A STUDY OF
LARGE SCALE VERTICAL MOTION
IN THE ATMOSPHERE

DISSERTATION

submitted by

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LIST OF CONTENTS

	Page
INTRODUCTION	(1)
DESCRIPTION OF METHODS	(5)
PRESENTATION OF RESULTS	(21)
INTER-COMPARISON OF RESULTS	(23)
CORRELATION WITH RAINFALL CHART	(26)
GENERAL DISCUSSION	(30)
GENERAL CONCLUSIONS	(39)
ACKNOWLEDGMENTS	(40)
REFERENCES	
APPENDIX I (TABLES AND FIGURES)	
APPENDIX II	

INTRODUCTION

The present study is concerned with one of the fundamental problems of meteorology - that of estimating large-scale vertical motion in the atmosphere, the term large-scale here implying motion on the synoptic scale.

Very appreciable vertical motion of course occurs on a small horizontal scale, notably in association with cumulonimbus clouds; such vertical currents are frequently of the order of 10 metres per second and are amply large enough to be measured directly. The vertical motion with which this study is concerned is some two orders of magnitude less than the above. This velocity is too small to admit of direct measurement in the presence of predominantly horizontal air motion which is normally several hundred times larger in magnitude: knowledge of such vertical motion may be obtained only by computation.

The type of motion studied here, though small in magnitude, is of fundamental importance in weather processes. Although the geostrophic wind approximation is a wide-spread and useful concept in synoptic meteorology, it is the a-geostrophic part of the motion - that associated with vertical motion - which is responsible for pressure changes, the deepening and filling of pressure systems, the formation of cloud and rain, or the dispersal of cloud.

Present study

Various methods have been proposed for the computation of the vertical wind component on the synoptic scale, some of them differing fundamentally one from the other, others differing only in detail. This study consists essentially of applying certain of these methods (those which appear most capable of useful application on a routine basis) in a selected situation and of comparing the results obtained. In order to facilitate inter-comparison, values of vertical velocity have been computed, where possible, at selected grid-points- 72 in all.

Selected situation

The particular situation under examination was that of September 3, 1956. Outwardly the situation was not at all remarkable. It was however selected because the rainfall which occurred over the northern half of the British Isles was a good deal more than was forecast, and it was desired to investigate whether the vertical motion computations might contain a hint which was missing from the other types of charts. At the same time, as already stated, the opportunity was taken to generalise the situation and to calculate the vertical velocity over a wide network of grid points, for comparison between the various methods.

An anticyclone to the north of Scotland moved slowly north, while a depression off south-west England, after extending its influence over the whole country, gradually filled. An occluded front moved northwards over the country with associated heavy falls of rain in Northern England and in Southern and Eastern Scotland. From 0900h to 2100h on the 3rd, Dyce Airport e.g., recorded 42mm of rain. The area under consideration extended from Iceland in the north to Italy in the south, and from Greenland in the west to Finland in the east. Raw data from some 60 meteorological radio-sonde stations and weather ships in this area, in the form of coded teleprinter messages, were obtained from the Meteorological Office, Dunstable, and used to form a complete picture of the weather situation at various pressure levels. The "Daily Weather Report"(both surface and upper air) was also used to help in processing the data. The corresponding surface chart (D.W.R. 0600h) is shown as fig.1 of Appendix I.

Processing of data

The teleprinter messages had to be decoded to give details of pressure, temperature and wind speed and direction. This proved to be a rather lengthy process. The details given were of the following nature:- pressure (millibars) ; heights of selected pressure surfaces (geopotential metres); temperature (degrees Centigrade); dew point (degrees Centigrade); wind speed (Knots); wind direction(10 degrees). (For the application of a kinematical method, the British winds reported to the nearest degree were obtained on request). This raw data was then processed in the forms required by the various methods of computation. This involved

(1) The construction of tephigrams for every station at 0200h and 1400h.

(2) The drawing of constant pressure charts for the following levels: 1000, 850, 700 and 500mb.

(3) The drawing of constant level charts of (a) pressure and (b) temperature, for the following levels: 1500, 3000, 6000 and 9000 geopotential metres. This latter process involved the drawing up of tables in which the arguments were pressure, height, and temperature.

It was very often necessary to estimate the wind vectors at points other than those covered by direct observations. The calculations were therefore made for the construction of geostrophic wind scales for use on individual constant-level charts at the various upper levels, and on all constant-pressure charts.

The isopleths on all charts were drawn as carefully as possible. A fair amount of interpolation and smoothing was required in regions of scanty observations.

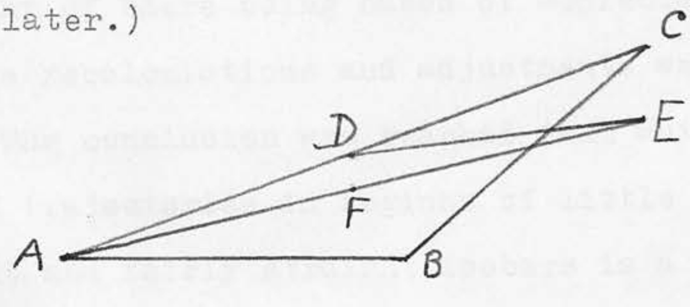
DESCRIPTION OF METHODS

(1) Graham's Adiabatic Method [1]

Graham's method is an adiabatic constant-level technique, which requires data in two main forms-constant-level charts of pressure and temperature, and tephigrams for all the various radio-sonde stations. Briefly the method is as follows. Starting at a radio-sonde station the twelve-hour trajectory of the air is found and the temperature and pressure of the air parcel at the end-point of the trajectory are determined from synoptic charts of these elements. This temperature and pressure are plotted on the tephigrams of the radio-sonde station at which the air parcel originated, and the adiabatic rise or fall of the air parcel obtained by the standard 'path-curve' method.

Construction of trajectories. The first problem is to find the horizontal path, during the twelve-hour period which separates successive radio-sonde observations, of an air parcel leaving a point above a particular radio-sonde station. This is done by the construction of a trajectory, this being a vector drawn to represent the horizontal component of the path of an air parcel over a certain period of time.

The following method was used to construct the trajectories required in this and a number of other methods (those of Longley and Panofsky(trajectory): a different technique was used in Sawyer's method, as described later.)



Let AB represent the horizontal component of the path of an air parcel, using the wind speed and direction measured at A at 0200h (say): the air parcel left A at 0200h and arrived (as a first approximation) at B twelve hours later. Now, using the wind speed and direction at B at 1400h (direct observations if available, otherwise the winds interpolated from the isobars drawn on constant-level charts), let BC be the path of the air parcel for the next twelve hours. Then AD = $\frac{1}{2}$AC is a second approximation to the twelve-hour trajectory of the air parcel which left A at 0200h. A closer (third) approximation can in theory be obtained by interpolating from a pressure contour chart the value of the wind vector at D at 1400h; representing this vector by DE (say) and using AF = $\frac{1}{2}$AE as the twelve-hour trajectory from A. After various trials, the second-approximation method referred to above was employed as a routine. As a check, however, the method was frequently applied in reverse i.e. starting at the newly-found end-point of a trajectory, one should arrive back at the radio-sonde station: in the event of there being cases of appreciable error, appropriate recalculations and adjustments were made.

The conclusion was reached that while the drawing of trajectories in regions of little pressure development and fairly straight isobars is a reasonably accurate process, it is not easy, when these two conditions are not satisfied, to eliminate from twelve-hour trajectories appreciable error.

Vertical Motion from the Tephigram. Once the end-point of the trajectory is located, its temperature and pressure are read off from the constant-level charts of these variables at the appropriate time. These are then plotted as a point on the tephigram for the start of the trajectory. The problem is now to find the point on the environment curve from which adiabatically rising (or falling) air would reach the final point already plotted on the tephigram. Care has to be taken with this if the air becomes saturated (or unsaturated) during its ascent (or descent). Multiple solutions, which do occur, are best dealt with by comparing them with those from nearby stations. The vertical motion over the twelve-hour period, is the difference in pressure between the initial point plotted on the tephigram and the final point just located above.

(2) Longley's Method [2]

Longley's method is similar in many ways to that of Graham. It too is an adiabatic constant-level technique employing constant-level charts as well as the tephigrams. These latter must have the humidity element represented by the wet-bulb temperature in place of the more usual representation of the dew point.

This method differs from that of Graham in that it makes use of the highly conservative element, the wet-bulb potential temperature (θ') in place of the dry bulb temperature. θ' can be used as an air-mass or air-parcel label since it remains unchanged in the presence of adiabatic ascent or descent, evaporation or precipitation. It is thus possible to identify a particular air parcel after six or twelve hours (say) have elapsed. This method of using wet-bulb potential temperatures was first proposed by Hewson.[3]

The initial part of the method involves the construction of twelve-hour trajectories at the various required levels, using the locations of radio-sonde stations as the starting points. The same procedure was adopted as with Graham's method. The value of θ' above the station for the level under consideration is found from the tephigram of the station, and in order to find the vertical motion, the height of an air parcel at the end of the twelve-hour trajectory having the same value of θ' as the initial air parcel, has to be found. This is done by examining the tephigrams of radio-sonde stations near the end-point of the trajectory. One station may be sufficient, but usually the values obtained from several 'surrounding' radio-sonde stations, suitably weighted according to their distance from the end-point, have to be averaged.

(3) Panofsky's Isobaric Trajectory Method [4]

This adiabatic method is based purely on the assumption that rising or falling air cools or warms at a known rate. The vertical motion is calculated from the rate of change of temperature of an air parcel moving along an isobaric surface, with the existing and adiabatic lapse-rates taken into account.

Derivation of equation used. The rate of change of temperature following the air motion ($\frac{dT}{dt}$) is expanded as

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \underline{V} \cdot \nabla_H T + w \frac{\partial T}{\partial z} \qquad \text{-------(1)}$$

where $\underline{V}$ = horizontal velocity vector,

w = vertical velocity,

z = vertical coordinate,

∇_H = horizontal differential operator,

$\frac{\partial T}{\partial t}$ = local rate of change of temperature.

$\frac{\delta T}{\delta t}$, the rate of change of temperature along a trajectory following a pressure surface, is similarly expanded as

$$\frac{\delta T}{\delta t} = \frac{\partial T}{\partial t} + \underline{V} \cdot \nabla_H T + w_p \frac{\partial T}{\partial z} \qquad \text{-------(2)}$$

where w_p = vertical velocity of the air parcel moving along a pressure surface.

Even in regions of high wind speed the slopes of isobaric surfaces are relatively small, and it may be shown by calculation in actual cases that the term $\omega_p \frac{\partial T}{\partial z}$ is in general negligible in comparison with the remaining terms.

Subtracting equation (1) from equation (2) and neglecting $\omega_p \frac{\partial T}{\partial z}$ gives

$$\frac{\delta T}{\delta t} - \frac{dT}{dt} = -\omega \frac{\partial T}{\partial z} \qquad \text{--------(3)}$$

It is readily shown that

$$\frac{dT}{dt} = -\omega \frac{g}{c_p} = -\omega \gamma_{ad} \qquad \text{--------(4)}$$

where c_p = specific heat at constant pressure,

g = acceleration of gravity,

γ_{ad} = adiabatic lapse-rate of temperature (saturated or unsaturated).

Thus equation (3) becomes, on substitution from equation (4)

$$\frac{\delta T}{\delta t} = -\omega(\gamma_{ad} - \gamma) \qquad \text{--------(5)}$$

Equation (5) is that applied in Panofsky's isobaric trajectory method.

<u>Evaluation of vertical motion</u>. The rate of change of temperature along isobaric trajectories (the horizontal vector being computed as described in Graham's method,)

is found from constant-pressure charts of temperature and divided by the average value of ($\gamma_{ad} - \gamma$) along the trajectory to give the vertical motion. Some difficulty arises over the choice of the appropriate height interval to be used to determine γ : too small a height interval is open to the objections of unrepresentativeness of small sections of the ascent curve and to over-straining of accuracy of the environment curve; the use of too large an interval may on the other hand lead to undue smoothing out of real features. The interval actually used varied somewhat according to the nature of the ascent curve, but did not exceed 100mb.

(4) Panofsky's Advective Method (5), (6)

This method takes into account the advection of cold or warm air over a station. Also, as in Panofsky's isobaric trajectory method, it assumes that rising or falling air cools or warms at a known rate.

The equation used is

$$\frac{\partial T}{\partial t} + \underline{V} \cdot \nabla_H T + \omega(\gamma_{ad} - \gamma) = 0$$

which is derived by the substitution of equation (4) into equation (1) of the method last described. In this equation only ω is unknown and can therefore be calculated. $\frac{\partial T}{\partial t}$ is the local change in temperature above the radio-sonde station over a period of twelve hours. $\underline{V} \cdot \nabla_H T$, the advective term, is found by multiplying the temperature gradient above the station by the wind-component parallel to the temperature gradient. The temperature gradient is found from isotherms drawn on constant-level charts, while the wind-component is the appropriate component (i.e. that parallel to the temperature gradient) of the wind observed at the station.

(5) Del Computer Method (7)

This kinematical method, employed by R.D. Graham, is a combination of the Bjerknes - Sandström streamline technique and the Bellamy objective technique.

Briefly, the method involves computing the horizontal divergence ($\nabla_H \cdot \underline{V}$), (hereafter divergence), using the "Del Computer", and calculating the vertical motion from the divergence. As well as computing $\nabla_H \cdot \underline{V}$, the "Del Computer" will give simultaneously values of ζ, the normal component of relative vorticity, (hereafter vorticity), which can also be used to calculate the vertical motion.

Theory of "Del Computer". The Bellamy technique is based on the following equations:-

$$\zeta = \text{Vorticity} = \oint \frac{\underline{V} \cdot \delta \underline{r}}{\sigma} \qquad \text{------(1)}$$

$$\nabla_H \cdot \underline{V} = \text{Divergence} = \oint \frac{\underline{k} \wedge \underline{V} \cdot \delta \underline{r}}{\sigma} \qquad \text{------(2)}$$

where $\underline{V}$ = wind velocity vector,

$\delta \underline{r}$ = vector line element,

$\underline{k}$ = unit vertical vector,

and $\oint$ represents integration round a closed curve of area σ. From these equations it can be shown that for any triangle ABC,

$$\zeta = \sum_{i=A}^{C} \frac{v_{ip}}{h_i},$$

$$\text{and } \nabla_H \cdot \underline{V} = \sum_{i=A}^{C} \frac{v_{in}}{h_i},$$

where h_i is the altitude at vertex i, and v_{ip}, v_{in}, are the wind components at vertex i parallel and normal to the opposite side of the triangle. These are the equations used by Bellamy in his objective method which employed observed winds at the vertices of the triangle. He applied this procedure to a fixed system of triangles formed by the network of radio-sonde stations over the United States. Graham saw that it would be advantageous to have a single triangle able to move to any part of the chart under consideration, and calculate the divergence there. To be able to do this, however, an isogon-isovel wind analysis is necessary, so that the wind speed and direction is theoretically known at every point of the chart. Graham's Del Computer method starts, therefore, with an isogon-isovel wind analysis (which is the first stage of the Bjerknes - Sandström technique), and then employs the Bellamy system and associated equations. The triangle ABC used is equilateral, of altitude 100 nautical miles, and when reading off wind speed and direction from the wind-analysis chart, it is always orientated such that BC is West-East and A is to the North. The values thus obtained are fed into the "Del Computer" which calculates the divergence and vorticity over the triangle ABC.

<u>Description and use of Del Computer</u>. The Del Computer has two sections - a STATOR, (see fig.2), which comprises a stationary protractor and a coordinate grid, and a transparent ROTOR, (see fig.2), which has three points A,B,C, marked symmetrically around its circumference. The rotor rests on top of the stator and can turn freely about a central pivot.

The vectors obtained from the wind-analysis chart are plotted in turn on the rotor, which has previously been rotated to that vector's direction. The ends of the vectors are marked by spots of ink, which can be wiped off after use, and the final values of divergence and vorticity read off from the coordinate grid of the stator below (see fig.2).

The mesh size of this grid and the lengths of the vectors are chosen to comply with the following. If the units of divergence and vorticity are $10^{-5}\sec^{-1}$, then the wind speed scale must be such that a length corresponding to 100 knots equals 27.8 units on the coordinate scale.

$$\left[\nabla_H \cdot \underline{V} = \sum \frac{v_{in}}{h_i} \;;\; \text{if } \sum v_{in} = 100 \text{ knots}, \nabla_H \cdot \underline{V} = 27.8 \times 10^{-5} \sec^{-1}\right]$$

When the divergence has been calculated, the vertical motion is obtained from the continuity equation, which reduces to

$$\frac{\partial w}{\partial z} = -\operatorname{Div} \underline{V} ,$$

if density changes are neglected: the boundary assumption is made that the vertical motion at the earth's surface is zero.

(6) <u>Sheppard's Method</u> (8)

The principles underlying this method are the same as for the Bellamy objective technique described under the Del Computer method. It was shown there that the divergence over any triangle ABC, is given by

$$\nabla_H \cdot \underline{V} = \sum_{i=A}^{C} \frac{v_i}{h_i}$$

where v_i is the component of the wind parallel to the altitude at vertex i, and h_i is the length of this altitude.

The eight radio-sonde stations in the British Isles form a network of seven triangles of roughly the same size, which afford an opportunity for the straightforward application of the above equation.

Once the divergence has been calculated at various levels, the vertical motion is found using the continuity equation and boundary condition as in the Del Computer method.

Sheppard's examination of the errors involved in this method led him to the statement that wind direction to a higher degree of accuracy than that normally given in weather broadcasts (nearest 10°) is required for the application of the method. British observations reported to the nearest degree were specially obtained on request.

(7) Sawyer's Dynamical Method (9)

Two main theoretical equations are used.

(a) the approximate vorticity equation

$$\mathrm{Div}\,\underline{V} = -\frac{1}{\zeta + \ell}\frac{D}{Dt}(\zeta + \ell) \qquad \text{------(1)}$$

and (b) the continuity equation, which may be written

$$\frac{\partial}{\partial z}(\rho w) = -\mathrm{Div}(\rho \underline{V}) - \frac{\partial \rho}{\partial t}. \qquad \text{------(2)}$$

In these equations, $\underline{V}$ = horizontal wind velocity vector; ζ = vertical component of vorticity vector; ℓ = the coriolis parameter; ρ = density; and w =vertical velocity.

The method is briefly as follows. The vertical component of vorticity, ζ , is calculated from the constant-pressure charts and used to construct charts of absolute vorticity, $\zeta + \ell$. From these $\mathrm{Div}\,\underline{V}$ is calculated using equation (1), and this, when substituted in equation (2), gives the vertical velocity.

Evaluation of ζ. Total vorticity is given by

$$\nabla \times \underline{V} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

and ζ = vertical component = $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

For geostrophic flow,

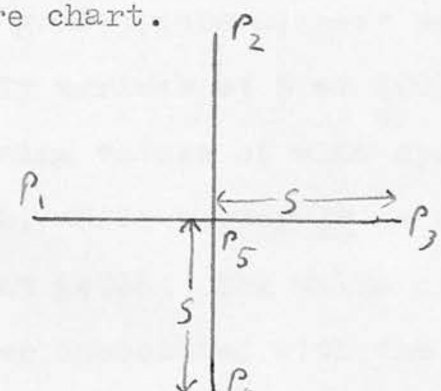
$$v = \frac{g}{\ell}\frac{\partial z}{\partial x} \quad \text{and} \quad u = -\frac{g}{\ell}\frac{\partial z}{\partial y}$$

hence $$\zeta = \frac{g}{\ell}\frac{\partial^2 z}{\partial x^2} + \frac{g}{\ell}\frac{\partial^2 z}{\partial y^2} - \frac{g}{\ell^2}\frac{\partial z}{\partial y}\frac{\partial \ell}{\partial y}$$

The last term in this equation can be shown to be very small except near the equator. Hence in mid and high latitudes,

$$\zeta = \frac{g}{\ell}\left[\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right] \qquad \text{------------(3)}$$

This equation has now to be used to obtain ζ from constant-pressure chart.

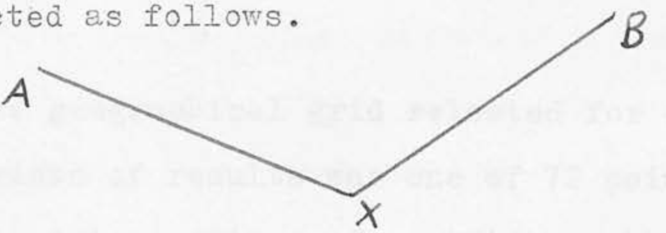


The overlay shown above can be used to evaluate $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial y^2}$ by reading the heights ($z_1, z_2, \cdots \cdots z_5$) at $P_1, P_2, \cdots P_5$ when it is placed over the constant-pressure chart. Equation (3) then becomes

$$\zeta = \frac{g}{\ell S^2}\left[z_1 + z_2 + z_3 + z_4 - 4z_5\right] \qquad \text{-----------(4)}$$

where S is the distance from the centre of the overlay to the points P_1, P_2, P_3 and P_4. Having obtained values of ζ from equation (4), the coriolis parameter ℓ is added and charts of $\zeta + \ell$, the absolute vorticity, constructed.

In order to evaluate $Div \underline{V}$ (equation(1)), the change in absolute vorticity along a horizontal trajectory is divided by the mean value of absolute vorticity along the trajectory. The trajectories are constructed as follows.



The air parcel leaves A at 0200h (say), reaches X (one of the reference grid-points already mentioned) at 0800h, and finally arrives at B at 1400h. The vector AX is obtained using values of wind speed and direction observed at 0200h, while vector XB is obtained using values observed at 1400h. The value of divergence calculated is then associated with the grid-point X.

Once the divergence has been calculated, equation (2) is used to compute the vertical motion. If the local time variation, and horizontal variation of density are neglected, and the normal boundary assumption made of no vertical motion at the earth's surface, equation (2) has the simple form already used in connection with the Del Computer and Sheppard's methods; i.e.

$$\frac{\partial w}{\partial z} = - \mathrm{Div}\, \underline{V}.$$

PRESENTATION OF RESULTS

The geographical grid selected for easier inter-comparison of results was one of 72 points, formed by the intersections of certain parallels of Longitude and Latitude. The grid did not cover the whole area for which raw data was available, but was restricted to a central section of the chart. In this way, any spurious results near the edges of the charts were eliminated.

In methods (1) - (4) (previous section), the grid was imposed only when the vertical motion calculations was complete. Calculation in terms of the grid was not possible at an earlier stage, since in all these (trajectory or advective) methods, the vertical motion is computed from actual data at radio-sonde stations. The computed values were transferred to the grid by interpolation of numerical values or by construction of isopleths where necessary. Although this process involved a certain smoothing of the results, it is not considered that any significant casual error was thereby introduced.

In methods (6) and (7) the grid was imposed from the beginning of the computations since those could be made at any point on the chart.

The values obtained in method (5) were not transferred to the grid, since, being dependent on winds to nearest degree, they covered only the British Isles.

Although the grid comprised 72 points, the total number of values of vertical motion for any one method, and at any one level, was always less than this number. This was due to various reasons including gaps in the raw observational data, and the scarcity of radio-sonde stations over the sea.

The unit of vertical motion selected as most appropriate for the present purpose was millibars per 12 hours: Pressure is perhaps the most convenient height parameter, and the use of the time interval of twelve hours - that between successive radio-sonde observations - rather than a shorter time interval helps to emphasise that the computed values are indeed averages over rather a long period. Comparison with the C.G.S. unit (centimetres per second) is made later in the paper.

INTER-COMPARISON OF RESULTS

The results obtained by the various methods (excluding that of Sheppard) were compared in two ways.

(1) Inter-correlation coefficients were calculated for computed values of vertical motion at individual grid points: this was done at each of the four pressure levels, so that four sets of correlation coefficient, r, were obtained (see Table I).

(2) The average magnitude of computed vertical motion, irrespective of sign, was calculated for each of six methods and at each of the four pressure levels, and plotted on a graph(see fig.3).

Inter-correlation

The problem of computing the standard error of the individual correlation coefficients shown in Table I is very difficult because of the lack of statistical independence of the correlated pairs of values (in theory numbering 72, but in practice numbering normally between 60 and 70 because of missing values at individual grid points). Nevertheless, the correlation method is perhaps the most convenient way of summarising the degree of agreement of computed vertical motion, in magnitude and sign, between the various methods.

In order to assess (very roughly) the measure of significance of individual correlation coefficients, we may perhaps assume one in three of the correlated pairs independent. The corresponding probable error of the calculated coefficient, though dependent on this latter value itself, is then approximately 0.10 to 0.15 (probable error = $\frac{2}{3}\frac{(1-r^2)}{\sqrt{n}}$) where n is the number of independent pairs from which r is calculated. On this very approximate basis, only those coefficients exceeding about 0.30 (three times the probable error) have any likely significance as to sign.

The following main points emerge from Table I.

(1) The measure of agreement obtained between the various methods is variable but on the whole very poor.

(2) Sawyer's method stands out as that method whose results are least in agreement with those of the other methods. In only one case (correlation with the Del Computer method as 300mb) was a fairly high positive value of r obtained. This seemed to indicate that a major systematic error had been made in the application of this method and initiated a detailed examination of the method, to be discussed later. (The indications of fig. 3 are that it is mainly the <u>sign</u> of the vertical motion that is involved, since Sawyer's method agrees well with most of the other methods in average magnitude of computed vertical motion).

(3) On the very rough assumption, referred to above, that values of $r < 0.30$ have no significance even as to sign, Table II has been drawn up from the results shown in Table I in order to summarise briefly the measure of agreement between the various results. From this it may be seen that the Del Computer and Sawyer's methods stand out as being in poor agreement both with each other and with the other methods. This does not of itself signify that either one of these may not best reflect the true picture of vertical motion, since it must be borne in mind that the other

methods included in Tables I and II have a good deal in common in being each of them based on the assumption of adiabatic motion.

Magnitude of Vertical Motion

The main facts that emerge from fig.3 are that four of the six compared methods are in fairly good agreement as to the average magnitude of vertical motion, irrespective of sign: an average value of about 60mb / 12hr (1.5 cm/sec) is indicated. Graham's method shows, however, values only about half as large as this, and the Del Computer twice as large.

There is a strong suggestion from fig. 3 that the computed values of vertical motion increase with height up to 300 mb. The agreement between the various methods is, in this sense, good.

CORRELATION WITH RAINFALL CHART

It was stated in the introduction that the choice of the particular weather situation under study was governed by the associated unexpectedly large rainfall. This aspect is studied here in relation to the vertical motion problem by the comparison of rainfall and computed vertical motion charts.

The depiction of all the 28 computed charts of vertical motion is laborious and hardly necessary for the present purpose. The field of motion at 700mb was selected as that most appropriate for comparison with the rainfall pattern, since on the day in question the Freezing Level over the British Isles was near 700mb. (It is safe to assume that vertically upward motion was occurring at this level(and probably at higher and lower levels also) in regions showing large amounts of rain). The computed vertical motion charts for the 700mb level are shown in Appendix I (figs.4 to 10).

Since routine rainfall measurements are made only at 09h and 21h, two rainfall charts were required to depict the conditions in the period 02h to 14h on 3rd September covered by the calculations of average vertical motion; those for the periods 21h on the 2nd to 09h on the 3rd and 09h on the 3rd to 21h on the 3rd. These charts are shown separately. An attempt has been made in a third chart to estimate the rainfall during the 02h to 14h period, by weighting the values measured at 09h and 21h in the ratio 7:5. This procedure makes implicitly the very rough approximation that the measured rainfall was evenly distributed with time . All three rainfall charts are shown in Appendix I (figs.11 to 13).

According to a formula developed by Bannon,[10] a rate of rainfall of 22mm/12hr when the temperature lapse rate follows the saturation adiabatic and the wet-bulb potential temperature is approximately 50°F requires an average upward vertical velocity of 7.6cm/sec (290mb/12hr) from 1000mb upwards to the tropopause. A value of this order in the main rain area is, on this basis, to be expected.

From a study of the rainfall chart shown in Appendix I (fig.13), the main area of upward motion would be expected over North-East Scotland, possibly extending well into the North Sea. In general the 700mb vertical motion charts show very poor agreement with this picture. Longley's method gives the best agreement, although his main area of upward motion is a little too far south and its maximum value too small. The methods of the Del Computer and Sheppard show areas of upward motion of the right magnitude, but they are too far to the south and do not extend their influence over North-East Scotland at all - indeed their results indicate downward motion over that area.

The methods of Graham and Panofsky (both advective and trajectory) show extensive areas of upward motion over the North Sea, but the magnitude of this motion is small especially over the area of maximum rainfall. Sawyer's method gives a completely different picture to that expected.

A more detailed consideration on the basis of vertical motion at all computed levels up to 500mb in the region of heaviest rainfall, showed that upward motion predominated for the Longley and Panofsky (advective) methods, to a lesser extent in the Graham and Panofsky (trajectory) methods, while downward motion was shown at all three levels (850, 700, 500mb) in the Sawyer and Del Computer methods.

Thus on the basis of these comparisons with the rainfall pattern, the Longley and Panofsky(advective) methods can be considered the most reliable, while the methods of Sawyer and the Del Computer are least reliable. As was stated above, however, no one method gave a really satisfactory correlation with the rainfall pattern.

GENERAL DISCUSSION

This discussion will be given under three headings based on the broad divisions of principle underlying the various methods:

(a) adiabatic,

(b) kinematic,

(c) dynamic.

(a) Adiabatic Methods

The accuracy of all these methods (Graham, Longley, Panofsky advective and trajectory) is very sensitive to the degree of accuracy of the original raw data. This is especially so in the methods of Graham and Longley where e.g. an error in the temperature of 1C° can lead in certain cases to large errors in the final vertical motion estimated from the tephigrams. Such cases occur when the existing lapse rate is near the dry adiabatic lapse rate. Since the observed temperatures play an important role in the calculation of vertical motion in the methods of Panofsky(advective and trajectory) the accuracy of these methods is also highly dependent on accurate observations.

The standard hours of radio-sonde observations

(02h and 14h) have different associated conditions of solar radiation in the high atmosphere. Radiation errors are known from the analysis by Kay [11] to be present in radio-sonde temperature observations. Though in more recent years the temperature data have been corrected for radiation effects, there exists a possibility that such corrections are systematically somewhat in error. If, in fact, there is a radiation error in the corrected values of the same sense as that present in the uncorrected - a tendency for too-high values at the 14h ascent - a systematic over-estimation of vertical downward motion and under-estimation of upward motion would be involved in the Graham, Panofsky advective and Panofsky trajectory methods. The presence of large casual and systematic errors in the analysis of these and the other methods, makes it impossible however, to determine whether or not there exists an effect such as that here mentioned.

Three of these adiabatic methods are dependent on the construction of trajectories. These are in every case somewhat approximate, especially so if the wind speed is great, or if the isobaric curvature is marked or changes rapidly with time - as in the case of vigorous development.

The problem of multiple solutions presents a real difficulty in the application of the Graham method, and to a much greater extent in the Longley method: in the latter, multiple solutions and indeterminate solutions (those cases in which the value taken from the tephigram at the starting point of the air does not intersect any of the ascent curves in the vicinity of the end point) amount to about 50% of the total - in such a case there is obviously difficulty in obtaining help from neighbouring values.

Since all adiabatic methods are based on the assumption of adiabatic changes, high accuracy is hardly attainable in the lower layers of the atmosphere, where radiation may be expected to have a very significant effect within a period of twelve hours. Since a low level heat <u>source</u> is mainly involved in the time interval from 02 hours to 14 hours, systematic errors of the same sense as those involved in radiation errors in temperature recording, are to be expected from this cause.

Further errors which may occur are, in the case of the adiabatic methods other than Longley's, neglect of latent heat, from which, for example, may arise changes in the potential temperature because of evaporation from precipitation falling through the layer under consideration.

A rough experimental determination of the probable error involved in individual computations of vertical motion by the adiabatic method suggests a value of about 30mb per 12 hours.

(b) Kinematic Methods

A conspicuous feature to which attention has already been drawn was the high average values of vertical motion (independent of sign) obtained with the Del Computer method: averaged over the small area of the British Isles application of Sheppard's method gave values slightly smaller than those obtained by the Del Computer over the same area.

A basic difference between the kinematic and other methods is that the former leads to an instantaneous value, averaged over an area, while the other methods give, in essence, values averaged over a period of time. It would therefore seem that whether or not the resulting values are strictly comparable with each other depends on the relative scale of area and time with respect to air motion over which the averages are taken. The use of a small area in the kinematic methods and of a large time interval in the adiabatic methods would imply the study of motion on two different scales - respectively small as opposed to large scale.

In the Del Computer and Sheppard methods, the vertical motion is computed from the horizontal divergence, which is calculated over triangles, the size of which may be selected - though in Sheppard's method within the limitations set by the locations of the radio-sonde stations. The kinematic methods were applied to a simple situation using triangles of different area. It was found that larger values of computed divergence, and also larger values of vertical velocity - with appreciable distortion of shape - were obtained with the smaller triangles. In the case of the Del Computer in which divergence is calculated over an equilateral triangle, the values of divergence obtained were found to be approximately inversely proportional to the length of altitude of the triangle.

Rough experimental determination was made of the probable error of a single computed value (Del Computer) of vertical motion, assuming a rough 'probable' error of wind vector and allowing for the fact that, on the size of grid chosen, points could be plotted only to the nearest 0.2 x 10^{-5} sec.$^{-1}$. The resulting estimated error in vertical motion was about 100mb per 12 hours.

(c) Dynamic Method

The results of the application of Sawyer's method were notably at variance with the other methods and also with the synoptic situation. An examination was therefore carried out to see whether some systematic error was made in the application.

First, an independent evaluation of vertical motion was made in a specially selected vigorously developing situation (November 4 1957). In this case, exactly the procedure described by Sawyer was used instead of the slight variation used in the 1956 case previously described. This involved the construction of charts of $\log_e(\zeta + l)$ both at 02h and at 14h and the use of these to measure the divergence graphically, instead of numerically, as previously described. The results of this trial are not reproduced. They were, however, unsatisfactory, since the expected field of vertical motion in the case of the vigorous depression with heavy rainfall was not obtained or even closely approximated.

The failure to obtain a realistic picture of vertical motion in a situation of vigorous development was considered disturbing, and also puzzling in view of the results quoted by Sawyer. The probable reason was, however, considered to be the 12-hour gap in radio-sonde ascents in the two tests referred to above as compared with the 6-hour gap in the data used by Sawyer. In order to verify that the method was being correctly interpreted, it was decided to apply the method in the first of the two cases quoted by Sawyer - that of March 17 1948. Precisely the method described by the author - including the graphical determination of the time rate of change of absolute vorticity was therefore applied to this situation.

Reproduced in fig.14 of Appendix I is a chart showing the resulting computed vertical motion, together with associated surface fronts. For purposes of comparison with Sawyer's results, interpolated values from both trials at twenty-four selected grid points are shown in Table III: the unit of vertical motion is in this case mb/sec x 10^{-4} to afford an easier comparison with Sawyer's results. The agreement obtained as regards the magnitude of the vertical motion is reasonably good, both with regard to upward and downward motion (independent of sign) and to the overall preponderance of upward motion over the area. There are, however, large differences in the overall picture. That shown on the chart gives a much worse fit with the classical frontal picture than does Sawyer's : maximum upward motion is behind and not in advance of the warm front while downward motion occurs at the surface cold front and not just behind as obtained by Sawyer. A small negative correlation coefficient is in fact obtained between the twenty-four corresponding interpolated values of Table III.

In the application of Sawyer's method, subjective processes arise mainly (a) in the drawing of original height contours of isobaric surfaces, and (b) in deciding on the trajectory of the air at the two levels.

(a) Available upper air data for the case in question was only moderate and some subjectivity no doubt arose here.

(b) The determination of the trajectory of the air at 1000mb was made difficult by the rapid increase with time of the curvature of the path of the air at that level. Repeat calculations were made assuming'possible' (i.e.less likely) trajectories at the two levels. The results obtained are contained in fig.15 in Appendix I in the form of a scatter diagram, the corresponding correlation coefficient between computed values at twenty-four grid points being +0.60. A reasonable assumption appears to be that differences in assumed trajectories in the two independent trials of the same situation are unlikely to have produced so bad a fit as to have yielded a <u>negative</u> correlation coefficient.

Since (a) and (b) have been rejected as being the source of any major lack of agreement*x*, only the matter of the choice of s in $\zeta = \frac{g}{\rho s^2}(z_1 + \cdots - 4z_5)$ is left open to doubt. (S being the length of the grid in the overlay used to calculate the vorticity). Overlays with different values of s were first tried on a simple artificially symmetrical trough of low pressure. The result obtained was that small values of s gave isolated large values of vorticity, while as the length of s increased the vorticity became smaller and more smoothed out over the area under consideration.

A larger value of s was therefore tried on a section only of the 1948 situation with the same result - a smoothing out of isolated high values of vorticity leading to some smoothing but also appreciable distortion of the vertical motion pattern. It therefore appears that the choice of s is not as arbitrary as might be thought, but is in fact very important, the vertical motion pattern being very much dependent upon the choice made.

The size of s presents the same basic physical problem as does the area of the triangle, discussed under 'kinematic methods'.

GENERAL CONCLUSIONS.

The conclusions of the results obtained in this study are such that no method of computing vertical velocity here tried inspires any real confidence, when applied under realistic 'field' conditions as they now exist in N.W.Europe/i.e. radio-sonde ascents at a time interval of 12 hours (though on a moderately close network), and upper-air winds at 6-hour intervals and to the nearest ten degrees of arc.

The study indicates that some at least of the methods based on the adiabatic assumption probably yield vertical motion patterns of reasonable accuracy over restricted areas and at higher levels. Trial of kinematic and dynamic methods produced extremely erratic results in terms of what appeared to be the physical reality of the test situation. The results of the calculation of probable errors of computed values or of independent trials in a given situation are such as to hold out little hope of the successful application of these methods in the present 'field' conditions described. In addition, basic difficulties related to the scale of the movement involved in the study arose with both these methods.

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Lastly, I wish to thank the Director General of the Meteorological Office for data given on request.

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APPENDIX I

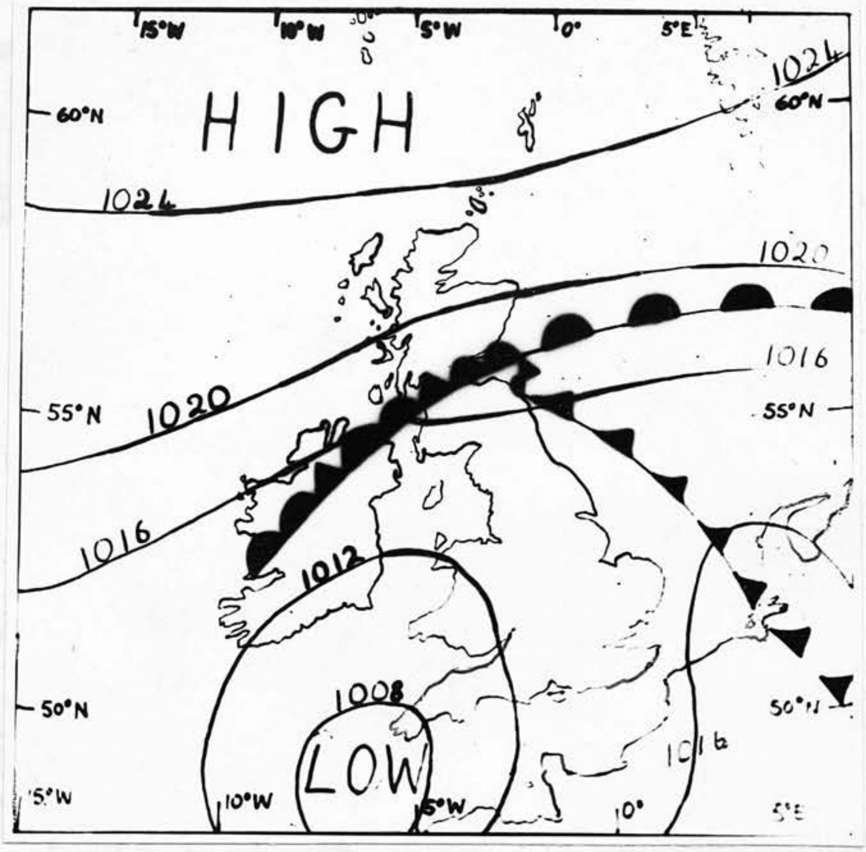

fig. 1

Surface synoptic chart for 0600h on September 3 1956.

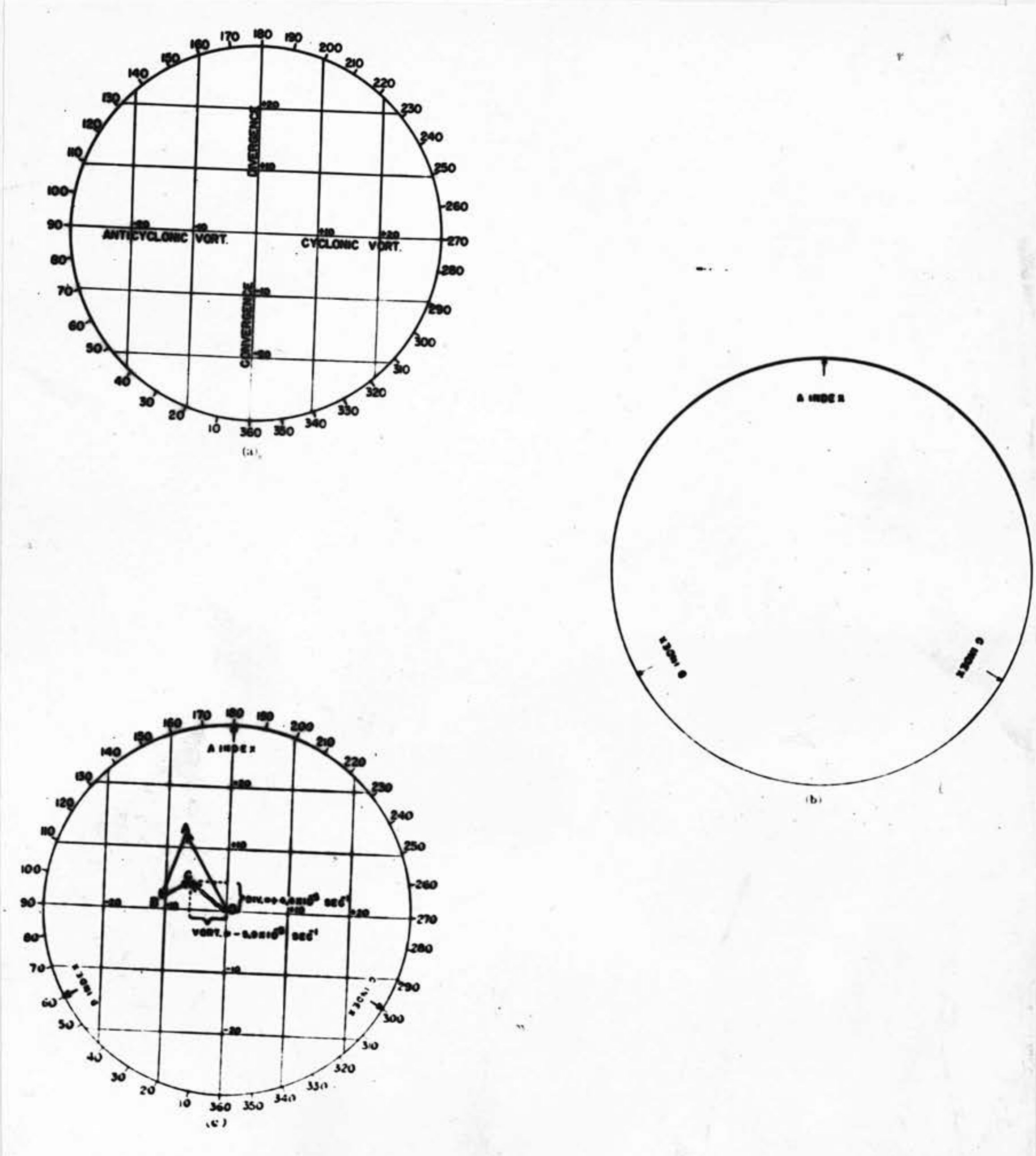

fig.2

Diagrams showing (a) STATOR of Del Computer (b) ROTOR of Del Computer and (c) example of calculation using the Del Computer.

TABLE I

TABLE OF CORRELATION COEFFICIENTS

Pressure Level	850mb	700mb	500mb	300mb
D & G	-0.05	+0.14	-0.01	---
D & L	+0.27	+0.06	-0.34	+0.01
D & Pa	+0.15	+0.25	+0.01	-0.05
D & Pt	+0.06	+0.23	-0.02	-0.05
D & S	-0.16	+0.02	-0.08	+0.32
G & L	+0.06	+0.06	+0.62	---
G & Pa	+0.35	-0.17	+0.12	---
G & Pt	+0.83	+0.56	+0.69	---
G & S	-0.02	-0.44	+0.05	---
L & Pa	+0.30	+0.35	+0.26	+0.40
L & Pt	+0.16	+0.35	+0.62	+0.89
L & S	-0.19	-0.32	+0.04	-0.18
Pa & Pt	+0.53	+0.46	-0.04	+0.40
Pa & S	-0.32	+0.08	-0.03	+0.07
Pt & S	-0.22	-0.32	+0.02	-0.10

D represents Del Computer Method

G do Graham's Method

L do Longley's Method

Pa do Panofsky's Advective Method

Pt do Panofsky's Trajectory Method

S do Sawyer's Method

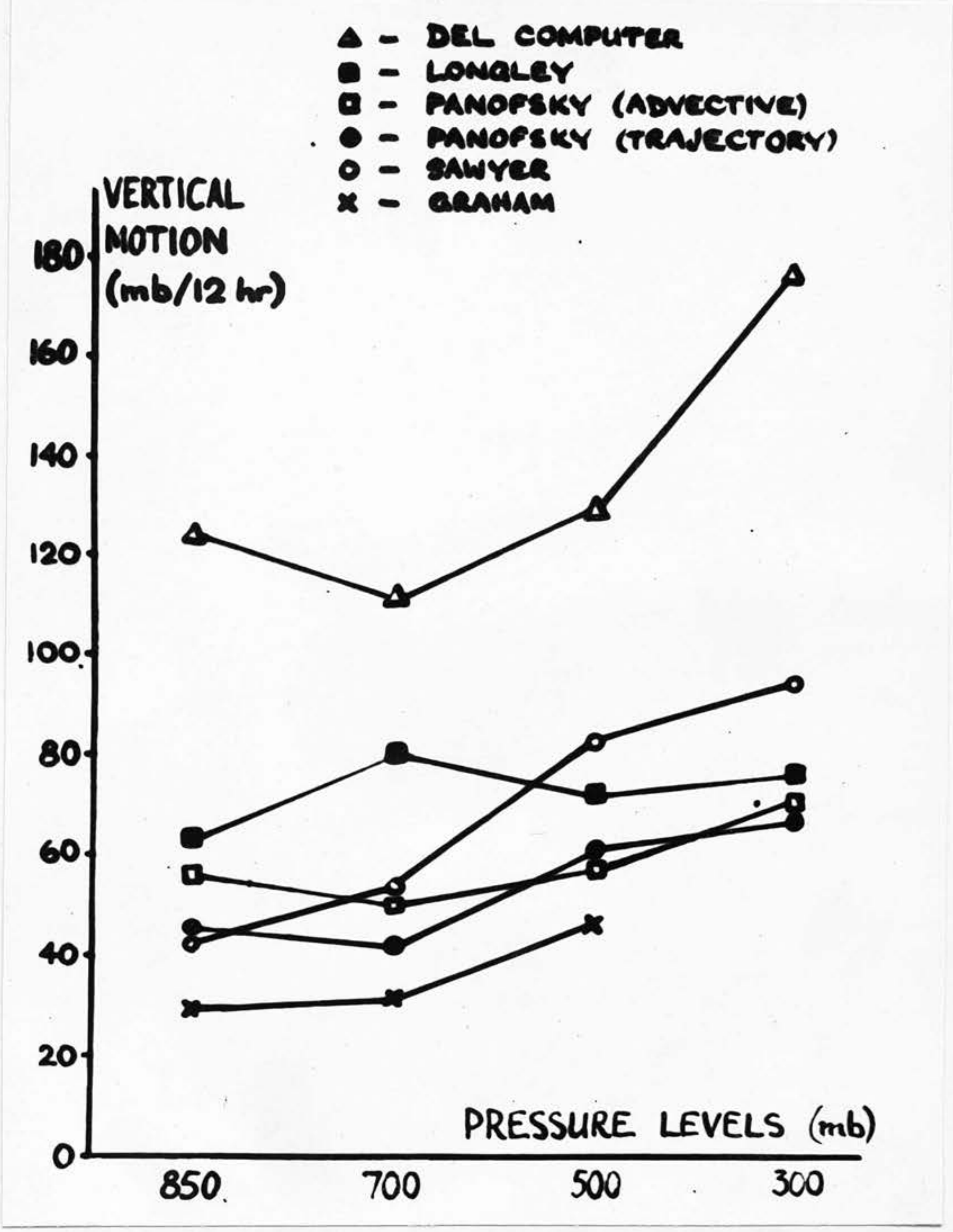

fig. 3

Graph showing the average magnitude of vertical motion (irrespective of sign) computed by the different methods at four pressure levels.

TABLE II

Distribution of frequencies of computed r

(a) $r > +0.3$ (b) $r < -0.3$ (c) $+0.3 > r > -0.3$.

Methods	D	G	L	Pa	Pt	S
(a)	1	5	7	7	9	1
(b)	1	1	2	1	1	4
(c)	17	9	10	11	9	14

The notation as regards the methods is the same as in Table I.

figs. 4 - 10

The following seven charts show vertical motion computed by the various methods for the 700mb pressure level. Units are mb/12hr, the +ve signs indicating upward motion and the -ve downward.

fig. 4.........Graham's Method

" 5.........Longley's Method

" 6.........Panofsky's (trajectory)Method

" 7.........Panofsky's (advective) Method

" 8.........Del Computer Method

" 9.........Sheppard's Method

" 10.........Sawyer's Method

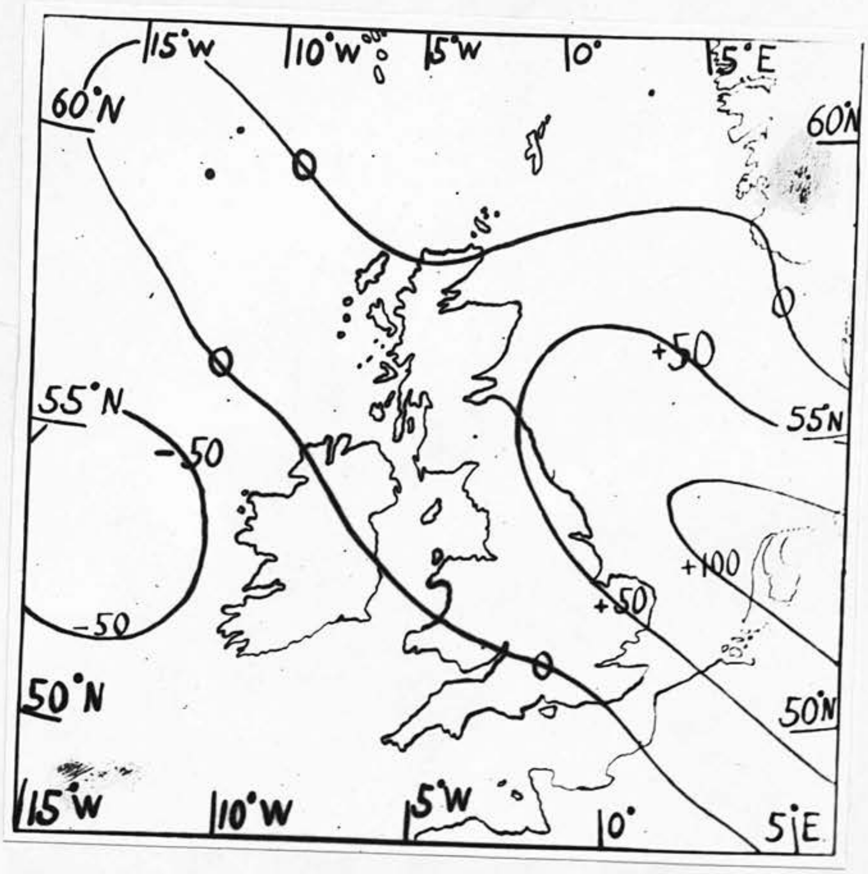

fig. 4

Graham's Method

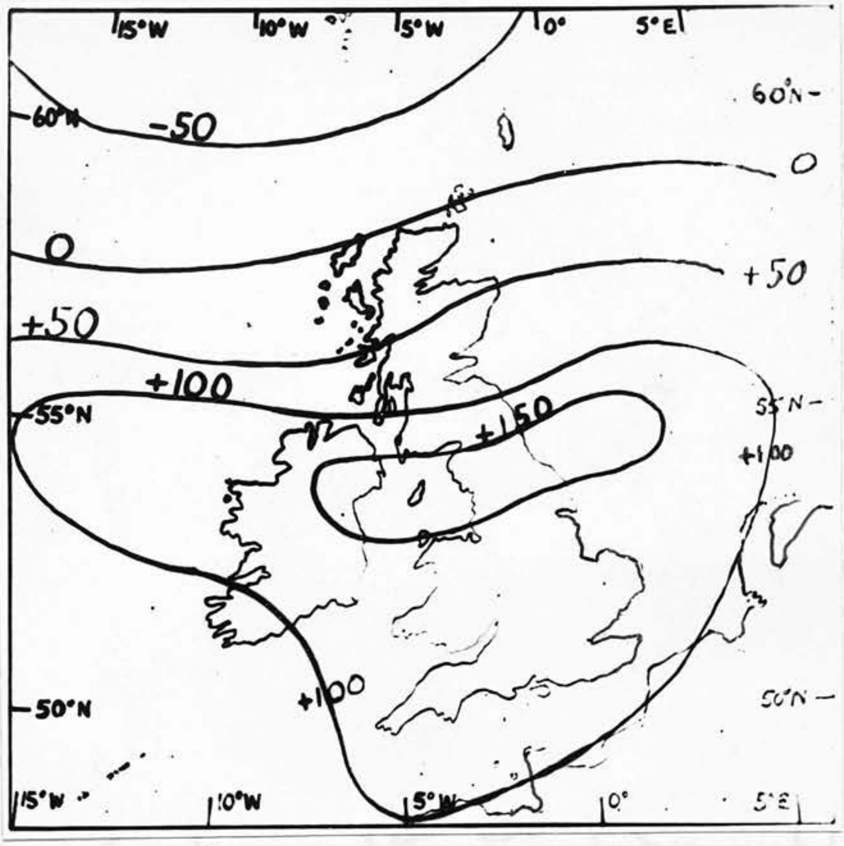

fig.5

Longley's Method

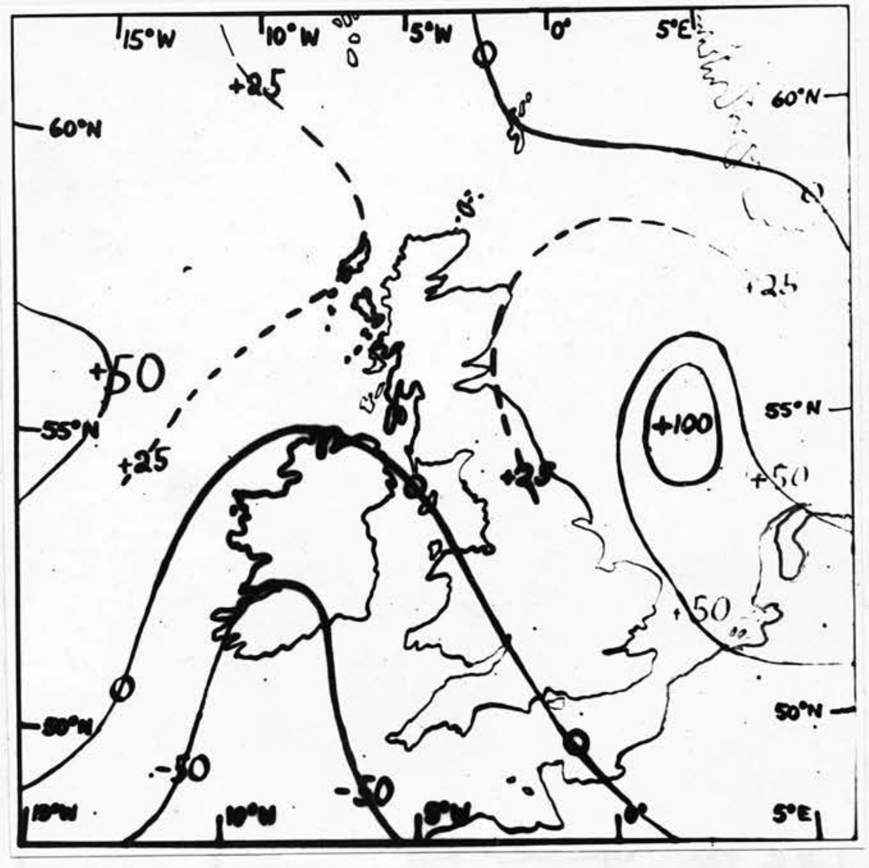

fig.6

Panofsky's (trajectory) Method

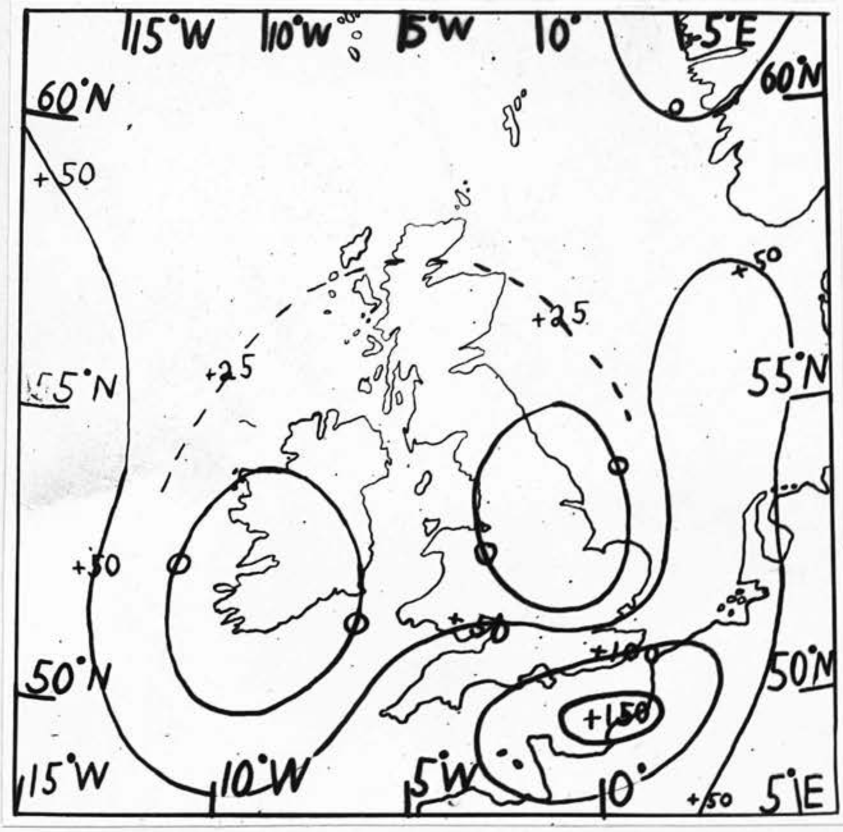

fig.7

Panofsky's (advective) Method

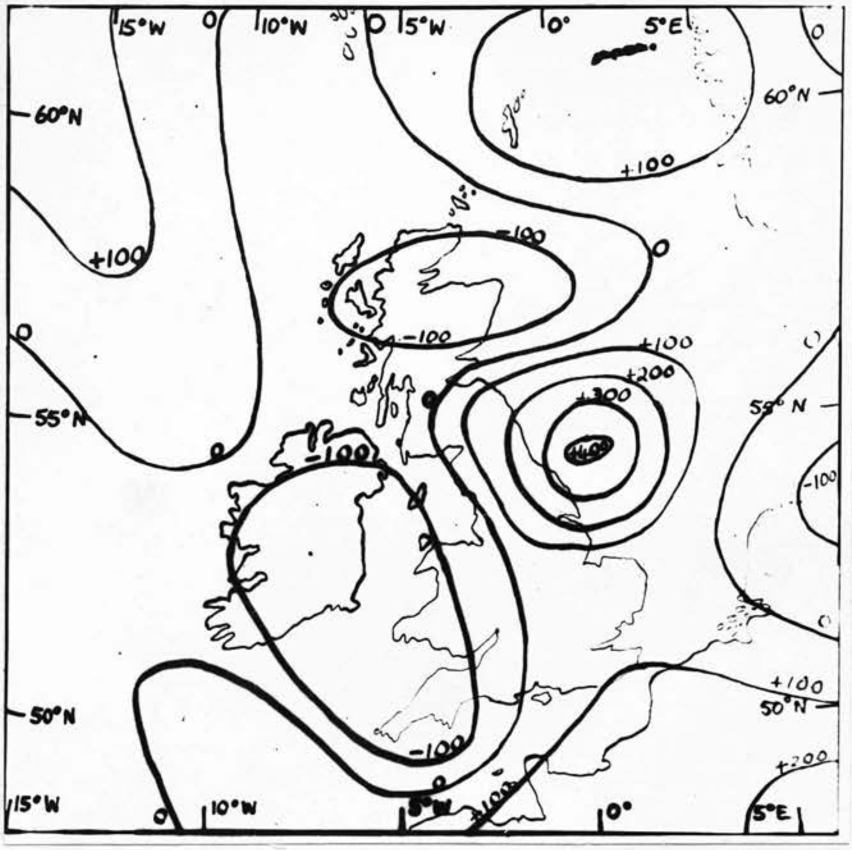

fig.8

Del Computer Method

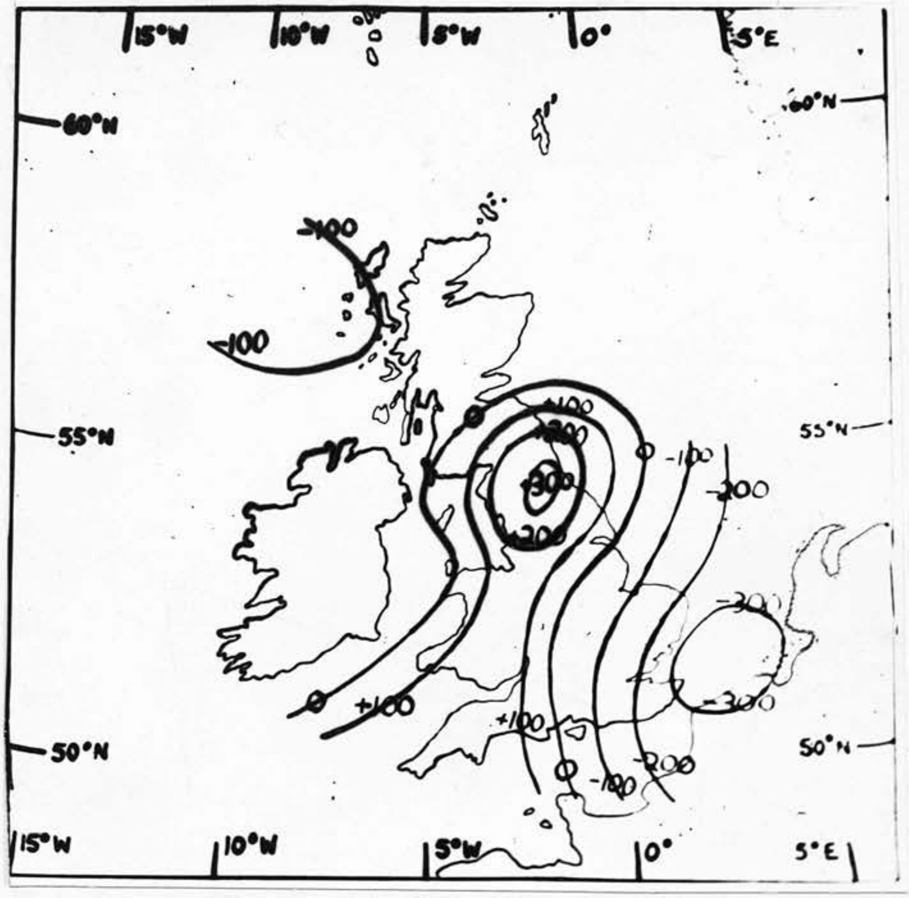

fig.9

Sheppard's Method

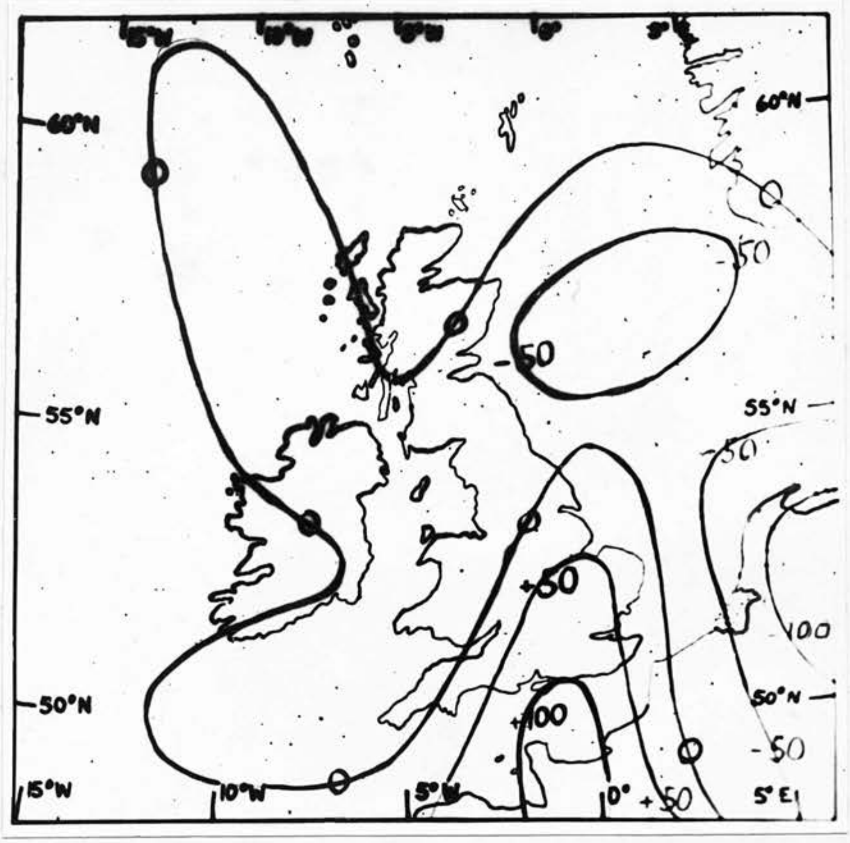

fig.10

Sawyer's Method

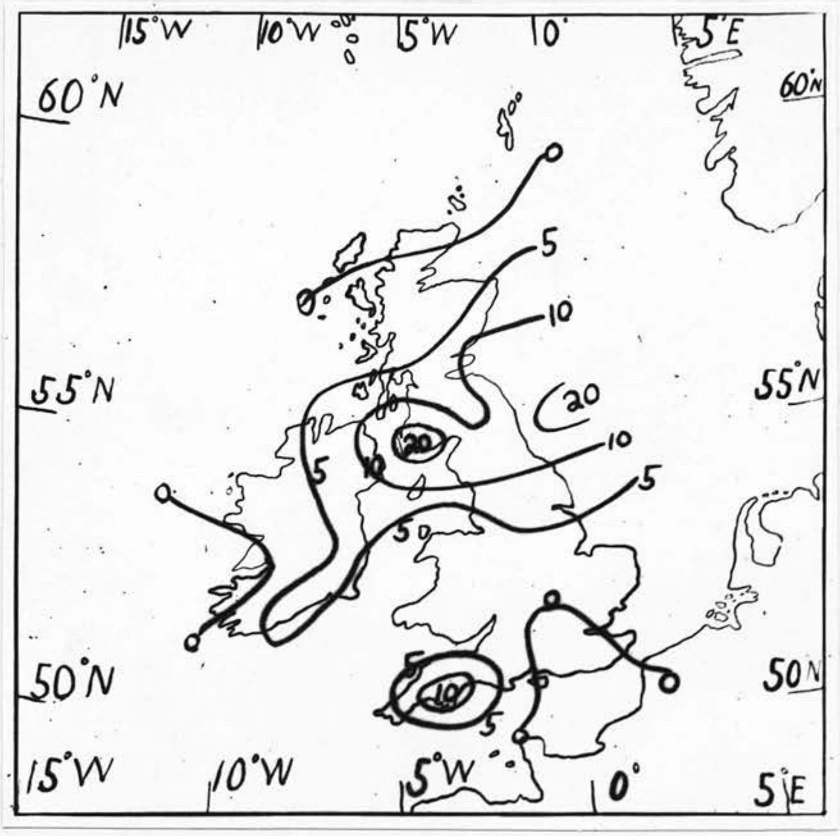

fig.11

Chart showing rainfall recorded from 21h on 2nd to 09h on 3rd September 1956 in millimetres.

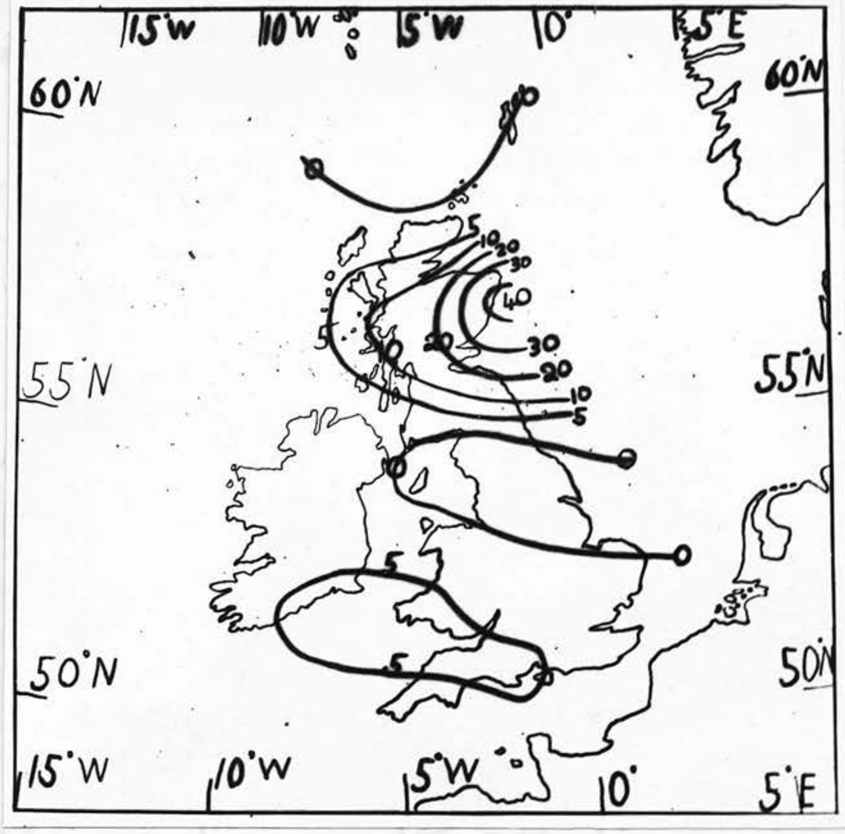

fig.12

Chart showing rainfall recorded (in mm.) from 09h to 21h on September 3 1956.

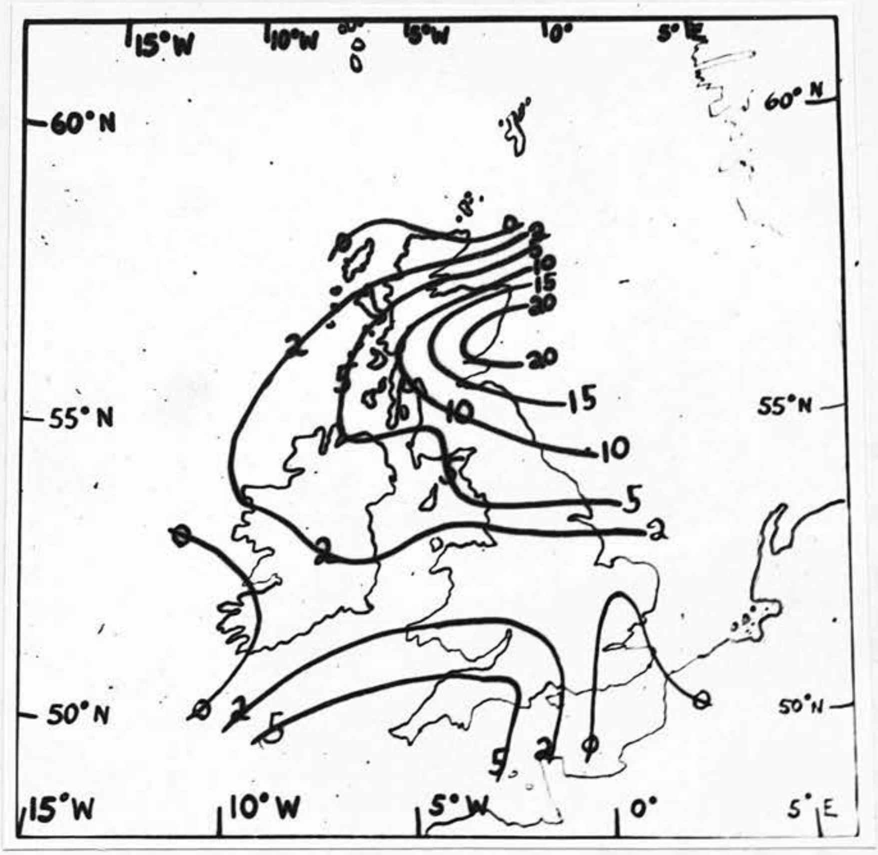

fig.13

Estimated rainfall pattern for the period 02h to 14h on September 3 1956. Isopleths in mm.

TABLE III

This table gives two sets of values of vertical motion, computed for the 700mb level at 18h on March 17 1948, at 24 selected grid points over the British Isles. The upper figure in each case is the value computed in the present study, while the bracketed figure is that interpolated from Sawyer's charted results. The units are mb/sec x 10^{-4}, +ve sign signifying ascending motion.

LAT. N	Longitude 8°W	6°W	4°W	2°W	0	2°E
55	+32	+20	0	+40	+40	+40
	(+40)	(+55)	(+70)	(+60)	(+50)	(+40)
53	+32	-16	-48	+40	+60	+48
	(0)	(+40)	(+80)	(+100)	(+100)	(+80)
51	-60	+16	+60	+140	+60	-16
	(+20)	(+20)	(0)	(-10)	(0)	(+60)
49	0	0	+48	+48	+48	+48
	(0)	(-20)	(-35)	(-35)	(-20)	(-20)

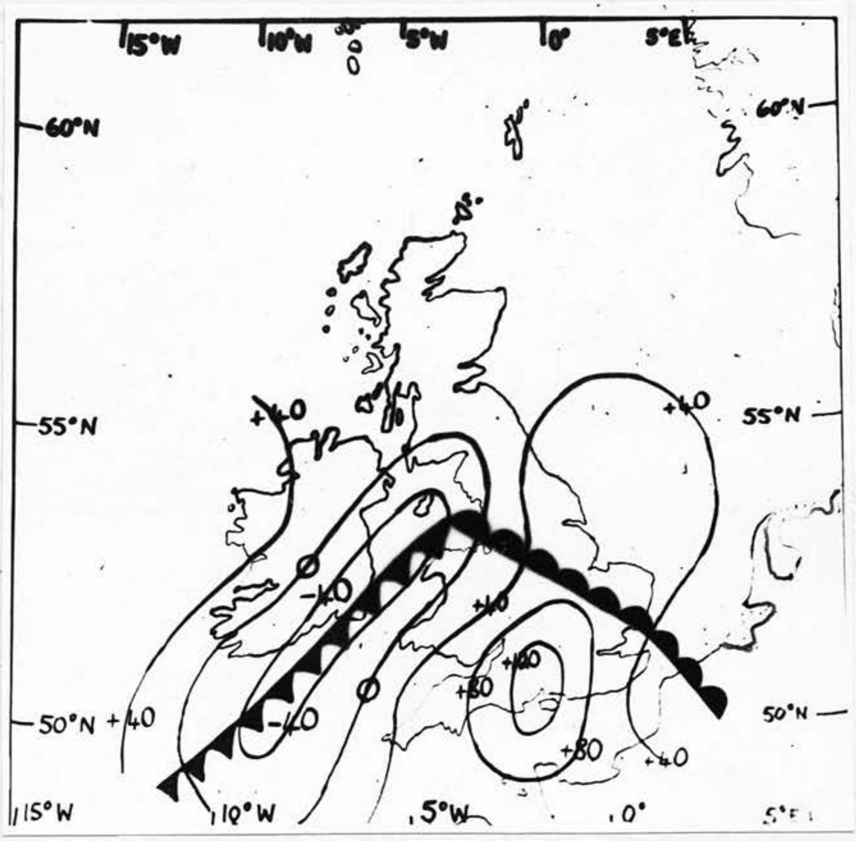

fig.14

Chart of computed vertical motion,
together with surface fronts for 18h on March 17 1948.
Units of vertical motion are mb/sec x 10^{-4}.

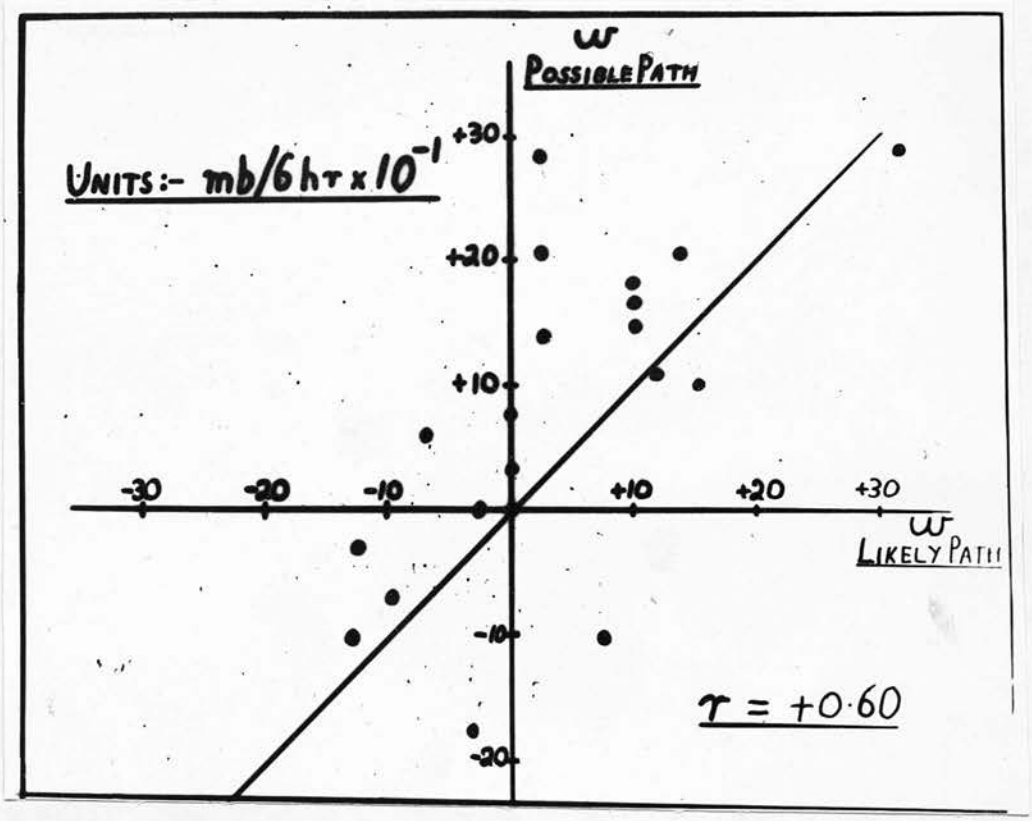

fig.15

Vertical velocity computed using 'possible' (i.e. less likely) trajectories plotted as a function of vertical velocity computed using 'likely' trajectories.

APPENDIX II

Other Methods

The methods which have been applied in this study are not the only ones which have been proposed for the computation of vertical motion.

Reference was made in the text to work by Bannon [10] who evolved a method of computation of vertical motion in those cases where this motion results in steady rainfall over a wide area. This method is based on a number of restrictive assumptions - such as the lack of influence of local instability effects - which greatly restrict its practical applicability.

Sheppard, Pettersen, Priestley, and Johanesson [12] described a method for calculating large scale vertical motion and applied it to the calculation of subsidence in the free atmosphere. The method is very similar to that of Panofsky (trajectory) and was not separately applied in the present study for this reason.

The total amount of measured ozone in the atmosphere is known to be highly dependent on the