal number of the sets. In the direct comparison condition, although the two rows have same length, they are different in number. In the comparison after hidden reproduction condition, although the (accurately reproduced) rows have same number, they are generally different in length. Under the assumption that Stage 1 children base their numerosity judgments on an estimation of the spatial size of the sets (length in particular), and that they develop in Stage 2 to compare the sets' numerical size using counting or one-to-one matching, I predict that either the children fail both conditions of comparison (e.g. direct and after hidden reproduction), using length as criterion, or they solve both task conditions, using cardinal number to make their numerosity judgments.

Hypothesis 2: The solution of the direct comparison task is concurrent with the solution of comparison after the hidden reproduction task.

Correct performance on comparison after hidden reproduction should be strongly associated with correct performance in the direct comparison task; incorrect performance on the comparison after visible reproduction task should be strongly associated with incorrect performance in the direct comparison task.

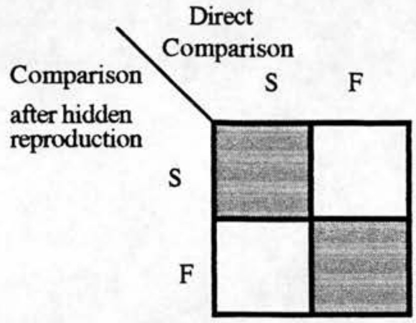

Fig. 7.4: Model of concurrency between responses to direct comparison and comparison after hidden reproduction according to hypothesis 2 (the white cells are the cells predicted to be empty).

The same response pattern (envisaged from the angle of the difference between Stage 1 and Stage 2) corresponds to the collective décalage between visible reproduction and direct comparison. In this case, the failure in the reproduction task predicts failure in the direct comparison task, whereas the opposite does not hold.

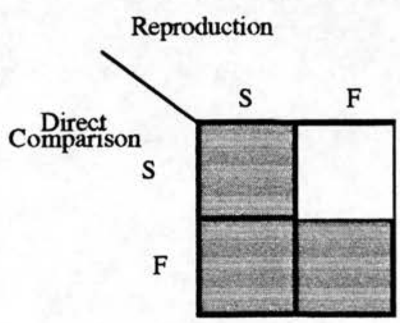

Fig. 7.5: Model of collective décalage between responses to reproduction and direct comparison according to hypothesis 2.

The third group of hypotheses deals with the acquisition of reproduction in the different task conditions. The weak décalage reported in Experiment 1 may underlie a tendency towards individual décalage, which may have been masked in that experiment by the larger number of children who solved both tasks. In Experiment 2, the two reproduction tasks are administered to a sample of younger children and the following hypothesis is tested:

Hypothesis 3: There is individual décalage between the solution of the hidden reproduction task and the solution of the visible reproduction task.

Correct performance on the hidden reproduction task should be equally associated with correct and incorrect performance in the visible reproduction task. Correct performance on the visible reproduction task should also be equally associated with correct and incorrect performance in the hidden reproduction task.

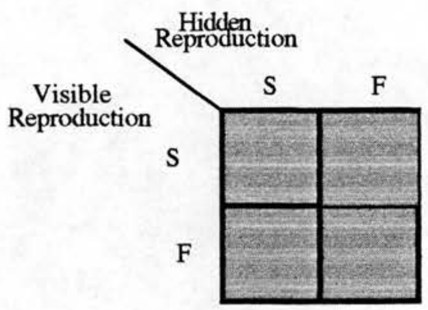

Fig. 7.6: Model of individual décalage between responses to visible reproduction and hidden reproduction according to hypothesis 3.

7.3.4 Design

Each child performed the five tasks:

1) <u>number reproduction</u>, visible condition: a row of objects, uniformly spaced, was laid down before the child. The child was asked to take the same number of similar objects from a box and to construct a row which had the same number of objects.

2) <u>number reproduction</u>, hidden condition: the experiment had put a number n of objects behind a screen. The child was asked to take a same number of objects n from a box and to construct a row which had the same number of objects. This task was an exact replication of the hidden unseen condition of reproduction used in Experiment 1.

3) <u>number comparison</u>, after visible reproduction condition: the child was asked whether the two rows had the same number of objects or whether one of them had more objects.

4) <u>number comparison</u>, after hidden reproduction condition: the screen hiding the model row was taken away. The child had the two rows before him and was asked

whether the two rows had the same number of objects or whether one of them had more objects.

5) direct comparison: the experimenter laid down two rows of objects, of which one had one element more than the other. The rows were constructed to have same length. The child was asked whether the two rows had the same number of objects or whether one of them had more objects.

The tasks of reproduction and comparison were presented in a fixed sequence of reproduction followed by comparison. The order of presentation of the direct comparison was also fixed. It systematically followed the two series of reproduction-comparison tasks, which were counterbalanced. Half of the children started with visible reproduction and half with hidden reproduction. For each reproduction-comparison pair, the number and the type of objects was changed so as to reduce repetition effects. The two orders were presented in the following schema:

Order A	Order B
Reproduction Visible	Reproduction Hidden
Comparison	Comparison
Reproduction Hidden	Reproduction Visible
Comparison	Comparison
Direct Comparison	Direct Comparison

The *independent variables* were:

- tasks:
  - reproduction visible, hidden;
  - comparison after visible reproductions visible, after hidden reproductions;
  - direct comparison.
- schooling/age group:
  - Nursery (between age 3,6 and 5);
  - Primary 1 (between age 5 and 6)
- order of presentation of the reproduction-comparison series:
  - visible reproduction first (e.g. order A);
  - hidden reproduction first (e.g. order B).

Dependent variable was the number of correct responses on individual tasks and pairs of tasks.

This experiment, like the previous one, was a within-subjects design, with subjects nested in age groups and order (A or B). Along the within-subjects dimension, I compared the children's performance across the whole set of tasks; while in the nested, between-subject dimension, I compared the performance of the age groups across tasks.

7.3.5 Statistical analysis

The same statistical tests used in the preceding experiment were also employed in Experiment 2. They are described in section 7.2.5.

7.3.6 Material

The same three sets of objects used in Experiment 1: wooden cylinders (green and red), small plastic animals (pigs and hippopotamuses), round sweets (orange and yellow). The cylinders and the animals were interchangeably employed in the two reproduction-comparison pairs. The objects were given to the child in two boxes, one containing the pigs and one the red cylinders. In the condition hidden of the reproduction task, the screen used to hide the model set was a red, rectangular cardboard. The round sweets were employed in the direct comparison task.

7.3.7 Procedure

The children were pre-tested on their counting abilities, as in Experiment 1, and the set sizes used were chosen within the children' counting competence span. Nursery children were generally tested with sets of between 4 and 6 objects; Primary 1 children with sets between 7 and 10. In the direct comparison task, I used three (or four) objects in one row and four (or five) objects in the second row with Nursery children; five (or six) in one row and six (or seven) in the second row. The pre-test was followed by the two series of reproduction and comparison tasks, and the final direct comparison task.

The presentation of the reproduction and comparison tasks was identical to that followed in Experiment 1 (see sections 7.2.7.1 and 7.2.7.2) apart from a more systematic request for checks, corrections and justifications in the comparison after hidden reproduction task. In the direct comparison task instead, the experimenter laid down the two rows of sweets, first putting one orange candy, then a yellow candy in front of it; a new orange, and then another yellow further apart, and so on, until two rows were created one with one object more than the other, though both are same length. The experimenter then said: "here I have made two lines (or rows) of sweets.

Are there (pointing to one row) the same number of sweets as there (pointing to the other row)? or is the number different?" Alternatively, the question was worded as "If you want to have a lot of sweets, which ones (or row) would you want?". After the child had answered, the experimenter asked if he knew some way to check whether they were the same. If the child did not attempt any checking procedure, the experimenter suggested counting the two rows "Why don't you try counting the rows. Do you think it's a good way to see if they are the same number?".

The interviews took place in a relatively quiet corner of the class-room. During the interviews the other children were kept away from the area, but could observe what was happening. I chose this setting to make the child feel as comfortable as possible. During the interview, the experimenter noted the responses and the strategies employed on an already made protocol-schema (see Appendix 7.3). Immediately after the interview, the notes were completed with some more general remarks about specific behaviours, comments or reactions. The testing session lasted between 10 and 20 minutes.

7.3.8 Measure

Children's performance is measured by the number of correct reproductions, and comparison. The scoring criteria are equivalent to those used in Experiment 1[41]. They are extensively described in section 7.2.8. In the new, direct comparison task, children are scored correct when they answer that the two collections have a different number of elements or when they revise an initial judgment of equinumerosity after counting or establishing a one-to-one correspondence and remark the difference.

7.3.9 Subjects

60 children from age 3,3 to age 5,11 years were tested. They were divided into two class-age groups of 40 Nursery children (mean age = 4,3 years; SD = .44) and 20 Primary 1 (mean age = 5,5 years; SD = .25). The Nursery children are from two different schools, from one of which the Primary 1 children also come from. The children are of a mixed social background. Four children were not native speakers of English.

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[41]In administering the tasks, my priority has been to stimulate as much as possible the use of checking procedures and the eventual revisions or corrections of the initial answer. The scoring criteria also try to take into account the checks and the corrections carried out and to go beyond a simple first answer criterion.

7.3.10 Results

As in Experiment 1, the analysis of the results was performed at three levels. The Group analysis examined the effects of the variable order of presentation and schooling/age group on the reproduction, comparison and direct comparison responses. The Task analysis examined the adequacy of the hypothesized order of difficulty of the tasks in predicting changes across tasks in the two age groups. The Hierarchical analysis determined whether the predicted across-tasks response patterns conformed well to the observed overall distributions of responses.

7.3.10.1 Order of task presentation

Tables 7.3.1 to 7.3.3 present the contingency tables of the frequency of correct reproductions (7.3.1), comparisons (7.3.2) and direct comparisons (7.3.3) for the two orders: visible reproduction first (order 1) and hidden reproduction first (order 2). Each table is followed by the χ^2 statistics computed on it to compare the number of correct responses who fall into each cell (e.g. observed frequencies) as against the numbers of correct responses we would expect to fall into each cell if there were in fact no differences between the two orders of presentation (e.g. expected frequencies). Also in Experiment 2 the reported values of χ^2 include Yates' correction for continuity.

Response \ Order	First	Second
C	23	25
I	7	5

(A) Reproduction condition visible

Response \ Order	First	Second
C	26	24
I	4	6

(B) Reproduction condition hidden

Table 7.3.1: Frequency of correct (C) and incorrect (I) *reproduction* responses for condition visible (A) and hidden (B) as a function of order of presentation (A: χ^2 (1, N=60) = 0.4, 0.7 > p > 0.5; B: χ^2 (1, N=60) = 0.48, 0.5 > p > 0.3).

Response \ Order	First	Second
C	23	25
I	7	5

(A) Comparison after visible Reproduction

Response \ Order	First	Second
C	11	10
I	19	20

(B) Comparison after hidden Reproduction

Table 7.3.2: Frequency of correct (C) and incorrect (I) *comparison* responses as a function of order of presentation of the initial visible reproduction (A: χ^2 (1, N=60) = 0.4, 0.7 > p > 0.5) and hidden reproduction (B: χ^2 (1, N=60) = 0.06, p = 0.8).

Response \ Order	First	Second
C	10	13
I	20	17

(A) Direct comparison after visible Reproduction

Response \ Order	First	Second
C	20	17
I	10	13

(B) Direct comparison after hidden Reproduction

Table 7.3.3: Frequency of correct (C) and incorrect (I) *direct comparison* responses as a function of order of presentation of the initial visible reproduction (A: χ^2(1, N=60) = 0.64, 0.5 >p > 0.3) and hidden reproduction (B: χ^2(1, N=60) = 0.64, 0.5 > p > 0.3).

As the χ^2 test does not reach the .05 significance level for any of the tasks, the null hypothesis of homogeneity of correct responses as a function of order of presentation of the initial reproduction task cannot be rejected. The χ^2 analyses thus indicate that the number of correct reproductions and comparisons of children starting with the visible condition of reproduction is not significantly different from that of children who start with the hidden condition of reproduction. Since the order of presentation of the task conditions does not appear to have any significant effect on the children's performance, this variable will be ignored for the rest of the discussion.

7.3.10.2 Age groups analysis

Tables 7.3.4 to 7.3.8 present the number of correct and failed responses to the five task conditions in the two age groups: Nursery and Primary 1. A χ^2 test is computed on each contingency table to assess the degree of correspondence between the observed and expected distributions of responses in the two age-groups and to determine whether the reproduction and comparison performance varies with age.

Resp \ Age	Nursery	Primary 1
Correct	28	20
Failure	12	0

Table 7.3.4: Correct and incorrect *reproduction* responses in the *visible* condition for Nursery and Primary 1 subjects (χ^2 (1, N=60) = 5.74, p=0.0165)

Resp \ Age	Nursery	Primary 1
Correct	32	18
Failure	8	2

Table 7.3.5: Correct and incorrect *reproduction* responses in the *hidden* condition for Nursery and Primary 1 subjects (χ^2 (1, N=60) = 0.375, p=0.54)

Resp \ Age	Nursery	Primary 1
Correct	28	20
Failure	12	0

Table 7.3.6: Correct and incorrect *comparison* responses after *visible* reproduction for Nursery and Primary 1 subjects (χ^2 (1, N=60) = 5.74, p=0.0165).

Resp \ Age	Nursery	Primary 1
Correct	10	11
Failure	30	9

Table 7.3.7: Correct and incorrect *comparison* responses after *hidden* reproduction for Nursery and Primary 1 subjects (χ^2 (1, N=60) = 4.04, p=0.0444)

Resp \ Age	Nursery	Primary 1
Correct	10	13
Failure	30	7

Table 7.3.8: Correct and incorrect *direct comparison* responses for Nursery and Primary 1 subjects (χ^2 (1, N=60) = 7.41, p=0.0064)

The null hypothesis of across age groups homogeneity is rejected for all tasks but one. The correct and failed responses of Nursery children differ significantly from those of Primary 1 children in visible reproduction, comparison after visible reproduction and after hidden reproduction as well as in the direct comparison task. They do not differ significantly in the hidden reproduction task (Table 7.3.5).

Notice also that the distribution of correct and wrong responses is identical in the tasks of visible reproduction and subsequent comparison and is similar in the tasks of comparison after hidden reproduction and direct comparison. This conforms well with the hypothesis that these two pairs of task are acquired concurrently. The hierarchical analysis below will provide firmer evidence of concurrency by determining whether these equivalent frequencies represent the same children, e.g. if the 48 children correct in the visible reproduction are the same 48 children who are correct in the comparison after visible reproduction, and whether the 21 children who compare correctly after the hidden reproduction are among the 23 children who compare correctly in the direct condition. But before carrying out the hierarchical analysis of response patterns, I briefly examine the accuracy of the proposed order of task complexity as predictor of response changes in the two age groups.

7.3.10.3 Analysis of task difficulty

Implicit in the hypotheses of order of acquisition is a hypothesis of order of complexity between the task conditions. The present analysis examines whether the children's performance varies according to the predicted order of complexity of the task conditions. The hypotheses of concurrency between a) visible reproduction and comparison after visible reproduction and b) comparison after hidden reproduction and direct comparison imply that the associated tasks are of equivalent complexity so that a significant proportion of children should give a correct response both in the former and the latter task. The hypothesis of collective décalage between hidden reproduction and comparison after hidden reproduction implies that hidden reproduction is easier than comparison after hidden reproduction and so a significant proportion of children should shift from correct reproduction responses to wrong comparison responses. The following orders of complexity, with the associated sets of coefficients adjusted to sum zero, are tested (see section 7.2.10.1.3 for a more detailed description):

Order A: Visible Repro = Comp after Visible Repro > Direct Comp
1 (-1) 1 (-1) 2 (+2)

Order B: Hidden Repro > Comp after Hidden Repro = Direct Comp
2 (-2) 1 (-1) 1 (-1)

The Marascuilo & McSweeney test is calculated to see whether the order of complexity of the task conditions is a good predictor of the variation of correct responses frequency in the different task conditions. Table 7.3.9 summarize the results of the normal curve z test obtained for the orders A and B with Nursery and Primary 1 children. The z scores significant at the level of $p < .05$ are marked by an asterisk.

Order \ Age	Nursery	Primary 1
Order A visible	2.45*	1.53
Order B hidden	2.54*	1.31

Table 7.3.9 Marascuilo & McSweeney Z values of the predicted orders A and B for Nursery and Primary 1 subjects

The orders of complexity of task conditions are accurate predictors of the children's responses only in the case of Nursery children. The null hypothesis that the responses to the tasks are unaffected by the degree of complexity of the task conditions is in fact rejected for the Nursery group (for order A, $z = 2.45$, $p = 0.007$; for order B, $z = 2.54$, $p = 0.005$), but not for the Primary 1 group (for order A, $z = 1.53$, $p = 0.063$; for order B, $z = 1.31$, $p = 0.095$). Younger children find a) visible reproduction as difficult as comparison after visible reproduction, and both task conditions easier than direct comparison (order A); b) hidden reproduction easier than comparison after hidden reproduction, which itself is as difficult as direct comparison (order B). Older children, on the other hand, tend to respond homogeneously throughout the two task series. The data confirm the finding of the difference between the two age groups overall performance indicated by the previous age group analysis.

7.3.10.4 Hierarchical analysis of task solutions

The central testing of the hypothesized patterns of concurrency, collective décalage or individual décalage in the solution of pairs of tasks is carried out using the prediction analysis of cross-classifications technique. Tables 7.3.10 to 7.3.15 present the contingency tables corresponding to the responses to each pair of tasks for which an ordering hypothesis has been formulated. Each table is followed a) by the value of the Del index corresponding to the improvement over chance produced by the relative triangular hypothesis, b) by the values of the z test calculated on the Del and c) by the values of the z test of the difference between the main hypotheses and the two existing alternative hypotheses.

7.3.10.4.1 Hypothesis 1a: Collective décalage between hidden reproduction and comparison after hidden reproduction

Repro hidden \ Comp	S	F
S	20	30
F	1	9

Table 7.3.10: Contingency table for reproduction and comparison in the *hidden* condition (the white cell is the cell predicted to be empty by Hypothesis 1a (Del = 0.71; z = 2.7, p = 0.003)

Collective décalage from hidden reproduction to comparison after visible reproduction predicts the significantly non-chance contingencies and is a significantly better predictor than concurrency or the reverse décalage (both giving z = 2.6, p = 0.004).

7.3.10.4.2 Hypothesis 1b: Concurrency between visible reproduction and comparison after visible reproduction

For the visible condition, strict concurrency exists between the solution of reproduction and comparison. As the error cells of each one of the three possible orders are empty, all three hypotheses give Del of 1, significant at p = 0.000. Implicit in hypotheses 1a and 1b is the order between the comparison conditions.

Repro visible \ Comp	S	F
S	48	0
F	0	12

Table 7.3.11: Contingency table for reproduction and comparison in the *visible* condition (Del = 1; z = e, p = 0.000)

Since the comparison after visible reproduction is expected to be concurrent with visible reproduction and visible reproduction is itself expected to be solved prior to comparison after hidden reproduction, it should follow that the two forms of comparisons are themselves solved in a fixed sequence, with collective décalage in favour of comparison after visible reproduction. This derived hypothesis is examined in the following table:

Comp hidden \ Comp visible	S	F
S	21	0
F	27	12

Table 7.3.12: Contingency table for comparison after *visible* and after *hidden* reproduction (Del = 1; z = e, p = 0.000)

For the two conditions of reproduction, there is décalage from comparison after visible reproduction to comparison after hidden reproduction. This model is a significantly better predictor than concurrency (z = 10.7, p < 0.0001) or the reverse décalage (z = 18.9, p < 0.0001).

7.3.10.4.3 Hypothesis 2: Concurrency between comparison after hidden reproduction and direct comparison

Strict concurrency exists between the solution of the two conditions of the comparison task. The three error cells have very low frequencies and all three hypotheses give Del close to 1 (Del =. 92, z=12.4 for décalage of comparison over direct comparison; Del = .79, z = 7.8 for the opposite décalage), significant at p < 0.00001 and not different from each other.

Direct Comp \ Comp hidden	S	F
S	20	3
F	1	36

Table 7.3.13: Contingency table for *direct* comparison and comparison after *hidden* reproduction (Del = 0.86; z = 12.3, p < 0.00001)

Since the direct comparison task is solved concurrently with the comparison after hidden reproduction, and the latter was solved only once comparison after visible reproduction had been acquired, direct comparison is also expected to be solved after comparison following visible reproduction. The relationship between these two tasks is examined in the following table:

Direct Comp \ Comp visible	S	F
S	23	0
F	25	12

Table 7.3.14: Contingency table for *direct* comparison and comparison after *visible* reproduction (Del = 1; z = e, p = 0.000)

Collective décalage in favour of comparison after visible reproduction is highly significant and a significantly better predictor than concurrency (z = 9.5, p < 0.00001) and collective décalage in the opposite direction (z = 4.4, p < 0.0003). This result together with that of table 7.3.12 clearly indicates that the development of generalized comparison competence goes through two steps: first children know to compare sets where number and length coincide and only later they acquire the capacity to compare also sets where number and length do not coincide. Either the two rows have same number but different length (e.g. comparisons after accurate hidden reproductions) or they have same length but different number (e.g. direct comparison).

7.3.10.4.4 Hypothesis 3: Individual décalage between visible reproduction and hidden reproduction

Repro visible \ Repro hidden	S	F
S	42	6
F	8	4

Table 7.3.15: Contingency table for visible reproduction and hidden reproduction tasks

The three Dels associated to the three order hypotheses are the following:
- concurrency: Del = 0.19; z = 1.3, p = 0.097;
-collective décalage from hidden to visible reproduction: Del = 0.18; z = 1.25, p = 0.1;
- collective décalage of visible over hidden reproduction: Del = 0.20; z = 1.26, p= 0.1.

As none of the Dels is significantly greater than the chance Del, nor significantly different from the other Dels (z values of comparisons between 0.24 and 0.26, p = 0.44), the individual décalage explanation appears to hold between visible and hidden reproduction. This was, of course, our finding on Experiment 1. Again, however, we are faced with an unbalanced distribution in which 70% of the children succeed at both conditions of reproduction, so that an even younger sample might reveal some order effects. Notice that an equivalent individual décalage exists between hidden reproduction and comparison after visible reproduction. Both results suggest that numerical representations based on one-to-one matching and counting develop with some independence.

Repro hidden \ Comp visible	S	F
S	42	8
F	6	4

Table 7.3.16: Contingency table for hidden reproduction and comparison after visible reproduction

7.3.10.5 Summary of the results

The analyses of the acquisition orders provide support for all three order hypotheses. As in Experiment 1, concurrent solution of visible reproduction and comparison after visible reproduction identifies a first stage of numerical competence (Stage 1). Again collective décalage between hidden reproduction and the subsequent comparison task identifies two ordered stages (Stage 1 and Stage 2) is confirmed. Moreover, the new direct comparison task which involves distracting length cues, is in fact solved only by children who already solve a reproduction task which involves only counting, but is solved concurrently with the comparison after hidden reproduction which again offers unhelpful length cues. The ability to carry out a correct numerical comparison in the direct condition, despite conflicting visual cues, is hence another demonstration of the numerical understanding characteristic of Stage 2 (and of the limitations typical of Stage 1).

The data here also confirm the earlier finding of individual décalage in the solution of the two conditions of reproduction, suggesting that children reach Stage 1 through two different paths: some children first solve the reproduction in the condition visible and fail the condition hidden, while other children do the opposite. The more detailed reconstruction of the developmental process which emerges from the results of Experiment 2 consists of three main stages:

Stage 0, the children cannot reproduce nor compare sets;

Stage 1, the children can reproduce sets and compare sets of same length and number;

Stage 2, the children compare sets' numerosities independently from spatial cues and hence can judge sets in which the numerical size does not coincide with spatial size.

Children developing from Stage 0 to Stage 1 through the elaboration either of the capacity to reproduce sets using spatial matching (e.g. visible reproduction) or of the capacity to reproduce using counting (e.g. hidden reproduction). Stage 1 is reached by age 5, the age of the older children in the Nursery group, while Stage 2 is reached by age 6, the age of the older children in the Primary 1 group. The description of the three stages in the development of cardinal representations is enriched and further specified by examining the strategies underlying the children's responses.

7.3.10.6 Qualitative analysis of the responses

The main behavioural patterns described in Experiment 1 are also found in Experiment 2. In this section, I present the solution procedures used in each task (e.g. strategies, checks, justifications and corrections) and classify them according to the three stages. This further analysis provides a qualitative description of the competence levels alongside success and failure in the tasks.

7.3.10.6.1 Stage 0

Children are classified at Stage 0 when they fail the reproduction task in both the visible and hidden conditions. The inaccurate visible reproductions derive from three basic strategies. The children use *all the objects* at their disposal. Either they construct a very long row

o o o o o

oooooooooooooo

or they create two or three rows whose end points match those of the model row;

o o o o o

ooooooo

ooooooo

Other children start with an initial accurate one-to-one matching and gradually shift to put the objects very close to each other until an object is in correspondence with the last object of the model row;

o o o o o

o o ooooo

The third procedure observed consists of an inaccurate one-to-one correspondence. The reproduction of Kev (4,3) offers a particularly clear illustration of this strategy. Kev creates a loose matching between the objects in the model and in the copy set, putting four objects in correspondence with five. He then remarks the difference and adds one object at the end of the row of four; this yields two rows of different length. After a short pause, Kev adds another object to the end of the shorter row which fills in the unmatched end of the row. The two rows now have 5 and 6 objects respectively, but same length.

The fourth strategy consists of putting down the objects in a line which continues the model row, without counting nor applying any other form of quantification or correspondence:

o o o o ooooo   or   o o o o ooooo

Also in the hidden reproduction task, some children use all the objects at their disposal. Other children place the objects so as to reproduce the full length of the screen. One child instead takes a handful of objects (three objects) out of the box and puts them down as a bunch, without apparent counting. The majority of mistaken solutions however, consists of *inaccurate counts*. Children put down five items for a model row of four, or three items where four are required. In these cases, the experimenter repeats the question ("Here there are 5 'objects'. Did you put down the same number of 'objects' there?"). Some children do not recount, while other children recount but do not seem to appreciate the difference, and when they do they do not seem to envisage corrections nor modifications.

In the subsequent comparisons, the children who fail the reproduction are divided between those who incorrectly confirm that the two rows are equinumerous and those who, after they are induced to count, remark the difference but cannot quantify the difference nor do anything to correct it or equate the two sets. Either they inaccurately respond that the two rows have the same number of objects, or they say correctly that the two rows are different, but cannot decide which is bigger or what to do to balance the difference. When they try to cancel the difference, they add a random number of objects. When the experimenter suggests counting, they either count only one row or they count the two rows as one, without stopping at the end of the first row and restarting the count with the second row (1/2/3/4/5 (first row) 6/7/8/9 (second row)). In the case of Cra (4,10), the two rows are counted separately. Cra finds out that one row has four objects and the other five, but does not attempt any action to cancel the difference. Fra (5,2) instead notices that the row he has created is different from the model row, and simply rearranges the two rows to have same length and similar distribution.

7.3.10.6.2 Stage 1

Children are classified at Stage 1 when they succeed at the reproduction tasks and the comparison with rows of same number and length, but fail at numerical judgment when number and length do not coincide.

In the visible condition, when the row to be reproduced is present before the child the accurate reproductions are always carried out by the systematic one-to-one matching of each object of the row with one object from the child's bunch. Four children who have originally provided inaccurate reproductions because of a loose correspondence, spontaneously correct their reproductions. Two of the children move the objects back into a precise one-to-one correspondence and put the remaining, unmatched, objects away. Cla (5,8) spontaneously counts the two rows, notices that the copy row has one object less and adds one object to it. Fra (5,2) ends up with a copy row with matching end-points and nine objects instead of eight. He counts, remarks the numerical difference but unexpectedly takes one object out of the smaller set (the model row), instead of the larger one (the copy row). The two rows now have different number (7 and 9) and different length. Fra then takes away the object, at the end of the larger row, which is unmatched and obtains two rows of 7 and 8 with same length. He recounts the rows by counting the objects two by two, that is pointing to one object of the upper row and to the corresponding object of the lower row. He counts 1 and 1, 2 and 2, 3 and 3, etc., and does not remark the difference, as he counts one object twice. He says "I put them together", puts the objects of the two rows in one to one correspondence and finds out that there is one object which cannot be matched and takes it away.

In hidden reproduction (where the children know only the numerosity of the model set), the correct solutions are based on counting out of the box a number of objects equivalent to the target cardinal number. Some children execute this operation accurately, while other children need a second count. In most cases this count appears spontaneously; in other cases only after the experimenter has reminded the child of the number of objects in the model set.

All the children who succeed the visible reproduction task also succeed the subsequent comparison. Some of them count the two rows, others simply justify their reply saying that "that's the same and that's the same" or "'cause they are four".

Among the children who shift from a correct hidden reproduction to a failed comparison, three main response patterns are observed. Some children identify the difference in the elements of one row extending beyond the limits of the other row. They indicate the difference by tracing an imaginary line connecting the unmatched end points with their finger:

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oooo

o o o o
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They generally cancel the difference either by adding some elements to the shorter row or by taking away some elements from the longer row. Gor (4,5) for instance says "it's diagonal", and proposes to "take a pig away" to cancel the difference.

Some children consider the rows as wholes, that is without explicitly identifying the locus of the difference. They take the longer row to be more numerous and operate, as before, additions or subtractions. More frequently, however, they change the density of a row as a whole, condensing or expanding it, to achieve equivalence of spatial size. Bob (4,9) says that one row is more numerous because "it's longer" and proposes to "squash them" to have equivalent rows. Ja (4,1) says that it is "more in the long" and to have the same to "put them back together".

Very interesting behaviours emerge when the experimenter asks the children to verify their numerical judgment and suggests counting. Some of the children who based their relative number judgment on the difference in the rows' length, count the two rows in a way that reflects their judgment. This means that when the longer row, judged to be more numerous, is counted after the shorter row, one of its elements is counted twice, so as to obtain a cardinal value larger than the shorter row. If the shorter row is counted after the longer row, the child skips one element of the row so as to obtain a smaller cardinal value than the longer row. In most cases, the children give the impression of being aware of the miscount as immediately after they have skipped (or counted twice) an element they look up at the experimenter, as if they expected a reaction from him. Children "cancel" the difference by taking one object away from the longer row or adding one object to the shorter row in a way consistent with their doctored count. In some of these cases, however, they do not carry out any correction. Ed (5,6) counts the shorter row correctly to seven and the longer row to eight, counting an element twice. Asked to make the two collections the same, he proposes to "take one away", but does not do it. After a pause, the experimenter repeats the child's proposal: "so if you take this object away, would the two lines have the same number?". Ed does not reply, nor does he undertake any action. Similarly, Ka (5,7) counts the longer row to six correctly and the shorter row, also of six, to five. She does say that the longer row has one object more but does not attempt any change to cancel the difference.

Among the children who had failed the hidden reproduction, there is only one case of correction in the subsequent comparison task. Aid (3,5) has put down three objects when the model set had four. When comparing them, he first claims that the two rows are equinumerous, then counts them to find out that they have a difference of one. he immediately adds an element to the copy row to create the equivalence.

Stage 2 children also fail the direct comparison task. These failures are the outcome of four main strategies. Some children respond that the two collections are equinumerous, and confirm the judgment after having counted the two collections separately (even when one is counted to five and the other to six) or together (they are counted up to eleven). Fra (5,2) says that the two collections have the same number of objects and, asked to check by counting whether they are equinumerous, counts the two rows together, up to eleven. Other children consider the two collections to be equinumerous and modify their counting of the two rows to conform to that judgment. On the basis of the count obtained in the row counted first, be it of five or of six, the second row is counted up to the same number, regardless of its true cardinality. As in the case of the comparisons after hidden reproductions, the children seem to be aware of this trick as they look at the experimenter after the count and sometimes pause for an instant before double counting or skipping an item.

Some children realize that the two collections are of different number, but cannot say which collection has more elements. When they are invited to count, either they count only one row, or they count the two rows as one. After such a count, they can certainly not say which collection is more numerous, what is the difference nor have they the grounds to equate the two collections. The corrections which are attempted consists of: two children suggest to "swap them (the two rows) round", two other children take one element away from the smaller array.

The most puzzling behaviours are those of four children who remark the difference (either from the start or after the counting check) but cannot cancel it nor equate the two sets in a way they consider satisfactory. They realize that one row has one element more than the second row and add a new element to the smaller row. This element however introduces a difference in length, that they cancel by adding a new element to the shorter row. After this action, they count the two rows again, remark the difference and add another object. The procedure is generally interrupted after a couple of attempts or when all the objects at the child's disposal have been used. Ro (3,11) and Alex (4,2), for instance, say that there are more yellow sweets and add one orange sweets. They look puzzled, pause for a moment and put another yellow sweet in front of the orange one, thus reestablishing the matching of end points. Before answering Kat (5,10) counts the two rows together, up to eleven, then recounts them to six and six and still unconvinced counts them again to find that one has six sweets and the other five. She concludes that the two collections have a different number of sweets. When asked to make them have the same number of objects, she takes away one object of the larger set and, asked whether now the two collections are equinumerous, after a pause answers that she does not know.

Consider finally the case of the three children who fail the comparison after hidden reproduction and succeed the subsequent direct comparison. In all three cases the children correct an initial judgment of equinumerosity after counting the two arrays, whereas in the previous comparison, they had counted the collections without finding or correcting the difference. Lou (5,10) considers the longer row as having more objects. As she is asked to quantify the difference, she counts the two rows, finds out that they have both six objects, but does not correct her previous judgment to conclude that they are equinumerous. Ed (5,6) takes the longer row to be more numerous and counts it to eight, where both rows have seven elements. He identifies the difference and cancels it only after the experimenter suggests putting the two arrays in one-to-one correspondence. In the subsequent direct comparison task he replies that the two rows are equinumerous, but then counts them to five and six to conclude that they differ of one element.

7.3.10.6.3 Stage 2

The Stage 2 children solve all the five tasks correctly. Among the children who succeed that reproduction and then confirm equinumerosity in the following comparison task:

a) some children justify the answer by referring to the previous task (Ch (4,3) "'cause I counted five");

b) some children spontaneously count the two rows before answering the question (Mar (5,4) counts the two rows to six and concludes that they have the same number; Eli (4,10) replies "'cause it's 2 and 2 here and 2 and 2 here".

c) other children first say that the longer row is more numerous and then check whether this is the case by counting or by establishing a one-to-one correspondence. In (5,2) initially says that the longer row is more numerous, then counts the two collections and after a brief pause concludes that they are the same. Similarly Ben (4,7) first considers the longer row more numerous, then moves the objects in one-to-one correspondence and concludes that they are the same.

The case of Cla (5;8) provides a particularly clear illustration of the kind of processes underlying the comparison of arrays of different length. Cla has correctly laid down six objects. As the screen is taken away, the model row appears to extend beyond the row that she has constructed in length at both ends.

Model row o o o o o o

Copy row oooooo

Cla says that there are more objects in the model row and adds three objects to the copy row in such a way that their end points now coincide. Asked again whether the two rows are equinumerous, she counts them to seven (for six) and to nine. She adds three objects to the model row, inserting them in some of the wide gaps existing between the objects:

Model row o oo o oo oo o

Copy row o o o o o o oo o

Cla seems to hesitate; the experimenter asks the comparison question a third time to see whether she is convinced of the equinumerosity. Cla replies that the two rows are different and adds two more objects to the model row, recounts the two rows and finally adds two more objects to the copy row to achieve equinumerosity between rows again (eleven objects each).

In the direct comparison task, the correct responses are obtained by two procedures. In one, children say that the two rows are equinumerous on the basis of the spatial extent of the collections, and then count the two rows. When they discover that they are different, they calculate the size of the difference and add or subtract the element which constitutes the difference. In the other procedure, before answering the comparison question, they count the two collections to conclude that they are different and that one of them has one object more than the other.

4.3.10.7 Summary of the qualitative analysis

The qualitative analysis of the children's solution strategies helps characterize the three competence levels identified in this experiment. In particular the analyses expose in some detail the progress that Stage 1 constitutes over Stage 0 and the difficulties that Stage 1 children experience with the comparison tasks solved at Stage 2. Stage 0 corresponds to a systematic failure in all tasks. In both conditions of reproduction Stage 0 children either use all the objects at their disposal or put down an unquantified bunch, reproducing the length of the model row or simply creating another unrelated row. The progress characteristic of Stage 1 is reflected in the accurate reproductions carried out through a one-to-one matching when the model set is visible and through counting out the required set in the hidden condition. At this stage there is a remarkable consistency between visible reproduction and the subsequent comparison.

The most interesting behaviours relative to Stage 1, however, emerge from the more difficult tasks of comparison after hidden reproduction and of direct comparison. Stage 1 children abandon the equinumerosity established in the initial correct reproduction and judge the two, now visible, rows as different in number once they see that the two rows have a different configuration. The longer row is systematically taken to be greater in number than the shorter row. At the same time, they take the two rows of same length (e.g. the direct condition of comparison) to be also equivalent in number, despite one has one element more than the other. The Stage 1 children appear to lack the competence to compare the cardinality of sets of objects, and to judge their numerosity systematically on the basis of spatial extent (actually length or the largest dimension). Accordingly, children's success in the comparison after visible reproductions does not constitute an expression of numerical competence, but rather the outcome of a length comparison which is correct because the spatial and numerical dimensions of the sets coincide.

In comparison tasks, then, number appears to be a non-criterial dimension, even though it is relevant and operational in the previous reproduction tasks. Stage 1 children thus use counting and counting information when they are asked to check their judgment. However, some children count both rows as one, others count the two rows separately to conclude that they are equinumerous, regardless of the different counts they obtain. Other children instead remark the difference but can neither quantify it precisely nor cancel it. The checking procedures identify also some behaviours which may be interpreted as expression of an internal conflict between space and number based judgments (e.g. counting is adjusted to fit the original judgment, attempts to achieve at the same time equivalence of number and length).

In some cases the conflict is resolved in favour of space and produces incorrect comparisons; in other cases it is resolved in favour of number and leads to correct comparisons; in the last examples instead, the conflict does not seem to be resolved as the children try to combine the numerical and spatial information about the sets. They either produce a dishonest count or attempt by a trial and error method to achieve equivalence of spatial size and of number.

Finally, Stage 3 children solve the full battery of tasks, making, testing and revising their numerical judgments. In the comparison tasks, they may start by judging the longer row to be more numerous than the shorter to then count the two rows (or match their elements) to check whether the longer row is indeed more numerous. Alternatively, they count before making their judgments or compare correctly and justify their response by reference to the reproduction they carried out before.

7.4 General discussion of Experiments 1 and 2

Experiments 1 and 2 have pointed out a clear developmental change in the child's understanding of cardinal number in the period which goes from the Nursery school years to Primary 2, that is, from age 4 to 7 years. Number development proceeds through four stages of increasing problem-solving ability from the failure to solve the tasks of set reproduction (Stage 0), to the capacity to reproduce sets (Stage 1), to make judgements of numerosity (Stage 2), to conserve number in the standard Piagetian task (Stage 3). Stage 1 is attained through two different paths. Some children first learn how to reproduce sets when this involves the one-to-one matching of objects (e.g. the visible reproduction task) and only later carry out reproductions using counting. Other children follow the opposite path.

Whereas I cannot say much about the nature of the cardinal number concept underlying Stage 0 since all the tasks seem to be beyond the child's competence[43], some hypotheses about the nature of the cardinal number concept at Stage 1, 2 and 3 can be advanced. These hypotheses are derived from the general theoretical framework proposed in the preceding chapters.

We have proposed that children have domain-specific structures specialized in processing particular kinds of information. In the appropriate circumstances, these structures are activated (possibly by something like a pattern-matching process) and produce an internal representation of the aspects of the situation relative to the domain. On these lines, Gelman & Meck (1986) advocate

> the idea that much of early cognitive development proceeds as a function of some domain-specific principles that define domains, focus attention on domain-relevant inputs, and play a central role in the selection and generation of the class of domain-appropriate behaviors (p.29).

In the course of development, we claim, children work out the relevance of these representations in the form of contributions towards drawing some inference useful to the solution of problems, the prediction of events and their classification. The process of "making information relevant" however is not envisaged as functioning on a case-by-case basis depending on the child's immediate circumstances. Instead it is envisaged as proceeding in a step-by-step fashion from simpler to more complex contents. The discovery of the import of some representations in a new, more complex, class of situations produces a generalization and reorganization of the domain-specific knowledge that the child already possesses. This conceptual restructuring permits new applications and serves as pre-requisite for further generalizations.

The developmental process itself is envisaged as the transition from a cognitive state in which the domain-specific representation is entertained as irrelevant, to an

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43 Tasks more appropriate to children younger than 4 should be devised and examined, like for instance Gelman's magic conservation (see Appendix 6.1), to evaluate the numerical competence of Stage 0.

intermediate phase corresponding to ambivalence towards the import of this representation, to the final discovery of its relevance and of the consequences that can be drawn from it. The outcome of the process is a more articulate and general understanding of the domain, as the domain-structure becomes relevant for a new class of (more complex) contents.

In the case of the cardinal number domain, the structure specialized for processing numerical information in the environment abstracts and produces representations of cardinality for sets of objects. Its basic functions are to identify entities such as *objects* and *collections of objects* and to establish relations of *one-to-one correspondence* between elements of collections. I would claim that this structure is present from very early in development and maybe biologically determined. Support for this claim is found in studies of infants and preschool children who display a range of quantitative abilities, such as magnitude discrimination, counting, estimation of numerosities. In fact, even infants are able to discriminate between small numerosities (Starkey & Cooper 1980, Starkey, Spelke & Gelman 1983, Strauss & Curtis 1984). These studies indicate a) that infants can discriminate between arrays of two and three items by age 10 months, and sometimes between three and four items[44]; b) that infants 6 to 8 months-old can detect intermodal numerical correspondences between a visible arrangement and a sequence of sounds[45].

However a large qualitative gap exists between these first demonstrations of quasi-perceptual numerical competence and the understanding and conceptual use of number in the tasks I have studied. While the infants' tasks deal with the capacity to abstract and match representations of numerosity, the tasks of reproduction, comparison and

[44]The technique used is the multiple-habituation technique. An infant is presented with several members of a category until habituation occurs. During the test phase, the infant is presented with two types of instances of the familiar category and with instances of a new category. The ability to categorize is inferred from the continued habituation to new instances of the invariant category and from the dishabituation to instances from a new category. In the case of number, the infant is habituated to a category of number (N) items varying in dimensions like length of the array, density, item type, size, position, etc. The dishabituation corresponds to instances of a category N + 1 or N - 1. If the infant generalizes to this new category, then it is inferred that infants are not capable of abstracting numerosity. If instead they don't, as it appears to be the case, they are attributed the capacity to discriminate numerosity against other dimensions.

[45]Infants are presented with a choice of looking at a picture containing either two or three objects while they heard either two or three drumbeats. Infants look longer at the visual display with the number of items that matched the number of drumbeats. From this result, the authors conclude that infants must be able to abstract away from the modality of presentation and the type of item to be enumerated.

conservation require that operations be carried out on these representations, inferences be drawn from them and precise quantifications arrived at. The progression from the initial forms of number knowledge to the later forms of reflected, explicit forms corresponds to the developmental process by which the child works out the relevance of the cardinal representations (produced by the number structure) to acting in real world situations, solving problems, inferring numerical properties and principles. The constraint under which this process functions is that it goes through a series of stages corresponding to the application of the structure to different and more complex contexts, i.e. the contexts for which the numerical representations are discovered to be appropriate and useful. Consider the stage sequence emerging from Experiment 1 and 2 from this perspective.

The reproduction task provides the first context of application for which we have evidence that the structure is operational and produces relevant representations. Stage 0 children do not attempt to establish any form of numerical relationship between the model set and the copy. On the contrary, Stage 1 children either partition the objects at their disposal to form a subset which they then reproduce via counting or one-to-one correspondence and equate to the model set. Or from the beginning they establish a one-to-one correspondence between the elements of the copy and model sets.

The second context for which the structure application becomes operational is the comparison task. Stage 1 children, though they can reproduce sets, fail the comparison tasks where number and length do not coincide because they judge the sets' numerosity on the basis of length. The collection is perceived as a whole, and not as constituted of a sum of individual elements, and no decomposition and matching are attempted. Stage 2 children instead look for the numerical relation between the two collections and check whether two sets are equinumerous or not either by counting them or by matching their elements one to one. This does not mean however that they never use spatial cues as a first hint into the sets' sizes. It means rather that they have the means to test whether an estimation based on spatial extent is accurate. Stage 1 children lack this second resource fundamental to make number judgments.

The third context is constituted by the Piagetian conservation of number task. Here Stage 2 children seem to regress to the spatial estimation characteristic of Stage 1 children's number judgments. When they witness the spatial transformation of one of the two equinumerous sets, they abandon the previous judgment and say that the rows have now different number. Interestingly, they do not attempt any verification as they generally do in the comparison tasks. Stage 3 children find the conservation problem easy and swiftly confirm the equinumerosity that they justify by reference to the starting equivalence or to the spatial nature of the transformation performed.

While the patterns of behaviour are similar, the three competence levels appear to differ in the nature of the objects over which the child has to operate with cardinality. The reproduction task bears on an *individual set* which has to be identified as such and quantified. A correspondence has then to be established between each of its elements and one element of the child's bunch. The operation of correspondence can be carried out with two means: the spatial matching and counting.

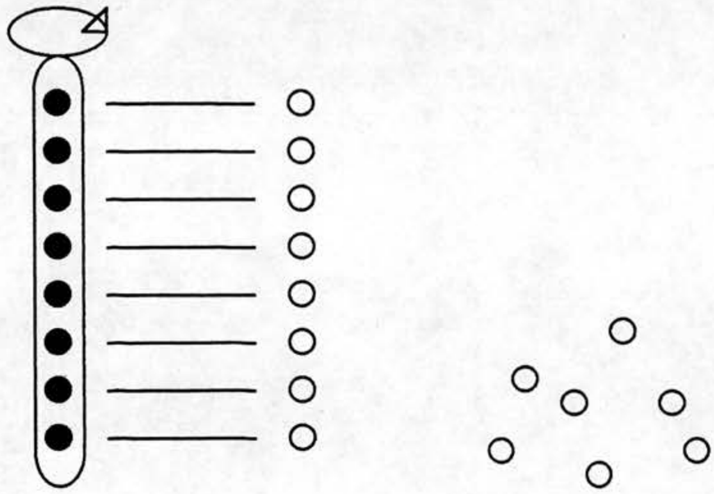


Fig. 7.7: Cardinal number structure applied to the set reproduction task (the black counters correspond to the model row, the white counters to the bunch the child has at his disposal to carry out the reproduction)

The comparison task (fig. 7.8) bears on a *pair of sets* which have to be identified as two separated entities (e.g. a step which can be problematic for those children who, when asked to verify their inaccurate judgment, count the two sets as one) and which have to be matched with respect to their numerical size. A correspondence has to be established between the elements of the two collections either via counting or spatial matching.Now consider the standard conservation task. Two facts strongly suggest that this task requires a more complex operation than establishing a numerical relationship between two sets. First, its solution appears after the solution of the comparison task with rows of different length and same number. If conservation demanded a simple comparison of the two post-transformation rows of different length, it should be solved concurrently with the comparison of rows.

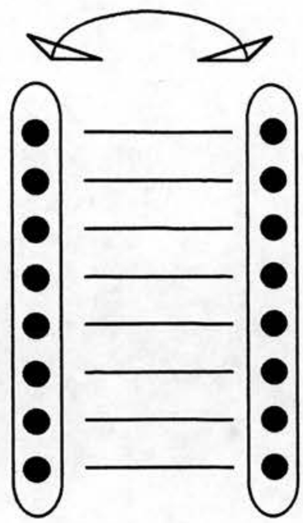


Fig. 7.8: Cardinal number structure applied to the set comparison task

Second, non-conserving children who systematically use counting and matching to solve the comparison tasks do not apply any of these strategies to solve the conservation problem (e.g. a common observation in conservation studies, see Gelman 1982). It appears then that because of its structure the conservation task does not invite empirical procedures (e.g. a second count of the two rows) to determine whether they are still equinumerous or not, but some form of inference from the initial equinumerosity and the transformation type.

Elkind (1967, see section 6.2.3.1) argues that the conservation task sets two requirements: the conservation of the number identity of the set which is transformed (i.e. A = A', the transformed set) and the transitive inference to go from the initial equinumerosity A = B to the conclusion A' = B, through the intermediate step A = A'. The children may thus know how to determine the numerosity relation between two sets and to conserve the numerosity of a single set, but still fail to derive the conservation principle as they do not have the logical competence to draw transitive inferences[46].

In the discussion of early conservations in the accidental-incidental tasks in Chapter 4, I have proposed an alternative interpretation of the requirements of the standard conservation task (see section 4.3.5). I suggested that to understanding the

[46]There is now evidence that children can draw transitive inferences from the age of 4. Although the debate about whether this competence corresponds to the fully abstract transitive reasoning described by Piaget at the concrete operational stage is still open, Elkind's interpretation may need to be reconsidered. I shall come back to this issue in the case of transitivity of numerical relations in the next Experiment 3.

conservation principle the child must relate the equinumerosity of the initial pair of sets to the numerical relationship of the post-transformation pair. So, while the number judgment task bears on a pair of sets, the conservation task bears *on a pair of pairs of sets*. It involves the application of the cardinal structure to a more complex class of objects, i.e. pairs of pairs of sets, to work out the conservation of equinumerosity. In other words, the understanding of the equivalence conservation principle involves a kind of second-order numerical representation over the first-order representation of the equinumerosity of two sets.

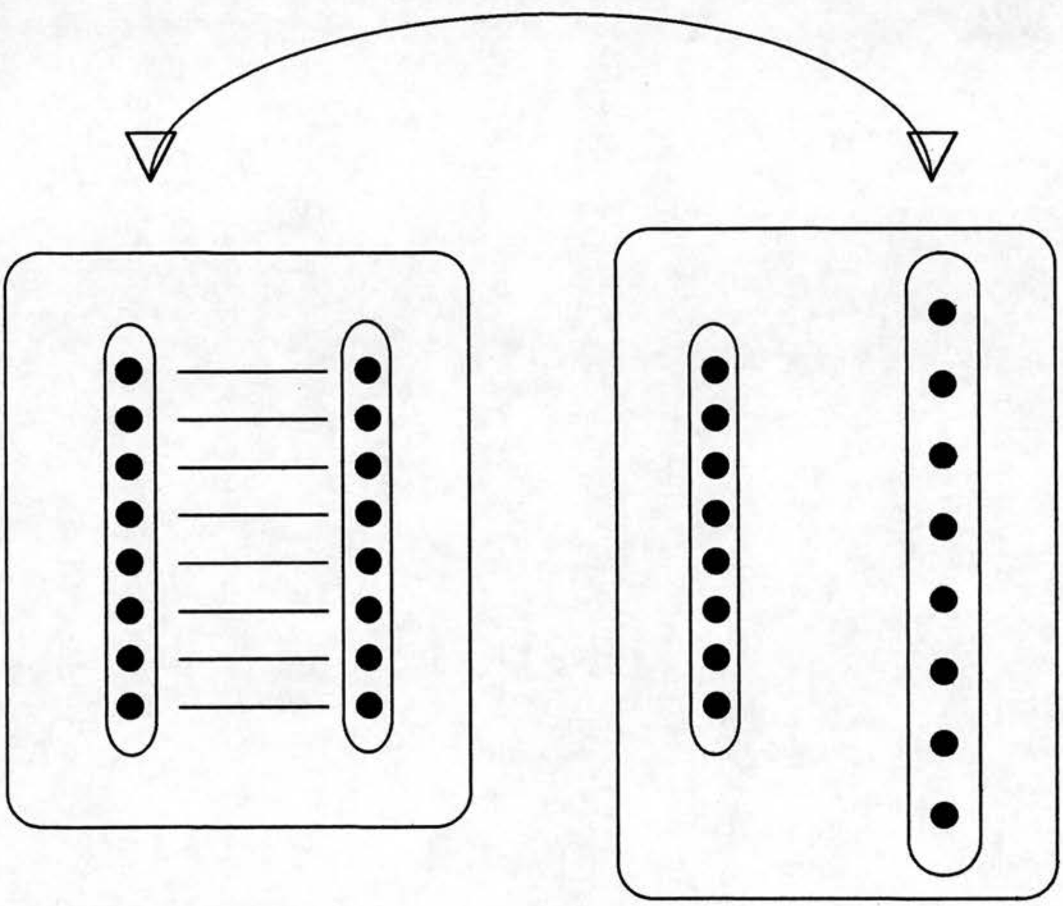


Fig. 7.9: Cardinal number structure applied to the number conservation task

To summarize, Stage 1 corresponds to the ability to operate with number on single sets; Stage 2 to operate with number on pairs of sets; Stage 3 to operate with number on pairs of pairs of sets. At all levels, the structure involved is the same, as it identifies objects and sets of objects and relates them by one-to-one correspondence, but its domain of application varies from individual sets, to pairs of sets, to pairs of pairs of sets. In Stage 1, the child discovers the relevance of the number structure application to quantifying individual sets precisely. The reproduction task in fact requires that one set be matched to another set to produce two equinumerous sets and cardinality is a much more accurate and reliable means to achieve that than estimations based on spatial extent. In Stage 2, the child discovers the relevance of the cardinal representation when one has to decide which set of a pair is more numerous. In this context too, number provides a much more reliable cue to size than spatial extent. In Stage 3 yields, the child learns the relevance of number to the equinumerosity of two sets. The child in fact works out the general property of invariance of two sets' equinumerosity when these are transformed in shape.

Each step of the process is a pre-requisite for the next. The elaboration of cardinality as a property of sets of objects is instrumental in working out the numerical relationship which may hold between two or more sets. Understanding cardinality as the basis of the numerical relationship between sets is instrumental in working out the principle of invariance as a general law of number, which itself can open the way to subsequent more articulate and coherent number concepts. Each new acquisition hence provides another building block for the elaboration of the subsequent concept and constitutes the basis for new abstractions (e.g. the conservation principle is understood only after the numerical relations between pairs of sets have been worked out).

This account of cardinal number development sheds some light on the precocious forms of conservation that I have discussed in the literature review (see section 6.2.3): identity conservation (section 6.2.3.1) and counted conservation (section 6.2.3.2). Consider identity conservation first. The task of identity conservation involves one row of objects which is spread out. After the transformation, the child is asked whether or not the number of objects in the row is still the same. The studies which compare the same children's performance in the identity and standard, equivalence conservation tasks report a tendency for identity conservation to be solved earlier.

I argue that this décalage can be explained by the fact that the identity task requires matching the pre-transformation set to the post-transformation set. This operation, according to the account presented above, can be performed at Stage 2, where the child has discovered that sets of objects are numerically related. Although the application of the number structure in the set comparison and the identity conservation tasks involves the same class of objects (a pair of sets), the latter task may introduce further difficulties as the two sets are not both present before the child at the same time. The child may thus match a memory of the pre-transformation set with the post-transformation set. Nevertheless, the nature of the operation is the same.

Now consider the counted conservation task. Recall that before the conservation question, the child is required to count the two rows. Under these conditions, the children can confirm the equinumerosity earlier than they do in standard conservation. I argue that this décalage can also be explained by the fact that the modified task requires the matching of two sets' numerosities, an operation which is understood by the child at Stage 2. In counted conservation in fact, the child judges two sets equinumerous, then one of the set is lengthened. At that moment, the child counts the two sets and only now is he asked whether their number is still the same or whether it has changed. The child can answer the question on the basis of his count alone without having to think back to the initial equinumerosity or to consider the relationship linking the pre- and post-transformation rows.

Two predictions follow from this alternative interpretation of the early forms of conservations. Identity conservation and counted conservation should be failed at Stage 1 and solved at Stage 2 concurrently with the comparison of sets whose length and number do not coincide. These hypotheses are examined in the next chapter where I report Experiment 3.

Chapter 8 Experiment 3

8.1 Introduction

Experiment 3 focuses on the period of development encompassing Stage 2 and 3 and examines the hypothesis that the difference between the competence underlying these two stages rests in the nature of the objects among which the children can recognize the equinumerosity: sets in Stage 2, sets of sets in Stage 3. The hypothesis is tested via the acquisition orders of the tasks of set comparison, standard number conservation and two modified conservation tasks which identify early forms of conservation: identity conservation and counted conservation. I have claimed that the modified conservation tasks require the competence to relate and match the cardinality of sets of objects, i.e. the competence characteristic of Stage 2. Accordingly, these tasks should be solved after set reproduction and before the standard number conservation task. In fact, if the modified conservation tasks, like the set comparison task, involve a numerical relationship between two sets, these tasks should be failed by some of the children who reproduce sets and who have the competence to operate on individual sets only (i.e. Stage 1). Similarly, if they involve a first-order cardinal relationship, some of the children who conserve in the modified tasks should fail to conserve in the standard task which involves a second-order cardinal relationship (i.e. Stage 3). Above all however, if the modified conservation tasks and the set comparison tasks track the same level of number competence, i.e. the ability to relate and to match the cardinality of sets of objects, then they should be solved concurrently.

Experiment 3 also examines the alternative interpretation of the standard conservation task proposed by Elkind (1967). Whereas I have suggested that understanding conservation consists of working out the cardinal relationship between the more complex class of objects 'pairs of pairs of sets', Elkind argues that the conservation task involves advanced logical reasoning. According to Elkind, non-conservation is due to the fact that the children lack the logical competence to draw transitive inferences and in particular to conclude from the initial equinumerosity $A = B$ and the number identity of the set pre- and post-transformation $A = A'$, that the sets $A' = B$ are equinumerous.

To provide some indications about whether Stage 2 children do indeed lack the logical competence to draw transitive inferences, Experiment 3 introduces a new comparison task involving three sets whose number and length do not coincide. The comparison of three sets can in fact be carried out by at least two kinds of procedures:

a) the sets are counted and compared two by two; the two larger sets are then compared between themselves to pick the largest of the three, if there is one;

b) other procedures involve some elementary forms of transitivity. A rather sophisticated use of ordinal information in the number words sequence underlies the simpler procedure of counting the three sets and picking out the set corresponding to the largest count as most numerous. A more explicit inference underlies the following procedure: two sets are counted, the larger is picked out and then compared with the third set. For instance, if A is found to be larger than B, A is then directly compared with C, to determine whether A or C is the largest of the three. This procedure omits one step, the comparison of C with B. Depending on the particular circumstances, the transitive inference takes the child from A is greater than B and C is greater than A, to the conclusion that C is also greater than B and is the largest of the three. Or alternatively it can take the child from A is equivalent to B, and C is greater than B to the conclusion that C is also greater than A and is the largest of the three.

8.2 Objectives

Experiment 3 has three basic objectives:

1. To replicate the findings of the previous experiments that the capacity to reproduce sets appears consistently before the capacity to compare sets and that this appears systematically before the capacity to conserve number;

2. to evaluate the prediction that identity conservation, conservation of counted sets and comparison of two and three sets are acquired concurrently at Stage 2 (with the two corollaries that these tasks are solved with a collective décalage over the preceding solution of the reproduction task and the following solution of the conservation task);

3. to examine the procedures used to solve the three-set comparison task. Since I claim that Stage 2 children have a concept of number as relation between sets as well as the inferential capacities to reason with it, I expect to find some evidence of transitive reasoning. While Stage 1 children tend to employ exhaustive counting and to match each pair of sets, Stage 2 children should compare two pairs of sets only and apply transitive reasoning

8.3 Hypotheses

Experiment 3 investigates two groups of response patterns. The first group is a replication of the findings of Experiment 1 and 2: 1. *Collective décalage* between hidden reproduction and comparison after hidden reproduction; 2. *Collective décalage* between comparison after hidden reproduction and conservation.

Hypothesis 1a: there is collective décalage between the solution of the reproduction task and the later solution of the comparison task.

Correct performance on the comparison should be more strongly associated with correct performance in the reproduction task than with incorrect performance in the reproduction task (see figure 7.2, p. 148).

Hypothesis 1b : there is collective décalage between the solution of the comparison task and the later solution of the conservation task.

Correct performance on the conservation task should be more strongly associated with correct performance on the comparison tasks than with incorrect performance on this task (see figure 7.1b, p. 147).

The second group of hypotheses deals with the relationship between the solution of the tasks of comparison of two sets, three sets and of identity and counted conservation. We predict that all four tasks will be solved at the same stage because they involve *cardinal representations of same order*, i.e. between pairs of sets. Before Stage 2, children cannot establish such representations for pairs of sets and base their number judgments on spatial extent and length. At Stage 2, children have elaborated such representations and can recognize and operate with the numerical relationships between two or more sets. They can thus solve all the tasks of number comparison (two- and three-set comparisons and counted conservation) as well as conserve number in the identity conservation by relating the cardinal representation of the pre-transformation set with that of the post-transformation set.

We also predict that these tasks will be mastered *after* set reproduction and *before* standard conservation. The décalage from reproduction to comparisons and modified conservations is explained by the fact that the operation of relating pairs of sets (required by the comparison and modified conservation tasks) cannot be handled with the Stage 1 number concept as property of individual sets. The décalage from comparisons and modified conservations to standard conservation is explained by the fact that the operation of relating pairs of pairs of sets (required by the standard conservation task) cannot be handled by the Stage 2 number concept as relation between sets. These two corollary hypotheses and the associated tables are presented in Appendix 8.1.

Hypothesis 2a : The solution of comparison after hidden reproduction is concurrent with the solution of three-set comparison task.

Correct performance on the comparison after hidden reproduction should be strongly associated with correct performance in the three-set comparison task; incorrect performance on the comparison after visible reproduction task should be strongly associated with incorrect performance in the three-set comparison task. The white cells are the cells predicted to be empty as shown in figure 8.1 below.

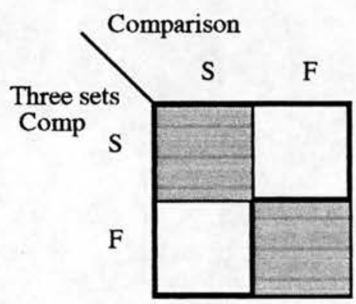


Fig. 8.1: Model of concurrency between *comparison* after *hidden* reproduction and *three-set comparison* according to hypothesis 2a.

Hypothesis 2b : The solution of comparison after hidden reproduction is concurrent with the solution of the identity conservation task.

Correct performance on the comparison after hidden reproduction should be strongly associated with correct performance in the identity conservation task; incorrect performance on the comparison after visible reproduction task should be strongly associated with incorrect performance in the identity conservation task.

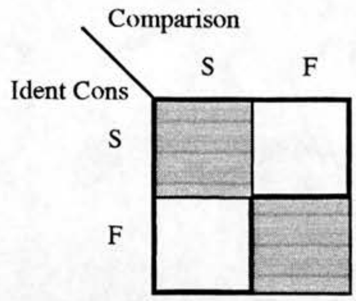


Fig. 8.2: Model of concurrency between comparison after *hidden* reproduction and *identity conservation* according to hypothesis 2b.

Hypothesis 2c : The solution of comparison after hidden reproduction is concurrent with the solution of the counted conservation task.

Correct performance on comparison after hidden reproduction should be strongly associated with correct performance in counted conservation; incorrect performance on comparison after visible reproduction task should be strongly associated with incorrect performance in counted conservation.

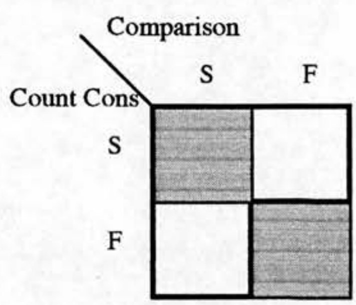


Fig.8.3: Model of concurrency between *comparison* after *hidden* reproduction and *counted conservation* according to hypothesis 2c.

Hypothesis 2d : The solution of three-set comparison is concurrent with the solution of counted conservation task.

Correct performance on three-set comparison should be strongly associated with correct performance in counted conservation; incorrect performance on three-set comparison should be strongly associated with incorrect performance in counted conservation.

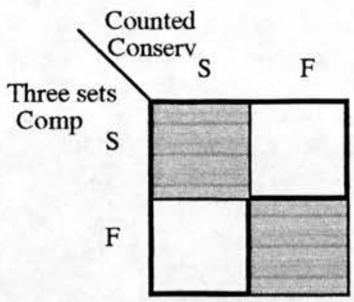


Fig. 8.4: Model of concurrency between three-*set comparison* and *counted conservation* according to hypothesis 2d.

Hypothesis 2e : The solution of three-set comparison is concurrent with the solution of the identity conservation task.

Correct performance on three-set comparison should be strongly associated with correct performance in identity conservation; incorrect performance on three-set comparison should be strongly associated with incorrect performance in identity conservation.

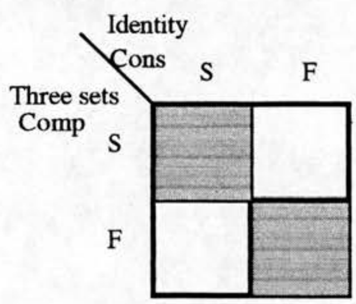


Fig. 8.5: Model of concurrency between *three-set comparison* and *identity conservation* according to hypothesis 2e.

Hypothesis 2f: The solution of counted conservation is concurrent with the solution of the identity conservation task.

Correct performance on counted conservation should be strongly associated with correct performance in identity conservation; incorrect performance on counted conservation should be strongly associated with incorrect performance in identity conservation.

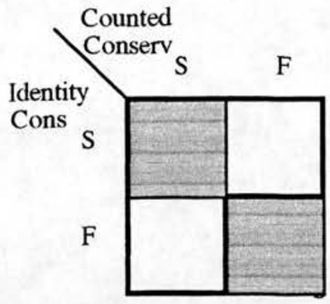


Fig. 8.6: Model of concurrency between *identity conservation* and *counted conservation* according to hypothesis 2f.

8.4 Design

Each child participated in six tasks:

1) number reproduction. This task was identical to the hidden condition of reproduction in Experiments 1 and 2;

2) number comparison (after the hidden reproduction condition) was again the task used in Experiments 1 and 2;

3) three-set comparison. The experimenter presented three rows of objects of different length, one of which had one element more than the other two. The child was asked whether the three rows had the same number of objects or whether one of them had more;

4) identity conservation. The experimenter presented a row of objects, then spread it out to form a longer row. The child was asked whether the row had the same number that it had had before or whether the number was now different;

5) counted conservation. The experimenter presented the child with two rows of objects of same number and length, asking him whether they had the same number of objects or not. One of the rows was then spread out to form a markedly longer row. The child was required to count the two rows and only after the count was asked whether the two rows were equivalent or not;

6) standard conservation,. The experimenter asked the child to reproduce a set placed before him. When the row had been created and the equinumerosity confirmed, the experimenter spread one of the two rows out and asked the conservation question: whether the two rows were still equinumerous or whether one had more elements.

Reproduction and comparison tasks were presented first, in a fixed sequence. The remaining tasks were presented in a balanced set of 24 orderings according to a Latin Square design (with three subjects per block). The levels of the *independent variables* are:

- <u>tasks</u>:
    - reproduction;
    - comparison after reproduction;
    - comparison of three-set;
    - identity conservation;
    - counted conservation;
    - standard conservation.
- <u>schooling/age group</u>:
    - Nursery (between age 3,6 and 5 years);
    - Primary 1 (between age 5 and 6 years);
    - Primary 2 (between age 6 and 7 years).

Dependent variable were the number of correct responses in each task and the contingencies between correct and incorrect responses in pairs of tasks.

8.5 Statistical analysis

The statistical methods employed here are identical to those used in Experiments 1 and 2, with one exception: no statistics are computed regarding the order of presentation given the balanced task orderings.

8.6 Material

The same collections of objects used in Experiment 1 were employed in the reproduction, comparison and counted, standard conservation tasks. They consisted of red, blue and green cylinders (1.7 cm high and 1.7 cm of diameter) and small plastic animals (hippopotamuses and pigs of size equivalent to the cylinders'). In the identity conservation task I used a collection of plastic frogs slightly bigger than the pigs and hippopotamuses. In the three-set comparison, three rows of colourful clown's faces (1.5 cm wide and 2 cm high) were presented on a white cardboard. They were stuck on with a weak glue which allowed them to be easily removed. The three rows were of different length, two of the three having same number, and the third one object more. The top row consisted of six faces (dominant colour black) and was 14 cm long. The mid row also contained six faces (dominant colour red) and was 24 cm long. The bottom row had seven faces (dominant colour yellow) and was 18 cm long. The three sets were presented as in the following configuration:

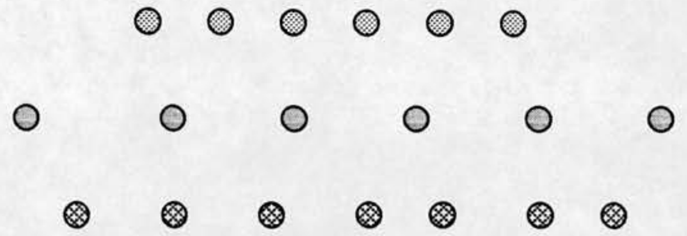


Fig. 8.7: The three-set comparison task display

8.7 Procedure

The reproduction, comparison and conservation tasks were exact replicas of the tasks used in Experiment 1. I refer to section 7.2.7 for a detailed description of their presentation. The only difference with respect to Experiment 1 is that, because of the balanced order of presentation, the conservation task did not systematically follow the reproduction-comparison pair. Hence it was always preceded by its own set reproduction (visible condition) task. The child was asked to take the same number of objects as there were in a row before him and to construct a second array in front of the first one. After the equinumerosity had been confirmed, the experimenter said "Look what I do now" and spread one of the two arrays out. He then asked the conservation question (see section 7.2.7.3). The child was asked to justify his response and, if he had answered that the two rows are different, first to say which row he thought had more elements, how many more and second to equate the two rows' number. Because of the long series of trials, the conservation task is presented only once with a spread out transformation.

The identity conservation task was presented in the context of a story to make it more plausible. The frogs were lined up very close together. The experimenter said: "You see these frogs, they are going for a walk all together". He then started moving the first frog to the end of the table, the second frog at some distance and so on for all the other frogs. In the meantime he said: "Some of the frogs walk much faster than the other frogs and are farther ahead. Some of the frogs are slow and stay a little behind. Do you think that there is still the same number of frogs in this long line as it was at the start of the walk?". If the child answered that the number was now different, the experimenter asked whether it was more or less and why it was. When the response was positive, the child was asked why he thought so.

To introduce the counted comparison task, the child was presented with two rows of objects of same length and number and asked: "Do these two rows have the same number of rounds, or does one of them have more rounds?". After the child had confirmed the equinumerosity, the experimenter lengthened one of the rows and asked the child to count the two rows "I would like you to count this line (pointing to one of the rows). Could you also count this other line? How many rounds are there? and how many rounds are there?". If a count was inaccurate, the experimenter intervened and suggested counting a second time. Finally the experimenter asked the conservation question and the usual justifications and clarifications.

Finally, in the three-set comparison, the child was presented with the three rows of clown faces and asked: "I would like you to tell me whether there is the same number of clowns here (pointing to the bottom row), here (pointing to the mid row) and here (pointing to the top row), or whether the number is different? You can either count them or move them around if you wish". If the child responded that the number was different, he was asked to indicate which row (or rows) had more objects and to quantify the difference. The experimenter systematically suggested to carrying out a control ("Do you know of a way to check that this line has more clowns?") either by counting the three rows ("Would counting help?") or by relocating the objects ("Would moving the faces help?").

8.8 Interviews

The interviews took place in the class-room in a corner separated by a low bookcase or by a desk and some chairs. The interviews lasted between 15 and 25 minutes. Notes were taken on a prepared sheet (see Appendix 8.2) by the experimenter.

8.9 Measure

Children's performance was measured by the number of correct reproduction, comparison and conservation responses. The scoring criteria were:

- in the <u>reproduction task</u>, the number of items of the copy set had to be equivalent to the number of items of the model set;
- in the <u>two sets comparison task</u>, the judgment of equinumerosity had to conform to the actual situation; in the case of incorrect reproductions, comparisons were scored correct when the child recognized the difference and corrected it; children who admitted that the two sets were different but did not know how to equalize them nor to quantify the difference were scored as failing the task;
- in the <u>three-set comparison task</u>, the numerosity judgement had to identify the row with seven faces as the largest. If an initial incorrect judgment was corrected after the experimenter's suggestion of checking it using counting or moving the objects, the child was scored correct;

- in the <u>counted, identity and standard conservation task</u>, children were scored correct when they answered that the collection had the same number of elements as it had before (i.e. identity conservation task) and that the two collections had the same number of objects (i.e. counted and standard conservation tasks). The scoring did *not* take into account the justifications (see footnote 37).

Beside scoring the responses to the three-set comparison in correct and failed, the order in which the rows were compared was analysed in detail (e.g. the three rows were counted one after the other, or were counted in pairs and compared two-by-two, some row was double counted).

8.10 Subjects

72 children from age 3,8 to age 7,4 years were tested. They were divided into three class-age groups of 24 Nursery children (mean age = 4,8 years; SD = .43), 24 Primary 1 (mean age = 5,8 years; SD = .24) and Primary 2 (mean age = 6,9 years; SD = .23). The children came from different classes of the same Edinburgh school and were from a mainly working class social background. All children were native speakers of English.

8.11 Results

The analysis of the results is carried out at the level of age group comparisons, of within age groups across tasks changes in response and of overall response patterns to pairs of tasks.

8.11.1 Age groups analysis

Tables 8.1 to 8.6 present the number of correct and failed responses to the six tasks in the three age groups: Nursery, Primary 1 and Primary 2. A χ^2 test is computed on each contingency table to assess the degree of correspondence between the observed and expected responses falling in the success and failure categories in the three age-groups and to evaluate the variation in performance across age. This analysis is complemented by comparisons of pairs of groups to identify the precise periods in which performance changes occur.

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	14	22	24
Failure	10	2	0

Table 8.1 Correct and failed reproduction responses for three age groups

Since the distribution of correct and failed reproductions yields three cells with expected frequencies lower than 5, I have collapsed the responses of the Primary children and computed a χ^2 test on this distribution : χ^2 (1, N=72) = 16.2, p < 0.001. The responses are homogeneous in the two Primary groups (91% of the Primary 1 children and 100% of the Primary 2 children reproduce correctly), whereas the Nursery children's performance is much less accurate (58% of correct reproductions). I have computed a second χ^2 test on the distribution of correct and wrong reproductions in the Nursery and Primary 1 groups to determine whether the performance in the reproduction task differs significantly in these two groups. The χ^2 ((1, N=48) = 6.8) gives a significant result at the level of .01 > p > .005.

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	9	17	21
Failure	15	7	3

Table 8.2 Correct and failed comparison responses for three age groups

The significant value of χ^2 ((2, N=72) = 13.72, p=0.001) indicates that the frequency of correct and failed comparison responses is not independent of the age group variable. The difference between the observed and the chance distribution is mainly concentrated around the Nursery and Primary groups. Whereas the primary groups give a comparable number of correct and wrong responses (χ^2 (1, N=48) = 2, .2 > p > .1), the Nursery and Primary 1 comparison responses (and consequently also Primary 2) are significantly different: χ^2 (1, N=48) = 5.36, .025 > p > .02.

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	6	13	19
Failure	18	11	5

Table 8.3 Correct and failed standard conservation responses for three age groups

The conservation responses are related to the age groups: χ^2 (2, N=72) = 14.15, p=0.0008. In particular the frequency of Nursery children's correct and failed conservation responses is significantly different from the Primary 1's (χ^2 (1, N=48) = 4.2, .05 > p > .025). The two Primary groups' conservation responses do not differ significantly: (χ^2 (1, N=48) = 3.32, .10 > p > .05).

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	7	16	21
Failure	17	8	3

Table 8.4 Correct and failed identity conservation responses for three age groups

Here χ^2 (2, N=72) = 17.65, p=0.0001. Also the identity conservation responses are not distributed homogeneously across age-groups. In particular, the performance difference is concentrated between the Nursery and Primary children. The χ^2 test computed on the frequency of correct and failed conservation responses in the Nursery and Primary 1 groups is equivalent to: χ^2 (1, N=48) = 6.8, .01 > p > .005. The two Primary groups' responses on the other hand distribute homogeneously (χ^2 (1, N=48) = 2.96, .10 > p > .05)

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	10	19	24
Failure	14	5	0

Table 8.5 Correct and failed counted conservation responses for three age groups

Here χ^2 (2, N=72) = 21.59, p=0.00002. The significant difference between observed and expected frequencies is particularly noticeable between Nursery and Primary 1 (χ^2 (1, N=48) = 7, .01 > p > .005).

Resp \ Age	Nursery	Primary 1	Primary 2
Correct	7	16	20
Failure	17	8	4

Table 8.6 Correct and failed three-set comparison responses for three age groups

In this case χ^2 (2, N=72) = 15.36, p=0.0004. The same pattern described for the Identity conservation table also underlies the distribution of correct and failed three-set comparisons. The response category and age variables are related and this relationship is more noticeable between Nursery and Primary 1 children (χ^2 (1, N=48) = 6.8, .01 > p > .005). No significant difference exists between Primary 1 and Primary 2 comparison responses.

8.11.1.1 Summary of the age groups analysis

The χ^2 analysis reveals that in all six tasks the number of correct and failed responses does not distribute uniformly across age groups and thus that performance changes with age. In the period which goes from Nursery to Primary 2, the number of correct responses appears to increase and the number of failed accordingly to decrease. The main difference is found between the Nursery and Primary children. The performance of Primary 1 and Primary 2 children instead appears to be comparable. The χ^2 analysis of Primary 1 and Primary 2 response distribution comes close to the significance level of $p < .05$ only in the identity and standard conservation tasks.

8.11.2 Analysis of task difficulty

The results of the analysis of across groups performance in each of the six tasks is complemented by the analysis of across tasks performance in each age group. The response patterns hypothesized underlie a specific order of complexity between tasks, from reproduction, to comparison and modified conservations, and finally to standard conservation. The expected order should be reflected in the children's shift from correct reproductions to failed comparisons and modified conservations in a first phase of development; from correct reproductions, comparisons and modified conservations to failed standard conservations in a second phase of development; to a third phase in which they solve all tasks.

The following statistical analysis examines whether the children's responses vary according to this predicted order of task complexity and whether the expected change occurs in all the three age groups or whether it is specific to some particular age group. The following order of complexity, with the associated sets of coefficients adjusted to sum zero, is tested:

Repro Vis >	2-sets Cp =	3-sets Cp =	Identity Cs =	Counted Cs >	Standard Cs
(-2)	(-1)	(-1)	(-1)	(-1)	(+6)

The Marascuilo & McSweeney test evaluates how the order of complexity of the task conditions fits with the variations from correct to failed responses across tasks. Table 8.7 summarizes the result of the normal curve z test for the Nursery, Primary 1 and Primary 2 groups. The z scores significant at the level of $p < .05$ are marked by an asterisk.

Age	Nursery	Primary 1	Primary 2
Order	2.68*	3.15*	2.7*

Table 8.7: Marascuilo & McSweeney Z values associated to the predicted order of task difficulty for Nursery, Primary 1 and Primary 2 children

The observed across-tasks response change fits well the predicted order of task difficulty. Contrary to Experiment 1, where Primary 2 children's responses were homogeneous across tasks, in Experiment 3 the response change observed is significant in all three age groups.

8.11.3 Hierarchical analysis of task solutions

The central testing of the hypothesized patterns of concurrency, collective décalage or individual décalage in the solution of pairs of tasks is carried out using the prediction analysis of cross-classifications technique. Tables 8.8 to 8.17 present the contingency tables of the frequency of correct and failed responses to each pair of tasks for which an ordering hypothesis has been formulated. Each table is followed a) by the value of the Del index corresponding to the improvement over chance produced by the associated triangular hypothesis, b) by the values of the z test calculated on the Del and c) by the values of the z test of the difference between the main hypotheses and the two alternative hypotheses of order.

8.11.3.1 Hypothesis 1a: Collective décalage between reproduction and comparison

Comp \ Repro	S	F
S	45	2
F	15	10

Table 8.8: Contingency table for *reproduction* and *comparison* (the white cell is the cell predicted to be empty by Hypothesis 1a) (Del = 0.99; z = 15.79, p < .00003)

Collective décalage from reproduction to comparison is significant and a significantly better predictor than concurrency (z = 2.6, p = .004) or décalage from comparison to reproduction (z = 7.83, p < .00003).

8.11.3.2 Hypothesis 1b: Collective décalage between comparison and conservation

Cons \ Comp	S	F
S	38	0
F	9	25

Table 8.9: Contingency table for *comparison* and *conservation* (Del = 1; z = e, p < .00001).

Collective décalage from comparison to conservation predicts the significantly non-chance contingencies and is a significantly better predictor than concurrency (z = 3.3, p = .0004) or the reverse décalage (z = 4.1, p < .00003).

The response patterns described in Experiment 1 are replicated. *Reproduction is solved before comparison*, as only two children who fail the reproduction can solve the comparison, while some among the children who can reproduce number, can also make accurate number judgments. *Comparison is solved before conservation*, and only one child who fails the comparison later succeeds the conservation. A subset of the children who make an accurate numerical judgments succeed in conserving number. As expected, the same collective décalage holds between the easiest reproduction task and the more complex conservation task, as the following table clearly indicates:

Cons \ Repro	S	F
S	38	0
F	22	12

Table 8.10: Contingency table for *reproduction* and *conservation* (Del = 1, z = e, p < .00001)

The next section examines the relationship between the solutions of comparison and of three-set comparison, identity and counted conservation. The predicted response pattern is one of concurrency as all these tasks are expected to track the number competence characteristic of Stage 2, that is, the capacity to recognize the equinumerosity of two or more sets.

8.11.3.3 Hypothesis 2a: Concurrency between two and three-set comparison

Comp \ Three sets Comp	S	F
S	42	5
F	1	24

Table 8.11: Contingency table for *two-set comparison* and *three-set comparison* (Del = 0.82; z = 11.8, p < .00003).

The concurrency pattern is significant and a significantly better predictor than collective décalage in favour of three-set comparison ($z = 2.05$, $p = .02$). It is not, however, significantly different from the collective décalage in favour of two-set comparison ($z = 1.56$, $p = .06$). Hypothesis 2a is only partially verified. Although 92% of the responses conform to the expected pattern and correspond to failure in both tasks and success in both tasks, 5 of the remaining 6 responses fall into the cell corresponding to success in the two-set comparison and failure in the three-set comparison. It appears then that the *concurrency* is accompanied by a few cases of prior solution of the two-set comparison over the three-set comparison (i.e. *collective décalage* from two-set comparison to three-set comparison).

8.11.3.4 Hypothesis 2b: Concurrency between set comparison and identity conservation

Comp \ Iden Cons	S	F
S	42	5
F	2	23

Table 8.12: Contingency table for *comparison* and *identity conservation* ($Del = 0.79$; $z = 10.5$, $p < .00003$)

The concurrency model yields a significant improvement over chance as also the two models of collective décalage do ($Del_{id} = .72$; $z = 7.1$, $p < .00003$; $Del_{comp} = .86$; $z = 10.1$, $p < .00003$). The three Dels are not significantly different from each other:

a) concurrency and décalage from identity conservation to comparison: $z= 1.3$, $p= .09$;

b) concurrency and décalage from comparison to identity conservation: $z= 1.07$, $p=.14$;

c) décalage from comparison and décalage from identity conservation: $z= 1.16$, $p= .12$;

Since all three hypotheses yield a significant improvement over chance at the same time and are not significantly different between each other, I conclude that the order in which the solutions of identity conservation and two sets comparison appear is of *weak concurrency.*

8.11.3.5 Hypothesis 2c: Concurrency between set comparison and counted conservation

Comp \ Count Cons	S	F
S	45	2
F	8	17

Table 8.13: Contingency table for *two-set comparison* and *counted conservation* (Del = 0.67; z = 7.2, p < .00003)

The concurrency model is significant and a significantly better predictor than collective décalage from comparison to counted conservation (z = 2.7, p = .0034). It is, however, a significantly worse predictor than *collective décalage from counted conservation to comparison* (z = -1.7, p = .044). As this latter pattern is also a better predictor than the collective décalage in favour of comparison (z = 2.05, p = .02), it constitutes the overall best predictor of performance. The hypothesis of concurrency is thus falsified. Although 86% of the children contribute to the concurrency pattern, either failing or succeeding at both tasks, there is a significant number of children who succeed the counted conservation task and fail the comparison task.

8.11.3.6 Hypothesis 2d: Concurrency between three-set comparison and counted conservation

Three sets Comp \ Count Cons	S	F
S	42	1
F	11	18

Table 8.14: Contingency table for *three-set comparison* and *counted conservation* (Del = 0.63; z = 6.8, p < .00003)

Here too, the concurrency model is significant and a significantly better predictor than collective décalage in favour of three-set comparison (z = 6.5, p < .00003) but is a significantly worse predictor than *collective décalage from counted conservation to three-set comparison* (z = -2.8, p = .0025). As this latter pattern is also a better predictor than the collective décalage in favour of comparison (z = 3.6, p = .00016), it appears to be the overall best predictor of performance. Thus, in the case of three-set comparison and counted conservation, the hypothesis of concurrency is falsified. Although 83% of the children contribute to the concurrency pattern and either fail or solve both tasks, 11 children (15%) solved counted conservation task and failed three-set comparison.

8.11.3.7 Hypothesis 2f: Concurrency between identity conservation and counted conservation

Identity Cons \ Count Cons	S	F
S	44	0
F	9	19

Table 8.15: Contingency table for *identity conservation* and *counted conservation* (Del = 0.72; z = 8.5, p < .00003)

Also in the third comparison involving counted conservation, the concurrency model is significant and a significantly better predictor than collective décalage in favour of identity conservation (z = 8.5, p < .00003) but is a significantly worse predictor than *collective décalage from counted conservation to identity conservation* (z = -3.3, p = .00048). As this latter pattern is also a better predictor than the collective décalage in favour of comparison (z = 4.2, p = .00003), it appears to be the overall best predictor of performance. For this third pattern, the hypothesis of concurrency relative to counted conservation is falsified. Again, the bulk of the children (88%) produced concurrent results, but nine children (12%) succeeded at the counted conservation task while failing at identity conservation.

Since collective décalage of similar strength holds from counted conservation, to comparison, to three-set comparison and to identity conservation (contrary to the hypothesis of concurrency), an intermediate competence level prior to the capacity to compare sets and to conserve a single set numerosity (Stage 2) has clearly emerged. Furthermore, collective décalage from reproduction to counted conservation has been found as the following table indicates:

Count Cons \ Repro	S	F
S	50	3
F	10	9

Table 8.16: Contingency table for *reproduction* and *counted conservation* (Del = 0.66; z = 4.05, p = .00003)

Collective décalage from reproduction to counted conservation predicts the significantly non-chance contingencies and is a significantly better predictor than concurrency (z = 1.94, p = .026) or the reverse décalage (z = 1.64, p = .050).

The ability to succeed at the counted conservation task appears thus after the ability to reproduce sets is in place (Stage 1) and before the ability to compare sets and conserve a set number has been elaborated (Stage 2). I propose to interpret these two décalages as evidence of a substage 2a, which will be further discussed in the concluding section.

8.11.3.8 Hypothesis 2e: Concurrency between three-set comparison and identity conservation

Three sets Comp \ Ident Cons	S	F
S	42	1
F	2	27

Table 8.17: Contingency table for *three-set comparison* and *identity conservation* (Del = 0.91; z = 18.4, p < 00003)

Since the model of concurrency as well as the two models of collective décalage are significant with Del values close to 1 (.94 for the collective décalage in favour of three-set comparison; .88 for the collective décalage in favour of identity conservation) and are not significantly different (the z test values range from .56 to .59 and are significant at the level of .29), I conclude that a *strict concurrency* exists in the solution of the three-set comparison and identity conservation tasks. The hypothesis 2e that the two tasks are acquired concurrently is verified.

8.11.4 Analysis of the procedures to solve the three-set comparison

The correct three-set comparison responses are the outcome of two basic strategies: a) the three rows are counted, b) the rows are compared two-by-two. The former strategy consists of counting the *three* rows, one after the other and of picking out the largest. The children conclude that "it's got one more", or "these have 6, this is more, it's 7". This strategy is employed more frequently (67%) both from the start (62%) and to check an initial judgment based on a spatial estimation of size (38%). In this case, the child first answers that the longer row is more numerous, and asked to check whether that row is indeed the more numerous, counts the three rows and revises his initial judgment.

The second strategy consists of counting *two* rows, always the top and the middle row, of noticing that they are the same number (e.g. 6 and 6), and then of counting the bottom row to 7 to conclude that this is the most numerous row. Alternatively, children count the top and middle rows, first, and the middle and bottom rows, then, to conclude that the latter is the largest of the three. The children justify their judgment by saying: "this one and this one are the same, and this is seven, one more" or pointing to the rows "top and middle are the same, bottom is bigger" or "there's one more (bottom) and these are less". These strategies are employed by 33% of the children (71% from the start and 29% as a check of an initial judgment based on length). Sco (5,2) for instance, takes the long row to be the largest, counts the top row to 6 and the long to 7, double counting one of its elements. He then counts the bottom row accurately to 7; counts again the long, middle row to 6 and the bottom row to 7 and concludes that the bottom row is the largest of the three. Lis (7,0) judges the long row to be more numerous, counts it to 6 and the bottom row to 7, then compares the top and the bottom rows and counts them to 6 and 7 and concludes that the bottom row is the largest of the three.

These observations and especially the responses based on the second type of strategy are indicative of the fact that Stage 2 children can carry out some simple transitive inference as they identify the largest row after two comparisons, rather than exhaustively examining all three possible pairings of sets. If the top and the middle rows are found to be equinumerous, then the third row is counted and compared to the two previous rows' cardinal number directly. Both strategies indicate that the Stage 2 children can use the order information embedded in the sequence of cardinal numbers and draw inferences on relationships of equivalence and difference between sets.

The Stage 1 children lack this competence as their errors in the three-set comparison task indicate. The incorrect responses are all based on a length comparison. The middle row is taken to be more numerous. As Cla (4,5) says: " here (top row) it's not much, here (mid row) it's more much and here (bottom row) it's more ". When verification by counting is suggested, 45% of the children count the three rows, and regardless of the different cardinal number they obtain, confirm the initial judgment. Ian (7,1) judges the three rows to be equinumerous and justifies the judgment saying that "this is a bit longer, but these two are more crowded". He then counts the three rows accurately to 6, 6 and 7 and concludes that the middle row (i.e. the long row) is the largest. 21% of the children count the three rows as if they were one (e.g. 1, 2, 3, 4, 5, 6 (top row), 7, 8, 9, 10, 11, 12 (mid row), 13, 14, 15, 16, 17, 18, 19 (bottom row)). The remaining children either adapt the count to the initial length based judgment (18%) or oscillate between a number and a length-based judgment. For example, Jac (6,1) count the three rows accurately and concludes that they are the same. When she is asked to justify her reply, she answers that one is bigger because it is the longest. The experimenter then asks her to remind him of how many elements there are in the three rows. Jac recounts and says that there are 6 in the top row, 6 in the middle row and 7 in the bottom row, and concludes that the bottom row is largest. The experimenter asks for a confirmation and Jac responds that the middle row is the most numerous.

8.12 Summary and discussion of the results

In this last section, I discuss the relationship between the results of Experiment 3 and of the previous Experiments 1 and 2, and in particular the place that the acquisition of identity and counted conservation takes in the three stages previously identified. I then evaluate the hypothesis that the modified conservation task should be solved concurrently with the set comparison tasks, since they involve the operation of matching sets characteristic of Stage 2. To further qualify Stage 2 number competence, I examine the strategies observed in the solution of the three-set comparison task and the evidence they provide of transitive reasoning on numerical relations.

Experiment 3 replicates the main results of the previous experiments: the collective décalages from reproduction to comparison, from reproduction to conservation and from comparison to conservation. The basic sequence of three stages of numerical competence described by Experiments 1 and 2 is thus confirmed. Children first acquire the capacity to reproduce sets, then to compare pairs of sets and finally to maintain the equinumerosity of two sets throughout spatial transformations. As expected, the conservation task is in fact solved with a constant collective décalage, after all the comparison and modified conservation tasks (see appendix 8.1).

Experiment 3 also provides some support for the characterization of the numerical competence underlying the three stages presented in the general discussion of section 7.4. According to this account, Stage 2 corresponds to the capacity to establish numerical relations (equivalence or difference of numerosity) between sets. I argued that the modified conservation tasks which deal with a single set which is spread out (identity conservation) and which require a count of the two sets before answering to the conservation question (counted conservation) both depend on the Stage 2 ability to relate sets' cardinal numbers. Experiment 3 has tested this hypothesis by examining whether the comparison and modified conservation tasks are solved concurrently.

The hypothesis is verified for three-set comparison and identity conservation tasks (strict concurrency), for comparison and identity conservation tasks (weak concurrency) and for three-set and two-set comparison tasks (concurrency together with collective décalage from two-set comparison to three-set comparison). On the other hand, the hypothesis is falsified with respect to all the response patterns involving counted conservation. Although counted conservation is systematically solved once the ability to reproduce is in place (collective décalage from reproduction to counted conservation), it is always solved before the comparison and identity conservation tasks (collective décalage from counted conservation to comparison and from counted conservation to identity conservation). I interpret this finding as evidence of an early form of the capacity to recognize the equinumerosity of sets whose numerical and spatial sizes do not coincide, induced by the specific conditions in which the numerical judgement is carried out in the counted conservation task.

In this task, the child is first asked whether or not two rows of same length and number are equinumerous. All the children examined answer that the two rows have the same number of elements. Then the experimenter spreads one of the rows out and asks the child to count the two rows separately and to repeat the cardinality of the first row and then of the second row. Only after these operations have been accurately carried out, is the conservation question asked. Some of the children answer that the two rows are different and choose the longer row as being the more numerous, regardless of the result of the previous count (i.e. the children who generally belong to Stage 1). Other children confirm that the two rows are equinumerous, but when confronting the comparison, identity conservation or three-set comparison tasks, they base their numerical judgment on the length of the rows (i.e. the children who are classified at Substage 2a). When asked to check their judgments by counting, these children make the errors already described for the previous experiments. Some of them adapt their count to fit the judgment. Others count the two rows as one and so do not obtain any relevant information.

The order in which the judgment and the count are carried out may be playing an important role. In counted conservation, the judgment is asked after the count has been performed on each row separately. Before comparing the sets then, the child has created a separate representations of each set's cardinality which is not affected by the length difference between the two rows or by the objective of comparing the two rows, which is set only later. In the comparison tasks and in the identity conservation task, on the other hand, the judgment is made first and only then is counting suggested. Before counting, the child may have already created a representation of the numerical relationship holding between the two sets and may not be willing to question its accuracy nor to revise it. For that reason the counting information is often irrelevant and sometimes manipulated to fit the previous judgment.

It makes sense, therefore, to propose an intermediate level corresponding to the solution of counted conservation (Substage 2a), in which the child has learned to rely on count information to establish the numerical relationship between two sets of objects *when he has this information*. When instead he is asked to judge the sets' numerosity, he does not spontaneously count, but favours spatial cues (as the Stage 1 child does) and does not see the import of the subsequent counts. This behavioural pattern has been extensively described by Michie (1984a, b) (see section 6.4 of the literature review chapter). The décalage between the solution of the counted conservation task and the later solution of the comparison and identity conservation tasks may thus identify a phase in the developmental process by which counting becomes the privileged means to create cardinal representations. In this phase of relative instability, when the representation based on counting is created first, the risk of interference from spatial information decreases. When instead counting comes in after a cardinal representation has been created, a conflict between spatial and count information eventually emerges and is generally solved in favour of spatial cues. Only at Stage 2, does counting work both as a strategy for comparing sets (e.g. when asked to make a number judgment, the child spontaneously counts the two rows and judges on the basis of the number reached) and as a method for verifying the accuracy of an estimate based on spatial extent (e.g. when asked to make a number judgment, the child picks out the longer row as the most numerous, he then counts the two rows and determines whether his hypothesis was correct or not).

Experiment 3 also has implications for our view of the logical competence characterizing Stage 2. The analysis of the strategies used in the three-set comparison gives some indication that children can combine the relations holding between pairs of sets (e.g. A = B and C > B) to infer new relationships on other pairs (e.g. C > A). Although this evidence is rather weak, it indicates a basic competence to reason on first-order relations of equinumerosity and difference. Further support can be found in the capacity to use the order relationships embedded in the number words sequence (e.g. the solution of the three-set comparison based on counting the three rows and identifying the largest) and to conserve number in the identity conservation task (e.g. the children justify the conservation by the fact that nothing has been added nor taken away, and that it is possible to go back to the initial configuration), .

In conclusion, Experiment 3 provides further support for the description of the development of cardinal representation as proceeding through three major stages and for the characterization of the three stages competence proposed in section 7.4. Stage 1 corresponds to the ability to reproduce sets and could be characterized by a concept of cardinal number as *property of sets of objects.* Stage 2 corresponds to the ability to make numerical judgments on two and three sets and to conserve the numerosity of a single set when this is transformed in length. For this reason, the Stage 2 child should understand cardinal number as a *relationship between sets.* This concept is first expressed in the context of the counted conservation task, that is when the child creates cardinal representations of each set with counting before matching them, and is then generalized to other situations like two- and three-set comparison and identity conservation. Stage 3 corresponds to the ability to conserve the equinumerosity of two sets when one of them is modified spatially. Now the child appears to have a concept of number as a *second order relationship between sets of sets.*

Chapter 9 A model of cardinal number development

9.1 Introduction

In this last chapter, I summarize the findings of Experiments 1, 2 and 3 and examine their contribution to the existing literature on the development of cardinal number representations. I then model the transition between each pair of stages using Richards' logical representations and algorithm. Since this applications is not very satisfying, I examine the reasons and suggest to constrain the domain of developmental phenomena that Richards' approach can model in a useful and meaningful way. Finally, I propose some new directions for empirical research and for modeling developmental processes.

According to the literature, the process of cardinal number development results in the shift from space to number based estimations and judgments of numerosity. The experiments I have carried out provide evidence that this shift does not occur *all at once* across all numerical tasks, but emerges *first* in the context of set reproduction problems, *second* in number judgment tasks and *third* in standard conservation of number tasks.

This finding challenges the two principal accounts of cardinal number development: Piaget's and Gelman's. Piaget's theory cannot account for the specific number concept underlying Stage 2, a concept which possesses some of the logical features (e.g. composition, reversibility, transitivity) that Piaget attributes only to the operational Stage 3 number concept. Gelman's theory, which envisages the development of cardinal number competence as learning to produce accurate and reliable representations of numerosity for increasingly larger sets and as abstracting general numerical properties, is challenged by the result that cardinal number develops in a stage-like way, independently from the cardinal numbers involved. The collective décalages clearly indicate that cardinality becomes operational in the reproduction of sets first, in the comparison of sets later and in the standard number conservation finally for set sizes that the child can count accurately.

I have proposed an alternative account of this stage sequence in the terms of the theoretical framework advanced in this thesis. According to this account, the development of the number concept consists of the abstraction of cardinality over increasingly more complex objects, i.e. individual sets, sets, sets of sets. The stage-like nature of the process reflects the fact that each abstraction is a *prerequisite* for the following conceptual elaboration: cardinality as a property of sets of objects is a prerequisite for the subsequent discovery of cardinality as a relation between sets; cardinality as a relation between sets is a prerequisite for the subsequent discovery of the principle of conservation of numerosity over sets of sets.

From this perspective, the transition between stages is envisaged as the process by which the relevance of cardinal number structure is discovered for new and more complex objects. The process is reflected in the shift from space-based to number-based estimations and judgments. Between Stage 0 and Stage 1, children move from inaccurate reproductions, e.g. the length of the row is reproduced, to accurate reproductions. Between Stage 1 and Stage 2, children move from numerosity judgments based on spatial dimensions of the rows, and especially length, to judgements based on the cardinal size of the rows. Between Stage 2 and Stage 3, children move from abandoning the equinumerosity holding between two rows when one of them is transformed into a longer row to confirming the equinumerosity, regardless of the differences in spatial extent introduced.

This quasi-recursive process conforms well to the uniform model of transition between stages proposed by Richards in the case of the object concept development. Richards' model represents each stage transition as a fixed sequence of three cognitive states each of which underlies an inferential system of different power. A semantic algorithm operates and strengthens the logical systems (e.g. the logic of first-degree entailment is pruned of the truth value *neither true not false* to yield the stronger Kalman logic) and brings about the conceptual change characteristic of the new stage. After having analysed the main articulations of the process of cardinal number development, I provide a model for it using the logical representation and algorithms proposed by Richards.

9.2 The import of the experimental results for the existing literature on the development of cardinal representations

The studies of cardinal number development presented in the literature review (Chapter 6) report analogous shifts from space based to number based estimations of the numerosity of sets of objects in tasks dealing with number reproduction, comparison and conservation. The series of experiments that I have run replicates this phenomenon and adds a critical new element regarding its unfolding. They indicate that the shift from space to number based estimations of numerosity does not occur all at once, but is spread over the age period from 3,6 to 7 years of age, in relation to the particular task requirements. First, the capacity to carry out accurate reproductions of sets is acquired (around age 5). Second, the capacity to make numerical judgments and to conserve a single set's numerosity across spatial transformations (around age 6) is acquired, and finally the capacity to conserve the equinumerosity of two collections across spatial transformations to one of them emerges (around age 7).

Let us examine the sequence of competence levels more closely. Stage 0 children fail the reproduction task mainly because they reproduce the length of the row instead of its cardinality. Nevertheless independent evidence suggests that even at Stage 0 the children have some understanding of cardinal number: a) conservation of numerical judgments in Gelman's magic experiment (see Appendix 6.1), b) ability to count and to make judgments on the basis of the order of the number words sequence (e.g. 2 is larger than 1, 3 is larger than 2), c) appreciation of the consequences that adding and subtracting an element from a set has on its numerosity (see Appendix 6.2), d) capacity to take out 'n' objects and to answer 'how many' questions. However, when the Stage 0 children are asked to abstract and precisely quantify the cardinality of a set, and on this basis create another equinumerous set, they systematically fail the task. The gap between possessing the competence previously described and putting it into practice in the reproduction task can be accounted for in terms of the complexity of the operations involved in that task. The cardinal size of the set has to be isolated from the other spatial properties of the set. The size has then to be precisely quantified and reproduced in a new set, either via counting or one-to-one matching. Finally, the set of objects that serve to carry out the reproduction has to be partitioned, and a subset of it has to be left unused.

Stage 1 children reproduce the set's number accurately, either by counting out the same number of items or by establishing a one-to-one spatial correspondence between elements of the model row and elements from the set they are given to carry out the reproduction. At the same time, when faced with the task of judging the numerosity of two or three sets, or the identity of the numerosity of a row which is spatially modified, Stage 1 children fall back on a space based estimation of size and generally take the longer row to be the more numerous. This phenomenon is particularly striking in the case of two-set comparisons which immediately follow set reproduction in the hidden condition. Stage 1 children solve the reproduction task and create a row which has the same number 'n' of elements as a model row 'n', hidden behind a screen. When the screen is taken away, two rows of different length are revealed. Despite the immediately preceding reproduction, Stage 1 children judge that the two rows have a different number of elements.

The difficulty with combining difference in length with equinumerosity (or difference in number with equal length, as in the static comparison task of Experiment 2) is overcome in Stage 2, where children base their numerical judgments on numerosity alone. They either start with a hypothesis based on spatial estimation that they then verify using one-to-one correspondence and, more frequently, counting; or they count the two or three rows from the start and isolate the most numerous row. Similarly, in the identity conservation task the operation of spreading out the rows' elements does not lead to the conclusion that the row's number has also changed. The children explain that it is still the same row and no elements have been added nor taken away from it. However, the Stage 2 number concept is still not completely independent from spatial features of the set. Faced with the standard conservation task, Stage 2 children fall back on an estimation of equinumerosity based on the difference in length introduced by the spatial transformation of one of the rows.

Only at Stage 3, do children solve the standard number conservation task as well. Since, to my knowledge, no tasks have been devised that involve more complex operations on the cardinal representation of sets of objects than the conservation task, Stage 3 constitutes the top level in the development of cardinal representations of sets.

To summarize, Experiments 1, 2 and 3 point out some important new elements in the development of cardinal representations. The stage-sequence is schematically presented in table 9.1 and further qualified in the following paragraphs:

1. The response patterns to a battery of nine tasks involving set reproduction, set comparison and number conservation identify three distinct competence levels: Stage 0 where no tasks are solved, Stage 1 where the set reproduction tasks are solved, Stage 2 where the set reproduction, two- and three set comparison, identity conservation and counted conservation (substage 2a) tasks are solved, Stage 3 where set reproduction, two- and three-set comparison, identity conservation, counted conservation and standard conservation are solved.

2. Stage 1 is attained at around age 5 (the older children in the Nursery school). Stage 2 is attained at around age 6 (towards the end of Primary 1). Stage 3 is attained at around age 6,6, 7 (during Primary 2).

3. At each stage, children have some numerical competence and can carry out complex operations. As Stage 2 has been explored more thoroughly, it provides some particularly interesting illustrations of sophisticated inferences on cardinal number. I found some indications of transitive inferences from relations of equinumerosity and difference in the three-set comparison, of articulate explanations in the identity conservation task, where children justify that the cardinality of the row has not been changed by the spatial transformation by the fact that no elements have been added nor taken away from the set. Both in this task and in the comparison tasks, children justify their judgments also by combining the length and the density of the rows (e.g. this one is longer, but more empty; this one is shorter but more crowded).

4. At each stage, the competence has characteristic limitations. Whereas the circumstances under which the children operate accurately are revealed by the tasks that the children can solve simultaneously, the limitations are identified by the response patterns of collective décalage. The Stage 1 children fail the two- and three-set comparison tasks, the identity conservation and the counted conservation tasks as well as the standard conservation task. The Stage 2 children fail the standard conservation task.

Stage	Age	Tasks solved	Notes
0	4	None of the tasks	accurate counts to 4-5 In set reproduction: use of all the objects arbitrary number inaccurate matching copy of length
1	5	Set reproduction visible, hidden	In set reproduction: accurate matching and counting In set comparison: longer = more numerous equal length = same number
2a	5-6	Counted conservation Set reproduction	In set comparison: if count first, accurate comp. if count after space-based judgment, inaccurate comp.
2	6	2-set comparison 3-set comparison Identity conserv Counted conserv Set reproduction	In set comparison: count precedes judgment or follows as a check In standard conservation: after transformation, sets are different and longer = more
3	7	All the tasks	In standard conservation: operational justifications

Table 9.1: The four Stages (with one Substage), with the corresponding approximate age, task solved and basic solution strategies, as reported in Experiments 1, 2 and 3.

5. Knowledge of the count words sequence, enumeration, and even counting do not form a sufficient condition for carrying out the numerical operations and inferences required by these diverse tasks. In all three experiments, in fact, care has been taken to examine children using set sizes that are in the range of their counting competence and to help children correct their counts in case of mistakes. Despite this precaution,

inaccurate reproductions and comparisons have not disappeared. The developmental process therefore cannot be the product of the increased reliability of children's counting. Nevertheless, accurate counting plays a crucial role in testing hypotheses about numerical relations between sets and in discovering specific properties and regularities. This clearly emerges from the early success in the count conservation task (where a count of the two rows is required before making the number judgment) as the first form of numerical comparison competence.

6. The two fundamental means of creating precise numerical representations of the sets' cardinality, that is counting and one-to-one spatial correspondence, develop with some independence. The response pattern of individual décalage between reproduction in the visible condition and reproduction in the hidden condition indicate that some children develop counting before matching, while other children do the opposite. In Stage 2, both strategies are present and are applied in a coordinated fashion to test for the kind of numerical relations holding between sets. The children can in fact invoke counting and matching in a interchangeable way or use them as double checks.

9.3 The import of this result on the main accounts of cardinal number development

These findings challenge both Piaget's and Gelman's accounts of number development. According to Piaget, an important milestone in the development of cardinal number is reached when a system is constructed that leads to number conservation: the realization that the equality of two collections of objects put in one-to-one correspondence and judged equal does not change when one of the collections is spaced out and the optical correspondence is destroyed. The system emerges from the reciprocal assimilation of the operational structures of classes and relations and guarantees the necessity of conservation through reversibility (i.e. the two sets are still equinumerous because it is possible to go back to the initial optical correspondence), identity (i.e. the two sets are still the same because nothing has been added nor taken away from them), composition (i.e. the two sets still have the same number of elements as one is longer, but more spaced, while the second is shorter, but more crowded). Before this system is in place, the child has an intuitive concept of number which is perception bound and irreversible.

When this characterization of number development is matched to the developmental sequence that I have described, the intuitive, pre-operational number concept corresponds to Stage 1. The child can reproduce a set using counting or one-to-one correspondence, but as he is faced with two rows of different length he abandons the equinumerosity (e.g. comparison after hidden reproduction, identity, counted and standard conservation) and makes a judgement based on the set's spatial extension. The operational number corresponds instead to Stage 3. The child can conserve the equinumerosity of two collections across their spatial transformations, and explain this using logical arguments of reversibility, identity and composition.

Where the Piagetian account in terms of two clear-cut stages fails is with respect to the Stage 2 number concept. Piaget's theory does not have the means to characterize the achievement of Stage 2 where the representation of the set's cardinality is clearly distinct from spatial size and have some operativity. In making numerical comparisons of rows in which length and number do not coincide (e.g. same length and different number, and same number and different length), Stage 2 children in fact go beyond spatial differences and recognize the rows' equinumerosity or different number. They justify their judgments using arguments of composition of length and density (e.g. this row looks bigger, but is more spaced. This row looks smaller, but is more crowded). In the identity conservation task, Stage 2 children maintain a set's number across spatial changes to it and justify the conservation again with arguments of composition, reversibility and also identity (e.g. You didn't add nor take away any). In the three-set comparison, beside picking out the more numerous array independently from the arrays' length, some of the Stage 2 children carry out some form of transitive reasoning (e.g. from the result that A is equal to B and that C is larger than B they conclude that C is the largest set, without needing to compare C with A). In sum, the Stage 2 number concept possesses the logical properties that Piaget attributes only to the Stage 3 concept. The Piagetian analysis cannot account for the coherence of the Stage 2 number concept and for the difference between the Stage 2 and Stage 3 concepts, a difference which is reflected in the Stage 2 children's failure to solve the standard number conservation task.

The finding that the development of cardinal number from set reproduction to number conservation proceeds through three fixed stages also constitutes a challenge to Gelman's analysis of number development and does so on two levels. According to Gelman, numerical inferences (what she calls number operators) appear only after the child has developed number estimators which provide reliable and accurate numerical representations. In the three experiments I have run, care has been taken to assess children always on set sizes that they can handle and count accurately (see the pretest to determine the size of the sets the child is asked to reproduce and compare). The finding that children fail the tasks of reproduction, comparison and conservation even when the size of the sets involved is within the child's counting range indicates that numerical development goes beyond the accurate representation of sets' cardinality and that counting is not a sufficient condition for numerical reasoning. Although advanced counting skills play a major role in discovering and testing hypotheses about numerical properties of sets and relations between sets, proficient counting per se does not directly bring about complex number concepts. Consider, for instance, the case of the children that in the comparison task modify their counts as a function of their original numerosity judgment. Their counting is proficient enough to be (dishonestly) adapted to confirm the initial hypothesis, if the child think that a longer row must indeed be more numerous than a shorter row. Thus extensive development in number competence goes on beyond the acquisition of advanced counting skills.

The second problematic aspect of Gelman's account of number development refers to her more general theoretical framework. Gelman bases her analysis of development on Rozin's theory (1976) that part of cognitive development involves an increasing ability to access the structures underlying early cognitive and perceptual abilities. Because of this growing general ability to gain access to underlying competences, the early and possibly innate abilities are used in wider and wider settings and combine to serve new abilities. To illustrate how this process works, Gelman gives the following example:

> Consider the case of reading. I know of no claims that the ability to read is innate. Many have argued that a child has to access the phonetic speech-stream if he is to master the sound-sight correspondence rules. But the ability to do this develops relatively late. In Rozin's terms, this is because the ability to produce speech only embeds an implicit ability to use the phonetic code. This ability is not an explicit one. With development, there is an increase in the ability to access the phonetic code and put it to work in the service of acquiring a new ability - to read. (Gelman 1982, p. 217).

Similarly in the case of number, with development the child gradually gains access to the principles underlying his counting practice. Access to these principles make possible more efficient and operational numerical reasoning (e.g. about number conservation). The finding that cardinal number development proceeds in an orderly manner through three stages goes against a simple learning view that this competence develops as a function of practice and of the child's encounters with many and diverse number situations. The stage sequence identified puts *supplementary constraints* (beyond Rozin's and Gelman's accounts) on the developmental process by which access is gained to implicit knowledge of numbers.

The stage-like nature of the developmental process can instead be expressed within the theoretical framework proposed in this thesis. The framework assumes that development is a stage-like process by which some structure specialized in processing particular kinds of information becomes relevant and operational over more and more complex classes of objects. From this perspective, an account can be given of the competence underlying the three stages in terms of a concept of number defined by the fundamental number-structure and by the content this structure applies to.

The Stage 1 number concept corresponds to the application of the number structure to individual sets and defines cardinal number as a property of individual sets. The child is granted a full logical competence to draw inference and carry out complex operations which involve the cardinality of individual sets. Unfortunately, the only illustration of this competence in the studies I have reported is provided by the solution of the set reproduction tasks.

The Stage 2 number concept corresponds to the application of the number structure to two or more sets and encompasses the cardinal relations of equinumerosity and different number between sets. The child is again granted a full logical competence to draw inferences and carry out complex operations which involve numerical relations between two or more sets. In the case of Stage 2, there is more evidence in support of this claim. The children work out the property of conservation of individual sets across spatial transformation by relating the initial configuration with the post-transformation configuration and by noticing that nothing has been added nor taken away (identity), that it is possible to go back to the initial configuration (reversibility) and that although the post-transformation row looks larger, its elements are more spaced than in the initial configuration (composition). Similarly, the indications of transitive inferences support the view that children can combine the equinumerosity between two sets (A = B) with the difference between one of these sets and a third set (C > B) to conclude that this set is also larger than the first one (C > A). The Stage 2 concept defines cardinal number as a first-order relation between sets together with all the inferences that can be drawn from this kind of relation.

The Stage 3 number concept corresponds to the application of the number structure to sets of sets, and in particular to the initial pair of sets of the number conservation task (A = B) and the post-transformation pair (A = B'). The application of the number structure to the more complex object pairs of sets permits the child to discover the principle of number conservation, that is the permanence of the equinumerosity of two sets across spatial transformations, as long as no elements are added nor taken away.

This alternative account also permits us to explain some of the results reported in the literature. First of all, the early conservations can be explained in terms of the competence characteristic of Stage 2. The modified conservation tasks are solved prior to the standard conservation task because they require the matching of the two post-transformation sets and not the more complex operation of matching the initial pair of sets with the post-transformation pair, as in Piaget's standard task. Halford & Boyle's (see Section 6.2.2.2.3) finding that before age 7 children cannot transfer a numerical judgment (e.g. two large rows are presented, the child estimates the row on the left as larger) across spatial transformations on one of the rows (e.g. the same child now judges the right row to be larger) can be explained in terms of the Stage 3 competence to relate pairs of sets. Only when the concept of cardinal number which expresses sets of sets is elaborated can the children carry the same judgment through the series of spatial transformations and give consistency to their numerical judgments. Before this stage, the children can only give independent judgments of each pair after each transformation, as Halford & Boyle report.

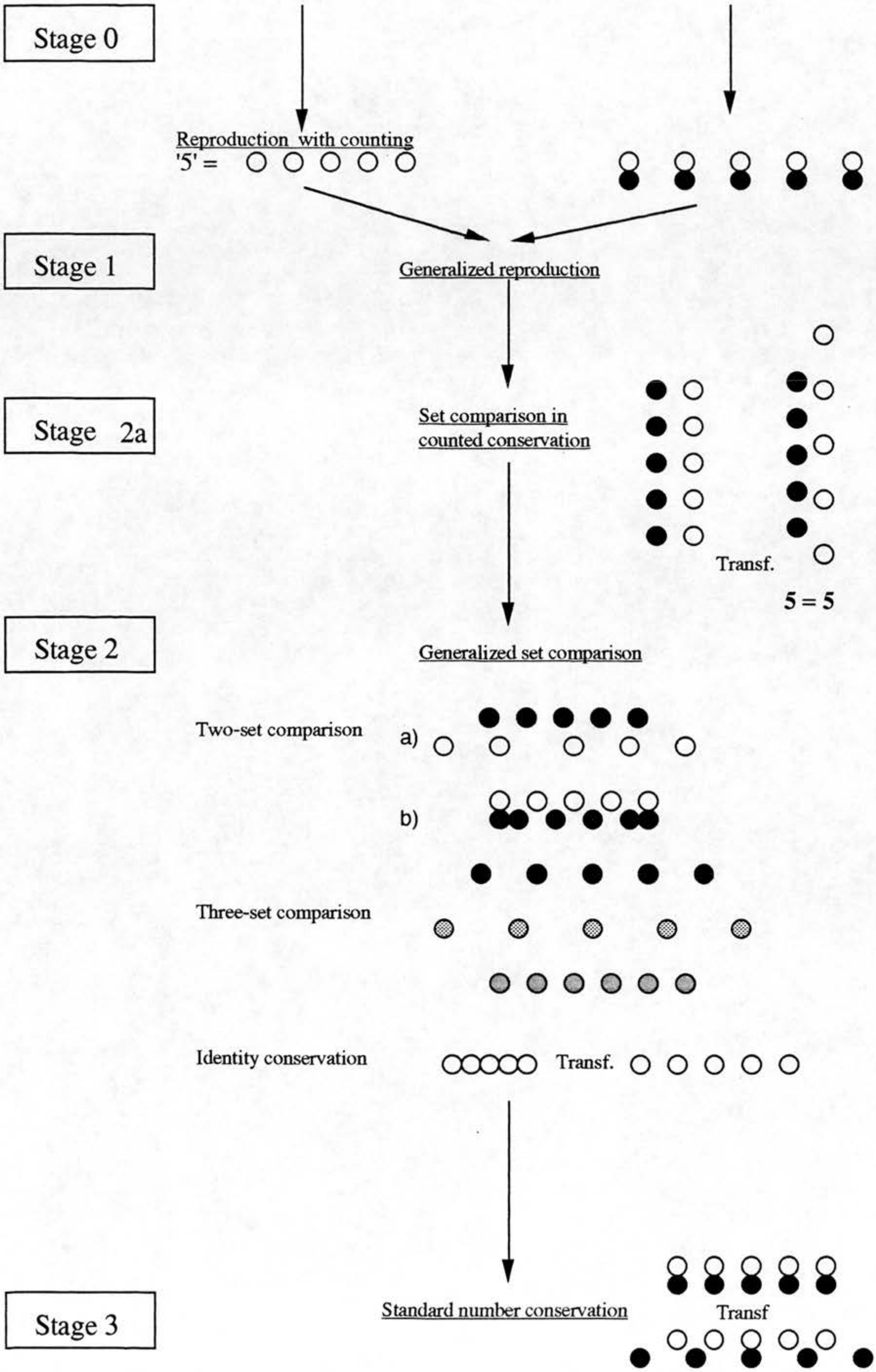
9.4 The development of cardinal number, a detailed description

As we have seen in the previous sections, the development of cardinal number from set reproduction to conservation of number develops through four stages. I now examine the paths that children take along this stage sequence.

Children reach Stage 1 two different ways, either by elaborating one-to-one matching first or counting first. Cardinal number structure is thus realized by two initially independent means. At the end of Stage 1, the two forms of cardinal operations are in place, and are favoured depending on the specific circumstances of the task. Between Stage 1 and Stage 2, children develop a first form of the competence to compare sets. When they have a precise representation of cardinality (e.g. through counting) at their disposal, they base their numerical judgments on this information. The counted conservation task is thus solved by children who at the same time fail the other comparison tasks, where the judgment is demanded before a count has been obtained. Children move then through to Stage 2 where they can perform different kinds of matching between sets. They compare two and three sets and they compare the initial and final configurations of a row which is spread out, concluding that the set has still the same number of elements. The two strategies for recognizing equinumerosity, counting and matching, become fully operational as a means to make judgments and to assess their accuracy, one as test of the other. Children move from Stage 2 to Stage 3 where they discover the principle of conservation of equinumerosity: if two sets are equinumerous, they remain so regardless of changes to their configuration as long as no element is added or taken away from them.

The steps of the developmental process are summarized in the following figure 9.2:

Fig. 9.2: Developmental paths along the stage sequence (tasks characteristic of stages are underlined and illustrated by arrays of circles: numerals appear when counting is required, 'transf' indicate that the row's length is transformed; arrows correspond to transitions between stages).


Stage 0
Reproduction with counting
'5' =
Stage 1
Generalized reproduction
Stage 2a
Set comparison in counted conservation
Transf.
5 = 5
Stage 2
Generalized set comparison
Two-set comparison
a)
b)
Three-set comparison
Identity conservation
Transf.
Stage 3
Standard number conservation
Transf

9.5 Modeling the process of development

The uniform developmental process of moving from space-based to number-based representations of numerosity can be captured using Richards' transition model (see Chapter 3). According to Richards' model, developmental transition occurs by reasoning and in particular by *overcoming contradiction.* In the case of the transition between successive stages of cardinal number competence, the contradiction is installed between number-based and spatial-based estimations (assumed to be more primitive measures of size than cardinal number) and is solved in favour of number, that is gradually recognized as the most reliable measure of set's size. This process yields the recognition of cardinality as the relevant dimension in the reproduction of sets first, in the comparison of sets second, and the discovery of new properties of the cardinal number of sets, like identity and equivalence conservations.

The transition is envisaged as proceeding uniformly through three cognitive states which reflect the interpretation given to the cardinal representation produced by the number-domain structure in the the task situation. Depending on the child's stage, this representation can be envisaged as irrelevant, paradoxical or relevant. Consider the case of a child who is confronted with a new problem which is beyond his competence (e.g. a Stage 1 child whose number concept is inadequate to establish the equinumerosity of two sets and fail the number comparison task with rows of same number and different length). According to Richards' model, in the initial phase, the child represents the equinumerosity (the cardinal representations of the two sets and of the one-to-one correspondence holding between their elements produced by the specialized number-domain structure), but *does not recognize its relevance* for the task at hand. The child solves the task by means other than numerical strategies (e.g. spatial estimations of the rows' length or density). In an intermediate phase, the child starts envisaging the import of the cardinal representation of the sets for the solution of the task. The result of an estimation based on number can be, however, in contradiction with the result of the previous estimation by space, as in the comparison of rows equivalent in number and different in length or different in number and equivalent in length. This leads the child into a *paradoxical state* where he tries to combine the new number-based estimation with the previous space-based estimation. The contradiction is overcome by envisaging the new number-based estimation as a possible way of out of the contradiction. The child project equinumerosity as a hypothesis, that is, as being either true or false, and proceeds to test it (e.g. by counting, matching) or to evaluate it by argument (e.g. the conservation responses which refer to the fact that nothing has been added nor taken away, or that it is possible to go back to the initial configuration).

A successful test yields the recognition of the relevance of the cardinal number structure in a new situation and the discovery of new facts about number (e.g. that two different looking sets can have the same number, or that two equinumerous sets, transformed in distribution, remain equinumerous).

9.5.1 The transition from Stage 0 to Stage 1

The first developmental transition brings the child from the failure in the reproduction task (e.g. all the objects are used or a reproduction of the length of the row instead of its number is carried out), to correct reproductions. This transition follows two paths. The individual décalage between counted reproduction and matched reproduction in fact indicates that the children acquire the two forms of correspondence independently. To model the failure characteristic of Stage 0 and the two paths followed to reach Stage 1, I invoke two propositions which express the conditions under which the model set and the reproduced set have the same number of objects. The discovery of the relevance of these two propositions brings the children from Stage 0 to Stage 1 and constitutes the basis of what will be the Stage 1 theory of cardinal number.

Consider the hidden reproduction task first. The model set (m) is placed behind a screen and is identified by its cardinal value (Count (m)). The child's task is to create a set (n) equinumerous with set (m). For set reproduction in condition hidden to be correct, the following complex proposition[47] must be true.

(1) [*Given Count (m), Number (n) is the same as Number (m) iff Count (n) is the same as Count (m)*]$_1$

Consider then the visible reproduction task. The model set (m) is placed before the child. For a set reproduction condition visible to be correct, the following proposition must be true[48].

(2) [*Given m, Number (n) is the same as Number (m) iff the objects in n can be matched one-one to the objects in m*]$_1$

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[47]To avoid tedious repetitions in the presentation of the model, I do not distinguish between propositions and propositional schemas. The model refers from the start to the level of generalized representations of knowledge. Each proposition is between square brackets and is numbered. Each proposition also has an index (subscript) corresponding to the Stage in which its truthness is discovered for a characteristic class of tasks and situations.

[48] Also proposition (1) would satisfy the conditions for an accurate reproduction, but, as the results of Experiments 1 and 2 indicate, counting is never used in this task.

The propositions (1) and (2) correspond to the application of the cardinal number structure to set (m) (see figure 7.7, p.210), either in its physical or in its symbolic (number word) realization, and express the internal representation of the cardinality of (m) produced. The performance in the reproduction tasks at the different stages is modeled in terms of the interpretation given to the two propositions (1) and (2).

In Stage 0, the failure in the two conditions of reproduction is represented by the *propositions (1) and (2) being entertained as irrelevant.* Although the number-domain structure yields a representation of the cardinality of the model set, the bearing of this representation on the problem is not appreciated. The children thus do not have the cognitive motivation to attempt to establish a one-to-one matching, nor to count out the same number (m) of objects out of their bunch. The behaviours observed reflect the non-numerical nature of the Stage 0 children's approach to the task and suggest that the children *do not grasp the task as bearing on a set's cardinality.* The children systematically use all the objects at their disposal; reproduce the length of the row (or of the screen in the hidden reproduction task); or create a second row, with an arbitrary number of elements. A more primitive quantitative comparison schema based on the set's spatial extension is often favoured over one-to-one correspondence, especially in the visible condition of reproduction.

Between Stage 0 and Stage 1, the child goes through an intermediate phase which is brought about, according to the model, by the intervention of the semantic algorithm which prunes the logical system in which the child is reasoning (i.e. the logic of first-degree entailment) of the truth value for neither true nor false. The pruning of this value leads to the reassignment of truth values to the propositions which are irrelevant: *the propositions (1), (2) are reinterpreted as both true and false* within Kalman logic. The corresponding cognitive state of paradox and ambivalence is reflected in the child's attempts to establish a one-to-one correspondence and at the same time use up all the objects. Some children thus create two or three rows of objects which are all in one-to-one correspondence with the model row. These attempts can be interpreted as indications of a search for a *compromise between the condition for equinumerosity* that the child starts envisaging as relevant *and the previous non-numerical solution strategies.*

The contradiction between the spatial and numerical solutions is solved by a second intervention of the semantic algorithm, which prunes the value both-true-and-false from the semantic environment in which the child is reasoning. The paradoxical *propositions (1), (2) are reinterpreted into relevant propositions* which are either true or false, and can function as hypotheses within a classical logic reasoning system. The child has now the cognitive motivation to carry out the action of counting out a same number of objects or of establishing a one-to-one correspondence. This action produces the accurate equinumerous set.

This transition does not proceed in parallel for the two conditions of accurate reproduction (counting and matching). The individual décalage in the solution of the two tasks indicate that some children first work out the relevance of proposition (2). They reproduce correctly in the visible condition of the reproduction task in which the model set is before the child. At the same time, they fail the hidden condition of the reproduction task in which proposition (1) has to be invoked. Since proposition (1) is irrelevant, the children cannot make any use of the information that the model set has 'n' element, and produce non-numerical reproductions. Some children follow the opposite pattern. They first work out the relevance of proposition (1). They can make use of the information that the model set has 'n' elements to carry out the reproduction. When instead in the visible reproduction task this information is not given, the child lacks a pointer to number and provides non-numerical reproduction.

The same basic conceptual relation, i.e. *one-to-one correspondence applied to the elements of one set to reproduce an exact copy of it*, is expressed by two means: spatial matching and counting. These two basic strategies which guarantee accurate reproduction of sets come together at Stage 1. The propositions (1) and (2) are simultaneously relevant and define the Stage 1 number theory:

(1) [*Given Count (m), Number (n) is the same as Number (m) iff Count (n) is the same as Count (m)*]$_1$

(2) [*Given m, Number (n) is the same as Number (m) iff the objects in n can be matched one-one to the objects in m*]$_1$

9.5.2 The transition from Stage 1 to Stage 2

The transition from Stage 1 to Stage 2 corresponds to the extension of the Stage 1 theory to situations of set comparisons. The two propositions (1) and (2) are in fact relevant only with respect to individual sets. When the Stage 1 child is faced with two rows which he has to judge in numerosity, his number concept is not developed enough to express the relationship between sets. This means that although the number structure yields a representation of the sets' cardinality and of the correspondence holding between their elements, this representation is initially interpreted as irrelevant. The situation is represented in terms of a new complex proposition that expresses the condition to recognize *equinumerosity between two sets* using either matching or counting (see figure 7.8, p.211).

(3) [*Number (m) is the same as Number (n) iff the objects in m can be matched one-one to the objects in n or Count (m) is the same as Count (n)*]$_2$

Since this proposition is irrelevant in Stage 1, the child relies on the primitive schema of estimation of quantity on spatial extension and judges as larger the set which is longer (or sometimes the set which is more crowded). On this basis, two rows which have same spatial extent are taken to be equinumerous, both when they have same number, and when their difference (e.g. one) is too small to create a perceptual difference in spatial size (e.g. length, density). Of two rows which differ in length, the longer row is generally taken to be the more numerous, regardless of the rows' cardinal numbers.

The transition from Stage 1 to Stage 2 proceeds through an intermediate stage where the reduction of the semantic environment in which the child reasons (i.e. the logic of first-degree entailment) yields to the reinterpretation of the proposition entertained as neither-true-nor-false as both-true-and-false. The paradoxical state, in which the proposition (3) is seen as at the same time true and false, explains the solutions which suggest an underlying conflict between the result of counting and the estimation of size on the basis of length. Some children in fact try to combine the two sources of information about size and attempt to achieve simultaneous equivalence of spatial size and equinumerosity. In the case of two rows of same number and different length, as we have seen, the children apply first the spatial criterion to conclude that the two rows have different number. They add some objects to cancel the spatial difference, count the two collections, obtain a different cardinal number, add the number of objects corresponding to the difference. This addition introduces a new length difference, a new conflict and a new addition to match the end points, and so on. They go on switching from one criterion to the other until all the objects are used.

Another illustration of the kinds of contradictions that the children experience is provided by those children who formulate a numerosity judgment using spatial size as the criterion and then bend their counting to fit the initial judgment. As I have remarked in the presentation of qualitative results in Experiments 2 and 3, the children seem to be aware of the trick they employ to make the count information fit the space-based judgment. The contradiction between the initial judgment and the result of the count is anticipated and solved by modifying the count.

The transition algorithm intervenes again to reduce the semantic environment and to transform the proposition (3) which was entertained as both true and false into relevant, that is either true or false. The child can now project the hypotheses that if the two sets are equinumerous, their counts are the same or alternatively that their elements can be matched. The hypotheses are tested either by counting the two sets or by matching their elements, or by carrying out both tests.

When the true proposition (3) is applied to the identity conservation problem, the child matches set (m) with the post-transformation set (m') to conclude that:

(4) [*If m is spatially transformed into m', then Number (m) is the same as Number (m')*]$_2$

The conclusion of equinumerosity can be reached empirically (when set (m) had been counted, set (m') is also counted and their cardinal values are the same; by matching a memory of set (m) with set (m') before the child), or by inference (the arguments that nothing has been added to nor taken away from set (m), that set (m') is longer, but its elements are more spaced than in the shorter set (m), and that it is possible to go back to the initial configuration). Both conceptual and empirical solutions rely on the ability to match the two sets (m) and (m') and to work out their cardinal properties (conservation after spatial transformations) and relations (equinumerosity).

The décalage between counted conservation and the other number comparison tasks of Stage 2 is explained by the fact that half of the right-hand side of the biconditional (3) is acquired first in the direction from equal counts to same number. The substage 2a is represented by the following relevant proposition:

(5) [*If Count (m) is the same as Count (n), then Number (m) is the same as Number (n)*]$_{2a.}$

In the subsequent Stage 2, the proposition (5) is extended and included within the more general proposition (3). The Stage 2 number theory corresponds to the Stage 1 number theory plus the propositions (3), (4) and (5), to give the following propositional network:

(1) [*Given Count (m), Number (n) is the same as Number (m) iff Count (n) is the same as Count (m)*]$_1$

(2) [*Given m, Number (n) is the same as Number (m) iff the objects in n can be matched one-one to the objects in m*]$_{1.}$

(5) [*If Count (m) is the same as Count (n), then Number (m) is the same as Number (n)*]$_{2a.}$

(4) [*If m is spatially transformed into m', then Number (m) is the same as Number (m')*]$_2$

(3) [*Number (m) is the same as Number (n) iff the objects in m can be matched one-one to the objects in n or Count (m) is the same as Count (n)*]$_{2.}$

9.5.3 The transition from Stage 2 to Stage 3

The Stage 2 number theory is still not sufficiently articulate and general to express the conditions for the permanence of equinumerosity in the standard conservation task. Stage 2 children in fact typically fail the conservation task. This situation is modeled in terms of the same two propositions referring to the context set by the number conservation task (see the index 3, which refers to the task characteristic of Stage 3, i.e. standard conservation) and to the conditions of equinumerosity of the set m and the set n', i.e. the row n modified by the spatial transformation. At Stage 2, the proposition (6) is irrelevant .

(6) [*Given two equinumerous sets (m) and (n), if m is spatially transformed into m', Number (m') is the same as Number (n)*

The irrelevance of the cardinal representation produced by the number structure when the tasks demands that two pairs of sets (m, n and m',n) be matched is reflected in the abrupt change from a judgment of equinumerosity to a spatially based judgment of difference once a spatial transformation on one of the rows is carried out. Whereas the Stage 2 theory led to accurate set comparisons, the same theory is in trouble when faced with judging the two sets again, after a spatial transformation has been performed and has introduced a difference in spatial size.

Elsewhere I have argued that the décalage between the solution of the set comparison tasks and the standard conservation task indicates that the latter task introduces completely new requirements, i.e. the matching of the initial pair of sets with the post-transformation pair (see figure 7.9, p.213). Since this new context for recognizing equinumerosity is outside the domain of the Stage 2 number concept, the child makes appeal again to the primitive schemas for estimation of quantity. He abandons the initial equinumerosity for a judgment of difference, where the longer row is taken to be the more numerous.

The reinterpretation of proposition (6) into both-true-and-false explains the kind of ambivalent responses that children give in a period which is generally considered intermediate between non-conservation and conservation. The children oscillate between a judgment of equinumerosity and a judgment of difference both in the case of a unique transformation of a pair of rows (e.g. firstly the child answers that the two rows are equinumerous, and asked to justify that, replies that in fact they are different) and in the case of different trials (e.g. the child confirms equinumerosity after a lengthening transformation and abandons equinumerosity after a shortening transformation).

Finally the proposition (6) becomes relevant and effective reasoning can be performed on it. The child is motivated to determine whether the propositions are true or false and can either carry out an empirical test of equinumerosity (i.e. count the two post-transformation rows or match them), or derive the equinumerosity from the premises that a) nothing has been added nor taken away, b) the difference bears only on length and c) it is possible to go back to the starting configuration.

The Stage 2 number concept is enriched by the inclusion of the proposition (6) and by all the new relations between propositions that emerge, to give the number theory of Stage 2:

(1) [*Given Count (m), Number (n) is the same as Number (m) iff Count (n) is the same as Count (m)*]$_1$

(2) [*Given m, Number (n) is the same as Number (m) iff the objects in n can be matched one-one to the objects in m*]$_{1.}$

(5) [*If Count (m) is the same as Count (n), then Number (m) is the same as Number (n)*]$_{2a.}$

(4) [*If m is spatially transformed into m', then Number (m) is the same as Number (m')*]$_2$

(3) [*Number (m) is the same as Number (n) iff the objects in m can be matched one-one to the objects in n or Count (m) is the same as Count (n)*]$_{2.}$

(6) [*Given two equinumerous sets (m) and (n), if m is spatially transformed into m', Number (m') is the same as Number (n)*

9.6 Evaluation of the model

As I argued at the end of my presentation of Richards' model, this model offers a general purpose structural description of stages and transition in the development of conceptual domains. The two main features of the description of cardinal number development that it provides are the following:

1. The structure underlying each competence level is represented explicitly as a network of propositions (e.g. Stage 2 number theory). The relation of inclusion between any two successive stage structures is captured by the fact that the propositional network of the lower stage is part of the propositional network of the following stage-structure, e.g. the Stage 2 structure is part of the Stage 3 structure. At the same time, the reorganization and generalization characteristic of the new stage is captured by the fact that the new stage structure contains new propositions which come to interact with the propositional network of the previous stage to give a new network of interconnections between propositions and new meanings.

2 - the transition from one stage to the next is represented in terms of the semantic algorithm that strengthens the logical framework in which the child reasons and brings about a reinterpretation of information about cardinality. The shift from space-based to number-based estimations and judgments of numerosity is explained as a fixed sequence of interpretations of the import of cardinal representations to a class of situations.

Although both the formalism for the stage sequence and the transition mechanism fit well the development of cardinal number representations, the model looses some of its explanatory power with respect to its previous application to the development of the object concept. In the case of the object concept in fact, beside defining the truth values of propositions that represent the child's understanding of the task at different stages, it was possible to model inferences which were characteristic of the different logical systems in which these propositions were embedded.

Consider again Substage 4 of the genesis of the object concept, which corresponds to the task of retrieving the object from under one of two covers A and B: cover A where the object had been found previously and cover B where the experiment has put it before the infant's eyes. Richards' model explains in terms of the weak logic in which the child is reasoning the fact that the Substage 4 infant typically looks under cover A, and not finding the object, still does not go and look under B. In the logic of first-degree entailment in fact, the disjunctive syllogism which would take the child from the negation of the first of two possible locations A and B (e.g. the infant does not find the object under A where he expected to find it) to looking in the second location (e.g. under B) is not a valid schema of inference. In the case of the analysis of cardinal number development that I have provided, no comparable situations were identified. Of the principle that children of different stages reason within different logical systems, only the truth values associated to the different systems have been used (e.g. propositions which are irrelevant, paradoxical) while none of their characteristic inferences have been identified.

From the application to cardinal number problems, it appears that the domain for which the model has some explanatory power has to be restricted to situations where the child's hypotheses and the verifications or falsifications of the hypotheses are distinct, that is to situations in which the child is required to make predictions and in which we can observe his reactions when the hypotheses are falsified by the circumstances (e.g. the object is not found under A). Since none of the experimental number situations investigated have this characteristic (e.g. they do not provide a direct falsification for inaccurate reproductions nor comparisons), the application of the model to the development of cardinal number remains somewhat trivial. Nevertheless the model contributes to make the account of the nature of the concept (i.e. the objects to which the specialized cardinal number structure are applied) and of the developmental process (i.e. the generalization of the number structure to increasingly more complex objects) more explicit.

9.7 Appraisal and new directions of research

Although the account of the nature of the number concept at the three different stages remains still very tentative and all its consequences have not been fully explored, the basic objective of this thesis has been satisfied. We can now go back to our 5-year-old child of the introduction and can provide a specific characterization of his understanding of conservation in the accidental and in the counted conservation tasks and his simultaneous failure to conserve in the standard number conservation task. The 5-year-old child is operating with a concept of number which can express and recognize the equinumerosity (or the numerical difference) between two sets of objects. This concept on the other hand, cannot express the numerical relations between sets of sets and cannot recognize the equinumerosity between two pairs of sets, an operation which is required in the standard conservation task.

Furthermore, the characterization of the nature of the number concept at the different stages has permitted us to devise new experimental situations, to ask new questions about existing data, to provide a uniform account for the new experimental results and to offer a reinterpretation of some of the most classic puzzles in developmental studies, like the early conservations.

The main interest of the account of number development proposed however emerges in the new questions it raises and in the new research it motivates. Firstly, the sequence of three stages of number competence can be extended both in the direction of earlier competence levels and of subsequent levels of competence. This question can be approached experimentally by looking for stages which systematically precede Stage 1 and by defining their characteristic competence. In the opposite direction, we can look for more advanced stages of cardinal number representation beyond Stage 3 of number conservation. These studies involve the creation of new tasks, i.e. tasks that are accessible to Stage 0 children and tasks which are critical for Stage 3 children and the exam of the developmental orders in the solution of these tasks, an analysis that can be carried out using the hierarchical analysis method. Beside extending the analysis of the development of the cardinal representations of sets of objects, new studies can be undertaken to investigate the relationship between the three stages of cardinal competence and other kinds of knowledge, such as a) the development of arithmetical knowledge, in the context of practical tasks involving exchanges of objects, additions, subtractions, etc.; b) the development of measure concepts, in the context of practical tasks involving the measure and comparison of length, weight and surface. The relationship between the genesis of cardinal representations and of arithmetics remains within the domain of number; whereas the study of the relationship between the development of cardinal number and of measure raises fundamental questions about the interactions between different domains of knowledge (e.g. in this case, knowledge of numbers and knowledge of the domain to be measured).

The hierarchical method can serve to examine the previous questions experimentally. Firstly, the development of arithmetical and measure competence can be investigated separately on the basis of the response patterns to a battery of tasks. Secondly, the stages identified in the genesis of the two concepts can be matched to the stages described in the case of number. The analysis of the response patterns across tasks dealing with cardinal number and arithmetic can provide evidence of the relationship between these two types of knowledge in development. Similarly, the response patterns across tasks on cardinal number and measure can provide information about the relationships between these two domains in the course of their development.

I take the more properly cognitive science objective of this dissertation to be achieving greater explicitness in characterizing the developmental processes invoked. I have expressed my reservations about the explanatory value of the application of Richards' model to an explicit account of cardinal number development. Alternative ways of modeling the developmental process have to be sought which can allow a much richer characterization of the stage structures and of their inferences, while preserving the original intuition of Richards' model that developmental change is a semantic process of working out *relevant applications for representations to overcome contradictions*. The limitations of Richards' model should not preclude the possibility of coordinating a more complex and articulate characterization of the content of stage competence with a formal characterization of their structure and of the transition process. The Artificial Intelligence literature provides some indications of how the representation of complex knowledge structures can be achieved (see Hayes 1985a, b) and of how revision and reorganization of knowledge structures can be modeled (see Martins & Shapiro 1984[49]). The semantic mechanism of Richards' may thus be embedded within more

[49]The SWM System of Martins & Shapiro is based, as Richards' model, on Relevance Logic. The basic feature of the SWM System is that it keeps track of the propositions which are used to derive any given proposition. When a contradiction is detected, the system can thus recover and identify exactly the premisses from which the contradictory proposition was derived.

detailed representations of stage knowledge and may operate upon parts of the stage structures. A formal system which would make it possible to represent the richness and complexity of the knowledge structure underlying a stage and the transition process which operates on it to bring about the structure of the following stage would in any case be of an extreme complexity and of difficult realization at present.

Appendix

Contents

Appendix of Chapter 6: Conservation before age 5

In this appendix, I briefly present two experimental paradigms used to assess young children's (3- and 4-year-old) understanding of number conservation: Gelman's magic paradigm and Mehler & Bever's conservation of small collections.

6.1 Gelman's magic experiment

Gelman (1972b) devises a new procedure to assess conservation of number which involves two phases. First the child is trained to make a number judgment between two arrays of two or three items. The child is shown two sets of objects (mice) on separate plates placed side by side and his/her task is to find the 'winner'. The plates are covered, shuffled, uncovered over a few trials so that the child comes to expect that each tray will have a specific number of mice, e.g. 'The winner has three; the looser has two'. Then the experimenter introduces unexpected modifications, such as a rearrangement of the display or the removal of items, and assesses how the child treats the surreptitious subtractions or displacements. Three measures are taken: 1) the confirmation or change of judgment, 2) the differential reactions (on a 3-point scale: 0 = no discernible surprise; 1 = minimal surprise; 2 = moderate to extreme surprise), 3) the comments to the change.

The finding that children change their judgment and express surprise equally after the rearrangement and the removal of items (and change their judgment) would confirm that young children are nonconservers and can not distinguish number relevant from number irrelevant transformations. If instead children change their judgment only after the removal of items, although they express surprise after both transformation, this would indicate that children take note of the transformations but can discriminate between number relevant and number irrelevant operations on sets and can conceive of number as a dimension which is independent from the spatial dimensions of the set.

Gelman examines 96 children between 3,6 and 5,10 years of age and observes that the majority of children conform to the latter pattern. Their identification of the winner array does not change after the displacement although they appear to have recognized that the array has been changed in shape (e.g. 'Still three just moved them'). Instead, the subtraction operation leads to surprise as well as a change of judgment (e.g. 'Not four, only one-two-three; took one'). This experiment has been replicated successfully by Gelman (1972b) and by Silverman, Rose & Phillis (1979) also when the arrays are presented simultaneously and when the displacement is performed before the child, rather than surreptitiously.

Gelman concludes that, from age 3, children appreciate that a small number of items in an array remains invariant through the operations of displacement but not addition and subtraction. They are capable of conserving number in the magic task; a conservation which is systematically supported by the use counting to evaluate the modification and figure out the answers. According to Gelman, these same preschoolers fail the standard number conservation task because they lack an explicit understanding of one-to-one correspondence, which remains inaccessible under most circumstances until age 6 and 7 years.

6.2 Mehler & Bever's conservation of small collections

Mehler & Bever's studies (1967, with Epstein, 1968) examine the implicit assumption that, if 4 year-olds do not conserve number, than also children younger than 4 should not conserve number. Mehler & Bever argue that young children may in fact possess an innate understanding of number that simply disappears at around age 4 to reappear at age 6 years, as reported by Piaget. This alternative hypothesis is directly tested by a study of number conservation among 2 to 4 year olds with a task adapted to the younger population in the number of items used. Four items are used instead of six or seven. Mehler, Bever & Epstein (1968) report 80% conservation responses at age 2, 50% at age 3 and 64% at 4, a pattern which conforms to the expected U-shaped distribution.

The subsequent replication studies (Beilin 1968, Piaget 1968, Rothenberg & Courtney 1968, Higgins-Trenk & Looft 1971, LaPointe & O'Donnell 1974, Calhoun 1971 and Hunt 1975), however, which control for factors such as the order in which the alternatives "same number" and "different number" are presented in the question and the role of the experimenter's expectations, do not confirm Mehler, Bever & Epstein's results. It was also remarked that the task, as it was presented, did not bear directly on the conservation of the equinumerosity as children were not asked to judge whether the two sets had initially the same number of elements. Moreover, the child was not asked to give justifications and was considered to be conserving only on the basis of "same" answers. Finally, Mehler & Bever's observation that children seem to be responding that the modified row is still the same row, and not that the two rows have the same number, was confirmed.

The results of the replications, hence, strongly undermine the original conclusion of Mehler & Bever that children younger than age 4 can conserve, an ability that is lost, to then reappear at age 6. Nevertheless, the series of studies on preschoolers' number competence did reveal some understanding of elementary numerical operations, such as the effect of adding or subtracting elements on sets' numbers.

Appendix of Chapter 7

7.1 Protocol form for Experiment 1

Class:
Name:
Age:

Reproduction visible

Here I have a line (or row) of 'objects', I give you these. I would like you to take the same number of 'objects' as there are here (or as I have) and make a line (or row) with them.

<u>Objects used and number</u>:

<u>Actions</u>

The objects of the model set are counted:	YES	NO
The objects are counted out of the box:	YES	NO

The objects are taken out of the box:

- one at a time
- in handfuls
- in one bunch

The objects are put on the table:

- each one in front of one of the models' objects	Near-Match	Look-Match
- matching end-points		
- global reproduction of shape		
- randomly all		
- bunch readjusted to match model	Global	One-to-one

<u>Result of reproduction</u>

- Correct			
- Incorrect	More	Less	How many?

<u>Reason of failure</u>

- unquantified bunch
- global copy of shape
- inaccurate matching
- inaccurate counting
- other

Comparison

Is there and there the same number of objects, or does one of the rows have more objects?

Answer: SAME DIFFERENT

Correct Wrong

If Different:

Where is more and how many more?

- Count
- Indication of exceeding elements

What can you do to have the same number here and there?

- change in the arrangement
- addition or subtraction

Conservation 1

Look what I do. (specify the type of transformation)

Is the number of objects the same here and there, or is the number different now?

Same Different

Why is it, or why do you think so?

If different, which has more?

Conservation 2

Look what I do. Is the number of objects the same here and there, or is the number different now?

Same Different

Why is it, or why do you think so?

If different, which has more?

Reproduction hidden seen

Could you count this row of 'objects'?

Count: Correct Wrong (suggestion)

Now I cover it up, so that you cannot see them. I would like you to take the same number of 'objects' as there are here and make a line with them here.

Objects and number:

Actions:

- unquantified bunch all the objects
- count one at a time out of the box
- in handfuls
- other

Result of reproduction

- Correct
- Incorrect More Less How many?

Reason of failure

- unquantified bunch
- all the objects are used
- inaccurate counting
- other

Comparison

The row is uncovered

Is there and there the same number of objects, or does one of the rows have more objects?

Answer: SAME DIFFERENT

Correct Wrong

If Different:

Where is more and how many more?

- Count
- Indication of exceeding elements

If Same:

How do you know that?

What can you do to have the same number here and there?

- change in the arrangement
- addition or subtraction

Conservation (same as above)

Reproduction hidden unseen

I'm making a line with 'n' objects here. I would like you take the same number of 'objects' from the box and make a row with them on this side.

(Actions, results and checks as in reproduction condition hidden seen)

Comparison (same as above)

Conservation (same as above)

7.2 Some a posteriori comparisons

In this appendix, I present the contingency tables of the pairs of tasks for which Experiment 1 did not formulate any specific hypothesis:

a) visible reproduction and hidden seen reproduction;

b) visible reproduction and hidden unseen reproduction;

c) hidden seen reproduction and hidden unseen reproduction;

d) visible reproduction and comparison after hidden seen reproduction;

e) visible reproduction and comparison after hidden unseen reproduction;

f) comparison after hidden seen reproduction and comparison after hidden unseen reproduction;

For each pair of tasks, the three developmental models of concurrency and collective décalage in the two directions were calculated and compared. This analysis provided some indications as to the order in which these tasks are acquired. The contingency tables and the results of the statistical tests are presented below.

7.2.1 Visible reproduction and hidden reproduction condition seen and unseen

Repro Visible \ Repro h seen	S	F
S	42	3
F	8	7

a

Repro Visible \ Repro h uns	S	F
S	39	6
F	10	5

b

Table A.7.1: Contingency table a) for *visible* and *hidden seen* reproduction and b) for *visible* and *hidden unseen* reproduction

Consider distribution a) first. The three Dels of the models of concurrency (Del_c) and collective décalage (Del_{d1} and Del_{d2}) are all significant:

$\mathrm{Del}_c = .45$ ($z = 3.25$, $p = .0006$)

$\mathrm{Del}_{d1} = .6$ ($z = 3.26$, $p = .0006$)

$\mathrm{Del}_{d2} = .36$ ($z = 2.69$, $p = .0035$)

The comparison indicate that the model of concurrency is a significantly better predictor of results than décalage from reproduction visible to reproduction hidden seen ($z = 2.02$, $p = 0.02$), but is not significantly different from décalage from reproduction hidden seen to reproduction visible ($z = 1.26$, $p < 0.10$). Since the two collective décalages are not themselves significantly different from each other ($z = 1.48$, $p = .069$), none of the models provides a better fit to the distribution. Nevertheless, as they are all significant, I can conclude that a weak form of concurrency exists between the solution of visible and hidden seen reproduction.

Consider now distribution b). None of the three Dels is significant:

$Del_c = .22$ ($z = 1.5$, $p = .06$)

$Del_{d1} = .27$ ($z = 1.5$, $p = .06$)

$Del_{d2} = .18$ ($z = 1.4$, $p = .08$)

Since the models are not significantly different form each other (z ranges between .76 and 1.01), individual décalage appears to hold between the solution of visible and hidden unseen reproduction. This result may be due also to a sampling problem as the majority of children (65%) solve both tasks.

7.2.2 Seen and unseen hidden reproductions

Repro h uns \ Repro h seen	S	F
S	46	3
F	4	7

Table A.7.2: Contingency table for *hidden seen* and *hidden unseen* reproduction

The three Dels of the models of concurrency (Del_c) and collective décalage (Del_{d1} and Del_{d2}) are all significant:

$Del_c = .75$ ($z = 8.8$, $p < .00003$)

$Del_{d1} = .68$ ($z = 5.5$, $p < .00003$)

$Del_{d2} = .81$ ($z = 9.05$, $p < .00003$)

They are not, however, significantly different from each other (z ranges from .87 to 1.1). I thus conclude that a weak form of concurrency exists between the solution of hidden reproduction in the conditions seen and unseen .

7.2.3 Visible reproduction and Comparison after hidden seen and unseen reproduction

Repro Vis \ Comp h uns	S	F
S	34	11
F	5	10

Repro Vis \ Comp h seen	S	F
S	36	9
F	7	8

Table A.7.3: Contingency table for *visible* reproduction and a) comparison after *hidden unseen* reproduction, b) and *hidden seen* reproduction

Consider table a) first.

Delc = .37 (z = 2.9, p = .0018)

Deld1 = -.01 (z = .8, p = .2)

Deld2 = .49 (z = 3.3, p = .0004)

The models of concurrency and of décalage from reproduction to comparison are significant, but not significantly different from each other. Since they both yield better predictions than the reverse décalage model, I conclude that the two tasks are solved with concurrency together with décalage from reproduction to comparison. The same conclusion applies to Table A.7.3 b (the Dels being: Delc = .65 (z = 2.6, p = .004), Deld1 = -.25 (z = .4, p = .3), Deld2 = .51 (z = 2.5, p = .006)

7.2.4 Comparison after hidden seen and after hidden unseen reproduction

Comp h seen \ Comp h uns	S	F
S	39	4
F	0	17

Table A.7.4: Contingency table for comparison after *hidden seen* reproduction and comparison after *hidden unseen* reproduction

The three Dels of the models of concurrency (Delc) and collective décalage (Deld1 and Deld2) are all significant:

$Del_c = .85$ ($z = 8.8$, $p < .00003$)

$Del_{d1} = .73$ ($z = 5.5$, $p < .00003$)

$Del_{d2} = 1$ ($z = e$, $p < .00000$)

The Del for décalage from comparison after hidden seen reproduction to comparison after after hidden unseen reproduction is a significantly better predictor of results than concurrency ($z = 2.07$, $p < 0.019$) or the reverse décalage ($z = 2.4$, $p < 0.008$). Notice however, that the great majority of responses (93%) correspond to success in both tasks or failure in both tasks. This suggests that concurrency does also provide a good fit to the data, accounting for 93% of the responses.

Appendix 7.3 Protocol form for Experiment 2

Class:

Name:

Age:

Reproduction visible

Here I have a line (or row) of 'objects', I give you these. I would like you to take the same number of 'objects' as there are here (or as I have) and make a line (or row) with them.

Number:

Actions

Model set counted:	YES	NO
Objects counted out of the box:	YES	NO

Near-Match

Look-Match

Bunch

Bunch readjusted to match model   Global   One-to-one

All

Result of reproduction

- Correct
- Incorrect   More   Less   How many?

Reason of failure

Inaccurate counting

Inaccurate matching

Matching end-points

Global reproduction of shape

Unquantified   all

Comparison

Is there and there the same number of objects, or does one of the rows have more objects?

Answer: SAME   DIFFERENT

Correct   Wrong

If Different:

Where is more and how many more?

- Count
- Indication of exceeding elements

What can you do to have the same number here and there?

- change in the arrangement
- addition or subtraction
- nothing

Reproduction hidden

I'm making a line with 'n' objects here. I would like you take the same number of 'objects' from the box and make a row with them on this side.

Number:

Actions:

Objects counted out of the box: YES NO

If No:

Bunch

All

If Yes:

Accurate Inaccurate Counting

Result of reproduction

- Correct
- Incorrect More Less How many?

Reason of failure

- unquantified bunch
- all the objects are used
- inaccurate counting
- other

Comparison

The row is uncovered

Is there and there the same number of objects, or does one of the rows have more objects?

Answer: SAME DIFFERENT

Correct Wrong

If Different:

Where is more and how many more?

- Count
- Indication of exceeding elements
- Suggested Count

If Same:

How do you know that?

What can you do to have the same number here and there?

- change in the arrangement
- addition or subtraction

Direct comparison

Here I have made two lines of sweets. Are there the same number of sweets as there (pointing to the other row)? or is the number different?"

Answer: SAME DIFFERENT

If Different:

Where is more and how many more?

- Count
- Indication of exceeding elements

Why don't you try counting the rows. Do you think it's a good way to see if they are the same or a different number?

What can you do to have the same number here and there?

- change in the arrangement
- addition or subtraction

If Same:

How do you know that?

But you see, they look the same.

Appendix of Chapter 8 Experiment 3

8.1 The corollary hypothesis of Experiment 3: there is collective décalage from identity, counted conservation, three-set comparison and standard number conservation

In Appendix 8.1, I present the results of the comparison of the children's performance in the tasks of identity conservation, counted conservation and three-set comparison, on the one hand, and standard number conservation, on the other hand. The corollary hypothesis of hypothesis 2 that a collective décalage exists from the solution of the Stage 2 tasks (identity conservation, counted conservation and three-set comparison) to the solution of the standard Piagetian conservation task is tested. Correct performance on the standard conservation task is thus expected to be more strongly associated with correct performance on the Stage 2 tasks than with incorrect performance on these tasks. The hypothesis is schematically presented in figure 8.A.1:

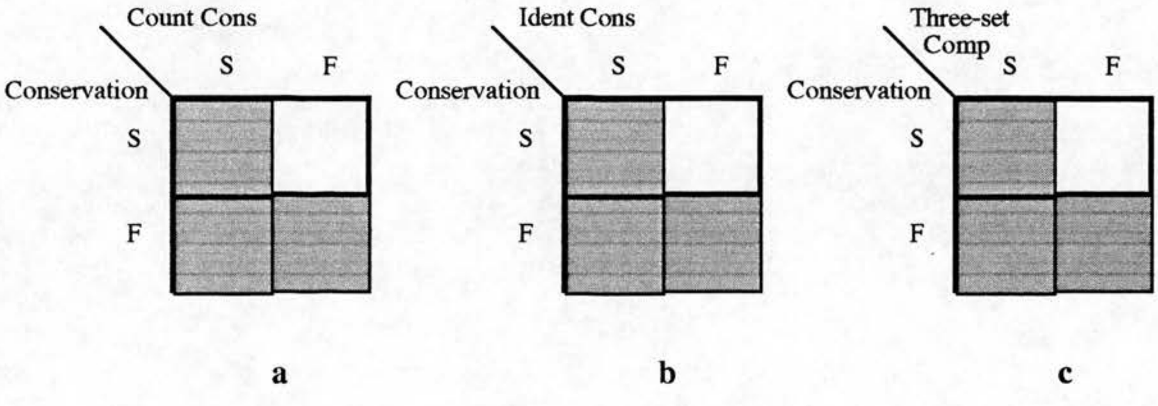

Fig. 8.A.1: Models of collective décalage between responses to the standard conservation task and a) counted conservation task, b) identity conservation task, c) three-set comparison task.

The results are summarized in the following contingency table 8.A.1 (the white cell is the cell predicted to be empty):

a

Cons \ Count Cons	S	F
S	38	0
F	15	19

b

Cons \ Ident Cons	S	F
S	37	1
F	7	27

c

Cons \ Three sets Comp	S	F
S	37	1
F	6	28

Table 8.A.1: Contingency table for standard conservation and a) counted conservation, b) identity conservation, c) three-set comparison.

In all three tables, collective décalage predicts the significantly non-chance contingencies in favour of:

a) counted conservation (Del = 1, z = e);

b) identity conservation (Del = .93, $z = 14.2$, $p < .00003$);

c) three-set comparison (Del = .93, $z = 14.2$, $p < .00003$);

it is also a significantly better predictor than concurrency:

a) $z = 4.8$, $p < .00003$

b) $z = 2.1$, $p = .018$

c) $z = 1.8$, $p = .036$

and is a significantly better predictor than the reverse décalage:

a) $z = 6.8$, $p < .00003$

b) $z = 2.4$, $p = .008$

c) $z = 2.1$, $p = .018$

These results corroborate the hypothesis that counted conservation, identity conservation and three-set comparison are acquired before standard conservation.

Appendix 8.2 Protocol form for Experiment 3

Class:
Name:
Age:

Reproduction hidden

I'm making a line with 'n' objects here. I would like you take the same number of 'objects' from the box and make a row with them on this side.

<u>Number:</u>

<u>Actions:</u>

Objects counted out of the box: YES NO

If No:

Bunch

All

If Yes:

Accurate Inaccurate Counting

<u>Result of reproduction</u>

- Correct
- Incorrect More Less How many?

<u>Reason of failure</u>

- unquantified bunch
- all the objects are used
- inaccurate counting
- other

Comparison

The row is uncovered

Is there and there the same number of objects, or does one of the rows have more objects?

<u>Answer</u>: SAME DIFFERENT

Correct Wrong

<u>If Different:</u>

Where is more and how many more?

- Count
- Indication of exceeding elements
- Suggested Count

If Same:

How do you know that?

What can you do to have the same number here and there?

- change in the arrangement
- addition or subtraction

Reproduction visible

Here I have a line (or row) of 'objects', I give you these. I would like you to take the same number of 'objects' as there are here (or as I have) and make a line (or row) with them.

Number:

Actions

Model set counted: YES NO

Objects counted out of the box: YES NO

All Near-Match Look-Match Bunch Bunch readjusted (Global - 1-to-1)

Result of reproduction

- Correct
- Incorrect More Less How many?

Conservation

Look what I do.

Is the number of objects the same here and there, or is the number different now?

Same Different

Why is it, or why do you think so?

If different, which has more?

Can you make them the same?

Addition Subtraction Length Nothing

Three-set Comparison

I would like you to tell me whether there is the same number of clowns here, here and here, or whether the number is different? You can either count them or move them around if you wish.

Same Different

- Count

- Length estimation
- Other

If Different: Which row has more objects?

How many more?

If no answer, Suggestion: Do you know of a way to check that this line has more clowns? Would counting help? Would moving the faces help?

If Same: How do you know that?

Suggestion: Do you know of a way to check that they have the same number of clowns?

Counted Conservation

Pre-transformation: Do these two rows have the same number of rounds, or does one of them have more rounds?

Post-transformation:

I would like you to count this line. Could you also count this other line?

Count: Correct Wrong Recount:

How many rounds are there? and how many rounds are there.

Number: Correct Wrong

Is the number of objects the same here and there, or is the number different now?

Same Different

Why is it, or why do you think so?

If different, which has more?

Can you make them the same?

Addition Subtraction Length Nothing

Identity Conservation

You see these frogs, they are going for a walk all together.

Transformation

Some of the frogs walk much faster than the other frogs and are farther ahead. Some of the frogs are slow and stay a little behind. Do you think that there is still the same number of frogs in this long line as it was at the start of the walk?

Same Different

If Same: Why is it? How do you know?

If Different: Is it more or is it less? Why is it?

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