A STUDY OF SOME PROBLEMS ARISING FROM SPONTANEOUS FLUCTUATIONS OF ELECTRICITY.

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by

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C O N T E N T S

CHAPTER I.

INTRODUCTION AND STATEMENT OF PROBLEMS INVESTIGATED.

1. 1 : Introductory Survey.

Interest in the phenomena of spontaneous fluctuations dates from the first observations by Brown[1][2] of the random motion of particles of microscopic size in fluid. Brown himself believed the motion to originate from an elementary form of life, but later workers in the nineteenth century accumulated evidence leading to the correct interpretation; the name "Brownian motion" is however now frequently applied to the general field of spontaneous fluctuations whether of mechanical or electrical systems.

It may fairly be said that Einstein[1] was the first to make a systematic mathematical study of the fundamental problems involved, and Smoluchowski[1,2] in particular later made very notable contributions to the theory. Perforce experimental verification and extension of the theory at this time was limited to consideration of mechanical systems of one sort or another and Svedberg[1], Westgren[1] and Fürth[1,2] for example worked in this field. One must, however, observe that Einstein[3] himself early derived an expression for thermal fluctuations of electricity to be expected in a circuit in thermal equilibrium in terms of the random transport of electric charge, Δq, in a time t, in a resistance, R : namely :-

$$\overline{\Delta q^2} = \frac{2kT}{R}.t \qquad \text{1-(1)}$$

Also, this theoretical consideration is developed a little further by Mrs. de Haas-Lorentz[1] where the problem of the inter-related electrical and mechanical spontaneous fluctuations of a galvanometer is also treated.

Some consideration of electrical fluctuations with particular reference to photo-electric phenomena was also made around 1907 by Schweidler[1] and others. The theory was examined by N.R. Campbell[1,2,3] who proved two valuable fluctuations theorems which will be mentioned below and are of value in a number of fields.

Later the possibility of experimental investigation in the electrical field provided by the possibility of obtaining very great amplification through the use of thermionic valves, whose development was accelerated by the Great War, led Schottky[1] to re-examine the problems in a well-known paper in 1918. Therein he obtained a rather more useful expression for the random fluctuation power due to thermal agitation in the input circuit of an amplifier. The expression, although of considerable practical value, was not yet in a form most amenable for treating the general thermal agitation problem in electrical circuits, and it remained for Nyquist (to be mentioned below) to give the most useful form.

Secondly as a much more generally known result he derived a specific expression for the fluctuations of current in a thermionic valve, under the assumption that the passage of individual electrons across the valve constitute independent events. In this case if the fluctuations be observed over a time τ then Schottky showed on a simple statistical basis that :

$$\overline{(i-I)^2} = \frac{eI}{\tau} \text{ - - - - - - - - - - - - } 1-(2) \quad (\text{Where } I = \bar{i}, \text{ the mean current})$$

This formulation of the phenomenon is inconvenient for experimental verification and more convenient and appropriate forms will be discussed below in detailed work.

The first attempt at experimental verification of the theory by determining the apparent value of e appears to have been made by Hartmann (1), and after correction of an error and a presentation of a simple theory yielding a result readily applicable to a specific electrical circuit by Fürth (3), reasonable agreement was obtained with the accepted value. Many later experiments with refined technique (e.g. Hull and Williams (1), Williams and Vincent (1), yielded excellent agreement with theory. In these cases care was taken to ensure that the current was temperature-limited and thus each electron transit could safely be regarded as independent.

When, however, this was not the case and the current was "space-charge limited", so that the full cathode emission was not drawn to the anode, it quickly became evident that the fluctuations observed were very considerably less than that predicted by Schottky's formula; (E.g. Hull and Williams [11]). From then until the present time the phenomenon has been the subject of much theoretical discussion and experimental work.

Various attempts to evolve an analysis applic-:able to the space-charge limited region have been put forward, which will be discussed in more detail below, but satisfactory quantitative agreement with experiment was not obtained until recently as the result of theory evolved variously by Schottky [3], D.O. North and Thompson [1], and A.J. Rack [1]. The "reduction factor" of the fluctuations is most frequently expressed as Γ^2.

Thus formally we may write, after Schottky:

$$\overline{(i-I)^2} = \frac{eI\Gamma^2}{\tau} \quad \text{------------------} \quad 1-(3a)$$

or in a form frequently more suitable for experimental work, to be justified below:

$$\overline{(i-I)^2} = \frac{1}{\pi}eI\Gamma^2\Delta\omega = 2eI\Gamma^2\Delta f \quad \text{------------} \quad 1-(3b)$$

where : $\Delta\omega$ is the portion of the angular frequency spectrum accommodated by the measuring instrument:

: Δf is the corresponding portion of the frequency spectrum.

At this point we observe that the analogous forms to (1) as stated by Nyquist [1] for the thermal fluctua-:tions of a resistance at temperature T are:

$$\overline{(v-V)^2} = \frac{2}{\pi}RkT\Delta\omega = 4RkT\Delta f \quad \text{------} \quad 1-(4a)$$ (Expressed as a voltage fluctu-:ation).

$$\overline{(i-I)^2} = \frac{2}{\pi}\frac{kT}{R}\Delta\omega = \frac{4}{R}kT\Delta f \quad \text{------} \quad 1-(4b)$$ (Expressed as a current fluctu-

Now as the result of a detailed analysis the various workers mentioned just previously derived the result finally that (strictly for a parallel-plane diode) :-

$$\overline{I^2} = \frac{2\theta k T}{I e} \cdot g_a \text{ --------- } 1\text{-}(5) \quad (T : \text{Cathode temperature})$$

where: $g_a \equiv 1/R_a$ and R_a is the "differential" or "slope" resistance of the valve. That is, $g_a = \frac{\partial I}{\partial V_a}$

and : θ is a factor which varies very slowly and may be taken $\simeq \cdot 64$ over the major operating region of the valve, falling rather abruptly to $\cdot 5$ when retarding field conditions enter. When (5) is insert:ed in (3b) a rather remarkable result emerges. We have, in fact :

$$\overline{(i-I)^2} = \frac{4}{R_a} k(\theta T).\Delta f \text{ ------ } 1\text{-}(6)$$

In other words, formally at any rate comparing (6) with (4b) we may regard a diode as generating thermal fluctuations at a temperature θT, where θ -although certainly a variable - suffers only very slow alteration over a considerable range of valve operation. This conclusion, regarded simply as an engineering formula, is most valuable in enabling rapid estimation of practical noise formulae in valves to be made. From the physical stand-point, however, (6) is obviously directly suggestive of a system in thermal equilibrium. Yet North himself is most emphatic in stating that in general no thermal argument may be applied to the problem, emphasising that "shot" and "thermal" fluctuations must be considered as two distinct phenomena. To quote : "........yet the two phenomena "must not be confused in concept. For, thermal "agitation is known to be a form of Brownian movement "and finds its origin in the equipartition of energy "among the various mechanical and electrical degrees "of freedom of a substance in thermal - i.e.kinetic or "statistical - equilibrium. The diode, on the other "hand, while clearly in a stationary state, cannot be "regarded as a system in thermal equilibrium so long as "there is a battery providing plate voltage and energy. "The mechanics of the two phenomena are, therefore, "distinct; the formulae alone exhibit a resemlance."

Such a resemblance, however, without an underlying unity of source seems unacceptable. The writer has always believed that a more general approach to the problem must be possible showing clearly this unity. Or, otherwise stated, a more general initial statement of the essentials must be possible than that adopted by North, although to obtain ultimately numerical results would probably require the same degree of detailed computation necessary to arrive at (6) or (5).

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1 : 2. Statement of problems investigated.

The detailed work of this thesis is divided into four sections. In Chapter 2., following this introductory section, a critical discussion of the various theories surrounding space-charge limitation of valve fluctuations is presented and an attempt is made to provide a unifying statement from which each theory may be derived, and also to show that thermal fluctuation is indeed to be regarded as the fundamental source of "shot" noise in its various forms. In Chapter 3., a detailed investigation, both theoreti:cal and experimental, of the characteristics of the retarding field region in diodes is undertaken with particular reference to the later measurement of the fluctuations generated in this regime. It is established that, for the purpose of fluctuation measurement, this region has not previously been enter:ed. It is felt that Chapter 2. will make evident why the retarding field region is to be regarded as of primary importance in general, but in particular it will be at once clear that under purely retarding conditions a very close connection must obtain between the valve fluctuations and purely thermal fluctuations on general physical principles. Chapter 4. deals with the measurement of the fluctuations in the retard:ing region over a quite wide range of valve currents, and also embodies some measurements of fluctuations on high vacuum photoelectric cells in the saturation region and adjoining space-charge limited region for various degrees of illumination whose general signifi- :cance is discussed. The final chapter, Chapter 5., presents some detailed statistical investigations of electrical fluctuations carried out on individual photographic records. This work was originally under- :taken in order to investigate particularly whether any significant difference was observable in the time-correlation of the records arising from a "thermal" source or a "shot" source. In géneral, however, the work has yielded confirmation of quite a wide range of theoretical work relating to the statistical behaviour of fluctuations which experimental confirmation has not previously been provided.

CHAPTER 2.

DISCUSSION OF SPACE-CHARGE LIMITATION OF VALVE FLUCTUATIONS.

2 : 1. Effect of finite "slope resistance" of valve.

J.B. Johnson [1] in 1925 first suggested that the only effect to be allowed for to account for the observed reduction in the fluctuation in the space-charge region was the shunting effect of the valve differential resistance, R_a. However, it was found that this effect, while clearly reducing the magnitude of the fluctuation observable externally, still result-:ed in considerable over-estimation of the effect. One may note, however, that the theory was still adhered to by Moullin and Ellis [5] several years later.

A fundamental problem emerges by the very introduction of R_a, a macroscopic valve parameter, as a variable in the fluctuation problem. This topic has been discussed by various writers, notably Rowland [1,2] and J.M. Whittaker [1], and the author feels that a certain amount of clarification is called for. For this purpose it is expedient to state briefly two fundamental fluctuation theorems by N.R. Campbell [1,2,3].

These may be simply derived as follows :

Let events of the same type (i.e. such that each event produces a correspondingly similar response in the measuring instrument) be occurring randomly in time at an average rate N. Then in an interval $(t; t+\Delta t)$ the expected number will be $N\Delta t$, while the observed number is, say, $x(t).\Delta t$. Let an event occurring at time t produce a response in the instrument at time T : $f(T-t)$. Then if further it be assumed that the effect of each event at any instant may be linearly superposed it is clear that in the limit $\Delta t \to 0$, the response at time T is:

$$\theta(T) = \int_{-\infty}^{T} x(t).f(T-t).dt \quad \text{------------} \quad 2\text{-}(1)$$

∴ the average response (i.e. averaged over a large number of observations), bearing in mind that the integral in (1) is approached as the limit of a sum, is given by :

$$\bar{\theta} = \int_{-\infty}^{T} \overline{x(t)}.f(T-t).dt$$

$$= N\int_{-\infty}^{T} f(T-t).dt$$

$$= N\int_{0}^{\infty} f(t).dt \quad \text{----------------} \quad 2\text{-}(2)$$

Secondly :

$$\theta'^2 \equiv (\theta-\bar{\theta})^2 = \int_{-\infty}^{T}\{x(\alpha)-N\}.f(T-\alpha).d\alpha\int_{-\infty}^{T}\{x(\beta)-N\}.f(T-\beta).d\beta$$

$$= \int_{-\infty}^{T}\delta(\alpha).f(T-\alpha).d\alpha\int_{-\infty}^{T}\delta(\beta).f(T-\beta).d\beta$$

[Where $\delta(\mu) = x(\beta) - N$]

Now let :

$$\alpha - \beta = \gamma$$

$$\therefore \overline{\theta'^2} = \int_{-\infty}^{T} d\alpha \int_{-\infty}^{\alpha-T} \overline{\delta(\alpha).\delta(\alpha-\gamma)}.f(T-\alpha).f(T-\overline{\alpha-\gamma}).d\gamma$$

Now $\overline{\delta(\alpha).\delta(\alpha-\gamma)}$ may be regarded as a Dirac function only non-vanishing at $\gamma = 0$ and such that

$$\int_{-\infty}^{\infty} \delta(\alpha).\delta(\alpha-\gamma).d\gamma = N \quad \text{----------} \quad 2\text{-}(3)$$

$$\therefore \overline{\theta'^2} = N\int_{-\infty}^{T}\{f(T-\alpha)\}^2.d\alpha$$

whence changing variable :

$$\overline{\theta'^2} = N\int_{0}^{\infty}\{f(t)\}^2.dt \quad \text{----------} \quad 2\text{-}(4)$$

Two features to be observed are the assumption of linear superposition of $f(t)$ and the specification of $f(t)$ in any particular case.

If, for example, we consider the case of a diode with an external impedance of resistance and capacity in parallel (see Fig.1), then the fundamental events constitute the passage of electrons across the diode and then subsequent effect upon the anode circuit. If we accept the validity of normal circuit laws for individual electrons, then :

$$f(t) = \frac{e}{C}\varepsilon^{-t/CR} \qquad 2-(5)$$

Immediately then :

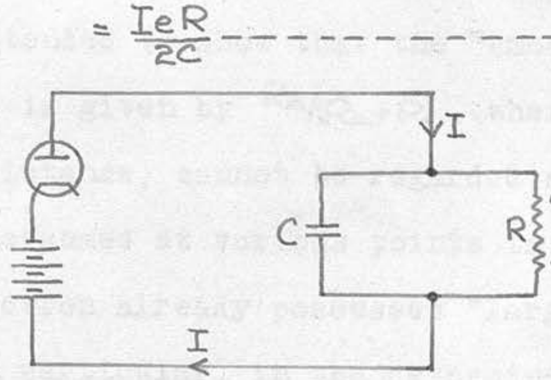
$$\overline{\theta} = \overline{v} = \frac{Ne}{C}\int_0^\infty \varepsilon^{-t/CR}dt$$

$$= NeR = IR \qquad 2-(6)$$

(where I, the mean current, = Ne).

and : $$\overline{\theta'^2} = \overline{(v-\overline{v})^2} = \frac{Ne^2}{C^2}\int_0^\infty \varepsilon^{-2t/CR}dt$$

$$= \frac{IeR}{2C} \qquad 2-(7)$$



(Fig.1).

In this form the theory has received ample confirmation. Rowland[(1)] and the writer have attempted to state the individual event in other forms which would avoid the assumption that electrons behave individually in the circuit as "large charges" but unsuccessfully, and it appears essential to postulate this behaviour in the classical theory. Other workers such as Campbell[(4)] himself and D.O. North[(1)] whose work will be discussed further below have accepted this assumption without comment. If we adopt such a course then the assumption of linear superposition must equally be adopted without further ado. In addition, however, a certain confusion of thought regarding the introduction of R_a appears to have existed. Both J.M. Whittaker[1,2] and Rowland[1] have attempted to justify the inclusion of the slope resistance as an independent problem. If, however, we accept that we may set $f(t) = \varepsilon^{-t/CR}$ (or similar expressions in other circuits which are confirmed by experiment) then there can be no further argument against the employment of the parameter R_a or the use of Helmholtz' theorem[1,2,3] or in fact any general network theorem in the fluctuation problem. Thus the ingenious analysis of Whittaker intended to show that the "smoothing factor" of the anode is given by $R_a/(R_a+R)$, where R is the external resistance, cannot be regarded as satisfactory, since it is assumed at various points in the analysis that the electron already possesses "large charge" qualities. In particular, in the determination of a certain constant towards the end of the analysis the statement appears: "When the incident current has its "average value ϕ_0 the effect of a single electron at "time τ is to diminish the anode current by an amount " $-\frac{1}{CR_a}\varepsilon^{-k(t-\tau)}$ ". The whole essence of the fact to be proved that R_a may be regarded in the problem as a resistance shunting the external circuit (and thus immediately giving rise to the factor $(R).\frac{R_a}{R_a+R}$) appears to lie in that type of assumption, making further proof unnecessary. Furthermore, on this basis, using the "Faltung" theorem of Fourier applicable to general network theory (e.g. Campbell and Francis [4]):

$$\int_0^\infty |F(\omega)|^2 . d\omega = \pi \int_0^\infty f(t)^2 . dt \text{ ---- } 2\text{-}(8) : \quad \text{where } F(\omega) = \int_0^\infty f(t) . \varepsilon^{-i\omega t} . dt$$

it may then be readily shown that, <u>from the point of view of the instrument measuring the fluctuations</u>, the fluctuations up to reasonable frequency limits may be regarded as having a uniform frequency spectrum. That is to say we may derive the rather more convenient form of the shot-fluctuation formula quoted in Chapter 1: viz:

$$\overline{(i-I)^2} = \frac{1}{\pi} eI\Delta\omega = 2eI\Delta f \text{ ------- } 2\text{-}(9)$$

and the corresponding formulae of Nyquist for the thermal fluctuation.

To the writer it appears that considerable unnecessary argument has taken place as to whether a "frequency spectrum" can be ascribed or not to fluctuations (e.g. T. C. Fry [1] and Campbell and Francis [4]). The use of these formulae essentially depends upon an application of Helmholtz' (Thévenin's) network theorem in either the "constant voltage" or "constant current" form and to quote specifically from Helmholtz' original statement :

"Wenn ein körperlicher Leiter mit constanten elektromotorischen Kräften in zwei bestimmten Punkten seiner Oberfläche mit beliebigen linearen Leitern verbunden wird, so kann man an seiner Stelle immer einen linearen Leiter von bestimmter elektromotorischer Kraft, und bestimmtem Widerstande substituiranode potential V_a and emission J.

I.e.:

$$I = I(J, V_a) \text{------------------} 2\text{-}(10)$$

$$\therefore \delta I = \frac{\partial I}{\partial J}.\delta J + \frac{\partial I}{\partial V_a}.\delta V_a \text{----------} 2\text{-}(11)$$

But we assume $\delta V_a \equiv 0 \therefore \delta I = \frac{\partial I}{\partial J}.\delta J$ ----- 2-(11a)

$$\therefore \overline{\delta I^2} = \left(\frac{\partial I}{\partial J}\right)^2 . \overline{\delta J^2} \text{-------------} 2\text{-}(12)$$

But certainly

$\overline{\delta J^2} = 2eJ\Delta f$ ---- (Equation (9) for shot fluctuation without space charge reduction).

$$\therefore \overline{\delta I^2} = 2eJ.\left(\frac{\partial I}{\partial J}\right)^2.\Delta f \text{-----------} 2\text{-}(13)$$

i.e. $\overline{\delta I^2} = 2eI.\Gamma^2\Delta f$ ------------ 2-(14)

where $\Gamma^2 = \frac{J}{I}.\left(\frac{\partial I}{\partial J}\right)^2$ ------------ 2-(15)

However, it may be shown that ($\partial I/\partial J$) and J/I as determined from the static valve characteristic are such as to make $\Gamma^2 \to 0$ in (15) as $I/J \to 0$ (extreme space-charge limitation), which is definitely contrary to experimental evidence. Llewellyn, aware of this, introduced an ad hoc hypothesis; namely, that to account for the "residual" noise under extreme space-charge limitation, one must include thermal fluctuation of the slope resistance R_a. By quite fallacious reasoning he stated that this must be taken as at the temperature T of the cathode.

This addition by Llewellyn has proved, in the writer's opinion, doubly unfortunate. First, if Llewellyn's analysis - or any similar thereto - is to be of worth then clearly it must account for <u>all</u> the valve fluctuation. We have, so to speak, examined in a little detail the mechanism of the fluctuation rather than trying to treat the whole valve as a closed system a priori. If, however, a discrepancy appears in our "detailed analysis" we must surely not suggest an additional source of noise arising from a treatment of the valve as a complete unit. Secondly, it seems quite certain that an attempted application of Nyquist's theorem in the general case to the tube as a whole must be doomed to failure since (and in this respect the writer certainly agrees) as North says the valve as a whole can in no sense be regarded in the general case as being in a state of thermal equilibrium.

One of the aims of the writer in this Chapter is to unify all analyses on the subject rather than, as has been the custom in the past, to discard each analysis in turn without further comment if it fails to agree with experiment. In the case of Llewellyn's analysis an error in fact exists in the transition from equation (10) to equation(12). First we must observe that in order that equation (11) (or 11a) may have any mathematical significance we must assume that I is a single-valued function of J . As we shall see later, it has in fact been necessary to discard this assumption to obtain final agreement of experiment with theory. However, granted this, J.M. Whittaker (2) in a later paper has discussed in effect this transition (i.e. from equation (10) to (12). We may quote :....."The "proper procedure in the case of a fluctuating stream "which for some reason is reduced in size is to "consider separately the factor which reduces the "average stream, and the factor which reduces the "fluctuations". Whittaker then says that if we call the former factor α, and the latter β, then the mean square fluctuations are reduced in the ratio $\alpha\beta^2$. We note also that Whittaker's argument while stated specifically by him in a form most suitable for application to this particular problem is essentially contained already in the mean square fluctuation theorem of Campbell discussed above :-

$$\overline{\theta'^2} = N\int_0^\infty \{f(t)\}^2 dt$$

If the actual number of events involved is altered from N to N', say, then clearly $\overline{\theta'^2}$ is only linearly altered by this factor, corresponding directly to Whittaker's factor α. On the other hand, any modification of the essential nature of the fluctuations as envisaged by Whittaker's factor, β, will be involved in the specification of $f(t)$ and thus $\overline{\theta'^2}$ will be altered in a quadratic manner. Considering, then, Llewellyn's first step :

$$I = I(J) \text{------------} 2\text{-}(10)$$

$$\delta I = \frac{\partial I}{\partial J}\cdot\delta J \text{----------} 2\text{-}(11a)$$

it is clear that we cannot immediately identify the factors α and β in the modifying factor $\frac{\partial I}{\partial J}$. However, it is quite clear that we must set $\alpha = \frac{I}{J}$ in any case and therefore we may re-write (11a) as :

$$\delta I = \left\{\frac{\partial I}{\partial J}\cdot\frac{J}{I}\right\}\cdot\frac{I}{J}\cdot\delta J \text{--------} 2\text{-}(11b)$$

Thus if in accordance with Whittaker we write :

$\delta I = (\beta.\alpha).\delta J$, then clearly we must identify the true fluctuation reducing factor, β , as :

$$\beta = \frac{\partial I}{\partial J}.\frac{J}{I}$$

Thus in place of (12) above we must now write :

$$\overline{\delta I^2} = \{\alpha.\beta^2\}.\overline{\delta J^2} \text{ - - - - - - - - - - 2-(12a), i.e.}$$

$$\overline{\delta I^2} = \left\{\frac{\partial I}{\partial J}.\frac{J}{I}\right\}^2.\frac{I}{J}.\overline{\delta J^2} \text{ - - - - - - - - - 2-(12b)}$$

But $\overline{\delta J^2} = 2eJ\Delta f$, as before,

whence $\overline{\delta I^2} = 2e\left\{\frac{\partial I}{\partial J}.\frac{J}{I}\right\}^2.I\Delta f$ - - - - - - - - 2-(13a)

thus immediately we may identify Γ^2 as :

$$\Gamma^2 = \left(\frac{\partial I}{\partial J}.\frac{J}{I}\right)^2 \text{ - - - - - - - - - - - - 2-(15a)}$$

This corrected expression for Γ^2 does not now tend to vanish for $\frac{I}{J} \rightarrow 0$; in fact on the assumption of a Maxwell-Boltzmann law for the valve it leads to the accepted value of unity, and it will be shown that this expression for Γ^2 agrees with later work on the subject and thus represents in the writer's opinion the simplest approach to an expression for Γ^2 based on the monovalued law.

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2. 3. Early analysis of Schottky.

W. Schottky (2) in 1938 proposed an analysis for space-charge reduction. In evaluating this he restricted himself to the assumption of a " $\frac{3}{2}$ -power " law for a region in the valve and a number of rather unnecessary assumptions are involved in the analysis;

the writer some time ago in an unpublished paper evolved an analysis on similar lines, at that time unaware of Schottky's analysis, and since this is very considerably shorter it will now be presented.

We know that :

$$\overline{(j-J)^2} = 2eJ\Delta f \text{ ------------ } 2\text{-}(16)$$

and that over a "long" period :

$$I = J\varepsilon^{\frac{eV_m}{kT}} \text{ ----------- } 2\text{-}(17)$$ where V_m is the potential minimum in the valve assumed parallel-plane and a Maxwellian emission distribution is assumed.

Now let us observe i over "short" periods τ, but long enough for a large number of electrons to flow. We then assume that a similar relation to (17) will hold but such that i, j, v_m are average values over τ and in general different from the "long term" averages I, J, V_m,

i.e. $$i = j\varepsilon^{\frac{ev_m(j)}{kT}} \text{ --------------- } 2\text{-}(18)$$

$$= j.\varepsilon^{\frac{ev_m(J+\delta J)}{kT}}$$ where δJ, the average deviation of the emission over τ, will be assumed $\ll J$.

Now on <u>this</u> basis, $v_m(j)$ will be a continuous single valued function of the emission (and of V_a which however is assumed constant - as above)

$$\therefore v_m(J+\delta J) \doteq V_m(J) + \delta J.\frac{\partial V_m}{\partial J}$$

$$\therefore i = j.\exp\left\{\frac{e.\delta J.\frac{\partial V_m}{\partial J}}{kT}\right\}.\exp\left\{\frac{e.V_m(J)}{kT}\right\}$$

whence, setting $\begin{cases} i = I + \delta I \\ j = J + \delta J \end{cases}$ we arrive readily at :

$$\delta I = \delta J\left\{1 + \frac{Je}{kT}.\frac{\partial V_m}{\partial J}\right\}.\frac{I}{J} \text{ -------- } 2\text{-}(19)$$, where $\partial V_m/\partial J$ is intrinsically negative.

i.e. $\delta I = \delta J.\beta.\alpha$ where $\alpha = \frac{I}{J}$; $\beta = 1 + \frac{Je}{kT}.\frac{\partial V_m}{\partial J}$

employing notation conformed to the previous problem.

Hence : $\overline{\delta I^2} = \alpha . \beta^2 . \overline{\delta J^2}$

$$= \frac{I}{J}\cdot\left(1+\frac{Je}{kT}\cdot\frac{\partial V_m}{\partial J}\right)^2 . 2eJ\Delta f$$

$$= 2eI\left(1+\frac{Je}{kT}\cdot\frac{\partial V_m}{\partial J}\right)^2 . \Delta f \text{ ------ } 2\text{-}(20)$$

Thus:

$$\Gamma^2 = \left(1+\frac{Je}{kT}\cdot\frac{\partial V_m}{\partial J}\right)^2 \text{ -------- } 2\text{-}(21)$$

This analysis presents clearly certain aspects. First, obviously (since $\frac{\partial V_m}{\partial J}$ is intrinsically negative) $0 < \Gamma^2 \leq 1$. Secondly when $\frac{\partial V_m}{\partial J} = 0$, $\Gamma^2 = 1$. That is to say, if the minimum potential in the valve inter-:space is invariant to changes of emission, which will occur if the potential minimum is situate at the cathode (saturated conditions) or at the anode (true retarding field conditions) which agrees quantitatively with experiment (including the work of this thesis). Finally, Γ^2 will pass through a minimum where $\frac{\partial V_m}{\partial J}$ has its greatest numerical value, which will be where the valve is most usefully employed as an amplifier. This latter conclusion agrees qualitatively with experimental fact.

Now, employing the Langmuir (1) and Fry (2) planar analysis it may be shown that :

$$\left(1+\frac{Je}{kT}\cdot\frac{\partial V_m}{\partial J}\right)^2 = \left(\frac{g_a kT}{Ie}\right)^2 \text{ ------ } 2\text{-}(22)$$

(The writer is indebted to Professor D.R.Hartree, F.R.S. for deriving (22)). Experiment, however, shows (cf. Introductory Chapter) that this expression consider-:ably underestimates the fluctuation observable. It appears then that the analysis suffers error in the transition from (17) to (18); which conclusion is inescapable since the remainder is pure algebra.

It appears in fact that we cannot recognise a departure of the emission j from its average value, (J), without concomitantly recognising statistical departure from the average (Maxwell-Boltzmann) law. That is to say, <u>only</u> departures (or <u>fluctuations</u>) from the "normal" (or average) law as expressed by (17) can give rise to random fluctuations in j of the fundamental type considered, or expressed otherwise, the fundamental fluctuations in j, with the resultant fluctuations in i and v_m, are only another aspect of the basic statistical fluctuations in the normal law of distri:bution of emission velocities on which (17) is based.

Before proceeding, however, to the more refined analysis based on this realisation, we may carry out a simple transformation of the last result and show its direct equivalence to Llewellyn's earlier analysis on the basis of the Maxwell-Boltzmann law.

We have :

$$I = J\varepsilon^{\frac{eV_m}{kT}} \text{ --------- } 2\text{-}(17)$$

$$\therefore \frac{dI}{dJ} = \varepsilon^{\frac{eV_m}{kT}} + \frac{eJ}{kT}.\varepsilon^{\frac{eV_m}{kT}}.\frac{dV_m}{dJ}$$ (regarding J as the independent variable)

$$\therefore \frac{dI}{dJ} = \frac{I}{J} + \frac{I}{J}\left\{\frac{eJ}{kT}.\frac{dV_m}{dJ}\right\}$$

whence :

$$\left\{1 + \frac{eJ}{kT}.\frac{dV_m}{dJ}\right\} = \frac{J}{I}.\frac{dI}{dJ} \text{ ---------- } 2\text{-}(23)$$

Comparing then (21) with (15a), using (23), identity is established.

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2. 4. Refined analysis of Schottky, Thompson and North, and Rack.

Following upon the indications contained in the last Section the above-mentioned workers have concentrated attention upon the detailed character of the emission fluctuations. The emission, j, is supposed divided up into elementary velocity classes according to the Maxwell-Boltzmann law; the fluctuations in each velocity-class are presumed independent of one another and to obey the simple shot fluctuation law. The resulting fluctuations of i in each velocity-class are calculated in detail assuming that one may employ the Langmuir-Fry law with small perturbations in each velocity-class in turn, meanwhile assuming the remaining velocity-classes to be unperturbed. Finally a summation (clearly an integration in the ~~minit~~ limit) is carried out over all velocity-classes, "weighted" according to the average Maxwell-Boltzmann law, to provide the overall resultant fluctuation in the space current. The general result obtained has already been stated in the Introduction, namely :

$$\Gamma^2 = \frac{2\theta kT}{Ie} \cdot g_a \text{------------} 2\text{-}(24)$$

$$\therefore \overline{(i-I)^2} = \frac{4}{R_a} \cdot k(\theta T) . \Delta f \text{-----------} 2\text{-}(25)$$

There seems little doubt that the details of this analysis are essentially correct, but the writer feels that the fundamental principles underlying the phenomenon are obscured. For example, the démarche of

North quoted in the Introductory Section is typical.

It is certainly clear as mentioned above that any attempt - and such attempts have been made in the past - to apply a thermodynamical analysis to the valve as a whole must be invalid. Difficulties of this nature are clearly evidenced when one considers the transition from (25), applicable to a diode, to the triode problem. North has shown, by introducing the concept of the "equivalent diode" with anode in the grid plane of the triode, that for triodes - at any rate with reasonably closely-spaced grids - whose control grids are negative :

$$\overline{(i-I)^2} = \sigma . 4 g_m . k(\theta T) . \Delta f \text{ - - - - - - - } 2\text{-}(26)$$

where :

$g_m = \frac{\partial I}{\partial V_g}$ (the "transfer") ("mutual") conductance of the triode, and V_g is the grid voltage.)

and :

σ is a constant factor for each valve mainly determined by the valve geometry such that $0 < \sigma \leq 1$ and generally $\sigma \geq \cdot 75$.

Thus we see that emphasis in this case has shifted from the anode-cathode terminals of the valve by inserting a grid, which act however would not be expected to disturb in so radical a manner a fundamental overall thermal equilibrium if such indeed existed.

....

2. 5. Unified statement of valve fluctuation problem.

The basic assumption underlying the following discussion is not claimed as entirely original; the writer believes that a contributor (?Dr. D. Gabor) to an unrecorded discussion of the Institute of Physics on Fluctuations led by Drs. P.B. Moon and Nimmo at Birmingham in 1943 made some verbal suggestion of this nature without quantitative development. We also feel that in effect this concept has underlain the more recent advanced work of Dr. Llewellyn (much of it unpublished (March,1946)); very probably other workers have thought on similar lines. As far as the writer is aware, however, a precise statement of the concept and the development to give a unified treatment of the phenomenon has not been stated elsewhere.

The basic assumption is simply that the retarding "cathode-barrier" region shall always be regarded as a region in thermal equilibrium at cathode temperature, T. Due, however, to the fact that the barrier surface is a "rectifying" plane - i.e. an accelerating field exists "to the right" of the barrier only one half precisely of the fluctuation ~~observed~~ expected is transmitted and observable. In other words, the barrier plane is like a "sticky" electrical plane in the random walk problem treated by Chandrasekhar (1) and others. Another rather striking way of expressing the significance of the barrier lies in the electro-dynamical solution of a retarding region. If at some plane a Maxwell-Boltzmann equilibrium distribution of density ρ_0 at temperature T exists, then at a rectifying plane negative to the initial plane by a voltage V, the density is given by $\rho_V = \frac{1}{2}\rho_0 \varepsilon^{\frac{eV}{kT}}$, if no intervening potential minimum exists. On the other hand if no "sticky" plane is permitted, the purely thermodynamical solution is $\rho_V = \rho_0 \varepsilon^{\frac{eV}{kT}}$.

It is, however, now most important to recognise that Nyquist's equation :

$$\overline{(i-I)^2} = \frac{4}{R} kT\Delta f \quad\text{------------}\quad 1\text{-}(4b)$$

only applies to an element assumed to have an unfluctuating potential difference across it (by the application of Helmholtz' theorem). That is, we have now somehow to take account of the effect of the barrier-anode region in relation to the cathode-barrier region in modifying the fluctuations observed externally. This is a formally identical process to the modification (by the reduction factor R_a/R_a+Z) of the overall valve fluctuations observed externally in an impedance in its relation to the overall valve resistance R_a.

Thus the basic principle of the fluctuations is expressed by the following equation for the short-circuit fluctuation of the cathode-barrier region:

$$\overline{(i-I)_b^2} = \frac{1}{2}\left(\frac{4}{R_b} . kT\Delta f\right) = \frac{2kT}{R_b} . \Delta f \quad\text{-----}\quad 2\text{-}(27)$$

To allow then for the modifying effect of the "feed-back" due to the barrier-anode region we write formally:

$$\overline{(i-I)_b^2} = \frac{2kT}{R_b}(F^2)\Delta f \quad\text{-----}\quad 2\text{-}(28) \qquad [0 < F^2 \leq 1]$$

If now we adopt the earlier concept (Sections 2 and 3 of this Chapter) of a "steady-state", one valued, equation connecting I, J and V_m, then F^2 can be immediately evaluated by straightforward theory. We have in fact :

$$F = \frac{R_b}{R_b + Z} \text{ --------- 2-(29)}$$ (See Figs. 2a and 2b.)

where : $Z = \frac{\partial |V_m|}{\partial I}$

This result follows directly from the fundamental "feed-back" equations applied to this problem :

$$\delta I' = \delta I + \delta I' \left\{ g_b \cdot \frac{\partial |V_m|}{\partial I} \right\} \text{ ... 2-(30)}$$

and : $$F = \frac{\delta I'}{\delta I} \text{ ----------- 2-(31)}$$

) with $g_b = \frac{\partial I}{\partial V_m} = \frac{1}{R_b}$

where :

δI is the "primary" or unmodified current-variation.

$\delta I'$ is the "resultant" or modified current-variation due to the "feed-back".

Such equations are, in fact, of value in the fundamental analysis of any problem involving "negative feed-back", such as a cathode-follower valve, for example.

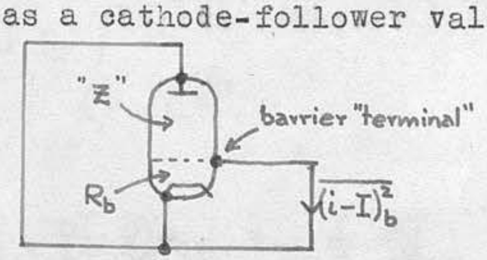

Fig. 2a

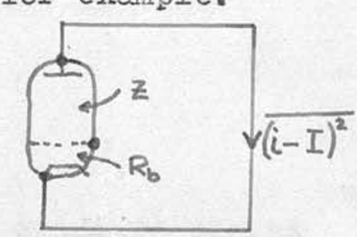

Fig. 2b.

Thus the basic formula for any analysis based on a steady-state relationship must be :

$$\overline{(i-I)^2} = \frac{2kT}{R_b} \left(\frac{R_b}{R_b + Z} \right)^2 \cdot \Delta f \text{ --------- 2-(32).}$$

It should be observed that no additional contribution arises in respect of " Z ", since no fluctuations can originate there fundamentally; only those that are injected from R_b can exist. Expressed otherwise, the impedance Z is always to be regarded as at zero temperature.

Alternatively to (32), in a parallel-plane structure, since $I = J\varepsilon^{\frac{eV_m}{kT}}$:

$$\therefore \frac{1}{R_b} \equiv \frac{\partial I}{\partial V_m} = \frac{eI}{kT}$$

$$\therefore \overline{(i-I)^2} = 2eI.\left(1+\frac{Z}{R_b}\right)^{-2}.\Delta f \text{ ---------- } 2\text{-}(33)$$

where we may write formally in this case:

$$\Gamma^2 = \left(1+\frac{Z}{R_b}\right)^{-2} \text{ --------- } 2\text{-}(34)$$

Now it is evident that in general $\left(\frac{\partial|V_m|}{\partial I}\right)_{V_a\,const.} \neq \left(1\Big/\frac{\partial I}{\partial V_a}\right)_{V_m\,const.}$ but nevertheless if we were to permit this relation and substitute in (34) we immediately arrive at the expression which Schottky (2) in his earlier work derived after rather lengthy analysis, namely :

$$\Gamma^2 = \left\{1+\left[\frac{\partial I}{\partial V_m}\Big/\left(\frac{\partial I}{\partial V_a}\right)_{V_m}\right]\right\}^{-2} \text{ -----}$$ (his equation (13) in our notation).

Further, if we postulate as before that we may relate I to J by :

$$I = J\varepsilon^{\frac{eV_m}{kT}}$$

then :

$$-\frac{Z}{R_b} = \frac{eI}{kT}\cdot\frac{\partial V_m}{\partial I} = 1 - \frac{I}{J}\cdot\frac{dJ}{dI}$$

, where we regard $\frac{dJ}{dI}$ as a full differential, $\therefore = 1\Big/\frac{dI}{dJ}$ since J is the independent variable, i.e. variations of V_a are excluded, which was effectively Schottky's error in arriving at his equation (13). Thus, we have on this basis :

$$(i-I)^2 = \frac{2eI\Delta f}{\left(\frac{I}{J}\cdot\frac{dJ}{dI}\right)^2} = 2eI\left(\frac{J}{I}\cdot\frac{dI}{dJ}\right)^2\Delta f \text{ ---- } 2\text{-}(35)$$

which we have shown previously is the corrected result of Llewellyn's early analysis. Since we have already shown that that expression is identical to that derived in the writer's early analysis we have completed the derivation of all the early analyses from our fundamental equation.

Finally, then experiment having shown these analyses to be in error we must clearly redirect our attention to the determination of $F = \frac{\delta I'}{\delta I}$. This then is essentially the problem that Schottky and North tackled ultimately (Section 4 of this Chapter). In view of the form of the basic valve analysis employed (that of Langmuir (1) and Fry (2)) it proves most simple to calculate F on the following basis, when using the velocity-class distribution concept.

Let I be the undisturbed mean current in the valve.

Let δI_v be the primary or unmodified current fluctuation in a given velocity class v to $v+dv$.

Let $\hat{I}$ be the computed "new" mean current in the valve resulting from the disturbance δI_v but not including δI_v itself.

Now using this concept it proves necessary to recognise two types of feed-back factor F ; one arises when the initial current fluctuation belongs to a velocity class, v , such that the electron energy is in fact insufficient to carry it across the barrier, and therefore δI_v itself does not occur in the "modified" current fluctuation $\delta I'$; this case we call "α", in conformity with North. In case "β", the electron energy is in fact sufficient, and δI_v does therefore contribute directly to $\delta I'$.

Thus for case α :

$$\delta I' = \hat{I} - I$$

$$\therefore F_v \equiv \frac{\delta I'}{\delta I_v} = \frac{\hat{I}-I}{\delta I_v} \text{ --------- } 2\text{-}(36)$$

for case β :

$$\delta I' = \hat{I} - I + \delta I_v$$

$$\therefore F_v \equiv \frac{\delta I'}{\delta I_v} = 1 + \frac{\hat{I}-I}{\delta I_v} \text{ ------ } 2\text{-}(37)$$

The detailed work of North and Schottky then rests in evaluating equations (36) and (37) for all v, and then determining a weighted average for F^2.

i.e. formally : (using equation (28))

where W is a dimensionless factor proportional to the energy of the electrons, E, occurring in the average statistical law of distribution

$$p(E)dE = A\varepsilon xp(-E/kT).dE$$

$$\overline{(i-I)^2} = \frac{2kT}{R_b}\left[\int_0^\infty F_v^2.p(W).dW\right].\Delta f \text{ ----- } 2\text{-}(38)$$

This is regarded by the writer as the fundamental mode of expression of the result. If we wish however to throw this formula into the shape of (25) then we must write :

$$\overline{(i-I)^2} = \frac{4kT}{R_a}.\left[\frac{1}{2}.\frac{R_a}{R_b}.\int_0^\infty F_v^2.p(W).dW\right].\Delta f \text{ ------- } 2\text{-}(39)$$

and : $\theta = \frac{1}{2}.\frac{R_a}{R_b}\int_0^\infty F_v^2.p(W).dW$ ---------- 2-(40)

Equation (40) then defines essentially a valve parameter, θ. Since in the space-charge limited region, generally, differential valve parameters tend to assume a nearly constant behaviour as evidenced by the quasi-linear nature of the characteristics it is perhaps not surprising that the valve parameter θ when evaluated does not vary rapidly over the same region. We observe however immediately that the insertion of a grid would radically alter the magnitude of θ by affecting very seriously the factor $\frac{R_a}{R_b}$, indicating why the application to a triode valve would necessitate a very severe change in θ so defined.

Naturally the numerical evaluation of (40) will not be simple because the fundamental solution of current-potential problems in thermionic valves is not simple in general, but it is believed that unification of outlook is offered in the foregoing discussion.

To conclude, the writer suggests therefore that in order to obtain a unified concept of the whole problem, one need only suppose a fundamental thermal fluctuation to exist in basic electrical elements, and that "space-charge reduction" simply results from the effect of having various impedances in parallel; naturally in some cases the detailed calculation of these is more difficult than in others. The writer feels that the term "(apparent) internal impedance reduction" describes the phenomenon more clearly than others employed; in particular perhaps the word <u>smoothing</u> should be avoided; the reduction on this concept is no more to be regarded as smoothing than the obvious reduction of current which arises in a given resistance fed from a constant current generating source when another resistance is placed in parallel.

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Appendix to Chapter 2.

True smoothing effect due to correlation.

To lay any claim to completeness in a survey of this problem it appears essential to give mention to the suggested theory that a valve operating outwith ultimate saturation conditions introduces a degree of correlation between the individual events and thus introduces true smoothing of the fundamental fluctua-:tions. Something of this sort may have been in the mind of Llewellyn in his first paper (1) discussed above when he says ".....then small fluctuations in the "density of the stream emitted by the filament are all "smoothed out in the space-charge region, and the current "reaching the plate contains no variations which result "from changes in the filament emission". The most pleasing analytical statement of this hypothesis appears in an appendix to a paper by Thatcher and Williams (1), in which they state ..."A theory proposed "by Uhlenbeck sought to explain this behaviour (i.e. "space-charge reduction) on the basis of statistical "correlation between instantaneous values of the anode "current". That is to say, it is suggested that the introduction of a correlation function which is not effectively a pure Dirac- δ-function of time would account for the phenomenon. We note that the employ:ment of the correlation concept applied to Brownian motion generally appears in 1917 in a paper by Professor L.S. Ornstein (1) and is also employed in specific electrical fluctuation problems later by Ornstein and Burger (2).

Since the results of the analysis are of great value in the statistical investigation of the fluctua:tion records generally, apart from this particular problem, it seems useful to give it here in some detail. This is now presented in a modified version which may then be used readily in conjunction with Schottky's basic "shot fluctuation" formula.

Let the mean current fluctuation measured over a very short interval τ be represented by its Fourier transform :

$$\Delta i_\tau = \int_{-\infty}^{\infty} F(\omega).\,\varepsilon^{i\omega t} d\omega \quad \text{, where } F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Delta i_\tau \varepsilon^{-i\omega t}.dt$$

$$\therefore \overline{\Delta i_\tau^2} = \overline{\int_{-\infty}^{\infty} F(\omega).\varepsilon^{i\omega t}.d\omega \int_{-\infty}^{\infty} F(\mu).\varepsilon^{i\mu t} d\mu}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{F(\omega).F(\mu)}.\varepsilon^{i(\omega+\mu)t} d\mu\, d\omega$$

, assuming the validity of the interchange of the "averaging" and integrating operations.

But

$$\overline{F(\omega).F(\mu)} = \overline{\frac{1}{4\pi^2}\int_{-\infty}^{\infty} \Delta i_\tau(t_1)\varepsilon^{-i\omega t_1} dt_1 \int_{-\infty}^{\infty} \Delta i_\tau(t_2)\varepsilon^{-i\mu t_2} dt_2}$$

Now let $t_1 = t_2 + T$

$$\therefore \overline{F(\omega).F(\mu)} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\Delta i_\tau(t_2).\Delta i_\tau(t_2+T)}\,\varepsilon^{-i(\omega+\mu)t_2}.\varepsilon^{-i\omega T}.dt_2.dT$$

Now if the fluctuations be assumed to be a stationary statistical process (i.e. such that a time-ensemble is equivalent to a virtual ensemble) then :

$\overline{\Delta i_\tau(t_2).\Delta i_\tau(t_2+T)} = \psi(T)$, a function of T alone, which we may call the unnormalised correlation of Δi_τ.

Thus :

$$\overline{F(\omega).F(\mu)} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} \varepsilon^{-i(\omega+\mu)t_2}dt_2 \int_{-\infty}^{\infty}\psi(T).\varepsilon^{-i\omega T}dT$$

Now the obvious barrier to further rigorous progress is the integral $\int_{-\infty}^{\infty}\varepsilon^{-i(\omega+\mu)t_2}.dt_2$ which is undefined and therefore apparently offers support to T.C. Fry's contention mentioned earlier that to discuss the spectrum of the noise current itself is strictly speaking unpermissible. However, the same integral occurs in a class of network problems (e.g. MacDonald (1)) and by approaching it as the limit of a Fourier summation it is found that it may be represented consistently by a Dirac-δ-function. i.e.

$$\int_{-\infty}^{\infty}\varepsilon^{-ixt}dt \triangleq 2\pi.\delta(x)$$ (where we indicate by $\triangleq$, "is equal by definition").

Thus :

$$\overline{F(\omega).F(\mu)} = \delta(\omega+\mu).\Phi(\omega)$$, where $\Phi(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi(T).\varepsilon^{-i\omega T}.dT$

Hence :

$$\overline{\Delta i_\tau^2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(\omega+\mu).\Phi(\omega).\varepsilon^{i(\omega+\mu)t}.d\mu.d\omega$$

Let now

$$\theta = \omega+\mu$$

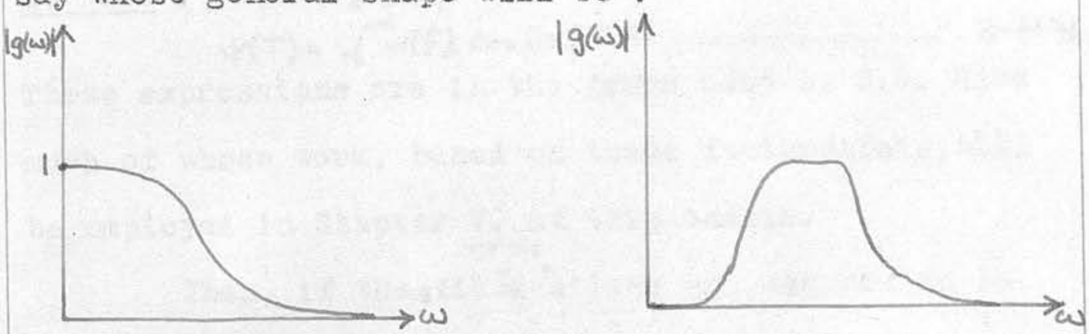
$$\therefore \overline{\Delta i_\tau^2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(\theta).\Phi(\omega).\varepsilon^{i\theta t}d\omega d\theta$$

$$= \int_{-\infty}^{\infty}\Phi(\omega)d\omega\int_{-\infty}^{\infty}\delta(\theta).\varepsilon^{i\theta t}d\theta$$

$$= \int_{-\infty}^{\infty}\Phi(\omega).d\omega$$

This integral as it stands will not be expected to con-:verge on classical theory although the introduction of quantum mechanical theory will cause it to do so. How-

:ever, in the practical case, we must recognise that we observe these fluctuations by some instrument whose frequency response in fact is limited. This may be taken account of by introducing a response factor $g(\omega)$ say whose general shape will be :



Thus for the observed mean square fluctuation we write:

$$\overline{\Delta i_\tau^2} = \int_{-\infty}^{\infty} g(\omega).\Phi(\omega).d\omega$$

And we remark that $\Phi(\omega)$ may be regarded as the power spectrum in angular frequency of the fluctuations. The very significant result, (due to Wiener[1] and Khintchine[1]), is then evident that the power spectrum and the correlation of the fluctuation are intimately related as Fourier transforms :

i.e. $$\Phi(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi(T).\varepsilon^{-i\omega T}dT \qquad 2\text{-}(41)$$

And therefore by the reciprocal property of the Fourier relation :

$$\psi(T) = \int_{-\infty}^{\infty}\Phi(\omega)\varepsilon^{i\omega T}d\omega \qquad 2\text{-}(42)$$

Since $\psi(T)$ must be an even function of T as a result of the stationary property of the fluctuation and $\Phi(\omega)$ must inevitably be an even function of ω, we may write:

$$\Phi(\omega) = \frac{1}{\pi}\int_{0}^{\infty}\psi(T).\cos\omega T dT \qquad 2\text{-}(41a)$$

$$\psi(T) = 2\int_{0}^{\infty}\Phi(\omega).\cos\omega T d\omega \qquad 2\text{-}(42a)$$

In terms of frequency, f, denoting by $w(f)$, the "power spectrum" (in frequency) of the fluctuation defined by

$$\overline{\Delta i_\tau^2} = \int_0^\infty g(f).w(f).df$$

we have : $w(f) = 4\int_0^\infty \psi(T).\cos 2\pi f T.dT$ ---------- 2-(43)

$\psi(T) = \int_0^\infty w(f).\cos 2\pi f T.df$ ---------- 2-(44)

These expressions are in the forms used by S.O. Rice much of whose work, based on these fundamentals, will be employed in Chapter V. of this thesis.

Then, if the fluctuations are assumed to be entirely random, $\psi(T)$ will only be non-zero over the very short interval τ and :

$$\Phi(\omega) \doteq \frac{1}{2\pi}.\overline{\Delta i_\tau^2}.\tau$$

But by Schottky's original result $\overline{\Delta i_\tau^2}.\tau = eI$ (vide 1-(2).)

Hence : $$\overline{\Delta i_\tau^2} = \frac{eI}{2\pi}\int_{-\infty}^{\infty} g(\omega).d\omega$$

$$= \frac{eI}{\pi}\int_0^\infty g(\omega).d\omega$$

which is the familiar shot fluctuation formula expressed in the Fourier form.

If, however, we suggest with Uhlenbeck that correlation may exist over larger intervals than the fundamental interval of observation, τ, then $\Phi(\omega)$ will be reduced in magnitude and $\overline{\Delta i_\tau^2}$ will in consequence also be reduced indicating true <u>smoothing</u> of the fluctuations. At the same time, of course, the "power spectrum" will no longer be uniform as was the case with vanishingly short correlation and in fact to produce anything approaching the degree of reduction observed experimentally, it would be necessary to postulate an extremely "broad" correlation function. This would then of necessity introduce a most evident decay of the frequency spectrum even at relatively low frequencies, which is in fact not observed.

Undoubtedly, of course, the correlation function cannot be an ideal δ-function, but there is no evidence of any correlation between individual events in thermionic valves furnished by experiment. On the other hand, of course, when the duration of an individual event becomes of significance then a reduction does display itself with the inevitable "decay" of the spectrum, and this is the basis of the fluctuation smoothing (the word is purposely employed in this context) due to "transit time" in valves evident at high radio frequencies and dealt with by Ballantine (I), Spenke (I) and A. J. Rack (I).

CHAPTER 3.

EXAMINATION OF THE RETARDING REGION OF THERMIONIC VALVES.

3. 1. General Statement.

If a parallel-plane diode has an emission J from a cathode at temperature T obeying the Maxwell-Boltzmann law for the distribution of the emitted electrons then we may readily determine the current I which flows if the anode potential is V_a and the field is wholly retarding from cathode to anode. Two limiting methods of analysis are possible. In the first, which we shall call the electro-dynamical (E-D) method, the electrons are supposed to interact in the inter-electrode space solely through the average potential V at any point as given by Poisson's equation :

$$\nabla^2 V = 4\pi\rho \qquad \text{3-(1)}$$

or in planar co-ordinates :

$$\frac{\partial^2 V}{\partial x^2} = 4\pi\rho(x) \qquad \text{3-(1a)}$$

and the trajectories are evaluated by assuming that at every point we may set :

$$\tfrac{1}{2}m(v_0^2 - v_x^2) = eV \qquad \text{3-(2)}$$

Expressed otherwise, the motion is supposed individually conservative throughout.

In the second, we assume that a state of thermal equilibrium effectively exists throughout the inter-space. This analysis, which we shall call the thermo-dynamical (T-D) method, must clearly certainly fail very close to the anode since a current is being drawn from the system (in other words the valve is supplying energy to the battery - in fact at a rate IV_a), but if this current is very small in comparison with the emission J, (or in other words that $I.V_a$ is very small compared with the total power "available" in the emission), the error involved in the analysis will be expected to be very small.

In the particular problem mentioned above the solutions are identical; in other evaluable instances small differences do exist as will be shown; on the other hand, in certain cases the T-D method yields results much more readily than the E-D methód and may therefore be of value on those occasions. In fact, the conditions will always be expected to be a compromise between the two treatments. Langmuir and Compton (2) suggest that the E-D treatment will always obtain in practice since individual "collisions" are unlikely except under exceedingly high space-charge densities, and the T-D treatment essentially implies maintenance of energy equipartition by continuous interaction. However, it is not a matter of certainty to define a significant collision. We may also note that in both cases, the solution becomes strictly invalid for very small space-charge densities due to the particle nature of the charge. This is pointed out by Laue (1) who has developed solutions for a number of cases based on the T-D treatment, and whose work will be used further below.

Returning to the immediate problem, the treatment is based on Boltzmann's equation :

$$\rho = \rho_0 \varepsilon^{\alpha V} \quad \text{3-(3)} \quad \left[\alpha = \frac{|e|}{kT}\right]$$

in conjunction with Poisson's equation. The solution on both treatments is readily shown to be :

$$I = J\varepsilon^{\alpha V} \quad \text{3-(4)}$$

This equation is of fundamental significance. We have :

$$\log I = \alpha V + \log J$$

$$\therefore \frac{d(\log I)}{dV} = \frac{|e|}{kT} \quad \text{3-(5)}$$

In this form it appears that the measurement of the slope of the log $I:V$ curve for a valve in the retarding region should yield a determination of the temperature, T, of the cathode. Alternatively, we may regard this equation, given a knowledge of T, as providing an experimental method of verifying the law of emission; the evidence in this case being whether or not the log $I:V$ characteristic does ultimately provide linearity. It is in this form (and in analogous form for cylindrical structures to be discussed below) that the main body of experimental work has been done. Familiar names in this field are Schottky [4,5], Germer and Davisson [1] and Rothe [1,2]. In general the results have been satisfactory although certain inconsistencies appeared. One general disadvantage however in all these experimental investigations is the use of a logarithmic plot for the determination; the use of logarithmic plotting can readily mask a slow "power" variation of a variable, and since it is absolutely essential to ensure that the retarding region has been entered this point is of some importance. An example of this was made very evident at a recent

Thermionics Colloquium (*) when it was suggested that very violent errors in the past (e.g. Kingdon's work) had occurred in the experimental determination of the emission constants in Richardson's equation as a consequence of gradual transition from one logarithmic plot to another due to the varying adsorption of oxygen with temperature when plotting log I/T^2 against $1/T$. The author at that time suggested that some other form of plotting or an alternative form of experimental procedure leading to the use of different significant variables not involving logarithmic variation would probably have clearly shown up the truly non-linear variation involved. As a second objection in this particular case, the author does not favour the measurement of voltage accurately.

To avoid these problems we consider :

$$I = J\varepsilon^{\alpha V} \quad \text{3-(4)}$$

$$\therefore \frac{\partial I}{\partial V} = \frac{1}{R_a} = \alpha J \varepsilon^{\alpha V} = \frac{|e|I}{kT} \quad \text{3-(5a)}$$

Thus if the true retarding region is reached we should have :

$$\frac{|e| I R_a}{kT} = 1 \quad \text{3-(6)}$$

That is, the product IR_a should remain strictly constant in that region. In this form, no measurement of voltage is called for and logarithmic variation does not enter into the problem. Earlier experimental workers were primarily concerned with determination of temperature and therefore, apart from problems of direct measurement, it was not a matter of great concern whether

(*: See Vick:(1))

very small currents had to be employed or in fact at what current value the retarding region was actually reached. Since, however, this work was primarily undertaken in connection with fluctuation measurements it became a matter of vital concern to consider the critical current value. The theoretical value may be readily determined in the parallel-plane case either by the E-D or T-D theory.

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3. 2. E-D theory of limiting current for parallel-plane case.

The general solution of the parallel-plane diode has been presented by Langmuir (1) and Fry (2) and in dimensionless co-ordinates may be written :

$$\left(\frac{d\eta}{d\xi}\right)^2 = \varepsilon^{\eta} - 1 - \left(\frac{2}{\sqrt{\pi}}\left[\eta^{1/2} - \varepsilon^{\eta}\,\mathrm{erf}\,\eta^{1/2}\right]\right) \text{------ } 3\text{-}(7) \quad \text{(See Fig. 3.)}$$

for the cathode-barrier region. $\mathrm{erf}\,t = \int_0^t \varepsilon^{-x^2}dx$

Now under the conditions within which we shall apply the analysis the ratio $\frac{I}{J}$ will be very small; that is to say η at the cathode will be very large. Therefore we consider this equation as $\eta \to \infty$.

$\eta = \frac{e}{kT}(V - V_m)$

$\xi = \left[4.\left(\frac{\pi}{2kT}\right)^{3/4}.m^{1/4}.e^{1/2}.J^{1/2}\right].(x-x_m)$

$= A.J^{1/2}(x - x_m)$

(where J is the current density).

$$\left(\frac{d\eta}{d\xi}\right)^2_{\eta\to\infty} \doteq 2\varepsilon^{\eta}$$

i.e.

$$\frac{d\eta}{d\xi} = -\sqrt{2}\,\varepsilon^{\eta}, \quad \therefore \frac{d\eta}{\varepsilon^{\eta}} = -\sqrt{2}\,d\xi$$

$$\therefore \sqrt{2}\,\xi = -\varepsilon^{-\eta} + C$$

i.e. as $\eta \to \infty$; ξ_c tends to a finite limit.

For $\frac{I}{J} \to 0; \xi_c \to -2.554$, while for $\frac{I}{J}$ as low as $7.4 (\varepsilon^2)$, $\xi_c = -2.013$

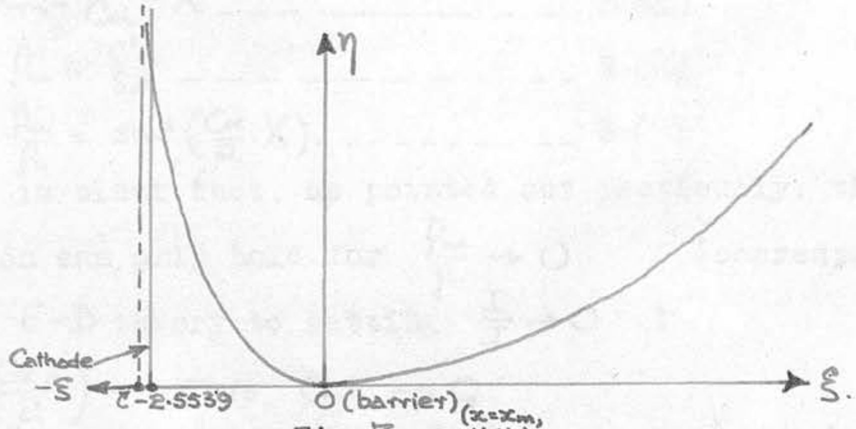


Fig. 3.

Thus save in very exceptional cases we may set, in general, :

$$\xi_c \doteq -2.5$$

i.e. $A . J^{1/2} . (x - x_m)^{1/2} \doteq -2.5$

But if the retarding region has just been reached then x_m co-incides with the anode surface and $x_m - x_c = d$, the electrode separation, and therefore the critical current density is given by

$$J \doteq \frac{(2.5)^2}{A^2 d^2} = \frac{(2.5)^2 (kT)^{3/2} . \sqrt{2}}{\pi^{3/2} . 8e\sqrt{m} . d^2} \qquad \text{-------- -- 3-(8)}$$

....

3. 3. T-D theory of limiting current for parallel-plane case.

$$\rho = \rho_0 \varepsilon^{\alpha V(x)}$$

$$\frac{\partial^2 V}{\partial x^2} = 4\pi\rho = 4\pi\rho_0 \varepsilon^{\alpha V(x)}$$

Solving we readily obtain :

$$\rho = \frac{A^2 \alpha}{8\pi} . \frac{1}{\sinh^2\left[\frac{A\alpha}{2}[X + x]\right]} \qquad \text{(Lame [1] - p.213)}$$

in the case that no potential minimum is supposed to exist,

or $$\rho = \frac{C^2 \alpha}{8\pi} . \frac{1}{\sin^2[C\alpha(X + x)]} \qquad \text{---------- 3-(9)}$$

(X, C : Integration constants.)

where a potential minimum exists for $\frac{C\alpha}{2}(X+x) = \frac{\pi}{2}$

i.e. $x_m = \frac{\pi}{C\alpha} - X$ ——— 3-(9a)

and : $\rho_m = \frac{C^2\alpha}{8\pi}$ ——— 3-(10)

i.e. $\frac{\rho_m}{\rho_0} = \sin^2\left(\frac{C\alpha}{2}.X\right)$ ——— 3-(11)

But it is clear that, as pointed out previously, the solution can only hold for $\frac{\rho_m}{\rho_0} \to 0$ (corresponding in the E-D theory to setting $\frac{I}{J} \to 0$)

$\therefore \left(\frac{C\alpha X}{2}\right)^2 \to 0$ i.e. $C\alpha X \to 0$

$\therefore$ from (9a)

$$C\alpha x_m \simeq \pi \text{ ——— 3-(9b)}$$

and $\therefore$ from (10) immediately :

$$\rho_m = \frac{\pi}{8\alpha x_m^2} = \frac{\pi}{8\alpha d^2} \text{ ——— 3-(12)}$$

Finally, on the assumption that thermal equilibrium exists "almost up to" the anode, the current density there is related to ρ_m by the well-known relation :

$$\mathcal{J} = \rho_m \sqrt{\frac{kT}{2\pi m}} \text{ ——— 3-(13)}$$

, based on the Maxwell-Boltzmann distribution laws.

$\therefore$ from (12) and (13).

$$\mathcal{J} = \frac{(kT)^{3/2}\,\pi^{1/2}}{8\sqrt{2}.e\sqrt{m}\,d^2} \text{ ——— 3-(14)}$$

3. 3. Comparison and application of results.

(8) and (14) then agree to a numerical factor in the limiting case ($I/J \to 0$) of :

$$\frac{\mathcal{J}_{T-D}}{\mathcal{J}_{E-D}} = \frac{\pi^{1/2}\;\pi^{3/2}}{\sqrt{2}\;\sqrt{2}(2.554)^2} = 0.76.$$

In any given case we should certainly expect $\mathcal{J}_{T-D} \leq \mathcal{J} \leq \mathcal{J}_{E-D}$.

It is immediately clear that if we wish $\mathcal{J}$ not to be very small, since fluctuation measurements become excessively difficult with very small currents, it will be necessary to use close spacing of the electrodes (i.e. d small). As an example let us consider the 6H6 type of diode. This is in fact of cylindrical construction as are all the diodes used in this work, but the formula should give a reasonable indication since the spacing is small.

We have :

$l = 0.6$ cm. (coated length of cathode)

$r_c \simeq 0.06$ cm. (cathode radius)

$r_a \simeq 0.1$ cm. (anode internal radius)

$\therefore d \simeq 0.04$ cm.

The practical form of (8) is :

$$J_{E.D} \doteqdot \frac{7 \times 10^{-12} \times T^{3/2}}{d^2} \text{ amps./sq.cm.} \qquad 3\text{-}(15)$$

and we find

$$I_{E.D} \simeq 30 \mu\text{amp.}$$

....

3. 5. Fundamental equation for cylindrical case.

When we turn to the cylindrical problem proper, we first consider what form equation (6) now takes. Schottky (4) has evolved on an E-D basis the analogous equation to (4) for the cylindrical case under the retarding regime. His equation is :

$$I = \frac{2}{\sqrt{\pi}} . J \left\{ \varepsilon^{\alpha V} \int_0^{\sqrt{\frac{\alpha V}{\eta}}} \varepsilon^{-(1-\eta)x^2} dx + \int_{\sqrt{\frac{\alpha V}{\eta}}}^{\infty} \varepsilon^{-x^2} dx \right\} \qquad 3\text{-}(16)$$

[where we set $\eta = 1 - \frac{r_c^2}{r_a^2}$.]

When this has been applied to the log I : V method in the past the approximation $\frac{r_c}{r_a} \to 0$ (i.e. $\eta \to 1$) is always made before the logarithmic differentiation. In our method however the general form of the formula may readily be retained, and this seems very advisable since the structures of the diodes used can hardly be regarded as limiting cylindrical cases.

From (16) :

Let $y = \sqrt{-\frac{\alpha V}{\eta}}$; $\therefore \frac{dy}{dV} = -\frac{\alpha}{2\eta} \Big/ \sqrt{-\frac{\alpha V}{\eta}}$

And

$$\frac{\partial I}{\partial V} \equiv \frac{1}{R_a} = \frac{2J}{\sqrt{\pi}}\left[\alpha \varepsilon^{\alpha V}\int_0^y \varepsilon^{-(1-\eta)x^2}dx - \frac{\alpha}{2\eta}\varepsilon^{\alpha V}\Big/\sqrt{-\tfrac{\alpha V}{\eta}}\cdot\varepsilon^{+\frac{(1-\eta)\alpha V}{\eta}} + \frac{\alpha}{2\eta}\Big/\sqrt{-\tfrac{\alpha V}{\eta}}\cdot\varepsilon^{\frac{\alpha V}{\eta}}\right] \text{------} 3\text{-}(17)$$

$$\therefore \frac{1}{\alpha R_a} = \frac{2J}{\sqrt{\pi}}\left[\varepsilon^{\alpha V}\int_0^y \varepsilon^{-(1-\eta)x^2}dx + \frac{1}{2\eta\sqrt{-\frac{\alpha V}{\eta}}}\left\{\varepsilon^{\frac{\alpha V}{\eta}} - \varepsilon^{\alpha V}.\varepsilon^{+\frac{(1-\eta)}{\eta}\alpha V}\right\}\right]$$

The second term vanishes identically and thus :

$$\frac{1}{\alpha R_a} = \frac{2J}{\sqrt{\pi}}\left[\varepsilon^{\alpha V}\int_0^y \varepsilon^{-(1-\eta)x^2}dx\right] \text{-------------} 3\text{-}(18)$$

$$\therefore I\alpha R_a \equiv \frac{IeR_a}{kT}^{*} = \left[\varepsilon^{\alpha V}\int_0^y \varepsilon^{-(1-\eta)x^2}dx + \int_y^\infty \varepsilon^{-x^2}dx\right]\Big/\varepsilon^{\alpha V}\int_0^y \varepsilon^{-(1-\eta)x^2}dx \text{---} 3\text{-}(19)$$

$$= 1 + \delta(\eta, V)$$

where

$$\delta(\eta, V) = \left\{\varepsilon^{-\alpha V}\int_y^\infty \varepsilon^{-x^2}dx\right\}\Big/\int_0^y \varepsilon^{-(1-\eta)x^2}dx$$

Now let $1-\eta = \lambda^2$; $t = \lambda x$

then

$$\delta(\eta, V) = \sqrt{1-\eta}.\varepsilon^{-\alpha V}\int_y^\infty \varepsilon^{-t^2}dt\Big/\int_0^z \varepsilon^{-t^2}dt \quad ; \quad \left(\text{where } y = \sqrt{-\tfrac{\alpha V}{\eta}},\ z = \sqrt{-\left(\tfrac{1-\eta}{\eta}\right)\alpha V}\right)$$

$$= \sqrt{1-\eta}\,\varepsilon^{-\alpha V}(1 - \operatorname{erf} y)/\operatorname{erf} z \text{--------} 3\text{-}(20)$$

$$\sim \frac{\sqrt{1-\eta}.\varepsilon^{-(1-\eta)y^2}}{\sqrt{\pi}.y}\left\{1 - \frac{1}{2y^2} + \frac{3}{4y^4} - \frac{15}{8y^6} + \dots\right\}\Big/\operatorname{erf} z \text{------} 3\text{-}(21)$$

$\delta(\eta, V)$ may quite readily be estimated in any particular case. Since the 6H6 type diode departs furthest from the planar condition among the three valve types investigated, we evaluate $\delta(\eta, V)$ for this case. It

*: We omit the modulus sign in future.

is quoted (*) that a reasonable value for the emission density of an oxide-coated cathode is 1mA./mm². This gives a value $J \simeq 30$mA. ; (which is a reasonable value), therefore a current of $I \simeq 30\mu$A. represents a ratio of $\frac{J}{I} \simeq 10^3:1 \simeq \varepsilon^7:1$ as a lower limit. Further $\eta \simeq 0.64$. Tabulating some values :

$\eta = 0.64$ $-\frac{eV}{kT}$	$\delta(\eta, V)$
6	.0038
8	.0010
10	.0003.

Thus it is quite evident that the correction term to be expected is very much less than 1%, and thus equation (6) may be used with confidence in this respect.

....

3. 6. Theory of limiting current in cylindrical case.

The problem of determining the theoretical value of the limiting current for a cylindrical structure is rather more complex. On the E-D theory it seems impossible to progress rigorously beyond a formulation of the space-charge integral; a tabulated solution of the cylindrical diode "away from" the retarding regime has been provided by Wheatcroft [1,2] and this paper illustrates well the complexity of the problem. Möller and Detels [1] in a paper - "Uber die Bestimmung der Glühfaden-temperatur in Elektronenröhren" - present an

*: See Millman:(1).

E-D analysis under the assumption $\frac{I}{J} \ll 1$; they assume that only radial velocities need be considered however, and this, on the face of it, appears immediately to restrict the analysis to filamentary cath:odes where at some considerable distance from the cathode the general movement will always appear to be practically radial. The formula derived by them is :

$$I \doteqdot \frac{(kT)^{3/2}\pi^{3/2} l}{4\sqrt{2m}\cdot e r_a \log^2\left(\frac{r_a}{r_c}\right)} \text{-------} 3\text{-}(22)$$

where l is the effective length of the valve.

$$\doteqdot \frac{40.10^{-12} \cdot T^{3/2} l}{r_a \cdot \log^2(r_a/r_c)} \text{ amps.} \text{-----} 3\text{-}(23)$$ (in practical units)

If we consider the approach to the parallel-plane case, setting $r_a = (1+x)r_c \quad (x \ll 1)$ and $d = x r_c$ then we have:

$$J \doteqdot \frac{6.4.10^{-12} \cdot T^{3/2}}{d^2} \text{ amps./sq.cm.} \text{----------} 3\text{-}(24)$$

which may be compared with equation (15).

In view of the limitations involved in the Möller-Detels analysis and the extreme difficulty of a rigorous E-D analysis the problem was also analysed on the T-D approach. We have :

$$\rho = \rho_1 \varepsilon^{\alpha V(r)} \text{---------------} 3\text{-}(25)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} = 4\pi\rho_1 \varepsilon^{\alpha V} \text{-----------} 3\text{-}(26)$$

The solution for the periodic case*, with the existence of a minimum, is :

$$V = -\frac{1}{\alpha}\{A + 2x + 2\log\sin Cx\}$$

$$\therefore \rho = \rho_1 \varepsilon^{-A} \cdot \varepsilon^{-2x} \cdot 1/\sin^2 Cx$$

$$\therefore \rho = C^2 / 2\pi\alpha r^2 \sin^2\left(C \log\frac{r}{r_0}\right) \text{-----} 3\text{-}(27)$$

($x = \log\frac{r}{r_0}$; noting that for $r = r_0$; $x = 0$ and $V \to -\infty$; hence the radius r_c of the cathode must be in fact greater than r_0).

$(A - \log\{4\pi$ [illegible]

* : see Laue:(1): p.226.

We may note that for anode and cathode very close and setting $r_a = r_0 + \delta$ where $\frac{\delta}{r_0} \ll 1$ we have :

$$\rho_a = \frac{B^2\alpha}{8\pi.\left(\frac{r_a}{r_0}\right)^2.\sin^2\left(\frac{B\alpha}{2}.\delta\right)} \text{ - - - - - - - - - - } 3\text{-}(28) \quad \left(\text{With } \frac{B\alpha}{2} = \frac{C}{r_0}\right)$$

which is identical with the previous result derived for the planar case apart from the factor $\left(\frac{r_a}{r_0}\right)^2$ which in the limit tends to unity.

In general, for a minimum of ρ (and of V) we must have :

$r\sin\left(C\log\frac{r}{r_0}\right)$ maximal,

i.e. $\sin\left(C\log\frac{r}{r_0}\right) + C\cos\left(C\log\frac{r}{r_0}\right) = 0$

i.e. $\tan\left(C\log\frac{r}{r_0}\right) = -C$

i.e. at the anode (radius r_a) we have to satisfy

$$\tan\left(C\log\frac{r_a}{r_0}\right) = -C \text{ - - - - - - - - - - - - } 3\text{-}(29)$$

leading to $\rho_a = (1+C^2)/2\pi\alpha r_a^2$ - - - - - - - - - - - 3-(30)

Now in an exactly analogous manner to that used in the planar case, since we are assuming $\frac{\rho_a}{\rho_c} \ll 1$ we may set $r_0 \simeq r_c$ in dealing with conditions <u>at the anode</u>.

Thus equation (29) immediately enables us to determine C, i.e.

$$\frac{\tan(C\gamma)}{C\gamma} = -\frac{1}{\gamma} \qquad \text{where } \gamma = \log\frac{r_a}{r_c}$$

We can thus determine C rapidly from a table of the function $\frac{\tan x}{x}$. (E.g. Janke and Emde - "Funktion: tafeln") (Addenda)

Then from (30) using the value of C thus determined we obtain ρ_a.

Finally if we employ Richardson's formula (assuming of course equilibrium "almost" right up to the anode

~~plane~~ surface) we have :

$$I = \rho_a \cdot S \cdot \sqrt{\frac{kT}{2\pi m}}$$, where S is the anode surface area.

i.e.

$$I = \frac{(1+C^2).(kT)^{3/2}.\ell}{\sqrt{2\pi m}.e.r_a} \text{------- } 3\text{-}(31)$$,where ℓ is the effective length of the valve.

We may evolve a useful approximation if $\frac{r_a}{r_c}$ is "not too large". If we may set $\log\left(\frac{r_a}{r_c}\right) \ll 1$ then :

$\frac{\tan(C\gamma)}{C\gamma}$ is very large and thus $C\gamma \simeq \frac{\pi}{2}$

$$\therefore C \simeq \frac{\pi}{2}\left(\frac{1}{\log \frac{r_a}{r_c}}\right)$$ (which is large)

$$\therefore I \simeq \frac{\pi^2 (kT)^{3/2}.\ell}{\sqrt{2\pi m}.4.e.r_a \log^2\left(\frac{r_a}{r_c}\right)} = \frac{(\pi kT)^{3/2}.\ell}{4.\sqrt{2\pi m}.e.r_a \log^2\left(\frac{r_a}{r_c}\right)} \text{--------- } 3\text{-}(32)$$

This is identical with the Möller-Detels result as given in equation (22), which appears rather surprising in view of the apparent limitation mentioned above.*

It may also be verified readily at this stage that for $\left(\frac{r_a}{r_c}\right)$ very close to unity (approaching the limiting parallel-plane case) equation (32) agrees with equation (14). In view of the agreement above noted we shall use the Möller-Detels result as a general rule, but discrepancy between the two results does become significant for "filamentary" structures. The factor of disagreement is seen to be :

$$\lambda = \frac{4}{\pi^2}(1+C^2).\left(\log \frac{r_a}{r_c}\right)^2$$

Examples : $\frac{r_2}{r_1} = 2$; $\lambda = 1.7$

$\frac{r_2}{r_1} = 30$; $\lambda = 7.1$

That is, for a filamentary structures the Möller-Detels result indicates considerably lower limiting currents

*: Although not entirely unexpected in view of the close agreement, already demonstrated, with the parallel plane [illegible]

than the T-D argument. The writer under such circumstances is further inclined to favour the Möller-Detels result throughout since the density will certainly fall off very rapidly from the cathode in cylindrical filamentary structures; an E-D argument is thus indicated in this case.

For future reference we shall now indicate the theoretical values of I_{lim} applicable to the valves used by the writer, at a standard temperature of 1,000°K.

Valve Type	(app.) r_c	(app) r_a	l	(approx) $I(\mu a.)$
6H6	0.06cm.	0.1cm.	0.6cm.	30
EA50	0.05cm.	0.09cm.	0.7cm.	40
CV140	0.06cm.	0.078cm.	0.6cm.	165

(It should be emphasised here that all these valves were of simple cylindrical structure, being simple diodes, and all with indirectly heated cathodes.)

....

3. 7. Critical survey of previous work on this problem.

Before proceeding to describe the experimental investigation of the author, it is felt advisable to discuss earlier work in this field. As far as the writer is aware only two workers previously have seriously attempted to enter the retarding region in connection with fluctuation measurements of this kind. D.O. North (2) approached the region with some measurements on a cylindrical diode ($r_a \simeq .13$cm. $r_c \simeq .064$cm; $l = 1.5$cm.), but did not proceed below currents $\simeq 25\mu A.$, while the theoretical value of $I_{lim} \simeq 50\mu A.$. However, as will be shown below by the author in general it appears necessary to go considerably lower than the theoretically indicated value before entering in fact the retarding region. This is also borne out by the fact that the product IR_a had not yet reached constancy in North's experiments.

F.C. Williams (1), using a Mazda AC/Pen Valve, conducted fluctuation measurements, which however suffered fundamentally from the disadvantage of the use of relatively low frequency for the receiver introducing "flicker-effect" (see Chapter 4 and 5 of this Thesis for further brief mention) as a disturbing factor. He believed himself to have examined the retarding region and suggests that for currents <u>below $50\mu A$</u>, this was the case. The writer obtained a drawing of the Mazda A/C Pen valve from the makers, Messrs.Edison Swan Electric Co.Ltd., to whom he is indebted, and this is reproduced as Figure 4. One must first observe that the structure departs considerably from ideal cylindricality particular- :ly in the very critical region close to the cathode; granting this, however, we must take as the limiting dimensions those of anode and cathode and we have :

Type	r_a	r_c	l
AC/PEN	1.0cm.	0.055cm.	3.6cm.

$$\left[\therefore \frac{r_a}{r_c} = 18.2\right]$$

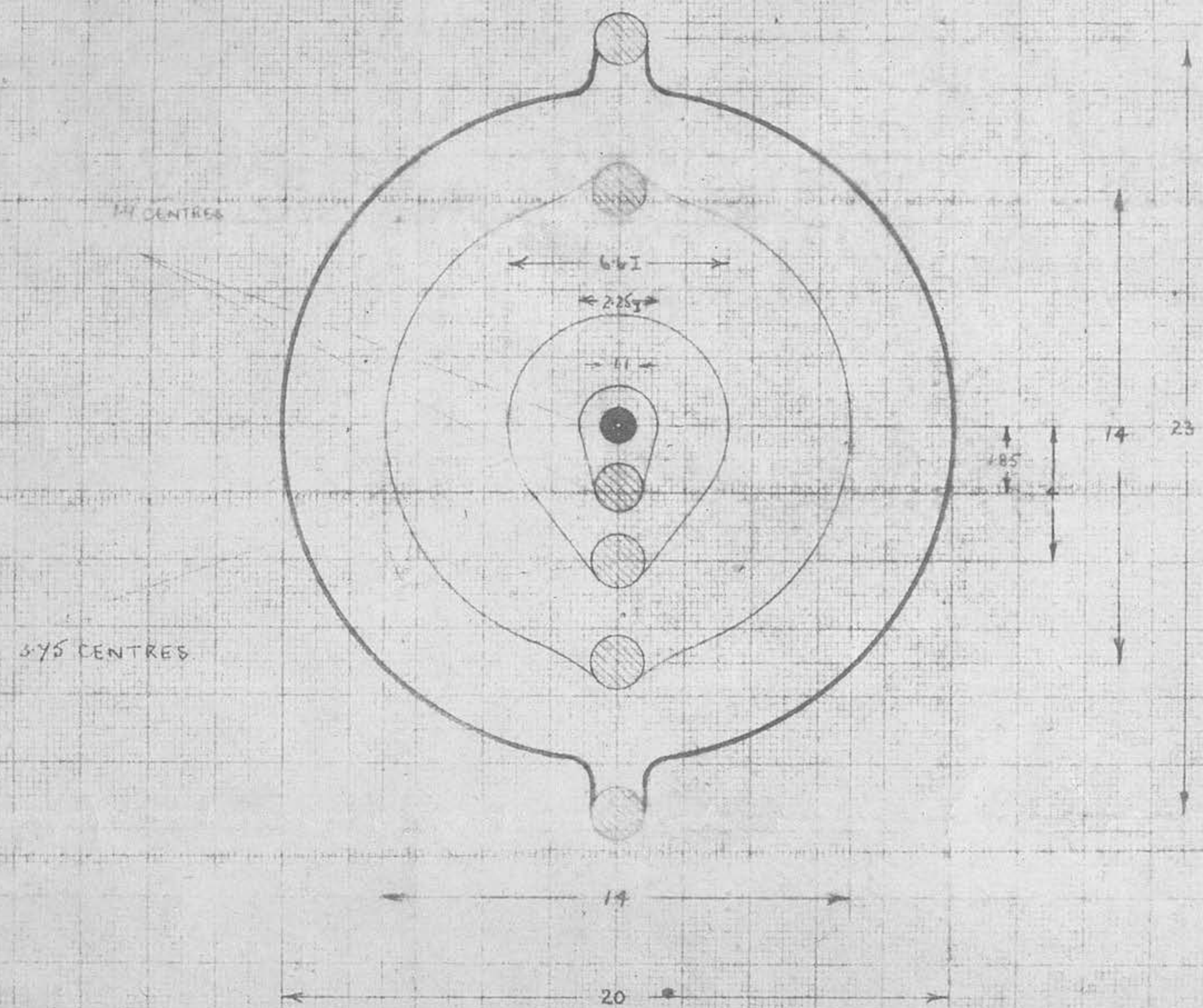
To get an estimate of I_{lim}, calculating on the M-D formula, we find, assuming $T \simeq 1{,}000°K$:

$$I = \frac{40.10^{-12}.10^{9/2}.3.6}{1.0.8.5} \text{ amps.}$$

$$= 0.54\ \mu A.$$

MAZDA VALVE TYPE AC/PEN

ELECTRODE ARRANGEMENT



G1 25 T.P.I. 38·6 HELIX LENGTH 0·004" DIA
G2 20 T.P.I. 38·25 HELIX LENGTH 0·006" DIA
G3 8 T.P.I. 39·9 HELIX LENGTH 0·007" DIA

CATHODE UNCOATED DIAMETER 1·1.
COATING LENGTH 36.

DIMENSIONS IN mms
EXCEPT WHERE STATED
OTHERWISE

Figure : 4.

Since Williams only carried his investigations as low as 12 μA, it is clear that he could not have even approached the true retarding region. This further is clearly brought out by the fact that the product IR_a was not even approximately constant, but fell steadily as the current was reduced. To quote Williams : "It may be "found from (his) Fig.5 that as I decreases from 0.05 to "0.012 mA., ρI ($\rho \equiv R_a$) decreases from 0.146 to 0.126 volt; "thus the characteristic is in substantial (sic) agreement "with (his) equation 10 ($\rho I = \frac{kT}{e}$) which requires ρI to "be constant". In fact it is quite evident the valve was operating at currents well above those necessary for the retarding region. This is further clearly shown by the fact that the mean indicated temperature from the equation $\frac{IeR_a}{kT} = 1$ was 1,600°K. He ascribes this to discrepancies which "other workers have observed". However, much detailed work (of which Williams and Moullin appear perhaps to have been unaware) has been done by investigators, especially by Möller and Detels (1), Adolf Demski (1), and Heinze and Hass (1) since the first attempts and if the retarding region is entered with certainty no discrepancies of this order should be observed in a valve of this type. Moullin (3), describing Williams' work, also mentions some other valve types on which similar measurements were carried out. These have each been investigated in a similar manner and where reasonable temperatures were obtained the indicated limiting currents suggest that entry into the retarding region would have been quite likely. However since no specific values of current limits investigated are furnished a verdict can naturally not be given with certainty.

A short table is provided summarising these results :

Valve Type	t_a (cm.)	t_c (cm.)	l (cm.)	Indicated Temperature obtained by Moullin and Williams.	Theoretical Limiting Current.
AC/PEN	1.0	0.055	3.6	1600°K	0.54 μA.
AC/HL	0.21	0.055	3.6	1160°K (Probable)	12 μA.
M.H.4.	(See	sketch	Fig. 5)	1160°K (Probable)	Say 7 μA.
"Special Cylindrical Diode"	0.5	0.01	7.9	Very improb-:able tem-peratures.	3.7 μA.

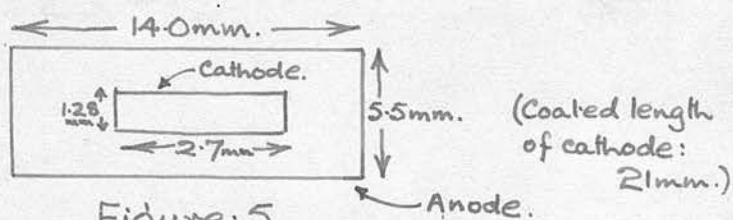


Figure: 5

For reference a drawing of the AC/HL valve is also provided at Figure 5a.

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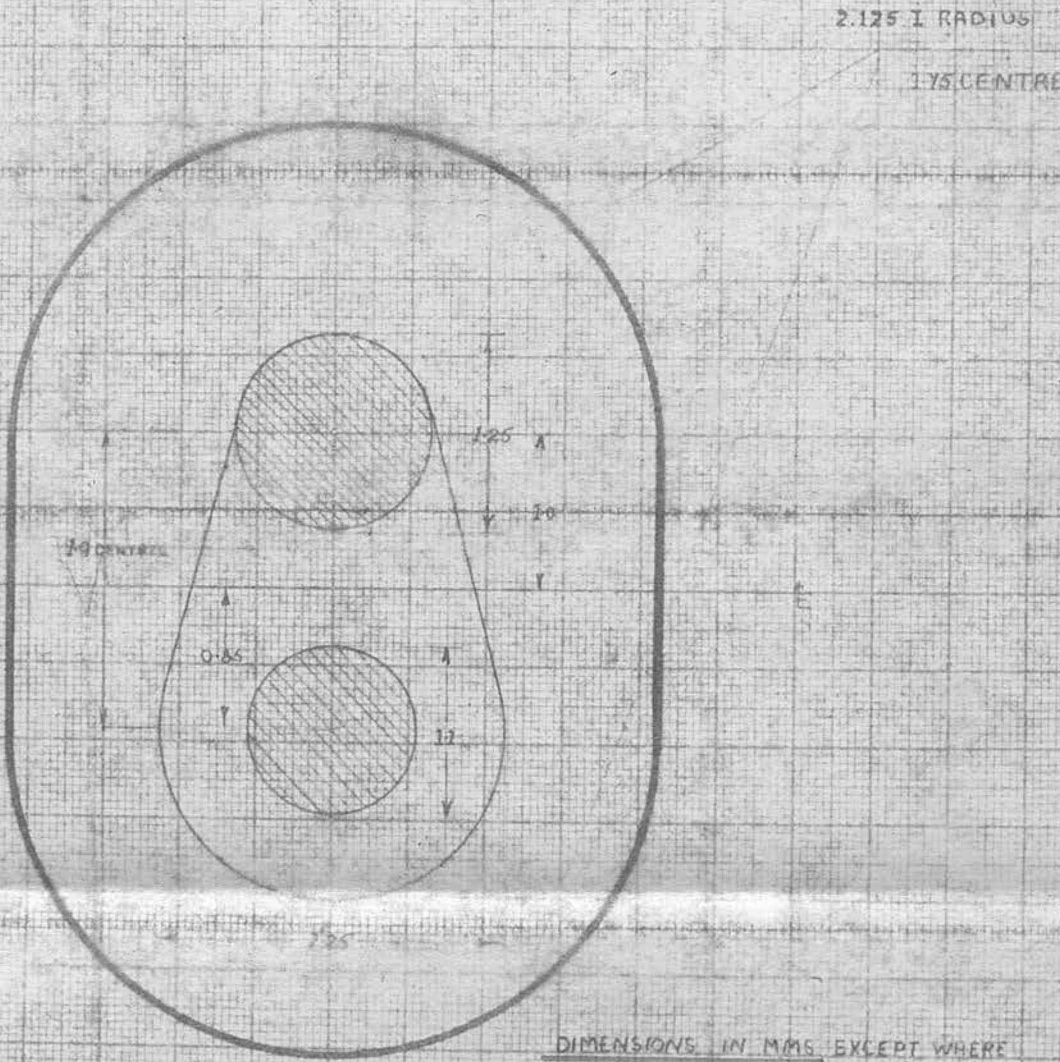
3. 8. Description and analysis of experimental technique.

The experimental layout finally adopted by the writer was a Wheatstone bridge arrangement indicated in Figure 6. The bridge is fed from a Beat Fre-:quency Oscillator, and frequencies between 1 Kc/s and 10 Kc/s. were variously employed. The fixed arms

MAZDA VALVE TYPE AC/HL

ELECTRODE ARRANGEMENT



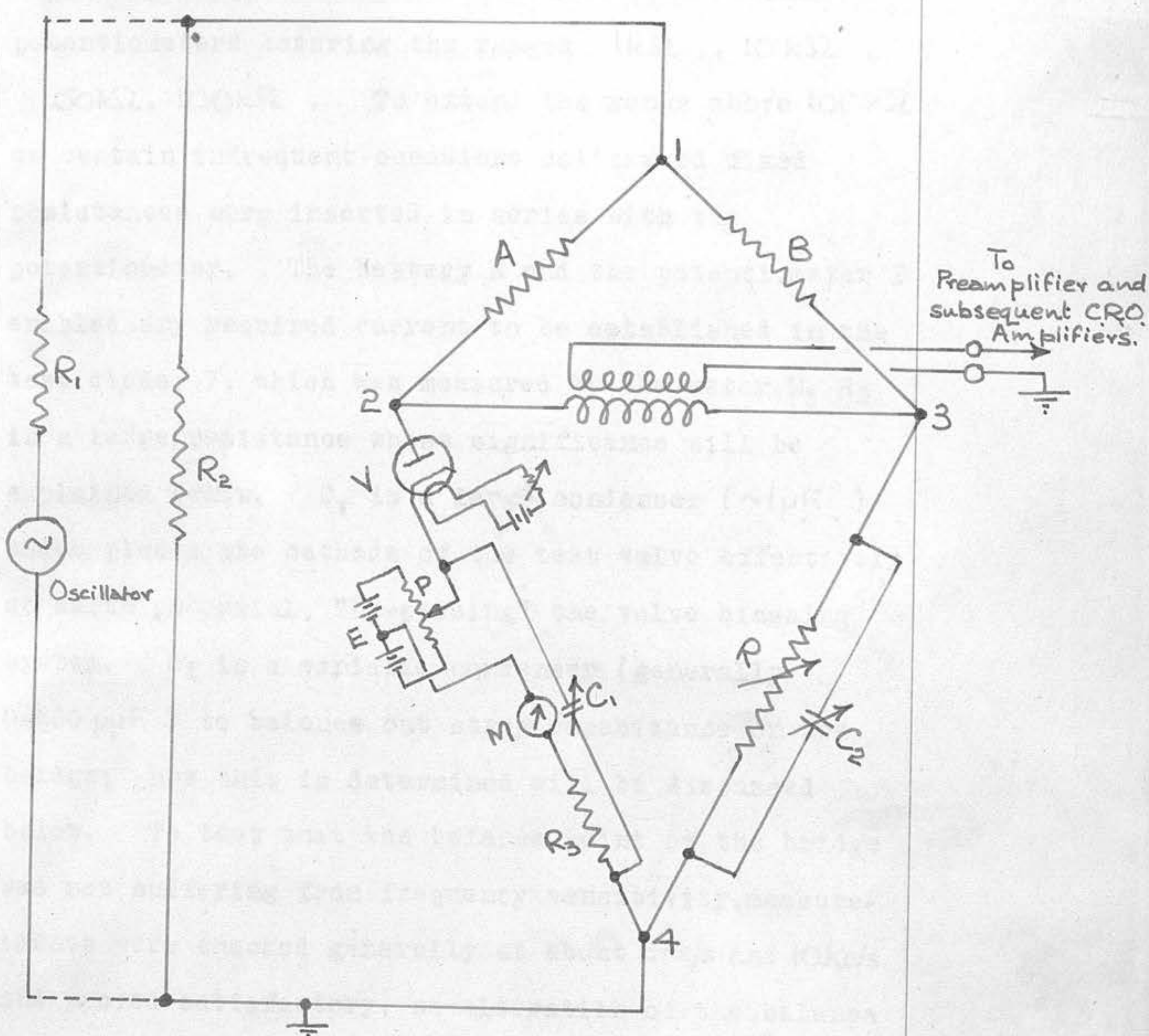
DIMENSIONS IN MMS EXCEPT WHERE OTHERWISE STATED.

CATHODE - 36mm COATING LENGTH

GRID 62.5 T.P.I. 40.5mm HELIX OF .0004" DIAM

Figure: 5a

Plate 1 (to follow p.57)



Detail of Bridge Circuit for

Slope Resistance Measurement.

Figure : 6.

A, and B were either $10 k\Omega$ or $100 k\Omega$. The variable arm R consisted of a set of calibrated wire-wound potentiometers covering the ranges $1 k\Omega$, $10 k\Omega$, $50 k\Omega$, $100 k\Omega$. To extend the range above $100 k\Omega$ on certain infrequent occasions calibrated fixed resistances were inserted in series with the potentiometer. The battery E and the potentiometer P enabled any required current to be established in the test diode, V, which was measured by the meter M. R_3 is a large resistance whose significance will be explained below. C_1 is a large condenser ($\sim 1 \mu F$) which places the cathode of the test valve effectively at earth potential, "by-passing" the valve biassing system. C_2 is a variable condenser (generally 0-200 $\mu\mu F$) to balance out stray capacitance on the bridge; how this is determined will be discussed below. To test that the balance point of the bridge was not suffering from frequency sensitivity, measurements were checked generally at about 2kc/s and 10kc/s and proved satisfactory, no alteration of the balance point being observed as the frequency was varied. Finally it will be shown below that the input must be limited to a rather small value; this was achieved by the use of the resistances R_1 and R_2 ($5 k\Omega$ and 5Ω) To avoid stray "pick-up" from the oscillator direct, R_1 was placed directly at the oscillator terminals; long screened leads were used from the oscillator, and R_2 was placed directly across the bridge terminals.

It was found necessary to insert a large resistance (R_3) in series with the test valve to prevent slow drift of the measured current over periods of the order of several seconds, such drift becoming evident when operating at very low space-currents. We now show that such an arrangement "will" stabilise the valve current, both for "external" battery fluctuations because of the particular region of operation of the valve, and also for "internal" fluctuations of valve current - due presumably, if existent, to slow "drift" variations of the cathode emission as a general principle.

Case 1 : Battery fluctuations.

Two extreme possibilities may be considered. In Figure 7 the resistance R is assumed to be very large compared with the apparent "d-c" valve resistance and therefore we may write :

$$I \doteq \frac{V_1}{R} \text{-------- } 3\text{-}(33)$$

$$\therefore \frac{dI}{I} = \frac{dV_1}{V_1} \text{-------- } 3\text{-}(34)$$

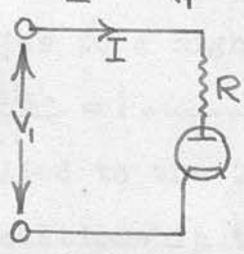


Fig. 7

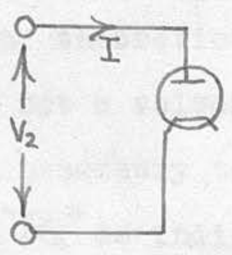


Fig. 8.

In Figure 8 no series resistance is present and for the same current I flowing and assuming as a sufficient approximation in this connection, the planar formula for the retarding region we have :

$$I = J\varepsilon^{\frac{eV_2}{kT}} \text{---------- } 3\text{-}(35)$$

$$\therefore \frac{dI}{I} = -\left(\log\frac{J}{I}\right)\cdot\frac{dV_2}{V_2} \text{ - - - - - - - - - - } 3\text{-}(36)$$

Assuming now proportional "drift" in V_1 and V_2, then $\frac{dV_1}{V_1} = \frac{dV_2}{V_2}$, and since under the conditions of operation $J \gg I$ i.e. $\log\frac{J}{I} \gg 1$, and $\therefore$ (34) will provide greatly improved stabilisation compared with (36). A further point to notice is that as I is reduced by increasingly retarding applied fields so $\log\frac{J}{I}$ will increase and the relative stabilisation will improve - a most useful advantage. This indication is confirmed in practice.

Case 2 : Emission fluctuations.

This case is self-evident. Regarding now J as the variable then from (33) :

$$\frac{dI}{I} \simeq 0 \text{ - - - - - - - - - } 3\text{-}(37)$$

while from (35) :

$$\frac{dI}{I} = \frac{dJ}{J} \text{ - - - - - - - - - } 3\text{-}(38)$$

Bridge Analysis.

As has been shown already, $g_a \equiv \frac{1}{R_a} = \frac{\partial I}{\partial V}$ ------ 3-(39) satisfies to a high accuracy the theoretical equation $\frac{IeR_a}{kT} = 1$ ------3-(40). If now a voltage $2v\sin\omega t$ be applied to the bridge it is necessary to examine what relationship the apparent "R_a" as indicated by the calibrated arm of the bridge at balance will bear to the value defined by (39). It will be assumed that equation (35) is of sufficient accuracy in determining possible sources of error and limits of accuracy. Let us assume the bridge is balanced; the voltage at terminal 3, with respect to earth, will be $v\sin\omega t$. The fundamental component of voltage across the diode (bearing in mind that the diode is a non-linear element) - (i.e. the voltage at terminal 2) will also be $v\sin\omega t$ at balance.

We now divide the problem of the valve bridge arm into two parts as shown in Figures 9 and 10., applicable to the direct - and alternating - components respectively.

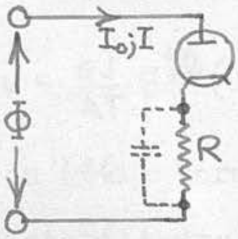


Figure 9 (D.C.)

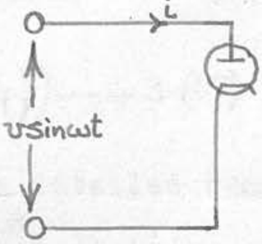


Figure 10 (A.C.)

In Figure 9 if the (mean) voltage across the valve at any time is V then :

$$V = \Phi - RI \text{-----} 3\text{-}(41)$$ (where I is the (mean) current).

Thus when the oscillatory voltage is applied :

$$I = J.\overline{\varepsilon^{\frac{e(V+v\sin\omega t)}{kT}}}$$ (where the "bar" indicates a time-average)

$$= J.\varepsilon^{\frac{eV}{kT}}.\overline{\left\{1+\frac{ev\sin\omega t}{kT}+\frac{e^2v^2}{2(kT)^2}.\sin^2\omega t+\dots\right\}}$$ (to terms of the 2nd. order).

$$= J.\varepsilon^{\frac{eV}{kT}}\left\{1+\frac{v^2}{4}\cdot\left(\frac{e}{kT}\right)^2\right\} \text{-------} 3\text{-}(42)$$

Initially we had $V_0 = \Phi - RI_0$ and $I_0 = J\varepsilon^{\frac{eV_0}{kT}}$

Let $V = V_0 + \delta$, thus from (41) and (42)

$$V = \Phi - RJ\varepsilon^{\frac{e(V_0+\delta)}{kT}}\left\{1+\frac{v^2}{4}\left(\frac{e}{kT}\right)^2\right\}$$

i.e.

$$\Phi - RJ\varepsilon^{\frac{eV_0}{kT}} + \delta \doteq \Phi - RJ\varepsilon^{\frac{eV_0}{kT}}\left\{1+\frac{\delta e}{kT}\right\}.\left\{1+\frac{v^2}{4}\left(\frac{e}{kT}\right)^2\right\}$$ (to terms of 1st. order in δ).

$$\therefore -RJ\varepsilon^{\frac{eV_0}{kT}} + \delta = -RJ\varepsilon^{\frac{eV_0}{kT}}\left\{1+\frac{\delta e}{kT}+\frac{v^2}{4}\left(\frac{e}{kT}\right)^2\right\}$$

$$\therefore \delta = -RJ\varepsilon^{\frac{eV_0}{kT}}.\frac{\delta e}{kT} - RJ\varepsilon^{\frac{eV_0}{kT}}.\frac{v^2}{4}\left(\frac{e}{kT}\right)^2$$

i.e. $$\delta = -RI_0\frac{v^2}{4}\left(\frac{e}{kT}\right)^2\Big/\left(1+\frac{RI_0e}{kT}\right) \text{----------} 3\text{-}(43)$$

Now since $R \gg R_a$; $\frac{RI_0 e}{kT} \gg 1$

$$\therefore \delta \doteq -\frac{v^2}{4}\left(\frac{e}{kT}\right)^2 \quad \text{------------} \quad 3\text{-}(44)$$

The current is thus given by :

$$I + i = J\varepsilon^{\frac{eV_0}{kT}}\left(1 - \frac{v^2}{4}\left(\frac{e}{kT}\right)^2\right)\cdot\left(1 + \frac{ev}{kT}\sin\omega t + \left(\frac{e}{kT}\right)^2\cdot\frac{v^2}{2}\left(\frac{1-\cos 2\omega t}{2}\right) + \cdots\right) \ldots 3\text{-}(45)$$

$$\doteq J\varepsilon^{\frac{eV_0}{kT}}\left(1 + \left\{1 - \frac{v^2}{4}\left(\frac{e}{kT}\right)^2\right\}\cdot\frac{ev}{kT}\sin\omega t\right)$$

$$\therefore I = J\varepsilon^{\frac{eV_0}{kT}} \quad \text{------------} \quad 3\text{-}(46)$$

And :

$$i = \frac{eJ}{kT}\cdot\varepsilon^{\frac{eV_0}{kT}}(v\sin\omega t)\left(1 - \frac{v^2}{4}\left(\frac{e}{kT}\right)^2\right) \text{-----} 3\text{-}(47)$$

Equation (46) forms, in fact, a detailed demonstration of the stabilising effect of the resistance R discussed more generally above, while (47) shows that the balance of the bridge will be given for a resistance such that :

$$\frac{1}{R} = \frac{eI}{kT}\left(1 - \frac{v^2}{4}\left(\frac{e}{kT}\right)^2\right) \text{----------} 3\text{-}(48)$$

That is, "R_a" as determined by the bridge will be increased over its true value by $\left\{1 - \frac{v^2}{4}\left(\frac{e}{kT}\right)^2\right\}^{-1}$. It therefore appears, at first sight, that comparing (48) with (40) accurate results will be obtained if :

$$\frac{v^2}{4}\left(\frac{e}{kT}\right)^2 \ll 1 \;;\; \text{i.e. } v \ll \frac{2kT}{e} \text{----} 3\text{-}(49)$$

E.g. for 1% accuracy $v = \frac{1}{5}\frac{kT}{e}$

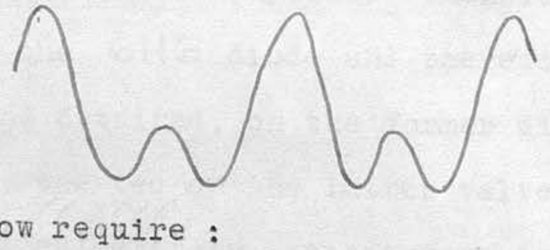
Thus for $T \simeq 1000°K$; $\frac{kT}{e} = \cdot 086$ Volt

$$\therefore v \doteq 17 mV.$$

A further disturbing feature however must be recognised. It is readily seen (equation(45)) that a component of 2nd. harmonic current flows as a result of the diode action. (To solve the bridge problem entirely rigorous-:ly it would have been necessary to consider in turn the effect of the "distortion current components" in modifying the resultant bridge voltage applied to the diode, but the error involved should be entirely negligible if we maintain v below the limits set above). This 2nd. harmonic current flowing through the bridge arm, A, will result in a (small) component of 2nd. harmonic voltage at terminal 2. This component will not of course be balanced out by the (purely) fundamental component at terminal 3, and must in fact become the dominant feature when close to balance. Thus if as a first approximation we neglect the effect of the trans:former between terminals 2 and 3 for a small deviation from balance, then under the assumption of an entirely symmetrical bridge condition $A = B \doteqdot R \doteqdot R_a$, the output voltage for 1% deviation from true balance is $\frac{a}{200}$, where a is the magnitude of the fundamental component across the valve. If then this component is to be legible in the presence of the 2nd. harmonic component at balance we must consider the ratio :

$$\frac{a}{200} : \left(\frac{e}{kT}\right)^2 . \frac{v^2}{4}$$

If one suggests equality of these components as the legible limit, giving a pattern on the Cathode Ray Tube:



we should now require :

$$\frac{ev}{kT} = 50\left(\frac{ev}{kT}\right)^2$$

i.e. $v = \frac{1}{50} . \frac{kT}{e}$, which for $T = 1000°K$ gives

$$\underline{v \simeq 2mV.}$$

In practice inputs ranging between these two suggested limits were employed with consistent results. It was found in fact that discrimination of the balance-point was extremely sensitive using the appearance of the 2nd. harmonic component, and this also readily indicated any lack of reactive adjustment by phase-shift (see Figures 11, 12, 13, 14). In the measurement the various theoretical indications above were amply confirmed. In particular, in earlier experimental work the severe limitation on injected voltage had not been appreciated and therefore the values of R_a deduced were too large (Equation (48)) and the indicated temperatures were therefore too great by some 20-30% in fact. To check the later figures experimental observations at ~17mV. input were often checked at ~2mV. input and showed no variation of indicated R_a.

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3. 9. Experimental results.

Experimental results are shown in Figures 15, 16 and 17. Experimental results on the EA50 diode are not included since the physical dimensions differ little from the 6H6 diode and therefore the results expected, and obtained, on the former differ little from those presented on the latter valve. The results are regarded as entirely satisfactory indicating strict constancy of the "IR_a" product after a certain point has been reached and therefore very strong presumption that the retarding region has been entered; furthermore in each case the critical current involved is less than

Plate ii : following p.64.

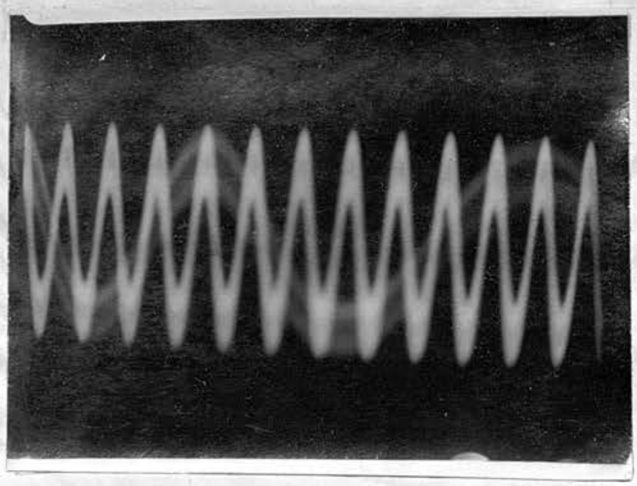


Fig. 11: Bridge unbalanced

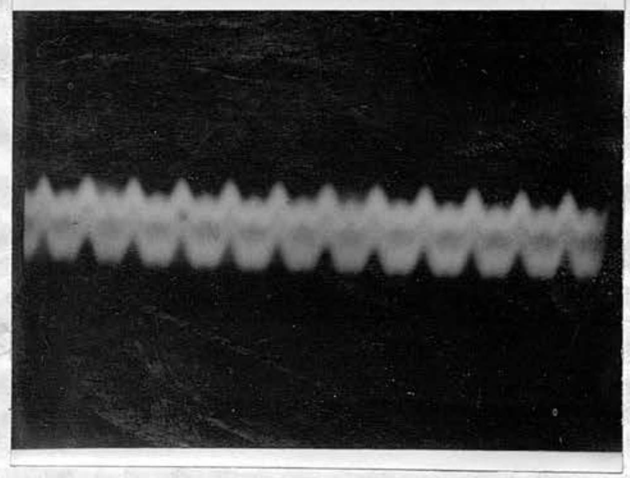


Fig. 12: Bridge near balance (Note evidence of 2nd harmonic).

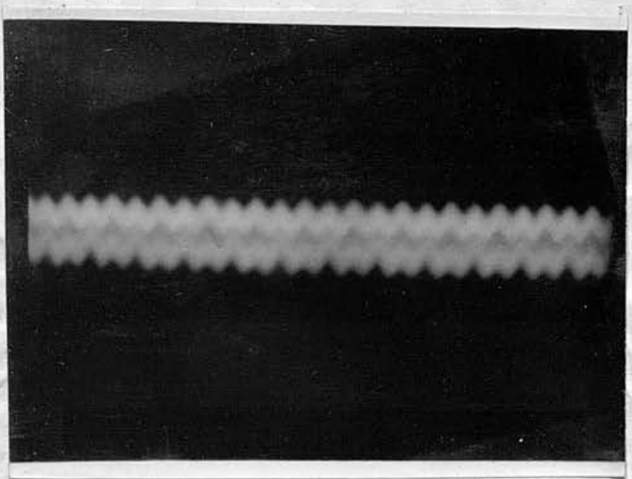


Fig. 13: Bridge balanced (Complete dominance of 2nd harmonic).

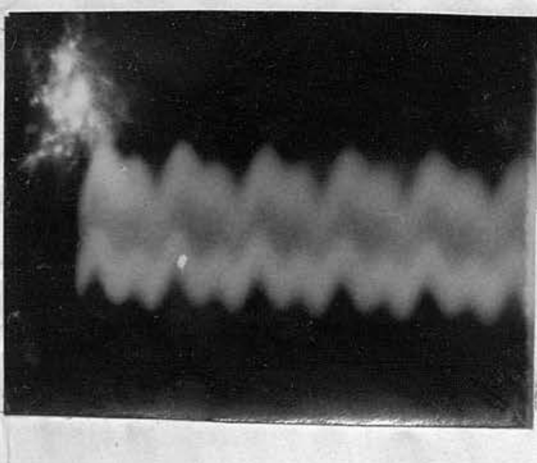
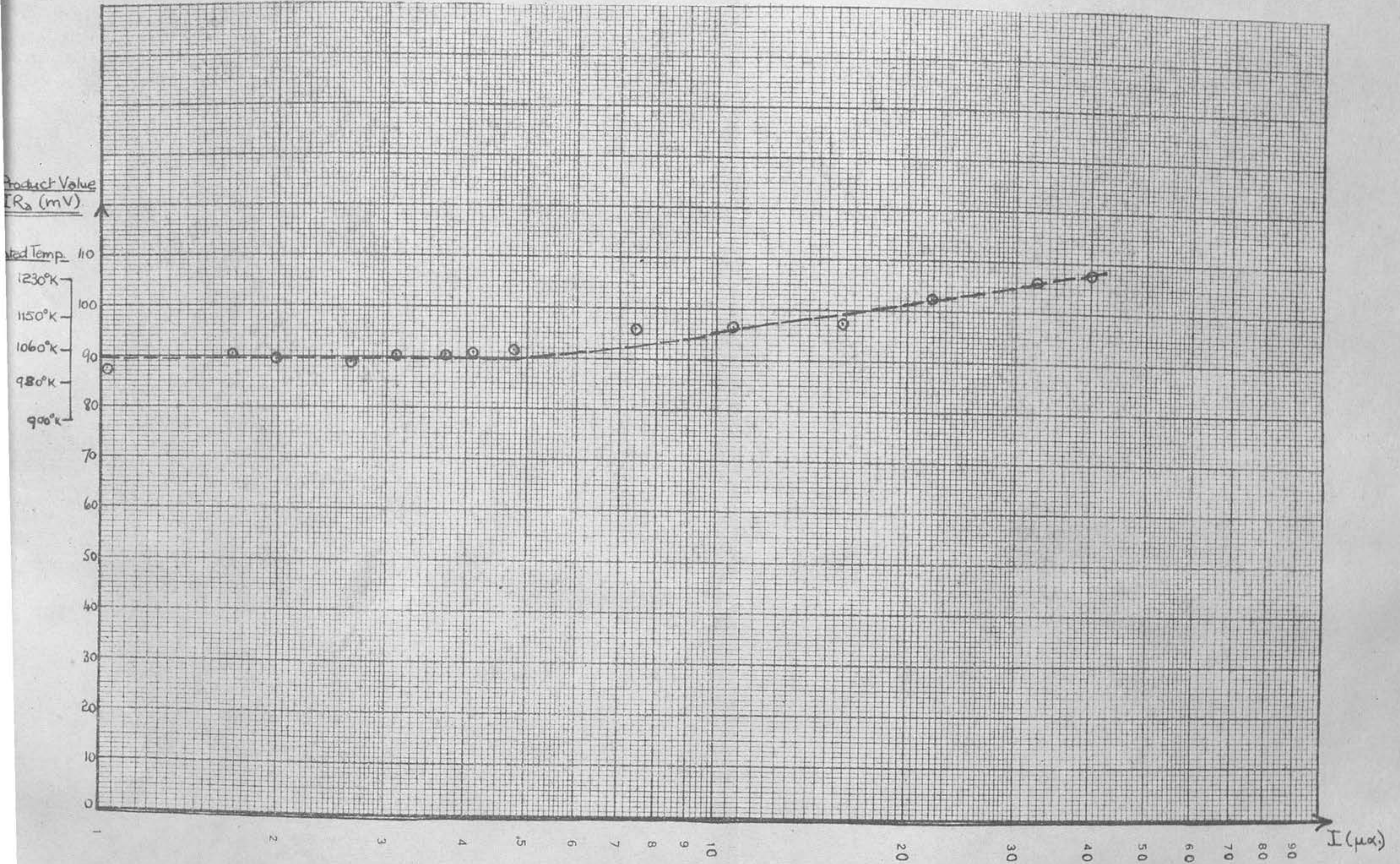


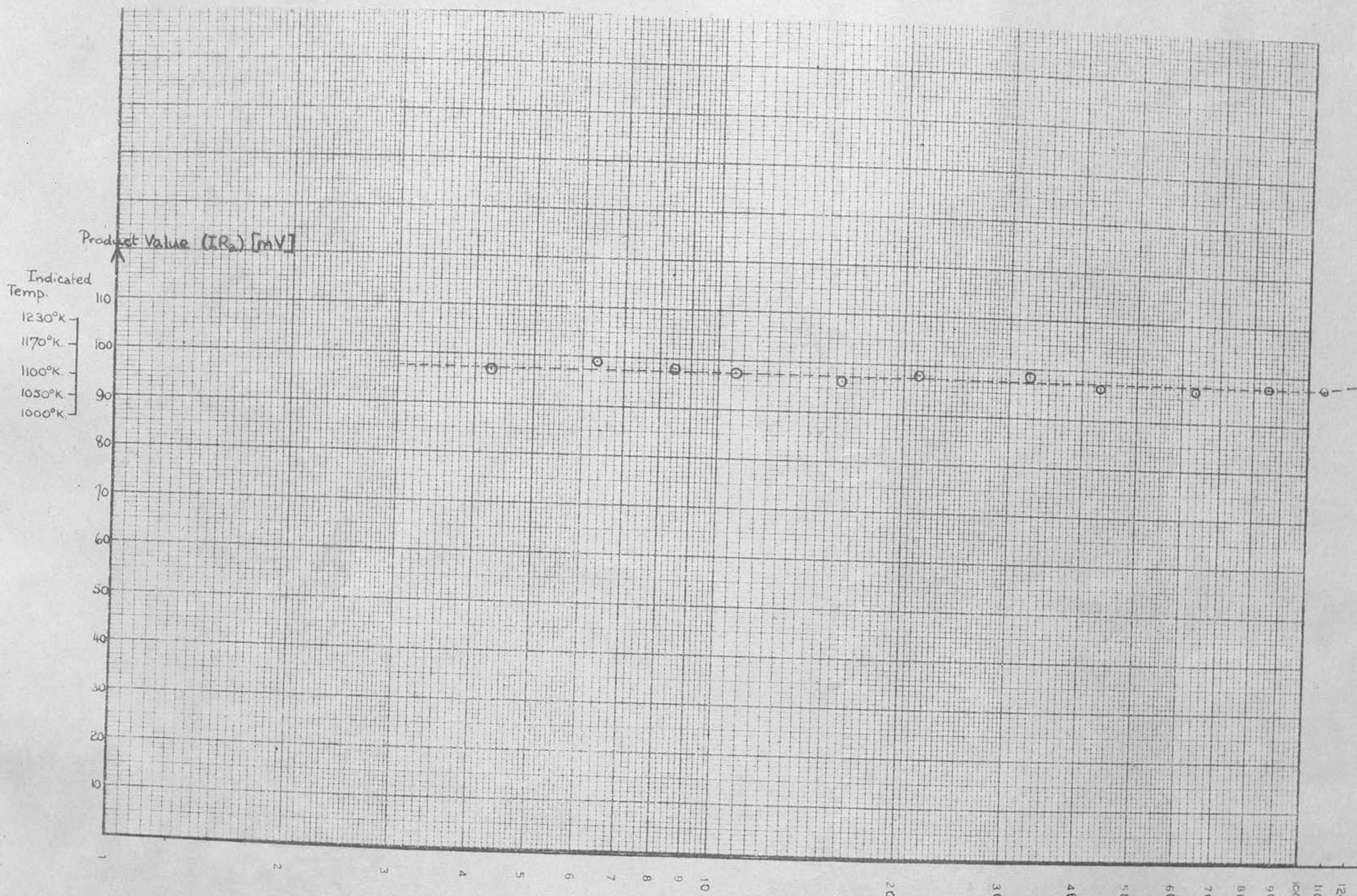
Fig. 14: Bridge near balance, but reactance incorrectly adjusted.



EXPERIMENTAL INVESTIGATION OF VALVE CHARACTERISTIC IN RETARDING FIELD REGION — TYPE 6H6 DIODE

[If = 0·3a]

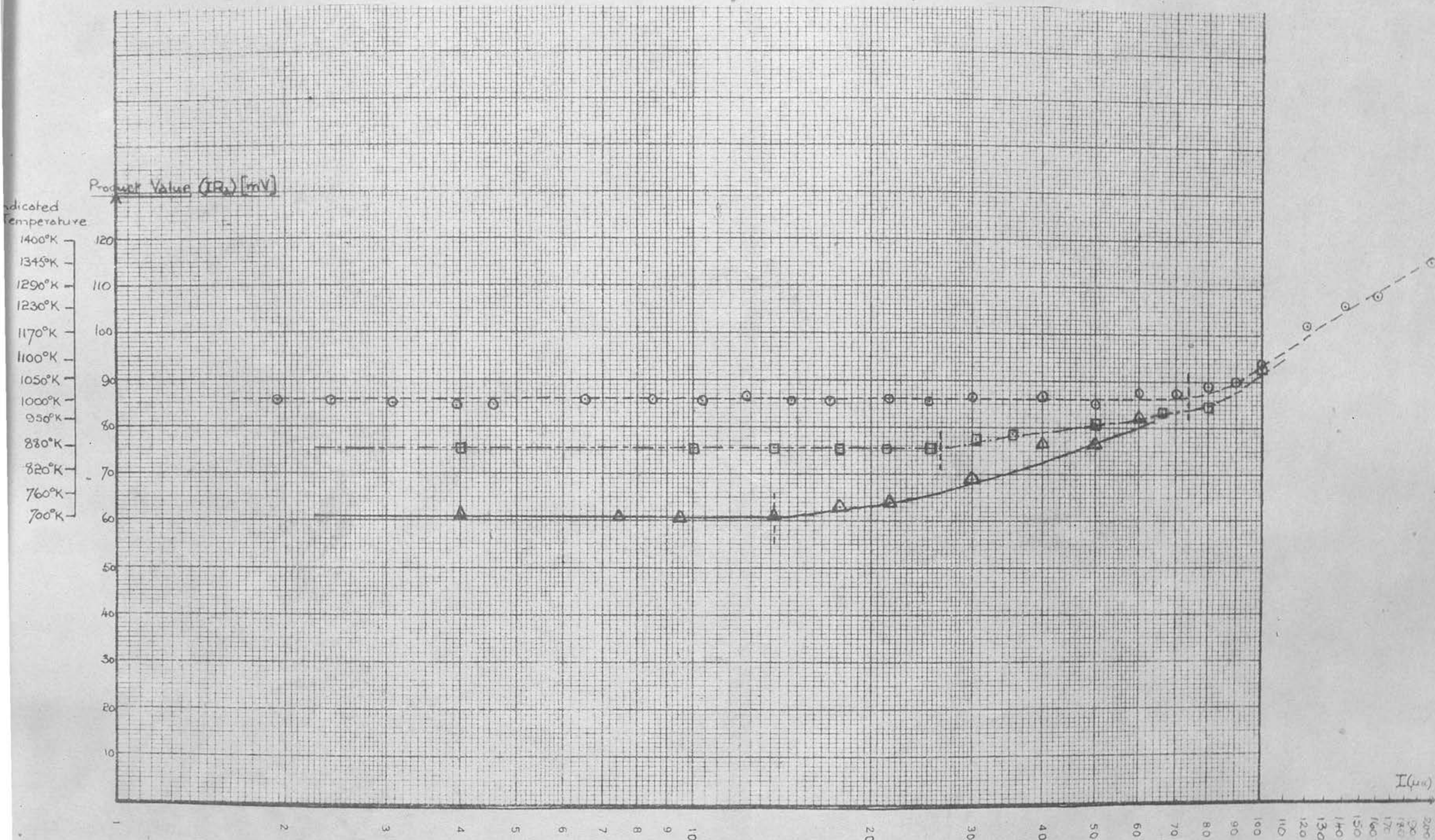
Figure : 15.



Examination of Valve Characteristic under retarding field conditions
Close-spaced Diode [Second Model] [Type CV140]

Figure : 16.

[$I_f = 0{\cdot}3\alpha$; $V_f = 6{\cdot}9$ V]



Examination of Valve Characteristic under retarding field conditions

Close-Spaced Diode [First Model] [Type CV140]

Figure : 17.

Legend

⊙: Experimental Points: 1st. Series [$I_f = 0{\cdot}3\alpha$; $V_f = 6{\cdot}1V$]

⊡: Experimental Points: 2nd. Series [$I_f = 0{\cdot}25\alpha$; $V_f = 4{\cdot}3V$]

△: Experimental Points: 3rd. Series [$I_f = 0{\cdot}2\alpha$; $V_f = 2{\cdot}65V$]

the theoretical value derived above. In particular the measurements on the CV140 valves are noteworthy, as the most satisfactory later fluctuations measurements were carried out on this type. Finally reasonable values of indicated temperature are obtained in all cases.

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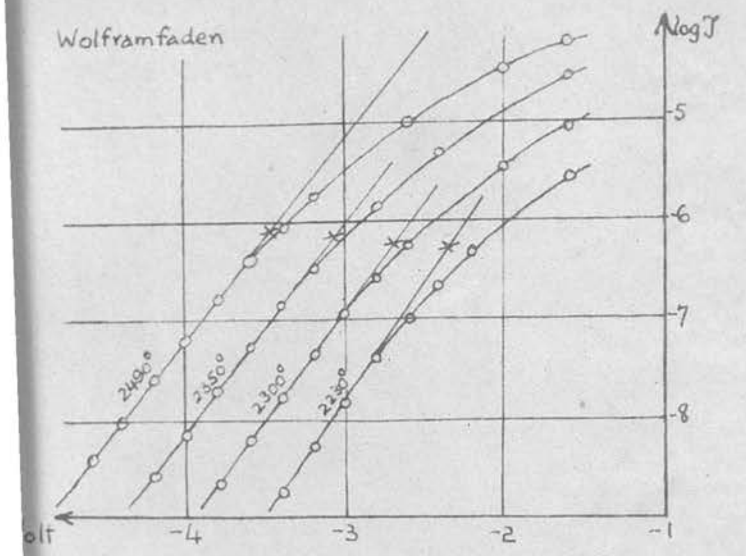
3. 10. Discussion of experimental results.

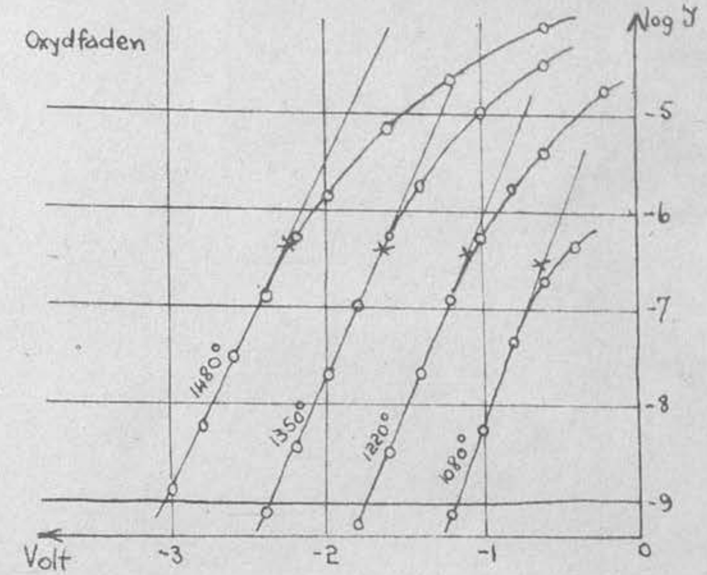
It should be observed that the evidence of the lower values of the experimental limiting current than the theoretical value are confirmed by the work of Möller and Detels (1). Figure 18 is reproduced from their paper and in all cases the experimental limit of current is lower than the theoretical value; furthermore it may be noted that calculating the theoretical value on the T-D analysis discussed above enhances the discrepancy. Also the experimental work of D.O. North (2) discussed above, provides similar evidence since the product IR_a had not reached constancy for $I \approx 25\mu A.$, while the theoretical value was $\sim 50\mu A.$.

The possibility that a quantum-mechanical analysis might modify the results was investigated. The problem of the planar diode has been discussed by N.H. Frank (2) on the basis of previous work with L.A. Young. (1)

In general terms the modification to classical electronic theories due to quantum mechanics rests in the denial of the classical assumption that an electron



(Fig. 3)



(Fig. 4)

[Calculated current values:-
(r_a = 0,15 cm. ; r_d = 0,0025 cm.)]

Tabelle IIa

T	2230	2300	2350	2500
log J	-6,24	-6,23	-6,22	-6,16

(r_a = 0,15 cm ; r_d = 0,005 cm.)

T	1080	1220	1350	1480
log J	-6,55	-6,47	-6,44	-6,35

Figure: 18.

of energy E will, - or will not, - surmount a potential energy barrier of magnitude V_0 precisely according as E is greater or less than V_0. In quantum mechanics, on the one hand, there is always a finite probability that for $E < V_0$ the electron will "penetrate" the barrier although for $E \ll V_0$ the probability becomes vanishingly small; on the other hand it also is not a certainty that the electron will surmount the barrier for $E > V_0$, although for E significantly exceeding V_0 the probability rapidly approaches unity. This is explained, or interpreted, by ascribing to the electrons a wave-like property rather than solely a particle attribute where the amplitude of the wave signifies the probability of an electron occupying a given position. The wave-length so attributed is generally referred to as the "de Broglie wavelength" and is dependent upon the energy of the electron in a given position.

In particular, in considering the penetration of a potential barrier, the "width" of the barrier is very significant since the probability wave for $E < V_0$ is rapidly attenuated within the barrier according to an expotential law. If the barrier is very "narrow" as at the surface of an emitting cathode with a high accelerating field applied as in Figure 19 :

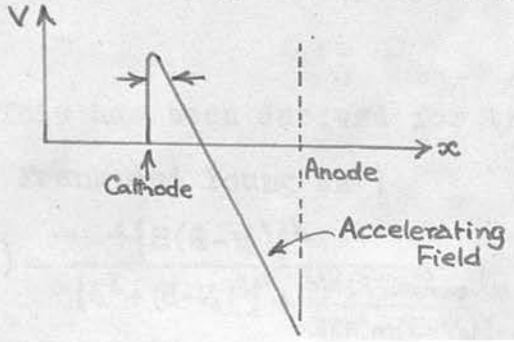


Figure: 19.

then a considerable degree of "penetration" will exist through the barrier where the width is narrow as indicated by the arrows. This effect explains results obtained,-"anomalous" on classical concepts,- under strong accelerating fields.

In our case with a solely retarding field from cathode to anode as in Figure 20:

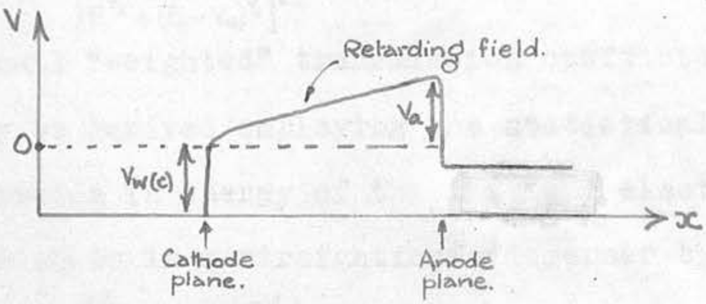


Figure:20

"penetration" of the barrier will be quite negligible for $E<V_a$ because of its great width. On the other hand it will be necessary to take account of the fact that the probability of an electron reaching the anode is not unity for all $E>V_a$.

By considering the boundary conditions to be satisfied by the "incident" electron-waves on the potential barrier and the "transmitted" electron-waves (such electrons constituting the observed current) a "transmission coefficient" $D(E)$ defining the probability of transmission may be derived for a given incident energy.

This has been derived for the case under considera-:tion by Frank and Young as :

$$D(E) = \frac{4\{E(E-V_0)\}^{1/2}}{\{E^{1/2}+(E-V_0)^{1/2}\}^2+\frac{h^2e^2(\partial V/\partial x)^2_{x=0}}{128\pi^2 m(E-V_0)^2}} \quad --------3-(50)$$

where the second term in the denominator may generally be neglected in practice. This expression is only valid for $E > V_0$ clearly. Since, however, we have shown that in this problem the case $E < V_0$ need not be considered, the expression is a sufficient approxima:tion. Granted this, and therefore setting :

$$D(E) = \frac{4\{E(E-V_0)\}^{1/2}}{\{E^{1/2}+(E-V_0)^{1/2}\}^2} \text{ --------------- } 3\text{-}(51)$$

an overall "weighted" transmission coefficient may readily be derived employing the statistical average distribution in energy of the incident electrons. It is thus shown in a straightforward manner by Frank that:

$$I = J\varepsilon^{\frac{eV_a}{kT}} \frac{\bar{D}(\beta - \frac{eV_a}{kT})}{\bar{D}(\beta)}$$

where $\bar{D}(\beta)$ is the overall transmission coefficient defined by :

$$\bar{D}(\beta) = 4\int_0^\infty \frac{(y+\beta)^{1/2} \cdot y^{1/2} \cdot \varepsilon^{-y} dy}{\{y^{1/2}+(y+\beta)^{1/2}\}^2}$$

where $\beta = \frac{eV_w}{kT}$ and V_w is the "work-function voltage". In general $\beta \gg 1$ and therefore the integral for $\bar{D}(\beta)$ may be expanded in powers of y/β (while the factor ε^{-y} should ensure convergence as $y \to \infty$).

Then :

$$\bar{D}(\beta) = \frac{2\pi^{1/2}}{\beta^{1/2}} - \frac{8}{\beta} + \frac{15\pi^{1/2}}{2\beta^{3/2}} - \frac{16}{\beta^2} + \text{ ------------ } 3\text{-}(52)$$

Further, Frank shows that if $-\frac{eV_a}{kT} \gg 1$ (in general the case in this work) and if $\beta \gg 1$ then :

$$\bar{D}(\beta - \frac{eV_a}{kT}) \doteqdot 1 - \left\{(1+\alpha V_1)^{1/2} - (\alpha V_1)^{1/2}\right\}^4 \cdot \left\{1 - \frac{2}{\beta}(\alpha V_1)^{1/2} \cdot (1+\alpha V_1)^{1/2}\right\} \text{ ----- } 3\text{-}(53)$$

where for convenience we have written $V_1 = |V_a|$ and $\alpha = \frac{1}{V_w}$.

Applying this analysis now to the problem in hand :

$\bar{D}(\beta)$ is a constant; if we assume for oxide-coated cathodes $V_W \doteq 1$ Volt and $T \doteq 10^3 K$, then we find from (52): $\bar{D}(\beta) \simeq 0.6$

Thus $I = A\varepsilon^{-\frac{eV_1}{kT}} f(V_1)$, where A is a constant

$$\therefore \frac{\partial I}{\partial V_1} = -\frac{Ae}{kT}\cdot\varepsilon^{-\frac{eV_1}{kT}} f(V_1) + A\varepsilon^{-\frac{eV_1}{kT}} f'(V_1)$$

$$\therefore g_a = \frac{1}{R_a} = -\frac{\partial I}{\partial V_1} = \frac{eI}{kT}\left\{1 - \frac{kT}{e}\frac{f'(V_1)}{f(V_1)}\right\} \text{------------ 3-(54)}$$

(Since we may see readily that $f(V_1)$ and $f'(V_1)$ are both intrinsically positive this equation immediately indicates a <u>qualitative</u> increase of the vital product IeR_a/kT over unity).

Now by performing the differentiation of (54) in (53) above :

$$\frac{\partial \bar{D}(\beta + \frac{eV_1}{kT})}{\partial V_1} \equiv f'(V_1) = 2\left\{(1+\alpha V_1)^{1/2} - (\alpha V_1)^{1/2}\right\}^3 \cdot \left\{\left[(1+\alpha V_1)^{1/2} - (\alpha V_1)^{1/2}\right]\cdot\frac{\alpha}{2\beta}\cdot\left[\left(\frac{1+\alpha V_1}{\alpha V_1}\right)^{1/2} + \left(\frac{\alpha V_1}{1+\alpha V_1}\right)^{1/2}\right]\right.$$

$$\left. + 2\left[1 - \frac{2}{\beta}\left\{(\alpha V_1).(1+\alpha V_1)\right\}^{1/2}\right]\cdot\frac{\alpha}{2}\cdot\left[(\alpha V_1)^{-1/2} - (1+\alpha V_1)^{-1/2}\right]\right\} \text{---- 3-(55)}$$

Considering the results in the case of the 6H6 type diode, let us take $I = 10\mu A.$; $J = 30 mA.$; $V_W = 1$ Volt;

$\therefore \alpha = 1; \beta = 11.6; V_1 = 8.\cdot 086 \simeq \cdot 69 V.$ $\left[\varepsilon^{+8} \simeq 3{,}000 = \frac{30 mA}{10\mu A}\right]$

Then :

$$f'(V_1) \doteq \cdot 0\ 7$$

And $f(V_1) \simeq 1$ (The accuracy of this term clearly need not be great).

Thus the correction term = $\frac{kT}{e}\left\{\frac{f'(V_1)}{f(V_1)}\right\} \simeq \cdot 006$

$= \underline{\cdot 6\%}$

This is then clearly insufficient to account for the actual increase of about 5 to 6% over the limiting value observed at this current (see Figure 15).

It thus appears that we cannot look in this direction for the source of the discrepancy.

North (3) has considered the possibility that electron reflection at the anode might produce anomalously large noise under normal space-charge conditions in a true diode. This factor has therefore been considered in connection with these measurements as a possible explanation of the low values observed for the limiting current, I_{lim}. It appears that data on the subject is rather scanty but that a paper by H.E. Farnsworth (1) is relevant. This has been consulted and it appears that as the primary energy diminishes the overall secondary/primary ratio falls; we remember that in the retarding field case under consideration the primary energies will be very low, and from Farnsworth's data we observe that for .6 volt primaries on a Nickel Anode the ratio is only about .12. On the other hand the relative amount of those secondary electrons whose energy is equal to that of the primaries increases and appears to approach unity.

Let us then consider the effect of reflection in the retarding region. Let us assume that the potential minimum is "near" to the anode but has not yet quite reached it. Reflected electrons may then either re-penetrate the barrier and return to the cathode or if they have very low energy may again return to the anode after approaching the potential barrier.

Considering the first case: if a fraction δ

of the "incident" current, I_1, to the anode is reflected without significant energy loss through the barrier then the <u>observed</u> current, $\bar{I}$, is given by :

$$\bar{I} = I_1 - \delta . I_1 \doteq (1-\delta).J\varepsilon^{\frac{eV_a}{kT}} \qquad 3\text{-}(56)$$

assuming that the reflection process has not disturbed the assumed Maxwell-Boltzmann distribution. In this case we should still retain $\frac{\bar{I}eR_a}{kT} = 1$ since δ, appears not to vary significantly with infinitesimal variations in $\bar{I}$. On the other hand, however, the effective space-charge density will be determined by :

$$\hat{I} = I_1 + \delta . I_1 \qquad 3\text{-}(57)$$

That is, the "arrival" of the potential minimum will be governed by $\hat{I}$ (<u>not</u> $\bar{I}$), and thus the observed current for the incidence of the true retarding regime will be smaller than the theoretical current by a factor $\frac{1-\delta}{1+\delta}$. Insertion of numerical values readily shows that the effect is of small significance in this problem and a similar conclusion is arrived at if one considers the effect of electron reflection with a significant loss of energy. In the latter case the factor of reduction is $\frac{1}{1+2\delta}$, under similar assumptions.

This effect may therefore be considered to contribute little to the phenomenon.

As a final point, it appeared just possible with the earlier valves that end-effects on the cylinder might "delay" the appearance of an overall retarding field. It seemed however quite certain that no such effect could exist with the very close-spaced diode type CV140 used in the latter measurements. In addition, certain data provided by the Electrotechnics Department of Manchester University on an experimental valve made by Mullard is of interest. This valve had very wide spacing using a tungsten filament and was intended to enable students to verify experimentally the Langmuir-Child - or "$^3/_2$ power" - law. To eliminate end-effects ("field-fringing") due to the large anode-cathode diameter ratio, "guard-anodes" were provided close to either end of the main anode and are of course during experiment maintained at the same potential as the main anode. For the original purpose of the valve only the main anode current is measured but some data around saturation is of interest in our problem:

V_a	Main Anode Current	"Guard"- -Anode Current.
50V	27mA.	23mA.
100V	67mA.	32mA.
150V	79mA.	34mA.
200V	86mA.	35mA.

Thus the fractional increase on the guard-anode from 100V-200V is $\simeq 3/33 = 1/11$, while on the main anode it is $\simeq 19/77 \doteq 3/11$. It thus appears that the effect of "field-fringing" (predominant on the guard anodes) is to cause <u>more rapid</u> saturation to be reached than where the field is truly cylindrical (on the main anode). That is, it suggests that the space-charge has less effect on the potential distribution when "fringing" occurs, and one might therefore similarly expect the retarding condition to be reached more rapidly with a degree of "fringing" than without. It thus appears that such an effect is unlikely to explain the discrepancy.

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3. 11. Conclusion.

The simplest explanation thus emerges that a potential barrier of some sort exists at the surface of the anode. Thus although theoretical calculation based on the inter-electrode space-charge distribution indicates a critical current I_{Lim}, the measured current would be reduced by $\varepsilon^{-\frac{eV_1}{kT}}$, where V_1 is the barrier potential. To account for a reduction of $\frac{1}{7}$ say would only require a factor $\sim \varepsilon^{-2}$, which for $T = 1000°K$ would give $V_1 \doteq \cdot 17$ Volt.

Such a potential barrier would be expected to differ from valve to valve explaining the varying divergencies of the observed limiting current from that predicted theoretically in differing valve specimens.

It is hoped at some later date to make further experiments with a valve comprising a Tungsten filament, so that temperatures may be independently determined accurately (which is not possible with oxide-coated cathodes), and a specially heat-treated anode of very small radius (~1mm.) provided with "guard-anodes"). The author is indebted to Messrs British Thompson Houston Ltd., for undertaking the construction of such a valve.

It is, however, felt that the results presented here on the diodes examined are quite conclusive and undoubtedly demonstrate entry into the true retarding field region for the fluctuation measurements to be described in the following chapter.

CHAPTER 4.

FLUCTUATION MEASUREMENTS ON ELECTRON VALVES.

4. 1: Measurements of fluctuations under retarding field conditions in thermionic valves.

4. 1. 1: Introduction.

Ample experimental work appears to have been carried out in the past to investigate the fluctuations in thermionic valves under normal space-charge limited operating conditions. Workers in this field are notably Llewellyn, Schottky, Williams, Pearson (1), Spenke (2) and North, whose work has been discussed in Chapters 1 and 2 of this thesis.

Generally speaking, the results show satisfactory agreement with equations (1-(5)) and (1-(6)) relevant to the normal space-charge region where thermionic valves are of greatest value in engineering applications. (See also addendum overleaf.) In consequence, the writer has not duplicated this work by operating his diodes under these conditions.

The particular problem which we wished to investigate was the behaviour of the fluctuations under true retarding field conditions. Reasons have been put forward in some detail in the previous chapter to establish that this region had not previously been investigated.

....

4. 1. 2: Survey of Experimental Technique.

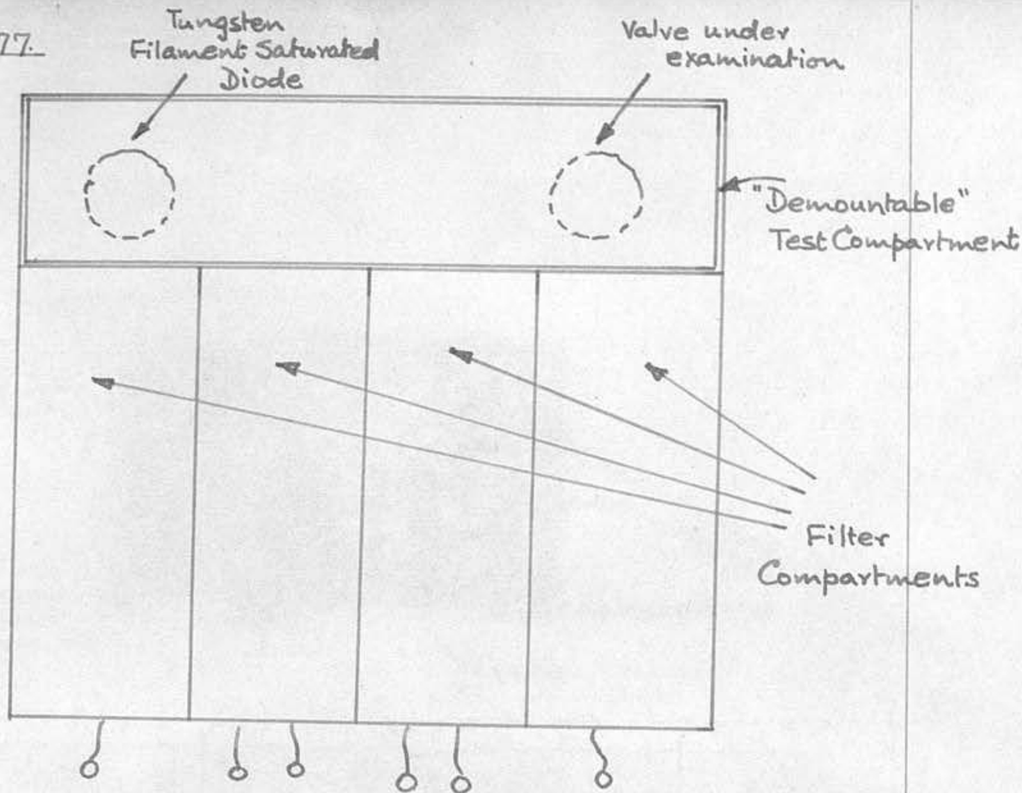
To avoid the contamination of "flicker effect", (and at the same time exclude any suggestion of "transit time smoothing") the lead of D.C. North was followed

Addendum to page (76) :

It should however be noticed that in the experimental work of D.O.North[4] in the general space-charge region on multi-electrode tubes operated as diodes and also on true diodes a lack of agreement with the theory is evidenced. This he ascribes to a degree of electron reflection at the anode under diode conditions, triodes showing very good agreement with theory. It must however be noticed that Pearson's results[2] obtained on diode-operated valves showed no such disagreement.

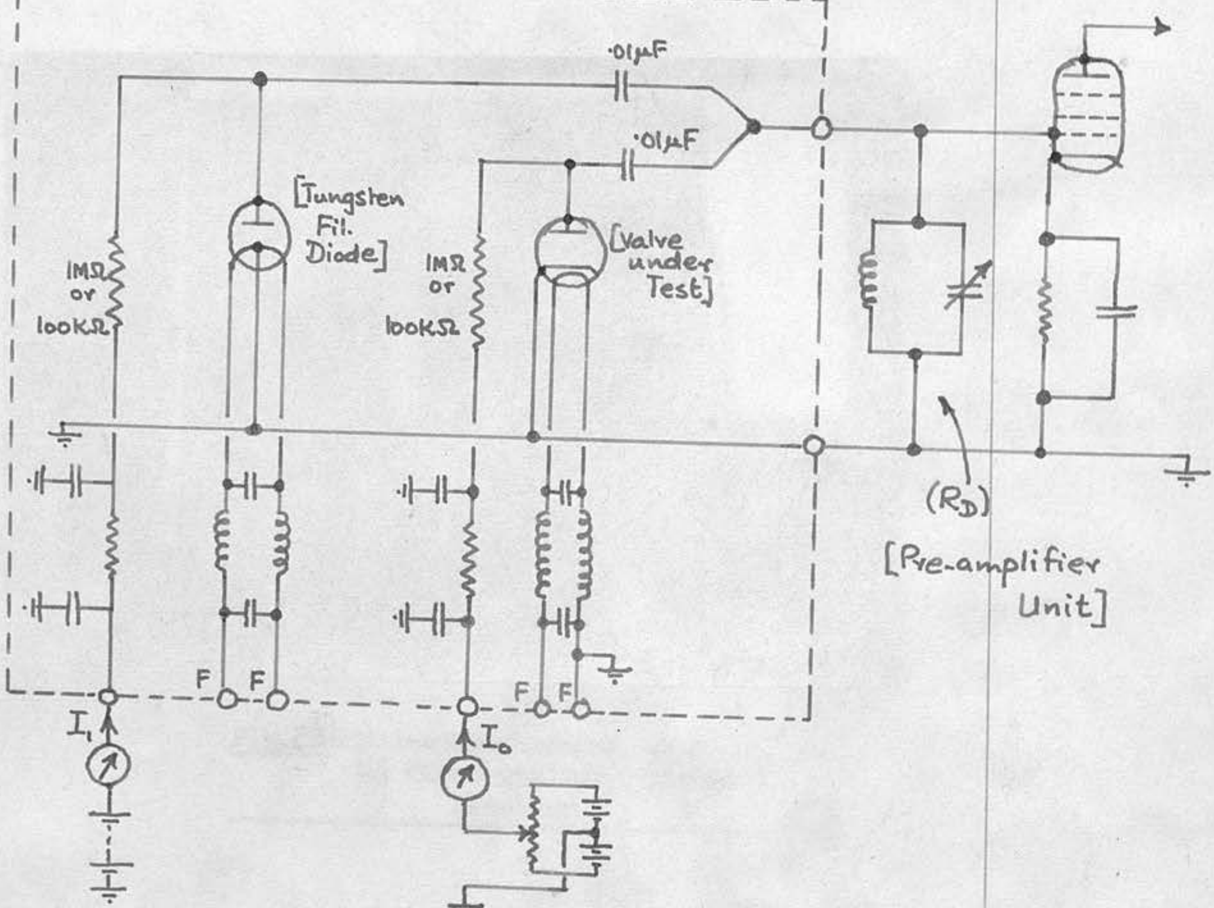
The writer feels himself to be in qualitative agreement with D.A.Bell[1] that North's explanation of discrepancy is not correct, although he does not agree with Mr. Bell's suggested diode fluctuation analysis. In particular, however, it should be noted that Dr.North's experimental results show closest agreement with theory when approaching the retarding region and that the writer's work has proved perfectly satisfactory within the retarding field region.

and a receiver operating in the one megacycle region was employed as the measuring instrument. It was desired in general to avoid any considerable construction work and after a preliminary test on the input circuit the Marconi Communication Receiver was employed for the work, and a supplementary unit was designed for the fluctuation measurements. A general sketch of the unit is given in Figure 21 , a detailed circuit in Figure 22 , and a photograph in Figure 23a. In the early experiments conducted on a 6H6 type diode the unit was connected directly to the grid of the first valve in the Marconi receiver. The unit was provided with a removeable section containing the valve under examination and the tungsten diode working under saturation conditions employed as the reference noise generator, so that the particular valve specimen under test could be changed readily. To allow for the added capacity of the unit, the relevant section of the main "ganged" tuning condenser was disconnected and the unit was then tuneable over a very limited range by means of the aerial "trimmer" condenser provided in the set. As output indicator in these measurements either a thermocouple or valve voltmeter was employed; the choice of indicator will be discussed in more detail below. A serious problem with this layout was to avoid "pick-up" of extraneous signals; to assist in detection of this effect the fluctuation measured was displayed at all times visually on a Cathode Ray Tube and aurally on head-phones or a loud-speaker. The unit was also



General Sketch of Noise Measurement Unit

Figure : 21.



Circuit Detail of Noise Measurement Unit

Figure: 22.

Plate IV: following p.77.

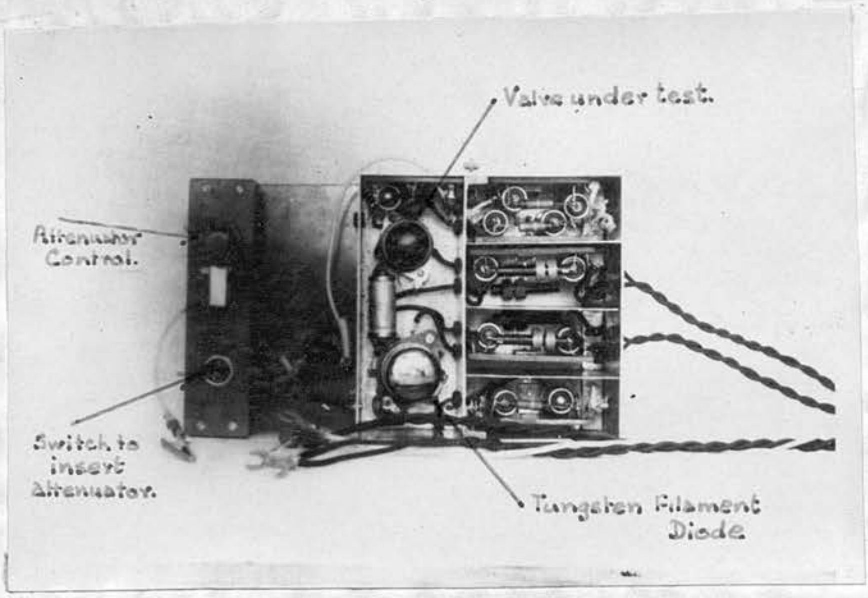


Fig.23a: Noise Measurement Unit

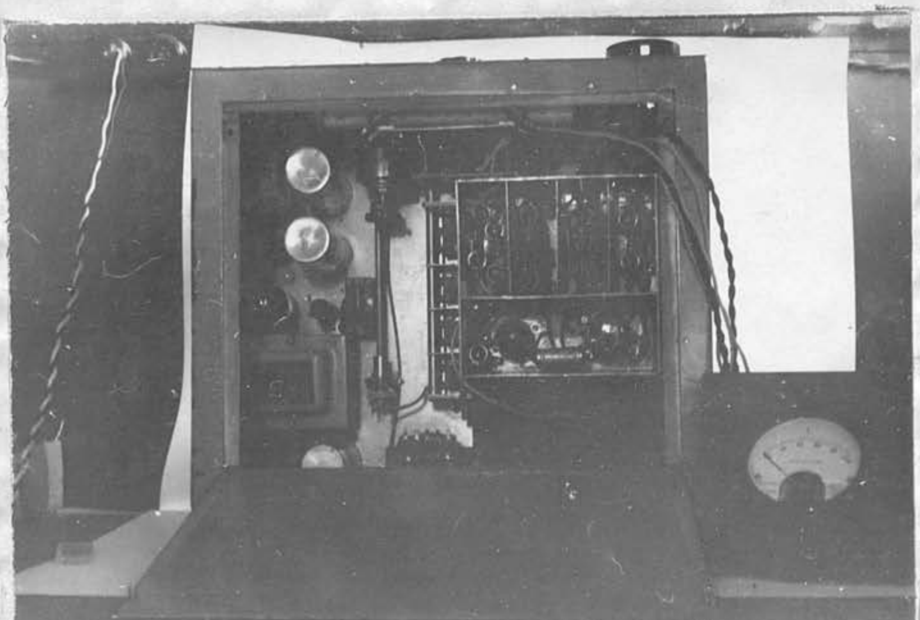


Fig.23b: Noise Measurement Unit
as first employed inside
Receiver

originally designed to fit inside the Marconi receiver (see Figure 23b) with as short a connecting lead as possible to assist in avoidance of this trouble.

The general results of a large number of measurements indicated that in the retarding region (currents up to 2-3μA.) the relevant fluctuation formula was

$$\overline{(i-I)^2_{exp}} = 2eI\Delta f \qquad 4-(1)$$

(i.e. $\Gamma^2 = 1$) but the results showed a considerable overall spread (about 0.7 to 1.3) and the measurement methods adopted (to be discussed below) involved accurate meter calibration, which proved difficult at these levels and also depended upon accurate determination of the dynamic resistance of the 1st., tuned circuit and of the slope resistance of the valve. One particular trouble was that the Cambridge "Mirror Spot" Galvanometer employed required frequent calibration; further "pick-up" troubles proved rather severe; measurements often having to be done on Sundays or in the late evenings to avoid local interference. Finally, when any fault developed the whole unit had to be disconnected and removed to trace the source of trouble.

Therefore a single stage external pre-amplifier was designed with particular attention paid to screening to operate in immediate conjunction with the "noise measurement unit". Grid and anode circuits were tune-able separately to allow for the varying capacity introduced by the fluctuation unit; a frequency range around 1.5 to 2 Mc/s. was now chosen to minimise interference.

The amplifier valve selected was a low-noise, high amplification pentode, Mullard type EF50, and the out-:put was taken from a low-impedance winding on the anode circuit via a low-impedance (80Ω) coaxial screened cable to the normal 80Ω (Dipole) input terminal of the Marconi receiver some distance away. The arrangement proved very satisfactory and free from external disturbance and very much more convenient than the first system, the noise unit being always completely accessible with its pre-amplifier. All heaters were fed from a large capacity accumulator and the high tension supply to the pre-amplifier was derived from a separate dry battery to eliminate mains interference. The high-tension to the main receiver was derived from an ancillary excellently smoothed mains fed power-pack which was available and no difficulty was experienced.

Although it was quite evident that the system was operating excellently and on the other the close-spaced diode type CV140 was now available so that much larger currents could be employed, the results were still unsatisfactory and it was therefore decided to examine the various possible methods of measurement to analyse the possible sources of error. The earlier methods employed were devised to avoid placing any dependence on the operation of output meter and to use it merely as a "fixed position" indicator.

The two methods were as follows :

Method 1:-

A switch and preset resistance across the last intermediate frequency amplifier stage were incorporated in the unit (see Figure 23a) which could reduce the gain of the set by .707:1 in voltage ($\frac{1}{2}$: 1 in power). The following method was then employed :

(a) : Test Valve and Tungsten Valve inoperative

$\therefore O_1 = G\Delta f\{4R_D kT_r + 4R_{eq}kT_r\}$ ----- 4-(2)

where (O: indicates an output meter observation.
(G: power gain of set.
(Δf: overall bandwidth of set.
(R_D: Dynamic resistance of 1st. circuit.
(R_{eq}: "Equivalent" noise resistance of set.
(T_r: Ambient temperature.

(b): Insert Switch and re-estab-:lish output by introducing current I_1, to tungsten diode (by controlling filament current).

$$\therefore O_2 = O_1 = \frac{G}{2}\Delta f\left\{\left(\frac{4}{R_D}kT_r + 2eI_1\right)R_D^2 + 4R_{eq}kT_r\right\} \text{ ---- } 4\text{-}(3)$$

(See Figure 24.)

(We observe that since the tungsten filament diode is operating under extreme saturation conditions the noise current generated is simply given by $\overline{(i-I_1)^2} = 2eI_1\Delta f$ and no correction is called for in practice to allow for its internal differential impedance (several megohms).

Whence :

$$4R_{eq}kT_r = 2eI_1R_D^2 - 4R_DkT_r \text{ ---------- } 4\text{-}(4)$$

(This incidentally has provided an explicit determina-:tion of the "inherent noise" of the set in terms of the "equivalent noise resistance R_{eq}" - a familiar measurement - alternatively expressible as the "Noise Figure" or "Noise Factor" (e.g. H.T. Friis (1) or D.K.C. MacDonald (2)).

(c): Open switch and cut off tungsten doide. Introduce desired current (I_0) into valve examined by battery control; make an observation :

Plate $\overline{V}$: following p.80.

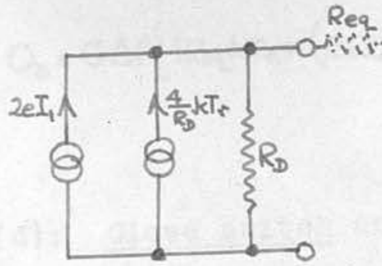


Fig: 24.

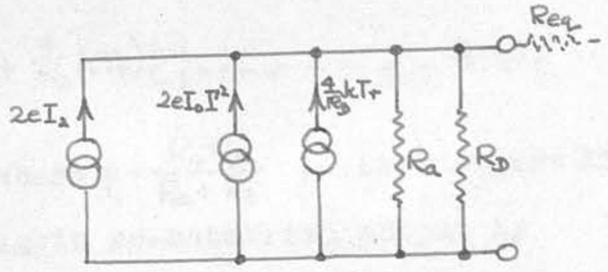


Fig: 25.

[The factor Δf is to be understood throughout]

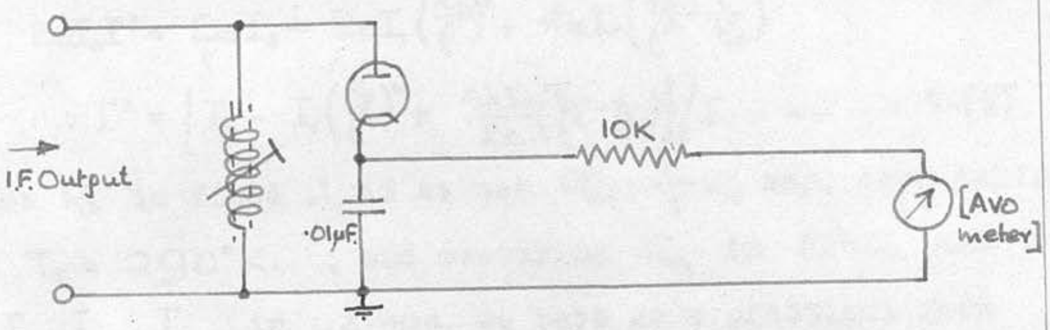
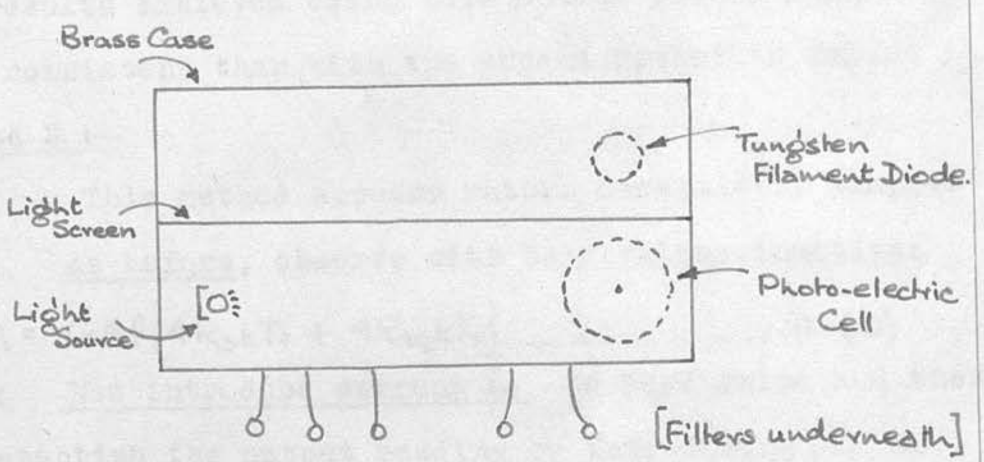


Fig: 26 : see page 83



Sketch of photo-electric cell unit.

Fig: 30 - see page 89.

$$O_3 = G\Delta f\left\{4R_{eq}kT_r + \left(2eI_0\Gamma^2 + \frac{4}{R_D}kT_r\right)\rho^2\right\} \text{---------} 4\text{-}(5)$$

(where $\rho = \dfrac{R_a R_D}{R_a + R_D}$) (See Figure 25.)

(d): Close switch and again re-establish output by introducing current I_2 into tungsten diode.

$$\therefore O_4 (= O_3) = \frac{G}{2}\Delta f\left\{4R_{eq}kT_r + \left(2eI_2 + 2eI_0\Gamma^2 + \frac{4}{R_D}kT_r\right)\rho^2\right\} \text{---} 4\text{-}(6)$$

$$\therefore \left\{2eI_0\Gamma^2 + \frac{4}{R_D}kT_r\right\}\rho^2 + 4R_{eq}kT_r = 2eI_2\rho^2 \text{---------} 4\text{-}(7)$$

But using (4):-

$$2eI_0\Gamma^2 = 2eI_2 - 2eI_1\left(\frac{R_D}{\rho}\right)^2 + 4kT_r\left(\frac{R_D}{\rho^2} - \frac{1}{R_D}\right)$$

$$\therefore \Gamma^2 = \left\{I_2 - I_1\left(\frac{R_D}{\rho}\right)^2 + \frac{4kT_r}{2e}\left(\frac{R_D}{\rho^2} - \frac{1}{R_D}\right)\right\}\Big/ I_0 \text{-----} 4\text{-}(8)$$

Now R_D is fixed ∴ if we set $R_a = \frac{1}{y}R_D$ say, then taking $T_r = 290°K.$, and measuring R_a in $10^3\Omega$ and I_0, I_1, I_2, in μamps. we have as a practical form of (8) :

$$\Gamma^2_{Exp} = \left\{I_2 - (1+y)^2 I_1 + \frac{50}{R_a}(2+y)\right\}\Big/ I_0 \text{---------} 4\text{-}(9)$$

The results achieved using this method proved rather more consistent than with the second method to follow :

Method 2 :

This method appears rather deceptively simple.

(a) : As before, observe with both valves inactive:

$$O_1 = G\Delta f\left\{4R_D kT_r + 4R_{eq}kT_r\right\} \text{---------} 4\text{-}(10)$$

(b) : Now introduce current I_0 to test valve and then re-establish the output reading by introducing I_1 to the saturated diode :

$$\therefore O_2 = O_1 = G\Delta f\left[\left\{2eI_0\Gamma^2 + 2eI_1 + \frac{4}{R_D}kT_r\right\}\rho^2 + 4R_{eq}kT_r\right] \text{----} 4\text{-}(11)$$

∴ from (10) and (11) :

$$\Gamma^2 = \left[\frac{4kT_r}{2e}\left(\frac{R_D}{\rho^2} - \frac{1}{R_D}\right) - I_1\right]\Big/ I_0 \text{-----------} 4\text{-}(12)$$

Or in practical units as before :

$$\Gamma^2_{Exp} = \left[\frac{50}{R_a}(2+y) - I_1\right] / I_0 \qquad 4\text{-}(13)$$

A practical difficulty becomes particularly evident, that the current I_1, tends to zero rather rapidly, but, the difference in the numerator is still very significant. In particular, both formulae (9) and (13) involve accurate determinations of R_a and R_D ; further the meter(s) used for measuring I_0, I_1, I_2 must be accurately calibrated. To remove some source of error in this respect, the same meter (or galvanometer) was used whenever possible to measure I_0, I_1, I_2, but since the currents are of different magnitudes calibration errors may easily play an important role. Thus a set of early measurements on a 6H6 Diode may be cited :

I_0 (μA.)	I_1 (μA.)	I_2 (μA.)	$\Gamma^2_{Exp.}$
4.35	0.96	8.00	1.10
0.58	0.98	1.74	1.07

The method employed by North (5) was therefore re-considered. In this method the "onus" is thrown on to the output meter, which must be capable of measuring relative outputs with accuracy. North (and the writer) have used thermocouples for this purpose; these are very useful in that an occasional impulse of interference does not cause appreciable disturbance to the output reading. In addition, in principle, they are inherently square-law devices which simplifies the measurement problem because the mean square fluctuations of statistically independent elements add linearly; this is of course a fundamental consequence of Campbell's theorem. In practice, however, this principle may prove deceptive as unless the input power is limited severely, making their operation rather insensitive, heat losses cause departure from idealised operation; this implies careful calibration nullifying a consider-:able part of the advantage supposedly accruing from the use of thermocouples.

On the other hand a sustained burst of local interference(Electric Drill, Pulse Generator, Trans-:mitter testing, etc.) will frequently destroy the thermocouple - as North says, it proves very satisfactory "until it leaves in a pouf". An alternative device is a Valve Voltmeter - this was employed on several measure-:ments - which should provide a linear output reading and is not damaged by interference. On the other hand a single impulse of interference usually gives full scale deflection, the instrument taking a considerable time to settle down once more due to the long time-constant involved. A crystal detector (frequently quoted as square-law) was also tried but the "law" of various crystals lay somewhere between 1.3 and 1.75 and therefore did not prove attractive.

A simple diode (current) detector system operating on a high voltage input - effectively a "power" detector - was ultimately found to be very useful. The output intermediate frequency signal from the Receiver was given further amplification (to provide ~10-100V.) and then applied to the simple system of Figure 26 (following p.80).

This provided clear, legible indications on a good milliameter (AVO type 7 or 40) with plenty of power available and the time-constant was such that no trouble due to interference was experienced. If the amplitude of the input signal is made sufficiently large such a system should give a linear output reading.

To check this and to examine the overall linearity of the system, a very small signal was injected at the pre-amplifier input, but just sufficiently large (say a ratio of 3 or 4 : 1) compared with the inherent set noise to make the "noise" ride" on top of the signal (cf. later statistical work - the overall distribution is now practically pure Gaussian). A linear scale of output against input from the signal generator should now result; and when extrapolated back should pass through the origin, or practically so - any small bias (arising from a small degree of residual asymmetry in the combined distribution) being in the sense of non-zero output for zero input. The chart of Figure 27 obtained for this system gave complete confidence in its operation.

The method for measurement is now as follows :

(a) : Short-circuit the input and read output (V_1)

$$V_1^2 = A.\Delta f(4R_{eq}kT_r) \text{------} 4\text{-}(14)$$

(A: proportional to power amplification of set).

(b) : Remove short-circuit and introduce required current I_o to test valve and read output (V_2)

$$V_2^2 = A.\Delta f\left(4R_{eq}kT_r + \left\{2eI_o\Gamma^2 + \frac{4}{R_D}kT_r\right\}\rho^2\right) \text{---------} 4\text{-}(15)$$

$$\rho = \frac{R_a R_D}{R_a + R_D}.$$

(c): Now introduce I_1 to saturated diode to give V_3 such that $V_3^2 = 2V_2^2 - V_1^2$, then :

$$V_3^2 = A.\Delta f\left(4R_{eq}kT_r + \{2eI_0\Gamma^2 + \frac{4}{R_D}kT_r + 2eI_1\}\rho^2\right) \text{------- } 4\text{-}(16)$$

and :

$$\therefore 2eI_0\Gamma^2 = 2eI_1 - \frac{4}{R_D}kT_r$$

$$\therefore \Gamma^2_{Exp} = \frac{I_1 - \frac{2kT_r}{e}.\frac{1}{R_D}}{I_0} \text{------------ } 4\text{-}(17)$$

For $R_D \doteqdot 16.6k\Omega$ (approximately the value in use) We have in practical units :

$$\Gamma^2_{Exp} = \frac{I_1 - 3}{I_0} \text{-------------- } 4\text{-}(18)\ (I_1; I_0 \text{ in } \mu A.)$$

This is the method adopted by North; using a thermo-:couple, however, the output readings ($O_1; O_2; O_3$.) are directly proportional to the square and thus the require-:ment in principle is simply $O_3 = 2O_2 - O_1$, although North found it necessary to calibrate his output system accurately. However, it was a very simple matter, taking $V_1 = 1$ (a convenient scale figure) to construct a chart to provide :

$$V_3 = \sqrt{2V_2^2 - 1}$$

Further:- $V_3 = \sqrt{2}.V_2\left(1 - \frac{1}{2V_2^2}\right)^{1/2}$,

$$= \sqrt{2}.V_2\left(1 - \frac{1}{4V_2^2} - \frac{1}{64}.\frac{1}{V_2^4}\cdots\right)$$

$\therefore$ if $V_2 = 2$ then :

$V_3 = \sqrt{2}V_2$ to an accuracy of about 6% and thus the chart is practically linear except for values of V_2 very close to unity.

The immediate advantage of the method is that no determination of R_a is necessary, the calculation is greatly simplified, and R_D only enters as a fixed "correction term", which for currents above, say $20\mu A$, becomes of very small significance. Even for currents as low as 10μA. the relative accuracy of determination need not be great. Finally for currents $> 10\mu A$. say, if Γ^2_{exp} is expected to turn out approximately unity then I_1 and I_0 will be of the same order; this fact was utilised by ensuring that the same meter was always employed to determine I_1 and I_0 and therefore very accurate calibration was quite unnecessary.

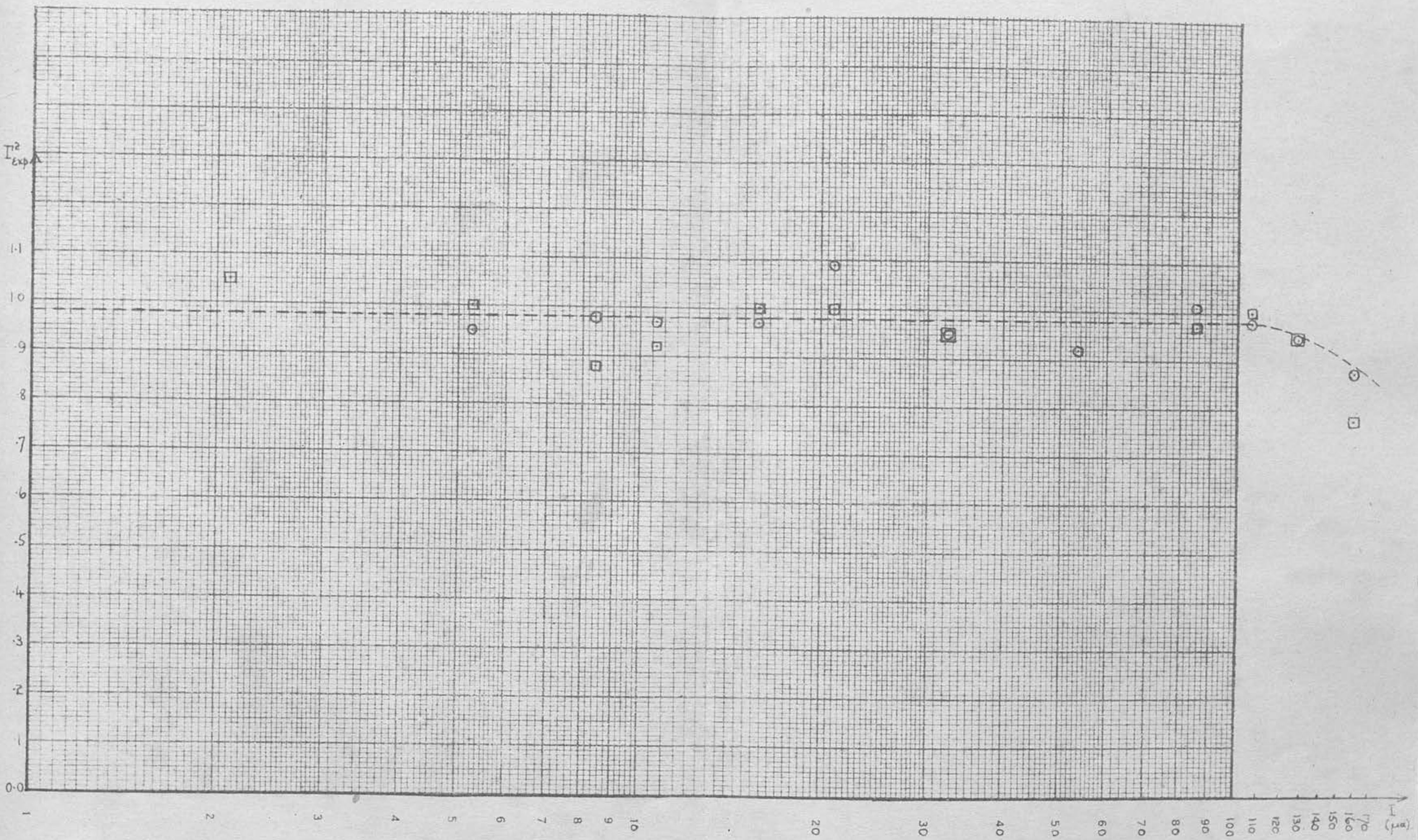
Since the results using this method on the close-spaced diode for currents ranging from $2 \rightarrow 150\mu A$ were very satisfactory, the earlier results on the 6H6 type diode will not further be quoted in detail.

....

4. 1. 3. Experimental Results.

The results of the measurements are presented as Figures 28 and 29 and demonstrate conclusively that Γ^2 in the retarding region may be accepted as unity throughout. There appears some evidence (cf. Figure 28 and Figure 16) that Γ^2 does reach unity at the same point that the valve enters the retarding region although since the transition is smooth an exact "point of entry" is not definable in either case.

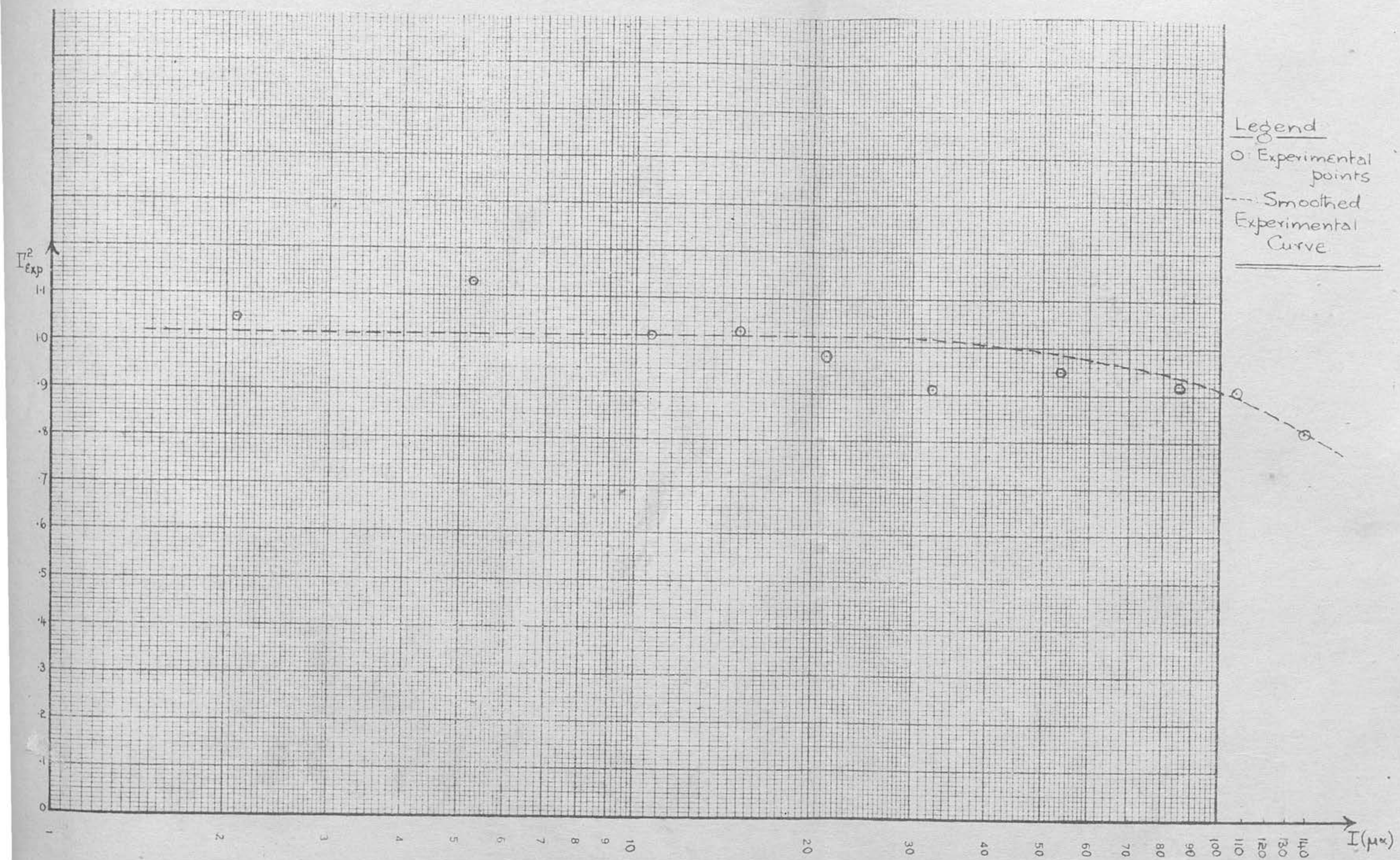
The measurement of R_D was carried out "in situ" by a direct substitution method using a signal generator and a known carbon resistance; further checks on the shunting effect of the test valve on the dynamic resist:ance with a given current through it (and therefore



EXPERIMENTAL DETERMINATION OF SPACE-CHARGE REDUCTION FACTOR – Γ^2_{Exp} – IN DIODE CV140 [Second Model] UNDER TRUE RETARDING FIELD CONDITIONS

LEGEND: ⊙ . OBSERVED VALUES FOR $I_f = 0{\cdot}3\alpha$ [MEAN : 0·98]
⊡ : OBSERVED VALUES FOR $I_f = 0{\cdot}25\alpha$ [MEAN : 0·97₅]

Figure: 28.



Experimental Determination of Space-Charge Reduction Factor (Γ^2) in Close-Spaced Diode [Second Model] (Oxide-Coated Cathode at low temperature) under true retarding field conditions

[Average value of Γ^2_{Exp} (to $33\mu\alpha$) = 1·02]
[I_f = 0·2α (rated value 0·3α)]

Figure: 29.

known R_a from the "bridge" measurements) revealed consistency although the "spread" of the inferred values of R_a by this method was generally rather greater than those obtained on the bridge, illustrating the value of that method.

....

4. 2: Fluctuation Measurements on Photo-Electric Cells.

4. 2. 1: Purpose of Measurements.

The following general reasons were initially envisaged for conducting some measurements on fluctuations of High-Vacuum photo-cells. First, the data available appears very scanty. Kingsbury (1) reports some measurements on saturated photo-cells which gave a value of the fundamental charge e some 25% too high. In other words in more familiar notation :

$$\overline{(i-I)^2} = 2eI\Gamma^2\Delta f \quad 4-(19) \text{ with } \Gamma^2 \doteqdot 1.25$$

W.A. Harris (1) states baldly that $\Gamma^2=1$ for photo-electric cells under saturation conditions (i.e. normal operating conditions when used as photo-detection devices) but does not quote any measurement data. Secondly it was believed that photo-electric cells entered the effective saturation region more "rapidly" than for example a tungsten filament, or rather that the "knee" on the characteristic evidencing the disappearance of an internal potential minimum, was more abrupt. Now the writer had for some time, as a complement to the retard-:ing region measurements, wished to determine whether or not Γ^2 became unity identically whenever the internal potential minimum disappeared or whether the transition was more gradual and asymptotic. The point at issue is this: Schottky's original derivation of the "shot" formula is based on the electron-transits constituting independent "events". The exact significance of the operative word <u>independent</u> does not appear to have been discussed. If we accept the "potential minimum" criterion inherent in current theories discussed earlier then the implication is simply that the occurrence of <u>one</u> event cannot entirely <u>prevent</u> the occurrence of any other event and Γ^2 should become and remain unity whenever the minimum disappears. On the other hand, if the implication is one of mutual interaction it might well appear that only when very high voltages were applied such that space-charge effects became negligible would Γ^2 strictly reach unity. Typical tungsten filament diodes (which can be saturated satisfactorily) which would appear suitable for such an experiment, however, do not in general provide a very sharp "knee" and the evidence appears rather inconclusive.

The writer after reading a paper by Thatcher and Williams (1) considered the following possibility. A tungsten filament tetrode could have a fairly "open" 1st. grid and a rather closely wound 2nd. grid. The 1st. grid would be operated at a high potential to draw a saturation current from the filament of which a fair portion would pass through the grid. The 2nd. grid would then be operated at low potentials around zero, while the anode would be operated at a normal positive potential. If on variation of the anode potential a strictly constant current was observed then it would be known that no potential minimum existed in the valve; further, in principle, it should be possible to obtain a strictly constant current because the "Schottky" field effect at the cathode would be nil, because of the screening action of the grids. In fact, the essence of the experiment is that the 1st.grid would be regardable as a saturation emitter entirely unaffected by Schottky field effect which it was presumed was the main factor in vitiating the desired sharp "knee" in the diode characteristic. The writer was indebted to Messrs. Ferranti Ltd., Moston, Manchester, for making some specimens to this general design. Under certain circumstances a reasonably sharp knee was obtained but not under the predicted conditions. As other work was pressing, this project was abandoned - for presentation in this thesis at any rate.

Finally it appeared of interest to see how far space-charge effects modified the fluctuations.

....

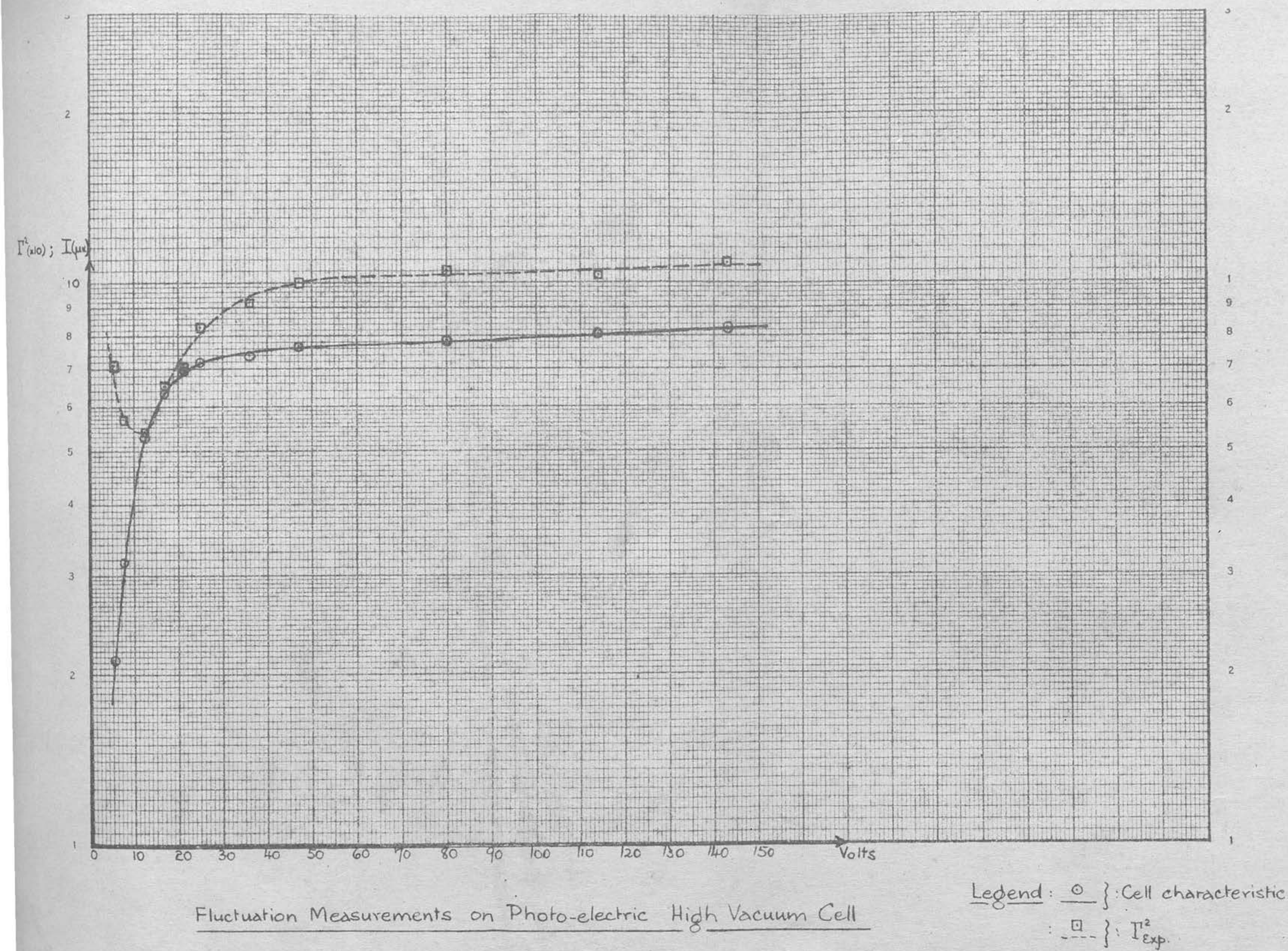
4. 2. 2: Measurement technique.

A measurement unit similar to that devised for the thermionic valve measurements was therefore designed of which a general sketch is shown in Figure 30A (following p.80). Some preliminary measurements showed that the device was, as expected, very sensitive to lamp current and it was therefore found advisable to determine the characteristic of the cel~~l~~ concomitantly with the noise measurements. This was readily done as a 1 MΩ resistance was inserted in series with the photo-electric cell internally and therefore a voltmeter could be attached externally (rather than attempting to use a valve voltmeter) to read the effective voltage directly at all times. Strictly speaking the characteristic should be modified by allowing for the small potential drop across the resistance, but the modification to the characteristics is negligible.

....

4. 2. 3: Experimental results.

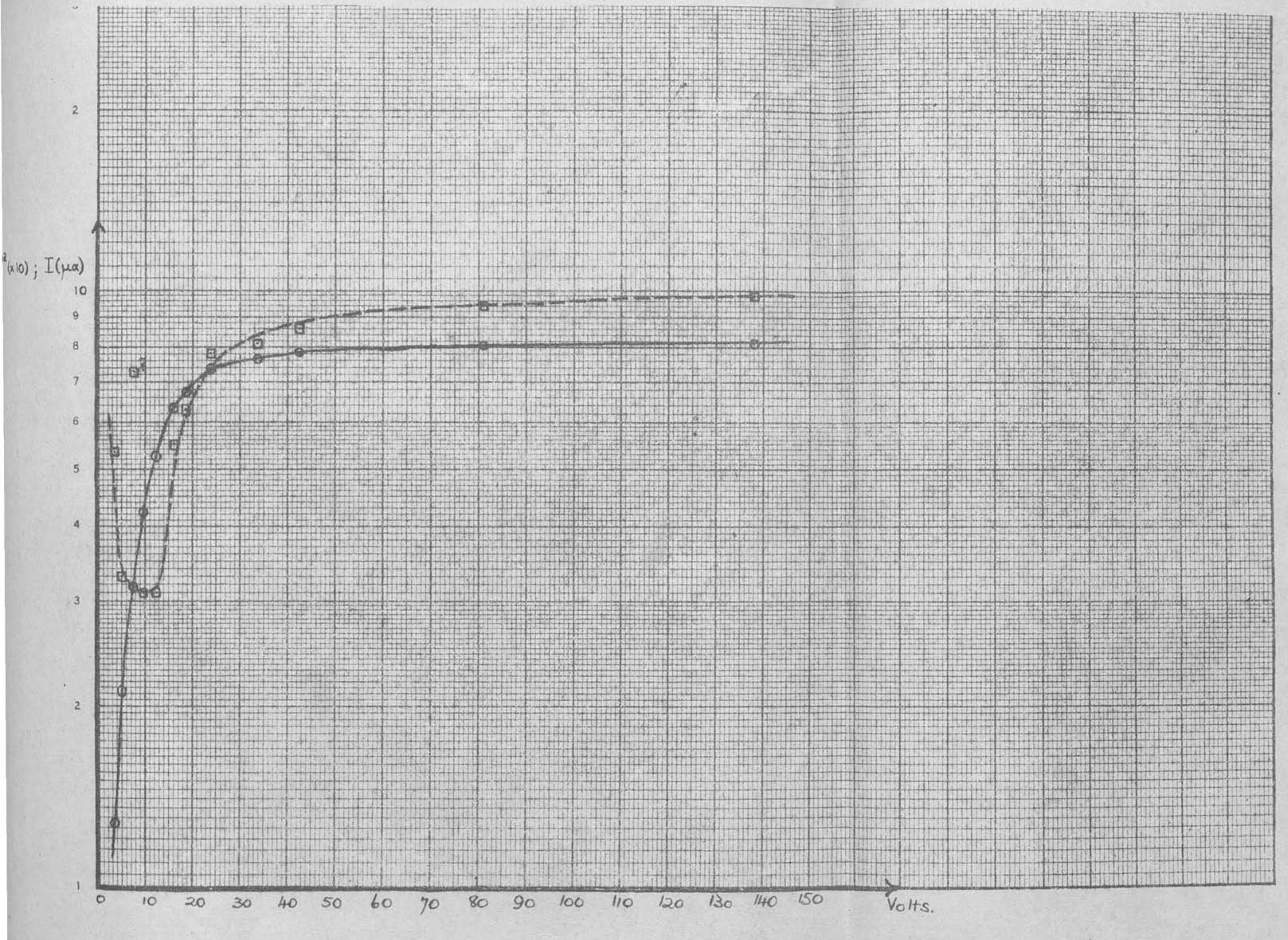
The results obtained for three different degrees of illumination are presented in Figures 31, 32 and 33. It appears quite clear that the value of Γ^2 under saturation conditions is at any rate very close to unity. On the other hand, in each case there appears a definite tendency for Γ^2 to rise very slowly (possibly towards an asymptotic value) with large voltages. A second point of observation is that with the greatest illumination the variation in Γ^2 appears to follow the "knee" in the cell characteristic very closely, but successively less closely at the lower illuminations. Thirdly, considerable space-charge reduction is evidenced in all three cases. This would certainly appear to add further confirmation to the modern "potential minimum" concept of fluctuation



Fluctuation Measurements on Photo-electric High Vacuum Cell

[Lamp Current : 0·28a.]

Figure: 31.

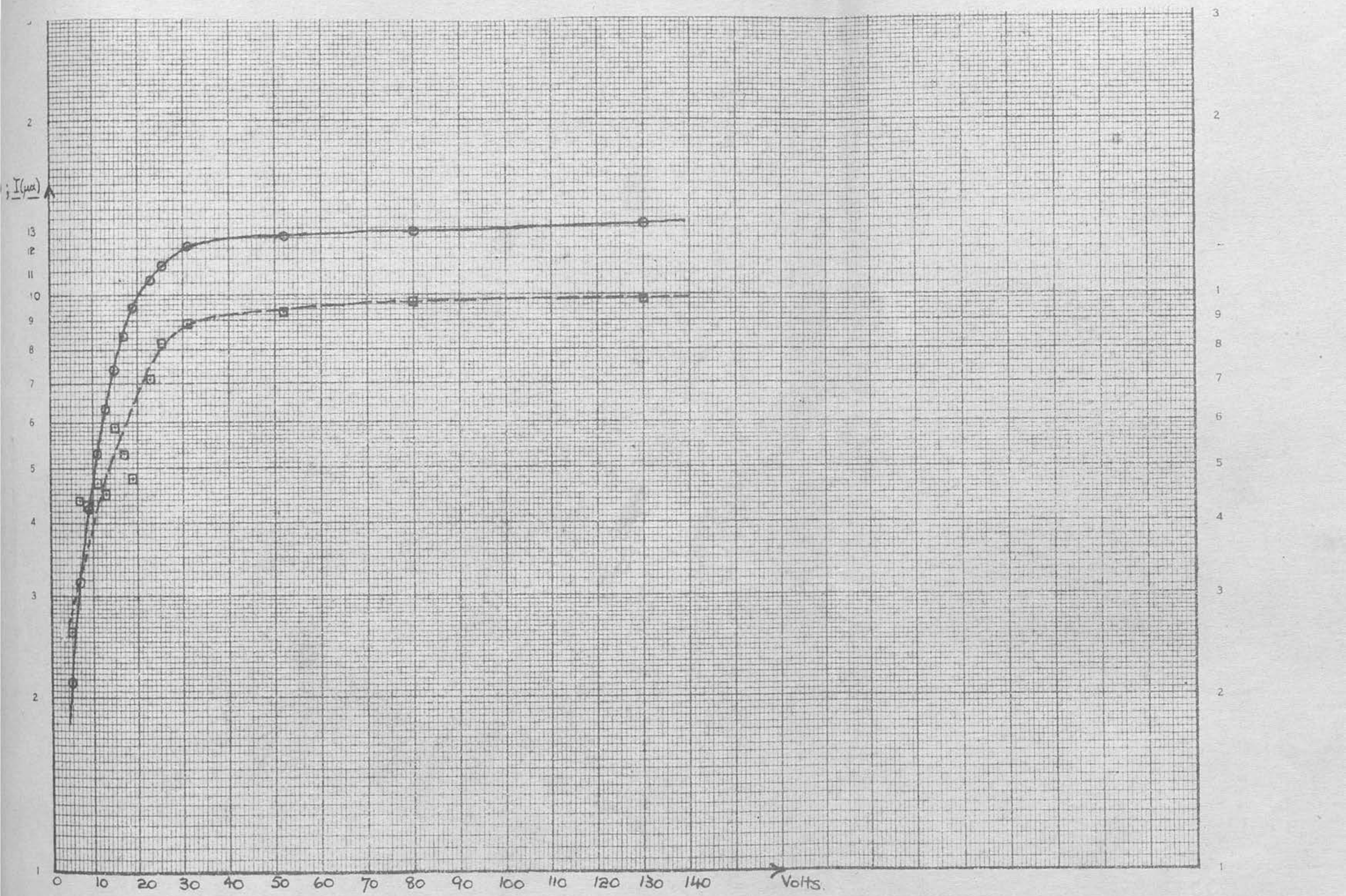


Legend: —⊙— } : Cell characteristic
—⊡— } : $I^2_{Exp.}$

Fluctuation Measurements on Photo-Electric High Vacuum Cell

[Lamp Current: 0·3a]

Figure: 32.



Fluctuation Measurements on Photo-Electric High-Vacuum Cell

[Lamp Current: 0.33a.]

Figure: 33.

reduction rather than the alternative concept of an inherent smoothing of fluctuations due to the very existence of the space-charge since in the photo-cell (particularly bearing in mind the large dimensioned spherical construction) the absolute value of the space-charge density must be very low indeed. Finally the existence of a minimum for Γ^2 is evident for the two lower values of illumination the value of the current for this diminishing as the illumination is increased and the minimum value of Γ^2 progressively decreasing. Since the most obvious effect of increased illumination is to increase the emission this behaviour is consistent with that of a thermionic valve. An interesting feature is that for the lower values of emission practically the whole variation of Γ^2 can readily be traced, varying from unity under saturation conditions (potential minimum at cathode) through a minimum value and rising (presumably) towards unity again as retarding field conditions (potential minimum at anode) are approached. The writer hopes in the future to be able to carry similar measurements into the true retarding field region for photo-cells.

....

4. 2. 4: "Effective temperature" indicated by photo-electric cell fluctuations.

4. 2. 4. 1: Purpose of measurements.

The fact that it has been shown and confirmed for thermionic valves working under space-charge limited conditions with I/J small that the space-charge reduction factor Γ^2 may be expressed as :

$$\Gamma^2 \simeq \frac{2k}{I e R_a}(\theta T) \text{ - - - - - } \begin{cases} 1-(5) \\ 4-(20) \end{cases}$$ (where θ is a "slowly varying" factor).

offers an interesting field of investigation in the photoelectric cell. If we assume that under reasonably space-charge limited conditions a similar relationship will hold for a photo-electric device then having measured Γ^2 it should be possible to obtain an indication of the effective temperature of the electron "cloud", writing :

$$(\theta T)_{Exp} = \frac{I e R_a . \Gamma^2_{Exp}}{2k} \text{ - - - - - - - - - - } 4-(21)$$

We should emphasise at this stage that only an <u>indication</u> is expected from these measurements for two general reasons. First, the expressions in (20) and (21) above have, of course, been derived for an emission obeying the Maxwell-Boltzmann law at temperature T ; it is not, however, a simple matter to state the statistical distribution under photo-electric emission although it is hoped to proceed further with this work in detail at a later date. Secondly, apart from this, the factor θ only assumes a practically constant value for I/J very small, and it has not yet been possible to investigate space-currents in the photo-electric cells so low as to make this assumption truly valid. On the other hand, if the statistical distribution of the emission can be established satisfactorily and a detailed theory evolved for the fluctuations in the future, then it should be quite possible to evaluate θ (or an analogous factor) for a <u>given</u> ratio I/J in that case.

A true Maxwell-Boltzmann distribution of the photo-electrons at the temperature of the lamp filament could, of course, only exist if thermal equilibrium were established between the electrons and the radiation of the illuminating source; in this case, of course, equation (21) should be strictly applicable and the indicated temperature should be that of the lamp filament. However, as will be shown in the next section, the effective temperature proved to be around 10,000°K - 20,000°K - i.e. much higher than the filament temperature of the lamp (say 2,500°K - 3,000°K), showing clearly that the photo-electrons are not in statistical equilibrium.

Nevertheless the electrons will have some sort of statistical distribution of energy due to individual variations in the energy of the incident photons and to the interaction of the electrons with the cathode. It is therefore suggested that we employ the above formula (21) to derive a value for a characteristic temperature T in which case "kT" will represent the average energy of the electrons in the immediate neighbourhood of the cathode surface, and this in turn may be set equal to hf where f represents an effective light frequency, namely:

$$hf \sim kT \text{--------- } 4\text{-}(22)$$

The maximum intensity of the incident radiation lies, of course, in the infra-red region of the spectrum. On the other hand, the sensitivity of the photo-cell increases with increase of f ; further the ultra-violet region of the spectrum is practically cut off by the glass walls of the bulb. Thus qualitatively we should expect the effective frequency f to be somewhere in the visible spectrum; this is further confirmed by quantitative response curves of photo-sensitive materials (e.g. E.F. Seiler (1)).

....

4. 2. 4. 2: Experimental results.

A fair value of R_a may readily be determined from the slope of the cell characteristics in Figures (31), (32) and (33); I and Γ^2_{Exp} are extracted for as low a value of current as convenient and where the experimental data is felt to be reasonably reliable.

The results for θT are tabulated below :

Illumination Lamp Current	I (μA.)	Γ^2_{Exp}	R_a ($10^6\Omega$)	θT (°K.)
0.28 amp.	2. 0	.71	2.67	~ 22,000
0.3 amp.	2.3	.33	2.22	~ 10,000
0.33 amp.	4.3	.43	1.91	~ 20,000

As indicated previously these temperatures immediately exclude any possibility of thermal equilibrium at the lamp temperature and it remains to see whether the indicated frequency corresponds to the part of the spectrum suggested.

If then we set :

$hf = kT$, and $\lambda = \frac{c}{f}$ for the wavelength of the incident light

then: $\lambda = \frac{hc}{kT}$

$$\doteqdot \left[\frac{3.10^{10}.6\cdot5.10^{-27}.10^{28}}{1\cdot4.10^{-16}.T}\right] \text{Å}$$

$$\doteqdot (1\cdot4 \times 10^8)/T. \text{ Å}$$

If then we take θ of order unity (bearing in mind that its asymptotic value for $\frac{I}{J}$ small in the thermionic problem is $\sim \cdot 64$) then we find for the three cases above :

Lamp Current	λ(Å)
0.28 amp.	6,400
0.3 amp.	14,000
0.33 amp.	7,000

....

4. 2. 4. 3: Conclusion.

This then shows, bearing in mind the approximate nature of this investigation, that, as expected, the effective wave-length lies somewhere in the visible part of the spectrum. No further quantitative conclusions can be drawn from the present experiments. In order to obtain more information about the fluctuations in photo-electric currents, especially in connection with the question whether the fluctuations are connected directly with the fluctuations in the primary radiation, more extensive experimental investigations would be necessary using monochromatic light.

The writer wishes to emphasise, however, that it is clear that the region of space-charge limitation is almost certainly the only profitable region in this respect, since in the saturated region interaction between the electrons as evidenced by their statistical distribution is specifically excluded. One can therefore only expect in the latter region to verify the simple statistical law

$\overline{(i-I)^2} = 2eI\Delta f$, as first derived by Schottky, which does not rest on the specific statistical distribution of the electrons which is of vital interest.

CHAPTER 5.

STATISTICAL EXAMINATION OF RECORDS OF ELECTRICAL FLUCTUATIONS.

5. 1: Introduction and Experimental considerations.

It was considered that it would be of value to obtain some records of electrical fluctuations so that detailed statistical analysis might be undertaken there-:on. So far as the writer is aware, apart from one rather obscure example quoted by S.O.Rice, whose work will be discussed further below, no investigation of this nature has been previously published.

A very considerable amount of work has been carried out on mechanical systems, such as that of Svedberg and Westgren on colloid statistics and that of Fürth later in similar fields. The closest analogous investigation to that presented here is that conducted by Eugen Kappler [1,2] on the random oscillation of a torsion balance. This was observed under varying degrees of air pressure and photographic records were obtained by using the familiar method of light beam amplification. One rather important feature underlying the results presented here should be emphasised; in Kappler's experiments, because of the very low natural frequency ($T \simeq 30$ secs) involved in the mechanical system, it was feasible to employ, practically speaking, all degrees of damping as represented by the surrounding air pressure and still obtain legible records without difficulty; that is to say records ranging from highly periodic to practically pure aperiodic conditions were obtainable. Expressed in language more adapted to an electrical system, relatively wide bandwidths (corres-

:ponding to the highly aperiodic case) could be readily employed. It will of course be appreciated that the greater the degree of damping employed, the more rapid does the observed rate of fluctuation of the system become; stated otherwise, the system can respond over a wider frequency spectrum to the primary Brownian impulses of the surrounding medium.

In our work it is not feasible to employ a very low natural frequency for the electrical system since for frequencies below about 100 Kc/s an alien disturbing fluctuation effect becomes evident to an increasing degree. This is known as "Flicker effect" and has been discussed, for example, by Schottky (6) and Johnson (2). It is presumed that this arises from some relatively gross, slow variation of the emissive properties of valve emitters (although one should note that a similar effect also makes itself evident in resistors) such as the migration perhaps of oxygen molecules over the surface of the cathode resulting in large local variations of the cathode emission. However, this not being a form of Brownian motion due to fundamental thermal agitation, it was not desired to include its effect in these records. The effective natural frequency employed in fact in the records throughout was about 465 Kc/s, the customary intermediate fre:quency of radio sets, although the primary fluctuations were generated in circuits with natural frequencies between about 100 Kc/s and 2 Mc/s.

The factor which then characterises the nature of the records is the extent of the frequency spectrum to which the receiver responds (i.e."Bandwidth"). This has a ready analogue in the theoretical expressions quoted by Kappler (3) for the significant quantities in the periodic case. The significant factor governing the decay rate of the correlation of the fluctuations is $\exp(-\frac{w}{2m}t)$ where w represents the damping (die Reibungskonstante) and m the mass of the system. The corresponding factor in an electrical oscillatory circuit of simple "lumped" components is $\exp(-\frac{R}{2L}t)$. However, it is known that the circuit bandwidth in a lightly damped resonant circuit is expressible as :

$$\Delta f' \doteqdot \frac{f_r}{Q}$$,where Q is the "quality factor" of the circuit.

and : $Q = \frac{2\pi f_r L}{R} \therefore \Delta f' = \frac{R}{2\pi L}$.

Thus the significant rate of fluctuation recorded - that is, in fact, the rate of fluctuation of the amplitude of the oscillations of the system (or the "envelope" as it is generally termed) - is governed by the bandwidth employed. The choice of bandwidth again is governed by a number of considerations. One possible hope originally entertained, and discussed later was to examine whether any distinction could be observed in the records between the correlation involved in pure "shot" noise and "thermal" noise (in this connection, the discussion concluding the introductory chapter is germane). For this purpose it would naturally be desirable to employ as wide a bandwidth as possible. Against the employment of very wide bandwidths however two considerations militate. First to obtain wide bandwidths it is necessary to employ rather high frequencies; in this case, however, the amplification per stage of the receiver will diminish and thus the relative contribution of the fluctuations generated at the input of the receiver to the noise arising in successive amplifiers will fall, making it therefore increasingly difficult to control and define the dominant source of the recorded fluctuation. In particular it becomes impossible to record predominantly thermal fluctuations arising from a tuned circuit, the noise arising from the following amplifier valve "swamping" that arising from the associated circuit. Expressed otherwise, the "noise figure" (references (a), (b), (c) and in this connection especially (d)) of the receiver deteriorates. [(a): see H.T. Friis (1) (c): see K. Fränz (1) (b): see D.K.C. MacDonald (2) (d): see E.W. Herold (1)]

Secondly, the speed of photography available must be considered. As a test example one or two exposures were taken on a receiver whose bandwidth was of the order of ½ megacycle/sec., and it was only attempted to record the approximate envelope of the fluctuations (as produced by a diode detector valve). This proved in fact fruitless as may be seen from Figure 34.

On the other hand too narrow a bandwidth is undesirable since first it affords no possibility of recording rapid fluctuations and secondly rather

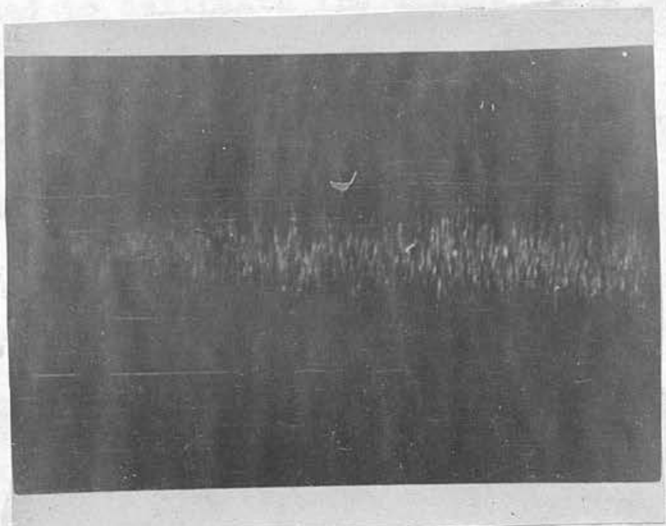


Figure: 34.

Test frame of rectified noise from wide bandwidth receiver.

long lengths of film would be necessary to provide sufficient data for analysis. The possibility of using a moving film was in fact considered, but the writer was not anxious to embark at the time on the mechanical work necessary to provide a high-speed moving film; secondly, in this case the "spot" on the cathode ray tube used to delineate the fluctuations would not traverse in the horizontal direction but would simply deflect vertically in time. To ensure therefore a good record it would be necessary to provide a screen with very rapid decay properties and in fact it seemed very unlikely that this method would prove satisfactory with the equipment available to the writer. On this latter point the writer is indebted to the staff of the British Thompson-Houston Research Laboratories for advice and information.

In consequence it appeared that the bandwidths provided on the Marconi Receiver mentioned in Chapter 3 - nominally 6 Kc/s., 3 Kc/s., 1.2 Kc/s., .3 Kc/s., - should prove a satisfactory compromise, and this has been the case.

The only further experimental problem to be overcome was the provision of a suitable "single-stroke" time base for the photography proper. The first attempt to provide this lay in assembling a simple one-valve linear time-base whose output was fed to the X_1 terminal of the Cossor "Double-Beam" Cathode Ray Oscillograph. While this worked excellently when no signal was being dealt with, when the fluctuation with its attendant high amplification was applied the time-base became severely distorted, presumably due to a small electrical surge generated when the switch of the time-base was closed being picked up on the amplifier. Some work was done to remove this but although the trouble was alleviated to some extent, it could not be entirely removed. Finally, a modification of the "trigger" facility already provided in the Cossor Oscillograph so as to provide true "single-stroke" action proved perfectly satisfactory. Generally a timing wave from a Beat Frequency oscillator at $250^c/_s$ $500^c/_s$, $1K^c/_s$ or $2\ K^c/_s$ was employed in the recording on the second Y plate of the Cossor oscillograph except on the occasions when it was desired to show two simultaneous records from different points in the receiver.

A selection of enlarged prints of the records are shown in Figures 35 to 39. Early in the work it was considered whether it might not be sufficient to record the detector output since, as discussed above, the envelope of the wave is the significant feature. Figure 39, however, displaying the detected output simultaneously with the fundamental fluctuation shows that "detail" is lost in this process; difficulty is also met with in avoiding drift of the mean level, and in providing a suitable filter to remove stray low-frequency (e.g. mains "hum" at $50^c/_s$) "pick-up" in such records. In recording the fundamental fluctuation a simple intermediate frequency filter permanently placed across the final terminals of the oscillograph sufficed

Plate VII : following p.103.

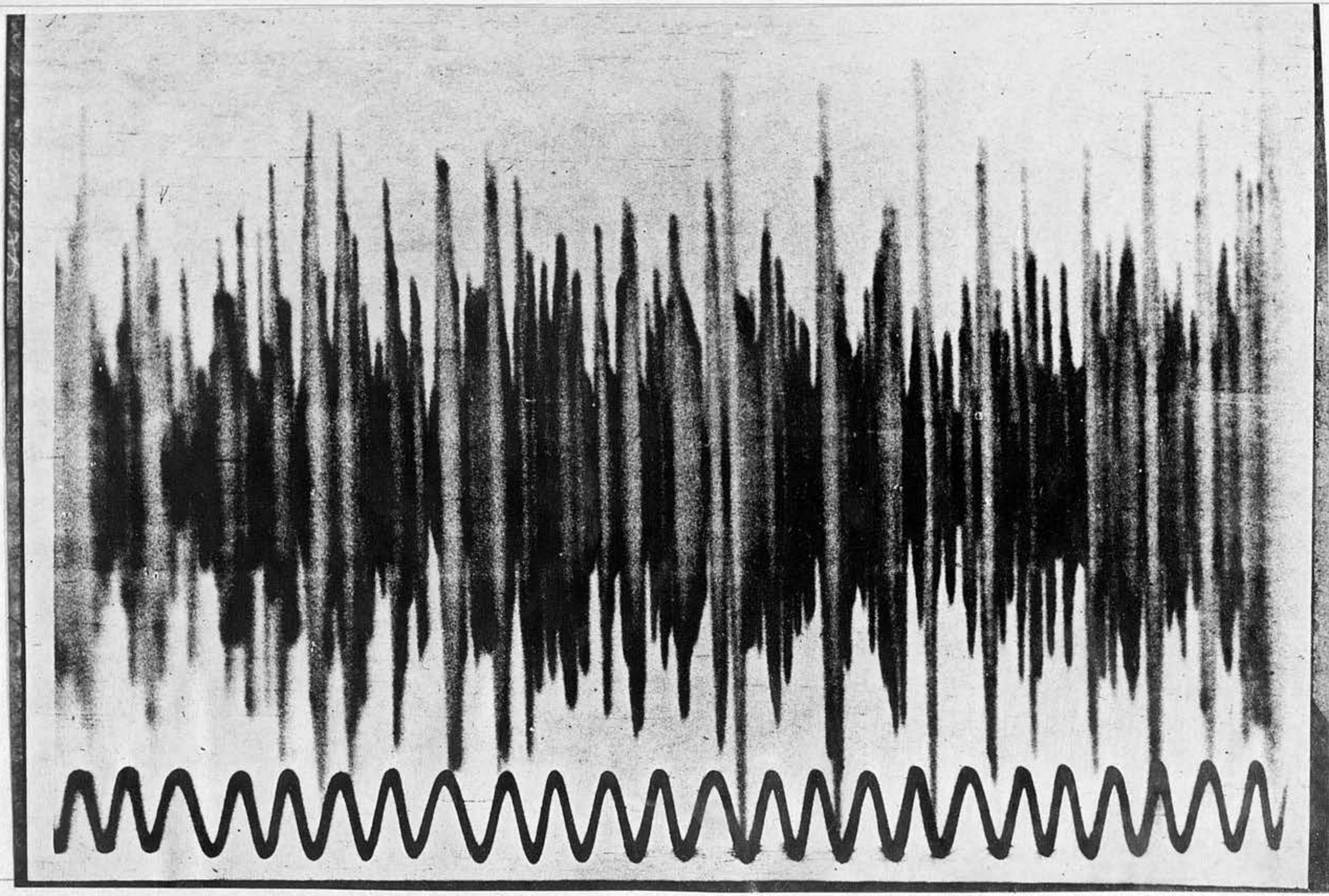


Fig:35: Frame 3:Series G
"Thermal" Fluctuation
Frequency: 125 Kc/s
Nominal Band-width:1.2Kc/s
Timing Wave: 250c/s

Plate VIII: following p.103.

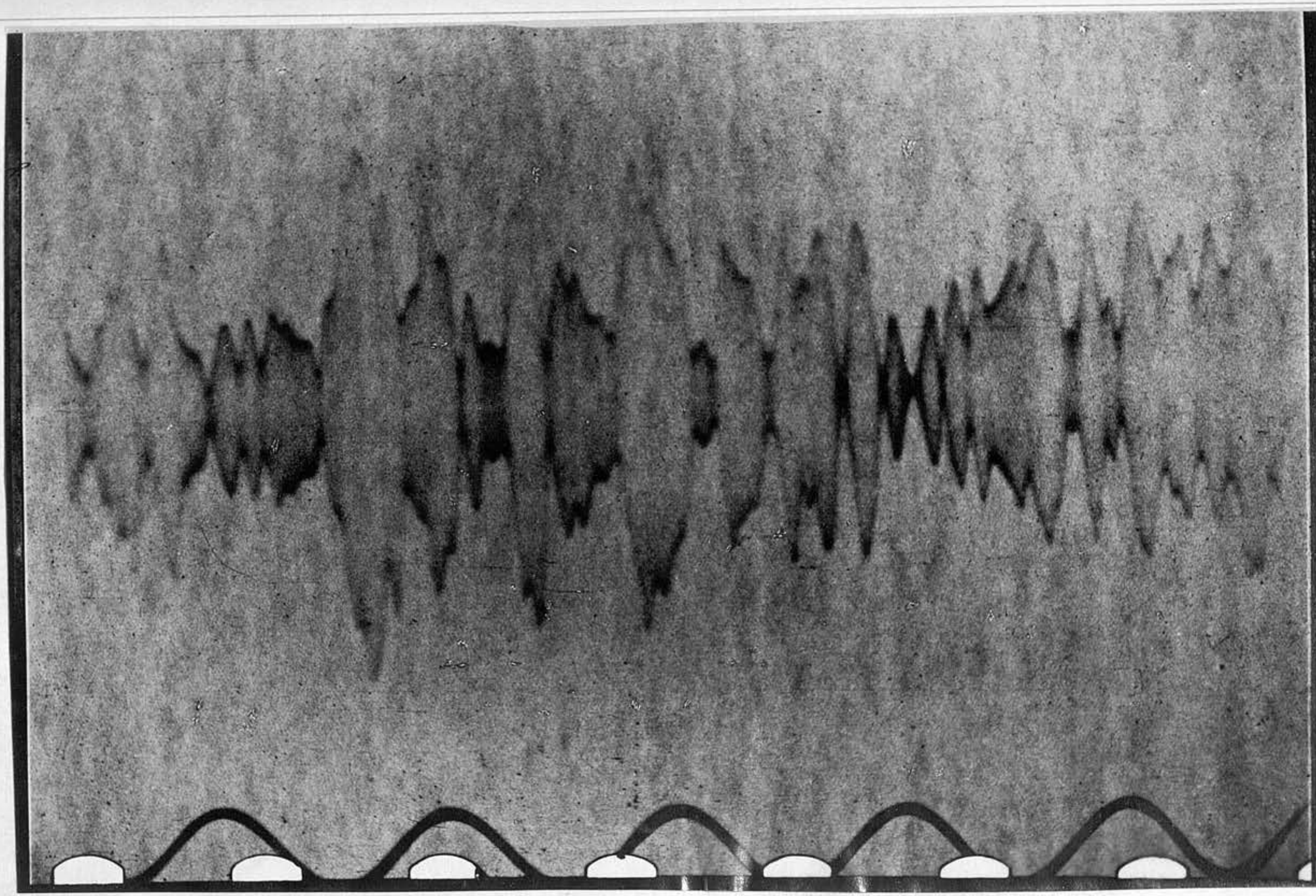


Fig:36: Frame 4: Series J
"Shot" Fluctuation
Frequency: 0.83 Mc/s
Nominal Band-width: 6Kc/s
Timing Wave: 1Kc/s

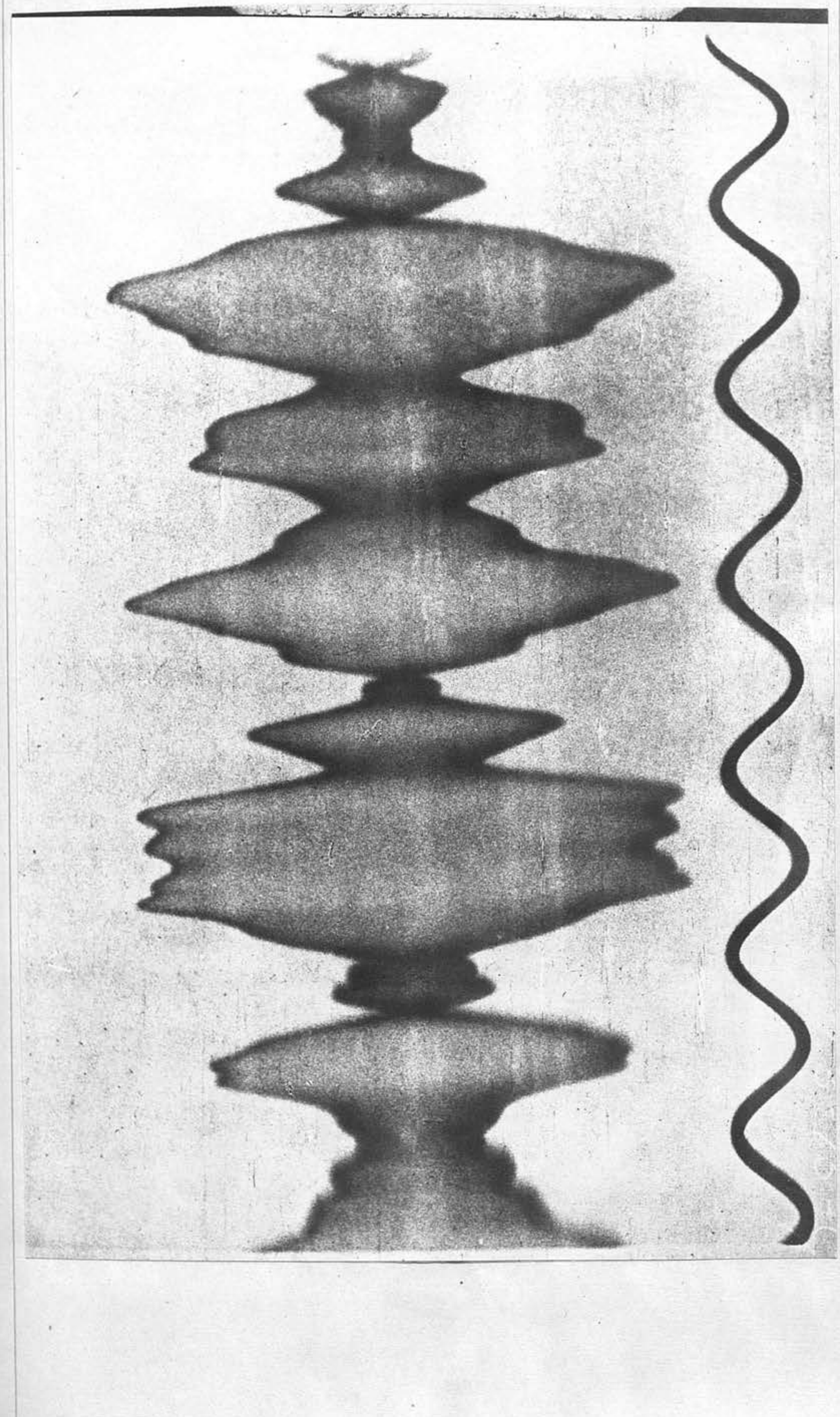


Fig:37: Frame 13; Series G
"Shot" fluctuation
Frequency; 95c/s
Nominal Bandwidth; 1.2Kc/s
Timing Wave; 500c/s

Plate X: following p.103.

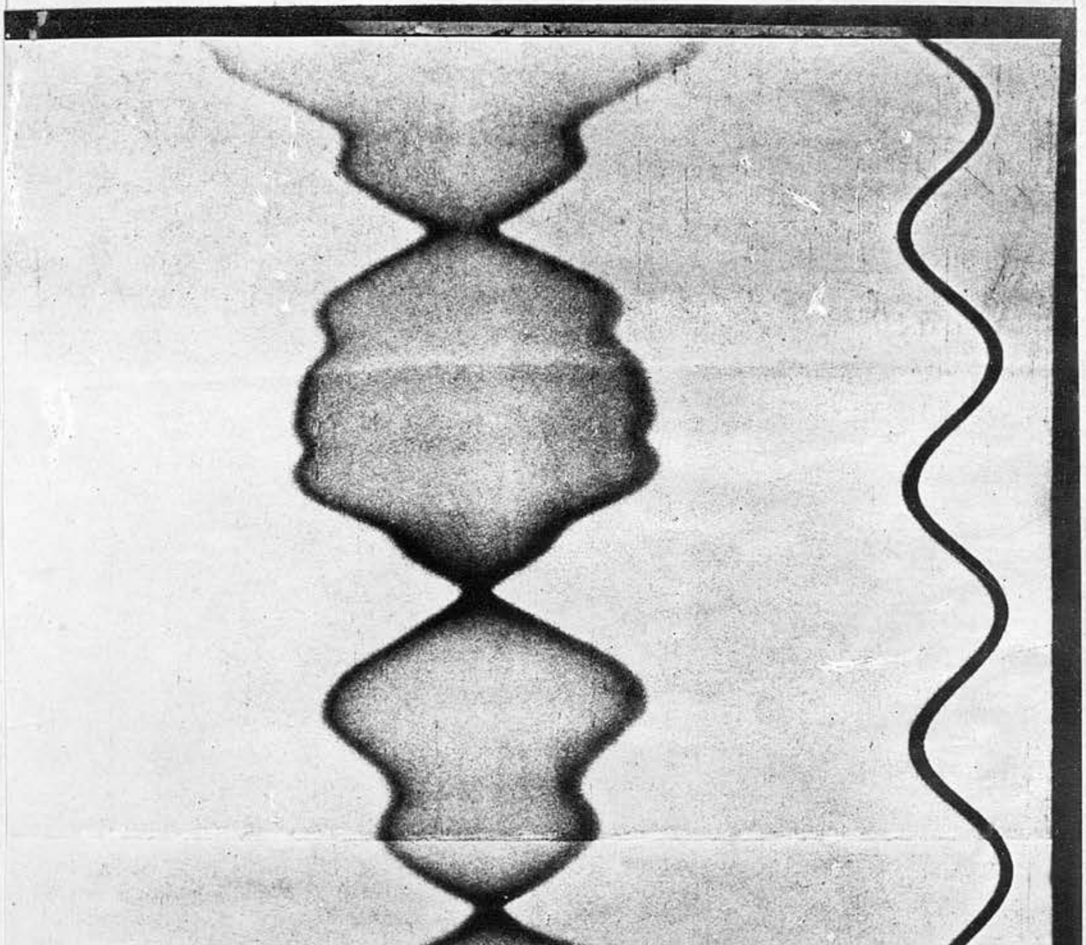


Fig:38: Frame 15:Series G
"Shot" fluctuation
Frequency: 95Kc/s
Nominal Band-width:1.2Kc/s
Time Wave: 1Kc/s

Plate XI: following p.103.

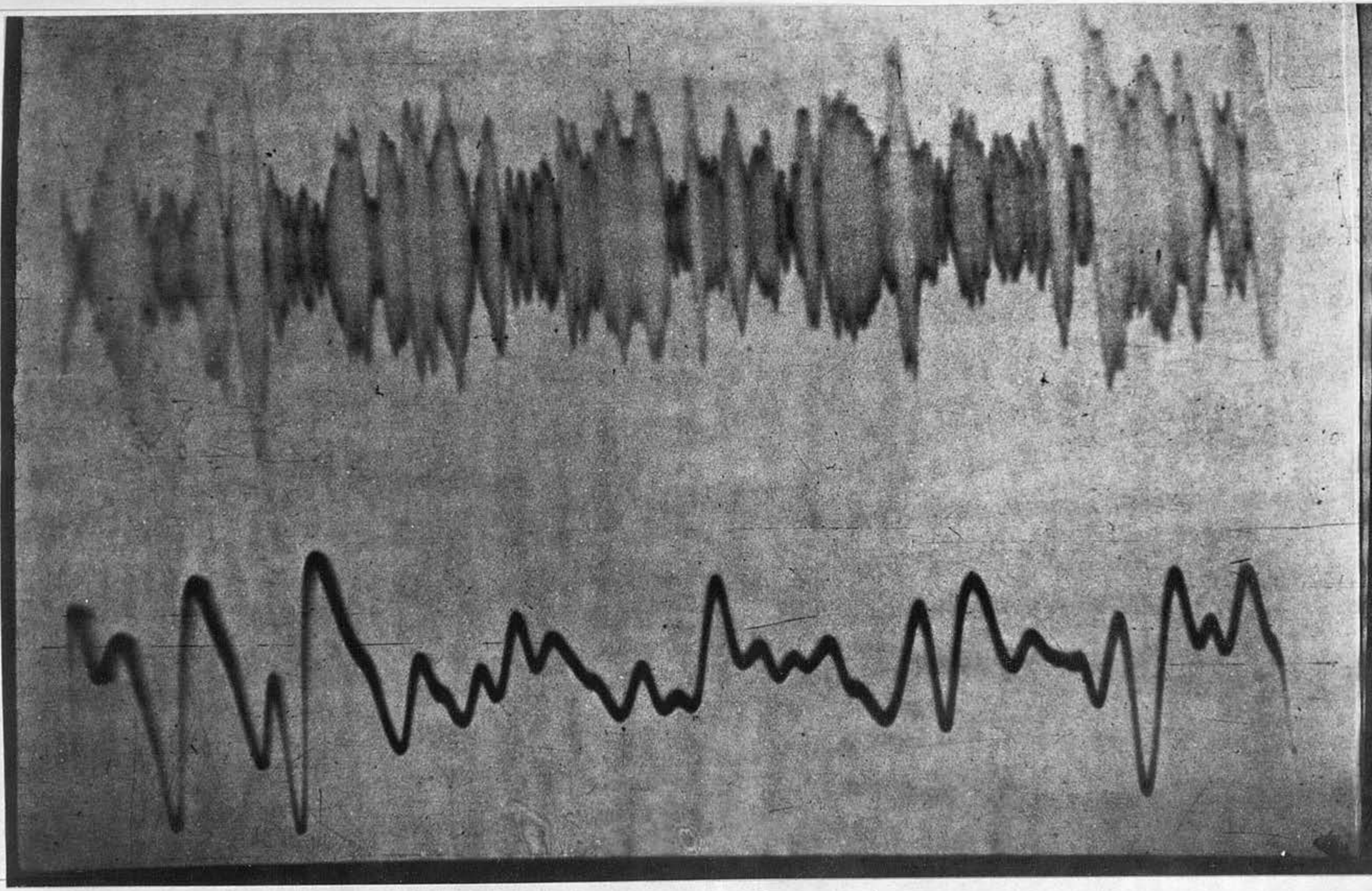


Fig :39: Frame 12: Series H
Thermal Fluctuation plus rectified output
Frequency : 1.1 Mc/s
Nominal Band-width : 6 Kc/s.

to remove any unwanted low-frequency disturbance.

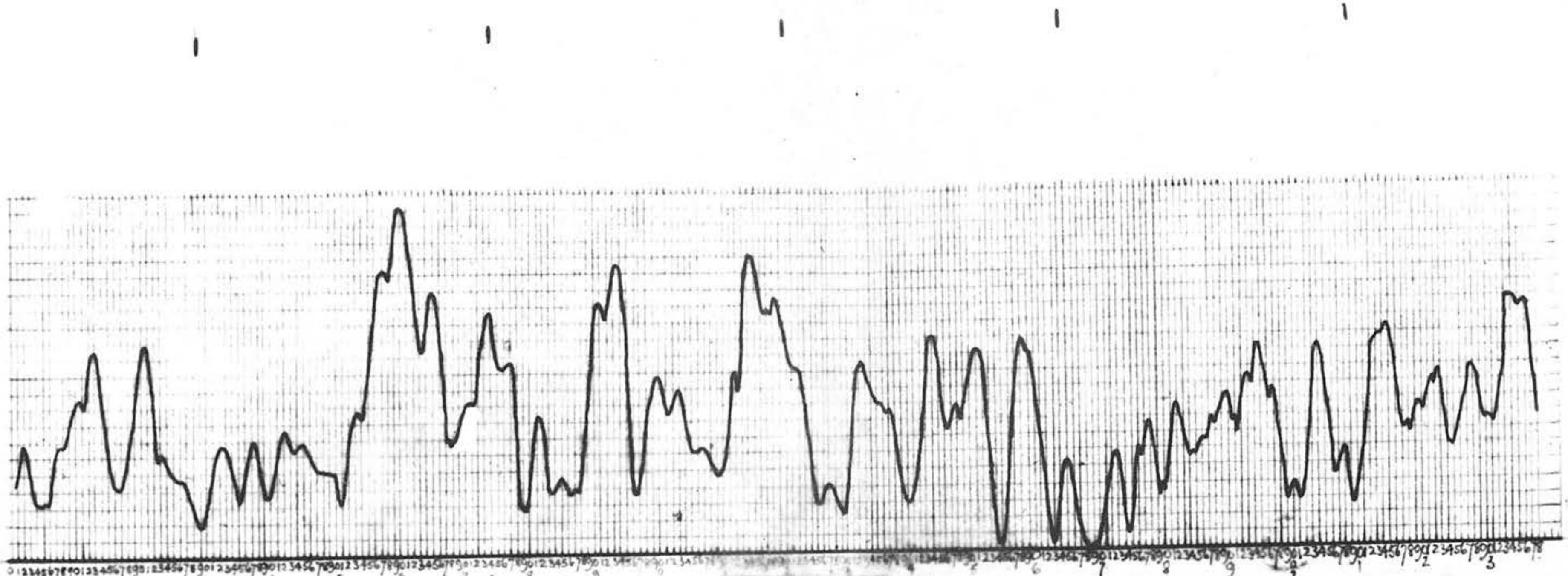
To carry out detailed analysis of the records the original negatives were placed in a standard Leitz Enlarger and the enlarged image projected on to a sheet of drawing paper. (The writer is indebted to Mr. K.J.R. Wilkinson, of the British Thompson-Houston Research Laboratories for advice on this point). It was then relatively straightforward to trace directly in a darkened room the image of the envelope with pencil and later to trace these pencil images over in Indian Ink at leisure; examples are shown in Figures 40 to 44 .

....

5. 2 : Analysis of Envelope Distribution.

5. 2. 1 : "Noise" alone.

The first analyses conducted were to examine the distribution of the envelope: $R(t)$. Theoretical work on the development of expressions relevant hereto has been carried out by several workers but for con:venience here the standard reference work used will be a set of integrating papers by S.O. Rice[1,2] which prove rather comprehensive on this subject. In future, for brevity, references to Rice's work will simply be given in the text as "Rice, paper...., page...." The theoretical expression for $p(R).dR$ may readily be developed as follows :



FRAME: 4 SERIES: J.

Figure: 40.

Detail:

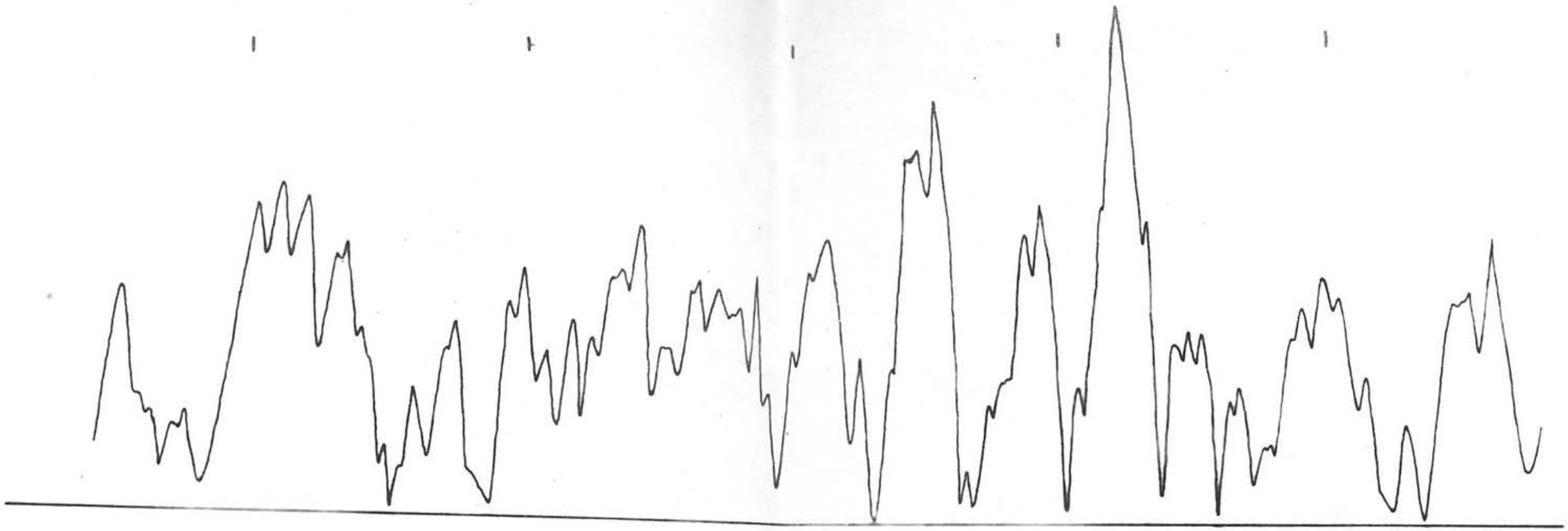
Date: 18 : XII : 45

Frequency 0.85 Mc/s

Subject: Shot Noise (50μa.)

Bandwidth: 6 Kc/s.

Timing:- 1 Kc/s. (1 ms. timing marks)



FRAME 8 SERIES J

Figure: 41.

Detail:-

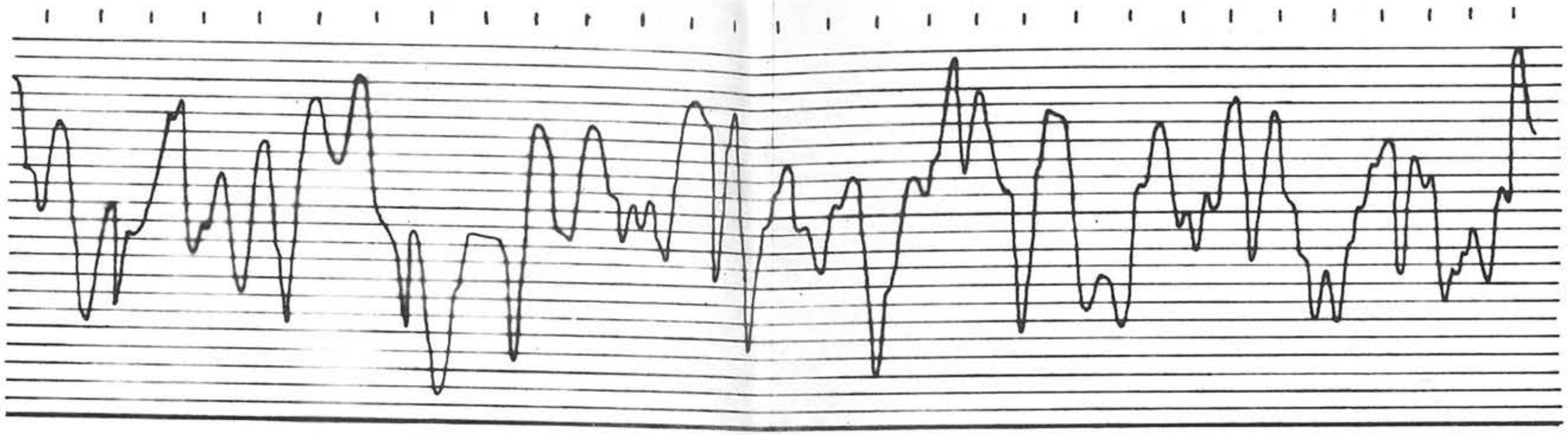
Date 18:XII:46

Subject Thermal Noise

Frequency: 1·05 Mc/s.

Bandwidth 6 Kc/s

Timing: 1 Kc/s (1 ms. timing marks)



FRAME 9 SERIES C.

Figure: 42.

Detail:

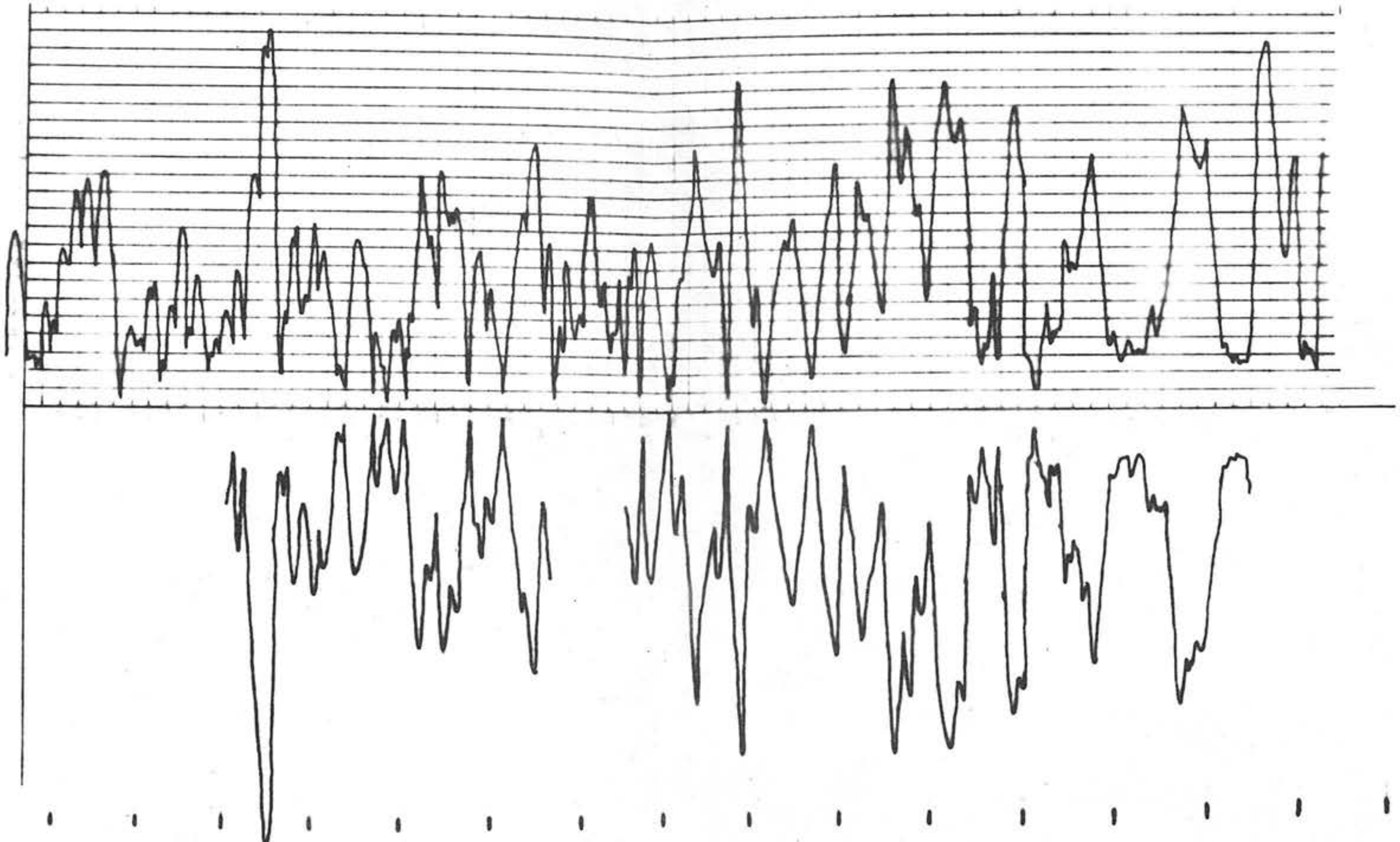
Date: 19:xi:45

Subject: Shot Noise + Carrier

Frequency: 100 Kc/s.

Bandwidth: 1·2 Kc/s.

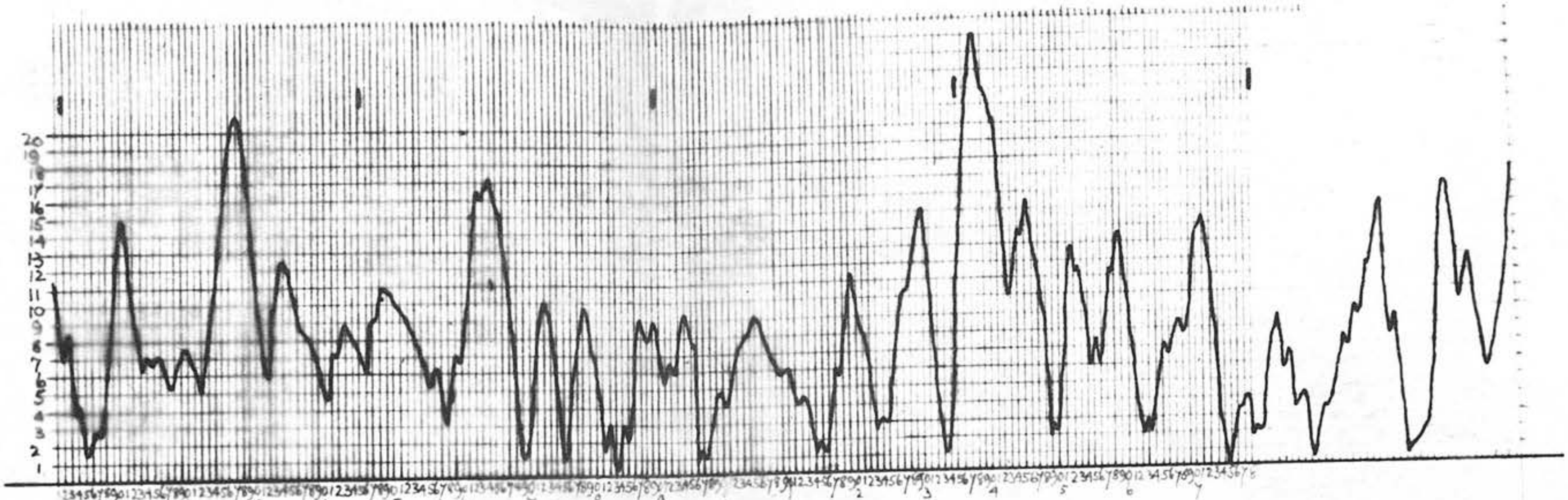
Timing: 500c/s (2ms. time-marks)



Detail: Date: 22:xi:45
Subject: Thermal Noise
Frequency: 1·05 Mc/s
Bandwidth: 3 Kc/s

FRAME 3: SERIES D

Figure: 43.



FRAME 16 : SERIES C.

Figure: 44.

Detail:

Date:	19: xi: 45
Subject:	Thermal Noise.
Frequency:	130 Kc/s.
Bandwidth:	6 Kc/s
Timing:	500 c/s (2 ms. timing marks)

Assume then that the fluctuation $I(t)$ through a network of "dominant" frequency f_o may be expressed as :

$$I(t) = a(t).\sin 2\pi f_o t + b(t)\cos 2\pi f_o t \text{ --------- } 5\text{-}(1)$$

where $a(t)$ and $b(t)$ are "slowly" changing functions.

Then we set :

$$R(t) = \sqrt{a^2(t) + b^2(t)} \text{ ------------------ } 5\text{-}(2)$$

It may readily be appreciated that a , b are normally distributed :

$$p(a).da = \frac{1}{\sqrt{2\pi\psi_o}}.\varepsilon^{-\frac{a^2}{2\psi_o}}da \; ; \; p(b).db = \frac{1}{\sqrt{2\pi\psi_o}}.\varepsilon^{-\frac{b^2}{2\psi_o}}.db$$

(where $\psi_o = \int_0^\infty w(f).df$, and $w(f)$ is the "power spectrum" of $I(t)$ as discussed in the appendix to Chapter 2.)

Whence:

$$p(R).dR = \frac{1}{2\pi\psi_o}\int_{a,b} \varepsilon^{-\frac{(a^2+b^2)}{2\psi_o}}.da.db$$

(Integrated over all values of a , b to maintain (a^2+b^2) constant).

To perform this integration set :

$$a = R\sin\theta \qquad b = R\cos\theta \qquad \therefore |J| = RdRd\theta$$

$$\therefore p(R).dR = \frac{1}{2\pi\psi_o}\int_\theta \varepsilon^{-\frac{R^2}{2\psi_o}} RdRd\theta$$

Clearly θ may vary from 0 to 2π :

i.e. $$p(R).dR = \left\{\frac{R}{\psi_o}.\varepsilon^{-\frac{R^2}{2\psi_o}}\right\}dR \text{ ------------ } 5\text{-}(3)$$

This result was verified by straightforward plotting for a number of fluctuation records; two examples are presented as Figures (45) and (46). In addition, however, a more stringent test was applied, as follows :

From : $p(R) = \frac{R}{\psi_o}\varepsilon^{-\frac{R^2}{2\psi_o}}$

we have : $$\log\left\{\frac{p(R)}{R}\right\} = -\frac{1}{2\psi_o}.R^2 - \log\psi_o \text{ --------- } 5\text{-}(4)$$

FRAME 15 : SERIES D : PROBABILITY DENSITY OF R

⊙ : EXPERIMENTAL POINTS ON 1ST HALF ; - - - - : SMOOTH CURVE DRAWN À PRIORI AS BEST CURVE THROUGH THESE POINTS

⊡ : " " " 2ND "

△ : " " " WHOLE CURVE TO SAME SCALE.

X : COMPUTED POINTS FROM $p(R) = \frac{R}{\psi_0} \varepsilon^{-\frac{R^2}{2\psi_0}}$ SCALED TO AGREE AT MAXIMUM POINT

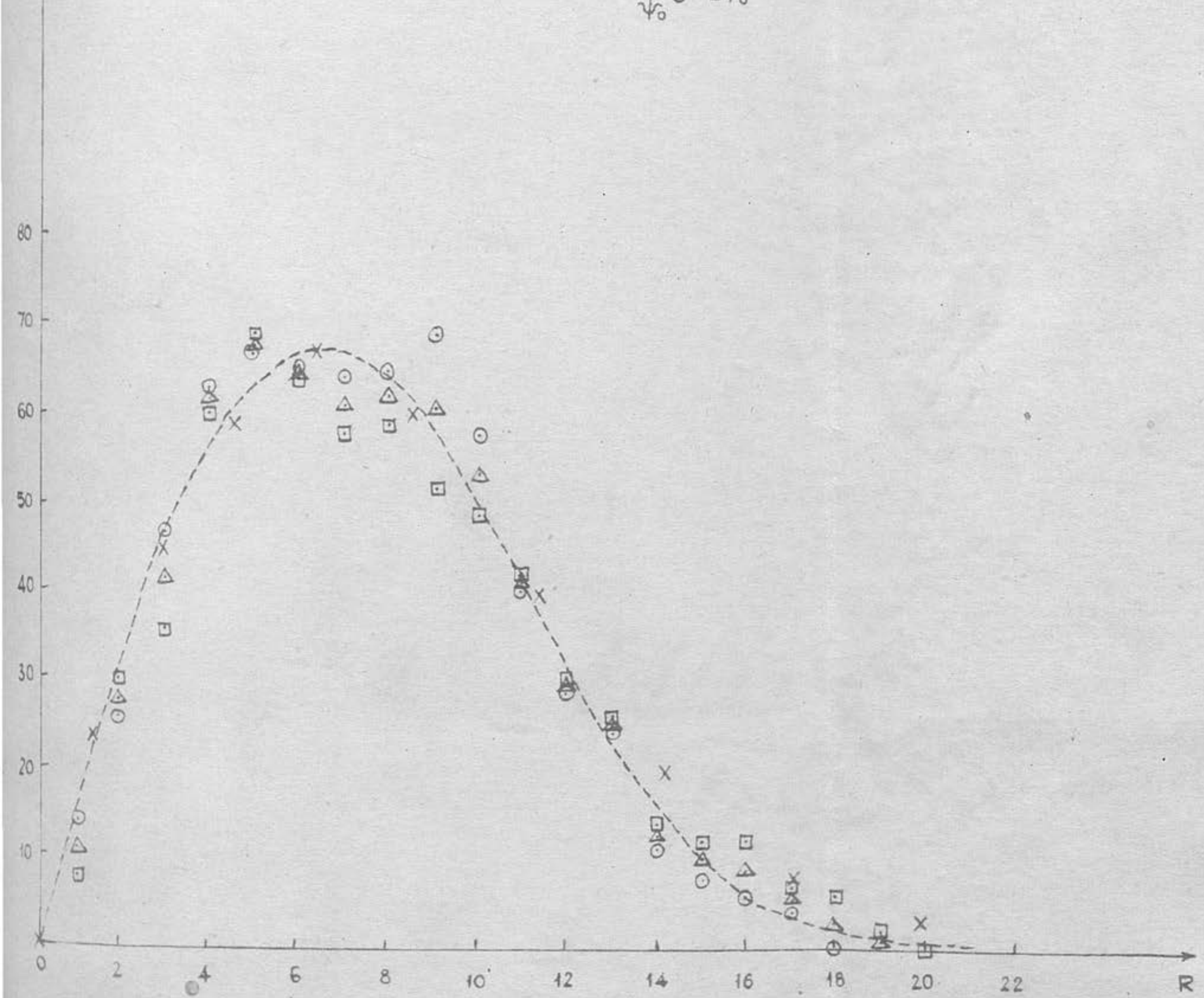


Figure: 45.

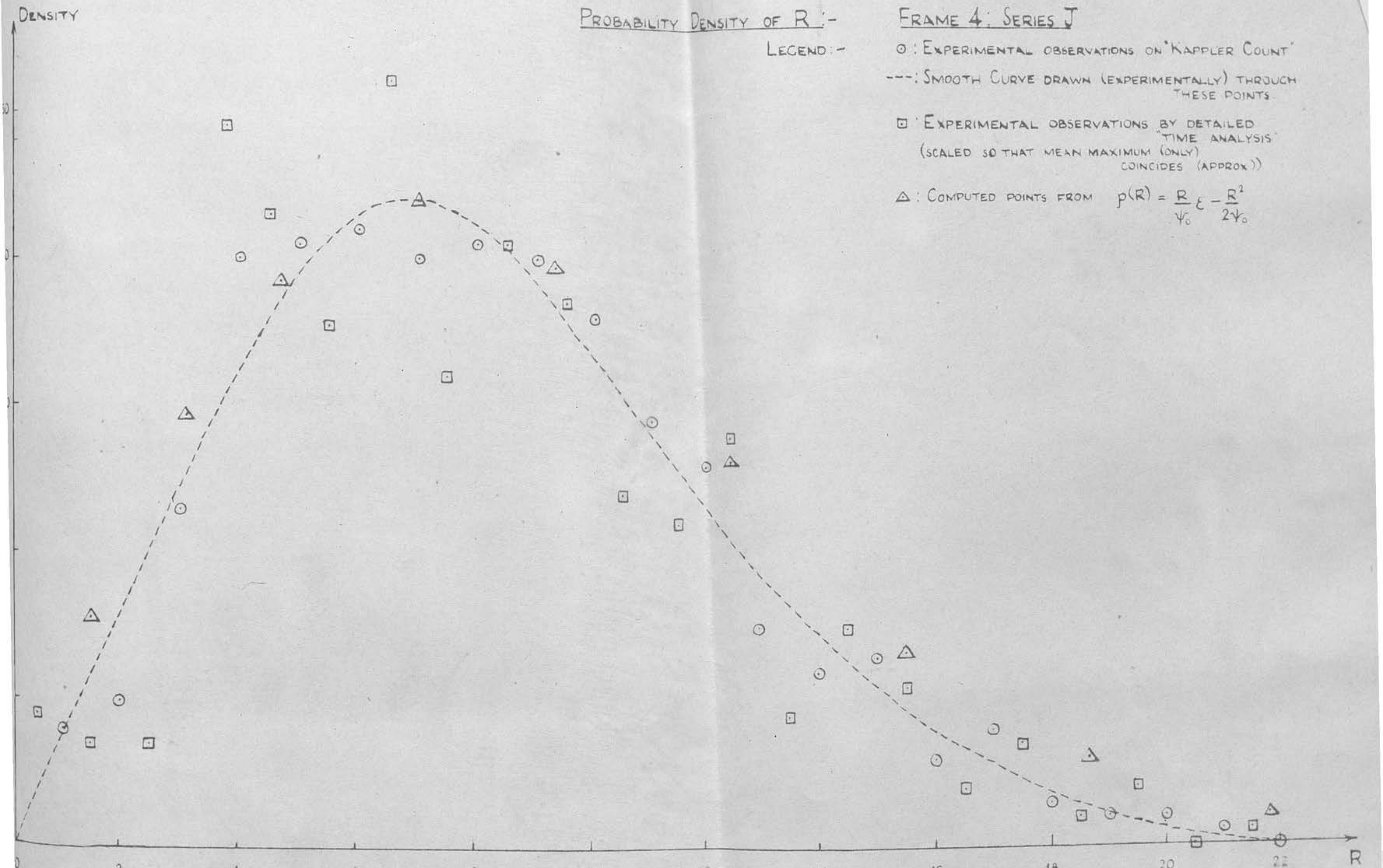

DENSITY
PROBABILITY DENSITY OF R :-
FRAME 4: SERIES J
LEGEND:-
⊙ : EXPERIMENTAL OBSERVATIONS ON 'KAPPLER COUNT'
---: SMOOTH CURVE DRAWN (EXPERIMENTALLY) THROUGH THESE POINTS.
⊡ : EXPERIMENTAL OBSERVATIONS BY DETAILED 'TIME ANALYSIS' (SCALED SO THAT MEAN MAXIMUM (ONLY) COINCIDES (APPROX))
△ : COMPUTED POINTS FROM $p(R) = \frac{R}{\psi_0} \varepsilon^{-\frac{R^2}{2\psi_0}}$
0
2
4
6
8
10
12
14
16
18
20
22
R

Figure: 46.

Thus if we plot log $\left\{\frac{p(R)}{R}\right\}$ against R^2 a straight line should result with a negative slope $-\frac{1}{2\psi_0}$. Further however we see that :

$$\overline{R^2} = \int_0^\infty R^2 \, p(R).dR = 2\psi_0$$

Hence computing $\overline{R^2}$ using closely-spaced ordinates on the record a strict check of the result may be obtained. This is presented as Figure (47). The slope derived is 80.1; the computed value of $\overline{R^2}$ was 81. The agreement is thus seen to be excellent, strongly confirming the theoretical equation.

In deriving $p(R)$, or rather in general a quantity $q(R)$ directly proportional to $p(R)$ ✱, first the method employed by Kappler was adopted and is referred to as a "Kappler Count". Here horizontal lines were drawn on the records at uniform vertical intervals, the number of intersections with the trace noted and plotted as proportional to the "density" at that point. Later, however, doubt was felt as to the strict validity of this operation since if a sine-wave for example be considered then it is readily seem that this process would give uniform density at all levels up to the limit of amplitude whereas the true density is proportional to $\frac{1}{\sqrt{1-R^2}}$. Thus a "detailed" distribution

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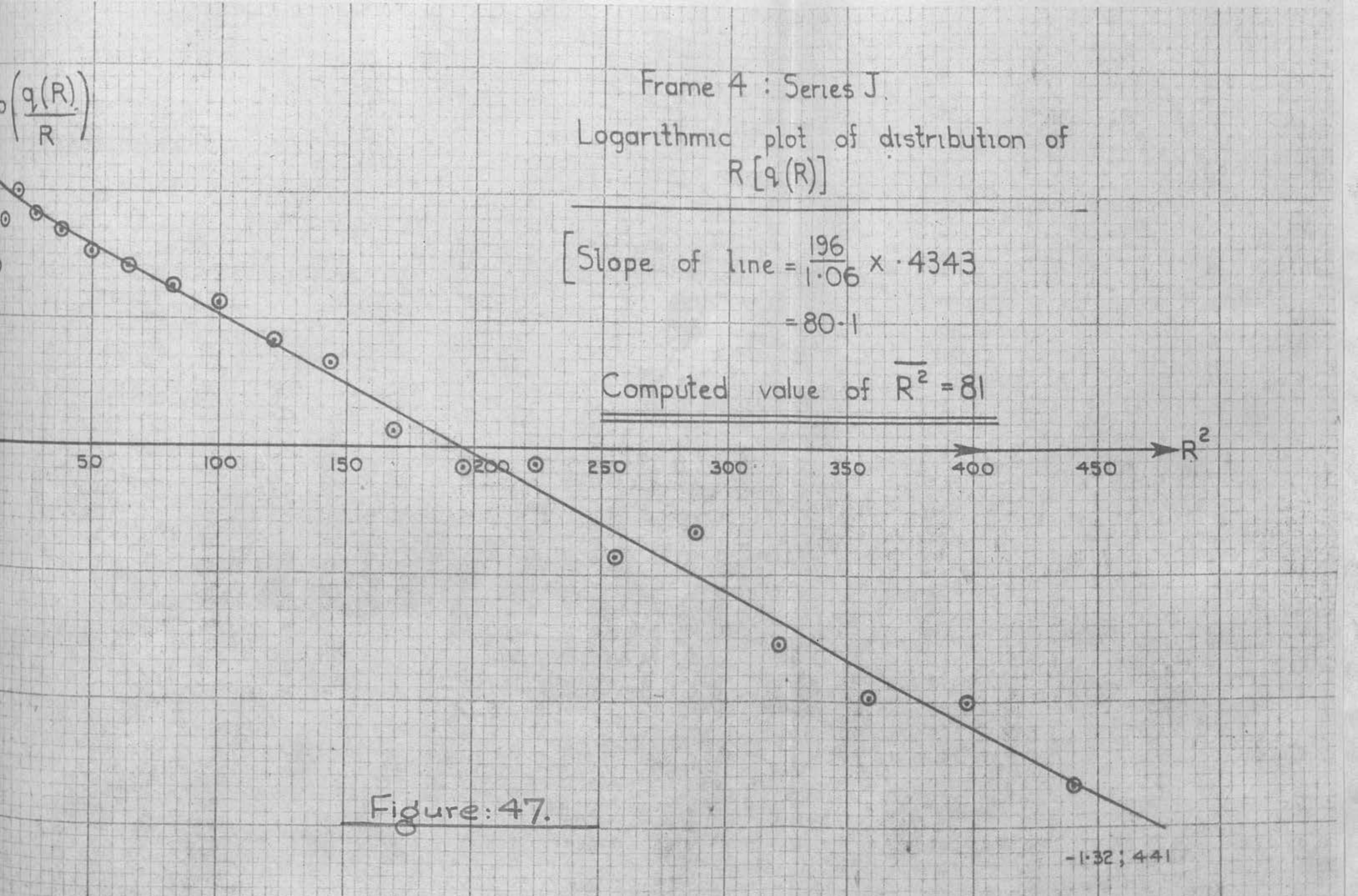
(Footnote) : ✱: In general in this work the quantity $q(R)$ is not <u>normalised</u>, while $p(R)$ - the true probability density - <u>is</u>, such that $\int_0^\infty p(R).dR = 1$.



(q(R)/R)
Frame 4 : Series J
Logarithmic plot of distribution of R[q(R)]
[Slope of line = 196/1·06 × ·4334
= 80·1
Computed value of $\overline{R^2}$ = 81
50
100
150
200
250
300
350
400
450
R^2
-1·32 ; 441

Figure: 47.

analysis was also carried out by measuring amplitudes at short equidistant time-intervals throughout the whole trace. An example is also plotted in Figure (46) and confirms the "Kappler Count" but with greater experimental "spread" of data. This is reasonable when one realises that the accuracy of measuring the amplitude of the rapidly varying trace is not extremely high, whereas the "Kappler Count" is very certain and embodies no ambiguity. R.E. Burgess, however, in an unpublished paper has shown that the number of times that the amplitude passes through a given level (either with positive or negative slope) is given by an expression :

$$N(R) = A\left[\frac{R}{\psi_0}\varepsilon^{-\frac{R^2}{2\psi_0}}\right] \text{------------} 5\text{-}(5)$$

This immediately confirms the validity of the "Kappler Count" since the form of (5) is identical with that of (3). When, further, one observes that Burgess' analysis depends on the assumption that the slope of the envelope at any point is completely <u>uncorrelated</u> with the amplitude at that point (i.e. $\overline{R.\frac{dR}{dt}} = 0$) (as opposed to the case of the sine wave or any regular waveform) then the result becomes entirely obvious, and the "Kappler Count" method emerges as of general application in this work.

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5. 2. 2: "Noise" plus regular signal.

If we inject a regular signal (from a signal generator) at the receiver input then we have to deal with the distribution of the envelope arising from the combined random fluctuation and the regular signal.

Let us set as the signal :

$$S(t) = S.\cos 2\pi f_0 t$$

then the overall input is given by :

$$I(t) = a(t).\sin 2\pi f_0 t + (b(t) + S).\cos 2\pi f_0 t$$

$$\therefore \ R(t) = \sqrt{a^2(t) + \{b(t) + S\}^2}$$

Then as before, setting $a = R\sin\theta$; $b + S = R\cos\theta$

$$\therefore p(R).dR = \frac{1}{2\pi\psi_0}\int_0 \varepsilon xp\left\{-\frac{(R\sin\theta)^2 + (R\cos\theta - S)^2}{2\psi_0}\right\} R\,dR\,d\theta$$

$$= \frac{R\,dR}{2\pi\psi_0}\int_0 \varepsilon xp\left\{-\frac{(R^2 + S^2 - 2RS\cos\theta)}{2\psi_0}\right\} d\theta$$

Again integrating over θ from 0 to 2π

$$p(R)dR = \frac{R\,dR}{\psi_0}.\varepsilon xp\left\{-\frac{R^2 + S^2}{2\psi_0}\right\}.I_0\left\{\frac{RS}{\psi_0}\right\} \text{---------} 5\text{-}(6)$$

(where $I_0(z) = J_0(iz)$ and J_0 is the Bessel function of the first order and zero order).

If now we set : $v = \frac{R}{\sqrt{\psi_0}}$; $g = \frac{S}{\sqrt{\psi_0}}$

we have :

$$p(v) = v.\varepsilon xp\left\{-\frac{v^2 + g^2}{2}\right\}.I_0(gv)$$

If then gv is "large" (i.e. reasonably large "signal-to-noise ratio") then we may replace the Bessel function by its asymptotic expression, namely :

$$I_0(z) \underset{z\to\infty}{\sim} \frac{\varepsilon^z}{\sqrt{2\pi z}}$$

and therefore :

$$p(v) \sim \left(\frac{v}{2\pi g}\right)^{1/2}.\varepsilon xp\left(-\frac{(v-g)^2}{2}\right)$$

As a further approximation we may obviously set $\frac{v}{g} \simeq 1$ in the first factor since the exponential factor will decay rapidly for v significantly different from under the condition gv "large", and thus derive :

$$p(v) \simeq \frac{1}{\sqrt{2\pi}}.\varepsilon^{-\frac{(v-g)^2}{2}} \text{-------------} 5\text{-}(7)$$

[Valid over the "significant" range of the function]

This then expresses the intuitive result mentioned earlier (page 84) that under these conditions the noise will "ride" on top of the signal, and the expected result of a Gaussian distribution with its maximum at signal level, and deviation determined by the noise, follows. These basic results are also presented by S.O. Rice (Paper B, pages 75/76 and 100/101), and a set of graphs (Paper B, page 102) of $p(v)$ for values of g from 0 to 5 shows how rapidly the limiting form of equation (7) above is sensibly achieved. Unfortunately when taking these particular records a specific measurement of the signal was not made at the time. The most useful measurements in fact would have been thermocouple observations of noise alone and noise plus injected signal to derive the ratio $g^2 = \frac{S^2}{\psi_0}$. The relevant values to draw a theoretical curve were, however, inferred as follows :

We have :

$$\overline{v^2} = g^2 + 2 \quad \text{------------- 5-(8)}$$

as a direct consequence of the fact that the regular signal and random noise are uncorrelated.

Allowing for a scale factor k :

$$\overline{V^2} = G^2 + 2k^2 \quad \text{-------- 5-(9)} \qquad \text{where} \begin{cases} V = kv \\ G = kg \end{cases}$$

$$\therefore k = \sqrt{\frac{\overline{V^2} - G^2}{2}} \quad \text{--------- 5-(9a)}$$

In addition we observe the point of maximum observed density, which gives G.

Figure (42) shows a typical record and by using closely-spaced ordinates we derive :

$$\overline{V^2} = 239.4$$

Also we observe : $G = 14{\cdot}3 \therefore G^2 = 204$

whence we determine : $k = \sqrt{17{\cdot}7} = 4{\cdot}2$

We thus plot, as the theoretical curve :

$$p(V) = \frac{1}{\sqrt{2\pi}.k}\cdot\varepsilon^{\frac{-(V-G)^2}{2k^2}} \qquad \text{------------} \quad 5\text{-}(10)$$

or allowing for the arbitrary scale-factor in plotting density from the "Kappler Count", and inserting the values determined :

$$q(V) = 55.\varepsilon^{\frac{-(V-14{\cdot}3)^2}{35{\cdot}4}} \qquad \text{------------} \quad 5\text{-}(10a)$$

This has been plotted along with the Experimental curve in Figure ($47\frac{1}{2}$) corresponding to the fluctuation record of Figure (42). On the same figure appears another experimental distribution curve for another value of signal-to-noise ratio.

....

5. 2. 3: "Stationary" character of fluctuation records.

In order to check that the records satisfied the following two requirements :

(i): that the fluctuation was statistically stationary,

(ii): that a possible burst of local interference had not vitiated the record,

the records were frequently divided into two halves and the distribution on each half computed to test whether these tallied with that obtained on the whole record. The "spread" of the "half" distributions will be expect:ed to be greater than that obtained on the whole record, but the smoothed curves should agree. Examples are shown in Figures 45 and $47\frac{1}{2}$. One record that was found unsatisfactory appears in Figure 43 and visual

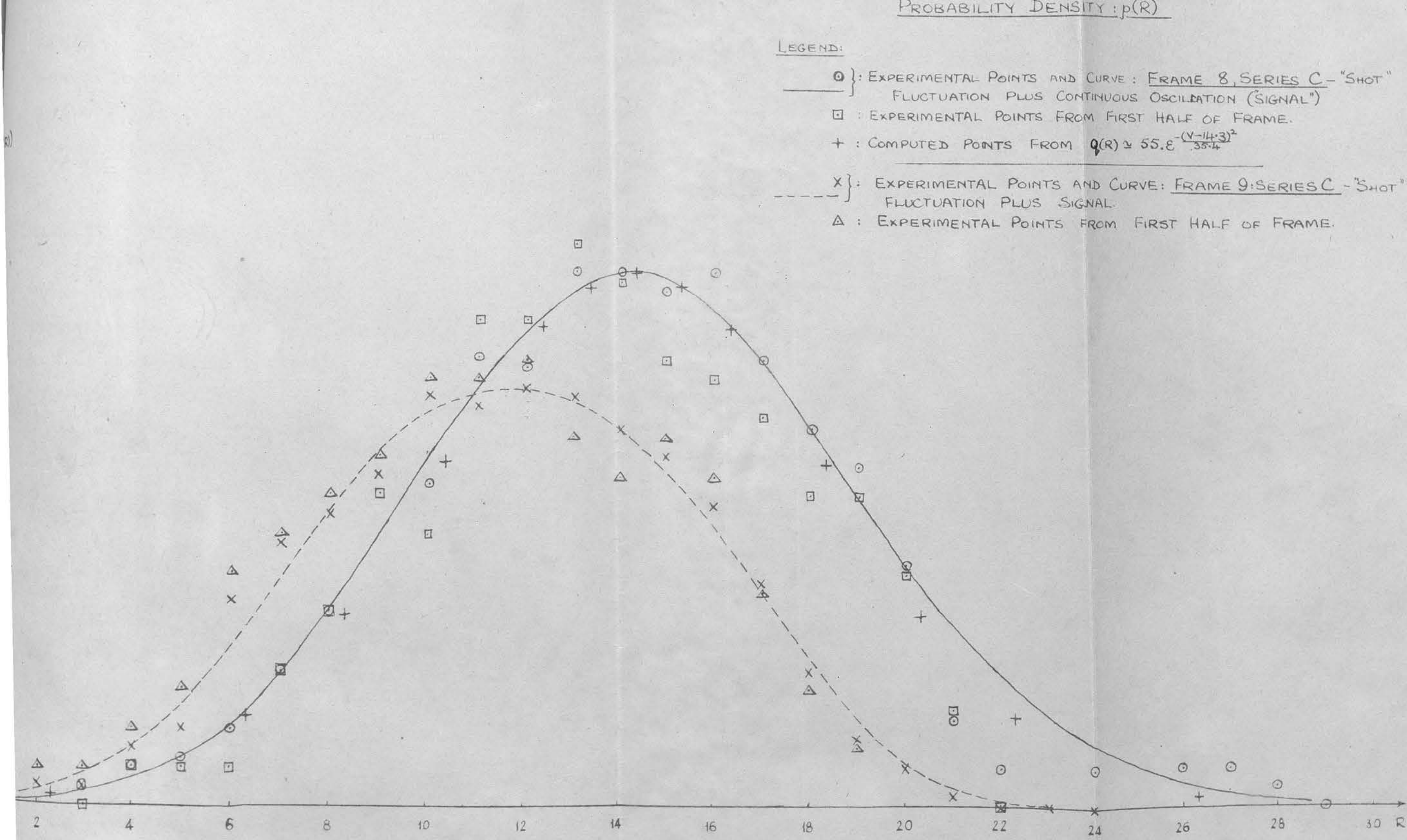

PROBABILITY DENSITY : p(R)
LEGEND:
⊙ : EXPERIMENTAL POINTS AND CURVE : FRAME 8, SERIES C – "SHOT" FLUCTUATION PLUS CONTINUOUS OSCILLATION ("SIGNAL")
⊡ : EXPERIMENTAL POINTS FROM FIRST HALF OF FRAME.
+ : COMPUTED POINTS FROM $Q(R) \simeq 55.\varepsilon^{-\frac{(V-14.3)^2}{35.4}}$
X : EXPERIMENTAL POINTS AND CURVE: FRAME 9: SERIES C – "SHOT" FLUCTUATION PLUS SIGNAL.
△ : EXPERIMENTAL POINTS FROM FIRST HALF OF FRAME.
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
R

Figure : 47½.

examination of the record shows a fundamental difference between the two halves, and suggests that interference spoiled the second half as was in fact confirmed by the plotted distribution.

....

5. 2. 4: Overall statistical examination.

Given that $p(R) = \frac{R}{\psi_0} \varepsilon^{-\frac{R^2}{2\psi_0}}$ for records of "pure noise" one may readily derive simple relationships between average values of powers of R, independent of the scale of measurement, using standard integrals of the form :

$$\int_0^\infty x^n \varepsilon^{-\lambda x^2} dx \qquad (n = 2, 3 \ldots .)$$

Thus :

$$\overline{R} = \frac{1}{\psi_0} \int_0^\infty R^2 \varepsilon^{-\frac{R^2}{2\psi_0}} dR = \sqrt{\frac{\pi\psi_0}{2}} \text{ -------- } 5\text{-}(11)$$

$$\overline{R^2} = 2\psi_0 \text{ ---------------- } 5\text{-}(12)$$

$$\overline{R^3} = 3\psi_0 \sqrt{\frac{\pi\psi_0}{2}} \text{ -------------- } 5\text{-}(13)$$

whence we find :

$$(\overline{R})^2 / \overline{R^2} = \frac{\pi}{4} \doteq \cdot 785 \text{ ------------- } 5\text{-}(14)$$

$$\overline{R^3} / (\overline{R} . \overline{R^2}) = 1.5 \text{ -------------- } 5\text{-}(15)$$

$$\overline{R^3} / (\overline{R^2})^{3/2} = \frac{3\sqrt{\pi}}{4} \doteq 1.33 \text{ ------------ } 5\text{-}(16)$$

One or two examples of experimental results are tabulated below for comparison.

Frame 16: Series D (Whole Frame).

$\overline{R_{Exp}} = 6.12$; $\overline{R^2_{Exp}} = 49.8$

$\therefore \left\{ \frac{(\overline{R})^2}{\overline{R^2}} \right\}_{Exp} = \cdot 754$; theoretical value $= \cdot 785$; $\left\{ \frac{Exp.V.}{The.V.} \right\} = \cdot 96_5$

Frame 16: Series D (First third of frame).

$\overline{R_{Exp}} = 1.66$; $\overline{R^2_{Exp}} = 3.74$; $\overline{R^3_{Exp}} = 10.51$ (In different units of measurement).

$\therefore \left\{ \frac{(\overline{R})^2}{\overline{R^2}} \right\}_{Exp} = \cdot 74$; theoretical value $= \cdot 785$; $\left\{ \frac{Exp.V.}{The.V.} \right\} = \cdot 94_4$

$$\left\{\frac{\overline{R^3}}{\overline{R}.\overline{R^2}}\right\}_{Exp} = 1.7$$; theoretical value $= 1.5$; $\left\{\frac{Exp.V.}{The.V.}\right\} = 1.13.$

$$\left\{\frac{\overline{R^3}}{(\overline{R^2})^{3/2}}\right\}_{Exp} = 1.46$$; theoretical value $= 1.33$; $\left\{\frac{Exp.V.}{The.V.}\right\} = 1.10$

Similarly one can derive the theoretical value of the maximum point of the probability distribution, R_f, although this does not provide a particularly good statistical test because of the inherent "flatness" of the maximum; this naturally provides a rather large additional field for experimental error.

In theory to determine R_f we have :

$$p(R) = \frac{R}{\psi_0}\cdot\varepsilon^{-\frac{R^2}{2\psi_0}}$$

$$\therefore \frac{\partial p(R)}{\partial R} = \frac{1}{\psi_0}\varepsilon^{-\frac{R^2}{2\psi_0}}\left(1-\frac{R^2}{\psi_0}\right)$$

$\therefore$ for R to be maximal, $= R_f$, we have :

$$1-\frac{R_f^2}{\psi_0} = 0 \quad \therefore R_f = \sqrt{\psi_0}$$

But we have shown $\overline{R} = \sqrt{\frac{\pi\psi_0}{2}}$

Hence : $\frac{R_f}{\overline{R}} = \sqrt{\frac{2}{\pi}} = \cdot 8$

or using equation (12) :

$$\frac{R_f}{\sqrt{\overline{R^2}}} = \frac{1}{\sqrt{2}} = \cdot 71$$

Taking Frame 4; Series J, for example, - since $\overline{R^2}$ has already been computed, - we consider the latter relation :

$\overline{R^2} = 81, \therefore \sqrt{\overline{R^2}} = 9$; By observation on the curve (see Figure 46)....

.... we find $R_f \doteqdot 7$

<u>Hence</u> :

$$\left\{\frac{R_f}{\sqrt{\overline{R^2}}}\right\}_{exp} \doteqdot \cdot 78$$; theoretical value $= \cdot 71.$ $\left\{\frac{Exp.V.}{The.V.}\right\} = 1.1$

....

5. 3: Distribution of Maxima in the fluctuation Envelope.

If one were dealing with true Brownian motion (in particular, say, the motion of a colloid particle in liquid) then it would be meaningless to discuss the maxima, or rather "turning points", of the observed trajectory. This is because in intervals of macroscopic observation the trajectory has undergone an immense number of microscopic fluctuations; what is observed is the "meaned" motion over the observation interval, and in general this is expressed by saying that the trajectory is non-differentiable. Thus, for example, the expression for the mean square displacement of a spherical particle in liquid, assuming the validity of Stoke's Law, given by Einstein (3) is :

$$\overline{\Delta^2} = \frac{kT}{\pi\eta a} \cdot t \quad \text{---- } 5\text{-}(17)$$

, where η = viscosity of the liquid.
a = radius of particle.
t = time interval from commencement of observation.

This has received ample experimental verification. On the other hand Einstein (2) also derives the well-known expression for the mean square velocity of a Brownian particle :

$$\overline{v^2} = \frac{3kT}{m} \quad \text{---- } 5\text{-}(18)$$

, where m = mass of the particle, and points out that such a velocity will not in fact be observable on a microscopic particle, because of the averaging of the great number of impulses over an observation interval. This is confirmed by the fact that experimental values of $\overline{v^2}$ are far lower than that predicted by (18).

The case, however, is different in the electrical fluctuation records when we deal with the envelope of the fluctuations produced by an electrical system of limited frequency response. If we admit the representation of $R(t)$ as given in equation (2) then clearly the rate of variation of $R(t)$ will be limited by the characteristics of the system itself. The problem has been discussed by Rice (Paper B, page 79) and the analysis and the resulting expression for the maxima distribution is rather complex. The general shape for a rectangular filter is shown in Figure 48.

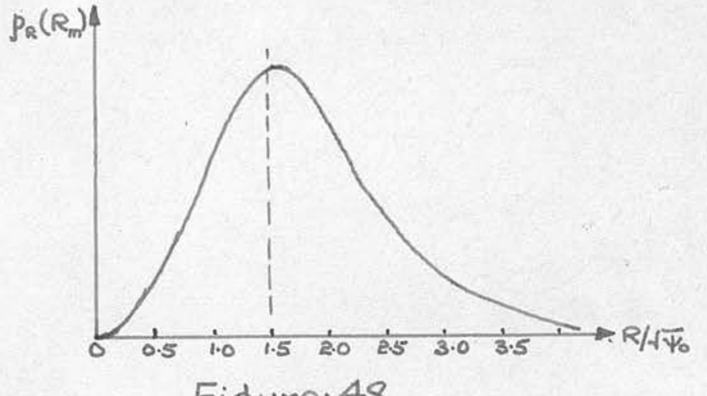


Figure: 48

The experimental distribution for the maxima was derived for a number of "frames", and an example is shown in Figure 49. It will be seen that the general shape agrees with Figure 48, as was so with the other investigations. Detailed computation was not carried out because of the complexity of the expression for the distribution; the analysis would have had to be further modified to allow for the actual shape of the frequency response of the system.

....

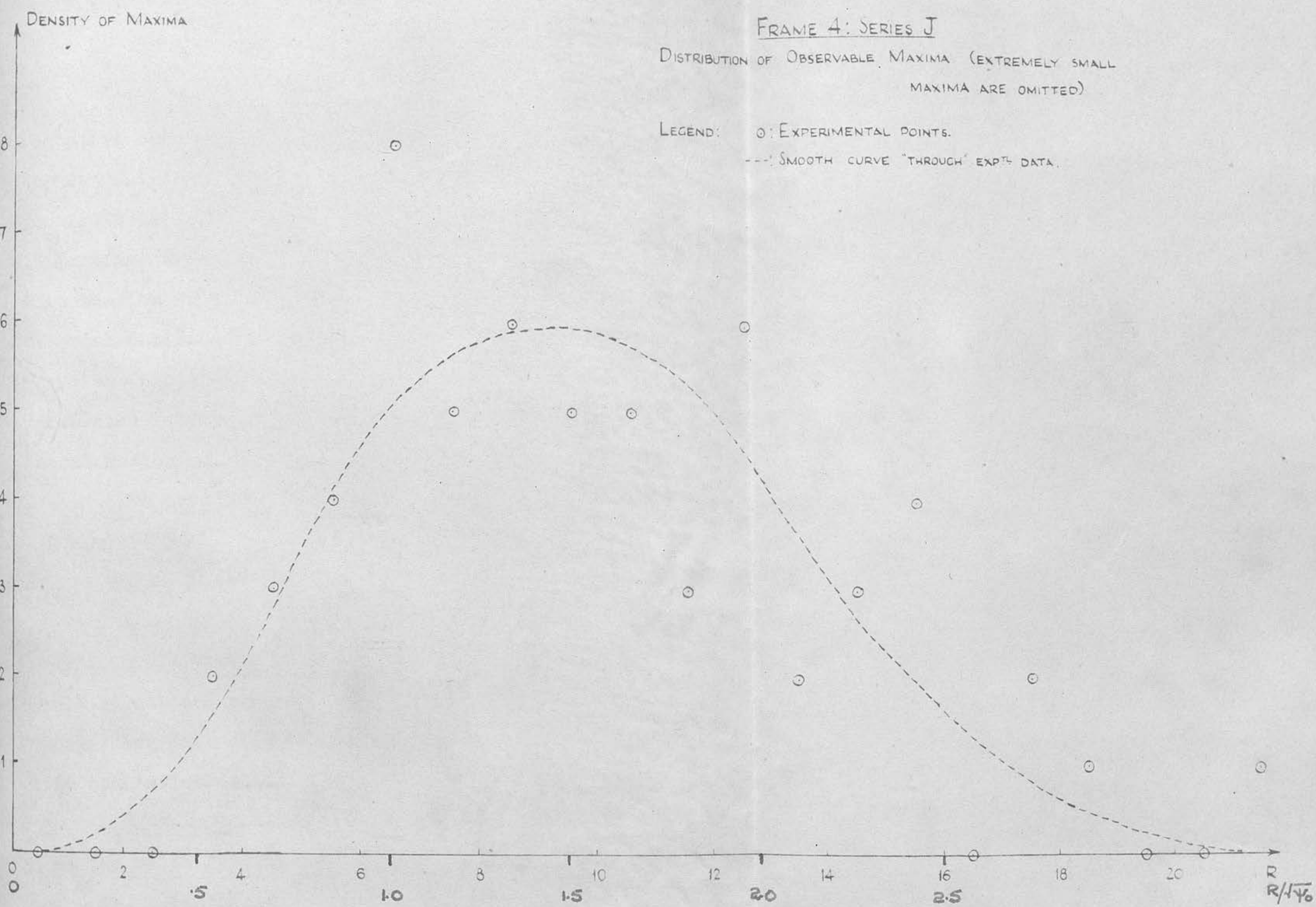

DENSITY OF MAXIMA
FRAME 4: SERIES J
DISTRIBUTION OF OBSERVABLE MAXIMA (EXTREMELY SMALL MAXIMA ARE OMITTED)
LEGEND: ⊙: EXPERIMENTAL POINTS.
---: SMOOTH CURVE "THROUGH" EXPTL DATA.
8
7
6
5
4
3
2
1
0
2
4
6
8
10
12
14
16
18
20
R
0
·5
1·0
1·5
2·0
2·5
R/√ψ₀

Figure: 49.

5. 4 : Correlation Analysis on Fluctuation Records.

5. 4. 1: Basic theory.

The original intention in carrying out a detailed correlation analysis on suitable records was to determine whether or not any significant difference could be detected between records arising from the two "limiting" types of fluctuation, (dominantly "shot" fluctuation from a saturated diode and dominantly "thermal "noise generated in the 1st. circuit). Some possible indication has in fact been observed to be discussed later, but further interesting experimental confirmation of derived theory has been obtained.

We have now to deal with quantities of the type $R(t+\tau) - R(t)$. We define, in what follows :

$$\Delta_\tau = \overline{|R(t+\tau) - R(t)|} \text{ --------- } 5\text{-}(19)$$

$$v_\tau = R(t+\tau) - R(t) \text{ --------- } 5\text{-}(20), \text{ or } \Delta_\tau = \overline{|v_\tau|}$$

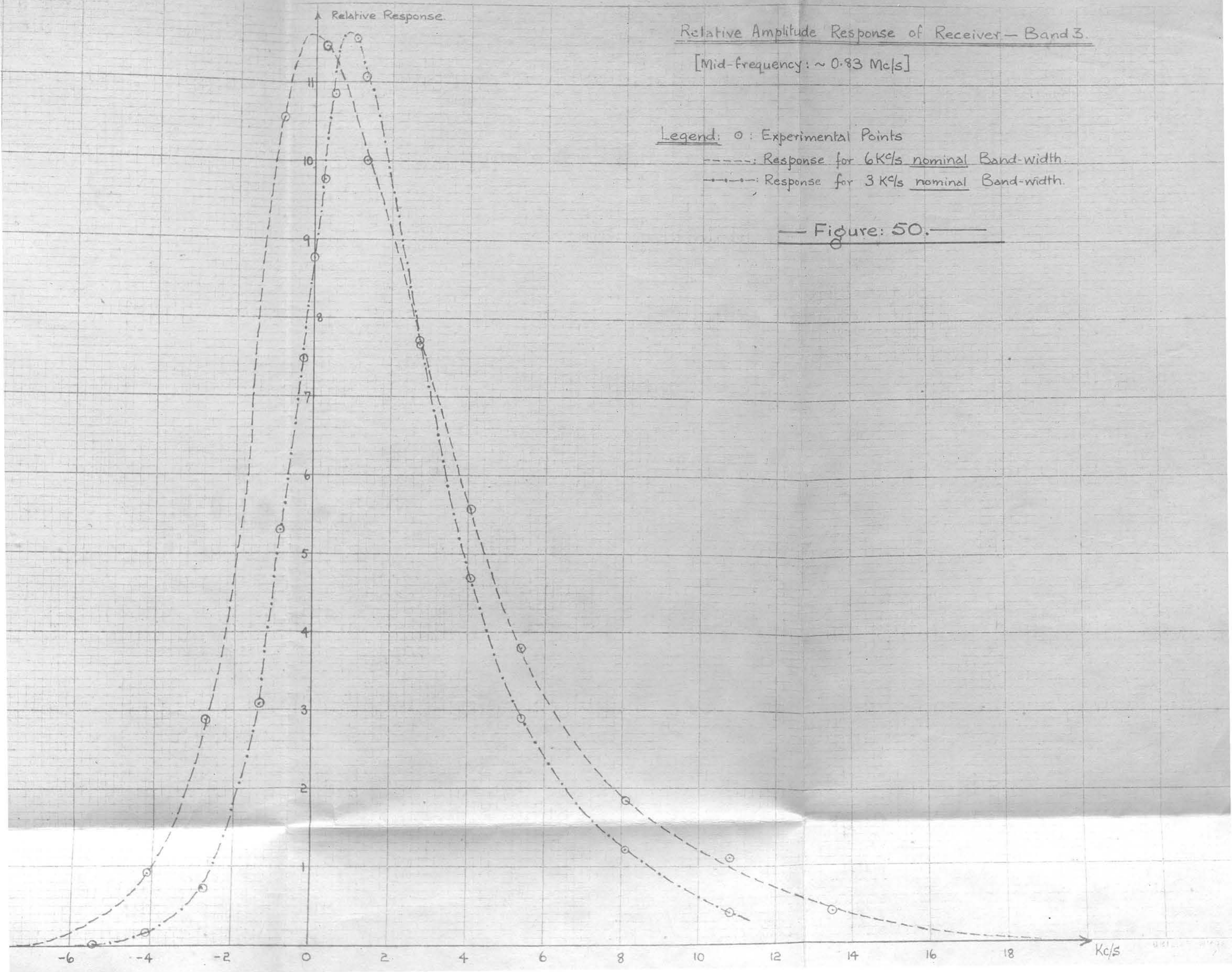
where in the latter expression the subscript to v_τ will be omitted when no confusion will arise. It would have been more convenient, as will be seen from the theoretical aspect, to have dealt with a parameter:

$$\Delta'_\tau = \sqrt{\overline{v_\tau^2}} \text{ ------------- } 5\text{-}(21)$$

but the modulus value was initially adopted for ease of quick computation without an intention of theoretical work and was adhered to later for uniformity.

Rice (Paper B, pages 77/78) derives an expression for the combined probability density that the envelope has a value R_1 , at time t , R_2 at time $t+\tau$, namely :

$$p(R_1 R_2) = \frac{R_1 R_2}{A} . I_0\left(\frac{R_1 R_2}{A} . \{\mu_{13}^2 + \mu_{14}^2\}^{1/2}\right) . \exp\left(-\frac{\psi_0}{2A}(R_1^2 + R_2^2)\right) \text{ ---- } 5\text{-}(22)$$



Relative Amplitude Response of Receiver — Band 3.
[Mid-frequency: ~ 0·83 Mc/s]
Legend: ⊙ : Experimental Points
- - - - -: Response for 6 Kc/s nominal Band-width.
-·-·-·-: Response for 3 Kc/s nominal Band-width.
— Figure: 50. —
Relative Response.
11
10
9
8
7
6
5
4
3
2
1
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
Kc/s

5. 4. 2 : Correlation as $\tau \to 0$.

We first consider the case of τ very small (i.e. R_1 and R_2 very close together). In this case we may set, using (27) and (29) :

$$\mu_{13} \simeq \psi_0\{1-2\pi^2\sigma^2\tau^2\} \text{ --------- } 5\text{-}(30)$$

$$A \simeq \psi_0^2\{4\pi^2\sigma^2\tau^2\} \text{ --------- } 5\text{-}(31)$$

Thus :

$$p(R_1R_2)_{\tau\to 0} = \lim_{\tau\to 0}\left[\frac{R_1R_2}{\psi_0^2(4\pi^2\sigma^2\tau^2)}\cdot I_0\left(\frac{R_1R_2}{4\pi^2\sigma^2\tau^2\psi_0} - \frac{R_1R_2}{2\psi_0}\right)\cdot\exp\left(-\frac{(R_1^2+R_2^2)}{2\psi_0\cdot 4\pi^2\sigma^2\tau^2}\right)\right] \text{---} 5\text{-}(32)$$

Now the asymptotic expression for $I_0(x)$ as $x\to\infty$ is :

$$I_0(x) \underset{x\to\infty}{\sim} \frac{\varepsilon^x}{\sqrt{2\pi x}}$$

Hence :

$$p(R_1R_2)_{\tau\to 0} \sim \lim_{\tau\to 0}\frac{\sqrt{R_1R_2}}{\sqrt{2\pi}\cdot\psi_0\sqrt{\psi_0}\,2\pi\sigma\tau}\cdot\exp\left\{-\frac{(R_1-R_2)^2}{2\psi_0\cdot 4\pi^2\sigma^2\tau^2}\right\}\cdot\exp\left\{-\frac{R_1R_2}{2\psi_0}\right\}$$

Hence :

$$p(R,v_\tau)dR\,dv_\tau{}_{\tau\to 0} \simeq \frac{R\,dR}{\psi_0}\cdot\exp\left(-\frac{R^2}{2\psi_0}\right)\cdot\frac{\exp\left(-\frac{v_\tau^2}{8\pi^2\psi_0\sigma^2\tau^2}\right)}{\sqrt{2\pi}\cdot 2\pi\sigma\tau\sqrt{\psi_0}}\cdot dv_\tau$$

Hence :

$$p(v_\tau)dv_{\tau\,(\tau\to 0)} \equiv dv_\tau\int_0^\infty p(R,v_\tau)\,dR$$

$$= \frac{\varepsilon^{-v_\tau^2/8\pi^2\psi_0\sigma^2\tau^2}\,dv_\tau}{\sqrt{2\pi}\cdot 2\pi\sigma\tau\sqrt{\psi_0}}\int_0^\infty\frac{R}{\psi_0}\cdot\varepsilon^{-\frac{R^2}{2\psi_0}}\,dR$$

The latter integral is equal to unity, whence :

$$p(v_\tau)_{\tau\to 0} = \frac{\varepsilon^{-v_\tau^2/8\pi^2\psi_0\sigma^2\tau^2}}{\sqrt{2\pi}\cdot 2\pi\sigma\tau\psi_0} \text{ --------- } 5\text{-}(33)$$

It is convenient to write :

$$p(v_\tau)dv_\tau = p(x)\cdot dx = \frac{1}{\sqrt{\pi}}\cdot\varepsilon^{-x^2}dx \text{ -------- } 5\text{-}(34)$$

where $x = \dfrac{v}{2\pi\sigma\tau\sqrt{\overline{R^2}}}$, using $\overline{R^2} = 2\psi_0$

We denote below the constant ratio $\frac{v}{x}$ by c

$$\therefore c = 2\pi\sigma\tau\sqrt{\overline{R^2}} \text{ ----------- } 5\text{-}(35)$$

We may now put this theory to the test of experiment. This has been done on a number of fluctuation records, using time-intervals as close together (1 mm.) as might be reasonably legible. The experimental results for the distribution of v in Frames 4 and 7, Series J, are displayed in Figure 52, and for Frame 16, Series C, in Figure 53. By determining the best fit of the theoretical curve given by equation (34) with the experimental data we derive a value for $\frac{v}{\tau} = c$ in each case.

For comparison we can also derive a theoretical value for c by means of equation (35). τ and $\sqrt{\overline{R^2}}$ are readily determinable from the fluctuation record, using for the former the timing wave recorded simultaneous-:ly with the fluctuation. In order to determine theoretical values of σ from the data available we have recourse to the frequency response curves obtained experimentally for the receiver operating under the various conditions. An example is shown in Figure 50. We wish to determine the best value of σ to fit the theoretical amplitude curve :

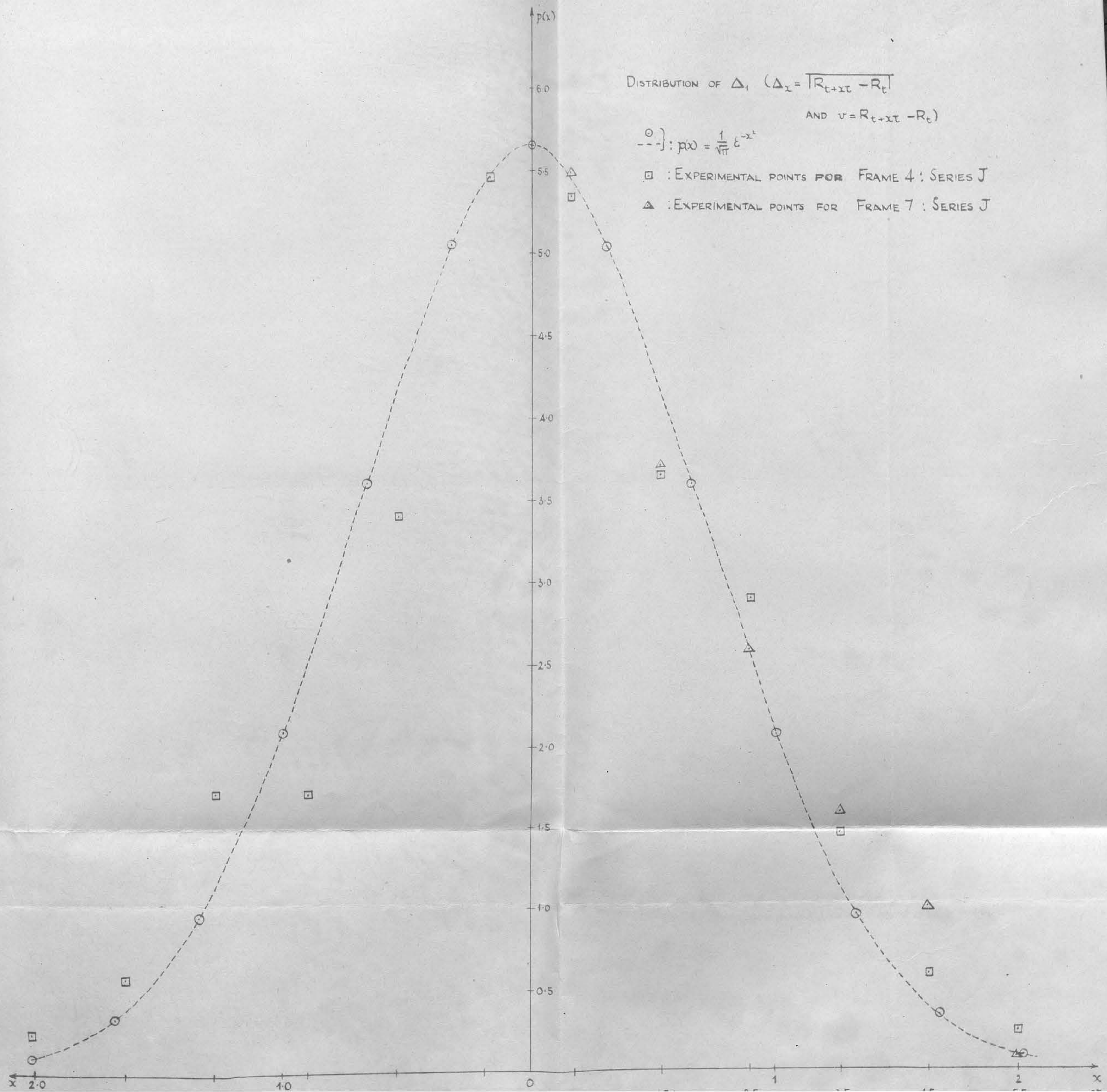
$$A(f) = A_0 \varepsilon^{-\frac{(f-f_0)^2}{4\sigma^2}} \quad \text{---------} \quad 5\text{-}(36)$$

corresponding to :

$$W(f) = W_0 \varepsilon^{-\frac{(f-f_0)^2}{2\sigma^2}} \quad \text{------} \quad 5\text{-}(37)$$ for the power spectrum.

The possibility of logarithmic plotting in this case to determine σ (cf. Section 5.2.1., equation (4)), was not proceeded with because two parameters, f_0 and σ, are involved and the curves are not exactly symmetrical about the maximum. After consideration it was felt that numerical integration should give the most reliable result giving a direct value of σ and accounting for the entire shape of the curve. Thus from (36), :

$$\text{Area} = \int_{-\infty}^{\infty} A(f).df = \underline{2A_0\sigma\sqrt{\pi}}$$



DISTRIBUTION OF Δ_1 ($\Delta_x = |R_{t+x\tau} - R_t|$
AND $v = R_{t+x\tau} - R_t$)
⊙ - - - } : $p(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}$
⊡ : EXPERIMENTAL POINTS FOR FRAME 4 : SERIES J
△ : EXPERIMENTAL POINTS FOR FRAME 7 : SERIES J
p(x)
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
x
2.0
1.0
0
1
2
x

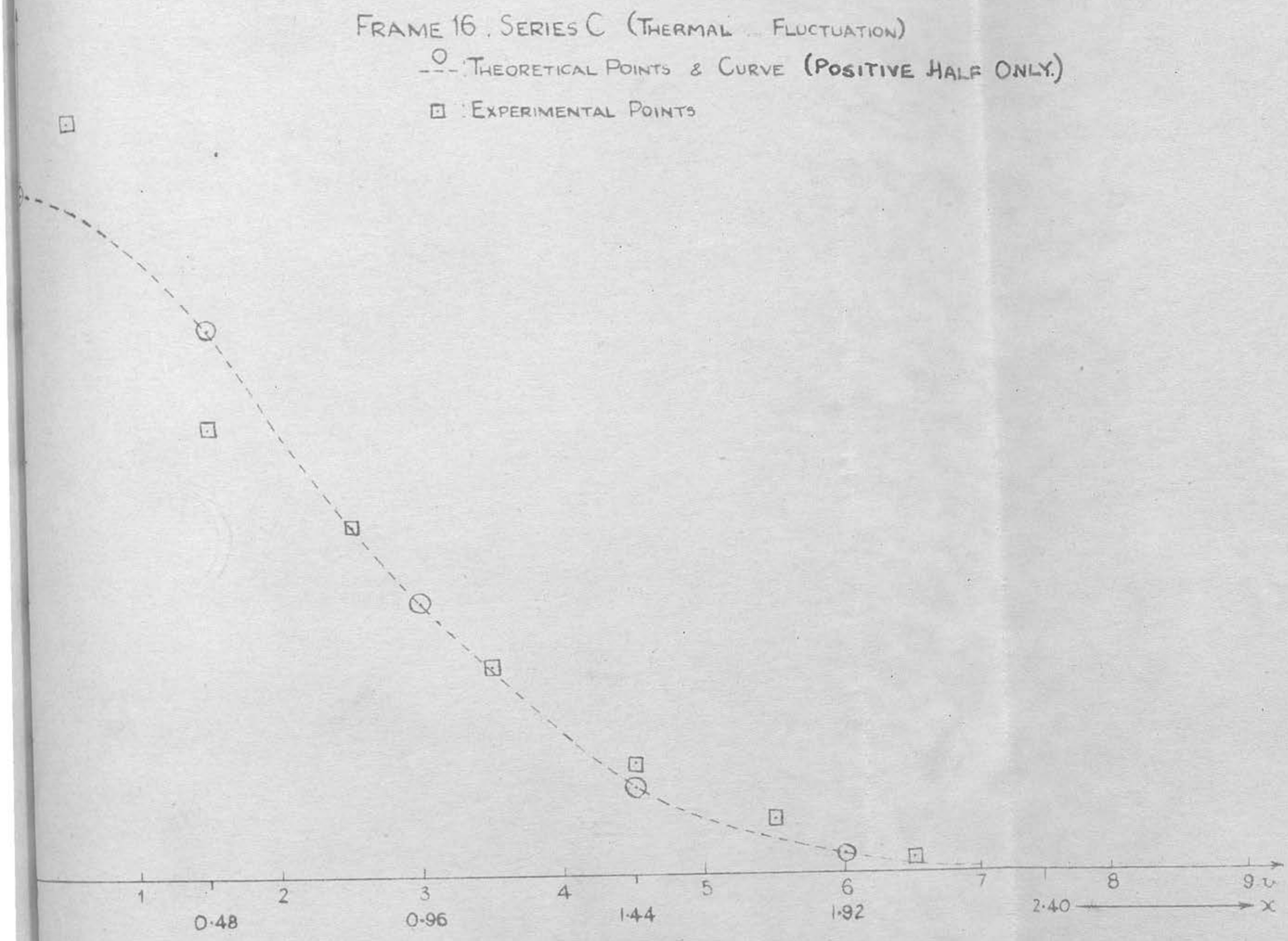

FRAME 16. SERIES C (THERMAL FLUCTUATION)
-○- THEORETICAL POINTS & CURVE (POSITIVE HALF ONLY.)
⊡ EXPERIMENTAL POINTS
1
2
3
4
5
6
7
8
9 v
0·48
0·96
1·44
1·92
2·40
x

Figure: 53.

In this manner corresponding to the three records above values of σ were derived.

The results of this work, leading to experiment-:al and calculated values of C for comparison, are tabulated below :

Frame	$\tau \equiv 1mm.$ $(10^{-5} sec.)$	$\sqrt{\overline{R^2}}$	σ $(10^3/sec.)$	$C_{th.}$	$C_{exp.}$	$C_{exp}/C_{th.}$
4: Series J.	2.2	9.0	1.95	2.42	2.77	1.14
7: Series J.	2.2_4	9.6	2.08	2.76	2.76	1.005
16: Series C.	4.5	9.4	1.19	3.14	3.26	1.04

The agreement, as signified by the last column, is considered to be very satisfactory.

From equation (33), which strictly defines a Dirac - δ -function as $\tau \to 0$, we see by evaluating :

$$\Delta_{\tau(\tau\to 0)} \equiv 2\int_0^\infty v.p(v).dv = 2\sqrt{\pi}\,\sigma\tau\sqrt{\overline{R^2}} \text{ ------ } 5\text{-}(38)$$

that, for small τ, Δ_τ is proportional to τ . Such a behaviour is clearly to be expected in general for sufficiently small τ since the fluctuation envelope has a finite rate of variation, as has previously been discussed (Section 5.3.), and is borne out by experiment-:al records.

....

5. 4. 3: Correlation as $\tau \to \infty$.

In this case, since R_1 and R_2 are obviously uncorrelated, we may immediately set :

$$p(R_1 R_2)dR_1 dR_1 = \frac{R_1 R_2}{\psi_0}.\varepsilon^{-\frac{R_1^2+R_2^2}{2\psi_0}}.dR_1 dR_2 \text{ ------ } 5\text{-}(39)$$

or we may observe that this follows from equation (22), noting that $\lim_{\tau\to\infty} \mu_{13} = 0$ and that $\lim_{x\to 0} I_0(x) = 1$.

Then setting $R_2 = R + v$ and $R_1 = R$ we see that :

$$p(v)dv = dv \int_0^\infty \frac{R(R+v)}{\psi_0^2} \varepsilon^{-\frac{R^2+(R+v)^2}{2\psi_0}} dR \quad \left[\begin{array}{l}\text{for } v>0\text{; and for } v<0 \\ \text{a mirror-symmetrical expression}\end{array}\right].$$

By straightforward reduction we find :

$$p(v) = \varepsilon^{-\frac{v^2}{4\psi_0}}\left\{\left[\int_0^\infty \frac{(R+\frac{v}{2})^2}{\psi_0^2} \cdot \varepsilon^{-\frac{(R+\frac{v}{2})^2}{\psi_0}} . dR\right] - \left[\frac{v^2}{4\psi_0}\int_0^\infty \varepsilon^{-\frac{(R+\frac{v}{2})^2}{\psi_0}} . \frac{dR}{\psi_0}\right]\right\}$$

and finally :

$$p(v) = \frac{1}{\sqrt{\psi_0}} . \varepsilon^{-\frac{v^2}{4\psi_0}} \left[\frac{1}{2} \cdot \frac{v}{2\sqrt{\psi_0}} . \varepsilon^{-\frac{v^2}{4\psi_0}} + \left(\frac{1}{2} - \frac{v^2}{4\psi_0}\right) . \frac{\sqrt{\pi}}{2} . \left(1 - \mathrm{erf}\,\frac{v}{2\sqrt{\psi_0}}\right)\right]$$

(Setting here : $\mathrm{erf}\,t = \frac{2}{\sqrt{\pi}} \int_0^t \varepsilon^{-x^2} dx$)

Finally, defining for convenience in computing :

$$x = \frac{v}{2\sqrt{\psi_0}}$$

we have :

$$p(x) = \varepsilon^{-x^2}\left\{x\varepsilon^{-x^2} + \sqrt{\pi}\left(\tfrac{1}{2} - x^2\right).\left(1 - \mathrm{erf}\,x\right)\right\} \text{ - - - - - - - 5-(40)}$$

(which may be readily calculated, using for example Janke and Emde: "Funktiontafeln".)

Since we know that $\overline{R^2} = 2\psi_0$, this expression may also be tested against experimental data. This is shown as an example for Frame 4: Series J in Figure 55.(*) We have $\overline{R^2} = 81$, the scale factor for the abcissa is $\frac{v}{x} = 12.7$. The agreement is seen to be excellent. To obtain v_τ $(\tau \to \infty)$, values of R at 30 units apart were employed. Since Δ_τ is found to reach a steady value for $\tau > 10$ mm. say, this should provide an ample separation.

We observed from the plot of $p(x)$ that it had a strong resemblance to a purely Gaussian curve. If we were to replace $p(x)$ as in equation (40) by a Gaussian curve $p'(x)$ which is to "fit" at $x = 0$ then it must satisfy $p'(0) = p(0) = \frac{\sqrt{\pi}}{2}$. Also clearly it must obey : $\int_{-\infty}^{\infty} p'(x).dx = 1$.

(*: N.B.: There is no Fig. 54.)

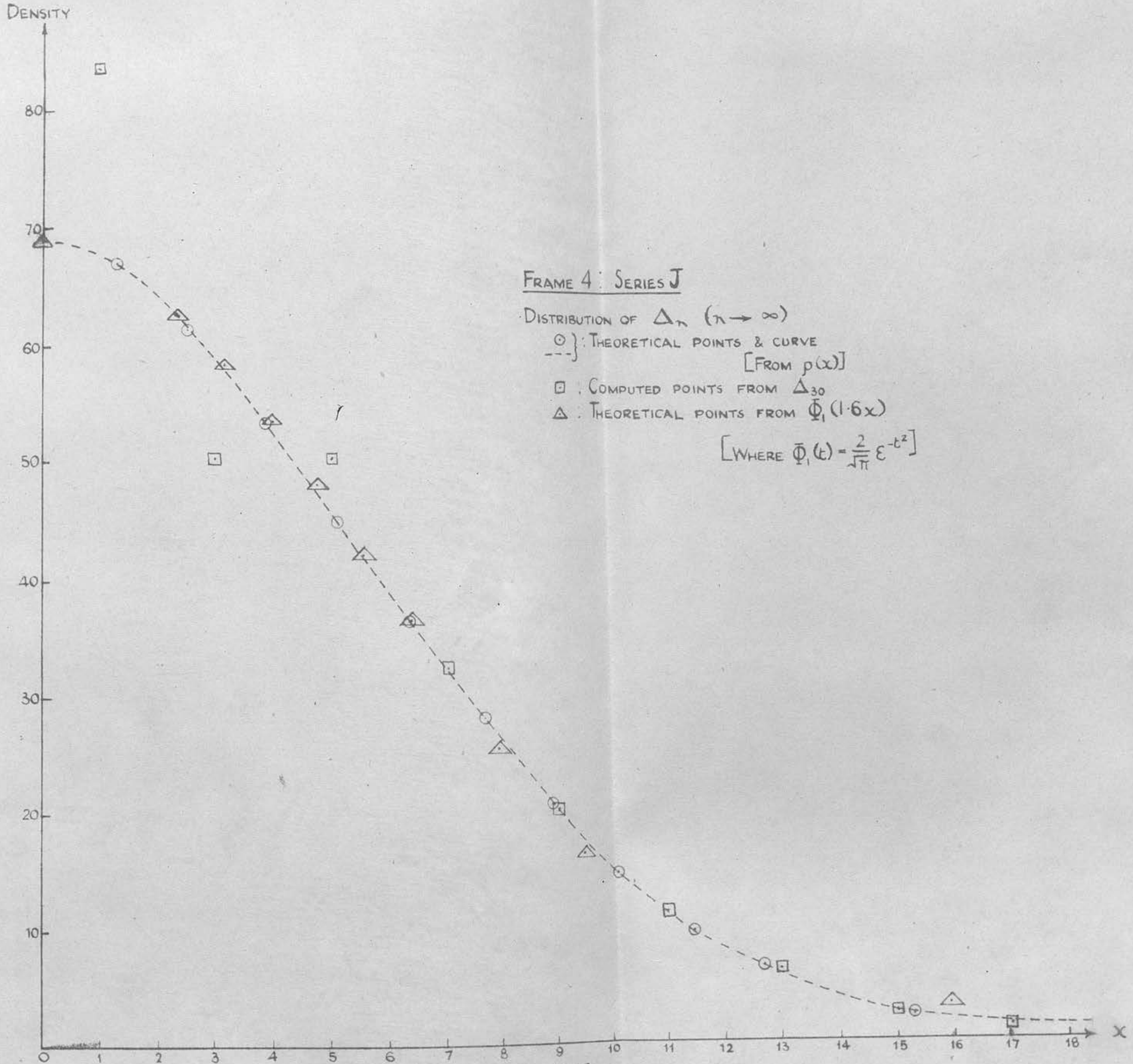

DENSITY
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
X
FRAME 4 : SERIES J
DISTRIBUTION OF Δ_n $(n \to \infty)$
⊙ } : THEORETICAL POINTS & CURVE
[FROM $p(x)$]
⊡ : COMPUTED POINTS FROM Δ_{30}
△ : THEORETICAL POINTS FROM $\Phi_1(1 \cdot 6x)$
[WHERE $\Phi_1(t) = \frac{2}{\sqrt{\pi}} \varepsilon^{-t^2}$]

Figure: 55.

And hence: $p'(x) = \frac{\sqrt{\pi}}{2}.\varepsilon^{-(1.57x)^2}$ ---- 5-(41) (Noting that $\frac{\pi}{2} \simeq 1.57$).

A reasonable fit was in fact found for $p'(x) \sim \varepsilon^{-(1.6x)^2}$ on the graph of Figure 55 (plotted on a linear scale of co-ordinates). As a more rigorous determination $p(x)$, as computed, was plotted logarithmically against x^2 and, as seen from Figure 56, an excellent straight line characteristic resulted with a slope .43, from which we derive as exponent factor : $\frac{1}{\sqrt{.43}} \doteq 1.53$.

....

5. 4. 4: General distribution of v_t.

The writer has also derived a general expression as follows for $p(v_t)$:

We have, as before, using $z = \mu_{13}/\psi_0$; $\mu_{14} \doteq 0$:-

$$p(v) = \frac{1}{\psi_0}\int_0^\infty \frac{R(R+v)}{\psi_0(1-z^2)}.\exp\left\{-\frac{R^2+(R+v)^2}{2\psi_0(1-z^2)}\right\}.I_0\left\{z.\frac{R(R+v)}{\psi_0(1-z^2)}\right\}dR$$

$$= \frac{\varepsilon^{-\frac{v^2}{2\alpha}}}{\psi_0}\int_0^\infty \frac{R(R+v)}{\alpha}.\exp\left\{-\frac{R(R+v)}{2\alpha}\right\}.I_0\left\{z.\frac{R(R+v)}{\alpha}\right\}.dR$$

(writing $\alpha = \psi_0(1-z^2)$)

Now let : $t = \frac{R(R+v)}{\alpha}$, $\therefore dt =$ etc...

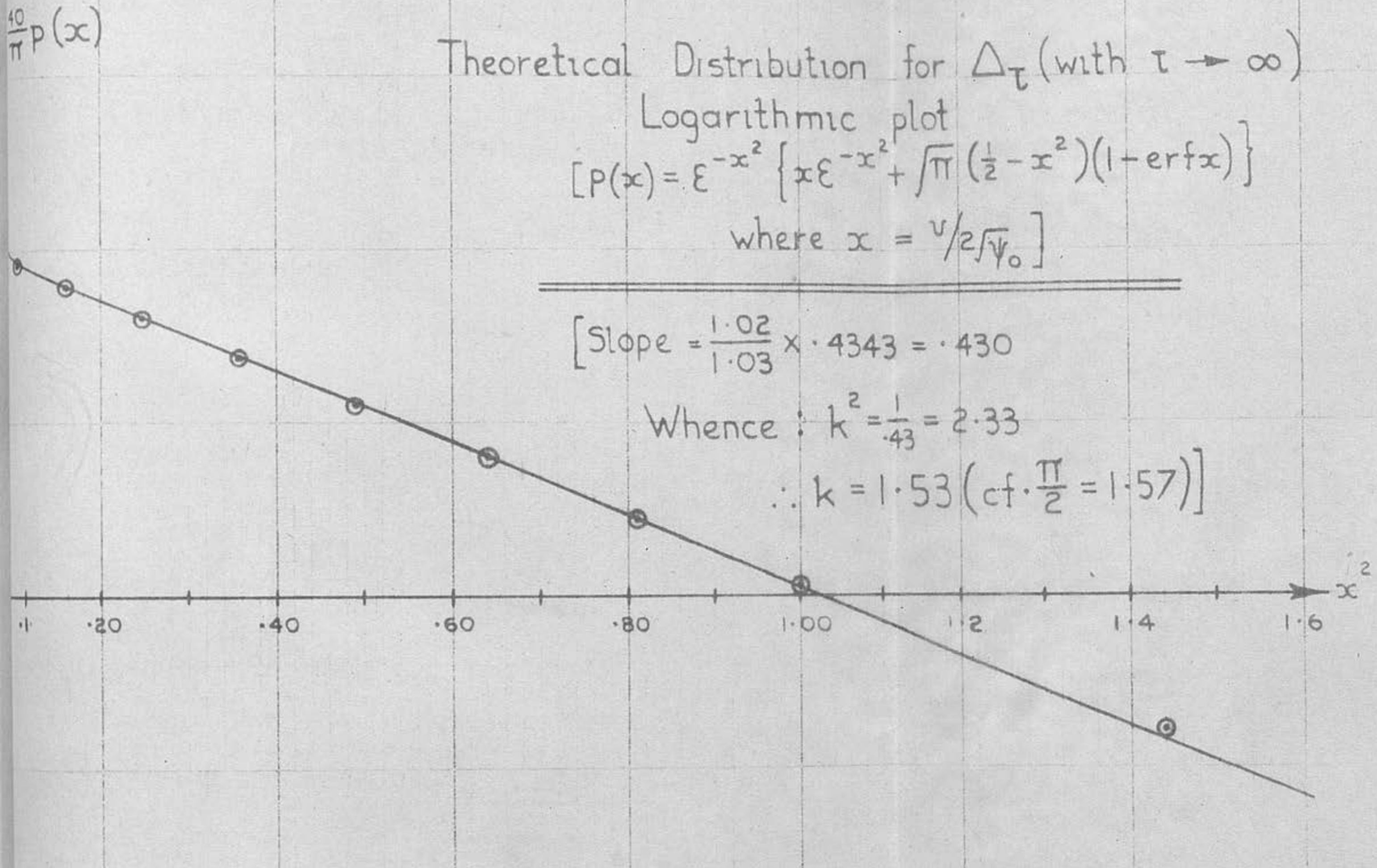
Then after some reduction :

$$p(v) = \frac{\sqrt{\alpha}}{2\psi_0}.\varepsilon^{-\frac{v^2}{2\alpha}}\int_0^\infty t.(t+\beta)^{-1/2}.\varepsilon^{-t}.I_0(zt).dt$$ ------, (setting $\beta = \frac{v^2}{(2\sqrt{\alpha})^2} = v^2/4\alpha$.)

We find that expanding $(t+\beta)^{-1/2}$ in powers of β/t rather than in powers of t/β causes the integral to converge :

$$\therefore p(v) = \frac{\sqrt{\alpha}}{2\psi_0}\varepsilon^{-\frac{v^2}{2\alpha}}\int_0^\infty\left(t^{1/2} - \frac{\beta}{2}t^{-1/2} + \frac{\beta^2}{8}t^{-3/2} - \dots\right)\varepsilon^{-t}.I_0(zt).dt \quad \text{---- 5-(42)}$$

$$= \frac{\sqrt{\alpha}}{2\psi_0}\varepsilon^{-\frac{v^2}{2\alpha}}\left[I_1 - \frac{\beta}{2}I_2 + \frac{\beta^2}{8}I_3 - \dots\right] \quad \text{--------- 5-(43)}$$



$\frac{40}{\pi}p(x)$
Theoretical Distribution for Δ_τ (with $\tau \to \infty$)
Logarithmic plot
$[p(x) = \varepsilon^{-x^2}\{x\varepsilon^{-x^2} + \sqrt{\pi}(\frac{1}{2} - x^2)(1 - \mathrm{erf}\,x)\}$
where $x = v/2\sqrt{\psi_0}]$
[Slope $= \frac{1 \cdot 02}{1 \cdot 03} \times \cdot 4343 = \cdot 430$
Whence: $k^2 = \frac{1}{\cdot 43} = 2 \cdot 33$
$\therefore k = 1 \cdot 53$ (cf $\cdot \frac{\pi}{2} = 1 \cdot 57$)]
x^2
·1
·20
·40
·60
·80
1·00
1·2
1·4
1·6

Figure: 56.

Now Whittaker and Watson(1) give the following integral, due to Hankel, :

$$\int_0^\infty \varepsilon^{-at}.J_\nu(bt).t^{\mu-1}.dt = \frac{\left(\frac{b}{2a}\right)^\nu.\Gamma(\mu+\nu)}{a^\mu.\Gamma(\nu+1)}.{}_2F_1\left(\frac{\mu+\nu}{2},\frac{\mu+\nu+1}{2};\nu+1;-\frac{b^2}{a^2}\right)$$

and we thus find :

$$I_1 \equiv \int_0^\infty t^{1/2}.\varepsilon^{-t}.I_0(zt)dt = \frac{\Gamma(3/2)}{\Gamma(1)}.{}_2F_1\{3/4,5/4;1;z^2\}$$

(Where F is the hypergeometric function.)

$$= \frac{\sqrt{\pi}}{2}.\left\{1+\frac{3.5}{4.4}z^2+\frac{3.5.7.9}{4^4.2^2}.z^4+\cdots\cdots\right\}$$

and :-

$$I_2 = \frac{\Gamma(1/2)}{\Gamma(1)}.{}_2F_1(1/4,3/4;1;z^2)$$

$$= \sqrt{\pi}\left\{1+\frac{3}{16}z^2+\frac{105}{1024}z^4+\cdots\cdots\right\}$$

etc.

Thus, except very near to $z=1$ (i.e. $\tau\to 0$ which has already been dealt with), we may write approximately :

$$p(v) \doteqdot \frac{\sqrt{\alpha}}{2\psi_0}.\varepsilon^{-\frac{v^2}{2\alpha}}\left\{\frac{\sqrt{\pi}}{2}\left(\left\{1+\frac{15}{16}z^2\right\}-\frac{v^2}{4\psi_0(1-z^2)}\left\{1+\frac{3}{16}z^2\right\}-\frac{v^4}{32\psi_0^2(1-z^2)^2}.\left\{1-\frac{1}{16}z^2\right\}\cdots\right)\right\}$$

It has not been felt of sufficient value to this work to examine this expression in further detail, but it appears reasonable to assume, partly from the form of the expression itself, and partly from the specialised work already performed for $\tau\to 0$ and $\tau\to\infty$, that $p(v)$ may with fair accuracy be replaced for all τ by a Gaussian distribution.

If this assumption, however, be granted we may then readily derive a general expression for Δ_τ, since theoretical analysis is amenable to dealing with $\overline{v_\tau^2}$ in general. Burgess, in unpublished work, has analysed the product correlation factor for general τ.

We have then :

$$\psi_\tau = \overline{R(t).R(t+\tau)}$$

But :

$$\overline{v_\tau^2} = \overline{\{R(t)-R(t+\tau)\}^2} = 2\overline{R^2} - 2\psi_\tau \quad ------- 5\text{-}(44)$$

Further, assuming Gaussian distribution throughout for v_τ we show readily that :

$$\overline{v_\tau^2} = \frac{\pi}{2}\left(\overline{|v_\tau|}\right)^2$$

$\therefore$ using (44) :

$$\psi_\tau = \overline{R^2} - \frac{\pi}{4}\Delta_\tau^2 \text{ ------------ } 5\text{-}(45)$$

or, normalising ψ_τ so that $\psi_0 = 1$, by dividing through by $\overline{R^2}$:

$$\psi_\tau = 1 - \frac{\pi}{4}\frac{\Delta_\tau^2}{\overline{R^2}} \text{ ------------ } 5\text{-}(46)$$

Burgess derives, starting from the general expression for $p(R_1R_2)dR_1dR_2$ (see equation (22)), that :

$$\psi_\tau = \frac{\pi}{4}\cdot {}_2F_1\left(-\tfrac{1}{2}, -\tfrac{1}{2}; 1; z^2\right) = \frac{\pi}{4}\left(1 + \frac{z^2}{4} + \frac{z^4}{64} + \dots\right)$$

, where F is the hyper-geometric function. and $z = \frac{\mu_{13}}{\psi_0}$

Thus for the case of a Gaussian filter as assumed here $z = \varepsilon^{-2\sigma^2\pi^2\tau^2}$; the powers of z thus rapidly tend to zero as $\tau \to \infty$.

Whence as an overall approximation to give $\psi_\tau = 1$ at $\tau = 0$ we set :

$$\psi_\tau \doteq \cdot 8\left\{1 + \frac{\varepsilon^{-4\sigma^2\pi^2\tau^2}}{4}\right\} \text{ -------- } 5\text{-}(47)$$

Expressions (46) and (47) have been compared (see Figure 57). The general agreement of shape is good, although the value of σ in (47) necessary to give a good fit throughout shows some divergence from that derived in the earlier work of this chapter;* this is in part due to the various approximations employed.

We also observe from (47) and (46) that this gives as an "expected" approximate expression for Δ_τ :

$$\Delta_\tau \doteq \sqrt{\frac{4\overline{R^2}}{5\pi}\left\{1 - \varepsilon^{-4\sigma^2\pi^2\tau^2}\right\}} \text{ ---- } 5\text{-}(48)$$

[*: The scale of the curve and the relatively small range of variation are such that no stress should be laid on the numerical value of σ in this case; the later comparison for Δ_τ is much more significant.]

(PRODUCT) NORMALISED CORRELATION FACTOR $\psi(\tau)$

FRAME 4: SERIES J
("SHOT" FLUCTUATION)

$$\psi(\tau) = \lim_{T\to\infty}\int_0^T R(t).\,R(t+\tau)\,dt \Big/ \lim_{T\to\infty}\int_0^T \{R(t)\}^2\,dt$$

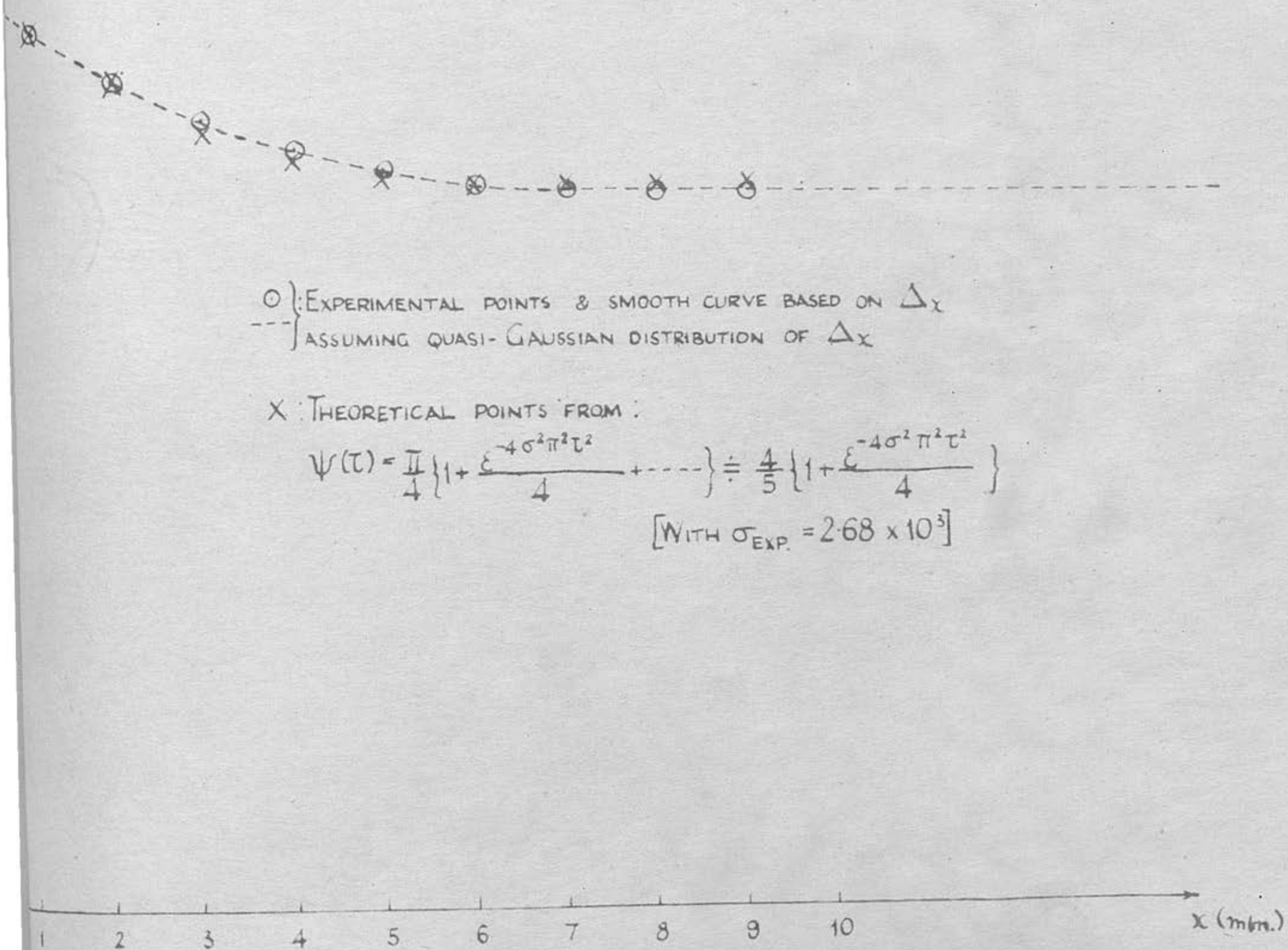


Figure: 57.

It follows in particular, as we should expect, that for $\tau \to 0$, $\Delta_\tau \propto \tau$, in fact :

$$\Delta_{\tau(\tau\to 0)} \doteqdot 2\sqrt{\cdot 8\pi}.\sigma.\tau\sqrt{\overline{R^2}}$$

$$\doteqdot 2(\cdot 9)\sqrt{\pi}\sigma\tau\sqrt{\overline{R^2}} \quad \text{- - - - - - - - - - - - 5-(49)}$$

which agrees closely with equation (38), previously derived.[φ] For $\tau \to \infty$ we find Δ_τ tending to a constant value, as we should also expect, namely :

$$\Delta_\infty \doteqdot \frac{2\sqrt{\overline{R^2}}}{\sqrt{5\pi}} \quad \text{- - - - - - - - - - - - - 5-(50)}$$

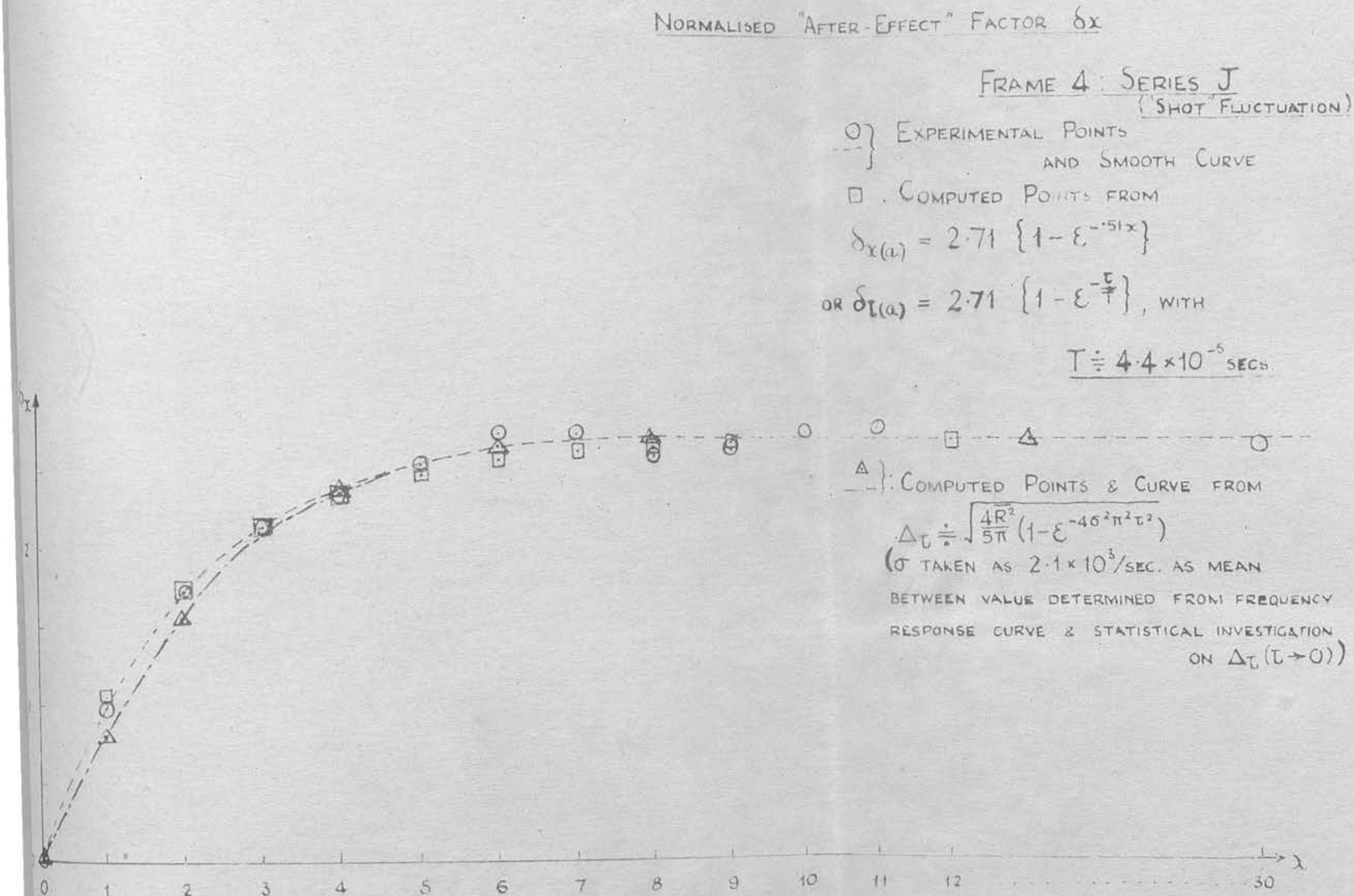
Experimental confirmation of the expression for $\Delta_{\tau\,(\tau\to 0)}$ has been furnished earlier and that for Δ_∞ will be provided below :

[φ: The approximations leading to (47) being in such a sense as slightly to reduce the coefficient of $\Delta_{\tau(\tau\to 0)}$, namely by ·9 approx.]

....

5. 4. 5: Experimental results for Δ_τ .

Two frames each of "thermal" and "shot" fluctuation were analysed in detail and the experimental data are presented as Figures 58, 59, 60, and 61. In order to remove the varying effects of receiver amplification etc., the "after-effect" factor Δ_τ has been "normalised". That is to say, all Δ_τ's have been divided by Δ_{τ_1} , where τ_1 is the unit of time-measurement and is almost identical in every frame dealt with in this section (2.2 to 2.27 x 10^{-5} secs). This normalised after-effect factor δ_x is thus defined as : $\delta_x = \frac{\Delta_{\tau_x}}{\Delta_{\tau_1}}$ ($\therefore$ $\delta_1 \equiv 1$ in all cases). The general shape of δ_x is the same in all frames, and a simple empirical expression for δ_x that might be of use in future work is found to be $\delta_{x(a)} = A\{1-\varepsilon^{-\frac{\tau}{T}}\}$.


NORMALISED "AFTER-EFFECT" FACTOR δx
FRAME 4: SERIES J
('SHOT' FLUCTUATION)
EXPERIMENTAL POINTS AND SMOOTH CURVE
COMPUTED POINTS FROM
$\delta_{x(a)} = 2\cdot71 \{1 - \varepsilon^{-\cdot51x}\}$
OR $\delta_{\tau(a)} = 2\cdot71 \{1 - \varepsilon^{-\frac{\tau}{T}}\}$, WITH
$T \doteq 4\cdot4 \times 10^{-5}$ SECS.
COMPUTED POINTS & CURVE FROM
$\Delta_\tau \doteq \sqrt{\frac{4\bar{R}^2}{5\pi}(1-\varepsilon^{-4\sigma^2\pi^2\tau^2})}$
(σ TAKEN AS $2\cdot1 \times 10^3$/SEC. AS MEAN BETWEEN VALUE DETERMINED FROM FREQUENCY RESPONSE CURVE & STATISTICAL INVESTIGATION ON $\Delta_\tau(\tau \to 0)$)
δx
λ
0
1
2
3
4
5
6
7
8
9
10
11
12
30
Figure: 58.

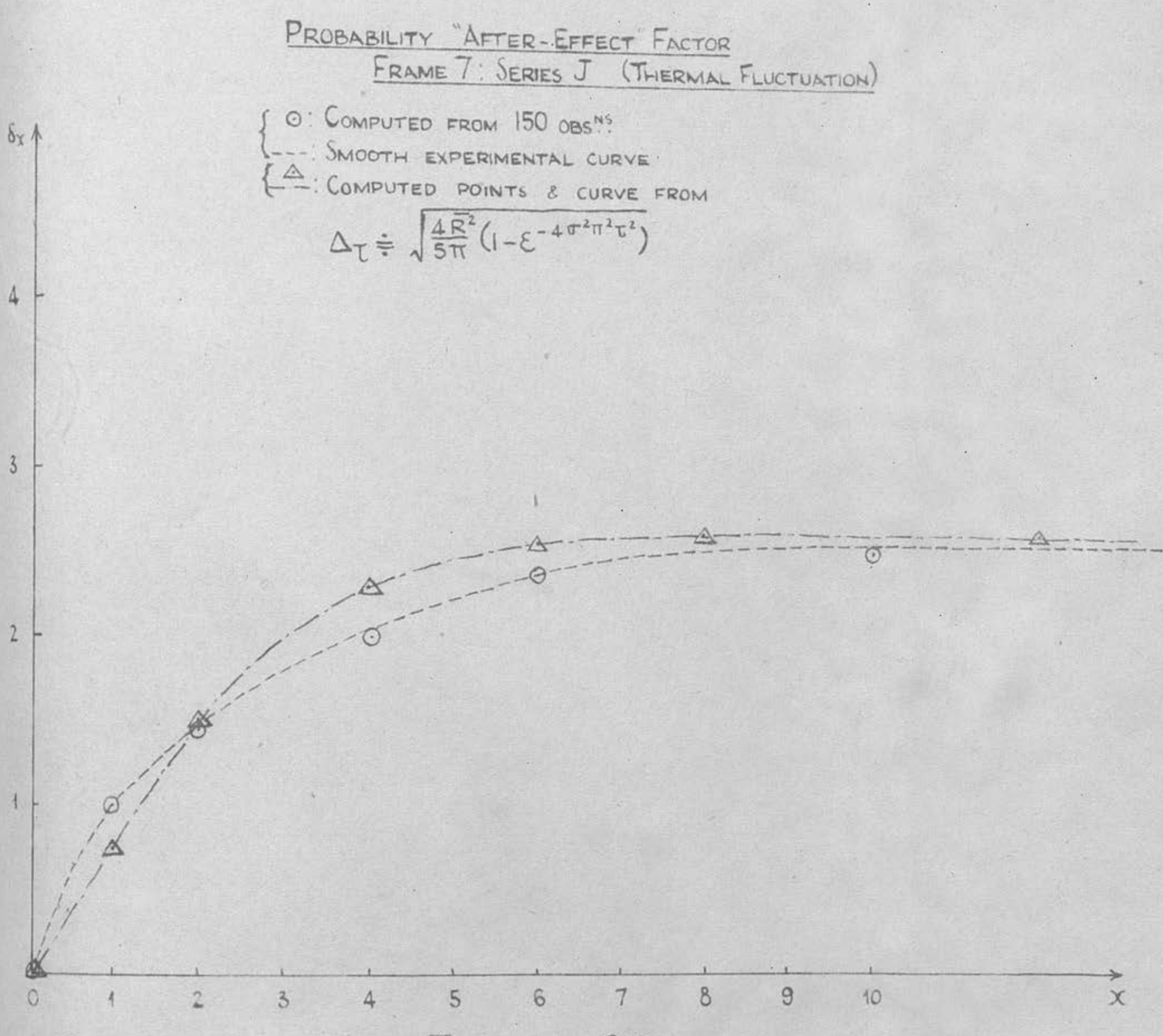

PROBABILITY "AFTER-EFFECT" FACTOR
FRAME 7: SERIES J (THERMAL FLUCTUATION)
⊙: COMPUTED FROM 150 OBS^NS.
----: SMOOTH EXPERIMENTAL CURVE
△ —·—: COMPUTED POINTS & CURVE FROM
$\Delta_\tau \doteqdot \sqrt{\frac{4\bar{R}^2}{5\pi}\left(1-\varepsilon^{-4\sigma^2\pi^2\tau^2}\right)}$
δ_x
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
x

Figure: 59.

Normalised "after-effect" factor : δ_x

Frame 6 : Series J ("Shot" Fluctuation).

---- ⊙ } Experimental points and smooth curve.

[Frequency : ~ 0·83 K c/s.

Nominal Band-width : 6 K c/s.

x in mm. ; 1 mm. ≡ $2{\cdot}27 \times 10^{-5}$ secs.]

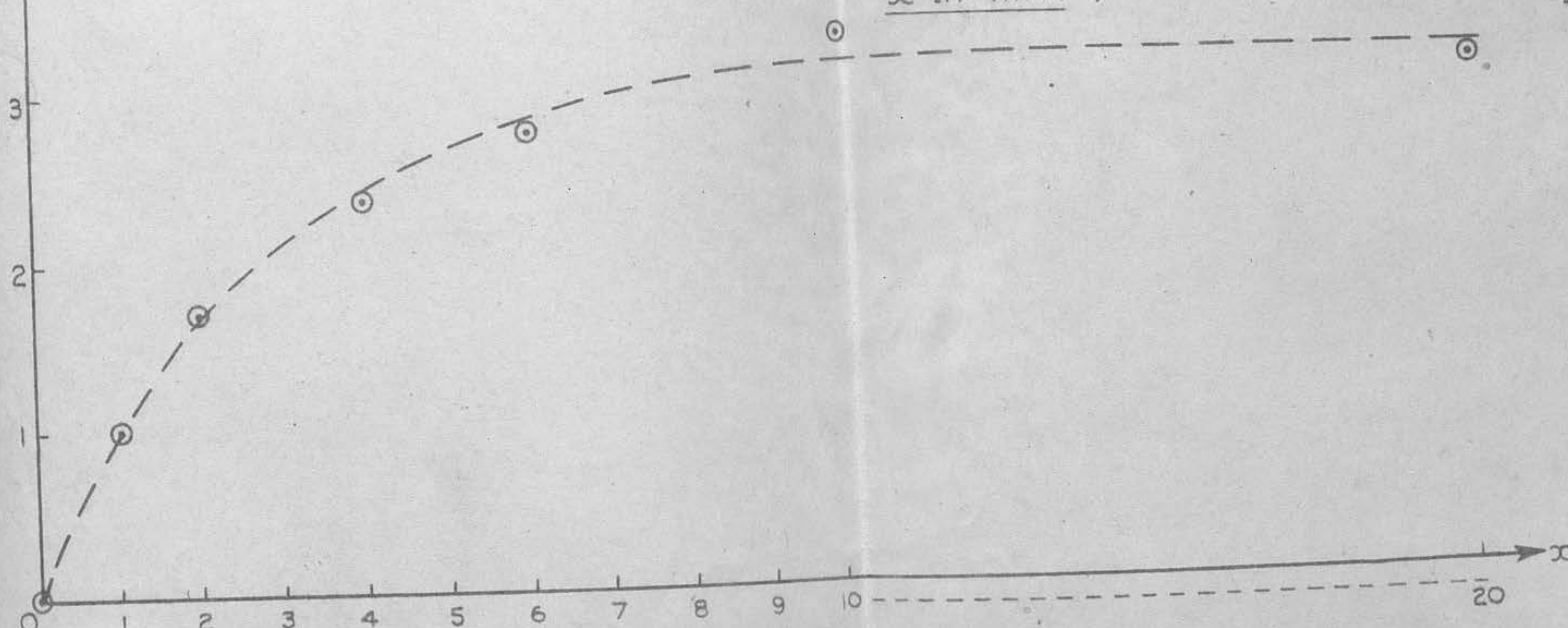


Figure: 60.

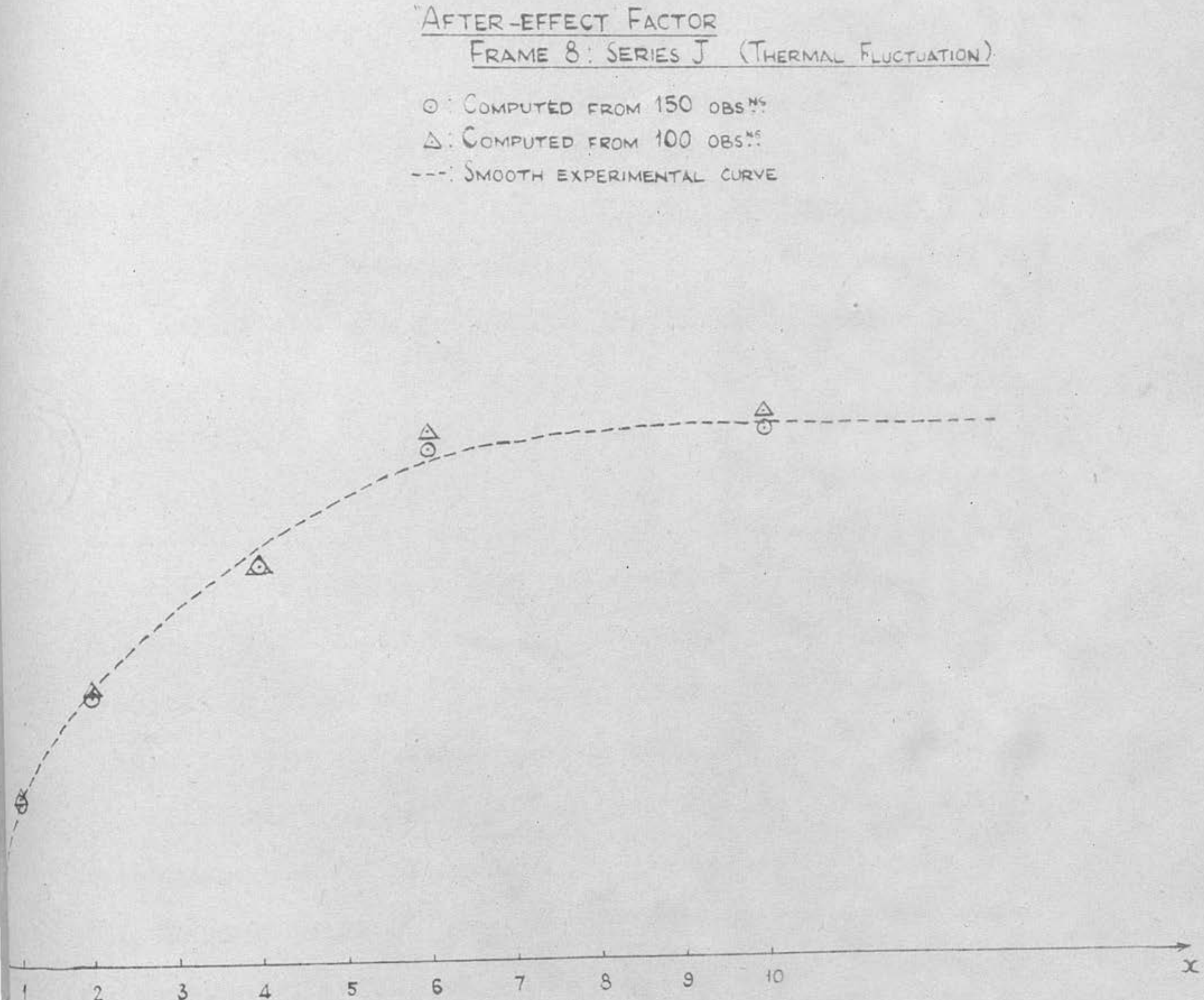

"After-effect Factor
Frame 8: Series J (Thermal Fluctuation)
⊙: Computed from 150 obsns.
△: Computed from 100 obsns.
---: Smooth experimental curve
1
2
3
4
5
6
7
8
9
10
x

Figure: 61.

Thus for Frame 4: Series J, for example, we find :

$$\delta_{x(a)} = 2.71\left\{1 - \varepsilon^{-\frac{\tau}{T}}\right\} \text{----}, \text{ with } T = 4.4 \times 10^{-5} \text{ sec.}$$

We have in addition already derived an approximate theoretical expression for Δ_τ as equation (48) based on the assumption of Gaussian distribution for all τ. We may readily compare Δ_∞ with experiment for one or two cases. These are tabulated below :

Frame	$\sqrt{\overline{R^2}}$	$(\Delta_\infty)_{th.}$	$(\Delta_\infty)_{Exp.}$	$\frac{(\Delta_\infty)_{Exp}}{(\Delta_\infty)_{th.}}$
4: Series J.	9.0	4.55	4.6	1.01
7: Series J.	9.6	4.85	4.6	.95

providing the confirmation of equation (50) promised above. δ_x as derived from equation (49) is also plotted on Figures 58, and 59, for these two Frames. The overall agreement ~~appears rather~~ is seen to be better for the "shot" fluctuation than for the "thermal" fluctuation. This statement ~~may~~ should be considered in conjunction with the discussion below on the variation of δ_2 between the two classes of fluctuation; one must, however, always bear in mind with caution the fact that alien disturbances are more likely to enter when observing thermal noise from a tuned circuit of (necessarily) high dynamic impedance than when observing "shot" noise by injection of a considerably increased inherent magnitude of fluctuation from a saturated diode; precautions have of course been taken, as discussed in Cap.4., to minimise any such disturbance; see also 5.2.3.

For record purposes an extracted sheet of detailed numerical data for a part of one Frame is given on page (126).

Extract of detailed numerical data on fluctuation envelope. Sheet 2 (observation 51-100 inclusive) relevant to Frame 6: Series J.

No	R	R_{+1}	Δ_1	R_{+2}	Δ_2	R_{+4}	Δ_4	R_{+6}	Δ_6	R_{+10}	Δ_{10}	R_{+20}	Δ_{20}
51	2·0	4·0	2·0	4·9	2·9	3·9	1·9	3·9	1·9	14·0	12·0	7·6	5·6
2	4·0	4·9	0·9	4·0	0·0	4·1	0·1	6·6	2·6	17·8	13·8	7·3	3·3
3	4·9	4·0	0·9	3·9	1·0	3·9	1·0	7·7	2·8	18·4	13·5	4·0	0·9
4	4·0	3·9	0·1	4·1	0·1	6·6	2·6	10·0	6·0	14·8	10·8	3·0	1·0
5	3·9	4·1	0·2	3·9	0·0	7·7	3·8	14·0	10·1	8·0	4·1	3·7	0·2
6	4·1	3·9	0·2	6·6	2·5	10·0	5·9	17·8	13·7	6·0	1·9	3·6	0·5
7	3·9	6·6	2·7	7·7	3·8	14·0	10·1	18·4	14·5	6·5	2·6	4·0	0·1
8	6·6	7·7	1·1	10·0	3·4	17·8	11·2	14·8	8·2	5·9	0·7	5·9	0·7
9	7·7	10·0	2·3	14·0	6·3	18·4	10·7	8·0	0·3	4·4	3·3	8·0	0·3
60	10·0	14·0	4·0	17·8	7·8	14·8	4·8	6·0	4·0	6·0	4·0	10·5	0·5
1	14·0	17·8	3·8	18·4	4·4	8·0	6·0	6·5	7·5	7·6	6·4	13·5	0·5
2	17·8	18·4	0·6	14·8	3·0	6·0	11·8	5·9	11·9	7·3	10·5	15·0	2·8
3	18·4	14·8	3·6	8·0	10·4	6·5	11·9	4·4	14·0	4·0	14·4	14·7	3·7
4	14·8	8·0	6·8	6·0	8·8	5·9	8·9	6·0	8·8	3·0	11·8	14·5	0·3
5	8·0	6·0	2·0	6·5	1·5	4·4	3·6	7·6	0·4	3·7	4·3	14·0	6·0
6	6·0	6·5	0·5	5·9	0·1	6·0	0·0	7·3	1·3	3·6	2·4	12·0	6·0
7	6·5	5·9	0·6	4·4	2·1	7·6	1·1	4·0	2·5	4·0	2·5	13·8	7·3
8	5·9	4·4	1·5	6·0	0·1	7·3	1·4	3·0	2·9	5·9	0·0	15·0	9·1
9	4·4	6·0	1·6	7·6	3·2	4·0	0·4	3·7	0·7	8·0	3·6	14·9	10·5
70	6·0	7·6	1·6	7·3	1·3	3·0	3·0	3·6	2·4	10·5	4·5	12·8	6·8
1	7·6	7·3	0·3	4·0	3·6	3·7	3·9	4·0	3·6	13·5	5·9	8·8	1·2
2	7·3	4·0	3·3	3·0	4·3	3·6	3·7	5·9	1·4	15·0	7·7	2·5	4·8
3	4·0	3·0	1·0	3·7	0·3	4·0	0·0	8·0	4·0	14·7	10·7	3·5	0·5
4	3·0	3·7	0·7	3·6	0·6	5·9	2·9	10·5	7·5	14·5	11·5	6·5	3·5
5	3·7	3·6	0·1	4·0	0·3	8·0	4·3	13·5	9·8	14·0	10·3	8·7	5·0
6	3·6	4·0	0·4	5·9	2·3	10·5	6·9	15·0	11·4	12·0	8·4	7·1	3·5
7	4·0	5·9	1·9	8·0	4·0	13·5	9·5	14·7	10·7	13·8	9·8	2·0	2·0
8	5·9	8·0	2·1	10·5	4·6	15·0	9·1	14·5	8·6	15·0	9·1	4·0	1·9
9	8·0	10·5	2·5	13·5	5·5	14·7	6·7	14·0	6·0	14·9	6·9	5·9	2·1
80	10·5	13·5	3·0	15·0	4·5	14·5	4·0	12·0	1·5	12·8	2·3	2·3	8·2
1	13·5	15·0	1·5	14·7	1·2	14·0	0·5	13·8	0·3	8·8	4·7	4·8	8·7
2	15·0	14·7	0·3	14·5	0·5	12·0	3·0	15·0	0·0	2·5	12·5	8·0	7·0
3	14·7	14·5	0·2	14·0	0·7	13·8	0·9	14·9	0·2	3·5	11·2	9·6	5·1
4	14·5	14·0	0·5	12·0	2·5	15·0	0·5	12·8	1·7	6·5	8·0	11·5	3·0
5	14·0	12·0	2·0	13·8	0·2	14·9	0·9	8·8	5·2	8·7	5·3	13·5	0·5
6	12·0	13·8	1·8	15·0	3·0	12·8	0·8	2·5	9·5	7·1	4·9	14·7	2·7
7	13·8	15·0	1·2	14·9	1·1	8·8	5·0	3·5	10·3	2·0	11·8	13·8	0·0
8	15·0	14·9	0·1	12·8	2·2	2·5	12·5	6·5	8·5	4·0	11·0	12·3	2·7
9	14·9	12·8	2·1	8·8	6·1	3·5	11·4	8·7	6·2	5·9	9·0	10·9	4·0
90	12·8	8·8	4·0	2·5	10·3	6·5	6·3	7·1	5·7	2·3	10·5	9·9	2·9
1	8·8	2·5	6·3	3·5	5·3	8·7	0·1	2·0	6·8	4·8	4·0	9·0	0·2
2	2·5	3·5	1·0	6·5	4·0	7·1	4·6	4·0	1·5	8·0	5·5	6·0	3·5
3	3·5	6·5	3·0	8·7	5·2	2·0	1·5	5·9	2·4	9·6	6·1	7·2	3·7
4	6·5	8·7	2·2	7·1	0·6	4·0	2·5	2·3	4·2	11·5	5·0	8·0	1·5
5	8·7	7·1	1·6	2·0	6·7	5·9	2·8	4·8	3·9	13·5	4·8	8·3	0·4
6	7·1	2·0	5·1	4·0	3·1	2·3	4·8	8·0	0·9	14·7	7·6	8·9	1·8
7	2·0	4·0	2·0	5·9	3·9	4·8	2·8	9·6	7·6	13·8	11·8	12·0	10·0
8	4·0	5·9	1·9	2·3	1·7	8·0	4·0	11·5	7·5	12·3	8·3	13·7	9·7
9	5·9	2·3	3·6	4·8	1·1	9·6	3·7	13·5	7·6	10·9	5·0	12·4	6·5
100	2·3	4·8	2·5	8·0	5·7	11·5	9·2	14·7	12·4	9·9	7·6	13·1	10·8
TOTAL			95·3		157·8		229·0		283·4		364·3		183·5
	R	R_{+1}	Δ_1	R_{+2}	Δ_2	R_{+4}	Δ_4	R_{+6}	Δ_6	R_{+10}	Δ_{10}	R_{+20}	Δ_{20}

On this sheet of data, we observe :

$$\delta_2 = \frac{157\cdot8}{95\cdot3} = 1\cdot65_5$$

$$\delta_4 = \frac{229\cdot0}{95\cdot3} = 2\cdot40 \text{ ------ , etc..}$$

As foreseen above, a possibly interesting feature emerged from the computations. While δ_x for large values of x varied considerably from sheet to sheet (say 50 observations) of data within one frame and varied from frame to frame, on the other hand δ_2 differed little from sheet to sheet, but showed rather interesting variations between the different frames.

Thus taking Frame 4: Series J (238 observations of R in all), we find the following :

Frame 4: Series J ("Shot" fluctuation):

Sheet	Δ_{τ_1}	Δ_{τ_2}	δ_2
1 (50 obs^{ns}.)	1.30	2.25	1.73
2 (50 obs.)	2.13	3.68	1.73
3 (50 obs.)	1.44	2.68	1.86
4 (50 obs.)	1.87	3.15	1.68_5
5 (38 obs.)	1.76	3.07	1.74_5

Overall δ_2 = 1.72.

and for the other frames examined :

Frame 6: Series J ("Shot" fluctuation):

Sheet	Δ_{τ_1}	Δ_{τ_2}	δ_2
1 (50 obs.)	92.2	160.2	1.74
2 (50 obs.)	95.3	157.8	1.65_5
3 (50 obs.)	73.8	121.0	1.64
4 (50 obs.)	132.3	217.4	1.64

Overall δ_2 = 1.67.

Frame 7: Series J ("Thermal" fluctuation).

Sheet	Δ_{τ_1}	Δ_{τ_2}	δ_2
1 (50 obs.)	82.9	126.0	1.52
2 (50 obs.)	94.1	124.5	1.32
3 (50 obs.)	98.8	151.1	1.53

Overall δ_2 = 1.46.

Frame 8: Series J ("Thermal" fluctuation).

Sheet	Δ_{τ_1}	Δ_{τ_2}	δ_2
1 (50 obs.)	101.5	156.1	1.54
2 (50 obs.)	125.2	212.9	1.70
3 (50 obs.)	146.7	227.5	1.55

Overall δ_2 = 1.6.

Although the variation in δ_2 between the first two (thermal fluctuation) and the last two (shot fluctuation) can only be regarded as a just possible significant indication, it is mentioned here, because as pointed out previously, the original incentive to this particular investigation was a desire to see whether any observable difference could be found in the correlation for the two limiting classifications of the fundamental phenomenon. It is very much hoped to carry this work further at a later date, but here we simply observe that a reduction in δ_2 means an inherently more rapid time-variation in the envelope; that this is so may readily be appreciated by considering the exaggerated sketches of figures (62) and (63).

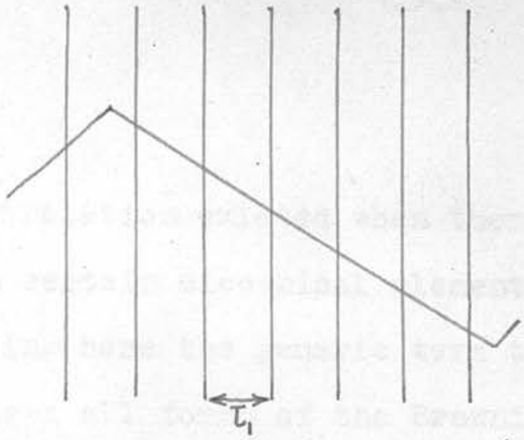


Figure (62)

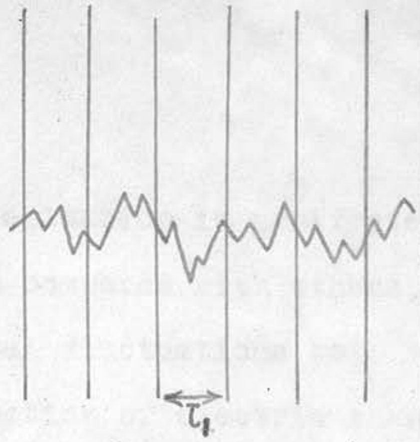


Figure (63).

It is clear that in (62) $v_{\tau_2} = 2v_{\tau_1}$ throughout except at the (relatively infrequent) "turning points" of the record, and ∴ in this case $\Delta_2 \doteqdot 2\Delta_1$ (i.e. $\delta_2 \doteqdot 2$). On the other hand in (63) Δ_2 will be little different from Δ_1, since the "turning-points" are relatively frequent compared with the fundamental interval of observation τ_1, i.e. in this case $\delta_2 \simeq 1$. Intermediate cases will, of course, be represented by the transition of δ_2 from 2 to 1.

We further remark that in the writer's opinion some further evidence of this effect exists in a visual examination of the actual fluctuation records (see for comparison Figures 40 and 41) and that this was observed in fact before the detailed numerical examination was carried out.

It must be observed that on the strict basis of the theory as developed by Rice, other writers and in this thesis, the correlation is solely determined by the macroscopic parameters of the system, such as σ, τ, etc. The fundamental events in the theory are simply regarded, whatever the prime source, throughout as entirely independent, instantaneous, 'shocks' to the system. If the possible indication here is later confirmed one would have to recognise that perhaps some greater degree of correlation existed when thermal agitation is manifested in certain electrical elements as compared with others, using here the generic term thermal fluctuations to cover all forms of the Brownian motion of electric charge. We should, however, re-emphasise that it is certainly not suggested that this would account for the very significant general "space-charge reduction" of valve fluctuations, this point having been particularly examined in the Appendix to Chapter 2. It <u>might</u>, however, be contributory, for example, to the observed "rounded-off" progression of Γ^2 to its limiting value under saturation conditions - rather than a relatively sudden transition - as indicated, to some extent, by the observations on the photo-electric cells partly directed to this end.

In closing this section, we emphasise that we well appreciate that if such small differences in are to be ultimately confirmed great care would have to be exercised in controlling the experiment under the two conditions to ensure that no extraneous disturbing effects could appear. In particular, of course, it would be very desirable to try to increase the band-:width employed, although this of course will com-:plicate the recording problem as discussed in the Introductory Survey to this Chapter.

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5. 5 : Analysis of detected fluctuation passed through filter of limited bandwidth.

The theoretical analysis of the passage, through a filter of limited response, of the output from a diode detector fed with the fundamental fluctuation was briefly considered, since facilities existed on the Marconi Receiver employed for such experimental records.

We assume here simply that the detector follows the envelope, $R(t)$, of the input. Since we already have an expression for the product correlation of we may derive the power spectrum of $R(t)$ by the theorem of Khintchine and Wiener discussed in the Appendix to Chapter 2.

I.e. :

$$W(f) = 4\int_0^\infty \psi_\tau . \cos 2\pi f\tau . d\tau \quad \text{and} \quad \psi_\tau = \int_0^\infty W(f) . \cos 2\pi f\tau . df$$

(see also Rice, Paper A., page 312).

where :

$\psi_\tau = \overline{R(t) . R(t+\tau)}$, where the bar indicates time-averaging, or, because of the stationary character of the fluctuations :

$$\psi_\tau = \int_0^\infty \int_0^\infty R_1(t) . R_2(t+\tau) \, p(R_1 R_2 \tau) \, dR_1 dR_2$$

Then following Burgess (see equation (47)), we derive :

$$\psi_\tau \doteqdot \frac{\pi}{4} \overline{R^2} \left\{ 1 + \frac{\varepsilon^{-4\sigma^2\pi^2\tau^2}}{4} \right\} \quad \text{(omitting higher order terms)}$$

$$\therefore W(f) \doteqdot \pi \overline{R^2} \left[\int_0^\infty \cos 2\pi f\tau . d\tau + \frac{1}{4} \int_0^\infty \varepsilon^{-4\sigma^2\pi^2\tau^2} . \cos 2\pi f\tau . d\tau \right]$$

Using then the expression discussed in the Appendix to

Chapter 2 for the first (undefined) integral, namely :
$\int_0^\infty \cos 2\pi f\tau \, d\tau = \frac{1}{2}\delta(f)$ (see also Rice, Paper A, p.314).
we have :

$$W(f) = \pi \overline{R^2}\left\{\frac{1}{2}\delta(f) + \frac{1}{16\sigma\sqrt{\pi}}\varepsilon^{-f^2/\sigma^2}\right\}$$

This indicates a power spectrum consisting of a d.c. term (represented by the impulse function) and a continuous Gaussian term.

The low-frequency filter available in the set had a practically Gaussian response (see Figure 64) centred about 1,000 $^c/_s$ with no response to d.c. Thus the correlation of the output fluctuation is given by :

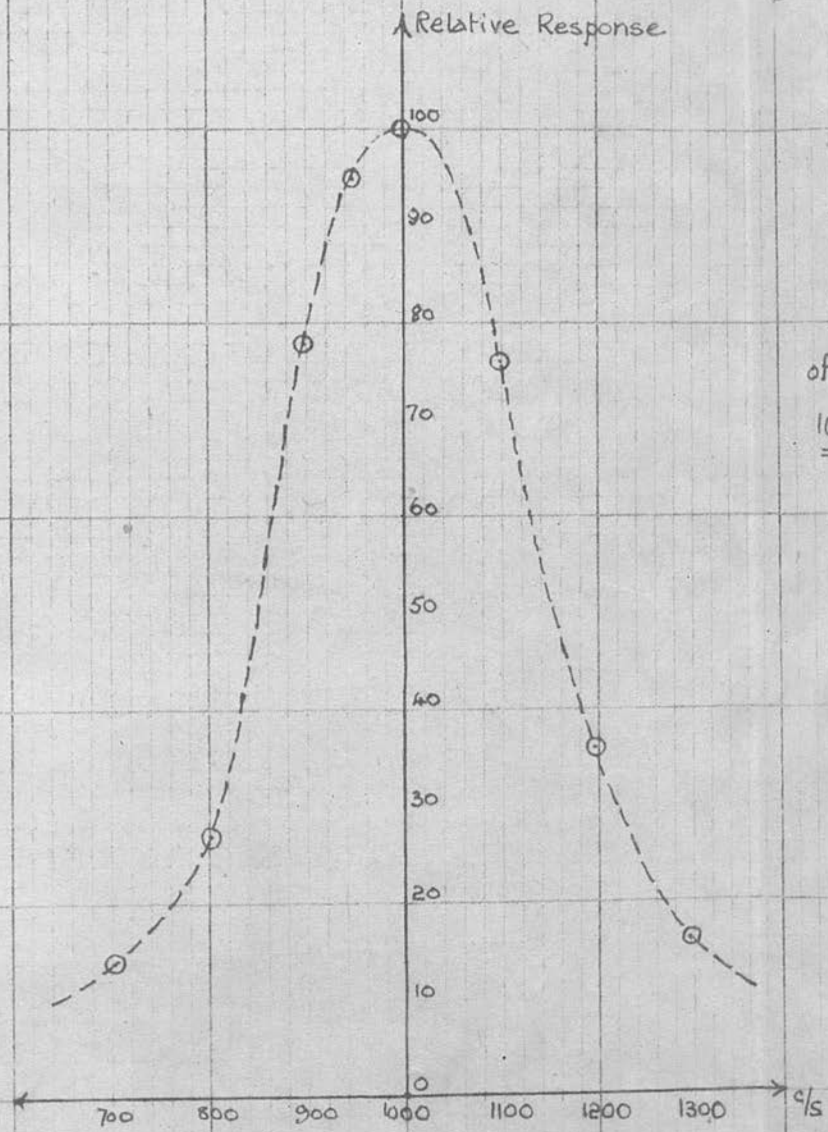
$$\Psi_\tau = \int_0^\infty W_1(f).W(f)\cos 2\pi f\tau \, df \quad \text{, where } W_1(f) = \frac{\psi_1}{\sigma_1\sqrt{2\pi}}.\varepsilon^{-\frac{(f-f_1)^2}{2\sigma_1^2}}$$

$$= \frac{\psi_1}{8\sqrt{2}\sigma_1\sigma}\int_0^\infty \varepsilon^{-(f-f_1)^2/2\sigma_1^2}\varepsilon^{-f^2/\sigma^2}\cos 2\pi f\tau \, df \quad (\text{and } f_1 = 1{,}000\,^c/_s)$$

In this case $\varepsilon^{-f^2/\sigma^2}$ may be considered as practically constant over the significant range of $\varepsilon^{-(f-f_1)^2/2\sigma_1^2}$, and we thus find readily :

$$\Psi_\tau \simeq \frac{\psi_1\sqrt{\pi}}{8\sigma}.\cos 2\pi f_1\tau.\varepsilon^{-2\pi^2\sigma_1^2\tau^2}.$$

We thus see that in this case the output will be a wave of sinusoidal character at 1,000 $^c/_s$ with an envelope fluctuating slowly at a rate solely dependent on the nature of the low frequency filter, and amplitude dependent on both filters. Further detailed numerical work was not embarked upon, as it appeared that no new results would be forthcoming; three photographs of records obtained are presented as Figures 65, 66 , and 67 . These are also of interest since they show a very strong analogy with the records obtained by



Relative Amplitude Response

of "Audio"-frequency nominal 100 c/s filter in Receiver (ii)

[$\sigma = f_0 - f_m / 1.66$

$f_0 - f_m = 151$ c/s

$\therefore \quad \sigma = \frac{151}{1.66}$ c/s

$\doteqdot 91$ c/s]

Figure: 64.

Plate: XII : following p.132.

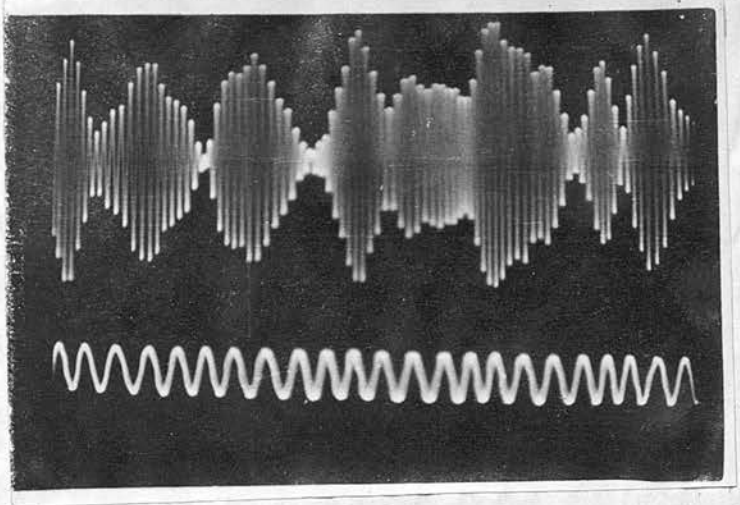


Fig.65: Rectified noise through low-pass filter.
Timing wave 250 c/s.

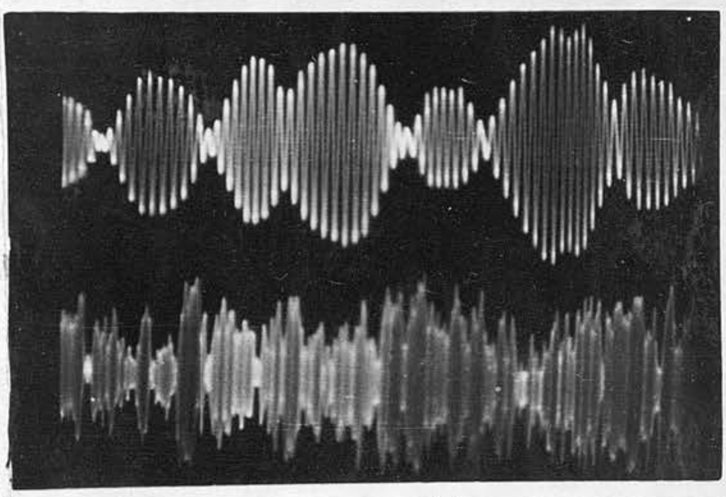


Fig.66: Rectified noise through low-pass filter with generating i.f. noise.

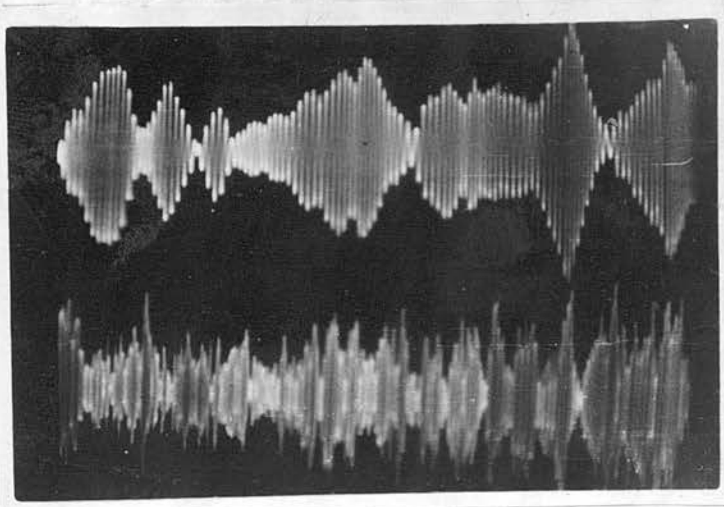


Fig.67: Rectified noise through low-pass filter with generating i.f. noise.

Kappler with very light damping on his mechanical system.

SUMMARY.

In this thesis we have studied a number of problems of electrical fluctuations whose common initial origin lay in the fundamental significance of "shot" and "thermal" fluctuations.

First, a theoretical and historical study was presented of the space-charge reduction of fluctuations in thermionic valves leading to a unification of the concepts of shot and thermal noise therein. In particular we observed that the common origin becomes most obvious when retarding field conditions obtain; a theoretical and experimental study of the retarding field region was therefore then undertaken. This included measurements to investigate and confirm the valve characteristics in that region and experimental observation of the fluctuations under true retarding conditions. The criteria relevant to entry into the region were also examined. As a by-product of the experimental fluctuation measurements, work was also undertaken on photo-electric cells and an interesting field of investigation is indicated by the results obtained.

Finally, detailed statistical examination was presented of fluctuation records with relevant derived theory in which satisfactory agreement was obtained; this provided also experimental confirmation of previously known theoretical results. A possible

indication of discrimination was observed between noise generated by a valve working under saturation conditions and that generated by thermal agitation in a tuned circuit such that it might possibly be accounted for by a certain degree of interaction between the primary events in the latter case.

We should emphasise however, particularly in view of Chapter 2, that we are convinced that all these phenomena should be classed as thermal agitation, the terms "shot noise " and "thermal noise" being employed solely as a convenient classification indicating the electrical element wherein the fundamental phenomenon is manifested.

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