THE ENTROPY OF DELTA - CODED SPEECH

by

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Abstract

This thesis presents a study of the information properties of delta-coded speech and a derivation and evaluation of applications of the results to channel encoding and signal detection problems.

First, a signal analysis of the coding technique is given, and performance characteristics are presented for a message approximant of realistic power spectral density.

A speech processing facility based on SC247 analogue and PDP-8 digital computers is then described, and details are given of the software for transition probability matrix assembly and the hardware for message time-scaling and DM† interfacing. The results of computations of the entropy of Markov process approximations to the source of order 1 - 9 are detailed, and the redundancy is found to be typically about one half.

Linear prediction of the source sequence by Wiener estimation is considered, and a predictor success probability of 0.72 is found to be typical, while optimal group codes for block lengths 2 - 6 are determined and evaluated for comparison with predictive coding. The optimal non-linear predictor structures for orders 1 - 7 are established, and the entropies of the sequences generated by modulo-2 addition of the predictions and source elements are computed and found to be

† Delta Modulation

much closer to the process entropies than the corresponding performance characteristics for group encoding.

The form of a practical near-optimal 6th order predictive coder is treated in detail, and a TTL* prototype which attains a typical success probability of 0.9 is described.

Encoding of the predictor error sequence to achieve bandwidth reduction is then considered, and 5 element group encoding, attaining a compression factor of 0.48, is found to be superior to run-length encodings. The combination IP† transformation achieves this compression with very considerably less hardware complexity than is required for equivalent performance by direct exact coding of blocks of source elements.

To determine the channel buffer requirements for transmission at uniform data rate, the statistical properties are analysed by an application of queueing theory, and a capacity of 270 words is shown to be sufficient for a 2 to 1 bandwidth compression.

Finally, it is shown that the redundancy of the message source can alternatively be exploited at a DM receiver, and a typical 40% error probability reduction is found to result from an easily implemented application of non-linear prediction to optimisation of the signal detection process.

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* Transistor - Transistor Logic

† Information - Preserving

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Principal symbols in order of use.

$m(t)$ - Message waveform

$\Phi(\omega)$ - Power spectral density

Δ - DM increment

f_s - Sampling frequency

R - Information rate

$\varphi(\tau)$ - Auto- or cross-correlation function

q - Quantiser interval

$E[\ ]$ - Expected value of variant

$n(t)$ - Noise waveform

S_i - Source state vector

N - Markov process order

H - Process entropy

B_i - Element block

G_N - Group entropy

F_N - Conditional entropy

T - Transformed element

S - Source element

E - Predictor element

P_s - Predictor success probability

H_T - Mod - 2 adder sequence entropy

η - Transmitter power saving

$E(s)$ - Estimator transfer function

$e(t)$ - Estimator impulse response

$m_i(t)$ - DM echelon signal

$w(\tau)$ - Optimal weighting function

λ - Signal generation time

$\epsilon(t)$ - Error waveform

σ_e - Rms estimator error

$m_e(t)$ - Estimate of message

$f(\tau)$ - Integral weighting function

N_i - Codeword length

R_B - Group encoder compression factor

R_M - Upper bound for group encoding

P_b - Element error probability

TH_T - Error sequence mod - 2 adder output entropy

r - Run - length

P_{gr} - Geometric distribution run - length probability

H_g - Geometric distribution entropy

L_g - Geometric distribution sequence length

H_{gr} - Geometric distribution relative entropy

P_r - Run probability

H_r - Relative entropy for run - length encoding

TG_N - Error sequence group entropy

TR_M - Upper bound for error sequence group encoding

TH - Error sequence entropy

TR_B - Error sequence group encoding compression factor

N_c - 5 element error sequence group encoding word length

N_m - Number of sources multiplexed

N_q - Buffer capacity

f_c - Channel data rate per source

N_a - Average codeword length

N_b - Average queue length

P_{ov} - Buffer overflow probability

Θ_i - Signal transmitted for symbol i

$\Lambda(x)$ - Likelihood ratio

K - Decision threshold

ρ - Signal cross-correlation coefficient

x_c - Correlator output

b - Detector bias level

δ - Rms noise level

M - Mean signal power

x_d - Decision boundary

α, β - Component error probabilites

P_{Tf} - Fixed bias error probability

P_{Tv} - Variable bias error probability

x_n - Envelope detector output

x_{dn} - Decision boundary for noncoherent signalling

T - Transition probability matrix

Chapter 1

Introduction

With the continuing expansion of world demand for interpersonal telecommunications facilities at a rate more than twice that of world population - and that is a growth described by demographers as an explosion - research activity is profitably directed to the development of improved ways of encoding messages, new methods of signalling the codes, new media for transmission of the signals, and new schemes for the detection and processing of the received information. In the field of circuit technology, the progress of the microelectronic era is altering profoundly the results of cost-effectiveness evaluations of complex digital signal processing systems, and motivates the further application of information theoretic principles to the study of specific message generating and transmitting situations in order to exploit these advances to augment the effective communications capability of existing links and ensure efficient utilisation of those envisaged.

Today, most speech communication traffic is transmitted by channels carrying signals which are frequency-division-multiplexed analogue transforms of the source messages, generated from them by the application of filtering for bandwidth restriction and frequency translation to spectrally adjacent

locations in the passband of the channel. But for several reasons it may be expected that in future an increasing proportion of this traffic will be transmitted by digital signals. First, a very rapid growth in the volume of machine-originated data communication is envisaged as a result of developments in computer science which allow multiple-access to large processors by many dispersed users, and it is economically unattractive to provide separate channel facilities for digital and analogue signals. Second, the switching process by which a channel is routed involves discrete signal processing, and hence can be conveniently integrated with the message signals if both are digital in form. Also, the conversion of many existing communications circuits to digital signal operation results in valuable increases in channel capacity, because proper decoding of these signals is less disturbed by the high attenuation and distortion which previously restricted the usable circuit bandwidth. Further, when applied to long multi-stage speech circuits, digital techniques result in improved and consistent transmission quality because with the use of regeneration at intermediate repeaters the effects of noise are not cumulative. Finally, in military applications, it is known that the encyphering of speech messages to preserve secrecy is most readily and effectively accomplished when the messages are in digital coded form.

Delta modulation (DM) is an analogue-digital conversion

scheme, with properties particularly appropriate to speech message encoding, first described by Libois[1] and since studied extensively in theory and practice in basic and variant forms*. The principle of DM is illustrated in Fig. 1.1.

In the encoder, the analogue message waveform m(t) is compared continuously with the output of an integrator connected to the channel line. The error signal amplitude is hard limited by a non-linear network, and the output of this clipper is sampled periodically to generate a train of positive and negative impulses for transmission. At the decoder, the impulse train is integrated as by the local integrator in the encoder, and the resultant echelon approximation to m(t) is then filtered to remove noise outside the spectrum of the message. Since in practice the channel signals are not impulses but have finite amplitude and duration, the encoder incorporates delaying gates or a master-slave flip flop at the output of the comparator in order that changes in error signal polarity during their transmission are inhibited.

As a delta-coder is logically simple, and does not require a precision divider network, nor a reference voltage supply, it can be readily realized by present techniques as a single-chip low-cost integrated microcircuit and is therefore an attractive solution to the source encoder problem in a digital speech communications network. Because the scheme is

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*For a comprehensive bibliography, see Reference (2).

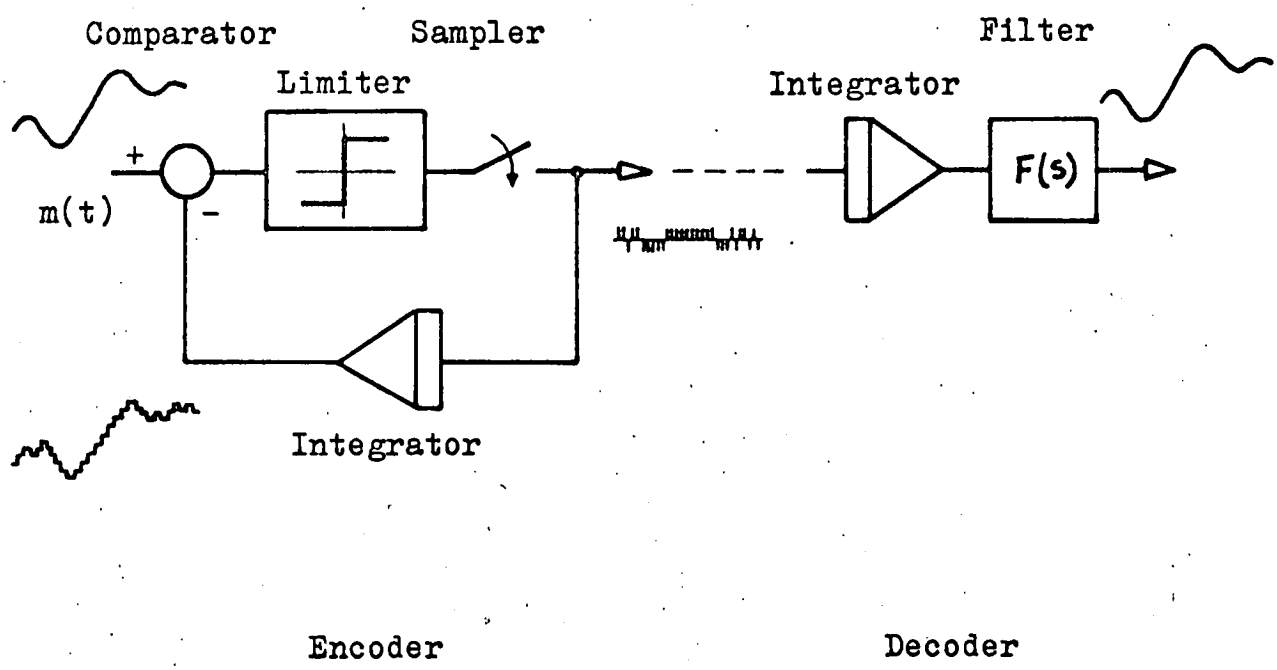

Fig. 1.1 Delta modulation

essentially a single-bit form of differential pulse code modulation (PCM), no word synchronisation is required between senders and receivers connected to the network, a feature which results in a valuable hardware simplification at multiplexing nodes and terminals.

It is known[3],[4] that for applications in which the required quality of speech transmission is not high, DM is a relatively efficient encoding scheme in the sense that the channel bit rate required is lower than that necessary for the same transmitted speech quality by other systems such as conventional PCM*. Where the speech quality required is high, however, the channel bit rate in the case of DM is greater, so that in this case if channel capacity is at a premium a requirement for more complex coding is indicated. Now the anticipated properties of the promising new media for overland channels provide inexpensive broad but dispersive bandwidths appropriate to the transmission of time-division-multiplexed digital signals. The channel bandwidth of an overmoded circular waveguide under development, for example, is 35 GHz, while the inherent capacity of a single laser beam exceeds that required for the total of present day telephone, radio and television world communications. Interest in DM for source encoding therefore parallels progress in the development of

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*Conventional PCM refers to a system of ADC whereby the message is bandlimited, then sampled at the Nyquist rate and the sample amplitudes are quantised and encoded as groups of binary digits.

these media.

But in typical communications situations it is necessary to integrate with the network circuits of a kind such that either the medium itself is expensive, as with submarine telephony, or the terminal facilities, as in troposcatter links, and intermediate hardware, as in satellite relays, are much more costly than average. Since the traffic volume carried by these links is only a small proportion of the total originated, it is economically inappropriate to equip every message source with a complex encoder whose greater efficiency is justified only during infrequent communication events involving high-cost channel capacity.

This problem can be solved in the manner shown in Fig. 1.2 if it is possible to provide, as part of the terminal equipment which processes signals for transmission over expensive major links, channel encoders which perform an information-preserving (IP) transformation* on incoming digital signals from source encoders. The output transform sequences, from which the original signals can be reconstructed by an inverse operation in the channel decoders, are generated at a lower bit rate and hence utilize the available channel capacity more efficiently. In conventional analogue telephony practice, the concept of employing special coding to achieve more efficient circuit

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*IP transformations are such that the entropies of the operand sequence and its transform are the same. This is sometimes called 'exact coding'.

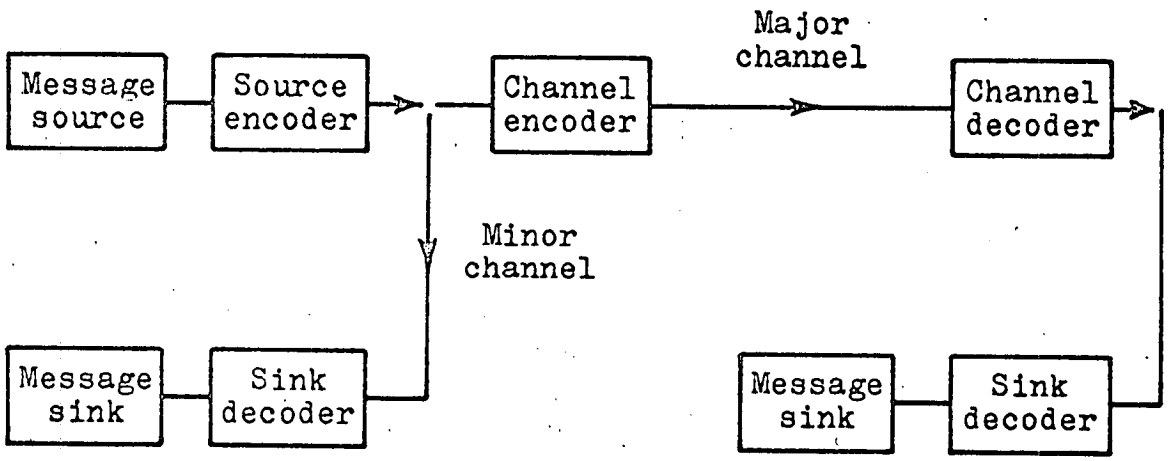

Fig. 1.2 Channel encoding

utilisation is well known and is applied extensively at the terminals of major channels. Such schemes are primarily of the TASI*(5) generic type which exploit individual source inactivity, because the application of transformations to active channels in the analogue case suffers the limitation that since in general the operations are not IP they incur an irrevocable degradation in fidelity, and this must be held to a level which is perceptually tolerable. The magnitude of the bandwidth compression attainable by IP transformation in a digital speech communication system is determined by the extent to which the source encoder sequences exhibit redundancy as a result of the nature of the encoding scheme and constraints on the properties of the message generating process.

This thesis presents a new study of the information properties of delta-coded speech with a derivation and evaluation of signal processing procedures which may be exploited in channel encoders to effect redundancy removal for the reduction of the channel capacity required for transmission. Coding systems for reducing the entropy of the distribution of the average signal element in the sequence generated by DM are described, and compared with computations of the relative entropy of the message generating process for orders up to 9. Linear prediction using Wiener estimation, and non-linear

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*TASI is an acronym. Time assignment speech interpolation.

prediction by optimal and practically convenient sub-optimal functions, is presented. Optimal group encoder structures are determined, and their performance compared with that of the predictive coding procedures. The compression of the transformed signal sequences generated by modulo-2 addition of the predictor outputs and source messages by run-length codings and group encoding is studied, and the effect on system performance of errors due to channel noise is determined. The buffer capacity required to allow transmission of the variable length code at a uniform data rate is then examined and operating characteristics are derived. Finally, the related problem of exploiting the redundancy of the source sequences at a DM receiver is treated, and a further application of non-linear prediction to optimise the signal detection process is presented.

Chapter 2

Signal analysis

The study of a continuous message generating process, and of connected information coding and transmission systems, involves a marriage of two major areas of statistical communication theory. There is first the field explored by Wiener[6] in his work on the spectral analysis of random processes and the theory of optimal filtering of signals and noise. In this approach, which is applied here to the problems of message analysis, linear estimation and signal detection, integral transform techniques are used extensively and primary manipulations are developed in the frequency domain. Complementing Wiener's correlation methods is the body of knowledge originated by Shannon[7] which deals quantitatively with the concept of information production and transmission rather than the analysis of the signals by which it is accomplished, and it is on this work that the major contributions of this thesis, the analyses of the relative entropies of delta-coded speech and redundancy-removing encoding systems, are based.

2.1 Message power spectral density

A non-deterministic approach to the signal analysis of DM speech commences with the selection of a rational function approximation $\Phi_{mm}(\omega)$ to the power spectral density of the exciting

speech message. For conversational speech, Dunn and White[8] have reported power spectral density measurements which indicate a maximum at 300 - 450 Hz and a fall of about 6 dB/octave over the following decade, while bandpass filtering is employed in communication practice to achieve concentration of the power in the range 300 - 3000 Hz as this results in improved intelligibility at the expense of an acceptable loss of naturalness. Since DM slope overload occurs when $|m'(t)| > \Delta.f_s$, the inverse square power spectrum contributes to the relative efficacy of the scheme for encoding speech, the transmitted power† for a given overload probability being greatest for a message with this spectral characteristic.

Tschebycheff polynomials may be employed to derive for the system function

$$H(s) = \frac{k s^6}{\prod_{i=1}^{7} (s^2 + 2\delta_i s + r_i^2)} \qquad (2.1.1)$$

a pole distribution which results in a denominator real coefficient set (Appendix 1) for

$$\Phi_{mm}(\omega) = \frac{k^2 \omega^{12}}{\sum_{i=0}^{14} A_{2i} \omega^{2i}} = |H(j\omega)|^2 \qquad (2.1.2)$$

such that the function is a 1 dB equi-ripple approximant in the message band and has rapid cut-off at upper and lower band limits, while k^2 is computed such that the message power $\overline{m^2(t)} = \int_{-\infty}^{\infty} \Phi_{mm}(\omega) d\omega = 1$. The power spectral density is shown in Fig. 2.1.

† i.e. the reproduced message power at a decoder.

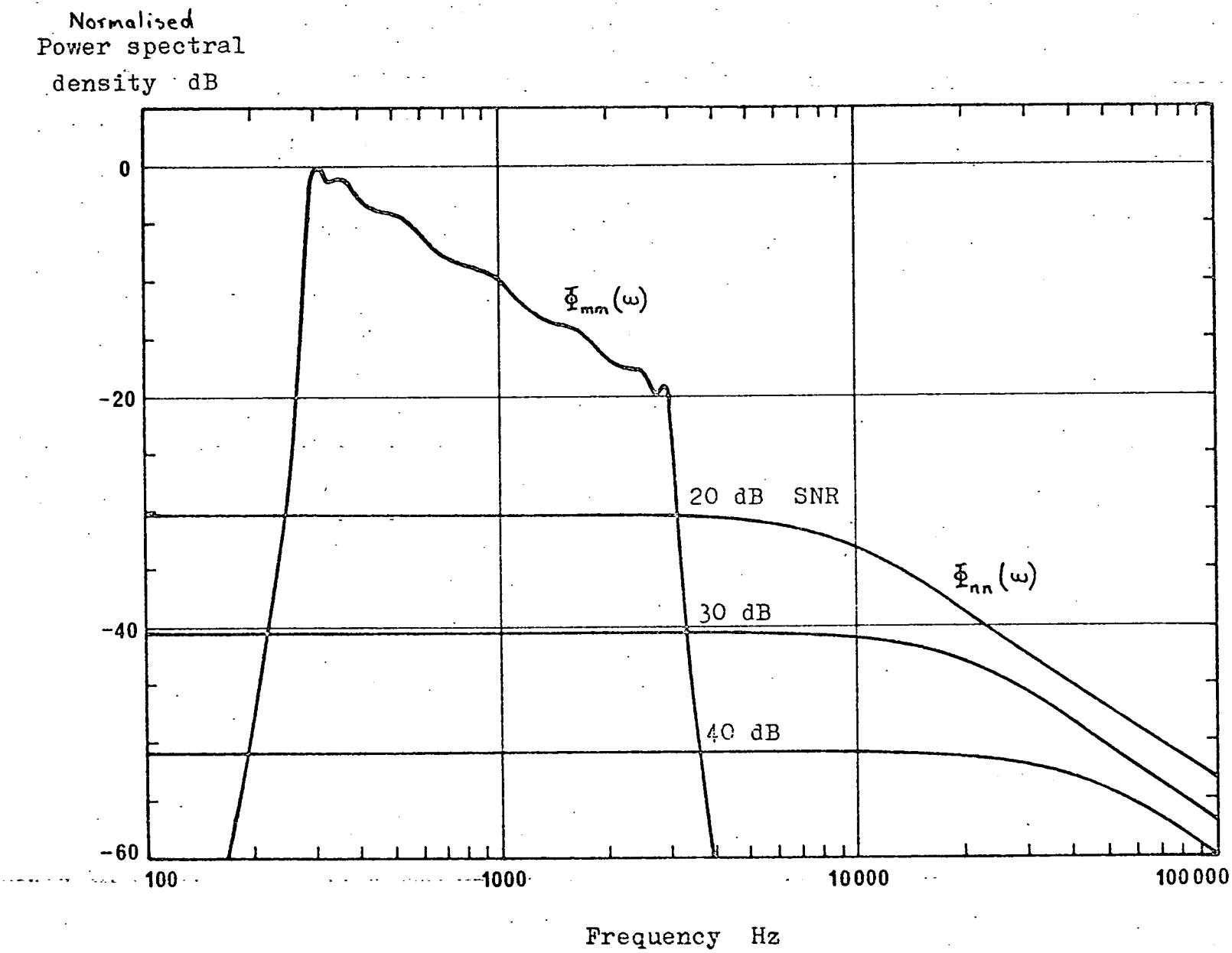

Fig. 2.1 Message and quantisation noise spectra

2.2 Information rate

Shannon[9] has demonstrated that a message with Gaussian amplitude probability distribution has higher entropy than those generated by other processes of the same power. Hence by considering excitation by a stationary Gaussian message an upper bound for the information transmission rate capability of DM for the derived $\Phi_{mm}(\omega)$ may be found. For if the power spectral density of the error due to coding is $\Phi_{nn}(\omega)$, the information rate[10]

$$R = \frac{1}{2\pi}\int_0^\infty \log \frac{\Phi_{mm}(\omega)}{\Phi_{ss}(\omega)}\, d\omega \qquad (2.2.1)$$

where $\Phi_{ss}(\omega) = \begin{cases} \Phi_{nn}(\omega) & \text{for } \Phi_{nn}(\omega) \leq \Phi_{mm}(\omega) \\ \Phi_{mm}(\omega) & \Phi_{nn}(\omega) > \Phi_{mm}(\omega) \end{cases}$

Because the contribution to the information rate of spectral components of the message power outside the region $\Phi_{nn}(\omega) \leq \Phi_{mm}(\omega)$ is precisely zero, the boundaries ω_1, ω_2 of this region will be termed the information band limits, and the signal-to-noise ratio is defined as the ratio of total message power to total noise power in the information band

$$SNR = \frac{\int_{\omega_1}^{\omega_2} \Phi_{mm}(\omega)\, d\omega}{\int_{\omega_1}^{\omega_2} \Phi_{nn}(\omega)\, d\omega} \qquad (2.2.2)$$

It will later be shown that the power spectrum of the quantisation noise introduced by delta-coding the message is flat within 0.3 dB over the information band for an SNR of

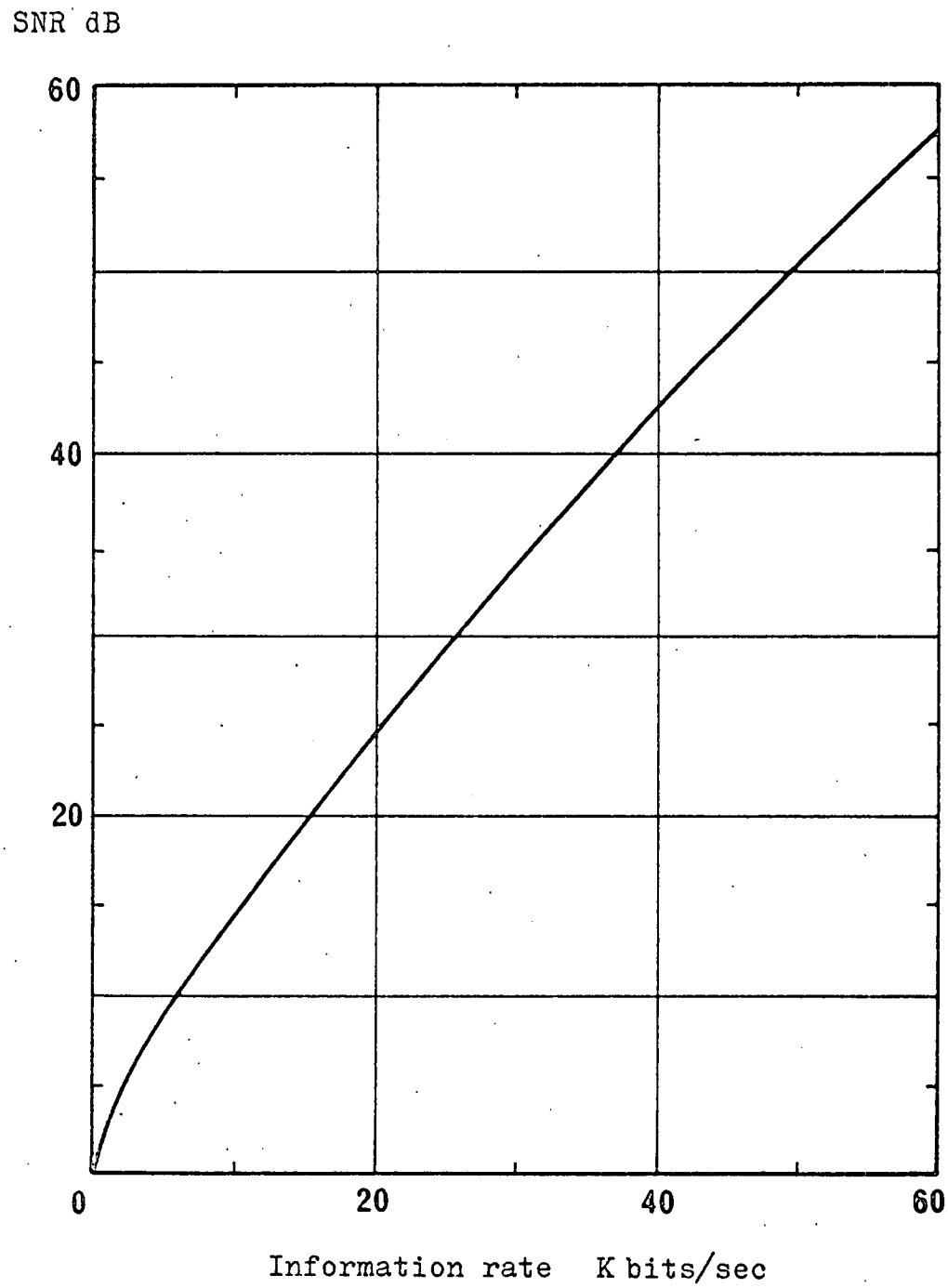

Fig. 2.2 Information rate

20 dB, and more nearly uniform for higher SNR, so that a white noise approximation may be taken. At an SNR of 20 dB, less than 0.05 % of the message power lies outside the information band. It then becomes convenient to iteratively adjust ω_1 and ω_2 to determine Φ_{nn} for a range of SNR and evaluate the corresponding information rates, shown in Fig. 2.2. This establishes the theoretical minimum channel bit rate required for the transmission of the message with a given SNR if the coding efficiency were 100 %, and allows the appraisal of possible improvements in known practical systems for which the actual channel bit rates necessary are significantly greater.

2.3 Message autocorrelation

Computation by inverse Fourier transformation of the message autocorrelation function

$$\varphi_{mm}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{mm}(\omega)\, e^{j\omega\tau}\, d\omega$$

$$= \frac{1}{\pi}\int_{0}^{\infty} \frac{k^2\omega^{12}}{\sum_{i=0}^{14} A_{2i}\,\omega^{2i}} \cos\omega\tau \; d\omega \qquad (2.3.1)$$

is fundamental to evaluation of the characteristics of DM quantisation noise and indicates also the temporal extent of the constraint on the message source output. Fig. 2.3 shows that, for a typical f_s = 96 KHz, there is strong correlation ($\varphi_{mm}(\tau) > 0.9$) over 7 consecutive samples and significant correlation over intervals of several milliseconds (hundreds of

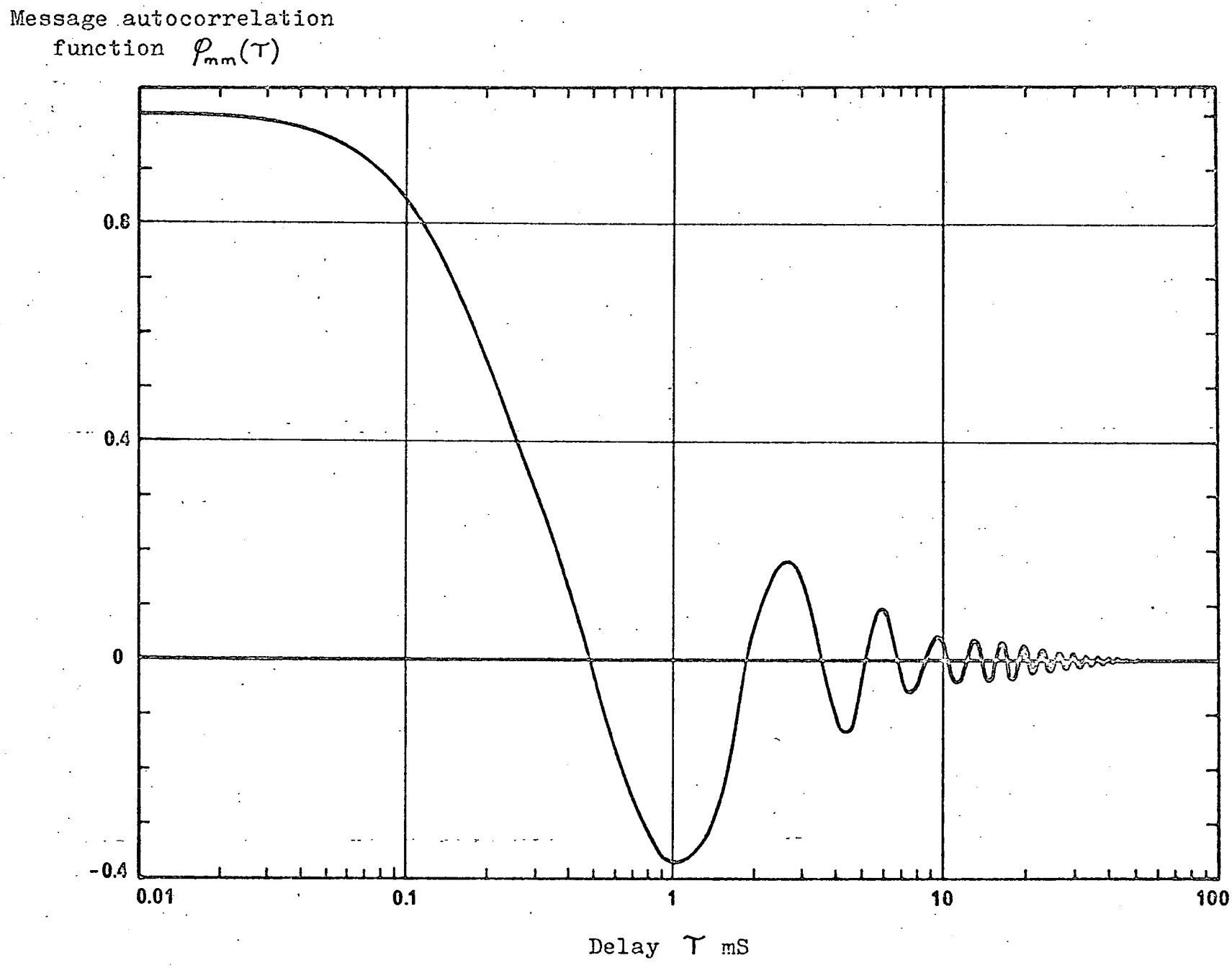

Fig. 2.3 Message autocorrelation

samples). The indicated statistical dependence of the message amplitude at any sampling instant on the amplitudes of many previous samples is evidence of redundancy in the message sequence, but it should be noted that Δ increment and sampling frequency for DM are such that there is sufficient correlation between successive sample amplitudes for their difference to be in general less than Δ and they are not encoded independently.

2.4 Quantisation noise

Typically, an equi-interval quantiser with step size q has a transfer characteristic such that it delivers an output kq for inputs x in the range $q(k-\frac{1}{2}) \leqslant x < q(k+\frac{1}{2})$; i.e. the input is approximated by the nearest integer multiple of the quantising interval. The echelon signal produced by delta-coding, however, has the special property that an increment $\pm\Delta$ occurs at every sampling instant, even when the new level is a poorer approximant to the message than the old. The process may therefore be represented as that of sampling and quantisation by the characteristic of Fig. 2.4, a unity gain quantiser with interval $q = 2\Delta$ and clock-synchronised origin shift function

$$a(t) = \frac{\Delta}{2}(\cos \pi f_s t + 1) \qquad (2.4.1)$$

shown in Fig. 2.5.

Applying Watts'[11] analysis of a generalised quantiser, the joint moment between the quantisation noises n_x, n_y of two correlated unit power Gaussian signals x, y separately

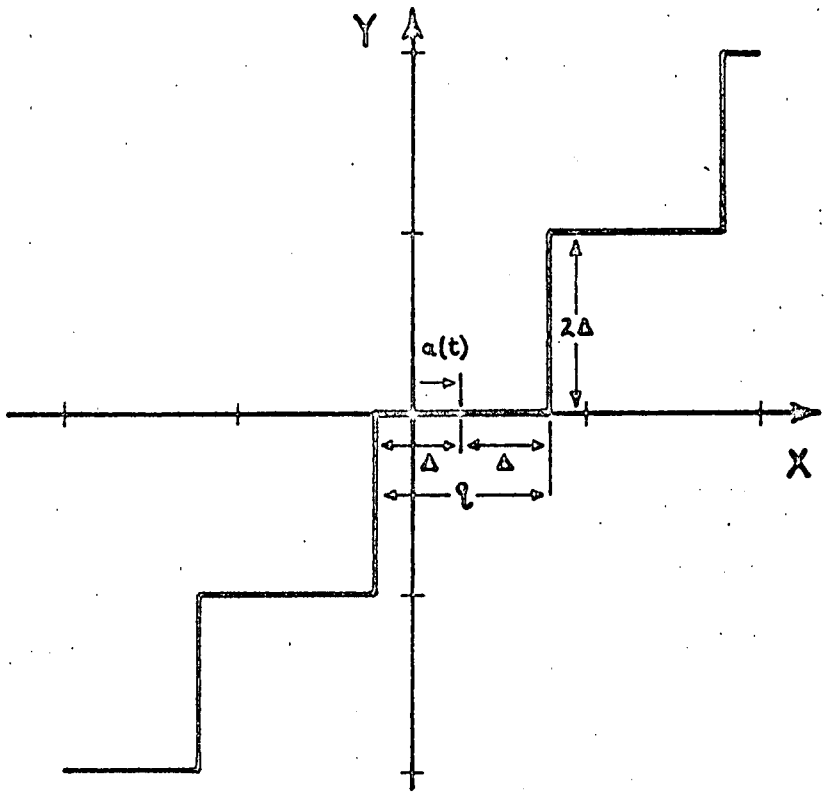

Fig. 2.4 Quantiser characteristic

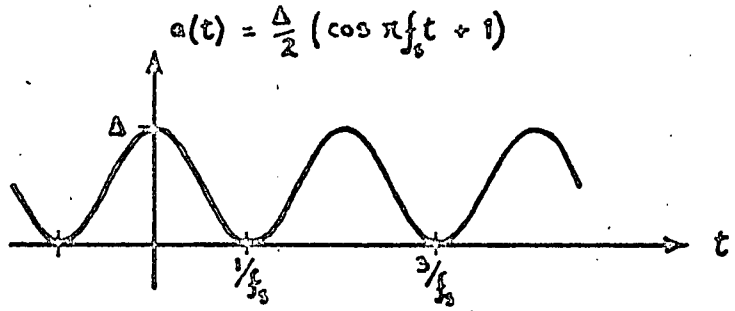

Fig. 2.5 Characteristic shift function

processed by the quantiser of Fig. 2.4 is found to be

$$E[n_x n_y] = \frac{1}{(-j2\pi)^2} \sum_{k\neq 0} \sum_{m\neq 0} e^{-j2\pi(k+m)a(t)} \cdot \frac{(-1)^{k+m}}{km} \cdot e^{-2\pi^2\left[\left(\frac{k}{q}\right)^2 + \frac{2km}{q^2} E[xy] + \left(\frac{m}{q}\right)^2\right]} \tag{2.4.2}$$

For $x = m(t)$, $y = m(t-\tau)$ (Fig. 2.6), the joint moment between x and y becomes the message autocorrelation function

$$E[xy] = \overline{m(t).m(t-\tau)} = \varphi_{mm}(\tau) \qquad \text{(m(t) ergodic)} \tag{2.4.3}$$

and the quantisation noise autocorrelation function referred to the quantiser output

$$\varphi_{nn}(\tau) = -\frac{\Delta^2}{\pi^2} \sum_{k\neq 0} \sum_{m\neq 0} e^{-j2\pi(k+m)a(t)} \cdot \frac{(-1)^{k+m}}{km} \cdot e^{-\frac{\pi^2}{2\Delta^2}\left[k^2 + 2km\varphi_{mm}(\tau) + m^2\right]} \tag{2.4.4}$$

Since $\varphi_{mm}(\tau) \leq 1$, the exponent of the last factor results in significant terms for $k = -m$ only, so that

$$\varphi_{nn}(\tau) = \frac{2\Delta^2}{\pi^2} \sum_{k=1}^{\infty} \frac{1}{k^2} \cdot e^{-\left(\frac{\pi k}{\Delta}\right)^2 (1-\varphi_{mm}(\tau))} \tag{2.4.5}$$

and the same noise autocorrelation function is found for a single fixed quantiser with interval $q = 2\Delta$.

The total quantisation noise power by this derivation

$$\overline{n(t)^2} = \varphi_{nn}(0) = \frac{q^2}{2\pi^2} \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{q^2}{12} \qquad \text{(Jolley}^{(12)}\text{)} \tag{2.4.6}$$

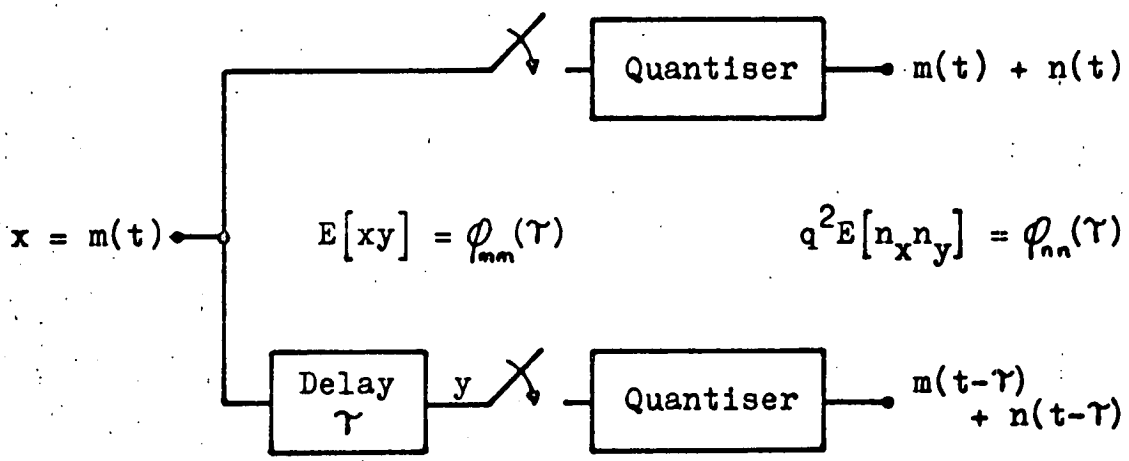

Fig. 2.6 Quantisation noise autocorrelation

which verifies a well known result.[13]

From (2.3.1) and (2.4.5) the quantisation noise power spectral density

$$\Phi_{nn}(\omega) = \int_{-\infty}^{\infty} \varphi_{nn}(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= \frac{4\Delta^2}{\pi^2} \int_0^{\infty} \sum_{k=1}^{\infty} \frac{1}{k^2}\, e^{-\left(\frac{\pi k}{\Delta}\right)^2 \left(1-\varphi_{mm}(\tau)\right)} \cdot \cos\omega\tau\, d\tau \qquad (2.4.7)$$

may be computed* for any increment Δ.

From the numerical results the relation

$$\log_{10} \Phi_{nn} = -5.288 + 3 \log_{10} \Delta \qquad (2.4.8)$$

has been derived and determines the power spectral density in the information band within 1% for the usual range of SNR.

The DM increments for Φ_{nn} values corresponding to the range of SNR considered in section 2.2 are therefore readily evaluated and noise power spectra for SNR = 20, 30, 40 dB are shown in Fig. 2.1. The spectra are flat in the information band and have an asymptotic rate of fall of 6 dB/octave; but in the proximity of the break frequencies, which increase as the quantisation is made finer, they are sharper than for a first order characteristic. Because of the higher sampling frequencies employed in DM, aliasing of noise power into the information band is negligible for the spectra shown, in contrast to the

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*Approximately 5 minutes central processor time per spectrum on an English Electric KDF9 computer.

large proportion of the total quantisation noise power which may be spectrally transposed downwards into the band as the lower sidebands of the fundamental and harmonic components of the sampling function in systems employing the Nyquist rate.

2.5 Message derivative

DM sampling frequencies are selected such that, for the Δ increment which satisfies the required SNR criterion, amplitude excursions of the message derivative exceeding $\Delta.f_s$ are infrequent. For the power spectral density (2.1.2), the variance of the message derivative

$$\overline{(m'(t))^2} = 2\int_0^\infty \omega^2 \Phi_{mm}(\omega)\, d\omega \qquad (2.5.1)$$

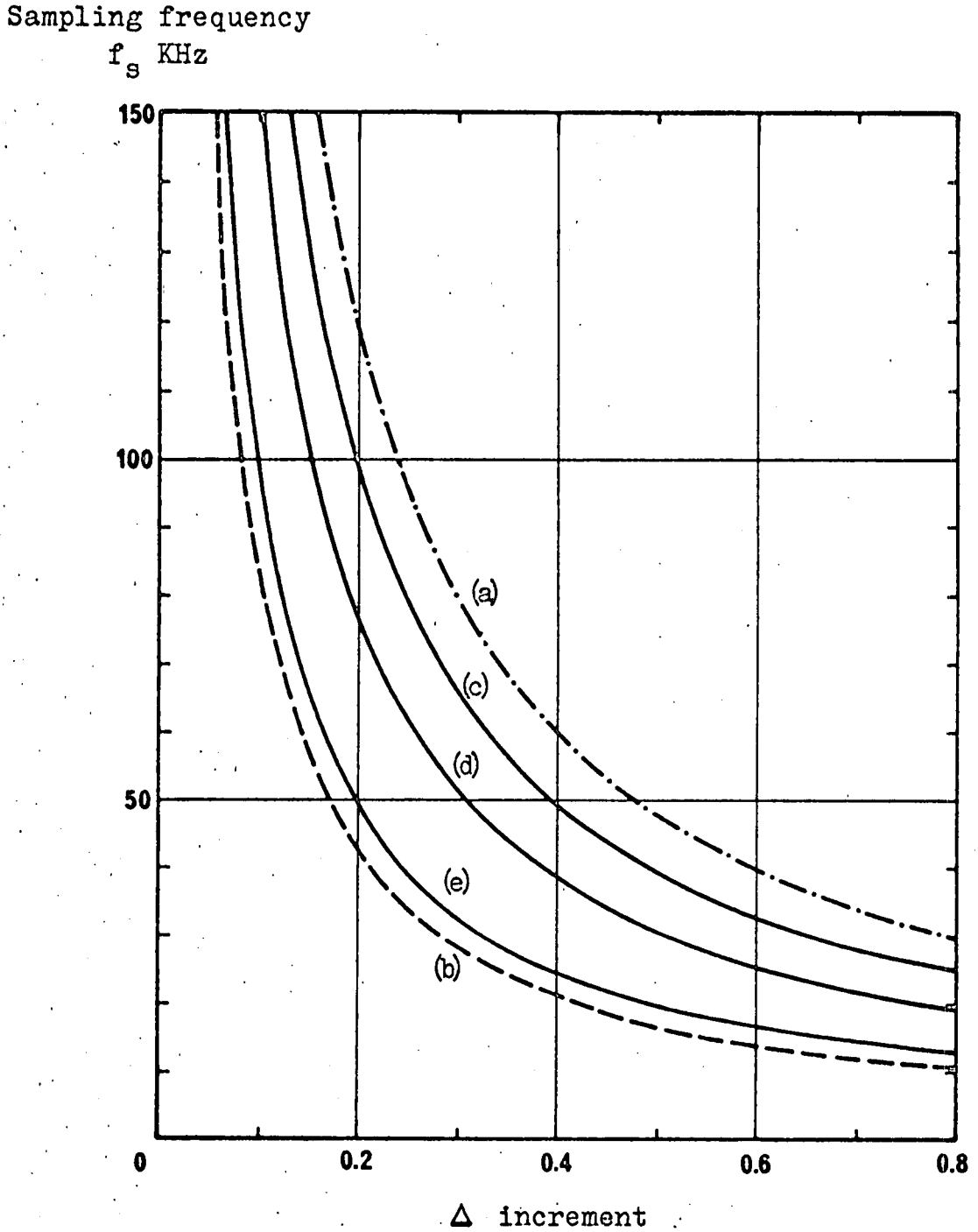
is computed as 3.553×10^7 S^{-2}

De Jager[14] and others have found it convenient to test DM systems with a sine wave of reference frequency such that overloading occurs simultaneously in corresponding encoders with amplitude and slope limitation, significant overloading being avoided if the message does not exceed the maximum amplitude of this test tone. This reference frequency ω_r is here given by

$$\omega_r^2 = \overline{(m'(t))^2} \qquad (2.5.2)$$

and is found to be 949 Hz, which compares with de Jager's observation of 800 Hz.

For a Gaussian message, $m'(t)$ is also Gaussian and



(a) d_d = $\frac{1}{4}$ rms overload

(b) De Jager test tone 949 Hz

(c) Overload probability 0.1%

(d) " " 1%

(e) " " 10%

Fig. 2.7 Sampling frequency

writing $\sigma_d = (\overline{m'(t)^2})^{\frac{1}{2}}$ one finds for this distribution

$$p\left(|m'(t)| > \Delta . f_s \right) = 1 - \sqrt{\frac{2}{\pi}} \int_0^{\frac{\Delta . f_s}{\sigma_d}} e^{-\frac{t^2}{2}} \, dt \qquad (2.5.3)$$

from which the sampling frequencies required for a range of Δ are shown in Fig. 2.7.

Parameter values assumed in earlier studies[13], [15] result in an rms overload level of $4\sigma_d$, which corresponds to rather light loading of the encoder and an overload probability of less than 0.007 %.

Chapter 3

Delta - coded speech processor

Some of the properties of delta modulation are revealed by the performance of the encoder with a deterministic input[16] (for which an excitation - dependent model is applicable), while the random signal analysis presented in chapter 2 determines the characteristics for an information - generating source of prescribed statistical properties. However, a detailed study of delta - coded speech requires the analysis, by a data processor with substantial rapid - access memory capacity, of the signal sequences generated when the exciting waveform is an actual speech message. Effective changes in processor structure, and also subsequent processing of results, are accomplished readily if the machine is a general purpose digital computer operating on the data by executing a stored instruction set. Such a facility has been established, based on the hybrid operation of Solartron SC247 analogue and DEC PDP-8 digital computers with an interface for delta - coded speech, and is described briefly in this chapter.

3.1 FM system

Frequency division of speech messages by a factor of 64 to the band 4.7 - 47 Hz scales the source spectrum appropriately for the analogue computer and with a PDP-8 core store cycle time of

1.5 μS allows the execution of 150 to 220 memory reference instructions during a minimum period of 667 μS for a real time sampling frequency of 96 KHz. Source message recordings prepared directly by Edinburgh University Phonetics Department are transferred with a frequency scale of $\frac{1}{2}$ to an Elliott Tandberg data recorder using a carrier frequency of 12 KHz. A transcription of this record with a scale of $\frac{1}{4}$ is replayed on an Akai M8 crossfield head machine with a final scale of $\frac{1}{8}$ and a tape speed of $1\frac{7}{8}$ i.p.s., so that the output carrier frequency is 375 Hz.

The FM signal is processed by the demodulator of Fig. 3.1, in which the message is extracted by the smoothing (by an FET 7th order Butterworth active LP filter with a rapid cut-off above 60 Hz) of an internally generated train of equi-energy pulses triggered by the input signal zero-crossings. Calibration tones at $\pm 50\%$ frequency deviation, corresponding to peak message excursion for the design deviation ratio of 4, are used with the SC247 digital voltmeter to set the DC amplifier gain for a range of effective values of Δ increment, the step size of the delta-coder being in fact fixed.

3.2 Analogue filter

Bandpass filtering of the message to the power spectrum of Fig. 2.1 is achieved by M-method synthesis of the transfer function H(s) (Sec. 2.1) with the time-scaled frequency variable and one additional zero at the origin, giving

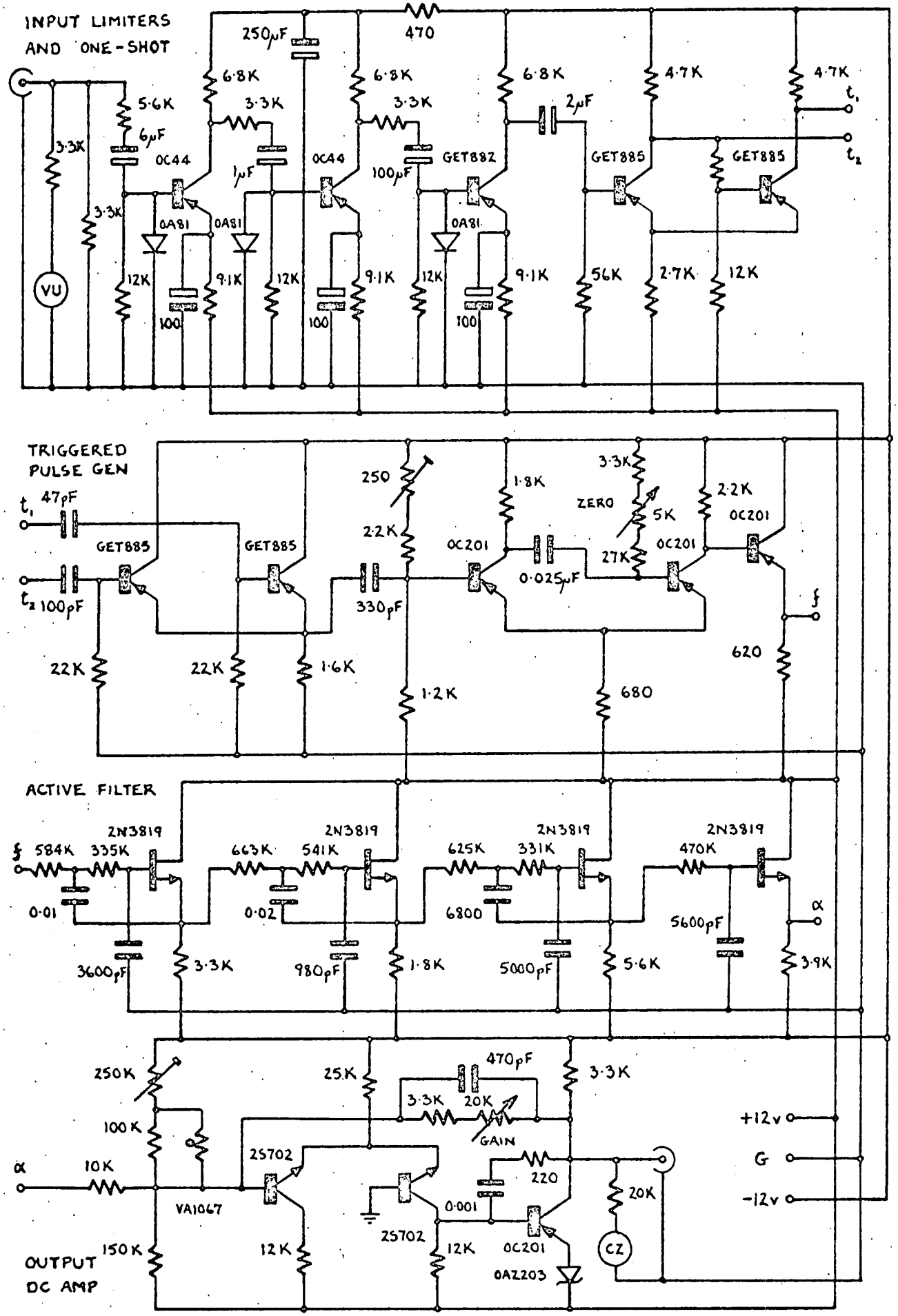


Fig. 3.1 FM speech demodulator

$$G(s) = \frac{Y}{X} = \frac{Ks^7}{\sum_{i=0}^{14} Q_i s^i} \qquad (3.2.1)$$

where the Q_i are computed from the product of quadratic factors in (2.1.1).

Letting

$$\frac{Y}{Ks^7} = \frac{X}{\sum_{i=0}^{14} Q_i s^i} = M \qquad (3.2.2)$$

and scaling for equal coefficients, appropriate computer variables are $(Q_i s^i M)$ so that the solution for the highest derivative is

$$Q_{14} s^{14} M = X - \sum_{i=0}^{13} Q_i s^i M, \qquad (3.2.3)$$

which is synthesised by the configuration detailed in Appendix 2.

3.3 Delta - coder

The precision DM unit shown in Fig. 3.2 replaces a dual inverter module in the SC247. Diode - isolated positive and negative increment pulses transmitted from the computer interface are integrated by one unit of the associated twin operational amplifier and the error between the message and its sampled and quantised approximant is extracted and limited by the second amplifier.

The translated error signal operates an IC differential

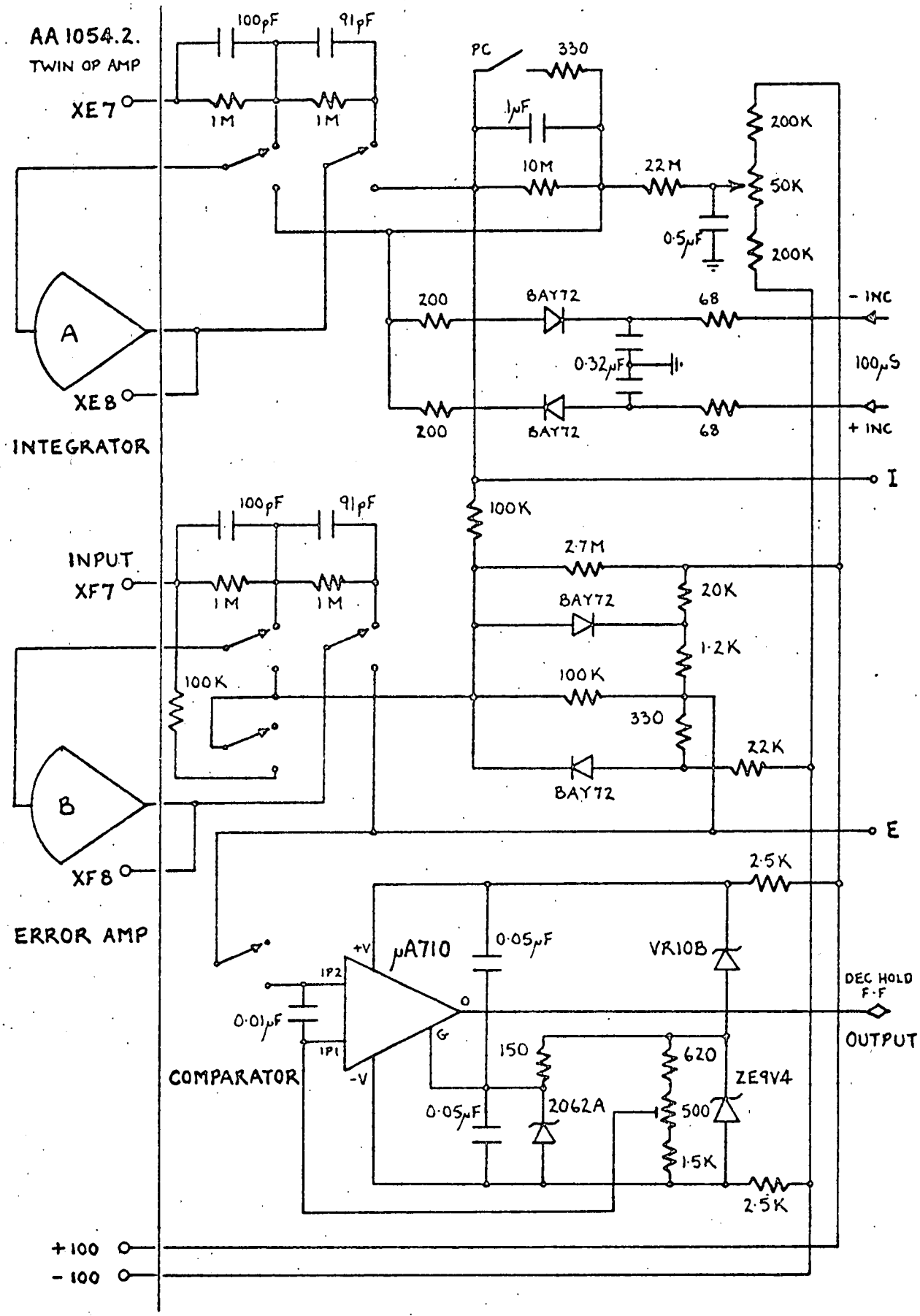


Fig. 3.2 Precision deltamodulator

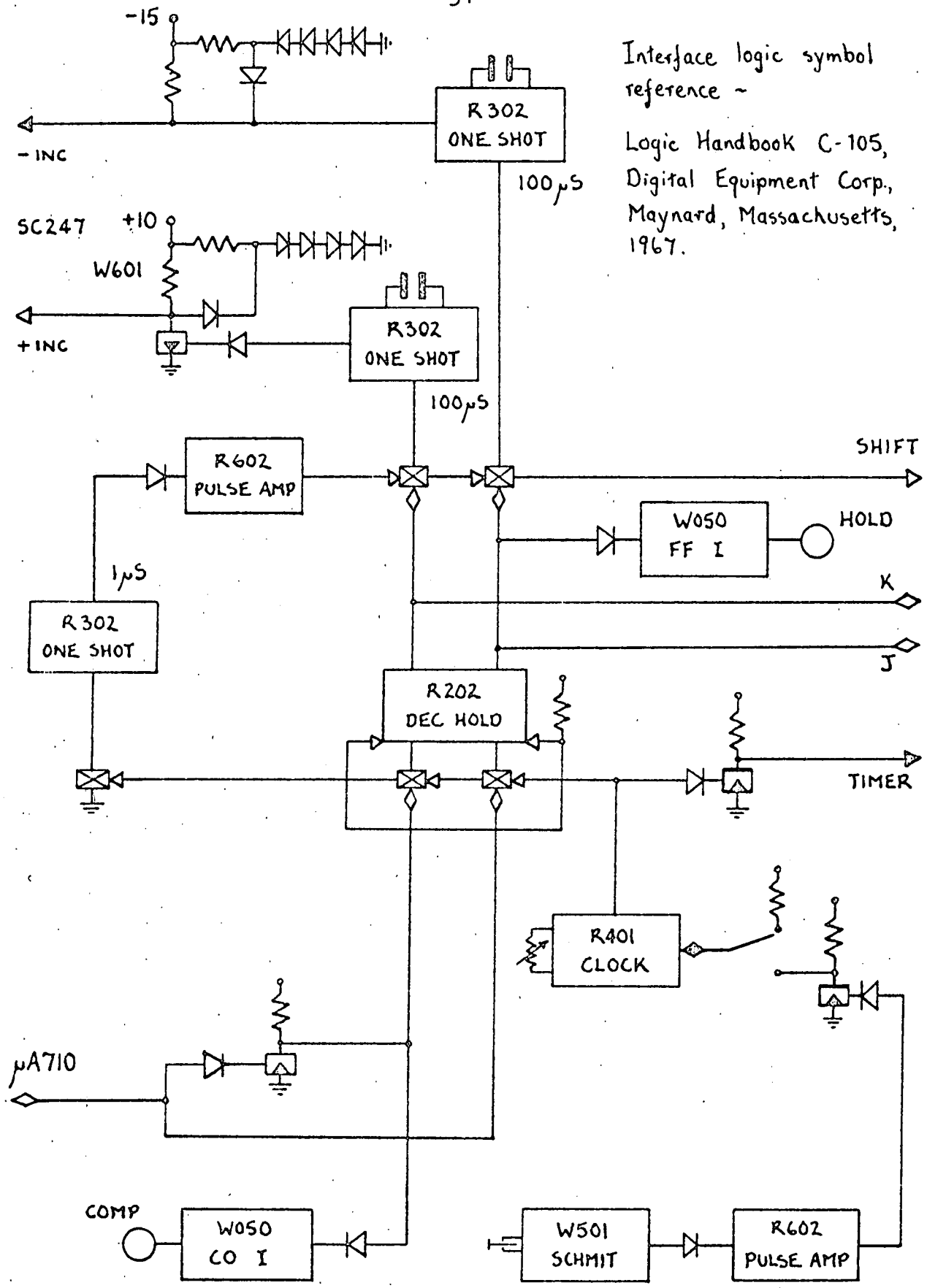
comparator, an arrangement which ensures that the resolution of the comparator (3mV), whose input range is restricted, is effective over the whole of the 300 V echelon signal range at the integrator output, and therefore remains fine relative to the DM step size of 1 V. The comparator is biased to the logic threshold for zero message and integrator output and arranged to deliver the inverted levels required by the interface DEC modules.

3.4 Computer interface

The detailed configuration of the interface logic is determined by the processing operations with the most stringent computer time requirements, increased program volume compensating for the sub-optimal data organisation in other cases.

In the analogue interface (Fig. 3.3) the variable clock, which is the primary timing source for DM sampling and data transfers, pulses DCD gates enabled in complement by the status of the delta-coder comparator. The gates are internally conditioned by the associated flip flop which holds the comparator decision during the sampling periods and enables the one-shot multivibrators which generate increment pulses of the appropriate polarity for return transmission to the SC247.

At the same time as the selected one-shot is pulsed by the delayed clock signal, the flip flop status is transferred



Interface logic symbol reference ~

Logic Handbook C-105, Digital Equipment Corp., Maynard, Massachusetts, 1967.

Fig. 3.3 Analogue interface

to the least significant slot of a 7 bit shift register parallel-connected to the PDP-8 memory buffer (MB) (Fig. 3.4). After this shift, but before a data transfer is initiated, two further input gates, enabled by a flip flop whose status is under program control, are pulsed to complement the least significant bit if an error is called for. During subsequent shift cycles the erroneous bit continues to propagate along the input buffer.

When data transfer is selected, the delayed complement pulse sets the break request flip flop (Fig. 3.5) which signals the computer to enter the break state after completion of the current instruction. The location in core memory for each transfer is specified by the content of the interface memory address (MA) register, the more significant 5 bits (selecting one of the available 32 pages of 128 words) being set by switch register and the remaining 7 bits by an up-counter.

When the break state has been entered, the address accepted pulse generated by the computer is used to clear the break request flip flop, increment the MA register and trigger a one-shot which grounds the program interrupt bus. Synchronisation between program execution and the DM clock is thus achieved by arranging that the effective starting address of the instruction set is 0001* and after completion of the processing initiated at a sampling instant the machine cycles

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*12 bit computer words and memory addresses are written throughout as 4 digit octal numbers.

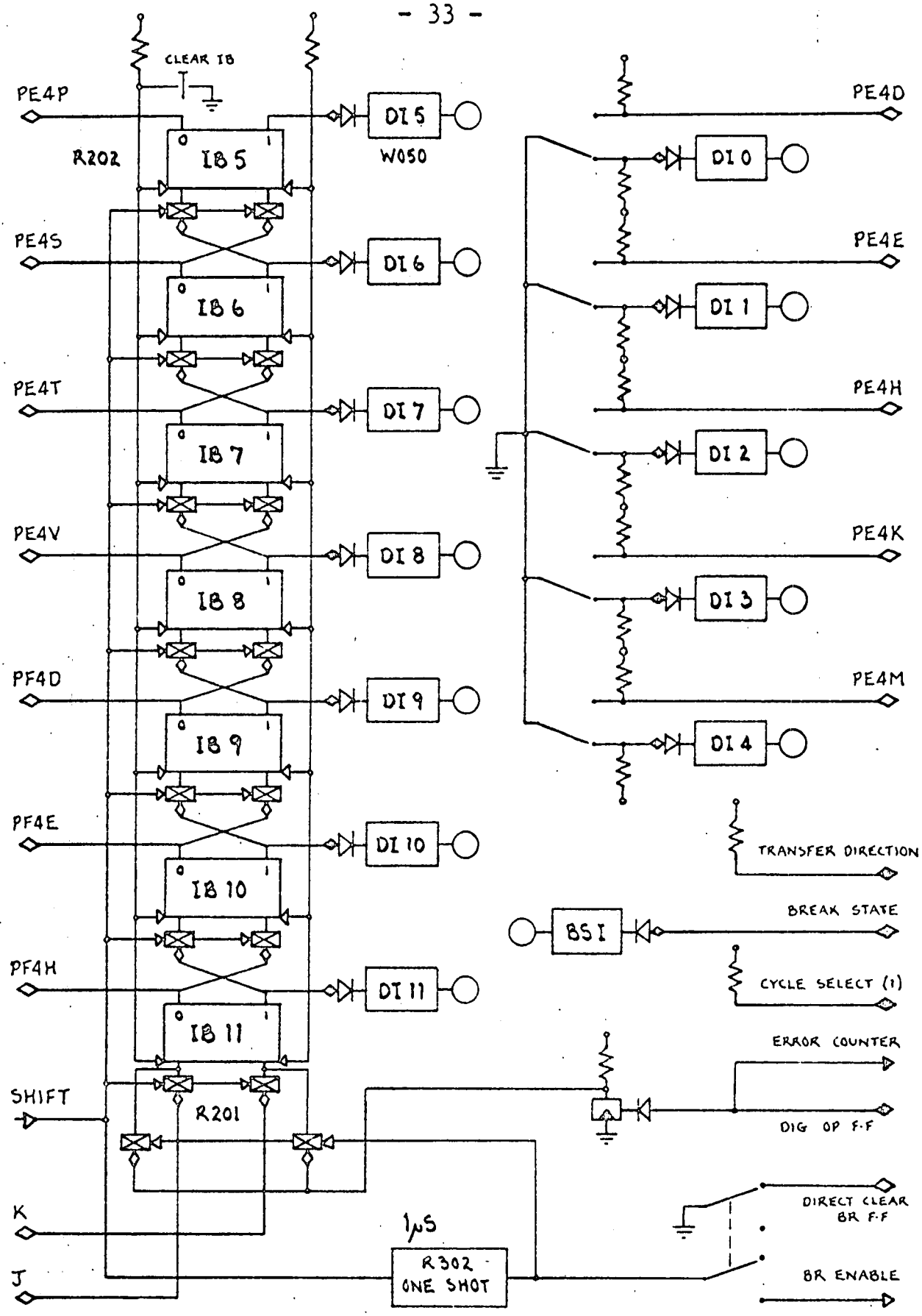


Fig. 3.4 MB register interface, data error control

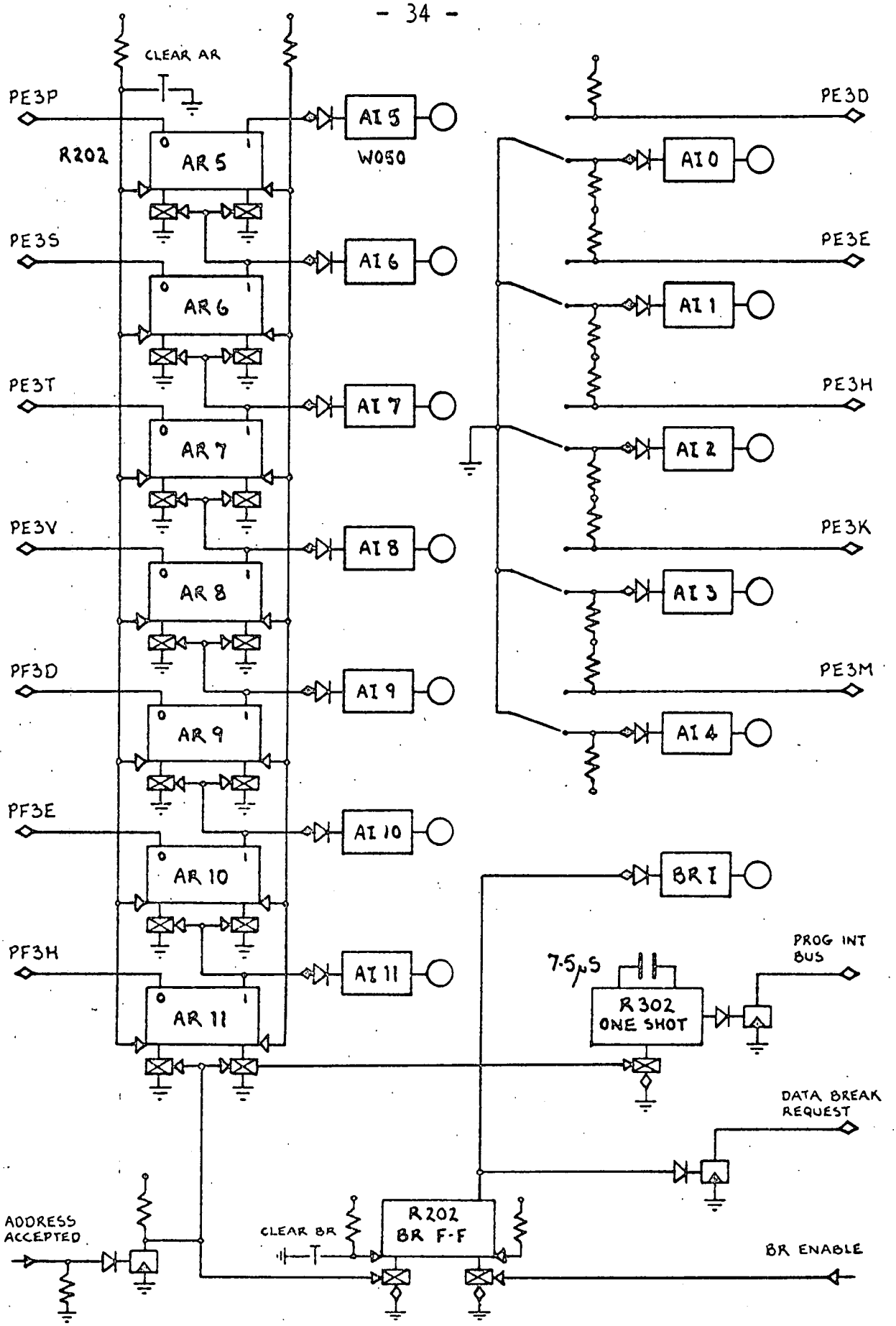


Fig. 3.5 MA register interface, data transfer control

in an instruction loop with the interrupt enabled until a data break is requested.

The interface thus allows any selected page of PDP-8 core memory to be operated as a continuously cycling data register in which can be accessed at any time the 134 most recent bits of the DM sequence. Each word on this page is composed of 5 preselected bits (the processing of which by program execution forms from the word an absolute machine address with data-specified within-page section) followed by 7 consecutive data bits of which the more significant 6 are written redundantly in the less significant locations of the next lower address, and the less significant 6 in the more significant locations of the group in the next higher, the last address of the page being followed functionally by the first of the same page rather than of the next page.

The delta-coded speech processor is shown in Figs. 3.6, 3.7.

Fig. 3.6 SC247 and PDP-8 computers

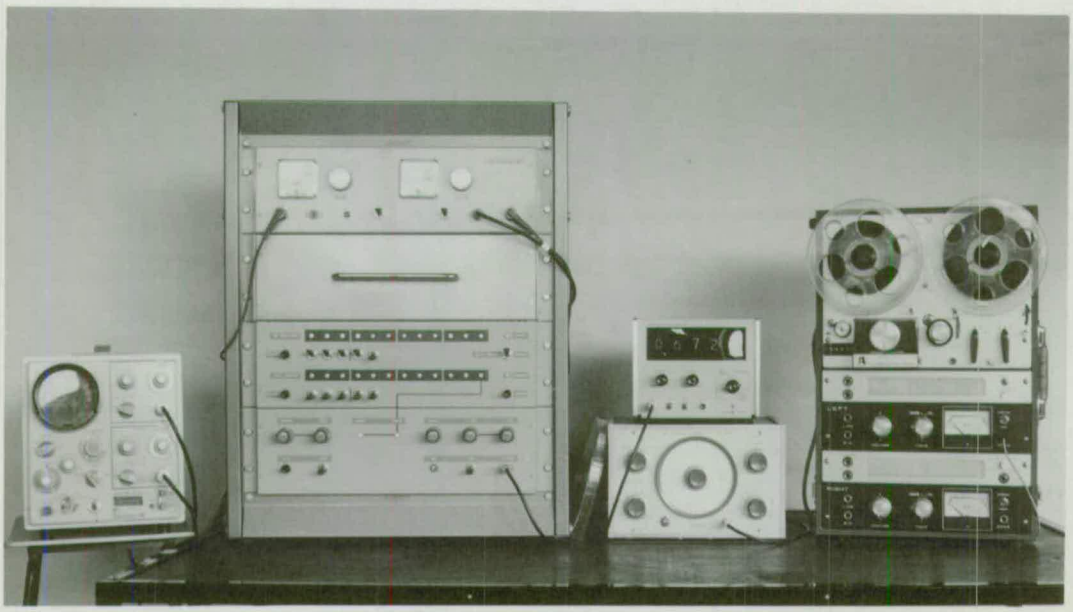


Fig. 3.7 Delta - coded speech interface

Chapter 4

Linear prediction

In introducing the discrete information source treatment of the signal generated by a delta-coder, the constituent 0 or 1 symbols of the binary output sequence are termed the elements of the message, while the state i of the source at any time is defined by a multi-dimensional vector whose components are a finite number of the symbols which have been generated prior to that time. Fig. 2.3 indicates that the significant dimensionality of the states exceeds 100, and N th order Markov process approximations to the source are studied for which an element is considered to have a statistical dependence limited to the preceding N symbols.

4.1 Entropy

The lemmas of Shannon identify three entropy measures relevant to a DM message. There is first the entropy* of the process

$$H = -\sum_i \sum_j p(i)\, p(j|i) \log p(j|i) \qquad (4.1.1)$$

in which $p(j|i)$ is the conditional probability of occurrence of state j following state i.

Second, there is the entropy of blocks B_i of N elements of

*Apart from explicit exceptions, entropy measures are per element of the signal sequence.

the signal sequence,

$$G_N = -\frac{1}{N}\sum_i p(B_i)\log p(B_i) \qquad (4.1.2)$$

And third, for an N th order approximation ((N+1) th order in Shannon's terminology) to the source in which the N element block B_i determines the conditional distribution from which the next symbol S_j is drawn, the entropy

$$F_N = -\sum_i \sum_j p(B_i, S_j)\log p(S_j|B_i) \qquad (4.1.3)$$

By the adoption of the finite state $(1 \leq i \leq 2^N)$ representation of the signal source, F_N may be equated with the process entropy H and evaluated by (4.1.1) which involves the conditional rather than the joint probabilities.

As the channel capacity required for direct transmission of a delta-coded message is equal to the average element entropy*, relative entropies are referred to the H = 1 bit per element value for a source for which the S_j are equiprobable and independent of preceding elements.

4.2 Predictive coding

Procedures for redundancy† reduction by predictive

---

*'Average element entropy' is used concisely throughout with the meaning 'entropy of the distribution of the average element' and should be clearly distinguished from the average of the entropies of the distributions of the elements, which is the entropy of the process.

†Redundancy is the complement of relative entropy.

coding[17] lower the entropy of the average element distribution by a transformation utilising such information about the conditional probability $p(S_j | B_i)$ as it is economically viable to compute at sender and receiver for limited size blocks B_i of past elements. For a delta-coded message, an estimator processing N previous elements can make a binary prediction of the next element in sequence and the transformation effected as shown in Fig. 4.1 by modulo-2 addition of the element and its prediction.

$$T = \overline{S}\cdot E + S\cdot\overline{E} \qquad (4.2.1)$$

At the receiver, the same prediction is generated by an identical estimator processing the same elements and the new element is determined by processing inversely.

For by De Morgan's laws

$$\overline{T} = \overline{(\overline{S}\cdot E)}\cdot\overline{(S\cdot\overline{E})} = (\overline{E} + S\cdot E)\cdot(\overline{S} + S\cdot E)$$

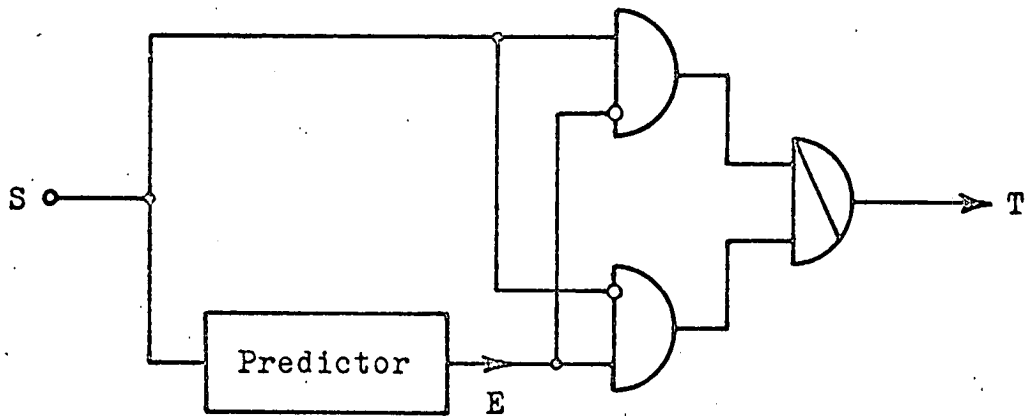
So that

$$\begin{aligned}\overline{T}\cdot E + T\cdot\overline{E} &= (\overline{E} + S\cdot E)\cdot(\overline{S} + S\cdot E)\cdot E + (\overline{S}\cdot E + S\cdot\overline{E})\cdot\overline{E} \\ &= S \qquad (4.2.2)\end{aligned}$$

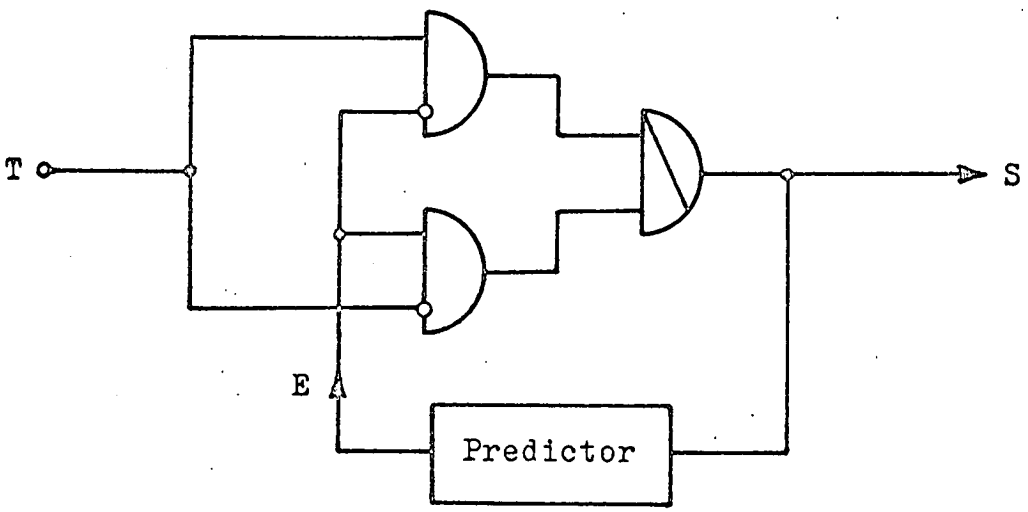
In words, the binary symbols which previously had the direct significance :

(a) { 0 : Decrement approximant by Δ
{ 1 : Increment " "

then assume the indirect function :



Transformation



Inversion

Fig. 4.1 DM predictive coding

0 : Estimator output correct, interpret it as (a)
1 : " " wrong, complement it and interpret as (a)

As the transformations are one - one the process entropy computed by (4.1.1) for $N \rightarrow \infty$, the weighted average of the entropies of the conditional distributions for all possible states, is the same for operand and transform sequences, but for a predictor success probability $P_s > 0.5$, the entropy

$$H_T = -\left[P_s \log P_s + (1 - P_s) \log (1 - P_s)\right] \quad (4.2.3)$$

of the average element of the new sequence is lower. While schemes for encoding the transformed sequence to achieve a reduction in channel bit rate are described later (chapter 8), it is noted here that direct transmission of the sequence in an application in which binary 1 is signalled by an output pulse, binary 0 by pulse absence, results in a transmitter power economy of

$$\eta = 100 (2P_s - 1) \% \quad (4.2.4)$$

4.3 Estimator function

Although processing of the same number of elements by a predictor function without a linearity constraint is in general advantageous for other than stationary Gaussian messages, linear prediction by a Wiener estimator is first described, for the hardware requirement increases exponentially with N in the former case but in direct proportion for the latter. Of course the linear predictor can also be synthesised without data

truncation as an analogue network rather than a digital circuit, but in this case errors due to component tolerances would be expected between encoder and decoder predictions when the estimates are near the binary decision threshold. The estimator, with transfer function E(s), corresponding impulse response e(t), is assumed to be excited by the echelon signal $m_i(t)$ formed at the integrator output and is required to produce an optimal estimate of the message with zero lag (i.e. neither delayed nor advanced) for comparison with the echelon signal itself as in a standard delta-coder to generate the logical prediction E for processing as shown in Fig. 4.1.

The method of spectrum factorisation[18] is first applied to the input message plus noise power spectral density

$$\Phi_{ii}(\omega) = \Phi_{mm}(\omega) + \Phi_{nn}(\omega), \qquad (4.3.1)$$

with corresponding autocorrelation function $\rho_{ii}(\tau)$, to compute factors

$$\Phi_{ii}^{+}(s) \cdot \Phi_{ii}^{-}(s) = \Phi_{ii}(s) \qquad (4.3.2)$$

such that $\Phi_{ii}^{+}(s)$ contains the 28 left half plane singularities, while $\Phi_{ii}^{-}(s)$ has poles and zeros in the right half plane only*.

---

*For most SNR, a combination procedure involving a modification of the Bernoulli method for approximate location and an extension of the Birge-Vieta algorithm to the complex plane for refinement of the estimate has been found to ensure certainty of convergence while avoiding the rapid growth of the numbers involved from iteration to iteration.[19]

The inverse transform of $\Phi_{ii}^{+}(s)$ is therefore zero for $t < 0$, so that

$$\Gamma_{ii}^{+}(s) = \frac{1}{\Phi_{ii}^{+}(s)}$$

(which also has singularities confined to the left half plane, the poles and zeros of $\Phi_{ii}^{+}(s)$ becoming the zeros and poles of its reciprocal) is a physically realizable transfer function which would convert the coded message to white noise.

For the case of desired estimator output equal to the message without lag or lead, the Wiener - Hopf* equation[20], derived by the application of a variational method to minimise the mean square error, takes the form

$$\varphi_{im}(\tau) = \int_{-\infty}^{\infty} e(\delta)\, \varphi_{ii}(\tau - \delta)\, d\delta, \qquad \tau \geq 0 \qquad (4.3.3)$$

so that writing

$$\Psi(s) = \frac{\Phi_{im}(s)}{\Phi_{ii}^{-}(s)} \qquad (4.3.4)$$

the transform

$$R(s) = \Phi_{ii}^{-}(s)\left[E(s).\Phi_{ii}^{+}(s) - \Psi(s)\right] \qquad (4.3.5)$$

has no left half plane singularities.†

The input signal - message cross power spectral density

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*The analyses of Wiener and Lee are developed in the λ plane, $\lambda = \omega + j\delta$, while the standard complex variable $s = \delta + j\omega$ is retained in this thesis. Hence functions with poles and zeros in the left half plane have the time domain properties of those with upper half plane singularities in the literature referenced.

†Note that the restriction $\tau \geq 0$ in (4.3.3) allows $R(s)$ right half plane poles.

in (4.3.4), which corresponds to the cross correlation function $\varphi_{im}(\tau)$ in (4.3.3), becomes

$$\Phi_{im}(\omega) = \Phi_{mm}(\omega) + \Phi_{nm}(\omega) \qquad (4.3.6)$$

$$= \Phi_{mm}(\omega)$$

on the assumption that Δ is sufficiently fine for the message - quantisation noise cross power spectral density to be zero.

There results some pole - zero cancellation with the right half plane input spectrum factor in (4.3.4) and the complex residues at the remaining 14 left half plane message spectrum poles are evaluated to allow the partial fraction expansion

$$\Psi(s) = \Psi^{+}(s) + \Psi^{-}(s) \qquad (4.3.7)$$

in which $\Psi^{+}(s)$ contains the terms with left half plane poles.

Inspection of

$$\frac{R(s)}{\Phi_{ii}^{-}(s)} = E(s).\Phi_{ii}^{+}(s) - \Psi^{+}(s) - \Psi^{-}(s) \qquad (4.3.8)$$

shows that for the estimator function to be physically realizable (left half plane poles only)

$$E(s) = \frac{\Psi^{+}(s)}{\Phi_{ii}^{+}(s)} \qquad (4.3.9)$$

Although the message is sharply bandlimited, $|E(j\omega)|$ does not decrease more rapidly than $1/\omega$ at high frequencies. This

characteristic necessarily results if the corresponding impulse response, formally

$$e(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{\Psi^+(s)}{\Phi_{ii}^+(s)}\, e^{st}\, ds, \qquad (4.3.10)$$

has its maximum at $t = 0$, so that the weighting function $w(\tau) = e(\tau)$, where $\tau = t - \lambda$ is the age at time t of the signal generated at time λ, weights the new datum most strongly.

(4.3.10) represents tandem processing by systems with the impulse responses

$$\psi^+(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi^+(\omega)\, e^{j\omega t}\, d\omega \qquad (4.3.11)$$

and

$$\gamma_{ii}^+(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\Phi_{ii}^+(\omega)}\, e^{j\omega t}\, d\omega \qquad (4.3.12)$$

Thus

$$w(\tau) = \int_{-\infty}^{\infty} \psi^+(\lambda)\ \gamma_{ii}^+(\tau-\lambda)\, d\lambda \qquad (4.3.13)$$

Optimal weighting functions are shown in Fig. 4.2, in which the steeper initial slope of $w(\tau)$ for higher SNR corresponds in the frequency domain to the use, in generating the estimate, of the information in the upper spectral components of the message which are less severely impaired than when the quantisation noise power is greater.

4.4 Estimator error

The estimator error distribution and DM quantising

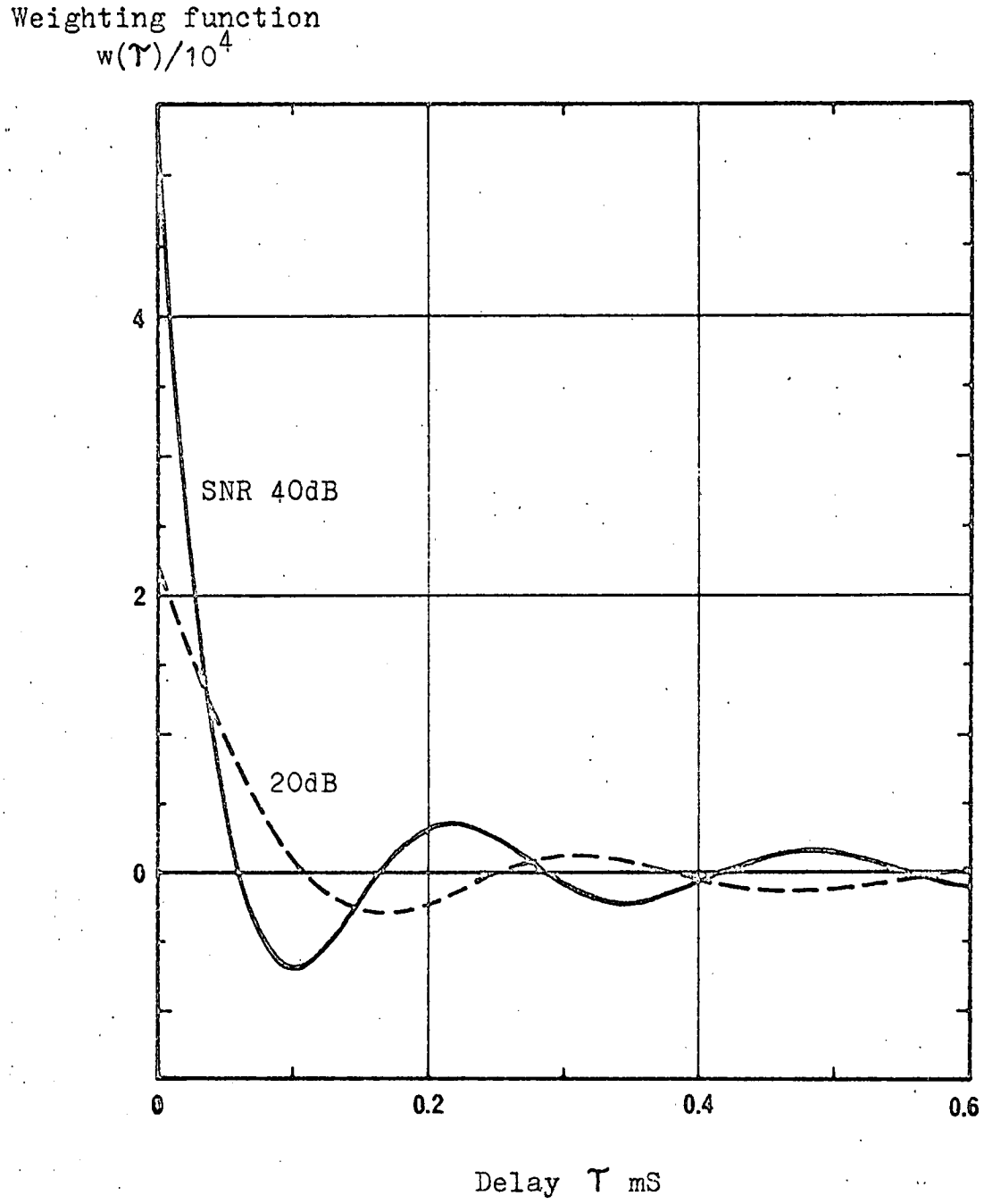

Fig. 4.2 Optimal weighting functions

interval determine the probability of successful binary prediction. For the estimate $m_e(t)$ whose value at the sampling instants has the probability density function $p(m_e)$ with mean (which is $m(t)$) displaced δ from the last quantised level, the probability of an individual prediction being correct is

$$p_s = \int_{-\delta}^{\infty} p(\epsilon)\, d\epsilon \dagger \qquad (4.4.1)$$

For reasonably fine quantisation, the distribution of δ over the set of $p(\epsilon)$ at the sampling instants may be considered uniform over the range $-\Delta < \delta < \Delta$, so that the predictor success probability is

$$P_s = \frac{\int_{\delta=-\Delta}^{0} \int_{\delta}^{\infty} p(\epsilon)\, d\epsilon\, d\delta}{\int_{\delta=-\Delta}^{0} d\delta} \qquad (4.4.2)$$

For $m(t)$ Gaussian, $\epsilon(t)$ is also normally distributed, and writing $\sigma_e^2 = \overline{\epsilon^2(t)}$,

$$P_s = \frac{1}{\Delta} \int_{\delta=-\Delta}^{0} \left[\frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_{0}^{-\frac{\delta}{\sqrt{2}\sigma_e}} e^{-u^2} du \right] d\delta \qquad (4.4.3)$$

$$= \frac{1}{2} + \frac{1}{2} \operatorname{erf} \frac{1}{\sqrt{2}} \left(\frac{\Delta}{\sigma_e}\right) + \frac{1}{\sqrt{2\pi}} \left(\frac{\sigma_e}{\Delta}\right) \left[e^{-\frac{1}{2}\left(\frac{\Delta}{\sigma_e}\right)^2} - 1 \right] \qquad (4.4.4)$$

$$\text{For } \left(\frac{\Delta}{\sigma_e}\right) > 4, \quad P_s \approx 1 - 0.399 \left(\frac{\Delta}{\sigma_e}\right) \qquad (4.4.5)$$

Fig. 4.3 shows how the predictor success probability, and corresponding average element entropy, vary with the

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† In which $p(\epsilon)$ is the probability density function of the error $\epsilon(t) = m_e(t) - m(t)$

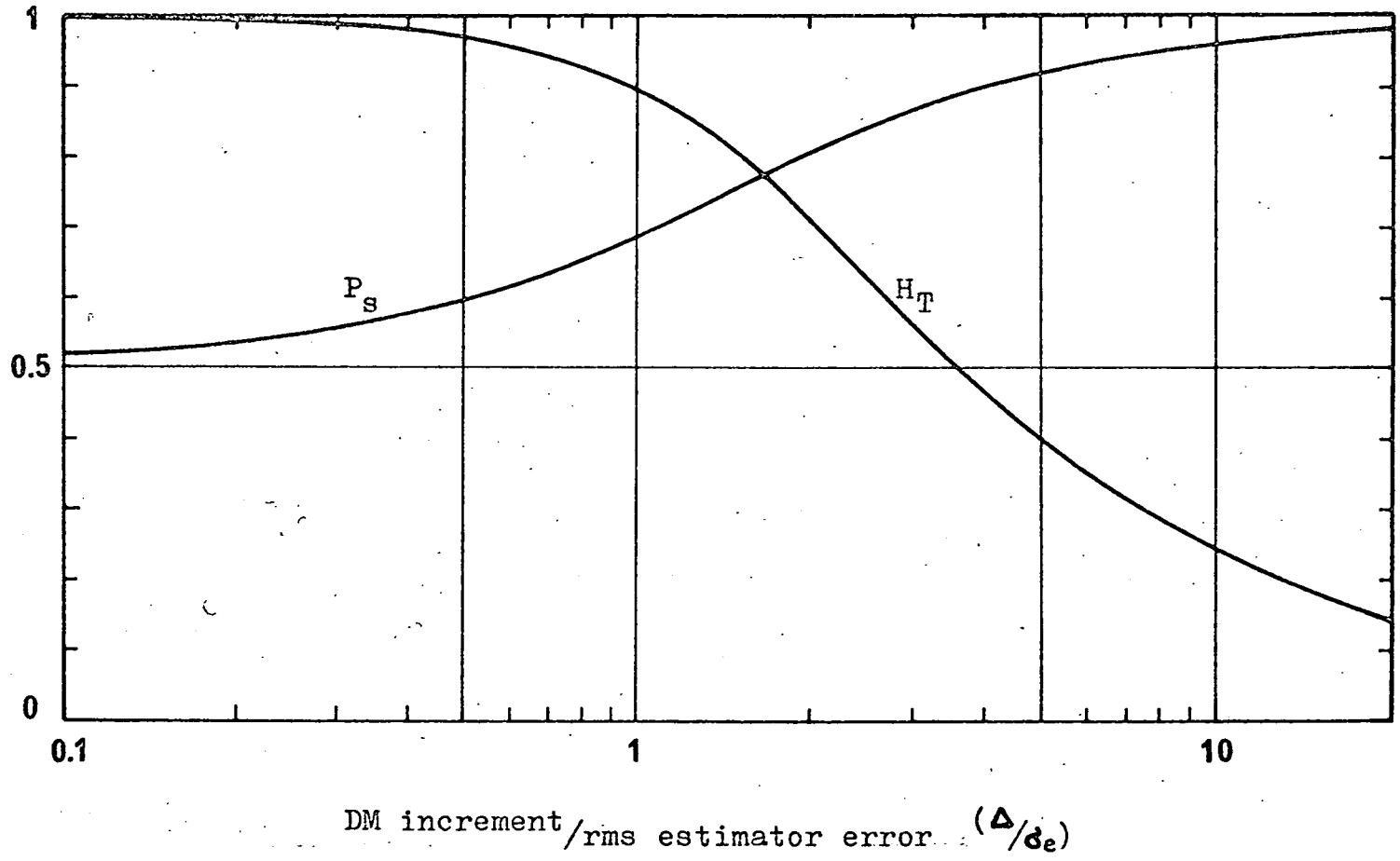

Fig. 4.3 Linear prediction success probability

relative magnitudes of Δ increment and rms estimator error.

For the estimator with the optimum transfer function given by (4.3.9), the error power is determined from (4.3.5), (4.3.7) and (4.3.11) by computing the integral square of the inverse transform of $\Psi^{+}(\omega)$ over an adequate time range

$$\overline{\epsilon^2(t)} = \varphi_{mm}(0) - \frac{1}{2\pi}\int_0^{\infty} \left[\psi^{+}(\tau)\right]^2 d\tau \qquad (4.4.6)$$

in which $\varphi_{mm}(0) = 1$ for unit message power.

The relation between relative estimator error and SNR is then determined from (2.2.2), (2.4.7) and (4.4.6) and shown in Fig. 4.4, from which it is found that the corresponding power saving η (4.2.4) indicated by the characteristics as attainable by predictive coding ranges from 59 % at low SNR for which DM is relatively satisfactory to over 70 % where the quality required is high and the inefficiency of conventional DM is greater.

4.5 Implementation

Convolution of the input signal and the weighting function given by (4.3.13) yields the estimate

$$m_e(t) = \int_0^{\infty} w(\tau)\, m_i(t-\tau)\, d\tau \qquad (4.5.1)$$

For a digital implementation with a finite duration signal sequence, $w(\tau)$ is truncated at $\tau = \theta$ and scaled to retain $\int_0^{\theta} w(\tau)\, d\tau = 1$.

$$\frac{\text{DM increment}}{\text{rms estimator error}} \quad (\Delta/\sigma_e)$$

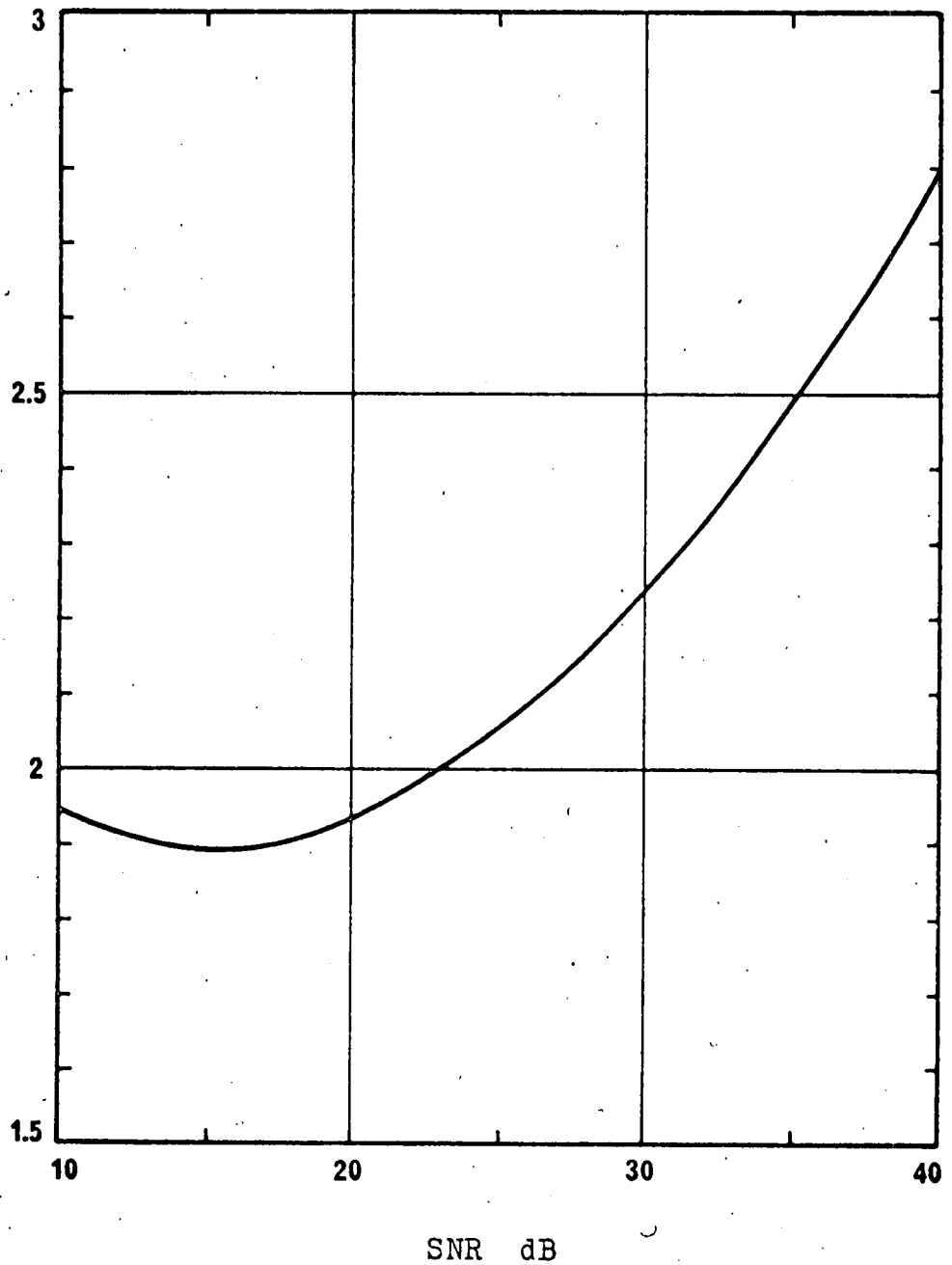

Fig. 4.4 Relative estimator error

Then approximately

$$m_e(t) = \int_0^{\theta} \omega_i(\tau)\, m_i(t-\theta+\tau)\, d\tau \qquad (4.5.2)$$

in which

$$w_i(\tau) = w(\theta-\tau) \qquad (4.5.3)$$

The multiply time of a digital computer (for PDP-8 360 μS by software ; or 21 μS with extended arithmetic unit) is too great for real time evaluation of (4.5.2), so we proceed by calculating the integrand product by parts.

$$m_e(t) = \left[m_i(t-\theta+\tau)\int \omega_i(\tau)\, d\tau\right]_0^{\theta} - \int_0^{\theta}\left[\frac{d}{d\tau}\, m_i(t-\theta+\tau)\right]\left[\int \omega_i(\tau)\, d\tau\right] d\tau$$

$$= m_i(t) - \int_0^{\theta}\left[\frac{d}{d\tau}\, m_i(t-\theta+\tau)\right]\left[f(\tau)\right] d\tau \qquad (4.5.4)$$

where the integral weighting function

$$f(\tau) = \int w_i(\tau)\, d\tau \qquad (4.5.5)$$

The logical prediction E being determined by the sign of $\left[m_e(t) - m_i(t)\right]$, we have

$$\begin{aligned} E = 1, \quad & \int_0^{\theta}\left[\frac{d}{d\tau}\, m_i(t-\theta+\tau)\right]\left[f(\tau)\right] d\tau \leq 0 \\ = 0, \quad & \text{"} \qquad\qquad\qquad\quad > 0 \end{aligned} \qquad (4.5.6)$$

At any time t, the function $\frac{d}{d\tau}\, m_i(t-\theta+\tau)$ is a signal sequence which is the time derivative of that portion of the input signal from $\lambda = t - \theta$ to the immediate datum $\lambda = t$. The sequence is a bipolar impulse train with value zero between the sampling instants, so that the relevant values of $f(\tau)$ form

a fixed ordinate set. Implementation of (4.5.6) by a data processor therefore involves for each prediction only the fast summation (3 μS per addition in two's complement arithmetic with directly addressed operand) of the members of the set with sign gated by the polarity of the corresponding impulses of the derivative sequence.

This procedure has been tested with PDP-8 for a speech message* of 43 sec. duration and parameters f_s = 48 KHz, SNR = 25 dB, and yields a predictor success probability of 0.72, increased to 0.75 with a refinement by which after an error the decision threshold is adjusted for subsequent data to a level such that the estimate would just have resulted in the complement prediction. Program organisation trades processor memory size for arithmetic capability by arranging that the appropriate 19 DM code blocks of 7 bits selected from the data page during a sampling period each address locations in core store containing the corresponding integral contributions for one segment of the weighting function, so that only 18 additions are executed although a 133 bit sequence is processed.

The integral contribution set is calculated by a larger KDF9 computer and converted in tape code and by the addition of

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*The reading of 'The North Wind and the Sun' which is the source message for the University of Edinburgh's PAT speech synthesiser and can also be generated by the Atlas computer of Manchester University.

pseudo instructions by PDP-8 to a source language program for translation by Macro-8 assembler to the binary format data sequence actually loaded into core memory. As the optimum weighting function $w(\tau)$ for $\tau > \theta$ is less than 1 % of $w(0)$ and the quantisation of the ordinates of $f(\tau)$ is fine (12 bit), an analogue estimator would be expected to have a similar performance.

Chapter 5

Relative entropy

Following the introduction of chapter 4, the signal information properties of delta-coded speech are now determined in detail. A stochastic state transition probability matrix source representation is used throughout, the matrix being 2^N square for an N th order approximation and containing two non-zero complementary elements per operand state column vector. Of course, all the matrix elements for an unconstrained symbol source have value 0.5, while greater and lesser transition probabilities from high probability operand states will correspond to significant redundancy in the sequences generated.

5.1 Transition matrix assembly

The matrix assembly software, a combined octal/symbolic listing of part of which is given in Appendix 3 (but treatment of some features is deferred), has the characteristics briefly described in the following.

With a typical test signal sequence in excess of 4 million bits (a 42 sec. duration message sampled at 96 KHz), the high probability states are entered more frequently than the maximum count which can be stored by one word of core memory, so that two consecutive locations are utilised per counter to store the

parts of upper and lower significance. Additionally, for each state entered, two counts are required to total the transition frequencies to both of the possible subsequent states, so that a memory allocation of 2^{N+2} words is required for an N th order approximation. Computations for $N \leq 9$ may therefore be performed by the 4096 - word PDP-8, and one half of the core memory is available for program instructions, literals, temporary registers, loading, initializing, list processing, code conversion and output routines and the one page cycling data register.

On entry, the counter and temporary storage registers and transfer flags are cleared, the data register is loaded with an alternate sequence of binary digits and the data and state address registers are initialized. The program interrupt is enabled and the machine cycles in an instruction loop until the interface generates the first data break request at the commencement of the speech message on magnetic tape. During the clock - synchronised program execution which is then initiated, the appropriate data page addresses are referenced to determine the current state and the subsequent transition, and the corresponding counters are incremented using the link as a carry register following accumulator overflow.

At the termination of a 46 minute test run, PDP-8 scans the pages of counters in sequence and generates the 9 element states corresponding to the addresses of the groups of 4 registers

which they have been allocated. A listing is produced by high speed paper tape punch of the state vectors, represented as 3 - digit octal numbers, followed by the transition totals for each of the possible following states, the double precision binary counts being converted to BCD before output in ASCII code. The counters are then scanned again and each state is masked to 8 elements only, the transitions from and to states which then become identical being summed and the limit of the output, but not of the register scan, being reduced by one half. This procedure continues repetitively with appropriate masks and limits to generate blocks of data for all Markov process orders N = 9 - 0, and the total bit count for the run is also accumulated and punched. A representative transition probability matrix for a 4th order approximation and typical DM parameters is given in Appendix 4.

5.2 Markov process entropy

To proceed with relative entropy computations based on the matrix element listings, the previous data and instruction set is over - written in core memory by the Fortran object time system by which are loaded interpretive code programs which operate on the data punched at the end of the test run for a range of analyses. For each process order N the entropy for every state is first computed and then the average H of all the entropies, each weighted in proportion to the probability of occurrence of the corresponding state, is determined in

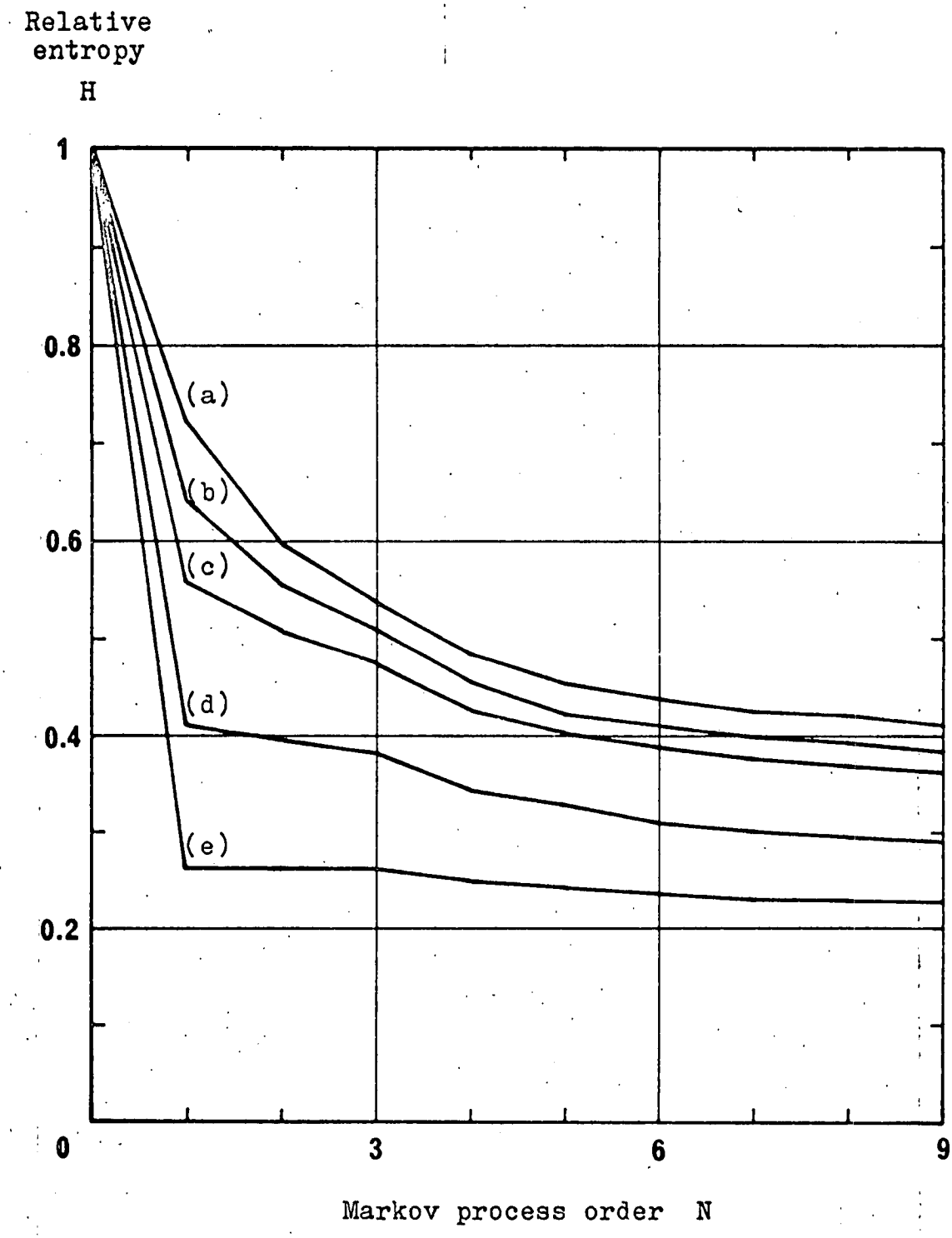
accordance with (4.1.1).

Fig. 5.1 presents the results of five test runs with f_s = 48 KHz and values of Δ increment ranging from coarse quantisation to that causing frequent overloading. We find that in all cases, the redundancy of the delta-coded message exceeds one half for $N \geqslant 4$. The characteristics define the lower bounds of the channel capacity required for transmission of the DM signal after IP transformation by an encoder processing N elements only.

Redundancy reduction studies endeavour to determine practically convenient schemes which achieve performance approaching the bound with acceptable hardware complexity. Were the entropy computation continued for large N, a value for the total message information could be obtained by the procedure described, and this might correspond to a rather low value of relative entropy per element. But the result would be more apposite to a study of phonetic constraint than communications engineering since the memory requirement of an encoder capable of processing many elements increases in proportion with that of the computer required for the investigation.

5.3 Parameter variation

Information properties found to be subject to wide variation with choice of DM design parameters would entail



Pk - pk message amplitude (a) 70Δ (b) 56Δ (c) 42Δ
(d) 28Δ (e) 14Δ

Sampling frequency f_s 48 KHz

Fig. 5.1 Relative entropy characteristics

elaborate presentation and be of limited application. Fortunately the relative entropy characteristics prove to be substantially independent of changes in sampling frequency and Δ increment provided their choice is appropriate (i.e. they are altered in inverse proportion). This invariance is shown by the constant overload probability groups of Fig. 5.2, which contain members for f_s = 32, 48, and 96 KHz, and comparison of the corresponding transition probability matrices shows that this property extends to the detailed predictor structures also. In general, parameter values f_s = 96 KHz, peak-peak message amplitude 128Δ are considered, these being typical for good communications quality with infrequent overloading. Agreement between repeat test runs is better than 0.1%, and adequate test message duration is indicated by differences of less than 0.5% between results for the complete passage and its halves processed separately.

5.4 Transform sequence entropy

In generating a complete element listing for all vector source states, the transition matrix assembler implicitly defines the forms of the optimal discrete fixed-structure predictors operating on blocks of signal elements of length N. † On identification of the current state, the next element is predicted by these functions such that the subsequent state is that for which the transition probability is known to be a maximum.

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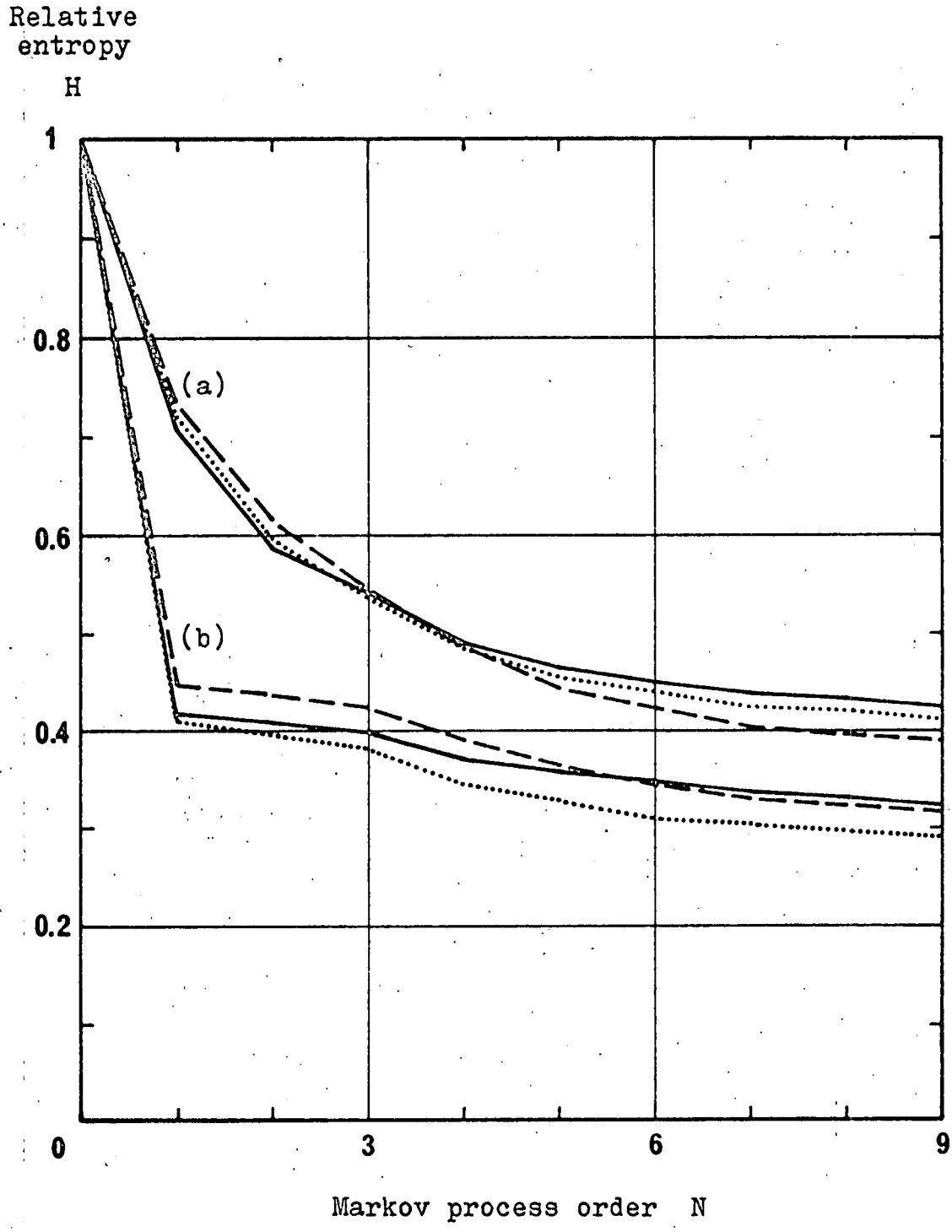
† The attention of the reader is drawn to this statement, which defines the non-linear prediction procedure. An example is given in Appendix 4.



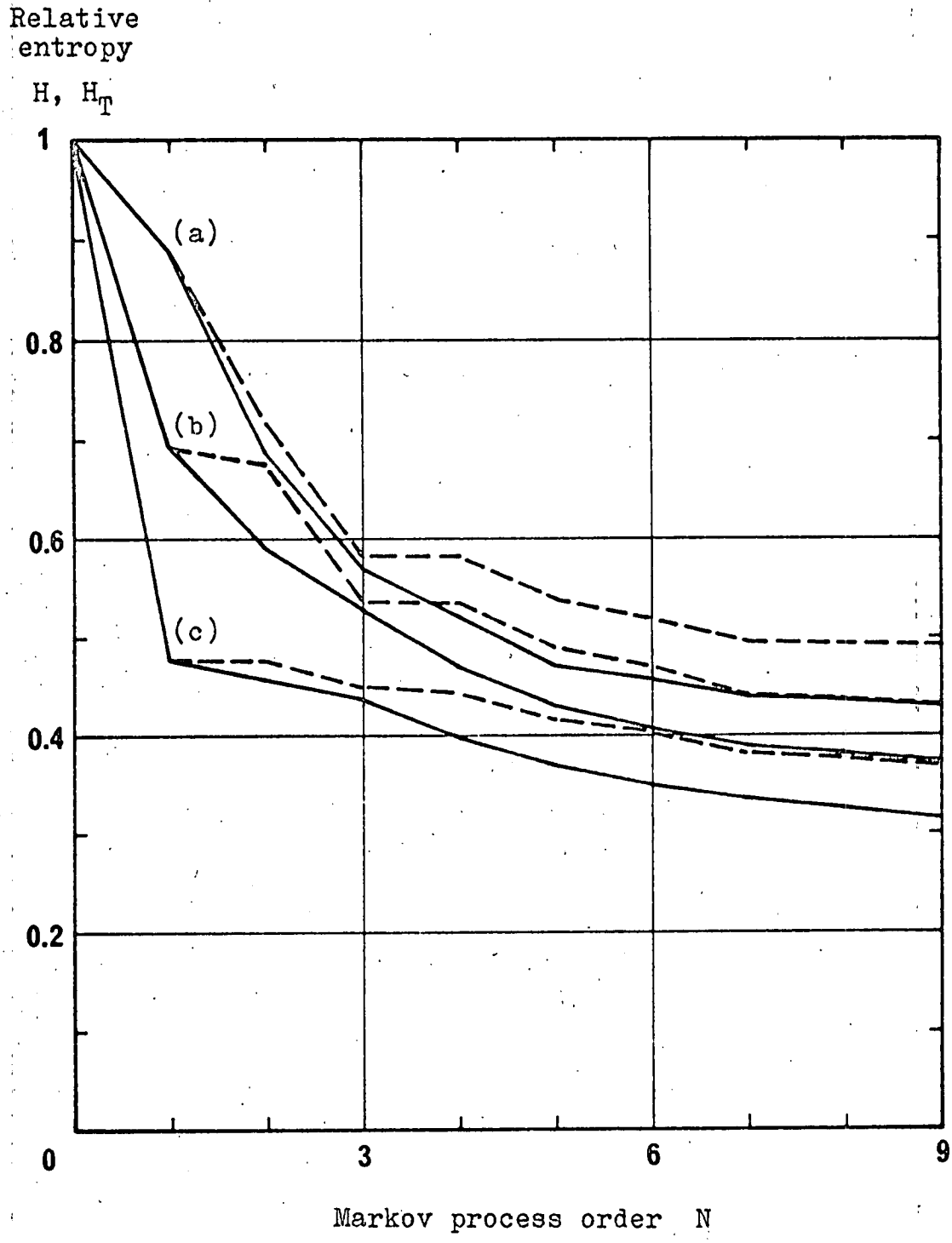
Constant Δf_s = Pk message amplitude × (1.39 group (a); 3.48 " (b))

Sampling frequency f_s (——— 32; ····· 48; - - - - - 96) KHz

Fig. 5.2 Process entropy for constant overload probability

Since the transformation of Fig. 4.1 is IP, the asymptotic process entropies for the DM source signal and modulo - 2 adder output sequence are the same, but the entropy of a zero order approximation to the latter, computed for three test runs and shown by the characteristics of Fig. 5.3, is much less than that of a zero order representation of the former. For a range of N and Δ, the average element entropy of the transformed sequence for N element prediction does not greatly exceed the process entropy of an N th order approximation to the delta - coded message source and for the typical median characteristic, an N = 6 predictor achieves $P_s = 0.9$, corresponding to a power economy $\eta = 80\%$. The average element entropy is within 14 % of the 6 th order process entropy, and the redundancy in both cases exceeds one half. The optimal predictors for this characteristic for N = 1 - 7 are given in Table 5.4.

While increase of N by one order doubles the number of distinguishable states, the optimal predictor structure and performance changes only if the transition probability distributions for two or more of the states previously considered together have maxima for complementary next elements. In the cases of characteristic (a) for N = 3 - 4 and (c) for N = 1 - 2 in Fig. 5.3, for example, this does not occur so that predictors operating on the shorter signal sequences are as effective as those processing the longer.



———) Relative entropy of process (order N)

- - - - -) Half adder output sequence entropy (zero order) with N element optimum predictor

Pk - pk message amplitude (a) 256Δ (b) 128Δ (c) 64Δ

Sampling frequency f_s 96 KHz

Fig. 5.3 Predictive coding sequence entropy (zero order)

TABLE 5.4 OPTIMAL PREDICTORS FOR DELTA CODED SPEECH

SAMPLING FREQ 96 KHZ PK-PK MESSAGE AMP 128 DELTA
(VECTOR STATES IN OCTAL)

N = 1

STATE	PRED
0	1
1	0

N = 2

0	0
1	0
2	1
3	1

N = 3

0	0	4	1
1	0	5	0
2	1	6	1
3	0	7	1

N = 4

00	0	10	1				
01	0	11	0				
02	1	12	1				
03	0	13	0				
04	1	14	1				
05	0	15	0				
06	1	16	1				
07	0	17	1				

N = 5

00	0	10	1	20	0	30	0
01	0	11	0	21	0	31	0
02	0	12	1	22	1	32	1
03	1	13	0	23	0	33	0
04	1	14	1	24	1	34	0
05	0	15	0	25	0	35	1
06	1	16	1	26	1	36	1
07	1	17	1	27	0	37	1

N = 6

00	0	10	1	20	0	30	1	40	0	50	1	60	0	70	0
01	0	11	0	21	0	31	0	41	0	51	0	61	0	71	0
02	0	12	1	22	1	32	1	42	0	52	1	62	1	72	1
03	1	13	0	23	0	33	0	43	0	53	0	63	0	73	1
04	0	14	1	24	1	34	1	44	1	54	1	64	1	74	0
05	0	15	0	25	0	35	1	45	0	55	0	65	0	75	1
06	1	16	1	26	1	36	1	46	1	56	1	66	1	76	1
07	0	17	1	27	0	37	1	47	0	57	1	67	0	77	1

N = 7

000	0	020	0	040	0	060	0	100	0	120	0	140	0	160	0
001	0	021	0	041	0	061	0	101	0	121	0	141	0	161	0
002	0	022	0	042	0	062	1	102	0	122	1	142	0	162	0
003	1	023	0	043	0	063	0	103	1	123	0	143	0	163	0
004	0	024	1	044	1	064	1	104	0	124	1	144	0	164	0
005	0	025	0	045	0	065	0	105	0	125	0	145	0	165	0
006	1	026	1	046	1	066	1	106	1	126	1	146	1	166	1
007	1	027	1	047	0	067	0	107	0	127	0	147	0	167	1
010	0	030	1	050	1	070	1	110	1	130	1	150	0	170	0
011	0	031	0	051	0	071	0	111	0	131	0	151	0	171	0
012	1	032	1	052	1	072	1	112	1	132	1	152	1	172	1
013	1	033	1	053	0	073	1	113	0	133	0	153	0	173	1
014	1	034	1	054	1	074	0	114	1	134	1	154	1	174	0
015	1	035	1	055	0	075	1	115	0	135	1	155	1	175	1
016	1	036	1	056	1	076	1	116	1	136	1	156	1	176	1
017	1	037	1	057	1	077	1	117	1	137	1	157	1	177	1

Chapter 6

Group encoding

Instantaneous group encoding procedures considered by Shannon[7], Fano[21] and Huffman[22] effect redundancy reduction by the assignment, to N element blocks B_i of the signal sequences generated by a source, of uniquely decipherable N_i element codewords in a manner determined by their occurrence probability set. For all procedures, the average channel capacity required for transmission of the encoded signal, expressed per element of the source output, approaches G_N (4.1.2) as N increases, while G_N converges to H for large N, so that the coding schemes are asymptotically completely efficient. But for finite N the Huffman approach yields group codes with optimal efficiency, and its application to delta-coded speech is therefore studied for a comparative evaluation with predictive coding.

6.1 Group entropy

To compute the relative entropy of groups of signal elements, the transition matrix element listings are again processed by a PDP-8 Fortran program. For each listing for source approximation order N, the occurrence frequencies of N + 1 element blocks are simply the separate transition totals from every state. The entropy contributions for each such group are totalled and

divided by $N+1$ to obtain the relative entropies per element of the encoded sequence and the results are shown in Fig. 6.1. The group entropies are compared with the Markov process and modulo - 2 adder output sequence entropies based on the same total number of signal elements, including the immediate elements in the last two cases, direct transmission corresponding to a zero order approximation or encoding in groups of one element. The characteristic defines a lower bound to the required channel capacity for signal transmission after group encoding by any procedure.

By optimal coding it is always possible to establish a transformation resulting in an average codeword length $\sum_i p(B_i)\, N_i$ within one element of the average block total entropy $N G_N$, so that an upper bound, also shown in Fig. 6.1, is set by

$$N R_B \leqslant N G_N + 1 = N R_M \qquad (6.1.1)$$

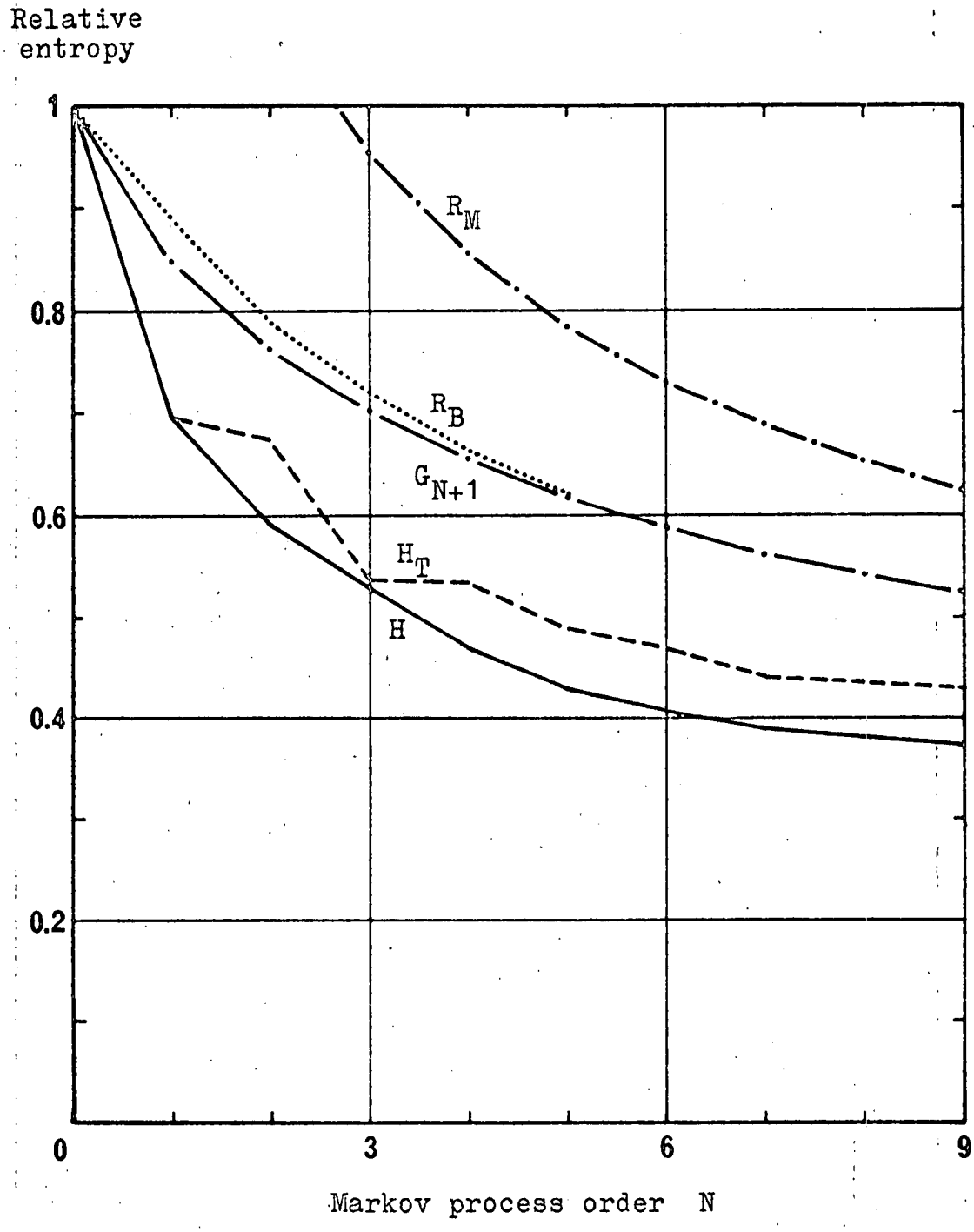
in which the ratio of the channel capacity required for the encoded signal to that for direct transmission

$$R_B = \sum_{i=1}^{z^N} \frac{p(B_i)\, N_i}{N} \qquad (6.1.2)$$

indicates the redundancy reduction produced.

6.2 Optimal group codes

Limited computer memory capacity dictates a two stage computational - topological technique for the determination of



R_M	Upper bound for group encoding
R_B	Huffman encoder characteristic
G_{N+1}	N+1 element group entropy
H_T	Mod - 2 adder sequence entropy
H	Process entropy

Fig. 6.1 Delta coded speech entropies

the optimal codes. First the structures of the trees of successive state subsets are defined by processing the matrix element listings ; then the trees are constructed and the codewords generated by the manual assignment of symbols to the intermediate branches.

For each group length, PDP-8 is programmed to initially scan the terminal nodes and locate the two states for which the sums of the transition probabilities to subsequent states are least. These states are printed and combined as a subset, described by the numerically lower and having probabilities equal to the sums of those of its member pair. After elimination of the higher state from the scan the process is repeated until the final two subsets are combined as the root. Then the state subset trees, of which Fig. 6.2 for 5 element groups is representative, are drawn from the intermediate node locations indicated by the state print-outs and the codewords are determined by tracing the unique path to each state via the annotated branches.

The optimal group codebooks for 2 to 6 signal elements are given in Table 6.3. Codes for larger blocks are unlikely to be of practical interest because of their complexity and the long word lengths associated with low probability states.

6.3 Code evaluation

To evaluate the efficiencies of the group codes, the codeword lengths N_i elements, indicated for each block by

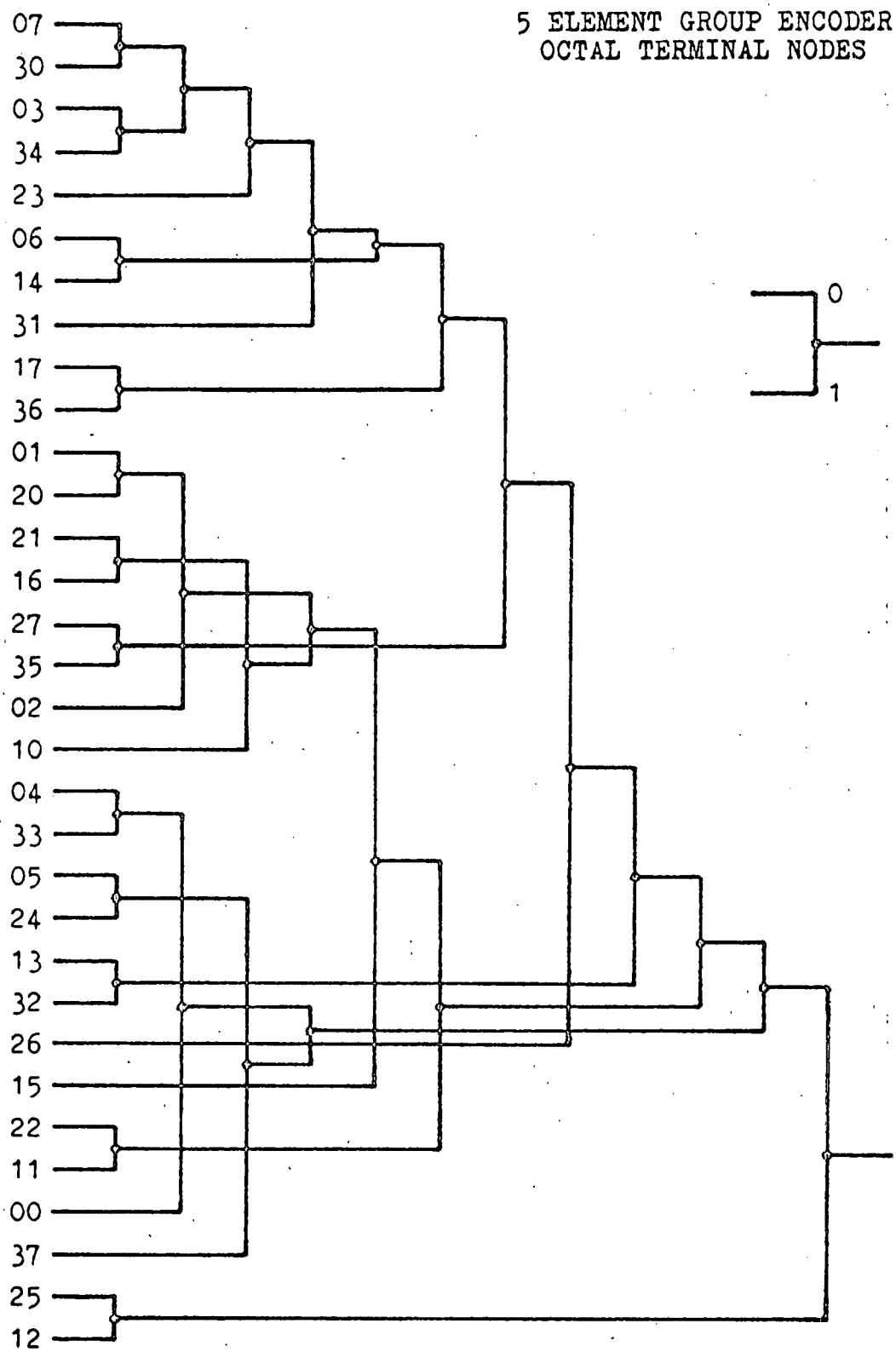


Fig. 6.2 State subset tree

Table 6.3, are loaded by a program which assigns them to the appropriate groups B_i of the source sequence. Each element is considered to be the first of a following group, so that the evaluation for each group length N is the average for N effective test runs with time origins at each of the elements of the leading DM code block. The element numbers for the effective runs are totalled and compared directly with the corresponding total message bit counts and values of R_B are shown in Fig. 6.1.

In appraising the group encoding results, it is noted that even for small N, the codes are efficient in the sense that R_B is near the lower bound set by the relative entropy of groups of N + 1 elements of the source sequences, and the performance characteristic converges to G_{N+1} much faster than does the upper bound R_M. But the convergence of G_N to the process entropy H is slow, so that large blocks must be encoded to achieve a high overall coding efficiency.

In comparison, the relative entropy (zero order) H_T of the modulo - 2 adder output sequence for an N element optimum predictor remains much closer to H for all N, so that a predictive coding approach is found to be superior to optimal group encoding for delta - coded speech. H_T for a 3 element predictor for example, which has a quite elementary structure (Table 5.4), is lower than the attainable R_B for blocks of 8 elements, although the group encoder for the latter is complex in form and generates very long codewords.

TABLE 6.3 OPTIMAL CODEBOOKS FOR DELTA CODED SPEECH

BLOCK LENGTH N+1 ELEMENTS

N = 1

0	001
1	1
2	01
3	000

N = 2

0	001 1
1	000 10
2	1
3	000 00
4	000 11
5	01
6	010 01
7	001 0

N = 3

00	000 1
01	000 001
02	010 1
03	000 000 01
04	011 0
05	10
06	010 01
07	000 011
10	000 010
11	010 00
12	11
13	011 1
14	000 000 00
15	001 0
16	000 000 1
17	001 1

N = 4

00	010 1
01	001 000 00
02	001 000 1
03	000 000 000 010
04	010 00
05	011 00
06	000 000 010
07	000 000 000 000
10	001 001 1
11	001 11
12	11
13	000 10
14	000 000 011
15	001 01
16	001 001 01
17	000 000 10
20	001 000 01
21	001 001 00
22	001 10
23	000 000 000 1
24	011 01
25	10
26	000 01
27	000 001 0
30	000 000 000 001
31	000 000 001
32	000 11
33	010 01
34	000 000 000 011
35	000 001 1
36	000 000 11
37	011 1

N = 5

00	000 11
01	001 010 010
02	000 001 00
03	001 000 100 001
04	000 000 1
05	001 000 100 01
06	001 000 100 000 001
07	001 000 000 001
10	001 001 1
11	001 011 0
12	010 11
13	000 000 001 0
14	001 000 100 000 1
15	001 000 100 1
16	001 000 100 000 000 01
17	000 000 000 100

(N = 5 CODEBOOK CONTINUED)

```
20  000 000 01
21  001 000 01
22  001 11
23  000 000 001 1
24  010 01
25  10
26  011 10
27  000 000 000 01
30  001 000 100 000 000 001
31  001 010 000 0
32  011 00
33  001 011 1
34  001 000 100 000 000 000 01
35  001 001 00
36  000 010 001
37  001 010 001
40  001 010 011
41  000 010 000
42  001 000 11
43  001 000 100 000 000 000 1
44  000 011
45  011 01
46  001 010 000 1
47  001 000 100 000 000 000 00
50  000 000 000 11
51  011 11
52  11
53  010 10
54  001 000 000 1
55  000 10
56  001 001 01
57  000 001 01
60  000 000 000 001
61  001 000 100 000 000 1
62  001 000 001 1
63  000 000 000 000
64  001 000 001 0
65  010 00
66  001 010 1
67  000 001 1
70  000 000 000 101
71  001 000 100 000 01
72  001 000 000 01
73  000 010 1
74  001 000 000 000
75  000 010 01
76  001 000 101
77  001 10
```

Chapter 7

Non-linear prediction

In developing the application of the general results of chapter 5 to the realization of a practical predictive coding procedure, attention is now restricted to a single typical case. Hardware complexity estimates based on Table 5.4 and attainable performance data indicated by Fig. 6.1 suggest that for delta-coded speech with peak-to-peak amplitude 128Δ and sampling frequency 96 KHz a 6 th order predictor offers substantial encoding gains ($\eta = 80\%$, or $H_T = 0.48$) for a small investment per channel and the form of such an encoder is now considered in detail.

7.1 Predictor function

As a consequence of the information source asymptotic equipartition property fundamental to Shannon's central theorems, the source state ensemble for N large comprises a 2^{NH} member set of high probability and a remaining set of low probability. While a distinct set boundary does not occur for $N = 6$, $H = 0.41$, it may be inferred that a number of the 64 states of the ensemble will have quite low probabilities, and this is confirmed by the transition matrix listing. Eight states have occurrence probabilities less than 0.01 %, and in a chain of over 4 million states, there are no occurrences at all

of 34_8, all transitions to 70_8 and 71_8 being from 74_8. This property may be exploited to derive practically convenient predictor functions which represent worthwhile simplifications at the expense of negligible performance degradation from that of the optimal structure.

If the predictor is implemented as a codebook with addressable memory, it is possible without seriously affecting the success probability to group together states with low probabilities of occurrence, and those with high occurrence probabilities but near equiprobable transition probabilities (high state entropies), and assign them an arbitrary prediction collectively. Alternatively, a more efficient simplification may be effected by the study of a Veitch - Karnaugh map representation of the optimal predictor function, which distinguishes those states for which allowing non - optimal prediction results in a direct logic hardware saving, and this is shown in Fig. 7.1. Correlation of the transition matrix with this approach indicates the numerous low probability states for which the simplest factoring is in fact achieved by optimal prediction.

Assertion of the 6 code elements processed is indicated in Fig. 7.1 by binary variables A, B, . . . F in order of generation time and the two dimensional array of predicted elements within the map is formatted to exhibit combinatorial symmetry for variable elimination about the central axes shown

N = 6 PREDICTOR
KARNAUGH MAP

ROW = 1ST 3 BIT GROUP
COL = 2ND 3 BIT GROUP

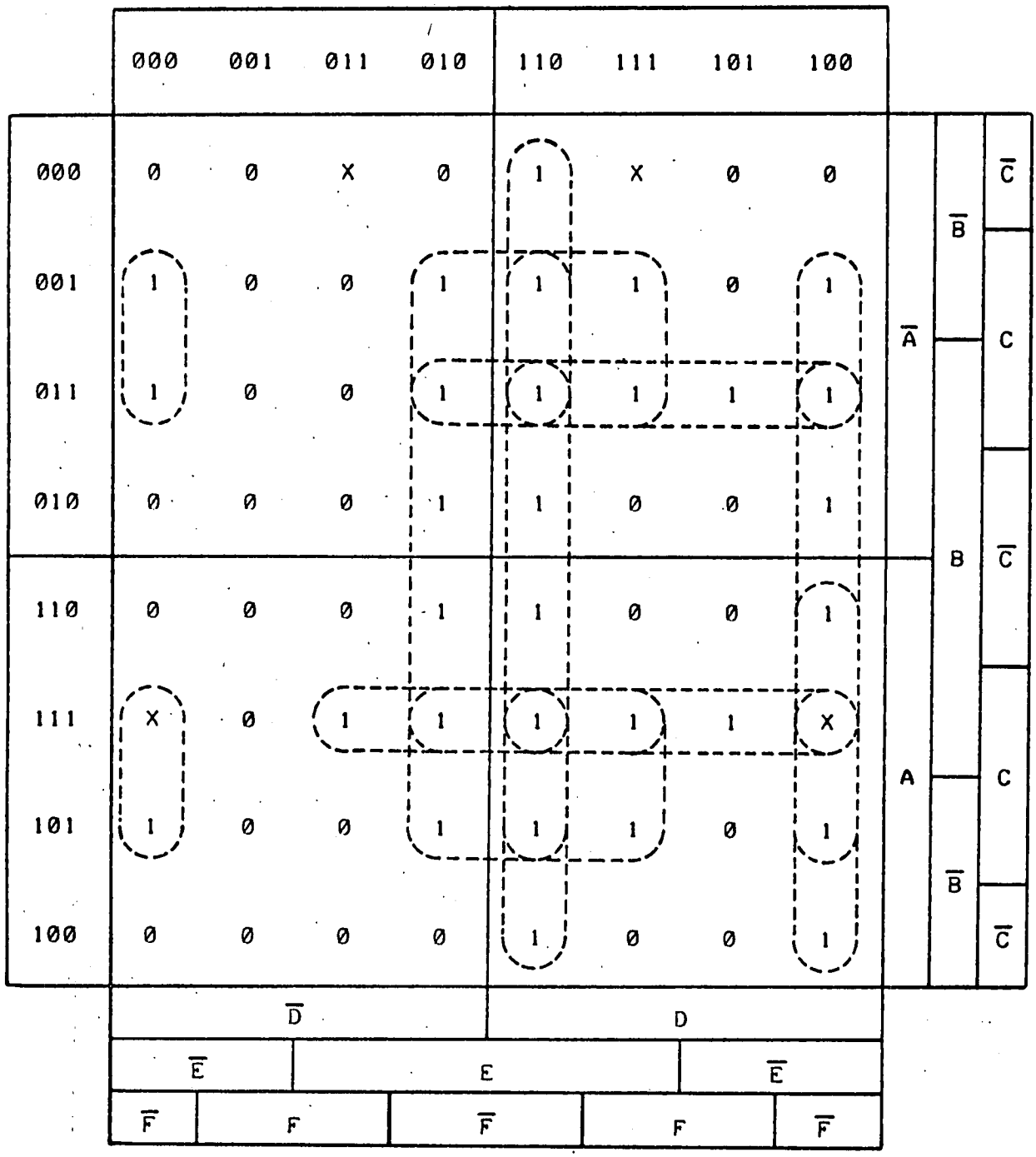


Fig. 7.1

LOGIC FUNCTION

$$A \cdot B \cdot C \cdot E + C \cdot \overline{F} + D \cdot E \cdot \overline{F} + B \cdot E \cdot \overline{F} + C \cdot D \cdot E + B \cdot C \cdot D + B \cdot D \cdot \overline{E} \cdot \overline{F} + A \cdot D \cdot \overline{E} \cdot \overline{F}$$

$$= A \cdot B \cdot C \cdot E + C \cdot \overline{F} + E \cdot \overline{F} \cdot (D + B) + C \cdot D \cdot (E + B) + D \cdot \overline{E} \cdot \overline{F} \cdot (A + B)$$

as well as within the 16 element quadrants. For the map area corresponding to each state, there are six 'adjacent' areas corresponding to states differing in one element only, and between which it may be eliminated by grouping; four from the same quadrant and two images in the opposite parity quadrants.

By assigning 'don't care' predictions X to certain areas, the 1's factoring may be carried out as shown in groups of four or fewer variables by combining four or more elements per term and the sub-optimal predictor function becomes

$$A \cdot B \cdot C \cdot E + C \cdot \overline{F} + E \cdot \overline{F} \cdot (D + B) + C \cdot D \cdot (E + B) + D \cdot \overline{E} \cdot \overline{F} \cdot (A + B) \qquad (7.1.1)$$

7.2 Processor form

The predictor function is compactly programmed for computer evaluation as the sequential application to the current state of a set of masks and tests based on the following properties implied by (7.1.1).

Prediction F: 4 states with C - F = 0, 4 with C - F = 1;
02 and its 5 element equivalent 42;
their 6 element complements 75 and 35;
04 and its complement 73.

Prediction $\overline{F}$: Remaining states, including 4 assigned 'don't cares'.

By prefixing the information analysis program by this test sequence as shown in Appendix 3, and incorporating modulo-2 addition of the prediction and source sequence before the

assembly routines, transition matrix element listings for the predictive coded message are generated which can be processed in the same way as those for the DM signal direct.

The results show that P_s for the former case and $N = 0$ is less than for the latter and $N = 6$ by 0.14 %, which only slightly exceeds the variation between repeat runs. So the performance of the sub-optimal predictor described is little degraded from that of the optimal structure by the simplifications allowed in its derivation.

7.3 IC predictive coder

Existing expertise in monolithic semiconductor technology permits the fabrication of the predictive coder as a single functional array integrated circuit, and to establish the logic configuration for such an array a high speed saturated transistor-transistor encoder design has been evolved as shown in Fig. 7.2.

The encoder gates the outputs of the shift register of three dual J-K master-slave flip flops which store the signal elements to be processed. Complement outputs drive triple 2-input positive NAND gates which OR element 5 with 2, 3 and 6 and then dual 4-input and triple 3-input units effecting logical AND, the outputs of which are OR'd by a single 5-input gate to generate the prediction. While a small logic simplification could be achieved by the integration of the mod-2

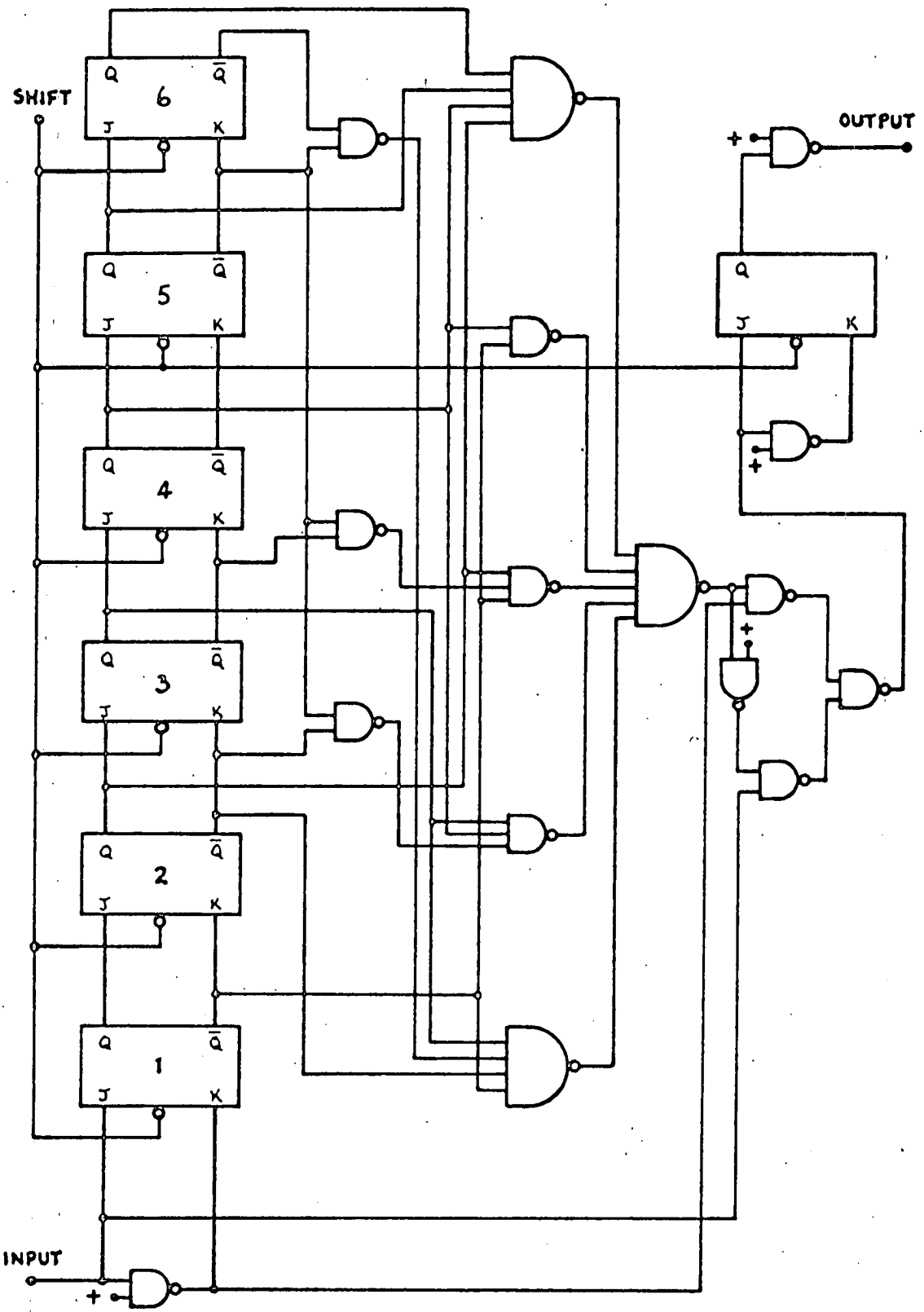


Fig. 7.2 Predictive coder configuration

adder with the prediction gating, it is convenient for system testing to preserve its separate identity and it is therefore assigned a distinct quad 2-input unit. The output of the adder conditions the encoder output flip flop on the arrival of the shift pulse which clocks into slot 1 of the shift register the most recent signal element with which the prediction has been compared. Obvious modifications of the structure form the corresponding decoder, in which the input signal drives the mod-2 adder but the output sequence enters the shift register.

The IC predictive coder is shown in Fig. 7.3. As PDP-8 is a negative logic system, it is powered from the positive bias channel and used in conjunction with level converting modules to exchange signals with the computer interface.

7.4 Data errors

In direct DM transmission, signal detection errors due to channel noise result in an output SNR reduction which has been found by Braun et al[23] to range from 1 dB for error probability P_e = 0.001 to 15 dB for P_e = 0.1. When the redundancy of the signals generated by an information source is reduced, the susceptibility of the message to mutilation by noise is increased. For the predictive coding of delta-coded speech, incorrectly detected elements cause the same immediate signal reconstruction errors on reception, but in addition P_s is reduced by the impairment of the decoder predictor's assessment

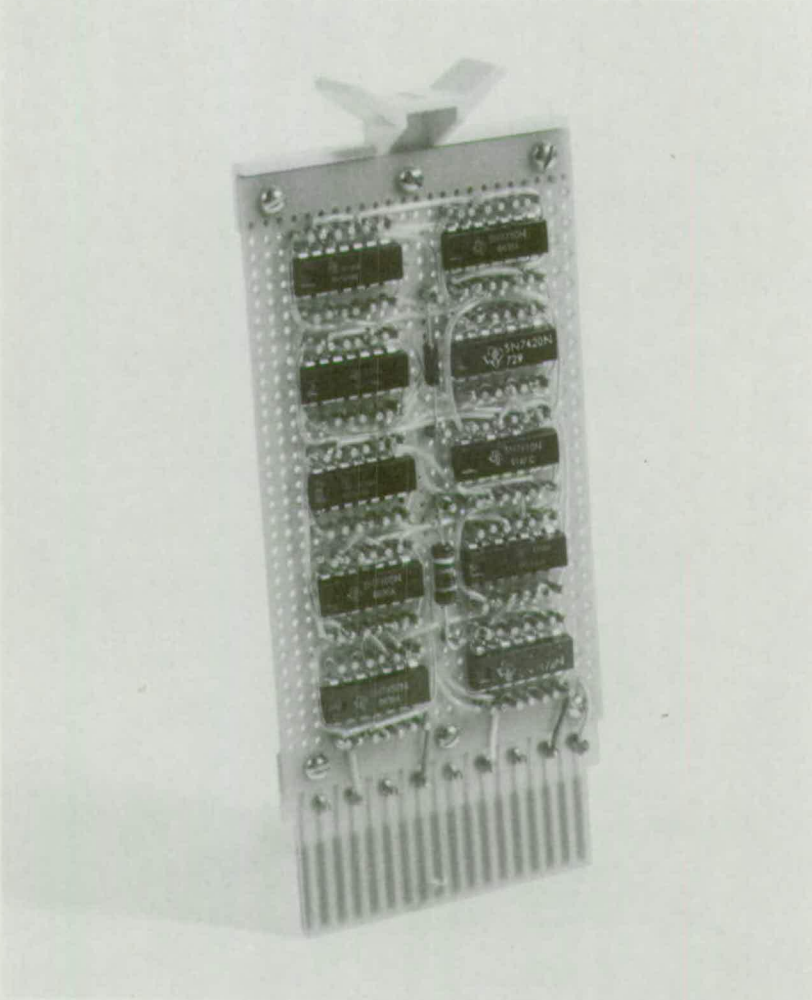


Fig. 7.3 TTL encoder for DM speech

of the subsequent N states and the increased entropy of the data sequence.

The sensitivity of the 6 th order predictor performance to data errors is determined by a series of processor runs in which random element errors are introduced with a range of controlled probabilities P_b by facilities included in the software listing Appendix 3. Routines generate members of a random number set uniformly distributed over 0000 - 7777 during each sampling interval and cause the transmission of complement pulses to the interface when the samples exceed a switch register specified threshold which is P_b as an octal fraction.

The degradation of P_s with increasing data error rate is shown in Fig. 7.4, with the corresponding rise in H_T as the sequence properties change from the constrained statistics of delta - coded speech to those of binary random noise. The predictor remains effective for error rates up to that causing a considerable reduction in output SNR. However, as with all differential transmission systems, it is advisable to inhibit error accumulation by the periodic clearing of encoder and decoder memories, in this case conveniently to 12_8 or 25_8.

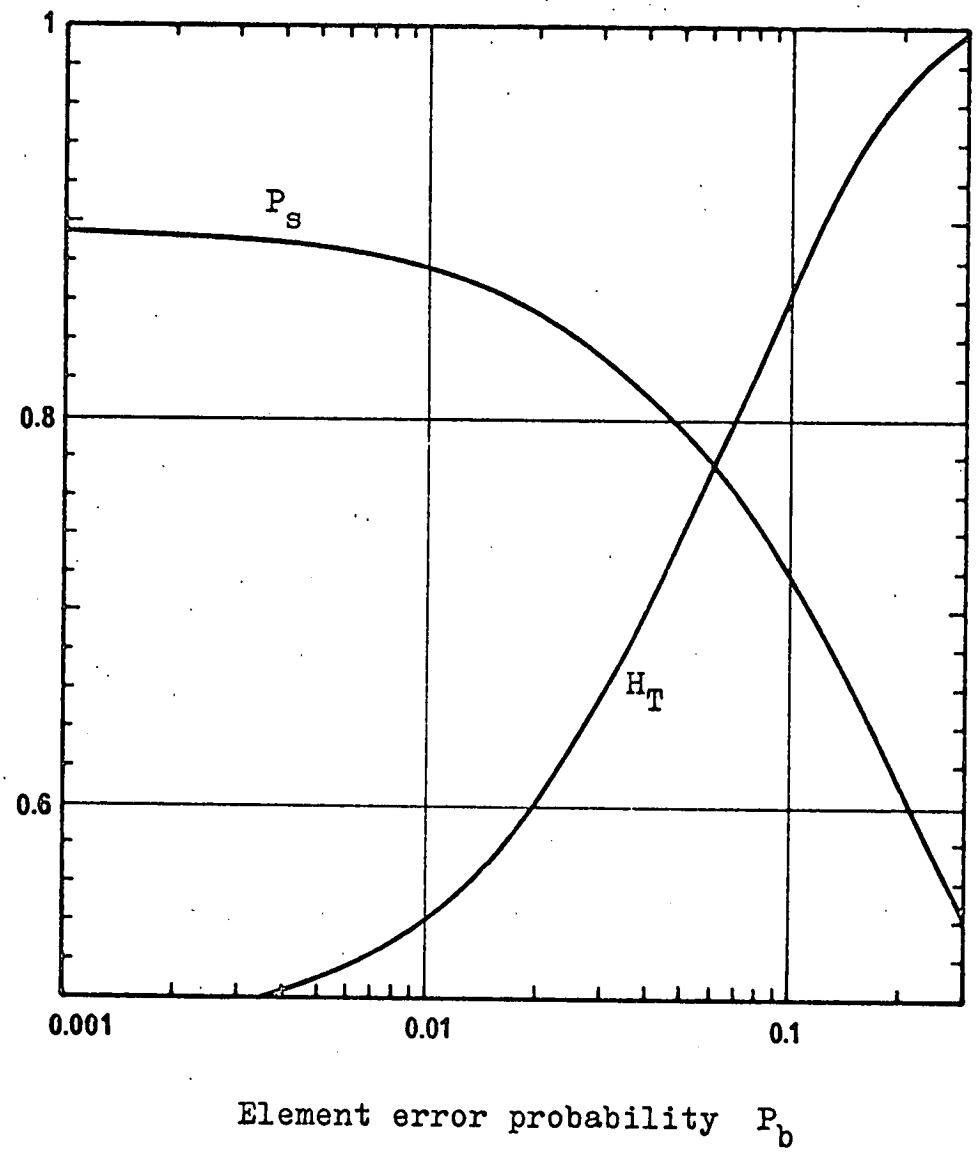


Fig. 7.4 N = 6 predictor error performance

Chapter 8

Error sequence encoding

Having established that the realization of a 6 th order near - optimal predictive coder capable of reducing the average element entropy of a delta - coded speech message by one half is practical, we now explore applications to channel encoding and transmitter power economy. The binary signal sequence generated by modulo - 2 addition of the DM message and predictor output is termed the encoder error sequence, for as detailed in Sec. 4.2 it has the function of indicating the errors in the decoder predictions. Three methods of variable length coding the error sequence to achieve channel bandwidth compression are evaluated and compared.

8.1 Direct error transmission

As was noted earlier, a transmitter power saving is achieved by direct transmission of the predictive coded sequence instead of the original DM message, and for the 6 th order predictor with $P_s = 0.9$ this has value $\eta = 80\%$. In a pack set application in which low level circuits are microminiaturized and the power input to the final transmitter stages is a large proportion of the total, operational life from a primary power source is therefore extended by a factor of several times.

In addition, use of this mode extends to pulse systems the operational advantages brought to analogue communications by suppressed carrier VOX working. For both techniques result in there being no output from a sender except during speech utterances, so that single channel conversational mode operation becomes possible among the contributing members of a communications net, bit synchronisation being established by the transmitting member and no frame synchronisation being required to decode the transmitted sequences.

The entropy characteristics of Fig. 6.1, and the corresponding predictor performances, were computed by processing a speech message generated by the continuous reading of a prepared text. In the case of duplex communications, the power saving (and also the detection immunity) compared with direct delta - coding are therefore further enhanced by the occurrence of intervals during which the transmitter output of a listening operator is zero, although his talk channel remains active for interruption of the sender at any time. The power saving for this case becomes

$$\eta = 100\,(1 - 2P_t + 2P_sP_t)\,\% \qquad (8.1.1)$$

in which P_t is the channel activation probability, so that an extension of operational life by a factor of 10 is indicated in this service for $P_t = 0.5$.

At fixed stations, primary power consumption is generally a minor consideration and for a transmitter limited by output device

mean dissipation the encoding gains are more usefully exploited to allow an increase in channel signal power. The thermal time constants of the dissipating structures of high-power tubes and semiconductors are typically sufficient to smooth the fluctuations during speech and allow close to a factor of $1/2(1 - P_s)$ increase (7 dB for the 6th order predictor), but they are not sufficient to average the dissipated power over active and inactive periods in duplex working. For the case of a peak power limited sender, and for other ways of exploiting the redundancy of delta-coded speech, we must consider further processing of the encoder error sequence.

8.2 Error sequence entropy

In the channel encoder situation, it is required to translate the attainable average element entropy reduction to a decrease in bit rate*, either to increase the size of the TDM group or (pursuing a total coding philosophy advocated in the literature) to cascade the operation of removal of the inefficient source redundancy with that of substitution by the check digits of an error-correcting code chosen specifically for the particular channel noise properties encountered.

By processing the transition matrix listings produced by the software predictive coder of Sec. 7.2 by the Fortran programs outlined in Sec. 5.2, the error sequence information properties

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*This may be termed a 'bandwidth compression' although the component digital signals are not associated with distinct spectral segments of their multiplex.

The TDM group situation is considered because the buffer requirement it demands is moderate.

summarized in Fig. 8.1 are determined. The process entropy characteristic given indicates the reduction of the statistical interdependence of sequential elements resulting from the coding, for the invariant distribution for the zero order case with element probabilities P_s, $1 - P_s$, has entropy not exceeding those computed from the sets of conditional distributions for $N = 1 - 9$ by more than 16%. The variation with N of TH_T, the average element entropy of the sequences generated by a second application of predictive coding, using optimal predictors of order N, to the error sequence for coding with the near-optimal 6 th order predictor, is even less over this range. In fact an extension of the initial predictor order by 1 achieves an entropy reduction which it requires a second application predictor order $N > 9$ to match.

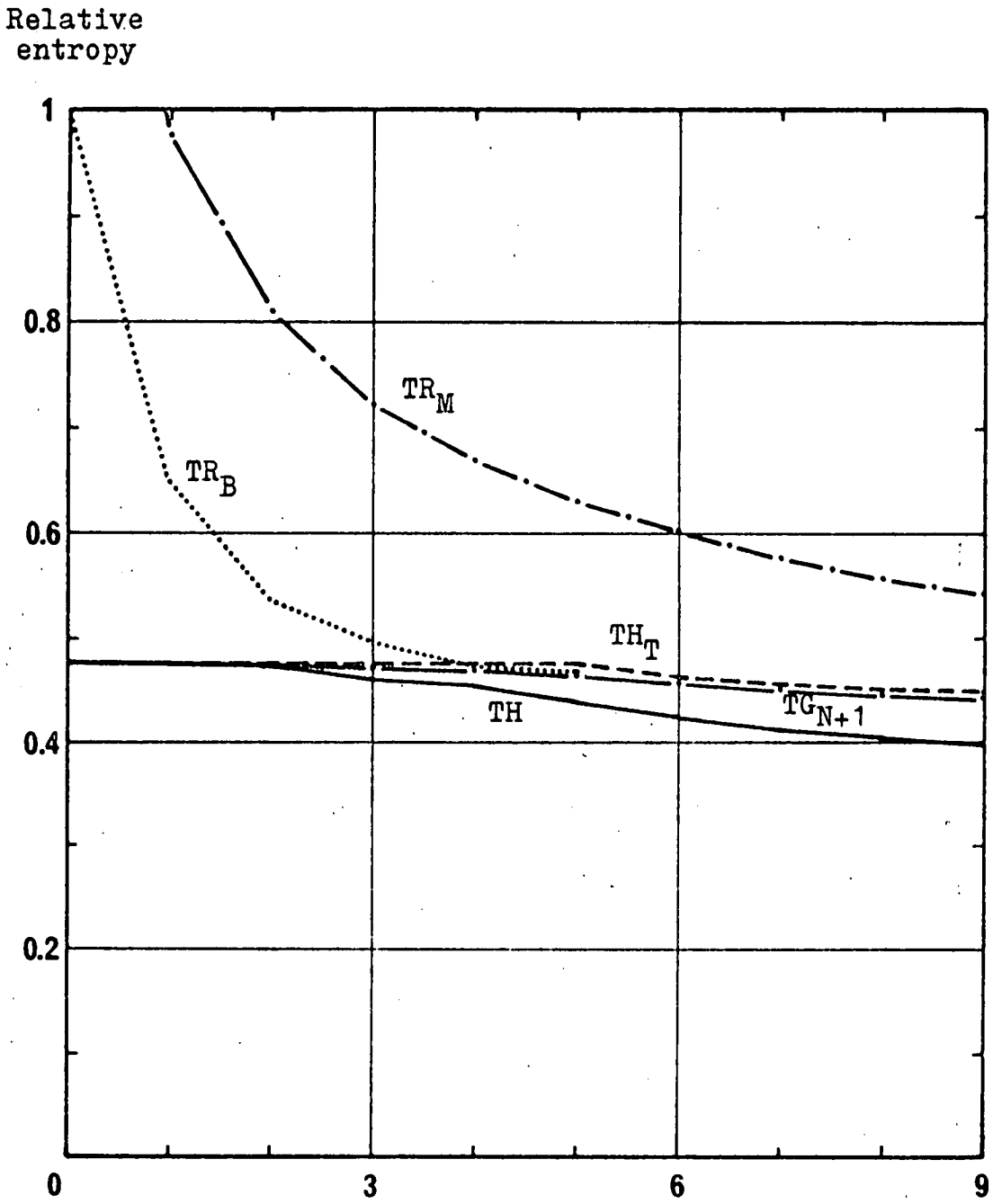
8.3 Run-length encodings

The zero order approximation to the error sequence suggested by the entropy results of Fig. 8.1 implies a geometric probability distribution for the lengths of runs of consecutive 0 symbols in the sequence. The probability of a run of length r ($r \geq 0$, so that every symbol 1 both terminates and starts a run) is

$$P_{gr} = P_s^r(1 - P_s) \qquad (8.3.1)$$

and the entropy per selection from the distribution,

$$H_g = \sum_{r=0}^{\infty} P_{gr} \log P_{gr}, \qquad (8.3.2)$$



TR_M	Upper bound for group encoding
TR_B	Huffman encoder characteristic
TG_{N+1}	N+1 element group entropy
TH_T	Mod - 2 adder sequence entropy
TH	Process entropy

Fig. 8.1 Error sequence entropies

has value 4.66 bits for the 6 th order predictor.
The average sequence length per run,

$$L_g = \sum_{r=0}^{\infty} P_{gr}(r+1) \qquad (8.3.3)$$

is 9.83 elements and the relative entropy per element $H_{gr} = {}^{H_g}/_{L_g} = 0.474$, corresponding to TH_T for $N = 0$.
Similar form relations derive from the actual run probabilities P_r.

Error sequence run-length distributions are determined by processor tests with a program assembled by Macro-8 which executes predictive coding of the DM speech input and, on termination of all run-lengths $r < 256$, increments the appropriate registers of double precision occurrence counts assigned to 4 pages of core memory. The run occurrence frequencies are punched out on paper tape and subsequently processed by Fortran programs. Fig. 8.2 compares the run-length distribution for the typical DM parameter case with the geometric distribution for the same P_s, and distributions for other cases are similar. Deviations for small r are caused by the previous element dependence of the error element distribution remaining after coding, and for $r \geq 7$, P_r is always less than P_{gr}.

From the run probability results, the average entropy per run is computed as 4.31 bits and the average sequence length per run as 9.75 elements so that the relative entropy per element $H_r = 0.442$ is rather lower than for the geometric distribution.

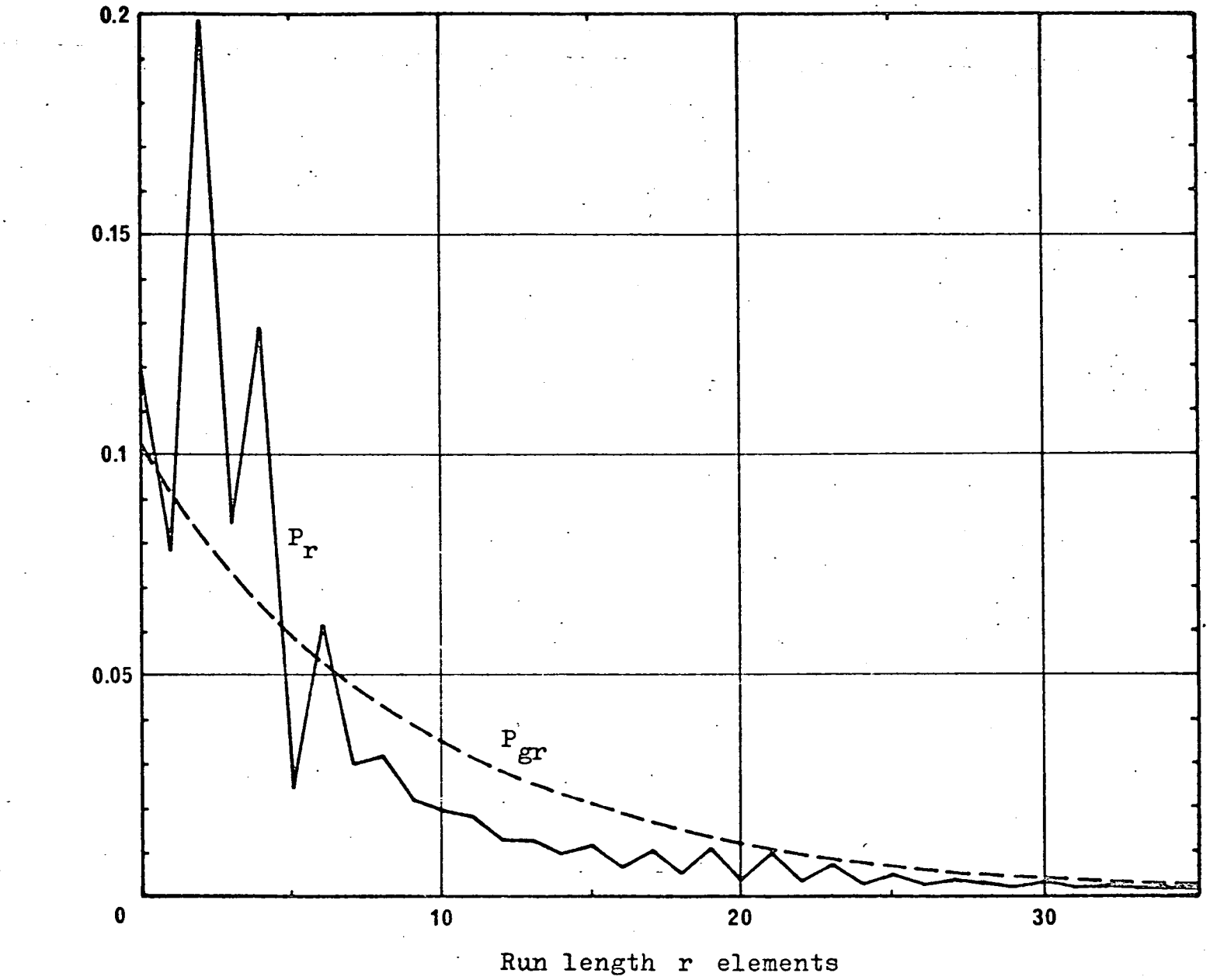


Fig. 8.2 Run length distributions

While Huffman encoding of the run-lengths in accordance with their occurrence probabilities allows this compression to be closely attained, the codebook is quite large (64 entries to include 98 % of the run-lengths) and the encoder, although very much simpler than that which would be required to achieve the same compression by a direct application to the source sequence (over 32K entries), is still unattractively complex.

Considering only comma-free codes for direct transmission through binary channels, a first practical alternative is fixed length binary number coding of the run-lengths, which of course is still a variable rate encoding procedure overall because of the distribution of element sequence lengths selecting each number. For this approach 6 bit coding yields a compression of 0.615, while encoding only run-lengths 0 - 31 (which includes 94 % of the total) by 5 bits results in a ratio of 0.513. The occurrence of the longest runs, which are associated with quiet intervals between spoken words, is a function of the system noise and reverberation prior to source encoding and is likely to be less frequent in the average communications situation than for the studio environment in which the test message is prepared and the precision laboratory equipment by which it is processed.

For a second procedure of intermediate complexity, the set of P_r may be approximated by a geometric distribution which Golomb[24] has noted is favourable to Huffman encoding for

integer values of

$$m = -\frac{\log 2}{\log P_s} \qquad (8.3.4)$$

For the 6 th order predictor, $m = 6$ is appropriate and the initial entries for the run-length codebook are given in Table 8.3, examination of the structure of which reveals the following encoding rule for a run-length of r elements :

By division of r by 6, obtain $6A + R$. (integers $A \geq 0$, $0 \leq R \leq 5$)

Encoding rule : Output A binary 1's, followed by

$$\begin{cases} \text{for } 0 \leq R \leq 1, \text{ the 3 bit binary representation of } R \\ \text{for } 2 \leq R \leq 5, \text{ the 4 bit binary representation of } R+2 \end{cases}$$

An equally straightforward decoding rule applies.

Evaluation of this encoding procedure for the actual run-length distribution of Fig. 8.2 indicates an average relative entropy (per element of the error sequence) of 0.486, reduced to 0.406 by truncation of the codebook at 32 entries. The approximation of the run-length statistics by a geometric distribution, and of P_s by a value giving m integer, thus yields a practical run-length encoding scheme with performance within 10 % of H_r.

8.4 Group error encoding

While predictive coding has been considered in earlier sections as an IP transformation lowering the source sequence

TABLE 8.3 RUN LENGTH CODEBOOK

M = 6 GEOMETRIC DISTRIBUTION

R	CODEWORD
0	000
1	001
2	010 0
3	010 1
4	011 0
5	011 1
6	100 0
7	100 1
8	101 00
9	101 01
10	101 10
11	101 11
12	110 00
13	110 01
14	110 100
15	110 101
16	110 110
17	110 111
18	111 000
19	111 001
20	111 010 0
21	111 010 1
22	111 011 0
23	111 011 1
24	111 100 0
25	111 100 1
26	111 101 00
27	111 101 01
28	111 101 10
29	111 101 11
30	111 110 00
31	111 110 01

average element entropy, significant reductions are effected in the entropies of blocks of N elements for $N \geqslant 2$ also. The group entropy characteristic TG_{N+1} and upper bound for the optimal coding TR_M shown in Fig. 8.1 are obtained for the encoder error sequence by the processing method described in chapter 6 for the DM signal prior to coding. In the present case, TG_{N+1} remains much closer to TH, even for N small, and is actually slightly less than TH_T.

As a third approach, the group codes are therefore constructed by the application, to the encoder error sequence transition matrix element listings, of the computational - topological procedure described earlier, and the optimal codebooks are given in Table 8.4. Evaluation of these codes yields the characteristic TR_B shown in Fig. 8.1 for the reduction in required channel capacity. Convergence to the lower bound occurs rapidly, an average bit rate reduction of one half being obtained with groups of 4 only, and TR_B being less than TH_T for block length $\geqslant 5$. An attractive case, compromising considerations of coding efficiency and hardware complexity, is the 5 element group code which achieves $TR_B = 0.479$.

Comparison of the most practical case performance for each of the three error sequence encoding procedures described is now possible and the compression factors attainable are summarized below.

TABLE 8.4 OPTIMAL ERROR SEQUENCE CODEBOOKS

BLOCK LENGTH N+1 ELEMENTS

N = 1

0	1
1	01
2	000
3	001

N = 2

0	1
1	010
2	001
3	000 10
4	011
5	000 00
6	000 11
7	000 01

N = 3

00	1
01	010 0
02	011
03	000 010
04	001
05	000 101
06	000 001
07	000 000 10
10	010 1
11	000 11
12	000 011
13	000 000 01
14	000 100
15	000 000 001
16	000 000 11
17	000 000 000

N = 4

00	1
01	011 1
02	010 1
03	010 000 00
04	001
05	010 000 1
06	000 000 1
07	000 001 010
10	011 0
11	000 01
12	010 001 1
13	000 001 011
14	010 001 0
15	000 000 000 000 00
16	000 001 000 1
17	000 000 010
20	000 1
21	010 010 0
22	010 011
23	010 000 01
24	000 001 1
25	000 000 000 1
26	000 000 001
27	000 000 000 010
30	010 010 1
31	000 001 000 0
32	000 000 000 011
33	000 000 000 000 01
34	000 001 001
35	000 000 000 001
36	000 000 011
37	000 000 000 000 1

N = 5

00	1
01	010 1
02	011 0
03	000 100 001
04	001 1
05	001 010 0
06	001 000 01
07	000 100 100 01
10	011 1
11	000 101
12	001 000 1
13	000 100 100 1
14	001 001 1
15	000 100 000 000 000
16	000 100 100 00
17	000 000 000 1

(N = 5 ERROR SEQUENCE CODEBOOK CONTINUED)

20	010 0
21	000 100 11
22	000 011
23	000 000 01
24	001 010 1
25	000 100 000 11
26	000 000 101 1
27	001 000 000 010
30	001 001 0
31	000 010 000 0
32	000 100 000 000 001
33	001 000 000 000 000 001 0
34	000 100 000 01
35	000 100 000 001 1
36	000 000 100 1
37	000 100 000 000 010
40	000 11
41	001 011
42	000 010 1
43	001 000 001
44	000 001
45	001 000 000 1
46	000 100 101
47	000 000 000 0
50	000 100 01
51	000 000 001
52	000 100 000 10
53	001 000 000 000 000 01
54	000 000 101 0
55	001 000 000 000 000 000 1
56	001 000 000 000 01
57	000 100 000 001 0
60	000 000 11
61	000 010 01
62	000 010 000 1
63	000 100 000 000 1
64	001 000 000 011
65	001 000 000 000 000 1
66	001 000 000 000 000 001 1
67	001 000 000 000 000 000 01
70	000 010 001
71	001 000 000 000 1
72	001 000 000 001
73	001 000 000 000 000 000 000
74	000 000 100 0
75	001 000 000 000 001
76	000 100 000 000 011
77	001 000 000 000 000 000 001

(a)	6 element binary number coding of run-lengths	0.615
(b)	Geometric distribution run-length encoding	0.486
(c)	5 element group encoding of error sequence	0.479

Method (a) achieves simplicity at the cost of reduced efficiency, while speed considerations suggest that method (c), which attains the best performance with increased memory capacity, is preferable to obtaining a similar compression with greater arithmetic logic by method (b).

Chapter 9

Channel buffer

In speech communication, as opposed to telegraphy, the rate of reconstruction of the message for the recipient cannot be varied from its rate of generation without distortion of the intelligence conveyed, and this requirement is easily met in the case of conventional DM transmission for which the necessary channel capacity is the fixed maximum for $N = 0$. Where redundancy reduction is effected by a statistical encoding procedure, however, the required channel capacity is proportional to the information content of the source messages. At channel encoders and decoders, buffer stores are therefore necessary to smooth the fluctuating data rate for transmission and reconstruct its variation with time on reception. The analysis of the channel buffer situation presented in this chapter utilizes the theory of queueing (see Fry[25]) developed by telephone traffic statisticians for considering trunking problems, and telephone switching system terminology is retained where it remains appropriate.

9.1 Queue organisation

For the specific numerical results, it is assumed that redundancy reduction is achieved by the 6th order predictor derived in chapter 7 followed by 5 element group encoding of the

error sequence by the code given in Table 8.4 ($N = 4$). Each DM source, with data rate f_s, thus selects for transmission codewords of length $N_c = 1 - 14$ elements (excluding 2 and 11) at a rate $f_s/5$.

The sequence of codewords generated by successive selections by the members of the multiplex of N_m sources is held in the buffer store, of limited capacity N_q words, and this queue is serviced in order by the sender which transmits corresponding signals over the channel at a bit rate $N_m f_c$. For its complete transmission, each codeword requires a 'holding time' proportional to its length, after which the channel becomes available to service the next codeword in the buffer.

When a codeword is generated, it encounters a number w of codewords ahead of it awaiting or undergoing transmission. With a probability which is small, w is zero and the codeword is processed immediately; while more frequently it experiences a delay which will have duration $> \tau$ if $w - 1$ or fewer codeword transmissions are completed during time τ. Buffer capacity and channel data rate require to be chosen such that τ rarely exceeds $5 N_q/f_s N_m$, so that the probability of buffer overflow* is small. The requirement for the receiver is

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*Blasbalg and Van Blerkom[26] suggest degrading the source fidelity to maintain transmission when buffer overflow occurs. Short codewords which approximate the longer sequences selected may be sent until the overflow clears.

identical, codewords transmitted by the channel entering the buffer store which receives sequential service from the group decoder.

9.2 Codeword length distribution

From the information analysis of the predictor error sequence described in Sec. 8.2 the total occurrence frequency of each 5 element state is found and, by grouping those states for which the corresponding codewords from Table 8.4 are equal in length, the probabilities of all N_c are computed. The resultant integral distribution function $p(>N_c)$ is given in Fig. 9.1 and the average codeword length

$$N_a = \sum_{N_c=1}^{14} p(N_c)\, N_c \qquad (9.2.1)$$

is 2.40 elements.

As shown, a negative exponential approximation to the distribution with the same average may be taken, for which

$$p(N_c) = \frac{1}{N_a} e^{-N_c/N_a} \qquad (9.2.2)$$

For N_m large, successive codewords in the buffer queue may be considered to have lengths independently selected in accordance with this probability set.

The average channel holding time is $N_a/N_m f_c$, and the probability of a holding time exceeding t,

$$p(>t) = e^{-t N_m f_c/N_a} \qquad (9.2.3)$$

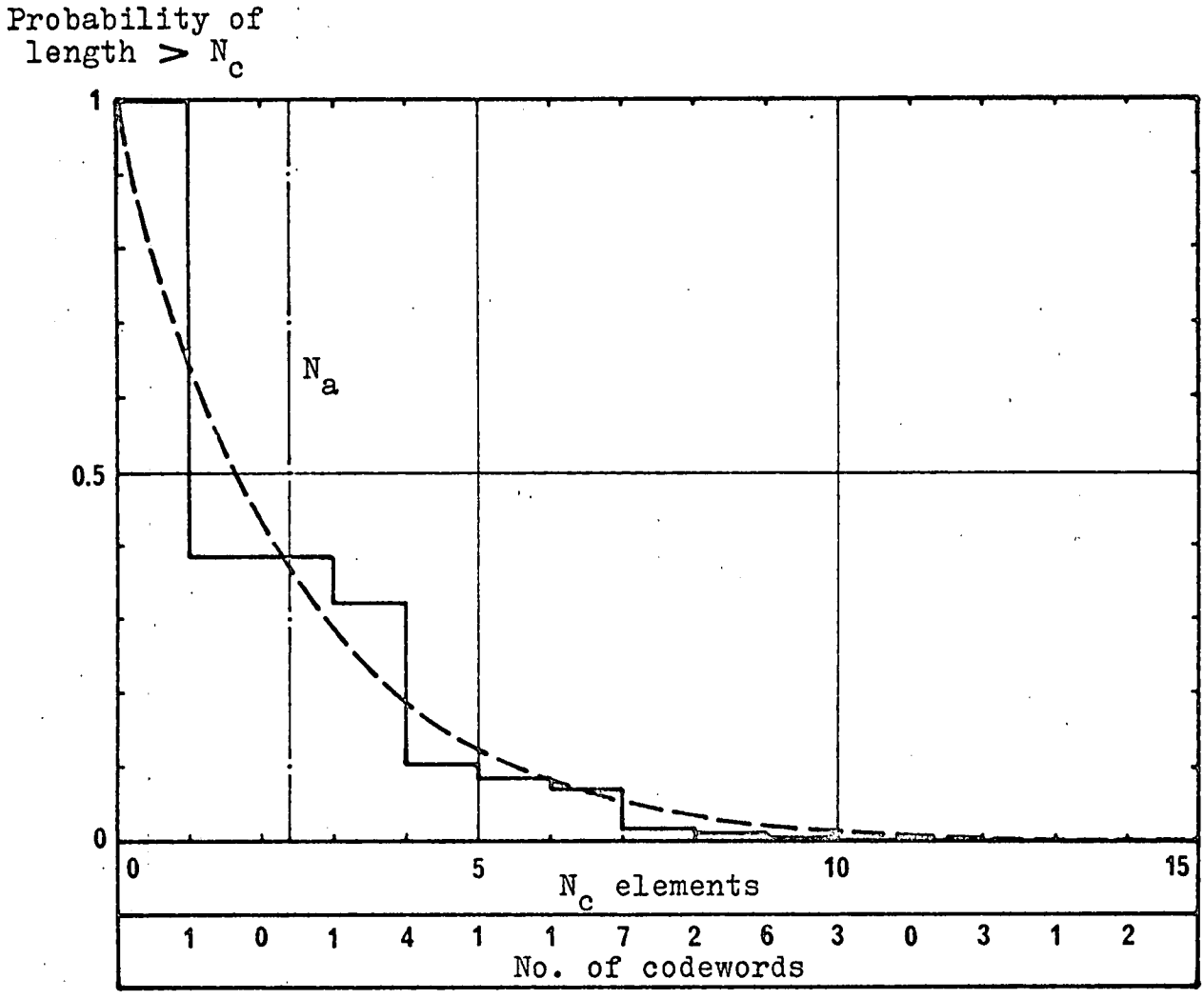

Fig. 9.1 Codeword length distributions

while a holding time between t and dt occurs with probability

$$p(t)\,dt = \frac{N_m f_c}{N_a} e^{-\frac{t N_m f_c}{N_a}} dt \qquad (9.2.4)$$

Thus the probability of the transmission of a codeword ending during dt,

$$\frac{p(t)\,dt}{p(>t)} = \frac{N_m f_c}{N_a} dt \qquad (9.2.5)$$

independent of the time of commencement of transmission.

Now the probability p_1 that one codeword transmission ends during an interval τ is the probability that none ends between 0 and t ($t < \tau$), one ends between t and t + dt, and none ends in the remaining time up to τ. From (9.2.3) and (9.2.5),

$$p_1 = \int_0^{\tau} e^{-\frac{t N_m f_c}{N_a}} \cdot \frac{N_m f_c}{N_a} \cdot e^{-\frac{(\tau - t) N_m f_c}{N_a}} dt$$

$$= \left(\frac{\tau N_m f_c}{N_a}\right) \cdot e^{-\frac{\tau N_m f_c}{N_a}} \qquad (9.2.6)$$

Similarly for 2 codewords,

$$p_2 = \int_0^{\tau} e^{-\frac{t N_m f_c}{N_a}} \cdot \frac{(\tau - t) N_m f_c}{N_a} \cdot e^{-\frac{(\tau - t) N_m f_c}{N_a}} \cdot \frac{N_m f_c}{N_a} dt$$

$$= \left(\frac{\tau N_m f_c}{N_a}\right)^2 \frac{1}{2} e^{-\frac{\tau N_m f_c}{N_a}} \qquad (9.2.7)$$

Extending this approach, the probability that transmission of w - 1 codewords is completed during time τ has the Poisson distribution

$$p_{w-1} = \left(\frac{\tau N_m f_c}{N_a}\right)^{w-1} \frac{1}{(w-1)!}\ e^{-\frac{\tau N_m f_c}{N_a}} \qquad (9.2.8)$$

9.3 Buffer capacity

From (9.2.5) and (9.2.8), the probability that a particular codeword is delayed in the buffer for an interval τ and then the channel becomes available during dt is

$$p_\tau = \sum_{w=1}^{\infty} p(w) \left(\frac{\tau N_m f_c}{N_a}\right)^{w-1} \frac{1}{(w-1)!}\ e^{-\frac{\tau N_m f_c}{N_a}}\ \frac{N_m f_c}{N_a}\ d\tau \qquad (9.3.1)$$

For a total rate of selection of codewords of $N_m f_s/5$, we have from the recurrence relations of queueing theory (Molina[27])

$$p(w) = p(0) \left(\frac{f_s}{5} \cdot \frac{N_a}{f_c}\right)^w \qquad (9.3.2)$$

For

$$\sum_{w=0}^{\infty} p(w) = 1, \qquad p(0) = \frac{1}{\sum_{w=0}^{\infty} \left(\frac{f_s N_a}{5 f_c}\right)^w}$$

$$= 1 - \frac{f_s N_a}{5 f_c} \qquad (9.3.3)$$

(Jolley[28])

Substituting in (9.3.1),

$$p_\tau = \left(1 - \frac{f_s N_a}{5 f_c}\right) \frac{N_m f_s}{5}\ e^{-\frac{\tau N_m f_c}{N_a}} \sum_{w=1}^{\infty} \frac{\left(\frac{\tau N_m f_s}{5}\right)^{w-1}}{(w-1)!}\ d\tau$$

$$= \left(1 - \frac{f_s N_a}{5 f_c}\right) \frac{N_m f_s}{5}\ e^{-\left(1 - \frac{f_s N_a}{5 f_c}\right)\frac{\tau N_m f_c}{N_a}}\ d\tau \qquad (9.3.4)$$

Hence the probability of a codeword being delayed

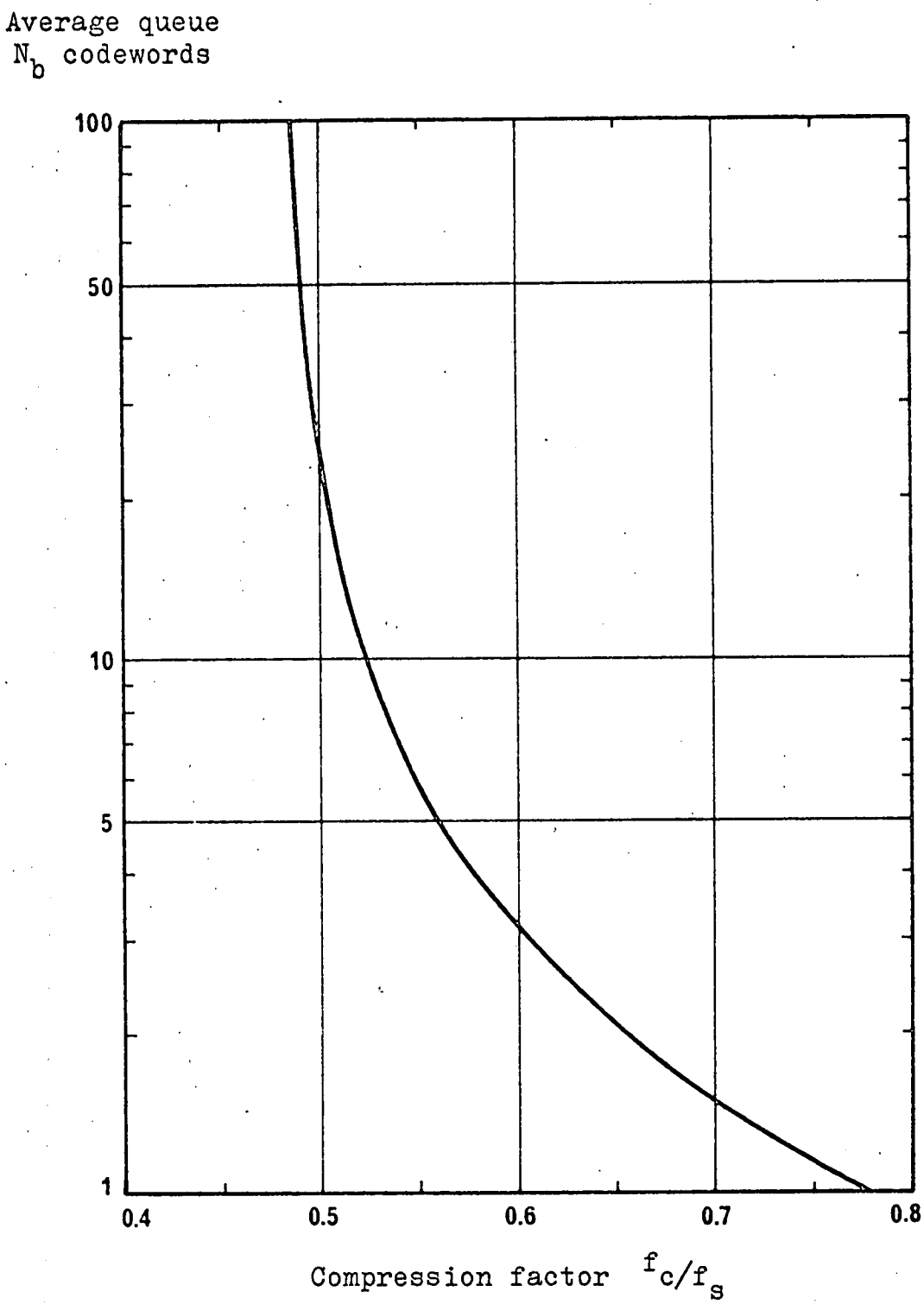

Fig. 9.2 Average queue length

greater than t before transmission

$$p(>t) = \frac{N_m f_s}{5}\left(1-\frac{f_s N_a}{5f_c}\right)\int_t^\infty e^{-\frac{t N_m f_c}{N_a}\left(1-\frac{f_s N_a}{5f_c}\right)}\,dt$$

$$= \frac{f_s N_a}{5f_c}\, e^{-\frac{t N_m f_c}{N_a}\left(1-\frac{f_s N_a}{5f_c}\right)} \qquad (9.3.5)$$

The average number of codewords stored in the buffer is then

$$N_b = \frac{N_m f_s}{5}\int_0^\infty t\,\frac{d}{dt}p(>t)\,dt$$

$$= \frac{(f_s N_a)^2}{(5f_c - f_s N_a)\,5f_c} \qquad (9.3.6)$$

which is independent of the number of sources multiplexed and is shown as a function of the compression factor f_c/f_s in Fig. 9.2. While queue length increases without limit as $(f_c/N_a)/(f_s/5) \to 1$, it falls rapidly to practical values as f_c is increased above the minimum permitted by TR_B.

From (9.3.5), the probability of buffer overflow becomes

$$P_{ov} = \frac{f_s N_a}{5f_c}\cdot e^{-N_q\left(\frac{5f_c}{f_s N_a}-1\right)} \qquad (9.3.7)$$

Fig. 9.3 gives representative channel buffer characteristics which allow the selection of an appropriate compromise between attainable compression factor and required buffer capacity for a range of overflow probabilities. For a 2 to 1 reduction in data rate, $P_{ov} < 10^{-5}$ is obtained with $N_q = 270$.

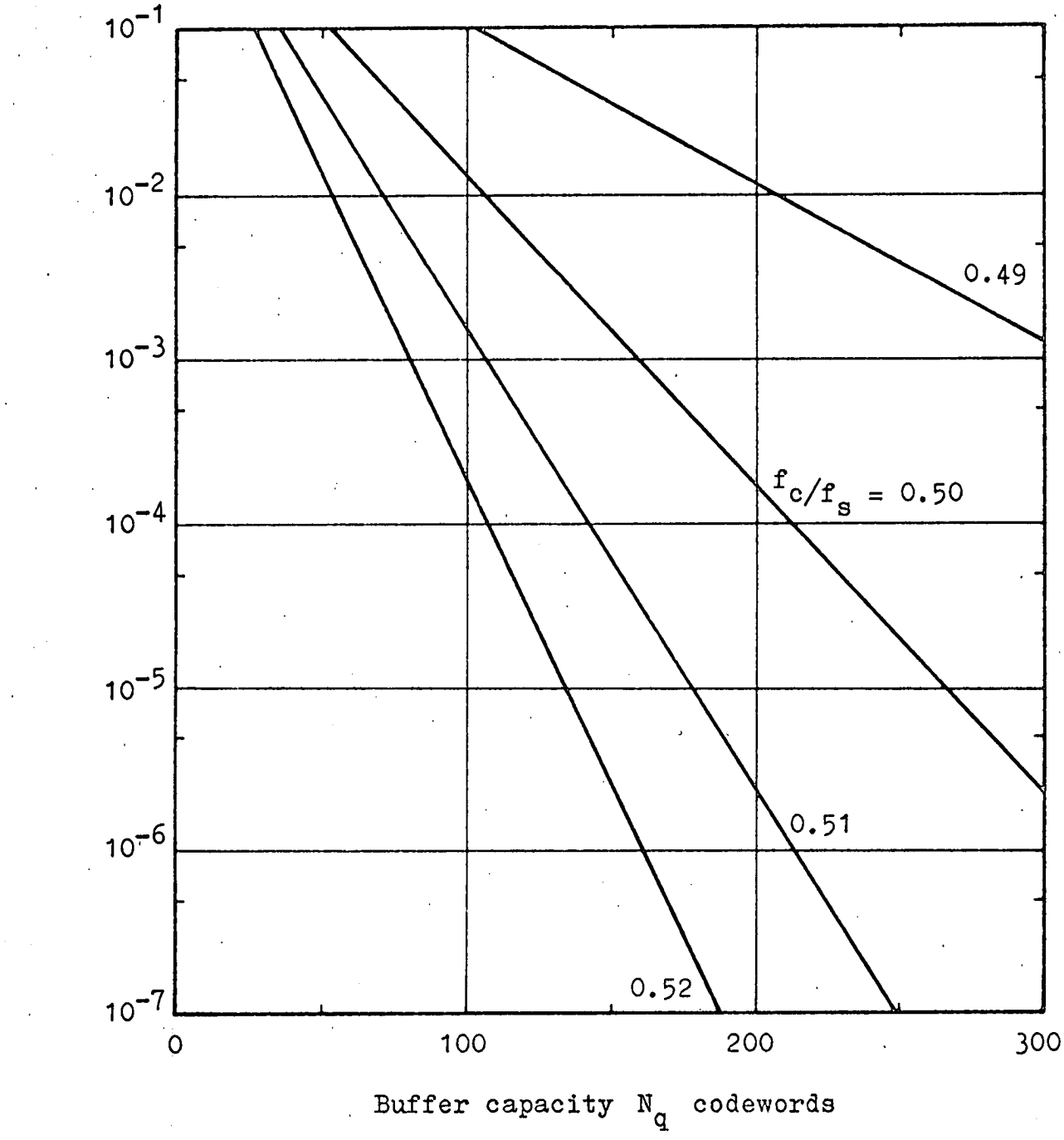

Fig. 9.3 Channel buffer characteristics

Chapter 10

DM signal detection

The previously considered applications of the relative entropy studies on delta-coded speech have been at the sending terminal of a communications link, to effect savings in required transmitter power or channel capacity. By an application of statistical decision theory it is also possible to exploit the knowledge of the message source information properties at a DM receiver to improve the signal detection performance. The gains attainable in this way are less than those for channel encoding, but the method can be very simply applied to an existing delta-coded speech circuit to secure a useful error rate reduction without any change of coding and signalling procedure.

A DM receiver improvement has been suggested by Tanaka et al[29], who show that for a sine wave signal an error rate reduction is achieved by replacing the channel signal by a 1 element prediction when the former is in the vicinity of the threshold. The receiver structure presented in this chapter employs switching of the decision threshold by the 6 element predictor described previously to achieve optimal (minimum error probability) detection for speech messages.

10.1 Optimal receiver structure

For a DM receiver performing bit - by - bit detection of signal elements from channel signals impaired by additive Gaussian noise, a Bayes' strategy[30] of minimizing the average risk is appropriate. Equal costs may reasonably be assigned to the errors occurring when the received signal x is judged to have been caused by transmitted signal θ_0 or θ_1 when the reverse is true and then the threshold K with which the likelihood ratio

$$\Lambda(x) = \frac{p(x \mid \theta_1)}{p(x \mid \theta_0)} \qquad (10.1.1)$$

is to be compared becomes simply

$$K = \frac{p(\theta_0)}{p(\theta_1)} \qquad (10.1.2)$$

Since in this case the total probability of error is minimized, the strategy is also that of Siegert's 'ideal observer'.[31]

In a typical optimum coherent discrete communications system, the signals with cross - correlation coefficient ρ representing the transmitted binary symbols are either orthogonal ($\rho = 0$) or antipodal ($\rho = -1$) and their replicas, either locally generated or stored as the impulse responses of two matched filters,* are cross - correlated in the Bayes'

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*For white noise, the filters have impulse responses of the form of the signals θ_0, θ_1 run backwards in time from T.

receiver structure with the signal delivered by the channel. To the difference x_c between the correlator outputs at the end of the signal interval T is added the negative of a bias level b corresponding to the Bayes' test decision boundary and the sum is limited and sampled to generate the detector output sequence.

Fig. 10.1 indicates this structure for the $\rho = 0$ case of on-off pulse transmission, in which case only one correlator is required as signal $\theta_0 = 0$, and the probability density distributions $p(x_c | \theta_i)$, $i = 0, 1$, are shown in Fig. 10.2. For channel noise power σ^2, average signal power M^*, $E[x_c]$ for θ_1 transmitted is

$$\mu = 2M(1 - \rho) \qquad (10.1.3)$$

and the conditional distributions, which remain Gaussian because of the linearity of the correlation detector, have

$$\sigma_c = \sigma\sqrt{2M(1 - \rho)} \qquad (10.1.4)$$

10.2 Decision boundary loci

From (10.1.1) and (10.1.2), the decision boundary x_d is found by setting

---

*i.e. 'mark' power = 2M. This definition is chosen so that the results are identical for the case of equi-energy orthogonal signalling (eg. coherent FSK) in which a second correlator is required for θ_0 and the distributions for x_c are translated for symmetry about the origin.

Synchronous crosscorrelator Variable threshold detector

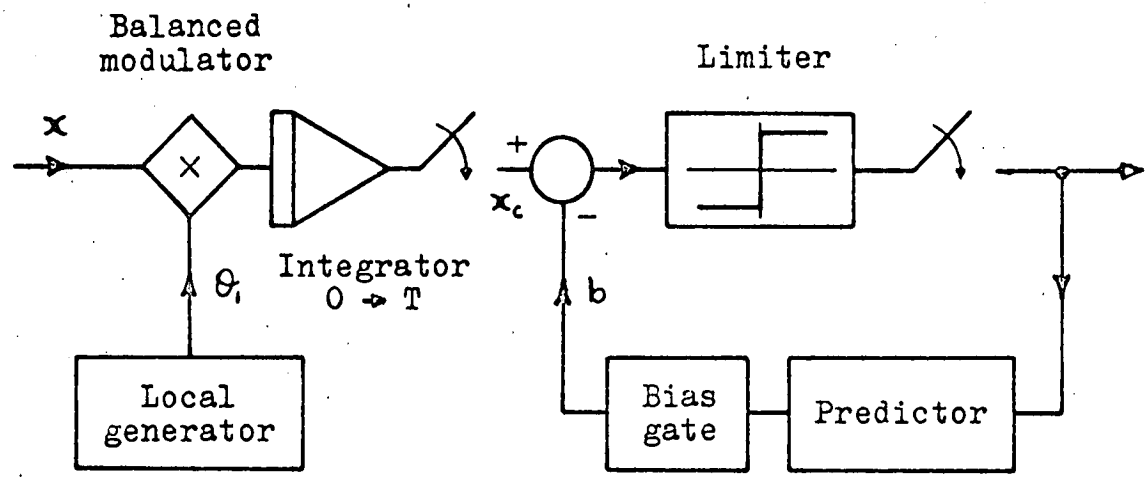


Fig. 10.1 Optimal receiver structure

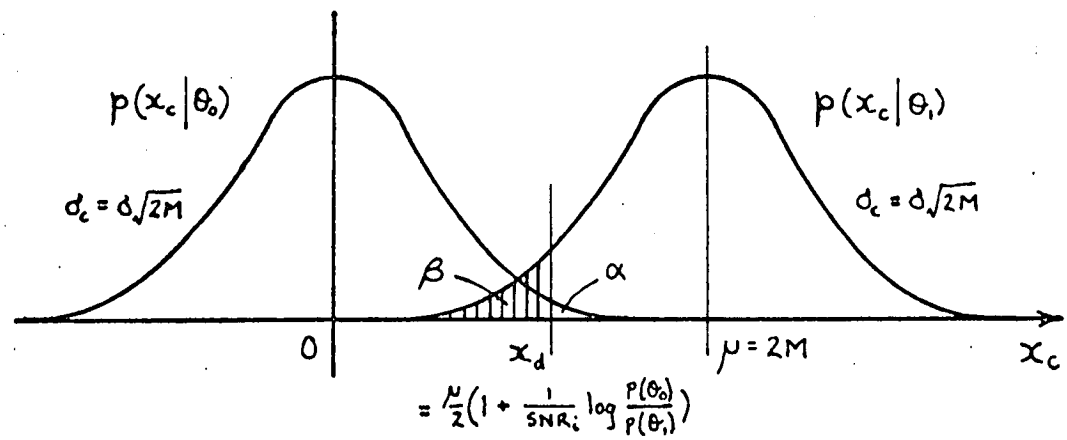


Fig. 10.2 Signal probability density distributions

$$K = \Lambda(x_d) = \frac{p(x_d|\theta_1)}{p(x_d|\theta_0)} = \frac{\frac{1}{\sqrt{2\pi}\,\sigma_c} e^{-\frac{(x_d-\mu)^2}{2\sigma_c^2}}}{\frac{1}{\sqrt{2\pi}\,\sigma_c} e^{-\frac{x_d^2}{2\sigma_c^2}}}$$

$$= e^{\frac{\mu}{\sigma_c^2}(x_d - \frac{\mu}{2})} \qquad (10.2.1)$$

from which

$$x_d = \frac{\mu}{2} + \frac{\sigma_c^2}{\mu} \log_e K \qquad (10.2.2)$$

Now it is possible from the results of chapter 5 to process groups of past detector output elements to generate predictions of $p(\theta_0)$, $p(\theta_1)$ for the following θ_i. Using this additional a priori information, the detector bias level can be switched so that the decision boundary location for each element corresponds to comparison of the likelihood ratio with the optimal threshold.

From (10.1.3), (10.1.4) and (10.2.2), the normalised boundary location for the orthogonal case is given by

$$\frac{x_d}{\mu} = \frac{1}{2} + \frac{1}{2\,SNR_i} \log_e \frac{p(\theta_0)}{p(\theta_1)} \qquad (10.2.3)$$

in which the input average signal-to-noise ratio

$$SNR_i = \frac{M}{\sigma^2} \qquad (10.2.4)$$

and decision boundary loci for a range of $p(\theta_0)$ and SNR_i are given in Fig. 10.3.

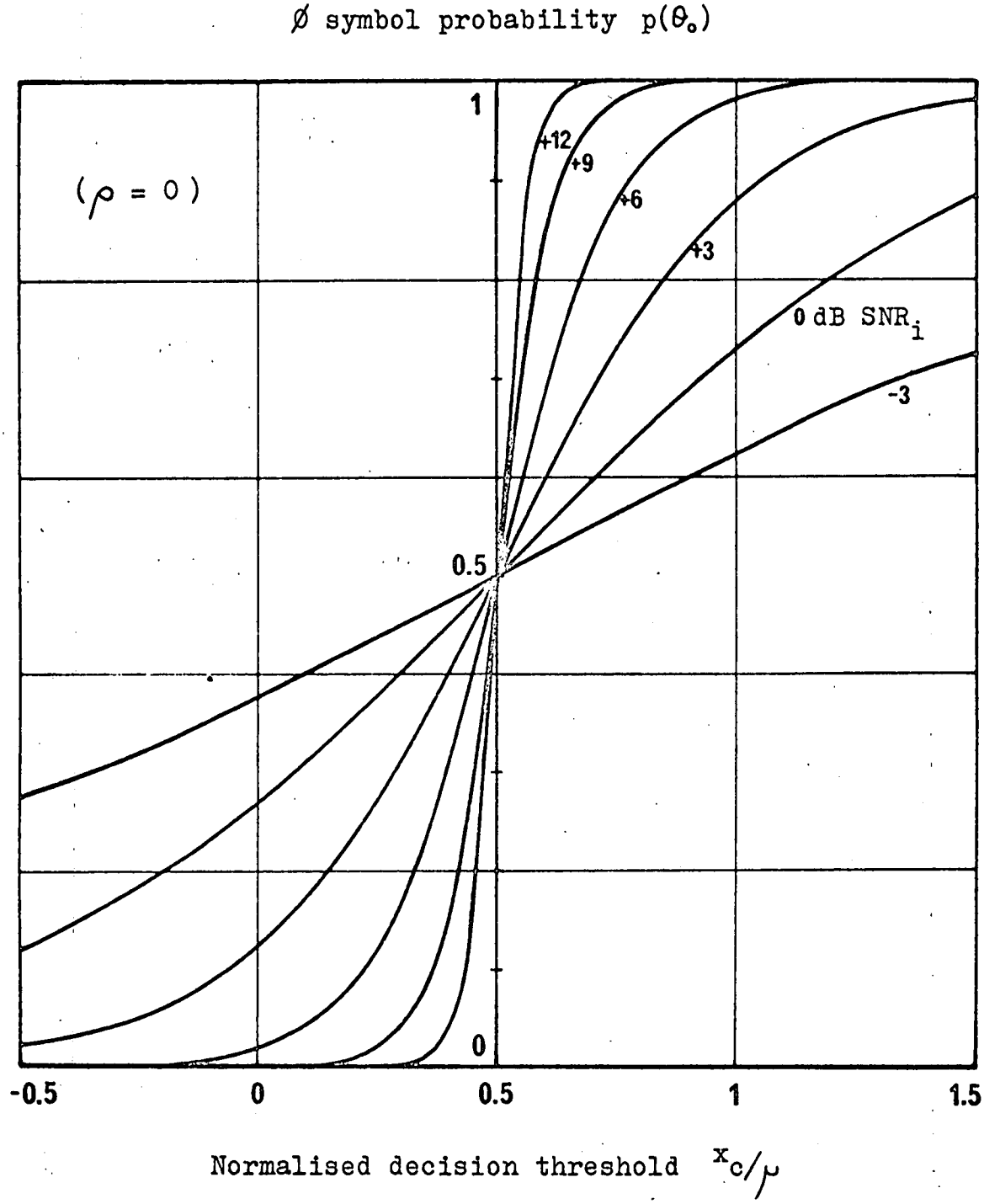


Fig. 10.3 Decision boundary loci

Restricting consideration to the practical case of the 6 element predictor described in chapter 7, which makes only a binary estimate of the more probable next symbol with average success probability P_s, the bias level is switched between values

$$\frac{b}{\mu} = \frac{1}{2} \pm \frac{1}{2\,SNR_i} \log_e \frac{P_s}{1-P_s} \qquad (10.2.5)$$

with sign + or - according as a $\emptyset$ or 1 symbol is predicted. The bias characteristics for $P_s = 0.898$ are shown in Fig. 10.4, in which the apparent rather critical dependence of the optimum levels on SNR_i is removed by considerations which follow.

10.3 Error rates

For transmitted signal θ_o, signal detection errors occur when $x_c > x_d$, which occurs with probability

$$\alpha = \int_{x_d}^{\infty} p(x_c|\theta_o)\,dx_c = \int_{\frac{N}{2}\left(1+\frac{1}{SNR_i}\log_e\frac{p(\theta_o)}{p(\theta_1)}\right)}^{\infty} \frac{1}{\sqrt{2\pi}\,\delta_c} e^{-\frac{x_c^2}{2\delta_c^2}}\,dx_c$$

$$= \frac{1}{2}\left[1 - erf\left(\frac{\sqrt{SNR_i}}{2} + \frac{1}{2\sqrt{SNR_i}}\log_e\frac{p(\theta_o)}{p(\theta_1)}\right)\right] \qquad (10.3.1)$$

While for θ_1, error probability

$$\beta = \frac{1}{2}\left[1 - erf\left(\frac{\sqrt{SNR_i}}{2} - \frac{1}{2\sqrt{SNR_i}}\log_e\frac{p(\theta_o)}{p(\theta_1)}\right)\right] \qquad (10.3.2)$$

The total error probability is then

$$P_T = p(\theta_o)\alpha + p(\theta_1)\beta \qquad (10.3.3)$$

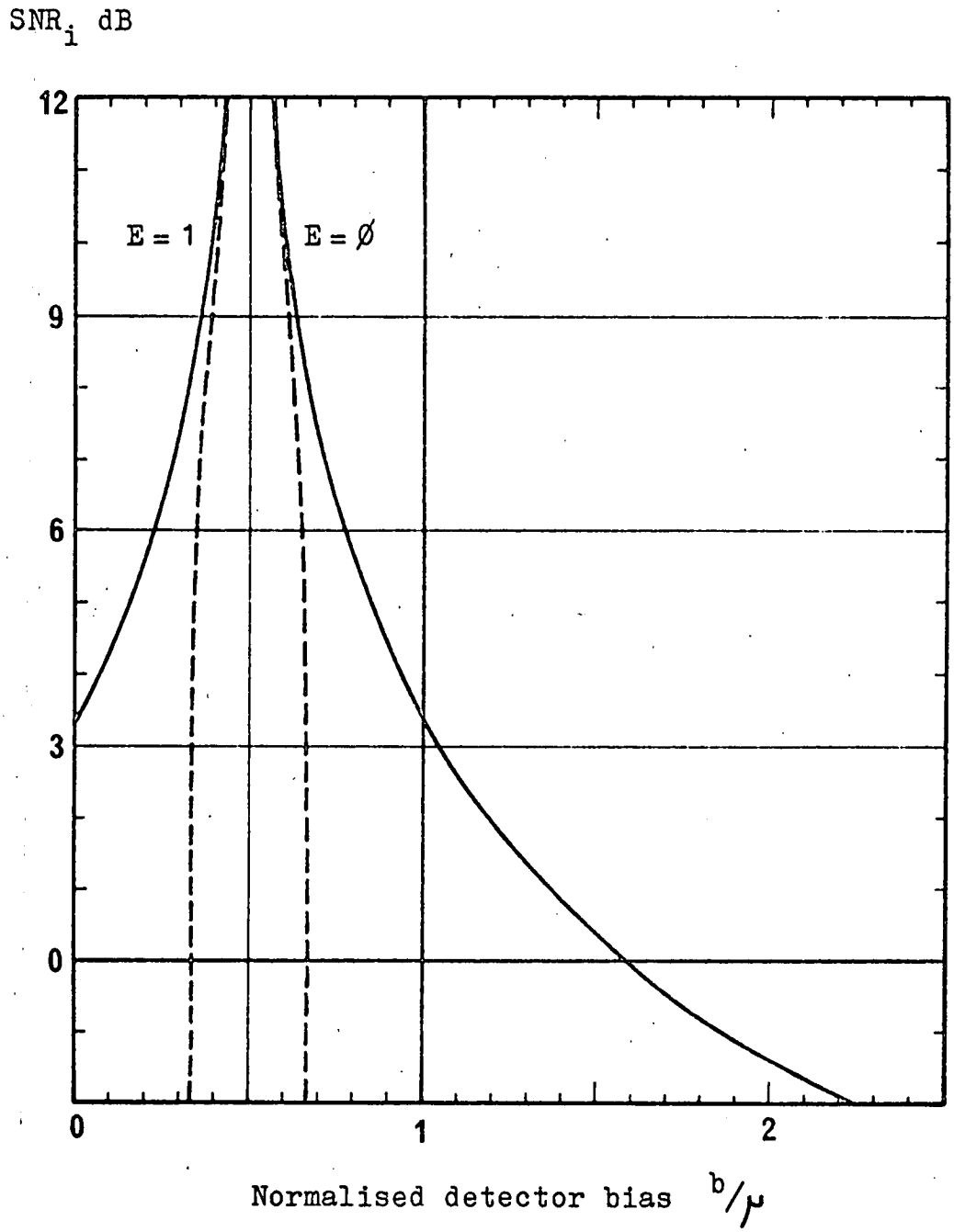


Fig. 10.4 Detector bias characteristics

For the case of fixed bias detection of a DM signal with $p(\theta_0)/p(\theta_i) = 1$, (10.3.3) becomes

$$P_{Tf} = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{\sqrt{SNR_i}}{2}\right)\right] \qquad (10.3.4)$$

which is shown in Fig. 10.5, together with the corresponding error characteristic for θ_i antipodal ($\rho = -1$ in (10.1.3) and subsequently).

Employing bias switching by (10.2.5), the total error probability becomes

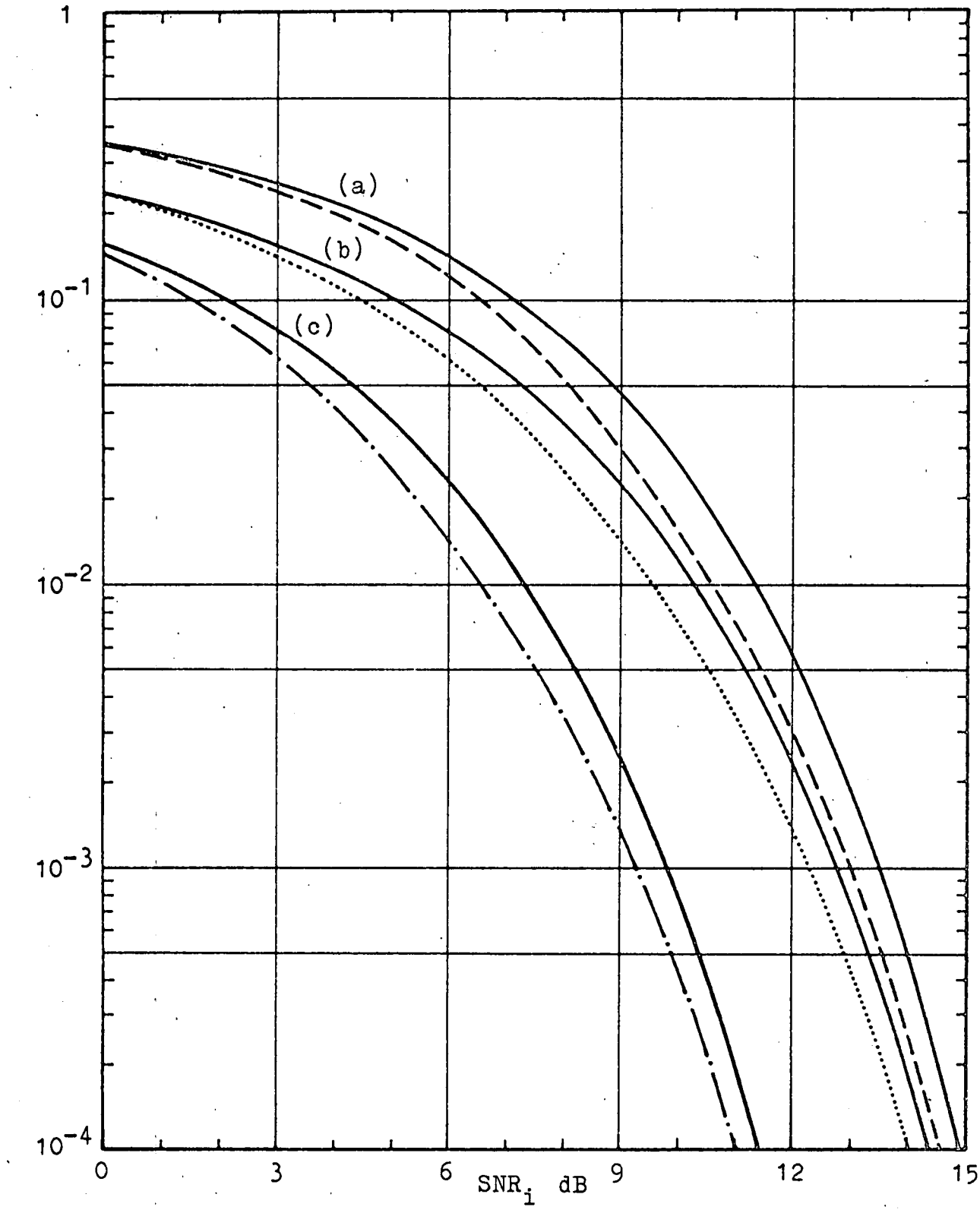
$$P_{Tv} = \frac{1}{2}\left[1 - P_s \operatorname{erf}\left(\frac{\sqrt{SNR_i}}{2} + \frac{1}{2\sqrt{SNR_i}} \log_e \frac{P_s}{1-P_s}\right) + (P_s - 1)\operatorname{erf}\left(\frac{\sqrt{SNR_i}}{2} - \frac{1}{2\sqrt{SNR_i}} \log_e \frac{P_s}{1-P_s}\right)\right] \qquad (10.3.5)$$

For the fixed 6th order predictor structure, P_s is a function of the data error rate which has been determined in Sec. 7.4. By a set of iterations for a range of channel SNR_i, bias levels and data error rates are computed from (10.2.5), (10.3.5) and the data of Fig. 7.4, and the results are shown in Figs. 10.4, 10.5.

The effect of the degradation in P_s at low SNR_i is to make the normalised decision boundary locations near constant, and detector bias switching between levels

$$\frac{b}{\mu} = 0.34,\ 0.66 \qquad (10.3.6)$$

is near optimal for a wide range of SNR_i. With these levels, an error rate reduction of 39 - 43 % results over the typical



(a) Noncoherent (b) Coherent orthogonal (c) Coherent antipodal
Continuous curves: fixed bias P_{Tf}. Broken curves: switched bias P_{Tv}

Fig. 10.5 Error probabilities

range in the case of both orthogonal and antipodal signalling.

10.4 Noncoherent signalling

In some digital communications situations noncoherent signalling is dictated by equipment instabilities or transmission path fluctuations. The optimal signal set is then restricted to an orthogonal pair, and typically on - off transmitter modulation is employed, while at the receiver bandpass filtering and envelope detection of the pulsed carrier is effected prior to the threshold comparator.

The probability density distribution for the envelope detector output x_n for a signal in noise has been studied by Rice[32] and

$$p(x_n|\theta_1) = \frac{x_n}{\delta^2} I_0\left(\frac{\sqrt{2}\, x_n \sqrt{SNR_i}}{\delta}\right) e^{-\left(\frac{x_n^2}{2\delta^2} + SNR_i\right)} \qquad (10.4.1)$$

while

$$p(x_n|\theta_0) = \frac{x_n}{\delta^2} e^{-\frac{x_n^2}{2\delta^2}} \qquad (I_0(0) = 1) \qquad (10.4.2)$$

Hence for the Bayes' criterion decision boundary x_{dn},

$$I_0\left(\sqrt{2}\,\frac{x_{dn}}{\delta}\sqrt{SNR_i}\right) e^{-SNR_i} = \frac{p(\theta_0)}{p(\theta_1)} \qquad (10.4.3)$$

from which x_{dn}/δ for switched bias detection may be determined by iterative solution of the approximation

$$\frac{e^{\nu}}{\sqrt{2\pi\nu}}\left(1 + \frac{1}{8\nu} + \frac{9}{2(8\nu)^2}\right) = \frac{1 \pm (2p_s - 1)}{1 \pm (1 - 2p_s)} e^{SNR_i} \qquad (10.4.4)$$

in which ν is the argument of the zero order modified Bessel

function of the first kind in (10.4.3) and signs + or - apply for a $\emptyset$ or 1 prediction as before.

The conditional error probabilities are then

$$\alpha = \int_{\frac{x_{dn}}{\delta}}^{\infty} \frac{x_n}{\delta} e^{-\frac{x_n^2}{2\delta^2}} \frac{dx_n}{\delta} = e^{-\frac{1}{2}\left(\frac{x_{dn}}{\delta}\right)^2} \qquad (10.4.5)$$

and

$$\beta = \int_{0}^{\frac{x_{dn}}{\delta}} \frac{x_n}{\delta} I_0\left(\sqrt{2}\frac{x_n}{\delta}\sqrt{SNR_i}\right) e^{-\left(\frac{x_n^2}{2\delta^2} + SNR_i\right)} \frac{dx_n}{\delta} \qquad (10.4.6)$$

in the computation of which the approximation $I_0(\nu) = e^{-\frac{\nu^2}{4}}$ may be used for the range of integration resulting in $\nu \leq 1$ and the expansion of (10.4.4) for the remainder.*

By repetitive application of a procedure by which, for decision boundary iterative solutions of (10.4.4) for trial values of P_s, total error probabilities are computed from (10.4.5) and (10.4.6) and used to revise the estimates by Fig. 7.4, the error characteristic for noncoherent signalling given in Fig. 10.5 is determined. The performance improvement is found to approach that attainable by coherent reception of the same transmitted signals, with less complexity than that necessary for a phase-locked demodulator and without imposing the equipment and path stability requirements of the latter.

$E\left[x_n \middle| \theta_i\right]$ in the noncoherent case is

---

*Both approximations incur an error of about 2% at $\nu = 1$.

$$\mu = \sigma\sqrt{\frac{\pi}{2}}\, e^{-\frac{SNR_i}{2}}\left[(1+SNR_i)\, I_0\left(\frac{SNR_i}{2}\right) + SNR_i\, I_1\left(\frac{SNR_i}{2}\right)\right], \qquad (10.4.7)$$

and when the optimal x_{dn} are normalised to this level it is again found that the decision boundary locations are not a sensitive function of SNR_i, and remain close to

$$\frac{b}{\mu} = 0.50,\ 0.71 \qquad E = 1, \emptyset \qquad (10.4.8)$$

over the typical range. Unlike those of (10.3.6), the bias levels are not symmetrical about $\mu/2$ because the Rayleigh distribution of the detector output for θ_0 has non-zero mean. To maintain minimum error probability reception during received signal strength fluctuations, the levels may readily be tapped in the ratios of (10.4.8) from a potential divider chain across which the mean detector output for θ_1 is developed by conventional gated AGC methods.

Chapter 11

Conclusion

In reviewing briefly the primary application areas of the knowledge gained by this study, the system performance data are selected from those presented in the thesis for representative characteristics and the most practically convenient procedures.

11.1 Transmitter applications

The information analysis of delta-coded speech by the data processing facility described has yielded central results for the entropies of this message source which indicate that the redundancy typically exceeds one half for process orders of practical interest. The efficacy of a predictive coding approach to the exploitation of this redundancy has been demonstrated by comparison of the entropy characteristics for Markov process approximations, for blocks of source elements and the corresponding optimal codes, and for the sequences generated by modulo-2 addition with the outputs of optimal predictors operating on past message elements.

IP transformations utilizing this property have been presented which may be employed at a sender to allow either a transmitter power saving or a reduction in required channel bandwidth without change to the message reconstructed by the

receiver. The common element of these techniques is a 6 th order digital predictive coder with near - optimal structure, of form appropriate to economical monolithic fabrication, and exhibiting a typical predictor success probability of 0.9. The encoder performance has been shown to be superior to that for an optimal linear predictor processing many more message elements, and its use in a DM pack set application permits an extension of operational life by a factor of up to 10.

To achieve bandwidth compression in a channel encoder, the predictive coder is followed by optimal 5 element group encoding (or other schemes evaluated) of the error sequence, a combination which realises the attainable 2 to 1 reduction in data rate with considerably less hardware complexity than is required for the direct application of any known exact coding procedure. The channel buffer capacity necessary for uniform output data rate is 270 words, independent of the number of sources multiplexed, and this requirement decreases rapidly for compression factors slightly exceeding the minimum.

11.2 Receiver applications

It has further been shown that the practical predictor structure derived may alternatively be used in signal detection schemes to exploit the knowledge of the statistical properties of the message source at a receiver of delta - coded speech. In this case, the additional a priori information provided by the predictor is used to switch the decision threshold to

minimize the average error risk, and in noncoherent and coherent systems using orthogonal or antipodal signals error probabilities are reduced by 40% by this easily implemented procedure.

11.3 Further work

With the exception of printed English, detailed entropy studies of actual message sources are rarer than their conceptual and practical value merits. While the quantitative information approach is founded on Shannon's treatise of 1948, and the encoding and detection problems which it illuminates are classical, its extensive application to real source analysis has been restricted by message time - scaling and on - line computing requirements.

The combination of the falling availability - cost of these facilities with the increasing capital investment involved in establishing the major new communications links assures a profitable future return on research effort which may be stimulated by this study embracing the parallel application of its philosophy and technique to other information sources of engineering interest.

Appendix 1

Message power spectral density

The system function H(s) (2.1.1) is defined by 12 zeros at the origin and 14 left half plane poles; the following 7 and their conjugates.

$-704.39 + j18774$
$-1903.8 + j15765$
$-2422.6 + j10692$
$-1742.3 + j5700.4$
$-717.36 + j3182.5$
$-268.00 + j2221.0$
$-70.936 + j1889.6$

Normalising $\omega_n = \omega/10^4$, the denominator coefficient set of $\Phi_{mm}(\omega_n) = \dfrac{k^2 \omega_n^{12}}{\sum_{i=0}^{14} C_{2i}\, \omega_n^{2i}}$ $(A_{2i} = 10^{8i}.C_{2i})$ is computed as

C0	$5.1101\ 10^{-7}$
C2	$6.0907\ 10^{-5}$
C4	$3.0233\ 10^{-3}$
C6	$8.1291\ 10^{-2}$
C8	1.2932
C10	1.2547 10
C12	$7.4548\ 10_2$
C14	$2.6778\ 10^2$
C16	$5.9052\ 10^2$
C18	$7.8729\ 10^2$
C20	$6.4275\ 10^2$
C22	$3.2005\ 10^2$
C24	9.4289 10
C26	1.5047 10
C28	1.0

with $k^2 = 1.7713\ 10^{-5}$ for unit message power $\int_{-\infty}^{\infty} \Phi_{mm}(\omega)\, d\omega$.

Appendix 2

Analogue filter

With the computer variable scaling chosen, (3.2.3) is represented by the configuration of Fig. A2.1 in which the required gain between consecutive output taps of Q_{i-1}/Q_i is realized by the integrator time constants shown and the following potentiometer settings.

Q	0.2447
P0	0.3559
P1	0.9919
P2	0.4725
P3	0.2287
P4	0.6765
P5	0.4308
P6	0.1101
P7	0.7892
P8	0.2018
P9	0.1286
P10	0.3794
P11	0.1842
P12	0.8735

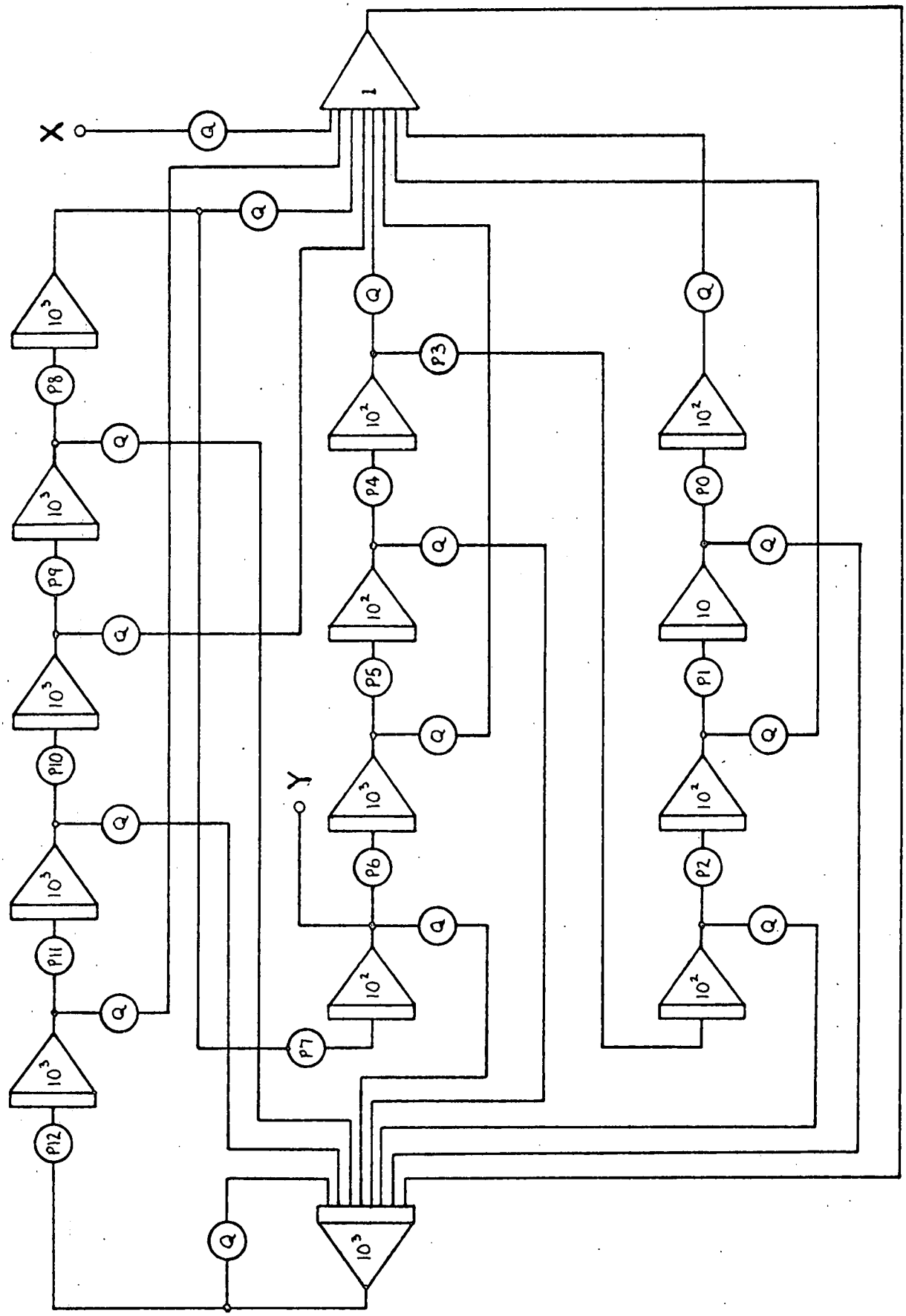


Fig. A2.1 Analogue message filter

Appendix 3

```
                /STOCHASTIC MATRIX ASSEMBLER 06HR

                PAGE 0

0000  0000  REG,    0
0001  7300          CLA CLL
0002  1525          TAD I CODAD     /LATEST 7 BIT TRANSFER
0003  7010          RAR
0004  3127          DCA STATW       /6 BIT STATE
0005  7004          RAL
0006  3130          DCA NUBIT

0007  1127          TAD STATW
0010  0377          AND (0017       /MASK SAVING 8-11
0011  7450          SNA
0012  5045          JMP PRED0       /LAST FOUR 0
0013  1376          TAD (7761
0014  7450          SNA
0015  5050          JMP PRED1       /LAST FOUR 1

0016  7200          CLA
0017  1127          TAD STATW
0020  0375          AND (0037       /MASK SAVING 7-11
0021  1374          TAD (7776
0022  7450          SNA
0023  5045          JMP PRED0       /02 OR 42
0024  1373          TAD (7745
0025  7450          SNA
0026  5050          JMP PRED1       /35 OR 75

0027  7200          CLA
0030  1127          TAD STATW
0031  1372          TAD (7774
0032  7450          SNA
0033  5045          JMP PRED0       /04
0034  1371          TAD (7711
0035  7450          SNA
0036  5050          JMP PRED1       /73

0037  7200          CLA             /COMPLEMENT 11
0040  1127          TAD STATW
0041  7010          RAR
0042  7430          SZL
0043  5045          JMP PRED0
0044  5050          JMP PRED1

0045  7200  PRED0,  CLA             /CL LINK FOR CORRECT PRED
0046  1130          TAD NUBIT
0047  5053          JMP .+4
0050  7200  PRED1,  CLA
0051  1130          TAD NUBIT
0052  7001          IAC
0053  7010          RAR
```

```
0054  7200          CLA             /FORM H-ADD OP SEQUENCE
0055  1124          TAD TEMDAT
0056  7004          RAL             /L TO 11
0057  0370          AND (1777
0060  3124          DCA TEMDAT      /STORE 10 BIT SEQ
0061  1124          TAD TEMDAT
0062  7120          STL
0063  7004          RAL
0064  1367          TAD (3400
0065  3126          DCA CNADD       /STORE LO ADD

0066  7100          CLL             /INCREMENT COUNT
0067  1526          TAD I CNADD
0070  7001          IAC
0071  3526          DCA I CNADD     /STORE LS BITS
0072  1126          TAD CNADD
0073  1366          TAD (7777
0074  3126          DCA CNADD       /STORE HI ADD
0075  7024          CML RAL         /DUBL COMP LINK TO BIT 11
0076  1526          TAD I CNADD
0077  3526          DCA I CNADD     /STORE MS BITS

0100  1125          TAD CODAD       /INCREMENT CODAD
0101  1365          TAD (0201
0102  7500          SMA             /TEST TO CYCLE
0103  1364          TAD (7600
0104  1364          TAD (7600
0105  3125          DCA CODAD

0106  7604          LAS             /TEST SR FOR T ENABLE
0107  7012          RTR
0110  7430          SZL
0111  5763          JMP TERMIN
0112  7200          CLA

0113  6662          OPGND
0114  4762          JMS RN          /GET RANDOM NO
0115  7100          CLL
0116  1154          TAD ERRBND
0117  7430          SZL             /L SET: BIT ERROR
0120  6664          OPNEG
0121  7200          CLA

0122  6001  HOLD,    ION             /WAIT FOR DATA BREAK
0123  5122          JMP .-1

0124  0000  TEMDAT,  0               /TEMPORARY REGISTERS
0125  7400  CODAD,   7400
0126  0000  CNADD,   0
0127  0000  STATW,   0
0130  0000  NUBIT,   0

0131  0000  STATEP,  0
0132  0000  STATET,  0
0133  0000  STATE0,  0
0134  0000  STATE1,  0
0135  0000  STATAP,  0
0136  3400  STATAT,  3400
```

```
0137  0000  C0H,     0
0140  0000  CN0HI,   0
0141  0000  CN0LO,   0
0142  0000  C0L,     0

0143  0000  C1H,     0
0144  0000  CN1HI,   0
0145  0000  CN1LO,   0
0146  0000  C1L,     0

0147  0000  SWITT,   0

0150  0000  TCNHI,   0
0151  0000  TCNLO,   0

0152  0000  TOTBHI,  0
0153  0000  TOTBLO,  0

0154  0000  ERRBND,  0

0155  0000  QUOTH,   0

            OPGND=6662          /CHANNEL ERROR PARAMETER ASSIGNMENT
            OPNEG=6664

            PAGE 1

0200  7200  ENTER,   CLA         /ENTER PROGRAM. SET D ADDS
0201  1377          TAD (7400   /CLEAR REGISTERS AND FLAGS
0202  3125          DCA CODAD   /INITIALIZE ROUTINES
0203  3124          DCA TEMDAT
0204  3152          DCA TOTBHI
0205  3153          DCA TOTBLO
0206  1376          TAD (T1
0207  4775          JMS TYPSTG
0210  1374          TAD (T4
0211  4775          JMS TYPSTG
0212  7200          CLA
0213  7402          HLT         /(LOAD SR WITH ERROR RATE)

0214  7604          LAS
0215  3154          DCA ERRBND
0216  1373          TAD (T5
0217  4775          JMS TYPSTG
0220  7402          HLT         /(CLEAR SR)

0221  4772          JMS INIT
0222  4771          JMS IR
0223  6662          OPGND
0224  6032          KCC
0225  6042          TCF
0226  6022          PCF
0227  7200          CLA

0230  6001          ION         /WAIT FOR FIRST DATA BREAK
0231  5230          JMP .-1
```

```
0232  0015  T1,     TEXT /@
                    @
0233  0012
0234  2324  ST
0235  1703  OC
0236  1001  HA
0237  2324  ST
0240  1103  IC
0241  4015   M
0242  0124  AT
0243  2211  RI
0244  3040  X
0245  0123  AS
0246  2305  SE
0247  1502  MB
0250  1405  LE
0251  2240  R
0252  6066  06
0253  1022  HR
0254  0015  @
                    @
0255  0012
0256  0401  DA
0257  2401  TA
0260  4024   T
0261  2201  RA
0262  1623  NS
0263  0605  FE
0264  2240  R
0265  2417  TO
0266  4067   7
0267  6460  40
0270  6000  0@
0271  1500  @
0272  1200  @
0273  1200  @
0274  0100  A/

0275  2305  T4,     TEXT /SE
0276  2440  T
0277  0522  ER
0300  2217  RO
0301  2240  R
0302  2201  RA
0303  2405  TE
0304  0015  @
                    @
0305  0012
0306  0001  @A
0307  0000  /
0310  0314  T5,     TEXT /CL
0311  0501  EA
0312  2240  R
0313  2322  SR
0314  0015  @
                    @
0315  0012
                    @
```

```
0316  0012
            @
0317  0012
0320  0001  @A
0321  0000  /

0322  0000  LDRTRL, 0              /10 INS 0200
0323  7200        CLA
0324  1370        TAD (7633
0325  3341        DCA LENGTH
0326  1367        TAD (0200
0327  4333        JMS PRINTC
0330  2341        ISZ LENGTH
0331  5326        JMP LDRTRL+4
0332  5722        JMP I LDRTRL

0333  0000  PRINTC, 0
0334  6026        PLS
0335  6021        PSF
0336  5335        JMP .-1
0337  7200        CLA
0340  5733        JMP I PRINTC

0341  7633  LENGTH, 7633

0367  0200  PAGE 2
0370  7633
0371  2200
0372  1600
0373  0310
0374  0275
0375  2400
0376  0232
0377  7400

0400  7200  TERMIN, CLA            /TERMINATE RUN
0401  3147        DCA SWITT
0402  3152        DCA TOTBHI
0403  3153        DCA TOTBLO
0404  1377        TAD (T3
0405  4776        JMS TYPSTG
0406  4775        JMS LDRTRL
0407  4775        JMS LDRTRL
0410  1374        TAD (MASKL
0411  3306        DCA MASKA     /SET MASK POINTER
0412  1373        TAD (271
0413  3307        DCA RUNDGT    /SET ORDER
0414  5234        JMP NEST-2

0415  7200  INITZ, CLA            /SET STATEP LIMITS
0416  1706        TAD I MASKA
0417  7041        CIA
0420  3310        DCA LIMIT
0421  1706        TAD I MASKA
0422  7001        IAC
0423  3311        DCA COLIM
0424  4775        JMS LDRTRL
```

```
0425  1372          TAD (T11
0426  4776          JMS TYPSTG
0427  1307          TAD RUNDGT
0430  6026          PLS
0431  6021          PSF
0432  5231          JMP .-1
0433  4771          JMS TYCR

0434  7200          CLA
0435  3131          DCA STATEP    /CLEAR

0436  1131  NEST,    TAD STATEP
0437  3132          DCA STATET

0440  7200          CLA           /FORM PRESENT STATE ADDRESS
0441  1131          TAD STATEP
0442  7006          RTL
0443  0370          AND (3774
0444  1367          TAD (3400
0445  3135          DCA STATAP

0446  1535          TAD I STATAP /LOAD TEMP REGISTERS
0447  3140          DCA CN0HI
0450  2135          ISZ STATAP
0451  1535          TAD I STATAP
0452  3141          DCA CN0LO
0453  2135          ISZ STATAP
0454  1535          TAD I STATAP
0455  3144          DCA CN1HI
0456  2135          ISZ STATAP
0457  1535          TAD I STATAP
0460  3145          DCA CN1LO

0461  1147          TAD SWITT
0462  7450          SNA
0463  5766          JMP TBCN

0464  7200  SEARCH, CLA
0465  2132          ISZ STATET
0466  1132          TAD STATET
0467  7004          RAL
0470  7006          RTL           /BIT 2 TO LINK
0471  7430          SZL
0472  5765          JMP OUTLNK    /STATES 000-777 SEARCHED
0473  7010          RAR
0474  7012          RTR

0475  0706          AND I MASKA   /MASK
0476  3312          DCA TRMST     /STORE TRIMMED STATE
0477  1131          TAD STATEP
0500  0706          AND I MASKA   /MASK
0501  7041          CIA
0502  1312          TAD TRMST
0503  7450          SNA           /STATES SAME?
0504  4764          JMS TRANSF    /YES: TRANSFER COUNTS
0505  5264          JMP SEARCH    /NO: CONTINUE
```

```
0506  0000  MASKA,  0
0507  0271  RUNDGT, 271
0510  7001  LIMIT,  7001
0511  1000  COLIM,  1000
0512  0000  TRMST,  0

0564  0600  PAGE 3
0565  1000
0566  1024
0567  3400
0570  3774
0571  2617
0572  0720
0573  0271
0574  0642
0575  0322
0576  2400
0577  0654

0600  0000  TRANSF, 0             /FORM REP STATE ADDRESS
0601  7200        CLA
0602  1132        TAD STATET
0603  7006        RTL
0604  0377        AND (3774
0605  1376        TAD (3400
0606  3136        DCA STATAT

0607  7300        CLA CLL         /ADD 0 COUNT
0610  2136        ISZ STATAT
0611  1536        TAD I STATAT
0612  1141        TAD CN0LO
0613  3141        DCA CN0LO
0614  1375        TAD (7777
0615  1136        TAD STATAT
0616  3136        DCA STATAT
0617  7024        CML RAL         /DUBL COMP LINK TO A11
0620  1536        TAD I STATAT
0621  1140        TAD CN0HI
0622  3140        DCA CN0HI

0623  1136        TAD STATAT      /ADD 1 COUNT
0624  1374        TAD (0003
0625  3136        DCA STATAT
0626  7300        CLA CLL
0627  1536        TAD I STATAT
0630  1145        TAD CN1LO
0631  3145        DCA CN1LO
0632  1375        TAD (7777
0633  1136        TAD STATAT
0634  3136        DCA STATAT
0635  7024        CML RAL         /DUBL COMP LINK TO A11
0636  1536        TAD I STATAT
0637  1144        TAD CN1HI
0640  3144        DCA CN1HI

0641  5600        JMP I TRANSF
```

```
0642  0777  MASKL,  777
0643  0377           377
0644  0177           177
0645  0077           077
0646  0037           037
0647  0017           017
0650  0007           007
0651  0003           003
0652  0001           001
0653  0000           000

0654  0015  T3,      TEXT /@
            @
0655  0012
0656  2405  TE
0657  2215  RM
0660  1116  IN
0661  0124  AT
0662  0504  ED
0663  0015  @
            @
0664  0012
            @
0665  0012
            @
0666  0012
            @
0667  0012
0670  1723  OS
0671  2440  T
0672  4024   T
0673  2201  RA
0674  1623  NS
0675  4060   0
0676  4040
0677  2422  TR
0700  0116  AN
0701  2340  S
0702  6100  1@
0703  1500  @
0704  1200  @

0705  1224  T
0706  1724  OT
0707  0114  AL
0710  4002   B
0711  1124  IT
0712  4003   C
0713  1725  OU
0714  1624  NT
0715  4040
0716  4000   @
0717  0100  A/

0720  0015  T11,     TEXT /@
            @
0721  0012
```

```
0722  7160  90
0723  0001  @A
0724  0000  /

0774  0003  PAGE 4
0775  7777
0776  3400
0777  3774

1000  4777  OUTLNK, JMS OUTPUT   /PRINT CYCLE

1001  1131         TAD STATEP   /TEST STATEP
1002  1776         TAD LIMIT
1003  7450         SNA          /LIMIT REACHED?
1004  5210         JMP NURUN    /YES: REDUCE ORDER AND RERUN
1005  1775         TAD COLIM    /NO: INCR AND REPEAT
1006  3131         DCA STATEP
1007  5774         JMP NEST

1010  4773  NURUN,  JMS TYCR
1011  2772         ISZ MASKA
1012  7200         CLA
1013  1771         TAD RUNDGT
1014  1370         TAD (7520
1015  7450         SNA          /RUNDGT 260? (0)
1016  5222         JMP CONCLD   /YES: EXIT
1017  1367         TAD (257     /NO: REDUCE BY 1 AND REPEAT
1020  3771         DCA RUNDGT
1021  5766         JMP INITZ

1022  4765  CONCLD, JMS LDRTRL
1023  7402         HLT

1024  7300  TBCN,   CLA CLL      /FORM STATE ENTERED TOTAL
1025  1141         TAD CN0LO
1026  1145         TAD CN1LO
1027  3151         DCA TCNLO
1030  7004         RAL
1031  1140         TAD CN0HI
1032  1144         TAD CN1HI
1033  3150         DCA TCNHI

1034  7300         CLA CLL      /INCREMENT TOTAL BIT COUNT
1035  1153         TAD TOTBLO
1036  1151         TAD TCNLO
1037  3153         DCA TOTBLO
1040  7004         RAL
1041  1152         TAD TOTBHI
1042  1150         TAD TCNHI
1043  3152         DCA TOTBHI

1044  1131         TAD STATEP   /TEST STATEP
1045  1364         TAD (7001
1046  7450         SNA          /IS IT 0777?
1047  5253         JMP TBOP
1050  1363         TAD (1000
1051  3131         DCA STATEP
1052  5774         JMP NEST
```

```
1053  1362  TBOP,   TAD (TOTBHI
1054  4761          JMS IDBBCD
1055  0153          TOTBLO
1056  4773          JMS TYCR
1057  2147          ISZ SWITT
1060  5766          JMP INITZ

1161  2000  PAGE 5
1162  0152
1163  1000
1164  7001
1165  0322
1166  0415
1167  0257
1170  7520
1171  0507
1172  0506
1173  2617
1174  0436
1175  0511
1176  0510
1177  1200

1200  0000  OUTPUT, 0             /ONE STATE OUTPUT SEQUENCE
1201  7200          CLA
1202  4777          JMS TYCR
1203  1131          TAD STATEP    /PRINT STATE OCTAL
1204  4776          JMS TYPOCT
1205  4775          JMS TYSP
1206  4775          JMS TYSP
1207  1374          TAD (CNØHI
1210  4773          JMS IDBBCD    /PRINT TOTAL TRANS TO STATE Ø
1211  0141          CNØLO
1212  4775          JMS TYSP
1213  4775          JMS TYSP
1214  1372          TAD (CN1HI
1215  4773          JMS IDBBCD    /PRINT TOTAL TRANS TO STATE 1
1216  0145          CN1LO

1217  7200          CLA
1220  5600          JMP I OUTPUT

1372  0144  PAGE 6
1373  2000
1374  0140
1375  2627
1376  1400
1377  2617

1400  0000  TYPOCT, 0             /3D OCTAL STATE FORMATION
1401  7006          RTL           /7 LEFT
1402  7006          RTL
1403  7006          RTL
1404  7004          RAL
1405  3225          DCA TEMOCT
1406  1377          TAD (7775
1407  3226          DCA CNOCT     /SET DIGIT COUNT
```

```
1410  1225          TAD TEMOCT
1411  0376          AND (0007     /MASK, SAVE 9-11
1412  1375          TAD (0260     /ASCII
1413  4227          JMS PRINTN
1414  1225          TAD TEMOCT
1415  2226          ISZ CNOCT     /DONE 3?
1416  5221          JMP MDIGIT    /NO: 3 LEFT AND REPEAT
1417  7200          CLA           /YES: EXIT
1420  5600          JMP I TYPOCT

1421  7006  MDIGIT, RTL
1422  7004          RAL
1423  3225          DCA TEMOCT
1424  5210          JMP TYPOCT+10

1425  0000  TEMOCT, 0
1426  0000  CNOCT,  0

1427  0000  PRINTN, 0             /PUNCH DIGIT SR
1430  6026          PLS
1431  6021          PSF
1432  5231          JMP .-1
1433  7200          CLA
1434  5627          JMP I PRINTN

1575  0260  PAGE 7
1576  0007
1577  7775

1600  0000  INIT,   0             /COUNT AND DATA REG INITIALIZATION
1601  7200          CLA
1602  1377          TAD (3400
1603  3240          DCA CTAD
1604  3640          DCA I CTAD
1605  1240          TAD CTAD      /TEST CTAD
1606  1376          TAD (0401
1607  7450          SNA           /IS IT 7377?
1610  5213          JMP ALT       /YES: DO DATA REGISTER
1611  1375          TAD (7400     /NO: INCREMENT CTAD 0001
1612  5203          JMP INIT+3

1613  7200  ALT,    CLA
1614  1375          TAD (7400
1615  3241          DCA DATAD
1616  1374          TAD (0125
1617  3641          DCA I DATAD   /LOAD 1010101
1620  1241          TAD DATAD
1621  7001          IAC
1622  3241          DCA DATAD
1623  1373          TAD (0052
1624  3641          DCA I DATAD   /LOAD 0101010
1625  1241          TAD DATAD     /TEST DATAD
1626  1372          TAD (0201
1627  7450          SNA           /IS IT 7577?
1630  5233          JMP FINI      /YES: EXIT
1631  1371          TAD (7600     /NO: INCREMENT DATAD 0001
1632  5215          JMP ALT+2
```

```
1633  7200  FINI,    CLA
1634  1370            TAD (T2
1635  4767            JMS TYPSTG
1636  7200            CLA
1637  5600            JMP I INIT

1640  0000  CTAD,     0
1641  0000  DATAD,    0

1642  0015  T2,       TEXT /@
                      @
1643  0012
1644  2324            ST
1645  0124            AT
1646  0540            E
1647  0317            CO
1650  2516            UN
1651  2423            TS
1652  4003             C
1653  1405            LE
1654  0122            AR
1655  0015            @
                      @
1656  0012
1657  0401            DA
1660  2401            TA
1661  4020             P
1662  0107            AG
1663  0540            E
1664  1417            LO
1665  0104            AD
1666  0504            ED
1667  0015            @
                      @
1670  0012
                      @
1671  0012
                      @
1672  0012
1673  0001            @A
1674  0000            /

1767  2400  PAGE 10
1770  1642
1771  7600
1772  0201
1773  0052
1774  0125
1775  7400
1776  0401
1777  3400

2000  0000  IDBBCD,  0               /DUBL PREC INT BIN TO BCD
2001  3263            DCA VALUEH     /ADDR H ORDER IP
2002  1663            TAD I VALUEH
2003  3265            DCA VALH
```

```
2004  3271           DCA DIGIT       /CLEAR
2005  1600           TAD I IDBBCD
2006  2200           ISZ IDBBCD
2007  3262           DCA VALUEL      /ADDR L ORDER IP
2010  1662           TAD I VALUEL
2011  3264           DCA VALL
2012  1377           TAD (-7         /SET COUNTERS
2013  3270           DCA CNTR1
2014  1376           TAD (TAD TENPH /SET TABLE ARROWS
2015  3232           DCA ARROW1
2016  1375           TAD (TAD TENPL
2017  3226           DCA ARROW2

2020  1264           TAD VALL        /COPY
2021  3266           DCA VL
2022  1265           TAD VALH
2023  3267           DCA VH

2024  7300           CLA CLL         /DUBL PREC SUB PWR OF 10
2025  1266           TAD VL
2026  1273  ARROW2,  TAD TENPL
2027  3266           DCA VL
2030  7004           RAL
2031  1267           TAD VH
2032  1272  ARROW1,  TAD TENPH
2033  7004           RAL
2034  7430           SZL             /RESULT STILL POSITIVE?
2035  5246           JMP READY       /NO: CANCEL
2036  7010           RAR             /YES: CONTINUE
2037  3267           DCA VH
2040  2271           ISZ DIGIT       /DEVELOP BCD DIGIT
2041  1266           TAD VL
2042  3264           DCA VALL
2043  1267           TAD VH
2044  3265           DCA VALH
2045  5224           JMP ARROW2-2    /LOOP

2046  7200  READY,   CLA             /TYPE BCD DIGIT
2047  1271           TAD DIGIT
2050  4774           JMS TDIG
2051  7200           CLA
2052  3271           DCA DIGIT
2053  2232           ISZ ARROW1      /ADVANCE TABLE ARROWS
2054  2232           ISZ ARROW1
2055  2226           ISZ ARROW2
2056  2226           ISZ ARROW2
2057  2270           ISZ CNTR1       /DONE EIGHT DIGITS?
2060  5220           JMP ARROW2-6    /NO: CONTINUE
2061  5600           JMP I IDBBCD    /YES: EXIT

2062  0000  VALUEL,  0
2063  0000  VALUEH,  0
2064  0000  VALL,    0
2065  0000  VALH,    0
2066  0000  VL,      0
2067  0000  VH,      0
```

```
2070  7771  CNTR1, -7
2071  0000  DIGIT,  0
2072  7413  TENPH,  7413          /-1000000
2073  6700  TENPL,  6700

2074  7747  DUBL   -0100000
2075  4540
2076  7775         -0010000
2077  4360
2100  7777         -0001000
2101  6030
2102  7777         -0000100
2103  7634
2104  7777         -0000010
2105  7766
2106  7777         -0000001
2107  7777
2110  7200  CLA

2174  2645
2175  1273
2176  1272
2177  7771

            /THE FOLLOWING SOFTWARE PACKAGES REQUIRE
            /TO BE APPENDED TO THIS PROGRAM:


            /PAGE 11
            /DECUS NO 5-25 PSEUDO RANDOM NUMBER GENERATOR

            /PAGE 12
            /DIGITAL-8-20-U-SYM CHARACTER STRING TYPEOUT

            /PAGE 13
            /DIGITAL-8-19-U-SYM TELETYPE OUTPUT SUBROUTINES
            /(MODIFIED FOR HS PUNCH)

            /PAGE 14
            /DIGITAL-8-6-U-SYM OCTAL MEMORY DUMP


            /DOUBLE PRECISION STATE COUNTS OCCUPY PAGES 16 - 35


            /DATA BREAK TRANSFERS ARE MADE DIRECT TO PAGE 36


            /PAGE 37 HOLDS DIGITAL-8-1-U RIM LOADER AND
            /DIGITAL-8-2-U BINARY LOADER



            PAUSE
```

Appendix 4

4th order transition probability matrix for delta - coded speech

Sampling frequency = 96 KHz
pk - pk message amplitude = 128 Δ

Prediction Example ~ On occurrence of group 1001 (11) predictor selects 0010 (02), the probability of transition to which exceeds that to 0011 (03).

↓	00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17
00	0.895	0	0	0	0	0	0	0	0.468	0	0	0	0	0	0	0
01	0.105	0	0	0	0	0	0	0	0.532	0	0	0	0	0	0	0
02	0	0.963	0	0	0	0	0	0	0	0.965	0	0	0	0	0	0
03	0	0.037	0	0	0	0	0	0	0	0.035	0	0	0	0	0	0
04	0	0	0.437	0	0	0	0	0	0	0	0.077	0	0	0	0	0
05	0	0	0.563	0	0	0	0	0	0	0	0.923	0	0	0	0	0
06	0	0	0	0.832	0	0	0	0	0	0	0	0.813	0	0	0	0
07	0	0	0	0.168	0	0	0	0	0	0	0	0.187	0	0	0	0
10	0	0	0	0	0.198	0	0	0	0	0	0	0	0.149	0	0	0
11	0	0	0	0	0.802	0	0	0	0	0	0	0	0.851	0	0	0
12	0	0	0	0	0	0.923	0	0	0	0	0	0	0	0.579	0	0
13	0	0	0	0	0	0.077	0	0	0	0	0	0	0	0.421	0	0
14	0	0	0	0	0	0	0.036	0	0	0	0	0	0	0	0.037	0
15	0	0	0	0	0	0	0.964	0	0	0	0	0	0	0	0.963	0
16	0	0	0	0	0	0	0	0.523	0	0	0	0	0	0	0	0.094
17	0	0	0	0	0	0	0	0.477	0	0	0	0	0	0	0	0.906

Notes 1) States are octal, probabilities decimal.

2) In preference to the more common transpose, the conditional probabilities in the above format are for transitions from operand state columns to transform state rows. The source states are then column rather than row vectors, but the order of matrix multiplication is the natural

$$S_{i+1} = T \; S_i$$

in which operator precedes operand.

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