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Extremal problem with variable quantity of points on rays.

Let $n, m, d \in \mathbb{N}$, $m = nd$. Let's consider a set of natural numbers $\{m_k\}_{k=1}^{n}$ such that

$$\sum_{k=1}^{n} m_k = m. \tag{1}$$

System points

$$A_{n,d} := \left\{a_{k,p} \in \mathbb{C} : \, k = \overline{1,n}, \, p = \overline{1,m_k}\right\},$$

the satisfied condition (1), the define generalized (n,d)-equiangular with variable quantity of points on rays, if at all $k = \overline{1,n}$ and $p = \overline{1,m_k}$ realize relation:

$$\begin{array}{l} 0 < |a_{k,1}| < \ldots < |a_{k,m_k}| < \infty; \\ \arg a_{k,1} = \arg a_{k,2} = \ldots = \arg a_{k,m_k} = \frac{2\pi}{n}(k-1). \end{array} \tag{2}$$

Let $r(B,a)$ – inner radius domain $B \subset \overline{\mathbb{C}}$ with respect to a point $a \in B$.

Subject of studying of our work are the following problems.

Problem 1. Let $n, m, d \in \mathbb{N}$, $m = nd$, $n \geq 2$. To find a maximum

$$\left(r\left(B_0, 0\right) \cdot r\left(B_\infty, \infty\right)\right)^{\frac{n^2}{4}} \cdot \prod_{k=1}^{n} \prod_{p=1}^{m_k} r(B_{k,p}, a_{k,p}),$$

$$r^{\frac{n^2}{4}}\left(B_0, 0\right) \cdot \prod_{k=1}^{n} \prod_{p=1}^{m_k} r(B_{k,p}, a_{k,p}),$$

$$r^{\frac{n^2}{4}}\left(B_\infty, \infty\right) \cdot \prod_{k=1}^{n} \prod_{p=1}^{m_k} r(B_{k,p}, a_{k,p}), \qquad \prod_{k=1}^{n} \prod_{p=1}^{m_k} r(B_{k,p}, a_{k,p}),$$

where $A_{n,d} = \{a_{k,p}\}$ – arbitrary generalized (n,d)-equiangular with variable quantity of points (2), and $\{B_{k,p}\}$ – arbitrary set partially non-overlapping domains, or, somewhat, partially non-overlapping domains, $a_{k,p} \in B_{k,p} \subset \overline{\mathbb{C}}$, and to describe all extremals ($k = \overline{1,n}$, $p = \overline{1,m_k}$).

Problem 2. Let $n, m, d \in \mathbb{N}$, $m = nd$, $n \geq 2$. To find a maximum

$$\left(r\left(D, 0\right) \cdot r\left(D, \infty\right)\right)^{\frac{n^2}{4}} \cdot \prod_{k=1}^{n} \prod_{p=1}^{m_k} r(D, a_{k,p}), \qquad r^{\frac{n^2}{4}}\left(D, 0\right) \cdot \prod_{k=1}^{n} \prod_{p=1}^{m_k} r(D, a_{k,p}),$$

$$r^{\frac{n^2}{4}}\left(D, \infty\right) \cdot \prod_{k=1}^{n} \prod_{p=1}^{m_k} r(D, a_{k,p}), \qquad \prod_{k=1}^{n} \prod_{p=1}^{m_k} r(B_{k,p}, a_{k,p}),$$

where $A_{n,d} = \{a_{k,p}\}$ – arbitrary generalized (n,d)-equiangular with variable quantity of points (2), and D – arbitrary open set, $a_{k,p} \in D \subset \overline{\mathbb{C}}$, and to describe all extremals ($k = \overline{1,n}$, $p = \overline{1,m_k}$).