Cooperative Diversity in Wireless Networks: Frameworks and Analysis

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Abstract

Mobile communications systems have proven to be very popular in the consumer market, allowing ubiquitous communications and data transfer. New technologies and techniques are being developed for deployment in future generation cellular networks to provide new and higher quality services. Of particular importance in future generation networks is actively combating fast-fading which causes random fluctuations in the power of a received signal over short times and distances. This leads to a property know as spatial diversity where independent fading signals can be received by antenna elements separated by a small distance. Combining of the received signals can lead to an increase in the information data rate and reliability of communications over the wireless channel.

Spatial diversity can be achieved in a cellular network by sharing information between Mobile Terminals (MTs) where one MT acts as a relay supporting the data transmitted from the source by forwarding information to the destination. Due to the cooperation between MTs this is termed cooperative diversity. Initially this thesis considers the effect of cooperative diversity in an environment where MTs are equipped with two antenna elements, effectively combining the fast-fading combating techniques of cooperative diversity and multiple-antennas.

Cooperative diversity transmission can be performed by a number of different protocols, which are termed Protocols I-V. Imposing system constraints on the network in order to make a fair comparison between the protocols, and in-particular the traditional single-hop channel, allows the benefits of cooperative diversity to fully be established. An information theory approach is taken using multiple antenna techniques to develop a framework for cooperative diversity. It is shown that cooperative diversity can offer significant improvements in terms of probability of outage and capacity. In-particular, an adaptive cooperative diversity protocol is developed to select the optimal protocol dependant on channel conditions which shows a 4.25dB increase in capacity, at the 5% outage level, for a single user.

Declaration of originality

I hereby declare that the research recorded in this thesis and the thesis itself was composed and originated entirely by myself in the Institute of Digital Communications at The University of Edinburgh.

Allan Jardine

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Acronyms and abbreviations

1G	First Generation cellular network
2G	Second Generation cellular network
3G	Third Generation cellular network
4G	Fourth Generation cellular network
AF	Amplify-and-Forward
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
CP	Cyclic Prefix
COST	Cooperation in the field Of Scientific and Technical research (Europe)
DF	Decode-and-Forward
DS	Direct Sequence
DSP	Digital Signal Processing
EP	Equal Power
FDD	Frequency Division Duplexing
FDMA	Frequency Division Multiple Access
FH	Frequency Hopping
FIR	Finite Impulse Response
GSM	Global System for Mobile communications
HF	High Frequency
HSDPA	High-Speed Downlink Packet Access
ICI	Inter Channel Interference

IEEE	Institute of Electrical and Electronics Engineers
IMT	International Mobile Telecommunications
IMTS	Improved Mobile Telephone Service
IP	Internet Protocol
ISI	Inter Symbol Interference
LEO	Low Earth Orbit
LHS	Left Hand Side
LOS	Light Of Sight
MA	Multiple Access
MAC	Medium Access Control
MIMO	Multiple In Multiple Out
MISO	Multiple In Single Out
MRDF	Minimum Rate Decode-and-Forward
MT	Mobile Terminal
MTS	Mobile Telephone Service
OFDM	Orthogonal Frequency Division Multiplexing
PDF	Probability Density Function
PR	Pseudo Random
PSTN	Public Switched Telephone Network
RA	Typical Rural (COST 207)
RHS	Right Hand Side
RRDF	Required Rate Decode-and-Forward
SDMA	Space Division Multiple Access
SIMO	Single In Multiple Out

SISO	Single In Single Out
SM	Spatial Multiplexing
SNR	Signal to Noise Ratio
STBC	Space-Time Block Code
STC	Space-Time Code
STTD	Space-Time Transmit Diversity
TC	Turbo Code
TCP	Transmission Control Protocol
TDD	Time Division Duplexing
TDMA	Time Division Multiple Access
TH	Time Hopping
TU	Typical Urban (COST 207)
V-BLAST	Vertical - Bell Labs Layered Space-Time
VAA	Virtual Antenna Array
WRC	World Radio Congress

List of principal symbols

$X \to Y$	Transmission between source X and destination Y
$\partial(\mathcal{J})$	Set of terminals taking part in the second phase
$\lvert\mathcal{X}\rvert$	Cardinality of the set $\mathcal{X}$
A_{p_m}	Source transmit power level in the first phase for protocol p in mode m
b	Collision control parameter in the second phase
B_{p_m}	Source transmit power level in the second phase for protocol p in mode m
C_{j,p_m}	Relay j transmit power level in the second phase for protocol p in mode m
$\mathcal{CN}(0, \sigma^2)$	Set of circularly symmetric complex Gaussian random variables, variance σ^2
D	Destination terminal
$\det(\mathbf{x})$	Determinant of matrix $\mathbf{x}$
E_{XY}	Average signal energy received at the destination terminal, Y, from source, X
e^x	Exponential of variable x
h_{XY}	Channel coefficient between terminals X and Y
$h_{XY,l}$	Channel coefficient between terminals X and Y for tap index l
$\mathbf{H}_p$	Channel coefficient matrix for protocol p
$\mathbf{H}_{p,j}$	Decomposed channel coefficient matrix for protocol p and relay j
$\mathbf{I}_x$	Identity matrix of size x
I_p^m	Mutual information for protocol p and mode m (MIMO or MISO)
$\mathcal{J}$	Set of potential relay terminals
j	Relay terminal index
k	OFDM tone index

L	Number of channel taps
L_{CP}	Length of OFDM tone's cyclic prefix
l	Tap index
N	Number of OFDM tones
N_0	Variance of additive Gaussian noise
n	Additive noise received at destination
$\mathbf{n}$	Additive noise vector received at destination
n_{Y_i}	Additive noise received at destination, Y, in phase i
P	Number of relaying protocols
p	Protocol index
$\mathrm{Pr}[x]$	Probability of occurrence of variable x
$\|\|\mathbf{i}\|\|$	Frobenius Norm of array i
R	Relay terminal
R_j	Relay terminal j
R	Spectral efficiency rate
R_i	Spectral efficiency rate in phase i
S	Source terminal
T	Transmit time period
SNR	Recorded Signal to Noise Ratio
x	Value of interest when considering a PDF or CDF
$\mathbf{x}$	Data vector transmitted over two phases
x_1	Data signal transmitted in first phase
x_2	Data signal transmitted in second phase
y_{Y_i}	Signal received at terminal Y in phase i

$y_{Y_{j,i}}$	Signal received at terminal Y in phase i decomposed for relay j
$[x, y)$	Uniform random variable in the range x to y, inclusive of x, but not of y

Chapter 1
Introduction

1.1 Introduction

The twentieth century has been one of unparalleled technological achievement and this is continuing unabated at the start of the twenty-first century. Technology is a fundamental building block to a modern sustainable society and is heavily relied upon for day-to-day life. This concept is exemplified by the role of telecommunications in contemporary modern society, which are almost ubiquitous in their deployment. At any location around the world a person can now expect high quality personal communications, leading to a society which is heavily dependant on telecommunications and the wide range of services it can provide. Increased user mobility has necessitated the development of radio telecommunications that can provide access to these services for those on the move. As consumers demand new and higher quality services, technology has had to progress and new avenues of research have opened.

The transfer of information by radio originated from the theoretical work carried out by Maxwell in 1864 [1, p.1], three decades after Morse developed the telegraph. The equations developed by Maxwell were confirmed experimentally by Hertz in 1888 [2, p.8], whose work in combination with many other researchers in this area inspired Marconi to develop the first commercial radio system, including life saving ship-to-shore wireless communications equipment. Development of wireless communications gathered pace in the early 1900s once it was realised that there was a commercial aspect to the technology, and was further helped by DeForest's invention of the triode vacuum tube which offered significant amplification of an electrical signal. High Frequency (HF) communications became the standard signalling frequency range due to the relative accessibility to the technology available and the discovery of ionospheric propagation by Kennelly and Heaviside allowing long distance communications.

The first land based mobile telecommunications service was installed and operated by the Detroit Police department in 1921 [2, p.25-26]. The police commissioner at the time believed that automobiles had given criminals an advantage in speed and that being able to deploy police cars using radio, which operated in the 2MHz band, countered this advantage. Several obstacles were encountered during this early deployment, including the sensitivity of the vacuum tubes and the operational battery life. Broadcast radio began to grow in popularity and regular programming became commonplace in the 1930's. However, development of commercial radio stalled during World War II, during which the military advantage of radio systems was aggressively pursued by research and development teams.

Technological advances made during World War II allowed higher frequencies to be exploited for radio communications, leading to the introduction of the Improved Mobile Telephone Service (IMTS) by Bell Laboratories in 1969. This system, operating initially in the 150MHz band with 11 channels, each with 30kHz bandwidth, offered considerable improvements over the original Mobile Telephone Service (MTS) by providing full duplexing support, and full dial access to and from the mobile station [3, p.3].

As the demand for wireless communications increased in the 1970's the requirements for more channels to be made available grew. In response to the fact that the electromagnetic spectrum is a finite resource, network designers have sought wireless deployment methods which utilise the fixed frequency allocations made by the radio spectrum licensing bodies in individual countries as efficiently as possible. First generation (1G) cellular networks were developed in response to this concern and were deployed in the early 1980's, where frequency reuse is employed to achieve much greater spectral efficiency[1] [4, p.12]. 1G cellular networks typically operated in the 800MHz-900MHz band, allowing a considerably higher number of channels to be made available, while still allowing 25kHz-30kHz of bandwidth per-channel, and were based on analogue techniques. There was little co-ordination between the development of 1G networks, with several companies promoting their own implementation, making roaming between networks deployed in different countries a practical impossibility.

The first generation of cellular networks quickly became very popular and demand for greater user capacity was again met by the wireless research and development community. The Groupe Spécial Mobile (GSM)[2] was formed to address the problems with 1G networks, and develop a specification for second generation (2G) cellular networks [5]. After several years of discussion and technical demonstrations a digital system was decided upon and the first signing of the GSM Memorandum of Understanding (MoU) was made in 1987 [6]. This was a key moment as GSM sought to create a pan-European cellular standard. Using digital techniques GSM

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[1] Cellular networks are discussed in more detail in chapter 2.

[2] Later known as the Global System for Mobile communications

ensures that the frequency spectrum available was employed more efficiently than that used by 1G networks, in-particular cell sizes were much smaller allowing for greater frequency reuse and the communications data could be compressed using digital techniques. Although other 2G networks were deployed (particularly in North America) GSM quickly became a global standard due to its popularity in Europe and its scalability. The success of mobile communications and GSM is illustrated by the fact that as of June 2006 worldwide GSM subscriptions stood at two thousand million subscribers [7].

Although 2G networks are well equipped to cope with the subscriber number demands placed on the system due to their scalability, consumer demand for higher quality voice communication, faster data transfers and new multimedia services has spurred development of third generation (3G) cellular networks. Based upon modern Digital Signal Processing (DSP) and manufacturing techniques 3G developers attempted to recreate the success of the GSM global standard through the creation of the International Mobile Telecommunications (IMT) family of standards [8, 9]. While generally successful, IMT describes a wide-range of standards as patent holders pushed for their techniques and technologies to be adopted. 3G networks typically operate in the 2GHz band and require a licence to operate. The initial expense of this licence was illustrated in the UK where the net revenue for the five available licences was £22 thousand million [10]. 3G network cell sizes are reduced, compared to their 2G counterparts, due to the increased free space loss at higher frequencies as the transmit power of the base stations is limited, leading to a large required deployment of 3G base stations to meet the required coverage standards expected by consumers. This has lead network operators to introduce 2G fall-back from 3G 'islands' where a 3G equipped mobile unit will use a 2G network if 3G coverage is not available.

3G networks introduced new services such as video conferencing and internet connectivity due to the higher data transfer rates available, however consumer demand has continued to grow for high data rate connectivity. The introduction of the IEEE 802.11 wireless specification as a short range communications protocol in 1997 and its rapid development thereafter has spurred this demand. This has lead research and development teams to consider future generation cellular networks, often called fourth generation (4G) cellular networks [11–13] or 'Beyond 3G'.

To provide higher data rates to mobile devices future generation networks will see a step up in standard carrier frequency, as observed in previous generation networks. It is typically expected that the unlicensed 5GHz band will be used, although use of the unlicensed 60GHz band has also been proposed. This step up in frequency will reduce the cell size further from the 3G cells due to propagation laws of radio waves, and this problem will be particularly acute at 60GHz. Therefore, 60GHz is not likely to be supported by the network operators due to the increased number of base stations that would be required for such a deployment. Global standardisation of the frequency bands used is an important part of the development of future generation networks in order to ensure roaming is possible and to reduce handset manufacturing costs. A meeting of the World Radio Congress (WRC) in 2007 will attempt to address the issue of spectrum allocation for future generation cellular networks [14].

Data transfer rates of up to 100Mbps for moving terminals and 1Gbps for low mobility terminals have been suggested for future generation networks [15]. Although these targets may be met by researchers they are not likely to translate directly into real world application, as observed with the 2Mbps promise of 3G networks [16, p.11]. However, increased data transfer rates are a primary goal of future generation networks and significant research is currently being carried out in this area.

As the cell size is expected to decrease with the move to a higher frequency band in future generation networks, the cost of deploying the network infrastructure becomes a significant concern, particularly in areas of low population density. To address this problem two different network architectures have been proposed, both of which are significantly different from current generation networks:

- Relaying
- High Altitude Platforms (HAPs)

In a relaying network, multi-hop communications links can occur between a base station and mobile devices, thereby increasing the coverage area of a single cell [17–19]. In such a network ad-hoc communications without the aid of a base station between terminals becomes a possibility, possibly reducing the demands on the base station and also allowing intra-cell communications between devices which are geographically close although not in the same cell. This is termed fuzzy edge cells. Such a relaying architecture is shown in Figure 1.1(a).

An alternative architecture to relaying is that of employing HAPs to create very large cells to give complete coverage over the area of interest, and deploying hot-spot islands with a more

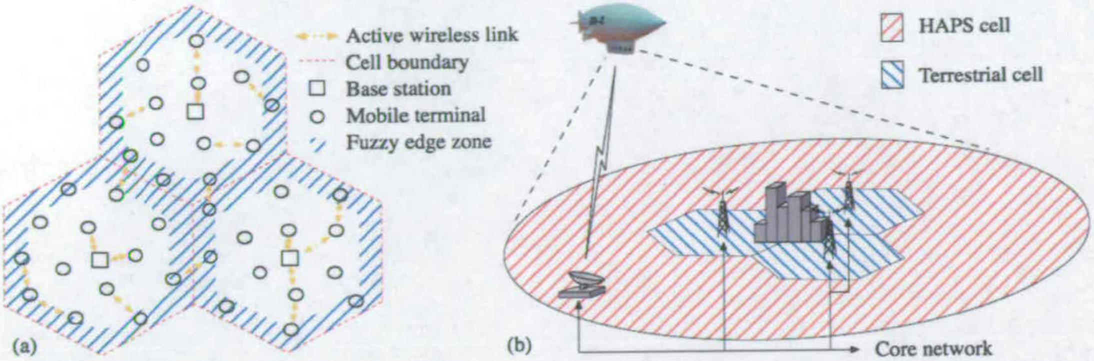


Figure 1.1: *Two possible architectures of future generation cellular networks: (a) Relaying based architecture to increase coverage area, (b) HAPs based network where wide area coverage is provided by a high altitude platform.*

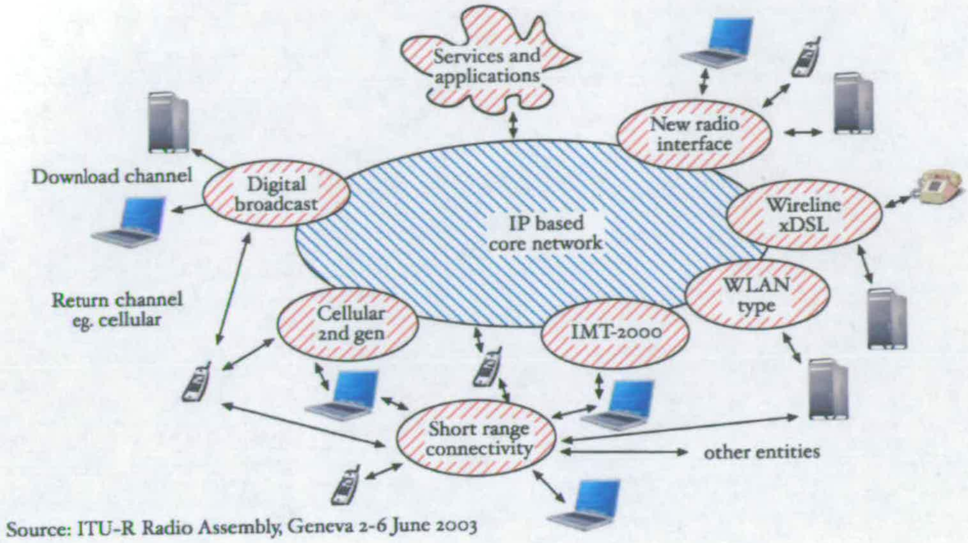
traditional cellular architecture in areas of high population density [20–22]. In such a system an unmanned platform would fly at altitudes of 17km-22km, which reduces the latency problems that would be encountered in trying to employ Low Earth Orbit (LEO) satellite communications systems as a solution. A broadband link such as a microwave communications channel would provide the HAP with a connection to the core network, Figure 1.1(b).

Both of the considered architectures are likely to use an Internet Protocol (IP) core to provide the backbone infrastructure for the network [23]. This is highly advantageous to the deployment of future generation networks as fast and reliable IP networks are already in place in many countries and it can be used as a gateway protocol to many other services and networks. This is illustrated in Figure 1.2.

1.2 Thesis structure

This chapter is a brief introduction to the background of wireless communications, highlighting the path to future generation cellular networks. The demands that are being placed on next generation networks were explored and network architectures which are currently in development, were introduced.

Chapter 2 continues the introduction into wireless communication networks by initially exploring the fundamental properties of the wireless channel. In-particular, the issues faced by



Source: ITU-R Radio Assembly, Geneva 2-6 June 2003

Figure 1.2: *IP core at the centre of future generations of mobile communications.*

designers of wireless networks, such as fast-fading, are characterised. Several of the available methods that can be employed to provide multi-user duplex access are then introduced. Following this, methods to combat fast-fading using the unique properties of the wireless channel are presented, including multiple antenna techniques. These techniques are then expanded to consider the situation where the number of antennas in a mobile device are limited and relaying is explored to combat fast-fading in such a situation. Specifically, cooperative diversity is considered. Orthogonal Frequency Division Multiplexing (OFDM) techniques are introduced as an alternative technique, which can also be employed to combat fast-fading. Finally, the aims of this thesis are presented against the background of research currently available.

Chapter 3 initially considers the Shannon capacity benefits of utilising cooperative diversity in a network, expanding on the body of work currently available. Cooperative diversity techniques in the situation where mobile terminals in an ad-hoc network are each equipped with two antenna elements are then explored. Constraints are placed on the network model such that the time, power and bandwidth of the transmitted signal are limited to be the same as the traditional single-hop transmission. This is done in order to make a fair and direct comparison between the techniques used in current generation networks and cooperative diversity networks. Several multiple antenna techniques are utilised to explore completely the benefits which can be obtained.

Chapter 4 is dedicated to developing an information theory framework for the situation where mobile terminals are equipped with only one antenna element and introduces the complete family of cooperative protocols, extending the system constraints in the previous chapter to this framework. Multiple antenna element techniques are employed to explore the different relaying methods available and develop an information theory approach to characterise this network. Of particular importance in this chapter is the power constraint on the system. An analytical approach is taken to optimise the transmit power levels for relaying protocols where this constraint is a strong limitation. Finally, a performance comparison between the relaying protocols is made.

Chapter 5 considers two methods to optimise further the performance of the cooperative diversity protocols. Initially, the effect of using cooperative diversity over OFDM tones and, in particular, the effect of collision in the second phase is explored. This is considered in a frequency-selective fading environment and the performance of the cooperative diversity protocols in such an environment are considered. Finally, an adaptive protocol is considered where the optimal protocol, dependant on channel conditions, is selected for transmission.

The objective of the final chapter is to summarise the work presented in this thesis and highlight its limitations. Future work based upon the information theory framework presented and the implications of the results obtained on the development of relaying techniques for future generation networks are also discussed.

1.3 Contributions of this thesis

The benefits of relaying for future generation cellular networks have not yet been fully characterised. Developing a framework which enabled such an assessment to be carried out is the primary goal of this thesis, under conditions which make a fair and direct comparison with the traditional single-hop channel. The system constraints which enable this comparison to be made are central to this thesis and are fully introduced in the following chapter.

In the following chapter the possibility of using a relay network where each terminal is equipped with multiple antenna elements is considered. Such a system offers the joint benefits of spatial diversity from multiple antenna elements, but also from the cooperative diversity protocol.

A single information theory framework is very important to be able to fully characterise the relay channel and the cooperative diversity protocols which it makes possible. This is one of the primary contributions of this thesis and is presented in chapter 4. Multi-carrier techniques are considered in a cooperative diversity context, in the first half of chapter 5, which offers the benefits from both methods and this is optimised by using the framework previously developed. Finally, an adaptive protocol is developed and presented, utilising the optimal transmission protocol based on instantaneous channel conditions.

Chapter 2
Spatial diversity for ad-hoc wireless networks

2.1 Introduction

In the previous chapter the need to increase the reliability of communications and the data capacity of wireless networks, particularly in cellular systems, was identified. In response to this need, it is proposed to

- Develop techniques to combat the challenges presented by the wireless channel to data communications (which is the main goal of this thesis), and subsequently
- Optimise the developed techniques to maximise the throughput achievable in a practical implementation of the proposed techniques.

The feasibility of this approach can be assessed by developing an information theory framework for the proposed techniques, which make use of one of the traditional pitfalls in wireless communications, fast-fading [24,25]. Results from this framework are then compared and contrasted with traditional wireless communications techniques, allowing conclusions to be drawn on the effectiveness of the proposed framework.

This chapter initiates the development of the proposed techniques by considering the fundamental properties of the wireless communications channel. In particular, the traditional cellular network is described with a view as to how it could be modified for future generation networks. The causes of signal degradation in a wireless network are explored and presented with a mathematical description.

Following this, the next section examines the relatively recent field of multiple antenna element transmission and reception systems which can offer considerable increases in the performance of a wireless communications channel [26,27]. This method has received a lot of attention recently as the benefits it offers come only at the cost of complexity, rather than additional power, time or bandwidth. These benefits come from a property which is unique to wireless channels called spatial diversity [9, p.174] which is fully explored in that section. Spatial diversity is the corner stone of the work presented in this thesis, as its properties can be used to combat problems that fast-fading causes for wireless communications.

Spatial diversity techniques are then developed for application in a wireless network where the number of antenna elements at a transmitter or receiver may be limited. This is done by introducing a paradigm shift for mobile networks, away from communication data generated by Mobile Terminals (MTs) being shared only with the network infra-structure (as is the case in current generation networks), to sharing information directly between MTs. This leads to the introduction of ad-hoc wireless networks, and in particular cooperative diversity [28–30].

This chapter is therefore structured as follows. Section 2.2 introduces the wireless communications channel both for the cellular environment and more generally for an ad-hoc communications situation. Spatial diversity and multiple antenna techniques are then considered in section 2.3 which is then expanded in section 2.4 to consider cooperative diversity techniques. Section 2.5 considers multi-carrier transmission as a different method to increase the data capacity of the wireless channel and section 2.6 discusses the contributions of this thesis. Finally, conclusions are drawn in section 2.7.

2.2 The wireless communications environment

The wireless channel is a much more hostile environment to communications than the wireline channel. Factors which are beyond the control of the network designers affect the reliability of the signal at the intended destination, including reflections from possibly moving objects and large obstructions such as buildings. In this section the wireless environment is explored through current generation wireless systems and fundamental properties of the wireless channel. The groundwork is subsequently laid for the consideration later in this thesis of future generation networks.

2.2.1 The cellular environment

Before considering a more analytical overview of the wireless channel, it is useful to have a clear view of how such a system might be used and deployed. As noted in the previous

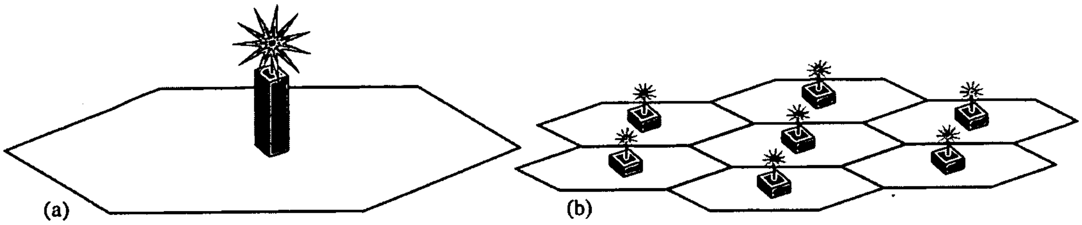


Figure 2.1: *Diagrammatic representation of wireless communication architectures. (a) Broadcast architecture with a single high power transmitter and a wide coverage area, (b) Cellular architecture with several low power transmitters each covering a small area.*

chapter, wireless communications have become ubiquitous over the last decade and reliable digital communication is expected to be available at all times by the general population. Traditionally, wireless systems were used for one way communications, such as television or radio. Since these networks transmit data common to all nodes in the network in a simplex manner from only one point with a very large coverage area (usually to the horizon) they are termed broadcast systems [5], Figure 2.1(a). Beyond the horizon another broadcast transmitter will provide coverage, using a different frequency to ensure that the is little or no interference between transmitters. This design requires the use of a high-power transmitter in a prominent geographical location (such as a hill top) and is not naturally suited for the two way communications required for data exchange. This is due to the low number of available channels per unit coverage area, resulting in poor use of the frequency spectrum available.

To overcome this problem, the cellular architecture was proposed [2] in the late 1940's, although not deployed until the 1980's with the advent of the first generation of cellular communication networks. In a cellular network, a large number of relatively low-powered transmitting base-stations (typically 20W [31]) are used with a much smaller coverage area[1], where each area is referred to as a cell. The carrier frequency of an individual cell can be used by another cell if they cause negligible interference between each other, i.e. if they are separated by at least one cell of a different carrier frequency. This is termed frequency reuse and leads to the traditional honey-comb pattern associated with cellular networks, Figure 2.1(b). The cellular network allows for a low frequency reuse distance[2] making them attractive for

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[1] Typically several kilometres in diameter

[2] The frequency reuse distance is the metric used to measure the distance between cells which make use of the same carrier frequency.

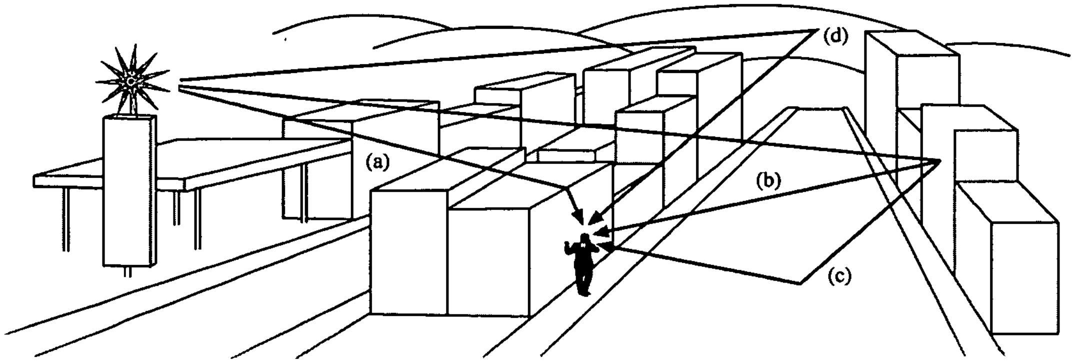

Figure 2.2: *Signal degradation mechanisms in the mobile wireless environment: (a) diffraction, (b) and (c) reflection, and (d) long time delays.*

deployment as they are much more efficient in their use of the frequency spectrum, due to the increased ratio of the available channels per unit coverage area, than broadcast infrastructure networks. Furthermore, cells can be of variable size, so in high population density areas (a city centre) the cell size would be smaller than that found in areas of low population density (the countryside). This technique is commonly known as cell splitting [32] and shows how flexible the cellular architecture can be.

As previously mentioned, the mobile radio environment is particularly hostile to reliable communications. There are many reasons for this, several of which are illustrated in Figure 2.2 which shows a conceptualised cell in a wireless network [33]. The difficulties that can be expected in wireless communications include

- Diffraction, (a), occurs typically when the radio wave passes over the edge of a building or through a small gap. Diffraction can occur multiple times for a single path.
- Reflection, (b) and (c), is the mechanism of the signal being reflected off surfaces such as buildings, buses or trees.
- Long time delays, (d), leading to a situation where signals can arrive at the receiver significantly out of phase causing Inter-Symbol Interference (ISI) as information from the previous data transmission may arrive at the receiver when it is expecting information about the next data transmission.

The received signal at the destination terminal is the phasor superposition of all signals received from the scattering paths listed above. This effect is known as multi-path propagation.

2.2.2 Mobile radio propagation

Using wireless signals for data communications means utilising certain frequencies in the electromagnetic spectrum to perform the intended function. The laws of electromagnetic wave propagation are well understood due in part to the pioneering experiments of Michael Faraday and the all-encompassing theory of electromagnetism developed by James Clerk Maxwell [34]. As a mobile terminal moves around in the wireless environment the received signal varies significantly. The changes in the environment which cause these variations can be broken into four distinct sections which are grouped by the distance of movement that the mobile terminal must undergo to observe the effect:

- **Fast-fading** is the observed effect over short distances and leads to independent fading signals being observed over half a wavelength in a high scattering environment, caused by the addition and subtraction of in- and out-of-phase signals at the receiver, Figure 2.3.
- **Frequency-selective fading** occurs when the bandwidth of the transmitted signal is greater than the coherence bandwidth of the channel or the delay spread is greater than the symbol period giving rise to noticeable time delays in the received multipath signal.
- **Shadowing (slow-fading)** is the effect on the received signal caused by large obstructions such as buildings in the urban environment or hills in the rural environment. This is a medium term variation (noticed over distances of hundreds of wavelengths).
- **Path-loss** is caused by the dissipation of energy as the distance for the transmitter increases, and as such is observable over large distances (several hundreds of wavelengths).

A number of deterministic and empirical models have been developed to provide tools for simulation of the wireless environment. The following sections describe, mathematically, the above causes of signal degradation at the receiver.

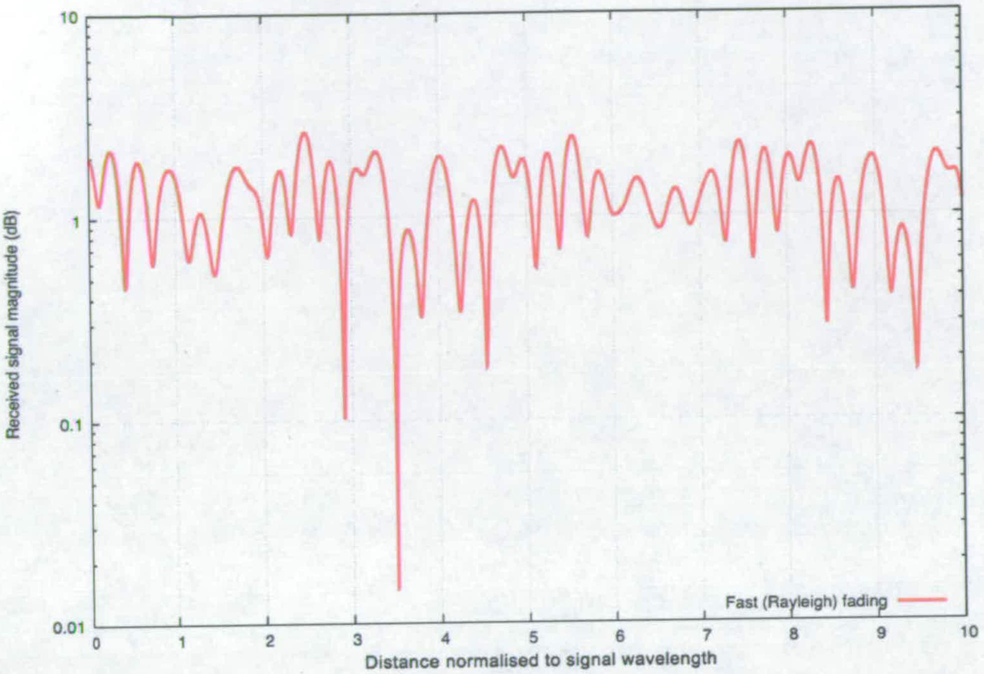

Figure 2.3: *Typical observed signal power at a receiving terminal showing the effect of fast-fading.*

2.2.2.1 A mathematical description of fast-fading (quasi-static fading)

As previously considered, fast-fading is caused by the super-position of the received signal which has taken different paths (rays) between the transmitter and receiver. Fast-fading can be considered to be quasi-static fading (i.e., constant over the time period of a single symbol) when the bandwidth of the transmitted signal is less than the coherence bandwidth of the channel or the delay spread is less than the symbol period. A mathematical model was proposed for this super-position of N complex (phase difference) waves at the receiver, where the received signal is given by

$$Z = \sum_{i}^{N} A_i \mathrm{e}^{-jkvt\cos\alpha_i} \tag{2.1}$$

where A_i is the complex amplitude of the ith multipath component, $k = 2\pi/\lambda$ is the wavenumber, v is the velocity of the receiving mobile terminal and α_i is the angle between the direction of movement and the azimuthal arrival angle of the ith multipath component. The received signal can be expressed as a Fourier series by using the substitution $A_i = R_i + jS_i$ as

$$
\begin{aligned}
Z &= X + jY && (2.2)\\
\text{where} \quad X &= \sum_{i}^{N} (R_i \cos \varepsilon_i + S_i \sin \varepsilon_i) && (2.3)\\
Y &= \sum_{i}^{N} (S_i \cos \varepsilon_i - R_i \sin \varepsilon_i) && (2.4)\\
\varepsilon_i &= kvt \cos \alpha_i && (2.5)
\end{aligned}
$$

where the real and imaginary parts of A_i (R_i and S_i respectively) are random variables.

The magnitude of the received signal (2.2) is therefore

$$|Z| = \sqrt{X^2 + Y^2} \tag{2.6}$$

As a consequence of the central limit theorem [35] for large N the two components X and Y can be considered to be Gaussian random processes [36]. Therefore the Probability Density Function (PDF) of the magnitude of the received signal (2.6) as the joint PDF of X and Y is given by the Rayleigh distribution function as

$$f(x) = \frac{x e^{\frac{-x^2}{2\sigma^2}}}{\sigma^2} \tag{2.7}$$

That fast-fading can be effectively modelled by the Rayleigh distribution has several empirical studies to back this claim up [37, 38]. Other fast-fading models also exist including Rician fading [39], which is similar to Rayleigh fading but has a strong Line-Of-Sight (LOS) component and Nakagami fading [40], of which Rayleigh fading is a special case. Nakagami fading is a useful model as it can fit a wide variety of fading statistics, such as multipath scattering with relatively large delay-time spreads, however it is harder to justify physically than Rayleigh or Rician fading. The primary aim of this thesis is to combat fast-fading and therefore an accurate model is required. Due to the supporting empirical evidence and relative simplicity of the Rayleigh fading model, this is the model that this thesis will consider.

2.2.2.2 A mathematical description of frequency-selective fading

Frequency-selective fading occurs when multi-path signals are spread significantly over time as well as space such that strong component signals with a significant time delay are recorded at the receiver after their transmit symbol period has passed, which gives rise to ISI. Wireless communication protocols are designed with this effect in mind and typically the symbol period of the network is designed to be longer than the expected delay spread of the transmitted signal. Therefore the received signals caused by time spreading have insignificant power and their interference effect can be discarded for the next symbol. However, this is not always the case, particularly in environments where large delay spreads can be expected and a short symbol period is used.

Frequency selective fading can be modelled by summing the received signals from all paths taken between the transmitter and receiver. This is the same as integrating over the power delay profile at the receiver, models for which are given in COST-207 [41, p. 137]. For example COST-207 gives the power delay profile for the typical rural (non-hilly) area as

$$P(\tau) = \begin{cases} \mathrm{e}^{-9.2\tau} & \text{for } 0 < \tau < 0.7 \\ 0 & \text{elsewhere} \end{cases} \tag{2.8}$$

where τ denotes the time delay.

To reduce the computational complexity of simulating a frequency-selective fading channel, a Finite Impulse Response (FIR) filter can be used with a set number of taps to simulate the recorded time delays, and tap weights to reflect the strength of the recorded signal, such that

$$P(\tau) = \sum_{i=1}^{I} P_i \delta(\tau_{i-1} * \Delta\tau) \tag{2.9}$$

where $P(\tau)$ is the total received power, τ is the delay, I is the number of taps, $\Delta\tau$ is the time spacing of the taps, P_i is the ith tap weight and δ is the Dirac-delta function. It should also be noted that each path in the multi-path propagation environment will undergo individual quasi-static Rayleigh fading.

More complex models have also been developed, as well as more environment specific models, such as indoor models at particular frequencies [42].

2.2.2.3 A mathematical description of shadowing

Medium term variation in the power of the received signal at a mobile terminal is termed shadowing due to the effects that cause it. Shadowing arises due to obstacles in the path of the radio signal's propagation such as hills, buildings and trees. Shadowing is also often referred to as slow-fading in literature due to the similarities to fast-fading, but is observed over longer distances. At 900MHz the effect of shadowing can be seen over distances of around 25-100m [43]. The effect of shadowing on a received signal depends heavily on the frequency of the carrier wave. For example, a communications signal using a low carrier frequency might diffract around a building giving the receiver a strong signal, however a signal using a much higher carrier frequency would not diffract around the building giving a poor received signal.

Empirical results have shown that the mean received power at the mobile terminal, $m(t)$, follows a lognormal distribution, i.e., $R \doteq \log m(t)$ is normally distributed with a standard deviation of approximately 6-8dB for an urban environment [9, 24, p.121]. The PDF of the lognormal distribution can be expressed as

$$f(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}\mathrm{e}{-}(\ln x - \mu)^2/2\sigma^2 \tag{2.10}$$

where μ and σ denote the mean and the standard deviation respectively.

2.2.2.4 A mathematical description of path-loss

Long term variation in the received signal power from the transmitter can be attributed to path-loss, which is caused by the dissipation of the transmitted signal as it radiates out from the transmitter in free-space. The free-space path loss is well known to be [43]

$$L_F = \left(\frac{\lambda}{4\pi d}\right)^2 g_T g_R \tag{2.11}$$

where g_T and g_R are the antenna gains of the transmitting and receiving antennas respectively, λ is the wave-length and d is the distance from the source. Expressing this in logarithmic terms gives the path-loss ($P_T - P_R$) as

$$L_F = -10\log g_T - 10\log g_R + 20\log f + 20\log d - 20\log\frac{c}{4\pi} \tag{2.12}$$

where P_T and P_R are the transmitted power and received power respectively and c is the speed of light.

There are many combinations of the above phenomenon, and multiple models for each. Due to the complexity of the wireless environment it is not possible to simulate every possible path in the multi-path scenario, leaving statistical analysis as presented above as a requirement to develop simple simulation models. This provides a very accurate method of ensuring simulations are realistic, assuming correct models for the environment are chosen.

2.2.3 Signal duplexing

To facilitate two way communications, wireless networks can either employ Frequency Division Duplexing (FDD) methods or Time Division Duplexing (TDD) methods. In FDD the channel is split into two distinct sub-bands, where one is assigned to be the uplink band, and the other is the downlink. This assignment does not necessarily have to be symmetrical, and often this is not in the case where data communications suggest that more data would be transmitted in one direction. An example of this is the viewing of web-pages. In this case a request for the web-page is made by the client to a server, which requires very little data transfer. The requested data is then sent from the server to the client, requiring large amounts of data transfer relative to the original request.

In the case of TDD all sub-bands will be assigned for uplink data at one time instance, and then downlink the next. This requires high precision timing synchronisation between the BS and all mobiles in the network. Like FDD, TDD can also be assigned to be asymmetric.

2.2.4 Multiple access methods

Mobile radio networks must provide services for multiple users to be able to simultaneously connect to the network for data exchange, either between mobile users in the same cell, or to an external cell or network (such as the Public Switched Telephone Network (PSTN). There are three fundamental methods which can be used to provide Multiple Access (MA) to the network [44]:

- Frequency Division Multiple Access (FDMA)
- Time Division Multiple Access (TDMA)
- Code Division Multiple Access (CDMA)
- Space Division Multiple Access (SDMA)

2.2.4.1 Frequency Division Multiple Access

In FDMA systems the available frequency spectrum is sub-divided into contiguous frequency bands which are then allocated to the mobile users as required. A small gap, called the guard band, is left between frequency bands such that they do not interfere with one another. FDMA is illustrated in Figure 2.4(a) [5].

2.2.4.2 Time Division Multiple Access

A TDMA system assigns the entire frequency spectrum available to a single mobile at any one time and rotates access between the mobiles in the network by allowing each terminal to access the network for only a limited time, Figure 2.4(b). Similar to FDMA, TDMA systems require a gap between the multiple access data carriers, in this case called a guard time.

2.2.4.3 Code Division Multiple Access

The CDMA system is significantly different from FDMA and TDMA as it allows multiple users to utilise the same carrier frequency at the same time, Figure 2.4(c). Although this appears initially counter intuitive, by modulating the transmitted signal using a high speed 'chip-code' to spread the signal over a wide frequency range[3], as shown in Figure 2.5, it is possible to use the inverse of the chip-code to recover the signal of interest at the receiver, with limited interference. This provides limited inherent security, since the original code is required to receive a single signal. For this reason CDMA initially found application in military

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[3] This gives rise to the term *spread spectrum.*

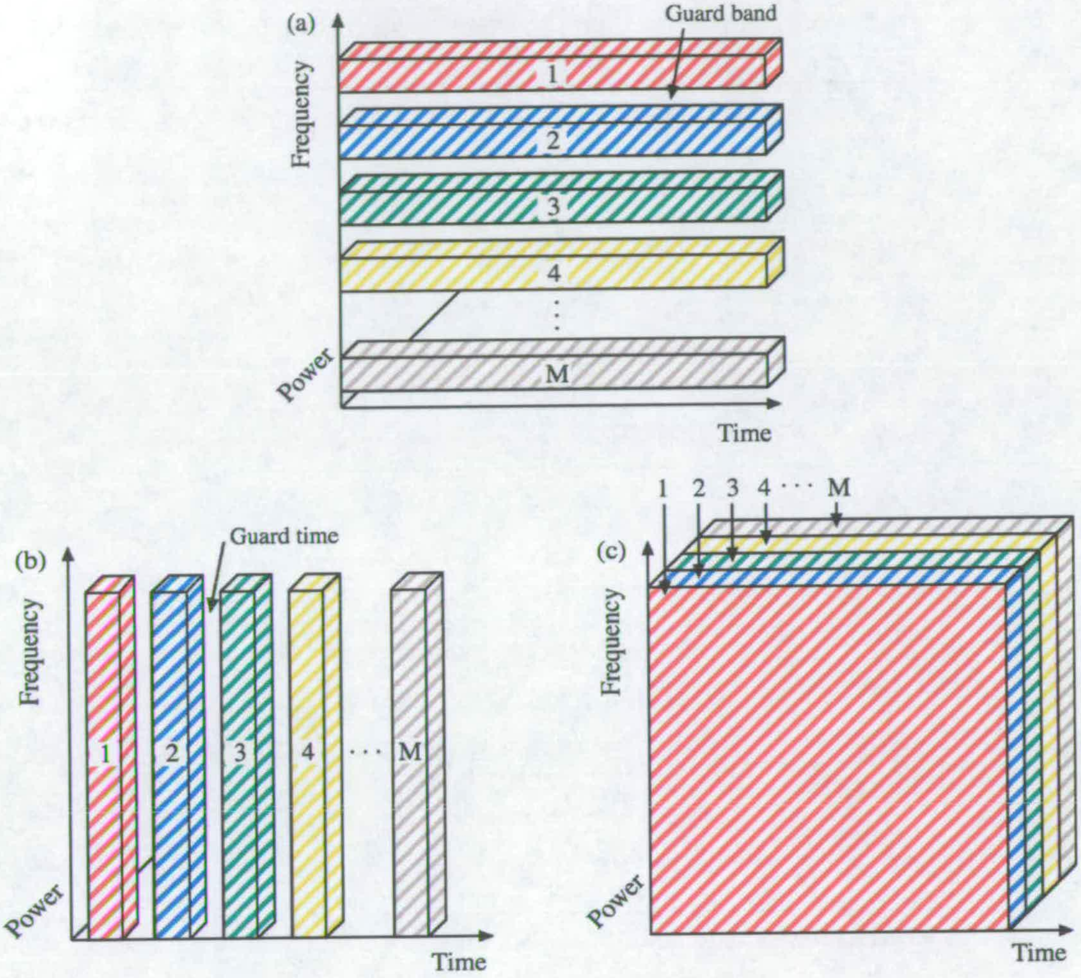

Figure 2.4: *Multiple access strategies for M users: (a) FDMA, (b) TDMA, (c) CDMA.*

communications, particularly missile/torpedo control [45]. There are a number of methods which can be employed to accomplish CDMA including, but not limited to:

- Direct Sequence (DS) spread spectrum - A data signal is spread over a wide frequency range by modulating it onto a carrier frequency with a Pseudo-Random (PR) high speed chip-code.
- Frequency Hopping (FH) spread spectrum - The available bandwidth is sub-divided into a large number of contiguous frequency bands similar to FDMA techniques, and the carrier frequency for a single data signal is changed rapidly between different frequencies in a

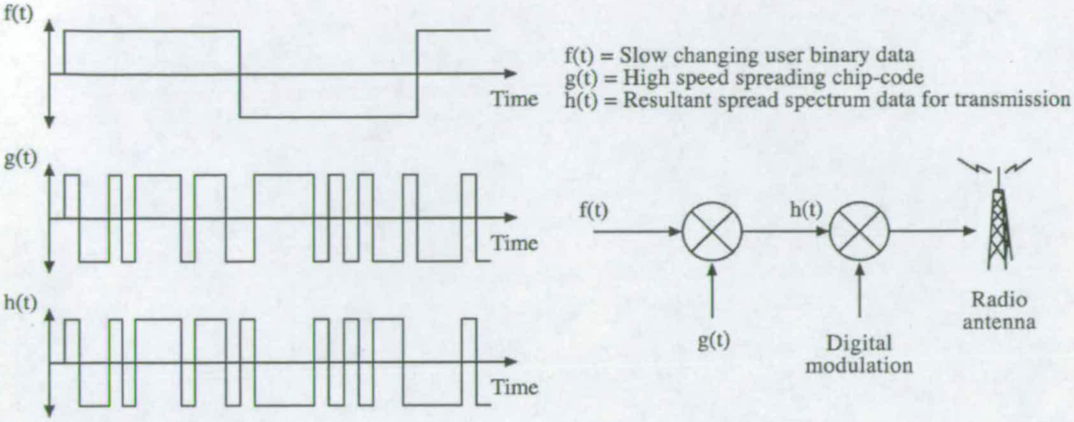

Figure 2.5: *Direct Sequence Code Division Multiple Access spread spectrum method to combine the high speed chip-code with user binary data resulting in a coded high rate data signal.*

PR fashion.

- Time Hopping (TH) spread spectrum - A short data pulse is transmitted having either a PR pulse duration or the data pulse is transmitted in a random time position relative to the input bit period.

2.2.4.4 Space Division Multiple Access

SDMA systems exploit the fact that transmitted signals have a limited range by assigning the entire frequency band that is available to a single node, which is separated by a large enough distance that its signal will not interfere significantly with other nodes. In practice this means that only one transmitting terminal can be located in an area, typically several kilometres in the case of GSM transmission. Despite this SDMA is a natural phenomenon in wireless communications, and allows frequency reuse, as discussed in the previous chapter.

2.2.4.5 Practical implementation

In practical deployment no single one of the above systems is suitable on its own, rather an appropriate combination must be developed into a standard protocol for deployment. GSM is an excellent example of such a system which has been very well received in the market place. In a GSM network FDD is used to provide two way communications, where approximately

25MHz in total is dedicated to both the uplink and the downlink separately[4] [31]. The link in each direction is then divided into channels of 200kHz in an FDMA fashion, each of which is then further divided into eight TDMA slots, each of which is 0.577mS long. This allows GSM to provide data-rates of up-to 13kbps for voice data. There is also considerable overhead used in the GSM system for timing synchronisation and resource allocation.

Newer networks such as 3G systems employ FDMA and TDMA techniques similar to the 2G GSM system, however they also provide access protocols for CDMA techniques, due to the flexibility it provides for the network operators, and the potentially increased efficiency of the use of the radio spectrum available.

2.3 Spatial diversity

As the previous section showed, one of the pitfalls of wireless communications is fast-fading, where the characteristics of a received signal can change dramatically over short distances and short time intervals. Although this effect can cause considerable problems for the designers of wireless communications systems and their protocols, it can also be leveraged to the advantage of communications as the signals that are received are separated in both space and time. This leads to independent signals which can be received by use of multiple receive antennas. Similarly multiple transmit antennas can be used to help mitigate the multi-path fast-fading effect and the two methods can also be combined in space [26, 27, 46] and space-time [47, 48] techniques. The use of multiple antennas at either the transmitter or receiver if often referred to as *smart antennas* due to their ability to use diversity and also potentially use interference cancellation techniques [49]. This phenomenon is termed spatial diversity.

The use of multiple antennas to achieve diversity dates back to Marconi and the early radio pioneers, however it was World War II which, as an engine of change, drove considerable research into the area of using antenna arrays for radar systems to exploit spatial diversity [50, p.1-8]. The 1970's saw a further interest in multiple antenna systems with the advent of DSP technologies, which helped to dramatically improve diversity and interference cancellation techniques [51]. The deployment of GSM in Europe in the 1990's and then around the world saw a large scale deployment of smart antennas where two transmit antenna elements are often

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[4] This discussion assumes that the 800MHz GSM band is being used. 900MHz and 1900MHz bands are also available for use in GSM networks.

used at the base-station to achieve spatial diversity. At the time, the extra cost and complexity was considered to be more affordable at the base-station than on a small mobile phone handset [49]. Recent advances in both the field of DSP and manufacturing mean that this is no longer a necessary constraint.

Importantly, to achieve spatial diversity and take advantage of the benefits offered by multiple element antenna arrays, the antenna elements must be separated by a large enough distance to ensure that the signal received by each element has no correlation with that received by the other elements. To ensure this independence of the received signals it is typical to space the antenna elements by several wave-lengths [28], although this independence can be obtained over half a wave-length in a high scattering environment. Therefore the required distance of separation changes with frequency. For the expected carrier frequency of 5GHz for next generation networks, this translates to a required separation of 2.5-10cm for 0.5-2 wave-length spacing for the MT antenna elements[5].

2.3.1 Exploiting multiple antenna systems

There are multiple configurations which can be used in multi-antenna element systems. Single-In Single-Out (SISO) is the traditional wireless link between a single transmit antenna and a single receive antenna. In the literature the Single-In refers to the transmitted signal which is injected into the wireless channel, and the Single-Out refers to the received signal coming out of the wireless channel. This is often referred to as a 1x1 channel to reflect the number of antenna elements. The capacity of the SISO channel was found in the landmark paper by Shannon in 1948 [52] as

$$I = \log_2(1 + \rho|h_{SD}|^2) \qquad \text{b/s/Hz} \tag{2.13}$$

where h_{SD} is the normalised complex channel gain between the transmitter and receiver, and ρ is the Signal to Noise Ratio (SNR) at the receive antenna element. Note that under the assumptions made in this thesis, the capacity of a communications channel can be termed either Shannon capacity or bandwidth efficiency, and both terms are regularly used in the

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[5] The MT can be assumed to be in a high scattering environment while in an urban location, however a base-station is usually located at the top of a building or spire, leading to reduced scattering at the base-station. In such a case the required separation will be on the order of 5-10 wave-lengths (25-50cm)

literature. Therefore, since the terms are mathematically identical in this thesis the terms are used interchangeably.

A Single-In Multiple-Out (SIMO) system has multiple receive antennas (M_R) and a single transmit antenna and is referred to as a $M_R \times 1$ channel. A SIMO channel has a capacity of

$$I = \log_2(1 + \rho \sum_{i=1}^{M_R} |h_{SD_i}|^2) \qquad \text{b/s/Hz} \tag{2.14}$$

where h_{SD_i} is the channel gain for antenna i. In this case increasing M_R results in only a logarithmic increase in capacity.

A Multiple-In Single-Out (MISO) system has a single receive antenna and multiple transmit antennas (M_T) in a $1 \times M_T$ system. The capacity of this configuration is given by [26] as

$$I = \log_2(1 + \frac{\rho}{M_T} \sum_{i=1}^{M_T} |h_{SD_i}|^2) \qquad \text{b/s/Hz} \tag{2.15}$$

where the normalisation by M_T ensures a fixed total transmitter power. As can be observed, (2.14) and (2.15) are very similar, with the addition of the transmit power constraint in the MISO case. This constraint ensures that additional transmit power at the transmitter is not used, when compared with the single antenna case.

The Multiple-In Multiple-Out (MIMO) system has multiple receive and transmit antenna elements in a $M_R \times M_T$ configuration. The MIMO capacity equation is found [26, 53] to be

$$I_{EP} = \log_2 \left(\det \left(\mathbf{I}_{\mathbf{M_R}} + \frac{\rho}{M_T} \mathbf{H}\mathbf{H}^* \right) \right) \qquad \text{b/s/Hz} \tag{2.16}$$

where * denotes the transpose-conjugate and $\mathbf{H}$ is the $M_R \times M_T$ channel matrix. I_{EP} denotes that in (2.16) Equal Power (EP) is used at each transmit antenna element. Note also that the assumption is made such that the covariance matrix of the transmitted signal matrix $\mathbf{x}$, $\mathbf{R_{xx}} = \varepsilon\{\mathbf{x}\mathbf{x}^H\}$ must satisfy $\text{Tr}(\mathbf{R_{xx}})$ in order to constrain the total average energy transmitted over a symbol period. Foschini [26] and Telatar [27] demonstrated that the capacity of the MIMO channel (2.16) grows linearly with min(M_R,M_T) as shown in Figure 2.6. Figure 2.6 also shows

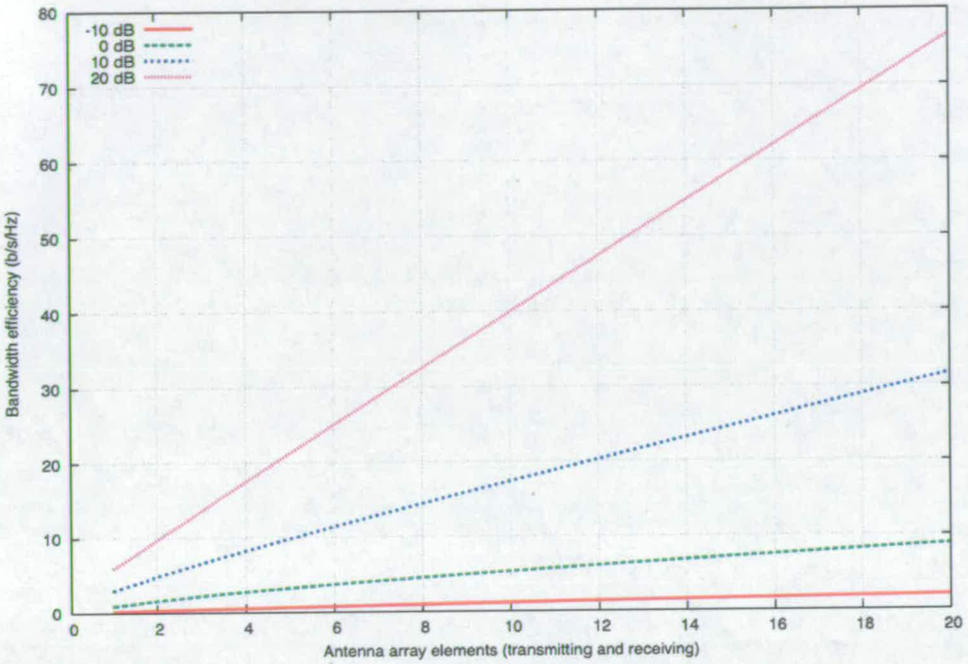

Figure 2.6: *MIMO bandwidth efficiency for various SNRs against the number of antenna elements in the transmitting and receiving arrays ($M_R = M_T$).*

the very large capacities which can be achieved with $M_R = M_T$. Part of the challenge now facing the wireless communications community is how these capacity benefits can be realised for mobile communications.

In summary all four antenna configurations are shown in Figure 2.7.

The benefits offered by the use of multiple antenna elements are due to

- Array gain
- Diversity gain
- Spatial Multiplexing Gain
- Interference Reduction

These four topics are now discussed in more detail.

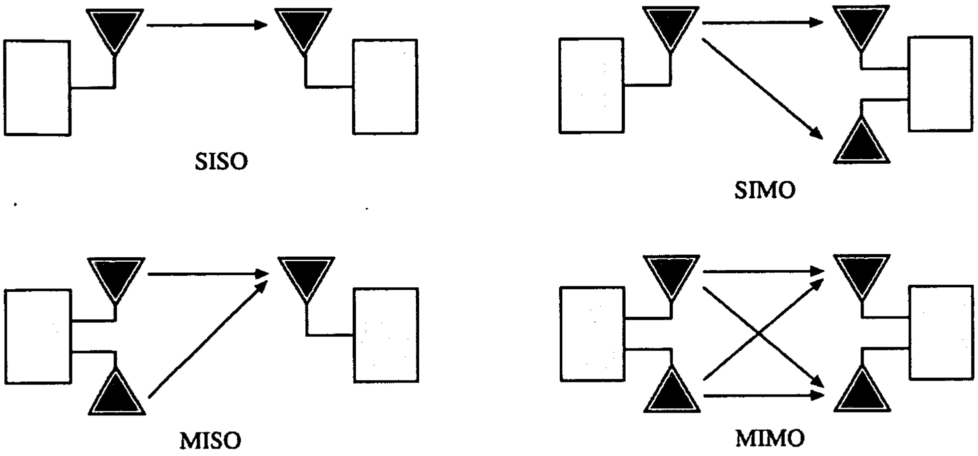

Figure 2.7: *Multiple antenna transmit and receiver configurations, with a maximum of two transmit or receive antennas.*

2.3.1.1 Array gain

Array gain refers to the gain that can be made available in SNR at the receiver due to signal processing at both the transmitter and receiver due to a coherent combining effect. For example in the SIMO channel, signals arriving at the receive antenna array have different amplitudes and phases. The signals can then be combined coherently so that the resultant signal is enhanced. For system architectures with multiple transmit antennas, knowledge of the channel is required to exploit the array gain, which implies feedback from the receiver in the transmission protocol.

2.3.1.2 Diversity gain

As has been previously discussed in this chapter, signal power at the receiver in a wireless system fluctuates randomly (Rayleigh fading). By using multiple antennas, information can be transmitted over $M_R \times M_T$ independent fading channels. The probability of all channels being poor decreases as the number of antenna elements considered increases, leading to improved communications reliability. Assuming that the $M_R \times M_T$ channels can be suitably combined at the receiver, the variability of the received signal's amplitude is greatly reduced, leading to a $M_R \times M_T$th order diversity gain. Extracting spatial diversity using multiple transmit antenna elements, with the absence of channel knowledge at the transmitter, is possible using suitably designed transmit signals known as Space-Time Codes (STC). Due to the potential benefits from realising such codes, STCs have attracted a lot of interest from the research community

[54–57].

2.3.1.3 Spatial Multiplexing gain

As demonstrated previously in the discussion of the capacity of the multi-antenna element configurations, the MIMO system can offer a linear increase in capacity as the number of transmit and receive antenna elements increase. This is termed the spatial multiplexing gain and can be realised in the MIMO channel, with no increase in transmit time, bandwidth or power. This is one of the main reasons why multi-antenna systems have attracted intense interest in previous years, due to the extremely high capacities that can be achieved (2.16).

In systems which employ Spatial Multiplexing, the bit stream to be transmitted is demultiplexed into M_T half-rate information streams (assuming M_T transmit antenna elements), and then modulated and transmitted simultaneously from each transmit antenna. The receiver can then extract both signals, and with knowledge of the channel conditions fully separate them and reconstruct the original information stream [51].

2.3.1.4 Interference reduction

Co-channel interference exists in wireless systems due to frequency reuse. Using multiple antenna techniques the receiver is able to differentiate between the spatial signatures of the desired signal and the co-channel signals using knowledge of the desired signal. Subsequently the receiver is able to reduce the interference from other users of the wireless network [58].

2.3.2 Practical implementation considerations

It is important to note that it is not possible to exploit all four of the above mentioned benefits of MIMO transmission at the same time due to conflicting demands on the degrees of freedom that are available ($M_T \times M_R$). The designers of a wireless network must select an appropriate combination of techniques based on the requirements of each individual network.

2.4 Cooperative diversity

In the wireless communications market, device size is an important constraint to be considered since the consumer must carry their device with them. Therefore a maximum of two antenna elements might be used on a device the size of a mobile phone at the target carrier frequencies (5GHz) for future generation cellular networks. A laptop computer or similar sized mobile device might be equipped with up to eight antenna elements. The challenge presented to designers of mobile networks is how to translate the advantages offered by multiple element antenna arrays into practical benefits of a deployed network where the number of antenna elements are limited.

Mobile terminals situated in different geographical locations are each able to receive the same signal and process it independently[6]. Although each terminal receives a different independent fading signal they have no method by which to make use of the diversity that exists between each terminal. By sharing information about the received signal between terminals, the benefits of spatial diversity can be utilised. For example, one MT might be in a deep fade in the signal sent by the original transmitter, and another might not, therefore enabling the second MT to forward the information on to the first. This effectively creates a *Virtual Antenna Array* (VAA) [59, 60]. The sharing of information represents a paradigm shift in cellular communications, since MTs can currently only communicate with the local base-station. Due to the sharing of information between MTs to benefit from diversity, this technique has been termed *cooperative diversity* [28, 29].

The sharing of information between MTs cannot be readily implemented in a cellular network for several reasons which need to be addressed by the research community. In-particular the benefits of deploying a cooperative diversity network in a situation similar to a cellular network must be fully quantified, which is a primary goal of this thesis. Battery life issues must also be considered since MTs acting as relays for other MTs will be required to transmit extra data, as well as their own, which is a battery intensive process. The extra transmission of shared data between MTs will also create extra interference which is an important consideration. Finally, the sharing of information between MTs implies that security must be an important factor in any cooperative diversity network.

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[6] Current generation wireless protocol inhibit this ability due to obvious security concerns

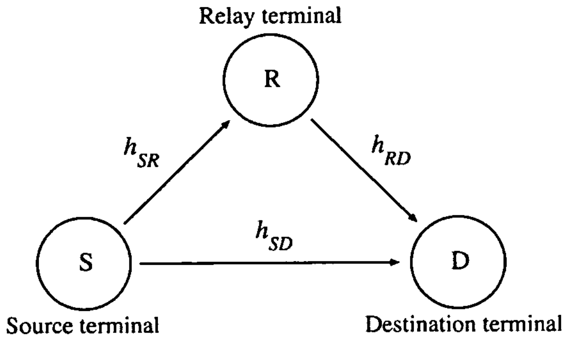

Figure 2.8: *Three terminal fading relay channel.*

2.4.1 Cooperative relay channels

The basic relay channel, and its extension into multiple supporting relays, is central to the study of cooperative diversity. The relay channel was originally introduced by van der Meulen [61, 62] has subsequently been studied by several other authors and is shown schematically in Figure 2.8. In the general relay channel the relay supports the source by recording the signal sent from the source, processing that signal, and then, possibly, sending some information to the destination. By necessity, cooperative diversity is split into distinct two transmission phases, each of which occurs at a different time giving rise to the term *time split* in cooperative diversity literature. This is required as the relay initially does not have information to share with the destination. In the first phase of transmission the source communicates with the relay and potentially with the destination. Following this in the second phase the relay will support the source in one of several configurations, while the source may potentially communicate with the destination[7]. Although it is possible to develop transmission protocols with higher numbers of transmission phases, the work in this thesis is generally restricted to two transmission phases, although this restraint is relaxed for discussion in chapter 6.

Cover and El Gamal [63] consider three configurations in which the relay can support the source:

- Facilitation: The relay assists the source by minimising the interference it creates in the received signal at the destination.
- Cooperation: The relay fully decodes the transmission from the source and re-transmits

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[7] Note that this gives rise to several possible transmission protocols which are described fully in Chapter 4.

some information about that signal to the destination. Specifically, the relay encodes the bin index of the previous source message, from a random binning of the source messages. The destination then suitably combines the two sets of information, coherently combining the received identical bin indexes.

- Observation [64]: The relay transmits a quantised version of its received signal and forwards it to the destination, which then averages its two received signals to reduce the noise of the received signal.

These three relaying techniques are fundamental to all methods which are used in the present literature. In the facilitation case the relay does not actively transmit to the destination, therefore this is effectively the traditional single-hop channel previously considered in this chapter. In the cooperation method the relay will decode the source message, potentially process the data and then transmit information on to the destination. This information may simply be a re-encoded version of the original source signal which would effectively remove the noise in the received signal[8]. Alternatively the relay could transmit parity bits on to the destination [63]. This type of relaying is broadly referred to as Decode-and-Forward (DF). The final method of observation, as termed by Laneman [64], comprises of the relay simply conveying a representation of its received signal on to the destination. This might take the form of quantising the received signal, amplification and then forwarding the signal [65], or possibly compressing the received signal before retransmitting it [63]. Of the observation methods, amplification is the most popular for implementation due to its simplicity, and is therefore known as Amplify-and-Forward (AF).

Each of the DF and AF methods has its own advantages and disadvantages. DF is popular as it removes the noise from the relay's received signal, ensuring that each signal the destination receives will only have noise induced by a single-hop channel (source to destination and relay to destination). However this gain comes at the cost of processing power at the relay as it is required to fully decode and then re-encode the signal.

The AF relaying method does not have this disadvantage, as it simply amplifies the received signal, which can readily be done. However, it does imply that any noise in the received signal at the destination is amplified and then forwarded to the destination. If the channel noise of the source to relay channel is very low compared to the relay to destination this is an insignificant

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[8] It should also be noted that while removing noise, it could also be enhancing noise if data corruption has occurred between the source and relay

factor and AF might be the preferred method. However, this is not a suitable assumption to be applied to the study of a wireless network where a relay may potentially move geographically as much as the destination terminal.

As DSP tools and processors become more effective and cheaper to install, the disadvantages of the DF method decrease. Furthermore, there has been some work on the relative merits of AF and DF relaying modes [30, 66] and combinations and adaptive protocols of such [30, 67]. DF is generally shown to offer a small advantage over AF, depending on the environment considered. Therefore, this thesis will concentrate primarily on DF methods.

2.4.2 Decode and forward

The information transmitted from the relay to the destination does not have to be exactly the same message as was decoded from the source, as previously mentioned. Hannerstroem et al. [68] considered Alamouti's Space-Time Block Code (STBC) [56], which has proven to be a popular avenue of research for cooperative diversity. Differential modulation techniques were considered by Tarasak et al. [69], and Chu et al. [70] developed relaying techniques based upon sending parity bits to the destination. One of the most promising areas of research stems from the land-mark paper by Berrou et al. [71] which introduced error correcting codes called Turbo Codes (TC) which could achieve near Shannon capacities in a communications channel. Due to the time split between the source and relay transmitting in cooperative diversity, TCs are a natural application to the encoded data the relay sends, and this has been the subject of extensive research [72–74].

In this thesis no assumption is made about the data which is transmitted by the relay, instead focusing on the Shannon capacity limit of the channel, to fully characterise the possible gains from utilising cooperative diversity. Not only are there different methods for which information the relay is to transmit but there are also two different methods by which the relay can be selected to actively support the source:

- Laneman et al. [30, 75, 76] which requires a particular information rate to be received at the relay.
- Nabar et al. [77, 78] which uses the relay at all times, limiting the initial rate to be the poorest of the source to relay link or the source to destination link.

2.4.2.1 Laneman et al. - Required Rate DF

The DF scheme proposed by Laneman et al. and used extensively in literature, requires that the relay (or set of relays) be able to decode the signal from the source in the first phase in order to repeat the information to the destination in the second phase. If a relay cannot fully decode the source's signal (i.e. the source to relay link is poor) the relay does not actively transmit in the second phase. In this thesis this method of DF is termed Required Rate DF (RRDF).

The basic cooperative diversity protocol which was considered by Laneman et al. [30, 76] is where the source communicates with both the relay and destination in the first phase, and then remains silent in the second phase. The relay transmits in the second phase. This is only one protocol of the complete family of cooperative diversity protocols presented by Nabar et al. [77], however it is the most popular in literature due to the fundamental work carried out by Laneman et al. The complete family of cooperative diversity protocols is considered further and extended in Chapter 4.

For this discussion of DF type relaying methods it is assumed that there is only one relay available to support the source. Furthermore it is assumed that all channels are quasi-static Rayleigh fading channels with fading coefficients being circularly symmetric zero mean complex Gaussian random variables. Finally, perfect timing synchronisation is assumed between transmissions and that the two transmission phases are equally split. Under these conditions the Shannon capacity of the source to relay link in the first phase is given by Laneman et al. [76] as

$$I_{SR} = \frac{1}{2}\log_2\left(1 + \frac{E_{SR}}{N_0}|h_{SR}|^2\right) \text{ b/s/Hz} \tag{2.17}$$

where I_{XY} is the mutual information exchanged between the transmitter X and the receiver Y, E_{XY} is the average signal energy over one symbol period and the scalar h_{XY} is the random, complex-valued, unit-power channel gain between the source and destination terminals. In this case $X = S$ (the source) and $Y = R$ (the relay). The factor of $1/2$ arises from the two equal duration time phases employed in cooperative diversity, since the source can only transmit information for exactly $1/2$ of the total time available.

Rearranging (2.17) to give the condition of the relay being able to decode the first phase transmission, and substituting the mutual information I_{SR} for the required rate, R, gives

$$|h_{SR}|^2 > \frac{2^{2R} - 1}{\mathrm{SNR}} \tag{2.18}$$

where $E_{SR}/N_0 = \mathrm{SNR}$. In the remainder of this chapter it is assumed that all network channel links have the same SNR, specifically $E_{XY}/N_0 = \mathrm{SNR}$ to simplify presentation. The case where the relay is able to support the source in the second phase is denoted δ, while the opposite scenario is denoted $\bar{\delta}$.

$$I_{RRDF} = \begin{cases} \frac{1}{2}\log_2\left(\mathrm{SNR}|h_{SD}|^2 + \mathrm{SNR}|h_{RD}|^2 + 1\right), & \delta \\ \frac{1}{2}\log_2\left(\mathrm{SNR}|h_{SD}|^2 + 1\right), & \bar{\delta} \end{cases} \tag{2.19}$$

where D denotes the destination terminal.

2.4.2.2 Nabar et al. - Minimum Rate DF

Nabar et al. [77,78] consider a different approach to DF relaying, where the relay is always active in the second phase. In this case, the source to relay link quality becomes a second limiting factor, as the transmission rate from the source cannot be higher than that which is supported by the source to relay link. The source to destination link quality in the first phase is the first limiting factor where the source cannot transmit at a rate higher than the source to destination link can support. For this reason the Nabar method of DF relaying is termed Minimum Rate DF (MRDF).

The Minimum Rate DF scheme can readily be shown to have a Shannon capacity of

$$I_{MRDF} = \min\left\{\frac{1}{2}\log_2\left(\mathrm{SNR}|h_{SD}|^2 + \mathrm{SNR}|h_{RD}|^2 + 1\right), \frac{1}{2}\log_2\left(1 + \mathrm{SNR}|h_{SR}|^2\right)\right\} \tag{2.20}$$

2.4.2.3 Comparison of DF techniques

As this thesis considers DF as the primary relaying technique it is important to consider the advantages and disadvantages of both the RRDF and MRDF schemes. This has not been covered in literature to date, therefore results are presented for the bandwidth efficiency of each

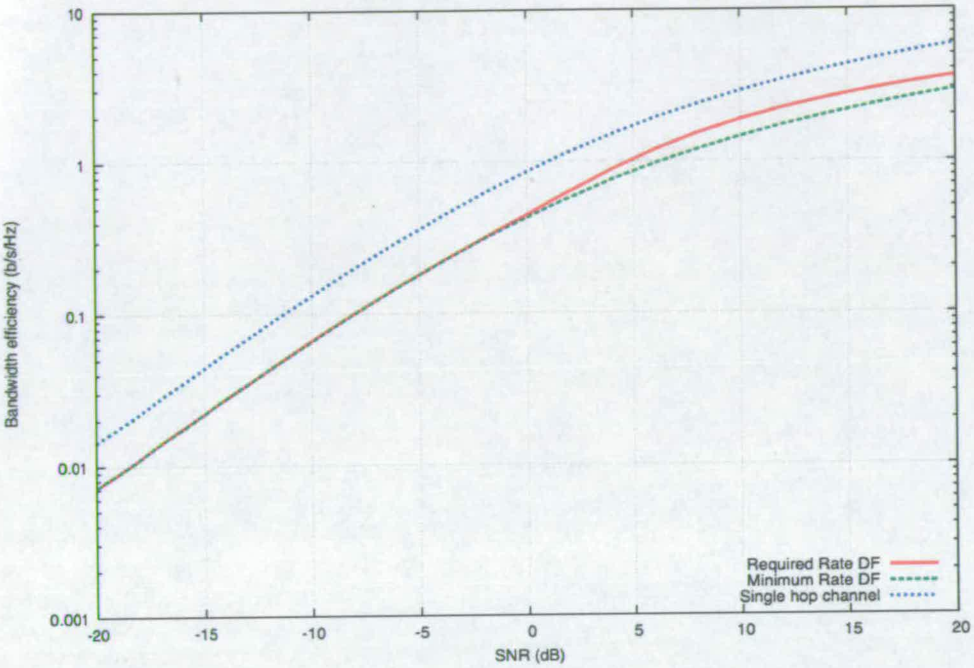

Figure 2.9: *Simulation results for the bandwidth efficiency of RRDF and MRDF relaying methods where SNR is equal at all receivers. The traditional single-hop channel is also shown for comparison.*

DF relaying technique for various SNR. As previously noted, the SNR for all channel links are assumed to be the same, and only Rayleigh fading is considered. The results are presented in Figure 2.9 where the required rate for the RRDF method is 1b/s/Hz.

From Figure 2.9 it can be seen that at low SNR both RRDF and MRDF give similar performance results, due to the poor relay channel. However at higher SNR, where the relay in the RRDF scheme starts to actively support the source as the source to relay link is strong enough to support the required rate, RRDF offers a higher capacity than the MRDF method. It is interesting to note from Figure 2.9 that the bandwidth efficiency of the single-hop channel is constantly greater than either cooperative diversity method. This is in part due to the factor of $1/2$ required for cooperative diversity in (2.17) and (2.20). Furthermore the single-hop channel outperforms the cooperative diversity methods as the bandwidth efficiency curve does not show the effect that the diversity offered by cooperative diversity has on the capacity of the link. This is fully explored in subsequent chapters.

In terms of practical deployment of each scheme the RRDF method is simpler to employ in a

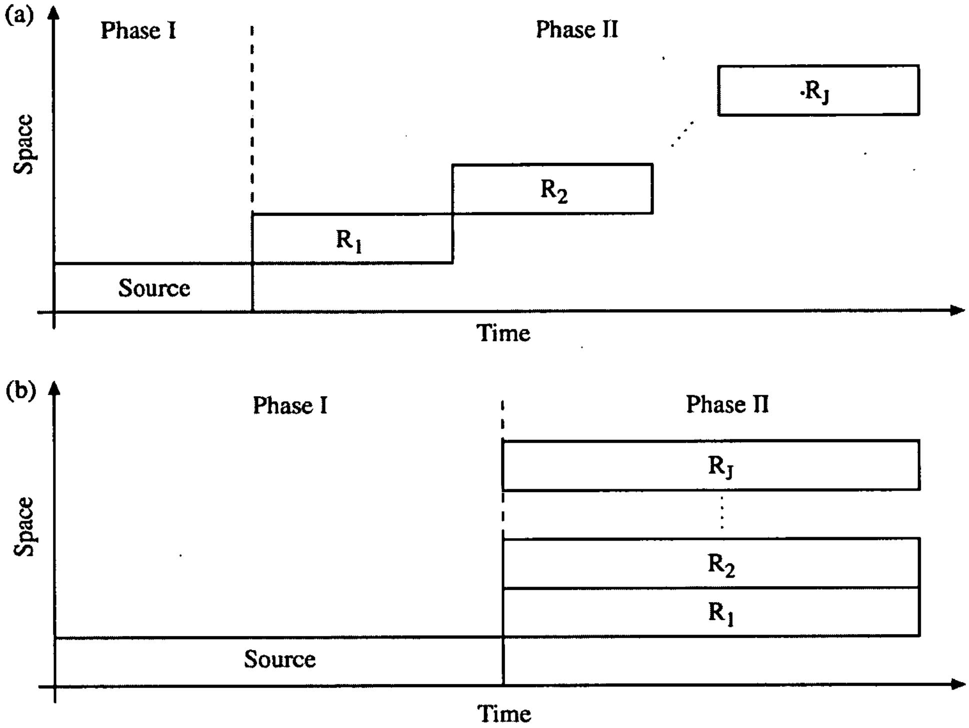

Figure 2.10: *Cooperative diversity methods shown in space/time with J available relays: (a) Repetition based cooperative diversity, (b) STC based cooperative diversity. Each block represents a transmitting terminal.*

networking protocol as the source does not require forward knowledge of the source to relay link as it does in the MRDF case, reducing the required information overhead of the protocol. In light of these results, RRDF is the DF method considered in this thesis.

2.4.3 Active research topics in cooperative diversity

As cooperative diversity offers the potential to combat Rayleigh fading in the wireless environment it has attracted considerable research and development interest since the work of Sendonaris et al. [28, 29] and Laneman [64]. One particular area of research interest lies in using multiple relays to support the source's communication with the destination. This was considered by Laneman et al. [76] where two methods were proposed for multiple relays. The first is termed repetition cooperative diversity where the transmission time is split equally between all transmitting terminals, Figure 2.10(a), and each terminal transmits only once in a designated time slot when no other terminals transmit.

In such a situation, if there are J transmitting terminals, the source is able to transmit for only $1/J$ of the available time, limiting the amount of information the source can transmit. To overcome this [76] also proposed an STC based cooperative diversity where the entire set of supporting relays would support the source at the same time and in the frequency band, Figure 2.10(b). The properties of STCs would then allow the receiver to fully separate the received signals and process them accordingly. This method allows both the source and relays to transmit for exactly half the available time over the two time slots.

Due to this advantage that the source can transmit information for exactly half of the available time STC cooperative diversity has been the main avenue for research and considerable effort has been expended in developing suitable STCs for the distributed environment of a cooperative network [78–81]. Furthermore, only STC cooperative diversity is considered in this thesis, although the single relay case can potentially be considered to be repetition cooperative diversity.

A research topic which is closely related to cooperative diversity is MIMO relay networks [82–84]. A MIMO relay network is a two-hop network (i.e. similar to a cooperative channel where the source has no direct communication link with the destination) where transmitting and receiving MTs have multiple antenna elements. This is important to cooperative diversity as the information theory frameworks introduced for MIMO relay networks can potentially be developed to include support for cooperative diversity networks with multiple transmit and receive antenna elements.

2.5 Orthogonal Frequency Division Multiplexing

As Rayleigh fading has such a significant impact on communications in the wireless environment it attracts a lot of attention from the research community. Relaying and multiple antenna element arrays are just two of the methods which have been proposed to combat this problem. Another method is to transmit data over multiple independent carrier frequencies. The concept behind this is very similar to the cooperative diversity method such that if the signal carried by one of the carrier frequencies is in a deep fade at the receiver, others might not be due to the frequency selective nature of the channel. Modulating data onto several carriers was used 60 years ago in the Collins Kineplex [85] system which used four-phase differential modulation for parallel data transmission over 20 sub-channels and achieved a data

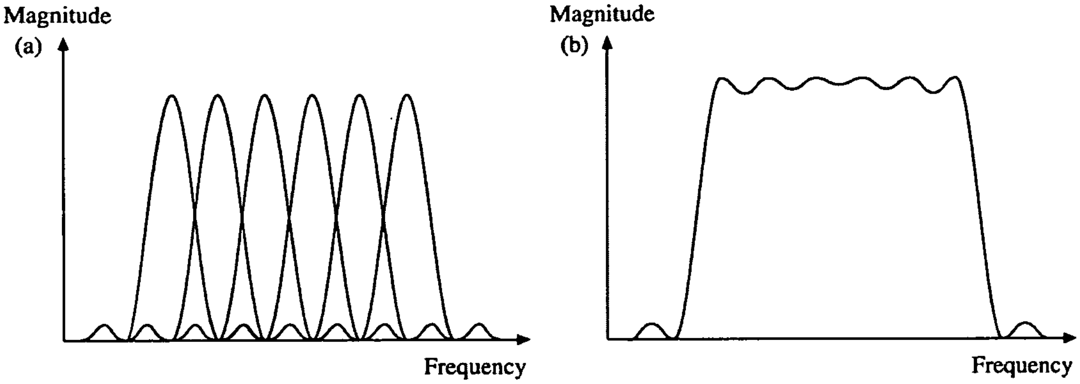

Figure 2.11: *OFDM multi-tone carriers (a) Individual tones, (b) Resultant frequency spectrum.*

rate of 3 kb/s.

Until recently multi-carrier transmission has only been of peripheral interest due to the guard-bands which were required to ensure Inter Channel Interference (ICI) did not occur. These guard bands were effectively wasted bandwidth which could not be used effectively [86]. Recent research has proposed a multi-carrier OFDM transmission system whereby the carrier frequencies are narrowly spaced such that the modulated data overlaps in frequency as shown in Figure 2.11. The carrier frequencies are selected to be orthogonal, such that, despite the overlapping of the spectra, detection of the signal in a single sub-channel gives no output from any other sub-channel, therefore achieving full separation [47, 87, 88].

As well as effectively combating Rayleigh fading, another attraction of OFDM is that it is possible to modulate each carrier with different data, giving a multiplexing gain. Due to its similarities to the MIMO system, OFDM in the context of spatial multiplexing has been proposed in [47, 87, 89, 90]. An extensive study of the capacity of the OFDM spatial multiplexing system was carried out by Bölcskei et al in [48] which showed benefits from both diversity and multiplexing gains.

2.6 Summary

This chapter has introduced the wireless communications environment and shown how this hostile environment causes problems for designers of wireless networks. In-particular, Rayleigh fading was considered as the result of multi-path fading, which is a problem inherent in wireless communications. Recently, spatial diversity techniques have been introduced to combat this problem and have lead information theory to show that dramatic increases in the available data transfer rate between terminals can be achieved using multiple antenna systems such as MIMO.

Antenna elements in a multiple antenna element array must be spaced by several wavelengths, leading to the fact that only two antenna elements could be used on a device the size of a mobile phone. This has lead researchers to develop cooperative diversity relaying techniques where a number of relays are able to support the source's transmission to the destination. There are a number of ways in which the relays can do this, and in this thesis Decode-and-Forward using the RRDF method will be used due to the relative benefits these techniques offer over the other methods as shown in the literature and in this chapter.

The benefits of a cooperative diversity network are explored in detail in the following chapter to characterise the potential benefits of cooperative diversity to a network where all terminals are equipped with two antenna elements. In the following chapters an information theory framework is developed, under the required system constraints, to fully explore all methods of cooperative diversity where multiple relays are available.

Chapter 3
Multiple-antenna cooperative diversity

3.1 Introduction

The previous chapter introduced cooperative diversity in a relay network. As was noted, cooperative diversity is based in the theory of spatial diversity and it therefore makes sense to apply other spatial diversity techniques to cooperative diversity to maximise the performance of the system. This chapter develops cooperative diversity strategies for the relay network with multiple antenna elements at each transmitting and receiving MT.

The first part of this chapter is devoted to expanding the work presented in [75] on MTs equipped with single antennas to include capacity measurement metrics. The main contribution, in the second part, is to introduce two methods for performing cooperative diversity in an environment where all mobile terminals are equipped with two antenna elements. These two methods are then analysed and compared to the basic direct transmission method, using the extended metrics, culminating in a method which can offer both reduced probability of outage and higher Shannon capacities.

This chapter is structured as follows: section 3.2 introduces the system model, the constraints placed upon it and the direct transmission method. Section 3.3 then presents single antenna STC based cooperative diversity, which will be used as the basis for comparison between the single antenna and dual antenna cases. Section 3.4 introduces two different dual antenna techniques which can be used for transmission in an STC cooperative network. Section 3.6 presents results and draws comparisons between the different cooperative diversity and transmission schemes and finally conclusions are drawn in section 3.7.

3.2 System model

To analyse the different relaying techniques, consider a wireless network with a set of transmitting terminals denoted $\mathcal{J} = \{1, 2, ..., j\}$. A source terminal, $S \in \mathcal{J}$ has information

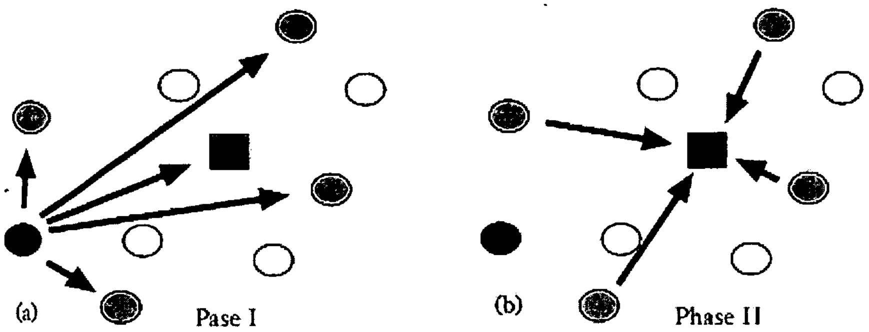

Figure 3.1: *Illustration of the two phase cooperative diversity method. In the first phase (a) the source transmits information in an omnidirectional manner to all terminals. In phase two (b), those terminals which can decode the signal then relay the signal on to the destination.*

to transmit to a single destination terminal $D \notin \mathcal{J}$, potentially using terminals $|\mathcal{J}| - \{S\}$ as relays to perform cooperative diversity, where $|\mathcal{J}|$ is the cardinality of the set $\mathcal{J}$. Therefore $|\mathcal{J}| - 1$ terminals may co-operate to support the source. The subset of $\mathcal{J} - \{S\}$ which can decode the signal according to the RRDF method is defined as $\partial(\mathcal{J})$, with cardinality $|\partial(\mathcal{J})|$.

The method described here of implementing cooperative diversity consists of two transmission phases, shown in Figure 3.1. During the first phase the source will transmit information to all available mobile terminals in the network. Those terminals will then attempt to decode the signal, and if successful they will take part as the decoding set in phase two, transmitting information to D.

In the system model below, (equations (3.2)-(3.4)) h_{XY} denotes the effects of multi-path fading between transmitter X and receiver Y. Statistically, h_{XY} is modelled as a zero-mean, independent, circularly-symmetric complex Gaussian random variable with variance λ_{XY}, so that the magnitudes $|h_{XY}|$ are Rayleigh distributed and the phases $\angle h_{XY}$ are uniformly distributed on $[0, 2\pi)$[1]. Furthermore, the coefficients $n_Y[n]$ are modelled as zero-mean mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_0. The scalar $n_Y[n]$ captures the effects of receiver noise and other forms of interference in the system. Also note that n denotes the time index. In this chapter it is assumed that quasi-static fading occurs where the fading coefficients are constant over the considered time and frequency. The scenario presented in this chapter assumes the fading coefficients are

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[1] Note that this notation indicates that 0 is inclusive in the set of random numbers, while 2π is not, ensuring uniform circular symmetry

known to the appropriate receivers but not known, or exploited, by the transmitters.

To make a fair comparison between the network models using cooperative diversity and the direct transmission between source and destination MTs, several constraints are placed upon the cooperative diversity model:

- The time used must not exceed that used by the direct transmission case (1 unit)
- The total of transmit power of the complete system (power used by source and the power used by the relays) must not exceed that used by the direct transmission case (1 unit)
- The bandwidth used must not exceed that used by the direct transmission case (1 unit)

The effect of the time and bandwidth constraints is that the transmission of the two phase cooperative diversity must take place in the same amount of time as the single-hop transmission. As noted in the previous chapter this usually involves a factor of 1/2 when considering the capacity of the system[2]. An important point that arises from this constraint is that the Rayleigh block-fading that is considered is now assumed to be quasi-static over the two time phases i.e., the fading coefficient is simply h_{SD} with no phase index. This is a reasonable assumption since the time constraint divides the transmit time of the original signal into two, which would otherwise be considered as quasi-static fading.

The power constraint, in the situation where this is a single relay supporting the source, has little impact on the transmit power levels of the source and relay. This is due to that fact that both the source and relay transmit for exactly half of the available time, ensuring only one transmitting antenna active at any one time. The transmitter can therefore transmit at full power (unity) during its allotted time slot. In the case where there is more than one supporting relay, the transmit power is shared equally among the relays in the second phase. Therefore each relay transmits with a power factor of $1/(|\mathcal{J}| - 1)$, since the source is not a part of the second phase of transmission[3].

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[2] The capacity of the system refers to the system Shannon capacity (also termed bandwidth efficiency as noted in the previous chapter) rather than another measure of capacity such as number of supported users. This qualification is carried though this thesis unless stated otherwise.

[3] Ideally the power constraint would be $1/(|\partial\mathcal{J}| - 1)$, however this is not possible as it would require the relays taking part in the second phase to know how many other relays are also taken part in the second phase, which cannot be known instantaneously. Rather a higher level protocol will govern how many relays *may* transmit in the second phase.

3.2.1 Channel model

The data signal transmitted by the source during the first and then by the individual relays during the second phases are denoted as $x_S[n]$ and $x_{R_j}[n]$, respectively. In the remainder of this chapter, symbol-by-symbol transmission is considered, subsequently the time index n of the transmitted symbols can be dropped, and are simply denoted x_S and x_{R_j}. The signal received at a destination terminal is given by the general expression

$$y_Y = \sqrt{E_{XY}} h_{XY} x + n_{Y_i} \tag{3.1}$$

where E_{XY} is the average signal energy received at the destination terminal, Y, over one symbol period from the transmitting source, X, having accounted for path-loss and shadowing, h_{XY} is the random, complex-valued, unit power channel gain as discussed in the previous section and $n_{Y_i} \sim \mathcal{CN}(0, N_0)$ is additive Gaussian white noise at the receiver Y during transmission phase i.

Specifically, during the first phase, each potential relay $R_j \in \mathcal{J} - S$ receives

$$y_{R_j} = \sqrt{E_{SR_j}} h_{SR_j} x_S + n_{R_{1,j}} \tag{3.2}$$

where y_{R_j} is the received signal at R_j during the first phase and $n_{R_{1,j}}$ is additive Gaussian white noise at the receiver at relay R_j in the first phase. If the relay can then decode the source transmission, R will support the source in the second phase, such that $R \in \partial(\mathcal{J})$.

The destination MT receives signals during both transmission phases. During the first phase, the received signal is modelled at D as

$$y_{D_1} = \sqrt{E_{SD}} h_{SD} x_S + n_{D_1} \tag{3.3}$$

in the appropriate channel, where y_{D_1} denotes the received signal by the destination in the first phase. During the second phase, the equivalent channel models are different for repetition based cooperative diversity and space-time coded schemes. For space-time coded cooperative diversity, all relay transmissions occur in the same channel and are combined at the destination, such that

$$y_{D_2} = \sum_{R_j \in \partial(\mathcal{J})} \sqrt{E_{RD_j}} h_{RD_j} x_{R_j} + n_{D_2} \tag{3.4}$$

in the appropriate channel, where y_{D_2} denotes the received signal by the destination in the second phase.

Two important parameters of the system are the received signal-to-noise ratio, SNR (dB) and the bandwidth efficiency, R (b/s/Hz). It is natural to define these parameters in terms of the continuous-time channel with non-cooperative diversity as a baseline. In this chapter, the simplistic ideal of normalising the attempted rate to 1 b/s/Hz, and assuming that all mobile terminals are equidistant from one another is assumed. This of course is unrealistic, but it serves as a baseline for comparing the results. All transmission schemes are constrained to transmit at a maximum power of unity, and transmit period T in line with the system constraints introduced in the previous section.

3.2.2 Direct transmission

In direct transmission, no extra mobile terminals are available to relay the signal in the second phase. This is termed the classical scheme, where the source transmits information at full power for all of the available time slot, Figure 3.2. The classical case is included to act as a baseline reference, for comparison of the advantages and disadvantages offered by the cooperative diversity schemes, with and without relays being able to support the source. The Shannon capacity of the direct transmission case is given as [52]

$$I_{dir} = \log_2(1 + \frac{E_{SD}}{N_0}|h_{SD}|^2) \quad \text{b/s/Hz} \tag{3.5}$$

3.3 Cooperative diversity - single antenna

To be able to add additional MTs to the network to act as relays for a particular target MT there is a need for a method which allows this addition to the network without sacrificing the amount of information which can be sent, while also meeting the time constraint. Figure 3.3 shows a network with three relay terminals contributing to the signal received at the intended destination, D. As introduced in the previous chapter STCs can exploit spatial diversity, which

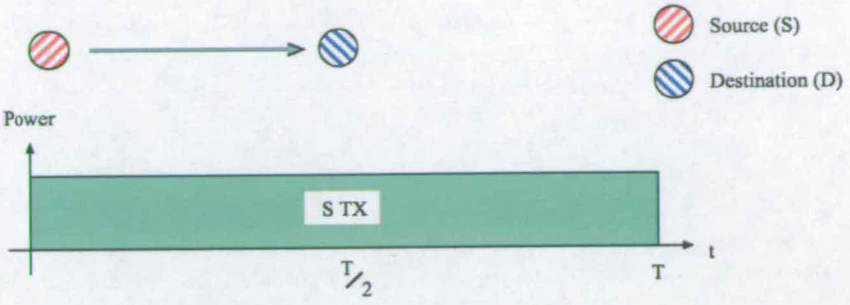

Figure 3.2: *Classical single-hop direct transmission between a source and destination terminal, each utilising a single antenna element.*

enables information from several sources to be transmitted to a single destination. The receiver can then combine the received signals to retrieve the original information. Since STCs use spatial diversity, all transmitters can transmit on the same frequency with the same modulation scheme at the same time.

It is not essential for STCs to be utilised for the destination to receive information from all of the relays. However, in order to fully exploit the spatial dimension cooperative diversity makes available and to facilitate the following analysis, it will be assumed thought out the remainder of this thesis that STCs will be employed in the second phase where more than two terminals transmit at the same time. Furthermore, note that rate 1 STCs do not exist for complex modulation schemes where more than two antenna elements are used. This leads to practical implementation issues in a network where more than two terminals transmit at the same time in the second phase, and must be considered in future work.

Using STCs for transmission during the second phase the relays transmit for $T/2$, however they are limited to transmit at $1/(|\mathcal{J}|-1)$ of full power due to the power constraint. The relays only retransmit the information that they received from the source, if they are able to decode the signal as required by the RRDF method. The mutual information for the STC signal antenna element case was derived by Laneman et al. [75].

With these constraints imposed on the STC case, STCs allow the power, time and bandwidth system constraints to all be met, while also allowing information from the source to be transmitted for half of the available time. Each transmission scheme is termed either full-time or half-time depending on how much of the information the source can transmit compared to the direct transmission case. For example, if the source can only transmit half the information

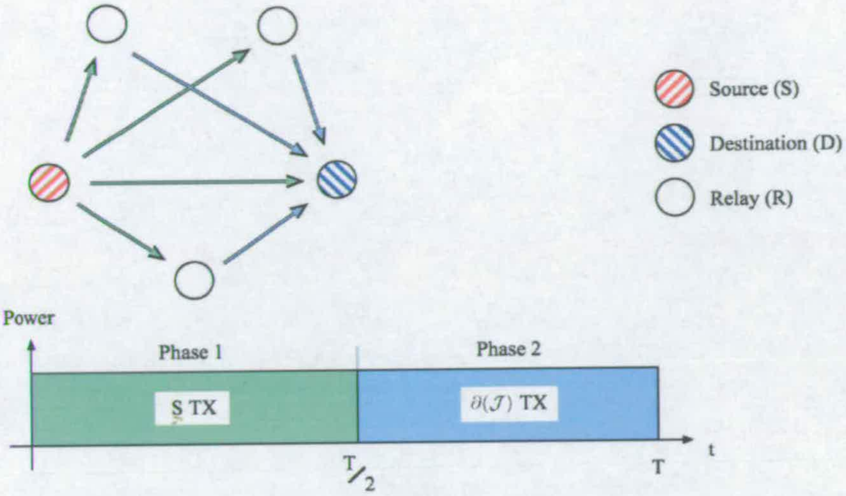

Figure 3.3: *Cooperative diversity model where multiple relays are available to support the source's transmission to the destination by transmitting information in the second phase, each utilising a single antenna element.*

the direct case would for a particular scheme, it will be termed half-time.

3.4 Cooperative diversity - dual antenna methods

It is likely that in future wireless systems it will be possible for mobile terminals the size of current hand held mobile phones to have two antenna elements, each of which will receive an independent signal from the transmitter in a high scattering environment. In the following two cooperative diversity schemes it is assumed that all MTs have two antenna elements available for transmission and reception, including the source. With MTs which are equipped with two antenna elements, transmission between MTs and between a base-station and MTs can use spatial diversity transmission techniques to help improve performance. Two different space-time transmit schemes are evaluated in this chapter for the dual antenna cooperative diversity case:

- Space-Time Transmit Diversity (STTD) [56].
- Vertical-Bell Laboratories Layered Space-Time Architecture (V-BLAST) Spatial Multiplexing (SM) [91].

In the proposed cooperative diversity schemes the same transmission technique is used in both phases, i.e. the STTD scheme uses STTD to transmit from the source to the relays and destination during phase one, and also from the relays to the destination during phase two. During phase two, STCs are used in a similar way as in the single antenna case to allow all relays to transmit at the same time, in the same channel.

3.4.1 STTD Half-time STC cooperative diversity

STTD is a multiple antenna transmission and reception technique which is used to provide robust communications channels. The information to be transmitted is split into two streams which are initially identical to the original information stream, each of which is then encoded to make them mutually orthogonal. In the two antenna STTD transmission case, the receiver will record

$$y_1 = h_1 x_1 + h_2 x_2 + n_1 \tag{3.6}$$

$$y_2 = h_1 x_2^* - h_2 x_1^* + n_2 \tag{3.7}$$

in both phases at each antenna (y_1 and y_2). Here h_i is defined as the channel coefficients, x_i the data symbol and n_i the additive noise. Note that as previously stated, symbol-by-symbol transmission is considered, subsequently a time index n is not shown in (3.6) or (3.7).

Each relay can then attempt to decode the STTD signal for retransmission. If it cannot decode either signal it will not take part in the second phase. In retransmitting, a relay will use STTD to transmit the two orthogonally separated information streams on to the intended destination, in combination with using its own STC to allow it to transmit at the same time as the other relays, Figure 3.4.

The mutual information for the channel is given by an extension of the mutual information equation derived by Laneman et al. [75] as

$$I_{sttd\_stc} = \frac{1}{2}\log_2\left(1 + \frac{E_{SD}}{N_0}\alpha_h + \frac{1}{(|\mathcal{J}|-1)}\sum_{R_j \in \partial(\mathcal{J})} \frac{E_{RD_j}}{N_0}\alpha_h\right) \tag{3.8}$$

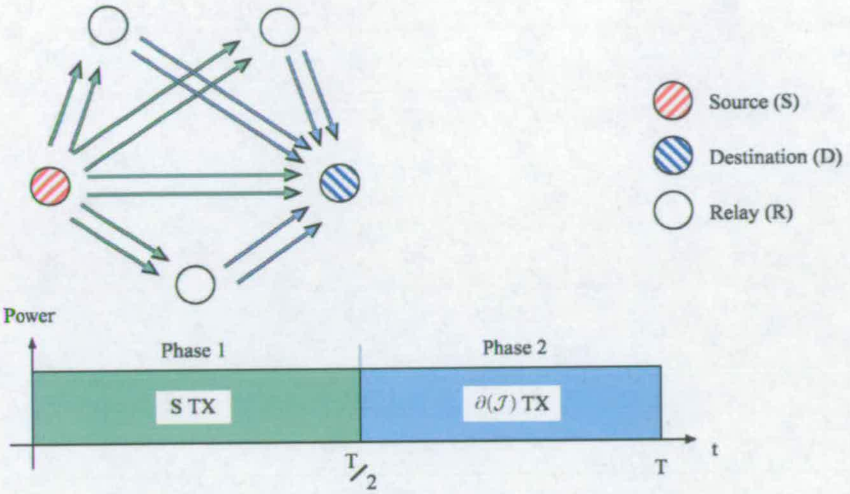

Figure 3.4: *Cooperative diversity model with multiple supporting relays, where each terminal is equipped with two antenna elements, using half-time STTD multiple antenna element transmission techniques.*

where

$$\alpha_h = \sum_{x=1}^{2}\sum_{y=1}^{2}\frac{|h_{xy}|^2}{2}$$

The scalar h_{xy} defines the channel coefficients between the xth transmit antenna element and the yth receive antenna element. In this case both x and y are limited to 2 since only two transmit/receiver antenna pairs elements are used in each MT.

In this scheme, the information between S and R for independent and identically distributed (i.i.d.) complex Gaussian code-books for each space-time coded information stream is given by

$$\frac{1}{2}\log_2\left(1+\frac{E_{SR_j}}{N_0}\alpha_h\right)$$

Under this rule the probability of a relay being able to decode the information stream as

$$\Pr[decode] = \Pr\left[\alpha_h > \frac{2^{2\mathrm{R}}-1}{E_{SD_J}/N_0}\right] \tag{3.9}$$

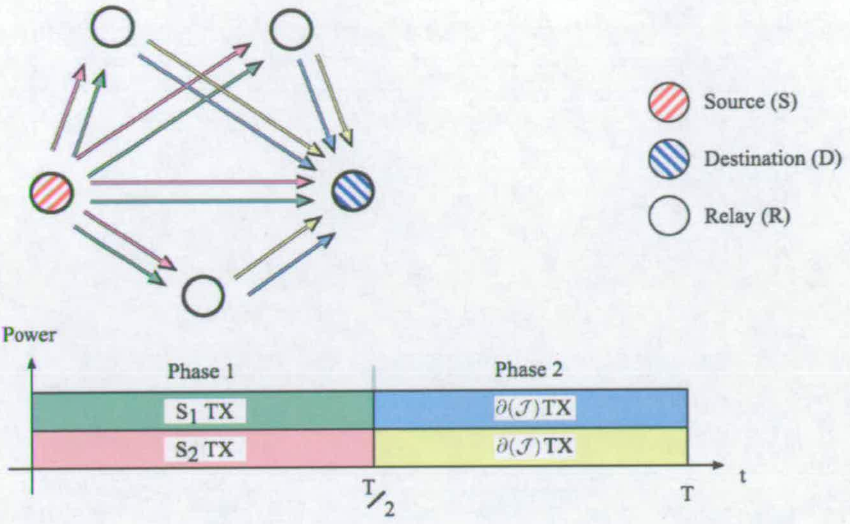

Figure 3.5: *Cooperative diversity model with multiple supporting relays, where each terminal is equipped with two antenna elements, using full-time SM transmission techniques to allow different information to be sent from each antenna element.*

3.4.2 SM Full-time STC cooperative diversity

Although STTD cooperative diversity, using two transmit antennas, can dramatically reduce the outage probability at the destination due to the increased SNR, it suffers from the fact that the source can only transmit information for a limited amount of the time. This leaves the direct transmission scheme with a distinct advantage, because it can always transmit new information, unless it needs to retransmit due to data-loss. In order to transmit all of the information that can be transmitted in the direct case, during phase one, it is possible to employ SM, Figure 3.5. Utilising SM in a MIMO relaying network has attracted considerable interest by the academic community [83, 84, 92]. In the following the current body of work is expended to consider the cooperative diversity scenario considered in this chapter.

Using SM, the original source data stream is demultiplexed into two data streams, which are each half the rate of the original. Each stream is then transmitted from a single antenna element, and received via the two antenna elements at the receiver. This transmission technique is known as V-BLAST. The mutual information channel average of a V-BLAST system, at the receiver, is given by [93] as

$$I_T = 2 * \min(I_1, I_2) \tag{3.10}$$

where $\min(I_1, I_2)$ is the minimum mutual information of the two information streams, I_T is the mutual information of the V-BLAST channel and

$$\begin{aligned} I_1 &= \log_2(1 + \frac{E_{SR_j,1}}{N_0}|h_{SR_j,1}|^2) \\ I_2 &= \log_2(1 + \frac{E_{SR_j,2}}{N_0}|h_{SR_j,2}|^2) \end{aligned}$$

where $\frac{E_{SR_j,k}}{N_0}$ denotes the SNR recorded from source S at destination R_j for each spatial multiplexing data stream k ($k = 1, 2, ..., N$). N is the number of transmit and receive antenna elements used, in this case $N = 2$. The notation for the channel fading is similarly extended, $h_{XY,k}$, for spatial multiplexing.

The destination will receive both information streams during the first phase, as will all relays. The sum capacity of each source to relay link is not calculated by the individual relays[4], rather the relays simply decode the received spatial multiplexing data streams, if it is possible to do so, and transmit the two separate spatially multiplexed information streams on with as high a rate as possible. Limited feedback is required to allow the relay to make the decision of which data stream should be transmitted from which antenna. In keeping with the RRDF protocol, if a relay cannot decode the transmission from the source in the first phase, it will not take part in the second phase transmission. This is done on a per stream basis (for example a relay might transmit both information streams received from the source, just one, or none).

It should be noted that the channel rate measurements would be made during a link set-up phase and this simplification was made for capacity calculation simulations. It is also noted that using $\min(I_1, I_2)$ of V-BLAST channel capacity reduces the feed-back required to the source from the destination, as the same modulation and coding is used for both antennas. This also reduces complexity in the relays since the same decoding and re-encoding is used for both information streams. However, using this V-BLAST algorithm reduces the link capacity as described in [94].

During the second phase, assume that the destination monitors the channel strength received from each transmit antenna and periodically uses limited feedback to the source and relays to

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[4] Calculating the sum capacity of the source to relay information streams would make it possible to recombine the information at the relay and then distribute it optimally between the antenna elements for second phase transmission.

allow them to choose which information stream to transmit from each antenna element. The relay will transmit the highest capacity stream it receives on the highest capacity link it has to the destination. The lower capacity stream will be transmitted on the slower link. If a stream cannot be decoded, like all the other cooperative diversity networks discussed in this chapter, it will not be transmitted on to the destination. In this case, to keep the scenario simple, no information is transmitted from the antenna element that would have been used if decoding was successful.

In a similar manner to the second phase in the half-time schemes, here MTs participating in the second phase will only be able to transmit at $1/|(|\mathcal{J}|-1)$ of full power. Again the source does not take part in the second phase.

For the SM full-time case the information capacity is given by

$$I_{sm\_stc} = 2 * \min\left(I_{sm\_1}, I_{sm\_2}\right) \tag{3.11}$$

$$I_{sm\_1} = \log_2\left(1 + \frac{E_{SD,1}}{N_0}|h_{SD,1}|^2 + \frac{1}{(|\mathcal{J}|-1)}\sum_{R_j \in \partial(\mathcal{J})} \frac{E_{RD_j,2}}{N_0}|h_{RD_j,1}|^2\right) \tag{3.12}$$

$$I_{sm\_2} = \log_2\left(1 + \frac{E_{SD,2}}{N_0}|h_{SD,2}|^2 + \frac{1}{(|\mathcal{J}|-1)}\sum_{R_j \in \partial(\mathcal{J})} \frac{E_{RD_j,2}}{N_0}|h_{RD_j,2}|^2\right) \tag{3.13}$$

The probability of each information stream being decoded is an extension of the direct transmission method. Since the realised mutual information between S and R_j for i.i.d. complex Gaussian code-books is given by

$$\frac{1}{2}\log_2(1 + \frac{E_{SR_j,2}}{N_0}|h_{SR_j}|^2)$$

Under this rule the probability of a relay being $R_j \in \partial(\mathcal{J})$ is given by

$$\Pr[R_j \in \partial(\mathcal{J})] = \Pr\left[|h_{SR_j}|^2 > \frac{2^{2\mathrm{R}}-1}{E_{SR_j,2}/N_0}\right] \tag{3.14}$$

3.4.3 Extensions to the proposed schemes

It is possible for the full-time scheme to be extended so that the source transmits different information to every relay in the network. This would allow the MIMO capacity limits to be approached, and will subsequently be termed max-time. However this would require that the both the source and destination have a number of antenna elements equal to the number of potential relays in the network, $|\mathcal{J}|$. As previously discussed this is not possible due to the size constraints of mobile terminals, and the requirements that antenna elements must be at least half a wavelength apart to maintain independent fading channels.

The max-time scheme would be applicable to mobile terminal where more than two antenna elements could be deployed, for example laptop computers or a remote fixed relay.

One possible extension of all the cooperative diversity schemes discussed above, instead of the relays simply decoding the information and then retransmitting it to the intended destination, is that the relays could transmit different information such as parity bits calculated from the received signal. Turbo coding [95] is also an option due to the parallel nature of the relaying signals. This could potentially dramatically decrease the probability of outage at the destination, however it is beyond the scope of this thesis.

3.5 Results and analysis

In order to discover the performance properties and practical suitability of the different cooperative diversity relaying techniques, Monte-Carlo simulations are presented, based on the capacity equations for each relaying scheme. The capacity of each is analysed in three different ways:

- Outage probability against SNR (required bandwidth efficiency fixed at 1 b/s/Hz)
- Bandwidth efficiency against SNR (outage fixed at 5%)
- Outage probability against bandwidth efficiency (SNR fixed at -5 dB).

Each combination of the three communication link characteristics has been included since analysing them all leads to a deeper insight. For example, the outage probability is expected to drop significantly for the repetition scheme when compared to the classic case. However, it would also be expected that the bandwidth efficiency of the link would drop significantly since redundant information is being transmitted. To readily present numerical results, in the remainder of this chapter it is assumed that all network channel links have the same SNR, specifically $E_{SR}/N_0 = E_{RD}/N_0 = E_{SD}/N_0 = \text{SNR}$.

For each relaying scheme, the results are compared directly to the single-hop single antenna direct transmission case, and then indirectly between each other. For the case where the bandwidth efficiency is fixed, 1 b/s/Hz has been chosen in order to make the calculations simpler, since it is a reasonable practical value. Note that this value is simply the threshold for outage in these results. Where the outage level is fixed, the 5% level has been chosen since this is likewise a practical value in a wireless communications network. Although a number of values for the case where the SNR at each receiver is fixed could have been selected, -5 dB was chosen since it is in the middle region of signal quality. Results for networks with up-to four relays in it ($|\mathcal{J}| = 5$) are presented. Note that where each terminal is equipped with a single antenna element $|\mathcal{J}| = 1$ is the equivalent of the classical link as no relays are able to support the source in the second phase.

3.5.1 Single antenna STC cooperative diversity

In single antenna STC based cooperative diversity, as the number of relays in the network is increased, although the transmit power for each relay is reduced to meet the power constraint, another spatial degree of freedom is added. It is clear that as the diversity degrees of freedom increase with $|\mathcal{J}|$, the probability of outage will reduce, Figure 3.6. This is clearly seen in Figure 3.6 where the SNR required to achieve a probability of outage of 0.01 changes from 15.4dB for a single relay to 10.8dB for four available relays. Due to the deceased transmit power as $|\mathcal{J}|$ increases, diminishing returns can be observed as more relays are added into the network. Note that at low SNRs Figure 3.6 shows that the classical scheme performs better. This is due to a combination of the time constraint, the power constraint and potentially poor source to relay link in the first phase[5]. If the source to relay link is poor then the relay will not be able to decode and transmit the information in the second phase, therefore the power of the transmission that would have been used by the relay in phase two is not used at any point in the transmission.

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[5] Note that this is equally likely as a poor source to destination link and a poor relay to destination link since it is assumed that all MTs are equally spaced.

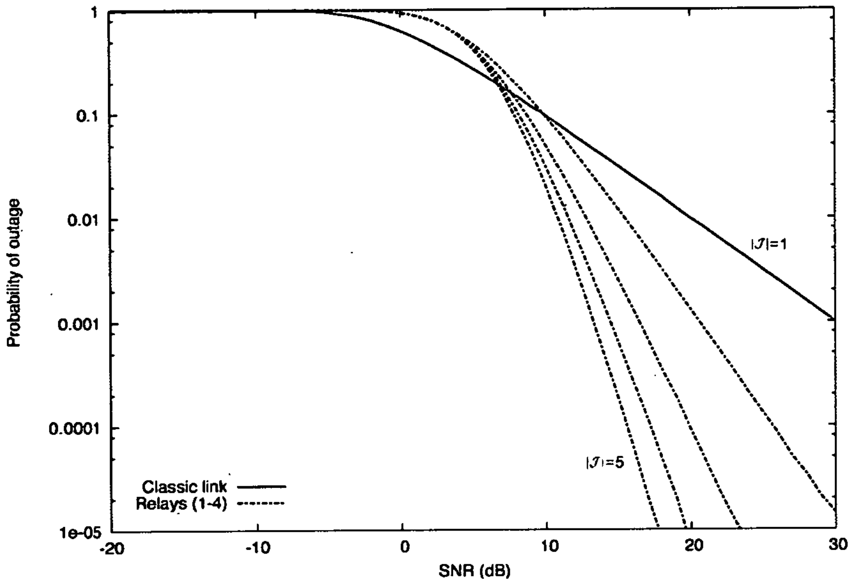

Figure 3.6: *Simulation results for single antenna STC based cooperative diversity scheme where the spectral efficiency is fixed at 1 b/s/Hz. SNR and outage probability are compared for* $|\mathcal{J}| = 1, 2...5$.

Figure 3.7 shows that the single antenna STC based cooperative diversity scheme offers improvements in outage probability when dealing with low Shannon capacities, compared to the classical case. This is due to the fact that redundant information is transmitted, and therefore is more likely to be received successfully. However, at higher capacities the classical transmission case begins to outperform the STC based scheme.

This effect is clearly seen in Figure 3.8 where at low SNRs the single antenna STC based scheme offers improvements in capacity over the classical case. However, at higher SNRs again, the classical case offers improved capacity.

3.5.2 Dual antenna STTD cooperative diversity

The dual antenna STTD half-time case builds on the single antenna cooperative diversity by adding another opportunity to use spatial diversity for transmission. The effect of using STTD on the probability of outage is clearly shown in Figure 3.9 where it can be seen that dual antenna STTD transmission greatly decreases the probability of outage at higher SNRs. Again there is a region at low SNRs where direct transmission is preferable.

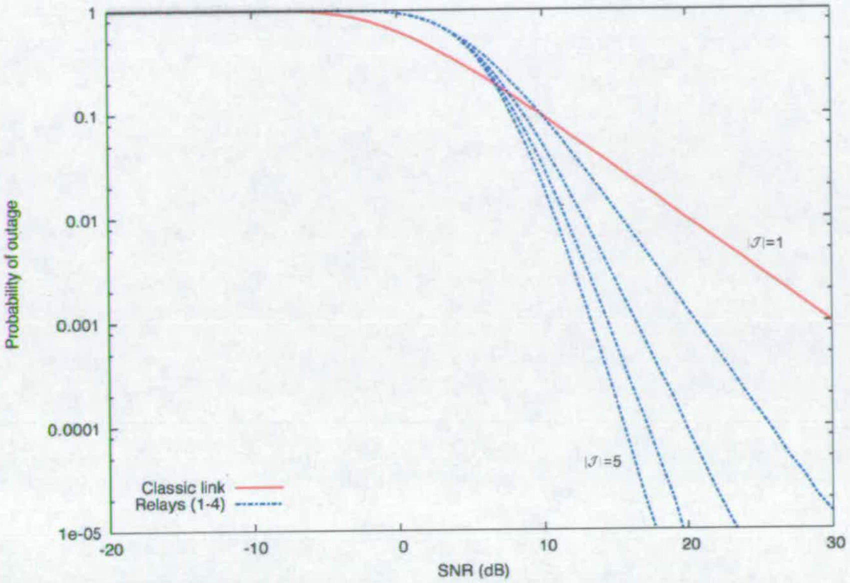

Figure 3.6: *Simulation results for single antenna STC based cooperative diversity scheme where the spectral efficiency is fixed at 1 b/s/Hz. SNR and outage probability are compared for* $|\mathcal{J}| = 1, 2 \ldots 5$.

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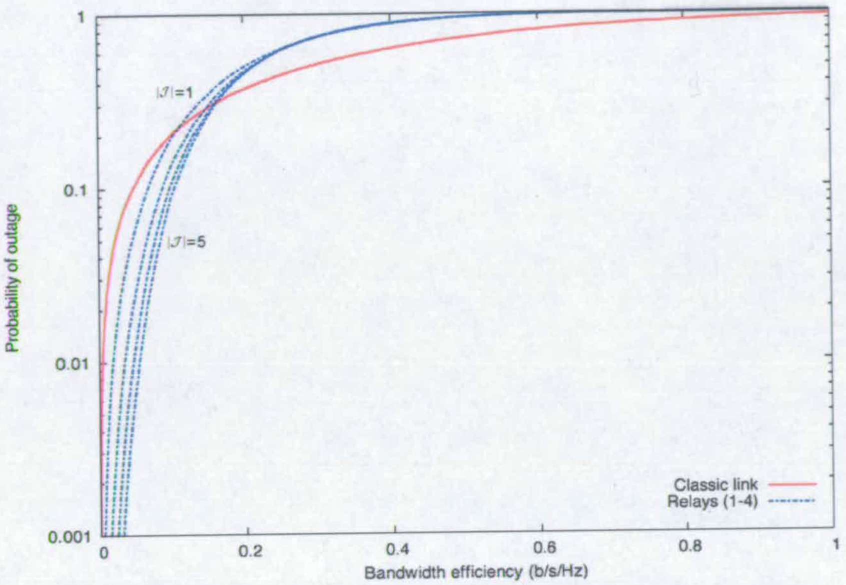

Figure 3.7: *Simulation results for the single antenna STC cooperative diversity scheme where the SNR is fixed at -5dB. Outage probability and bandwidth efficiency are compared for* $|\mathcal{J}| = 1, 2...5$.

The much lower probability of outage that dual antenna STTD can offer translates into being able to handle higher data rates than the classical and single antenna cases at a given outage probability, Figure 3.10. Figure 3.11 shows that the capacity of a cooperative diversity network with dual antenna STTD transmission also benefits. It is interesting to note that Figure 3.11 shows that using direct STTD transmission from the source to the destination is preferable over using one relay at low SNR. However, adding more than one relay into the network gives performance benefits over the direct STTD transmission case. This does not hold true at high SNR where the direct STTD transmission case out-performs the relay configurations, due to the split time phases required for cooperative diversity.

3.5.3 Dual antenna SM cooperative diversity

The dual antenna STTD half-time scheme suffers from not being able to transmit the same amount of original information over the same limited amount of time as the classical case. Full-time SM cooperative diversity attempts to overcome this problem by using SM from the source in the first phase as well as from the relays in the second phase. Two co-operating sub-sets are then created to perform the phase two transmission. This is done at the cost of

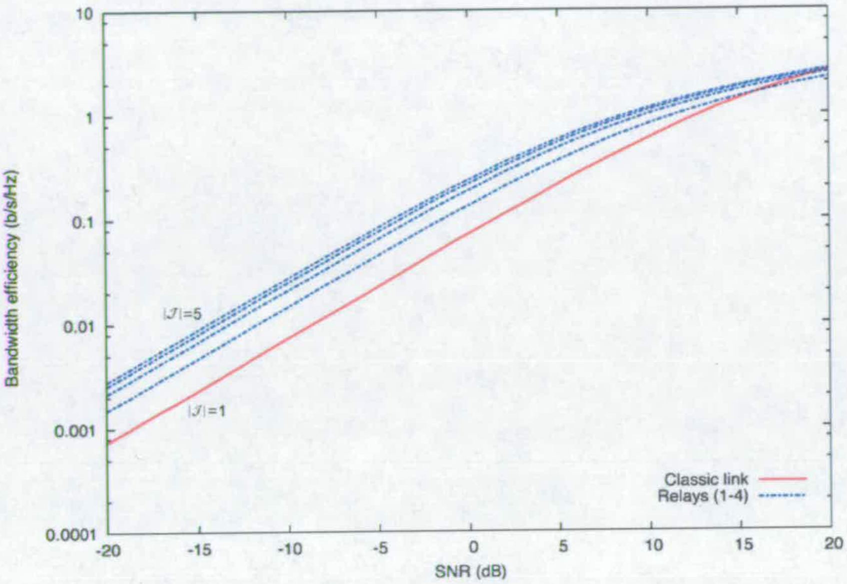

Figure 3.8: *Simulation results for single antenna STC cooperative diversity scheme where the outage level is fixed at 5%. Bandwidth efficiency and SNR are compared for* $|\mathcal{J}| = 1, 2...5$.

splitting the available transmit power at the source between two antenna elements.

Figure 3.12 clearly shows the huge benefit to the probability of outage offered by the full-time scheme. There is still a small region at low SNRs (lower than 5dB) where classical transmission would be preferred. This is due to the full power transmission that the source uses in the classical case. Again the diminishing returns for higher number of relays, due to the relay power transmission being constrained by $1/(|\mathcal{J}| - 1)$, can be observed.

When the outage probability against the bandwidth efficiency of STTD, shown in Figure 3.10, is compared with with the results obtained for SM, Figure 3.13, it can be seen that the half-time dual antenna STTD scheme offers large improvements when compared with the SM dual antenna case. However, when attempting higher capacities, direct transmission using SM would be preferred. This is again due to the split transmission power at the source.

Despite this, when the capacity is compared with the SNR in Figure 3.14, it can been seen that the full-time cooperative diversity scheme matches or betters the classical transmission case, and offers large improvements at SNRs lower than 15dB.

The results presented in this chapter confirm the results of [96] where it was found that STTD

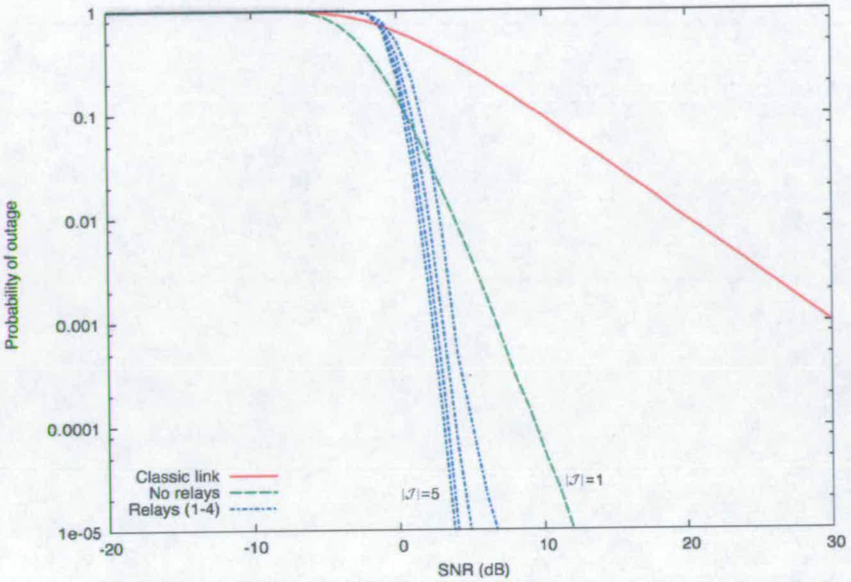

Figure 3.9: *Simulation results for dual antenna STTD STC based cooperative diversity scheme where the spectral efficiency is fixed at 1 b/s/Hz, with the single-hop single antenna classical link included for comparison. SNR and outage probability are compared for* $|\mathcal{J}| = 1, 2...5$.

transmission can be preferred to SM transmission under certain circumstances.

3.5.4 Degrees of freedom

One result that can be observed from the calculated results is that the largest performance gain does not necessarily come from the addition of the first relay in the next work, rather it comes from the first additional degree of freedom introduced to the network. The degrees of freedom considered in this chapter, due to the three constraints imposed on the system, come from spatial diversity. This can be spatial diversity introduced by using multiple antenna elements at the transmitter and receiver or it could stem from using other MTs as spatially separated relays for cooperative diversity. The results presented show that additional degrees of freedom that are introduced to the network give diminishing returns in performance, as previously noted for additional relays. This can be observed from all of the results of the scenarios presented, however, it can be seen particularly well in Figure 3.9, Figure 3.10 and Figure 3.11 where the first degree of freedom (using STTD transmission instead of the classical scenario) provides the biggest performance increase. Using cooperative diversity to add additional degrees of

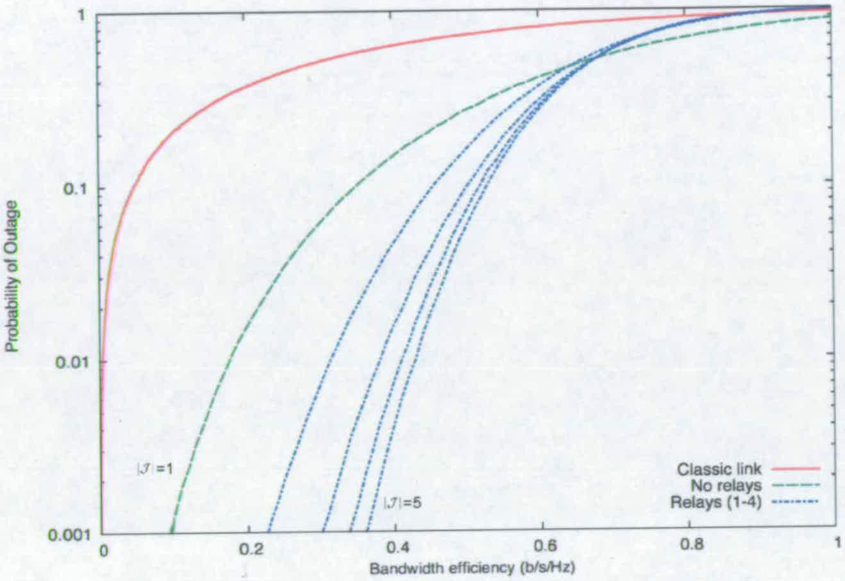

Figure 3.10: *Simulation results for dual antenna STTD STC based cooperative diversity scheme where the SNR is fixed at -5dB, with the single-hop single antenna classical link included for comparison. Outage probability and bandwidth efficiency are compared for* $|\mathcal{J}| = 1, 2...5$.

freedom, although it increases the performance of the network, does not provide significant additional performance. This would explain why STTD provides the best results, as it has the highest degrees of freedom of the scenarios considered in this chapter.

3.6 Conclusions

This chapter has introduced dual antenna elements for MTs in a cooperative diversity network and presented two different schemes for using the antenna elements to benefit wireless communications. Through Monte Carlo simulation utilising the derived capacity equations for each scheme it was observed that, as expected, the use of two antenna elements in MTs can add considerable capacity and outage benefits over the single antenna case.

Cooperative diversity is intrinsically limited by the need to have two transmission phases, and the limit of two antenna elements in any hand sized mobile terminal. Using the two antenna STTD or SM techniques presented in this chapter it is possible to overcome these problems and increase the capacity of the network. It has been shown that both in terms of probability

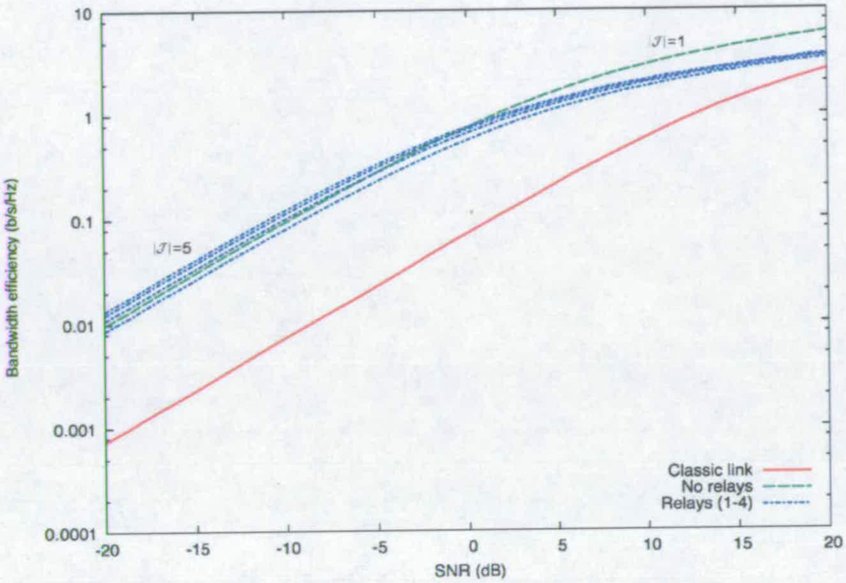

Figure 3.11: *Simulation results for dual antenna STTD STC based cooperative diversity scheme where the outage level is fixed at 5%, with the single-hop single antenna classical link included for comparison. Bandwidth efficiency and SNR are compared for* $|\mathcal{J}| = 1, 2...5$.

of outage and potential spectral efficiency STTD based cooperative diversity is preferred over SM. It is also noted that the largest gains come from the first supplementary degree of freedom, and diminishing returns are observed for additional degrees of freedom.

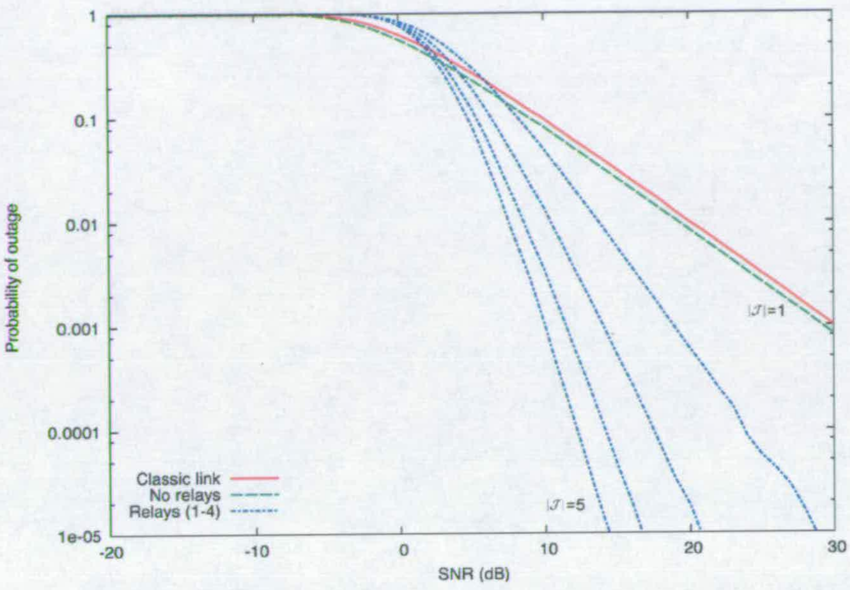

Figure 3.12: *Simulation results for dual antenna SM STC based cooperative diversity scheme where the spectral efficiency is fixed at 1 b/s/Hz, with the single-hop single antenna classical link included for comparison. SNR and outage probability are compared for* $|\mathcal{J}| = 1, 2...5$.

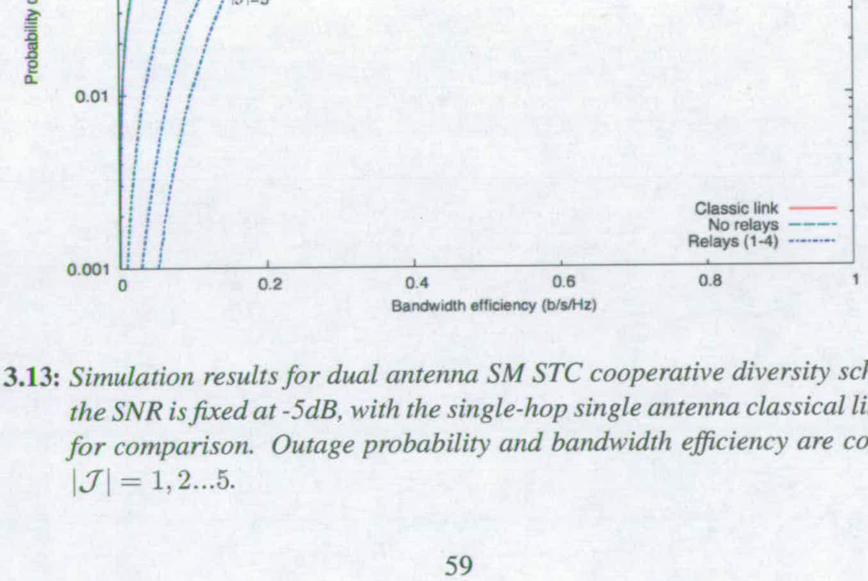

Figure 3.13: *Simulation results for dual antenna SM STC cooperative diversity scheme where the SNR is fixed at -5dB, with the single-hop single antenna classical link included for comparison. Outage probability and bandwidth efficiency are compared for* $|\mathcal{J}| = 1, 2...5$.

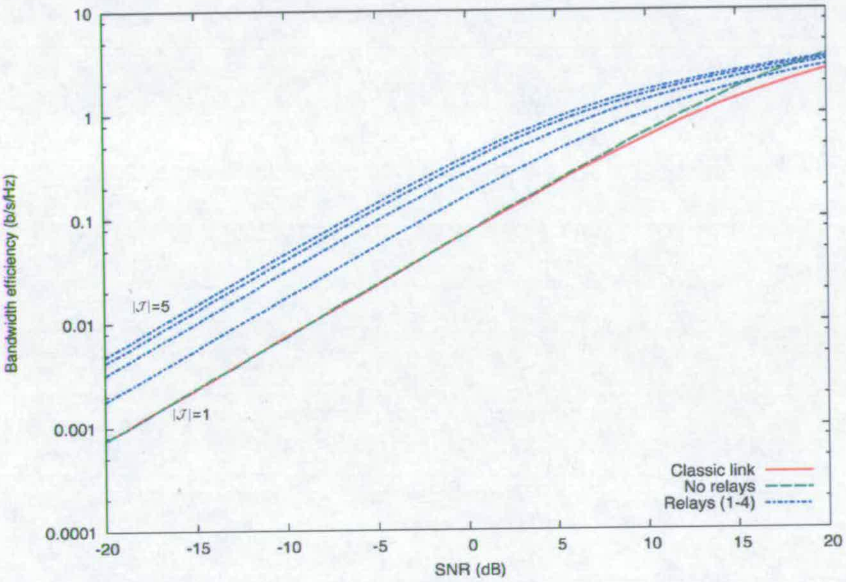

Figure 3.14: *Simulation results for dual antenna SM STC cooperative diversity scheme where the outage level is fixed at 5%, with the single-hop single antenna classical link included for comparison. Bandwidth efficiency and SNR are compared for* $|\mathcal{J}| = 1, 2 ... 5$.

Chapter 4
Relay network frameworks

4.1 Introduction

The previous chapter made use of a single relaying protocol to perform cooperative diversity. Clearly other relaying protocols are possible in the three terminal ad-hoc network, consequently Nabar et al. [77, 78] introduced two additional relaying protocols to complete the family of cooperative diversity protocols where one supporting relay is considered.

This chapter develops a framework for three available cooperative diversity protocols using principles and techniques from MIMO and introduces two new protocols to the relay network to be used as performance comparisons; the two-hop relay channel and the single-hop channel. The two additional relaying protocols are important as they can show whether the cooperative diversity protocols offer performance benefits over the current paradigm of non-cooperative communications. It can then be determined if the cooperative protocols are suitable for inclusion in future generation wireless specifications.

All relaying protocols in this chapter are subject to the system constraints introduced in the previous chapter to ensure that a fair comparison is made between each protocol, and specifically the single-hop link, although the power constraint is relaxed for discussion in section 4.2.1. Furthermore RRDF is employed as the relay forwarding method. Each protocol is considered in terms of MIMO (where possible) and MISO based transmission techniques and the developed framework is able to support both transmission types. The developed framework includes multiple cooperative relays which can assist the source in its communication with the destination, expanding on previous work in this field to consider multiple relays for all cooperative protocols.

To develop fully the relaying framework, section 4.2 introduces the signal and channel models alongside the cooperative diversity relaying protocols. Section 4.3 expands the analysis to consider the DF relaying method and investigates optimal transmit power levels for the probability of outage of the communications channel. Section 4.4 completes the framework by

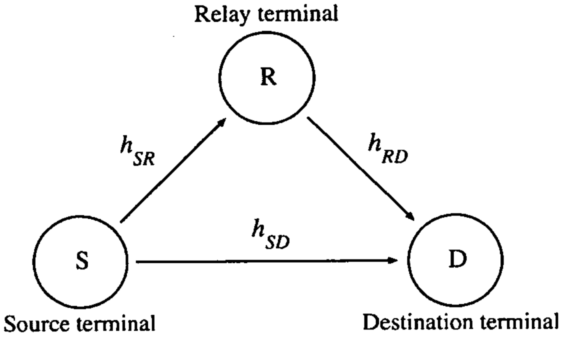

Figure 4.1: *Three terminal fading relay channel.*

analysing the performance of the various protocols in terms of outage, bandwidth efficiency and diversity. Finally section 4.5 draws conclusions for the work presented in this chapter.

4.2 Cooperative diversity relaying protocols

The dual antenna techniques developed in chapter 3 focused on a single cooperative diversity protocol which was originally proposed by Laneman et al. [76]. Two additional protocols were proposed by Nabar et al. [78] to complete the cooperative diversity family. In the following we extend this family to a generic relaying framework, with several potential relays and impose system constraints to provide consistency with single-hop links.

4.2.1 Protocol descriptions

Consider initially the fading relay channel shown in Figure 4.1 with three mobile terminals. Data is transmitted from the source terminal, S, to the destination terminal, D, potentially with the assistance of the relay, R. All terminals in this chapter are considered to have a single transmit and a single receive antenna element. Also a terminal cannot simultaneously transmit and receive information.

In each relay assist mode, it is possible to describe five different transmit protocols, each of which implements varying degrees of broadcast and receive collision in the relay network. The broadcast degree is given by the number of terminals listening to a single transmission at the same time (i.e., in the same phase), for example, one if only the destination listens to the source in the first phase, or two if both the destination and relay listen. Similarly the receive collision is given by the number of terminals transmitting at the same time. The full family of relay transmission protocols is now described using the same protocol naming terminology as [78] and extending it with two final protocols to complete the family.

Protocol I: In the first phase the source terminal signals to both the relay and destination. In the second phase both the source and relay communicate with the destination. This is the only protocol to utilise the maximum degrees of broadcast and receive collision.

Protocol II: This was the original cooperative diversity protocol proposed by Laneman et al. [76], and was used extensively in the previous chapter. The source communicates with both the relay and destination in the first phase, but remains silent in the second phase, where only the relay signals to the destination.

Protocol III: The final cooperative diversity protocol to utilise both the single-hop link and the two-hop link. Here the source transmits to the relay in the first phase but not to the destination. Both the source and relay then communicate with the destination in the second phase.

Protocol IV / relay channel: The source never communicates directly with the destination, instead transmitting only to the relay in the first phase. In the second phase the relay then communicates with the destination.

Protocol V/direct channel: For completeness of the family of relaying protocols, the single-hop channel is included here. The source communicates directly with the destination in both phases never utilising the relay as an extra transmitter.

The protocols described above are summarised in Table 4.1 and are shown diagrammatically with further information in Appendix A. Protocol's I, II and III are termed cooperative diversity protocols, in keeping with earlier publications in this field, since both source and relay can potentially communicate directly with the destination. Protocol IV is termed the two-hop relay channel and Protocol V is the classical single-hop channel.

It is possible to see that each relaying protocol may be preferred under certain channel and system conditions. For the following discussion it is assumed that each transmitting terminal transmits with the maximum power available to it (for example Protocol I would use 1.5 times more power than Protocol V). A tighter constraint is made on transmit power allocation later in this chapter.

Transmission phase	Protocol				
	I	II	III	IV	V
1	$S \rightarrow R, D$	$S \rightarrow R, D$	$S \rightarrow R$	$S \rightarrow R$	$S \rightarrow D$
2	$S \rightarrow D, R \rightarrow D$	$R \rightarrow D$	$S \rightarrow D, R \rightarrow D$	$R \rightarrow D$	$S \rightarrow D$

Table 4.1: *Relay channel transmission protocols. The Source, Relay and Destination terminals are denoted by S, R and D respectively. Communication between terminals is signified by $X \rightarrow Y$.*

As was previously noted, Protocol I utilises the full degrees of broadcast and receive collision and would be the preferred transmission protocol if all channels are available at all times. Protocol II makes use of the source transmission in only the first phase and therefore the source would be free in the second phase to transmit to a different destination terminal in the wider network. The converse of this is seen in Protocol III where by the destination terminal might use the first phase to transmit to another terminal, which would be useful if it were acting as a relay for another cooperative diversity channel, or in a multi-hop network. If the direct source to destination channel is particularly poor then Protocol IV might offer the best performance, and likewise if either the source to relay or relay to destination links are poor Protocol V might be preferred. Focussing on transmission power, it is possible to see that Protocols II, IV and V make use of only one transmitting terminal during each transmission phase, however Protocols I and III potentially use more than one transmitting terminal in each phase. This suggests that Protocols I and III are less power efficient. Note that although Protocols I and III potentially use more power than the other protocols, the capacity benefits of such a strategy may outweigh the cost in terms of power. However, as previously stated in the system constrains, the transmit power of all protocols in this thesis is limited to that of the single-hop case, and a relaxation of this constraint is left open for future research.

4.2.2 Channel and signal models

For the remainder of this chapter the channel fading model is assumed to be quasi-static Rayleigh fading channels with fading coefficients being circularly symmetric zero mean complex Gaussian random variables. No channel knowledge is assumed at the transmitters, however perfect channel knowledge is known to the receivers for the reverse channel. Perfect timing synchronisation is assumed, which is particularly important in the two phase network.

The complete signal model for the relay channel depends on the type of relaying used (AF or DF), therefore only general notes of the channel model are made here and the full DF mode channel is developed later in this chapter[1]. As in the previous chapter, symbol-by-symbol transmission is considered for the data signal transmitted during the first and second phases and is denoted x_1 and x_2, respectively. The signal received at a destination terminal during phase i is given by

$$r_{B_i} = \sqrt{E_{XY_i}} h_{XY_i} x_i + n_{Y_i} \tag{4.1}$$

where E_{XY_i} is the average signal energy received at the destination terminal over one symbol period through the $X \rightarrow Y$ link in the i^{th} phase, having accounted for path-loss and shadowing, h_{XY_i} is the random, complex-valued, unit power channel gain between terminals X and Y and $n_{Y_i} \sim \mathcal{CN}(0, N_0)$ is additive Gaussian white noise at the receiver Y.

Statistically, h_{XY_i} is modelled as a set of zero-mean, independent, circularly-symmetric complex Gaussian random variable with variance λ_{XY}, so that the magnitudes $|h_{XY_i}|$ are Rayleigh distributed and the phases $\angle h_{XY_i}$ are uniformly distributed on $[0, 2\pi)$. Furthermore, the additive noise n_{Y_i} is modelled as zero-mean mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_0. The scalar z_{Y_j} is assumed to capture the effects of receiver noise and other forms of interference in the system. Finally note that for the data symbols transmitted is it assumed that $\mathcal{E}\{x_i\} = 0$ and $\mathcal{E}\{|x_i|^2\} = 1$.

When considering the complete relay channel signal model, it can be seen that it resembles a MIMO system with two antenna elements at both the transmitter and receiver. The fading relay channel from Figure 4.1 is redrawn in Figure 4.2 to highlight the MIMO characteristics by splitting the destination into the two time phases, and adding an extra subscript to the channel notation to denote the phase of transmission. This is a schematic presentation only and the destination terminal actually uses only one antenna element. However, its access of the channel is split over the two time phases.

The relay channel, when considered in a MIMO fashion, can be seen to have a similar matrix structure of channel coefficients as a 2 × 2 MIMO channel. The primary difference is that the

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[1] It should be noted that it is possible using the material presented here to construct a similar framework for AF mode relaying.

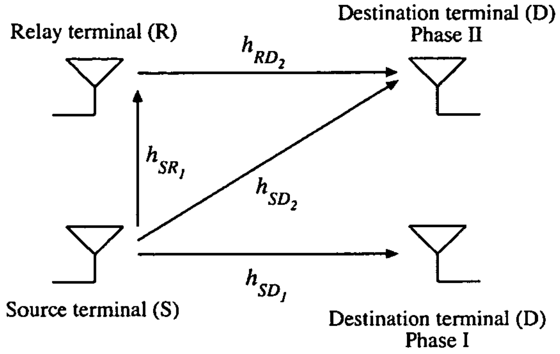

Figure 4.2: *Relay fading channel shown as a MIMO channel.*

relay cannot transmit in the first phase since it doesn't have any information to repeat. This leads to the MIMO channel matrix having a zero in it for the channel coefficient that represents the relay transmission in the first phase. The MIMO channel matrix considers one column for each transmitting antenna element, and one row for each receiving antenna element. This is extended in this analysis to one column for each transmitting terminal, and one row for each transmission time phase, in the case considered this leads to the following 2 × 2 channel matrix

$$\mathbf{H}_p = \begin{bmatrix} S \rightarrow D_1 & 0 \\ R \rightarrow D_2 & S \rightarrow D_2 \end{bmatrix} \tag{4.2}$$

where $X \rightarrow Y_i$ denotes transmission from terminal X to Y in phase i, and $\mathbf{H}_p$ is the generic channel matrix at the destination for the family of relaying protocols. Furthermore the transmitted data is given by the data transmitted in the first phase, x_1, and the data transmitted in the second phase, x_2[2], as

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4.3}$$

It follows from (4.2) and (4.3) that the input-output relation of the relay channel can be summarised as

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[2] Note that when no information is transmitted in the second phase from the source $x_2 = 0$

Protocol		
I	II	
$\mathbf{H}_1 = \begin{bmatrix} S \longrightarrow D_1 & 0 \\ R \longrightarrow D_2 & S \longrightarrow D_2 \end{bmatrix}$	$\mathbf{H}_2 = \begin{bmatrix} S \longrightarrow D_1 & 0 \\ R \longrightarrow D_2 & 0 \end{bmatrix}$	

Protocol		
III	IV	V
$\mathbf{H}_3 = \begin{bmatrix} 0 & 0 \\ R \longrightarrow D_2 & S \longrightarrow D_2 \end{bmatrix}$	$\mathbf{H}_4 = \begin{bmatrix} 0 & 0 \\ R \longrightarrow D_2 & 0 \end{bmatrix}$	$\mathbf{H}_5 = \begin{bmatrix} S \longrightarrow D_1 & 0 \\ 0 & S \longrightarrow D_2 \end{bmatrix}$

Table 4.2: *Channel matrices for the five relay channel transmission protocols. The Source, Relay and Destination terminals are denoted by S, R and D respectively. Communication between terminals is signified by $X \longrightarrow Y$, and $\mathbf{H}_p$ denotes the channel matrix for channel p. These are the channel matrices that give the received signal at the destination terminal.*

$$\mathbf{y} = \mathbf{H}_p\mathbf{x} + \mathbf{n} \tag{4.4}$$

where $\mathbf{y}$ is the received vector by the destination and $\mathbf{n}$ is additive white Gaussian noise. Table 4.2 shows $\mathbf{H}_p$ for $p = 1, 2...5$.

Of note in the above discussion is that the channel matrix does not consider the source to relay link in the first phase and therefore assumes that the relay is available in the second phase. While this assumption is made here, it does not hold true for all scenarios, and is discussed fully later in this chapter. Also of note is that the transmission of the source is not represented solely in the first column, rather it resides on the matrix diagonal. This is required due to the breakdown of the transmission from the source into two time phases, such that $\mathbf{H} \times \mathbf{H}^*$ gives the same channel matrix as the single-hop channel when only the source is considered. Furthermore, due to the transmission of information over two phases, when considering the capacity[3] of the relay network in a MIMO manner, a factor of 1/2 must be considered.

---

[3] As in the previous chapter the capacity refers to the Shannon capacity (also termed bandwidth efficiency) unless stated otherwise.

4.2.3 Part-time and full-time cooperative transmission

In the previous chapter, the concept of part-time and full-time cooperative diversity was introduced. In part-time transmission, only part of signal that could be transmitted in a single-hop case (i.e., exactly one-half due to the equal split of the two transmission time phases) is actually transmitted. Similarly, full-time transmission was considered whereby all information that could be transmitted in the single-hop case could also be transmitted in the cooperative diversity case. The following introduces a similar notion for the single antenna case considered here.

Protocol I and Protocol III offer the opportunity to introduce full-time cooperative diversity since the source transmits in both the first and second phase. Rather than having the source simply repeating the same data as it transmitted in the first phase in the second phase, it is possible to have it transmit different information, thus completing the requirement set out in the previous chapter to be considered full-time cooperative diversity.

When transmitting in full-time mode, Protocol I and Protocol III are termed MIMO cooperative diversity due to their similarity to the complete MIMO system. Their counterparts whereby the source repeats its first phase information are termed MISO, where by spatial diversity is offered by the relay[4]. Note that Protocol II can operate only in the MISO mode since it does not use source transmission in the second phase.

Although full-time transmission through MIMO techniques can potentially offer increased benefits over MISO transmission in terms of capacity, it also has higher complexity, particularly at the destination. It is possible that for the MIMO system with different information being transmitted in the second phase from the source, the destination will require two antenna elements to successfully decode both signals. Alternatively, if the destination receives information from the source in the first phase (Protocols I, II and V) it might be possible to successfully decode the signals in the second phase by using V-BLAST style subtraction. Although this is beyond the scope of this thesis, it would be an interesting avenue of future research.

This completes the family of cooperative diversity protocols and their transmission modes.

---

[4] The MISO term comes from considering a perfect source to relay link, leading to two spatially separated signals in the second phase to the signal antenna of the destination

4.2.4 Relaying system constraints

One of the aims of this work is to research the impact of using one or more of the cooperative diversity protocols in place of a single-hop channel. To make a fair comparison between the network models using cooperative diversity and the single-hop case several constraints are placed upon the set of relay channel models. These constraints were introduced in the previous chapter and ensure that the cooperative network cannot exceed the transmit time, power or bandwidth of the single-hop channel. As previously discussed the time and bandwidth constraints lead to a factor of 1/2 when considering the capacity of the channel.

The effect of the transmit power constraint is particularly important when considering Protocol I and Protocol III since they use three transmissions (the source in both phases and the relay in the second phase) and must therefore have limited power factors introduced. In order to do this, denote the transmit power factor from the source in the first phase, the second phase and the relay in the second phase by A_p, B_p and C_p respectively, where p denotes the protocol being used. This leads to the illustrative channel matrix below

$$\mathbf{H}_p = \begin{bmatrix} A_p * (S \rightarrow D_1) & 0 \\ C_p * (R \rightarrow D_2) & B_p * (S \rightarrow D_2) \end{bmatrix} \tag{4.5}$$

To constrain the transmit power to be the same as the single-hop case, first recall that the transmission in the relaying network has been split into two phases, therefore it follows that

$$A_p + B_p + C_p \leq 2 \tag{4.6}$$

A transmit power constraint is also placed upon each transmitting terminal such that it cannot broadcast with more power than the single-hop case would (unity), i.e.,

$$\begin{aligned} A_p &\leq 1 \\ B_p &\leq 1 \\ C_p &\leq 1 \end{aligned} \tag{4.7}$$

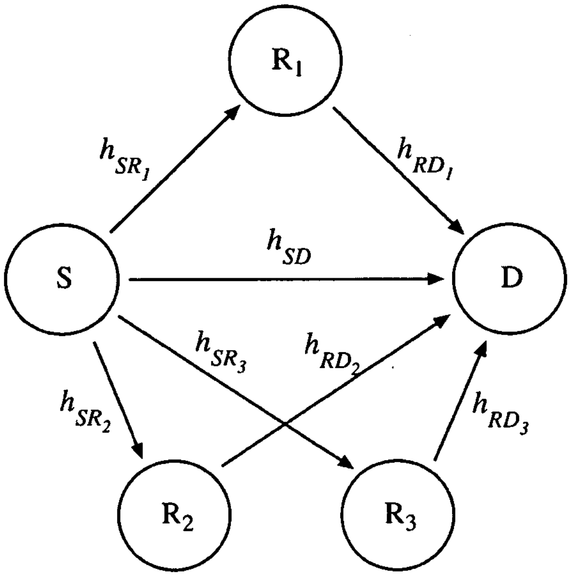

Figure 4.3: *Relay fading channel with three supporting relays.*

This gives a optimisation problem for what the optimal values of A_p, B_p and C_p are, which is considered later in this chapter.

4.2.5 Multiple supporting relay terminals

As noted in the previous chapter, work by Laneman et. al. [76] has shown that using more than one relaying terminal can be beneficial as it provides additional spatial diversity. The following extends the channel model framework introduced in this chapter to include multiple supporting relays, and also Laneman's analysis to the full family of cooperative diversity protocols.

In keeping with the MIMO channel model techniques previously used, note that additional relaying terminals simply add extra strength to the relay signal in the second phase, Figure 4.3. The addition of more than one relay terminal in the network does not alter the relay channel family of protocols as all relays have only the same signal to retransmit, thus there are two main types of transmitting terminals, the source and the relays.

As in the previous chapter, consider a relay channel with a set of transmitting terminals denoted $\mathcal{J} = \{1, 2, ..., j\}$. The source terminal, $S \in \mathcal{J}$, transmits to the destination terminal $D \notin \mathcal{J}$, potentially using $|\mathcal{J}|$ relays to support the signal[5]. The channel coefficients for the relays are given an additional subscript h_{XY_j} to indicate which relay is being addressed, which can be seen in Figure 4.3. The subset of $\mathcal{J}$ which can decode the signal according to the RRDF method is defined as $\partial(\mathcal{J})$, with cardinality $|\partial(\mathcal{J})|$.

The channel matrix can now be considered as a three dimensional structure; the source and relays (one column each), number of transmit phases (rows) and number of supporting relays (depth). Consider the relay channel shown in Figure 4.3 with $|\partial(\mathcal{J})|$ relays (in this case 3), the channel matrix is given by the $2 \times 2 \times 3$ matrix

$$\begin{aligned}
\mathbf{H}_{p,1} &= \begin{bmatrix} A(S \rightarrow D_1) & 0 \\ 0 & B(S \rightarrow D_2) \end{bmatrix} \\
\mathbf{H}_{p,2} &= \begin{bmatrix} 0 & 0 \\ C_1(R_1 \rightarrow D_2) & 0 \end{bmatrix} \\
\mathbf{H}_{p,3} &= \begin{bmatrix} 0 & 0 \\ C_2(R_2 \rightarrow D_2) & 0 \end{bmatrix} \\
\mathbf{H}_{p,4} &= \begin{bmatrix} 0 & 0 \\ C_3(R_3 \rightarrow D_2) & 0 \end{bmatrix}
\end{aligned} \tag{4.8}$$

where $\mathbf{H}_{p,j}$ indicates the generic channel matrix for protocol p and relay j, R_j denotes relay j and $C_{p,j}$ denotes the relay power allocation for relay j operating in protocol p with the power constraint[6]. The channel matrix presented in (4.8) is specifically the Protocol I matrix, however the channel matrices for the other protocols can be obtained by zeroing different terms where no information is transmitted. Specifically, the Protocol II channel matrix is obtained by zeroing the second phase source to destination term ($B(S \rightarrow D_2)$), the Protocol III channel matrix is obtained by zeroing the first phase source to destination term ($A(S \rightarrow D_1)$), the Protocol IV channel matrix is obtained by zeroing both source to destination terms, and the Protocol V channel matrix is obtained by zeroing all relay to destination terms.

---

[5] In this chapter it is possible for the source to transmit in the second phase, unlike the previous chapter.

[6] Note that to meet the power constraint (4.6) expanded to $A + B + \sum C_{p,j} \leq 2$ and also $C_{p,j} \leq 1$

To be able to exploit the maximum order of diversity offered by the multiple relay channel, each transmitting terminal in the second phase must transmit a different signal so they can be separated at the destination. Since multiple terminals are (potentially) communicating with the destination in the second phase at the same time there could be significant co-channel interference. Therefore STCs are used to separate (ideally) fully the received signals at the destination, similar to the multiple relay case in [76]. Each relay transmits using the signal x_{j+1} since the source must use the two initial data signals, x_1 and x_2, therefore giving a multi-dimensional structure similar to (4.8). Again consider the relay channel shown in Figure 4.3 with $|\partial(\mathcal{J})|$ transmitters (in this case four), the data vector for each transmitting terminal is given by

$$\begin{aligned} \mathbf{x}_1 &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \mathbf{x}_2 &= \begin{bmatrix} x_3 \\ 0 \end{bmatrix} \\ \mathbf{x}_3 &= \begin{bmatrix} x_4 \\ 0 \end{bmatrix} \\ \mathbf{x}_4 &= \begin{bmatrix} x_5 \\ 0 \end{bmatrix} \end{aligned} \tag{4.9}$$

Subsequently from (4.8) and (4.9) the input-output relation of the relay channel with $|\partial(\mathcal{J})|$ relays can be summarised as

$$\mathbf{y} = \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \mathbf{H}_{p,j}\mathbf{x}_j + \mathbf{n} \tag{4.10}$$

4.2.6 Relay channel capacity

In the following, an ergodic block-fading channel model with independent blocks is employed, and i.i.d. Gaussian codebook with covariance matrix $\mathbf{R}_{xx} = \mathcal{E}\{xx^H\} = \mathbf{I}_2$ is assumed. Also, note again that the destination is considered to have perfect channel knowledge of all channels in the network. The mutual information for the relay channel in MIMO mode is therefore obtained from (4.10) and (4.8) as

$$I_p^{MIMO} = \frac{1}{2}\log_2 \det\left(\mathbf{I}_2 + \frac{1}{N_0}\sum_{\mathcal{J}_j \in \partial(\mathcal{J})}(\mathbf{H}_{p,j})\sum_{\mathcal{J}_j \in \partial(\mathcal{J})}(\mathbf{H}_{p,j}^H)\right) \text{b/s/Hz}, \qquad p = 1, 2 \ldots 5 \tag{4.11}$$

where I_p^{MIMO} denotes the MIMO mutual information for protocol p. It is important to note that although (4.11) can apply to all five relaying protocols, only Protocols I, III and V can take full advantage of the MIMO nature of the channel, since those protocols are able to transmit different information from the source in the second phase. This is not possible in Protocols II and IV and when (4.11) is used with the corresponding channel matrix for these protocols the vector $\mathbf{x}$ becomes the scalar x_1 indicating only first phase information can be successfully transmitted. This distinction is also made clear in Appendix A which shows the relaying protocols graphically. The time and bandwidth system constraints applied are accounted for by the factor of 1/2 and the power constraint by the power allocation factors introduced to the channel matrix (4.5).

Similar to the MIMO mutual information, the MISO mutual information for the relay channel protocols can be defined by the sum of the transmitting terminals, which is given by taking the Frobenius Norm of the channel matrix such that

$$I_p^{MISO} = \frac{1}{2}\log_2 \det\left(\mathbf{I}_2 + \frac{1}{N_0}||\sum_{\mathcal{J}_j \in \partial(\mathcal{J})}(\mathbf{H}_{p,j})||^2\right) \text{b/s/Hz}, \qquad p = 1, 2 \ldots 5 \tag{4.12}$$

As previously noted, all protocols can transmit in MISO mode, which is shown in (4.12). Once more, note that (4.11) and (4.12) assume a perfect source to relay link. This constraint is relaxed and fully explored in the following sections of this chapter where the full mutual information expressions are explored for each protocol. Finally, it is also important to note that when operating in MISO mode, those protocols which transmit information from the source in the second phase must use STCs to enable the destination to separate the signals from the relays and that from the source.

In order to make a direct comparison with the single-hop channel, Protocol V is assumed to operate in MIMO mode in the remainder of this chapter, as this is equivalent of the single-hop channel. Protocol V in MISO mode is only included here for completeness.

4.3 Decode and Forward framework

Following the previous section introduction of a general relay channel framework, this section considers RRDF mode relaying. This section initially completes channel model for the DF model and then explores the optimal transmit power levels to meet the power constraint for Protocol.I and Protocol III.

4.3.1 Channel model

The DF mode channel model is now completed based on the general channel model introduced in previous sections of this chapter. Consider initially Protocol I as the other protocols can be considered to be derivatives of Protocol I. The signal received by the destination during the first phase is given by (4.1) as

$$y_{D_1} = \sqrt{A_1 E_{SD}} h_{SD} x_1 + n_{D_1} \tag{4.13}$$

Similarly the signal received by each relay terminal in the first phase is given by

$$y_{R_{j,1}} = \sqrt{A_1 E_{SR_j}} h_{SR_j} x_1 + n_{R_j} \tag{4.14}$$

where $y_{R_{j,1}}$ denotes the signal received by relay R_j in the first phase. In the second phase the destination terminal receives a superposition of the signal transmitted by the source and from all relays in the network which can fully decode the received first phase signal which is given by

$$y_{D_2} = \sqrt{B_1 E_{SD}} h_{SD} x_2 + \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \sqrt{C_{j,1} E_{RD_j}} h_{RD_j} x_{j+1} + n_{D_2} \tag{4.15}$$

where $\partial(\mathcal{J})$ denotes the subset of relay terminals which can fully decode the first phase signal and are therefore available to support the source in the second phase. In terms of the three dimensional channel matrix (4.8), wherever a relay is not able to decode the source's first phase information a zero is inserted.

The general channel matrix for Protocol I with a single supporting relay is then given by

$$\mathbf{H}_1 = \begin{bmatrix} \sqrt{A_1 E_{SD}} h_{SD} & 0 \\ \sqrt{C_{j,1} E_{RD_j}} h_{RD_j} & \sqrt{B_1 E_{SD}} h_{SD} \end{bmatrix} \tag{4.16}$$

Moreover, as noted in section 4.2.5, the channel matrix for Protocol II is obtained by zeroing the $\sqrt{B_1 E_{SD}} h_{SD}$ term since no information is transmitted from the source to destination in the second phase. A similar procedure is followed to obtain the channel matrices for the remaining protocols.

4.3.2 Information theoretic analysis

Since the channel mutual information, I_p, is a function of the random fading coefficients, the mutual information is also a random variable. The event $I_p < \mathrm{R}$ is defined as the channel being in outage and therefore $\Pr[I_p < \mathrm{R}]$ is referred to as the outage probability of the channel. Moreover, as $\partial(\mathcal{J})$ is a random set of relays, the probability of outage conditioned on each possible relay set can be summed to give the total probability of outage for the channel. The total probability law can be employed to write

$$\Pr[I_p < \mathrm{R}] = \sum_{\partial(\mathcal{J})} \Pr[\partial(\mathcal{J})] \Pr[I_p < \mathrm{R} | \partial(\mathcal{J})] \tag{4.17}$$

This condition for inclusion in the relaying set satisfies the channel matrix and mutual information formulas (4.11) and (4.12). Note also that if there are no relays with a suitably strong signal to decode the information and then retransmit it to the destination in the second phase, the channel matrix takes the form of the single-hop channel (Protocol V).

In the following section Protocol I is the primary protocol to be considered since all other protocols can be considered to be derivatives of it. From the channel matrix for Protocol I given in Table 4.2 and equations (4.8) and (4.11) the mutual information for Protocol I operating in MIMO mode and conditioned on the relay set $\partial(\mathcal{J})$ can be derived for a single relay by

$$\begin{aligned}
I_{1_M} =& \frac{1}{2}\log_2\det\left(\mathbf{I}_2 + \frac{1}{N_0}\sum_{R_j\in\partial(\mathcal{J})}(\mathbf{H}_{1,j})\sum_{R_j\in\partial(\mathcal{J})}(\mathbf{H}_{1,j}^H)\right) && (4.18)\\
=& \frac{1}{2}\log_2\det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right.\\
&+ \left.\begin{bmatrix} \sqrt{A_{1_M}E_{SD}}h_{SD} & 0 \\ \sqrt{C_{1_M}E_{RD}}h_{RD} & \sqrt{B_{1_M}E_{SD}}h_{SD} \end{bmatrix}\begin{bmatrix} \sqrt{A_{1_M}E_{SD}}h_{SD} & 0 \\ \sqrt{C_{1_M}E_{RD}}h_{RD} & \sqrt{B_{1_M}E_{SD}}h_{SD} \end{bmatrix}^H\right) && (4.19)\\
=& \frac{1}{2}\log_2\det\left(\begin{bmatrix} A_{1_M}E_{SD}h_{SD}^2 + 1 & \sqrt{A_{1_M}C_{1_M}E_{SD}E_{RD}}h_{SD}h_{RD} \\ \sqrt{A_{1_M}C_{1_M}E_{SD}E_{RD}}h_{SD}h_{RD} & B_{1_M}E_{SD}h_{SD}^2 + C_{1_M}E_{RD}h_{RD}^2 + 1 \end{bmatrix}\right) && (4.20)\\
=& \frac{1}{2}\log_2\left(\frac{A_{1_M}B_{1_M}E_{SD}^2}{N_0}|h_{SD}|^4 + \frac{A_{1_M}E_{SD}}{N_0}|h_{SD}|^2\right.\\
&+ \left.\frac{B_{1_M}E_{SD}}{N_0}|h_{SD}|^2 + \frac{C_{1_M}E_{RD}}{N_0}|h_{RD}|^2 + 1\right) && (4.21)
\end{aligned}$$

where the power transmit levels are denoted A_{1_M}, B_{1_M} and C_{1_M} to indicate that they are used for Protocol I MIMO transmission.

More generally for multiple relays, the mutual information for Protocol I in MIMO mode can be written as

$$\begin{aligned}
I_{1_M} =& \frac{1}{2}\log_2\left(\frac{A_{1_M}BE_{SD}^2}{N_0}|h_{SD}|^4 + \frac{A_{1_M}E_{SD}}{N_0}|h_{SD}|^2\right.\\
&+ \left.\frac{B_{1_M}E_{SD}}{N_0}|h_{SD}|^2 + \sum_{\mathcal{J}_j\in\partial(\mathcal{J})}\frac{C_{j,1_M}E_{RD_j}}{N_0}|h_{RD_j}|^2 + 1\right)\text{b/s/Hz} && (4.22)
\end{aligned}$$

Equally the MISO mutual information for Protocol I conditioned on the relay set $\partial(\mathcal{J})$ can be found from (4.12) as

$$I_{1_S} = \frac{1}{2}\log_2\left(\frac{A_{1_S}E_{SD}}{N_0}|h_{SD}|^2+\frac{B_{1_S}E_{SD}}{N_0}|h_{SD}|^2+\sum_{\mathcal{J}_j\in\partial(\mathcal{J})}\frac{C_{j,1_M}E_{RD}}{N_0}|h_{RD_j}|^2+1\right)\text{b/s/Hz} \tag{4.23}$$

where the power transmit levels are denoted A_{1_S}, B_{1_S} and C_{1_S} to indicate that they are using for Protocol I MISO transmission.

From (4.22) and (4.23) by inspection the MIMO case will always outperform the MISO case under identical channel and power constraint conditions due to the extra $|h_{SD}|^4$ term in the equation. However, this gain comes at the expense of extra decoding complexity and, as was previously noted, this is a prime topic for future research.

The mutual information equations for the remaining protocols can be found from the individual channel matrices in Table 4.2 and are shown in Appendix A along side the channel matrices and a diagrammatic view of the protocol's relay channel.

4.3.3 Optimal transmit power allocation

4.3.3.1 Protocol I MISO analysis

To optimise the transmit power levels A_{1_S}, B_{1_S} and C_{1_S} so the probability of outage is minimised at a particular SNR while meeting the power constraint on the network, initially consider the Protocol I MISO transmission case with one potential relay. Using a single relay in the network ensures that the results can be visualised in two dimensions and potentially extended to the full $\mathcal{J}$ set of relays in future. Furthermore, the MISO transmission case is considered prior to the MIMO case as it can readily be described analytically. In the remainder of this chapter it is assumed that all network channel links have the same SNR, specifically $E_{SR}/N_0 = E_{RD}/N_0 = E_{SD}/N_0 = \text{SNR}$.

Since A_{1_S},B_{1_S} and C_{1_S} are limited by (4.6), by varying A_{1_S} and B_{1_S} between 0 and 1 independently, the entire range of results are explored. Expanding (4.17) for the single relay case gives

$$\begin{aligned} \Pr\left[I_{1_{MISO}} < \mathrm{R}\right] &= \Pr\left[\partial(\mathcal{J}_0)\right]\Pr\left[I_{1_S} < \mathrm{R}|\partial(\mathcal{J}_0)\right] \\ &+ \Pr\left[\partial(\mathcal{J}_1)\right]\Pr\left[I_{1_S} < \mathrm{R}|\partial(\mathcal{J}_1)\right] \end{aligned} \tag{4.24}$$

where $\partial(\mathcal{J}_q)$ denotes the decoding set that has q relays in it. Substituting the required mutual information equations into (4.24) and rearranging so the random variables are on the Left Hand Side (LHS) of each individual probability inequality gives

$$\begin{aligned} \Pr\left[I_{1_S} < \mathrm{R}\right] =& \Pr\left[|h_{SR}|^2 < \frac{2^{2\mathrm{R}}-1}{A_{1_S}\mathrm{SNR}}\right]\Pr\left[|h_{SD}|^2 < \frac{2^{2\mathrm{R}}-1}{(A_{1_S}+B_{1_S})\mathrm{SNR}}\right] \\ &+\Pr\left[|h_{SR}|^2 > \frac{2^{2\mathrm{R}}-1}{A_{1_S}\mathrm{SNR}}\right]\Pr\left[(A_{1_S}+B_{1_S})|h_{SD}|^2 + C_{1_S}|h_{RD}|^2 < \frac{2^{2\mathrm{R}}-1}{\mathrm{SNR}}\right] \end{aligned} \tag{4.25}$$

The probability inequalities with a single channel coefficient on the LHS can be solved analytically as they are single Rayleigh random variables for which the Cumulative Distribution Function (CDF) is well known as $[1-\exp(-x)]$ when the standard deviation is 1 (as it is in this case). Moreover x for the Rayleigh CDF is given by the Right Hand Side (RHS) for each probability inequality.

The CDF of the more complex joint distribution of the two random variables $(A_{1_S} + B_{1_S})|h_{SD}|^2 + C_{1_S}|h_{RD}|^2$ is not so well known. To find an analytical expression for the probability of outage conditioned on a single relay an expression for the CDF of the joint random variables must be found. This is done by convolving the PDF of each random variable to obtain the joint PDF, which is then integrated to obtain the required CDF. To start this process note that the PDF of $(A_{1_S} + B_{1_S})|h_{SD}|^2$ is given by

$$\frac{1}{A_{1_S}+B_{1_S}} e^{\frac{-x}{A_{1_S}+B_{1_S}}}. \tag{4.26}$$

The PDF of $C_{1_S}|h_{RD}|^2$ is found in a similar fashion. Convolving these PDFs leads to

$$
\begin{aligned}
(f*g)(x) &= \int_0^x \frac{1}{A_{1_S}+B_{1_S}} \mathrm{e}^{\frac{-v}{A_{1_S}+B_{1_S}}} . \frac{1}{C_{1_S}} \mathrm{e}^{\frac{-x+v}{C_{1_S}}}\, dv \\
&= \frac{1}{C_{1_S}(A_{1_S}+B_{1_S})} \int_0^x \mathrm{e}^{\frac{-v}{A_{1_S}+B_{1_S}} - \frac{x}{C_{1_S}} + \frac{v}{C_{1_S}}}\, dv \\
&= \frac{1}{C_{1_S}(A_{1_S}+B_{1_S})} \frac{1}{\frac{1}{C_{1_S}} - \frac{1}{A_{1_S}+B_{1_S}}} \left[\mathrm{e}^{\frac{-x+v}{A_{1_S}+B_{1_S}} \cdot \frac{-v}{C_{1_S}}} \right]_0^x \\
&= \frac{1}{A_{1_S}+B_{1_S}-C_{1_S}} \left(\mathrm{e}^{\frac{-x}{C_{1_S}}} - \mathrm{e}^{\frac{-x}{A_{1_S}+B_{1_S}}} \right) \mathrm{e}^{-x \frac{A_{1_S}+B_{1_S}+C_{1_S}}{(A_{1_S}+B_{1_S})C_{1_S}}}
\end{aligned} \tag{4.27}
$$

Finally integrating the resulting PDF (4.27) to give the CDF

$$
\begin{aligned}
&\int_0^x \frac{\left(\mathrm{e}^{\frac{-x}{C_{1_S}}} - \mathrm{e}^{\frac{-x}{A_{1_S}+B_{1_S}}} \right) \mathrm{e}^{-x \frac{A_{1_S}+B_{1_S}+C_{1_S}}{(A_{1_S}+B_{1_S})C_{1_S}}}}{A_{1_S}+B_{1_S}-C_{1_S}} = \\
&\qquad \frac{A_{1_S}(1-\mathrm{e}^{\frac{-x}{A_{1_S}+B_{1_S}}}) + B_{1_S}(1-\mathrm{e}^{\frac{-x}{A_{1_S}+B_{1_S}}}) - C_{1_S}(1+\mathrm{e}^{\frac{-x}{C_{1_S}}})}{A_{1_S}+B_{1_S}-C_{1_S}}
\end{aligned} \tag{4.28}
$$

It can been seen that (4.28) does not apply if $A_{1_S} + B_{1_S} = C_{1_S}$ due to the zero term in the denominator. This special case, due to the power constraints placed on the system, is only possible when $A_{1_S} + B_{1_S} = C_{1_S} = 1$. Under these conditions the combination of the fading coefficients becomes $h_{SD} + h_{RD}$ which is a Chi-Squared random variable with four degrees of freedom. Therefore from the Chi-Squared law it can be shown that the CDF for this special case is given by

$$
1 - \mathrm{e}^{-x(x+1)}, \qquad A_{1_S} + B_{1_S} = C_{1_S}. \tag{4.29}
$$

From the Rayleigh CDF functions and (4.28), and also letting $x = (2^{2\mathrm{R}} - 1)/\mathrm{SNR}$, an analytical expression can now be written for (4.24) such that

$$\Pr\left[I_{1_S} < \mathbb{R}\right] = \begin{cases} \left(1 - \mathrm{e}^{\frac{-x}{A_{1_S}}}\right)\left(1 - \mathrm{e}^{\frac{-x}{A_{1_S}+B_{1_S}}}\right) + \left(\mathrm{e}^{\frac{-x}{A_{1_S}}}\right) \cdot \\ \qquad \left(\dfrac{A_{1_S}(1 - \mathrm{e}^{\frac{-x}{A_{1_S}+B_{1_S}}}) + B_{1_S}(1 - \mathrm{e}^{\frac{-x}{A_{1_S}+B_{1_S}}}) - C_{1_S}(1 + \mathrm{e}^{\frac{-x}{C_{1_S}}})}{A_{1_S} + B_{1_S} - C_{1_S}}\right) \\ & A_{1_S} + B_{1_S} \neq C_{1_S} \\ \left(1 - \mathrm{e}^{\frac{-x}{A_{1_S}}}\right)\left(1 - \mathrm{e}^{\frac{-x}{A_{1_S}+B_{1_S}}}\right) \\ \quad + \left(\mathrm{e}^{\frac{-x}{A_{1_S}}}\right)\left(1 - \mathrm{e}^{-x}(x+1)\right), & A_{1_S} + B_{1_S} = C_{1_S}. \end{cases} \tag{4.30}$$

4.3.3.2 Numerical results

To conclude the discussion of optimal transmit power levels for MISO Protocol I, results are presented from numerical calculation of the outage probability (4.30) derived in the previous section. This is done by calculating the probability of outage for a high accuracy range of possible combinations of A_{1_S}, B_{1_S} and C_{1_S}, specifically varying A_{1_S} and B_{1_S} between 0 and 1 with an incremental step of 0.001, to find the lowest outage level for specific values of SNR at a particular spectral efficiency. Figure 4.4 and Figure 4.5 show the probability of outage surface obtained at 0dB and 10dB for an RRDF required spectral efficiency of 1b/s/Hz, respectively. Contours are displayed with the surface to easily identify the surface gradient.

Optimal power levels for A_{1_S}, B_{1_S} and C_{1_S} results are presented in Figure 4.6 for a required spectral efficiency of 1b/s/Hz.

As the results show the initial transmit power level from the source in the first phase, A_{1_S}, is always preferred to utilise full transmit power and therefore the remaining power is split between the two second phase transmitting terminals. Figure 4.6 shows that below approximately 5dB SNR it is preferred that the relay not transmit and all of the available power is used by the source in both transmission phases. This is due to the fact that at low SNR the required spectral efficiency is less likely to be met by the relay link than the single-hop link. At higher SNR it can be seen that the transmit power levels B_{1_S} and C_{1_S} tend towards an asymptote.

Between 5dB and 6dB at 1b/s/Hz spectral efficiency there is a sharp change away from only the

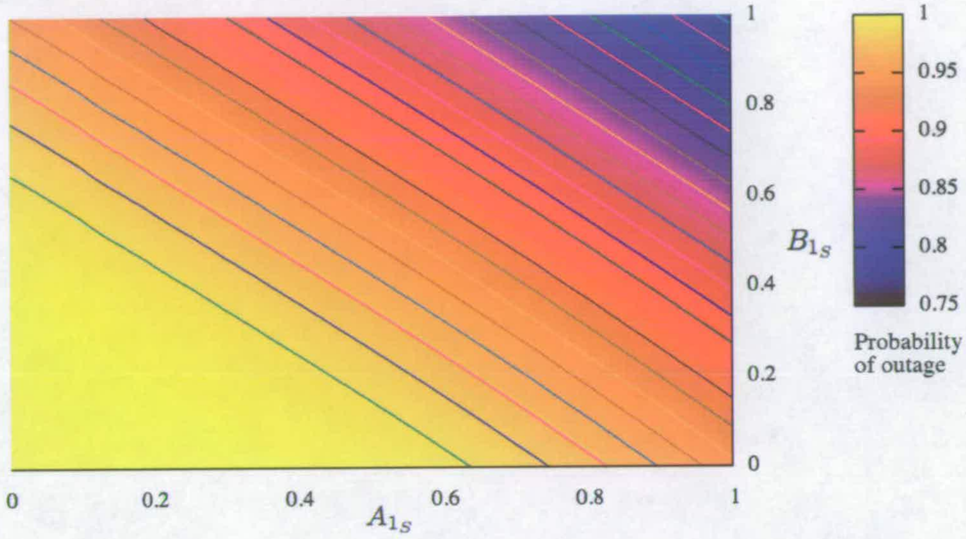

Figure 4.4: *Probability of outage surface for Protocol I MISO mode DF relaying with power control levels A_{1_S} and B_{1_S} for 0dB SNR and an RRDF required spectral efficiency of 1b/s/Hz, with contours shown for clarity of the surface shape.*

source transmitting in both phases to high power levels used by the source. At this point is can be seen that the relay link starts to offer the required spectral efficiency and the protocol starts to make use of the extra diversity offered by the cooperative relay.

Due to the fact that at all SNRs, A_{1_S}, is preferred to transmit at the full power available, the optimal transmit levels can be characterised by either B_{1_S} or C_{1_S}. Figure 4.7 presents a general overview of the optimal power level for B_{1_S} where the spectral efficiency constraint is relaxed. Therefore Figure 4.6 can be seen as a cross section of Figure 4.7. As can been seen in Figure 4.7 at lower capacities the sharp change between use only of single-hop transmission and use of the available relay moves to lower SNRs, while the opposite is true at high SNRs.

4.3.3.3 Protocol I MIMO analysis

Following the previous section, now consider Protocol I operating in MIMO transmission mode. Similar to the expanded probability equation for MISO mode relaying, (4.24) is now expanded for MIMO transmission such that

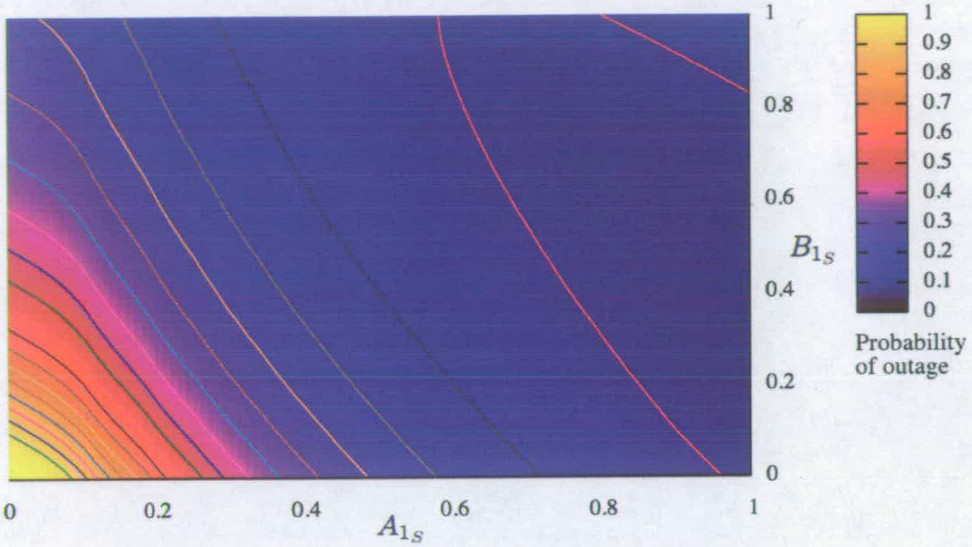

Figure 4.5: *Probability of outage surface for Protocol 1 MISO mode DF relaying with power control levels A_{1_S} and B_{1_S} for 10dB SNR and an RRDF required spectral efficiency of 1b/s/Hz, with contours shown for clarity of the surface shape.*

$$\begin{aligned}
&\Pr\left[I_{1_M} < \mathtt{R}\right] \\
&\quad = \Pr\left[|h_{SR}|^2 < \frac{2^{2\mathtt{R}}-1}{A\mathrm{SNR}}\right] \Pr\left[A_{1_M}B_{1_M}\mathrm{SNR}|h_{SD}|^4 + (A_{1_M}+B_{1_M})|h_{SD}|^2 < \frac{2^{2\mathtt{R}}-1}{\mathrm{SNR}}\right] \\
&\quad + \Pr\left[|h_{SR}|^2 > \frac{2^{2\mathtt{R}}-1}{A_{1_M}\mathrm{SNR}}\right] \\
&\qquad\qquad \Pr\left[A_{1_M}B_{1_M}\mathrm{SNR}|h_{SD}|^4 + (A_{1_M}+B_{1_M})|h_{SD}|^2 + C_{1_M}|h_{RD}|^2 < \frac{2^{2\mathtt{R}}-1}{\mathrm{SNR}}\right]
\end{aligned} \tag{4.31}$$

As can be seen (4.31) is considerably more complex than (4.25) due to the additional $|h_{SD}|^4$ term for the probability of outage whether there is a relay available or not. Due to its complexity an analytical expression is fully explored in Appendix B. As is shown in Appendix B.1 a closed form solution cannot be found for the probability of outage (the CDF) when one relay is available to support the source. However, the closed form solution for the PDF where one relay is available can be evaluated using numerical integration methods to find the optimal transmit power levels. This expression is available in Appendix B.1.

Results from the numerical evaluation are shown in Figure 4.8 for a required spectral efficiency

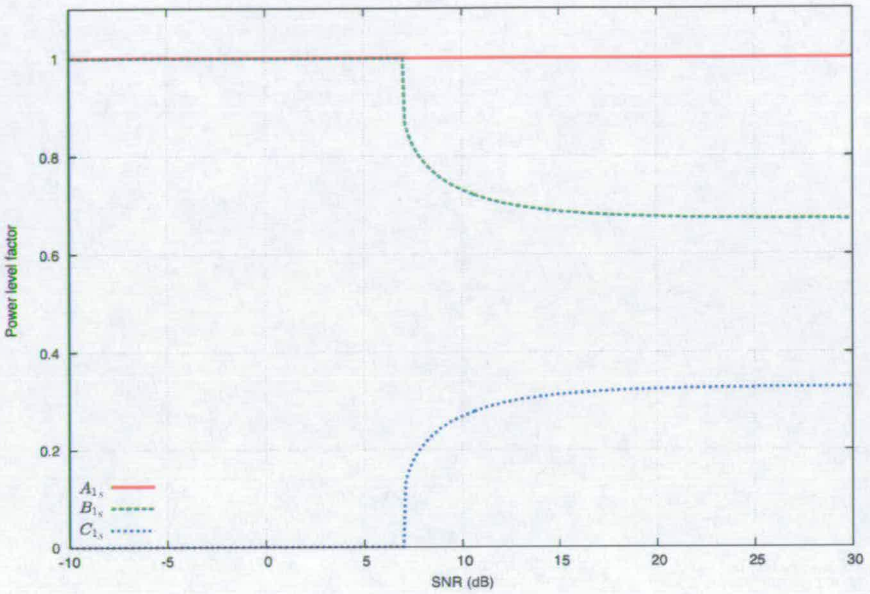

Figure 4.6: *MISO Protocol I optimal power control levels A_{1_S}, B_{1_S} and C_{1_S} against SNR for an RRDF required spectral efficiency of 1b/s/Hz.*

of 1b/s/Hz and show a similar trend to the MISO mode transmit power levels. Below approximately 6dB SNR only the source is preferred to transmit since the relay cannot support the required spectral efficiency and all of the available power is dedicated to the source's transmission over both time phases. Above the 6dB point the relay power allocation increases sharply while the second phase source transmission power level falls off proportionally and both approach an asymptote at high SNRs. Note that the transmit power levels for B_{1_M} are lower than that used in the MISO mode for a comparable SNR, which is due to the $|h_{SD}|^4$ term being dependant on B_{1_M}.

It is important to consider that the numerical integration used to calculate the optimal transmit power levels for Protocol I operating in MIMO mode is very computationally intensive. For this reason a similar graph to Figure 4.7 for Protocol I in MIMO mode is is not presented, although from Figure 4.8 it is possible to say that B_{1_M} will be very similar to the surface for B_{1_S} in Figure 4.7, but slightly shifted to the left due to the slightly higher SNR fall off seen in Figure 4.8.

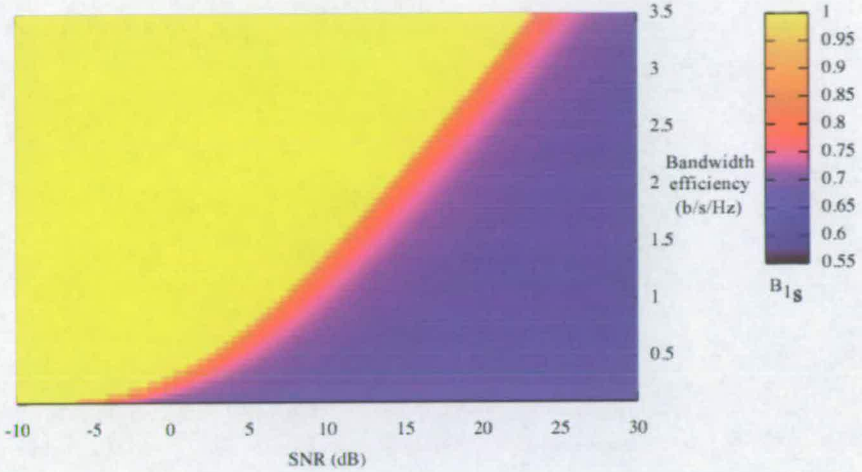

Figure 4.7: *MISO Protocol I optimal power control level B_{1_S} against SNR and bandwidth efficiency.*

4.3.3.4 Protocol III analysis

The final protocol to be considered for the optimal transmit power levels is Protocol III. From Appendix A it can be seen that while Protocol III can transmit in both MISO and MIMO modes, the mutual information for the two modes is exactly the same. This suggests that the optimal power level for Protocol III transmission is the same for both MISO and MIMO transmission. Furthermore, in this scenario MISO would generally be preferred as the decoding at the destination would be simpler.

As for Protocol I (4.24) is expanded to give the probability of outage for Protocol III such that

$$\begin{aligned}\Pr\left[I_3 < \mathrm{R}\right] = {} & \Pr\left[|h_{SR}|^2 < \frac{2^{2\mathrm{R}}-1}{A_3\mathrm{SNR}}\right]\Pr\left[|h_{SD}|^2 < \frac{2^{2\mathrm{R}}-1}{B_3\mathrm{SNR}}\right] \\ & + \Pr\left[|h_{SR}|^2 > \frac{2^{2\mathrm{R}}-1}{A_3\mathrm{SNR}}\right]\Pr\left[B_3|h_{SD}|^2 + C_3|h_{RD}|^2 < \frac{2^{2\mathrm{R}}-1}{\mathrm{SNR}}\right]\end{aligned} \tag{4.32}$$

Similar to the analytical approach for Protocol I MISO transmission, the variables $B_3|h_{SD}|^2 + C_3|h_{RD}|^2$ can be combined through convolution of the respective PDFs to obtain the combined PDF and then integration to give the CDF. Due to similarity of the analysis previously presented the full working of this approach is not reproduced here but is available in Appendix B.3. The combined CDF conditioned on one relay being available can be written as

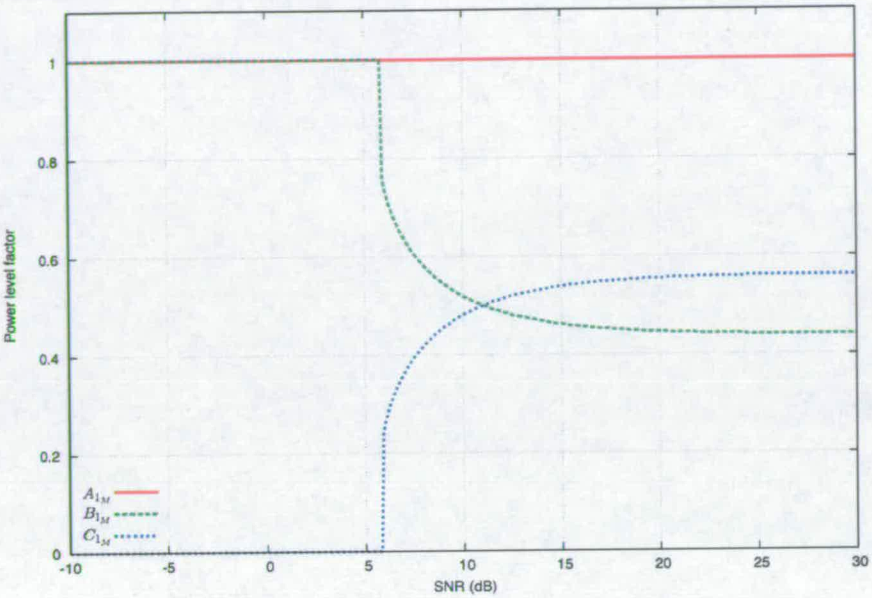

Figure 4.8: *MIMO Protocol I optimal power control levels* A_{1_M}, B_{1_M} *and* C_{1_M} *against SNR for an RRDF required spectral efficiency of 1b/s/Hz.*

$$\Pr\left[I_3 < \mathtt{R} \middle| \partial(\mathcal{J}_1)\right] = \begin{cases} \frac{1}{B_3 - C_3}\left(\mathrm{e}^{-x\frac{B_3+C_3}{B_3C_3}}(B_3 - C_3) - B_3\mathrm{e}^{\frac{x}{C_3}} + C_3\mathrm{e}^{\frac{x}{B_3}}\right) \quad \mathrm{e}^{-x\frac{B_3+C_3}{B_3C_3}}, & \\ & B_3 \neq C_3 \\ 1 - e^{\frac{-x}{B_3}}\left(\frac{x}{B_3} + 1\right), & B_3 = C_3 \end{cases} \tag{4.33}$$

Following this the full analytical approach can be found by combining (4.32) and (4.33) to give

$$\Pr\left[I_3 < \mathtt{R}\right] = \begin{cases} \left(1 - \mathrm{e}^{\frac{-x}{A_3}}\right)\left(1 - \mathrm{e}^{\frac{-x}{A_3}}\right) + \left(\mathrm{e}^{\frac{-x}{A_3}}\right) & \\ \qquad \left(\frac{1}{B_3 - C_3}\left(\mathrm{e}^{-x\frac{B_3+C_3}{B_3C_3}}(B_3 - C_3) - B_3\mathrm{e}^{\frac{x}{C_3}} + C_3\mathrm{e}^{\frac{x}{B_3}}\right)\mathrm{e}^{-x\frac{B_3+C_3}{B_3C_3}}\right), & \\ & B_3 \neq C_3 \\ \left(1 - \mathrm{e}^{\frac{-x}{A_3}}\right)\left(1 - \mathrm{e}^{\frac{-x}{A_3}}\right) & \\ \qquad + \left(\mathrm{e}^{\frac{-x}{A_3}}\right)\left(1 - e^{\frac{-x}{B_3}}\left(\frac{x}{B_3} + 1\right)\right), & B_3 = C_3. \end{cases} \tag{4.34}$$

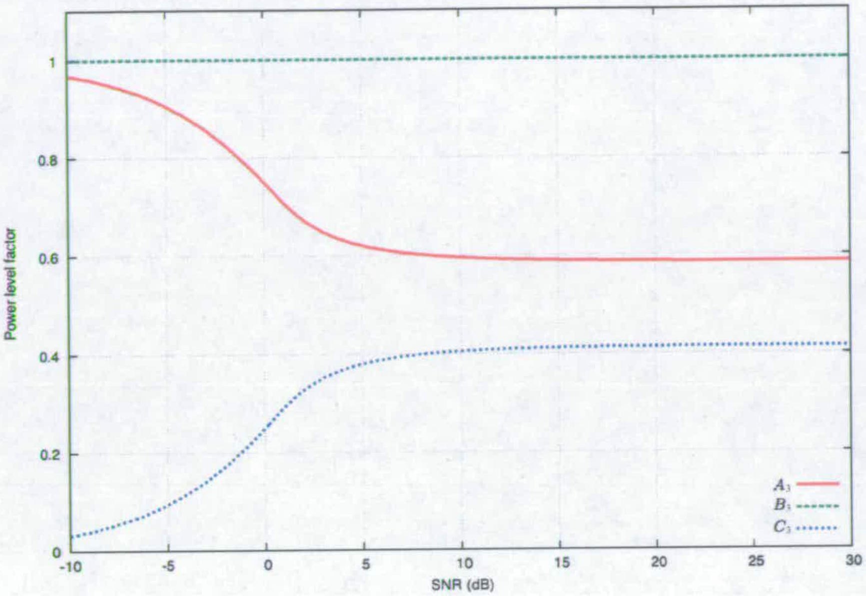

Figure 4.9: *Protocol III optimal power control levels A_3, B_3 and C_3 against SNR for an RRDF required spectral efficiency of 1b/s/Hz.*

Numerical results for the optimal transmit power levels for Protocol III at a required spectral efficiency of 1b/s/Hz are shown in Figure 4.9. The trends shown in Figure 4.9 are significantly different to those observed for both MISO and MIMO Protocol I, the most striking of which is that the first phase source transmit power level is not constant at 1. Rather the second phase source transmit level is constant using the full transmit power available. This is due to the fact that in Protocol III the source only communicated directly with the destination in the second phase. This emphasises the importance of the single-hop link in cooperative diversity channels.

Furthermore, there is no sharp charge in the values of the transmit power levels as previously observed for Protocol I, but a smooth transmission between the low SNR regime where the source is primarily used, and the high SNR regime where the relay and source share the available transmit power in the second phase. Note also that unlike the Protocol I power levels for the relay in the second phase, C_3 for Protocol III is less that B_3. Finally, is can also be seen that at both high and low SNR the transmit power levels A_3 and C_3 tend towards asymptotes.

The characteristics noted above can also be observed over several capacities as shown in Figure 4.10. The surface shown in Figure 4.10 is the value of A_3 since, as was previously noted, B_3 is preferred to always use the maximum power available.

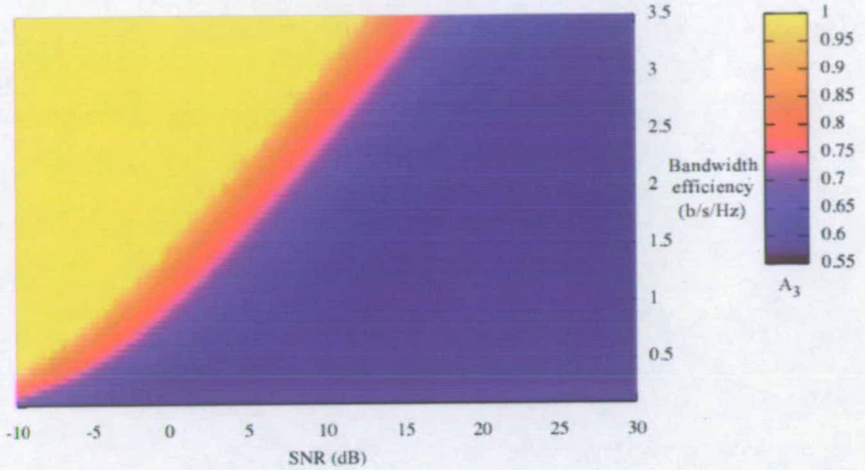

Figure 4.10: *Protocol III optimal power control level A_3 against SNR and bandwidth efficiency.*

4.4 Performance results

In the following section the performance of the complete set of relaying transmission modes are analysed in terms of outage, diversity and bandwidth efficiency. Initially, the performance is evaluated by inspection of the information theoretic framework developed earlier in this chapter. Numerical results are then presented and discussed as the conclusion of this work.

4.4.1 Information theoretic performance

The system constraints placed on the system were made to ensure that all relaying protocols can be directly compared to the single-hop (Protocol V) case. For this reason we take the single-hop scenario as the base line for analysing the performance of the other protocols. In the following discussion of the bandwidth efficiency of the protocols, the optimal transmit power levels for 1b/s/Hz as previously presented are used. Moreover, it is important to note that the ordering of the protocols in terms of bandwidth efficiency does not consider the diversity offered by the cooperative diversity protocols. Rather, this situation is considered later in this chapter.

Protocol I operating in MIMO mode would generally be expected to outperform Protocol I operating in MISO mode as it transmits different information in the second phase, and this would be expected to be particularly noticeable at high SNR. At low SNR note that Protocol I operating in MIMO mode would be expected to match the single-hop case to approximately

7dB SNR since the source transmits at full power during both phases ($A_{1_S} = B_{1_S} = 1$) and the relay is not utilised. Above 7dB SNR due to the split of power between the source and relay, the single-hop case will outperform the Protocol I MIMO case. Similarly Protocol I in MISO mode will benefit from the additional diversity and subsequently is expected to outperform the other cooperative diversity protocols, but not the single-hop link.

Next Protocol II is considered. Protocol I will always outperform Protocol II due to the additional transmission from the source in the second phase. However, Protocol II will outperform Protocol III due to the power constraint on Protocol III, which does not apply to Protocol II due to the destination listening to the source in the first phase rather than the second. Protocol IV is always outperformed by the cooperative diversity family of protocols since Protocol IV has no single-hop link from the source to destination in either phase. Finally, note the single-hop link is expected to outperform all other protocols since the source not required to share power with any available relays and is available to transmit different information in each phase. This can be summarised as

$$I_5 \geq I_{1_{MIMO}} \geq I_{1_{MISO}} \geq I_2 \geq I_3 \geq I_4 \tag{4.35}$$

4.4.2 Comparison from a diversity point of view

Considering the relaying protocols from a diversity point of view, the simplification that the $S \rightarrow R$ link is very good (i.e. the link supports the required rate R) is made. Defining $\beta_2 = \min\{E_{SD}/N_0, E_{RD}/N_0\}$ it is a simple matter to verify that for large β_2 the outage probability for Protocol II can be upper-bounded by

$$\Pr[I_2 < \mathrm{R}] \leq \left(\frac{2^{2\mathrm{R}} - 1}{\beta_2}\right)^{|\partial(\mathcal{J})|+1} \tag{4.36}$$

which demonstrates that Protocol II extracts full diversity in the effective SNR β_2. In this context, full diversity refers to the fact that the outage probability decays proportional to $1/\mathrm{SNR}^{|\partial(\mathcal{J})|+1}$ at high SNR. It is likewise straightforward to verify that Protocol I and Protocol III also extract full diversity. Protocol IV however extracts diversity of order $|\partial(\mathcal{J})|$ since there is no single-hop link between the source and destination. Finally, it is well known that a single-hop link extracts only first order diversity.

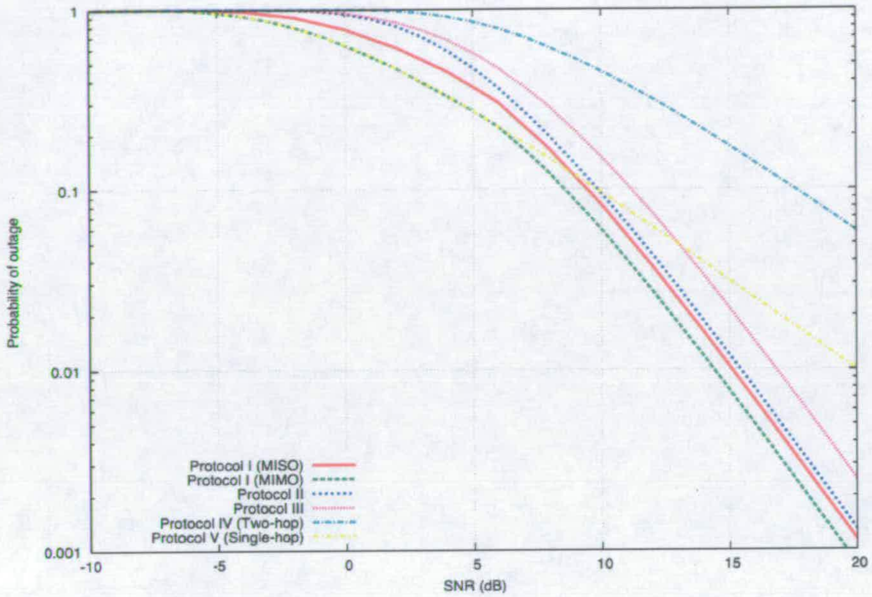

Figure 4.11: *Simulation results for cooperative diversity where the spectral efficiency is fixed at 1 b/s/Hz. SNR (fixed for all links) and outage probability are compared for all relaying protocols including both MISO and MIMO based transmission for Protocol I.*

4.4.3 Numerical results

To conclude the discussion of the performance of the complete family of relaying protocols, numerical results are presented for the probability of outage, the bandwidth efficiency and the capacity at a fixed level of outage for each individual protocol. The assumption that all terminal links have the same SNR is continued here, as is the assumption that there is only one relay potentially available. These assumptions are made for simplicity of presenting the results graphically, but can be readily relaxed using the framework developed earlier in this chapter.

Consider initially the probability of outage metric for comparison of each relaying protocol. Figure 4.11 shows the probability of outage against SNR, where the SNR for all links is the same, for all five protocols at a fixed required rate of 1b/s/Hz. As was previously noted, the optimal power levels which were obtained earlier in this chapter are optimal in terms of outage (rather than capacity), which can be seen by the ordering in Figure 4.11. Protocol I while operating in MIMO mode matches the performance of the single-hop link in terms of probability of outage at low SNR but, due to the additional diversity offered by the relay in the second phase, it outperforms the single-hop case at high SNR. Since Protocol I utilises

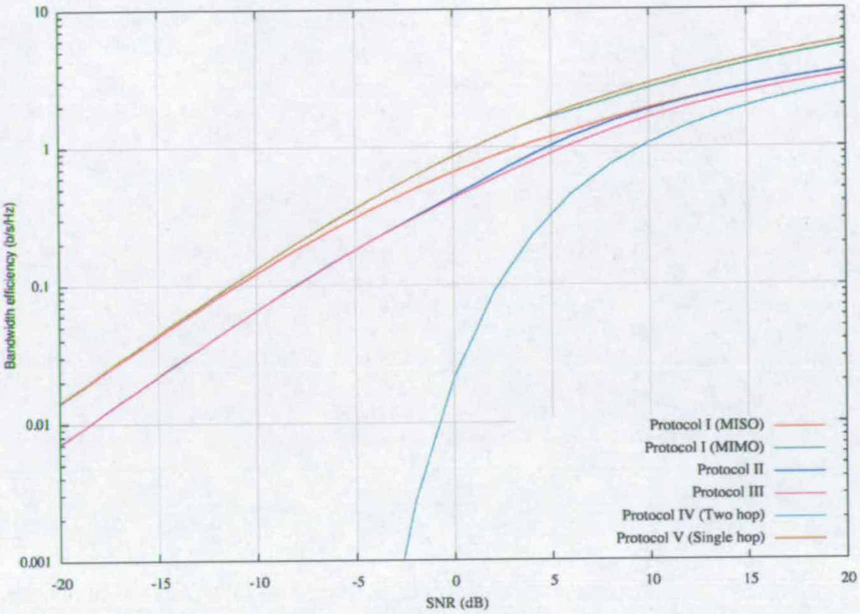

Figure 4.12: *Simulation results for bandwidth efficiency of cooperative diversity. SNR and bandwidth efficiency are compared for all relaying protocols including both MISO and MIMO based transmission for Protocol I.*

full transmit and receive collision its it outperforms all other protocols in outage terms at high SNR and therefore is the preferred protocol to be implemented in a wireless network where the smallest possible probability of outage is critical.

Figure 4.11 also clearly shows the diversity benefit offered by the cooperative diversity protocols over the single-hop and two-hop links. At high SNR all cooperative diversity protocols outperform the non-cooperative links due to the extra diversity offered by the relays. This shows again how using cooperative diversity in a Rayleigh fading wireless environment can be beneficial to the quality of the communications available. Also note that Protocol II outperforms Protocol III at all times due to the power level transmit constraint of Protocol III, even although the two protocols are similar. This suggests that Protocol III would only ever be used in a situation where the destination is not available to receive information in the first phase.

In terms of bandwidth efficiency performance, the ordering of (4.35) can be observed in Figure 4.12. Since the transmit power levels are optimised in terms of probability of outage, they can't necessarily be directly applied to the bandwidth efficiency performance metric. However,

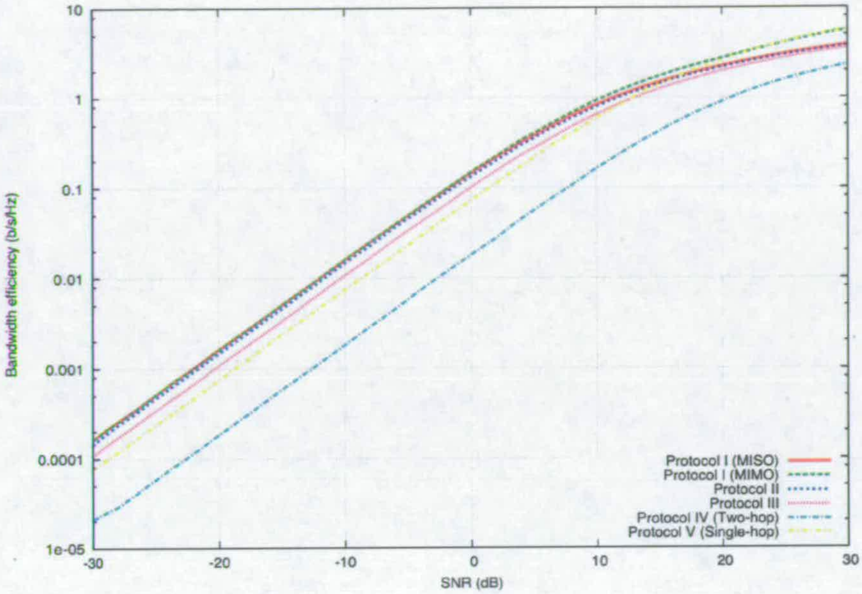

Figure 4.13: *Simulation results for bandwidth efficiency of cooperative diversity at the 5% outage level. SNR and bandwidth efficiency are compared for all relaying protocols including both MISO and MIMO based transmission for Protocol I.*

it is illustrative to consider the ordering of the results, and as such the bandwidth efficiency is measured using the optimal transmit power levels as calculated for 1b/s/Hz. In-particular Protocol I (in both MIMO and MISO modes) matches the capacity of the single-hop link at low SNR[7] as was suggested by the fact that only the source transmits in both phases due to the power constraint.

Figure 4.13 is of particular interest for consideration in a practical wireless communication network where a certain level of signal outage is tolerated, as it shows the achievable capacity for a set SNR at the 5% outage level. This simulation was made by calculating the optimal transmit power levels for each protocol, at each SNR and attempted capacity, on the fly. This is an ideologically similar method to pre-computing a range of optimal power levels and using a 'look-up table' type approach to finding the correct power levels for certain channel characteristics. In terms of practical implementation this is the most likely approach due to the computational complexity of computing the optimal transmit power levels on the fly.

As can be seen from Figure 4.13 the cooperative diversity protocols offer a considerable

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[7] Note that MIMO converges to the single-hop link at higher SNR than the MISO mode

increase in the capacity of a link, with the extra diversity offered by the relay effectively combating the Rayleigh fading at low SNR. A significant gain of 3.4dB can be observed when comparing the achievable capacities of Protocol I (MIMO mode) with the single-hop link at low SNR. Protocol I (MISO mode) offers a similar increase in capacity at low SNR, however cannot match Protocol I (MIMO mode) or the single-hop link at high SNR due to the lack of new information in the second phase of transmission. Protocol II can obtain an increase in capacity of 3dB at low SNR over the single-hop case. Protocol III can only offer a 1.7dB increase in capacity over the single-hop case, which is due to the power constraint placed on the protocol, despite its similarity to Protocol II. It can therefore be concluded that Protocol I (MIMO mode) it preferred over the other available protocols, where the SNR between all terminals is equal.

From this discussion it can be seen that there is a trade-off between optimising for probability of outage as has been pursued in this chapter and optimising for capacity at a specific outage level. The following chapter explores the possibility of combining the protocols optimised for probability of outage as derived in this chapter, in such a way that the protocol that is optimal for the instantaneous channel conditions is the one that is employed.

4.5 Conclusions

In this chapter a general framework for the complete family of relaying protocols was introduced for an ad-hoc network which consists of a single source, a destination terminal and $|\mathcal{J}|$ potential supporting relays under three basic system constraints: time, bandwidth and power. The proposed framework was then specialised for decode-and-forward relaying and the complete set of mutual information equations were presented. Further to this, an analytical approach was taken to derive the optimal transmit power levels for Protocol I (MISO mode) and Protocol III and a numerical approach was used to find the optimal transmit power levels for Protocol I (MIMO mode). Finally, the complete family of relaying protocols were compared in terms of bandwidth efficiency, outage performance, capacity and diversity. Protocol I (MIMO mode) was found to be the optimal protocol in terms of outage, where the SNR between all terminals is equal, when averaged over a large number of independent fading samples as found by Monte Carlo simulation.

Chapter 5
Optimisation strategies for relay networks

5.1 Introduction

This chapter considers two optimisation strategies for the family of relay networks introduced in the previous chapter, which maximise the potential benefits offered by relaying. This optimisation is performed by expanding the framework previously developed, initially to frequency selective fading channels and then to consider adaptive protocol selection based on limited feedback from the receiver.

Multi-carrier transmission methods were introduced in chapter 2 as a method for combating the adverse effect Rayleigh fading has on wireless communications by transmitting data over multiple channels separated in frequency. This is made efficient in terms of spectrum use by utilising orthogonal transmission methods which ensure that the carriers can be closed spaced and is consequently termed OFDM. Methods which combine the diversity offered by relaying methods and OFDM have attracted considerable interest as this can lead to much more reliable communications [83, 97, 98]. This has also lead to research involving cooperative diversity methods and OFDM transmission [99, 100].

In the first part of this chapter, the framework previously developed for the relay network is used to answer the question of what effect a frequency-selective fading environment has on the performance of the cooperative diversity protocols. This is done by modelling the fading channel as a set of OFDM tones, each of which is subjected to appropriate signal fading to simulate the effect of frequency-selective fading. Since the signal received by the destination, for each tone, may have arrived by several paths the received signal is modelled by a finite-impulse response filter with an appropriate set of tap weights to model a suitably correlated signal. In this chapter the COST-207 channel model is considered with the corresponding tap weights. Furthermore, each path modelled for each OFDM tone is subjected to independent Rayleigh fading, as would be expected from the properties of the wireless channel.

Before the question of how the cooperative diversity protocols perform in the frequency-selective channel can be answered, another important question must be posed and answered first. As was noted in the previous chapter Protocol I and Protocol III make use of full receive collision (in frequency) at the destination in the second phase and it is not known if this is optimal for the OFDM case. Other scenarios which are fully explored later in this chapter include no-collision in the second phase and partial collision. As in the previous chapter, Protocol I (MIMO mode) is the primary relaying protocol considered since all others are effectively derivatives of it and it was shown to have the best performance of the cooperative diversity protocols in a quasi-static fading environment when averaged over a large number of independent fading samples. The three system constraints of time, bandwidth and power are also continued in this chapter to make a comparison with the single-hop case fair and therefore the optimal power level methods derived in the previous chapter are used through the remainder of this work for Protocols I and III.

The second part of this chapter considers an adaptive method to select the optimal protocol, in terms of capacity, based on the instantaneous channel conditions. As was noted in the previous chapter, the transmit power levels considered were optimal in terms of outage rather than capacity, so to see the full benefit of using cooperative diversity to combat Rayleigh fading, now consider optimising the relay channel in terms of capacity. This is done by using limited feedback from the destination terminal, which has knowledge of all reverse channels, to the source. This approach is already used in practical wireless communication systems to make the best possible use of the channel based on instantaneous channel conditions, such as the High-Speed Downlink Packet Access (HSDPA) system.

This chapter is structured as follows: section 5.2 expands the channel models and framework developed in the previous chapter to consider the frequency-selective fading environment. In section 5.3 the effect of collision in frequency (signal) space is examined. The effect of the frequency-selective environment on the cooperative diversity protocols is then considered in section 5.4. Section 5.5 considers an adaptive method to select the optimal relaying protocol to use based on current channel conditions and finally conclusions are drawn in section 5.6.

5.2 Frequency selective cooperative diversity strategies

5.2.1 Channel model

In this section the quasi-static fading channel model used in the previous chapter is extended to consider a frequency-selective fading model. To take account of the frequency-selective fading, the channel transfer coefficient for the kth tone is given by the Discrete Fourier Transform (DFT) [101, p.243] as

$$h_{XY}(k) = \sum_{l=0}^{L-1} h_{XY,l} e^{\frac{-2\pi lk}{N}} \quad k = 0, 1, \ldots, N-1 \text{ carriers} \tag{5.1}$$

where the subscript h_{XY} denotes the channel between source X and destination Y, and $h_{XY,l}$ is the time-domain channel impulse response coefficient recorded at discrete delay l. The frequency-selective fading model considered is restricted to quasi-static Rayleigh fading channels with fading coefficients $h_{XY,l}$ being circularly symmetric zero mean complex Gaussian random variables. Furthermore, the channel coefficients are assumed to be uncorrelated across taps.

Each phase of transmission employs OFDM with N tones and the length of the Cyclic Prefix (CP) satisfies $L_{cp} \geq L$, which guarantees that each of the frequency-selective fading channels de-couples into a set of parallel quasi-static fading channels.

From the previous assumptions (4.13), (4.14) and (4.15) can be rewritten for the kth tone as

$$y_{D_1}(k) = \sqrt{A_1 E_{SD_k}} h_{SD}(k) x_{1,k} + n_{D_1} \tag{5.2}$$

$$y_{R_{j,1}}(k) = \sqrt{A_1 E_{SR_{j,k}}} h_{SR_j}(k) x_{1,k} + n_{R_1} \tag{5.3}$$

$$y_{D_2}(k) = \sqrt{B_1 E_{SD_{j,k}}} h_{SD}(k) x_{2,k} + \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \sqrt{C_{j,1} E_{RD_{j,k}}} h_{RD_j}(k) x_{j+1,k} + n_{D_2} \tag{5.4}$$

where (5.2) is the received signal by the destination in the first phase for the kth tone, (5.3) is the signal received by the relay j in the first phase for the kth tone and (5.4) is the signal received by the destination in the second phase from the source and all available transmitting relays for the kth tone. Furthermore E_{XY_k} is the average signal energy received by the destination terminal over one symbol period through the $X \rightarrow Y$ link on the kth tone. Similarly, $E_{XY_{j,k}}$ denotes the average signal energy received by the destination Y as transmitted by the jth relay, and $x_{j,k}$ denotes the information symbol transmitted in the jth transmitting terminal on the kth tone.

Moreover, the Protocol I channel model can now be summarised in matrix notation similar to (4.16) as

$$\mathbf{H}_1(k) = \begin{bmatrix} \sqrt{A_{1_M} E_{SD}} h_{SD}(k) & 0 \\ \sqrt{C_{1_M,j} E_{RD_j}} h_{RD_j}(k) & \sqrt{B_{1_M} E_{SD}} h_{SD}(k) \end{bmatrix} \tag{5.5}$$

5.2.2 Multiple access scheme

To fully analyse the effect that signal collision in the second phase has on the performance of the cooperative relaying protocols, consider a MA scheme similar to the MIMO MA scheme used in [97]. The MA channel is simplified significantly in this work since each terminal is assumed to have only one transmit/receive antenna. Furthermore, both the source and the set of relays are assumed to make up the MA channel, so the number of terminals communicating with the destination is $|\partial(J)|$. The information rates that the MA channel can support are split into two; R_1 denotes the information transmitted in the first phase (x_1) and R_2 denotes the information transmitted in the second phase (x_2). The sum of the information transmitted over the two phases of transmission is called the sum rate and denoted R_{sum}.

The family of MA schemes used is obtained by assigning each OFDM tone $k = 0, 1, ..., N-1$ to a subset of the transmitting terminals, $\mathcal{U}_k$, in the second phase. During the first phase all tones are assigned to the source and none to the relay, hence there can be no collision in frequency in the first phase. The full collision-based MA scheme where all tones are assigned to both the source and the relay during the second phase (i.e., $\mathcal{U}_k = \{S, R\}$ for $k = 0, 1, ..., N-1$). Conversely, the no collision case is defined by the tone assignment pattern such that each tone has one or less (zero) transmitting terminals assigned to it, which satisfies $|\mathcal{U}_k| \leq 1$ for $k = 0, 1, ..., N-1$. The variable amount of collision employed in the second phase transmission is shown graphically in Figure 5.1. It should be noted that unlike most cooperative diversity

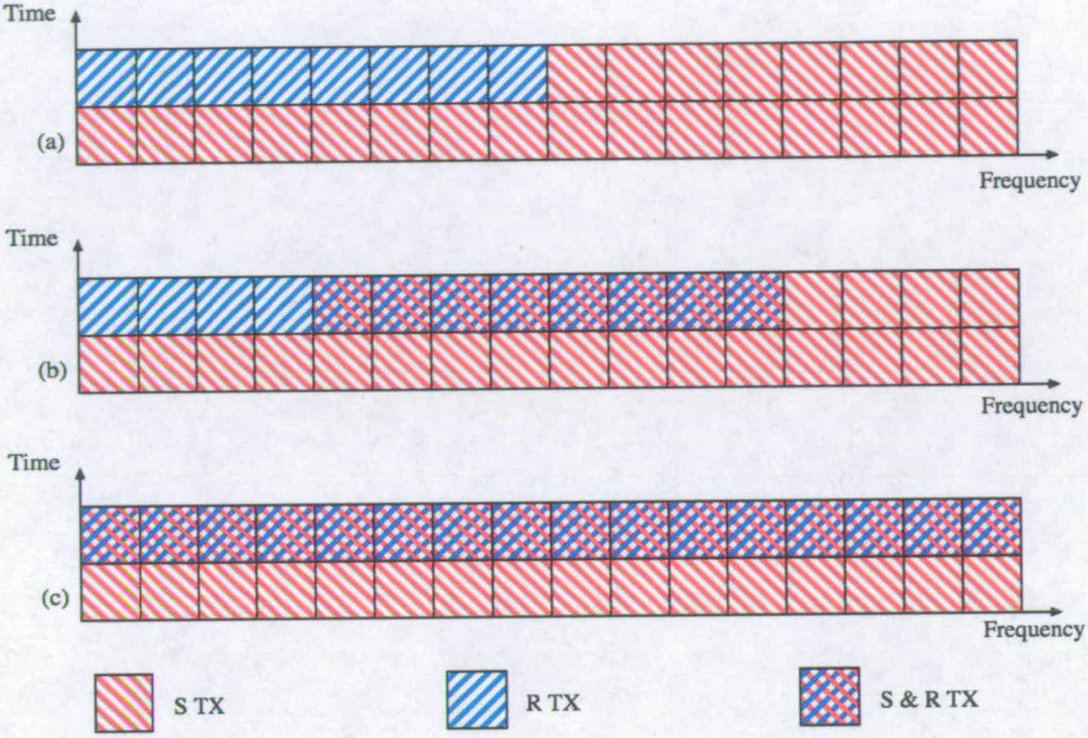


Figure 5.1: *Variable amount of transmission collision in individual OFDM tones in the second phase, in frequency where (a) shows the no collision case, (b) shows the part collision case and (c) shows the full collision case.*

figures in literature, time is on the vertical axis in Figure 5.1, to clearly show the OFDM tones on the horizontal axis.

By considering the terminals transmitting on each tone over the two transmission phases in Figure 5.1 it can be seen that each tone can be split into one of the five relaying protocols introduced in the previous chapter. In particular

- Tones where there is no collision in the second phase and the relay transmits information in the second phase can be considered to be Protocol II.
- Tones where there is no collision in the second phase but only the source transmits can be considered to be two single-hop transmissions from source to destination (Protocol V).
- Tones where there is collision can be considered to be Protocol I as the full degree of receive collision is used.

Figure 5.2: *Variable amount of collision in the second phase (Figure 5.1(b)) split into the component rates using the collision detailed 5.1(b). (a) shows the component* R_1*, (b) shows the component* R_2 *and (c) shows the full sum rate,* R_{sum}*. The type of relaying for each pair of tones is also shown at the top of the figure.*

The information rate R_1 is obtained from all tones in the first phase and the tones in the second phase where the relay transmits, including the collision tones. Similarly, the rate R_2 is obtained from only the tones in the second phase where the source transmits, including the collision tones, and the sum rate, R_{sum}, is obtained from all tones regardless of the transmission type. This is shown graphically in Figure 5.2 which considers the part collision case shown in Figure 5.1(b), and it can therefore be seen that each MA channel consists of (possibly) multiple relaying protocols.

5.2.3 Bandwidth efficiency

To analyse the bandwidth efficiency region of the fading relay channel, the case where all available OFDM tones are used and assigned to either the source, relay or to both terminals is investigated. Both the source and relay have the same number of tones $N_u \geq \frac{N}{2} \in \mathbf{N}(u = \{S, R\})$ out of which bN tones with $b = \frac{2i}{N}, i \in \{0, 1, ..., N/2\}$ are assigned to both transmitting terminals in the second phase. Consequently, the case of $b = 0$ corresponds to full collision and $b = 1$ describes no collision in the second phase. The transmit power used by the terminals is unity in the non-collision tones, and uses the Protocol I (MIMO mode) power control factors, as found in the previous chapter, in collisions tones. Under these assumptions, bN tones are assigned to both the source terminal and the set of relaying terminals, leading to collision in signal space. Subsequently these tones are termed collisions tones. Both the source and the set of relays and are assigned $\frac{1-b}{2}$ tones independently, where no collision occurs (termed non-collision tones). Subsequently both the source and set of relaying terminals are each assigned $\frac{1+b}{2}$ tones.

Subsequently the following MA rates are obtained

$$\begin{aligned}\mathrm{R}_1 \leq\, & b \log_2 \left(1 + \frac{A_{1_M} E_{SD}}{N_0} |h_{SD}|^2 + \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \frac{C_{1_M} E_{RD}}{N_0} |h_{RD}|^2 \right) \\ & + \frac{1-b}{2} \log_2 \left(1 + \frac{E_{SD}}{N_0} |h_{SD}|^2 \right) \\ & + \frac{1-b}{2} \log_2 \left(1 + \frac{E_{SD}}{N_0} |h_{SD}|^2 + \frac{E_{SD}}{N_0} |h_{SD}|^2 \right)\end{aligned} \tag{5.6}$$

$$\begin{aligned}\mathrm{R}_2 \leq\, & b \log_2 \left(1 + \frac{B_{1_M} E_{SD}}{N_0} |h_{SD}|^2 \right) \\ & + \frac{1-b}{2} \log_2 \left(1 + \frac{E_{SD}}{N_0} |h_{SD}|^2 \right)\end{aligned} \tag{5.7}$$

$$
\begin{aligned}
\mathrm{R}_1+\mathrm{R}_2 \leq b\log_2 \Bigg(1 &+ \frac{A_{1_M} B_{1_M} E_{SD}^2}{N_0} |h_{SD}|^4 + \frac{A_{1_M} E_{SD}}{N_0} |h_{SD}|^2 \\
&+ \frac{B_{1_M} E_{SD}}{N_0} |h_{SD}|^2 + \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \frac{C_{1_M,j} E_{RD_j}}{N_0} |h_{RD_j}|^2 \Bigg) \\
+ \frac{1-b}{2} \log_2 \Bigg(1 &+ \frac{E_{SD}}{N_0} |h_{SD}|^2 + \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \frac{E_{RD}}{N_0} |h_{RD}|^2 \Bigg) \\
+ (1-b) \log_2 \Bigg(1 &+ \frac{E_{SD}}{N_0} |h_{SD}|^2 \Bigg)
\end{aligned}
\qquad (5.8)
$$

where R_1 is the maximum information rate that can be transmitted by the source in the first phase, Figure 5.2(a), R_2 is the maximum information rate that can be transmitted by the source in the second phase, Figure 5.2(b) and R_{sum} is the maximum information that can be obtained from the MIMO channel that is formed, Figure 5.2(c). Furthermore A_{1_M}, B_{1_M} and C_{1_M} are the Protocol I (MIMO mode) transmit power levels as found in the previous chapter.

Channel rates (5.6), (5.7) and (5.8) are presented in a fully expanded form (rather than in channel matrix form) to avoid ambiguity between tones which have the power control factors A_{1_M}, B_{1_M} and C_{1_M}, and those that do not.

5.3 Effect of second phase collision

To conclude the discussion of the effect of second phase collision on the capacity regions of the cooperative relay network in an OFDM framework, numerical results are presented as recorded from a Monte Carlo simulation. It is assumed in the simulation that there is a maximum of one relay terminal potentially available to support the source's transmission. Furthermore it is assumed that all terminal links have the same SNR. As in the previous chapter, these assumptions are made for the simplicity of presenting the results, but can readily be relaxed to the more general case used in the developed framework.

As the achievable channel capacity is bound by (5.6), (5.7) and (5.8) the capacity results are presented as an envelope of the achievable rates. Specifically the envelope boundary between admissible and impossible rate combinations is found by numerical simulation. Furthermore, note that higher envelope boundary values are preferable, as this indicates that higher channel

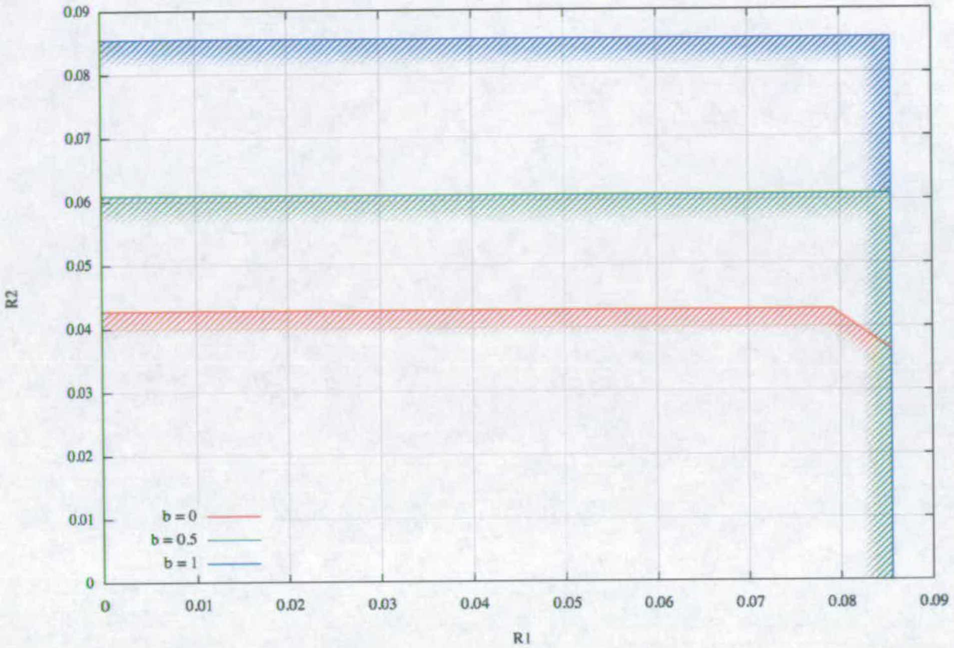


Figure 5.3: *Multiple-Access bandwidth efficiency regions for no, half and full collision (b=0, b=0.5 and b=1 respectively) at -10dB SNR. The rate achievable in the first phase,* $\mathtt{R}_1$*, is shown plotted against the rate achievable in the second phase,* $\mathtt{R}_2$*.*

capacities can be achieved.

Figure 5.3 and Figure 5.4 show the MA channel rates for -10dB and 10dB SNR respectively. As can be readily seen from both figures, full collision in the second phase is the preferred method of MA in order to maximise the bandwidth efficiency. Moreover, this is true not only for the sum rate, but also for $\mathtt{R}_1$ and $\mathtt{R}_2$. It is interesting to note from Figure 5.3 that the effect of collision in the second phase has very little impact on $\mathtt{R}_1$ at low SNR.

The benefit of using full signal collision in the second phase is also seen in Figure 5.5 which shows the capacity region for the sum capacity region at various SNR and collision levels. It is clear that full collision is always preferred in the second phase in an OFDM environment for the relay channel.

]

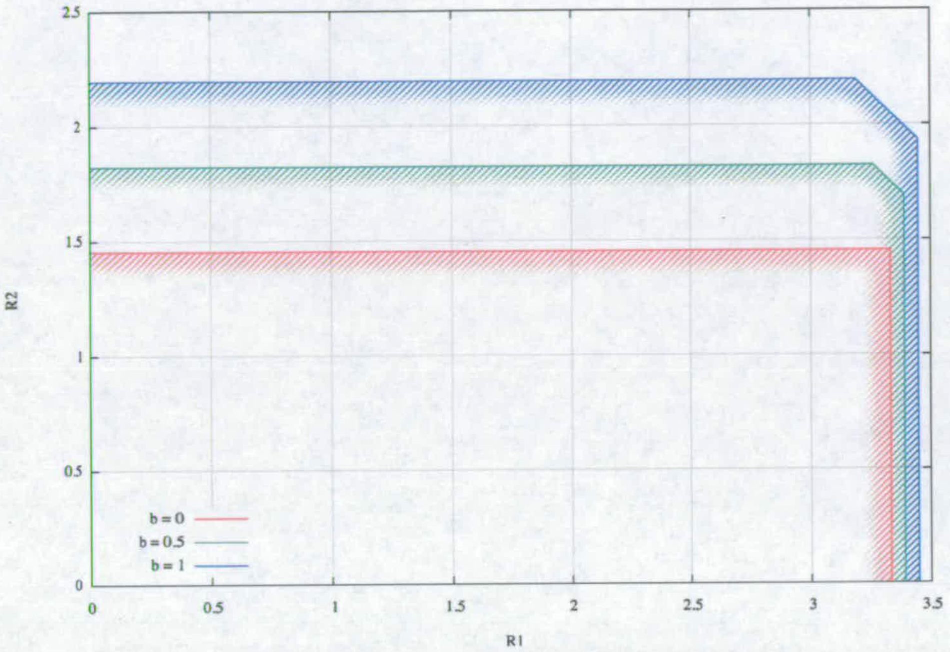


Figure 5.4: *Multiple-Access bandwidth efficiency regions for no, half and full collision (b=0, b=0.5 and b=1 respectively) at 10dB SNR. The rate achievable in the first phase,* R_1*, is shown plotted against the rate achievable in the second phase,* R_2.

5.4 Frequency-selective fading relay network analysis

In the previous section, the frequency-selective channel model was introduced, (5.1), and it was shown that full collision is the preferred method of cooperative transmission in the second phase over the OFDM channel. This is now used to explore the effect that frequency-selective fading has on the performance of the cooperative relay protocols.

To model the effects of the frequency-selective channel the COST 207 model is employed [41] which states the channel tap weights that should be used to simulate several environments, including the typical rural environment (RA) and the typical urban (non-hilly) environment (TU). The tap weights for these two environments are given in Appendix C. COST 207 does provide other tap weight specifications for several other environments, however only the RA and TU scenarios are considered in this chapter as these are two common environments where wireless communications are used, and also to reduce the complexity of presenting several other frequency-selective environments.

Note that the performance ordering of the cooperative relay protocols does not change in

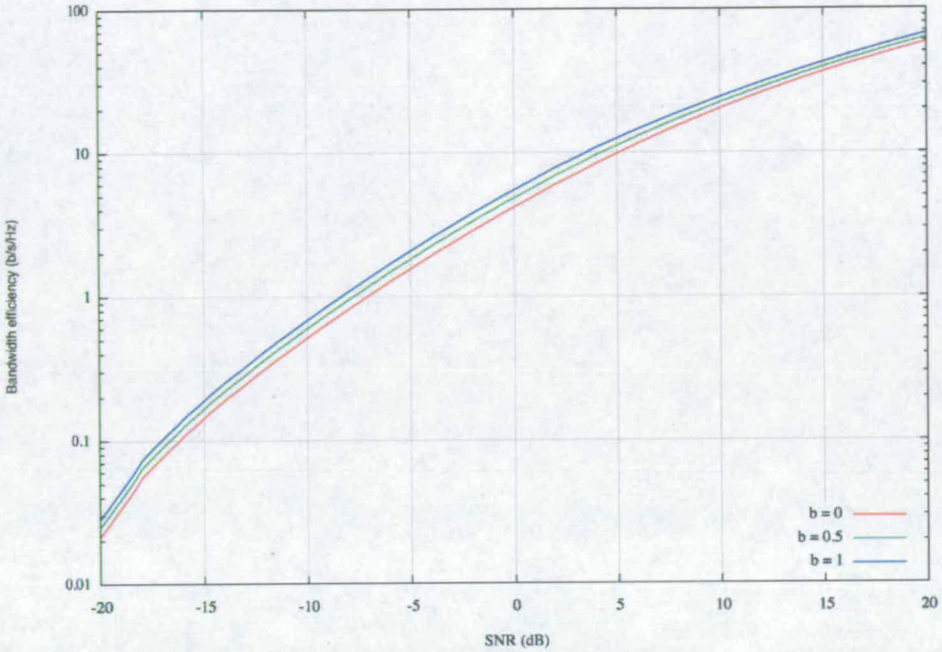


Figure 5.5: *Simulation results for the bandwidth efficiency region for the sum capacity region of the MA channel with one potential relay, for no, half and full collision (b=0, b=0.5 and b=1 respectively).*

the frequency-selective environment from the quasi-static fading case derived in the previous chapter, for both bandwidth efficiency (4.35) and diversity (4.36). Rather, what is of particular interest is the numerical results of the protocols. Furthermore, although the ordering of the protocol's diversity performance does not change, the order of diversity is now not only dependent on the number of available relays, but also on the inverse of the normalised coherence bandwidth, which is modelled by multiple taps in the COST 207 model.

5.4.1 Numerical results

To fully investigate the effect of a frequency-selective environment on the relay protocols, numerical results from a Monte Carlo simulation are presented. A channel with 64 OFDM tones is considered and the DF decision as to whether a relay can take part in the second phase or not is made on a per-tone basis. Both COST 207 RA and TU environments are considered and the bandwidth efficiency results as a function of SNR are presented in Figure 5.6 and Figure 5.7 respectively.

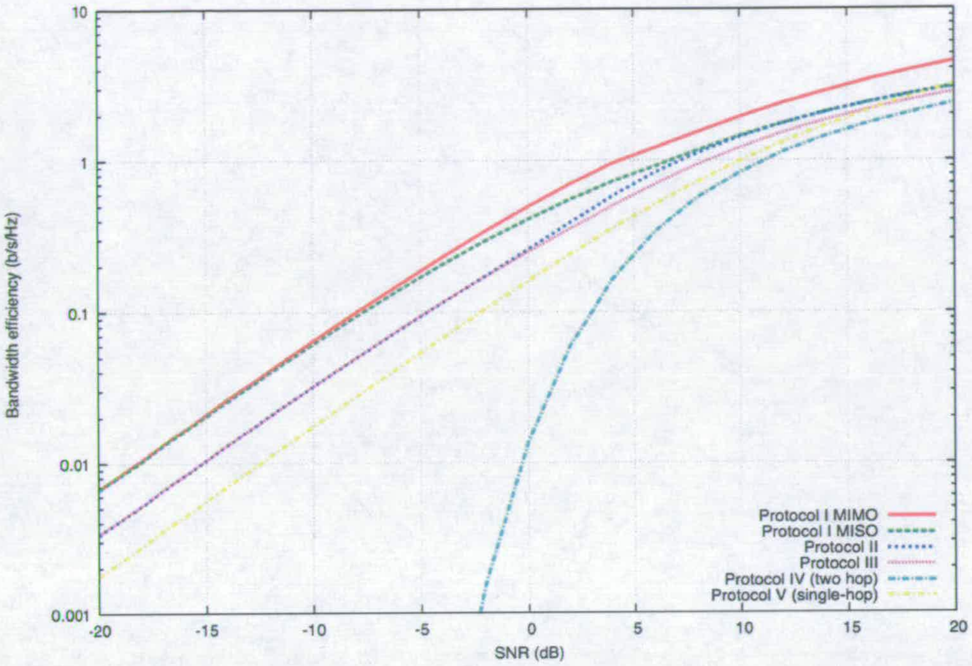


Figure 5.6: *Simulation results for bandwidth efficiency of cooperative diversity in a frequency-selective environment using the COST 207 RA model. SNR and capacity are compared for all relaying protocols including both MISO and MIMO based transmission for Protocol I.*

Figure 5.6 and Figure 5.7 show that the ordering of the capacity does not change due to the frequency-selective fading, although a large decrease in the achievable capacity is noticeable when compared to the non-frequency selective case. At low SNR for Protocol I (MIMO mode) the decrease in the achievable capacity is 3.3dB for the RA case, and 9.1dB for the TU case compared to the non-frequency selective fading case considered in the previous chapter in Figure 4.12. As would be expected, all available protocols perform considerably worse in terms of capacity in the TU scenario.

Similarly, the outage results as a function of SNR for a required spectral efficiency of 1b/s/Hz are presented in Figure 5.8 and Figure 5.9 respectively.

The effect of the additional diversity offered by the multi-path frequency-selective fading is particularly apparent in Figure 5.8 and Figure 5.9. The additional diversity offered by the supporting relay can be observed clearly in the RA case, but it is not so clear in the TU case, which already has 12th order diversity for the single-hop case due to the number of paths

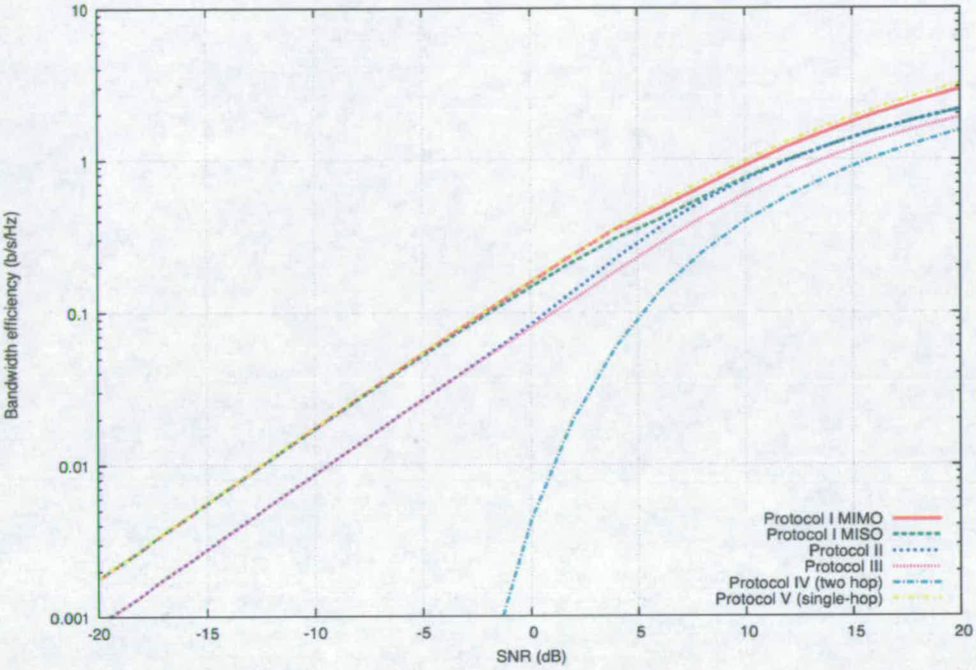


Figure 5.7: *Simulation results for bandwidth efficiency of cooperative diversity in a frequency-selective environment using the COST 207 TU model. SNR and capacity are compared for all relaying protocols including both MISO and MIMO based transmission for Protocol I.*

modelled. The diversity ordering derived in (4.36) can also be observed here.

5.5 Adaptive optimal relaying protocol selection

When selecting which relaying protocol to utilise it is important to consider the trade-off between the complexity of the system and the advantages that each protocol can offer. In a situation where it is desired to get the maximum possible data transfer rates that can be achieved, the system designer might be willing to use Protocol I (MIMO mode) despite its complexity. However, in a situation where the wireless communication devices must be kept as simple as possible (for example for battery life reasons), Protocol II might be sufficient. In both of these scenarios the relay transmission protocol is fixed from design to implementation, and the transmission type based on wide-ranging general assumptions about the channel conditions that will be encountered during the network's operation.

In the previous chapter it was noted that under certain channel conditions one protocol might

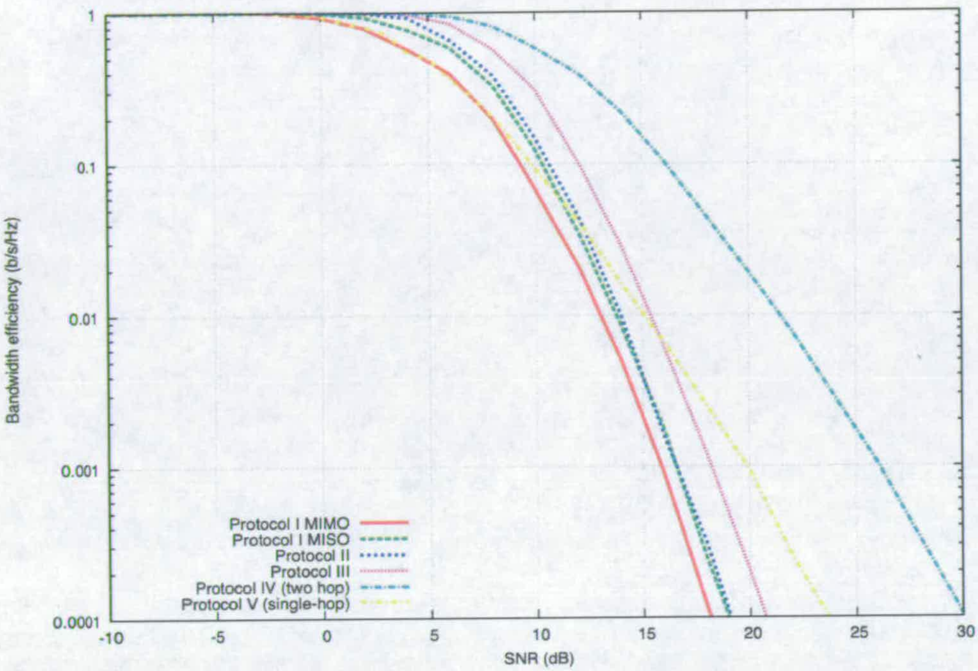


Figure 5.8: *Simulation results of cooperative diversity outage in a frequency-selective environment using the COST 207 RA model where the spectral efficiency is 1 b/s/Hz. SNR and outage probability are compared for all relaying protocols including both MISO and MIMO based transmission for Protocol I.*

be preferred over another, for example if the source to relay signal is very poor in comparison to the source to destination signal, it would be wise to transmit using the single-hop (Protocol V) method only. Due to the fact that the relay network has $2|\mathcal{J}| + 1$ random fading channels for $|\mathcal{J}|$ relays, an adaptive method to select the optimal protocol based on the current channel conditions can be envisaged. Such a method would require feedback from the destination to the source, containing information about the current state of the channel links which lead from the source to the destination. This information would be contained in a control or header packet sent from the destination to the source which is used to maintain the communication between the two terminals, for example a TCP packet [102]. Such a method for selecting the optimal transmission technique is already used in HSDPA [103] and certain 801.11 [104] communication systems, which change the modulation used to carry data based on available channel measurements.

For the purposes of the development of this adaptive relaying protocol, consider only a Rayleigh fading channel, since this chapter has previously shown that full collision is preferred in the

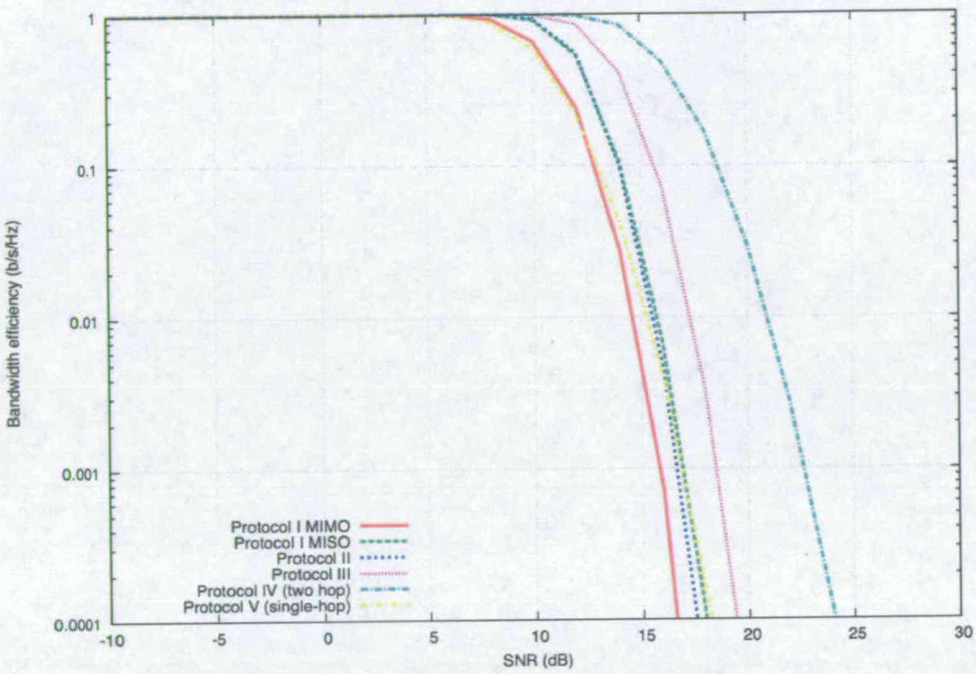


Figure 5.9: *Simulation results of cooperative diversity outage in a frequency-selective environment using the COST 207 TU model where the spectral efficiency is 1 b/s/Hz. SNR and outage probability are compared for all relaying protocols including both MISO and MIMO based transmission for Protocol I.*

second phase, which is the same as a narrowband system which uses full receive collision in the second phase. Furthermore, the system constraints previously placed on the relay network are continued in this section.

5.5.1 Protocol selection

Before developing an analytical solution to the selection of which protocol should be used at any one time, consider which protocols might be preferred under certain channel conditions. Protocol II will always outperform Protocol III due to the power limitation on Protocol III. Protocol II will also always outperform Protocol IV as it has the advantage of utilising the single-hop source to destination link in the first phase as well as the two-hop relay link, which Protocol IV is limited to. Similarly the relay link (Protocol IV) will always be outperformed by Protocol II due to the additional single-hop transmission in Protocol II. Furthermore, although Protocol I in both MIMO and MISO modes would generally be expected to outperform Protocol II, this generalisation cannot be made here due to the system transmit power constraint as this prevents Protocol I (in either MIMO or MISO mode) from transmitting at full power, therefore reducing the achievable rate. Equally, Protocol I (MIMO mode) would generally be expected to outperform Protocol I (MISO mode), however again this generalisation cannot be made. Finally, the single-hop link (Protocol V) would be expected to be the optimal protocol if either the source to relay or relay to destination link is poor. In summary the four protocols which can be considered in the selection of the optimal protocol for adaptive relaying are:

- Protocol I - MIMO mode
- Protocol I - MISO mode
- Protocol II
- Protocol V (single-hop)

The analysis of which protocol is optimal under certain channel conditions is complicated considerably by the fact that the optimal power levels are not fixed, but depend on the current link's SNR and the attempted spectral efficiency. Due to this, the analysis is implemented by a simulation similar to those carried out in section 4.4.3. Specifically the simulation considers the capacity achievable at the 5% outage level. Consider a relay channel with one relay potentially available to assist the source. For each set of channel conditions it is possible to calculate the capacity that each of the four candidate adaptive protocols can achieve. The percentage of time that each protocol is used can then be analysed for each SNR, the results of which are shown in Figure 5.10.

The results shown in Figure 5.10 are particularly interesting, although not necessarily surprising. It is shown that Protocol I in either MIMO or MISO mode is never the optimal protocol to be considered. Although it might be expected that Protocol I in MIMO mode would outperform the other protocols, due to the power constraint on the system this situation never arises. This would not be the case if the power constraint was not in place, as shown in [78]. It can also be seen that generally single-hop transmission is the preferred option, although Protocol II is used a significant amount of the time, particularly at low SNRs. This is due to the extra diversity offered by the relay, and the roughly 1/3 to 2/3 split that can be observed between Protocols II and V might be expected as there are three randomly fading channels. However, as the signal quality improves the single-hop case becomes dominant due to that fact that it utilises full-time transmission, while Protocol II can use only half-time transmission.

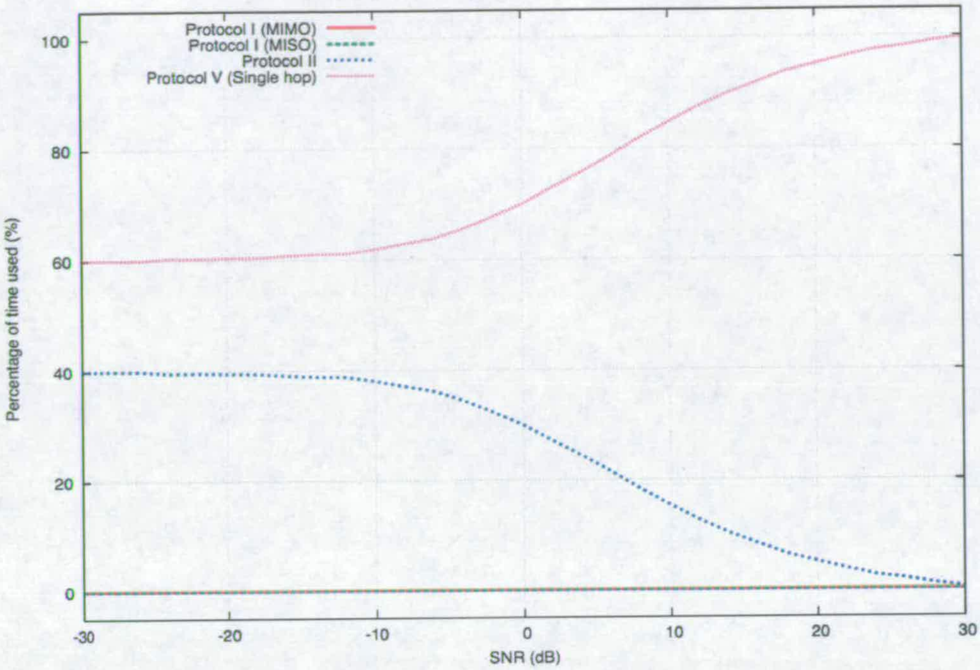


Figure 5.10: *Simulation results of the percentage of time each of the four candidate bandwidth efficiency optimal adaptive protocols are used for 5% outage level.*

An important issue which arises from the fact that only Protocol II and Protocol V are used in the adaptive optimal protocol is that the overall complexity of the system in considerably reduced when compared to a system which employs Protocol I in either MISO or MIMO mode. This is due to the fact that neither Protocol II or V utilise receive collision at the destination (unless there is more than one supporting relay), therefore making the decoding at the receiver easier.

5.5.2 Analytical selection

Due to that fact that only Protocols II and V are used in the adaptive transmission protocol, an analytical selection algorithm can readily be obtained, and used in a practical implementation. Initially it can be said that if the mutual information achievable by Protocol V is greater than that which can be achieved by Protocol II, then Protocol V should be used. Likewise the converse is true. Therefore the inequality $I_5 > I_2$ can be written. This can be expanded and simplified as follows

$$
\begin{aligned}
I_5 &> I_2 && (5.9) \\
\log_2\left(1+\mathrm{SNR}|h_{SD}|^2\right) &> \frac{1}{2}\log_2\left(1+\mathrm{SNR}|h_{SD}|^2+\mathrm{SNR}|h_{RD}|^2\right) && (5.10) \\
\left(1+\mathrm{SNR}|h_{SD}|^2\right)^2 &> 1+\mathrm{SNR}|h_{SD}|^2+\mathrm{SNR}|h_{RD}|^2 && (5.11) \\
1+2\mathrm{SNR}|h_{SD}|^2+\mathrm{SNR}^2|h_{SD}|^4 &> 1+\mathrm{SNR}|h_{SD}|^2+\mathrm{SNR}|h_{RD}|^2 && (5.12) \\
|h_{SD}|^2\left(1+\mathrm{SNR}|h_{SD}|^2\right) &> |h_{RD}|^2 && (5.13)
\end{aligned}
$$

Furthermore, it is important to note that (5.13) assumes that the relay can successfully decode the signal transmitted at the attempted spectral efficiency from the source in the first phase, which is not always true. This decision bound comes directly from the RRDF framework that the considered relaying network employs, and is given by

$$
\begin{aligned}
\log_2\left(1+\mathrm{SNR}|h_{SR}|^2\right) &< \mathrm{R} && (5.14) \\
1+\mathrm{SNR}|h_{SR}|^2 &< 2^{2\mathrm{R}} && (5.15) \\
|h_{SR}|^2 &< \frac{2^{2\mathrm{R}}-1}{\mathrm{SNR}} && (5.16)
\end{aligned}
$$

If either inequality, (5.13) and (5.16), is true, then Protocol V is the optimal protocol to use. However, if both inequalities are false, Protocol II should be used. These inequalities are relatively trivial in complexity, and can be readily implemented and calculated on-the-fly, by the processors of a mobile terminals, which receive feedback from the destination. In this way it can switch into and out of cooperative mode when it is attractive to do so. It is also important to note that using the adaptive protocol presents interference issues which are synomonous with cooperative diversity relaying networks and must be considered before practical implementation, although this is beyond the scope of this thesis.

5.5.3 Numerical results

To conclude the discussion of an adaptive protocol which makes use of both Protocol II and Protocol V, depending on the instantaneous channel conditions, results are presented which compare the adaptive protocol to several other relaying protocols. The performance of the

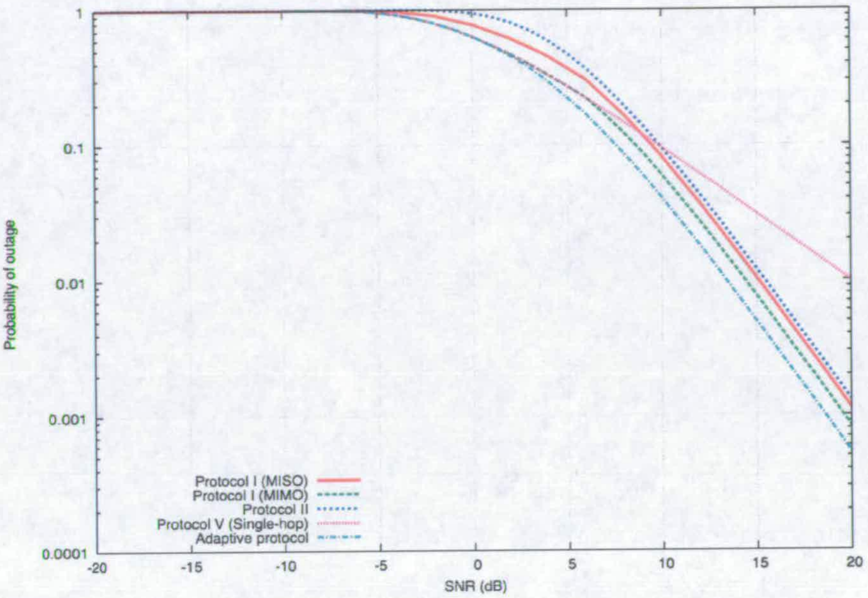


Figure 5.11: *Simulation results for cooperative diversity protocols, including the adaptive protocol, where the spectral efficiency is fixed at 1 b/s/Hz. SNR and outage probability are compared for all four candidate protocols.*

adaptive protocol is measured for the probability of outage and the capacity at a fixed level of outage.

Although it was shown in the previous chapter that Protocol I (MIMO mode) is preferred if only a single transmission protocol is used in the relaying network, it would be expected that the adaptive protocol would outperform all other protocols, including Protocol I (MIMO mode). This is clearly shown in Figure 5.11 which directly compares the adaptive protocol with the four candidate protocols, where the attempted spectral efficiency is fixed at 1 b/s/Hz. At high SNR the adaptive protocol outperforms Protocol I (MIMO mode) by 1.92dB, making it very attractive for high reliability wireless communications. The diversity offered by the supporting relay is also clearly observable, which allows the adaptive protocol to considerably outperform the traditional single-hop link, particularly at high SNR.

As in the previous chapter, the results where the achievable capacity, where a certain level of outage is tolerated, are of particular interest from the point of view of how the adaptive protocol might perform in a practical wireless network. These results are presented in Figure 5.12. As can be seen, the adaptive protocol offers a significant increase in capacity at low SNR over all other considered relaying protocols. Most noticeable is that the adaptive protocol offers a gain

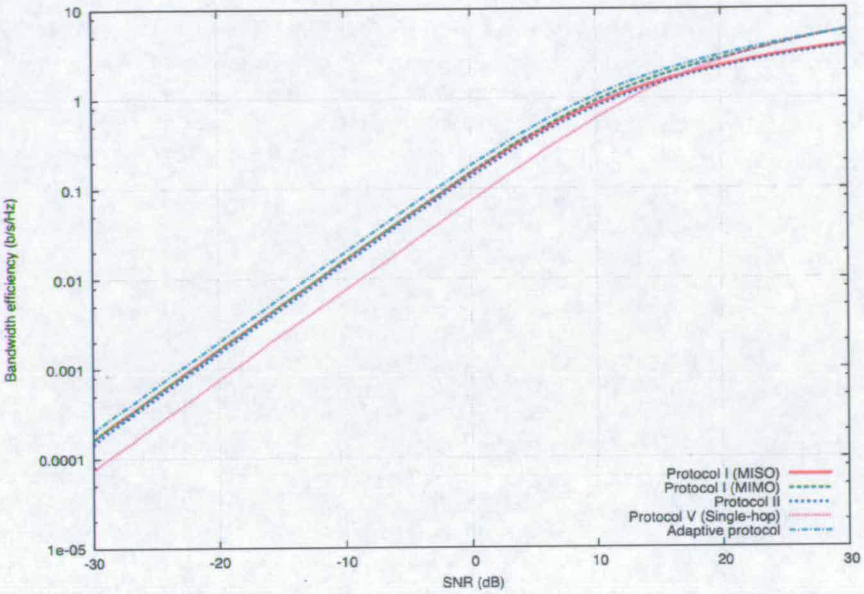


Figure 5.12: *Simulation results for the bandwidth efficiency of the cooperative diversity protocols, including the adaptive protocol, at the 5% outage level. SNR and bandwidth efficiency are compared for all four candidate protocols.*

of 4.25dB over the traditional single-hop case at low SNRs, compared to the 3.4dB offered by Protocol I (MIMO mode). At high SNR, due to the fact that the single-hop case is usually preferred as shown in Figure 5.10, the capacity approaches that of the single-hop case.

The results presented here show that an adaptive cooperative diversity protocol which can switch between Protocol II and Protocol V dependant on instantaneous channel conditions can offer a significant increase in the performance of the channel.

5.6 Conclusions

This chapter has considered two methods to optimise the family of relaying protocols introduced in the previous chapter. Initially a frequency-selective fading environment was considered and then an adaptive protocol which uses the optimal protocol based on instantaneous channel conditions was developed.

Initially the channel model previously used was extended to consider an OFDM channel with N tones and L taps at the receiver to model the multi-path channel. The effect of second phase collision upon the bandwidth efficiency was then considered by varying the amount of signal collision in the OFDM tones at the receiver through a control parameter b. It was shown that the different combinations of collision types on each tone across the two time phases could be considered as either Protocol I (MIMO mode), Protocol II or Protocol V. Protocol I (MIMO mode) was the main protocol considered as it was previously shown to offer the greatest benefits in both terms of capacity and outage. The conclusion drawn here is that full collision is preferred to maximise the bandwidth efficiency at all SNRs.

The effect of frequency-selective fading on the full family of relaying protocols was then considered by using the COST 207 multi-path model. It was shown that the capacity and outage ordering derived in the previous chapter holds for the frequency-selective scenario and numerical results were presented for two COST 207 channel models.

The second part of this chapter considered choosing a protocol to maximise the capacity of the network by using information from the receiver fed-back from the destination to the source, about the status of each individual channel in the network. Four relaying protocols were considered as candidate protocols which might be preferred under certain channel conditions, however it was shown that only Protocol II and Protocol V (single-hop) were ever preferred. An analytical approach was then introduced to select the optimal protocol on-the-fly, using a method which could be readily implemented in a practical wireless system. Finally, numerical results were presented showing that the adaptive protocol can offer significant benefits over the other relaying protocols, in-particular over the traditional single-hop link.

Chapter 6
Conclusions

This final chapter summarises the work that has been conducted in each of the previous chapters and draws conclusions from the results and contributions presented. The limitations of this work and several avenues for future research are also considered.

6.1 Summary

Chapter 1 gave an introduction to the history of wireless communications and an overview of the currently deployed cellular networks. It was shown that as wireless telecommunications technology has progressed there have always been demands for increased capacity of the network. In early development and until the deployment of second generation cellular networks this demand was for increased user capacity of the networks, primarily due to poor frequency reuse. This concern has been addressed in 2G networks as seen by the large numbers of subscribers to the GSM protocol. The demand for capacity has now turned to demand for increased data capacity for higher quality communications and new services such as music and video downloads and full internet access. Although this has been met to some extent by the development and deployment of 3G networks, consumers have continued to require higher data rates as services evolve. Meeting this demand is highlighted as a key goal for future generation networks and is subject to several avenues of research, including relaying, which was considered in this thesis.

To continue a more rigorous discussion of technologies, which harness the wireless channel, and methods to offer increased capacities, chapter 2 initially presented the properties and effects observed in the wireless channel. It was shown that the signal received by a mobile terminal is the superposition of all the paths (rays) that the transmitted signal will take between the transmitter and receiver due to scattering. In a high scattering environment, such as a city centre, this superposition can be modelled accurately by the Rayleigh distribution probability density function, and gives rise to a phenomenon known as fast-fading. Fast-fading makes the wireless channel uniquely hostile for communications, but also offers diversity in the channel.

By utilising this diversity using multiple antenna element techniques, each antenna element can receive an independent fading signal, which has been shown to offer considerable potential for increasing data capacities.

As the number of antenna elements in a mobile device is limited, several authors have proposed that MTs could co-operate to form a VAA, effectively combating fast-fading through sharing information in a cooperative diversity protocol. In an ad-hoc cooperative diversity scenario the source's transmission will be supported by at least one potential relay, which will forward information about the signal from the source to the destination. By necessity, transmission in a cooperative network is split into two time phases whereby the relay will receive information from the source in the first phase, before transmitting it to the destination in the second phase. Several methods have been proposed for how the relay can support the source, and research has primarily focused on the RRDF method. MRDF has been proposed as an alternative and a comparison of bandwidth efficiency showed that RRDF offers greater benefits in terms of capacity, and was used throughout the remainder of this thesis.

Chapter 3 initially expanded the work available from cooperative diversity literature, where a single antenna is used, to consider the effect on the capacity of the network as well as the probability of outage. Following this, the advantages of cooperative diversity where each MT is equipped with two antenna elements, which is the maximum number that is suitable for application in a device the size of a mobile phone, were characterised. This approach effectively combines the diversity advantage of multiple antenna elements with that of cooperative diversity relaying. The protocol used involved the source transmitting in the first phase but not the second, while the relay listens in the first and transmits information to the destination in the second assuming the RRDF criterion is met. Three system constraints were introduced to enable a fair and direct comparison with the traditional single-hop channel to be made; transmit time, power and bandwidth must not exceed that of the single-hop case. The concept of full-time and part-time cooperative diversity was also introduced and explored. In part-time transmission the amount of information the source can transmit, compared to the single-hop case is limited to 1/2 due to the system constraints, while in full-time transmission the source is able to transmit the full amount of information that it would be able to in the single-hop case. Spatial-Multiplexing methods were considered, in particular V-BLAST, as a method of achieving full-time transmission, effectively allowing the source to transmit twice the information in the first phase as the single antenna case, recovering the 1/2 constraint factor.

STTD was also considered as a transmission method, and is typically more suited to improving the reliability of the communications channel.

The numerical results presented in chapter 3 show that although cooperative diversity is limited by the necessity of having two transmission phases, it can offer considerable benefits in terms of capacity in a Rayleigh fading environment. Using the techniques presented it is possible to counter the effects of the Rayleigh fading channel effectively, by employing a combination of the diversity offered by the relays and that which is offered by the multiple antenna elements. It was shown that although SM can achieve full-time transmission, the half-time STTD method is preferred due to its use of data redundancy in the Rayleigh fading environment.

Chapter 4 focused on developing an information theory framework which could be used to fully characterise the cooperative diversity channel. The entire family of cooperative diversity protocols, initially presented by Nabar et al., was introduced and an information theory framework suitable for RRDF relaying was developed. This framework also supports the single-hop and two-hop channels as special cases, and allows multiple relays to support the source. Furthermore, the three system constraints placed on the system in chapter 3 were continued in this chapter and throughout the remainder of this thesis. The introduction of the additional cooperative diversity protocols presented the opportunity to apply MIMO techniques to the cooperative channel, in particular where the source transmits to the destination in the second phase. In this case the source could send extra information rather than repeating the first phase information similar to the relay's transmission. Subsequently, Protocol I and Protocol III, where this condition was met, could be considered to be full-time cooperative diversity, termed MIMO cooperative diversity in this chapter for clarity of transmission technique. The remaining protocols, including Protocol II, which was also considered in chapter 3, are only able to transmit using half-time cooperative diversity techniques, and therefore termed MISO cooperative diversity.

The effect of the power constraint on the system was noted to be particularly important for those protocols, which transmit information from the source in both time phases. In order to ensure that the power constraint is met, the transmitting terminals must reduce their transmit power in a manner which is optimal in terms of probability of outage (the channel not being able to support the required rate). An analytical approach was taken where possible to find the optimal transmit power values, and numerical results were presented. It was found that a closed form expression could not be derived for Protocol I in MIMO mode and a numerical approach was taken to find the optimal transmit power levels for this case. Capacity and probability of outage simulations were presented, which show that Protocol I in MIMO mode offers 3.4dB gain in capacity over the single-hop case at low SNR, while also offering full diversity.

Chapter 5 considered two approaches to further optimise the cooperative diversity channel. Initially OFDM transmission is considered to analyse the effect of second phase collision on the capacity of the channel. As Protocol I in MIMO mode out-performed the others in chapter 4, and also utilises the full degrees of collision in the second phase, this was the main protocol to be considered. The framework developed in chapter 4 was extended to consider the frequency selective OFDM channel, whereby the amount of collision in the second phase was controlled by the parameter b. That full collision is preferred to maximise the capacity of the channel was shown by numerical results.

The second approach that was taken in chapter 5 was to develop an adaptive cooperative diversity protocol, which uses the optimal protocol in terms of capacity based on instantaneous channel conditions. Four protocols were considered from the family of relaying protocols, including Protocol I MIMO, Protocol I MISO, Protocol II and Protocol V (single-hop). Initially a numerical approach was taken to analyse the frequency of which each protocol was used. This showed that only Protocol II and Protocol V were ever used, the proportion of which is dependant on the SNR modelled. An analytical method was then derived to select which protocol should be used. The capacity and probability of outage results for the individual protocols from chapter 4 were compared with the adaptive protocol, which shows that the adaptive protocol, as would be expected, offers improved performance over the other protocols. Significantly, at the 5% outage level the adaptive protocol gives a 4.25dB improvement in performance over the single-hop case.

6.2 Conclusions

A number of conclusions can be drawn from the work presented in this thesis. Most importantly is that it has been shown that under the constraints imposed upon the network to make a direct comparison with the single-hop case, the cooperative diversity protocols can offer benefits in terms of both probability of outage and capacity. This is particularly evident in chapter 3 where each MT is equipped with two antenna elements. The combination of the diversity offered by the multiple antenna elements and the VAA show clear advantages of using STTD based half-time cooperative diversity.

The cumulative effect of work in chapter 4 and chapter 5 is particularly interesting as it shows that although Protocol I in MIMO offers the greatest benefits in terms of a single protocol, an adaptive protocol consisting of Protocol II and Protocol V would be preferred. This is important as Protocol I in MIMO mode is comparatively complex both in terms of decoding the signal at the receiver in the second phase and also the transmit power allocation. As a numerical approach cannot be taken to calculate the Protocol I MIMO mode transmit power levels they would have to be pre-calculated in a quantised manner and stored in a look up table at each transmitter. The advantage of the protocols, which make up the adaptive protocol, is that they only have a single terminal transmitting at any one time (assuming that there is only one supporting relay). This means that each transmitting terminal can transmit at the full power available to it, thereby reducing the complexity of the protocol. Also there is no collision at the destination during the second phase making decoding of the second phase signal much simpler.

As the adaptive protocol gives a gain of 4.25dB over the single-hop case in the scenario considered in this thesis, the assumption that relays would be provided in a cellular environment by subscriber units would have to be closely considered. If two subscriber units co-operate together to support the transmission from the source, any gain greater than 3dB would represent an increase in capacity which is shared between each user. In this case, a gain in capacity of 1.23dB would be recognised by each user. Due to the extra signalling overhead required by the cooperative diversity protocol it is unclear as to whether this benefit is enough to see an increased capacity in a deployed product. Note also that cooperative diversity can effectively be used to increase the coverage area of a cell as the capacity that is achievable at any particular SNR for all links would require a much higher SNR for the source to destination link in the single-hop case.

One possibility for cooperative diversity deployment is to use fixed relays, which are installed and managed by the network operators, in hotspot areas, such as city centre offices or cafes, and support a number of MTs in a limited geographical location. This would effectively increase the coverage area of the cell and remove the security concern of sharing information with other users. Furthermore, this method could increase the battery life of an MT as, under certain channel conditions, during the uplink transmission from the subscriber unit to the base station the MT would only transmit for half of the time slot, while the fixed position relay would transmit for the second half. The benefit to users of the network would be a considerable increase in the network capacity due to the abilities of the adaptive cooperative diversity method to combat Rayleigh fading and the benefit to network operators would be the increased coverage of the cell.

6.3 Future work

Although the benefits of using cooperative diversity are significant, there are a number of areas that future research can pursue using the information theory framework developed in this thesis. In particular, the numerical results in this paper have generally assumed that the SNR of all channel links in the network are identical. This is impractical in any realistic wireless environment (particularly where there is more than one relay) and a full study where the relay is mobile in the ad-hoc environment is needed. It is interesting to note that the method used in this thesis, where links have the same SNR, is non-optimal and therefore the capacity benefits of the adaptive cooperative diversity protocol may be considerably higher than reported here. An example of this is if the relay is positioned exactly between the source and destination terminals, then E_{SR}/N_0 and E_{RD}/N_0 are likely to be much higher than E_{SD}/N_0. Under the conditions of which transmission method the adaptive protocol must use, (5.16) and (5.13), the probability of the relay being actively used becomes much greater, thereby offering the benefits of the additional diversity to the communications channel.

The analysis of the case where E_{SR}/N_0, E_{RD}/N_0 and E_{SD}/N_0 are not all equal can readily be carried out using the information theory framework developed in this thesis. The assumption $E_{SR}/N_0 = E_{RD}/N_0 = E_{SD}/N_0 =$ SNR made in section 4.3.3.1 must simply be dropped. Also note that when this constraint is relaxed, the relative merits of each protocol may change significantly and is an interesting avenue for future research.

Similarly, it was assumed that the relays available to support the source were able to transmit with up to the same transmit power as the source. Although this might be true in the uplink channel where the source and relays are both mobile terminals, it is not a reasonable assumption for the downlink where a mobile terminal may be acting as a relay, or even if the fixed relay deployment method presented in the previous section is utilised, to support the transmission from a base-station. In a cellular network it would be expected that a base-station would be able to transmit at much higher power than the relays. This is a further power constraint on the system which must be considered in future work.

Further to the consideration of the constraints placed on relay transmit power, as stated in chapter 3, throughout this thesis it was assumed that all relays transmit with $1/(|\mathcal{J}| - 1)$ of the full power available. In the case where the link quality is known, such as is required for the adaptive protocol, power could be allocated in a manner which would maximise the signal power at the receiver. One possible method which could be used is the well known water filling power allocation technique [105].

It has been assumed during the discussion of the adaptive cooperative diversity protocol that the source's knowledge of the forward channels in the network is perfect and that it will always make the optimal decision. Relaxing this constraint and exploring the effect of the level of channel knowledge required is an important consideration for deployment of an adaptive cooperative diversity network. As previously noted HSDPA makes use of such a system already, modifying the modulation used depending on channel conditions, and use of the techniques used by HSDPA may prove fruitful for a cooperative diversity study. Furthermore, another important constraint placed on the system is that all networks are perfectly synchronised. Again this is not a practical assumption in a deployed wireless network, and the effect of relaxing this constraint to include timing synchronisation data in the protocol must be fully explored. A suitable signalling protocol must also be developed for the cooperative diversity protocols, with particular emphasis on developing suitable STCs. An evaluation of the signalling overhead that the cooperative diversity protocols require in order to perform effectively, is also required, considering the impact on the gain in capacity that can be achieved.

Furthermore, Protocols I-V describe the full range of possible transmit protocols in a two phase transmission system. However, note that the number of possible protocols increases dramatically as the number of transmit phases increases. As an example consider a protocol under a three phase transmission, in phase one and two the source transmits to both relay and destination, in the third phase the relay repeats a combination of the two signals it received previously (sending only parity bits is one possibility to reduce the data requirements for the third phase). Along with the increased complexity of such a system, previous work has shown that repetition based cooperative diversity, which splits the available transmit phases time between each transmitting terminal in the network, is spectrally inefficient due to the limited information that can be transmitted in such a network [76]. However the benefit of extra diversity in a high scattering environment is significant for high reliability communications and this is an area where future research may prove useful, particularly in an ad-hoc environment where many relays are available such as in a military theatre. Note that the information theory framework developed in this thesis can readily be expanded to include multiple transmission phases by increasing the number of rows in the channel matrix (4.8).

As well as the possibility of increasing the number of transmission phases in a cooperative diversity network, it is also possible to consider a situation where the duration of each transmission phase is unequal. Throughout this thesis it was assumed that the two transmission phases utilised were always of equal duration, however unequal transmission phases may prove to provide additional benefits. An example of this is the situation where the source and relay have a very good link, while neither the source nor relay have a good link with the destination. In such a situation, a short transmission from the source to the relay, followed by a longer second phase might prove advantageous.

It is clear that cooperative diversity and, in particular, an adaptive protocol, which uses cooperative diversity techniques, offer the possibility of significant enhancements to wireless networks. This is done by using the properties of the wireless channel to the advantage of communications through spatial diversity techniques. There is considerable research and development interest in this area and with further development it can be concluded that cooperative diversity is a suitable candidate, in parallel with several other techniques, for supplying the required high data capacities in future generation cellular networks.

Appendix A
Relaying protocols

This appendix presents the full family of relaying protocols used in Chapter 4, in both channel matrix and diagrammatic form. The capacity equations for both MIMO and MISO modes are also presented where one relay is available. Note that the subscript to identify which protocol each power control level (A, B and C) applies to is not shown in this appendix as the the tabular structure makes these subscripts irrelevant.

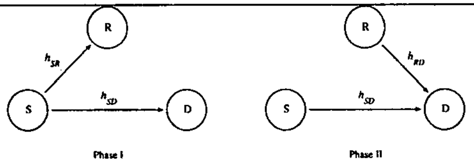
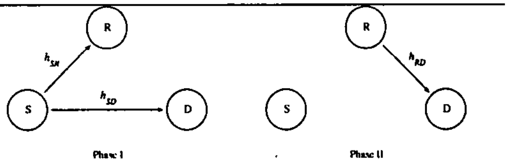
	Protocol	
	I	II
Channel model		
Channel matrix	$\mathbf{H}_1 = \begin{bmatrix} \sqrt{AE_{SD}}h_{SD} & 0 \\ \partial(\mathcal{J}) & \sqrt{BE_{SD}}h_{SD} \end{bmatrix}$	$\mathbf{H}_2 = \begin{bmatrix} \sqrt{AE_{SD}}h_{SD} & 0 \\ \partial(\mathcal{J}) & 0 \end{bmatrix}$
MIMO capacity (b/s/Hz)	$\frac{1}{2}\log_2\left(\frac{ABE_{SD}^2}{N_0}\|h_{SD}\|^4 + \frac{AE_{SD}}{N_0}\|h_{SD}\|^2 + \frac{BE_{SD}}{N_0}\|h_{SD}\|^2 + \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \frac{C_j E_{RD}}{N_0}\|h_{RD_j}\|^2 + 1\right)$	N/A
MISO capacity (b/s/Hz)	$\frac{1}{2}\log_2\left(\frac{AE_{SD}}{N_0}\|h_{SD}\|^2 + \frac{BE_{SD}}{N_0}\|h_{SD}\|^2 + \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \frac{C_j E_{RD}}{N_0}\|h_{RD_j}\|^2 + 1\right)$	$\frac{1}{2}\log_2\left(\frac{AE_{SD}}{N_0}\|h_{SD}\|^2 + \sum_{\mathcal{J}_j \in \partial(\mathcal{J})} \frac{C_j E_{RD}}{N_0}\|h_{RD_j}\|^2 + 1\right)$

Table A.1: *Mutual information equations and channel descriptions for Protocols I and II relay channel cooperative diversity and DF mode for both MIMO and MISO realisations conditioned on the relay set* $\partial(\mathcal{J})$.

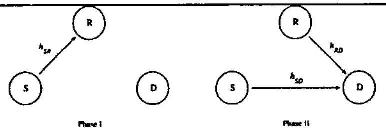
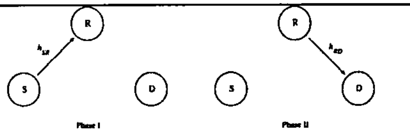
	Protocol	
	III	IV
Channel model		
Channel matrix	$\begin{bmatrix} 0 & 0 \\ \partial(\mathcal{J}) & \sqrt{BE_{SD}}h_{SD} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ \partial(\mathcal{J}) & 0 \end{bmatrix}$
MIMO capacity (b/s/Hz)	$\frac{1}{2}\log_2\left(\frac{BE_{SD}}{N_0}\|h_{SD}\|^2 + \sum_{\mathcal{J}_j\in\partial(\mathcal{J})}\frac{C_jE_{RD}}{N_0}\|h_{RD_j}\|^2+1\right)$	N/A
MISO capacity (b/s/Hz)	$\frac{1}{2}\log_2\left(\frac{BE_{SD}}{N_0}\|h_{SD}\|^2 + \sum_{\mathcal{J}_j\in\partial(\mathcal{J})}\frac{C_jE_{RD}}{N_0}\|h_{RD_j}\|^2+1\right)$	$\frac{1}{2}\log_2\left(\sum_{\mathcal{J}_j\in\partial(\mathcal{J})}\frac{C_jE_{RD}}{N_0}\|h_{RD_j}\|^2+1\right)$

Table A.2: *Mutual information equations and channel descriptions for Protocols III and IV relay channel cooperative diversity and DF mode for both MIMO and MISO realisations conditioned on the relay set* $\partial(\mathcal{J})$.

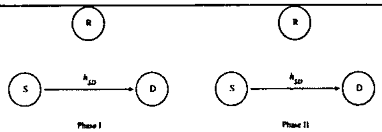
	Protocol
	V
Channel model	
Channel matrix	$\begin{bmatrix} \sqrt{AE_{SD}}h_{SD} & 0 \\ 0 & \sqrt{BE_{SD}}h_{SD} \end{bmatrix}$
MIMO capacity (b/s/Hz)	$\frac{1}{2}\log_2\left(\frac{ABE_{SD}^2}{N_0}\|h_{SD}\|^4 + \frac{AE_{SD}}{N_0}\|h_{SD}\|^2 + \frac{BE_{SD}}{N_0}\|h_{SD}\|^2 + 1\right)$
MISO capacity (b/s/Hz)	$\frac{1}{2}\log_2\left(\frac{AE_{SD}}{N_0}\|h_{SD}\|^2 + \frac{BE_{SD}}{N_0}\|h_{SD}\|^2 + 1\right)$

Table A.3: *Mutual information equations and channel descriptions for Protocol V relay channel cooperative diversity and DF mode for both MIMO and MISO realisations.*

Appendix B
Joint distribution functions

This appendix presents the derivation of the PDF for the channel fading random variable for Protocol I in MIMO mode, and the CDF of the channel fading random variable for Protocol III.

B.1 Derivation of joint PDF for Protocol I (MIMO mode)

As was discussed in Chapter 4, the derivation of the PDF, and subsequently the CDF, is considerably more complex in the Protocol I MIMO case than for either Protocol I in MISO mode or Protocol III. This is due to the additional h_{SD}^4 term in the capacity equation. The probability for outage of the relay channel, operating with a single relay available, is given by

$$\Pr\left[A_{1_M} B_{1_M} \mathrm{SNR} |h_{SD}|^4 + (A_{1_M} + B_{1_M})|h_{SD}|^2 + C_{1_M}|h_{RD}|^2 < \frac{2^{2\mathrm{R}} - 1}{\mathrm{SNR}}\right] \tag{B.1}$$

Before a PDF can be derived for the joint h_{SD} and h_{RD} terms of (B.1) a single equation must be found for the h_{SD} term. Begin by defining

$$Y = A_{1_M} B_{1_M} \mathrm{SNR} |h_{SD}|^4 + (A_{1_M} + B_{1_M})|h_{SD}|^2 \tag{B.2}$$

which shows that Y is a function of the single random variable h_{SD}. To find the PDF of a function of a random variable employ (5-15) and (5-16) as presented by Papolius and Pillai [106] where the equation $Y = g(X)$ is solved. Denoting the real roots by X_n,

$$y = g(X_1) = \ldots = g(X_n) \tag{B.3}$$

Papolius and Pillai show that the PDF of a single random variable is given by

$$f_Y(Y) = \frac{f_X(1)}{|g'(X_1)|} + \frac{f_X(2)}{|g'(X_2)|} + \ldots + \frac{f_X(n)}{|g'(X_n)|} \tag{B.4}$$

where $f_X(X)$ is the function of the single random variable in question, and $g'(X_1)$ is the derivative of $g(X)$. For the case in question let $X = |h_{SD}|^2$ to give

$$g(X) = A_{1_M}B_{1_M}\text{SNR}X^2 + (A_{1_M} + B_{1_M})X - Y \tag{B.5}$$

To find the roots of (B.5) use the standard quadratic equation where $a = A_{1_M}B_{1_M}\text{SNR}$, $b = A_{1_M} + B_{1_M}$ and $c = -Y$.

$$\begin{aligned} 0 = \quad & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{(B.6)} \\ & \frac{-(A_{1_M} + B_{1_M}) \pm \sqrt{(A+B)^2 + 4A_{1_M}B_{1_M}Y\text{SNR}}}{2A_{1_M}B_{1_M}\text{SNR}} && \text{(B.7)} \\ & \frac{\sqrt{(A_{1_M} + B_{1_M})^2 + 4A_{1_M}B_{1_M}Y\text{SNR}} - (A_{1_M} + B_{1_M})}{2A_{1_M}B_{1_M}\text{SNR}} && \text{(B.8)} \end{aligned}$$

where in (B.8) discard the negative term since the interest lies only in probabilities (i.e. between 0 and 1 inclusive) leaving only one root. Furthermore the derivative $g(x)$ is found as

$$g'(x) = A_{1_M}B_{1_M}\text{SNR}X + A_{1_M} + B_{1_M} \tag{B.9}$$

By combining (B.4), (B.8) and (B.9) the general PDF is given by

$$f(Y) = \frac{\Pr\left[\frac{\sqrt{(A_{1_M}+B_{1_M})^2+4A_{1_M}B_{1_M}Y\text{SNR}}-(A_{1_M}+B_{1_M})}{2A_{1_M}B_{1_M}\text{SNR}}\right]}{2A_{1_M}B_{1_M}\text{SNR}\left(\frac{\sqrt{(A_{1_M}+B_{1_M})^2+4A_{1_M}B_{1_M}Y\text{SNR}}-(A_{1_M}+B_{1_M})}{2A_{1_M}B_{1_M}\text{SNR}}\right) + (A_{1_M} + B_{1_M})} \tag{B.10}$$

$$= \frac{\Pr\left[\frac{\sqrt{(A_{1_M}+B_{1_M})^2+4A_{1_M}B_{1_M}Y\mathrm{SNR}}-(A_{1_M}+B_{1_M})}{2A_{1_M}B_{1_M}\mathrm{SNR}}\right]}{\sqrt{(A_{1_M}+B_{1_M})^2+4A_{1_M}B_{1_M}Y\mathrm{SNR}}} \tag{B.11}$$

Since h_{SD}^2 is a Rayleigh distributed random variable, the PDF of which is given by Pr[x] $= \mathrm{e}^{-x}$, the PDF of the joint h_{SD} term is given by

$$\frac{\exp\left(\frac{\sqrt{(A_{1_M}+B_{1_M})^2+4A_{1_M}B_{1_M}x\mathrm{SNR}}-(A_{1_M}+B_{1_M})}{2A_{1_M}B_{1_M}\mathrm{SNR}}\right)}{\sqrt{(A_{1_M}+B_{1_M})^2+4A_{1_M}B_{1_M}x\mathrm{SNR}}} \tag{B.12}$$

where the value of interest is denoted x.

Before deriving the final combined PDF, note that the PDF of $C_{1_M}|h_{SD}|^2$ is given by the RHS of (B.1) as

$$\frac{1}{C_{1_M}}\mathrm{e}^{\frac{-x}{C_{1_M}}} \tag{B.13}$$

Under the above conditions x is given by

$$\frac{2^{2R}-1}{\mathrm{SNR}} \tag{B.14}$$

Therefore the joint PDF is found by convolving the two PDFs according to

$$(f*g)(x) = \frac{\exp\left(\frac{\sqrt{(A_{1_M}+B_{1_M})^2+4A_{1_M}B_{1_M}x\mathrm{SNR}}-(A_{1_M}+B_{1_M})}{2A_{1_M}B_{1_M}\mathrm{SNR}}\right)}{\sqrt{(A_{1_M}+B_{1_M})^2+4A_{1_M}B_{1_M}x\mathrm{SNR}}} \cdot \frac{1}{C_{1_M}}\mathrm{e}^{\frac{-x+v}{C_{1_M}}}\,dv \tag{B.15}$$

As can be seen by inspection of (B.15) this is a non-trivial integration. However, symbolic computer algebra can be employed to find a suitable integral which is given as

$$(f*g)(x) = \frac{\sqrt{\pi}e^{-\frac{4xA_{1_M}B_{1_M}\mathrm{SNR}+{A_{1_M}}^2+2A_{1_M}B_{1_M}+{B_{1_M}}^2-2C_{1_M}A_{1_M}-2C_{1_M}B_{1_M}+{C_{1_M}}^2}{4A_{1_M}B_{1_M}C_{1_M}\mathrm{SNR}}}}{2A_{1_M}B_{1_M}C_{1_M}\mathrm{SNR}\sqrt{-\frac{1}{A_{1_M}B_{1_M}C_{1_M}\mathrm{SNR}}}} \left(erf\left(\frac{csgn\left(A_{1_M}+B_{1_M}\right)A_{1_M}+csgn\left(A_{1_M}+B_{1_M}\right)B_{1_M}-C_{1_M}}{2A_{1_M}B_{1_M}C_{1_M}\mathrm{SNR}\sqrt{-\frac{1}{A_{1_M}B_{1_M}C_{1_M}\mathrm{SNR}}}}\right) + erf\left(\frac{-\sqrt{4xA_{1_M}B_{1_M}\mathrm{SNR}+{A_{1_M}}^2+2A_{1_M}B_{1_M}+{B_{1_M}}^2}+C_{1_M}}{2A_{1_M}B_{1_M}C_{1_M}\mathrm{SNR}\sqrt{-\frac{1}{A_{1_M}B_{1_M}C_{1_M}\mathrm{SNR}}}}\right)\right) \tag{B.16}$$

where $erf(z)$ is the Error Function and $csgn(z)$ is the sign function is used to determine in which half-plane (left or right) the complex-valued number z lies. It is defined by

$$csgn(z) = \begin{cases} 1 & \text{if Re(z) > 0 or Re(z) = 0 and Im(z) > 0} \\ -1 & \text{if Re(z) < 0 or Re(z) = 0 and Im(z) < 0} \end{cases} \tag{B.17}$$

It can be seen that the derived PDF for the joint distribution (B.16) is very complex. To find the CDF of the joint distribution it is necessary to integrate this expression again, however it is not of a standard form and a closed for solution can not be readily found. Due to this, Runge-Kutta numerical integration methods are used to approximate the integral.

B.2 Derivation of CDF for Protocol I (MIMO mode) with no available relays

To be able to fully calculate the probability of outage for Protocol I operating in MIMO mode, consideration must also be made for the situation where no relays are available to support the source in the second phase, for example when the source to relay signal is itself in outage. The previous section of this appendix has already derived the PDF of the h_{SD} term as (B.12). To find the CDF (B.12) simply needs to be integrated. Again using symbolic computer algebra the CDF of the probability of outage where there are no relays available for Protocol I (MIMO mode) is shown to be

$$
\begin{aligned}
&\Pr\left[A_{1_M}B_{1_M}\mathrm{SNR}|h_{SD}|^4 + (A_{1_M} + B_{1_M})|h_{SD}|^2 < \frac{2^{2\mathrm{R}}-1}{\mathrm{SNR}}\right] = \\
&\frac{1}{\sqrt{\mathrm{e}^{\frac{csgn\left(A_{1_M}+B_{1_M}\right)\left(A_{1_M}+B_{1_M}\right)}{A_{1_M}B_{1_M}\mathrm{SNR}}}}}\left(\mathrm{e}^{\frac{A_{1_M}+B_{1_M}}{2A_{1_M}B_{1_M}\mathrm{SNR}}}\right. \\
&\qquad \left. -\,\mathrm{e}^{\frac{-\sqrt{{A_{1_M}}^2+2A_{1_M}B_{1_M}+{B_{1_M}}^2+4xA_{1_M}B_{1_M}\mathrm{SNR}}+A_{1_M}+B_{1_M}}{2A_{1_M}B_{1_M}\mathrm{SNR}}}\cdot\sqrt{\mathrm{e}^{\frac{csgn\left(A_{1_M}+B_{1_M}\right)\left(A_{1_M}+B_{1_M}\right)}{A_{1_M}B_{1_M}\mathrm{SNR}}}}\right)
\end{aligned}
\tag{B.18}
$$

The necessary components to fully construct (4.31) are now available using numerical integration of (B.16) and the closed form solution (B.18).

B.3 Derivation of joint CDF for Protocol III

This section presents the derivation of the CDF used for the Protocol III probability of outage where one relay is available to support the source.

$$
\Pr\left[B_3|h_{SD}|^2 + C_3|h_{RD}|^2 < \frac{2^{2\mathrm{R}}-1}{\mathrm{SNR}}\right] \tag{B.19}
$$

Start by noting that the PDF of $B_3|h_{SD}|^2$ is given by

$$
\frac{1}{B_3}\mathrm{e}^{\frac{-x}{B_3}}. \tag{B.20}
$$

The PDF of $C_3|h_{RD}|^2$ is found in a similar fashion. Convolving these PDFs gives the joint PDF as

$$
\begin{aligned}
(f*g)(x) &= \int_0^x \frac{1}{B_3}\mathrm{e}^{\frac{-x+v}{B_3}}.\frac{1}{C_3}\mathrm{e}^{\frac{-x}{C_3}}dv \\
&= \frac{1}{B_3-C_3}\left(C_3\mathrm{e}^{\frac{x}{C_3}} - B_3\mathrm{e}^{\frac{x}{B_3}}\right)\mathrm{e}^{-x\frac{B_3+C_3}{B_3C_3}}, \qquad B_3 \neq C_3
\end{aligned}
\tag{B.21}
$$

Finally integrating the resulting PDF (B.21) to give the CDF as

$$\begin{aligned}&\int_0^x \frac{1}{B_3 - C_3}\left(C_3 \mathrm{e}^{\frac{x}{C_3}} - B_3 \mathrm{e}^{\frac{x}{B_3}}\right) \mathrm{e}^{-x\frac{B_3+C_3}{B_3C_3}} = \\ &\quad \frac{1}{B_3 - C_3}\left(\mathrm{e}^{-x\frac{B_3+C_3}{B_3C_3}}(B_3 - C_3) - B_3 \mathrm{e}^{\frac{x}{C_3}} + C_3 \mathrm{e}^{\frac{x}{B_3}}\right) \mathrm{e}^{-x\frac{B_3+C_3}{B_3C_3}}, \quad B_3 \neq C_3. \qquad \text{(B.22)}\end{aligned}$$

It can been seen that (B.22) does not apply if $B_3 = C_3$ due to the zero term in the denominator. This special case, due to the power constraints placed on the system in Chapter 4, is only possible when $C_3 = 1$. In this case the probability of outage is given by

$$\Pr\left[|h_{SD}|^2 + |h_{RD}|^2 < \frac{2^{2\mathrm{R}} - 1}{B_3 \mathrm{SNR}}\right] \qquad \text{(B.23)}$$

As was shown in Chapter 4, the combination of the two independent rayleigh fading variables, $h_{SD} + h_{RD}$, are Chi-Squared randomly distributed with four degrees of freedom. Therefore from the Chi-Squared law it can be shown that the CDF for this special case is given by

$$1 - e^{\frac{-x}{B_3}}\left(\frac{1}{B_3} + 1\right), \qquad B_3 = C_3 \qquad \text{(B.24)}$$

Appendix C
COST 207 tap weights

Tap Number	Tap weight	Power (dB)
1	0.575	0
2	0.362	-2
3	0.057	-10
4	0.006	-20

Table C.1: *COST 207 tap weights for the typical rural environment (RA).*

Tap Number	Tap weight	Power (dB)
1	0.090	-4
2	0.113	-3
3	0.226	0
4	0.142	-2.6
5	0.113	-3
6	0.072	-5
7	0.045	-7
8	0.072	-5
9	0.056	-6.5
10	0.029	-8.5
11	0.018	-11
12	0.023	-10

Table C.2: *COST 207 tap weights for the typical urban (non-hilly) environment (TU).*

Appendix D
Published papers

- A. Jardine, S. McLaughlin, and J. Thompson. Comparison of space-time cooperative diversity relaying techniques. *IEEE 61st Vehicular Technology Conference*, 4:23742378, May 2005.
- A. Jardine, J. Thompson, and S. McLaughlin. MIMO cooperative diversity strategies for frequency selective fading relay channels. *IEEE 64th Vehicular Technology Conference*, August 2006.
- A. Jardine, J. Thompson, and S. McLaughlin. Dual antenna cooperative diversity techniques. *IEE Communications Proceedings*, 153:4 556-564, September 2006.
- A. Jardine, S. McLaughlin, and J. Thompson. MIMO Cooperative Diversity in a Transmit Power Limited Environment. *IEEE International Conference on Communications*, June 2007 - *Submitted*

Comparison of space-time cooperative diversity relaying techniques

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Abstract—**In this paper a number of techniques which can be used for performing cooperative diversity in a cellular based network are presented. Initially the idea of cooperative diversity, and the constraints on the system are introduced. Two different implementations using space-time techniques are suggested and compared with the non-cooperative case. It is shown that cooperative diversity can dramatically improve performance of a network in terms of outage probability.**

I. INTRODUCTION

One of the biggest problems faced by designers of mobile telecommunication systems is the multi-path nature of the communications channel. The very multi-path that creates difficulties also offers us *diversity* [1]. This can help mitigate the problem as a result of the transmission of redundant information over independent fading channels in conjunction with a suitable receiver. There are three types of diversity which can be used in cellular communications systems [2]:

- Time diversity - Due to multi-path fading the communication channel will change its characteristics over time, therefore transmitting at different times leads to diversity.
- Frequency diversity - Channels at different frequencies have different characteristics due to frequency selective fading, hence transmitting at different frequencies leads to diversity.
- Space diversity - Channels used from different points in space have their own unique fading characteristics, leading to diversity if transmission occurs from several different points.

In recent years space diversity has been suggested as a route to improve capacity of cellular systems through the use of multiple antennas. Multiple Input Multiple Output (MIMO) communication systems have the transmitter and intended destination equipped with antenna arrays [3]. Through this method information can be sent through more than one independent fading channel, if the antenna elements are separated by a suitable distance and sufficient multi-path diversity exists in the channel. This can potentially result in either very high spectral efficiencies or a low probability of incorrectly decoding any data.

Although the equipment at the base station in a cell can readily be equipped with a multi-element antenna array, physical size limitations of cellular mobile terminals preclude fitting more than two antenna elements. In general it would be preferable to use only one element at the mobile terminal to keep the complexity and hence cost to a minimum. The challenge that is now presented to wireless network designers is how to achieve the high capacities offered by MIMO in a cellular or ad-hoc environment.

One proposal for this was made by Laneman and Wornell [4], where they proposed a scheme using available mobile terminals as relays; forwarding the original message from the source to the intended destination. The relays co-operate together to form a virtual antenna array, hence the name *cooperative diversity* [5]. Where this method suffers is that although it can decrease the outage probability considerably, as it will be shown later, it does so at the expense of the data transfer rate that can be achieved. In this paper two methods for performing cooperative diversity are analysed and compared to the basic direct transmission method. Section II introduces cooperative diversity and the different relaying schemes used in the paper. Section III then presents results and draws comparisons between the different schemes. Finally conclusions are drawn in section IV.

II. COOPERATIVE DIVERSITY - RELAYING TECHNIQUES

To analyse the different relaying techniques, consider a wireless network with a set of transmitting terminals denoted $\mathtt{M} = \{1, 2, \ldots, m\}$. A source terminal, $s \in \mathtt{M}$ has information to transmit to a single destination terminal $d(s) \in \mathtt{M}$, potentially using terminals $\mathtt{M} - \{s\}$ as relays. Thus there are m co-operating terminals to $d(s)$. All of our relaying schemes require that the relay can fully decode the source signal if it is to relay it on to the destination. The subset of $\mathtt{M} - \{s\}$ can decode the signal is defined as $\partial(s)$.

The method described here of implementing cooperative diversity consists of two transmission phases shown in Figure 1. During the first phase the source will transmit information to all available mobile terminals in the network. Those terminals will then attempt to decode the signal, and if successful they will take part as the decoding set in phase two, transmitting information to $d(s)$.

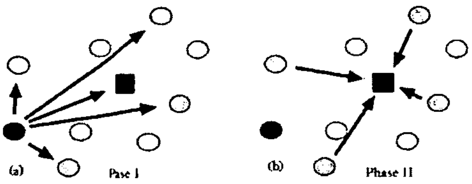

Figure. 1. Illustration of the two phase cooperative diversity method. In the first phase (a) the source transmits information in an omnidirectional manner to all terminals. In phase two (b), those terminals which can decode the signal then relay the signal on to the destination.

In the system model below (equations (1)-(4)) $a_{i,j}$ can capture the effects of path-loss, shadowing and multi-path fading. However, only multi-path fading is considered in this paper to keep calculations simple. Statistically, $a_{i,j}$ is modelled as a zero-mean, independent, circularly-symmetric complex Gaussian random variable with variance $\lambda_{i,j}$, so that the magnitudes $|a_{i,j}|$ are Rayleigh distributed and the phases $\angle a_{i,j}$ are uniformly distributed on $[0, 2\pi)$. Furthermore, $z_j[n]$ is modelled as zero-mean mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_0. The scalar $z_j[n]$ captures the effects of receiver noise and other forms of interference in the system. In this paper it is assumed that quasi-static fading where the fading coefficients are constant over the example time and frequency occurs. The scenario considered here assumes the fading coefficients are known to the appropriate receivers, but not known, or exploited by the transmitters.

During the first phase, each potential relay $r \in M - s$ receives

$$y_r[n] = a_{s,r} x_s[n] + z_r[n] \qquad (1)$$

in the appropriate channel, where $x_s[n]$ is the source transmitted signal and $y_r[n]$ is the received signal at r. If the relay can then decode the source transmission, r will serve as a relay for the source, so that $r \in \partial(s)$.

The destination receives signals during both transmission phases. During the first phase, the received signal is modelled at $d(s)$ as

$$y_{d(s)}[n] = a_{s,d(s)} x_s[n] + z_{d(s)}[n] \qquad (2)$$

in the appropriate channel. During the second phase, the equivalent channel models are different for repetition based cooperative diversity and space-time coded schemes. For repetition based, the destination receives separate re-transmissions from each of the relays, where the received signal $d(s)$ is

$$y_{d(s)}[n] = a_{r,d(s)} x_r[n] + z_{d(s)}[n] \qquad (3)$$

in the appropriate channel, and $x_r[n]$ is the transmitted signal of relay r. For space-time coded schemes, all relay transmissions occur in the same channel and are combined at the destination, so that

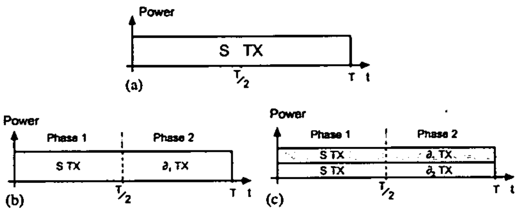

Figure. 2. Cooperative diversity relaying methods, with MACs and illustrative transmission diagrams. (a) The direct transmission scenario (the classical case) where there are no relays, (b) half-time space-time coded cooperative diversity where the relays all transmit in the same sub-channel during phase two and (c) full-time space-time coding where the source transmits using space-time coding and two decoding sets are formed.

$$y_{d(s)}[n] = \underset{r \in \partial(s)}{} a_{r,d(s)} x_r[n] + z_{d(s)}[n] \qquad (4)$$

in the appropriate channel.

Two important parameters of the system are the received signal-to-noise ratio, SNR (dB) and the bandwidth efficiency, R (b/s/Hz). It is natural to define these parameters in terms of the continuous-time channel with non-cooperative diversity as a baseline. In this paper, the simplistic ideal of normalising the attempted rate to 1b/s/Hz, and assuming that all mobile terminals are equidistant from one another is assumed. Obviously this is unrealistic, but it serves as a baseline for comparing the results. All transmission schemes are constrained to transmit at a maximum power of unity, and transmit period T.

There are a number of different methods which can be employed to implement cooperative diversity. The Medium Access Control (MAC) used in each method compared in this paper is shown in Figure 2.

A. Direct transmission

In direct transmission, no extra mobile terminals are available to relay the signal in phase two. This is termed the classical scheme where the source transmits information at full power for all of the available time slot. The classical case is included to act as a baseline reference, for comparison of the advantages and disadvantages offered by the cooperative diversity schemes. The mutual information equation for the direct transmission case was found by Shannon [6] to be

$$I_{dir} = \log_2(1 + SNR_{s,d(s)} |a_{s,d(s)}|^2) \qquad (5)$$

B. Half-time space-time coded cooperative diversity

Space-time coding allows the transmission of different information in the same sub-channel, at the same time. In the case of half-time space-time coded cooperative diversity this means that during the second phase, all of the participating relays can transmit in the same sub-channel.

In this scheme, during phase one the source is able to transmit at full power for $T/2$ of the time (hence half-time). During the second phase the relays can also transmit for $T/2$

to make up the full time slot, however they are limited to transmit at $1/(m-1)$ of full power due to the power constraint. Note that the fraction is one less than m because s does not take part in the second phase. The relays do not add any new information to $d(s)$: they do however act to decrease the probability of outage. The mutual information for this scheme is given by

$$I_{hc\_stc} = \frac{1}{2}\log_2(1+SNR_{s,d(s)}|a_{s,d(s)}|^2 + \frac{1}{m-1}\sum_{r\in\partial(s)} SNR_{r,d(s)}|a_{r,d(s)}|^2) \quad (6)$$

The mutual information between s and r for i.i.d. complex Gaussian code-books here is given by

$$\frac{1}{2}\log_2(1+SNR_{s,r}|a_{s,r}|^2)$$

under this rule the probability of a relay being $r\in\partial(s)$ is given by

$$P[r\in\partial(s)] = P\ |a_{s,r}|^2 > \frac{2^{2R}-1}{SNR_{s,r}} \quad (7)$$

C. Full-time space-time coded cooperative diversity

Although half-time cooperative diversity can potentially dramatically reduce the outage probability at $d(s)$ due to the added SNR, it suffers from the fact that the source can only transmit information for a limited amount of the time. This leaves the direct transmission scheme with a distinct advantage, because it can always transmit new information, unless it needs to retransmit due to data-loss. In order to transmit all of the information that can be transmitted in the direct case, during phase one, space-time coding techniques can be used in a similar manner as used in phase two of the half-time case.

Orthogonal Transmit Diversity (OTD) can be employed to send information over two spatially orthogonal channels. Through this method the source can transmit the full set of data during phase one (hence full-time). However, it must transmit at half power from each antenna element to meet the power constraint.

The destination will then receive

$$y_1 = a_1x_1[n] + a_2x_2[n] + z_1[n] \quad (8)$$

$$y_2 = a_1x_2^*[n] + a_2x_1^*[n] + z_2[n] \quad (9)$$

during phase one [7], as will each of the relays. Again a_i is defined as the channel coefficients, $x_i[n]$ the data symbol and $z_i[n]$ the additive noise. Each relay can then attempt to decode the two signals and pick which one to retransmit. If it cannot decode either signal it will not take part in the second phase. In retransmitting a relay will join similar relays to form one of two decoding sets, $\partial_1(s)$ and $\partial_2(s)$. If a relay can decode both signals it will pick which the stronger.

In a similar manner to phase two in the half-time scheme, in this scheme mobiles participating in phase two will only be able to transmit at $1/(m-1)$ of full power.

For the OTD full-time case the mutual information is given by

$$\begin{aligned} I_{otd\_stc} = {} & \frac{1}{4}\log_2\ 1 + SNR_{s,d(s),1}|a_{s,d(s),1}|^2 \\ & + \frac{1}{(m-1)}\sum_{r\in\partial_1(s)} SNR_{r,d(s),1}|a_{r,d(s),1}|^2 \\ & + \frac{1}{4}\log_2\ 1 + SNR_{s,d(s),2}|a_{s,d(s),2}|^2 \\ & + \frac{1}{(m-1)}\sum_{r\in\partial_2(s)} SNR_{r,d(s),2}|a_{r,d(s),2}|^2 \end{aligned} \quad (10)$$

The mutual information between s and r for i.i.d. complex Gaussian code-books here is given by

$$\frac{1}{4}\log_2(1+SNR_{s,r}|a_{s,r}|^2)$$

and therefore the probability of a relay being able to reach each of the two information streams is given by

$$P[decode_1] = P\ |a_{s,r_1}|^2 > \frac{2(2^{2R}-1)}{SNR_{s,r_1}} \quad (11)$$

$$P[decode_2] = P\ |a_{s,r_2}|^2 > \frac{2(2^{2R}-1)}{SNR_{s,r_2}} . \quad (12)$$

III. RESULTS AND ANALYSIS

In order to discover the performance properties and practical suitability of the two cooperative diversity relaying techniques, Monte-Carlo simulations are presented, based on the capacity equations for each of the relaying schemes. The capacity equation of each is analysed in three different ways:

- Outage probability against SNR (bandwidth efficiency fixed at 1b/s/Hz)
- Bandwidth efficiency against SNR (outage fixed at 5%)
- Outage probability against bandwidth efficiency (SNR fixed at 10dB).

Each combination of the three communication link characteristics has been included since analysing them all leads to a deeper insight. For example, the outage probability is expected to drop significantly for the half-time scheme when compared to the classic case. However, it would also be expected that the capacity of the link would drop significantly since redundant information is being transmitted.

For each relaying scheme the results are compared directly to the classical case of direct transmission, then indirectly between each other. For the case where the bandwidth efficiency is fixed, 1b/s/Hz has been chosen in order to make the calculations simpler, and since it is a reasonably practical value. Where the outage level is fixed, the 5% level has been chosen since this is likewise a practical value in a wireless communications network. Although a number of values for

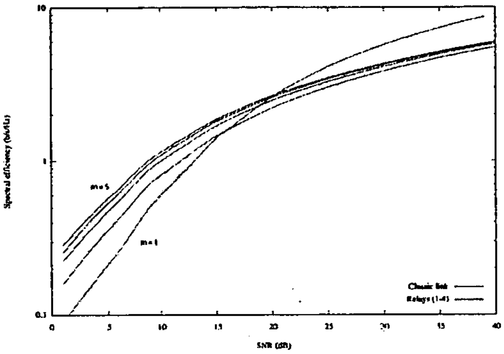

Figure. 3. Simulation results for the half-time space-time coding based cooperative diversity scheme where the outage level is fixed at 5%. Spectral efficiency and SNR are compared for $m = 1,2. .5$.

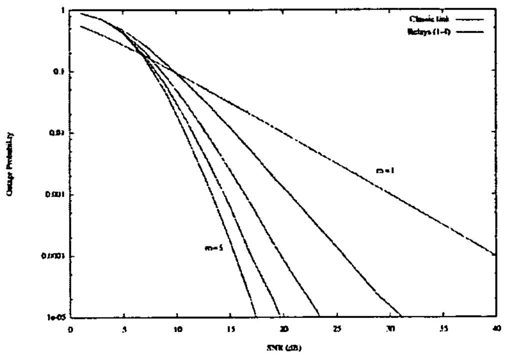

Figure. 5. Simulation results for the half-time space-time coding based cooperative diversity scheme where the spectral efficiency is fixed at 1 b/s/Hz. SNR and outage probability are compared for $m = 1,2. .5$.

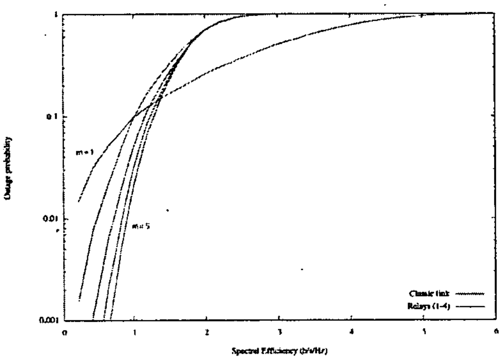

Figure. 4. Simulation results for the half-time space-time coding based cooperative diversity scheme where the SNR is fixed at 10dB. Outage probability and spectral efficiency are compared for $m = 1,2. .5$.

the case where the SNR at each receiver is fixed could have been selected, 10dB was chosen since it is in the middle region of signal quality. Results for networks with up-to four relays ($m = 5$) are presented.

A. Half-time space-time coded cooperative diversity

In half-time space-time coded cooperative diversity the source will transmit information for exactly of half of the available time, the impact of which can clearly been seen in Figure 3. The half-time scheme offers improved spectral efficiency over the classical case at low SNRs, but is out performed at high SNRs. This is due to the fact that at high SNRs the probability of outage in the classical transmission is low enough to not suffer serious signal degradation and subsequently the source can transmit more information over the available time. At low SNRs the half-time scheme offers better performance since extra information is transmitted and the probability of outage is much lower, Figure 4.

Figure 5 shows that it is advantageous to use as many relays as possible to achieve a lower outage probability for a given SNR. Low SNRs are an exception, where the classical case out performs the half-time scheme. At higher SNRs diminishing returns for each relay added is noted. This is due to the fact that each relay's transmission power is constrained by $1/(m - 1)$.

B. OTD Full-time STC cooperative diversity

The half-time cooperative diversity scheme suffers from not being able to transmit the same amount of original information over the same limited amount of time as the classical single transmit antenna case. Full-time space-time coded cooperative diversity attempts to overcome this problem by using OTD transmission from the source in the first phase as well space-time coded transmission from the relays in the second phase. Two co-operating sub-sets are then created to perform the phase two transmission. This is done at the cost of splitting the available transmit power at the source between two antenna elements.

Figure 6 clearly shows the benefit to the probability of outage offered by the full-time OTD scheme. In the region below 12dB it would be preferable to transmit using a direct OTD link between the source and destination. This is because with no relays the source is free to transmit at full power all of the time. Again the diminishing returns for higher number of relays, due to the relay transmission being constrained by $1/(m - 1)$, can be observed.

When the outage probability is compared with the spectral efficiency in Figure 7, it can be seen that the full-time scheme offers large improvements when compared with the half-time and repetition schemes. This again translates into improved spectral efficiency over the direct transmission case as the SNR varies Figure 8.

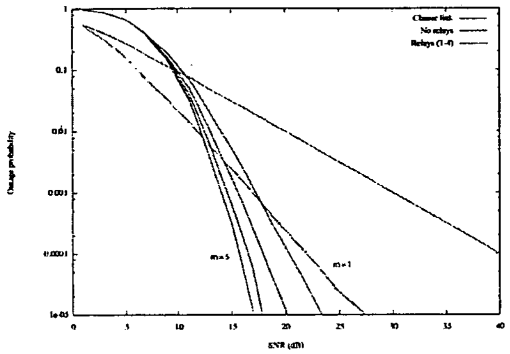

Figure. 6. Simulation results for the OTD full-time space-time coding based cooperative diversity scheme where the spectral efficiency is fixed at 1 b/s/Hz. SNR and outage probability are compared for m = 1,2. .5.

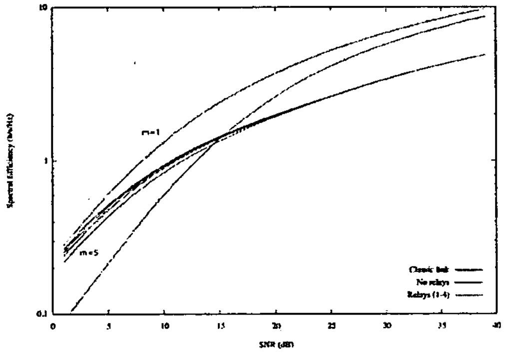

Figure. 8. Simulation results for the OTD full-time space-time coding based cooperative diversity scheme where the outage level is fixed at 5%. Spectral efficiency and SNR are compared for m = 1,2. .5.

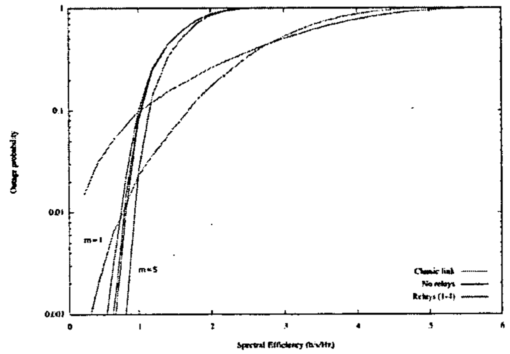

Figure. 7. Simulation results for the OTD full-time space-time coding based cooperative diversity scheme where the SNR is fixed at 10dB. Outage probability and spectral efficiency are compared for m = 1,2. .5.

IV. FURTHER WORK

Full-time cooperative diversity cannot only be performed using OTD, but also through Space Time Transmit Diversity (STTD) and Spacial Multiplexing (SM). SM in particular is likely to increase the capacity of the channel and this will be the focus of future work in this area.

The simulation results presented in this paper were based on a large number of simplifications, and were intended only to characterise the cooperative diversity channel. The analysis presented in this paper will is useful as a comparison for what can be achieved under ideal conditions, therefore, taking this work further will involve creating a more realistic simulation through the use of a channel model incorporating shadowing to compare with a realistic classical transmission scheme.

It is expected that expansion on this work to characterise pools of multi-hop relays through an idea called Mobile Adhoc Trunking (MAT) to extend the range of terrestrial cells, or a purely ad-hoc network, will be possible.

V. CONCLUSION

This paper has introduced cooperative diversity and presented two different schemes for performing it in a communications network; half-time cooperative diversity and OTD based full-time cooperative diversity. Through simulations of the derived capacity equations for each scheme it was observed that both methods offer advantages over the traditional direct link method. In particular the OTD based full-time scheme introduced in this paper, where space-time techniques are used in both phases, can offer significant improvements.

Cooperative diversity is intrinsically limited by the need to have two transmission phases, and the limit of two antenna elements in any hand sized mobile terminal. Due to these two factors the techniques fall short of achieving the capacity offered by MIMO theory.

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MIMO Cooperative Diversity Strategies for Frequency Selective Fading Relay Channels

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Abstract— **This paper presents a family of cooperative diversity strategies for the fading relay channel in an initially frequency flat fading environment. The relay channel used in this paper operates in what is termed as MIMO cooperative diversity mode, where the source transmits to both relay and destination terminals in the first instance. Both the source and relay then transmit to the destination in the second instance. Initially the current cooperative diversity frameworks are extended to consider system constraints to make a direct and fair comparison with the single-hop case. In-particular a power constraint is put on the system and the optimal transmit power levels are presented. The framework is then extended to consider the frequency selective channel by consider an Orthogonal Frequency Division Multiplexing (OFDM) framework. The amount of collision in frequency can be varied and then analysed. This paper shows that full collision is the preferred transmission scheme.**

I. INTRODUCTION

One of the biggest problems faced by designers of mobile telecommunication systems is the multi-path nature of the communications channel. The very multi-path that creates difficulties also offers us *diversity* [1]. This can help mitigate the problem as a result of the transmission of redundant information over (ideally) independent fading paths (in time/frequency/space) in conjunction with a suitable receiver. Spatial diversity techniques are particularly attractive as they provide diversity gain while incurring no penalty of extra transmission time or bandwidth. Spatial diversity techniques such as Multiple-Input Multiple-Output (MIMO) wireless systems [2] have been shown to significantly increase the spectral efficiency of point-to-point wireless links, including cooperative diversity relay channels [3].

Contributions and relation to previous work. The first part of this paper is devoted to extending the framework developed in [4] to consider the system constraints introduced in [5] to make a fair and direct comparison between the cooperative diversity relay channel and the single-hop channel. This section of the paper develops an optimal power control based relaying system. The second part of the paper extends the cooperative diversity framework to a frequency selective environment. The model used in this paper implements a variable amount of collision in frequency between the source and relay by assigning (potentially overlapping) subsets of the available Orthogonal Frequency Division Multiplexing (OFDM) [6] tones to the two different transmitting terminals [7]. The ergodic capacity region for variable amounts of collision in the framework is then analysed and the results presented.

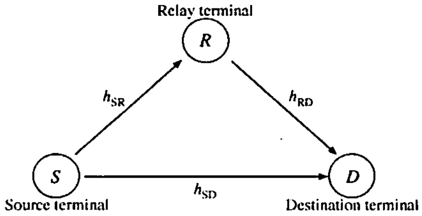

Figure. 1. Relay fading channel

Notation. The superscripts ^{T} and ^{H} stand for transposition and conjugate transpose of a matrix, respectively and ε denotes the expectation operator. The random variable Z is a circularly symmetric complex Gaussian random variable where $Z = X + jY \sim \mathcal{CN}(0,\sigma^2)$, where X and Y are independent identically distributed (i.i.d.) $\mathcal{N}(0,\sigma^2/2)$. Finally $A \rightarrow B$ denotes transmission between terminal A and terminal B.

II. PROTOCOL DESCRIPTIONS AND SYSTEM CONSTRAINTS

A. Protocol Description

Consider the fading relay channel shown in Figure 1. Data is transmitted from the source terminal, S, to the destination terminal, D, potentially with the assistance of the relay, R. All terminals in this paper are considered to have single transmit and single receive antenna elements although this constraint can potentially be relaxed. Additionally, a terminal cannot simultaneously transmit and receive information. Although it is possible for the relay to assist in either an Amplify-and-Forward (AF) or Decode-and-Forward (DF) manner, only DF is considered in this paper. In DF mode the relay demodulates and decodes the signal before re-encoding the signal and retransmission occurs.

Due to the constraint of terminals not being able to transmit and receive information at the same time, the transmission of information from the source to destination, in a cooperative diversity network, is broken into two time slots, termed phase

Transmission phase	Protocol		
	I	II	III
1	S → R, D	S → R, D	S → R
2	S → D, R → D	R → D	S → D, R → D

TABLE I

COOPERATIVE DIVERSITY PROTOCOLS. THE SOURCE, RELAY AND DESTINATION TERMINALS ARE DENOTED BY S, R AND D RESPECTIVELY.

I and phase II [8]. This paper considers the scenario where the source transmits different information to the destination in the second phase than that which was transmitted during the first phase. The destination can therefore potentially receive two different information streams during the second phase and this scenario is termed *MIMO cooperative diversity* due to the similarity to MIMO wireless systems. Note that it may be necessary for the destination terminal to have two antenna elements to be able to fully decode the second phase transmission, however this area is left open for future research and it is assumed in this paper that the receiver can fully separate the two information streams.

The relaying framework extended in this paper was derived by Nabar et al [4] and considers three cooperative diversity protocols, shown in Table I and are now described.

Protocol I: In the first phase the source terminal signals to both the relay and destination. Following this in the second phase both the source and relay communicate with the destination. This is the only protocol available to utilise both the maximum degrees of broadcast and receive collision.

Protocol II: The source communicates with both the relay and destination in the first phase, but where only the relay signals to the destination in the second phase [9].

Protocol III: Here the source transmits to the relay in the first phase but not to the destination. Both the source and relay then communicate with the destination in the second phase.

B. System constraints

To be able to fairly and directly compare the performance of the cooperative diversity network with the single-hop channel, several constraints are placed upon the channel model:

- The time used must not exceed that used by the single-hop transmission case (1 unit)
- The bandwidth used must not exceed that used by the single-hop transmission case (1 unit)
- The total transmit power of the complete system (power used by the source and relay for transmission) must not exceed that used by the single-hop case (1 unit)

The effect of the time and bandwidth constraints is that the transmission of the two phase cooperative diversity must take place in the same amount of time as the single-hop system. This typically involves a factor of 1/2 when considering the spectral efficiency of the system. An important point that arises from this constraint is that the Rayleigh block-fading is now considered to be flat over the two time phases due to the 1/2 factor.

The effect of the transmit power constraint is particularly important when considering Protocols I and III since they use three transmissions (the source in both the first and second phase and the relay in the second phase only) and are therefore must reduce their transmit power to the power constraint. Protocol II however uses only one terminal transmitting in each phase and is therefore free to transmit at the full power available to it.

- A - first phase transmission from the source
- B - second phase transmission from the source
- C - second phase transmission from the relay

To constrain the transmit power to be the same as the single-hop case, first recall that the transmission in the relaying network has been split into two phases, it therefore follows that

$$A+B+C=2 \tag{1}$$

A transmit power constraint is also placed upon each transmitting terminal such that it cannot broadcast with more power than the single-hop case would (1 unit), i.e.,

$$A \leq 1 \qquad B \leq 1 \qquad C \leq 1. \tag{2}$$

C. Channel Model

Throughout this paper it is assumed that the receivers have perfect channel state information of all the reverse channels and that timing synchronisation in the network is ideal. It is also assumed that there is limited feedback to the source and relay from the destination of the average channel Signal to Noise Ratio (SNR) which is required to perform power control for Protocols I and III.

The signals transmitted by the source during the first and second phases are denoted $x_1[n]$ and $x_2[n]$ respectively. Symbol-by-symbol transmission is considered so the time index n can be dropped to simply give x_1 and x_2. Note also that for the data symbols transmitted is it assumed that $\mathcal{E}\{x_i\}=0$ and $\mathcal{E}\{|x_i|^2\}=1$ for $i=1,2$. The signal received at the destination terminal in the first phase is given by

$$y_{D,1}=\sqrt{AE_{SD}}h_{SD}x_1+n_{D,1} \tag{3}$$

where $y_{Y,1}$ is the received signal at the destination (Y) in the first phase, E_{XY} is the average signal energy over one symbol period. The scalar h_{XY} is the random, complex-valued, unit-power channel gain between the source and destination terminals, and $n_{Y,1} \sim \mathcal{N}(0,N_0)$ is the additive white noise for transmitting terminal X and received terminal Y, in this case S and D respectively. Note that E_{XY} and h_{XY} does not have a phase subscript due to the earlier assumption of flat fading across the two transmission phases. Similarly the signal received at the relay in the first time slot is given by

$$y_{R,1}=\sqrt{AE_{SR}}h_{SR}x_1+n_{R,1} \tag{4}$$

Finally assuming that the relay can fully decode the signal from the first phase source transmission, the signal received by the destination in the second phase is given by

$$y_{R,2} = \sqrt{BE_{SD}}h_{SD}x_2 + \sqrt{CE_{RD}}h_{RD}x_2 + n_{D,2} \qquad (5)$$

The input-output relationship can now be summarised as

$$\mathbf{y} = \mathbf{H}_1\mathbf{x} + \mathbf{n} \qquad (6)$$

where $\mathbf{y} = [y_{D,1}y_{D,2}]^T$ is the received signal vector, $\mathbf{H}_1$ is the Protocol I $2x2$ channel matrix

$$\mathbf{H}_1 = \begin{bmatrix} \sqrt{AE_{SD}}h_{SD} & 0 \\ \sqrt{CE_{RD}}h_{RD} & \sqrt{BE_{SD}}h_{SD} \end{bmatrix} \qquad (7)$$

$\mathbf{x} = [x_1x_2]^T$ is the transmitted signal vector and $\mathbf{n}$ is additive white Gaussian noise. The channel matrixes for Protocol II and Protocol III are denoted H_2 and H_3 and can be found by zeroing the second h_{SD} term for Protocol II and by zeroing the first h_{SD} term for Protocol III.

D. *Information-Theoretic performance*

In the following the transmission rates over the first and second phases are denoted as R_1 and R_2 respectively. For the relay to be able to fully decode the signal from the source in the first phase, R_1 must satisfy

$$R_1 \leq \log_2\left(1 + \frac{AE_{SR}}{N_0}|h_{SR}|^2\right) \qquad (8)$$

Further to this R_1 must also satisfy

$$R_1 \leq \log_2\left(1 + \frac{AE_{SD}}{N_0}|h_{SD}|^2 + \frac{CE_{RD}}{N_0}|h_{RD}|^2\right) \qquad (9)$$

as the achievable rate over the source to destination (first phase) and relay to destination (second phase) links, assuming that (8) is satisfied. The rate R_2 is then given by the achievable rate over the source to destination (second phase) channel, specifically

$$R_2 \leq \log_2\left(1 + \frac{BE_{SD}}{N_0}|h_{SD}|^2\right) \qquad (10)$$

Finally the sum rate for Protocol I is given from the vector notion as

$$R_1 + R_2 \leq \log_2\left(1 + \frac{1}{N_0}\mathbf{H}_1\mathbf{H}_1{}^H\right) \qquad (11)$$

Note that the sum rate for Protocol II and Protocol III can be derived using the channel matrixes $\mathbf{H}_2$ and $\mathbf{H}_3$ respectively.

Following the notation used in previous literature we denote the right-hand side of (8), (9), (10) and (11) as R^{max}_{relay}, R^{max}_1, R^{max}_2 and R^{max}_{total}. Due to the double constraint of R_1 under certain channel conditions the sum rate R^{max}_{total} is not always achievable. Following this if the second phase source to destination link cannot support the required rate to make up the sum rate, the maximum achievable rate is limited by the source to relay (first phase) and source to destination (second phase) rates. This is given by

$$R_{sum} = \begin{cases} R^{max}_{total}, & R^{max}_{relay} \geq R^{max}_{total} - R^{max}_2 \\ R^{max}_{relay} + R^{max}_2, & R^{max}_{relay} < R^{max}_{total} - R^{max}_2. \end{cases} \qquad (12)$$

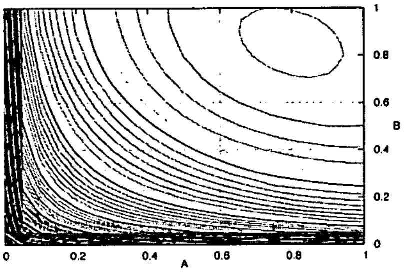

Figure. 2. Power control levels showing A against B for 26dB SNR.

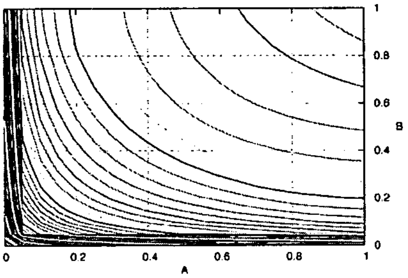

Figure. 3. Power control levels showing A against B for 30dB SNR.

E. *Power Control Analysis*

Consider now the problem of maximising the channel rates as functions of the power control levels A, B and C. An analytical analysis of this maximisation problem is non-trivial, and is the subject of current research. A Monte Carlo simulation, however, provides interesting insight to the behaviour of the three power control variables and initial results.

Since A, B and C are limited by (1), by varying A and B between 0 and 1 independently, the entire range of results are explored. The results presented here are considered at the 5% outage level and are found from a Monte Carlo simulation, Figure 2 and Figure 3. For simplicity the SNR level between all terminals is considered to be equal. Note that the key has been omitted from these figures since the interest lies in the maximum of the surface rather than the specific values. Results for various levels of SNR are also shown in Table II.

From the results presented it can been seen that at high SNR levels (> 30) it is preferable put all the available power into the single-hop source to destination link. Also of interest is that just below 30dB SNR the power levels move quickly

SNR (dB)	A	B	C
-30	0.715	0.875	0.41
-20	0.715	0.875	0.41
-10	0.715	0.875	0.41
0	0.72	0.87	0.41
5	0.72	0.85	0.43
10	0.72	0.83	0.45
15	0.72	0.815	0.46
20	0.725	0.81	0.465
25	0.745	0.805	0.45
30	1	1	0

TABLE II

THE OPTIMAL POWER LEVELS FOR A, B AND C TO MEET THE UNITY POWER SYSTEM CONSTRAINT AT VARIOUS SNR LEVELS WITH AN ERROR OF 0.0025.

away from the single-hop case and settle towards the low SNR levels. At SNR levels below 30dB it can been seen that the gradient near the maxima is very low, so one can therefore conclude that small changes in the power control levels A, B and C would result in only a very small decrease in performance. Finally note that at low SNRs (< 10) the power control levels stay constant, which is in line with the capacity curve expected from such a network.

III. FREQUENCY SELECTIVE COOPERATIVE DIVERSITY STRATEGIES

A. *Channel Model*

In the remainder of this paper the flat fading channel model is extended to consider a frequency selective fading model. To take account of the frequency selective fading, the channel transfer coefficient is modified according to

$$h_{XY}(k) = \sum_{l=0}^{L-1} h_{XY,l} e^{\frac{-2\pi lk}{N}} \quad k = 0, 1, \ldots, N-1 \text{ carriers} \tag{13}$$

where h_{XY} is the channel coefficient between source X and destination Y, and $h_{XY,l}$ is the channel coefficient recorded at tap l. The frequency selective fading model considered is restricted to Rayleigh block-fading channels with fading coefficients $h_{XY,l}$ being circularly symmetric zero mean complex Gaussian random variables. Furthermore, the channel coefficients are assumed to be uncorrelated across taps. Although it is trivial to rewrite the channel transfer equations (3) to (5) they do not appear here due to space constraints.

Each phase of transmission employs OFDM with N tones and the length of the Cyclic Prefix (CP) satisfies $L_{cp} \geq L$, which guarantees that each of the frequency-selective fading channels de-couples into a set of parallel frequency-flat fading channels.

B. *Multiple Access Scheme*

The family of MA schemes used is obtained by assigning each OFDM tone $k = 0, 1, ..., N-1$ to a subset of the transmitting terminals, $\mathcal{U}_k$, in the second phase. During the first phase all tones are assigned to the source and none to the relay, hence there can be no collision in frequency in the first phase. A fully collision-based MA scheme where all tones are assigned to both the source and the relay during the second phase (i.e., $\mathcal{U}_k = \{S, R\}$ for $k = 0, 1, ..., N-1$) is referred to as Code Division Multiple Access (CDMA). Conversely, Frequency Division Multiple Access (FDMA) is defined by the tone assignment pattern satisfying $|\mathcal{U}_k| \leq 1$ for $k = 0, 1, ..., N-1$.

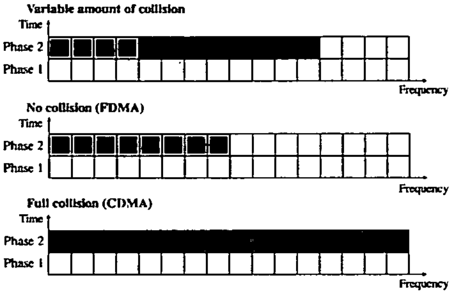

Figure. 4. Variable amount of collision in the second phase in frequency.

The variable amount of collision employed in the second phase transmission is shown graphically in Figure 4. From Figure 4 it can been seen that in the tones where there is no collision in the second phase and the relay transmits information in the second phase can be considered to be Protocol II. Equally for the tones where there is no collision in the second phase but only the source transmits can be considered to be two single-hop transmissions from source to destination. Finally the section where there is collision can be considered to be Protocol I as the full degree of receive collision is used.

C. *Information theoretic performance*

To analyse the ergodic capacity region of the fading relay channel, the case where all OFDM tones available are used and assigned to either the source, relay or to both terminals is investigated. Both the source and relay have the same number of tones $N_u \geq \frac{N}{2} \in \mathbb{N}(u = \{S, R\})$ out of which bN tones with $b = \frac{2i}{N}, i \in \{0, 1, ..., N/2\}$ are assigned to both transmitting terminals in the second phase. Consequently, the case of $b = 0$ corresponds to FDMA and $b = 1$ describes CDMA in the second phase. The transmit power used by the terminals is unity in the non-collision tones, and uses the factors A, B and C as found previously in this paper in collisions tones. Under these assumptions, bN tones are assigned to both transmitting terminals and each receives $\frac{1-b}{2}$.

The notion of R_1 and R_2 being the rate transmitted in the first and second phase respectively and $R_1 + R_2$ being the maximum sum rate achievable over the two phases is continued in this section. Subsequently the the following rates are obtained

$$R_1 \leq b\log_2\left(1+\frac{AE_{SR}}{N_0}|h_{SR}|^2\right) + \frac{1-b}{2}\log_2\left(1+\frac{E_{SR}}{N_0}|h_{SR}|^2\right) \tag{14}$$

$$\begin{aligned} R_1 \leq\ & b\log_2\left(1+\frac{AE_{SD}}{N_0}|h_{SD}|^2+\frac{CE_{RD}}{N_0}|h_{RD}|^2\right) \\ &+\frac{1-b}{2}\log_2\left(1+\frac{E_{SD}}{N_0}|h_{SD}|^2\right) \\ &+\frac{1-b}{2}\log_2\left(1+\frac{E_{SD}}{N_0}|h_{SD}|^2+\frac{E_{SD}}{N_0}|h_{SD}|^2\right) \end{aligned} \tag{15}$$

$$R_2 \leq b\log_2\left(1+\frac{BE_{SD}}{N_0}|h_{SD}|^2\right) + \frac{1-b}{2}\log_2\left(1+\frac{E_{SD}}{N_0}|h_{SD}|^2\right) \tag{16}$$

$$\begin{aligned} R_1+R_2 \leq\ & b\log_2\left(1+\mathbf{H}_1\mathbf{H}_1^H\right) \\ &+\frac{1-b}{2}\log_2\left(1+\frac{E_{SD}}{N_0}|h_{SD}|^2+\frac{E_{RD}}{N_0}|h_{RD}|^2\right) \\ &+(1-b)\log_2\left(1+\frac{E_{SD}}{N_0}|h_{SD}|^2\right) \end{aligned} \tag{17}$$

Note that (14), (15), (16) and (17) are again denoted as R_{relay}^{max}, R_1^{max}, R_2^{max} and R_{total}^{max} and that (12) still applies.

D. Numerical Results

To analyse the performance of different levels of collision in the second phase results are presented from Monte Carlo simulations at the 5% outage level. Full collision, half collision and no-collision is considered and the results are presented in Figure 5 and Figure 6 for -20dB and 20dB SNR respectively as a multiple access channel for R_1, R_2 and R_{sum}. From the results it can be seen that full collision is the preferred transmission technique.

IV. CONCLUSION

This paper has presented a cooperative diversity framework which is directly comparable with the direct transmission case. In particular the power constraint was considered and values for the optimal transmit power control values were found for several SNR levels. The cooperative diversity framework was then extended to the frequency selective fading channel to consider the effect of signal collision in the second phase. It was shown that full collision is the preferred method of transmission.

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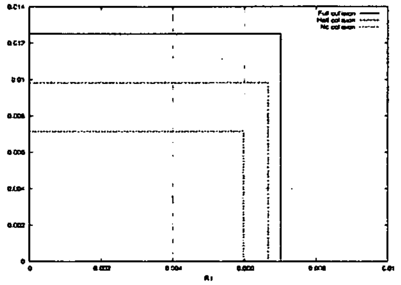

Figure. 5. Access rates for full, half and no collision (b=1, b=0.5 and b=0 respectively) at -20dB and 5% outage level.

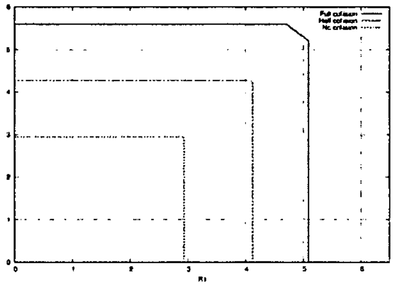

Figure. 6. Access rates for full, half and no collision (b=1, b=0.5 and b=0 respectively) at 20dB and 5% outage level.

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MIMO WIRELESS AND MOBILE COMMUNICATIONS

Dual antenna cooperative diversity techniques

A.J. Jardine, J.S. Thompson, S. McLaughlin and P.M. Grant

Abstract: This paper presents the application of two spatial transmission techniques which can be used in a cooperative diversity wireless network, where each mobile terminal has two antenna elements. Initially the current body of work in single antenna cooperative diversity is expanded to include the effect of cooperative diversity upon the capacity of a link. This is to serve as a baseline model for comparison with the dual antenna methods. This paper then introduces two techniques which can be used for cooperative diversity where mobile terminals are each equipped with two antenna elements. Using the same metrics as the single antenna case, a comparison is made between the results from simulations of each technique. It is shown that dual antenna elements offer considerable advantage over the single antenna element case, and that two antenna element space-time transmit diversity (STTD) based cooperative diversity is preferred at low SNRs.

1 Introduction

One of the biggest problems faced by designers of mobile telecommunication systems is the multi-path nature of the communications channel. The very multi-path that creates difficulties is also an advantage as it offers diversity [1]. Diversity can help mitigate the problems caused by multi-path as a result of the transmission of redundant information over independent fading channels in conjunction with a suitable receiver. There are three types of diversity which can be observed in wireless communications systems [2]:

Time diversity – owing to multi-path fading the communication channel will change its characteristics over time, therefore transmitting at different times results in diversity.

Frequency diversity – channels at different frequencies have different characteristics owing to frequency selective fading, hence transmitting at different frequencies results in diversity.

Space diversity – channels between different points in space have their own unique fading characteristics, leading to diversity if transmission occurs from several different locations.

In recent years, space diversity has been suggested as a route to improve capacity of wireless communication networks through the use of multiple antennas. Multiple-input multiple-output (MIMO) communication systems have both the transmitter and intended destination equipped with antenna arrays, an area of research which was initiated by [3–5]. Through this method, information can be sent through more than one independent fading channel, if the antenna elements are separated by a suitable distance and sufficient multi-path diversity exists in the channel. This can potentially result in very high spectral efficiencies [4].

In a cellular network, although the equipment at the base station can readily be adapted with a multi-element antenna array, physical size and complexity limitations of mobile terminals in both cellular and ad-hoc networks typically preclude fitting more than two antenna elements. In general, it would be preferable to use only one or two elements in the mobile terminal to keep the complexity and hence cost to a minimum. The challenge that is now presented to wireless network designers is how to achieve the high capacities offered by MIMO in a cellular or ad-hoc environment.

Sendonaris et al. [6] introduced cooperative diversity as a method for increasing the uplink capacity in a network, using a single relay. This area has been the subject of a great deal of research and Sendonaris et al. expanded on their original proposal in [7, 8] for cellular networks. Laneman et al. in [9–12] extended the proposal to include multiple relays and found the outage limits for repetition-based cooperative diversity and space-time code (STC)-based cooperative diversity. Boyer et al. extend the previously proposed cooperative diversity schemes to the multi-hop case [13].

Cooperative diversity makes use of available mobile terminals as relays, splitting the available time slot into two phases. The source transmits during the first phase, and the relay(s) transmit during the second phase, sending a version of the received signal to the intended destination. The relays co-operate together to form a virtual antenna array (VAA) [14], hence the name cooperative diversity. It was shown in [9] that full diversity can be achieved by both amplify-and-forward and decode-and-forward cooperative techniques in the relays. Nabar et al. completed the cooperative diversity family of protocols in [15]. Research on these protocols is currently proving fruitful as it is shown that the capacity of the network is increased if the source transmits a different symbol in the second phase [16, 17].

When more than one relay is used, special modulation and error correction techniques are required, namely STC-based cooperative diversity. Specifically co-ordination among the relays is required to form co-operation groups and transmit simultaneously during the second phase. Space-time coding and co-ordination for more than one

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relay is still an open topic of research [11, 18]. As more relays are added, the work presented in this paper and from previous studies show that diminishing returns are observed. Subsequently node selection is the subject of considerable research [19, 20].

1.1 Contributions and relation to previous work

The first part of this paper is devoted to expanding the work presented in [9] on mobile terminals equipped with single antennas to include capacity measurement metrics. The main contribution, in the second part, is to introduce two methods for performing cooperative diversity in an environment where all mobile terminals are equipped with two antenna elements. These two methods are then analysed and compared to the basic direct transmission method, using the extended metrics, culminating in a method which can offer both reduced probability of outage and higher capacities.

1.2 Organisation of the paper

Section 2 introduces the system model, the constraints placed upon it and the direct transmission method. Section 3 then presents single antenna space-time code (STC)-based cooperative diversity, which will be used as the basis for comparison between the single antenna and dual antenna cases. Section 4 introduces two different dual antenna techniques which can be used for transmission in an STC cooperative network. Section 5 presents results and draws comparisons between the different cooperative diversity and transmission schemes. Future research is discussed in Section 6 and finally conclusions are drawn in Section 7.

2 System model

To analyse the different relaying techniques, consider a wireless network with a set of transmitting terminals denoted $M = \{1, 2, \ldots, m\}$. A source terminal, $s \in M$ has information to transmit to a single destination terminal $d(s) \in M$, potentially using terminals $M \ \{s\}$ as relays to perform cooperative diversity. If m is the cardinality of the set M, there are $m \ 1$ terminals co-operating with $d(s)$. All of the relaying schemes presented in this paper require that the relay can fully decode the source signal if it is to relay it on to the destination. The subset of $M \ \{s\}$ which can decode the signal is defined as $\mathcal{D}(s)$.

The method described here of implementing cooperative diversity consists of two transmission phases, shown in Fig. 1. During the first phase the source will transmit information to all available mobile terminals in the network. Those terminals will then attempt to decode the signal, and if successful they will take part as the decoding set in phase two, transmitting information to $d(s)$.

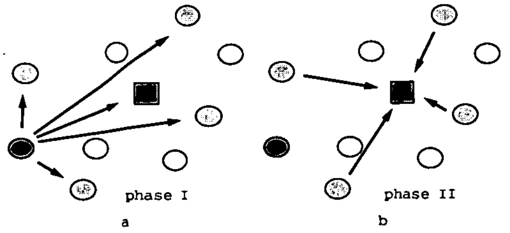

Fig 1 Illustration of the two-phase cooperative diversity method
In the first phase (a) the source transmits information in an omnidirectional manner to all terminals. In phase two (b), those terminals which can decode the signal then relay the signal on to the destination

In the system model below (equations (1)–(4)) $a_{i,j}$ can capture the effects of path-loss, shadowing and multi-path fading between transmitter i and receiver j. However, only multi-path fading is considered in this paper to keep the calculations simple. Statistically, $\{a_{i,j}\}$ is modelled as a set of zero-mean, independent, circularly-symmetric complex Gaussian random variable with variance $1_{i,j}$, so that the magnitudes $|a_{i,j}|$ are Rayleigh distributed and the phases $\angle a_{i,j}$ are uniformly distributed on $[0, 2\pi)$. Furthermore, the coefficients $z_j[n]$ are modelled as zero-mean mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_0. The scalar $z_j[n]$ captures the effects of receiver noise and other forms of interference in the system. We also note that n denotes the time index. In this paper it is assumed that quasi-static fading occurs where the fading coefficients are constant over the considered time and frequency. The scenario presented in this paper assume the fading coefficients are known to the appropriate receivers, but not known, or exploited by the transmitters.

During the first phase, each potential relay $r \in M \ s$ receives, at time n

$$y_r[n] = a_{s,r} x_s[n] + z_r[n] \qquad (1)$$

where $x_s[n]$ is the source transmitted signal and $y_r[n]$ is the received signal at r. If the relay can then decode the source transmission, r will serve as a relay for the source, so that $r \in \mathcal{D}(s)$.

The destination mobile terminal (MT) receives signals during both transmission phases. During the first phase, the received signal is modelled at $d(s)$ as

$$y_{d(s)}[n] = a_{s,d(s)} x_s[n] + z_{d(s)}[n] \qquad (2)$$

in the appropriate channel. During the second phase, the equivalent channel models are different for repetition based cooperative diversity and space-time coded schemes. For repetition based, the destination receives separate re-transmissions from each of the relays, where the received signal $d(s)$ is

$$y_{d(s)}[n] = a_{r,d(s)} x_r[n] + z_{d(s)}[n] \qquad (3)$$

in the appropriate channel, and $x_r[n]$ is the transmitted signal of relay r. For space-time coded schemes, all relay transmissions occur in the same channel and are combined at the destination, so that

$$y_{d(s)}[n] = \sum_{r \in \mathcal{D}(s)} a_{r,d(s)} x_r[n] + z_{d(s)}[n] \qquad (4)$$

in the appropriate channel.

Two important parameters of the system are the received signal-to-noise ratio, SNR (dB) and the bandwidth efficiency, R (b/s/Hz). It is natural to define these parameters in terms of the continuous-time channel with non-cooperative diversity as a baseline. In this paper, the simplistic ideal of normalising the attempted rate to 1 b/s/Hz, and assuming that all mobile terminals are equidistant from one another is assumed. Obviously this is unrealistic, but it serves as a baseline for comparing the results. All transmission schemes are constrained to transmit at a maximum power of unity, and transmit period T.

To make a fair comparison between the network models using cooperative diversity and the direct transmission between source and destination MTs, several constraints are placed upon the cooperative diversity model:

The time used must not exceed that used by the direct transmission case (1 unit).

The total of transmit power of the complete system (power used by source and the power used by the relays) must not exceed that used by the direct transmission case (1 unit).

The bandwidth used must not exceed that used by the direct transmission case (1 unit).

2.1 Direct transmission

In direct transmission, no extra mobile terminals are available to relay the signal in phase two. This is termed the classical scheme, where the source transmits its information at full power for all of the available time slot, Fig. 2. The classical case is included to act as a baseline reference, for comparison of the advantages and disadvantages offered by the cooperative diversity schemes. The mutual information equation for the direct transmission case was found by Shannon [21] to be

$$I_{dir} = \log_2(1 + SNR_{s,d(d)}|a_{s,d(d)}|^2) \qquad (5)$$

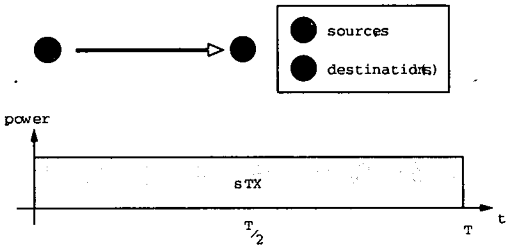

Fig 2. Direct transmission with no cooperative diversity

3 Cooperative diversity – single antenna

To be able to add additional MTs to the network to act as relays for a particular target MT there is a need for a method which allows this addition to the network without sacrificing the amount of information which can be sent, while also meeting the time constraint. Figure 3 shows a network with three relay terminals contributing to the signal received at the intended destination, d(d). Space-time coding (STC) is a technique to utilise spatial diversity, which enables information from several sources to be transmitted to a single destination. The receiver can then combine the received signals to retrieve the original information. Since space-time codes (STCs) use spatial diversity, all transmitters can transmit on the same frequency with the same modulation scheme at the same time.

With the transmission of information split into the two cooperative diversity phases, transmission from the source can occur for exactly one half of the available time ($T/2$), and relaying from available MTs takes place in the second half of the time slot, Fig. 3.

Using STCs for transmission during the second phase the relays can also transmit for $T/2$ to make up the full time slot, however they are limited to transmit at $1/(m-1)$ of full power owing to the power constraint. Note that the fraction is one less than m because s does not take part in phase two. The relays only retransmit the information that they received from the source, if they are able to decode the signal. The mutual information for the STC signal antenna element case was derived by Laneman et al. [9].

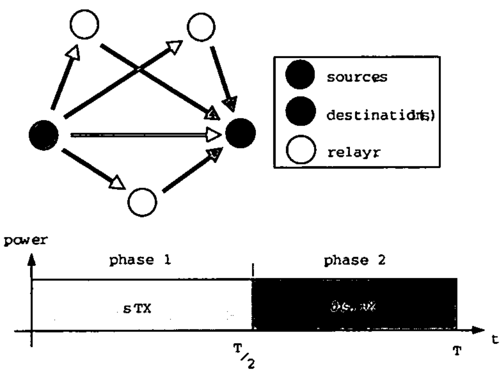

Fig 3 Signal antenna based cooperative diversity

With these constraints imposed on the STC case, STCs allow the power, time and bandwidth system constraints to all be met, while also allowing information from the source to be transmitted for half of the available time. Each transmission scheme is termed either full-time or half-time depending on how much of the information the source can transmit compared to the direct transmission case. For example, if the source can only transmit half the information the direct case would for a particular scheme, it will be termed half-time.

4 Cooperative diversity – dual antenna methods

It is likely that in future wireless systems it will be possible for mobile terminals the size of current hand held mobile phones to have two antenna elements, each of which will receive an independent signal from the transmitter in a high scattering environment. In the following two cooperative diversity schemes it is assumed that all MTs have two internal antenna elements available for transmission and reception, including the source. With MTs which are equipped with two antenna elements, transmission between MTs and between a base-station and MTs can use spatial diversity transmission techniques to help improve performance. Two different space-time transmit schemes are evaluated in this paper for the dual antenna cooperative diversity case:

space-time transmit diversity (STTD)

vertical Bell Laboratories layered space-time architecture (V-BLAST) spatial multiplexing (SM).

In the proposed cooperative diversity schemes the same transmission technique is used in both phases, i.e. the STTD scheme uses STTD to transmit from the source to the relays and destination during phase one, and also from the relays to the destination during phase two. During phase two STCs are used in a similar way as in the single antenna case to allow all relays to transmit at the same time, in the same channel.

4.1 STTD half-time STC cooperative diversity

STTD is a multiple antenna transmission and reception technique that is used to provide robust communications channels. The information to be transmitted is split into two streams which are initially identical to the original information stream, each of which is then encoded to make them mutually orthogonal. In the two antenna STTD transmission case, the receiver and relays will record

$$y_1(n) = a_1(n)x_1(n) + a_2(n)x_2(n) + z_1(n) \qquad (6)$$

$$y_2(n) = a_1(n)x_2^*(n) - a_2(n)x_1^*(n) + z_2(n) \qquad (7)$$

during phase one at each antenna ($y_1(n)$ and $y_2(n)$). Here $a_i(n)$ is defined as the channel coefficients, $x_i(n)$ the data symbol and $z_i(n)$ the additive noise. In the following, symbol-by-symbol transmission is considered, subsequently the time index n can be dropped from the transmitted symbols, channel coefficients, received signals and additive noise.

Each relay can then attempt to decode the STTD signal for retransmission. If it cannot decode either signal it will not take part in the second phase. In retransmitting, a relay will use STTD to transmit the two orthogonally separated information streams on to the intended destination, in combination with using its own STC to allow it to transmit at the same time as the other relays, Fig. 4.

In this scheme MTs participating in phase two will only be able to transmit at $1/(m-1)$ of full power. The mutual information for the channel is given by an extension of the mutual information equation derived by Laneman et al. [9] as

$$I_{std\_stc} = \frac{1}{2}\log_2\left(1 + SNR_{s,d(s)}a_h + \frac{1}{(m-1)}\sum_{r \in D(s)} SNR_{r,d(s)}a_h\right) \qquad (8)$$

where

$$a_h = \sum_{i=1}^{2}\sum_{j=1}^{2}\frac{|h_{i,j}|^2}{2}$$

The scalar $h_{i,j}$ defines the channel coefficients between the ith transmitter and the jth receiver antenna elements. In this case both i and j are limited to 2 since only two transmit/receiver antenna pairs elements are used in each MT. We also note that it is assumed that receiving terminals have full channel knowledge of their backwards channels. This assumption extends throughout this paper.

In this scheme, the information between s and r for independent and identically distributed (i.i.d.) complex Gaussian code-books for each space-time coded information stream is given by

$$\frac{1}{2}\log_2(1 + SNR_{s,r}a_h)$$

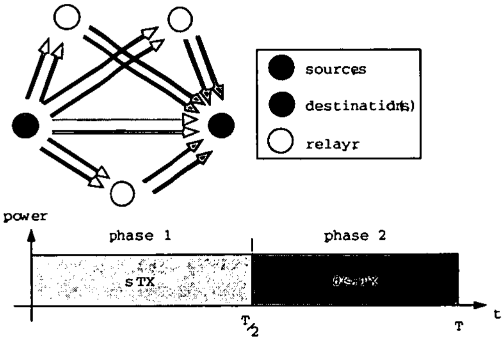

Fig 4 Half-time STTD-based cooperative diversity

Under this rule the probability of a relay being able to decode the information stream as

$$P(\text{decode}) = P\left[a_h \geq \frac{2^{2R} - 1}{SNR_{s,r}}\right] \qquad (9)$$

4.2 SM full-time STC cooperative diversity

Although STTD cooperative diversity, using two transmit antennas, can dramatically reduce the outage probability at $d(s)$ owing to the increased SNR, it suffers from the fact that the source can only transmit information for a limited amount of the time. This leaves the direct transmission scheme with a distinct advantage, because it can always transmit new information, unless it needs to retransmit owing to data-loss. In order to transmit all of the information that can be transmitted in the direct case, during phase one, it is possible to employ SM, Fig. 5.

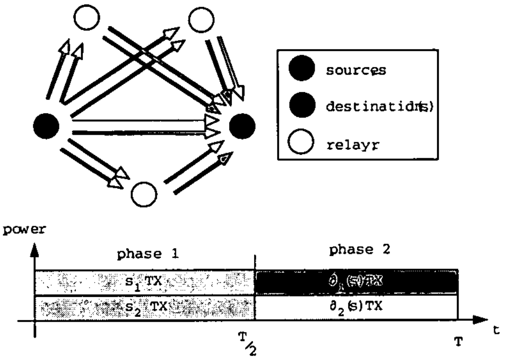

Fig 5 Full-time STC-based cooperative diversity

Using SM, the original source data stream is demultiplexed into two data streams, which are each half the rate of the original. Each stream is then transmitted from a single antenna element, and received via the two antenna elements at the receiver. This transmission technique is known as V-BLAST. To use V-BLAST and still meet the imposed power constraint on the cooperative diversity network, each transmission antenna element must now transmit at half power. The mutual information channel average of a V-BLAST system, at the receiver, is given by [22] as

$$I_T = 2\min(I_1; I_2) \qquad (10)$$

where $\min(I_1; I_2)$ is the minimum mutual information of the two information streams, C_T is the mutual information of the V-BLAST channel and

$$I_1 = \log_2(1 + SNR_{s,r,1}|a_{s,r,1}|^2)$$

$$I_2 = \log_2(1 + SNR_{s,r,2}|a_{s,r,2}|^2)$$

where $SNR_{i,j,k}$ denotes the SNR recorded from source i at destination j for each spatial multiplexing data stream k ($k = 1; 2; \ldots; N$). N is the number of transmit and receive antenna elements used, in this case $N = 2$. The notation for the channel fading is similarly extended, $a_{i,j,k}$, for spatial multiplexing.

The destination will receive both information streams during phase one, as will all relays. The sum capacity of each source to relay link is not calculated by the individual relays [Note 1], rather the relays simply decode the received spatial multiplexing data streams, if it is possible to do so, and transmit the two separate spatially multiplexed information streams on with as high a rate as possible. The relays therefore add their signal power to what the destination receives. It is not until the end of the second phase that the destination mobile terminal makes the decision of which information stream carries the minimum information and therefore the capacity of the overall cooperative channel.

It should be noted that the channel rate measurements would be made during a link set-up phase and this simplification was made for capacity calculation simulations. It is also noted that using $\min(C1; C2)$ of V-BLAST channel capacity, reduces the feedback required to the source from the destination, as the same modulation and coding is used for both antennas. This also reduces complexity in the relays since the same decoding and re-encoding is used for both information streams. However, using this V-BLAST algorithm reduces the link capacity as described in [23].

During the second phase we assume that the relays all have perfect channel knowledge and can choose which information stream to transmit from each antenna element. The relay will transmit the highest capacity stream it receives on the highest capacity link it has to the destination. The lower capacity stream will be transmitted on the slower link. If a stream cannot be decoded, like all the other cooperative diversity networks discussed in this paper, it will not be transmitted on to the destination. In this case, to keep the simulation simple, no information is transmitted from the antenna element that would have been used if decoding was successful.

In a similar manner to phase two in the half-time schemes, here mobiles participating in phase two will only be able to transmit at $1/(m-1)$ of full power. Again the source does not take part in the second phase.

For the SM full-time case the information capacity is given by

$$I_{sm\_sc} = 2 \min(I_{sm\_1}; I_{sm\_2}) \quad (11)$$

$$I_{sm\_1} = \log_2\left(1 + SNR_{s,d(s),1}|\alpha_{s,d(s),1}|^2 + \frac{1}{(m-1)}\sum_{r\in D(s)} SNR_{r,d(s),1}|\alpha_{r,d(s),1}|^2\right). \quad (12)$$

$$I_{sm\_2} = \log_2\left(1 + SNR_{s,d(s),2}|\alpha_{s,d(s),2}|^2 + \frac{1}{(m-1)}\sum_{r\in D(s)} SNR_{r,d(s),2}|\alpha_{r,d(s),2}|^2\right) \quad (13)$$

The probability of each information stream being decoded is an extension of the direct transmission method. Since the realised mutual information between s and r for i.i.d. complex Gaussian code-books is given by

$$\frac{1}{m}\log_2(1 + SNR_{sr}|\alpha_{sr}|^2)$$

Under this rule the probability of a relay being $r \in D(s)$ is given by

$$P[r \in D(s)] = P\left[|\alpha_{sr}|^2 > \frac{2^{mR}-1}{SNR_{sr}}\right] \quad (14)$$

4.3 Extensions to the proposed schemes

It is possible for the full-time scheme to be extended so that the source transmits different information to every relay. This would allow us to approach the MIMO capacity limits, which will be termed max-time. However this would require that the both the source and destination have a number of antenna elements equal to the number of relays in the network, $m-1$. As previously discussed this is not possible owing to the size constraints of mobile terminals, and the requirements that antenna elements must be at least half a wavelength apart to maintain independent fading channels.

The max-time scheme would be applicable to a mobile terminal where you could fix more antenna elements, for example laptop computers, or a remote fixed relay.

One possible extension of all the cooperative diversity schemes discussed above, instead of the relays simply decoding the information and then retransmitting it to the intended destination, is that the relays could transmit different information such as parity bits calculated from the received signal. Turbo coding [24] is also an option owing to the parallel nature of the relaying signals. This could potentially dramatically decrease the probability of outage at the destination, however it is beyond the scope of this paper.

5 Results and analysis

In order to discover the performance properties and practical suitability of the different cooperative diversity relaying techniques, Monte Carlo simulations are presented, based on the capacity equations for each relaying scheme. The capacity equation of each is analysed in three different ways:

outage probability against SNR (bandwidth efficiency fixed at 1b/s/Hz)

bandwidth efficiency against SNR (outage fixed at 5%)

outage probability against bandwidth efficiency (SNR fixed at 5dB).

Each combination of the three communication link characteristics has been included since analysing them all leads to a deeper insight. For example, the outage probability is expected to drop significantly for the repetition scheme when compared to the classic case. However, it would also be expected that the capacity of the link would drop significantly since redundant information is being transmitted.

For each relaying scheme, the results are compared directly to the classical case of direct transmission, and then indirectly between each other. For the case where the bandwidth efficiency is fixed, 1b/s/Hz has been chosen in order to make the calculations simpler, and since it is a reasonably practical value. Where the outage level is fixed, the 5% level has been chosen since this is likewise a practical value in a wireless communications network. Although a number of values for the case where the SNR at each receiver is fixed could have been selected, 5dB was chosen since it is in the middle region of signal quality. Results for networks with up-to four relays in it $(m = 5)$ are presented.

Note 1: Calculating the sum capacity of the source to relay information streams would make it possible to recombine the information at the relay and then distribute it optimally between the antenna elements for second phase transmission.

Note also that it would be possible, using the analysis presented by Laneman et al. [9], to calculate the asymptotes of the outage probability. Owing to lack of space we have not repeated the techniques used, which can be readily extended to the scenarios considered in this paper.

5.1 Single antenna STC cooperative diversity

In single antenna STC based cooperative diversity as the number of relays in the network is increased, although the transmit power for each relay is reduced to meet the power constraint, another spatial degree of freedom is added. It is clear that as the diversity degrees of freedom increase with m, the probability of outage will reduce, Fig. 6. Owing to the decreased transmit power as m increases, diminishing returns can be observed as more relays are added into the network. Note that at low SNRs Fig. 6 shows that the classical scheme performs better. This is due to a combination of the time constraint, the power constraint and potentially poor source to relay link in the first phase. Note that this is equally likely as a poor source to destination link and a poor relay to destination link since it is assumed that all MTs are equally spaced. If the source to relay link is poor then the relay will not be able to decode and transmit the information in the second phase, therefore the power of the transmission that would have been used by the relay in phase two is not used at any point in the transmission.

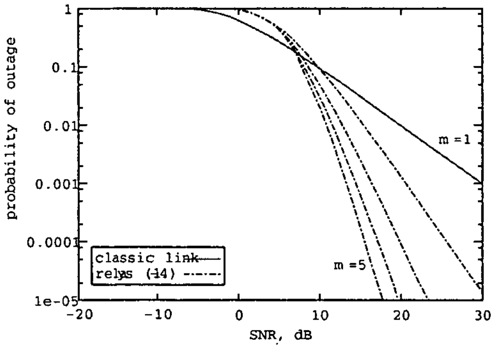

Fig 6 Simulation results for single antenna STC-based cooperative diversity scheme where the spectral efficiency is fixed at 1 b/s/Hz
SNR and outage probability are compared for m ¼ 1,2,y ,5

Figure 7 shows that the single antenna STC-based cooperative diversity scheme offers improvements in outage probability when dealing with low spectral efficiencies, compared to the classical case. This is due to the fact that redundant information is transmitted, and therefore is more likely to be received successfully. However, at higher spectral efficiencies the classical transmission case begins to outperform the STC based scheme.

This effect is clearly seen in Fig. 8 where at low SNRs the single antenna STC-based scheme offers improvements over the classical case. However, at higher SNRs again, the classical case offers improved spectral efficiency.

5.2 Dual antenna STTD cooperative diversity

The dual antenna STTD half-time case builds on the single antenna cooperative diversity by adding another opportunity

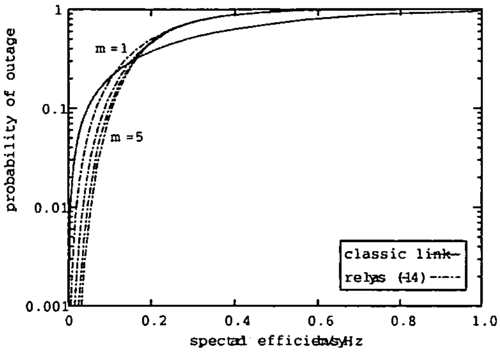

Fig 7 Simulation results for the single antenna STC cooperative diversity scheme where the SNR is fixed at 5dB
Outage probability and spectral efficiency are compared for m ¼ 1;2;...;5

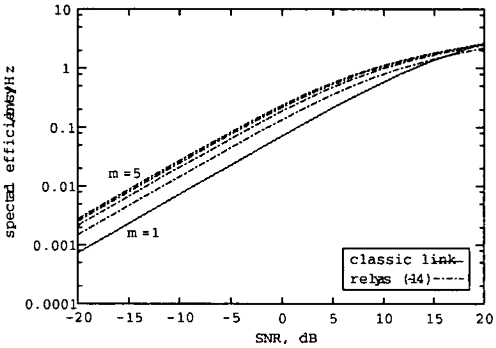

Fig 8 Simulation results for single antenna STC cooperative diversity scheme where the outage level is fixed at 5%
Spectral efficiency and SNR are compared for m ¼ 1;2;...;5

to use spatial diversity for transmission. The effect of using STTD on the probability of outage is clearly shown in Fig. 9 where it can be seen that dual antenna STTD transmission greatly decreases the probability of outage at higher SNRs. Again there is a region at low SNRs where direct transmission is preferable.

The much lower probability of outage that dual antenna STTD can offer translates into being able to handle higher data rates than the classical and single antenna cases, Fig. 10. Figure 11 shows that the capacity of a cooperative diversity network with dual antenna STTD transmission also benefits. It is interesting to note that Fig. 10 shows that using direct STTD transmission from the source to the destination is preferable over using one relay. However, adding more than one relay into the network gives performance benefits over the direct STTD transmission case.

5.3 Dual antenna SM cooperative diversity

The dual antenna STTD half-time scheme suffers from not being able to transmit the same amount of original information over the same limited amount of time as the classical case. Full-time SM cooperative diversity attempts to overcome this problem by using SM from the source in

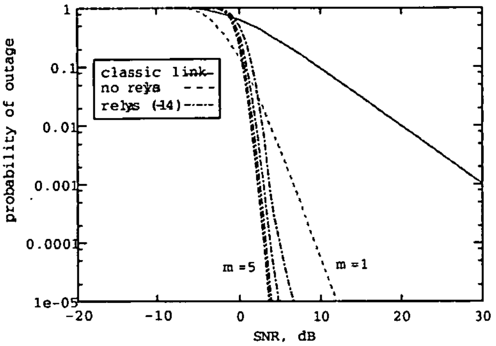

Fig 9 Simulation results for dual antenna STTD STC-based cooperative diversity scheme where the spectral efficiency is fixed at 1b/s/Hz
SNR and outage probability are compared for m = 1;2;...;5

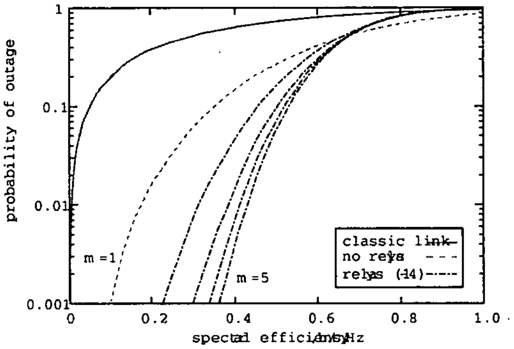

Fig 10 Simulation results for dual antenna STTD STC-based cooperative diversity scheme where the SNR is fixed at 5dB
Outage probability and spectral efficiency are compared for m = 1;2;...;5

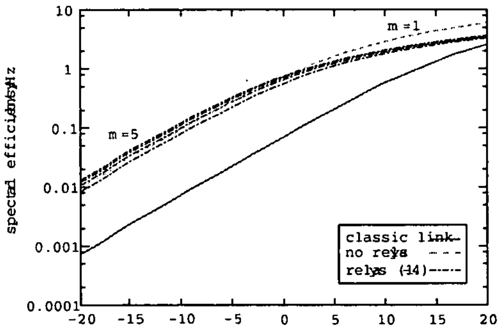

Fig 11 Simulation results for dual antenna STTD STC-based cooperative diversity scheme where the outage level is fixed at 5%
Spectral efficiency and SNR are compared for m = 1;2;...;5

the first phase in addition to from the relays in the second phase. Two co-operating sub-sets are then created to perform the phase two transmission. This is done at the cost of splitting the available transmit power at the source between two antenna elements.

Figure 12 clearly shows the huge benefit to the probability of outage offered by the full-time scheme. There is still a small region at low SNRs (lower than 5dB) where classical transmission would be preferred. This is due to the full power transmission that the source uses in the classical case. Again the diminishing returns for higher number of relays, owing to the relay transmission being constrained by $1/(m-1)$ can be observed.

When the outage probability is compared with the spectral efficiency in Fig. 13, it can be seen that the half-time dual antenna STTD scheme offers large improvements when compared with the SM dual antenna and STC single antenna cases. However, when attempting higher spectral efficiencies, direct transmission using SM would be preferred. This is again due to the split transmission power at the source.

Despite this, when the spectral efficiency is compared with the SNR in Fig. 14, it can been seen that the full-time cooperative diversity scheme matches or betters the classical transmission case, and offers large improvements at SNRs lower than 15dB.

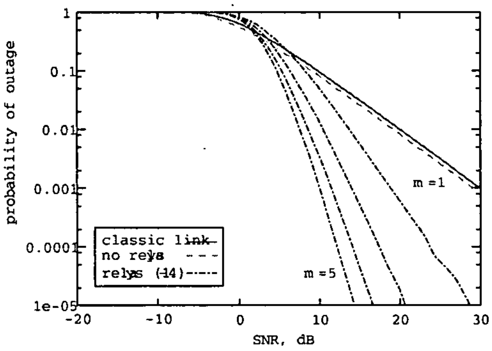

Fig 12 Simulation results for dual antenna SM STC-based cooperative diversity scheme where the spectral efficiency is fixed at 1b/s/Hz
SNR and outage probability are compared for m = 1;2;...;5

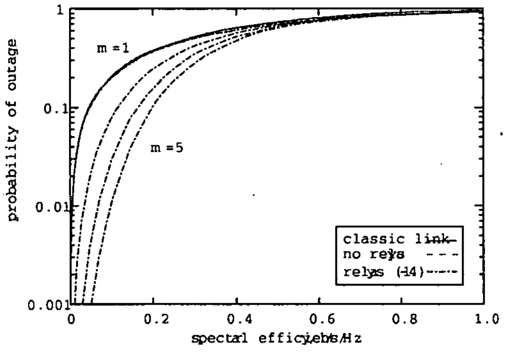

Fig 13 Simulation results for dual antenna SM STC cooperative diversity scheme where the SNR is fixed at 5dB
Outage probability and spectral efficiency are compared for m = 1;2;...;5

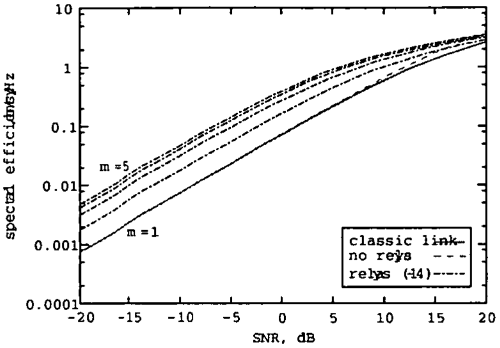

Fig 14 Simulation results for dual antenna SM STC cooperative diversity scheme where the outage level is fixed at 5%
Spectral efficiency and SNR are compared for m ¼ 1;2;...;5

The results presented in this paper confirm the results of [25] where it was found that STTD transmission can be preferred to SM transmission under certain circumstances.

5.4 Degrees of freedom

One result that can be observed from the calculated results is that the largest performance gain does not necessarily come from the addition of the first relay in the network, rather it comes from the first additional degree of freedom introduced to the network. The degrees of freedom considered in this paper, owing to the three constraints imposed on the system, come from spatial diversity. This can be spatial diversity introduced by using multiple antenna elements at the transmitter and receiver or it could stem from using other MTs as spatially separated relays for cooperative diversity. The results presented show that additional degrees of freedom that are introduced to the network give diminishing returns in performance, as previously noted for additional relays. This can be observed from all of the results of the scenarios presented, however, it can be seen particularly well in Figs. 9–11 where the first degree of freedom (using STTD transmission instead of the classical scenario) provides the biggest performance increase. Using cooperative diversity to add additional degrees of freedom, although it increases the performance of the network, does not provide significant additional performance. This would explain why STTD provides the best results, as it has the highest degrees of freedom of the scenarios considered in this paper.

6 Further work

The simulation results presented in this paper were based on a large number of simplifications, and were intended only to characterise the cooperative diversity channel when using dual antenna elements for transmission and reception. The analysis presented in this paper will is useful as a comparison for what can be achieved under ideal conditions, therefore, taking this work further will involve creating a more realistic simulation through the use of a channel model incorporating shadowing to compare with a realistic classical transmission scheme. STCs have also been extensively used in this paper to allow the destination terminal to decode the signals from the different transmitting terminals. Note however, that the design and use of distributed STCs for a relay situation such as the one described here is not a solved problem, and could prove a fruitful area of future research. Nabar et al. [15] have considered basic space-time signal design for the cooperative diversity they introduce.

It is expected that expansion on this work to characterise pools of multi-hoop relays through an idea called mobile ad-hoc trunking (MAT) to extend the range of terrestrial cells, or a purely ad-hoc network, will be possible.

7 Conclusions

This paper has introduced dual antenna elements for MTs in a cooperative diversity network, and presented two different schemes for using the antenna elements to benefit wireless communications. Through simulations of the derived capacity equations for each scheme it was observed that the use of two antenna elements in MTs can add considerable capacity and outage benefits over the single antenna case.

Cooperative diversity is intrinsically limited by the need to have two transmission phases, and the limit of two antenna elements in any hand-sized mobile terminal. Using the two antenna STTD or SM techniques presented in this paper it is possible to overcome these problems and increase the capacity of the network. It has been shown that both in terms of probability of outage and spectral efficiency STTD based cooperative diversity is preferred over SM. It is also noted that the largest gains come from the first supplementary degree of freedom, and diminishing returns are observed for additional degrees of freedom.

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MIMO Cooperative Diversity in a Transmit Power Limited Environment

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***Abstract*— This paper considers a fading relay channel where the total transmit power used is constrained to be equal to that of the standard single-hop channel. The relay channel used operates in what is termed as MIMO cooperative diversity mode, where the source transmits to both relay and destination terminals in the first instance. Both the source and relay then transmit to the destination in the second instance. Initially the cooperative diversity framework is introduced to consider system constraints so a direct and fair comparison with the single-hop case can be made. In-particular a power constraint is placed on the system and the optimal transmit power levels are derived and presented. The derived technique for finding the optimal transmit power levels is then used to demonstrate the advantages of using cooperative diversity in a wireless network. The results presented show that MIMO cooperative diversity offers a 3.4dB increase in spectral efficiency at 5% outage, with no additional cost incurred in transmit time, power or bandwidth.**

I. INTRODUCTION

One of the biggest problems faced by designers of mobile telecommunication systems is fast-fading of the received signal due to the multi-path nature of the communications channel. The very multi-path that creates difficulties also offers us *diversity* [1]. This can help mitigate the problem as a result of the transmission of redundant information over (ideally) independent fading paths (in time/frequency/space) in conjunction with a suitable receiver. Spatial diversity techniques are particularly attractive as they provide diversity gain while incurring no penalty of extra transmission time or bandwidth. Spatial diversity techniques such as Multiple-Input Multiple-Output (MIMO) wireless systems [2] have been shown to significantly increase the spectral efficiency of point-to-point wireless links, including cooperative diversity relay channels [3] which are designed to take advantage of spatial diversity where Mobile Terminals (MTs) are limited to a single transmit/receive antenna.

Contributions and relation to previous work. This paper considers the advantage of using cooperative diversity in an environment where the total transmit power is limited to that of a single-hop communication channel [4]. This allows a fair and direct comparison between the cooperative diversity relay channel and the single-hop channel to be made. The power constraint placed on the system is an important factor to be considered for any practical deployment of a cooperative diversity network, both due to interference concerns [5] and to ensure battery life prolongation of the relay terminal.

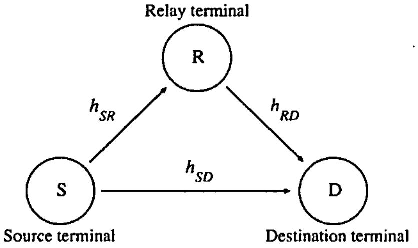

Figure. 1. Relay fading channel

The first part of this paper introduces the cooperative diversity protocol and channel model framework, originally developed by Nabar et al. [6] and later extended in [7]. A jointly analytical-numerical method to optimise the transmit power levels of each transmitting terminal within the power constraint, in terms of probability of outage, is then developed. This extends the work presented in [7] to consider an alternative Decode-and-Forward (DF) relaying scheme and introduces an analytical approach to optimising the transmit power levels. Results for the optimal power levels are then introduced. The second part of this paper presents a comparison between the cooperative diversity protocol considered and the single-hop case. Finally conclusions are drawn in the final section.

II. PROTOCOL DESCRIPTIONS AND SYSTEM CONSTRAINTS

A. Protocol Description

Consider the ad-hoc fading relay channel shown in Figure 1 with three MTs. Data is transmitted from the source terminal, S, to the destination terminal, D, potentially with the assistance of the relay, R. All terminals in this paper are considered to have single transmit and single receive antenna elements. Additionally, a terminal cannot simultaneously transmit and receive information. Although it is possible for the relay to

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assist in either an Amplify-and-Forward (AF) or Decode-and-Forward (DF) manner, only DF is considered in this paper. In DF mode the relay demodulates and decodes the signal before re-encoding the signal and retransmission occurs. The DF mode that is used in this paper requires the relay to receive a required information rate, R, before it can take part in the second phase, which is denoted $R \in \mathrm{D}$, where D is the set of supporting relays[1]. If the relay is unable to decode the first phase signal from the source, it does not transmit any information in the second phase [9], which is denoted $R \notin \mathrm{D}$.

Due to the constraint of MTs not being able to transmit and receive information at the same time, the transmission of information from the source to destination, in a cooperative diversity network, is broken into two time slots, termed phase I and phase II [8]. This paper considers the cooperative diversity scenario where the source transmits in both time phases and the relay only in the second having (potentially) received the first phase transmission from the source. Furthermore the source transmits different information to the destination in the second phase than that which was transmitted during the first phase. The destination can therefore potentially receive two different information streams during the second phase.

This cooperative diversity protocol was originally introduced by Nabar et al. [6] as one member of a family of cooperative diversity protocols. The subject protocol is referred to as *Protocol I* in literature, however as it is the only protocol to be considered in this paper, any mention of cooperative diversity refers to this protocol unless otherwise specified.

When considering the complete cooperative diversity model, it can be seen that it bears a strong resemblance to a MIMO system with two antenna elements at both the transmitter and receiver. The fading relay channel from Figure 1 is redrawn in Figure 2 to highlight the MIMO characteristics by splitting the destination into the two time phases, and adding an extra subscript to the channel notation to denote the phase of transmission. This is a schematic presentation only and the destination terminal actually uses only one antenna element. However, its access of the channel is split over the two time phases. This scenario is therefore termed *MIMO cooperative diversity* due to the similarity to MIMO wireless systems.

B. System constraints

To be able to fairly compare the performance several constraints are placed upon the channel model:

- The time used must not exceed that used by the single-hop transmission case (1 unit)
- The bandwidth used must not exceed that used by the single-hop transmission case (1 unit)
- The total transmit power of the complete system (power used by the source and relay for transmission) must not exceed that used by the single-hop case (1 unit)

The effect of the time and bandwidth constraints is that the transmission of the two phase cooperative diversity must take place in the same amount of time as the single-hop system. This typically involves a factor of $1/2$ when considering the spectral efficiency of the system. An important point that arises from this constraint is that the Rayleigh block-fading is now considered to be unchanged over the two time phases.

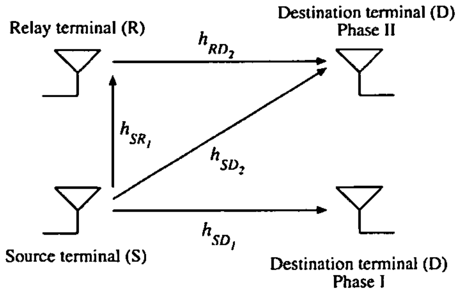

Figure. 2. Relay fading channel shown as a MIMO channel

The effect of the transmit power constraint is particularly important for the cooperative diversity protocol considered in this paper since it uses three transmissions (the source in both the first and second phase and the relay in the second phase only) and therefore must reduce their transmit power to the power constraint. The following transmit power constraint variables are placed upon the system:

- A - first phase transmission from the source
- B - second phase transmission from the source
- C - second phase transmission from the relay

To constrain the transmit power to be the same as the single-hop case, first recall that the transmission in the relaying network has been split into two phases, it therefore follows that

$$A + B + C = 2 \tag{1}$$

A transmit power constraint is also placed upon each transmitting terminal such that it cannot broadcast with more power than the single-hop case would (1 unit), i.e.,

$$A \leq 1 \qquad B \leq 1 \qquad C \leq 1. \tag{2}$$

III. CHANNEL AND SIGNAL MODELS

Throughout this paper the channel fading model is assumed to be frequency-flat, Rayleigh block-fading channels with fading coefficients being circularly symmetric zero mean complex Gaussian random variables. Perfect timing synchronisation is assumed. Furthermore it assumed that the receivers have perfect channel state information of all the reverse channels. It is also assumed that there is limited feedback to the source and relay from the destination of the average channel Signal to Noise Ratio (SNR) which is required to perform power control for MIMO cooperative diversity.

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[1] In this paper the maximum cardinality of D is limited to 1 since there is only one supporting relay, however [8] considered multiple relays

The signals transmitted by the source during the first and second phases are denoted $x_1[n]$ and $x_2[n]$ respectively. Symbol-by-symbol transmission is considered so the time index n can be dropped to simply give x_1 and x_2. Note also that for the data symbols transmitted is it assumed that $\mathcal{E}\{x_i\} = 0$ and $\mathcal{E}\{|x_i|^2\} = 1$ for $i = 1, 2$. The signal received at the destination terminal in the first phase is given by

$$y_{D,1} = \sqrt{AE_{SD}}h_{SD}x_1 + n_{D,1} \tag{3}$$

where $y_{Y,1}$ is the received signal at the destination (Y) in the first phase, E_{XY} is the average signal energy over one symbol period. The scalar h_{XY} is the random, complex-valued, unit-power channel gain between the source and destination terminals, and $n_{Y,1} \sim \mathcal{N}(0, N_0)$ is the additive white noise for transmitting terminal X and received terminal Y, in this case S and D respectively. Note that E_{XY} and h_{XY} does not have a phase subscript due to the earlier assumption of flat fading across the two transmission phases. Similarly the signal received at the relay in the first time slot is given by

$$y_{R,1} = \sqrt{AE_{SR}}h_{SR}x_1 + n_{R,1} \tag{4}$$

Finally assuming that the relay can fully decode the signal from the first phase source transmission, the signal received by the destination in the second phase is given by

$$y_{R,2} = \begin{cases} \sqrt{BE_{SD}}h_{SD}x_2 + \sqrt{CE_{RD}}h_{RD}x_2 + n_{D,2} & R \in \mathrm{D} \\ \sqrt{BE_{SD}}h_{SD}x_2 + n_{D,2} & R \notin \mathrm{D} \end{cases} \tag{5}$$

The input-output relationship can now be summarised as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{6}$$

where $\mathbf{y} = [y_{D,1}y_{D,2}]^T$ is the received signal vector, $\mathbf{H}$ is the MIMO cooperative diversity $2x2$ channel matrix

$$\mathbf{H} = \begin{cases} \begin{bmatrix} \sqrt{AE_{SD}}h_{SD} & 0 \\ \sqrt{CE_{RD}}h_{RD} & \sqrt{BE_{SD}}h_{SD} \end{bmatrix} & R \in \mathrm{D} \\ \\ \begin{bmatrix} \sqrt{AE_{SD}}h_{SD} & 0 \\ 0 & \sqrt{BE_{SD}}h_{SD} \end{bmatrix} & R \notin \mathrm{D} \end{cases} \tag{7}$$

$\mathbf{x} = [x_1x_2]^T$ is the transmitted signal vector and $\mathbf{n}$ is additive white Gaussian noise.

A. *Information-Theoretic performance*

The DF protocol used in this paper states that the rule for the relay to be considered part of the decoding set is that the relay must be able to support the required spectral efficiency, R. Since the source/relay channel mutual information, I_{SR}, is a function of the random fading coefficients, the mutual information is also a random variable. The event $I_{SR} < \mathrm{R}$ is defined as the source/relay channel being in outage and therefore $\Pr[I_{SR}] < \mathrm{R}]$ is referred to as the outage probability of the channel. Moreover this definition is extended from the relay inclusion condition to the general channel such that the event $I < \mathrm{R}$ is considered channel outage, where I is the total mutual information of the cooperative channel.

The probability of the channel being in outage when the relay is available to support the source can be written as

$$\Pr[R \in \mathrm{D}]\Pr[I < \mathrm{R}|R \in \mathrm{D}] \tag{8}$$

Furthermore the probability of the channel being in outage when the relay is not available can be written as

$$\Pr[R \notin \mathrm{D}]\Pr[I < \mathrm{R}|R \notin \mathrm{D}] \tag{9}$$

Subsequently the total probability of the channel being in outage is given by the sum of (8) and (9) as

$$\begin{aligned} \Pr\big[I < \mathrm{R}\big] &= \Pr\big[R \in \mathrm{D}\big]\Pr\big[I < \mathrm{R}|R \in \mathrm{D}\big] \\ &+ \Pr\big[R \notin \mathrm{D}\big]\Pr\big[I < \mathrm{R}|R \notin \mathrm{D}\big] \end{aligned} \tag{10}$$

The mutual information between the source and relay in the first phase, is given by Shannon's capacity equation, with a factor of 1/2 due to the two equal transmission time phases, as

$$I_{SR} = \frac{1}{2}\log_2\left(1 + \frac{AE_{SR}}{N_0}|h_{SR}|^2\right) \text{ b/s/Hz} \tag{11}$$

Rearranging (11) to give the condition of the relay being able to decode the first phase transmission, and substituting the mutual information I_{SR} for the required spectral efficiency, R, gives

$$|h_{SR}|^2 > \frac{2^{2\mathrm{R}} - 1}{\mathrm{SNR}} \tag{12}$$

where $E_{SR}/N_0 = \mathrm{SNR}$. In the remainder of this paper it is assumed that all network channel links have the same SNR, specifically $E_{SR}/N_0 = E_{RD}/N_0 = E_{SD}/N_0 = \mathrm{SNR}$ to simplify presentation and simulation results. This assumption can be readily relaxed for the more general case.

Finally, the mutual information for the MIMO cooperative diversity channel is obtained from (6) and (7) as

$$I = \frac{1}{2}\log_2\det\left(\mathbf{I}_2 + \frac{1}{N_0}\mathbf{H}\mathbf{H}^H\right) \text{ b/s/Hz} \tag{13}$$

which is expanded to

$$I = \begin{cases} \frac{1}{2}\log_2\Big((AB\mathrm{SNR})^2|h_{SD}|^4 + A\mathrm{SNR}|h_{SD}|^2 & \\ \quad + B\mathrm{SNR}|h_{SD}|^2 + C\mathrm{SNR}|h_{RD}|^2 + 1\Big) & R \in \mathrm{D} \\ \frac{1}{2}\log_2\Big((AB\mathrm{SNR})^2|h_{SD}|^4 + A\mathrm{SNR}|h_{SD}|^2 & \\ \quad + B\mathrm{SNR}|h_{SD}|^2 + 1\Big) & R \notin \mathrm{D} \end{cases} \tag{14}$$

IV. POWER CONTROL ANALYSIS

Consider now the problem of minimising the probability of outage of the cooperative diversity channel as a function of the power control levels A, B and C. Since A, B and C are limited by (1), by varying A and B between 0 and 1 independently, the entire range of results are explored, and subsequently the minimum probability of outage can be found.

To begin an analytical approach to this problem, consider the channel probability of outage (10), substitute the required mutual information equations (11) & (14) and rearrange so the random variables are on the Left Hand Side (LHS) of each individual probability inequality. This is written as

$$\begin{aligned}&\Pr\left[I_{1_M} < \mathrm{R}\right] = \\ &\left(\Pr\left[|h_{SR}|^2 < \frac{x}{A}\right]\right. \\ &\qquad \left.\Pr\left[AB\mathrm{SNR}|h_{SD}|^4 + (A+B)|h_{SD}|^2 < x\right]\right) \\ &+\left(\Pr\left[|h_{SR}|^2 > \frac{x}{A}\right]\right. \\ &\qquad \left.\Pr\left[AB\mathrm{SNR}|h_{SD}|^4 + (A+B)|h_{SD}|^2 + C|h_{RD}|^2 < x\right]\right) \end{aligned} \tag{15}$$

where $x = (2^{2\mathrm{R}} - 1)/\mathrm{SNR}$.

The probability inequalities with a single channel coefficient on the LHS (probability of relaying being able to support the source) can readily be solved analytically as they are single Rayleigh random variables for which the Cumulative Distribution Function (CDF) is well known as $[1-\exp(-x/A)]$ when the standard deviation is 1 as it is in this case. Moreover x for the Rayleigh CDF is given by the Right Hand Side (RHS) for each probability inequality.

A. Derivation of CDF with relay available

The CDF of the more complex joint distribution of the two random variables $AB\mathrm{SNR}|h_{SD}|^4+(A+B)|h_{SD}|^2+C|h_{RD}|^2$ is not so well known. To find an analytical expression for the probability of outage conditioned on the relay being available, an expression for the CDF of the joint random variables must be found. This is done by convolving the Probability Density Functions (PDF) of each random variable to obtain the joint PDF, which is then integrated to obtain the required CDF.

The probability of outage of the relay channel, operating with the relay available, is given by

$$\Pr\left[AB\mathrm{SNR}|h_{SD}|^4 + (A+B)|h_{SD}|^2 + C|h_{RD}|^2 < x\right] \tag{16}$$

Before a PDF can be derived for the joint h_{SD} and h_{RD} terms of (16) a single equation must be found for the h_{SD} term. Begin by defining

$$Y = AB\mathrm{SNR}|h_{SD}|^4 + (A+B)|h_{SD}|^2 \tag{17}$$

which shows that Y is a function of the single random variable h_{SD}. To find the PDF of a function of a random variable employ (5-15) and (5-16) as presented by Papolius and Pillai [10] where the equation $Y = g(X)$ is solved. Denoting the real roots by X_n,

$$y = g(X_1) = \ldots = g(X_n) \tag{18}$$

Papolius and Pillai show that the PDF of a single random variable is given by

$$f_Y(Y) = \frac{f_X(1)}{|g'(X_1)|} + \frac{f_X(2)}{|g'(X_2)|} + \ldots + \frac{f_X(n)}{|g'(X_n)|} \tag{19}$$

where $f_X(X)$ is the function of the single random variable in question, and $g'(X)$ is the derivative of $g(X)$. For the case in question let $X = |h_{SD}|^2$ to give

$$g(X) = AB\mathrm{SNR}X^2 + (A+B)X - Y \tag{20}$$

To find the roots of (20) use the standard quadratic equation where $a = AB\mathrm{SNR}$, $b = A + B$ and $c = -Y$.

$$0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{21}$$

$$\frac{-(A+B) \pm \sqrt{(A+B)^2 + 4ABY\mathrm{SNR}}}{2AB\mathrm{SNR}} \tag{22}$$

$$\frac{\sqrt{(A+B)^2 + 4ABY\mathrm{SNR}} - (A+B)}{2AB\mathrm{SNR}} \tag{23}$$

where in (23) discard the negative term since the interest lies only in probabilities (i.e. between 0 and 1 inclusive) leaving only one root. Furthermore the derivative of $g(x)$ is found as

$$g'(x) = AB\mathrm{SNR}X + A + B \tag{24}$$

By combining (19), (23) and (24) the general PDF is given by

$$f(Y) = \frac{\Pr\left[\dfrac{\sqrt{(A+B)^2 + 4ABY\mathrm{SNR}} - (A+B)}{2AB\mathrm{SNR}}\right]}{\sqrt{(A+B)^2 + 4ABY\mathrm{SNR}}} \tag{25}$$

Since h_{SD}^2 is a Rayleigh distributed random variable, the PDF of which is given by $\Pr[\mathrm{x}] = \mathrm{e}^{-x}$, the PDF of the joint h_{SD} term is given by

$$\frac{\exp\left(\dfrac{\sqrt{(A+B)^2 + 4ABx\mathrm{SNR}} - (A+B)}{2AB\mathrm{SNR}}\right)}{\sqrt{(A+B)^2 + 4ABx\mathrm{SNR}}} \tag{26}$$

where the value of interest is denoted x.

Before deriving the final combined PDF, note that the PDF of $C|h_{SD}|^2$ is given by the RHS of (16) as

$$\frac{1}{C}\mathrm{e}^{\frac{-x}{C}} \tag{27}$$

Therefore the joint PDF is found by convolving the two PDFs.

This is a non-trivial integration. However, symbolic computer algebra can be employed to find a suitable integral

$$(f * g)(x) = \frac{\sqrt{\pi} e^{-\frac{4xAB\mathrm{SNR}+A^2+2AB+B^2-2CA-2CB+C^2}{4ABC\mathrm{SNR}}}}{2ABC\mathrm{SNR}\sqrt{-\frac{1}{ABC\mathrm{SNR}}}}$$
$$\left(\mathrm{erf}\left(\frac{\mathrm{csgn}\,(A+B)\,A+\mathrm{csgn}\,(A+B)\,B-C}{2ABC\mathrm{SNR}\sqrt{-\frac{1}{ABC\mathrm{SNR}}}}\right)\right.$$
$$\left.+\,\mathrm{erf}\left(\frac{-\sqrt{4xAB\mathrm{SNR}+A^2+2AB+B^2}+C}{2ABC\mathrm{SNR}\sqrt{-\frac{1}{ABC\mathrm{SNR}}}}\right)\right) \tag{28}$$

where $\mathrm{erf}(z)$ is the Error Function and $\mathrm{csgn}(z)$ is the sign function is used to determine in which half-plane (left or right) the complex-valued number z lies.

It can be seen that the derived PDF for the joint distribution (28) is very complex. To find the CDF of the joint distribution it is necessary to integrate this expression again, however it is not of a standard form and a closed form solution can not be readily found using either standard methods or symbolic computer algebra. Due to this, Runge-Kutta numerical integration methods are used to approximate the integral.

B. Derivation of CDF with relay not available

To be able to fully calculate the probability of outage for MIMO cooperative diversity, consideration must also be made for the situation where no relays are available to support the source in the second phase, for example when the source to relay signal is itself in outage. The previous section derived the PDF of the h_{SD} term as (25). To find the CDF (25) simply needs to be integrated. Again using symbolic computer algebra the CDF of the probability of outage where the relay is not available is shown to be

$$\Pr\left[AB\mathrm{SNR}|h_{SD}|^4+(A+B)|h_{SD}|^2<\frac{2^{2\mathrm{R}}-1}{\mathrm{SNR}}\right]=$$
$$\frac{1}{\sqrt{\mathrm{e}^{\frac{\mathrm{csgn}(A+B)(A+B)}{AB\mathrm{SNR}}}}}\left(\mathrm{e}^{\frac{A+B}{2AB\mathrm{SNR}}}\right.$$
$$\left.-\,\mathrm{e}^{\frac{-\sqrt{A^2+2AB+B^2+4xAB\mathrm{SNR}}+A+B}{2AB\mathrm{SNR}}}\sqrt{\mathrm{e}^{\frac{\mathrm{csgn}(A+B)(A+B)}{AB\mathrm{SNR}}}}\right) \tag{29}$$

The necessary components to fully construct (15) are now available using numerical integration of (28) and the closed form solution (29), however the trivial re-writing this equation in full is omitted in this paper due to space constraints.

C. Numerical results

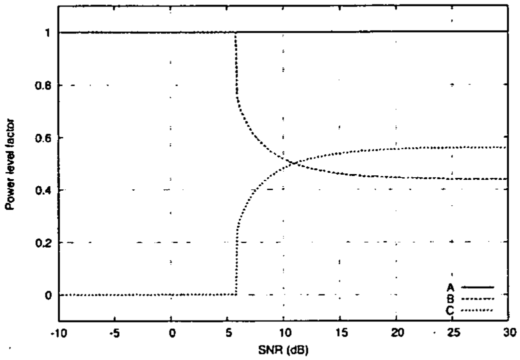

Figure. 3. MIMO cooperative diversity optimal power control levels A, B and C against SNR for spectral efficiency of 1b/s/Hz.

Results from the numerical evaluation of (15) are shown in Figure 3 for a spectral efficiency of 1b/s/Hz. Below approximately 6dB SNR only the source is preferred to transmit and all of the available power is dedicated to the source's transmission over both time phases. This is due to the fact that for SNRs lower than this the source to relay link is not suitable for the required 1b/s/Hz spectral efficiency. Above the 6dB point the relay power allocation increases sharply while the second phase source transmission power level falls off proportionally and both approach an asymptote at high SNRs. Interestingly at high SNR it is optimal in terms of outage for the relay to use a greater power allocation than the source in the second phase. This is due to the extra diversity that the relay offers, once it is able to take part in the second phase.

At lower required spectral efficiencies it would be expected that the relay would be be allocated transmit power in the second phase at lower SNRs than that recorded for 1b/s/Hz. This can be seen to be true by inspection of (12), which suggests that at low spectral efficiencies the SNR for a similar probability of inclusion of the relay in the second phase will also be lower. Similarly at higher spectral efficiencies the SNR level where the relay would actively be used would be higher.

V. PERFORMANCE COMPARISON

Figure 4 shows the probability of outage against SNR for MIMO cooperative diversity, the single-hop case and the two-hop channel (which is included for reference) at a fixed required rate of 1b/s/Hz. At low SNR it can be seen that MIMO cooperative diversity tracks the probability of outage of the single-hop channel, as would be expected from Figure 3, where A and B are shown to be equal to 1, which is the same as the single-hop case. At higher SNRs when the relay terminal is allocated transmit power, the extra diversity offered by having two paths available to the destination makes a significant impact on decreasing the probability of outage.

Figure 5 is of particular interest for consideration in a practical wireless communication network where a certain level of signal outage is tolerated, as it shows the achievable spectral efficiency for a set SNR at the 5% outage level. This

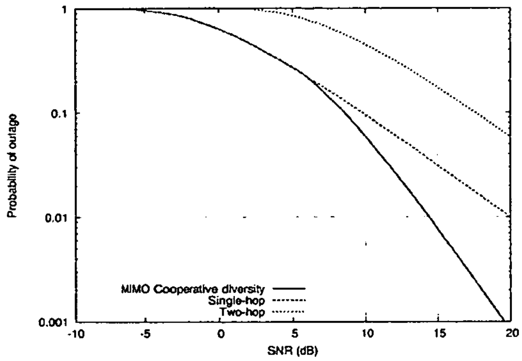

Figure. 4. Simulation results for probability of outage where the spectral efficiency is fixed at 1 b/s/Hz. SNR and outage probability are compared for MIMO cooperative diversity, the single-hop link and the classical relay (two-hop) link.

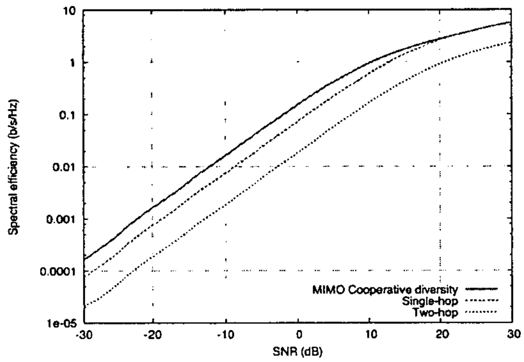

Figure. 5. Simulation results for spectral efficiency at the 5% outage level. SNR and spectral efficiency are compared for MIMO cooperative diversity, the single-hop link and the classical relay (two-hop) link.

simulation was made by calculating the optimal transmit power levels for each protocol, at each SNR and attempted spectral efficiency, on the fly.

As can be seen from Figure 5 MIMO cooperative diversity offers a considerable increase in the spectral efficiency of the channel, with the extra diversity offered by the relay effectively combating the Rayleigh fading at low SNR. A significant gain of 3.4dB can be observed over the single-hop link at low SNR. At high SNR the spectral efficiency of the MIMO cooperative diversity protocol converges with the single-hop case.

VI. CONCLUSION

This paper has presented a framework for MIMO cooperative diversity which is directly comparable with single-hop transmission. In-particular the power constraint was analytically considered and values for the optimal transmit power control values were presented. It was shown that at low SNR MIMO cooperative diversity can offer a considerable advantage over the single-hop case, due to the extra spatial diversity of the relay, with no additional cost incurred in transmit time, power or bandwidth.

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