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Mathematical Reasoning in Plato's Epistemology

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Declaration

- (a) this thesis is my own work
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Jane Orton

05/09/2013

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Abstract

According to Plato, we live in a substitute world. The things we see around us are shadows of reality, imperfect imitations of perfect originals. Beyond the world of the senses, there is another, changeless world, more real and more beautiful than our own. But how can we get at this world, or attain knowledge of it, when our senses are unreliable and the perfect philosophical method remains out of reach? In the Divided Line passage of the *Republic*, Plato is clear that mathematics has a role to play, but the debate about the exact nature of that role remains unresolved.

My reading of the Divided Line might provide the answer. I propose that the ‘mathematical’ passages of the *Meno* and *Phaedo* contain evidence that we can use to construct the method by which Plato means us to ascend to knowledge of the Forms. In this dissertation, I shall set out my reading of Plato’s Divided Line, and show how Plato’s use of mathematics in the *Meno* and *Phaedo* supports this view. The mathematical method, adapted to philosophy, is a central part of the Line’s ‘way up’ to the definitions of Forms that pure philosophy requires. I shall argue that this method is not, as some scholars think, the geometric method of analysis and synthesis, but *apagōgē*, or reduction. On this reading, mathematics is pivotal on our journey into the world of the Forms.

Introduction

Throughout history, mathematics has influenced those at the forefront of other fields, and philosophy is no exception. Writing about the great philosopher Thomas Hobbes, Aubrey captures the massive impact that the achievements of mathematics can have on a thinker:

He was 40 years old before he looked on Geometry; which happened accidentally. Being in a Gentleman's Library, Euclid's Elements lay open, and twas the 47 EL. Libri I. He read the Proposition. *By G-* sayd he (he would now and then swear an emphaticall Oath by way of emphasis) this is impossible! So he reads the Demonstration of it, which referred him back to such a proposition; which Proposition he read. That referred him back to another, which he also read...that at last he was demonstrably convinced of that truth. This made him in love with Geometry (Cited in Stillwell [1989] p 13).

Consider also the words of Bertrand Russell:

At the age of eleven I began Euclid...This was one of the great events of my life, as dazzling as first love. I had not imagined there was anything so delicious in the world (Cited in Stillwell [1989] p 26).

This sentiment is not limited to philosophers: writing of himself, here is what Abraham Lincoln has to say about the same subject:

He studied and nearly mastered the six books of Euclid since he was a member of Congress. He began a course of rigid mental discipline with the intent to improve his faculties,

especially his powers of logic and language. Hence his fondness for Euclid, which he carried with him on the circuit till he could demonstrate with ease all the six books (Cited in Stillwell [1989] p 26).

No doubt Plato would have approved of the study of geometry by a statesman; much of his *Republic* is taken up with a plea for the rulers of his ideal state to undergo rigorous mathematical training. Plato has his own love affair with mathematics, and, as I shall try to argue, his dialogues can tell us about the effect it had on his philosophy.

Plato's relationship with mathematics, it has already been argued,¹ changed his entire philosophical approach. This dissertation explores one aspect of this change: the hypothetical method in the so-called 'middle dialogues,' with particular reference to the use of imagery. Plato is famously critical of images² and although much work has been done to rehabilitate the status of the image in Plato's political thought (Nehamas [1999] Ch. 12-13), it remains the black sheep of Platonic epistemology. Yet the dialogues are full of images, as Plato himself is not unaware.³ Indeed, the use of images, along with the use of hypothesis, is a distinguishing mark in the science that Plato so much admires: the mathematics of his time. In the divided line passage of the *Republic*, Plato notes that the use of hypotheses and the use of diagrams are both characteristics of *dianoia*, or mathematical reasoning. This project will explore the influence of mathematics on Plato's philosophy.

¹ See especially Vlastos (1991), pp. 107-131 for a discussion of the influence of mathematics on Plato.

² See books IX and X of *Republic* for Plato's criticism of images.

³ Socrates refers to such issues at *Phaedo* 100a.

Aims of the Thesis

Plato's theory of Forms is one of the most distinctive aspects of his philosophy. Forms are central to his metaphysics, epistemology and moral and political theory. The Philosopher Kings of the *Republic* are to spend years in training to attain knowledge of the Forms before they are to rule. In the divided line passage of the *Republic* (509d-511e), Plato speaks of there being two realms: the intelligible (containing the Forms) and the visible, and each realm has two states of mind associated with it. *Dianoia* is the state of mind that Plato links to mathematics: I shall argue that it is a kind of reasoning that Plato himself uses in the middle dialogues, and it is the second highest kind of reasoning in Plato's scheme. The ultimate aim for the philosopher is to attain the highest state of mind, *noēsis*, but the problem for Plato is how to do that when most of us spend our lives contemplating the visible world. In fact, as Plato notes in the *Phaedo* (66cd) we are tied to the physical world, so how can we release ourselves from these tethers to contemplate the intelligible?

The primary aim of the thesis is to show how Plato can use the mathematical method to provide a solution. I shall suggest that Plato's divided line can be seen as an epistemological scale to be ascended by the philosopher, and that the mathematical method provides a pivotal role in ascending this scale. In fact, it is by using this method that we may escape the prison of the senses, and begin to contemplate the intelligible. By examining Plato's remarks about mathematics, and the structure of some of the important arguments in the middle dialogues, we shall see that Plato himself uses the mathematical method to do exactly this: when Socrates' friends are stuck in *pistis*, the state of mind that uses physical objects as tools of inquiry, Socrates sometimes uses *dianoia* to ascend the epistemological scale and reach for knowledge of the Forms. I shall argue that he does this in the *Meno* and *Theaetetus*, and that his friends initiate this in the *Phaedo*.

I also aim to explain in detail how the history of mathematics influences Plato's thought, by arguing that, in addition to the important role that mathematics plays in Plato's epistemology as a whole, there is a specific mathematical method that Plato uses as a way of ascending the epistemological scale. I shall build on the work of Karasmanis ([1987] see especially pp. 44-52 and 303-307), who points out that, whereas Plato is often supposed to be using the method of analysis and synthesis in the mathematical passages (often called the hypothetical method), it is probable that he is using the method of *apagōgē*, or reduction. I aim to support this claim, and also to build upon it: I will suggest that *apagōgē* is also the method used in ascending part of the epistemological scale as described in the *Republic*: I shall argue that it is used in both *dianoia* and the initial stages of *noēsis*. In addition, I shall argue that a closer examination of the text reveals a more exact match to this mathematical method than has been suggested before, particularly in the *Meno*, as well as suggesting that the structure of Plato's arguments often reflects his epistemological scheme.

A further aim will be to examine the characteristics of *dianoia* as described in the *Republic*, with particular reference to the role of images in Plato's epistemology. This study will suggest that to Plato the image is not an ideal epistemological tool, but it is useful in the initial stages of the philosopher's quest. In the famous 'Divided Line' passage (510cd), the *Republic* tells us that dianoetic reasoning has two distinctive qualities: the use of hypotheses and the reliance on imagery. Plato is not specific about how the relationship between the use of diagrams and the use of hypotheses came to be, yet he seems to want to say that the connection is philosophical, rather than historical— that is, it is the nature of mathematical thought itself that causes hypotheses and images to be used in this connection.⁴

Although dianoetic reasoning and the hypothetical method are not the same, we shall see that *dianoia* comprises the preliminary stages of this method. I shall argue that the

⁴ See Burnyeat's discussion ([2000] pp. 37-38) for the view that hypotheses are 'intrinsic to the nature of mathematical thought' (p. 37).

hypothetical method spans both the *dianoia* and *noēsis* segments of the line, so an investigation into the properties of dianoetic reasoning can tell us a great deal about the hypothetical method as a whole. We shall use Plato's own dialogues to examine the role that imagery plays as the hypothetical method is deployed.

Finally, my proposed solution to Plato's problem uses Plato's conception of definitions and their role in epistemology. I aim to show that Platonic definitions, which I shall be calling *ti estis*, are needed to attain knowledge of the Forms, and needed for *noēsis*: Plato's conception of these and the role they play will be discussed in the sections on the *Republic* and *Meno*. From the early dialogues, Socrates insists on finding the *ti esti* of a thing before we can inquire into its properties. However, without a *ti esti*, it can be very difficult to get the discussion started. As Meno remarks to Socrates, "How on earth are you going to set up something you don't know as the object of your search?" (*Meno* 80d). The problem is that, without the *ti estis* we need for *noēsis*, it is difficult to know how to escape *pistis*, a state of mind connected with contemplation of the sensible world. I propose that the characteristics of *dianoia* can provide the means for the mind to ascend the epistemological scale. Hypotheses and images, I will suggest, can be used as proxies for *ti estis*, so that the inquiry can continue until a Platonic definition is attained.

Statement of Terminology

I have spoken about Plato's use of mathematical reasoning in his dialogues, as well as his use of images in connection with mathematical thought. I now want to summarise my use of the relevant terminology, and how I will use these terms throughout the thesis. Firstly, when I say 'mathematical reasoning' I mean reasoning modelled on mathematical practice, *ie*, *dianoia* or the hypothetical method. As I have noted, *dianoia* is not the same thing as the

hypothetical method, although there is an overlap: I shall suggest that *dianoia* makes up the initial stages of the hypothetical method, and this process is Plato's way of ascending the epistemological scale. I shall argue that Plato has modelled this process on the mathematical method of *apagōgē*, as opposed to analysis and synthesis, so when I speak about the mathematical method, this is the method I mean.

Throughout the thesis 'hypothesis' and 'assumption' are translations of the same Greek word; I shall prefer to use 'hypothesis,' except when citing a translation that uses the word 'assumption.' In addition, when discussing scholarship that uses both 'hypothesis' and 'assumption,' I shall need to use that terminology also. For example, when I discuss Robinson's scholarship on this issue in the section on Imagery and Hypothesis, below, I shall use statements like 'the hypothesis is treated as an assumption.' By this, I mean to stress that the hypotheses are treated as strictly hypothetical, and that the user of the hypothetical method acknowledges that the hypotheses still need confirmation.

I shall also want to bring the reader's attention to the influence mathematics in general has on Plato's work, for example his many references to mathematics in the *Phaedo* outside of the hypothetical passage. Mathematics for Plato is arithmetic (things "to do with number", as he says in *Republic* 525a), plane geometry, the study of plane surfaces (*Republic* 528ab), and solid geometry, "the treatment of dimension and depth" (*Republic* 528d). Plato also mentions astronomy and harmonics as mathematical sciences (*Republic* 528e-531d). Therefore, when I talk about mathematics in general and its influence on Plato, I mean these features of the discipline.

Plato himself is not consistent in his use of terminology regarding the divided line; see for example, his different uses of terms at *Republic* 511de and 534a. Plato himself says that he is not keen to fix on rigid technical terms (*Republic* 533e). However, we should be clear about our use of the term 'image', as I want to use it in quite a broad sense. Plato seems

to want to use the term for both visible diagrams or pictures and analogies in discourse (which I shall also call ‘verbal images’). When I am speaking of *dianoia*, ‘image’ will mean visible diagrams in those cases when the philosopher or mathematician actually uses a visible figure (for example, when Socrates draws the squares on the sand in the *Meno*’s slave boy passage). The term will refer to analogies in discussion when Socrates and his interlocutors are using analogies as verbal instantiations of a Form to inquire about its properties (for example, Simmias and Cebes’ harmony and tailor images in the *Phaedo*). Of course, Plato says that images are also used in *eikasia*, the state of mind that is the lowest on his divided line. Once again, I shall argue that eikasiastic images can be either verbal or visible: in the *Republic* section, I shall suggest that Plato means the term image to refer to both poetry and painting. For example, in book X of the *Republic*, Plato links the two: he says that a painting is only a representation of a physical object (596de), and this is compared to the poet creating a likeness of physical things (598e-599a).

In the case of *dianoia*, the use of the term ‘image’ to refer to both verbal and visible images is supported by the fact that Plato uses the same term for both, as well as by the fact that they perform the same role in dianoetic discussion. In the *Phaedo*, Plato makes Simmias and Cebes use physical things as images in the discussion; these are specifically called images, *eikones* (87b), just as the figures of the mathematician are described as images at *Republic* 510b. I shall argue that the images used by Simmias and Cebes in the *Phaedo* are part of the process of *dianoia* described by Socrates in the *Republic*. This, I shall argue, justifies the use of the term ‘image’ for both visible diagrams and analogies in discussion.

In addition, my broader understanding of the term ‘image’ is supported by the way in which the image works in the discussion. I shall argue that verbal images allow Socrates and his friends to make philosophical progress in the same way that the visible diagram often does. For example, it allows the group to inquire into something for which they have no

definition, in this case, the soul. In the same way, in the *Meno*, Socrates and the slave use the visible diagram of a square to inquire into its properties without first defining what it is. I shall argue that Simmias' and Cebes' use of their images allows them to break Socrates' 'say only what you believe' rule without resorting to sophistry. This, I shall argue in the *Meno* section, is also an important feature of *dianoia*, and, as such, warrants the inclusion of analogies in discussion under the term 'images.' Finally, these verbal images allow us to perform operations on a proxy of a Form; as I shall explain in the *Republic* section, Plato is concerned about mathematicians performing operations on things that are eternal and unchangeable (*Republic* 527ab). I shall suggest that Plato's dianoetic images allow the philosopher to get around this problem: this is true for both visible and verbal images, and is a further reason for including both visible figures and analogies under the term 'image.'

Development of Plato's Methodology⁵

Traditionally, Plato's dialogues have been split into the following categories: early, (transitional), middle and late. These divisions have been made according to stylistic and philosophical groupings that occur in the corpus. The argument is that we can reconstruct a development in philosophical method. In the early dialogues, we see the deployment of the Socratic *elenchus*, which the historical Socrates is believed to have used. Vlastos has convincingly argued that, after Plato's return from Sicily, where he learned new techniques from the Pythagoreans, Plato abandons the method of his old teacher and brings in the hypothetical method. We see this in the *Meno*, *Phaedo* and *Republic* (Vlastos [1991] Ch. 4, pp 107-131; cf. Karasmanis [1987] especially pp. 303-307, who makes a similar point). Later, in dialogues like the *Phaedrus*, this method is eclipsed by the method of collection and

⁵ The dialogues are not treatises; when we say that Plato has a 'method,' we mean simply the structure of the argument.

division (White [1987] Ch. 5, pp 117-125 and Kahn [1998] Ch. 10, pp 292-328). Note that these changes in methodology do not exactly correspond to the division between early, middle and late – *Phaedrus*, for example, is usually classed as a middle dialogue - but this point of view does imply the development of ideas in Plato associated with the traditional distinctions. The purpose of this section will be to examine this development and briefly state our position.

In terms of doctrine, the conventional standpoint is that the early dialogues are characterised by Socratic eudaimonism (the idea that virtue is sufficient for happiness) (*Apology* 28bd; 38a) and the Socratic paradox that no-one errs voluntarily (*Protagoras* 329c-333b). Crafts and skills are characterised as branches of knowledge (*Charmides* 165e; *Euthydemus* 281a; *Ion* 532c; *Protagoras* 356de; *Republic I*), innate knowledge is implied by the *elenchus* and its ‘say only what you believe’ requirement (*Phaedrus* 275b; *Meno* 71d; *Theaetetus* 171d) and *akrasia*, or weakness of will is impossible.

In the middle period, when Plato abandons the Socratic *elenchus* in favour of the hypothetical method, the idea of innate knowledge is made explicit in the theory of recollection (*Meno* 81-86c; *Phaedo* 73c-75c). Forms are introduced, and the introduction of the tripartite soul in *Republic IV* (also *Phaedrus* 246) allows for the possibility of *akrasia*. In later dialogues, it is argued by some that Plato abandons his theory of Forms, having identified several problems with it in the *Parmenides*.⁶ There is also a significant shift in style in later works, with Socrates taking less of a prominent role, which has led some scholars to suggest that Plato became disillusioned with the dialogue form.⁷

More recent scholarship has questioned the traditional reading. Kahn (1998, especially pp. 38-48) has argued that, although stylistic analysis can give us some clue as to

⁶ Notably, although not restricted to, the problem of participation (especially *Parmenides* 131a-133a). The question is not immediately relevant to this paper, but it should be noted that this author does not support this reading of *Parmenides*. Rather, the theory of forms is not abandoned here, but subjected to a high level of scrutiny that requires immense intellectual honesty on Plato’s part.

⁷ This view is not universally accepted. See Rowe (2007), especially pp 266-276.

the chronology of the dialogues, we cannot find a sharp break between the early dialogues and the metaphysical doctrine of *Phaedo* and *Republic*. Kahn thinks that Socrates was an influence on Plato's moral theory, but this is not localised in the early dialogues. He sees a continuity in Plato's thought throughout the corpus and reminds us that Plato's works are literary as well as philosophical; Plato deliberately 'holds back,' maintaining a psychological distance between himself and his audience.

In a similar vein, Rowe (2007) points out that Plato is aware of how strange his ideas are, and that he is reluctant to reveal the most radical points of his philosophy at first. Agreeing with the threefold stylistic division of Plato, Rowe tries to reconcile the themes from all three parts – so Socrates' agnosticism in *Apology* is not at odds with his dogmatism in *Phaedo*, for example (ibid, Ch. 3, pp.122-142).

The so-called 'developmentalist' debate lies outside the scope of the present study, but any reading of the individual dialogues is unavoidably affected by one's conception of its place in the development of Plato's thought. Therefore, we should at least be clear about our own assumptions regarding the relevant points of chronology to our task. For this reason, I shall now briefly state the position assumed by the current project.

There is an argument that Plato's style can be used to order the dialogues, however much disagreement exists over their exact order or the development of doctrine. This study will loosely follow Vlastos' (1991, pp. 46-47) ordering of the dialogues, with the notable exception that *Republic I* is not taken to be a separate work from the rest of the dialogue.⁸ We shall retain the idea of a 'transitional' period, also following Vlastos.

The study therefore follows the following chronological scheme:

⁸ See Kahn (1993) pp. 131-142 for an excellent discussion on this.

Early dialogues (not in chronological order): *Apology*, *Charmides*, *Crito*, *Euthyphro*, *Gorgias*, *Hippias Minor*, *Ion*, *Laches*, *Protagoras*

Transitional dialogues (not in chronological order): *Euthydemus*, *Hippias Major*, *Lysis*, *Menexenus*, *Meno*

Middle dialogues: *Cratylus*, *Phaedo*, *Symposium*, *Republic*, *Phaedrus*, *Parmenides*, *Theaetetus*

Late dialogues: *Timaeus*, *Critias*, *Sophist*, *Politicus*, *Philebus*, *Laws*

The dialogues are literary works, not treatises, so the extraction of doctrine from the text must be undertaken with caution. The dialogue form allows Plato to present views for scrutiny without necessarily having to endorse them, and allows the reader to examine his own beliefs (Frede [1998], pp. 253-269) - both important for Plato's dynamic conception of philosophy.⁹ In addition, Plato is aware of how strange his ideas are, and it is understandable that he would want to introduce his reader to them gradually, rather than being explicit from the beginning. We cannot even rule out the idea that Plato went back to rewrite some of his early work. In this way, the developmentalist position is difficult to prove, given that Plato never makes his own position explicit through dramatisation, and may often be holding back.¹⁰

However, there is considerable evidence that events in Plato's life did influence his philosophy, and caused a radical departure from the thought of his teacher Socrates. We can also see that new techniques are introduced at different stages in the corpus, and it has been convincingly argued that at least some of these find parallels in the intellectual climate to

⁹ At *Phaedrus* 274-276 and *Seventh Letter* 344 (if it is genuine) Plato is cautious about the written form, noting that it lacks the ability of speech to respond to ideas as they are proposed.

¹⁰ For example, can we see a suggestion of the tripartite soul in *Phaedo* 81d-84a, or *Symposium* 207-209? These dialogues seem to concentrate on the division between body and soul. Certainly there is a division of the appetitive, the timocratic and the philosophic, but until the explicit exposition in *Phaedrus* and *Republic*, it is difficult to say exactly when the idea of the divided soul was formulated.

which Plato was exposed.¹¹ This is especially evident in Plato's first visit to Sicily. Cicero tells us that, here, Plato became intimate with Archytas the mathematician-statesman, and learned about mathematics and the transmigration of the soul from the Pythagoreans. Because he loved Socrates, he interwove Socratic and Pythagorean ideals in the dialogues (Cicero, *De Republica* I.X.16). We are told that the historical Socrates discouraged the study of mathematics that has no practical application (Xenophon, *Memorabilia* 4.7, 6-10), whereas Plato sees it as essential part of the training for his philosopher-rulers (*Republic* VII).

Bluck argues that *Meno* was written soon after Plato's return from Sicily, on the grounds that it uses Pythagorean beliefs and adapts these ideas (Bluck [1961] pp.111-116; cf. Vlastos [1991] pp. 107-131, who makes a similar point). With that in mind, we can see evidence of Plato trying to import the methods of his exciting new studies into his dialogues. Vlastos argues convincingly for the importance of Plato's mathematical studies in his departure from Socratic thought. He notes that, in *Gorgias*, Plato makes Socrates confident that the elenctic method is the final arbiter of truth, whereas after this, he seems to lose faith in it. Vlastos argues that Plato's studies of mathematics caused the change:

That Plato's encounter with geometry was to prove no passing infatuation, but a love-match, a life-long attachment as deep as it was intense, is not hard to understand. We know how susceptible he was to beauty. Is any product of the human imagination more beautiful than are some of the proofs in Euclid? The elenchus is a messy business by comparison (Vlastos [1991] p.130).

¹¹ For example, by Karasmanis (1987): see especially pp. 286-308.

This is a highly credible account of the development in Plato's thought.¹² Therefore, while remaining cautious about the development of *doctrine* in Plato, this study accepts the traditional account of development in *method*, as outlined below.

The Socratic elenchus is introduced in the early dialogues as a method of testing for falsehood by refutation. Usually, a question is put forward about the definition of an ethical term – in *Euthyphro*, for example, the question is, what is holiness? (5d). The interlocutor is instructed to say only what he believes. He gives his primary answer ('what is agreeable to the gods is holy, and what is not agreeable is unholy' - 7a) and Socrates, feigning ignorance, asks a series of questions, to which the natural answer is 'yes' (for example, in *Euthyphro*, Socrates asks whether, if the gods dispute with one another, these disputes are over what is just and unjust, fine and despicable or good and bad - 7be). This leads to the identification of a flaw in the initial definition ('Then the same things would be both holy and unholy according to this account' - 8a). The elenchus can be seen as a form of moral education¹³ - once the false opinion is removed, the interlocutor is in a better position than when he started, now that he knows that he does not know (cf. *Meno* 84ab, which makes a similar point). The early 'Socratic' dialogues tend to end in *aporia* – a state of puzzlement where the initial problem remains unsolved.

Although recent scholarship has done much to overturn the traditional view that the historical Socrates did not hold any positive views (Vlastos [1991] pp107-131; Frede [1998] pp 254-269), *elenchus* works primarily by eliminating falsehoods, rather than generating truths.

In the middle dialogues, we see the introduction of the hypothetical method, which aims to prove, rather than disprove. The method involves the setting down of a hypothesis

¹² Note that even writers like Kahn accept that there is such a development in methodology. On this point, see Kahn (1998) Ch. 10 pp 292-328.

¹³ Robinson (1953) pp 7-19. For a discussion of presocratic use of elenchus as a method of establishing truth by eliminating falsehood, and its early links with morality, see Furley (1989) pp. 1-12.

and using it in two ways. Firstly, there is the ‘downwards’ path, from the premiss to the conclusion and secondly, there is an ‘upwards’ path towards the premiss or prior questions. The method is substantially different from the *elenchus* in that it requires debating an unasserted premiss, whereas the *elenchus* requires that the interlocutor says only what he believes (Vlastos [1991] Ch. 4 pp. 107-131). It is discussed in *Meno*, *Phaedo* and *Republic*, and although there are considerable differences between these passages, it has been convincingly argued that we can speak of these as different aspects of the same method (Karasmanis [1987] pp. 303-307; Kahn [1998] Ch. 10 pp. 292-328). We shall outline these passages in the following section.

Later, the hypothetical method is replaced by the method of ‘collection and division’, or ‘synthesis and analysis,’¹⁴ a method that involves the ‘collection’ of things that share some particular similarity and then division of these things in their subcategories according to their differences. For example, at *Phaedrus* 265-270, Socrates ‘collects’ a range of behaviours characterised as ‘madness’ then distinguishes between the different kinds of madness, according to their variations. We also see applications of this method in *Sophist*, *Statesman*, and *Philebus*.¹⁵ It has been argued that this method is closer to the hypothetical method than has previously been supposed, which is worth noting, although such a question lies outside the scope of the current study.¹⁶

The Hypothetical Method and its Application in Plato

We shall now outline the passages that speak about the hypothetical method in *Meno*, *Phaedo* and *Republic*, before going on to discuss possible applications of the method in the dialogues.

¹⁴ At least, the hypothetical method is not mentioned again after *Parmenides*; for this point, see Kahn (1998) Ch 10 pp. 292-328.

¹⁵ See especially, *Sophist* 232b-264b and *Philebus* 16-17; on the same point, see Matthews (1972) pp. 39-40.

¹⁶ Sayre ([1969] Ch. 4, pp. 216-238): Sayre argues that these methods are close in the goal of achieving knowledge in the form of definitions.

The hypothetical method is introduced in *Meno* 86dff. In order to answer a question about virtue, Socrates insists that we must first be clear about what virtue is and decides to investigate by the method of hypothesis. The question is whether or not virtue can be taught, so the hypothesis is put forward that virtue is a kind of knowledge. Socrates considers arguments both for and against this position. This form has been compared with the hypothetical syllogism,¹⁷ with the hypothesis being put forward, and arguments being found both for and against it. In the *Meno* section, I shall be arguing that the argument against the hypothesis is not a part of the hypothetical method, but that it is a separate, empirical argument.

The next explicit statement of the hypothetical method we see is in *Phaedo* 99bff. Socrates is about to reply to Cebes' objection to the immortality of the soul by presenting his theory of causation, but first explains his frustration with pre-Socratic explanations. He has previously described his disappointment with the failure of Anaxagoras to keep his promise to explain causation in terms of intelligent design. He does not feel able to produce the best kind of explanation himself, so he goes on to give an account of a *deuteros plous*, a second voyage or a second-best method, like taking up the oars in a boat once the wind has failed (according to Bostock [1986] p.157; originally from the poet Menander Fr 241). The metaphor here does not imply that the philosopher will not eventually arrive at the destination, but Socrates does feel that he has had to resort to a more laborious method in the absence of the account he had hoped for.

Socrates says that he first lays down¹⁸ the theory that seems to be the least vulnerable, and assumes to be true whatever agrees with it, assuming anything else to be untrue. In this passage, he uses the hypothetical method to formulate his theory of causation. He assumes the existence of 'Beauty in itself and Goodness and Largeness and all the rest of them'

¹⁷ Robinson (1953) p. 119; on this point, see also Aristotle, *Prior Analytics* 50a.

¹⁸ ὑποθέμενος, the verb related to the noun hypothesis.

(101b). So if someone asked Socrates why something is beautiful, he would disregard explanations that refer to its colour or shape or other attributes, and clings simply to the explanation that it is by Beauty that beautiful things are beautiful.

If anyone should question the hypothesis itself, Socrates would not answer until he had considered whether the consequences were mutually consistent or not. Then he would substantiate the hypothesis by assuming a ‘higher’ hypothesis until he reached a satisfactory one – he would not ‘mix the two things together by discussing both the starting point and its consequences, like one of these masters of contradictions – that is, if you wanted to discover any part of the truth’ (101e).

The next appearance of the hypothetical method is in *Republic VI*. Socrates has been explaining his ‘divided line’ allegory, which is introduced after the allegory of the sun. The latter has given the interlocutors a concept of two things: those that can be seen and those that can be understood (509d). We are told to imagine a line divided into two unequal parts, with one section representing the class of visible things, and the other one representing the intelligible. Each section is divided again into two subsections in the same proportion: the first subsection is a state of mind that uses images, the second is a state of mind that uses sensible things, the third is a state of mind that uses mathematical thought, and the final subsection is a state of mind that uses the forms. These states of mind are respectively: *eikasia*, or conjecture, *pistis*, or belief, *dianoia*, or mathematical reasoning and *noēsis*.¹⁹

In the description of dianoetic, or mathematical reasoning, we are told that the mathematicians use hypotheses, but are unable to go further than these. This kind of inquiry

¹⁹ The exact translation of these terms is highly controversial, and will be discussed in a later chapter. For now, we leave *noēsis* untranslated and the translations of the other terms remain highly provisional. Some translators translate *dianoia* as ‘understanding’ and *noēsis* as ‘intelligence.’ *Noēsis* itself is often translated as ‘understanding,’ but due to recent academic discussions over this word, we should prefer to avoid it. The divisions between contemporary epistemologists and philosophers of science over whether there exists such a thing called ‘understanding’ distinct from knowledge has been echoed in Plato scholarship and lies beyond the scope of the current project. For an example of the view in philosophy of science that the two are not distinct, see Lipton (2004) p. 30; for the view in epistemology that the two are distinct, see Kvanvig (2003) Ch. 8 pp. 185-204 and Pritchard (2007, SEP). For the corresponding debate in ancient philosophy, see Fine (1990) pp.85-115 and Burnyeat (1981) 97-139.

uses as images those things which are themselves the originals of the images in the lowest subsection (511a). In the highest subsection, these hypotheses are treated not as first principles but literally as hypotheses ‘like stepping stones and jumping-off places’ (501b). In this way, the dialectician proceeds to an unhypothesised first principle. Then he may investigate what follows from it to arrive at a conclusion ‘not using anything perceptible at all but proceeding by means of forms themselves, through forms. And it ends with forms’ (501c).²⁰

I shall argue that, although the dialectician improves upon the mathematician, dianoetic reasoning is not a completely separate endeavour. In fact, *dianoia* actually forms the initial stage of the hypothetical method, and is part of Plato's use of the mathematics of his day in philosophy.

What is being taken over from mathematics, I shall suggest, falls into two categories: tools and methodology. *Dianoia*, as Plato explicitly states at *Republic* 511, uses hypothesis and imagery, which are the tools of the mathematician. The 'higher hypothesis' is Plato's own invention, and it would be difficult to find an equivalent in geometry. We shall explore this concept in the *Phaedo* section: I shall describe it as Plato's way of ascending the epistemological scale, but it is not something that Plato has taken from the mathematicians of his day. I shall also argue that Plato adopts part of the mathematical method itself. As I shall argue in the *Meno* section, Plato adopts a method of *apagōgē* (as opposed to analysis and synthesis) as his model of mathematical reasoning.

The implications of the 'hypothetical' passages are very much disputed, but they are commonly cited as the most explicit expositions of the hypothetical method we find in Plato

²⁰ In this way, the dialectician improves upon the use of hypotheses of the mathematician. However, we should not take this to be a criticism of the discipline: mathematics plays a vital role in Platonic epistemology, and forms a major part of the education of the philosopher-rulers. As Burnyeat notes, ‘mathematics is not criticised but *placed*.’ (2000) p. 42. The virtue of mathematics according to Burnyeat’s view is not, in fact, its rigorous procedures, but its ability to tell us how the world is, objectively speaking.

(however infuriatingly little this tells us). There is another small passage in *Cratylus* which is thought to hint at the method. We shall return to this passage throughout the project:

It is just so sometimes in geometrical diagrams; the initial error is small and unnoticed, but all the numerous deductions are wrong, though consistent. Everyone must therefore give great care and great attention to the beginning of any undertaking, to see whether his foundation is right or not. If that has been considered with proper care, everything else will follow (*Cratylus*, 436cd).

Various arguments have been put forward about when and how Plato applied the method. Robinson thinks that ‘the hypothetical method is less used in the dialogues than it is abstractly discussed’ (Robinson [1953] p. 209). He does acknowledge that it is immediately practised after its introduction in *Meno*, and says that it is put to use more thoroughly in *Phaedo* than in any other dialogue – although he thinks that this dialogue does not illustrate all of the extensions of the method.²¹ He thinks that much of *Republic* is in fact alien to the hypothetical method. Finally, he concedes that we may interpret chains of elenchus in *Cratylus*, *Parmenides* and *Theaetetus* as leading to the intuition of a beginning, which Robinson identifies as the procedure of the ‘upward path’ (we shall dispute this interpretation). However, he concludes that there is little exemplification of the method on the whole in the middle dialogues or elsewhere in Plato.

Karasmanis ([1987] pp. 119-183) disagrees with the traditional interpretation that the hypothetical method is limited in its application in the *Phaedo*: indeed, he thinks that if we take the dialogue as a whole, we can see all extensions of the method exhibited. He agrees with other scholars that the first part of the method, hypothesizing a proposition and positing

²¹ In particular, Robinson argues that we do not see an example of the appeal to a second, or if needs be a third higher hypothesis, until an adequate hypothesis is reached in *Phaedo* (1953) p 203.

as true whatever agrees with it, and false whatever disagrees with it, can be found in 100b-107a, as well as other parts of the dialogue. As Bailey ([2005] pp. 95-115) shows, there is a difficulty here in understanding what Socrates means by this. What Socrates says seems to require something more than consistency but less than entailment.²² Karasmanis says that the second part of the method, examining whether any contradiction arises out of the hypothesis, is seen in the refutation of Simmias' 'attunement' theory.

Karasmanis challenges the view that the third part of the method, hypothesising a 'higher' hypothesis in order to give an account of a lower one, is not exhibited in *Phaedo*. Instead, he argues that all three parts of the method applied throughout the whole dialogue, especially the first and third parts. He does this by tracing the application of the method throughout the development of the dialogue, and consideration of linguistic analysis of the relevant passages.

Matthews ([1972] pp. 38-39) thinks that some of the clearest examples of the use of the method can be seen in *Theaetetus*, although this dialogue gives us little in terms of actual discussion of the method. She also thinks that *Parmenides*, in addition to giving us a demonstration of the use of hypotheses, makes further points concerning the method, notably the necessity of considering the contradictory of the hypothesis and its consequences. It also explores the possibility that absurd consequences can result from acceptance of both the hypothesis and its contradictory. This observation has led some²³ to suggest that this observation makes Plato abandon the hypothetical method altogether after *Parmenides*.

²² Bailey ([2005] pp. 95-115, especially pp. 95-98) shows that, if we understand Socrates' remarks about agreement and disagreement to mean 'be consistent with,' we run into problems. He says, "but for any hypothesis, there will be an infinite number of propositions which are consistent with it, but which we have no independent reason to assert. Surely we are not to set these down as true simply because they bear a relation as weak as consistency to the hypothesis. It sounds ludicrous – and of course it results in disaster. For among this infinite number of propositions there will be pairs of propositions inconsistent with one another" (p.97). However, if we take Socrates to mean entailment, this also has problematic results, Bailey says, as 'it is also ludicrous to put down as false all the propositions which are not entailed by the hypothesis' (p.98). I examine Bailey's solution to the problem in Section Three, Chapter One iv d.

²³ Notably Robinson (1953) Ch. 13 pp. 223-280.

Imagery and Hypotheses

We should note that the hypothetical method is not identical to dianoetic reasoning, as the former extends into dialectic. In the divided line passage, Plato is explicit about the fact that the dialectician should go beyond the mathematician in two ways. Firstly, he should treat the hypotheses as statements still needing confirmation, not absolute beginnings, and secondly, he should not rely on the evidence of the senses, in the reference to images.

This study will argue that the dialectician's improvement on *dianoia* does not exclude dianoetic processes from the initial stages of the hypothetical method. That is, although he is expected to question the initial hypothesis and abandon the use of imagery in the final stages, the initial stage does not require such a step. This will involve close inspection of *Republic* 511, and a difficult passage in *Phaedo*, 99e-100a, which speaks ambiguously about the use of images in theory.

In the divided line passage, we see that *dianoetic* reasoning has two distinctive qualities: the use of hypotheses and the reliance on imagery. In the debate about the significance of each level of the Line for Plato's epistemology as a whole, various suggestions have been made about the connection between the two features. Plato is not specific about how the relationship between the use of diagrams and the use of hypotheses is any more than a historical fact, yet he seems to want to say that the connection is a philosophical necessity, and comes from the nature of *dianoia* itself.²⁴ This project aims to explain the connection as philosophically important.

The most prominent suggestion as to the philosophical weight of the connection between hypotheses and imagery in dianoetic reasoning comes from Robinson (1953 pp. 152-156), who acknowledges that Plato is not explicit about the existence of a philosophical

²⁴ See also (Burnyeat [2000] pp. 1-81) for this view.

connection between the two marks. He points out that Plato does wish to present thought [*sc. dianoia*] as a distinct type of reasoning: “If these two marks were connected only historically, ‘thought’ would not be a real species of mental activity, but a conjunction of two real species” (ibid p. 155).

Robinson suggests that the connection lies in the need for mathematicians to appeal to spatial intuition to support their postulates. Due to the fact that the mathematician’s hypotheses are assumptions, not certainties, they use diagrams which appear to support them. In this way, they fail to recognise that their hypotheses are assumptions, and do not question them, or see the need to give an account of them. Robinson’s suggestion is grounded in his wider conception of the importance of intuition in Plato’s philosophy: for example, he prefers the ‘intuition theory’ of the upward path and makes use of the ‘divine spark’ passage of the Seventh Letter.

This explanation is supported by Cross and Woosley ([1986] pp. 244-246) who add that Greek mathematics of Plato’s time aspired to be an idealised description of the spatial world, and say that Euclid, writing later, upheld postulates that had their roots in common experience.²⁵ Cross and Woosley try to support this by giving an account of *dianoia* as a parallel mental state to *eikasia*, the lowest level of the divided line. They think that people in the state of *eikasia* take the image for an original, not realising that it is a copy. Mathematicians, however, in the state of *dianoia*, take hypotheses as truths, without recognising them as assumptions. In this way, for Cross and Woosley, both *eikasia* and *dianoia* are dogmatic, given to taking sense experience as truth.

The weakness of the Robinson/Cross/Woosley view is that it does not take into account the way in which Plato employs hypotheses and images in the dialogues. Robinson has a whole chapter devoted to explaining away the use of analogy and imagery, and has to

²⁵ Here, Cross Woosley cite Blumenthal (1961) p. 16.

say that the dialogues are teaching devices, not models for how philosophy is to be done. Imagery is not good as a means of discovery, but good as a means of teaching. He uses evidence from *Phaedrus* and the *Seventh Letter* to support this view. However, Plato's methodology could be much more efficiently and informatively explained if the descriptions of the conversations in the dialogues were taken as illustrations of Platonic method.²⁶ There are several passages in the middle period where hypotheses and imagery interact, and these could be instrumental in shedding light on the description of dianoetic reasoning in the divided line passage.

This project will criticise the explanation mentioned above of the link between hypotheses and imagery, while retaining the idea that the relationship is philosophically useful (rather than historically factual). Using the critical passages from *Phaedo* and the *Meno* where the images are used along with hypotheses, this study will explain the link by showing that it is, like the hypothesis, a proxy for the *ti esti* - which I shall describe as a Platonic definition - that we do not have at this stage in the inquiry.

A *ti esti* is a Platonic definition: that is, a definition that has the attributes that Plato thinks are needed for high-level philosophy. Particularly in the early dialogues, Plato makes Socrates ask for the *ti esti*, or definition, of the thing to be investigated as a priority. As we shall see in the *Meno* section, a *ti esti* must both identify the thing to be investigated, and also be a heuristic tool. In the absence of it, a hypothesis or image can be used as proxy.

In this way, I shall argue, *dianoia* is a pivotal part of the philosopher's ascent to knowledge.²⁷ As we shall see, the idea of an epistemological ascent is found at *Republic* 511 and 517b, and also in the *Symposium* 201a-212c. In the latter, it is the perception of sensibles that

²⁶ Of course Plato could have chosen to write treatises; see my comments above. My point is not that Plato used the dialogue form because it was the best way to illustrate the Platonic method; rather, I mean that the images Plato uses can help us to understand how his arguments work.

²⁷ As opposed to Scott ([1995] pp. 15-52)a, who does not see the path to philosophy as an ascent from ordinary learning. Rather, Scott says that, for Plato, philosophical discoveries are startling revisions of ordinary ways of thinking.

leads us to the Forms: through the love of beautiful bodies, one can come to contemplate beauty itself, and become a lover of knowledge. What I suggest in this study is that the mathematical stage of the divided line acts as a way of getting from the perception of sensibles to the contemplation of Forms. We need a *ti esti* to reason about the Forms, but, when only perceiving sensibles, we do not have one of these. The tools of *dianoia* provide a proxy until a *ti esti* can be attained.

We shall explore the conception of imagery as a part of a thought experiment, using the *aporetic* passages from the *Meno* to explain the psychological basis for Plato's theory. We shall see that the hypothetical method has a distinct psychological element; indeed, it needs this to prevent it from slipping into eristic and to go beyond the *elenchus*. In this way, traditional Socratic concepts like *aporia* are an important part of the method; it is a necessary condition for any research, and we shall see it manifest in different ways according to the nature of the project in hand.

This explanation has the advantage of explaining Platonic texts in terms of Plato's own theory. It also allows a theory-building role to the hypothetical method, and supports Plato's criticism of the reliance on *reductio ad absurdum* arguments of the sophists. Although 'proof by contradiction' has arguably been used successfully in mathematics by Plato's time, Plato takes the point that the opposition of contradicting concepts as an epistemological method can lead to fallacy: in *Euthydemus*, he makes Socrates point out that truth can lie outside of the opposites that frame the question. In giving cognitive weight to the image in dianoetic reasoning, Plato allows for the development of argument in a way that could not result from the argument from opposites. Although he certainly does not abandon the latter as a method, it is clear that he sees the need for an epistemological device that plays a more positive role in philosophical debate.

We shall see that Robinson is right to stress the role of intuition in the upward path,²⁸ but he ignores the role of the interlocutor in the dialogues. As recent scholarship suggests,²⁹ the interlocutor does not always contradict Socrates with a direct opposite, but sometimes indicates the weakness in his theory in other ways. This study will argue that the image plays an important part in theory-building in the early stages, and aims to illustrate this using the ‘harmony’ passage in *Phaedo*.

This conception will allow us to build upon scholarship that identifies the application of the hypothetical method in *Phaedo* as a unified whole, outlined above. We shall see the vital role played by imagery especially in the passage where the initial stage of the hypothetical method is displayed. We shall see that, at this stage, the image plays a theory-building role in *dianoia* that enables the transition to the later stages. In this way, we can explain the link between hypotheses and images implied by *Republic VI* in a way that is consistent with his use of them in *Phaedo*.

The image and the hypothesis are philosophically connected by their shared provisional nature, and the sense in which they are both ‘copies’ in some way. However, far from being negative, as some scholarship suggests, this provisional nature is necessary, and, when it follows the psychological state of *aporia*, allows the argument to advance rather than jeopardising its integrity. They are provisional, because they are to be replaced by *ti estis* as the thinker acquires them. For this reason, the two actually complement each other, with the image often prompting new avenues of enquiry that the hypothesis in itself cannot.

In the first section of this study, we shall examine the allegories of the sun, divided line and cave of the *Republic*, focusing particularly on the *dianoia* passage of the divided line. After introducing the allegories, I shall discuss them in the context of the wider

²⁸ See the ‘divine spark’ passage of the *Seventh Letter* 340: “If the hearer has the divine spark which makes philosophy congenial to him and fits him for its pursuit, the way described to him appears so wonderful that he must follow it with all his might if life is to be worth living.”

²⁹ Alex Long (2013) especially pp. 67-74; also Beversluis (2000) pp. 18-57.

epistemology of the *Republic* as a whole, with a particular focus on Gail Fine's controversial interpretation, and the response of her critics. I shall then go on to give my own reading of the allegories, arguing that we can see the divided line as an epistemological scale, to be ascended by the philosopher, with *noēsis* being the ultimate goal, but *dianoia* a necessary step towards that goal. Not only is it a scale, I shall say, but it is a *continuum*, with the later stages of *dianoia* being progressively closer to *noēsis*. The tools of *noēsis*, I shall say, are *ti estis*, or Platonic definitions, which, I shall argue, have a particular set of requirements that Plato indicates in the *Meno* and other dialogues. In the absence of these, *dianoia* uses hypothesis and imagery to make progress.

I shall then go on to take a closer look at *dianoia*, arguing that the dianoetic image is different from the eikasiastic image, which is how we normally think about images. The dianoetic image, I shall argue, is an intelligible instantiation of a Form: it may have a visible part, perhaps in the diagram that is drawn, but its properties are most fully contained in the mind of the thinker. This means that Robinson's solution, that the image provides the mathematician with the visual appeal to his intuition that he needs, is redundant. Of course, there is an element of 'seeing' that something is the case if we can see it represented visually, but this does not do anything like the work that Plato's dianoetic image needs to do. I shall argue for this point throughout my readings of the *Meno*, *Phaedo* and *Republic*.

After examining the dianoetic image, I shall then go on to examine the hypothesis, which of course extends from *dianoia* into *noēsis*. I shall argue that there is a role for intuition in the 'upward path' as Robinson calls it, but that this is far more limited than Robinson would have it. Finally in the *Republic* section, I shall briefly discuss how hypothesis and imagery might work together in *dianoia*, by looking at the *Theaetetus* as a case study.

My next section will be on the *Meno*. I shall begin by looking at what the dialogue can tell us about Platonic definitions, or *ti estis*, because it is these that I am saying the hypotheses and images substitute in *dianoia*. I shall consider the idea that Plato is looking for a series of closer approximations to the right answer, and that the *ti esti* is not necessarily going to be a concise answer; but nor does it have to be an exhaustive description of a Form. I shall go on to examine the *aporia* of the middle part of the *Meno* as preparation for a new method, before examining the hypothetical passage as Plato's modelling philosophy on mathematical method. I shall recognise that there are problems with many attempts to do this, because usually, it is assumed that the method Plato has in mind is analysis and synthesis, which does not fit with the text. By looking to a different method, and seeing the end of the passage as being a little earlier than is usually supposed, we can see a much closer fit between mathematics and philosophy.

The hypothetical passage and the slave-boy experiment in the *Meno*, I shall argue, are examples of the exercise of *dianoia*. In my *Phaedo* section, I shall argue that the objections of Simmias and Cebes, which also use hypothesis and imagery, are also examples of the exercise of *dianoia*. I shall then go on to say that Socrates' 'intellectual autobiography,' which follows the objections, takes us through an ascent of part of the epistemological scale described in the divided line passage of the *Republic*. The hypothetical passage, which comes straight after this, occupies the section on the scale which takes us from *dianoia* to *noēsis*. In my discussion of this passage, I shall consolidate what I said in the *Republic* section about the 'upwards' path, arguing that the hypotheses can be any part of the eventual *ti esti*; there are no particular requirements for what kind of statement a hypothesis has to be. This, I shall say, is because of the nature of the epistemological ascent. Because a *ti esti* will include the 'account' generated by the hypothetical process, there is no one kind of statement we need for our hypothesis.

The main body of my study will focus on the texts of the relevant passages from the *Meno*, *Phaedo* and *Republic*. My reading will be based on what Plato explicitly says about importing the mathematical method into philosophy, as well as what he actually does when Socrates and his friends try to apply it. However, a study like this needs to appeal to the history of mathematics to support the claims being made, so I shall do this at various points. For example, when following Karasmanis in arguing that the method of *apagōgē* is used in the *Meno* and *Phaedo*, rather than that of analysis and synthesis, I shall appeal to the history of geometric methods. Finally, in the appendix, I suggest some possible avenues of future research: for example, an alternative reading to the view that the role of the diagram in geometry was to appeal to spatial intuition, by looking at the avoidance of completed infinities in Greek mathematics. On this reading, there is no need to split off so much of what Plato says from what he does.

Section One: The *Republic*

According to Plato, we live our day-to-day lives in a substitute world. The things we see around us are shadows of reality, imperfect imitations of perfect originals. Beyond the world of the senses, there is another, changeless world, more real and more beautiful than our own. But how can we get at this world, or even attain knowledge of it, when our senses are useless and the perfect philosophical method remains out of reach? In the divided line passage of the *Republic*, Plato is clear that mathematics has a role to play. In passages in the *Meno* and *Phaedo*, Socrates is forced to adopt a ‘second-best’ mathematical alternative to pure philosophy. The role of mathematics, according to Plato, is pivotal in the philosopher’s initial journey into the world of Forms, but the debate about exactly what that role is remains unresolved.

In this section, I suggest that my reading of the divided line might provide the answer, in addition to solving the problem of the roles of hypothesis and imagery in *dianoia*. This is the central problem that this project aims to solve, set out in the divided line passage of the *Republic*. The line is one of three allegories that Plato makes Socrates give to explain his theory of Forms. It is also, along with the allegories of the cave and the sun, one of the most beautiful and radical innovations in the history of philosophy. Notoriously ambiguous, the allegories lend themselves to a vast range of often contradictory interpretations, so a consensus upon Plato's exact meaning remains elusive.³⁰

Essentially, this study asks an epistemological question: what is the connection between hypothesis and imagery in mathematical reasoning as conceived by Plato, and what are their respective roles? However, the solution to this problem needs to connect this kind of reasoning to truth, as this is the aim of knowledge, so we need to take account of Plato's metaphysical system. The connection between Plato's metaphysics and his epistemology is so intimate that some have claimed that there is no divide between the two.³¹ Plato's metaphysical and epistemological claims are mutually reinforcing, and mutually dependent. The highest kind of knowledge must be knowledge of Forms. In order to know about Justice, we must not confine ourselves to the examination of just acts, but we must study the Form of Justice itself.

However, in dianoetic reasoning, the Forms are not directly examined, but *dianoia* is still conceived by Plato as a kind of knowledge, or at least the foundations of knowledge. For our solution to work, the image and hypothesis must be in some sense veridical. What makes them veridical? The answer, I shall argue, is to be found in the metaphysics of the divided line, and supported by a connected reading of certain passages of the *Phaedo*. I am not

³⁰ See Annas (1981) pp. 245- 246 for allegations of Plato's mysticism. Annas, claiming that Plato does not make clear how the Good makes things known in the way that the sun makes things known to the eye, writes: 'Plato's Good, which he refuses to clarify here, becomes a byword for obscurity' p 246.

³¹ For example, see White (1992) pp.277-310.

claiming that any image or hypothesis is veridical, only those that are successfully chosen. As we shall see in the discussion of Simmias' objection, some images are not veridical: as an objection to Socrates' theory (*Phaedo* 86ad; 87a-88b), Simmias proposes that the soul could be like an attunement of a musical instrument, which is also invisible, but which ceases to exist once the instrument is broken (*Phaedo* 73d8-9). Socrates is able to evaluate the fit of the attunement hypothesis as a model for the soul, concluding that 'there is no justification for our saying that the soul is a kind of attunement' (94e-95a). Although some images are not veridical, I argue those that are veridical are connected to the Forms.

One of the things I want to say about the metaphysics of the line is that the objects associated with *dianoia* are not just mathematical objects. I cover this in Chapter Three ii c of this section, in which I argue that *dianoia* can be used to reason about moral truths as much as mathematical ones. That is what Plato is doing in the passages of the *Meno* and *Phaedo* that we shall cover in the following sections.

Chapter One: Introducing the Sun, Line and Cave

Grounded in the preceding discussion about knowledge and belief in book V of the *Republic*, the allegories of the sun, line and cave provide the context for the main question of this study. In this chapter, I shall introduce the allegories, picking out the relevant points for discussion in later chapters. The traditional reading of the allegories takes them to be an illustration of Plato's Two Worlds theory, both metaphysically and epistemologically. By this, I mean that the traditional reading takes Forms to exist in an intelligible realm separate from our own (ie, as *universalia ante rem*, universals independent of things, as opposed to *universalia in rebus*, universals in things).³² In addition, this reading seems to suggest that rejection of true belief

³² See Price ([2001] pp. 20-41) for a good discussion of the distinction.

as a basis for knowledge, and the two worlds theory, are mutually reinforcing. The traditional ‘objects analysis’ sees the objects of knowledge and belief as distinct. Knowledge is knowledge of Forms; belief is belief about sensibles.

In addition to outlining this reading and pointing out the extent to which I think Plato is committing to it, I shall also pick out some other issues to which I shall return in later chapters. Namely, I wish to highlight the fact that Socrates feels that his own account is inadequate, and that there is a better way of giving an account that lies beyond his current resources; I shall return to this in the *Phaedo* section. In addition, I want to introduce the idea of the divided line as an epistemological scale, to which I shall return in the chapter explaining my own reading of the allegories. I also want to point out the shift I see in the way that Plato explains the four states of mind of the divided line when he comes to *dianoia*, which needs to be taken into account when assessing the traditional ‘objects’ reading of the allegories.

When Plato introduces the allegories of the sun, divided line and cave, Socrates is describing the qualities that a good philosopher must have, and the education that will produce philosophers with these qualities. The philosopher ‘must work as hard at his intellectual training as his physical’ (504cd), in order to reach the highest form of knowledge. At this point, Adeimantus is surprised that Socrates should think that there is anything higher than justice and the other qualities they discussed, and says that Socrates cannot ‘escape cross-questioning about what you call the highest form of knowledge and its object’ (504e).

Socrates says, ‘the highest form of knowledge is knowledge of the form of the good, from which things that are just and so on derive their usefulness and value...the good is the end of all endeavour’ (505ad). However, Socrates thinks that a satisfactory explanation of the

good is beyond the reach of their present inquiry: instead, he tells the group about something that he calls ‘a child of the good’ (506e).³³

First, Socrates reminds the group that they have agreed that there are particular things, as well as ‘things-in-themselves’ (Forms). The particulars are objects of sight but not of intelligence; the Forms are objects of intelligence but not of sight (*Republic* 507ac). However, sight and the visible need a third element to enable us to see: light, from the sun. The sun is neither identical with sight, nor with the eye, but it is the cause of sight, and is seen by the sight it causes. This is what Socrates calls the child of the good:

...the good has begotten it in its own likeness, and it bears the same relation to sight and visible objects in the visible realm that the good bears to intelligence and intelligible objects in the intelligible realm (508bc).

Now, Socrates says, apply the analogy to the mind:

When the mind’s eye is fixed on objects illuminated by truth and reality, it understands and knows them, and its possession of intelligence is evident; but when it is fixed on the twilight world of change and decay, it can only form opinions, its vision is confused and opinions shifting, and it seems to lack intelligence...then what gives the objects of knowledge their truth and the knower’s mind the power of knowing is the form of the good (508de).

The form of the good enables knowledge and truth to be, but is not identical with them, in fact it is more splendid than them (*Republic* 508e-509a). Pursuing the analogy further, Socrates points out that the sun not only makes the objects of sight visible, but

³³ See also 507a, where Plato also uses this phrase.

‘causes the process of generation, growth and nourishment, without itself being such a process’ (509b). So:

The good therefore must be said to be the source not only of the intelligibility of the objects of knowledge, but also of their being and reality; yet it is not itself that reality, but is beyond it, and superior to it in dignity and power (509b).

Socrates says that he has not nearly finished his account, and will have to leave a lot out. We shall go on to discuss the implications of Socrates’ analogy in the following section, but for now we should note that Socrates has hinted at an account of the process of generation that has as its root the form of the good: this is not the ‘secondary account’ that results from the ‘second best method’ of the *Phaedo* (see the following chapters), but nor is it one that Socrates feels able to give at this point. He does seem to want to say here that the form of the good is the source of knowledge and reality; the highest kind of knowledge must be in some sense connected to it, even though he does not feel able to do this himself at this point.

For now, Socrates decides to do his best to fill in everything he can about his account under the present circumstances. This is where he introduces his divided line analogy (*Republic* 509d-511e), which is what I want to argue is Plato’s epistemological scale. It is also Plato’s connection of the levels of his metaphysics to the stages of his epistemology. He begins from the point of the ‘two powers’ - that is, the form of the good and the sun - of which they have spoken (*Republic* 509d). The form of the good is supreme over the intelligible realm; the sun over the visible.

Then Socrates asks the group to suppose that there is a line divided into two unequal parts, which are divided again into two unequal parts in the same ratio. The upper two parts represent the intelligible realm and the lower two parts represent the visible realm. The lowest subsection of the line stands for ‘images,’ by which Socrates means ‘shadows then

reflections in the water and other close-grained, polished surfaces, and all that sort of thing' (510a). The next subsection of the line, the upper of the two 'visible' sections, stands for the objects that are the originals of the images: 'the animals around us, and every kind of plant and manufactured object' (510a). Socrates says that these subsections differ in that one is genuine and one is not, and the relation of the image to the original is the same as that of the realm of opinion to that of knowledge (*Republic* 510a).

As for the intelligible part of the line:

...in one subsection the mind uses the originals of the visible order in their turn as images, and has to base its inquiries on assumptions and proceed from them not to a first principle but to a conclusion: in the other, it moves from assumption to a first principle which involves no assumption, without the images used in the other subsection, but pursuing its inquiry solely by and through forms themselves (510b).

This is by far the most important passage for the present study, and it is one that we shall return to both in this and the following sections. For now I want to note a shift in the way that Plato is describing the parts of the line (at least if we take the traditional 'objects' reading of the line that I am going to describe below). For the first two subsections of the line, Plato, according to the traditional reading, has been listing the objects of inquiry: images and physical things. For the next two sections, he seems to be speaking about the process by which the mind inquires, that is, what the soul does, and the objects that the mind uses to inquire, not the objects of inquiry themselves.³⁴ For now, all I want to note is that, although the 'objects' reading of the line is not incompatible with what Plato says here, it is certainly not something he is committing to at this point.

³⁴ By comparing the upper and lower divisions of the line, we should be able to get a better idea of what goes on in the upper division, as this is the part that is most difficult to describe.

What Socrates says next serves to highlight the fact that he is talking about the tools of inquiry, rather than the objects of it. To clarify his theory, Socrates gives the following example:

I think you know the students of geometry and calculation and the like begin by assuming there are odd and even numbers, geometrical figures and the three forms of angle, and other kindred items in their respective subjects; these they regard as known, having put them forward as basic assumptions³⁵ which it is quite unnecessary to explain to themselves or anyone else on the grounds that they are obvious to everyone. Starting from them, they proceed through a series of consistent steps to the conclusion which they set out to find (510cd).

He goes on:

You know too that they make use of and argue about visible figures (*eidos*) though they are not really thinking about them, but about the originals which they resemble; it is not about the square or diagonal which they have drawn that they are arguing, but about the square itself or diagonal itself, or whatever the figure may be. The actual figures they draw or model, which themselves cast their shadows and reflections in water – these they treat as images only, the real objects of their investigation being invisible except to the eye of reason (510d-511a).³⁶

In this passage, Plato distinguishes between the objects of investigation and the tools that the mind uses in this kind of inquiry: images and hypotheses. He says, ‘this type of thing

³⁵ ὑποθέμενοι, or hypothesizing.

³⁶ This is Lee’s translation, where ‘eye of reason’ translates τῆι διανοίᾳ. Grube (1974) translates this passage: “You also know that they use visible figures and talk about them, but they are not thinking about them but about the models of which these are likenesses; they are making their points about the square itself, the diameter itself, not about the diameter which they draw, and similarly with the others. These figures which they fashion and draw, of which shadows and reflections in the water are images, they now in turn use as images, in seeking to understand those others in themselves, which one cannot see except in thought.”

I called intelligible,' which by no means limits this kind of inquiry to geometry; it also happens in 'kindred sciences' ('*technai*,' or brother arts, in this case other branches of mathematics). As for the images it uses, these are 'the very things which in turn have their images and shadows on the lower level' (511a) – ie, the physical objects are in turn used as images in the lowest intelligible subsection of the divided line.

When Socrates goes on to elaborate on the other intelligible subsection, he focuses on the process of argument, not the objects of inquiry:

...it treats assumptions not as principles in the true sense, that is, as starting points and steps in the ascent to something which involves no assumption and is the first principle of everything; when it has grasped that principle it can again descend, by keeping to the consequences that follow from it, to a conclusion. The whole procedure involves nothing in the sensible world, but moves solely through forms to forms, and finishes with forms (511bc).

Again, this vital passage will be discussed in greater detail in the following chapters and sections, but the thing I want to note is that Plato has not said that Forms are the objects of the procedure, but that the procedure moves *through* Forms. I am not arguing that Forms are not the objects of this procedure – indeed, only Forms are mentioned, and this passage is compatible with the traditional reading. What I do want to stress is that Plato wishes us to consider here the tools used by the mind at least as much as, if not more than, the objects of inquiry.

The divided line passage finishes with a summary of Socrates' remarks, which also gives us some additional information about the states of the mind in each section. The upper part of the intelligible section of the line (he says) has more clarity than the lower. Although the latter uses reason rather than sense-perception, its use of assumption places it between opinion and intelligence. Socrates concludes that there are four states of mind, *pathēmata en*

tēi psuchēi, corresponding to each of the four sections of the line. The *episteme* part of the line is divided into *noesis* and *dianoia*, of which *noesis* is the bigger part. The *doxa* part of the line is divided into *pistis* and *eikasia*. The ratio *pistis:eikasia* is equal to that of *noesis:doxa*. This representation is based on Plato’s statement of the division at 511de.³⁷ In fact, Plato’s statement at 534a is slightly different. At this point:

<i>noēsis</i> covers the top two sections of the line, and includes:	<i>epistēmē</i>
	<i>dianoia</i>
<i>doxa</i> covers the bottom two sections of the line, and includes:	<i>pistis</i>
	<i>eikasia</i>

In this scheme, *epistēmē* is restricted to the top subdivision, and *noēsis* covers the top two sections of the line. *Dianoia* is, in both cases used for the mathematical subdivision. Plato is not consistent in his terminology, and in fact suggests that he is not keen to fix on rigid technical terms.³⁸ Foley ([2008] pp.1-24), who examines another controversy in Plato’s two statements of the divided line,³⁹ has a reflection that may also apply to this case. He says, “By knowingly giving a contradictory analogy for his epistemology, and perhaps his ontology as well, Plato reveals that the reader must ultimately be willing to devote herself to philosophical enterprise...Socrates’ demurrer at 534a – precisely the point that he should be wrapping up his comments about the line – takes on added meaning. The passage shows that

³⁷ For the point that 511d2 is an interpolation, see Slings (2005) pp.113-119. Slings thinks that *καίτοι νοητῶν ὄντων μετὰ ἀρχῆς* is an interpolation, because *καίτοι* as a modifying participle is hardly ever used by classical authors; to a lesser extent, he thinks that the use of the genitive is suspect. Slings also feels that the clause is too compact, and even that it is harsh Greek for such an important point. The significance is that it makes mathematical objects intelligibles in the wider sense of the term, making Plato seem more positive about mathematics than he would be without this clause. However, as Slings points out, the basic concepts of mathematics are *νοητά* anyway, and what the mathematicians have could ultimately be converted into knowledge.

³⁸ ‘...I don’t think we should quarrel about a word – the subject of our inquiry is too important for that...we shall be content to use any term provided it conveys the degree of clarity of a particular state of mind’ (*Republic* 533e). See Lee’s notes to his (1987) translation of the *Republic*, pp 249-251 and 283-284.

³⁹ The question on which Foley writes is whether the middle two subsections of the line are equal in length. The question of epistemological terminology is different, but Foley’s point about the possibility of Plato leaving their resolution to his readers can apply to both.

Plato is not willing to set forth his views on the further complexities that have emerged. It is a task that he intentionally leaves for his readers” (p. 23).

Throughout this thesis, I have chosen to use the terminology of the earlier version, for several reasons. Firstly, the earlier version is the one in which Plato’s focus is on the *pathēmata en tēi psuchēi*, which relates to the central question of my thesis: how does one ascend the mental states in Plato’s epistemology? The second reason I have chosen to use this terminology is due to the debate about knowledge in contemporary scholarship. For example, although Gail Fine ([1999] pp. 215-246) uses the word ‘understanding’ as a translation for *noēsis*,⁴⁰ she disagrees with Burnyeat’s ([1981] pp. 97-139) distinction between knowledge and understanding, which links explanation and understanding, but not explanation and knowledge. Instead, Fine prefers to think of ‘richer and deeper kinds of knowledge.’⁴¹ This debate is reflected in contemporary epistemology and philosophy of science over whether there exists such a thing called ‘understanding’ distinct from knowledge.⁴² However, this is a modern problem, and we should be wary of letting it affect our reading of Plato. I chose to use the terminology of the earlier version partly to avoid the problem of letting modern debates about epistemology create artificial distinctions in Plato’s thought.

Others may wish to adopt the terminology of the later version. If one were to argue that *dianoia* has the same scope as the sphere of knowledge and at the same time argue that *dianoia* falls short of *epistēmē*, the terminology of the statement at 534a is a good fit. This confines *epistēmē* to the top subdivision, with the result that *noēsis* covers the whole of the top section, although it continues to use *dianoia* for the mathematical subdivision. This version allows *dianoia* to be a form of *noēsis* while achieving something less than knowledge. However, my reading presents *dianoia* as the foundations of knowledge, but still

⁴⁰ Others translate *dianoia* as ‘understanding’ and *noēsis* as ‘intelligence.’

⁴¹ Gonzalez ([1996] p 258) has yet another perspective on this distinction, saying that ‘knowledge here is understanding and acquaintance, understanding achievable only in direct acquaintance with certain objects.’

⁴² As mentioned in the introduction to this thesis, Lipton (2004) p. 30 thinks that knowledge and understanding are not distinct, Kvanvig (2003) Ch. 8 pp. 185-204 and Pritchard (2007, SEP) believe that they are.

inferior to *noēsis*. We shall see that *noēsis* is less removed from truth (because of its tools, which I shall argue are the *ti estis* that the mind uses in this particular mental state) and more certain (because of the testing of hypotheses). As I shall argue in Section One, Chapter Three ii, Plato says that the mind uses different kinds of things to reason with (tools) in each section of the divided line: the tools of *noēsis* are *ti estis*, those of *dianoia* are hypotheses and dianoetic images. *Pistis* uses perception of physical objects and the tools of *eikasia* are images of physical things.

The earlier version of the line is the one in which Plato tells us about the tools the mind uses in each mental state and I argue that there is much value in looking at the question from this aspect. In particular, it will allow me to suggest a solution to the problem of how we can progress through the mental states (which, at *Republic* 532ab, Plato explicitly mentions). I propose this solution in Section One, Chapter Three ii and Four of this thesis. It also allows for the division between *dianoia* and *noēsis* that Plato specifies, because although hypotheses are used at both levels, the dianoetic image is restricted to *dianoia* and the *ti esti* is restricted to *noēsis*.

The reference to the allegories in the later statement at 534c and connection of the education of the philosophers to the line at 533e-534a show the correspondence between the two statements. With this in mind, I shall continue to refer to *noēsis* as the top subsection of the line, and *epistēmē* as the upper two subsections, on the understanding that Plato is inconsistent in his use of terminology elsewhere.

Finally, Socrates goes on to describe the allegory of the cave, which he says will illustrate the enlightenment or ignorance of the human condition. We are asked to imagine a cave, in which men have been imprisoned since they were children, with their heads fixed so they can only see the wall. Behind them is a fire, and between the fire and the prisoners runs a road, in front of which is a curtain wall. Men carry things beyond the curtain wall,

projecting figures of men and animals onto the wall that the prisoners can see. The prisoners believe that the shadows on the wall are the real things, until one is released from his bonds and able to look around the cave. At first, he is dazzled by the fire, and finds it difficult to see or regard as real the originals of the shadows he used to see. If he were dragged up the ascent to the world outside, he would be so dazzled by the sunlight that he would not be able to see any one of the things he was now told were real. He would have to become accustomed to the light, then begin by looking at shadows and reflections, then objects and the heavenly bodies at night. Finally, 'he would be able to look at the sun itself, and gaze at it without using reflections in water or any other medium, but as it is in itself' (516b).

The allegory has much to say about the psychology of learning and the pragmatics of politics that we do not have time to consider here. For now, I just want to mention Socrates' remarks on how the allegory of the cave is connected with the line and the sun:

The realm revealed by sight corresponds to the prison, and the light of the fire in the prison to the power of the sun. And you won't go wrong if you connect the ascent into the upper world and the sight of the objects there with the upward progress of the mind into the intelligible region (517b).

Mapping the stages of the prisoner's journey onto the divided line is not such an easy task as Socrates' remarks might suggest. Do the mathematical objects of the traditional reading of the divided line correspond to the reflections of the physical objects in the outside world of the cave analogy? This is one problem that we need to discuss, along with the question of how far the metaphor goes: can we take seriously the mathematical consequences of the ratios in the divided line? We shall also need to look more closely at what Plato has to say about *dianoia*, as he makes some remarks in Book Seven which can tell us more about

the mathematical sciences. First, though, we should examine the debate about how Plato's epistemology works, as it is described here.

Chapter Two: Readings of the Allegories in Context

The three allegories are neither complete nor freestanding expositions of Plato's epistemology, but grounded in the wider discussion of books V to VII of the *Republic* and further informed by Plato's remarks on education in book VII. Academic readings of the allegories tend to be built on a wider theory that tries to incorporate Plato's discussion of knowledge and belief in *Republic* V, and is supported by epistemological passages of other middle dialogues, such as the *Meno* and *Phaedo*. Notably, interpretations of the allegories need to be consistent with a convincing reading of what knowledge and belief are 'set over,' according to Plato's remarks in books V-VII. In this chapter, we shall discuss Gail Fine's challenge to the traditional reading, and the response of her critics, all of which attempt to produce such an interpretation. I want to make clear what I endorse about Fine's reading (her concerns about the impossibility of knowledge of sensibles), what I reject (her propositional reading and rejection of the Two Worlds metaphysics), what I would amend (her coherentism), and what problems my account shares with hers (my tendency to rationalize Plato).

i. Knowledge, Belief and Gail Fine

By far the most controversial article to have been published on this topic is Gail Fine's 'Knowledge and Belief in Republic V-VII.'⁴³ In it, she challenges the traditional 'Two

⁴³ (1999) pp. 215-246.

Worlds' reading of Plato 'according to which there is no knowledge of sensibles, but only of Forms, and no belief about Forms, but only about sensibles' (Fine [1999] p215). Fine means that she thinks the traditional interpretation of the divided line holds the objects of inquiry to be what we have labelled as the 'metaphysics' side of the line in the diagram of the previous chapter.

Fine's problems with the traditional reading of Plato's epistemology in his middle work are connected with her careful reading of the middle dialogues and a reinterpretation of what Aristotle can tell us about Plato.⁴⁴ She thinks that, if we say that for Plato the objects of knowledge and belief are disjoint, we also need to say that Plato radically rejects the *Meno*'s account of knowledge, in which true beliefs become knowledge when they are adequately bound to an explanatory account. She says, 'for the *Meno*, knowledge implies true belief; on TW (Two Worlds), knowledge excludes true belief' (Fine [1999] p 216).

Fine also thinks that the implications of this cause a big problem for Plato. On a practical level, one cannot first believe that the sun is shining, then come to know it is. The consequences for the political theory of the *Republic*, then are remarkable: the philosopher-ruler, can know the Form of Justice, but he cannot know what actions are just (because he has no knowledge of actions in the sensible world). She says that, in this case, it is strange that the *Republic* should be an attempt to convince us that the philosophers should rule, since their knowledge is inapplicable in the sensible world.

Fine ([1999] pp. 218-219) calls the traditional reading the 'objects' reading of the line, because she says that this reading takes *Republic* V to mean that Plato specifies the objects of knowledge and belief. As a consequence of this reading, we can only know what exists (so there can be no knowledge of things such as Father Christmas) and we can only have beliefs about what does not exist. Fine thinks that, on the contrary, Plato is referring to

⁴⁴ For the latter, see Fine (1995), especially pp. 23-29 and pp. 36-65.

the contents of knowledge and belief, so one can only know true propositions, and one can believe both true and false propositions. She calls her reading the ‘contents’ analysis.

To summarize, Fine wants to avoid the traditional ‘Two Worlds’ reading because it leads to the absurdity that we cannot know facts about the physical world, it contradicts Socrates’ claim to have only belief about the Form of the Good, and it contradicts the claim that the philosopher who returns to the cave can have knowledge of the sensibles.

Fine’s solution to the problem is to argue that *Republic* 5-7 is not committed to the Two Worlds reading of Plato’s metaphysical epistemology (TW). In fact, she thinks that Plato is never committed to TW. She thinks TW contradicts the text of the *Republic*, because at 506c, Plato says that he has no knowledge of, but beliefs about the Form of the Good, and at 520c, he says that the philosopher who returns to the cave will know the things there.⁴⁵ She agrees with the aspect of the traditional reading that correlates knowledge with Forms and beliefs with sensibles, but she says that this need not imply TW. According to Fine, Plato argues only that knowledge requires (but is not restricted to) Forms, and that one can at most achieve belief if restricted to sensibles.

Fine strengthens the claim she is making by saying that Plato is a coherentist, rather than a foundationalist, about justification. She suggests that, for Plato, no beliefs are self-evident or self-justified; to be known, they must be justified in terms of other beliefs. Her reading of Plato’s epistemology relies on the claim that Plato is talking about propositional knowledge. She rejects the ‘objects analysis’ that reads Plato as holding knowledge to be restricted to certain kinds of objects in favour of a ‘contents analysis’ that correlates knowledge to certain sorts of propositions. This is the most vulnerable part of Fine’s theory: Plato never explicitly frames knowledge in this way, and it is on this point that Fine receives the most criticism.

⁴⁵ Fine also thinks she has found other instances of Plato allowing for knowledge of the sensibles, at *Meno* 71b, 97a9 and *Theaetetus* 201ac.

We shall examine Fine's argument and the criticisms against her in detail before I go on to explain my own interpretation of Plato's epistemology in the *Republic*. I am not going to endorse her reading, but I do sympathise with some of the points she raises about the traditional reading of the line and Plato's epistemology as a whole. The biggest problem I see with Fine's work is that she needs to restrict Plato to talking about 'propositional' knowledge, when it is not clear from the text that Plato had any such distinction in mind. This is a common criticism of Fine, as we shall see, so finally I want to see if we can salvage anything from what Fine is trying to do by explaining my own reading, which allows us to have knowledge of the sensibles without some of the problems that Fine has to deal with.

Fine examines⁴⁶ Plato's remarks in *Republic* 5, in which he offers an account of how knowledge differs from belief. Fine makes a lot of the fact that the argument is intended to persuade the 'sight-lovers', so should not begin from any premiss that the sight-lovers should dispute. She says that, when Plato speaks of knowledge and belief as being 'set over' certain things, we can interpret him as meaning (in the case of knowledge, for example) any of the following:

- a. Knowledge is set over what exists.
- b. Knowledge is set over what is *F* (for some predicate *F* to be determined by the context).
- c. Knowledge is set over what is true.

Fine explains that, on the (a) and (b) readings, the objects of knowledge and belief are specified. On the (c) reading, it is the propositions that are the contents of knowledge and belief that are specified. Because Fine sets so much store by the fact that the sight-lovers are

⁴⁶ 1990 pp.85-115 or 1999 pp. 215-246; 1995 pp. 23-29, 36-65; 2004 pp. 48-81; 2005 pp. 200-226; 2007 pp. 331-367 and 2010 pp. 125-152.

the supposed audience for this passage, she thinks she can argue that readings (a) and (b) violate the dialectical requirement, according to which Plato cannot begin from a premiss that the audience would readily accept. These two premisses separate the objects of knowledge and belief, which the sight-lovers cannot be expected to agree to from the outset. Fine also thinks that the sight-lovers would have a problem with the idea of knowledge as involving some kind of acquaintance, as (a) implies, or that it is impossible to know something that is equal and unequal, as (b) implies.

She thinks that premiss (c) satisfies the dialectical requirement, as it says only that knowledge entails truth. She also wants to qualify this interpretation to talk about sets of propositions, so that Plato's conception of belief entails that the set of propositions that can be believed includes some truths and some falsehoods. She thinks that this means that all we have been told so far is that knowledge, but not belief, is truth-entailing, which is quite compatible with the claim that there can be knowledge and belief about the same objects.

Plato is clear that knowledge and belief are different capacities, but, as Fine says, this does not mean that they must be set over different objects. She says, 'Husbandry and butchery, for instance, do different work; but they are both set over the same objects – domestic animals' (Fine [1999] p 220).

Fine has good reason to draw our attention to this apparent problem in Plato: in spite of the fact that the analogy of the state is introduced as a way of examining on a larger scale the properties of justice in the soul (*Republic* 368d-369a), the *Republic* does seem to want to argue for a specific political theory that has philosophers as rulers. As such, it would be very strange if they could not apply their knowledge to the physical world. Moreover, Socrates says he has beliefs about, but no knowledge of, the Form of the Good (*Republic* 506c) and the philosopher who returns to the cave will know things about sensibles (*Republic* 520c). As

Fine says, at this point the only thing that Plato has committed himself to so far is that knowledge and belief are different capacities, and that knowledge must be truth-entailing.

In her reading of *Republic* 6-7, Fine builds on her theory with an extensive reference to the sun, cave and divided line allegories. For Plato, she says, there are two sorts of knowledge and two sorts of belief. The best sort of knowledge requires knowledge of the Form of the Good. Moreover, she points to Plato's remark that the Form of the Good is not an *ousia* at all, but, says Fine, it is the formal and final cause of all Forms. The Form of the Good is 'the teleological structure of things; individual Forms are its parts, and particular sensible objects instantiate it' (Fine [1999] p 228).

This, says Fine, embodies her point that Plato is a holist about knowledge: 'Full knowledge of anything requires knowing its place in a system of which it is a part, or which it instantiates; we do not know things in the best way if we know them only in isolation from one another' (Fine [1999] p 229).⁴⁷

All this has implications for Fine's interpretation of the divided line, which she correlates to the allegory of the cave and which we shall examine now before I explain my own interpretation in the following section. At level one, *eikasia*, which Fine translates as 'imagination', the prisoner is unable to distinguish between images and their objects. It is not that he only sees images, because 'even if they were confronted with a physical object, they would remain at level one, so long as they could not systematically discriminate between images and their objects, and could not tell that the objects are more real than the images, in that they cause images' (Fine [1999] p 232). Fine explains Plato's remark that most of us are at level one by saying that it is not because we only see images, but because we have level one moral beliefs.

⁴⁷ Fine thinks this makes middle Plato more coherent with the holist conception we find in later dialogues; She also argues this in Fine (1979) pp.70-80. She also thinks it is more coherent with Plato's creation myth in the *Timaeus*; see also Fine (1999) p 229 for this view.

At level two, *pistis*, which Fine translates as ‘confidence,’ the prisoner learns to distinguish between images and their objects. This represents the first application of *elenchus* or dialectic, in which the prisoner can discriminate between physical objects and images, but not explain their differences. Again, Fine rejects the ‘objects analysis’ by saying that one does not need to be confronted with a physical object to be at level two: if one is lacking an *aitias logismos*, a satisfactory account, of the sort necessary for knowledge, ‘he remains at a belief state, though at a better one than he was before’ (Fine [1999] p 234).

At level three, *dianoia*, which Fine translates as ‘thought,’ the prisoner’s attention is turned to Forms. The two key differences between levels three and four are that, at level three, one uses sensibles as images of Forms, and proceeds from a hypothesis to various conclusions. Fine argues (and this is one of the points upon which I shall agree with her) that level three reasoning is not restricted to mathematics. She writes:

...any reasoning that satisfies the more general features [ie, the use of hypothesis and imagery] belongs at L3. Indeed, it seems reasonable to suggest that that although Socrates (in the Socratic dialogues and *Meno*) places himself at L2 in his moral reasoning, Plato in the *Republic* places himself at L3 (Fine [1999] p 236).

This means that it is possible to reason dianoetically and noetically about mathematical objects, which is a problem for the ‘objects’ analysis of the divided line, but important for the solution to the main problem of this dissertation. Fine backs up this point by pointing out that, in the *Republic*, Plato uses images all the time. For example, he explains justice in the soul through the analogies of health and justice in the city; the analogy of the ship is used to illustrate the nature of democracy. So the moral reasoning in the *Republic* satisfies the dianoetic feature of using imagery, says Fine.

Fine's argument that 'L3 is not restricted to mathematics' also includes a reference to *Phaedo* 100ff, which, she points out, 'is plainly not restricted to mathematical reasoning' ([1990] p. 106). I will also argue that this passage uses the same method as Fine's L3 and part of L4 of the divided line in Section Three, Chapter One and Chapter Two ii. The link between the two passages is one argument in favour of extending L3 reasoning beyond mathematics. I also make a similar link (although Fine may not necessarily agree with this) between this kind of reasoning and the hypothetical passage of the *Meno* in Section Two, Chapters Three and Four. In both the *Meno* and the *Phaedo*, I will argue, Plato hints that mathematics can be used as a model for other kinds of philosophy both in the hypothetical passages and in other places in the texts.

Moreover, Fine says that the partial account of virtue in book four of the *Republic* is a 'mere outline that requires a longer way' (Fine [1999] p 236).⁴⁸ The longer way would involve relating those virtues to the Form of the Good, which is what makes the account in book four *dianoia* rather than *noesis*. Fine thinks this satisfies the feature of *dianoia* that it 'proceeds from a hypothesis to various conclusions' (Fine [1999] p 235-6).⁴⁹

Fine makes clear that she thinks *dianoia* is not always deductive. According to her, the moral reasoning of the *Republic* is also an example of level three/dianoetic reasoning. This is not a claim that I am making in this study; as I made clear in the introductory section, I am looking only at the images that are clearly part of the argument, that the interlocutor is given a chance to amend, not those that are used as illustrations. Fine's definition of *dianoia*

⁴⁸ Cf *Republic* 435d and 504c9-e2.

⁴⁹ On the point that the analogical reasoning in Book IV is to be identified as equivalent to the mathematical reasoning of the divided line, Fine refers us to Irwin ([1977] pp. 222-223). Here, Irwin's point is that 'the definitions of the virtues in *Republic* IV are hypotheses, not resting on a full account of the good, but assuming some beliefs about it and drawing conclusions about the virtues – that is why they are only sketches, not completed definitions (504d6-8)' (Irwin [1977] p.222). In Chapter Three ii.c and d of this section, I shall give my reading of how hypotheses might stand in for definitions as sketches, which does allow for the identification of the analogical reasoning in Book IV to be identified with the mathematical reasoning of the divided line.

is a lot wider than (but not incompatible with) mine, although I share her view that it is not restricted to mathematics.⁵⁰

Fine's account is a coherentist one. Rather than the foundationalist claim that the regress of knowledge stops owing to self-justified or self-evident beliefs, Fine's coherentist reading of Plato holds that the regress is finite but has no end: 'I explain p in terms of q , and q in terms of r , and so on until, eventually, I appeal again to p ; but if the circle is sufficiently large and explanatory, then it is virtuous, not vicious' (Fine [1999] p 240-241).⁵¹

This is quite a controversial claim for Fine to make, but I am sympathetic. It is controversial because, as Fine admits, Plato is typically counted as a foundationalist. Plato seems to reject coherentism at *Republic* 533c, where he says, 'For if one's starting point is something unknown, and one's conclusion and intermediate steps are made up of unknowns also, how can the resulting consistency ever by any manner of means become knowledge?'⁵²

There is also a passage in *Cratylus*, in which Plato seems to endorse foundationalism:

⁵⁰ As a result of her reading of Plato's conception of knowledge, Fine realises that at this point, she must provide an account of the famous justification regress. She thinks that knowledge requires at least two things. One, she calls KL, meaning that knowledge requires an account, or *logos*, in addition to true belief. So it is not enough to believe, truly, that Edinburgh is north of London; we must also be able to provide an account of why this is true. She also thinks that, for Plato, KBK, ie, knowledge must be based on knowledge (so 'I know a thing or proposition only if I can provide an account of it which I also know' Fine [1999] p 238). The regress occurs because, to know something, one must provide an account. Given KBK, one must also know this account. Given KL, I must also provide an account of it, which, given KBK, I must also know, and so on *ad infinitum*. In addition, Fine needs to account for the implication in Plato that, to know anything, one must know Forms.

Fine's solution to this problem is to suggest that the hypotheses and conclusions used by the mathematician might be mutually reinforcing: 'For the hypotheses and proofs used to derive the conclusions might reasonably be thought to constitute an account of – an explanation of, and so an adequate justification for believing – them' (Fine [1999] p 239). This does not escape the regress, because if the hypotheses are not known, it violates KL and KBK. However, Fine thinks, this does not preclude justifying the hypothesis in the course of the inquiry (Fine [1999] p 240). As Fine goes on to say, there does seem to be a circle here: for KBK to be satisfied, the conclusions must also be known; but they may not be known, because, although KL is satisfied in this case, KBK is not, because the hypothesis is not also known. However, Fine writes, 'although there is a circle here, it is not a vicious one. The hypotheses are justified in terms of the conclusions, and the conclusions in terms of the hypotheses' (Fine [1999] p 240).

⁵¹ Cf Kvanvig 2007 pp185-204 for a modern perspective.

⁵² Cf White (1976) p 113 n 50, in which White comments that the method of hypothesis is a way of gaining consistency, but that this consistency is not turned into knowledge.

The name-giver might have made a mistake at the beginning and then forced the other names to be consistent with it. There would be nothing strange in that. It is just that way sometimes in geometrical constructions: given the initial error, small and unnoticed, all the rest that then follow are perfectly consistent with one another. That's why every man must think a lot about the first principles of anything and investigate them thoroughly to see whether or not it's correct to assume them. For if they have been adequately examined, the subsequent steps will plainly follow from them (*Cratylus* 436cd).⁵³

At level four, *noēsis*, which Fine translates as 'understanding,'⁵⁴ the prisoner reaches an unhypothetical first principle, which Fine conceives of as 'a definition of, and perhaps further propositions about, the Form of the Good' (Fine [1999] p 242). She thinks that when

⁵³ Cf *Republic* 377a: '...the first step, as you know, is always what matters most.' Lee in his (1987) translation of the *Republic* (p 69) points out that this refers to the Greek proverb, 'The beginning is everything.' A more literal translation would be, 'the beginning is half of the whole,' although 'well begun is half done' also conveys the sense of the Greek phrase. None of this troubles Fine. She says that the passages do seem to commit Plato to KBK, but that her coherentist account means that one may come to know the starting-point through deriving conclusions from it. She says that the passage might suggest that consistency is insufficient for knowledge 'but any self-respecting coherentist would agree' (Fine [1999] p 241). The consistent beliefs must be mutually supporting or explanatory, and form a sufficiently large group; moreover, such coherence is only sufficient for justification; knowledge requires truth. At this point, we are talking about coherentist justifications in mathematics and *dianoia* as a whole, so on the face of it, Fine's interpretation may seem strange. It is common to associate the mathematics, with its extensive use of axioms, with foundationalist accounts. Euclid's *Elements of Geometry* has been described as exemplary of a foundationalist system (Newman [2010]), as it begins with a foundation of first principles (definitions, postulates and axioms) to construct further propositions. Euclid's system even inspired Descartes' approach of building knowledge from first principles.

So why should we seriously consider Fine's claim that Plato's *dianoia* is coherentist? Certainly, it is a very elegant solution to the problem, but it is difficult to reconcile it with the common reading of the passages from *Republic* and *Cratylus* cited above. I admire the elegance of Fine's position, but I think it would take more work to provide a convincing case for coherentism at the level of *dianoia*. In fact, it is not something I am going to attempt to resolve here; it is beyond the scope of the current project, given that the solution I propose does not require a coherentist account.

I would suggest that Fine would need to give an account of Plato's reading of mathematics in order to support her claim. In the *Meno* section, I consider the possible mathematical methods that Plato might be using, and there is a case to be made for suggesting that *apagōgē* does satisfy the conditions that Fine wants for her argument to work. This is a method that reduces the problem to a series of lemmas, or premises, until we arrive at a conclusion that is independently known of the thing sought. In this case, it does look like Greek mathematics (and Plato's reading of it) would support a foundationalist, rather than a coherentist, theory of knowledge, but Fine could try to argue that one could know the hypothesis and conclusion simultaneously; the conclusion would satisfy KL and KBK for the hypothesis, and the hypothesis would satisfy KL for the independently known conclusion. Of course, Fine's claim is actually stronger than this: she wants to say that there is no need to have an independently known conclusion, but the evidence for this claim is not immediately apparent.

⁵⁴ At least, that is her translation here (1999) pp. 215-246. However, she does not agree with Burnyeat's ([1981] pp. 97-139) distinction between knowledge and understanding, which links explanation and understanding, but not explanation and knowledge. Instead, Fine prefers to think of 'richer and deeper kinds of knowledge.'

one can relate the hypotheses to the Form of the good, then the hypotheses cease to be mere hypotheses and are known in a noetic sense, rather than a dianoetic sense. Moreover, in *noēsis*, one thinks of Forms directly, not through images of them, and uses only Forms. Crucially, Fine thinks that one can apply this knowledge to the sensibles. She compares this capacity with Aristotle's assertion that one can define various species and genera without reference to particular instances of them, but once one has done this, one can apply the definitions to particulars in such a way that gives knowledge of them.

The problem that Fine faces in her account of level four reasoning is that the hypotheses are explained by relating them to the Form of the Good. But how is the Form of the Good known?⁵⁵ Fine goes further: 'Indeed, Plato's coherentism may require L4 type knowledge of sensibles to be possible' (Fine [1999] p 245). This is one of her strongest and most controversial claims: usually Plato is seen as very strongly rejecting the sensibles as a means to knowledge. Some people criticise this aspect of Fine's work, saying that she is imposing a sort of implicit empiricist approach onto Plato. They say that Plato follows Parmenides in being committed to the nonreality of the phenomenal world, so for Plato, being is prior to becoming.⁵⁶

Although I am not endorsing Fine's coherentism here, the use of sensibles as a means to knowledge is something that I am going to have to deal with, because I want to say that *dianoia* is connected to the sensibles as it uses them as images through which to reason about

⁵⁵ There is nothing more fundamental than the Form of the Good that can be used to explain it, so how can we avoid violating KL or KBK? Fine rejects the kind of solution that involves connecting *noēsis* with vision or acquaintance, because she thinks it does not satisfy KL: 'it claims that knowledge does not require an account after all, but only a vision' (Fine [1999] p 243).

Fine's solution is again to appeal to coherence. She stresses that dialectic, not acquaintance, must be the route to *noēsis*. One's claims about the Form of the Good are justified not in terms of anything more fundamental, but in terms of its explanatory power. Fine sees the Form of the Good as the teleological structure of things. Claims about it can be justified 'by showing how well it allows us to explain the natures of, and interconnections between, other Forms and sensibles' (Fine [1999] p 244).

This kind of coherence, Fine claims, is superior to the level three kind, because it integrates reality into a 'synoptic whole,' rather than being restricted to individual branches of knowledge like *dianoia*. The coherentist explanations are fuller and richer in *noēsis* than in *dianoia*, which is what makes it a better sort of knowledge.

⁵⁶ I am grateful to Simon Trépanier for his remarks on this subject; cf Kahn (1998) pp. 345-346, who covers the same ground.

intelligibles. This means that I do not have the work of supporting Fine's coherentist reading, or arguing that Plato means particularly knowledge of propositions, but I do need to argue that Plato's epistemology is a scale and involves an ascent that is not purely intuitive. Accounts that use intuition as a mechanism for ascending the Platonic epistemological scale, like Robinson's, portray Plato as less rational than my account. To be clear, my account does leave room for intuition in some sense, but it is not at all the most important factor in my reading of Plato's middle epistemology.

This means that I need to argue for an ascent in Platonic epistemology that is not largely accidental.⁵⁷ The ascent passage in the *Symposium* provides a picture of an ascent to higher principles, but it lies outside the scope of the current study to provide a full exposition of this (although I would point to it as part of an appeal to wider evidence in Plato). This is a problem I shall tackle in two ways: first, in this section, I shall produce my own reading of the divided line that explains the path to knowledge as an upward ascent from sensibles-based reasoning, although I am not suggesting that it is the sense faculties that are involved in *dianoia*. Secondly, in the following chapters, my reading of the passages from *Meno* and *Phaedo* will highlight text-based evidence of this kind of ascent in Plato.

All in all, Fine's account of Plato's epistemology makes him extremely modern, in that she thinks he conceives knowledge in terms of explanation and interconnectedness, not in terms of certainty or vision. According to Fine, Plato thinks that one knows more to the extent that one can explain more; but she thinks 'this is only to say that, for him justification typically requires explanation' (Fine [1999] p 246).

⁵⁷ Again, I am grateful to Simon Trépanier for his constructive criticism on this subject.

ii. Propositions or Objects: Gonzalez on Fine

Fine's work has been much debated and criticised. The most controversial issues that have been identified with Fine's reading are her reliance on propositions to explain Plato's theory of knowledge and her rejection of the division of the objects of knowledge and belief. One of the most definitive criticisms of Fine is that of Gonzalez (1996) pp. 245-275, who attacks Fine's rejection of the 'objects analysis' of the divided line and argues that her 'propositional' reading is untenable. Gonzalez does this mainly through his reading of *Republic V*, which takes an 'existential-predicative' reading, which seems to demand knowledge by acquaintance, rather than propositional knowledge. He also accuses Fine of not abiding by her own dialectical requirement. A third controversial aspect of Fine's reading is her 'coherentist account' of Platonic knowledge, especially *dianoia*. Gonzalez does not tackle this aspect, but, as we have discussed it above, we shall now turn to Gonzalez' account.

As we saw, Fine's use of the dialectical requirement to argue against the existential or predicative readings relies on their alleged contentiousness as opening premiss. Gonzalez argues that, if the text is read correctly, there is actually nothing controversial in the existential or predicative readings. These two readings, he says, are mutually supportive and consistent with the text; they also fulfil Fine's dialectical requirement. Why should it be controversial to equate knowledge with perception? If, Gonzalez says, we are trying to persuade the sight-lovers, surely the opposite is true, and there is no good reason to prefer propositional knowledge. In this case, we should be thinking about Platonic knowledge as knowledge by acquaintance.

In Gonzalez' favour, we can find passages in middle Plato that do seem to give examples of knowledge by acquaintance. For example, in *Meno*, Socrates speaks of knowing Meno and being able to recognise him, and his thought experiment about the way to Larissa

has also been interpreted as supporting a theory of knowledge by acquaintance (or at least holding this as an unarticulated assumption).

In addition, later works such as the *Timaeus* sometimes seem to support the objects of belief and knowledge as being diverse:

If intelligence and opinion are different in kind, then these ‘things-in-themselves’ certainly exist, forms imperceptible to our senses... now there is no doubt that the two are different, because they differ in origin and nature (*Timaeus* 51dff).

This in itself is not conclusive; other passages in middle Plato conversely seem to support the idea of knowledge and belief sharing objects, in that knowledge is true belief plus something. There is the lengthy discussion in the *Theaetetus* of the hypothesis that knowledge is ‘correct judgement and an account’ (202d). This question is not resolved, but it is an argument that Plato seems to take seriously. Moreover, Fine herself uses the Larissa example of the *Meno* to support *her* case.⁵⁸ As we shall discuss in the *Meno* section, she sees the passage as further proof that Plato is not committed to the Two Worlds theory in the middle period. She thinks that the *Meno* contains Plato’s first suggestion that knowledge is true belief plus something.⁵⁹

In this case, we need something more than an initial intuitive preference for knowledge by acquaintance in Plato in order to defeat Fine on this point, and we need to at least make sure that this reading is consistent with the text. Gonzalez thinks that he can also show that both his existential-predicative reading and knowledge by acquaintance are philosophically coherent with each other, and that they are consistent with the text.

⁵⁸ Fine (2004) pp. 48-81 thinks that the passage contains Plato’s suggestion that there are similarities between knowledge and true belief. They are both equally good guides (to Larissa and in action) and can have as their objects at least some of the same things (Larissa and how to double the area of a square).

⁵⁹ See section two of this thesis for more discussion of this topic.

He points out that we need not interpret the predicative reading as absurdly meaning that one cannot know that something is both F and not- F . We can see it as claiming that knowledge of what F is cannot be 'set over' what is both F and not- F . He refers to Charles Kahn's (1996) observation that 'to speak of *what F is* or *what is (truly) F* for Plato, is to speak of the same thing.'⁶⁰ Predication, for Plato, is not predication in the modern sense: 'In the predication 'x is F ,' what is primarily referred to is not the x, but the F ' (Gonzalez [1996] p 254). Gonzalez thinks that he has found support for this view at *Republic* 524a, which he reads as setting perception over 'not the sensible object per se as a substance distinct from its properties, but the properties themselves' (Gonzalez [1996] p 255).⁶¹ This, thinks Gonzalez, puts the predicative reading on at least an even footing with Fine's predicative reading as a noncontroversial opening premiss, especially as he thinks that the parallel to knowing and perceiving is natural to the sight-lovers.

Gonzalez points to the debate about the meaning of *doxa*. *Doxa* has been connected to judgement by some scholars, whereas others maintain that it is 'more analogous to perception, being essentially *intuitive* and even nonpropositional' (Gonzalez [1996] p 256).⁶² Gonzalez' position is to remark that perception can be the basis for statements of what F is, and if that perception is confused, those statements can be true or false:

It is undeniable that Plato sees the person whose cognitive state is *doxa* as believing that certain things are the case and thus as affirming numerous true propositions...However, it is nevertheless possible that when Socrates speaks of *doxa* he is referring primarily not to the

⁶⁰ Cited in Gonzales (1996) p 254.

⁶¹ To Gonzalez, this is the most important point of response to Fine's theory. His argument relies on the complementary natures of the existential and predicative readings, so he needs to prove that Socrates does not mean that we cannot believe something to be both equal and unequal. This is one of Fine's points against the objects analysis, but I do not think it is as important as her main argument about the possibility of having beliefs about Forms and knowledge about sensibles.

⁶² Cf Bluck (1963) pp 30-43, who cites the example of finding the way to Larissa as relating to the role of intuition in *doxa*.

assertion that something is F, but rather to the confused intuition of F itself that underlies and guides this assertion (Gonzalez [1996] p 256).

He argues that perception of beauty, for example, is for sight-lovers confined to sensible objects that are as ugly as they are beautiful, is equally 'set over' ugliness and therefore cannot clearly separate the two. Gonzalez believes that this suggests doxa is perceptual rather than *propositional*. This depends very much on his interpretation of the predicative reading, in which knowledge and belief are assigned to objects. On his view, the predicative reading complements the existential reading. He weighs in on the debate about understanding by saying, 'knowledge here is understanding *and* acquaintance, understanding achievable only in direct acquaintance with certain objects; doxa is the failure to achieve such understanding on account of the character of the objects with which it is acquainted' (Gonzalez [1996] p 258).

Gonzalez then turns to Fine's rejection of what she thinks is a nonsensical notion of degrees of existence, a debate which itself has attracted a lot of interest. Fine thinks it is nonsensical to say that a thing has degrees of existence: something has to either exist or not exist; it cannot half-exist, or vary in degrees, the way that its properties can.

Fine's view is supported by Annas ([1981] pp. 195-199) and Vlastos ([1973] pp. 58-75), both of whom agree that the notion makes no sense. Gonzalez remarks that the Greeks did not possess our concept of existence, and claims that Vlastos' fixation on contemporary usage is beside the point. According to him, Plato does not distinguish a thing's 'substance' from its 'properties' in the way that Fine thinks. Gonzalez argues that, for Plato, existence is always *einai ti*, being something; there is no concept no concept of existence as such, for subjects of an indeterminate nature.

The distinction between the predicative and existential readings of the word 'is' which Fine imposes onto Plato, says Gonzalez, makes it look as though Plato is confused. Both the existential and predicative readings can be found in the text, but that is not because Plato is confused, but because, for Plato, they are simply different aspects of the same meaning. Gonzalez thinks he has shown that they do not violate the dialectical requirement, and the two kinds of knowledge they require ('knowledge what' and 'acquaintance') are compatible and complementary.

It is Fine's reading, Gonzalez argues, that is incompatible with the text. He says that the combined existential-predicative reading is compatible with the objects analysis and incompatible with Fine's veridical reading, which itself is compatible with the contents analysis. The latter is, according to Gonzalez, incompatible with the text for the following reasons:

First, it is difficult to square with the language of the argument that strongly suggests objects; Socrates' claims seem to assign knowledge to what exists and ignorance to what does not exist. Second, it is difficult to fit Fine's reading into the broader context of the argument: the dispute with the sight-lovers concerns the knowledge of and existence of beauty itself, not of propositions. Their dispute with the philosophers is about what objects can be known, in which case it is difficult to fit in Fine's reading with the argument. Gonzalez concedes that these two points are not decisive, and does not elaborate. The next two points, though, he says are major difficulties:

Gonzalez' third point is to find a problem with Fine's idea of knowledge and belief being different capacities set over the same things. Gonzalez⁶³ points out that the analogy with sights and sounds would break down if Fine's reading were correct: we cannot see sounds, or hear sights. He thinks that Fine is wrong in saying that the word *epi* can be set

⁶³ Cf Stokes (1992) pp. 103-132, who makes a similar point.

over contents as well as objects, saying that there is no textual basis for such a use.⁶⁴ Moreover, even if we ignore the problems Gonzalez sees with *epi*, he says that there are claims made in the text are inconsistent with Fine's interpretation.

At 478b3-4, Socrates says that what is known is 'what is' so what is believed must be something 'other than (ἄλλο ἢ) what is.' For Fine's reading to work, we need to interpret ἄλλο ἢ as 'not only, but also,' rather than 'other than.' This is because Fine wants to say that what is believed is what is false in addition to what is true. Gonzalez thinks that we cannot justify such an interpretation, when the text can be much more economically explained by saying that the province of belief is a third thing distinct from the provinces of knowledge and ignorance. He thinks that the decisive passage comes at 478e1-2, where the objects of belief are said to participate in both being and non-being. As a thing is not identical with that in which it participates (Gonzalez [1996] p 268), belief needs to be different from both being and non-being, and cannot be a mere combination. Gonzalez thinks that the mention of participation makes it 'as clear as possible' that belief is set over sensible objects (Gonzalez [1996] p 268).

Moreover, Gonzalez continues, Socrates says that it is impossible to believe what is not, so if Fine reads 'what is not' as meaning 'what is false', then she is making Socrates claim that all beliefs are true: 'but if all beliefs are necessarily true, then the only distinction Fine recognises between belief and knowledge is collapsed' (Gonzalez [1996] p 268-9).

Gonzalez' fourth point is that Fine violates her own dialectical requirement. Her interpretation requires two assumptions. Firstly, 'there is just one property, the *F*, the same in all cases, in virtue of which all and only *F* things are *F*.' Secondly, 'we cannot know anything about *F* things unless we can define what *F* is' (Gonzalez [1996] p 270).⁶⁵ These assumptions

⁶⁴ This claim is extended by Gonzalez, who argues that this reading also incorrectly reduces the *epi* to the difference between the fallibility and infallibility of the work that belief and knowledge do and thus eliminates it as a distinct criterion; see Gonzalez (1996) p 267. However, Gonzales does not elaborate on his point here.

⁶⁵ Referring to Fine (1990) pp. 85-115.

are necessary in order to argue that all knowledge of sensibles requires knowledge of Forms. However, Gonzalez says, 'it is hard to imagine assumptions that would be more controversial than these to the lovers of sights and sounds' (Gonzalez [1996] p 270). In this case, Fine violates her own dialectical requirement, in a way more serious than the one with which she charges rival interpretations and Fine's argument is refuted on its own terms.⁶⁶

As for Gonzalez own theory, he admits that Fine has legitimate concerns about the Two Worlds theory in Plato. He thinks that the existential/predicative reading can provide a solution. He says that we can still have justified true belief about the sensibles, but that this is simply not what Socrates means by *epistēmē*: the sensibles are simply not objects relevant to knowledge.

Moreover, he says, we need to recognise that the relation between Forms and sensibles is not one between two completely distinct worlds, because sensibles are instantiations of the Forms. In this case, recognising beautiful objects needs some awareness of the Form of Beauty, but it is a 'dream-like' awareness that fails to distinguish the Form from the sensibles. In the sense that the Form is deficiently grasped through the sensibles, *doxa* can be about the Form; but the cognition of *doxa* is still set over the sensibles. The deficiency of *doxa* lies in its indirectness. 'Therefore, when Socrates claims to have only *doxa* concerning the Form of the Good, he means that he is in some sense confined to sensible images (such as the sun) in his understanding of the good, that the good is *not* a direct, explicit object of his cognition' (Gonzalez [1996] p 273).

By the same reasoning, Gonzalez thinks he can explain the philosopher's descent into the cave: 'the philosopher knows the sensibles *precisely as imperfect images* of forms that transcend them...because this knowledge is *not* of sensibles, but of forms...it can reveal

⁶⁶ However, I think that Gonzalez' reading of Fine's dialectical requirement makes her claim too strong; she allows Plato to build in some more controversial assumptions and concedes herself that the One over Many assumption and Priority of Knowledge of a Definition assumption might be controversial to the sight-lovers (1999 n. 13 and 15 pp 222-223).

sensibles for what they are' (Gonzalez [1996] p 273). It is not the shadows on the cave wall that the philosopher knows, but the fact that they are imitations of sensible things.

Gonzalez' account has the virtue that the bridge between the Two Worlds is provided by the objects themselves. In this case, knowledge and belief are not divorced completely, but related (like dreaming and waking) as their objects are related. Can this be what Socrates means? Gonzalez' reading is supported by his reading of *Republic* 520, in which Socrates tries to persuade the philosophers to return to the cave:

For once accustomed [to the darkness] you will see with infinitely greater clarity than those down there, and you will know what each of the images is, and what it is an image of, because you have seen the truth concerning the beautiful, the just and the good (*Republic* 520c3-6).⁶⁷

Gonzalez takes this to mean that it is not the sensibles of which the philosopher has knowledge, but their relationship to Forms. It is a very elegant reading and, if he is correct, it could be a good starting point for an answer to the Two Worlds problem. I say 'starting point' because Gonzalez devotes very little space to the solution, whereas what he (or, at least, Plato) needs is a fuller metaphysical account of the relationship of Forms and sensibles, and how this correlates to his epistemological explanation.

The distinction between the dream-state of belief and the wakened state of knowledge does seem to come down at least in part to the ability to know what each thing is a shadow of, as Socrates continues, 'and so our state and yours will be really awake, and not merely dreaming like most societies today...' (520c). However, this passage does not explicitly limit the philosopher to the sort of awareness-knowledge that Gonzalez would like. Lee's translation has the philosophers able to 'distinguish the various shadows, and know what they

⁶⁷ Cited in Gonzalez (1996) p 273.

are shadows *of*,⁶⁸ which could be taken to mean that the knowledge is of sensibles as well as Forms. Moreover, is only initially that the returned philosophers cannot compete with the prisoners 'before his eyes got used to the darkness' (Republic 517a).

With this in mind, the above passage is not conclusive evidence for Gonzalez' reading. In addition, he needs to explain how his solution answers the problem that he is attempting to solve. How should the knowledge of what the sensible object is an image *of* equip the philosopher to rule any better than the others? On Gonzalez' reading, the only difference between the philosopher and the other citizens is that he knows that a just act is a reflection of the form of Justice; but how does this furnish him with the tools he needs to rule?

One solution would be to argue that Plato does not mean that the philosophers are better rulers because they have knowledge of the sensibles, but for some other reason. Socrates emphasizes another reason for wanting the philosophers to return to the cave: 'The truth is that if you want a well-governed state to be possible, you must find for your future rulers some way of life they like better than government...what we need is that the only men to get power should be men who do not love it, otherwise we shall have rivals' quarrels' (520e-521b).

But does Plato really want the philosophers to undergo decades of training for the sole purpose of turning the future rulers into selfless politicians? Undoubtedly, Plato sees their unwillingness to rule as an advantage, but this is not all: the philosophers are 'better and more fully educated than the rest and better qualified to combine the practice of philosophy and politics' (520b). This suggests that the expertise of the philosopher bridges the gap between the sensible and the intelligible, which could at least in part be explained by Gonzalez' idea of knowledge-awareness if he expanded his idea a little. However, Gonzalez would still need

⁶⁸ Lee's translation of the *Republic* (1897) p. 263; my italics.

to account for the fact that *Republic* 520 does seem to suggest knowledge of the sensibles as well as their relationships to the Forms they imitate.

To summarize, the main charges that Gonzalez levels against Fine are that Fine violates her own dialectical requirement⁶⁹ and that her idea of knowledge and belief being different capacities set over the same things is inconsistent with the text. The second point boils down to her rejection of the Two Worlds theory and her related view of Platonic knowledge as propositional, which seems to be the sticking point of her theory.⁷⁰ The problem is that Plato never explicitly says that knowledge is propositional, so even if her reading can be made consistent with the text, it is difficult to see why we should prefer it to the others.

On the other hand, should we endorse any reading that seeks to separate out the various functions of *einai* so starkly? Is Gonzalez justified in saying that the reading should be only existential-predicative and not propositional? In Greek, *einai* can be existential ('x exists'); veridical ('x is true'); predicative or copulative ('x is F,' with F being some property) or the 'is' of identity. Plato was certainly aware of its different functions, and the potential chaos it can cause in philosophical argument.⁷¹ But can we assume that Plato consciously wanted to work such a distinction into his philosophy?⁷² Surely knowledge of a

⁶⁹ All I would say about this is that the dialectical requirement is probably more important to Fine than it is to Plato, so her own violation of it is serious only to the extent to which her argument relies on it; Plato never explicitly says that his opening premises must be intuitive to sight-lovers. I have already noted that I think Gonzalez' statement of Fine's requirement is too strong, and only applies to the arguments opening premises. This still leaves us with the question of whether the chosen reading of what knowledge and belief are set over is controversial (ie, is it knowledge by acquaintance or propositional knowledge?), but does not necessarily apply to the two points that Gonzalez mentions in his concluding argument. In fact, her argument does not rely on the dialectical requirement entirely, but also draws upon her readings of the *Meno*, *Theaetetus*, Aristotle and other passages of the *Republic*.

⁷⁰ Fine's point is a controversial one. Here, she rejects only the epistemological division of the Two Worlds, ie, that we cannot have knowledge of sensibles and beliefs about Forms. She does not reject the metaphysical separation of Forms and sensibles here; for this, see Fine (1995) pp. 44-65.

⁷¹ See *Euthydemus* 283b-d; or *Parmenides* 132; Cf Waterfield's comments in his (1987) translation of *Euthydemus* 284c for his remarks on the implication in Greek that that things we think are 'somethings,' which exist and are true. See also Kahn (1993) pp 131-142 on the same point.

⁷² Gonzalez would say that we need to assume that he does make such a distinction in order to avoid incompatibility with the text. However, this rests on an assumption that the deeper implications of Plato's theory

Form leads to true descriptions of it, and these are included in the Platonic conception of knowledge?⁷³ So even if Plato does take the ‘objects’ reading, it should still be possible to make true propositions about the things you know.

Much remains unresolved, and given that there are a few big fights I want to pick, I must leave aside those points that do not directly affect my argument. This said, there are several major issues raised here, which my own account will need to address, when it comes to placing it in the debate. Fine’s work is one of the most controversial (but well-known) readings of Plato’s epistemology in recent times; I do not endorse everything she says, but there are some points she makes that I think deserve to be taken seriously. In this section, I have tried to make clear what these are. Namely, I do not think that Plato makes the distinction between propositional knowledge and object-based knowledge that Fine (or indeed Gonzalez!) proposes. Moreover, although I think that her coherentist explanation is very elegant, I think she needs to appeal more to the history of mathematics to make her case. Given that I think my account will be consistent with either her coherentist reading, or a foundationalist account, I shall not explore this aspect further.

However, I do think that Fine makes a valuable point in that we should not so readily reject the possibility that Plato allows for knowledge of the sensibles and beliefs about the Forms. As she says, it would be strange if the philosopher-ruler can know the Form of Justice, but he cannot know whether his actions are just ([1999] pp. 215-246). In this case, there are serious implications for Plato’s political theory if there can be only knowledge of Forms, and Fine thinks we might reasonably question whether we can see the *Republic* as an attempt to convince us that the philosophers should rule.

were philosophically coherent (his third point against Fine), and even if we use the principle of charity, we cannot be sure that Plato has this worked out and leaves it implicit.

⁷³ Cf White (1993) Chapter One pp. 1-33. White proposes that true propositions can be deduced from definitions, surely the result of knowledge of a Form, although he does go on to give an alternative reading in Chapter Two pp. 35-61.

This said, there is a problem in treating the objects of the lower part of the line as objects of knowledge in the strict sense, because the objects of the sensible world are constantly changing. One solution might be to say that the Philosopher-rulers use knowledge of the Forms to guide their decisions in the sensible world. If we adopt the 534a version of the line, *dianoia* could be a form of *noēsis* but also something less than knowledge; I have already stated my reasons for not doing this (Section One, Chapter One). On my reading, *dianoia* can still provide the foundations of knowledge while still falling short of the epistemic achievements of the top segment of the line. Whereas Fine's account entails that there must be knowledge of perceptibles, my account does not necessarily need to argue for this. Even if knowledge and belief are set over different objects, this does not have to mean that each of the four sections of the line have their own objects. In *Republic* 510d-511a, Plato distinguishes between the objects of investigation (Forms, according to the traditional reading) and the tools that the mind uses in this kind of inquiry: the tools of *dianoia* are images and hypotheses; those of *noēsis* are *ti estis*. I argue that what differentiates *noēsis* and *dianoia* are their tools, not their objects, so my reading does not imply that mathematicians are not talking about Forms.

What I mean is this: the objection to Platonic knowledge about sensibles and beliefs about Forms is that it contradicts Plato's remarks in *Republic* V, that knowledge and belief are 'set over' different things. The four mental states that are represented by each subsection of the line are not introduced here, so there is no reason for saying that *dianoia* and *noēsis*, both kinds of knowledge, cannot be set over the same thing. That is, even if we accept that there is a difference between the objects of knowledge and belief, we still have no reason to assign different objects to each of the mental states that compose them. Of particular importance for this project, I do think that Plato means that we can reason dianoetically about Forms. That is, we can use the tools of hypothesis and imagery, which characterise *dianoia*,

to make inquiries about Forms. I shall try to illustrate the fact that Plato himself does this in the *Meno* and the *Phaedo* in the relevant sections. First, though, I shall need to present my own interpretation of the divided line, which allows for this reading of Plato.

Chapter Three: My Reading

The purpose of this chapter is to present my own reading of the epistemology of the *Republic*, focusing on the divided line, but drawing on the issues raised in books V-VII. I shall argue that the line does represent an epistemological scale for Plato, but even if we say that knowledge and belief have distinct objects, *noēsis* and *dianoia* may concern the same things. After discussing the extent to which the allegories may be taken literally, I shall go on to present my reading of each of the states of mind and the kinds of tools that each state of mind uses to do its work.

i. How Seriously Should We Take the Allegories?

Socrates himself admits that the allegories are not a satisfactory account of what he is trying to say, calling it a ‘poor, blind, halting display,’ (*Republic* 506c17-d1) and we will, in Chapter Three ii. b of this section, discuss Plato’s famous criticisms of the use of images. On the other hand, they do seem to provide the most detailed account of Plato’s epistemology in the middle dialogues, and the prominence Plato gives to them would suggest that he means us to take them seriously. However, some people have gone so far as to make claims about Plato’s epistemology based on the mathematical proportions of the line rather than what Plato says; others have tried to map the epistemology of the line exactly onto that of the cave.

We should be wary about Plato's use of imagery to make a philosophical point, given his own reservations and the general difficulties with the use of analogy in philosophical argument. One person who has been notoriously critical of Plato's use of analogy in this passage is Julia Annas. She writes,

The imagery is apt to get overloaded, as happens with the Line, because Plato is trying to do two things at once with it. And the detail of the imagery tempts us to ask questions that cannot be satisfactorily answered within the terms of the imagery; if we treat it with philosophical seriousness, the image turns out incoherent (Annas [1981] p 252).⁷⁴

Annas means that she thinks the line is not just an extended analogy to complement that of the sun, but also an ambitious classification of the epistemological states of mind in its own right: 'as often happens with Plato, his eagerness to use analogy and images leads him into intellectual unclarity' (Annas [1981] p 249). In addition to distinguishing the visible and intelligible realms, says, Annas, the line also puts them on a continuous scale of epistemological achievement. She also thinks that *eikasia* seems not to correspond to anything significant in our lives, being only there for the sake of the analogy, complains that *dianoia* seems to be too restrictive as it is confined to mathematical thinking, and remarks that the Form of the Good, so important in the simile of the sun, cannot fit happily into the line. Finally, she complains that it is unclear whether the cognitive states of the line are classified by their methods or by their objects.

Given the debate over the 'objects' and 'contents' analysis discussed above, Annas is right in that it is not immediately clear whether Plato's line is divided according to the objects of knowledge, but I think my interpretation can answer some of her other concerns. My

⁷⁴ Cf Iris Murdoch (1977) p 68, which Annas also cites: 'The Theory of Forms, when read in conjunction with the explanatory tropes of the Line and the Cave...can certainly produce some blazingly strong imagery in the mind which may well in the long run obstruct understanding.'

reading will suggest that, if we take *eikasia* to include images in the widest sense, it is more significant in our (and Greek) life than Annas thinks; moreover, I will argue that *dianoia* in the line is not restricted to mathematics, but can also be mathematical reasoning applied to philosophy. Finally, I will try to show that the line really is a continuum, and the four parts of it are not homogeneous. The early stages of *dianoia* look different from the later stages, as mathematical reasoning gradually turns the mind towards *noēsis*. Moreover, it is only at the later stages of *noēsis* that the Form of the Good comes into play. In this case, Plato's use of the line analogy does not cause us the problems that Annas alleges.

However, there is no denying that the allegories have caused a lot of confusion, especially with regard to how literally to take them, and how they work together. My position is that we cannot expect them to be accurate with respect to all aspects of Plato's theory, given that they are allegories rather than exhaustive descriptions of his thought. However, with regard to the epistemological points that Plato was trying to make, we can take them to be quite precise, and although we may not be able to map each directly on to the others, Plato does mean for them to work together. I certainly think, for example that the cave has a lot more to tell us about the line than has previously been thought.

The major problem of how the line fits in with the sun has already been mentioned: Plato is not specific about how the Form of the Good fits in to the line.⁷⁵ I shall have some suggestions for how this might work when I give my own reading, but here I want to say more about the correspondence with the cave. Along with the ascent passage in the *Symposium*, the cave allegory is one of the most powerful depictions of an epistemological ascent in Plato; as I shall want to say that the line can be seen as an epistemological ascent, it would be useful to see what the cave might have to tell us about this.

⁷⁵ Another issue is that, in the allegory of the sun, the visible is used as an analogy for the intelligible, whereas in the line passage, this analogy is discarded; however, as Plato is clear about where the analogy ends, it does not present any major problem for the relationship between the two. See Raven ([1953] pp. 22-32).

Traditionally, it is thought that Plato meant the two figures to be parallel. This reading⁷⁶ has it that there are four cognitive levels in the cave, corresponding to the four sections of the line, just as there are two ‘realms’ of the cave, corresponding to the two major parts of the line. These are:

The intelligible, corresponding to epistēmē:

Contemplation of the objects of the world above, corresponding to *noēsis*;

Viewing of the reflections of objects in the world above, corresponding to *dianoia*.

The visible, corresponding to doxa:

Seeing the objects in the cave, corresponding to *pistis*;

Watching the shadows on the cave wall, corresponding to *eikasia*.

There are unquestionably four parts of the divided line, but Plato does not explicitly say that there are four stages to the cave allegory, so how can we be sure that he meant to map it onto the four stages of the divided line? Moreover, some think that Plato actually had two incompatible interpretations of the cave.⁷⁷ If so, it is difficult to argue for such a close correspondence. In fact, some think that Plato never intended such a correspondence at all.⁷⁸

We could say that the cave represents the educational progress of the soul, and the line represents states of mind; in this case there is no need to try to argue for such a close correspondence between the two allegories. On the other hand, Plato’s comments at *Republic* 517b would certainly seem to suggest that he wishes us to make such a connection, as we

⁷⁶ As described by Malcolm ([1962] pp. 38-45), although I do not retain Malcolm’s substitution of *epistēmē* for *noēsis*, and his modification of the list given at 511d .

⁷⁷ See Ross (1951) pp. 37-82, especially pp. 72-76.

⁷⁸ In the mid-20th Century, the traditional view began to be challenged; see Robinson (1953) pp.180-196. This resulted in a series of responses trying to argue for the unity of the three allegories; see especially Raven (1953) pp. 22-32, Malcolm ([1962] pp.38-45) and Ferguson (1963) pp.188-193. The discussion does not end here: even later scholars who think that harmonizing the line and cave allegories is problematic think that, ‘we would perhaps be tempted to harmonize Line and Cave even without Plato’s encouragement, since there is an obvious continuity of interest in the relation of image to the original’ Annas (1981) p 254.

mentioned in the above paragraphs. Moreover, there is a continuous development throughout the entire passage which some argue is often overlooked.⁷⁹ The divided line is introduced as a way of ‘completing the analogy of the sun’ (509c)⁸⁰ and the analogy of the cave picks up from the second division of the divided line into states of the mind. In this case, if there are no serious clashes between the allegories, we should take seriously the idea that Plato wants us to see the correspondence.

One of the main problems with drawing the comparison seems to be with the lowest stage, at which the prisoner sees nothing but shadows. The criticism is that a strict correspondence with the line would require this to be *eikasía*, ‘...and yet surely it is not true. Who lives all his life at this level?’ (Ross [1951] p 34). Moreover, it is argued, this would mean that *pistis*, which should mean conviction or confidence, bears no relation to the prisoner’s attitude immediately after his release, which is described as bewilderment. In this case, even if Plato himself asserts that his cave is parallel to his line, we must assert that it is not.⁸¹

However, this argument is certainly not conclusive: as Malcolm (1962 pp. 38-45) points out, there is no indication that the prisoner is to stay bewildered. Moreover, mapping the cave allegory onto the line need not involve precision in every aspect, including the psychological; we can say that the correlation is an epistemological, and perhaps also a metaphysical one.

⁷⁹ See Raven (1953) p 32: ‘Just as...the Divided Line continues and expands the analogy of the sun, so the allegory of the Cave continues and expands the Divided Line.’ Raven thinks that this continuity is overlooked due to Plato’s momentarily using the visible world as an analogy for the intelligible: In the last section of book 5 of the *Republic*, Plato draws, in considerable detail, the contrast between opinion on the one hand, which concerns itself only with the particulars of the sense world and refuses to recognise the single Idea underlying the many, and knowledge and philosophy on the other, which always concentrate on the unique underlying Idea...When, however, we come to the analogy of the sun itself...the contrast is momentarily relegated into the background...[In book VI] he is reverting to the earlier contrast.’ (1953) pp. 30-31.

⁸⁰ Especially 509c1-2; Cf Raven (1953) pp. 22-32 for his remarks on this. Raven also points out that at 509d, Plato’s summary of the conclusions of the sun analogy confirms this point.

⁸¹ Robinson (1953) pp. 180-196 expresses this view.

Another problem that has been identified with correlating the sections of the line with stages in the cave centres on the moment at which the philosopher emerges from the cave:

...when he emerged into the light his eyes would be so dazzled by the glare of it that he wouldn't be able to see a single one of the things he was now told were real...Because, of course, he would need to grow accustomed to the light before he could see things in the upper world outside the cave. First he would find it easier to look at shadows, next at the reflections of the men and other objects in water, and later on at the objects themselves (516a).

This problem that people find with this passage is that the shadows and reflection in the world outside the cave do not seem to correspond to anything. Some people have tried to get around this, by suggesting that they correspond to mathematical objects,⁸² but, as I shall argue, it is not clear that Plato means mathematical objects (such as numbers and geometric shapes) to have their own ontological category.

Some people want to make the divided line correspond to the cave, but not to the sun. From the beginning of the divided line passage, Bedu-Addo argues, Plato does not mean to identify the upper section of the line with the intelligible world of the sun passage. The grading of the objects of the line corresponds exactly to the grading of the symbols in the cave, because the objects of *pistis* and *dianoia* are the same. The equality of the two middle subsections of the divided line is intended to represent the ontological identity of the objects of *pistis* and *dianoia*.⁸³ The difference is that, in *dianoia*, the mind 'dreams' about the Forms 'in the sense that it is quite unable to distinguish them in its thinking from sensible particulars which are the images of Forms' (Bedu-Addo [1979] p 97).

⁸² For example, Miller (2007) pp.310-342.

⁸³ These are not 'intermediates' because Plato is clear that the objects of *dianoia* are the same as those of *pistis*, but clearer. See Chapter Four of this section.

On this reading, Plato does not offer four grades of reality on the cave, but three, and these correspond to the line:

Line:	Cave:
Forms (<i>noēsis</i>)	Physical Objects in the Outer World
Images of Forms (<i>pistis</i> and <i>dianoia</i>)	Shadows and Reflections in the Outer World; Puppets and Statues in the Cave
Images of Images of Forms (<i>eikasia</i>)	Shadows of Statues and Puppets

According to Bedu-Addo, it is not to the allegory of the sun that we should be looking for an explanation of the distinction between *noēsis* and *dianoia*. Rather, we should look to book V of the *Republic*. When he describes the mental states of the divided line, Plato is relating all this to the metaphor of waking and dreaming. *Dianoia* is dreaming because it is unable to distinguish the original and its image. It wakes in *noēsis* when it distinguishes between Forms and particulars.

I see two main weaknesses of Bedu-Addo's account. First, if the divided line could be fitted to the cave so closely, we might also look for parallels between the line and the sun. To be fair to Bedu-Addo, he is not basing his argument on the presupposition of symmetry between the line and the cave; but he does rely on a break in the continuation between the sun and the line which, as we have seen, is not entirely justified.⁸⁴ Secondly, the 'Images of Forms' that Bedu-Addo says make the objects of *pistis* and *dianoia* are not ontologically the same, neither in the cave, nor in Plato's epistemology. While I agree that they are both

⁸⁴ Cf Raven (1953) pp. 22-32, discussed above.

images of Forms, in the cave, they each belong to clearly distinct worlds, and in the line, the treatment of sensibles is very different in *dianoia* to that in *pistis*.

My account of the line will suggest that the shadows and reflections outside the cave may be the hypotheses and images of the divided line. My claim is not so strong as to argue that Plato explicitly intended this correlation (although this may in fact be the case); I am simply saying that it fits.

This is as far as I am prepared to go in using the cave to support my reading of the line: where Plato is specific about the connection, such as at *Republic* 517b, we can take him literally. Otherwise, any apparent correlation between the allegories must be taken as supporting evidence only; it must not be the driving force of the interpretation of the line. Even those who argue that there is no happy correlation between the allegories acknowledge their appeal to the imagination, even if they can be philosophically frustrating.⁸⁵ In this case the allegories should be used to inform each other, but not at the expense of disregarding other evidence.

Some people have argued for a reading of the allegories that asserts the correspondence purely for the sake of preserving the symmetry. I shall be avoiding this; in addition, I also want to avoid readings that are driven by properties of the allegories that are possibly accidental, or that Plato may not necessarily see as relevant to what he wants to say.

For example, Foley (2008 pp. 1-23) tries to argue that we can read more than Plato says into the mathematical proportions of the line. He points out that, although Plato never explicitly mentions the fact, the two middle subsections of the line are mathematically equal. However, in what he calls the overdetermination problem, he says that, according to the best interpretation of the passage, they are actually unequal, because ‘they represent mental states of unequal clarity, and possibly also objects with unequal degrees of reality’ (Foley [2008] p

⁸⁵ See especially Annas (1981) pp. 245- 246.

1).⁸⁶ Plato was certainly aware of this, says Foley, and intended it to be discovered: ‘in Plato’s day and ours, only curiosity and rudimentary skills are required to discover it’ (Foley [2008] p 3). Moreover, Foley thinks that this is an intentional feature of the divided line analogy.⁸⁷ He goes on to give his own theory about why Plato chose to do this, but given that Plato never specifically mentions the problem, he relies on the idea that it was intended to be built into Plato’s theory implicitly.

Was this an intended feature of the line at all? Perhaps this is a minor unintended implication of the division procedure. Foley calls this the Gaffe Interpretation, saying that it suffers from a disadvantage from the outset, because ‘it is preferable, wherever possible, to prefer a consistent, successful interpretation’ (Foley [2008] p 12). This seems unfair, as Foley would surely not want to say that we should prefer a reading that made Plato’s argument consistent with other implications of his metaphors: if the sun will die one day, what does this mean for the analogy with the Form of the Good, which is said to be immortal? Foley seems to think that we need to argue that Plato is unaware of the mathematical consequences of his instructions, but this need not be the case. We need not claim that Plato was ignorant of the mathematical properties of the line, only that he viewed certain properties of it as irrelevant to his point.⁸⁸

In contrast to those who take such a literal reading of the divided line, Bedu-Addo ([1978] pp. 111-127) argues that the mind of the dianoetic mathematician cannot grasp the mathematical Forms or any other supra-sensible entities at this stage. Plato describes him as dreaming about reality, and having *doxa*, rather than *epistēmē* (*Republic* 533b and 534c). It is only for the purpose of the divided line that Plato regards the *doxa* of *dianoia* as a lower

⁸⁶ This is not the first statement of the problem. See Brumbaugh (1954) pp. 91-104.

⁸⁷ Cf Denyer (2007) pp. 284-309, who notes the same thing.

⁸⁸ Foley also rejects the view that the subsections of the divided line are merely demarcation devices, as Reeve (1988) suggests: ‘This equality is simply not a functioning part of the simile, as the green of a shamrock was not a functioning part of saint Patrick’s use of it as a simile for the trinity.’ Foley dismisses this interpretation on the grounds that Plato compares *pistis* and *dianoia*, and that *dianoia* is arguably a clearer mental state.

grade of knowledge. Rather, we should interpret the four states of mind in the light of the metaphor of being awake and dreaming in relation to images and their originals.

In fact, my position is very close to Foley's position, emphasising the importance of mathematical thought in Plato, in that Foley thinks that Plato is committing himself to the importance of thinking mathematically in philosophical activity. To be fair to Foley, he does try to support his reading by reference to the importance that Plato gives to mathematics in the middle books of the *Republic*, and thinks that the line is an ontological taxonomy rather than an analogy *per se* (Foley [2008] p 20). However, he relies too much on implicit properties of the line, which Plato has himself acknowledged, is not a perfect account of his theory.⁸⁹ Foley's own suggestion, that Plato intended the equality of these two subsections to prompt debate outside the dialogue on the comparable qualities of the states of mind, is as far as I am prepared to go in terms of drawing conclusions from the mathematical proportions of the line; drawing conclusions about doctrine requires much more evidence than this.

To summarise, although we should be wary when trying to use these allegories to provide a literal and coherent account of Plato's theory, they can, when used carefully, have a lot to tell us about Plato's theory. The allegories should not be forced into an exact correspondence, but they can certainly be used to inform each other, and their implicit properties should be seen as additional avenues of investigation, rather than the driving force of interpretation.

⁸⁹ Other scholars go deeply into the mathematical properties of the divided line. See, for example Balashov (1994) pp. 283-295, who discusses the division of the line into the mean and extreme ratio. I am certainly not against investigation into the divided line's mathematical properties. In fact, I wholeheartedly agree with Foley that it is a virtue of this passage that the reader is encouraged to 'interact dialogically with Plato's writings, then leave them behind and think for herself' (2008) p 23. My point is simply that we cannot use these implications to argue for a certain reading of Plato's theory without more textual evidence than Foley seems to think necessary.

ii. *Ascending the Scale*

I said that I would try to show that the divided line really is a continuum, and the four parts of it are not homogenous. This point is at the heart of my reading of the divided line, and is important to my solution to the problem of how hypotheses and imagery relate to each other. I am going to suggest that the early stages of *dianoia* look different from the later stages, as mathematical reasoning gradually turns the mind towards *noēsis*. The hypotheses and images are gradually replaced by *ti estis* (which, I shall explain, are Platonic definitions; see especially section two, chapter one for this) as the mind becomes more experienced in dianoetic thought, until at the later stages, it uses no images and reaches for the higher hypotheses of *noēsis*. I will try to show that arguments do not generally involve only one concept, so we may have a mixture of *ti estis*, hypotheses and images in the same discussion, which would place it in the higher stages of *dianoia*, below *noēsis*. It is beyond the scope of this project to resolve the debate about the nature of the Form of the Good, but I shall suggest that it is only at the later stages of *noēsis* that this is grasped by the philosopher.

Inevitably, I shall need to argue for rather a wide scope to the states of mind. *Eikasia*, I shall argue, is not restricted to the images of physical things (either as contents or as objects), but includes the images made by artists that shape Greek moral life; *pistis* includes all physical things and *dianoia* is mathematical reasoning in philosophical arguments, as well as the investigations of the mathematicians.⁹⁰ For the points about *eikasia* and *pistis*, I shall be appealing to books II, III, VI, VII and X of the *Republic*. For the point about *dianoia*, I shall appeal to the *Republic*, but also try to support this reading with additional textual evidence in the *Meno* and *Phaedo* sections.

When I introduced the allegories, I pointed out that, when he comes to *dianoia*, there is a shift in the way that Plato is describing the parts of the divided line, at least if we take the

⁹⁰ Cf Fine (1999) p 233: ‘To be sure, the Line (unlike the Cave) is not an allegory. It describes literal examples of cognitive conditions – but they are only illustrative, not exhaustive, examples.’

traditional ‘objects’ reading. According to this reading, for Plato the objects of inquiry are the first two subsections of the line: images and physical things. For the next two sections, he seems to focus more on the process by which the mind inquires, and the objects that the mind uses to inquire. He says that, in *dianoia*, the mind uses images and hypotheses; in *noēsis*, it proceeds from assumption to a first principle...pursuing its inquiry solely by and through forms themselves’ (510b).

Forms must be the object of *noēsis*: even Gail Fine’s ‘contents analysis’ of the line allows for this; she just thinks that Forms are not exclusively the subject-matter of *noēsis*. However, no-one would want to argue that hypotheses and imagery are the *objects* of *dianoia*; even those who support the ‘objects analysis’ of the line would want to say that the objects of *dianoia* are Forms (mathematical or otherwise) or mathematical intermediates, if they support that particular reading.⁹¹

Of course, this does not mean that the divided line passage contradicts the objects analysis; it is perfectly compatible with what Plato says in this section. It does mean that, if you want to support that particular reading, you need to admit that Plato is switching focus here, unless you want to say that Plato is talking about the tools that the mind *uses* for the first two subsections, instead of, or in addition to, the objects of inquiry. This is what I am going to suggest. As far as I am aware, my reading will be compatible with both the objects analysis, and the contents analysis, although it does share some things in common with Fine’s reading. What defines the states of mind of the divided line, I propose, is not the objects of inquiry, but the process of inquiry. In this, I am saying something similar to Fine, but also something less radical: I am not committing myself to the fact that *epistēmē* and *doxa* have distinct objects at this point.

⁹¹ This is supported by *Republic* 510-511: “it is not about the square or diagonal which they have drawn that they are arguing, but about the square itself or diagonal itself, or whatever the figure may be.”

So what defines each state of mind? It is not, I shall argue, the objects of their inquiry, nor is it an awareness of the nature of these objects. That is, the fact that *pistis*, for example, may draw conclusions about physical things does not make it *pistis*; nor does an awareness that these physical things are different from intelligibles or images define *pistis* as a state of mind, although both of these things may be features of it. What defines *pistis* is that it uses physical things to draw conclusions. Each of the *pathēmata en tēi psuchēi* is what it is not because of its subject-matter, but because of the tools it uses to do its work.

a. *Pistis*

Pistis, like *eikasia*, has a specific set of objects connected to it, which in this case is the set of physical things. Traditionally, this has been read as Plato emphasizing that *pistis* is *about* physical things, rather than that it *uses* them in its work. Undoubtedly, Plato introduces the divided line as an extension of the division that he has drawn in the allegory of the sun between the visible and intelligible worlds (*Republic* 509d). In 509e-510b, he simply seems to be talking about what kind of objects there are in each realm, without saying whether they are the tools or the subject-matter of each state, but in the descriptions of *dianoia* and *noēsis*, he is specifically talking about their tools. What if Plato meant for his descriptions of each subsection to be continuous throughout the passage? This would mean that, in *eikasia*, the mind uses images of sensibles with which to reason, and in *pistis*, it uses physical things.

If we think about *pistis* in this way, we can place the method of the natural scientists in this scheme.⁹² Although nothing that would satisfy our definition of an ‘experiment’ existed in Greece at the time, there have been some attempts to use and manipulate the

⁹² That is, insofar as they were natural scientists at all. In Plato’s day, the natural sciences were in their infancy. The Presocratic philosophers did make some advances in the field, but there was no developed methodology, and their experiments were more like demonstrations. There was no demarcation of the natural sciences from the other disciplines, and the early scientists were really early philosophers as much as scientists.

physical world in order to discover facts about it.⁹³ Moreover, this seems to be the kind of thing to which Socrates refers in his criticism of Anaxagoras in his autobiography in the *Phaedo*. I will examine this in greater detail in the *Phaedo* section, and justify the connection I make between the autobiography and the divided line but for now I just want to mention it as an illustration of *pistis*.

In his intellectual autobiography in the *Phaedo*, Socrates hopes to have found an authority on causation in Anaxagoras. However, these hopes are quickly dashed, when he finds that Anaxagoras uses physical entities to explain phenomena: things like air, ether and water ‘and many other oddities’ (*Phaedo* 98bc). Anaxagoras is using the physical world as a tool for inquiry: he wants to know about causation, so he looks at physical things in order to produce an account.

Socrates’ problem with this kind of explanation is that it is no more use than saying that Intelligence is the reason for everything Socrates does, but then explaining his actions in terms of bones, sinews and other physical properties of the body. As Socrates points out, the reason his sinews and bones are in a prison in Athens, and not in some other place, is that he felt it was better to remain and submit to the penalty that had been imposed upon him (*Phaedo* 98e-99a). His major argument against Anaxagoras is that he is unable to distinguish between ‘the reason for a thing and the condition without which the reason couldn’t be operative’ (99b). That is, Socrates’ physical characteristics are required for him to be in a cell in Athens, but the reason for his being there is that he has decided it is better that way. We shall return to this in Section Three, Chapter Two i and ii.

⁹³ In order to support his argument that void does not exist, Anaxagoras inflates a wineskin with air, and tortures it to demonstrate that the air offers resistance. He encloses air inside a water-thief to show that the air assists in moving the water, in the style of a pipette (Aristotle, *Physics* 213a22-213b). Empedocles is convinced that there is no such thing as void and refers to the operation of a clepsydra, which, like Anaxagoras’ water-thief, lifts quantities of water out of the river using trapped air (Aristotle, *On Youth, Old Age, Life and Death and Respiration* 473a15. Other observations/experiments of Empedocles include the investigation of what the modern scientist would call centrifugal force with water in a cup, to develop theories about the motion of the heavens. See Aristotle, *On the Heavens* 295a15-22).

Pistis need not be limited to investigation into the natural world. In fact, I would suggest that many of Socrates' interlocutors have this state of mind when they investigate moral entities. Often, when Socrates first poses a question about the *ti esti* of a thing, his interlocutor replies by listing instances of it from the physical world. In *Euthyphro*, when Socrates asks Euthyphro what holiness is, Euthyphro replies by giving particular examples from the sensible world:

Well, I say that holiness is what I am doing now, prosecuting a criminal either for murder or for sacrilegious theft or for some other such thing, regardless of whether that person happens to be one's father or one's mother or anyone else at all, whereas not to prosecute is unholy. Take a look, Socrates, and I'll show you clear evidence of divine law... (*Euthyphro* 5de).

Meno does the same. When Socrates initially asks him for a definition of virtue, he provides a list of examples, drawn from his experience of the sensible world: a man's capability of managing the city's affairs; a woman's skill in being a good housewife, her care with her stores and obedience to her husband (*Meno* 71e, 72a). Theaetetus' response to Socrates' question about knowledge is to list the kinds of knowledge that he knows from the sensible world, rather than thinking about knowledge itself: geometry, cobblery 'and so on' (*Theaetetus* 1146cd).

Pistis, then, is drawing conclusions from observations of the physical world, whether that is from natural phenomena, or observations of human actions to draw conclusions about morality. The tools of *pistis* are physical things, but this does not mean that its subject-matter need be limited to the physical world; at this point, we are not committed either way to the objects or contents analysis of the divided line.

b. *Eikasia*

With respect to *eikasia*, we are told little of the contents (or objects, depending on which reading you choose it take) of this section of the line, just that it is ‘one subsection of images...shadows, then reflections in the water and other close-grained, polished surfaces, and all that sort of thing, if you understand me’ (*Republic* 510a). If we take this to be an exhaustive list of what *eikasia* involves, it does seem rather obscure, as Annas complains (Annas [1981] p 250). However, I want to suggest that Plato might have in mind something that played very large role in Greek life: making claims based on poetry and the arts.

As we can see from books III and X of the *Republic*, art and poetry contain reflections that are a third remove from reality. These are images, just as the shadows and reflections in the water and polished surfaces are images. In this way, I want to argue, we can legitimately make comparisons (in terms of ‘degrees of clarity’) with ‘all that sort of thing’ that Plato means when he refers to the shadows and reflections at *Republic* 510a.

Plato describes poetry as *mimēsis* in book III of the *Republic*. He asks, ‘is not to assimilate oneself to another person in speech or manner to ‘represent’ the person to whom one is assimilating oneself?’ (393c). This kind of representation, says Plato, has far-reaching implications, and he goes on to discuss the effect of it on the citizens of the ideal state.

In book X, Socrates explains the ‘real nature’ of such representations as poetry,⁹⁴ an explanation that also applies to art. The group’s procedure is that ‘we usually postulate a single form for each set of particular things, to which we apply a single name’ (596a). So there is one Form for a bed, and one for a table. Next, we have the physical instantiations of the bed and the table. A painter can create a bed in some sense, but this only a representation of the physical object; this is even compared to holding a mirror up to the world to ‘make’ a reflection (596de). The same goes for the (tragic) poet: ‘if his art is representation, (it) is by

⁹⁴ At least, the poetry that he classes as ‘representation’ in book III.

nature a third remove from the throne of truth; and the same is true of all other representative artists' (597e). In fact, the artist does not even represent physical things as they are, but as they appear, for if you look at a physical bed from a different angle, it will look different. Plato then goes on to examine the implications of this for poetry:

We must go on to examine the claims of the tragedians and their chief, Homer. We are told that they are masters of all forms of skill, and know all about human excellence and defect and about religion; for – so the argument runs – a good poet must, if he is to write well, know all about his subject, otherwise he can't write about it. We must ask ourselves whether those who have met the poets have, when they see or hear their works, failed to perceive that they are representations at the third remove from reality, and easy to produce without any knowledge of the truth, because they are appearances and not realities; or are they right, and do good poets really know about the subjects on which the public thinks they speak so well? (598e-599a).

The answer to the final question is negative. If a man was capable of producing an original, surely he would not devote himself to the manufacture of copies (*Republic*). The tragedian does not really know about the things he represents:

We may assume, then, that all poets from Homer downwards have no grasp of truth but merely produce a superficial likeness of any subject they treat, including human excellence (599a).

Plato hints at the connection of this passage to the divided line. A painter may paint a picture of a bridle and bit, but these are made by a harness-maker and smith. The makers do not *know* what the bridle and bit ought to be like (or how to make a set); this knowledge

belongs to the horseman, who knows how to use them (*Republic* 601c). The maker of an implement has *πίστιν ὀρθήν*: The user has direct experience, which gives him knowledge (*Republic* 601c-602a).⁹⁵

What Plato is doing here is evaluating the epistemic merit of reasoning through images of sensibles. When we read the works of a poet, we are studying something that is a long way from truth; we are engaging with images of sensibles, and our state of mind is *eikasia*.

As Lee points out in his translation of the *Republic*, ‘the claims made for the poets by Greek opinion were often extravagant. They treated the works of Homer and the poets as their Bible, and in Plato’s *Ion* Homer is claimed as a teacher of everything from carpentry to morals and generalship’ (Lee [1987] p 359). In this case, *eikasia* is not as insignificant as Annas claims.⁹⁶

On this reading of *eikasia*, we see how prominent it was in Greek life, and it becomes clear why Plato saw fit to give it a place on the divided line, and devote so much of *Republic* X in discussion of it. I would even suggest that we see hints of *eikasia* in the dialogues. In *Protagoras* 339a-347, Socrates and Protagoras engage on a discussion of Simonides’ poetry. This is Protagoras’ suggestion: he thinks that to become an authority on poetry is the most important part of a young man’s education. The passage forms part of a discussion on the nature of virtue, and the poetry is used to try to support the positions taken by Socrates and Protagoras in the dialogue. Socrates indulges Protagoras, but at the end of the passage suggests that they leave the subject of poetry:

⁹⁵ This is a potential stumbling-block for Fine’s rejection of knowledge by acquaintance in Plato.

⁹⁶ It has been pointed out to me that the word *eikasia* itself does not necessarily involve mistaking an image for reality, and that there is no suggestion that a reflection in water is not recognised as a reflection. Here is one of the points at which I think the line and cave allegories diverge: the fact that the prisoners in the cave fail to recognise that the shadows are not the true reality does not mean that everyone exercising *eikasia* is in the same position. For example, we might want to draw conclusions about morality by reading Homer, but that does not mean that we do not recognise Homer as a poet. As I will argue, whether or not one is in a particular state of mind does not depend on this kind of awareness.

Conversation about poetry reminds me too much of the wine-parties of second-rate and commonplace people. Such men...(are) too uneducated to entertain themselves as they drink by using their own voices and conversational resources... (*Protagoras* 347cd).⁹⁷

Socrates stresses the indirectness of this route to truth, although it forms such a large part of the Greek educational system. Having said this, in books II and III of the *Republic*, he does allow for the use of poetry as an educational tool: children learn from imitation, but adults are not exposed to it. Initially incapable of understanding, children can develop a taste for beauty and goodness (*Republic* 402a).⁹⁸ This reinforces the idea of the divided line as an epistemological scale: *eikasia*, although the least preferable state of mind, can form the initial stages of education. Imitation is worthless as a source of knowledge, but may constitute a form of ‘play.’⁹⁹ *Eikasia*, then, is drawing conclusions based on images, whether that is conclusions about a physical object based upon a reflection or shadow of it, or about morality, based upon the ‘reflections’ in the arts.

c. *Dianoia*

I have argued for a reading of *pistis* and *eikasia* that takes the objects associated with them, physical things and images of physical things, to be the tools the mind uses when it is in that particular state. Although Plato is not specific about the fact that these are tools in the *pistis* and *eikasia* passages, I have grounded my reading in the remarks Plato makes about poetry

⁹⁷ Simonides is a lyric poet, so he escapes the harshest criticisms levelled at the poets in book III of the *Republic*. In lyric poetry, the poet speaks in his own person, so he is not producing the kind of *mimēsis* of which the tragic, comic and epic poets are guilty. On the other hand, drawing conclusions from a discussion of lyric poetry is still an indirect route to truth (*Republic* 393c-394c).

⁹⁸ Cf Nehamas (1999) pp. 251-269, who discusses the relevance of poetry in the early stages of development.

⁹⁹ See *Republic* 424-426: *paidia* is only to be forbidden if it is harmful. See Nehamas (1999) pp.251-269 for a discussion on this.

and the natural sciences in the *Republic*, *Protagoras* and *Phaedo*, as well as the actual practice of some of Socrates' interlocutors in the dialogues. This, I have argued, gives us good evidence for taking the 'tools' reading of the passages when the text is unclear. In itself, this reading does not mean that we have to reject the 'objects' reading, because we could still say that, in *pistis*, for example, the mind uses the tools of physical objects to reason about physical objects.

What it does mean, though, is that Plato's main concern in the divided line passage is the tools that the mind uses in each subsection of the divided line. This is explicitly clear when Plato turns to his description of *dianoia*. Here, as I stressed when I introduced the allegories, Plato's remarks are about what the mind uses in inquiry. In this section, I take the *dianoia* passage to be very much about the methodology, a view which has support from other scholars, such as Benson. However, unlike Benson's 'methodological' reading, I am going to argue that the method of mathematics and the dianoetic method are the same, although the latter has a much wider scope in terms of subject matter. That is, it can be applied to ethics, mathematics and any other philosophical subject.

Benson distinguishes between the 'mathematical' and 'methodological' view of the line (2010 pp. 188-208; 2011 pp. 1-34). The mathematical view, endorsed most famously by Burnyeat ([2000] pp. 1-81), is that the mathematical method is identical to the dianoetic method. Benson ([2011] p.2) describes this view as follows: "Plato is not distinguishing between a correct and incorrect application of the same method (or even between a better and worse method...Rather dianoetic is the correct application of the same method applied by dialectic. Dialectic is simply reserved for a further inquiry - an ontological inquiry - not pursued by dianoetic." My reading has many similarities with this view, although I arrive at my reading through rather different means. The methodological view, Benson says, is that Plato does not identify mathematics and *dianoia*. Rather, mathematics as it should be done is

identical to dialectic, and the dianoetic method is rather the misapplication of the mathematical method by some mathematicians.

Both arguments have strong support. I am committed to the view that, when Socrates says that he is going to apply the practice of the geometers in the *Meno*, and when we reach the hypothetical passage in the *Phaedo*, he is committed to applying the method of mathematics to philosophy. Moreover, this is a second-best method, identical with the dianoetic method of the divided line. I shall make this argument in the *Meno* and *Phaedo* sections of this thesis; here, I want to look at the debate in the context of the *Republic*.

Myles Burnyeat, whose reading falls into what Benson would call the ‘mathematical’ reading, distinguishes between two possible ways in which mathematics could be good for the soul. Isocrates, Plato’s arch-rival, argued that the value of mathematics is not the knowledge you gain in doing it, but process of acquiring it. Burnyeat likens this to a ‘dry-as-dust classicist for whom the value of learning Greek had nothing to do with the value of reading Plato or Homer,’¹⁰⁰ but rather it has to do with the rigorous discipline that helps train the mind.

On the other hand, Alcinous, a later Platonist, says that mathematics provides the precision needed to focus on real beings: ‘mathematical objects can only be grasped by precise definition, not otherwise, so there is good sense in the idea that precision is the essential epistemic route to a new realm of beings.’¹⁰¹ This view, Burnyeat likens to ‘enlightened classicists (who) promote Greek and Latin as means of access to a whole new realm of poetry and prose which you cannot fully appreciate in translation.’¹⁰²

Burnyeat says that the latter view seems more satisfactory. However, he argues that it does not go far enough. The comparison for his view ‘would be with a classicist who dared

¹⁰⁰ Burnyeat (2000) p 3.

¹⁰¹ Burnyeat (2000) p 5; Annas (1981) pp 238-9, 250-1 and 272-3 also comments on this point.

¹⁰² Burnyeat (2000) p 5.

claim that embodied in the great works of antiquity is an important part of the truth about reality and the moral life' (Burnyeat [2000] p 5).

Burnyeat points out that the goal of the mathematical curriculum is knowledge of the Good.¹⁰³ We need to remember that, for Plato, the 'real' objective world is the world of Forms. The idea that we do not see the world as it is actually is not strange to the modern reader of Plato; but for us, the 'world as it actually is' is, according to scientists and many philosophers, a world of forces and particles, stripped of morality and values. For Plato, Burnyeat reminds us, mathematics can tell us about how things are objectively speaking, which is 'precisely what enables us to understand goodness.'¹⁰⁴

Burnyeat's point that mathematics' value is not purely instrumental is evident in book VII of the *Republic*. Socrates says that mathematics 'is really necessary to us, since it so obviously compels the mind to use pure thought in order to get at the truth' (*Republic* 526ab).¹⁰⁵ Burnyeat admits that Plato also likes the fact that arithmetic makes you quicker at other studies ('transferable skills,' as Burnyeat puts it [2000 p 9] ¹⁰⁶) and the fact that its demanding nature makes it a good test of moral calibre.¹⁰⁷ However, Burnyeat argues that the transferable skills and practical application of mathematics (eg, the practical application of geometry in war¹⁰⁸) are by-products, coming second in importance to pure theoretical knowledge. He cites several passages to support this,¹⁰⁹ but the decisive passage is *Republic* 527de:

¹⁰³ Referring to *Republic* 526de, 530e, 531c, 532c.

¹⁰⁴ Burnyeat (2000) p 8. Moreover, says Burnyeat, 'It is this concept of impersonal, objective goodness that links the epistemology and metaphysics of the Republic to its politics' (2000) pp 8-9.

¹⁰⁵ This point is also made at *Republic* 524d-526b.

¹⁰⁶ Supported by *Republic* 526b, 522c.

¹⁰⁷ See also *Republic* 503ce, 535a-537d for this.

¹⁰⁸ Also found at *Republic* 526d.

¹⁰⁹ *Republic* 525bc, in which the philosopher needs mathematics for a different reason than the generals; 526de, in which Socrates says that we should be thinking about whether geometry will help us to know the Form of the Good.

‘You amuse me,’ I said, ‘with your obvious fear that the public will disapprove if the subjects you prescribe don’t seem useful. But it is in fact no easy matter, but very difficult for people to believe that there is a faculty in the mind of each of us which these studies purify and rekindle after it has been ruined and blinded by other pursuits, though it is more worth preserving than any eye since it is the only organ by which we perceive truth.’

Burnyeat stresses: ‘The benefit of mathematics does not reside in its rigorous procedures’ (Burnyeat [2000] p 13). Rather, it turns the mind away from the sensible world towards the intelligible (*Republic* 527bc). Burnyeat argues that the mathematician does not study Forms directly, as we saw above, but that it is an intermediate epistemic state, which plays a pivotal, highly positive role in education. Mathematics is ‘the lowest-level articulation of the world as it is objectively speaking’ (Burnyeat [2000] p 42).

Moreover, Burnyeat argues, ‘mathematics and meta-mathematics prescribed for the future rulers is much more than instrumental training for the mind. They are somehow supposed to bring an enlargement of ethical understanding’ (Burnyeat [2000] p 46). Burnyeat’s solution to the problem of how mathematics may perform this role is to appeal to the ethical value of concord and attunement. Burnyeat says that the concord, unity and proportion that we learn about through mathematics are valued because they create and sustain unity.

Burnyeat appeals to Plato’s work in the *Timaeus* to support his reading. Concordant intervals sound good to the ear because of their imitation of the divine attunement (*Timaeus* 80ab; 67ac).¹¹⁰ There is ethical value in concord and attunement, not least because of the strong emphasis placed on it in the *Republic*.¹¹¹ Mathematical harmonics, argues Burnyeat, gives the philosophers ‘an abstract, principles understanding of structures they will want to

¹¹⁰ Burnyeat also makes this point in (2000) pp. 47-53 and 64-67.

¹¹¹ See also *Republic* 401cd; 410a-412b, 441e-442a; 430e, 431e; 432ab, 442cd; 443de; 522a; cf Burnyeat (2000) pp. 53-54 for further explanation.

create and sustain when they return to the cave...Thus, knowing what numbers are concordant, and why, has a very great deal to do with the tasks of government, because concord is an important structural value at the lower level of ethics and politics' (Burnyeat [2000] p 56).

A satisfactory account of *dianoia* should explain how it can lead to *noēsis*. One way of doing this, as Burnyeat does, is to show that mathematics turns the mind towards intelligible things. The main virtue of Burnyeat's account is that, along with accounting for the high level of respect that Plato has for the mathematical disciplines, he also provides a mechanism for ascending the epistemological scale using mathematics, which means that *dianoia* fits coherently into the divided line.

For Burnyeat, mathematics is crucial in turning the mind away from the sensible world and towards the intelligible. He thinks that *Republic* 524d-526b is actually the first example of how it might do so. Here, Plato discusses the education of the philosopher, of which a major part is mathematics. Initially, the philosophers are to be trained in literature and music, which is to be complemented by physical training, but now Socrates is looking for a subject that will draw the minds of the philosophers from the world of change towards reality. He says that the discipline that all occupations make use of, but that, unlike other disciplines, leads to *noēsis* is mathematics.

According to Socrates, there are some things which have the drawing-power and some which do not:

You see, there are some perceptions which don't call for thought (*noēsis*), because sensation can judge them adequately, but others which demand the exercise of thought, because sensation cannot give a trustworthy result...By perceptions that don't call for thought I mean those that don't simultaneously issue in a contrary perception; those that do call for thought

are those that do so issue in the sense that in them sensation is ambiguous between two contraries, irrespective of distance (*Republic* 523ab).

A finger, says Socrates, is an example of a perception that does not call for thought: 'Each of them looks as much a finger as any other... There is nothing here to force the mind of the ordinary man to ask further questions or to think what a finger is; for at no stage has sight presented the finger to it as being also the opposite of a finger' (523d).

The problem for Plato is that he needs to distinguish between the various mental representations here: he wants to say that the unit can somehow result in a higher kind of thought than the finger, but how are these objects of thought different? We are only given a hint at the answer: Plato describes how the 'perception' of the unit 'by sight or any other sense' (524d) does not draw the mind towards reality any more than the perception of a finger. However, when seen in conjunction with a plurality, as happens in arithmetic, it naturally 'calls for the exercise of judgement and forces the mind into a quandary in which it must stir itself to think, and ask what unity itself is; and if that is so, the study of the unit is among those that lead the mind on and turn it to the vision of reality' (524d). The difference here is analogous to the difference between thinking 'I am hungry' and thinking about the nature of hunger itself.

Philosophers are thus asked to study arithmetic until they come to understand 'by pure thought,' (525c) the nature or metaphysics of numbers. In this way, the mind is forced to argue about 'numbers in themselves' (525d). Burnyeat describes the ascent thus:

It is the most elementary example of the intellect (the instrument of the soul) being forced to turn towards something non-sensible and abstract. The next step is to go beyond counting and calculating to begin a systematic study of what Socrates calls 'the nature of the numbers' (525c) or 'the numbers themselves' (525d). Note the plural. This is number theory as we find

it in Books VII-IX of Euclid's Elements. Recall the variety of kinds of number that Euclid sets out for study: even-times even, even-times odd, odd-times odd, prime number, numbers prime to one another, composite number, numbers composite to each other, perfect number. As you leave behind the everyday practice of counting and calculating (whether for trade or military purposes), a whole new realm of abstract objects opens to the eye of the soul (Burnyeat [2000] pp 75-76).

Miller ([2007] pp. 310-342) provides a similar, fuller account of how mathematics might turn the soul towards the Forms, which accords with my idea of the divided line as a continuum. At *Republic* 394d, Socrates says that there is a 'longer and fuller way' to get the 'best possible view' of the soul and its virtues, but does not take it here. At 504c, he suggests that the 'longer way' is the educational process of the philosopher-kings. The five mathematical studies (arithmetic, plane geometry, solid geometry, astronomy and harmonics), taken as a sequence, form a 'purgative ascent' (Miller [2007] p 320) to the threshold of the Forms. Each stage of the mathematical curriculum in book VII purges a little more of the visible and sensible world. In calculation and arithmetic, we use pebbles in our visible models and many instantiations of pure units, 'but it is by means of their spatial arrangements – in expanding triangles, squares and oblongs – that their defining kinds, the series of integers and of odds and evens, are collected and distinguished for thought' (Miller [2007] p 321). In geometry, 'we drop the pebbles and the units they represent in order to let the figures that they compose emerge in their own right and come to stand as proper objects' (Miller [2007] p 321). In harmonics, by way of astronomy, we make another purgation: 'we drop these figures in order to let the ratios they express emerge in their own right and come to stand as proper objects' (Miller [2007] p 321). These ratios are neither visible nor spatial. Step by step, we have purged the sensible aspect of our studies, and prepared ourselves to turn to dialectic.

The mathematical studies, says Miller, allow us to ‘understand the intelligible as the *very structure of the sensible*.’ Or, in Socrates’ ontological terms, even as we come to understand ‘being in its irreducible difference from and priority to ‘becoming,’ we also come to understand it as the very being of that which becomes. Accordingly, the ‘conversion should be understood as a process not just of departure, but, rather, of departure that is also return; in bringing the soul to the pure ‘understanding of being, philosophical education will bring it to the ‘understanding’ of becoming as well, in its dependence on being’ (Miller [2007] p 322-323). This account appeals to me, because it provides the bridge between the intelligible and the sensible that I said Gonzalez needed, when he tries to explain the application of knowledge of the Forms to the sensible world. It is a very elegant account, as it allows Plato’s metaphysics to do some of the work of his epistemology.

Miller’s and Burnyeat’s explanations of the return of the mind is close to my own in that he makes mathematics the stepping stone to the world of the Forms, after being confined to the physical objects of *pistis*. However, I do not see that such an ascent should be made solely through the subject matter of mathematics, when Plato makes it clear that the mathematical method can be applied to other areas of philosophy. We see this in the passages of the *Meno* and *Phaedo* that will be discussed in the following sections, as well as in the ascent passage of the *Symposium*, which we mentioned before. My own reading makes not mathematics, but the *mathematical method*, the stepping stone, something I have in common with the ‘methodological’ approach.

Benson ([2011] pp. 1-34) calls Burnyeat’s reading the ‘mathematical reading,’ as opposed to his own ‘methodological reading’. He admits that Burnyeat has made an impressive defence of this method, but says that, in spite of the many things to recommend it, Burnyeat’s reading is incorrect. Benson thinks that the dianoetic method is not identical to the mathematical method, but rather the misapplication of it by a subset of practising

mathematicians. In this sense, says Benson, Plato is not criticising mathematics, but mathematicians. The mathematical method and the philosophical method are identical. It is their subject matter that differs.

As Benson concedes, the mathematical method has much to recommend it. Plato approves of the mathematical disciplines, and recommends the study of them, as we can see from *Republic* VII. Benson acknowledges that Plato's remarks here are hardly critical of mathematics, and agrees with Burnyeat that the mathematician is forced, *anagkazetai*, to use hypotheses; he is not finding fault with this, as it is a necessary part of mathematics. The difference between *noēsis* and *dianoia*, says Benson, is not *dianoia*'s use of hypothesis, because dialectic uses these too. Rather, it is the use of images in *dianoia*, and the mistaking of the hypothesis for first principles by the dianoetician.

Burnyeat anticipates the claim that it could be the mathematicians, not mathematics, that Plato is criticising, remarking that the mathematician makes epistemological, rather than methodological, mistakes. The claim to 'know' the hypothesis, if it is ever made, is the problem, not the fact that the hypothesis is used. But, claims Benson, Plato is not talking about good mathematicians; *dianoia* is the practice of bad mathematicians.

Dianoia must be the practice of a select group of mathematicians who incorrectly apply the mathematical method, says Benson. We know that Plato distinguishes between method and its practitioners from *Euthydemus*:

Socrates: My dear Crito, don't you realize that in every pursuit most of the practitioners are paltry and of no account whereas the serious men are few and beyond price? For instance, doesn't gymnastics strike you as a fine thing? And money making and rhetoric and the art of the general?

Crito: Yes, of course they do.

Socrates: Well, then: in each of these cases, don't you notice that the majority give a laughable performance of their respective tasks?

Crito: Yes indeed – you are speaking the exact truth.

Socrates: And just because this is so, do you intend to run away from all these pursuits and entrust your son to none of them?

Crito: No, this would not be reasonable, Socrates.

Socrates: Then don't do what you ought not to, Crito, but pay no attention to the practitioners of philosophy, whether good or bad. Rather give serious consideration to the thing itself: if it seems to you negligible, then turn everyone from it, not just your sons. But if it seems to you what I think it is, then take heart, pursue it, practise it, both you and yours, as the proverb says (*Euthydemus* 307ac).

Here, Plato is clearly distinguishing between philosophy and its practitioners, and claiming that this distinction could be made in every pursuit, which presumably includes mathematics. This, says Benson, should make us more comfortable with the idea that Plato distinguishes between mathematics and the mathematicians in the *Republic*. In support of Benson's claim, I would concede that Plato makes the same distinction between philosophy and its practitioners in the *Republic*. Plato remarks,

But far the most damaging reproach to philosophy is brought on by those who pretend to practise it, and whom your critic has in mind when he says that most people who resort to it are vicious, and the best of them useless – a criticism with which I agreed, did I not?... (But) it's not philosophy's fault (489de).

However, I would stress that this in itself is no reason to suppose that Plato is making the same distinction in the divided line passage. My reading of the divided line is as a

classification of the different states of mind, when the mind pursues certain types of inquiry, not as comparing good and bad practice of the same method. Even if we were to allow that Plato does use such a distinction in this passage, why should we not say that the difference is between better and worse philosophers, rather than mathematicians? That is a step closer (although not identical) to my reading of *dianoia* as the second best philosophical method.

Having made the point that Plato would distinguish between mathematics and the mathematicians, Benson goes on to consider the mathematicians about whom Plato might be speaking. Of course, Plato is familiar with talented mathematicians like Theaetetus and Eudoxus, but surely also with less accomplished mathematicians like Hippias, whose thinking seems to be characterized by the flaws of *dianoia*: Hippias is prone to thinking he knows things he does not, thus taking hypotheses to be *archai*. Moreover, his work in harmonics might suggest an inappropriate appeal to sensible objects. Benson points out that Plato associates Hippias with mathematical expertise in three different dialogues. Hippias professes to be, or is reported as professing to be, an expert in arithmetic (*Hippias Minor* 366cd; *Hippias Major* 285bd; *Protagoras* 318d), astronomy (*Hippias Major* 285bd; *Protagoras* 318d), geometry (*Hippias Major* 285bd; *Protagoras* 318d) and music (*Hippias Major* 285bd; *Protagoras* 318d). Benson argues that, although we think of Hippias as a sophist, these passages suggest that Plato also viewed him as a mathematician, albeit a self-professed one. Also, Theodorus, described by Socrates as an expert in geometry, astronomy and music (*Theaetetus* 147d-148b), is recognized as a less accomplished mathematical practitioner than Theaetetus.

Benson's point is that Plato may fail to criticise arithmetic and geometry in the course of describing his educational scheme, but this does not mean that they are being practised correctly. To say that they would be 'both prima facie implausible and in conflict with Plato's claim in the *Euthydemus* that in every pursuit most of its practitioners practise it

incorrectly, as well as with Plato's assessment of Hippias and perhaps Theodorus' (Benson [forthcoming] p 23). Once again, I think it is too strong to say that such a reading conflicts with Plato's distinction between mathematics and mathematicians; Plato is talking about the mathematicians of his day when he makes the distinction in the *Euthydemus*, but surely the mathematical education of the philosophers will not be vulnerable to such a charge. Benson has yet to show that Plato's acknowledgement of bad mathematicians is evidence for the connection between *dianoia* and bad mathematical practice.

The other failing of *dianoia*, notes Benson, is the use of diagrams in geometry, or even, he concedes, thought experiments that use imagery in moral inquiry. However, Benson thinks that it is unclear why this should make it inferior to dialectic. He says that, if we think about Plato's comments about the difference between making *logoi* about sensible objects and making *logoi* for the sake of Forms, we might suggest that 'Plato is emphasizing the indirect nature of dianoetic. Dianoetic seeks to know or think about the forms by in some way using or thinking about the things that are images of forms. Unfortunately, Plato provides very little guidance on the nature of this indirection' (Benson [forthcoming] p 14).

Given the extensive attention that Plato gives to the relationship of Forms to their images in book X of the *Republic*, it is difficult to agree that Plato does give little guidance. As we have seen, the (sensible) image is three times removed from reality, and does not contain full information about the original, so Plato has plenty of scope for being suspicious of it. As we shall see in the following chapter, the dianoetic image is different from the sensible image, but shares with it the fact that it is removed (although not as extensively removed) from reality. This means that even a method that uses diagrams correctly is always, like *dianoia* and the *deuteros plous*, a second-best method. In addition, at *Republic* 510de, Plato says that the geometers use models, but they are not thinking of these but of the square

or diagonal itself. In light of this, it is difficult to agree that mathematicians are thinking about things that are images of Forms.

Patterson ([2007] pp.1-33) argues that Plato recognises that diagrams are useful, and they have many important uses in mathematical proofs. They can be examples, both clarifying (for example to check that the slave knows what a square is in the slave-boy passage of the *Meno* 82 b) or probative. They can also provide a notational system, encoding information that we do not get from the proof or anywhere else; moreover they are temporally enduring, extending memory over time and providing a synoptic view of how we get from premiss to conclusion. They can also be dynamic, by being revised or enlarged if necessary. Finally, they play a role in mathematical discovery and creativity, which could have arisen in the slave-boy experiment of the *Meno* (which I discuss in Section Two, Chapter One ii b) if we imagine him noticing on his own the potential answer, when looking at the line drawn from opposite corners of the square.

Given the many uses of the diagram both in Greek and modern mathematics, and as it emerges from the dialogues, it would seem futile and misguided of Plato to object to the mathematicians' use of diagrams in general, says Patterson. He thinks that Plato's actual concern with diagrams is firstly that they can be mistaken for the true subject matter of geometry, and given position and shape and so on. Secondly, the habit of imagistic thinking can lead the mathematician into error about the nature of their own cognition. The mathematician is using Forms in some sense, even if she does not recognise the fact. Thus, *dianoia* is limited in that it fails to appreciate the metaphysical and epistemological foundations of mathematics. 'None of this entails that the dialectician should try to prove geometrical theorems without using diagrams...They may even, at the end of a hard day of dianoetic theorem proving, safely retire to enjoyment of the divine Homer...The important thing is not to avoid the use of diagrams in proving theorems, but to appreciate the reality and

nature of separate mathematical Forms, and to understand how they are fundamental to mathematical truth and mathematical cognition' (Patterson [2007] p 33).

Patterson's reading has much in common with Benson's, in that it allows for both the correct and incorrect application of the mathematical method. However, Patterson stresses the validity of the use of diagrams, which I think is a strong point against Benson's reading. The use of diagrams is not a mark of a bad mathematician, as opposed to a good one. Rather, it simply means that he is reasoning *dianoetically*, rather than *noetically*. The distinction between the two states lies in the tools used by the practitioner.

As Benson concedes, at *Republic* 527b and 533d, Plato equates mathematics with *dianoia*. Indisputably, in the divided line passage, the use of the image and hypothesis is tied to the mathematical method, and, as we have seen, the use of images *is* a part of the correct mathematical method. The virtue of Benson's approach is that it allows for the use of hypothesis when reasoning noetically, which the divided line passage implies is the case. However, his account provides no mechanism for ascent to *noēsis*, as the 'mathematical' reading does, and cannot account for the correct use of imagery in mathematics.¹¹²

My reading of *dianoia* is 'methodological' in the sense that I think that what defines *dianoia* is the use of particular tools. I retain the association of mathematics with *dianoia*, but argue that it is that mathematical method, whether applied to mathematics or philosophy, that denotes this particular state of mind. When one uses the tools of imagery and hypothesis, one is in *dianoia*. The use of hypothesis in *noēsis* can be explained by the fact that the divided

¹¹² Consider Ross' (1951) remarks: "It is his conviction that geometry consists not in deducing, by pure logic alone, conclusions from propositions taken as starting points, but in apprehending the implications of figures which we draw. The drawn 'square' is not that which the geometer is reasoning about, but a mere image or approximation to it; yet he would not be able to deduce the properties of the genuine square if he did not see the way in which the elements of a seen or imagined square fit together. He needs an intuition of spatial figures, as well as his axioms, definitions and postulates" (pp. 48-49). Ross cites Aristotle's observation that geometers make their discoveries by dividing figures (*Met* 1051a22), and says that this was undoubtedly the method of the Greek mathematicians. In this way, the ascent to knowledge begins in the sensible realm and proceeds - via mathematics - into the intelligible.

line is actually a continuum, and it is the hypotheses themselves that lift the mind from *dianoia* to *noēsis*. This is because the hypothesis and the image act as proxies for the tools of *noēsis* when we do not have them: *ti estis*, or Platonic definitions. As the mind ascends the epistemological scale into *noēsis*, the hypotheses and images are gradually replaced by *ti estis*. This reading has the virtue of accounting for Plato's respect for mathematics and allowing for the correct use of imagery, like the 'mathematical reading,' but also the generalization of *dianoia* to other kinds of philosophy provides a mechanism for epistemological ascent to knowledge of the Forms in a much wider sense. I shall expand on this reading in the following chapter, but first I want to justify my account of *ti estis* as the tools of *noēsis*.

d. Noēsis

Noēsis, we are told, 'moves from assumption to a first principle which involves no assumption, without the images used in the other sub-section, but pursuing its inquiry solely by and through forms themselves' (*Republic* 510b). This would suggest that the tools of *noēsis* are Forms, and perhaps initially hypotheses as the mind moves away from *dianoia*. Plato goes on, saying that the mind

...treats assumptions not as principles, but as assumptions in the true sense, that is, starting points and steps in the ascent to something which involves no assumption and is the first principle of everything; when it has grasped that principle it can again descend, by keeping to the consequences that follow from it, to a conclusion. The whole procedure involves nothing in the sensible world, but moves solely through forms to forms, and finishes with forms (511b).

Again, Plato says that *noēsis* proceeds through Forms, suggesting that these are the tools of this particular mental state. But how exactly does one proceed through Forms? I suggest that we do so by attaining *ti estis*, or definitions, of the Forms. A *ti esti* is a Platonic definition, that is a definition that has the attributes that Plato thinks are needed for high-level philosophy. As we shall see in the *Meno* section, a *ti esti* must both identify the thing to be investigated, and also be a heuristic tool: we shall examine these points more closely in the *Meno* section, but for now, we shall examine the place of definitions in Plato's wider philosophy.

Aristotle highlights the link between the importance placed on the search for a definition and the theory of Forms:

For as a young man Plato was originally an associate of Cratylus and Heraclitean opinions, to the effect that all perceptible things were in a permanent state of flux and that there was no knowledge of them, and these things he also later on maintained. But when Socrates started to think about ethics and not at all about the whole of nature, but in ethics seeking universals and first seeing the importance of definitions, by accepting him as such he thought that this could apply also to other things and not to the objects of perception. For a general definition was impossible of any of the sensible things, which were constantly changing. He then called such entities Forms, and he said that all sensible things were spoken of in accordance with them (*Metaphysics* 987a32-b9).¹¹³

This in itself is not sufficient to convince us that the theory of Forms came about as a result of the Socratic/Platonic emphasis on definition.¹¹⁴ However, unlike in the case of

¹¹³ See also *Metaphysics* 1078b17-32 and 1086b2-7 for similar statements; Cf Irwin (1977) pp. 144-148 for a discussion on the same point.

¹¹⁴ See p. 120 n 135 of this thesis for remarks about Aristotle as a historian of philosophy. Here, he offers us a very scant idea of what the link might be. The only hint he gives us would involve rejecting the 'contents'

mathematical intermediates, we have extensive evidence in the dialogues to support the link between definitions and the theory of Forms. This is something that I shall cover in more detail in the *Meno* section, so here I shall focus on how a *ti esti* might fit into Plato's scheme.

I shall argue that my reading of *noēsis* helps to explain the connection between imagery and hypothesis in *dianoia*: I shall do this by focusing on the tools used by each state of mind, rather than their objects. Again, that is not to deny that there are distinct objects of knowledge and belief; I just want to say that we need to ask a different question about the divided line if we are to answer the question about the connection between the two marks. I shall argue that, for *noēsis*, the mind's tools are *ti estis*. A *ti esti*, as we shall see in the following section, is not an exhaustive account of a Form, but it is something by means of which we are able to study the Form. This explains the role played by hypothesis and imagery in *dianoia*. If we do not have a *ti esti*, we need a proxy, which should not only identify the thing to be investigated, but also be a heuristic tool; something that allows us to make progress in the quest for knowledge. In *dianoia*, lacking these *ti estis*, the hypothesis and the image act as such proxies.

This means that we can assign a different kind of tool to each section of the divided line:

Section:	Subsection:	Tool:
<i>epistēmē</i>	<i>noēsis</i>	<i>ti estis</i> ¹¹⁵
	<i>dianoia</i>	hypotheses and dianoetic images
<i>doxa</i>	<i>pistis</i>	physical objects
	<i>eikasia</i>	images of physical things

analysis of the divided line, as he says that there is no knowledge of perceptibles. It does not, however, conflict with my reading of the divided line. Kahn (1996) pp. 329-363 provides an opposing view of this.

¹¹⁵ That is, *ti estis* are the main tool of *noēsis*. In the early stages of *noēsis*, one may also employ hypotheses – provisional assumptions that await confirmation – as a way of moving the argument forward, but this is part of the process of moving from *dianoia* to *noēsis*. To remain in *noēsis* means to eventually confirm these and move solely through *ti estis*.

So why am I justified in saying that the tool associated with *noēsis* is the *ti esti*, and what should a *ti esti* look like? I shall tackle the second question in the *Meno* section, but here I want to say a few things about the role of the *ti esti* in the dialogues, arguing that Plato's Socrates' aim is always this, as it is the closest we can get to the Forms in our lifetimes. To study a Form through a *ti esti* is the most direct way of studying the Form at all.

The first clue to this can be found in the huge emphasis on the *ti esti* in the dialogues. As we shall see in the following section, Socrates often refuses to inquire into the properties of something, often to the exasperation of his interlocutors, without first saying what it is. In *Meno*, for example, he says, 'how can I know a property of something when I don't even know what it is? Do you suppose that somebody entirely ignorant who Meno is could say whether he is handsome and rich and well-born or the reverse? Is that possible, do you think?' (*Meno* 71b).¹¹⁶

Socrates makes the point more emphatically in the *Hippias Major*:

...during a discussion in which I was condemning some things as contemptible but praising others as fine, I was rudely interrupted with a question which went somewhat as follows: 'Socrates,' I was asked, 'what makes you an expert on what sorts of things are fine and contemptible? I mean, could you tell me what fineness is?' Now, I'm not up to this kind of thing, so I got confused and couldn't make a proper reply. After we'd parted company, I was angry with myself, told myself off, and swore that as soon as I had bumped into any of you experts, I would return to my inquisitor to renew the battle, with instruction, teaching and study to back me up (*Hippias Major* 286cd).

¹¹⁶ The early 'Socratic' dialogues centre on the search for definition: in the *Euthyphro*, we ask, 'what is holiness?'; in *Charmides*, 'what is *sophrosune*?'; in *Laches*, 'what is bravery?'; in *Hippias Major*, 'what is a fineness?'; in *Lysis*, 'what is friendship?'; Cf Benson (2012) pp. 1-37, who argues that the Socrates of the Socratic dialogues takes the answers to these definition-questions to have a special epistemic status. Benson agrees that knowledge of what *F-ness* is itself is essential for any other knowledge of F-ness.

Socrates emphasises this point at the end of the dialogue, saying of his constant inquisitor:

‘And yet,’ he continues, ‘how can you know whose speech or other action is finely formed, if you’re ignorant about fineness?’ (*Hippias Major* 304de).¹¹⁷

The early ‘Socratic’ dialogues are emphatic about the need for a definition in order to pursue inquiry, and the middle dialogues are clear that knowledge needs (and, according to the ‘objects’ interpretation, is exclusively about) Forms, as we have seen so far. In this case, it would be reasonable to suppose that definitions somehow allow us to study Forms. But in what way do they do this?

White ([1976] pp. 30-53) suggests two possibilities. Either definitions of Forms are premisses from which further statements can be deduced, or they allow us to look to and examine Forms, and by that examination, we can somehow observe features of the Form that are not inferable from the definition. White describes the difference thus: ‘Very roughly put, the difference is like the one between telling that Socrates is a teacher by learning that he is the teacher of Plato and therefrom inferring that he is a teacher, and learning that he is the most snub-nosed man in the market place, finding him on the basis of that description, observing his behaviour, and telling from that observation that he is a teacher’ (White [1976] p 38).

The second alternative would be knowledge by acquaintance, and would seem to be supported by Socrates’ example of knowing who *Meno* is in order to know what he is like, which I mentioned above. Some people would also want to use the example of finding the

¹¹⁷ See also *Charmides* 176ab and *Lysis* 223b, in which similar remarks are made.

way to Larissa, which comes at the end of the *Meno* (98a), to support this view: knowledge of the way to Larissa comes from travelling it, through direct acquaintance with the way to Larissa.

Some readings of *Euthyphro* 11ab would support this reading. Here, Plato draws the distinction between the essence, *ousia* of holiness and the affection, *pathos*, of it. Here, Socrates says,

...perhaps, Euthyphro, when asked what the holy is, you don't want to point out the essence for me, but to tell me of some attribute which attaches to it, saying that holiness has the attribute of being approved by the gods; what it is, you've not yet said. So if you don't mind, don't keep me in the dark, but tell me again from the beginning what on earth the holy is, whether it gets approved by the gods or what happens to it (as it's not over this that we disagree) (*Euthyphro* 11ab).

The use of this passage to support the second alternative would rely on the assumption that it is a forerunner to the *ti esti/hopoion* distinction that we find in the *Meno*,¹¹⁸ which I have tacitly assumed here. Then one could use the idea of knowing the essence of holiness to argue that knowledge must be by acquaintance.

However, this does not settle the question, because, in the *Euthyphro*, Plato displays the same laxness of terminology that we have mentioned before. That is, he uses 'knowing holiness' and 'knowing what holiness is' interchangeably.¹¹⁹ This overlap occurs in other dialogues (*Meno* 75b5 with c5, d6; 79d7-8, c1, 4 with c8-9), so it could be argued (as Gail

¹¹⁸ White thinks that Plato had not fully worked out his views on this, because Socrates does not seem to be saying that knowledge of *ousia* must precede knowledge of *pathos* (White, 1976 pp. 40-53). This is part of a tradition which tries to argue against the generality of the priority of definition, or even to disregard it altogether. See Benson (2012) pp. 1-37 for a discussion of this.

¹¹⁹ *Euthyphro* 15c11-12: the necessity of discovering 'what holiness is' as opposed to *Euthyphro* 15e1 and e6-7: the need to 'know holiness.' Cf White (1976) p 45: Plato 'tends to describe his epistemological efforts quite indifferently as being efforts to know *X* and also being efforts to know what *X* is.'

Fine might) that to know the form of Holiness is simply to know true propositions about it (Cf Fine [1990] pp. 85-115). In this way, the passage could be used to support the first alternative, in which the definition of a Form can be used to derive true propositions about it.

In further support of this view, it is said that *Meno* 86ff suggests that the answer to questions such as ‘is virtue teachable?’ can be deduced from the definition (White [1976] pp. 35-53 also has a good discussion of this). On this point, I would say that, here, we are talking about a hypothesis rather than a *ti esti*, the latter being much more extensive, as we have said. White thinks that, at least at this stage, Plato does not give us any clear indication of which alternative he has in mind, but it is clear that Plato thinks that definitions are necessary in our quest for knowledge of the Forms.

What I have to say is compatible with either alternative. I am saying that the hypothesis and image allow us to study the Forms indirectly in *dianoia*, by acting as proxies for the *ti estis* that we do not have. The *ti estis* will either give us knowledge of the Forms by allowing us to be acquainted with the Forms, or by deducing true propositions about the Forms from the definition. My reading does not *prima facie* exclude either knowledge by acquaintance; nor does it exclude epistemological ascent being partially by intuition, because a *ti esti* could enable us to gain intuition of the Form. In this case *dianoia* and *noēsis* are distinct because of the different tools they use; *noēsis* is ‘clearer’ than *dianoia* because its tools enable us to study the Forms in a more direct way. If we take knowledge of the Forms to be knowledge by acquaintance, then we have a further distinction between the two states: in this case, knowledge is deduced from the hypothesis or image, whereas knowledge is by acquaintance in *noēsis*.

Plato hints at the difference between the work of the mathematicians and the dialecticians in *Euthydemus*:

No art of actual hunting, he said, extends any further than pursuing and capturing: whenever the hunters catch what they are pursuing they are incapable of using it, but they and the fishermen hand over their prey to the cooks. And again, geometers and astronomers and calculators (who are hunters too, in a way, for none of these make their diagrams; they simply discover those which already exist), since they themselves have no idea how to use their prey but only how to hunt it, hand over the task of using their discoveries to the dialecticians – at least, those of them do so who are not completely senseless (*Euthydemus* 289e-290c).

In this case, the objects of the philosophers and the mathematicians (or at least the domain of their subjects) are not distinct. What is distinct is the work they do. In the *Meno* section, I argue that often one of the roles of the image is to answer the identification question. This part of the work of mathematical thought: to hunt down the concept in question. In the *Phaedo* section, I suggest that the criticism given by Socrates of Simmias' choice of image is also part of this process: we need to check if what we have got hold of is in fact an instantiation of the Form we are looking for.

Dianoetic thought 'hunts' these Forms indirectly through the tools of hypothesis and imagery, in the same way as a hunter might have to throw a spear at his prey. *Noēsis* gets much closer to the Forms: its tools, the *ti estis*, allow the practitioner of *noesis* to study the Forms directly, just as a cook's knife allows him to work much more closely with the object handed over by the hunter.

The problem remains that, according to my reading, *noēsis* uses *ti estis* to study Forms, but I want to say that it studies Forms directly. Surely, then, *noēsis* studies forms indirectly, through the medium of *ti estis*? I want to say that, in fact, the study of a Form through a *ti esti* is the most direct way of studying it that is possible in our lifetimes. We are told in *Phaedo* that in life, we are tied to the body, actually a hindrance to rational inquiry. This means that probably, 'either it is totally impossible to acquire knowledge, or it is only

possible after death, because it is only then that the soul will be isolated and independent of the body' (*Phaedo* 66e-67a). I shall look at this idea quite extensively in the *Phaedo* section, so here, I just want to point out that, in life, we need something like a *ti esti* to reach beyond the sensibles and grasp the Forms. In the absence of these, *dianoia* gives us a proxy via the hypothesis or image. It is pivotal on our journey into the world of the Forms.

iii. *Noēsis* and the Role of Definition in Plato's Epistemology

I shall argue in the *Meno* section (Chapters One ii and Chapter Two, especially Chapter Two iv) that the *aporia* that results from a failed search for definition is Plato's psychological foundation for *dianoia*, and that dianoetic reasoning is introduced in the absence of a *ti esti*. I also want to point out that book one of the *Republic* is actually a failed search for the definition of justice. Various definitions of justice are proposed.¹²⁰ However, the book ends with Socrates complaining that 'we have left the original object of our inquiry, the definition of justice, before we had discovered it' (534b).¹²¹ Socrates decides to try to 'find justice on a larger scale in the larger entity' (368e-369a) by examining justice in the state – in the absence of a definition of justice. As we see in the earlier dialogues, Socrates is uneasy about inquiring into the properties of a thing without knowing its definition.¹²² The use of dianoetic reasoning is helpful to Plato because its hypotheses and images allow the philosopher to make progress without a *ti esti*, but also without resorting to eristic (see Section Two, Chapter Two of this thesis). However, the *ti esti* remains an aim of the philosopher, and, I shall argue,

¹²⁰ For example, Simonides' idea that justice is rendering what is owing (332a) and Thrasymachus' idea that justice is 'that which is advantageous to the stronger' (338c).

¹²¹ Instead, says Socrates, the group 'went off to consider whether it is a vice and ignorance, or wisdom and virtue. Then another argument appeared, to the effect that injustice is more profitable than justice. And I could not refrain from leaving what we were at for this further point, so that now the result of our conversation is that I know nothing. For when I do not know what justice is I am hardly likely to know whether it is a virtue or not, or whether he that possesses it is unhappy or happy' (*Republic* 354b).

¹²² See my discussion of Gail Fine's Priority of Knowledge What in Section Two, Chapter One ii.b

once she has attained a sufficient number of *ti estis*, the philosopher can make even more progress towards an understanding of the Form of the Good.

The hypothesis or image of *dianoia* needs to do two things: firstly, it should identify the thing to be investigated (for example at *Meno* 82b, where Socrates uses the image of a square to confirm that a square is ‘a figure like this’). Secondly, it should be heuristic: the use of it should allow us to work out things about the Form we are studying (for example, Socrates’ drawing of the square and extra lines in the *Meno* allows the slave to work out that the area of the square is half of what it would be if it were drawn on the diagonal). As I shall argue in the *Phaedo* Section (Chapter Three), during the process of using dianoetic reasoning, the philosopher can generate a series of other statements about the Form we are studying.¹²³ These statements form at least one approximation (which can be improved and made fuller by further argument) of the *ti esti* we wanted.

A *ti esti*, then, retains both qualities we said belong to the image and hypothesis of *dianoia*. It will be heuristic, allowing us to work out other things about the Form, and it will identify the Form. However, it will have an additional degree of certainty, having gone through the process of reduction of the initial problem to a hypothesis, and the subsequent (usually deductive) downward step. Each reduction provides us with additional information about the Form that we are studying.

The main focus of this thesis is *dianoia*, and the role that images and hypotheses play in Plato’s epistemological scheme. However, my reading also has implications for the role of Platonic definitions, because I am saying that the tools of *dianoia* act as proxies for those of *noēsis*. I want to suggest that *ti estis* also have a role in Plato’s epistemological ascent. That

¹²³ For example, dianoetic reasoning in the *Phaedo* generates these statements about the Soul: Soul must be present in a body to make it alive (105dc); when soul takes possession of a body, it always brings living with it (105d); soul will never admit the opposite to that which accompanies it (105d); the soul is undying (105e); when death approaches, the soul retires and escapes unharmed and indestructible; our souls will really exist in the next world (106e-107a).

is, once the philosopher has her *ti esti*, there remains more work to be done in her philosophical quest to gain an understanding of the Form of the Good.

As I will argue in Section Three, Chapter Four, the immediate value of attaining the *ti esti* is that it allows the philosopher to arrive at more true statements about the Form. For example, in the *Phaedo*, the statement that ‘our souls will really exist in the next world’ (106e-107a) is both a result of the hypothetical argument and part of the set of statements about the soul that form (at least an approximation of) a *ti esti*. Not only does this give the immediate result of answering the *hypoion* question of whether or not the soul survives after death, but it also has a wider implication: once the philosopher has generated sufficient *ti estis*, she may begin to form a teleological account of the kind that Socrates wants in the *Phaedo*, because these *ti estis* will allow Socrates to see connections between the properties of the Form, and its connection to other Forms (and ultimately to the Form of the Good).

Imagine the philosopher applies the hypothetical method to several *hypoion* problems about the soul, the body and the afterlife, leading to several *ti estis* belonging to related Forms. The generation of true statements about the definiendum results will allow her to make connections between the definiendum and other Forms. After many such investigations, she begins to build a teleological account that links all *ti estis* to the Form of the Good. I propose that the teleological structure of the *ti estis* is the end of the ‘upward path’ in the *Republic*’s method of dialectic.

As we shall see in Chapter Four ii of this Section, Annas ([1981] pp. 291-293) complains of Plato’s divided line that there is nothing left for the way down after the triumph of the ascent through Plato’s states of mind of the divided line. Annas complains, “The Line clearly suggests (511bc) that there is not only a ‘way up’ to the unhypothetical first principle, but also a ‘way down.’ What is the difference between them?...” On my reading, Plato can avoid this complaint, because the ‘way down’ is not symmetrical to the upward path. Rather

than using *ti estis* to answer *hypoion* problems, on the ‘way down,’ the philosopher provides the kind of teleological accounts that Socrates is ideally looking for in the *Phaedo*, giving a completely different kind of explanation, this time with reference to the Form of the Good.

This thesis aims to show that the tools of *dianoia* allow Plato’s characters to make progress in philosophical discussions in a way that the *elenchus* of the early dialogues could not. They can be heuristic devices used to generate a positive statement, in contrast to the *elenchus*’ purely purgative role (see my Section Two, Chapter Two). Once this has been done, and we arrive at our *ti esti*, we may be able to answer our initial *hypoion* question. However, there remains more work to be done in the philosopher’s overall epistemic journey, starting with the gathering of more *ti estis*, if she is to attain Plato’s ultimate goal of knowledge of the Form of the Good.

Chapter Four: A Closer Look at Dianoia

Having sketched my reading of the divided line, I shall now go on to have a closer look at *dianoia*. To summarise the claims that I am making about *dianoia* specifically, my main aim is to show that the hypothesis and the image are used in dianoetic thought as proxies for *ti esti* answers that we do not have, whether that is in mathematics, or in mathematical reasoning applied to philosophy. The ‘hypothetical method’ is not identical to *dianoia*, although its early stages are a kind of *dianoia*. The hypothetical method is distinct from the Socratic *elenchus* (contra Fine). We can have *dianoia* without hypotheses, which usually uses images, for example the slave-boy passage in the *Meno*. Sometimes, the hypothesis *is* an image, as in the objections of Simmias and Cebes in the *Phaedo*. We can also have hypotheses without images, as we see in the ‘hypothetical passage’ of the *Meno*.

When we come to the upper part of the line, Plato explicitly states that he is talking about the tools the mind uses to draw conclusions, and in the case of *dianoia*, these tools are hypothesis¹²⁴ and imagery. In this section, I want to examine each of them, explaining how I think my reading provides a mechanism for ascent to *noēsis*, which I said was so critical in the previous chapter. I shall examine first the dianoetic image and then the hypothesis, before I go on to give an example of the method in action.

We sometimes see the hypothesis and the image used in conjunction, which is what is happening in the *Theaetetus*. I shall be expanding on my claims concerning the *Meno* and *Phaedo* in the relevant sections, so in the final part of this chapter, I want to illustrate what I mean by saying that the *Theaetetus* shows *dianoia* using hypotheses and imagery. The main scope of the project concerns the *Meno*, *Phaedo* and *Republic*, but we should also be aware of other dialogues, especially those that have such a strong connection to the issues at stake.

i. *The Dianoetic Image*

In *dianoia*, reasoning employs hypothesis and imagery. However, the imagery of *dianoia* is not the same as the imagery of *eikasia*. In *dianoia*,

...the mind uses the originals of the visible order in their turn as images... (*Republic* 510b).¹²⁵

The difference between the images of *eikasia* and *dianoia* is that the image of *eikasia* is an image of a sensible object, whereas the image of *dianoia* is a sensible object used as an image. This distinction is not always acknowledged. Cross and Woosley ([1964] pp. 238-

¹²⁴ Cornford ([1932] pp. 39-41) points out that there are, according to Aristotle's *Posterior Analytics* 1 10, two kinds of hypothesis in mathematics: those relative to the pupil (something he tries to prove) and the hypotheses of the science itself (*archai*, basic truths). Plato means the former kind. Cornford thinks that Plato seems not to include definitions in hypotheses, but assumptions of existence: the odd and the even etc.

¹²⁵ My emphasis.

244) think that *dianoia* is a parallel mental state to *eikasia*, and often the two states are linked by scholars due to their apparently common use of imagery. However, we should ask whether Plato meant for this to be the case, or whether he had something else in mind for the images of *dianoia*. We also need to engage with the debate over the existence of mathematical ‘intermediates’: did Plato mean that there are objects (rather than tools) of mathematics that correspond with the *dianoia* part of the line, just as the Forms correspond with *noēsis*? I shall argue that the two questions are distinct, but linked in this way: the question of what the dianoetic image is refers to one of the two tools of *dianoia*, whereas the question of the existence of intermediates refers to the subject-matter of *dianoia* (on the traditional reading).¹²⁶ The questions are, what does the dianoetician use, and what does he study? The answers, I suggest, are that he uses hypotheses and *dianoetic* images, to study Forms.

So what kind of image is a tool of *dianoia*? Even if the dianoetic image is different from that of *eikasia*, some maintain that we should see a connection between the two states. Cross and Woosley’s view of *eikasia* is formed partly on the basis of the alleged connection with *dianoia*. They think that, if we are to maintain an exact parallelism between the cave and the divided line, we cannot say that *eikasia* is conjecturing about originals through their reflections. They stress that there is no linguistic objection to this. Instead, they try to argue for the parallel between *eikasia* and *dianoia* based on the properties of the divided line itself. They note that, when we look at the ratios, *eikasia* and *dianoia* are related: *eikasia* is to *pistis* what *dianoia* is to *noēsis*.

They think that this leads us to another parallel in the mental states, suggesting ‘that as the mathematician in some sense takes the likeness for an original, not realising it is a likeness, so the man in the state of *eikasia* does the same...likeness is accepted as reality, without any realisation that it is a likeness...and it would then be parallel to the state of mind

¹²⁶ The dianoetician’s tool is an image of a sensible thing, whereas its subject-matter is (at least sometimes, according to Fine; always, according to the traditional reading) an intelligible thing.

of the prisoners in the cave' (Cross and Woozley [1964] pp 219-220). Cross and Woozley think that *eikasia* in the moral spheres corresponds to accepting the imitations of justice made by the rhetoricians, sophists and politicians in law-courts as real.¹²⁷ In this case, say Cross and Woozley, *dianoia* is parallel to *eikasia* in that it accepts its hypothesis without question, just as *eikasia* accepts the images without recognising that they are nothing else but images (Cross and Woozley [1964] pp 243-244). However, it is difficult to reconcile this with the claim at *Republic* 510de that those who reason dianoetically “make use of and argue about visible figures though they are not really thinking about them, but about the originals which they resemble; it is not about the square or diagonal which they have drawn that they are arguing, but about the square itself or diagonal itself, or whatever the figure may be.”

Klein ([1965] pp. 112-125) thinks that *dianoia* and *eikasia* are connected in a different way, and writes of the ‘dianoetic extension of *eikasia*’ (Klein [1965] p 115). Klein sees *eikasia* as the faculty to see an image as an image, and thinks that we should not overlook the crucial importance of it; he sees it as particular to human beings, our ‘prerogative.’¹²⁸ The *dianoetic eikasia* exercised by *dianoia*, according to Klein, ‘consists in understanding visible things in terms of their intelligible foundations’ (Klein [1965] p.120). Klein thinks that, in *dianoia*, we interpret the things and properties of the physical world as ‘images of invisible νοητά’ (Klein [1965] p 119). So arithmeticians and geometers use figures and pebbles for their demonstrations, but they do not have these ‘in mind.’ Klein rightly points out that this

¹²⁷ Cross and Woozley’s reading is similar to mine in that they take a wider reading of *eikasia* to include the moral sphere, but I do not think that *eikasia* needs to include a lack of awareness of the nature of the image: rather it is the use of the image to draw conclusions that defines *eikasia*. The term *eikasia* occurs in his text only when Plato is talking about the Line (not in connection with the prisoners in the Cave, who really do lack an awareness of the nature of the image).

¹²⁸ Klein (1965) p 114-115. His view is in opposition to Fine, among others, who believe that those in *eikasia* are unable to make such a distinction.

kind of *eikasia* is ‘different from the one we exercise in the domain of visible things and *their* images’ (Klein [1965] p 119).¹²⁹

I agree with Cross and Woozley that *dianoia* and *eikasia* are connected in that their ratios with the states of mind directly above them on the line are the same. However, this does not necessarily mean that they use similar tools, nor that their awareness of the nature of these tools is what assigns them to these particular states. Plato is simply telling us that *dianoia* stands in relation to *noēsis* as *eikasia* does to *pistis* in terms of their respective clarity. If we were to extend this reasoning to the tools or objects of each state, then we would say that the dianoetic image is proportional to, but not identical to the image of *eikasia*.

Moreover, much of Cross and Woozley’s reading stems from the implication that we should look to the Cave analogy to inform our translation of *eikasia*. The term *eikasia* occurs in the text only when Plato is talking about the Line. Until they are released, the prisoners in the Cave have no experience of the world except the shadows they see on the wall. They have no experience of their originals, so they have no reason to doubt the reality of the shadows. At 515c5, Plato calls their mental state ἀφοροσύνη, 'mindlessness' or 'delusion' – not *eikasia*. In this case, we need not impose such a strict correlation between the line and the cave by suggesting that *eikasia* involves such a delusion. We should continue to be careful about translating *eikasia* in the same way. As I discuss in Chapter Three, segment ii b of this section, *eikasia* is not such a strange state of mind as this correlation would imply.

Note that my reading does not differentiate between the mental states in terms of awareness of the nature of the tools or objects as I explained in the previous chapter. What makes *dianoia* a state of mind is the (self-aware) employment of hypothesis and imagery as tools of inquiry. As I shall argue Socrates and his friends do this in *Meno* and *Phaedo*; they are using dianoetic reasoning, but this does not mean that they are unaware of the nature of

¹²⁹ Although I would not use term ‘*eikasia*’ here without the specific qualification that it means looking at something and drawing conclusions about the thing it represents, not that it means ‘illusion.’”

the hypothesis and image.¹³⁰ In addition, Plato's remarks at 510de do imply that *dianoia* involves being aware that one is not really thinking about the images, but about "the originals which they resemble."

As for Klein's view, that *dianoia*, understands visible things in terms of their intelligible foundations, I would suggest that *dianoia* does not always employ visible images, and that of course hypotheses, their other tools, are not visible at all. For example, in the *Phaedo* section, I shall argue that Simmias' harmony image and Cebes' tailor image are dianoetic images; but we would not call them visible images. Rather they, like the diagrams of the mathematicians, are physical objects used as images of Forms. In this way, the dianoetic image is different from that of *eikasia*; the tools of those two states are completely distinct. Plato affirms this himself: in *dianoia*, the mind uses 'as illustrations the very things which in turn have their images and shadows on the lower level' (*Republic* 511a). That is, the mind uses the physical objects as images, it does not use images of the physical objects in *dianoia*.

If this is what the dianoetician uses, what does she study? I wanted to divide the question in this way, because I am proposing that there are a number of ontologically different things involved in what I want to say, and the debate about mathematical intermediates has so many similarities to my theory about the tools of *dianoia* that it would be easy to confuse the two things.

The distinction is between mathematical objects (such as numbers and shapes) and the objects of mathematical thought, including mathematical thought applied to philosophy. It is

¹³⁰ Unlike Cross and Woosley, who stress the lack of awareness of those in *eikasia* that the things they study are only images, Klein argues that it is exactly such an awareness that is distinctive about *eikasia*. Many other scholars use this idea of awareness of the real nature of objects to define the different states of mind: Gail Fine says that the prisoners in the cave are in *eikasia* (or L1, as she calls it) 'because they cannot systematically discriminate between images and the objects they are images of.' The philosopher who returns to the cave will not lapse back into L1, because he knows the images there (that is, he knows that they are images; Cf *Republic* 520c). My reading does not depend on awareness of the nature of the objects (either tools or subject matter) for the distinction between mental states. It is possible to engage in *eikasia* while being aware that the tools you are using are only images, for example. This is what Socrates does when he uses the poetry of Simonides in the passage of Protagoras, referred to above.

possible, I suggest, to reason noetically about mathematics. There is such a thing as the Form of the Number Two, and the Form of the Square, and this is what *noēsis* is about. *Dianoia*, when one is reasoning dianoetically in the sphere of mathematics, can also be about the Form of the Square or the Forms of the Number Two, but indirectly, *through instantiations of these forms in theory*. These instantiations are not ‘intermediates’ as they have been described by previous scholars. They do not exist as a category, until they have been instantiated in the mind of the thinker.

In this case, some of the things I am going to suggest look very similar to the theory of intermediates, because I am going to say that hypotheses and images instantiate Forms. However, the difference is that these are not the objects of *dianoia*, but rather the tools used by the dianoeticians. The objects of *dianoia* could still be Forms, but in *dianoia*, they are studied indirectly; in *noēsis*, they are studied directly. What is ‘intermediate’ about my reading is the medium of theory in which the Forms are instantiated. This may be the same thing as calling these instantiations ‘intermediate,’ but they are certainly not intermediates in the same sense as it is traditionally argued.

I am saying that the image and the hypothesis are ontologically different from the tools in the other mental states: *ti estis*, physical things and eikasiastic images. Leaving aside for the moment Fine’s view, that *dianoia* can be about Forms or sensibles, the traditional ‘objects’ reading tells us that *pistis*, *eikasia* and *noēsis* have as their objects images, physical things and Forms respectively. The debate about the existence of mathematical ‘intermediates’ centres on the discussion at *Republic* 510d-511a, in which *dianoia* is discussed. The question at the heart of the debate is whether *dianoia* is about Forms, like *noēsis*, or whether it is about a distinct class of objects called ‘intermediates.’

This could mean one of several things. It could mean that intelligible mathematical entities form their own class of objects, but that these are Forms in every other sense. This

view would say that mathematical Forms are the subject of *dianoia*, and it is their subject-matter, not their ontological status that defines them. As I have rejected the ‘mathematical’ reading of the line, which limits the subject-matter of *dianoia* to mathematics, I shall say very little about this; I just want to mention it now in contrast to the other views.

The second thing it could mean is that images in the sensible world are copies of physical objects, which are copies of mathematical intermediates, which are copies of Forms. These correspond exactly to the mental states of *eikasia*, *pistis*, *dianoia* and *noēsis*. If we reject the idea of intermediates and give Plato only a threefold ontology of images, physical objects and Forms, we lose the symmetry.¹³¹ As I rejected readings of the line that are driven by the mathematical properties of the line and the wish for symmetry, rather than by Plato’s more explicit philosophical concerns, I shall not focus on this here.

The strongest arguments for the existence of intermediates involve the idea of ‘perfect exemplars.’ If the proponent of intermediates agrees that numbers are Forms, we end up with a distinction between ‘mathematical numbers’ and ‘ideal numbers.’ Wedberg¹³² agrees that mathematical numbers are intermediates between ideal numbers and sensible things, or collections of sensible things. On his reading, mathematical numbers are perfect exemplifications of ideal numbers, and collections of sensible things are imperfect exemplifications of ideal numbers. Geometrical intermediates exist for Plato, consciously or unconsciously, in a way that is analogous to numerical intermediates.¹³³

¹³¹ In this case, we can have two ways of looking at the same type of object: we can have *pistis* about physical objects, or we can use them as images of Forms in *dianoia*. Foley (2008) pp.1-23, who thinks that this is the only way to deal with the rejection of intermediates, thinks that this still leaves this interpretation open to the overdetermination problem of the line on epistemological grounds: *dianoia* is still a clearer state of mind than *pistis*.

¹³² (1955), especially pp. 65-67 and Appendix D; Cf Wedberg in Vlastos (1971) pp 28-52. In Appendix D (of 1955), Wedberg presents the evidence for his reading of intermediates in Plato’s dialogue, which are reviewed by Annas (1975) pp. 146-166.

¹³³ In a similar reading, Miller (2007) pp. 310-342 says that Plato distinguishes between a Form and its perfect exemplar, for example, the Form of Triangularity, as opposed to the perfect Triangle. The perfectly triangular triangle presents itself to the mathematician in thought, but it is the Form of Triangularity that acts as a tacit standard of this. The role of perfection, then is to provide the context in which the perfectly intelligible figure can come to mind. Transcending all exemplars, perfection itself is indeterminate, so it allows us to understand

Plato never explicitly says that there is an ontologically separate class of mathematical objects; but there is a tradition that says that he endorses them implicitly. People who take this view draw on Aristotle's reading of Plato. Aristotle attributes to Plato the view that there exist mathematical objects that are ontologically intermediate between sensibles and Forms:

Again, in addition to sensible objects and Forms, they said that mathematical objects existed between them, differing from the sensibles in that they were eternal and unchanging and from the Forms in that there were many similar ones but only one Form of any kind (*Metaphysics* 987b14-17).¹³⁴

He adds,

Plato's doctrine was that Forms and mathematical are two substances and that the third substance is that of perceptible bodies... (*Metaphysics* 1028b18-21).

Aristotle is writing as a philosopher, not a doxographer, and he is notoriously problematic as a historian of philosophy.¹³⁵ Of course, this in itself does not mean that he is wrong here, but we have other reasons for being suspicious. Firstly, Plato's own silence on the subject should make us reluctant to attribute such a doctrine to him; those who wish to

the goodness of the Good as perfection. In this way, we can understand the Good as responsible for the existence of all Forms and mathematical. As I noted in the previous chapter, Miller's reading has the virtue of providing a way of progressing from mathematics to teleology, and therefore to moral theory.

¹³⁴ Sir David Ross's ([1924] pp. 165-168), in his note on *Metaphysics* 987b 14, confesses that he cannot find evidence for intermediates in the divided line (or elsewhere), although he does believe that 'the logic of the simile requires that the objects of *dianoia* should be a distinct class of entities' (pp. 167-168). Ross thinks that Aristotle might have heard Plato speak about them. I have already argued that we should not ascribe otherwise unsupported views to Plato in order to preserve or create symmetry in the divided line. See my Section One, Chapter Three i for my discussion of this.

¹³⁵ See Cherniss ([1964] pp. 347-404) and Guthrie ([1957] pp. 35-41), who highlight this issue. I am not saying that my scepticism to Aristotle's attitude is an accusation that Aristotle is wilfully misrepresenting or even misunderstanding Plato; rather Aristotle's concern is to present his own ideas, rather than to preserve those of others.

argue for it need to try to find it in various passages in which Plato is concerned with other matters. Secondly, it is probably the case that Aristotle is attributing intermediates to Plato's theory because he feels that Plato needs it to resolve a problem implicit in his theory.

As Aristotle is not specific about what kind of entity intermediates might be or why he thinks Plato might need them, we need to seek the solution in for ourselves. Annas (1975) thinks that the answer lies at *Metaphysics* 987b14-18, quoted above. For Plato, Forms are eternal and unchanging, and this is a property shared by intermediates. Aristotle is at least clear that, for Plato, numbers are forms,¹³⁶ so the intermediates are something other than numbers:

Now, if you buy the Theory of Forms, then at least numbers explain your entities (after a fashion). The idea is that each of the numbers is a Form and that the Form is a cause of being for other entities (*Metaphysics* 1090a46).

Annas thinks that this fits in with the problem for Plato of adding two and two together to make four. If the Form of two is unique, we should not be able to add it to itself. The same applies to geometry: mathematicians can draw two circles intersecting, whereas the Form of the circle is unique. However, mathematical operations cannot refer to the physical world, so Plato needs intermediates in order to resolve this. Annas calls this the Uniqueness Problem ([1975] p 151) and says that it is the only line of argument suggested by Aristotle's reading:¹³⁷ Forms (including number-Forms) are unique, whereas intermediates are not. This

¹³⁶ Annas also cites *Phaedo* 101 and 103-105 as passages in which Plato treats numbers as Forms, although he is not explicit here. Annas (1975) makes a similar point on p 150.

¹³⁷ Annas also thinks that Aristotle's objections to the idea of intermediates supports her view that Aristotle thinks that intermediates are a solution to the Uniqueness Problem. His first objection is that he disagrees with the theory of Forms anyway, so the theory of intermediates irritates him even more. Secondly, there is no real difference between Forms and intermediates, which suggests that intermediates were only introduced to solve a problem. Finally, Aristotle says that the situation for non-mathematical things for which there are Forms is not parallel to that for mathematical things because 'they blithely posit the mathematical between the Forms and

is the only hint that Aristotle gives us as to the distinction between Forms and intermediates, so Annas thinks that it must be Aristotle's reason for attributing the intermediates to Plato's theory.

However, Annas says that the Uniqueness Problem is Aristotle's reason for thinking that Plato needs the intermediates, not Plato's: when Annas examines the passages in which she thinks Plato does hint at intermediates, she finds that they do not contain the same line of thought as Aristotle. Annas does not deny that Plato endorses intermediates, but she does think that Aristotle is making a different argument, and one that does not necessarily refer to what we find in the dialogues.¹³⁸

As for the passages in which Plato supposedly hints at a theory of intermediates, they fall into two categories according to Annas (1975): 'inconclusive' and those that can be treated as serious attempts to establish the existence of intermediates. Annas does a good job of arguing that the 'inconclusive' passages should not be taken as evidence for the existence of intermediates, and we shall, in any case, be referring to them later in the dissertation, so I shall not attempt to give my own criticisms of this reading of these passages here.¹³⁹

Annas does accept that Aristotle's claim, that Plato's mathematical reasoning does not concern Forms, is plausible when you think about the discipline itself: 'mathematics talks about circles and lines, not about the physical diagrams that illustrate them, nor about the

perceptibles, as being Third Entities between the Forms and the things round here' (*Metaphysics* 1059bff); so the intermediates are needed to maintain the higher status of mathematics over science.

¹³⁸ Annas rightly thinks this reflects a more general feature of Aristotle's treatment of Plato. She says, 'where what Aristotle says about a thesis of Plato's does not square with the dialogues, we should not assume that there is conflict and hasten to resolve the problem on that assumption. When we examine the argument on both sides, we may find that they are completely different' Annas (1975) p 166.

¹³⁹ These passages are: *Euthydemus* 290b, in which Plato likens the mathematicians to hunters; *Theaetetus* 198a, in which Plato distinguishes the thought that 11 things are 12 things from the thought that 11 is 12; *Phaedo* 101b, in which it Plato seems to distinguish numbered groups from pure numbers; *Cratylus* 432ab in which Plato distinguishes the logic of pictures from the logic of numbers and *Timaeus* 50c, in which geometrical figures seem not to be identified with either Forms or physical objects. Annas (1975), pp. 156-160, argues that Plato is concerned with a number of different things here: there is no single concern with intermediates that unites them and any reference to intermediates is implicit at best.

unique Form of Circle and Form of Line' (Annas [1981] p 251). She sees more reason to take seriously the idea of Platonic intermediates in passages in the *Republic* and *Philebus*.

In *Republic* 525c-526b, Annas thinks that Plato seeks to replace groups of physical objects with more dependable objects counted by philosophers and mathematicians. These are indivisible units or 'ones': 'satisfactory counting and calculating require a whole different range of objects distinct from the objects around us that we count' (Annas [1975] p 162). This is not the same as the Uniqueness Problem, because, as we learn from *Philebus*, Plato's dissatisfaction lies in the dissimilarity and divisibility of groups of physical objects, not in the uniqueness of Forms (*Philebus* 56ce).¹⁴⁰ In the divided line passage, Annas affirms that people want to posit intermediates because they wish to assign intermediates as objects of *dianoia*.

Given Annas' successful criticism of Aristotle's reading of Platonic intermediates, it seems that the doctrine only holds if we say that the logic of the divided line demands them, or if we take the passages from *Philebus* and *Republic* 525c-526b to suggest them. As my reading of the divided line does not require them to complete the symmetry,¹⁴¹ I shall now turn to the latter point.

I examine the *Republic* passage in my discussion of the application of *dianoia* to spheres other than mathematics, so here I shall restrict my remarks to the points relevant to the discussion of intermediates.

Wedberg thinks that, although the distinction between ideal and mathematical numbers is not explicitly made in the *Republic*, we do see the implication that the arithmetician needs sets of Ideal numbers (intermediates) in the following passage:

¹⁴⁰ Cf Annas (1975) p 161, who refers to this distinction.

¹⁴¹ I have repeatedly said that symmetry should not be the driving force here; in addition, my 'tools' reading could provide the symmetry if it were required.

‘... You must know how the experts in the subject, if one tries to argue that the unit is itself divisible, won’t have it, but make you look absurd by multiplying it if you try to divide it, to make sure that their unit is never shown to contain a multiplicity of parts.’

‘Yes, that is quite true.’

‘What do you think they would say, Glaucon, if one were to say to them, “This is very extraordinary – what are these numbers you are arguing about, whose constituent units are, so you claim, all precisely equal to each other, and at the same time not divisible into parts?” What do you think their answer would be to that?’

‘I suppose they would say that the numbers they mean can be apprehended by reason, but that there is no other way of handling them’

‘You see therefore,’ I pointed out to him, ‘that this study looks as if it were really necessary to us, since it so obviously compels the mind to use pure thought in order to get at the truth.’

‘It certainly does have that effect,’ he agreed.

(*Republic* 525d-526b).

Wedberg thinks that this passage implicitly assumes the existence of intermediates as sets of Ideal numbers: ‘In the numbers of which he is speaking every unit is exactly equal to every other unit and, further, each unit is in itself absolutely without parts’ (Wedberg [1955] p 124). He thinks this thesis reappears in the *Philebus*:

SOCRATES: The most precise sciences, however, are those we recently called essential.

PROTARCHUS: I suppose you mean arithmetic and the other sciences you mentioned along with it.

SOCRATES: I do. Here again, however, Protarchus, oughtn’t we to speak of two sets of sciences, not one? What do you think?

PROTARCHUS: Which sets do you have in mind?

SOCRATES: Take arithmetic first: shouldn't we distinguish between the common and the philosophical variety?

PROTARCHUS: What's the criterion for distinguishing these two kinds of arithmetic?

SOCRATES: The boundary between them is clearly visible, Protarchus. Some arithmeticians operate with unequal units: for example, they add two armies together, or two cows, or two things one of which might be the smallest and the other the largest thing in the world. Others, however, would never follow their example unless every unit, no matter how many there are, is taken to be identical to every other unit.

PHILEBUS: You put that very well: since arithmeticians clearly fall into two classes, it makes sense for there to be two kinds of arithmetic.

(*Philebus* 56ce).

This passage is supposed by Annas and Wedberg to shed light on the previous passage from the *Republic*: philosophical units are equal, whereas the units of the arithmeticians are not equal in all respects: an army is itself divided into 'units;' cows may be unequal in respect of size. As we have seen, Annas differs from Wedberg in saying that this is not the same line of argument that Aristotle applies to Plato (ie, it does not concern the Uniqueness Problem), but she thinks that this passage from the *Philebus* elaborates on the argument at *Republic* 525c-526b. The problem with which Annas thinks that Plato is concerned is the dissimilarity and divisibility of groups of physical objects, not the problem of duplicating a Form in calculations.

However, if this is the case, there is no need to postulate intermediates without the Uniqueness Problem: collections of sensibles are dissimilar and divisible, but Forms are not. So the mathematician needs to calculate with homogeneous units; but there can only be one Form of the Unit (or Form of Unity), so they exemplify the unit perfectly multiple times as intermediates and so on.

Can this be what Plato means in the *Philebus*? Socrates goes on to confirm that ‘...there are two techniques of arithmetic, two techniques of measurement, and so on’ (*Philebus* 57d). But why can this not mean that there is one technique for collections of sensibles, and another for Forms? The passage from *Philebus* suggests a dichotomy like this; not the trichotomy that we would need to postulate intermediates: for this, we need to distinguish between counting sensibles, performing calculations on sets of ideal numbers (or intermediates) and contemplating Ideal numbers (or number-Forms). Moreover, Annas concedes that to postulate intermediates contradicts directly *Republic* 510d, in which mathematicians talk about ‘the square itself’ and ‘the diagonal itself’ (‘surely Forms,’ says Annas [1981] p 251), as well as the depiction of mathematics as the best introduction to the contemplation of Forms (although she does not retract her position).

The passages that have been most convincingly used to argue for the existence of intermediates are those from *Republic* in conjunction with the *Philebus* quoted above, yet these passages do not make the distinctions necessary to uphold the doctrine. We have already rejected the evidence from Aristotle as being inconclusive and the ‘intermediates’ doctrine contradicts Plato’s explicit words at *Republic* 510d. We still have the problem that the ‘ones’ or units that the mathematicians study are many, whereas there can only be one Form of One; likewise, they use more than one of the same geometrical figure in their operations, where there can only be one Form for each.¹⁴² However, at no point in the dialogues does Plato propose the existence of intermediates as a solution to this problem.

Confining ourselves to the divided line passage, the strongest evidence against the existence of intermediates is the divided line passage itself. Annas admits that Plato’s

¹⁴² This is the Uniqueness Problem, which, in spite of Annas’ claims that it does not concern Plato, seems to be the enduring snag of his theory; See also Cross and Woosley (1964) pp 236-7 for this argument. Of course, we could say that there is a contradiction in Plato’s theory: he does not endorse intermediates, but without them, there is a problem with his theory of Forms: we cannot perform mathematical operations without contradicting the uniqueness of the Forms we use. However, I suggest that this is not a problem unique to mathematics, but to calculations in all spheres.

reference to ‘the square itself’ and ‘the diagonal itself’ conflict with the idea that the geometers are not concerned with Forms. Here, Plato says that they are arguing about ‘the square itself or diagonal itself, or whatever the figure may be.’ (*Republic* 510d). Clearly, here Plato is saying that the subject-matter of *dianoia* is Forms; its tools are images and hypotheses.¹⁴³ As we shall see, this reading has been challenged by Burnyeat, but only to the extent to say that it is not decisive.

On first glance, the only reason, based on the divided line passage alone, that you would want to postulate the existence of intermediates would be for the sake of assigning a different object to each subsection of the line.¹⁴⁴ However, I suggest that, if we read the divided line as distinguishing between the mental states on the basis of the tools they use, rather than their subject-matter, this problem does not exist: as images are assigned to *eikasia* and physical things to *pistis*, hypothesis and the dianoetic images are assigned to *dianoia*. *Noēsis* has its own tools, which we shall examine below. In this case, each mental state has a different tool or tools, meaning that there is no need to postulate the existence of intermediates on the grounds of the internal logic of the divided line. I have tacitly assumed the ‘objects’ analysis in this section, although a similar argument could apply to the ‘contents’ analysis.

So far, we have made little headway in resolving the debate. Aristotle’s reading of Platonic intermediates gives by far the best reason for wanting to assign them to Plato (the Uniqueness Problem), but we find little in the text to support them. In fact, as Annas argues,

¹⁴³ Looking at the debate from the angle of Plato’s scheme for higher education, Cornford ([1968] pp. 61-95, especially pp. 77-65) says that nothing here points to a class of mathematical numbers and figures intermediate between forms and sensibles. The only Forms that figure in the scheme of education are the moral (507b) and the mathematical (510d). Mathematical forms can be apprehended by *noēsis*; they do not form a lower class, apprehended by *dianoia*. According to Cornford, at *Republic* 511d, mathematical objects are seen in the connection with the first principle.

¹⁴⁴ Not everyone thinks that the objects of *dianoia* are intelligible at all. Bedu-Addo (1978) pp. 111-127, who, as we saw thinks that it is only for the purpose of the divided line that Plato regards the *doxa* of *dianoia* as a lower grade of knowledge, says that from *Republic* 510e and 511a that it is not only physical models and objects that fall into the metaphysical subsection of this part of the line. Ontologically, diagrams are the same as these. The objects of *pistis* and *dianoia* are the same: ‘those very objects (*auta*)’ of *pistis* are clearer in *dianoia*. They are the same objects, but, in *dianoia*, the mind treats them as images of the Forms, albeit unconsciously.

Plato seems to have something entirely different in mind. Moreover, we have the problem that Plato speaks of the square ‘itself’ in *dianoia*, and seems to be talking about Forms. This seems to be a problem that, in the dialogues at least, Plato leaves unresolved.

Burnyeat’s¹⁴⁵ contribution to the case for intermediates is the most convincing. According to Burnyeat, Plato does not settle the exact ontological status of mathematical objects in the *Republic*, a point with which I agree. Burnyeat thinks that this is because Plato was primarily interested in conveying his point about mathematics and the Good, and did not see the need to go into this. He thinks that Plato makes the distinction between *dianoia* and *noēsis* on the grounds of the further ontological investigation pursued by the latter. He points out that, in the *Republic*, Socrates is intentionally leaving aside the distinction (if any) between the objects of *dianoia* and *noēsis*. His translation of the relevant passage, Plato’s second discussion of the divided line at *Republic* 534a, reads, ‘Let’s leave aside the proportion exhibited by the objects of these states when the opinionable and the intelligible are each divided in two. Let us leave this aside, Glaucon, lest it fills up with many times more arguments/ratios than we have already.’¹⁴⁶

Burnyeat goes on to give his own statement of the uniqueness problem, pointing out that the *Republic* tells us that mathematicians talk about pluralised, idealised entities that are not Forms, and that if we take an example of a theorem from Euclid, which refers to multiple shapes and numbers, we can see that this is true. Burnyeat goes on,

Plato may have thought that the mathematicians’ multiple non-sensible particular numbers and figures...could ultimately be derived from Forms, so that in the end mathematics would turn out to be an indirect way of talking about Forms. Perhaps the mathematical entities are the ‘divine reflections’ outside the cave (532c 1) dependent on the ‘real things’ they image.

¹⁴⁵ Burnyeat (2000) pp 33-35, although the groundwork for this is laid in Burnyeat (2000) pp 29-33.

¹⁴⁶ *Republic* 534a, trans. Myles Burnyeat in Burnyeat (2000) p 33-34.

But whatever Plato thought, or hoped to show, Greek mathematics is certainly not a direct way of talking about forms (Burnyeat [2000] p 34).

The main strength of Burnyeat's argument lies in his refutation of the apparently decisive support for the argument against intermediates at *Republic* 510de and 525de, in which Plato says that the arguments of the mathematicians are pursued for the sake of 'the square itself' and 'the diagonal itself' and in which he says that if someone tries to divide 'the one itself', the mathematicians laugh and do not allow it. Burnyeat points out that the word 'itself' should not be decisive on its own, because that would mean that we have to admit that the Socrates means the Form of Thirst when he speaks of 'thirst itself' in book IV.¹⁴⁷

Burnyeat is convincing in his suggestion that we should not take the word 'itself' to be decisive, but, once again, this gives us no reason to prefer the argument for intermediates over any other. Burnyeat's other appeal is to the uniqueness problem, which, as we have seen, was not a concern of Plato's, so the argument remains unresolved if we let it rest on this point.

In fact, I am going to leave the question of mathematical intermediates unresolved. As Burnyeat says, it is simply not Plato's concern in the divided line passage to resolve it, and the evidence in the dialogues is too patchy to be sure. The related problem that Plato does need to tackle - and that relates to the question of what the mind uses in each mental state - is that mathematical objects are already abstractions, so it is difficult to talk about the difference between the Form of Two and a given two. Plato does seem to want to say that the Forms are unique, so he needs some way of being able to perform these calculations without sacrificing that. On the other hand, Plato would not want to say that the Forms are confined to the mind of the thinker. Rather, I will suggest, the operations that take place in the mind of the thinker

¹⁴⁷ Burnyeat gives other examples, the most convincing of these being that of *Phaedo* 66a 1-3, in which we would have to say that one Form studies another if we took 'itself' to signify a Form: Plato recommends using 'pure thought itself by itself to try to hunt down each pure being itself by itself.'

involve instantiations of the Form, but the thinker cannot actually perform operations upon the Form itself (*Republic* 526ab).

As Patterson ([2007] pp. 1-33) points out, even if there were intermediates, they could not be the ultimate ground of the truth of geometry: the Forms are. Thus, the unique Forms are still the foundations of geometry.¹⁴⁸ Of course, one could extend this reading to the other mental states: physical things are instantiations of Forms, so really, any truth about the physical world is underpinned by the Forms, too. If we take the ‘objects’ reading of the line, this means that the Forms underpin *pistis*; on the ‘contents’ reading, it means that the Forms underpin *noēsis* applied to the sensibles.

I want to say that we should be focusing on a different aspect of the question. I introduced the distinction between what the dianoetician uses and what he studies. Usually, the debate about intermediates focuses on the latter: mathematicians study mathematical intermediates. However, I want to argue that Plato does recognise a more generalized and implicit version of the Uniqueness Problem, but that mathematical intermediates (whether they exist or not) are not his answer, although they may be implicit in the answer. I am going to say that the Uniqueness Problem is not resolved by mathematical intermediates as objects of *dianoia*, but by the tools of *dianoia* as distinct from those of *noēsis*.

I have already covered the usual passages that are supposed to support mathematical intermediates in Plato; I shall not repeat them here. Instead, I want to examine another passage from the *Republic*. Speaking about geometry, Socrates notes that the subject is in fact the opposite of what is implied by the terms of its practitioners:

The terms are quite absurd, but they are hard put to it to find others. They talk about ‘squaring’ and ‘applying’ and ‘adding’ and so on, as if they were doing something

¹⁴⁸ Cf *Phaedo* 96e ff, where Socrates points to the ‘simple but safe’ explanation that the Forms are the only cause of things.

and their reasoning had a practical end, and the subject were not, in fact pursued for the sake of knowledge...(yet) the objects of that knowledge are eternal and not liable to change and decay (*Republic* 526c 527 a6-b1, b5-6).

Aristotle says that Platonic Forms are unique and unchanging; Platonic intermediates are unchanging but not unique. What I am about to propose is not the same as Aristotle's account of intermediates, but draws on this distinction, in conjunction with the above passage of the *Republic*. The reason geometers (and other mathematicians, and as, I shall argue, some other philosophers who reason dianoetically, using hypotheses and images) are 'hard put to it' to find other terms for their operations is that they do not recognise that they are speaking about multiple instantiations of the same changeless Form, rather than making changes to the Form.

Both those who propose Plato's endorsement of mathematical intermediates¹⁴⁹ and those who do not¹⁵⁰ note that mathematical operations involve the use of more than one instance of the same Form. Socrates' slave-boy experiment in the *Meno* involves several squares; geometers talk of two circles bisecting; one is added to one to make two. This is what gives rise to the problem of Uniqueness. However, this is not what Plato is talking about in the above passage of the *Republic*: he is talking about the implicit claim of geometers to change the unchangeable.¹⁵¹ Plato would have the same problem with adding the area of a square to a rectangle. This is not the Uniqueness problem, because here, we are not talking about two of the same Form, rather, the problem here is that we are talking about changing the changeable.

¹⁴⁹ For example, Burnyeat (2000) pp.1-81.

¹⁵⁰ For example, Annas (1975) pp. 146-166.

¹⁵¹ Plato is also presumably concerned by the fact that some geometers do not realise the value of geometry in leading us to a higher state of mind, because they see only its practical use.

Plato's solution is that *dianoia* instantiates the Form in theory. What I mean by this is that *dianoia* provides instances of Forms, and therefore gives us access to them. It does this by allowing us to access Forms through images taken from the phenomenal world. As we shall see, in the *Phaedo*, Socrates speaks of the need for the *deuteros plous* to study reality through 'some other medium' lest it injures the eyes (99de). This method takes recourse to theories, so the Forms are instantiated here through hypotheses and images. In this way, the dianoetic slave-boy experiment instantiates several squares in the course of the argument, in order to 'double' the area. The image that Socrates draws on the ground acts as a substitute *ti esti*, firstly by helping to identify what a square is ('a figure like this' - *Meno* 82b). Secondly, we are able to draw conclusions about properties of the Form of Square by instantiating it several times throughout the course of the investigation. (for example, that the area of the square is half of that it would be if it were drawn on the diagonal).

As I have been arguing, dianoetic reasoning includes the mathematical method applied to philosophy in other spheres, so the objects of mathematical thought, or dianoetic thought, would include numbers and shapes, when a mathematician studies them with hypotheses and images and, crucially, moral concepts like virtue and entities such as the soul, when that method is applied to philosophy. So in the *Republic*, the Form of Justice is instantiated in theory, through philosophical images of the soul and the state. We draw conclusions about the nature of the Form of Justice by comparing the relationships between parts of a just soul and a just state, so we use two instantiations of the Form of Justice, even though there can only be one Form.

This is an intelligible instantiation, and a more direct instantiation of a Form than the eikastic image. What makes the (correctly chosen) dianoetic image veridical is its connection to the Form. *Dianoia* is, on this reading, metaphysically connected to *noēsis*

because they are respectively indirect and direct ways of studying the same thing.¹⁵² I shall go on to expand on the nature of the dianoetic image, and its connection to truth, in the *Meno* and *Phaedo* sections, and my concluding chapter. For now, I just want to summarize what I have said about it here.

Firstly, it is not the same as an eikastic image. It is a sensible object used as an intelligible instantiation (an intelligible representation or a dianoetic image) of a Form. As I will argue in the following chapter, even a diagram has intelligible qualities, because the information it contains is not entirely sensible, but is partially contained in the mind of the dianoetician. When Socrates draws his figures on the ground in the *Meno*, he cannot represent perfect figures, and the square he draws on the diagonal will not be exactly double the area of the original square. The image includes the diagram plus information like this that the diagram does not contain. It is intelligible, in that its properties exist in their fullest sense in the mind of the thinker.

The dianoetic image is more veridical than that of *eikasia* because it is less removed from the Form. Once again, it is not an image of a sensible, which would make it ‘thrice removed’ from reality like the eikastic images, but an image of a Form. We saw that the problem with the images of *eikasia* was that they do not contain all the information about the thing they represent:

If you look at a bed, or anything else, sideways or endways, or from some other angle, does it make any difference to the bed? Isn’t it merely that it looks different, without being

¹⁵² This could also be said to provide a mechanism for ascent up the epistemological scale. In the *Republic*, Socrates says that he has only *doxa* about the Form of the Good (*Republic* 506c). By using an intelligible image in the form of the allegory of the sun, he is able to draw conclusions about it in a way that suggests *dianoia*. In this way, the intelligible image ‘lifts’ the mind from *doxa* into *epistēmē*.

different?...When the painter makes his representation, does he do so by reference to the object as it actually is or to its superficial appearance? (*Republic* 598ab).¹⁵³

The information contained in the eikasiastic image is limited to the angle from which it was made. The dianoetic image, however, being intelligible, is heuristic: we can use information it contains to draw conclusions about the Form we study. The tools of *dianoia* are ‘intermediate’ in a sense, in that *dianoia* uses instantiations of Forms in theory, but not in the traditional sense of the word. I have left the debate about traditional intermediates unresolved, partially because we have insufficient evidence to resolve it, partially because, like Burnyeat, I think that Plato’s focus was elsewhere, and partly because, like Patterson, I think that Forms ultimately underpin the tools of *dianoia*, and it is that which makes the image (or hypothesis) veridical.

ii. *The Hypothesis*

I shall say a good deal more about the hypothesis in *dianoia* when we come to the *Phaedo* section, and a good deal of what I have already said of the dianoetic image applies to hypotheses. The hypothesis performs a similar role to the image in that it stands in for a *ti esti*, and this similarity will be examined more closely in the *Meno* section. As I have argued, the *ti esti* is the most direct way of studying the Forms during our lifetimes. Although the hypothesis and image perform similar roles, there are important differences between the two. The main difference is that the dianoetic image is only found at *dianoia*, whereas the hypothesis appears at both *dianoia* and *noēsis*. A second difference is that a hypothesis is not an image of a Form. Rather it is a *provisional statement about a Form*. Finally, I said that the

¹⁵³ Cf *Cratylus* 432b, in which Plato points out that no image can be perfectly like its original, inside as well as out, because if it were, it would not be an image but a replica or double. Robinson (1953 p 218) takes this to mean that, for Plato, we should learn through realities rather than images.

roles played by the hypothesis and image were similar, not that they were the same. As we shall see, the hypothesis has a greater role to play in the ascent of the epistemological scale.¹⁵⁴

This last point has to do with the involvement of the hypothesis in ‘the upward path,’ which, since Robinson, has tended to be ascribed to the move from a hypothesis to an unhypothesized beginning.¹⁵⁵ There are two passages in the *Republic* that give us reason to ascribe an ‘upward path’ to Plato. At *Republic* 511b, Plato seems to imply that the hypothesis can be stepping stones to an unhypothesized beginning:

Then when I speak of the other sub-section of the intelligible part of the line you will understand that I mean that which the very process of argument grasps by power of dialectic; it treats assumptions not as principles, but as assumptions in the true sense; that is, as starting points and steps in the ascent to something which involves no assumption and is the first principle of everything; when it has grasped that principle it can again descend, by keeping to the consequences that follow from it, to a conclusion (*Republic* 511b).

This is the passage from *noēsis* in the divided line, which we examined before. As I have been arguing, the divided line represents an epistemological ascent for Plato; the mental states are arranged in a hierarchy, which is clear from Plato’s remarks at *Republic* 511e: the

¹⁵⁴ Although I do suggest that the dianoetic image can lift the thinker from *pistis* to *dianoia*.

¹⁵⁵ Robinson is not the first to use the idea of ascent and decent in Plato’s epistemology. For example, see Adam ([1902] pp.168-179), who uses these words in his Appendix iii Book VII of the *Republic* in his commentary. Adam links the idea of ascent to examples in the early dialogues of what he thinks is progression from one hypothesis to another. In addition, *Laws* 626b5-627a10, has been suggested to me as an example of ascent and descent in the application of the method of analysis in a non-mathematical context. Here, ‘the same test holds good’ for comparing cities with cities, as comparing villages with villages. An original definition of the well-constituted city as the city that ‘must be so equipped as to be victorious over its rivals in warfare’ is replaced by another in which it is described as the city in which the better rule the worse. The progression is made upwards (from city via village and household to individual), then downwards via the same route, with the claim by Clinias that the position is made all the more incontestable by reducing the method to first principles. As the downward step is made as a summary, and does not involve an ascent to the Form of the Good, I would not say that this is the method described in the divided line as a whole, although it may be said to have been inspired by the method of analysis and synthesis (it cannot be reduction per se as, in this process, the way up and the way down are not separated like this).

states of mind are arranged ‘on a scale, (and we may) assume that they have degrees of clarity corresponding to the degree of truth possessed by their subject-matter.’ Not only is it an ascending scale of progressively improving epistemological states, I want to say, but it is also a continuum: there will be a point at which the mind leaves *dianoia* and enters *noēsis*; this passage is describing that point.

Plato explicitly says that the mind progresses through the mental states (*Republic* 532a), and the implication is that we proceed through each of the states and build on them. On this reading, *dianoia* is an indispensable part of the epistemological ascent. This is in direct contradiction to Benson’s reading, which I described above, which says that *dianoia* is the misapplication of the method of *noēsis* and therefore not the foundation for it.

On my reading, the so-called hypothetical method of the *Republic* extends from *dianoia* into *noēsis*, and the hypotheses we find in *dianoia* are the foundations for *noēsis*. On this point, I also differ from Burnyeat, because I am saying that the same hypotheses from *dianoia* can be substantiated in *noēsis*. Plato tells us a little about how the ‘upward path’ might look at *Republic* 533c, where he speaks of the dialectical method as destroying hypotheses to the very beginning, in order to receive confirmation:

Dialectic, in fact, is the only procedure which proceeds by the destruction of assumptions to the very first principle, so as to give itself a firm base. When the eye of the mind gets really bogged down in a morass of ignorance, dialectic gently pulls it out and leads it up, using the studies we have described to help it in the process of conversion (533cd).

Robinson tackles the two questions of what receives confirmation, and what it means to destroy a hypothesis: ‘destroying’ a hypothesis cannot mean either refuting or confirming, as there are bound to be some true, and some false, hypotheses. Robinson thinks that it is the hypothetical character of the hypotheses that are being destroyed, not the proposition itself:

‘To destroy hypotheses is not to prove the falsity of certain propositions which we formerly hypothesized (nor to prove their truth), but to cease hypothesizing certain propositions which we formerly hypothesized, and take another attitude towards them’ (Robinson [1953] p 161). The problem is that this seems to be implied by the ‘downward’ path anyway: after the dialectician has done his work, the proposition considered will either be a certain truth or a certain falsehood.

It is not the deductive process that results in certainty: at least, not the absolute certainty that Plato is looking for. He says: ‘for if one’s starting point is something unknown, and one’s conclusion and intermediate steps are made up of unknowns also, how can the resulting consistency ever by any manner of means become knowledge?’ (*Republic* 533c). In this case, deduction can only give certainty if the hypothesis has already been substantiated, which is the role of the upward path.

But how exactly does one make this epistemological ascent? Robinson distinguishes between the synthesis theory of the upward path, the mathematical theory, the ‘*Phaedo*’ theory and the intuition theory. He rejects the synthesis theory and the mathematical theory, has sympathies with the ‘*Phaedo*’ theory and supports the intuition theory. I also reject the synthesis theory (for different reasons from Robinson), but as I am proposing what Robinson might class as a mathematical theory, I shall now go on to state my reasons for differing from Robinson.

The synthesis theory is that there are two processes involved in dialectic: the upward path towards the principle and the downwards path away from the principle. The upward path is a process of generalization (as we might see from the ascent passage of the *Symposium*) and the downward path is a process of classification or division. This would mean that the upward and downward paths of the *Republic* are similar to the method of collection and division of the *Phaedrus*.

Robinson rejects this reading on the grounds that the Form of the Good is not, for Plato, a *summum genus*. How, asks Robinson, can finding the genus of a group of species be ‘destroying the hypothesis?’ Robinson thinks that for the upward path to be generalization, it would have to be empirical,¹⁵⁶ whereas dialectic proceeds without the senses. Moreover, identifying the upward path with collection means identifying the downward path with division; but the downward path should more properly be thought of as proof.¹⁵⁷ The activities of collection and division might aid the dialectician, but this is not what Plato means by the upward and downward paths of the divided line.

As I will explain in the *Phaedo* section, I do not think that the epistemological ascent is driven by generalization in this sense; although knowledge of the genus and that of the species are mutually reinforcing. So, for example, knowledge of the species lion (*panthera leo*) will tell us something about the genus *panthera*, and vice versa. Plato is not thinking primarily of biological examples here, but the lion example serves to illustrate Robinson’s point: to find that the tiger, lion, jaguar and leopard all belong to the genus *panthera* cannot substantiate the hypothesis. Obviously, there will be some features that these animals have in common, but Plato thinks that a hypothesis can be made about a feature of a species that is not common to the genus. If we want to hypothesize, ‘lions live in prides,’ there is no way that this hypothesis can be substantiated by finding the genus, because living in prides is a property of lions, not of the other cats in the genus *panthera*.

The same can be said of other disciplines. As we shall see, the hypothesis can be any kind of proposition about the thing we are investigating: it does not have to be a definition or statement of existence. So if we want to hypothesize, ‘the diagonal of a square is

¹⁵⁶ Robinson thinks that *Phaedrus* 249bc supports his point, in which ‘It is impossible for a soul that has never seen the truth to enter human shape; it takes a man to understand by the use of universals, and to collect out of the multiplicity of sense-impressions a unity arrived at by the process of reason.’ Here, sense-impressions are important in the process of generalisation, as the collection of the former enables the latter (although the soul has experienced the Forms before entering the body).

¹⁵⁷ In fact, *Republic* 534bc seems to Robinson to show division as arriving at the Form of the Good, so would therefore come in the first part of dialectic.

incommensurable with its side,' generalization to the properties of shapes is not going to destroy the hypothesis. That is not to say that generalization will not come into it at some point; in fact, knowledge of the properties of shapes is fundamental to geometry, and it is our knowledge of geometric principles that allows us to prove the incommensurability of the diagonal with the side. What I am saying is that this generalization is not what Plato is talking about at this stage: rather, it comes in at a higher level of *noēsis*, where teleology (Plato's 'best method') comes into play.

I have said that the divided line is a continuum: the early stages of *noēsis* are a progression on the later stages of *dianoia*, so they will look different to the later stages. When the mind ascends to *noēsis* from *dianoia*, it substantiates the hypothesis from which it has derived its conclusions by deriving this from a 'higher' hypothesis. In the later stages of *noēsis*, these hypotheses are supported by the teleological account that the mind has attained. On this reading, the method that appears in the *Phaedo*, the *deuteros plous*,¹⁵⁸ is *dianoia* plus the early stages of *noēsis*, before the teleological account has been produced.

Robinson has some sympathy for the '*Phaedo*' theory of the upward path, according to which the upward path is the hypothetical method described in the *Phaedo*. According to Robinson, this means first using the *elenchus* to check the hypotheses for consistency, which Robinson thinks is probably what Plato did mean in the divided line passage.¹⁵⁹ However, Robinson argues that, in spite of first appearance, the hypothetical method of the *Phaedo* does not exhaust the upward path of the *Republic*, because it cannot give the infallible certainty of the 'unhypothesized beginning' which is emphasized in the *Republic*. Robinson thinks that the method of the *Phaedo* is probably a part of the upward path of dialectic, because, as the downward path sounds like a deduction of theorems from axioms, it is likely

¹⁵⁸ See Section Three, Chapter Two segment ii, for a discussion of *deuteros plous* as 'second best.'

¹⁵⁹ In Book VII (which Robinson says is 'universally recognized to be the expression of the same ideas) we find *elenchus*. Robinson cites *Republic* 534bc, in which the words ἐλέγχων (c1) ἐλέγχειν (c3) are found.

that the upward path would proceed from as yet unproved theorems to as yet uncertified axioms.

However, Robinson's conception of the 'higher' hypothesis in the *Phaedo* does not make it higher in the sense that it takes the argument 'upward,' as the divided line passage seems to require: 'it is much more likely to be 'on the same level', since it is intended to fulfil the same function' (Robinson [1953] pp 171-172). This is not my reading of the 'higher' hypothesis in the *Phaedo*, as I will explain in this section. However, Robinson, while sympathetic to the idea that the method of the *Phaedo* is connected to that of the *Republic*, does not think that it is enough to explain the 'upward' movement of dialectic.

Robinson champions the 'intuition-theory' of the upward path. He agrees that the method of the *Phaedo* plays a part in the epistemological ascent, but thinks that, in order to achieve the certainty of the unquestionable first principle, something else is needed. This 'something extra' is not an additional method: 'he merely claims that the man who competently and conscientiously practises this hypothetical and elenctic procedure will, or may, one day find himself in the possession of an unhypothetical certainty' (Robinson [1953] p 172-173). After many months of testing, it will dawn on him that one particular hypothesis will have endured every possible test. It is the intuition, not proof or demonstration that makes this hypothesis a certainty. The process strengthens the philosopher's mental vision. Robinson thinks that his account is supported by the ascent to knowledge on the cave allegory of the *Republic*, the *Symposium* and the *Seventh Letter*. In this way, thinks Robinson, the upward path of the *Republic* is the method of the *Phaedo* with the added claim that this method can produce certainty. The confirmation of the proposition, on this reading, occurs long after its conception.

I agree with Robinson in that Plato seems to be talking about the same thing in both the 'hypotheses' passages of the divided line and the hypothetical passage of the *Phaedo*. I

also agree that intuition has a role to play in the epistemological ascent. However, on my reading, this role is limited to the ‘upward’ heuristic process of finding an appropriate ‘higher’ hypothesis (or dianoetic image: it is not the appeal to intuition that makes the image convincing, as Robinson says, but rather, it is the intuition of the Form that allows the practitioner of *dianoia* to choose an appropriate image). This intuitive function is part of the application of the mathematical method to philosophy that I will try to demonstrate in the following sections. I will argue that the image and hypothesis play heuristic roles, but that finding the appropriate hypothesis or image is itself a heuristic process, and it is here that intuition comes in. The intuitive part according to Robinson is also in the heuristic reach for the hypothesis, but he also thinks that it is the intuitive certainty about a hypothesis that makes the ascent; I do not.

Robinson is also correct in saying that the method of the *Phaedo* does not exhaust the method of dialectic described in the *Republic*. As I have said, the *deuteros plous* of the *Phaedo* represents *dianoia* and the early stages of *noēsis*. I am using the term ‘method’ quite loosely here. What I mean is that the teleological account that we find in the later stages of *noēsis* is a different kind of explanation produced by the later stages of the same method. I shall explain this more in the *Phaedo* section. For now, I just want to point out that we should not be expecting the *deuteros plous* to exhaust dialectic: as I shall take great pains to point out in the *Phaedo* section, the *deuteros plous* is a second-best approach and, as such, it is by no means the end of the story.

So far, I’ve agreed with Robinson that the ‘upward’ path is linked to the method of the *Phaedo* and that intuition plays a role in the heuristic ascent, but I’ve disagreed with the main thrust of his argument, that it is intuition that gives us the true testing of an unhypothesized beginning. My own reading falls under what Robinson would describe as the ‘mathematical’ theory of the upward path, which Robinson rejects. He associates this with

the method of geometrical analysis, which I shall discuss in the *Meno* section. Robinson argues that there is little incentive to read the method into the divided line because ‘it would have to mean that dialectic hypothesises some proposition and goes on deducing its consequences until it arrives at the Idea of the Good, which it independently knows to be true. It then (and this synthesis would be the downward path) asserts the Idea of the Good, and deduces consequences therefrom in the reverse order until it arrives at the original hypothesis, which is thus established’ (Robinson [1953] p 166).

The problem with this, Robinson thinks, is that the Form of the Good cannot be independently known to be true; it is the job of the upward path to confirm it. We could try to say that the upward path (analysis) involves making propositions about the Form of the Good and the downward path (synthesis) involves proving our definition from the known fact at the end of the analysis. However, this is inconsistent with the divided line passage, because the Good comes at the beginning, not at the end, of the downward path.

As I shall argue, the method that Plato applies to philosophy in the *Meno* and *Phaedo* is not analysis and synthesis, but *apagōgē* (reducing the problem to another that is easier to solve). I follow Karasmanis (1987) in this, but in his reading of the divided line, Karasmanis thinks that the dialectical method of the *Republic* is that of analysis and synthesis (pp. 257-307). This is where I differ from Karasmanis. The main difference between the two methods is that, in the latter, the ‘way up’ and the ‘way down’ are separated, whereas in the former, every step up is followed by a step downwards. The other difference is that, in *apagōgē*, the problem is reduced to a hypothesis, whereas in analysis, the thing sought is the hypothesis and starting point. Because *apagōgē* is a forerunner to analysis, the two methods present some similarities.

As I explain in Section Two, Chapter Three ii, Plato uses the method of reduction at *Meno* 87b-89a (and Aristotle seems to confirm this in *Prior Analytics* 69a).¹⁶⁰ Reduction starts from the enunciation of a problem, then ‘reducing’ it to a problem that is easier to solve, through the use of lemmas (premisses assumed in establishing something else).

Enunciation of the problem → lemma1 → lemma2 → lemma3 → ... Conclusion

In the *Meno*, Socrates reduces the problem of whether virtue is teachable to the problem of whether virtue is knowledge. He enunciates the problem with the theorem ‘If virtue is knowledge, it can be taught’ (87c) and the diorismos is ‘this is the condition on which virtue is teachable’ (87c). During the passage, lemma 1, ‘virtue is knowledge’ is reduced to lemma 2: ‘virtue is good’ (87d).

Unlike Robinson, Karasmanis thinks that the dialectical ascent of the *Republic* is analysis and synthesis. He thinks it cannot be *apagōgē*, because the (a) two processes, up and down, are clearly separated and (b) the dialectical ascent arrives at the Form of the good, the unhypothetical first principle. Against Robinson’s objection that geometrical analysis is always deductive, whereas the dialectical ascent is not, Karasmanis argues that this is not necessarily true (a claim that I shall explore in the *Meno* section). He says that the two methods are not identical, and that Plato is aware of the differences between a mathematical proof and a philosophical argument. However, he thinks that the general form of the dialectical ascent in the *Republic* and the method of analysis and synthesis is the same.

In the sections that follow, I shall be building on Karasmanis’ claim that *apagōgē* is the method that Plato has in mind in the *Meno* and *Phaedo*. I have already said that the *deuteros plous* of the *Phaedo* covers *dianoia* and the early stages of *noēsis* in the divided line. In the previous chapter, I described hypotheses as proxies for *ti estis*, saying that they are not themselves definitions, but propositions about Forms. In the *Meno* section, I argue that a *ti*

¹⁶⁰ I also argue that what follows 89a, is not part of the method of reduction, but two separate, empirical arguments.

esti is not an exhaustive account of a Form, but it is sufficiently extensive to allow us to work out other things about the Form we are studying.

On my reading, the move from *dianoia* to *noēsis* is made as we use the method of *apagōgē*. We have a problem (mathematical or otherwise) and reduce it to a hypothesis-lemma (an ‘upwards’ step). The consequences of this are checked for coherence (a downwards step). The hypothesis is reduced to a ‘higher’ hypothesis (which, as I shall argue, is identical with the method of the *Phaedo*), which involves an ‘upwards’ move from *dianoia* to *noēsis*, as this is equivalent to the state of mind that ‘treats assumptions not as principles, but as assumptions in the true sense’ (*Republic* 511b).¹⁶¹ The reach for this higher hypothesis is an upwards step, but the reduction of the previous hypothesis is a (probably deductive) downwards step. So far, the transition from *dianoia* to *noēsis* resembles *apagōgē*.

A *ti esti*, I shall argue, needs more than just the proposition about a Form provided by its proxy, the hypothesis. But indisputably, hypotheses are ‘starting points and steps’ to the unhypothesized first principle, so I propose that the method of reduction provides us with the extra properties that a *ti esti* needs (*Republic* 511b). It needs at least some degree of certainty, plus enough information to enable us to work out other things about the Form: it cannot be limited to the existential or propositional statement that we get from a hypothesis. The *ti esti* is able to appear in *noēsis* because at least some degree of certainty is provided by the reduction of the initial problem to a hypothesis, and the subsequent (usually deductive) downward step. Each reduction provides us with additional information about the Form that we are studying. This provides an account, which, I shall argue in the *Meno* and *Phaedo* sections, is the second thing we need to make the claim a *ti esti*. So, after the initial stages of *noēsis*, *ti estis* appear. The nature of these, on which I shall expand in the following sections, allows us to build the teleological account that Socrates seeks in the *Phaedo*, that goes

¹⁶¹ As I have stressed throughout this chapter, it is not the *awareness* of the hypothesis or image that determines the mental state of the thinker, but the *treatment* of them.

beyond the *deuteros plous* to the first principle of everything, which is the end of the upward path in the *Republic*'s method of dialectic.

Once we arrive at this principle, we begin the descent. One of Robinson's problems with the mathematical theory of the dialectic ascent is that it is apparently inconsistent with the divided line passage, because the Good comes at the beginning, not at the end, of the downward path. On my reading, which does not try to make analysis and synthesis the format of the whole process, this is not a problem. The ascent moves from hypotheses to *ti estis* to teleology and the Form of the Good. This enables us to give the kind of teleological accounts that Socrates wants in the *Phaedo* on the way down. This reading has the added bonus of avoiding Annas' ([1981] pp. 291-293) problem that there is nothing left for the way down after the triumph of the ascent: on my reading, the way down builds on the way up to give a completely different kind of explanation.

Of course, as I have taken the *deuteros plous/dianoia* to *noēsis* transition to be *apagōgē*, the initial stages of the 'way up' actually begin with successive steps up and down, but the overall movement on this view would be upwards, and my reading fits in nicely with Plato's description of the *dianoia* to *noēsis* transition as 'steps and sallies' at *Republic* 511b.¹⁶² Plato does not mention here a transition as such from the lower to the higher division. However, Plato explicitly says that the mind progresses through the mental states (*Republic* 532ab) and speaks of progress to 'the summit of the intellectual realm' (532b). My reading suggests a solution to the problem of how Plato expects us to do this. It also allows for the division between *dianoia* and *noēsis* that Plato specifies, because although hypotheses are used at both levels, the dianoetic image is restricted to *dianoia* and the *ti esti* is restricted to *noēsis*.¹⁶³

¹⁶² It is the hypotheses in the uppermost section of the Line that are called 'steps and sallies.' However, that does not exclude the possibility of 'downward' steps at the beginning of *dianoia* (in fact, as I argue in Section Two, Chapter 3.ii, this is what happens). Moreover, it is consistent with what I have to say about the extension of the use of hypotheses into *noēsis*.

¹⁶³ Scott (1995) pp.15-23 and 38-52 has an opposing view to mine. Rather than thinking that we ascend to knowledge of the Forms, as I have suggested, he distinguishes between two kinds of learning in Plato.

iii. Theaetetus: *Hypothesis and Image in the Search for a Definition*

The main focus of this project is dianoetic thought as described and practised in the *Meno*, *Phaedo* and *Republic*. There are many more passages in middle Plato (and beyond) that refer to mathematics, but it is beyond the scope of this project to give exhaustive accounts of all of them. However, it has been said that we see, in *Theaetetus*, the clearest example of the hypothetical method in action (Matthews [1972] pp. 38-39), and the dialogue's affinity to the *Meno* as a search for definition that resorts to hypothesis¹⁶⁴ means that it merits at least some attention. Although I am not attempting to give a full exposition of this dialogue here, I do want to suggest that this is one instance of hypotheses being used in conjunction with images in dianoetic reasoning in Plato. I also want to show that the image does not have to be a hypothesis, but because, as I argued above, most arguments require the use of more than one concept, we can see the image being used as proxy for a *ti esti* that we do not yet have.

According to Scott, Plato excludes ordinary learning from the scope of recollection: Scott thinks that Plato has an empiricist theory of this: according to him, ordinary learning is acquired by habit. On this reading, most people do not go beyond ordinary learning to recollect, but philosophical discoveries are startling revisions of ordinary ways of thinking. We forget our knowledge of the Forms at birth and regain it by the process of recollection.

¹⁶⁴ The search for a definition is a feature of many early dialogues, but *Theaetetus* and *Meno* share a particular feature of introducing the hypothetical method after the initial failure of the first few attempts. In both *Meno* and *Theaetetus*, the Socratic interlocutors offer initial definitions as answers to the question posed by Socrates. In both cases, these definitions are rejected on the basis of the form of these definitions. Theaetetus' and Meno's first definitions consist of lists of examples without reference to anything that unites all instances (*Meno* 71e, 72ab; *Theaetetus* 1146cd). The other problem with Theaetetus' definition, and with Meno's other two definitions, is that they incorporate concepts that rely on the things to be defined (*Meno* 73d, 77b; *Theaetetus* 146c). In both dialogues, the interconnectedness of definitions is remarked upon (*Meno* 75bc; *Theaetetus* 196de).

Firstly, I will argue that the discussion in *Theaetetus* is heavily influenced by mathematics. This in itself should suggest that the *Theaetetus* contains at least some mathematics applied to philosophy, and my reading of the argument will show that that the three hypotheses are used in conjunction with images, making *Theaetetus* a paradigm of dianoetic reasoning.

Theaetetus is heavily influenced by mathematics; Socrates' interlocutors are themselves mathematicians, making them ideally placed to apply mathematical reasoning to philosophy, especially Theaetetus, who is young enough to pick up new ways of speaking. The mathematical example is introduced after the rejection of Theaetetus' initial definition of knowledge. Theaetetus says that he had learned of the irrationality of $\sqrt{3}$, $\sqrt{5}$... $\sqrt{17}$ from Theodorus. He saw that there were evidently an infinite number of square roots, and tried with a friend to gather them all under a single heading. They call numbers that can be the product of multiplying some number by itself 'square' numbers, and those that cannot 'oblong' numbers. The lines which form the side of a square whose area is a square number is a 'rational length', and those which form the sides of a square whose area is an oblong number is an 'irrational root' (*Theaetetus* 148ab).

Theaetetus points out that a similar distinction can also be made for solid figures. Theaetetus sees that Socrates wants the same kind of answer to his question about knowledge, and thus takes mathematics as a model for philosophy. This sentiment is echoed by Socrates, when he says,

You showed the way well, just now, so take your answer about irrational roots as a model. What you must try to do is give a single account of the many branches of knowledge, in the same way that you gathered together the plurality of irrational roots under a single concept (*Theaetetus* 148d).

Mathematics is again referred to later in the dialogue to set the standards of sound method. The fact that geometers would be considered worthless if they relied on probability is cited as a reason to prefer logical proof to probability (*Theaetetus* 162e-163a).¹⁶⁵

Another instance of mathematics being used as a model for philosophy in *Theaetetus* is in the discussion on false belief. Socrates and Theaetetus have already included mathematical concepts like ‘odd’ and ‘even’ in the question about whether there is a specific organ with which the mind perceives features that cannot be grasped by the senses (*Theaetetus* 185cd). Then, in the analysis of the wax tablet image, a mathematical example is given to argue that error can occur even when we think about things that are already known. We can know the numbers five and seven, but erroneously conclude that they are eleven when added together. Theaetetus adds,

...and the greater the number, the greater the chance of error. I mention larger numbers because I take you to be talking about any number (*Theaetetus* 196b).

In the analysis of the aviary image, arithmetic is again used as an example, this time of a skill that affords control over pieces of numerical knowledge (*Theaetetus* 198ab). These examples not only highlight the virtue of mathematics as a discipline that produced universal rules to cover all instances, but also commend it as a model for other types of inquiry.

Theodorus recommends that Socrates ‘treat the theory like a geometrical problem and look into it by ourselves’ (*Theaetetus* 180c) and this, I suggest, is how the group proceeds. The method of the mathematicians is applied to philosophy throughout the dialogue, as the bulk of the discussion is engaged in the hypothetical method. We are not given a theoretical account of the method, as we are in the *Meno*, but we do see the method in action. Socrates

¹⁶⁵ *Phaedo* 92d also seems to make this point.

and Theaetetus are searching for a *ti esti* for knowledge; in the absence of this, they use hypotheses.

Theaetetus suggests three hypotheses which could serve as answers to the question, ‘what is knowledge?’ and they investigate each in turn. The first hypothesis is, ‘knowledge is perception’ (151e). The second is, ‘knowledge is true belief’ (187b), and the third is, ‘knowledge is true belief accompanied by a rational account’ (201c). They are treated as hypotheses throughout the text,¹⁶⁶ and they are treated differently from the definitions. Rather than being objected to on account of their form, the hypotheses are expounded. This happens either by being supplemented with an additional theory (in the case of flux), or by images (in the case of the aviary and wax tablet images). Following this, the elaborated hypotheses are subject to analysis before being finally rejected.

This, I am saying, is an example of *dianoia*, where hypotheses are adopted in the absence of a *ti esti*. However, as Theaetetus and Socrates investigate, they need to use other concepts, for which they are also missing *ti estis*. It is here that the image, the other tool of *dianoia*, is used.

As Socrates and Theaetetus investigate the hypothesis ‘true belief is knowledge,’ they find that they need to find out how there can be false belief. Socrates says that he thinks he has found a way in which the inquiry can continue. It is possible that error can occur by something unknown being thought to be something known. They thought this to be impossible before, because it made the known things unknown. However:

...for the sake of argument, imagine that our minds contain a wax block, which may vary in size, cleanliness and consistency in different individuals, but in some people is just right...whenever we want to remember something we’ve seen or heard or conceived on our own, we subject the block to the perception or the idea and stamp the impression onto it, as if

¹⁶⁶ For example, at 165d, Theaetetus says, Ἀλλά λογίζομαι ὅτι τὰναντία οἷς ὑπεθέμην.

we were making marks with signet-rings. We remember and know anything imprinted, as long as the impression remains in the block; but we forget and do not know anything which is erased or cannot be imprinted (*Theaetetus* 191ce).

Here, the image stands in for a *ti esti* that we do not have, for ‘memory.’ We said that an image standing in for a *ti esti* should be a heuristic tool; something that allows us to make progress in the quest for knowledge. This image is heuristic because it allows Socrates to move the argument on, and explain false belief as ‘cross belief’ (193d; cf 194b6). The image is eventually found not to apply to all cases, and the assertion of false belief as cross belief is dropped (196d).¹⁶⁷ However, progress has still been made, because it has allowed Socrates to identify another problem: they do not have a *ti esti* for ‘knowing.’ Socrates is frustrated about the need for more terms that they have not yet defined:

Then doesn’t it strike you as dishonourable for us to assert what knowing is like, when we are ignorant about knowledge? But in fact, Theaetetus, our conversation has been contaminated by impurities for a long time! Countless times we’ve said ‘We know’ or ‘We do not know’, ‘We have knowledge’ or ‘We have no knowledge’, as if we could understand each other in the slightest, as long as we remain ignorant about knowledge. And never mind the past; we’re at it again at the moment! We’ve just used the words ‘ignorant’ and ‘understand’, as if we had the right to use them while knowledge eludes us (*Theaetetus* 196de).

Here, Socrates puts his finger on a problem that many scholars overlook when thinking about definition in the dialogues. If we want to prioritise definition, which Socrates evidently does, we cannot avoid using other, related terms in our quest for the definition we seek. Socrates and Theaetetus want to find the definition of knowledge; they cannot help

¹⁶⁷ At 194b6: εἰς πλάγια καὶ σκολιὰ.

using words like ‘knowing,’ ‘ignorant’ and ‘understand’; but they lack definitions for these terms, too.

Aware that he cannot avoid using these terms, Socrates mentions what he calls a ‘topical description of knowing’ (197a). This is, ‘the having of knowledge’ (197b). This is not the only place we find this description. Dionysodorus gives us the same one in *Euthydemus*:

Tell me, isn’t learning the acquisition of the knowledge of what one learns?

Cleinius agreed.

And what about knowing? He said. Is it anything except having knowledge already?

He agreed.

Then not knowing is not yet having knowledge?

(*Euthydemus* 277bc).

This may well be a sophistic definition of knowledge,¹⁶⁸ which could explain Socrates’ reference to it as ‘topical.’ At any rate, Socrates is not happy with the description (it is not a satisfactory *ti esti*) and wants to change it to ‘the possession of knowledge’ (197b). For possessing and having, says Socrates, are different. This prompts Socrates to use another image:

Think of the analogy of someone who has tracked down some wild birds (pigeons or whatever) and keeps them in a pigeonry he’s constructed. Surely we would say that in a sense he always has them, because he possesses them, wouldn’t we? (*Theaetetus* 197cd).

This allows him to make a step towards the *ti estis* he needs:

¹⁶⁸ See Waterfield’s note 1 in his translation of *Theaetetus* (1987).

And we call transmission ‘teaching’, reception ‘learning’, and having, through processing in our hypothetical aviary, we call ‘knowing’ (*Theaetetus* 198b).

Using the aviary image, Socrates is able to explain how things that were learned some time ago can be re-learned, and to distinguish the possession of knowledge from having knowledge.

Section Two: The *Meno*

Recent scholarship on the *Meno* has acknowledged the dialogue’s ambition and scope. Although nominally an inquiry into the nature of virtue, *Meno* also offers complex discussion on definition, inquiry and knowledge. It has been suggested that it is a mistake to search for a single main theme; rather, the dialogue has ‘complex unity’ and a ‘dramatised conflict of interests.’¹⁶⁹ Importantly for this project, it also presents mathematics as a model for philosophy, and the hypothetical method is used by Socrates for the first time here.

The *Meno* opens with a conversation between Meno and Socrates, in which Meno asks Socrates whether or not virtue can be taught. Socrates answers that he does not even know what virtue is; so he cannot possibly know what its properties are; this is typical of Socrates’ preference for a *ti esti* before inquiry into a *hypoion*. After three failed attempts to define virtue, Meno confesses to *aporia*. Moreover, he says that he does not even know how to inquire into it, as it is impossible to recognise something that he does not know, even when he has found it.

¹⁶⁹ Scott (2006) pp.1-7; Cf Fine (2007) pp.331-367, who also acknowledges the complexity of the dialogue.

As a response to Meno's paradox, Socrates introduces the theory of recollection. He conducts an experiment with Meno's slave boy asking him how to double the area of a square. The boy thinks he knows, but is mistaken. After a similar experience of *aporia* to Meno's, he discovers the answer. Socrates does not give him information; instead, he questions the slave until he arrives at the solution himself. Socrates thinks that this proves that the slave already possessed the knowledge himself. What is interesting for us is that, in this passage, there is no *ti esti* for what a square is; instead, Socrates uses a diagram.

What follows is the hypothetical passage (*Meno* 86-89), which draws upon the method used by geometers in their inquiries. Meno still wants to inquire into the *hypoion* question of whether virtue can be taught, whereas Socrates prefers to ask the *ti esti* question of what virtue itself is. However, Socrates reluctantly agrees to investigate whether or not virtue can be taught, providing they adopt the hypothetical method. This is done by using the hypothesis, 'virtue is knowledge.'

However, the dialogue still ends in *aporia*. After the hypothetical passage follows an empirical discussion which seems to show that virtue is not knowledge, and then a discussion of the difference between knowledge and true belief. Socrates says that they will not get to the bottom of the matter until they have agreed upon what virtue is. Therefore our interpretation of the hypothetical passage in the *Meno* needs to account for the apparent lack of progress on the issue, if we are to say that mathematics provides a step up the epistemological ladder.

The dialogue is often seen as a 'transitional' work between Plato's early 'Socratic' period and his middle works, in which he moves away from the Socratic *elenchus* and towards the more positive (rather than refutational) hypothetical method.¹⁷⁰ It is sometimes said to be a dialogue of two parts. The first part attempts to find a definition of virtue, and is

¹⁷⁰ See Vlastos (1991) pp. 107-131 and the introduction to this dissertation.

usually seen to be similar to the early Socratic dialogues; the second part, 80 to the end is where Plato introduces his more ‘Platonic,’ ‘middle-period’ ideas.¹⁷¹ However, some people deny the existence of such a sharp break; some say that we could see the whole dialogue as Platonic (Karasmanis [2006] pp. 129-141). The first part of the dialogue is the failed search for a *ti esti* for virtue, and the second part employs the hypothetical method, so the relationship between the two is crucial for our solution to the research question. This solution is that hypotheses (and images) are used in the absence of *ti esti* answers in inquiry at a certain point on Plato’s epistemological scale, ie, the section of the scale which uses mathematical reasoning.

The problem of the *Meno* for this project is this: if mathematics is not important in a general epistemological sense, or if is not imported into the philosophical method to a significant degree, and with positive results, the solution proposed by the project is undermined. We need to show that mathematics is an integral part of Plato’s epistemological scale, that he imports its methodology into his philosophy to a significant degree, and that hypotheses (and images, although this will be more important in the *Phaedo* chapter) are used as substitutes for *ti esti* answers.

The purpose of this section will be to show importance of mathematical reasoning in Plato’s *Meno*, arguing that the hypothetical method is adopted in response to the absence of a *ti esti* for virtue. The failed search for a definition in the first part has cohesion with the second, in that it justifies the resort to the hypothetical method, and also gives us the psychological preparation we need to use it and some epistemological signposts for how to do so (chapters one and two). We shall see that the hypothetical method does exist as distinct from the *elenchus*, and it succeeds at least partly in importing mathematical methodology into philosophy (chapter three). Moreover, it produces positive results, as the *aporia* at the end of

¹⁷¹ For example by Robinson (1953) pp.10-15, 114-122; Sharples (1985) pp.3-4.

the dialogue is a result of the empirical arguments that follow it, not of the hypothetical method itself (Chapter Four).

Chapter One: Definition in the *Meno*

Socrates insists that he cannot answer Meno's question, 'can virtue be taught?' until he knows what virtue is, so the first part of the *Meno* is spent searching for a definition, a *ti esti*, of virtue. I want to explain the hypothetical method in Plato's middle works as something the philosopher does when he does not have a *ti esti*. It is clear that Socrates thinks that a *ti esti* should be heuristic: in order to solve the problem of whether virtue is teachable, we need a *ti esti*: a definition for Plato is not just something done for its own sake, but this, or something like it is a necessary condition for inquiry. Therefore, the problem for this chapter will be to explain what, according to the *Meno*, a *ti esti* answer involves and how it might help us to gain knowledge.

The first subsection will examine what Socrates appears to want from a definition, looking at his objections to Meno's answers, and his own attempts at definitions. We shall examine Karasmanis' view that, in the *Meno*, Plato presents us with an almost complete theory of definition, and Charles' ([2006] pp.110-128) view that there are two questions: the signification question and the essence question. This should tell us what a *ti esti* should look like. Then we will look at how *ti esti* answers might fit into Plato's epistemological scheme. We shall examine Fine's view that inquiry in the *Meno* is conducted through a series of approximations, using this idea to place the *ti esti* in Plato's scale of reasoning.

i. What does Socrates want from a ti esti?

The *Meno* is usually considered to be a transitional dialogue, divided into two parts: 70-9 and 80 to 100c. The first part attempts to find a definition of virtue, and is usually seen to be similar to the early Socratic dialogues. The second part is sometimes seen (for example by Vlastos) as Plato's first attempt to employ his new, positive mathematical methods. However Karasmanis ([2006] pp.129-141) argues that the whole dialogue should be seen as Platonic, as the first part presents an almost complete theory of definition, not simply the refutation of three bad definitions of virtue. We shall examine the definitions given in the first part of the dialogue, and try to see what Plato tells us about what a good definition should offer.

There are six attempts at definitions in the first part of the *Meno*: three from Meno about virtue, and three mathematical definitions from Socrates. All three of Meno's definitions are judged to be inadequate, but Socrates' objections can tell us a lot about what a good definition should be like: we shall examine those first (a). Socrates has more success with his own mathematical definitions, so we shall go on to examine these (b). Finally, I want to put forward my own theory about what these definitions tell us about Plato's requirements for a *ti esti* response (c).

a. Meno's Definitions:

Meno's initial reaction when Socrates asks for a definition of virtue is to provide a list of examples of virtue:

First of all, if it is manly virtue you are after, it is easy to see that the virtue of a man consists in managing the city's affairs capably, and so that he will help his friends and injure his foes while taking care to come to no harm himself. Or if you want a woman's virtue, that is easily described. She must be a good housewife, careful with her stores and obedient to her husband. Then there is another virtue for a child, male or female, and another for an old man, free or

slave as you like; and a great many more kinds of virtue, so that no one need be at a loss to say what it is. For every act and every time of life, with reference to each separate function, there is one virtue for each of us, and similarly, I should say, a vice (*Meno* 71e-72a).

Socrates is not happy with this definition. He gives Meno the example of bees: insofar as they are bees, they do not differ from each other at all. If they differ, it is in respect to some other quality like size or beauty. He puns, 'I wanted one virtue and I find that you have a whole swarm of virtues to offer' (72ab). Socrates asks Meno to think about virtue in the same way:

Even if they are many and various, yet at least they all have some common character which makes them virtues. That is what ought to be kept in view by anyone who answers the question: 'What is virtue?' (*Meno* 72cd).

Socrates goes on to explain that, just as health, size and strength are the same in a man, a woman 'and the rest,' in their 'characters,' so virtue 'in its character' must be the same in each one of us, and Karasmanis thinks that we now have the following requirements for a good definition: it should give the common characteristic of all instances,¹⁷² and this characteristic must be 'because of which they are virtues.' Karasmanis thinks that this means the definition should refer to the essential nature of a thing.¹⁷³ I also want to note that Meno's definition is composed of particular instances from the physical world, which would place him firmly in the *pistis* phase of the divided line on the *Republic*.

¹⁷² Ie, 'one over many' or the unity assumption.

¹⁷³ Here, Karasmanis refers to *Meno* 72c8.

Meno's second definition does have the right form: it is not *hopoion* (that is, the definition aims to say what virtue *is*, rather than picking out one of its properties) and not by enumeration. The definition is:

It must be simply the capacity to govern men, if you are looking for one quality to cover all the instances (*Meno* 73d).

However, this is dismissed as not applying to all instances, as it does not apply to the virtue of a child or a slave. Karasmanis says that this definition and its refutation by a counterexample tells us:

- a. When we have a definition in the right form, we need to check for counterexamples.
- b. If the extension of the definiens is larger than the definiendum, we can revise by adding a qualification (as Socrates adds that 'justly' to the 'capacity to govern men').
- c. We should not identify a part with the whole.
- d. Enumeration of species of the genus is not the correct form.

Meno's final attempt, which comes after Socrates' mathematical definitions, is to quote a poet. According to this, virtue is:

... 'to rejoice in fine things and have power' and I define it as desiring fine things and being able to acquire them (*Meno* 77b).

This definition is dismissed after Socrates' introduction of what is known as the Socratic Paradox. Socrates argues that everyone desires good things; those who desire evil

things do not recognise that they are evil. We cannot discuss the Paradox here.¹⁷⁴ Socrates says that, according to this definition, no one is better than his neighbour (78b). He asks if Meno would add that the fine things must be acquired justly and righteously, and Meno agrees (78d). In this case, says Socrates, Meno has ignored what Socrates asked him to do and broken virtue up into fragments, as justice is itself a part of virtue (*Meno* 78e-79b). The question remains unresolved and Meno confesses to *aporia*.

Karasmanis says that we have now learnt that definitions:

- a. Should use the economy principle, as the definition is reduced to ‘the ability to get good things.’
- b. Can be refuted with a counterexample in problems of extension, as in this case, the extension of definiens is larger than the definiendum, so we restrict the extension of the definiens (‘... with justice or other parts of virtue’).

b. Socrates’ Definitions:

In order to help Meno to understand what he wants from a definition, Socrates gives three definitions of his own. He uses geometry as a model for the kind of thing he wants. When Meno says that ‘justice is virtue’ (73d), Socrates points out that it is not virtue, but *a* virtue, and uses geometry to illustrate this:

Take something quite general. Take roundness, for instance. I should say that it is a shape, not simply that it is shape, my reason being that there are other shapes as well (*Meno* 73e).

¹⁷⁴ See especially Scott (2006) Ch.4 pp. 46-59.

Meno still does not understand what Socrates wants, so Socrates points out that he does not want the kind of answer that gives a plurality:

Seeing that you call these many particulars by the same name, and say that every one of them is a shape, even though they are the contrary of each other, tell me what this is which embraces round as well as straight, and what you mean by shape when you say that straightness is a shape as much as roundness (*Meno* 74de).

He reiterates: 'I am looking for what is the same in all of them' (75a).

Meno wants Socrates to give his own definition first, so Socrates defines shape as 'the only thing that always accompanies colour' (75b).

However, Meno objects that this definition is 'naïve': if we do not know what colour is, we are no better off with this definition (75c). After checking that Meno understands the terms 'limit,' 'surface' and 'solid,' Socrates gives his second geometrical definition: shape is 'the limit of a solid' (76a).

Finally, Meno asks how Socrates would define colour, so, with reference to Empedocles' theory of effluences, Socrates says that 'colour is an effluence from shapes commensurate with sight and perceptible by it' (76d).

Karasmanis says that all three of Socrates' definitions are good according to the theory that Karasmanis says is implicit in his refutation of Meno's definitions: for all of them, the extension of definiendum is the same as the definiens, there is one characteristic common to all cases, and the definitions reveal the reason for something being X. As Karasmanis says, the third definition is the worst (possibly because this is drawn from the natural sciences): Socrates says that he is convinced that the definition of shape is better, so if the theory is

refuted, the definition collapses.¹⁷⁵ Karasmanis adds that it is actually an explanation of how we perceive colours, rather than a definition.

Which of the first two definitions is better? The answer to this can tell us a great deal about what Plato wants from a definition. Karasmanis says that most people think the second definition is best, and if one is better, we can suppose that one refers to the essence, the other to what the name signifies. However, says Karasmanis, if they are equally good, we need to admit that Plato permits two (best) definitions for the same thing: but that means that the same thing could have two essences, if the definition is supposed to reveal the essence of the thing defined.

Karasmanis has identified a lot of important features that Plato used in his evaluation of the six definitions that he proposes. However, I would hesitate to say that he gives us a complete theory of definition, or even an almost complete theory. That is not to say that this means the first part of the *Meno* is Socratic, rather than Platonic, because Plato does have a lot to say about what he is looking for in answers to a *ti esti*. However, as I shall argue below, he never explicitly endorses one particular type of definition over another, even if we can be reasonably sure that he does prefer one of the definitions over the others.

I will also argue that Plato might have a preferred answer, but that does not mean that this is the answer to the essence question, while the second-best is the answer to the signification question. Rather, (following Fine) it is just a better approximation. Moreover, if the definition is supposed to reveal the essence of a thing, two good but different ‘essence’ definitions of a thing do not mean two different essences, but two different means of revealing the same thing.

c. What is Plato’s preferred answer?

¹⁷⁵ *Meno* 76e. The implication is that both definitions of shape are better than the definition of colour.

I want to argue that Socrates does have a ‘best definition’ but this is because it is the best of a set of possible answers, and its superiority does not derive from the type of definition it is. In short, I want to say that Plato is looking for the best definition possible *at this stage in the inquiry*. I shall go on to build on Fine’s (2005 and 2010) idea, that we can inquire by drawing upon our true beliefs about a thing, by putting forward my own theory about how it is that we do that, drawing on evidence from later passages in the *Meno*. To do this, we first need to look more closely at what Plato wants from a *ti esti* answer.

We should examine whether there is a good reason to think that Plato is asking two different questions when he makes Socrates ask, ‘what is virtue?’ After some additional remarks on the Socrates’ definitions, which are, after all, his models for the answer to the virtue question, we shall consider David Charles’ careful argument that Plato is asking two questions about virtue: the signification question (ie, what does the word ‘virtue’ mean?) and the essence question (what is the *eidos*, or Form, of virtue?). Again following Fine, we shall argue that there is no evidence that Plato does make this distinction in the *Meno*. Moreover, we shall say, it is not clear that a *ti esti* for Plato needs to specify an *eidos*, at least not at this stage of the inquiry. What Plato wants from a *ti esti*, we shall argue, is the means by which to study the Form; but that need not be a full exposition of the Form itself.

David Charles ([2006] pp.110-128) points to the three kinds of definition that critics have identified when talking about Socrates’ responses to Meno. He says that these critics say that Socrates fails to determine which of these types of definition is to be preferred. The critics distinguish between:

(a) *real definitions*: propositions which give the essence of the thing to be defined (for example, colour is the effluence of shapes, commensurate with and perceptible to sight).

(b) *conceptual definitions*: true propositions about the thing to be defined, known a priori by anyone who understands the concept (for example, shape is the limit of solid).

(c) *factual claims which identify the phenomenon* (for example, shape is the only thing which always accompanies colour) (Charles [2006] p 110).¹⁷⁶

The problem is that none of these definitions is an ideal candidate for Plato's ideal definition. Type (a) seems to be a good choice at first, as Plato seems to want to get at the essence of a thing; sometimes his reference to the thing that makes a bee a bee at 72b is seen as an early reference to the Theory of Forms. It seems clear that, to be epistemologically useful, a *ti esti* must have some kind of relation to the Forms; whether this means that it must specify an *eidos* or whether it is simply a means to study the *eidos* will be discussed below. However, the particular example we are given of this kind of definition sheds doubt on whether Plato did prefer it to the others.

The definition of colour as an effluence from shapes is an example from Empedocles' theory of effluences, from the natural sciences.¹⁷⁷ As we have discussed in Section One, Chapter Three ii, and as we shall go on to discuss in Section Three, Chapter Two i and ii, Plato doesn't think that the natural sciences by themselves can furnish us with the truth: it would be odd if he chose to use them as a model for an ideal definition. Moreover, the textual evidence does not support the argument that this is Plato's favourite definition.

¹⁷⁶ Cf. Robinson (1953) pp. 49-60 who also discusses Plato's types of definition.

¹⁷⁷ The natural sciences in Plato's day were not advanced. There was no reliable way of demarcating the natural sciences from philosophy and early natural scientists had no systematic methodology or tradition of experiments. However, with these qualifications, we can see Empedocles as an early natural scientist. Empedocles believes that intelligence can overcome the limitations of sense-data (Sextus Empiricus, *Against the Logicians* I.125), and appeals to demonstrations that use physical objects, although these are more like observations than experiments. Empedocles is convinced that there is no such thing as void and uses these observations of the physical world to prove it. He refers to the operation of a clepsydra, which lifts quantities of water out of the river using trapped air (Aristotle, *On Youth, Old Age, Life and Death and Respiration* 473a15). He also investigates what the modern scientist would call centrifugal force with water in a cup, to develop theories about the motion of the heavens (Aristotle, *On the Heavens* 295a15-22). Moreover, Aristotle, in *Poetics* 1447b17-20 gives an account of Empedocles as a natural scientist rather than a poet.

Firstly, the definition is not presented as a model definition, but an answer *à la* Gorgias (76c). Socrates asks, ‘you and he (Gorgias) believe in Empedocles’ theory of effluences, do you not?’ (76c). But Socrates himself never himself commits to the theory. He says that, in the theory, there are passages to which and from which the effluences make their way, into which some of the effluences fit, whereas other effluences are too coarse or too fine (76cd). That colour is an effluence from shape commensurate with and perceptible by sight is what follows ‘from these notions,’ (76d) but at no point does Socrates maintain that the theory itself is true. In fact, although Meno likes this definition, Socrates is ‘convinced the other is better’ (76e), a clear show of preference for the geometrical answer over that from the natural sciences.

Type (c) definitions are often dismissed by critics as Plato’s preferred answer because of the forms they take. As Karasmanis (2006) points out, this seems strange to the modern reader: ‘we would expect a definition by a genus and differentia and we find a totally different pattern’ (Karasmanis [2006] p 136). This in itself need not lead us to rejecting it as a good definition: Meno objects to it for different reasons, these being that ‘colour’ is an as yet undefined term. Perhaps Meno would have been satisfied with the definition if Socrates had defined ‘colour’ beforehand, but it seems that the next definition, type (b), is taken both by Meno and Socrates to replace the first, so we should take this to mean that this is at least as good as the other.¹⁷⁸

Perhaps Plato is not intending to make a general argument about types of definitions at all. We can argue that *the example* of type (b) is Plato’s preferred definition on the grounds that, after Socrates and Meno use this definition to replace the type (c) definition Socrates

¹⁷⁸ Some people have puzzled over why Plato thought that shape was the limit of a solid and not the converse, as we find in Euclid. Karasmanis argues that at the time Plato wrote *Meno*, geometry had not been axiomatized. In fact, we could argue that Plato’s conversion of genus and species is evidence that this axiomatization had yet to be done: if the axioms we find in Euclid had been established, Plato would certainly have been aware of them from his studies in Sicily. The first axiomatization of geometry, Karasmanis argues, takes place within the Academy. See Karasmanis (2006) p 138 n 33 for this.

originally gave, then he goes on to say that it is to be preferred to the type (a) definition (*Meno* 76e). However, the type (a) definition is rejected on the grounds of its foundation in the natural sciences, and the type (b) definition is never actually rejected as being useless in inquiry. It's possible that the type (a) definitions could be perfectly acceptable if the terms are all properly defined beforehand, or that type (b) definitions would be acceptable if they were grounded in something other than the natural sciences.

We shall see below that one possible reason for preferring the particular type (b) definitions is that the examples we have of types (a) and (c) seem to cancel each other out, as they result in circularity. However, this is particular to these examples, and there is certainly no need to say that this criticism is applicable to all definitions in this category. It is likely that the critics who try to put Socrates' examples into these categories are making a mistake, and that each definition is judged on its own merits, and not on the type of definition it is. Socrates certainly never says that each of his definitions represent a particular type.¹⁷⁹ In this case, Plato's preference for any one of these definitions should not be taken as his affirmation of the priority of any one type of definition.

We should be clear that all of these definitions are attempts to answer the same question. For this reason, we shall examine Charles' argument that they are answers to two different questions, before going on to argue that the quest for a *ti esti* is an epistemological ascent for Plato.

Charles claims that Socrates is not asking just one question. He says that the answer to the (essence) question, 'What is virtue?' should have at least these two characteristics:

- i. It should specify the *eidos*, or form, which all virtues possess.
- ii. It should specify the *eidos*, by being which, all virtues are virtues.

¹⁷⁹ Moreover it is not clear that type (a) definitions are actually used at all: this would mean saying that the essence of shape is a kind of efflux, when it is not at all clear that this is how Plato conceives of essence.

He says that Socrates sometimes asks a different question (the signification question):

iii. 'What is it to which the name x applies?'

This is less demanding, but Charles thinks that there is linguistic evidence that Socrates means this.

We shall consider Charles' claims about the answer to an essence question needing to specify an *eidos* in the following chapter, but first, let us examine his claim that Socrates sometimes asks two different questions.

The number of definitions as answers to the question, 'what is virtue?' in the *Meno* has sometimes puzzled people: we saw Karasmanis' attempt to construct a single theory of definition from them above. Charles says that, instead of there being 'one good question, several distinct and conflated answers,' there are 'two good questions, each with an appropriate answer'. These are, *What is virtue?* (essence question) and *What is 'virtue?'* (signification question). Charles uses three kinds of evidence to support this claim. He cites linguistic evidence: the fact that Socrates sometimes asks what virtue is, sometimes to what the name 'virtue' refers. Secondly, he cites the fact that the accounts of shape and colour don't specify the real essence of shape or colour, but are good answers to the signification question. Finally, he cites Socrates' alleged confusion of the signification and essence questions in his formulation of and reply to Meno's paradox.¹⁸⁰

It would be difficult to argue that Socrates is asking two distinct questions on purely linguistic grounds: but Charles does provide some linguistic evidence, citing various places in which he thinks Plato is asking two different questions: When he asks, 'what is virtue?' at 77b9, Charles thinks that he is specifically interested in the *eidos* of virtue. However, when he goes on to define 'shape,' he asks, 'do you call something a boundary?' at 75e1, and 'what

¹⁸⁰ The Paradox goes from [A] 'How can you search for something (e.g. for what virtue is) if you do not know at all what virtue is?' (80d5-6) to [B] 'A person cannot search for what he knows because he already knows it nor can he search for what he does not know, since he does not know what he is searching for.' Charles thinks that Socrates conflates the two questions: ie, Meno is asking the signification question, saying that you need it in order to answer the essence question.

is that of which there is the name 'shape'? In this way, Charles would argue that 'shape (is the name of) the limit of a solid' (Charles [2006] p 112).

However, without additional evidence to support this, we could just cite stylistic variation for these differences. For example, Gail Fine ([2010] pp. 125-152) says that all of Socrates' questions are just different ways of asking the essence question. She says that there is 'one good question, with one correct sort of answer, as well as others that approximate to it.'¹⁸¹ We shall examine Fine's alternative reading below.

As for Charles' point that the accounts of shape and colour don't specify the real essence of shape or colour, but are good answers to the signification question, we should remember why Socrates is giving us these definitions in the first place. They are models for Meno's definition of virtue, which Charles says should be an answer to the essence question. He uses the example of there being shapes other than roundness to point out one of Meno's mistakes: 'Just as I could name other shapes if you told me to, in the same way, mention some other virtues' (74a). Subsequently, the attempt to define shape is 'practice for the question about virtue' (75b). It would be very strange if Socrates sets these definitions up as models for the answer to the virtue question, when in fact they are answers to a completely different kind of question.

Charles points out that his reading avoids the circularity in Socrates' definitions. As has sometimes been said, if we combine two of the definitions, we arrive at a circular statement. So:

¹⁸¹ Fine also successfully refutes Charles' view that Socrates confuses the signification and essence questions in his formulation of a reply to Meno's paradox. She agrees with Charles that Socrates takes Meno to think there are just two possibilities. Meno thinks that either one knows, or else one doesn't know: 'But this needn't involve confusion between the essence and signification questions.' Fine suggests that Socrates thinks Meno assumes that, for any x, one either knows all there is to know about x or is in a complete cognitive blank with respect to x: 'It's true that someone who holds that view doesn't believe one can know what 'F' signifies without knowing the essence of F. But it's not that the person conflates the two issues.' Fine's solution is that Plato tries to show that lack of knowledge does not have to mean being in a complete cognitive blank. As we shall see below, this is all part of Plato's epistemological scale.

‘Shape is that which always accompanies colour’

and,

‘Colour is the effluence of shape’

Gives us the combined answer,

‘Shape is that which always accompanies the effluence of shape’

or,

‘Colour is the effluence of that which always accompanies colour’¹⁸²

As we said above, one reason for preferring the alternative definition of shape (shape is the limit of a solid) is that it avoids the circularity. If we say that this is Plato’s preferred definition, we have no need to adopt Charles’ distinction between the signification and essence questions to avoid this, because this second definition of shape produces no circularity when combined with the definition of colour. The definition of colour is only introduced after Socrates has replaced the first definition of shape, so there is no question of Socrates wanting to combine it with the first definition.

In conclusion, there is no need to say that there are two separate questions when Socrates asks, ‘what is virtue?’ If there were, it would mean that Socrates’ definitions are not supposed to be models for Meno’s answer, which is inconsistent with what he actually says in the text. There is only one question, with several attempts to answer it. Plato thinks that some answers are better than others, but that does not mean that he is endorsing any particular ‘type’ of definition, at least not as the critics have defined them.

¹⁸² See Charles (2006) p 115, although many others have remarked on this apparent circularity.

What we are left with is a set of different answers to the question, ‘what is virtue?’ Up until now, I have tacitly assumed that this is what Charles would call the essence question, but this needs to be qualified to avoid Karasmanis’ problem of a thing having two essences if there are two good ‘essence-type’ definitions of something. I have said that Plato prefers the definition ‘shape is the limit of a solid,’ so he does have a preferred definition; but this does not exclude the possibility of there being a case in which there are two equally good definitions, since I have also argued that Plato does not endorse any particular type of definition. I shall now go on to tackle these problems.

ii. What role do definitions play in Plato’s epistemological scale?

I want to explain *ti esti* answers as having a role in the ascent up Plato’s epistemological scale. Any attempt at a *ti esti* is a good start, and can be improved upon by a series of better approximations. As I shall argue in the rest of the *Meno* chapter, failure to produce a satisfactory *ti esti* is often followed by *aporia* and the hypothetical method, but what we eventually want from a *ti esti* is for it to reveal the Form. After saying a few words about *ti estis* and essences, we shall consider Gail Fine’s concept of inquiry in the *Meno*, endorsing her idea that we can proceed through a series of good approximations. I want to build on that by putting forward my own theory about how it is that the approximations help us in the epistemological ascent. Finally, we shall see what this means for our concept of a *ti esti*, and their wider role in Plato’s epistemology.

a. Definition and Essence

We said that there is only one question about virtue in the first part of the *Meno*, and so far I have tacitly assumed that this is at least some kind of essence question. We also said that we wanted a *ti esti* to help to reveal a Form. A *ti esti* cannot actually be a Form; at the very most it could be a literally true and exhaustive description of a Form. However, it is not clear that Plato is asking for even this much. The geometrical definitions certainly do not provide this, and we said that these definitions were model answers. It could be that a *ti esti* allows us to examine a form and then make inferences about it.¹⁸³

However, this does not necessarily mean that a *ti esti* is an exhaustive account of a Form, merely that it is something by means of which we are able to study the Form. It should at least identify the Form; that is, it should answer the signification question, and also allow us to say other things about it. It is possible that what have been called conceptual definitions could enable us to do this. By giving true propositions about the thing to be defined (known *a priori* by anyone who understands the concept) we could arrive at other true propositions about it. As Socrates says, ‘when a man has recalled a single piece of knowledge,...there is no reason why he should not find out all the rest’ (*Meno* 81d). Through this process, we would eventually arrive at an account, which is the kind of thing Socrates wants when he asks ‘what is *x*’ type questions.

We traditionally expect Socrates to be looking for a short, perhaps sentence-long answer for a *ti esti*, but this may not be the case. The longest description of a single Form we have occurs in the *Symposium*; it is not limited to a single proposition or statement. This longer account enables us to see connections between the properties of the Form, and its connection to other Forms, which is what Plato is ultimately looking for, and the best shot we have of providing a teleological account.¹⁸⁴

¹⁸³ Cf White (1976) pp. 35-38, in which he discusses the role of definitions in enabling us to examine Forms.

¹⁸⁴ For example, in the *Symposium* (210e-211b; cf. 211c-212a) we are given the most extensive description of a Form than we find anywhere else in Plato. Is any one of these statements a clear candidate for the only *ti esti* of

At the very least, we need our *ti esti* to enable us to arrive at other true propositions about the thing defined. If we are going to allow a concise answer, the preferred definition of shape is a good candidate for this, as it specifies the relation of shape to solid, so we can go in to ask more questions about the nature of a limit, what shape must be like to perform this relation, etc, whereas the definition, ‘shape is the only thing which always accompanies colour’ tells us nothing about the relation of shape to colour and gives us less scope for further questions.

This would explain why Plato prefers the definition, ‘shape is the limit of a solid,’ because we might arrive at other true statements about shape by building on this definition. At the same time, there could be another, equally good definition that allows us to do the same thing. In this way, we can have more than one good *ti esti* about the same thing without having to admit to there being two essences of that thing.

b. Gail Fine and the Meno

In her response to David Charles, Fine elaborates on her previous work on epistemology in the *Meno*. She thinks that, in the *Meno*, there is ‘one good question (about virtue), with one correct sort of answer, as well as others that approximate to it’ (2010 pp. 125-152). We have already said that there is only one question about virtue (at least in the first part of the *Meno*). We also said that there was a preferred answer, although this is not necessarily something that can be reached by knowing a particular format to look for in advance (ie, Plato does not give us an ideal type of definition).

Fine draws attention to the importance of ‘helpful steps’ in the quest for knowledge. In her evaluation of Charles’ work, she points to the first definition of shape. We said that this

Beauty? Could we derive several good but different *ti estis* from the passage? On my reading, a *ti esti* is not the concise statement we traditionally expect, but rather, a longer account like this.

seemed like an odd definition, because it simply gave a feature true of all and only shapes, without attempting to answer the essence question or being in the form we are used to (definition by a genus and differentia). Fine thinks that Socrates views this as helpful step in the effort to find the essence of shape. She refers us to 75b11-c1, where Socrates says he would be pleased if Meno could say what virtue is in a way analogous to saying that shape is the only thing that always follows colour.

Fine takes him to mean that he would be pleased if Meno could give an analogous answer to the essence question ‘What is virtue?, ‘not because it would be ultimately satisfactory, but because it would be better than anything Meno has come up with so far - a helpful step in the effort to answer the essence question.’

This idea of ‘helpful steps’ is an extension of Fine’s earlier work on inquiry in the *Meno* (2005) which is in turn an extension of her wider project to refute the ‘two worlds’ reading of Plato’s middle dialogues (1990 and 1995). I do not intend to endorse Fine’s wider project of refuting the ‘two worlds’ reading, and I made it clear exactly how my reading differs from Fine’s in the *Republic* chapter. However, I do share Fine’s reading of the *Meno* in the sense that inquiry is a kind of ascent to knowledge, made through a series of approximations.

We shall examine Fine’s reading more closely, before identifying exactly what it is I want to endorse. I shall then go on to put forward my own explanation about how each step in the scale is helpful.

Fine acknowledges the importance of *ti esti* answers in Platonic inquiry. Like me, she does not think that this prevents the inquiry from making progress. She argues that Socrates’ insistence on having a *ti esti* answer for inquiry, which she calls the Priority of Knowledge What (PKW), does not mean that we need knowledge before we even begin to inquire (not even recollection). She says that it is not clear that PKW says that one needs to know the

essence of something to know anything else about it. It simply says that one needs to know what (*ti*) something is, to know what it is like (*poion*). According to Fine ([2005] pp.200-226), PKW means that if one does not know the definition of something, one cannot know anything at all about it; but one can have true beliefs and these can guide inquiry.

In addition to her comments on the ‘helpful steps’ in the first part of the dialogue, Fine thinks that the second part displays this kind of inquiry. This, she says, is the elenctic reply to Meno’s paradox. I’m going to propose a slightly different reading of the second part of the *Meno* below: I want to say that what follows from the paradox is actually a departure from the elenchus. However, I do agree with Fine’s point that Plato’s solution to the paradox does not end with the theory of recollection. Moreover, I think that the parts of Fine’s reading I want to endorse can support my conception of an epistemological ascent in Plato, so now is a good point to explain which parts these are.

Let us first examine the paradox. After Meno admits to *aporia*, Socrates urges him to carry on the inquiry into what virtue is. However, Meno thinks he can convince Socrates that his inquiry is impossible, producing the following paradox:

But how will you look for something when you don’t in the least know what it is? How on earth are you going to set up something you don’t know as the object of your search? To put it another way, even if you come right up against it, how will you know that what you have found is the thing you didn’t know? (*Meno* 80d).

The implications of the paradox are that gaining new knowledge is impossible: in fact, it is impossible to begin inquiry at all. As part of his response to Meno, Socrates then introduces the theory of recollection. He says that he has heard that the soul of man is immortal:

Thus the soul, since it is immortal and has been born many times, and has seen all things both here and in the other world, has learned everything that is...for seeking and learning are in fact nothing but recollection (*Meno* 81cd).

In order to illustrate this, Socrates carries out the slave-boy experiment. He draws a square in the sand and asks one of Meno's slaves if he is familiar with the figure. He then asks the slave-boy about the properties of the square: the boy has never studied mathematics before, but Socrates wants to know if the boy knows how to double the area of the square. The boy initially gets the answer wrong, but after Socrates questions him more closely, the boy arrives at the correct answer. Socrates claims that he has answered only with his own opinions, and he has gone from not knowing to knowing. He says that his 'opinions were somewhere in him' (*Meno* 85c). In this way, all learning is actually recollection.

The theory of recollection is sometimes seen as Plato's response to Meno's Paradox: if we admit that acquiring new knowledge is impossible, claiming that learning is actually remembering existing knowledge avoids the problem. However, it does not tell us how this existing knowledge is accessed. Fine thinks that, for this reason, the theory of recollection cannot be the whole reply to the paradox.

According to Fine, the slave-boy experiment illustrates that the slave lacks knowledge, but has true beliefs, so that enables him to inquire.¹⁸⁵ She says that recollection is important, but it is not the answer to the paradox on its own: the method of elenchus is the key. According to Fine, the difference between knowledge and true belief is crucial to the elenctic reply: the slave-boy can inquire into geometry in the absence of knowledge, because he has true beliefs about the subject. Although she acknowledges that recollection is important, she disagrees with Nehamas' ([1985] pp.5-6) objection that *Meno* 85c7, in which Socrates' points

¹⁸⁵ In fact, Socrates does say that the slave will have to repeat the exercise several times before he can be said to have knowledge.

to the slave's true opinions, is just an intermediate step. Fine points out that Plato returns to true belief later in the dialogue, saying that it is a good guide for action: thus, it is not just an intermediate step, but the reason that inquiry is possible.¹⁸⁶

In this way, lacking knowledge of something does not mean having a complete cognitive blank, because we still have true beliefs, and it is this that provides the 'helpful steps' in inquiry. Fine draws heavily on the distinction between knowledge and true belief that we find in the final stages of the *Meno*, taking seriously the metaphor in which true belief about the way to Larissa is as good a guide as knowledge. We shall examine this more closely below.

But do we need to rely so heavily on true belief to argue that inquiry is made through a series of approximations? Unarguably, true belief is important. The Socratic *elenchus* requires the interlocutor to 'say only what he believes,' which implicitly assumes that the truth is in him already.¹⁸⁷ True belief is important in the generation of propositions to be investigated (that is, it is important in the context of discovery as well as the context of justification).¹⁸⁸ Then, the theory of recollection that we find in the *Meno* and other middle-period works makes this assumption explicit: 'So a man who does not know has in himself true opinions without having knowledge.' (*Meno* 85c).

However, in the following chapter, I will argue that this 'say only what you believe' rule takes a back seat in the second part of the dialogue, when the hypothetical method is introduced. In addition, Socrates seems to be relaxing this rule in the first part of the dialogue. He begins by insisting on the rule: when Meno says that Gorgias knows what virtue is, he says, 'Then let's leave him out of it, since after all he isn't here. What do you yourself

¹⁸⁶ See the section on knowledge and true belief in the *Meno*, below.

¹⁸⁷ Cf Vlastos ([1988] pp. 362-396), who also discusses this.

¹⁸⁸ See Reichenbach's ([1938] pp. 6-7 and 381-382) famous distinction between the contexts of discovery and justification. Burnyeat [(2000) pp.6-8] discusses Reichenbach's distinction in relation to Plato.

say virtue is?’ (*Meno* 71d). However, Socrates himself does not believe at least one of his own definitions: as we pointed out, he never commits to believing his definition of colour.

True belief is definitely important in the slave-boy experiment, although it is not clear whether it is equally important in the rest of the dialogue. It could be that inquiry in the *Meno* works just as well if the interlocutor proposes a series of approximations that have no connection to what he believes, but that are necessary to propose in order to solve the problem: that’s something we shall examine in the third chapter of this section. For now, let us examine Fine’s broader idea of ‘helpful steps’ as a route to knowledge.

The slave-boy experiment is a demonstration of the doctrine of recollection, which most people agree is at least Plato’s initial response to the paradox. Fine thinks that the experiment is also the ‘elenctic response’ in that it shows how we may recollect: through the use of the *elenchus*. I think that the hypothetical method that follows the passage is Plato’s abandonment of the *elenchus*, so I think it would be wrong to say that Plato is proposing the *elenchus* as the only way to recollect, but, with this in mind, I want to otherwise endorse Fine’s reading, and explain how the slave-boy experiment fits in with Plato’s epistemological ascent.

At first, the only concern is that the boy speaks Greek (82b): he has never studied mathematics before, although he is able to recognise the image of a square when Socrates draws one (82b). Socrates does not ask him what a square is. This is the first point I want to make about what happens when Socrates allows inquiry without an accepted definition. In this case, *Socrates does not ask for the ti esti of a square, but draws an image of a square on the sand*. This project argues that, at least in middle-Plato, if we are to inquire about something without a *ti esti* and arrive at knowledge, we need to use something to stand in for that *ti esti*, and, in pure dianoetic thought, that is going to be either an image or a hypothesis. The slave-boy experiment is evidence for this, and gives us an additional suggestion: if the

investigation is about a mathematical object, the likelihood is that the ‘stand-in’ is going to be a mathematical diagram, as it is in this case.¹⁸⁹

Socrates asks the slave: Are the four sides equal? Are the two lines which intersect in the middle of the square equal? Could the square be either larger or smaller? If the area were two feet in one direction and only one in the other, would it not be two feet taken once? Is the area actually twice two feet? (82cd) The boy answers truly in the affirmative, then works out that the area is four feet. Socrates is asking carefully chosen questions, but the boy is answering with his own opinions: so far, Fine’s point that it is the slave’s true beliefs that enable him to inquire is not contradicted.

Socrates then poses the following problem for the boy: how can we draw a square with double the area? The slave correctly believes that the area will be eight feet, but incorrectly thinks that the sides will be four feet long (82de). Socrates introduces what follows as ‘the proper way to recollect’ (82e), which supports Fine’s argument that the experiment is an elaboration of the ‘recollection’ response to the paradox.

What Socrates does next is elenctic:¹⁹⁰ he proceeds to question the boy on his beliefs, demonstrating that they lead to a falsehood: ‘doubling the side has not given us a double but a fourfold figure’ (83b). They decide that ‘the side of the eight-foot figure must be longer than two feet but shorter than four’ (83d).

Next, the slave says that the side will be three feet long. Again, Socrates repeats the elenctic process of showing the boy that his beliefs are incorrect through a method of question and answer and the boy admits to *aporia*. In the passage that follows, Socrates continues to only ask questions until they arrive at the answer.

¹⁸⁹ Cf Patterson (2007) pp. 1-33, who also sees the diagram as part of an abbreviated definition of a square.

¹⁹⁰ The slave-boy passage is often seen as a paradigm of the Socratic elenchus; for example by Robinson (1953) pp. 10-12 and Sharples (1985) pp.8-9.

The ‘notorious objection’ is that ‘the vast majority of his questions contain the correct answer. They take the form, *such and such is true, is it not?*’¹⁹¹ The problem is that Socrates already knows the answer, so we are inclined to accuse him of feeding the boy information surreptitiously. In support of Fine’s view, we do have a series of approximations leading to the correct answer: first the slave gives two completely wrong answers (82e and 83e); then he assents to a good approximation¹⁹² before finally arriving at the correct answer (85b).

However, the Socratic question-and answer method is not sufficient on its own to arrive at the answer. In the introduction, we noted that the *elenchus* is a method for eliminating falsehood, but in this case, we want to solve a particular problem: how to double the area of a square. We might get away with saying that Socrates’ *x- is-true- is-it-not* questions are a valid part of the *elenchus*, but we should remember that the puzzle is solved by Socrates drawing in extra lines in the diagram. That is, the heuristic success of the experiment relies on the mathematical image. Let us examine the role played by the diagram in finding a solution:

1. After the *aporia*, Socrates points to the original square of four feet (84d).
2. He then draws in three identical squares, to make a single large square of sixteen feet. The boy agrees it is four times the original (84de).
3. He draws in the diagonal of each small square, and the boy agrees that they cut each of the squares in half (85a).
4. He asks the boy how many halves there are in the large figure, and the boy answers, ‘four.’ (85a).

¹⁹¹ Scott (2006) p 101. Scott’s reading of this is that Socrates is not teaching the boy, but allowing him to crystallise views that he already holds. There is also the possibility that Socrates is using the word teach in a narrower sense than we might: maybe teaching for Socrates implies a certain level of passiveness on the part of the learner.

¹⁹² ‘...the side of the eight-feet figure must be longer than two feet but shorter than four.’ *Meno* 83d.

5. He asks how many halves are in the original figure, and the boy answers, ‘two’ (85b).
6. Socrates says, ‘What is the relation of four to two?’ and the boy answers, ‘Double’ (85b).
7. Socrates says, ‘How big is the figure then?’ and the boy answers, ‘Eight feet’ (85c).
8. Socrates asks, ‘On what base?’ and the slave answers, ‘this one,’ meaning the diagonal of the original square (85b).

Steps one to three are taken up with Socrates adding in lines on the diagram and checking that the slave understands their relationship to the original square. Steps four to eight are only possible because of these additions. The additional lines are heuristic, and enable the problem to be solved. This is a different function from that of the original diagram, which was initially used to check that the boy knew ‘that a square is a figure like this’ (82b). To use Charles’ terminology, it is initially used as an answer to the signification question, *what is it to which the term ‘square’ applies?*

Extra lines on the diagram are used by Socrates to purge the boy of his false beliefs: Socrates actually draws the squares of three-foot and eight-foot sides, when the boy suggests them as solutions, to illustrate that they are the wrong sizes. However, the diagrams play a vital heuristic role in the final part of the experiment, when the boy comes up with a solution.¹⁹³ We said that a *ti esti* response should at least answer the signification question, and also allow us to say other things about it; derive true propositions and say what its

¹⁹³ I have left room in my interpretation for the view that Socrates is ‘cheating’ by feeding the boy information, but the view I have taken is also compatible with a more charitable reading. It is still possible to argue, as Scott ([2006] pp.98-120) does, that the additional lines in the diagram are vital but allow the boy to understand for himself. What’s important about my interpretation is that, without a *ti esti* for a square, the diagram is essential in solving the problem.

properties are. We do not have a *ti esti* for the square in this particular experiment, so the diagram performs that role.

It should be noted that, in this case, the problem requires a diagram for the solution. The question is, ‘what will be the side of a double-sized (square)?’ (82d) The answer is that it will be the length of the diagonal of the original two-foot sided square. However, these do not share a common measure, so the only way for the boy to answer the question is to point to the line on a diagram.

As we saw in the introduction, the discovery of the incommensurability of the side of a square and its diagonal had already played a part in Greek intellectual history, and Plato was well aware of its implications for philosophical discourse. He is also interested in the mathematician’s reliance on the diagram in his inquiries, as we discussed in the *Republic* section. Plato’s use of this particular problem in the slave-boy experiment allows him to highlight the role played by diagrams in mathematical investigation, but it also allows us to see Plato’s conception of dianoetic inquiry. We saw from the first part of the dialogue that a *ti esti* needs to answer the signification question, and also allow us to further investigate the properties of the thing defined: that is, it has a heuristic role to play. In the slave-boy passage, we do not have a *ti esti*, so the diagram performs that role.

In this way, Fine’s argument about true beliefs enabling inquiry in the absence of knowledge, through a series of ‘helpful steps’ is not a complete explanation: we also need a heuristic mechanism, in the form of a *ti esti*, if one is available, or a diagram in the case of the slave-boy passage. As we will see below, even the true beliefs can even be dispensed with when we employ the hypothetical method: in this case, we can replace beliefs with the ability to posit lemmas. I would qualify Fine’s explanation in this way, while retaining her conception of inquiry in the *Meno* as a series of helpful steps and approximations towards knowledge.

In conclusion, we have seen that a *ti esti* is necessary to inquire about something. If we do not have one, we need something to stand in for it, like the diagram in the slave-boy experiment. A *ti esti* or its proxy should not only identify the thing to be investigated, but also be a heuristic tool; something that allows us to make progress in the quest for knowledge. The heuristic value of the diagram in the slave-boy passage allowed an epistemological ascent for the boy: he now has *dianoia* kind of knowledge, whereas before he only had *pistis*. Meno begins the inquiry in a state of *pistis*; he looks to instances in the physical world for an answer to Socrates' questions; however Socrates thinks that a *ti esti* for virtue can allow us to make headway in our investigations into what it is like. In this way, a *ti esti* or its proxy is a vital step in Plato's epistemological scale.

Chapter Two: *Aporia* and the Psychology of Mathematics

We saw that Fine's account of true belief as a guide to inquiry in the absence of knowledge was incomplete in that we need a heuristic mechanism to help us to inquire. Preferably, this is a *ti esti*, which is why the first part of the dialogue is spent searching for one. In the slave-boy experiment, a diagram was used as a substitute, but Meno still insists on wanting to inquire into the properties of virtue, when he is still in a state of *aporia* about its *ti esti*. Socrates agrees to do this only if they use the hypothetical method, a clear indication that this is something we do in the absence of a *ti esti*. In this chapter, I want to present the hypothetical passage as a progression of the epistemological journey started at the beginning of the dialogue: it builds upon the *aporia* established by three failed definitions of virtue.

The hypothetical method is often seen as a way out of *aporia*, a state of perplexity often induced by Socrates which characterises many of the early dialogues. In *Meno*, there are instances of *aporia* experienced by Meno (79e-80b) and the slave (84a) respectively, but

we shall see that the *Meno* gives us a different presentation of *aporia* than we find in the earlier dialogues. This chapter will present *aporia* in the *Meno* as a necessary state of mind for research: it is psychological preparation for the hypothetical method and a bridge between *pistis* and *dianoia*. In doing so, we shall see that the hypothetical method is something very much distinct from the *elenchus*, more evidence for the place of mathematical reasoning as an integral part of the epistemological scale.

v. *What is aporia?*

Socratic *aporia* is the state of puzzlement that results from the interrogation of one's beliefs by Socrates. The Socratic interlocutor gives his opinion on a subject, only to find that his opinion leads to contradictory beliefs. He then finds himself in a state of *aporia*, perplexity, about the truth of the matter in question.¹⁹⁴

Meno experiences *aporia* after the failure of his attempts to define virtue. He admits to having previously thought that he knew what it was, often speaking about it in front of large audiences (80b), before being perplexed by Socrates (80ab) and now having nothing to reply (80b). The slave boy undergoes the same sequence of emotions when Socrates questions him. The boy thought he knew the answer and Socrates playfully remarks (84b) that he thought he could, like Meno, speak well and fluently on the problem on many occasions before large audiences, before being perplexed by Socrates and having nothing to reply (82d).

vi. *What good is aporia?*

¹⁹⁴ See Vlastos ([1988] pp. 362-396) and Scott ([2006] pp. 69-74) for a further discussion of this.

On the traditional reading, the *aporia* we see in the early (Socratic) dialogues plays the role of purging the interlocutor of his false beliefs, making him more likely to pursue knowledge as a result. There is a purgative effect; Meno no longer has the same conviction about his initial definitions of virtue (71e-79e); the slave boy no longer believes that doubling the length of the side will double the area of a square (82e-84a). In other dialogues, Charmides is disencumbered of his false definitions of self-control (*Charmides* 160bd-161ab;162ab); Laches is disencumbered of his false definitions of bravery (*Laches* 190e-193e).

We also have a motivational effect. The horse (a metaphor for the Athenian) of the *Apology* is stimulated into action (*Apology* 30e); *Laches* closes with everybody keen to recommence the discussion in the morning (*Laches* 201bc); Charmides resolves to turn himself over to Socrates for more training (*Charmides* 176bd). The presentation of *aporia* in *Meno* is the same in this respect. Socrates points out that the slave boy is now aware of his own ignorance (84a); he is in a better position than he was before, and no harm has been done to him by the experience (84b). In fact, now he is more likely to search for what he does not know and the process was good for him (84c).

This might be what Socrates would have us believe, but it is not always clear from the dialogues that the result is so positive. Consider Beversluis' comments on *Laches*:

In the end, everyone remains exactly as they were. All they can do is marvel at the time they have wasted, and, with their appetites inexplicably whetted for more of the same, resolve to reconvene the next day... Why bother?¹⁹⁵

Myles Burnyeat argues that it is not until *Theaetetus* that *aporia* is treated as a productive state (Burnyeat [1977]). He thinks that it is the first stirring of creative thought, whereas in

¹⁹⁵ Beversluis (2000) p 134. Not everyone has such a negative view of *aporia*: see Scott (2006) pp.69-74.

the early dialogues it is valued only for its purgative functions.¹⁹⁶ In *Theaetetus*, we are given the metaphor of Socrates as midwife, and *aporia* represents the labour pains of an idea struggling to be born (*Theaetetus* 148e; 151ab). Burnyeat thinks that we should take the metaphor seriously, and wishes to draw a line between the *aporia* of *Theaetetus* and that of the early dialogues – including *Meno*. We shall see how the presentation of *aporia* in *Meno* is also a departure from that of the earlier dialogues; here we are given another metaphor, which should also be taken seriously.

vii. *What is distinct about aporia in Meno?*

Meno likens the experience of *aporia* to being stung by a stingray: not only does Socrates resemble the stingray in outward appearance, but he also has the same effect on those he meets; he renders them numb, into a state of torpor (νάρκη; *Meno* 80ab). Socrates protests that the metaphor succeeds only if the stingray renders himself numb at the same time; in the discussion about virtue (although not in the slave boy passage), Socrates admits to being just as perplexed as his interlocutor (80cd).

This aspect of *aporia* is quite different from the one we see in *Apology*. Here, Socrates claims to be like a gadfly, stinging a large thoroughbred horse. The horse (which in this case represents the city) is lazy and in need of stimulation. Socrates does not cease to settle on the horse, ‘rousing, persuading, reproving’ so that before long the city may awake from its drowsing (*Apology* 30e-31a). This is a very different image from the one we are

¹⁹⁶ We should not limit our conception of *aporia* only to these two functions in the early dialogues. See, for example, MacKenzie ([1998] pp. 351-38) on *aporia* as self-consciousness in *Charmides* and Politis ([2006] pp. 88-109) on the different kinds of *aporia* in early Plato.

given in *Meno*; in *Apology*, Socrates tries to shake people out of their state of torpor; in *Meno*, he seems to induce it.¹⁹⁷

Of course this is not the only point of divergence in *Meno* from the earlier dialogues. As we noted in the introduction, *Meno* is often seen as a ‘transitional’ dialogue, where the Socratic *elenchus* is abandoned, and Plato experiments with a philosophical method that produces positive results.¹⁹⁸ The early ‘Aporetic’ dialogues are products of the *elenchus*, which serves to eliminate falsehood, but has no means of generating truths. As Ryle puts it,

...Socrates has, like a gadfly, to sting Athens into wakefulness, with almost nothing to show what she is to be wakened to, save to the existence of the gadfly (Ryle [1966] p 177).

Meno, in the hypothetical passage, tentatively attempts to say something positive about virtue.¹⁹⁹ To do this, we need a quite different approach, and this is where the hypothetical method comes in.

The introduction of this method in the *Meno* is significant to the different presentation of *aporia* we find there. There is nothing incompatible about the *aporia* purging one’s false beliefs, motivating one to learn, and yet leaving one in a state of torpor – in fact, we find all three present in *Meno*. But certainly here the emphasis is different; the *νάρκη* induced by *ἡ πλατεία νάρκη ἢ θαλαττία* is central to the presentation. For the rest of this chapter, we shall see that this state of mind is important psychological preparation for those wishing to undertake hypothetical investigation.

viii. *Aporia and hypothesis*

¹⁹⁷ Other scholars have noticed this difference before; see Scott (2006), Ch. 6 pp. 69-74 and Cavini (2009) pp. 159-187.

¹⁹⁸ Cf. Vlastos (1988) pp. 362-396; Karasmanis (2006) pp 129-141 and Scott (2006) pp. 129-133, who support this view.

¹⁹⁹ Cf. Robinson (1953) pp.114-122, who supports this view.

The hypothetical passage in *Meno* is an inquiry without a definite starting-point. We are given hints throughout the dialogue that an inquiry with a definite starting point would look very different. We are told that once a man has recalled a single piece of knowledge, there is nothing to stop him from finding out all the others (81d); and in presenting the essential nature of virtue as the main question (86d), Plato implies that knowing the nature of virtue itself would allow us to deduce other things about it.²⁰⁰

Meno himself notices the interconnectedness of definitions in geometry and, by extension in philosophy. When Socrates defines shape as the only thing which always accompanies colour, Meno points out that we need to know what colour is in order for the definition to be acceptable (75bc). Without a solid definition of virtue, in which all terms are agreed upon, the hypothetical method is adopted as a second best approach to inquiry.²⁰¹

The hypothetical approach is often seen as a way out of the *aporia* of not knowing what virtue itself is, but *aporia* in *Meno* is actually important preparation for using the method. In particular it is this conception of being in a ‘frozen’ state of mind that is important when we undertake mathematical investigation, or, as in the *Meno*, philosophical investigation modelled on a mathematical approach.

The stingray is a literary metaphor; it is not picked apart explicitly in the dialogue for philosophical merit. Why, then, should we take it seriously as having important philosophical meaning? There are two justifications for doing so. Firstly, as we have previously noted, Socrates returns to Meno’s comments in the slave boy passage, pointing out that the numbing process has been good for him (84ab). This tells us that Plato wishes to emphasise the

²⁰⁰ Cf. Bluck (1964) pp. 320-321, who supports this point, and thinks that Plato intended his readers to see all of this.

²⁰¹ That is not to say that an inquiry with a definite starting point would exclude deductive influence, but it would proceed from principles that are not hypothetical, and have already been agreed upon. In short, there is no need to use the hypothetical method in such an inquiry. I would say that the difference between the two kinds of inquiry is akin to the difference between *dianoia* and the later stages of *noēsis* in the Divided Line passage of the *Republic*, but that is another conversation.

importance of *aporia*, in the sense described by Meno. Secondly, the conception of *aporia* in *Meno* is significant in the shift in the psychology of learning that we see there and that in turn informs the psychology of mathematics for which we argue here.

Let's note a difference between the hypothetical method and the *elenchus*. One of the requirements of the *elenchus* is that the interlocutor says only what he believes (*Charmides* 161c; *Phaedrus* 275b; *Meno* 71d; *Theaetetus* 171d). Consider (the early) Socrates' frustration with Protagoras, in the dialogue of the same name. The latter says that he is willing to assume for the sake of argument that justice is holy and holiness just, even though he does not really believe it:

“Excuse me,” I said. “It isn't this ‘if you like’ and ‘if that's what you think’ that I want to examine, but you and me ourselves. What I mean is, I think the argument will be most fairly tested, if we take the ‘if’ out of it” (*Protagoras* 331c).²⁰²

This is quite unlike the hypothetical method, where making an assumption is central.²⁰³ The ‘say only what you believe requirement’ of the *elenchus*, does at least implicitly carry the idea of innatism, as it presupposes that one has the beliefs in him already. Recollection, as conceived in *Meno*, is a species of innatism, but seems to require something more than the *elenchus* to bring it about. We need some way of generating positive statements that the *elenchus* cannot provide. Socrates uses more than the *elenchus* to help the slave boy; although he does not tell the boy the answer, he does draw in the additional lines

²⁰² See also, for example, *Charmides* 161c.

²⁰³ We do not have to assume the developmentalist position to take the point about this difference in approach. We see Socrates' concern for the real beliefs of his interlocutors in the *Meno* itself (*Meno* 71d) and in later dialogues (for example, *Theaetetus* 171d). What is important is that the hypothetical method, used for the first time in *Meno*, actually requires the philosopher to do what is prohibited by the *elenchus* that is so prominent in the early dialogues.

on the diagram, that enable the boy to find it.²⁰⁴ In the subsequent inquiry into virtue, in which Socrates does *not* already know the answer, the hypothetical method is adopted.

Fine ([2007] pp. 331-367) argues that the hypothetical method does not violate any Socratic practices or assumptions. She points out that, in the early dialogues, we do not always have to know what something is in order to inquire into what it is like, and cites *Crito*, in which the question of whether it is just for Socrates to flee is discussed, without a definition of justice being offered. Fine acknowledges that the method may be new, but she still thinks that the dialectical requirement is formulated at 75c8-d7 and 79d.

However, we should remember that the dialectical requirement as Fine understands it is an assurance that the interlocutor understands the terms, rather than the ‘say only what you believe’ rule. The formulation at 75cd, Socrates says that, in a more confrontational dispute, he would say, ‘You have heard my answer. If it is wrong, it is for you to take up the argument and refute it’ (75c). However, in a friendly conversation, such as the one with Meno, ‘one’s reply must be milder and more conducive to discussion. By that I mean that it must not only be true, but must employ terms with which the questioner admits he is familiar’ (75d). The formulation at 79d emphasises the need to avoid terms which are still in question.

The first formulation does specify truth, but it does not specify that one must be committed to the truth of each statement one makes. This is implied at 71d, when Socrates asks Meno to give his own opinion and leave Gorgias out of it, and also at 85bc, in which Socrates says that the slave-boy’s opinions were all his own. However, all of these instances are firmly distinct from the hypothetical passage, which breaks the rule, so the method does represent a break from the *elenchus*.

²⁰⁴ This is not to argue that the *elenchus* is not used in the slave boy passage, only that it is not sufficient by itself. Many scholars see the slave boy passage as a paradigm of the elenctic method; see Irwin ([1977] pp.133-147) and Nehamas ([1985] pp. 1-30), for example. I am grateful to Vlastos ([1988] pp.362-396) for his discussion of this point. For an opposing view, see Fine ([2005] pp.200-226).

Kahn notices a further difference in *Meno*. He calls the hypothetical method that follows the *aporia* in *Meno*, ‘the earliest known theoretical account of deductive inference’ (Kahn [1998] p. 309). He thinks it is the first text to distinguish sharply between the truth of the premiss and the validity of the inference. This is quite a difference from the elenctic method of only beginning from premisses we believe to be true.

This is one of the main differences between the *elenchus* and the hypothetical method. The term *hypothesis* is used in the early dialogues,²⁰⁵ to describe a position expressed by the interlocutor, but it is not until *Meno* that the hypothesis is severed so explicitly from actually held beliefs.²⁰⁶

However, there is a danger to abandoning the ‘say only what you believe’ rule, and arguing for both sides. Think about *Euthydemus*, which aims to show Socratic argument as distinct from eristic. Dionysodoros tells Socrates that ‘whichever answer the lad gives, he will be proved wrong’ (*Euthydemus* 275e). When Socrates later asks Dionysodoros if he is merely arguing for arguments sake, or if he really believes what he says, Dionysodoros says, ‘Just try to refute me’ (286de). The goal for the Sophist is not truth, but to beat his opponent – in this case by proving the truth of the opposing argument.²⁰⁷

The last thing that Plato wants is to associate Socrates with the Sophists. They are mentioned later on in the dialogue, where Plato takes thinly-disguised shots at their behaviour and standing in the community. Anytus’ reaction when Socrates mentions them as teachers of virtue emphasises this attitude:

²⁰⁵ *Euthyphro* 11c; *Gorgias* 454c and Kahn (1998) p 310.

²⁰⁶ Opposing sides of an argument are also investigated earlier than *Meno*, for example in *Charmides* 167b-175a; see Politis ([2008] pp. 1-34). However, again, we do not see the separation between the proposition and the interlocutor, which we see in *Meno*.

²⁰⁷ Socrates later describes this as ‘light-hearted dancing...initiation’ into the sophistic mysteries (277de). Contrast this with the Plato’s use of the idea of initiation in the description of *aporia* in *Meno*. See Lloyd ([1992] pp. 166-183) on the idea of initiation in the latter dialogue.

Good heavens, what a thing to say! I hope no relative of mine or any of my friends, Athenian or foreign, would be so mad as to go and let himself be ruined by those people. That's what they are, the manifest ruin and corruption of anyone who comes into contact with them (*Meno* 91c).

Socrates says that he finds it unbelievable that men like Protagoras 'took in the whole of Greece, corrupting his pupils and sending them away worse than when they came to him, for more than forty years' (91e). However, Anytus insists that the Sophists are so corrupt, blaming the young men who pay them and the cities that allow them in (92ab). He has no experience of them himself, but he knows their 'kind' (92c). This is the kind of attack from which Plato wants to insulate Socrates.

Plato needs to legitimise the *Meno*'s departure from Socrates' 'say only what you believe' rule; he does not want to reduce the hypothetical method to a kind of eristic. The 'frozen' kind of *aporia* allows for the use of hypotheses on a psychological level. We are disabused of our false beliefs: rather than the stumbling bewilderment of the earlier presentations, this *aporia* emphasises the suspension of our beliefs and judgement. In earlier dialogues, Socrates and his interlocutors are 'woozy with the argument' (*Lysis* 222c); made fools of (*Lysis* 223b, *Charmides* 176a). But the *aporia* of *Meno* prevents the interlocutor from being made a fool of, as would happen in an eristic display and even in the earlier dialogues. It provides for a way forward without commitment to untested propositions, introducing the argument as a thought experiment rather than just an argumentative display. If the hypothesis follows any other state of mind than this 'frozen' *aporia*, it jeopardizes the integrity of the argument.

The *aporia* on this reading also has the virtue of making the hypothesis hypothetical. As we shall see, Plato says of the mathematicians in *Republic VI* that they proceed from assumptions that they regard as known, without feeling the need to explain them (510cd). The philosopher must go beyond the mathematician. One of the first steps, we are told, is to

treat ‘assumptions not as principles, but as assumptions in the true sense, that is, as starting points and steps’ (511b).

The hypothetical method in *Meno*, we shall see, is an exercise in the application of mathematical method to philosophy. If the *Meno* tries to show that discovery is recollection, then *aporia* plays an important role in the process. The presentation of it we are given is not incompatible with previous conceptions, but gives us a new perspective, along with the hypothetical method introduced in the dialogue. The *aporia* of *Meno* gives us the psychological preparation we need: disencumberment of our false beliefs and the suspension of attachment to new ones. It is Plato’s first step towards a psychology of mathematics.

It is also a clear sign that Plato is about to do something different. Socrates is about to abandon the ‘say only what you believe’ rule of the *elenchus* but wants to avoid the mean tricks of eristic. This is why, *aporia*, although a negative state of mind for Meno is a positive one for Socrates: it is a necessary condition for research.²⁰⁸ We need to ensure that the inquiry maintains its integrity, even when it resorts to the provisional tool of hypothesis. In this way, *aporia* in *Meno* is a way of marking the difference between *pistis* and *dianoia*, and the foundation for the hypothetical method that Socrates goes on to use.

Chapter Three: The Hypothetical Passage and Mathematical Method in the *Meno*

Rowe ([2007] pp.131-134) downplays the importance of mathematics in Plato’s epistemology. Some people think that we cannot be made to ‘see’ truths about virtue in the same way that we can be made to ‘see’ mathematical truths, so mathematical demonstrations like the slave-boy experiment have limited implications for Plato’s epistemology as a whole. Also, there is a perceived looseness of fit between Plato’s hypothetical passages and

²⁰⁸ For this comment, I am grateful to Walter Cavini, in the unpublished paper he gave in Bologna, *Aporie, Paradossi e Misteri (Platone, Menone 79e5-81e2)* 21.9.09. See also Frede (1998) pp. 253-269, who covers similar ground.

mathematical method, which Plato does not seem to acknowledge; Plato has also been accused of imprecise use of mathematical terminology by Mueller (2005). Taken on its own, this could shed doubt on the extent to which mathematical reasoning is an important part of Plato's epistemological scale, a vital element of the solution proposed by this project: I am going to argue that, on the contrary, mathematics is pivotal.

Plato's application of the mathematical method in the dialogues should be taken piece by piece, so this chapter will focus on the passages we find in the *Meno*. In this chapter, we shall briefly summarise the *Meno*'s presentation of mathematics as a model for philosophy. This account should at least make us think seriously before devaluing the role that mathematics plays in Plato's epistemology, and suggests that Plato gives mathematical reasoning a role in ethical inquiry. Next, we shall examine the hypothetical passage in *Meno*. We shall make particular reference to the claim that Plato does not fully import the mathematical method, arguing that there is a closer fit between mathematics and the passage than previous scholars have allowed. Finally, we shall examine the passage in the context of the dialogue as a whole.

i. Mathematics as a model for philosophical investigation

In the earlier attempts to define virtue, a geometrical example is used as a model (*Meno* 73e), and later (75b-76a). Socrates gives Meno two definitions of 'shape' and asks him to model his definition of virtue on these geometrical examples (77ab). The qualities that Socrates looks for in a philosophical definition are the same as those required in geometry.²⁰⁹ Note also that when Socrates appeases Meno by using a definition from the natural sciences,

²⁰⁹ The definition should not employ terms that are still in question (*Meno* 75c; 79d) and should look for a single quality to cover all instances (*Meno* 73e; 74a).

drawing upon Empedocles' theory of effluence (76cd), he qualifies it with the comment that the geometrical answer is better (76e).

We also saw that the slave-boy experiment uses a geometrical example to illustrate Socrates' point. This is not an accidental choice; Socrates takes care to point out that the same principles which apply to the mathematical example apply to all other kinds of knowledge. The experiment is intended to show that *all* learning is recollection, not just mathematical learning (81d). When the experiment is concluded, Socrates explains that the boy will come to know things by recovering them for himself. This is not limited to the sphere of mathematics. Socrates emphasises, 'he will behave in the same way with all geometrical knowledge, and all other kinds of knowledge' (85e).

Moreover, the method that Socrates uses in the slave-boy experiment is faithful to the mathematical method. Patterson ([2007] pp. 1-33) notes that Socrates' proof for doubling the square in *Meno* is idiosyncratic, but it does bear significant resemblance to the Euclidian version in that it starts with just the given square then (ignoring the two false starts and the *aporia*) supplies the diagonal and the square on the diagonal, and concludes by showing that the constructed square does have the double area required.²¹⁰

Finally, we are given a mathematical example as a model for the hypothetical method (87a). The example is obscure,²¹¹ but involves whether it is possible to inscribe a given area into a circle. We are told that the geometer makes a hypothesis that if a particular condition is fulfilled, one result follows; if not, the result is different. Then the two proceed to investigate

²¹⁰ On the other hand, it differs from the second type of Euclidian proof in that the latter 'gives away' in its Proposition the solution that the slave recollects for himself.

²¹¹ It is beyond the scope of this section to resolve this, but the exact nature of the problem to which Plato refers is unclear. Heath ([1960] pp. 297-303) gives an overview and comments on the number of different interpretations, while other discussions have been provided by Sternfeld and Zyskind ([1977] pp. 206-211), Lloyd ([1992] pp.166-183). Guthrie ([1956] pp.103-114) comments in his translation of *Meno* that it is not necessary to understand the example in order to grasp the hypothetical method, but given the lack of consensus on the details of the hypothetical method in *Meno*, any clues we could glean from knowing the exact geometric example would be useful. See Karasmanis's (1987) thesis for an example of how an understanding of Greek geometry can inform our conception of Plato's use of hypotheses. See also Mueller ([2005] pp.170-199) and Knorr ([1986] pp. 86-88).

the question of whether virtue can be taught using this method, beginning from the hypothesis that ‘if virtue is knowledge, it is teachable, if it is not knowledge, it is not teachable’²¹² from which the investigation proceeds.²¹³

Clearly then, Plato wishes to model at least part of his philosophical investigation on mathematics. In addition to the numerous references to mathematics in the dialogue, the slave-boy experiment and the geometrical example are intended to be exemplars for inquiry. Therefore we should take seriously Socrates’ claim that he is using the method of the mathematicians in the hypothetical passage.

iii. *The Hypothetical Method in the Meno*

The problem for this chapter is to gain a better understanding of how the method works, and to explain mathematical reasoning as productive at this stage in the dialogue. The challenge is to show that, although the fit between mathematics and philosophy is not exact, it is close enough to argue for the constructive role of mathematical thought in the dialogue. We will examine Mueller’s discussion of the alleged ‘looseness of fit’ between the hypothetical method and the method of the geometers, then go on to question whether Plato is using the method of geometrical analysis, as Mueller assumes, or whether Plato is using a different mathematical method (the method of reduction is a much closer fit than that of analysis).²¹⁴

²¹² Or, on an alternative reading, ‘virtue is knowledge.’ For discussions on whether or not the hypothesis is bi-conditional, see Scott (2006) pp221-224; Zyskind and Sternfeld ([1976] pp. 206-211); Karasmanis ([1987] pp. 77-93) and Rose ([1970] pp.1-8) and section three, below.

²¹³ We should note that this is a kind of second best in the absence of $\tau\acute{\iota}\ \acute{\epsilon}\sigma\tau\iota$ knowledge. Socrates says that he would prefer to find out what virtue itself is, but since Meno is succeeding in governing his actions (*Meno* 86d), he agrees to answer the question that Meno would like. This is consistent with Plato’s portrayal of the hypothetical method as the second best method in *Phaedo*; Cf Shipton ([1979] pp. 33-53) and the *Phaedo* chapter of this dissertation).

²¹⁴ I am grateful to Karasmanis’ ([1987] pp. 19-60 and 211-315 especially) unpublished DPhil thesis for his excellent discussion of these methods. Although other scholars have contributed much to the debate, Karasmanis discussion has had a huge impact, in spite of being unpublished. See Vlastos ([1991] pp. 123-124 n. 67 and 72) for his acknowledgement of the influence of Karasmanis’ thesis on his own discussion of mathematical influences on Plato.

Ian Mueller ([2005] pp.170-199) argues that Plato does try to unite smooth-working mathematics with the ‘rough and tumble’ of Socratic examination, but there are clear discrepancies between his method, and that of the mathematicians. Moreover, he says that Plato ignores the differences between mathematical method and his own adaptation of it. Mueller does not want to disparage Plato’s accomplishments, but he says that we should not lose sight of the imperfect fit. We shall now examine this claim, arguing that there is a much closer fit than Mueller supposes between mathematical method and the hypothetical passage.

The hypothetical method in *Meno* is widely supposed to be based on the geometrical method of analysis and synthesis (Mueller [2005] pp.170-199; Beaney [2009] SEP). According to Michael Beaney, the influence of the method of analysis, in particular, is evident in *Meno*. Socrates ‘hypothesizes’ the supposedly prior proposition, that virtue is knowledge. By means of this, the proposition under consideration, that virtue is teachable, can be demonstrated. However, Beaney acknowledges that, like geometrical analysis, it is not clear what the relationships are supposed to be between the various elements involved, and that there are important differences between geometrical analysis and the method of hypothesis (Beaney [2009]).

It is argued that Plato tries to apply this method to philosophy, even if he is not absolutely precise in its application. Mueller argues that, in *Meno*, the method of analysis is imperfectly applied. Firstly, says Mueller, Socrates does not use mathematical terminology precisely in the geometrical example, nor does he employ the terms in the hypothetical passage. Moreover, says Mueller, ‘the dialogue ends with Socrates arguing against both the hypothesis...and the teachability of virtue’ (Mueller [2005] p 179). He thinks this is a reflection of the practical difference between mathematics and philosophy. Greek geometrical analysis is a method of successful analysis rather than a method of searching and, according to Mueller, this may also explain ‘why, in the *Meno*, no attempt is made to relate the

subsequent refutation of the claim that virtue is knowledge to the mathematician's investigation from hypothesis' (Mueller [2005] p 179-80). Mueller says that this is because the hypothesis will more frequently be found questionable in philosophy than in mathematics.

We shall examine Mueller's two points: that Plato does not apply mathematical concepts in the passage in a precise way and that he argues against the hypothesis in the end. We shall argue that Muller assumes that Plato tries to apply the method of geometrical analysis, and that he does not take into account the fact that there is another possible method that Plato is trying to apply - ἀπαγωγή, or reduction. This is a precursor to analysis and a likely candidate for the method Plato was trying to use as a model for the hypothetical method in the *Meno*. If we take Plato to be trying to use this method, and take 89a or even 89c as the end of the hypothetical passage, there is actually a much closer fit between the *Meno* passage and the method than Mueller supposes. Moreover, if we think about the mathematical terminology Mueller mentions in terms of Plato's enunciation of the problem to be solved, it is not clear that Plato is being so imprecise.

Mueller refers to Philodemus's history of the Platonic school, which lists analysis and the lemma concerning *diorismoι* as being created under Plato's general directorship of the Academy.²¹⁵ Analysis is commonly called the 'analysis and synthesis,' to reflect the reverse process in the second part of the method. Mueller then goes on to explain these terms:

Analysis can be thought of as the process of looking for the proof of an assertion P by searching for propositions that imply P, propositions that imply those, and so on until one reaches propositions already established; in synthesis, one simply writes down the proof

²¹⁵ In fact, the method is said by Proclus to have been invented by Plato and handed over to Leodamus. However, it is likely that Plato did not actually invent the method, and that it was actually in use before him. Probably Proclus confused Plato's invention of the method of collection and division with that of analysis. His contribution in terms of geometrical analysis was to observe the importance from the point of view of logical rigour. Cf Heath (1949) p 272 and (1960) p 290-292.

discovered by analysis, that is, one goes through the steps of analysis in reverse order (Mueller [2005] p 175).

A *diorismos* is usually explained as the determination of the necessary and sufficient conditions for the solution of a problem or the truth of a proposition (Mueller [2005] p 175).

The term ‘lemma is frequently predicated of any premiss assumed in establishing something else...But in geometry a lemma is specifically a premiss which needs verification (Proclus, *Commentary on the First book of Euclid’s Elements*, cited in Mueller [2005] p 176).

Mueller says that Plato does not use these words in their technical sense, but in the *Meno*, invokes mathematical precedent. He cites the geometrical example that precedes the hypothetical passage in the dialogue:

What I mean by ‘from a hypothesis’ is like the way in which the geometers often consider some question someone asks them, for example, whether it is possible for this area to be inscribed in this circle as a triangle. Someone might say, ‘I don’t yet know whether this is such that it can be inscribed, but I think I have a certain hypothesis, as it were, which is useful for the question, as follows: if this area is such that, when one places it alongside its given line, it falls short by a figure similar to the one that was placed alongside, I think one result will follow, and another, on the other hand, if this cannot happen to it. Making a hypothesis, then, I am willing to tell you the result concerning the inscribing of it in a circle, whether it is possible or not (*Meno* 86e-87b, cited in Mueller [2005] p 177-178).

Mueller then goes on to say that a geometer like Euclid would reformulate the situation using the following terms (this would be what geometers call an enunciation of the problem):

Problem: To inscribe a triangle of a given area in a given circle.

Diorismos: Thus it is necessary that, 'if one places the area alongside its given line, it falls short by a figure similar to the one that was placed alongside.'

Theorem: If the area of a triangle inscribed in a circle is 'placed alongside its given line, it falls short by a figure similar to the one that was placed alongside' (Mueller [2005] p 178).

Mueller says that Socrates' presentation does not make clear whether he thinks that the *diorismos* or the theorem is the hypothesis, although it depends on both. When Socrates returns to the topic of virtue, the method becomes less clear still. Mueller cites Socrates' presentation of that problem:

Similarly then concerning virtue, since we don't know either what it is or what sort of thing it is, let's make a hypothesis and consider whether it is teachable or not, as follows: what sort of thing among those connected with the soul would virtue be to make it teachable or not teachable? First, if it is different from or like knowledge, is it teachable or not?...Or is this at least clear to everyone, that a person isn't taught anything other than knowledge?

But if virtue is some sort of knowledge, it's clear that it will be teachable.

Then we've quickly finished with this point: if virtue is of one sort it's teachable, and, if of another, not (Mueller [2005] p 178).

Mueller says that Socrates does not make a *diorismos*, but performs an analysis: he reduces the question of whether virtue is teachable to the claim that virtue is knowledge. So we now have:

Hypothesis-theorem: If virtue is knowledge, then it is teachable (Mueller [2005] p 179).

This is of use only if one can establish:

Hypothesis-lemma: Virtue is knowledge (Mueller [2005] p 179).

Mueller points to the scholarly disagreement about which of these hypotheses Socrates uses. He thinks that the most explicit text implies the hypothesis-theorem, although the hypothesis-lemma still needs to be shown. He goes on to question whether the further hypothesis, 'virtue is good' is conceived as a theorem or a lemma. He says that the method of geometrical analysis would use the hypothesis theorem, and concludes that Plato's imprecise use of vocabulary is part of the reason we cannot expect a 'perfect fit' between mathematics and philosophy.

Let's examine this claim, before we go on to examine Mueller's other assertion, that the end of the argument precludes the possibility of a perfect fit.

The method of analysis, which Mueller and other scholars claim to be the method that Plato tries to use in *Meno*, takes its name from ἀνάλυσις or ἀναλύειν. So the word takes its meaning from λύειν, to loosen, untie or set free, which when combined with the preposition ἀνά, gains meaning of 'upwards', 'backwards' and sometimes 'strengthening' or 'repetition.' We also find a sense of 'taking apart,' closer to our modern use of the word, 'analyse.'²¹⁶

The method is described by Pappus of Alexandria (c.290-c.350 CE) as follows:

²¹⁶ Cf Aristotle, *Prior Analytics*, 49a, 51a, 47a, 50ab, 51a; *Nicomachean Ethics* 1112b. I am grateful to Karasmanis for his (1987) discussion of this. Szabo ([1974] pp. 118-130) argues that ἀνάλυσις does not have this meaning in ancient times, but Karasmanis points to these parts of Aristotle, which do seem to use the word in the sense of 'taking apart.'

(A) Analysis is the way from what is sought – as if it were admitted – through its concomitants in order to reach something admitted in synthesis. (B) For in analysis we suppose what is sought to be already done and we inquire from what it results, and again what is the antecedent of the latter, until we on our backward way light upon something already known or having the position of a beginning. And we call such a method analysis, as being a solution backwards.²¹⁷

Pappus then goes on to define synthesis:

In synthesis, on the other hand, we suppose what was reached last in analysis to be already done, and arranging in their natural order as consequents the former antecedents and linking them one with another, we in the end arrive at the construction of the thing sought. And this we call synthesis.²¹⁸

He distinguishes between two types of analysis: theoretical and problematical.

Theoretical analysis ‘seeks the truth’:

...we suppose the thing sought as being and as being true, and then we pass through its consequences in order, as though they were true and existent by hypothesis, to something admitted; then, if that which is admitted is true, the thing sought is true, too, and the proof will be the reverse if the analysis. But if we come upon something admittedly false, the thing sought will be false, too.²¹⁹

Problematical analysis, Pappus says, ‘serves to carry out what was desired to do’:

²¹⁷ Pappus *Reference Collectio* VII Praef. 1-3 in Karasmanis (1987) pp.23-24; Cf Mansfield (1998) pp 9-14, who makes a similar point.

²¹⁸ Pappus *Reference Collectio* VII Praef. 1-3 in Karasmanis (1987); Cf Mansfield (1998) pp 9-14.

²¹⁹ Pappus *Reference Collectio* VII Praef. 1-3 in Karasmanis (1987); Cf Mansfield (1998) pp 9-14.

...we suppose the desired thing to be known, and then we pass through its consequences in order, as though they were true, up to something admitted. If the thing admitted is possible or can be done, that is, if it is what the mathematicians call given, the desired thing will also be possible. The proof will again be the reverse of the analysis. But if we come upon something admittedly impossible, the problem will also be impossible.²²⁰

The other definition of analysis we have preserved from antiquity is from
Euclid:

Analysis is an assumption of that which is sought as if it were admitted (and the arrival) by means of its consequences at something admitted to be true (Euclid, *Elements* XIII in Heath [1908] p 442).

Synthesis is an assumption of that which is admitted (and the arrival) by means of its consequences at something admitted to be true (Euclid, *Elements* XIII in Heath [1908] p 442).

Aristotle also describes the method of analysis. He likens it to a the method of a doctor, who does not deliberate about the end (whether or not he shall cure a man) but about by what means he can reach it:

Having set the end, they consider how and by what means it is to be attained; and if it seems to be produced by several means they consider by which it is most easily and best produced, while if it is achieved by one only they consider how it will be achieved by this and by what

²²⁰ Pappus *Reference Collectio* VII Praef. 1-3 in Karasmanis (1987); Mansfield (1998) pp. 9-14 also refers to this.

means *this* will be achieved, till they come to the first cause, which in the order of discovery is last. For the person who deliberates seems to inquire and analyse in the way described as though he were analysing a geometrical construction.²²¹

As in Pappus's description, Euclid and Aristotle have a hypothetical starting point, and the last step in analysis is the first step in synthesis. Here also, analysis is an upward movement and the process is heuristic.²²²

Karasmanis ([1987] pp. 19-59) distinguishes between what he calls the classical view of Greek geometrical analysis and a second view. According to the classical view, analysis is deductive, a method of discovering either proofs or propositions, or the solution to geometrical problems. It was followed by synthesis, the confirmation of the analysis, which is also deductive and the actual proof or solution for the sake of which analysis was undertaken. According to the second (Cornford's) view, analysis is an upward movement to prior assumptions, from which the original assumption follows.

Robinson ([1969] pp. 1-15) defends the classical view. On his reading, analysis proceeds as follows: I want to prove a proposition (1). I assume (1) is true. (1) implies (2). (2) implies (3). (3) implies (4). I continue on in this way until I reach a proposition, say (5) that I know to be true independently of (1).

For the method to work, says Robinson, the implications must be reciprocal. This means that the chain 1-2-3-4-5 must be convertible, with symmetrical relations between the propositions: (2) must imply (1), just as (1) implies (2). The synthesis tests the analysis by confirming that the chain 5-4-3-2-1 makes a necessary inference when taken in order. Robinson also thinks that we can widen the account of analysis to say that if (5) is proved

²²¹ Nichomachean Ethics 1112b15ff. Cf Karasmanis (1987) p 27, who cites Hintikka and Remes as pointing out the similarities with Pappus' both in vocabulary and structure.

²²² There is not necessarily reversible movement between premise and conclusion, although this is debated in the literature.

false, we can show (1) to be false without the aid of any synthesis; so analysis is also a way of showing the impossibility of proving a given proposition.²²³

According to the second interpretation, analysis is not deductive, ie, we do not try to see what follows from the original assumption, but from what the original assumption follows; then we proceed backwards until we reach a proposition independently known to be true. Synthesis deduces the original assumption through this proposition and thus proves it. On this interpretation, only synthesis is deductive, not analysis.

Cornford²²⁴ defends this view. He says that analysis proceeds as follows: I want to prove a proposition (1). (2) implies (1). Do I know that (2) is true? If so, the analysis is over. If not, search for a proposition (3) that implies (2). (4) implies (3). Something I do know, say (5), implies (4).

According to Cornford, the synthesis is deductive: (5) is true, and (5) implies (4) implies (3) implies (2) implies (1). Cornford thinks that his reading is supported by the difference between Pappus (A) (from what is sought through its concomitants) and (B) (suppose what is sought to be already done and inquire from what it results) in the first citation above.²²⁵ However, as Robinson rightly points out, Pappus is looking at analysis as existing for the sake of synthesis, so the second of his expressions (B) is that of synthesis (1969).

Karasmanis is sympathetic to this objection, but he thinks that the classical view, that we draw conclusions from the theorem to be proved until we arrive at the thing known, is misleading. He says we should pay attention to the role that intuition, experience and mental

²²³ This means that *reductio ad absurdum* is a special case of analysis. Karasmanis (pp 42-43) rejects this, because analysis is always followed by synthesis, *reductio ad absurdum* never; analysis leads to affirmation, *reductio* to negative proof.

²²⁴ (1967) pp. 61-95; Cf Robinson ([1969] pp.1-15) for his representation of Cornford's views. According to Robinson, Cornford's analysis is a series of upwards intuitions.

²²⁵ As additional support for his interpretation, Cornford thinks that 'you cannot follow the same series of steps first one way, then the opposite way, and arrive at logical consequences in both directions' (1967). According to Robinson ([1969] pp.1-15), we should marvel at this. He says that it is, at least in ordinary language, false. He cites a counterexample: (1) $3x=4y$ (2) $3x+y=5y$ (3) $3x+2y=6y$. He also cites an example from Euclid XIII which he takes to conform to his conception of the method, and goes on to note that false premises can give rise to true conclusions, so on Cornford's account, it is possible for (5) to be false and (1) to be true.

ability of the geometer play in the process. According to Karasmanis, ‘the main problem of the method of analysis and synthesis is not so much whether the process is deductive or not, but the process of thought of the geometer when he practises analysis.’²²⁶

I shall follow the classical view, with an acknowledgement to Karasmanis’ point that there is a psychological side to the process that also must be taken into account. In this case, the method of analysis is as follows: In the enunciation of the problem, we have a thing given (*δεδομένον*) and a thing sought (*ζητούμενον*). From this, we move (usually deductively) from a hypothesis, using other known propositions, to a conclusion that is independently true of the thing sought.²²⁷ Synthesis starts from this conclusion and moves backwards to the thing sought.²²⁸

In this case, the method of geometrical analysis follows the following process:

Enunciation of the problem → Analysis: (1, the thing sought) implies (2) → (2) implies (3) → (3) implies (4)...M implies N (independently known) → Synthesis: N implies M...(4) implies (3) → (3) implies (2) → (2) implies (1).

Mueller has assumed that Plato has modelled the hypothetical passage in the *Meno* on the method of analysis, but there was a forerunner to this method. This is *apagōgē*, or reduction. This was practised by Hippocrates of Chios (c.470-c.410 BCE), as described by Proclus (c.412-c.485 CE):

²²⁶ (1987) p 39. He says that the ‘heuristic secret’ of the method is to use B itself to find a proof for it, so there is also a creative/choice element to the method. At every step, we have a lot of choices both in auxiliary constructions and consequences. He notes that, in analysis, ‘we have two procedures bound together: a) drawing conclusions and b) looking for premises’ (1987) p 38.

²²⁷ The end point may be something impossible, something constructible, or failed analysis (ie, it does not help us to resolve/reject the problem), but it is always the starting point for synthesis.

²²⁸ So, says Karasmanis, there is a clear differentiation between analysis and synthesis: analysis goes upwards or backwards and is heuristic. Synthesis goes downwards and is demonstrative. Cf Lakatos (1976) p 9 n1 and 106 n3 for a discussion of the method of analysis as heuristic and the importance of ‘guessing’ theorems before arriving at them deductively. See especially chapter one for Lakatos’ theory of proofs.

Reduction is a transition from a problem or theorem to another, which, if known or constructed, will make the original problem or theorem evident. For example, to solve the problem of doubling the cube geometers shifted their inquiry to another on which this depends, namely, the finding of two mean proportionals; and henceforth they devoted their efforts to discovering how to find two means in continuous proportion between two given straight lines. It is reported that the first to effect reduction of difficult constructions was Hippocrates of Chios who also squared the lunes (Proclus, *In Euc*, 213, cited in Mueller [2005]).

According to this, reduction is a method that starts from an enunciation of the problem (eg, that of doubling the cube), then ‘reducing’ it to another that is easier to solve (eg, that of finding of two mean proportionals). This constitutes the principle for the solution of the original problem. This process can be repeated until we arrive at a conclusion that can be proved. These ‘principles’ are the same as Proclus’ lemmas. We said that a lemma is a premiss that needs verification, but Proclus also tells us that

The term ‘lemma’ is frequently predicated of any premiss assumed in establishing something else, as when people say they have made a proof from so and so many lemmas...we assume them directly without proof to verify other things.²²⁹

In this case, *apagōgē* is a method that reduces the problem to a series of lemmas, until we arrive at a conclusion that is independently known of the thing sought:

Enunciation of the problem → lemma1 → lemma2 → lemma3 → ... Conclusion

²²⁹ Proclus, *In Euc*, 211, cited in Mueller (2005 pp. 170-199).

Some people think that this method is the same as the first stage of the method of analysis (the ‘analysis’ part of analysis and synthesis),²³⁰ but we can see it to be an early forerunner of the method. One of the main differences between the two methods is that the method of analysis and synthesis is a series of steps ‘up’ followed by a corresponding series of steps ‘down,’ whereas in *apagōgē*, each step ‘up’ is immediately followed by a step ‘down’ in the form of a check for coherence.

In addition, *apagōgē* is not simply the first stage of analysis: it is not just breaking an argument into smaller steps. Pappus does not seem to relate the two terms methodologically.²³¹ In fact, rather than simply breaking up the argument into smaller steps, *apagōgē* has an extra heuristic dimension: the original problem is reduced to a different one that is easier to solve.

When people talk about the fit of the mathematical method to Plato’s philosophy, they usually assume that it is analysis and synthesis, rather than *apagōgē* that he is using as a model.²³² *Apagōgē*, I am going to argue, is a much better fit, but this idea is often overlooked in Ancient Philosophy. Karasmanis’ DPhil thesis champions the idea of *apagōgē* in the *Meno* and *Phaedo*, but because this is unpublished, the idea has not been widely circulated. The method is certainly much discussed in the history of mathematics, but so far, this discussion has not been widely extended to Plato scholarship.²³³

This gives us a choice between the two possible methods as models for the hypothetical passage in Plato. I want to argue firstly that *apagōgē* is a better fit for the passage, and secondly that chronologically, it makes more sense for Plato to be using that

²³⁰ Karasmanis ([1987] pp. 19-59) thinks that the fundamental difference in their structure is that in *apagōgē*, each reduction simultaneously involves an analytic ‘step up’ and a synthetic ‘step down.’

²³¹ Nor, according to Otte and Panza ([1997] p. 201) do any of the other ancients.

²³² For example, Benson ([2010] pp.188-208); Mueller ([2005] pp.170-199); Robinson ([1953] pp.120-122). Menn ([2002] pp. 193-223) conflates the two, ascribing the invention of analysis to the originator of *apagōgē*, Hippocrates. He thinks that it is the method of analysis to which the geometrical example refers in the *Meno*; but his account of analysis is closer to reduction than those of other scholars. Interestingly, Menn identifies the reduction to an *archē* with direct perception.

²³³ See, for example, Knorr (1986) Chapter Three pp. 49-88; Szabó (1978) Part One, especially pp 48 ff.

method at the time of writing the *Meno*. We shall consider whether Plato uses analysis in other dialogues in the chapter on the *Phaedo*, but here, we shall argue that *apagōgē* makes much more sense. Following this, I shall argue that we save Plato from some of the charges of conceptual imprecision by looking at his enunciation of the problem.

As I shall argue below, we can take the hypothetical passage as finishing at 89a (passage B of the part ii). First, I want to present the passage as the method of reduction applied to philosophy.

When Socrates has described the geometrical example, he says, ‘Let’s do the same about virtue’ (*Meno* 87b). He then gives an enunciation of the problem:

Socrates: We shall say: ‘What attribute of the soul must virtue be, if it is to be teachable or otherwise?’ Well, in the first place, if it is anything else but knowledge, is there a possibility of anyone teaching it – or, in the language we used just now, reminding someone of it? We needn’t worry about which name we give to the process, but simply ask: will it be teachable? Isn’t it plain to everyone that a man is not taught anything except knowledge?

Meno: That would be my view.

Socrates: If on the other hand virtue is some sort of knowledge, clearly it could be taught.

Meno: Certainly.

Socrates: So that question is easily settled; I mean, on what condition virtue would be teachable.

Meno: Yes.

Socrates: The next point, then, I suppose, is to find out whether virtue is knowledge or something different (*Meno* 87bd).

If we take this whole passage to be the enunciation of the problem, we have all the elements we would expect from a mathematical method. We have the thing given, *δεδομένον* (nothing is teachable except knowledge) and the thing sought *ζητούμενον* (is virtue teachable?). We also have the theorem (if virtue is knowledge, it can be taught) and the *diorismos* ('if virtue is knowledge, it can be taught') is the condition on which virtue is teachable).²³⁴

This in itself does not mean that the method is either analysis or *ἀπαγωγή*, but it does clear Plato from some of the charges of confusing mathematical concepts. Socrates is not confusing the theorem and *diorismos*, nor, as Mueller thinks, is he imposing the condition given by the *diorismos* and relying on the theorem (Mueller [2005] p 178). He is simply stating the two as part of the enunciation.

Mueller thinks that the 'hypothesis-theorem (If virtue is knowledge, it is teachable.)' is only of use if one can establish the 'hypothesis-lemma(Virtue is knowledge.). The lack of clarity about which of the two is supposed to be the actual hypothesis (which I discuss later in Section Two, Chapter Three. ii) leads Mueller to suppose that Plato has conflated the terms.

However, if we think of the method of *apagōgē*, the lemma, 'virtue is knowledge' is not brought in because the hypothesis-theorem is too similar to the *diorismos*, but because it is the next step in the process. The problem, to prove that 'virtue is teachable' is reduced to, 'is virtue knowledge?' (Lemma 1: 'virtue is knowledge). So Lemma 1 entails the thing sought (as stated in the *diorismos*). The next step, as Socrates says, is to find out if virtue is knowledge, so this problem is reduced to the second lemma, 'virtue is good.' Lemma 2 entails Lemma 1:

²³⁴ See Euclid VI 28 on the incorporation of *diorismoι* into enunciation.

1. Virtue is good (87d).
2. If there is something good that is different from/not associated with knowledge, virtue is not necessarily a form of knowledge (87d).
3. If knowledge embraces everything that is good, virtue is knowledge (87d).
4. All good things are advantageous (87e).
5. So virtue must also be advantageous (87e).
6. Health, strength, good looks and wealth are all advantageous (87e).
7. But these things sometimes do harm (88a).
8. The thing that determines whether these things are advantageous or harmful is right use (88a).
9. Also consider spiritual qualities like temperance, justice, courage, quickness of mind memory, nobility of character (88a).
10. Also learning and discipline is profitable in conjunction with right use, but without it, harmful (88b).
11. Everything that the human spirit undertakes is useful when guided by wisdom, harmful when not (88c).
12. So if virtue is an infallibly beneficial attribute of the spirit, it must be wisdom (88cd).
13. The same applies to the first class of things: right use makes them advantageous (88d).
14. The right user is the wise man (88d).
15. Goodness of non-spiritual assets depends on our spiritual character – and the goodness of that depends on wisdom (88d).
16. The advantageous element must be wisdom (89a).
17. Virtue is advantageous (89a).
18. So virtue is wisdom, either the whole or the part of it (89a).

If we take the argument to end there, we have the structure of *apagōgē*, consisting of two reductions:

Enunciation:

Is virtue teachable (87b)? [THING SOUGHT].

Nothing is teachable except knowledge (87c) [THING GIVEN].

If virtue is knowledge, it can be taught (87c) [THEOREM].

This ('if virtue is knowledge, it can be taught') is the condition on which virtue is teachable (87c) [DIORISMOS].

Lemma 1: Virtue is knowledge (87c) Reduced to:

Lemma 2: Virtue is good (87d).

Mueller says: 'there can be no question of a perfect fit with the method of mathematical analysis since the dialogue ends with Socrates arguing against both the hypothesis-lemma and the teachability of virtue' (Mueller [2005] p 179). However, if we take the following passages to be separate arguments, there is no reason to suppose that Socrates' arguments against the hypothesis-lemma and the teachability of virtue undermine a fit with mathematical method, merely that Socrates has used different arguments to get different results. We will argue for taking the argument to end here in the next part, but now we should note that Mueller's point partly depends on taking the hypothetical passage to extend into the rest of the dialogue.

Historical sources also point to the acceptance in antiquity of the fact that Plato was using *ἀπαγωγή*. Here is Aristotle's description of the method:

By reduction, we mean an argument in which the first term clearly belongs to the middle, but the relation of the middle to the last term is uncertain though equally or more probable than the conclusion; or again, in an argument in which the terms intermediate between the last term and the middle are few. For in any case, it turns out that we approach more nearly to knowledge.

(Prior Analytics 69a).

Aristotle then goes on to cite an example remarkably close to that which we find in *Meno*:

For example, let A stand for what can be taught, B for knowledge, C for justice. Now it is clear that knowledge can be taught: but it is uncertain whether virtue is knowledge. If now the statement BC is equally or more probable than AC, we have a reduction: for we are nearer to knowledge, since we have taken a new term, being so far without knowledge that A belongs to C. Or again suppose that the terms intermediate between B and C are few: for thus, too we are nearer knowledge. For example let D stand for squaring E, for rectilinear figure, F for circle. If there were only one term intermediate between E and F (viz. that the circle is made equal to a rectilinear figure by the help of lunules), we should be near to knowledge. But when BC is not more probable than AC, and the intermediate terms are not few, I do not call this reduction: nor again when the statement BC is immediate: for such a statement is knowledge.

(Prior Analytics 69a).

It is likely that Aristotle does have the hypothetical passage from the *Meno* in mind when he writes this.²³⁵

²³⁵ Cf Bluck (1964) pp. 76-79, who also links the two passages.

So far, we have argued that the thing sought, the thing given, theorem and diorismos can all be found in Plato's enunciation of the problem, and that his method more closely resembles ἀπαγωγή than geometrical analysis; evidence from Aristotle bears this out.²³⁶ We have also seen that, if we take the argument to end at 89a, Mueller's point about Socrates arguing against the hypothesis does not affect the fidelity of the passage to the method, when the passage is taken on its own.

This reading also avoids the problem that Plato is not clear about what the hypothesis actually is. It could be either, 'virtue is knowledge' or the bi-conditional 'if virtue is knowledge it is teachable, if it is not knowledge it is not teachable.'²³⁷ I endorse Scott's argument for the hypothesis being, 'virtue is knowledge,'²³⁸ but the bi-conditional reading is not incompatible with the argument that Plato is closely following the method of reduction, rather than loosely following the method of geometrical analysis. What is important is Plato's use of (what Mueller concedes is) a lemma, 'virtue is knowledge' as a heuristic device. This (Mueller again concedes) is directly taken from mathematics, and, if we think about the history of geometry and reduction as a forerunner to the method of analysis, we can see that the hypothetical passage is far closer to the mathematics of Plato's day than Mueller allows. Historical evidence linking the *Meno* passage to reduction bears this out.

²³⁶ Once again, I am grateful to Karasmanis' wonderful thesis for this.

²³⁷ We cannot give full attention to the argument here, but the question has been much discussed. See Scott ([2006] pp. 221-224); Zyskind and Sternfeld ([1976] pp. 130-134); Karasmanis ([1987] pp.103-118) and Rose ([1970] pp. 1-8) for discussions. Briefly, the debate can be summarized: the geometrical example would indicate that the hypothesis is the bi-conditional, if we assume that everything that follows the words 'the following hypothesis' at 87a3 is the hypothesis, which would mean that 87c8-9 would be a full statement of the bi-conditional hypothesis about virtue. However, as Scott (2006) points out, we could take 'virtue is knowledge' in apposition to 'the hypothesis' at 89c2-4. Moreover, Scott continues, the bi-conditional interpretation cannot do justice to the methodological remarks Socrates makes at 86de: Socrates is trying to broker a deal with Meno, so the hypothesis must be provisional, or the method is a concession, not a compromise. The bi-conditional cannot do justice to these remarks, so Scott thinks that 'rules it out of court' (2006) p 224.

²³⁸ Summarized in the preceding note. As I have taken the passage to end at 89a, the main textual evidence for the bi-conditional actually falls outside the hypothetical passage, which would support this reading. As I shall argue below, 89a-89c actually forms a separate empirical argument, so when Meno speaks of an inescapable conclusion from the assumption 'if virtue is knowledge, it is teachable,' at 89c, he is actually referring to the assumption of 89a, not whatever assumption was made when the method was introduced.

The most controversial thing I have done so far is to say that the passage ends at 89a. Some people think that the hypothetical passage takes up most of the second part of the dialogue,²³⁹ whereas others think it ends at 89c. In the following chapter, I shall argue for my interpretation, but for now, we should note that we have everything we need for the method of reduction up to 89a, and that even if we take the passage to end at 89c, Mueller's point about Socrates going on to argue against the hypothesis does not effect the passage's fidelity to the method when taken in itself.

Given this interpretation, we can see the role of the hypothetical method in the absence of a *ti esti* for virtue. Taking the lemma, 'virtue is knowledge' as a hypothesis, we can see that the lemma plays a similar role to the diagram in the slave-boy experiment: it acts as proxy for the *ti esti*. We said that a *ti esti* needs to answer the signification question, and also allow us to further investigate the properties of the thing defined as its wider heuristic role.

It is clear that the lemma(s) in both the hypothetical passage and the method of reduction are heuristic mechanisms. It is also clear that, in the hypothetical passage, the lemma is adopted as a result of the absence of a *ti esti*, as we have argued in chapters one and two of this section. It is less clear that the lemma answers the signification question in the same way that the diagram does in the slave-boy experiment. However, it is used as a proxy until such an answer has been attained.²⁴⁰

If we take the hypothesis to be the lemma 'virtue is knowledge' (which we shall do from now on), we can say that the role of the hypothesis in this passage is to act as proxy for

²³⁹ For example, Vlastos ([1991] pp. 123-125) puts forward this view.

²⁴⁰ We gain a glimpse of what that answer might be within the passage itself. Socrates says that spiritual qualities such as temperance, justice, courage, quickness of mind, memory, nobility of character and others may be harmful as well as advantageous (88ab). These qualities are profitable in conjunction with wisdom, but without it harmful (88b). Socrates goes on to provisionally identify wisdom with virtue. If we rephrase that, we have: 'wisdom (of which virtue is some sort) is the only thing that always accompanies the advantageous results of spiritual qualities' which is the same form as Charles' paradigm signification answer, 'shape is the only thing that always accompanies colour.' The problem with this is that, at 97a, Socrates says that it was a mistake to suppose that knowledge was a *sine qua non* for right leadership. In any case, the lemma, 'virtue is knowledge' is used as a substitute until we arrive at a satisfactory signification answer.

the missing *ti esti*, and work as a heuristic device in the progression towards knowledge. If we take it to be the bi-conditional, we can say that the role of the hypothesis is to generate lemmas that do this, although, as we shall see in the *Phaedo* chapter, it makes much more sense to say that the hypothesis *is* the lemma.

In conclusion, we can see that the slave-boy passage and the hypothetical passage in *Meno* are both good examples of each of the characteristics of dianoetic reasoning described in the divided line passage of the *Republic*. The diagram of the slave-boy passage and the hypothesis-lemma of the hypothetical passage both act as proxies to the absent *ti esti*, and are both heuristic devices used to generate a positive statement, in contrast to the *elenchus*' purely purgative role. So far, this supports our solution to the problem of the relationship between the two characteristics. However, we still need to explain how *dianoia* can be part of an epistemological ascent in the *Meno* when the dialogue nevertheless seems to end in *aporia*. Moreover, we need to further justify the fact that we took the hypothetical passage to end at 89a. This will be the purpose of the following chapter.

Chapter Four: The Hypothetical Passage in the Context of the Inquiry

The problem of the *Meno* for us is that the dialogue still ends in *aporia*: Socrates still does not know what virtue is, or whether or not it can be taught. After the demonstration that virtue is knowledge, we seem to have another passage showing that virtue is not knowledge. As we want to say that mathematics provides a step up the epistemological ladder, we need to account for the apparent lack of progress on the issue. Socrates says that they will not get to the bottom of the matter until they have agreed upon what virtue is, but we should at least have arrived at some kind of provisional truth if the hypothetical method is to be of any use of all. This chapter aims to show that the *aporia* (if it is *aporia*) is actually a result of two

empirical arguments that follow the hypothetical passage, not of the hypothetical method itself.

We said that the dialogue ends in *aporia*, but I'd like to qualify this by pointing out that Socrates' final '*aporia*' is nothing like the *aporias* of Meno and the slave-boy in the middle of the dialogue. We said that *aporia* in these passages is like being stung by a stingray: he renders them numb, into a state of torpor: Meno and the slave-boy previously thought they had knowledge, Meno often speaking about it in front of large audiences (80b), before being perplexed by Socrates (80ab; 84b) and now having nothing to reply (80b; 82d; 84b). The numbness also implied a sense of helplessness in not knowing which way to turn. None of these traits are present in the final passage. Socrates never claimed to know whether virtue could be taught; he claimed that it was impossible to answer that question without a *ti esti*, a Platonic definition, for virtue and this itself gives him the next step if he wants to carry on the inquiry. The similarity between Socrates' *aporia* and Meno's can be found in Socrates' suspension of judgement until we have a *ti esti*:

If all we have said in this discussion, and the questions we have asked, have been right, virtue will be acquired neither by nature nor by teaching. Whoever has it gets it by divine dispensation without taking thought...But we shall not get to the truth of the matter until, before asking how men get virtue, we try to discover what virtue is, in and by itself' (*Meno* 100b).

However, this is quite different from the suspension of judgement that Meno is forced into by his *aporia*. In Meno's case, he is unwilling to commit to any proposition, because he is perplexed by Socrates. In Socrates' case, there is simply more work to do to complete the inquiry. The argument is inconclusive, rather than aporetic in the strict sense. How much of this inconclusiveness or *aporia* is due to the hypothetical method? We shall examine the

structure of the second part of *Meno*, arguing that this *aporia* is due to the empirical arguments that follow the hypothetical passage; the hypothetical passage itself merely renders the argument inconclusive rather than perplexing in the aporetic sense.

In the discussion from 98d to the end, Socrates summarises the argument of more or less the whole second part of the dialogue (87b-98c). It is useful to note the main points in his summary, as we can see how the conclusions have been reached. Here are the steps in the argument, as summarised by Socrates (call this passage A):

1. 98d. Socrates says: assuming that there are men good and useful to the community, it is not only knowledge that makes them so, but also right opinion, and neither of these comes by nature but both are acquired (referring to 96e-98d). That being so, goodness is a matter of teaching, if virtue is knowledge and conversely, if it can be taught, it is knowledge (he is referring to 87bd).
2. 98de. Socrates says: Next, we decided that if there were teachers of it, it could be taught, but not if there were none (he is referring to 89d).
3. 98e. Socrates says: But we have agreed that there are no teachers of it, and so that it cannot be taught and is not knowledge (he is referring to 89e-96d).
4. 98e. Socrates says: At the same time, we agreed that it is something good, and that to be useful and good consists in giving right guidance (he is referring to 87d-89a).
5. 99a. Socrates says: And that these two, true opinion and knowledge, are the only things which direct us aright (he is referring to 96d-98c).
6. 99ab. Socrates says: Since virtue cannot be taught, we can no longer believe it to be knowledge, (he is referring to 89e-96d again) so that is not the guide in public life (95b).

7. 99c. Socrates says: The alternative is that it is well-aimed conjecture,²⁴¹ which is no different from divine revelation.
8. 99e. He concludes: On our present reasoning, whoever has virtue gets it by divine dispensation.

How much of this argument is taken up with dianoetic reasoning and specifically the hypothetical passage? Meno formulates the problem that the argument is trying to solve at 86cd: ‘are we to pursue virtue as something that can be taught, or do men have it as a gift of nature or how?’ That is, Meno wants to know if virtue is (a) teachable, (b) a gift of nature or (c) got by some other means. In the hypothetical passage, Socrates says that he is responding specifically to point (a): ‘let us use a hypothesis in investigating whether it is teachable or not’ (87b).

I want to argue that the argument ends at 89a. This would make the structure of the hypothetical argument (including the enunciation at 87bc, but excluding the geometrical model at 86e-87b) as follows (call this passage B):

1. 87b. Virtue is knowledge.
2. 87c. Nothing is teachable except knowledge.
3. 87c. If virtue is knowledge, it can be taught.
4. 87c. This is the condition on which virtue is teachable.
5. 87c. Is virtue knowledge?
6. 87d. Virtue is good.
7. 87d. If there is something good that is different from/not associated with knowledge, virtue is not necessarily a form of knowledge.

²⁴¹ Guthrie (1956) chooses the translation ‘conjecture.’ See the *Republic* section for a discussion of this.

8. 87d. If knowledge embraces everything that is good, virtue is knowledge.
9. 87e. All good things are advantageous.
10. 87e. So virtue must also be advantageous.
11. 87e. Health, strength, good looks and wealth are all advantageous.
12. 88a. But these things sometimes do harm.
13. 88a. The thing that determines whether these things are advantageous or harmful is right use.
14. 88a. Also consider spiritual qualities like temperance, justice, courage, quickness of mind memory, nobility of character.
15. 88b. Also learning and discipline are profitable in conjunction with the right use, but without it, harmful.
16. 88c. Everything that the human spirit undertakes is useful when guided by wisdom, harmful when not.
17. 88cd. So if virtue is an infallibly beneficial attribute of the spirit, it must be wisdom.
18. 88d. The same applies to the first class of things: right use makes them advantageous.
19. 88d. The right user is the wise man.
20. 88d. Goodness of non-spiritual assets depends on our spiritual character – and the goodness of that depends on wisdom.
21. 89a. The advantageous element must be wisdom.
22. 89a. Virtue is advantageous.
23. 89a. So virtue is wisdom, either the whole or the part of it.

This would mean that the hypothetical passage only covers steps one and four of Socrates' summary. That is, goodness would be a matter of teaching, if virtue were

knowledge and if it could be taught, it would be knowledge; also virtue is something good, and that to be useful and good consists in giving right guidance.

Following the hypothetical passage as I want to define it, there is a short passage that supports the argument (call this passage C). This correlates to the second part (b) of Meno's formulation: whether virtue is a gift of nature:

1. 89a. Socrates points out that, if his conclusion that virtue is wisdom is true, good men cannot be good by nature.
2. 89b. There is another point: If good men were good by nature, there would be experts among us who recognised them from an early age, and removed them from bad influence.
3. 89bc. If goodness does not come by nature (as in 89a), it is got by learning.
4. 89c. Meno: this is an inescapable conclusion from the assumption: if virtue is knowledge, it is teachable.

Most people think that this passage is part of the hypothetical passage: step four is usually tied to the original hypothesis. However, I want to argue that it is actually a short, but separate, empirical point that is meant support the hypothetical argument. In any case, it is strange that the argument should take such an empirical turn here and we should definitely not see this as superseding the previous passage. Rather, it is a separate response to a *different part* of Meno's formulation of the problem at 86cd.

To show that it really is a separate point, as Socrates claims, and not a continuation of the hypothetical argument, we should think about the method of reduction. In this case, we can see that passage C is superfluous to the hypothetical passage (passage B). Steps one to four of passage B set out the problem, and the conditions for which it is solvable

(enunciation); steps five-twenty three demonstrate the two lemmas, ‘virtue is knowledge’ and ‘virtue is good.’ When he speaks at 89c (step three of passage C), Meno means that the conclusion of 89bc, that ‘if goodness does not come by nature, it is got by learning’ is an inescapable conclusion from the assumption ‘if virtue is knowledge, it is teachable.’ That is, step one of passage C is not another lemma that requires demonstration in a series of reductions, but a separate observation inferred from the enunciation of the problem.

We said that the form of *apagōgē* was as follows:

Enunciation of the problem → lemma1 → lemma2 → lemma3 → ... Conclusion

Each lemma must be a reduction of the preceding problem to one that’s easier to solve, so in the case of the hypothetical passage in *Meno*, the problem, is *virtue teachable?* is reduced from ‘virtue is teachable’ to Lemma 1, ‘virtue is knowledge,’ which is in turn reduced to Lemma 2, ‘virtue is good.’ The conclusion, ‘virtue is advantageous (and thus good) supports Lemma 2, which in turn supports Lemma 1, and thus the statement that virtue is teachable. There is no need for a subsequent lemma, because the Lemma 2 is demonstrated by the conclusion. Therefore, step twenty-three of passage B is the end of the argument and step one of passage C is the beginning of a new argument.

This means that we can safely say that the argument is passage B and ends at 89a for two reasons: first, passage C focuses on a different part of the problem, and second, it is superfluous to the argument of passage B.

The other distinctive feature of passage C is that, although it is inferred from the assumption ‘if virtue is knowledge, it is teachable,’ Socrates supports it with an empirical observation (step 2 in passage C). This should strike us as odd, partly because it is such a bad argument: it is not at all clear that if good men were good by nature, ‘there would probably be

experts among us who could recognise the naturally good at an early stage,' and remove them from bad influence (89b and step 2 of passage C). Secondly, it is not clear why Socrates should resort to an empirical argument at all.

The argument takes a stranger turn in the subsequent passage. Socrates does not withdraw the claim that, if virtue is knowledge, it must be teachable, returns to the problem formulated before the hypothetical passage at 86cd: is virtue teachable? This is done in another empirical argument. Let's call the following passage D:

1. 89de. If virtue (or anything) is teachable, there must be teachers and students of it (and conversely, if there are no teachers or students of it, it cannot be taught).
2. 89e-96c. There are no teachers of virtue.
3. 96c. Virtue is not teachable (and therefore not knowledge).

Again, this seems strange, partly because of the poor quality of the argument.

Firstly, it is not at all clear that if something is teachable, there must exist teachers of it (89de and step one of passage D). Secondly, the establishment of step two is sloppy and based on *ad hominem* attacks on the Sophists, which we will briefly summarize:

Socrates begins cautiously, saying,

All I can say is that I have often looked to see if there are any (teachers of virtue), and in spite of all my efforts I cannot find them, though I have had plenty of fellow-searchers, the kind of men especially whom I believe to have most experience in such matters (*Meno* 89d).

Following this, Anytus joins the group. Socrates thinks him qualified to answer the question, as he is 'a man of property and good sense, who...earned (his fortune) by his own

brains and hard work.’ He also shows himself to be ‘a decent and modest citizen, with no arrogance or bombast or offensiveness about him.’ He also brought up his own son well, and the Athenian people appreciate that. Socrates says, ‘this is the right sort of man with whom to inquire whether there are any teachers of virtue, and if so, who they are’ (90ab).

Socrates and Anytus agree that, if you wish to learn an art, it would be foolish not to go to those who undertake to teach the art and are paid for it (90ce). Those who profess to teach virtue and are paid for it in Athens are the Sophists. We have already mentioned Plato’s attitude to the Sophists and his wish to disassociate Socrates from them, and Anytus’ reaction when Socrates refers to them as teachers of virtue. Anytus decided that any Athenian individual would be a better teacher of virtue than any Sophist, and they in turn learned it from other individuals (92e-93a).

Then Socrates does a curious thing. In response to Anytus’ question, whether he would deny that there have been many good men in Athens, he says, yes, there have been many; but that is not the question. The question is,

...have they (the good statesmen of Athens) also been good teachers of their own virtue? That is the point we are discussing now...whether virtue can be taught. It amounts to the question of whether the good men of this and former times have known how to hand on to someone else the goodness that was in themselves, or whether on the contrary it is not something that can be handed over, or that one man can receive from another.

(*Meno* 93ab).

According to this, the question of whether virtue can be taught amounts to the question of whether good men have been able to teach it or whether it is teachable; again it is not clear that these are the only two options. Worse, Socrates goes on to give particular examples of Themistocles and Lysimachus, Pericles and Thucydides, all of whom would have taught their

sons to be good men if it were possible, and goes on to conclude from this that virtue cannot be taught (93c-94e).

He also speaks of a contradiction in the poetry of Theognis, which apparently shows that the poet contradicts himself as to whether virtue is teachable or not ('for fine men's teaching to fine ways will win thee' as opposed to 'no teachers' skill can turn to good what was created ill' 95d-96a). When people are so confused about a subject, says Socrates, they cannot be teachers in a true sense. In that case:

...if neither the Sophists nor those who display fine qualities themselves are teachers of virtue, I am sure no one else can be, and if there are no teachers, there can be no students either...and we have agreed that a subject of which there were neither teachers nor students was not one which could be taught...so it would appear that virtue cannot be taught.

(*Meno* 96bc).

Let us enumerate the strange things about this passage. First, Socrates seems to endorse the claim that the question of whether virtue can be taught amounts to the question of whether it has been taught or whether it is teachable, when obviously there is the alternative that virtue could be teachable, but untaught.²⁴² Second, Plato seems to want to influence the reader against the Sophists as teachers of virtue, in an *ad hominem* attack from Anytus.²⁴³ Thirdly, he picks out examples of apparently good men, who apparently would have their sons tutored in virtue if it were possible – but the reader has no chance of evaluating this claim. Finally, he makes Socrates appeal to a contradiction in poetry, as though it were evidence one way or another about the teachability of virtue.²⁴⁴

²⁴² The fallacy of false alternatives, if we are to be uncharitable.

²⁴³ *Meno* 91c-92c, quoted above.

²⁴⁴ Cf *Protagoras* 338e-347a, in which Socrates and Protagoras spar over poetry.

It is not controversial to say that passage D is not part of the hypothetical passage²⁴⁵: it is almost universally agreed to be an empirical argument that contradicts the claim that virtue is teachable. We have already argued that the passage ends at 89a, so on our reading, it is very much separate from the passage.

On this reading, the second part of the *Meno* (87b-96c) runs as follows:

1. The problem is formulated by Meno at 86cd: ‘are we to pursue virtue as something that can be taught (a), or do men have it as a gift of nature (b) or how (c)?’
2. (Dianoetic) reply to the problem (passage B 87b-89a, including a model for the answer at 86e-87b and an enunciation in the style of the geometers at 87bc): virtue is knowledge, and therefore teachable.
3. First empirical reply to the problem of 86cd (passage C, 89ac): virtue is not got by nature.
4. Second empirical reply to the problem of 86cd (passage D, 89d-96c): virtue is not teachable.

The first and second empirical arguments focus on two different parts of the question. The first seeks to demonstrate that virtue cannot be a gift of nature; the second seeks to demonstrate that it cannot be taught. Note that, although Socrates wonders if the lemma ‘virtue is knowledge’ is correct at the beginning of the second empirical reply, that is not actually what is investigated. That is, is virtue teachable?²⁴⁶ I do not deny that Socrates thinks this amounts to the same thing: he says that nothing can be taught except knowledge (87b)

²⁴⁵ Cf Robinson ([1953] pp.114-122), who points out that, after 89c, neither the word ‘hypothesis’ nor any other methodological remark occurs in the rest of the dialogue.

²⁴⁶ To formulate the question, Socrates says, ‘if anything – not virtue only – is a possible subject of instruction, must there not be teachers and students of it?’ (89d); the question of whether virtue can be taught amounts to whether it has been taught (93b); there are neither teachers nor students of virtue, so it cannot be taught (96c). It is the *teachability* of virtue that is being investigated here, not its being knowledge.

and retains his commitment to, ‘if virtue is knowledge, it must be teachable’ (89d). However, what we can say is that the second empirical reply is not a direct response to the hypothetical passage, but a completely fresh response to the original problem at 86bc.

In the following passage (96d-98d, passage E, discussed below), we are told that the only alternative to acquiring virtue through nature or teaching is divine dispensation (99be) so ‘on our present reasoning, then, whoever has virtue gets it by divine dispensation’ (100b). However, this is not the final word, on the matter, as we still need a *ti esti* in order to inquire about virtue.

I want to say that the *aporia* in this case is not the result of the hypothetical method, but of the juxtaposition of the two empirical arguments. On the reading I have presented, we have a single problem presented at 86bc and three completely separate responses to it. The hypothetical passage is an attempt to say something positive, and gives us the provisional truth that virtue is knowledge, and therefore teachable. The two empirical arguments that follow it seek to expose falsehood. It is their refutation of virtue as teachable and virtue as given by nature that leaves us with the inconclusive result.²⁴⁷

Here, Plato shows us that it is empirical reasoning - not hypothetical reasoning - that results in *aporia* or inconclusiveness. It could be that Plato is trying to subtly show us the dangers of empirical arguments, and this certainly fits with the reading of the divided line passage that is presented in this project. What I do want to stress is that the apparent inconclusiveness of the *Meno* does not undermine the value of the hypothetical method; we have said something positive, if provisional, even though we need a *ti esti* to ‘get to the truth of the matter’ (100b). In this way, we do immediately see the virtue of Plato’s new method, and its potential to make progress in inquiry.

²⁴⁷ If we do not see these arguments as separate, we run the risk of exposing Plato to the charge of denying the antecedent (that is, saying, if P, then Q. Not P. Therefore not Q.) Plato is NOT saying, ‘if virtue is knowledge it is teachable. It is not knowledge. Therefore it is not teachable.’ The principle of charity supports reading the arguments as separate, in addition to the textual reasons given above.

Chapter Five: Knowledge and True Belief in the *Meno*

There is another relevant section of the *Meno* that we have yet to examine. This comes after the second empirical passage. It exploits the difference between knowledge and true belief, something that we will examine to a greater extent elsewhere, and relates to comments made in the hypothetical passage. Here, I just want to make two brief points: that the passage supports the reading that the *Meno* employs reduction, and that certainty is needed for a *ti esti*, and therefore for knowledge.

After the second empirical response, the dialogue takes another turn. Socrates thinks that there is something that they have failed to perceive. Call this passage E:

1. 96e. It is not only under the guidance of knowledge that human action is well and rightly conducted (as we said it was at 88a and 89bc in the hypothetical passage).
2. 96e. Good men must be profitable and useful.
3. 97a. They will be profitable and useful if they conduct our affairs aright.
4. 97a. We were mistaken in insisting that knowledge was a *sine qua non* for right leadership (again, as opposed to 88a and 89bc).
5. 97b. True belief is as good a guide for the purpose of right activity.
6. 97bc. That is what was left out of the discussion on virtue (which took place at 88a-89a).
7. 98b. Therefore, knowledge and true belief are different.
8. 98b. When true belief governs any course of action, it produces as good a result as knowledge.

9. 98bc. So for practical purposes, right opinion is no less useful than knowledge, and the man who has it is no less useful than one who knows.

This passage seems to be a footnote to the hypothetical passage. That is, we said that knowledge was a *sine qua non* for right leadership, but in fact, it looks like true belief is also a good guide. In this case, some of the reasoning that follows the second lemma in the hypothetical passage is incomplete. In this case, the second lemma, ‘virtue is good’ is not an exact reduction of the first ‘virtue is knowledge,’ because virtue could still be good if it were true belief. This supports the fact that the method of the hypothetical passage is supposed to be reduction, because it shows that, at the time, Socrates meant the second lemma to be a reduction of the first.

This does not refute the hypothetical argument *tout court*, but we still need something to show that ‘virtue is knowledge’ after all, which I suggest would be the *ti esti* that Socrates refers to at 100b. This supports my reading of the text, that the original lemma used in the hypothetical passage is a proxy for a *ti esti*, and can make some progress, but it is only a provisional step in the ascent to the highest form of knowledge.

The second thing that emerges from this passage is the distinction between knowledge and true belief. The distinction is made at 97ab, in which Socrates says that true opinion can be as good a guide as knowledge:

If someone who knows the way to Larissa, or anywhere else you like, then when he goes there and takes others with him he will be a good and capable guide...but if a man judges correctly which is the road, though he has never been there and doesn't know it, will he not also guide others aright?...And as long as he has correct opinion about which the other has knowledge, he will be just as good a guide, believing the truth but not knowing it. (*Meno* 97ab)

Then we have a discussion about why knowledge is valued more highly than true belief, and what the difference might be (97c-98a). At first, Meno thinks that ‘the man with knowledge will always be successful, and the right opinion only sometimes’ 97c). However, Socrates points out that he will always be successful as long as he has the right opinion, and proposes his own solution, using the statues of Daedalus as an example:

Perhaps you have not observed them in your country...if no one ties them down, (they) run away and escape. If tied, they stay where they are put (*Meno* 97d).

This, explains Socrates, is how we account for the value of knowledge, as opposed to true belief:

If you have one of his works untethered, it is not worth much: it gives you the slip like a runaway slave. But a tethered specimen is very valuable, for they are magnificent creations (*Meno* 97e).

True opinions, says Socrates

will not stay long. They run away from a man’s mind, so they are not worth much until you tether them by working out the reason. Once they are tied down, they become knowledge, and are stable...What distinguishes one from the other is the tether’ (*Meno* 98a).

What I do want to say is that, apart from being a footnote to the hypothetical passage as we argued above, this passage touches upon another relevant aspect of our inquiry. We see a kind of epistemological ascent from belief to knowledge, which will be helpful in placing mathematics on Plato’s epistemological scale. I said that a *ti esti* required certainty and an

account. On my reading of this passage, the ‘tethering’ is an important part of attaining this certainty, achieved by the mathematical method of reduction applied to philosophy. If we combine this reading of the *Meno* with my readings of the *Republic* and *Phaedo*, we see that mathematical reasoning is a way of progressing from opinion to knowledge.

Section Three: The *Phaedo*

Phaedo recounts a conversation between Echecrates and Phaedo in a remote Peloponnesian township, in which Phaedo describes Socrates’ last day in prison and death. Socrates chooses to spend his final hours in philosophical reflection, in particular ‘inquiring into our views about the future life, and trying to imagine what it is like’ (*Phaedo* 61e). His companions are a group of friends, including Phaedo, Simmias and Cebes. Plato is notable by his absence.

Ostensibly, *Phaedo* can be read as a discourse on the immortality of the soul, in which Socrates attempts to persuade his friends of the fact of this immortality. In this dialogue, his interlocutors are largely sympathetic, wishing Socrates to be right, but critical, putting forward their own objections to safeguard the integrity of the argument. The internal dialogue begins with Socrates claims that the philosopher welcomes death (60a-69e). Suicide is to be avoided, because ‘we men are in the care of the gods’ (62b), but the philosopher does want to isolate himself from the body, as it is a source of hindrance to the philosopher (65b ff). Beauty itself and Goodness itself are apprehended by the soul, and in this, the body only acts as a restraint (67c). Therefore, death, as the release of the soul from the body, is to be welcomed.

Cebes challenges Socrates to inquire into the verity of his claim that the soul continues to exist after death and retains its force of intelligence (70b). In response, Socrates offers the Argument from Opposites (70c-72e), in which he argues that everything comes to

be from its opposite. For something to become bigger, it must first have been smaller; being dead is the opposite of living, so they must come from one another. As dying is certain, not to admit that life follows from it is to contradict a law of nature. Therefore the souls of the dead exist. This argument is supplemented by the Theory of Recollection (72e-77a), in which it is argued that the soul exists prior to this life, on the grounds that we can recognise the standard of equality (or 'all such things' - 75c) even though all sensible objects fall short of it. Socrates says that if we combine the Argument from Opposites with the Theory of Recollection, we have proof that the soul will exist after death no less than before birth (77cd).

Cebes is not satisfied with this as a proof, so there follows the Argument from Affinity (78b-79e). Socrates says that we need to consider what sort of thing it is that would naturally suffer the fate of being dispersed (78b). That is, naturally compound objects would suffer this fate, but the soul is more akin to uniform, inconstant and invisible entities. However, this argument still does not satisfy Simmias and Cebes, who each have an objection to Socrates' theory (86ad; 87a-88b): Simmias proposes that the soul could be like an attunement of a musical instrument, which is also invisible, but which ceases to exist once the instrument is broken; Cebes proposes that maybe the soul is like a tailor, who outlasts the cloaks he makes for himself (with the exception of the last cloak), but who does die sooner or later.

Socrates replies to Simmias that he would have to reject the theory of recollection in order for his account to be 'harmonious,' and that he has not picked a good analogy for the soul (92ac; 93ae);²⁴⁸ his response to Cebes is more complex. He gives an account of his intellectual development as a young man (96a-100a), and his disappointment in the failure of the natural sciences to produce a teleological account of causation. Lacking a perfect solution

²⁴⁸ Cf Scaltsas ([1990] pp. 110-119), for a discussion of this.

himself, he devised his own secondary approach to the problem, the hypothetical method, which he describes, and then applies to the question of the immortality of the soul (100a-107a). Finally, Socrates gives a myth-like account of the nature of the universe and the soul's prospects after death, before his final moments and death (107a-114d; 115b ff).

This is a summary of the apparent form and purpose of the *Phaedo*, but the dialogue has a lot more to tell us about method; in particular, the hypothetical method and the use of imagery in connection with it. Imaginative thought is not only appealed to,²⁴⁹ but also encouraged and praised.²⁵⁰ Moreover, not only do we have an extensive description of the method, but also examples of its application. We are told of the psychological elements of the method and given a description of the method by Socrates. Equally valuably, we see its application in *Phaedo*, and the interaction between imagery and hypotheses in the dialogue.

We should note that the hypothetical passage in the *Phaedo* is generally recognised to run from 100a-102e. However, some of its most important features are discussed and elaborated upon in other parts of the dialogue. Notably, the need for hypotheses to accord or harmonise with each other is discussed in 92b-c and the hypothetical method as a second best method is discussed at 85cd, using the same imagery.²⁵¹ Moreover, the method can be seen to be applied throughout the dialogue as a whole.²⁵² In particular, I want to argue that dianoetic reasoning takes place in the discussion before the 'hypothetical passage,' and the clearest example of the interaction between hypothesis and imagery occurs with the objections of Simmias and Cebes (*Phaedo* 84c-88a; 91c-d).

²⁴⁹ I refer to the famous eschatological myth and the two images that are presented at objections by Simmias and Cebes; Cf Cebes' comments at 87b: '...like Simmias I must have recourse to an image.'

²⁵⁰ I acknowledge the controversial nature of this comment. I refer to Socrates' comments at 60e-61c, in which he describes how he has been practising arts other than philosophy in prison. More importantly, see again Socrates' comments at 61e, where he recommends that someone in his position 'inquires into the future, and tries to imagine what it is like.' Finally, see the role of imagery in recollection, below.

²⁵¹ See Gallop's (1980) translation of the *Phaedo* pp. 176-177. He notes of 99b6-d3 that here, Socrates is not expecting the kind of teleological explanation he once expected of Anaxagoras; rather this is an inferior kind of explanation; inferior, that is, to the teleological kind he expected from Anaxagoras, but superior to the kind that Anaxagoras actually provides, Gallop notes (and I wholeheartedly agree) that this passage should be taken in the spirit of Simmias' remarks at 85cd.

²⁵² Karasmanis ([1987] pp. 119-183) has a good discussion of this.

These objections employ both hypotheses and images, fitting the criteria of *dianoia*, as described in the *Republic*, and also the initial stages of the hypothetical method as described in the ‘hypothetical passage’ of the *Phaedo*. This section will be split into three chapters: the first part will argue that it is *dianoia* that is employed in the objections of Simmias and Cebes; the second chapter will examine Socrates’ autobiography, arguing that Socrates’ rejection of the natural sciences also correlates with the divided line. The final chapter will examine the ‘hypothetical passage’ in more detail, and the implications for the connection between hypothesis and imagery in dianoetic reasoning.

Chapter One: *Dianoia and the Objections of Simmias and Cebes*

The purpose of this chapter is to argue two points: firstly that Plato intends parts of the *Phaedo* to be applications of the hypothetical method; secondly, that the objections of Simmias and Cebes fit Plato’s description of *dianoetic* reasoning in the divided line passage of the *Republic* and the early stages of the hypothetical method in the *Phaedo*. We shall consider the textual evidence that Plato wishes to present parts of the *Phaedo* as dianoetic reasoning. We shall point out that mathematics is presented in the dialogue as a model for philosophy, which is evidence for this because dianoetic reasoning in Plato’s epistemology is the method of the mathematicians applied to philosophy. However, it is also a second best to *noēsis*, and we shall also see that the dialogue is full of references to the method of the *Phaedo* as second best. There are also explicit comments made by Simmias that show that Plato sees at least Simmias and Cebes’ objections as the initial stages of the hypothetical method.

i. Plato Wishes to Illustrate Mathematical Reasoning in the Phaedo

Like the *Meno*, the *Phaedo* is full of references to mathematics as a model for philosophy. The hypothetical passage is the most extensive of these, and will be discussed below, but we should first mention the others, as they give us a good idea of Plato's confidence in the value of mathematics as an archetype in the dialogue as a whole.

When the Theory of Recollection is introduced by Cebes, Simmias asks him to 'remind' the group of the proof of the theory (72e-73a). Cebes makes two quick points before Socrates takes over the discussion: that someone could come to a correct conclusion (which requires perfect knowledge) if he was questioned in the right way, and that the way someone reacts when they are confronted with a diagram also proves this theory (73ab). Although mathematics is not directly mentioned, the use of τὰ διαγράμματα immediately calls the discipline to mind. The combination of this with the reference to the method of questioning to draw out a correct conclusion strongly recalls the slave-boy passage of the *Meno*, in which Socrates uses a combination of the diagram and interrogation to draw out correct answers from the slave-boy (*Meno* 82b-85b). As we have seen, this passage in the *Meno* is part of the dialogues' wider endeavour to promote mathematics as a model for philosophical inquiry.

Recollection is to be achieved (as least initially, given the absence of a more perfect method) through mathematical reasoning, as the hypothetical passage will explain later in the dialogue. This is consistent with the presentation in *Meno*, and anticipates mathematical thought as the beginnings of knowledge, as opposed to belief, in the Divided Line allegory in the *Republic* (509d-511e). Note in this passage that when Socrates takes up Cebes' argument for Recollection in *Phaedo*, he expands upon the idea of visual prompts for recollection. His point starts with an appeal to the fact that when someone sees, hears or otherwise notices something, and becomes conscious of something else, he can be said to be reminded of that thing (73cd). When Socrates expands this point, he takes up the example of seeing, and says

that one could be reminded of a person by seeing an object that they own, or even a picture, ἰδόντα, of someone (or even to be reminded of something dissimilar - 73cd). Further, it is from seeing equal objects like stones and sticks that the notion of equality comes to mind (74b). Once again, recollection and mathematical reasoning begin with a sensible object and are realised through images.

The other reference to mathematics as a model for philosophy in *Phaedo* comes in the discussion of Simmias' image of the soul as an attunement. Once again, the reference is linked to the Theory of Recollection. Socrates points out that Simmias would not wish to say that the attunement existed before the musical instrument, so this means that the Attunement hypothesis does not 'harmonise' with the Theory of Recollection, because this theory states that the soul existed before the body *Phaedo* (92ac). Simmias agrees that he needs to reject one of the two claims, and rejects the attunement hypothesis, accepting the Theory of Recollection and giving the following reason:

The other appealed to me, without any proof to support it, because it came with a certain likelihood and attractiveness; which is why it appeals to most people. But I realise that theories which rest their proof on likelihood are imposters, and unless you are on your guard, they deceive you properly, *both in geometry and everywhere else. (Phaedo 92d).*

This has clear parallels with the *Meno*, in which Socrates illustrates the recollection of knowledge, which applies to 'geometrical knowledge, and every other subject' (*Meno* 85e).²⁵³ Once again, the principles that apply to mathematical knowledge are used as a model for other kinds of knowledge. There are also parallels with the *Theaetetus*, when it is acknowledged that geometers would be considered worthless if they relied on likelihood is cited as a reason to prefer logic to probability (*Theaetetus* 162e-163a). In this passage also,

²⁵³ Plato also does this at *Meno* 81d.

the standards of mathematics are applied to philosophy, to determine what is acceptable as an argument.

ii. The Hypothetical Method as a Second Best Method

a. The Phaedo and Epistemological Pessimism

Unlike the *Meno* and *Theaetetus*, which unambiguously stress the importance of the aporetic state for inquiry, *Phaedo* acknowledges the dangers of perplexity, while retaining some of the most important aspects of the psychological value of *aporia*. Plato acknowledges the vulnerability of the human psyche: it is as though within us there is a fearful child who needs reassurance. He makes this point when Socrates teases Simmias and Cebes for being afraid, ‘as children are,’ (*Phaedo* 77d) that the soul will be scattered after death. Cebes replies, ‘...not so much that it’s our true selves who are afraid – perhaps there’s a kind of child with this kind of fear hidden in us too. Try to convince him not to be afraid of death as though it were a bogey’ (77e).

The *Phaedo* also introduces the concept of becoming ‘misologic’ (89b-91c). This is introduced after the objections of Simmias and Cebes. Socrates is quick to recognise how this turn of discussion has affected the group (89a), which feels that its convictions have been upset and its confidence ‘not only in what had been said already, but also in anything that was to follow later’ (88c), and fears ‘that either we are incompetent judges, or these matters themselves are inherently obscure’ (88c). Even Echecrates, listening to Phaedo’s account of the conversation, shares their misgivings. He says, ‘How can we believe in anything after that?’ (88d).

Recognising the psychological impact of these objections on the group, Socrates warns them against the danger of becoming ‘misologic’ in the same way that some people become misanthropic. After repeated disappointments in arguments, at the hands of his friends, a man might become irritated and begin to dislike everybody, and to lose faith in the existence of sincerity anywhere. The same thing can happen with a man’s faith in argument. If he believes that an argument is true without possessing any skill in logic, then later decides that it is false, then he repeats this pattern over and over, he might come to believe that there is nothing stable or dependable in arguments, and that everything fluctuates. Those who spend their careers arguing both sides of an argument, like the Sophists, are especially vulnerable to this charge,²⁵⁴ and may begin to believe that they are wiser than everyone else. Socrates urges the group not to ‘let it enter our minds that there may be no health in argument. On the contrary we should recognise that we ourselves are still intellectual invalids...’ (90de; compare this with 68a).

These passages are extremely telling in a number of ways when we think about the hypothetical method. The idea of a blank slate of *aporia* is still recognised as having cognitive value in the *Phaedo*, but Plato is acknowledging here that extreme doubt can also have negative effects. Given the fearful, childlike instincts we have when approaching arguments of vital importance, and given the human tendency to become mistrustful of argument, we should proceed with great self-awareness and an awareness of epistemological limitations. Bound as we are to the sensible world, we may never have the ability to produce infallible results, if we do not live as philosophers, as described in the opening section of the dialogue (63b-69e). An absolute truth about the matter may exist, but we ourselves may lack the capacity to reach it. Therefore, we should look towards improving ourselves, rather than giving up on the capacity of reason to find it. As in the *Meno*, Plato takes care here to

²⁵⁴ Plato exposes this in *Euthydemus*.

distinguish between Socrates' activity, and the Sophists, portraying the former as genuinely in search of the truth.

Again and again, the impossibility of certainty in earthly inquiry is alluded to. Near the beginning of the dialogue, when the group discusses the philosopher as avoiding suicide but welcoming death, Socrates admits that 'even my information is based on hearsay' (61d). However, he goes on to develop the case that, in life, we are tied to the body, and with it, the experiences of the senses, which are actually a hindrance to rational inquiry. This is why the philosopher seeks to isolate himself from bodily concerns (65cd). In fact,

It's likely...that the wisdom which we desire and upon which we profess to have set our hearts will be attainable only when we are dead, and not in our lifetime...either it is totally impossible to acquire knowledge, or it is only possible after death, because it is only then that the soul will be isolated and independent of the body.

(Phaedo 66e-67a).

This theme is continued throughout the dialogue, and is picked up by other speakers. When Simmias introduces his objection, he begins by saying,

I think, just as you do, Socrates, that although it is very difficult if not impossible in this life to achieve certainty about these questions, at the same time it is utterly feeble not to use every effort in testing the available theories, or to leave off before we have considered them in every way, and come to the end of our resources.

(Phaedo 85e).

After the argument for the immortality of the soul is concluded, Simmias retains his doubts about the possibility of certainty, even in the face of a convincing argument. He says,

I have no doubts myself now, in view of what you have just been saying. All the same, the subject is so vast, and I have such a poor opinion of our weak human nature, that I can't help still feeling some misgivings.

(*Phaedo* 107ab).

Socrates praises Simmias for this remark, saying that he is right to still have some misgivings, and that he and his friends must continue to search for the truth, 'in so far as it is possible for the human mind to attain it' (107b). He later reaffirms his own inability to speak with certainty on his theories about the earth (108de).

Although extolling the virtue of reason, the *Phaedo* is nevertheless pessimistic about the possibility of pure rational inquiry in a man's life, as least insofar as it yields certain results. It is often forgotten that the dialogue frequently appeals to divine revelation as superior to rational thought. This should seem strange to us, given that rational inquiry is Socrates' preferred occupation during his last day on earth, and he holds it in such high esteem.

Yet divine revelation is constantly cited as a greater source of certainty than rational thought. Socrates admits to writing poetry in obedience to a dream (60e-61b); Simmias refers to divine revelation as a surer means of ascertaining the facts than human intelligence (85c). Perhaps most strikingly, Socrates cites divine revelation as the basis for his belief in the immortality of the soul, in spite of the fact that he spends the day in rational discussion of the matter. In fact, divine revelation makes him invulnerable to the doubt that the rational objections that Simmias and Cebes might arouse. He says,

Evidently you think that I have less insight into the future than a swan... [They] know the good things that await them in the unseen world...I consider that I am no worse endowed with prophetic powers by my master²⁵⁵ than they are, and no more disconsolate at leaving this life (*Phaedo* 84e-85b).

Although Socrates goes on to provide a response to his friends' objections, his message is clear: their argument relies on uncertain means, and the divine revelation that he has received from Apollo supersedes the conclusions of rational thought, at least in the context of the present discussion. Reassured that what they have to say will not upset Socrates, 'in his present misfortune,' Simmias and Cebes are happy to make their objections (84d; 85b).

We should be aware of the controversial nature of this claim: the *Phaedo* is often seen as a dialogue in which the virtues of rationality are valued above everything; after all the rational, philosophical life is upheld as the best preparation for death, and in fact the best way to live in the opening scenes of the internal dialogue. However, again and again we are cautioned against placing too much faith in the conclusions of such inquiry: until we either find an infallible method, or are blessed with divine revelation, all our conclusions are provisional.

The *Phaedo* does not even seem to share the *Meno*'s optimism about the capacity for reason belonging to all people. While the slave-boy experiment of the *Meno* demonstrates Socrates' conviction that we are all able to 'recollect' knowledge, the *Phaedo* seems to be more hesitant about attributing the capacity for rational inquiry to all people. Socrates says that 'ordinary people' do not seem to understand the philosopher's endeavour (64a) and urges the group to 'dismiss them and talk among ourselves' (64c).

²⁵⁵ Apollo.

This is a step towards the claim that Plato's Socrates makes in the *Republic*, that not all of us have the right character to lead the philosophical life. Here, if we take Socrates seriously, we are not all cut out for the kind of reasoning that the philosopher needs to do. On the other hand, the *Republic* does seem to abandon the epistemological pessimism of the *Phaedo*: if the Philosopher Kings are to use their knowledge to rule wisely, we have to suppose that such knowledge is attainable during their lifetimes. Perhaps it is attainable to them because of the training they receive,²⁵⁶ which would not contradict the claim in the *Phaedo* that certain knowledge is unattainable to Socrates and his interlocutors, because they have not received such training.

In the *Republic* section, I stressed that my reading of Plato's epistemology gives us the most direct way of studying Forms that we can have in our lifetimes, and I promised that, in this section, I would expand on my point about Plato's pessimism about the possibility of direct contact with the Forms before death. Here, I have tried to show that the presentation we are given in the *Phaedo* is that the group is unable to attain certain knowledge, and must resort to a more provisional, 'second-best' method, which is what I shall focus on next. On my reading of the *Republic*, this method provides the foundation for the best possible method that we are able to achieve in our lifetimes. *A ti esti* is the most direct way of studying a Form that we can achieve in our lifetimes, and a part of the teleological account that the *Phaedo* seems to require. Because our souls are bound by the senses in life, we need to proceed through a second-best way before we can achieve even this.

b. The Second Sailing

²⁵⁶ Or perhaps they have access to the method that Socrates was hoping to find in the works of Anaxagoras; see *Phaedo* 96a-99d for Plato's hints about this.

This, then, is the psychological setting for the hypothetical method in *Phaedo*. The possibility of achieving certain knowledge is exceedingly slim, yet it would be ‘utterly feeble not to use every effort in testing the available theories (85c). The group must resort to a secondary approach, referred to explicitly and with strikingly similar imagery in two places in the dialogue.

Simmias describes this first. He says that it is extremely difficult, if not impossible to achieve certainty about what happens to the soul after death, so we must do one of two things: ‘either ascertain the facts...or, if that is impossible, to select the best and most dependable theory that human intelligence can supply, and use it as a raft to ride the seas of life – that is, assuming that we cannot make our journey with greater confidence and security by the surer means of a divine revelation’ (85cd).

Socrates’ use of the idea comes after his explanation of his disappointment with Anaxagoras’ idea of intelligence. As a young man, Socrates was keen to study causation, and had hoped to find some answers from a book of Anaxagoras’, which argued that Intelligence was the reason for everything. However, on procuring the book, Socrates was disappointed to find that Anaxagoras adduced reasons other than Intelligence for things, and that his explanation was quite insufficient. In the absence of a satisfactory explanation, Socrates has worked out his own ‘secondary approach (99cd)’ *deuteros plous*, a secondary approach, or literally a ‘second sailing’ when there is no firm starting point to attain knowledge. This approach is the hypothetical method.

The striking similarity of Socrates and Simmias’ remarks cannot be overlooked: both refer explicitly to a second best method and both use the same metaphor of sailing. Simmias makes his comments by way of introduction to his and Cebes’ objections, showing that these objections employ the same methodology that Socrates describes.

Second Sailing as Second Best?

In noting Plato's apparent epistemological pessimism in the *Phaedo*, we should be careful not to overlook Plato's steps towards epistemological progress: after all, the *Phaedo* does portray Socrates as eager to make as much progress towards knowledge as possible. In this segment, I shall expand upon what I mean by saying that the second sailing is a 'second best' method – as I go on to explain in Section Three, Chapter Two ii, some people (for example Byrd [2007 pp.141-158]) reject this translation, so here, I shall explain what I mean.

The first thing to say is that Socrates' secondary approach is secondary not to Anaxagoras' disappointing theory, but to the approach that Socrates had initially hoped to find. That is, Socrates feels that his method is an improvement on that of Anaxagoras.²⁵⁷ His metaphor of *deuteros plous* or 'second sailing' implies a more laborious means of getting to one's destination (although it does not necessarily imply that there is less likelihood of getting to one's destination).²⁵⁸ We also have contextual evidence to show that Socrates feels his method is 'second best': he tells Cebes that "what you require is no light undertaking, Cebes. It involves a full treatment of the reasons for generation and destruction (95e-96a)." He had hoped for this from Anaxagoras, but was disappointed (96a-99d). Having been unable to discover the best approach, Socrates adds, "I've worked out my own secondary approach to the problem" (99cd). I argue in Section Three, Chapter Two i that Socrates is looking for a teleological explanation as the best account. I also propose that teleological explanations can

²⁵⁷ I have already (in the introduction to Section Three) referred to and agreed with Gallop's (1980) translation of the *Phaedo* (pp. 176-177) in which Gallop remarks that Socrates' secondary approach is not the kind of explanation he once expected of Anaxagoras. Rather, it is an inferior kind of explanation to the teleological kind he expected from Anaxagoras, but superior to the kind that Anaxagoras actually provides. See also Burger ([1984] pp. 144-147) for another view that supports the reading of Socrates' *deuteros plous* as the 'second-best alternative' (p. 144): Burger argues that it is a complementary, not compromising alternative.

²⁵⁸ According to Liddell and Scott's Greek-English Lexicon, *deuteros plous* is 'the next best way.' This does not necessarily imply failure to reach the destination, rather a more laborious means of arriving. Dorter (1982) also agrees that *deuteros plous* "suggests that Socrates' ultimate destination remains the same, the discernment of the teleological cause, and only the means of approaching it has changed... the alternative approach turns out to be the metaphor of hypothesis..." (p. 120).

be built with the *ti estis* that result from the hypothetical method, so Socrates' secondary approach, although more laborious, and secondary in the sense that the explanations it initially produces are not teleological, does not prohibit the philosopher from reaching the desired destination in the end.

I argue that Socrates' method is the initial stages of dialectic, and covers the *dianoia-noēsis* transition. This is not Socrates' 'best method,' but it allows the philosopher to get to the destination in a way in which it can be done in the absence of the best method. As a result of this method, the philosopher will have the tools she needs to arrive at the destination for which Socrates initially set out. That is, she will have attained an understanding of teleology. Having done so, she may then use the 'best' method, to approach questions like the one Cebes asks Socrates about the immortality of the soul, and which Socrates restates at *Phaedo* 95bd. The *deuteros plous*, I argue, allows Socrates to generate *ti estis* about the thing to be investigated. Sufficient *ti estis* will enable the philosopher to gain an understanding of teleology. Therefore, although the *deuteros plous* is not the direct approach that Socrates had hoped for, it by no means prohibits the philosopher from getting to the desired destination.

This process has two phases: firstly, the use of the *deuteros plous*, or the hypothetical method, generates true statements about the Form we are investigating, the kind of definition upon which the Socrates of the early dialogues famously insists. We actually see this happening in the *Phaedo*. As I shall discuss in the following segment, from *Phaedo* 102a – 107a, during the process of reducing the problem of whether the soul is immortal to the hypothesis that Forms exist, and in the subsequent steps, Socrates generates a series of other statements about the soul: Soul must be present in a body to make it alive (105dc); when soul takes possession of a body, it always brings living with it (105d); soul will never admit the opposite to that which accompanies it (105d); the soul is undying (105e); when death approaches, the soul retires and escapes unharmed and indestructible; our souls will really

exist in the next world (106e-107a). Socrates' 'second best' method has thus generated at least one approximation (which can be improved and made fuller by further argument) of a *ti esti* for the soul.

The second phase (which goes beyond the hypothetical method but which has been made possible because of it) is the generation of the teleological account. As I argued in Section One, Chapter 4. ii, the nature of the *ti estis* collected in the initial stages of dialectic allows us to build the teleological account that Socrates seeks in the *Phaedo*. This is because a good *ti esti* will allow Socrates to see connections between the properties of the Form, and its connection to other Forms (and ultimately to the Form of the Good), especially once this method has been applied to several problems, leading to several, interconnected *ti estis*. The generation of true statements about the definiendum results in understanding, and will allow us to make connections between the definiendum and other Forms.²⁵⁹ This is comparable to Fine's account of teleology: she describes the Form of the Good as 'the teleological structure of things; individual Forms are its parts, and particular sensible objects instantiate it' (Fine [1999] p 228).

This process of collecting *ti estis* will lead to the teleological account that Socrates is looking for. That is, the end of the 'upward path' in the *Republic*'s method of dialectic. The ascent moves from hypotheses to *ti estis* to teleology and the Form of the Good. The 'way down,' I argued (and will continue to argue in Chapter Three of this section) is not symmetrical to the upward path, but comes after reaching Socrates' unhypothetical beginning (the Form of the Good): the philosopher provides the kind of teleological accounts that Socrates is ideally looking for in the *Phaedo*. This reading also avoids the problem that there is nothing left for the way down after the triumph of the ascent: on my reading, the way down builds on the way up to give a completely different kind of explanation (the 'best' method),

²⁵⁹ For example, the longest description of a Form we have is that of the Form of Beauty in the *Symposium* (210e- 211e). I already mentioned in the *Republic* section that this account enables us to see connections between the properties of the Form, and its connection to other Forms, for example the Form of Love.

this time with reference to the Form of the Good. In this way, although I am limiting my reading of the *deuteros plous* to the initial stages of dialectic, the method does not fall short of reaching the desired destination, because it eventually results in a collection of *ti estis*, which will allow us to build the teleological account that Socrates initially wanted. It is ‘second best’ because it is more laborious, and because the initial explanations it produces are not teleological, but Plato’s epistemological scheme allows the philosopher to progress to this kind of explanation as she surpasses the initial stages of dialectic.

iii. More Textual Evidence for Dianoia in Phaedo

As we have seen, Simmias’ comments when he introduces the Attunement Hypothesis anticipate Socrates’ remarks at 99cd. Socrates’ response to Simmias involves a discussion of what is required for the harmonious coexistence of separate hypotheses. The metaphor of the Attunement hypothesis ‘harmonising’ with the Theory of Recollection is key here, because it anticipates the metaphor of accord in the discussion of the hypothetical method in the following passages (100a; 101d). In the latter, Socrates says,

...I first lay down the theory which I judge to be the least vulnerable; and then whatever seems to agree (συμφωνεῖν) with it... (*Phaedo* 100a).

Then,

If anyone should question the hypothesis itself, you would ignore him and refuse to answer until you could consider whether its consequences were mutually consistent (συμφωνεῖν) or not (*Phaedo* 101d).

Both Socrates' response to Simmias' Attunement hypothesis and his remarks on συμφωνεῖν, or accord, in the hypothetical passage, relate to the question of how two claims can coexist with each other. The latter, as a response to Cebes' Tailor Theory, is an expansion on the former. Therefore, it is more evidence that Plato intends for these objections to be the early stages of the hypothetical method, and therefore, *dianoia*.

We should also not forget that Simmias and Cebes are Pythagoreans. As such, they are dedicated to the pursuit of mathematics, and, according to Plato's epistemology, must base their reasoning on the use of hypothesis and imagery. Most tellingly, 87b allows the inference that both the Attunement and Tailor are images; Attunement is explicitly called a hypothesis (94b) and the Tailor also involves a hypothesis.

This tells us that what Simmias and Cebes are about to embark upon are the initial stages of the hypothetical method: *dianoia*. This is a secondary, imperfect method that we should use in the absence of certain knowledge, until we have 'come to the end of our resources' (85c). In the initial stages, we shall argue, in which there are no *ti esti* answers to help us in our inquiries, these resources might include such imperfect tools as imagery.

iv. The Objections of Simmias and Cebes and the Divided Line

I suggested that Plato intends some of the arguments from the *Phaedo* to be examples of *dianoia*. That is, strong references to mathematics as a model for inquiry, the epistemological pessimism about attaining absolute knowledge and explicit textual references that link the Attunement and Tailor Hypotheses to the initial stages of *dianoia*. The purpose of this section will be to examine these passages in detail, showing how the arguments follow the criteria set

out for *dianoia* in the divided line passage. Following this, we shall examine the implications of this for the role played by the image in hypothetical reasoning. Firstly, we need an account of the passages in question.

Simmias' objection is that the things that have been agreed upon about the soul could also be said of the attunement of a musical instrument. It is something invisible, incorporeal, splendid and divine, and located in the instrument, as the soul is located in the body. The body, on the other hand, is like the instrument: corporeal, composite, earthly and closely related to what is mortal. However, if the instrument is broken, or if the strings are cut, or snapped, the attunement is destroyed. Simmias says that if the soul really is such an attunement (as contemporary theories said it was, and as is consistent with Socrates' reasoning from the Argument from Affinity), it must be a blending of those things that keep the body in tension ('hot and cold, dry and wet, and the like' [86a]). It then follows that the soul will be destroyed when the body dies (85e-86d). The question of whether there are varying degrees of attunement is unresolved.

Cebes' objection is a little different. He is satisfied with the proof that the soul existed before birth, and willing to concede that it is stronger and more durable than the body. However, this does not mean that the soul is immortal:

Suppose an elderly tailor has just died. Your theory would be just like saying that the man is not dead, but still exists somewhere safe and sound; and offering as proof the fact that the cloak which he had made for himself and was wearing was still intact.

Cebes says that the same analogy could apply to the relation of the soul to the body: the soul could be superior to the body, and longer-lived, even 'wearing out' a number of

bodies, but at some point, one of these bodies will be its last.²⁶⁰ In this case, no-one has a right to face death ‘with any but a fool’s confidence, unless he can prove that the soul is absolutely immortal and indestructible’ (88b).

These objections make a big impression on the group. The interlocutors are quite depressed after hearing them, and Echebrates, hearing Phaedo’s account, interjects to say that he sympathises with their misgivings. He asks for another proof, ‘right from the beginning,’ once again emphasising the ‘blank slate’ element involved in using hypotheses, which we noted in the Chapter on the *Meno*. Socrates’ first response is to say that Simmias’ objection does not ‘harmonise’ with what has previously been agreed upon, but then he goes on to respond in a way that tells us a great deal about Plato’s use of imagery in this kind of argument.

Socrates evaluates the fit of the attunement hypothesis as a model for the soul.²⁶¹ He identifies the following properties of an attunement: it should not be in a condition different from its composite elements; it should not control its elements, but should follow their lead; it should not conflict with its elements in any way; there are varying degrees of attunement and there is no such thing as a ‘good’ or ‘bad’ attunement in the moral sense. Socrates and Simmias agree that the varying degrees of attunement cannot correlate to the quantity of goodness or badness in the soul. Moreover, the soul does not possess the other properties associated with attunement – it does not follow the lead of its elements, for example, so ‘there is no justification for our saying that the soul is a kind of attunement’ (94e-95a).

Socrates’ response to Cebes’ objection is that a full treatment of the reasons for generation and destruction is required. He goes on to give an intellectual autobiography and an account of the hypothetical method, but he does not criticise Cebes’ choice of image as a bad model for the soul as it has been conceived at this point. Also, he does not say that it

²⁶⁰ In the examination of this argument, the definition of death changes from ‘the separation of the soul from the body’ to ‘the extinction of the soul’ (see *Phaedo* 87e and 95d).

²⁶¹ Scaltsas ([1990] pp.110-119) discusses this point.

contradicts any other hypothesis that has already been accepted in the way that the Attunement hypothesis contradicts the Theory of Recollection (we shall discuss the fact that the latter is also a hypothesis in the following section).

In the rest of this section, we shall examine the Attunement and Tailor hypotheses more closely, looking at the role the images play in the argument. We shall identify these passages as part of the beginnings of dianoetic thought, as described in Plato's divided line of the *Republic*, and as such, we shall begin to understand why sometimes the philosopher must resort to images at this stage, as he does not have access to the *ti esti* definitions that are available to him as he ascends the epistemological hierarchy.

When we summarised the divided line passage, we saw that *dianoia* is intelligible, but has the following features: the mind is forced to use the objects of *pistis* as illustrations, it uses assumptions, it does not proceed to a first principle, and it is coherent. We shall see that the objections of Simmias and Cebes share all of these properties.

a. Originals Used as Images

As we saw, the one of the first things that Socrates tells us about *dianoia* in the *Republic* is that 'the originals of the visible order' are used as images, and that its inquiries are based on assumptions²⁶² proceeding not to a first principle but to a conclusion (*Republic* 510b).

If we look at the metaphysics of the *Phaedo*, we see that Simmias and Cebes are doing exactly this: using the originals of the visible order as images. Forms have already been introduced in the *Phaedo* in Socrates' defence of his readiness to die. There is such a thing as justice itself, beauty itself and goodness itself – but these things are not apprehensible to the senses, which is why the philosopher attempts to proceed through the intellect (65d-66a).

²⁶² ὑπόθεσις.

This is developed in the Argument from Affinity, in which ‘the actual nature of things’ is described as inapprehensible ‘except by the workings of the mind’ (79a3).

However, it is not Forms that are used in the objections of Simmias and Cebes, but images of the sensibles. The musical instrument, tailor and coat are all things belonging to the sensible world.²⁶³ The group is not actually investigating these images, rather, the images are used in the discussion to investigate what is “invisible except to the eye of reason” (*Republic* 511a).²⁶⁴

b. Hypotheses as Starting Points

The next feature of *dianoia* mentioned by the divided line is that the mathematician starts from the hypothesis that the mathematical objects exist, without feeling the necessity to explain them. In the *Republic* section, we argued that ‘explaining them’ would begin with giving *ti esti* answers to the question of what they are; something that is absent from *dianoia*. Again, note the similarity with the methodology in the *Phaedo*: we do not have a *ti esti* answer to the question, ‘what is the soul?’ Thus, we are forced to begin from hypotheses: that the soul is like an attunement, or like a tailor.

Recalling conversations in previous dialogues, in which Socrates is reluctant to investigate the properties of an object without such an answer, it seems strange that Socrates is happy to spend his last day on earth investigating this question at all.²⁶⁵ From a dramatic point of view, we could note that, with just one day remaining in his life, Socrates does not have time to investigate both the *ti esti* and *hypoion* properties of the soul.

²⁶³ Attunement is a little different, given that it is ‘invisible and incorporeal’ (85e) (the reason, in fact, that it was picked for the example in the first place). However it nonetheless falls short of ‘attunement itself’ because we are arguing about a particular example of an attuned instrument, not ‘the actual nature of things.’

²⁶⁴ As we noted in Section one, this is Lee’s translation of τῆτι διανοίαι.

²⁶⁵ For Socrates’ wish for a *ti esti*, see *Meno* 70b-71d, 80cd, 86ce.

Some have suggested that Plato nevertheless implicitly makes the point that arguing from property to property without a *ti esti* results in a total lack of conviction, and that this is the function of the Argument from Affinity (Tarrant [1993] p 136). Whatever the reason, by the time Simmias and Cebes come to make their objections, there is still no agreement upon what the soul actually *is*. As Cebes says, ‘It seems to me that the argument is just where it was’ (86e). Simmias has already acknowledged the need to employ the second best method (later to be called the hypothetical method), and his ‘harmony’ image is twice called a hypothesis (93c and 94b). In the absence of a *ti esti* for the soul, Simmias and Cebes must begin from assumption and, like the dianoeticians of the *Republic*, they ‘must have recourse to an image’ (87b)²⁶⁶

c. Proceeds to a Conclusion

Next, says the divided line passage, the mathematician proceeds ‘through a series of consistent steps to the conclusion which they set out to find’ (*Republic* 510d). If we look at the structure of the arguments, we shall see that these objections do proceed downwards to a conclusion, but not ‘from assumption to a first principle which involves no assumption’ (510b), as would happen in *noesis*.

First of all, Simmias’ objection:

1. The attunement and the soul share the properties of invisibility, being incorporeal, splendid and divine, and being located in the tuned instrument (*Phaedo* 85e-86a).
2. The musical instrument and the body share the properties of being material, corporeal, composite, earthly and closely related to what is mortal (*Phaedo* 86a).

²⁶⁶ Compare this with *Republic* 510d.

3. Suppose that the instrument is broken, or its strings cut or snapped (*Phaedo* 86a).
4. According to the Argument from Affinity (78b-79e), the attunement must still exist (*Phaedo* 86a).
5. The body is taken to be a balance of the extremes of the body combined in the right proportion (in the Greek world) (*Phaedo* 86bc).
6. But an attunement, like all things that are a balance of physical constituents, is the first thing to be destroyed in such a case (*Phaedo* 86cd).
7. Therefore the soul is the first thing to be destroyed when the body dies (*Phaedo* 86d).

From the hypothesis that the soul is a kind of attunement, we proceed to the conclusion that it is the first thing to be destroyed when the body dies; but the argument does not (at this point) go past the hypothesis to a first principle which involves no hypothesis. Next, Cebes problem:

1. Suppose that an elderly tailor has just died (*Phaedo* 87b).
2. According to the Argument from Affinity, we would have to say that the tailor exists somewhere safe and sound, offering as proof the fact that his coat is still intact (*Phaedo* 87b).
3. The proof would rest on the fact that tailors are generally more enduring than coats (*Phaedo* 87bc).
4. But the tailor may outlast any number of coats; nevertheless he perishes before the last one; this does not mean that he is lowlier or frailer than a coat *Phaedo* (87cd).

5. If we apply this analogy to the relation of the soul to the body, even if we admit that the soul may 'wear out' a number of bodies, there is no justification for any confidence in the view that the soul continues to exist after death (*Phaedo* 87de).

Again, Cebes' argument begins from the hypothesis that the soul shares the same properties as the tailor, and proceeds to the conclusion that that, at some point, the soul will perish. However, the argument does not go beyond this hypothesis to a higher principle.

We should note that these hypotheses are true hypotheses in the sense that the interlocutors are allowed to break the (early/historical) Socrates' 'say only what you believe' rule. We noted that, in the *Meno*, Socrates and Meno proceed by assuming hypotheses to which they do not necessarily subscribe. In the *Phaedo*, also, the 'say only what you believe' rule is abandoned. Simmias and Cebes do not actually subscribe to the Attunement and Tailor hypotheses; they offer them as problems that need to be solved in order to make Socrates' argument sound. Simmias even admits that the Attunement hypothesis appealed to him without any proof to support it (92cd); Cebes puts forward the Tailor hypothesis to illustrate a weakness in Socrates argument without having to admit to it as an accurate description of the soul (86e-87a).

In fact, Cebes' objection makes it clear that the image is a valuable thought experiment regardless of how much of the theory one subscribes to: he says, 'suppose that one conceded even more to the exponent of your case, granting not only that our souls existed before our birth, but also that some of them may still exist when we die...' (88a). This shows that the method does not depend upon starting from secure knowledge; we may proceed from an hypothesis to the conclusions, without actually believing in that hypothesis in the first place.

Alex Long believes that the breaking of this rule plays an important role in the nature of the *Phaedo*. He says that here, Socrates' interlocutors take it upon themselves to speak for positions that they themselves do not endorse (or, in Simmias' case, do not wholeheartedly endorse). The difference here, Long believes, is that, as collaborative critics, they are entitled to do this, and break the 'say only what you believe' rule of the early dialogues.²⁶⁷ As we noted in the *Meno* section, the breaking of this rule is a central part of the hypothetical method in general and *dianoia* in particular, and further illustration of the method applied in *Phaedo*.

d. Consistency

The passage in the *Republic* says that the mathematician proceeds 'through a series of *consistent* (my emphasis) steps to the conclusion which they set out to find.' (510d.). The idea of consistency is fundamental to the hypothetical method, and an important feature in how the objections of Simmias and Cebes fit in with what Plato is trying to do. We have already noted, above, that the metaphor of *συμφωνεῖν*, accord, occurs twice in the hypothetical passage of the *Phaedo*, and is strongly connected to Socrates' refutation of Simmias' 'harmony' objection in the preceding arguments. This is one of the features that illustrate Plato's intention to show dianoetic reasoning in the objections of Simmias and Cebes. We shall see that the image actually plays a role in evaluating the consistency of propositions in the method.

Firstly, let us consider what Plato means for the steps to accord or harmonise, looking at both passages. Richard Robinson argues that there are only two conceivably interpretations for the metaphor: either 'to accord with' means 'to be consistent with' or 'is deducible

²⁶⁷ Long ([2008] pp. 45-59).

from/entailed by.’²⁶⁸ He thinks that, even though we are not justified in accepting every proposition that is consistent with our hypothesis, this is better than setting down as false every proposition that is not deducible from it. Therefore, he treats the metaphor as meaning, ‘to be consistent with’ in the literal sense.

However, Bailey has another interpretation.²⁶⁹ He thinks that consistency is a necessary condition for a proposition to justify the use of συμφωνεῖν, but this is not sufficient. If a proposition is entailed by a hypothesis, this is sufficient, but not necessary. He says that συμφωνεῖν lies between consistency and entailment.

In proposing a solution, Bailey takes the musical metaphor seriously, underlining that, in Greek musical theory, musical systems are structured significantly. The pitches in a harmony come together to form a unity. When the notes are in the ratio (*logos*) 2:1, 3:2 or 4:3, they come together to make accord. The notes in a harmony that do not have this ratio do not form a unity. Bailey thinks that we can make an analogy between this relationship in music and that which is implicated in the metaphor of accord. Rather than a musical unity, the hypothetical method requires a theoretical unity. Bailey supports his case with literary and historical evidence,²⁷⁰ as well as a careful philosophical examination of the text and themes. He thinks that this explanation means that the method is genuinely heuristic, as it means that you can proceed without having the right answers.

Elsewhere, Plato links the idea of accord with mathematics. In the *Republic* section, I quoted this passage from the *Cratylus* when I was discussing foundationalism in Greek mathematics:

The name-giver might have made a mistake at the beginning and then forced the other names to be consistent (*symphōnein*) with it. There would be nothing strange in that. It is just that

²⁶⁸ Robinson ([1953] Ch. 9 pp. 123-145); See also Kahn ([1996] pp. 313-320) for a similar argument.

²⁶⁹ Bailey ([2005] pp.95-115); Cf Gentzler ([1991] pp. 265-276) for a similar point.

²⁷⁰ He mentions the many references to Apollo, for example. See *Phaedo* 58c, 60d, 85b.

way sometimes in geometrical constructions: given the initial error, small and unnoticed, all the rest that then follow are perfectly consistent (*homologeîn*) with one another. That's why every man must think a lot about the first principles of anything and investigate them thoroughly to see whether or not it's correct to assume them. For if they have been adequately examined, the subsequent steps will plainly follow from them.

(*Cratylus* 436cd).

In this case, Plato is associating the mathematical process of drawing conclusions with the metaphor of accord. In this case, he needs a process that can apply to names and the connection between them (which would imply consistency) and geometry (which would apply deduction), as well as to other subjects.²⁷¹ In which case, I would argue, Plato does not require a fixed intermediate between consistency and entailment, but a flexible one, to allow for the difference in subject-matter. As we have seen, Plato's terminology is notoriously vague.²⁷² What is clear is that, although *symphōneîn* is important, it is only part of the process, and its value is dependent on the *archai* that produce it.

At this point in the dialogue, we do not have a Theory of the Soul, at least in the sense that Plato would wish, because for Plato, a theory requires a *ti esti* definition.²⁷³ The images of Cebes and Simmias provide a substitute for this. By using images, they are able to postulate that the soul behaves in a particular way, because the properties of the image provide the next steps in the argument in a way that 'harmonises' with the hypothesis. So Simmias uses the properties of an attunement to show that, on this model, the soul would be destroyed along with the body.

In addition, the images provide us with the means to criticise them as suitable theories: we can see what does not accord with them. Socrates is able to show that the Theory

²⁷¹ See White ([1976] pp. 74-75) for the connection between Forms and names in the *Phaedo* and the *Cratylus*, and the argument for a 'semantic consideration' in *Phaedo*.

²⁷² See Sayre ([1969] pp.15-40, especially pp.39-40) for the view that Plato is deliberately vague here.

²⁷³ See the *Republic* section.

of Recollection does not accord with the hypothesis that the soul is a kind of attunement, because an attunement does not have the property of existing before the instrument.²⁷⁴ Cebes' image allows him to show that Socrates' remarks on the longevity of corpses in Egypt (80c) do not add anything to his proof of the immortality of the soul. He uses the tailor's property of being more enduring than his coats to show that being more enduring does not mean being immortal.

When we look at the 'hypothetical passage' of the *Phaedo*, we shall come across the idea of postulating a higher hypothesis, which moves the argument on and, as we argued in the *Republic* section, is a way of moving from *dianoia* to *noesis*. Images do not perform this role, but they do stand in as substitutes for *ti esti* answers and theories. They are, like hypotheses, means of 'testing the available theories,' (85c) as well as provisional means of exploring a topic for which a theory, or a *ti esti*, is not yet available.

Chapter Two: Socrates' 'Intellectual Autobiography'

Socrates' answer to Cebes' objection, and his description of the hypothetical method, is introduced by the story of Socrates' own philosophical journey. In one of Plato's most touching passages, we are taken from Socrates youthful passion for (and subsequent disappointment in) the natural sciences to his own formulation of a 'second best' method.

As a response to Cebes' objection, Socrates says that what is required of a response to Cebes is no light undertaking:

²⁷⁴ See Karasmanis ([1987] pp. 119-139) and Gallop ([1965] pp.113-131) for support for the argument that this feature makes the passage part of the hypothetical method.

It involves a full treatment of the reasons for generation and destruction. If you like, I will describe my own experiences in this connection; and then, if you find anything helpful in my account, you can use it to reassure yourself about your own objections.

(*Phaedo* 96a).

Socrates is not presenting his response as a universal answer to Cebes' concerns; rather his reply is a form of intellectual autobiography, which Cebes may use to rebut his own objections. Plato's character Socrates places himself in intellectual history before going on to tell us about his new method. Socrates' intellectual autobiography draws heavily on mathematical thought, and has been compared to Descartes' *Discourse on the Method* as an attempt to pioneer a new philosophical approach.

The problem for this chapter will be to explain how this approach fits into Plato's epistemological scheme as we have described it. After rejecting the natural sciences as inadequate to attain knowledge, Socrates advocates an approach inspired by mathematics as a stepping-stone to the perfect method. I shall argue that this represents a move from *pistis* to *dianoia* on the divided line. As I argued in the *Republic* section, *pistis* is drawing conclusions from observations of the physical world, whether that is from natural phenomena, or observations of human actions to draw conclusions about morality. The tools of *pistis* are physical objects, and it is this reliance on the physical world to produce explanations that Socrates is criticising here. Socrates' intellectual autobiography is also the story of his mathematical journey away from *pistis*, into the world of the Forms.

i Early Enthusiasm and Disappointment

Socrates begins by giving an account of his own youthful enthusiasm for the natural sciences, describing his 'extraordinary passion' for them, and his hopes that they would

explain to him the reasons for which each thing comes and ceases and continues to be. Menn ([2010] pp. 37-68) points out that this introduces the first of two groups of objections to the natural sciences: Socrates' initial investigations into the natural sciences, and his objections to Anaxagoras. The initial investigations begin from Socrates curiosity about the physical world:

I was constantly veering to and fro, puzzling primarily over this sort of question: 'Is it when heat and cold produce fermentation, as some have said, that living creatures are bred? Is it with the blood that we become aware, or with the air or the fire that is in us? Or is it none of these, but the brain that supplies our senses of hearing and sight and smell; and from these that memory and opinion arise, and from memory and opinion, when established, that knowledge comes?' Then again, I would consider how these things are destroyed, and study celestial and terrestrial phenomena, until at last I came to the conclusion that I was uniquely unfitted for this form of inquiry (*Phaedo* 96ac).

Some people think that these kinds of questions already show a particular focus on the human psyche, the overarching concern of the *Phaedo*.²⁷⁵ However, it occurs to me that that the kind of questions Socrates is asking span epistemology as well as the natural sciences: he is asking if knowledge comes from memory and opinion. Note that, after initially inquiring about physical phenomena, he's asking if memory and opinion themselves arise from the senses, and if these arise from the brain. That is, how does the *physical world* explain knowledge? In this way, Socrates' questions are framed in a way that makes them accessible to the methods of the natural sciences. This supports our reading of the *Republic* section, in which we said that *pistis* is not limited to investigation into the natural world. Rather,

²⁷⁵ Burger (1984) pp. 135-137; Cf Menn ([2010] pp. 37-68) who implies the same.

Socrates' description of the natural sciences, as with his description of *pistis* in the divided line passage, allows for their ambition to reach beyond explanations of physical things.

The young Socrates, then, seeks to understand the world using physical things as explanations. However, he soon becomes frustrated with his lack of progress in this line of inquiry. He had previously 'understood some things plainly before in my own and other people's estimation; but now I was so befogged by these speculations that I unlearned even what I had thought I knew, especially about the reason for growth in human beings' (96c). Previously, Socrates had thought it obvious to anybody that growth in humans was due to eating and drinking; so that 'when, from the food which we consume, flesh is added to flesh and bone to bone, and when in the same way the other parts of the body are augmented by their appropriate particles, the bulk which was small is now large; and in this way the small man becomes a big one' (96d).²⁷⁶

This seemed reasonable to the young Socrates, and also to Cebes, at whose objections the autobiography is directed (96d). The young Socrates complacently extends this kind of reasoning to other areas:

I had been content to think, when I saw a tall man standing beside a short one, that he was taller by a head; and similarly in the case of horses. And it seemed to me even more obvious that ten is more than eight because it contains two more; and that two feet is bigger than one because it exceeds it by half its own length (*Phaedo* 96de).

However, now Socrates is not so sure:

²⁷⁶ This explanation is 'specifically Anaxagoras' explanation of growth, that each of the homoeomerous substances is imperceptibly present in the nutriment, from which they are separated out and added to homereromerous substances in us' according to Menn ([2010] pp.37-68). Bostock (1986) argues that the theory contains hints of Anaxagoras but this is 'really muddling' (p 136); Plato just means the common sense view.

Why, upon my word, I am very far from supposing that I know the explanation of any of these things. I cannot even convince myself that when you add one to one either the first or the second one becomes two, or they both become two by the addition of one to the other....Nor can I now persuade myself that I understand how it is that things become one; nor, in short, why anything else comes or ceases or continues to be, according to this method of inquiry. So I reject it altogether, and muddle out a haphazard method of my own (*Phaedo* 96e-97b).

Is Socrates describing his own experience of *aporia*? Usually the term is associated with being perplexed by Socratic examination, so it could well apply to Socrates being perplexed by himself. Certainly, Socrates' experience does resemble that of Meno and the slave-boy, who had also previously thought they had knowledge (*Meno* 80b; 82d), before being perplexed by Socrates himself (*Meno* 80ab; 84b), although instead of Meno's 'frozen kind of *aporia*, Socrates tries to find his own method.

The puzzles presented here are what Menn would call the first set of objections, in the autobiography. These have been seen as 'meta-puzzles' rather than puzzles: the problem for the modern reader is to figure out how anyone could have been puzzled by them in the first place. Vlastos²⁷⁷ argues that Socrates' mistake is to confuse the mathematical operation of addition with the physical process of joining one thing to another, and to try to give physical causes for what can only be given conceptual or logical reasons.

However, Menn thinks that this is untenable, because Socrates is trying to explain a physical truth, not a mathematical fact. In this case, 'it would be strange for Plato to represent

²⁷⁷ ([1969] pp. 291-325). Vlastos points out that *aitia* has a much more general meaning than the word 'cause' in English speech. Socrates' disappointment in the physical philosophers stems from the fact that all they can offer is material *aitiai*, whereas he is convinced that only teleology provides true or real *aitiai* of natural phenomena. Unable to discover it for himself, he falls back on the *deuteros plous*. Bostock ([1986] pp. 135-153) provides a discussion of this and Sedley ([1991] pp. 359-83) proposes that Socrates' 'safe explanation' provides the key to the first objections. It is safer to cite only the Form itself, and not other proposed causes, because otherwise, there is fear of contradiction.

this assumption as an elementary ‘howler’ that Socrates would now be blaming on his early material enthusiasm’ (Menn [2010] p 41).

Menn thinks that the purpose of the first objections is to show that ‘when we see how the soul functions in explanations...we can eliminate the Presocratic ways of conceiving of soul that makes it seem possible that a soul could perish’ (Menn [2010] p 62). He thinks that it is clear that Socrates is referring to problems about growth in a wider sense. At least one problem Plato is raising, according to Aristotle, is that of identity through-time,²⁷⁸ and Menn thinks that Aristotle is reading the *Phaedo* correctly: ‘Plato is not denying that human beings persist through time when they grow...but he denies that an explanation of growth through addition can explain this’ (Menn [2010] p 45). While it is unproblematic that a human body can persist through growth, no number can persist through the addition of a unit to make a larger number.

Following Furley ([1976] pp. 61-86), who argues that Anaxagoras thinks that something is *F* by participating in the *F* in the sense of containing within itself some portion of *F*, Menn proposes that Plato is taking over the language of predication from Anaxagoras. According to Menn, Plato thinks that Anaxagoras’ account breaks down where *F* is ‘odd’ or ‘even’ or ‘beautiful.’²⁷⁹ He thinks that, in order to fix this, participation must occur in some non-spatial way. Menn adds that the same difficulty arises for qualitative as well as quantitative change.

Menn’s solution is thoroughly worked out, with a careful consideration of the place of the autobiography in the history of science. However, I do not think that there is any need to posit such a complicated reading. Menn is correct that the *Phaedo* is concerned with persistence through time; he is also right that the autobiography is a criticism of Presocratic science. However, as I pointed out when I introduced the passage, the criticism focuses on

²⁷⁸ Menn (2010) p 43; Cf Aristotle, *On Generation and Corruption* 321a34-35. Menn cites Epicharmus fr 276 as evidence that Plato is developing existing concerns about this.

²⁷⁹ Menn (2010) p 49; Cf *Hippias Major* 300b-302b.

the reliance of sensible things for an explanation, when we should be looking to the intelligible, which is also a major concern of the *Phaedo*. Socrates is looking for a unified theory of growth. In introducing us to the inadequacy of physical explanations, the first objections prepare us for the *deuteros plous* as a stepping stone to the intelligible.

Socrates goes on to explain that he heard about a book that Anaxagoras has written, which says that Intelligence is the reason for everything, an explanation which pleased him. He says:

Somehow it seemed right that Intelligence organizes things and is the reason for everything; and I reflected that if this is so, in the course of its arrangement Intelligence sets everything in order and arranges each individual thing in the way that is best for it (*Phaedo* 97c).

Anticipating a teleological explanation for the way things are, Socrates speaks of his delight in having ‘found an authority on causation who was after my own heart – Anaxagoras’ (97d). He is prepared to learn about a great number of things and ‘give up hankering after any other kind of reason’ (97d). He emphasizes his hopefulness, saying, that he ‘would not have parted with my hopes for a great sum of money. I lost no time in procuring the books, and began to read them as quickly as I possibly could’ (98b).

However, Socrates is soon disappointed: ‘It was a wonderful hope...but quickly dashed’ (98b). The problem is that Anaxagoras’ explanation is again that he uses physical entities to explain phenomena – things like air, ether and water ‘and many other oddities’ (98bc). Socrates’ problem with this kind of explanation is that it is no more use than saying that Intelligence is the reason for everything Socrates does, but then explaining his actions in terms of bones, sinews and other physical properties of the body. As Socrates points out, the reason his sinews and bones are in a prison in Athens, and not in some other place, is that he

felt it was better to remain and submit to the penalty that had been imposed upon him (98e-99a).

His major argument against Anaxagoras is that he is unable to distinguish between ‘the reason for a thing and the condition without which the reason couldn’t be operative’ (99b). He is criticising Anaxagoras’ reversion to mechanism.²⁸⁰ That is, Socrates’ physical characteristics are necessary for him to be in a cell in Athens (‘the condition without which a thing couldn’t be operative’), but the reason for his being there is that he has decided it is better that way (‘the reason for a thing’). This mistake, Socrates says, applies to all theories of natural science:

That is why one person surrounds the earth with a vortex, and so keeps it in place by means of the heavens; and another props it up with a pedestal of air, as though it were a wide platter. As for a power which keeps things ever in the best position, they neither search for it nor believe that it has any remarkable force; they imagine that they will someday find a more mighty and immortal and all-sustaining Atlas; and they do not think that anything is really bound and held together because goodness requires it (*Phaedo* 99bc).²⁸¹

These remarks should tell us two important things about how we can fit the epistemological system of the intellectual autobiography into the divided line. Firstly, that Socrates is looking for a teleological explanation in an ideal account: goodness is the underlying reason for all things. Secondly, the natural sciences explain things by means of physical objects. These physical objects might provide condition without which a thing could not be operative, but that is not the same as the *reason* for it.

²⁸⁰ Cf Burger (1984) pp. 139-144, who argues that Socrates thinks Anaxagoras ‘could present only the means, without revealing their status as mere means’ (p. 142).

²⁸¹ Surely Socrates refers to Anaximenes’ theory when he speaks of a ‘pedestal of air.’ For suggestions about the other theories, see Tarrant’s (1993) note 158 on the *Phaedo* pp. 226-227.

When we discussed the divided line, we said that *pistis* involves the use of physical things to build its explanations. To this extent then, Socrates' description of the natural sciences in his intellectual autobiography can be correlated to *pistis*. I have not argued for an exact fit here, just a meaningful correlation. In fact, although Socrates seems to present two kinds of objections to the natural sciences, they are both different aspects of the same one: explanation by means of physical things confines us to *pistis*. As in the *Republic*, we need to turn our minds to the intelligible for a satisfactory method.

ii. Socrates' 'Secondary Approach'

As we noted, the best approach would be teleological, and Socrates would like to learn about the workings of such an explanation. However, he admits that he has been 'unable either to discover it for myself or learn it from another' (99c). In this case, he has worked out his own secondary approach:

...when I was worn out with my investigations into reality, it occurred to me that I must guard against the same sort of risk which people run when they watch and study an eclipse of the sun; some of them, you see, injure their eyes, unless they study its reflection in water or some other medium. I conceived something like this happening to myself, and I was afraid that by observing objects with my eyes and trying to comprehend them with each of my other senses I might blind my soul all together. So I decided that I must have recourse to theories, and use them in trying to discover the truth about things. Perhaps my illustration is not quite apt; because I do not entirely agree that an inquiry by means of theory employs 'images' any more than one which confines itself to facts (*Phaedo* 99d-100a).

Socrates goes on to explain this approach in more detail, as we shall see in the following chapter. His explanation is highly focused on the use of hypotheses, and has little to say about imagery. However, in this passage Socrates denies that the *deuteros plous* has no more recourse to images than what we have described as *pistis*. Note that he does not deny that this approach uses images at all; moreover, he does not even say that the *deuteros plous* uses images any *less* than *pistis* either. In the *Republic* section, we noted that the tools of both *dianoia* and *pistis* are images of Forms: if, as I am arguing, the *deuteros plous* includes *dianoia*, then this fits in with what Socrates has to say about it in the divided line.

What is described in the autobiography is the early part of *dianoia*, which, as in Simmias' and Cebes' objections and the slave-boy passage of the *Meno*, includes images. The hypothetical passage that follows the autobiography covers the later stage of *dianoia* and the beginning of *noēsis*. I shall cover the hypothetical passage in the following chapter, but for now, I want to extend the claim I made previously about the autobiography of the *Phaedo* corresponding to the divided line of the *Republic*.

I am not the first to propose a correlation between the two passages, and here I want to consider (and amend) one very good argument for the correlation, to defend the main thrust of the argument.²⁸² Byrd ([2007] pp.141-158) argues that Socrates' intellectual development in the *Phaedo* is parallel to the description of ascent in the divided line in the *Republic* (I shall endorse this). She says that Socrates' autobiography at *Phaedo* 96a-100e illustrates his application of the method (I shall not endorse this). Thus, says Byrd, while the hypothetical method and dialectic are not identical, one can apply information about the hypothetical method to the remarks about the upward use of hypothesis in the *Republic* (I

²⁸² Some commentators point to the similarity between the method of hypothesis in the *Phaedo* and dialectic in the *Republic*, but most scholars reject the possibility that they are the same method for two reasons. Firstly, the 'stopping point' in the two dialogues is different, and secondly, the hypothetical method is a *deuteros plous*. Those who stress the similarity of the hypothetical method to dialectic include: Bedu-Addo ([1979] pp. 89-109); Bluck in his (1955) translation of the *Phaedo* pp. 21-31. Those who stress the difference of the two methods: Robinson ([1953] pp.61-113); Rose ([1966] pp.464-473); Sayre ([1969] pp. 3-56). See also Byrd ([2007] pp. 141-158) for a discussion of this.

shall amend this view, saying that the hypothetical method does not correspond to dialectic, but to some or all of *dianoia* plus early *noēsis*).

Byrd says that, according to *Phaedo* 100a and 101de, there are three injunctions:

1. Put forth the hypothesis that seems strongest to you and accept as true what agrees with it.
2. Deduce conclusions from the hypothesis and check them for consistency.
3. Provide an account of the hypothesis by deducing it from higher hypotheses until you reach something adequate.²⁸³

She thinks that step three begs comparison with the upward path described at *Republic* 510c-511b, which I have already noted in the *Republic* section of this project. In contrast with students of geometry, who do not go beyond their hypotheses, those who engage in dialectic use hypotheses as stepping stones to the unhypothetical first principle. She writes, ‘Thus dialectic and the method of hypothesis seem to have two characteristics in common: both involve the pursuit of higher hypotheses until one reaches an adequate stopping point, and, in both, higher hypotheses entail lower ones’ (Byrd [2007] p 144). Byrd suggests that the hypothesis ‘above’ might be higher in the sense that they are more general and self-evident, or in the sense that they entail the lower hypotheses.²⁸⁴ I have already argued, through my correlation of the method with *apagōgē* in the *Republic* and *Meno* sections, that a hypothesis is ‘higher’ in the sense that the other one is derived from it. It is possible to have more than one hypothesis in the same argument, but this need not mean that one entails or is entailed by

²⁸³ Byrd appeals to Robinson (1953) p 134, Ross (1951) p 28, Shorey (1933) p 179, Sayre (1969) p 30, Rose (1966) p 466 and Bostock (1986) p 170 for support of her reading.

²⁸⁴ Against the view that the stopping point in the *Phaedo* is provisional, and not a true first principle (as Robinson [1953] pp.123-145; Burger [1984] pp. 147-150 argue) Byrd points out that the necessity of accounting for the hypothesis does not depend on there being an objector, and that the account needs to be epistemologically, rather than persuasively adequate. Cf *Phaedo* 107b.

the others: for example, the Recollection hypothesis and the Harmony hypothesis in Simmias' objection are not derived from each other. 'Higher' in this sense means a further reduction in the argument.

Throughout this dissertation, I have taken care to stress that the method described in the *Phaedo* is a *deuteros plous*, a second best method, a fact that has been used by others to try to dissociate the method from dialectic. Byrd, on the other hand, objects to the traditional translation of *deuteros plous* as 'second best,'²⁸⁵ suggesting alternatives: 'taking to the oars when the wind has failed' and 'making a second safer journey. Moreover, she says, the referent of *deuteros plous* might not, in fact, be the hypothetical method. Rather, says Byrd, it may simply apply to the particular hypothesis under consideration in the dialogue. So the hypothetical method is not inferior to dialectic, but the description of causes based on Forms given in the *Phaedo* is inferior to a teleological account based on the Form of the Good.

In this case, says Byrd, we have no reason to prefer the reading that makes the hypothetical method completely distinct from dialectic. Moreover, she thinks that Socrates' autobiography, which precedes the hypothetical method, gives us positive evidence for the method of hypothesis as dialectic. This argument rests on two points. Firstly, says Byrd, the autobiography is proleptic in the sense that Plato illustrates the method of hypothesis described, and secondly, that the description of Socrates' intellectual development parallels the ascent of the divided line. I am going to argue that Byrd is right that the autobiography parallels the ascent of the divided line, but that the illustration of the *deuteros plous* actually comes later in the autobiography than Byrd thinks. I have already argued that 'second best' in this context should be taken to mean the initial stages of dialectic, rather than fully realized dialectic: the *deuteros plous* covers the *dianoia-noēsis* transition, which is by no means

²⁸⁵ As Grube (1977) and Hackforth ([1955] pp. 27-29) translate it.

dialectic in its completion. As I have tried to show, we do not have direct access to the Forms: to study them in the most direct way possible, we must build on our previous work.

Byrd would have it that Socrates' initial hypothesis is that causal explanations are given in terms of sense perception. So, for example, at *Phaedo* 96d, Socrates assumes that the cause of a tall man being taller than another is 'by a head.' However, this leads to contradictory results, because a bigger thing could be bigger, and a smaller thing smaller, by the same thing. As a result of Socrates' discovery of these contradictions, he rejects his original hypothesis and, after his failure to produce a teleological account, posits a second hypothesis of Forms as causes.

This, says Byrd, means that the autobiography itself has illustrated her stages one and two of the hypothetical method. Stage three, she says, is not at first glance illustrated in the *Phaedo*. However, if we allow for a higher hypothesis to be posited in order to resolve contradictions to which the original hypothesis led, we can say that it has been illustrated in the *Phaedo*. That is, Socrates revised hypothesis of Forms as causes is higher in the sense that it revises the original hypothesis of causal explanations as sense-perceptible.²⁸⁶

To connect her reading to the divided line, Byrd connects Plato's description of 'summoners' at *Republic* 522e-525a to his remarks at *Phaedo* 102be: 'Just as the index finger appears to be both tall and short when viewed along with the thumb and middle finger, Simmias appears to be tall and short when viewed alongside Phaedo...The summoner provokes one to ascend from the sensible to the intelligible portion of the divided line' (Byrd [2007] p 154).

This is an instance of the summoner elevating the mind from *pistis* to *dianoia*. At a higher level, Byrd thinks that summoners also elevate the mind from *dianoia* to *nous*.²⁸⁷ In trying to

²⁸⁶ Byrd appeals to Dorter's ([1982] pp. 115-140, especially pp. 133-138) suggestion that the purpose of higher hypotheses can be revision as well as demonstration.

²⁸⁷ Or *noēsis*, to be consistent with my use of Plato's rather confused terminology. See Section One, Chapter One for a discussion of this.

resolve the contradiction of a failed hypothesis, the mind needs to identify the plurality involved, separate out these elements, and relate them, using the new hypothesis, in such a way that no contradiction is involved. In doing this, an account of these elements is provided, and the higher hypothesis will be higher in that it explains more. This process will eventually lead to the first principle.

As I have argued, Byrd's reading contradicts the reading of 'higher hypothesis' that I have been presenting. In addition to this, I think that Byrd is wrong for three reasons. Firstly, Socrates' initial investigations are not hypotheses. They are things he believes, pre-aporetic, which are no different from the statements examined in ordinary *elenchus*, which, as I argued, is not the same as examining hypotheses. This uncovers contradictions and identifies falsehood, which is the same as what is happening in the initial objections. Of course, the hypothetical method has in common with *elenchus* the discovery of contradiction; in this way, the *elenchus* can even be said to be part of the hypothetical method. That said, this does not make Socrates' pre-aporetic investigations hypothetical.

The second reason I disagree with Byrd is that, in the autobiography, Socrates is clear that he devised the *deuteros plous* after his *aporia*, which supports my reading rather than Byrd's. In this case, the first part of the biography is *pistis* plus *elenchus* ending in *aporia*; the *deuteros plous* is *dianoia* plus the initial stages of *noēsis*. Finally, as I have said, a higher hypothesis substantiates a correct hypothesis. In order to ascend, the initial hypothesis must be verified, and the higher one provides evidence for it.

I have disagreed with Byrd's claim that the *deuteros plous* is illustrated in the first part of the autobiography. In what follows, I will try to show that it is actually illustrated just after Byrd thinks, in the use of hypotheses in the argument about the soul. However, I think that Byrd is correct in drawing a parallel between the autobiography and the divided line. As I

have indicated, I do think that the autobiography parallels the divided line in taking us from *pistis* to *dianoia* and beyond.

Chapter Three: The Hypothetical Passage

Socrates has told us that his ‘second best’ approach starts off by appealing to theories, which, although it uses images, does not do so in a way more serious than the natural sciences. This method proceeds by laying down hypotheses, and the things which follow from them:

In every case, I first lay down the theory which I judge to be the least vulnerable; and then whatever seems to agree with it – with regard to reasons or anything else – I assume to be true, and whatever does not, I assume to be untrue (*Phaedo* 100a).

By way of illustration, Socrates applies this method to the theory of causation, in order to help Simmias understand how it might answer his objection. Call this Passage A:

[Stage One]

1. Grant the existence of the Forms: Beauty in Itself, Goodness, Largeness ‘and all the rest of them’ (*Phaedo* 100bc).
2. Whatever else is beautiful apart from Beauty itself is beautiful because it partakes of that Beauty, and for no other reason. Call this the ‘safe answer.’ (*Phaedo* 100 c).²⁸⁸
3. All other kinds of reason are to be disregarded, except those which appeal to the forms (*Phaedo* 100ce).

²⁸⁸ Vlastos ([1969] p.306) argues that ‘it is the logical function of the metaphysical entity that does the explanatory work of the ‘safe’ *aitia*.’ For Vlastos, Plato’s logical *aitia* is the same as a metaphysical one.

4. So if one man is taller than another, it is by virtue of tallness; and if one man is shorter than another, it is by virtue of shortness (*Phaedo* 101ab).
5. By the same reasoning, ten is not more than eight because of two; one added to one is not two in virtue of the addition (*Phaedo* 101bc).
6. There is no other way in which a given attribute can come to be except by sharing the essential nature of the thing it has a share of (*Phaedo* 101c).
7. So there is no other reason, for example, for something coming to be two than sharing in duality (*Phaedo* 101c).
8. If anyone should question the hypothesis itself, you would ignore him and refuse to answer until you have considered whether the consequences of the hypothesis are consistent or not (*Phaedo* 101d).

[Stage Two]

9. To substantiate the hypothesis itself, you would proceed in the same way, assuming whatever more basic hypothesis commended itself to you, until you reached one that is satisfactory (*Phaedo* 101de).
10. You would not mix the two things together by discussing both the starting points and its consequences (*Phaedo* 101e).

Stage one of this method is applied to the argument about the soul. Call this passage

B:

1. Begin by accepting that various Forms exist, and the reason that other things are called after the Forms is that they share in them (*Phaedo* 102ab).
2. In us, there is both tallness and shortness, but the tallness in us never admits shortness, just as the shortness in us declines to be tall (*Phaedo* 102a-103a).

3. The general principle is that an opposite can never be opposite to itself (*Phaedo* 103c).
4. Heat and cold are different from snow and fire, but snow can never admit heat and still remain snow, just as fire can never admit cold and remain fire (*Phaedo* 103cd).
5. So in cases like these, the name of the Form is not only applicable to the Form itself, but also to something else, which is not the Form but invariably possesses its distinguishing characteristic (*Phaedo* 103e).
6. So three must always be odd, even though three and odd are not the same thing, just as two and four are not identical with even, but are always even (*Phaedo* 103e-104a).
7. Just as opposites themselves do not admit one another, those things which always possess an opposite quality do not take on the opposite Form to that which is in them, but on its approach either cease to exist or retire before it (*Phaedo* 104bc).
8. Yet two and three are not opposites, so it is not only the opposite Forms that cannot face one another's approach, but also things which compel whatever they get a hold of to assume not only their own Form, but invariably also some other Form which is an opposite (*Phaedo* 104cd).
9. So when the Form of Three gets hold of a group of objects, it compels them to be odd as well as three (*Phaedo* 104d).
10. Into such a group the opposite Form to the character which has this idea can never enter (*Phaedo* 104d).
11. We can build sophisticated answers by going beyond the 'safe answer' of step 2, passage A, above: for instance by saying that fire must be present in a body to

make it hot, rather than giving the simplistic answer of heat; unity, rather than oddness, must be present in a number to make it odd (*Phaedo* 105bc).

12. Soul must be present in a body to make it alive (*Phaedo* 105cd).
13. When soul takes possession of a body, it always brings living with it (*Phaedo* 105d).
14. Dying is opposite to living (*Phaedo* 105d).
15. Soul will never admit the opposite to that which accompanies it (*Phaedo* 105d).
16. We call that which does not admit dying ‘undying’ (*Phaedo* 105de).
17. So the soul is undying (*Phaedo* 105e).
18. What is undying is imperishable, so it is impossible that it should cease to be at the approach of death (*Phaedo* 106ae.)
19. So when death approaches, the soul retires and escapes unharmed and indestructible; our souls will really exist in the next world (*Phaedo* 106e-107a).

The group agrees that they have no criticisms, and no doubt about the truth of Socrates’ argument. However, Simmias, although he agrees with Socrates’ argument, admits: ‘All the same, the subject is so vast, and I have such a poor opinion of our weak human nature, that I can’t help feeling some misgivings’ (107ab). The argument is coherent in itself and follows from the hypothesis, but in this passage, the application of the argument has not gone beyond Stage one of passage A: the original hypothesis has not been substantiated.

Stage two has possibly already occurred earlier in the dialogue, at *Phaedo* 74-76, in which Socrates argues for the existence of Forms. Call this passage C:

1. Admit that there is such a thing as absolute equality beyond that of stick to stick and stone to stone (*Phaedo* 74a).

2. We do not get knowledge of this by looking at sensible things (*Phaedo* 74b).
3. So equal things are not the same as equality (*Phaedo* 74c).
4. But equal things bring to your mind the thought of absolute equality (*Phaedo* 74c).
5. Then this must be a case of recollection (*Phaedo* 74cd).
6. Sensible things fall short of actual equality (*Phaedo* 74d).
7. So we must have some previous knowledge of equality (*Phaedo* 74e-75b).
8. We must have attained this knowledge before birth (*Phaedo* 75c).
9. So things-in-themselves exist in such a way that we can attain knowledge of them before birth (*Phaedo* 75c-76e).

Immediately after passage C, Simmias says that he is happy that the argument proves the existence of Forms. However, after the argument for the immortality of the soul (passage B) Socrates himself admits to the need to substantiate it. He says,

...even if you find our original assumptions convincing, they still need more accurate consideration. If you and your friends examine them closely enough, I believe that you will arrive at the truth of the matter, in so far as it is possible for the human mind to attain it; and if you are sure that you have done this, you will not need to inquire further (*Phaedo* 107b).

Socrates means that, even if the original hypothesis of the argument (the existence of Forms) is convincing, it still needs further work. In the context of the dialogue, *Phaedo* 74-76 could play that role. In this case, stage one of the *deuteros plous* of the *Phaedo* is illustrated at 100b-101e (passage B) and stage two is illustrated at 74-76 (passage C).

This is a plausible reading of the illustration of the *deuteros plous* in the dialogue. More importantly, I now want to look a little more closely at the exact procedure described in

passage A. Here, we have seen that Socrates' description gives us two stages, and some scholars give each of the stages two procedures, based on what emerges from this in conjunction with the divided line passage from the *Republic* and the hypothetical passage from the *Meno* (Benson [2010]). In the first stage, the philosopher identifies a hypothesis, from which an answer one seeks to know can be derived (1a), then shows how this can be taken to answer the question (1b). In the second stage, the philosopher seeks to confirm the truth of the hypothesis by identifying a further hypothesis from which the original hypothesis can be derived (2a), and testing the consequences of this hypothesis (2b). In this way, the method that Socrates is describing has a proof stage and a confirmation stage, in the same way that the method of analysis has as we described it in the previous section. Robinson describes these stages as the 'upward' and 'downward' paths respectively.²⁸⁹

Benson argues that the tradition that has emerged since Robinson, of describing the hypothetical method as consisting of both an upward path and a downward path is potentially equivocal (Benson [2010] pp.108-208). He says that both stages of the method could be described as consisting of an upward and a downward path. According to him, the upward paths of both stages consist in identifying relevant hypotheses (1a and 2a) from which either the original question is to be derived (1a) or the original hypothesis is to be derived (2a). The downward path, says Benson, consists of the proof from the original hypothesis or hypotheses (1b) or testing the consequences of the further hypothesis or hypotheses to see if they agree with each other.

Benson argues that the procedures of the proof and confirmation stages are not symmetrical: he says that, while the 'upward' paths of the two stages are merely different tokens of the same type, both consisting of identifying higher hypotheses, the 'downwards' paths are quite different: 'The downward path of the first stage amounts to providing or

²⁸⁹ See the *Republic* section.

displaying a proof of the answer to the original question (the conclusion or *teleuté*), while the downward path of the second stage amounts to a second confirmation of the procedure of the hypothesis' (Benson [2010] p 6).

Benson thinks that this failure to distinguish between these different downward paths undermines the comparison of the method of hypothesis with the method of analysis and synthesis. He says that the method of analysis and synthesis is primarily restricted to the first (proof) stage of the method of hypothesis, whereas there is nothing in the former to correspond to the second (confirmation) stage. In this case, thinks Benson, if we want to identify the method of analysis with the method of hypothesis, we need to either omit the second confirmation procedure or conflate it with synthesis. However, we cannot do this, Benson thinks, because it is explicit at *Phaedo* 101d, and exemplified at *Meno* 89c-96d and *Republic* 487a-502c.²⁹⁰

I have argued in the *Meno* section that the comparison with analysis and synthesis is unnecessary, since, as Karasmanis ([1987] pp. 73-93) points out, Plato's method is actually drawn from its forerunner, *apagōgē*. Analysis and synthesis, we said, moves from a hypothesis, which is the thing sought, using other known propositions, to a conclusion that is independently true of the thing sought. Synthesis starts from this conclusion and moves backwards to the thing sought:

Enunciation of the problem → Analysis: (1, the thing sought) implies (2) → (2) implies (3) → (3) implies (4)...M implies N (independently known) → Synthesis: N implies M...(4) implies (3) → (3) implies (2) → (2) implies (1).

²⁹⁰ Cf Mueller ([2005] pp.170-199), who argues along the same lines as Benson.

However, *apagōgē* reduces the problem to a series of lemmas, which, if known or constructed, will make the original problem or theorem evident, until we arrive at a conclusion that is independently known of the thing sought:

Enunciation of the problem → lemma1 → lemma2 → lemma3 → ... Conclusion

Following Karasmanis, I maintain that, as in the *Meno*, the hypothetical method of the *Phaedo* resembles *apagōgē*. We start from the problem, reduce it to another problem (the hypothesis) on which it depends. We deduce the original problem from this, so it is solved hypothetically (stage one of passage A). For a real solution, we need to solve the second problem by reducing it to a higher hypothesis (stage two of passage A). Continue this process until we arrive at an independently known conclusion. In this process, the ways up (heuristic) and down (deductive) are done at each step; separate them out, and we have the method of analysis and synthesis. In this case, the asymmetry that Benson worries about is not a problem for us: the *apagōgē* of the *deuteros plous* is not intended to cover the whole of dialectic; dialectic's 'way down' is a different process entirely.

I have already argued that we should take this reading into account: the method of analysis and synthesis may have been known to Plato, but it is *apagōgē* that he has in mind when he applies the mathematical method to philosophy. I go a step further than Karasmanis in saying that *apagōgē*, not analysis, is a part of the dialectic of the *Republic*. Karasmanis ([1987] pp. 251-268) thinks that dialectic taken as a whole is closer to analysis and synthesis, whereas I think that dialectic is *apagōgē* initially plus something else. The upward path, in the sense of a heuristic process, can still be seen as a part of this process, which is where the intuitive leap that I promised comes in. I am not the only one to see the search for a new

hypothesis as an intuitive leap.²⁹¹ My reading allows for the role of intuition that Robinson says is implied by the *Phaedrus* and *Seventh Letter*, but within the mechanism I am proposing for ascending the epistemological scale. I shall now expand on how that mechanism works, in the light of the hypothetical passage in the *Phaedo*.

I argue that the ‘higher hypothesis’ is the way to ascend the epistemological scale. In the *Republic* section, I described the ‘upward path,’ as it has become known, as a way out of *dianoia* and into *noēsis*. I argued that the *deuteros plous* of the *Phaedo* represents *dianoia* and the early stages of *noēsis*, with each ‘reduction’ taking us further up the scale. I agreed with Robinson’s linking of the upward path with the *Phaedo* in the *Republic* section, but here I want to highlight an important difference between my reading and Robinson’s.

Stage two of the *deuteros plous* requires us to posit a higher hypothesis ‘until you come to something adequate.’ Robinson (1953) thinks that this means ‘adequate to satisfy your objector.’ He thinks that we only need posit a higher hypothesis if someone questions the original. Stage one of the method is the internal, negative and *elenctic* process of checking for contradiction. Stage two, the positing of a higher hypothesis in the *Phaedo* is, according to Robinson, only done when someone objects to the original, and the metaphor of ‘above’ does not mean something more universal.

Robinson admits that his view sounds strange, but says that the only real test of the hypothesis is deducing the consequences, which is the same as the procedure in stage one: ‘If the hypothesis fails to imply the conclusion, or if it develops an internal contradiction, it will

²⁹¹ See also Karasmanis (1987 pp. 139-145). To clarify, Robinson does not think that the deductive part of the hypothetical method is intuitive (although he thinks that entailment method is fundamentally intuitive), but, like me, sees intuition as having a role to play in the choosing of hypotheses (1953 p 109). The difference between my view and Robinson’s is the extent to which we think intuition plays a part. I allow the image to have far less intuitive sway in *dianoia* than Robinson, and do not cite intuition as the reason for Plato’s extensive use of imagery in the dialogues. My position on the role of intuition in hypothesis is almost identical to that of Robinson: ‘What the method does is rather to *economize* intuition, by restricting it almost entirely to the intuition of a logical implication performed again and again, and to criticize and dispose of other intuitions by examining their mutual consistency. The unique beauty of Euclidean geometry down to the nineteenth century was that it seemed to combine in perfect harmony complete logical rigour and the complete satisfaction of all our intuitions about the subject-matter’ Robinson (1953) p 109.

have to be abandoned anyhow; and there would be no point in trying to defend it against outside objections by deducing it from a higher hypothesis. If it could be so deduced, the higher hypothesis would itself involve the contradiction involved by the lower' (Robinson [1953] p 141). My reading avoids the strangeness of Robinson's. By seeing the higher hypothesis as a reductive step in *apagōgē*, the higher hypothesis is given a positive role: it makes progress in the epistemological ascent.

Traditionally, epistemological ascent in Plato is seen as a feature of the 'upward' path. Annas distinguishes between the 'upwards' and 'downwards' paths by placing much greater importance on the 'upwards' path. She says that every step 'upwards' is the way to progressively greater understanding, treating the hypothesis of each science (especially mathematics) as literally hypothetical. It examines first the problem, then the concept, its status and position in the science, the science itself, and finally the whole of human knowledge and the nature of goodness. However, says Annas, this leaves little for the way down: there is nothing that a further downward path can add to our understanding.²⁹²

Benson thinks that the solution is to say that dianoetic confines itself to the proof stage, whereas dialectic embraces both the proof and the confirmation stages. He cites Plato's remarks that 'if one's starting point is something unknown, and one's conclusion and

²⁹² One solution to this problem would be to cast the role of the downward path as presenting and expounding what has become intelligible. Annas draws on Aristotle's *Posterior Analytics* for this explanation. However, Annas thinks that this solution would have to go with an Aristotelian conception of the relationship of philosophy with the special sciences. Annas says: 'For Aristotle, philosophy is continuous with science and like it in being a growing and continuous body of knowledge to which individuals make their contribution, relying on what has gone before...but Plato does not believe in such a fixity of philosophical achievement. It runs up against his repeated insistence that philosophical truth is something that each individual has to discover for himself or herself, and not something that can be handed on without difficulty' (Annas [1981] p 292). In this case, a true Platonist could not hold the 'downward' path to be a way of presenting and expounding the intelligible. If Plato is serious about the way down, Annas thinks, he has not thought his way clearly through the role of the downward path, nor how dialectic can be both eternal and personal. The practitioner of dialectic must actively seek the truth instead of relying on the opinions of even the wisest.

intermediate steps are made up of unknowns also, how can the resulting consistency ever by any manner of means become knowledge?’²⁹³

We should not lumber Plato with problems that arise from priorities that he does not have. As I pointed out in the *Republic* section, by far the most important distinguishing features of *dianoia* are the use of hypothesis and imagery, and that of *noēsis* is its procedure through Forms (by way of *ti estis*, according to my interpretation). The upwards and downwards paths are not what distinguishes *noēsis* from *dianoia*, although it may be a feature of the hypothetical method. The emphasis on the symmetry between the upwards and downwards paths belongs not to Plato, but to scholars writing after Robinson.

That said, it would be problematic to make the downward path redundant in Plato’s epistemology. The philosophical work we do after reaching the unhypothetical beginning should be richer, not poorer, and it is here that the downward path begins. Even if the two paths are not symmetrical, they should both be important. In my reading, the value of the way down lies in providing the kind of teleological accounts that Socrates is ideally looking for in the *Phaedo*. On my reading, the way down is able to provide this because of what happens on the way up.

I have said a lot about how we move from *dianoia* to *noēsis*. I have already said that this ascent should provide us with what we need for the *ti estis* we lacked in the first stages of *dianoia*. In the *Republic* section, I said that the *ti esti* is able to appear in *noēsis* because at least some degree of certainty is provided by the reduction of the initial problem to a hypothesis, and the subsequent downward step. Each reduction provides us with additional information about the Form that we are studying, which gives us the account we need for a *ti esti*.

²⁹³ *Republic* 533c. Cf Benson (2010) p 9 and Burnyeat (2000) p 23 n 3. As we observed in the *Republic* section, Benson proposes this solution as part of his wider theory about the distinction between *dianoia* and dialectic. He thinks that *dianoia* is the incomplete or erroneous application of the method of hypothesis, whereas dialectic is its correct application. The goal of dialectic is to provide hypotheses from which the answer to the original question can be derived, and which themselves are derivable from the unhypothetical first principle.

In order to show this, I need to qualify the connection I am making between hypothesis and definition. I mentioned in the *Republic* section, and argued in the *Meno* section, that, in a *ti esti*, Socrates is looking for something more than a concise statement about a thing. He is looking for more information; something like an account.²⁹⁴ I am saying that a hypothesis is a stand-in for a *ti esti*, but it is not simply a definition awaiting confirmation; it does not have to even be in the form of a definition. Rather, it is a proposition about the Form to be defined, and the use of it will give us the degree of certainty and account we need to generate the *ti esti*.

Robinson ([1953] pp. 100-105) against whose reading I have been arguing, also argues against the view that the hypothesis was always a definition, and the popular view that, according to Plato, the hypothesis is always a statement of existence. He provides some examples of statements that are not definitions:

There is a beautiful itself by itself and a good and great and all the others (*Phaedo* 100b).

Likeness exists (*Parmenides* 136b).

Likeness does not exist (*Parmenides* 136b).

Robinson goes on to give examples of those that are not existential:

Piety is what all the gods love (*Euthyphro* 9d).

Knowledge is perception (*Theaetetus* 165d).

Finally, Robinson gives examples of hypotheses that are neither definitions nor existential:

²⁹⁴ Cf Gallop ([1975] pp. 113-131) for the view that giving a definition requires giving an account.

The geometers hypothesis (*Meno* 87a).

Temperance is noble (*Charmides* 160d).

Virtue is good (*Meno* 87d).

Not-being must not partake either of one or of the many (*Sophist* 238e).

According to Robinson, a hypothesis is the sort of thesis which does not define a term but makes an assertion, but there is no special restriction on the kind of proposition that may be hypothesized. I agree with this point. As I have argued, what we are looking to do is to generate other true statements about the Form, which will eventually lead to a *ti esti*.

In passage B, the problem of whether the soul is immortal was reduced to the hypothesis that Forms exist, and in the subsequent steps, we generated a series of other statement about the soul: Soul must be present in a body to make it alive; when soul takes possession of a body, it always brings living with it; soul will never admit the opposite to that which accompanies it; the soul is undying; when death approaches, the soul retires and escapes unharmed and indestructible; our souls will really exist in the next world. The certainty we need in a *ti esti* is given to us by the process of *apagōgē*; the ‘account’ is provided by the statements it generates. What is the soul, on this account? The presence in a body that brings life, that retires when death approaches it - and so on. As I argued in the *Meno* section (Chapter One ii.a), there is not one ‘correct’ statement of a full *ti esti* of any given Form. The hypothetical argument has allowed us to generate at least one approximation, which, as we saw in the *Meno* section, can be improved and made fuller by further argument.

Because of the philosophical work that Plato is expecting a *ti esti* to do, we should expect the answer to the ‘what is x?’ question to have a number of different answers, with the

best approximation being the most complete. As the most direct way of studying the Form, the *ti esti* allows us to study its *eidos*, which will in turn allow us to build our teleological account, at which point we should have reached the unhypothetical first principle. The way down involves giving teleological accounts of what has previously been explained by the ‘safe’ answer, so Socrates would be able to explain that it was best for him to be in the prison cell in Athens, and not safe in some other place.

Conclusion

Throughout this thesis, I have suggested that Plato’s divided line should be read primarily as an epistemological scale, organised in an ascending order of progressively more epistemologically sound mental states. Not only is it a scale but it is also a continuum: so each subsection of the line representing a mental state is not homogeneous. The early stages of *dianoia*, for example, are not the same as the later stages and so on. The epistemological transition from one stage to the other is also continuous. I have shown that, when discussing *dianoia*, Plato distinguishes between the objects of investigation and the tools that the mind uses in this kind of inquiry (images and hypotheses). In fact, I have demonstrated that it is the tools of inquiry that define the mental states: *pistis*, for example, is drawing conclusions from observations of the physical world, but, crucially, this does not mean that its subject-matter need be limited to the physical world. It may draw on natural phenomena, or on observations of human actions to make claims about morality.

A particular problem that relates to *dianoia* is, what does Plato see as the connection between the tools of *dianoia*, hypothesis and imagery? The solution, I have proposed, is that both are proxies for *ti estis*, or Platonic definitions. My explanation is stronger than Robinson’s and Cross and Woosley’s view that the connection is that both hypotheses and

images appeal to intuition, because it takes into account the way in which Plato employs hypotheses and images in the dialogues. In this way, my account does not need to explain away (as Robinson does) the use of analogy and imagery by speculating that they are teaching devices.

I have shown that the dianoetic image is not the same as the imagery of *eikasia*, arguing against scholars like Klein, who I suggest rely too much on the idea of awareness to distinguish between mental states. The difference is that the image of *eikasia* is an image of a sensible object, whereas the image of *dianoia* is a sensible object used as an image. Moreover, in a similar way to the argument I made for the other mental states, dianoetic reasoning includes the mathematical method applied to philosophy in other spheres, so the objects of dianoetic thought would include numbers and shapes and, crucially, moral concepts like virtue and entities such as the soul, when that method is applied to philosophy. So in the *Republic*, we draw conclusions about the nature of the Form of Justice by comparing the relationships between parts of a just soul and a just state, so we use two instances of the Form of Justice, even though there can only be one Form. I have agreed with scholars like Fine here, who also thinks that different mental states can share objects, although, like many other scholars, I do not support Fine's 'contents' reading of the line.

I also demonstrated that, although the dianoetic image can sometimes have a sensible element, its qualities are primarily intelligible. Even a diagram has intelligible qualities, because the information it contains is not entirely sensible, but is partially contained in the mind of the dianoetician. When Socrates draws his figures on the ground in the *Meno*, he cannot represent perfect figures, and the square he draws on the diagonal will not be exactly double the area of the original square. The image includes the diagram plus information like this that the diagram does not contain. It is intelligible, in that its properties exist in their fullest sense in the mind of the thinker. The dianoetic image is more veridical than that of

eikasia because it is less removed from the Form. The information contained in the eikasiastic image is limited to the angle from which it was made. The dianoetic image, however, being intelligible, is heuristic: we can use information it contains to draw conclusions about the Form we study.

The hypothesis performs a similar role to the image in that Plato uses it in the absence of a *ti esti*. For example, in the *Meno*, Plato is explicit about the fact that he is adopting the hypothetical method because Socrates and Meno have failed to agree on a definition of virtue (*Meno* 86de). Although the hypothesis and image perform similar roles, there are important differences between the two. The main difference is that the dianoetic image is only used when engaging in *dianoia*, whereas the hypothesis appears when engaging in both *dianoia* and *noēsis*. A second difference is that a hypothesis is not an image of a Form. Rather, it is a provisional statement about a Form.

I also wanted in this thesis to suggest a solution to the problem of how Plato expects us to ascend the epistemological scale. I have shown that the hypothesis of *dianoia* has an important role to play here. Like Burnyeat, I think that mathematics has a vital role to play in the philosopher's approach to knowledge. However, I have shown that Burnyeat does not go far enough in the role that he ascribes to mathematics: it is not just the subject matter of mathematics, but the mathematical method itself that is so important in Plato's thought. I have also demonstrated that Benson's view, that the dianoetic method is the misapplication of the mathematical method, is wrong. This is because, at *Republic* 527b and 533d, Plato equates mathematics with *dianoia*, and *dianoia* is an indispensable part of the epistemological ascent.

Plato explicitly says that the mind progresses through the mental states (*Republic* 532a), and the implication is that we proceed through each of the states and build on them. On my reading, *dianoia* is an indispensable part of the epistemological ascent. The so-called

hypothetical method of the *Republic* extends from *dianoia* into *noēsis*, and the hypotheses we find in *dianoia* are the foundations for *noēsis*. When the mind ascends to *noēsis* from *dianoia*, it substantiates the hypothesis from which it has derived its conclusions by deriving this from a ‘higher’ hypothesis. I have also demonstrated that the method that appears in the *Phaedo*, the *deuteros plous*, is *dianoia* plus the early stages of *noēsis*.

I consider that one important contribution this thesis makes to our understanding of Plato is my work on the influence of the mathematical method on Plato’s epistemological ascent. Scholars like Mueller and Robinson have tended to downplay the extent to which Plato imported the mathematical method into his philosophy. These scholars tend to assume that the mathematical method Plato had in mind was analysis and synthesis. Building on the work of Karasmanis, I have shown that it was not; rather Plato modelled his ideas on the method of *apagōgē*, or reduction. The main difference between the two methods, we said, is that, in the latter, the ‘way up’ and the ‘way down’ are separated, whereas in the former, every step up is followed by a step downwards. The other difference is that, in *apagōgē*, the problem is reduced to a hypothesis, whereas in analysis, the thing sought is the hypothesis and starting point. Because *apagōgē* is a forerunner to analysis, the two methods do present some similarities. *Apagōgē* is a method that starts from an enunciation of the problem (eg, that of doubling the cube), then ‘reducing’ it to another that is easier to solve (eg, that of finding of two mean proportionals). This constitutes the principle for the solution of the original problem. This process can be repeated until we arrive at a conclusion that can be proved. My work builds upon that of Karasmanis, because I have been able to show that Plato wants us to use this method in the *dianoia/noēsis* segments of the divided line (not just in the hypothetical passages of the *Meno* and *Phaedo*).

I have shown that the tools of *noēsis* are *ti estis*, or Platonic definitions, of Forms, noting the early ‘Socratic’ dialogues’ emphasis on the need for a definition in order to pursue

inquiry, and the middle dialogues' claim that knowledge needs Forms. I said that the hypothesis and image allow us to study the Forms indirectly in *dianoia*, by acting as proxies for the *ti estis* that we do not have. The *ti estis* will either give us knowledge of the Forms by allowing us to be acquainted with the Forms, or by deducing true propositions about the Forms from the definition. In this case, *dianoia* and *noēsis* are distinct because of the different tools they use; *noēsis* is 'clearer' than *dianoia* because its tools enable us to study the Forms in a more direct way.

The most important contribution I consider this thesis to make is that my reading also gives us a way of ascending the scale of mental states. The tools of *dianoia* and *noēsis*, hypotheses, images and *ti estis*, are an important part of this ascent. A *ti esti* needs more than just the proposition about a Form provided by its proxy, the hypothesis. But indisputably, hypotheses are 'starting points and steps' to the unhypothesized first principle, so I proposed that the method of reduction provides us with the extra properties that a *ti esti* needs. It needs at least some degree of certainty, plus enough information to enable us to work out other things about the Form: it cannot be limited to the existential or propositional statement that we get from a hypothesis. The *ti esti* is able to appear in *noēsis* because at least some degree of certainty is provided by the reduction of the initial problem to a hypothesis, and the subsequent (usually deductive) downward step. Each reduction provides us with additional information about the Form that we are studying, guided by the *ti estis* themselves.

It is only after we arrive at the form of the Good that we begin the descent. One potential problem with the 'mathematical theory' of the dialectic ascent (that is, the theory that Plato modelled his method on a mathematical method) is that it is apparently inconsistent with the divided line passage, because the Good comes at the beginning, not at the end, of the downward path. On my reading, which does not try to make analysis and synthesis the format of the whole process, this is not a problem. The ascent moves from hypotheses to *ti*

estis to teleology and the Form of the Good. This enables us to give the kind of teleological accounts that Socrates wants in the *Phaedo* on the way down. This explains how Socrates' *deuteros plous* can be 'second best' without decreasing the likelihood of arriving at one's desired destination. It has the added bonus of avoiding Annas' problem that there is nothing left for the way down after the triumph of the ascent: on my reading, the way down builds on the way up to give a completely different kind of explanation, this time with reference to the Form of the Good.

Of course, as I have taken the *deuteros plous/dianoia* to *noēsis* transition to be *apagōgē*, the initial stages of the 'way up' actually begin with successive steps up and down, but the overall movement on this view would be upwards, and my reading fits in nicely with Plato's description of the *dianoia* to *noēsis* transition as 'steps and sallies' at *Republic* 511b. In the absence of the *ti estis* of *noēsis*, *dianoia* gives us a proxy via the hypothesis or image. It is in virtue of this that we make our ascent.

Throughout this dissertation, I have tried to provide an account of the contribution of mathematics to Plato's epistemology, based especially on the *Meno*, *Phaedo* and *Republic*. In the course of this I have explained my wider conception of Plato's epistemology, although due to the limitations of the scope of the project, there remains a great deal more to say. For example, I have indicated that I think the *ti esti* can lead us to the teleological account we need for the 'way down,' and I have indicated how and expansion of this topic might be approached. For example, Miller's ([2007] pp. 310-342) work on the idea of perfection in mathematics is a rich field, and is compatible with the reading I presented here. I think that my reading is compatible with a number of different readings, and there is much more I would have to say elsewhere on the topic.

The mathematical approach is indispensable for Plato on the ascent to teleology, and essential to his epistemology as a whole. I have supported Vlastos' work on the importance

of mathematics in Plato's philosophy, but I wanted to go beyond that by proposing a solution for exactly how Plato means us to use the mathematical method in epistemology. I have shown that the application of mathematical methods to philosophy is a part of Plato's epistemological ascent, and that we can see this approach in the dialogues. This, I have said, can explain the connection that Plato makes between hypothesis and imagery in the *Republic*. At certain points in inquiry, we lack the tools necessary for the highest form of reasoning. At this point, we use the hypothesis and image proxies. It is in this way that, for Plato, mathematics provides the stepping stone to the highest form of philosophical reasoning.

Appendix: Possible Future Research

I have focused heavily on the text of the dialogues in this dissertation, but a future avenue of research could develop Plato's theory of diagrams in the context of the history of mathematics.

i. Plato's theory of diagrams

As I have argued, dianoetic images do not exist in nature: they are made by the thinker. Plato says in the *Republic* that, in *dianoia*, the mind takes the sensibles as images of the Forms: without the mind doing this, they are not dianoetic images. The musical instrument and the cloak of the *Phaedo* are not dianoetic images until Simmias and Cebes use them as such. The diagram of the mathematician is a dianoetic image because it is taken to be so. We know that Plato thinks that the eikasiastic image is deceptive because it does not contain all the information of the original²⁹⁵; the dianoetic image is different, we said, because it is more closely underpinned by the thinker's relationship with the Forms. In the case of the dianoetic image, the properties of the image in are most fully instantiated in the mind of the thinker.

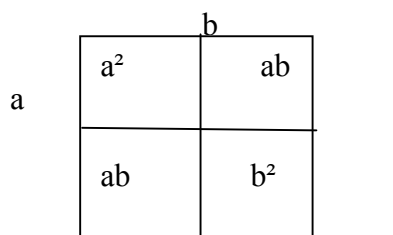
Think about Cebes's remark in the *Phaedo*, that a diagram can aid recollection:

²⁹⁵ See my comments on *Republic* 598ab and *Cratylus* 432b, in the *Republic* section.

One very good argument, said Cebes, is that when people are asked questions, if the question is put in the right way they can answer everything correctly, which they could not possible do unless they were in possession of knowledge and a correct explanation. Then again, if you confront people with a diagram, or anything like that, the way in which they react provides the clearest proof that the theory is correct (*Phaedo* 73ab).

Plato is not specific about what he means by ‘the way in which they react,’ but it is clear that the diagram should in some way jog our memories of the Forms, because Cebes’ comments are made in support of the Recollection argument. Gallop²⁹⁶ notes that the word translated as ‘diagrams’ could mean ‘proofs,’ citing *Cratylus* 436d2, the passage that we have already discussed, as evidence. Gallop thinks that ‘diagrams’ is a good fit, and that, in this case, ‘anything else of that sort’ would refer to solid models. However, he thinks that either word is consistent with the text.

This passage has been taken to mean that the diagram appeals to our intuition, which at first seems logical given that we are remembering something we once knew, so seeing something that resembles it should appeal to our intuition of it. This idea is supported by the way we think about diagrams in modern mathematics: because we are used to thinking about diagrams as helping us in certain ways, it seems natural to assume that Plato meant the same thing. Sometimes diagrams can help us to ‘see’ truths that we have difficulty in grasping. For example, we want to show that the sum of two squares cannot be factored using real numbers: $(a+b)(a+b) = a^2 + 2ab + b^2$. We can show this with a diagram:



²⁹⁶ ([1965] pp. 113-131).

To someone who knows little about mathematics, the diagram can show why it must be the case that $(a+b)(a+b) = a^2 + 2ab + b^2$; without it, it is a common mistake among those who are not used to thinking mathematically to wrongly assume that $(a+b)(a+b) = a^2 + b^2$. This is an example of a diagram usefully illustrating a mathematical truth. I am not denying that this is the case, and I do not think that Plato would either; I just mean that this is not what Plato has in mind when he talks about the mathematician's reliance on images in the divided line passage.

Taking this a step further, some philosophers of mathematics even think that a diagram can prove a theorem. Brown²⁹⁷ argues that, although it is generally thought that pictures prove nothing, in fact, they have a role in proofs that go beyond the heuristic. Pictures, according to Brown, are crucial: they provide the independently known consequences for testing the hypotheses of arithmetization.²⁹⁸ This is further than Plato would wish to go, but he would agree that we should take seriously their role in the mathematical method. Again, Brown's example is not what Plato has in mind in the *Republic*.

As we have seen, a common approach to images in mathematics is part of the view that diagrams allow us to 'see' truths by visualizing something that we have trouble grasping or proving using only words. However, I want to propose another solution, which I think is more in keeping with Plato's conception of recollection, intuition, and the history of Greek mathematics. I am going to argue that the image provides an intelligible instantiation of a Form, and the reason it helps us in recollection is not its appeal to the visual, but the fact that it is a particular instance of a Form.

Throughout this dissertation, I have argued that the role of intuition in the upward path is found in the heuristic grasp for a 'higher' hypothesis in *apagōgē*. I said that it is not

²⁹⁷ (2008) pp. 26-50.

²⁹⁸ For an example of how a picture can do this, see Brown's ([2008] Chapter Three, especially pp. 26-31) discussion of Bolzano's intermediate value theorem.

the visual appeal to intuition that makes the image convincing, as Robinson says, but rather, it is the intuition of the Form that allows the practitioner of *dianoia* to choose an appropriate image or hypothesis. Does this intuition of a Form allow the practitioner of *dianoia* to choose one instantiation of the Form for an image to be investigated? If we do not have a *ti esti*, a direct grasp on the Form itself, then the study of an intelligible instantiation provides the stepping stone; for the mathematician's diagram it is at least partly instantiated intelligibly, and Simmias' and Cebes' images wholly so.

The use of the diagram in the slave-boy experiment is essential for a geometrical solution. Some have argued that Plato indicates that this solution is inferior, but that is not the only possible explanation. I have already argued in the *Meno* section for the heuristic role of the diagram, and in the *Republic* section, I said that the diagram is not just a sensible object, but reaches into the intelligible. One possible avenue of future research could explore whether the diagram's *intuitive* worth lies not in its visual appeal, but in the fact that it provides a *particular* for study, when the Form itself is unattainable.

ii. Plato and the history of mathematics

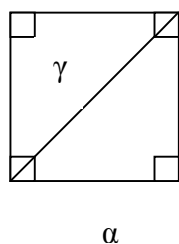
The problem of the slave-boy experiment is connected to the problem of incommensurables: the reason that the slave needs to point to the diagram to give the correct answer is that there is no common measure between the diagonal and its side. Usually, that is seen as a failing of geometry, and is sometimes cited as a reason why Plato might have held the discipline in lower esteem. In fact, it could be argued that Plato does not see geometry as having less value, and its use of diagrams are appropriate for the place that geometry, and *dianoia* in general, hold on the epistemological scale.

Rather than taking this as an isolated problem, we can regard the discovery of irrationals (arithmetic) and incommensurables (geometry) as a part of this 'fear of infinity.'

Aristotle hints at the proof when he speaks of the proof for the irrationality of $\sqrt{2}$ and the incommensurability²⁹⁹ of the side of a square with its diagonal:

...the diagonal is incommensurable because if it is put as commensurable, then odd numbers become equal to even ones. It *deduces* that odd numbers become equal to even ones, then, but it *proves* the diagonal to be incommensurable from an assumption since a falsehood results by means of its contradiction (*Prior Analytics* 41a26-32).

The discovery of irrationals and incommensurability go hand in hand, if we know Pythagoras' theorem:³⁰⁰



Use Pythagoras' theorem (on any right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides) to find $\alpha^2 + \beta^2 = \gamma^2$. If α and β each measure one unit, $\gamma^2 = 2$. So $\gamma = \sqrt{2}$. If $\sqrt{2}$ is irrational, the diagonal is incommensurable with the side.

We may prove the incommensurability of $\sqrt{2}$ with unity by the method that Aristotle speaks of in the above passage of *Prior Analytics*: proving the original when something impossible results from its contradiction. This is supported by the first Scholium on Book X of the *Elements*, which credits the Pythagoreans with the discovery of the irrational.³⁰¹ The Appendix to Book X sets a method for proving incommensurability of $\sqrt{2}$ with unity, but does not link this method with the Pythagoreans. It seeks to prove that AB, the diagonal of a

²⁹⁹ 'Those magnitudes are said to be commensurable which are measured by the same measure, and those incommensurable which cannot have any common measure.' Euclid, *Elements* Book X Definition 1.

³⁰⁰ We cannot explore the discovery of the Pythagoras' theorem here. It is likely that it was Euclid who refined the theorem, building upon earlier Pythagorean work. See Euclid, *Elements* I.47; Cf Proclus' *Summary* in Thomas (1939) p 185. See also Heath's translation of the *Elements* pp 352-356 for discussion. To make our point, we need not argue that the Pythagoreans give the proof as it appears in Euclid, only that they know of the rule.

³⁰¹ Euclid, *Elements* X, Scholium I in Thomas (1939) p 215.

square, is incommensurable with its side, AC. Therefore, we should investigate the result of the opposing hypothesis, that AB is commensurable with AC. In this case, we should be able to express their ratio in its lowest terms $\gamma:\alpha$. So $\gamma>\alpha$ and therefore $\gamma>1$. $AB^2:AC^2=\gamma^2:\alpha^2$. According to Euclid I.47, $AB^2=2AC^2$, so $\gamma^2=2\alpha^2$. Therefore, γ^2 is even, so γ is even. Since $\gamma:\alpha$ is in its lowest terms, α must be odd. For some number, β , $\gamma=2\beta$. Therefore, $4\beta^2=2\alpha^2$ or $\alpha^2=2\beta^2$. So α^2 and therefore α is even. But α was also odd, which is impossible.³⁰² We may identify this as the probable method of the Pythagoreans given the evidence of the *Prior Analytics*.³⁰³

The irony is that it is the Pythagorean interest in ‘principles from the beginning’ leads to the discovery of incommensurables. This crisis undermines the basic Pythagorean doctrine, ‘all is number’ because they wish to say that all things in the world can be expressed as integers, or as a ratio of integers, which is impossible with incommensurables. The one who made this known is said to have drowned at sea in a shipwreck, surrounding which there is great controversy.³⁰⁴

The problem of incommensurables can be linked to the idea of infinite divisibility. This causes serious asymmetry in the Quadrivium, because what can be said of multitudes cannot be said of magnitudes:

...for though the unit is a common measure of all numbers they [the Pythagoreans] could not find a common measure of all magnitudes. The reason is that all numbers, of whatsoever kind leave some least part which will not suffer further division; but all magnitudes are divisible

³⁰² Heath’s translation of Euclid’s *Elements* Vol. 3 p 2.

³⁰³ von Fritz ([1945] pp. 242-264) thinks that the discovery of incommensurables was probably made by Hippasus in the last quarter of the fifth century. Wasserstein ([1958] pp.165-179) thinks that Fritz has confused the story of Hippasus’ drowning at sea as a punishment for divulging the Pythagorean secret of how to inscribe a dodecahedron in a sphere (Iamblichus, *Life of Pythagoras* XVIII) with the legend we mentioned about the divulger of incommensurables suffering the same fate. Fritz is not confused; he uses Hippasus’ interest in the sphere of twelve pentagons to devise an alternative way of discovering incommensurables. However, there is no textual evidence to support this, so our account of the discovery being made by the use of opposing claims, being based upon textual evidence, is the most likely.

³⁰⁴ Euclid, *Elements* X, Scholium I in Thomas (1939) p 217.

ad infinitum and do not leave some part which will not admit of further division, but that the remainder can be divided *ad infinitum*; and in sum, magnitude partakes in division of the principle of the infinite, but in its entirety of the principle of the finite, while number in division partakes finite, but in its entirety of the infinite...³⁰⁵

The severity of the discovery of incommensurables has been argued to have affected the prestige of geometry in the long term. Heath ([1960] Chapter 3 pp. 65-117) says that the Pythagoreans allocate the discovery to the realm of geometry, citing the fact that Euclid X speaks in terms of straight lines and areas, and that Proclus speaks of irrational straight lines. Certainly the discovery of incommensurables took some time to overcome, and were a difficulty even by Euclid's time: the *Elements* postpones the theory of proportion, which avoids the problem of incommensurables, until Book V, and uses the gnomon to solve problems for which a modern geometer would use similitude.³⁰⁶

However, Stillwell ([1989] pp. 37-47) has shown that this reaction was part of a general rejection of infinite processes, and has nothing to do with the perceived inferiority of geometry to arithmetic. Stillwell points to the paradoxes of Zeno as the beginning of this trend, which led to the avoidance of completed infinities and limits in Greek mathematical proofs. Eudoxus' theory of proportions was designed to enable lengths and other geometric quantities to be treated precisely as numbers, while admitting only the use of rational numbers. So $\lambda = \lambda_a$ if any rational length $< \lambda$ is also $< \lambda_a$ and vice versa. Likewise, $\lambda < \lambda_a$ if there is a rational length $> \lambda$ but $< \lambda_a$. According to this theory, the infinite set of rational

³⁰⁵ Euclid, *Elements* X, Scholium I in Thomas (1939) p 215-217.

³⁰⁶ *Elements* V; Cf Coolidge (1963) Chapter II §3 pp. 34-38, who has a good discussion of the application of areas, which handled theorems proved by proportion in the meantime. See *Elements* II.5 for the first application of the gnomon; Pythagorean use of the gnomon, which we cannot discuss here, is testimony to the wish to express the world in terms of integers, which explains their delight that a monad added to a gnomon produces a square number.

lengths $< \lambda$ is present in spirit, but Eudoxus avoids using it explicitly by speaking of an arbitrary rational length $< \lambda$.³⁰⁷

This theory was successful in avoiding an arithmetic approach to irrational numbers, and Stillwell has a theory about why the geometrical approach was more intuitive. He writes:

The theory of proportions was so successful that it delayed the development of a theory of real numbers for 2000 years. This was ironic, because the theory of proportions can be used to define irrational numbers just as well as lengths. It was understandable, though, because common irrational lengths, such as the diagonal of the unit square, arise from constructions that are intuitively clear and finite from a geometric point of view. Any *arithmetic* approach to $\sqrt{2}$, whether by sequences, decimals or continued fractions, is infinite and therefore less intuitive (Stillwell [1989] p 39).

Stillwell goes on to say that this intuitive superiority of geometry seemed a good reason for considering geometry a better foundation for mathematics than arithmetic. Eudoxus' method of exhaustion is a generalization of the theory of proportions: known figures are used to determine unknown quantities by approximation. For example, an approximation of the circle is determined by inner and outer polygons. Stillwell argues that this is another way of avoiding completed infinities: 'Notice that 'exhaustion' does not mean using an infinite sequence of steps...rather, one shows that any disproportionality can be refuted in a finite number of steps (by going to a suitable P). This is typical of the way in which exhaustion arguments avoid mention of limits and infinity' (Stillwell [1989] p 42).

On Stillwell's reading, geometry's ability to avoid the infinite and provide particular, finite instances of a problem gives the subject its intuitive worth. By extension, we can say

³⁰⁷ There is a consensus among historians of mathematics that Eudoxus' theory was a reaction to the discovery of incommensurables. See van der Waerden (1983) pp. 88-91 and Heath (1960) pp. 325-329. See Szabó (1978) for the view that it was not Eudoxus' definition that made the construction of a mean proportional possible; in fact this must have been known at least as early as Hippocrates.

that the diagram in geometry, or image in general dianoetic thought, is a part of the same process. In the absence of a *ti esti* grasp on the Form, the image provides us with a particular.

Plato was certainly aware of the discovery of incommensurables and its implications. In the slave-boy experiment, the problem relies on drawing the diagonal of a square that has sides of two feet long. Socrates is asking for the length of a square that is double the area of the initial one. He says, ‘If you don’t want to count it up, just show us on the diagram’ (*Meno* 83e-84a). As the side and the diagonal are incommensurable, the boy would have been unable to give the length of the diagonal in integers, so pointing it out would have been the only sensible option. The image, in this case, enables the boy to give a finite answer, something that would have been impossible without it.

Some have argued that Plato’s language itself suggests a criticism of geometry for being subject to this problem.³⁰⁸ Another avenue of possible future research would be to the passage in the context of the history of mathematics and Plato’s scheme in the divided line and remarks about the *deuteros plous*.

³⁰⁸ See, for example, Malcolm Brown (1971) pp. 198-242. Brown relies on the fact that when ἀλλά is used in a conditional sentence to introduce the apodosis, where a command is expressed and the protasis is negative, the substitute is inferior: ‘if you don’t want to count it up (arithmetic), just show us on the diagram (geometry)’ *Meno* 24a. According to Brown, the geometric alternative is inferior to the arithmetic.

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