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Extremal problem on $(2n, 2m-1)$-system points on the rays.

For fix number $n \in \mathbb{N}$ system points

$$A_{2n,2m-1} = \{a_{k,p} \in \mathbb{C} : k = \overline{1, 2n},\ p = \overline{1, 2m-1}\},$$

we will called on the $(2n, 2m-1)$-system points on the rays, if at all $k = \overline{1, 2n}$, $p = \overline{1, 2m-1}$ the relations are executed:

$$\begin{aligned} &0 < |a_{k,1}| < \ldots < |a_{k,2m-1}| < \infty; \\ &\arg a_{k,1} = \arg a_{k,2} = \ldots = \arg a_{k,2m-1} =: \theta_k; \\ &0 = \theta_1 < \theta_2 < \ldots < \theta_n < \theta_{n+1} := 2\pi. \end{aligned} \tag{1}$$

Let's consider system of angular domains:

$$P_k = \{w \in \mathbb{C} : \theta_k < \arg w < \theta_{k+1}\}, \quad k = \overline{1, 2n}.$$

Let D, $D \subset \overline{\mathbb{C}}$ – arbitrary open set and $w = a \in D$, then $D(a)$ the define connected component D, the contain point a. For arbitrary $(2n, 2m-1)$-system points on the rays $A_{2n,2m-1} = \{a_{k,p} \in \mathbb{C} : k = \overline{1, 2n},\ p = \overline{1, 2m-1}\}$ and open set D, $A_{2n,2m-1} \subset D$ the define $D_k\left(a_{s,p}\right)$ connected component set $D\left(a_{s,p}\right) \bigcap \overline{P_k}$, the contain point $a_{s,p}$, $k = \overline{1, 2n}$, $s = k, k+1$, $p = \overline{1, 2m-1}$, $a_{n+1,p} := a_{1,p}$.

The open set D, $A_{2n,2m-1} \subset D$ satisfied condition meets the condition of unapplied in relation to the system of points $(2n, 2m-1)$-system points on the rays $A_{2n,2m-1}$ if a condition is executed

$$D_k(a_{k,s}) \bigcap D_k(a_{k+1,p}) = \varnothing, \tag{2}$$

$k = \overline{1, 2n}$, $p, s = \overline{1, 2m-1}$ on all corners $\overline{P_k}$.

The define $r(B; a)$ inner radius domain $B \subset \overline{\mathbb{C}}$ with respect to a point $a \in B$.

Subject of studying of our work are the following problem.

Problem. Let $n, m \in \mathbb{N}$, $n \geq 2$, $m \geq 2$, $\alpha \in \mathbb{R}_+$. Maximum functional be found

$$I = \prod_{k=1}^{n} \prod_{p=1}^{m} r^{\alpha}\left(D, a_{2k-1,2p-1}\right) \cdot \prod_{k=1}^{n} \prod_{p=1}^{m-1} r\left(D, a_{2k-1,2p}\right) \times$$

$$\times \prod_{k=1}^{n} \prod_{p=1}^{m-1} r^{\alpha}\left(D, a_{2k,2p}\right) \cdot \prod_{k=1}^{n} \prod_{p=1}^{m} r\left(D, a_{2k,2p-1}\right),$$

where $A_{2n,2m-1}$ – arbitrary $(2n, 2m-1)$-system points on the rays, satisfied condition (1), D – arbitrary open set, the satisfied condition (2), $a_{k,p} \in D \subset \overline{\mathbb{C}}$, and all extremal the describe ($k = \overline{1, 2n}$, $p = \overline{1, 2m-1}$).