Towards a Conceptual Foundation of

Activity-Based Costing: Theory and a Simulation Experiment

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Declaration

I hereby declare that this thesis is entirely my own composition, has been based on an independent research of my own, and has not been submitted for any other degree or professional qualification.

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Abstract

This study extends existing literature on the theoretical foundations of activity-based costing (ABC). This is done in two principal ways. Firstly, it identifies the conditions that support the construction of an aggregate activity output, i.e. the conditions under which a single measure of output can be used to accurately determine cost object incremental costs. This is a significant issue which has not been explored in the management accounting literature. However, as this study demonstrates, it does impose important conditions on the technological specifications of situations where ABC can generate decision relevant costs. Two conditions are jointly necessary and sufficient. The first one is the linear homogeneity property associated with each cost object production function. This condition ensures that marginal costs are constant, which is essential if the cost reported by an ABC system is also to be a relevant cost for decision-making. The second is that the marginal cost corresponding to a unit of cost driver used by a cost object is equal for all cost objects within a cost pool. This condition guarantees that the aggregated cost function at a given activity level depends on only one cost driver. Secondly, this study derives the short run structure of ABC. Based on the finding that ABC, as a basis for decision relevant costs, is only compatible with both linearly homogeneous technologies and activities operating with excess capacity, an analytical representation of the short run equation of capacity is presented. This is one of the highest profile innovations of ABC systems (Cooper and Kaplan, 1992). The study then develops existing product costing theory by investigating the consequences of relaxing the two above conditions. Firstly, it considers situations where technologies are not linearly homogenous. Two types of

technologies are explored: homogeneous and non-homothetic technologies. The reason for choosing these two technologies is that they accommodate a great number of non-linear input-output relationships. Overall, the distortions arising from the application of average cost driver rates to cost outputs, a fundamental procedure underlying both conventional and ABC systems, increase as the elasticity of each input demand with respect to output deviates from one, that is to say, when we depart from a linearly homogeneous technology. Secondly, and on the assumption that cost object technologies are linearly homogeneous, this study develops a simulation experiment with the objective of both testing the existence of a single cost driver and evaluating an accounting procedure that specifically accommodates the existence of multiple cost drivers. The simulation experiment serves also to introduce the question of uncertainty in input usage, another issue that has been neglected in the product costing literature.

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Finally, I dedicate this dissertation to the memory of my mother and Karen and to those who love me.

CHAPTER I – INTRODUCTION

1.1. Background

Activity-based costing has received considerable attention since its emergence in the late eighties, well-demonstrated by the significant number of articles published in professionally oriented journals and, to a less extent, in academic accounting journals (Lukka and Granlund, 2002, and Bjornenak and Mitchell, 2002).

Only a few, however, have focused on the theoretical foundations of ABC. From this perspective, Noreen (1991) constitutes the first significant example. He has focused the analysis of the theoretical foundations of ABC on the conditions related to cost functions and has derived three necessary and sufficient conditions for ABC systems to measure relevant costs for decision-making[1]. These are that (1) total costs can be divided into independent cost pools, each of which depends on one and only one activity, (2) the cost in each cost pool is strictly proportional to the level of activity in that cost pool and (3) the volume of an activity is simply the sum of activity measures utilised by the individual products.

Christensen and Demski (1995), Bromwich (1995, 1997), Bromwich and Hong (1999, 2000) have supplemented the work of Noreen (1991) by developing a more fundamental analysis of the theoretical foundations of ABC, in the sense that they

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[1] In this study, decision relevant is taken to mean relevant to decision making on final output variation, e.g. the expansion or reduction of output of existing products, the introduction of a new product, make or buy (outsourcing) decisions or special orders. This is consistent with Noreen (1991).

consider technology, apart from input prices, as the primary determinant of cost functions. This constitutes a perspective which, although well recognised in the production economics literature (e.g. Chambers, 1988 and Varian, 1991), had been systematically absent in the management accounting literature.

In an ABC context, Christensen and Demski (1995) have interpreted concepts already familiar in the production economics literature, namely cost function separability and linearity of the cost function. Cost function separability ensures the aggregation of inputs into independent activity cost pools or the definition of activity cost functions that are all independent of each other. Linearity of the cost function ensures that the cost reported by an ABC system, an average cost, is also a relevant cost for decision-making. Bromwich and Hong (1999, 2000) have investigated the technological conditions that support ABC systems capable of measuring incremental costs. Their analysis has investigated the conditions related to technology that more generally satisfy the three conditions derived by Noreen for ABC systems to measure decision relevant output costs. Specifically, they have derived the following conditions (see also Lucas, 2003, for a review of the conditions derived by Bromwich and Hong): (1) non-jointness, to rule out the existence of economies or diseconomies of scope either within or between cost pools, (2) production function homotheticity, in order to represent the inputs used in a cost pool by a single aggregate cost driver, (3) production function separability, to ensure the independence between cost pools.

1.2. Problem Statement

Although the above management accounting literature provides a deeper analysis of the theoretical foundations of ABC and thus an extension of the work of Noreen (1991), it has two major gaps.

The first gap is that it is implicitly (Christensen and Demski, 1995) or explicitly (Bromwich and Hong, 1999, 2000) founded on the assumption that it is possible to represent a multi-output technology as a single output technology. It is worth analysing the meaning of this assumed equivalence between a multi-output technology and a single output technology.

In single output technologies an activity cost pool is usually interpreted as an intermediate input that is used to produce the single final output[2]. In a multi-output technology, the first reason for aggregating say m technologies in an activity cost pool is the fact that they depend on the same vector of inputs, which are separable among the m outputs (cost objects). Existing ABC literature assumes that it is possible to construct a single or aggregate measure of output that fully captures the cost of the resources used by the various cost objects within a cost pool. Such an aggregate measure of output means that a multi-output technology is in fact equivalent to a single output technology. In the production economics literature this

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[2] One thing is the final output (or final good output), another thing is the output of an activity, such as the number of set-ups, the number of deliveries or even an intermediate output. Note also that cost objects are not necessarily final outputs. For example, a cost object can be an activity, when the output of an activity is the input of another activity.

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is called output separability and allows an output vector to be expressed by a single output index (Bromwich and Hong, 1999, p. 47).

The point that must be emphasized here is that the ABC literature has taken output separability for granted, without deriving the necessary and sufficient conditions that support the construction of an aggregate activity output. One of the consequences of this gap is that the precise analytical articulation that necessarily exists between a cost driver, the technology of different cost objects and activity costs is not clear. In other words, the duality between the multi-output technology and costs in ABC has not been derived. Moreover, any complete analysis of the conditions that support the aggregation of the m technologies in an activity cost pool cannot be fully derived without the representation of the relationships that exist between the technology of different cost objects, the single measure of output and activity costs. This derivation is the first and primary purpose of this study.

The second major gap in the literature on the theoretical foundations of ABC is that it has only considered a long run perspective, where all inputs are variable with the activity output. However, one of the major recognised innovations of ABC systems is the introduction of the distinction between the cost of resources used and the cost of resources supplied, where the difference between the two is given by the cost of resources not used (Cooper and Kaplan, 1992). It is in the short run, where some inputs are fixed, that this analysis has to be carried out. The investigation of the restrictions on cost variability and thus the analysis of the short run activity cost function is the second purpose of this study.

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As will be shown, the two jointly necessary and sufficient conditions supporting the construction of an aggregate activity output are (i) the linear homogeneity of each cost object production function and the fact that (ii) the marginal cost of a unit of cost driver used by a cost object is equal for all cost objects within a cost pool. While the first condition ensures that cost functions are linear with output, which is essential if the cost reported by an ABC system is also to be a relevant cost for decision-making, the second guarantees that the aggregated activity cost function depends on only one cost driver. These conditions constitute the core requirements for decision relevant ABC.

By clarifying the technological foundations of an aggregate of activity output and therefore of ABC, this study extends existing product costing literature in two further ways.

The first extension is the consideration of situations where technologies are not linearly homogenous. From this perspective, and while considering the case where technologies are homogeneous of a degree different from one, a more general case, the so-called non-homothetic one, is derived.

A conventional accounting procedure, underlying the architecture of traditional and ABC systems, is the application of average cost driver rates to cost outputs. As will be shown, this procedure can only be justified when technologies are linearly homogeneous. Otherwise, some product cost distortion will occur. The major

purpose underlying the analysis of situations where technologies are not linearly homogeneous is the estimation of the product cost distortions arising from the application of average cost driver rates to cost outputs.

The second extension is the consideration of situations where, although cost object production functions are linearly homogeneous, the aggregated activity cost function depends on more than one cost driver, that is, the second condition supporting the construction of an aggregate activity output does not hold. In this case, the possibility of developing an accounting procedure that accommodates, in one or another way, the existence of multiple cost drivers is investigated.

Both the above developments are in line with the idea contained in the work of Noreen (1991, p. 160), when he claims that:

> "Traditional costing systems are merely simplified, and perhaps poorly designed, special cases of activity-based costing, just as activity-based costing is a special case of more general costing systems that could be devised".

In fact, it will be possible to visualize ABC as a particular case of a more general cost system derived in this study (see chapters 4 and 7).

1.3. Research Method

Along with the analytical developments that the pursuit of the four major purposes of this study originate, simulations will be used with the objective of both corroborating/illustrating some theoretical findings and hypothesis testing.

Simulations as a research method have been widely used in management science (e.g. Pidd, 1984). In the product costing literature, they have been increasingly used, particularly with the purpose of assessing the robustness of various accounting methods and policies both in allocating costs and in pricing and capacity decisions (see Balakrishnan and Sivaramakrishnan, 2002, who provide an excellent review of the management accounting literature that uses simulations as a research method).

1.4. Organisation of the Thesis

This study is organised as follows. Chapter two reviews critically the literature on the theoretical foundations of ABC. Chapter three derives both the technological foundations of an aggregate activity output and the structure of the short run activity cost function. Chapter four focus on technologies that are not linearly homogeneous. While chapter five addresses some issues that the empirical work in the area of product costing poses, chapter six concentrates on the description of the research method. In a context where cost object technologies are linearly homogeneous, chapter seven develops a simulation experiment to address both the question of testing the (non) existence of a single cost driver and the development of an

accounting procedure that specifically accommodates the existence of multiple cost drivers. Moreover, the simulation experiment serves also to incorporate into the analysis one important issue that existing management accounting literature has neglected: the stochastic attribute of the technology. Finally, chapter eight presents the conclusions of this study.

CHAPTER II – TECHNOLOGY AND COST FUNCTIONS

2.1. Introduction

In economics, the importance of technology tends to rely less on its technical relationship between inputs and outputs but more on the restrictions that it might impose on the economic behaviour of agents. As Chambers (1988, p. 7) observes, when the technical aspects of production do not impose restrictions on the economic behaviour of agents, the technology has only an ancillary importance to economists. In fact, the study of supply and input demand relationships has attracted most of the attention of economists. In spite of this, the investigation of technical relationships between inputs and outputs has also received considerable attention in the production economics literature (see Chambers, 1988, p. 1-5, for a brief historical review of this issue). A common feature in most production economics studies is that they examine the behaviour of aggregated production and cost functions. A general input definition found in many production economics studies is the aggregation of all inputs into four input categories: capital, labour, energy and materials. The Berndt-Wood (1975) sample (a time series of 25 yearly observations on capital, labour, energy and materials for U.S. manufacturing) is a good example (see, for example, Pollak and Wales, 1987, 1992, who have investigated various technological relationships using this sample).

The distinctive feature in many ABC studies, however, compared with many production economics studies, is that they are developed at a much more

disaggregated level. Many ABC studies concentrate on a particular firm and on part of its cost structure (see, for example, Foster and Gupta, 1990, Datar *et al.*, 1993, or, for an even more disaggregated study, Maher and Marais, 1998). While many production economics studies investigate, for example, the cost function for a particular industrial sector, many ABC studies investigate the cost function for a specific activity undertaken in a given firm (the paper of Maher and Marais, 1998, is a good example). More important, and as will be shown, the relationship between inputs and outputs that technology imposes, which is not always crucial to economists, as observed above, is a fundamental one to the understanding of an ABC system.

The major objective of this chapter is to develop a critical review of the management accounting literature that uses the production economics literature as a basis to derive the theoretical foundations of ABC.

This chapter begins with a brief explanation of the relationship between cost functions and production functions. In the production economics literature this is called duality. This concept is developed in section 2.2. Section 2.3 presents one particular type of cost functions: cost functions *weakly separable in the extended partition*. According to the existing ABC literature, these functions have two important properties that ABC systems should have if they were to measure incremental costs. Firstly, it allows the construction of cost pools that are all independent of each other. Secondly, it requires cost functions multiplicatively separable in input prices and output. One of the most important conclusions of this

chapter, which strongly contrasts with existing ABC literature, is that not all cost functions multiplicatively separable in input prices and output are compatible with ABC. Section 2.4 addresses the technological conditions that support the first property of cost functions *weakly separable in the extended partition*. Section 2.5 focuses on the technological conditions related to the second property. It shows what is the precise technology that supports an ABC system. Section 2.6 introduces the concepts of volume and non-volume drivers, one of the most recognised innovations of ABC systems. Section 2.7 characterizes non-joint technologies. Finally, section 2.8 presents the conclusions.

2.2. Duality between Costs and Technology

In the production economics literature, technology and input prices are the two major determinants of cost functions. Technology, in its basic form, is simply a relationship between inputs and output. This relationship can be represented as:

(1) $\qquad y = f(x)$

Where y is one vector of outputs and x one vector of inputs. The relationship between inputs and outputs is represented by the functional form f, which reflects a given state of technology.

Given the technology and the input price set, what the rational firm does is to find the minimum cost of producing a given level of output, that is, the cost function. But this process can be reversed. This means that, given the cost function, it is also possible to determine the technology that generated that cost function. That is to say, the cost function contains essentially the same information the production function contains. In the production economics literature this is called duality. The production function has a dual definition in terms of cost function as this has its dual in terms of production function (Varian, 1991, p. 81).

The process of finding the minimum cost of producing a given level of output can be decomposed in two stages. In the first stage, all efficient combinations of factors capable of producing a given level of output are derived. In the second stage, the

combination that gives the least cost of obtaining the desired level of output is determined.

To proceed with a formalisation of the cost minimising problem, it is necessary to introduce some notation (see Chambers, 1988, chapter 7). Let $y = (y_1, ..., y_m)$ represent an m-dimensional vector of outputs and $x = (x_1, ..., x_n)$ an n-dimensional vector of inputs. Given the technology, the production possibilities set T denotes all feasible input and output combinations and is represented by $(x, y) \in T$. It is also convenient to represent the input requirement set, which means the set of all input combinations capable of producing output bundle y and is represented by $V(y) = \{x: (x, y) \in T\}$. Finally, let $w = (w_1, ..., w_n)$ represent an n-dimensional vector of input prices. Now, the cost function can be written as:

(2) $c(w, y) = \min\{w\,x: x \in V(y)\}$[1]

The cost function gives the minimum cost of producing output bundle y. It is assumed that $c(w, y)$ is (i) nonnegative for $w > 0$ and $y > 0$, (ii) non-decreasing in w and y, (iii) positively linearly homogeneous in w, (iv) concave and continuous in w, and (v) equal to zero when $y = 0$, that is, there are no fixed costs[2]. These are the sufficient conditions for a cost function to be a representation of some technology. These are also a complete list of the implications of cost-minimising behaviour (Varian, 1991, p. 84-85).

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[1] $w\,x$ is the inner product: $\sum_{i=1}^{n} w_i\,x_i$.

[2] This means that we are in the long run, where, by definition, all inputs are variable.

2.3. Cost Functions Weakly Separable in the Extended Partition

This section analyses the properties of cost functions *weakly separable in the extended partition*. These cost functions are well recognised in the production economics literature (see Chambers, 1988, p. 113). It will be shown that the general representation of an ABC system might be interpreted in the light of this type of cost functions (see Bromwich and Hong, 1999). In this context, a parallelism between cost functions in productions economics and activity cost functions will be established.

To proceed with an explanation, it is necessary to introduce further definitions. Let $\mathbf{I} = (\mathbf{I}^1, \mathbf{I}^2, ..., \mathbf{I}^r)$ represent the partition of the n-dimensional vector of inputs previously defined. This means that the n inputs are grouped in r groups. Each of these groups might be interpreted as a cost pool that aggregates say p inputs. But this will be explained after introducing the concepts of cost functions *weakly separable* and cost functions *weakly separable in the extended partition*.

In what follows, y is a single output (and not a vector of outputs). The cost function c(w, y) is called *weakly separable* in the partition $\mathbf{I}$ if (Chambers, 1988, p. 111):

(3)
$$\frac{\partial}{\partial w_k} \left(\frac{\partial c(w, y)/\partial w_i}{\partial c(w, y)/\partial w_j} \right) = 0, \quad i, j \in \mathbf{I}^t, \quad k \notin \mathbf{I}^t$$

Expression (3) means that the ratio of the change in the total cost with respect to an alteration in the price of one input ($\partial c(w, y)/\partial w_i$) relative to the change in the total

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cost with respect to an alteration in the price of another input belonging to the same group ($\partial c(w, y)/\partial w_j$) is not affected by alterations in input prices not belonging to that group (w_k). Expression (3) might be interpreted in terms of the isocost curve. The isocost curve shows how one input price increases (decreases) while another input price decreases (increases) in order to maintain costs constant. A cost function is *weakly separable* if the shape of the isocost curve[3] associated with group t does not change when input prices belonging to other groups alter. This signifies that w_i and w_j ($i, j \in \mathbf{I}^t$) are separable from inputs that do not belong to $\mathbf{I}^t$. In these circumstances, the cost function for a group of inputs is independent of the cost function of other groups of inputs.

By the *Lemma* of Shephard, the derived demand for input i, x_i, equals the derivative of the cost function with respect to the price of the same input, w_i[4]:

$$(4) \qquad \frac{\partial c(w, y)}{\partial w_i} = x_i$$

Expression (3) can then be rewritten in two equivalent forms:

$$(5) \qquad \frac{\partial}{\partial w_k}\left(\frac{x_i}{x_j}\right) = 0, \quad i, j \in \mathbf{I}^t, \quad k \notin \mathbf{I}^t$$

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[3] That is, the rate at which one input price increases (decreases) while another input price decreases (increases) without altering costs.

[4] Let x^* represent the cost-minimising input bundle that produces y at prices w^*. Let also define the function $g(w) = c(w, y) - w\, x^*$. Since $c(w, y)$ gives the minimum cost of producing y, $g(w)$ is always less than or equal to zero. The maximum of $g(w)$ is zero when $w = w^*$. First-order conditions for the existence of a maximum imply:

$\dfrac{\partial g(w^*)}{\partial w_i} = \dfrac{\partial c(w^*, y)}{\partial w_i} - x_i^* = 0$ or $\dfrac{\partial c(w^*, y)}{\partial w_i} = x_i^*$ (see Varian, 1991, p. 74).

(6) $$\frac{\partial x_i / x_i}{\partial w_k / w_k} = \frac{\partial x_j / x_j}{\partial w_k / w_k}, \quad i, j \in I^t, \quad k \notin I^t$$

Expression (5) shows that the input mix of one group is not affected by alterations in input prices belonging to another group. To illustrate consider the case of the bicycle industry. Expression (5) means that the input mix necessary to produce frames is not affected by any alteration in the price of inputs used to produce pedals or brakes, for example. The price elasticity of input demand shows the ratio of the relative change in the demand of input i to the relative change in the price of input k, ceteris paribus. Expression (6) signifies that the derived-demand elasticities for all inputs in a group with respect to a price from a separate group are equal, signifying that each group of inputs has a common economic structure (Christensen and Demski, 1995, p. 16). A *weakly separable* cost function can be written as (Chambers, 1988, p. 111-115):

(7) $$c(w, y) = C(y, c^1(w^1, y), ..., c^r(w^r, y))$$

Expression (7) might be interpreted as if the cost minimisation process followed two stages. In the first stage, the cost of producing a single unit of an aggregate input, the amount of which depends on the level of output, is minimised ($c^t(w^t, y)$). In the second stage, these aggregate inputs are combined in a cost-minimising way to produce output y. Let us consider again the case of the bicycle industry. In the first phase, inputs are combined in a way (depending on the technology, the input price set and on the number of bicycles to be produced) that results in the cost of producing a certain number of intermediate components (such as wheels, pedals, brakes or frames) being minimised. In the second phase, these intermediate

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components are combined in a way that minimise assembly costs. In these circumstances, expression (7) represents the minimum cost of producing a given number of bicycles.

It should be noted that each aggregate input price depends on the price of inputs belonging to that group and on the output, $c^t(w^t, y)$. If, however, each aggregate input price depends only on input prices, $c^r(w^r)$, the cost function is *weakly separable in the extended partition*. This signifies that w_i and w_j $(i, j \in \mathbf{I}^t)$ are separable not only from other inputs (as when the cost function is *weakly separable* – see expressions (3) and (7)) but also from y in the cost function. Consequently, it is possible to construct an aggregate input price that is independent of y. Formally, w_i and w_j $(i, j \in \mathbf{I}^t)$ are separable from y if:

$$(8) \qquad \frac{\partial}{\partial y}\left(\frac{\partial c(w, y)/\partial w_i}{\partial c(w, y)/\partial w_j}\right) = \frac{\partial}{\partial y}\left(\frac{x_i}{x_j}\right) = 0, \quad i, j \in \mathbf{I}^t$$

Equivalently, w_i and w_j $(i, j \in \mathbf{I}^t)$ are separable from y if:

$$(9) \qquad \frac{\partial x_i/x_i}{\partial y/y} = \frac{\partial x_j/x_j}{\partial y/y}$$

Expression (8) means that the input mix of one group is independent of the output. In the example presented above, this signifies that the input mix necessary to produce an intermediate input (a frame, for example) is independent of the number of bicycles produced. The elasticity of input demand with respect to output shows the

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ratio of the relative change in the demand of input i to the relative change in output, ceteris paribus. Expression (9) signifies that all derived-demand elasticities with respect to output are equal. Cost functions *weakly separable in the extended partition* can be written as:

$$(10) \qquad c(w, y) = C(y, c^1(w^1), \ldots, c^r(w^r))$$

It is now time to translate these results to the case of ABC. To start with, it seems clear that each partition $\mathbf{I}^t$ is no more than a group of inputs that are grouped together in say activity cost pool t. Secondly, cost functions *weakly separable in the extended partition* have two important properties that must exist in an ABC system compatible with the generation of incremental costs. The first one is that it is permissible to construct cost pools independent of other cost pools (see expressions (3) and (7)). In our example, this means that the input mix necessary to produce frames is not affected by alterations in the price of inputs necessary to produce wheels or pedals. The second one is that it allows the construction of activity cost functions separable in input prices and output (see expressions (8), (9) and (10)). However, and as will be demonstrated in section 2.5, not all cost functions separable in input prices and output are compatible with ABC. This result strongly contrasts with existing ABC literature.

2.4. Weakly Separable Technologies

This section investigates the technological foundations of the first property of cost functions *weakly separable in the extended partition*, that is, the property that allows the construction of cost pools that are all independent of each other. This property is associated with the concept of input separability.

Input separability means that the input mix of one activity is not affected by the level of inputs used in other activities (see Bromwich, 1997, p. 25-27 and Bromwich and Hong, 1999, p. 52). This property is possessed by what Chambers (1988, p. 41-48) calls *weakly separable* production functions. In an ABC context, the activity production function $f^t(x^t)$ is separable from the activity production function $f^p(x^p)$ if:

$$(11) \qquad \frac{\partial}{\partial x^p_k} \left(\frac{\partial f^t(x^t)/\partial x^t_i}{\partial f^t(x^t)/\partial x^t_j} \right) = 0, \ x^t_i, \ x^t_j \in \mathbf{I}^t, \ x^p_k \in \mathbf{I}^p$$

Where $x^t = (x^t_1, x^t_2, ..., x^t_p)$ is the vector of input quantities used at activity t. This means that the marginal rate of technical substitution between x^t_i and x^t_j ($MRTS_{i,j} = \partial x^t_j / \partial x^t_i$)[5], which belong to cost pool t, is independent of all inputs that are not elements of that cost pool.

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[5] $MRTS_{i,j}$ measures the rate at which x^t_i may be substituted for x^t_j without altering output.

Graphically, a production function is *weakly separable* if the shape of the isoquant[6] associated with activity t does not change with alterations in the level of inputs belonging to other activities. So *weakly separable* production functions imply that each activity cost pool is separable from other activity cost pools.

2.5. Homothetic and Linearly Homogeneous Technologies

This section investigates the technological foundations of the second property of cost functions *weakly separable in the extended partition*, that is, the property that allows the construction of cost functions separable in input prices and output. As will be shown, all homothetic technologies give rise to cost functions separable in input prices and output. However, only linearly homogenous technologies, a special case of homothetic technologies, are compatible with an ABC system. This result is in marked contrast with Bromwich and Hong (1999).

While input separability allows the treatment of the inputs of one cost pool as independent of the inputs of other cost pools, output separability permits the treatment, within a cost pool, of each output as independent of other outputs. Here, the important point is that output separability allows the representation of a vector of outputs through some aggregate or index, as if it was as a single output (see Chambers, 1988, p. 285; see also Bromwich and Hong, 1999, p. 47). Thus, output separability permits the output vector $y = (y_1, ..., y_m)$ to be expressed by a single

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[6] An isoquant is a curve along which output is the same. It shows different combinations of inputs that produce the same output.

output $g(y) = \sum_{j=1}^{m} g_j(y_j)$. In the next chapter, the very essence of such a single measure of output in ABC will be investigated. In this section, we are going to take it for granted. Alternatively, and in order to avoid this problem, we might assume that $g(y)$ is an intermediate input which is used by the various products. That is to say, each activity is by assumption a single output technology. The cost function at activity t can now be written as follows:

(12) $c^t(w^t, g^t(y)) = \min\{w^t x^t : x^t \in V(g^t(y))\}$

Where x^t is a vector of input quantities and w^t the corresponding vector of input prices. The cost function $c^t(w^t, g^t(y))$ gives the minimum cost of producing output $g^t(y)$ at activity t.

A technology is homothetic if it can be written as $H^t(x^t) = h^t(f^t(x^t))$, where $\partial h^t(f^t(x^t))/\partial f^t(x^t) > 0$ and $f^t(x^t)$ is linearly homogeneous (see Takayama, 1985, p. 146). All homothetic technologies ensure that cost functions are separable in input prices

and output. That is, all homothetic technologies guarantee condition (8)[7]. Moreover, the cost function dual to a homothetic technology takes the following form[8]:

(13) $$c^t(w^t, g^t(y)) = \varphi^t(g^t(y))\, \phi^t(w^t)$$

Where $\varphi^t(g^t(y)) = h^{t-1}(g^t(y))/h^{t-1}(1)$. $\varphi^t(g^t(y))$ is thus an increasing transformation of $g^t(y)$, the single measure of output. $\phi^t(w^t)$ is an aggregate input price which depends only on input prices, i.e., it is independent of the output. Moreover, and as usually assumed in the production economics literature, $\phi^t(w^t)$ is concave and linearly homogeneous in input prices.

A conventional accounting procedure, underlying the architecture of both conventional and ABC systems, is the application of average cost driver rates to cost outputs. This procedure can only be justified when cost functions are linear with output. Only in this case average and marginal costs are constant and equal. The only case where this occurs is when $\varphi^t(g^t(y)) = g^t(y)$ (since this implies that average cost = marginal cost = $\phi^t(w^t)$). Furthermore, $\varphi^t(g^t(y)) = g^t(y)$ if and only if

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[7] This can be demonstrated as follows. The cost-minimization problem, expressed in terms of the *Lagrange's method*, is: $L = \sum_{i=1}^{p} w_i^t x_i^t - \mu\,(h^t(f^t(x^t)) - g^t(y))$, where μ is a *Lagrange* multiplier. Cost-minimisation implies the following (first-order conditions):
$\frac{\partial L}{\partial x_i^t} = w_i^t - \mu\, \frac{\partial h^t(f^t(x^t))}{\partial f^t(x^t)}\, \frac{\partial f^t(x^t)}{\partial x_i^t} = 0$ or $\frac{\partial f^t(x^t)/\partial x_i^t}{\partial f^t(x^t)/\partial x_j^t} = \frac{w_i^t}{w_j^t}$. But, if $f^t(x^t)$ is homogeneous of degree one $\partial f^t(x^t)/\partial x_i^t$ is homogeneous of degree zero $(\partial f^t(\lambda\, x^t)/\partial x_i^t = \partial f^t(x^t)/\partial x_i^t)$. Thus an increase of λ in all the p inputs does not change $\frac{\partial f^t(x^t)/\partial x_i^t}{\partial f^t(x^t)/\partial x_j^t}$. Therefore, given the input price set, if the first-order conditions are fulfilled by the input combination $(x_1^t, ..., x_p^t)$ they must also be fulfilled by the combination $(\lambda\, x_1^t, ..., \lambda\, x_p^t)$. This implies that $\frac{\partial}{\partial g^t(y)}\left(\frac{\partial c^t(w^t, g^t(y))/\partial w_i^t}{\partial c^t(w^t, g^t(y))/\partial w_j^t}\right) = \frac{\partial}{\partial g^t(y)}\left(\frac{x_i^t}{x_j^t}\right) = 0$.

[8] See appendix I for a formal proof.

the underlying technology is linearly homogenous. Finally, note that a linearly homogenous technology is a special case of a homothetic technology, where $h^t(f^t(x^t)) = f^t(x^t)$ [9].

Suppose now that a given product increases the output of activity t from say a^t to b^t. The incremental (product) cost is given by:

$$(14) \qquad \int_{a^t}^{b^t} \frac{\partial c^t(w^t, g^t(y))}{\partial g^t(y)} \, dg^t(y) = \phi^t(w^t) \, (b^t - a^t)$$

That is, the incremental cost is equal to the constant (average and marginal) cost driver rate ($\phi^t(w^t)$) times the activity usage (($b^t - a^t$)). It must be remarked that this result takes place if and only if the technology is linearly homogeneous.

Bromwich and Hong (1999, p. 56-57, italics are not in original) conclude that:

> "A strong version of this type of separability appears where the production function satisfies the conditions of (weak) homothetic separability (discussed earlier). *This means that each group of inputs has a homothetic technology and the overall technology is separable. These are the characteristics which were said earlier in this article to be of the essence of ABC and CVO (cost proportional with output volume).* The assumption of homothetic technology allows price indices to be written as independent of volume whereas with weakly separable cost functions[10] local cost functions are also a function of volume. Cost functions separable in input prices and output are sufficient to provide a price index which allows the cost function for a cost pool to be written as a function of an aggregate input and of an aggregate price index. *This is what is required for cost functions in ABC and CVO.* Thus weak

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[9] The cost function dual to a homothetic technology takes the form $c^t(w^t, g^t(y)) = \varphi^t(g^t(y)) \, \phi^t(w^t)$, where $\varphi^t(g^t(y)) = h^{t-1}(g^t(y))/h^{t-1}(1)$. This implies that the cost function dual to a linearly homogeneous technology is $c^t(w^t, g^t(y)) = g^t(y) \, \phi^t(w^t)$.

[10] This is exactly the weakly separable cost function concept defined in section 2.3.

separability in input prices does not generate separable cost functions compatible with ABC and CVO because with weak separability local cost functions are functions of volume *whereas ABC and CVO require that the unit cost for each cost poll must be invariant with volume*".

Thus Bromwich and Hong (1999) clearly state that all cost functions separable in input prices and output are compatible with ABC. That is, it is claimed that all homothetic technologies give rise to cost function compatible with ABC. It is true that ABC requires that the unit cost for each cost poll must be invariant with volume (since only in this case average and marginal costs are constant and thus equal). However, the fact that it is possible to construct an aggregate price index that is independent of the output ($\phi^t(w^t)$) does not imply that the unit cost for each cost poll is invariant with volume. The unit cost for each cost pool will be invariant with volume if and only if $\varphi^t(g^t(y)) = g^t(y)$, i.e. if the cost function is linear with output. Otherwise, the unit cost will change with output. Moreover, the result that activity cost functions have to be linear with output is a fundamental result contained in the work of Noreen (1991). The only case where $\varphi^t(g^t(y)) = g^t(y)$ is when the technology is linearly homogeneous, a special case of a homothetic technology. To sum up, not all homothetic technologies give rise to cost functions compatible with ABC. We will return to this point in chapters three and four.

2.6. Volume and Non-Volume Drivers

At this point, one issue should be observed. The analysis developed in the previous sections does not consider the existence of non-volume drivers. A volume driver is one where the relationship between activity usage (or the output of an activity) and

product output volume (or final good output) is strictly proportional (Bromwich and Hong, 1999, p. 43). Each activity cost function defined in expression (13) depends on a vector of input prices (w^t) and on a single measure of output, $g^t(y)$, which depends itself on a vector of final good outputs, y. This means that it is possible to express the activity output as a function of the final good output vector. This is not the case, however, with non-volume driver activities. By definition, the output of a non-volume driver activity is independent of volume changes. Consequently, the output of a non-volume driver activity cannot be expressed as a function of the final good output vector.

Miller and Vollmann (1985, p. 144) recognize the importance of non-volume drivers (transaction drivers in their terminology) in explaining overhead costs:

> "… in the "hidden factory", where the bulk of manufacturing overhead costs accumulates, the real driving force comes from transactions, not physical products. These transactions involve exchanges of the materials and/or information necessary to move production along but do not directly result in physical products".

Many ABC studies that try to test if non-volume drivers affect overhead costs, control volume changes to ascertain if overhead costs are correlated with non-volume drivers (Kaplan, 1993, p. 2).

Foster and Gupta (1990) have investigated the effect of volume-based, complexity-based and efficiency-based drivers on manufacturing overhead costs of thirty-seven facilities of an electronics company. They have concluded about a strong empirical association between volume-based drivers and overhead costs. However,

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they have only found a limited empirical support for complexity-based and efficiency-based drivers in explaining overhead costs behaviour. Two reasons were identified for this finding. Firstly, in the setting studied, it was difficult to identify variables that adequately captured the complexity-based and efficiency-based concepts. Secondly, the complexity-based and efficiency-based variables were less consistently measured in each facility, comparing with volume variables (Foster and Gupta, 1990, p. 331).

In contrast, Banker and Johnston (1993), in a study of cost drivers in the U.S. airline industry, have concluded both volume and operations-based drivers related to product diversity and production process complexity to be statistically significant.

Similarly, Banker *et al* (1995), in a study of a sample of 32 plants, have found evidence that overhead costs were explained by volume and non-volume variables. Moreover, they have concluded that most of the variations in overhead costs were explained by non-volume variables.

Datar *et al.* (1993), in a study of quality costs in a manufacturing facility for lamp assemblies for automobiles and trucks, have also found empirical support for the hypothesis that non-volume drivers affect manufacturing overhead costs.

Finally, Macarthur and Stranahan (1998) have concluded both volume and complexity variables to be statistically significant in explaining hospital overhead costs.

In most production economics texts, however, it seems that there is only one driver and only one cost object, which is the final good output. Expression (2) represents the typical cost function depicted in a production economics textbook.

Christensen and Demski (1995) assume Leontief and Cobb-Douglas technologies and compute the minimum cost of producing a given level of output. They only have considered the existence of volume drivers.

> "Several points emerge. First, the objective function in programme [C] is parametrized by the quantity (q) and price (p) vectors. Holding price constant, each cost expression varies with q. *There are no non-volume cost drivers*" (Christensen and Demski, 1995, p. 18-19, italics are not in original).

Bromwich and Hong (1999) have also considered only the case of volume drivers. They explicitly write the cost function for a cost pool as depending on a vector of final good outputs and on a vector of input prices.

To proceed with the analysis, let us consider a numerical example. Suppose a company that produces two types of bicycles (P_1 and P_2) and imagine three possible cost pools of its ABC system. The first one is a direct cost pool, corresponding to a direct labour activity. The second one is an overhead cost pool that corresponds to a set-up activity. The number of set-ups is the cost driver. The third one is also an overhead cost pool that represents a product support activity. The cost driver is the

number of products[11]. This information, as well as benchmark costs, resource usage and total costs, is presented in the following table:

Table 1: Values for Benchmark Costs, Resource Usage and Total Costs

Panel A: Benchmark Costs

Activity	Cost driver	Cost driver rate
Direct labour	Hours of direct labour	£5/hour
Set-up	Number of set-ups	£10/set-up
Product support	Number of bicycle types	£200/bicycle type

Panel B: Resource Usage

Product	Production	Hours of direct labour (per unit)	Number of set-ups
P_1	100 units	2 hours	20 set-ups
P_2	150 units	3 hours	10 set-ups

Panel C: Total Costs

Activity	Activity output	Total cost (P_1)	Total cost (P_2)
Direct labour	650 hours (1)	£1 000 = £5 × 100 × 2	£2 250 = £5 × 150 × 3
Set-up	30 set-ups (2)	£200 = £10 × 20	£100 = £10 × 10
Product support	2 products (3)	£200 = £200 × 1	£200 = £200 × 1
		£1 400	£2 550

(1) 650 hours = 100 units × 2 hours (P_1) + 150 units × 3 hours (P_2)
(2) 30 set-ups = 20 set-ups (P_1) + 10 set-ups (P_2)
(3) 2 products = 1 (P_1) + 1 (P_2)

Assuming that activity technologies are linearly homogeneous, the marginal cost of an activity equals its average cost driver rate (see section 2.5). It must be noted that the only activity whose output can be expressed as a function of volume is the direct labour activity. Consequently, the incremental cost of producing an additional unit of P_1 or P_2 can only be referred to the direct labour activity. The incremental costs are £10 (£5/hour × 2 hours) for P_1 and £15 (£5/hour × 3 hours) for P_2. Concerning the set-up and product support activities, it is only possible to determine the incremental

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[11] Or number of bicycles type. It is assumed that each bicycle type gives rise to an equal increase in terms of resources acquired and used in the product support activity. In practice, however, it is doubtful if this is the case. Normally, the resources devoted to these kinds of activities are inherently joint and non-separable among products.

cost of a set-up, in the first case, or the incremental cost of introducing a new product, in the second[12]. However, the cost of these activities can be distributed between P_1 and P_2 (see panel C of table 1).

In order to distinguish different categories of cost drivers, let us represent the vector of outputs by $q^r = (q^r_1, ..., q^r_m)$, where q^r_j is the volume of output r associated with cost object j. For example, q^1_j might be the units produced of product j while q^2_j the number of set-ups of the same product. As observed, output separability allows a vector of outputs to be expressed by a single measure of output. Let $y^{t,j}$ denote the total units of cost driver used by cost object j at activity t. That is, $y^{t,j}$ is the units of the single measure of output associated with cost object j at activity t. Moreover, each $y^{t,j}$ depends on the volume of output r of cost object j, i.e, $y^{t,j} = g^t_j(q^r_j)$. Therefore, the aggregate single measure of output can be written as $g^t(y^t) = \sum_{j=1}^m y^{t,j}$ $= \sum_{j=1}^m g^t_j(q^r_j)$, where $y^t = (y^{t,1}, ..., y^{t,m})$. For example, if the activity output is the total number of set-ups then $g^t(y^t) = \sum_{j=1}^m y^{t,j} = \sum_{j=1}^m q^2_j$, where q^2_j is the number of set-ups of product j $(g^t_j(q^r_j) = q^r_j = y^{t,j})$.

The crucial issue in the definition of q^r_j is the identification of the relevant cost object, that is, the object whose existence produces an increment in the cost at

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[12] Observe that, given the technology specification, the cost per set-up is independent of the number of units of P_1 or P_2 per set-up.

activity t. The cost object might be a product[13], a profit centre, an investment centre or even an activity[14], for example.

In the example under consideration, the output of the direct labour activity is $g^t(y^t) = \sum_{j=1}^{2} y^{t,j} = \sum_{j=1}^{2} q^1_j a^t_j$, where q^1_j is the number of units produced of P_j and a^t_j the direct labour hours per unit of P_j. Only in this case it is possible to express the activity output as a function of volume. The output of the set-up activity is $g^t(y^t) = \sum_{j=1}^{2} y^{t,j} = \sum_{j=1}^{2} q^2_j$, where $y^{t,j} = q^2_j$ is the number of set-ups of P_j. Finally, the output of the product support activity is $g^t(y^t) = \sum_{j=1}^{2} y^{t,j} = 2$, since $y^{t,j} = q^3_j = 1$. That is, P_j gives rise to an output of 1 (independent of the number of units produced of P_j or the number of set-ups of P_j).

The distinction between different kinds of drivers is one of the distinguishing features of ABC. The hierarchy of activities proposed by Cooper and Kaplan (1991, 1998) (unit-level, batch-level, product-sustaining and facility-sustaining activities) is simply based on the recognition of different types of outputs (drivers)[15]. The recognition of different types of drivers, including volume and non-volume drivers, has not been accepted without difficulty. Christensen and Demski (1995, p. 22) claim that:

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[13] But not necessarily the final good output (see the case of the set-up activity or of the product support activity in table 1).

[14] Consider the case of the cost of secondary activities assigned to primary activities (cost objects).

[15] Unit-level activities are performed for every unit of product or service produced. Batch-level activities are performed for each batch or set-up of work performed. Product-sustaining activities are performed to enable the production of individual products or services. Facility-sustaining activities provide general production and sales capabilities (e.g. administrative staff).

"In a sense, output is measured with error and the additional variables, the non-volume drivers, are used to deal with that error".

A major difficulty in supporting different types of drivers is that, frequently, volume and non-volume drivers will be correlated. For example, the number of units produced and the number of set-ups are normally to some extent correlated, as increases in units produced create the need for set-ups. If no products are produced no set-ups are carried out. If the production dramatically increases it is likely that the number of set-ups will exhibit a similar increase, especially if there is not a change in the number of units per set-up. If batch level activity costs vary with the number of set-ups and there is a strong correlation between the number of units produced and the number of set-ups, then a significant correlation will exist between those (batch) costs and the number of units produced. It might be argued that, in technological terms, the number of set-ups drives batch costs but, in spite of that, the existence of a strong correlation between the two drivers (set-ups and output volume) might support, without a major loss of accuracy, the substitution of the number of set-ups by the output volume, in the batch cost function.

In the management accounting literature, Ittner *et al.* (1997) empirically observed that measures related to unit, batch and product-sustaining activities had considerable correlations. They found evidence that the production policies followed by organizations might account for this fact. A good example occurs when organizations implement economic order quantity (EOQ) models, where an optimal batch size is determined. In these cases, the number of batches and the total number of units produced are strongly correlated, since the number of batches equals the total units

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produced divided by the optimal batch size. In the example of Table 1, this signifies that the vectors (q^1_1, q^1_2) and (q^2_1, q^2_2) are, at an extreme, linearly dependent $((q^1_1, q^1_2) = \lambda (q^2_1, q^2_2)$, where λ is the number of units per set-up). This type of analysis suggests that, apart from other issues that the design of cost systems raise, the correlations between different cost drivers (and thus between activity costs and cost drivers) should be investigated, in order to decide which drivers should be chosen and to evaluate the trade-offs between the cost and benefit of refining cost systems.

2.7. Non-Joint Technologies

Bromwich and Hong (1999) show that if ABC systems are to measure incremental costs technologies have to be non-joint.

Non-jointness between outputs in their use of inputs means that there are no cost complementarities or diseconomies between inputs (Bromwich and Hong, 1999, p. 45). In the case presented in table 1 of section 2.6, non-jointness signifies, for example, that the total hours of direct labour used by P_1 is not affected by or is independent of the total hours of the same input used by P_2.

The technology is non-joint if the two following conditions are presented (see also Bromwich and Hong, 1999, p. 45-46 and Hall, 1973, p. 884-885). Firstly, it is possible to specify for each cost object j and for each activity t the two following production functions:

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(15) $\quad\quad\quad f_j^t(x^{t,j}) = f_j^t(x_{1,j}^t, x_{2,j}^t, \ldots, x_{p,j}^t) = q_j^r$

(16) $\quad\quad\quad f^t(x^t) = f^t(x_1^t, x_2^t, \ldots, x_p^t) = q^r$

Where $f_j^t(x^{t,j})$ is the individual production function for cost object j; $x^{t,j} = (x_{1,j}^t, x_{2,j}^t, \ldots, x_{p,j}^t)$ the vector of inputs used by the same cost object, $f^t(x^t)$ the overall production function at activity t, $x^t = (x_1^t, x_2^t, \ldots, x_p^t)$ the vector of inputs used at the same activity and $q^r = (q_1^r, \ldots, q_m^r)$ the vector of outputs (see previous section). While expression (15) shows the production function for cost object j when it is produced separately, expression (16) shows the production function when the m cost objects are produced together. Secondly, non-jointness imposes that the total quantity of input i used in the overall production function equals the sum of the quantities used by each cost object in the individual production functions:

(17) $\quad\quad\quad x_i^t = \sum_{j=1}^m x_{i,j}^t$

Non-jointness might be defined in a different but equivalent away. Let $c_j^t(w^t, q_j^r) = c^t(w^t, 0, \ldots, 0, q_j^r, 0, \ldots, 0)$ represent the stand-alone cost of q_j^r, that is the cost of producing output $q_j^r > 0$ when $q_k^r = 0$ ($\forall$ k $\neq$ j). A necessary and sufficient condition for non-jointness is that the cost of producing output q_j^r is separable from the output q_k^r:

(18) $\quad\quad\quad c^t(w^t, q_1^r, \ldots, q_m^r) = \sum_{j=1}^m c_j^t(w^t, q_j^r)$

Expression (18) applies to individual cost pools and means that the cost of producing the m outputs together (left-hand side of expression (18)) is the sum of the costs of producing them separately (right-hand side of expression (18)). In the case presented above, the total cost of the direct labour activity is the sum of the costs of producing P_1 and P_2 separately. Applied across cost pools, expression (18) can be written as:

$$(19) \qquad \sum_{t=1}^{r} c^t(w^t, q^r_1, ..., q^r_m) = \sum_{t=1}^{r} \sum_{j=1}^{m} c^t_j(w^t, q^r_j)$$

The cost of producing output q^r_j is independent of the outputs q^r_k and q^p_h if the inputs necessary to produce output q^r_j do not alter with the outputs q^r_k and q^p_h. To exemplify, consider the previous case. Under non-jointness, the cost of producing P_1 in the direct labour activity is independent not only of the units produced of P_2 in the same activity but also of the number of set-ups of P_1 or P_2 in the set-up activity.

Non-jointess signifies that there are no economies or diseconomies of scope either within or between cost pools. A necessary and sufficient condition for non-jointness is that the marginal cost of cost object j at activity t is not affected by the marginal cost of cost object k (which uses the resources aggregated at activity t) or by the marginal cost of cost object h (which uses the resources aggregated at activity p) (see also Hall, 1974, p. 885). Formally, we have:

$$(20) \qquad \frac{\partial}{\partial q^r_k}\left(\frac{\partial c^t(w^t, q^r_1, ..., q^r_m)}{\partial q^r_j}\right) = 0$$

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Expression (20) applies to individual cost pools. Applied across cost pools, expression (20) can be written as:

$$(21) \qquad \frac{\partial}{\partial q^p_h} \left(\frac{\partial c^t(w^t, q^r_1, \ldots, q^r_m)}{\partial q^r_j} \right) = 0$$

It might be said that under non-jointness the marginal cost of a set-up of P_1 is independent not only of the marginal cost of a set-up of P_2 (in the set-up activity – expression (20)) but also of the marginal cost of a unit of P_1 or P_2 (in the direct labour activity – expression (21)).

Production function separability and non-jointness are two different requirements. As Lucas (2003, p. 210) observes:

> "Whereas production function separability concerns whether inputs for different activities can, in principle, be specified independently of each other, non-jointness concerns whether activities are, in fact, performed separately. A profit maximising firm will perform activities jointly if this costs less than performing them separately. Costing systems should not, therefore, treat them as separable cost pools – the joint cost will be less than the sum of the incremental costs of the separate activities."

Thus non-jointness signifies that the different activities, as well as the production of the various cost object volumes, are performed independently of each other. This occurs if and only if the cost of performing the different activities separately (and, within each activity, the various cost object volumes) equals the cost of performing them together. It was demonstrated in the section 2.5 that ABC is only compatible

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with linearly homogeneous technologies. This, together with output separability (that is, the ability to represent a multi-output technology by a single measure of output), ensures that activity cost functions can be written as $c^t(w^t, g^t(y^t)) = g^t(y^t) \phi^t(w^t)$, where $g^t(y^t) = \sum_{j=1}^{m} y^{t,j}$ and $y^{t,j} = g^t_j(q^r_j)$ (see section 2.5). Observe now that this implies the verification of conditions (20) and (21), that is, the condition of non-jointness holds both within and between cost pools[16].

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[16] $\dfrac{\partial}{\partial q^r_k} \left(\dfrac{\partial c^t(w^t, g^t(y^t))}{\partial q^r_j} \right) = \dfrac{\partial}{\partial q^r_k} \left(\dfrac{\partial}{\partial q^r_j} \left(g^t(y^t) \phi^t(w^t) \right) \right) = \dfrac{\partial}{\partial q^r_k} \left(\dfrac{dg^t_j(q^r_j)}{dq^r_j} \phi^t(w^t) \right) = 0 = \dfrac{\partial}{\partial q^p_h} \left(\dfrac{\partial c^t(w^t, g^t(y^t))}{\partial q^r_j} \right).$

2.8. Conclusions

This chapter has reviewed critically the ABC literature. According to the literature, ABC systems should possess the properties of what in the production economics literature is known as cost functions *weakly separable in the extended partition.*

On the one hand, cost functions *weakly separable in the extended partition* allow the construction of cost pools that are all independent of each other. As was shown, this property is possessed by *weakly separable* technologies. On the other hand, cost functions *weakly separable in the extended partition* permit the construction of activity cost functions multiplicatively separable in input prices and output. It was shown that this property is possessed by homothetic technologies.

However, it was also shown that not all cost functions multiplicatively separable in input prices and output are compatible with ABC. This is in marked contrast with the existing ABC literature, more precisely Bromwich and Hong (1999). A conventional accounting procedure, underlying the architecture of more traditional and ABC systems, is the application of average cost driver rates to cost outputs. This procedure can only be justified when cost functions are linear with output. It was demonstrated that only linearly homogeneous technologies, a special case of a homothetic technologies, give rise to cost functions linear with output.

This chapter has also considered one of the most recognisable innovations of ABC systems: the distinction between volume and non-volume drivers. In the production

economics literature, however, it seems that there is only one type of cost driver and only one cost object: final good output. Finally, non-joint technologies were characterised. It was demonstrated that the assumptions of linearly homogeneous technology and output separability ensure that the condition of non-jointness holds both within and between cost pools.

Appendix I

Derivation of Expression (13) (Section 2.5)

If $H^t(x^t)$ is homothetic then $H^t(x^t) = h^t(f^t(x^t))$, where $\partial h^t(f^t(x^t))/\partial f^t(x^t) > 0$ and $f^t(\lambda x^t) = \lambda f^t(x^t)$. The following developments take place:

$$c^t(w^t, 1) = \min w^t x^t : h^t(f^t(x^t)) \geq 1$$

$$c^t(w^t, 1) = \min w^t x^t : f^t(x^t) \geq h^{t-1}(1)$$

$$c^t(w^t, 1) = \min w^t x^t : f^t\!\left(\frac{x^t}{h^{t-1}(1)}\right) \geq 1 \text{ (since } f^t(\lambda x^t) = \lambda f^t(x^t))$$

$$c^t(w^t, 1) = \min w^t x^t : h^{t-1}(g^t(y))\, f^t\!\left(\frac{x^t}{h^{t-1}(1)}\right) \geq h^{t-1}(g^t(y))$$

$$c^t(w^t, 1) = \min w^t x^t : f^t\!\left(\frac{h^{t-1}(g^t(y))}{h^{t-1}(1)} x^t\right) \geq h^{t-1}(g^t(y)) \text{ (since } f^t(\lambda x^t) = \lambda f^t(x^t))$$

$$c^t(w^t, 1) = \min w^t x^t : h^t\!\left(f^t\!\left(\frac{h^{t-1}(g^t(y))}{h^{t-1}(1)} x^t\right)\right) \geq g^t(y)$$

$$c^t(w^t, 1) = \frac{h^{t-1}(1)}{h^{t-1}(g^t(y))}\ \min w^t \frac{h^{t-1}(g^t(y))}{h^{t-1}(1)} x^t : h^t\!\left(f^t\!\left(\frac{h^{t-1}(g^t(y))}{h^{t-1}(1)} x^t\right)\right) \geq g^t(y)$$

$$c^t(w^t, 1) = \frac{h^{t-1}(1)}{h^{t-1}(g^t(y))}\ \min w^t z^t : h^t(f^t(z^t)) \geq g^t(y)$$

$$c^t(w^t, 1) = \frac{h^{t-1}(1)}{h^{t-1}(g^t(y))}\ c^t(w^t, g^t(y))$$

Or $c^t(w^t, g^t(y)) = \dfrac{h^{t-1}(g^t(y))}{h^{t-1}(1)}\ c^t(w^t, 1) = \varphi^t(g^t(y))\, \phi^t(w^t).$ ∎

(see Jehle, 1991, p. 234)

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CHAPTER III – ACTIVITY-BASED COSTING AND AGGREGATION IN MULTI-OUTPUT TECHNOLOGIES

3.1. Introduction

The purpose of this chapter is twofold. Firstly, it investigates the technological foundations of an aggregate activity output. The objective is to establish the analytical structure linking cost object technologies, the aggregate measure of output and activity costs, i.e. the duality between the multi-output technology and costs in ABC.

Secondly, it investigates the structure of the short run activity cost function. In the long run, all inputs are variable. In the short run, however, some inputs are fixed. The significance of the restrictions on cost variability imposed by the fact that some inputs are fixed and the implications for ABC are therefore investigated.

This chapter is organised as follows. Section 3.2 investigates the technological foundations of an aggregate activity output. Section 3.3 concentrates on the structure of the short run activity cost function and the final section presents the conclusion.

3.2. Technological Foundations of an Aggregate Activity Output

This section investigates the analytic foundations of an aggregate activity output. It derives the necessary and sufficient conditions that support the construction of an aggregate activity output in ABC. It assumes a long run perspective, where all inputs are variable with the activity output.

To begin the analysis, suppose that p inputs are aggregated at activity t, which are separable among m cost objects. The production function for cost object j at activity t might be defined as follows:

(1) $$f^t_j(x^{t,j}) = f^t_j(x^t_{1,j}, \ldots, x^t_{p,j}) = y^{t,j}$$

Where $x^{t,j} = (x^t_{1,j}, \ldots, x^t_{p,j})$ is a vector of strictly positive input quantities used by cost object j at activity t and $y^{t,j}$ the units of cost driver used by cost object j at the same activity (also assumed to be strictly positive). In ABC, an aggregate activity output is equal to $g^t(y^t) = g^t(y^{t,1}, \ldots, y^{t,m}) = \sum_{j=1}^{m} y^{t,j}$. For example, $y^{t,j}$ can be the number of set-ups of product j or the number of deliveries of product j while $g^t(y^t)$ the total number of set-ups or the total number of deliveries.

The question to which we should first direct attention concerns the way the vector of inputs $(x^t_{1,j}, \ldots, x^t_{p,j})$ is obtained. This involves the calculation of the minimum cost of producing output $y^{t,j}$, which is determined through the resolution of the following cost minimisation problem:

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(2) $c^t_j(w^t, y^{t,j}) \equiv$

Minimise $\sum^p_{i=1} x^t_{i,j} w^t_i$

Such that $f^t_j(x^{t,j}) = y^{t,j}$

Where $w^t = (w^t_1, \ldots, w^t_p)$ is a vector of strictly positive input prices. It is assumed that problem (2) is a well-behaved optimisation exercise. A regular production function for cost object j at activity t ensures that. It is useful to represent problem (2) in terms of the *Lagrangian function*:

(3) $L(x^{t,j}, \mu) = \sum^p_{i=1} x^t_{i,j} w^t_i - \mu (f^t_j(x^{t,j}) - y^{t,j})$

Where μ is a *Lagrange multiplier*. Cost minimisation implies the following first-order conditions:

(4) $\dfrac{\partial L}{\partial x^t_{i,j}} = w^t_i - \mu \dfrac{\partial f^t_j(x^{t,j})}{\partial x^t_{i,j}} = 0 \implies \dfrac{\partial f^t_j(x^{t,j})/\partial x^t_{i,j}}{\partial f^t_j(x^{t,j})/\partial x^t_{u,j}} = \dfrac{w^t_i}{w^t_u}$ [1]

Expression (4) is the well know result that at an optimum the marginal rate of technical substitution of input i for input u ($MRTS^{t,j}_{i,u}$) equals the ratio of the corresponding input prices[2].

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[1] And $\dfrac{\partial L}{\partial \mu} = 0$, i.e. $f^t_j(x^{t,j}) = y^{t,j}$.

[2] If $f^t_j(x^{t,j})$ is *strictly quasiconcave* then the second order-conditions will be satisfied (see, for example, Chiang, 1984, p. 387-404).

A conventional accounting procedure, underlying the architecture of ABC and more traditional cost systems, is the application of average cost driver rates to cost outputs. This procedure can only be justified when cost functions are linear with output. Otherwise, average and marginal costs will differ (see section 2.5). If this is the case, the cost reported by an ABC system does not measure incremental costs.

Lemma Average and marginal costs for cost object j at activity t are constant and equal to $\phi^t_j(w^t) = \sum_{i=1}^{p} \frac{w^t_i}{\alpha^t_{i,j}(w^t)}$ if and only if $f^t_j(x^{t,j})$ is linearly homogeneous.

Proof.

If $f^t_j(x^{t,j})$ is homogeneous of degree one $\partial f^t_j(x^{t,j})/\partial x^t_{i,j}$ is homogeneous of degree zero $(\partial f^t_j(\lambda\, x^{t,j})/\partial x^t_{i,j} = \partial f^t_j(x^{t,j})/\partial x^t_{i,j})$. Thus an increase of λ in all the p inputs does not change the $\mathrm{MRTS}^{t,j}_{i,u}$. Therefore, given the input price set, if the first-order conditions are fulfilled by the input combination $(x^t_{1,j}, ..., x^t_{p,j})$ they must also be fulfilled by the combination $(\lambda\, x^t_{1,j}, ..., \lambda\, x^t_{p,j})$. Moreover, if the input combination $(x^t_{1,j}, ..., x^t_{p,j})$ is associated with the production of output $y^{t,j}$, the input combination $(\lambda\, x^t_{1,j}, ..., \lambda\, x^t_{p,j})$ is associated with the production of output $\lambda\, y^{t,j}$ (since $f^t_j(\lambda\, x^{t,j}) = \lambda\, f^t_j(x^{t,j})$). This implies the condition $y^{t,j} = \alpha^t_{1,j}(w^t)\, x^t_{1,j} = ... = \alpha^t_{p,j}(w^t)\, x^t_{p,j}$, where $\alpha^t_{i,j}(w^t)$ is a constant for a given input price set and for a given linearly homogeneous technology. Now, it can be shown that average and marginal costs are constant and equal to $\phi^t_j(w^t) = \frac{c^t_j(w^t, y^{t,j})}{y^{t,j}} = \frac{\partial c^t_j(w^t, y^{t,j})}{\partial y^{t,j}} = \sum_{i=1}^{p} \frac{\partial x^t_{i,j}(w^t, y^{t,j})}{\partial y^{t,j}} w^t_i = \sum_{i=1}^{p} \frac{w^t_i}{\alpha^t_{i,j}(w^t)}$.

Additionally, by the envelope theorem, $\mu = \phi^t_j(w^t)$. Observe that the condition above

also implies that the elasticity of input demand with respect to output is constant and equal to one for all the p inputs. It is obvious that the cost function will be curvilinear if there is at least one input for which the output elasticity of demand is different from one. Thus the condition that the production function is linearly homogeneous is necessary and sufficient if the dual cost function is to be linear with output. ∎

The condition that cost object technologies are linearly homogeneous is necessary for the construction of an aggregate activity output in ABC. It is not sufficient, however. The construction of an aggregate activity output presupposes that a second condition is also verified. To derive it, it is necessary to introduce the following definition.

Definition 1 Assume that cost object technologies are linearly homogeneous. $y^{t,j}$ and $y^{t,k}$, $j \neq k$, are the same cost driver at activity t if and only if $\phi_j^t(w^t) = \phi_k^t(w^t)$, that is, if the marginal cost of a unit of cost driver used by cost object j equals the marginal cost of a unit of cost driver used by cost object k.

Proposition $g^t(y^t) = \sum_{j=1}^{m} y^{t,j}$ is a single measure of output that accurately identifies cost object incremental costs if and only if $y^{t,j}$ and $y^{t,k}$, $\forall j \neq k$, are the same cost driver.

Proof.

Sufficiency The *Lemma* implies that $c^t_h(w^t, y^{t,h}) = y^{t,h} \phi^t_h(w^t) = \sum_{i=1}^p x^t_{i,h}(w^t, y^{t,h}) \, w^t_i$.

Now, if $y^{t,j}$ and $y^{t,k}$, $\forall \, j \neq k$, are the same cost driver then $\phi^t_j(w^t) = \phi^t_k(w^t) = \phi^t(w^t)$. The cost allocated to cost object h under ABC is equal to:

$$y^{t,h} \frac{\sum_{j=1}^m \sum_{i=1}^p x^t_{i,j}(w^t, y^{t,j}) \, w^t_i}{\sum_{j=1}^m y^{t,j}} = y^{t,h} \frac{\sum_{j=1}^m y^{t,j} \phi^t_j(w^t)}{\sum_{j=1}^m y^{t,j}} = y^{t,h} \frac{\phi^t(w^t) \sum_{j=1}^m y^{t,j}}{\sum_{j=1}^m y^{t,j}} =$$

$$y^{t,h} \phi^t_h(w^t) = \sum_{i=1}^p x^t_{i,h}(w^t, y^{t,h}) \, w^t_i.$$

That is, the cost allocated to cost object h equals its incremental cost. ∎

Necessity Suppose that $\phi^t_j(w^t) < \phi^t_k(w^t)$ and $\phi^t_k(w^t) = \phi^t(w^t)$, $\forall \, k \neq j$. Let also $y^{t,h} > 0$, $\forall$ h. The cost allocated to cost object h under ABC is now equal to:

$$y^{t,h} \frac{\sum_{i=1}^p x^t_{i,j}(w^t, y^{t,j}) \, w^t_i + \sum_{k \neq j} \sum_{i=1}^p x^t_{i,k}(w^t, y^{t,k}) \, w^t_i}{\sum_{j=1}^m y^{t,j}} = y^{t,h} \phi^t(w^t, y^{t,j}, g^t(y^t_{k \neq j}))$$

Where $\phi^t(w^t, y^{t,j}, g^t(y^t_{k \neq j})) = \dfrac{y^{t,j} \phi^t_j(w^t) + g^t(y^t_{k \neq j}) \, \phi^t(w^t)}{g^t(y^t)}$ and $g^t(y^t_{k \neq j}) = \sum_{k \neq j} y^{t,k}$

Since $y^{t,h} > 0$, $\forall$ h, and $\phi^t_j(w^t) < \phi^t(w^t)$ then $\phi^t_j(w^t) < \phi^t(w^t, y^{t,j}, g^t(y^t_{k \neq j})) < \phi^t(w^t)$. This implies that $y^{t,h} \phi^t(w^t, y^{t,j}, g^t(y^t_{k \neq j})) \neq y^{t,h} \phi^t_h(w^t) = \sum_{i=1}^p x^t_{i,h}(w^t, y^{t,h}) \, w^t_i$. That is, the cost allocated to cost object h distorts its incremental cost. Therefore, the condition that $y^{t,j}$ and $y^{t,k}$, $\forall \, j \neq k$, are the same cost driver is a necessary and sufficient

condition for the construction of a single aggregate activity output that accurately measures cost object incremental costs. ∎

Corollary 1 The aggregated cost function at activity t can be written as:

$$c^t(w^t, y^{t,1}, ..., y^{t,m}) = \sum_{j=1}^{m} c^t_j(w^t, y^{t,j}) = \sum_{j=1}^{m} y^{t,j} \phi^t_j(w^t) = c^t(w^t, g^t(y^t)) = g^t(y^t) \phi^t(w^t).$$

Corollary 2 If $\exists$ $j \neq k$: $\phi^t_j(w^t) \neq \phi^t_k(w^t)$, the aggregated cost function at activity t depends on more than one cost driver[3]. Therefore, the application of a single average cost driver rate to allocate costs distorts cost object incremental costs.

It is worth now analyzing the full technological implications of the *Proposition* above. For that, it is necessary to introduce the following definition.

Definition 2 $y^{t,j}$ and $y^{t,k}$, $j \neq k$, are the same type of output if $f^t_j(x^{t,j})$ and $f^t_k(x^{t,k})$ are identical, that is, if $f^t_j(x^{t,j}) = f^t_k(x^{t,k})$, $\forall$ $x^{t,j} = x^{t,k}$. [4]

The previous definition signifies that all cost object volumes represent various levels of the same output. It was previously demonstrated that when technologies are linearly homogeneous the cost minimisation input-output relationships for cost object

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[3] For example, if $\phi^t_j(w^t) \neq \phi^t_k(w^t)$ and $\phi^t_k(w^t) = \phi^t(w^t)$, $\forall$ $k \neq j$, the aggregated cost function at activity t depends on two cost drivers. More formally, $c^t(w^t, y^{t,j}, g^t(y^t_{k \neq j})) = y^{t,j} \phi^t_j(w^t) + g^t(y^t_{k \neq j}) \phi^t(w^t)$, where $g^t(y^t_{k \neq j}) = \sum_{k \neq j} y^{t,k}$.

[4] Equivalently, $f^t_j(x^{t,j})$ and $f^t_k(x^{t,k})$ are identical if (i) isoquants for cost objects j have the same shape as those for cost object k ($MRTS^{t,j}_{i,u} = MRTS^{t,k}_{i,u}$), and (ii) equal positioned isoquants (for cost object j and for cost object k) in the input-output space are associated with the same output.

j at activity t implies that $y^{t,j} = \alpha^t_{i,j}(w^t) x^t_{i,j}$, $\forall$ i. Additionally, if all cost object technologies are identical then $\alpha^t_{i,j}(w^t) = \alpha^t_{i,k}(w^t) = \alpha^t_i(w^t)$, $\forall$ i, k $\neq$ j and $\forall$ w^t, which also implies that $\sum^m_{j=1} y^{t,j} = \sum^m_{j=1} \alpha^t_{i,j}(w^t) x^t_{i,j}$ or $g^t(y^t) = \alpha^t_i(w^t) x^t_i$, where $x^t_i = \sum^m_{j=1} x^t_{i,j}$. That is, if $f^t_j(x^{t,j})$ and $f^t_k(x^{t,k})$ are identical $\phi^t_j(w^t)$ and $\phi^t_k(w^t)$ are also identical ($\phi^t_j(w^t) = \phi^t_k(w^t)$, $\forall$ w^t). Thus when cost object technologies are not only linearly homogeneous but also identical the marginal cost of a unit of cost driver used by a cost object is both constant and equal for all cost objects within a cost pool[5]. In this case, any input aggregated at activity t can be used as a single measure of output. More formally, if input u is used as a measure of the output, the average and marginal cost driver rate at activity t is:

(5)
$$\phi^t(w^t)_u = \sum^p_{i=1} \frac{\partial x^t_i}{\partial x^t_u} w^t_i = \alpha^t_u(w^t) \sum^p_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)} \;^6$$

However, it must be noted that even when $y^{t,j}$ and $y^{t,k}$ are not the same type of output the marginal cost of a unit of cost driver used by a cost object might still be equal for all cost objects within a cost pool. In other words, the *Proposition* above does not necessarily imply that all (linearly homogenous) cost object technologies are identical.

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[5] Note that we are assuming a long run perspective, where all inputs are variable with output. In fact, the implication that the marginal cost of a unit of cost driver used by a cost object is equal for all cost objects within a cost pool when cost object technologies are both linearly homogeneous and identical cannot be established if, in the short run, activities are operated above capacity. This point will be analysed in the next section.

[6] Thus *Corollary 1* can be rewritten as $c^t(w^t, x^t_{u,1}, ..., x^t_{u,m}) = \sum^m_{j=1} c^t_j(w^t, x^t_{u,j}) = \sum^m_{j=1} x^t_{u,j} \phi^t_j(w^t)_u = c^t(w^t, x^t_u) = x^t_u \phi^t(w^t)_u$, where $\phi^t_j(w^t)_u = \sum^p_{i=1} \frac{\partial x^t_{i,i}}{\partial x^t_{u,j}} w^t_i = \alpha^t_{u,j}(w^t) \sum^p_{i=1} \frac{w^t_i}{\alpha^t_{i,j}(w^t)}$.

To illustrate this latter point consider that activity t is a set-up activity that aggregates three inputs. Assume also that there are two products (P_1 and P_2). The production function for P_j can be represented as $f_j^t(x_{1,j}^t, x_{2,j}^t, x_{3,j}^t) = y^{t,j}$, where $x_{i,j}^t$ is the quantity of input i used by P_j and $y^{t,j}$ the number of set-ups of P_j. Additionally, $f_j^t(x_{1,j}^t, x_{2,j}^t, x_{3,j}^t) = \min(\alpha_{1,j}^t x_{1,j}^t, \alpha_{2,j}^t x_{2,j}^t, \alpha_{3,j}^t x_{3,j}^t)$, that is, the technology supporting P_j is Leontief, a special case of a linearly homogeneous technology that does not allow any substitution between inputs. The cost per set-up of P_j is $\phi_j^t(w^t) = \sum_{i=1}^{3} \frac{w_i^t}{\alpha_{i,j}^t}$. It is obvious that $\phi_1^t(w^t) = \phi_2^t(w^t)$ when $\alpha_{i,1}^t = \alpha_{i,2}^t, \forall i$. However, it is no less obvious that even when $\alpha_{i,1}^t \neq \alpha_{i,2}^t, \forall i$, we might have $\phi_1^t(w^t) = \phi_2^t(w^t)$ (at least for some input price set). That is to say, it is possible that P_1 and P_2 use different input mixes while the cost per set-up of P_1 is still equal to the cost per set-up of P_2. If this is the case, the total number of set-ups is in fact a single measure of output that accurately identifies incremental product costs ($y^{t,1}$ and $y^{t,2}$ are the same cost driver). Suppose now that we consider using as an allocation base either the total number of set-ups or the total quantity of input u. It might be the case that $y^{t,1}$ and $y^{t,2}$ are not the same cost driver ($\phi_1^t(w^t) \neq \phi_2^t(w^t)$) while $x_{u,1}^t$ and $x_{u,2}^t$ are the same cost driver ($\phi_1^t(w^t)_u = \phi_2^t(w^t)_u$)[7]. For example, assume that $w_1^t = w_2^t = w_3^t = £1$ and $1/\alpha_{1,1}^t = 20$, $1/\alpha_{2,1}^t = 60$, $1/\alpha_{3,1}^t = 40$, $1/\alpha_{1,2}^t = 65$, $1/\alpha_{2,2}^t = 70$, $1/\alpha_{3,2}^t = 5$. Hence $\phi_1^t(w^t) = £120 \neq \phi_2^t(w^t) = £140$ and $\phi_1^t(w^t)_2 = \phi_2^t(w^t)_2 = £2$. This means that if the total number of set-ups is the allocation base activity costs are incorrectly distributed

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[7] $\phi_j^t(w^t)_u = \alpha_{u,j}^t \sum_{i=1}^{3} \frac{w_i^t}{\alpha_{i,j}^t}$.

between P_1 and P_2. However, if input 2 is the allocation base activity costs are accurately distributed between P_1 and P_2 (the same cannot be concluded when input 1 or input 3 is used as the allocation base)[8]. It would be possible to imagine many other situations where P_1 and P_2 use completely different input mixes while activity costs are still accurately distributed between P_1 and P_2 when the total number of set-ups (or the total quantity of input u) is the allocation base.

The discussion undertaken in the previous paragraph implies that even when product technologies are heterogeneous, that is, even when the various products use different inputs mixes, product cost distortions might be small. Hwang *et al* (1993) observe that product cost distortions, due to the use of an allocation base to distribute the cost of the inputs aggregated in a cost pool among the various products, increase when product technologies are significantly different. Although this can be accepted as a general observation, the preceding analysis serves to illustrate that high product technology heterogeneity does not necessarily lead to high product cost distortions.

To sum up, the two jointly necessary and sufficient conditions supporting the construction of an aggregate activity output are (i) the linear homogeneity property associated with each cost object production function and the fact that (ii) the marginal cost corresponding to a unit of cost driver used by a cost object is equal for all cost objects within a cost pool. While the first condition ensures that marginal

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[8] Observe that, given the specific (heterogeneous) product technologies, the conclusion that $y^{t,1}$ and $y^{t,2}$ (or $x^t_{u,1}$ and $x^t_{u,2}$) are or not the same cost driver depends on the input price set. In other words, it might be the case (or not) that for some input price set $y^{t,1}$ and $y^{t,2}$ (or $x^t_{u,1}$ and $x^t_{u,2}$) are the same cost driver. However, when product technologies are identical, $y^{t,1}$ and $y^{t,2}$ (and $x^t_{u,1}$ and $x^t_{u,2}$) are the same cost driver for all the input price sets. We will return to this point in chapter seven.

costs are constant, the second ensures that the aggregated cost function at a given activity depends on only one cost driver. Finally, it should be noted that these two conditions automatically exclude the existence of economies or diseconomies of scope either within or between cost pools. That is to say, they exclude the existence of any joint technologies, a necessary condition for an ABC system to measure incremental costs (see section 2.7). It is also being implicitly assumed that inputs are traded on a perfect marked. As Bromwich and Hong (1999, p. 53) observe:

> "A perfect input market is necessary to ensure that input prices are linear with either output or activity output, otherwise identical inputs may have different prices".

The following table summarises the technological foundations of an aggregate activity output.

Table 1 – Technological Foundations of an Aggregate Activity Output

(Cost object) Production function linearly homogeneous
$$f^t_j(\lambda\, x^t_{1,j}, \ldots, \lambda\, x^t_{p,j}) = \lambda\, y^{t,j}$$

$\downarrow$

(Cost object) Cost function linear with output
$$c^t_j(w^t, y^{t,j}) = y^{t,j}\, \phi^t_j(w^t)$$

$+$

Marginal cost of a unit of cost driver used by a cost object equal for all cost objects within a cost pool
$$\phi^t_1(w^t) = \ldots = \phi^t_m(w^t) = \phi^t(w^t)$$

$\downarrow$

(Activity) Cost function linear with the aggregate activity output
$$c^t(w^t, g^t(y^t)) = g^t(y^t)\, \phi^t(w^t), \text{ where } g^t(y^t) = \sum_{j=1}^{m} y^{t,j}$$
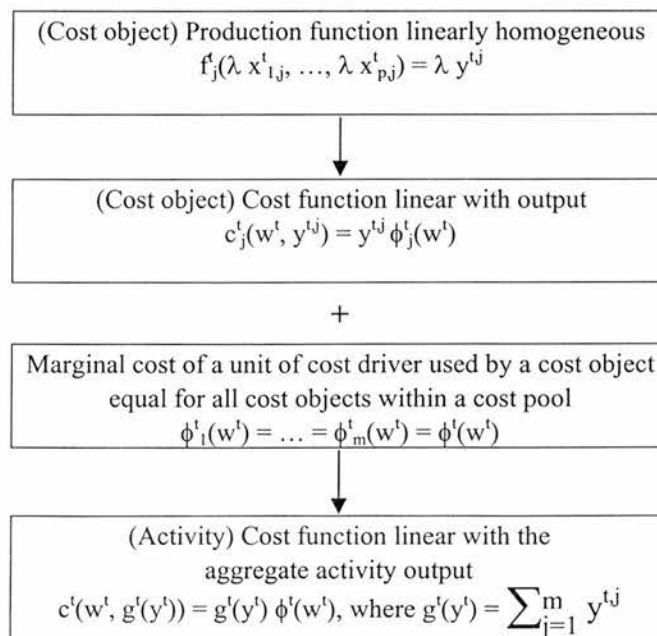
These results are new in the ABC literature and are in marked contrast with the findings of Bromwich and Hong (1999).

Firstly, Bromwich and Hong (1999, p. 56-57) observe that cost functions compatible with ABC require that the unit cost for each cost pool should be invariant with output and claim that homothetic technologies ensure that. However, the results previously derived show that only linearly homogeneous technologies, a special case of homothetic technologies, guarantee that the unit cost for each cost pool is invariant with output. That is, only linearly homogeneous technologies give rise to cost functions linear with output (see the demonstration of the *Lemma*; see also section 2.5).

Secondly, Bromwich and Hong (1999, p. 48-49) also observe that the aggregation of a multi-output technology requires that a constant input mix is common to all products irrespective of volume. Conversely, the above results show that even where the various cost objects use different input mixes (which are constant for a given input price set) it might be possible to construct an aggregate activity output with an ABC system. What is necessary is that the marginal cost corresponding to a unit of cost driver used by a cost object is both constant and equal for all cost objects within a cost pool. It is sufficient to ensure this result that all cost object technologies are not only linearly homogeneous but also identical (which automatically implies that a constant input mix is common to all cost objects irrespective of volume). However, this is not necessary. That is, even when the various cost objects use different input mixes it might be possible, at least in theory, to represent the aggregated cost

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function at activity t as depending on only one cost driver (at least for some input price set). Nevertheless, it should be recognized that it is a very strong assumption that $y^{t,j}$ and $y^{t,k}$, $\forall\, j \neq k$, are the same cost driver when all (linearly homogeneous) cost object technologies are not identical.

3.3. Short Run Activity Cost Function

The analysis undertaken in the last section has assumed a long run perspective, where all inputs are variable with the activity output. In the short run, however, some inputs are fixed. This section investigates the structure of the short run activity cost function.

To incorporate the distinction between variable and fixed inputs, let represent the vector of inputs supplied at activity t in the following manner: $x^t_{(supplied)} = (x^t_{f1}, \ldots, x^t_{fu}, x^t_{u+1}, \ldots, x^t_p)$, where u inputs (i = 1, ..., u) are fixed and (p - u) inputs (i = u+1, ..., p) are variable in the period in consideration[9].

Assumption $y^{t,j}$ and $y^{t,k}$, $\forall\, k \neq j$, are the same type of output (see *Definition 2*).

As was defined in the previous section, $y^{t,j}$ and $y^{t,k}$, $\forall\, k \neq j$, are the same type output if all cost object technologies are identical. In other words, all cost object volumes represent various levels of the same output. This permits the visualization of the activity output as an intermediate input that is used by the various cost objects. The

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[9] x^t_{fi} means that the input quantity i is fixed.

analysis undertaken in this section explicitly assumes this[10]. The short run cost

minimisation problem can now be defined as follows:

(6)   Minimise $\sum_{i=1}^{p} x_i^t w_i^t$

Such that $f^t(x^t) = f^t(x_1^t, \ldots, x_p^t) = g^t(y^t)$ and $x_i^t \leq x_{fi}^t$

Where $f^t(x^t)$ is the production function at activity t, $x^t = (x_1^t, \ldots, x_p^t)$ a vector of

strictly positive input quantities used at the same activity and $g^t(y^t)$ a strictly positive

output.

Observe that when all inputs are variable and all cost object technologies are both

linearly homogeneous and identical then $g^t(y^t) = \alpha_i^t(w^t) \, x_i^t$, $\forall$ i (see previous section).

That is, the constant $\alpha_i^t(w^t)$ results from the resolution of the long run cost

minimisation problem. Moreover, assuming that input i is fixed and all the other

inputs are variable, the fixed quantity x_{fi}^t gives rise to a (maximum) output of say

$g_{fi}^t(y^t) = x_{fi}^t \alpha_i^t(w^t)$, compatible with the equality between the long run and short run

solutions to the cost minimisation problems[11, 12]. To proceed with an analysis of the

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[10] As was demonstrated in the last section, although the assumption that $y^{t,j}$ and $y^{t,k}$, $\forall$ k ≠ j, are the same type of output, together with the necessary condition that cost object technologies are linearly homogenous, is sufficient to construct an aggregate activity output, it is not necessary. Only for the sake of analytical simplicity in the analysis of the short run cost minimisation problem it is assumed here that cost object technologies are identical.

[11] Note that when $g^t(y^t) > g_{fi}^t(y^t)$ the long run solution imposes that $x_i^t > x_{fi}^t$. This, however, violates the short run restriction $x_i^t \leq x_{fi}^t$. Therefore, when $g^t(y^t) > g_{fi}^t(y^t)$ the short run solution imposes that the restriction $x_i^t \leq x_{fi}^t$ binds in problem (6). In this case, the long and short run solutions to the cost minimisation problem will be different.

[12] Observe that without the assumption that cost object technologies are identical it was not possible to define the constant $g_{fi}^t(y^t) = x_{fi}^t \alpha_i^t(w^t)$. Instead, it would only be possible to establish that $\sum_{j=1}^{m} \frac{y_{fi}^{t,j}}{\alpha_{i,j}^t(w^t)} = x_{fi}^t$, In this case, there are multiple vectors $(y_{fi}^{t,1}, \ldots, y_{fi}^{t,m})$ which satisfy this equality.

short run cost minimisation problem it is now necessary to introduce a new definition.

Definition 3 An activity is operated with excess capacity if $\forall \, x^t_{fi} \in (x^t_{f1}, \ldots, x^t_{fu})$: $g^t(y^t) \leq g^t_{fi}(y^t)$ or simply $g^t(y^t) \leq \min (g^t_{fi}(y^t))$. It is operated above capacity if $\exists \, x^t_{fi} \in (x^t_{f1}, \ldots, x^t_{fu})$: $g^t(y^t) > g^t_{fi}(y^t)$ or $g^t(y^t) > \min (g^t_{fi}(y^t))$.

That is, an activity is operated with excess capacity if all the fixed inputs are not fully used. It is operated above capacity if at least one fixed input is fully used.

Case 1 (excess capacity): $g^t(y^t) \leq \min (g^t_{fi}(y^t))$

When an activity is operated with excess capacity all the fixed inputs are not fully used, implying that the restrictions $x^t_i \leq x^t_{fi}$ slack in problem (6). Consequently, the solutions to the long and short run cost minimisation problems coincide. Therefore, the vector of inputs $(g^t(y^t)/\alpha^t_1(w^t), \ldots, g^t(y^t)/\alpha^t_p(w^t))$ minimises the short run cost of resources used (and the long run cost of resources used and supplied as well). In these circumstances, the short run cost function can be represented as:

(7) $\qquad c^t(w^t, g^t(y^t)) = \sum_{i=1}^{u} \dfrac{w^t_i}{\alpha^t_i(w^t)} (g^t_{fi}(y^t) - g^t(y^t)) + \sum_{i=1}^{p} \dfrac{w^t_i}{\alpha^t_i(w^t)} g^t(y^t)$

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Proof.

It has been shown that when activity t is operated with excess capacity, cost object technologies are linearly homogeneous and $y^{t,j}$ and $y^{t,k}$, $\forall\, k \neq j$, are the same type of output the cost minimisation input-output relationships imply that $g^t(y^t) = \alpha^t_i(w^t)\, x^t_i$, $\forall\, i$. Additionally, $g^t_{f\,i}(y^t) = \alpha^t_i(w^t)\, x^t_{f\,i}$, $\forall\, x^t_{f\,i}$. The following developments take place:

$$c^t(w^t, g^t(y^t)) = \sum\nolimits^u_{i=1} x^t_{f\,i}\, w^t_i + \sum\nolimits^p_{i=u+1} x^t_i\, w^t_i =$$

$$\sum\nolimits^u_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}\, g^t_{f\,i}(y^t) + \sum\nolimits^p_{i=u+1} \frac{w^t_i}{\alpha^t_i(w^t)}\, g^t(y^t) =$$

$$\sum\nolimits^u_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}\, (g^t_{f\,i}(y^t) - g^t(y^t)) + \sum\nolimits^u_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}\, g^t(y^t) + \sum\nolimits^p_{i=u+1} \frac{w^t_i}{\alpha^t_i(w^t)}\, g^t(y^t) =$$

$$\sum\nolimits^u_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}\, (g^t_{f\,i}(y^t) - g^t(y^t)) + \sum\nolimits^p_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}\, g^t(y^t). \quad\blacksquare$$

In ABC terminology, expression (7) is the cost of resources supplied, which can be split into the cost of resources not used and the cost of resources used (Cooper and Kaplan, 1992). The cost of resources not used is $\sum\nolimits^u_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}\, (g^t_{f\,i}(y^t) - g^t(y^t))$, while the cost of resources used is $\sum\nolimits^p_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}\, g^t(y^t)$. Note that the cost of resources used changes at a rate of $\sum\nolimits^p_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}$, which is independent of the output. Moreover, while the component $\sum\nolimits^u_{i=1} \frac{w^t_i}{\alpha^t_i(w^t)}$ represents an increase in the cost of resources used but not in the cost of resources supplied, the component $\sum\nolimits^p_{i=u+1} \frac{w^t_i}{\alpha^t_i(w^t)}$ represents an

increase in the cost of resources used and supplied. Only this latter component generates relevant costs for decision-making.

In the short run, unless activity t is operated in perfectly efficient conditions, the cost reported by an ABC system, $\phi^t_{SR}(w^t, g^t(y^t))$, does not coincide with the long run average and marginal cost, $\phi^t(w^t)$. To see why observe that:

(8)
$$\phi^t_{SR}(w^t, g^t(y^t)^*) = \phi^t_{SRNI}(w^t, g^t(y^t)^*) + \phi^t_{SRI}(w^t) = \phi^t(w^t)$$

If and only if $g^t(y^t)^* = \alpha^t_i(w^t)\, x^t_{fi}$

$$\phi^t_{SRNI}(w^t, g^t(y^t)^*) = \frac{\sum_{i=1}^{u} x^t_{fi}\, w^t_i}{g^t(y^t)^*} = \sum_{i=1}^{u} \frac{w^t_i}{\alpha^t_i(w^t)}$$

$$\phi^t_{SRI}(w^t) = \frac{\sum_{i=u+1}^{p} x^t_i\, w^t_i}{g^t(y^t)^*} = \sum_{i=u+1}^{p} \frac{w^t_i}{\alpha^t_i(w^t)}$$

$$\phi^t(w^t) = \sum_{i=1}^{p} \frac{w^t_i}{\alpha^t_i(w^t)}$$

Proof.

Note again that $g^t(y^t) = \alpha^t_i(w^t)\, x^t_i,\ \forall\, i$. The cost of resources supplied changes at a rate of $\phi^t_{SRI}(w^t) = \sum_{i=u+1}^{p} \frac{\partial x^t_i}{\partial g^t(y^t)}\, w^t_i = \sum_{i=u+1}^{p} \frac{w^t_i}{\alpha^t_i(w^t)}$, which is independent of the output. The component $\phi^t_{SRNI}(w^t, g^t(y^t))$ reported by an ABC system depends on the output, since $\sum_{i=1}^{u} x^t_{fi}\, w^t_i$ is fixed. This component will reflect optimal input usage if and only if $g^t(y^t)^* = \alpha^t_i(w^t)\, x^t_{fi},\ \forall\, x^t_{fi}$. In this case, $\phi^t_{SRNI}(w^t, g^t(y^t)^*) = \sum_{i=1}^{u} \frac{w^t_i}{\alpha^t_i(w^t)}$.

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Finally, the long run average and marginal cost is equal to $\phi^t(w^t) = \sum_{i=1}^{p} \frac{\partial x_i^t}{\partial g^t(y^t)} w_i^t =$

$\sum_{i=1}^{p} \frac{w_i^t}{\alpha_i^t(w^t)}$.[13] ∎

Condition (8) shows that activity t is being operated neither with excess capacity nor above capacity, in the sense that all resources supplied are being used. It signifies that the short run vector of inputs supplied and used $(x_{f1}^t, \ldots, x_{fu}^t, x_{u+1}^t, \ldots, x_p^t)$ is the long run cost minimisation vector of producing output $g^t(y^t)^*$. So when condition (8) takes place activity t is being operated in perfectly efficient conditions.

Case 2 (above capacity): $g^t(y^t) > \min(g_{fi}^t(y^t))$

In this case, the vector of inputs $(g^t(y^t)/\alpha_1^t(w^t), \ldots, g^t(y^t)/\alpha_p^t(w^t))$ is not the short run cost minimisation vector anymore. The fact that some inputs are fixed and $g^t(y^t) > \min(g_{fi}^t(y^t))$ means that one or more scarce (fixed) inputs have to be substituted by the variable inputs[14]. That is, the input mix will change. Consequently, the long run cost of resources used and supplied of producing output $g^t(y^t)$ will not be achieved[15]. The precise linearly homogeneous technology will determine the major or minor degree of substitution between inputs[16]. At one extreme, if it is a Leontief technology, no substitution between inputs will be permitted. In this case problem (6)

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[13] Note that if $\exists\, x_{fi}^t: g^t(y^t) < \alpha_i^t(w^t)\, x_{fi}^t$, then $\phi_{SRNI}^t(w^t, g^t(y^t)) > \sum_{i=1}^{u} \frac{w_i^t}{\alpha_i^t(w^t)}$ and $\phi_{SR}^t(w^t, g^t(y^t)) > \phi^t(w^t)$.

[14] Scarce (fixed) inputs might also be substituted by other fixed inputs, as long as they are not fully used. This might happen if $\exists\, x_{fi}^t, x_{fu}^t: \alpha_i^t(w^t)\, x_{fi}^t \neq \alpha_u^t(w^t)\, x_{fu}^t$.

[15] See footnote 11.

[16] This is given by the $MRTS_{i,u}^t$.

does not have a solution so the short run maximum output will be $g^t(y^t) = \min(g^t_{fi}(y^t))$. At another extreme, if it is a linear technology, inputs are perfect substitutes for each other and problem (6) will have a solution. In the cases in between, the major or minor degree of substitution between inputs that the technology allows will determine if the output $g^t(y^t)$ can be achieved or not. In the short run, the following condition will be observed:

$$(9) \qquad \sum_{i=1}^{p} x^t_i w^t_i \geq \phi^t(w^t) g^t(y^t)$$

This means that the short run cost of resources used (equal or lower than the cost of resources supplied) at activity t will be higher than or equal to the long run cost of resources used and supplied. It will be higher than the long run cost if activity t is being operated above capacity. This is because when $g^t(y^t) > \min(g^t_{fi}(y^t))$ the rate at which the cost of resources used increases will no longer be constant, but will increase at a progressively higher rate, as the input mix changes (and differs from the long run input mix). Thus $y^{t,j}$ and $y^{t,k}$, $j \neq k$, will no longer be the same cost driver. That is, the increase in the cost of resources used when a unit of cost driver is used by cost object j will be different from the increase in the cost of resources used when a unit of cost driver is used by cost object k[17]. Therefore, ABC is only compatible with activities operating with excess capacity.

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[17] This contrasts with the long run analysis undertaken in the last section. As was demonstrated, when cost object technologies are both linearly homogeneous and identical, the increase in the cost of resources used (and supplied) when a cost object uses a unit of cost driver is equal for all cost objects within a cost pool. However, this does not occur when, in the short run, activities are operated above capacity.

A corollary of condition (9) is that the short run cost reported by an ABC system will be higher than or equal to the long run average and marginal cost $(\phi^t_{SR}(w^t, g^t(y^t)) \geq \phi^t(w^t))$. It is not hard to prove this claim. It has been shown that when $g^t(y^t) \leq \min (g^t_{fi}(y^t))$ the short run cost of resources used will be equal to the long run cost of resources used and supplied. This implies either (i) $\phi^t_{SR}(w^t, g^t(y^t)) = \phi^t(w^t)$, if all the resources supplied are being used or (ii) $\phi^t_{SR}(w^t, g^t(y^t)) > \phi^t(w^t)$, if not all the resources supplied are being used (see condition (8)). If $g^t(y^t) > \min (g^t_{fi}(y^t))$ then $\phi^t_{SR}(w^t, g^t(y^t)) > \phi^t(w^t)$, as the short run cost of resources used will be higher than the long run cost of resources used and supplied.

3.4. Conclusions

This chapter has first identified the necessary and sufficient conditions that support the construction of an aggregate activity output, compatible with costs being directly proportional to the level of that output. Two conditions were derived. The first is that (i) cost object production functions are linearly homogeneous. This condition ensures that marginal costs are constant, which is essential if the cost reported by an ABC system, an average cost, is also to be a relevant cost for decision-making. The second condition is that (ii) the marginal cost of a unit of cost driver used by a cost object is equal for all cost objects within a cost pool. This condition ensures that the aggregated activity cost function depends on only one cost driver. These two conditions are jointly necessary and sufficient for the construction of an aggregate activity output in ABC.

It has been shown that when cost object technologies are not only linearly homogeneous but also identical the marginal cost of a unit of cost driver used by a cost object is both constant and equal for all cost objects within a cost pool. The contrary, however, is not true, meaning that even when cost object technologies are heterogeneous the marginal cost of a unit of cost driver used by a cost object might still be equal for all cost objects within a cost pool. This is why high product technology heterogeneity does not necessarily lead to high product cost distortions. It has also been shown that when condition (ii) does not hold the aggregated activity cost function depends on more than one cost driver. In this case, the application of a single average cost driver rate to allocate costs distorts product costs.

Second, the short run structure of ABC was introduced. In the short run, it is necessary to distinguish between the case where an activity is operated with excess capacity and the case where it is operated above capacity. An activity is operated with excess capacity if all the fixed inputs are not fully used. It is operated above capacity if at least one fixed input is fully used. This concept of capacity has been neglected in the management accounting literature, in general, and in the ABC literature, in particular. However, it is fundamental to represent the short run equation of capacity, one of the highest profile innovations of ABC systems (Cooper and Kaplan, 1992).

The rate at which the cost of resources used changes is constant only when activities are operated with excess capacity. This rate can be split into two components. While one component denotes an increase in the cost of resources used but not in the cost of resources supplied, the other denotes an increase in the cost of resources used and supplied. Only this latter component generates relevant costs for decision-making. The rate at which the cost of resources used changes will no longer be constant when activities are operated with excess capacity, since, in this case, the input mix will change and differ from the long run input mix. Thus ABC is only compatible with activities operating with excess capacity.

CHAPTER IV – HOMOGENEOUS, NON-HOMOTHETIC TECHNOLOGIES AND PRODUCT COSTING

4.1. Introduction

It was demonstrated in the last chapter that the two necessary and sufficient conditions supporting the construction in ABC of an aggregate activity output are (i) the linear homogeneity property associated with each cost object production function and the fact that (ii) the marginal cost corresponding to a unit of cost driver used by a cost object is equal for all cost objects within a cost pool. These two conditions ensure that the activity cost function is linear with respect to an aggregate activity output or cost driver. Under these circumstances, the typical product cost reported by an ABC system, an average cost, would also be a relevant cost for decision-making.

The linear homogeneity property of cost object production functions signifies that an equal and simultaneous increase or decrease in all inputs is reflected in the same way in the activity output. This constitutes a very strong condition to impose on real word cost systems and for this reason it is worth pursuing the investigation of situations where this property does not apply. The non-existence of the linear homogeneity property of cost object production functions will imply that average product costs will differ from marginal product costs, that is, ABC systems will not generate relevant costs for decisions involving output variation.

This chapter explores the relaxation of the linear homogeneity property of cost object production functions. The analysis is based on the duality between production and cost functions. As in the study of the short run activity cost functions in section 3.3, it is also assumed here that cost object technologies are identical. Thus all cost object volumes represent various levels of the same output. This permits the visualization of the activity output as an intermediate input that is used by the various cost objects. Two types of non-linear technologies are considered. The first one is the case of homogeneous technologies, a particular category of homothetic technologies. The second one concerns the case of non-homothetic technologies. These two types of technologies are sufficiently flexible to cover a great variety of situations in terms of non-linear input-output relationships. In the case of homogeneous technologies all inputs always change in the same way. This amounts to saying that when the activity output increases, all inputs equally vary by a greater (decreasing returns to scale), equal (constant returns to scale) or smaller (increasing returns to scale) proportional change[1]. In many real world situations, however, this rigid pattern of input change is unlikely to occur. Some inputs will change more than proportionally, others less than proportionally and yet others in the same way as the activity output. The characterisation of the non-homothetic attribute provides a basis for addressing these possible types of input behaviour.

The chapter is organised as follows. Section 4.2 addresses the structure of homogeneous technologies. The major advantage of assuming this type of technology is its simple analytical structure, as it gives rise to cost functions

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[1] Noreen and Soderstrom (1994, 1997), for example, find evidence, in a study of overheads costs in a hospital, for the existence of increasing returns to scale.

multiplicatively separable in input prices and output. This multiplicatively separable structure presupposes that the input mix is constant along the expansion path (homothetic property). However, this assumption is not valid when normal and inferior inputs are combined. An inferior (normal) input is one whose quantity increases (decreases) with output. Section 4.3 introduces a procedure developed by Pollak and Wales (1992) in the production economics literature that transforms a homogeneous technology into a non-homothetic one. In the section 4.4, this procedure is applied to the Leontief technology in order to derive the non-homothetic Leontief cost function. In the section 4.5 a model is specified while in section 4.6 some simulations are performed in order to ascertain the major implications of the results derived in section 4.5. Finally, section 4.7 presents the conclusions.

4.2. Homogeneous Technologies and Cost Functions

Homogeneous technologies are a particular case of homothetic technologies. If fact, all homogeneous technologies are homothetic. The contrary, however, is not true, as it possible to identify homothetic technologies that are not homogeneous. An example of a homothetic technology that is not homogeneous will be presented later in this section.

The usual rationality for assuming homotheticity is that it gives rise to cost functions multiplicatively separable in input prices and output. That is to say, under homotheticity it is possible to construct an aggregate input price that is independent of the level of output. A homogeneous technology is a special case of a homothetic technology, where $h^t(f^t(x^t)) = (f^t(x^t))^{\beta^t} = g^t(y^t)$. It was demonstrated in section 2.5 that the cost function for a homothetic technology can be written as $c^t(w^t, g^t(y^t)) = \varphi^t(g^t(y^t))\, \phi^t(w^t)$, where $\varphi^t(g^t(y^t)) = h^{t-1}(g^t(y^t))/h^{t-1}(1)$. This implies that the cost function for a homogeneous technology takes the following form:

$$(1) \qquad c^t(w^t, g^t(y^t)) = g^t(y^t)^{v^t}\, \phi^t(w^t)$$

Where $v_t = 1/\beta^t$. As previously observed, the output $g^t(y^t)$ is as an intermediate input that is used by the various cost objects. The cost function (1) is homogeneous of degree v^t in output, while the associated production function is homogeneous of degree $\beta_t = 1/v^t$. When the production function is homogeneous of degree one the cost function becomes $c^t(w^t, g^t(y^t)) = g^t(y^t)\, \phi^t(w^t)$. An ABC system designed to

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generate relevant costs presupposes such a structure, where average and marginal costs are constant and equal to $\phi^t(w^t)$ (see chapter 2). It just signifies that costs vary linearly with the activity output.

Expression (1) shows that when $v^t < 1$ (or $\beta_t > 1$) we have increasing returns to scale, while when $v^t > 1$ (or $\beta_t < 1$) we are in the presence of decreasing returns to scale. Increasing returns to scale means that when all inputs increase in proportion by a certain amount the activity output shows a more than proportional increase. This implies that the rate at which costs increase reduces with the level of output. In other words, marginal costs are decreasing. Simultaneously, decreasing returns to scale signifies that when all inputs increase in proportion by a given amount the activity output shows a less than proportional increase. Consequently, costs increase at a progressively higher rate, i.e., marginal costs increase with output. This can be easily demonstrated if we derive from (1) the marginal cost (MC_t):

$$(2) \qquad MC_t = \frac{\partial c^t(w^t, g^t(y^t))}{\partial g^t(y^t)} = v^t \, g^t(y^t)^{v^t - 1} \, \phi^t(w^t)$$

From (2) it is apparent that the marginal cost is an increasing function when $v^t > 1$, but a decreasing function when $v^t < 1$. Similarly, the average cost (AC_t) is:

$$(3) \qquad AC_t = \frac{c^t(w^t, g^t(y^t))}{g^t(y^t)} = g^t(y^t)^{v^t - 1} \, \phi^t(w^t)$$

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Both the marginal and average costs depend on the level of output. The only case when the marginal and average costs are independent of the level of output is when the production function is linearly homogeneous ($\beta_t = v^t = 1$). From (2) and (3), it can be shown that marginal and average costs of homogeneous functions are related as follows:

$$(4) \qquad MC_t = v^t AC_t = v^t \frac{\sum_{i=1}^{p} x_i^t w_i^t}{g^t(y^t)}$$

This means that the marginal cost can be obtained if we multiply the average cost by the degree of homogeneity of the cost function. Note, however, that the average cost depends on the level of output, as does the marginal cost. This means that the determination of the marginal cost for a given output through (4) presupposes that the average cost associated with the same level of output is known. This corresponds to a situation where costs for different level of outputs can be accumulated. Furthermore, it is clear from expression (4) that the marginal cost of homogeneous functions will be either strictly above or strictly below the average cost, depending on the degree of homogeneity of the cost function, v^t. In other words, $MC_t > AC_t$ when $v^t > 1$, but $MC_t < AC_t$ when $v^t < 1$.

In the product costing literature, the only attempt to incorporate into the analysis non-linearity issues are Christensen and Demski (1997, 2003). However, they only make use of the result that increasing (decreasing) returns to scale imply that average costs will be above (below) marginal costs. They state (1997, p. 83):

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"It is well known that d(x) strictly concave implies marginal cost is strictly increasing in output. This implies average cost is strictly below marginal cost" (d(x) is the production function).

In this study, however, the analysis is founded on the duality between production and cost functions, and while deriving the cost function dual to a homogeneous technology, a more general case, the non-homothetic one (see sections 4.3 and 4.4), is also derived.

Finally, it must be observed that in the case of homothetic technologies that are not homogeneous the relationship between average and marginal costs is not generally constant. Consider the example $h^t(f^t(x^t)) = Ln\ f^t(x^t) = g^t(y^t)$, where $f^t(x^t) = x_1^{t\ \alpha} x_2^{t\ 1-\alpha}$ and $f^t(x^t) \geq 1$ (since $g^t(y^t) \geq 0$). This technology is homothetic but not homogenous. In this case, $MC_t = g^t(y^t)\ AC_t$, meaning that the relation between marginal and average costs is not constant, as it depends on the level of output[2].

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[2]$Ln\ f^t(x^t)$ is homothetic because (i) $f^t(x^t)$ is homogeneous of degree one and (ii) $\dfrac{d\ h^t(f^t(x^t))}{d\ f^t(x^t)} = \dfrac{1}{f^t(x^t)} > 0$.

It is not homogeneous, however, since $Ln\ f^t(\lambda\ x^t) = Ln\ (\lambda\ f^t(x^t)) = Ln\ \lambda + Ln\ f^t(x^t) \neq \lambda^{\beta^t} Ln\ f^t(x^t)$. The cost function dual to a homothetic technology is $c^t(w^t, g^t(y^t)) = \varphi^t(g^t(y^t))\ \phi^t(w^t)$, where $\varphi^t(g^t(y^t)) = \dfrac{h^{t-1}(g^t(y^t))}{h^{t-1}(1)}$. Therefore, $c^t(w^t, g^t(y^t)) = \dfrac{e^{g^t(y^t)}}{e} \phi^t(w^t)$. This implies that $MC_t = \dfrac{e^{g^t(y^t)}}{e} \phi^t(w^t)$, $AC_t = \dfrac{1}{g^t(y^t)} \dfrac{e^{g^t(y^t)}}{e} \phi^t(w^t)$ and $MC_t = g^t(y^t)\ AC_t$.

4.3. Non-Homothetic Technologies and Cost Functions

In economics, the concept of non-homotheticity is associated with the combination of normal inputs with inferior inputs. An inferior input is one whose quantity falls as output increases, while a normal input is one whose quantity increases with output (Chambers, 1988, p. 69). Homothetic technologies have the property that the quotient between pair of inputs does not change with output (see section 2.5). This signifies that all the inputs always change in the same way. When one input increases or decreases in proportion by a given amount, all the others change by the same amount. In the case of a non-homothetic function, however, the ratio between pair of inputs changes with the level of output. It might be the case that as output increases some inputs increase more than proportionally while others increase less than proportionally. This section introduces a procedure developed by Pollak and Wales (1992) that transforms a homogeneous technology into a non-homothetic one.

Consider again expression (1), which shows the structure of a cost function homogeneous of degree v^t in output. Since $\phi^t(w^t)$ is homogeneous of degree one in inputs prices (see section 2.5), expression (1) can be rewritten as:

$$(5) \qquad c^t(w^t, g^t(y^t)) = g^t(y^t)^{v^t} \phi^t(w^t) = \phi^t(w^t_1 \, g^t(y^t)^{v^t}, \, \ldots, \, w^t_p \, g^t(y^t)^{v^t})$$

If we allow the v^t's corresponding to different inputs to differ, we have:

$$(6) \qquad c^t(w^t, g^t(y^t)) = \phi^t(w^t_1 \, g^t(y^t)^{v^t_1}, \, \ldots, \, w^t_p \, g^t(y^t)^{v^t_p}) = \phi^t(h^t_1, \, \ldots, \, h^t_p)$$

Except in certain degenerate special cases, the technology corresponding to specification (6) is non-homothetic (Pollak and Wales, 1992, p. 213)[3]. It follows from (6) that the marginal and average costs are:

$$(7) \qquad MC_t = \frac{\partial c^t(w^t, g^t(y^t))}{\partial g^t(y^t)} = \frac{\sum_{i=1}^{p} x^t_i \, w^t_i \, v^t_i}{g^t(y^t)}$$

Proof:

$$\frac{\partial c^t(w^t, g^t(y^t))}{\partial g^t(y^t)} = \sum_{i=1}^{p} \frac{\partial \phi^t(\cdot)}{\partial h^t_i} \frac{\partial h^t_i}{\partial g^t(y^t)} = \sum_{i=1}^{p} \frac{\partial \phi^t(\cdot)}{\partial h^t_i} w^t_i \, v^t_i \, g^t(y^t)^{v^t_i - 1}.$$ Moreover, by

the *Lemma* of Shephard $\dfrac{\partial c^t(w^t, g^t(y^t))}{\partial w^t_i} = x^t_i = \dfrac{\partial \phi^t(\cdot)}{\partial h^t_i} \dfrac{\partial h^t_i}{\partial w^t_i} = \dfrac{\partial \phi^t(\cdot)}{\partial h^t_i} g^t(y^t)^{v^t_i}.$ Thus

$$MC_t = \frac{\sum_{i=1}^{p} x^t_i \, w^t_i \, v^t_i}{g^t(y^t)} \cdot \blacksquare$$

$$(8) \qquad AC_t = \frac{c^t(w^t, g^t(y^t))}{g^t(y^t)} = \frac{\sum_{i=1}^{p} x^t_i \, w^t_i}{g^t(y^t)}$$

From (7) and (8) it can be shown that marginal and average costs are related in the following manner:

$$(9) \qquad MC_t - AC_t = \frac{\sum_{i=1}^{p} x^t_i \, w^t_i (v^t_i - 1)}{g^t(y^t)}$$

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[3] Pollak and Wales (1992, p. 219) show that the Cobb-Douglas technology is a degenerative case, in which distinct v^t's imply a homogeneous technology, i.e., a homothetic technology.

When the technology is homogeneous, $v_i^t = v^t$ ($i = 1, ..., p$). In this case, expression (9) results in expression (4), case in which the marginal and average costs are exactly related by the degree of homogeneity of the cost function.

From (9), it can be shown that $x_i^t w_i^t (v_i^t - 1) > 0$, if $v_i^t > 1$ and $x_i^t w_i^t (v_i^t - 1) < 0$, if $v_i^t < 1$. This attribute is not observable in the context of any non-linear technology. It is observable, however, in the context of non-homothetic technologies, since, here, some inputs might increase more than proportionally while others might increase less than proportionally, when output increases. Finally, it is interesting to observe that this specific characteristic might reduce the difference between marginal and average costs (as it reduces the absolute value of the numerator of expression (9)). However, note that because the ratios $x_i^t / g^t(y^t)$ change with output so does the difference between marginal and average costs, i.e., the value of expression (9) depends on the level of output[4].

4.4. The Non-Homothetic Leontief Cost Function

In this section we apply the procedure introduced in the previous section to the Leontief technology. Two reasons support the choice of this technology. Firstly, it constitutes a natural extension of the linearly homogeneous Leontief technology that has been widely used in the management accounting literature (e.g. Balakrishnan and

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[4] Since $\dfrac{x_i^t}{g^t(y^t)} = \dfrac{\partial \phi^t(\cdot)}{\partial h_i^t} g^t(y^t)^{v_i^t - 1}$.

Sivaramakrishnan, 2002). Secondly, it is a relatively straightforward matter to represent it in terms of the duality between costs and technology.

The non-homothetic Leontief technology for activity t can be represented as follows:

(10) $\qquad g^t(y^t) = \min (\alpha^t_1 x^t_1{}^{\beta^t_1}, \ldots, \alpha^t_p x^t_p{}^{\beta^t_p})$

At the same time, the minimum cost of producing a given level of output implies that:

(11) $\qquad g^t(y^t) = \alpha^t_i x^t_i{}^{\beta^t_i} \quad \text{or} \quad x^t_i = \left(\dfrac{g^t(y^t)}{\alpha^t_i}\right)^{1/\beta^t_i} = \left(\dfrac{g^t(y^t)}{\alpha^t_i}\right)^{v^t_i}$

Observe that the v^t_i's corresponding to different inputs are different. The essence of the non-homothetic feature lies precisely here. It can be shown that v^t_i is the elasticity of input demand with respect to output, that is, the ratio of the relative change in the demand of input i to the relative change in output, ceteris paribus:

(12) $\qquad \in_{D,O} = \dfrac{dx^t_i/x^t_i}{dg^t(y^t)/g^t(y^t)} = v^t_i$

Proof:

$$\in_{D,O} = \frac{dx^t_i}{dg^t(y^t)} \frac{g^t(y^t)}{x^t_i} = v^t_i \left(\frac{g^t(y^t)}{\alpha^t_i}\right)^{v^t_i} \frac{1}{g^t(y^t)} \frac{g^t(y^t)}{\left(\frac{g^t(y^t)}{\alpha^t_i}\right)^{v^t_i}} = v^t_i. \blacksquare$$

The cost function for the non-homothetic Leontief technology is given by:

$$(13) \qquad c^t(w^t, g^t(y^t)) = \sum_{i=1}^p x_i^t w_i^t = \sum_{i=1}^p \left(\frac{g^t(y^t)}{\alpha_i^t}\right)^{v_i^t} w_i^t$$

From (13), it can be seen that the cost function is not multiplicatively separable in input prices and output, as when the technology is homothetic. The marginal and average costs are:

$$(14) \qquad MC_t = \frac{\partial c^t(w^t, g^t(y^t))}{\partial g^t(y^t)} = \sum_{i=1}^p v_i^t \left(\frac{g^t(y^t)}{\alpha_i^t}\right)^{v_i^t} \frac{w_i^t}{g^t(y^t)}$$

$$(15) \qquad AC_t = \frac{c^t(w^t, g^t(y^t))}{g^t(y^t)} = \sum_{i=1}^p \left(\frac{g^t(y^t)}{\alpha_i^t}\right)^{v_i^t} \frac{w_i^t}{g^t(y^t)}$$

At the same time, the difference between marginal and average costs is given by:

$$(16) \qquad MC_t - AC_t = \sum_{i=1}^p (v_i^t - 1) \left(\frac{g^t(y^t)}{\alpha_i^t}\right)^{v_i^t} \frac{w_i^t}{g^t(y^t)}$$

As previously observed, the central feature of a non-homothetic technology is the fact that the ratio between pair of inputs changes with the level of output. Thus we can make use of the concept of elasticity of input substitution with respect to output to ascertain how the ratio between pair of inputs varies with output. If we apply it, we obtain:

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$$(17) \qquad \in_{S,O} = \frac{d(x_i^t/x_u^t)/(x_i^t/x_u^t)}{dg^t(y^t)/g^t(y^t)} = v_i^t - v_u^t$$

Proof:

Note first that $\dfrac{x_i^t}{x_u^t} = \left(\dfrac{1}{\alpha_i^t}\right)^{v_i^t} \left(\dfrac{1}{\alpha_u^t}\right)^{-v_u^t} g^t(y^t)^{v_i^t - v_u^t}$. Thus $\in_{S,O} = \dfrac{d(x_i^t/x_u^t)}{dg^t(y^t)} \dfrac{g^t(y^t)}{x_i^t/x_u^t} =$

$= \left(\dfrac{1}{\alpha_i^t}\right)^{v_i^t} \left(\dfrac{1}{\alpha_u^t}\right)^{-v_u^t} (v_i^t - v_u^t)\, g^t(y^t)^{v_i^t - v_u^t - 1} g^t(y^t) \left(\dfrac{1}{\alpha_i^t}\right)^{-v_i^t} \left(\dfrac{1}{\alpha_u^t}\right)^{v_u^t} g^t(y^t)^{-v_i^t + v_u^t} =$

$= v_i^t - v_u^t.$ ∎

The sign of $v_i^t - v_u^t$ indicates if the technology is input i or input u biased. That is to say, it biased in relation to the input which increases by a relative by large amount when output increases. If $v_i^t > v_u^t$ the technology is input i biased (input i increases relatively by more than input u when output increases). If $v_i^t < v_u^t$, the technology is input u biased. If $v_i^t = v_u^t$ the quotient between pair of inputs is constant (the elasticity of input substitution is zero), just as it is when the technology is homothetic. In the next section, a formal model is specified, in order to provide a basis for the evaluation of the implications of the technology characteristics discussed so far.

4.5. Model

Consider a setting where m products are produced. Production only takes place when a new order arrives, since there is no production for stock. Additionally, orders are processed according to a FIFO discipline. The production process encompasses three sequential activities. The first activity, A_1, aggregates two inputs (x^1_1 and x^1_2, with prices w^1_1 and w^1_2, respectively) and is a typical batch activity, where the number of set-ups of each of the m products is the (non-final good) output. The second activity, A_2, corresponds to the incorporation of a single direct input (x^2_1, with price w^2_1) in each of the individual units of the m products. This is the case where inputs are measured individually and, as such, no aggregation of inputs occurs. The third activity, A_3, aggregates two inputs (x^3_1 and x^3_2, with prices w^3_1 and w^3_2, respectively) and is also final good output driven.

The vector of outputs is represented as (q^r_1, ..., q^r_m), where q^r_j is the volume of output r associated with cost object j (see section 2.6). In the model, (q^1_1, ..., q^1_m) is the vector of final good outputs, where q^1_j is the number of units of product j. This vector drives the consumption of resources in activities A_2 and A_3. In the case of activity A_1, (q^2_1, ..., q^2_m) is the corresponding vector of outputs. Here, q^2_j represents the number of set-ups of product j[5]. As previously defined, $y^{t,j}$ denotes the total units of cost driver used by cost object j at activity t. This variable can be expressed as a function of q^r_j. Accordingly, $y^{t,j} = q^r_j a^t_j$, where a^t_j is the units of cost driver used by

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[5] Implied in this analysis is the fact that the total number of set-ups is separable among the m products, i.e., each set-up is associated with one and only one product. If a set-up involves, for example, two products (which might be represented by $q^2_{j,k}$, where $j \neq k$), its cost is not separable between them. In this case, we have an inherently joint cost.

one unit of product j, in activities A_2 and A_3 $(g^t(y^t) = \sum_{j=1}^{m} q^1_j a^t_j, \ t = 2, 3)$, or by one

set-up of product j, in activity A_1. In this last case, $a^1_j = 1$, as the activity output is

exactly the total number of set-ups $(g^1(y^1) = \sum_{j=1}^{m} q^2_j a^1_j = \sum_{j=1}^{m} q^2_j)$. In activity A_2, a^2_j

is the units of input 1 per unit of product j.

Now the question of technology can be considered. Datar and Gupta (1994) in their study of aggregation, specification and measurement errors when determining product costs have explicitly concentrated on linear cost functions. Aggregation errors occur when costs caused by different cost drivers are aggregated in a single cost pool. Specification errors are caused by inexactitudes in cause-and-effect relationships and result from choosing the wrong cost driver[6]. In practice, aggregation and specification errors are correlated. If an activity wrongly includes costs from other activities then we observe simultaneously both aggregation and specification errors. At the same time, if the dependent and independent variables (activity cost and activity cost driver) are "correctly" defined, there are no aggregation and/or specification errors. In the context of our model, we assume the inexistence of aggregation and specification errors. We also consider that the dependent and independent variables are measured without error. We assume the inexistence of aggregation, specification and measurement errors in order to

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[6] These two definitions implicitly assume that it is possible to design a cost system in such a way that for each cost pool there is one and only one cost driver. Both conventional and ABC systems are based on this assumption. Note also that, in this section, it is being explicitly assumed that each activity depends on one and only one cost driver (a single intermediate input). This might not be the case, however. As was shown in the last chapter, even assuming that technologies are linearly homogeneous, it might be the case that an activity depends on more than one cost driver (see *Corollary 2*, section 3.2). If this is the case, a specification error occurs if we represent the activity cost function as depending on only one cost driver.

introduce and isolate what can be called a functional form error. The is because even when the dependent and independent variables are "correctly" defined, the functional form relating one with the other might be not. For example, the usual linearity assumption might simply prove to be invalid. Normal cost accounting procedures may calculate a cost driver rate that is invariant with the activity output. Consequently, if the functional form underlying an activity production function is not linearly homogeneous then a difference will exist between marginal and average costs so implying that cost accounting numbers do not measure incremental costs.

The functional form feature is specifically analysed in the context of the activity A_3, where we simulate two types of non-linearly homogeneous technologies: the homogeneous and the non-homothetic Leontief technology (see expression (13)). Concerning activities A_1 and A_2, we assume the usual linearly homogeneous Leontief structure. Considering earlier definitions, the cost driver rates for the three activities are determined as follows:

Table 1 – Average Cost Driver Rates

Activity A_1	Activity A_2	Activity A_3
$\phi^1(w^1) = \dfrac{x^1_1 w^1_1 + x^1_2 w^1_2}{\sum_{j=1}^{m} q^2_j}$	$\phi^2(w^2) = \dfrac{x^2_1 w^2_1}{\sum_{j=1}^{m} q^1_j a^2_j} = w^2_1 \ ^*$	$\phi^3(w^3, g^3(y^3)) = \dfrac{x^3_1 w^3_1 + x^3_2 w^3_2}{\sum_{j=1}^{m} q^1_j a^3_j}$

* Since $g^2(y^2) = \sum_{j=1}^{m} q^1_j a^2_j \equiv x^2_1$

Later we will discuss in more detail the determination of the cost driver rates associated with activities A_1 and A_3. For now, note only that while the average cost driver rates for activities A_1 and A_2 are constant ($\phi^1(w^1)$ and $\phi^2(w^2)$, respectively –

given the assumed linearly homogeneous structure), the average cost driver rate for activity A_3 is not constant, as it depends on the level of output ($\phi^3(w^3, g^3(y^3))$).

The question that we should now direct attention to concerns the incremental cost of a new order. To undertake this analysis, consider that a given order increases the output of activity t from a^t to b^t. Therefore, its incremental cost is:

$$(18) \qquad \int_{a^1}^{b^1} \frac{\partial c^1(w^1, g^1(y^1))}{\partial g^1(y^1)}\, dg^1(y^1) + \int_{a^2}^{b^2} \frac{\partial c^2(w^2, g^2(y^2))}{\partial g^2(y^2)}\, dg^2(y^2) +$$

$$+ \int_{a^3}^{b^3} \frac{\partial c^3(w^3, g^3(y^3))}{\partial g^3(y^3)}\, dg^3(y^3)$$

After simplification, expression (18) results in:

$$(19) \qquad \phi^1(w^1) + w^2{}_1 (b^2 - a^2) + \sum_{i=1}^{2} \left(\frac{b^3}{\alpha^3{}_i}\right)^{v^3{}_i} w^3{}_i - \sum_{i=1}^{2} \left(\frac{a^3}{\alpha^3{}_i}\right)^{v^3{}_i} w^3{}_i$$

Proof:

$$\int_{a^1}^{b^1} \frac{\partial c^1(w^1, g^1(y^1))}{\partial g^1(y^1)}\, dg^1(y^1) + \int_{a^2}^{b^2} \frac{\partial c^2(w^2, g^2(y^2))}{\partial g^2(y^2)}\, dg^2(y^2) + \int_{a^3}^{b^3} \frac{\partial c^3(w^3, g^3(y^3))}{\partial g^3(y^3)}\, dg^3(y^3) =$$

$$= \int_{a^1}^{b^1} \phi^1(w^1)\, dg^1(y^1) + \int_{a^2}^{b^2} \phi^2(w^2)\, dg^2(y^2) + \int_{a^3}^{b^3} v^3{}_1 \left(\frac{g^3(y^3)}{\alpha^3{}_1}\right)^{v^3{}_1} \frac{w^3{}_1}{g^3(y^3)}\, dg^3(y^3) +$$

$$+ \int_{a^3}^{b^3} v^3{}_2 \left(\frac{g^3(y^3)}{\alpha^3{}_2}\right)^{v^3{}_2} \frac{w^3{}_2}{g^3(y^3)}\, dg^3(y^3) =$$

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$$= [\phi^1(w^1)\, g^1(y^1)]_{a^1}^{b^1} + [w^2{}_1\, g^2(y^2)]_{a^2}^{b^2} + \left(\left(\frac{g^3(y^3)}{\alpha^3{}_1}\right)^{v^3{}_1} w^3{}_1\right)_{a^3}^{b^3} + \left(\left(\frac{g^3(y^3)}{\alpha^3{}_2}\right)^{v^3{}_2} w^3{}_2\right)_{a^3}^{b^3} =$$

$$= \phi^1(w^1) + w^2{}_1\,(b^2 - a^2) + \sum_{i=1}^{2}\left(\frac{b^3}{\alpha^3{}_i}\right)^{v^3{}_i} w^3{}_i - \sum_{i=1}^{2}\left(\frac{a^3}{\alpha^3{}_i}\right)^{v^3{}_i} w^3{}_i$$

Observe that an order gives rise to one set-up, which increases the output of activity A_1 from a^1 to $b^1 = a^1 + 1$ ($b^1 - a^1 = 1$). Note also that while the marginal costs at activities 1 and 2 are constant (given the assumed linearly homogeneous structure), the marginal cost at activity 3 (which is derived directly form expression (14)) is not constant. ∎

Conventional cost accounting practice, based on an average cost driver rate, "correctly" identifies the incremental cost at activities A_1 and A_2. This is patent in the first two terms of expression (19), where the incremental cost at activities A_1 and A_2 is simply the average cost driver rate times the increase in the activity output ($b^t - a^t$). This is not the case with activity A_3, however. Here, the difference between the activity's incremental cost (third and fourth terms of expression (19)), and the cost based on an average cost driver rate, $\phi^3(w^3, g^3(y^3))$, might be represented as:

$$(20) \qquad \sum_{i=1}^{2}\left(\frac{b^3}{\alpha^3{}_i}\right)^{v^3{}_i} w^3{}_i - \sum_{i=1}^{2}\left(\frac{a^3}{\alpha^3{}_i}\right)^{v^3{}_i} w^3{}_i - \phi^3(w^3, g^3(y^3))\,(b^3 - a^3)$$

The fact that the technology associated with activity A_3 is not linearly homogenous, either it is homogeneous of a degree different from 1 or it is non-homothetic, signifies that the average cost driver rate depends on the level of output considered,

$\phi^3(w^3, g^3(y^3))$. Conventional cost accounting procedures suggest that the determination of $\phi^3(w^3, g^3(y^3))$ is derived from a mean activity output (per period), which depends itself on the mean demand of each of the m products. Accordingly, $E(g^3(y^3)) = \sum_{j=1}^{m} E(q^1_j) a^3_j$, where $E(g^3(y^3))$ is the mean activity output (per period), $E(q^1_j)$ the mean demand for product j (per period) and a^3_j, as already defined, the units of cost driver used per unit of product j. Specific simulations are undertaken in the next section.

4.6. Simulations

For simulation purposes, we consider the existence of only two products, P_1 and P_2. The specific parameters assumed for each of the three activities are presented in table 2.

Table 2 – Activity Parameters

Activity A_1		Activity A_2		Activity A_3	
$1/\alpha^1_1$	$1/\alpha^1_2$	-		$1/\alpha^3_1$	$1/\alpha^3_2$
2	1	-		4	2
v^1_1	v^1_2	-		v^3_1	v^3_2
1	1	-		3 technologies (*)	
w^1_1	w^1_2	w^2_1		w^3_1	w^3_2
£2	£3	£3		£4	£2
a^1_1	a^1_2	a^2_1	a^2_2	a^3_1	a^3_2
1	1	2	3	1	2

(*) T_1: ($v^3_1 = 0{,}8$; $v^3_2 = 0{,}8$); T_2: ($v^3_1 = 1{,}2$; $v^3_2 = 1{,}2$); T_3: ($v^3_1 = 0{,}8$; $v^3_2 = 1{,}2$)

The linearly homogeneous Leontief structure of activity A_1 implies that the cost driver rate is $\phi^1(w^1) = \sum_{i=1}^2 \frac{w_i^t}{\alpha_i^t} = £2 \times 2 + £3 \times 1 = £7$ [7]. This is exactly the cost per set-up. The cost driver rate for activity A_2 is $\phi^2(w^2) = w_1^2 = £3$.

With respect to activity A_3, we simulate three alternative technologies, as stated in table 2. Technology T_1 gives rise to a cost function homogeneous of degree 0,8 in output, while T_2 supports a cost function homogeneous of degree 1,2. In the first case the marginal cost is a monotonically decreasing function, whereas in the second it is a monotonically increasing function. Technology T_3 is non-homothetic, since $v_1^3 \neq v_2^3$. In activity A_3, we compute a cost driver rate assuming a mean activity output (per period) of 14 units of cost driver. Therefore, the cost driver rate $\phi^3(w^3, g^3(y^3))$ presented in table 1 is determined as follows: $\phi^3(w^3, g^3(y^3)) = \sum_{i=1}^2 \left(\frac{g^t(y^t)}{\alpha_i^t}\right)^{v_i^t} \frac{w_i^t}{g^t(y^t)} = \frac{£4}{14}(4 \times 14)^{v_1^3} + \frac{£2}{14}(2 \times 14)^{v_2^3}$ (see expression (15)). Direct calculations show that $\phi^3(w^3, g^3(y^3)) = £9,2$ $(T_1: (v_1^3 = 0,8; v_2^3 = 0,8))$, $\phi^3(w^3, g^3(y^3)) = £43,6$ $(T_2: (v_1^3 = 1,2; v_2^3 = 1,2))$ and $\phi^3(w^3, g^3(y^3)) = £14,9$ $(T_3: (v_1^3 = 0,8; v_2^3 = 1,2))$. Table 3 provides information concerning the (cumulative) output of the three activities in a given period.

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[7] $\phi^1(w^1)$ is derived directly from either expression (14) (or expression (15)), noting that $v_1^1 = v_2^1 = 1$.

Table 3 – Activity Output

Product	Order Number	Units per Order	Activity A_1 (q^2_i)	Activity A_2 $(q^1_i a^2_i)$	Activity A_3 $(q^1_i a^3_i)$
P_1	1º	1	1	1×2	1×1
P_2	2º	4	$2\,(1+1)$	$14\,(2+4\times3)$	$9\,(1+4\times2)$
P_1	3º	3	$3\,(2+1)$	$20\,(14+3\times2)$	$12\,(9+3\times1)$
P_2	4º	1	$4\,(3+1)$	$23\,(20+1\times3)$	$14\,(12+1\times2)$
P_2	5º	2	$5\,(4+1)$	$29\,(23+2\times3)$	$18\,(14+2\times2)$

Table 3 should be read in the following way. The 2º order of the period (4 units of P_2) increased the output of activity A_1 from 1 to 2 (since it gave rise to one set-up), the output of activity A_2 from 2 to 14 ($2+4\times3$) and the output of activity A_3 from 1 to 9 ($1+4\times2$). The remainder of table 3 should be read in the same fashion.

The objective of the simulations is the evaluation of the distortions arising from the use of an accounting procedure that computes and utilises an average cost driver rate. As has been observed, distortions arise whenever cost functions are not linear with output. In the model, the total cost error, the difference between the incremental cost of an order and the cost that is based on an average cost driver rate, is given by expression (20). Bearing in mind that expression (19) is the incremental cost of an order, the percentage cost error is defined as the quotient of expression (20) to expression (19).

The following chart shows the percentage cost error for the five orders presented in table 3, under the three alternative technologies postulated for activity A_3.

Chart 1 – Percentage Cost Error under Three Alternative Technologies

Series1: $T_1(v^3_1 = 0,8; v^3_2 = 0,8)$

Series2: $T_2(v^3_1 = 1,2; v^3_2 = 1,2)$

Series3: $T_3(v^3_1 = 0,8; v^3_2 = 1,2)$
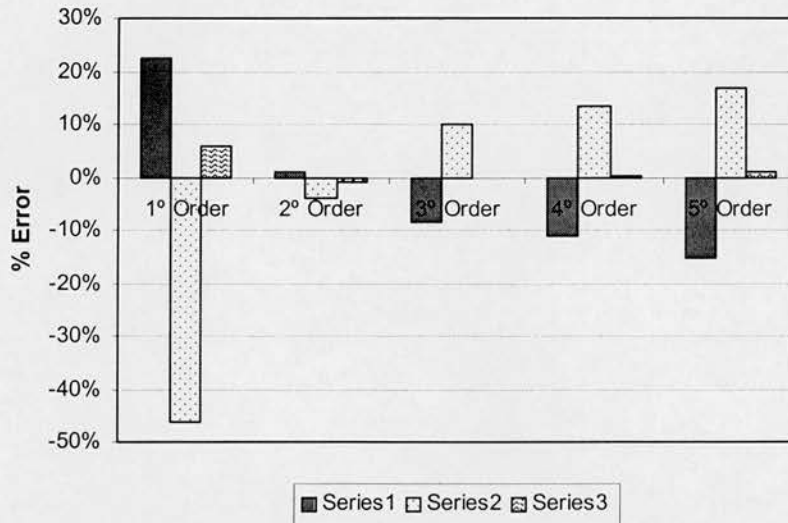
Series1 corresponds to a cost function homogeneous of degree 0,8 in output, implying that the marginal cost strictly decreases with output (and the percentage cost error as well). Observe that the percentage error for the first order is around + 22% (the incremental cost of the 1° order is higher than the cost allocated to the same order), while for the fifth order is almost –15% (the incremental cost for the 5° order is lower than the cost allocated to the same order). Chart 2 helps to visualize how the total cost error changes with the activity output, for this technology.

Chart 2 – Activity A_3: T_1 (v^3_1 = 0,8; v^3_2 = 0,8)

Marginal Cost versus Average Cost Driver Rate (£9,2)
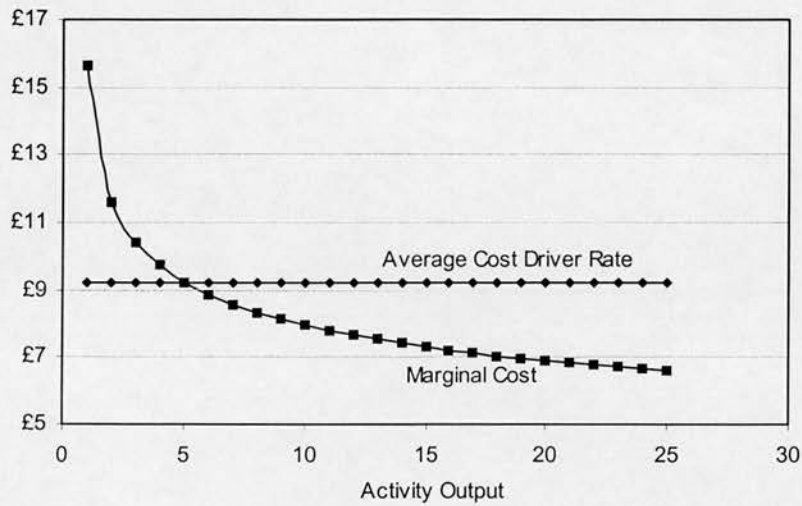
According to table 3, the first order increases the output of activity A_3 from 0 to 1. Graphically, the total cost error is the cumulative difference between the marginal cost curve and the average cost driver rate line, when the output changes from 0 to 1. The second order increases the activity output from 1 to 9. Observe that when the output increases from 1 to 5 the cumulative difference between the marginal cost curve and the average cost driver rate line is positive (as the marginal cost is higher than the average cost driver rate). However, that difference becomes negative when the output is higher than 5. So the fact that the cumulative difference between the marginal cost and the average cost driver rate is positive when the activity output goes from 1 to 5 while negative when it goes from 5 to 9 explains the low percentage cost error for the second order (around 1%).

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Series2 of chart 1 characterizes the percentage cost error for a cost function homogeneous of degree 1,2 in output. In this case, the marginal cost strictly increases with output (and the percentage cost error as well). Chart 3 helps to visualize this.

Chart 3 – Activity A_3: T_2 (v^3_1 = 1,2; v^3_2 = 1,2)
Marginal Cost versus Average Cost Driver Rate (£43,6)
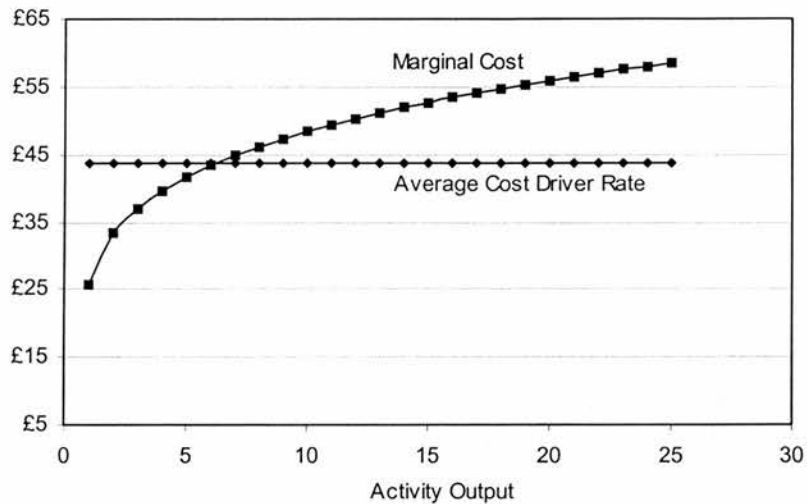
Among the three alternative technologies under consideration, the non-homothetic technology (T_3) gives rise, overall, to a smaller percentage error (see chart 1). Actually, the percentage error varies from 6% (1° order) to just about 1% (5° order). The explanation of this fact lies again in the curvature of the marginal cost, which is presented in chart 4.
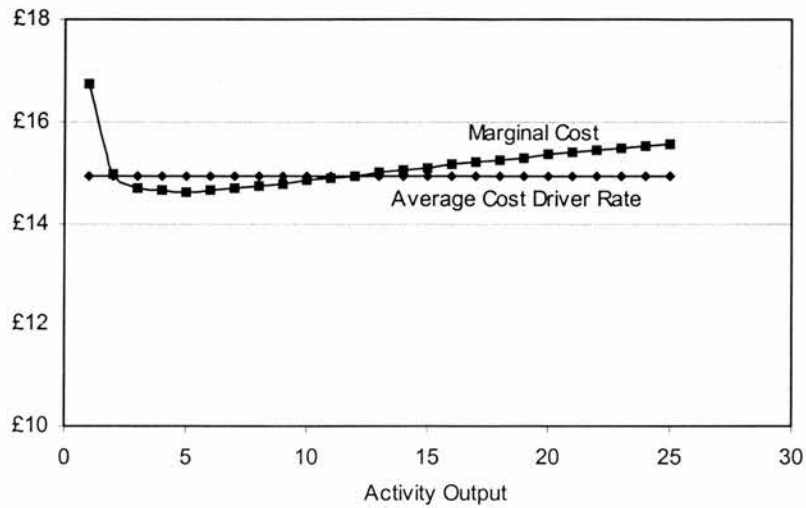
Observe that the marginal cost decreases between 0 and approximately 5, but increases in a very smooth way after that level of output. At the same time, the cumulative difference between the marginal cost curve and the average cost driver line is very low, which explains the small percentage error across the five orders. In this particular case, a cost accounting system founded on an average cost driver rate does not distort to a great extent incremental costs.

The magnitude of the distortion is deeply associated with the parameters $v^3{}_1$ and $v^3{}_2$ of the cost function. When both converge to one the percentage error tends to zero, since that results in a cost function linear with output, where average and marginal costs coincide. The reverse is also true, i.e., the magnitude of the distortion increases as the parameters $v^3{}_1$ and $v^3{}_2$ diverge from one.

The two following charts show how the percentage cost error for the first two orders changes when v^3_1 and v^3_2 vary from 0,5 to 1,5 (appendix II shows the charts for the remaining orders). Both charts are cut with a plane parallel to the $v^3_1 v^3_2$ plane, corresponding to the set of points (v^3_1, v^3_2) for which the percentage cost error for each order is zero.

Chart 5 – Percentage Cost Error for the 1° and 2° Orders

$$(0,5 \leq v^3_i \leq 1,5)$$

$$x \equiv v^3_1 \text{ and } y \equiv v^3_2$$
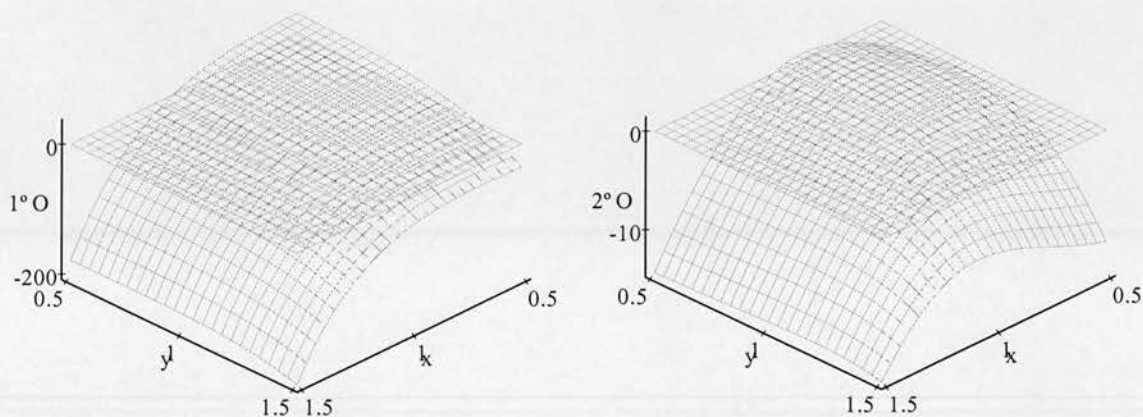
As already noted, when the cost function is not linear with output the percentage error changes from one order to another, that is to say, the percentage error depends on the level of output. However, a common attribute of both figures is the fact that each of them converges to zero when the elasticity of each input demand tends to one (since this results in a cost function linear with output).

4.7. Conclusions

This chapter has concentrated on the relaxation of one of the most important properties underlying the structure of an ABC system: the linear homogeneity property of cost object production functions. It was assumed that all cost object technologies were identical. More specifically, it was explicitly supposed that the activity output was an intermediate input used by the various cost objects.

It was considered that aggregation, specification or measurement errors, in the terms they have been defined by Datar and Gupta (1994), were absent. However, another type of error, designated as functional form error, was introduced. This type of error was identified from the investigation of the distortions arising from the use of an accounting procedure based on an average cost driver rate, in a context where cost functions were not linear with output. On the whole, distortions increase as the elasticity of each input demand diverges from one. That is, distortions increase as the input-output relationships depart from the linear case. But distortions also change with output. This means that, depending on the output interval, the application of a single average cost driver rate might undercost, overcost or even approximate incremental costs.

The above results demonstrate that a mathematical analysis can be applied to facilitate the assessment of the decision suitability of cost systems information outputs. They show that the distortions arising from the adoption of cost functions linear with output can be identified quantitatively. This contributes to the

ascertainment of the costs and benefits of costing systems and is one aspect that should be given attention when designing a new cost system or evaluating an existing one.

Appendix II

Percentage Cost Error for the 3°, 4° and 5° Orders (Section 4.6)

$$(0,5 \leq v^3_i \leq 1,5)$$

$$x \equiv v^3_1 \text{ and } y \equiv v^3_2$$
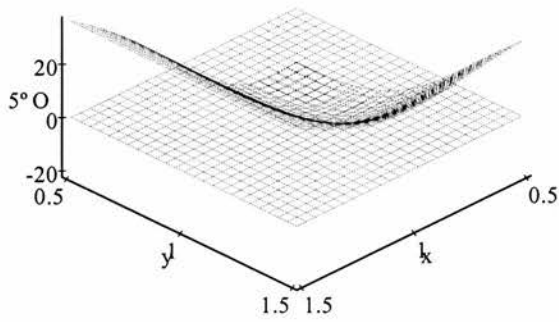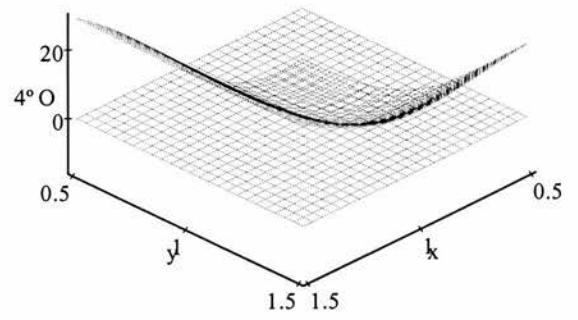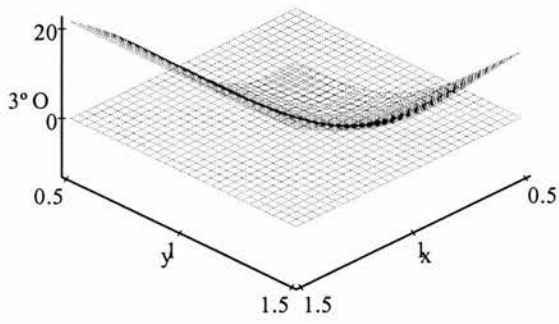
CHAPTER V – EMPIRICAL ISSUES

5.1. Introduction

The purpose of this chapter is to address some issues that the empirical work in the area of product costing poses. It should be recognised that, in general, it is quite challenging to develop credible empirical work in this area. The fact that the researcher has usually little control over variables that might interfere directly on the validity of the results creates the challenge. The objective of this chapter is to identify and review these variables.

Although the motivation of this chapter is not to justify the choice of the research method used later (next chapter provides a justification), the difficulties that the empirical work poses also serves to explain, together with the specific purposes of this study, the use of simulation as a research method in chapter seven.

The main difficulties raised by empirical work are caused by the fact that production and cost functions are not usually known in practice. Section 5.2 analyses how researchers in the area of product costing have dealt with this difficulty. But, even if production and cost relationships are known, there might exist difficulties related to the measurement of the variables. With respect to this point, particularly problematic is the case of the, so-called, committed resources, where large differences can exist between the cost of resources used and the cost of resources supplied (Cooper and Kaplan, 1992). Section 5.3 concentrates on the difficulties that this poses in terms of

the statistical analysis of ABC data. Sections 5.4 and 5.5 examine two further difficulties which, unless their effects are adequately controlled, would seriously reduce the validity of the statistical analysis of ABC data. Specifically, section 5.4 focuses on the problem of input prices change, while section 5.5 analyses the problem of technical change. Finally, section 5.6 presents the conclusions.

5.2. Functional Form Specification

Since the emergence of ABC, the investigation of the factors that cause overhead costs has attracted a great deal of attention on the part of management accounting researchers. Particular attention has been given to the role of non-volume or complexity variables in explaining overhead costs (see section 2.6).

Without *a priori* any specific knowledge concerning the technology and (given the duality between cost and technology) the cost functions driving overhead costs, most of these studies postulate a given functional form. The choice of a particular type of cost function might reflect the purpose of the research, data issues, such as sample size, among other factors. With regard to the factors supporting the choice of the functional form it is worth considering here two or three examples from the literature.

Banker *et al* (1995) specify a (log) linear cost function, with a Cobb-Douglas structure. They note that:

> "...this specification is not intended to represent an economic cost function reflecting optimal allocation of resources given prices ... we choose this form because it is parsimonious, requiring fewest number of parameters to be estimated given the volume and transactional variables, thus preserving the degrees of freedom for our sample of only 32 plants" (Banker *et al*, 1995, p. 123).

Noreen and Soderstrom (1994, 1997) specifically test whether overhead costs were proportional to activity output, an implicit assumption of conventional cost

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accounting procedures, which implies the equality between marginal and average costs. They assume a log (linear) cost function, consistent with a Cobb-Douglas structure. They observe that:

> "The log transformation is not entirely *ad hoc* in the context of estimating cost functions; it has a long tradition in economics and, as previously noted, is consistent with a Cobb-Douglas production function" (Noreen and Soderstrom, 1994, p. 265).

Ittner *et al* (1997) use principal component analysis to investigate to what extent a wide variety of manufacturing measures were associated with the different levels of the ABC cost hierarchy (unit, batch, product and facility). Here, the absence of knowledge regarding the structure of cost functions supported the use of a procedure that sought to associate the different manufacturing measures with the ABC cost hierarchy levels.

Overall, these studies are important for driver identification, as they provide evidence concerning the effect of different drivers on overhead costs. Testing the two basic properties supporting an ABC system (the linear homogeneity property associated with each cost object production function and the fact that the marginal cost corresponding to a unit of cost driver used by a cost object is equal for all cost objects within a cost pool) requires, however, finer specification and, most importantly, much more detailed data, which hardly ever is available in practice. We will return to the issue of testing ABC in chapter seven. Specifically, the question of testing the second condition supporting the construction of an aggregate activity

output (the condition under which the marginal cost of a unit of cost driver used by a cost object is equal for all cost objects within a cost pool) will be addressed.

5.3. Resources Supplied versus Resources Used

The problem analysed here concerns the measurement of the cost of resources used. Financial systems measure the cost of resources supplied. ABC systems, however, measure the cost of resources used. The problem of measuring the cost of resources used is particularly problematic in the case of the committed resources (see Cooper and Kaplan. 1992, 1998), where large differences can exist between the cost of resources supplied and the cost of resources used[1]. Under these circumstances, and unless the unused capacity can be identified and measured, simply regressing the cost of resources supplied on measures of the activity output will not produce reliable statistical evidence on cost causality.

To overcome the difficulties in measuring the cost of resources used, researchers cannot rely on readily available data. As Kaplan (1993, p. 3) observes:

> "Instead of using readily available data, the researcher can look closely at internal company events and data so that the unused and even overused capacity for individual resources and activities can be identified and measured". Alternatively, "They (researchers) can choose organizations where the cost of unused capacity are not likely to be high, because of high growth situations"

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[1] Committed resources are acquired before they are used. The expenses of supplying these resources are incurred whether the resources are used or not. In contrast, flexible resources are acquired as needed.

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So the key aspect of the research process is the analysis of the organization context and careful construction of data. Only then it will be possible to undertake credible statistical analysis and inference. For example, it was the analysis of the organizational context, specifically interviews with people, that gave Foster and Gupta (1990) insights into the process of understanding why they found a limited empirical support for complexity-based and efficiency-based drivers in explaining overhead costs behaviour in thirty-seven facilities of an electronics company (see section 2.6).

5.4. Input Prices Change

Unless production and cost relationships are known, testing empirically the two basic properties underlying an ABC system requires that it is necessary to control for the effect of input prices change. Suppose that cost object technologies are both linearly homogeneous and identical. Further, consider that $C^t_{(n)}$ is the total cost at activity t in period n and $x^t_{u\,(n)} = \sum_{j=1}^{m} x^t_{u,j\,(n)}$ the quantity of input u used at the same activity in period n. The average and marginal cost driver rate (when input u is used as a measure of the activity output) might then be obtained if the following specification is estimated:

(1) $\qquad C^t_{(n)} = \delta^t + \phi^t_u\, x^t_{u\,(n)} + \varepsilon^t_{(n)}$

Where δ^t is the intercept and $\varepsilon^t_{(n)}$ a disturbance term. In the absence of fixed costs, the constant δ^t should not be statistically different from zero. Since cost object

technologies are not known (although it is assumed that they are both linearly homogeneous and identical), at best we obtain the (estimated) cost driver rate for a given input price set[2]. However, the possibility of the time-series data containing input prices change might distort the estimator of ϕ^t_u.

To sum up, if production and cost relationships are not known, the best the researcher can obtain is the (estimated) cost driver rate for a given input price set. However, if little control is exerted over the effects of input prices change, we risk estimating correlations that are entirely misleading.

5.5. Technical Change

The analysis so far has assumed that the state of technology is constant over time. It is doubtful that this is the case, however. The possibility of the time-series data containing technical change might also distort the estimator of ϕ^t_u, in the same way input prices change does.

In the real world, technical change often takes the form of a completely new technology. Supposing that $f_{(n)}(x_{(n)})$ represents the production function in period n, it might be the case that $f_{(n)}(x_{(n)})$ and $f_{(n')}(x_{(n')})$ $(n \neq n')$[3] are two distinct functional forms, where the vector of inputs $x_{(n)}$ and $x_{(n')}$ are not necessarily equal, as new

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[2] Remember that the constant (average and marginal) cost driver rate is conditional to both a given input price set and a given linearly homogeneous technology (see section 3.2).
[3] This might be seen as the production function for a given activity. However, technical change might also signify that activities are not stable over time, as new activities arise and/or old activities give rise to new ones.

inputs are also part of the innovation process. Unfortunately, treating analytically this kind of technical change is very difficult. So, although production economists have considered in the statistical analysis of time series data the evolution of technology, they have concentrated on what in the literature is referred to as disembodied technical change. A particularly popular definition is that introduced by Hicks (1963), who defined technical change in terms of the relative input utilization. More specifically, a production function is *Hicks neutral* if it can be written as (see Chambers, 1988, Chapter 6):

(2) $\qquad f(g(x), n)$

Expression (2) implies that time (represented by n) is separable from the vector of inputs (represented by x) in the production function. To see why, note that the marginal rate of technical substitution between say input i and input j ($MRTS_{i,j}$) does not depend on time since:

(3) $\qquad MRTS_{i,j} = \dfrac{\partial f(g(x), n)/\partial x_i}{\partial f(g(x), n)/\partial x_j} = \dfrac{\partial f(g(x), n)/\partial g(x)\ \partial g(x)/\partial x_i}{\partial f(g(x), n)/\partial g(x)\ \partial g(x)/\partial x_j} = \dfrac{\partial g(x)/\partial x_i}{\partial g(x)/\partial x_j}$

Expression (3) shows that the rate at which two inputs are substituted for each other is independent of time. That is, technical change might shift isoquants, but not their shape. In another way, *Hicks neutrality* does not change the degree of substitution between inputs. This is why this type of technical change is referred to as disembodied technical change.

Although analytically appealing, disembodied technical change is very limited in terms of describing many real world innovations. As observed, technical change might give rise to a completely new production function. If the time-series data contains technical change of this type we risk identifying erroneous correlations. So empirical work in the area of product costing estimation should take into consideration, apart from the problem of input prices change, the problem of technical change. Moreover, it might happen that input prices change and technical change are intertwined, as many innovations can reflect the effect of input price changes.

5.6. Conclusions

This chapter has addressed some difficulties that the empirical work in the area of product costing poses. Together they pose a significant challenge for empirical researchers testing the operation of ABC systems in the real world.

Section 5.2 has concentrated on the specification of the functional form underlying production and cost functions. Section 5.3 has discussed the difficulties associated with measuring the cost of resources used, a particularly relevant question in the case of the committed resources, where significant differences can exist between the cost of resources used and the cost of resources supplied. Sections 5.4 and 5.5 have concentrated on the questions of input prices change and technical change.

It has been shown that special attention should be directed to the analysis and construction of time series data, before any statistical inference is undertaken. For example, if the time series data contains the effects of input prices change and/or technical change (particularly disembodied technical change), we risk identifying flawed correlations and obtaining erroneous estimators of the activity cost driver rate. Improving the cost estimation art, however, cannot be achieved without incorporating these issues into the analysis. To obtain the new real world data in a suitably adjusted form for analysis is thus a major difficulty for ABC researchers.

CHAPTER VI – RESEARCH METHOD

6.1. Introduction

The purpose of this chapter is to describe the research method that supports the analysis conducted in the next chapter. As will be shown, chapter seven explores two fundamental issues relating to the second condition supporting the construction of an aggregate activity output, i.e. the condition under which the marginal cost of a unit of cost driver used by a cost object is equal for all cost objects within a cost pool (henceforth condition (ii)). Firstly, it is necessary to deal with the question of testing the (non) existence of this important property of ABC. Secondly, it is worth investigating the possibility of designing an accounting procedure that accommodates, by some means, its non-existence. Simulations will be used as a means for addressing these two questions.

6.2. The Comparative Advantages of Simulations

Simulations as a research method have been extensively used in management science (e.g. Pidd, 1984). To simulate is to replicate or mimic the characteristics of a real system or phenomenon. Depending on the characteristics of the problem, management scientists might use simulations, direct experimentation or mathematical modelling. Developing a direct experiment on a real system to estimate, for example, the effect of various conditions is usually time-consuming and expensive to put into effect. It might, however, be possible to simulate months or

even years in a computer in order to evaluate and compare a whole range of conditions. Distinct versions of the problem can therefore be reproduced and the effects of different variables or policies analysed. A mathematical model might not always satisfactorily cope with all the features of the problem. For example, it might be necessary to impose some assumptions to make the model analytically tractable. Thus its general applicability is reduced. Alternatively, and instead of imposing additional assumptions on the model, simulations can be used in order to study its properties and estimate the optimal solution.

6.3. Simulations in Cost Accounting

Accounting researchers have usually combined mathematical modelling with simulations (Balakrishnan and Sivaramakrishnan, 2002). The mathematical model serves to derive the optimal solution, which is used as a benchmark. In practice, however, the optimal solution can be expensive to implement. This may occur because its adoption is informationally demanding. The problem is often not the impossibility of deriving analytically the optimal solution (as in many management science studies), but the fact that it is costly to operationalise it. Thus some sub-optimal or heuristic solution is used. The heuristic can be an accounting method or policy. The analytic and the heuristic solutions are then compared in order to determine the magnitude of the economic loss that results from using a non-optimal solution. Simulations are apposite in this context because the objective is to assess the robustness of various accounting procedures, which are ideally used as surrogates for the optimal solution (although not as expensive to implement as the analytic

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solution). For example, both Balakrishnan and Sivaramakrishnan (2001) and Burgstahler and Noreen (1997) use simulations to examine the economic loss resulting from the use of a full-cost-based procedure in setting product prices (see also Banker and Hansen, 2002). Banker and Potter (1993) also use simulations to investigate the economic implications of using single cost driver systems (see also Hwang *et al*, 1993).

The design of a more or less refined cost system depends on cost-benefit considerations. At one extreme, inputs can be measured individually, i.e. we can create a cost pool for each individual input and associate each of them with the various products. This would permit the calculation of the true or benchmark cost of a product. In practice, however, cost-benefit issues preclude such detailed disaggregations. This is why the aggregation of two or more inputs in a cost pool and the distribution of them between two or more products, using as an allocation base the quantity of a particular input, is a common practice. Simulations are appropriate here because the objective of the study is to determine the magnitude of the economic loss that results from the use of a particular allocation method. Additionally, the question of designing an accounting procedure that hypothetically accommodates the non-existence of condition (ii) would not be possible to address, except in rare circumstances, with real data. This is because the benchmark cost is not usually known, which makes it impossible to compute the economic loss arising from the use of a particular allocation method. This point constitutes the first major justification for the use of simulations in the next chapter. A second and natural reason is the possibility of developing controlled experiments to facilitate an

exploration of the effects of specific variables on the performance of the proposed accounting procedures.

6.4. Validity

Experiments in general, and simulation experiments in particular, are suitable when the research question involves the investigation of causal relations between variables. In these circumstances, the major advantage of developing a simulation experiment is that it increases the internal validity of the research study. The internal validity is defined in terms of how well researchers can exclude rival explanations for their results (Schulz, 1999, p. 29). Increasing the internal validity reduces, however, the external validity. The external validity should be interpreted here in the sense of the so-called *mundane realism* (Schulz, 1999, p. 30, referencing Brownell, 1995). This refers to the extent to which the experimental setting is equivalent to the real-world setting. Experiments rarely satisfy mundane realism. For example, one important assumption behind the construction of the simulation model in the next chapter is that cost measurement errors are distributed independently of the quantity of each input used by the various cost objects in a cost pool (the explanatory variables). This serves essentially to ensure that the assumption in the classical regression model of zero covariance between the disturbance term and the independent variables is satisfied (otherwise the properties of the OLS estimators do not hold true). It is, however, doubtful if this is the case in a real-world production environment. Thus some generality in the applicability of the results is lost, that is to say the external validity is reduced.

6.5. Deterministic and Stochastic Simulations

Chapter four has used deterministic simulations. Chapter seven, however, will be based on stochastic simulations. A deterministic system is one whose behaviour is entirely predictable (Pidd, 1984, p. 17-18). In the simulation model developed in chapter four the extent of the economic loss, defined as the difference between the marginal and the average cost, was calculated (taking the demand as fixed) as depending on the elasticity of each input demand with respect to output, i.e. on the specific deterministic technology. Many systems, however, are not entirely predictable. The simulations performed in chapter seven are based on two sorts of stochastic elements: the demand of each product in a given period and the technology. More precisely, the demand for each product and the quantity of each input per unit of output are random variables.

The usual steps performed in a simulation experiment in accounting research are as follows (Balakrishnan and Sivaramakrishnan, 2002, p. 23-26; see also Render *et al.*, 2003).

1. Define the mathematical model of the problem. In chapter seven, this is the analytic model linking product technologies, input prices and benchmark product costs.

2. Specify parameter values (e.g. input prices). For random variables specify probability distributions. As will be shown in the next chapter, these are the

quantity produced of each product, the quantity of each input per unit of output and the cost measurement error.

3. Generate one random number for each (random) variable in the model.

4. Determine the optimal solution (the benchmark product cost).

5. Determine the heuristic solutions (the cost allocated to a product under the proposed accounting procedures).

6. Compute the difference between the optimal solution (step 4) and the heuristics (step 5), i.e., the magnitude of the economic loss associated with each heuristic.

7. Repeat steps 3 to 6 to obtain a distribution of the economic loss of the proposed heuristics. The analysis developed in chapter seven is based on the generation of 5000 random numbers for each variable.

8. Repeat steps 2 to 7 for alternative parameter values. For example, in chapter seven, a scenario where the uncertainty in terms of input usage is uniform across all inputs will be compared with a scenario where such uniformity does not hold.

These steps are followed in this study and the next chapter will give more details of each step performed in the simulation experiment.

CHAPTER VII – SIMULATION EXPERIMENT

7.1. Introduction

It was demonstrated in chapter three that the two necessary and sufficient conditions supporting the construction of an aggregate activity output, and therefore constituting the very essence of ABC, are (i) the linear homogeneity property associated with each cost object production function and the fact that (ii) the marginal cost of a unit of cost driver used by a cost object is equal for all cost objects within a cost pool.

While chapter four concentrated on non-linear technologies, in a context where the activity output was an intermediate input used by the various cost objects, this chapter assumes the verification of condition (i) so as to specifically explore the relaxation of condition (ii). That is, a setting will be considered where cost object technologies are linearly homogeneous but where the aggregated activity cost function depends on more than one cost driver. This chapter develops a simulation experiment to explore three main issues. Firstly, it addresses the issue of testing the verification/non-verification of condition (ii). Secondly, we develop an accounting procedure that specifically accommodates the existence of multiple cost drivers, i.e. the non-verification of condition (ii). The simulation experiment serves to test the robustness of the accounting procedure proposed. Finally, the simulation experiment serves also to introduce the question of uncertainty in input usage.

The chapter is organised into four sections. Section 7.2 specifies the model that supports the simulation experiment developed in the sections 7.3 and 7.4. Section 7.3 is based on the assumption that the technology is deterministic, whereas section 7.4 is founded on the assumption that the technology is stochastic. Section 7.5 presents the conclusions.

7.2. Model Specification

7.2.1. Technology

Assume an activity that aggregates p inputs, which are separable among m products. As was previously noted, it is assumed that product technologies are linearly homogenous. Therefore, taking into consideration the analysis developed in chapter three, it can be stated that the minimum cost of producing a given output implies the following condition (see the demonstration of the *Lemma* in section 3.2):

$$(1) \qquad q_{j\,(n)} = \alpha^t_{1,j}(w^t)\, x^t_{1,j\,(n)} = ... = \alpha^t_{p,j}(w^t)\, x^t_{p,j\,(n)}$$

Where $q_{j\,(n)}$ is the quantity of product j produced in period n and $x^t_{i,j\,(n)}$ the quantity of input i used by the same product at activity t. The ratio $x^t_{i,j\,(n)}/q_{j\,(n)} = 1/\alpha^t_{i,j}(w^t)$ represents the quantity of input i used per unit of product j. If the technology is both linearly homogenous and deterministic this ratio is a constant for a given input price set. Condition (1) supposes this. While section 7.3 specifically assumes this structure, section 7.4 presumes that the quantity of input i used per unit of product j is stochastic. This reflects variations in the input usage efficiency.

7.2.2. Accounting Procedure 1 – $CS^{t,k}_{1,u\,(n)}$

One way of "correctly" distributing the cost of the p inputs among the m products when activity t depends on more than one cost driver is the creation of p cost pools,

one for each input. This corresponds to a situation where the various inputs are measured individually. In practice, however, cost-benefit issues preclude such detailed disaggregations. This is why some aggregation usually takes place. It will be assumed that only one input, say input u, is measured individually. Moreover, the total quantity of input u used by the various products at activity t will be the allocation base to distribute the cost of the other (p – 1) inputs among the m products. This is consistent with the practice of using, for example, direct labour hours or machine hours as a basis to compute an (overhead) absorption rate for a cost pool. Since the marginal cost of a unit of cost driver used by a product is not equal for all products within a cost pool, this implies that some product cost distortion will arise (see *Corollary 2*, section 3.2). For example, the (activity) cost change when one hour of labour (or one machine hour) is used by product j might be different from the (activity) cost change when one hour of labour (or one machine hour) is used by product k. That is, the hours of labour (or of machine) used by product j and the hours of labour (or of machine) used by product k might be two different cost drivers. To compute later the distortion arising from the use of the total quantity of a given input as an allocation base, first define the benchmark cost of product k in period n:

$$(2) \qquad C^t_{k\,(n)} = \sum_{i=1}^{p} x^t_{i,k\,(n)}\, w^t_i$$

Where w^t_i is the price of input i at activity t. The cost at activity t in period n is given by:

$$(3) \qquad C^t_{(n)} = \sum_{j=1}^m \sum_{i=1}^p x^t_{i,j\,(n)}\, w^t_i + \varepsilon^t_{(n)}$$

Where $\varepsilon^t_{(n)}$ is a random cost measurement error at activity t. It will be assumed that the $\varepsilon^t_{(n)}$'s are normally distributed with average zero and positive variance $(\varepsilon^t_{(n)} \sim N(0, \sigma^{2\,t}_\varepsilon))$. The quantity of input i used at activity t in period n is:

$$(4) \qquad x^t_{i\,(n)} = \sum_{j=1}^m x^t_{i,j\,(n)}$$

Suppose now that the quantity of input u used at activity t in period n is the allocation base to distribute the total cost at activity t among the m products. The cost allocated to product k at activity t in period n is:

$$(5) \qquad CS^{t,k}_{1,u\,(n)} = x^t_{u,k\,(n)}\, \frac{C^t_{(n)}}{x^t_{u\,(n)}} = x^t_{u,k\,(n)}\, \phi^t_u$$

Where ϕ^t_u is say the overhead absorption rate when input u is the allocation base. The way ϕ^t_u is computed implicitly treats $x^t_{u,j\,(n)}$ and $x^t_{u,k\,(n)}$, $j \neq k$, as the same cost driver. However, it is being explicitly assumed that $x^t_{u,j\,(n)}$ and $x^t_{u,k\,(n)}$ are not the same cost driver. Therefore, some product cost distortion will result when expression (5) is used to allocate costs. Hereafter expression (5) will be referred to as accounting procedure one.

Most conventional accounting allocations are precisely founded on the procedure defined in expression (5). Hwang et al (1993) assume a set of Leontief technologies

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and compute the product cost distortion as a function of (1) production technology heterogeneity, (2) unit input costs and (3) product mix. The product cost distortion equation they derive arises from the use of an accounting procedure equivalent to that defined in expression (5), where each unit of input u weights exactly the same, whether it is used by one product or by another. This accounting procedure introduces systematic product cost distortions when the aggregated cost function at a given activity depends on more than one cost driver.

7.2.3. Accounting Procedure 2 – $CS^{t,k}_{2,u\,(n)}$

The question to which it is now worth directing attention concerns the possibility of designing an accounting procedure that specifically takes into consideration the existence of more than one cost driver. To pursue an answer to this question it is necessary to turn back to expression (5). As noted, expression (5) implicitly assumes that $x^t_{u,j\,(n)}$ and $x^t_{u,k\,(n)}$ are the same cost driver. More formally, $x^t_{u,j\,(n)}$ and $x^t_{u,k\,(n)}$ are the same cost driver if and only if the following condition occurs (see *Definition 1*, section 3.2):

$$(6) \qquad \frac{\partial C^t_{(n)}}{\partial x^t_{u,j\,(n)}} = \frac{\partial C^t_{(n)}}{\partial x^t_{u,k\,(n)}}, \; j \neq k$$

Expression (6) signifies that the cost change at activity t when input u increases one unit is the same whether it is used by product j or by product k. Given condition (1), the cost minimization input-output relationships, the cost change at activity t in period n when one unit of input u is used by product k is:

$$(7) \qquad \frac{\partial C^t_{(n)}}{\partial x^t_{u,k\,(n)}} \equiv \frac{\partial C^t_{j\,(n)}}{\partial x^t_{u,k\,(n)}} = \sum_{i=1}^{p} \frac{\partial x^t_{i,k\,(n)}}{\partial x^t_{u,k\,(n)}} w^t_i = \sum_{i=1}^{p} \frac{\alpha^t_{u,k}(w^t)}{\alpha^t_{i,k}(w^t)} w^t_i = \phi^t_{u,k}$$

Condition (6) takes place automatically $\forall$ u (inputs) and $\forall$ j ≠ k (cost objects) when cost object technologies are identical[1]. However, even when cost object technologies are not identical condition (6) might still occur for some inputs[2]. If condition (6) is verified in the case of input u and for $\forall$ j ≠ k then the aggregate measure $x^t_{u\,(n)}$ corresponds in fact to the sum of different quantities of the same cost driver. In this case, if input u is used as the allocation base, accounting procedure one does not distort the distribution of the total cost at activity t among the various products. Thus the following result can be established:

(8)   If condition (6) takes place for input u and for $\forall$ j ≠ k then:

$$CS^{t,k}_{1,u\,(n)} = C^t_{k\,(n)}$$

That is, the cost allocated to product k under accounting procedure one equals its benchmark cost.

Proof:

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[1] This is because when cost object technologies are identical all the products use the same input mix. That is, $\dfrac{\alpha^t_{u,j}(w^t)}{\alpha^t_{i,j}(w^t)} = \dfrac{\alpha^t_{u,k}(w^t)}{\alpha^t_{i,k}(w^t)}$, $\forall$ i, u (inputs) and $\forall$ j ≠ k (cost objects) (see section 3.2).

[2] Since even when $\dfrac{\alpha^t_{u,j}(w^t)}{\alpha^t_{i,j}(w^t)} \neq \dfrac{\alpha^t_{u,k}(w^t)}{\alpha^t_{i,k}(w^t)}$ we might still have $\sum_{i=1}^{p} \dfrac{\alpha^t_{u,j}(w^t)}{\alpha^t_{i,j}(w^t)} w^t_i = \sum_{i=1}^{p} \dfrac{\alpha^t_{u,k}(w^t)}{\alpha^t_{i,k}(w^t)} w^t_i$, j ≠ k. This point will be explored in sub-section 7.4.4.

First suppose that there are no cost measurement errors and the technology is deterministic. Condition (7) implies that $C^t_{k(n)} = \sum_{i=1}^{p} x^t_{i,k(n)} w^t_i = x^t_{u,k(n)} \phi^t_{u,k}$. From expression (5) we have:

$$CS^{t,k}_{1,u(n)} = x^t_{u,k(n)} \frac{C^t_{(n)}}{x^t_{u(n)}} = x^t_{u,k(n)} \frac{\sum_{j=1}^{m} \sum_{i=1}^{p} x^t_{i,j(n)} w^t_i}{\sum_{j=1}^{m} x^t_{u,j(n)}} = x^t_{u,k(n)} \frac{\sum_{j=1}^{m} x^t_{u,j(n)} \phi^t_{u,j}}{\sum_{j=1}^{m} x^t_{u,j(n)}} .$$

If condition (6) takes place for input u and for $\forall j \neq k$ then $\phi^t_{u,j} = \phi^t_{u,k} = \phi^t_u$.

Therefore, $CS^{t,k}_{1,u(n)} = x^t_{u,k(n)} \dfrac{\phi^t_u \sum_{j=1}^{m} x^t_{u,j(n)}}{\sum_{j=1}^{m} x^t_{u,j(n)}} = x^t_{u,k(n)} \phi^t_{u,k} = \sum_{i=1}^{p} x^t_{i,k(n)} w^t_i = C^t_{k(n)}.$

■

To sum up, even when cost object technologies are not identical (i.e. even when the different products use different input mixes), result (8) opens the possibility of having no cost distortions when input u is used as the allocation base[3]. Note that the demonstration of result (8) is similar to the demonstration of the *sufficiency* of the *Proposition* in section 3.2.

The question on which we should now focus concerns the case where input u is the allocation base but where $\exists j \neq k$ such that $\phi^t_{u,j} \neq \phi^t_{u,k}$. In this situation, accounting procedure one introduces systematic product cost distortions. To propose an accounting procedure that takes into account the fact that $\phi^t_{u,j} \neq \phi^t_{u,k}$ consider that based on a time-series regression of $n = 1, ..., N$ observations the following model is estimated:

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[3] This point will be illustrated in sub-section 7.4.4.

$$(9) \qquad C^t_{(n)} = \delta^t + \sum_{j=1}^{m} \phi^t_{u,j} \, x^t_{u,j \,(n)} + \varepsilon^t_{(n)}$$

As already defined, $C^t_{(n)}$ is the total cost at activity t while $\varepsilon^t_{(n)}$ is a cost measurement error. Assume now that the structure generating the N observations is still the same in period (N+1). This amounts to saying that the input price set and the state of technology in period (N+1) are the same that generated the $n = 1, ..., N$ observations[4]. Let us then define the following accounting procedure:

$$(10) \qquad CS^{t,k}_{2,u \,(n)} = x^t_{u,k \,(n)} \, \hat{\phi}^t_{u,k} \, \frac{C^t_{(n)}}{\sum_{j=1}^{m} \hat{\phi}^t_{u,j} \, x^t_{u,j \,(n)}} = x^t_{u,k \,(n)} \, \hat{\phi}^t_{u,k} \, \lambda^t_{(n)}$$

where $\hat{\phi}^t_{u,j}$ is the estimator of the parameter $\phi^t_{u,j}$ in model (9). The essence of expression (10), henceforth designated accounting procedure two, is the monetary homogenisation of each unit of input u. That is, each unit of input u weights differently, depending on the product that uses it. The weights are exactly the estimated cost change at activity t when one unit of input u is used by cost object j ($\hat{\phi}^t_{u,j}$). The rationality behind expression (10) can be fully explored if we imagine a situation where there are (i) no cost measurement errors and (ii) the technology is deterministic. Under these circumstances we have:

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[4] In conformity with previous assumptions, the random measurement error in period (N+1) is normally distributed with average zero and variance $\sigma^{2\,t}_{\varepsilon}$.

(11) $\qquad CS^{t,k}_{2*,u\,(n)} = C^t_{k\,(n)}$

Where $CS^{t,k}_{2*,u\,(n)} = x^t_{u,k\,(n)}\,\phi^t_{u,k}\,\dfrac{C^t_{(n)}}{\sum^m_{j=1}\phi^t_{u,j}\,x^t_{u,j\,(n)}}$.

Proof:

Note first that given (i) and (ii) we have $C^t_{(n)} = \sum^m_{j=1}\sum^p_{i=1} x^t_{i,j\,(n)}\,w^t_i =$

$\sum^m_{j=1}\phi^t_{u,j}\,x^t_{u,j\,(n)}$. This signifies that the total cost at activity t, $C^t_{(n)}$, is equal to the

sum of m separable and deterministic effects, each of which are equal to

$C^t_{k\,(n)} = \sum^p_{i=1} x^t_{i,k\,(n)}\,w^t_i = \phi^t_{u,k}\,x^t_{u,k\,(n)}$. This implies the following:

$$CS^{t,k}_{2*,u\,(n)} = \frac{\sum^p_{i=1}\sum^m_{j=1} x^t_{i,j\,(n)}\,w^t_i}{\sum^m_{j=1}\dfrac{\sum^p_{i=1} x^t_{i,j\,(n)}\,w^t_i}{x^t_{u,j\,(n)}}\,x^t_{u,j\,(n)}}\,\frac{\sum^p_{i=1} x^t_{i,k\,(n)}\,w^t_i}{x^t_{u,k\,(n)}}\,x^t_{u,k\,(n)} = C^t_{k\,(n)}. \quad \blacksquare$$

Thus, in an "ideal world", accounting procedure two "correctly" distributes the total

cost at activity t among the various products. This is because in equation (9) the

independent variables, $x^t_{u,j\,(n)}$, fully explain the dependent variable, $C^t_{(n)}$. In a more

realistic situation, when there are cost measurement errors and the technology is

stochastic, accounting procedure two introduces some product cost distortion. Even

though, it is expected that it produces better product cost estimates than accounting

procedure one, specially when the aggregated activity cost function depends on more

than one cost driver. This point will be analysed in the next two sections.

7.2.4. Accounting Procedure 3 – $CS^{t,k}_{3,u\,(n)}$

Accounting procedure two ensures that the total cost at activity t in period n (including some cost measurement error) is allocated among the m products. Alternatively, if a standard costing system is used, the cost allocated to product k is (hereafter designed accounting procedure three):

$$(12) \qquad CS^{t,k}_{3,u\,(n)} = \hat{\phi}^t_{u,k}\, x^t_{u,k\,(n)}$$

In this case, the difference between the actual cost (AC) and the standard cost (SC) is not distributed among the various products:

$$(13) \qquad \Delta^t_{AC\text{-}SC\,(n)} = C^t_{(n)} - \sum_{j=1}^{m} \hat{\phi}^t_{u,j}\, x^t_{u,j\,(n)}$$

In the absence of input price changes and cost measurement errors, expression (13) might be interpreted as an efficiency variance in the input utilization. That is, expression (13) reflects the stochastic attribute of technology.

7.2.5. Testing the Existence of a Single Cost Driver

Equation (9) can be used to test the hypothesis of the existence of a single cost driver at activity t. Specifically, we want to test the hypotheses:

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$$H_0: \phi^t_{u,j} = \phi^t_{u,k} \text{ or } \phi^t_{u,j} - \phi^t_{u,k} = 0$$

$$H_1: \phi^t_{u,j} \neq \phi^t_{u,k} \text{ or } \phi^t_{u,j} - \phi^t_{u,k} \neq 0$$

That is to say, we want to test if the two slope coefficients $\phi^t_{u,j}$ and $\phi^t_{u,k}$ are equal. Under the classical assumptions, it can be shown that the statistic:

$$t = \frac{\hat{\phi}^t_{u,j} - \hat{\phi}^t_{u,k}}{\sqrt{\text{var}(\hat{\phi}^t_{u,j}) + \text{var}(\hat{\phi}^t_{u,k}) - 2\,\text{cov}(\hat{\phi}^t_{u,j}, \hat{\phi}^t_{u,k})}}$$

follows the t distribution with $n - (m + 1)$ degrees of freedom, where n is the number of observations and $(m + 1)$ the number of parameters estimated (Gujarati, 1988, p. 227). Note that $(m - 1)$ tests have to be performed. If, at least for one of these tests, the null hypothesis is rejected we can conclude that activity t depends on more than one cost driver. Note also that if the null hypothesis is not rejected we cannot conclude that all cost object technologies are identical. That is, although it is true that when product technologies are identical (as well as linearly homogeneous) the marginal cost corresponding to a unit of input u used by a product is equal for all products, the contrary is not true (see section 3.2). Thus, if the null hypothesis is not rejected, we can only conclude that, statistically speaking, the cost change when the usage of input u increases one unit is the same whether it is used by one or another product. Therefore, accounting procedure one does not introduce major product cost distortions.

7.2.6. Multicollinearity

At this point one potentially problematic aspect should be observed. In practice, there might be problems in estimating equation (9), particularly when the output mix is relatively constant from one period to another. This is a serious problem when the objective is obtaining reliable estimates of the parameters because with multicollinearity the standard errors of the estimators increase significantly (Gujarati, 1988, p. 283-309). Unfortunately, this is likely to happen in practice, particularly in organizations that produce a stable output mix. Given our purposes, this makes the second and third accounting procedures difficult to use with some credibility. If the output mix is correlated, only the following model can be estimated:

(14) $$C^t_{(n)} = \delta^t + \phi^t_u \, x^t_{u\,(n)} + \varepsilon^t_{(n)}$$

Equation (14) introduces, however, a specification error, unless $\phi^t_{u,j} = \phi^t_u$, $\forall \, j$, case in which specifications (9) and (14) are equivalent (that is, result (8) takes place). From the theory of aggregation bias (Maddala, 1977, p. 207-217), the specification error can be calculated if we compute the regression coefficients of the variables $x^t_{u,j\,(n)}$ on the variable $x^t_{u\,(n)}$:

(15) $$x^t_{u,j\,(n)} = a^t_{u,j} \, x^t_{u\,(n)} + \gamma^t_{u,j\,(n)}$$

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The disturbance terms $\gamma^t_{u,j\,(n)}$ are assumed to be independent and identically distributed, with zero mean and constant variance. Substituting expression (15) into expression (9), we obtain:

$$(16) \qquad C^t_{(n)} = \delta^t + \sum\nolimits^m_{j=1} \phi^t_{u,j} \left(a^t_{u,j}\, x^t_{u\,(n)} + \gamma^t_{u,j\,(n)} \right) + \varepsilon^t_{(n)} =$$

$$= \delta^t + x^t_{u\,(n)} \sum\nolimits^m_{j=1} \phi^t_{u,j}\, a^t_{u,j} + \sum\nolimits^m_{j=1} \phi^t_{u,j}\, \gamma^t_{u,j\,(n)} + \varepsilon^t_{(n)}$$

If $\hat{\phi}^t_u$ is the estimator of ϕ^t_u then $E(\hat{\phi}^t_u) = \sum\nolimits^m_{j=1} \phi^t_{u,j}\, a^t_{u,j}$. This signifies that the expected value of $\hat{\phi}^t_u$ is a weighted average of the parameters $\phi^t_{u,j}$. Moreover, noting that $x^t_{u\,(n)} = \sum\nolimits^m_{j=1} x^t_{u,j\,(n)} = \sum\nolimits^m_{j=1} \left(a^t_{u,j}\, x^t_{u\,(n)} + \gamma^t_{u,j\,(n)} \right) = x^t_{u\,(n)} \sum\nolimits^m_{j=1} a^t_{u,j} + \sum\nolimits^m_{j=1} \gamma^t_{u,j\,(n)}$, the sum of weights, $\sum\nolimits^m_{j=1} a^t_{u,j}$, equals 1 and the sum of disturbance terms, $\sum\nolimits^m_{j=1} \gamma^t_{u,j\,(n)}$, equals 0. To sum up, equation (14) is in line with accounting procedure one, as it treats in the same way any unit of input u, whether it is used by one or another product. That is to say, it implicitly assumes the existence of a single cost driver. It thus introduces a specification error when the aggregated cost function at a given activity depends on more than one cost driver.

7.3. Deterministic Technology

7.3.1. Simulation Parameters

For simulation purposes, consider an activity that aggregates three inputs, which are used by three products. The simulation study undertaken in this section assumes a deterministic technology and the existence of cost measurement errors. The analysis is based on the generation of 5000 random numbers for each variable.

It is assumed that the output of product j in period n is uniformly distributed, i.e. $q_{j\,(n)} \sim U(a_j, b_j)$. The random number generation process follows two steps. Firstly, the random number generator of Excel is used to generate 5000 random numbers when $a_j = 0$ and $b_j = 1$. Secondly, the output of product j is determined through the expression $q_{j\,(n)} = a_j + U(0, 1) (b_j - a_j)$. That is to say, the random numbers generated on the interval [0, 1] are converted into random numbers on the interval [a_j, b_j]. All simulations performed in this and in the next section presuppose that $q_{j\,(n)} \sim U(1, 20)$, $j = 1, 2, 3$. Thus the output of each product in a given period is uniformly distributed between 1 and 20.

Under the assumption of deterministic technology, the quantity of input i used per unit of product j at activity t in period n is constant for a given input price set. That is, $x^t_{i,j\,(n)}/q_{j\,(n)} = 1/\alpha^t_{i,j}(w^t)$. The specific parameters assumed are as follows:

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Table 1 – Parameters $1/\alpha^t_{i,j}(w^t)$

$1/\alpha^t_{i,j}(w^t)$	$j = 1$	$j = 2$	$j = 3$
$i = 1$	400	300	300
$i = 2$	500	200	800
$i = 3$	600	100	400

Observe that while product 1 uses relatively more of input 3, product 2 uses relatively more of input 1 and product 3 uses relatively more of input 2. Thus the three products use the three inputs in distinct manners. Finally, it is assumed that the input price set is $w^t = (w^t_1, w^t_2, w^t_3) = (£1, £1, £1)$.

Sub-section 7.3.2 assumes a deterministic technology and the absence of cost measurement errors, implying that the relationship between activity costs and cost drivers is perfect. In sub-section 7.3.3, however, the existence of cost measurement errors will be introduced. Specifically, it will be assumed that the cost measurement error $\varepsilon^t_{(n)}$ is normally distributed with average zero and standard deviation equal to 3750 ($\varepsilon^t_{(n)} \sim N(0, 3750^2)$). This value corresponds approximately to 10% of the expected value of the cost at activity t in period n[1]. The random number generation process follows again two steps. Firstly, the random number generator of Excel is used to generate 5000 random numbers assuming the standardized normal distribution ($z^t_{(n)} \sim N(0, 1)$). Secondly, the $\varepsilon^t_{(n)}$'s are determined through the expression $\varepsilon^t_{(n)} = 3750\, z^t_{(n)}{}^2$.

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[1] $3750 \cong 10\%\ E(C^t_{(n)})$, where $E(C^t_{(n)}) = E\left(\sum_{j=1}^{3} \sum_{i=1}^{3} x^t_{i,j\,(n)} w^t_i + \varepsilon^t_{(n)}\right) =$

$= \sum_{j=1}^{3} \sum_{i=1}^{3} \dfrac{a_j + b_j}{2} \dfrac{w^t_i}{\alpha^t_{i,j}(w^t)}$ $\left(\text{since } E(x^t_{i,j\,(n)}) = E(q_{j\,(n)}) \dfrac{1}{\alpha^t_{i,j}(w^t)} \text{ and } E(q_{j\,(n)}) = \dfrac{a_j + b_j}{2}\right)$.

[2] Note that $z^t_{(n)} = \dfrac{\varepsilon^t_{(n)} - \mu}{\sigma} = \dfrac{\varepsilon^t_{(n)}}{3750}$.

7.3.2. Absence of Cost Measurement Errors

The case analysed here assumes that the technology is deterministic and that there are no cost measurement errors. Under these circumstances, the marginal cost of a unit of a cost driver used by the various products (henceforth designated as product cost driver rates) can be computed directly from Table 1 and the input price vector w^t = (£1, £1, £1). Table 2 shows the calculations.

Table 2 – Product Cost Driver Rates*

Panel A: Allocation Base: Input 1

$$\phi^t_{1,1} = \sum_{i=1}^3 \frac{\alpha^t_{1,1}(w^t)}{\alpha^t_{i,1}(w^t)} w^t_i = \frac{400 \times £1 + 500 \times £1 + 600 \times £1}{400} = £3,75$$

$$\phi^t_{1,2} = \sum_{i=1}^3 \frac{\alpha^t_{1,2}(w^t)}{\alpha^t_{i,2}(w^t)} w^t_i = \frac{300 \times £1 + 200 \times £1 + 100 \times £1}{300} = £2,00$$

$$\phi^t_{1,3} = \sum_{i=1}^3 \frac{\alpha^t_{1,3}(w^t)}{\alpha^t_{i,3}(w^t)} w^t_i = \frac{300 \times £1 + 800 \times £1 + 400 \times £1}{300} = £5,00$$

Panel B: Allocation Base: Input 2

$$\phi^t_{2,1} = \sum_{i=1}^3 \frac{\alpha^t_{2,1}(w^t)}{\alpha^t_{i,1}(w^t)} w^t_i = \frac{400 \times £1 + 500 \times £1 + 600 \times £1}{500} = £3,00$$

$$\phi^t_{2,2} = \sum_{i=1}^3 \frac{\alpha^t_{2,2}(w^t)}{\alpha^t_{i,2}(w^t)} w^t_i = \frac{300 \times £1 + 200 \times £1 + 100 \times £1}{200} = £3,00$$

$$\phi^t_{2,3} = \sum_{i=1}^3 \frac{\alpha^t_{2,3}(w^t)}{\alpha^t_{i,3}(w^t)} w^t_i = \frac{300 \times £1 + 800 \times £1 + 400 \times £1}{800} = £1,875$$

Panel C: Allocation Base: Input 3

$$\phi^t_{3,1} = \sum_{i=1}^3 \frac{\alpha^t_{3,1}(w^t)}{\alpha^t_{i,1}(w^t)} w^t_i = \frac{400 \times £1 + 500 \times £1 + 600 \times £1}{600} = £2,50$$

$$\phi^t_{3,2} = \sum_{i=1}^3 \frac{\alpha^t_{3,2}(w^t)}{\alpha^t_{i,2}(w^t)} w^t_i = \frac{300 \times £1 + 200 \times £1 + 100 \times £1}{100} = £6,00$$

$$\phi^t_{3,3} = \sum_{i=1}^3 \frac{\alpha^t_{3,3}(w^t)}{\alpha^t_{i,3}(w^t)} w^t_i = \frac{300 \times £1 + 800 \times £1 + 400 \times £1}{400} = £3,75$$

* See expression (7), sub-section 7.2.3.

It is obvious that when the technology is deterministic and there are no cost measurement errors the only uncertainty in the estimation of the linear regression model (9) introduced in sub-section 7.2.3 is caused by round-off errors. In this case,

the OLS method would "correctly" estimate the various parameters $\phi^t_{u,j}$, i.e.

$\hat{\phi}^t_{u,j} \cong \phi^t_{u,j}$ so implying that $\hat{\phi}^t_{u,j} x^t_{u,j\,(n)} \cong C^t_{j\,(n)} = \sum^3_{i=1} x^t_{i,j\,(n)} w^t_i$. Of course, we would

have to assume that the output mix was not correlated, so the OLS method could

identify the separable effect of each independent variable on the dependent variable.

In fact, the existence of some correlation between the output of the various products

would preclude the possibility of obtaining reliable estimators of $\hat{\phi}^t_{u,j}$, as a

multicollinariety problem would arise (see sub-section 7.2.6).

7.3.3. Existence of Cost Measurement Errors

The case analysed in this sub-section still assumes that the technology is

deterministic, but introduces the existence of cost measurement errors, which are

normally distributed with average zero and standard deviation equal to 3750

$(\varepsilon^t_{(n)} \sim N(0, 3750^2))$. As was observed, the simulation study is based on the

generation of 5000 random numbers for each variable (the $q_{j\,(n)}$'s and $\varepsilon^t_{(n)}$). The

Ordinary Least Square (OLS) results for model I are presented in the following

table[3]:

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[3] Model I corresponds exactly to equation (9) introduced in sub-section 7.2.3.

Table 3 – Deterministic Technology and Existence of Cost Measurement Errors[*]

Model I: $\hat{C}^t_{(n)} = \hat{\delta}^t + \sum_{j=1}^{3} \hat{\phi}^t_{u,j} x^t_{u,j\,(n)}$			
Independent Variables		Input	
	u = 1	u = 2	u = 3
Intercept	-164,89	-164,89	-164,89
	(0,3745)	(0,3745)	(0,3745)
$x^t_{u,1\,(n)}$	3,76	3,01	2,51
	(0,0001)	(0,0001)	(0,0001)
$x^t_{u,2\,(n)}$	2,04	3,06	6,13
	(0,0001)	(0,0001)	(0,0001)
$x^t_{u,3\,(n)}$	4,98	1,86	3,74
	(0,0001)	(0,0001)	(0,0001)
Adjusted R^2	0,91	0,91	0,91
Prob.(F-Statistic)	0,0001	0,0001	0,0001

[*] Significance levels in parenthesis

The fact that the cost measurement error $\varepsilon^t_{(n)}$ is by construction independent of the $x^t_{u,j\,(n)}$'s explains why the estimators $\hat{\phi}^t_{u,j}$ are not statistically different from the non-stochastic counterparts (see previous sub-section)[4]. Thus we obtain reliable estimators of the parameters $\phi^t_{u,j}$. As will be shown in the next section, this is essential to the performance of accounting procedures two and three.

The preceding analysis suggests that, at least when the technology is deterministic, the third accounting procedure performs better than the second procedure. This is because in the case of the second procedure the cost allocated among the various products at activity t is not the benchmark cost but the benchmark cost plus some

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[4] A t test shows precisely that, where $t = \dfrac{\hat{\phi}^t_{u,i}\,(\text{Table 3}) - \phi^t_{u,i}\,(\text{Table 2})}{\sigma(\hat{\phi}^t_{u,j}\,(\text{Table 3}))}$.

cost measurement error[5]. As was noted, the $\hat{\phi}^t_{u,j}$'s in table 3 are not statistically different from the case when there are no cost measurement errors (sub-section 7.3.2), which, as observed in the last sub-section, are approximately equal to the $\phi^t_{u,j}$'s. Therefore, $CS^{t,k}_{3,u\,(n)} = \hat{\phi}^t_{u,j}\, x^t_{u,j\,(n)} \cong C^t_{j\,(n)}$. That is, the cost allocated to product j is approximately equal to its benchmark cost.

Finally, it is necessary to interpret of the intercept in model I. Given the assumptions of the model, the intercept should not be different from zero. The fact that it is different from zero reflects the inability of the linear regression model to explain the cost measurement error. In other words, no independent variable can explain the cost measurement error. There is also an explanation for the fact that the intercepts of the three regressions are equal. This is because product technologies are deterministic[6] and the three regressions are based on the same sample, i.e. the random measurement error is exactly the same in the three cases.

Table 4 presents an example of product cost calculation under the three accounting procedures. Information concerning benchmark costs is also provided. The vector of final good outputs is $(q_{1\,(n)}, q_{2\,(n)}, q_{3\,(n)}) = (11, 15, 9)$. The activity cost is $C^t_{(n)} = £41\,000$, which is equal to the benchmark cost (£39 000) plus the cost measurement error (£2 000). Input 1 is used a basis for allocating activity costs

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[5] Note that, in practice, the benchmark cost is not known, but only the benchmark cost plus some cost measurement error.

[6] Which implies that the input usage correlations (between $x^t_{1,j\,(n)}$ and $x^t_{2,j\,(n)}$, $x^t_{1,j\,(n)}$ and $x^t_{3,j\,(n)}$ or $x^t_{2,j\,(n)}$ and $x^t_{3,j\,(n)}$) are perfect.

(under the three accounting procedures). The percentage cost error of each product[7] is the criterion used to compare the performance of the three accounting procedures.

Table 4 – Product Costs

Panel A: Benchmark Costs *

$$C^t_{1\,(n)} = q_{1\,(n)} \sum_{i=1}^{3} \frac{w^t_i}{\alpha^t_{i,1}(w^t)} = 11 \times (400 \times £1 + 500 \times £1 + 600 \times £1) = £16\ 500$$

$$C^t_{2\,(n)} = q_{2\,(n)} \sum_{i=1}^{3} \frac{w^t_i}{\alpha^t_{i,2}(w^t)} = 15 \times (300 \times £1 + 200 \times £1 + 100 \times £1) = £9\ 000$$

$$C^t_{3\,(n)} = q_{3\,(n)} \sum_{i=1}^{3} \frac{w^t_i}{\alpha^t_{i,3}(w^t)} = 9 \times (300 \times £1 + 800 \times £1 + 400 \times £1) = £13\ 500$$

Panel B: $CS^{t,k}_{1,u\,(n)}$ **· **

$$x^t_{1,j\,(n)} = \frac{q_{j\,(n)}}{\alpha^t_{1,j}(w^t)}, \quad x^t_{1,1\,(n)} = 11 \times 400, \quad x^t_{1,2\,(n)} = 15 \times 300, \quad x^t_{1,3\,(n)} = 9 \times 300$$

$$x^t_{1\,(n)} = 4\ 400 + 4\ 500 + 2\ 700 = 11\ 600$$

$$\phi^t_{1\,(n)} = \frac{C^t_{(n)}}{x^t_{1\,(n)}} = \frac{£41\ 000}{11\ 600} = £3,53$$

$$CS^{t,1}_{1,1\,(n)} = \phi^t_{1\,(n)}\, x^t_{1,1\,(n)} = £3,53 \times 4\ 400 = £15\ 532 \quad (+5,86\%)$$

$$CS^{t,2}_{1,1\,(n)} = \phi^t_{1\,(n)}\, x^t_{1,2\,(n)} = £3,53 \times 4\ 500 = £15\ 885 \quad (-76,50\%)$$

$$CS^{t,3}_{1,1\,(n)} = \phi^t_{1\,(n)}\, x^t_{1,3\,(n)} = £3,53 \times 2\ 700 = £9\ 531 \quad (+29,40\%)$$

Panel C: $CS^{t,k}_{2,u\,(n)}$ **· **

$$\sum_{j=1}^{3} \hat{\phi}^t_{1,j}\, x^t_{1,j\,(n)} = 3,76 \times 4\ 400 + 2,04 \times 4\ 500 + 4,98 \times 2\ 700 = £39\ 170$$

$$\lambda^t_{(n)} = \frac{C^t_{(n)}}{\sum_{j=1}^{3} \hat{\phi}^t_{1,j}\, x^t_{1,j\,(n)}} = \frac{£41\ 000}{£39\ 170} = 1,05$$

$$CS^{t,1}_{2,1\,(n)} = \lambda^t_{(n)}\, \hat{\phi}^t_{1,1}\, x^t_{1,1\,(n)} = 1,05 \times £3,76 \times 4\ 400 = £17\ 371 \quad (-5,27\%)$$

$$CS^{t,2}_{2,1\,(n)} = \lambda^t_{(n)}\, \hat{\phi}^t_{1,2}\, x^t_{1,2\,(n)} = 1,05 \times £2,04 \times 4\ 500 = £9\ 639 \quad (-7,10\%)$$

$$CS^{t,3}_{2,1\,(n)} = \lambda^t_{(n)}\, \hat{\phi}^t_{1,2}\, x^t_{1,2\,(n)} = 1,05 \times £4,98 \times 2\ 700 = £14\ 118 \quad (-4,58\%)$$

Panel D: $CS^{t,k}_{3,u\,(n)}$ **· **

$$CS^{t,1}_{3,1\,(n)} = \hat{\phi}^t_{1,1}\, x^t_{1,1\,(n)} = £3,76 \times 4\ 400 = £16\ 544 \quad (-0,26\%)$$

$$CS^{t,2}_{3,1\,(n)} = \hat{\phi}^t_{1,2}\, x^t_{1,2\,(n)} = £2,04 \times 4\ 500 = £9\ 180 \quad (-2,00\%)$$

$$CS^{t,3}_{3,1\,(n)} = \hat{\phi}^t_{1,2}\, x^t_{1,2\,(n)} = £4,98 \times 2\ 700 = £13\ 446 \quad (+0,40\%)$$

* $1/\alpha^t_{i,j}$ (see Table 1); $(w^t_1, w^t_2, w^t_3) = (£1, £1, £1)$
** $\hat{\phi}^t_{1,1}$, $\hat{\phi}^t_{1,2}$ and $\hat{\phi}^t_{1,3}$ (see Table 3); *** Product cost errors in parenthesis

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[7] $\dfrac{C^t_{k\,(n)} - CS^{t,k}_{h,u\,(n)}}{C^t_{k\,(n)}} \times 100.$

Table 4 shows that accounting procedure three (Panel D) performs relatively better than accounting procedure two (Panel C), while this performs significantly better than accounting procedure one (Panel B), in accordance with our previous conjectures.

7.4. Stochastic Technology

7.4.1. Simulation Parameters

Again consider an activity that aggregates three inputs, which are used by three products. The output of product j in period n is yet uniformly distributed between 1 and 20 ($q_{j\,(n)} \sim U(1, 20)$). It will also be supposed that the cost measurement error at activity t is normally distributed with average zero and standard deviation equal to 3750 ($\varepsilon^t_{(n)} \sim N(0, 3750^2)$). The technology, however, is now stochastic. Specifically, the quantity of input i used per unit of product j at activity t in period n is normally distributed with average $\mu^t_{i,j}$ and variance $\sigma^{2\,t}_{i,j}$ ($\pi^t_{i,j\,(n)} \sim N(\mu^t_{i,j}, \sigma^{2\,t}_{i,j})$). There is support in the management accounting literature for this construction. Consider the case of standard costing. The existence of a positive (negative) material usage variance, when standard and actual usages are compared, is no more than the manifestation of the stochastic attribute of the technology. Given that the normal distribution is symmetric the probability of having a positive variance is equal to the probability of having a negative variance. The random numbers for the variable $\pi^t_{i,j\,(n)}$ are generated using a similar approach to that described in the previous section for the variables $q_{j\,(n)}$ and $\varepsilon^t_{(n)}$.

Sub-section 7.4.2 assumes that the $\sigma^t_{i,j}$'s are equal to 10% of the $\mu^t_{i,j}$'s. Sub-section 7.4.3 supposes that the standard deviation of input 1 is 20% of the average, while the standard deviations of input 2 and 3 are 5% of the corresponding averages. Finally, sub-section 7.4.4 addresses some particular issues arising when product technologies

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are heterogeneous but where it is possible to express the aggregated activity cost function as depending on only one cost driver (see result (8), p. 113).

7.4.2. Uniform Uncertainty in Input Usage

The parameters of the distributions of $\pi^t_{i,j\,(n)}$ are presented in table 1:

Table 1 – Parameters $\mu^t_{i,j}$ and $\sigma^t_{i,j}$

	$j = 1$		$j = 2$		$j = 3$	
	$\mu^t_{i,1}$	$\sigma^t_{i,1}$	$\mu^t_{i,2}$	$\sigma^t_{i,2}$	$\mu^t_{i,3}$	$\sigma^t_{i,3}$
$i = 1$	400	40	300	30	300	30
$i = 2$	500	50	200	20	800	80
$i = 3$	600	60	100	10	400	40

The analysis developed in this and in the next two sub-sections proceeds as follows. Firstly, based on the generation of a first set of 5000 random numbers for the variables $q_{j\,(n)}$, $x^t_{i,j\,(n)}$ and $\varepsilon^t_{(n)}$, models I and II are estimated (see sub-section 7.3.2). Secondly, the estimators $\hat{\phi}^t_{u,j}$ are used to define the second and third accounting procedures. Finally, a second set of 5000 random numbers for the same variables is generated so as to evaluate the performance of the three accounting procedures.

As observed in the previous section, when (i) the technology is deterministic, (ii) the cost measurement error at activity t, $\varepsilon^t_{(n)}$, is not correlated with the independent variables, $x^t_{u,j\,(n)}$, and (iii) the independent variables are also not correlated with one another, the third accounting procedure produces better product cost estimates than the second procedure. The efficiency of the estimators $\hat{\phi}^t_{u,j}$ together with the fact that

the second procedure allocates among the various products not only the benchmark cost at activity t but also the cost measurement error $\varepsilon^t_{(n)}$ explain this result. However, this result cannot be extended to the case of stochastic technology, since the variance of the estimators $\hat{\phi}^t_{u,j}$ in this later case will be higher than when the technology is deterministic. That is, it will be the magnitude of the cost measurement error $\varepsilon^t_{(n)}$ together with the major or minor efficiency of the estimators $\hat{\phi}^t_{u,j}$ that determine if $CS^{t,k}_{3,u\,(n)}$ (third accounting procedure) produces or not better product cost estimates than $CS^{t,k}_{2,u\,(n)}$ (second accounting procedure). More specifically, under the third accounting procedure the cost of product k is overcosted (undercosted) if $\hat{\phi}^t_{u,k} > \phi^t_{u,k}$ $(\hat{\phi}^t_{u,k} < \phi^t_{u,k})$. Under the second accounting procedure the cost measurement error $\varepsilon^t_{(n)}$ overcosts (undercosts) the cost of product k if $\varepsilon^t_{(n)} > 0$ $(\varepsilon^t_{(n)} < 0)$. Similarly, the estimator $\hat{\phi}^t_{u,k}$ overcosts (undercosts) the cost of product k if $\hat{\phi}^t_{u,k} > \phi^t_{u,k}$ $(\hat{\phi}^t_{u,k} < \phi^t_{u,k})$ (but the fact that $\hat{\phi}^t_{u,j} \neq \phi^t_{u,j}$, $j \neq k$, also distorts the cost of product k under accounting procedure two). Together, these errors determine if the cost of product k is overcosted or undercosted. However, the fact that we might have errors of opposite signs when accounting procedure two is selected[1] implies that nothing can guarantee that the product cost error under accounting procedure three is lower than the product cost error under accounting procedure two.

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[1] For example, $\varepsilon^t_{(n)} > 0$ overcosts the cost of product k while $\hat{\phi}^t_{u,k} < \phi^t_{u,k}$ undercosts the cost of the same product. That is, these two errors are of opposite signs.

The estimation results for models I and II[2] and for the first set of 5000 random numbers are presented in table 2:

Table 2 – Estimation Results [a]

Model I: $\hat{C}^t_{(n)} = \hat{\delta}^t + \sum_{j=1}^{3} \hat{\phi}^t_{u,j} x^t_{u,j\,(n)}$				Model II: $\hat{C}^t_{(n)} = \hat{\delta}^t + \hat{\phi}^t_u x^t_{u\,(n)}$			
Independent Variables	Input			Independent Variable	Input		
	u = 1	u = 2	u = 3		u = 1	u = 2	u = 3
Intercept	1 597,12	949,81	1 237,55	Intercept	1 466,15	3 565,47	4 829,24
	(0,0001)	(0,0001)	(0,0001)		(0,0001)	(0,0001)	(0,0001)
$x^t_{u,1\,(n)}$	3,57	2,89	2,42	$x^t_{u\,(n)}$	3,45	2,17	2,86
	(0,0001)	(0,0001)	(0,0001)		(0,0001)	(0,0001)	(0,0001)
$x^t_{u,2\,(n)}$	1,87	2,86	5,64	Adjusted R^2	0,79	0,85	0,83
	(0,0001)	(0,0001)	(0,0001)	Prob.(F-Statistic)	0,0001	0,0001	0,0001
$x^t_{u,3\,(n)}$	4,84	1,86	3,67				
	(0,0001)	(0,0001)	(0,0001)				
Adjusted R^2	0,87	0,89	0,88				
Prob.(F-Statistic)	0,0001	0,0001	0,0001				
$H_0: \phi^t_{u,2} = \phi^t_{u,1}$ [b]	0,0001	0,5953	0,0001				
$H_0: \phi^t_{u,3} = \phi^t_{u,1}$ [b]	0,0001	0,0001	0,0001				

[a] Significance levels in parenthesis
[b] Significance levels for the test of equality of two regression coefficients

The cost distortion of product k when accounting procedure h is selected and input u is the allocation base is:

$$ (1) \qquad ES^{t,k}_{h,u\,(n)} = \frac{C^t_{k\,(n)} - CS^{t,k}_{h,u\,(n)}}{C^t_{k\,(n)}} $$

Where $C^t_{k\,(n)}$ is the benchmark cost of product k and $CS^{t,k}_{h,u\,(n)}$ the cost allocated to product k (see section 7.2).

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[2] Note that models I and II correspond exactly to equations (9) and (14) introduced in the previous section.

In order to evaluate the $ES^{t,k}_{h,u\,(n)}$'s a second set of 5000 random numbers is generated. However, and as noted, $CS^{t,k}_{h,u\,(n)}$, h = 2, 3 (and $ES^{t,k}_{h,u\,(n)}$ as well), is still based on the estimators $\hat{\phi}^{t}_{u,j}$ for the first set of random numbers. A t-test shows that the estimators $\hat{\phi}^{t}_{u,j}$ for the second set of random numbers are not statistically different from those obtained in the first set.

In addition to analysing each cost distortion measure individually, the different measures are compared. This comparison is undertaken in two ways. Firstly, given the accounting procedure selected, the relative performance of using one and another input as the allocation base are compared. Secondly, given the input used as the allocation base, the relative performances of the three accounting procedures are compared.

Given both the sample size and the Central Limit Theorem, the sample mean $\hat{\mu}(ES^{t,k}_{h,u\,(n)})$ is approximately normally distributed. Therefore, a t-test for the equality of two means is used to compare the various cost distortion measures.

Table 3 presents the descriptive statistics (sample mean and sample standard deviation) as well as the significance levels for the test of equality of two means.

Table 3 – Descriptive Statistics ($ES^{t,k}_{h,u\,(n)}$) and Hypothesis Testing

	$ES^{t,k}_{1,1}$	$ES^{t,k}_{1,2}$	$ES^{t,k}_{1,3}$	$ES^{t,k}_{2,1}$	$ES^{t,k}_{2,2}$	$ES^{t,k}_{2,3}$	$ES^{t,k}_{3,1}$	$ES^{t,k}_{3,2}$	$ES^{t,k}_{3,3}$
$\hat{\mu}(ES^{t,1}_{h,u\,(n)})^{a}$	0,0360	0,1895	-0,3338	-0,0003	0,0076	-0,0040	0,0460	0,0366	0,0334
$\hat{\sigma}(ES^{t,1}_{h,u\,(n)})^{b}$	0,1669	0,1365	0,2349	0,1436	0,1397	0,1440	0,0863	0,0800	0,0742
$H_0: \mu(ES^{t,1}_{1,1\,(n)}) = \mu(ES^{t,1}_{1,2\,(n)})^{c}$	0,0001			0,0055			0,0001		
$H_0: \mu(ES^{t,1}_{1,1\,(n)}) = \mu(ES^{t,1}_{1,3\,(n)})^{c}$		0,0001			0,2067			0,0001	
$H_0: \mu(ES^{t,1}_{1,u\,(n)}) = \mu(ES^{t,1}_{h,u\,(n)})^{c}$				0,0001	0,0001	0,0001	0,0001	0,0001	0,0001
$\hat{\mu}(ES^{t,2}_{h,u\,(n)})^{a}$	-0,8067	0,1888	0,4413	0,0173	0,0170	0,0208	0,0654	0,0472	0,0577
$\hat{\sigma}(ES^{t,2}_{h,u\,(n)})^{b}$	0,3151	0,1452	0,1085	0,1464	0,1499	0,1588	0,0593	0,0813	0,0978
$H_0: \mu(ES^{t,2}_{1,1\,(n)}) = \mu(ES^{t,2}_{1,2\,(n)})^{c}$	0,0001			0,9275			0,0001		
$H_0: \mu(ES^{t,2}_{1,1\,(n)}) = \mu(ES^{t,2}_{1,3\,(n)})^{c}$		0,0001			0,2470			0,0001	
$H_0: \mu(ES^{t,2}_{1,u\,(n)}) = \mu(ES^{t,2}_{h,u\,(n)})^{c}$				0,0001	0,0001	0,0001	0,0001	0,0001	0,0001
$\hat{\mu}(ES^{t,3}_{h,u\,(n)})^{a}$	0,2769	-0,2960	0,1094	-0,0164	-0,0222	-0,0156	0,0302	0,0088	0,0211
$\hat{\sigma}(ES^{t,3}_{h,u\,(n)})^{b}$	0,1306	0,2086	0,1611	0,1507	0,1390	0,1478	0,0980	0,0587	0,0918
$H_0: \mu(ES^{t,3}_{1,1\,(n)}) = \mu(ES^{t,3}_{1,2\,(n)})^{c}$	0,0001			0,0468			0,0001		
$H_0: \mu(ES^{t,3}_{1,1\,(n)}) = \mu(ES^{t,3}_{1,3\,(n)})^{c}$		0,0001			0,7807			0,0001	
$H_0: \mu(ES^{t,3}_{1,u\,(n)}) = \mu(ES^{t,3}_{h,u\,(n)})^{c}$				0,0001	0,0001	0,0001	0,0001	0,0001	0,0001

[a] sample mean; [b] sample standard deviation
[c] significance levels for the test of equality of two means (two-sided),

$$\frac{\hat{\mu}(ES^{t,k}_{h,i\,(n)}) - \hat{\mu}(ES^{t,k}_{p,u\,(n)})}{\sqrt{\left(\frac{1}{N_1} + \frac{1}{N_2}\right)\left(\frac{(N_1-1)\hat{\sigma}^2(ES^{t,k}_{h,i\,(n)}) + (N_2-1)\hat{\sigma}^2(ES^{t,k}_{p,u\,(n)})}{N_1 + N_1 - 2}\right)}} \sim t_{\alpha/2}(N_1 + N_2 - 2)$$

(e.g. Hogg and Tanis, 2001, p. 453-459)

It was shown in sub-section 7.2.2 that accounting procedure one implicitly assumes that $x^{t}_{u,j\,(n)}$ and $x^{t}_{u,k\,(n)}$ are the same cost driver, i.e. the cost change when the quantity of input u increases one unit is the same whatever is the product that uses it.

Given the estimation results presented in table 2, it can be concluded that, statistically speaking, $x^{t}_{u,j\,(n)}$ and $x^{t}_{u,k\,(n)}$, $j \neq k$, are not the same cost driver. Note that although in the case of input 2 the null hypothesis for the test of equality of two regression coefficients is not rejected when product 1 and product 2 are compared, it is rejected when product 1 and product 3 (or product 2 and product 3) are compared.

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Therefore, whichever input is used as an allocation base, the first accounting procedure introduces significant product cost distortions. This is patent in the first three columns of table 3. For example, if input 2 is used as the allocation base, the average product cost distortion is around + 20% for products 1 and 2 (both are undercosted) whereas the average product cost distortion for product 3 is approximately − 30% (it is overcosted). The fact that $\hat{\phi}^{t}_{2,j} > \hat{\phi}^{t}_{2}$ when j = 1, 2 but $\hat{\phi}^{t}_{2,3} < \hat{\phi}^{t}_{2}$ explains that (see table 2)[3]. That is, the average cost driver rate $\hat{\phi}^{t}_{2}$ is lower than the "true cost driver rate" of products 1 and 2, but higher than the "true cost driver rate" of product 3.

Comparing accounting procedures one and two, it is possible to observe that the second procedure produces better product cost estimates than the first procedure. While $\hat{\mu}(ES^{t,k}_{2,u\,(n)})$ varies from - 0,03% to + 2,22%, $\hat{\mu}(ES^{t,k}_{1,u\,(n)})$ varies from - 80,67% to + 29,60%. The fact that $CS^{t,k}_{2,u\,(n)}$ accommodates the existence of multiple cost drivers, but $CS^{t,k}_{1,u\,(n)}$ does not, explains the accuracy of the product cost estimates under the second accounting procedure.

Another interesting feature is that under the second and third accounting procedures choosing input i or input u as the allocation base does not introduce major product cost distortion differences. For example, under the second procedure the various average product cost distortions ($\hat{\mu}(ES^{t,k}_{2,1\,(n)})$, $\hat{\mu}(ES^{t,k}_{2,2\,(n)})$ and $\hat{\mu}(ES^{t,k}_{2,3\,(n)})$) are not

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[3] Note that model II is in line with accounting procedure 1 (see sub-section 7.2.6).

statistically different at a significance level of 4%[4]. The reason behind this result will be explored in the next sub-section.

Finally, it is also interesting to compare the standard deviation of the $ES^{t,k}_{h,u\,(n)}$'s. Overall, the estimated standard deviation of $ES^{t,k}_{2,u\,(n)}$ is lower than $ES^{t,k}_{1,u\,(n)}$, but higher than $ES^{t,k}_{3,u\,(n)}$. Thus the second procedure not only produces better product cost estimates than the first procedure but also reduces the variability of the estimates. Lastly, the fact that the cost allocated to product k under the third procedure depends only on $\hat{\phi}^{t}_{u,k}$ (and $x^{t}_{u,k\,(n)}$ as well), but not on the total cost at activity t, which includes the cost measurement error $\varepsilon^{t}_{(n)}$ (and not on the $\hat{\phi}^{t}_{u,j}$'s, $j \neq k$, as in the case of accounting procedure two), explains why the standard deviation of $ES^{t,k}_{3,u\,(n)}$ is lower than the standard deviation of $ES^{t,k}_{2,u\,(n)}$ (see table 3).

7.4.3. Non-Uniform Uncertainty in Input Usage

The scenario explored in this sub-section is similar to that of the last sub-section. The only difference concerns the parameters of the distribution of the variable $\pi^{t}_{i,j\,(n)}$, which are presented in the following table:

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[4] Except in the case of product 1 and when input 2 is used as the allocation base $\mu(\hat{ES}^{t,1}_{2,2\,(n)})$.

Table 4 – Parameters $\mu^t_{i,j}$ and $\sigma^t_{i,j}$

	j = 1		j = 2		j = 3	
	$\mu^t_{i,1}$	$\sigma^t_{i,1}$	$\mu^t_{i,2}$	$\sigma^t_{i,2}$	$\mu^t_{i,3}$	$\sigma^t_{i,3}$
i = 1	400	80	300	60	300	60
i = 2	500	25	200	10	800	40
i = 3	600	30	100	5	400	20

Observe that the $\mu^t_{i,j}$'s are still equal to those assumed in the previous sub-section. This is not the case with the $\sigma^t_{i,j}$'s. While the $\sigma^t_{i,j}$'s in the last sub-section are all equal to 10% of the $\mu^t_{i,j}$'s, now $\sigma^t_{i,j}$ is 20% of $\mu^t_{i,j}$ in the case of input 1 but 5% in the case of inputs 2 and 3. It was concluded in the last sub-section that when the second and third procedures are used, selecting input i or input u as the allocation base does not produce significant differences in terms of product cost distortion. The fact that the various $\sigma^t_{i,j}$'s are equal to 10% of the $\mu^t_{i,j}$'s explains this result. Conversely, when the uncertainty in terms of input usage is not uniform across all inputs, choosing one or another input as the allocation base might not be indifferent. The specific purpose of this sub-section is to explore this issue.

As in the previous sub-section, based on a first set of 5000 random numbers models I and II are estimated. The following table presents the results:

Table 5 – Estimation Results [a]

Model I: $\hat{C}^t_{(n)} = \hat{\delta}^t + \sum_{j=1}^{3} \hat{\phi}^t_{u,j} x^t_{u,j\,(n)}$				Model II: $\hat{C}^t_{(n)} = \hat{\delta}^t + \hat{\phi}^t_u x^t_{u\,(n)}$			
Independent	Input			Independent	Input		
Variables	$u=1$	$u=2$	$u=3$	Variable	$u=1$	$u=2$	$u=3$
Intercept	184,64	539,06	572,83	Intercept	4 239,44	3 144,31	4 340,51
	(0,0001)	(0,0067)	(0,0047)		(0,0001)	(0,0001)	(0,0001)
$x^t_{u,1\,(n)}$	3,33	2,94	2,45	$x^t_{u\,(n)}$	3,20	2,20	2,89
	(0,0001)	(0,0001)	(0,0001)		(0,0001)	(0,0001)	(0,0001)
$x^t_{u,2\,(n)}$	1,88	2,99	5,96	Adjusted R^2	0,77	0,85	0,84
	(0,0001)	(0,0001)	(0,0001)	Prob.(F-Statistic)	0,0001	0,0001	0,0001
$x^t_{u,3\,(n)}$	4,36	1,85	3,69				
	(0,0001)	(0,0001)	(0,0001)				
Adjusted R^2	0,83	0,89	0,88				
Prob.(F-Statistic)	0,0001	0,0001	0,0001				
$H_0: \phi^t_{u,2} = \phi^t_{u,1}$ [b]	0,0001	0,3827	0,0001				
$H_0: \phi^t_{u,3} = \phi^t_{u,1}$ [b]	0,0001	0,0001	0,0001				

[a] Significance levels in parenthesis

[b] Significance levels for the test of equality of two regression coefficients

Similarly, based on a second set of 5000 random numbers, the variable $ES^{t,k}_{h,u\,(n)}$ is evaluated. Table 6 presents the descriptive statistics (sample mean and sample standard deviation) as well as the significance levels for the test of equality of two means.

Table 6 – Descriptive Statistics ($ES^{t,k}_{h,u(n)}$) and Hypothesis Testing

	$ES^{t,k}_{1,1}$	$ES^{t,k}_{1,2}$	$ES^{t,k}_{1,3}$	$ES^{t,k}_{2,1}$	$ES^{t,k}_{2,2}$	$ES^{t,k}_{2,3}$	$ES^{t,k}_{3,1}$	$ES^{t,k}_{3,2}$	$ES^{t,k}_{3,3}$
$\hat{\mu}(ES^{t,1}_{h,u(n)})^a$	0,0433	0,1886	-0,3321	0,0041	0,0039	-0,0001	0,1172	0,0212	0,0179
$\hat{\sigma}(ES^{t,1}_{h,u(n)})^b$	0,1954	0,1315	0,2235	0,1770	0,1346	0,1345	0,1378	0,0667	0,0639
$H_0: \mu(ES^{t,1}_{1,1(n)}) = \mu(ES^{t,1}_{1,2(n)})^c$		0,0001			0,9537			0,0001	
$H_0: \mu(ES^{t,1}_{1,1(n)}) = \mu(ES^{t,1}_{1,3(n)})^c$			0,0001			0,1837			0,0001
$H_0: \mu(ES^{t,1}_{1,u(n)}) = \mu(ES^{t,1}_{h,u(n)})^c$				0,0001	0,0001	0,0001	0,0001	0,0001	0,0001
$\hat{\mu}(ES^{t,2}_{h,u(n)})^a$	-0,7924	0,1819	0,4399	-0,0513	-0,0216	-0,0212	0,0755	-0,0042	-0,0034
$\hat{\sigma}(ES^{t,2}_{h,u(n)})^b$	0,3555	0,1479	0,1050	0,1959	0,1614	0,1634	0,1006	0,1109	0,1141
$H_0: \mu(ES^{t,2}_{1,1(n)}) = \mu(ES^{t,2}_{1,2(n)})^c$		0,0001			0,0001			0,0001	
$H_0: \mu(ES^{t,2}_{1,1(n)}) = \mu(ES^{t,2}_{1,3(n)})^c$			0,0001			0,0001			0,0001
$H_0: \mu(ES^{t,2}_{1,u(n)}) = \mu(ES^{t,2}_{h,u(n)})^c$				0,0001	0,0001	0,0001	0,0001	0,0001	0,0001
$\hat{\mu}(ES^{t,3}_{h,u(n)})^a$	0,2801	-0,2972	0,1116	0,0236	-0,0055	-0,0018	0,1342	0,0126	0,0166
$\hat{\sigma}(ES^{t,3}_{h,u(n)})^b$	0,1579	0,2075	0,1486	0,1789	0,1323	0,1349	0,1421	0,0482	0,0583
$H_0: \mu(ES^{t,3}_{1,1(n)}) = \mu(ES^{t,3}_{1,2(n)})^c$		0,0001			0,0001			0,0001	
$H_0: \mu(ES^{t,3}_{1,1(n)}) = \mu(ES^{t,3}_{1,3(n)})^c$			0,0001			0,0001			0,0001
$H_0: \mu(ES^{t,3}_{1,u(n)}) = \mu(ES^{t,3}_{h,u(n)})^c$				0,0001	0,0001	0,0001	0,0001	0,0001	0,0001

[a] sample mean; [b] sample standard deviation
[c] significance levels for the test of equality of two means (two-sided)

In contrast with the case analysed in the previous sub-section, it is not now indifferent using one or another input as an allocation base. Under the second and third accounting procedures, the average product cost distortions are statistically lower when input 2 or 3 is the allocation base[5]. To understand why first note that when input u is the allocation base it is in fact being used to estimate the usage of the other two inputs. Now observe that when the technology is deterministic, $x^t_{1,j(n)}$, $x^t_{2,j(n)}$ and $x^t_{3,j(n)}$ are perfectly correlated. At the same time, in the case of the previous sub-section, all correlations are approximately equal to 0,95. That is, all correlations are similar (although not being perfect, as in the case of deterministic technology). This is why using one or another input as a basis for allocating the total

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[5] Except in the case of product 1 and under the second accounting procedure.

cost of the three inputs does not produce major product cost differences. Moreover, this only occurs because the uncertainty in input usage is similar for the three inputs (see table 1, sub-section 7.4.2). In the case presented in this sub-section, however, the correlations between the various inputs are not similar. The fact that the uncertainty in terms of input usage is higher in the case of input 1 than in the case of input 2 or 3 (while the standard deviations of these two are similar – see table 4) implies that the correlation between $x^t_{2,j(n)}$ and $x^t_{3,j(n)}$ is higher than the correlation between $x^t_{1,j(n)}$ and $x^t_{2,j(n)}$ (or $x^t_{1,j(n)}$ and $x^t_{3,j(n)}$). The following table shows precisely this.

Table 7 – Input Usage Correlations [*]

	$x^t_{1,1(n)}$	$x^t_{3,1(n)}$	$x^t_{1,2(n)}$	$x^t_{3,2(n)}$	$x^t_{1,3(n)}$	$x^t_{3,3(n)}$
$x^t_{2,1(n)}$	0,9092 (0,0001)	0,9883 (0,0001)	-	-	-	-
$x^t_{1,1(n)}$	1,0000	0,9096 (0,0001)	-	-	-	-
$x^t_{2,2(n)}$	-	-	0,9129 (0,0001)	0,9882 (0,0001)	-	-
$x^t_{1,2(n)}$	-	-	1,0000	0,9131 (0,0001)	-	-
$x^t_{2,3(n)}$	-	-	-	-	0,9115 (0,0001)	0,9885 (0,0001)
$x^t_{1,3(n)}$	-	-	-	-	1,0000	0,9145 (0,0001)

* Significance levels in parenthesis, $\dfrac{\hat{\rho}\sqrt{N-2}}{\sqrt{1-\hat{\rho}^2}} \sim t_{\alpha/2}(N-2)$

(e.g. Hogg and Tanis, 2001, p. 515-519)

Therefore, under the second and third accounting procedures, if input 2 (input 3) is used as the allocation base input 3 (input 2) is reasonably well distributed among the various products, at least comparing with input 1. This, together with the fact that

input 2 and 3 represent more than 50% of the total cost of any product (see table 4), explains why it is advantageous to use input 2 (or input 3) as an allocation base.

The preceding analysis suggests that when the uncertainty in the input utilization is not uniform across all inputs and the inputs characterized by a lower uncertainty of usage account for a significant proportion of the total cost, it is advantageous to choose as the allocation base one input with low variance of usage.

7.4.4. Case of Equivalence between $CS^{t,k}_{1,u\,(n)}$ and $CS^{t,k}_{2,u\,(n)}$

This sub-section explores a case where, although product technologies are heterogeneous, using the first accounting procedure might not introduce, at least when one particular input is used as the allocation base, major product cost distortions. In other words, a situation compatible with result (8) of sub-section 7.2.3 is analysed. The parameters of the distribution of the variable $\pi^{t}_{i,j\,(n)}$ are as follows:

Table 8 – Parameters $\mu^{t}_{i,j}$ and $\sigma^{t}_{i,j}$

	j = 1		j = 2		j = 3	
	$\mu^{t}_{i,1}$	$\sigma^{t}_{i,1}$	$\mu^{t}_{i,2}$	$\sigma^{t}_{i,2}$	$\mu^{t}_{i,3}$	$\sigma^{t}_{i,3}$
i = 1	400	40	300	30	300	30
i = 2	500	50	200	20	350	35
i = 3	600	60	100	10	400	40

The parameters in table 8 are sill the same as those assumed in the sub-section 7.4.2, except in the case of $\mu^{t}_{2,3}$ (and $\sigma^{t}_{2,3}$). All the $\sigma^{t}_{i,j}$'s are yet 10% of the $\mu^{t}_{i,1}$'s. The parameter $\mu^{t}_{2,3}$ has been intentionally changed from 800 to 350 so as to reproduce a

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situation where if input 2 is used as the allocation base the first accounting procedure performs as good as the second accounting procedure (although the three products do not use the same input mix).

As before, models I and II are estimated. The estimation results for the first set of 5000 random numbers are as follows:

Table 9 – Estimation Results [a]

Model I: $\hat{C}^t_{(n)} = \hat{\delta}^t + \sum_{j=1}^{3} \hat{\phi}^t_{u,j} x^t_{u,j\,(n)}$				Model II: $\hat{C}^t_{(n)} = \hat{\delta}^t + \hat{\phi}^t_u x^t_{u\,(n)}$			
Independent Variables	Input			Independent Variable	Input		
	$u=1$	$u=2$	$u=3$		$u=1$	$u=2$	$u=3$
Intercept	833,26	746,05	632,26	Intercept	111,74	643,54	3 467,33
	(0,0001)	(0,0003)	(0,0016)		(0,6381)	(0,0009)	(0,0001)
$x^t_{u,1\,(n)}$	3,65	2,96	2,46	$x^t_{u\,(n)}$	3,13	2,94	2,55
	(0,0001)	(0,0001)	(0,0001)		(0,0001)	(0,0001)	(0,0001)
$x^t_{u,2\,(n)}$	1,94	2,88	5,69	Adjusted R^2	0,81	0,86	0,84
	(0,0001)	(0,0001)	(0,0001)	Prob.(F-Statistic)	0,0001	0,0001	0,0001
$x^t_{u,3\,(n)}$	3,41	2,89	2,59				
	(0,0001)	(0,0001)	(0,0001)				
Adjusted R^2	0,86	0,86	0,86				
Prob.(F-Statistic)	0,0001	0,0001	0,0001				
$H_0: \phi^t_{u,2} = \phi^t_{u,1}$ [b]	0,0001	0,2171	0,0001				
$H_0: \phi^t_{u,3} = \phi^t_{u,1}$ [b]	0,0001	0,1042	0,0001				

[a] Significance levels in parenthesis
[b] Significance levels for the test of equality of two regression coefficients

Observe that when input 2 is used as the allocation base the parameters $\phi^t_{2,1}$, $\phi^t_{2,2}$ and $\phi^t_{2,3}$ are not statistically different. Consequently, models I and II have the same explanatory power, as can be concluded by the fact that both have an adjusted R^2 of 86%. That is, in this particular case, $x^t_{2,1\,(n)}$, $x^t_{2,2\,(n)}$, $x^t_{2,3\,(n)}$ are in fact different quantities of the same cost driver. Therefore, they can simply be added together

without introducing significant product cost distortions. If input 2 is then used as the allocation base, the first and second accounting procedures produce similar results.

Table 10 presents the descriptive statistics (sample mean and sample standard deviation) as well as the significance levels for the test of equality of two means.

Table 10 – Descriptive Statistics ($ES^{t,k}_{h,u(n)}$) and Hypothesis Testing

	$ES^{t,k}_{1,1}$	$ES^{t,k}_{1,2}$	$ES^{t,k}_{1,3}$	$ES^{t,k}_{2,1}$	$ES^{t,k}_{2,2}$	$ES^{t,k}_{2,3}$	$ES^{t,k}_{3,1}$	$ES^{t,k}_{3,2}$	$ES^{t,k}_{3,3}$
$\hat{\mu}(ES^{t,1}_{h,u(n)})$[a]	0,0164	0,0032	-0,1587	0,0024	-0,0086	-0,0021	0,0246	0,0166	0,0174
$\hat{\sigma}(ES^{t,1}_{h,u(n)})$[b]	0,1489	0,1596	0,2172	0,1601	0,1617	0,1614	0,0882	0,0816	0,0754
H_0: $\mu(ES^{t,1}_{1,1(n)}) = \mu(ES^{t,1}_{1,2(n)})$[c]		0,0001			0,0007			0,0001	
H_0: $\mu(ES^{t,1}_{1,1(n)}) = \mu(ES^{t,1}_{1,3(n)})$[c]			0,0001			0,1582			0,0001
H_0: $\mu(ES^{t,1}_{1,u(n)}) = \mu(ES^{t,1}_{h,u(n)})$[c]				0,0001	0,0002	0,0001	0,0001	0,0001	0,0001
$\hat{\mu}(ES^{t,2}_{h,u(n)})$[a]	-0,5660	0,0021	0,5146	0,0055	0,0141	0,0302	0,0304	0,0406	0,0493
$\hat{\sigma}(ES^{t,2}_{h,u(n)})$[b]	0,2799	0,1717	0,0994	0,1649	0,1699	0,1710	0,0615	0,0819	0,0987
H_0: $\mu(ES^{t,2}_{1,1(n)}) = \mu(ES^{t,2}_{1,2(n)})$[c]		0,0001			0,0099			0,0001	
H_0: $\mu(ES^{t,2}_{1,1(n)}) = \mu(ES^{t,2}_{1,3(n)})$[c]			0,0001			0,0001			0,0001
H_0: $\mu(ES^{t,2}_{1,u(n)}) = \mu(ES^{t,2}_{h,u(n)})$[c]				0,0001	0,0005	0,0001	0,0001	0,0001	0,0001
$\hat{\mu}(ES^{t,3}_{h,u(n)})$[a]	0,1050	0,0016	-0,1045	0,0017	0,0103	-0,0055	0,0252	0,0357	0,0146
$\hat{\sigma}(ES^{t,3}_{h,u(n)})$[b]	0,1642	0,1637	0,2090	0,1669	0,1625	0,1642	0,0853	0,0805	0,0750
H_0: $\mu(ES^{t,3}_{1,1(n)}) = \mu(ES^{t,3}_{1,2(n)})$[c]		0,0001			0,0093			0,0001	
H_0: $\mu(ES^{t,3}_{1,1(n)}) = \mu(ES^{t,3}_{1,3(n)})$[c]			0,0001			0,0304			0,0001
H_0: $\mu(ES^{t,3}_{1,u(n)}) = \mu(ES^{t,3}_{h,u(n)})$[c]				0,0001	0,0081	0,0001	0,0001	0,0001	0,0001

[a] sample mean; [b] sample standard deviation
[c] significance levels for the test of equality of two means (two-sided)

As suggested, both the first and second accounting procedures produce low product cost distortions when input 2 is used as the allocation base. This is because the fundamental assumption behind the construction of $CS^{t,k}_{1,2(n)}$, i.e., the assumption under which $x^t_{2,1(n)}$, $x^t_{2,2(n)}$ and $x^t_{2,3(n)}$ are the same cost driver, holds in this particular case. If, however, input 1 or 3 is the allocation base, accounting procedure two

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produces statistically and significantly lower average product cost distortions than accounting procedure one.

7.5. Conclusions

The results derived in this chapter are based on the appraisal of two accounting procedures that specifically accommodate the non-existence of the second condition supporting the construction of an aggregate activity output in ABC, i.e. the condition under which the marginal cost of a unit of cost driver used by a product is equal for all products within a cost pool. The assumption that product technologies are linearly homogeneous, the other property supporting an ABC system, has been maintained. Section 7.2 specifies the technology as well as the accounting procedures supporting the analysis undertaken in sections 7.3 and 7.4. While section 7.3 is based on the assumption that the technology is deterministic, section 7.4 relaxes this and explicitly assumes that the quantity of each input per unit of output is stochastic.

It was shown that even when in a cost pool only one input is measured individually, and this input is used as a basis for allocating the cost of the other inputs, it is possible to design an accounting procedure that takes into account the fact that different products use different input mixes. In a context where there are cost measurement errors and the technology is stochastic, the simulation results show that the accounting procedures proposed generate relatively unbiased estimates of product costs, in contrast with a conventional accounting method that systematically and significantly distorts product costs.

The conventional accounting procedure implicitly assumes that the (activity) cost change when one unit of input u is used by product j equals the (activity) cost change

when one unit of the same input is used by product k. That is, it assumes that the quantity of input u used by product j and the quantity of the same input used by product k are the same cost driver.

In contrast, under the accounting procedures proposed, each unit of input u weights differently, depending on the product that uses it. In other words, they explicitly accommodate the existence of multiple cost drivers. Their design presupposes two stages. Firstly, a linear regression model provides estimates where the dependent variable is the activity cost and the independent variables are the quantity of input u used by the various products. The coefficient estimates of this model show the estimated activity cost change when the different products use one unit of input u. Secondly, each unit of input u is weighted by those coefficients, in the construction of an homogeneous (monetary) measure of output.

The simulation experiment has also served to introduce the stochastic attribute of technology. The uncertainty was defined in terms of the quantity of the various inputs necessary to produce a given output. When the uncertainty in the input usage is uniform across all inputs, using one or another input as a basis for allocating costs among the various cost objects does not produce, under the accounting procedures proposed, major product cost differences. However, this is not the case when the uncertainty in the input usage is not uniform across all inputs. The results derived suggest that when the uncertainty in the input utilization is not uniform across all inputs and the inputs characterized by a lower uncertainty of usage account for a

significant proportion of the total cost, it is advisable to choose as an allocation base one input with low variance of usage.

The above results have implications for practice. The first justification for the construction of more refined cost systems is the distinction between different categories of cost drivers (unit-level, batch-level, product-sustaining and facility-sustaining), one of the most significant innovations of ABC systems (Cooper and Kaplan, 1992). A second justification lies in the two conditions supporting the construction of an aggregate activity output. As was shown, the two conditions ensure that the marginal cost of a unit of cost driver used by a cost object is both constant and equal for all cost objects within a cost pool. Assuming that cost object technologies are linearly homogeneous, the art in designing an ABC system is in aggregating inputs in such a way that, together with an adequate selection of allocation bases, the marginal cost of a unit of cost driver used by a cost object is approximately equal for all cost objects in a cost pool. In other words, the aggregation of inputs and the choice of allocation bases should guarantee, as far as possible, that the aggregate measure of output in a cost pool corresponds in fact to the sum of different quantities of the same cost driver. Alternatively, and given the impossibility of ensuring that a cost pool depends on only one cost driver, the proposed accounting procedures might constitute a viable way of accommodating the existence of multiple cost drivers.

It must be recognised that two major difficulties may exist in using the proposed accounting procedures (apart from those described is chapter 5). Firstly, their

adoption presupposes that there is some historical data in order to estimate the linear regression model supporting their design. Secondly, it is necessary that the output mix is not constant over time. Otherwise, a multicollinearity problem will arise, making it virtually impossible to use them with some degree of credibility.

CHAPTER VIII – CONCLUSIONS

8.1. Results

The design of more refined cost systems, aimed at improving product cost accuracy, has received considerable attention in the management accounting literature, especially after the emergence of ABC. However, with an ABC system, designed to generate relevant costs for decision-making, strong assumptions are implicit in respect of the nature of costs. Whereas Noreen (1991) concentrates on assumptions relating to cost functions, Christensen and Demski (1995), Bromwich (1995, 1997) and Bromwich and Hong (1999, 2000) explore the assumed characteristics of technology. The assumption that technology, apart from input prices, determines costs is well established in the production economics literature (Chambers, 1988, is a classic reference). It was not until the work of Christensen and Demski (1995), Bromwich (1995, 1997) and Bromwich and Hong (1999, 2000) that this analysis was incorporated into the product costing literature through their work on ABC.

Although the above extensions of the work of Noreen (1991) provide a deeper analysis of the theoretical foundations of ABC, they have two major limitations.

The first limitation is that they take for granted the equivalence between a multi-output technology and a single output technology. More specifically, they assume that it is possible to construct a single or aggregate measure of output that totally captures the cost of the resources used by the various cost objects within a cost pool.

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However, the necessary and sufficient conditions supporting the construction of such an aggregate measure of output have not been derived. One consequence of this gap is that the precise analytical articulation that exists between the single or aggregate measure of output, the technology of different cost objects and activity costs remains unclear. That is to say, the duality between the multi-output technology and costs in ABC has not been derived. This derivation was the primary purpose of this study. The derivation of this completes the mapping of the theoretical foundations for ABC and so specifies the key characteristics of technology, costs and outputs which have to exist if product costs are to be decision relevant.

The second gap of the ABC literature is that only the long run has been considered. From this time perspective, all inputs are variable with output. In the short run, however, some inputs are fixed. It is in the short run that one of the most recognised innovations of ABC, the distinction between the cost of recourses supplied and the cost of resources used (Cooper and Kaplan, 1992), has to be incorporated in the costing analysis. The short run analysis of ABC was the second major purpose of this study.

The specific contributions of this study are outlined below in terms of the four particular results obtained.

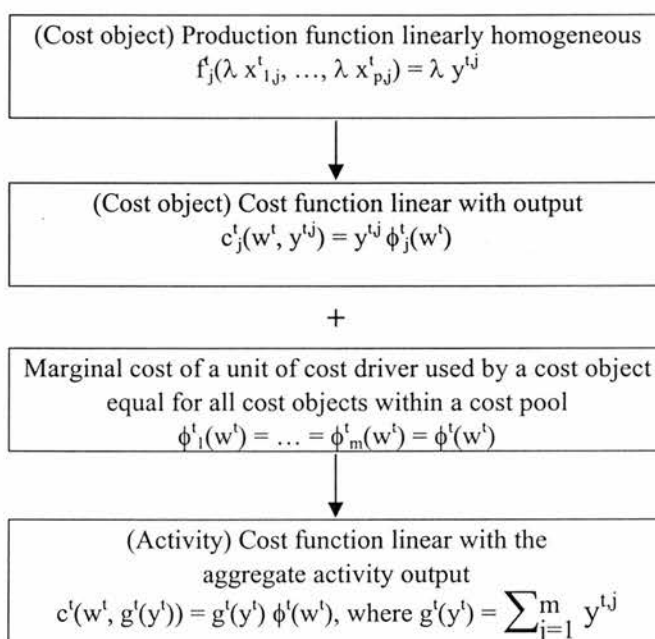## Result 1 – Technological foundations of an aggregate activity output

It was demonstrated that two conditions are jointly necessary and sufficient for the construction of an aggregate activity output, compatible with costs being directly proportional to the level of that output. The first is that (i) cost object production functions are linearly homogeneous. This condition ensures that marginal costs are constant, which is essential if the cost reported by an ABC system, an average cost, is also to be a relevant cost for decision-making. The second condition is that (ii) the marginal cost of a unit of cost driver used by a cost object is equal for all cost objects within a cost pool. This condition ensures that the aggregated cost function at a given activity depends on only one cost driver.

Condition (i) shows that only linearly homogeneous technologies, a special case of homothetic technologies, give rise to cost functions compatible with ABC. This finding contrasts with Bromwich and Hong (1999), who claim that homothetic technologies give rise in general to cost functions compatible with ABC. Condition (ii) is automatically ensured when all (linearly homogeneous) cost object technologies are identical. However, and at least in theory, condition (ii) might still occur when cost object technologies are heterogeneous. That is, condition (ii) might take place when the various cost objects use different input mixes within a cost pool (see section 3.2). This implies that the condition that all cost objects use the same input mix in a cost pool is not a necessary condition for the construction of an aggregate activity output in ABC. This also contrasts with Bromwich and Hong (1999), who state that a constant input mix has to be common to all products in a cost

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pool. It shows why even when the various products use the inputs within a cost pool in different ways product cost distortions might be small. That is, high product technology heterogeneity does not necessarily imply high product cost distortions. Finally, it has been shown that when condition (ii) does not hold the aggregated activity cost function depends on more than one cost driver. This is why using a single average cost driver rate to allocate costs distorts product costs.

Table 1 summarises the technological foundations of an aggregate activity output.

Table 1 – Technological Foundations of an Aggregate Activity Output

$$(\text{Cost object) Production function linearly homogeneous}$$
$$f^t_j(\lambda\, x^t_{1,j}, \ldots, \lambda\, x^t_{p,j}) = \lambda\, y^{t,j}$$

$\downarrow$

$$(\text{Cost object) Cost function linear with output}$$
$$c^t_j(w^t, y^{t,j}) = y^{t,j}\, \phi^t_j(w^t)$$

$+$

$$\text{Marginal cost of a unit of cost driver used by a cost object}$$
$$\text{equal for all cost objects within a cost pool}$$
$$\phi^t_1(w^t) = \ldots = \phi^t_m(w^t) = \phi^t(w^t)$$

$\downarrow$

$$(\text{Activity) Cost function linear with the}$$
$$\text{aggregate activity output}$$
$$c^t(w^t, g^t(y^t)) = g^t(y^t)\, \phi^t(w^t), \text{ where } g^t(y^t) = \sum_{j=1}^{m} y^{t,j}$$

Result 2 – Short run activity cost function

In the short run, it is necessary to distinguish between the case where an activity is operated with excess capacity and the case where it is operated above capacity. An activity is operated with excess capacity if all the fixed inputs are not fully used. It is operated above capacity if at least one fixed input is fully used. This concept of

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capacity has been neglected in the management accounting literature, in general, and in the ABC literature, in particular. However, it is fundamental to represent the short run equation of capacity, one of the highest profile innovations of ABC systems (Cooper and Kaplan, 1992).

It was shown that the rate at which the cost of resources used changes is constant only when activities are operated with excess capacity. This rate can be split into two components. While one component denotes an increase in the cost of resources used but not in the cost of resources supplied, the other denotes an increase in the cost of resources used and supplied. Only this latter component generates relevant costs for decision-making. The rate at which the cost of resources used changes will no longer be constant when activities are operated with excess capacity (even assuming that technologies are linearly homogenous), since, in this case, the input mix will change and differ from the long run input mix. This presumes, however, that the technology allows some degree of substitution between inputs. The only case where an activity never operates above capacity is when the technology is Leontief, an extreme case of a linearly homogeneous technology that does not allow any substitution between inputs, i.e. inputs are combined in completely fixed proportions (see Chambers, 1988, p. 15-17).

A conventional accounting procedure, behind the construction of both traditional and ABC systems, is the application of average cost driver rates to cost outputs. This procedure can only be justified when activities are operated with excess capacity. Otherwise, the cost of resources used does not change linearly with output. A

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corollary of this is that ABC implicitly assumes that activities are operated with excess capacity.

By characterising the theoretical foundations of ABC, this study further extends existing product costing literature as it considers situations where either production functions are not linearly homogeneous or the aggregated activity cost function depends on more than one cost driver (while still assuming that production functions are linearly homogeneous).

Result 3 – Homogeneous, non-homothetic technologies and product costing

The first extension concentrates on technologies that are not linearly homogeneous. Given the duality that exists between costs and technology, this implies that cost functions are not also linear with output. It was assumed that the activity output was an intermediate input that is used by the various cost objects. Two types of non-linear technologies were considered: homogeneous and non-homothetic technologies. The reason for choosing these two technologies is that they accommodate a great number of non-linear input-output relationships. The objective was to evaluate the distortions arising from the application of average cost driver rates to cost outputs.
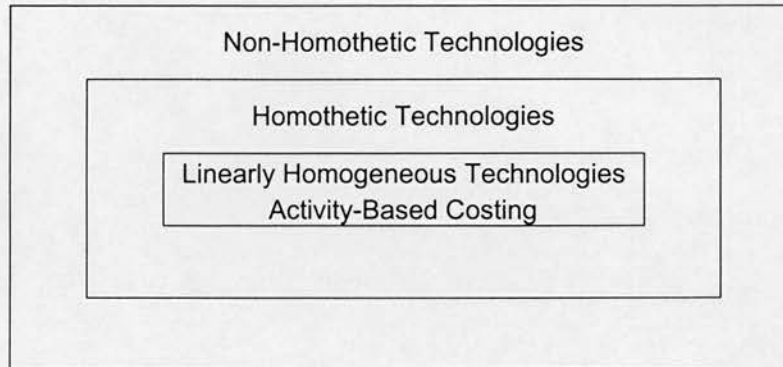
The results derived can be described in terms of the elasticity of input demand with respect to output, which shows the ratio of the relative change in the demand of a given input to the relative change in output, ceteris paribus. It was shown that

distortions increase as the elasticity of input demand with respect to output diverges from one. But distortions also change with the level of output. This amounts to saying that, depending on the output interval, the application of a single average cost driver rate might undercost, overcost or even approximate marginal costs.

In the product costing literature, the only researchers who have attempted to incorporate non-linearity issues into the analysis of costing are Christensen and Demski (1997, 2003). However, they only make use of the result that increasing (decreasing) returns to scale imply that average costs will be above (below) marginal costs. In contrast, in this study, the analysis is founded on the duality between production and cost functions, and while deriving the cost function dual to a homogeneous technology, a more general case, where cost functions are not multiplicatively separable in input prices and output, the non-homothetic one, was derived.

ABC can be seen as a particular case of a more general cost system that can be derived. ABC is a case where the elasticity of input demand with respect to output is constant and equal to one for all inputs within a cost pool (linearly homogenous technology). A more general case is where such elasticity, although being equal for all inputs within a cost pool, is different from one (homothetic technology). An even more general case is where the elasticity of input demand with respect to output is not equal for two or more inputs within a cost pool (non-homothetic technology) (Table 2).

Table 2 – Non-Homothetic, Homothetic, Linearly Homogeneous

Technologies and Activity-Based Costing

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┌─────────────────────────────────────────────────────────┐
│              Non-Homothetic Technologies                 │
│  ┌───────────────────────────────────────────────────┐  │
│  │              Homothetic Technologies              │  │
│  │  ┌─────────────────────────────────────────────┐  │  │
│  │  │     Linearly Homogeneous Technologies       │  │  │
│  │  │            Activity-Based Costing           │  │  │
│  │  └─────────────────────────────────────────────┘  │  │
│  │                                                   │  │
│  └───────────────────────────────────────────────────┘  │
│                                                          │
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Result 4 – Heterogeneous (cost object) technologies, uncertainty in input usage and product costing

The second extension of the product costing literature made in this study concentrates on situations where, although cost object technologies are still linearly homogeneous, the aggregated activity cost function depends on more than one cost driver. While analysing the question of testing the (non) existence of a single cost driver, an accounting procedure that specifically accommodates the existence of multiple cost drivers was introduced. Its adoption presupposes two stages. Firstly, a linear regression model was estimated in which the dependent variable is the activity cost and the independent variables are the quantity of say input u used by the different cost objects. The coefficient estimates of this model show the (estimated) activity cost change when a given cost object uses one unit of input u. Secondly, a weighted aggregate measure of output, where the weights are exactly the estimated coefficients, was constructed.

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This construction is in contrast with a conventional accounting procedure, which uses the total quantity of a given input used in a cost pool as a measure of the output. As was shown, this conventional procedure implicitly assumes that the activity cost change when one unit of input u is used by say cost object j equals the activity cost change when one unit of the same input is used by cost object k. That is, it assumes that there is only one cost driver.

In the product costing literature, Hwang *et al* (1993) assume a set of Leontief technologies and compute the product cost distortion as a function of (1) production technology heterogeneity, (2) unit input prices and (3) product mix. The product cost distortion equation they derive results from the use of the above conventional accounting procedure.

The accounting procedures proposed in this study permit, however, and in a setting where product technologies are linearly homogeneous, the accommodation of the existence of more than one cost driver. The results of the simulation experiment developed in chapter seven show that the accounting procedures proposed, which specifically accommodate the existence of multiple cost drivers, produce relatively unbiased estimates of product costs, in accordance with the theoretical conjectures. In contrast, the conventional accounting procedure systematically and significantly overestimates (underestimates) product costs.

The simulation experiment developed in chapter seven also introduced the stochastic attribute of technology, an aspect that has been neglected in the product costing

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literature. Uncertainty was defined in terms of the quantity of the various inputs necessary to produce a given output. It was shown that when the uncertainty in the input usage is uniform across all inputs, using one or another input as a basis for allocating costs among the various cost objects does not produce, under the accounting procedures proposed, major product cost differences. However, this is not the case when the uncertainty in the input usage is not uniform across all inputs. The results derived are in accordance with our intuition that it is generally better to use as a basis for allocating costs one input with low uncertainty in terms of usage.

8.2. Theoretical Foundations of Activity-Based Costing

Noreen (1991) was the first author to focus on the analysis of the theoretical foundations of ABC (see Table 3). He has concentrated on conditions related to cost functions and has derived three necessary and sufficient conditions for ABC systems to measure relevant costs for decision-making. These are that (1^c) total costs can be divided into independent cost pools, each of which depends on one and only one activity, (2^c) the cost in each cost pool is strictly proportional to the level of activity in that cost pool and (3^c) the volume of an activity is simply the sum of activity measures utilised by the individual products.

This study, together with the work of Christensen and Demski (1995), Bromwich (1995, 1997) and Bromwich and Hong (1999, 2000), has developed a deeper analysis of the theoretical foundations of ABC, as it is based on the (production economics) assumption that technology and input prices are the two primary determinants of cost

functions. Condition (1^c) is what, in the production economics literature, is known as cost function separability – *weakly separable cost function* (see section 2.3). In the management accounting literature, Christensen and Demski (1995) were the first researchers to observe this point. In terms of technology, cost function separability is ensured if technologies are also separable (1^t) – *weakly separable technology* (see section 2.4). This point has been emphasised by Bromwich and Hong (1999) (and implicitly by Christensen and Demski, 1995). Conditions (2^c) and (3^c) are ensured when (2^t) production functions are linearly homogenous and (3^t) the marginal cost of a unit of cost driver used by a product is equal for all products within a cost pool. This last property is determined by the simultaneous effect of technology and input prices (see the case where product technologies are heterogeneous but this condition still holds – section 3.2). Conditions (2^t) and (3^t) constitute exactly the essence of Result 1 of this study (see pages 151-152). A further contribution of this study is the introduction of the short run structure of ABC. With respect to this point, it was shown that ABC is only compatible with activities operating with excess capacity. The following table summarizes the theoretical foundations of ABC (long run).

Table 3 – Theoretical Foundations of Activity-Based Costing (Long Run)

Cost Functions	Technology
(1^c) Total costs can be divided into independent cost pools (Noreen, 1991)	(1^t) Production functions weakly separable (Christensen and Demski, 1995, Bromwich and Hong, 1999)
(2^c) The cost in each cost pool is strictly proportional to the level of activity in that cost pool (Noreen, 1991)	(2^t) Production functions linearly homogeneous (This study)
(3^c) The volume of an activity is simply the sum of activity measures utilised by the individual products (Noreen, 1991)	(3^t) Marginal cost of a unit of cost driver used by a product equal for all products within a cost pool (This study)

It should be noted that the above conditions exclude the existence of any jointness either within or between cost pools, a necessary condition for an ABC system to measure incremental costs (see section 2.7). This amounts to saying that the cost of performing the different activities separately (and, within each activity, the various product volumes) equals the cost of performing them together, that is, there are no economies or diseconomies of scope either within or between cost pools. Finally, it should be observed that the conditions in table 3 also implicitly assume that inputs are traded on a perfect market, otherwise input prices might vary with the activity output as identical inputs might have different prices (Bromwich and Hong, 1999).

8.3. Implications for the Design of Cost Systems

In theory, it is always possible to develop very disaggregated costing systems. At one extreme, inputs can be measured individually, i.e. we can create a cost pool for each individual input and associate each of them with the various cost objects. In practice, however, cost-benefit issues preclude such detailed disaggregations. So the aggregation of two or more inputs in a cost pool and the distribution of them between two or more cost objects is a common practice. Assuming that cost object technologies are linearly homogeneous, the art in designing an ABC system is in aggregating inputs in such a way that, together with an adequate selection of allocation bases, cost pools depend on only one cost driver, i.e. the marginal cost of a unit of cost driver used by a cost object is approximately equal for all cost objects within a cost pool.

With regard to the way of dealing with the non-verification of the two conditions constituting the very essence of an aggregate activity output and therefore of ABC, the following can be considered.

Firstly, it should be observed that even assuming that technologies are linearly homogeneous, it is a very strong assumption that the marginal cost of a unit of cost driver used by a cost object is equal for all cost objects within a cost pool. That is, it is a very strong assumption that the aggregated cost function at a given activity depends on only one cost driver. In the impossibility of ensuring that a cost pool depends on only one cost driver, the accounting procedures proposed in section 7.2 might constitute a way of accommodating the existence of more than one cost driver.

Secondly, it has been shown that when cost functions are not linear with output or when technologies are not linearly homogeneous, the application of an average cost driver rate to allocate costs distorts product costs (see sections 4.2 and 4.3). Accounting numbers are, at best, refined approximations. One way of approximating marginal costs might be the accumulation of costs and the definition of average cost driver rates for different output intervals. Using this approach the accuracy of product costs will increase as the size of each interval reduces (since at the limit, when each intervals tends to zero, the marginal cost is obtained). This would partially address the issue of different resource usage intensities, the essence of non-linearity and so might constitute a viable way of deriving adequate surrogates of marginal costs, particularly in a setting where production and cost relationships are not known.

In short, the distortions arising from the application of average cost driver rates to cost outputs (final outputs or not), an elementary procedure underlying both ABC and traditional systems, increase in two fundamental cases. Firstly, distortions increase when the input-output relationships are other than linear, that is, when technologies are not linearly homogeneous. Secondly, distortions increase when, even assuming that technologies are linearly homogenous, the marginal cost of a unit of cost driver used by a cost object is not equal for all cost objects, that is, when activity cost functions depend on more than one cost driver. These two basic cases constitute rules of thumb for testing whether ABC will be appropriate in a particular setting.

Finally, it should be noted that cost systems have to be re-evaluated over time, as new products are introduced or the characteristics of existing products change. For example, suppose that m products are aggregated at activity t and conditions (1^t), (2^t) and (3^t) hold (see table 3). Assume now that a new product say m+1 that uses the same separable inputs is introduced. The fact that either the underlying technology is not linearly homogeneous or the marginal cost of a unit of cost driver used by this product is different from that of the other products will distort automatically the distribution of the total cost among the m+1 products. That is to say, as new products are introduced or the attributes of existing products change, the aggregation process is no longer error free. Such changes suggest that cost systems have to be continually reassessed as product lines alter. This involves evaluating the way inputs are aggregated across cost pools as well as the choice of allocation bases.

8.4. Future Work

The opportunities for future work that the analysis developed in this study raises are threefold.

Firstly, it is necessary to test empirically the two basic conditions supporting an ABC system. The empirical investigation of the cost distortions arising in situations where those conditions do not apply will be also of particular relevance to those who wish to assess the utility of an existing costing system or who wish to design a new system. One of the challenges facing this work will be the ability to deal with variables such as the effects of input prices change and technical change (particularly, disembodied technical change) or the measurement of used and unused capacity (see chapter 5). In fact, the validity of the results derived through the empirical work will depend on the major or minor control that the researcher exerts over these variables.

Secondly, conventional and ABC systems assume that it is possible to represent a multi-output technology as a single output technology. That is to say, it is assumed that it is possible to create a single measure of output or cost driver that fully captures the cost of the resources used by the various cost objects within a cost pool. In a context where technologies are linearly homogeneous, an accounting procedure that accommodates the existence of multiple cost drivers was derived. At the same time, the incorporation into the analysis of technologies that are not linearly homogeneous was undertaken in a context where the activity output was an

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intermediate input used by the various cost objects. That is, each activity was by assumption a single output technology. The analysis of situations where one or more cost object production functions are not linearly homogenous (and are not identical) and do not therefore permit the creation of an undistorted single measure of output, provides possibly a more interesting case, which will be worth investigating.

Finally, the simulation experiment developed in chapter seven constitutes a first attempt to integrate into the product costing theory the question of uncertainty in input usage. Future work could expand this analysis, particularly through analytical modelling.

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