A STUDY OF THE LATERAL STRENGTH OF BRICKWORK PANELS WITH OPENINGS

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To the memory of my father

ABSTRACT

Brickwork panels are often required to resist out-of-plane lateral loading due to wind. In many cases these panels contain window openings. Although extensive research has been done on the lateral strength of walls without openings and the bending moments coefficients have been given in the British Standard Code of practice for the use of masonry, little is known about the behaviour of brickwork panels with window openings subjected to wind loading.

The work described in this thesis, therefore is mainly confined to the lateral strength of unreinforced brickwork wall panels with window openings subjected to uniformly distributed loading simulating the wind pressure. The experimental work was carried out on half-scale single-leaf rectangular panels having window openings. The investigation relies on extensive experimental work to gather data on the behaviour of unreinforced brickwork at cracking and at failure. Tests done on 160 wallettes, 24 cross-beams and 16 walls are presented in Chapters 3, 4 and 5. The material properties were defined from the prisms and wallettes tests. The cross-beams were tested to obtain the behaviour of masonry in bi-axial bending

A moment interaction diagram at cracking in bi-axial bending has been presented which is subsequently used to predict the experimental cracking pressures of the cross-beams and panels. An apparent increase in the flexural tensile strength perpendicular to the bed-joints was found in bi-axial bending compared to the ultimate flexural tensile strength in uni-axial bending. The proposed cracking criterion, when combined with the finite element analysis for orthotropic plates, predicts cracking pressure reasonably well for the walls having window opening.

The strip method and yield-line analysis, both, were used to predict the ultimate failure pressure of the panels. Extensive theoretical yield-line solutions for the panels of various boundary conditions containing window openings subjected to wind loading have been presented in Chapter 5. The yield-line method seems to give good correlations with the experimental results. The strip method overestimates some of the test results, hence can not be recommended for the design of brickwork panels containing window openings.

The failure pressures for the tested panels were also obtained by using the equations presented in the Chapter 5 in conjuction with the flexural tensile strength values given in BS 5628 British Standard Code of practice for use of masonry and compared with the test results. The predicted pressures in all cases are lower than the experimental results, hence the yield-line method may safely be recommended for the design of such panels.

DECLARATION

This thesis is the result of research work undertaken in the Department of Civil Engineering and Building Science at the University of Edinburgh, for the degree of Doctor of Philosophy.

I declare that all the work in this thesis has been carried out by myself unless otherwise stated, and the thesis has been composed by myself under the supervision of Dr. B. P. Sinha.

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NOTATION

a	distance between the supports and the line loading
d	maximum deflection
E	modulus of Elasticity
E_{yi}	initial tangent modulus of brickwork perpendicular to the bed joints (vertical direction)
E_{ys}	secant modulus of brickwork perpendicular to the bed joints
f_{cx}	compressive strength of brickwork parallel to the bed joints
f_{cy}	compressive strength of brickwork perpendicular to the bed joints
f_{tx}	flexural tensile strength parallel to the bed joints
f_{ty}	flexural tensile strength perpendicular to the bed joints
L_x	length/horizontal span
L_y	height/vertical span
I_x, I_y	second moment of area
G	shear modulus
m	moment per unity width perpendicular to the bed joints
M_d	design moment
M_x	moment along the horizontal direction
M_y	moment along the vertical direction
M_{xu}	ultimate moment along the horizontal direction (parallel to the bed joints)
M_{yr}	residual moment in the vertical direction after cracking
M_{yu}	ultimate moment along the vertical direction (perpendicular to the bed joints)
m_{xy}	torsional moment
P	point load
P_c	cracking load
P_x	total reaction in the horizontal direction
P_y	total reaction in the vertical direction
P_u	ultimate point load

Z	section modulus
w	uniformly distributed load
w_x	uniformly distributed load supported by the horizontal strips
w_y	uniformly distributed load supported by the vertical strips
$\alpha, \beta, \lambda, \gamma$	factors
δ	unit displacement due to the applied load
ε_{long}	longitudinal strain
ε_{lat}	lateral strain
θ_x	angle of the yield-line with the horizontal axis
θ_y	angle of the yield-line with the vertical axis
γ_m	material partial safety factor
γ_f	partial safety factor for load
μ	strength orthotropy (f_{tx}/f_{ty})
ν	Poisson's ratio

CONTENTS

CHAPTER 1

1.1 INTRODUCTION

Wind loading is resisted by the external walls in a masonry building by bending and transferred through floor slabs to cross and shear walls. In the old buildings the resistance to wind forces was provided by the massiveness of their construction, which did not pose any problem for their lateral resistance. The problem of the lateral resistance of these walls appeared only with the development of modern thin masonry-wall construction. In framed structures, the masonry walls tended to be relegated to the relatively secondary role of providing cladding to the buildings. With the development of tall buildings and the need to reduce dead weight, the non load-bearing masonry walls have became thinner and have to be designed to resist lateral loading by flexural action. Hence, a rational assessment of the resistance of masonry walls subjected to lateral pressure is necessary, since we can not depend on massive wall construction with their inefficient use of material and space.

The prediction of failure load of brickwork wall panels, simply supported on two opposite sides offers no difficulty. In a panel simply supported on top and bottom the failure in bending happens when the bond strength between the brick and the mortar in the bed joint is exceeded. In the horizontal and strongest direction, failure is much more complex. It involves bond strength in perpendicular joints, mortar or brick strength in tension and torsional shear in bedjoints where bricks overlap. The problem increases in complexity when brickwork wall panels span in two directions as a plate. As these panels are statically indeterminate structures, it is very difficult to find the distribution of moments in the main orthogonal directions. The analytical methods that have been used till now can be classified into three categories:

- empirical, like the Strip Method;
- elastic plate bending, using mainly the finite element analysis; and
- plastic, like the Yield-line Method.

In fact, all these methods can be classified as empirical methods when applied to unreinforced brickwork, since they have been used more to fit experimental results rather than used to explain the behaviour of brickwork in bi-axial bending.

Rationally, the elastic plate bending theory should be able to predict initial cracking load and also ultimate load if there is no reserve of strength after initial cracking. But this method does not explain the markedly non-linear behaviour and also the considerable reserve of strength after initial cracking of some type of wall panels. Beside, the method seems to underestimate not only the failure, but also the cracking pressure of some test walls.

In yield-line theory, the material is idealized as rigid and perfectly plastic. This means that the structural element made of this material does not deform till yielding and deforms plastically carrying the constant moment after yielding. As brickwork is a brittle material this assumption can not be maintained. Cracks have no tensile strength and can not resist constant maximum bending moment. Nevertheless, as the crack patterns observed at failure resemble very much like yieldline patterns and the method has been predicting ultimate test pressures reasonably, it has been used for the design of brickwork solid walls.

A rational assessment of the lateral resistance of masonry brickwork wall panels is still in question, and is the main aim of this investigation to contribute to elucidate its behaviour in two-way bending. In this thesis an extensive experimental work to measure the properties of brickwork in lateral bending was performed. New testing procedure are presented, gathering new information about the load distribution on statically determinate brickwork structures, providing a firm basis for understanding the flexural behaviour of unreinforced brickwork wall panels spanning in two directions. Although the lateral load tests were performed only on wall panels having openings, the findings of this investigation are applicable to all types of brickwork walls.

A literature review is presented in Chapter 2, to summarize all the previous work done on determining the flexural properties and lateral load behaviour of unreinforced brickwork panels. The experimental work is presented in Chapters 3, 4 and 5. Chapter 3 presents the tests performed on wallettes and small specimens to determine the mechanical properties of brickwork required for the analysis in this investigation. Chapter 4 describes the tests performed on cross beams, in order to study the distribution of load and the cracking criterion for brickwork in bi-axial bending. Lateral load test on half-scale walls with window openings are presented in Chapter 5, with theoretical analysis of the results and a comparison between the experimental and theoretical pressures. Theoretical work include the application of the finite element method for orthotropic plates, development of equations for yield-line analysis for panels containing window openings and application of strip method. The results show that the yield-line equations may be used to predict the flexural strength of the tested panels. An output from the computer program, some of the finite element meshes used in this analysis and an example how to use the cracking criterion are presented in the Appendices.

For convenience, the tables, the drawings and the photographs are inserted into their respective chapters.

CHAPTER 2

LITERATURE SURVEY

2.1 GENERAL

It seems that bricks have been used widely for more than 5000 years. Brickwork or stonework provided the structure of the most important buildings built in the past, and the lateral stability was provided by the mass of the structure. Rule-of-thumb methods dominated the art of construction in masonry and this situation was no longer acceptable due to the rapid progress of the technology of construction of steel or concrete framed buildings, in which thin brickwork was used as cladding panels. Recent years masonry structures have been built with thin walls based on the same design principles as steel or concrete. Hence, there has been urgency to investigate the lateral resistance of masonry walls, often not required previously in traditionally built structures.

In the following section the work done in the past will be presented in a chronological order. Emphasis will be given to the work done in the later years, because some good literature review was presented in work done till 1982.

2.2 HISTORICAL BACKGROUND

The first lateral load tests on unreinforced masonry were done at the National Bureau of Standards in Washington, D.C., in 1925-1926, by Stang, Parsons and Foster[1]. Research in America was continued by Kelch[2] in 1931, Richart, Moorman and Woodwath[3] in 1932. All these tests with brickwork or blockwork were involved mainly with the flexural behaviour in the vertical direction, examining the effects of the properties of bricks, blocks and mortar.

In 1936, Royen[4] published a paper suggesting that the horizontal moment of resistance could be calculated by summing the individual torsional moment of resistance of each overlapping bed joint along the failure line.

The influence of using bricks with a low suction rate with mortars of a high water retentivity on the interface bond strength was presented in two papers in America by Whittlemore, Stang and Parsons[5] in 1938 and Parsons[6] in 1939.

In 1939, Plummer and Reardon[7] presented correlations between the tensile bond strength and the suction of the bricks, the initial flow and the water retentivity of the mortar. They concluded that the tensile bond strength decreased as suction of the bricks increased but increased with the increase in initial flow and water retentivity of the mortar.

Unreinforced masonry walls spanning in two directions were first tested at the Commonwealth Experimental Building Station in Australia, in 1947, by Tasker[8], and he found that the behaviour of the two test walls were markedly plastic. In 1948, Isaacs[9] tried unsuccessfully to apply the yield line method in an attempt to analyse the flexural behaviour of the two walls tested by Tasker. In the same year, Nerlich[10] applied the elastic method also to analyse the behaviour of masonry walls under lateral loading. Both researchers were concerned with the method of design of such masonry walls.

In 1950 and 1952 two papers were presented by Davey et al[11] and Davey[12] giving the test results of laterally loaded masonry wall panels. They observed that the walls at failure presented a pattern of cracks similar to the yield lines which develop in a reinforced concrete slabs at failure. They also mentioned that some walls developed increased ultimate lateral load resistance due to "dome" action.

In 1952, Hummel[13] pointed out the importance of compressive in-plane forces in increasing the vertical tensile flexural strength of masonry walls.

In 1953, Thomas[14] confirmed that the crack patterns at failure of wall panels were similar to yield lines and pointed out that the increase of ultimate lateral load resistance is due to the "dome" action. He tested full size single and cavity walls applying the load by means of hydraulic jacks at 16 points. In the same year, cavity walls were tested by Goalwin[15], who found that the two leaves did not work together due to failure of the bond of the tie.

In 1954, Monk[16] presented one of the most important reports which identified the effect of the properties of bricks and mortar and the workmanship on the flexural behaviour of masonry walls. Brick properties, which affected the strength were: suction rate at the time of laying, water absorption, surface roughness, texture and coring. Similarly, the mortar characteristics which affected the strength were flow, water retentivity, workability, curing and age. The workmanship factors were considered such as filling of head joints, non-furrowing of bed joints and tooling of joints. He also performed lateral load field test on full scale walls with and without openings

Mc Dowell, Mc Kee and Sevin[17], in 1956, tested vertically spanning wall panels built between rigid supports and proposed an arching action theory for laterally loaded masonry walls subjected to precompression. According to them the wall derives the lateral resistance due to internal forces built up as a consequence of the crushing of the material at midspan and at the two end supports. The tensile resistance of masonry in flexure, was ignored. The theory predicts very conservative pressures when compared with the experimental results. Cohen and Lang[18] modified this theory by a simplified method based on a plastic stress distribution utilising a rectangular stress block.

In 1958 three papers on the lateral strength of masonry walls were presented. Cox and Ennenga[19] stated that the modulus of rupture in horizontal bending was larger than in vertical bending for concrete walls. Benjamin and Williams[20] pointed out that the flow and water retentivity test for mortar were the most important factors for the interface bond strength between bricks and mortar. The water cement ratio plays a secondary role. Allen[21] presented a comprehensive bibliography on lateral load resistance of unreinforced masonry walls.

Hedstrom[22] performed a series of tests in 1961, which investigated the lateral load behaviour of masonry panels built in stack and running bond, and calculated the modulus of rupture by averaging the tensile strength of brick and mortar.

In 1962, Falconer[23] suggested moment coefficients for laterally loaded wall panels spanning in two directions,using elastic theory, for the New Zealand Building Code for masonry. The code recommended that walls with openings should be designed as several strips spanning in one direction only, with the strips bordering the openings.

The Structural Clay Products Research Foundation[24,27,28,29,31,35] started a research program in masonry in 1964, in which tests in small specimen and full-size wall panels were done. The reports presenting results of lateral loading of brickwork were published between 1964 to 1969. Among some of the conclusions of this research program, it was stated that the bond strength is not directly influenced by the compressive strength of mortar, the modulus of elasticity of brickwork in flexure and compression is approximately the same and the lateral strength of brickwork decreases as the bed-joint thickness increases.

Two investigations were published in 1965 by Krone and Pollitz[25] and Bradshaw and Entwistle[26]. The formers proposed some charts to design masonry walls spanning in two-directions assuming that the joints were able to support only shear stresses, due to the imposed vertical stresses. The latter proposed permissible tensile stresses for brickwork in both directions. In the vertical direction the maximum stress up to 0.07 MPa was suggested and in the horizontal direction up to 0.14 MPa. They also suggested site tests to measure the flexural strength of brickwork in both directions. An approximate method of design was proposed based on bending moment coefficients obtained from the elastic theory, similar to those used for reinforced concrete slabs spanning in two directions with torsional resistance.

In 1966, beside the two reports already cited[28,29], the Norwegian Building Research[30] Institute performed some vertical and horizontal flexure tests on small specimens. It confirmed the earlier findings that the initial rate of absorption of the bricks had strong influence upon the flexural strength of brickwork, i.e., bricks with either very high or very low initial rate of absorption resulted in low flexural strength.

In 1967, three papers were presented at the International Conference on Masonry Structural System in Texas. Youl and Foster[32] showed the importance of the use of workable consistency for the mortar to get high flexural strength of brickwork. They also confirmed the importance of the use of mortar with high initial flow and high water retentivity in order to provide brickwork with higher flexural strength. Greenley[33], working with special mortars, obtained high flexural bond strengths (up to 2.7 MPa) and Isberner[34] called attention to the importance of curing of masonry for the compressive and flexural strengths of masonry. The latter showed that brickwork specimens cured in wet conditions had considerable higher tensile flexural strength than specimens cured in air.

In 1969, both in America[36] and Australia[37], the permissible stress method was proposed for the design of masonry walls subjected to lateral loading. A 33% increase in permissible stress was proposed by Monk in America, if the stresses were solely due to wind.

In the CIB Symposium, held in Warsaw in the same year, Losberg and Johansson[38] presented a report about lateral load tests on masonry wall panels. Although they agreed that the observed crack lines were not true yield lines, they supported the use of the yield line method because of good correlation between the predicted and experimental pressures. In the same symposium, Hallquist[39] presented more work done at the Norwegian Building Research Institute, supporting the use of orthotropic elastic plate theory as a design method to predict the first crack, with suitable factors of safety.

Later, in the same year, Francis[40] presented a paper recommending the same moment coefficients used by Bradshaw and Entwistle[26] as a design guidance for the Australian Code.

In 1970, Nilsson and Losberg[41], from Sweden, pointed out that the cracking load of unreinforced and reinforced panels can be predicted by the elastic theory. They suggested the use of yield line theory for the prediction of ultimate strength.

Hendry, Sinha and Maurenbrecher[42] carried out lateral load tests on cavity brickwork walls, with and without returns. They showed that the strength of strip walls with precompression can be calculated by the simple arching theory. They also presented a simple analysis to take into account the effect of returns on the lateral strength of walls, provided that the L/H ratio is greater than 0.75, but for aspect ratios lower than this the actual strength of a wall is less than that indicated by this theory.

The majority of the work published in 1971[43,44,45] were concerned with the behaviour of laterally loaded wall panels with vertical precompression. These papers dealt with ultimate failure pressure and the method used in most of the cases was the yield line theory.

Baker[46,47], from Australia, published two works on flexural behaviour of unreinforced brickwork. Working with small specimens, he found a linear load deflection relationship almost to failure for wallettes bending in the vertical direction. Wallettes bending in the horizontal direction had the load-deflection curve divided in two sections: an initial linear section followed by another linear section of reduced stiffness. He also performed lateral loading test in wall panels and found that elastic theory provided good estimates of the cracking loads of some panels, but always underestimated the failure loads. Yield line theory consistently overestimated the failure loads. Baker, then, applied the strip method[48] to his experimental results and concluded that this empirical method corresponded better than all the previous ones.

Also in 1972, at the University of Edinburgh, Satti[49] tested small specimens and brick walls. He found that the ratios of orthogonal strengths of wallettes varied approximately from 3 to 9 (there were great variability in his results). Elastic and yield line theories were applied to analyse the experimental failure pressures of the panels. He concluded that elastic theory corresponded well with the experimental failure pressures. His investigation was reworked later by Hendry[50], who found that mistakes were made with the application of this theory, and both, in fact, underestimated the experimental pressures. Hendry suggested that his experimental results were better predicted using moment coefficients based on a horizontally spanning strip. He also pointed out the main difficulties in an elastic analysis approach for laterally loaded walls due to non-linear behaviour of the test walls, the uncertainty involved in obtaining the elastic constants and the lack of knowledge about the criterion of failure of brickwork in bi-axial bending. In the same paper, he presented a comprehensive review of the work done in the United Kingdom about flexural behaviour of brickwork, calling attention that more information was necessary on the following areas:

a) the theoretical behaviour of non-loadbearing panels supported on three and on four sides;
b) the relationship between brickwork flexural strength and wall strength;

c) the effects of openings;

d) the effects of continuous support; and,

e) the influence of practical support conditions such as damp-proof courses.

Morton[51] presented a Ph.D. thesis dealing with lateral pressure due to gas explosion. But his work was concerned only with precompressed brickwork wall panels.

Hellers and Sahlin[52], from Sweden, carried out lateral loading test on masonry foundation walls in 1972. They concluded that the elastic theory predicted the cracking loads and the reserve of strength could be calculated on the basis of the yield line theory. Also from Sweden, Magdalinski[53], in the same year, presented comparisons between elastic and yield line theories. He supported the use of both methods. The elastic method was only considered suitable in cases where the cracking pressure agreed with the ultimate pressure. Due to the flexibility of the yield line method, he also suggested it for the design of walls with openings.

Lindsay[54] presented, in 1973, a review of Isaac's work with the yield line method for two-way-spanning wall panels. He recommended the use of elastic analysis for the design of laterally loaded unreinforced masonry walls, using permissible stresses twice higher than those prescribed by the Australian code.

Baker presented two papers in Australia, also in 1973. The first, with Base[55], reported tests on small specimens of brickwork. The second[56] reported lateral load tests on brickwork walls and recommended the strip method for the design of such walls.

The British Ceramic Research Association started a program investigating the flexural behaviour of brickwork. The first results were published in 1973, at the Third International Brick Masonry Conference, by West, Hodgkinson and Webb[57], and Haseltine and Hodgkinson[58]. Tests on small specimens and wall panels with different boundary conditions and materials were performed and it was found that both theories, elastic and yield line, underestimated the failure pressures.

At the same conference, three more papers were presented by Cajdert and Losberg[59], from Sweden, and by Baker[60]. The former performed lateral load tests on unreinforced masonry walls and pointed out that elastic theory was in good agreement with the cracking pressure. Yield line theory was also applied averaging the orthotropic strength by considering the two orthogonal directions and an inclined direction, but no justification was presented for this new averaging of orthogonal strength. When applied in the ordinary way, yield line analysis overestimated the experimental failure pressures. Baker presented some results of his thesis published earlier. Satti and Hendry[61] reported tests on the modulus of rupture of brickwork. They tested specimens with the bending plane inclined to 0^o, 45^o and 90^o to the bed joints.

Baker[62,63] presented two more papers in 1974. In the first paper, he tested brickwork wall panels supported on four sides and insisted in his previous observations that cracking pressures are well predicted by the elastic theory while ultimate pressures agreed closely with the strip theory. In the second paper, he applied statistical formulation to demonstrate that the flexural tensile strength of brickwork is dependent of the number of joints in the span and the shape of the applied bending moment diagram. He analysed some of his previous experimental results[46] by computer simulation and found good agreements.

The International Symposium on Load Bearing Brickwork, held in 1974, had several papers dealing with lateral load behaviour. Sinha and Hendry[64] pointed out that the orthogonal strength ratio was more dependent of the vertical flexural tensile strength than the horizontal flexural tensile strength, as the former presents a lot more variation. Due to this variation, the orthogonal flexural tensile strength could not be a constant as recommended by the British Standard CP111.

West and Hodgkinson[65] and Haseltine[66] reported the research program of the British Ceramic Research Association and, based on lateral load tests carried out on small specimens and full-scale walls, proposed a design method for non-loadbearing unreinforced masonry walls. They compared the experimental failure pressures with predicted pressures obtained by elastic and yield line theories. Yield line theory gave better predictions of the failure pressures and was found conservative, hence it was suggested as design method. Although Haseltine did not explain the real statical behaviour of laterally loaded masonry walls and provided no justification why the yield line theory gave so good agreement, he pointed out the practical advantages of this approach for the design. It was also pointed out that the flexural tensile strength was dependent on the total water absorbed by the bricks during 5 hours of boiling test. A good relationship between the length of storey-height walls and the logarithm of the lateral load resistance between the range of length varying from 2.44 to 5.50m was found. This relationship did not work so well for walls 1.52m long.

In 1974, Lawrence and Morgan[67] presented their test results of the strength and modulus of elasticity of brickwork in two directions. Tests were done as simply supported beams with three or four line loadings. They observed the same change in stiffness of the load deflection curve as found by Baker.

James[68], in 1975, in Australia, carried out lateral load tests on small specimens and wall panels. He also obtained variation of the orthogonal strength ratio. It was reported that the strip method gave good correlations with the experimental failure pressures for walls supported on four sides, but underestimated the results for walls with one free edge.

In the same year, Lawrence[69] suggested an empirical relationship between the flexural tensile strengths of brickwork in two directions.

Kheir[70] presented his thesis in 1975 investigating the flexural behaviour of cladding brickwork walls. He found that the yield-line method gave better correlations between the experimental and predicted failure pressures. Elastic theory and strip method were also applied to predict experimental pressures.

Sinha and Hendry[71] presented a paper in 1975, which was concerned mainly with lateral load test on bearing walls with analysis of results. Assuming that the bottom and top supports provided moments of resistance of aproximately 25% and 60%, they found that yield line theory agreed with the experimental results.

In 1975, the Swedish Building Code for Masonry[72] recommended that the lateral strength of masonry may be calculated by means of two alternative methods: elastic plate theory or ultimate state design. Arching action in both direction was also allowed. Isotropic properties were allowed as long as they were considered safe.

During 1975, Cajdert and Losberg[72] presented a paper justifying the use of yield line theory for the design of unreinforced masonry walls. The reasons given were: i) there is possibility to have some moment redistribution at the joints due to its "plasticity"; ii) eccentric compressive forces can be developed in a cracked wall supported on three sides, giving some moment resistance across the cracked section.

Mayers and Clough[74] presented a survey of literature on masonry, including for tensile and bond strength, in 1975.

In 1976, at the Fourth International Brick Masonry Conference, four papers concerned with the flexural behaviour of masonry were presented. Hendry and Kheir[75] performed lateral load tests on wall panels built with one-sixth scale model bricks and analysed the experimental results using elastic, yield line and strip method. The yield line approach, using the moduli of rupture determined in test on small specimens, provided the best correlation with the experimental pressures. Although they pointed out that there was no rational justification for the application of this theory, they suggested its use as it always underestimated the ultimate pressure.

Baker and Franken[76] carried out tests on small specimens and, using statistical approach, suggested that the flexural strength of a specimen is dependent of the number of bed joints in the span, the number of units in the width of the specimen and the distribution of the applied bending moments, i. e. , the way the loading is applied on the specimen.

Hodgkinson, West and Haseltine[77] performed lateral load tests to assess arching action in order to incorporate this in design. To obtain full arching, they built a stiff reinforced concrete frame, strong enough to withstand a horizontal load of 30 tons per unit metre. West[78] also presented a paper dealing with the flexural strength of small specimens.

Anderson[79], at the Polytechnic of South Bank , performed lateral load tests on blockwork walls. He stated that walls simply supported at the vertical edges with the upper edge free had little reserve of strength after initial cracking, with the exception of the wall with a length to height ratio of two. Walls with the vertical sides built in had significant reserve of strength after the initial cracking. Anderson justified this behaviour due to arching action. In the same year, Anderson and Bright[80], reported more lateral load tests on blockwork walls built in a special steel frame, bolted on the floor and restrained laterally. Again the walls were supported on three sides only with the upper edge free. They compared these results with the previous tests and concluded that the cracking pressures were unaffected, but the ultimate pressure were increased, in some cases, to more than twice due to arching action.

The Australian Department of Construction[81] published a design note recommending the use of the strip method, proposed by Baker[46], to calculate the ultimate strength of laterally loaded masonry walls. Although some researchers[61,64] have called attention to the great variation in the orthotropy strength, a ratio of 3 was recommended for the design purposes.

Four papers reporting work done at the British Ceramic Research Association were presented in 1977: West, Hodgkinson and Haseltine[82,84] and Haseltine, West and Tutt[83,85], summarised the contents of the work done previously at the British Ceramic Research Association, which formed the basis of the amendments to the Britsh masonry code[86] concerning lateral load design of unreinforced brickwork walls. The first reported tests on small specimens and on full-sized masonry walls. It was recommended that the characteristic flexural tensile strength should be considered dependent on the water absorption of the units and, to some degree, to the composition of the mortar, which was contrary to the findings of others. The second, based on the results presented by the first, compared the experimental failure pressure with failure pressures predicted by the elastic and yield line theory. The authors recommended the use of moment coefficients based on the yield line theory for the design of unreinforced masonry walls. It was also suggested that walls with openings should be designed by splitting the panel into a number of sub-panels, making the calculation easier.

Baker presented three papers at the Sixth International Symposium on Load Bearing Brickwork. The first paper[87], supported by experimental work investigating the flexural strength of vertically spanning panels, suggested four possible criteria for failure of brickwork in vertical bending: brittle, successive cracking, partially plastic and fully plastic. The brittle criterion postulates that failure may occur by brittle crack propagation when the ultimate strength of the weakest joint in the bed joint is equal or smaller than flexural tensile stress. The successive cracking criterion means that ultimate failure may occur only after successive cracking of the weakest of the bed joints, eventually resulting in an overall reduced load capacity. The partially plastic criterion suggests that the ultimate strength of the bed joint is reached only after all points in the bed joint have plastically achieved their ultimate strength. Finally, the fully plastic criteria prescribe that the ultimate strength of the bed joint is reached only after several adjacent joints have plastically achieved their ultimate strength. Baker concluded that the partially plastic failure criterion predicted better the mean, the coefficient of variation and the failure mechanism of panel strength in vertical bending.

The second paper by Baker[88] presented a theoretical method supported by experimental investigation for the estimation of the increase in cracking strength due to arching action.

In the third paper, Baker[89] reported a study based on visits to various research centres concerned with lateral behaviour of masonry. A subjective judgement of the relative factors that influence of secondary effects was presented by comparing the experimental and theoretical pressures. The factors identified are: scale effects, methods of loading, self weight, arching, rotational restraint at supports, translational yielding of supports and the methods used to measure the flexural properties of small specimens.

Hatzinikolas, Longworth and Warwaruk[90] also presented a paper at the same conference, introducing a new method of measuring the tensile bond strength by the use of centrifugal force. They tested the new method in ninety joints and obtained a highly variable result.

In 1978, Schoner[91], from Germany, reported lateral load tests with and without precompression. These tests were performed on walls spanning vertically and simply supported on four sides. Experimental failure pressures were compared with predicted pressures by yield line and strip method. Both methods were shown to give reasonable agreement.

Sinha[92], in 1978, presented a theory to analyse the flexural behaviour of unreinforced brickwork walls. He postulated that yield line theory overestimates the failure pressures because the loading is distributed according to the flexural stiffness in both directions. To take account of bending stiffness, the ultimate moment of resistance was divided by the ratio of the moduli of elasticity in the two orthogonal directions. The rest of yield line assumptioms were kept in his theory. Good agreement between the predicted and experimental failure pressures was obtained.

In 1978, at the North American Masonry Conference, Huizer and Ward[93] proposed a test to determine the flexural strength of masonry, and Lawrence[94] presented more data to support his empirical relationship between the vertical and horizontal flexural tensile strengths.

Nine relevant papers were presented at the Fifth International Brick Masonry Conference in 1979. Baker[95,96,97] presented three papers dealing with flexural behaviour. The first paper was an investigation about the variation of flexural strength with age. Baker pointed out that the flow of mortar had an important effect on flexural bond strength. Tests results were highly variable. The same variation was also observed in similar tests done by Matthys and Grimm[98]. The second paper by Baker reported an analysis about thirteen different types of standard tests to measure the vertical flexural tensile strength of masonry. He pointed out that these tests had different number of joints in the specimens and the loading was applied in different ways, hence, they give different values for the same material. Baker suggested that the vertical flexural tensile strength should be standardised as the strength of joints, either measured from just one type of test or derived theoretically from beam tests. In the third paper, Baker confirmed experimentally the failure criterion for brickwork in lateral bending that he had presented two years before[87]. Although he had few results and tested specimens having only single joints, he presented an interaction diagram between the failure stresses in vertical and horizontal bending. He proposed an elliptical failure criterion for combined vertical and horizontal flexural tensile stresses and pointed out that the usual assumption of no interaction between these stresses is unconservative. From his work, it was suggested that the applied compressive stress increased both the vertical and horizontal bending strength and the combined action point was also on the elliptical failure criterion. Therefore, the increase in horizontal modulus of rupture was such that the orthogonal strength ratio remained the same, with or without the applied compressive stress.

Akio, Katsuro and So[99], from Japan, studied the effect of the fineness modulus of sand on the flexural strength of masonry. They found that the use of sands with small fineness modulus tended to increase the flexural strength of masonry.

The British Ceramic Research Association presented three papers at the same conference, in 1979. Moore, Haseltine and Hodgkinson[100] performed lateral load tests on walls with continuity over supports and they observed an increase in the flexural resistance. West, Hodgkinson and Haseltine[101] also performed lateral load tests on walls with one vertical free edge and applied the yield line method to predict the experimental failure pressure, with good agreement with the experimental failure pressures. West, Hodgkinson, Goodwin and Haseltine[102] tested several walls built with calcium silicate bricks. They used untreated bricks, oven dried, docked and saturated bricks in order to study the influence of these factors on the flexural tensile strength of masonry in two directions. They found that untreated and docked bricks gave higher flexural strengths than oven dried and soaked bricks. Also, no relevant correlation was found between either compressive and flexural strength or water absorption and flexural strength. All test walls had the failure pressure compared with predicted pressures using the moment coefficients given in the British Code[86]. The failure pressure was underestimated by the code.

Lawrence[103] also presented a paper at the Fifth International Brick Masonry Conference, reporting some full scale lateral load tests. He applied elastic, yield line and strip theories to predict the failure pressures, and concluded that strip theory gives better correlations. He called attention to the importance of the stiffness of the frame because masonry wall panels are very sensitive to deformations of the supports. These deformations could have an important role on the distribution of the bending moments in the walls, affecting the ultimate pressures. An interaction between the test wall and its supporting frame was considered too complex to quantify.

Sinha, Loftus and Temple[104] reported an investigation on lateral load behaviour comparing elastic, yield line and fracture line theory. They concluded that elastic theory underestimates and yield line overestimates the experimental failure pressures. Good agreement was obtained using the fracture line theory for panels supported on four sides.

Brincker[105], from Denmark, presented in 1979 an approach to the use of the yield line theory for unreinforced brickwork walls.. Working with small specimens, he studied the influence of deformations and material properties on a section, and the forces developed in a horizontal and an oblique yield-line. Assuming that the eccentric compression forces, the self weight and applied compression forces could resist the moment along a horizontal yield line, he obtained good agreement with his experimental results. To study the strength of the oblique yield lines that followed the horizontal and vertical mortar joint, he performed combined torsion and bending tests. Controlling the deformations accurately, he obtained good stress-deformations curves showing typical elastic-plastic behaviour for brickwork. He presented equations to calculate the yield moments per unit length for both cases. Although Brincker concluded that unreinforced brickwork subjected to lateral loads has an elastic-plastic behaviour at the ultimate stage, he did not present any solution to extend his approach to walls with top free edge and the other edges simply supported.

Lawrence[106] also presented, in 1979, a literature review of lateral loading of masonry infill panels.

In 1980, Sinha[107] carried out lateral load tests on third scale brickwork rectangular panels with openings, triangular panels and octagonal panels. Good agreement was found between the experimental failure pressures and the predicted pressures based on his fractured line method presented earlier[91], though the crack patterns were different than the theoretical paths. He concluded that this method can be used for the design of laterally loaded unreinforced brickwork panels with some confidence.

Also in 1980, Cajdert[108] published his thesis dealing with lateral load behaviour of unreinforced and reinforced masonry. Following Brincker's investigation he presented a comprehensive study of joint behaviour in flexure, by using strain gauges on and at the neighbourhood of the mortar joints. He found that there is successively increasing joint plasticity, which seems to be especially enhanced from about half the cracking load and above. This joint plasticity occurs due to deformations, at or in the immediate neighbourhood of the interface between mortar and brick or block, before any visible cracking can be noticed. Cajdert also measured the elastic moduli in the horizontal and in the vertical directions, and found that they were stress dependent having their ratio of vertical to horizontal modulus reduced with the increasing stresses. Brickwork and block wall panels tested under lateral loading possess a markedly elasto-plastic behaviour. Because of that, elastic isotropic analysis underestimates the cracking pressures. Yield line analysis underestimated the experimental failure pressure and Cajdert attributed this to the action of secondary effects, such as arching, support restraint, self-weight and crack pattern deviating from the theoretical paths.

In 1981 Baker[109] presented his Ph. D. thesis emphasising a new method of analysis based on the calculation of elastic principal moments. This method, based on elastic principal moment and the use of Monte Carlo simulation, allows to repeatedly assign strengths at random for comparison with those ones previously calculated. He obtained good agreement for both; cracking and ultimate pressures. A good statistical approach was used to estimate the mean and coefficient of variation of cracking and ultimate pressures. The method is rational and is the only one that takes account of random variation in flexural strengths of brickwork, but it is burdensome to be used as a design method and still needs more comparison with test data to prove its usefulness.

In 1982, at the Sixth International Brick Masonry Conference, in Rome, several papers dealing with the flexural behaviour of unreinforced brickwork were presented. Baker[110] refined his previous theory for the analysis of principal moments,taking into account principal moments inclined to the bed joints. He also presented results of his thesis in a second paper[111], where comparison with experimental test results of 30 walls were made showing good agreement.

De Vekey, Anderson, Beard and Hodgkinson[112] presented an investigation of the test method for evaluation of the flexural tensile strength of brickwork recommended by the British Code[85] and found a high level of variability. Some suggestions related with equipments, calibration, specimen storage and test procedures were made, though no comments were extended about the theoretical methods used to predict the failure pressures.

West[113] reported an investigation about the influence of docking and draining bricks on the flexural strength, but no conclusive results were obtained, though some results were higher than the characteristic flexural strength given in the BS 5628[86].

West, Haseltine, Hodgkinson and Tutt[114] described lateral loading tests of 38 cavity walls. These walls had one leaf built with bricks and the other leaf either with a different type of bricks or concrete blocks. They compared the experimental failure pressures with the predicted pressures obtained from BS 5628[86] and by yield line theory. It was concluded that it is safe to use both methods, though they recognised that yield line theory did not have a proper theoretical basis for its use in a brittle material like unreinforced masonry. They also suggested the possibility of composite action between the two leaves of the wall, which might have an effect on enhancing the strength of some walls, but no justification was given to that.

Also in 1982 Seward[115], presented a theoretical approach to investigate the flexural behaviour of unreinforced brickwork using elastic principal moments and a method of tracing to determine the point of failure. The main difference from Baker's theory is that this method does not consider the random variation in flexural tensile strengths of brickwork. Seward claimed that his method predicts closer correlations with experimental failure pressures than yield line theory, and suggested that design coefficients given in BS 5628[85] might be unsafe in some cases.

In 1983, Lawrence[116] presented his Ph.D. thesis dealing with flexural behaviour of unreinforced brickwork walls. He carried out lateral load tests on 32 full-scale walls and compared the results with predicted pressures given by elastic plate theory, yield line theory, strip method and Monte Carlo simulation. He found that elastic theory can predict the cracking pressures. Failure pressures were overestimated by the yield line theory, underestimated by Monte Carlo simulation and reasonably predicted by the strip method. Although he recognised that strip method does not have a proper theoretical basis, he suggested that this method can be used with some confidence for the design of unreinforced brickwork walls. He also carried out tests on small specimens to determine the material properties of brickwork.

Another investigation on horizontal and oblique yield lines was done by Brincker[117] in 1984. He worked with small specimens and four combinations of materials (two types of bricks and two types of mortars) and found that torsional and bending moments were kept constant, independent of the slopes of the oblique yield lines. The bending strain at failure in an oblique yield line was assumed to be a linear function of the compressive stress applied to the specimens. As in his previous investigation, he supported the application of the yield line theory as a design method for laterally loaded unreinforced brickwork walls.

A computer program, based on a non-linear finite element technique using the bi-axial failure criterion in flexure, was presented by Ma and May[118] in 1984. Although based on a failure criterion similar to the one presented by Baker[110], they did not make use of random variation of strength of brickwork.

At the 7th International Brick Masonry Conference held in Melbourne, in 1985, seven papers were presented dealing with lateral strength of unreinforced brickwork.

Baker, Gairns, Lawrence and Scrivener[119] presented a state-of-the-art report on the flexural behaviour of masonry panels in 1985. They pointed out the need for more research, specially dealing with the behaviour of panels containing openings.

Anderson[120], in 1985, performed six lateral loading tests on vertical spanning masonry walls. The walls had a stiff return at one end and a line loading along the free vertical edge at the other end. After cracking the failed joints were repaired by injecting epoxy resin and re-tested again. Yield line theory showed to be a satisfactory method for analysing the flexural behaviour of the walls under a line loading. Anderson attributed this to secondary effects like in-plane membrane action, self-weight and partial arching action along the supports, which can induce eccentric compression forces across the cracks. A theoretical justification to quantify any of these secondary effects was not presented. The characteristic value of flexural strength of brickwork given in Table 3 of BS 5628[86] was found to be very conservative.

A study about the effect of mortar composition on the flexural strength of brickwork, using three different types of mortar, was performed by de Vekey[121]. He concluded supporting the Code[86] prohibition of the use of plasticising agents for mortar designation (i) for masonry designed to resist lateral loading.

Drysdale and Gazzola[122] found that there is no basis for relating tensile bond strength to compressive strength of mortar cubes or brick prisms. They also drew the attention to the high variability of the bond tensile strength of masonry.

Gazzola, Drysdale and Essawy[123] presented a failure criterion for blockwork subjected to bi-axial bending. This failure criterion for blockwork is similar to the failure criteria presented by Baker[110] for brickwork, i.e. it also assumes an elliptical interaction between bending moments in the two main directions. Data were obtained by testing wallettes and prisms in uni-axial bending at different orientations to the bed joints, creating different combinations of flexural tensile stresses normal and parallel to the bed joints. It was found that shear interaction with tensile stresses along the mortar joints influences very much the failure mode. Good agreement with experimental wall failure pressures was achieved.

Grimm and Tucker[124], in 1985, performed an experimental investigation of the effects of quality of workmanship, method of loading and number of mortar joints in the span on the flexural strength of masonry.

Two papers were presented in the Journal of Masonry International in 1986 dealing with flexural behaviour of masonry. Scrivener and Gairns[125] investigated the effect of humidity, temperature and exposure state on the modulus of rupture of concrete masonry specimens. Sinha and Mallick[126] performed lateral load tests on 12 brickwork walls against a concrete frame. The uniformly distributed loading due to the wind was simulated by a series of jacks applying the loading on 12 points. They prudently suggested that due to the brittle nature of brickwork cracking load should be considered as the ultimate load, specially for design purposes.

The First International Masonry Conference held in London, in 1986, had four papers dealing with flexural behaviour. Sise, Shrive and Jessop[127] , in an investigation using five different types of units (concrete bricks made of lightweight, semi-lightweight and normal weight concrete, a three-holes pressed clay brick and a ten holes extruded clay brick) and five different types of mortars remarked that joint thickness is the most relevant factor affecting bond strength. They also concluded that units properties, such as porosity, pore size distribution in the units, moisture contents and absorption, are more important on bond strength than mortar properties.

A comparative investigation of experimental techniques for determining the flexural resistance of masonry, done with six different units and four types of mortar, was performed by Fried, Anderson and Gairns[128]. The concluded that the mean of the joint flexural strength is greater than mean of the wallette flexural strength, because the joint flexural strength is the mean of a complete sample of joints and the wallette flexural strength is the mean of a sample of the weakest joints of the wallettes. They also recommended conversion factors for deriving mean wallette strengths from the results of an alternative test procedure, assuming that all samples sets have a coefficient of variation of 25%.

Ma and May[129] proposed a failure criterion for brickwork under bi-axial stress, covering tension-tension, tension-compression and compression. They incorporated an equation governing the failure criterion in a non-linear finite element computer program. Predicted displacements using this computer program were not far from the experimental ones since a non-linear behaviour was assumed, but no comments were made about the comparison between predicted and experimental failure pressures. Only one panel was analysed.

A comparison between yield line theory and arching theory using five full sized walls with openings was presented by Southcombe and Tapp[130]. The openings had neither doors nor windows, only holes. It was found that yield line theory gave better prediction of the failure pressure, though no theoretical calculation was presented.

In 1987 Edgell[131] presented a paper about the effects of initial rate of suction and the effects of docking on the flexural strength of brick masonry. Comments, about Table 3 of BS 5628 were made and he concluded that more research is needed on the effect of docking of bricks with a high initial ratio of absorption.

Anderson[132] carried out lateral load tests, also in 1987, using an awkward technique to apply the load. The load source was a double acting servo-controlled hydraulic actuator pulling an assembly of cables, distribution beams and wires which sub-divided the load into a number of point loads to simulate an uniformly distributed loading. He criticized the value given for the characteristic flexural strength recommended in BS 5628[86] but found that yield line theory gave good prediction of the failure pressure for two-way spanning walls with simple edge supports or edges with returns.

The 8th International Brick/Block Masonry Conference, in 1988, brought up seven relevant papers on lateral load behaviour. Gairns and Scrivener[133] performed lateral load tests on brickwork and blockwork wall panels comparing the experimental pressures with predicted pressures using elastic plate theory, yield line theory and strip method. They found that all analysis underestimated the hollow blockwork results. In case of two brickwork walls, the cracking pressures were underestimated by the elastic theory. Failure pressures were underestimated by strip method but overestimated by yield line theory. In trying to analyse the effect of unit size on wall strength, Baker's Mason computer program was used. The predictions obtained using Mason for the test brickwork walls underestimated actual ultimate strengths by 40 to 49%, not surprising as this is based on elastic theory.

Blockwork wall panels were tested under lateral loading against a flexible steel frame by Dawe and Seah[134]. The authors suggested that the inclusion of arching action in yield line analysis improves predicted failure pressures over that obtained using conventional yield line analysis. Although they tested solid walls without openings, the authors, based on the supposition that central panel strips are not as effective as perimeter strips in developing arching action, concluded that central openings do not reduce the arching strength significantly, which is contrary to those obtained by Southcombe and Tapp[130].

An investigation on the behaviour of laterally loaded masonry walls spanning in just one direction was conducted by Thurliman and Guggisberg[135]. The authors used a conventional test rig in which the loading could be applied in two directions independently. A failure criterion was proposed in which an elliptical interaction relationship between the failure moments in both directions was used. However the authors did recognise that the failure criterion will have to be substantiated by more research.

An analysis of the experimental results of 107 full size wall test panels reported by several researchers was presented by Candy[136], together with a new method to calculate the flexural strength of laterally loaded walls. In this analysis three methods were used: the yield line theory, the empirical strip method and a new proposed energy line method. In this comparision the yield line theory was employed on all analysis using an orthogonal strength ratio of 3 as recommended in BS 5628[86], giving the worst results compared to the others. Besides this, the author did not present a theoretical basis of his proposed energy line method, which assumes that vertical bending moments are zero along all "energy lines", taking no account of orthogonal strength ratio.

Fried, Anderson and Smith[137] presented an analysis of work done by different researchers. Attention was called on the use of specific methods of determining the flexural properties of masonry when comparing analytical methods for predicting the resistance of walls against lateral loading. A comparison between the British and Australian Codes showed that the yield line theory resulted in overestimation of failure pressure compared to the strip method for walls with various boundary conditions.

An investigation about the pressure that causes the first cracks on laterally loaded brickwork panels was presented by Lawrence and Cao[138]. Using Monte Carlo simulation combined with isotropic elastic plate analysis, bending moments were evaluated on the plate in bi-axial bending for two configurations of supports: one with four sides simply supported and the other with two vertical sides and bottom simply supported and the top edge free. The analysis was done assuming the behaviour of brickwork panels as geometrically linear and neglecting the orthogonal strength of brickwork. Consequently, the analysis did not give reliable predictions and the authors concluded that random variation in flexural tensile strength was the reason.

Results of a comparision of the load carrying capacity of masonry panels with and without openings were presented by May, Bishop and Ma[139], and a non-linear finite element analysis was used. The authors suggested that modifications should be made when using an yield line approach to take into account the lack of ductility of masonry. For panels with openings, the yield line method overestimated the collapse pressure when the boundaries were assumed fully fixed.

Lovegrove[140] presented a paper in 1988, on the use of yield line theory to unreinforced masonry. He suggested replacing the necessary plastic hinges by the energy needed to produce a crack. Recognising that the use of yield line theory in BS 5628[86] is based solely on experimental results, he tried to justify the Code suggesting that the failure criterion for masonry might be obtained by considering the energy, rather than the bending moment, required to produce unit length of crack.

In 1991, at the Conference on Computer Methods in Structural Masonry, held in Swansea, two papers were presented on lateral load behaviour of masonry. Lawrence and Lu[141] applied two failure criteria to analyse the flexural behaviour of masonry. They considered no interaction between vertical and horizontal moments and the principal moment criterion. The lateral pressure at which the first crack occurs in the wall, was done by a computer program using the finite element method combined with a Monte Carlo simulation. This approach takes account of random variation in flexural tensile strength of masonry. The program was tested first on experimental results of solid walls and, once the results were satisfactory, it was later extended to walls containing openings. The comparison with the solid walls showed that the no-interaction criterion provides better agreement than the principal moment criterion. The authors concluded that there is an urgent need for more experimental data to verify both analytical approaches, mainly because the principal moment criterion developed by Baker[110] is supported by few experimental results. The theoretical method was applied to walls with holes to simulate door and window openings. He did point out an urgent need for experimental investigation to clarify the effect of different sizes and location of openings. The analysis did not take into account the line loadings which develop at the edges of the window due to the pressure applied on that area.

The second paper was presented by Chong, May, Southcombe and Ma[142], also reporting a computer analysis of lateral load tests on masonry walls. The computer program used an isotropic non-linear analysis in conjunction with the bi-axial failure criterion. Solid walls tested by Haseltine, West, Hodgkinson and Tutt[83] and walls with openings tested by Tapp and Southcombe[130] were analysed, and good correlation between the experimental and ultimate pressures was found. Random variation of brickwork was not simulated in this analysis. The authors also used yield line theory and concluded that it tends to overestimate the ultimate pressures in some cases, particularly of panels having high aspect ratio (H/L).

2.3. SCOPE OF THIS RESEARCH PROJECT

From the literature review it became very clear that the work done previously on the behaviour of brickwork panels subjected to lateral pressure, was either concerned with solid walls or walls with holes. In walls with window openings the line loading will develop at the edges of the window opening as a result of wind pressure, which has been ignored or not researched. Therefore, this investigation was carried out to study the flexural behaviour of brickwork panels containing window openings subjected to lateral pressure. The scope of this investigation is summarised as follows:

i) To establish the cracking criteria in bi-axial bending of unreinforced brickwork with an unique test method.

ii) To study the ultimate strength of panels having window openings. The variables considered in this investigation were:
- aspect ratios,
- disposition of the window in the panels, and
- boundary conditions.

iii) The development of the yield-line equations for the panels with openings as no standard equations are available for such cases.

The experimental failure pressures were also compared with theoretical results obtained from finite element, strip method and yield-line analysis for the cross-beams and panels tested in this project. An assessment of BS 5628 British Code of Practice of the use of unreinforced masonry was done in the light of the results obtained for the brickwork panels with openings for the design.

In addition, tests on small specimens were performed to obtain the material properties of brickwork.

CHAPTER 3

MATERIAL PROPERTIES

3.1 INTRODUCTION

This Chapter briefly presents the results of tests which have been carried out on bricks, mortar and small specimens of brickwork to determine their mechanical properties required in this investigation. These tests were performed according to the relevant British standards.

During the investigation the same type of bricks and mortar was used. All test specimens, from small wallettes to half-scale panels, were built in the laboratories of the Civil Engineering Department and cured at the same place covered with plastic sheets. The specimens were built by the same bricklayer for the entire project.

3.2 PROPERTIES OF BRICKS AND MORTAR

3.2.1 Properties of bricks

Half-scale bricks were used and tested according to BS 3921[86] to obtain the dimensions, compressive strength, initial rate of suction and water absorption. Each kind of test requires an amount of at least ten units and the results of the tests are summarised in Tables 3.1 and 3.2.

Table 3.1
Coordinating size of the bricks

Length(mm)	Width(mm)	Height(mm)
113.7	56.2	37.9

The compressive strength of bricks is not a relevant property to study the flexural behaviour of brickwork, though work done at the SCPRF[24] found some correlation between the flexural strength of brickwork and the compressive strength of brick, mortar and brickwork. As the bricks are classified by the compressive strength in most of the Codes around the world and it has become to a great extent the measure of the quality of a brick, this test was included in this investigation.

Table 3.2
Properties of bricks

	Compressive strength (N/mm^2)	Initial rate of suction ($gms/mm^2/min$)	Water absorption (% by weight at 5hr. boiling)
Average	36.0	1.91	14.74
Range	28.7-41.1	1.4-2.41	14.1-15.97
Standard deviation	3.9	0.33	0.54

The characteristic flexural strength of clay brickwork is prescribed in BS 5628[86] according to the water absorption of bricks, however several researchers have pointed out that this is more dependent on the initial rate of suction. In addition, recent investigation[109] also called attention to the unrealistic relationship recommended in this Code between the water absorption and characteristic flexural strength of brickwork. Consequently, it was felt that both results of the tests should be included in this investigation.

The constituents of the mortar used were mixed by volume in proportion of 1:3 (rapid-hardening cement : sand), with the water/cement ratio of 1.06. This water cement ratio was established for the workable mix in accordance to the bricklayer. As the strength of bond between brick and mortar depends on the workability of the mortar, it was kept constant during the whole experimental programme. The sand used was a clean pit sand and was dried in the oven before being used, and it came always from the same batch. The average mortar cube strength varied from 11.5 - 22.9 Nmm^{-2}, with the characteristic strength of 10.9 Nmm^{-2} at 14 days. Three mortar cubes were tested for each test wall and also for a group of cross beams. The results of the compressive strength tests of mortar cubes are given in Table 3.3.

Table 3.3
Compressive strength of mortar

No. of Specimens	Compressive strength (N/mm^2)	Average Compressive Strength (N/mm^2)	Corresponding Wall
1 2 3	21.8 20.0 20.0	20.6	1
4 5 6	14.6 14.4 14.1	14.4	2
7 8 9	18.8 18.0 17.2	18.0	3
10 11 12	16.2 16.4 16.8	16.5	4
13 14 15	14.1 14.3 14.3	14.2	5
16 17 18	19.1 22.9 20.1	20.7	6
19 20 21	13.0 13.5 12.0	12.8	7
22 23 24	14.0 13.8 13.7	13.8	8
25 26 27	15.5 12.7 13.0	13.8	9
28 29 30	15.0 16.9 17.8	16.6	10
31 32 33	11.5 12.4 11.6	11.8	11
34 35 36	16.0 14.7 15.5	15.4	12
37 38 39	18.1 18.0 17.9	18.0	13
40 41 42	15.0 16.7 15.4	15.7	14
43 44 45	12.5 12.2 12.2	12.3	15
46 47 48	13.2 13.0 13.5	13.2	16

3.3 DETERMINATION OF YOUNG'S MODULUS AND POISSON'S RATIO

3.3.1 Introduction

The elastic analysis assumes that the load acting on a panel is distributed according to the stiffness in respective directions. Brickwork panels exhibit stiffness orthotropy in the two main directions, mainly due to the shape of the units and the way as the joints are aligned. Hendry[50] pointed out that the degree of accuracy to predict pressure using elastic analysis based on plate bending theory is dependent on the chosen material properties. As such in this study, the Young's moduli and Poisson's ratios in two orthogonal directions were needed for the analysis. Since flexural tests on wallettes were performed in this investigation, Young's modulus was determined using the same set up. However, the determination of Poisson's ratios requires different arrangements in the two orthogonal directions, hence, compression tests were also done.

3.3.2 Elastic constants obtained from the flexural tests

Sections of uncracked brickwork from the tested Walls no. 9 and 10 were removed. From these sections, two types of wallettes, very similar to those ones subjected to flexural tests, were cut out. The dimensions of these wallettes were determined by the following characteristics:

- wallettes to obtain the flexural tensile strength in the horizontal directions (x) were four bricks long in the main direction and four bricks wide; and
- wallettes for the determination of vertical flexural tensile strength were eight or nine bricks course high and two bricks wide.

These wallettes were tested as simply supported beams subjected to two line loads. Because they represent horizontal and vertical sections of the test walls and were tested as simply supported beams, as such they are designated as horizontal and vertical beams. Fig. 3.1 shows the configurations of the beams. The distance between the two line loads contained at least three mortar joints.

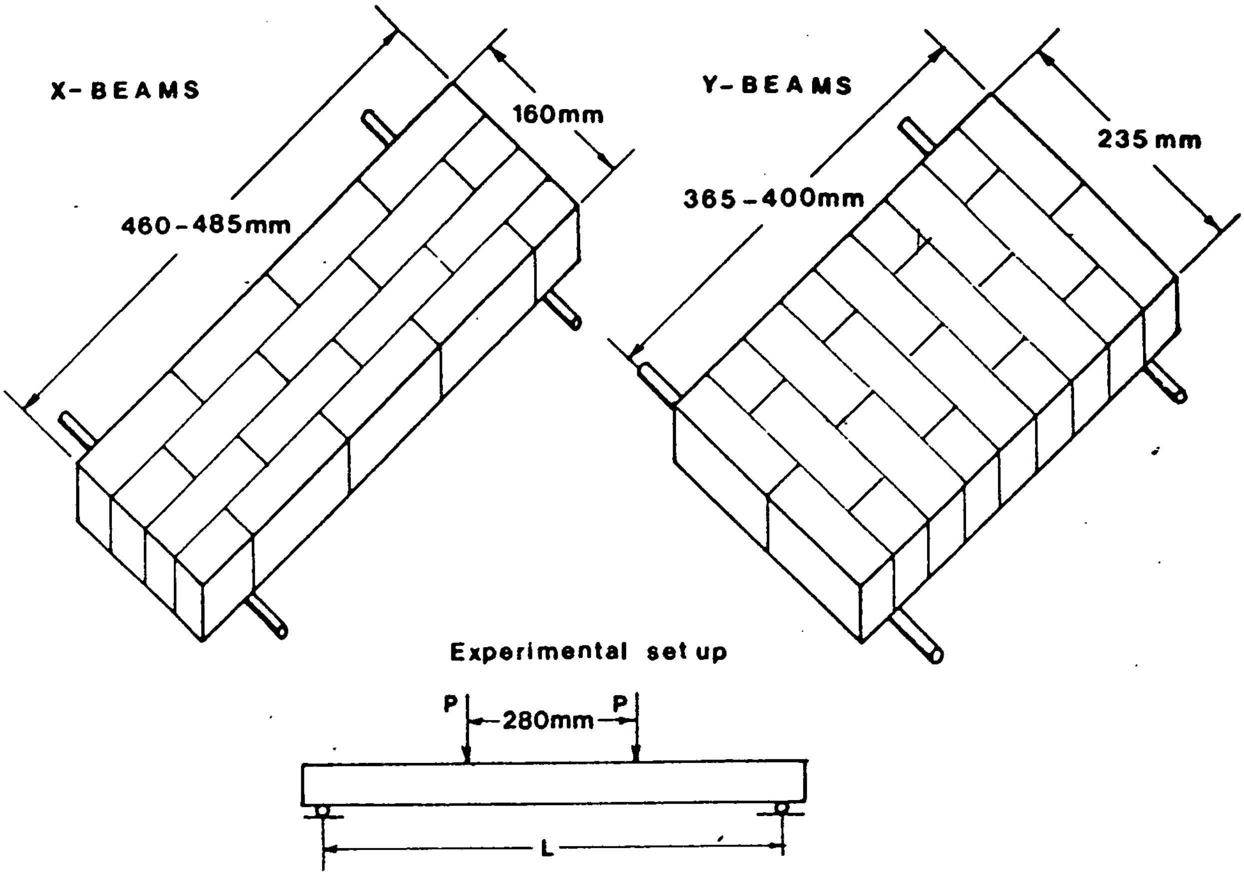

Figure 3.1
Beam configuration and experimental set up

The loading was applied in increments of approximately 250N using an 'Instron' testing machine recording every step of the loading. The loading rate was kept constant during each loading increment. Three mechanical dial gauges with a resolution of 0.002mm were used to measure the displacements, fixed on an independent frame. This procedure was changed later on, when just one dial gauge was utilised mounted on an independent frame supported at three points directly over the test specimen supports.. The Young's modulus in the x and y direction was calculated using the following expression:

$$E = \frac{PL^3}{6I\delta\left(\frac{3a}{4L} - \left(\frac{a}{L}\right)^3\right)} \qquad (3.1)$$

where E is the Young's modulus;

P is each of the line loading;

L is the total span;

a is the distance between the support and the line loading ;

δ is the deflection; and

I is the second moment of area.

Fig. 3.2 to 3.9 show the load-deflection relationship of both types of beams. These results are presented individually because the beams had small differences of span. From those figures the values of the initial tangent elastic moduli and the secant moduli at time of rupture of all beams were calculated, using equation (3.1). Tables 3.4 and 3.5 show these results. Fig. 10 and 11 show two failed specimens.

Table 3.4

Initial tangent modulus; and the secant modulus at time of failure of vertical beams

Beam	E_{yi}(N/mm^2)	E_{ys}(N/mm^2)
1	11,580	4,050
2	15,760	7400
3	11,380	------
4	11,360	5,890
Mean	12,520	5,780

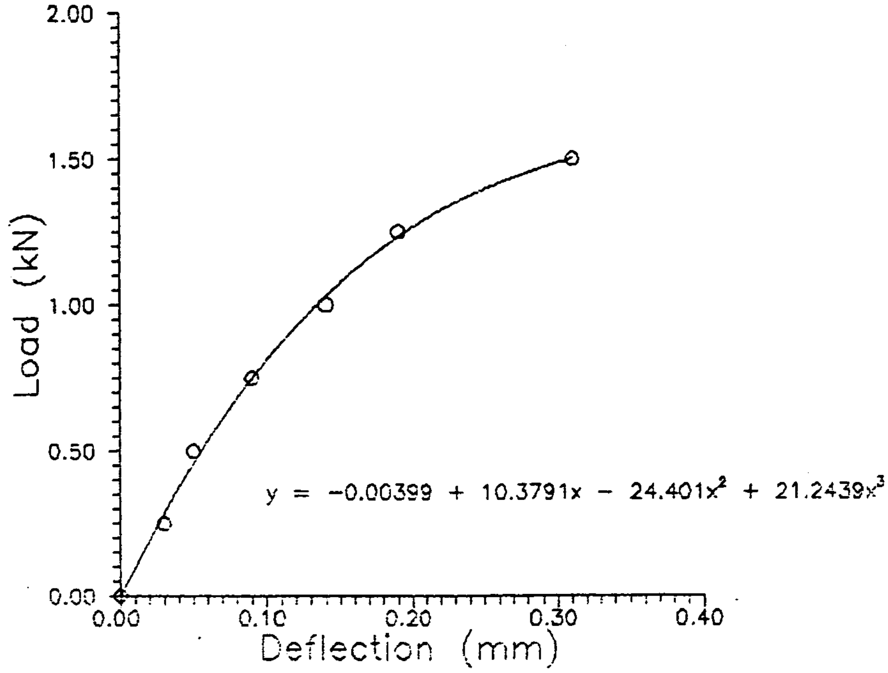

Fig. 3.2
Load-deflection relationship of horizontal wallette (x-beam 1)

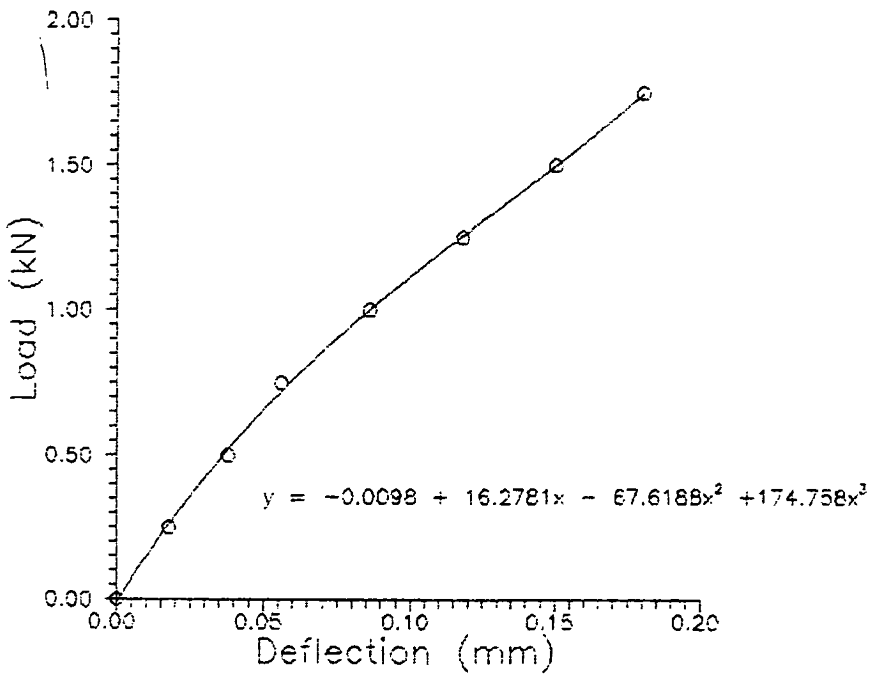

Fig. 3.3
Load-deflection relationship of horizontal wallette (x-beam 2)

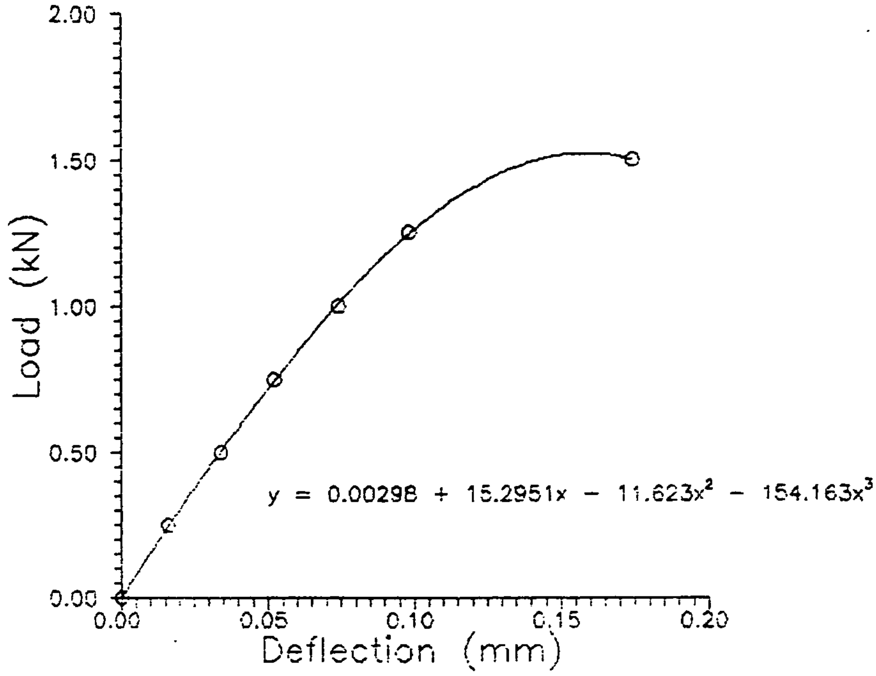

Fig. 3.4
Load-deflection relationship of horizontal wallette (x-beam 3)

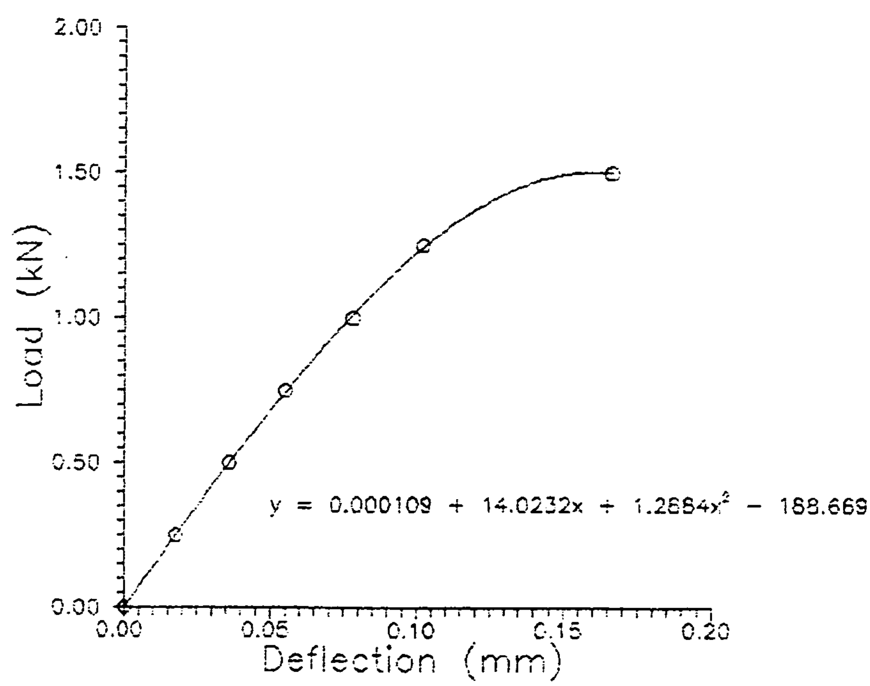

Fig. 3.5
Load-deflection relationship of horizontal wallette (x-beam 4)

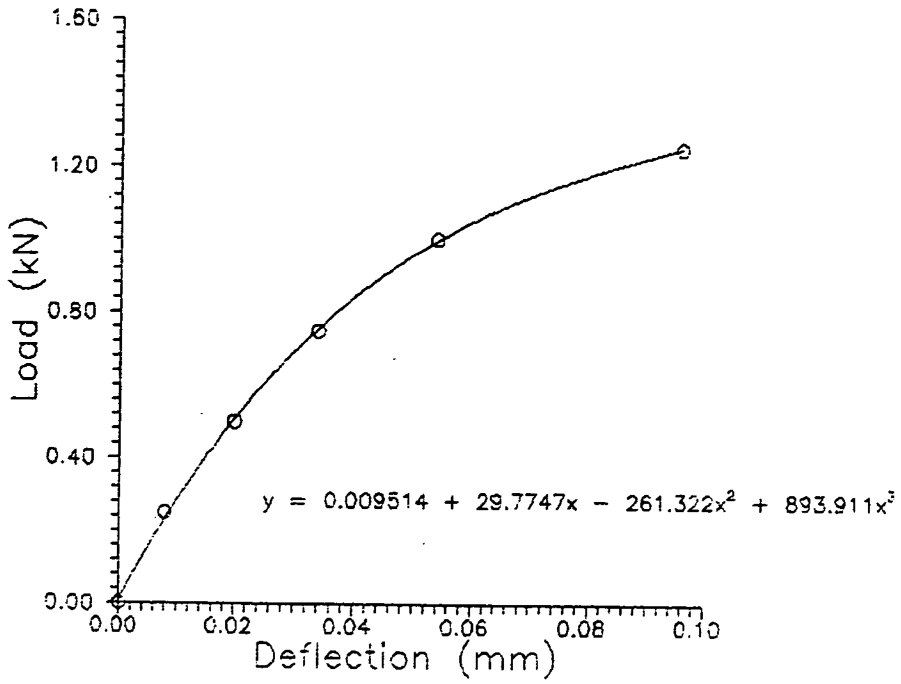

Fig. 3.6

Load-deflection relationship of vertical wallete (y-beam 1)

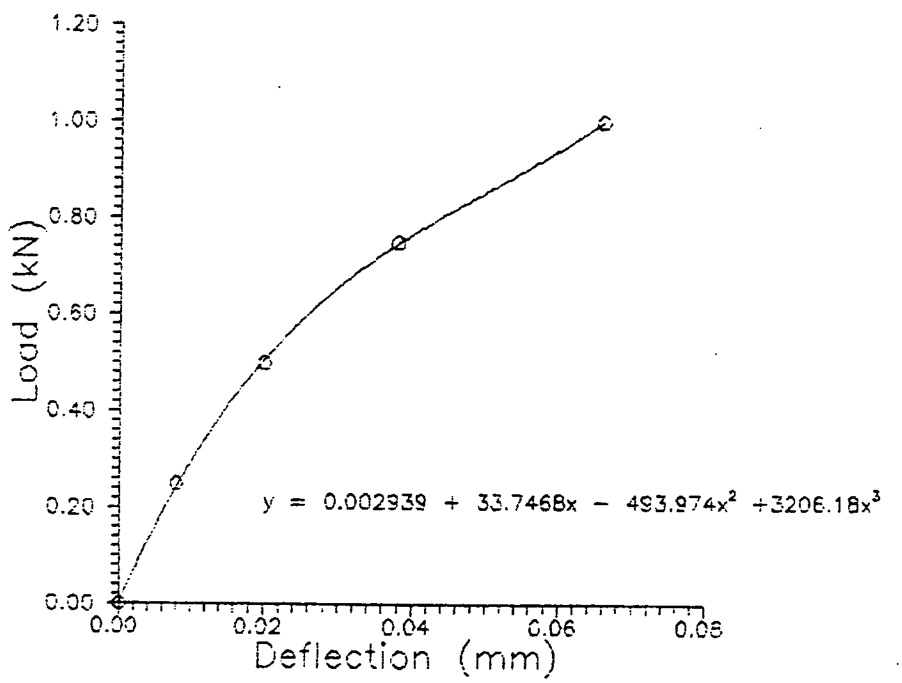

Fig. 3.7

Load-deflection relationship of vertical wallette (y-beam 2)

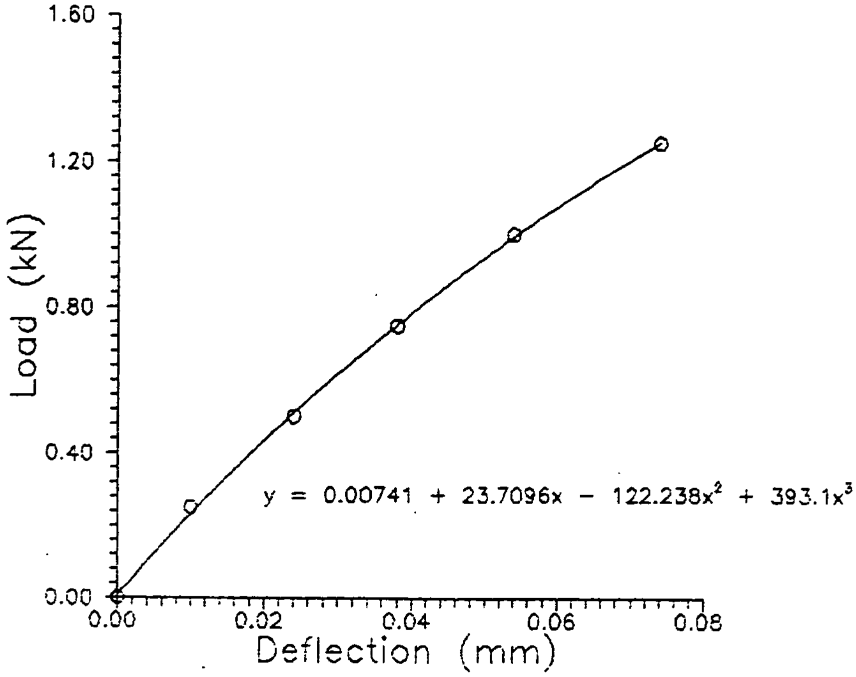

Fig. 3.8
Load-deflection relationship of vertical wallette (y-beam 3)

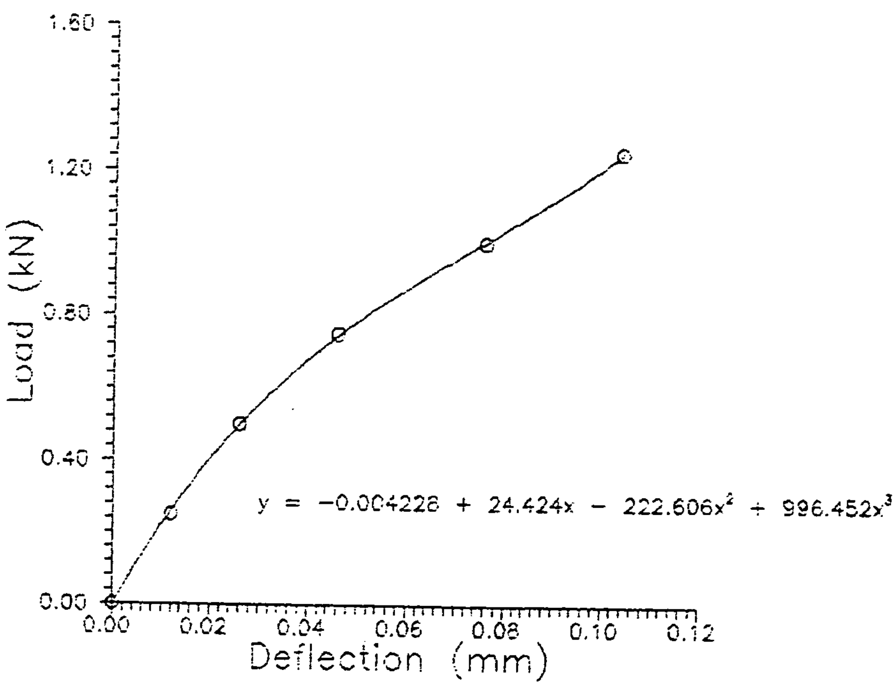

Fig. 3.9
Load deflection relationship of vertical wallette (y-beam 4)

Fig. 3.10
Vertical beam after failure

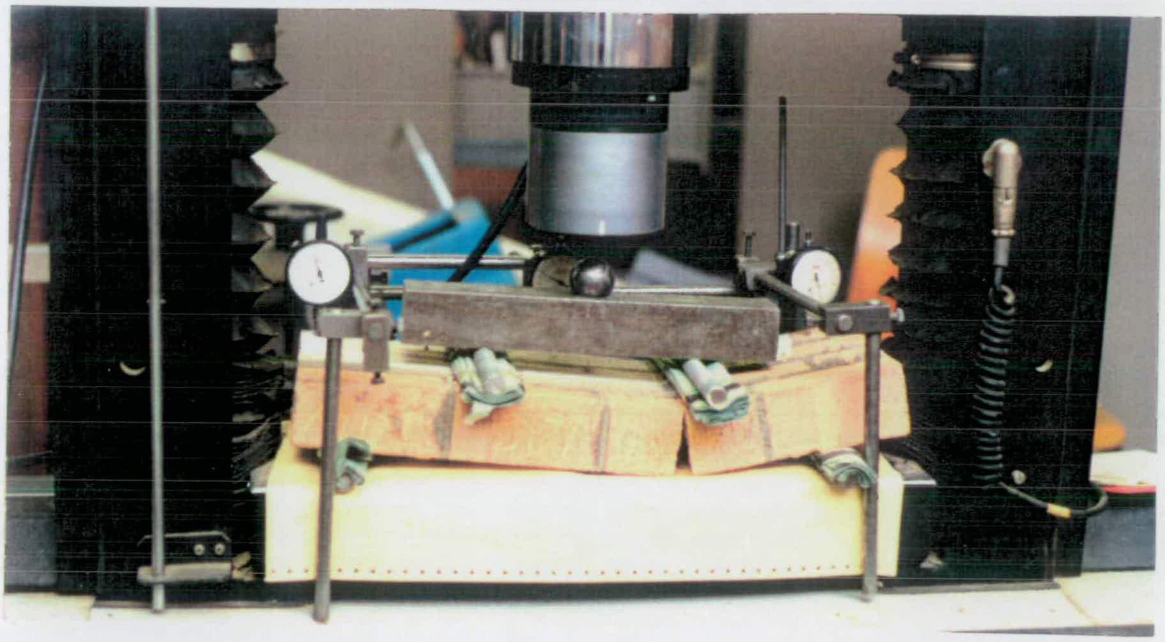

Fig. 3.11
Horizontal beam after failure

Table 3.5
Initial tangent modulus; and the secant modulus at time of failure of horizontal beams

Beam	E_{xi}(N/mm^2)	E_{xs}(N/mm^2)
1	12,270	5000
2	15,360	10820
3	17,230	7,400
4	16,940	-------
Mean	15,450	7,740

As can be seen from the Figures 3.2 to 3.9, the load-deflection relationship was non-linear. This non-linearity was more apparent with the horizontal than with the vertical beams.

3.3.3 Elastic constants obtained from the compression tests

Compression tests were done on square prisms extracted from the test walls. Compression tests have two main advantages compared with flexural tests. First, it is possible to obtain not only the Young's modulus in both directions but also the Poisson's ratios. Secondly, the equipment available was more accurate to apply small compressive stress and measure very small strain, obtaining the compressive load behaviour at the linear range.

Before the elastic constants were determined the ultimate compressive strengths parallel and perpendicular to the bed joints were obtained by testing the same type of specimen. Three square prisms were tested in compression till failure in each direction. Results are presented in Table 3.6. These tests were performed to ensure that the compressive stresses applied to determine the Young's moduli and Poisson's ratio were very small compared to the ultimate compressive stresses.

The values of Young's modulus and Poisson's ratio of brickwork in the directions parallel and perpendicular to the bed joints were both determined. A thin plywood sheet was used on the four edges of the wallettes to apply uniform load. The uniform in-plane compressive force was ensured by adjusting the packing till the measured strains on either sides of the wallette were approximately the same.

Table 3.6

Compressive strength of brickwork

Specimen	f_{cx}(N/mm^2)	f_{cy}(N/mm^2)
1	17.22	20.66
2	20.96	15.07
3	21.41	17.24
mean	19.86	17.66

The strain gauge used were acoustic type having a length of 63.5 mm, allowing to span over two horizontal joints and a brick course. In the other direction it spanned over only one vertical joint. Fig. 3.12 shows the position of the vibrating wire gauges on a prism.

The compressive loading was applied in four steps; 6, 10, 15 and 20 kN, respectively, with the strains measured in each increment. The prisms were first tested with the bed joints horizontal and the strain gauges vertical. After testing, the gauges were then placed horizontally and the same loading procedure was carried out. The results gave E_y and ν_{xy}. The prisms were then tested with the bed joints in a vertical direction, the strain gauges now being placed horizontally and the same loading procedure repeated. These results produced E_x and ν_{yx}. Tables 3.7 and 3.8 show the complete test results.

Fig. 3.13 and 3.14 show a typical stress-strain plot for the two orthogonal directions. The values of E_x and E_y were obtained from the best fit curve of Fig. 3.13 and 3.14 and the values of ν_{xy} and ν_{yx} are an average of the values presented in Tables 3.7 and 3.8. These values are:

$$E_x = 16{,}165 \text{ N/mm}^2; \ \nu_{yx} = 0.11;$$
$$E_y = 12{,}042 \text{ N/mm}^2 \text{ and } \nu_{xy} = 0.15.$$

Table 3.7

Elastic constants parallel to the bed joint obtained in compression tests

Specimen	Stress (N/mm^2)	ε_{long}	E_x (N/mm^2)	ε_{lat}	ν_{yx}
1	0.482	2.596×10^{-3}	16,840	4.050×10^{-6}	0.156
	0.720	4.429×10^{-3}	16,258	6.085×10^{-6}	0.155
	1.080	6.732×10^{-3}	16,043	1.037×10^{-5}	0.154
	1.440	8.943×10^{-3}	16,102	1.341×10^{-5}	0.150
2	0.466	3.023×10^{-3}	15,417	4.565×10^{-6}	0.151
	0.776	5.062×10^{-3}	15,329	7.644×10^{-6}	0.151
	1.165	7.754×10^{-3}	15,024	1.186×10^{-5}	0.153
	1.553	1.029×10^{-3}	15,086	1.574×10^{-5}	0.153
3	0.450	2.563×10^{-3}	17,558	3.947×10^{-3}	0.154
	0.750	4.290×10^{-3}	17,482	6.564×10^{-6}	0.153
	1.125	6.431×10^{-3}	17,494	9.839×10^{-6}	0.153
	1.501	8.695×10^{-3}	17,303	1.301×10^{-5}	0.150

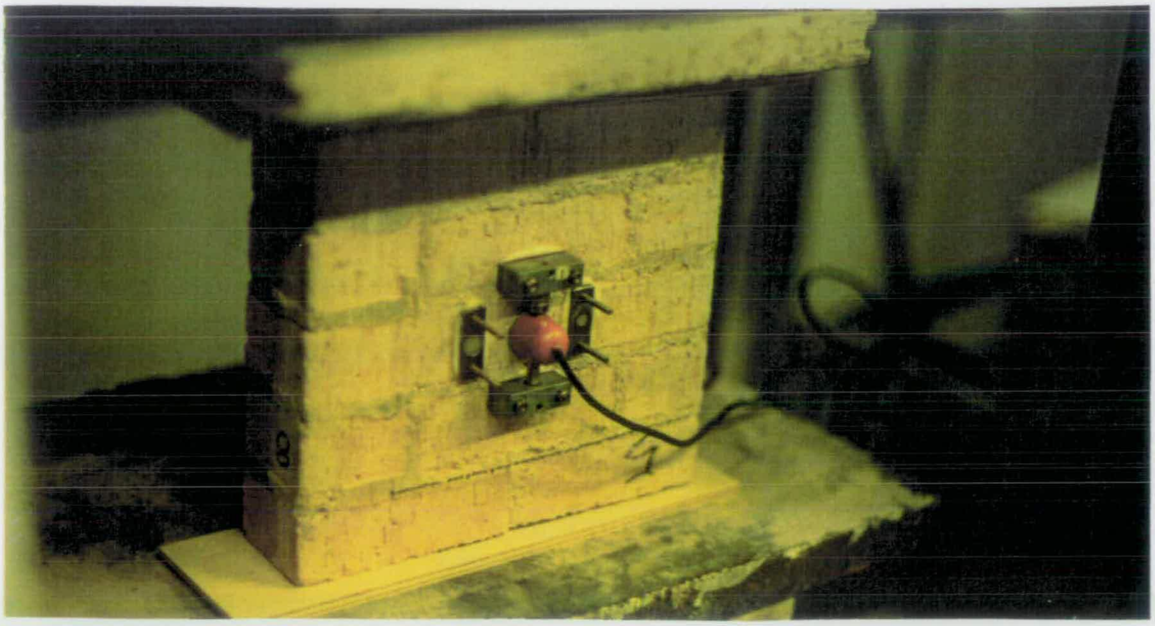

Fig. 3.12

Determination of Young's moduli and Poisson's ratio

Table 3.8
Elastic constants perpendicular to the bed joints obtained in compression tests

Specimen	Stress (N/mm^2)	ε_{long}	E_y (N/mm^2)	ε_{lat}	ν_{xy}
1	0.411	2.832×10^{-5}	14,573	3.105×10^{-6}	0.110
	0.684	4.653×10^{-5}	13,926	5.094×10^{-6}	0.109
	1.026	7.636×10^{-5}	13,437	8.396×10^{-6}	0.110
	1.368	1.021×10^{-4}	13,402	1.083×10^{-5}	0.106
2	0.480	3.994×10^{-5}	12,018	4.234×10^{-6}	0.106
	0.801	6.764×10^{-5}	11,842	7.305×10^{-6}	0.108
	1.201	1.011×10^{-4}	11,876	1.092×10^{-5}	0.108
	1.602	1.401×10^{-4}	11,438	1.485×10^{-5}	0.106
3	0.443	3.448×10^{-5}	12,848	3.689×10^{-6}	0.107
	0.738	5.821×10^{-5}	12,679	6.287×10^{-6}	0.108
	1.107	9.209×10^{-5}	12,021	9.946×10^{-6}	0.108
	1.476	1.262×10^{-4}	11,694	1.376×10^{-5}	0.109

3.3.4 Discussion of Experimental Results

The moduli of elasticity were calculated using two different approaches. The first approach was from testing beams, which produced a non-linear correlation of the load-deflection relationship and this can be attributed to the fact that the moduli of elasticity in both directions were calculated considering the gross-sectional area of the specimens assuming a homogeneous material with the same gross section properties. As all test wallettes were extracted from the undamaged parts of the failed walls it is possible that hair cracks occurred during the testing of the walls may have reduced the inertia of the sections affecting the determination of the Young's moduli. The results obtained from the compression tests are not affected by these hair cracks perpendicular to the direction of the applied load. In addition, the applied compressive stresses were very small compared to the ultimate compressive stresses and the load-deflection relationship was measured only at the linear range.

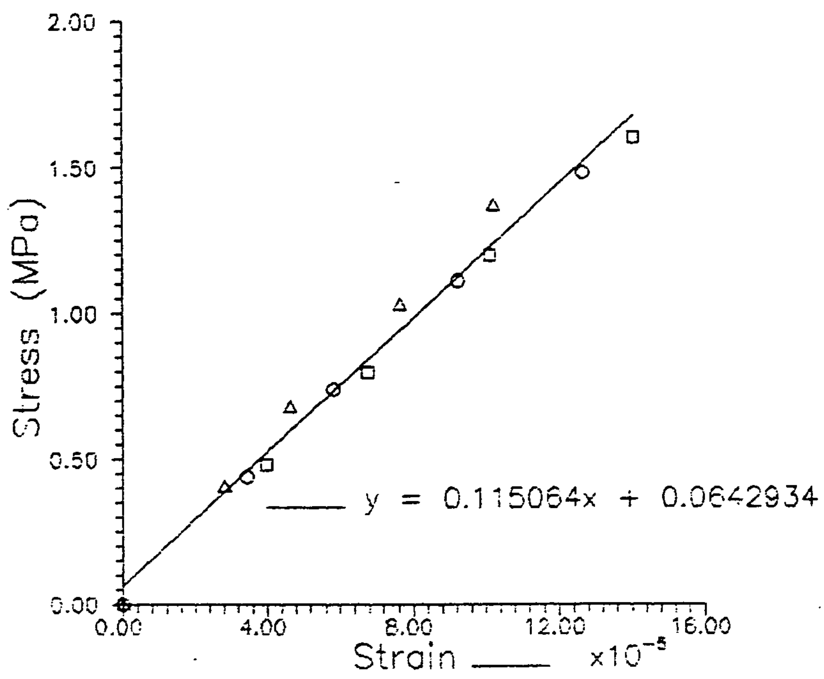

Fig. 3.13
Stress-strain relationship of prism:'y-direction ' (three specimens tested)

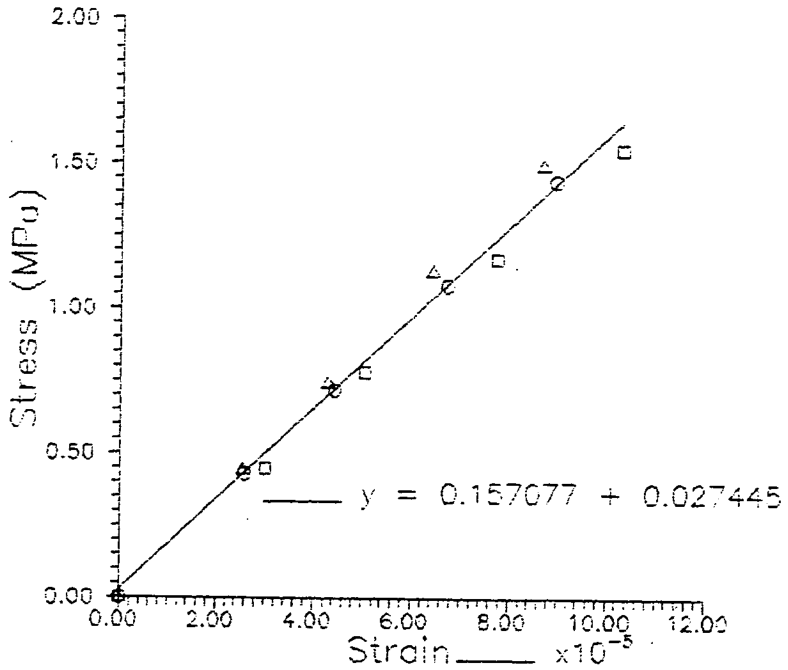

Fig. 3.14
Stress-strain relationship of prism: 'x-direction' (three specimens tested)

3.4 FLEXURAL STRENGTH OF WALLETTES

3.4.1 Introduction

Wallettes are tested in bending because they represent strips of the actual walls and it is necessary to know their flexural tensile strength in both directions to predict the wall strength. As they represent strips of the walls subjected to bending in one direction, they have been designated as vertical beams (y-beams) when bending moment acts along the vertical direction, i.e. perpendicular to the bed joints, and horizontal beams (x-beams) when the bending moment is applied along the horizontal direction, i.e. parallel to the bed joints.

These beams were tested to failure to obtain the modulus of rupture, which is calculated by dividing the ultimate moment of the beam by its sectional modulus.

Vertical beams use to fail at the interface of the mortar joint. In this project, all vertical beams followed this failure mechanism. Hence, the interface bond strength of the joints will determine the vertical flexural strength. It has been reported by various researchers that the interface bond strength is very variable and this was also found in this project as can be seen in Tables 3.9 and 3.11.

Horizontal beams have three different modes of failure: firstly, failure may occur through perpendicular to the mortar bed joints in zig-zag fashion; secondly, the crack lines may pass through the bricks and perpendicular joints and thirdly, the combination of the previous two. All the three modes of failure occurred in this investigation.

Similar test specimens as recommended in BS 5628[86] were used for the determination of the flexural tensile strengths. The tests were done laid flat as simply supported beams and not vertically as in BS 5628. The dead weight of the specimens varies from zero to maximum from top to bottom supports if the BS 5628 method of testing was adopted, which will result in some rotational restraint at the bottom. This in turn will have an effect on the flexural tensile strength. By testing flat in this investigation, this rotational restraint was eliminated and the dead weight of the specimens was accounted for in calculating the flexural tensile strength.

Initially, it was decided that three wallettes spanning in both directions built alongside each test wall should be tested in order to get the flexural tensile strength. But, only for Walls nos. 1, 2 and 3 wallettes were built alongside with them. It was found, later, that both types of wallettes could be extracted from the undamaged parts of the test walls. Hence, all other wallettes including for Walls 1, 2 and 3 were obtained in this manner. The results of the wallettes built alongside or extracted from the Walls 1, 2 and 3 are similar as can be seen from Table 3.9, 3.10 and 3.11. Hence it was decided not to build wallettes alongside the test walls anymore and only extract wallettes from the undamaged parts of the failed walls. This was decided in order to save labour time and material consumption.

3.4.2 Experimental results

Results of the wallettes built alongside the test walls and results of the wallettes extracted from the undamaged parts of the failed walls are presented in Tables 3.9, 3.10 and 3.11.

Table 3.9
Flexural tensile strength of wallettes built alongside the test walls

Wallettes	f_{tx}(N/mm^2)	f_{ty}(N/mm^2)
1	1.91	0.74
2	2.48	0.96
3	2.68	0.86
4	2.21	0.82
5	1.79	0.81
6	2.08	0.70
7	1.32	0.66
8	1.89	0.74
9	2.40	0.52
mean	2.08	0.76
Standard deviation	0.41	0.13

Table 3.10

Flexural tensile strength parallel to the bed joints of wallettes extracted from the test walls

Wallette	$f_{tx}(N/mm^2)$	Wallette	$f_{tx}(N/mm^2)$	Wallette	$f_{tx}(N/mm^2)$
1	2.10	29	1.61	57	1.87
2	2.41	30	2.24	58	1.89
3	2.35	31	2.01	59	1.35
4	1.59	32	2.08	60	2.91
5	2.24	33	2.48	61	1.63
6	2.07	34	2.15	62	2.37
7	1.83	35	2.97	63	2.20
8	1.23	36	2.46	64	2.15
9	1.77	37	2.33	65	2.06
10	1.83	38	2.19	66	1.40
11	1.95	39	2.00	67	2.01
12	1.76	40	2.47	68	1.64
13	2.02	41	2.27	69	1.38
14	1.90	42	1.81	70	1.24
15	2.30	43	2.26	71	2.01
16	2.20	44	1.98	72	2.14
17	1.89	45	2.12	73	1.74
18	2.15	46	2.07	74	1.63
19	2.61	47	1.74	75	1.70
20	2.51	48	2.66	76	1.81
21	2.12	49	2.09	77	2.02
22	2.55	50	2.43	78	1.80
23	2.21	51	1.66	79	2.26
24	2.33	52	2.07	80	1.60
25	1.85	53	1.60	81	1.30
26	1.48	54	2.13	82	2.00
27	2.18	55	2.16	-	-
28	3.11	56	1.91	-	-

i) mean = 2.03 N/mm^2;

ii) standard deviation = 0.38.

Table 3.11
Flexural tensile strength perpendicular to the bed joints of wallettes extracted from the test walls

Wallette	f_{ty} (N/mm^2)	Wallette	f_{ty} (N/mm^2)	Wallette	f_{ty} (N/mm^2)
1	0.83	21	0.77	41	0.79
2	0.61	22	0.82	42	1.22
3	0.64	23	1.07	43	0.88
4	0.60	24	1.10	44	0.81
5	0.54	25	1.01	45	0.69
6	0.50	26	1.62	46	0.53
7	0.81	27	1.53	47	0.99
8	0.47	28	0.62	48	0.89
9	0.48	29	1.07	49	1.13
10	0.89	30	1.42	50	0.83
11	0.59	31	0.60	51	1.02
12	0.89	32	0.66	52	0.84
13	0.89	33	0.63	53	1.58
14	0.86	34	0.93	54	0.79
15	0.57	35	1.48	55	0.52
16	0.55	36	0.75	56	0.61
17	0.67	37	0.94	57	0.97
18	0.65	38	0.98	58	0.37
19	0.89	39	0.78	59	0.88
20	0.78	40	0.44	-	-

i) mean = 0.84 N/mm^2;
ii) standard deviation = 0.28.

3.4.2.1 Discussion of experimental results

A statistical comparison has been made between the flexural tensile strength in two directions from wallettes extracted from the test walls or built separately during its construction.

From Tables 3.9, 3.10 and 3.11 it is very clear that there is practically no difference between the flexural tensile strength in two directions obtained from the wallettes extracted from the test walls or built separately. A comparison has been made using the "t distribution" for both data. The hypothesis was that the wallettes built separately do not belong to the population of the wallettes extracted from the undamaged parts of the failed walls and a .01 probability of error was chosen. This hypothesis was rejected and it is statistically proved, with a probability of error of .01, to suppose that both kind of wallettes belong to the same population. Hence it is possible to assume that the flexural tensile strength normal and perpendicular to the bed joints obtained from the wallettes built independently or extracted from the undamaged portion of the tested walls are similar.

The results of flexural tensile strength were compared with the recommended values of characteristic flexural tensile strength contained in Table 3 of BS 5628[86] and the characteristic flexural tensile strength calculated from the values given in Tables 3.10 and 3.11 (wallettes extracted from the undamaged portion of the test walls). Normally the designer will use the published values of flexural tensile strength recommended by BS 5628 instead of carrying out the flexural test on wallettes. According to this Code the characteristic flexural tensile strengths depend on the water absorption, which for the bricks used in this research is 14.74 % by weight after 5 hours of boiling. For such kind of bricks the allowable values prescribed by the Code are $f_{ty}=0.4$ N/mm^2 and $f_{tx}=1.2$ N/mm^2. The characteristic flexural tensile strengths of the wallettes extracted from the undamaged portion of the test walls are $f_{ty}=0.42$ N/mm^2 and $f_{tx}=1.41$ N/mm^2 and for wallettes built separately are $f_{ty}=0.52$ N/mm^2 and $f_{tx}=1.36$ N/mm^2. These values are similar to the recommended values of BS 5628.

3.5 CONCLUSIONS

Following conclusions can be drawn from the experimental work presented in this chapter:

i) The initial tangent moduli obtained from flexural tests are very similar to the ones obtained from the compression tests;

ii) The flexural tensile strengths normal and perpendicular to the bed joints obtained from the wallettes built independently or extracted from the undamaged portion of the tested walls are similar;

iii) The characteristic flexural tensile strengths of the wallettes are similar to the characteristic values of flexural tensile strengths recommended in BS 5628 for such type of bricks.

CHAPTER 4

CROSS-BEAM TESTS

4.1 INTRODUCTION

There is no difficulty in the determination of the distribution of moments and the failure criterion for brickwork spanning in just one direction. When it spans in the vertical direction, failure occurs if the tensile bond strength is exceeded at the interface of the bed joint. For brickwork spanning in the horizontal direction, the overlapping bricks in alternate courses may force the bricks to fail in tension or the horizontal joints to fail in torsional shear together in tension. The problem is more complicated when brickwork is subjected to bi-axial bending, i.e. vertical and horizontal bending moments acting simultaneously.

Normally the brickwork wall panels subjected to wind pressure can be treated as a two-way spanning slab, and as such are highly redundant structures. The degree of redundancy is influenced by the degree of fixity at the supports and sometimes can be difficult to quantify exactly. The theoretical distribution of moments throughout such structures can be determined either by elastic or plastic analysis. In a very simple approach, in elastic analysis the maximum moments are determined by the elastic orthotropy, i. e., the ratio between the Young's moduli in the two main directions. Plastic analysis assumes that at failure the distribution of moments is determined by the strength ratio, i.e. the ratio between the flexural tensile strengths in the two main directions while the structure is collapsing. The problems encountered with the use of both theories lie in the fact that elastic analysis underestimates the failure pressures and plastic analysis is not suitable to unreinforced brickwork walls because of elastic orthotropy and, for a brittle material, it is not possible to satisfy the required conditions of such theory, mainly the ability of cracked sections to support constant moments as happens in a ductile material like steel.

A lot of research has been done to determine the lateral strength of unreinforced brickwork. For panels that have the cracking pressures similar to the failure pressure, elastic analysis is supposed to predict results with good agreement with the experimental pressures. For panels that exhibit some reserve of strength after initial cracking, elastic analysis fails in predicting the failure pressures. Baker[46] pointed out that the reserve of strength after initial cracking may be due to an elastic redistribution of moments from the weaker vertical direction to the stronger horizontal direction. In this circumstance, elastic analysis should predict cracking pressures accurately, but it is very difficult to visualise the correct value of the moment when the first hair crack appears. Lawrence[116] attempted in his experimental investigation a sophisticated apparatus to detect the first cracks by using acoustic equipment, but some doubts still remained. Therefore conventional elastic analysis has been seen as inappropriate to predict both pressures at cracking and failure.

The main problem involved in investigating the bi-axial flexural behaviour of wall panels is to determine the real distribution of moments in a highly redundant structure like a plate. Because it is very difficult to apply and measure simultaneously the vertical and horizontal moments to a specimen of brickwork, Baker[110] attempted to explain the bi-axial behaviour by applying these moments in "single joints". As these "single joints", as represented in Fig. 4.1, could reproduce the behaviour of brickwork subjected to horizontal and vertical moments, he assumed that they could also reproduce the real behaviour of a brickwork wall panel subjected to horizontal and vertical moments simultaneously. He disregarded some factors that affect the behaviour of a real structure, like the influence of the degree of redundancy and deformation characteristic of the material. The result was a failure criterion, as is shown in Fig. 4.2, in which it was suggested that there is an elliptical interaction between the moments of resistance in the two orthogonal directions. Using this failure criterion and a statistical approach, the lateral strength of walls can be predicted provided that a correction is applied to take account for variability in joint strength within the wall. Nevertheless, it has been proved that this failure criterion also underestimates the failure pressure of laterally loaded brickwork walls[116,141].

Following Baker, an elliptical failure criteria was suggested by Gazzola, Drysdale and Essawy[123] for blockwork , without much experimental evidence. These researchers found it difficult to apply bending moments in the two orthogonal directions simultaneously, hence they tested wallettes as simply supported beams with horizontal joints inclined at different angles to the axis of the beams.

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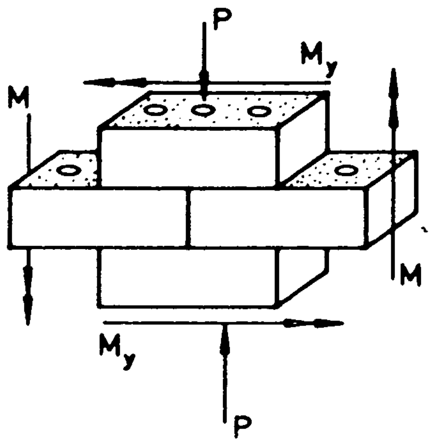


Fig. 4.1
Baker's joint specimen subjected to vertical and horizontal moments

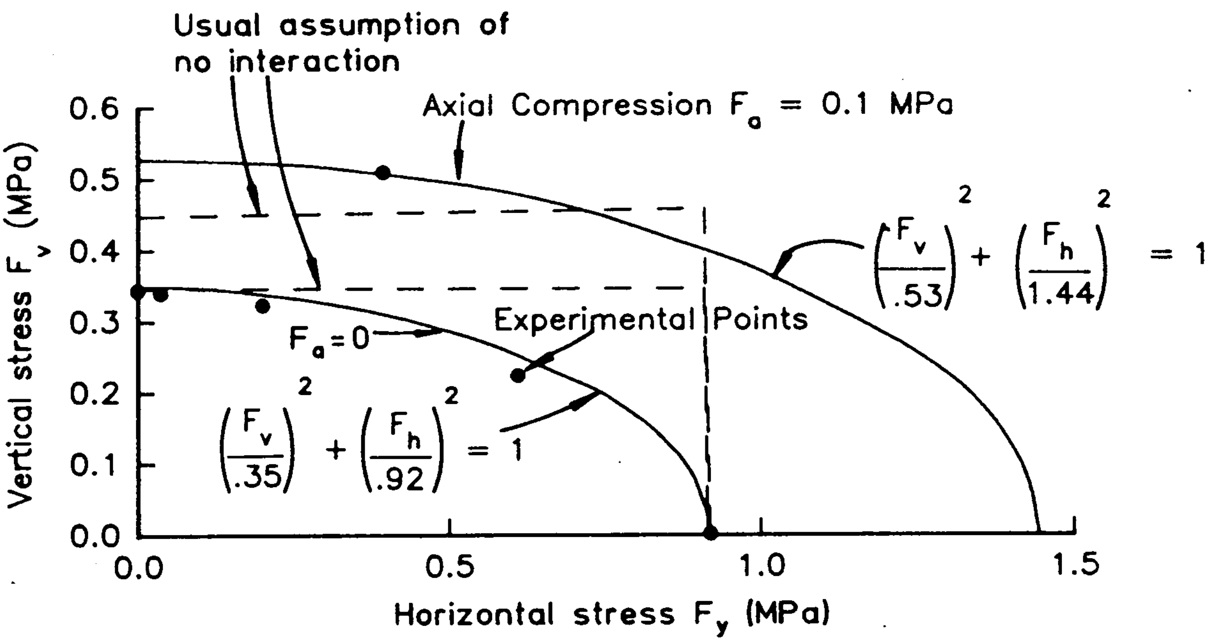


Fig. 4.2
Baker's criterion of failure of brickwork in bi-axial bending. Dashed lines indicate "no interaction" criteria

In this chapter, a method to apply horizontal and vertical moments simultaneously in specimens of brickwork is presented. The specimens in a shape of a cross was suggested by Sinha and some tests similar to Series I specimens were tested in Edinburgh in early eighties[143]. The details of the cross-beams and testing methods used for this investigation are described in the following sections.

4.2 EXPERIMENTAL PROGRAMME

The experimental programme was designed to measure in each cross-beam the reactions at the supports, thus, to establish the behaviour in bi-axial bending and also to establish the moment interaction diagram.

4.2.1 Specimen details

Three different types of cross-beams were tested, as described below:

i) Series I - these brickwork cross-beams with aspect ratio equal to one were built having all the joints filled with the same mortar described in Chapter 3. Three specimens having approximately the same aspect ratio ($L_x/L_y=1$) have been tested. Fig. 4.3 shows the test specimens;

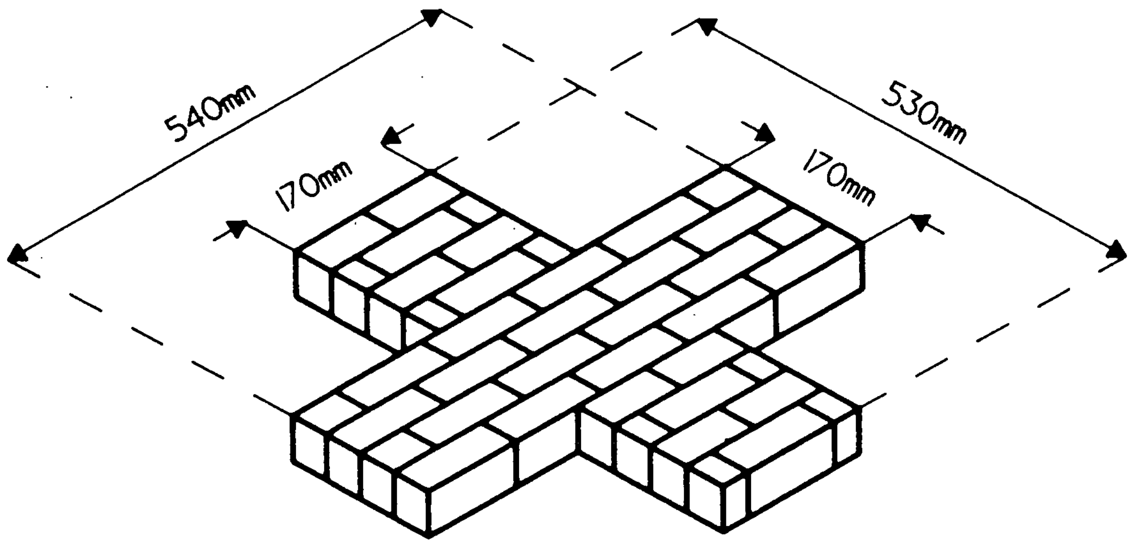


Fig. 4.3
Cross-beams of Series I (built using 1:3 cement sand mortar)

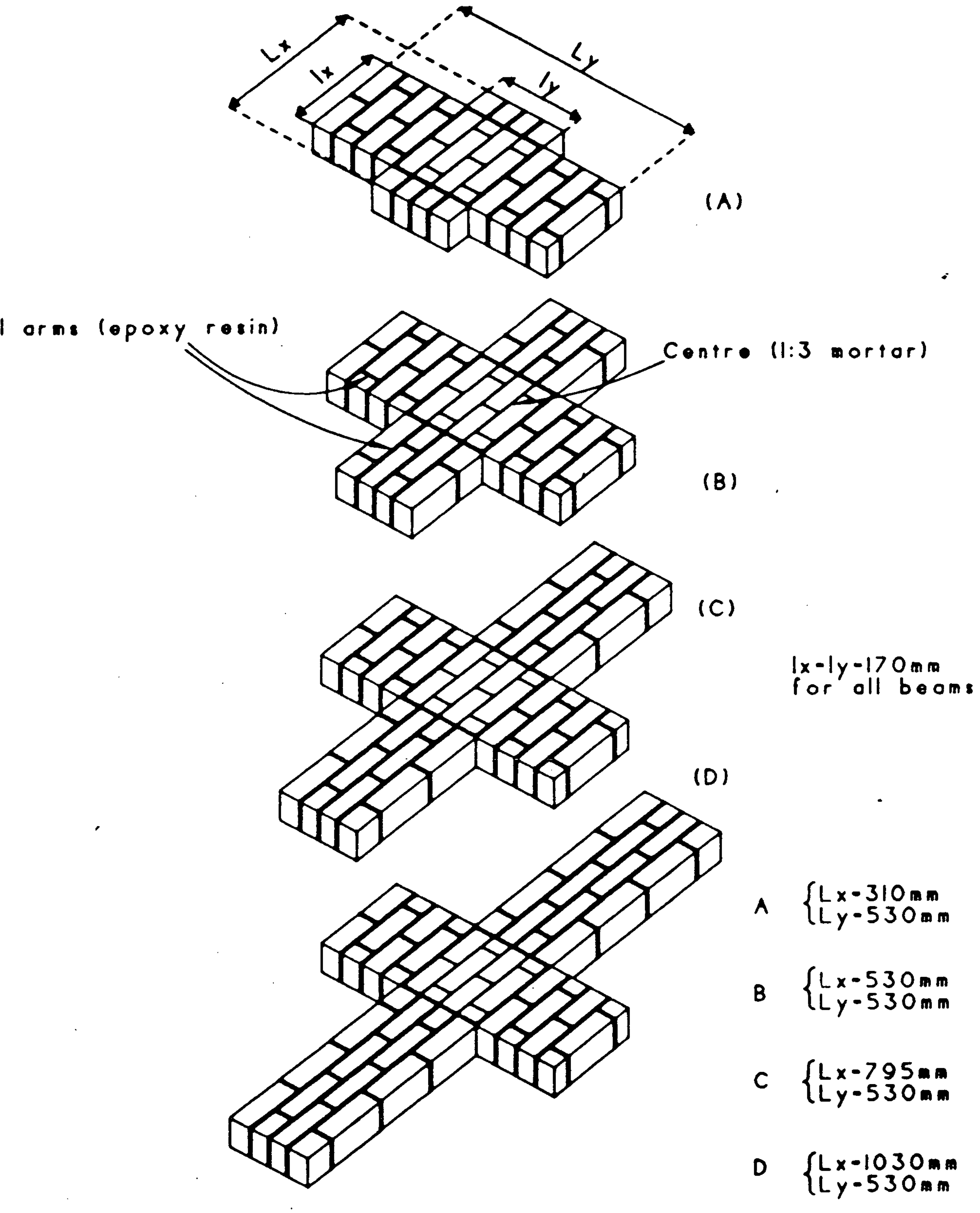


Fig. 4.4

Cross-beams of Series II (the centres were built with 1:3 mortar and all the arms were glued with epoxy resin)

ii) Series II - the central part of these cross-beams use the same cement mortar. The arms were constructed using a high strength epoxy resin, to prevent premature failure of the arms in shear or flexure. This was done intentionally to force the failure in the central part. Four aspect ratios were tested; L_x/L_y = 0.6, 1, 1.5 and 2. Fig. 4.4 shows the cross-beams of various aspect ratios. To obtain the strengths in uniaxial bending, wallettes similar to those described in section 3.4 were made with epoxy resin and tested. The results are shown in Table 4.6;

iii) Series III - these brickwork specimens had the central part built in cement mortar .The arms were combed like structures made with epoxy resin to prevent again premature failure in shear or bending. The mortar and the glue were the same as used before. Three aspect ratios have been used; 1, 1.5 and 2. Fig. 4.5 shows all test specimens.

Each series of cross beams built had at least three companion mortar sample moulded for compression tests, the results of which are presented in Table 4.1.

Table 4.1
Compressive strength of mortar

Mortar cubes	Compressive strength (N/mm^2)	Average compressive strength (N/mm^2)	Corresponding Series
1	16.3		
2	16.5	16.5	I
3	16.6		
4	14.8		
5	14.8		
6	14.5	14.1	II
7	13.1		
8	13.6		
9	13.7		
10	18.2		
11	17.8	18.0	III
12	18.0		

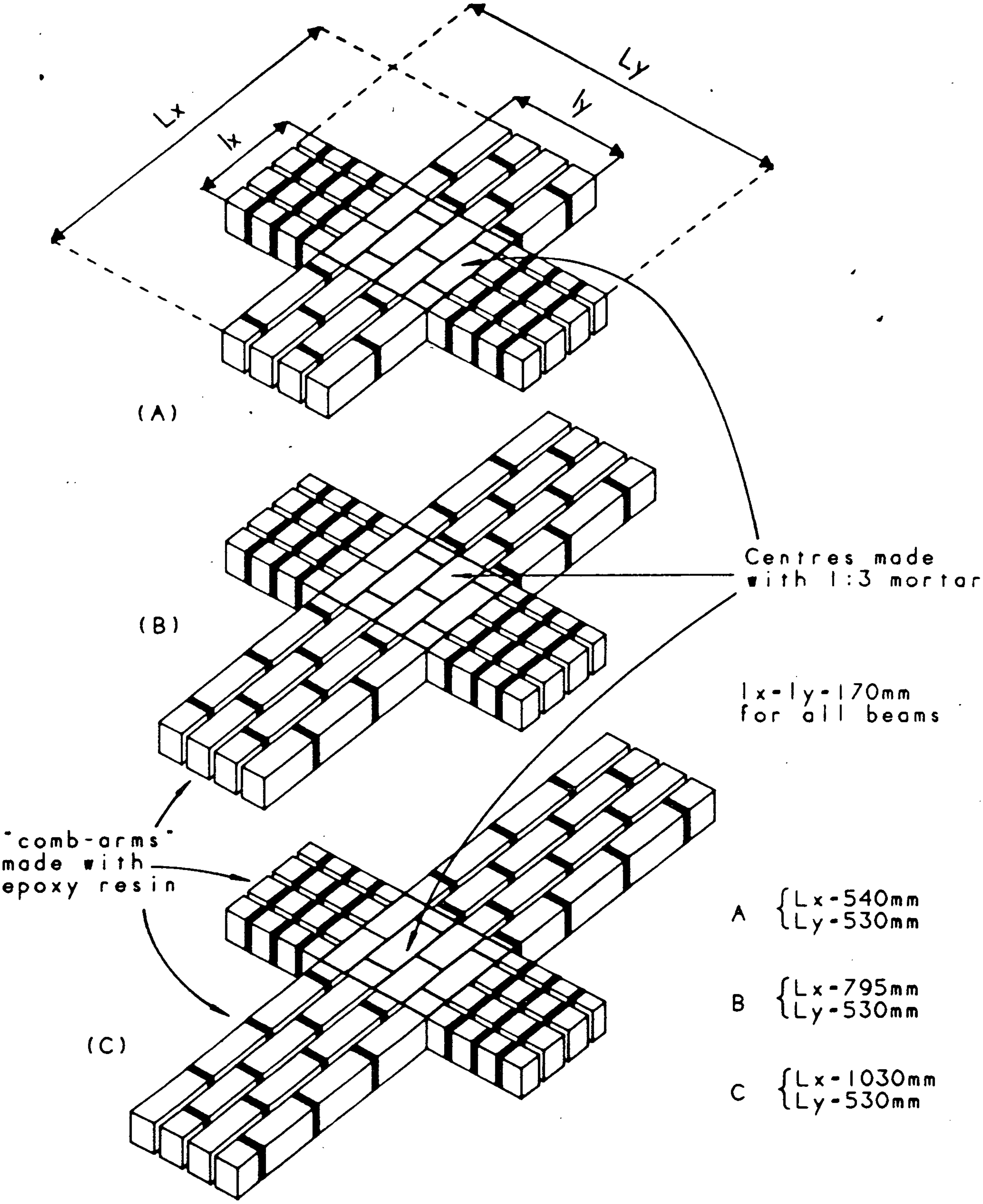


Fig. 4.5

Cross-beams of Series III (the centres were built with 1:3 mortar and the "comb-arms" were glued with epoxy resin)

All cross beams were single leaf construction and had the same half-scale bricks used for the test walls which were discussed in Chapter 3. Cross beams of Series I were built vertically with the horizontal arms being supported during the process of construction and fourteen days after that. Series II and III had the centres built separately from the arms, which were glued on the centres fourteen days after their construction. The cross beams of Series I and the centres of Series II and III were covered by a polythene sheet during these fourteen days. The same bricklayer that built the test walls made the cross beams to provide consistency of workmanship.

4.2.2 Test arrangements for the Cross-beams

The tests were designed to measure the reactions at the four supports and the applied load for all cross-beams of different series . The applied bending moments in two orthogonal directions, thus, could be established from the measured reactions in both directions. Three types of load cells were used for the tests. A 3-tonne load cell measured the jacking load. Two 1000-kgf load cells were placed under the supports receiving larger reactions in the cross-beams. Two 500-kgf load cells were positioned under the supports of the cross-beams in the weaker direction. Thus the support reactions were measured by the four load cells of 1000 kgf and 500 kgf capacities respectively. These reactions provided the experimental values of the loads P_x and P_y. All load cells were calibrated before the tests.

The jack was fixed on a specially built steel frame bolted to the strong floor of the Structural Laboratory. Fig. 4.6 and 4.7 show the set up for a test. The point load at the centre of the cross was applied via a 50 mm diameter iron disc. The disc was bedded on the cross with "dental plaster" and used for all tests. The four edges of the cross beams were supported on 30 mm square steel bars, on the top of the load cells. A layer of "dental plaster" was used over the bars to avoid any irregularities in support height. The distribution of the applied load could be affected, if the supports were not at the same level. The load was applied in small increments manually by a hydraulic jack. An electrical deflection gauge was positioned near the centre of the crosses to detect cracking by sudden and excessive changes in deflection. A continuous plot of the applied load and reactions was recorded using a five channels pen-chart recorder. There were no discrepancies between summing the four support reactions to the actual load applied. Three

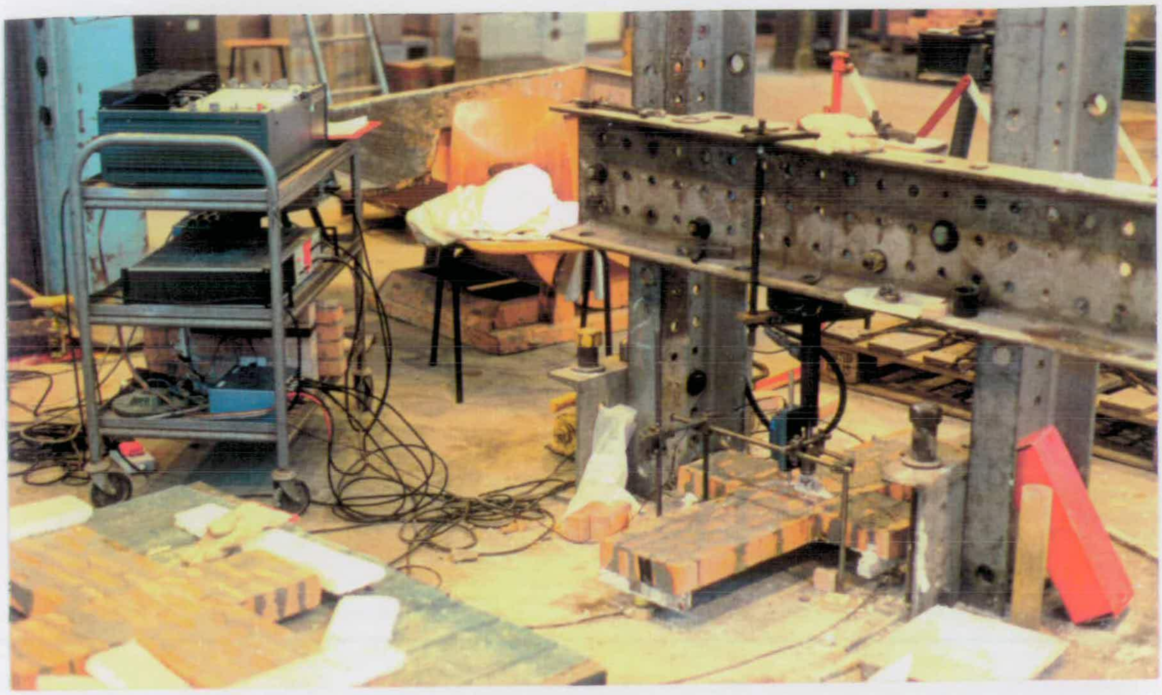


Fig. 4.6

Arrangements of equipment for a test on a cross-beam with aspect ratio 1:5

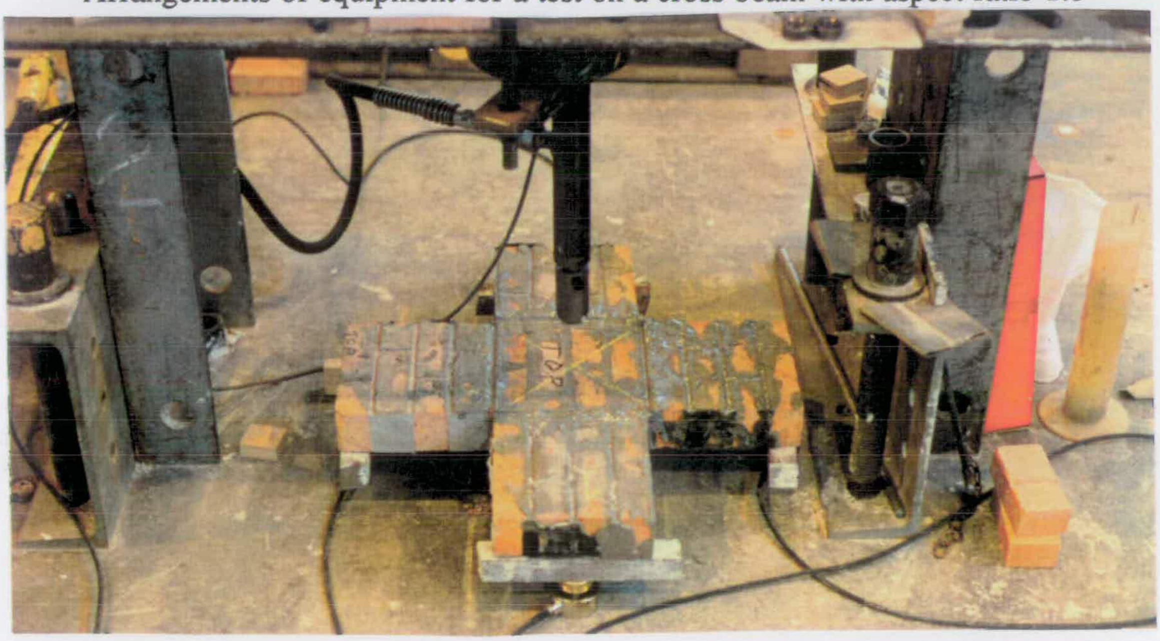


Fig. 4.7

Showing the position of the load cells used to measure the reactions cross-beams of Series III had electrical strain gauges fixed on both sides of the specimens to measure the strains to double check the load distribution.

4.3 EXPERIMENTAL RESULTS

The results of the various test are given in Tables 4.2 to 4.4. These tables contain the applied load, the measured reactions and the calculated moments from these reactions. It was decided to present the results in terms of moments rather than in terms of stresses as cracking could reduce the thickness of the sections, affecting the sectional moduli, thus the correct values of stresses at failure.

4.3.1 Discussion of the results of the Cross-beams of Series I

These specimens failed prematurely in shear along the bed joint connecting one of the vertical arms with the centre. The maximum flexural tensile stress develops at mid span and not near the junction of the arm where it failed. It is very likely that these parts were the weakest bed joints, due to the configuration and method of construction of the beams. Once failure happened in the vertical direction, all the applied load was resisted in bending only by the horizontal arms. Fig. 4.8 shows the three collapsed specimens. After the failure of the vertical arm the cross was not subjected to bi-axial bending any more, and the reactions dropped to zero as measured by the two load cells positioned at the edges of the beams. The failure of the vertical direction was followed by large displacements and the transference of the load to the horizontal strips. The failure of the beams happened only when the flexural tensile strength in the horizontal direction reached its ultimate strength. Table 4.2 presents the results of the tests and Fig. 4.9 shows a typical plot of the distribution of the applied load in both directions of the cross beams, as measured by the four load cells positioned at the edges. The ultimate moments in both directions are presented in that table together with the load that caused the failure in the vertical and horizontal directions. It is very clear that the beams exhibited reserve of strength after the failure in the y-direction. On average, the ultimate load is approximately 70% higher than the cracking load.

Table 4.2
Results of Series I flexural tests

	Cracking Loads (N)				Calculated Moments (N.mm)		
Specimens (Lx/Ly=1)	Applied Load: P_c	Measured Reactions		Ultimate loads: Pu (N)	Cracking Moments		Ultimate Moments
		Px	Py		Mx	My_u	Mx_u
1	1,183	706	477	1,947	1,103	307	2,780
2	1,070	586	484	1,875	889	290	2,642
3	917	545	372	1,444	859	241	2,228
Mean	1,057	612	444	1,755	941	279	2,550

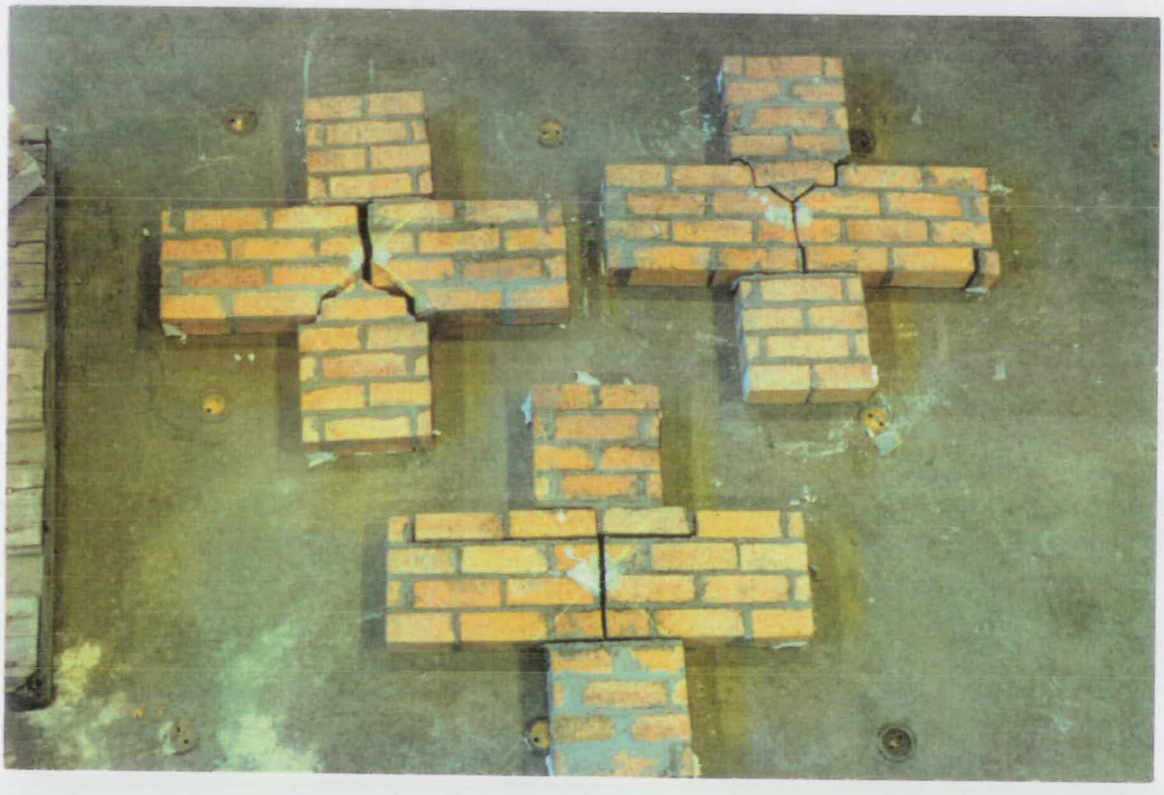


Fig. 4.8
Failure pattern of the cross-beams of Series I (aspect ratio 1)

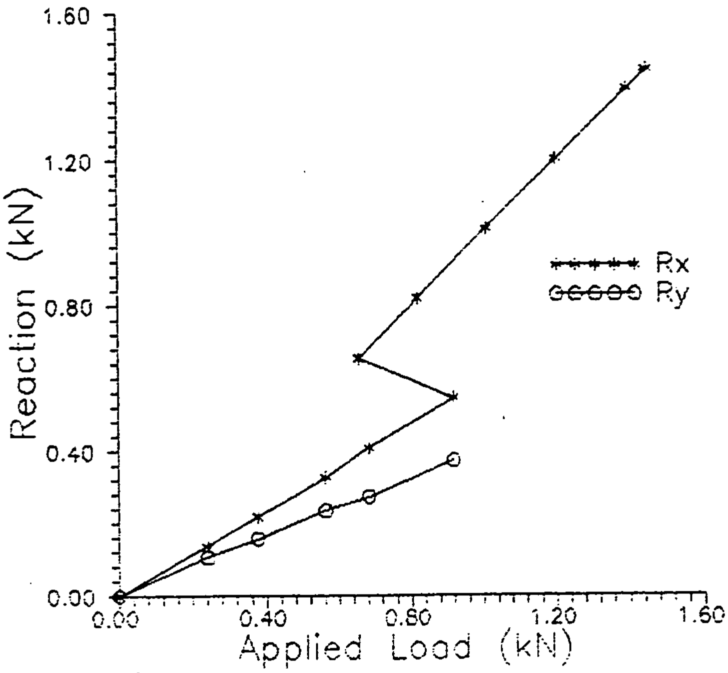


Fig. 4.9
Typical distribution of the applied load on both directions of Series I cross-beams.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

4.3.2 Discussion of the results of the Cross-beams of Series II

As explained earlier, the tests done in this series were an attempt to overcome the problems of failure by shear of the bed joint connecting the vertical arms with the centres of the cross beams. It was also intended to check the application of the yield line theory to analyse the lateral load behaviour of the cross beams, forcing the 'yield lines' to follow the ideal crack patterns assumed for this configuration of the structure.

As can be seen from Fig. 4.10 and 4.11, the failure happened always at the centres of all test beams through a diagonal, in a zig-zag fashion. The calculated moments using the measured reactions in both directions were many times higher than the ones obtained by using the test results from wallettes in uniaxial bending, given in Tables 3.10 and 3.11. Table 4.3 shows the results. The reason for these high moments in both directions may be due to the fact that the arms surrounding the centres act like a strong square ring, preventing not only cracking in the vertical and horizontal direction, but forcing it to fail in zig-zag fashion. The cross beams behaved as they were precompressed simultaneously in both directions. Fig. 4.12 to

4.13 show plots of the distribution of the load in both directions for the beams with aspect ratio 0.6 and 1. It can be seen, that these beams have in common the higher reaction in the horizontal direction and very little redistribution of the load near the collapse. The ultimate moment was reached first in the vertical direction. After this, it could not resist any further increment of load, though there was some reserve of strength in the horizontal direction. Therefore, all further applied load was resisted by the horizontal strip. In some cases, there were some reductions in the reactions measured by the load cells positioned under the edges of the vertical arms.

Table 4.3
Results of Series II Flexural Tests

Specimens	Applied Loads (N)	Measured Reactions (N)		Calculated Moments (N.mm)	
		Px	Py	Mx_u	My_u
A1	4,465	3,706	759	5,023	554
A2	5,784	5,300	484	2,310	355
A3	8,335	7,439	896	3,064	659
Average of Lx/Ly=058	6,125	5,452	713	3,465	523
B1	2,930	1,799	1,131	1,427	847
B2	3,204	1,772	1,432	1,406	1,071
B3	2,947	1,744	1,203	1,380	899
Average of Lx/Ly=1	3,027	1,772	1,255	1,404	941
C1	3,065	1,240	1,825	1,181	1,474
C2	2,779	994	1,785	946	1,432
C3	3,083	1,216	1,867	1,155	1,495
Average of Lx/Ly=1.5	2,976	1,150	1,826	1,094	1,467
D1	3,065	517	2,548	632	1,882
D2	3,443	595	2,848	727	2,101
D3	2,725	504	2,221	632	1,678
Average of Lx/Ly=2	3,078	539	2,539	664	1,887

Fig. 4.10
Showing the failure pattern of the cross-beams with aspect ratio 0.6

Fig. 4.11
Showing a group of failed cross-beams with aspect ratios 1, 1.5 and 2

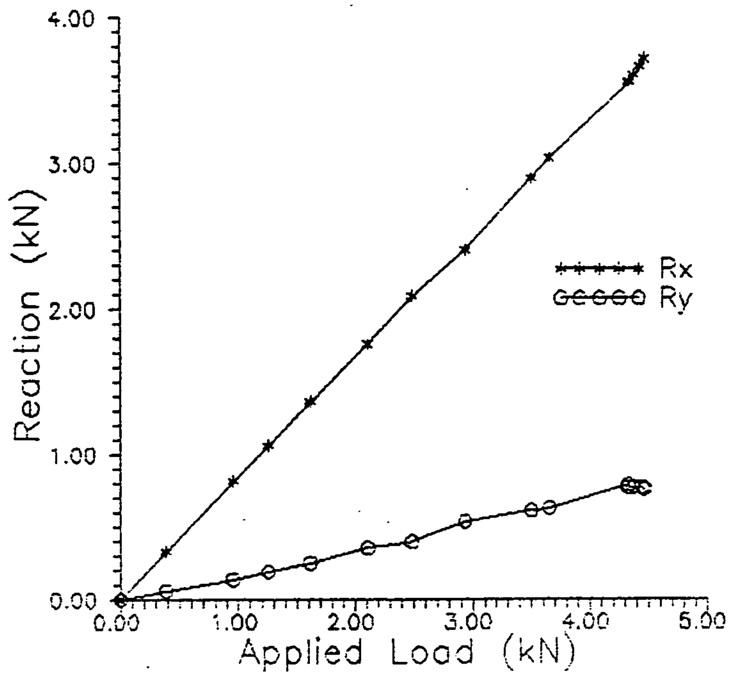


Fig. 4.12
Typical distribution of the applied load on both directions of Series II cross-beams.
Aspect ratio H/L=0.56.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

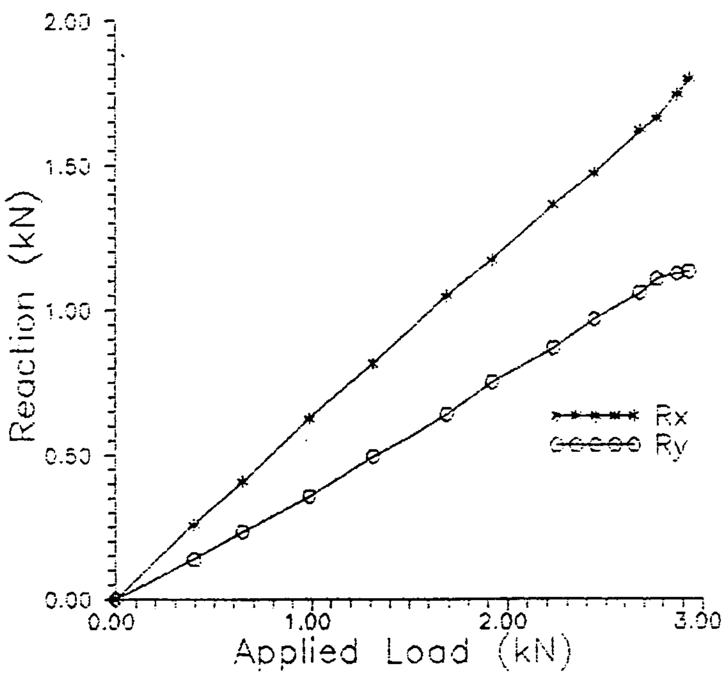


Fig. 4.13
Typical distribution of the applied load on both directions of Series II cross-beams.
Aspect ratio H/L=1.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

Cross-beams with aspect ratio 1.5 and 2 had different pattern distribution of the applied load compared to the specimens of aspect ratio 1. The measured reactions of the vertical arms of these specimens were higher than the horizontal arms. There was no redistribution of the load from one direction to the other compared to the specimens of aspect ratio 1, and failure was reached simultaneously in both directions. Fig. 4.14 to 4.15 show plots of the distribution of the load in the two orthogonal directions, as measured by the load cells.

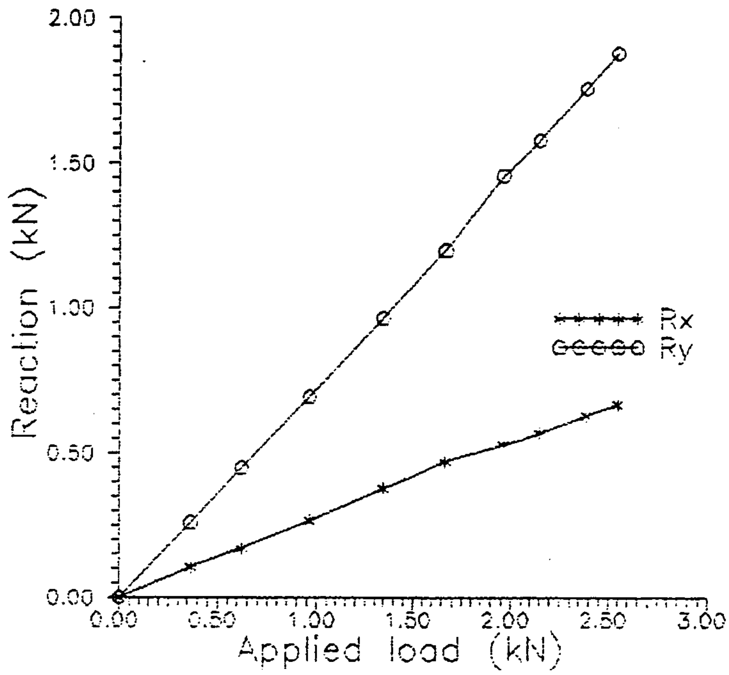


Fig. 4.14
Typical distribution of the applied load on both directions of Series II cross-beams.
Aspect ratio H/L=1.5.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

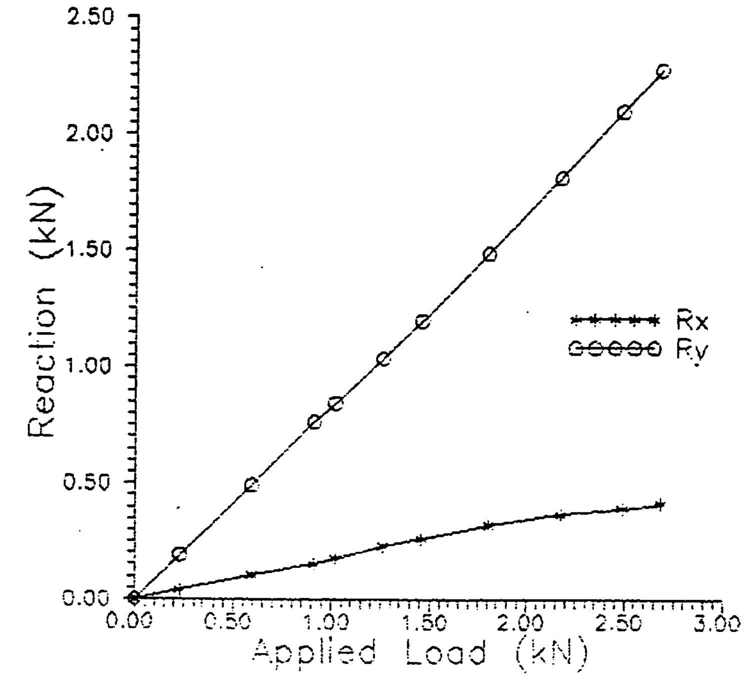


Fig. 4.15
Typical distribution of the applied load on both directions of Series II cross-beams.
Aspect ratio H/L=1.94.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

4.3.3 Discussion of the results of the Cross-beams of Series III

These cross-beams with arms like 'comb' did not present any initial cracks at the junction of the vertical arms with the centres. All specimens in this series cracked first in the centres along one of the horizontal joints, and, after this initial cracking, the load cells positioned under the edges of the vertical arms kept recording some reactions till the other orthogonal direction also failed. This was also confirmed by measurements taken with the electrical strain-gauges in both directions. Fig. 4.16 shows a typical result obtained using the rosette type strain-gauge on specimen A3. This residual strength of the vertical arms was measured on the three types of specimens tested in this Series and an average residual moment of 29% of the ultimate was obtained. Once the ultimate flexural tensile strength was reached in the vertical direction, the measured strains in this direction dropped dramatically as a consequence of the reduction of stiffness in the y-direction. At this time the jacking load also dropped due to large displacements. These displacements were measured by the electrical dial gauge positioned near the geometrical centre of the specimens. After the cracking in the y-direction all further applied load was supported by the horizontal arms, though the vertical arms kept supporting some load. An exception was specimen A3. Fig. 4.17 to 4.19 show the distribution of the applied load in the orthogonal directions of specimens having aspect ratio 1 (A1 to A3) and Fig. 4.20 shows a failed specimen. The residual stresses in the y-direction at the moment of failure of the x-direction were approximately 32% of the modulus of rupture of these specimens.

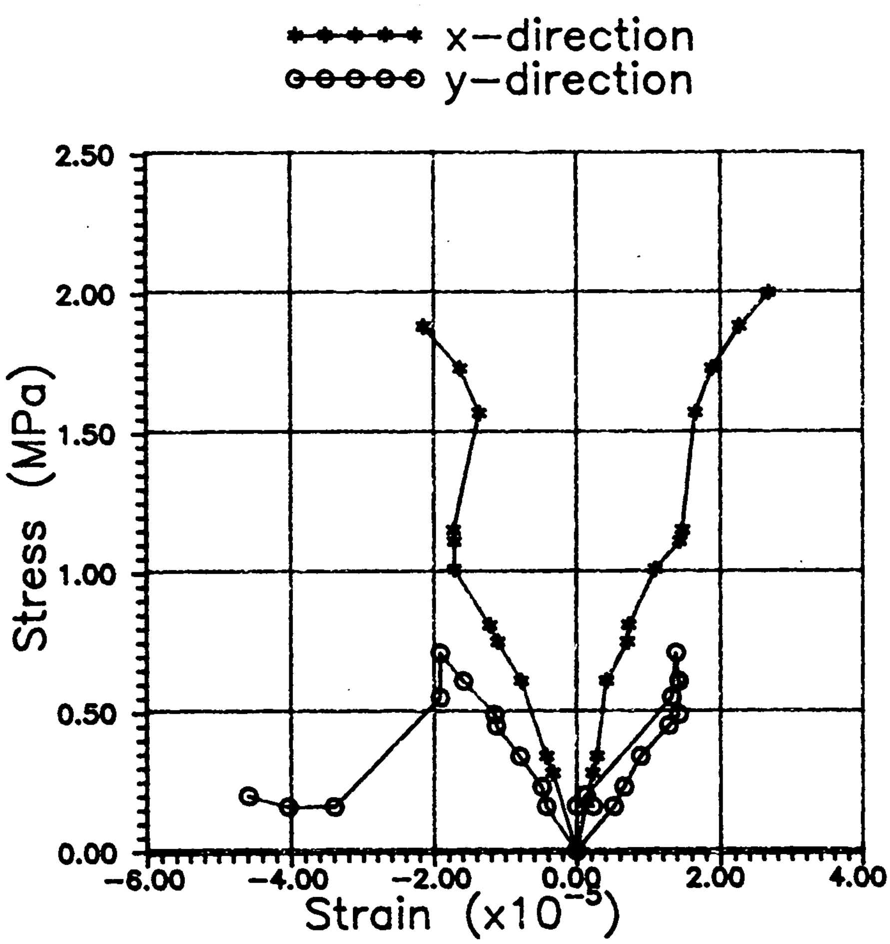


Fig. 4.16
Measured strains at the bottom (tensile) and at the top (compressive) of cross-beam A3

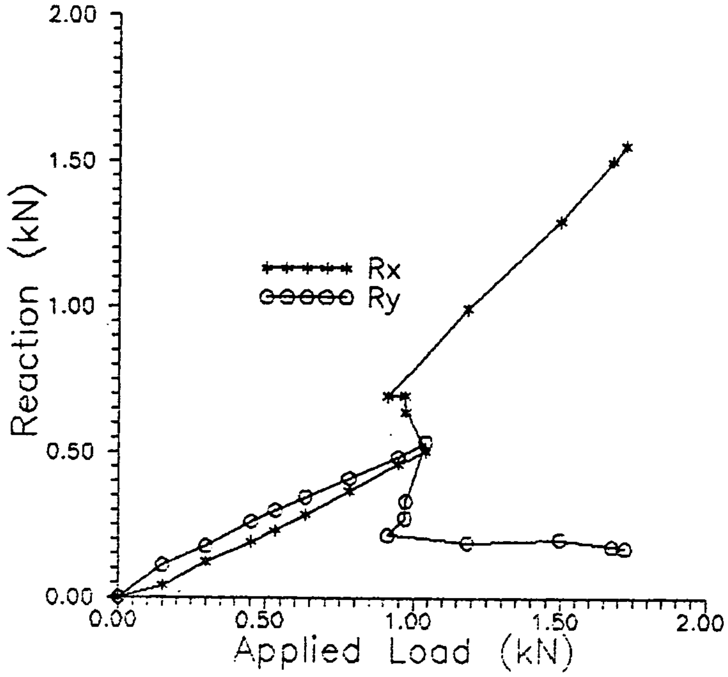


Fig. 4.17
Distribution of the applied load on both directions of Specimen A1: Series III
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

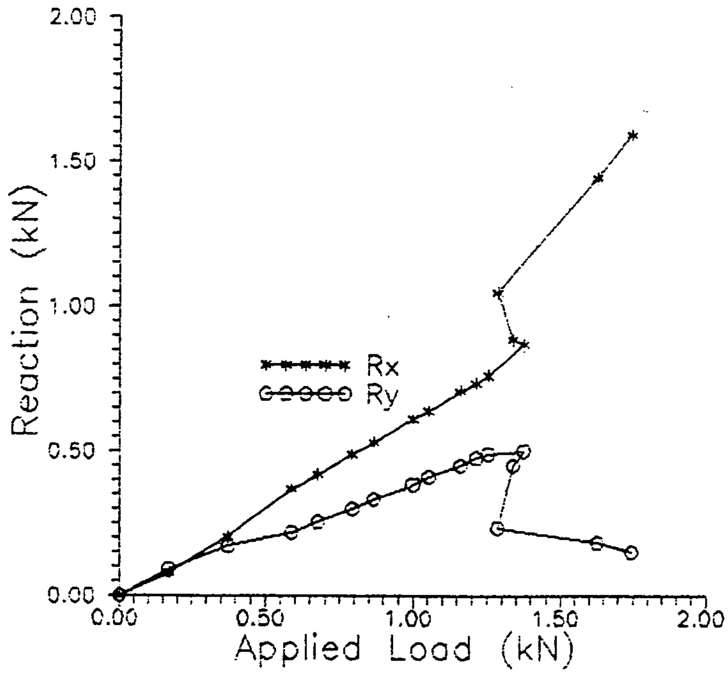


Fig. 4.18
Distribution of the applied load on both directions of specimen A2: Series III
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

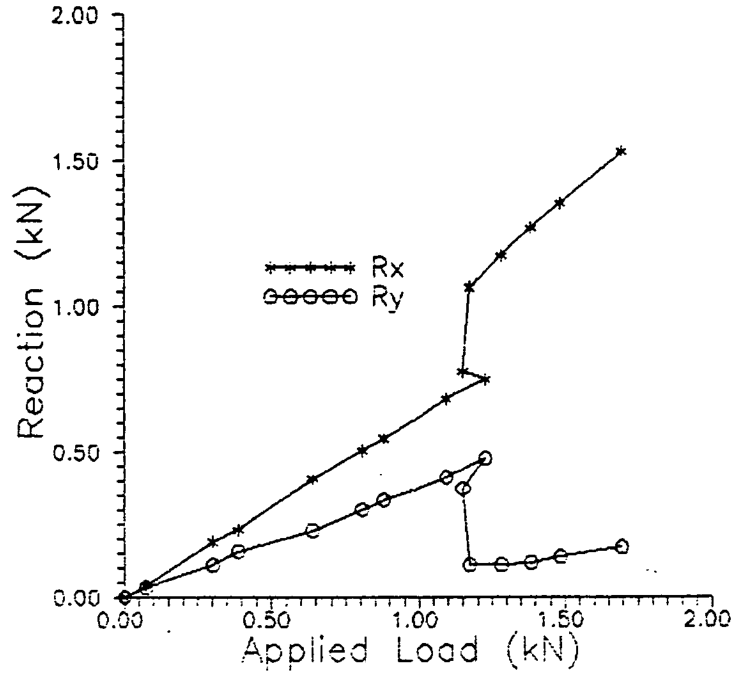


Fig. 4.19
Distribution of the applied load on both directions of specimen A3: Series III
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

Comparing the test results of Series I and III with aspect ratio 1, it can be seen, from the Tables 4.2 and 4.4, that the average ultimate load of specimens of Series I is slightly higher than specimens of Series III. Beside that, specimens of Series I had premature failure due to shear compared to specimens of Series III, hence the reserve of strength could not be easily compared. The average ultimate load of specimens A1 to A3 in Series III is 42% (Table 4.4) higher than the cracking load, confirming the great reserve of strength in horizontal direction. The average ultimate flexural tensile strength in the horizontal direction of specimens Series I and III has been calculated in order to study this difference. These averages were calculated assuming no reduction in the second moment of area of the specimens, and the values obtained are 2.36 and 2.25 MPa, respectively. This difference is not significantly higher than ones obtained testing wallettes in just one-direction, as shown in Chapter 3 (Tables 3.11 and 3.12).

Specimens B1 to B3 are the ones having aspect ratio approximately 1.5. For this configuration of cross-beams the vertical direction supported higher reaction compared to the x-direction. The collapse of these specimens happened almost immediately after the collapse of the vertical arms. As soon as the vertical arms failed, the load shifted almost immediately and reached the ultimate strength of the horizontal direction, provoking the collapse of the specimens. Only specimen B3 could resist a further increment of load after the failure of the vertical arms. Specimen B2 after the failure in the y-direction failed at the connection of one of the horizontal arms with the centres and, for this reason, it was not possible to record the stress on both directions after the initial collapse of the vertical direction. Fig. 4.21 to 4.23 present the reactions at the supports. The vertical direction carried moments equal to 12% of the ultimate moment in that direction even after cracking. This remained so till the final failure of the specimens in other direction, i.e. horizontal direction. Results are contained in Table 4.4.

Fig. 4.20
Collapsed specimen of Series III with aspect ratio 1

Table 4.4 Results of Series III Flexural Tests

Specimens	Experimental Loads (N)						Calculated Moments (N.mm)			
	At Cracking (N)			At Failure (N)			At Cracking		At Failure	
	Applied Load	Px	Py	Applied Load	Px	Py	Mx	My_u	Mx_u	My_r
A1	1,040	504	536	1,723	1,553	170	397	408	1,176	141
A2	1,375	872	503	1,744	1,594	150	669	382	1,207	125
A3	1,226	749	477	1,696	1,526	170	601	387	1,202	146
Average of Lx/Ly = 1	1,214	708	506	1,721	1,558	163	556	392	1,195	137
B1	996	301	695	909*	844	65	382	564	1,019	136
B2	1,282	451	831	1,282	451	831	556	669	-	78
B3	772	281	491	883	804	79	355	408	972	-
Average of Lx/Ly = 1.5	1,017	344	672	1,025	700	325	431	547	996	71
C1	742	170	472	743	634	109	282	476	993	115
C2	1,164	183	981	860*	778	82	303	758	1,213	125
C3	699	249	450	792	765	27	402	382	1,192	52
Average of Lx/Ly = 1.94	868	201	667	798*	726	72	329	539	1,133	97

*Load could not be metered after cracking.

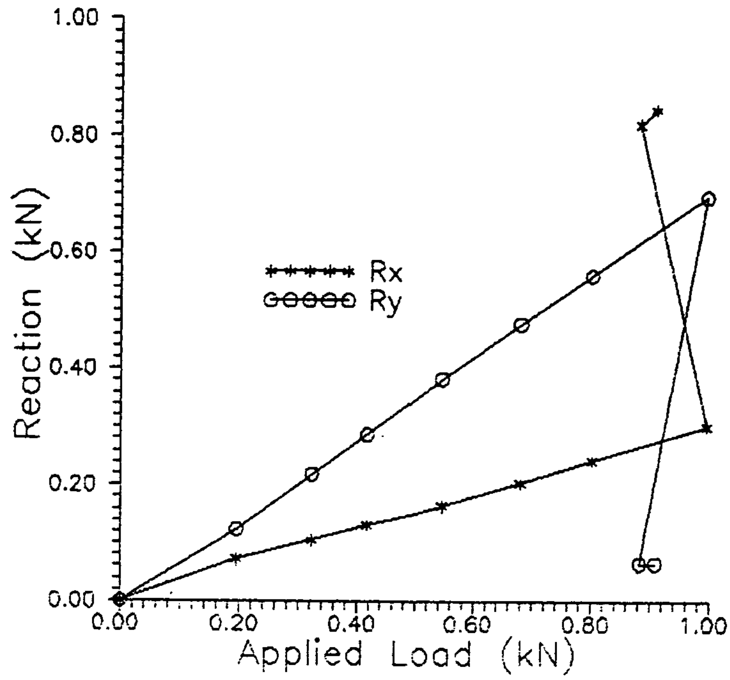


Fig. 4.21
Distribution of the applied load on both directions of specimen B1: Series III.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

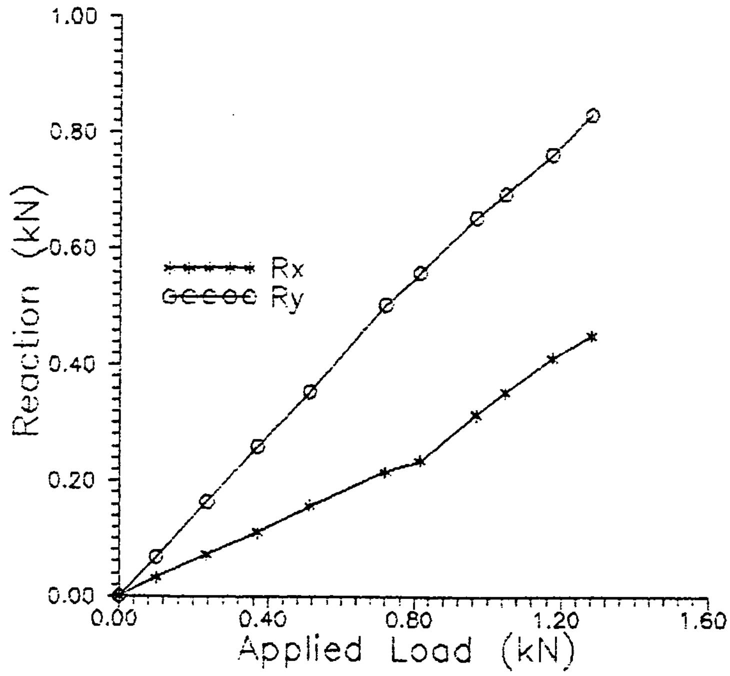


Fig. 4.22
Distribution of the applied load on both directions of specimen B2: Series III.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

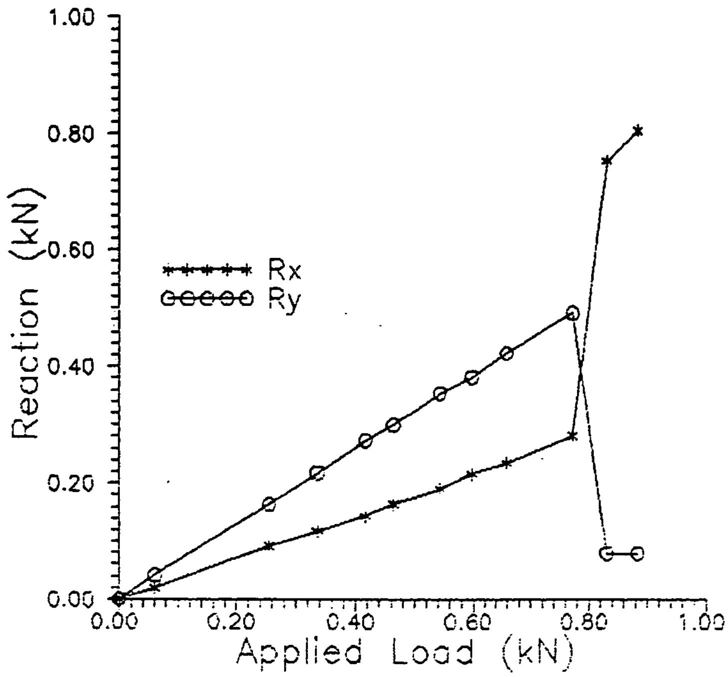


Fig. 4.23
Distribution of the applied load on both directions of specimen B3: Series III.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

Specimens C had the aspect ratio of 1.94. These cross-beam had the average final applied load lower than the average of the cracking load. Cracking in the vertical direction released energy that was picked up by the horizontal direction, causing the final collapse. Hence no reserve of strength was exhibited after the cracking of the y-direction. Only load redistribution was recorded. Specimen C3 is the only exception. This can be seen in Table 4.4 and Fig. 4.24 to 4.26.

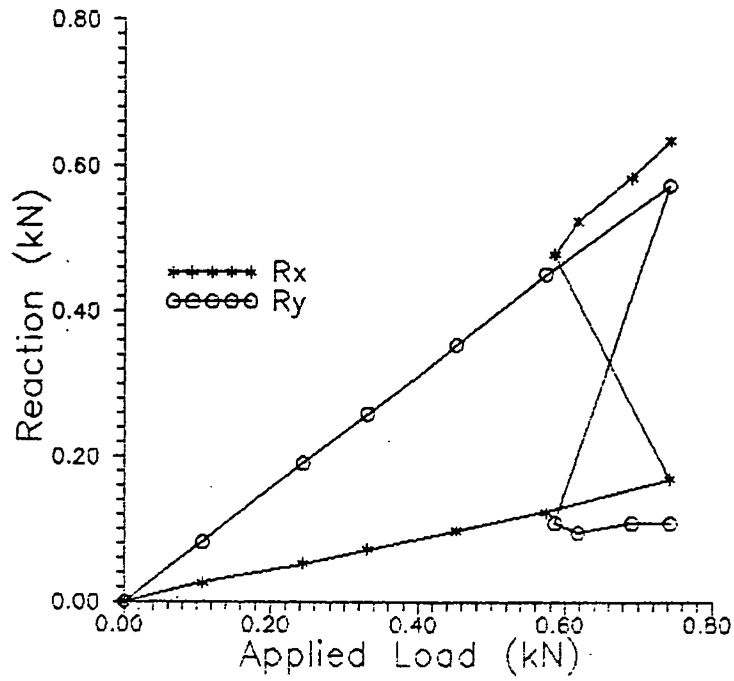


Fig. 4.24
Distribution of the applied load on both directions of specimen C1: Series III.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

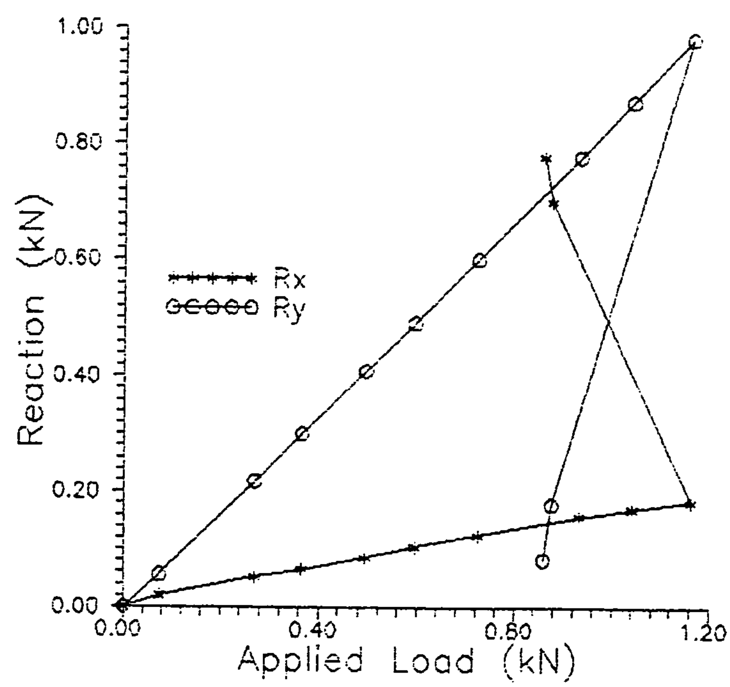


Fig. 4.25
Distribution of the applied load on both directions of specimen C2: Series III.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

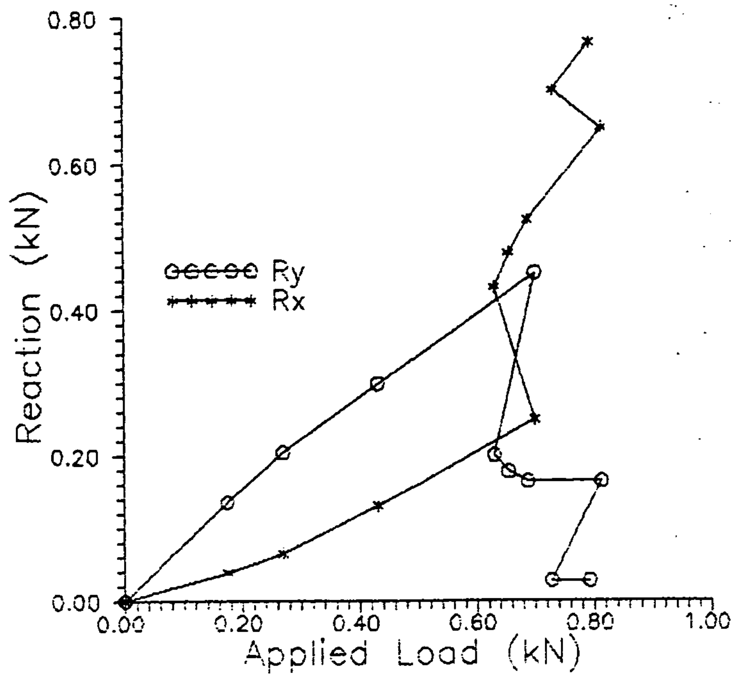


Fig. 4.26
Distribution of the applied load on both directions of specimens C3: Series III.
Rx=reaction in the horizontal direction
Ry=reaction in the vertical direction

In the tests performed by Baker[110] only single joints were subjected to bi-axial bending. Once these single joints failed in any of the two directions, the specimens also collapsed. This does not happen in some type of brickwork walls, like the ones simply supported along the four edges, which exhibit significant reserve of strength after the cracking pressure is reached. Consequently, those specimens tested by Baker do not reproduce the behaviour of brickwork subjected to combined horizontal and vertical moments simultaneously, as it was assumed. For the same reason, the wallettes tested by Gazzola et al[123] also do not reproduce the behaviour of brickwork in bi-axial bending, though the stresses were applied in different orientations to the bed joints. In those tests, as long as one of the horizontal joint failed the specimen also failed.

It is very difficult to extrapolate quantitatively this reserve of strength after the initial cracking of specimens having different configuration than the ones tested in this project. As the cracks did not perforate the full section this reserve of strength is due to membrane action.

4.3.4 Distribution of the applied load

From Table 4.5, it can be seen that to a large extent the applied load was distributed in the two orthogonal directions according to the stiffness orthotropy (E_x/E_y) as determined experimentally in Chapter 3. Some exceptions happened with specimens of Series III due to the uneven surface of the strong floor and the configuration of the 'comb arms'. As the load cells were very accurate, any small difference of level of the strong floor had to be corrected by using 'dental plaster' to fill these gaps between the test specimens and the supports, otherwise the four edges of the specimens would not rest over the supports altogether. Specimens of Series III with increasing aspect ratio were the most difficult ones to adjust. This can be seen from Fig. 4.18 and 4.26. Table 4.5 presents a comparison of the theoretical and experimental cracking load of some specimens having different aspect ratios. The theoretical cracking loads were calculated using the equations 4.5 and 4.6 presented in section 4.4.2. The experimental cracking loads are an average of the test results for a particular specimen.

Table 4.5
Distribution of the applied load

Aspect ratio	Series	P_x (N) (experimental)	P_x (N) (theoretical)	P_y (N) (experimental)	P_y (N) (theoretical)
0.58	II	5,482	5,274	713	851
1	I	612	606	445	451
1.5	III	344	289	672	727
2.0	III	200	125	668	743

4.3.5 Moments Interaction Diagram

The two set of specimens of Series II and III contain combinations of vertical and horizontal moments applied simultaneously to a brickwork specimen. It has been shown in the previous sections that the failure of these specimens did not happen in the same way as the failure of the one-direction wallettes presented in Chapter 3. A reserve of strength in the vertical direction was measured after cracking and the specimens only failed after the ultimate moment in the horizontal direction was reached.

Specimens of Series II had the flexural tensile strength higher compared to all the rest of the tests, while specimens of Series III had the values of the ultimate bending moments in both directions similar to the values obtained from the wallettes extracted from the undamaged part of the failed walls. Using the test results from Tables 4.3 and 4.4, two interaction diagram of moments are presented in non-dimensional form in the Fig. 4.27 and 4.28. The vertical axes of these diagrams are the ratio of the applied vertical moment and the ultimate moment in the vertical direction, i.e. M_y/M_{yu}. The horizontal axes are the ratio of the applied horizontal moment and the ultimate moment in the same direction, i.e. M_x/M_{xu}.

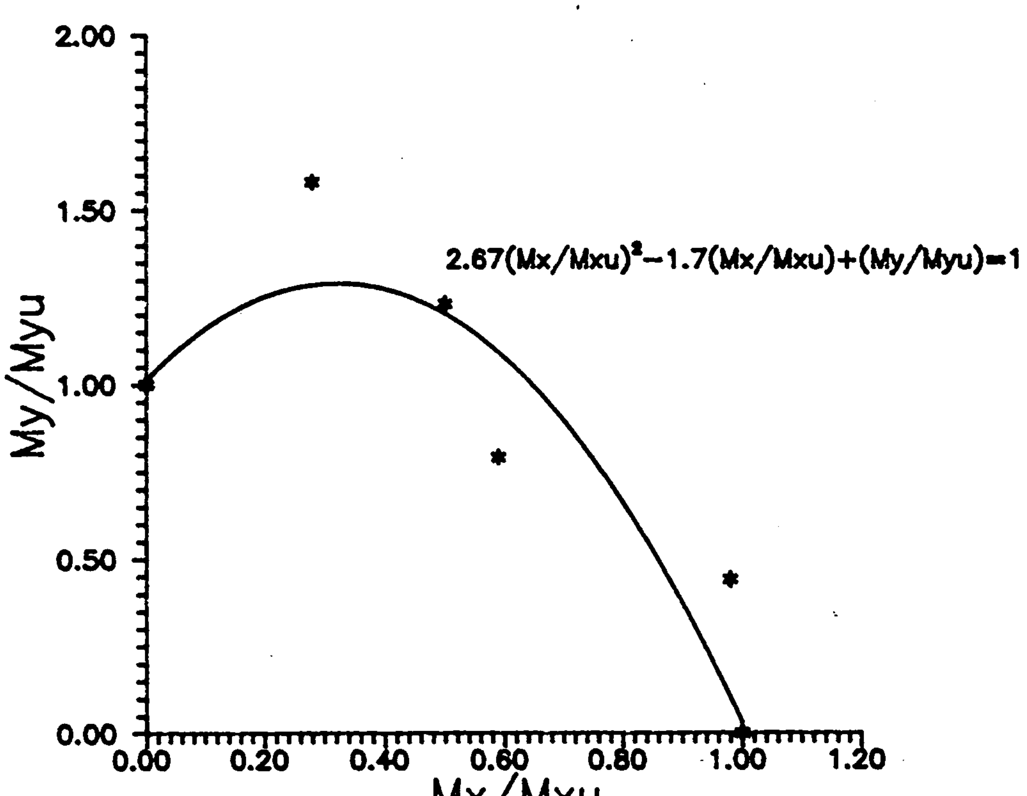


Fig. 4.27
Interaction of vertical and horizontal moments
(Series II tests)

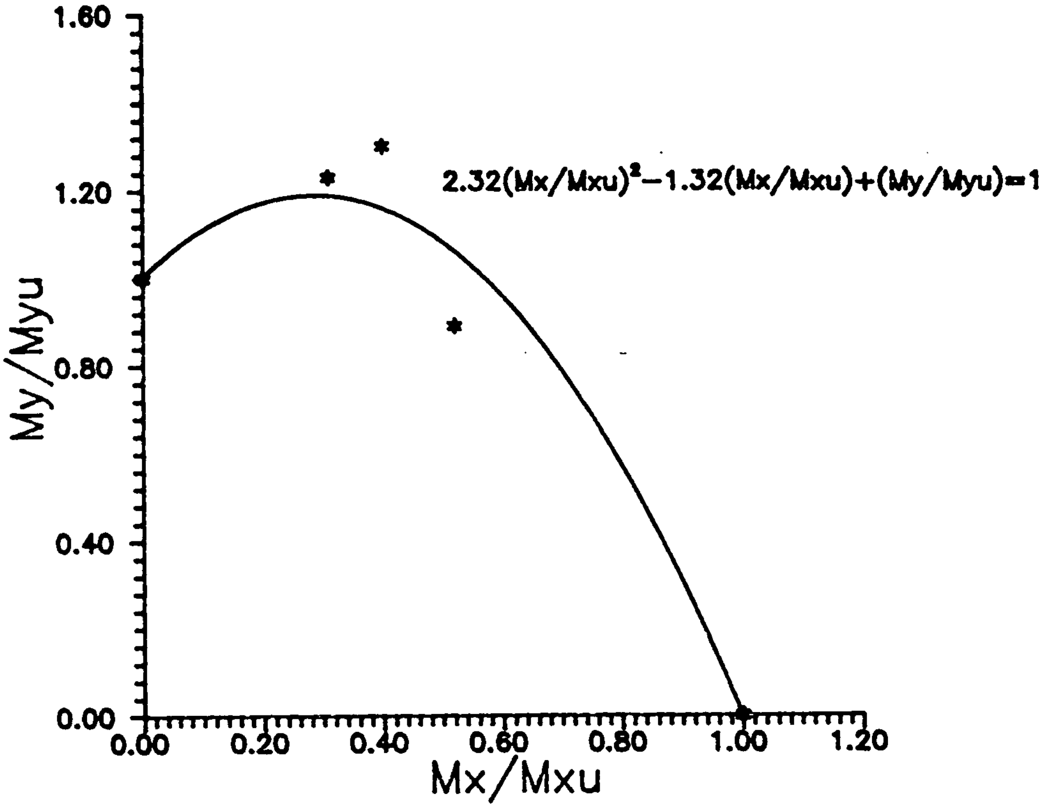


Fig. 4.28
Interaction of vertical and horizontal moments.
Cracking criterion of brickwork.
(Series III tests)

As the calculated moments of specimens of Series II were affected by the use of the epoxy resin in the arms, which forced failure in diagonal fashion, resulting higher moments in both orthogonal directions, wallettes were made and tested in the same way as the wallettes presented in Fig. 3.1. However, instead of the 1:3 cement mortar, the same epoxy resin utilised to make the arms of the specimens of Series II and III was used to join the bricks. Results are presented in Table 4.6. These results provided the uni-axial vertical and horizontal moments used in the moments interaction diagram of Series II.

The ultimate vertical and horizontal moment for specimens of Series III were calculated from the uniaxial flexural tests described in Chapter 3 and presented in Tables 3.10 and 3.11.

Table 4.6

Flexural tensile strength of wallettes built with the epoxy resin

Wallettes	f_{tx} (N/mm^2)	f_{ty} (N/mm^2)
1	4.48	2.29
2	4.22	2.17
3	4.98	2.38
mean	4.56	2.28

From the test results, the experimental interaction of moments in the two orthogonal directions at cracking is approximately represented by a second degree polynomial equation:

i) For Series II specimens,

$$2.67\left(\frac{M_x}{M_{xu}}\right)^2 - 1.67\frac{M_x}{M_{xu}} + \frac{M_y}{M_{yu}} = 1 \qquad (4.1)$$

ii) For Series III specimens,

$$2.32\left(\frac{M_x}{M_{xu}}\right)^2 - 1.32\frac{M_x}{M_{xu}} + \frac{M_y}{M_{yu}} = 1 \qquad (4.2)$$

The bi-axial flexural tests performed in the two different sets of cross-beams (Series II and III) resulted in similar diagrams of interaction of bending moments in the horizontal and vertical directions. Both diagrams show an increase of the flexural tensile strength in the vertical direction for the cross beams with aspect ratio 1.5 and 2.

The tests performed on the specimens of Series III showed in some cases significant reserve of strength after the cracking load was reached. Because of that, equation (4.2) represents the cracking criterion of brickwork subjected to bi-axial bending and not the failure criterion. The final collapse happens only after the applied moment reach the ultimate moment in the horizontal direction.

It is possible to estimate the penetration of the crack in the section working the residual moment in the vertical direction calculated from the measured reactions after cracking. The depth of the crack works out to be 28 mm for an average residual moment of 29% of the ultimate moment. Therefore it is possible to assume that half the section remained working till the collapse of the horizontal direction.

4.4 THEORETICAL ANALYSIS OF THE RESULTS

4.4.1 Introduction

The flexural behaviour of the cross-beams has been analysed by elastic and yield-line theory. The cross-beams are statically determinate structures, hence there was no necessity of using the finite element method, but a package program was used to determinate the moment distribution accurately.

4.4.2 Elastic analysis

Consider a simply supported cross-beam, as shown in Fig. 4.3, subjected to an applied point load P in the centre. This load will be shared by the strips in the x and y direction as:

$$P_x + P_y = P \qquad (4.3)$$

The deflection at the centre must be the same, which is given by:

$$\frac{P_x L_x^3}{48 E_x I_x} = \frac{P_y L_y^3}{48 E_y I_y} \qquad (4.4)$$

As $I_x = I_y$,

$$P_x = P_y \left(\frac{L_y}{L_x}\right)^3 \frac{E_x}{E_y} \qquad (4.5)$$

Substituting the value of P_x from equation (4.5) into (4.3)

$$P_y = \frac{P}{\left(1 + \left(\frac{L_y}{L_x}\right)^3 \frac{E_x}{E_y}\right)} \qquad (4.6)$$

Having calculated the load in the y-direction, the moments will be given by:

$$M_y = \frac{P_y L_y}{4} \qquad (4.7)$$

and

$$M_x = \frac{P_x L_x}{4} \qquad (4.8)$$

For the cross-beams of various aspect ratios, the theoretical load and cracking moments can be calculated from the equations (4.6) and (4.7). The values of the elastic moduli were those described in Chapter 3, section 3.3.3. The cracking moment was obtained from:

$$M_{y(\text{cracking})} = f_{ty}\, Z \qquad (4.9)$$

Hence, from (4.7) and (4.9)

$$P_y = \frac{4\, f_{ty}\, Z}{L_y} \qquad (4.10)$$

and

$$P_x = P_y \left(\frac{L_y}{L_x}\right)^3 \frac{E_x}{E_y} = \frac{4\, f_{ty}\, Z}{L_x^3} . L_y^2 . \frac{E_x}{E_y}$$

The theoretical values for Px and Py were calculated from equations (4.10) and (4.11) and compared with those measured for the cross-beams in Table 4.5. In calculating the values the dead weight of the cross-beams was accounted for.

The failure of the cross-beams was assumed to happen, when the moment exceeded the failure moment in the x-direction with no interaction of the moment in the y-direction.

At failure, the load is given by:

$$P_u = f_{tx}\, Z \qquad (4.12)$$

However, the load calculated this way will be lower than the experimental, as y-direction carried the residual bending moment even after cracking. From the measured reactions, an average of 29% of the ultimate moments in the vertical directions was calculated (Table 4.4), when ultimate strength in the horizontal direction was reached. Hence, for all specimens that have the failure load lower or equal to the cracking load, the failure load can be obtained by:

$$P_u = 4\left(\frac{M_x}{L_x} + \frac{0.29M_y}{L_y}\right) = 4\,Z\left(\frac{f_{tx}}{L_x} + 0.29\frac{f_{ty}}{L_y}\right) \qquad (4.13)$$

which takes into account the reserve of strength.

4.4.3 Finite Element Analysis

The results were analysed using the finite element technique. An eight noded plate element has been used to simulate brickwork. Each node has three degrees of freedom; one axial displacement and two rotations. The integration rule uses a 9x9 point Gauss quadrature. The mesh divided the centre and the four arms of each cross-beams in 16 elements, resulting an amount of 80 elements per each specimens.

The elastic constants used were the ones presented section 3.3.3, and from those the shear modulus was calculated: G=5,565 N/mm^2. The output of the programs was analysed considering two approaches to calculate the ultimate moments:

i) The usual assumption of no interaction between bending moments in the two orthogonal directions, i. e. the ultimate moments were calculated using the flexural tensile stresses obtained performing the tests on the one-directional wallettes extracted from the undamaged parts of the failed walls.
ii) The cracking criterion shown in Fig. 4.28.

4.4.4 Yield-line analysis

The yield-line analysis has been done using the principle of minimising the work done. The patterns of cracks at failure is shown in Fig. 4.29.

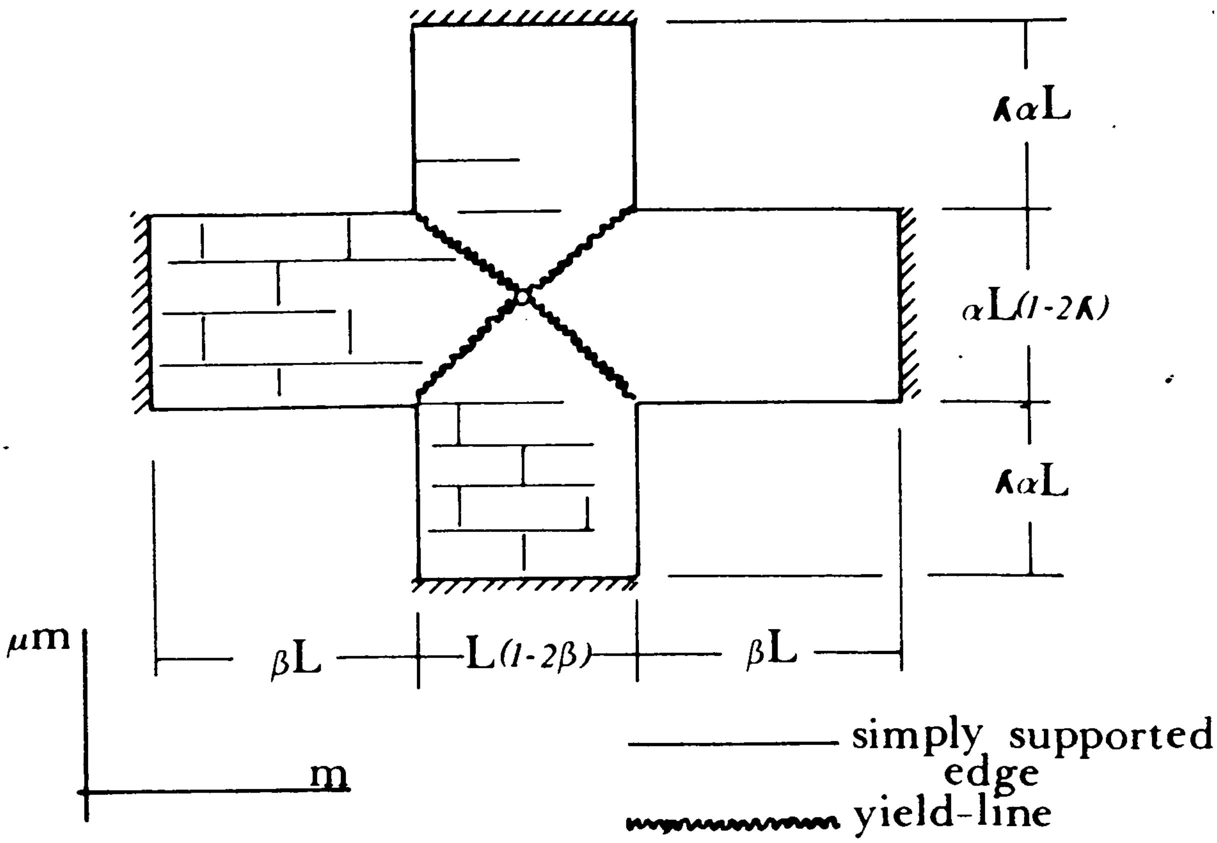


Fig. 4.29

Crack pattern for yield-line analysis of the cross-beam

If a virtual deflection of unity is given to the central point of the cross beam, the external work done by the applied point load (P) is given by:

$$\text{External work done} = P \times 1$$

The internal dissipation of energy along the yield lines crossing the beam in two diagonals is equal to $\Sigma(mL_x\theta_x + \mu mL_yq_y)$. For unit deflection, it is possible to assume that $\tan\theta_x = \theta_x$ and $\tan\theta_y = \theta_y$. As $\tan\theta_x = 2/\alpha L$ and $\tan\theta_y = 2/L$, the internal work done along the "x" and "y-axis" by the diagonal yield lines is given by

$$\Sigma M_x\theta_x = \frac{4m}{\alpha}(1 - 2\beta) \text{ and } \Sigma M_y\theta_y = 2\mu m\alpha L(1 - 2\lambda).$$

The total internal dissipation of energy is

Table 4.7
Comparison between predicted and experimental loads for Series I and III Specimens

	Cracking Loads (N)					Ultimate Loads (N)				
	Experimental		Theoretical			Experimental		Theoretical		
Aspect Ratio (Lx/Ly)	Series III	Series I	Finite Element	Finite Element using the Cracking Criterion	Elastic Theory	Series III	Series I	Elastic Theory	Elastic Theory using the Failure Criterion	Yield Line
1	1,214	1,057	1,116	1,129	1,318	1,721	1,755	1,396	1,532	2,042
1.5	1,017	-	834	914	766	1,025	-	943	1,022	1,488
2	868	-	742	839	636	798	-	730	762	1,277

Table 4.8
Comparison between predicted and experimental loads for Series II Specimens

Aspect ratio (Lx/Ly)	Experimental Ultimate Load	Yield Line
0.58	6,125	7,339
1	3,027	4,862
1.5	2,976	4,639
2	3,078	3,538

$$\Sigma(mL_x\theta_x + \mu mL_yq_y) = 4m\alpha\left(\frac{1-2\beta}{\alpha^2} + \mu(1-2\lambda)\right)\text{.........}(4.14)$$

As the external work done by the central point load must be equal to the internal dissipation of energy, the predicted failure point load is given by

$$P = 4m\alpha\left(\frac{1-2\beta}{\alpha^2} + \mu(1-2\lambda)\right)\text{..}(4.15)$$

This equation has been used to calculate the predicted failure load of the cross-beams of Series II and III. The orthotropy strength (f_{tx}/f_{ty}) and the ultimate moment were taken from the average of the three specimens tested in each set.

4.5 COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL LOADS

In this section a comparison between the results of the theoretical analysis and experimental work is presented. As each set of cross beams having the same configuration had only three specimens, all comparisons were done using the average of the cracking and the failure loads of each set. This comparison is presented in Table 4.7 for specimens of Series I and III and in Table 4.8 for specimens of Series II.

4.5.1 Comparison between experimental and theoretical cracking loads

Elastic analysis using Grashoff-Rankine theory and the finite element method have been used to predict only the cracking loads (P_c), as they do not take into account any reserve of strength exhibited by the other orthogonal direction. The cracking criterion has also been applied combined with the finite element method.

From the result presented in Table 4.6 it is clear that all methods overestimate the cracking loads of specimens of Series I, confirming the premature cracking at the junction of the vertical arms with the centres. It is also clear that the elastic theory predicts reasonably the loads only for specimens with aspect ratio 1. The other two sets of specimens are underestimated. It has been assumed in this analysis that failure happens if the ultimate bending moment of any one direction is reached.

On the other hand, the finite element method offers reasonable agreement with the experimental cracking loads.

The cracking criterion, established in section 4.3.5, combined with the use of the finite element method improves these agreements in all cases, particularly for specimens having aspect ratio 1.5 and 2, as it accounts for the increase in strength of the vertical direction of these specimens.

4.5.2 Comparison between theoretical and experimental ultimate loads

Elastic and yield line analysis have been used to predict the failure loads. This comparison is presented in Tables 4.7 and 4.8. From these tables it can be seen that elastic theory underestimates the failure load of specimens having aspect ratio 1, but it reasonably predicts for specimens having aspect ratio 1.5 and 2. This underestimation is probably due to the high values of flexural tensile strength in the x-direction obtained from the specimens with aspect ratio equal to one. These high values are not unusual and similar test results were obtained with some specimens in uni-axial bending, as it can be seen in Table 3.10.

Because the reactions at the supports were measured, the bending moments could be calculated along a cracked section of unreinforced brickwork. These residual bending moments have been taken into account to predict the failure loads of the specimens tested in this investigation in equation (4.15). The use of this equation combined with elastic analysis improves the correlation between the experimental failure loads, as it accounts for the reserve of strength in the vertical direction after the cracking.

Yield-line analysis assuming the strength orthotropy obtained from the wallettes' tests presented in Tables 3.10 and 3.11 overestimates the failure load of all set of specimens analysed. This strength orthotropy was from uniaxial bending tests. In case of cross-beams, residual moments ranging from 23 to 37% were calculated from the measured reactions after cracking in the y-direction till failure. Hence, at the time of failure the average strength orthotropy was greater than 2.42 as calculated from the wallettes' tests. The strength orthotropy at failure works out to be 8.46 from the test results of the cross-beams. If this strength orthotropy is taken into account for calculating the failure load, the yield-line equation (4.15) agrees well, as it can be seen in Table 4.9.

Table 4.9

Analysis of cross-beams (Series III) using different strength orthotropy

Aspect ratio (Lx/Ly)	Experimental ultimate load (N)	Yield-line ultimate load (N) $\mu=2.42$	Yield-line ultimate load (N) $\mu=8.46$
1	1,721	2,042	1,614
1.5	1,025	1,488	1,086
2	868	1,277	864

5.5 CONCLUSIONS

On the basis of the tests described in this Chapter, the following conclusions can be drawn:

i) the applied point load was distributed and transferred to the supports according to the stiffness orthotropy experimentally measured using the prisms extracted from the test walls (section 3.3.3);

ii) unreinforced brickwork is a brittle material. After the cracking load is reached in one direction, there in no yielding of the material as the bending moment is not kept constant. Nevertheless, bending moments were calculated from the measured reactions in the direction perpendicular to the cracks. This reserve of strength after cracking may be due to membrane action, as the cracks did not perforate the full depth of the section. Even for specimens that were forced to crack in diagonal fashion (Series II), predictions of ultimate loads using yield-line analysis overestimated the experimental failure loads;

iii) the cracking criterion for unreinforced brickwork in bi-axial bending has been developed and is given by equation (4.2)

$$2.32\left(\frac{M_x}{M_{xu}}\right)^2 - 1.32\frac{M_x}{M_{xu}} + \frac{M_y}{M_{yu}} = 1$$

Due to moment interaction this cracking criterion shows that the flexural tensile strength perpendicular to the bed joints (along the vertical direction) can be enhanced beyond its ultimate value obtained performing flexural tests on wallettes spanning in just one direction. This cracking criterion combined with the use of the finite element method for orthotropy plates predicts accurately the cracking loads of the test specimens;

iv) for all specimens that have the failure load lower or equal to the cracking load, failure can be predicted by equation (4.13)

$$P_u = 4\left(\frac{M_x}{L_x} + \frac{0.29M_y}{L_y}\right) = 4\ Z\left(\frac{f_{tx}}{L_x} + 0.29\frac{f_{ty}}{L_y}\right)$$

This equation takes into account the reserve of strength after the cracking of the vertical direction.

CHAPTER 5

BRICKWORK PANELS WITH OPENINGS SUBJECTED TO WIND LOADING

5.1 INTRODUCTION

Brickwork cladding panels are subjected to wind loading. These panels often contain window openings. The lateral strength of brickwork panels without openings has been the subject of investigation for long time, but panels containing openings have not been investigated to any extent. Some tests on panels with openings are available[130,139], but those which are available have ignored the line loading which develops naturally at the edges of a window opening as a result of wind pressure. Also, no definitive mathematical solution is available at present for panels with window openings subjected to wind loading. The only suggestion is presented in BS 5628: Part 1, Appendix D[86], to divide the panels into sub-panels and then to design each part either in accordance with the rules given in clause 36 or by the yield-line or elastic analysis. As there was no experimental data available for the design of such panels, the present investigation was undertaken.

The behaviour and lateral strength of brickwork panels with openings depend on various factors. In this investigation, the following factors were considered:
i) aspect ratios;
ii) boundary conditions; simply supported on three or four sides; and
iii) disposition of the window openings; symmetrical and unsymmetrical.

The results were analysed and compared with the elastic plate bending theory, the strip method and the yield-line theory. The results were compared with the provisions of the BS 5628 and some recommendations are made for the design of panels subjected to wind pressure.

5.2 EXPERIMENTAL DETAILS

5.2.1 Wall configuration

Half-scale bricks were used to build the 16 test walls in 1:3 (rapid-hardening cement:sand) mortar. As the flexural strength is very variable, the brickwork panels tested were replicated. Properties of bricks and mortar were already presented in section 3.2.1. The same bricklayer was used during the entire experimental programme. The dimensions of the wall panels and the position of the window openings are shown in Fig. 5.1. The shortest panels were 1200mm x 1200mm and the longest ones were 1200mm x 1800mm. With the exception of Walls 11, 12, 13 and 14 (Fig. 5.1), the rest of the walls had the window opening positioned in the centre. The panels were simply supported on four sides or simply supported on three sides and free on vertical or top edge. The position of the window openings and the boundary conditions covered a wide range of shapes of walls found in buildings.

A ply board sheet was used to represent the closed window which transferred the wind pressure to the edges of the window opening. It was found that owing to the different deformation properties of brickwork and the ply board sheet, the load was transferred as point loads at the corner of the opening. Hence, in order to improve the modelling for the theoretical analysis, it was decided to transfer the pressure from the ply board as four equal point loads through four wooden studs fixed at the corners of the test walls, which gave the exact determinate values of the reactions.

5.2.2 Test arrangements

The wall panels were tested in a special steel frame built on the strong floor of the Structural Laboratory. The testing apparatus consists of:

i) a frame that provided the supports for the test walls;

ii) a loading system that provided a uniform distribution of the load and control of the loading; and,

iii) necessary instrumentation for the measurements of the deflections.

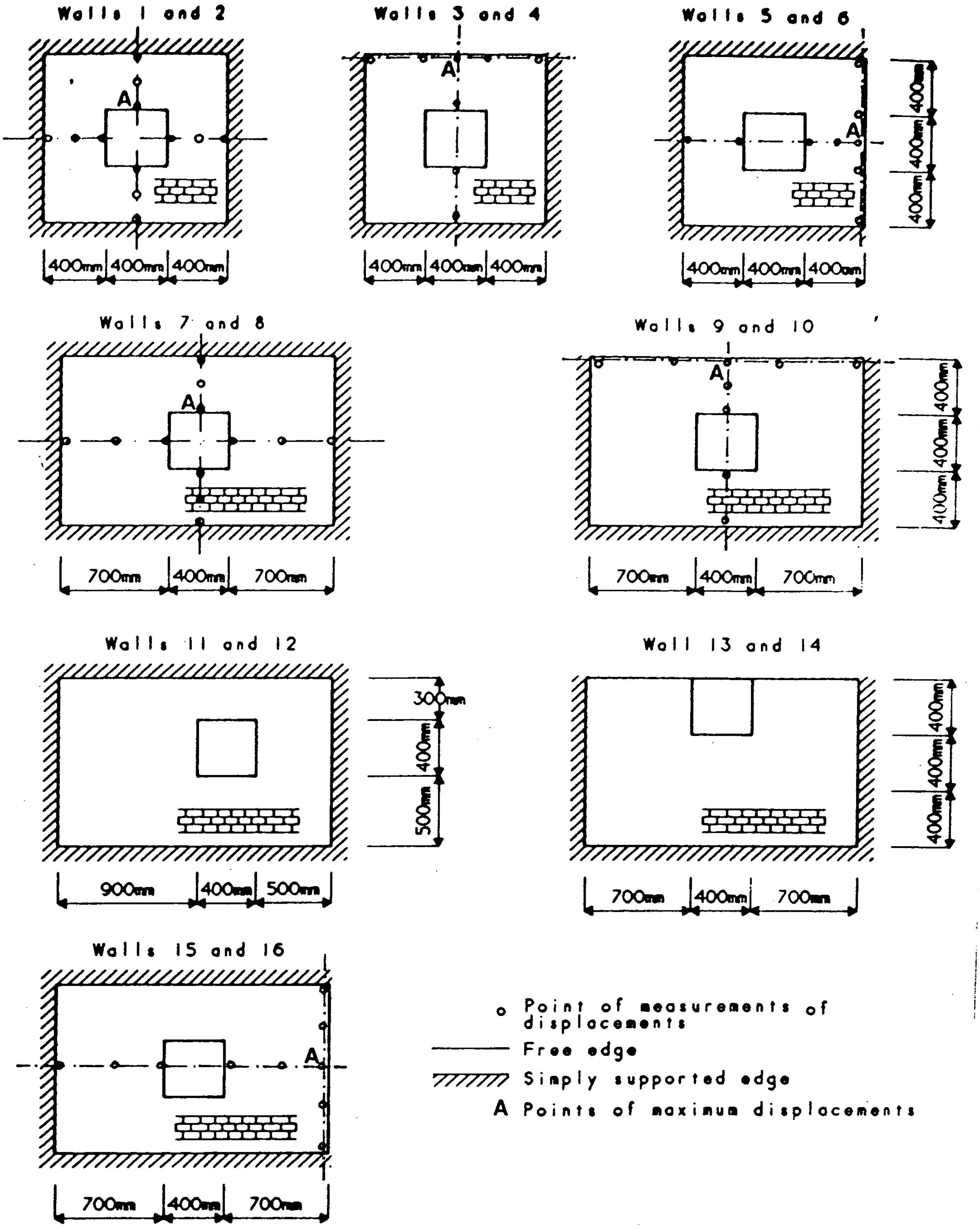


Fig. 5.1
Wall configurations

The frame was stiff enough to avoid the test results to be affected by any deflection due to the lateral loading inducing secondary forces. Another required condition was to suit all the boundary conditions and panel dimensions to be tested. Fig. 5.2 shows an outline of a testing frame and the panel.

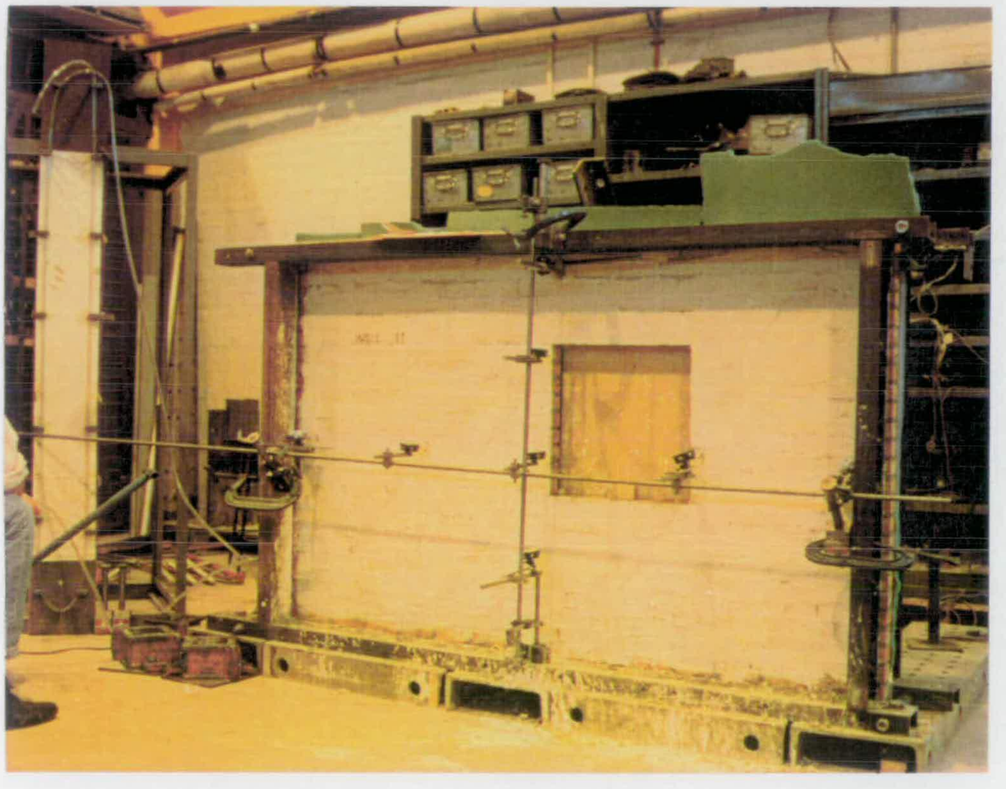


Fig. 5.2
The set up of a wall test: manometer, frame and disposition of the dial-gauges

The panels were built against the reaction frame and the lateral loading was applied in steps of 0.4 kN/m^2 through an air-bag sandwiched between the test wall and the loading frame. The air-bag had no divisions and two sizes of air-bags were used, according to the dimensions of the test walls. Before the start of every test the air-bag was inspected in order to detect any air leak which could affect the loading system. As the whole experimental program on the walls took almost two years, some parts of the air-bags became brittle and repairs and replacements had to be done. However the same type of air-bag using polythene sheets was used in all tests. The pressure was supplied by an air-compressor and measured by the water manometer. This system proved to be satisfactory for the application and control of the loading.

For every step of loading, the displacements at various points on the walls were measured by dial gauges. All displacement readings used to take approximately 1 min. The points at which the displacements were measured are given in Fig. 5.1 for some of the test walls. The instrumentation also accounted for deflections of the supports, as can be seen in Fig. 5.1. This configuration provides measurements of deflections on a vertical and horizontal profiles through the mid-height and mid-length of the test wall respectively. The number of dial gauges varied from 7 to 12, according to the dimensions and boundary conditions of the test walls. Before the start of the tests every wall was painted in white using a mixture of 'dental plaster' and lime, for better detection of the cracks. Cracking was detected visually and by sudden changes in the loading-deflection relationship. After failure, undamaged parts of the walls were used to obtain wallettes to determine the flexural tensile strengths in both directions.

5.2.3 Experimental results

The results are given in Table 5.1. There is some variation between some of the replicates, which is no way unusual, and it was also found by others[116] .

Table 5.1
Wall test results

Walls	Cracking pressure (kN/m^2)	Ultimate pressure (kN/m^2)
1	5.0	7.9
2	5.2	10.2
3	---	7.8
4	---	7.2
5	4.0	7.4
6	3.2	6.8
7	2.6	5.4
8	4.0	6.4
9	---	3.1
10	---	3.9
11	5.6	6.6
12	4.2	7.2
13	---	4.4
14	---	3.0
15	1.8	2.6
16	2.2	2.6

5.2.3.1 Deflections

Some typical load-deflection relationships are presented in Fig. 5.3 to 5.14. Vertical and horizontal profiles are included together with the wall behaviour at the points of maximum displacements. In addition, displacements predicted by a computer program using the finite element method for the cracking pressures are also shown along with the experimental results. As can be seen from Fig. 5.3 to 5.8 the load-deflection relationship of all walls is non-linear. A line connecting each points where the dial-gauges were positioned on the walls presents the load-deflection behaviour in the horizontal and vertical directions in Fig. 5.9 to 5.14. This method was chosen instead of joining the points with a smooth curve.

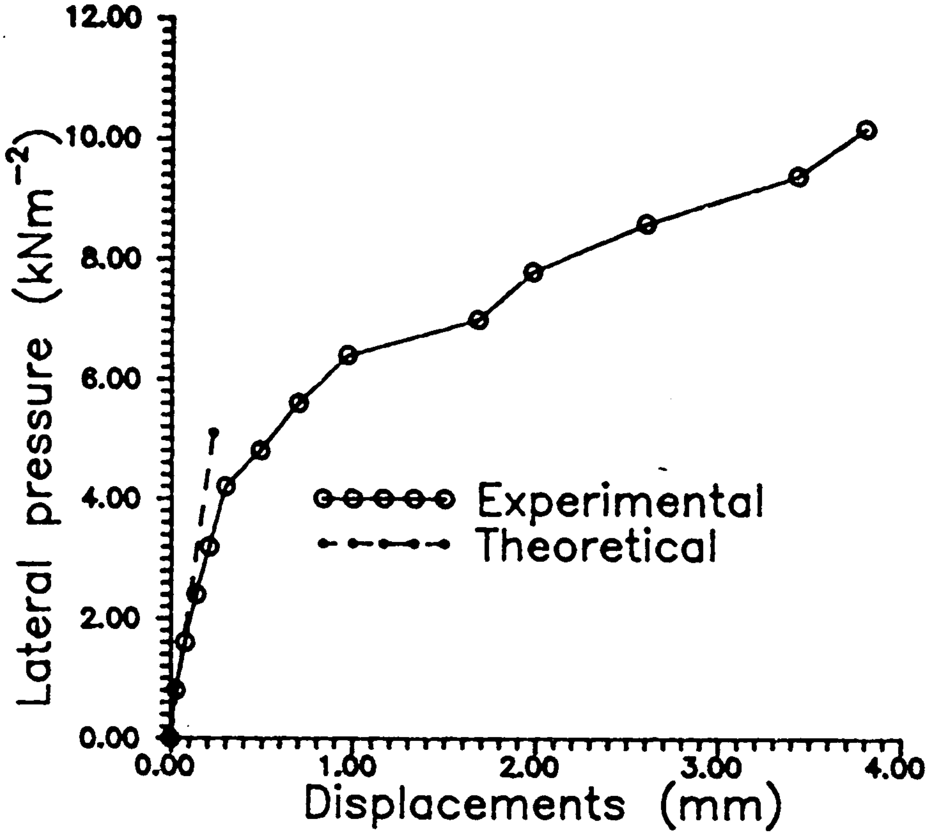


Fig. 5.3

Load-deflection relationship of Wall 2 at point A (aspect ratio 1:1)

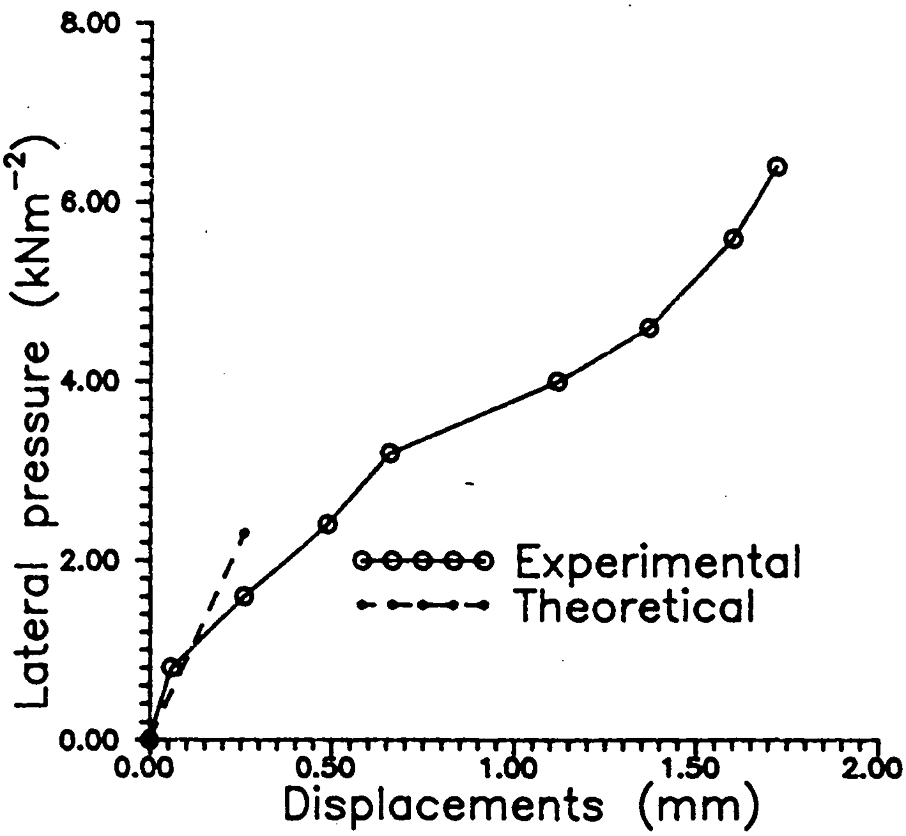


Fig. 5.4

Load-deflection relationship of Wall 8 at point A (aspect ratio 1:1.5)

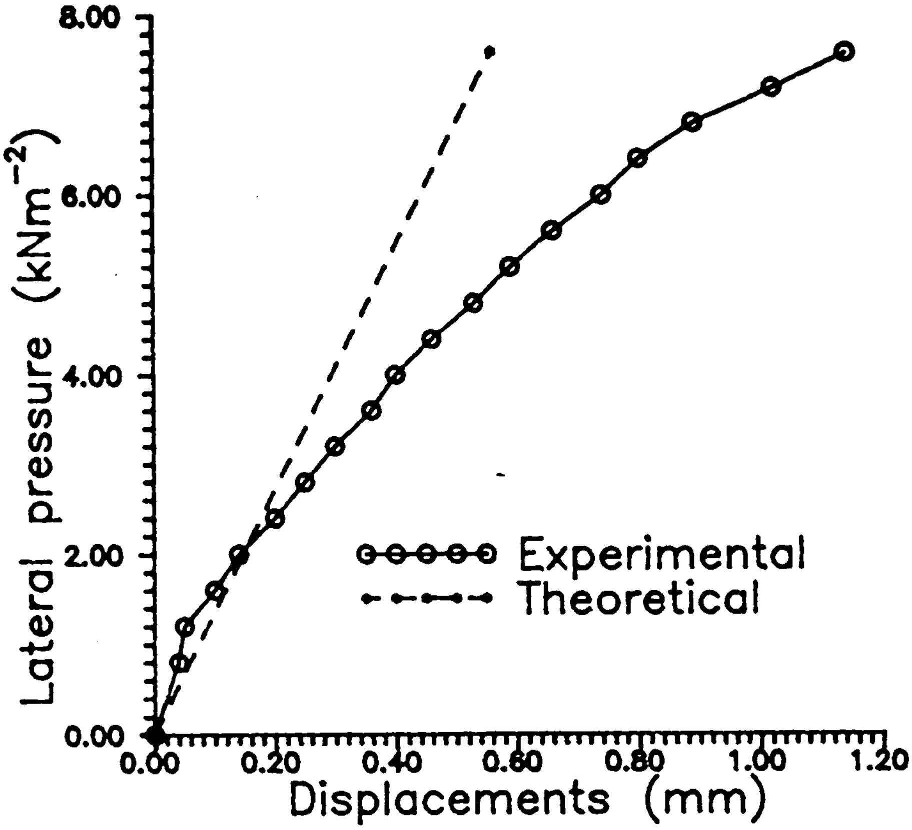


Fig. 5.5

Load-deflection relationship of Wall 3 at point A (aspect ratio 1:1)

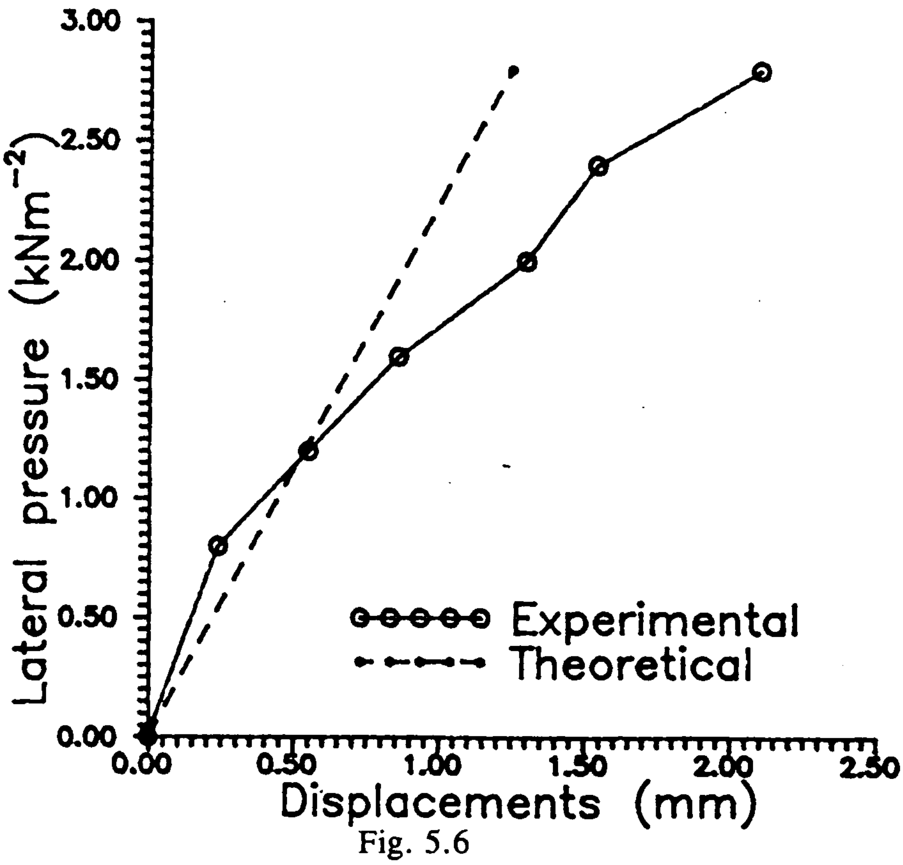


Fig. 5.6

Load-deflection relationship of Wall 9 at point A (aspect ratio 1:1.5)

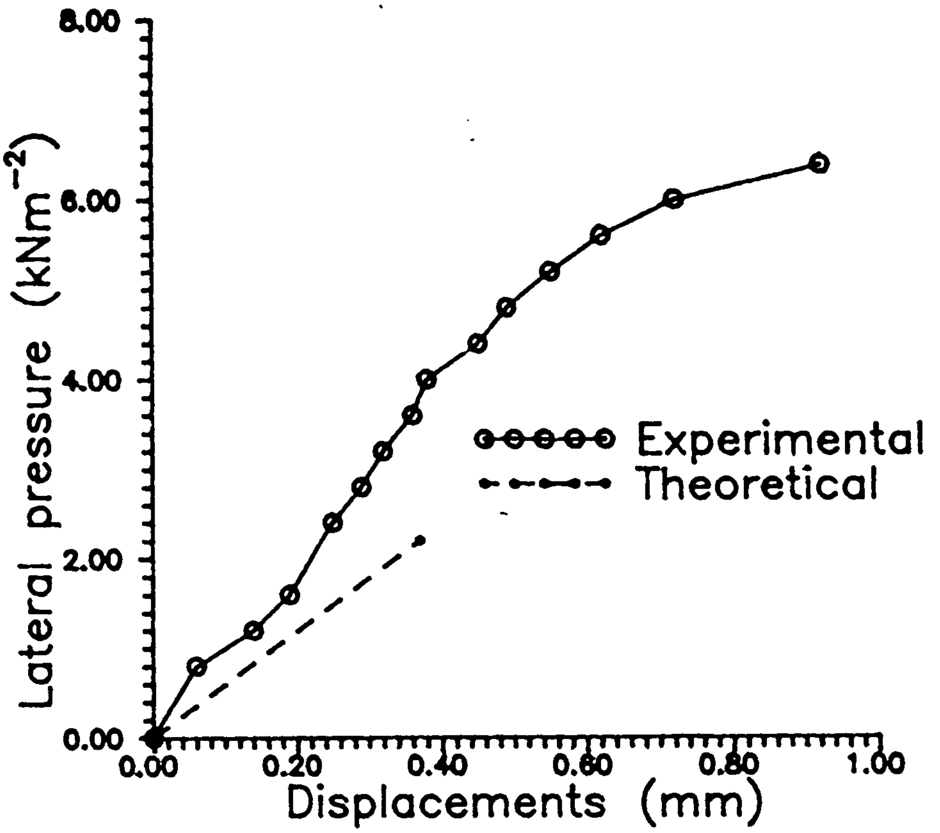


Fig. 5.7

Load-deflection relationship of Wall 5 at point A (aspect ratio 1:1)

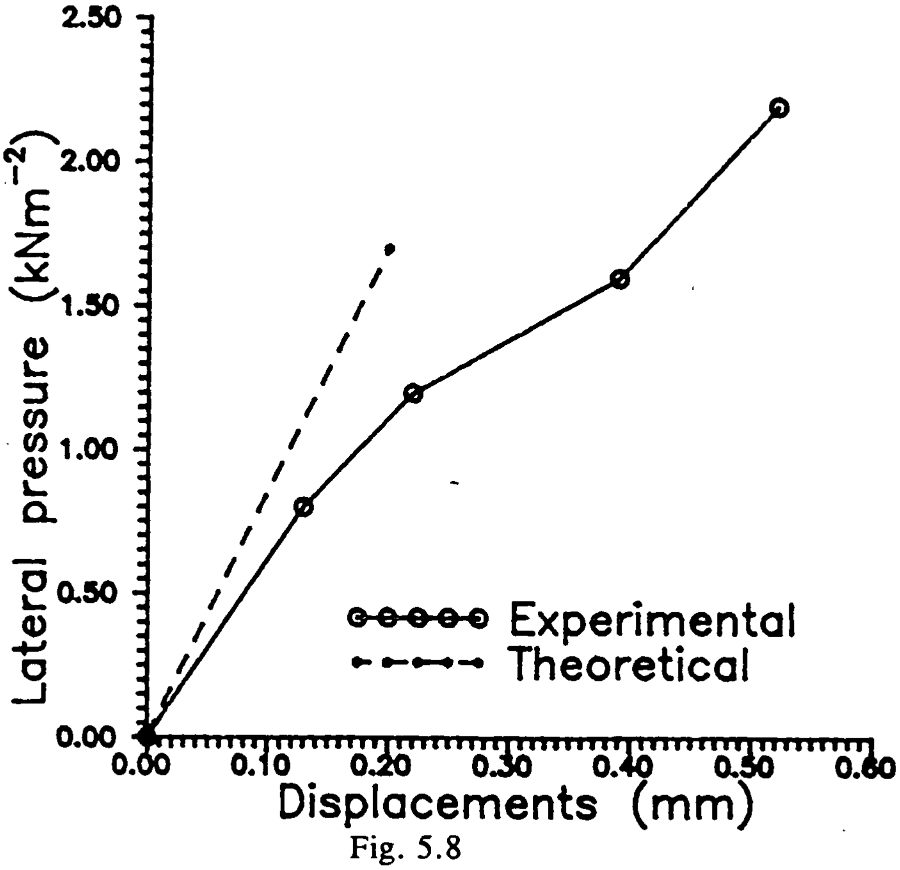
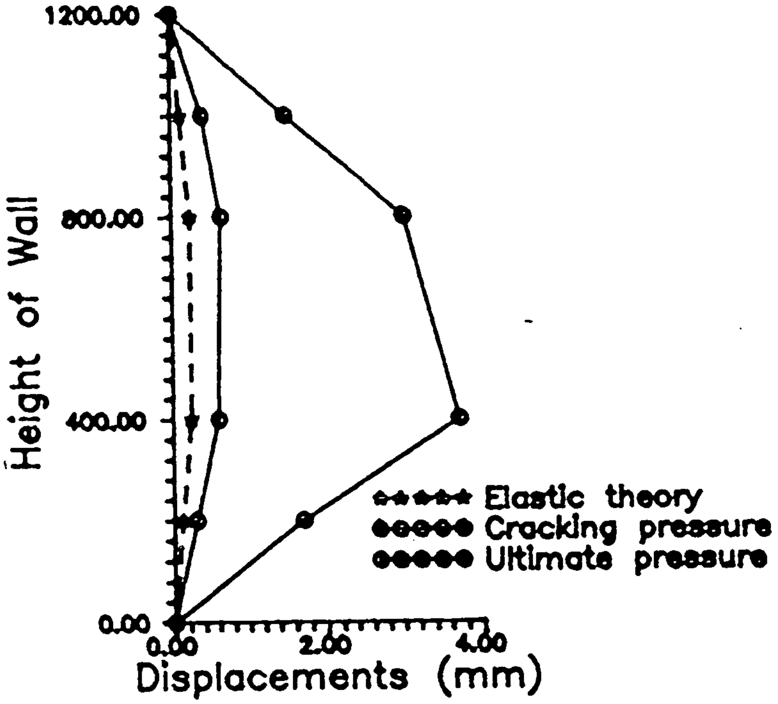


Fig. 5.8

Load-deflection relationship of Wall 16 at point A (aspect ratio 1:1.5)

Deflections along the vertical direction



Deflections along the horizontal direction

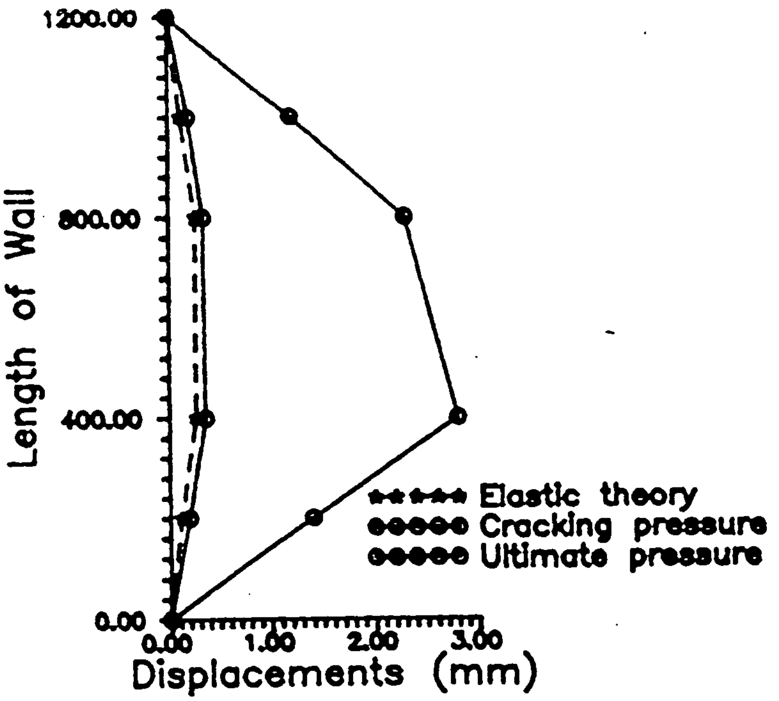
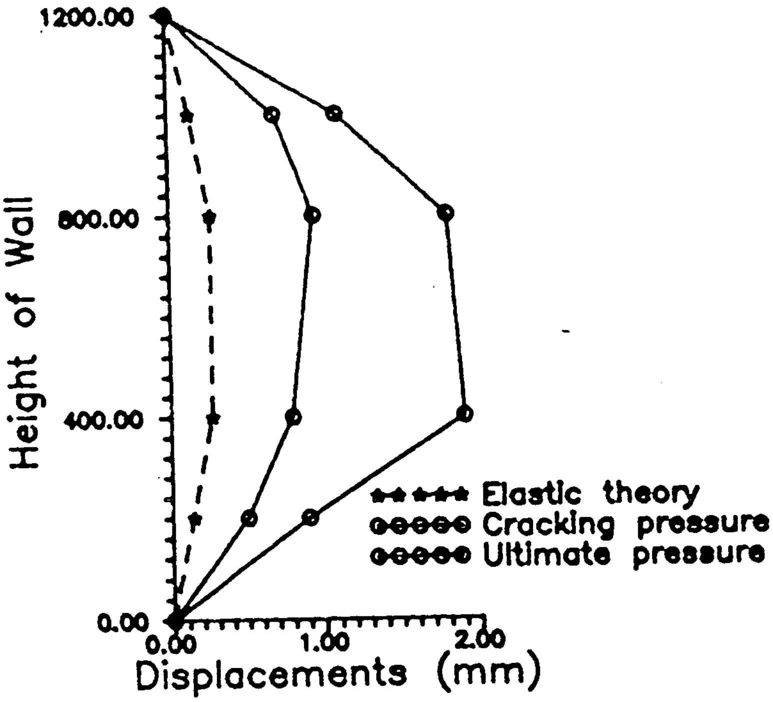


Fig. 5.9

Deflections of Wall 2 in horizontal and vertical direction

Deflections along the vertical direction



Deflections along the horizontal direction

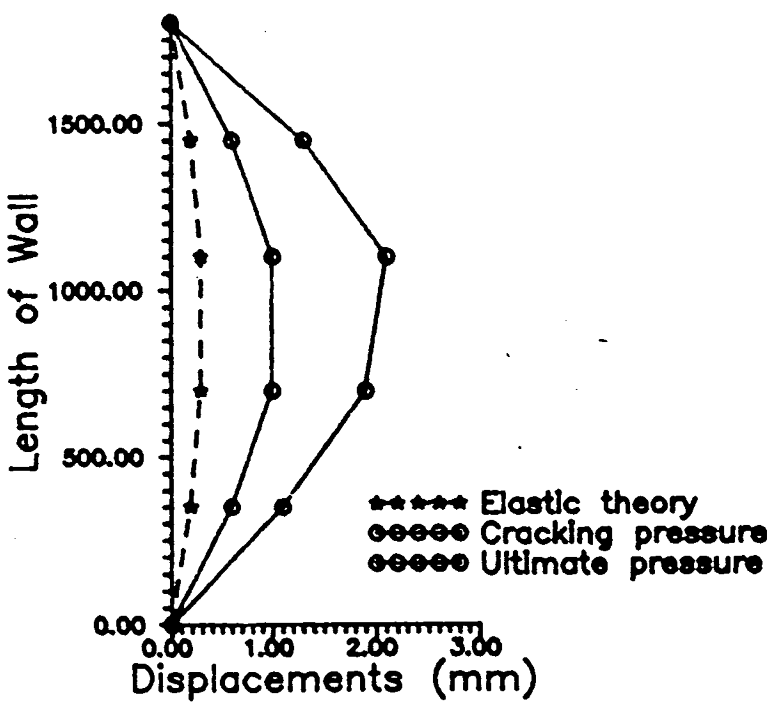
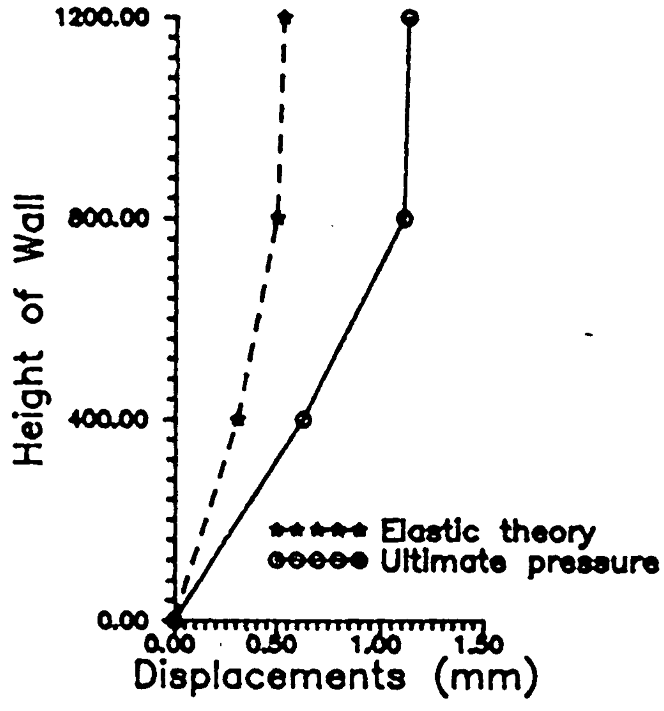


Fig. 5.10

Deflections of Wall 8 in horizontal and vertical direction

Deflections along the vertical direction



Deflections along the horizontal direction

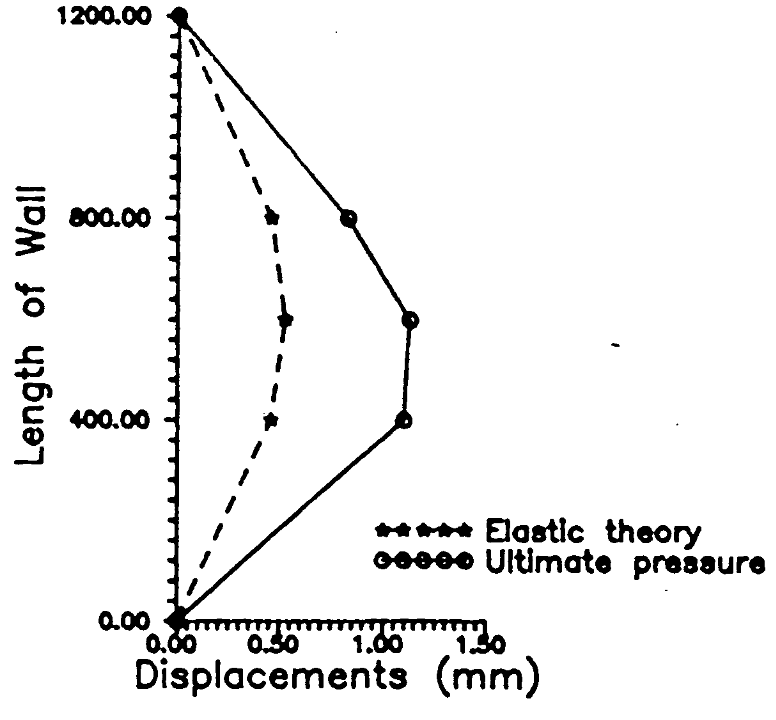
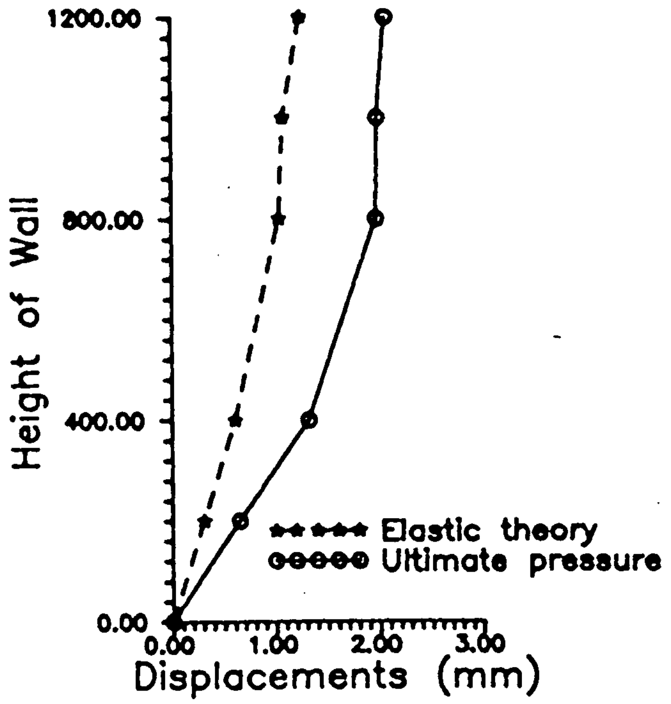


Fig. 5.11

Deflections of Wall 3 in horizontal and vertical direction

Deflections along the vertical direction



Deflections along the horizontal direction

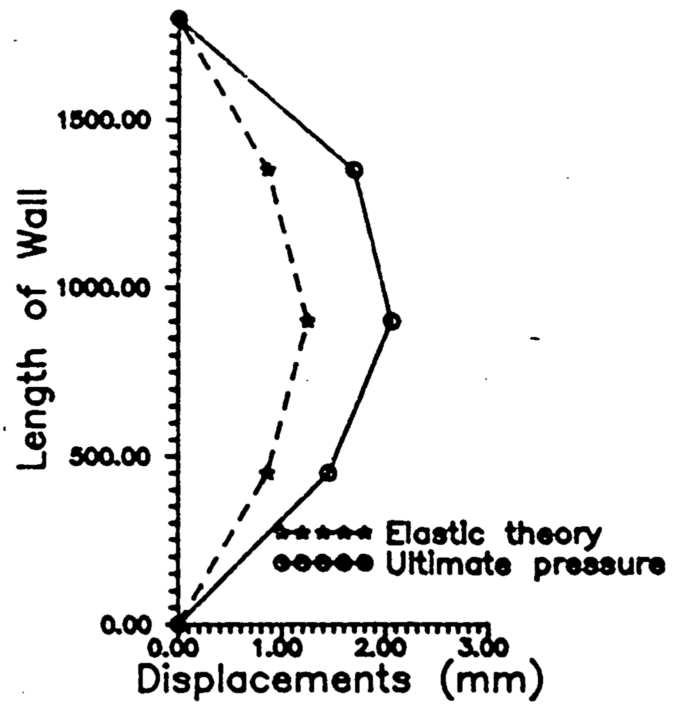
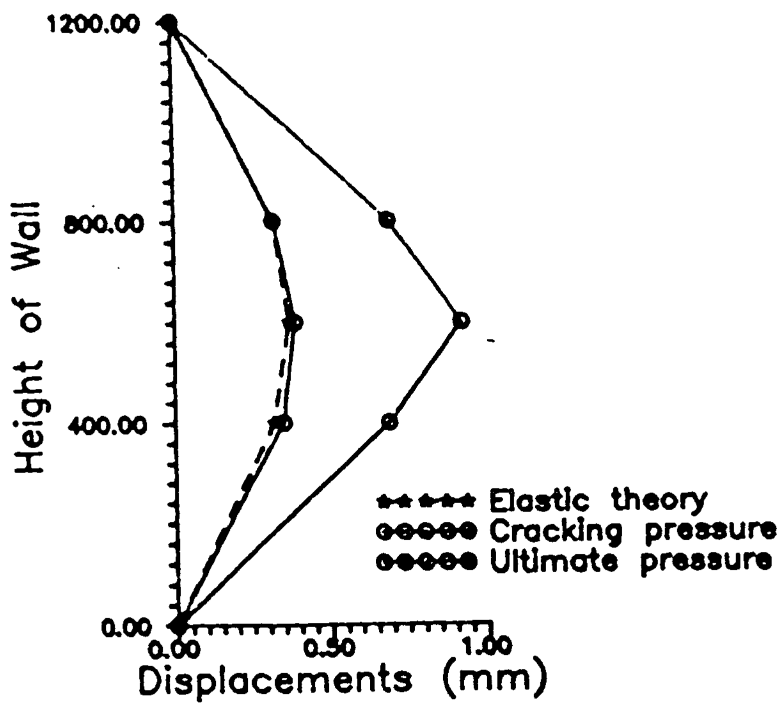


Fig. 5.12

Deflections of Wall 9 in horizontal and vertical direction

Deflections along the vertical direction



Deflections along the horizontal direction

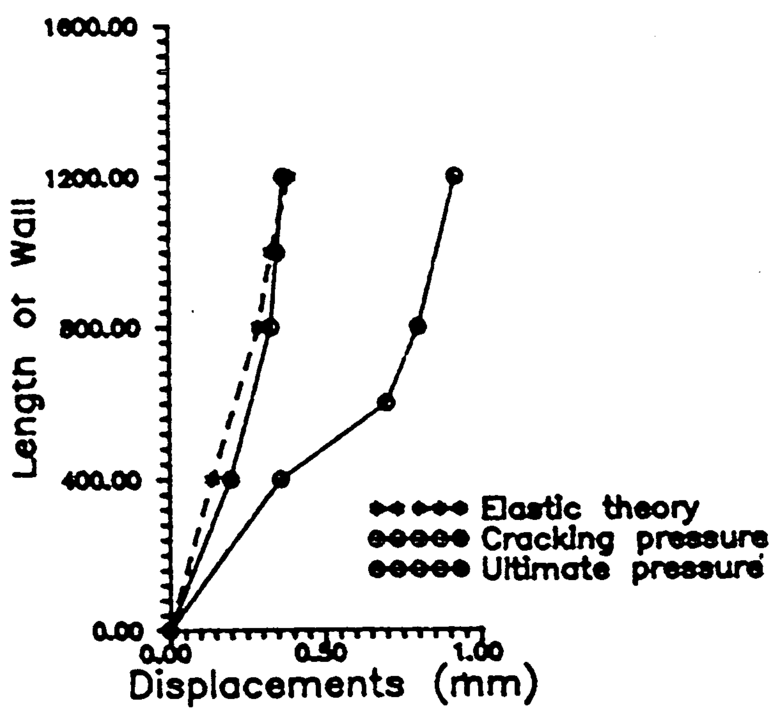
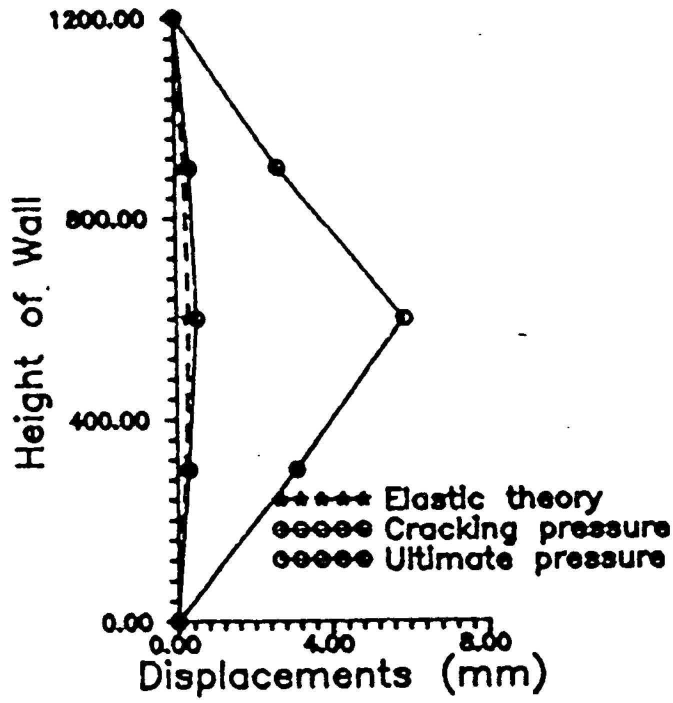


Fig. 5.13

Deflections of Wall 5 in horizontal and vertical direction

Deflections along the vertical direction



Deflections along the horizontal direction

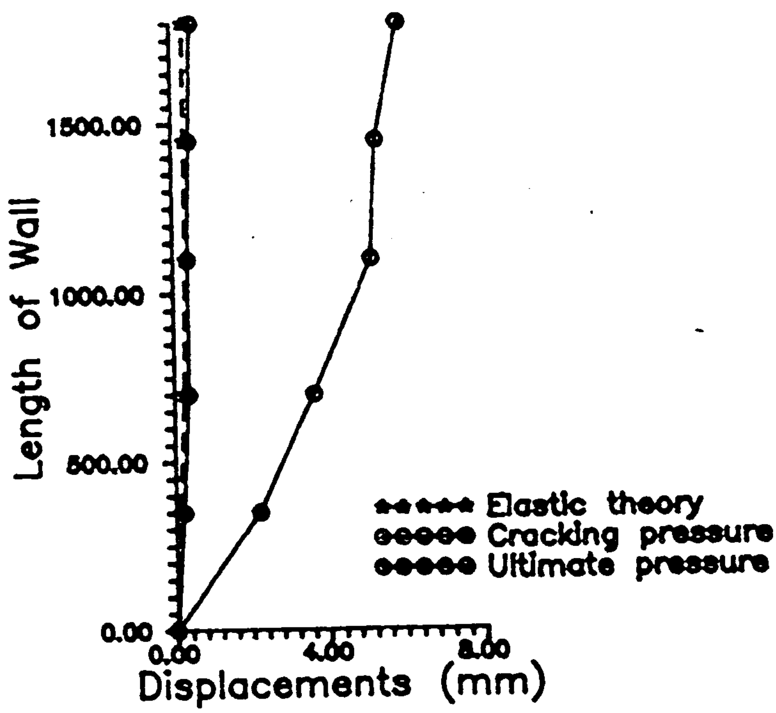


Fig. 5.14

Deflections of Wall 16 in horizontal and vertical direction

From Fig. 5.9, it can be seen that the deflections are different at 400 mm from the vertical and the horizontal supports. In a symmetrical isotropic panel with aspect ratio 1:1 they should be equal. This shows and confirms that the load distributes according to the elastic orthotropy as found in the cross-beam tests. Hence, brickwork panels should not be treated as isotropic for analysis. The elastic orthotropy was obtained from the prism extracted from the undamaged parts of the test walls (section 3.3.3).

5.2.3.2 Cracking

Before failure, initial horizontal hair cracks were noticed in the walls simply supported on four sides. It is difficult to establish the exact moment that the first hair crack appeared and in some cases this was done with the help of the dial-gauges, mainly the ones positioned near the areas where the maximum displacements were measured. This helped to detect the first cracks by watching sudden changes of the pointers of the dial-gauges. These horizontal cracks seem to divide these panels into two sub-panels, having three sides simply supported and one unsupported side (along the crack). Walls having one free vertical edge; 5, 6, 15 and 16 (see Fig. 5.1) also behaved in a similar way. They cracked horizontally in the vertical strip along the unsupported edge and it was also difficult to detect visually the first hair crack in some cases. Walls having the three sides simply supported and the top edge free did not show any sign of cracking . These walls; 3, 4 ,9 and 10 (see Fig. 5.1) behaved like a strip spanning horizontally at the top, and the failure happened immediately after the development of vertical cracks at ultimate pressure. There was no redistribution of loading from one direction to another after the failure of the strong horizontal strip. Wall 13 and 14 also behaved in a similar way, though the cracks were diagonal and they were the only panels having the window opening positioned along the edges.

5.2.3.3 Failure

All panels simply supported on four edges or supported on three sides and free along one of the vertical edges kept resisting the applied loading until the development of a full crack pattern forming a mechanism. The margin of pressure between first cracking and the formation of the full crack pattern was substantial.

In some cases, after the initial cracking, more cracks developed horizontally and also diagonally before a mechanism was formed. Cracking tended to develop mainly along the horizontal joints and, for this reason, in few cases these full crack patterns resembled the resulting pattern of cracks formed in a concrete plate under uniformly distributed load at failure. Walls with top edge free collapsed as soon as the initial cracks appeared. Failure was sudden for these walls. Fig. 5.15 to 5.31 show the crack patterns of the test panels.

The ultimate pressures of panels of aspect ratio 1:1 with three sides simply supported and the vertical or top edge free were similar. This could only be possible if the strengths in the vertical and horizontal directions were similar, i.e. the panels exhibited strength isotropy. It could be possible that the experimental pressures of such walls may have been enhanced due to membrane action. This enhancing of strength may also be the reason of the differences in flexural strength in some of the replicates panels, like Walls 1 and 2, and 13 and 14, as the flexural tensile strength obtained extracting wallettes from those walls did not present such variation.

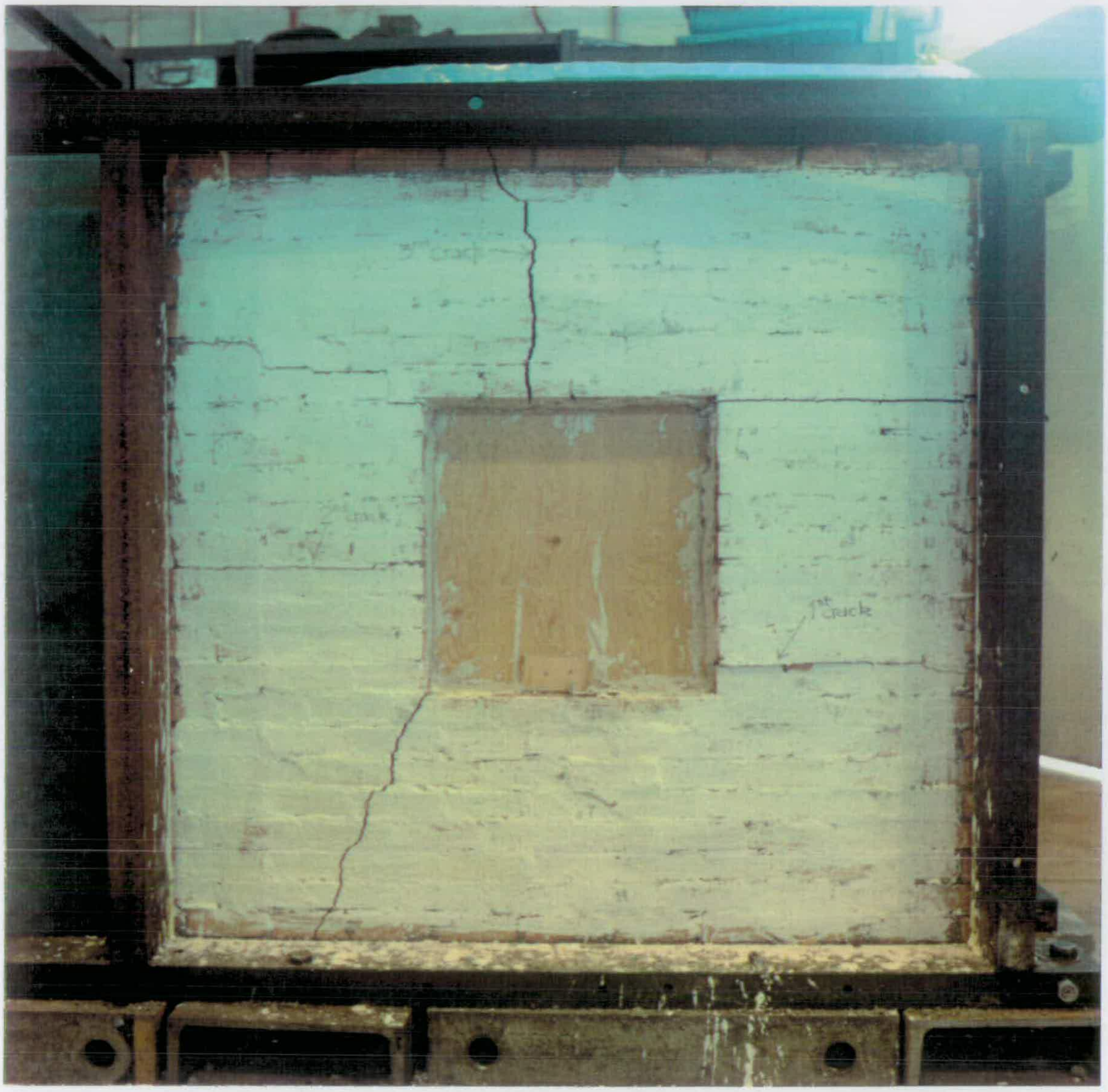


Fig. 5.15
Crack pattern of Wall 1

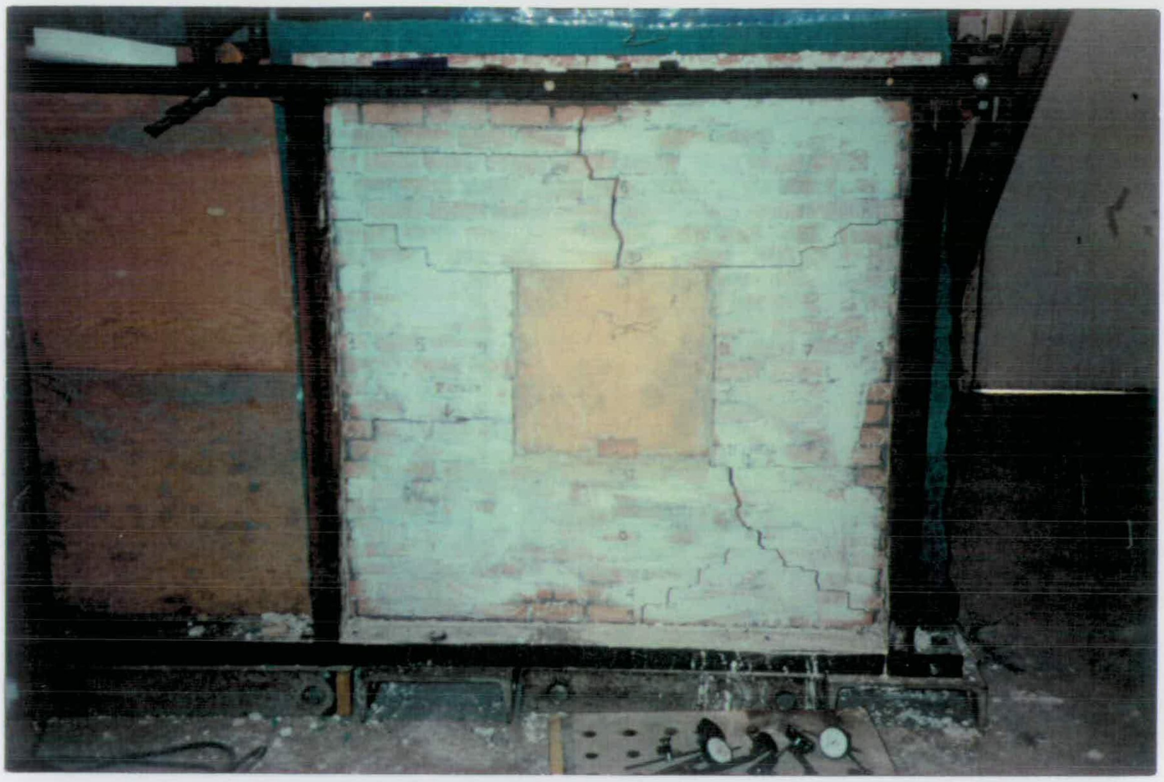


Fig. 5.16
Crack pattern of Wall 2

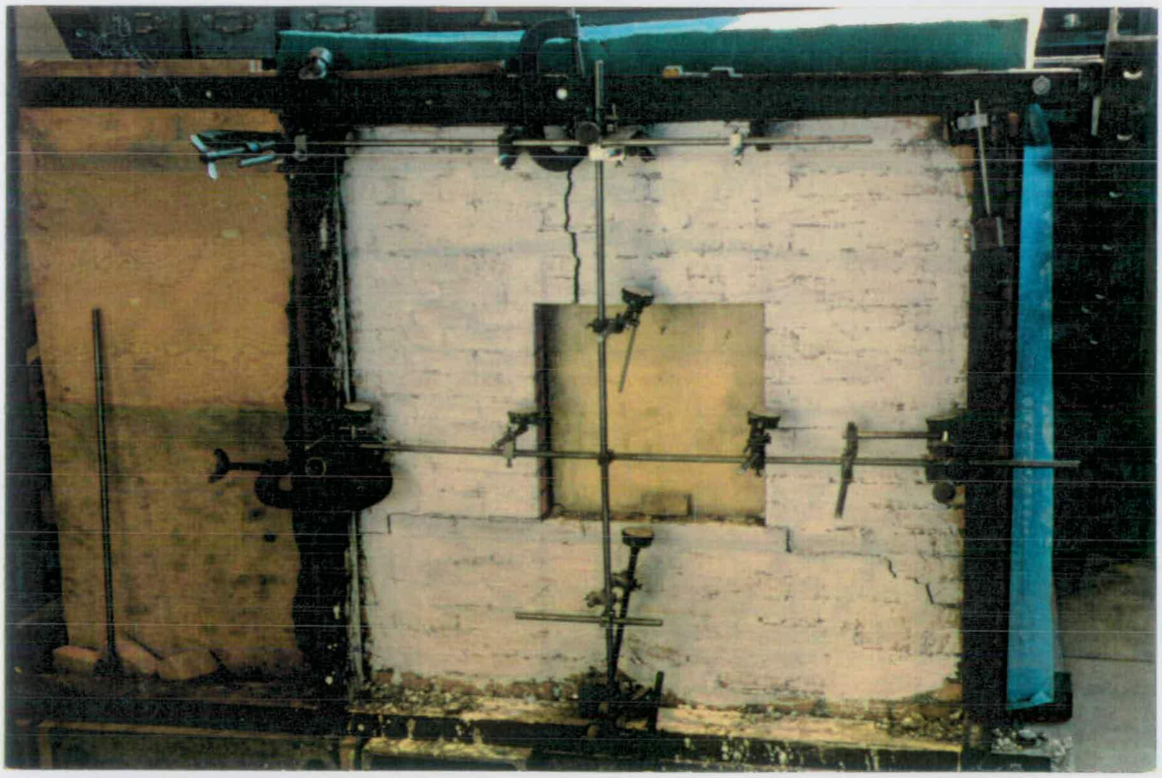


Fig. 5.17
Crack pattern of Wall 3

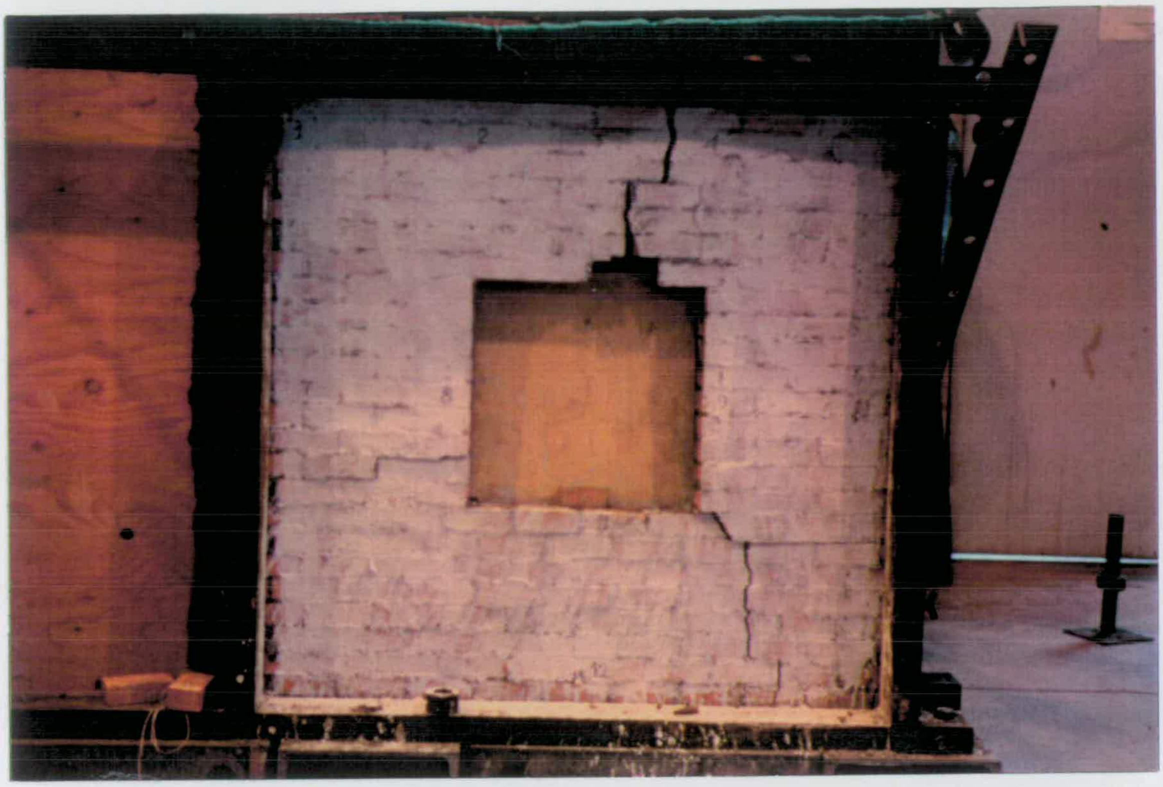


Fig. 5.18
Crack pattern of Wall 4

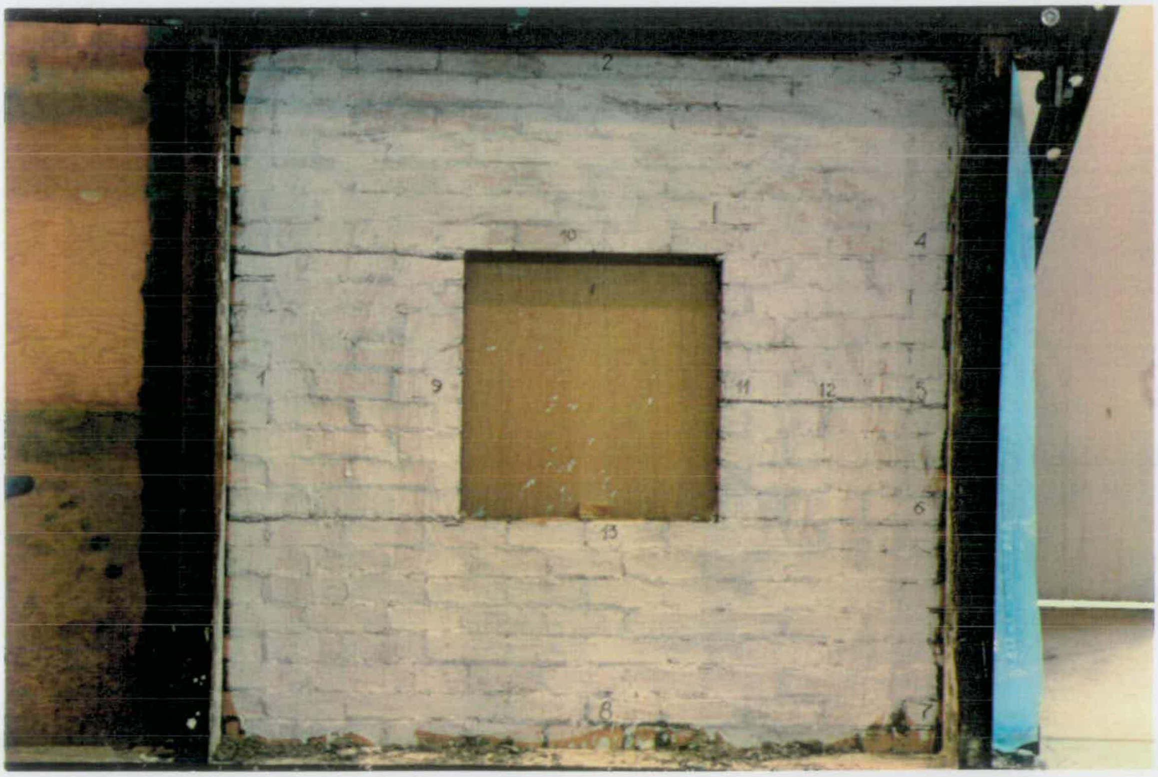


Fig. 5.19
Crack pattern of Wall 5

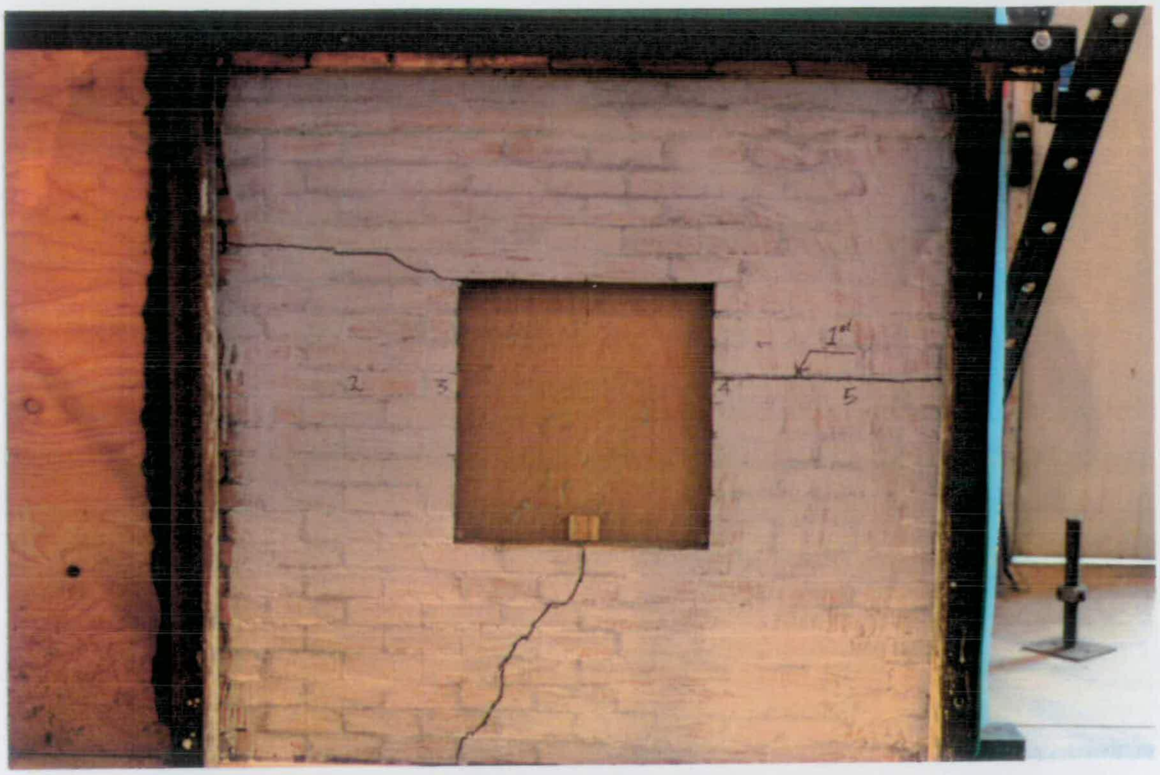


Fig. 5.20
Crack pattern of Wall 6

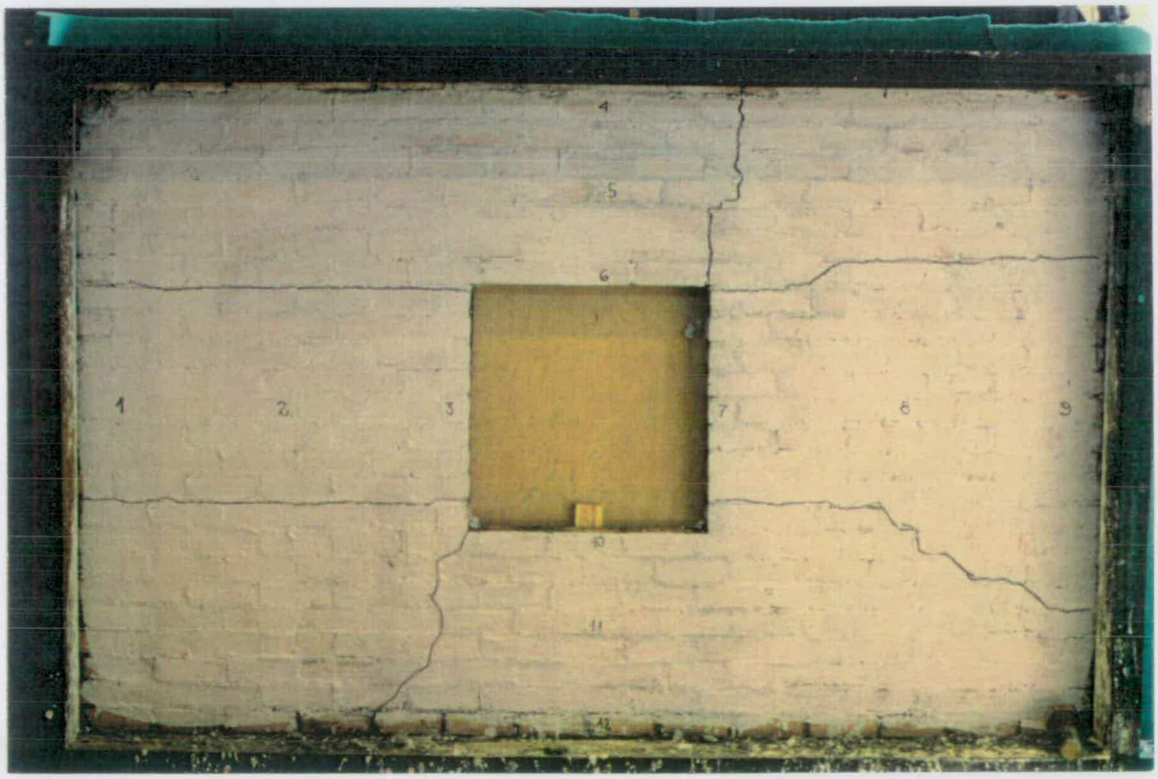


Fig. 5.21
Crack pattern of Wall 7

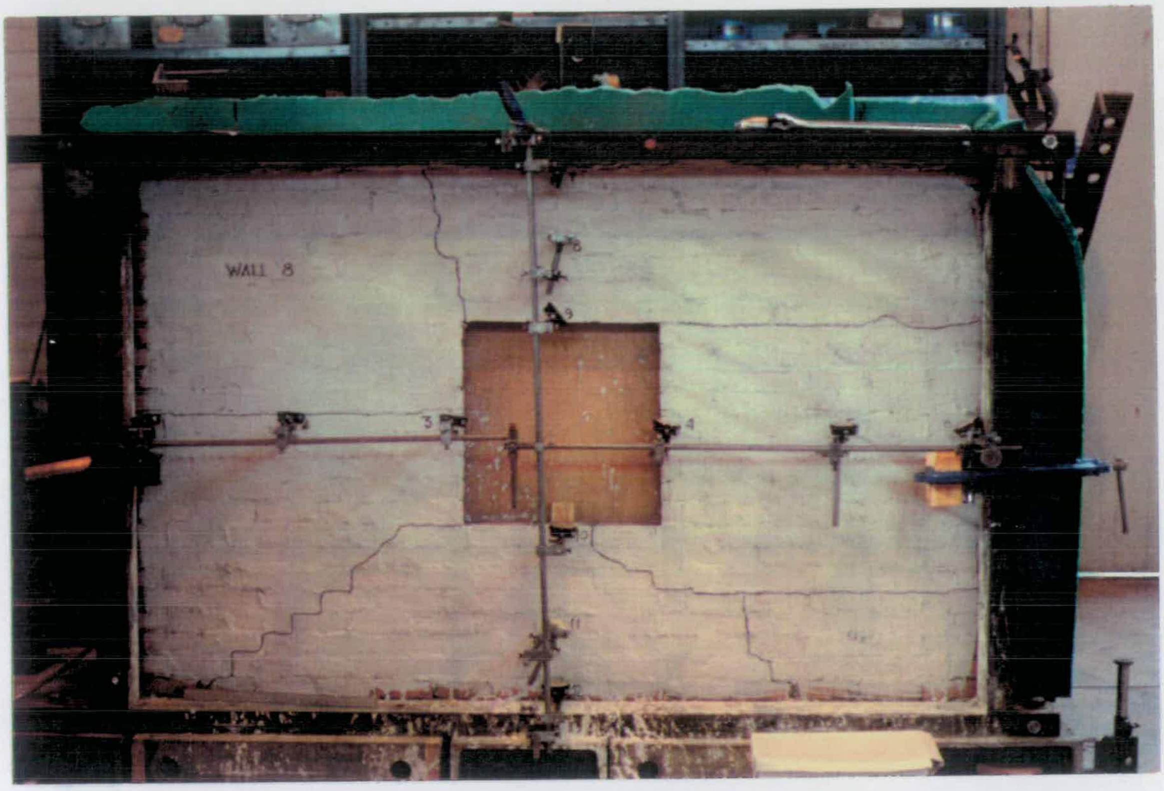


Fig. 5.22
Crack pattern of Wall 8

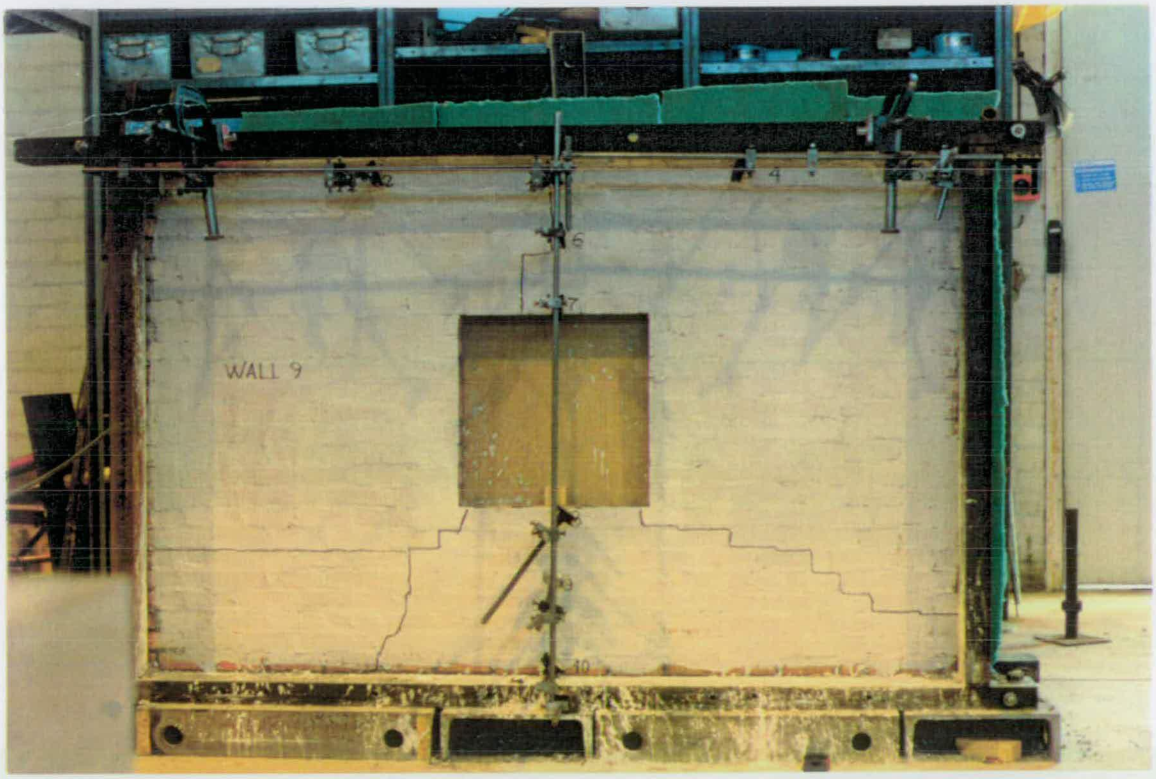


Fig. 5.23
Crack pattern of Wall 9

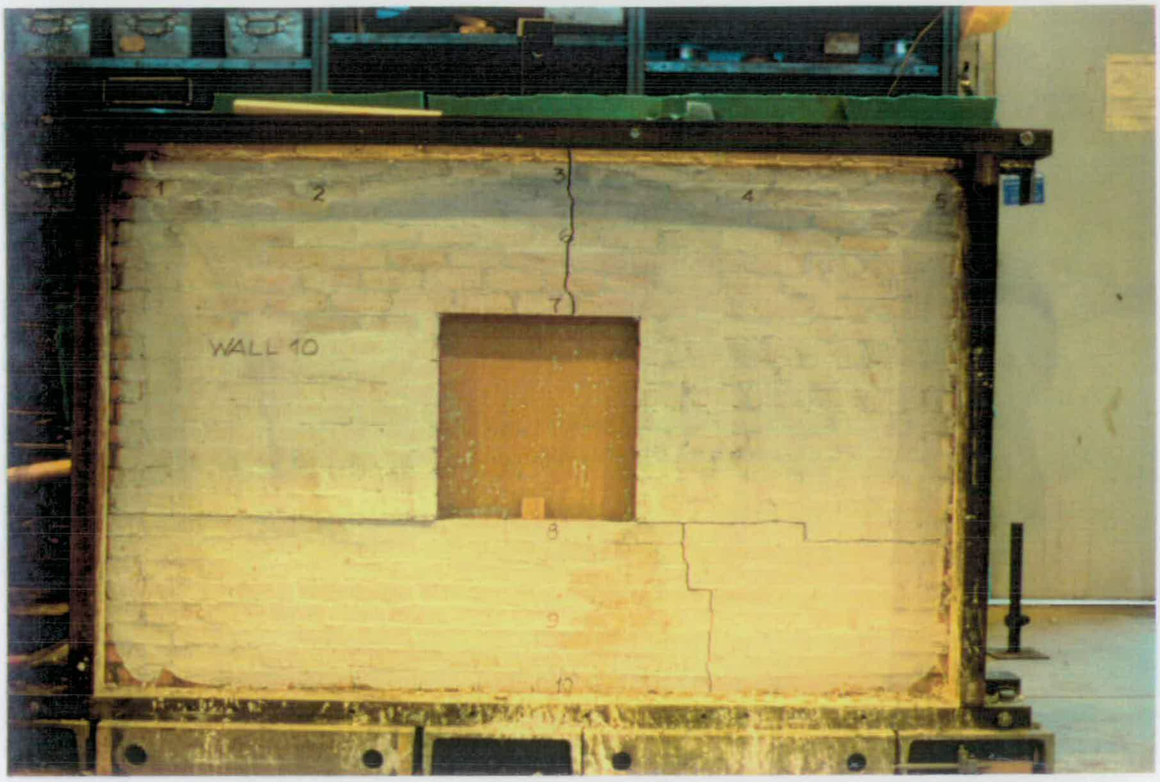


Fig. 5.24
Crack pattern of Wall 10

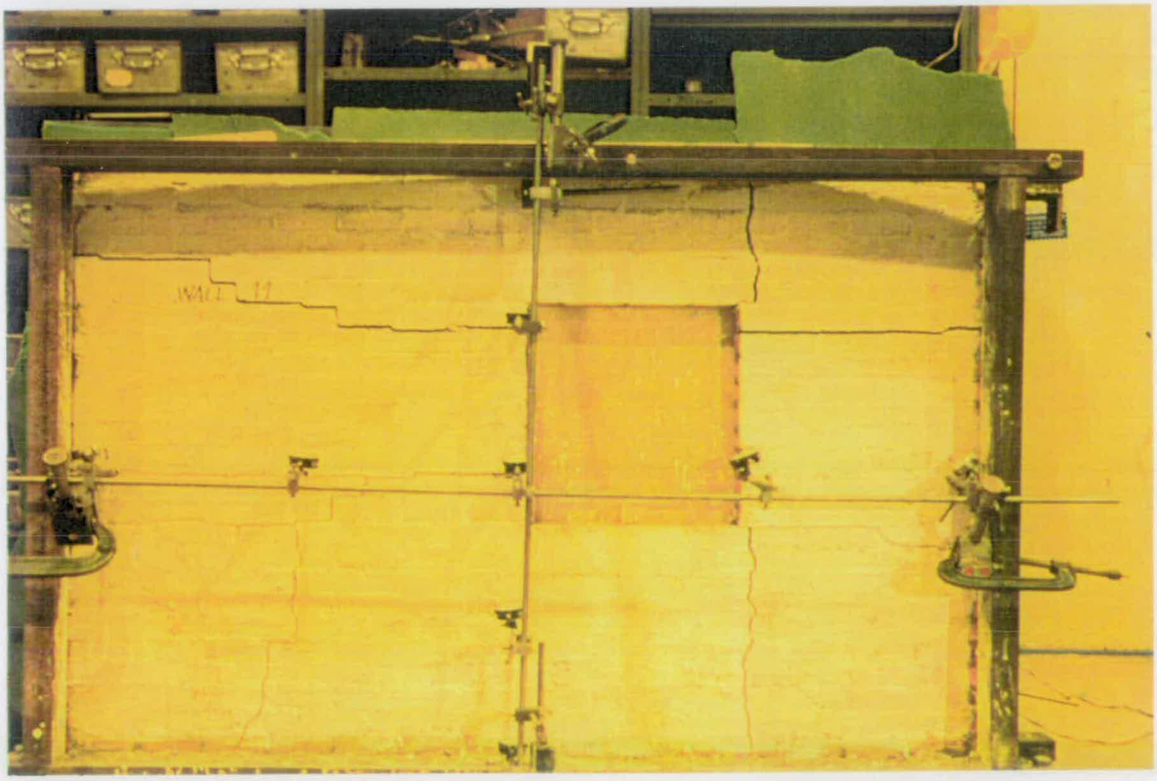


Fig. 5.25
Crack pattern of Wall 11

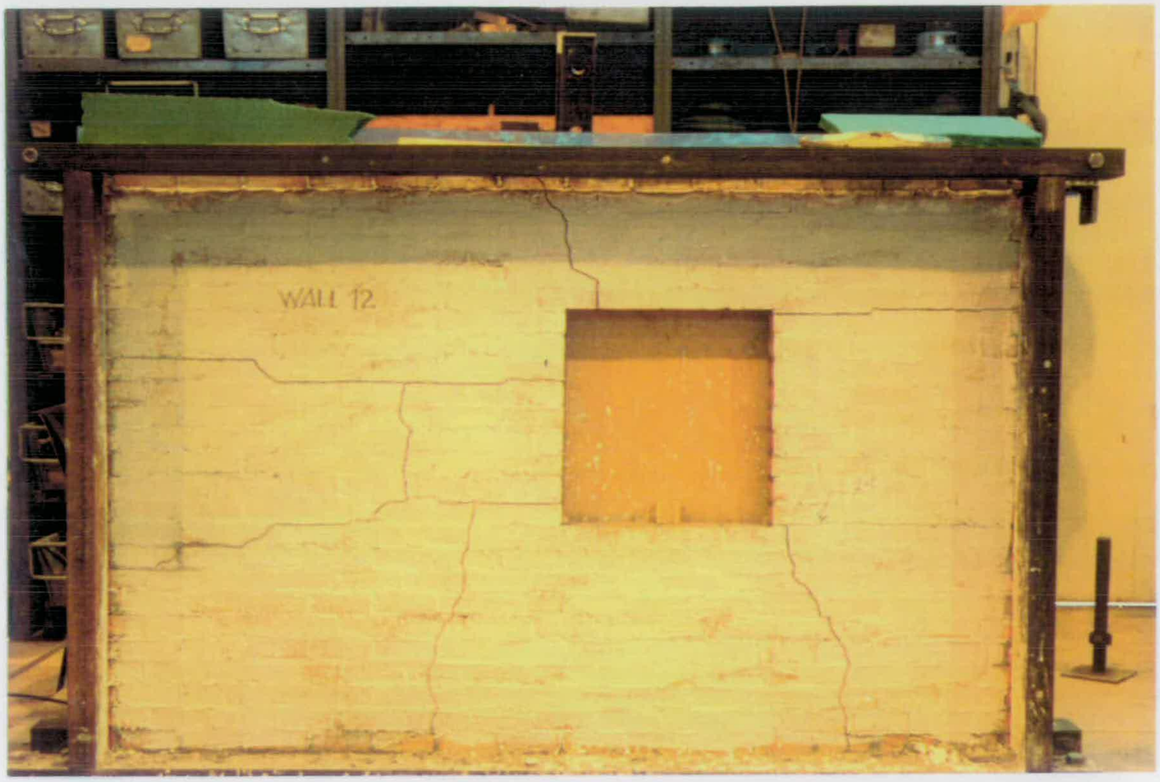


Fig. 5.26
Crack pattern of Wall 12

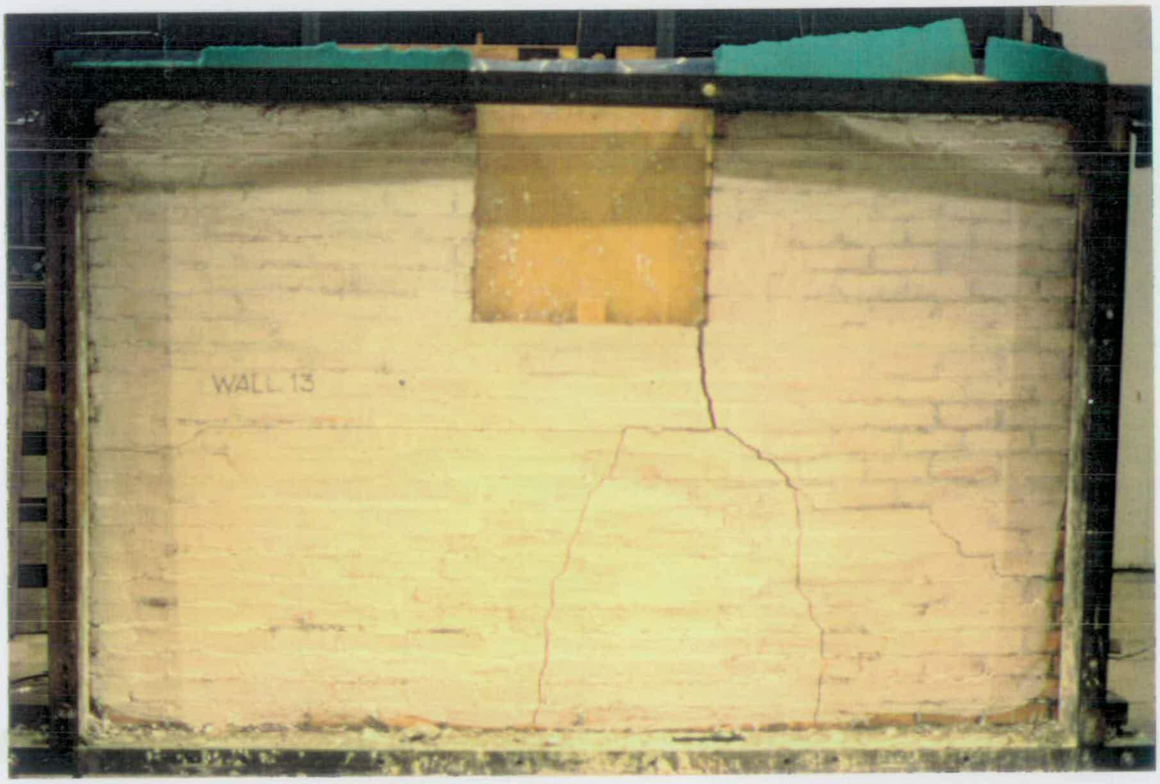


Fig. 5.27
Crack pattern of Wall 13

Fig. 5.28
Crack pattern of Wall 14

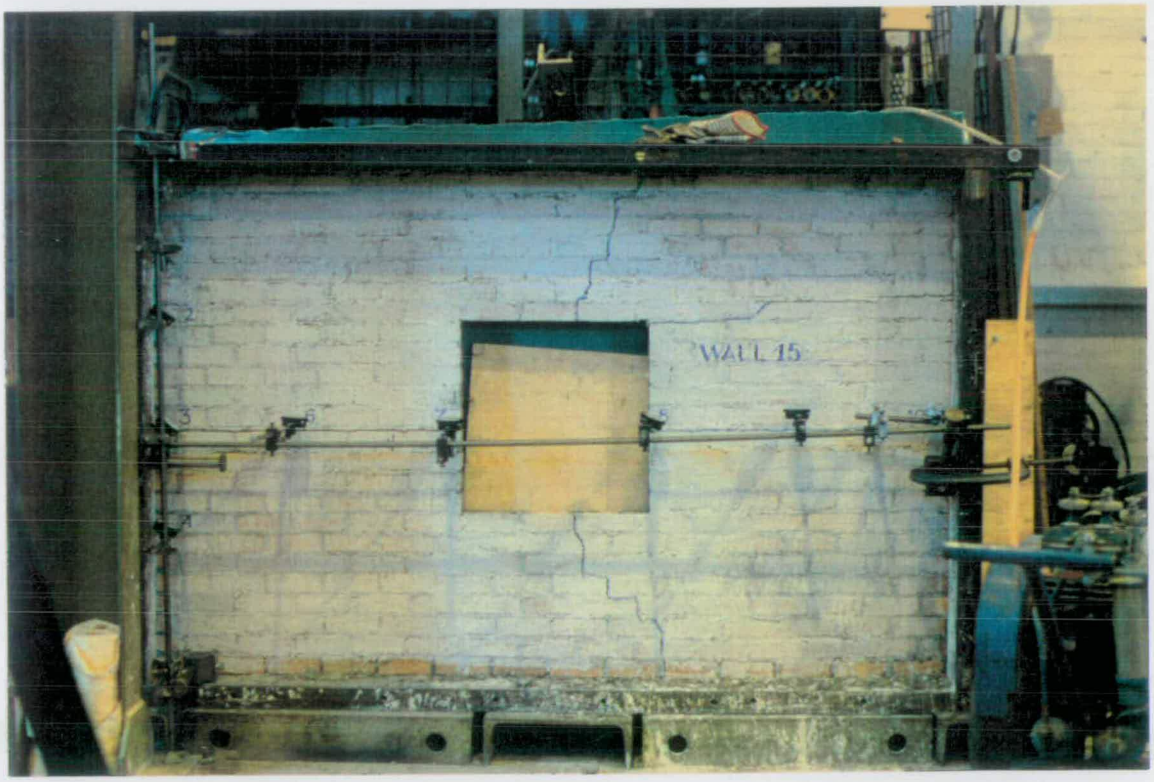


Fig. 5.29
Crack pattern of Wall 15

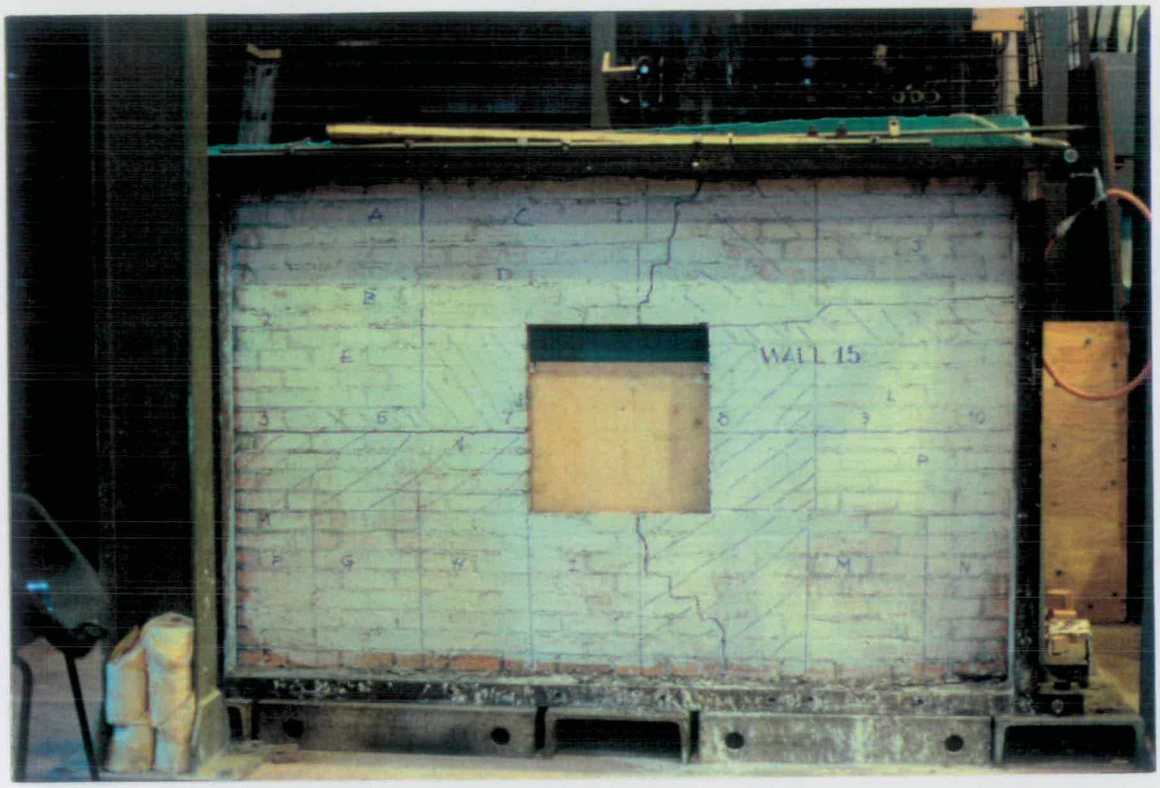


Fig. 5.30
Demarcation of a wall into wallettes after the end of a test.

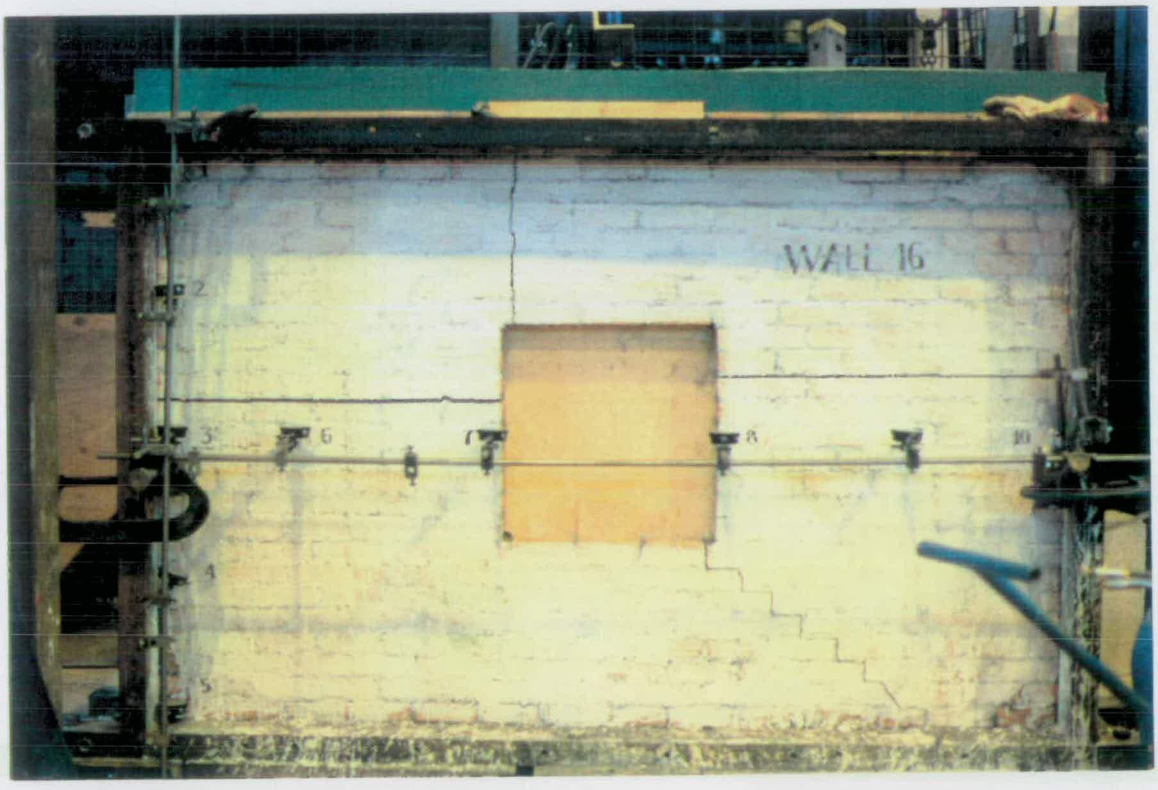


Fig. 5.31
Crack pattern of Wall 16

5.3 THEORETICAL METHODS OF ANALYSIS

5.3.1 Introduction

There are two main problems in analysing the flexural strength of two-way spanning wall panels subjected to lateral pressure: the calculations of bending moments in a highly indeterminate structure and the failure criterion to be used. Researchers seem to be divided between two main theories: elastic and plastic. Both, in some way, have not elucidated completely the flexural behaviour in bi-axial bending of unreinforced brickwork. Elastic theory, which is the method that sounds theoretically right, due to the brittle behaviour of unreinforced brickwork, has failed in predicting cracking and failure pressures. The yield-line theory, which is a method based on the plastic behaviour of the material, has produced better agreement with the experimental failure pressure.

Following Hillerborg strip method, an empirical method was proposed by Baker[109] and will also be studied in this investigation. The strip method, which does not have a proper theoretical basis, has been said to give good agreement with experimental failure pressures for solid brickwork walls. This is being used by the Australians, however, at the moment there is no rational justification for its use.

Another method of analysis have been proposed by other researchers, like the fracture line method by Sinha[107], which seems to be more a variance of the yield-line method to take into account the stiffness orthotropy of brickwork and will not be discussed further in this investigation. Therefore, only the first three methods discussed above will be applied to analyse the lateral resistance of the walls tested in this work.

As the designer will use the published values of flexural tensile strengths recommended by BS 5628 instead of test values, a comparison is also made using the prescribed values of the characteristic flexural tensile strengths and the wall's test results.

5.3.2 Elastic analysis

Elastic analysis is based on plate bending theory. For wall panels having irregular shape like the ones presented in Fig. 5.1, elastic analysis is hopelessly complicated and a solution is beyond the ability of even the best mathematician, unless a numerical method like the finite element method and a computer is available. Computer programs based on the finite element method have overcome the problem of the mathematical solutions making possible the analysis of complex structures. Since then, the finite element method has become part of the tools of the structural engineers and has been advocated by several researchers as the only method that has a proper theoretical basis to analyse the flexural behaviour of unreinforced brickwork due to its inherent brittle nature.

There are different ways of modelling the flexural behaviour of brickwork in a finite element analysis. One could be modelling each brick and mortar individually, with their own Young's moduli and Poisson's ratio. This has the disadvantage of using a large amount of elements required. Another way, more simple, is to analyse the wall simulating the behaviour of brickwork (bricks and mortar together) adopting a single mesh for them. In this investigation, the second method has been followed.

An eight noded plate bending element has been used to simulate brickwork. Each node has three degrees of freedom, one axial displacement and two rotations. The integration rule uses a 9x9 point Gauss quadrature. A convergence test had to be performed due to the continuous increase of the stresses near the corner of the windows. These areas are places where the stresses change directions and are highly stressed. Besides that, due to the transferring of loading from the windows to the wall through these nodes, the Gauss points near the window corners are the critical ones. Successive refinements of the mesh led to closer positioning of the Gauss point nearest to the node receiving the point load and, consequently, to higher stresses around the internal corners of the windows. Therefore, it was established that the refinement of the mesh should stop after no significant change of the displacements of the nodes receiving the point loads (simulating the wooden bolts on the windows) over 0.01 mm was found. A mesh containing elements of 100mm x 100mm at the neighbourhood of the nodes receiving the point loads was chosen to be used in all analyses. Otherwise, the predicted pressure would have continued to drop.

5.3.3 Yield-line analysis

Yield-line theory was developed by Johansen[144] to analyse the behaviour of under-reinforced concrete slabs, hence shear and bond failures in bending must be prevented when applying this theory to unreinforced brickwork. The method is supposed to give results on the unsafe side, i.e. based on the upper-bound theorem of the theory of plasticity. It assumes that failure occurs when a certain function of the stresses achieved certain limiting value, then failure will start at a single point of the plate under transverse loading. A brittle material like brickwork should lose its capacity to withstand the stresses at failure, and the stresses must be borne by the adjoining sections. However, these adjoining sections are already stressed almost to the failure point. When these additional stresses are imposed, these adjoining sections will also lose their capacity to take up the extra stresses, as well as the original stresses, and so on. Consequently, failure immediately spreads over an extensive area, which loses its capacity to bear stresses and can be extended to the whole of the plate's carrying capacity, causing its final collapse. The condition of failure is that the maximum bending moment corresponds to the ultimate load. The assumed collapse mechanism is defined by a pattern of yield lines.

Once the correct yield-line pattern of failure is predicted, the ultimate resistance moment along the yield lines can be calculated and by analysing the plate at failure conditions, the value of the load which is in equilibrium with these moments can be found. Two methods of analysis can be used in order to find the relation between the ultimate resistance moment and the ultimate load: the equilibrium method and the principle of virtual work. For practical calculations the virtual work method is easier to be applied and has been chosen in this investigation.

Yield-line theory has the major advantage that makes possible to work out with the orthogonal strength in both directions and can be applied to all types of wall panels, irrespective to their geometric shape and boundary conditions like the walls presented in Fig. 5.1. Therefore, the virtual work method of analysis has been chosen to be applied to all cases dealt in this thesis and the orthogonal strength ratio (μ=2.42) is considered as the ratio of the flexural tensile strength in the horizontal and stronger direction (f_{tx}=2.03 N/mm^2) over the flexural tensile strength in the vertical and weaker direction (f_{ty}=0.84 N/mm^2).

The elastic analysis showed that the internal window corners are highly stressed areas, due to the opening and the point load. Hence the yield line patterns were chosen in order to take into account this characteristic of the walls, i. e. the window corners are areas where the yield lines should start. Nevertheless, other possible collapse mechanisms have also been tried.

The mathematical solutions presented for various walls later in this section are of general nature, which could be applied to any size of walls or window openings with similar boundary conditions and not restricted only to the walls tested in this investigation.

5.3.3.1 Failure mode for walls with the four edges simply supported containing a central window

If a virtual deflection of unity is given to the four corners, "cdef", while the panel in Fig. 5.32 is collapsing, the external work done ($\Sigma w\delta$) on part "abcd" by the uniformly distributed load can be obtained by dividing the rigid area into two triangles and one rectangle, for simplification of the calculation.

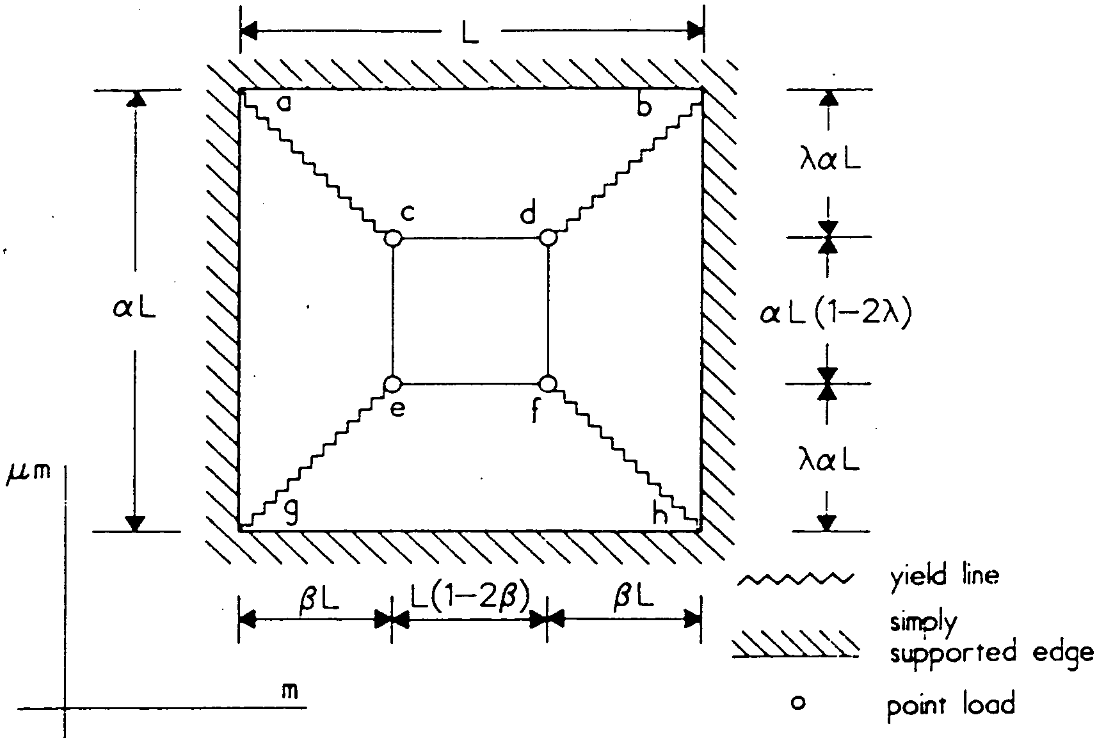


Fig.5.32
Failure mode for walls simply supported on four edges with a central window opening

$$\text{External work done on 'abcd'} = \frac{1}{3} w\beta\lambda\alpha L^2 + \frac{1}{2} w\lambda\alpha L^2(1 - 2\beta) \quad \text{....(5.1)}$$

Due to symmetry, the external work done on part "aceg" can be obtained by the same way.

$$\text{External work done on 'aceg'} = \frac{1}{3} w\beta\lambda\alpha L^2 + \frac{1}{2} w\beta\alpha L^2(1 - 2\lambda) \quad \text{.....(5.2)}$$

The pressure applied over the area of the window is transmitted to the panel through the four wooden bolts. The displacement of the window is unity, the external work done by the pressure applied on the window opening is given by:

$$\Sigma w\delta_{cdef} = w\alpha L^2(1 - 2\beta - 2\lambda + 4\beta\lambda) \quad \text{.....(5.3)}$$

From equations (5.1 to 5.3), and considering the symmetry, the total external work done can be calculated for the entire panel.

$$\text{Total external work done} = \frac{w\alpha L^2}{3}(3-3\beta-3\lambda+4\beta\lambda) \quad \text{.....(5.4)}$$

The internal dissipation of energy along the yield lines "ac, bd, eg and fh" in the y-direction is given by $M_y\theta_y$. M_y is equal to $4\mu m\lambda\alpha L$ and as θ_y is very small it is possible to assume that $\tan\theta_y = \theta_y$. As $\theta_y = 1/\beta L$, the dissipation of energy is given by:

$$M_y\theta_y = 4\mu m\lambda\alpha/\beta \quad \text{.....(5.5)}$$

Similarly, the internal dissipation of energy along the x-direction can be obtained. $M_x = 4m\beta L$. As θ_x is very small it is also possible to assume that $\tan\theta_x = \theta_x$ and hence $\theta_x = 1/\lambda\alpha L$. Therefore,

$$M_x\theta_x = 4m\beta/\lambda\alpha \quad \text{.....(5.6)}$$

As a result, the internal dissipation of energy along the yield lines is given by

$$\Sigma(M_x\theta_x + M_y\theta_y) = 4m\alpha\left(\frac{\beta}{\alpha^2\lambda} + \frac{\mu\lambda}{\beta}\right) \quad \text{.....(5.7)}$$

For equilibrium, the external work done by the uniformly distributed load applied over the area of the wall (including the window) must be equal to the internal dissipation of energy. Hence , external work done = internal work done, or

$$\frac{w\alpha L^2}{3}(3\text{-}3\beta\text{-}3\lambda+4\beta\lambda) = 4m\alpha\left(\frac{\beta}{\alpha^2\lambda}+\frac{\mu\lambda}{\beta}\right)$$

and the ultimate pressure can be obtained from:

$$w = \frac{12m\left(\frac{\mu\lambda}{\beta}+\frac{\beta}{\alpha^2\lambda}\right)}{L^2(3\text{-}3\beta+\beta\alpha\text{-}3\lambda)} \qquad \text{...(5.8)}$$

Another alternative solution is shown in Fig. 5.33. This alternative is presented having in mind a tall wall. The solution is obtained by giving a virtual displacement to the yield lines "bd" and "ef".

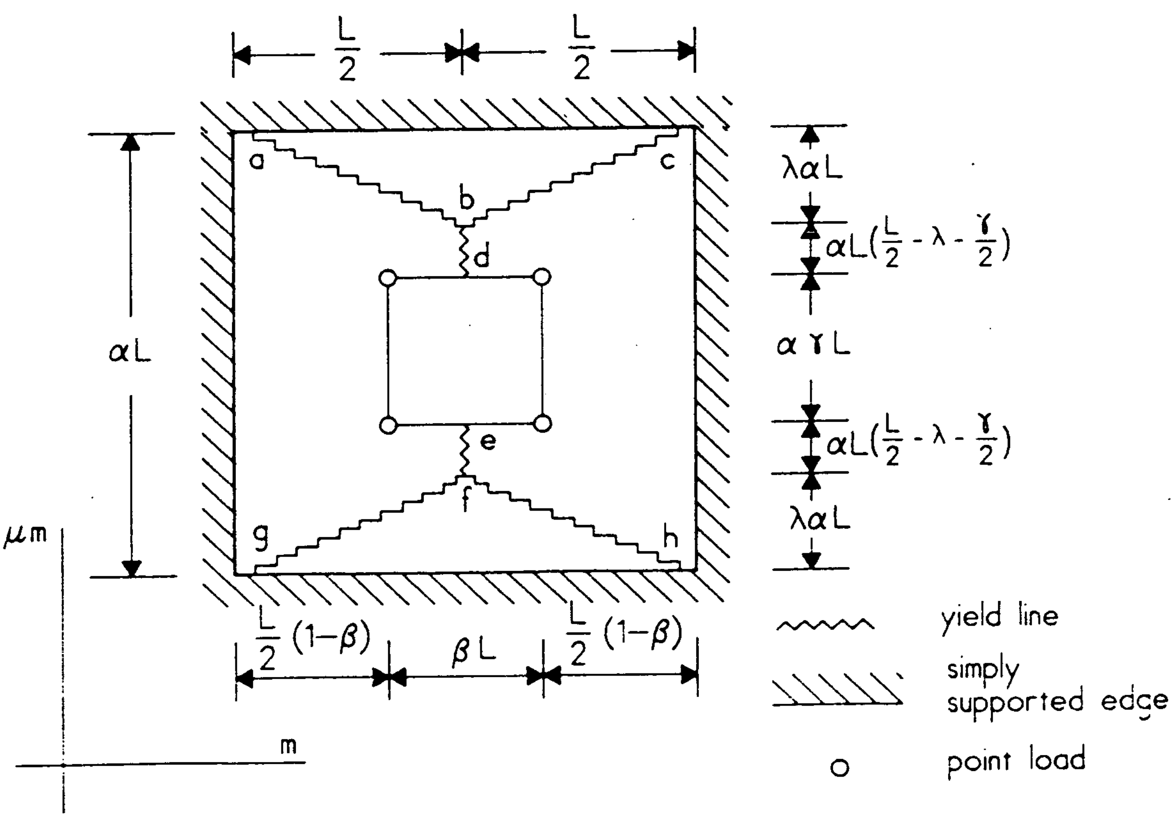


Fig. 5.33

Alternative failure mode for walls simply supported on four sides having a central window opening

To calculate the external work done by the pressure applied over the area of the panel, including the window, a simple solution can be obtained taking into account symmetry and by dividing the panel into triangles and rectangles having the following areas:

$L/2 \times \lambda\alpha L$, $L/4 \times \lambda\alpha L$ (2), $\alpha L(1/2-\lambda-\gamma/2) \times L/2$ (2) and $\alpha\gamma L \times L/2(1-\beta)$ (2).

The load applied on these areas and the window is given by:

$$\Sigma w = w\alpha\lambda L^2 + w\alpha\lambda L^2/2 + \alpha L^2(1/2-\lambda-\gamma/2) + \alpha\gamma L^2(1-\beta) + w\alpha\beta\gamma L^2$$

The displacements of the C.G. of these parts of the panel are: 1/3, 1/3, 1/2, $(1-\beta)/2$ respectively, and the window is $(1-\beta)$. Therefore the external work done by the applied pressure is given by:

$$\text{External work done} = \frac{w\alpha L^2}{6}(3-2\lambda+3\beta^2\gamma) \qquad (5.9)$$

The internal dissipation of energy is given by $\Sigma(M_x\theta_x + M_y\theta_y)$, where: $M_x=mL$ and $M_y=\mu m\alpha L(1-\gamma)$. As θ_x and θ_y are both very small, it is possible to assume that $\tan\theta_x$ and $\tan\theta_y$ are equal to θ_x and θ_y. In that case $\theta_x= 1/\alpha\lambda L$ and $\theta_y= 2/L$. Thus,

$$\Sigma(M_x\theta_x + M_y\theta_y) = 2m\left(\frac{1}{\alpha\lambda} + 2\mu\alpha(1-\gamma)\right) \qquad (5.10)$$

From equations (5.9) and (5.10),

$$\frac{w\alpha L^2}{6}(3-2\lambda+3\beta^2\gamma) = 2m\left(\frac{1}{\alpha\lambda} + 2\mu\alpha(1-\gamma)\right), \text{ therefore}$$

$$m = \frac{w\alpha L^2(3-2\lambda+3\beta^2\gamma)}{12\left(\frac{1}{\alpha\lambda} + 2\mu\alpha(1-\gamma)\right)} \qquad (5.11)$$

For minimum collapse pressure or maximum value of moment $d(m/w)/d\lambda=0$, thus

$$\lambda = \frac{\sqrt{16 + 48\mu\alpha^2(1 - \gamma - \beta^2\gamma + \beta^2\gamma^2)} - 4}{8\mu\alpha^2(1-\gamma)} \quad \text{.....................(5.12)}$$

The value of λ can be substituted in equation (5.11) to obtain the relationship between the failure moment and the load.

5.3.3.2 Failure mode for walls with one edge free and three other edges simply supported having a central window

For these type of walls one pattern of cracks can be obtained by given a virtual deflection of unity to the vertical yield-line joining points "b and d" in Fig. 5.34.

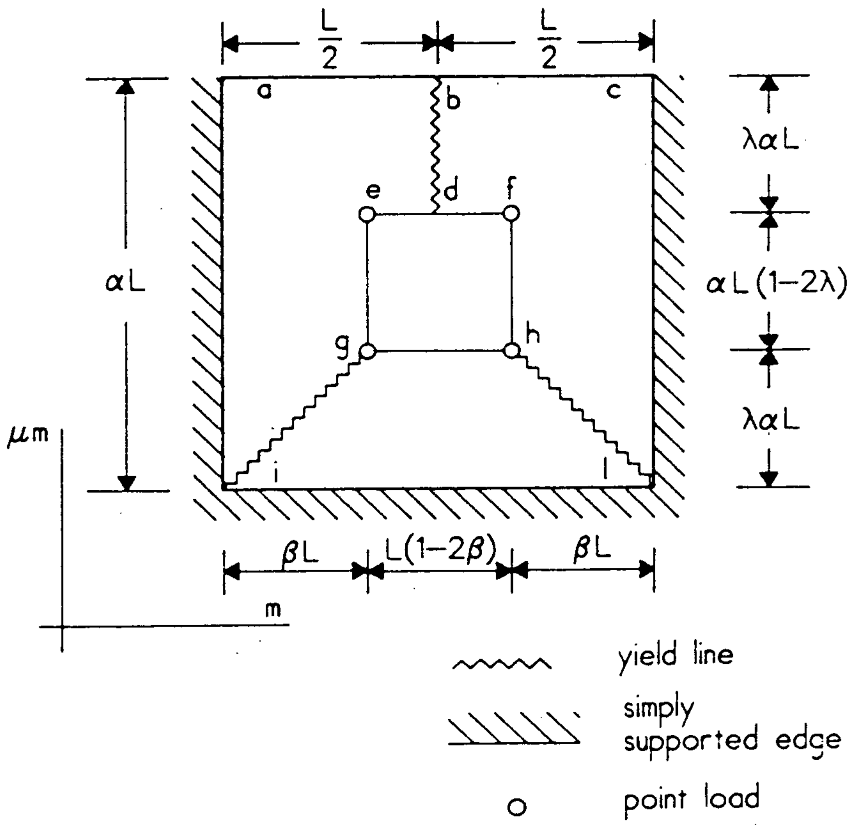


Fig. 5.34

Failure mode for walls with one edge free and three other edges simply supported having a central window

For simplification of the calculation, this panel can also be divided into several triangles and rectangles. Therefore, the panel has been divided into 5 rectangles having the following dimensions: $L/2 \times \lambda\alpha L$ (2), $\beta L \times L(1-2\lambda)$ (2) and $L(1-2\beta) \times \lambda\alpha L$, and 4 triangles with dimensions $\beta L \times \lambda\alpha L$. The load applied on these areas and the window is equal to

$$2w\alpha\lambda L^2 + 2w\alpha\beta L^2(1-2\lambda) + w\alpha\beta\lambda L^2(1-2\beta) + 2w\alpha\beta\lambda L^2 + w\alpha L^2(1-2\beta)(1-2\lambda)$$

and the displacements of the C.G. of each of these parts of the panel are: 1/2, β, β, $2\beta/3$ and 2β, respectively. Therefore the work done by the external uniformly distributed load applied over the whole wall is equal to:

$$\text{External work done} = \frac{w\alpha L^2}{6}(3\lambda+20\beta^2\lambda-18\beta\lambda+12\beta-12\beta^2) \quad \text{........(5.13)}$$

The assumptions made in the previous example related to the internal dissipation of energy are also valid here. Hence,

$$\theta_x = 2\beta/\lambda\alpha L \text{ and } \theta_y = 2/L.$$

The moment along the yield-lines "bd,ig and hl" in the x and y-direction is given by:

$$My = \mu m2\lambda\alpha L \text{ and } Mx = m2\beta L.$$

Therefore the internal dissipation of energy along these yield lines can be obtained:

$$\Sigma(M_x\theta_x + M_y\theta_y) = 4m\alpha\left(\frac{\beta^2}{\lambda\alpha^2}+2\mu\lambda\right) \quad \text{............................(5.14)}$$

The lowest failure pressure is obtained by equating the external work with the internal dissipation of energy. As

$$\frac{w\alpha L^2}{6}(3\lambda+20\beta^2\lambda-18\beta\lambda+12\beta-12\beta^2) = 4m\alpha\left(\frac{\beta^2}{\lambda\alpha^2}+2\mu\lambda\right)$$

then,

$$w = \frac{24m\left(\frac{\beta^2}{\lambda\alpha^2}+2\mu\lambda\right)}{L^2(3\lambda+20\beta^2\lambda-18\beta\lambda+12\beta-12\beta^2)} \quad \text{.........................(5.15)}$$

An alternative solution can be attempted by considering the failure pattern of cracks shown in Fig. 5.35.

This solution is obtained by considering a virtual displacement of unity of the yield lines "bc" and "df". Taking into account symmetry, the wall is divided into three rectangles and four triangles. The pressure(w) applied over these parts and the window is:

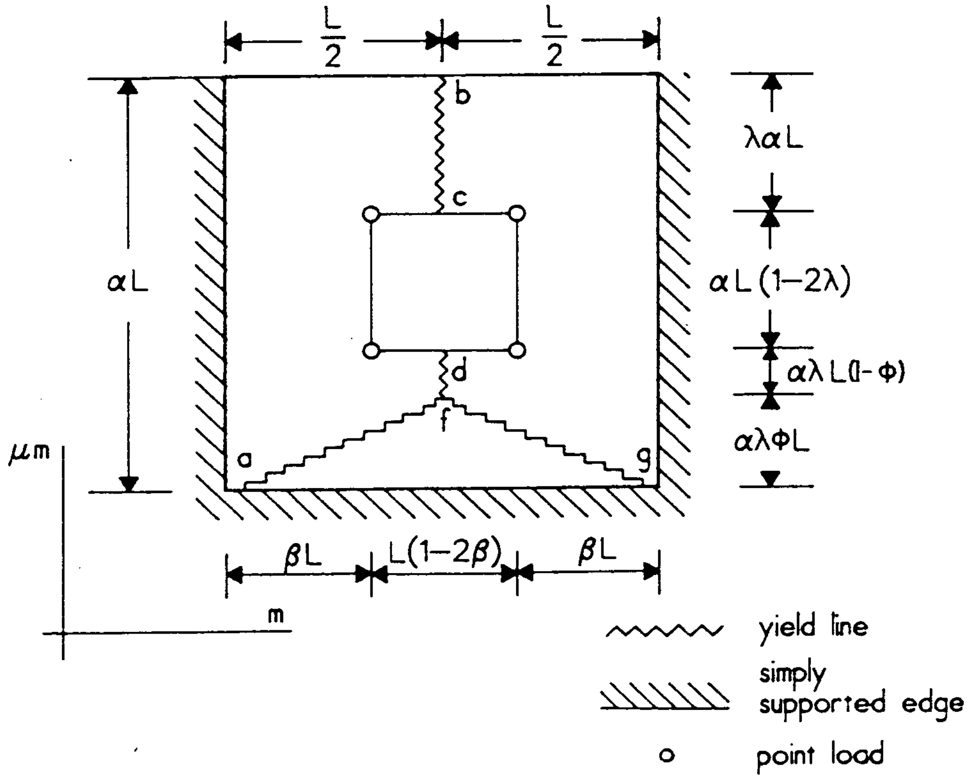


Fig. 5.35
An alternative failure mode for walls with one edge free and three other edges simply supported having a central window

$$w\alpha\lambda L^2/2 + w\alpha\beta L^2(1-2\lambda) + w\alpha\lambda L^2(1-\phi)/2 + w\alpha\lambda\phi L^2/4 + w\alpha L^2(1-2\beta-2\lambda+4\beta\lambda)$$

The displacements (δ) of the C.G. of each of these parts of the panel are: 1/2, β,1/2, 1/3 and 2β. Therefore, the external work done is given by:

External work done $= \frac{w\alpha L^2}{6}(6-6\beta-6\lambda+12\beta\lambda-6\beta^2+12\beta^2\lambda-\lambda\phi)$.......(5.16)

The moment along the yield-lines in the x and y-direction caused by the applied pressure is given by:

$$My = 4\mu m\alpha\lambda L \text{ and } Mx = m/\alpha\lambda\phi$$

As $\theta_y=2/L$ and $\theta_x=1/\alpha\lambda\phi L$, the internal dissipation of energy is

$$\Sigma(M_x\theta_x + M_y\theta_y) = m\alpha\left(\frac{1}{\alpha^2\phi\lambda} + 8\mu\lambda\right) \quad \text{.............................(5.17)}$$

To have equilibrium, equations (5.16) and (5.17) must be equal, then the failure pressure can be obtained.

$$w = \frac{6m\left(\frac{1}{\alpha^2\phi\lambda} + 8\mu\lambda\right)}{L^2(6-6\beta-6\lambda+12\beta\lambda-6\beta^2+12\beta^2\lambda-\lambda\phi)} \quad \text{.........................(5.18)}$$

This equation must be differentiated with respect to ϕ in order to get the minimum value of m/w. Therefore,

$$\phi = \frac{\sqrt{4\lambda^2 - 192\mu\alpha^2\lambda^3(1-\beta-\lambda-2\beta\lambda-\beta^2+2\beta^2\lambda)} - 2\lambda}{16\mu\alpha^2\lambda^3} \quad \text{...........(5.19)}$$

Substituting the value of ϕ in equation (5.18) it is possible to calculate the lowest pressure for walls having a high aspect ratio (H/L) or low value of orthotropy strength or a combination of both cases.

5.3.3.3 Failure mode for walls with one edge free and the three other edges simply supported having the window along the free edge

Two collapse mechanism have been considered for these type of walls. The first is presented in Fig. 5.36.

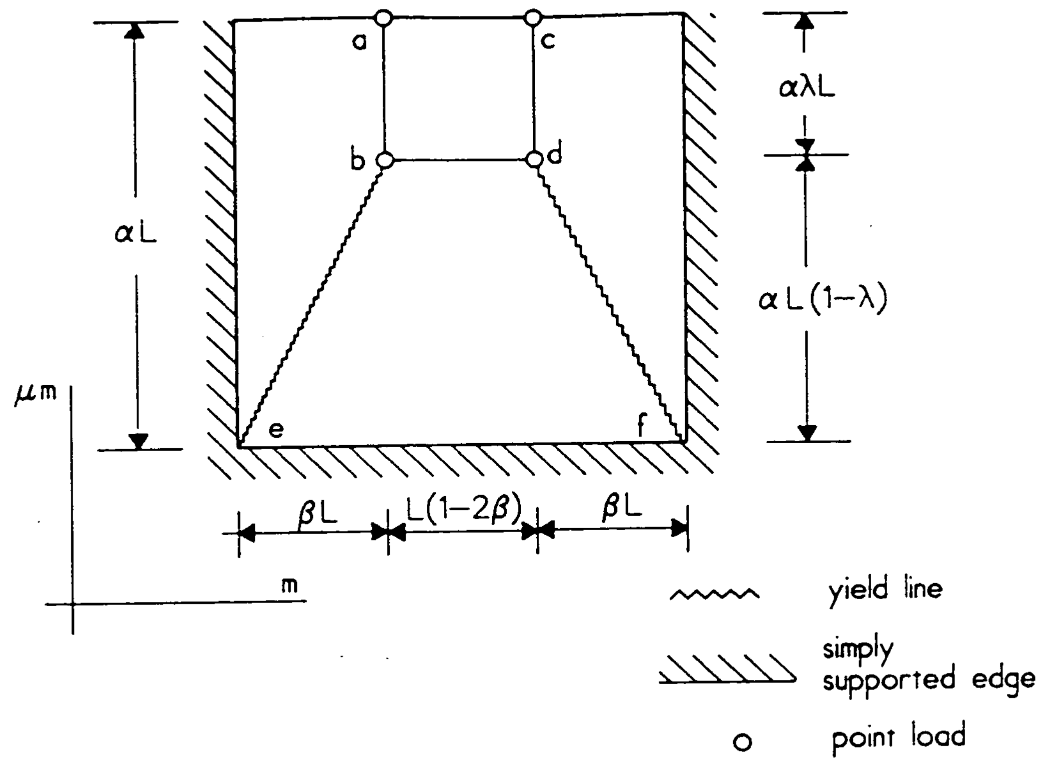


Fig. 5.36
Failure mode for walls with one edge free and the three other edges simply supported having the window along the free edge

The solution of this is obtained by giving a virtual deflection of unity to the four corners "abcd" of the window, while the wall is collapsing. For simplification of the calculation the wall has been divided into 3 rectangles and 4 triangles having the following dimensions:

$$\beta L \times \alpha\lambda L\ (2),\ \alpha L(1\text{-}\lambda) \times L(1\text{-}2\beta)\ (1) \text{ and } \beta L/2 \times \alpha L(1\text{-}\lambda)\ (4)$$

The displacements of these parts of the wall, including the window, is: 1/2, 1/2 and 1/3. Hence the external work done by the uniformly distributed load applied over the area of the wall is given by

$$\text{External work done} = \frac{w\alpha L^2}{6}(3\text{-}2\beta\text{-}4\beta\lambda+3\lambda) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.20)$$

As a result, the moments in the y and x-direction caused by the load are:

$$M_y=2\mu m\alpha L(1-\lambda) \text{ and } M_x=2m\beta L$$

As $\theta_x = 1/\alpha L(1-\lambda)$ and $\theta_y = 1/\beta L$, the internal dissipation of energy along the yield-lines "be" and "df" is given by:

$$\Sigma(M_x\theta_x + M_y\theta_y) = 2m\alpha\left(\frac{\beta}{\alpha^2(1-\lambda)}+\frac{\mu(1-\lambda)}{\beta}\right). \qquad \text{......................(5.21)}$$

By equating the external work done with the internal dissipation of energy on yield-lines "be" and "df", the failure pressure is obtained

$$w = \frac{12m\left(\frac{\beta}{\alpha^2(1-\lambda)} + \frac{\mu(1-\lambda)}{\beta}\right)}{L^2(3-2\beta-4\beta\lambda+3\lambda)} \qquad \text{..(eq. 5.22)}$$

The second collapse mechanism is shown in Fig. 5.37. In this case the solution can be obtained by giving a virtual deflection of unity to the yield line connecting the points "ab". Using the same procedures employed in the previous analysis, the panel has been divided into rectangles and triangles having the following dimensions:

βL x $\alpha\lambda L$ (2), $L(1-2\beta)$ x $\alpha L(1-\lambda)$ (1) and $\beta L/2$ x $\alpha L(1-\lambda)$ (4)

The displacements of the C.G. of these areas, and the window, are: 1/2, 1/2, 1/3 and 1. As a result, the work done by the external pressure is given by:

$$\Sigma w\delta = \frac{w\alpha^2L^2}{6}(3-12\beta\lambda+12\beta^2\lambda+3\lambda-\gamma) \qquad \text{.......................(5.23)}$$

The moment along the yield-lines in the x and y-direction is given by:

$$M_y=2\mu m\alpha L(1-\lambda) \text{ and } M_x=mL.$$

The angles of rotation of these rigid parts of the wall are:

$$\theta_y=2/L \text{ and } \theta_x=1/\alpha\gamma L$$

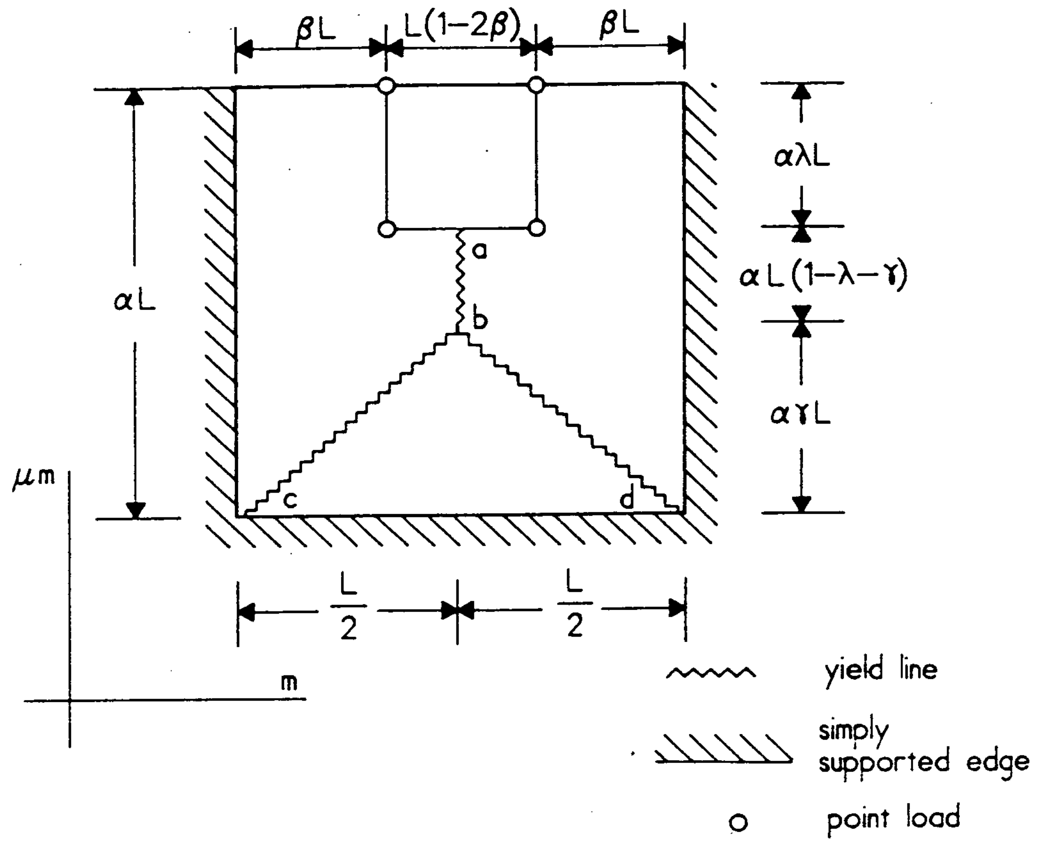


Fig. 5.37
An alternative failure mode for walls with one edge free and the three other edges simply supported having the window along the free edge

Consequently, the dissipation of energy along the yield lines is given by

$$\Sigma(M_x\theta_x + M_y\theta_y) = m\alpha\left(\frac{1}{\gamma\alpha^2} + 4\mu(1\text{-}\lambda)\right) \quad\text{.....................(5.24)}$$

By equating the external work done and the internal dissipation of energy along the yield lines the lowest pressure is obtained, then

$$w = \frac{6m\left(\frac{1}{\gamma\alpha^2} + 4\mu(1\text{-}\lambda)\right)}{L^2(3\text{-}12\beta\lambda+12\beta^2\lambda+3\lambda\text{-}\gamma)} \quad\text{....................................(5.25)}$$

For minimum collapse pressure or maximum value of moment $d(m/w)/d\gamma=0$, thus

$$\gamma = \frac{2 + \sqrt{4-48\mu\alpha^2(4\beta^2\lambda^2-4\beta\lambda^2-4\beta^2\lambda+4\beta\lambda+\lambda^2-1)}}{8\mu\alpha^2(1-\lambda)} \qquad (5.26)$$

The value of γ can be substituted in equation (5.25) to obtain the relationship between the failure moment and the lowest load.

5.3.3.4 Failure mode for walls simply supported on the four edges with eccentrically placed window opening

Due to the asymmetry of these type of walls, a general solution like the previous ones presented would involve several differentiations, making more difficult the search for the failure pressure. An easier approach is to draw some failure patterns of cracks satisfying the boundary and equilibrium conditions and then choose the one that gives the lowest failure pressure. This crack pattern is shown in Fig. 5.38.

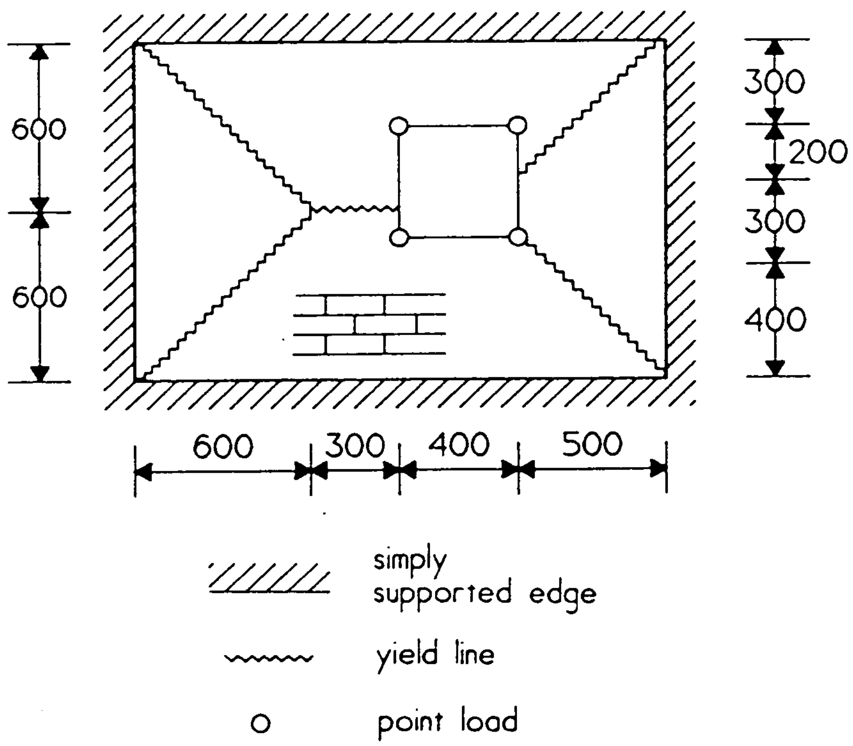


Fig. 5.38
Failure mode for walls simply supported on the four edges with eccentrically placed window opening

5.3.4 Strip method

The yield-line theory is a method of design based on the upper-bound solution - the correct or higher failure load - while the strip method is based on the lower-bound theorem of plasticity. Hence, this method should provide results on the safe side. It was developed by Hillerborg[48] for the design of concrete slabs and applied to unreinforced masonry walls by Baker[109]. The later also presented some changes to the method to allow for the design of wall panels with one free edge, as it is not usual to reinforce the free edges of walls like reinforced concrete slabs. The original assumption, presented by Hillerborg, consists that a strip along the unsupported edge takes a greater load per unit area than the actual strip acting, i.e. that the strip along the free edge behaves partially as a support for the strips at right-angles. Basically, the modification introduced by Baker consists of doubling the span in the direction perpendicular to the unsupported edge and, after that, replace it by a simply supported edge. Once there is equilibrium, the moments can be calculated with the increased span.

The method was called for the first time as simple strip method for the particular case in which the twisting moment was given a value of zero. In a simple way the method can be explained as a variant of the equilibrium theory, in which the entire calculation of the moments in the panel is converted to the calculation of the moments in a series of simple slab strips spanning in just one way. In the equilibrium equation for a panel, irrespective of the form in which it is presented, there are three different moments; two bending moments and one torsional moment. The equilibrium equation gives a relationship between these three moments. It is possible to fulfil the equilibrium equation by choosing two of the moments as functions of the co-ordinates of the panel and solving the third of the moments from the equilibrium equations.

The basis for the simple strip method as it is applied to unreinforced brickwork walls is that the torsional moment is chosen equal to zero. The equilibrium equation is given by

$$\frac{d^2m_x}{dx^2} + \frac{d^2m_y}{dy^2} - 2\frac{d^2m_{xy}}{d_x d_y} = -w \quad (5.27)$$

Neglecting the torsional moments,

$$\frac{d^2m_x}{dx^2} + \frac{d^2m_y}{dy^2} = -w \qquad (5.28)$$

which is valid for orthogonal as well as skew co-ordinates.

Equation (5.28) can also be split into two parts:

$$\frac{d^2m_x}{dx^2} = -w_x \qquad (5.29)$$

and

$$\frac{d^2m_y}{dy^2} = -w_y \qquad (5.30)$$

with the inter-relationship $w_x + w_y = w$.

One solution of the equilibrium equation for the panel can thus be obtained by dividing the load w into two parts w_x and w_y, after which these two latter equations are used to determine m_x and m_y.

In addition to the equilibrium equation, the boundary conditions must be satisfied. For edges parallel to the y-axis the following conditions must be met:

- simple support, $m_x = 0$;
- free edge, $m_x = 0$ and $dm_x/dx = 0$.

Both, the equilibrium equation and the boundary conditions for m_x are exactly the same as for a beam with a load w_x. It is thus possible to treat each strip of the panel parallel to the x-axis as a beam loaded with the strip load w_x. These conditions are also valid for edges parallel to the x-axis.

The Simple Strip Method does not take into account either stiffness orthotropy or strength orthotropy in the way the strips are designed, as long as boundary and equilibrium conditions are satisfied. For corners formed by the junctions of simply supported edges at 90°, preferably the strips have to be drawn following a line of 45° with both edges.

The strip method has been applied to solid unreinforced walls and seems to reasonably agree with the experimental ultimate pressures[109,116]. The calculations follow these steps:

i) the wall panel is divided into parts by means of boundary conditions at which it has been assumed that the average shearing force is zero, i.e. the average moment is maximum. Each part in which the wall panel is divided must be supported along an edge;
ii) the moments along the edges of these parts are determined in a way that, each part, or each group of parts which act together to support the load in a certain direction, is in equilibrium and that there is continuity regarding the sum of the moments corresponding to the bending moment resistance acting in each direction;
iii) the bending moment resistance in each direction is determined by the flexural tensile strength of this direction, which was obtained from the wallettes extracted from the undamaged parts of the failed walls; and
iv) the calculated moments in each directions are averaged and equated with the bending moment resistance in each directions. The failure pressure is then obtained.

Slight differences in the way as the moments are distributed in both directions do not affect considerably the ultimate pressures. Hillerborg[48] pointed out that "we have stressed several times that reasonable changes in the assumed lateral distribution of moments (and thus in the distribution of reinforcement) are unlikely to influence safety significantly. We may therefore select, within quite wide limits, a reasonable lateral distribution for the total moments necessary for the equilibrium of the element expressed as average values of design moments (maximum moments)". Hence, by following this procedure the wall panels were divided into strips. Different results could be obtained if different lateral distribution of moments was assumed. Fig. 5.39 shows the way the wall panels are divided in areas in order to calculate the moments along those strips.

5.4 Comparison between experimental and predicted pressures

5.4.1 Cracking pressures

The only method available for predicting the cracking pressures is elastic plate theory. As explained earlier, due to the complexity of the mathematical solutions of these type of panels, an approach is only possible by applying the finite element method, which overcome the problems of dealing with a large amount of equations.

First, an analysis was done in order to study the influence of the stiffness orthotropy on the flexural strength of brickwork. Most of the work done before[109,114,116,141,142] have considered isotropic properties without doing an analysis of the flexural behaviour. Orthotropic material properties have not been used to analyse the flexural strength of walls with openings. Some work done[141,142] before have first applied the finite element method for isotropic plates to solid walls and then, extended the same isotropic material properties to analyse walls with openings. Hence, it was decided to investigate its effect on the flexural strength by carrying out an analysis on isotropic and orthotropic panels. This was done employing two similar walls simply supported along the four edges; the first solid and the second with a central window, like Walls 1 and 2. Both were subjected to an uniformly distributed load. The walls have been analysed using two different approaches: the first considering isotropic elastic properties and the second orthotropic elastic properties.

i) Two isotropic analysis have been done using both values of moduli of elasticity determined in two main orthogonal directions. The material properties of the first analysis are $E=16,165$ N/mm^2, $\nu=0.15$ and $G=7,028$ N/mm^2. The material properties employed in the second analysis are $E=12,042$ N/mm^2, $\nu=0.11$ and $G=5,474$ N/mm^2. The shear modulus has been calculated using this expression

$$G = \frac{E}{2(1+\nu)} \quad \text{.........(5.31);}$$

ii) The material properties of the orthotropic analysis are the ones presented in section 3.3.3 and $G=5,565$ N/mm^2. The shear modulus has been calculated

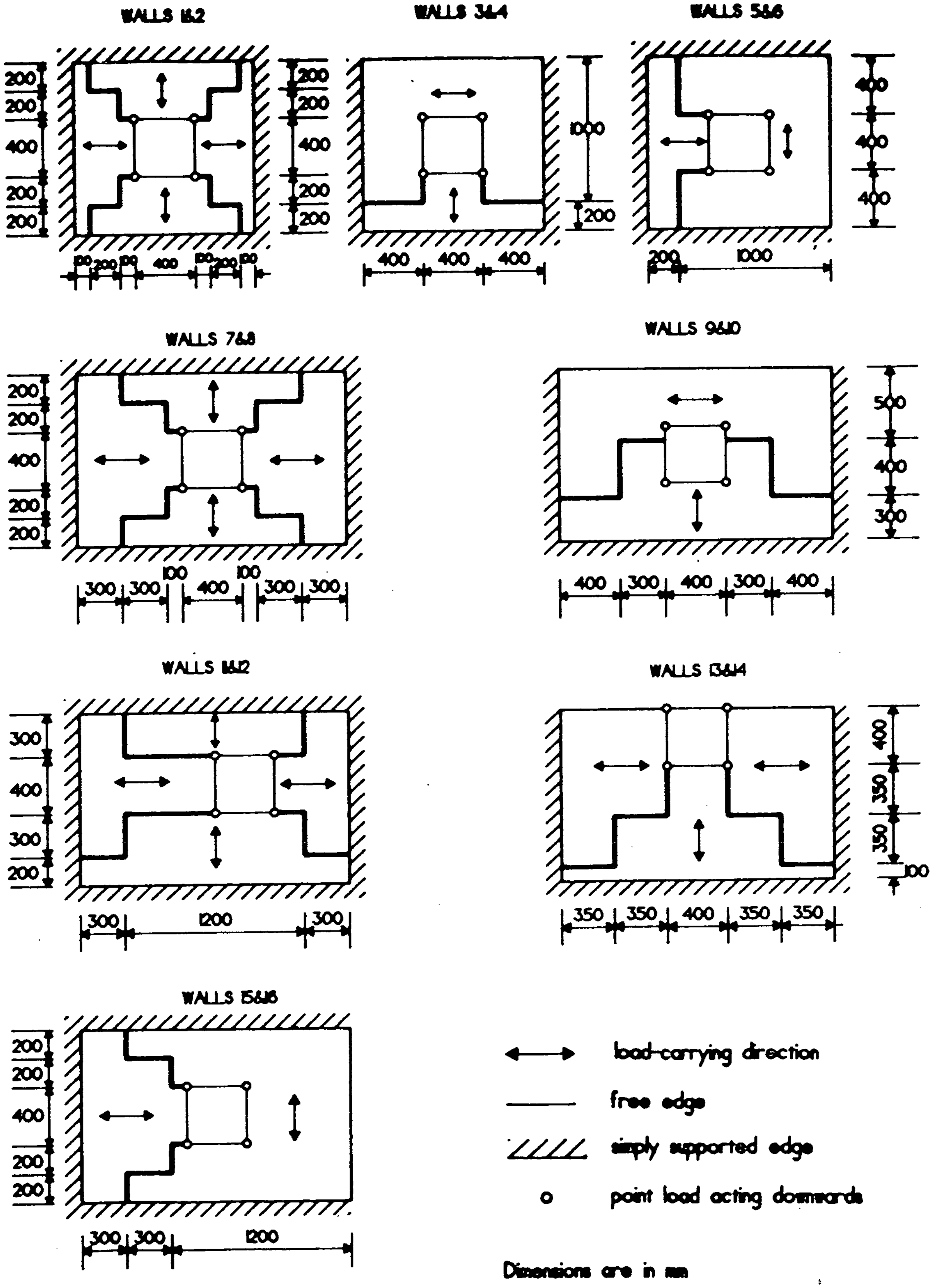


Fig. 5.39

Demarcation of walls into areas to apply the strip method using this expression

$$G = \frac{E_y}{2(1+\nu_{xy}\frac{E_y}{E_x})} \qquad (5.32)$$

Results of the comparison are presented in Table 5.2. The two isotropic analysis for the solid wall and the wall with opening resulted in similar cracking pressures.

Table 5.2
Comparison between isotropic and orthotropic material properties

Walls	Material properties	Cracking pressure (kN/m^2)
Solid	isotropic	7.5
Solid	orthotropic	8.1
with opening	isotropic	4.0
with opening	orthotropic	5.1

From this comparison of cracking pressures it is obvious that the use of orthotropic material properties is more important for the analysis of walls with openings than solid walls, and should not be disregarded. This is due to the fact that walls containing openings present the distribution of moments different than solid walls. Due to the openings the maximum bending moments are not in the central area of the wall but at the neighbourhood of the window corners. The distribution of moments follows a peak shape at the corners. The combination of higher modulus of elasticity and strength in the x-direction helps to increase the bearing capacity of brickwork walls, because more load is shared by the stronger x-direction than supported by the weaker y-direction.

The linear elastic analysis using isotropic and orthotropic material properties shows that the cracking pressure resisted by the wall (aspect ratio 1:1) is reduced substantially by the insertion of a window opening, as can be seen from Table 5.2.

The cracking pressure was achieved when the stress in any of the two main directions reached its ultimate flexural tensile strength as obtained from the one way spanning wallettes presented in Tables 3.12 (x and y-beams). The results of analysis are presented in Table 5.3, together with the experimental cracking pressures.

The cracking criterion presented in section 4.3.5 has also been used to predict the cracking pressures. The worked example how to use the cracking criterion is presented in Appendix C.

An output of one of the finite element analysis (Walls 3 and 4) is presented in Appendix A. In Appendix B, some of the finite element meshes used are given.

The deflections of the uncracked panels have also been predicted using elastic analysis. These are shown for some panels together with the experimental deflections; at cracking and at failure, in Fig. 5.3 to 5.9. It is clear that the deflections of the panels at various points along the horizontal and vertical centre-lines are greater than the predicted values obtained by elastic analysis (Wall 5 is an exception), as the walls showed markedly non-linear behaviour.

From Table 5.3 it is also very clear that there is a lot of variation between the observed experimental cracking pressure of the walls. This variation is higher compared to the variation of the failure pressure. This may be due to the fact that it is difficult to detect the first hair cracks in due time during the tests. Hence, it may be possible that some test walls have cracked long before it was noticed. Therefore, the comparison between experimental and theoretical cracking pressures has been done considering always the lowest measured experimental cracking pressure from each pair of identical test walls, and not their average.

The relationship between the experimental cracking pressure of the test walls and those predicted by the finite element method for orthotropic plates and the line of equality is given in Fig. 5.40. In an ideal situation all test results should lie on the line of equality. In this investigation, almost all the test results are above the line of equality, hence it is safe to use the elastic analysis with the cracking criterion of brickwork presented in section 4.3.5 for predicting the cracking pressures of brickwork panels with openings.

Table 5.3

Comparison between experimental and theoretical cracking pressures

Walls	Experimental cracking pressure (kN/m^2)	Elastic cracking pressure (kN/m^2)	Cracking Criterion (kN/m^2)
1 2	5.0 5.2	5.1	5.2
5 6	4.0 3.2	2.2	2.5
7 8	2.6 4.0	2.3	2.6
11 12	5.6 4.2	2.1	2.3
15 16	1.8 2.2	1.7	1.9

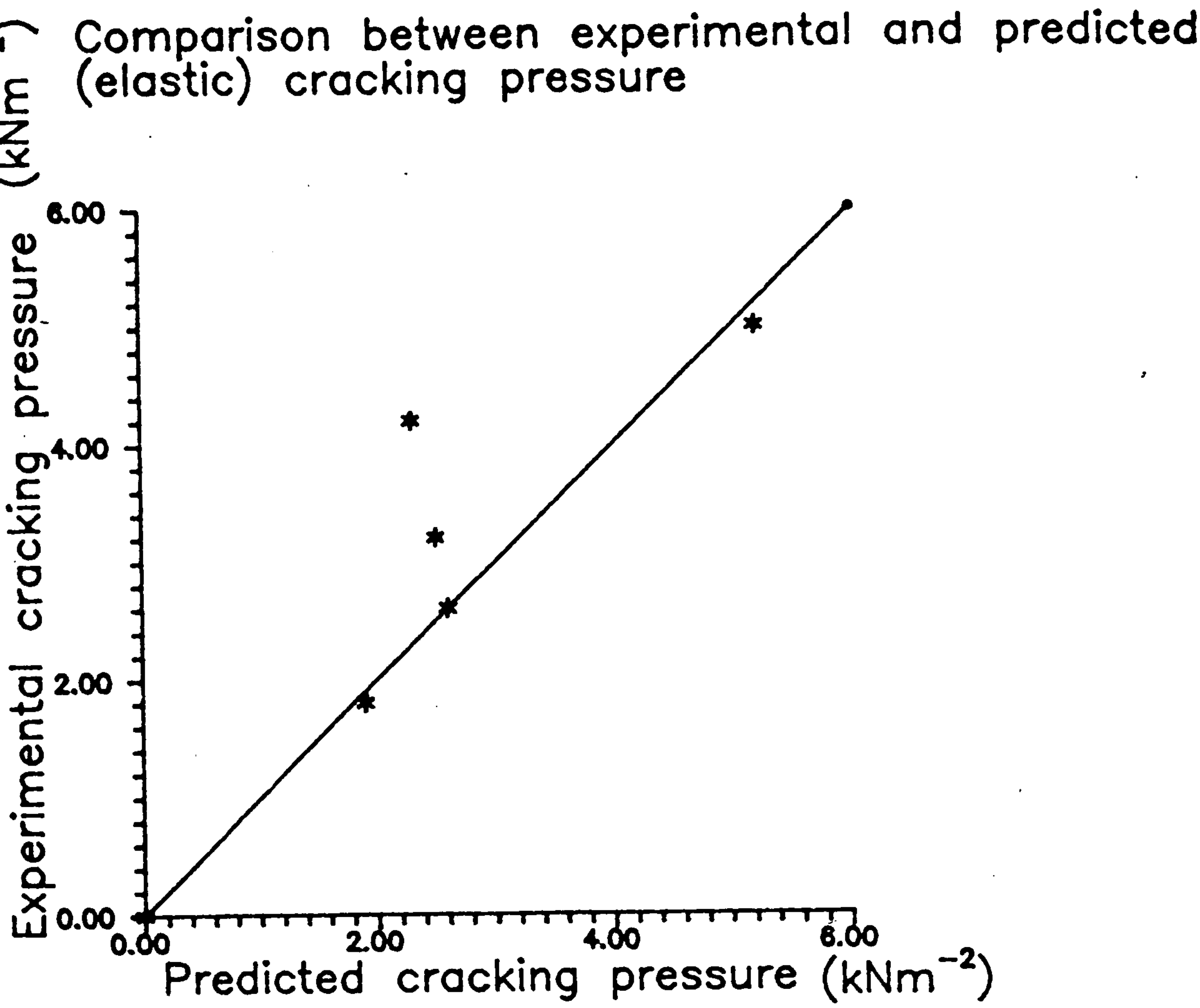


Fig. 5.40

Comparison between experimental and predicted cracking pressures

5.4.2 Ultimate pressures

5.4.2.1 Yield-line analysis

The yield-line equations have been presented in sections 5.3.3.1 to 5.3.3.4. The ultimate pressure of Walls 1, 2, 7 and 8 (Fig. 5.1) was predicted using equation (5.8). All walls; 3, 4, 5, 6, 9, 10, 15, and 16 with one free edge and having the window positioned at the centre, had the ultimate pressure predicted using equation (5.15). The equation (5.22) has been used to predict the failure pressure of Walls 13 and 14. The crack pattern which gives the lowest pressure for Walls 11 an 12 (Fig. 5.39) was used for comparison with the experimental values.

The theoretical failure pressures obtained from the equations mentioned above are compared with the experimental results in Table 5.4.

The relationship between the experimental failure pressures and those predicted by the yield-line method and the line of equality are presented in Fig. 5.41. It can be seen from Table 5.4 and Fig. 5.41 that all, except one test result, lie below the line of equality, which suggests that the yield-line method may be safer for the design of brickwork panels with openings subject to lateral pressure.

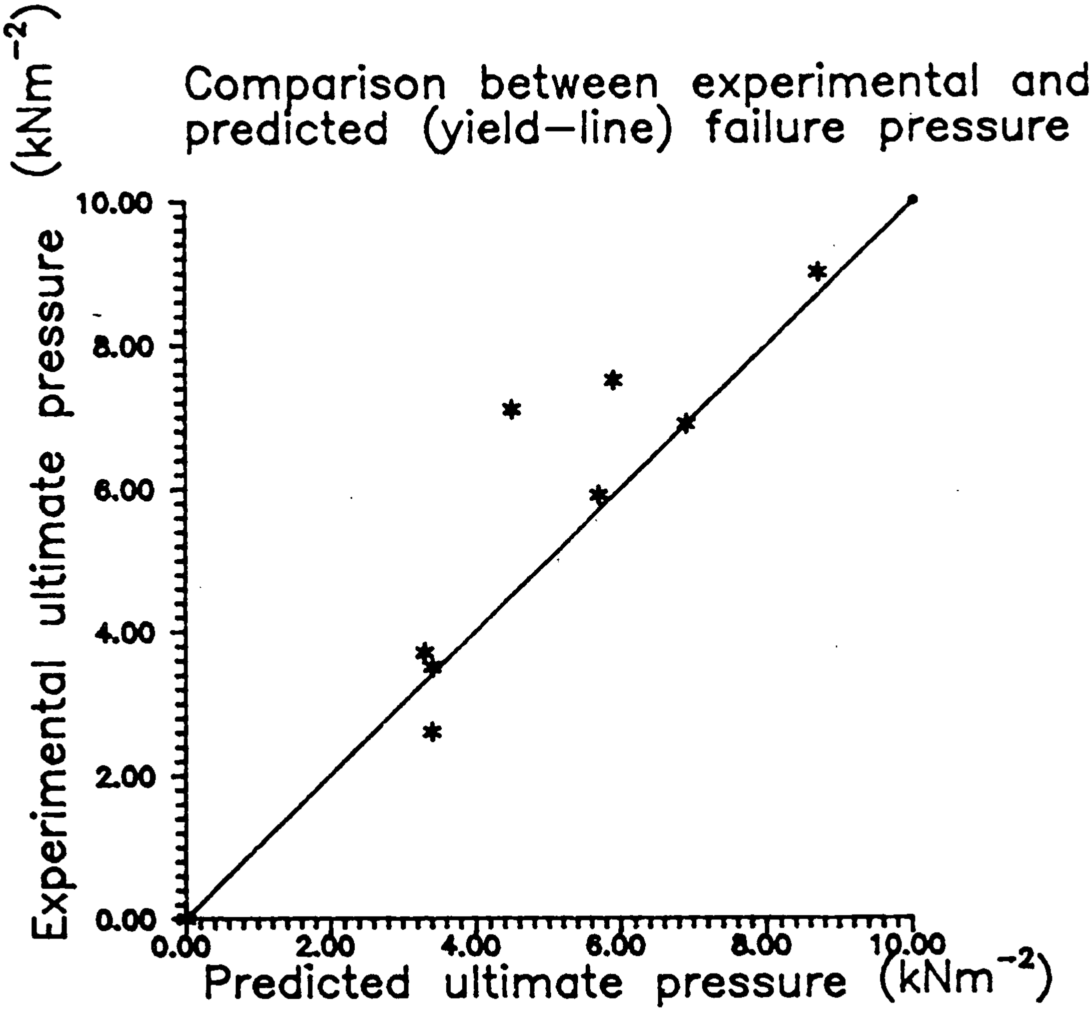


Fig. 5.41

Comparison between experimental and predicted (yield-line) failure pressure

5.4.2.2 Strip method

Worked examples of the application of the strip method are not presented as they involve only the well-known methods of calculation of moments in simply supported beams subjected to uniformly distributed and point loads. Results are presented in Table 5.4.

The relationship between the experimental failure pressures and those predicted by the strip method and the line of equality are presented in Fig. 5.42. From Table 5.4 and Fig. 5.42 it can be seen that 50% of the test results are lower than predicted by the strip method, hence it would not be safe to use this method for the design of unreinforced brickwork panels subjected to lateral pressure.

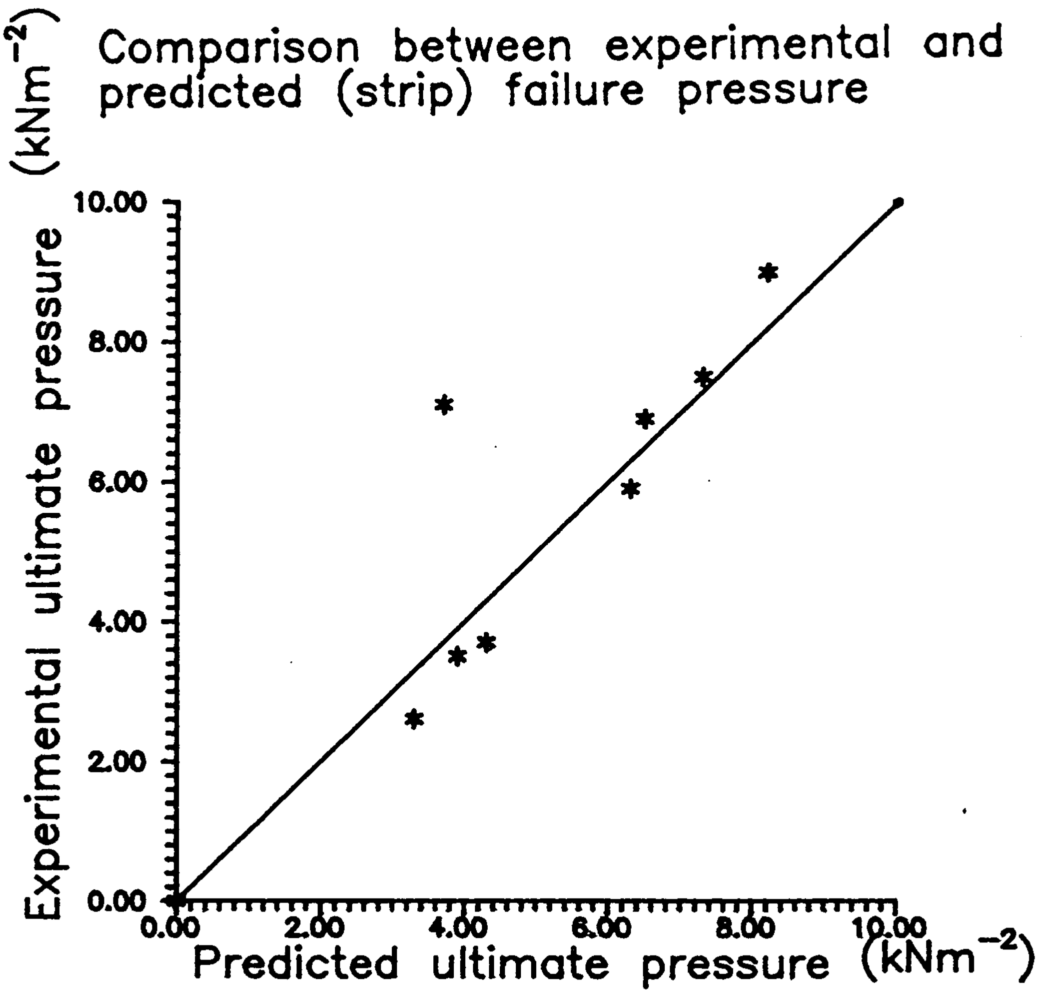


Fig. 5.42

Comparison between experimental and predicted (strip method) failure pressure

5.4.2.3 Code of practice for Structural use of masonry - BS 5628: Part 1

Extensive lateral load tests on panels without openings[83,85] formed the basis of the recommended design bending moment coefficients in BS 5628[86]. These bending moment coefficients are similar to those that can be obtained by yield-line analysis applied to under-reinforced concrete slabs. For the design of panels with openings the suggestion made in the Code, Appendix D[86], is to divide the panels into sub-panels and then to design each part either in accordance with the rules given in clause 36 or by the yield-line or elastic analysis. No experimental data for the lateral load design of panels with openings were available to support the contention of BS 5628. Hence, it was felt useful to make an assessment of BS 5628 in the light of the experimental results.

This has been done by comparing the test results with the Code of practice BS 5628. As the comparison is made with the failure pressure, the material partial safety factor is assumed as 1. The moment of resistance is given by:

$$M_{ult} = f_{ty}.Z \qquad (5.33)$$

The value recommended by Code for the characteristic flexural tensile strength is $f_{ty}=0.4$ N/mm^2 and the strength orthotropy is $\mu=3$. If this value is used the ultimate moment of resistance of the panels works out to be 209.07 Nmm. This ultimate moment of resistance has been used to obtain the failure wind pressure using the equations given in section 5.3.3.1 to 5.3.3.4 and the crack pattern given in Fig. 5.33. The results are presented in Table 5.4.

Table 5.4

Comparison between experimental and predicted failure pressures

Walls	Experimental failure pressure (kN/m^2)	Theoretical failure pressures (kN/m^2)		
		Yield-line	Strip method	BS 5628
1 and 2	9.0	8.7	8.2	4.8
3 and 4	7.5	5.9	7.3	3.4
5 and 6	7.1	4.5	3.7	2.4
7 and 8	5.9	5.7	6.3	3.0
9 and 10	3.5	3.4	3.9	1.9
11 and 12	6.9	6.9	6.5	3.8
13 and 14	3.7	3.3	4.3	1.8
15 and 16	2.6	3.4	3.3	1.7

The predicted failure pressure are in all cases lower than the experimental results. The ratio between the experimental failure pressure and the ultimate pressure given by BS 5628 ranges between 1.53 to 2.96, which suggests that it is safe to design the panels by the yield-line method using the Code provisions. The design will be conservative.

5.5 CONCLUSIONS

i) The finite element method using isotropic material properties for brickwork underestimates the cracking pressures of walls with window openings .

ii) The use of the finite element method with orthotropic material properties combined with the cracking criterion, developed in this work, predicts reasonably the experimental cracking pressures of the brickwork walls with window openings tested in this project. In service limit state, this method may be recommended for the design.

iii) The strip method does not correlate well with the experimental failure pressure, hence can not be recommended for the design of brickwork panels with window openings.

iv) The yield-line method predicts reasonably well the ultimate pressure of unreinforced brickwork walls with window openings.

v) The yield-line method in conjunction with the material properties recommended in BS 5628 gives a conservative estimate of the lateral pressure for walls with window openings, hence it can be used for the ultimate load design with some confidence.

CHAPTER 6

CONCLUSIONS

6.1 GENERAL CONCLUSIONS

This thesis presents the results of an investigation which has been carried out into the lateral strength of unreinforced brickwork wall panels with window openings. The findings of this thesis are supported by experimental work performed on 160 wallettes, 24 cross-beams and 16 half-scale wall panels with window openings. On the basis of the work done in this study the following conclusions can be drawn:

i) In the linear range the flexural and compression tests give the same value for the initial tangent modulus of brickwork in the two main orthotropic directions.

ii) The flexural tensile strengths of brickwork normal and parallel to the bed joints of brickwork are different and present a random variation, but there is no statistical difference whether the values of strengths are obtained from wallettes built independently or extracted from the undamaged parts of walls.

iii) The BS 5628 suggests a correlation between the characteristic flexural tensile strengths of brickwork and the water absorption of bricks. In this investigation the characteristic flexural tensile strength obtained for the bricks and type of mortar used in the experimental programme is similar to the value recommended by the Code.

iv) The load distribution was studied using different specimens of brickwork (cross-beams and half-scale walls). It was found that orthotropic material property is important as the load distributes according to the stiffness orthotropy determined experimentally.

v) Due to moment interaction the flexural tensile strength perpendicular to the bed joint can be enhanced beyond its ultimate value than the ones obtained performing flexural tests on wallettes spanning in just one direction. This cracking criterion developed in this thesis is best idealised by a polynomial expression.

i) The ultimate load of specimens of unreinforced brickwork subjected to bi-axial bending can be predicted by the elastic analysis using the residual moments in the cracked directions. Good correlations between the predicted and experimental failure loads were obtained for the cross-beams, using the obtained expression.

ii) Also, the yield-line method using the final strength orthotropy predicted accurately the failure loads of the cross-beams. Therefore, this method can also be applied to analyse the strength of brittle materials, provide the ultimate strength orthotropy is used.

iii) Brickwork wall panels show clear markedly non-linear behaviour at increasing lateral loading. As a consequence, predictions of the load-deflection behaviour using elastic analysis underestimates most of the wall's deflections.

ix) The finite element method using isotropic material properties underestimates the cracking pressure of unreinforced brickwork walls with window openings.

x) The finite element method with orthotropic material properties combined with the use of the cracking criterion developed in this investigation reasonably predicts the cracking pressure of unreinforced brickwork walls. In service limit state, this method may be recommended for the design.

xi) The strip method does not correlate well with the experimental pressures, hence can not be recommended for the design of brickwork panels with window openings.

xii) The yield-line method gives a good correlation between the theoretical and experimental ultimate pressures of unreinforced brickwork walls having window openings, hence it can be used for ultimate load design.

6.2 SUGGESTION FOR FURTHER RESEARCH

As it was shown in the literature review, a lot of work has been done to establish the flexural behaviour of unreinforced masonry walls. This thesis has elucidated the flexural behaviour at the uncracked stage, showing that, due to moment interaction, the bending moments perpendicular to the bed joints can be enhanced over the ultimate bending moment values determined performing flexural tests on one-direction wallettes. Also methods have been suggested for the design of brickwork panels on service and ultimate limit state.

However, further work needs to be done as suggested below.

i) Flexural tests on cross-beams having 'comb-arms' to gather more data, as the results presented are an average of three tests for each aspect ratio.

ii) Testing of cross-beams of different aspect ratios than those performed in this study. The aspect ratios that can led to simultaneously cracking in both directions or first cracking in the horizontal direction will be particularly interesting.

iii) Study of the influence of applied vertical precompression forces on the interaction of bending moments in unreinforced brickwork, simulating the in-plane forces due to the dead-weight of the building.

iv) Measurements of the residual strength after cracking in structures more redundant than cross-beams, i. e. wall panels, to determine the extent of the membrane action on such type of structures,.

REFERENCES

1. Stang, A. H., D. E. Parsons and H. D. Foster. "Compressive and Transverse Strength of Hollow-tile Walls". Tech. Paper of the Bureau of Standards 20, 347, 1925-1926.

2. Kelch, N.W. "Methods Used in Testing Masonry Specimen for Bending, Tension and Shear". Journal of American Ceramic Society 14(2), 1931, p125-132.

3. Richart, F. E., R. P. B. Moorman and P.M. Woodwarth. "Strength and Stability of Concrete Masonry Walls". University of Illinois Bulletin 251, 1932, p36.

4. Royen, N. "Design of Masonry Walls, Accounting for Friction and Bond". Hafet 9, Byggmastaren, Stockholm 1936, pp 105-108.

5. Whittemore, H. L., A. H. Stang and Parsons, D. E. "Building, Materials and Structures Reports Nos 5, 21, 22, 23, 24, 32, 38 and 53. National Bureau of Standards, Washington, 1938-1941.

6. Parsons, D. E. "Watertightness and transverse Strength of Masonry Walls". Structural Clay Products Institute, McLean, Virginia, 1939.

7. Plummer, H. and Rearden, L. "Principles of Brick Engineering, Handbook of Design". Structural Clay Products Institute, Washington DC, 1939.

8. Tasker, H. E. "Simulated Wind Pressure Tests". Technical Study No 13, Commonwealth Experimental Building Station, Sydney, 1947.

9. Isaacs, D.V. "Second Interim Analysis of the Strength of Masonry Walls". Special Report No 1. Commonwealth Experimental Building Station, Sydney, 1948.

10. Nerlich, W. "The Design of Load-bearing Brick walls to Resist Wind Pressure". Bauplan, Vol2, No9, 1948, pp271-272.

11. Davey, N. and Thomas, F.G. "The Structural Uses of Brickwork". Proceedings of Institution of Civil Engineers, Structural Building Paper No24, February 1950.

12. Davey, N. "Modern Research on Loadbearing Brickwork". National Federation of Clay Industries, The Brick Bulletin, Vol2, 1952.

13. Hummel, A. "Research on Mortar for Free Standing Chimneys". Comentry on Research of TH Aachen IfB, 1952-1953.

14. Thomas, F. G. "The Strength of Brickwork". The Structural Engineer, Vol31, No2, February 1953, pp35-46.

15. Goalwin, D. S. "Properties of Cavity Walls". Building Materials and Structures. Report No136. National Bureau of Standards, Washington 1953.

16. Monk Jr., C. B. "Transverse Strength of Masonry Walls". ASTM Special Technical Publication No166, Symposium on Methods of Testing Building Constructions, Chicago, June 1954, pp.21-50.

17. McDowell, E. L., McKee, K. E. and Sevin, E. "Arching Action Theory of Masonry Walls". Proceedings of ASCE Journal, Structural Division, Vol82, No St2, March 1956.

18. Cohen, E. and Lang, E. "Discussion of Reference 22". Proceedings of ASCE, Vol82, NoSt5, September 1956.

19. Cox, F. W. and Ennenga, J. L. "Transverse Strength of Concrete Block Walls". Journal of American Concrete Institute, Vol29, No11, May 1958, pp951-960.

20. Benjamin and Williams. "The Behaviour of One-Storey Brick Shear Walls". Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Paper 1723, St4, July 1958.

21. Allen, D. E. "Annotated Bibliography on Lateral Loads on Unreinforced Masonry Walls". Division of Building Research, National Research Council of Canada, Ottawa, 1958.

22. Hedstrom, R. O. "Load Tests on Paterned Concrete Masonry Walls". A.C.I. Journal Proceedings, Vol57, P.C.A. Development Department Bulletin D.41 , April 1961.

23. Falconer, B. H. "Engineering Design in Brickwork". Clay Products Bulletin No22. New Zealand Pottery and Ceramics Research Association, Lower Hutt, 1962.

24. "Compressive, Flexural and Diagonal Tensile Testing of Small-scale Four-inch Brick Masonry Specimens". National Testing Program Progress Report No1. Structural Clay Products Research Foundation, McLean, Virginia, 1964.

25. Krone, R. H. and Pollitz, R. N. "Spacing of Lateral Supports for Masonry Walls". Journal of the American Concrete Institute, February 1965, pp231-237.

26. Bradshaw, R. E. and Entwistle, F. D., "Wind Forces on Non-loadbearing Brickwork Panels". Technical Note, Vol1, No6, Clay Products Technical Bureau, London, May 1965.

27. "Compressive and Transverse Tests of Five-inch Brick Walls". Research Report No8. Structural Clay Products Research Foundation, McLean, Virginia, 1965.

28. "Compressive , Transverse and Racking Strength Tests of Four-inch Brick Walls". Research Report No9. Structural Clay Products Research Foundation, McLean, Virginia, 1966.

29. "Compressive and Transverse Strength Tests of Eight-inch Brick Walls". Research Report No10, Structural Clay Products Research Foundation, McLean, Virginia, 1966.

30. Hallquist, A. "Wind Loads on Cavity Walls". Tegl. Nos2 and 4. Journal from Teglverkens Forskninginstitut, Forskningsveien 3b, Blindern, Norway, 1966.

31. "Compressive, Transverse and Racking Strength Tests on Four-inch Structural Clay Facing Tile Walls". Research Report No11. Structural Clay Products Research Foundation, McLean, Virginia, 1967.

32. Youl, V. A. and Forster, P.K. "Miniature Tensile and Flexure Properties of Brickwork". Designing Engineering and Construction with Masonry Products. Gulf Publishing Company, Houston, Texas, 1969.

33. Grenley, D. G. "Study of the Effects of Certain Mortars on Compressive and Flexural Strength of Masonry". Proceedings of the First International Brick Masonry Conference, Houston, 1967, pp28-33.

34. Isberner, A. W. "Properties of Masonry Cement Mortars". Designing and Construction with Masonry Products. Gulf Publishing Company, Houston, Texas, 1969.

35. "Compressive, Transverse and Shear Strength Test of Six and Eight-inch Single-wythe Walls Built with Solid and Heavy-duty Hollow Clay Masonry Units. Research Report No16. Structural Clay Products Research Foundation, McLean, Virginia, 1969.

36. "Building Code Requirements for Engineering Brick Masonry". Structural Clay Products Institute, McLean, Virginia, August 1969.

37. Standards Association of Australia, "S.A.A. Brickwork Code AS CA 47, 1967.

38. Losberg, A. and Johansson, S. "Sideway Pressure on Masonry Walls of Brickwork". CIB Symposium on Bearing Walls, Warsaw, June 1969.

39. Hallquist, A. "Lateral Loads on Masonry Wall". CIB Symposium on Bearing Walls, Warsow, June 1969.

40. Francis, A. J. "The SAA Brickwork Code: The Research Background". Civil Engineering Transations of Institution of Engineers, Australia, VolCE11, No2, October 1969, pp165-176.

41. Nilsson, I. H. E. and Losberg, A. "The Strength of Horizontally Loaded Prefabricated Brick Panel Walls". 2nd International Brick Masonry Conference, Stoke-on-Trent, 1970.

42. Hendry, A. W., Sinha, B. P. and Maurenbrecher, A. H. P. "Full-scale Tests on the Lateral Strength of Brick Cavity Walls. Proceedings of the 4th Symposium on Loadbering Brickwork, London, 1971.

43. Thomas, K. "Structural Brickwork - Materials and Performance". The Structural Engineer, Vol49, No10, October 1971.

44. West, H. W. H., Hodgkinson, H. R. and Webb, W. F. "The Resistence of Clay Brick Walls to Lateral Loading". Proceedings of the 4th Symposium on Loadbearing Brickwork, London, 1971.

45. Morton, J. and Hendry, A. W. "A Theoretical Investigation of the Strength of Brick Walls with Precompression". Proceedings of the 4th Symposium on Loadbearing Brickwork, London, 1971.

46. Baker, L. R. "Brickwork Panels Subjected to Face Wind Loads". MEngSc thesis, University of Melbourne, 1972.

47. Baker, L. R. "Manufacture and Testing of Model Brickwork Wind Panels". Structural Models Conference, Institution of Engineers, Australia, Sydney, May 1972.

48. Hillerborg, A. "Strip Method of Design". Viewpoint Publications, London, 1975 (1974), pp256.

49. Satti, K. M. H. "Model Brickwork Walls Panels Under Lateral Loading". PhD thesis, Edinburgh, 1972.

50. Hendry, A. W. "The Lateral Strength of Unreinforced Brickwork". The Structural Engineer, Vol51, No2, February 1973, pp43-50.

51. Morton, J. "A Theoretical and Experimental Investigation of the Static and Dynamic Lateral Resistence of Brickwork Panels with Reference to Damage by Gas Explosion". PhD thesis, Edinburgh, 1972.

52. Hellers, B. G. and Sahlin, S. "Loading Tests on Foundation Walls Unsupported at the Top Edge". National Swedish Building Research, Report No36, 1972.

53. Magdalinski, J. "How to Design Brick Walls for Wind Pressure". Translation No1660. Building Research Station, United Kingdom, 1972.

54. Lindsay, D. "Notes on a Method of Analysis of Brick Panel Walls Subjected to Out of Plane Loading". Commonwealth Department of Works, June 1973.

55. Base, G. D. and Baker, L. R. "Fundamental Properties of Structural Brickwork". Journal of the Australian Ceramic Society, Vol.9, No1, May 1973, pp1-6.

56. Baker, L. R. "Structural Action of Brickwork Panels Subjected to Wind Loads". Journal of the Australian Ceramic Society, Vol.9, No1, May 1973, pp7-13.

57. West, H. W. H., Hodgkinson, H. R. and Webb, W. F. "Lateral Loading Tests on Walls with Different Boundary Conditions". Third International Brick Masonry Conference, Essen, 1973, pp180-186.

58. Haseltine, B. A. and Hodgkinson, H. R. "Wind Effects on Brick Panel Walls - Design Considerations. Third International Brick Masonry Conference, Essen, 1973.

59. Cajdert, A. and Losberg, A. "Laterally Loaded Light Expanded Clay Block Masonry. The Effect of Reinforcement in Horizontal Joints". Third International Brick Masonry Conference, Essen, 1973.

60. Baker, L. R. "Flexural Strength of Brickwork Panels". Third International Brick Masonry Conference, Essen, 1973, pp378-383.

61. Satti, K. M. H. and Hendry, A. W. "The Modulus of Rupture of Brickwork". Third International Brick Masonry Conference, Essen, 1973, pp155-160.

62. Baker, L. R. "Wind Loading on Brickwork Buildings". Brickwork Seminar Papers, B.D.R.I., University of Melbourne, May 1974.

63. Baker, L. R. "Variation in Flexural Strength of Brickwork with Beam Span and Loading". Journal Australian Ceramic Sociaty, Vol10, No2, August 1974, pp25-38.

64. Sinha, B. P. and Hendry, A. W. "Tensile Strength of Brickwork Specimens". Proceedings of the British Ceramic Society, No24, 1975 (1974), pp91-100.

65. West, H. W. H. and Hodgkinson, H. R. "The Lateral Load Resistence of Brickwork Without Precompression". Proceedings of the British Ceramic Society, No24, September 1975, pp101-113 (1974).

66. Haseltine, B. A. "The Design of Laterally Loaded Wall Panels". Proceedings of the British Ceramic Society, No24, September 1975 (1974).

67. Lawrence, S. J. and Morgan, J. W. "Strength and Modulus of Elasticity in Lateral Bending". Proceedings of the British Ceramic Society No24, 1975 (1974), pp79-90.

68. James, J. A. "An Investigation of the Lateral Load Resistence of Walls of Unreinforced Brickwork Without Precompression". Report W/LAT/1. Building Develpment Laboratories, Perth, 1975.

69. Lawrence, S. J. "Flexural Strength of Brickwork Normal to and Parallel to the Bed Joints". Journal of the Australian Ceramic Society, Vol11, No1, 1975, pp5-6.

70. Kheir, A. M. A. "Brickwork Panels Under Lateral Loading". M.Phil. thesis, University of Edinburgh, 1975, p75.

71. Sinha, B. P. and Hendry, A. W. "Tests on Full-scale Cavity Walls Under Lateral Loading". Second International Symposium on Bearing Walls, Warsaw, 1975.

72. Swedish Building Code for Masonry. SBN 19754, Chapter 4.

73. Cajdert, A. and Losberg, A. "Basement Masonry Walls of No-fires Concrete Blocks". Design for Earth Pressure, Third Nordic Masonry Symposium, Copenhagen, 1975.

74. Mayers, R. L. and Clough, R. W. "A Literature Survey: A Compressive Tensile, Bond and Shear Strength of Masonry". Report No EERC 75-15, College of Engineering, University of California, June 1975.

75. Hendry, A. W. and Kheir, A. M. A. "The Lateral Strength of Certain Brickwork Panels". Fourth International Brick Masonry Conference, Brussels, April 1976.

76. Baker, L. R. and Kheir, A. M. A. "The Lateral Strength of Certain Brickwork Panels". Fourth International Brick Masonry Conference, Brussels, April 1976.

77. Hodgkinson, H. R., West, H. W. H. and Haseltine, B. A. "Preliminary Tests on the Effect of Arching in Laterally Loaded Walls". Fourth International Brick Masonry Conference, Brussels, April 1976.

78. West, H. W. H.,"The Flexural Strength of Clay Masonry Determined from Wallette Specimens". Fourth International Brick Masonry Conference, Brussel, 1976, Paper 4.a.6.

79. Anderson, C. "Lateral Loading Tests on Concrete Block Walls". The Structural Engineer, Vol54, No7, July 1976, pp239-246.

80. Anderson, C. and Bright, N. J. "Behaviour of Non-loadbearing Block Walls Under Wind Loading". Concrete, Vol10, No9, September 1976, pp27-30.

81. Commonwealth Department of Works. "Limit State Strength of Brick Walls Subjected to Lateral Loading". Technical Information T1 108 SE. Melbourne 1977.

82. West, H. W. H., Hodgkinson, H. R. nad Haseltine, B. A. "The Resistence of Brickwork to Lateral Loading: Part 1 - Experimental Methods and Results of Tests on Small Specimens and Full-sized Walls". The Structural Engineer, Vol55, No10, October 1977, pp411-421.

83. Haseltine, B. A., West, H. W. H. and Tutt, J. N. "The Resistence of Brickwork to Lateral Loading: Part 2 - Design of Walls to Resist Lateral Loads". The Structural Engineer, Vol55, No10, October 1977, pp422-430.

84. West, H. W. H., Hodgkinson, H. R. and Haseltine, B. A. "The Lateral Resistence of Brickwork Walls Without Precompression". Proceedings of the Sixth International Symposium on Load Bearing Brickwork, December 1977, London.

85. Haseltine, B. A., West, H. W. H. and Tutt, J. N. "The Design of Laterally Loaded Wall Panels". Proceedings of the Sixth International Symposium on Load Bearing Brickwork, December 1977, London.

86. British Standard Institution. "Code of Practice for Structural Use of Masonry: Part 1 - Unreinforced Masonry. BS 5628, Part 1, 1978.

87. Baker, L. R. "The Failure Criterion of Brickwork in Vertical Flexure". Proceedings of the Sixth International Symposium on Load Bearing Brickwork, December 1977, London.

88. Baker, L. R. "Precracking Behaviour of Laterally Loaded Brickwork Panels with In-Plane Restraints". Proceedings of the Sixth International Symposium on Load Bearing Brickwork, December 1977, London.

89. Baker, L.R. "The Lateral Strength of Brickwork - An Overview". Proceedings of the Sixth International Symposium on Load Bearing Brickwork, December 1977, London.

90. Hatzinikolas, M., Longworth, J. and Warwaruk, J. "The Use of Centrifugal Force for Determining the Tensile Bond Strength of Masonry". Proceedings of the Sixth International Symposium on Load Bearing Brickwork, December 1977, London.

91. Schoner, W. "The Ultimate Strength of Masonry Walls Subjected to Combined Lateral and Compressive Loading". Technical University, Heft 41, Hannover 1978.

92. Sinha, B. P. "A Simplified Ultimate Load Analysis of Laterally Loaded Model Orthotropic Brickwork Panels of Low Tensile Strength". The Structural Engineer, Vol56B, No4, December 1978, pp81-84.

93. Huizer, A. and Ward, M. A. "A Simplified Flexural Bond Test for Clay Brick Masonry". Proceedings of the North American Masonry Conference, Boulder, 1978

94. Lawrence, S. J. "The Flexural Behaviour Brickwork". Proceedings of the Noth American Masonry Conference, Boulder, 1978.

95. Baker, L. R. "Some Factors Affecting the Bond Strength of Brickwork". Fifth International Brick Masonry Conference, Washington, 1979.

96. Baker, L. R. "Measurement of the Flexural Bond Strength of Masonry". Fifth International Brick Masonry Conference, Washington, 1979.

97. Baker, L. R. "A Failure Criterion for Brickwork in Bi-axial Bending". Fifth International Brick Masonry Conference, Washington, 1979.

98. Matthys, J. A. and Grimm, C. T. "Flexural Strength of Non Reinforced Brick Masonry with Age". Fifth International Brick Masonry Conference, Washington, 1979.

99. Akio, B., Katsuro, K. and So, K. "Design for Mix Proportion of Joint Mortar and Bond Strength". Fifth International Brick Masonry Conference, Washington, 1979.

100. Moore, J. F. A., Haseltine, B. A. and Hodgkinson, H. R. "Edge Restraint Provided by Continuity of Panel Walls". Fifth International Brick Masonry Conference, Washington, 1979.

101. West, H. W. H., Hodgkinson, H. R. and Haseltine, B. A. "The Lateral Resistence of Walls with One Free Vertical Edge". Fifth International Brick Masonry Conference, Washington, 1979.

102. West, H. W. H., Hodgkinson, H. R., Goodwin, J. F. and Haseltine, B. A. "The Resitence to Lateral Loads of Walls Built of calcium Silicate bricks". Fifth International Brick Masonry Conference, Washington, 1979.

103. Lawrence, L. R. "Full-scale Tests of Brickwork Panels Under Simulated Wind Load". Fifth International Brick Masonry Conference, Washington, 1979.

104. Sinha, B. P., Loftus, M. D. and Temple, R. "Lateral Strength of Model Bricwork Panels". Proc. of the Institution of Civel Engineers, 67, 1979, pp. 191-198.

105. Brincker, R. "The Lateral Strength of Masonry Walls. An Investigation of the Physical Properties of Masonry". Structural Research Laboratory, Technical University of Denmark, Report No R111, 1979, p217.

106. Lawrence, S. J. "Lateral Loading of Masonry Infill Panels - A Literatury Review". Technical Record 454. Experimental Building Station, Sydney, 1979.

107. Sinha, P. B. "An Ultimate Load Analysis of Laterally Loaded Brickwork Panels". International Journal of Masonry Construction, Vol1, No2, 1980.

108. Cajdert, A. "Laterally Loaded Masonry Walls". Chalmers University of Technology, Publication 80:5, Goteborg, 1980, p283.

109. Baker, L. R. "The Flexural Action of Masonry Structures Under Lateral Load". PhD thesis, Deakin University, July 1981, p389.

110. Baker, L. R. "A Principal Stress Failure Criterion for Brickwork in Biaxial Bending". Sixth International Brick Masonry Conference, Rome, May 1982, pp121-130.

111. Baker, L. R. "An Elastic Principal Stress Theory for Brickwork Panels in Flexure". Sixth International Brick Masonry Conference, Rome, May 1982, pp523-530.

112. De Vekey, R. C., Anderson, C., Beard, R. and Hodgkinson, H. R. "A Collaborative Evaluation of the BS 5628 Wallette Test for Measuring the

Flexural Strength of Brickwork". Sixth International Brick Masonry Conference, Rome, May 1982, pp131-142.

113. West, H. W. H. "The Influence of Time of Docking and Draining on the Properties of Bricks and Brickwork Specimens". Sixth International Brick Masonry Conference, Rome, May 1982, pp283-289.

114. West, H. W. H., Haseltine, B. A., Hodgkinson, H. R. and Tutt, J. N. "The Lateral Resistence of Cavity Walls with Dissimilar Leaves". Sixth International Brick Masonry Conference, Rome, May 1982, pp781-792.

115. Seward, D. W. "A Developed Elastic Analysis of Lghtly Loaded Brickwork Walls with Lateral Loading". International Journal of Masonry Construction, V2, N3, 1982, pp129-134.

116. Lawrence, S. J., "Behaviour of Brick Masonry Walls Under Lateral Loading". Ph.D. thesis, University of New South Wales, November 1983, p.577.

117. Ma, S. Y. A. and May, I. M. "Masonry Panels under Lateral Loads". University of Warwick, Rep. 3, 1984.

118. Brincker, R. "Yield-Line Theory and Material Properties of Laterally Loaded Masonry Walls". International Journal of Masonry Construction, No1, April 1984.

119. Baker, L. R., Gairns, D. A., Lawrence, S. J. and Scrivener, J. C. "Flexural Behaviour of Masonry Panels - A State of the Art". 7th International Brick Masonry Conference, Melbourne, February 1985, pp20-27.

120. Anderson, C. "Test on Walls Subjected to Uniform Lateral Loading and Edge Loading". 7th International Brick Masonry Conference, Melbourne, February 1985, pp889-900.

121. De Vekey, R. C. "The Influence of Mortar Type on the Flexural Strength of Masonry". 7th International Brick Masonry Conference, Melbourne, February 1985, pp915-926.

122. Drysdale, R. G. and Gazzola, E. "Influence of Mortar Properties on the Tensile Bond Strength of Brick Masonry". 7th International Brick Masonry Conference, Melbourne, February 1985, pp927-938.

123. Gazzola, E., Drysdale, R. G. and Essawy, A. "Bending of Concrete Masonry Wallettes at Different Angles to the Bed Joints". Proceedings of the Third North American Masonry Conference, Arlington, June 1985, pp 27.1-14.

124. Grimm, C. T. and Tucker, R. L. "Flexural Strength of Masonry Prisms Versus Wall Panels". Journal of Structural Engineering, Vol 111, September-December 1985, pp2021-2032.

125. Scrivener, J. C. and Gairns, D. A. "Curing and Modulus of Rupture of Concrete Masonry". International Journal of Masonry Construction, No 7, March 1986.

126. Sinha, P. and Mallick, S. K. "Behaviour of Model Brickwork Facade Wall under Lateral Loading". International Journal of Masonry Construction, No 8, July 1986, pp

127. Sise, A., Shrive, N. G. and Jessop, E. L. "Flexural Bond Strength of Masonry Stack Prism". 1st International Masonry Conference, London, December 1986, pp103-107.

128. Fried, A., Anderson, C. and Gairns, D. A. "A Comparative Study of Experimental Techniques for Determining the Flexural Resistence of Masonry". 1st International Masonry Conference, London, December 1986, pp98-102.

129. Ma, S.Y. A. and May, I. M. "A Complete Biaxial Stress Failure Criterion for Brick Masonry". 1st International Masonry Conference, London, December 1986, pp115-117.

130. Southcombe, C. and Tapp, A. "An Investigation of Laterally Loaded Brickwork Panels with Openings". 1st International Masonry Conference, London, December 1986, pp112-114.

131. Edgell, G. J. "Factors Affecting the Flexural Strength of Brick Masonry". International Journal of Masonry Construction, Vol 1, No 1, 1987, pp

132. Anderson, C. "Lateral Strength From Full-Sized Tests Related to the Flexural Properties of Masonry". International Journal of Masonry Construction, Vol1, No 2, 1987, pp

133. Gairns, D. A. and Scrivener, J. C. "The Effect of Masonry Unit Characterist on Panel Lateral Capacity". 8th International Brick/Block Masonry Conference, Dublin, September 1988, pp230-241.

134. Dawe, J. L. and Seah, C. K. "Lateral Load Resistence of Masonry Panels in Flexible Steel Frame". 8th International Brick/Block Masonry Conference, Dublin, September 1988, pp606-616.

135. Thurliman, B. and Guggisberg, R. "Failure Criterion for Laterally Loaded Masonry Walls: Experimental Investigations". 8th International Brick/Block Masonry Conference, Dublin, September 1988, pp699-706.

136. Candy, C. C. E. "The Energy Line Method for Masonry Panels under Lateral Loading". 8th International Brick/Block Masonry Conference, Dublin, September 1988, pp1159-1170.

137. Fried, A., Anderson, C. and Smith, D. "Predicting the Transverse Lateral Strength of Masonry Walls". 8th International Brick/Block Masonry Conference, Dublin, September 1988, pp1171-1183.

138. Lawrence, S. J. and Cao, H. T. "Cracking of Non-loadbearing Masonry Walls under Lateral Forces". 8th International Brick/Block Masonry Conference, Dublin, September 1988, pp1184-1194.

139. May, I. M., Bishop, N. W. M. and Ma, S. Y. A. "The Analysis and Design of Masonry Panels with Openings". 8th International Brick/Block Masonry Conference, Dublin, September 1988, pp1467-1475.

140. Lovegrove, R. "A Discussion of 'Yiellines' in Unreinforced Masonry". The Structural Engineer, Vol 66, No 22, November 1988, pp371-375.

141. Lawrence, S. J. and Lu, J. P. "An Elastic Analysis of Laterally Laoded Masonry Walls with Openings". Conference on Computer Methods in Structural Masonry, Swansea, April 1991, pp39-48.

142. Chong, V. L., May, I. M., Southcombe, C. and Ma, S. Y. A. "An Investigation of Laterally Loaded Masonry Panels Using Non-linear Finite Element Analysis". Conference on Computer Methods in Structural Masonry, Swansea, April 1991, pp49-61.

143. Mullholland, K. W. "Lateral Strength of Brickwork Crosses". Final Year Project, University of Edinburgh, 1980.

144. Johansen, K. "Yield-line formulae for slabs". Cement and Concrete Association, London, 1972.

APPENDIX A

FINITE ELEMENT ANALYSIS OF ORTHOTROPIC PLATE

A1 INPUT OF DATA

The cross-beams and the wall panels were analysed by using a standard computer programme based on the finite element method. An output of the analysis of Walls 5 and 6 is presented. For this analysis, the input data are:

- Moduli of Elasticity in the vertical and horizontal directions (N/mm^2);
- Poisson's ratios in the vertical and horizontal directions (ν_{xy} and ν_{yx});
- Shear modulus (N/mm^2);
- thickness of the section (mm);
- dimensions of the panel (mm);
- uniformly distributed load (N/mm^2); and
- point load acting at the window corners (N).

A2 OUTPUT OF THE COMPUTER PROGRAMME

An exemple of the output is presented for Walls 5 and 6. The mesh is presented in Appendix B. The applied uniformly distributed load is $0.0044N/mm^2$ and the point loads applied at each window corner is 176N.

parede5

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PROGRAM PBIP

FINITE ELEMENT ANALYSIS OF MINDLIN PLATES
USING LINEAR, QUADRATIC OR CUBIC ISOPARAMETRIC ELEMENTS

THIS RUN DONE USING PROGRAM PBIP5

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AUTHOR: J.M. ROTTER, UNIVERSITY OF SYDNEY
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THE DATA FILE WAS parede5.DAT
THE OUTPUT FILE WAS parede5.OUT

TOTAL NO. OF NODAL POINTS = 125
TOTAL NO. OF ELEMENTS = 32
NO. OF RESTRAINED NODES = 36
NO. OF LOAD CASES = 1
ELEMENT TYPE = 4
NO. OF NODES PER ELEMENT = 8
DEGS OF FREEDOM PER NODE = 3
NO. OF DIFFERENT MATERIALS = 1
NO. OF PROPERTIES PER MATL = 6
ORDER OF GAUSSIAN INTEGN = 3
NO. OF COORD DIMENSIONS = 2
NO. OF STRESS RESULTANTS = 5
NO. OF IND VARS PER ELEM = 24

ELEMENT	NODE NUMBERS								PROPERTY
1	1	10	15	16	17	11	3	2	1
2	3	11	17	18	19	12	5	4	1
3	5	12	19	20	21	13	7	6	1
4	7	13	21	22	23	14	9	8	1
5	15	24	29	30	31	25	17	16	1
6	17	25	31	32	33	26	19	18	1
7	19	26	33	34	35	27	21	20	1

8	21	27	35	36	37	28	23	22	1
9	31	38	42	43	44	39	33	32	1
10	33	39	44	45	46	40	35	34	1
11	35	40	46	47	48	41	37	36	1
12	42	49	53	54	55	50	44	43	1
13	44	50	55	56	57	51	46	45	1
14	46	51	57	58	59	52	48	47	1
15	53	60	64	65	66	61	55	54	1
16	55	61	66	67	68	62	57	56	1
17	57	62	68	69	70	63	59	58	1
18	71	75	81	82	83	76	73	72	1
19	73	76	83	84	85	77	64	74	1
20	64	77	85	86	87	78	66	65	1
21	66	78	87	88	89	79	68	67	1
22	68	79	89	90	91	80	70	69	1
23	81	92	98	99	100	93	83	82	1
24	83	93	100	101	102	94	85	84	1
25	85	94	102	103	104	95	87	86	1
26	87	95	104	105	106	96	89	88	1
27	89	96	106	107	108	97	91	90	1
28	98	109	115	116	117	110	100	99	1
29	100	110	117	118	119	111	102	101	1
30	102	111	119	120	121	112	104	103	1
31	104	112	121	122	123	113	106	105	1
32	106	113	123	124	125	114	108	107	1

NODAL POINT COORDINATES

NODE	X	Y
1	.0000000D+00	.0000000D+00
2	.0000000D+00	1.0000000D+02
3	.0000000D+00	2.0000000D+02
4	.0000000D+00	2.5000000D+02
5	.0000000D+00	3.0000000D+02
6	.0000000D+00	3.5000000D+02
7	.0000000D+00	4.0000000D+02
8	.0000000D+00	5.0000000D+02
9	.0000000D+00	6.0000000D+02
10	1.0000000D+02	.0000000D+00
11	1.0000000D+02	2.0000000D+02
12	1.0000000D+02	3.0000000D+02
13	1.0000000D+02	4.0000000D+02
14	1.0000000D+02	6.0000000D+02
15	2.0000000D+02	.0000000D+00
16	2.0000000D+02	1.0000000D+02
17	2.0000000D+02	2.0000000D+02
18	2.0000000D+02	2.5000000D+02

19	2.0000000D+02	3.0000000D+02
20	2.0000000D+02	3.5000000D+02
21	2.0000000D+02	4.0000000D+02
22	2.0000000D+02	5.0000000D+02
23	2.0000000D+02	6.0000000D+02
24	3.0000000D+02	.0000000D+00
25	3.0000000D+02	2.0000000D+02
26	3.0000000D+02	3.0000000D+02
27	3.0000000D+02	4.0000000D+02
28	3.0000000D+02	6.0000000D+02
29	4.0000000D+02	.0000000D+00
30	4.0000000D+02	1.0000000D+02
31	4.0000000D+02	2.0000000D+02
32	4.0000000D+02	2.5000000D+02
33	4.0000000D+02	3.0000000D+02
34	4.0000000D+02	3.5000000D+02
35	4.0000000D+02	4.0000000D+02
36	4.0000000D+02	5.0000000D+02
37	4.0000000D+02	6.0000000D+02
38	5.0000000D+02	2.0000000D+02
39	5.0000000D+02	3.0000000D+02
40	5.0000000D+02	4.0000000D+02
41	5.0000000D+02	6.0000000D+02
42	6.0000000D+02	2.0000000D+02
43	6.0000000D+02	2.5000000D+02
44	6.0000000D+02	3.0000000D+02
45	6.0000000D+02	3.5000000D+02
46	6.0000000D+02	4.0000000D+02
47	6.0000000D+02	5.0000000D+02
48	6.0000000D+02	6.0000000D+02
49	6.5000000D+02	2.0000000D+02
50	6.5000000D+02	3.0000000D+02
51	6.5000000D+02	4.0000000D+02
52	6.5000000D+02	6.0000000D+02
53	7.0000000D+02	2.0000000D+02
54	7.0000000D+02	2.5000000D+02
55	7.0000000D+02	3.0000000D+02
56	7.0000000D+02	3.5000000D+02
57	7.0000000D+02	4.0000000D+02
58	7.0000000D+02	5.0000000D+02
59	7.0000000D+02	6.0000000D+02
60	7.5000000D+02	2.0000000D+02
61	7.5000000D+02	3.0000000D+02
62	7.5000000D+02	4.0000000D+02
63	7.5000000D+02	6.0000000D+02
64	8.0000000D+02	2.0000000D+02
65	8.0000000D+02	2.5000000D+02

66	8.0000000D+02	3.0000000D+02
67	8.0000000D+02	3.5000000D+02
68	8.0000000D+02	4.0000000D+02
69	8.0000000D+02	5.0000000D+02
70	8.0000000D+02	6.0000000D+02
71	8.0000000D+02	.0000000D+00
72	8.0000000D+02	5.0000000D+01
73	8.0000000D+02	1.0000000D+02
74	8.0000000D+02	1.5000000D+02
75	8.5000000D+02	.0000000D+00
76	8.5000000D+02	1.0000000D+02
77	8.5000000D+02	2.0000000D+02
78	8.5000000D+02	3.0000000D+02
79	8.5000000D+02	4.0000000D+02
80	8.5000000D+02	6.0000000D+02
81	9.0000000D+02	.0000000D+00
82	9.0000000D+02	5.0000000D+01
83	9.0000000D+02	1.0000000D+02
84	9.0000000D+02	1.5000000D+02
85	9.0000000D+02	2.0000000D+02
86	9.0000000D+02	2.5000000D+02
87	9.0000000D+02	3.0000000D+02
88	9.0000000D+02	3.5000000D+02
89	9.0000000D+02	4.0000000D+02
90	9.0000000D+02	5.0000000D+02
91	9.0000000D+02	6.0000000D+02
92	9.5000000D+02	.0000000D+00
93	9.5000000D+02	1.0000000D+02
94	9.5000000D+02	2.0000000D+02
95	9.5000000D+02	3.0000000D+02
96	9.5000000D+02	4.0000000D+02
97	9.5000000D+02	6.0000000D+02
98	1.0000000D+03	.0000000D+00
99	1.0000000D+03	5.0000000D+01
100	1.0000000D+03	1.0000000D+02
101	1.0000000D+03	1.5000000D+02
102	1.0000000D+03	2.0000000D+02
103	1.0000000D+03	2.5000000D+02
104	1.0000000D+03	3.0000000D+02
105	1.0000000D+03	3.5000000D+02
106	1.0000000D+03	4.0000000D+02
107	1.0000000D+03	5.0000000D+02
108	1.0000000D+03	6.0000000D+02
109	1.1000000D+03	.0000000D+00
110	1.1000000D+03	1.0000000D+02
111	1.1000000D+03	2.0000000D+02
112	1.1000000D+03	3.0000000D+02

113	1.1000000D+03	4.0000000D+02
114	1.1000000D+03	6.0000000D+02
115	1.2000000D+03	.0000000D+00
116	1.2000000D+03	5.0000000D+01
117	1.2000000D+03	1.0000000D+02
118	1.2000000D+03	1.5000000D+02
119	1.2000000D+03	2.0000000D+02
120	1.2000000D+03	2.5000000D+02
121	1.2000000D+03	3.0000000D+02
122	1.2000000D+03	3.5000000D+02
123	1.2000000D+03	4.0000000D+02
124	1.2000000D+03	5.0000000D+02
125	1.2000000D+03	6.0000000D+02

RESTRAINED NODES

NODE	CODE	FIXED VALUES		
1	101	.0000000D+00	.0000000D+00	.0000000D+00
2	101	.0000000D+00	.0000000D+00	.0000000D+00
3	101	.0000000D+00	.0000000D+00	.0000000D+00
4	101	.0000000D+00	.0000000D+00	.0000000D+00
5	101	.0000000D+00	.0000000D+00	.0000000D+00
6	101	.0000000D+00	.0000000D+00	.0000000D+00
7	101	.0000000D+00	.0000000D+00	.0000000D+00
8	101	.0000000D+00	.0000000D+00	.0000000D+00
9	111	.0000000D+00	.0000000D+00	.0000000D+00
10	001	.0000000D+00	.0000000D+00	.0000000D+00
14	110	.0000000D+00	.0000000D+00	.0000000D+00
15	001	.0000000D+00	.0000000D+00	.0000000D+00
23	110	.0000000D+00	.0000000D+00	.0000000D+00
24	001	.0000000D+00	.0000000D+00	.0000000D+00
28	110	.0000000D+00	.0000000D+00	.0000000D+00
29	001	.0000000D+00	.0000000D+00	.0000000D+00
37	110	.0000000D+00	.0000000D+00	.0000000D+00
41	110	.0000000D+00	.0000000D+00	.0000000D+00
48	110	.0000000D+00	.0000000D+00	.0000000D+00
52	110	.0000000D+00	.0000000D+00	.0000000D+00
59	110	.0000000D+00	.0000000D+00	.0000000D+00
63	110	.0000000D+00	.0000000D+00	.0000000D+00
70	110	.0000000D+00	.0000000D+00	.0000000D+00
71	001	.0000000D+00	.0000000D+00	.0000000D+00
75	001	.0000000D+00	.0000000D+00	.0000000D+00
80	110	.0000000D+00	.0000000D+00	.0000000D+00
81	001	.0000000D+00	.0000000D+00	.0000000D+00
91	110	.0000000D+00	.0000000D+00	.0000000D+00
92	001	.0000000D+00	.0000000D+00	.0000000D+00
97	110	.0000000D+00	.0000000D+00	.0000000D+00

98	001	.0000000D+00	.0000000D+00	.0000000D+00
108	110	.0000000D+00	.0000000D+00	.0000000D+00
109	001	.0000000D+00	.0000000D+00	.0000000D+00
114	110	.0000000D+00	.0000000D+00	.0000000D+00
115	001	.0000000D+00	.0000000D+00	.0000000D+00
125	110	.0000000D+00	.0000000D+00	.0000000D+00

MATERIAL PROPERTIES
NUMBER THICKNESS YOUNG MOD XX YOUNG MOD YY
POISSON XY POISSON YX SHEAR MOD
1 5.6000000D+01 1.6165000D+04 1.2042000D+04 1.5300000D-01
1.1397620D-01 5.5650000D+03

LOAD CASE NO. 1
udl
NO. OF NODAL POINT LOADS = 1
64 1.7600000D+02 .0000000D+00 .0000000D+00
NO. OF MATERIAL TYPES CARRYING UNIFORMLY DISTRIBUTED LOADING = 1
MATL U.D.LOAD
1 4.4000000D-03

MAXIMUM FRONT WIDTH USED = 48

MAXIMUM PIVOT ENCOUNTERED = 3.223E+09
MINIMUM PIVOT ENCOUNTERED = 1.177E+05

DISPLACEMENTS

NODE	DISP	XZ ROT	YZ ROT
1	.000000E+00	9.230184E-04	.000000E+00
2	.000000E+00	8.923976E-04	.000000E+00
3	.000000E+00	8.031092E-04	.000000E+00
4	.000000E+00	7.398902E-04	.000000E+00
5	.000000E+00	6.604950E-04	.000000E+00
6	.000000E+00	5.705941E-04	.000000E+00
7	.000000E+00	4.692314E-04	.000000E+00
8	.000000E+00	2.443098E-04	.000000E+00
9	.000000E+00	.000000E+00	.000000E+00
10	9.218422E-02	9.068435E-04	.000000E+00
11	8.018413E-02	7.880879E-04	-1.157788E-04
12	6.583766E-02	6.440696E-04	-1.686835E-04
13	4.668687E-02	4.551308E-04	-2.108846E-04
14	.000000E+00	.000000E+00	-2.468167E-04

15	1.821721E-01	8.820381E-04	.000000E+00
16	1.759951E-01	8.518539E-04	-1.215979E-04
17	1.582591E-01	7.651315E-04	-2.321949E-04
18	1.451526E-01	6.948857E-04	-2.881212E-04
19	1.292612E-01	6.176261E-04	-3.388889E-04
20	1.113086E-01	5.276893E-04	-3.796891E-04
21	9.122735E-02	4.310148E-04	-4.142033E-04
22	4.740309E-02	2.216964E-04	-4.561891E-04
23	.000000E+00	.000000E+00	-4.818823E-04
24	2.690407E-01	8.589618E-04	.000000E+00
25	2.336411E-01	7.324610E-04	-3.675535E-04
26	1.892623E-01	5.721040E-04	-5.115170E-04
27	1.326346E-01	3.899670E-04	-6.101890E-04
28	.000000E+00	.000000E+00	-6.956777E-04
29	3.551716E-01	8.606599E-04	.000000E+00
30	3.434937E-01	8.274325E-04	-2.353916E-04
31	3.063836E-01	6.981147E-04	-5.185489E-04
32	2.772987E-01	6.193135E-04	-6.148936E-04
33	2.441489E-01	5.153364E-04	-6.957440E-04
34	2.079717E-01	4.298031E-04	-7.452501E-04
35	1.693949E-01	3.453350E-04	-7.885055E-04
36	8.719481E-02	1.743549E-04	-8.464658E-04
37	.000000E+00	.000000E+00	-8.842796E-04
38	3.750495E-01	5.675711E-04	-8.124527E-04
39	2.919479E-01	4.435396E-04	-8.599364E-04
40	2.015946E-01	2.983811E-04	-9.449431E-04
41	.000000E+00	.000000E+00	-1.048970E-03
42	4.303838E-01	4.434263E-04	-9.744778E-04
43	3.814927E-01	4.251913E-04	-9.707041E-04
44	3.328575E-01	3.647525E-04	-9.941053E-04
45	2.821337E-01	3.124885E-04	-1.033330E-03
46	2.292976E-01	2.547113E-04	-1.078630E-03
47	1.175442E-01	1.301635E-04	-1.145722E-03
48	.000000E+00	.000000E+00	-1.191420E-03
49	4.523731E-01	3.852069E-04	-1.024826E-03
50	3.503094E-01	3.291719E-04	-1.041301E-03
51	2.415035E-01	2.336808E-04	-1.135264E-03
52	.000000E+00	.000000E+00	-1.255223E-03
53	4.708106E-01	3.250628E-04	-1.039183E-03
54	4.187940E-01	3.158022E-04	-1.043435E-03
55	3.660106E-01	2.939643E-04	-1.080127E-03
56	3.107292E-01	2.565864E-04	-1.131391E-03
57	2.527230E-01	2.130232E-04	-1.185264E-03
58	1.296717E-01	1.113513E-04	-1.263160E-03
59	.000000E+00	.000000E+00	-1.314930E-03
60	4.849291E-01	2.553582E-04	-1.015937E-03
61	3.797492E-01	2.557703E-04	-1.100436E-03

62	2.629348E-01	1.930710E-04	-1.229750E-03
63	.000000E+00	.000000E+00	-1.369900E-03
64	4.950979E-01	1.774170E-04	-9.106824E-04
65	4.459094E-01	2.093884E-04	-1.023082E-03
66	3.919586E-01	2.229078E-04	-1.118558E-03
67	3.339489E-01	2.043644E-04	-1.196154E-03
68	2.721607E-01	1.726723E-04	-1.267958E-03
69	1.399510E-01	9.265421E-05	-1.360566E-03
70	.000000E+00	.000000E+00	-1.420706E-03
71	5.848193E-01	8.040937E-05	.000000E+00
72	5.792360E-01	8.001607E-05	-2.195295E-04
73	5.624795E-01	8.833241E-05	-4.381653E-04
74	5.344209E-01	1.123978E-04	-6.651819E-04
75	5.894039E-01	1.013353E-04	.000000E+00
76	5.674400E-01	1.120595E-04	-4.382357E-04
77	5.027228E-01	1.571283E-04	-8.544627E-04
78	4.024620E-01	1.911668E-04	-1.122007E-03
79	2.803727E-01	1.543219E-04	-1.301221E-03
80	.000000E+00	.000000E+00	-1.466644E-03
81	5.949935E-01	1.217738E-04	.000000E+00
82	5.896032E-01	1.227141E-04	-2.146216E-04
83	5.735071E-01	1.257324E-04	-4.266052E-04
84	5.467266E-01	1.372179E-04	-6.352749E-04
85	5.100517E-01	1.488802E-04	-8.234241E-04
86	4.645535E-01	1.644395E-04	-9.870731E-04
87	4.112841E-01	1.671674E-04	-1.123529E-03
88	3.520823E-01	1.576711E-04	-1.233868E-03
89	2.877120E-01	1.374820E-04	-1.328402E-03
90	1.484203E-01	7.584372E-05	-1.438936E-03
91	.000000E+00	.000000E+00	-1.508923E-03
92	6.014854E-01	1.377521E-04	.000000E+00
93	5.801361E-01	1.405360E-04	-4.238877E-04
94	5.175644E-01	1.508787E-04	-8.136464E-04
95	4.193886E-01	1.543238E-04	-1.128256E-03
96	2.942877E-01	1.256364E-04	-1.352868E-03
97	.000000E+00	.000000E+00	-1.547428E-03
98	6.087771E-01	1.536792E-04	.000000E+00
99	6.034180E-01	1.537871E-04	-2.124808E-04
100	5.875010E-01	1.542820E-04	-4.221817E-04
101	5.612349E-01	1.551976E-04	-6.213480E-04
102	5.252233E-01	1.558064E-04	-8.120823E-04
103	4.801327E-01	1.533917E-04	-9.804496E-04
104	4.269196E-01	1.480029E-04	-1.134707E-03
105	3.666389E-01	1.355159E-04	-1.262520E-03
106	3.003097E-01	1.171887E-04	-1.373723E-03
107	1.554180E-01	6.433529E-05	-1.501405E-03
108	.000000E+00	.000000E+00	-1.580817E-03

109	6.258783E-01	1.905357E-04	.000000E+00
110	6.044836E-01	1.878593E-04	-4.247917E-04
111	5.417679E-01	1.784342E-04	-8.204387E-04
112	4.420305E-01	1.572632E-04	-1.159736E-03
113	3.120125E-01	1.187901E-04	-1.421998E-03
114	.000000E+00	.000000E+00	-1.652467E-03
115	6.469529E-01	2.322792E-04	.000000E+00
116	6.414512E-01	2.309358E-04	-2.167411E-04
117	6.251507E-01	2.269963E-04	-4.306587E-04
118	5.981764E-01	2.200789E-04	-6.367166E-04
119	5.610581E-01	2.099111E-04	-8.344584E-04
120	5.142984E-01	1.964424E-04	-1.016281E-03
121	4.586522E-01	1.787989E-04	-1.185323E-03
122	3.950292E-01	1.571698E-04	-1.331330E-03
123	3.243567E-01	1.310249E-04	-1.460433E-03
124	1.685140E-01	7.074620E-05	-1.611516E-03
125	.000000E+00	.000000E+00	-1.706108E-03

REACTIONS

NODE	FORCE	XZ MOMENT	YZ MOMENT
1	-6.080078E+01	.000000E+00	8.789985E+02
2	-2.554675E+02	.000000E+00	1.185577E+04
3	-1.045431E+02	.000000E+00	1.023994E+04
4	-9.235277E+01	.000000E+00	1.511366E+04
5	-7.026488E+01	.000000E+00	9.783136E+03
6	-9.949839E+01	.000000E+00	2.010053E+04
7	-4.440459E+01	.000000E+00	1.754547E+04
8	-1.426345E+02	.000000E+00	5.110104E+04
9	1.701713E+01	-1.294258E+04	1.341051E+04
10	.000000E+00	.000000E+00	1.550321E+04
14	-1.408779E+02	-5.282249E+04	.000000E+00
15	.000000E+00	.000000E+00	1.277939E+04
23	-3.450208E+01	-2.442086E+04	.000000E+00
24	.000000E+00	.000000E+00	4.343456E+04
28	-2.226544E+02	-4.224406E+04	.000000E+00
29	.000000E+00	.000000E+00	9.872608E+03
37	-1.005177E+02	-1.973861E+04	.000000E+00
41	-2.057544E+02	-3.302732E+04	.000000E+00
48	-1.051027E+02	-1.056452E+04	.000000E+00
52	-1.019352E+02	-1.338940E+04	.000000E+00
59	-7.506072E+01	-6.161153E+03	.000000E+00
63	-1.143127E+02	-1.109966E+04	.000000E+00
70	-7.830722E+01	-5.150819E+03	.000000E+00
71	.000000E+00	.000000E+00	1.326356E+04
75	.000000E+00	.000000E+00	5.138020E+04

80 -1.325363E+02 -9.448148E+03 .000000E+00
81 .000000E+00 .000000E+00 2.572153E+04
91 -7.051756E+01 -4.019657E+03 .000000E+00
92 .000000E+00 .000000E+00 5.065732E+04
97 -1.623318E+02 -7.997445E+03 .000000E+00
98 .000000E+00 .000000E+00 3.784047E+04
108 -8.834388E+01 -5.696130E+03 .000000E+00
109 .000000E+00 .000000E+00 1.012059E+05
114 -3.302911E+02 -1.341503E+04 .000000E+00
115 .000000E+00 .000000E+00 2.565867E+04
125 -1.760049E+02 -3.943637E+03 .000000E+00

STRESSES

GP X COORD Y COORD X MOMENT Y MOMENT XY MOMENT XZ VERT SH YZ VERT SH MAX PR MOM MIN PR MOM ANGLE

ELEMENT NO 1
1 22.5403 22.5403 3.7326D+01 2.8893D+01 2.3395D+01 2.1743D+00-8.3018D-02 5.6882D+01 9.3377D+00 3.9892D+01
2 22.5403 100.0000 3.6934D+01 2.7156D+01 9.8062D+01 2.0607D+00 2.2492D-02 1.3023D+02-6.6139D+01 4.3573D+01
3 22.5403 177.4597 3.5004D+01 2.5244D+01 1.7004D+02 2.0562D+00-8.5920D-02 2.0023D+02-1.3999D+02 4.4178D+01
4 100.0000 22.5403 6.6494D+01 1.1711D+02 2.3572D+01 1.1400D+00-3.1177D-01 1.2638D+02 5.7217D+01 6.8516D+01
5 100.0000 100.0000 6.4694D+01 1.0939D+02 9.7874D+01 1.0664D+00 1.8382D-01 1.8744D+02-1.3350D+01 5.1431D+01
6 100.0000 177.4597 6.1356D+01 1.0150D+02 1.6949D+02 9.5972D-01-2.6966D-01 2.5210D+02-8.9244D+01 4.8377D+01
7 177.4597 22.5403 9.5715D+01 2.0566D+02 2.3579D+01 1.5813D+00-4.5292D-01 2.1051D+02 9.0871D+01 7.8392D+01
8 177.4597 100.0000 9.2507D+01 1.9197D+02 9.7517D+01 1.5013D+00 4.7534D-01 2.5171D+02 3.2773D+01 5.8511D+01
9 177.4597 177.4597 8.7760D+01 1.7811D+02 1.6877D+02 1.2459D+00-2.8062D-01 3.0764D+02-4.1774D+01 5.2492D+01

ELEMENT NO 2
1 22.5403 211.2702 3.6850D+01 2.6667D+01 1.9655D+02 2.0092D+00 8.1684D-02 2.2838D+02-1.6486D+02 4.4258D+01
2 22.5403 250.0000 4.0688D+01 2.5517D+01 2.3290D+02 1.9176D+00 5.3085D-02 2.6613D+02-1.9992D+02 4.4067D+01
3 22.5403 288.7298 3.7908D+01 2.3613D+01 2.6799D+02 2.0363D+00-7.5826D-02 2.9884D+02-2.3732D+02 4.4236D+01
4 100.0000 211.2702 6.4203D+01 1.0759D+02 2.0253D+02 1.0250D+00 1.0483D-01 2.8959D+02-1.1779D+02 4.8057D+01

5 100.0000 250.0000 6.8696D+01 1.0106D+02 2.3462D+02 1.1178D+00
1.6108D-02 3.2005D+02-1.5029D+02 4.6973D+01
6 100.0000 288.7298 6.6571D+01 9.3780D+01 2.6544D+02 8.0863D-01-
5.1765D-01 3.4597D+02-1.8562D+02 4.6467D+01
7 177.4597 211.2702 9.1702D+01 1.8947D+02 2.0952D+02 1.4504D+00-
2.7104D-01 3.5574D+02-7.4564D+01 5.1567D+01
8 177.4597 250.0000 9.6850D+01 1.7756D+02 2.3735D+02 1.8668D+00-
3.6077D-01 3.7796D+02-1.0355D+02 4.9825D+01
9 177.4597 288.7298 9.5380D+01 1.6490D+02 2.6391D+02 1.2690D+00-
1.2403D+00 3.9633D+02-1.3605D+02 4.8752D+01

ELEMENT NO 3
1 22.5403 311.2702 3.6560D+01 2.4263D+01 2.8321D+02 2.0452D+00-
1.4135D-01 3.1368D+02-2.5286D+02 4.4378D+01
2 22.5403 350.0000 3.5798D+01 2.2242D+01 3.1186D+02 1.8531D+00-
1.2160D-03 3.4095D+02-2.8292D+02 4.4377D+01
3 22.5403 388.7298 3.1623D+01 1.9832D+01 3.3898D+02 1.7103D+00
1.6679D-02 3.6476D+02-3.1331D+02 4.4502D+01
4 100.0000 311.2702 6.5030D+01 9.0364D+01 2.8436D+02 6.9205D-01-
2.7349D-01 3.6234D+02-2.0695D+02 4.6275D+01
5 100.0000 350.0000 6.3231D+01 8.1578D+01 3.0849D+02 6.8816D-01-
4.3812D-02 3.8103D+02-2.3622D+02 4.5852D+01
6 100.0000 388.7298 5.8020D+01 7.2403D+01 3.3108D+02 4.1793D-01-
3.5642D-01 3.9637D+02-2.6595D+02 4.5622D+01
7 177.4597 311.2702 9.2005D+01 1.4669D+02 2.8552D+02 1.0674D+00
1.4215D-01 4.0617D+02-1.6748D+02 4.7735D+01
8 177.4597 350.0000 8.9169D+01 1.3113D+02 3.0511D+02 1.2516D+00-
1.4592D-01 4.1599D+02-1.9568D+02 4.6967D+01
9 177.4597 388.7298 8.2920D+01 1.1520D+02 3.2318D+02 8.5369D-01-
1.3963D+00 4.2264D+02-2.2452D+02 4.6429D+01

ELEMENT NO 4
1 22.5403 422.5403 2.7488D+01 1.3098D+01 3.5658D+02 1.4484D+00-
2.6276D-01 3.7694D+02-3.3636D+02 4.4422D+01
2 22.5403 500.0000 1.9045D+01 9.6293D+00 3.8199D+02 7.3780D-01-
6.0052D-02 3.9636D+02-3.6769D+02 4.4647D+01
3 22.5403 577.4597 5.5389D+00 5.5833D+00 4.0343D+02 1.4863D-01-
1.7416D-01 4.0899D+02-3.9787D+02 4.5002D+01
4 100.0000 422.5403 4.9171D+01 4.8386D+01 3.4841D+02 3.7233D-01-
8.6565D-01 3.9719D+02-2.9964D+02 4.4968D+01
5 100.0000 500.0000 3.2153D+01 3.5326D+01 3.7110D+02 4.3379D-01-
1.4684D-01 4.0484D+02-3.3736D+02 4.5122D+01
6 100.0000 577.4597 1.0071D+01 2.1689D+01 3.8980D+02 1.4826D-01-
8.3361D-01 4.0573D+02-3.7396D+02 4.5427D+01
7 177.4597 422.5403 7.0510D+01 8.1423D+01 3.3781D+02 8.2955D-01-
1.0036D+00 4.1382D+02-2.6188D+02 4.5463D+01

8 177.4597 500.0000 4.4917D+01 5.8772D+01 3.5775D+02 9.9383D-01-4.8393D-02 4.0967D+02-3.0598D+02 4.5555D+01
9 177.4597 577.4597 1.4260D+01 3.5543D+01 3.7372D+02 3.4266D-01-1.5875D+00 3.9878D+02-3.4897D+02 4.5816D+01

ELEMENT NO 5
1 222.5403 22.5403 9.9838D+01 2.4764D+02 1.6311D+01-4.2752D-01 8.4929D-02 2.4941D+02 9.8059D+01 8.3777D+01
2 222.5403 100.0000 8.6797D+01 2.4017D+02 8.8952D+01 1.0873D+00 3.3896D-01 2.8093D+02 4.6040D+01 6.5383D+01
3 222.5403 177.4597 1.0232D+02 2.3597D+02 1.7595D+02 1.1579D+00-1.6238D-01 3.5736D+02-1.9066D+01 5.5398D+01
4 300.0000 22.5403 6.8929D+01 3.0663D+02 1.1725D+01 1.4050D+00 1.1549D+00 3.0721D+02 6.8352D+01 8.7183D+01
5 300.0000 100.0000 7.9831D+01 3.3297D+02 9.7850D+01 1.0423D+00-6.9366D-01 3.6638D+02 4.6418D+01 7.1146D+01
6 300.0000 177.4597 1.1930D+02 3.6256D+02 1.9833D+02 1.8784D+00 6.2950D-01 4.7359D+02 8.2732D+00 6.0760D+01
7 377.4597 22.5403 3.9308D+01 3.7405D+02 1.3602D+01-5.2286D-01 8.8818D-01 3.7460D+02 3.8756D+01 8.7677D+01
8 377.4597 100.0000 7.4153D+01 4.3418D+02 1.1321D+02-9.9551D-01-2.0181D+00 4.6682D+02 4.1512D+01 7.3917D+01
9 377.4597 177.4597 1.3757D+02 4.9757D+02 2.2718D+02 2.3737D+00 2.1745D+00 6.0741D+02 2.7722D+01 6.4196D+01

ELEMENT NO 6
1 222.5403 211.2702 1.1032D+02 2.3329D+02 2.2148D+02 8.3692D-01-4.2913D-01 4.0167D+02-5.8053D+01 5.2757D+01
2 222.5403 250.0000 1.1061D+02 2.1606D+02 2.4628D+02 8.9186D-01-7.5374D-02 4.1520D+02-8.8530D+01 5.1042D+01
3 222.5403 288.7298 1.2401D+02 2.0032D+02 2.6856D+02 1.4058D+00-8.1267D-01 4.3342D+02-1.0909D+02 4.9043D+01
4 300.0000 211.2702 1.2499D+02 2.9615D+02 2.3097D+02 1.6005D+00-1.4047D+00 4.5689D+02-3.5753D+01 5.5165D+01
5 300.0000 250.0000 1.3048D+02 2.6858D+02 2.6366D+02 1.1516D-01 2.6182D-01 4.7209D+02-7.3023D+01 5.2338D+01
6 300.0000 288.7298 1.4910D+02 2.4251D+02 2.9383D+02 3.0255D-01 1.4655D-01 4.9332D+02-1.0171D+02 4.9516D+01
7 377.4597 211.2702 1.3899D+02 3.5466D+02 2.4514D+02 2.8393D+00-4.0311D+00 5.1463D+02-2.0990D+01 5.6872D+01
8 377.4597 250.0000 1.4969D+02 3.1675D+02 2.8572D+02 4.5320D-01-1.3217D+00 5.3090D+02-6.4454D+01 5.3148D+01
9 377.4597 288.7298 1.7352D+02 2.8035D+02 3.2377D+02 9.5351D-01-1.0848D+00 5.5508D+02-1.0121D+02 4.9684D+01

ELEMENT NO 7

1 222.5403 311.2702 1.2889D+02 1.7835D+02 2.8519D+02 1.3518D+00-5.1615D-01 4.3988D+02-1.3264D+02 4.7478D+01
2 222.5403 350.0000 1.2682D+02 1.6096D+02 3.0248D+02 8.5731D-01-3.3433D-01 4.4685D+02-1.5907D+02 4.6615D+01
3 222.5403 388.7298 1.1913D+02 1.4294D+02 3.1978D+02 1.1400D+00-1.2364D+00 4.5103D+02-1.8897D+02 4.6066D+01
4 300.0000 311.2702 1.5251D+02 2.0839D+02 2.9087D+02 5.3862D-01-2.1023D+00 4.7266D+02-1.1176D+02 4.7743D+01
5 300.0000 350.0000 1.4492D+02 1.9041D+02 2.9720D+02 2.4777D-01-6.5850D-01 4.6573D+02-1.3040D+02 4.7188D+01
6 300.0000 388.7298 1.3169D+02 1.7179D+02 3.0353D+02 2.1264D-01-2.9621D-01 4.5593D+02-1.5245D+02 4.6889D+01
7 377.4597 311.2702 1.7131D+02 2.0693D+02 2.9281D+02 1.5168D+00-1.4496D+00 4.8247D+02-1.0423D+02 4.6740D+01
8 377.4597 350.0000 1.5819D+02 1.8835D+02 2.8817D+02 9.1762D-01-6.9957D-01 4.6184D+02-1.1529D+02 4.6498D+01
9 377.4597 388.7298 1.3944D+02 1.6914D+02 2.8354D+02 5.2695D-02-1.0288D+00 4.3822D+02-1.2964D+02 4.6499D+01

ELEMENT NO 8
1 222.5403 422.5403 1.0124D+02 9.8124D+01 3.3041D+02 9.6764D-01-1.2806D+00 4.3009D+02-2.3073D+02 4.4865D+01
2 222.5403 500.0000 6.3576D+01 7.0998D+01 3.4432D+02 6.8134D-02-1.0687D-01 4.1162D+02-2.7705D+02 4.5309D+01
3 222.5403 577.4597 1.9411D+01 4.3131D+01 3.5728D+02-9.2191D-02-1.8192D+00 3.8875D+02-3.2621D+02 4.5951D+01
4 300.0000 422.5403 1.0929D+02 1.1266D+02 3.0785D+02 1.7637D-01-1.8693D+00 4.1883D+02-1.9688D+02 4.5157D+01
5 300.0000 500.0000 6.8721D+01 8.3161D+01 3.1772D+02 1.0608D-02-3.2498D-01 3.9375D+02-2.4186D+02 4.5651D+01
6 300.0000 577.4597 2.1649D+01 5.2925D+01 3.2665D+02-1.7620D-02-1.9243D+00 3.6431D+02-2.8974D+02 4.6370D+01
7 377.4597 422.5403 1.1672D+02 1.2314D+02 2.8442D+02-6.6152D-02-1.6440D+00 4.0437D+02-1.6451D+02 4.5323D+01
8 377.4597 500.0000 7.3247D+01 9.1276D+01 2.9025D+02 2.6230D-01-2.3189D-01 3.7265D+02-2.0813D+02 4.5889D+01
9 377.4597 577.4597 2.3269D+01 5.8671D+01 2.9514D+02 1.2665D-01-2.2209D+00 3.3664D+02-2.5470D+02 4.6716D+01

ELEMENT NO 9
1 422.5403 211.2702 3.4020D+02 3.1302D+02 3.5384D+02 1.1176D+01-1.5689D+00 6.8072D+02-2.7489D+01 4.3900D+01
2 422.5403 250.0000 2.7130D+02 2.7600D+02 3.3308D+02 5.1535D-01-1.0989D+00 6.0674D+02-5.9436D+01 4.5202D+01
3 422.5403 288.7298 2.1470D+02 2.4037D+02 3.2274D+02-1.7422D+00-2.4725D+00 5.5053D+02-9.5458D+01 4.6139D+01

4 500.0000 211.2702 2.9893D+02 1.0181D+02 2.3346D+02 1.0198D+01
4.3499D+00 4.5378D+02-5.3045D+01 3.3556D+01
5 500.0000 250.0000 2.4672D+02 1.1180D+02 2.4590D+02-4.1127D-01
6.4948D-01 4.3425D+02-7.5727D+01 3.7330D+01
6 500.0000 288.7298 2.0682D+02 1.2320D+02 2.6877D+02-1.4801D+00-
2.0433D+00 4.3701D+02-1.0700D+02 4.0579D+01
7 577.4597 211.2702 2.7442D+02 2.0002D-01 1.1962D+02 8.3769D+00
6.1035D-01 3.1927D+02-4.4646D+01 2.0551D+01
8 577.4597 250.0000 2.3891D+02 5.7215D+01 1.6527D+02-1.2869D+00-
4.5409D-01 3.3666D+02-4.0530D+01 3.0601D+01
9 577.4597 288.7298 2.1570D+02 1.1563D+02 2.2134D+02-2.7215D-01
2.3406D+00 3.9260D+02-6.1262D+01 3.8631D+01

ELEMENT NO 10
1 422.5403 311.2702 1.8755D+02 1.9392D+02 2.7357D+02-7.9616D-01-
4.5594D-01 4.6432D+02-8.2849D+01 4.5334D+01
2 422.5403 350.0000 1.6257D+02 1.7780D+02 2.6858D+02-5.8759D-01-
5.1627D-01 4.3887D+02-9.8496D+01 4.5812D+01
3 422.5403 388.7298 1.4235D+02 1.6223D+02 2.6660D+02-1.8711D-01-
1.4154D+00 4.1907D+02-1.1450D+02 4.6068D+01
4 500.0000 311.2702 1.9492D+02 1.7226D+02 2.3553D+02-1.7292D-01
2.7304D-01 4.1939D+02-5.2206D+01 4.3623D+01
5 500.0000 350.0000 1.6456D+02 1.6856D+02 2.3553D+02-3.6294D-01-
3.0189D-01 4.0210D+02-6.8978D+01 4.5244D+01
6 500.0000 388.7298 1.3894D+02 1.6540D+02 2.3854D+02 7.8550D-02-
8.9190D-01 3.9108D+02-8.6737D+01 4.6588D+01
7 577.4597 311.2702 2.0350D+02 1.5843D+02 1.9247D+02 1.4567D+00
9.1782D-02 3.7474D+02-1.2819D+01 4.1661D+01
8 577.4597 350.0000 1.6773D+02 1.6715D+02 1.9746D+02 1.8143D-01-
5.1172D-01 3.6490D+02-3.0018D+01 4.4958D+01
9 577.4597 388.7298 1.3673D+02 1.7641D+02 2.0546D+02-2.2671D-02-
3.0657D-01 3.6299D+02-4.9852D+01 4.7758D+01

ELEMENT NO 11
1 422.5403 422.5403 1.1940D+02 1.2729D+02 2.6725D+02-1.0354D-02-
1.4477D+00 3.9062D+02-1.4393D+02 4.5423D+01
2 422.5403 500.0000 6.9652D+01 9.3930D+01 2.7225D+02 3.3432D-02-
1.5402D-01 3.5431D+02-1.9073D+02 4.6277D+01
3 422.5403 577.4597 2.1524D+01 6.0751D+01 2.7694D+02 1.6387D-02-
2.3607D+00 3.1877D+02-2.3650D+02 4.7026D+01
4 500.0000 422.5403 1.1495D+02 1.3312D+02 2.3769D+02 2.5930D-01-
1.5473D+00 3.6190D+02-1.1383D+02 4.6094D+01
5 500.0000 500.0000 6.7473D+01 9.9354D+01 2.4336D+02 8.8724D-02-
1.8168D-01 3.2730D+02-1.6047D+02 4.6874D+01
6 500.0000 577.4597 2.1614D+01 6.5774D+01 2.4873D+02 7.0616D-03-
2.3998D+00 2.9340D+02-2.0601D+02 4.7536D+01

7 577.4597 422.5403 1.1054D+02 1.3922D+02 2.0894D+02 2.6152D-02-
1.6829D+00 3.3431D+02-8.4549D+01 4.6963D+01
8 577.4597 500.0000 6.5335D+01 1.0505D+02 2.1528D+02-1.3932D-01-
2.1138D-01 3.0139D+02-1.3100D+02 4.7635D+01
9 577.4597 577.4597 2.1746D+01 7.1072D+01 2.2132D+02-6.6128D-02-
2.4069D+00 2.6910D+02-1.7628D+02 4.8179D+01

ELEMENT NO 12
1 611.2702 211.2702 2.7285D+02-1.5625D+01 1.0632D+02 5.7702D+00-
1.1401D+00 3.0780D+02-5.0578D+01 1.8198D+01
2 611.2702 250.0000 2.6501D+02 5.9293D+01 1.4730D+02 3.0702D-01-
1.0714D+00 3.4181D+02-1.7506D+01 2.7537D+01
3 611.2702 288.7298 2.1437D+02 1.2933D+02 1.9085D+02-2.1483D-01
3.7886D+00 3.6738D+02-2.3674D+01 3.8720D+01
4 650.0000 211.2702 2.7819D+02-2.0638D+01 6.4874D+01 3.9529D+00
1.1626D+00 2.9166D+02-3.4114D+01 1.1735D+01
5 650.0000 250.0000 2.6789D+02 5.9568D+01 1.0487D+02 6.1191D-01-
7.1242D-01 3.1154D+02 1.5922D+01 2.2597D+01
6 650.0000 288.7298 2.1480D+02 1.3490D+02 1.4744D+02 2.3243D-01
2.5556D+00 3.2761D+02 2.2094D+01 3.7419D+01
7 688.7298 211.2702 2.8807D+02 4.0751D+00 2.2301D+01 2.4231D+00
9.0019D-01 2.8981D+02 2.3346D+00 4.4628D+00
8 688.7298 250.0000 2.7533D+02 8.9568D+01 6.1318D+01 1.0507D+00-
1.0728D+00 2.9374D+02 7.1153D+01 1.6716D+01
9 688.7298 288.7298 2.1979D+02 1.7018D+02 1.0291D+02 6.6003D-01
2.4492D+00 3.0084D+02 8.9131D+01 3.8225D+01

ELEMENT NO 13
1 611.2702 311.2702 1.8493D+02 1.5800D+02 1.6272D+02 5.1601D-01-
6.0875D-02 3.3474D+02 8.1821D+00 4.2635D+01
2 611.2702 350.0000 1.5972D+02 1.7065D+02 1.7628D+02 2.9993D-01-
6.3517D-01 3.4155D+02-1.1176D+01 4.5887D+01
3 611.2702 388.7298 1.3549D+02 1.8341D+02 1.8813D+02-8.7515D-03-
2.2853D-01 3.4910D+02-3.0206D+01 4.8629D+01
4 650.0000 311.2702 1.8615D+02 1.7495D+02 1.4290D+02 6.3633D-01
1.2429D-01 3.2357D+02 3.7540D+01 4.3878D+01
5 650.0000 350.0000 1.6038D+02 1.8387D+02 1.5763D+02 4.6134D-01-
5.5997D-01 3.3020D+02 1.4056D+01 4.7131D+01
6 650.0000 388.7298 1.3558D+02 1.9290D+02 1.7067D+02 2.3864D-01-
4.9497D-01 3.3730D+02-8.8194D+00 4.9767D+01
7 688.7298 311.2702 1.8767D+02 1.9378D+02 1.2309D+02 6.9231D-01
2.1800D-01 3.1385D+02 6.7600D+01 4.5711D+01
8 688.7298 350.0000 1.6132D+02 1.9897D+02 1.3899D+02 5.5839D-01-
4.6028D-01 3.2040D+02 3.9885D+01 4.8856D+01
9 688.7298 388.7298 1.3595D+02 2.0426D+02 1.5320D+02 4.2169D-01-
6.2100D-01 3.2707D+02 1.3143D+01 5.1285D+01

ELEMENT NO 14

1 611.2702 422.5403 1.0866D+02 1.4293D+02 1.9512D+02 2.4903D-02-
1.6873D+00 3.2167D+02-7.0073D+01 4.7509D+01
2 611.2702 500.0000 6.1656D+01 1.0764D+02 2.0419D+02 2.6153D-01-
1.8537D-01 2.9013D+02-1.2083D+02 4.8212D+01
3 611.2702 577.4597 2.0518D+01 7.3020D+01 2.1095D+02 1.1473D-01-
2.4668D+00 2.5935D+02-1.6581D+02 4.8547D+01
4 650.0000 422.5403 1.0861D+02 1.5052D+02 1.7950D+02 1.8358D-01-
1.7008D+00 3.1028D+02-5.1152D+01 4.8329D+01
5 650.0000 500.0000 6.1753D+01 1.1274D+02 1.9080D+02 1.7565D-01-
1.1329D-01 2.7974D+02-1.0525D+02 4.8805D+01
6 650.0000 577.4597 2.0764D+01 7.5635D+01 1.9979D+02 5.5868D-02-
2.6253D+00 2.4987D+02-1.5347D+02 4.8909D+01
7 688.7298 422.5403 1.0877D+02 1.5946D+02 1.6397D+02 2.8518D-01-
1.9587D+00 3.0003D+02-3.1797D+01 4.9394D+01
8 688.7298 500.0000 6.2059D+01 1.1921D+02 1.7750D+02 5.7611D-02-
1.1608D-01 2.7042D+02-8.9155D+01 4.9572D+01
9 688.7298 577.4597 2.1219D+01 7.9615D+01 1.8873D+02-1.0251D-02-
2.6892D+00 2.4139D+02-1.4055D+02 4.9397D+01

ELEMENT NO 15

1 711.2702 211.2702 3.0745D+02 3.8842D+01 1.4633D+01-5.2577D-02
2.4586D+00 3.0824D+02 3.8047D+01 3.1090D+00
2 711.2702 250.0000 2.6281D+02 1.0713D+02 5.7046D+01 2.7301D+00-
4.0500D-01 2.8148D+02 8.8466D+01 1.8118D+01
3 711.2702 288.7298 2.2666D+02 1.7639D+02 7.5338D+01 9.0309D-01
1.3687D+00 2.8095D+02 1.2210D+02 3.5774D+01
4 750.0000 211.2702 3.5176D+02 1.6782D+02-1.0120D+02-1.6597D+00
4.2962D+00 3.9654D+02 1.2305D+02-2.3868D+01
5 750.0000 250.0000 2.7940D+02 1.8076D+02-1.6912D+01 2.2525D+00
7.2144D-01 2.8222D+02 1.7794D+02-9.4633D+00
6 750.0000 288.7298 2.1552D+02 1.9466D+02 4.3256D+01 1.9476D+00-
1.5146D+00 2.4959D+02 1.6059D+02 3.8221D+01
7 788.7298 211.2702 4.0922D+02 3.8271D+02-2.2366D+02-2.1095D+00-
2.2880D+00 6.2002D+02 1.7191D+02-4.3304D+01
8 788.7298 250.0000 3.0913D+02 3.4029D+02-9.7499D+01 2.0256D+00-
1.2396D+00 4.2344D+02 2.2597D+02-4.9539D+01
9 788.7298 288.7298 2.1752D+02 2.9883D+02 4.5446D+00 2.3361D+00-
2.1513D+00 2.9908D+02 2.1727D+02 8.6811D+01

ELEMENT NO 16

1 711.2702 311.2702 2.0659D+02 2.1453D+02 9.3437D+01-1.9841D-01-
5.1390D-02 3.0408D+02 1.1704D+02 4.6215D+01
2 711.2702 350.0000 1.6529D+02 2.1436D+02 1.2024D+02 4.2163D-01-
5.0678D-01 3.1254D+02 6.7101D+01 5.0766D+01
3 711.2702 388.7298 1.3405D+02 2.1533D+02 1.4295D+02 4.2623D-01-
6.7539D-01 3.2330D+02 2.6076D+01 5.2936D+01

4 750.0000 311.2702 1.9556D+02 2.5453D+02 7.5597D+01 1.0389D+00-7.0157D-01 3.0619D+02 1.4390D+02 5.5653D+01
5 750.0000 350.0000 1.6122D+02 2.4627D+02 1.0381D+02 8.4623D-01-6.4828D-01 3.1592D+02 9.1564D+01 5.6138D+01
6 750.0000 388.7298 1.3694D+02 2.3915D+02 1.2791D+02 5.0344D-01-8.6963D-01 3.2579D+02 5.0302D+01 5.5889D+01
7 788.7298 311.2702 1.8385D+02 2.9012D+02 6.0580D+01 1.4687D+00-8.7108D-01 3.1757D+02 1.5641D+02 6.5627D+01
8 788.7298 350.0000 1.5648D+02 2.7377D+02 9.0194D+01 8.4940D-01-5.8254D-01 3.2271D+02 1.0754D+02 6.1517D+01
9 788.7298 388.7298 1.3916D+02 2.5857D+02 1.1570D+02 5.4535D-01-1.1301D+00 3.2907D+02 6.8667D+01 5.8647D+01

ELEMENT NO 17
1 711.2702 422.5403 1.0710D+02 1.6502D+02 1.5673D+02 3.4568D-01-2.0787D+00 2.9544D+02-2.3328D+01 5.0235D+01
2 711.2702 500.0000 6.2274D+01 1.2327D+02 1.7085D+02 3.2814D-01-1.0714D-01 2.6632D+02-8.0774D+01 5.0061D+01
3 711.2702 577.4597 2.1723D+01 8.2011D+01 1.8186D+02 1.0402D-01-2.7667D+00 2.3621D+02-1.3247D+02 4.9706D+01
4 750.0000 422.5403 1.1024D+02 1.7605D+02 1.4208D+02 3.3618D-01-2.1992D+00 2.8898D+02-2.6990D+00 5.1520D+01
5 750.0000 500.0000 6.4246D+01 1.3082D+02 1.5795D+02 1.9525D-01-5.7770D-02 2.5895D+02-6.3888D+01 5.0950D+01
6 750.0000 577.4597 2.2528D+01 8.6069D+01 1.7072D+02 4.5320D-02-2.9713D+00 2.2795D+02-1.1935D+02 5.0271D+01
7 788.7298 422.5403 1.1355D+02 1.8822D+02 1.2732D+02 3.9506D-01-2.5112D+00 2.8356D+02 1.8203D+01 5.3171D+01
8 788.7298 500.0000 6.6393D+01 1.3950D+02 1.4494D+02 1.0089D-01-5.8654D-02 2.5243D+02-4.6538D+01 5.2077D+01
9 788.7298 577.4597 2.3507D+01 9.1264D+01 1.5947D+02-4.6979D-03-3.0848D+00 2.2042D+02-1.0564D+02 5.0997D+01

ELEMENT NO 18
1 811.2702 11.2702 1.3439D+01 7.7852D+02 3.2857D+00 3.9896D-01-1.8506D-01 7.7853D+02 1.3425D+01 8.9754D+01
2 811.2702 50.0000-1.9377D+00 7.7379D+02-4.2626D+00 4.1212D-01-7.1701D-01 7.7381D+02-1.9611D+00-8.9685D+01
3 811.2702 88.7298-7.7348D+00 7.7015D+02-1.2663D+01-1.4218D-01-1.4371D+00 7.7036D+02-7.9409D+00-8.9068D+01
4 850.0000 11.2702 1.9301D+01 7.7933D+02-2.7055D+00 8.7308D-02-1.2102D-01 7.7934D+02 1.9291D+01-8.9796D+01
5 850.0000 50.0000 1.7457D+01 7.7430D+02-1.2731D+01-9.4372D-02 1.0401D-01 7.7451D+02 1.7243D+01-8.9037D+01
6 850.0000 88.7298 2.5193D+01 7.7036D+02-2.3609D+01-4.0035D-01 2.4342D-02 7.7110D+02 2.4446D+01-8.8187D+01

7 888.7298 11.2702 2.3235D+01 7.6754D+02-4.0220D+00 4.5785D-02-
1.0913D-01 7.6756D+02 2.3214D+01-8.9690D+01
8 888.7298 50.0000 3.4925D+01 7.6221D+02-1.6525D+01 3.0858D-01
9.0945D-02 7.6259D+02 3.4549D+01-8.8699D+01
9 888.7298 88.7298 5.6194D+01 7.5797D+02-2.9880D+01 8.9022D-01-
1.3024D-01 7.5924D+02 5.4924D+01-8.7566D+01

ELEMENT NO 19
1 811.2702 111.2702-6.9202D-01 7.6614D+02-1.7519D+01 3.6655D-01-
1.9051D+00 7.6654D+02-1.0921D+00-8.8692D+01
2 811.2702 150.0000 6.9247D+01 8.1258D+02-9.5762D+01 5.1188D-01-
1.7606D+00 8.2472D+02 5.7109D+01-8.2776D+01
3 811.2702 188.7298 1.9809D+02 8.6573D+02-1.9306D+02-3.2522D+00
8.1475D-01 9.1754D+02 1.4628D+02-7.4978D+01
4 850.0000 111.2702 2.2862D+01 7.3891D+02-2.0767D+01-3.0614D-01
5.9566D-01 7.3951D+02 2.2260D+01-8.8340D+01
5 850.0000 150.0000 5.4461D+01 7.3973D+02-6.1062D+01-3.6071D-01-
9.0203D-02 7.4513D+02 4.9063D+01-8.4948D+01
6 850.0000 188.7298 1.4497D+02 7.4726D+02-1.2042D+02-1.5997D+00-
9.5177D-01 7.7044D+02 1.2179D+02-7.9103D+01
7 888.7298 111.2702 5.2489D+01 7.5136D+02-3.4812D+01 3.2630D-01-
6.2520D-01 7.5309D+02 5.0759D+01-8.7155D+01
8 888.7298 150.0000 4.5748D+01 7.0657D+02-3.7158D+01-1.4047D+00
3.2321D-01 7.0865D+02 4.3665D+01-8.6792D+01
9 888.7298 188.7298 9.7913D+01 6.6849D+02-5.8564D+01-1.5951D+00-
1.5106D+00 6.7444D+02 9.1964D+01-8.4200D+01

ELEMENT NO 20
1 811.2702 211.2702 1.9097D+02 4.6534D+02-1.4658D+02-3.6665D+00-
5.7586D+00 5.2892D+02 1.2740D+02-6.6552D+01
2 811.2702 250.0000 2.0650D+02 4.1775D+02-7.9249D+01-1.6917D-01-
2.0122D+00 4.4417D+02 1.8008D+02-7.1560D+01
3 811.2702 288.7298 2.1391D+02 3.6923D+02-1.6939D+01 1.2113D+00-
1.3856D+00 3.7106D+02 2.1209D+02-8.3848D+01
4 850.0000 211.2702 1.6159D+02 5.5013D+02-1.0838D+02-1.0382D+00-
4.7438D+00 5.7832D+02 1.3340D+02-7.5421D+01
5 850.0000 250.0000 1.8164D+02 4.9220D+02-5.7048D+01 1.2859D+00-
9.6152D-01 5.0235D+02 1.7149D+02-7.9914D+01
6 850.0000 288.7298 1.9357D+02 4.3333D+02-1.0731D+01 1.1177D+00-
9.8519D-01 4.3381D+02 1.9309D+02-8.7443D+01
7 888.7298 211.2702 1.2775D+02 6.0575D+02-6.8090D+01-4.0386D-01-
2.3021D+00 6.1526D+02 1.1824D+02-8.2049D+01
8 888.7298 250.0000 1.5232D+02 5.3747D+02-3.2746D+01 1.0344D+00-
2.9528D-01 5.4024D+02 1.4955D+02-8.5175D+01
9 888.7298 288.7298 1.6877D+02 4.6827D+02-2.4214D+00-3.9533D-01-
2.7807D+00 4.6829D+02 1.6875D+02-8.9537D+01

ELEMENT NO 21
1 811.2702 311.2702 2.0195D+02 3.1768D+02 3.5563D+01 1.1580D+00-
1.2542D+00 3.2774D+02 1.9190D+02 7.4213D+01
2 811.2702 350.0000 1.7262D+02 2.9545D+02 7.3778D+01 8.4603D-01-
7.6026D-01 3.3003D+02 1.3804D+02 6.4887D+01
3 811.2702 388.7298 1.3974D+02 2.7280D+02 1.0710D+02 3.5670D-01-
1.4604D+00 3.3235D+02 8.0188D+01 6.0925D+01
4 850.0000 311.2702 1.8358D+02 3.6624D+02 2.5738D+01 6.8529D-01-
2.0776D+00 3.6979D+02 1.8002D+02 8.2131D+01
5 850.0000 350.0000 1.6160D+02 3.3425D+02 6.0722D+01 7.5610D-01-
1.1157D+00 3.5347D+02 1.4238D+02 7.2438D+01
6 850.0000 388.7298 1.3607D+02 3.0186D+02 9.0813D+01 4.8540D-01-
2.0171D+00 3.4192D+02 9.6011D+01 6.6195D+01
7 888.7298 311.2702 1.6453D+02 4.1032D+02 1.8958D+01-1.0020D+00-
1.9187D+00 4.1177D+02 1.6307D+02 8.5615D+01
8 888.7298 350.0000 1.4990D+02 3.6858D+02 5.0711D+01-1.3202D-01-
7.6669D-01 3.7977D+02 1.3871D+02 7.7559D+01
9 888.7298 388.7298 1.3173D+02 3.2644D+02 7.7571D+01 2.3230D-01-
2.1470D+00 3.5356D+02 1.0460D+02 7.0727D+01

ELEMENT NO 22
1 811.2702 422.5403 1.0784D+02 1.9478D+02 1.2091D+02 2.2161D-01-
2.6139D+00 2.7980D+02 2.2819D+01 5.4887D+01
2 811.2702 500.0000 6.4613D+01 1.4429D+02 1.3855D+02 2.4305D-01-
1.9353D-02 2.4861D+02-3.9709D+01 5.3021D+01
3 811.2702 577.4597 2.3655D+01 9.4058D+01 1.5223D+02 8.1421D-02-
3.1839D+00 2.1510D+02-9.7392D+01 5.1510D+01
4 850.0000 422.5403 1.0488D+02 2.0755D+02 1.0747D+02 3.2995D-01-
2.6920D+00 2.7531D+02 3.7117D+01 5.7767D+01
5 850.0000 500.0000 6.3172D+01 1.5296D+02 1.2667D+02 2.3641D-01
9.2643D-02 2.4246D+02-2.6321D+01 5.4758D+01
6 850.0000 577.4597 2.3731D+01 9.8633D+01 1.4192D+02 6.4666D-02-
3.4218D+00 2.0796D+02-8.5596D+01 5.2391D+01
7 888.7298 422.5403 1.0212D+02 2.2163D+02 9.4389D+01 2.0702D-01-
3.0193D+00 2.7359D+02 5.0162D+01 6.1168D+01
8 888.7298 500.0000 6.1930D+01 1.6294D+02 1.1516D+02 9.9450D-02
1.1729D-01 2.3818D+02-1.3310D+01 5.6841D+01
9 888.7298 577.4597 2.4006D+01 1.0451D+02 1.3198D+02 1.8537D-02-
3.5850D+00 2.0224D+02-7.3721D+01 5.3481D+01

ELEMENT NO 23
1 911.2702 11.2702 4.0668D+01 7.6246D+02-1.2252D+00 2.0339D-02-
1.0669D-01 7.6246D+02 4.0666D+01-8.9903D+01
2 911.2702 50.0000 3.9994D+01 7.5513D+02-5.4617D+00-2.1332D-02-
6.3139D-02 7.5517D+02 3.9952D+01-8.9562D+01
3 911.2702 88.7298 4.1763D+01 7.4809D+02-9.7674D+00-1.3337D-01-
4.7752D-01 7.4822D+02 4.1628D+01-8.9208D+01

4 950.0000 11.2702 4.0752D+01 7.5906D+02-1.0951D+00-1.2029D-02-1.3494D-01 7.5906D+02 4.0751D+01-8.9913D+01
5 950.0000 50.0000 4.1509D+01 7.5175D+02-4.0109D+00-1.0921D-01-5.5949D-02 7.5177D+02 4.1486D+01-8.9676D+01
6 950.0000 88.7298 4.4708D+01 7.4472D+02-6.9959D+00-1.6375D-01-4.4436D-01 7.4479D+02 4.4638D+01-8.9427D+01
7 988.7298 11.2702 4.1003D+01 7.5675D+02-4.7316D-01-1.5987D-02-2.5365D-01 7.5675D+02 4.1003D+01-8.9962D+01
8 988.7298 50.0000 4.3190D+01 7.4946D+02-2.0683D+00-1.0142D-01-7.1619D-02 7.4946D+02 4.3184D+01-8.9832D+01
9 988.7298 88.7298 4.7820D+01 7.4244D+02-3.7326D+00-3.1208D-02-3.6646D-01 7.4246D+02 4.7800D+01-8.9692D+01

ELEMENT NO 24
1 911.2702 111.2702 5.1057D+01 7.5353D+02-2.4571D+01-2.2538D-01-1.3853D+00 7.5439D+02 5.0199D+01-8.7999D+01
2 911.2702 150.0000 6.8486D+01 7.0321D+02-3.3380D+01-2.3202D-01 2.1157D-01 7.0496D+02 6.6736D+01-8.6998D+01
3 911.2702 188.7298 8.6614D+01 6.5297D+02-3.6282D+01-3.0476D-03-1.4972D+00 6.5529D+02 8.4299D+01-8.6349D+01
4 950.0000 111.2702 5.0008D+01 7.3210D+02-1.4712D+01-2.1446D-01-8.2773D-01 7.3241D+02 4.9690D+01-8.8765D+01
5 950.0000 150.0000 6.3668D+01 6.9413D+02-1.9766D+01-1.8877D-01 8.2829D-02 6.9475D+02 6.3049D+01-8.8206D+01
6 950.0000 188.7298 7.8026D+01 6.5625D+02-1.8912D+01 1.0485D-01-1.5044D+00 6.5686D+02 7.7408D+01-8.8129D+01
7 988.7298 111.2702 5.0144D+01 7.1842D+02-6.8014D+00-9.8580D-02-9.7306D-01 7.1848D+02 5.0075D+01-8.9417D+01
8 988.7298 150.0000 6.0035D+01 6.9280D+02-8.0995D+00-3.0697D-01-2.6751D-01 6.9291D+02 5.9931D+01-8.9267D+01
9 988.7298 188.7298 7.0625D+01 6.6727D+02-3.4906D+00-2.1512D-01-1.2519D+00 6.6729D+02 7.0604D+01-8.9665D+01

ELEMENT NO 25
1 911.2702 211.2702 1.0193D+02 6.1747D+02-4.5442D+01-3.2648D-01-1.3832D+00 6.2144D+02 9.7952D+01-8.5001D+01
2 911.2702 250.0000 1.2763D+02 5.5018D+02-2.3577D+01-9.0492D-01-1.0970D-01 5.5150D+02 1.2632D+02-8.6816D+01
3 911.2702 288.7298 1.3910D+02 4.8128D+02 4.6806D+00-6.7268D-02-3.2737D+00 4.8134D+02 1.3904D+02 8.9216D+01
4 950.0000 211.2702 9.0264D+01 6.2111D+02-2.2078D+01 2.6896D-02-1.5567D+00 6.2202D+02 8.9348D+01-8.7623D+01
5 950.0000 250.0000 1.1293D+02 5.6731D+02-8.2001D+00-2.1869D-01-1.3830D-01 5.6746D+02 1.1279D+02-8.8966D+01
6 950.0000 288.7298 1.2136D+02 5.1189D+02 1.2071D+01 2.9309D-01-2.2831D+00 5.1226D+02 1.2099D+02 8.8231D+01

7 988.7298 211.2702 7.7533D+01 6.1776D+02-4.6900D-01-1.9501D-01-1.6811D+00 6.1776D+02 7.7533D+01-8.9950D+01
8 988.7298 250.0000 9.7166D+01 5.7745D+02 5.4213D+00-3.4789D-01-5.5148D-01 5.7751D+02 9.7105D+01 8.9353D+01
9 988.7298 288.7298 1.0256D+02 5.3552D+02 1.7705D+01-4.0211D-01-2.1108D+00 5.3624D+02 1.0184D+02 8.7662D+01

ELEMENT NO 26
1 911.2702 311.2702 1.3613D+02 4.2862D+02 2.1921D+01 3.1640D-01-1.8146D+00 4.3026D+02 1.3449D+02 8.5738D+01
2 911.2702 350.0000 1.2941D+02 3.8446D+02 4.8295D+01 3.3200D-02-5.4596D-01 3.9330D+02 1.2058D+02 7.9629D+01
3 911.2702 388.7298 1.1569D+02 3.3950D+02 7.4276D+01-1.7509D-02-2.0199D+00 3.6191D+02 9.3283D+01 7.3213D+01
4 950.0000 311.2702 1.1731D+02 4.5345D+02 2.5303D+01 5.6008D-01-2.1026D+00 4.5535D+02 1.1542D+02 8.5719D+01
5 950.0000 350.0000 1.1498D+02 4.0894D+02 4.6699D+01 4.1767D-01-6.7402D-01 4.1618D+02 1.0774D+02 8.1187D+01
6 950.0000 388.7298 1.0564D+02 3.6363D+02 6.7701D+01 1.8347D-01-2.0420D+00 3.8031D+02 8.8951D+01 7.6154D+01
7 988.7298 311.2702 9.7619D+01 4.7254D+02 3.0211D+01-2.6091D-01-2.1622D+00 4.7496D+02 9.5200D+01 8.5422D+01
8 988.7298 350.0000 9.9668D+01 4.2768D+02 4.6629D+01-5.3721D-02-9.3038D-01 4.3418D+02 9.3168D+01 8.2064D+01
9 988.7298 388.7298 9.4707D+01 3.8201D+02 6.2652D+01-2.6261D-01-2.5489D+00 3.9508D+02 8.1639D+01 7.8218D+01

ELEMENT NO 27
1 911.2702 422.5403 8.9936D+01 2.2876D+02 9.1385D+01 1.4582D-02-3.1778D+00 2.7410D+02 4.4590D+01 6.3609D+01
2 911.2702 500.0000 5.9267D+01 1.6873D+02 1.1116D+02 1.6859D-02 1.8295D-01 2.3790D+02-9.9037D+00 5.8107D+01
3 911.2702 577.4597 2.4664D+01 1.0825D+02 1.2716D+02 5.7479D-03-3.6025D+00 2.0030D+02-6.7392D+01 5.4097D+01
4 950.0000 422.5403 8.0800D+01 2.4127D+02 8.3665D+01 1.5215D-01-3.3470D+00 2.7695D+02 4.5114D+01 6.6900D+01
5 950.0000 500.0000 5.4404D+01 1.7764D+02 1.0204D+02 2.1186D-01 2.6269D-01 2.3522D+02-3.1764D+00 6.0563D+01
6 950.0000 577.4597 2.4074D+01 1.1356D+02 1.1663D+02 7.6183D-02-3.7900D+00 1.9374D+02-5.6104D+01 5.5494D+01
7 988.7298 422.5403 7.1540D+01 2.5297D+02 7.6776D+01-2.3049D-01-3.6907D+00 2.8109D+02 4.3411D+01 6.9878D+01
8 988.7298 500.0000 4.9417D+01 1.8574D+02 9.3745D+01 1.1372D-01 6.7186D-02 2.3349D+02 1.6728D+00 6.3010D+01
9 988.7298 577.4597 2.3360D+01 1.1807D+02 1.0694D+02 8.0542D-02-4.3534D+00 1.8767D+02-4.6243D+01 5.6942D+01

ELEMENT NO 28
1 1022.5403 11.2702 3.1836D+01 7.5578D+02 3.8402D-01-3.5392D-01-1.7838D-01 7.5578D+02 3.1835D+01 8.9970D+01
2 1022.5403 50.0000 3.2804D+01 7.4829D+02 8.6638D-01-2.6754D-01-4.0346D-02 7.4829D+02 3.2803D+01 8.9931D+01

3 1022.5403 88.7298 3.5928D+01 7.4104D+02 1.3381D+00-3.9777D-01-3.8279D-01
7.4104D+02 3.5925D+01 8.9891D+01
4 1100.0000 11.2702 2.3220D+01 7.5891D+02 1.1786D+00 1.0601D-01-2.3653D-02
7.5891D+02 2.3218D+01 8.9908D+01
5 1100.0000 50.0000 2.3696D+01 7.5131D+02 3.9145D+00 1.1970D-01-4.3010D-02
7.5134D+02 2.3675D+01 8.9692D+01
6 1100.0000 88.7298 2.6326D+01 7.4397D+02 6.6398D+00 1.1614D-01-5.4576D-01
7.4403D+02 2.6265D+01 8.9470D+01
7 1177.4597 11.2702 1.5141D+01 7.6554D+02 1.6446D+00-2.9036D-01-3.0433D-01
7.6555D+02 1.5137D+01 8.9874D+01
8 1177.4597 50.0000 1.5124D+01 7.5784D+02 6.6340D+00-3.9430D-01-2.6343D-01
7.5790D+02 1.5064D+01 8.9488D+01
9 1177.4597 88.7298 1.7261D+01 7.5039D+02 1.1613D+01-3.1614D-01-7.0884D-01
7.5058D+02 1.7077D+01 8.9093D+01

ELEMENT NO 29
1 1022.5403 111.2702 3.6213D+01 7.1615D+02 2.0513D+00-4.1699D-01-8.7364D-01
7.1616D+02 3.6206D+01 8.9827D+01
2 1022.5403 150.0000 4.2623D+01 6.9389D+02 4.4470D+00-3.6372D-01-2.1764D-01
6.9393D+02 4.2593D+01 8.9609D+01
3 1022.5403 188.7298 5.1160D+01 6.7188D+02 6.8710D+00-5.4399D-01-1.0143D+00
6.7196D+02 5.1083D+01 8.9366D+01
4 1100.0000 111.2702 2.6413D+01 7.2312D+02 9.5123D+00 1.3551D-01-5.8582D-01
7.2325D+02 2.6283D+01 8.9218D+01
5 1100.0000 150.0000 3.0466D+01 7.0072D+02 1.3934D+01 1.9298D-01-1.0200D-01
7.0101D+02 3.0177D+01 8.8810D+01
6 1100.0000 188.7298 3.6646D+01 6.7857D+02 1.8385D+01 2.1361D-01-1.0631D+00
6.7909D+02 3.6120D+01 8.8361D+01
7 1177.4597 111.2702 1.7009D+01 7.3269D+02 1.5366D+01-3.3524D-01-1.3094D+00
7.3302D+02 1.6679D+01 8.8771D+01
8 1177.4597 150.0000 1.8705D+01 7.1014D+02 2.1814D+01-4.9339D-01-8.3698D-01
7.1083D+02 1.8018D+01 8.8195D+01
9 1177.4597 188.7298 2.2528D+01 6.8784D+02 2.8291D+01-4.9167D-01-1.8017D+00
6.8904D+02 2.1327D+01 8.7569D+01

ELEMENT NO 30
1 1022.5403 211.2702 5.0586D+01 6.1985D+02 1.2219D+01-5.9397D-01-1.4168D+00
6.2011D+02 5.0324D+01 8.8771D+01
2 1022.5403 250.0000 5.7560D+01 5.8253D+02 2.1681D+01-5.1395D-01-3.8107D-01
5.8343D+02 5.6666D+01 8.7639D+01
3 1022.5403 288.7298 6.5402D+01 5.4531D+02 3.1467D+01-5.5667D-01-1.7540D+00
5.4737D+02 6.3347D+01 8.6264D+01
4 1100.0000 211.2702 3.6565D+01 6.3882D+02 2.3032D+01 2.4827D-01-1.2240D+00
6.3970D+02 3.5686D+01 8.7813D+01
5 1100.0000 250.0000 4.1283D+01 6.0265D+02 3.1833D+01 3.4306D-01-1.6248D-01
6.0445D+02 3.9484D+01 8.6765D+01
6 1100.0000 288.7298 4.6868D+01 5.6658D+02 4.0959D+01 3.9535D-01-1.4208D+00
5.6979D+02 4.3660D+01 8.5521D+01
7 1177.4597 211.2702 2.1704D+01 6.5230D+02 3.2170D+01-5.0298D-01-2.2834D+00
6.5393D+02 2.0067D+01 8.7087D+01
8 1177.4597 250.0000 2.4166D+01 6.1728D+02 4.0310D+01-6.2239D-01-1.5372D+00
6.2000D+02 2.1439D+01 8.6130D+01
9 1177.4597 288.7298 2.7495D+01 5.8235D+02 4.8776D+01-7.0406D-01-3.0221D+00
5.8661D+02 2.3240D+01 8.5014D+01

ELEMENT NO 31

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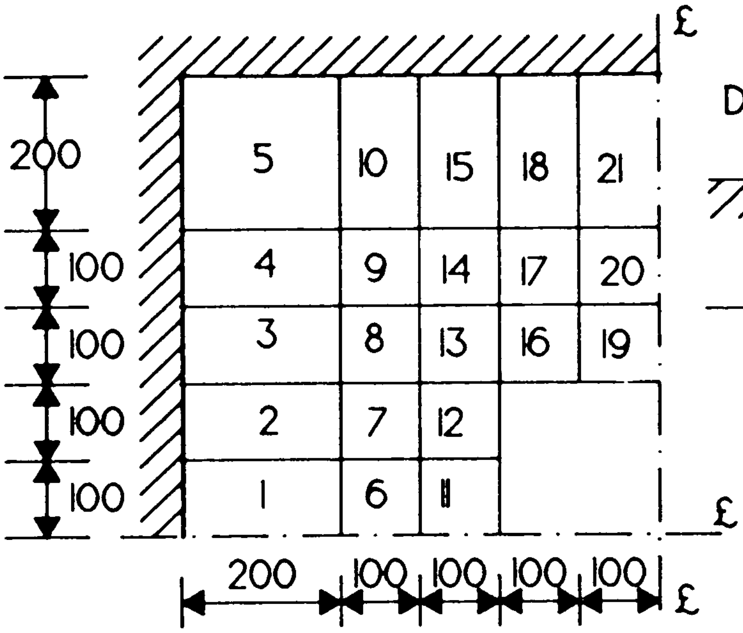
4.8872D+02 5.8333D+01 8.4389D+01
2 1022.5403 350.0000 6.2572D+01 4.3918D+02 5.7333D+01-5.8950D-01-5.2286D-01
4.4772D+02 5.4037D+01 8.1533D+01
3 1022.5403 388.7298 6.1739D+01 3.9365D+02 7.2710D+01-4.5190D-01-2.1020D+00
4.0888D+02 4.6510D+01 7.8170D+01
4 1100.0000 311.2702 4.4640D+01 5.1303D+02 4.7092D+01 3.9410D-01-1.7482D+00
5.1771D+02 3.9952D+01 8.4315D+01
5 1100.0000 350.0000 4.5902D+01 4.6742D+02 5.9353D+01 3.8420D-01-2.6853D-01
4.7562D+02 3.7704D+01 8.2136D+01
6 1100.0000 388.7298 4.6207D+01 4.2170D+02 7.1543D+01 4.0412D-01-1.6803D+00
4.3487D+02 3.3037D+01 7.9570D+01
7 1177.4597 311.2702 2.5120D+01 5.3025D+02 5.3103D+01-6.8916D-01-3.2870D+00
5.3577D+02 1.9598D+01 8.4063D+01
8 1177.4597 350.0000 2.7519D+01 4.8446D+02 6.2176D+01-6.1851D-01-2.3152D+00
4.9277D+02 1.9210D+01 8.2388D+01
9 1177.4597 388.7298 2.8962D+01 4.3856D+02 7.1178D+01-6.0653D-01-4.2545D+00
4.5058D+02 1.6945D+01 8.0418D+01

ELEMENT NO 32
1 1022.5403 422.5403 4.2755D+01 2.5978D+02 8.4856D+01-3.8268D-01-3.3740D+00
2.8902D+02 1.3516D+01 7.0988D+01
2 1022.5403 500.0000 3.1434D+01 1.9134D+02 1.0111D+02-2.8722D-01 6.1585D-01
2.4029D+02-1.7518D+01 6.4167D+01
3 1022.5403 577.4597 1.9381D+01 1.2280D+02 1.1536D+02-8.0873D-02-3.8828D+00
1.9751D+02-5.5328D+01 5.7073D+01
4 1100.0000 422.5403 2.8222D+01 2.7780D+02 7.9199D+01 3.5432D-01-3.3849D+00
3.0081D+02 5.2111D+00 7.3799D+01
5 1100.0000 500.0000 2.3905D+01 2.0580D+02 9.3211D+01 1.7185D-01 9.6637D-01
2.4509D+02-1.5377D+01 6.7148D+01
6 1100.0000 577.4597 1.8855D+01 1.3372D+02 1.0521D+02 3.2465D-02-3.7203D+00
1.9616D+02-4.3580D+01 5.9315D+01
7 1177.4597 422.5403 1.3016D+01 2.9142D+02 7.6139D+01-5.3663D-01-5.8059D+00
3.1088D+02-6.4461D+00 7.5661D+01
8 1177.4597 500.0000 1.5703D+01 2.1588D+02 8.7907D+01-2.8645D-01-1.6392D+00
2.4900D+02-1.7421D+01 6.9353D+01
9 1177.4597 577.4597 1.7657D+01 1.4025D+02 9.7667D+01-6.0975D-02-7.0599D+00
1.9426D+02-3.6356D+01 6.1056D+01
□

APPENDIX B

MESHES USED IN THE FINITE ELEMENT ANALYSIS

WALLS 1&2



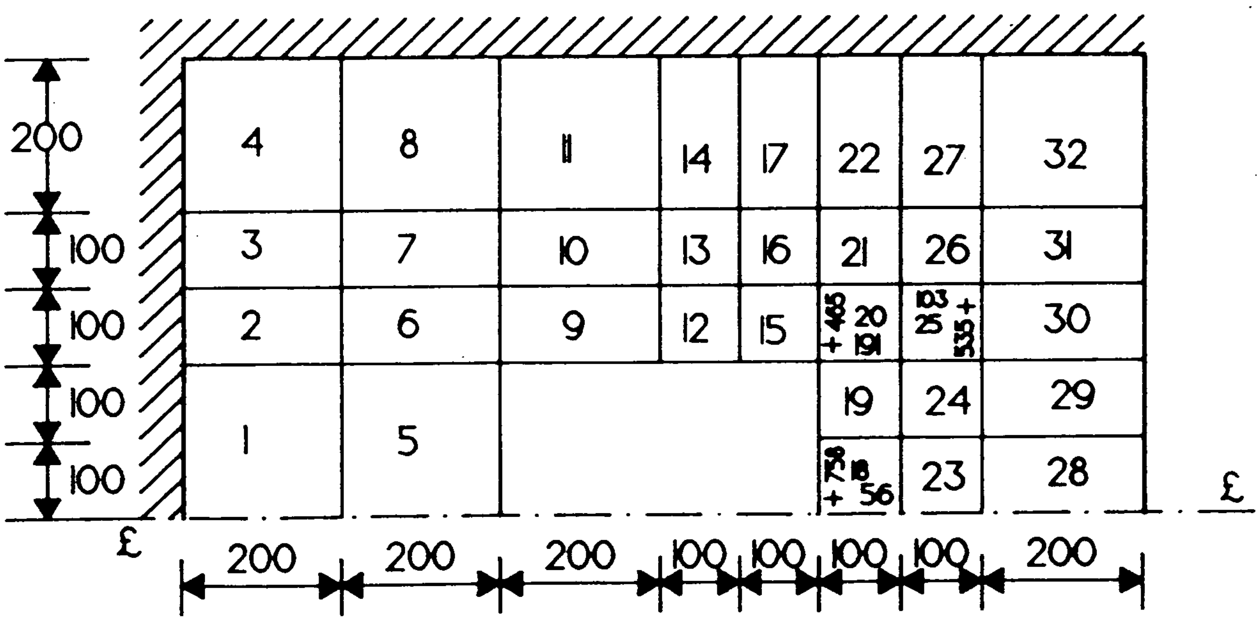
Dimensions are in millimetres

simply supported edge

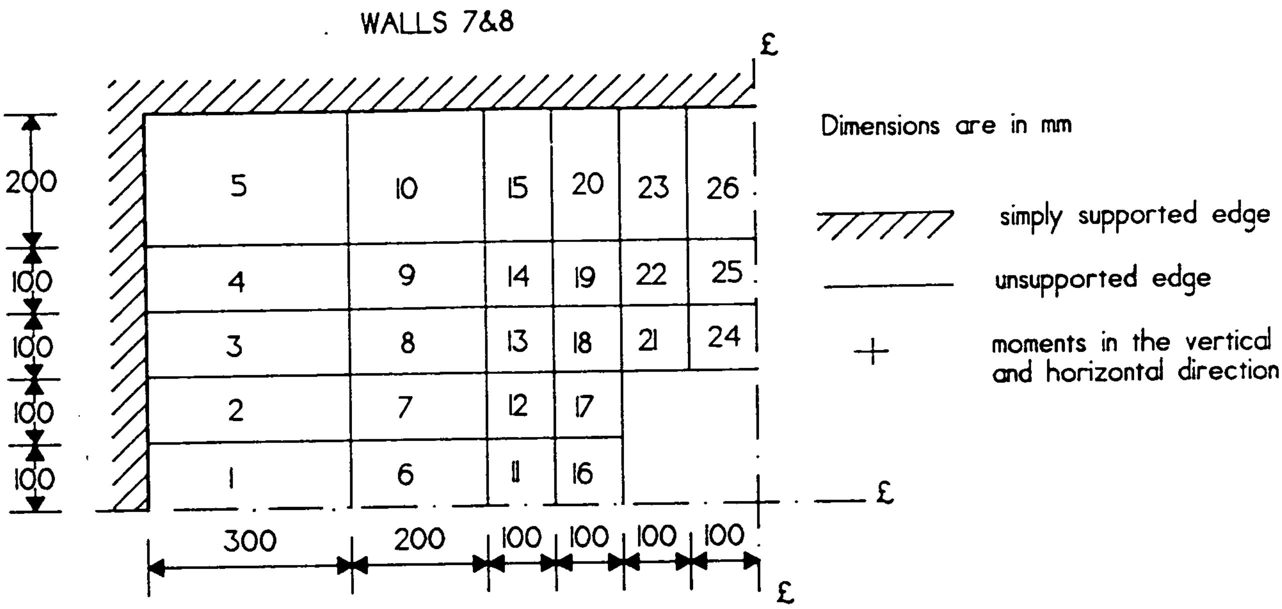
unsupported edge

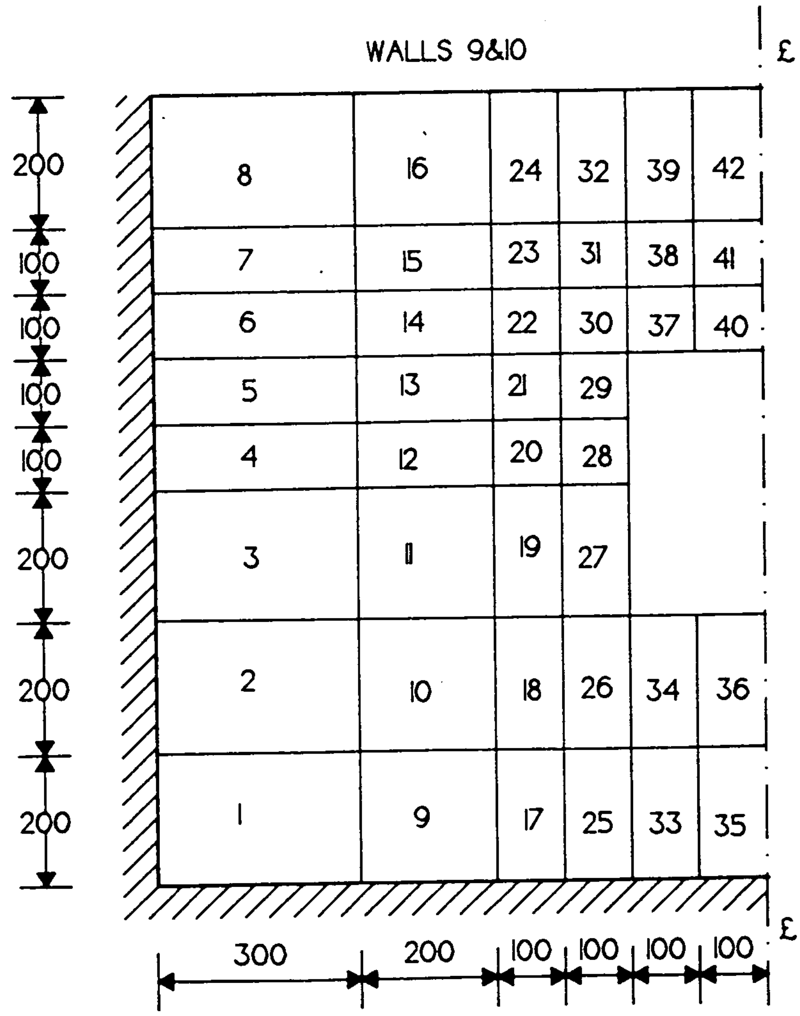
\+ moments in the vertical and horizontal directions

WALLS 5&6



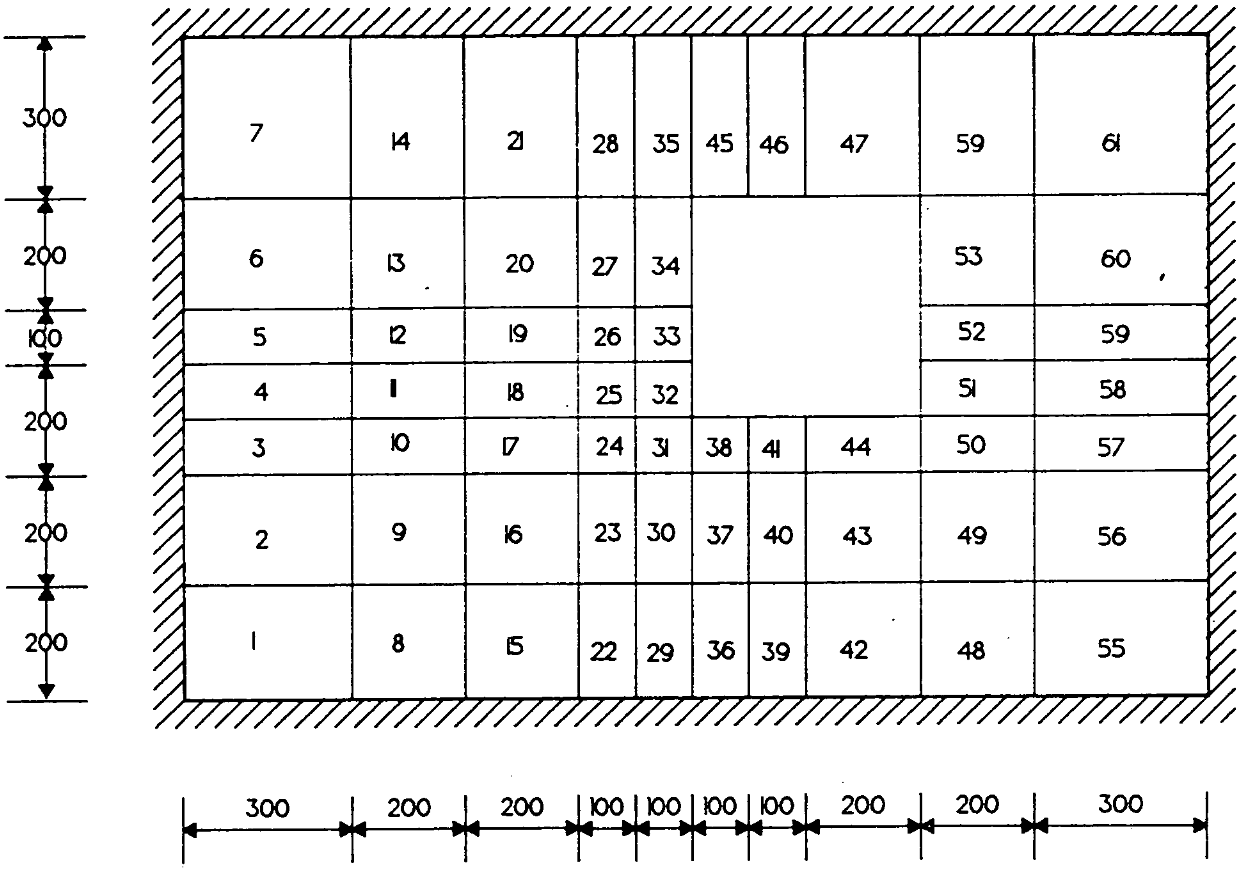
Mesh used in the Finite Element program



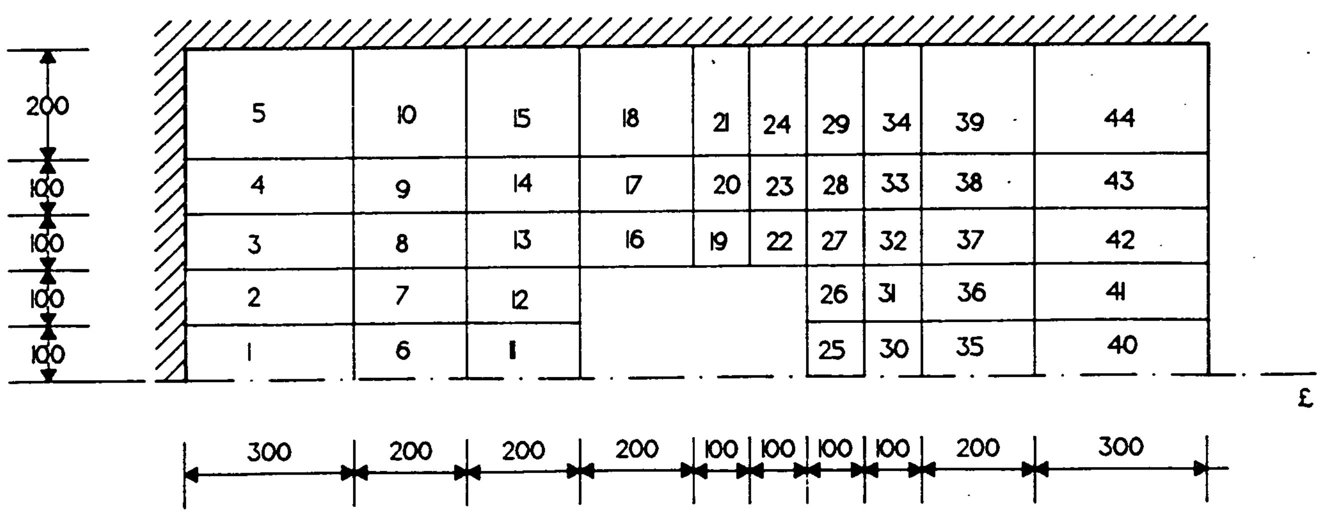


Mesh used in the Finite Element program

WALLS 1&12



WALLS 15&16



Dimensions are in mm

simply supported edge

unsupported edge

moments in the vertical and horizontal directions

Mesh used in the Finite Element program

APPENDIX C

HOW TO USE THE MOMENT INTERACTION DIAGRAM

An example is presented to show how the cracking pressure can be obtained using the moment interaction diagram presented in Fig. 4.28. Walls 5 and 6, for which the output of the finite element analysis for orthotropic plate is given in Appendix A, have been chosen.

The bending moments at three elements of Walls 5 and 6 shown in Appendix B have been selected. The applied uniformly distributed load taken as 0.0044 $N.mm^{-2}$:

i) Element 18 - m_x=56 N.mm and m_y=758 N.mm;

ii) Element 20 - m_x= 191 N.mm and m_y=465 N.mm; and

iii) Element 25 - m_x=103 N.mm and m_y=535 N.mm.

To find the coordinates of these points in the moment interaction diagram, the ratio between m_x/m_{xu} and m_u/m_{yu} must be calculated, hence:

- for Element 18

$$\frac{m_x}{m_{xu}} = \frac{56}{1061} = 0.053 \text{ and } \frac{m_y}{m_{yu}} = \frac{758}{439} = 1.73$$

To bring these coordinates to the polynomial equation of the moment interaction diagram, they have to be divided by a factor of 1.7. Then, the coordinates are:

$$\frac{0.053}{1.7} = 0.03 \text{ and } \frac{1.73}{1.7} = 1.02$$

The cracking pressure at the Gauss points presented in the mesh of Wall 5 is obtained by also dividing the previous applied load to the same factor 1.7. Thererefore,

$$\frac{0.0044 \text{ N.mm}^{-2}}{1.7} = 0.0026 \text{ N.mm}^{-2} \ (2.6 \text{ kN/m}^2)$$

- for Element 20

$$\frac{m_x}{m_{xu}} = \frac{191}{1061} = 0.18 \text{ and } \frac{m_y}{m_{yu}} = \frac{465}{439} = 1.06$$

To align these coordinates with the moment interaction diagram they have to be divided by a factor of 1.11. The cracking pressure is also obtained by dividing the applied load by the same factor 1.11. The cracking pressure is

$$\frac{0.0044}{1.11} = 0.0040 \text{ N.mm}^{-2} \text{ (4 kN/m}^2\text{)}$$

- for Element 25

$$\frac{m_x}{m_{xu}} = \frac{103}{1061} = 0.097 \text{ and } \frac{m_y}{m_{yu}} = \frac{535}{439} = 1.22$$

These coordinates have to be divided by 1.1 to be aligned with the curve of the moment interaction diagram, and the cracking pressure is obtained by dividing the previous applied pressure by the same factor. Therefore, the cracking pressure at this Gauss point is given by

$$\frac{0.0044 \text{ N.mm}^{-2}}{1.1} = 0.0040 \text{ N.mm}^{-2}) \text{ (4 kN/m}^2\text{)}$$

Cracking in the wall will start at the Gauss point reached by the lowest pressure obtained from the moment interaction diagram. Therefore the cracking pressure is 2.6 kN/m^2.

APPENDIX D

PUBLISHED PAPERS

The following papers were published by the author in collaboration with Dr. B. P. Sinha, during the course of this thesis:

"Lateral Strength of Brickwork Panels with Openings". Proceedings of the Institution of Civil Engineers, Structures & Buildings, Paper no 9884, November 1992, pp 397-402

"Laterally Loaded Brickwork Wall Panels with Openings". Proceedings of the 4th International Seminar on Structural Masonry for Developing Countries, Tata McGraw Hill, Madras, India, December 1992, pp 64-69.

Lateral strength of brickwork panels with openings

Proc. Instn Civ. Engrs Structs & Bldgs, 1992, 94, Nov., 397–402

Paper 9884

Written discussion closes 15 January 1993

R. B. Duarte, EngCivil, MSc, and *B. P. Sinha*, BSc, DBS, PhD, MICE, FIStructE

The Paper presents the results of an experimental investigation into the behaviour of brickwork panels subjected to lateral pressure. Twelve panels with window openings built with half-scale bricks were tested to failure. The variables considered were aspect ratios and boundary conditions. The experimental failure pressures were compared with those obtained by the yield-line and elastic analysis.

Notation

q	applied external pressure
m	ultimate moment per unit length along a yield-line
μ	strength orthotropy
ϕ_x and ϕ_y	rotation of the yield-line along the x and y axes
L_x and L_y	projection of the yield-line over the x and y axes

Introduction

Brickwork cladding panels are subjected to wind loading. These panels often contain openings. They resist load on account of plate bending, and their load-carrying capacities depend on the flexural tensile strengths normal and perpendicular to the bed-joint. For the design of panels without openings, BS 5628[1] gives the values of the bending moment coefficients similar to those that can be obtained by yield-line analysis applied to under-reinforced concrete slabs.[2] Strictly speaking, the application of the yield-line analysis to a brittle material such as unreinforced brickwork is questionable. However, the code gives these coefficients based on some test results as design guidance without explicitly acknowledging the sources for these coefficients. No such guidance is available for the design of panels with openings. The suggestion is made in the code,[1] Appendix D, to divide the panels into sub-panels and then to design each part either in accordance with the rules given in clause 36 or by the yield-line or elastic analysis. Some test results of lateral strengths of brickwork panels containing openings are available.[3,4] These have ignored the line loading which develops naturally at the edges of a window opening as a result of wind pressure. Also, no definitive mathematical solution is available at present for panels with window openings subjected to wind loading. Hence, an experimental investigation was carried out on panels with window openings to study the behaviour under lateral pressure. The variables considered in this study were: aspect ratio (h/l); the boundary conditions. The window was positioned in the centre of the panels in every case.

Experimental procedure

Panel details

2. Half-scale bricks were used to build the 12 test walls in 1 : 3 (rapid-hardening cement : sand) mortar. The average cube strength of the mortar varied from 10–18 N/mm^2, with the characteristic strength of 10·8 N/mm^2 at 14 days. The dimensions of, and the positions of openings in the test walls are shown in Fig. 1. A plyboard sheet was used to represent the closed window which transferred the wind pressure to the edges of the window opening. It was found that, owing to the different deformation properties of brickwork and the plyboard sheet, the load was transferred as point loads at the corner of the opening. Hence, in order to improve the modelling for the theoretical analysis, it was decided to transfer the pressure from the plyboard as four equal corner loads through four wooden studs fixed at the corner of the test wall, which gave the exact determinate values of the reactions.

3. The lateral loading in steps of 0·4 kN/m^2 was applied until failure by an air-bag sandwiched between the test wall and the loading frame. The pressure was measured by the water manometer. The deflections at various points were measured by dial gauges. The points at which the deflections were measured are given in Fig. 1.

Determination of flexural tensile strengths and elastic properties

4. The flexural tensile strengths normal and perpendicular to bed joints were obtained by testing wallettes, as shown in Fig. 2. These wallettes were built along with the test walls. In addition, wallettes were extracted from the undamaged portion of the failed walls for obtaining the flexural tensile strengths normal and perpendicular to the bed joint. These wallettes were tested to identify any differences in the strengths compared to the wallettes built along with the test walls.

5. The moduli of elasticity and the Poisson's ratios were obtained by testing wall-

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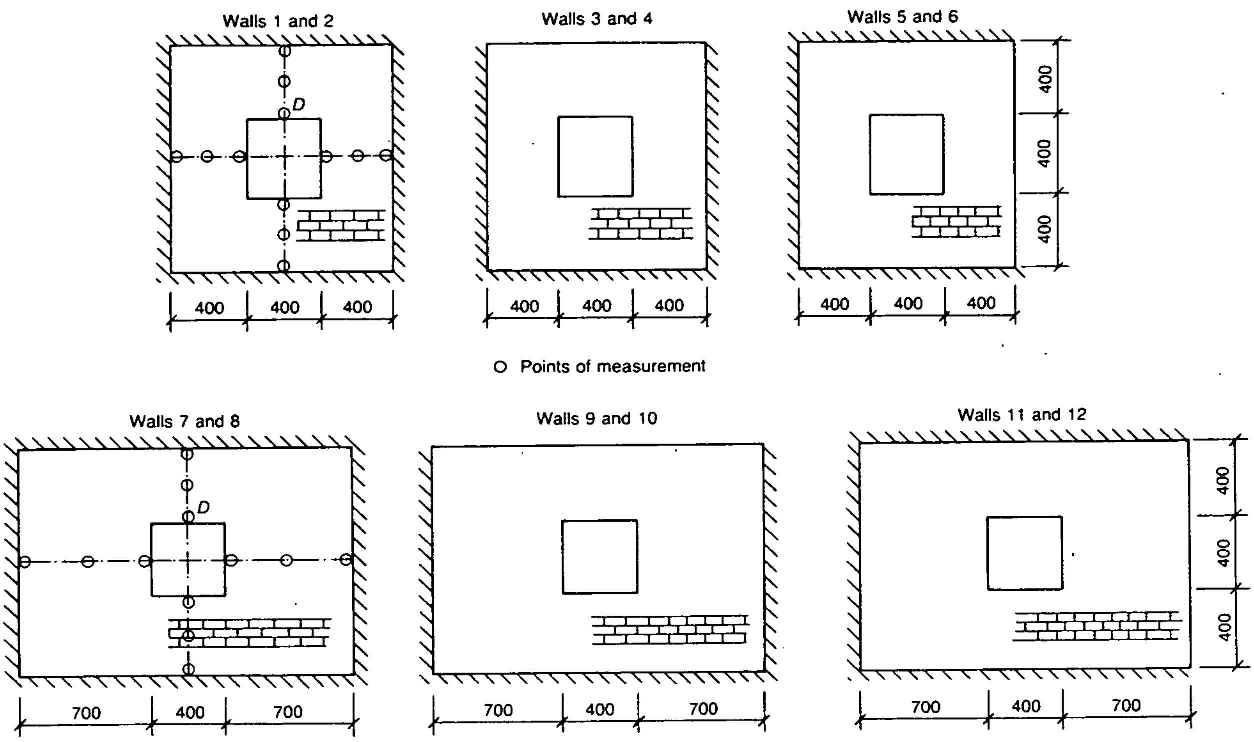


Fig. 1. Wall configurations (all dimensions in mm)

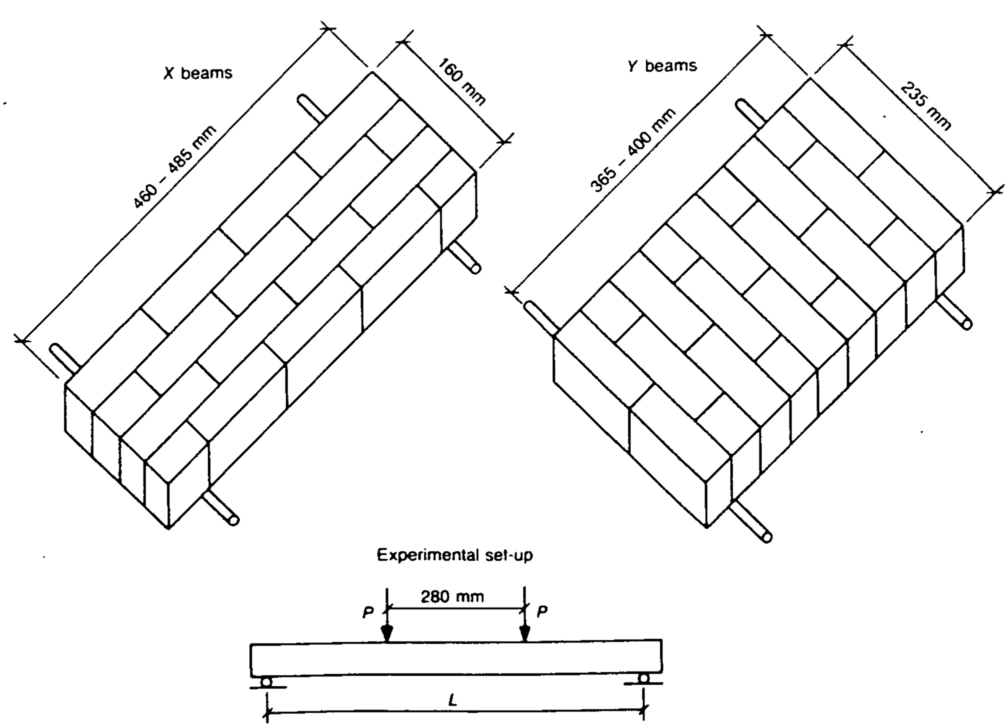


Fig. 2. Beam configuration and experimental set-up

ettes in compression. The compressive strain was measured by using the vibrating wire gauges. The values of moduli of elasticity were also obtained in bending (Fig. 2) and compared with those obtained in compression, and no significant difference was recorded. The average values of the tangent moduli of elasticity and the Poisson's ratio were

$$E_x = 17\,750\ \text{N/mm}^2,\ \upsilon_{yx} = 0{\cdot}11$$

$$E_y = 13\,500\ \text{N/mm}^2\ \text{and}\ \upsilon_{xy} = 0{\cdot}15$$

These values have been used for the elastic analysis.

Theoretical analysis

6. A standard computer program was used for the elastic analysis. The yield-line analysis was carried out for each of the walls. The work method[5] has been used for all test cases dealt with in this Paper. The idealized yield-line pattern, giving the lowest failure pressure, is shown in Figs 3–5 for each case.

Walls with four edges simply supported containing a central opening

7. If a virtual deflection of unity is given to the four corners, cdef, while the panel in Fig. 3 is collapsing, the external work done by the uniformly distributed and line loads is given by

$$\frac{q\alpha L^2}{3}(3\alpha + 3\beta - 8\beta\lambda) + (1 - 2\beta - 2\lambda l + 4\beta\lambda)$$

$$= \frac{q\alpha L^2}{3}(3 - 3\beta - 3\lambda + 4\beta\lambda) \quad (1)$$

The internal dissipation of energy on yield-lines ac, bd, fh and ge is equal to $\Sigma(mL_x\theta_x + \mu mL_y\theta_y)$, where

$$\theta_x = 1/\lambda\alpha L \quad \text{and} \quad \theta_y = 1/\beta L$$

Hence, the internal dissipation of energy is equal to

$$4m\alpha\left(\frac{\beta}{\alpha^2\lambda} + \frac{\mu\lambda}{\beta}\right) \quad (2)$$

Equating the external and internal work done gives

$$q = \frac{12m\left(\dfrac{\mu\lambda}{\beta} + \dfrac{\beta}{\alpha^2\lambda}\right)}{L^2(3 - 3\beta + 4\beta\alpha - 3\lambda)} \quad (3)$$

Walls with upper edge free and three other edges simply supported

8. The solution is obtained by giving a virtual deflection of unity to the vertical yield-line joining points b and d in Fig. 4. Equating the dissipation of internal energy and the external work done gives

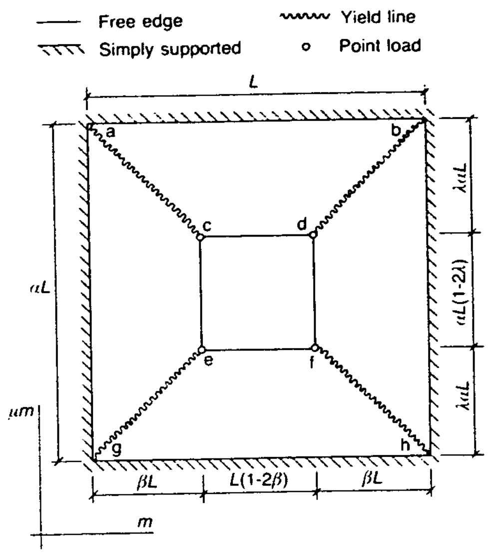


Fig. 3. Failure mechanism for walls with four edges simply supported

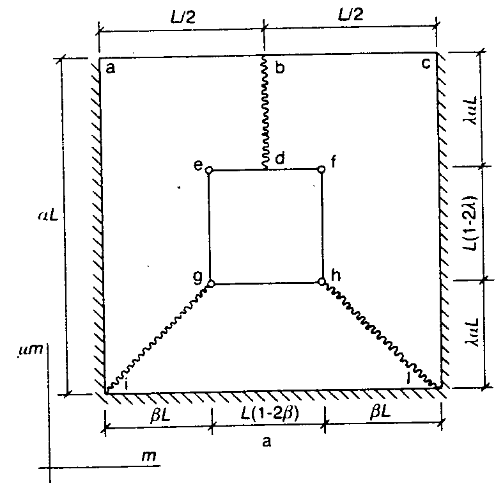


Fig. 4. Failure mechanism for walls with top edge free

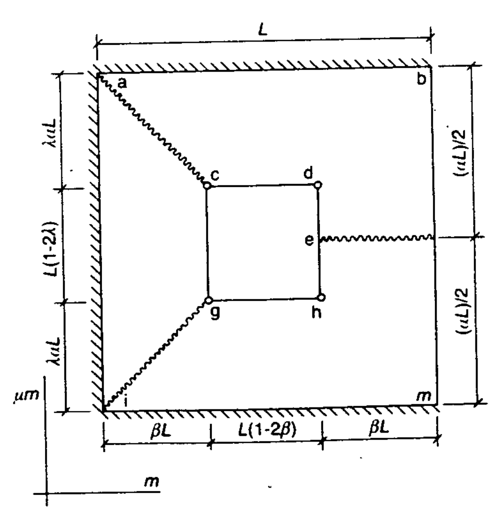


Fig. 5. Failure mechanism for walls with one lateral edge free

$$q = \frac{24m\left(\dfrac{\beta^2}{\lambda\alpha^2} + 2\mu\lambda^2\right)}{L^2(3\lambda + 20\beta^2\lambda - 18\beta\lambda + 12\beta - 12\beta^2)} \qquad (4)$$

Walls with one vertical edge free and three other edges simply supported

9. A similar solution is obtained by giving a virtual deflection of unity to the horizontal line between points e and f in Fig. 5. The predicted failure pressure is given by

$$q = \frac{24m\left(\dfrac{2\beta}{\alpha^2} + \dfrac{\mu\lambda^2}{\beta}\right)}{L^2(3\beta + 20\beta\lambda^2 - 18\beta\lambda + 12\lambda - 12\lambda^2)} \qquad (5)$$

The theoretical failure pressures obtained from equations (3)–(5) are shown in Table 1.

Discussion of results

10. The results are given in Tables 1, 2 and 3. From Tables 2 and 3, it is very clear that there is practically no difference between the flexural tensile strengths in two directions obtained from the wallettes extracted from the test walls after failure or built separately during its construction. This is also confirmed by statistical analysis of the results. There is a great deal of variation within the results, which has been reported in the literature[6] and which, as found in this test, is in no way unusual.

11. Some typical load–deflection relationships of panels 2 and 8 with the aspect ratios of 1 : 1 and 1 : 1·5 are shown in Fig. 6. The elastic analysis underestimates the deflections of the uncracked panels even at a very low pressure. At both low and failure pressures, the deflections of the panels at various points along the horizontal and vertical centre-lines (Fig. 6) are different compared with the predicted values

Table 1. Test results of walls

Test walls	Experimental pressure: kN/m²			Theoretical pressure: kN/m²		
	Cracking	Failure	Average failure	Elastic	Yield-line	BS 5628
1	5·0	7·9				
2	5·2	10·2	9·1	8·7	8·9	4·5
3	—	7·8				
4	—	7·2	7·5	5·2	6·1	3·1
5	6·8	7·4				
6	3·2	6·8	7·1	2·3	4·1	2·3
7	2·6	5·4				
8	4·0	6·4	5·9	2·6	5·7	2·9
9	—	3·1				
10	—	3·9	3·5	3·3	3·3	1·7
11	1·8	2·6				
12	2·2	2·6	2·6	2·1	3·3	1·7

Table 2. Wallettes built alongside test walls—flexural tensile strength

Wallettes	f_{tx}: N/mm²	f_{ty}: N/mm²
1	1·91	0·74
2	2·48	0·96
3	2·68	0·86
4	2·21	0·82
5	1·79	0·81
6	2·08	0·70
7	1·32	0·66
8	1·89	0·74
9	2·40	0·52
Mean	2·08	0·76
Standard deviation	0·41	0·13

Table 3. Wallettes extracted from the test walls—flexural tensile strength

Wallettes	f_{tx}: N/mm²	f_{ty}: N/mm²
1	2·10	0·83
2	2·41	0·61
3	2·35	0·64
4	1·59	0·60
5	2·24	0·54
6	2·07	0·50
7	1·83	0·81
8	1·23	0·47
9	1·77	0·48
10	1·83	0·89
11	1·95	0·59
12	1·76	0·89
13	2·02	0·89
14	1·90	0·86
15	2·30	0·57
16	2·20	0·55
17	1·89	0·67
18	2·15	0·65
19	2·61	0·89
20	2·51	0·78
21	2·12	0·77
22	2·55	0·82
23	2·21	1·07
24	2·33	1·10
25	1·85	1·01
26	1·48	1·62
27	2·18	1·53
28	3·11	0·62
29	1·61	1·07
30	2·24	1·42
31	2·01	—
32	2·08	—
33	2·48	—
34	2·15	—
35	2·97	—
36	2·46	—
37	2·33	—
38	2·19	—
39	2·00	—
40	2·47	—
41	2·27	—
42	1·81	—
43	2·26	—
Mean	2·14	0·82
Standard deviation	0·36	0·30

obtained by the elastic analysis. The analysis of the experimental deflections at 400 mm from the support in the symmetrical panel of the aspect ratio 1 : 1 suggests that the load distributes according to the flexural stiffness (i.e. stiffness orthotropy). In the conventional yield-line analysis as applied to reinforced concrete slabs, the question of the elastic orthotropy does not arise. Strictly speaking, ignoring this in obtaining the failure pressure of brickwork panels by the yield-line method may not be justified, as it violates the equilibrium condition and may explain the difference between the theoretical and experimental results. Fig. 7 shows the typical load–deflection relationship of point D (Fig. 1) which is non-linear for the test panels.

12. Before failure, initial cracks (Table 1) were noticed in the walls simply supported on four sides and in those simply supported on three sides with the vertical edge free. Walls 3, 4, 9 and 10 (Table 1), with three sides simply supported and the top edge free, did not show sign of cracking: they tended to behave like a strip spanning horizontally at the top, and the failure happened immediately after the development of vertical cracks at ultimate failure pressure.

13. The elastic analysis underestimates the failure pressure of the walls tested in this investigation. It also fails to predict the cracking pressure (Table 1). In the elastic method, it is assumed that the failure happens as soon as flexural strength in any one direction is reached. Thus no redistribution of moments can take place and, therefore, the strength orthotropy is neglected.

14. Although the typical crack patterns of the tested walls (Figs 8 and 9) were different and deviated from the idealized yield-lines, the correlation between theoretical predicted and experimental failure pressure (Table 1) was much better.

15. The experimental failure pressures for panels of aspect ratio 1 : 1 with three sides simply supported and with the vertical or top edge free were similar. This could be possible only if the strengths in the vertical and horizontal directions were the same, i.e. the panels exhibited strength isotropy. This is contrary to those failure pressures predicted theoretically by the yield-line method.

16. Figure 10 shows the relationship between the experimental failure pressure and those failure pressures predicted by both the yield-line method and the line of equality. In an ideal situation, all test results should lie on the line of equality. In this investigation, five test results of walls lie under this. However, in the case of the mean test results, all except one will be above the line of equality, which suggests that it is safer to use the yield-line method with all the shortcomings mentioned earlier for the

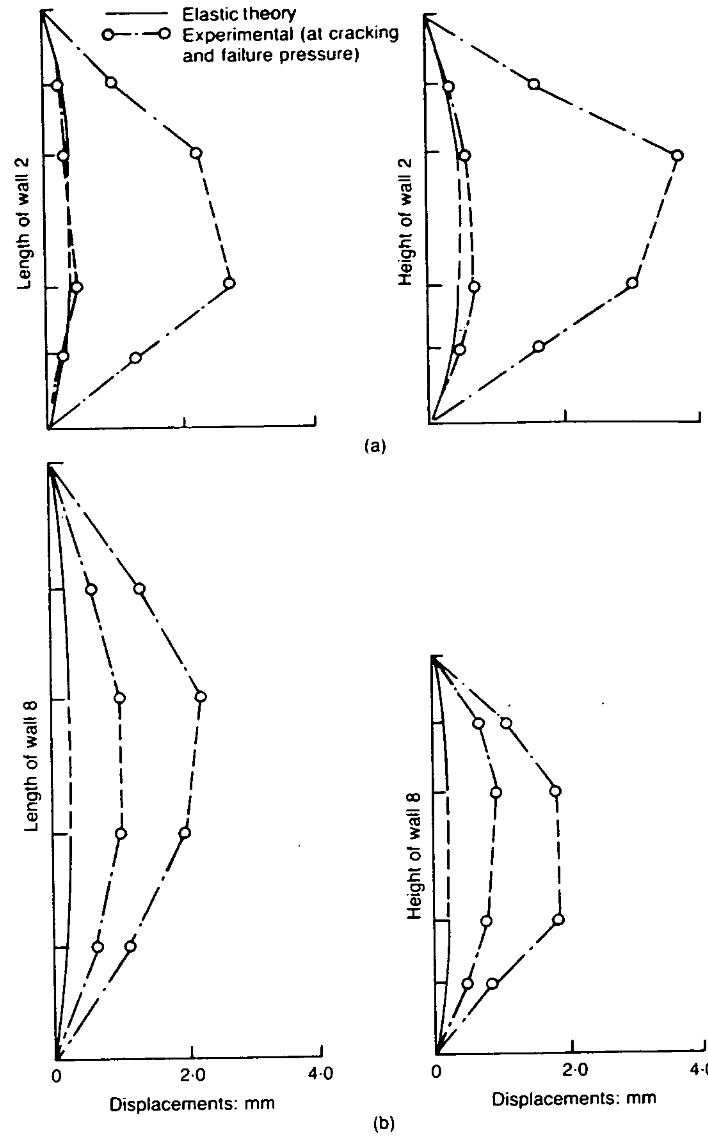


Fig. 6 (above). Load–deflection relationship of: (a) wall 2 (aspect ratio 1 : 1); (b) wall 8 (aspect ratio 1 : 5)

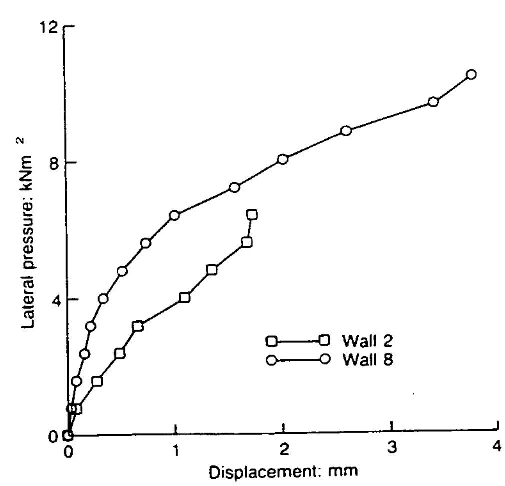
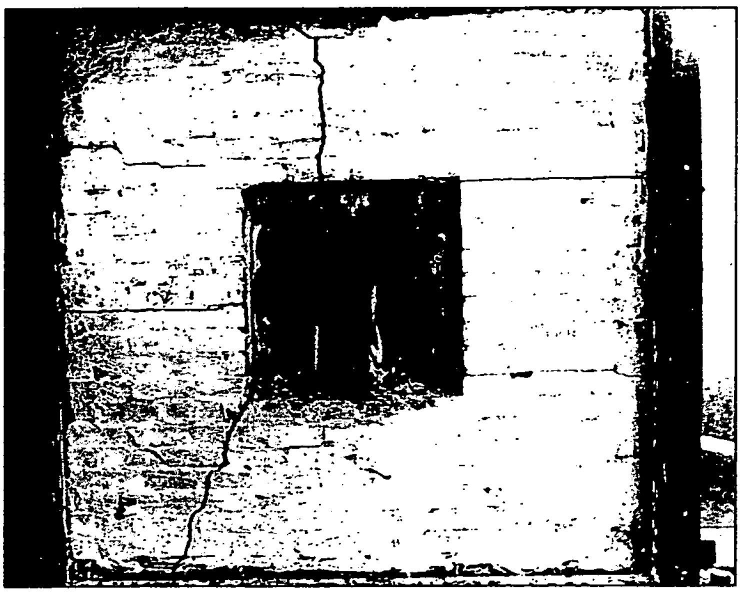


Fig. 7 (left). Load–displacement relationship of point D on walls 2 and 8 (see Fig. 1)



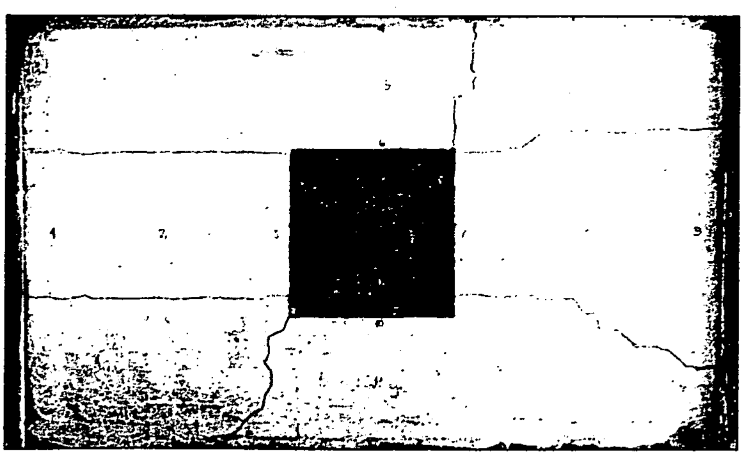


Fig. 8 (top). Cracks after ultimate failure of wall 1 (simply supported on four sides and with aspect ratio of 1 : 1)

Fig. 9 (above). Cracks after ultimate failure of wall 8 (simply supported on four sides and with aspect ratio of 1 : 5)

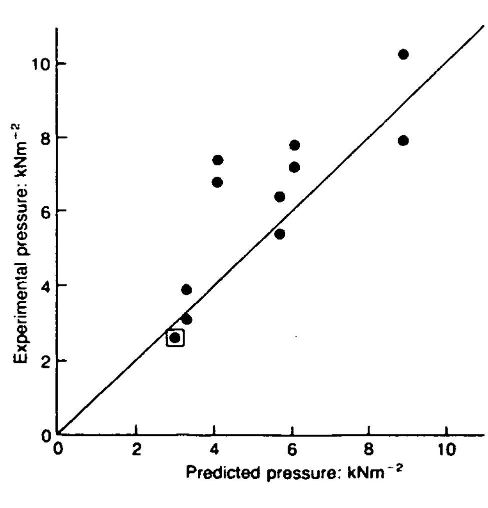


Fig. 10 (right). Comparison between experimental and predicted (yield-line) failure pressure for laterally loaded walls

design of brickwork panels, with openings subjected to lateral pressure.

17. Normally, the designer will use the published values of flexural strengths recommended by BS 5628[1] instead of test values; hence in Table 1, a comparison is made using the prescribed values of the characteristic flexural tensile strengths and the wall's test results. Because the ultimate failure pressures are being compared with the code,[1] the material partial safety factor has been assumed as one. The predicted failure pressures were many times lower than the experimental results. According to BS 5628,[1] the characteristic flexural strengths depend on water absorption; hence for these walls the allowable values are 0·4 N/mm^2 and 1·2 N/mm^2, which cause this underestimation of the pressure. The provision in the code of decreasing flexural strengths with increasing water absorption of bricks[7] seems obscure and may need revision in future as more data become available.

Conclusions

18. The flexural tensile strengths normal and perpendicular to the bed-joint obtained from the wallettes built independently or extracted from the undamaged portion of the tested walls are similar.

19. Compared with the elastic method, the yield-line method with all its limitations offers a better solution for predicting the lateral strength of brickwork panels with openings, and hence can be used with some confidence for the design of panels.

Acknowledgements

20. The Authors wish to thank the CNPq/ Conselho Nacional de Desenvolvimento Cientifico e Tecnologico/Brazil and CIENTEC/ Fundacao de Ciencia e Tecnologia do Estado do Rio Grande do Sul/Brazil for their financial support.

References

1. British Standards Institution. *Code of practice for the use of masonry, structural use of unreinforced masonry.* BSI, London, 1978, BS 5628: Part 1.
2. Haseltine B. A. *et al.* Design of walls to resist lateral load—Part 2. *Struct. Engr*, 1977, **55**, No. 10, Oct.
3. Southcombe C. and Tapp A. An investigation of laterally loaded brickwork panels with openings. *Proc. Br. Masonry Society*, 1988, No. 2, Apr.
4. May I. M. *et al.* The analysis and design of masonry panels with openings. *Proc. 8th Int. Brick/Block Masonry Conf., Dublin*, 1988.
5. Johansen K. *Yield-line formulae for slabs.* Cement and Concrete Association, London, 1972.
6. Sinha B. P. An ultimate load analysis of laterally loaded brickwork panels. *Int. J. Masonry Constr.*, 1980, **1**, No. 2.
7. West H. W. H. *et al.* The resistance of brickwork to lateral loading—Part 1. *Struct. Engr*, 1977, **55**, No. 10, Oct.

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LATERALLY LOADED BRICKWORK WALL PANELS WITH OPENINGS

R.B. DUARTE and B.P. SINHA

Abstract

Brickwork cladding panels resist wind loading in bending. These panels, supported on three of four sides bend in both directions. In this investigation 6 half-scale wall panels with window openings having different boundary conditions were tested up to failure and their strength compared with predicted failure pressure using elastic and yield line theories. An assessment of the Indian Code of Practice for the design of such panels has been done in the light of experimental results.

1. Introduction

In a masonry or infilled framed structure, the external walls or claddings have to resist wind loading. These wall panels resist the load in bending and thus, rely upon the flexural tensile strengths in two directions, perpendicular and parallel to the bed-joint. Often, these panels contain door or window openings. Some advanced code of practices(1) have given the bending moments coefficients for the design based on the the test results for panels without openings. The B. S. 5628(1) suggests the use of either yield-line or elastic analysis for the design of panels with openings. Extensive lateral load tests on panels without openings(2) formed the basis of these recommended design bending moment coefficients but no such data were available for the panels with openings. The problem is much more complicated as some of the lateral load from the window or door in the panel will be transferred as line loading on the edge of the openings. As no experimental data for the lateral load design of panels with openings were available to support the contention of the B. S. 5628, an investigation was carried out on 6, half-scale panels with window openings. The earlier part of this paper describes the test and compares the result of the failure pressures with those obtained by the yield line and elastic theories. An assessment of the Indian Code of Practice(3) for the design of laterally loaded panels is also done in the light of the experimental results.

2. Experimental work

The six test walls are shown in Fig. 1. The walls were built with half-scale bricks having a characteristic strength of 29.7 N/mm^2 in 1:3 (rapid hardening cement:sand) mortar. The average cube strength of the mortar varied from 11.5-23.6 N/mm^2 at 14 days, with characteristic strength of 10.7 N/mm^2.

The window openings were represented by a plywood sheet transferring the wind pressure to the walls through four wooden bolts positioned at each corner of the windows.

The wind pressure leading up to the failure of the wall was applied in steps of 4×10^{-4} N/mm^2 by an air bag sandwiched between the test wall and the loading frame; the pressure being recorded by a water manometer. Deflections were measured using mechanical dial gauges at various points on the walls.

The flexural tensile strengths normal and perpendicular to the bed joints were obtained by testing wallettes extracted from the undamaged parts of the test walls, and the tests were performed according to B. S. 5628. It has been shown(4) that the flexural tensile strengths obtained either by wallettes extracted from the undamaged parts of the walls or built alongside the test walls are not different.

3. Experimental results

A minimum of at least three wallettes were extracted to determine the flexural tensile strength normal (f_{tx}) and perpendicular (f_{ty}) to bed joints of each test wall. Fig. 2 shows the wallette configuration. The mean results are presented together with the wall test results in Table 1.

Walls 1,2,5 and 6 cracked horizontally before reaching the respective failure pressures. The hair-line cracks were noticed at the same time in the walls when the measured deflections increased enormously. Walls 3 and 4 did not show any sign of cracking. The walls collapsed due to the development of cracks starting from the window corners moving straight to the intersections of the supports,i.e. the corners of the wall 'e' and 'f'. A typical failure is shown in Fig. 3.

A typical load-deflection relationship at the point of maximum displacement is shown in Fig. 4.

For the elastic analysis the Young's moduli in both directions and the Poisson's ratio are required, which were obtained by compression tests on brickwork prisms extracted from the walls. The results are:

E_x=17,750 N/mm^2; ν_{yx}=0.11; E_y=13,500 N/mm^2 and ν_{xy}=0.15.

4.Theoretical methods

The yield line(5) and the elastic theory were applied to predict the failure and cracking pressures of the six test walls. Results are presented in table 2. The yield line analysis was used for calculating the theoretical failure pressures for the three types of walls. A typical example using the work method has been given in the paper. For Walls 3 and 4 the pattern of failure that gives the lowest failure pressure (w) is shown in Fig. 5.

4.1. Yield line analysis

If a virtual deflection of unity is given to the four corners "abcd" of the window (Fig.5), while the wall is collapsing, the external work done by the uniformly distributed load (w) applied over the area of the wall (including the window) is given by :

$$\frac{w\gamma L^2}{6}(3-2\beta-4\beta\lambda+3\lambda)\ldots\ldots\ldots\ldots(i)$$

The internal dissipation of energy along the yield lines "be" and "cf" is given by $\Sigma(mL_x\theta_x+\mu mL_y\theta_y)$, where $\theta_x=1/\gamma L(1-\lambda)$ and $\theta_y=1/\beta L$.

Hence,the total internal dissipation of energy is equal to

$$2m\gamma\left[\frac{\beta}{\gamma^2(1-\lambda)}+\frac{\mu(1-\lambda)}{\beta}\right]\ldots\ldots(ii)$$

By equating the external work done with the internal dissipation of energy on yield lines,we get

$$w=\frac{12m\left[\frac{\beta}{\gamma^2(1-\lambda)}+\frac{\mu(1-\lambda)}{\beta}\right]}{L^2(3-2\beta-4\beta\lambda+3\lambda)}\ldots(iii)$$

The same procedure was used to obtain the failure pressure of the other walls.

5.Discussions

From table 2 it can seen that the elastic analysis based on plate bending theory failed in predicting both cracking and failure pressures with the exception of Wall 5. The finite element program used in this analysis assumes that failure happens as soon as the flexural strength in any of the two directions is reached, thus not allowing any redistribution of moments. It must be pointed out that the test walls were assumed to have cracked at a pressure when enormous increase in deflections were measured by the mechanical dial guages along with the development of the noticeble hair-line cracks. However,it might be possible that small cracks would have developed,which is difficult to detect by the naked eye long before the change in the stiffness and thus the behaviour of the walls.

The yield line method gives a better correlation with the test walls, although there seems no theoretical justification for its application to a brittle material like unreinforced brickwork. Once the cracks develope, it is highly unlikely that the craked sections will be capable of resisting constant moments. Hence, its use in BS 5628(1) can only be justified on empirical basis in line with the experimental results.The test results were compared in table 1 with the BS 5628 by putting the material partial safety factor γ_m equal to one. From table 1 it can be seen that the predicted failure pressures for the walls by BS 5628 are lower than the experimental results.

5.1. Comparison of the results with the Indian Code of Practice for Structural use of Unreinforced Masonry

The Indian Code of Practice gives bending moment coefficients for the design of panels without openings supported on four edges or three edges with top edge free having varying aspect ratios (0.3 to 1.75) in tables 13 & 14 of Appendix D. The origin of the coefficients is not mentioned, but it is exactly the same as one can obtain from the yield line analysis. The design moment is based on the length of the panel, which is equated to the moment of resistance using the flexural tensile strength parallel to the bed-joint. This is exactly similar to the British Code of Practice BS 5628. Having established the theoretical basis of the coefficients for panels without openings in the Indian Code, it was felt prudent to compare these results of the panels with openings using the flexural tensile strength and the orthotropy given in this code. The results are shown in Table 2. The factor of safety ranges from 8.7 to 22.0, which is unusually high. This is due to the fact that the flexural tensile strengths in two directions in the Indian Code are very low compared to those obtained from wallette tests. It is, therefore, suggested that tests should be carried out with local materials to establish some realistic values of flexural tensile strengths in two directions for the Indian Code of Practice.

6. Conclusions

On the basis of this work following conclusions can be drawn:

i) The elastic analysis underestimates the failure pressure of brickwork anels with openings subjected to wind loading.

ii) The yield line analysis predicts closely the failure pressure of brickwork panels with openings and may be used for the design.

iii) The Indian Code of Practice seems very conservative for the design of panels with openings subjected to lateral pressure and may need revision.It is suggested that some realistic values for the flexural tensile strengths in two directions should be determined and incorporated in the code by testing the local materials.

References

1.BRITISH STANDARD INSTITUION, Code of Practice for the use of masonry, Structural use of unreinforced masonry, BS 5628:Part 1, 1978.

2.WEST, H. W. H., HODGKINSON, H. R. and HASELTINE, B. A., The Resistance of Brickwork to Lateral Loading - Part 1. The Structural Engineer, No. 10, Vol. 55, October, 1977.

3.INDIAN STANDARD, Code of Practice for Structural Use of Unreinforced Masonry,IS:1905-1987,Manak Bhawan, New Delhi.

4.DUARTE, R. B. and SINHA, B. P., Lateral Strength of Brickwork Panels with Openings, Proc. of the Inst. of Civil Engineers, Structures and Buildings,(In press).

5.JOHANSEN, K. W., Yield-Line Formulae for Slabs. Cement and Concrete Association, London, 1972.

6.HENDRY, A. W., SINHA, B. P. and DAVIES, S. R., Loadbearing Brickwork Design. Ellis Hoorwood Ltd.,Chichester, 1987.

Notations:

Free edge;

Simply supported;

Yield-line;

μ Strength orthotropy;

L_x and L_y Projection of the yield lines over the "x" and "y" axes.

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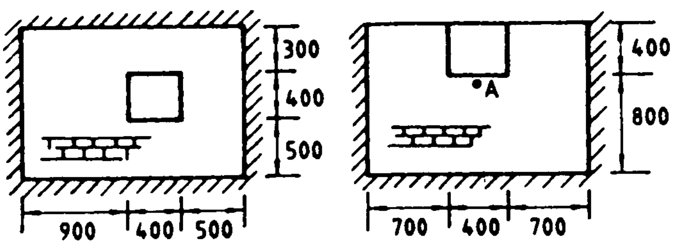
The authors wish to thank the CNPq/Conselho Nacional de Desenvolvimento Cientifico e Tecnologico/Brazil and CIENTEC/Fundacao de Ciencia e Tecnologia do Estado do Rio Grande do Sul/Brazil for their financial support.

Wall no.	Flexural tensile strength (N/mm^2)		Experimental pressures (kN/m^2)		Predicted pressures (kN/m^2)	
	f_{tx}	f_{ty}	Cracking	Failure	Yield line	Elastic
1	2.11	0.86	5.6	6.6	7.1	2.8
2	1.99	0.84	4.2	7.2	6.8	2.7
3	2.05	0.85	-	4.4	3.3	2.6
4	1.53	1.07	-	3.0	2.9	1.9
5	1.86	0.72	1.8	2.6	2.9	1.8
6	1.79	0.63	2.2	2.6	2.6	1.6

Table 1. Wall Results

Wall nos.	Failure pressure (kN/m^2)	Design pressure (kN/m^2)-BS5628	$\frac{\text{Failure pressure}}{\text{Design pressure BS5628}}$	Design pressure (kN/m^2) IS:1905	$\frac{\text{Failure pressure}}{\text{Design pressure IS:1905}}$
1	6.6	3.6	1.83	0.5	13.2
2	7.2	3.6	2.0	0.5	14.4
3	4.4	1.7	2.59	0.2	22.0
4	3.0	1.7	1.76	0.2	15.0
5	2.6	1.7	1.53	0.3	8.7
6	2.6	1.7	1.53	0.3	8.7

Table 2: Comparison of the experimental failure pressure of the walls with the Codes of Practice.



All dimensions are in mm.

Figure 1 Wall configurations

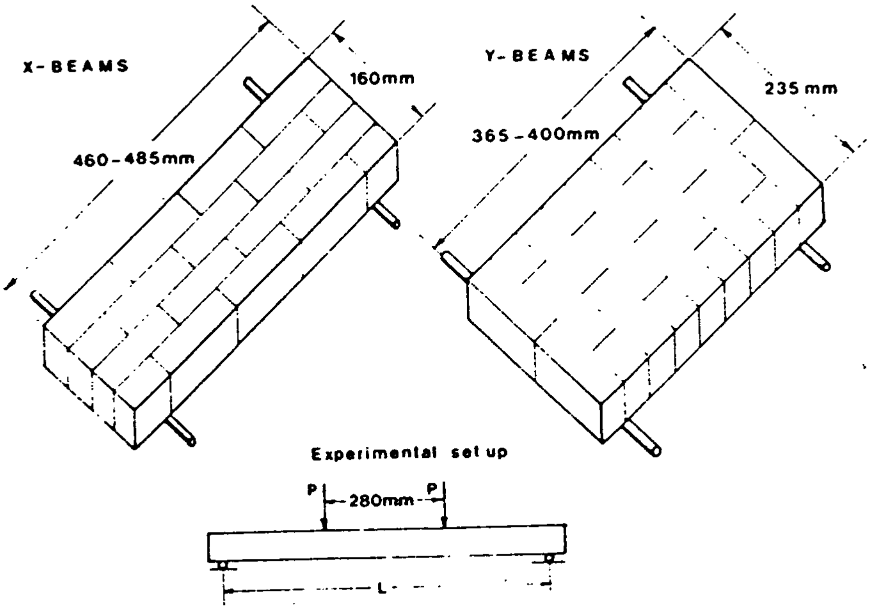


Fig. 2. Beam configuration and experimental set up.

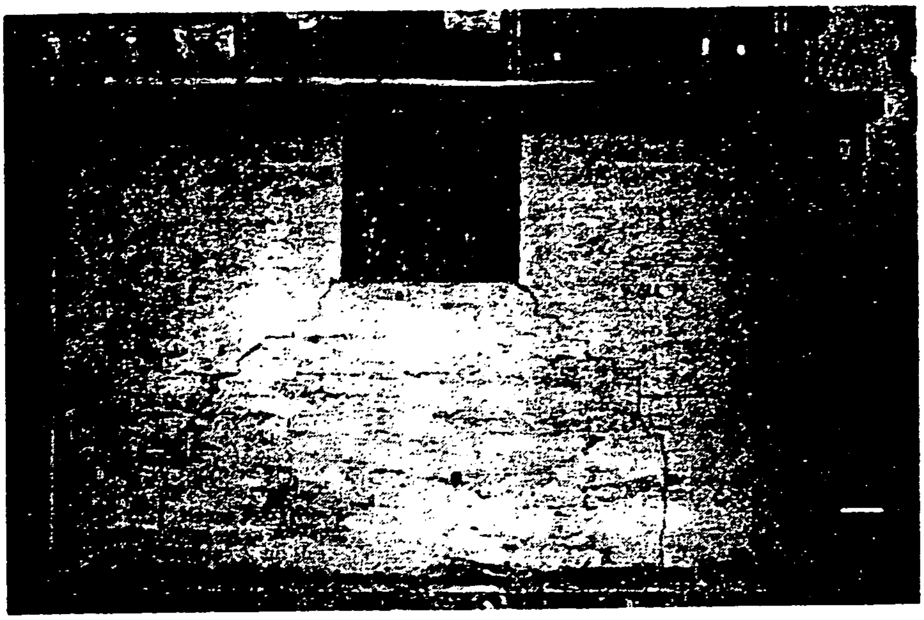


Fig.3. Showing the failure of Wall 4, simply supported on three sides with the top edge free.

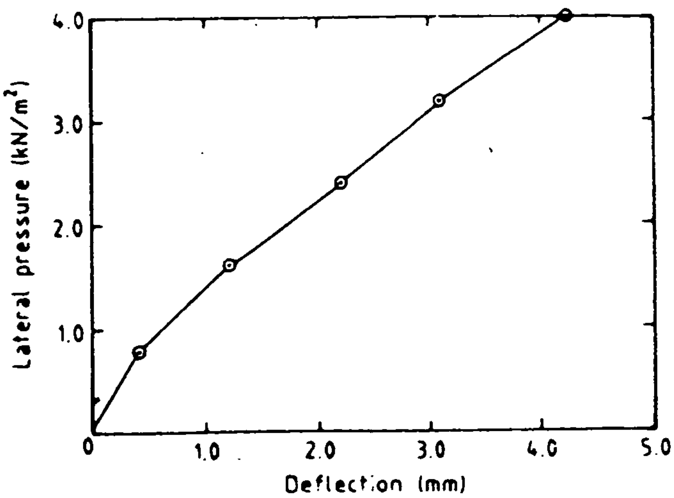


Figure 4 Load-deflection relationship of Wall 3 at the point A.

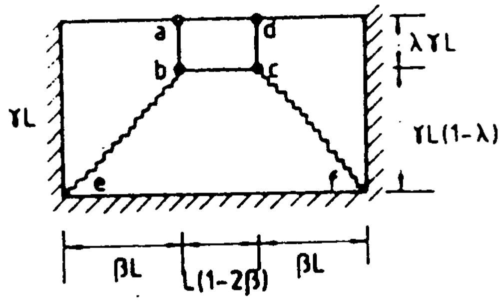


Figure 5 Pattern of cracks of walls 3 and 4