ON REMOVABILITY OF SINGULARITIES FOR DISCRETE OPEN MAPPINGS WITH CONTROLLED p-MODULE

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This talk continues our research of the generic properties of mappings with integrally bounded distortions. We consider discrete open mappings $f : D \to \overline{\mathbb{R}^n}$, $n \geq 2$, of domains $D \subset \mathbb{R}^n$, satisfying the inequality controlling the distortion of p-module ($p > 1$) by

$$\mathcal{M}_p(f(\Gamma(S_1, S_2, A))) \leq \int\limits_A Q(x)\, \eta^p(|x - x_0|)\, dm(x), \tag{1}$$

when $A \subset D$ are spherical rings $A = A(r_1, r_2, x_0) = \{x \in D : r_1 < |x - x_0| < r_2\}$, $0 < r_1 < r_2 < r_0 := \text{dist}\,(x_0, \partial D)$, and η is arbitrary measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that

$$\int\limits_{r_1}^{r_2} \eta(r)dr \geq 1,$$

while $Q : D \to [0, \infty]$ in (1) is a given measurable function. The point x_0 is fixed in D.

The purpose of the talk is to discuss the removability of isolated boundary points when the mappings are subject to (1). It was established by the third author, under rather general assumptions, that if a discrete open mapping $f : D \setminus \{x_0\} \to \overline{\mathbb{R}^n}$, $n \geq 2$, omits the values which range over some set of positive p-capacity and obeys (1), then f has a (finite or infinite) limit at x_0 when x approaches x_0. In this case, one has to assume that $\text{cap}_p\left(\overline{\mathbb{R}^n} \setminus f(D \setminus \{x_0\})\right) > 0$.

We are focussed to the case $n - 1 < p < n$, and show, first, that the same removability phenomenon holds also for such p, i.e. an essential singularity also cannot occur, and secondly, that the assumption $\text{cap}_p\left(\overline{\mathbb{R}^n} \setminus f(D \setminus \{x_0\})\right) > 0$ now can be omitted. This fact reveals an essential difference between the cases $n - 1 < p < n$ and $p = n$. In the last case, for example, the holomorphic mapping $f(z) = e^{1/z}$, $z \in \mathbb{C}$, has nonremovable singularity at the origin and satisfies the inequality (1) at every point of the complex plane with $Q(x) \equiv 1$. So we establish a removability of singularities without an expected requirement $\text{cap}_p\left(\overline{\mathbb{R}^n} \setminus f(D \setminus \{x_0\})\right) > 0$.

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