A PRACTICAL STUDY OF THE ERRORS

AFFECTING

SURVEYING OPERATIONS

by

R. McAdam, B.Sc., A.M.I.M.E.

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PREFACE.

All the material of this thesis has been published, the following being a list of the publications in question. It is presented in this form for the sake of uniformity.

The Accuracy of Linear Measurements.
Colliery Guardian, Vol. CXLVIII, pp. 339 - 341, and pp. 385 - 387.

A Preliminary Investigation of the Accuracy of Tacheometry.
Trans. Inst. Mine Surveyors, Vol. 13, Part 1, pp. 26 - 36.

Self-Reducing Tacheometers.
Trans. Inst. Mine Surveyors, Vol. 13, Part 4, pp. 128 - 145.

The Precision of Tacheometrical Levelling.
Colliery Guardian, Vol. CXLVII, pp. 575 - 577.

The Errors Affecting Plumblines in Shaft Connections.
Colliery Guardian, Vol. CXLVII, pp. 1197 - 1200.

Grateful acknowledgment is due to Professor Henry Briggs for his valuable help and advice; to Messrs Cooke, Troughton & Simms Ltd., Messrs E.R. Watts & Son Ltd., and Messrs Carl Zeiss Ltd. for the loan of instruments; and to the many mining students of the Heriot-Watt College for their willing assistance in taking field observations.

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THE ACCURACY OF LINEAR MEASUREMENTS.

The errors affecting linear measurements are so numerous, of such a variable nature and so inter-connected, that the ultimate accuracy of measuring distances is more difficult to assess than that of any other surveying operation. The greatest source of trouble is the pronounced interference of cumulative errors due principally to using an incorrect chain or tape, disregarding slope, poor alignment, the unevenness of the ground and the effect of sag.

The care with which the measurements are made is also an important factor, and in many cases, in mine surveying particularly, the operation of measuring is left in the hands of unskilled workmen, and appreciable marking and reading errors are intro:
:duced.

Standard Correction.-

It is generally assumed that a steel tape registers the correct length when it is at the standard tension and temperature. The length of a tape increases with use, however, that is to say, it is subject to an inelastic extension and, for this reason, the 'field' tapes are always checked against 'reference' tapes before and after every Geodetic Triangulation Base measurement, and any correction to be applied is called the Standard Correction.

Such a correction is unnecessary in normal topographical and mine surveying practice when using steel tapes, but should certainly be applied when using steel chains. A chain extends due to the wear on its eight hundred rubbing surfaces and the opening of its four hundred joints to a much greater extent than to the inelastic extension of the steel itself. The wear is inversely proportional to the thickness of the wire forming the chain, but even the heaviest chains are liable to extend due to wear. For example, the 100-foot chain used for the measurement of the Ordnance Survey Triangulation Base at Romney Marsh consisted of forty links, each ½-inch square in section, yet the correction to be applied for half the ascertained wear of the chain was 3.282 ins.*

The/

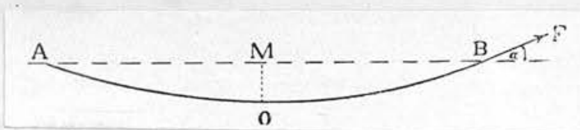
* Surveying, by John Whitelaw (Crosby Lockwood & Son), p. 426.

the ground is specially prepared it is rarely perfectly flat; and as the tape or chain normally rests on the surface of the ground another positive cumulative error is introduced, its magnitude depending on the degree of unevenness of the ground.

Correction for Sag.-

If a tape is unsupported at any part, a positive cumulative error is introduced due to the sagging of the tape. Exact formulae giving the correction to be applied for sag have been derived by Thompson* and others, but they are unwieldy. The following formula is much simpler and gives approximate solutions quite accurate enough for practical purposes.

Assume that the ends of the tape, A and B, are supported at the same level, or nearly so.



Let O = the centre of the tape
L = the length of the tape = AOB
d = the distance AMB
a = the amount of sag = OM
F = the standard pull in lbs.
w = the weight of the tape per linear foot.

a being small compared with d, the curve may be taken as a parabola instead of a catenary.

The length of a parabola from its origin O to any point xy on it is, with sufficient accuracy, when x is small:

$$x + \frac{2y^2}{3x}$$

The length OB is therefore $\frac{d}{2} + \frac{2a^2}{3\frac{d}{2}}$

$$\text{or } \frac{L}{2} = \frac{d}{2} + \frac{4a^2}{3d}$$

$$\therefore \quad L = d + \frac{8a^2}{3d}$$

and the correction, $L - d = \frac{8a^2}{3d}$ ....(1)

Now/

---

* "Improved Systems of Chaining", Proc. Inst. Civil Engrs., Vol. XCII.

Now consider the relation between a, w and F.

The equation of the parabola, with origin O, and axis OM, is:

$$x^2 = ky$$

where k = a constant.

Now the curve passes through the point B whose ordinates are $\frac{d}{2}$ and a

$$\therefore \frac{d^2}{4} = k\,a$$

$$\text{or} \quad k = \frac{d^2}{4a}$$

Therefore the equation of the parabola is

$$x^2 = \frac{d^2}{4a}\,y.$$

The pull, F, acts tangentially to the curve at B, and the equation representing the tangent to any curve at a point x_1, y_1, is:

$$y - y_1 = \left(\frac{dy}{dx}\right)_{x_1 y_1} (x - x_1)$$

$$\frac{dy}{dx} = \frac{8\,ax}{d^2}$$

and B is $\frac{d}{2}$, a

Therefore the equation of the tangent at B is

$$y - a = \frac{8\,ad}{2\,d^2}\left(x - \frac{d}{2}\right)$$

$$\text{or} \quad y - a = \frac{4a}{d}\left(x - \frac{d}{2}\right)$$

Let α equal the angle this tangent makes with the horizontal, then

$$\tan\alpha = \frac{4a}{d}$$

$$\text{or} \sin\alpha = \frac{4a}{\sqrt{16\,a^2 + d^2}}$$

Assumingthat the total weight of the tape (wL) acts at its centre point O, and resolving forces along OM, we get/

get

$$2F \sin \alpha = wL$$

or $$2F \frac{4a}{\sqrt{16 a^2 + d^2}} = wL$$

$$\therefore F = \frac{wL \sqrt{16 a^2 + d^2}}{8 a}$$

a is small compared with d, therefore a^2 is very small compared with d^2, therefore $\sqrt{16 a^2 + d^2}$ may be written as d.

$$\therefore F = \frac{wLd}{8a}$$

$$\text{or } a = \frac{wLd}{8F}$$

Substituting for a in Equation (1) we get:

$$L - d = \frac{8w^2L^2d^2}{64F^23d} = \frac{w^2L^2d}{24F^2}$$

or, near enough,

$$L - d = \frac{w^2L^3}{24F^2} \quad \text{.........................(2)}$$

The error due to sag given by Equation 2 is represented graphically in Fig. 1 for distances up to 100 feet, for a Chesterman steel tape 3/7 of an inch wide and .0145 of an inch thick, with a weight of .397 ozs. per linear foot; and some idea of the magnitude of the error is obtained from the graph.

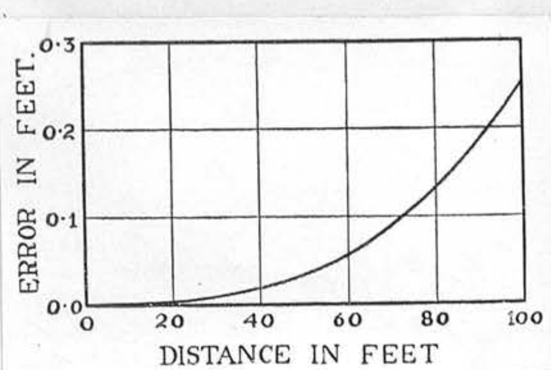


Fig. 1. Error introduced by sag.

Correction/

Correction for Tension.-

The length of a tape varies with the tension applied, and it is essentialto maintain the standard tension throughout the measurement. A spring balance should, of course, be used, but spring balances as supplied by the makers are adjusted to read correctly when held vertically, and as the surveyor uses his balance in a horizontal position a small error is often introduced. In an old balance the error frequently varies at different parts of the scale and the most satisfactory solution of this difficulty is to test the pull in a horizontal direc:tion and note the index error affecting the standard pull. Another source of error is the reduced strain on the tape, even when the balance is indicating the correct pull, due to the frictional resistance of the ground on the tape.

The effect of a varying tensional pull can be shown in a useful manner by calculating the <u>extra tension required to stretch a sagging tape to make it indicate the correct length</u>.

Let L_o = the length of the tape under no tension in feet.

$L - L_o$ = the extension due to the normal pull F, in feet.

F_1 = the pull necessary to make the tape indicate the correct length (lbs.).

S = the cross-sectional area of the tape in square inches.

W = the total weight of the tape.

= wL (lbs.).

E = Young's modulus of elasticity for the steel.

The Strain under pull F $= \frac{L - L_o}{L_o}$

The Stress under pull F $= \frac{F}{S}$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{FL_o}{S(L - L_o)}$$

$$\therefore L - L_o = \frac{FL_o}{SE}$$

Similarly $L_1 - L_o = \frac{F_1 L_o}{SE}$

$\therefore$/

$$\therefore L_1 - L = \frac{L_o}{SE}(F_1 - F)$$

where $F_1 - F$ is the extra tension required.

Let $F_1 - F = f$, then

$$L_o = \frac{L\,S\,E}{SE + F}$$

$$\therefore L_1 - L = \frac{LSE}{SE + F} \cdot \frac{f}{SE} = \frac{Lf}{SE + F} \text{ ....(3)}$$

We have now the condition that the tape of the length L, measured along the curve, under a ten::sion F_1, stretched between two points at a distance L apart, therefore equation (2) becomes

$$L_1 - L = \frac{W^2L}{24F_1^2}$$

From this and equation (3):

$$\frac{W^2}{24\,F_1^2} = \frac{f}{SE + F}$$

$$\therefore \frac{W^2}{24(F+f)^2} = \frac{f}{SE + F}$$

$$\therefore \frac{W^2SE + W^2F}{24} = f^3 + 2F\,f^2 + F^2f \text{ ......(4)}$$

where f is the extra tension required.

The extra tension required to stretch a sagging tape to make it indicate the correct length, as expressed by equation (4), is shown graphically in Fig. 2, for distances up to 100 feet, for a Chesterman steel tape 3/7 of an inch wide and .0145 inch thick; s = .006215 sq. ins.; w = .397 ozs. per ft. run; E = 30,850,000.

The extra tension required becomes so great as the distance increases, however, that this method of overcoming sagging errors soon ceases to be of practical use. For example, an extra pull of 10 lbs. is only sufficient to overcome sagging error on a length of 30 feet, and for one chain length an extra 22 lbs. is required, making a total pull of 32 lbs. The importance of supporting the tape as much as possible and of applying the correct tensional pull will therefore be evident.

Correction/

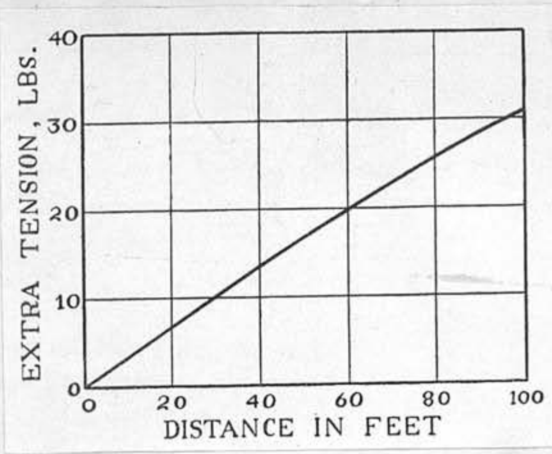


Fig. 2. Extra tension required to counteract sagging error.

Correction for Temperature.-

Up till 1932, steel tapes were standardised at a temperature of 62° Fahrenheit, and a tension of 10 lbs. when supported throughout. In 1932, however, the British Standards Institution and the International Committee of Weights and Measures agreed that the standard temperature should be 68°F., and accordingly manufacturers of steel tapes and bands are now producing tapes standardised at 68°F.

The coefficient of expansion of steel varies slightly, but the average coefficient of expansion of the famous Chesterman steel tapes is given by the makers as .0000062 per degree F., so the temperature correction for new Chesterman steel tapes is $\pm .0000062\ L\ (t^o - 68^o)$

where L = the length in feet

and t^o = the temperature of the tape in degrees Fahrenheit.

When t^o is greater than 68°, the correction is positive;

when t^o is less than 68°, the correction is negative.

Although the error introduced by temperature changes can be expressed by a simple mathematical equation, the temperature correction is one of the most difficult to assess truly as it is almost impossible to determine the correct temperature of the tape itself. For accurate surface work measurements should be made in dull weather or preferably at night as the effect of the sun's rays on a tape increases the temperature rapidly, and if the sun is clouded over/

over the temperature drops again just as rapidly. I have observed a rise in the temperature of a tape from 68°F to 84°F. in a few minutes due to the sun's rays heating on the tape and a corresponding drop in temperature just as quickly when clouds intervened. A slight wind is also an objectionable factor due to its rapid cooling effect as well as the distortion it causes.

Reading and Marking Errors.-

When using a chain and reading to the nearest link only, the error in marking the chain lengths is practically negligible compared with the error in reading provided proper care is taken in setting the arrows. In mine surveying, unfortunately, the chain lengths are very often recorded by making heavy chalk lines on the pavement, and this introduces appreciable marking errors.

If the chain is read to the nearest link, the reading error will vary between the limits of zero and .5 of a link, and, as there is an equal chance of any size of error between those limits, the average error of reading will be ± .25 of a link. So, if a line is measured along a perfectly flat surface with a chain of the correct length, by two skilled chainmen, the average error of measurement will be ± .25 of a link for any distance less than one chain length - the error increasing very slowly as further applications of the chain are made due to small marking errors. Actual measurements under favourable conditions gave the following results:

Number of Applications.	Average error of measurement in links.	
	66 foot chain	100 foot chain.
1.	± .25	± .22
2.	± .28	± .24
3.	± .27	± .25
4.	± .28	± .28
5.	± .30	± .28
6.	± .30	± .30
7.	± .31	
8.	± .32	
9.	± .35	

When/

When using a steel tape graduated to hundredths of a foot and reading to the nearest division, the reading error will be

$$\pm\sqrt{\left(\frac{1}{400}\right)^2 + \left(\frac{1}{400}\right)^2} = \pm .0035 \text{ of a foot,}$$

taking into account the fact that both ends of the tape are read.

When more than one application of the tape is necessary, marking errors have to be considered. The magnitude of the marking error naturally depends on the type of mark used to fix the terminal points, and as marking includes reading the tape as well as fixing the terminal point the marking error is considerably greater than the reading error. A marking error equal to three times the reading error was, indeed, accepted by Professor Briggs as an average figure for favourable conditions*, and the results of this investigation support that value. The error of measurement due to reading one end of the tape and marking the other end will therefore be equal to

$$\pm\sqrt{\left(\frac{1}{400}\right)^2 + \left(3.\frac{1}{400}\right)^2} = \pm .008 \text{ of a foot.}$$

If several applications of the tape are necessary, each is affected by the above error, so that the average error in the total length will be $\pm .008 \sqrt{n}$, where n is the number of applications. Thus, if a distance L is to be measured by a steel tape 100 feet long, there will be L ÷ 100 applications and the average error of reading and marking will be $\pm .0008 \sqrt{L}$.

Results of Tests.-

Owing to the complexity of errors affecting linear measurements the only feasible method of ascertaining the absolute accuracy of the different methods of measuring is by actual experiment. An extensive series of tests was accordingly carried out with 66 foot and 100 foot chains, and 100 foot and 300 foot steel tapes under various conditions both on the surface and underground. Thirty independent measurements were made of each length and the average fractional error of measurement calculated. The reference value for each length was obtained by measuring each section accurately with the tape supported throughout and applying the necessary corrections for slope and temperature.

The results of the tests with the Gunter chain are shown graphically in Fig. 3, for ideal con:
:ditions, reading the chain to the nearest link and also/

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* Effects of Errors in Surveying, (Griffin, London), 1912, p. 70.

also estimating the length to the nearest .5 and .1 of a link. It will be observed that the finer estimation of the length gives more consistent and more accurate readings, as one would expect.

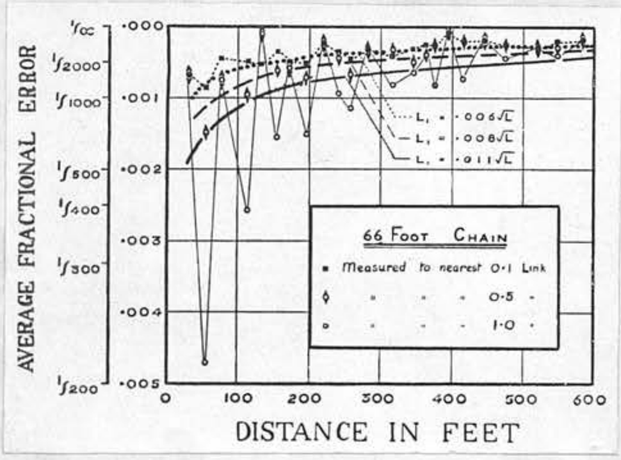

Fig. 3. Average Error Curves for 66 foot chains.

The general method of expressing error in linear measurements is to assume that the error is equal to $\pm K\sqrt{L}$, where K is a coefficient, the value depending on the character of the work, and L is the length. Although this is not perfectly accurate, due principally to the interference of cumulative errors, the expression is simple and easy to handle, and it is near enough for all practical purposes. The curve of the equation $L_1 = \pm K\sqrt{L}$ has therefore been added to the graphs with the appropriate value of K affixed.

The average error curve for the 100 foot chain is shown similarly in Fig. 4. The accuracy, when reading to the nearest link, is lower with the

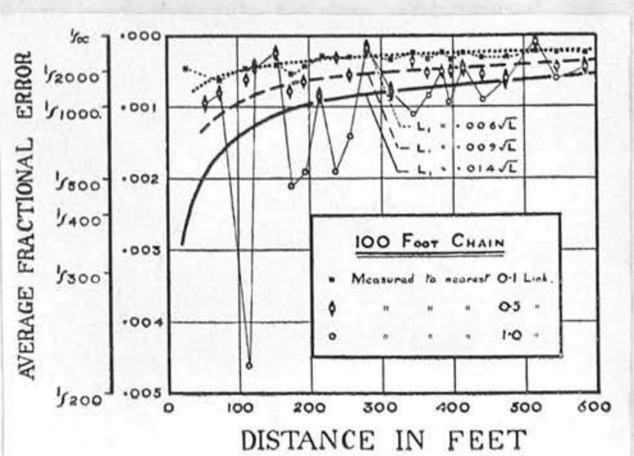

Fig. 4. Average Error Curves for 100 foot chains.

100 foot chain than with the 66 foot chain, again indicating that the longer the unit of measurement used the greater is the error obtained. When estimating to .5 or .1 of a link, however, the accuracy is practically the same as with the 66 foot chain because fewer applications of the chain are required to cover a given distance, and therefore the number of marking errors are smaller.

The results of the tests on the 100 foot steel tape are shown in Fig. 5. The tape was read to

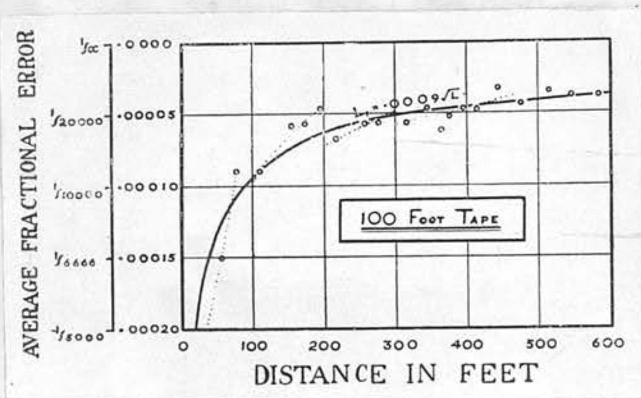

Fig. 5. Average Error Curve for 100 foot steel tapes.

the nearest .01 of a foot; the standard tension was applied by spring balance, but no temperature corrections were made. Furthermore the tests were again made under ideal weather conditions and over approximately level ground.

It will be observed that the value of the coefficient K, i.e., .0009, is very little higher than the theoretical reading and marking error mentioned above, viz: .0008, showing the absence of large cumulative errors due to sag, temperature, etc.

Another point to be observed is the step down in accuracy at every 100 feet length due to the marking error. It is indeed for this reason that long tapes are used for important work, for, as already pointed out, the error is directly proportional to the square root of the number of applications of the tape.

The graph representing the average error of measurement with a 300 foot steel tape, Fig. 6, shows the decrease in accuracy at the change-over point in a more pronounced manner than the 100 foot tape.

The points on the lower curve indicate the average fractional error of measurement with a 300 foot steel tape with the tape supported at 10 foot intervals/

intervals, when applying the standard tensional pull and estimating the reading to the nearest .001 of a

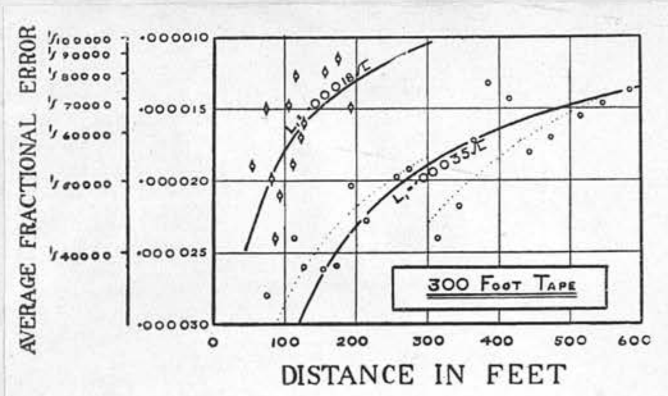

Fig. 6. Average Error Curves for 300 foot steel tapes.

foot. No temperature corrections were made, but during the measurements the temperature rarely, if ever, deviated from the standard temperature of 62°F by more than 10 degrees.

The top curve represents the average error of measuring with a 300 foot steel tape, with the tape supported on the correct grade at 5 foot intervals, when applying the standard tension with a spring balance and estimating to the nearest .001 of a foot. The temperature of the tape was read to the nearest degree Fahrenheit and the necessary temperature corrections applied.

The above results may be accepted as the best that can be obtained in their classes, and are the results to be aimed at in practice. Measurements were also made over rough ground on the surface, and also over different types of underground roadways and faces, and the following table summarises the results obtained for the different classes of work.

L = the distance measured

L_1 = the error in measurement

$$L_1 = \pm K\sqrt{L}$$

E_a = the Average Error of measuring.

	K	E_a @ 400 ft.
<u>66 Foot Chain</u>.		
Level ground, such as main surface roads; chain read to nearest link...........	0.011	$\pm \frac{1}{1,800}$
Level ground; chain read:ing estimated to 0.5 link.	0.008	$\pm \frac{1}{2,500}$
Level ground; chain read:ing estimated to 0.1 link.	0.006	$\pm \frac{1}{3,300}$
Rough ground, such as 3rd class roads, good under:ground roadways, etc.; chain read to nearest link.	0.015	$\pm \frac{1}{1,300}$
Very rough ground; such as reasonably flat fields.....	0.025	$\pm \frac{1}{800}$
<u>100 Foot Chain</u>.		
Level ground; chain read to nearest link...........	0.014	$\pm \frac{1}{1,400}$
Level ground; chain reading estimated to 0.5 link......	0.009	$\pm \frac{1}{2,200}$
Level ground; chain reading estimated to 0.1 link......	0.006	$\pm \frac{1}{3,300}$
Rough ground; chain read to nearest link................	0.016	$\pm \frac{1}{1,200}$
Very rough ground...........	0.025	$\pm \frac{1}{800}$
<u>100 Foot steel Tape</u>.		
Level ground; standard tension applied; tape read to nearest .01 ft...	0.0009	$\pm \frac{1}{22,000}$
Rough ground; standard tension applied; tape read to nearest .01 ft..	0.0022	$\pm \frac{1}{9,000}$
Very rough ground; standard tension applied; tape read to nearest .01 ft....................	0.0043	$\pm \frac{1}{4,600}$

a chain measurement, and for a steel tape measurement under unfavourable conditions .00621.* Accepting Professor Lorber's values of .00172 and .00239 for the steel tape and chain respectively, Professor Briggs deduced:

(the weight of a steel tape measurement) : (the weight of a chain measurement) .. $2\frac{1}{4}$:1

a considerable difference to 44 : 1.

An examination of Professor Lorber's figures shows, however, that he claims an average error of 1/8000 when using the chain, i.e., an error of only .05 of a link per 400 links. That accuracy, I think, it must be admitted, is abnormally high and very rarely attained. Professor Briggs, indeed, disregards the above values later in his treatise, and in working out several examples uses the coefficients .0063 for a steel tape measurement underground and .0096 for a 100 foot chain measurement when estimating the distance to .1 of a foot. While still retaining the $2\frac{1}{4}$: 1 ratio he comes much nearer the mark with the values of the coefficient K for the values obtained in this investigation are K = .0043 for a 100 foot steel tape on good underground roadways, and K = .006 for a 100 foot chain on level ground when estimating to .1 of a foot.

It is evident, however, that no definite values can be assigned to the coefficient K without taking into account the nature of the ground to be measured over and the size of the unit of measurement used.

* Briggs, Effects of Errors in Surveying, (Griffin, London), p. 72 and 75.

A PRELIMINARY INVESTIGATION OF THE ACCURACY OF TACHEOMETRY.

A PRELIMINARY INVESTIGATION OF THE ACCURACY OF TACHEOMETRY.

The accuracy of Tacheometric measurements depends on:

(1) The characteristics of the telescope used;

(2) The stability of the instrument;

(3) The effects of refraction on the lines of sight;

(4) The reading of the intercept on the staff;

(5) The manner of supporting the staff

and (6) The vertical angle.

1. The characteristics of the telescope.-

In tacheometry, all distances and heights are computed from the stadia intercept and the vertical angle, and the accuracy of spacing the stadia lines is the most important factor governing the overall accuracy of these determinations. The actual distance between the stadia lines depends on the size and optical combination of the telescope and it is generallyset to embrace an angle of 1 in 100, 1 in 50, or 1 in 200, when the telescope is at solar focus. The setting of these lines depends entirely on the instrument maker and all makers of repute will now give a guarantee with their glass diaphragms, such as:-

(a) The accuracy of the spacing of the lines is within 0.0001 in.;

(b) The thickness of the lines is about 0.00015 in.

Consider a glass diaphragm such as the above, and assume that the theoretical distance between/

between the webs should be 0.0715 in. (approximately that of the theodolites used in the following experiments). If the statement (a) above means that the maximum permissible error is 0.0001 in., then we may take the average actual error as

$$\frac{.0001}{2} = \frac{1}{20,000} \text{ in.,}$$

and as this affects two lines the average error of spacing

$$= \pm \frac{1}{20,000}\sqrt{2}.$$

Therefore <u>the Average Fractional Error due to mal-adjustment of the lines</u>

$$= \frac{\text{error of spacing}}{\text{distance between the webs}}$$

$$= \pm \frac{\frac{1}{20,000}\sqrt{2}}{.0715} = \pm \frac{1}{1000}$$

This error is indirectly proportional to the distance between the webs, or, in other words, the Average Error due to mal-adjustment of the webs decreases as the distance between the webs increases, <u>i.e.</u>, as the size of the telescope increases.

All distances computed from the stadia intercepts are referred to the anallatic point of the telescope used, and when this does not coincide with the centre of the instrument a stadia correction must be applied.

In the case of the external focusing telescope, consisting of anobjective and an eyepiece only, the addition of the well-known (f + c) constant automatically refers all distances to the centre of the instrument.

Professor J. Porro, of Milan, in 1823 introduced the telescope which now bears his name, and, by adding the so-called anallatic lens in the body of the telescope, brought the anallatic point into coincidence with the centre of the instrument. It is essential, however, in the Porro telescope, that the principal rays, after being refracted by the object glass, should cross before coming to the primary anallatic point, otherwise an error is introduced. This error is appreciable on sights under 50 feet.

Modern telescopes are generallyof the inter:

:nal/

internal focusing type, consisting of an objective, a focusing (negative) lens and an eyepiece. In this case, when the telescope is refocused from solar focus on a near object, the two optical separations (object glass to negative lens, and negative lens to reticule), the equivalent focal length of the telescope, and the positions of all the principal optical points are altered. As the location of the anallatic point is therefore variable the so-called Stadia Constant becomes a variable function, and if a fixed Stadia Constant is used for all distances, as is done in practice, an error is introduced on short sights, this being +ve or -ve according to the optical construction of the telescope.

The following curve (Fig. 7), taken from Mr. E. Wilfred Taylor's paper, The Tacheometric Telescope (Proceedings of the Optical Convention, 1926, Part II), shows the separation between the anallatic point and the centre of the instrument for an 11-inch telescope with the object at different distances. The effect of this varying separation

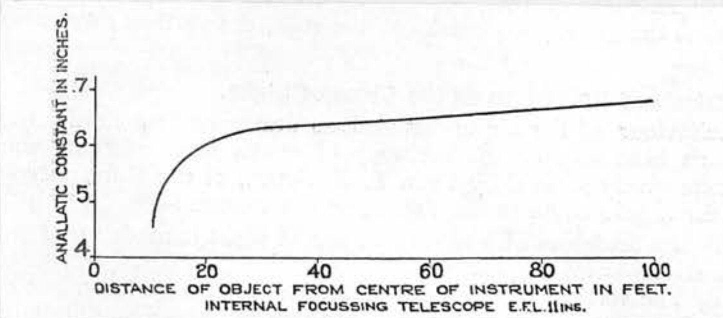

Fig. 7. Curve giving the separa:
:tion between the anallatic point
and the centre of the instrument
for an 11 in. telescope with the
object at different distances.

may cause an error of considerably more than $\frac{1}{1000}$, depending on the length of sight, and, to obviate this, instrument makers have introduced the internal focusing anallatic telescope. In this type the stadia constant can be neglected altogether as the error

$$\frac{\text{theoretical correction}}{\text{distance}}$$

is less than $\frac{1}{1000}$ for all distances down to 20 feet or thereabouts. Mr. E.W. Taylor points out (loc. cit.) that this $\frac{1}{1000}$ tolerance can be reduced to $\frac{1}{5000}$

at/

at the expense of the very short sights only, say those under 30 ft.

The following figures, taken from Messrs Cooke, Troughton & Simms' catalogue, give the corrections applicable to their new 4.5 inch Theodolite, fitted with the internal focusing anallatic lens:

Length of Sight	Correction
Infinity	+ 0.7 ins.
500 ft.	+ 0.67 "
250 "	+ 0.64 "
100 "	+ 0.56 "
50 "	+ 0.41 "
25 "	+ 0.15 "
20 "	- 0.05 "

It will be seen, therefore, that for all practical purposes the stadia correction may be neglected with the moderntype of telescope.

2. <u>The Stability of the Instrument</u>.-

Under normal conditions the effect of the wind in inducing vibration can be neglected, but on very windy days the shake on the instrument may be considerable. This shake, for any given wind velocity, is almost entirely dependant on the rigidity of the tripod. A well-designed tripod, strongly made, and with all sliding parts fitting closely and tightly clamped, will reduce the shake on the instrument to a negligible quantity on all but the very windy days.

The horizontal movement of one of the vertical webs on the diaphragm was measured on a fixed horizontal staff at a distance of 350 feet, on a fairly windy day, with the following results:

With an old-fashioned, adjustable, wooden tripod, the amplitude of this horizontal movement was 0.10 feet on the staff, with a rigid wooden tripod 0.001 feet, and with an adjustable tubular steel tripod of recent design 0.001 feet.

The effect of this shake, when reading on a vertical staff, is to decrease the distinctness of the graduation/

graduation marks on the staff, the vertical movement being practically nil.

3. <u>The Effects of Refraction on the Lines of Sight</u>.-

The behaviour of the air under various atmospheric conditions and the effect of such behaviour on the accuracy of stadia measurements was carefully and extensively studied by Professor L.S. Smith, of the University of Wisconsin, U.S.A., who came to the following conclusions:*

"(i) Unsteadiness of a terrestrial line of sight is made up of both vertical and horizontal vibrations.

(ii) The vertical vibrations are made up of two systems, one of large amplitude and slow movement, and the other of short amplitude and relatively fast movement.

(iii) This vertical vibration may seriously affect the accuracy of any single stadia reading, but from its nature it must cause a compensating error and therefore is of only secondary import:ance.

(iv) The time of maximum vibration is about the middle of the forenoon, or when the maximum <u>difference</u> of temperature between ground and air occurs.**

(v) Long sights (<u>i.e.</u>, those whose inter:cepts about equal the length of the rod) should either not be read in the hot parts of the day, or else should be read by half intervals on the upper part of the rod.

(vi) The vertical vibration is accompanied by an abnormal refraction in the stratum of air within 3 or 4 feet of the ground, which in all past work has caused large accumulating errors, this limiting the accuracy of stadia work to about 1/700."

The/

* "An Experimental Study of Field Methods which will insure to Stadia Measurements greatly increased accuracy"; <u>Bulletin of the University of Wisconsin</u>, Engineering Series, Vol. 1, No. 5.

** "The unsteadiness of the air over the ice of Lake Mendata the present spring (1895) has been observed by the writer to be as great as in the middle of the Yuma Desert with a temperature of 118° in the shade."

The effect of the unsteadiness of the line of sight can be neglected, not only because of its compensating nature, but because in this country with lines of sight under 600 feet in length it is only occasionally appreciable, such as when the sun shines brightly after a shower of rain.

The effect of refraction, however, is very important, and it is well to note here that in tacheometric measurements we are concerned not with <u>absolute</u> refraction, as in the case of Geodetic observations, but with <u>differential</u> refraction, <u>i.e.</u>, the varying amounts that the three lines of sight are refracted by the air. It is well known that if the three horizontal webs are read on a vertical staff, the top half-interval does not always equal the bottom half-interval, this being almost entirely due to differential refraction. This variation is greater on bright sunny days than on cloudy days, and it is also greater in the middle of the day than in the morning or evening. The following table is typical of the results obtained from low sights taken in calm sunny weather, the sights being sufficiently nearly of the same length to allow of mean values being struck.

	Face Left		Face Right	
	Bottom half-interval	Top half-interval	Bottom half-interval	Top half-interval.
	1.79	1.78	1.77	1.79
	1.79	1.78	1.78	1.78
	1.79	1.77	1.78	1.78
	1.85	1.85	1.86	1.85
	1.84	1.85	1.87	1.85
	1.71	1.72	1.72	1.70
	1.71	1.70	1.71	1.71
	1.71	1.70	1.70	1.71
	1.77	1.76	1.78	1.76
	1.77	1.77	1.77	1.77
Mean Values	1.773	1.768	1.774	1.770

The whole question of refractionis based on the principle that when a ray of light passes through a medium of varying density it is deflected or refracted in the direction of the denser part. Generally speaking, the density of the air decreases as the distance from the surface of the earth in:creases and therefore rays of light are bent down:wards/

downwards towards the earth; so in sighting an object it appears higher than it really is and a negative correction for refraction is required.

The above also is the case in tacheometric sights when the ground temperature is lower than the temperature of the air. On very sunny days, however, the stratum of air in contact with the ground may be heated by radiation and reflection of the sun's rays from the earth, and so for a relatively short distance from the ground the density of the air will increase as the distance from the surface increases. This will cause the lower line of sight, should it fall within this stratum of air, to be refracted upwards, and the bottom half-interval will therefore be smaller than the top one.

It is interesting to note that, as early as 1778, Wm. Green, an optician of London (who may be given the honour of introducing the sub-tense method of measuring distances by means of a telescope) pointed out that, if the staff was held horizontally, refraction errors would be cancelled, and the accuracy correspondingly increased.

4. The Reading of the Staff.-

The primary consideration is the magnification of the telescope, and Messrs Cooke, Troughton & Simms give the following details in one of their numerous pamphlets on surveying instruments (Publication No. 586).

"With normal eyesight and good lighting "conditions the .01 foot of a staff should be "resolved as follows:

Aperture	Distance.
1.125 in.	700 to 800 ft.
1.5 in.	900 to 1000 ft.
1.65 in.	1000 to 1100 ft.
1.8 in.	1100 to 1200 ft.
2.8 in.	1200 to 1400 ft."

In this country, however, the practical limit for Tacheometry is generally recognised as 600 ft., when using the standard tacheometer with an equivalent focal length of 9.25" and an aperture of 1.65". American writers claim 800 ft.

With the Rand theodolite used by me (aperture of object glass 1.125 ins., eyepiece magnification 20), the intercept on the staff can be read to 0.01 feet and estimated to 0.001 feet up to a distance of/

of 100 or 125 feet. At 350 to 400 feet the stadia lines appear about as wide as a 0.01 ft. division. At this distance and over, readings to 0.01 ft. depend on estimation, and the success of estimation itself depends on the distinctiveness of the mode of graduation of the staff. If sights are kept under this distance any clearly graduated staff is satisfactory, but for longer sights a clear, open-reading staff is necessary.

To ascertain the relative clearness and the adaptability of various staffs for tacheometric work, a series of comparative readings was taken on four types at various distances by different observers, and they were unanimously placed in the following order of clearness and ease of reading:

1st....The New Staff
2nd....Older type of Tacheometric Staff
3rd....Gayer's Pattern
4th....Sopwith.
(Fig. 8).

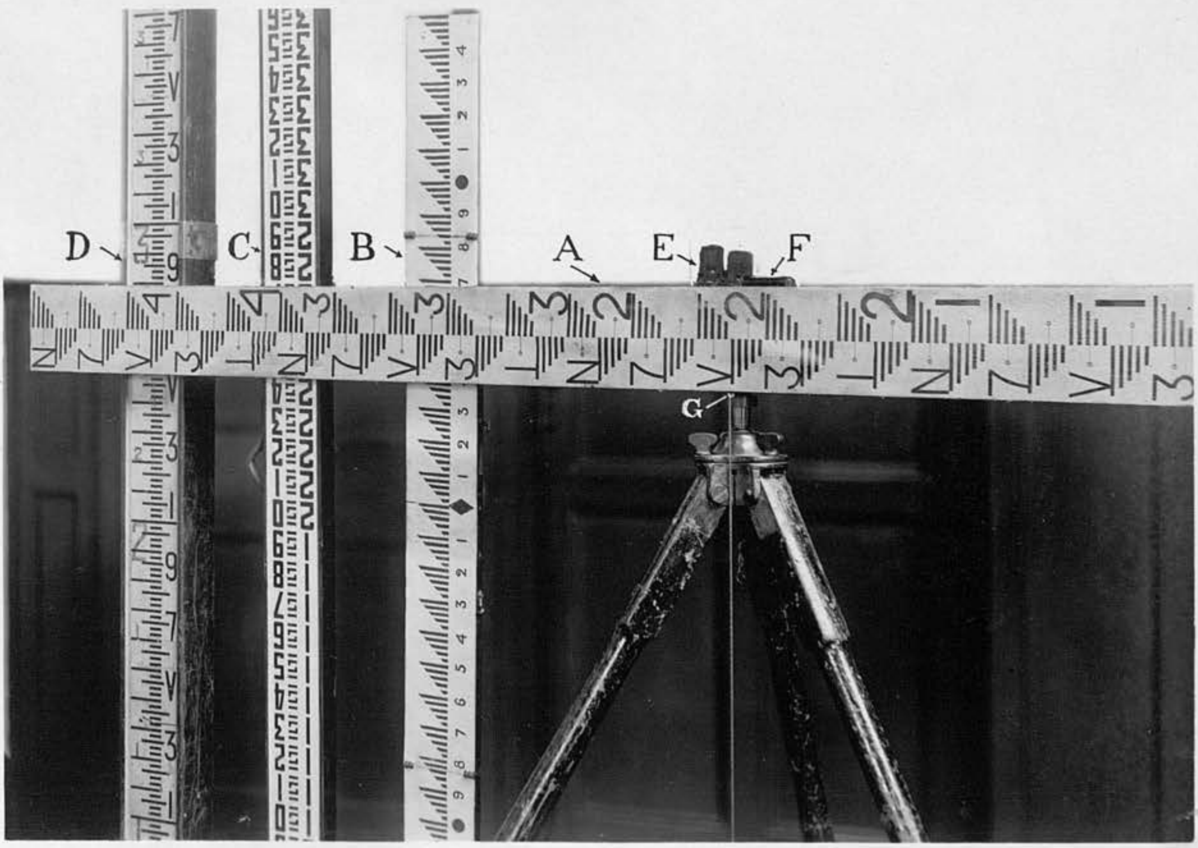

Fig. 8. New staff A mounted horizontally, with the older type of tacheometric staff B, the Gayer staff C, and the Sopwith staff D behind it. The sights E, the spirit level F, and the plumb-bob bracket G are also shown.

The so-called 'New Staff' used throughout these tests is that shown mounted horizontally in Fig. 8; it was designed and made in the Mining Department, and is an improvement on an older staff found suitable for underground tacheometry.* The 'foot' figures are in red and kept on one side of the staff. The decimal figures are black and arranged so that there can be no possibility of confusing any of the numbers when reading, viz., 1 3 V 7 N. The staff is also fitted with sights, a spirit level, and a bracket flush with the face of the staff. From this bracket a plumb-bob can be suspended directly under the 2.50 foot mark, thus enabling it to be used as a horizontal staff.

5. The Manner of Supporting the Staff.-

In order to ascertain the accuracy that could be obtained by using an ordinary theodolite fitted with stadia lines, and to determine the relative accuracy of the vertical and the horizontal staff, a series of observations was made and the results carefully analysed.

Readings were taken by four observers, and the work was carried out in all kinds of weather ranging from calm sunny days in July with a shade temperature of 65°F. and over, to windy wintry days in November with a shade temperature of 42°F. and under. Every result obtained was recorded and used in the calculations; the results are therefore fairly representative of practical field conditions in this country.

Preparation of the Lengths to be Measured.-

The work was carried out on the fairly level stretch/

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* (1) "Tacheometry as a Method of Underground Survey:ing", by Prof. Henry Briggs, D.Sc.,Ph.D., and James Cooper, Assoc. H.W. Coll., F.R.S.E., Trans. Inst. Min. Engrs., Vol. LXV (see Figs. 6 and 7) p. 34, and

(2) "Tacheometry as Applied to Underground Surveying" by James Cooper, Trans. Inst. Mine Surveyors, 1922.

stretch of ground used as the main base for the Burdiehouse Triangulation Survey undertaken yearly by the Mining students of theUniversity and Heriot-Watt College, Edinburgh. The end stations consist of ½" iron pipes sunk 4 ft. or so into the ground, and 30 wooden pegs, 12" x 2" x 2", were placed in a straight line between them and driven into the ground at approximately 20 ft. intervals. A theodolite was set up, centered, and levelled accurately at one of the end stations and directed on the other end station. Three-quarter inch wire nails were then lined in on the intermediate pegs and hammered down till there was only 1/8" protruding, thus marking the intermediate stations exactly.

Instruments used.-

Two theodolites were used, namely:

(a) The M.M. (Optical-micrometer)Theodolite, manufactured by Messrs E.R. Watts & Son Ltd. Designed by Carl Zeiss of Jena; internal focusing telescope; magnification 20 - 22 diameters; objective 1.3 ins. effective diameter; stadia interval = 100; stadia constant = 0.5 ft.; diaphragm etched with horizontal and vertical stadia lines.

(b) Rand Mining Theodolite, manufactured by Messrs Cooke, Troughton & Simms Ltd., Standard design; internal focusing telescope; magnification 20 diameters; objective 1.125 ins. diameter; stadia interval = 100; stadia constant, nil; diaphragm etched with horizontal and vertical stadia lines.

The staff used has already been described.

Method of Operation.-

One of the theodolites was set up accurat ely over one of the end stations and sighted on the other. The staff, held vertically, was then set up at the nearest intermediate station with the centre of the graduated face touching the nail, and a staff pill-box level wasused to insure that the staff was truly vertical. The telescope was focussed on the staff, making sure that there was no parallax effect and the three webs and the vertical angle read off. This was repeated twice, setting the centre web on a different mark each time. This procedure was repeat:ed at each station up to a distance of 400 ft. or thereabouts when reading the staff became a matter of estimation /

estimation.

The whole operation was repeated several times using both theodolites, and taking an equal number of observations on each theodolite with the telescope 'Face Left' and 'Face Right'. In all, 30 sights were taken to each of 20 pegs (15 face-left and 15 face-right), from both ends of the base, making a total of 1200 independant sights with the vertical staff.

The horizontal staff, mounted on a tripod (Fig. 8) was centered so that the plumb-bob hanging directly under the 2.5 ft. mark, and in the same plane as the face of the staff, was centered over the nail. The staff was levelled by means of the spirit level on top and placed at right angles to the line of sight by sighting the instrument through the collapsible slit and window sights attached to the staff, the whole operation of setting up taking less than 15 seconds. Four readings were taken at each set up, F.L., F.R., F.L., F.R., setting the centre horizontal web on the bottom of the staff, and 1120 independant sights were taken on the various stations using the horizontal staff.

In order to determine the levels of the staff-points, the height from the point to the bottom of the staff must be measured and recorded by the staff holder, thus laying more responsibility on him.

The height of the instrument was measured at every plant.

Additional notes were made in all cases regarding the time taken, atmospheric conditions, etc. for comparative purposes.

Finally the distances between the stations were measured six different times by steel tape, applying the correct tensional pull by means of a spring balance. The temperature of the tape was taken to the nearest degree Fahrenheit, and the difference in level between the stations was deter:mined by means of a dumpy level. The horizontal distances between the stations were then calculated, due allowance being made for slope and temperature.

A comparison between the total length of the base so obtained and measurements of former years justifies the assumption that the average fractional error of measurement is under 1 in 50,000, so these lengths may be taken as the standard to which the tacheometric measurements may be referred.

<u>Results of Observations</u>.-

The/

The horizontal distance of each sight was determined using the fundamental equations for inclined sights.

Vertical Staff:

$$D = 100\ g \cos^2 \alpha = G \cos^2 \alpha \ldots\ldots\ldots (5)$$

$$H = 100\ g \sin \alpha \cos \alpha = \frac{G}{2} \sin 2\alpha \ldots\ldots (6)$$

Horizontal Staff:

$$D = 100\ g \cos \alpha = G \cos \alpha \ldots\ldots\ldots\ldots (7)$$

$$H = 100\ g \sin \alpha = G \sin \alpha \ldots\ldots\ldots\ldots (8)$$

The results were grouped according to length of sight, giving 30 values for each length in the case of the vertical staff, and 28 in that of the horizontal staff. The actual error of each sight with reference to the standard length was next obtained, and the average error of a single observation was then determined for each group of sights by adding up these errors, irrespective of sign, and dividing the sum by the total number of sights.

In order to determine the relative accuracy of measurement, the error is best expressed in the form of the Average Fractional Error, viz:

$$\frac{\text{The average error of a single observation}}{\text{The length of the sight}}$$

$$= \frac{1}{x}.$$

These errors are plotted against their respective lengths in the graph, Fig. 9, where the abscissae represents the length of sight in feet, and the ordinates the average fractional error. AB represents the average curve for the horizontal staff, and CD represents that for the vertical staff. In this graph the ordinates are plotted on a distorted scale to exaggerate the variations in the errors according to the points observed.

The average curves for the horizontal and vertical staffs are shown on a straight scale in the graph, Fig. 10.

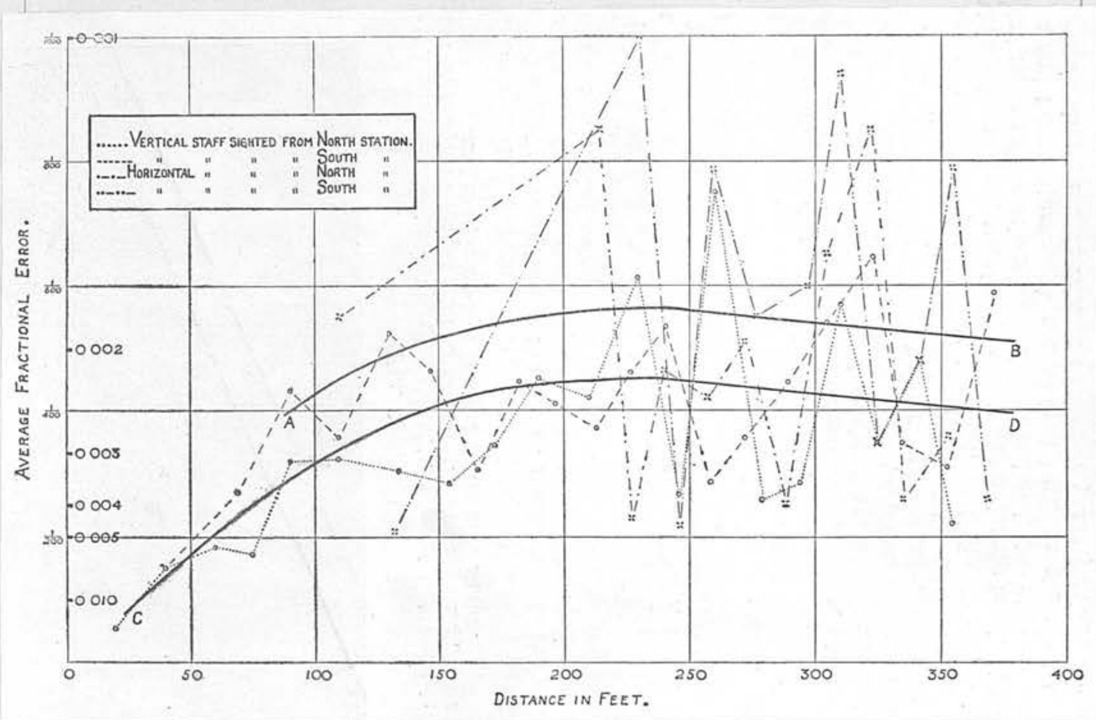

Fig. 9. Graph representing the average error of a single sight for various distances.

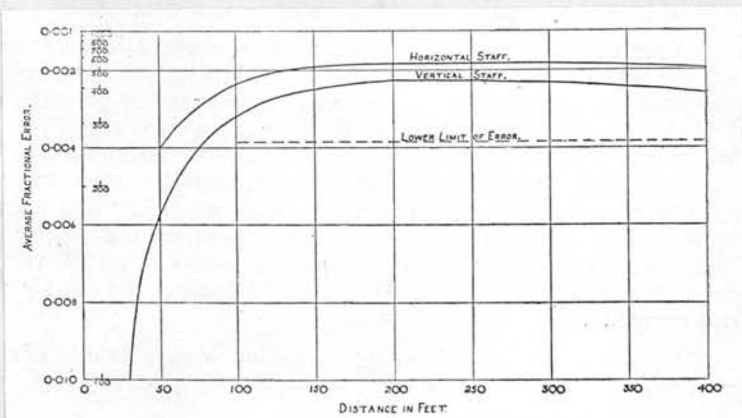

Fig. 10. Mean curves representing the average error of a single sight for the horizontal and vertical staffs.

Observations on Results.-

(1) The accuracy obtained when using a horizontal staff is approximately 20% greater than that obtained when using a vertical staff.

(2) The accuracy increases rapidly as the distance increases up to 200 feet.

At/

At this distance the Average Fractional Error when using a horizontal staff is approximately $\pm \frac{1}{550}$, and when using a vertical staff $\pm \frac{1}{450}$.

This increase is to be expected for, as the distance increases, the denominator of the fractional error increases. For example, an error of reading of the staff of, say, 0.005 ft. at 50 ft. is a fractional error of $\frac{1}{100}$, whereas the same error at 200 feet is $\frac{1}{400}$. The increasing denominator is the principal cause, but a study of the 'Curve of Errors' (Fig. 11) for sights under 100 ft. in length also

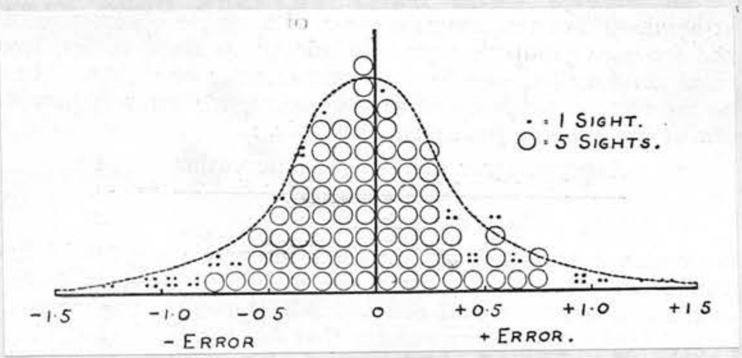

Fig. 11. Curve of errors for 427 sights under 150 feet in length, errors in feet.

shows that there is a slight under correction of the stadia constant on the short sights. The 'Curve of Errors' is obtained by plotting the actual errors of measurement according to their sign and magnitude as abscissae, against their frequency of occurrence as ordinates.

(3) <u>As the distance increases beyond 200 feet or thereabouts, the accuracy decreases slowly.</u>

This is probably due to a combination of several factors, the chief being decreased distinctness of the graduations on the staff; inability to read the staff exactly, and increased differential refraction effects since the lower line of sight comes nearer the ground.

(4) <u>The time taken for observation and computation when using a horizontal staff is 1.2 of that taken when using a vertical staff.</u>

As the time taken in making the actual field observations depends principally on the distances between the various points to be observed, and the number of staff holders employed, no fixed comparison is/

is possible, and my estimate in this connexion applies only to the special circumstances obtaining during the tests. In this investigation the personn: :el consisted of one instrument man, one man for book: :ing results, and one staff holder, and the time actually taken per 100 measurements when using a vertical and a horizontal staff was 70 and 85 minutes respectively. To speed up the office work, two people were employed, and the time taken for computa: :tion of vertical and horizontal staff measurements was 85 and 100 minutes respectively per hundred measurements.

(5) With regard to inclined sights, the accuracy obtained when using a horizontal staff is very much greater than when using a vertical staff.

The following section is included in support of this conclusion:-

6. The Vertical Angle.-

(a) Inclined Sights on a Vertical Staff:

When taking an inclined sight, a .01 ft. division subtends at the eye an angle which is cos α times that for a horizontal sight. The uncertainty of reading will therefore be increased by the proportion sec α.

So, if g_1^o is the average error in the staff-interval, g, for a given horizontal distance,

and g_1 is the average error for an inclined sight at the same distance from the instrument,

then $g_1 = g_1^o \sec \alpha$.

Let D_1, G_1 and α_1 represent the average errors in the horizontal distance, the inclined length and the vertical angle respectively. From equation (5):

$$D_1 = \pm \sqrt{\left[(G_1 \cos^2\alpha)^2 + (2G \sin\alpha \cos\alpha . \alpha_1)^2 \right]} \; *$$

(i.e. the average error affecting the product $G \cos^2 \alpha$)

$$\therefore \frac{D_1}{D} = \pm \sqrt{\left[\left(\frac{G_1}{G}\right)^2 + \frac{4 \cos^2 \alpha \sin^2 \alpha}{\cos^4 \alpha} . \alpha_1^2 \right]}$$

* See Briggs, Effects of Errors in Surveying, (Griffin, London) 1912, p. 24.

$$\therefore \frac{D_1}{D} = \pm \sqrt{\left[\left(\frac{g_1}{g}\right)^2 + 4\,\alpha_1^2 \tan^2 \alpha\right]}$$

$$= \pm \sqrt{\left[\left(\frac{g_1^o}{g}\right)^2 \sec^2 \alpha + 4\,\alpha_1^2 \tan^2 \alpha\right]}$$

$$= \pm \sec \alpha \sqrt{\left[\left(\frac{g_1^o}{g}\right)^2 + 4\,\alpha_1^2 \sin^2 \alpha\right]} \quad ..(9)$$

(b) Inclined Sights on a Horizontal Staff:

When looking through the telescope at a horizontal staff the graduations do not appear foreshortened, so the average error of reading is the same for an inclined as for a level sight of the same length = g_1^o .

From equation (7):

$$D_1 = \pm \sqrt{\left[(100\, g_1^o \cos \alpha)^2 + (-100 g \sin \alpha\, \alpha_1)^2\right]}$$

$$\therefore \frac{D_1}{D} = \pm \sqrt{\left[\left(\frac{g_1^o}{g}\right)^2 + \alpha_1^2 \tan^2 \alpha\right]} \quad ....(10)$$

The error expressed by equation (10) is less than that of equation (9), therefore the horizontal staff is superior in point of accuracy to the vertical staff for inclined sights also.

The accuracy obtained with inclined sights of approximately 200 ft. in length is shown in the graph (Fig. 12) for inclinations up to ± 60° from the

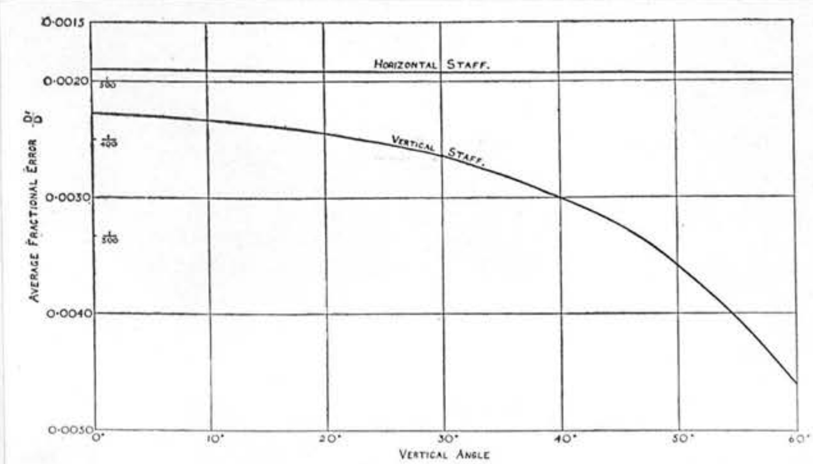

Fig. 12. Average error curves for the horizontal and vertical staffs with inclined sights.

the horizontal, assuming an average error of $\pm$ 30 seconds ($10^{-4} \times 1.45$ radian) in the vertical angle itself. This graph indicates that the fractional error $\frac{D_1}{D}$ increases considerably as the angle of inclination increases when using a vertical staff; whereas the fractional error is practically the same for inclined as for horizontal distances when a horizontal staff is used. In the latter case, the error in the vertical angle, α_1, is negligible if $\alpha_1 \tan \alpha$ is equal to, or less than,

$$\frac{1}{3} \frac{g_1^o}{g}$$

Let $\frac{g_1^o}{g} = \frac{1}{550}$, then the error in the vertical angle is negligible if $\alpha_1 \tan \alpha$ is equal to, or less than $\frac{1}{3} \times \frac{1}{550}$.

For example, when

$$\alpha = 45^o, \tan \alpha = 1$$

$$\therefore \alpha_1 = \pm \frac{1}{1650}$$

$$= \pm 0.000606 \text{ radian}$$

$$= \pm 125 \text{ seconds,}$$

i.e., when the angle of inclination is 45^o, an error of $\pm$ 2 minutes is negligible.

SELF-REDUCING TACHEOMETERS.

SELF-REDUCING TACHEOMETERS.

The purpose of this section is to give a brief account of the theory, construction and precision of three self-reducing tacheometers of different types, namely:

(1) The Watts-Szepessy Tacheometer
manufactured by
Messrs E.R. Watts & Son Ltd.

(2) The Jeffcott Tacheometer
manufactured by
Messrs Cooke, Troughton & Simms Ltd.

and (3) The Bosshardt-Zeiss Tacheometer
manufactured by
Messrs Carl Zeiss Ltd.

The method adopted to determine the accuracy of tacheometric measurements was described fully in the previous section: it consists, briefly, incomparing the tacheometric measurements with care::fully measured and corrected steel tape measurements. Thirty independant sights are made on each point and the 'average error' calculated. The 'average error' is then plotted against the respective horizontal distance in order to present the results of the tests as clearly and conveniently as possible:

(1) The Watts-Szepessy Tacheometer.-

The Watts-Szepessy tacheometer, invented by Szepessy, a Hungarian engineer, is based on the tangential principle. The honour of introducing the tangential principle in the first place, however, is given to Barcenas, an early Spanish writer on tacheometry. The principle may be briefly explained thus:

Consider a theodolite, T, (Fig. 13) sighted first on a point A, and then on a point B, on a vertical staff S.

Let D = the horizontal distance between the theodolite and the staff.

a/

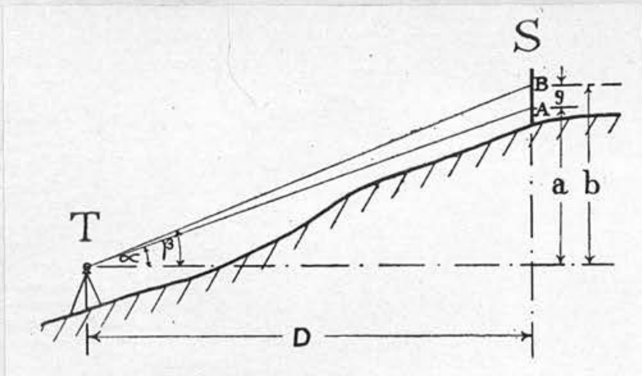

Fig. 13. Illustrating Barcena's Tangential Principle.

a = the vertical height between the horizontal axis of the theodolite and the point A.

b = the vertical height between the horizontal axis of the theodolite and the point B.

g = the staff intercept AB = (b - a).

α = the vertical angle to point A.

β = the vertical angle to point B.

Then:

$$a = D \tan \alpha \qquad (11)$$

and

$$b = D \tan \beta \qquad (12)$$

$$\therefore (b - a) = D (\tan \beta - \tan \alpha).$$

Now if the angles α and β are chosen such that

$$(\tan \beta - \tan \alpha) = 0.01$$

then

$$(b - a) = .01\, D$$

or

$$D = 100\, (b - a)$$

$$= 100\, g$$

i.e., the Horizontal Distance is equal to 100 times the staff intercept.

Now let us assume that

α/

$\alpha = 1^{\circ}08'45"$: $\tan\alpha = 0.02$

and $\beta = 1^{\circ}43'06"$: $\tan\beta = 0.03$

then the vertical height a

$= D \tan\alpha = 100 (b - a) \tan\alpha$

$= 100 (b - a) \times .02$

$= 2 (b - a)$

$= 2 g$

Similarly b $= 3 (b - a)$

$= 3 g$

i.e., the Vertical Heights a and b are 2 and 3 times the staff intercept respectively.

In the Watts-Szepessy tacheometer, Fig. 14,

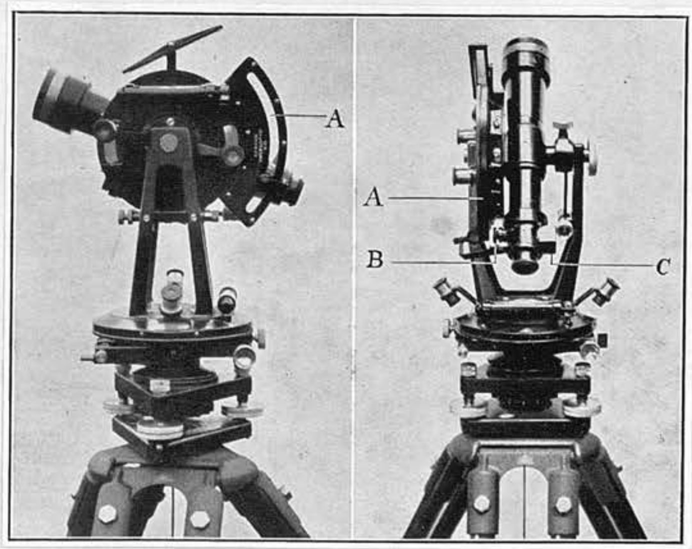

Fig. 14. Watts-Szepessy Tacheometer.

a glass arc A, with a tangent scale marked onit, is fixed to the vertical circle guard of a standard theodolite, and the graduations on the scale are reflected into the field of view of the eyepiece by means of a magnifying microscope B and reflecting prism situated inside the guard C. The tangent scale is numbered and marked at every 0.01, and marked at every 0.005 to the nearest second of arc.

In/

In sighting the staff therefore, the horizontal diaphragm line is brought into coincidence with one of the tangent scale marks and the staff is read opposite this mark to determine the height of the line of collimation, and also opposite the graduation marks immediately above and below to determine the staff intercept for an angle whose tangent is 0.01. Fig. 15 depicts the appearance in

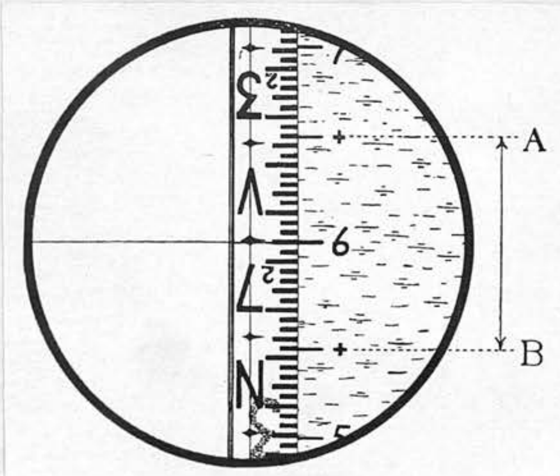

Fig. 15. Field of view in eye-piece of Watts-Szepessy Tacheometer.
Horizontal Distance = 100 x AB
= 100 x .441 = 44.1 ft.
Vertical Height = 6 x AB
= 6 x .441 = 2.646 ft.

the field of view of the telescope, and in this example

the horizontal distance = 100 x AB = 100 x .441
= 44. 1 ft., and

the vertical height = 6 x AB = 6 x .441
= 2.646 ft.

The first tests with the instrument gave a minus error of approximately 1 in 200 and the magnification of the tangent scale divisions had to be increased by rotating the microscope B. The instrument was then tested fully and the results are shown graphically in Fig. 16.

The mean curve representing the Average Error rises sharply to a maximum value of 1/300 at 125 ft. and thereafter decreases steadily.

The/

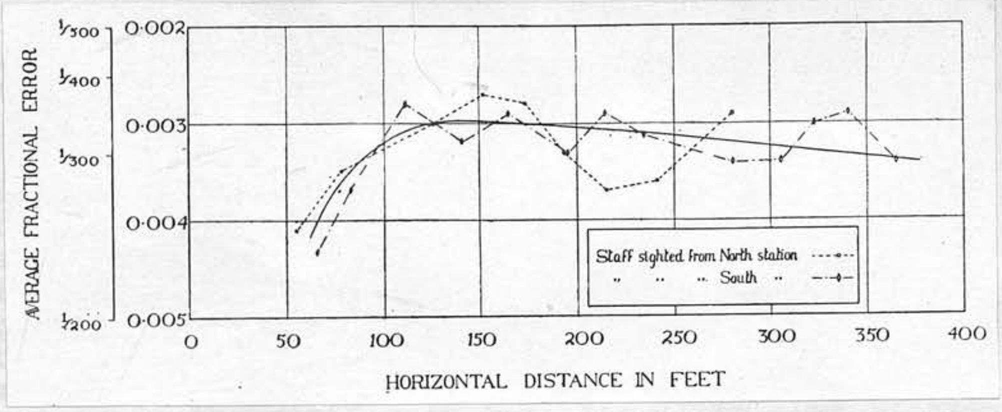

Fig. 16. Average Error curve for Watts-Szepessy Tacheometer.

The reason for the rapid fall in the accuracy, apart from reasons already discussed, is due principally to defects in the graduation of the tangential scale. The majority of the readings were taken with approximately horizontal sights, i.e., with scale readings within the limits of ± 5, and there was a decided negative error opposite the -1 graduation, and also to a lesser extent opposite the -3 graduation as shown in the following Table. The effect of these erroneous graduations has reduced the accuracy of the Szepessy tacheometer below average theodolite tacheometry.

Horizontal Distance in feet.	Scale Reading	Average Error in feet	Average Fractional Error.	
177	0	± 0.39	± .0022	± 1/450
	-1	-* 0.82	-*.0046	-*1/210
	-2	± 0.36	± .0020	± 1/490
	-3	-* 0.53	-*.0030	-*1/330

Atmospheric conditions of course affect optical methods of measuring distances to a great extent, and it must be stated here that the tests on the Szepessy tacheometer were carried out in September, October and November, and the weather was not ideal for tacheometry. Taking everything into consideration/

* All errors have minus sign.

consideration, however, I would hesitate to place the average fractional error of a single observation with the Watts-Szepessy tacheometer lower than 1/400.

In the previous description sights were stated to be made to points on the staff, such that the difference between the tangents of the vertical angles was .01. Owing to the construction of the instrument, however, sights could be taken to points on the staff subtending a larger angle, and the multiplication factor would be reduced propor: :tionally. For example, if sights are made to points on the staff such that the difference between the tangents of the vertical angles is .05 or 5/100, then the multiplication factor will be 100/5 or 20, and as the reading error will only be multiplied by 20 instead of 100, the accuracy will be considerably increased. Other multiplication factors could, of course, be used, but a limit is set by the length of the staff. Thus with a 16 ft. staff at a distance of 120 ft. it would be possible to set the horizontal diaphragm line opposite, say, the -2 graduation mark, and then opposite the +8 graduation mark, and get the staff intercept for the angle whose tangent is 0.1. The horizontal distance would then be 10 times the staff intercept. It is obvious, however, that a multiplication factor of10 could not be used with a 16 ft. staff for distances greater than 160 ft.

The results of the first tests on the Szepessy Tacheometer suggest that the magnifying microscope requires to be readjusted slightly for different observers to give the required spacing of the graduations in the field of view of the eyepiece. If several observers are using the same instrument this readjusting becomes impractical and consequently a personal reading error is introduced. With the second method of using the Szepessy tacheometer, however, this personal error is eliminated because only one ray of light, transmitted along the optical axis of the microscope, is used in place of the two diverging rays required by the standard method of using the instrument.

Horizontal Distance in feet.	No. of Scale Divisions used	Multiplication Factor	Average Fractional Error	
110	1	100	±0.0030	±1/350
	2	50	±0.0016	±1/600
	5	20	±0.0008	±1/1250

The/

The above Table shows the mean results of three groups of sixty sights taken with multiplication factors of 100, 50 and 20, and it is apparent that a greater accuracy can be obtained by reducing the multiplication factor in this way.

The extra time taken in setting the horizontal diaphragm line opposite a second graduation mark is practically negligible, and no extra work is involved in determining the vertical height.

(2) The Jeffcott Tacheometer.-

The Jeffcott tacheometer, Fig. 17, was designed by Professor H.H. Jeffcott, and as it is also manufactured by a British firm it is essentially a British tacheometer, and in this respect it is unique for the majority of self-reducing tacheometers were either invented abroad or are improvements on foreign designs.

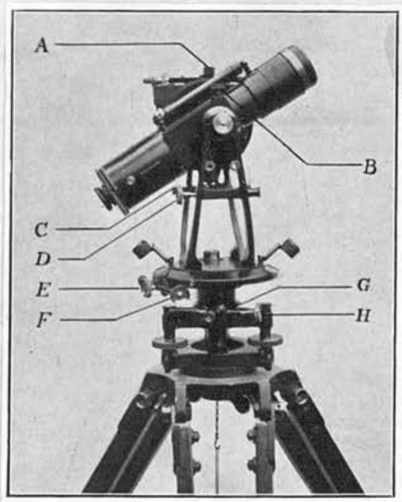

Fig. 17. Jeffcott Tacheometer.

Several publications have appeared during the past seventeen years on the theory and construction of this instrument, the principal one being "A Direct-Reading Tacheometer"*, written by the inventor himself; and as the Jeffcott tacheometer is already widelyknown in the surveying world only a brief description of the instrument need be given.

The/

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* Transactions of the Institution of Civil Engineers of Ireland, 1915.

The horizontal circle is 5 ins. in diameter and is provided with two verniers, one viewed from the eye-piece side of the instrument reading to 1 minute, the other diametrically opposite reading to 20 seconds. By placing the one-minute vernier almost under the eyepiece end of the telescope instead of at right angles to this position to conform with the standard design of theodolites, it is possible for the observer to make all the necessary field observations from one position. The time saved by not having to walk round the instrument is quite considerable in the course of a day's work, and the simple alteration of the position of the verniers is one of the note:worthy details in the constructionof the Jeffcott tacheometer.

The horizontal movement of the telescope is controlled by the body clamp G, and its tangent screw H, also by the plate clamp E and tangent screw F.

A 5-inch vertical circle fitted with two verniers reading to 20 seconds can be provided, but, as this instrument is essentiallya self-reducing tacheometer, the vertical circle is not a necessity and it is therefore omitted on the standard model.

The vertical movement of the telescope is controlled by the clamp B and tangent screw D.

The horizontal plate clamp and the vertical movement clamp, as well as the telescope focusing screw, are all conveniently arranged on the right-hand side of the instrument when viewed from the eye-piece end. This arrangement allows the observer to manipulate the instrument with his right hand only, leaving his left hand free to hold the field book, and is a further example of the forethought and attention paid to constructional details.

The telescope is anallatic; magnification 18 diameters; objective 1.65 ins.; distance stadia interval 100; height stadia interval 10; stadia correction nil.

A central collimation point and two stadia points are provided in place of the customary glass diaphragm. The central collimation point is fixed and the distance and height stadia points are mounted on the ends of two levers pivotted on the body of the telescope. The other ends ofthe levers are held in contact with two cams by means of light springs, as shown in Fig.18. The shape of the cams is so arranged that as the telescope is tilted the stadia points move so that the staff intercept between the collimation/

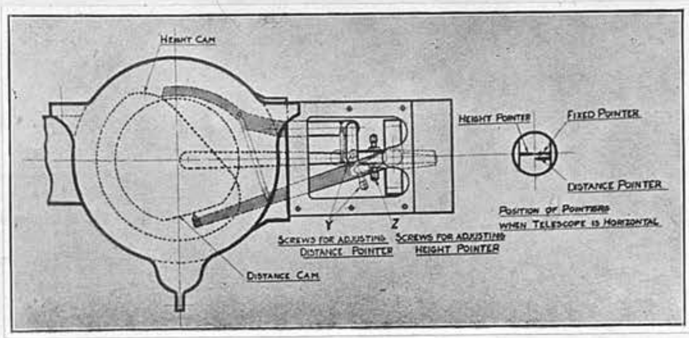

Fig. 18. Cam and Lever Arrangement in Jeffcott Tacheometer.

collimation point and the distance stadia point multiplied by 100 gives the horizontal distance; and the staff intercept between the collimation point and the height stadia point multiplied by 10 gives the difference in height between the axis of the teles: :cope and the collimation point sighted to on the staff. The cams are machined to within one-thousandth part of an inch of their theoretical shape, and, in order to preserve the mechanical accuracy as far as possible, the cam and lever arrangement is totally enclosed to prevent dirt adhering to the face of the cams. Furthermore, the cams are made of bell metal or rustless steel to withstand the effects of mois: :ture.

A spirit level is attached to the cams, and if that level is not horizontal when the instrument is levelled the necessary adjustment can be made by means of a fine adjusting screw C, the bubble being seen from the eye-piece end of the telescope through the prism A.

The results of the tests on the Jeffcott tacheometer are shown graphically in Fig. 19, the mean value of the Average Error being 1/600.

The tests were carried out during the months of July and August, and ideal weather conditions prevailed. The above figure can not be accepted, therefore, as a mean value for working conditions throughout the year, but, from past experience of tacheometrical surveying, I have no hesitation in stating that the precision of the Jeffcott self-reducing tacheometer is of the same order as that of good theodolite tacheometry with a vertical staff, i.e., an Average Error of 1/450 to 1/500 is obtainable.

The/

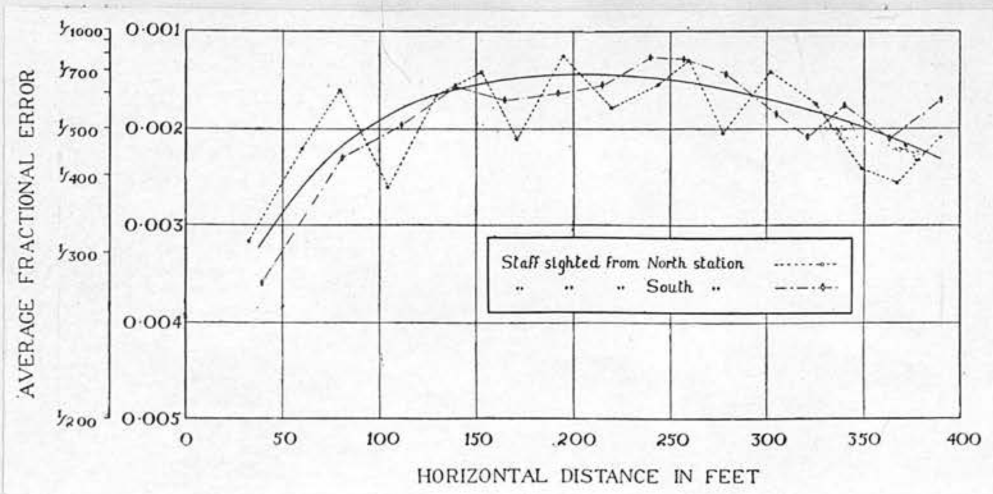

Fig. 19. Average Error curve for Jeffcott Tacheometer.

The convenient and serviceable construction of the instrument has already been mentioned, and it gave entire satisfaction except inone point, namely, that a certain amount of eyestrain was experienced after using the instrument continuously for some time. During the tests readings were taken for an hour at a time with practically no rest interval between read:ings, and the conditions were therefore more rigorous than normal working conditions; nevertheless, the eye-strain is present, and it is a decided disadvantage. It is caused by the stadia points rocking on the ends of the levers and swinging up and down in a circular path, thereby deviating from the principal focus of the eyepiece.

(3) The Bosshardt-Zeiss Reduction Tacheometer.-

This remarkable instrument, Fig. 20, was designed by Mr. R. Bosshardt, and is manufactured by Messrs Carl Zeiss of Jena. It was designed essentially as a precise tacheometer, but it can also be used as an ordinary theodolite.

The horizontal circle, 10.5 cms., (4.13") in diameter, and the vertical circle, 8 cms. (3.15") in diameter, are totally enclosed, and all readings are brought optically to the reading microscope, O, placed convenientlyalongside the telescope. Only one circle is seen at a time in the reading microscope, the other being brought into view by turning the stud, K.

The horizontal circle is graduated in degrees and thirds of a degree, and a micro-scale placed in the field of view reads to the nearest two minutes. Diametrically/

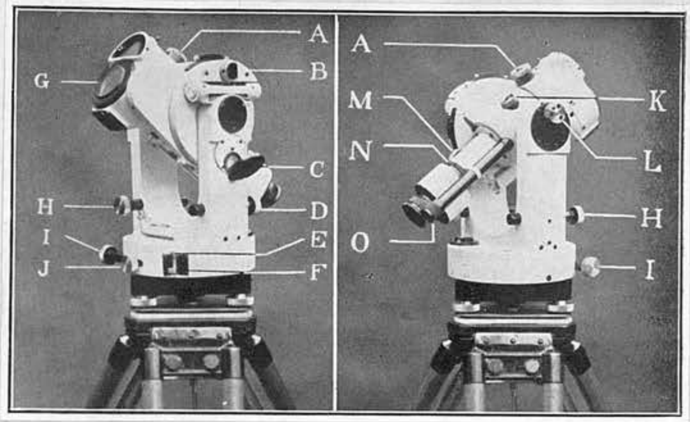

Fig. 20. Bosshardt-Zeiss Tacheometer.

Diametrically opposite, parts of the horizontal circle are seen, similar to the usual optical micrometer theodolite.

The Bosshardt tacheometer follows the principle of the Direction theodolite, and not the usual Repeating theodolite, inasmuch as there is but one vertical spindle and one clamp, J, and tangent screw I controlling the horizontal movement of the telescope. There is, however, a spring catch, F, to enable angles to be read by repetition, or to facilitate taking a back-sight at zero or any other desired reading. When the repetition lever F is depressed, the horizontal circle is clamped to the alidade, the lever being held in position by the locking catch E. If the body clamp J is then loosened, the telescope can be rotated and the reading of the horizontal circle remains unaltered. To release the circle it is only necessary to press down the locking catch E.

The vertical circle is also graduated in degrees and thirds of a degree and provided with a two-minute micro-scale. Only one part of the vertical circle is shown, however, the other half of the image being the tangent of the angle of inclination, as represented in Fig. 21.

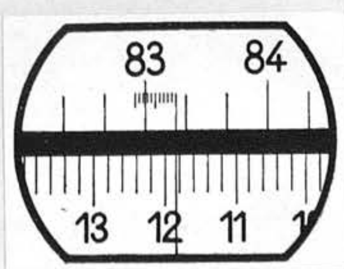

Fig. 21 Field of view in reading microscope of Bosshardt-Zeiss Tacheometer. Zenith Angle =83°15'. Tangent of Angle of Altitude = 0.1185.

The movement of the telescope in a vertical plane is controlled by the clamp L, and the tangent screw H.

The reflector C illuminates both circles.

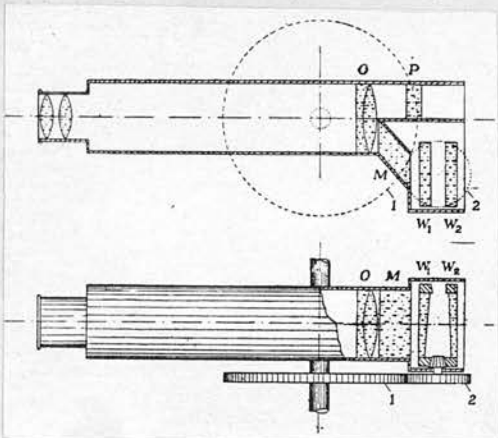

Fig. 22. Sections through telescope of Bosshardt-Zeiss Tacheometer.

The internal focussing telescope, with a magnification of twenty-four diameters is the unique part of the instrument. It is essentially a double image range finder and is an improvement on the Richard range finder. In the latter a glass wedge is placed in front of one half of the objective and in this way two superimposed images of the same object are seen, the distance between the images being proportional to the distance between the range finder and the object sighted. In the Bosshardt tacheometer there are two similar achromatised wedges, W_1 and W_2 in Fig. 22, and the rays of light pass through the wedges and are deflected by the rhomboid prism, M, into the objective, O, of the telescope. Furthermore, the two images are separated, one occupying the top half of the field of view, and the other the bottom half. This eliminates all confusion of the images and reduces eye strain to a minimum.

The automatic reduction of all distances to the horizontal is also brought about by the two wedges. The magnitude of the refracting angle of the wedges depends on the multiplication constant desired. For a constant of 1:100, the angle of parallax required is 63.6 minutes, and when the two wedges lie with their principal sections horizontal the deviation is accordingly 63.6 minutes. If the two wedges are rotated in opposite directions through 90 degrees, however/

however, the deviation becomes nil. The wedges are therefore mounted on ball bearings, and as the telescope is tilted they are made to move in opposite directions by means of the gear wheels 1 and 2 in Fig. 22. Gear-wheel No. 1 is mounted on the horizontal trunnion axis of the telescope and remains stationary. When the telescope is tilted No. 2 gear-wheel revolves round No. 1 and controls the movement of the wedges so that they move in opposite directions through the same angle as the angle of elevation of the telescope, thereby automatically reducing the inclined lengths to the horizontal distances.

The reduction affects the distance between the staff and the vertex of the angle of parallax, but does not affect the distance from the vertex (between the wedges) to the horizontal axis of the telescope, a distance of 88 mms. The error so induced varies for angle of elevation and depression as the wedges are situated about 22 mms. below the horizontal axis of the telescope. As the distance affected is a constant, and as the correction is proportional to the tilt of the telescope, the correction is marked on the outside of the vertical circle guard, and can be applied when necessary.

A special horizontal staff, Figs. 23 and 24,

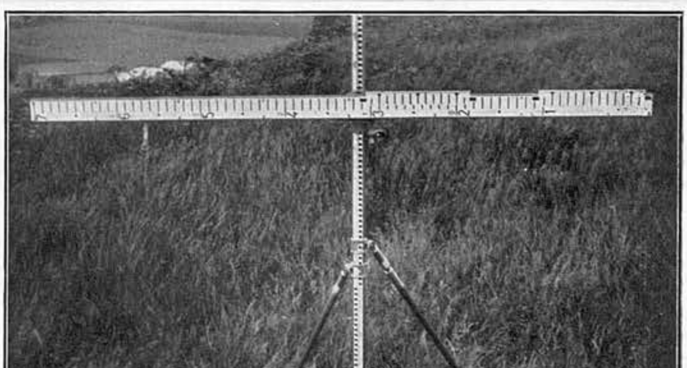

Fig. 23. Horizontal Staff for
Bosshardt-Zeiss Tacheometer.

is used with the Bosshardt-Zeiss tacheometer, a horizontal staff being essential when a high standard of accuracy is required in order to eliminate errors caused by differential refraction and secondary aberration. The staff is of an open reading type and canbe obtained graduated in feet or metres. The 'foot'/

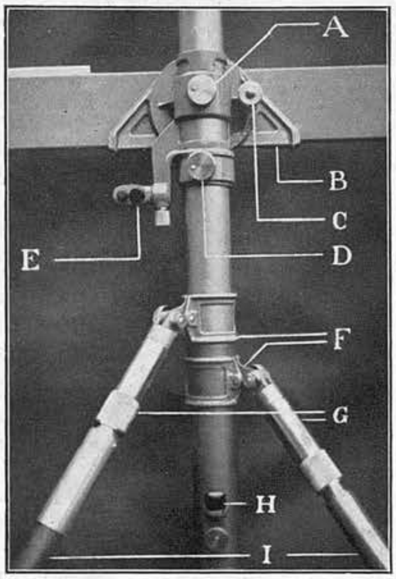

Fig. 24. Staff details.

'foot' staff is seven feet long and is divided into tenths of a foot. Two fixed verniers are provided, one between the zero end of the staff and the 1-foot mark, and the other nearer the centre of the staff between the 2 and 3-foot marks. The former is used when taking sights of 250 ft. or over, and the latter for shorter sights. The large figures on the staff apply to the vernier at the zero end and the small figures to the inner vernier.

Theoretically the two principal rays of light from the instrument to the staff should be equidistant on either side of a line joining the instrument to the staff support. The error induced by an asymmetrical reading is small enough to be neglected, however, if the two rays are approximately symmetrical, and it is for this purpose that two verniers are provided - the inner vernier for short sights and the outer one for long sights.

The staff support consists of a metal tube with a flat graduated face to facilitate setting the staff to any desired height. The centre of the graduated face is marked with a straight white line and this serves as a centre line to set the vertical web of the diaphragm on when measuring the horizontal angle.

Two strutts, I, are provided to steady the staff. These can be moved up and down, and can be clamped to the tubular support by the clamps, F.

A/

A pill-box level H is attached to the back of the support.

In setting up the staff, the foot of the support is put on the mark, the support is held vertically, and the struts pressed into the ground. By means of the fine adjusting extension screws G on the struts the staff holder can then bring the bubble H to the centre of the level and the support is then vertically above the mark. The staff is inserted in the staff-holder B and secured in place by a clamp. By loosening the stud C the staff can be swung up into the horizontal position, when it is automatically locked. The small turntable device which permits the staff to be swung from a vertical to a horizontal position, and vice versa, is very convenient, as the staff can be turned down to lie alongside its support when being carried about.

The staff can be set at any desired height above the mark and held there by means of the clamp D. With the clamp A loose, the staff is now rotated until it is at right angles to the line of sight. The staff-holder adjusts this by sighting the instrument through the diopter sight, E, and the clamp, A, is then tightened.

The accuracy of setting the staff at right angles to the line of sight can be checked by the instrument man by sighting the collimator on the diopter. If the staff is correctly aligned a vertical white line is visible in the collimator; if incorrect, no line is visible. See Figs. 23 and 25.

A deflection of the staff from the correct position produces a positive error as follows:

Deflection of staff $= \alpha$ Degrees	Error $= +\left(\frac{1}{\cos \alpha} - 1\right)$	
½	.00004	1/25,000
¾	.00009	1/11,100
1	.00015	1/ 6,670
2	.00061	1/1,640
3	.00137	1/ 730
4	.00244	1/ 410
5	.00382	1/ 260
6	.00551	1/ 180
7	.00751	1/ 135

The/

The collimator is accordingly adjusted so that the white line is visible when the staff is aligned to within half a degree of its true position. When the deflection exceeds three-quarters of a degree the collimator appears black, and between those limits the image is crescent-shaped.

The method of taking a tacheometric reading is as follows: Centre and level the instrument, and verify the level adjustment of the wedges. To facilitate the latter, a spirit bubble is attached to the stationary gear wheel operating the wedges, and the position of the bubble can be seen in the prism B from the eyepiece end of the instrument. It is most important that the bubble should always be in the centre of its run, and a fine-setting screw D is provided to adjust this when necessary.

The telescope is directed on the staff, focussed by means of the focusing screw, M, and adjusted in height so that the horizontal line dividing the two images appears to be about the centre of the staff. The staff then appears to be divided in two as in Fig. 25; the top half is seen directly through the objective, the bottom half being displaced laterally by the wedges.

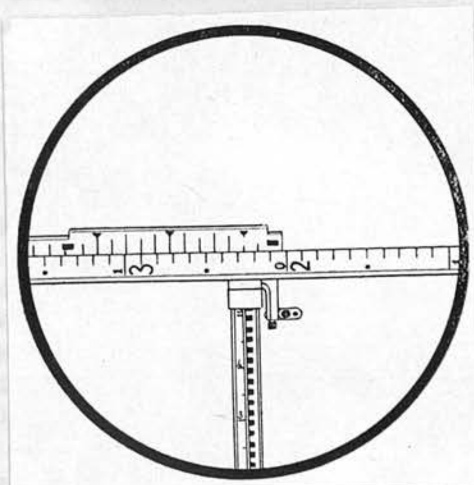

Fig. 25. Field of view in eyepiece of Bosshardt-Zeiss Tacheometer, after bringing graduations into coincidence. Horizontal distance = 227 ft. by vernier, + .xy by drum scale = 227.xy ft.

As the apparent displacement is equal to one-hundredth/

one-hundredth part of the horizontal distance, it is only necessary to read the distance the vernier appears to have moved along the staff. The vernier reads to one-hundredth part of a foot, thereby giving the horizontal distance to the nearest foot. To continue the reading to the second place of decimals, the nearest staff and vernier divisions are brought into coincidence by means of a rocking plate in front of the top half of the objective: a method now commonly used in precise levels. The rocking plate, P, (Fig. 22), is controlled by the drumscrew A (Fig. 20) and a lever transmission gear. The side of the drumscrew is graduated into ten divisions so that the first decimal place is read directly and the second place estimated. For example, the horizontal distance registered in Fig. 25 is 227 ft. by the staff and vernier, plus .xy by the drumscrew scale.

The addition of the rocking plate adds considerably to the efficiency of the instrument, principally because of the increased accuracy obtained by reading the drumscrew instead of estimating beyond the reading of the vernier, but also because the parallel plate can be displaced, re-set, and the drum screw re-read as often as desired, thereby obtaining a series of independant measurements.

After the horizontal distance is noted, a glance is taken through the telescope to see that the vertical web on the diaphragm is over the white line on the support. The eye is moved over to the reading microscope O and the horizontal angle is noted. The stud K is now rotated, bringing the vertical circle into view, and the tangent of the vertical angle is noted. See Fig. 21.

The vertical angle need not be read as the horizontal distance multiplied by the tangent of the angle of inclination gives the difference of height between the horizontal axis of the instrument and the point sighted to on the staff.

In determining the difference of elevation, it is advisable to set the bottom of the staff at a height equal to the height of the instrument, and, before reading the tangent of the angle of inclination, to bring the horizontal line dividing the two images down to the foot of the staff. In this way there is no uncertainty in sighting to the required height on the staff.

Before giving the results of the tests made on this instrument there are further points of refinement about the Bosshardt Tacheometer and staff which merit special note.

In/

In the first place, the zero points of the verniers are displaced 0.24 mms. inwards to allow for three corrections. First, there is the 88 mms. between the vertex of the angle of parallax and the horizontal axis of the instrument. Secondly, owing to the construction of the staff, the face of the staff is 36 mms. in front of the centre of the staff support. Thirdly, when the drumscrew is at zero, the sighting line is shifted 1 mm. to one side to allow for the parallel plate being inclined at an angle to the line of sight. To compensate for the abwve factors, the zero points of the verniers are displaced $\frac{88 + 36}{100} - 1 = 0.24$ mms. inwards and in this way all three corrections are applied automatic: :ally.

In order to overcome the mal effect of changing temperatures on the telescope objective, Mr. Bosshardt has selected the optical glass for the wedges and the material for the staff graduations, such that any change in temperature produces a similar effect in both, and consequently the multiplication constant remains correct.

Since the advent of the optical micrometer theodolite, investigations on sighting errors have shown that one observer takes a different pointing and reading from another observer, but the difference between the two observations is approximately of the same sign and magnitude at all times. Similarly, in bringing the staff and vernier divisions into coincidence the same systematic personal errors are made. To allow for the automatic correction of these errors the multiplication factor can be altered by rotating the glass disc G in front of the wedges, and the addition constant can be varied by rotating the drum-screw A relative to its index mark.

Furthermore, if the drum-screw is always turned in the same direction when bringing the marks into coincidence, steadier and more accurate readings are obtained. Attention should also be paid to the two pairs of marks on either side of the coinciding pair as the error of coincidence is twice as great there and therefore more easily seen.

The glass disc G is slightly tapered and its rotation causes a variation of the deflection angle and consequently the multiplication factor is altered. A scale engraved on the face of the disc and an index mark on the barrel of the telescope indicate the amount turned through and this can be noted for future reference.

In/

telescope in a vertical position must therefore be clamped tightly to prevent the objective end of the telescope from swinging down, and I feel that this clamp is called upon to do an undue amount of work. Instead of placing the responsibility on a clamp of holding an unbalanced telescope in position, I would prefer to balance the telescope about its horizontal axis by means of a small cylindrical counterpoise weight round the eyepiece end, and so relieve the clamp of the extra duty.*

The Bosshardt wedge arrangement can be cut out and the instrument used as an ordinary theodolite by simply turning the sleeve N on the barrel of the telescope. The diaphragm then shows a vertical and a horizontal line and two horizontal stadia lines with a stadia interval of 1:100 so that tacheometric measurements can be taken with an ordinary vertical staff.

There is no doubt whatsoever that measure::ments with the Bosshardt tacheometer are superior in accuracy to chain measurements, and the following table, taken from a short pamphlet on "The Economic Aspects of the Optical Measurement of Distances in Surveying"** by R. Bosshardt, shows the economic superiority also. The data embodied in the Table was taken from the Swiss tariff for land-registration surveys, prepared by the Swiss Federal Survey Depart::ment and the Swiss Surveyors Association, and shows a 40% saving in cost in favour of tacheometry.

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* This matter has now been rectified.

**Geo. 78(e), Carl Zeiss.

	Percentage of cost of survey by the chain-and-offset method.		Percentage of economy of total survey-costs by the use of the optical method
	Chain-and-offset method.	Optical method.	
A. Traversing:			
Marking out the traverse...	7	7	0
Measuring traverse angles.	12	12	0
Measuring traverse lines..	15	1.5	13.5
Calculation of co-ordin: :ates and levels, plotting, etc......................	16	16	0
B. Detail-survey:			
Detail-survey proper......	29	12.8	16.2
Checking the traverse.....	14	11.2	2.8
Making sketch-plans.......	7	7	0
	100	67.5	32.5
Add economy resulting from 15 per cent. reduction in the number of traverse-points required = 7.5 per cent. of the total cost of survey...........			7.5
Total percentage of economy in favour of optical method.........................			40.0

THE PRECISION OF TACHEOMETRICAL LEVELLING:

A PRACTICAL AND THEORETICAL INVESTIGATION.

THE PRECISION OF TACHEOMETRICAL LEVELLING: A PRACTICAL AND THEORETICAL INVESTIGATION.

Although tacheometry is accepted as a speedy and satisfactory method of levelling when great accuracy is not required, the actual accuracy of the method, and the low standard of accuracy obtained when inclined sights are taken, are not generally realised. In order to obtain definite values for the precision of tacheometrical levelling, observations were made to points at different distances and the levels of the observed points calculated, using the fundamental equations of tacheometry as applied to inclined sights.

Vertical Staff:

$$H = 100\ g \sin\alpha \cos\alpha = \frac{G}{2} \sin 2\alpha \ldots\ldots(13)$$

Horizontal Staff:

$$H = 100\ g \sin\alpha = G \sin\alpha \ldots\ldots\ldots(14)$$

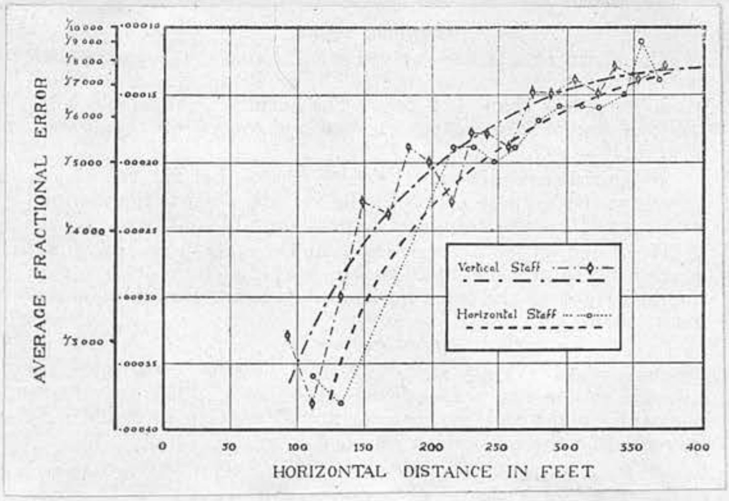
where H = the vertical height
g = the staff intercept
G = the inclined length = 100 g
α = the angle of inclination.

Two theodolites were used - one of standard design fitted with verniers reading to 20 seconds, the other, an optical micrometer instrument reading to single seconds. The vertical angles were read on each theodolite to the nearest minute only, therefore the average error of reading with both theodolites may be taken as similar and equal to approximately $\pm$ 20 seconds.

Thirty independant sights were made to each point and the Average Fractional Error of each group of sights was calculated, the standard of reference being the mean heights obtained from exacting measurements made with a dumpy level.

The results are shown graphically in Fig. 27, where the Horizontal Distances are plotted as abscissae against the Average Fractional Errors as ordinates.

The /



<u>Fig. 27.</u> Average Error curves for Vertical Heights with vertical and horizontal staves.

The accuracy obtained with the vertical staff under the conditions of the above tests was a little higher than with the horizontal staff, but both curves have similar characteristics, <u>i.e.</u>, the accuracy increases fairly rapidly as the distance increases up to 250 feet or thereabouts where the mean value for the average fractional error is 1/5000 and then con: :tinues to increase at a somewhat modified rate as the distance increases up to the practical limit of reading the staff.

<u>The Effects of Refractinn.-</u>

The effect of refraction on the lines of sight can be shown by arranging the sights in groups according to the height sighted to on the staff, and drawing up the Frequency Curves for these groups. The curves are shown in Fig. 28, and although they are only approximate, owing to the limited number of sights in each group and because the sights are of different lengths, it is nevertheless apparent that with low sights there is a large negative cumulative error, and that the error decreases as the height of the point sighted to increases up to 4 feet or so when the axis of the curve approaches very closelyto the normal <u>Y</u> axis of the graph.

Fig. 29 depicts the path of the line of collimation ray of light whenit traverses the stratum of/

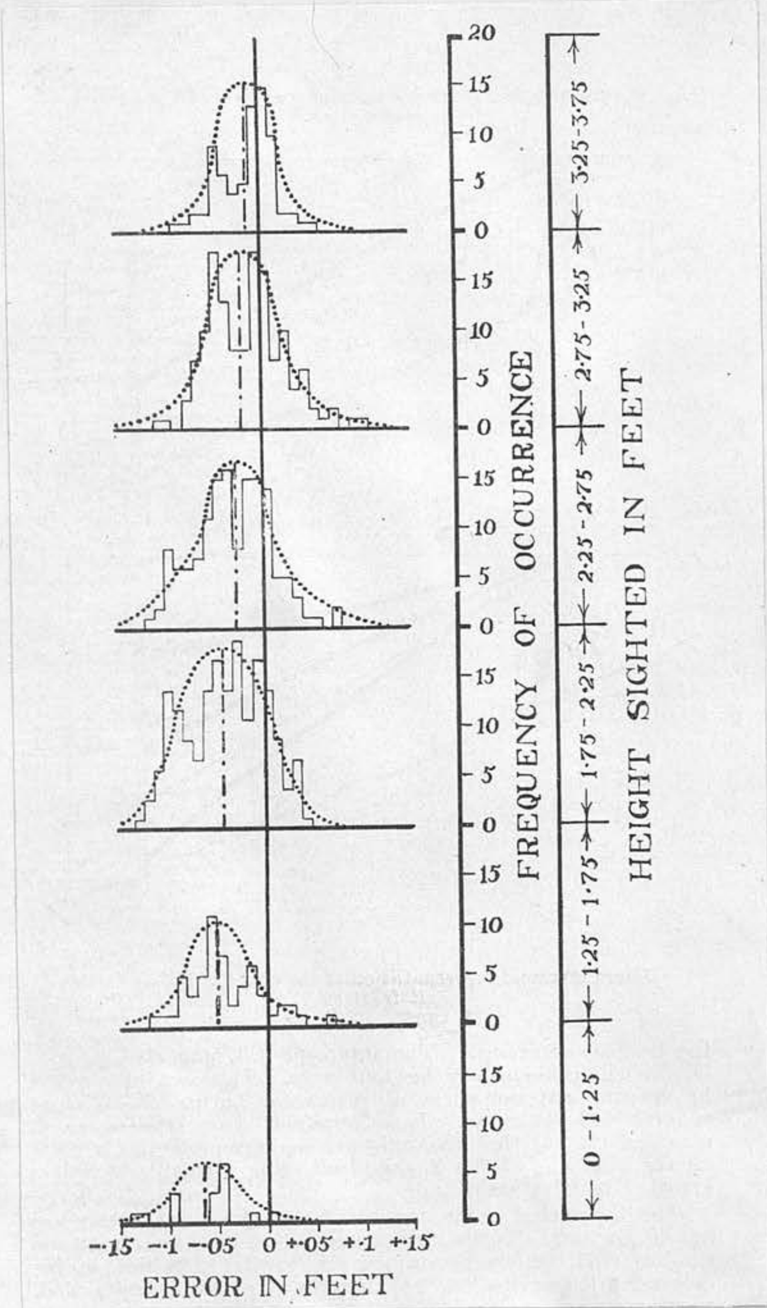


Fig. 28. Frequency curves, grouped according to the height sighted on the staff.

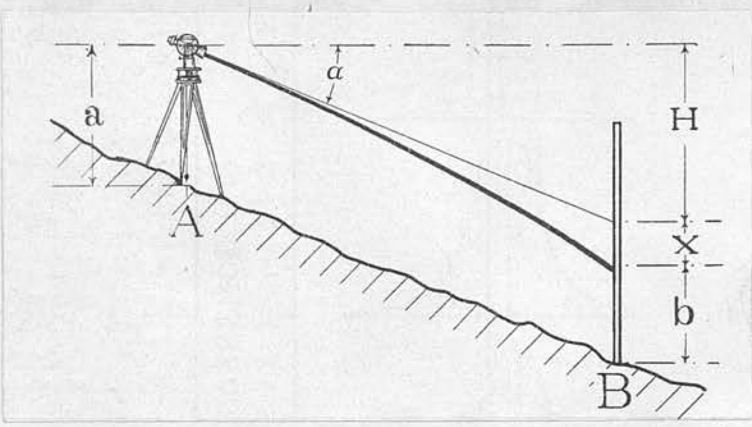


Fig. 29. Diagrammatical representation of the effect of Terrestrial Refraction.

of air lying within 4 ft. of the ground.

The calculated difference of elevation between A and B = a - (H + b) and

The true difference of elevation between A and B = a - (H + b + x),

i.e., the portion x is omitted in practice and constitutes the negative terrestrial refraction error.

The paths of the three principal rays of light, when using a vertical staff, are represented diagrammatically in Fig. 30. The lowest line of sight is refracted downwards more than the other two,

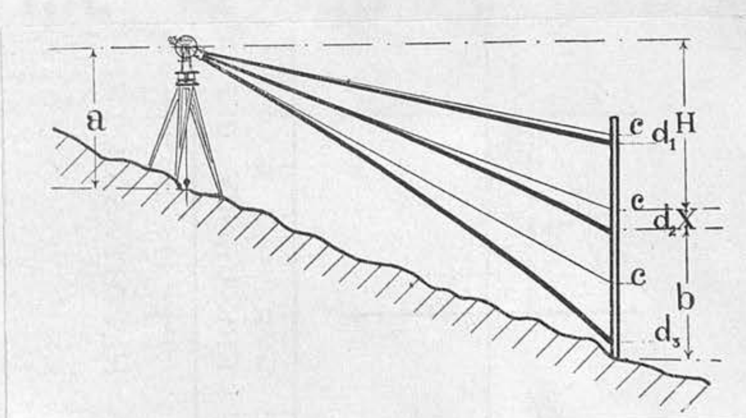


Fig. 30. Diagrammatical representation of the effect of Differential Refraction.

and therefore the bottom intercept d_2d_3 is greater than/

than the top intercept d_1d_2, a statement which agrees with the field observations.* The intercept d_1d_3 and the inclined length will consequently be too large. This can be verified by drawing out the Curve of Errors for Horizontal Distances of over 150 ft. or so. It is apparent from the curve so obtained, Fig. 31, that the axis of the asymptote lies a little on the plus side of the Y axis, indicating a small cumulative error of positive magnitude.

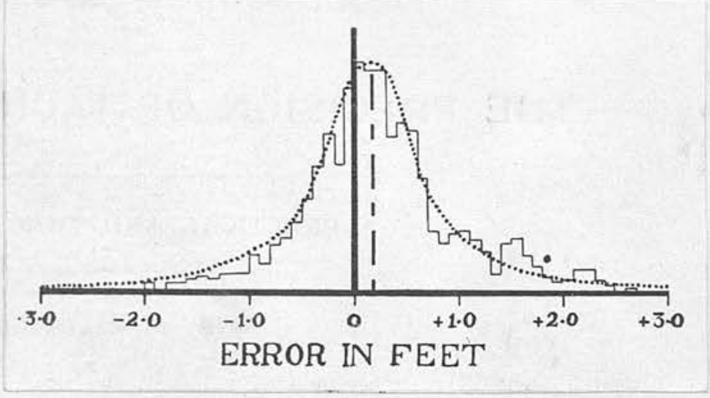


Fig. 31. Frequency curve for 1360 sights of over 150 ft. in length.

The theoretical value for the accuracy of tacheometrical levelling with the horizontal staff is higher than that with the vertical staff, chiefly because of the greater precision in the determination of the inclined lengths. In this investigation, however, the height of the horizontal staff was always in the neighbourhood of 20 inches, whereas with the vertical staff the height sighted on the staff varied from 1 to 4 feet, the average height being between $2\frac{1}{2}$ and 3 feet. The results of the observations show that the greater refraction error with low sights was sufficient to reduce the overall accuracy of the horizontal staff below that of the vertical staff.

It is obvious, therefore, that in tacheometrical levelling, either with a vertical or a horizontal staff, all sights should be kept more than 4 feet from the surface of the ground to eliminate terrestrial refraction errors as much as possible.

Inclined Sights.-

The above results are derived from actual observations/

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* See "A Preliminary Investigation of the Accuracy of Tacheometry", p. 24.

error of reading is the same for an inclined as for a level sight of the same length, i.e., = g_1^o .

From Equation (14):

$$H_1 = \pm\sqrt{\left[(100\ g_1^o\ \sin\alpha)^2 + (100\ g\ \cos\alpha.\alpha_1)^2\right]}$$

$$\frac{H_1}{D} = \pm\sqrt{\left[\left(\frac{g_1^o}{g}\right)^2 \tan^2\alpha + \alpha_1^2\right]} \ldots\ldots\ldots\ldots(16)$$

Equations 15 and 16 are represented graphically in Fig. 32 for inclinations up to $\pm$ 25° from the horizontal, assuming

$$\alpha_1 = \pm 20'' \ (10^{-5} \times 9.69 \text{ radians})$$

and

$$\left(\frac{g_1^o}{g}\right) = \frac{1}{550} \text{ for the horizontal staff and}$$

$\frac{1}{450}$ for the vertical staff, these being the values obtained for $\left(\frac{g_1^o}{g}\right)$ from previous tests.*

It is apparent from the graph that the

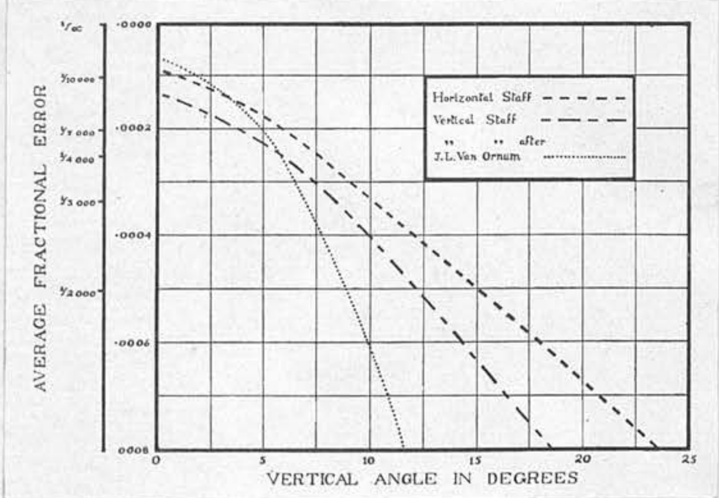

Fig. 32. Average Error curves for Vertical Heights with Inclined Sights.

horizontal staff is more accurate than the vertical staff/

* "A Preliminary Investigation of the Accuracy of Tacheometry", page 32.

staff, and, in fact, the difference is greater than indicated because the factor $\frac{(g^o)}{\frac{(1)}{(g)}}$ decreases as the angle of elevation increases when using a vertical staff, and remains practically constant when using a horizontal staff.* With both staffs, however, the accuracy decreases so rapidly as the angle of inclination increases, that for vertical angles of over 10 or 15 degrees, this method of levelling is almost too inaccurate for most practical purposes.

J.L. Van Ornum analysed the errors obtained in the Mexico-United States boundary survey,** and the following excerpt deals with tacheometrical levelling:

> "For the purpose of determining the error of closure in elevation relative to the degree of vertical angle, the author grouped the lines according to the size of those angles and plotted the corresponding errors as ordinates".
>
> "The equation of the curve so formed, where α represents the average vertical angle in degrees is..................expressed in feet per mile

$$e = \frac{30\ (\alpha \tan \alpha) + 8}{19}\ ." \qquad \text{...........(17)}$$

Equation 17, expressed as a fractional error, is also represented graphically in Fig. 32. The curve is similar to the theoretical curve for the vertical staff and as the accuracy decreases even more rapidly than the theoretical value it is obvious that tacheometrical levelling is, at the best, only an approximate method when inclined sights are used.

<u>Self-reducing Tacheometers.-</u>

The Jeffcott and the Bosshardt-Zeiss self-reducing tacheometers were tested in a similar manner and the Average Error Curves for these two instruments together with the mean Average Error Curve for tacheometrical levelling with a vertical staff are shown in Fig. 33.

The curve representing the Average Fractional Error of the Jeffcott tacheometer is almost identical with that of theodolite tacheometry with a vertical/

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* *Ibid*. See Fig. 12, p. 34.

**"Topography on the Survey of the Mexico-United States Boundary", *Proc. Am. Soc. Civ. Engs.*, 1895.

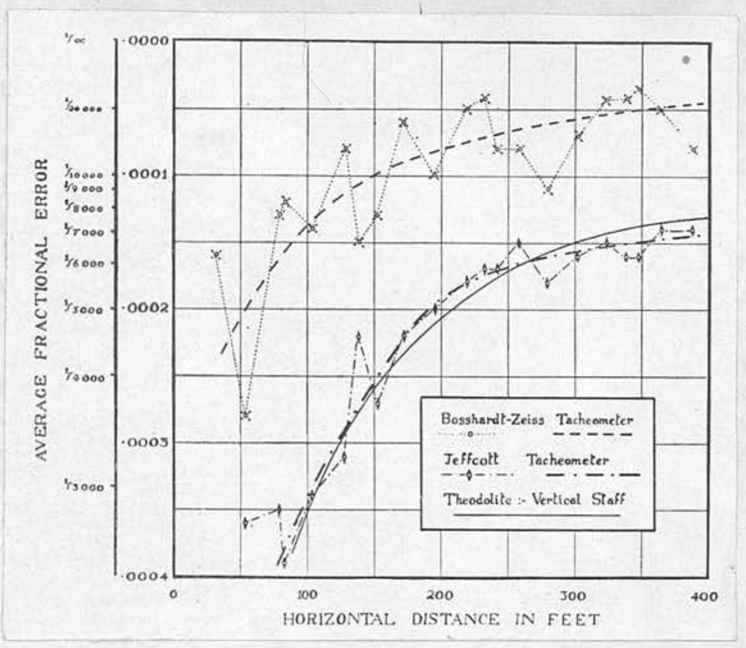

Fig. 33. Average Error curves for Vertical Heights with the Jeffcott and Bosshardt-Zeiss self-reducing tacheometers.

vertical staff. One would expect that levelling with the Jeffcott tacheometer would be more accurate than with theodolite tacheometry, as the height stadia intervals are 10 and 100 respectively. A scrutiny of the field-book, however, reveals the fact that the readings on the staff were all close to the ground when using the Jeffcott tacheometer and accordingly a large refraction error was introduced. On practically all sights of over 250 ft. in length, at least one of the principal rays of light traversed the stratum of air lying within 2 ft. of the ground, and this doubtless explains why the mean Average Error Curve of the Jeffcott tacheometer falls below that of the theodolite from 250 to 400 ft.

The accuracy obtained with the Bosshardt-Zeiss tacheometer was much higher than with the other instruments. At first sight, the points on the curve representing the Average Fractional Error of Levelling are surprisingly erratic considering the uniform arrangement obtained for the Average Error of determining Horizontal Distances.* As the difference in/

*"Self-Reducing Tacheometers", Fig. 26, page 55 .

in elevation is obtained by multiplying the horizontal distance by the tangent of the angle of elevation, and as the horizontal distances were unvaryingly equal, it would appear that a very fine estimation of the vertical angle tangent scale is required. The tangent scale is marked in successive steps of .002, but readings must be estimated much finer than this. For example, at a distance of 400 ft., a tangent scale reading of + .004 would represent a rise of 1.60 ft., whereas estimating to + .0041 would give a rise of 1.64 ft., i.e., a difference of .04 ft. in 400 ft. or 1/10,000. Thus a slight alteration of the tangent value makes an appreciable difference in the final result and this accounts for the large variation in the magnitude of the Average Error of levelling with the Bosshardt-Zeiss tacheometer.

THE ERRORS AFFECTING PLUMBLINES IN SHAFT CONNECTIONS.

THE ERRORS AFFECTING PLUMBLINES IN SHAFT CONNECTIONS.

Introduction.-

The orientation of an underground survey is the most important problem that a mine surveyor is called upon to undertake, and no pains should be spared in order to ensure as high a degree of accuracy as possible. When the correlation is affected by hanging two or more plumblines in one shaft, and the base common to surface and underground surveys is very short, the most rigorous precautions should be taken as the smallest deflection of the plumblines causes an appreciable error in the **azimuth** of the underground base.

The deflection of a long plumbline from the absolute vertical position, due to the effects of magnetic attraction, air currents, etc. is the bête noire of this otherwise admirable method of correlating surface and underground surveys. It would appear, a priori, that the deflection of a plumbline is indirectly proportional to the weight of the plummet, and a plumbline suspending a plummet of infinite weight would hang absolutely vertical. So, if observations are made on a plumbline, and if the weight of the plummet is increased between each series of observations, and the mean position of each series is plotted against the reciprocal of the weight of the plummet, then a graph should be produced passing through the true vertical position of the plumbline opposite the zero mark on the 'reciprocal of the weight' axis.

The following series of observations was undertaken to test this hypothesis, using plummets of different weights and a swinging plumbline.

Description of Observations.-

The plumbline was totally enclosed and shielded from draughts at the beginning of the experiments, by suspending it inside a 40 foot column of 4 inch diameter piping. The column was hung from the roof of a three storey building, plumbed carefully, and the bottom was let into a wooden box 4 feet high by/

by 2 feet square, and securely clamped. Glass windows were set in three sides of the box to let in light to illuminate the scale and mirrors, and to enable the movement of the wire to be observed. A disc of wood was placed on top of the column, and the whole construction was thus made reasonably airtight.

Shielding a plumbline from the mal effect of air currents by suspending it inside a column of piping has been tried in practice in shallow shafts, for example, the Mersey tunnel shafts, but obviously this cannot be practised in deep shafts. In the case of the Mersey tunnel shafts, the method of testing if the wire was swinging clear of the pipe was to electrify the wire and test with an electrometer. An easier method, however, and the one adopted in this investigation, was to determine the time taken for one complete swing of the plumbline and compare it with the theoretical time required for one complete swing of a pendulum of equal length, i.e., $\pi \sqrt{L/g}$ or $0.554\sqrt{L}$ seconds, where L equals the length of the pendulum in feet. If the actual time taken agrees with the calculated time, and the plumbline is swinging without an abnormal amount of jerkiness, it may safely be assumed that the wire is swinging clear of the pipe.

The wire used for the plumbline was No. 28 gauge piano wire with a breaking load of approximately 100 lbs.

The plummet consisted of cast-iron cheese weights. They were held in position by stringing them on an 18 inch bolt, and a tin shield 18 inches high was placed round the weights to ensure that the surface area of the plummet, exposed to draughts later in the experiments, would remain constant.

Effect of Magnetic Attraction.-

Iron plummets are frequently used in plumbing shafts, but these may give rise to serious errors due to being deflected from the true vertical position by magnetic attraction. The error so induced can be overcome, however, by employing lead or brass plummets.

An extreme example of deflection due to magnetic attraction was observed by Mr. O. Brathuhn when plumbing a shaft 130 yards deep.* "In a south-westerly direction from the shaft a cross-cut had been driven, in which, in addition to the ordinary rails/

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* A Treatise on Mine Surveying, by Bennet H. Brough and Harry Dean, page 306.

rails laid down, a large number of spare rails were stored so close to the shaft that one plumb:line hung in close proximity to the northern end of the rails. By the inducedmagnetism of the rails the plumbline was drawn from its perpen:dicular position to such an extent that the distance between the two plumblines was 7.5 millimetres greater underground than at the surface, and that the line connecting the plumb:lines at the points of suspension formed an angle of 6 minutes with that at the bottom of the shaft. The error was eliminated by the employ:ment of brass plumb-bobs".

Effect of Air Currents.-

The ventilating current is one of the chief factors that causes a plumbline to deviate from the vertical position, and several instances are on record where the air current was strong enough and sufficient:ly steady to cause a constant deflection of the plumbline.

H.M. Lane, writing about the plumbing of the shafts of the Tamarack mine,* cited the following results obtained in No. 5 shaft in September, 1901. The length of the plumbline was approximately 4440 feet. The distance between the plumblines at the top of the shaft was 17.58 feet, and at the foot of the shaft 17.65 feet, a difference of 0.07 of a foot. Lead plummets were substituted in place of the 50 lb. iron plummets originally used, but the divergence between the plumblines remained the same.

Professor F.W. McNair subsequently carried out experiments at the Tamarack Mine in January, 1902, and proved conclusively that the divergence of the plumblines was due to the effect of air currents.** He used plumblines consisting of No.20 B. & S. gauge phosphor bronze wire and 60 lb. lead plummets. The wires were hung as nearly as possible in the same positions as before, and the distance between them was 16.709 ft. at the top of the shaft, and 16.850 ft. at the bottom, giving a divergence of 0.141 of a foot. The latter divergence, it will be noted, is greater than that observed in September.

At that time No. 5 Shaft was a five-compart:ment shaft, and, until the mine holed through to another shaft, the three East compartments were used as/

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*"Plumbing Deep Shafts of the Tamarack Mine", Mines and Minerals, Vol. XXII, pages 247, 248.

** "The Divergence of Long Plumblines at the Tamarack Mine", The Engineering and Mining Journal, Vol. LXXIII, pages 578 - 580.

as the downcast side and the two West ones as the upcast. There were numerous openings in the casing between the downcast and upcast compartments, and air rushed through these openings. At first it was not considered possible that the draught so formed could be sufficiently strong and sufficiently steady to cause the observed deflection, but it was eventually decided to put the matter to the test. Accordingly, the openings in the casing between No. 2 and No. 3 compartments were closed up, and the shaft mouth was covered over to prevent circulation as much as possible. Even with these precautions, however, convection currents still circulated in the shaft. The wire hung in the upcast compartment was then transferred to one of the downcast compartments as a final precaution. The distance between the wires at the top then measured 11.994 feet, and at the bottom 11.962 feet, giving a divergence of only 0.018 of a foot, a decided improvement on the previous figure, namely 0.141 of a foot.

The remaining divergence, i.e., 0.018 of a foot, was accounted for by the convection currents still circulating in the shaft, and the difference between the divergence of the steel wires in September and the bronze wires in January was accounted for by the increased circulation of air in January, and also that the bronze wires were thicker than the steel, and so exposed a greater surface area to the draught.

Mr. George C. McFarlane made a useful series of tests in a shaft 700 feet deep, using a 10 lb. brass plummet and No. 28 gauge steel wire.* He determined the vertical position of the plumbline when the mine fan was stopped and all the ventilation doors were closed. The fan was then run at different speeds, and the apparent vertical position of the plumbline was noted. The results of the experiments showed that "the deflection from the vertical is

> always toward the wall of the shaft, and is practically proportional to the velocity of the air current".

It is evident, therefore, that air currents constitute a serious source of error in shaft plumbing operations. Even when the fan is stopped, there generally remains sufficient natural ventilation in a mine shaft to affect a hanging plumbline, and the only way to reduce the effect further is to close up the mouth of the shaft and all ingresses and egresses to and from it. This entails a considerable amount/

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* "Air Currents and Shaft Plumb Lines", <u>The Engineer:</u>
<u>:ing and Mining Journal</u>, Vol. XCI, pages 318, 319.

amount of time and labour, and the results of the present investigation show that such drastic measures are unnecessary if readings are taken when using plummets of different weights, and the correct position of the plumbline is determined for a plummet of infinite weight.

Effect of Eccentricity of the Plummet.-

The necessity of using a perfectly symmetrical plummet was not realised at first, but the results of the observations proved that this was essential for accurate work. A plumbline may be considered as a pendulum equal in length to the distance between the point of suspension and the centre of gravity of the plummet. Furthermore, it is the centre of gravity of the plummet that takes up a position vertically under the point of suspension, or swings symmetrically on either side of the vertical position, and, unless the centre of gravity of the plummet is on the continuation of the line of the plumbline, the mean position obtained from the swing of the plumbline will be incorrect.

When the plumbline is twisting it is easy to detect whether the plummet is hanging properly or not. If it is, the mean position as determined from the limiting positions of the swings will remain constant; if not, the mean position will vary in a uniform manner. This variation is shown in the graph, Fig. 34, where the mean positions on the scale

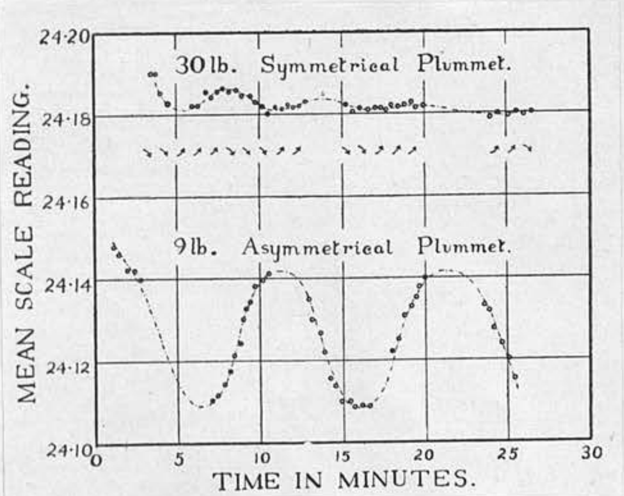

Fig. 34. Graph showing the oscillation of the mean position of a swinging plumbline with asymmetrical plummets.

as/

as determined from three swings are plotted as co-ordinates, against the time from the beginning of the test as ordinates, for an approximately symmetrical 30 lb. plummet, and for an asymmetrical 9 lb. plummet. The direction of the twist of the wire in the first example, as determined from the observation of a match attached to the wire, is indicated by small arrows underneath the curve, and the connection between the twist and the movement of the mean position is clear.

Shaft plumbing experiments were carried out by Mr. G.R. Thompson in a rectangular shaft 660 feet deep.* He suspended two 0.02" diameter tempered steel wires in the shaft and noted the swing of the wires against '2 chain to the inch' scales placed behind them. The scale readings obtained for the experiments on one wire are given in Tables 1 and 2 of his paper, and the mean position taking the readings three at a time are set out in the following table:

Weight of Plummet. (lbs)	6	13	19	6	13	19
MEAN	1.420	1.137	1.117	.502	.502	.512
	1.235	1.125	1.112	.501	.500	.510
	1.092	1.147	1.050	.492	.480	.502
SCALE	1.130	1.140	1.070	.490	.475	.505
		**				
	1.117	1.112	1.107	.487	.497	.495
	1.130	1.092	1.110	.467	.505	.465
READING.		1.070	1.095		.497	**
		1.077			.502	.475
					**	
		1.140			.480	
					.487	
					.497	
					.477	

Mr. Thompson states that "the variation is irregular and not due to a constant deflecting force", and I venture to suggest that the irregularity was chiefly due to using asymmetrical plummets - a small pulley for the 13 lb. plummet, and a shaft coupling for the 19 lb. one - notwithstanding his remarks that "the plummet may be of any shape". The first set indicates/

* "The Connection of Underground and Surface Surveys", _Trans. Inst. Min. Engrs._, Vol. XXII, p. 519 - 535.

**Reading missed.

indicates the undulating results obtained in the present investigation when using an asymmetrical plummet, although not so regular as outlined in Fig. 34. The second set indicates a gradual movement of the mean position in one direction as the plummet tends to take up its true vertical position during the first few minutes of swing.

On the whole it is rather a pity that the experiments were not carried out over a longer period so that the further behaviour of the plumbline might have been studied.

The practice of using any convenient weight irrespective of its shape should be strongly condemned, and readings should be continued until the plumbline settles into its normal swing, and the mean position, taking the readings three at a time, remains steady.

Effect of the Rotation of the Earth.-

The plumbline must be swung in a plane parallel to that of the scale, and at right angles to the line of sight, otherwise the mean position as determined from the scale readings will be incorrect. The divergence from the correct mean position is indicated in Fig. 35. The full lines represent

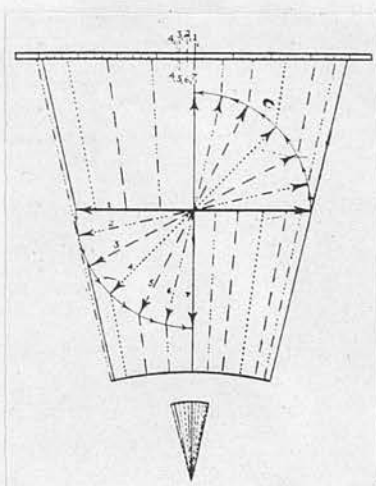

Fig. 35. Diagram showing the alteration in the mean position of a swinging plumbline, when the plumbline deviates from the plane of the scale.

lines/

lines of sight to the limiting positions of a plumb: line swinging parallel to the scale and at right angles to the line of sight, giving the correct mean scale reading 1. As the plumbline deviates from the correct position and swings in the planes 2, 3, 4, etc., the mean scale readings also deviates and gives erroneous scale readings of 2_1, 3_1, 4_1, etc., as indicated.

Owing to the rotation of the earth, the direction of swing of the plumbline will make one complete revolution, relative to the earth, every 24 hours, i.e., 15 degrees per hour. As a deviation of 3 or 4 degrees from the correct position causes no appreciable error, I recommend that the plumbline should be set swinging in a direction 5 degrees East of the true position to begin with. If it is then left swinging for ten minutes to let the plummet adjust itself, and to let any vibration of the wire be damped down, there remains twenty minutes in which to take observations, and the deviation of the plumb: :line from the true plane will only vary between the limits of $\pm 2\frac{1}{2}$ degrees.

In order to ensure that the plumbline was swinging parallel to the scale when the limiting positions of the swing were being read, the following simple optical device was used. Two mirrors, A and B in Fig. 36, 12 inches long by $1\frac{1}{2}$ inches high, were

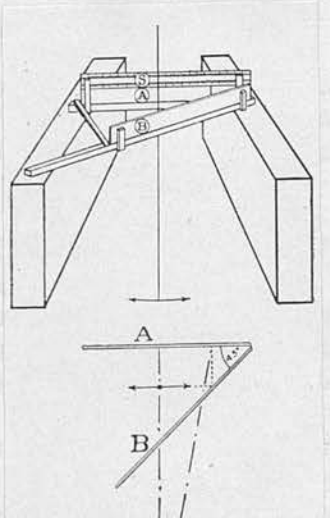

Fig. 36. Optical device used to verify that the plumbline was swinging parallel to the scale.

supported/

supported in a vertical position, and clamped at an angle of 45 degrees to each other, the reflecting surfaces facing inwards. The plotting scale, S, placed directly above mirror A, was fixed in a position perpendicular to the line of sight. When the plumbline was swung parallel to the scale, the image of the wire was reflected by mirror B on to mirror A, and as long as the plumbline was swinging in a plane parallel to the plane of the scale, the reflected image of the wire as seen in mirror A remained stationary, and its position could be read on the scale above. When the wire started to move away from the correct plane the image thrown on mirror A began to move also, and the angle of deviation could be determined from the amplitude of the swing parallel to mirror A as read directly on the scale S, and the amplitude of the swing at right angles to the scale as measured on the scale from the movement of the reflection on mirror A.

The effect of the earth's rotation can be seen in the graph representing the mean position of swing for a 9 lb. plummet, Fig. 34.

The plumbline was set swinging parallel to the scale at the beginning of the test, and it is apparent from the graph that the mean position is gradually moving along the scale; in this particular instance the scale reading is diminishing.

Vibration of the Plumbline.-

The slightest quiver of the plumbline makes accurate reading impossible, but vibration can be considerably reduced if the following precautions are taken.

As the effective length of the pendulum formed by the plumbline and plummet is equal to the distance from the point of suspension to the centre of gravity of the plummet, it follows that when tying up the plumbline the thread should be attached at a point opposite the centre of gravity of the plummet. If the thread is attached to the plummet too high up or too low down, a combination of two pendulums is formed - a long one from the point of suspension of the plumbline to the junction of the plumbline and the plummet, and a short one equal in length to the length of the plummet. When the plumbline is set free, these two pendulums start off on their respective swings with the consequence that the relatively short plummet imparts a decided quiver to the plumbline .

The/

The plumbline should also be tied up and left for ten minutes to let it steady before the thread is burned, and, of course, a weak thread should be used to ensure a smooth and steady release.

Results of Experiments.-

Representative results of the tests are shown graphically in Fig. 37. Each point on the graphs represents the mean position of the swing of the plumbline as determined from the readings on the scale, taking them three at a time for a period of ten minutes. Readings were taken with a 9 lb.

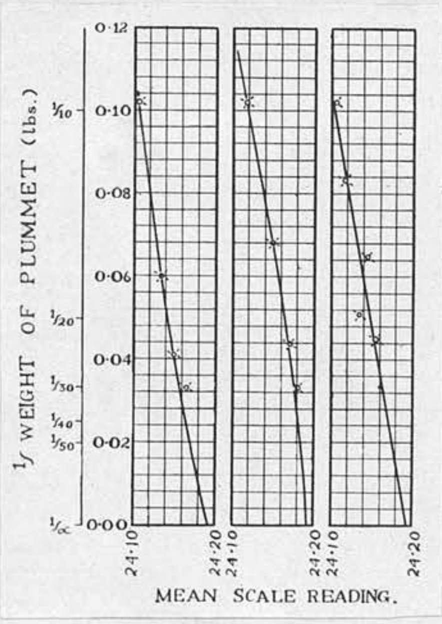

Fig. 37. Graphical determination of the true vertical position of a deflected plumbline.

plummet first, then the weight of the plummet was increased by adding more cheeseweights and the plumb: :line was again set in motion and the scale readings observed. Four different weights were used, the plummet was then unloaded, and the whole operation repeated for the next series.

The first two series were obtained with the plumbline totally enclosed, and the third was obtained when the sides were removed from the box at the foot of the pipe column, and the wind was allowed to play on the plummet. It is apparent from the graphs that the first two series are reasonably uniform, and each indicates that the true vertical position is opposite the/

the scale reading 24.19. The third series is not so uniform as one would expect, although the probable true vertical position is again 24.19 as indicated.

The reason for the slight deflection from a straight line graph in the first two sets is, in all probability, the asymmetrical nature of the plummet, as already explained. The reason for the deflection of the mean position from 24.11 with a weight of 9 lbs. to 24.16 with a weight of 30 lbs., and to the probable position of 24.19 with an infinite weight, was principally due to magnetic attraction as there was about one ton of iron lying within five feet of the plummet, while the obser: :vations were being made. The substitution of lead weights in place of the cast iron ones reduced the observed deviation.

The Wilson Plumbwire Locator.-

The Wilson Plumbwire Locator, Figs. 38 and 39, is manufactured by Messrs E.R. Watts & Son Ltd., and it is a compact and very simple device for determining the mean position of a swinging plumbline and clamping the wire in that position.

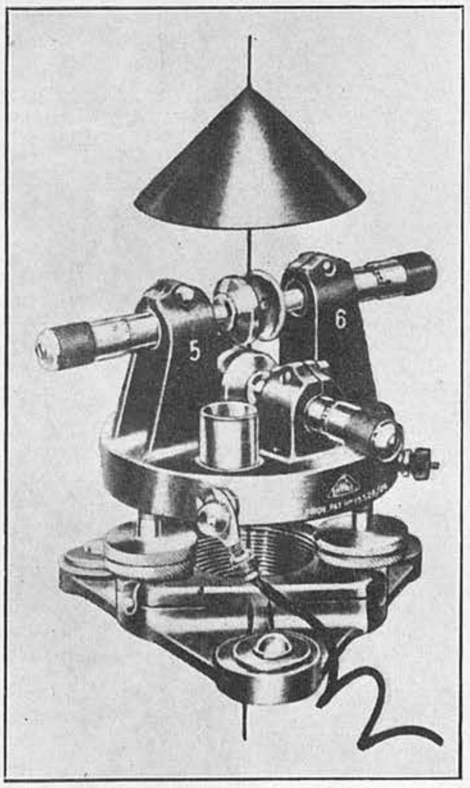

Fig. 38. The Wilson Plumbwire Locator.

It/

It consists essentially of four contact studs A attached to rods with micrometer heads B, reading to .001 inch. The framework supporting the contact rods is mounted on a three-screw levelling stage provided with a pill-box level D, and holes are bored in the stage to facilitate the fixing of the instrument on a plank at the foot of the shaft. The plumbline passes through the instrument and the supporting plank, and the plummet dips into a bucket of salt water.

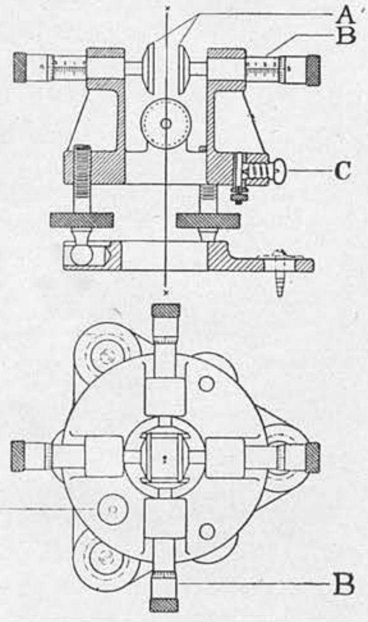

Fig. 39. Plan and Section of the Wilson Plumbwire Locator.

When the plumbline is set swinging, the micrometer rods are adjusted until the wire just touches the contact studs and the reading of the micrometer heads is noted. In order to indicate when the plumbline touches the contact studs, an electric bulb, C, is screwed on to the instrument, and an electric circuit is formed from the bulb to a battery, and from there to the pail containing the salt solu: :tion into which the plummet dips, and thence to the plumbline as indicated in Fig. 40. When the plumb: :line touches one of the contact studs on the instru: :ment, the circuit is completed and the bulb flashes.

Messrs E.R. Watts & Son Ltd., with their customary willingness, kindly loaned me two of these instruments, and I had the pleasure of using them on a short test plumbline. The Wilson Plumbwire Locator is essentially an instrument for use with long plumb: :lines, however, say those over 500 feet in length, as/

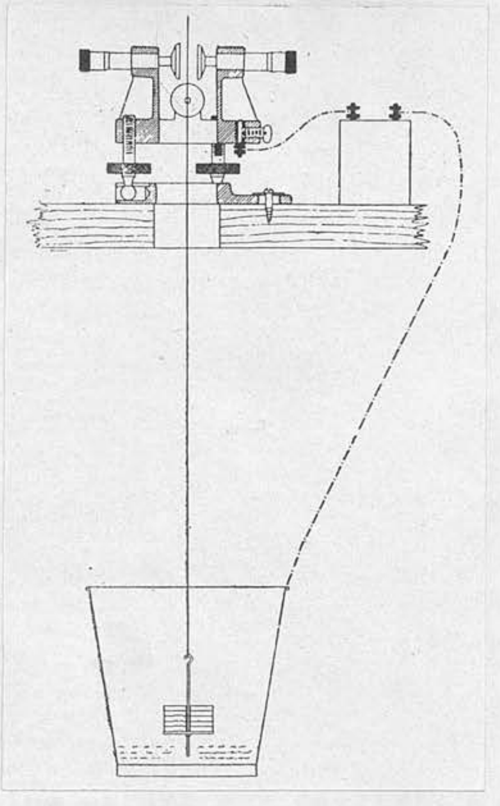

Fig. 40. Wiring diagram for the Wilson Plumbwire Locator.

as the timeof oscillation of a short plumbline is too small to enable the micrometer heads to be read after every swing. Again, only the tip of the plummet, or preferably a narrow projection of it, should be immersed in the salt solution, in order to prevent damping the magnitude of the oscillation as much as possible.

The instrument has the advantage that it is simpler and quicker to set up than scales and theodol:ites, but it does not give the same degree of accuracy, as it is impossible to adjust the contact studs with such fineness as one can read the limiting position of swing on a plotting scale with a theodolite.

Furthermore, in a mine shaft, a plumbline rarely, if ever, swings in a uniform manner. Every now and then, the amplitude of swing increases on one or both sides, and the Wilson Plumbwire Locator can not record these deviations as the contact studs can not be withdrawn quickly enough.

SUMMARY OF RESULTS.

Linear Measurements by Chains and Steel Tapes.-

It is of the utmost importance to check and adjust chains regularly to compensate for wear.

Two simple formulae are derived, the first giving the correction to be applied to counteract sagging error, the second giving the extra tension required to stretch a sagging tape to make it indicate the correct length (pp. 4 - 8).

Temperature corrections cannot be assessed with absolute accuracy owing to the difficulty in determining the exact temperature of the tape, and, when a high standard of accuracy is required, measure:ments should be made at night or in dull weather.

Marking errors are negligible in comparison with reading errors when using chains, but marking errors are at least three times as great as reading errors when using tapes.

When reading a chain to the nearest link only, greater accuracy can be obtained with a 66 foot chain than with a 100 foot chain; when estimating to .5 or .1 of a link the accuracy is the same with both chains.

Average error curves are given showing the error in measuring distances with 66 foot and 100 foot chains (p. 12), and 100 foot and 300 foot steel tapes (pp. 13 and 14).

Values are given for the coefficient K in the formula $L_1 = \pm K\sqrt{L}$ for various conditions (pp. 15 and 16).

Theodolite Tacheometry.-

The average fractional error in marking stadia lines on glass diaphragms is approximately $\pm\frac{1}{1000}$ for a theodolite telescope with a focal length of 7 or 8 inches.

With an internal focussing telescope the 'stadia constant' is a variable function, but with the new internal focussing anallatic telescope the 'stadia constant' can be neglected as the correction to/

to be applied is less than one thousandth part of the ascertained length.

Terrestrial refraction seriously affects the accuracy of tacheometric observations when the principal rays of light traverse the stratum of air within four feet of the ground. The effect is most pronounced when the difference of temperature between ground and air is greatest and the usual result is that the stadia intercept is too large.

A clear open-reading staff is essential when long sights are taken, and a new system of graduation is shown (p. 26).

The accuracy obtained when using a horizontal staff is greater than that obtained when using a vertical staff. The accuracy increases rapidly as the distance increases up to 200 feet and then decreases slowly. At a distance of 200 feet and thereabouts the average fractional error in determining horizontal distances is $\pm \frac{1}{550}$ when using a horizontal staff, and $\pm \frac{1}{450}$ whenusing a vertical staff.

Average error curves are given showing the error in ascertaining horizontal distances by tacheometry with vertical and horizontal staffs (p. 31).

When inclined sights are taken, the accuracy obtained when using a horizontal staff is practically the same as with horizontal sights; when using a vertical staff, the accuracy diminishes rapidly.

Average error curves are given showing the error in determining horizontal distances by tacheo:metry with vertical and horizontal staffs when inclin:ed sights are taken (p. 34).

Self-Reducing Tacheometers.-

The mean value of the average fractional error in determining horizontal distances with the Watts-Szepessy tacheometer is $\pm \frac{1}{300}$. Greater accur:acy can be obtained with short sights by employing the alternative method of using the instrument explained on page 42.

The mean value of the average fractional error in determining horizontal distances with the Jeffcott tacheometer was $\pm \frac{1}{600}$, and the mean value for working conditions throughout the year is estimat:ed as $\pm \frac{1}{450}$ to $\pm \frac{1}{500}$.

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The mean value of the average fractional error in determining horizontal distances with the Bosshardt-Zeiss tacheometer is $\pm \frac{1}{12000}$ with normal atmospheric conditions.

Average error curves are given showing the error in ascertaining horizontal distances with the Watts-Szepessy, the Jeffcott and the Bosshardt-Zeiss tacheometers on pages 41, 46 and 55 respectively.

A saving of cost up to 40% can be obtained by employing tacheometry when a great deal of detail has to be surveyed.

Tacheometrical Levelling.-

Average error curves are given showing the error in determining levels by tacheometry with vertical and horizontal staffs (p. 60).

It is emphatically demonstrated that refraction causes appreciable errors if the lines of sight traverse the stratum of air within 4 feet of the ground, the error being greatest near the ground and diminishing as the height from the ground increases (pp. 60 - 63).

When inclined sights are used the accuracy of tacheometrical levelling decreases so rapidly as the angle of elevation increases that this method of levelling is too inaccurate for most practical purposes.

Average error curves are given showing the error in determining levels with the Bosshardt-Zeiss and Jeffcott tacheometers (p. 67).

Shaft Connections by Plumb-lines.-

Lead or brass plummets should be used to overcome errors due to magnetic attraction.

Air currents cause plumblines to deviate from the vertical position by appreciable amounts.

Plummets should be perfectly symmetrical.

The plumbline should be swung in a plane perpendicular to the line of sight and parallel to the scale.

Vibration of the plumbline can be minimised if, when tying up the plummet, the thread is attached at a point opposite the centre of gravity of the plummet.

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