



# THE UNIVERSITY *of* EDINBURGH

This thesis has been submitted in fulfilment of the requirements for a postgraduate degree (e.g. PhD, MPhil, DClinPsychol) at the University of Edinburgh. Please note the following terms and conditions of use:

This work is protected by copyright and other intellectual property rights, which are retained by the thesis author, unless otherwise stated.

A copy can be downloaded for personal non-commercial research or study, without prior permission or charge.

This thesis cannot be reproduced or quoted extensively from without first obtaining permission in writing from the author.

The content must not be changed in any way or sold commercially in any format or medium without the formal permission of the author.

When referring to this work, full bibliographic details including the author, title, awarding institution and date of the thesis must be given.

# Papers on Organisational Governance and Strategy

A thesis presented for the degree of  
Doctor of Philosophy



**Stuart Andrew Craig Baumann**

School of Economics

College of Humanities and Social Sciences

University of Edinburgh

June 12, 2017

# Contents

<b>Contents</b>	<b>1</b>
<b>List of Figures</b>	<b>4</b>
<b>List of Tables</b>	<b>5</b>
<b>Acknowledgements</b>	<b>7</b>
<b>Declaration of own work</b>	<b>8</b>
<b>0 Introduction and lay summary</b>	<b>9</b>
Lay Summary: Putting it off for later . . . . .	9
Lay Summary: Comparative Advertising . . . . .	13
Lay Summary: It's good to be bad . . . . .	15
<b>1 Putting it off for later: Procrastination and end of fiscal year spending spikes</b>	<b>19</b>
1.1 Introduction . . . . .	19
1.2 Government Spending in the United Kingdom . . . . .	21
1.2.1 Northern Ireland Spending . . . . .	22
1.2.2 UK Central Government Spending . . . . .	23
1.2.3 The quality of end of fiscal year spending . . . . .	24
1.3 Procrastination . . . . .	25
1.3.1 A deterministic model of procrastination . . . . .	27
1.3.2 A calibrated procrastination model . . . . .	29
1.4 Discussion . . . . .	32
1.4.1 The Precautionary Savings Mechanism . . . . .	33
1.4.2 Model comparison . . . . .	34
1.5 Policy Responses to heightened end of year spending . . . . .	37
1.5.1 Time-variant budgetary tax . . . . .	38
1.5.2 Calibrated time variant budgetary tax . . . . .	39
1.6 Conclusion . . . . .	42

1.7	References . . . . .	43
	Appendices . . . . .	46
1.A	Proofs . . . . .	46
1.B	Data . . . . .	48
1.C	Curvature of production function robustness checks . . . . .	55
1.D	Econometric Robustness checks - Section 1.4.2 . . . . .	55
1.E	Time Series: Final Month Cash-outlays and Variance . . . . .	62
1.F	Further discussion . . . . .	66
1.G	Derivation of optimal tax for calibrated model in deterministic setting . . . . .	73
1.H	Social Cost and Optimal Budgeting . . . . .	74
1.I	5% and 10% discounting calibrations . . . . .	75
1.J	Precautionary Savings model Monte Carlos . . . . .	79
<b>2</b>	<b>Comparative Advertising: The role of prices</b>	<b>85</b>
2.1	Introduction . . . . .	85
2.2	Literature Review . . . . .	88
2.3	The Model . . . . .	90
2.3.1	Separating Equilibrium without price comparative advertising . . . . .	90
2.3.2	Separating Equilibrium with price comparative advertising . . . . .	92
2.4	Producer and Consumer Surplus . . . . .	101
2.5	Extensions . . . . .	104
2.5.1	Cost heterogeneity in high firms . . . . .	104
2.5.2	Monopolist supplier for high quality good . . . . .	105
2.6	Conclusion . . . . .	107
2.7	References . . . . .	108
	Appendices . . . . .	110
2.A	Heterogeneity Main Results . . . . .	110
<b>3</b>	<b>It's Good to be Bad: A Model of Low Quality Dominance in a Full Information Consumer Search Market</b>	<b>117</b>
3.1	Introduction . . . . .	117
3.2	Background . . . . .	120
3.3	The Model . . . . .	121
3.3.1	The asymmetric information subgame . . . . .	123
3.3.2	The full information subgame . . . . .	124

3.3.3	The disclosure decision . . . . .	131
3.3.4	Welfare . . . . .	131
3.4	Extensions & Discussion . . . . .	132
3.4.1	Endogenous Quality . . . . .	132
3.4.2	Signalling . . . . .	132
3.4.3	Multi-product Firms . . . . .	133
3.4.4	Many Firms . . . . .	134
3.5	Conclusion . . . . .	135
3.6	References . . . . .	136
	Appendices . . . . .	138
3.A	Proof of lemma 3 . . . . .	138
3.B	Equilibrium in a competitive market . . . . .	140
3.C	Derivation of Equilibrium when $a_T > 1$ . . . . .	141
3.D	Multiproduct firms . . . . .	141
3.E	For Online Publication: Many Firms in a full information consumer search	144
3.F	For Online Publication: Three levels of quality in the market . . . . .	146

# List of Figures

1	Northern Ireland departmental spending over the fiscal year . . . . .	10
2	UK Central government final month spending over time . . . . .	11
1.1	Average monthly spending in Northern Ireland 2008 - 2013 . . . . .	23
1.2	End of year spending from 1990-1991 to 2013-2014 . . . . .	24
1.3	The exponential and hyperbolic models calibrated to Northern Ireland departmental spending data . . . . .	32
1.D.1	Volatility and Renormalised Spikes . . . . .	56
1.E.1	$\Delta^{12}\ln(\text{Cash Outlays})_t$ and estimated conditional variances . . . . .	64
1.I.1	The exponential and hyperbolic models calibrated to Northern Ireland departmental spending data . . . . .	76
1.I.2	The exponential and hyperbolic models calibrated to Northern Ireland departmental spending data . . . . .	79
1.J.1	Time-variant budgetary taxes in a precautionary savings model . . . . .	82
1.J.2	Expected renormalised spending spike and renormalised volatility in the two models. . . . .	82
1.J.3	t-stats on Monte Carlos simulating the regressions of table 1.2 under the two models . . . . .	84
2.1	Profits in the no advertising fully separating equilibrium . . . . .	93
2.1	Equilibrium with advertising . . . . .	100
2.1	Surplus generated in each equilibrium case . . . . .	104
2.1	Monopolist supplier demand curves and profits . . . . .	107
3.1	Consumer search paths in the full information setting for the consumer with taste $a_i$ . . . . .	123
3.1	Equilibrium demand curves . . . . .	129

# List of Tables

1.1	Calibrated Parameters . . . . .	31
1.2	Volatility and Renormalised Spike . . . . .	36
1.3	Calibrated Time Variant Budgetary Tax . . . . .	41
1.B.1	Regressions on Northern Ireland Spending . . . . .	50
1.B.2	Northern Ireland department summary statistics . . . . .	52
1.B.3	Northern Ireland department abbreviations . . . . .	53
1.B.4	Spending Series And Rollover Regime . . . . .	54
1.C.1	Robustness checks for the gains from imposition of a tax for the 5% exponential discounting case. . . . .	55
1.D.1	Alternative Volatility Measure and Renormalised Spike . . . . .	56
1.D.2	Volatility and Renormalised Spike - Seperate Regressions by budget . . . . .	57
1.D.3	Volatility and Renormalised Spike - Seperate Regressions by budget and no year FEs . . . . .	57
1.D.4	Volatility and Renormalised Spike - Including Remaining Budget . . . . .	58
1.D.5	Volatility and Renormalised Spike - 5 million pounds provision threshold . . . . .	58
1.D.6	Volatility and Renormalised Spike - All 12 months of positive spending . . . . .	59
1.D.7	Volatility and Renormalised Spike - 5 years of spending data . . . . .	59
1.D.8	Volatility and Renormalised Spike - 4 months for calculating volatility . . . . .	60
1.D.9	Volatility and Renormalised Spike - 8 months for calculating volatility . . . . .	60
1.D.10	Volatility and Renormalised Spike - Spending as proportion of outturn . . . . .	61
1.D.11	Volatility and Renormalised Spike - Spending as proportion of provision . . . . .	61
1.D.12	Volatility and Renormalised Spike - Spending as proportion of funds inc. rollover . . . . .	61
1.D.13	Volatility and Renormalised Spike - First Differences and Random Effects . . . . .	62
1.E.1	Regression for Spending Spike Vs Variance . . . . .	65
1.E.2	UK Cash outlays GARCH models . . . . .	65
1.E.3	Regression for Renormalised Spike Vs Volatility & AltVolatility . . . . .	66
1.I.1	All 5% and 10% discounting calibrations . . . . .	77
1.J.1	Precautionary Savings calibrated parameters . . . . .	79

1.J.2 Time-variant budgetary taxes in a Precautionary Savings model . . . . . 81

3.1 Indifferent Consumers . . . . . 125

3.1 Profits and surpluses in each equilibria . . . . . 131

3.F.1 Indifferent Consumers for model with three quality levels . . . . . 147



# Acknowledgements

I am grateful to many people who helped me along the way in writing the papers of this thesis. My principal supervisor Prof. Philipp Kircher was truly brilliant and always supported me throughout my PhD study. In particular I am grateful for how generous Philipp was with his time and this thesis is much better for Philipp's advice and encouragement. I am also indebted to my secondary supervisor Prof. Ludo Visschers who was always a great source of advice when I got stuck on something.

Thank you to Margaryta Klymak for working me on this thesis' third paper and for her friendship. Thank you also to Craig, Victoria, Angus and James Baumann for your support and for flying all the way over to Scotland to visit me occasionally.

I thank also the various other people that have given me advice on various papers of this thesis: Andrew Clausen; Jan Eeckhout; Mike Elsbj; Patrick Harless; José V. Rodríguez Mora; Jozsef Sákovics; Sandro Shelegia; Andy Snell. Thank you also to my examiners Régis Renault and Ina Taneva for their advice on this thesis during my viva.

Finally thank you to all of my colleagues in the PhD program for making this PhD an enjoyable experience including Aspasia Bizopoulou, Johannes Eigner, Mingye Ma, Yulia Moiseevia, Sergei Plekhanov, Daniel Schäffer, Carl Singleton, Alessandro Spiganti and Tomasz Sulka.

# Declaration of own work

I declare that this work was written by myself and is the result of my own work unless clearly stated and referenced. Chapter 3 of this thesis is co-authored with Margaryta Klymak of Trinity College Dublin. I made substantial contributions to this chapter. All parts of this thesis are original and have not been submitted for any other degree or professional qualifications.

Stuart Baumann

June 12, 2017

## Chapter 0

# Introduction and lay summary

The papers of this thesis all look at different aspects of organisational governance and strategy. In particular these papers look at organisations that seem to be behaving in counterintuitive ways. For instance all around the world governments often spend disproportionately large amounts of money in the few months at the end of the fiscal year and in the private sector firms often advertise against their rivals even though by doing so they may face greater competition from these rival firms.

In these papers I look into whether these behaviours are as a result of a strategy or perhaps reflect some form of a problem in organisational governance. I try to analyse the effects on market efficiency and what steps a government or regulator might take to improve the outcome of the market. The approach is generally theoretical but in the case of the first paper on government spending I calibrate a theoretical model to Northern Ireland spending data.

In the rest of this document see non-technical abstracts for my three papers. Note that in order to avoid maths I had to simplify papers considerably so these nontechnical abstracts should not be cited.

## **Putting it off for later - Procrastination and end of fiscal year spending spikes**

The first paper of this thesis considers the issue of end of fiscal year government spending. Governments regularly spend a large portion of their annual budgets in the last few months of the fiscal year. As an example 16% of UK government capital expenditure is done in the final month of the fiscal year. The Northern Ireland government spends more than 3 times as much on capital goods in the final month as compared to the average month. In 2014-2015 UK government capital

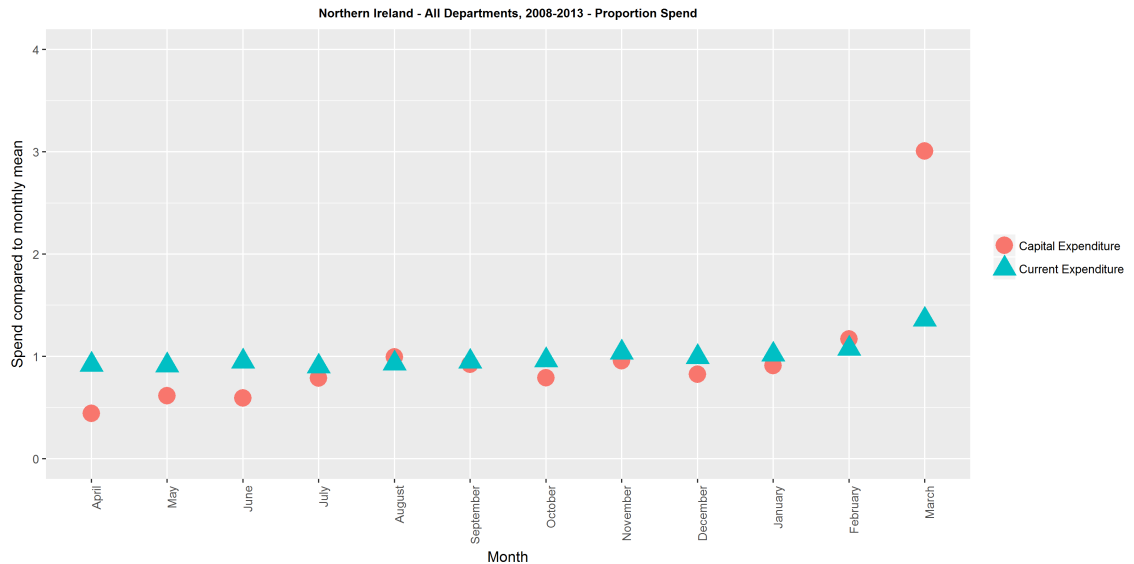


Figure 1: Northern Ireland departmental spending over the fiscal year<sup>0.1</sup>

spending rose in the last month of the year by £3.9 billion which is more than the cost of the U.S.S. Ronald Reagan Supercarrier.

This kind of spending is regularly criticized for being wasteful, for instance a 1980 US Senate subcommittee estimated that 2% of US government contract expenditure was wasted due to rushing at the end of the fiscal year. Recent research by Liebman and Mahoney (2013) has also backed up this view using more modern data on the value of US government IT spending.

But what is causing it? We can note that any explanation needs to have two features. The first is that there is some force which encourages departments to hold back some money in the early months of the year. The second is some force which encourages departments to spend this held back money at the end of the year.

The Liebman and Mahoney (2013) view is that this kind of spending is caused by government departments building up a rainy day fund (or “precautionary savings”) to use in case of sudden expenses that occur. In this model the force for holding back funds is uncertainty. The force to spend the money at the end of the fiscal year comes from expiring budgets that means that departments cannot save money from one fiscal year to the next. The Liebman and Mahoney (2013) solution to the problem is to allow government departments to “rollover” funds from one fiscal year to the next. They argue that with this policy in place departments will build up a rainy day fund once and then hang onto it and thus there will be no end of fiscal year spending spikes.

This is an interesting paper and the explanation for end of fiscal year spending that they propose is intuitive. My analysis suggests however that precautionary savings and expiring budgets provide at best a partial explanation. The first bit of evidence comes from the UK which did implement rollover budgeting starting in the 1998-1999 fiscal year. Heightened end of fiscal year spending did

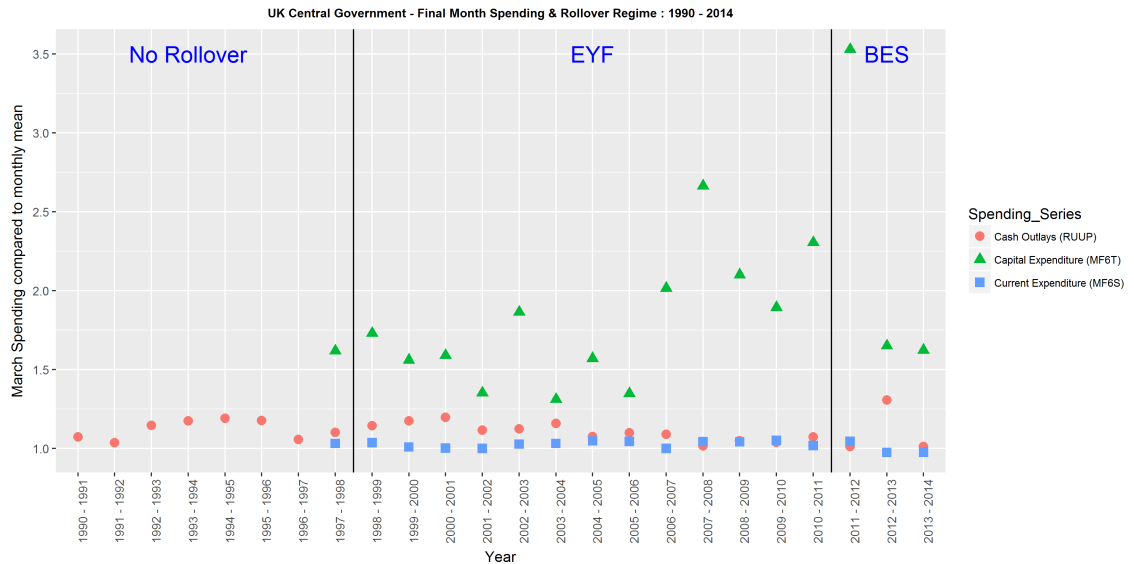


Figure 2: UK Central government final month spending over time

not seem to change as a result. Indeed in every fiscal year since final month government capital spending has been greater than the monthly average (it is usually a bit more than 50% higher). This is quite significant as it suggests that if rollover does not abate end of year spending then expiring budgets are not underlying the problem. This can be seen in figure 2.<sup>0.2</sup>

The second bit of evidence I assemble comes from a cross section of Northern Ireland government departments from 2008-2013. The Liebman and Mahoney (2013) theory would suggest that departments that have the most uncertainty would build up the most precautionary savings. These departments would then exhibit the greatest spending spikes at the end of the fiscal year. I test this with my Northern Ireland government departmental spending data sample and find that actually the departments with the least uncertainty have the greatest spending spikes which is opposite to the implication of the Liebman and Mahoney (2013) model. The third bit of evidence comes from the amount of uncertainty you need to have in order to explain the amount of money departments hold back to spend at the end of the year. When this is examined closely the implication is that government departments face demand for their services which is many times less predictable than inflation, unemployment or the prevailing interest rate. The implied level of uncertainty seems unrealistic in the real world.

I propose a different explanation based around procrastination. Here public servants hold back some money because there is effort in spending it and they would prefer to put off the effort. At the

<sup>0.2</sup>Here you can see UK final month spending for every year from 1990-1991 to 2013-2014. On the y axis of this figure is March spending divided by the average month of each fiscal year while on the x axis are all of the fiscal years. There are three spending series: government cash outlays; capital expenditure and current expenditure. There are three budgetary regimes in this period: No rollover where money could not be saved into the next fiscal year; End of Year Flexibility (EYF) where full rollover was allowed and finally Budgetary Exchange System (BES) which allowed rollover but with more restrictions than the EYF system. The key takeaway from this figure is that the implementation of rollover budgeting did not seem to impact end of fiscal year spending spikes.

end of the fiscal year they are judged (in part) by their usage of their budget and so they rush to spend money in order to look good for their managers. I show this explanation is more consistent with the three features of spending noted in the preceding two paragraphs than the precautionary saving model.

Whether precautionary savings or procrastination is the correct explanation is important for determining how to fix the problem. While the precautionary savings explanation would suggest end of year spending could be disincentivised by allowing government to rollover their funds between fiscal years this approach will not help if end of year spending is caused by procrastination. So instead I propose a more direct way in which a parliament can encourage departments to not procrastinate.

My solution is a kind of tax on departmental spending that increases throughout the fiscal year.<sup>0.3</sup> This means that it is relatively cheaper for government departments to spend in the early months of the fiscal year relative to the last months of the fiscal year. To give a simple example consider a government department with a budget of £120 that faced a tax of 0% in the first six months of the year and 20% in the second six months of the year. This means that if the government department spent their entire budget in the first six months of the year they could buy £120 worth of stuff but if they spent it in the second six months of the year they could only buy £100 worth of stuff. This gives an incentive for these departments to avoid procrastination. It also means that if departments do procrastinate then at least some of this rushed spending is returned to treasury through these “taxes”.

So how beneficial could such a tax be? This is a hard question as there are two things to consider. One is some money wasted at the end of the fiscal year could be saved with a mechanism like this tax. The second is that some value might be getting lost if departments have a good spending opportunity but the parliament will not give funds to the department for fears they will procrastinate and subsequently spend the money poorly. That is to say there is an opportunity cost of end of year spending in that it deters parliaments from giving budgets when it would be better for them to do so (in the absence of end of year spending problems).

To try and get an idea of the total value that a tax could deliver I calibrated an economic model of departmental procrastination with spending data from Northern Ireland. I then inserted my tax into this model and saw how government departments reacted to it. What I found is that the value of government spending increased substantially in every variation of the model that I considered. Thus the recommendation of this paper is that such a tax should be implemented to deter end of fiscal year spending spikes and encourage more even spending throughout the fiscal

---

<sup>0.3</sup>Of course this is not really a tax as the taxed money goes from a government department’s budget into general treasury. It could alternatively be thought of as a budgetary reduction through time but I personally find that thinking about it kind of like a tax is more intuitive.

year.

## Comparative Advertising: The role of prices

This paper investigates some of the strategic aspects related to a firm's decision to engage in comparative advertising. Comparative advertising is where firms compare their good to a rival good in their advertising. There are a few papers on this in economics (and many more in marketing) that looked at comparative advertising as disclosing the attributes of goods as compared to the attributes of rival goods.

For instance one paper (Barigozzi et al., 2009) argues that firms engage in comparative advertising in order to show that they are high quality. By comparing their good's quality to a rival good in their advertising, a firm opens itself up to litigation if the claims they make are unreasonable. This means that only a high quality firm would do this and hence comparative advertising allows firms with high quality goods to separate themselves from lower quality firms.

The other key paper in this field (Anderson and Renault, 2009) thinks about comparative advertising as showing individual consumers what the best good is for them. By doing this a weaker firm may be able to ensure that some consumers come to them. This mechanism is probably clearer with a simple example. Consider that you know that 70% of people like coke more than pepsi and 30% of people like pepsi more. You do not know if you will like coke or pepsi more however. If you were to buy a good however, you would pick coke because there is a 70% chance that you are one of the people that like coke more. This is bad for Pepsi because everyone that doesn't know what they like can reason with the same logic and buy coke so Pepsi makes no sales. But what if Pepsi could engage in comparative advertising which tells everyone all of the features of coke and pepsi? Then people would know whether they are a coke person or a pepsi person before they go to buy the good. This will mean that Pepsi will get 30% of the market which is much better than the 0% they could manage without comparative advertising.<sup>0.4</sup>

To me however it seems like there was a significant amount of comparative advertising that could not be explained by these previous papers. There are certain examples of comparative advertising that contrasted a firm's price with an external firm's price for essentially the same (or very similar) goods. Here the focus is not on contrasting good characteristics because often no large difference exists.

Take for instance the advertisements of Progressive Direct, an American auto insurance company that gives prospective consumers the prices offered by competitors for comparable insurance

---

<sup>0.4</sup>Please note that in order to avoid maths this paragraph does simplify this paper considerably, particularly by avoiding talk about the strategic pricing choices of coke and pepsi.

plans. They air advertisements promising “we compare our direct rates side by side to find you a great deal, even if its not with us”. Another example is provided by Amazon who allow competing firms to sell products on their site.<sup>0.5</sup>

As a final example consider the online travel agent Skyscanner which allows searchers for flights to automatically get prices from competing services like Expedia. This behaviour seems counterintuitive. Considering that a consumer is currently on Skyscanner’s webpage, why are they making it easier for that consumer to go to a rival website?

My explanation of this is built around the general tendency of consumers to judge quality by price. If a firm sets a high enough price for an item then a consumer can reason “If it were a low quality good that was cheap for the firm to produce then the firm would be want to sell it for a low price (with high sales quantity). The price is high though which must mean the good must cost alot to produce and it is a high quality good”. Thus setting a high price can be a method through which a firm can show consumers that their good is high quality.

Often however the price that is required to communicate this high quality is so high that the firm is not making much money because the quantity they sell is so low. If the firm could communicate their high quality another way they would like to do so they could drop their price to a lower level (with higher sales volume).

This is the basic insight through which I explain price comparative advertising. Firms sell the same goods as their rivals and consumers can realise that as the goods sold are identical, their quality must also be identical. Instead of charging a high price to signal high quality a firm can instead charge a lower price whilst engaging in price comparative advertising against a rival firm. The consumers can see the high price charged by the rival firm and realise that the good is high quality and buy from the advertising firm (which has the lower price).

The rest of this paper is comprised of analysing how this market will operate in equilibrium. Clearly if all firms were advertising like this there would not be any high pricing firms left to convince consumers the good was high quality. As it turns out advertising against rivals means that a firm can signal their high quality more efficiently than by setting a high price: the downside is that this firm faces greater price competition from the firm they advertise against. This means that in equilibrium there will be firms setting a high price and avoiding competition as well as firms that face greater price competition whilst advertising. This conforms to the intuition that the firms offering price checks tend to be the firms that are the lower pricing firms in the market.

There are other things I find that are less intuitive though. One is that firms do not earn any greater profit from advertising than from setting a high price. Whilst they do not need to set such

---

<sup>0.5</sup>Amazon also receives a portion of the revenues from these external sellers on their website. Whilst these revenues no doubt play some role in Amazon’s decision to operate this marketplace it is also true that this allows Amazon customers to compare Amazon’s prices with other vendors.



a high price (with low sale quantity) and thus signal more efficiently, any extra profits from this are completely eroded from additional price competition. Thus in equilibrium, firm profit will be unchanged from the case where there is no advertising. The effect on consumers however is very positive as they benefit from lower prices both from firms not price signalling as much as well as from greater price competition in the market.

Whilst regulators in Europe and the United States have long allowed comparative advertising it is still banned in countries like Saudi Arabia and China. In Japan it is allowed but seldom used as it is considered impolite by consumers. The conclusion of this research is that market efficiency and prices for consumers could both be improved by more widespread comparative advertising. Thus the recommendation of this paper is that comparative advertising should be legalised and encouraged by legislators and regulators.

## **It's good to be bad: A Model of Low Quality Dominance in a Full Information Consumer Search Market (with Margaryta Klymak)**

This paper starts with a simple question: Why do some firms tell consumers that they are low quality? That may seem a strange question but if you look at some advertising that is around that certainly seems to be part of the message they are trying to communicate. Take Ryanair for example, a self proclaimed “no frills” airline. Ryanair regularly communicates that they are low quality through the way they approach the media. As an example in 2011 they publicly floated the idea of charging for toilet use on a plane. Their CEO has also made many statements to the media suggesting low quality like when he said *“Anyone who thinks Ryanair flights are some sort of bastion of sanctity where you can contemplate your navel is wrong. We already bombard you with as many in-flight announcements and trolleys as we can. Anyone who looks like sleeping, we wake them up to sell them things.”*

Looking at the market for lawyers in America reveals a similar story. There are no less than 3 American lawyers who call themselves “the hammer”. In Texas there is a lawyer named Bryan Wilson who advertises by calling himself the “Texas Law Hawk”, riding jetskis and throwing sticks of dynamite in his commercials. This kind of advertising is strongly discouraged by the American Bar Association who warn that “lawyers should consider that the use of inappropriately dramatic music, unseemly slogans, hawkish spokespersons, premium offers, slapstick routines or outlandish settings in advertising does not instill confidence in the lawyer or the legal profession”.

It certainly seems that these firms want to communicate that they low quality to the market or

at the very least they are unconcerned with being perceived that way. Why would a firm want to disclose their low quality however? One potential explanation is that maybe this kind of advertising gets free media coverage. If this coverage meant that many consumers knew about the firm but judged it to be low quality however then it is not clear the firm would be better off.

This paper presents a different explanation in a consumer search model. Consumer search theory is an area of economic theory & industrial organisation where it is possible to get really counterintuitive results. This is great from one perspective as counterintuitive results can be quite interesting. It also means however that it can be difficult to explain a result without resorting to a mathematical model. For this reason I have to use a little bit of maths in what follows to make it make sense.

Consider a market where there is a low quality firm and a high quality firm each of which makes a good costlessly. The consumers are a diverse bunch with some that are willing to pay a lot to get better quality (“high taste consumers”) while some others are nearly indifferent and would not pay much to upgrade from the low quality good to the higher quality good (“low taste consumers”).<sup>0.6</sup> Now consider that every day every consumer walks to one of the firms. Each firm now needs to determine a price to offer all of the consumers that approach them. The consumers can accept that price and buy the good; walk to the other firm with a search cost (or bootleather cost) of \$1 or go home and get a utility worth \$0.

So what price should a firm choose to charge? One thing that a profit maximising price must satisfy is that one of the consumers should be just about indifferent to buying and taking their outside option (of either walking to the other firm or going home). If this were not the case then the firm could increase their price and get a higher margin without losing any consumers because no one is close to being indifferent.<sup>0.7</sup> This is the key aspect of the firm’s pricing decision that we will use to figure out what happens in this market.

Let’s look at the problem for the high firm. Let’s say they choose a midtaste consumer to be indifferent. This means the firm would like to sell to this consumer and all of the consumers with a higher taste than her. Now to determine the price that the high firm should charge this consumer. This is largely determined by the outside option of walking to the other firm to buy there. She anticipates that she can get a utility of \$5 if she bought the low firm good (after paying the low firm’s price). Let us also say that she values the high good at \$10. We can now work out that her outside option is worth \$4 and so to be made indifferent the firm will charge her (and everyone with a higher taste) \$6 which will give her a utility of \$4. Now consider what happens the next

---

<sup>0.6</sup>You can think of the market for flights: everyone prefers more legroom to less legroom but very tall people care a lot and very short people don’t care very much.

<sup>0.7</sup>By contrast if a consumer is about indifferent to buying the good than a price increase could tip them to prefer their outside option and the firm would lose this consumer.

day. She can anticipate that if she goes to the high firm again she will end up with a utility of \$4. If she goes to the low firm initially however she will end up with a utility of \$5. So she will go to the low firm initially.

Now they have lost her the high firm wants to make a slightly higher taste consumer indifferent to maximise their profit. Lets consider the case of this new consumer to be made in different. This consumer anticipates that he can get a utility of \$5 from buying from the low firm (after paying the low firm's price) and values the high good at \$11. He also faces the same \$1 search cost. Now his outside option is worth \$4, the high firm should charge him a price of \$7 to make him indifferent and he will end up with a utility worth \$4. Now consider what happens the next day. He can anticipate that if he goes to the high firm again he will end up with utility worth \$4 . If he goes to the low firm initially however he will have ended up with a utility of \$5. It is again better to go to the low firm initially.

Now it should be clear that there is some unravelling going on. Every day the low firm manages to peel off those high firms consumers that care the least about quality. This will continue until the high firm has no consumers left. The low quality firm will achieve market domination. Note that a key thing that is necessary for this to work is that consumers choose what firm to approach when they enter the market. If consumers did not know the quality of the two firms then this mechanism would not work and the high quality firm would be able to keep a decent amount of consumers. Thus this unravelling result can have the effect of encouraging low quality firms to disclose their low quality and this disclosure can drive the high quality firm out of the market.

Another key element is that a firm cannot promise to set a particular price before consumers visit the firm. If firms could commit to a certain fixed price then the unravelling could not continue. A key element of the unravelling is that every time the high firm loses a consumer who cares about quality relatively little, they increase their price because their remaining consumers care more about quality. If a price could be committed to then this could not continue. This means that clearly this model is not appropriate for markets where prices are set and posted, however it would be appropriate for other markets where prices are generally unknown or set after a consumer visits a firm.

The policy implication of this paper is that the market can be made more efficient by banning low quality firms from advertising or communicating to the market. This is a problematic implication as it could be argued that consumers and firms have a moral right to free communication before entering into a transaction. Nonetheless there may be some cases where it is feasible and desirable to ban low quality disclosure. The best example that I know of is the American market for lawyers where the American Bar Association (ABA) banned all lawyers from advertising from

---

1907 until 1977 (when the Supreme court overturned that advertising ban on freedom of speech grounds). This paper implies that by allowing advertising the market is lead to a state where low quality lawyers get a large amount of business with negative impacts on consumers and high quality lawyers. Thus this paper suggests an additional reason in support of the ABA's argument lawyers should not advertise.

# Chapter 1

## Putting it off for later: Procrastination and end of fiscal year spending spikes

**Abstract:** Many governments internationally exhibit heightened end of the fiscal year spending. These end of fiscal year spending spikes often concern policy makers due to their tendency to result in lower quality spending. This paper uses UK data to offer evidence against the precautionary savings explanation for spending spikes. An alternate explanation is offered with procrastination driving heightened end of fiscal year spending. A new technique of time variant budgetary taxes is calibrated to the model and shown to be effective for smoothing spending and improving spending efficiency throughout the fiscal year.<sup>1,1</sup>

**JEL Codes:** H11, H50, H61

**Keywords:** Government spending; fiscal year distortions

### 1.1 Introduction

Heightened spending at the end of the fiscal year is a salient feature of government spending. It has been observed in a variety of contexts including Canadian Military spending [Hurley et al., 2013], United States Government Procurement [Liebman and Mahoney, 2013], West German local

---

<sup>1,1</sup>Stuart.Baumann@ed.ac.uk, I would also like to thank for useful comments Philipp Kircher, Ludo Visschers, Mike Elsby, Javier Gómez Biscarri, Margaryta Klymak, Sergei Plekhanov, José V. Rodríguez Mora, Daniel Schäffer, Carl Singleton and Andy Snell. For providing their spending data I would like to thank the Northern Ireland government. This work was supported by the Economic and Social Research Council. Notwithstanding the advice I have received from many sources, any errors here are my own. An earlier version of this paper has been circulated as Edinburgh School of Economics discussion paper 260

government job training programs [Fitzenberger et al., 2014] as well as in the Australian government [Mannheim, 2012]. It can also be seen in the U.K. where 16% of governmental capital spending occurs in the last month of the fiscal year. In the 2011-2012 fiscal year this heightened spending represented an extra £9.5 billion of expenditure relative to the average month of this year.

This concentration of spending in the later months of the fiscal year has presented a problem for policy makers due to the tendency for rushed spending to be of lower quality. Some evidence for this comes from the United States where as far back as 1980 a senate subcommittee provided a conservative estimate that 2% of contract expenditure is wasted at the end of the fiscal year due to spending being rushed [US Senate subcommittee on oversight of government management, 1980]. A more recent analysis is provided by Liebman and Mahoney [2013] who analysed data on US government IT procurement and found that projects commissioned at the end of a fiscal year are 2.2 to 5.6 times more likely to receive a quality score of “low quality”.

In light of this inefficiency, this paper uses UK and Northern Ireland data to look at two key research questions. The first is what causes government departments to defer spending until the end of the fiscal year and the second is what policy responses are effective in counteracting this problem and improving the efficiency of government spending. To the former question, there are two putative explanations that are often offered but prove spurious. The first is that in many countries (but not currently the UK) if the departments do not spend their whole budget they lose the unspent portion back to general treasury. The second is that if they do not spend their whole budget then their budget for future fiscal years can be reduced. While both of these may be arguments for spending the entire budget in a fiscal year they are not arguments for deferring spending until the final months of the fiscal year. There must be some other mechanism that encourages departments to defer spending until the end of the fiscal year.

As of yet the most developed economic model to offer such a mechanism is that of Liebman and Mahoney [2013] who explained end of fiscal year spending through precautionary savings coupled with annually expiring budgets. Government departments build up a rainy day fund to insure themselves against stochastic shocks. At the end of the fiscal year they cannot carry over this fund to the subsequent fiscal year and hence they spend it all. The clear policy implication is to allow the use of “rollover” policies where unspent appropriations can be saved to be spent in future years.

This paper shows however that there are solid reasons for believing that this mechanism cannot fully explain the spending spikes seen in practice. Amongst other indicators this paper shows that heightened end of fiscal year spending persisted when the United Kingdom implemented a rollover policy to allow departments to save funds between fiscal years. There is no sign that the

implementation of this policy had any effect in smoothing fiscal year spending.

This paper offers an alternative explanation of spending spikes being caused by procrastination.<sup>1,2</sup> Departmental performance is primarily measured on a fiscal year basis and discounting provides an incentive for departments to delay spending money (and incurring disutility from the associated effort of spending it wisely) until later in the fiscal year. It is shown that the testable predictions of this model are more consistent with the UK and Northern Ireland experience than the precautionary savings model. This distinction between models is important as both models yield substantially different policy prescriptions.

In this model there are two ways in which spending spikes can be inefficient. The first is that the diminishing returns of government spending suggest that measures to smooth spending over the fiscal year can increase the efficiency of government spending. The second is that there are production shocks to government departments that are i.i.d. throughout the fiscal year. Heightened spending at the end of the fiscal year suggests that governments are not varying spending optimally in response to shocks with beneficial shocks earlier in the year being underutilized.

A new budgetary mechanism, time variant budgetary taxes, is suggested to smooth spending throughout the fiscal year. With this mechanism spending worth  $x$  pounds would cost  $x$  pounds for a department in month 1 and  $x\theta$  pounds in month 12 (with  $\theta > 1$ ), thus making procrastination more costly for the department. This tax is then calibrated to the model which shows they are suitable for smoothing spending over the fiscal year and can lead to greater value spending.

This paper proceeds as follows. Section 1.2 demonstrates fiscal year spending patterns and highlights a few aspects of the UK budgetary system that are relevant for the evaluation of testable model predictions. The procrastination model of government spending is then presented and calibrated to the data in section 1.3. Section 1.4 briefly outlines the precautionary savings model of Liebman and Mahoney [2013] before comparing the testable predictions of each model in the UK experience. Policy implications to smooth spending are then discussed and calibrated in section 1.5 before section 1.6 concludes.

## 1.2 Government Spending in the United Kingdom

The United Kingdom operates on a fiscal year running from April to March. Budgets are split by the purpose of the funds including current expenditure and capital expenditure. Current expenditure budgets are intended for the payment of operational expenses such as wages, rent and

---

<sup>1,2</sup>The term “procrastination” is often taken to imply time inconsistency in economics (for instance see Prelec [2004]). The model presented in this paper does not require time inconsistency. Two calibrations are done in section 1.3, one with time inconsistent discounting with the other using exponential discounting in a completely rational framework.

transfer payments. Capital expenditure budgets refer to money which is allocated for investment in durable assets that will benefit the department over an extended period. This may include computers, furniture, vehicles & building renovations.<sup>1.3</sup>

The United Kingdom government traditionally budgeted on a fiscal year basis with unspent funds being withdrawn from departments at the end of each fiscal year. This was changed with the introduction of a rollover mechanism in the 1998-1999 fiscal year [Crawford et al., 2009]. Called the “End of Year Flexibility” (EYF) scheme, departments were able to retain an entitlement to all of their unspent appropriation from previous years. Whilst treasury encouraged departments to make rollover available to all managers within each department, there is some evidence of cases where rollover did not cascade to lower level budget holders [Hyndman et al., 2007].

Concerned at the levels of “savings” that departments were accruing under the EYF scheme, the incoming coalition government overhauled this system in the 2010 spending review. The overhaul amounted to the implementation of a more restrictive rollover system as well as the loss of all savings accrued under the previous system [HM Treasury, 2010, Section 1.17]. The new “Budget Exchange System” (BES) came into force for the 2011-2012 fiscal year and included set limits of between 0.75% (for larger agencies) and 4% (for smaller agencies) on how much expenditure could be carried forward [HM Treasury, 2013, Chapter 2]. In addition the Budget Exchange System included measures to prevent savings from being accumulated over long periods of time by only allowing money to be rolled over once.

### 1.2.1 Northern Ireland Spending

The primary dataset for this study consists of monthly spending for various departments of the Northern Ireland (NI) government for the fiscal years 2008-2009 to 2012-2013.<sup>1.4</sup> This data is segmented by capital and current expenditure and is disaggregated for all NI government departments.<sup>1.5</sup> In addition data is available showing how much each department carried between fiscal years.

The NI government is a good case study as it provides a wide range of governmental services and consists of departments of varying size. In addition it is generally subject to the same budgetary restrictions as the wider UK government. Thus this NI dataset offers variation between departments without variation in budgetary framework or general environmental conditions.

Figure 1.1 shows average monthly spending (aggregated across all Northern Ireland departments

---

<sup>1.3</sup>Budgets are further split by the planning horizon of the funds. This division is less important for the purposes of this paper and discussion is deferred for online appendix 1.B.

<sup>1.4</sup>This data as well as the UK central government current & capital expenditure series (detailed in section 1.2.2) are based on accrual accounting meaning expenditure is recorded in the month when the underlying economic transaction took place rather than the month in which cash was exchanged.

<sup>1.5</sup>Summary statistics on a departmental basis are provided in tables 1.B.2 and 1.B.3 in online appendix 1.B.



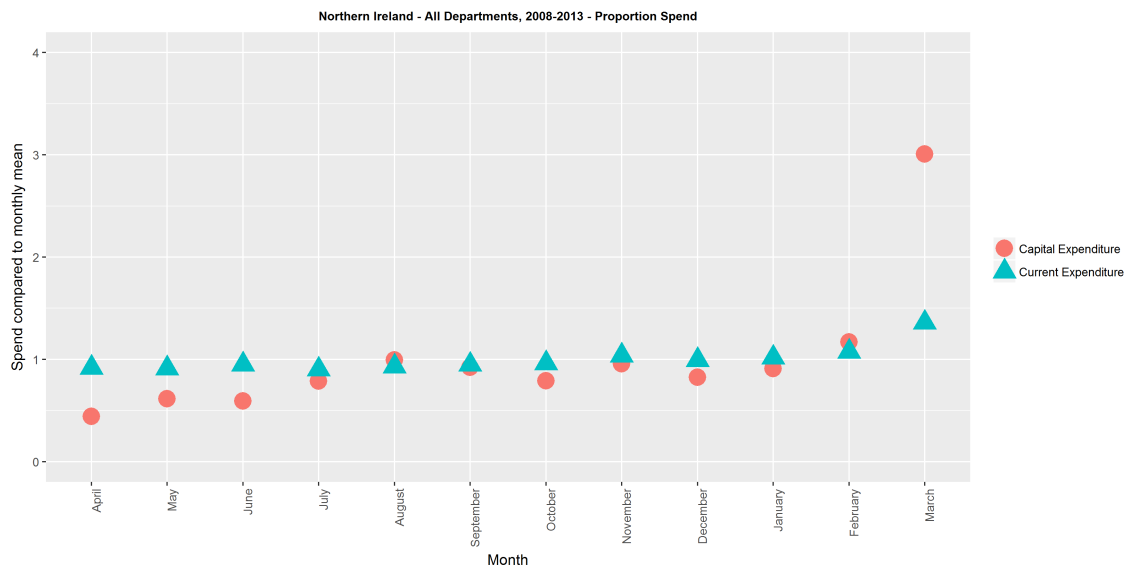


Figure 1.1: Average monthly spending in Northern Ireland 2008 - 2013

and the fiscal years 2008 - 2013) divided by the annual average monthly spending level, a measure of in-year spending that will be termed the spending ratio. A month in which spending is typically equal to the average annual monthly expenditure would have a spending ratio of 1. There is a small spending spike for current expenditure, the average final month spending ratio being 1.38. This is not particularly surprising as wages, transfer payments and other routine expenditures are inherently regular expenses. Capital expenditure is generally more discretionary and it can be seen that they exhibit heightened spending at the end of the fiscal year with a final month spending ratio of 3.06 over the years in the sample.<sup>1.6</sup>

There is some variation in the size of each year’s spending spike but there does not appear to be any consistent pattern governing this variation. There does not appear to be any consistent cointegrating relationship between the capital and current expenditure spending ratio series for each department. In neither the case of Northern Ireland nor the UK central government is there any evidence of a link between elections and the pattern (rather than level) of spending over the fiscal year.

## 1.2.2 UK Central Government Spending

The secondary dataset for this research consists of UK central government aggregate spending data split by current or capital expenditure. To give a relative idea of the size of these two categories of spending, for the 2013-2014 fiscal year capital expenditures contributed 5.5% of the spending

<sup>1.6</sup>The size of capital spending spikes relative to current expenditure spending spikes is consistent with the finding (using USA data) of Liebman and Mahoney [2013, pages 2-3] of greater spending spikes for “maintenance and repair of buildings, furnishings and office equipment, and I.T. services and equipment” all of which are typically capital expenditure items.

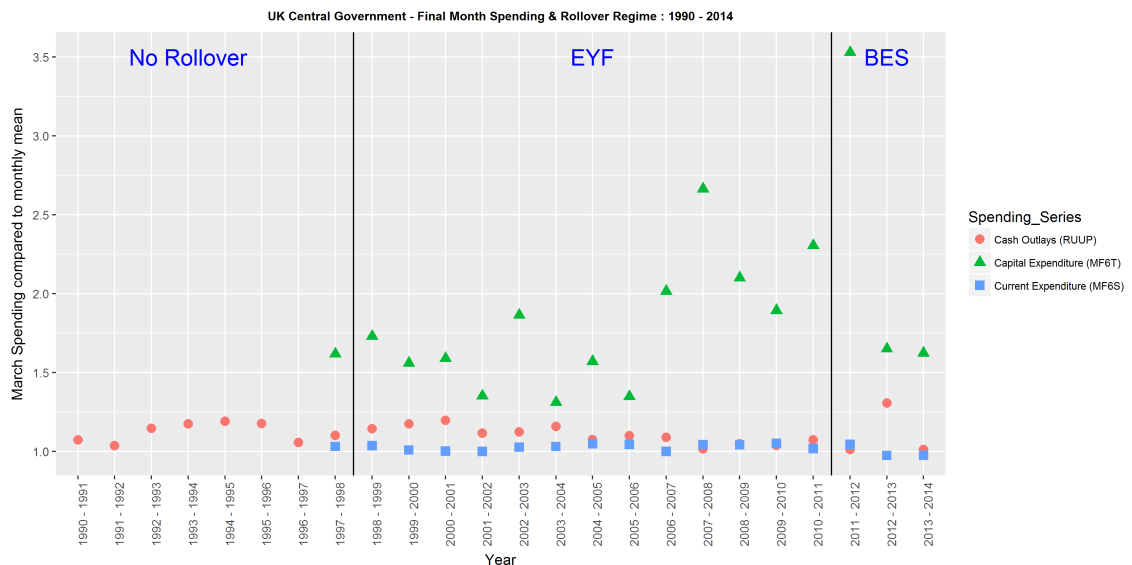


Figure 1.2: End of year spending from 1990-1991 to 2013-2014

total. One additional series is available for net central government cash outlays, a rough proxy for general government expenditure.

The spending ratios for the final month of each fiscal year from 1997-1998 to 2013-2014 are presented in figure 1.2. It can be seen that in every fiscal year capital spending was significantly heightened in the last month of the fiscal year. This figure also displays when the rollover systems were changed. The first change was from no rollover to the EYF system for the 1998-1999 fiscal year. The second change is the replacement of EYF by BES for the 2011-2012 fiscal year. It can be seen that neither the implementation of the EYF scheme or its abolition and replacement with a weaker scheme had any sizable effect of the size of end of fiscal year spending spikes (see also Crawford et al. [2009, Figure 3.2]). As will be discussed in section 1.4.2, this is an important testable prediction of the models to be presented in sections 1.3.1 and 1.4.1.

### 1.2.3 The quality of end of fiscal year spending

An important consideration is the quality of end of fiscal year spending. Some qualitative evidence comes from Hyndman et al. [2007] who conducted interviews with various budget managers in the UK public sector. Some interviewees argued that the potential for waste was abated by departmental controls, primarily including a reserve list of nonessential spending items which can be used if leftover funds are available. Other interviewees were more critical of end of fiscal year spending and referred to this practice pejoratively as “flag-pole painting”. It is suggested that items from this reserve list were of questionable quality and furthermore that such spending is often “over the odds” in terms of the prices contractors charge. Whilst some departments tried to prevent this

by getting contractors to commit to fixed prices for whole fiscal years, these contractors are often fully engaged at the end of year leaving the department having to find a different contractor at a higher price anyway.

Whilst no quantitative research is available based on UK data, Liebman and Mahoney [2013] use data on United States IT procurement controls to examine the quality of year-end spending. As they note IT spending is ubiquitous across government agencies and the quality of IT spending may be a valid proxy for spending more generally. They have access to some ordered categorical data series for the quality, timeliness and cost of IT projects commissioned at various points of the fiscal year. They find that projects commissioned at the end of a fiscal year are between 2.2 and 5.6 times more likely to be rated as low quality. An earlier paper was that of McPherson [2007] who interviewed various US Department of Defence managers with budgets ranging from \$200 000 to \$20 billion. 95% of these managers agreed that end of year spending was a problem with the managers estimating that 24% of spending from the last two months of the fiscal year goes to low priority projects and 8% is “at least partially wasted [McPherson, 2007, page 40]”.

It is plausible that these conclusions from the United States would be valid in the United Kingdom. Whilst the political systems of each country differ, the principal-agent relationship between legislative bodies that appropriate budgets and a nonpolitical public service that spend them is similar. A key constraint in end of year spending is that some production inputs such as departmental time and resources cannot be scaled up to the same extent as spending which leads to diminishing returns. This is as true for the United Kingdom as for the United States.

As was the case in Liebman and Mahoney [2013] this paper models production as having diminishing returns to spending.<sup>1.7</sup> A further mechanism for inefficiency is present in the stochastic model (section 1.3.2) where end of year spending leads to inefficient variation in spending over the fiscal year in response to i.i.d. production shocks.

### 1.3 Procrastination

Numerous explanations could be offered for heightened end of fiscal year spending. This section will discuss procrastination whilst section 1.4 discusses the precautionary savings model of Liebman and Mahoney [2013].<sup>1.8</sup> A recent paper of Frakes and Wasserman [2016] used a micro level dataset to find evidence of procrastination in the public sector. They examine US patent office workers who are given a quota of patent applications to review with deadlines occurring every two weeks. It was found that these workers completed about half of their tasks on the last day of the quota

<sup>1.7</sup>Further discussion of the diminishing returns assumption is available in online appendix 1.F.

<sup>1.8</sup>Alternate and less plausible explanations of spending spikes are briefly discussed in online appendix 1.F.

period. Consistent with the view that this was caused by procrastination, this backloading of effort was exacerbated when the patent office's telecommuting program was implemented and workers were subject to less supervision.

The model of procrastination that this paper proposes is motivated by two key observations. The first is that there are effort costs to departments from efficiently spending money. There often exists an extensive procedure for spending public funds, implemented to ensure probity, transparency and value for money. Spending money in a way that satisfies all such requirements is likely to be a strenuous activity. For instance in order to employ a contractor to perform a maintenance operation on a bridge, a public sector worker would have to advertise and operate a tender process before regularly monitoring quality, safety and traffic control while the works are underway. Outsourcing this activity to private sector project management firms typically costs between 7-15% of the project value [Bryne, 1999] which gives an idea of the effort costs involved.

The second observation is that ministers, managers and departments are often (formally and informally) judged on their performance to a substantial degree on a fiscal year basis. At a ministerial level some evidence for this comes from Hyndman et al. [2007], where many interviewees mentioned political pressure to avoid potentially embarrassing underspends. One interviewee recounted pitching a three year programme to a minister who refused it in favor of a one year plan because he had the political need to show results each year to other politicians [Hyndman et al., 2007, p.230-231]. Performance evaluation is also important for managers with impacts on both the bonuses paid to these managers as well as career prospects. The former of these is observable and it can be seen that these bonuses are structured around the fiscal year being distributed in October for performance in the preceding fiscal year [Hope, 2012]. These bonuses can be significant with managers in the Serious Fraud Office taking £3900 in both the 2008 – 2009 and 2009 – 2010 fiscal years [Serious Fraud Office, 2010].

This annual pattern of monitoring is also evident at a departmental level with all regular oversight reporting (both from parliamentary committees and the audit office) as well as general financial reporting being produced and published on a fiscal year basis. A key part of informal performance evaluation comes from the media and politicians who judge departments to a large extent based on this fiscal year reporting. Hyndman et al. [2007, p.234] write that even with the possibility of rollover “there might still be pressure to spend up to the budget rather than carry forward unspent surpluses because of the potential media, or perhaps political, pressures”. Similar points have been made in the United States where the United States General Accounting Office [1980, p.10] reported that “in spite of large and increasing unobligated balances, agency personnel seem impelled to obligate all available funds in the year appropriated, without distinguishing

between funding periods, to avoid agency reprogramming actions or congressional inquiries about unspent appropriations". In these cases a lack of rollover ability is not the culprit as these funds were not linked to a specific fiscal year.<sup>1.9</sup>

This feature of performance being evaluated at fiscal year intervals is easily rationalised. Performance is often measured in cost per output terms and therefore cannot be examined independently of expenditures. Thus there are economies of scale in examining performance at the same intervals as the fiscal year (budget) interval. This suggests that delayed utility from expenditure is likely a feature of other budgetary systems where spending spikes are observed.<sup>1.10</sup>

This paper will argue that the measurement of performance at discrete intervals can lead to a spike of activity towards the end of the interval. In the same way students often exert heightened effort in the weeks before their exams, departments spur to action in the lead up to the end of the fiscal year when performance is evaluated. This burst in activity explains end of fiscal year spending spikes.

### 1.3.1 A deterministic model of procrastination

Here a simple model will be developed in order to illustrate how procrastination can lead to end of fiscal year spending spikes. This is done in a deterministic setting and without loss of generality departments are not allowed to rollover funds.<sup>1.11</sup> Aside from the tractability advantages of a deterministic setting, this highlights the differences between this model and the precautionary savings model (to be outlined in section 1.4.1) where uncertainty is entirely responsible for heightened end of fiscal year spending.

There are two time periods and two agents, a representative department and the parliament. The representative department chooses when to spend in the fiscal year subject to an annual budget of  $B$ . Spending in period  $t$  is denoted  $x_t$ . A department receives positive utility from its performance, a monotonically increasing function of its expenditure denoted  $v(x_t)$  - however departmental performance is only realised at the end of the fiscal year.<sup>1.12</sup> A department realizes disutility from effort which is realised immediately. Effort in a period is a continuously differentiable increasing function of spending and is denoted  $e(x_t)$ . Discounting applies between periods via a

<sup>1.9</sup>A recent international example of this kind of political pressure comes from Russia where the governor of Saint Petersburg, Georgy Poltavchenko, publicly denounced public sector managers and denied them bonuses as they collectively spent only 92.4% of the budget allocated to them in the 2014 fiscal year, an amount he thought insufficient [NTV News, 2015].

<sup>1.10</sup>More discussion of the UK's performance monitoring system as well as some evidence of similar behaviour in other countries is available in online appendix 1.F

<sup>1.11</sup>Note there is no loss of generality in not allowing rollover in a model lacking stochastic elements and with concave utility functions. Discounting and the lack of precautionary savings ensure departments would never want to save even if they were allowed to.

<sup>1.12</sup>One example may be that departments get utility from their production, but this is only recognised at the end of the fiscal year. In this case we could have  $v(x_t) = g(x_t)$ . Another case is that departments are assessed solely on how much they spend by the end of the fiscal year (as may occur when production is unobservable).

parameter  $D < 1$ .

Subject to an annual budget of  $B$ , a department chooses an expenditure level each period. A department has the choice variables of  $x_1$  and  $x_2$  to maximise:

$$\begin{aligned} \max_{x_1, x_2 \geq 0} \quad & D[v(x_1) + v(x_2)] - e(x_1) - De(x_2) \\ \text{s.t.} \quad & x_1 + x_2 \leq B \end{aligned} \quad (1.1)$$

Here it can be seen that performance in the first two periods  $v(x_1)$  and  $v(x_2)$  is realised in the second period. Effort is always realised in the period in which spending occurs however.

Production is denoted  $g(x_t)$  and the department's optimal spending level (from the maximisation of equation 1.1) as a function of the budget is denoted  $x_t(B)$ . The parliament gains utility from production realized instantly. The parliament also realizes disutility (instantly) from the social cost of the funds denoted  $\lambda x_t$ . For simplicity the government discounts within the fiscal year at the same rate as the department. Subject to the autonomy of a department in selecting spending levels in each period, a parliament has the choice variable of the budget,  $B$ , to maximise:<sup>1.13</sup>

$$\max_B g(x_1(B)) - \lambda x_1(B) + D[g(x_2(B)) - \lambda x_2(B)] \quad (1.2)$$

The following conditions are placed on the performance, production and effort functions:

$$g(0) = 0 \qquad v(0) = 0 \qquad e(0) = 0 \quad (1.3)$$

$$g'(0) > 0 \forall x \qquad v'(x) > 0 \forall x \qquad e'(x) > 0 \forall x \quad (1.4)$$

$$v''(x) < e''(x) \forall x \quad (1.5)$$

$$g''(x) < 0 \forall x \quad (1.6)$$

$$Dv'(0) > e'(0) \quad (1.7)$$

The first two conditions (1.3, 1.4) are elementary and simply state that production, performance and effort are zero when spending is zero and rise as spending increases. The third condition (1.5) ensures that the marginal utility for performance for a department approaches the marginal effort. This assumption is necessary to avoid corner solutions where all funds are expended in the last period.<sup>1.14</sup> The fourth condition (1.6) is necessary so that the government prefers an even spending

<sup>1.13</sup>This model expands upon the model of Liebman and Mahoney [2013] and reduces to the same model if  $e(x_1) = e(x_2) = 0 \forall x$ , performance is recognised immediately and if there are stochastic shocks in each period. As will become apparent however the mechanism behind this procrastination model is very different to the precautionary savings that drive the Liebman and Mahoney [2013] model and both models lead to different testable predictions.

<sup>1.14</sup>For instance if both functions are linear, but  $v'(x) > e'(x)$ , all spending will occur in the last period.

pattern (Proposition 2). The fifth condition (1.7) represents a weakened inada condition for  $v(x)$ .

First order conditions yield the following simple results (proofs in appendix 1.A)

**Proposition 1.** *For any given budget departments will choose strictly higher spending in the second period relative to the first.*

The intuition for this result is that the department expects the same reward from spending in either the first or second period. In planning their year however they face a lower effort cost (due to discounting) from spending in the second period. As a result the total utility from spending increases with time and departments spend more in the second period.

**Proposition 2.** *Constant spending in each period is optimal for the parliament at some level which will be denoted  $x_{Par}$ .*

The parliament realises the social cost of spending and the utility from production instantaneously. There do not exist any time distortions and hence the same level of spending that is optimal in every period.

**Proposition 3.** *There is no budget that the parliament can set to achieve its optimal level of spending,  $x_{Par}$ , in each period.*

This is immediate from propositions 1 and 2.

### 1.3.2 A calibrated procrastination model

The previous discussion in a deterministic environment was useful in presenting the central mechanism through which procrastination can lead to delayed spending. In this section a procrastination based model is calibrated to the data to show that the procrastination mechanism is robust to the presence of shocks and is capable of describing the full size of spending spikes seen in the data.

The model is calibrated to the Northern Ireland departmental capital spending dataset.<sup>1.15</sup> As this data is monthly the model is expanded to encompass 12 periods per year. It is necessary to provide functional forms of various functions. The form of the performance function  $v(x_m)$  is given by  $x_t^\delta$  where  $\delta < 1$ . The form of the effort function  $e(x_m)$  is linear and is given by  $\Omega x_m$ .<sup>1.16</sup>

Rollover is allowed in the model as was the case for Northern Ireland departments over this

<sup>1.15</sup>For a discussion of the data see online appendix 1.B. The subset of the data used for the calibration was departments with an annual budget of more than £1 million over the fiscal years 2009-2010, 2010-2011 and 2011-2012. The annual budget for each department is estimated based on the amount spent in each month as well as the amount saved into the next fiscal year. Monthly spending is then divided by the annual budget so that annual spending and net rollover accrued in a year sum to one.

<sup>1.16</sup>Imposing linearity on the effort function is not a quantitatively important restriction as flexibility is allowed in  $v(x_m) = x_m^\delta$  and it is the net of this curvature against the curvature of the effort function which dictates how departmental utility from spending changes with time and hence the pattern of spending.

period. In each period the problem of the department is:

$$V_m(B_{m,y}, \alpha_{m,y}) = \max_{B_{m,y} \geq x_{m,y} \geq 0} D_{12-m} \alpha_{m,y} x_{m,y}^\delta - \Omega x_{m,y} + D_1 E_{m,y} [V_{m'}(B_{m',y'}, \alpha_{m',y'})] \quad (1.8)$$

Subject to:

$$B_{m',y'} = B_{m,y} - x_{m,y} + \kappa_m \quad (1.9)$$

$$B_{m,y} \geq 0 \forall m, y \quad (1.10)$$

Where  $m, y$  give the current month and year and  $m', y'$  give the month and year in one months time.<sup>1.17</sup>  $D_n$  is the discounting  $n$  periods into the future. Note that the performance function is discounted between the current month and the end of the fiscal year (when  $m = 12$ ).  $\kappa_m$  is the receipt of the annual budget. This is taken to be 1 if  $m = 1$  and 0 otherwise.<sup>1.18</sup>

The  $\alpha_{m,y}$  shocks are drawn from a normalised log normal distribution:

$$\alpha_{m,y} \sim \frac{LN(0, \sigma^2)}{E[LN(0, \sigma^2)]} \quad (1.11)$$

Two calibrations are performed, one for exponential discounting and one for hyperbolic discounting. Exponential discounting is done with the form:

$$D_n = \beta^n \quad (1.12)$$

with  $\beta$  taking the value of 0.996 (5% annual discounting<sup>1.19</sup>). The second model has quasi-hyperbolic discounting [Laibson, 1997] of the form:

$$D_n = k_n \beta^n \quad (1.13)$$

Where  $k$  is a constant if  $n \geq 1$  and 1 if  $n = 0$ . This model is calibrated to the values  $k = 0.975$ ,  $\beta = 0.998$ .<sup>1.20</sup>

These two models are calibrated in the following way. The value functions are first found

<sup>1.17</sup>This system for indexing time is convenient for reasons of modular arithmetic. For instance if it is currently  $m = 12, y = 3$  then the next month is  $m' = 1, y' = 4$  and the month after that is  $m'' = 2, y'' = 4$ .

<sup>1.18</sup>In choosing a constant budget there is an implicit simplifying assumption here that the parliament does not choose a budget conditional on departmental savings but instead choose a stationary budget that maximises long run expected utility. Discussion of this assumption is included in online appendix 1.H.

<sup>1.19</sup>There have been a number of papers who have found evidence of discounting rates in excess of 5%, including Warner and Pleeter [2001] and Harrison et al. [2002]. Whilst here we impose a discounting rate of 5% it is shown in online appendix 1.I that a similar model fit is available with a discounting rate of 10%.

<sup>1.20</sup>These parameters are chosen so that total annual discounting  $k\beta^{12}$  is 5% with half of the discounting coming from the hyperbolic parameter and half from exponential discounting.



numerically for each of the twelve months. The first order condition of (1.8) with respect to  $x_t$  is then taken and rearranged to give:

$$\alpha_{m,y} = \frac{\Omega - D_1 E_{m,y}[V'_{m'}(B_{m',y'})]}{D_{12-m} \delta x_{m,y}^{\delta-1}} \quad (1.14)$$

$x_{m,y}$ ,  $B_{m,y}t$  and  $m$  can be input to this equation to give the  $\alpha_{m,y}$  shock that would have caused the spending decision,  $x_{m,y}$ . The lognormal pdf is then used with these  $\alpha_{m,y}$  shocks to calibrate the model with maximum likelihood.

	Exponential discounting model	Quasi-hyperbolic discounting model
<b>Estimated Parameters</b>		
$\delta$	0.993	0.992
$\Omega$	0.536	0.257
$\sigma$	0.010	0.011
<b>Discounting (Imposed parameters)</b>		
$\beta$	0.996	0.998
k	1.000	0.975
<b>Moments</b>		
Shocks Mean	0.001	0.001
Shocks Standard Deviation	0.011	0.012
Final Month spending ratio	2.584	3.554
RSS	3.139	0.478
Log Likelihood	1310.602	1281.764
<small>Here the shocks mean and standard deviation refer to the first 2 moments of the implied shocks after they are transformed to the normal distribution. They should be equal to 0, <math>\sigma</math> to match the shock process anticipated by departments in the model. The RSS refers sum of squared differences between the predicted monthly spending values (from the long term average of simulations) and the average spending from the data over the fiscal year. This is also summarised in figure 1.3.</small>		

Table 1.1: Calibrated Parameters

The calibrations can be seen in figure 1.3<sup>1.21</sup> as well as in table 1.1. The nature of exponential discounting means that spending increases are constrained to be increasing regularly throughout the year. Thus while this model performs reasonably well in describing the slow increase in the early months it cannot describe the large increase at the end of the fiscal year. The hyperbolic model performs better in this regard.

In both calibrations the curvature of the performance function is close to the (linear) curvature of the effort function. This indicates that in order for modest amounts of discounting to explain the size of observed spending spikes, departments need to have a weak preference for evenly spending money throughout the fiscal year. There also exists a tradeoff between the curvature of the performance function and discounting. It is shown in online appendix 1.I that it is possible to calibrate 10% (total) discounting versions of the exponential and hyperbolic discounting models that achieve almost exactly the same model fit as the 5% discounting models. These calibrations require more curvature of the performance function (a lower  $\delta$ ) to compensate for the departments greater desire to delay spending. If the same performance function was used with this greater

<sup>1.21</sup>Note that the data differs slightly from table 1.1 due to the exclusion of data from 2009 & 2012 and the removal of outliers. This is as discussed in footnote 1.15 on page 29.

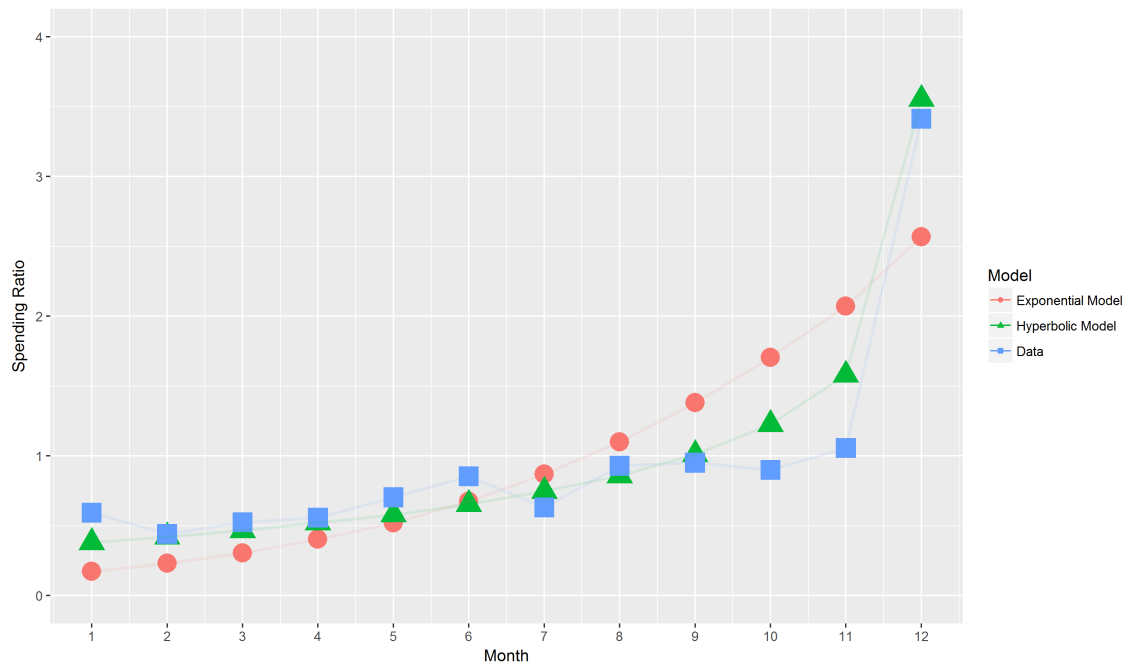


Figure 1.3: The exponential and hyperbolic models calibrated to Northern Ireland departmental spending data

discounting rate departments would delay too much spending leading to excessively large spending spikes. There also exists a link between the curvature of the performance function and the variance of the shock process. If the performance function is more concave the departments have a stronger preference towards smoothing spending and a greater volatility shock process is necessary to explain deviations from the average spending pattern over the year.

One interesting feature of simulations that were performed was the relationship between shock volatility and the size of spending spikes (holding other parameters constant). The procrastination model predicts that more volatility would induce more even expenditure and smaller end of year spending spikes. The intuition for this is simple: when the shock variance is greater the incentive for departments is to time spending to take advantage of beneficial shocks rather than timing towards the end of the year to delay the expenditure of effort. As shocks are i.i.d. across each month this is a force which evens out spending over the year.<sup>1.22</sup> This is a key testable prediction of the procrastination model which is examined in section 1.4.2.

## 1.4 Discussion

A natural question is how the procrastination model compares to alternate models of government spending. Currently the most developed model in the literature is the precautionary savings model of Liebman and Mahoney [2013]. This section will first briefly recount the precautionary savings

<sup>1.22</sup>This is shown in online appendix 1.I.

model before moving to a discussion of its performance against the procrastination model.

### 1.4.1 The Precautionary Savings Mechanism

Liebman and Mahoney [2013] present a model to explain end of fiscal year spending spikes based on precautionary savings. In their model the value of spending is stochastic and as a result departments build up precautionary savings throughout the year to spend in the case of positive productivity shocks. At the end of the year these savings stocks are then expended as no rollover of unspent funds is allowed.

Government departments want to maximize the value of their production without regard for the social cost of this spending. Their objective function can be written as:

$$V_m(B_{y,m}|\alpha_{y,m}) = \max_{B_{y,m} \geq x_{y,m} \geq 0} \alpha_{y,m}g(x_{y,m}) + \beta E_{y,m}[V_{m'}(B_{y',m'}|\alpha_{y',m'})] \quad (1.15)$$

The remaining budget is given by

$$B_{y',m'} = \begin{cases} B_{y,m} - x_{y,m} & \text{if } m < M \\ B_{y+1} + \chi(B_{y,M} - x_{y,M}) & \text{if } m = M \end{cases} \quad (1.16)$$

Where  $y, m$  are the current year and month and  $y', m'$  are the year and month in one month's time.  $x_{y,m}$  refers to monthly spending,  $g(x_{y,m})$  is a monotonically increasing concave production function,  $B_{y+1}$  is the next year's budget,  $B_{y,m}$  is the available funds at the beginning of period  $y, m$ ,  $V_m(B_{y,m}|\alpha_{y,m})$  is the value function for the department and  $\alpha_{y,m}$  is a monthly stochastic parameter without persistence.  $M$  is the number of months in the fiscal year.  $\chi$  is a parameter describing the amount of rollover into the next fiscal year, where  $\chi = 1$  describes full rollover and  $\chi = 0$  implies that no rollover is allowed. In the benchmark case no rollover is allowed.

The mechanism for end of year spending spikes here is precautionary savings driven by the  $\alpha_{y,m}$  parameter. At the start of the year agencies accrue a rainy day fund to allow sufficient funds to spend in the event of a high future  $\alpha_{y,m}$ . At the end of a year this cannot be carried over and so agencies spend this money. The concavity of the production function means that this concentration of funds in the end of the year represents inefficient spending.

The clear policy implication is to allow agencies to save unspent funds and spend these savings in future years. This would eliminate the spending spikes and prevent the associated low value spending. While this mechanism sounds plausible and may provide a partial explanation in some cases, there are reasons for believing that the majority of the effect is left unexplained by this mechanism.

## 1.4.2 Model comparison

### The UK's changes between budgetary rollover mechanisms

The U.K. traditionally had expiring budgets where unspent money was returned to treasury at the end of every fiscal year. A full rollover system (the EYF system) was then implemented for the 1998-1999 fiscal year before it was repealed and replaced with a more restrictive rollover system (the BES system) for the 2011-2012 fiscal year. The prediction of the precautionary savings model in this case is straightforward. The size of spending spike should have abated while the EYF scheme was in effect.<sup>1.23</sup> The new BES scheme should have resulted in spending spikes greater than existed under the EYF scheme as it is more difficult for some departments to access rollover.<sup>1.24</sup>

These predictions are not backed up by the data however. The size of the final month spending ratio can be seen in figure 1.2. It appears that neither the implementation of the EYF scheme nor its replacement by the BES scheme had any significant impact on the size of end of year spending spikes.

To more rigorously test this conclusion a series of t-tests were done for each spending series. Each t-test tested the null hypothesis that the vector of final month spending ratios for a given regime and series came from a distribution with the same mean as the vector of final month spending ratios coming from the other two regimes for that series. In no case was there sufficient evidence to reject the null hypothesis of no difference. The p-values from these tests along with the means and standard deviations for every series & regime are offered in table 1.B.4 (in online appendix 1.B).

Whilst inconsistent with the precautionary savings model, the lack of an impact for rollover is completely consistent with the procrastination model where rollover or the lack thereof does not play a crucial role.

### Volatility and Spending spikes

The precautionary savings model holds that spending spikes are a result of volatility in the production process for public goods. Therefore an implication of the model would be that higher levels of volatility would lead to greater spending spikes. This is the opposite prediction from the procrastination model where the shock process is a force which evens spending throughout the fiscal year. This is because departments must balance the utility benefit (through discounting) of

---

<sup>1.23</sup>Indeed simulations show that when rollover is applied to the procrastination model spending spikes should be completely eliminated (see online appendix 1.J).

<sup>1.24</sup>Note that while a particularly high spending spike in 2010-2011 might be expected due to the planned abolition of stocks, measures were taken to prevent departments from accessing these funds [The National Assembly for Wales, 2010]

procrastination with the exploitation of beneficial shocks.<sup>1.25</sup>

This can be tested using a dataset of large Northern Ireland departments for the five financial years from 2008 – 2009 to 2012 – 2013. The subset chosen for this regression was all department-budget-years with an annual budget of at least 10 million pounds.<sup>1.26</sup>

For each department-budget-year a volatility measure was constructed by looking at the relative difference between a monthly spending number and the average of the preceding and following month. The intuition is that a “low volatility” spending sequence should have monthly spending figures that are approximately the average of the two adjacent spending values. This annual volatility measure is calculated as:

$$\text{Volatility}_{y,d,b} = \frac{1}{4} \sum_{m=2}^5 \left| \frac{x_{m,y,d,b} - \frac{1}{2}(x_{m-1,y,d,b} + x_{m+1,y,d,b})}{\frac{1}{2}(x_{m-1,y,d,b} + x_{m+1,y,d,b})} \right| \quad (1.17)$$

This measure is calculated using spending values from the first 6 months of every fiscal year. As volatility early in the year may indicate a different budget position going into the second half of the year, a renormalised spending spike is used. This is the proportion of funds available as of the start of the 7th month that are spent in the final month:

$$\text{Renormalised Spike}_{y,d,b} = \frac{x_{12,y,d,b}}{B_{7,y,d,b}} \quad (1.18)$$

Where  $B_{7,y,d,b}$  describes the funds remaining to the department as of the start of month 7 in year  $y$  for department  $d$ , budget  $b$ . Nominal values were used for  $x_{12,y,d,b}$  and  $B_{7,y,d,b}$  meaning that these spending values and budgets are measured in pounds rather than the proportion of available funds spent.<sup>1.27</sup>

These volatility variables are split to allow for different relationships between volatility and renormalised spike mean for capital expenditure and for current expenditure.<sup>1.28</sup> The following regression equation with various levels of fixed effects was estimated:

$$\begin{aligned} \text{Renormalised Spike}_{y,d,b} = & \beta_0 + \beta_1(\text{Current Volatility})_{y,d,b} + \beta_2(\text{Capital Volatility})_{y,d,b} \\ & + \beta_3(\text{Current Expenditure})_{y,d,b} + \text{Year FE} + \text{Other FE} \end{aligned} \quad (1.19)$$

Aside from the year fixed effects, three levels of fixed effects are used to control for departmental

<sup>1.25</sup>These model implied relationships between volatility and spending spikes are shown from Monte Carlo results in appendices 1.J and 1.J.

<sup>1.26</sup>This lower threshold is necessary as the use of relative error as a measure of volatility is problematic when monthly spending figures can be zero (which often occurs for smaller departments). Robustness checks on this monetary threshold can be found in online appendix 1.D.

<sup>1.27</sup>This is to avoid annual budgets from being used in the calculation of both the volatility measure and the renormalised spike which would complicate the interpretation of the regression. Robustness checks with alternate spending measures is available in online appendix 1.D.

<sup>1.28</sup>Specifically  $(\text{Current Volatility})_{y,d,b} = (\text{Volatility})_{y,d,b}(\text{Current Expenditure dummy})_{y,d,b}$  and the capital volatility variable is coded analogously.

heterogeneity. The first is no fixed effects, the second is departmental fixed effects and the third is department-budget fixed effects (where only variation within a department-budget is used and the current expenditure dummy drops out). The regression results can be seen in table 1.2. While the pooled regression has a positive coefficient on the volatility of current expenditure, this significance disappears once time invariant heterogeneity is controlled for. In all cases there is a significantly negative relationship between the volatility of the capital spending series and the renormalised spike.<sup>1.29</sup> As end of fiscal year spending spikes are largest for capital expenditure this can be interpreted as evidence for the procrastination model and against the precautionary savings model.<sup>1.30</sup>

Table 1.2: Volatility and Renormalised Spike

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.110*** (0.034)	0.076 (0.052)	-0.064 (0.052)
Volatility (Capital)	-0.068*** (0.025)	-0.092** (0.037)	-0.109*** (0.038)
Budget	-0.232*** (0.033)	-0.245*** (0.034)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	115	115	115
R <sup>2</sup>	0.425	0.459	0.148
Adjusted R <sup>2</sup>	0.388	0.344	-0.170
F Statistic	11.320*** (df = 7; 107)	11.399*** (df = 7; 94)	2.404** (df = 6; 83)

Notes:

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

\* Significant at the 10 percent level.

Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

### The uncertainty required to generate the data

When the baseline CRRA calibration of the model of Liebman and Mahoney [2013] is examined it is immediately clear that it requires a high variance shock process to fit the observed magnitude of spending spikes. The distribution of the stochastic parameter,  $\alpha_{y,m}$  is calibrated to have a log normal standard deviation of 1.73.<sup>1.31</sup> This calibration is done in order to hit a final month spending ratio of 2.18, meaning that final month spending is 2.18 times the annual average. Changes in the distribution of the  $\alpha_{y,m}$  parameter would have two effects however, the first is in uncertainty and hence the amount of precautionary savings generated. The second is the extent to which spending changes month to month. A more comprehensive calibration strategy would take this into account calibrating the variance of  $\alpha_{y,m}$  to fit both aspects of the data.

It should be noted that variation in the effectiveness of government spending is not precisely what is needed for precautionary savings - what is needed is unpredictability. In this way the

<sup>1.29</sup>A range of robustness checks are included in online appendix 1.D.

<sup>1.30</sup>Intertemporal variation in the UK central government spending series was used to reach a similar conclusion in online appendix 1.E.

<sup>1.31</sup>Due to the multiplicative way  $\alpha_{y,m}$  enters equation (1.15) this distribution would indicate that the top end of the middle 66% (The shock greater than 83.3% of the distribution and less than 16.7% of the distribution) would result in 28 times more output than the bottom end of the middle 66%. The top end of the middle 95% of this distribution would result in 834 times more output than the bottom end of the middle 95%.

variance of the  $\alpha_{y,m}$  could alternatively be thought of as uncertainty about future production rather than variation in future production. Whilst  $\alpha_{y,m}$  is an abstract model parameter, it may be possible to get an idea of the plausibility of its calibrated variance by comparing it to the observable unpredictability of fundamentals in the economy.

Stark [2010, Tables 1-4] has published estimates of the errors of the survey of professional forecasters over the period 1985 Q1 to 2007 Q4 in predicting real output growth, inflation, the unemployment rate as well as the interest rate on 10-year Treasury bonds. The one quarter ahead Root Mean Square Errors (RMSEs) of these 4 variables are 1.65, 0.95, 0.25 and 0.53 respectively. By contrast on a month to month basis the implied RMSE of the calibration shocks is 19.4.<sup>1.32</sup> This reliance on an amount of uncertainty that might not be credible is a shortcoming of the precautionary savings model that is not shared by the procrastination model where shocks do not play the crucial role.<sup>1.33</sup>

## 1.5 Policy Responses to heightened end of year spending

From Proposition 3 the government is not able to induce its desired spending levels in every period. Clearly procrastination could be avoided with continuous performance evaluation however this may not be feasible with the annuality of budgeting, fiscal reporting and parliamentary oversight being entrenched in how the public sector operates.<sup>1.34</sup> Even if institutional inertia were not a factor it is possible that continuous performance evaluation would not be organisationally feasible or cost effective to implement.

This section examines another measure that is available to even spending across the fiscal year within an annual budgeting framework. Throughout this section it is assumed that it is possible for the desired level of spending to be achievable in any singular period through offering a sufficiently high budget.<sup>1.35</sup> By making this restriction attention is focused on the problem of inducing even expenditure (the shape of spending) over the year rather than adjusting the absolute level of spending over the year.

<sup>1.32</sup>Calculated by taking  $E[\alpha_{y,m}]$  as the forecast and a sample of 10 billion random draws of the distribution as the realized values.

<sup>1.33</sup>One solution to this criticism might be to interpret the uncertainty as being i.i.d. amongst low level departmental managers. In this case excessively large month to month changes in spending would not be seen in aggregate data. This assumption would be very tenuous however as shocks to different departmental managers are likely to be highly correlated as they service the same population and buy goods and services in the same economy at the same time. It would also raise the question why managers in a department could not collectively insure each other against these shocks.

<sup>1.34</sup>For instance when asked about moves away from annuality (such as EYF) one public sector manager gives a vivid description of this entrenchment telling Hyndman et al. [2007, p.231] that “Annuality is a useful mechanism because, to be blunt, I don’t know any other ... It’s the culture I work in. I haven’t had any experience of any other culture ... It’s difficult for me because I can’t really compare it.”

<sup>1.35</sup>This implies that the marginal utility from spending at the parliamentary optimal level is positive for the department in every period. Were this not the case the parliament would need to increase  $v(x_t)$  (for instance through increasing performance pay) or decrease  $e(x_t)$  (for instance through hiring more public servants) in order to induce the parliamentary optimal level of spending.

It is useful to first discuss what features are attractive in a feasible policy solution that could actually be implemented. The first property is that any policy measure should be of low cost. Aside from rollover, auditing is the most relied upon technique for preventing low value spending. A former Australian finance minister has suggested the only way to prevent excessive spending is to apply external scrutiny [Mannheim, 2012]. A key disadvantage of auditing is that it can be expensive which limits its potential for its widespread use.<sup>1.36</sup>

Due to the strong reticence of elected representatives to give up control of spending [McPherson, 2007, page 28], a policy would be more likely to be implemented if it does not result in a loss of power for elected officials. The loss of parliamentary control associated with rollover budgeting is the leading reason the UK’s rollover system was tightened in 2010.<sup>1.37</sup> A successful measure should also be simple to understand and implement. Furthermore it should be robust to interference by different levels of the bureaucracy.<sup>1.38</sup>

### 1.5.1 Time-variant budgetary tax

In this section we propose a new technique of time variant budgetary taxes to achieve even spending using the simple deterministic model introduced in section 1.3.1. With a time-variant budgetary tax (hereafter simply referred to as a tax), spending in later months of the year is more expensive relative to earlier months of the fiscal year. These taxes are not taxes in the normal sense but represent reductions on a department’s appropriated budget at a higher level than what the department actually spent. The implementation of these taxes would not necessarily mean lower overall spending as the department’s budget could be increased to compensate for these taxes being levied on purchases made later in the year.

Let  $\theta$  describe a tax rate by which expenditures in the second period are more expensive. The budget constraint faced by a department in equation 1.1 is replaced by:

$$x_1 + \theta x_2 \leq B \tag{1.20}$$

otherwise the department’s problem is unchanged. Note here that both  $\theta$  and  $B$  are choice variables for the parliament and hence taxes give parliaments more power to control spending in the fiscal year.

**Proposition 4.** *Time-variant budgetary taxes are sufficient to induce the parliamentary optimal expenditure level,  $x_{Par}$ , in each period.*

<sup>1.36</sup>Analysis of the effectiveness of audits in this context is deferred to the online appendix 1.F.

<sup>1.37</sup>In a letter to the author former UK chief secretary to the treasury Danny Alexander said of the EYF system: “departments were able to build up unlimited stocks of underspends which they were free to draw down and spend at any time. This posed a clear risk to effective control of public spending.”

<sup>1.38</sup>For instance in the case of rollover Hyndman et al. [2007] found cases where budget managers did not have rollover as managers higher up in the department wanted control of unspent funds each year.



The tax that induces optimal spending is shown in the proof (see appendix 1.A) to be given by:

$$\theta = \frac{Dv'(x_{\text{Par}}) - De'(x_{\text{par}})}{Dv'(x_{\text{Par}}) - e'(x_{\text{par}})} \quad (1.21)$$

This is the ratio of marginal utility from spending in the second period divided by marginal utility of spending in the first period with both evaluated at the parliamentary optimal spending level. This optimal  $\theta$  may differ between every department leading to the parliament taxing each department at different rates. If for any reason a parliament was constrained to implement a common  $\theta$  for several different departments it is simple to show that choosing the minimum  $\theta$  rate across all departmental specific  $\theta$  values would bring each department closer to parliamentary optimal spending relative to the no taxation case.<sup>1.39</sup>

## 1.5.2 Calibrated time variant budgetary tax

With the calibrated parameters of section 1.3.2, the long term spending behaviour of departments is well defined. In order to assess time variant budgetary taxes in a stochastic setting however a production function must be assumed.<sup>1.40</sup> We assume a function of:

$$\text{Production}(\alpha_{y,m}, x_{y,m}) = \alpha_{y,m} x_{y,m}^{\gamma} \quad (1.22)$$

Where  $\alpha_{y,m}$  are the same shocks that enter the department's utility function<sup>1.41</sup> and  $\gamma$  is a curvature parameter to be discussed.

With the assumption of a production function and the calibration of departmental behaviour with a budget of 1 we can numerically estimate the social cost that would have lead to an optimal budget of 1 using the equation (derivation in online appendix 1.H):

$$\lambda = \frac{\frac{\partial \text{Expected Annual Production}(B)}{\partial B}}{\frac{\partial \text{Expected Annual Expenditure}(B)}{\partial B}} \Bigg|_{B=1} \quad (1.23)$$

The functional form taxes will take over the year needs to be chosen.<sup>1.42</sup> For the exponential discounting model, a deterministic version of the calibrated model has an analytical solution for

<sup>1.39</sup>This may not be the constrained optimal  $\theta$  for the parliament however. To find an optimal common  $\theta$  level the inefficiency of excessive early spending (from a too high  $\theta$ ) would have to be balanced with the inefficiency of excessive late spending (from a too low  $\theta$ ).

<sup>1.40</sup>One option would be to assume the production function is congruent to the calibrated performance function. This is a different way of saying that performance reviews simply sum up all production done in the year. This is not done in this paper as the performance function is calibrated relative to the imposed linear effort function.

<sup>1.41</sup>The assumption of a shared shock process is reasonable in the principal-agent context of the parliament-department relationship. If shocks were uncorrelated with parliamentary utility then parliaments would seek to contract departments in such a way as to align shocks.

<sup>1.42</sup>More effective taxes could be found by optimising 11 different tax rates and the budget however this is computationally not feasible.

the tax rates that will yield even spending.

$$\theta_m = \frac{\delta D_{11} x_{\text{par}}^{\delta-1} - \Omega D_{m-1}}{\delta D_{11} x_{\text{par}}^{\delta-1} - \Omega} \quad (1.24)$$

Where  $\theta_m$  gives the tax in month  $m$ . The annual budget is then set so that the department can afford to spend  $x_{\text{par}}$  every period while paying the tax:

$$B = x_{\text{par}} \left[ \sum_{m=1}^{12} \theta_m \right] \quad (1.25)$$

Where  $x_{\text{par}}$  is a deterministic approximation of parliamentary optimal spending found by taking a first order condition for parliamentary utility resulting in the expression:

$$x_{\text{par}} = \left[ \frac{\lambda}{\gamma} \right]^{\frac{1}{\gamma-1}} \quad (1.26)$$

For the quasi-hyperbolic discounting model the taxation function (1.24) performed poorly in smoothing spending and so tuning parameters,  $\nabla$  and  $\Upsilon$  were added to give a form of  $\theta_m = \frac{\nabla \delta D_{11} x_{\text{par}}^{\delta-1} - \Omega D_{m-1}}{\nabla \delta D_{11} x_{\text{par}}^{\delta-1} - \Omega}$  for  $m \leq 11$  and  $\theta_{12} = \frac{\nabla \delta D_{11} x_{\text{par}}^{\delta-1} - \Omega D_{m-1}}{\nabla \delta D_{11} x_{\text{par}}^{\delta-1} - \Omega} + \Upsilon$  for the final month.<sup>1.43</sup>

An assumption also needs to be made for either the value of  $\gamma$  or the value of  $\lambda$ , the remaining parameter being able to be found from equation (1.23). A conservative approach is to choose a  $\gamma$  that will tend to underestimate the gains from the imposition of a tax. A near linear production function may be thought to be a conservative choice in that there are lesser diminishing returns during spending spikes. However a near linear production function also means that the underutilisation of beneficial shocks early in the year is particularly inefficient as such shocks could have been exploited to a greater degree.

With the prior belief that a near linear production function would be a conservative choice it was chosen to impose a curvature value of  $\gamma = 0.95$ , implying a modest level of diminishing returns. Robustness checks were then carried out which found that this assumption is not critical and other choices lead to quantitatively similar percentage gains from tax implementation.<sup>1.44</sup> These checks show that the forces described in the preceding paragraph roughly compensate for each other as  $\gamma$  changes within the range of credible curvature parameters. These robustness checks are detailed in online appendix 1.C.

For each of the two models a perfect agency simulation is done with funds being spent exactly

<sup>1.43</sup>Exponential discounting is used for  $D_{m-1}$  in the functional form equation (1.24). The quasi-hyperbolic form was not used after a calibration with this form performed worse than  $D_{m-1}$  with an exponential form. These tuning parameters were optimised by maximising parliamentary expected utility for both parameters with budgets being set according to equation 1.25.

<sup>1.44</sup>Although the production function imposed does change the absolute level of parliamentary utility before and after the imposition of a tax.

	5% Exponential discounting model	5% Quasi-hyperbolic discounting model
<b><math>\gamma</math> and <math>\lambda</math></b>		
$\gamma$	0.95	0.95
$\lambda$	1.046	1.047
<b>No Tax Model</b>		
$B$	1	1
Average $x_1$	1.4%	3.18%
Average $x_{12}$	21.5%	29.6%
Average Annual Production	1.101	1.102
Average Annual Spending	1	1
Parliament Utility	0.055	0.055
<b>Tax Model</b>		
$B$	1.788	1.752
Tuning Parameters ( $\nabla, \Upsilon$ )	NA	0.491, 0.022
Final Month Tax ( $\theta_{12}$ )	1.049	1.048
Average $x_1$	8.98%	9.09%
Average $x_{12}$	7.52%	9.55%
Average Annual Production	1.881	1.869
Average Annual Spending	1.747	1.726
Parliament Utility	0.064	0.062
<b>Perfect Agency</b>		
Average Annual Production	1.961	1.953
Average Annual Spending	1.781	1.773
Parliament Utility	0.098	0.098
Gains from tax	16.1%	12.6%
Maximum possible gain	78.8%	76.7%

Note that start month and end month spending amounts are not listed for the perfect agency case as monthly spending will be even at 8.3% ( $\frac{1}{12}$  of annual spending). Utility gains from the tax and from the perfect agency models are relative to the no tax model. The tax equation formula can be found in equation 1.24 for the exponential model, the taxation formula for the quasi-hyperbolic model is as described on page 40.

Table 1.3: Calibrated Time Variant Budgetary Tax

in accordance with the parliament's utility function. This is done to give the maximum possible gains from implementing some policy mechanism in this stochastic environment. The output of simulations for both models along with simulations for when taxes are applied and when perfect agency occurs is presented in table 1.3.

Even with this simple tax formula it can be seen that quite large gains are possible. The exponential and quasi-hyperbolic models have available utility benefits of 16.1% and 12.6% respectively. These gains are realized in three ways. The first is that heightened spending in the final months of the fiscal year is less likely to encounter inefficiency from diminishing returns. The second is that more even expenditure better utilises shocks which are i.i.d. throughout the fiscal year. The third is that the parliament can offer larger budgets as less spending takes place at low marginal value.

It can be seen that the assumed functional forms were moderately successful in smoothing spending. The exponential model has long run spending decreasing from a ratio of 1.08 in the first month to 0.90 in the last month. The quasi-hyperbolic model has its smallest average spending ratio of 0.91 in the 7th month with its greatest spending ratio at 1.15 in the last month of the fiscal year. Both tax formulas used were simple however and taxes calibrated with more parameters

could likely improve upon the results reported in table 1.3.

The maximum gains from the perfect agency simulations are larger than the benefits possible with a tax at 78.8% and 76.7%. In both models there is little difference between annual average spending in the tax case and the perfect agency case. In conjunction with the reasonably even spending in the tax case this suggests that the reason the tax case underperforms the perfect agency case is in less efficient exploitation of shocks.

Two calibrations with 10% discounting were also performed (presented in online appendix 1.I) which lead to greater gains from a tax than their 5% discounting counterparts. As the higher discounting rates indicate a stronger preference for delayed spending these 10% calibrations offset this tendency by having a more concave performance function which encourages more even spending. This means that the curvature of the departmental utility function is closer to the curvature of the parliament's utility function which leads to departmental spending choices closer to the parliamentary optimum. When the tax is calibrated there are utility benefits of 45.3% and 25.1% possible for the exponential and quasi-hyperbolic versions which compare to the perfect agency benefits of 64.3% and 62.9%. These calibrations indicate that the benefits from taxes have some level of robustness to the discounting rates and time consistency applied.

These gains are also robust to what  $\gamma$  value is imposed with every tax simulation performed in this research<sup>1.45</sup> achieving a taxation benefit of more than 10% over the no taxation case.<sup>1.46</sup>

## 1.6 Conclusion

This paper has detailed the governmental end of fiscal year spending spike phenomena using evidence from the United Kingdom. The key contributions of this paper are threefold. The first contribution is the development of a model for heightened end of year spending based on procrastination. The second is the model comparison of the procrastination model against the precautionary savings model which demonstrates the poor performance of the precautionary savings model in the UK experience. The third contribution is in the invention of time variant budgetary taxes as a policy tool to even spending and facilitate more efficient spending from government departments.

Time variant budgetary taxes are the recommendation of this paper. They are likely to be of low cost, easy to implement and understand, do not take power away from elected politicians and

<sup>1.45</sup>All calibrations are detailed in table 1.3, online appendix 1.I or online appendix 1.C.

<sup>1.46</sup>One potential concern is that if a model were written with persistent shocks, time variant budgetary taxes might discourage prudence by encouraging early spending before persistent shocks are revealed. This scenario has not been estimated due to the curse of dimensionality making the computation too difficult (as persistent shocks introduce another state variable to departmental value functions). However even in this case it is likely that taxes could be calibrated to make some welfare gains. In the worst case tax rates could be set to one so welfare losses should never result with a properly calibrated tax. Whilst proper calibration might be difficult where there is ex ante limited information about the model specification or parameters, it is anticipated that a government could act conservatively by implementing taxes at a low level and adapting them in later years based on realised spending and performance data.

are shown to be capable of smoothing spending throughout the fiscal year. Based on calibration results this paper estimates that the surplus that departments deliver for the parliament (and taxpayers) could be increased with the imposition of a time variant budgetary tax.

## 1.7 References

- John Bryne. Project management: how much is enough? <http://www.pmi.org/learning/library/project-management-much-enough-appropriate-5072>, 1999.
- Rowena Crawford, Carl Emmerson, and Gemma Tetlow. A survey of public spending in the UK. *Institute for Fiscal Studies, Briefing Note BN43*, pages 272–290, 2009.
- Bernd Fitzenberger, Marina Furdas, Olga Orlanski, and Christoph Sajons. End-of-year spending and the long-run effects of training programs for the unemployed. Working paper, University of Freiburg, 2014. URL <http://www.hha.dk/nat/larss/CAFE2014/BFitzenberger.pdf>.
- Michael D. Frakes and Melissa F. Wasserman. Procrastination in the workplace: Evidence from the U.S. patent office. *NBER Working Paper Series*, 22987, 2016.
- Hubert Hall. *The Red Book of the Exchequer*. Printed for Her Majesty’s Stationary Office by Eyre and Spottiswoode, 1896.
- Glenn Harrison, Morten Lau, and Melonie Williams. Estimating individual discount rates in denmark: A field experiment. *American Economic Review*, 92(5):1606–1617, 2002.
- HM Treasury. Public expenditure 2007-2008. [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/221495/1\\_10\\_.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/221495/1_10_.pdf), 2008.
- HM Treasury. Spending review 2010. [https://www.direct.gov.uk/prod\\_consum\\_dg/groups/dg\\_digitalassets/@dg/@en/documents/digitalasset/dg\\_191696.pdf](https://www.direct.gov.uk/prod_consum_dg/groups/dg_digitalassets/@dg/@en/documents/digitalasset/dg_191696.pdf), 2010.
- HM Treasury. Consolidated budgeting guidance from 2013-14. [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/207689/consolidated\\_budgeting\\_guidance\\_201314.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/207689/consolidated_budgeting_guidance_201314.pdf), 2013.
- Christopher Hope. Civil servants paid tens of millions of bonuses during recession. <http://www.telegraph.co.uk/news/politics/9756028/Civil-servants-paid-tens-of-millions-of-bonuses-during-recession.html>, 2012.
- House of Commons Scrutiny Unit. Review of departmental annual reports. <http://www.parliament.uk/documents/upload/darreviews09.pdf>, 2009.

- W.J. Hurley, Jack Brimberg, and Brent Fisher. Use it or lose it: On the incentives to spend annual defence operating budgets. *Defence and Peace Economics*, 25(4):401–413, 2013.
- Jane Hutt. Written statement - new end-year flexibility arrangements for devolved administrations. <http://wales.gov.uk/about/cabinet/cabinetstatements/2011/5185695/?lang=en>, 2011.
- Noel Hyndman, Rowan Jones, and Maurice Pendlebury. An exploratory study of annuality in the uk public sector: Plus ça change, plus c'est la même chose. *Financial Accountability & Management*, 23(2):0267–4424, 2007.
- David Laibson. Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics*, 112(2):443–478, 1997.
- Jeffrey B. Liebman and Neale Mahoney. Do expiring budgets lead to wasteful year-end spending? evidence from federal procurement. Working Paper 19481, National Bureau of Economic Research, September 2013. URL <http://www.nber.org/papers/w19481>.
- John Major. Speech to the conservative central council. <http://www.johnmajor.co.uk/page2017.html>, 1991.
- Markus Mannheim. If finance loosens reins, public service spend-up could stop. <http://www.theage.com.au/federal-politics/if-finance-loosens-reins-public-service-spendup-could-stop-20120609-202p0.html>, 2012.
- Michael McPherson. An analysis of year-end spending and the feasibility of a carryover incentive for federal agencies. *Masters diss. Naval Postgraduate School.*, 2007.
- Northern Ireland Assembly Research and Library Service. Comparative analysis of draft budget 2008-2011. <http://archive.niassembly.gov.uk/io/research/2008/4908.pdf>, 2008.
- Northern Ireland Department of Finance and Personnel. Main estimates 2009-2010. <https://www.finance-ni.gov.uk/sites/default/files/publications/dfp/main-estimates-2010-2011.pdf>, 2009.
- Northern Ireland Department of Finance and Personnel. Main estimates 2010-2011. <https://www.finance-ni.gov.uk/sites/default/files/publications/dfp/main-estimates-2010-2011.pdf>, 2010.
- Northern Ireland Department of Finance and Personnel. Northern ireland estimates 2012-2013. <http://www.dfpni.gov.uk/main-estimates-2012-13.pdf>, 2012.

Northern Ireland Department of Finance and Personnel. Spring supplementary estimates 2012-2013. <https://www.finance-ni.gov.uk/sites/default/files/publications/dfp/spring-supplementary-estimates-2012-13.pdf>, 2013.

Northern Ireland Department of Finance and Personnel. Main estimates 2014-2015. <https://www.finance-ni.gov.uk/sites/default/files/publications/dfp/main-estimates-2014-15.pdf>, 2014.

Northern Ireland Executive. Budget 2008-2011. <http://www.northernireland.gov.uk/finalbudget.pdf>, 2008.

Northern Ireland Executive. Budget 2011-2015. [http://www.northernireland.gov.uk/revised\\_budget\\_-\\_website\\_version.pdf](http://www.northernireland.gov.uk/revised_budget_-_website_version.pdf), 2011.

NTV News. Poltavchenko deprives budget managers of their bonuses (translated from Russian). <http://www.ntv.ru/novosti/1295836/>, 2015.

Nehal Panchamia and Peter Thomas. Public service agreements and the prime minister's delivery unit. <http://www.instituteforgovernment.org.uk/sites/default/files/case%20study%20psas.pdf>, 2010.

Drazen Prelec. Decreasing impatience: A criterion for non-stationary time preference and "hyperbolic" discounting. *The Scandinavian Journal of Economics*, 106(3):511-532, 2004.

Serious Fraud Office. How much was paid out in bonuses to your departments staff? <http://www.sfo.gov.uk/media/165070/bonuses.pdf>, 2010.

Tom Stark. Realistic evaluation of real-time forecasts in the survey of professional forecasters. <https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files/error-statistics>, 2010.

Justine Stephen, Petr Bouchal, and David Bull. Whitehall monitor 2013. <http://www.instituteforgovernment.org.uk/sites/default/files/publications/3046%20IFG%20-%20Whitehall%20Monitor%20A4.pdf>, 2013.

Annie Galvin Teich. Year-end k-12 spending survey: Use it or lose it. <http://education.mchdata.com/refmat/year-end-k-12-spending-survey-use-it-or-lose-it/>, 2013.

The National Assembly for Wales. Finance committee proceedings - monday, 22 november 2010. [http://www.assembly.wales/Committee%20Documents/FIN\(3\)-20-10%20%20Transcript%20\(PDF,%20431KB\)-22112010-205122/cyllid20101122fv-fin\\_3\\_-20-10-English.pdf](http://www.assembly.wales/Committee%20Documents/FIN(3)-20-10%20%20Transcript%20(PDF,%20431KB)-22112010-205122/cyllid20101122fv-fin_3_-20-10-English.pdf), 2010.

UK Cabinet Office. Permanent secretaries' objectives 2013 to 2014. <https://www.gov.uk/government/publications/permanent-secretaries-objectives-2013-to-2014>, 2013.

UK Government. The public contracts regulations 2006. [http://www.legislation.gov.uk/uksi/2006/5/pdfs/uksi\\_20060005\\_en.pdf](http://www.legislation.gov.uk/uksi/2006/5/pdfs/uksi_20060005_en.pdf), 2006.

United States General Accounting Office. Government agencies need effective planning to curb unnecessary year-end spending. <http://www.gao.gov/products/PSAD-80-67>, 1980.

US Office of Personnel Management. Senior executive service - compensation. <https://www.opm.gov/policy-data-oversight/senior-executive-service/compensation/>, 2015.

US Senate subcommittee on oversight of government management. Hurry-up spending, 96th congress, 2nd session, 1980.

John T. Warner and Saul Pleeter. The personal discount rate: Evidence from military downsizing programs. *American Economic Review*, 91(1):33–53, 2001.

## Appendices

### 1.A Proofs

The important feature of the procrastination model is that departmental utility from spending is time dependent. For ease and clarity of the proofs we define and use a concept of departmental time dependent utility.

A time dependent departmental utility function ( $u_t(x_t)$ ) in which in each period satisfies:

$$\frac{\partial u_t(x_t)}{\partial x_t} > 0 \quad \forall t \quad \text{when } x_t = 0 \quad (1.A.1)$$

$$\frac{\partial^2 u_t(x_t)}{\partial x_t^2} < 0 \quad \forall t, x \quad (1.A.2)$$

$$u_t(x) > u_T(x) \quad \forall x \quad \text{for } t > T \quad (1.A.3)$$

$$\frac{\partial u_t(x_t)}{\partial x_t} > \frac{\partial u_T(x_T)}{\partial x_T} \quad \forall x \quad \text{for } t > T \quad (1.A.4)$$

Clearly the procrastination model above is an example of such a model with  $u_t(x_t) = Dv(x_t) - D_t e(x_t)$  where  $D_1 = 1$  and  $D_2 = D < 1$ .

As discussed in the first paragraph of section 1.5 it is assumed that the marginal utility of spending is positive at the parliament optimal level of spending in any period of the year. This



assumption translates to the following expression<sup>1.47</sup>:

$$\frac{\partial u_t(x_{\text{par}})}{\partial x} > 0 \quad \text{for } t = t_{\min} \quad (1.A.5)$$

Where  $t_{\min}$  is the start of the fiscal year.

### Proof of Proposition 1

We can write a Lagrangian for the department:

$$V = u_1(x_1) + u_2(x_2) + \Lambda[B - x_1 - x_2]$$

$$\frac{\partial V}{\partial x_1} = \frac{\partial u_1(x_1)}{\partial x_1} - \Lambda \quad (1.A.6)$$

$$\frac{\partial V}{\partial x_2} = \frac{\partial u_2(x_2)}{\partial x_2} - \Lambda \quad (1.A.7)$$

Note that if the budget constraint doesn't bind  $\Lambda = 0$ , if it does  $\Lambda$  is positive. In either case the department will spend in each period until the marginal value of spending is equal to the shadow price  $\Lambda$ . From condition (1.A.4) when the marginal value at time 1 is  $\Lambda$ , the marginal value at time 2 is higher. Hence the department will spend more in the second period.

### Proof of Proposition 2

Suppose the parliament could choose spending in each period. We write the government objective function as:

$$G = g(x_1) - \lambda x_1 + D[g(x_2) - \lambda(x_2)]$$

$$\frac{\partial G}{\partial x_1} = g'(x_1) - \lambda \quad \frac{\partial G}{\partial x_2} = g'(x_2) - \lambda$$

Condition 1.6 implies a unique solution for both FOCs at  $x_1 = x_2$ .

### Proof of Proposition 3

From Proposition 1 we have the strict inequality  $x_2 > x_1$ . Therefore it is not possible to get  $x_2 = x_1 = x_{\text{Par}}$ .

<sup>1.47</sup>If this was violated the parliament would need to adjust  $v_t(x_t)$  in order to induce their desired spending level.

**Proof of Proposition 4**

We assume the parliament will set a budget such that it will be fully expended<sup>1.48</sup>:

$$V = u_1(x_1) + u_2\left(\frac{B - x_1}{\theta}\right) \quad (1.A.8)$$

This gives the first order conditions

$$\frac{\partial V}{\partial x_1} = \frac{\partial u_1(x_1)}{\partial x_1} - \theta^{-1} \frac{\partial u_2\left(\frac{B - x_1}{\theta}\right)}{\partial x_2} \quad (1.A.9)$$

For this to be 0 when  $x_1 = x_2 = x_{\text{Par}}$

$$\theta = \frac{\frac{\partial u_2(x_{\text{Par}})}{\partial x_2}}{\frac{\partial u_1(x_{\text{Par}})}{\partial x_1}} \quad (1.A.10)$$

**1.B Data**

Aside from current and capital expenditure, budgets are further split by the planning horizon of the funds in two categories: Annual Managed Expenditure (AMEs) and Departmental Expenditure Limits (DELs). AMEs are set annually and are intended for demand driven expenses including welfare payments, hospital admissions and public sector pension scheme payments. DELs are set in the spending review process and are intended for expenses which can be planned for in advance including wages and rent costs. Spending reviews occur every three years [Crawford et al., 2009] and each time set DELs for the coming three years. The purpose of this system is for departments to know their budgets for the coming fiscal years and be able to plan accordingly with AME funds being available on a shorter horizon to cover demand driven expenses that are more difficult to plan for.

**Northern Ireland Data**

Monthly Northern Ireland spending figures were accessed from a Freedom of Information request to the Northern Ireland government. This data is based on accrual accounting and only includes DEL data. The restriction to DEL data is due to the division of responsibilities between the UK government and the Northern Ireland government (who supplied the data). Although AMEs constitute about 40% of spending in Northern Ireland [Northern Ireland Department of Finance and Personnel, 2012] this restriction is not particularly important. Firstly the data most relevant for spending spikes is the capital spending data (See footnote 1.6 on page 23). Almost all Northern

<sup>1.48</sup>This will be true of any parliament optimally setting a budget and tax. Time variant budgetary taxes will be ineffective if the budget constraint never binds.

Irish capital spending is DEL spending and as such is represented in our data.<sup>1.49</sup> The second reason is that DELs are spending items that are driven by departmental decisions rather than being demand driven and outside of departmental hands. A restriction to DEL expenditures therefore focuses attention to those areas of government spending that are more discretionary and thus where the underlying principal agent relationship of interest occurs. This is in contrast to AMEs which are demand driven and largely include transfer payments such as public sector pension schemes.

Devolved administrations have certain privileges that Whitehall departments do not. The extent of these depends on the budgetary system and therefore changed after the 2010-2011 financial year. An important privilege is that devolved administrations do not need to notify treasury in advance of any planned underspends (and associated rollover) [Hutt, 2011]. Taken in total the differences in budgetary restrictions are relatively minor however.

The data is split by current and capital expenditure for every year except for the fiscal years 2011-2012 and 2012-2013 when “Non Ringfenced Resource Expenditure” and “Ringfenced Resource Expenditure” were summed up to get current expenditure. In addition this data was adjusted in three ways. Firstly these monthly spending figures are as realised by departments and reported to UK treasury in May of each year (2 months after the end of the fiscal year). The total department’s total outturn figure is then officially recorded in August each year with no new monthly series being created. The sum of the monthly spending series can differ from the final figure reflecting accounting adjustments between May and August. To correct for this the monthly spending figures are scaled such that the sum reflects the final outturn figure. The second adjustment accounts for the the differing number of days in each month. All monthly spending figures are normalised to their proportion of annual total spending (after taking account for rollover). Thirdly some months (about 2% of observations) record negative spending figures reflecting asset sales being recorded along with expenditures. In the cases of NIAUR 2012-2013 Current Expenditure and DOE 2008 Capital Expenditure, these department-budget-years were dropped in their entirety as annual spending was negative (revenue from asset sales exceeded capital expenses). In other cases negative months were dropped.<sup>1.50</sup>

Over and underspend figures were attained from a freedom of information request for the years 2009-2010, 2010-2011 and 2011-12. For 2012-2013 provision estimates were taken from Northern

---

<sup>1.49</sup>The Department for Employment and Learning is the only notable exception receiving about 80% of its capital budget from AMEs with all other departments receiving less than 5% of their capital budget from AMEs. The qualitative conclusions of the calibrations are robust to the omission this department.

<sup>1.50</sup>These decisions on dealing with negative values are relevant for the calibration and for the regressions of section 1.4.2. In the case of the calibration, the model presented requires positive spending values and hence it is not possible to attain a likelihood for negative spending values. The effect in the regression case is likely to be minimal. If the regressions in 1.4.2 are repeated only with department-budget pairs that have 60 observations (no negative values in a 5 year period) the results become more significant. This regression is shown in section 1.D.

Ireland Department of Finance and Personnel [2013] while outturn figures were taken from Northern Ireland Department of Finance and Personnel [2014]. For 2008-2009 provision and outturn figures are taken from Northern Ireland Department of Finance and Personnel [2009] and Northern Ireland Department of Finance and Personnel [2010] respectively.<sup>1.51</sup> Between these fiscal years each department's balance of funds can be found using the EYF and BES rollover rules as applicable. Figures for the exact level of rollover for each department going into 2008-2009 were not available and hence these were estimated by taking the total rolledover funds for Northern Ireland as of the start of the 2008-2009 fiscal year the total funds for Northern Ireland were taken from HM Treasury [2008, Table 6] and attributing them to each department based on the combined savings of each department as accumulated in the 2005-2006 and 2006-2007 fiscal years [Northern Ireland Assembly Research and Library Service, 2008, Table 6.1,6.2].

A regression of current and capital spending on month dummies can be seen in table 1.B.1. The observations in this case include monthly departmental spending for each department for each of the 5 years.

Table 1.B.1: Regressions on Northern Ireland Spending

	Capital Expenditure	Spending Current Expenditure
	(1)	(2)
April	0.012 (0.016)	0.073*** (0.003)
May	0.043*** (0.016)	0.075*** (0.003)
June	0.041*** (0.016)	0.080*** (0.003)
July	0.044*** (0.016)	0.075*** (0.003)
August	0.056*** (0.016)	0.080*** (0.003)
September	0.064*** (0.016)	0.074*** (0.003)
October	0.066*** (0.016)	0.081*** (0.003)
November	0.057*** (0.016)	0.085*** (0.003)
December	0.084*** (0.016)	0.080*** (0.003)
January	0.063*** (0.016)	0.080*** (0.003)
February	0.123*** (0.016)	0.087*** (0.003)
March	0.347*** (0.016)	0.130*** (0.003)
Observations	1,008	972
R <sup>2</sup>	0.402	0.919
Adjusted R <sup>2</sup>	0.395	0.918
Residual Std. Error	0.145 (df = 996)	0.025 (df = 960)
F Statistic	55.774*** (df = 12; 996)	904.508*** (df = 12; 960)

*Notes:*

\*\*\*Significant at the 1 percent level.

\*\*Significant at the 5 percent level.

\*Significant at the 10 percent level.

Northern Ireland's NIAUR (Energy Regulator) current expenditure observations were omitted as being extreme outliers.

<sup>1.51</sup>Note that underspend can be calculated by subtracting outturn from provision. Overspend refers to the case when underspend is negative. Overspend is comparatively rare and departments are generally sanctioned in such cases for spending more than they were authorised to.

Table 1.B.2 shows summary statistics by department while department abbreviations are shown in table 1.B.3.

### **The UK Government**

The UK Central Government Spending data was taken from the Office of National Statistics as the series: MF6S - Central Government: Total current expenditure: (£m) and MF6T - Central Government: Total capital expenditure: (£m). These series are accrual based meaning that expenditure is recorded in the month the underlying economic transaction took place rather than the month in which cash was exchanged.<sup>1.52</sup> All raw spending figures have been adjusted to reflect the differing number of days in each month. These series do not divide between DEL and AME expenditure but represent aggregate spending.

The UK Central Government cash outlays series was taken from the Office of National Statistics as the series: RUUP - Central Government: Cash outlays (£m). All raw spending figures have been adjusted to reflect the differing number of days in each month.

Table 1.B.4 shows summary statistics and t-tests relating to the three series on figure 1.2 and is discussed on page 34.

---

<sup>1.52</sup>This is imperfect however with several large capital receipts relating to the sales of 3G licenses in 2000 and the decommissioning of British Nuclear Fuels in 2005 being ascribed to particular months. For this reason some months of the 2000-2001 fiscal year have been omitted and the remaining months spending ratios renormalised.

Table 1.B.2: Northern Ireland department summary statistics

DeptCode	Budget	Staff	Invoices	Last_Month_No.Obs	Last_Month_Mean	Last_Month_SD	Other_Months_No.Obs	Other_Months_Mean	Other_Months_SD	test
AOCC	Capital	-	-	5	16	15	55	2	6	0.0913*
DARD	Capital	2879	27580	5	7642	2050	55	2542	4527	0.00157***
DCAL	Capital	276	4016	5	13010	7308	55	2523	1862	0.0323***
DE	Capital	592	2818	5	37400	23720	55	13290	13900	0.0847*
DEL	Capital	1981	7489	5	13610	9510	55	1889	2109	0.0508*
DETI	Capital	439	3650	5	13400	7768	55	6412	24620	0.168
DFP	Capital	3207	38619	5	7303	2522	55	1510	1721	0.00588***
DHSSPS	Capital	561	4219	5	70100	15520	55	14270	16250	0.000697***
DOE	Capital	2587	27468	5	3264	1512	55	215	1300	0.00898***
DOJ	Capital	3641	43336	3	23150	2592	33	4220	1875	0.00453***
DRD	Capital	2167	32898	5	92450	23550	55	37740	22630	0.00491***
DSD	Capital	6864	13950	5	70190	22770	55	10410	7418	0.00403***
FSA	Capital	-	-	5	13	17	55	0	3	0.181
NIA	Capital	-	-	5	186	222	55	96	126	0.416
NIAO	Capital	-	-	5	128	61	55	8	17	0.0113***
NIAUR	Capital	-	-	5	5	10	55	1	3	0.393
OFMDFM	Capital	346	6075	5	2790	908	55	478	767	0.00362***
PPS	Capital	529	10151	3	175	96	33	28	50	0.115
All	Capital	-	-	86	20090	30200	946	5462	13450	2.5e-05***
AOCC	Current	-	-	5	196	54	55	134	30	0.0625**
DARD	Current	2879	27580	5	37680	13760	55	17120	2848	0.0287***
DCAL	Current	276	4016	5	15800	1683	55	9781	3181	0.000227***
DE	Current	592	2818	5	197700	71610	55	166000	42790	0.382
DEL	Current	1981	7489	5	76230	23890	55	69840	26010	0.595
DETI	Current	439	3650	5	39850	4694	55	14280	10240	6.81e-06***
DFP	Current	3207	38619	5	27430	7145	55	15360	4776	0.0181**
DHSSPS	Current	561	4219	5	483200	249600	55	359400	22580	0.33
DOE	Current	2587	27468	5	21430	6045	55	10510	2501	0.0151**
DOJ	Current	3641	43336	3	148400	14390	33	103400	7474	0.0299***
DRD	Current	2167	32898	5	58580	7621	55	36180	11170	0.00219***
DSD	Current	6864	13950	5	74130	35410	55	41100	7403	0.105
FSA	Current	-	-	5	723	111	55	704	207	0.752
NIA	Current	-	-	5	5351	1124	55	3602	278	0.0251***
NIAO	Current	-	-	5	165	985	55	659	231	0.00765***
NIAUR	Current	-	-	5	2297	1609	55	-194	1625	0.0227***
OFMDFM	Current	346	6075	5	12580	2204	55	5376	2480	0.00099***
PPS	Current	529	10151	3	3848	1029	33	2764	337	0.209
All	Current	-	-	86	66440	129400	946	47300	89250	0.183

All values to 4 significant figures and are in units of thousands of pounds. All figures are adjusted for the length of each month. Note that for each series t-tests (R's t.test function) were performed with the vector of final month spending amounts for each department-budget against the vector of spending amounts from all other months. The p value for the tests is presented. For some smaller departments data on invoices and staffing levels are not available.

Table 1.B.3: Northern Ireland department abbreviations

DeptCode	Department
AOCC	Assembly Ombudsman Commissioner for Complaints
DARD	Department of Agriculture and Rural Development
DCAL	Department of Culture, Arts and Leisure
DE	Department of Education
DEL	Department for Employment and Learning
DETI	Department of Enterprise, Trade and Investment
DFP	Department of Finance and Personnel
DHSSPS	Department of Health, Social Services and Public Safety
DOE	Department of Environment
DOJ	Department of Justice
DRD	Department for Regional Development
DSD	Department for Social Development
FSA	Food Standards Agency
NIA	Northern Ireland Assembly
NIAO	Northern Ireland Audit Office
NIAUR	Northern Ireland Authority for Utility Regulation
OFMDFM	Office of First Minister and Deputy First Minister
PPS	Public Prosecution Service
All	All

Table 1.B.4: Spending Series And Rollover Regime

	NR No.Obs	NR Mean	NR SD	NR t	EYF No.Obs	EYF Mean	EYF SD	EYF t	BES No.Obs	BES Mean	BES SD	BES t
Cash Outlays (RUUP)	8	1.121	0.05998	0.625	13	1.105	0.05442	0.68	3	1.1120	0.17010	0.995
Capital Expenditure (MF6T)	1	1.619	-	-	13	1.794	0.40340	0.563	3	2.2680	1.09200	0.522
Current Expenditure (MF6S)	1	1.032	-	-	13	1.028	0.01909	0.351	3	0.9996	0.04005	0.336

For each series t-tests (F's t-test function) were performed with the vector of final month spending ratios for each regime against the vector of final month spending moment from the other regimes (collectively). This p-value for the t-tests is presented. In no case is the test significant at the 20% level. Due to a limited number of observation, t-tests and variances are unable to be calculated in some cases.

Abbreviations: NR = No Rollover; EYF = End of Year Flexibility; BES = Budgetary Exchange System; No.Obs = Number of Observations; SD = Standard deviation; t = t-test p-value



## 1.C Curvature of production function robustness checks

Table 1.3 and the discussion of section 1.5.2 concentrates on the case of  $\gamma = 0.95$ . Here a robustness check was carried out to assess the extent to which the benefits of a tax change when a different  $\gamma$  value is assumed. This was done for the 5% exponential discounting model. For a range of  $\gamma$  values from 1 to 0.5 time implied social cost ( $\lambda$ ) was found from equation 1.23.  $x_{\text{Par}}$  was then found using equation 1.26. For the purposes of the  $\gamma = 1$  case a  $\gamma$  value of 0.95 was used for the purposes of equation 1.26.

$\gamma$	$\lambda$	$x_{\text{Par}}$	Benefits From Tax	Benefits from Perfect Agency
1.00	1.007	0.310	68.629	$\infty$
0.95	1.046	0.145	1.161	1.790
0.90	1.086	0.152	1.109	1.847
0.85	1.128	0.152	1.103	1.838
0.80	1.171	0.149	1.107	1.809
0.75	1.229	0.144	1.116	1.756
0.70	1.288	0.139	1.123	1.696
0.65	1.36	0.131	1.127	1.614
0.60	1.426	0.123	1.125	1.529
0.55	1.229	0.144	1.116	1.756
0.50	1.288	0.139	1.123	1.696

All of these benefits are for the 5% exponential calibration. For the  $\gamma = 1$  estimation for the purposes of equation 1.26, a  $\gamma$  value of 0.95 was chosen in finding the parliament desired spending level. Furthermore in this case the utility improvement proportion should be interpreted with caution as a result of this assumption as well as the fact that utility in the no tax case was close to zero and numerical imprecision can have a large relative impact when calculating utility gains.

Table 1.C.1: Robustness checks for the gains from imposition of a tax for the 5% exponential discounting case.

## 1.D Econometric Robustness checks - Section 1.4.2

This appendix will present a series of robustness checks for the regressions of table 1.2 to show that the conclusion of these regressions is similar if alternate choices are made in model specification or data preprocessing.

### Alternate Volatility Measure

Defining average spending for department  $d$ , budget  $b$ , in month  $m$  as  $\bar{x}_{m,d,b} = \frac{1}{5} \sum_{y=2008}^{2012} x_{y,m,b}$  I use the following alternate volatility measure:

$$\text{AltVolatility}_{y,d,b} = \frac{1}{6} \left[ \sum_{m=1}^6 \left| \frac{x_{m,y,d,b} - \bar{x}_{m,d,b}}{\bar{x}_{m,d,b}} \right| \right] \quad (1.D.1)$$

The three regressions from table 1.2 are repeated with this alternate volatility measure with the results presented in table 1.D.1.

Figure 1.D.1 shows the volatility and renormalised spike for each department-budget-year as well as the plot for this alternate volatility measure. The left hand side data is that used for three regressions of table 1.2 whilst the right hand side data was used for table 1.D.1. The top of the

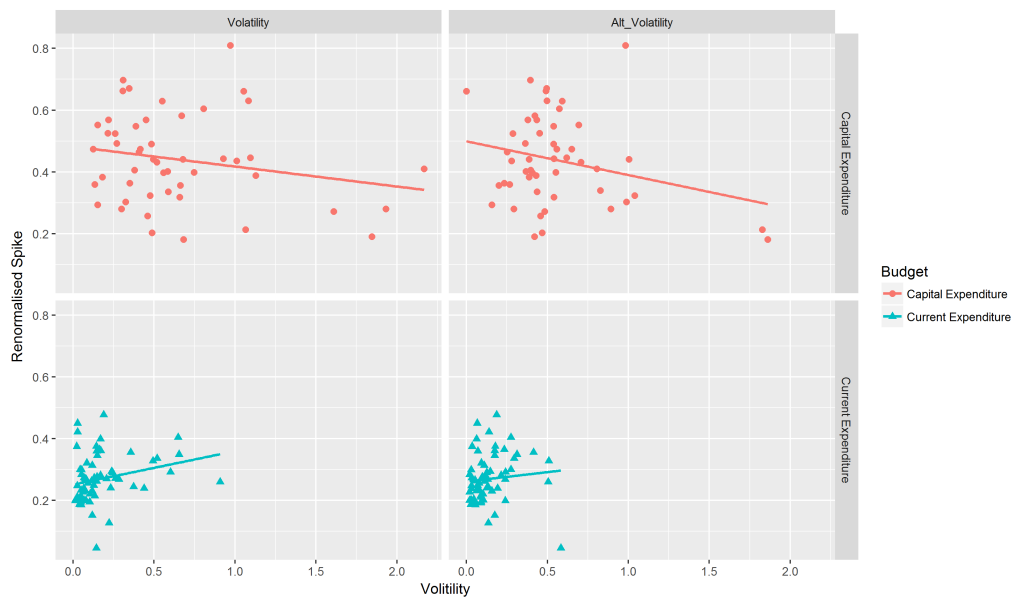


Figure 1.D.1: Volatility and Renormalised Spikes

Table 1.D.1: Alternative Volatility Measure and Renormalised Spike

	RenormalisedSpike		
	(1)	(2)	(3)
AltVolatility (Current)	0.088 (0.128)	-0.087 (0.160)	-0.200* (0.115)
AltVolatility (Capital)	-0.107** (0.042)	-0.120*** (0.037)	-0.185*** (0.039)
Budget	-0.240*** (0.035)	-0.224*** (0.032)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	116	116	116
R <sup>2</sup>	0.423	0.449	0.192
Adjusted R <sup>2</sup>	0.386	0.333	-0.106
F Statistic	11.318*** (df = 7; 108)	11.058*** (df = 7; 95)	3.328*** (df = 6; 84)

Notes: \*\*\*Significant at the 1 percent level.  
 \*\*Significant at the 5 percent level.  
 \*Significant at the 10 percent level.  
 Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

figure shows capital expenditure volatilities and renormalised spikes while the corresponding points for the current expenditure series are on the bottom.

### Separate Regressions

One potential concern is that the year fixed effects are not well suited for either current or capital expenditure due to the presence of the other expenditure series. As a result separate regressions for each type of expenditure were performed which are presented in table 1.D.2.

Table 1.D.2: Volatility and Renormalised Spike - Seperate Regressions by budget

	RenormalisedSpike			
	(1)	(2)	(3)	(4)
Volatility (Current)	0.110*** (0.032)	-0.059 (0.041)		
Volatility (Capital)			-0.060** (0.024)	-0.100*** (0.038)
Fixed Effects	No	Department	No	Department
Fiscal Year FE	Yes	Yes	Yes	Yes
Observations	66	66	49	49
R <sup>2</sup>	0.226	0.267	0.069	0.148
Adjusted R <sup>2</sup>	0.162	-0.013	-0.040	-0.278
F Statistic	3.509*** (df = 5; 60)	3.431** (df = 5; 47)	0.633 (df = 5; 43)	1.110 (df = 5; 32)

Notes: \*\*\*Significant at the 1 percent level.  
\*\*Significant at the 5 percent level.  
\*Significant at the 10 percent level.  
Standard errors are clustered at the (Department), (Department x Budget), (Department), (Department x Budget) levels respectively.

While the coefficient on capital volatility is significantly negative in the third and fourth regressions of table 1.D.2 these regressions fail the F-test of significance. This is a result of the increased degrees of freedom caused by budget specific year fixed effects. This can be seen by repeating the regressions of table 1.D.2 without including year fixed effects. This can be seen in table 1.D.3 where it can be noted that the departmental FE model for capital expenditure now passes the F-test of model significance.

Table 1.D.3: Volatility and Renormalised Spike - Seperate Regressions by budget and no year FEs

	RenormalisedSpike			
	(1)	(2)	(3)	(4)
Volatility (Current)	0.108*** (0.035)	-0.067 (0.064)		
Volatility (Capital)			-0.065** (0.028)	-0.107*** (0.039)
Fixed Effects	No	Department	No	Department
Fiscal Year FE	No	No	No	No
Observations	66	66	49	49
R <sup>2</sup>	0.063	0.017	0.044	0.126
Adjusted R <sup>2</sup>	0.049	-0.253	0.024	-0.166
F Statistic	4.315** (df = 1; 64)	0.872 (df = 1; 51)	2.163 (df = 1; 47)	5.173** (df = 1; 36)

Notes: \*\*\*Significant at the 1 percent level.  
\*\*Significant at the 5 percent level.  
\*Significant at the 10 percent level.  
Standard errors are clustered at the (Department), (Department x Budget), (Department), (Department x Budget) levels respectively.

### Controlling for the remaining budget

Another potential concern is that the use of renormalised spikes does not perfectly remove budget constraint issues from the regression. As a robustness check then we repeat the three regressions of

table 1.2 whilst controlling for the remaining budget as of the seventh month,  $B_{7,y,d,b}$ . This can be seen in table 1.D.4. The coefficients are similar to table 1.2 and in addition the coefficients on the log remaining budget are not generally significant which indicates that the benchmark specification is successful in separating budget issues from the volatility-spending spike relationship of interest.

Table 1.D.4: Volatility and Renormalised Spike - Including Remaining Budget

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.074** (0.032)	0.040 (0.057)	-0.068 (0.053)
Volatility (Capital)	-0.077*** (0.026)	-0.092** (0.036)	-0.107*** (0.034)
Budget	-0.214*** (0.037)	-0.183*** (0.054)	
Log(Remaining Budget)	-0.015* (0.009)	-0.035 (0.023)	-0.018 (0.063)
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	115	115	115
R <sup>2</sup>	0.442	0.475	0.150
Adjusted R <sup>2</sup>	0.400	0.356	-0.181
F Statistic	10.505*** (df = 8; 106)	10.511*** (df = 8; 93)	2.073* (df = 7; 82)

Notes:

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

\* Significant at the 10 percent level.

Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

### Varying the inclusion criteria - Annual Budget

The set criteria for inclusion was that the department-budget had a budget of 10 million pounds in a given year. If this is loosened such that a threshold budget of 5 million pounds were set then significance is reduced but still significant at 10% in the specifications with fixed effects. This can be seen in table 1.D.5.

Table 1.D.5: Volatility and Renormalised Spike - 5 million pounds provision threshold

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.062 (0.051)	0.049 (0.049)	-0.041 (0.048)
Volatility (Capital)	-0.036 (0.032)	-0.067* (0.036)	-0.083** (0.037)
Budget	-0.220*** (0.033)	-0.230*** (0.033)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	130	130	130
R <sup>2</sup>	0.446	0.454	0.099
Adjusted R <sup>2</sup>	0.414	0.342	-0.210
F Statistic	14.045*** (df = 7; 122)	12.701*** (df = 7; 107)	1.767 (df = 6; 96)

Notes:

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

\* Significant at the 10 percent level.

Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

### Varying the inclusion criteria - Monthly positive spending

A small number of zero and negative spending values were disregarded when estimating volatility due to the relative volatility measure being used sensitive to these cases. Thus some department-budget-years had volatility calculated by taking an average of fewer than 4 month-volatilities in a

way analogous to equation 1.17. We test the impact of this decision by repeating the benchmark regressions on the subset of department-years with all positive spending values. The significance is maintained in this specification as can be seen in table 1.D.6.

Table 1.D.6: Volatility and Renormalised Spike - All 12 months of positive spending

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.097 (0.071)	0.063 (0.089)	-0.110 (0.083)
Volatility (Capital)	-0.049** (0.020)	-0.052*** (0.016)	-0.066*** (0.011)
Budget	-0.195*** (0.028)	-0.202*** (0.025)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	95	95	95
R <sup>2</sup>	0.426	0.484	0.193
Adjusted R <sup>2</sup>	0.379	0.345	-0.185
F Statistic	9.210*** (df = 7; 87)	9.917*** (df = 7; 74)	2.552** (df = 6; 64)
Notes:	***Significant at the 1 percent level. **Significant at the 5 percent level. *Significant at the 10 percent level. Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.		

### Varying the inclusion criteria - Years of data

In the benchmark specification no thresholds were set for the number of years of data that were present. This may have an impact as some departments have more years of data with which to set fixed effects. If a threshold is introduced such that 5 years of data are necessary for inclusion into the regression (and thus the panel is balanced) then significance is maintained. This can be seen in table 1.D.7.

Table 1.D.7: Volatility and Renormalised Spike - 5 years of spending data

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.102*** (0.036)	0.024 (0.057)	-0.067 (0.049)
Volatility (Capital)	-0.079*** (0.014)	-0.077*** (0.009)	-0.072*** (0.010)
Budget	-0.220*** (0.033)	-0.216*** (0.030)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	95	95	95
R <sup>2</sup>	0.470	0.501	0.183
Adjusted R <sup>2</sup>	0.427	0.383	-0.098
F Statistic	11.017*** (df = 7; 87)	10.890*** (df = 7; 76)	2.608** (df = 6; 70)
Notes:	***Significant at the 1 percent level. **Significant at the 5 percent level. *Significant at the 10 percent level. Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.		

### Calculating Volatility and renormalised spike with other than 6 months.

In the benchmark setting 6 months were taken to calculate the volatility (with equation 1.17) and the final 6 months were used for calculating the renormalised spending spike with equation 1.18. Here we repeat the benchmark regressions with the first 4 months and the first 8 months used for calculating the volatility in tables 1.D.8 and 1.D.9 respectively. In these cases the renormalised

spending spikes were calculated as  $\frac{x_{12,y,d,b}}{B_{5,y,d,b}}$  and  $\frac{x_{12,y,d,b}}{B_{9,y,d,b}}$  respectively. The benchmark regression coefficients are much less significant when 4 months used to calculate volatility as there only exists months 2 and 3 for calculating volatility. The regressions with volatility from the first 8 months remains strongly significant however.

Table 1.D.8: Volatility and Renormalised Spike - 4 months for calculating volatility

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.072* (0.039)	0.081* (0.046)	-0.004 (0.055)
Volatility (Capital)	-0.008 (0.024)	-0.026 (0.020)	-0.061* (0.036)
Budget	-0.191*** (0.038)	-0.211*** (0.032)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	108	108	108
R <sup>2</sup>	0.445	0.492	0.126
Adjusted R <sup>2</sup>	0.406	0.375	-0.231
F Statistic	11.435*** (df = 7; 100)	12.037*** (df = 7; 87)	1.823 (df = 6; 76)

Notes: \*\*\*Significant at the 1 percent level.  
 \*\*Significant at the 5 percent level.  
 \*Significant at the 10 percent level.  
 Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

Table 1.D.9: Volatility and Renormalised Spike - 8 months for calculating volatility

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.189** (0.077)	0.045 (0.086)	-0.012 (0.097)
Volatility (Capital)	-0.059*** (0.017)	-0.064*** (0.020)	-0.084*** (0.027)
Budget	-0.243*** (0.030)	-0.228*** (0.028)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	113	113	113
R <sup>2</sup>	0.384	0.416	0.137
Adjusted R <sup>2</sup>	0.342	0.289	-0.194
F Statistic	9.332*** (df = 7; 105)	9.346*** (df = 7; 92)	2.140* (df = 6; 81)

Notes: \*\*\*Significant at the 1 percent level.  
 \*\*Significant at the 5 percent level.  
 \*Significant at the 10 percent level.  
 Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

## Alternate Spending Series

In the main regressions the spending and budget variables were expressed in pounds. In addition the budget value used to renormalise the spending spike included rollover funds. In this section I use three other spending and budget series to repeat the regressions of table 1.2. The first is a spending value expressed as a proportion of annual *outturn* with the complementary  $B_{7,y,d,b}$  value describing the proportion of outturned funds unspent as of the start of the seventh month. Any rollover funds are ignored in this specification. These regressions are presented in table 1.D.10. The second is similar but with spending being as a proportion of annual *provision* and the complementary  $B_{7,y,d,b}$  being the proportion of the allowed budget being unspent as of the start of the seventh month. Again rollover funds are ignored. These regressions are presented in table 1.D.11. The third spending series gives spending as a proportion of *annual provision plus saved*

rollover funds as of the start of the year with the complementary  $B_{7,y,d,b}$  being the proportion of total available funds (provision + rollover) being unspent as of the start of the 7th month. Rollover is accounted for in this regression. These regressions are presented in table 1.D.12.

Table 1.D.10: Volatility and Renormalised Spike - Spending as proportion of outturn

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.110*** (0.041)	0.120** (0.048)	-0.054 (0.052)
Volatility (Capital)	-0.080*** (0.028)	-0.095** (0.038)	-0.107*** (0.037)
Budget	-0.244*** (0.035)	-0.259*** (0.033)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	115	115	115
R <sup>2</sup>	0.455	0.492	0.157
Adjusted R <sup>2</sup>	0.419	0.384	-0.158
F Statistic	12.748*** (df = 7; 107)	12.995*** (df = 7; 94)	2.579** (df = 6; 83)

Notes: \*\*\*Significant at the 1 percent level.  
\*\*Significant at the 5 percent level.  
\*Significant at the 10 percent level.  
Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

Table 1.D.11: Volatility and Renormalised Spike - Spending as proportion of provision

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.110*** (0.034)	0.076 (0.052)	-0.064 (0.052)
Volatility (Capital)	-0.068*** (0.025)	-0.092** (0.037)	-0.109*** (0.038)
Budget	-0.232*** (0.033)	-0.245*** (0.034)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	115	115	115
R <sup>2</sup>	0.425	0.459	0.148
Adjusted R <sup>2</sup>	0.388	0.344	-0.170
F Statistic	11.320*** (df = 7; 107)	11.399*** (df = 7; 94)	2.404** (df = 6; 83)

Notes: \*\*\*Significant at the 1 percent level.  
\*\*Significant at the 5 percent level.  
\*Significant at the 10 percent level.  
Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

Table 1.D.12: Volatility and Renormalised Spike - Spending as proportion of funds inc. rollover

	RenormalisedSpike		
	(1)	(2)	(3)
Volatility (Current)	0.051 (0.039)	0.051 (0.043)	-0.060* (0.037)
Volatility (Capital)	-0.057** (0.026)	-0.060** (0.025)	-0.051** (0.025)
Budget	-0.087*** (0.034)	-0.089*** (0.031)	
Fixed Effects	No	Department	Department x Budget
Fiscal Year FE	Yes	Yes	Yes
Observations	115	115	115
R <sup>2</sup>	0.241	0.300	0.280
Adjusted R <sup>2</sup>	0.192	0.151	0.012
F Statistic	4.858*** (df = 7; 107)	5.758*** (df = 7; 94)	5.391*** (df = 6; 83)

Notes: \*\*\*Significant at the 1 percent level.  
\*\*Significant at the 5 percent level.  
\*Significant at the 10 percent level.  
Standard errors are clustered at the (Department), (Department), (Department x Budget) levels respectively.

## Alternate Panel Data Techniques

If first differences or random effects are used in place of fixed effects significance of the capital volatility variable is maintained. This can be seen in table 1.D.13.

Table 1.D.13: Volatility and Renormalised Spike - First Differences and Random Effects

	RenormalisedSpike	
	(1)	(2)
Volatility (Current)	-0.083 (0.056)	0.096*** (0.032)
Volatility (Capital)	-0.131*** (0.047)	-0.072*** (0.026)
Budget		-0.233*** (0.034)
Panel Structure	First Differences	Random Effects
Fiscal Year Dummies	Yes	Yes
Observations	89	115
R <sup>2</sup>	0.206	0.405
Adjusted R <sup>2</sup>	0.148	0.367
F Statistic	3.549*** (df = 6; 82)	10.424*** (df = 7; 107)

*Notes:* \*\*\*Significant at the 1 percent level.  
\*\*Significant at the 5 percent level.  
\*Significant at the 10 percent level.  
Standard errors are robust.

## 1.E Time Series: Final Month Cash-outlays and Variance

### Modelling variance with GARCH models

We can also examine this relationship using intertemporal variation for the cash outlays series over the period from the 1984-1985 fiscal year to the 2013-2014 fiscal year.<sup>1.53</sup> The following regression was run (with each year being an observation):

$$\text{Final Month Cash-Outlay Ratio}_y = \beta_0 + \beta_1 \text{Variance}_y + \epsilon_y \quad (1.E.1)$$

where  $y$  is a year index. In order to create the  $\text{Variance}_y$  variable, two GARCH models were estimated. Due to the sensitivity of equation 1.E.1 to the final month spending value, each model was calculated with the omission of the final month of each year. As the final month of the year is March and March is dropped the month of April comes immediately after February in this 11 month year.<sup>1.54</sup> This has the effect of estimating variance using only the residuals from the first 11 months of the fiscal year to ensure that final month spending does not impact both sides of equation 1.E.1.<sup>1.55</sup> The following models of cash outlays were estimated (with monthly data of

<sup>1.53</sup>A similar methodology was attempted for the other two spending series but with only about half as many years being available estimates of equation 1.E.1 were insignificant and did not pass the F test of model significance.

<sup>1.54</sup>Thus when it is the first month of the year then  $t - 1$  is the 11th month of the previous year and  $t - 11$  is the first month of the previous year.

<sup>1.55</sup>The dropping of the 12th month is not critical for these results. The variance coefficients in table 1.E.1 are actually more strongly negative when all 12 months are used. These regressions are omitted for brevity but are available on request from the author.



this 11 month year):

$$\Delta^{11}\ln(\text{Cash Outlays})_t = \theta_0 + \sum_{i=1}^4 \theta_i \Delta^{11}\ln(\text{Cash Outlays})_{t-i} + \theta_{11} \Delta^{11}\ln(\text{Cash Outlays})_{t-11} + \epsilon_t \quad (1.E.2)$$

$$\sigma_t^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \gamma_3 \sigma_{t-1}^2 + \gamma_4 \sigma_{t-2}^2$$

$$\Delta^{11}\ln(\text{Cash Outlays})_t = \theta_0 + \sum_{i=1}^{11} \theta_i \Delta^{11}\ln(\text{Cash Outlays})_{t-i} + \epsilon_t \quad (1.E.3)$$

$$\sigma_t^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \gamma_2 \sigma_{t-1}^2$$

These two models were then used to estimate the conditional variance throughout the period.<sup>1.56</sup> The raw Cash Outlays<sub>t</sub>, the  $\Delta^{11}\ln(\text{Cash Outlays})_t$  series as well as the estimated conditional variances from each model and final month cash-outlay ratios can be seen in figure 1.E.1. In this figure the conditional variance for model 1.E.2 is in black and the conditional variance for model 1.E.3 is in blue. The regression outputs are shown in table 1.E.2.<sup>1.57</sup>

For each model, predictions of the conditional variance,  $\sigma_t^2$ , were estimated for each month (in the 11 month year). These monthly estimated variances were then averaged to give an annual series to match the annual final month cash-outlay ratio data so each observation is for a year. The regression of equation 1.E.1 was estimated using the variances from each model as well as the logs of these variances. These regressions are presented in table 1.E.1 which lead to similar conclusions as the Northern Ireland cross sectional regressions of section 1.4.2. Higher variance is associated with lower spending spikes.<sup>1.58</sup>

<sup>1.56</sup>The 1984 - 1985 and 1985 - 1986 fiscal year were used in the calculation of each GARCH process and then dropped for the estimation of equations 1.E.1 to ensure sufficient data was available beforehand to estimate the conditional variance series.

<sup>1.57</sup>In this figure the conditional variance for model 1.E.2 is in black and the conditional variance for model 1.E.3 is in blue

<sup>1.58</sup>Note that whilst the coefficients both models are significantly negative the size of the estimated variance coefficients is different for each model. This is not an issue for this regression. Variances are estimated based on model residuals which depend on model specifications. A more complex model will generally have lower residuals and lower variance of the residuals. It is the changes in variance over time for a given model that matter for these regressions and not the (model dependent) magnitudes of the variance.

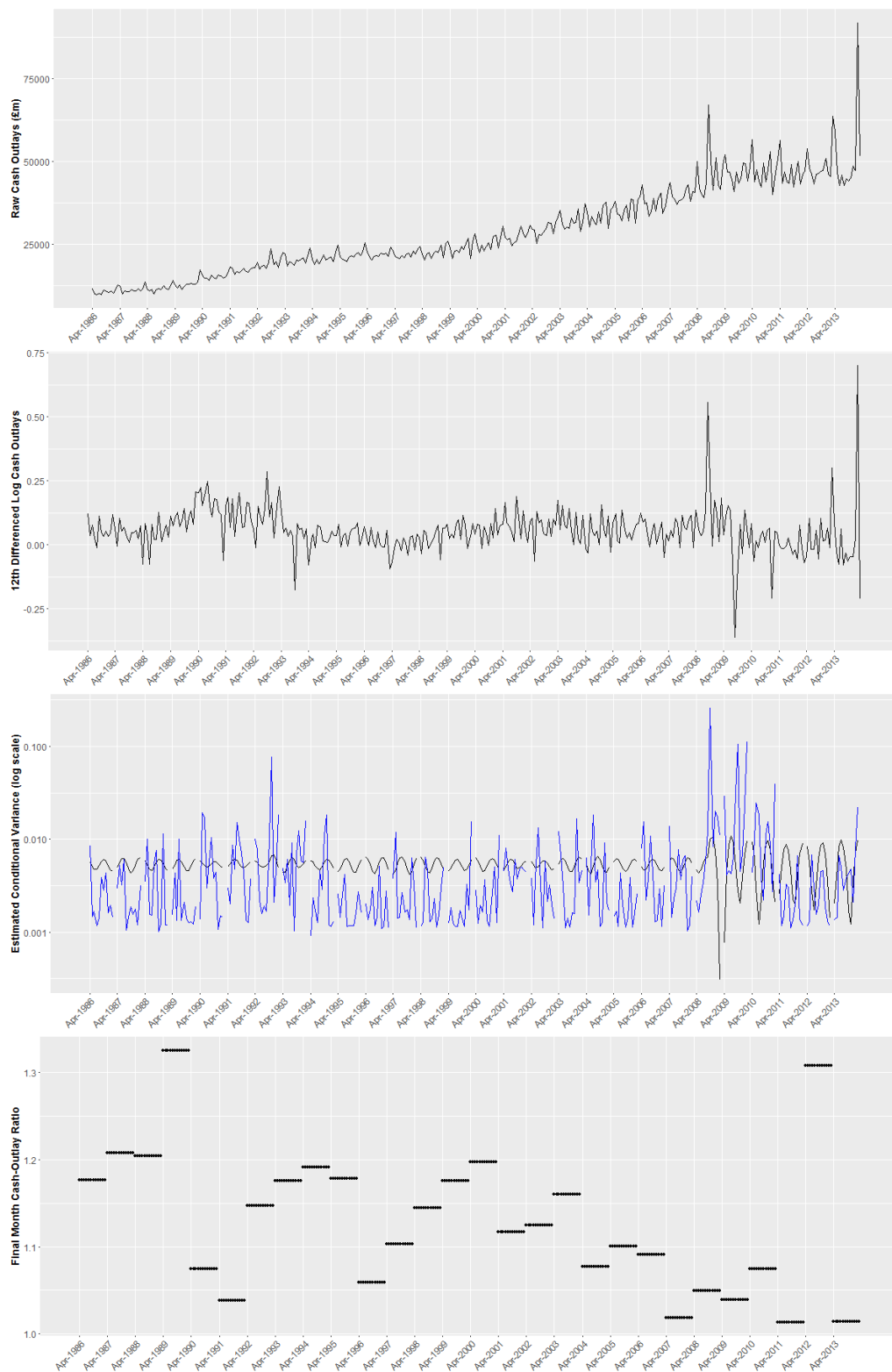


Figure 1.E.1:  $\Delta^{12}\ln(\text{Cash Outlays})_t$  and estimated conditional variances

Table 1.E.1: Regression for Spending Spike Vs Variance

	Dependent variable:			
	Final Month Cash-Outlays Ratio		Second Model	
	(1)	(2)	(3)	(4)
Variance	-213.357*** (70.771)		-3.938*** (0.975)	
Log Variance		-1.164*** (0.395)		-0.045*** (0.015)
Constant	2.284*** (0.388)	-4.947** (2.055)	1.153*** (0.020)	0.883*** (0.074)
Observations	28	28	28	28
R <sup>2</sup>	0.191	0.189	0.126	0.162
Adjusted R <sup>2</sup>	0.160	0.158	0.093	0.130
Residual Std. Error (df = 26)	0.075	0.075	0.078	0.076
F Statistic (df = 1; 26)	6.125**	6.074**	3.764*	5.026**

Note: \* p < 0.1; \*\* p < 0.05; \*\*\* p < 0.01  
Heteroskedastic consistent standard errors are used.

Table 1.E.2: UK Cash outlays GARCH models

	(1) Model 1.E.2	(2) Model 1.E.3
$\frac{\Delta^{11}\ln(\text{Cash Outlays})_t}{\Delta^{11}\ln(\text{Cash Outlays})_{t-1}}$	0.295*** (0.0797)	0.236*** (0.0396)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-2}$	0.00994 (0.0889)	-0.0236 (0.0416)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-3}$	0.392*** (0.0808)	0.457*** (0.0449)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-4}$	-0.00930 (0.0782)	0.0831* (0.0471)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-5}$		0.248*** (0.0537)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-6}$		-0.0242 (0.0444)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-7}$		-0.0493 (0.0450)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-8}$		0.264*** (0.0442)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-9}$		-0.154*** (0.0470)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-10}$		0.178*** (0.0432)
$\Delta^{11}\ln(\text{Cash Outlays})_{t-11}$	-0.264*** (0.0940)	-0.433*** (0.0488)
Constant	0.0318*** (0.00707)	0.00867*** (0.00277)
$\frac{\sigma_t^2}{\epsilon_{t-1}^2}$	0.0187*** (0.00552)	1.330*** (0.368)
$\sigma_{t-1}^2$	1.316*** (0.0100)	-0.0205 (0.0182)
$\sigma_{t-2}^2$	-1.012*** (0.0173)	
Constant	0.00369*** (0.000394)	0.00117*** (0.000213)
N	308	308
p	3.03e-09	1.20e-101

Standard errors in parentheses

The p value for each model refers to a wald test of model significance

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

### Modelling variance with the Volatility and AltVolatility metrics

Further regressions were performed using the volatility metrics from equations 1.17 and 1.D.1 and the renormalised spike described by equation 1.18. These can be seen in tables 1.E.3 for the Cash Outlays spending series.<sup>1.59</sup> Here it can be noted however that the two of these regressions that pass the F-test establish a significantly negative relationship between volatility and the renormalised spending spike (at the 10% and 5% levels).

Table 1.E.3: Regression for Renormalised Spike Vs Volatility & AltVolatility

	<i>Dependent variable:</i>			
	Renormalised Spending Spike			
	(1)	(2)	(3)	(4)
Volatility	-0.011 (0.115)			
Log Volatility		0.001 (0.005)		
Alt Volatility			-0.219* (0.115)	
Log Alt Volatility				-0.011** (0.005)
Constant	0.183*** (0.006)	0.186*** (0.016)	0.191*** (0.005)	0.145*** (0.016)
Observations	30	30	30	30
R <sup>2</sup>	0.0003	0.001	0.129	0.131
Adjusted R <sup>2</sup>	-0.035	-0.035	0.098	0.100
Residual Std. Error (df = 28)	0.012	0.012	0.011	0.011
F Statistic (df = 1; 28)	0.009	0.032	4.151*	4.224**

*Note:* \* p<0.1; \*\* p<0.05; \*\*\* p<0.01

## 1.F Further discussion

### Assumption of disutility from effort in spending money

A crucial assumption for the procrastination model is that there is some level of disutility that is increasing with spending. The project management industry gives some basic evidence for this. It is also possible however to motivate this assumption by starting with a performance function incorporating effort and money and with a cost of effort function involving only effort.

Denoting  $P(e_t, x_t)$  as the performance in a period from spending  $x_t$  and using effort  $e_t$  and denoting  $k(e_t)$  as the disutility from effort level  $e_t$  the problem for a department is:

$$\begin{aligned} \max_{x_1, x_2, e_1, e_2} \quad & D [P(e_1, x_1) + P(e_2, x_2)] - k(e_1) - Dk(e_2) \\ \text{s.t.} \quad & x_1 + x_2 \leq B \end{aligned} \tag{1.F.1}$$

<sup>1.59</sup>Similar regressions were attempted for the capital expenditure and current expenditure series however due to a lack of sufficient data these failed the F-test of model significance. These are available on request.

In any period we can use first order conditions to get a equation for optimal effort as a function of spending in the current period. As there is no intertemporal constraint on effort only the current period's spending will enter this function.

$$e_t = f(x_t) \tag{1.F.2}$$

We can substitute this into the performance and functions as:

$$P(e_t, x_t) = P(f(x_t), x_t) \qquad k(e_t) = k(f(x_t)) \tag{1.F.3}$$

$$= v(x_t) \qquad = e(x_t) \tag{1.F.4}$$

Substituting back into equation 1.F.1 yields equation 1.1.

### **Assumption of performance monitoring on a fiscal year basis**

A key assumption for the procrastination model is that performance is monitored on a fiscal year basis. This is a reasonable assumption when the UK is considered. While the performance frameworks of devolved administrations (such as Northern Ireland) will not be explicitly discussed they can be considered (in the timing dimension) to mirror the central government framework.

Since at least the time of the red book of the exchequer was written around the year 1230 [Hall, 1896, page 855] fiscal reporting has been organised around annual increments. This annual increment was maintained as performance-related pay was beginning to be introduced in 1991 by the Major government [Major, 1991]. An early implementation was in local government where a number of performance metrics were introduced to encourage greater public accountability [Pan-chamia and Thomas, 2010]. A broader based framework was introduced with the incoming Labour government in 1998 with the Public Service Agreement System where departments were given measurable performance targets (termed PSAs) along with completion dates. Despite the presence of in year completion dates and bimonthly Prime Ministerial meetings with department heads these performance targets were inexorably linked to the fiscal year. With the passage of the 2000 Government Resource Accounting Act the public spending committee was tasked with monitoring progress towards PSAs by requiring departments to provide performance reporting alongside spending figures which were published on a fiscal year basis. Furthermore the negotiation of PSAs was “explicitly tied to the spending review process, which increased the potential for the treasury to incentivise departments to deliver on government’s overall objectives in return for appropriate

funding. [Panchamia and Thomas, 2010, page 3]”.

There exists further evidence of performance and spending being scrutinised together in practice from the reports of the House of Commons Scrutiny Unit [2009, page 4]. In 2010 the incoming coalition government replaced the PSA system with the Departmental Business Plans system [Stephen et al., 2013]. These documents were updated on an annual basis and included regular goalsetting and performance measurement in a similar fashion to the PSA system. One new feature was the public release of individual performance targets for senior public servants on an annual (fiscal year) basis [UK Cabinet Office, 2013].

Regular performance appraisal is also common in other countries. The United States Office of Personnel Management describes the performance award framework for the US government with awards for performance awarded on a period basis similarly to the UK US Office of Personnel Management [2015].

### **Assumption of diminishing returns from spending**

In this paper the low quality of spending at the end of fiscal years is interpreted as being a result of diminishing returns to spending. Diminishing returns from spending emerge as a credible way to model spending when one considers what mechanisms could explain the link between heightened spending and low quality. One way this could occur is that managers have less time to spend on each spending project and projects could blow out costs due to mistakes being made. Whilst money expended (as a factor of production) can be scaled up, manager’s time is another factor of production that cannot be scaled up as easily. Another may be frivolous purchases that occur while senior managers cannot check on all of the purchases made by more junior budget holders. In this way heightened spending may lead to a higher proportion of funds being spent on items which may increase personal utility for these budget holders but do not deliver value to the organisation (for instance new office furniture or highend computer monitors). Crowding out of suppliers may be a further mechanism - due to higher demand at the end of the fiscal year suppliers raise their prices (Some evidence for this type of mechanism comes from Hyndman et al. [2007, p.227]). A further mechanism is that shorter timeframes for contracting as well as more contracts available for suppliers to quote on result in fewer quotes per project (a mechanism which finds direct support in the IT project analysis of Liebman and Mahoney [2013, page 24]). In addition there exists the possibility that departments simply run out of profitable spending opportunities and need to fall back on less valuable spending opportunities.

There may be alternative ways of modeling why fiscal-year ends have spending of low quality. The end of year is not an inherently special time however and so it is difficult to devise a feasible mechanism for this low quality that does not depend upon the heightened level of spending in a

way that is isomorphic to diminishing returns.

Note that this assumption of diminishing returns from spending is not important for showing how heightened end of year spending can be caused by procrastination. It is important in a deterministic setting however for establishing that there are welfare gains from eliminating these spending spikes however. As is discussed in section 1.5.2 in a stochastic setting end of year spending spikes can lead to inefficiencies even with a constant returns production function.

### **Alternate explanations of end of fiscal year spending spikes**

Two explanations frequently came up at early seminars of this paper and in discussions with public servants. The first is that if money is not spent by the end of the fiscal year it is lost by a government department. This has not been the case in the UK since 1998, nonetheless even if rollover were not available this is still only an argument for spending an entire budget within a fiscal year and is not an argument for spending the funds in the final months of a fiscal year. A further feature is needed in any model to explain why spending is delayed until late in the year. The second explanation offered is that if funds are not spent in a fiscal year then the budget for the next fiscal year is reduced by an amount corresponding to the underspend. This is subject to the same criticism as the first explanation in that it doesn't explain why spending is delayed until the end of the fiscal year. In addition it is likely that spending spikes at the end of the fiscal year would be a bad signal to the parliament which sets the budget. This mechanism may therefore actually work to push spending away from the end of the fiscal year towards more even spending. Furthermore Danny Alexander<sup>1.60</sup>, a former UK secretary of the treasury, indicated that he was aware of this argument but did "not believe it is entirely the case". He cites that even after allowing for the rolling over of funds, "since 2010 departments have on average underspent by an average of £5 billion a year".

UK procurement rules require that all calls for proposals be listed for a minimum period (52, 37 or 15 days depending on the case) before contracts can be awarded UK Government [2006]. End of year spending spikes might be explained by multiple calls for proposals being due before the end of a fiscal year with poorer quality explained by whether these calls are listed for a shorter duration due to the proximity of the end of the fiscal year. While this is plausible in the no rollover case this explanation cannot explain why end of year spending persists with rollover. In this case departments could save money from one fiscal year to pay for a call for proposal that is due in the next fiscal year.

Another potential mechanism is the release of some funds (Departmental Unallocated Provisions (DUPs)) that are typically authorised and available for departments in January. There are a few

---

<sup>1.60</sup>Source: Communications with the author

reasons to believe that this effect would have a minor impact on spending spikes. The first is that as these funds are awarded by January it would be expected that the effect of these funds would be realised in February and March. As can be seen in figure 1.1 however March spending is much greater than February. The second is that these funds are approved as DUPs at the start of the fiscal year and there exists an expectation that they would be approved for spending if they are required. In the absence of ambiguity about whether these funds would be approved these funds would be anticipated and funding more evenly dispersed. The third is that heightened spending is found in every Northern Irish department not all of which receive additional funding provision within the fiscal year.

Two further putative explanations can be dismissed as an explanation for end of year spending spikes. One is a general growth in the demand for and cost of government services as a result of population growth and inflation. The size of the spending spikes observed is far too large to be accounted for in this way. The second is measurement error where spikes are exaggerated by imperfect accrual accounting which tends to record a number of expenditure items in the last month of the fiscal year. Whilst it is credible that this could occur to some degree the survey data collected by Liebman and Mahoney [2013], Teich [2013] and McPherson [2007] support the existence of a spending spike independently of any evidence of substantial measurement error. Furthermore even if the last month is excluded it can be seen that spending is increasing over the first 11 months of the fiscal year for both Northern Ireland and the UK central government.

As a final point consider that government spending over time is naturally modelled in economics as a standard consumption smoothing problem where an agent maximises some utility function subject to an intertemporal budget constraint. In this framework explanations of end of fiscal year spending spikes can be modelled as predominantly coming from a feature of the budget constraint or from the utility function being time dependent. In such a setting precautionary savings can be seen as being driven by the budget constraint (in the absence of rollover) whilst procrastination comes about due to a time-dependent feature of the utility function. The allowance of budgetary rollover in the UK implies a lack of fiscal year distortions which means that a model relying on time-dependent utility is more credible than budget constraints for modelling end of fiscal year spending. Procrastination is the most credible explanation that results in utility from spending being time dependent. There may be other explanations possible however that also predict spending spikes resulting from time-dependent utility. To a large extent the procrastination model of this paper would be qualitatively isomorphic to other such explanations.<sup>1.61</sup> Thus even if there were some other time dependent utility force that existed alongside procrastination, the procrastination model

---

<sup>1.61</sup>Indeed all proofs of this paper are proved in terms of an arbitrary time-dependent utility function rather than the specific functional form assumed in the paper.



is likely to inform useful policy measures to address the problem. Indeed the (simple) proof for time dependent budgetary taxes only depends on time variant utility.

### Auditing to deter end of fiscal year spending

In this section we examine the use of auditing to achieve even spending using the simple deterministic model introduced in section 1.3.1. We first assume there exists a penalty that is incurred by a department in the event of them being found guilty of poor spending. All spending in the fiscal year is audited with a continuously differentiable increasing convex function<sup>1.62</sup>  $q(x)$  which gives the expected utility loss from audits as a function of spending.<sup>1.63</sup> This function indicates that high spending levels (at low marginal value) increase the chance of being penalised (or the penalty itself conditional on being caught) in an audit.

Thus the problem of the department becomes:

$$U = [Dv(x_1) - e(x_1) - Dq(x_1)] + [Dv(x_2) - De(x_2) - Dq(x_2)] \quad (1.F.5)$$

We can now get the following result:

**Proposition 5.** *Auditing with imperfect monitoring and constant penalties is insufficient to induce even expenditure*

*Proof.* We can note that the periodic utility is:

$$u_t(x_t) = Dv(x_t) - D_t e(x_t) - D_t q(x_t) \quad (1.F.6)$$

where  $D_1 = 1$  and  $D_2 = D < 1$

This satisfies the conditions (1.A.1), (1.A.2), (1.A.3) and (1.A.4) such that the proof of Proposition 1 is applicable.  $\square$

The reason that a constant level of auditing does not change the fact that late spending is relatively more attractive than early spending. To fix this a time variant strategy would be needed to target late spending and make spending in both periods equally attractive.

Now consider the case where the auditing frequency or penalties increase throughout the year. Thus there are two  $q(x)$  functions where the second period function  $q_2(x)$  is greater than  $q_1(x)$  for all  $x$ . This means that for any given spending level the expected utility loss from auditing is higher

<sup>1.62</sup>Specifically the conditions:

$$q(0) = 0 \qquad q'(0) = 0 \qquad q''(x) > 0 \quad \forall x$$

<sup>1.63</sup>This can be thought of as a penalty  $P$  for being caught spending poorly multiplied by a function that gives the probability of being caught as a function of spending,  $n(x)$ . In this case the expected utility loss from audits is  $q(x) = Pn(x)$ .

in the second period than the first. This relationship between  $q_1(x)$  and  $q_2(x)$  will be formalised as:

$$q_2(x) = \eta q_1(x) \quad (1.F.7)$$

for  $\eta > 1$ . With this relationship we get the following proposition.

**Proposition 6.** *There exists  $\eta > 1$  that is sufficient to induce even spending.*

*Proof.* We have the objective function:

$$V = u_1(x_1) - Dq_1(x_1) + u_2(x_2) - Dq_2(x_2) \quad (1.F.8)$$

We assume  $u_1(x_1) - Dq_1(x_1)$  and  $u_2(x_2) - Dq_2(x_2)$  are such that equation 1.A.5 holds.

We get the following first order conditions

$$\frac{\partial V}{\partial x_1} = u'_1(x_1) - Dq'_1(x_1) \quad (1.F.9)$$

$$\frac{\partial V}{\partial x_2} = u'_2(x_2) - Dq'_2(x_2) \quad (1.F.10)$$

These are equal at the optimal point:

$$u'_1(x_1) - Dq'_1(x_1) = u'_2(x_2) - Dq'_2(x_2) \quad (1.F.11)$$

Substituting in  $q'_2(x) = \eta q'_1(x)$  we want to find  $\eta$  such that both sides are equalised when  $x_1 = x_2 = x_{Par}$ . This solves to get:

$$\eta = 1 + \frac{u'_2(x_{Par}) - u'_1(x_{Par})}{Dq'_1(x_{Par})} \quad (1.F.12)$$

□

This  $\eta$  that induces optimal spending is given by:

$$\eta = 1 + \frac{(1 - D)e'(x_{Par})}{Dq'_1(x_{Par})} \quad (1.F.13)$$

By implementing harsher auditing throughout the year spending in later periods can be made less attractive then it was. A sufficiently harsh second period audit is enough to make spending in either period equally attractive which neutralises the incentive for departments to defer spending and leads to a constant level of spending throughout the fiscal year. Note that the parliament

could induce time variant auditing either through increasing the probability of being caught or through increasing the penalty from getting caught.

While a time variant auditing strategy can induce even expenditure such a strategy may not be practical in some cases. Variable penalties for the same behaviour throughout the year are unlikely to be tenable to many governments.<sup>1.64</sup> Increasing auditing frequency for end of year spending is another option but not an attractive one on the basis of cost. By contrast it is likely that time variant budgetary taxes would be close to costless.

### Proof of Proposition 6

We have the objective function:

$$V = u_1(x_1) - Dq_1(x_1) + u_2(x_2) - Dq_2(x_2) \quad (1.F.14)$$

We assume  $u_1(x_1) - Dq_1(x_1)$  and  $u_2(x_2) - Dq_2(x_2)$  are such that equation 1.A.5 holds.

We get the following first order conditions

$$\frac{\partial V}{\partial x_1} = u'_1(x_1) - Dq'_1(x_1) \quad (1.F.15)$$

$$\frac{\partial V}{\partial x_2} = u'_2(x_2) - Dq'_2(x_2) \quad (1.F.16)$$

These are equal at the optimal point:

$$u'_1(x_1) - Dq'_1(x_1) = u'_2(x_2) - Dq'_2(x_2) \quad (1.F.17)$$

Substituting in  $q'_2(x) = \eta q'_1(x)$  we want to find  $\eta$  such that both sides are equalised when  $x_1 = x_2 = x_{Par}$ . This solves to get:

$$\eta = 1 + \frac{u'_2(x_{Par}) - u'_1(x_{Par})}{Dq'_1(x_{Par})} \quad (1.F.18)$$

## 1.G Derivation of optimal tax for calibrated model in deterministic setting

As analytical results are not possible a deterministic approximation was used to give a functional form for a time variant budgetary tax.

<sup>1.64</sup>For two departments that make the same infraction (albeit in different periods) it would be inequitable to have them receive different penalties.

The departments have a problem represented by the lagrangian:

$$L = \sum_{m=1}^{12} D_{11} [x_m^\delta - \Omega D_{m-1} x_m] + \Lambda \left[ B - \sum_{m=1}^{12} \theta_m x_m \right] \quad (1.G.1)$$

Where  $\Lambda$  is the shadow price and  $D_i$  is the discounting  $i$  periods into the future. Taking the first order condition and setting it equal to zero we get:

$$\Lambda = \frac{\delta D_{11} x^{\delta-1} - \Omega D_{m-1}}{\theta_m} \quad (1.G.2)$$

Combining this equation for two different months  $x_m$  and  $x_{\bar{m}}$  we get:

$$\frac{\theta_{\bar{m}}}{\theta_m} = \frac{\delta D_{11} x_{\bar{m}}^{\delta-1} - \Omega D_{m-1}}{\delta D_{11} x_m^{\delta-1} - \Omega D_m} \quad (1.G.3)$$

Setting  $\theta_1 = 1$  and choosing the other tax rates such that  $x_m = x_{\bar{m}} = x_{\text{par}} \forall m, \bar{m}$  we get the expression:

$$\theta_m = \frac{\delta D_{11} x_{\text{par}}^{\delta-1} - \Omega D_{m-1}}{\delta D_{11} x_{\text{par}}^{\delta-1} - \Omega} \quad (1.G.4)$$

The annual budget is then set so that the department can afford to spend  $x_{\text{par}}$  every period while paying the tax:

$$B = x_{\text{par}} \left[ \sum_{t=1}^{12} \theta_t \right] \quad (1.G.5)$$

## 1.H Social Cost and Optimal Budgeting

We find the social cost ( $\lambda$ ) that implies an optimal budget choice of 1. This has the implicit assumption that the parliament does not choose a budget conditional on departmental savings but instead choose a stationary budget that maximises long run expected utility.

This is justified by the fact that this data refers to departmental expenditure limits which are decided upon in four year cycles (although departments still receive annual budgets. See appendix 1.B for a general discussion of this feature of governmental budgeting as well as Northern Ireland Executive [2008] and Northern Ireland Executive [2011]). In this framework it is not possible for parliaments to budget conditional on departmental savings accrued within a spending review period. Parliaments could only budget conditional on long term department savings (accrued between spending review periods). Thus this is a mild assumption. Aside from being close to the reality this assumption also offers a large computational advantage as the behaviour of the

department can be determined independently of the parliament as they are not optimising against the actions of the other party.

We assume that budgets are chosen to maximise the expected annual surplus without taking current departmental savings into account. The expected production is given by:

$$\text{Expected Production}(B) = \int_S E[P(B, S_y)] d\mu(S_y) \quad (1.H.1)$$

Where  $B$  is the budget,  $S$  is the state space of departmental savings,  $\mu(S_y)$  is the measure of each savings level and  $E[P(B, S_y)]$  is the expected production for a department with budget  $B$  and savings  $S_y$ . Expected expenditure will be equal to:

$$\text{Expected Expenditure}(B) = \int_S E[X(B, S_y)] d\mu(S_y) \quad (1.H.2)$$

Where  $E[X(B, S_y)]$  is expected expenditure for a department with budget  $B$  and savings  $S_y$ .

When the parliament is optimising we have:

$$\frac{\partial \text{Expected Production}(B)}{\partial B} = \lambda \frac{\partial \text{Expected Expenditure}(B)}{\partial B} \quad (1.H.3)$$

$$\lambda = \frac{\frac{\partial \text{Expected Production}(B)}{\partial B}}{\frac{\partial \text{Expected Expenditure}(B)}{\partial B}} \quad (1.H.4)$$

It is possible to evaluate this equation using numerical estimates of the gradients  $\frac{\partial \text{Expected Production}(B)}{\partial B}$  and  $\frac{\partial \text{Expected Expenditure}(B)}{\partial B}$

## 1.I 5% and 10% discounting calibrations

The spending pattern of these four calibrations against the Northern Ireland spending data can be seen in figure 1.I.1. Note that when the spending averages are considered the two exponential and the two hyperbolic discounting models appear to lie over the top of each other as changes in  $\delta$  compensate for the differing discount factors. On other measures however such as the percentage gains from a time variant tax the 5% models differ from their corresponding 10% model.

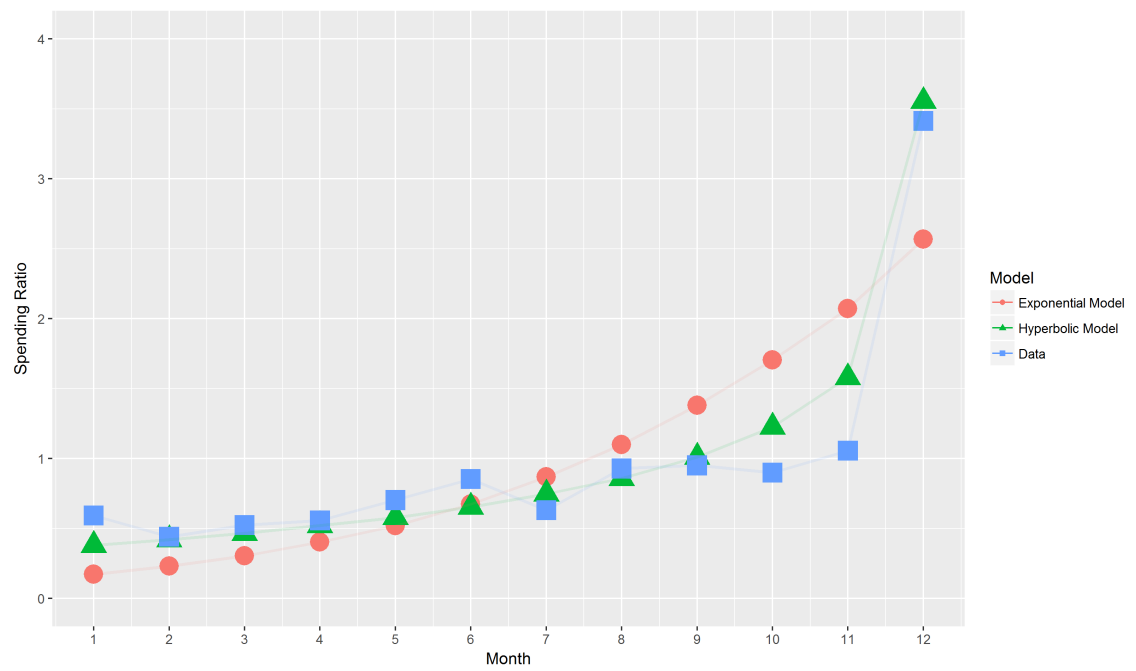


Figure 1.I.1: The exponential and hyperbolic models calibrated to Northern Ireland departmental spending data

	5%	10%	5%	10%
	Exponential	Exponential	Quasihyperbolic	Quasihyperbolic
<b>Calibrated Parameters</b>				
$\delta$	0.993	0.986	0.992	0.984
$\Omega$	0.536	0.523	0.257	0.251
$\sigma$	0.010	0.020	0.011	0.022
<b>Discounting</b>				
$\beta$	0.996	0.992	0.998	0.996
k	1.000	1.000	0.975	0.950
<b>Moments</b>				
Shocks Mean	0.001	0.002	0.001	0.002
Shocks Standard Deviation	0.011	0.021	0.012	0.023
Final Month Spending Ratio	2.584	2.555	3.554	3.601
RSS	3.139	3.119	0.478	0.457
Log Likelihood	1310.602	1018.531	1281.764	997.181
<b><math>\gamma</math> and <math>\lambda</math></b>				
$\gamma$	0.95	0.95	0.95	0.95
$\lambda$	1.046	1.054	1.047	1.055
<b>No Tax Model</b>				
$B$	1	1	1	1
Average $x_1$	1.4%	1.5%	3.18%	3.2%
Average $x_{12}$	21.5%	21.3%	29.6%	30.0%
Average Annual Production	1.101	1.101	1.102	1.111
Average Annual Spending	1	1	1	1
Parliament Utility	0.055	0.055	0.055	0.056
<b>Tax Model</b>				
$B$	1.788	1.597	1.752	1.532
Tuning Parameters ( $\nabla, \Upsilon$ )	NA	NA	0.491, 0.022	0.470, 0.059
Final Month Tax ( $\theta_{12}$ )	1.049	1.112	1.048	1.112
Average $x_1$	8.98%	9.14%	9.09%	9.45%
Average $x_{12}$	7.52%	7.45%	9.55%	7.61%
Average Annual Production	1.881	1.665	1.869	1.638
Average Annual Spending	1.747	1.515	1.726	1.487
Parliament Utility	0.064	0.069	0.062	0.070
<b>Perfect Agency</b>				
Average Annual Production	1.961	1.823	1.953	1.824
Average Annual Spending	1.781	1.643	1.773	1.643
Parliament Utility	0.098	0.091	0.098	0.091
Gains from tax	16.1%	24.3%	12.6%	25.1%
Maximum possible gain	78.8%	64.3%	76.7%	62.9%

Here the shocks mean and standard deviation refer to the first 2 moments of the implied shocks after they are transformed to the normal distribution. They should be distributed  $(0, \sigma)$  to match the shock process anticipated by departments in the model. The RSS refers sum of squared differences between the predicted monthly spending values (from the long term average of simulations) and the average spending from the data over the fiscal year. This is also summarised in figure 1.3. Note that start month and end month spending amounts are not listed for the direct parliament case as monthly spending will be even spending in all cases at 8.3%. Utility gains from the tax and from the perfect agency models are relative to the no tax model. The tax equation formula can be found in equation 1.24 for the exponential models, and in the associated text for the quasi-hyperbolic models.

Table 1.1.1: All 5% and 10% discounting calibrations

**The impact of shock variance on the calibrated model**

The model calibrated in section 1.3.2 includes a stochastic  $\alpha_{y,m}$  parameter. While it is counter intuitive it turns out the stochastic process has the effect of moderating the increase in spending throughout the year. The reason is that shocks are iid throughout the year which acts to encourage even spending in order to better exploit beneficial shocks. With rollover being available this moderating force is greater than any precautionary savings motive that might tend towards an increasing profile of spending.

This can be seen by examining the spending profile of the four calibrations of table 1.I but with  $\sigma = 0$ . The outcome of these simulations is plotted in figure 1.I.2. The sharper increase in spending can be seen by comparing this figure to figure 1.I.1.



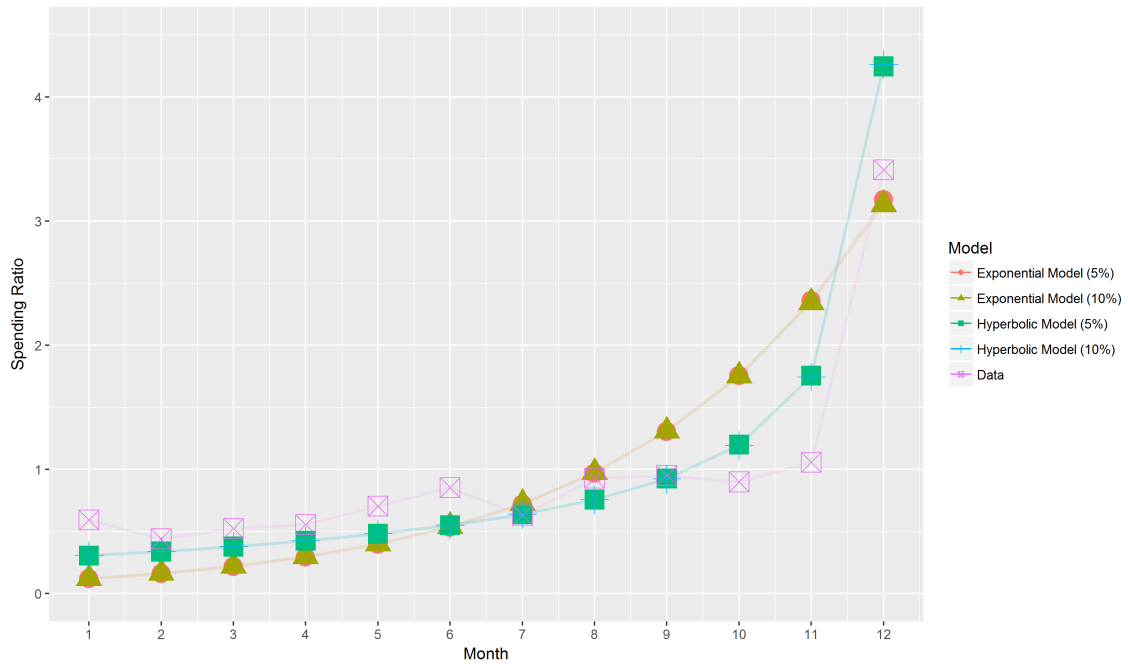


Figure 1.I.2: The exponential and hyperbolic models calibrated to Northern Ireland departmental spending data

## 1.J Precautionary Savings model Monte Carlos

A number of Monte Carlos were performed using the calibrated CRRA model of Liebman and Mahoney [2013]. In this model we have (in the notation of equation 1.15)  $g(x_{y,m}) = \frac{x^{1-\gamma}-1}{1-\gamma}$  and the stochastic parameter is drawn from a log normal distribution  $\alpha_{y,m} \sim \text{LN}(0, \sigma^2)$ . The calibrated parameter values are as described in table 1.J.1. In contrast to the procrastination model, Liebman and Mahoney [2013] have the same production function enter both the department and parliament's utility functions, a feature that I replicate here.

Their approach to calibration is to normalise  $\lambda$  to 1 and then endogenously determining the budget that the parliament (congress) will set by maximising an analogous equation to equation 1.2. Thus the budget was set by:

$$B = \arg \max_B E_0 \left[ \sum_{t=1}^{12} (\alpha_t \frac{x(B)_t^{1-\gamma}}{1-\gamma} - \lambda x(B)_t) \right] \quad (1.J.1)$$

Parameter	Value
$\gamma$	3.02
$\sigma$	1.73
$\lambda$	1

Table 1.J.1: Precautionary Savings calibrated parameters

With the parameters of table 1.J.1 the optimal budget comes out to be 15.5.

### Time Variant Budgetary Taxes

Monte Carlos were conducted examining the effectiveness of a time variant budgetary tax in a precautionary savings model. For these Monte Carlos rollover was not allowed so that welfare gains from time variant taxes can be seen in isolation.<sup>1.65</sup>

In contrast to the procrastination case, there is no obvious functional form to choose for the time variant budgetary tax. As a result an exponential form was chosen with  $\nabla$  and  $\Upsilon$  being inserted as tuning parameters to be optimised:<sup>1.66</sup>

$$\theta_m = \begin{cases} e^{\nabla t} & t \leq 11 \\ e^{\nabla t} + \Upsilon & t = 12 \end{cases} \quad (1.J.2)$$

An optimisation was then performed for the parliament choosing  $B$ ,  $\nabla$  and  $\Upsilon$  to maximise their expected utility:

$$\{B, \nabla, \Upsilon\} = \arg \max_{\{B, \nabla, \Upsilon\}} E \left[ \sum_{t=1}^{12} (\alpha_t \frac{x_t(B, \nabla, \Upsilon)^{1-\gamma}}{1-\gamma} - \lambda x_t(B, \nabla, \Upsilon)) \right] \quad (1.J.3)$$

The outcome of this optimisation is shown in table 1.J.2 and the profile of spending throughout the year can be seen in figure 1.J.1. Note that one interpretation of the welfare gains associated with time variant taxes in a precautionary savings is that a parliament can achieve the same level of production while spending 2% less.

### Spending spikes and volatility

Monte Carlos for both models were performed in order to ascertain the relationship between the two volatility measures (as represented by equations 1.17 and 1.D.1) and the renormalised spending spike (as defined in equation 1.18). The expected volatility and renormalised spending spike can be seen in figure 1.J.2. To create this figure Monte Carlo simulations were run for the procrastination model (as calibrated in section 1.3.2) and the precautionary savings model (as described above) with varying  $\sigma$  values. Two versions of each model were undertaken with and without rollover. The  $\sigma$  values that generate a certain volatility and renormalised spending spike (in expectation) are indicated on the figure by the numbers in black on the endpoints of the line (intermediate points on the line are generated by intermediate  $\sigma$  values).<sup>1.67</sup>

First considering the models without rollover in figure 1.J.2. Note that when  $\sigma$  is very low the procrastination model has a volatility close to zero but with relatively high renormalised spending spikes (This can be seen in figure 1.J.2). The precautionary savings model on the other hand

<sup>1.65</sup>See their paper for an analysis of the welfare gains from rollover in their model.

<sup>1.66</sup>Like in section 1.5.2 no claim can be made that the chosen function form is optimal. It is likely that a more flexible functional form for taxes could outperform the taxes calibrated here.

<sup>1.67</sup>The results of only a small number of  $\sigma$  values are shown in the figure for clarity. In preparing the figure many more were computed however.

Parameter	Value
<b>No Tax</b>	
Budget	15.5
Expected Production in Year	18.8
Expected Expenditure in Year	15.5
Parliament Utility	3.34
<b>Tax - Calibrated Parameters</b>	
Budget	19.8
$\nabla$	0.0188
$\Upsilon$	1.677
<b>Tax</b>	
Spending Ratio (m=1)	0.986
Spending Ratio (m=11)	1.02
Spending Ratio (m=12)	1.175
Expected Production in Year	18.8
Expected Expenditure in Year	15.2
Parliament Utility	3.76
Gains relative to no tax	12.4%
<b>Perfect Agency</b>	
Expected Production in Year	19.5
Expected Expenditure in Year	14.1
Parliamentary Utility	5.38
Gains relative to no tax	61.1%

Table 1.J.2: Time-variant budgetary taxes in a Precautionary Savings model

exhibits completely flat spending in the absence of shocks and hence the volatility is close to zero while the renormalised spending spike is around  $\frac{1}{6}$ . As  $\sigma$  increases volatility increases for both models. The difference is the change in the renormalised spending spike. It falls for the procrastination model reflecting smoother spending in the last months of the fiscal year so that the department can better exploit iid shocks. On the other hand it rises for the precautionary savings model as departments hedge against the volatility by holding a greater amount of precautionary savings that they then expend in the last month of the fiscal year.

For the models where rollover is allowed the procrastination model displays very similar behaviour to the no rollover case. On the other hand the precaution model shows no relationship between volatility and the renormalised spending spike as spending is flat across the fiscal year regardless of the variance of the shock process.

### Spending spikes and volatility - in small samples

Now that the expected relationships between volatility and the normalised spending spikes have been ascertained, we look at what relationships should be able to be witnessed in a small sample such as that provided by the NI departmental dataset. We take the case where no rollover is allowed.<sup>1.68</sup> This will be done by sampling  $\sigma$  values from a uniform distribution in the range

<sup>1.68</sup>This is a fiction for the case of Northern Ireland but is the only way to generate spending spikes with the precaution model. This is done as the goal of this exercise is to find alternate evidence for the cause of spending spikes (aside from rollover).

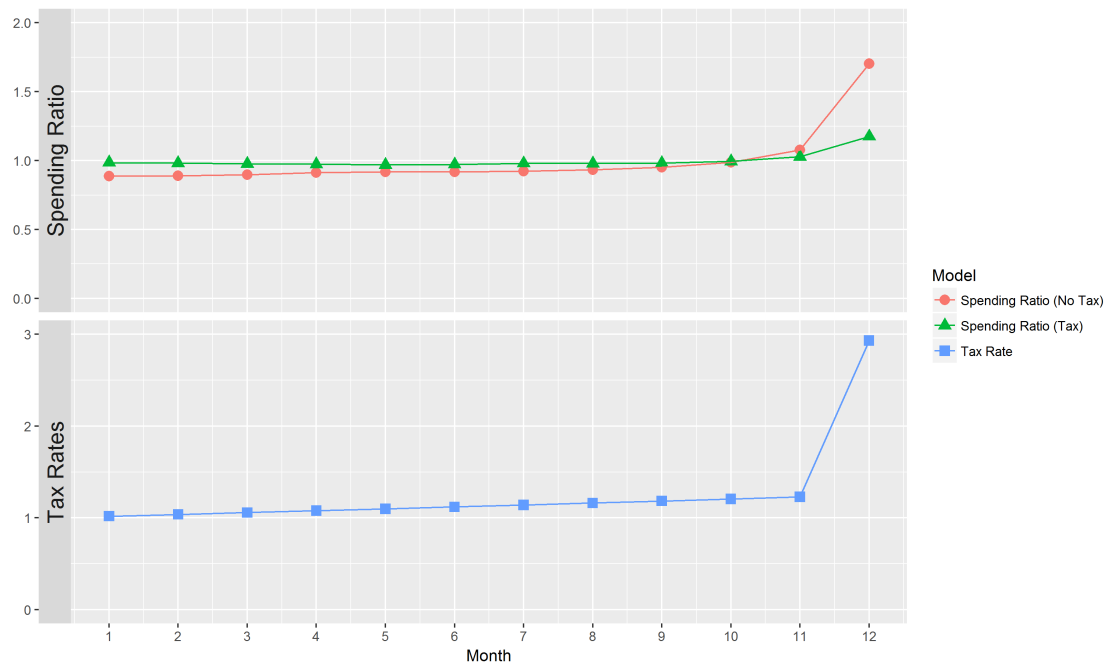


Figure 1.J.1: Time-variant budgetary taxes in a precautionary savings model

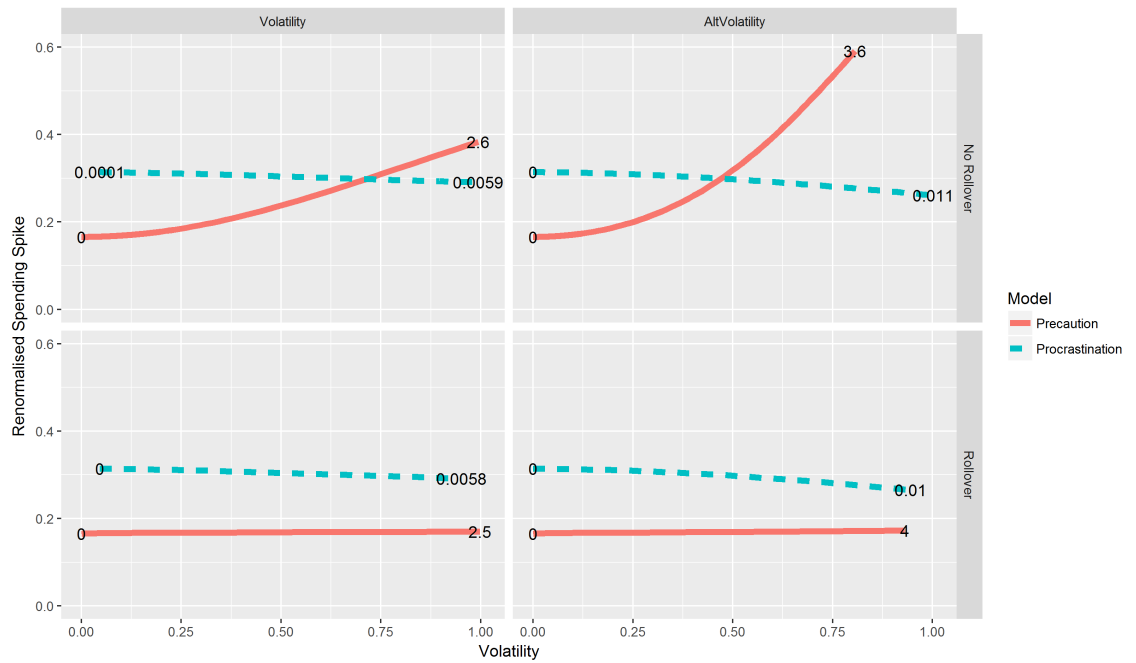


Figure 1.J.2: Expected renormalised spending spike and renormalised volatility in the two models.

$[0, 0.0059]$  for the procrastination model and in the range  $[0, 2.6]$  for the precautionary savings model. These ranges were chosen as from the top left panel of figure 1.J.2 they generate expected volatilities in the range  $[0, 1]$  which is the range of most of the volatilities in the data as can be seen in figure 1.D.1.<sup>1.69</sup> Twelve departmental  $\sigma$  values are then sampled from each uniform distribution and 5 years worth of simulated spending is taken for each department.<sup>1.70</sup> Volatilities and renormalised spikes are found for each department-year and these 60 observations are used to estimate:

$$\text{Renormalised Spike}_{d,t} = \beta_0 + \beta_1 \text{Volatility}_{d,t} + \epsilon_{d,t} \quad (1.J.4)$$

No fixed effects were used reflecting the lack of time invariant heterogeneity in the Monte Carlo's data generation process (with the exception of different  $\sigma$  values, the effect of which should be accounted for by the volatility metric). No distinction is made in this Monte Carlo between current and capital expenditure (as is the case in both models). This procedure was done repeatedly so that the distribution of  $\beta_1$  that would be expected if the true data generation process is procrastination or precautionary savings could be found. Two versions were performed, one where shocks were completely non correlated and one where they were perfectly correlated although of potentially different variance.<sup>1.71</sup>

The t-stat from the estimation of  $\beta_1$  that were taken were taken and can be seen in figure 1.J.3. This figure also marks the smallest magnitude t-stat from table 1.2 of  $-2.48$ . This provides a suggestive indication that the t-stats from this table are unlikely to be generated from a sample generated by the precautionary savings model but are relatively likely to be generated by the procrastination model.

---

<sup>1.69</sup>This assumption of a uniform distribution and the chosen endpoints are admittedly arbitrary, but it is likely that this exercise will be useful in giving a general idea of the t-stats and  $\beta$  estimates that would be expected from a finite sample of data generated from the two models.

<sup>1.70</sup>For these simulations the calibrated parameter values are used for each model with the exception of the shock variance for each model which is sampled as discussed. The shock

<sup>1.71</sup>Specifically a set of quasi-random values were generated from the uniform distribution on  $[0, 1]$ . Shocks were then found by putting these uniform quasi-random values into the inverse log normal cdfs for each department's  $\sigma$  value. Putting this mathematically for each  $\epsilon$  drawn from  $UD[0, 1]$ , a corresponding shock was then found from  $\alpha = F^{-1}(\epsilon, 0, \sigma)$  where  $F^{-1}$  is the inverse log normal cdf with mean 0 and variance  $\sigma$  and  $\epsilon$  is the input to this inverse cdf.

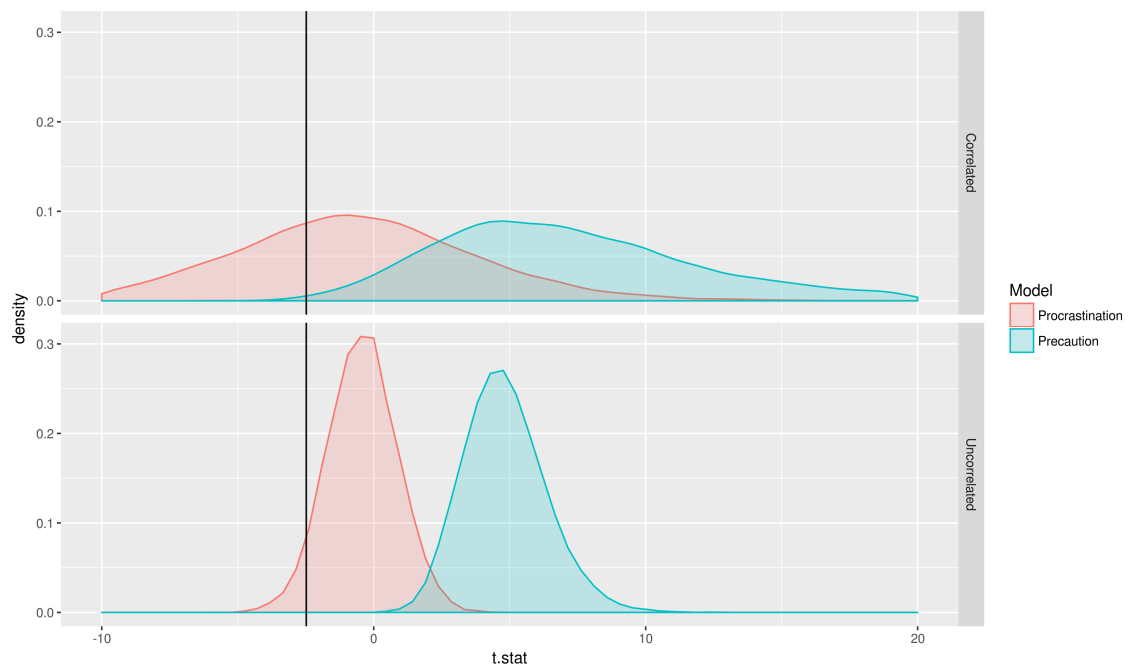


Figure 1.J.3: t-stats on Monte Carlo simulations of the regressions of table 1.2 under the two models

## Chapter 2

# Comparative Advertising: The role of prices

**Abstract:** In markets where firms sell similar goods to their competitors, firms may be able to free-ride off the costly price signalling of competitor firms by engaging in price comparative advertising. As the goods are similar, consumers can reason that if one good is high quality (revealed through price signalling) then so is the other. This paper models this phenomenon and finds that in equilibrium there will be firms price signalling as well as freeriding firms that signal through price comparative advertising. Welfare is strictly higher in markets where advertising firms are active relative to pure price signalling markets. In some cases advertising markets can be even more efficient than full information markets as advertisers surrender market power to avoid costly price signalling.<sup>2,1</sup>

**JEL Codes:** D43, D82, D83, M37

**Keywords:** Comparative advertising, Price Signalling

### 2.1 Introduction

In many markets firms have more information regarding the quality of their goods than potential consumers. As a result firms with high quality goods can signal the true quality of their goods to consumers with prices and warranties being common instruments for this. In many of these markets some of this uncertainty is common to goods offered by different firms. An example is cable television where two providers may offer sets of channels with substantial overlap. An alternate example is package holidays where the utility from holidays to the same location from different but similar tour companies are likely to be similar. In instances like this it may be possible for firms to earn greater profits from free-riding on the costly signalling of other firms.

---

<sup>2,1</sup>For useful discussions I would like to thank Philipp Kircher, Ludo Visschers, Jan Eeckhout, Margaryta Klymak, Régis Renault, Sandro Shelegia, Alessandro Spiganti and Ina Taneva. The author was visiting at Universitat Pompeu Fabra for part of the writing of this article. This work was supported by the ESRC postgraduate funding scheme. Notwithstanding any advice I have received any errors here are my own.

This paper investigates the potential for firms to engage in this kind of signal free-riding through comparative advertising. Comparative advertising occurs when a firm advertises by contrasting the price and features of its good as compared to those of rival firms. In the previous economic literature comparative advertising has been seen as directly informing consumers of the difference in vertical quality between two goods [Barigozzi et al., 2009] or of the difference in horizontal (seller specific) match utility a consumer would get if he bought a good from a competing firm as compared to buying from the advertising firm [Anderson and Renault, 2009]. The role of the disclosure of the *prices* of rivals in this context has received less attention.<sup>2.2</sup>

There are many examples of comparative advertising however where the disclosure of price information is a major part of the message. A basic example is offered in the advertisements of Progressive Direct, an American auto insurance company that gives prospective consumers the prices offered by competitors for comparable insurance plans [Yu, 2013]. They also air advertisements promising “we compare our direct rates side by side to find you a great deal, even if its not with us”. Three, a major UK phone carrier, similarly advertises with a webpage that asks shoppers on their website to “see how our prices compare” with a price comparison of Three against the prices of all of the other major phone carriers in the market [Three Mobile, 2016]. The online travel agent Skyscanner allows visitors to automatically replicate their flight searches on competing services such as Expedia and eDreams. Another example is provided by Book Depository, a leading UK online bookseller [Charlton, 2009] which for every product sold presented a link to the corresponding Amazon page for that item.<sup>2.3</sup>

This paper examines a vertically differentiated market in which consumers cannot directly observe the quality of goods. In such a market when high quality firms have higher costs, there is a literature that shows these firms can separate themselves from low quality firms by proposing a high price, often above the monopoly price they would charge if their quality were known. This is called in the literature “price signalling” as the price transmits information about the underlying quality of the good. The problem for high quality firms in such markets is that the high price that enables signalling may result in lower profits than would be available by pricing at the (lower) monopoly price in a full information setting. In this setting we show that price comparative advertising has a clear role: By showing an identical product from a rival firm at a high price, a firm can signal to its customers that its product must also be of high quality, even if the firm itself does not charge such a high price. In doing so a firm may be able to increase profits by pricing closer to its monopoly

---

<sup>2.2</sup>To the best of my knowledge this is the first paper to examine the informational content of prices as a component of comparative advertising or the firm strategy of providing competitor prices to their consumers more generally.

<sup>2.3</sup>A further example is provided by Amazon itself which has a marketplace which allows competitors to compete against Amazon on Amazon’s website. Amazon also receives a portion of the revenues from these external sellers on their website. Whilst these revenues no doubt play some role in Amazon’s decision to operate this marketplace it is also true that this allows Amazon customers to compare Amazon’s prices with other vendors.



price and freeriding on the rival firm's price signal.<sup>2.4</sup>

Clearly the logic of this argument does not immediately carry over to equilibrium behaviour. If all firms revert to pricing at the lower level rather than price signalling, free-riding possibilities may be lost. One of the questions in this paper is whether this simple logic extends in some form to an equilibrium setting. Indeed, only some insights of the previous setting carry over once equilibrium forces come into play. In equilibrium there will be firms that engage in price comparative advertising and firms that remain pricing at the signalling level. The advertising firms face increased price competition from the firms they advertise against and thus many will price at levels below the monopoly price. Thus while in normal price signalling markets asymmetric information has the effect of increasing prices, the equilibrium exhibiting price comparative advertising will have some firms pricing above and some below the monopoly level. Total welfare is improved by price comparative advertising relative to the asymmetric information equilibrium where no advertising is allowed. Firms do not earn higher profits however due to this additional price competition and the additional surplus is all accrued by consumers through lower prices. In some cases the welfare of an asymmetric information market with price comparative advertising can be greater than under full information as firms surrender market power in order to achieve more efficient signalling.

A number of extensions of the basic model are examined. When there is heterogeneity in marginal costs among high quality firms I find the intuitive result that it will be the lower cost firms that will be pricing lower while engaging in price comparative advertising. This result suggest another avenue through which price comparative advertising increases welfare as high price (and higher cost) advertisers lose sale quantity to low price (and lower cost) advertisers. An alternate extension is the special case where all high quality firms source their goods from a monopoly supplier.<sup>2.5</sup> Here it was found that the possibility of advertising can induce the monopoly supplier to reduce the price they charge reselling firms in the hope of increasing the quantity they sell. These extensions all point to increased welfare from the use of price comparative advertising. These results on welfare are considerably more positive for comparative advertising than the previous literature that examined the comparative advertising of product attributes and found that total welfare could be decreased when there was a large vertical quality difference between rival firms [Anderson and Renault, 2009].

Comparative advertising used to be relatively uncommon in developed countries but was legalised in the United States and Europe in 1979 [Federal Trade Commission, 1979] and 1997 [Council of European Union, 1997] respectively. Elsewhere in the world however comparative ad-

---

<sup>2.4</sup>The analogue in the traditional Spence [1973] signalling model of the labour market is that an uneducated worker goes to a firm with his educated identical twin. As both individuals are identical and this is observable the firm will reason that if one sibling is educated and has a high capability then so does the other.

<sup>2.5</sup>For instance there is a monopoly supplier of Samsung smartphones and Iron Maiden albums.

vertising is still subject to restrictions. In China and Hong Kong it is banned while in Japan it is allowed but seldom used due to it being perceived as impolite by Japanese consumers [Singh, 2014]. While comparative advertising has recently been legalised in Turkey [Gürkaynak et al., 2015] it remains banned in Saudi Arabia and Kuwait. In other countries such as Qatar the situation surrounding comparative advertising is simply ambiguous with no regulations or case law to determine its legality [Bradley, 2014]. The policy implications of this paper are clear: comparative advertising delivers lower prices to consumers, is welfare improving and should be supported and encouraged by governments and regulators.

This paper first provides an outline of the surrounding literature in section 2.2. The model is then presented in section 2.3 with welfare implications examined in section 2.4 and model extensions in section 2.5 before section 2.6 concludes.

## 2.2 Literature Review

This paper has strong links to two key literatures: the price signalling literature and the literature on comparative advertising.<sup>2.6</sup> In addition this paper is related to the marketing literature on reference pricing as well as recent work on search deterrence.

An early paper to examine the economic consequences of consumers judging quality by price is that of Scitovszky [1944]. Since that time, the advent of signalling theory has led to a number of papers applying price signalling to markets with imperfect information regarding product quality. A common mechanism for signalling is to have higher quality firms producing at a higher marginal cost than lower quality firms [Wolinsky, 1983, Bagwell and Riordan, 1991, Daughety and Reinganum, 2008]. In this way high quality firms have a higher optimal price than low quality firms which makes it comparatively less expensive for them to charge a high price, thus allowing signalling. This paper will use this feature of marginal cost increasing in product quality to model a market exhibiting price signalling.

Moving onto the comparative advertising literature, Barigozzi et al. [2009] argue that firms engage in comparative advertising as a means of signalling the vertical quality of their good. By directly informing consumers of the vertical quality of their good compared to a competitor's good the firm opens itself up to litigation expenses if claims made about the comparison of goods are unreasonable. Only a firm with a high quality good would engage in this practice as low quality firms would face a high expected loss from litigation. In this way comparative advertising serves as a signal for good quality as well as a vehicle for direct disclosure of quality. While they do not consider the impact of comparative advertising on total welfare they conclude that it should be

---

<sup>2.6</sup>The comparative advertising literature can further be considered a branch of the informative advertising literature. See Renault [2016] for a survey.

supported by regulators as it allows easier market access for new entrants. The authors note that this mechanism for comparative advertising is reliant on an effective court system which suggests comparative advertising would be less effective in countries with weaker institutions.

Another comparative advertising paper is that of Anderson and Renault [2009] who examine a duopolistic market where each firm sells a horizontally and vertically differentiated good. Vertical quality is readily apparent to consumers and so advertising focuses on horizontal differentiation rather than signalling vertical quality. Firms can engage in advertising by disclosing the match utility a consumer would get with their firm or comparative advertising by disclosing the match utility a consumer would get from both firms. They find that in cases where there is a small difference in vertical quality both firms will engage in advertising of their own match utility to benefit from the higher price generated by additional product differentiation. When there is a large quality differential however weaker firms will generally be the firms using comparative advertising to disclose both matches in order to increase their demand. They find in this setting that consumers and lower quality firms are better off when comparative advertising is allowed. So much damage is done to the higher quality firm profits however that total welfare falls as a result of comparative advertising. Related ideas are examined by Koessler and Renault [2012] and Celik [2014] who look at the conditions under which disclosure of quality attributes will occur.

This paper is different from the previous literature on comparative advertising in that it is the disclosure of *prices* of competitor firms rather than the direct disclosure of horizontal or vertical product attributes that is important. Indeed to highlight this channel, the model I present will exhibit firms advertising against competing firms offering identical products with no horizontal differentiation existing.<sup>2.7</sup> Before introducing this model however I will briefly mention two other literatures that are related to this paper.

The reference pricing strand of the marketing literature is related to the notion of price signalling. An example of reference pricing is a \$200 price crossed out and replaced with “\$100 for a limited time only!”. The idea is that this \$200 is suggestive of the quality of the good however the key problem with this strategy is credibility [Grewal and Compeau, 2002, Compeau et al., 2002, Kan et al., 2013]. Consumers will often doubt that the reference price is ever charged or is representative of the quality of the item. While this paper uses an economic signalling approach, its message might otherwise be motivated by external offers providing credible reference prices to consumers.

Finally the mechanism described by this paper represents an interesting contrast to the mechanism described by Armstrong and Zhou [2015] in a paper on search deterrence. That paper has

---

<sup>2.7</sup>Of course in the real world firms may advertise against competing products that are differentiated but share some common value component. In this case it is likely that comparative advertising plays the price-comparative role outlined in this paper along other roles of comparative advertising explored in the literature.

a model where a shopper is presented with a good of known quality but has an unknown outside option. It is shown that where possible a seller can increase profits by committing to an exploding offer where the consumer has to buy before seeing the outside option. This is in contrast to the current paper where the additional external information can efficiently signal the quality of the current good and thereby increase seller profits. When the good is high quality, this provision of information is good for the selling firm as they can freeride on the price signalling of other firms.

## 2.3 The Model

There is a high quality good and a low quality good in the market each of which is sold by a unit mass of firms. The quality of a good is observable to firms but not to consumers. Firms selling low quality goods have a marginal cost of  $c_L$  while firms selling high quality goods have a higher marginal cost of  $c_H$  with  $c_H > c_L$ . A fraction  $\lambda$  of firms sell high quality goods.

There is a unit mass of consumers, each of whom gets a utility equal to  $Q - P_f$  for buying good  $Q \in \{H, L\}$  from firm  $f$  at price  $P_f$ . Consumers have a heterogeneous outside option denoted by  $\Omega_k$  for customer  $k$ . This outside option is logconcave distributed with a pdf denoted  $\gamma(\Omega)$  and cdf denoted  $\Gamma(\Omega)$ .

The timing of this singleshot game is as follows. A firm is approached by one random consumer. The firm offers that consumer a price and the consumer can either buy at that price or leave the firm in favour of their outside option. Denoting the consumer's perceived quality level by  $\hat{Q}$ , the condition for a consumer to buy is:

$$\hat{Q} - P_f \geq \Omega_k \tag{2.1}$$

Thus the maximum  $\Omega_k$  consumer that will buy the good will have an  $\Omega_k$  value of  $\hat{Q} - P_f$  and hence from the firm's perspective the probability of a sale is given by  $\Gamma(\hat{Q} - P_f)$ .

Signalling equilibria are refined by the intuitive criterion [Cho and Kreps, 1987]. Beliefs are formalised by a function  $\mu(P)$  that gives the believed probability of a good being high quality given a price of  $P$ . Finally as this paper examines the potential use of price comparative advertising<sup>2.8</sup> as a signalling tool alongside price signalling, I will consider only fully separating equilibria.

### 2.3.1 Separating Equilibrium without price comparative advertising

In this section the fully separating equilibrium for this basic model lacking price comparative advertising will be examined. First defining the equilibrium concept:

<sup>2.8</sup>To be defined in terms of the model in section 2.3.2.

**Definition 1 PBNE (without advertising).** *A pure strategy Perfect Bayesian Nash Equilibrium (PBNE) in this model without advertising will be described by low and high firm pricing strategies  $P_L, P_H$  as well as a belief function  $\mu(P)$ , such that:*

$$\pi_L(P_L) \geq \pi_L(P) \quad \forall \quad P \in \mathfrak{R}^+ \quad (2.1)$$

$$\pi_H(P_H) \geq \pi_H(P) \quad \forall \quad P \in \mathfrak{R}^+ \quad (2.2)$$

and the belief function  $\mu(P)$  is derived in accordance with bayes rule and player strategies for all prices charged with positive probability in equilibrium.

As in a standard price signalling separating equilibrium we get the result that firms selling low quality goods will price at their monopoly price (to be denoted  $P_L$ ) which maximises their profit:

$$P_L = \arg \max_{P \in \mathfrak{R}^+} (P - c_L)\Gamma(L - P) \quad (2.3)$$

With the corresponding profit being denoted  $\pi_L^M = (P_L - c_L)\Gamma(L - P_L)$ . The prices that high quality firms can charge to differentiate themselves from the low quality firms are those prices  $P^*$  that satisfy:

$$\pi_L^M \geq (P^* - c_L)\Gamma(H - P^*) \quad (2.4)$$

The price which makes this expression bind with equality (The Riley [1979] price) is denoted by  $P^S$ . With the intuitive criterion applied beliefs will satisfy  $\mu(P^S) = 1$ . A natural way to extend these beliefs over all prices is thus:

$$\mu(P) = \begin{cases} 1 & P \geq P^S \\ 0 & P < P^S \end{cases} \quad (2.5)$$

In any fully separating equilibrium, high quality firms will charge the maximum of  $P^S$  or the high firm's monopoly price (which will be denoted  $P_H^M$ ) which maximises their profit in the absence of asymmetric information:

$$P_H^M = \arg \max_{P \in \mathfrak{R}^+} (P - c_H)\Gamma(H - P) \quad (2.6)$$

With the corresponding profit being denoted  $\pi_H^M = (P_H^M - c_H)\Gamma(H - P_H^M)$ . Henceforth we assume that this market is one where price signalling is costly and hence  $P_H^M < P^S$ . The profit of the high firm when signalling is denoted by  $\pi_H^S$  and can be expressed as:

$$\pi_H^S = (P^S - c_H)\Gamma(H - P^S) < \pi_H^M \quad (2.7)$$

To restrict attention to separating equilibria we assume that high firms prefer signalling to being mistaken for low firms. This condition is

$$\pi_H^S > (P - c_H)\Gamma(L - P) \quad \forall \quad P \in \mathfrak{R}^+ \quad (2.8)$$

Finally we can show that  $P_H^M > P_L$  by first obtaining an expression for the optimal monopolist price from first order conditions of  $\pi(P) = (P - c_Q)\Gamma(Q - P)$ . This results in:

$$P = c_Q + \frac{\Gamma(Q - P)}{\gamma(Q - P)} \quad (2.9)$$

As  $\Gamma(x)$  is logconcave there is a well-defined solution to this problem and  $\frac{\Gamma(x)}{\gamma(x)}$  is a monotonically increasing function [Bagnoli and Bergstrom, 2005]. From this we can see that the solution price is increasing in cost and in perceived quality. Hence we will always have  $P_H^M > P_L$ .

**Proposition 1.** *Without the possibility of advertising, a PBNE exists where  $L$  and  $H$  firms price at  $P_L$  and  $P^S$  respectively and consumer belief formation is as per equation 2.5.*

*Proof.* Equations 2.3, 2.4 and 2.8 ensure that high and low firms cannot get higher profits from deviating. The belief function described in equation 2.5 is consistent with this equilibrium.  $\square$

The prices charged and profits earned by each firm in equilibrium along with the high firm full information profits (that are unattainable under asymmetric information) are shown in figure 2.1.

### 2.3.2 Separating Equilibrium with price comparative advertising

In this section firms are allowed to engage in price comparative advertising at no cost. As motivation for this, consider that a high quality firm deviates from the no advertising signalling equilibrium described in the preceding section. They charge the monopolist price  $P_H^M$ . At this price however the consumer cannot tell if the good is of high or low quality. The firm can show the offer of another firm offering this same (high quality) good. This other price will be  $P^S$  as all other firms selling the high quality good charge this price. The consumer knows that both goods are the same and so will be convinced that the good is high quality after seeing this other price.

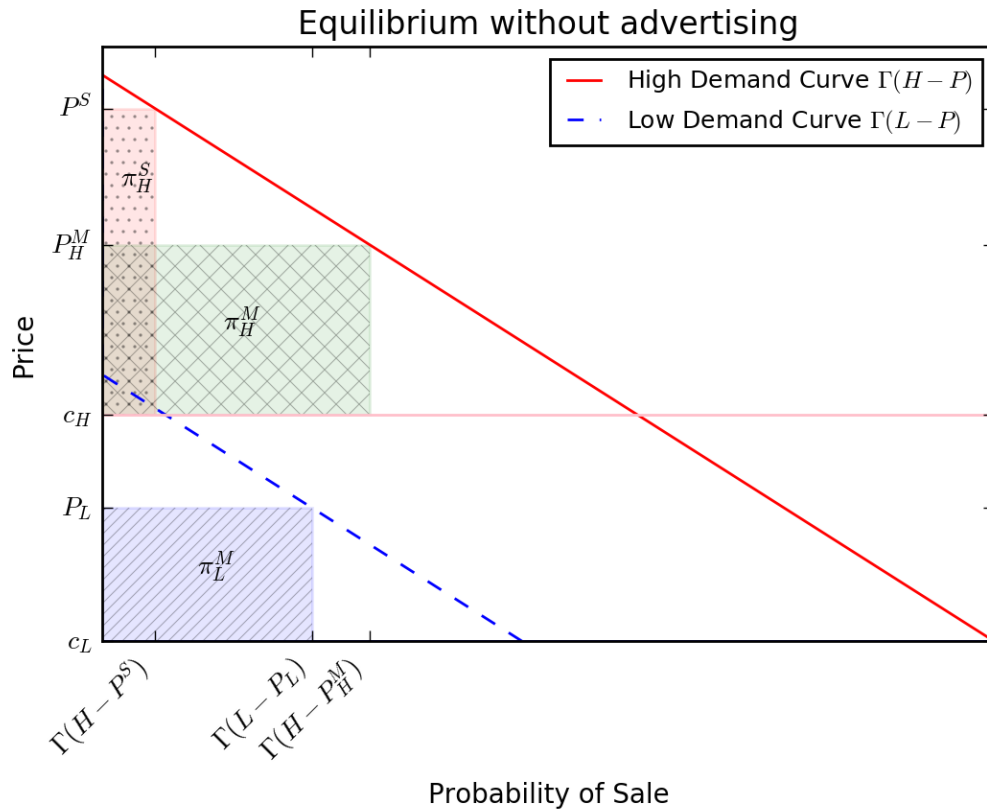


Figure 2.1: Profits in the no advertising fully separating equilibrium

The firm will earn  $\pi_H^M > \pi_H^S$  and hence this is a profitable deviation. This section considers how the possibility of this profitable deviation changes the equilibrium outlined in proposition 1.

It is assumed that when the consumer enters the market and meets a firm, that firm can costlessly show the consumer an offer from a competing firm that sells the same good.<sup>2.9</sup> As both offers concern the same good if one is high quality then so is the other. Henceforth the term *advertiser* will be used to describe a firm that provides an external offer. Firms that do not provide an external offer will be referred to as *non-advertisers*.

The assumed timing is that each firm decides on their own price and whether or not to show a competitors price simultaneously. A key assumption is made that advertising is undirected - firms cannot condition their advertising on the price offered by the firm they advertise against. This assumption can be simply motivated by considering firms to commit separately and simultaneously to their strategy of prices and advertising.<sup>2.10</sup>

For simplicity an advertising firm only shows one price from an external firm. A consumer

<sup>2.9</sup>There is an assumption here prohibiting a high/low firm from showing an offer from a low/high firm. This can be justified by the presence of many goods in the real world (while in the model there is one high and one low quality good). Thus a consumer shown an unrelated good cannot use this information to infer information about the good at hand and there is no incentive for firms to show these unrelated goods to the consumer.

<sup>2.10</sup>This assumption of undirected price comparison seems to be a good representation of progressive direct; three; skyscanner and book depository from the introduction.

observes a price from the external firm but not the advertising strategy of this external firm. A consumer at this point can decide to buy from either the advertising firm or the external firm (at no extra cost). The tie-breaking rule is adopted that when both firms price at an equal amount is that it is assumed that the consumer will buy from each with 50% probability.

The beliefs of a consumer who has seen one price will be represented by the function  $\mu(P)$  while the beliefs of consumer who has been shown two prices will be given by a function taking two arguments,  $\mu(P, P_E)$ . The convention will be maintained that the first price is that of the advertising firm and the second price is the price from an external firm. This paper will examine a fully separating mixed pricing PBNE. The equilibrium price domain of type  $Q \in \{L, H\}$  firms with strategy  $s \in \{A, N\}$  for advertising and non-advertising will be denoted by the set  $D_{Q,s}$ .

**Definition 2 PBNE (with advertising).** *A fully separating PBNE in this model with advertising will be described by pricing strategies and equilibrium profits denoted  $\hat{\pi}_L, \hat{\pi}_H$ , such that:*

$$\pi_{H,N}(P) = \hat{\pi}_H \geq \pi_{H,N}(P') \quad \forall \quad P \in D_{H,N}, P' \in \mathfrak{R}^+ \setminus D_{H,N} \quad (2.1)$$

$$\pi_{H,A}(P) = \hat{\pi}_H \geq \pi_{H,A}(P') \quad \forall \quad P \in D_{H,A}, P' \in \mathfrak{R}^+ \setminus D_{H,A} \quad (2.2)$$

$$\pi_{L,N}(P) = \hat{\pi}_L \geq \pi_{L,N}(P') \quad \forall \quad P \in D_{L,N}, P' \in \mathfrak{R}^+ \setminus D_{L,N} \quad (2.3)$$

$$\pi_{L,A}(P) = \hat{\pi}_L \geq \pi_{L,A}(P') \quad \forall \quad P \in D_{L,A}, P' \in \mathfrak{R}^+ \setminus D_{L,A} \quad (2.4)$$

*The belief functions  $\mu(P), \mu(P, P_E)$  are in accordance with the intuitive criterion, Bayes rule and player strategies for all information sets reached with positive probability in equilibrium.*

The logic behind simple price signalling for a firm that does not advertise still applies and hence  $\mu(P)$  will be as set in equation 2.5. Beliefs when advertising is undertaken shall be in accordance with Bayes rule at all points within the domain of prices charged in equilibrium. Thus:

$$\mu(P_A, P_E) = \frac{\lambda \text{Prob.}(P_A, P_E | \text{Good is H})}{\lambda \text{Prob.}(P_A, P_E | \text{Good is H}) + (1 - \lambda) \text{Prob.}(P_A, P_E | \text{Good is L})} \quad (2.5)$$

As we are restricting attention to fully separating equilibria we restriction attention to the case where there is no price point  $P^S > P > P_L$  that is charged by a positive mass of high firms and low firms.

**Lemma 1.** *In any fully separating equilibrium, no low firms offer a price  $P > P_L$ .*

*Proof.* We can first show there will be no advertisers pricing above  $P_L$ . Consider a putative equilibrium where there were advertisers pricing above  $P_L$  with price dispersion. The highest pricing advertiser would make no sales and be better off monopolising at  $P_L$ .

Consider a putative equilibria where there are low quality advertisers at a masspoint above  $P_L$ . As there are no high firms at this price (by the restriction to fully separating equilibria), these



firms will be seen as being of low quality. Hence beliefs cannot worsen from undercutting and there is a profitable deviation for a firm to undercut this masspoint. For low firms that do not advertise, as pricing at  $P_L$  dominates pricing higher given the beliefs in equation 2.5 there will be no low firms pricing above  $P_L$  either.  $\square$

This lemma makes it possible for a high firm to signal high quality by advertising whilst pricing at less than  $P^S$  but more than  $P_L$ . This leads to the first proposition which states that in any equilibrium there will exist advertising firms.

**Proposition 2.** *In any equilibrium there will be a positive mass of high firm advertisers.*

*Proof.* In the event of all mass of high firms being non-advertisers at  $P^S$ , consider a high firm's option of deviating to price at  $P_H^M$  and advertising against another firm selling the same good. If the customer accepted this firm was high quality the deviating firm would be better off. By contrast a low firm attempting to emulate high quality by doing the same would be worse off even if they were believed to be high quality as they would lose the sale to another low firm (that would be pricing at  $P_L$ ). Thus this is a profitable deviation as the high firm could use the intuitive criterion to price closer to their monopoly price whilst convincing consumers of their high quality. As this profitable deviation remains whilst there is a zero measure of advertisers we get the proposition.  $\square$

One implication of this proposition is that the equilibrium described in Proposition 1 is no longer an equilibrium where advertising is allowed. Whilst beliefs of  $\mu(P, P_E) = 0 \forall P, P_E$  could support this equilibrium such beliefs would not be supported by the intuitive criterion.

**Lemma 2.** *At all prices  $P > P_L$ , the equilibrium price distributions of high quality advertisers is atomless.*

*Proof.* If there were an atom at a price exceeding marginal cost then one of those firms could undercut the others. With a similar intuitive criterion argument as presented in the proof of proposition 2, the undercutting firm could convince consumers of their high quality and hence could get a discontinuous jump in expected profits. If there were an atom at a price equal to marginal cost [Bertrand, 1883] then profits are zero and the firms at this price would be better off monopolising to earn  $\pi_H^S$ .  $\square$

This lemma is similar in spirit to Varian [1980, Proposition 3] or Stahl [1989, Lemma 1] and reflects the fact that if there were a mass of firms offering a certain price then one of those firms could get a discontinuous jump in expected profits by undercutting the others. Note that non-advertising firms are monopolists and hence there is no similar restriction on mass points in the pricing distribution of these firms. This lemma is truncated to prices above  $P_L$  to reflect the possibility of adverse beliefs for advertisers pricing at or below  $P_L$  which would prevent undercutting.

We have now established that no low firms will price above  $P_L$ . The next two lemmas show that all low firms will in fact not be advertising and will be charging a price of  $P_L$ .

**Lemma 3.** *If the equilibrium price distribution of high firm advertisers is atomless at prices  $P \leq P_L$  then in equilibrium there will be a mass of non-advertising low firms offering a price of  $P_L$ .*

*Proof.* For low firms that do not advertise, Pricing at  $P_L$  dominates any other price.

If all firms were advertising with price dispersion the firm with the highest price would get no profit and hence would be better off not advertising whilst setting a price of  $P_L$ . If all low firms were advertising at a certain price less than  $P_L$  then from equation 2.5 (and the lack of high firm masspoints below  $P_L$ ) they would be considered low quality and would be better off not advertising at a price of  $P_L$ .  $\square$

**Lemma 4.** *If the equilibrium price distribution of high firm advertisers is atomless at prices  $P \leq P_L$  then in equilibrium all low firm will be non-advertisers at a price of  $P_L$ .*

*Proof.* Lemma 1 shows that no low firm will price at  $P > P_L$  and not-advertising at  $P_L$  dominates not-advertising at any other price. Thus it suffices to show no low firms will advertise at any price  $P \leq P_L$ . Considering the possibility of advertising at a price  $P \leq P_L$  the possible price distributions are a continuous distribution of prices, a masspoint of low firm advertisers at certain discrete prices or a distribution with both of these features. We shall show that these distributions are not sustainable in equilibrium.

We can first show that no low firm advertiser masspoints can survive in equilibrium at prices less than  $P_L$ . If there were an advertiser masspoint at a price  $P < P_L$  then from equation 2.5 (and the lack of high firm masspoints below  $P_L$ ) they would be considered low quality and would earn strictly higher profits undercutting other firms in this masspoint. Thus no masspoints of low firm advertisers can survive in equilibrium.

Now consider a putative equilibria where there is no atom of high firm advertisers at  $P_L$  but a mass of low firm non-advertisers at  $P_L$  (as per lemma 3) and a distribution of low firm advertisers on the domain  $[\bar{P}, \underline{P}]$  with  $P_L \geq \bar{P} > \underline{P}$ . The total mass of low firm advertisers is some number  $0 < \nabla < 1$ . Consider the advertiser pricing at  $\bar{P}$ . It matches with a firm pricing at  $P_L$  with probability  $(1 - \nabla)$  and matches with another (lower priced) advertiser with probability  $\nabla$ . When  $P_A = \bar{P}$ ,  $P_E = P_L$  are subbed into the beliefs equation 2.5 the numerator of this expression  $P(\bar{P}, P_L | \text{Good is H})$  will be zero as there is no mass of high firms charging  $P_L$ . The denominator is nonzero as there is a mass of low firms offering this price. Thus the advertiser at  $\bar{P}$  is recognised as being low quality and earns:

$$\pi_{L,A}(\bar{P}) = (1 - \nabla)\bar{P}\Gamma(L - \bar{P}) < \pi_{L,M}(P_L) \quad (2.6)$$

Thus the advertiser would be strictly better off being a non-advertiser at  $P_L$ .

In addition note that due to the assumed tiebreaking rule an advertiser at  $P_L$  is worse off than a non-advertiser at the same price. Thus if the equilibrium price distribution of high firm advertisers is atomless at a price  $P \leq P_L$  there can be no low firm advertisers in equilibrium.  $\square$

We have now established that if the equilibrium price distribution of high firm advertisers is atomless at a price  $P \leq P_L$  then all low firms will not advertise while setting a price of  $P_L$ . We can now shift attention to examining the behaviour of high firms. Assuming an atomless high firm pricing distribution, As no low firm will ever price at a price other than  $P_L$  high firms will be able to use the intuitive criterion to price at any level that represents a profitable deviation for them and still be recognised as high quality. We can thus write the profit function for high firm advertisers when the price distribution of high firm advertisers is atomless:

$$\pi_{H, \text{Advertiser}}(P) = (P - c_H)\Gamma(H - P) [(1 - \eta) + \eta G(P) + \eta G(P)] \quad (2.7)$$

Where  $G(P)$  is the survival function of the prices charged by high quality advertisers and  $\eta$  is the proportion of high quality advertisers. The terms in the square brackets account for the probabilities of matching with a non-advertiser, matching with an advertiser and an external advertiser matching with the firm respectively.

Now denoting  $g(P)$  to be the pdf of the advertiser price distribution (which is defined when the advertiser pricing distribution is atomless) we can show that the pricing distribution of high firms will be gapless:

**Lemma 5.** *If the equilibrium price distribution of high firm advertisers is atomless then there are no equilibria exhibiting gaps of positive measure in the price distribution of advertising high firms. That is for any  $P, P + \epsilon$  with  $\epsilon > 0$  and with  $g(P) > 0, g(P + \epsilon) > 0$  then  $\int_{\epsilon}^{P+\epsilon} g(p)dp > 0$ .*

*Proof.* Note from Lemma 4 that atomlessness in equilibrium implies low firms never advertise and an advertiser will always be recognised as high quality. Consider the case if such a gap did form between prices  $P$  and  $P + \epsilon$  with  $\epsilon > 0$ . Consider in particular the firm pricing at  $P$ . This firm could increase its price to  $P + \epsilon$  which would increase  $(P - c_H)\Gamma(H - P)$  whilst not changing  $[(1 - \eta) + \eta G(P) + \eta G(P)]$ . Thus there is a profitable deviation.  $\square$

We can now find that there will be high firm non-advertisers and in equilibrium all high firms will earn the same profits they would make price signalling at  $P^S$ . Intuitively this occurs because the possibility of not advertising while charging  $P^S$  puts a lower bound on how much Bertrand competition among advertisers can reduce their profits.

**Lemma 6.** *If the equilibrium price distribution of high firm advertisers is atomless then there is a positive mass of high firms selling at the signalling price  $P^S$  and not providing additional offers to consumers.*

*Proof.* If all mass of high firms were advertising in an atomless distribution then the top pricing firm would make no profits and be strictly better off offering a price of  $P^S$  without advertising.  $\square$

**Corollary 7.** *If the equilibrium price distribution of high firm advertisers is atomless then in equilibrium all high firms earn profits of  $\pi_H^S$ .*

*Proof.* Immediate from lemma 6, proposition 2 and definition 2.  $\square$

We can now use the profit function from equation 2.7 and the equilibrium profit from Corollary 7 to find the domain of high firm advertiser pricing and the proportion of advertisers.

**Lemma 8.** *If the equilibrium price distribution of high firm advertisers is atomless, The bottom pricing advertiser will charge  $P_B$  where  $P_B$  is the solution to:*

$$(P_B - c_H)\Gamma(H - P_B) = \frac{\pi_H^S}{1 + \eta} \quad (2.8)$$

*Proof.* Substituting equilibrium profit and that for the bottom pricing firm  $G(P_B) = 1$  into equation 2.7 yields this expression.  $\square$

**Lemma 9.** *If the equilibrium price distribution of high firm advertisers is atomless then the top pricing advertiser will charge  $P_H^M$ .*

*Proof.* This comes from equation 2.7. In the event the top pricing firm had a price less than  $P_H^M$  they could raise their price and increase  $(P - c_H)\Gamma(H - P)$  whilst the fraction lost to other firms  $[(1 - \eta) + 2\eta G(P)]$  stayed the same.  $\square$

**Lemma 10.** *If the equilibrium price distribution of high firms is atomless, the proportion of advertising firms in equilibrium is:*

$$\eta = 1 - \frac{\pi_H^S}{\pi_H^M} \quad (2.9)$$

*Proof.* To see this consider equation 2.7 for the advertiser offering a price of  $P_H^M$ . As  $G(P_H^M) = 0$ , their profit is  $\pi_H^M(1 - \eta)$  which from lemma 7 must be equal to  $\pi_H^S$  in expectation for a firm to price at this level. The lemma follows immediately from this equality.  $\square$

Now now split the analysis into two different cases. The first is where  $P_B > P_L$  and hence the atomlessness of the high firm advertiser pricing distribution is assured from lemma 2. The second is the complementary case where atomlessness is not assured.

**Case A:**  $P_B > P_L$

From Lemma 8 all high firm advertisers will price at least at  $P_B > P_L$ . Thus from lemma 2 the advertiser price distribution will be atomless and all of the preceding results are applicable.

**Proposition 3.** *There will exist a PBNE for this game. All low firms will price at  $P_L$  and earn  $\pi_L = (P_L - c_L)\Gamma(L - P_L)$  whilst all high firms will earn  $\pi_H^S$  as defined by equation 2.7. A proportion  $\eta$  as defined by equation 2.9 of high firms will advertise with a pdf of prices as given by:*

$$g(P) = \frac{\pi_H^S}{2\eta} \left[ \frac{\Gamma(H - P) - (P - c_H)\gamma(H - P)}{(P - c_H)^2\Gamma(H - P)^2} \right] \quad P_B \leq P \leq P_H^M \quad (2.10)$$

A proportion  $1 - \eta$  of high firms will not advertise and will set a price of  $P^S$ .

Beliefs in this PBNE will satisfy:

$$\mu(P) = \begin{cases} 1 & P \geq P^S \\ 0 & P < P^S \end{cases}$$

$$\mu(P, P_E) = \begin{cases} 1 & \text{for } P > P_L \quad \text{and} \quad \forall P_E \\ 0 & \text{for } P \leq P_L \quad \text{and} \quad \forall P_E \end{cases}$$

*Proof.* This pricing distribution can be obtained by noting that in equilibrium the price distribution  $G(P)$  must satisfy:

$$(P - c_H)\Gamma(H - P)[1 - \eta + 2\eta G(P)] = \pi_H^S$$

$$[1 - \eta + 2\eta G(P)] = \frac{\pi_H^S}{(P - c_H)\Gamma(H - P)}$$

$$G(P) = \frac{1}{2\eta} \left[ \frac{\pi_H^S}{(P - c_H)\Gamma(H - P)} - 1 + \eta \right] \quad P_B \leq P \leq P_H^M \quad (2.11)$$

The associated pdf can be found by differentiating the survival function and multiplying by negative one. Equation 2.11 is a valid survival function being decreasing in price with endpoints of  $G(P_H^M) = 0$  and  $G(P_B) = 1$ , thus this price distribution is feasible.

These beliefs will be satisfied in this equilibria as all high firms price at more than  $P_L$  and all low firms price at  $P_L$ . There is no profitable deviations for high firms who earn  $\pi_H^S$  at any point in the pricing domain and cannot earn higher profits outside this domain. Likewise low firms cannot profitably deviate.  $\square$

From its construction we can note that the equilibrium pricing distribution in proposition 3 is unique in the class of fully separating equilibria.<sup>2.11</sup> This equilibrium can be seen in figure 2.1 where the advertising support line shows the prices and corresponding sale quantities of advertising firms. At prices close to  $P_M^H$  the advertising support line sits beneath the high firm demand curve,  $\Gamma(H - P)$ , as they lose a share of their customers to the firms they advertise against. At lower prices it sits above the non-advertiser demand curve as they retain most of their customers and also

<sup>2.11</sup>Although this equilibrium pricing distribution could be sustained by different out of equilibrium beliefs.

gain customers from competing advertising firms. While the advertisers at  $P_H^M$  retain a proportion of  $1 - \eta$  of the firms they encounter, those pricing closer to  $c_H$  retain or win a greater total proportion of  $1 + \eta$  firms. This demonstrates the demand shifting taking place with the lowest pricing advertisers selling  $\frac{(1+\eta)\Gamma(H-P_B)}{(1-\eta)\Gamma(H-P_H^M)}$  times more goods than the highest pricing advertiser. All advertisers earn the same equilibrium profits however with expected profit the same as for the price signalling non-advertiser firms.

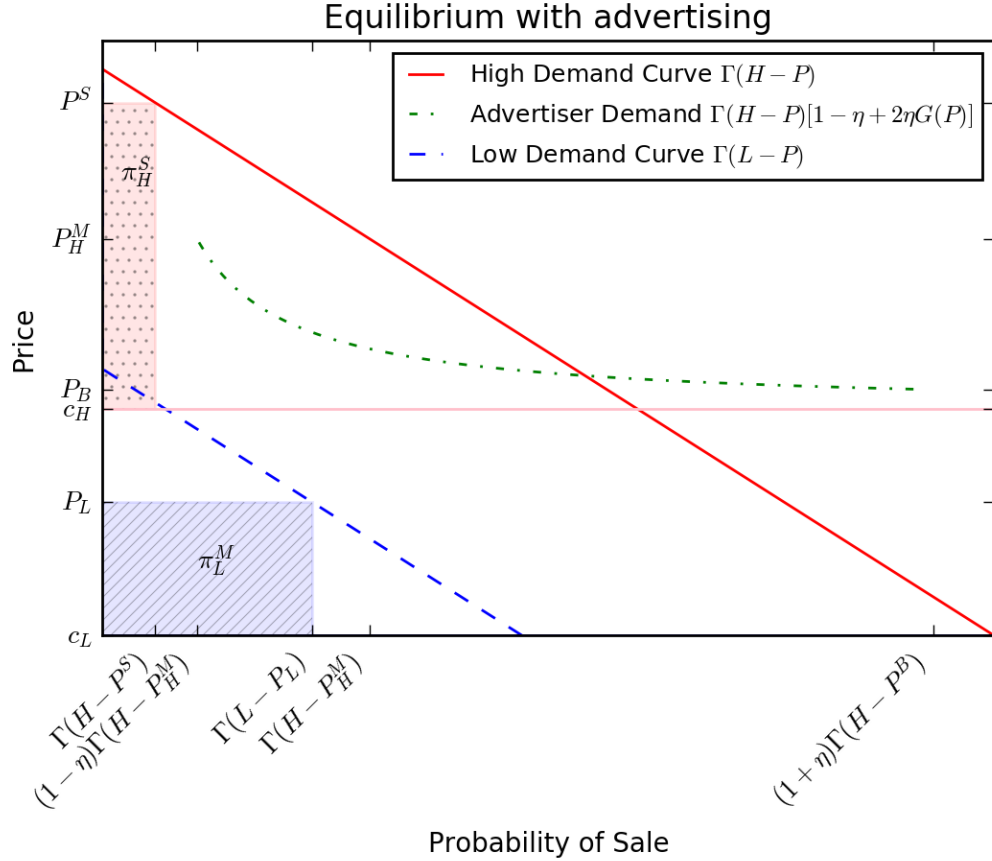


Figure 2.1: Equilibrium with advertising

**Case B:**  $P_B \leq P_L$

Considering the case when  $P_B < P_L$  is more problematic as lemma 2 cannot be used to obtain atomlessness of the pricing distribution. The equilibrium explored in this section will be one where the price distribution of high firms is guessed to be atomless at all prices (this guess will be verified later on). As a result of this however no claim can be made as to the uniqueness of the resulting equilibrium among the class of fully separating equilibria. Note that this assumption of atomlessness at any price is sufficient such that all of the previous supporting lemmas hold. In particular this conjecture results in an equilibrium closely related to the equilibrium expressed in proposition 3. This equilibrium will have the same functions to describe firm pricing decisions

however  $P_L$  is cut out of this distribution

**Proposition 4.** *There will exist a PBNE for this game. All low firms will price at  $P_L$  and earn  $\pi_L = (P_L - c_L)\Gamma(L - P_L)$  whilst all high firms will earn  $\pi_H^S$  as defined by equation 2.7. A proportion  $\eta$  as defined by equation 2.7 of high firms will advertise with a pdf of prices given by:*

$$g(P) = \begin{cases} \frac{\pi_H^S}{2\eta} \left[ \frac{\Gamma(H-P) - (P-c_H)\gamma(H-P)}{(P-c_H)^2\Gamma(H-P)^2} \right] & \text{for } P_B \leq P \leq P_H^M, P \neq P_L \\ 0 & \text{for } P = P_L \end{cases} \quad (2.12)$$

A proportion  $1 - \eta$  of high firms will not advertise and will set a price of  $P^S$ .

Beliefs in this PBNE will satisfy:

$$\mu(P) = \begin{cases} 1 & P \geq P^S \\ 0 & P < P^S \end{cases} \quad (2.13)$$

$$\mu(P, P_E) = \begin{cases} 1 & \text{for } P > P_L \quad \text{and} \quad \forall P_E \\ 1 & \text{for } P \leq P_L \quad \text{and} \quad P_E \neq P_L \\ 0 & \text{for } P \leq P_L \quad \text{and} \quad P_E = P_L \end{cases} \quad (2.14)$$

*Proof.* The proof of this proposition largely follows that of proposition 3. The key difference is the omission of  $P_L$  (a zero measure set) from the advertiser pricing function. The pricing distribution is atomless (which verifies the atomless guess) and all of the previous lemmas hold. Thus low firms will all not advertise while setting a price of  $P_L$  as per lemma 4 and beliefs for advertisers are the same as in proposition 3 for any set price.  $\square$

While this is an equilibrium with beliefs that are robust to the intuitive criterion it relies on discontinuous beliefs at a certain point. This may be less credible as a model for some markets than the previous case where  $P_B > P_L$ . For instance this equilibrium may not be robust if the marginal cost of low firms is continuously heterogeneous in some interval as there would then be a continuum of  $P_L$  prices.

## 2.4 Producer and Consumer Surplus

The total surplus generated in a separating equilibria when a consumer visits a firm is  $Q - c_Q$  if that consumer buys the good and is that consumer's outside option otherwise. The problem for the consumer on the other hand is to choose from the maximum of  $Q - P_f$  and  $\Omega_k$ . Clearly this implies that the closer is price to marginal cost the greater surplus generated in the market.

In order to evaluate the impact of advertising we can note that the low firms do not change their price from the full information case or the no advertising case (presented in proposition 1)

and so welfare is unchanged for consumers visiting low firms. When consumers visiting high firms are considered, equilibrium advertiser prices are less than both the full information monopoly price or the signalling price. Specifically we can write the following expressions for the surplus generated from consumers visiting high firms in the full information (FI) equilibrium, pure price signalling (PS) equilibrium as well as the surplus from a single advertiser (SA) and surplus from the advertising equilibrium (AE) respectively:

$$S_{\text{FI}} = (H - c_H)\Gamma(H - P_H^M) + \int_{H - P_H^M}^{\bar{\Omega}} \Omega\gamma(\Omega)d\Omega \quad (2.1)$$

$$S_{\text{PS}} = (H - c_H)\Gamma(H - P^S) + \int_{H - P^S}^{\bar{\Omega}} \Omega\gamma(\Omega)d\Omega \quad (2.2)$$

$$S_{\text{SA}} = (1 - \eta) \int_{P_B}^{P_H^M} \left[ (H - c_H)\Gamma(H - P) + \int_{H - P}^{\bar{\Omega}} \Omega\gamma(\Omega)d\Omega \right] g(P)dP \quad (2.3)$$

$$+ \eta \int_{P_B}^{P_H^M} \left[ (H - c_H)\Gamma(H - P) + \int_{H - P}^{\bar{\Omega}} \Omega\gamma(\Omega)d\Omega \right] \bar{g}(P)dP$$

$$S_{\text{AE}} = (1 - \eta)S_{\text{PS}} + \eta S_{\text{SA}} \quad (2.4)$$

Where the distribution  $\bar{g}(P)$  is the pdf of the first order statistic from two draws from the advertiser pricing distribution. This arises when a consumer has prices from two advertisers and will pick the lower price.

Here it can be seen that the surplus generated by a single advertiser is higher than the surplus generated by a price signalling firm or a firm in the full information equilibrium as the price an advertiser offers is always lower. This implies that when the fraction of advertisers is sufficiently high, it is possible for the advertising equilibrium to deliver greater surplus than the corresponding full information equilibrium. As the above expressions are analytically intractable this is shown numerically but first stating these implications in a proposition.

**Proposition 5.** *In any asymmetric information market, surplus is always weakly greater in the fully separating equilibrium exhibiting advertising relative to the fully separating equilibrium where no advertising is allowed. In some cases surplus can be higher in asymmetric information markets with advertising than in the corresponding full information markets.*

*Proof.* Proof for the first statement is provided by the fact consumers always choose the maximum of  $\Omega_k$  and  $H - P_f$ , whilst surplus is maximised by them taking the maximum of  $\Omega_k$  and  $H - c_H$ . The price distribution in the price signalling case weakly stochastically dominates the price distribution where advertising occurs and hence delivers weakly lesser surplus (strictly if  $\pi_H^S < \pi_H^M$ ). The proof of the second statement is by example 1.  $\square$

**Example 1.** *We define a uniform distribution of outside options in the space  $[0, 1]$ . Thus we have*



$\Gamma(x) = x$  in this domain. We assume that  $H = 1$ ,  $c_L = 0$  and  $L = \frac{8}{17}$ .<sup>2.12</sup> We can use these to get the following expressions for monopoly prices and profits in terms of  $c_H$ :

$$\begin{aligned} P_L &= \frac{4}{17} & P_H^M &= \frac{1 + c_H}{2} & P^S &= \frac{16}{17} \\ \pi_L &= \frac{16}{289} & \pi_H^M &= \left(\frac{1 - c_H}{2}\right)^2 & \pi_H^S &= \frac{1}{17}\left(\frac{16}{17} - c_H\right) \end{aligned}$$

From these prices and profits expressions for the proportion of advertisers and the bottom price can be obtained:

$$\eta = 1 - \frac{\pi_H^S}{\pi_H^M} \quad P_B = \frac{1 + c_H}{2} - \frac{\sqrt{(1 - c_H)^2 - 4\frac{\pi_H^S}{1 + \eta}}}{2}$$

Finally we can write the survival function and pdf of the advertiser pricing distribution as well as an expression for  $\bar{g}(P)$ .<sup>2.13</sup>

$$G(P) = \frac{\pi_H^S}{2\eta(P - c_H)(1 - P)} + \frac{1}{2} - \frac{1}{2\eta} \quad g(P) = \frac{\pi_H^S(1 + c_H - 2P)}{2\eta(P - c_H)^2(1 - P)^2}$$

$$\bar{g}(P) = 2G(P)g(P)$$

Now examining the bounds of feasible  $c_H$  values we focus on the case of a separating equilibria with costly signalling. Hence the maximum  $c_H$  value we consider is  $\frac{15}{17}$  as at this cost level the signalling price is equal to the monopoly price for the high firm. The lower limit  $c_H$  value is zero as by construction it is at a marginal cost of  $c_L$  when a firm is indifferent to selling with an expected value of  $L$  at the low monopoly price or  $H$  at the signalling price.

It can be seen in figure 2.1 that when  $c_H$  is low it implies that  $\pi_H^M$  is high compared to  $\pi^S$  and thus there are many advertisers in the market. Intense competition between these firms depresses prices closer to marginal cost. This results in surplus in the advertising equilibrium being higher than the full information equilibrium.

On the other hand when  $c_H$  is high  $\pi_H^M$  is not much higher than  $\pi^S$  and thus there are fewer advertisers in the market with less intense competition between them. In this case surplus is higher in the full information equilibrium than the advertising equilibrium.

<sup>2.12</sup>The values given  $H$ ,  $c_L$  are chosen so that the demand curve is downward sloping across all feasible prices. The value given  $L$  is chosen as it is approximately halfway up the interval and exploits the pythagorean triple (8, 15, 17) to get a rational signalling price. In general a higher  $L$  value leads to full information delivering higher surplus for all  $c_H$  whilst lower  $L$  values lead to the advertising equilibrium being more efficient for all  $c_H$ . Figures 2.1 and 2.1 were constructed with these parameters and  $c_H = 0.4$ .

<sup>2.13</sup>For discussion on finding the pdf of an order statistic see for instance Blitzstein and Hwang [2015, Theorem 8.6.4]

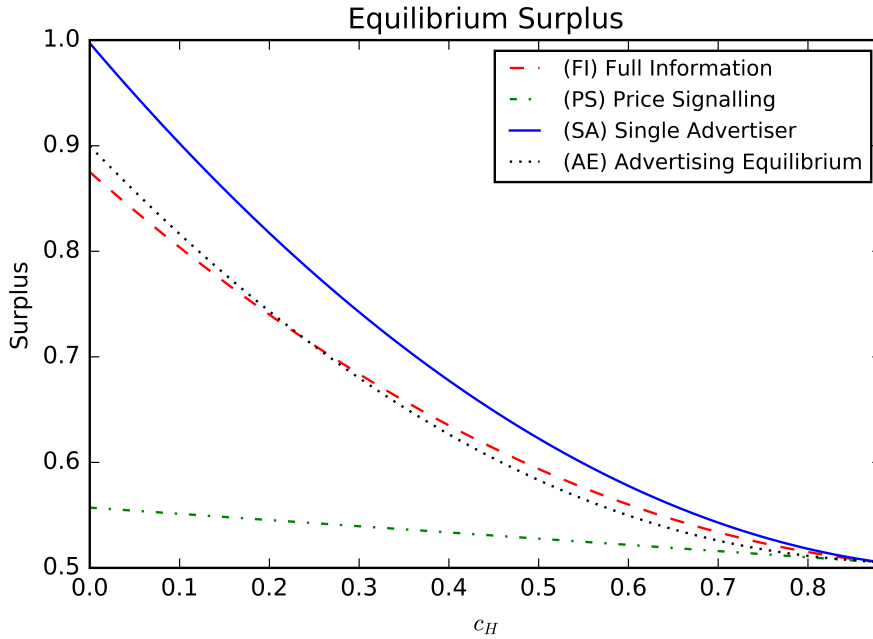


Figure 2.1: Surplus generated in each equilibrium case

## 2.5 Extensions

### 2.5.1 Cost heterogeneity in high firms

As noted in Shelegia [2012] even small differences in marginal cost can lead to firms randomising over different ranges of prices in a mixed strategy pricing equilibria such as those presented in this paper. This observation may have welfare implications in this paper as advertising results in a shift of quantity from higher pricing advertisers to lower price advertisers. If low cost firms are also low price firms this means that advertising can boost aggregate surplus by awarding larger quantity to lower cost firms.

This section examines this possibility by augmenting the model of section 2.3 with high firms with heterogeneous costs. Rather than having a homogeneous group of high firms we assume half are  $\alpha$  firms with a marginal cost of  $c_\alpha$  and half are  $\beta$  firms with a higher marginal cost of  $c_\beta > c_\alpha$ . Both types of firms sell high quality goods and a firm cannot selectively advertise against one of the two classes of firms. In all other aspects the model is unchanged. As the analysis for the most part follows that performed in section 2.3 it is deferred for appendix 2.A but two key implications are here stated.

**Proposition 6.** *In any fully separating equilibrium exhibiting advertising:*

- (a) *All  $\alpha$  firms will price equal or less than all  $\beta$  firms.*
- (b)  *$\beta$  firms will never earn more than their signalling profits however  $\alpha$  firms may earn more.*

*Proof.* See appendix 2.A. □

The intuition for the point (a) is that  $\alpha$  firms have a lower monopoly price which means price signalling is relatively more expensive for them as compared to  $\beta$  firms. This leads them to be more likely to advertise and more likely to offer lower prices while advertising. For a simple example of point (b) consider a case where  $c_\alpha$  is low enough such that the monopoly price of  $\alpha$  firms,  $P_\alpha^M$ , is less than the marginal cost of  $\beta$  firms,  $c_\beta$ . In this case an  $\alpha$  firm could price at  $P_\alpha^M$  and advertise losing at most half of their consumers to other  $\alpha$  firms and still gain some customers from advertising against  $\beta$  firms. In some cases this fraction of the monopoly profit will exceed the signalling profit.

Thus in the presence of cost heterogeneity, advertising can add to market efficiency as it shifts demand away from higher cost firms towards lower cost firms. Firms with lower costs will position themselves as lower pricing firms within the advertising equilibrium. Thus there is an efficiency gain from demand being shifted towards firms with a lower marginal cost.

## 2.5.2 Monopolist supplier for high quality good

Now we consider the special case where the common good sold in the market is provided by a single supplier firm that behaves as a monopolist and produces the product costlessly. This will often be the case where the product is copyrighted or patented. We will refer to these supplying firms as suppliers and the firms that buy from the suppliers as merchants. We assume full information exists between suppliers and merchants but consumers do not know the quality level of a good unless it is signalled to them.

In analysing this case we will first write an expression for the  $c_H$  level which equalises the monopoly price of high firms (which does depend on  $c_H$ ) with the signalling price (which does not depend on  $c_H$ ). This cost is denoted  $c_{\text{sig}}$  and is defined as the cost level which solves the following equation.

$$P^S = \arg \max_{P \in \mathfrak{R}^+} (P - c_{\text{sig}}) \Gamma(H - P) \quad (2.1)$$

where  $P^S$  is as defined in section 2.3.1. We can now write the profit function for the supplier as a function of the price they charge merchants. In this expression we will write the advertiser price pdf as  $g_c(P)$ , the bottom price as  $P_B(c)$  and the monopoly price as  $P_H^M(c)$  reflecting the fact that

these are affected by  $c$ :

$$\pi_{\text{Supplier}}(c) = \begin{cases} c\Gamma(H - P_H^M(c)) & c > c_{\text{Sig}} \\ c\Gamma(H - P^S) & c \leq c_{\text{Sig}}, \quad \text{without advertising} \\ c \left[ (1 - \eta)\Gamma(H - P^S) + \eta \int_{P_B^M(c)}^{P_H^M(c)} \Gamma(H - P)g_c(P)dP \right] & c \leq c_{\text{Sig}}, \quad \text{with advertising} \end{cases} \quad (2.2)$$

The benchmark price signalling model without advertising is first considered. We get the result that suppliers will always price at least  $c_{\text{Sig}}$  such that  $P_H^M \geq P^S$ .

**Proposition 7.** *In markets where advertising is not allowed the price charged by suppliers to merchants will never be less than  $c_{\text{Sig}}$*

*Proof.* Given that merchants will need to charge at least the signalling price to ensure low quality firms will not emulate high quality, all firms will price at  $P^S$  for all levels of  $c$  below a critical level to be denoted  $c_{\text{Sig}}$ . As the price charged by merchants is the minimum of the signalling and their monopoly price, this critical level  $c_{\text{Sig}}$  is such that these prices are equal as defined in equation 2.1.

Therefore in the absence of advertising a supplier will never charge less than  $c_{\text{Sig}}$  as they would sell the same quantity of  $\Gamma(H - P^S)$  whilst earning a lower price.  $\square$

The intuition here is that the supplier profit strictly increases as  $c$  increases until  $c$  reaches  $c_{\text{Sig}}$ . This is because merchants charge  $P^S$  at all of these cost levels and hence as the supplier increases  $c$  the price the supplier receives increases whilst the quantity stays constant.

If advertising is allowed when  $c \geq c_{\text{Sig}}$  then  $P^S \leq P_H^M$  and hence no advertising will be undertaken.<sup>2.14</sup> On the other hand if advertising is allowed when  $c < c_{\text{Sig}}$  then  $P^S > P_H^M$  and hence advertising will be undertaken. This observation raises the following possibility:

**Proposition 8.** *In some markets where advertising is allowed it may be optimal for suppliers to reduce their price below  $c_{\text{Sig}}$  and thus induce advertising to increase their sale quantity.*

*Proof.* Proof is by example 2.  $\square$

**Example 2.** *This example follows on from example 1 except for  $c_H$  now being endogenously determined by a monopolist supplier who produces the good costlessly. All other parameters and the outside option distribution are identical. We can note that in this case setting  $P_H^M = P^S$  obtains  $c_{\text{Sig}} = \frac{15}{17}$ :*

*The demand curve faced by the monopolist supplier in the advertising and no advertising case along with areas representing optimal profits are shown in figure 2.1.<sup>2.15</sup> It can be seen that where no*

<sup>2.14</sup>From equation 2.1 we have  $\pi_H^S = \pi_H^M$  when  $c = c_{\text{Sig}}$ . As  $c_{\text{Sig}}$  increases above this level the signalling profit drops below the monopoly profit. Hence there is no incentive to drop price and advertise.

<sup>2.15</sup>Note that equations for these demand curves are as described in equation 2.2 (once the  $c$  term giving the margin per item is removed)

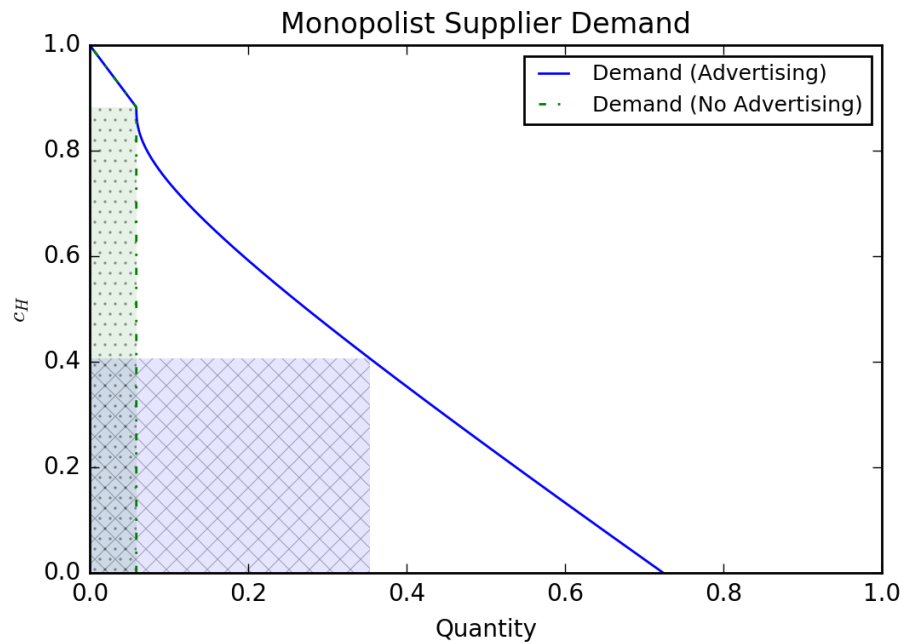


Figure 2.1: Monopolist supplier demand curves and profits

advertising is allowed the monopolist supplier prices at  $c_{sig}$ . When advertising is allowed however a lower  $c$  of approximately 0.41 is set as the greater sales volume obtainable is sufficient to make up for the loss in margin.

## 2.6 Conclusion

This paper has examined comparative advertising from the perspective of disclosing differences in prices. While at first glance this strategy seems counter-intuitive as it would result in greater pricing competition it is found that it can act as an alternative method of signalling quality than price signalling.

In the fully separating equilibria that this paper presents some firms will remain price signalling whilst other firms will lower their price closer to the monopoly level and instead signal by price comparative advertising against rival firms. While an advertiser may be able to achieve greater sales quantity by reducing their price, they also face competition from the firm they advertise against. Advertising has the effect of decreasing the equilibrium distribution of prices offered in the asymmetric information setting which increases consumer surplus. Firm profits on the other hand do not change from the case where no advertising is allowed. While firms do manage to price closer to their monopoly price any additional profits are lost through increased competition with other advertising firms.

A number of extensions were examined including the possibility of high quality firms having

heterogeneous marginal costs. In this case advertising can play a role in shifting demand from higher marginal cost firms to lower cost firms. The case of a monopoly supplier who provides goods to many reselling firms was also examined. It is found that advertising can result in suppliers reducing the price they charge reselling firms. As a large fraction of reselling firms no longer need to price signal (at a high price with low sale quantity) the supplier can reduce their price to incentivise advertising and boost sales volume. If the increase in quantity makes up for the decrease in margin then this can be more profitable.

These implications for welfare are substantially more clearcut and supportive of comparative advertising than previous papers that model it as the disclosure of differences in product features. This may indicate that the form of comparative advertising matters from a policy perspective. Anderson and Renault [2009] find that comparative advertising of horizontal good attributes can deteriorate total welfare when there is a sufficiently large quality gap between rival firms. By contrast this paper implies that price comparative advertising will increase total welfare and in addition no agent's surplus is decreased by the legalisation of comparative advertising. In the basic model this increase in welfare comes entirely from lower prices to consumers and more efficient signalling for firms. When the extensions are considered however there are additional vehicles for surplus to increase including the shifting of quantity to lower cost producers of the high quality good and the inducing of a monopolist supplier into decreasing the price they charge reselling firms. All of these insights present the clear and unambiguous implication that price comparative advertising is beneficial for welfare and should be supported by legislators and regulators.

## 2.7 References

- Simon Anderson and Regis Renault. Comparative advertising: disclosing horizontal match information. *RAND Journal of Economics*, 40(3):558–581, 2009.
- Mark Armstrong and Jidong Zhou. Search deterrence. *The Review of Economic Studies*, 83(3):1–32, 2015.
- Mark Bagnoli and Ted Bergstrom. Log-concave probability and its applications. *Economic Theory*, 26(2):445–469, 2005.
- Kyle Bagwell and Michael Riordan. High and declining prices signal product quality. *American Economic Review*, 81(1):224–239, 1991.
- Francesca Barigozzi, Paolo Garella, and Martin Peitz. With a little help from my enemy: Comparative advertising as a signal of quality. *Journal of Economics and management strategy*, 18(4):1071–1094, 2009.

- Joseph Bertrand. Book review of *theorie mathematique de la richesse sociale* and of *recherches sur les principes mathematiques de la theorie des richesses*. *Journal de Savants*, 67:499–508, 1883.
- Joeseeph Blitzstein and Jessica Hwang. *Introduction to Probability*. CRC Press, 1st edition, 2015. ISBN 9781466575578.
- Cedar White Bradley. Navigating Qatari law. *World Trademark Review*, April/May, 2014.
- Levent Celik. Information unravelling revisited: Disclosure of horizontal attributes. *Journal of Industrial Economics*, 62(1):113–136, 2014.
- Graham Charlton. Q&A: Kieron smith of the book depository. <https://econsultancy.com/blog/3497-q-a-kieron-smith-of-the-book-depository>, 2009.
- In-Koo Cho and David Kreps. Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102:179–221, 1987.
- Larry Compeau, Dhruv Grewal, and Rajesh Chandrashekar. Comparative advertising: Believe it or not. *The Journal of Consumer Affairs*, 36(2):284–294, 2002.
- Council of European Union. Directive 97/55/EC of European Parliament and of the Council of 6 October 1997 amending Directive 84/450/EEC concerning misleading advertising so as to include comparative advertising, 1997.
- Andrew Daughety and Jennifer Reinganum. Imperfect competition and quality signalling. *RAND Journal of Economics*, 39(1):163–183, 2008.
- Federal Trade Commission. Statement of policy regarding comparative advertising. <https://www.ftc.gov/public-statements/1979/08/statement-policy-regarding-comparative-advertising>, 1979.
- Dhruv Grewal and Larry Compeau. Comparative price advertising: Informative or deceptive? *Journal of Public Policy & Marketing*, 11(1):52–62, 2002.
- Gönenç Gürkaynak, İlay Yılmaz, and Burak Yeilaltay. Turkey green-lights comparative advertising. <http://www.mondaq.com/turkey/x/444016/advertising+marketing+branding/Turkey+GreenLights+Comparative+Advertising>, 2015.
- Cristina Kan, Donald R. Lichtenstein, Susan Jung Grant, and Chris Janiszewski. High and declining prices signal product quality. *Journal of Consumer Research*, 40:1078–1096, 2013.
- Frederic Koessler and Regis Renault. When does a firm disclose product information. *RAND Journal of Economics*, 43(4):630–649, 2012.

- Regis Renault. Advertising in markets. In Simon Anderson, David Stromberg, and Joel Waldfogel, editors, *Handbook of Media Economics*. Elsevier, 2016.
- John. G. Riley. Informational equilibrium. *Econometrica*, 47(2):331–359, 1979.
- Tibor Scitovszky. Some consequences of the habit of judging quality by price. *The Review of Economic Studies*, 12(2):100–105, 1944.
- Sandro Shelegia. Asymmetric marginal costs in search models. *Economic Letters*, 116:551–553, 2012.
- Meghna Singh. Comparative advertising effectiveness with legal and cross culture framework. *International journal for research in management and pharmacy*, 3(3), 2014.
- Michael Spence. Job market signalling. *The Quarterly Journal of Economics*, 87(3):355–374, 1973.
- Dale Stahl. Oligopolistic pricing with sequential consumer search. *American Economic Review*, 79(4):700–712, 1989.
- Three Mobile. Three store - pay as you go price plans. [http://www.three.co.uk/Store/Pay\\_As\\_You\\_Go\\_Price\\_Plans](http://www.three.co.uk/Store/Pay_As_You_Go_Price_Plans), 2016.
- Hal. R. Varian. A model of sales. *The American Economic Review*, 70(4):651–659, 1980.
- Asher Wolinsky. Prices of product quality. *Review of Economic Studies*, 50(4):647–658, 1983.
- Eric Yu. Price anchoring to optimize your pricing strategy. <http://www.priceintelligently.com/blog/bid/181199/Price-Anchoring-to-Optimize-Your-Pricing-Strategy>, 2013.

## Appendices

### 2.A Heterogeneity Main Results

Following on from the extension to the model introduced in section 2.5.1, equilibrium in this context can be defined as:

**Definition 3.** *A fully separating PBNE in this model will be described by pricing strategies and*



equilibrium profits  $\hat{\pi}_L, \hat{\pi}_\alpha, \hat{\pi}_\beta$ , such that:

$$\pi_{L,N}(P) = \hat{\pi}_L \geq \pi_{L,N}(P') \quad \forall \quad P \in D_{L,N}, P' \in \mathfrak{R}^+ \setminus D_{L,N} \quad (2.A.1)$$

$$\pi_{L,A}(P) = \hat{\pi}_L \geq \pi_{L,A}(P') \quad \forall \quad P \in D_{L,A}, P' \in \mathfrak{R}^+ \setminus D_{L,A} \quad (2.A.2)$$

$$\pi_{\alpha,N}(P) = \hat{\pi}_\alpha \geq \pi_{\alpha,N}(P') \quad \forall \quad P \in D_{\alpha,N}, P' \in \mathfrak{R}^+ \setminus D_{\alpha,N} \quad (2.A.3)$$

$$\pi_{\alpha,A}(P) = \hat{\pi}_\alpha \geq \pi_{\alpha,A}(P') \quad \forall \quad P \in D_{\alpha,A}, P' \in \mathfrak{R}^+ \setminus D_{\alpha,A} \quad (2.A.4)$$

$$\pi_{\beta,N}(P) = \hat{\pi}_\beta \geq \pi_{\beta,N}(P') \quad \forall \quad P \in D_{\beta,N}, P' \in \mathfrak{R}^+ \setminus D_{\beta,N} \quad (2.A.5)$$

$$\pi_{\beta,A}(P) = \hat{\pi}_\beta \geq \pi_{\beta,A}(P') \quad \forall \quad P \in D_{\beta,A}, P' \in \mathfrak{R}^+ \setminus D_{\beta,A} \quad (2.A.6)$$

The belief functions  $\mu(P)$  and  $\mu(P, P_E)$  are derived in accordance with Bayes rule and player strategies for all information sets reached with positive probability in equilibrium.

It can be seen that all of the results from lemma 1 to lemma 6 carry over without modification to this case. Thus all low firms will not advertise while pricing at  $P_L$  and all advertisers will be believed as being of high quality. It can be noted that the signalling price,  $P^S$  of both firms will be identical and as described in equation 2.4. Profits at the signalling price will be denoted by  $\pi_\alpha^S$  and  $\pi_\beta^S$ . The profit of a type  $d \in \{\alpha, \beta\}$  advertiser can be written as:

$$\pi_{d,A}(P) = (P - c_d)\Gamma(H - P)[1 - \eta + 2\eta G(P)] \quad (2.A.7)$$

Where  $\eta$  is the proportion of all firms that are advertising and  $G(P)$  is the advertiser price distribution. For brevity in some proofs this will be written as:

$$\pi_{d,A}(P) = (P - c_d)Q(P) \quad (2.A.8)$$

with  $Q(P) = \Gamma(H - P)[1 - \eta + 2\eta G(P)]$ .

The monopoly prices of  $\alpha$  and  $\beta$  firms are denoted  $P_\alpha^M, P_\beta^M$  respectively with the corresponding profits denoted  $\pi_\alpha^M$  and  $\pi_\beta^M$ . To ensure that from lemma 2 the advertiser price distribution is atomless we assume that the bottom pricing advertiser will price above  $P_L$ . This condition will be formalised later on.

**Lemma 11.** *No equilibrium exhibits an  $\alpha$  firm not advertising while a  $\beta$  firm does advertises.*

*Proof.* To see this consider the case where a  $\beta$  firm advertises at some price  $P \in D_{\beta,A}$  while an  $\alpha$

firm is not advertising. For the  $\beta$  firm we must have for some advertising price  $P$ :

$$\begin{aligned} (P - c_\beta)Q(P) &\geq (P^S - c_\beta)\Gamma(H - P^S) \\ (P - c_\alpha)Q(P) + (c_\alpha - c_\beta)Q(P) &\geq (P^S - c_\alpha)\Gamma(H - P^S) + (c_\alpha - c_\beta)\Gamma(H - P^S) \\ (P - c_\alpha)Q(P) - (P^S - c_\alpha)\Gamma(H - P^S) &\geq (c_\alpha - c_\beta) [\Gamma(H - P^S) - Q(P)] \end{aligned}$$

Note that as  $c_\alpha < c_\beta$  and  $\Gamma(H - P^S) < Q(P)$  the right hand side is positive. This putative case also implies for  $\alpha$  firms:

$$\begin{aligned} (P^S - c_\alpha)\Gamma(H - P^S) &\geq (P - c_\alpha)Q(P) \\ (P - c_\alpha)Q(P) - (P^S - c_\alpha)\Gamma(H - P^S) &\leq 0 \end{aligned}$$

A contradiction. Thus no equilibrium exhibits an  $\alpha$  firm not advertising while a  $\beta$  firm does advertise.  $\square$

**Corollary 12.** *In any equilibrium there will be a positive mass of  $\alpha$  firms advertising.*

*Proof.* From proposition 2 there must be a positive mass of advertisers. From lemma 11 these advertisers cannot all be  $\beta$  firms unless all firms are advertisers which would contradict lemma 6.  $\square$

**Lemma 13.** *In any equilibrium:*

1. *A positive mass of  $\beta$  firms will not advertise while setting a price at  $P^S$ ;*
2.  *$\beta$  firms earn  $\pi_\beta^S$  in equilibrium. Thus  $\hat{\pi}_\beta = \pi_\beta^S$ .*

*Proof.* From lemma 6 in any equilibrium there exists a positive mass of  $\alpha$  non-advertisers and/or  $\beta$  non-advertisers and one or both of the following equalities will hold:

$$\pi_\alpha = \pi_\alpha^S \qquad \pi_\beta = \pi_\beta^S \qquad (2.A.9)$$

Application of lemma 11 shows that no equilibrium with only  $\alpha$  advertisers can exist. Thus in equilibrium there must be a positive mass of  $\beta$  firms not advertising while setting a price of  $P^S$ . As a result of this all  $\beta$  firms must earn  $\pi_\beta^S$ .  $\square$

**Proposition 9.** *In any equilibrium all  $\alpha$  firms will price lower than all  $\beta$  advertisers.*

*Proof.* First considering the case for advertisers. Consider a putative equilibrium where  $\beta$  firms weakly prefer pricing at  $P$  than pricing at  $P'$  and  $\alpha$  firms weakly prefer pricing at  $P'$  than  $P$  with

$P < P'$ . Then for  $\beta$  firms:

$$(P - c_\beta)Q(P) \geq (P' - c_\beta)Q(P') \quad (2.A.10)$$

$$(P - c_\alpha)Q(P) + (c_\alpha - c_\beta)Q(P) \geq (P' - c_\alpha)Q(P') + (c_\alpha - c_\beta)Q(P') \quad (2.A.11)$$

$$(P - c_\alpha)Q(P) - (P' - c_\alpha)Q(P') \geq (c_\alpha - c_\beta)[Q(P') - Q(P)] \quad (2.A.12)$$

Note that as  $c_\beta > c_\alpha$  and  $Q(P) > Q(P')$  the right hand side is positive. Now consider the case of the  $\alpha$  firm:

$$(P' - c_\alpha)Q(P') \geq (P - c_\alpha)Q(P) \quad (2.A.13)$$

$$(P - c_\alpha)Q(P) - (P' - c_\alpha)Q(P') \leq 0 \quad (2.A.14)$$

A contradiction. Hence there is no equilibrium where  $\alpha$  firms advertise at a price higher than  $\beta$  firms. Now considering non-advertisers from lemma 11 there can never be  $\alpha$  non-advertisers whilst there are  $\beta$  firms advertising.  $\square$

At this point we introduce the bottom advertising price (analogously to  $P_B$  in equation 2.8).

**Lemma 14.** *The bottom price will be  $P_{B,\alpha}$  which will be defined by:*

$$(P_{B,\alpha} - c_\alpha)\Gamma(H - P_{B,\alpha}) = \frac{\pi_\alpha}{1 + \eta} \quad (2.A.15)$$

*Proof.* From proposition 9 the bottom pricing advertiser will be an  $\alpha$ . The bottom price is the lowest price that delivers this firm the equilibrium profit level for  $\alpha$  firms.  $\square$

The condition for all advertisers to price more than  $P_L$  is thus  $P_{B,\alpha} > P_L$ . From this point onwards we focus on proving the existence of equilibrium in the special case where there are some  $\beta$  firms advertising.<sup>2.16</sup>

**Lemma 15.** *If  $\pi_\beta^M \geq 2\pi_\beta^S$  then there will be a positive mass of  $\beta$  advertisers in equilibrium.*

*Proof.* If this did not hold then a  $\beta$  firm could price at  $P_\beta^M$  and would win against all other  $\beta$  firms to earn profits of at least  $\frac{\pi_\beta^M}{2}$ . With the assumption  $\pi_\beta^M \geq 2\pi_\beta^S$  this is strictly more than the profits attainable by not advertising with a price of  $P^S$ .  $\square$

**Lemma 16.** *If  $\pi_\beta^M \geq 2\pi_\beta^S$  the top advertiser price will be  $P_\beta^M$*

*Proof.* The top pricing advertiser is a  $\beta$  firm. With similar arguments to lemma 9 they will charge their monopoly price.  $\square$

<sup>2.16</sup>Whilst there is in principle no impediment to analysing the alternate case where all  $\beta$  firms (and potentially some  $\alpha$  firms) monopolise, this restriction is in order to show the notable result where  $\alpha$  firms earn above signalling profits as discussed in section 2.5.1.

**Lemma 17.** *If  $\pi_\beta^M \geq 2\pi_\beta^S$  then  $\eta$  is given by:*

$$\eta = 1 - \frac{\pi_\beta^S}{\pi_\beta^M} \quad (2.A.16)$$

*Proof.* The top pricing  $\beta$  firm charges  $P_\beta^M$  where  $G(P_\beta^M) = 0$  and must earn  $\pi_\beta^S$ . The expression follows immediately from substituting these factors into equation 2.A.7.  $\square$

**Lemma 18.** *If  $\pi_\beta^M \geq 2\pi_\beta^S$  then there is a unique price charged by both types of advertisers  $\bar{P} \equiv D_{\beta,A} \cap D_{\alpha,A}$  which is given by:*

$$\bar{P} = \frac{c_\beta \hat{\pi}_\alpha - c_\alpha \hat{\pi}_\beta}{\hat{\pi}_\alpha - \hat{\pi}_\beta} \quad (2.A.17)$$

*Proof.* At the unique price  $\bar{P}$  where there are advertisers from both types of firm the profits are:

$$\pi_{\beta,A}(\bar{P}) = (\bar{P} - c_\beta)\Gamma(H - \bar{P})[1 - \eta + 2\eta G(\bar{P})] \quad (2.A.18)$$

$$\pi_{\alpha,A}(\bar{P}) = (\bar{P} - c_\alpha)\Gamma(H - \bar{P})[1 - \eta + 2\eta G(\bar{P})] \quad (2.A.19)$$

And thus:

$$\frac{\pi_\beta(\bar{P})}{\pi_\alpha(\bar{P})} = \frac{\bar{P} - c_\beta}{\bar{P} - c_\alpha} \quad (2.A.20)$$

Rearranging this equation and noting at this point they make their equilibrium profits yields the lemma.  $\square$

**Proposition 10.** *If  $\pi_\beta^M \geq 2\pi_\beta^S$  then  $\alpha$  firms will earn more than their signalling profits in equilibrium.*

*Proof.* First recounting equation 2.A.20 and noting that at  $\bar{P}$  both high firm types earn their equilibrium profits.

$$\frac{\hat{\pi}_\beta}{\hat{\pi}_\alpha} = \frac{\bar{P} - c_\beta}{\bar{P} - c_\alpha} \quad (2.A.21)$$

Now examining signalling profits:

$$\pi_\beta^S = (P^S - c_\beta)\Gamma(H - P^S) \quad (2.A.22)$$

$$\pi_\alpha^S = (P^S - c_\alpha)\Gamma(H - P^S) \quad (2.A.23)$$

And thus:

$$\frac{\pi_\beta^S}{\pi_\alpha^S} = \frac{P^S - c_\beta}{P^S - c_\alpha} \quad (2.A.24)$$

As  $\frac{P-c_\beta}{P-c_\alpha}$  is a monotonic function for of  $P$  in the region  $[c_\beta, 1]$ .

$$\frac{\hat{\pi}_\beta}{\hat{\pi}_\alpha} < \frac{\pi_\beta^S}{\pi_\alpha^S} \quad (2.A.25)$$

And substituting in  $\hat{\pi}_\beta = \pi_\beta^S$

$$\pi_\alpha^S < \hat{\pi}_\alpha \quad (2.A.26)$$

So profits above signalling profits are made.  $\square$

At this point all of the results presented in section 2.5.1 have been shown to hold. The last remaining task is to show that an equilibrium will exist.

The profit functions for  $\alpha$  and  $\beta$  firms are:

$$\pi_{\beta,A}(\bar{P}) = (\bar{P} - c_\beta)\Gamma(H - \bar{P})[1 - \eta + 2\eta G(\bar{P})] \quad (2.A.27)$$

$$\pi_{\alpha,A}(\bar{P}) = (\bar{P} - c_\alpha)\Gamma(H - \bar{P})[1 - \eta + 2\eta G(\bar{P})] \quad (2.A.28)$$

And after rearranging to get the required  $G(P)$ :

$$G(P) = \begin{cases} \frac{1}{2\eta} \left[ \frac{\pi_\beta}{(P-c_\beta)\Gamma(H-P)} - 1 + \eta \right] & \text{for } \bar{P} < P \leq P_\beta^M \\ \frac{1}{2\eta} \left[ \frac{\pi_\alpha}{(P-c_\alpha)\Gamma(H-P)} - 1 + \eta \right] & \text{for } P_{\alpha,B} \leq P \leq \bar{P} \end{cases} \quad (2.A.29)$$

**Proposition 11.** *If  $\pi_\beta^M \geq 2\pi_\beta^S$  then the equilibrium described by a proportion  $\eta$  of  $\beta$  firms (as described in equation 2.A.16) monopolising at  $P^S$  and all other firms advertising at prices described by the survival functions in equation 2.A.29 and the beliefs described by*

$$\mu(P) = \begin{cases} 1 & P \geq P^S \\ 0 & P < P^S \end{cases} \quad (2.A.30)$$

$$\mu(P, P_E) = \begin{cases} 1 & \text{for } P > P_L \quad \text{and} \quad \forall P_E \\ 0 & \text{for } P \leq P_L \quad \text{and} \quad \forall P_E \end{cases} \quad (2.A.31)$$

is a PBNE.

*Proof.* Similar arguments as were made in lemma 4 show that these beliefs will be robust in equilibrium and no low firms will attempt to emulate high quality.

The  $G(P)$  function described in equation 2.A.29 is feasible, being decreasing in price and ranges between 0 and 1 when price changes from  $P_\beta^M$  to  $P_{\alpha,B}$ .

The case of low firms is unchanged to that described in proposition 3 with them unable to convincingly advertise. Hence there is no profitable deviation for these firms.

All  $\alpha$  firms earn the same profit at any price  $P \in D_{\alpha,A} \equiv [P_{\alpha,B}, \bar{P}]$ . From proposition 10 they earn more than their signalling profit. From proposition 9 they also earn more than is possible at any point in  $D_{\beta,A}$  and hence there are no profitable deviations for these firms.

All  $\beta$  firms earn the same profit at any price  $P \in D_{\beta,A} \equiv [\bar{P}, P_{\beta}^M]$ . From proposition 13 they earn their signalling profit. From lemma 9 they also earn more than is possible at any point in  $D_{\alpha,A}$  and hence there are no profitable deviations for these firms.  $\square$

## Chapter 3

# It's Good to be Bad: A Model of Low Quality Dominance in a Full Information Consumer Search Market (with Margaryta Klymak)

**Abstract:** This paper examines a consumer search market exhibiting vertically differentiated firms, heterogeneous consumers and endogenous consumer market entry. In an asymmetric information setting high and low quality firms make equal sales and profit in this market. Conversely when there is full information, search frictions induce an unravelling mechanism that leads to a unique refined equilibrium where all consumers approach low quality firms and high quality firms make no sales or profit. This presents a rationale for why low quality firms may disclose their quality and high quality firms may not even when disclosure is costless.<sup>3.1</sup>

**JEL Codes:** D82, D83, L15

**Keywords:** Quality Disclosure, Consumer Search

### 3.1 Introduction

The literature on asymmetric information in markets is largely focused on cases where higher quality firms may seek to differentiate themselves from lower quality firms. The role of low quality

---

<sup>3.1</sup>Baumann (Corresponding author): Stuart.Baumann@ed.ac.uk, School of Economics, University of Edinburgh, 30 Buccleuch Pl, Edinburgh EH8 9JT, United Kingdom. Klymak: klymakm@tcd.ie, Department of Economics, Trinity College Dublin, College Green, Dublin 2, Ireland. This work was supported by the ESRC postgraduate funding scheme (for Baumann) and the Grattan Scholarship scheme (for Klymak). For useful comments we would like to thank Philipp Kircher, Ludo Visschers, Andrew Clausen, Patrick Harless, Régis Renault, József Sákovics, Carl Singleton and Ina Taneva as well as seminar participants at the University of Edinburgh PhD seminar (2016) and internal seminar (2017), Royal Economic Society (2017), Irish Economic Society (2017).

firms has received less attention in this context. Looking at some marketing materials reveals however that some firms go to lengths to communicate that they are low quality to potential customers. A prominent example is Europe's budget airline Ryanair which has been proactive in building a low quality reputation as part of their "no frills" strategy. Ryanair CEO Michael O'Leary has stated to the media that "*Anyone who thinks Ryanair flights are some sort of bastion of sanctity where you can contemplate your navel is wrong. We already bombard you with as many in-flight announcements and trolleys as we can. Anyone who looks like sleeping, we wake them up to sell them things*" [The Telegraph, 2016].<sup>3.2</sup> Other examples are discount stores that specialise in selling "factory seconds" such as Australia's "Reject Shop". These stores can often be seen airing advertisements with phrases that are vague but suggestive of low quality such as "like new", "discontinued", "second-hand" or "refurbished". These advertisements are perplexing because even though some laws may require disclosure of quality information before purchase, no law requires firms to air advertisements disclosing this information to potential customers before they even approach the firm.

Looking at the advertising decisions of lawyers in the United States shows a similar story with many lawyers advertising in undignified ways that are likely to undermine confidence in their ability. The United States has at least three attorneys that call themselves "the hammer" in their advertisements [AboveTheLaw.com, 2012]. In Texas a criminal defence attorney has been advertising by calling himself the "Texas Law Hawk" and performing stunts like doing wheelies on a dirt-bike or throwing sticks of dynamite [Wilson, 2016]. This kind of advertising is strongly discouraged by the American Bar Association who warn that "*lawyers should consider that the use of inappropriately dramatic music, unseemly slogans, hawkish spokespersons, premium offers, slapstick routines or outlandish settings in advertising does not instil confidence in the lawyer or the legal profession*" [American Bar Association, 2016], a view that finds support in the marketing literature [Trebbi et al., 1999].<sup>3.3</sup> This again presents the question of why lawyers would choose forms of advertising which are likely to undermine consumer confidence in their ability.

This paper seeks to explain why firms may disclose that they are of low quality. A model is presented with a high quality firm and a low quality firm. Consumers have heterogeneous marginal utility from quality ("taste") and can choose what firm to go to upon entering the market. The key mechanism presented is one of unravelling. Considering a putative equilibrium where high taste consumers go to the high quality firm and low taste consumers go to the low quality firm, the high quality firm will price to make one of the consumers approaching it indifferent to buying and

<sup>3.2</sup>This has gone so far as the CEO of Ryanair publicly announcing that extra charges for toilet use were under consideration from the airline [The Economist, 2013]. These charges were never actually implemented.

<sup>3.3</sup>In addition the American Bar Association had previously banned all lawyer advertising in 1907 however this ban was overturned by the supreme court in 1977 on freedom of speech grounds [US Supreme Court, 1977].



searching at the low quality firm. The diamond paradox applies to this marginal consumer and they make no surplus. When a consumer can anticipate however that when they arrive at a high quality firm they will be this marginal consumer then that consumer would be better off (by the extent of the search cost) going to the low quality firm ex ante where there will be some lower taste consumer who will be made indifferent to buying and taking their outside option. This will result in fewer and higher taste consumers visiting the high quality firm which will result in this firm raising its prices to make a higher taste consumer indifferent to buying and searching at the low quality firm. This new marginal consumer will again be better off going to the low quality firm ex ante. Through this mechanism we get a full information unravelling result where the sole refined equilibrium is one where all consumers go to a low quality firm and no consumers will visit a high quality firm. Clearly search frictions are key to this result, however the only crucial assumptions over that of a standard consumer search model are that consumers can choose what firm to go to upon entering the market and that consumers have heterogeneous marginal utility from quality.

Aside from presenting a rationale for low quality firms to disclose their quality, a second contribution is to the growing literature considering a firm's choice between signalling and disclosing quality. Our model shows a force which acts to deter high quality firms from disclosing their quality as a known high quality firm is visited by no consumers in equilibrium. Signalling which occurs after a consumer visits a high quality firm remains a feasible strategy however. Low quality firms on the other hand will choose to disclose their low quality before consumers approach firms but will never want to disclose their quality after consumers have approached firms. This conflict between disclosure and signalling is different from the previous literature that emphasised strategic competitive pricing considerations [Jansson and Roy, 2015] or disclosure costs [Daughety and Reinganum, 2008]. A third contribution is that the model presented has an interesting contrast with Akerlof [1970]. Whilst in Akerlof asymmetric information causes markets to unravel with only low quality goods remaining, in this paper the opposite result is shown with unravelling only occurring in the full information setting.

An important insight of the model is that disclosure can adversely impact market efficiency by resulting in the suboptimal equilibrium where all consumers visit the low quality firm. In addition search frictions can encourage firms to decrease the quality of their good offerings even when it is costless to produce a higher quality good. To avert these outcomes it may thus be desirable to implement minimal quality standards in markets exhibiting search frictions. In addition it may be welfare improving for regulators to prevent low quality firms from voluntarily disclosing their quality. For instance this paper would provide an argument in support of the view that legal bar associations should have the ability to restrict forms of advertising that allow lawyers to disclose

their low quality to the market.

## 3.2 Background

There is an extensive literature examining the disclosure decisions of vertically differentiated firms where consumers cannot directly observe quality. Many early papers conclude that an unravelling result will prevail in such markets [Grossman, 1981, Milgrom, 1981]. This occurs from the highest quality firm in a putative pooling equilibria wanting to differentiate itself which lowers the expected value of the remaining pool, leading to the next highest quality firm wanting to differentiate itself from the pool and so on. That mandatory disclosure laws are thought necessary in the face of this unravelling result presents somewhat of a puzzle and so later papers look at where this result can fail [Dranove and Zhe Jin, 2010].

Board [2009] presents a duopolistic model where firms are heterogeneous in quality and consumers have heterogeneous marginal utility from quality. He finds cases where disclosure of quality information can result in intensified price competition and lower firm profits. This presents a rationale for mandatory disclosure to increase consumer surplus at the expense of firm profits through better sorting as well as the intensification of price competition among firms. Levin et al. [2009] present a model with two firms offering horizontally and vertically differentiated products. They also find that disclosure can intensify price competition thus deterring firms from disclosing quality information.

There have also been papers that have tried to include signalling and disclosure decisions in a unified framework. Daughety and Reinganum [2008] create a unified model where firms choose between disclosure and signalling. They find that when disclosure is costless all firms will disclose but if disclosure is sufficiently costly firms may signal. Another paper to include signalling and disclosure is that of Caldieraro et al. [2011]. Interestingly this paper shows cases where it can be optimal for low quality firms to disclose their quality. Their model includes the possibility of high quality firms signalling their quality by depressing their price which intensifies price competition with the low quality firms. Both high and low quality firms can avert this by disclosure to increase the proportion of consumers that can recognise quality. By informing these consumers the incentives for high quality firms to depress their price for signalling are eroded and both firms are better off while consumer surplus is reduced.

A recent paper to look at the choice between disclosure and signalling in a duopolistic market is that of Jansson and Roy [2015]. They examine firms that can be high or low quality (defined by an exogenous probability) where firms interact in a two stage game. In the first stage each firm can credibly disclose their quality to the market whilst in the second stage each firm offers a

consumer a price. The consumer then buys from the firm that offers the higher utility after taking into account perceived quality and price. They find equilibria where firms with high quality goods decide to signal rather than disclose due to the strategic effects of disclosure on the other firm's price. When firms price without knowing the quality of their competitor, price competition is less intense. Thus the effects of price competition deter disclosure.

This paper seeks to build upon the literature in a few respects. Whilst Jansson and Roy [2015] offer a mechanism where high quality firms are deterred from disclosure even when it is costless, their mechanism relies on market power and hence would not generalise to markets where there are many firms. On the other hand the mechanism presented in this paper relies on search frictions and hence may be more generalisable to such markets.<sup>3,4</sup> A second contribution is that in our model, low quality firms proactively disclose that their goods are of low quality. Whilst this possibility has been examined by Caldieraro et al. [2011] two points of difference should be drawn. Caldieraro et al. [2011] present a model where it is beneficial for *all* firms to disclose quality while this paper presents a case where only low quality firms would like to disclose and this disclosure harms high quality firms. Finally the mechanism for low quality firms to disclose in Caldieraro et al. [2011] requires firms to signal high quality by *reducing* their price. While this is supported in their model and may occur in certain markets, it is more intuitive to believe that in general consumers perceive high price to signal high quality. This paper on the other hand presents a rationale for low quality firm disclosure that stems from search fractions and thus may better describe some markets.

### 3.3 The Model

There are two firms in a market, one of which sells a product of quality  $H$  (the “high firm”) and one of which sells a good with a lower quality of  $L$  (the “low firm”). Both firms produce their goods costlessly. There is a unit measure of consumers. Consumers have heterogeneous marginal utility from quality described by a “taste” parameter. A consumer with a taste parameter of  $a_i$ , with an offer for a good with an expected quality of  $Q$  and a price of  $P$  gets an expected utility from purchase of:

$$a_i Q - P \tag{3.1}$$

---

<sup>3,4</sup>Whilst for simplicity the model of this paper analyses the case of two firms, in an extension (section 3.4.4) it is shown that the result where low quality firms benefit from full information extends to the setting of many firms.

The taste parameter is uniformly distributed on  $[0, 1]$  with a cdf given by:<sup>3.5</sup>

$$\text{Prob}[a_i < x] = x \quad \text{for} \quad x \in [0, 1] \quad (3.2)$$

Either firm can choose to disclose the qualities of both firms to the market in which case both qualities are observable and there is full information.<sup>3.6</sup> If neither firm chooses to disclose then there is asymmetric information and both goods appear identical to consumers. In the case of full information a fraction  $1 - \psi$  of “*directed*” consumers (orthogonal to taste) can choose to approach either the low or high quality firm upon entering the market. The complementary fraction,  $\psi$ , of consumers are “*undirected*” and approach either firm with 50% probability. Upon entering the market undirected and directed consumers are identical.

The timing is as follows. Firms choose whether or not to disclose and the information setting is realised. Consumers then proceed to approach one of the two firms. Firms offer a price to all consumers with no price discrimination possible. Consumers can then decide to buy, search at the other firm with a search cost of  $s$  or leave the market to get an exogenous outside option of value 0. We assume  $L > 0$  and thus a low quality firm recognised as being low quality will still be able to make some sales if visited by consumers with sufficiently high taste. We adopt the indifference rule that where  $a_i Q - P_Q < 0$  for both firms for a consumer with taste  $a_i$  that consumer will approach the firm with a higher value of  $a_i Q - P_Q$ .

We assume that all consumers enter the market costlessly.<sup>3.7</sup> Consumers face a search cost for visiting the other firm once they are in the market however. We assume that this search cost is strictly nonzero and positive but sufficiently small such that the best alternative to buying for the marginal consumer at the high firm is to buy from the low firm. This assumption depends on equilibrium pricing decisions and thus will be formalised later on.<sup>3.8</sup>

Note that the mass of undirected consumers ensures that each firm is always approached by a positive mass of consumers. This ensures that in the putative equilibrium where all consumers visit

<sup>3.5</sup>The simple form of the utility from consumption  $a_i Q - P$  and the uniform distribution of taste parameters are useful for tractability reasons, however the central unravelling mechanism (Proposition 1) extends to other taste distributions (including discrete distributions and distributions exhibiting gaps) and utility function forms,  $u$ , exhibiting  $\frac{\partial u}{\partial a} > 0$ ,  $\frac{\partial u}{\partial Q} > 0$ ,  $\frac{\partial^2 u}{\partial a \partial Q} > 0$ .

<sup>3.6</sup>Alternatively this can be thought of as one firm disclosing the relative qualities of its good in comparison to the other.

<sup>3.7</sup>One justification is that consumers may benefit from the experience of visiting a single firm but get bored thereafter. For instance the first test-ride of car at a dealership may be enjoyable whilst the process of looking for similar cars at other dealerships may be dull. To take an alternate example in the legal services market, a consumer facing criminal charges may benefit from hearing one lawyer's opinion on their case (while that lawyer provides a quote) but gets no additional information from visiting other lawyers (and getting other quotes). An alternate justification of this assumption may include undirected consumers randomly running into shops as they go about their normal day while directed consumers may be those able to plan their movements to choose what stores they run into. For instance a directed consumer intending to visit a beach may choose to go to one near a low/high quality shopping centre in order to visit a low/high quality firm at the same time. In this setting an undirected consumer is one that lives near only one beach so they cannot choose what firm to visit for free.

<sup>3.8</sup>This is done on page 125 just before lemma 3.

one of the firms the beliefs of the other firm regarding the taste of deviating consumers visiting them out of equilibrium do not need to be established. The methodology of analysing this game will be to first examine the Perfect Bayesian Equilibrium (PBEs) at any given level of  $\psi$ . For the equilibrium that we will call refined we will then take the limit of a sequence of PBEs as  $\psi \rightarrow 0$ .<sup>3.9</sup>

These equilibrium concepts can be formally described as:

**Definition 1 (Equilibrium Concepts).** *A Perfect Bayesian Equilibrium (PBE) in this game is defined as a pricing and disclosure strategy for firms, a search strategy for consumers and quality beliefs of consumers such that no consumers or firms have a profitable deviation and all beliefs are supported by Bayes rule in equilibrium.*

*A Refined Perfect Bayesian Equilibrium (RPBE) is the limit of a sequence of PBEs as  $\psi \rightarrow 0$ .*

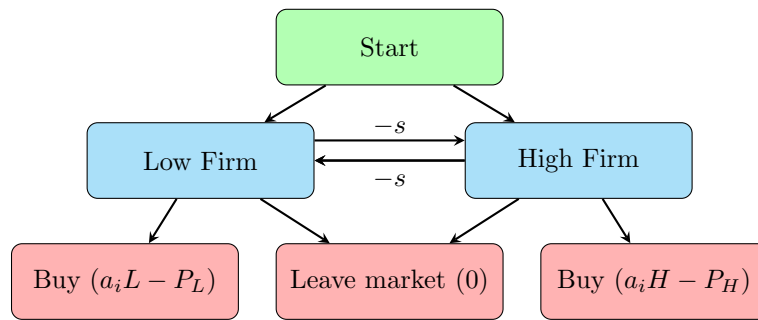


Figure 3.1: Consumer search paths in the full information setting for the consumer with taste  $a_i$

The possible search paths open to consumers in the full information setting are summarised in figure 3.1. This paper will first analyse the full asymmetric information subgame in section 3.3.1 before the full information subgame is examined in section 3.3.2. The disclosure decision of firms is then examined in section 3.3.3. Extensions are considered in section 3.4 before section 3.5 concludes.

### 3.3.1 The asymmetric information subgame

In the asymmetric information case, both firms appear identical so both directed and undirected consumers approach firms at random, each of which has an expected quality of  $\frac{H+L}{2}$ . In equilibrium each firm will be approached by a mass of  $\frac{1}{2}$  with tastes uniformly distributed on the unit interval. The condition for a consumer to buy is:

$$a \frac{H+L}{2} - P \geq \max(0, a \frac{H+L}{2} - P^E - s) \quad (3.1)$$

<sup>3.9</sup>Thus the refined equilibrium we present can be interpreted as a trembling hand refinement [Selten, 1975] of an equilibrium without undirected consumers, with the start of search strategy being trembled.

Where  $P^E$  is the consumer's expected price from the other firm. From the Diamond [1971] paradox result the right hand side of this expression will equal 0 for the consumer made (ex post) indifferent in equilibrium. Hence profit for an individual firm can be written as:

$$\pi(P) = \frac{1}{2}P \left[ 1 - \frac{P}{\frac{H+L}{2}} \right] \quad (3.2)$$

From the first order conditions the optimal price and profit for each firm is:

$$P = \frac{\frac{H+L}{2}}{2} \quad \pi = \frac{\frac{H+L}{2}}{8} \quad (3.3)$$

Which implies that both firms sell only to consumers with a taste greater than  $\frac{1}{2}$ .

### 3.3.2 The full information subgame

We open our analysis of the full information subgame with a lemma and a corollary that help to narrow the range of putative equilibria considerably:

**Lemma 1.** *When  $\hat{a} > a_*$  and  $H > L$  there will not exist any equilibrium where a taste  $a_*$  directed consumer goes to the firm with expected quality  $H$  whilst a taste  $\hat{a}$  directed consumer goes to a firm with expected quality  $L$ .*

*Proof.* In the  $a$  space the utility from each good is linear and given by  $aL - P_L$  and  $aH - P_H$ . This implies a single crossing condition that ensures that all consumers with a taste above the intercept point will approach the high firm and all consumers with a lower taste will approach the low firm. The assumed indifference rule ensures this holds in the case of low taste consumers where both  $aL - P_L$  and  $aH - P_H$  are less than 0.  $\square$

**Corollary 2.** *In any equilibrium, the set of directed consumers that choose to visit firms of a particular quality level will be convex in the "taste" dimension. We will denote the ex ante indifferent consumer's taste as  $a_A = \frac{P_H^E - P_L^E}{H - L}$ , hence  $[0, a_A]$  consumers will visit the low firm and  $[a_A, 1]$  consumers will approach the high firm.*

*Proof.* The ex ante indifferent consumer has equal expected utility from either firm and thus:

$$a_A H - P_H^E = a_A L - P_L^E \quad (3.1)$$

$$a_A = \frac{P_H^E - P_L^E}{H - L} \quad (3.2)$$

$\square$

By setting up expressions analogous to equation 3.1 and rearranging, we can derive expressions for the indifferent taste consumers at each firm. These can be seen in table 3.1.<sup>3.10</sup>

<sup>3.10</sup>It should be noted that at the high firm consumers use their expectation of  $P_L$  and at the low firm consumers

Notation	Location of consumer	Indifferent between	Formula
$a_H$	High firm	Buying and going to low firm	$\frac{P_H - P_L^E - s}{H - L}$
$a_{HL}$	High firm	Buying and leaving market	$\frac{P_H}{H}$
$a_{HLM}$	High firm	Going to low firm and leaving market	$\frac{P_L^E + s}{L}$
$a_L$	Low firm	Buying and leaving market	$\frac{P_L}{L}$
$a_T$	Low firm	Buying and going to high firm	$\frac{P_H^E - P_L + s}{H - L}$

Table 3.1: Indifferent Consumers

These can be used to write the demand function of the high firm:<sup>3.11</sup>

$$\begin{aligned}
Q_{\text{High}}(P_H) = & \overbrace{\frac{\psi}{2} [1 - \max(a_{HL}(P_H), a_H(P_H))]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi}{2} \left[ 1 - \max\left(a_A + \frac{s}{H - L}, a_{HL}(P_H), a_H(P_H)\right) \right]_+}^{\text{Undirected runoff from the } L \text{ firm}} \\
& + \overbrace{(1 - \psi) [1 - \max(a_A, a_{HL}(P_H), a_H(P_H))]_+}^{\text{Directed}}
\end{aligned} \tag{3.3}$$

In this demand function it can be seen that the high firm receives all directed consumers with a taste greater than  $a_A$  and undirected consumers from the low firm with a taste more than  $a_A + \frac{s}{H - L}$  (if any exist). Note also that  $a_A$  is taken exogenously by the high firm knowing that consumers use their expectation of  $P_H$  (rather than the high firm's choice of  $P_H$ ) to determine their search path. The high firm sells to all consumers that approach it with tastes between 1 and the maximum of  $a_H$  and  $a_{HL}$ .

The demand function for the low firm can be written as:

$$\begin{aligned}
Q_{\text{Low}}(P_L) = & \overbrace{\frac{\psi}{2} [\min(1, a_T(P_L)) - a_L(P_L)]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi}{2} \left[ \min\left(a_A - \frac{s}{H - L}, a_T(P_L)\right) - \max(a_{HLM}, a_L(P_L)) \right]_+}^{\text{Undirected Runoff from the } H \text{ firm}} \\
& + \overbrace{(1 - \psi) [\min(a_A, a_T(P_L)) - a_L(P_L)]_+}^{\text{Directed}}
\end{aligned} \tag{3.4}$$

In this case the low firm receives all directed consumers with a taste less than  $a_A$  and undirected consumers from the high firm with tastes between  $a_H$  and  $a_{HLM}$  (if any exist). The low firm sells to all consumers that approach it with tastes between  $a_T$  and  $a_L$ .<sup>3.12</sup>

At this point we recall the assumption that search costs are sufficiently small such that the best alternative to buying for the marginal consumer at the high firm is to go to the low firm. Mathematically this translates to the requirement that search costs are sufficiently low that in every

---

use their expectation of  $P_H$ . Before directed consumers enter the market they use their expectations of both prices. As pricing is simultaneous, each firm prices using an expectation of the other firm's price.

<sup>3.11</sup>We use here and throughout the notation  $[x]_+ = \max(0, x)$ .

<sup>3.12</sup>The fact that  $H > L > 0$  ensures that tastes higher than  $a_T$  strictly prefer to leave for the high firm and tastes lower than  $a_L$  strictly prefer to leave the market and consumers in the interim prefer to buy.

equilibrium we always have  $a_H > a_{HL}$  which implies that we must have  $s < L \left( \frac{P_H}{H} - \frac{P_L}{L} \right)$ . We can prove that for a sufficiently low search cost, all equilibria will have the property that equilibrium prices satisfy  $\frac{P_H}{H} > \frac{P_L}{L}$  and hence it is possible to find a positive search cost that satisfies this condition.

**Lemma 3.** *For a sufficiently low search cost, there are no equilibria where  $\frac{P_H}{H} \leq \frac{P_L}{L}$*

*Proof.* See appendix 3.A. □

Mathematically we must have  $a_T > a_A > a_H$  and  $a_{HLM} > a_L$ . As any equilibria will have the property  $\frac{P_H}{H} > \frac{P_L}{L}$ , we can additionally infer that  $a_L > a_{HL}$ ,  $a_T > a_L$  and  $a_H > a_{HLM}$ .<sup>3.13</sup> Putting these together leaves the only remaining taste ordering of  $a_T > a_A > a_H > a_{HLM} > a_L > a_{HL}$ .

At this point it can be noted that the marginal consumer at the high firm has a taste of  $a_H$  which is strictly less than the taste of the ex ante indifferent consumer  $a_A$ . This has profound implications for the equilibrium as highlighted in the following proposition:

**Proposition 1 (Unravelling of equilibrium without undirected consumers).** *In the special case where there are no undirected consumers ( $\psi \equiv 0$ ) there cannot exist equilibria where a positive measure of consumers approach both the high and low quality firms.*

*Proof.* A necessary condition for equilibrium in the absence of undirected consumers is that  $a_H = a_A$ . If we had  $a_H > a_A$  then consumers in the interval  $[a_A, a_H)$  will not buy at the high firm and would be better off going to the low firm ex ante. If we had  $a_H < a_A$  then the high firm sells to consumers in the interval  $[a_A, 1]$  whilst setting a price to make a consumer  $a_H$  indifferent. If this firm increased its price to make  $a_A$  indifferent it could maintain its quantity at a higher price. □

One way to think about this result is as an unravelling mechanism. Consider if the firm received all consumers with tastes in the interval  $[x, 1]$  and  $x$  is sufficiently high that the high firm wants to price to sell to all consumers with a taste greater than  $x$ .<sup>3.14</sup> The firm should optimally price to make the consumer with taste  $x$  indifferent to buying and walking away to the low firm. This implies a price of  $x(H - L) + P_L^E + s$  which leaves the consumer with a taste  $x$  as getting utility of  $Lx - P_L^E - s$ . This consumer would be strictly better off going to the low firm initially however as she would get an expected payoff of  $Lx - P_L^E$ . If this consumer (and others of similar taste) deviate to the low firm, then the high firm will be approached by consumers with tastes in the interval  $[x', 1]$  where  $x' > x$ . It will again be optimal for the high firm to price high enough so that the consumer with taste  $x'$  would be better off ex ante going to the low quality firm. This unravelling would continue until the high firm has no mass of consumers remaining.

<sup>3.13</sup>All of these inequalities come immediately from simple algebraic manipulation of the formulae in table 3.1.

<sup>3.14</sup>In the complementary case when  $x$  is low there cannot be an equilibrium as some consumers would not buy from the high firm and would instead go to the low firm - thus these consumers would be better off going to the low firm ex ante.



This proposition does not hold when there are undirected consumers however as the high firm may set a price to make an undirected consumer with a taste lower than  $a_A$  indifferent. The effect of  $a_H > a_A$  still has a substantial impact on the resulting equilibrium however as it implies that the optimal price for the high firm will be quite high. To see this informally note that the first order conditions of the general profit equation  $\pi = PQ(P)$  imply that the optimal price satisfies  $P = \frac{Q(P)}{Q'(P)}$ . When there are few undirected consumers then  $Q'(P)$  is quite low which implies a high optimal price.

We will show this formally by first rewriting the demand function (3.3) to incorporate the taste ordering discussed following lemma 3 and noting that the marginal consumer will have a taste between 0 and  $a_A$ :

$$Q_{\text{High}}(P_H) = \underbrace{\frac{\psi}{2} [1 - a_H(P_H)]}_{\text{Undirected - own}} + \underbrace{\frac{\psi}{2} \left[ 1 - \left( a_A + \frac{s}{H-L} \right) \right]}_{\text{Undirected runoff from the } L \text{ firm}} + \underbrace{(1 - \psi) [1 - a_A]}_{\text{Directed}} \quad (3.5)$$

Taking first order conditions for  $\pi_{\text{High}} = P_H Q_{\text{High}}(P_H)$  and rearranging yields:

$$P_H = \frac{H a_A \psi - 2H a_A + 2H - L a_A \psi + 2L a_A - 2L + P_L^E \psi}{2\psi} \quad (3.6)$$

Now looking at the low firm's demand function it is not possible to determine if the firm will lose undirected consumers to the high firm. That is it is unclear if  $a_T < 1$  in equilibrium. In the succeeding analysis we will assume  $a_T < 1$  and do the complementary case in appendix 3.C. First rewriting the demand function for the low firm (3.4):

$$Q_{\text{Low}}(P_L) = \underbrace{\frac{\psi}{2} [a_T(P_L) - a_L(P_L)]}_{\text{Undirected - own}} + \underbrace{\frac{\psi}{2} \left[ a_A - \frac{s}{H-L} - a_{HLM} \right]}_{\text{Undirected Runoff from the } H \text{ firm}} + \underbrace{(1 - \psi) [a_A - a_L(P_L)]}_{\text{Directed}} \quad (3.7)$$

Taking first order conditions for  $\pi_{\text{Low}} = P_L Q_{\text{Low}}(P_L)$  and rearranging yields:

$$P_L = \frac{H L a_A \psi - 2H L a_A + H L a_{HLM} \psi - L^2 a_A \psi + 2L^2 a_A - L^2 a_{HLM} \psi - L P_H^E \psi}{2H\psi - 4H - 4L\psi + 4L} \quad (3.8)$$

In equilibrium we will have  $P_H^E = P_H$  and  $P_L^E = P_L$ . Substituting this,  $a_{HLM} = \frac{P_L + s}{L}$  and

$a_A = \frac{P_H - P_L}{H - L}$  into equations 3.6 and 3.8 and solving for equilibrium prices yields:

$$P_H = \frac{2(H - L)(4H - 2L - H\psi + 2L\psi - \psi s)}{8(H - L) + H\psi(2 - \psi) + 2L\psi(1 + \psi)} \quad (3.9)$$

$$P_L = \frac{(H - L)(4L - \psi^2 s - 2\psi s)}{8(H - L) + H\psi(2 - \psi) + 2L\psi(1 + \psi)} \quad (3.10)$$

$$\pi_H = \frac{2\psi(H - L)(H\psi - 4H - 2L\psi + 2L + \psi s)^2}{(8(H - L) + H\psi(2 - \psi) + 2L\psi(1 + \psi))^2} \quad (3.11)$$

$$\pi_L = \frac{(H - L)(2H - H\psi + 2L\psi - 2L)(-4L + \psi^2 s + 2\psi s)^2}{2L(8(H - L) + H\psi(2 - \psi) + 2L\psi(1 + \psi))^2} \quad (3.12)$$

We can also write an expression for the ex ante indifferent consumer and the marginal undirected consumer who leaves the low firm for the high:

$$a_A = \frac{8(H - L) + 2\psi(2L - H) + \psi^2 s}{8(H - L) + H\psi(2 - \psi) + 2L\psi(1 + \psi)} \quad (3.13)$$

$$a_T = \frac{8H^2 - 2H^2\psi + 6HLL\psi - 16HLL + 2H\psi^2 s - 2H\psi s - 8Hs - 4L^2\psi + 8L^2 - 3L\psi^2 s - 2L\psi s + 8Ls}{(H - L)[8(H - L) + H\psi(2 - \psi) + 2L\psi(1 + \psi)]} \quad (3.14)$$

At this point we can state this paper's second proposition establishing equilibriums for the full information subgame:<sup>3.15</sup>

**Proposition 2.** *For any given  $\psi$  satisfying  $a_T < 1$  (with  $a_T$  given by equation 3.14) there exists a PBE of the full information subgame with firms pricing  $P_H$ ,  $P_L$  according to equations 3.9 and 3.10 respectively and directed consumers with tastes  $[0, a_A]$  approaching the low firm and consumers with taste  $[a_A, 1]$  approaching the high firm with  $a_A$  according to equation 3.13. Firms have correct beliefs over taste distribution of consumers approaching them and consumers have correct beliefs over the quality of goods.*

*Of the undirected consumers that initially approach the high firm, consumers with taste  $[a_H, 1]$  will buy; consumers with taste  $[a_{HLM}, a_H]$  will go the low firm and buy and consumers with taste  $[0, a_{HLM}]$  will leave the market. Of the undirected consumers that initially approach the low firm, consumers with taste  $[a_T, 1]$  will go to the high firm, consumers with taste  $[a_L, a_T]$  will buy from the low firm and consumers with taste  $[0, a_L]$  will leave the market.*

There are a few interesting features of this equilibrium. First we will consider the special case where there are no directed consumers ( $\psi = 1$ ). In this case we get the intuitive result that the high quality firm earns strictly more than the low firm. Dividing equation 3.11 by equation 3.12 shows that the high firm's profit is  $\frac{4L(3H-s)^2}{H(4L-3s)^2}$  times higher than the profit of the low firm. For a small search cost (in the limit as  $s \rightarrow 0$ ) this approaches  $\frac{9H}{4L}$ . This is a smaller ratio of high to low firm profits however than occurs in the competitive market setting where the high firm earns  $\frac{4H}{L}$

<sup>3.15</sup>For the equilibrium values in the alternative case where  $a_T > 1$  see appendix 3.C.

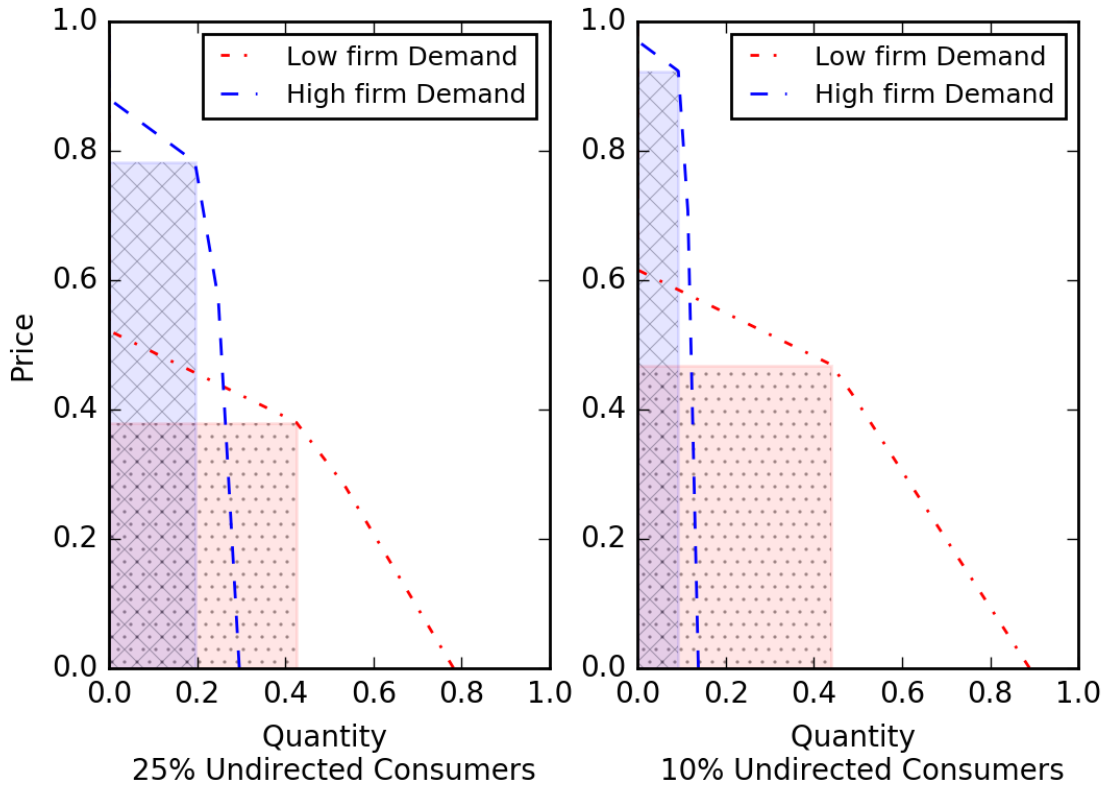


Figure 3.1: Equilibrium demand curves

times more than the low firm.<sup>3.16</sup>

Firm profits change markedly however when there are directed consumers in the market. This is illustrated by figure 3.1 which shows the demand curves faced by the high and low quality firms under two sets of parameters with their optimal prices, quantities and profits being indicated by the shaded rectangles. In both cases there is a search cost of  $s = 0.001$  and product qualities of  $L = 1$  and  $H = 1.5$  for the low and high firms respectively. In the left hand panel<sup>3.17</sup> 25% of consumers are undirected and so  $\psi = 0.25$  while on the right hand panel<sup>3.18</sup> there are 10% undirected consumers and so  $\psi = 0.10$ . Considering first the left hand panel it can be seen that both demand curves have a flatter segment at high prices where firms sell to their directed consumers and undirected consumers with the same taste as their directed consumers. In these flatter segments the high firm sells to consumers with tastes in their interval  $[a_A, 1]$  while the low quality firm sells to consumers with tastes in the interval  $[a_L, a_A]$ . As price is lowered however the firms have eventually sold to all consumers in these intervals and start selling to other undirected consumers. The slope of each demand curve becomes steeper reflecting the fact that there are fewer undirected consumers

<sup>3.16</sup>See appendix 3.B for analysis of the competitive market and comparison with search market profits.

<sup>3.17</sup>The equilibrium quantities end up being  $P_L = 0.379$ ,  $P_H = 0.781$ ,  $\pi_L = 0.161$ ,  $\pi_H = 0.152$ ,  $a_A = 0.805$  and  $a_T = 0.806$ .

<sup>3.18</sup>The equilibrium quantities end up being  $P_L = 0.468$ ,  $P_H = 0.922$ ,  $\pi_L = 0.208$ ,  $\pi_H = 0.085$ ,  $a_A = 0.908$  and  $a_T = 0.909$ .

approaching them. The high firm sets its price so that a consumer with a taste of  $a_A - \frac{s}{H-L}$  is indifferent to buying and going to the low firm. This price and quantity is slightly to the right and below the point at which the flatter and steeper segments of the demand curve meet which is that point at which consumers with taste  $a_A$  are indifferent to buying.

Looking now at the right panel, the only change to the parameters is that the population of undirected consumers has been reduced. In accordance with equation 3.13,  $a_A$  shifts upwards which results in the high firm receiving fewer directed consumers and the flat segment of the demand curve is narrower as a result. As the low firm receives more high taste consumers it raises its price which deteriorates the outside option for directed consumers at the high firm. As a result the high firm can sell to their directed consumers at a higher price. Overall however their profit has deteriorated relative to the case where more consumers were undirected.

The key takeaway from this figure is that even in full information markets where a significant fraction of consumers enter the market randomly, the low firm can earn greater profits than the high firm. In the left panel the low firm earns profits that are around 6% higher than the high firm whilst in the right panel the low firm earns 142% more. The reduction in the number of undirected consumers allows the low firm to capture market share from the high quality firm. This observation leads to the following proposition for the refined equilibrium of this subgame as the proportion of undirected consumers approaches zero:

**Proposition 3.** *The sole refined equilibrium of the full information subgame is one where no consumers approach the high firm who make no profits. The low firm sells a quantity of  $\frac{1}{2}$  at a price of  $\frac{L}{2}$  for a profit of  $\frac{L}{4}$ .*

*Proof.* Equation 3.13 describing the ex ante indifferent consumer taste ( $a_A$ ) is continuous in  $\psi$  and is 1 when  $\psi = 0$ . This indicates that as  $\psi \rightarrow 0$  we will have  $a_T = a_A + \frac{s}{H-L}$  exceeding 1. Once  $\psi$  is sufficiently high that  $a_T > 1$  the relevant equation describing  $a_A$  is as given in equation 3.C.6 which also exhibits  $a_A \rightarrow 1$  as  $\psi \rightarrow 0$ .

As in the limit no directed consumers approach the high firm and no undirected consumers exist the high firm will make no profit. The low firm's price, quantity and profit can be found by taking the limit as  $\psi \rightarrow 0$  for equations 3.C.3 and 3.C.5.  $\square$

As a final point before the full game is considered note that in the special case when  $\psi \equiv 0$  (as opposed to the refined equilibrium which is a limit as  $\psi \rightarrow 0$ ) it is also possible to establish all consumers visiting the low quality firm as an equilibrium of the subgame.<sup>3.19</sup> In this case however it would be necessary to state beliefs of the high firm over the tastes of consumers who deviate to

<sup>3.19</sup>For appropriate low firm beliefs on deviating consumers it is also possible to establish an equilibrium of the subgame (with  $\psi \equiv 0$ ) where all consumers approach the high quality firm. As can be seen in the preceding analysis however this alternate equilibrium would not survive trembling hand refinement through trembling the initial market entry strategy of consumers choosing to approach the low or high firm.

visit it out of equilibrium. The assumption that the high firm could observe the taste of a deviating consumer before pricing (and assuming a finite amount of consumers so each is of positive mass) or that the high firm believes that deviating consumers are of taste 1 would be sufficient to sustain this equilibrium.

### 3.3.3 The disclosure decision

In order to ascertain whether either firm will choose to disclose quality to the market it is necessary to consider each firm's profit in the asymmetric information and full information cases. Considering the refined equilibrium where all consumers visit the low firm and the high firm makes zero profits it is clear that the high firm will not choose to disclose when  $\psi$  is low. Whilst this firm can make  $\frac{H+L}{16}$  in the asymmetric information market its profit tends to zero in the full information case as  $\psi \rightarrow 0$ . The condition for the low firm to prefer to disclose is found by comparing its profit in the full information case which tends to  $\frac{L}{4}$  as  $\psi \rightarrow 0$  to its profits in the asymmetric information case  $\frac{H+L}{16}$ . This shows that the low firm will strictly prefer to disclose if  $H < 3L$ .

**Proposition 4.** *If  $H > 3L$  then the sole refined equilibrium is one where no firm discloses and the asymmetric information equilibrium described in section 3.3.1 results. If  $H < 3L$  then the sole refined equilibrium is one where the low quality firm discloses and the full information refined equilibrium described by proposition 2 results. If  $H = 3L$  then the two aforementioned equilibria are possible.*

*Proof.* See the paragraph preceding this proposition. □

### 3.3.4 Welfare

The profits and surpluses corresponding to the refined PBEs from proposition 4 can be seen in table 3.1 along with the profits and surpluses that would result from the asymmetric information subgame. It can be seen that the full information refined equilibrium delivers lower surplus than occurs under asymmetric information. Indeed the full information refined equilibrium delivers the same surplus as would occur in a monopolistic market containing only the low quality firm.

	Full Information Refined Equilibrium	Asymmetric Information Refined Equilibrium
Average Purchased Quality	$L$	$\frac{L+H}{2}$
Low firm Profit	$\frac{L}{4}$	$\frac{L+H}{16}$
High firm Profit	0	$\frac{L+H}{16}$
Sale Quantity	$\frac{1}{2}$	$\frac{1}{2}$
Producer Surplus	$\frac{L}{4}$	$\frac{L+H}{8}$
Consumer Surplus	$\frac{L}{8}$	$\frac{L+H}{8}$
Total Surplus	$\frac{3L}{8}$	$\frac{L+H}{4}$

Table 3.1: Profits and surpluses in each equilibria

## 3.4 Extensions & Discussion

### 3.4.1 Endogenous Quality

In the benchmark model quality is exogenous. In this extension we consider the case where quality is endogenous and firms are able to choose their quality level. We assume that firms can costlessly choose any quality level  $Q \in [0, \bar{Q}]$  where  $\bar{Q}$  is some upper limit set by technology. We consider only the full information subgame where firms choose their quality level concurrently with their pricing decision. Otherwise the game proceeds as per the model of section 3.3 and we will consider only refined equilibria.

At this point we can state a proposition describing the quality choices in this setting:

**Proposition 5.** *In all possible refined equilibria of the full information subgame, both firms will choose the minimum quality level 0 and no surplus will be generated in the market.*

*Proof.* Denoting the first and second firm's qualities by  $Q_1, Q_2$ , no equilibrium can exist with  $Q_1 > Q_2 \geq 0$ . In this setting, the second firm could increase their profits by increasing their quality closer to  $Q_1$ . By symmetry no equilibrium can exist with  $Q_2 > Q_1 \geq 0$ .

In any putative equilibrium with  $Q_1 = Q_2 > 0$ , one firm can earn a discontinuous increase in profits by reducing their quality by an epsilon and disclosing qualities to the market. This possibility exists until one of the firms offers a quality of zero.

Considering the putative equilibrium where  $Q_1 = Q_2 = 0$ . Neither firm can increase their profits given the strategy of the opposing firm and this is an equilibrium.  $\square$

The intuition here is that in a mechanism similar to the Bertrand [1883] argument, firms can capture the whole market if they undercut the other firm on *quality*. This potential to earn a discontinuous increase in profits through undercutting will persist until one firm has a quality of zero and no further undercutting is possible.

### 3.4.2 Signalling

The model presented does not include the possibility of signalling. This section will discuss how the possibility of signalling can affect the outcome of the model. We will consider signalling to occur at the point of the firm (for instance price signalling) rather than occurring before the consumer enters the search market (for instance money-burning advertising).<sup>3.20</sup>

The addition of signalling at the point of the firm would be a nontrivial change to the model and would require a beliefs refinement as well as cost differences (or some other difference between

<sup>3.20</sup>This temporal relationship where signaling can occur closer to the point of purchase than disclosure is similar in spirit to the assumptions of the model of Jansson and Roy [2015]. Signalling which occurs before the consumer enters the search market would be quite similar to disclosure in our model.

high and low quality firms) to allow for signalling. For this study on disclosure however only one implication of an expanded model is important. The low quality firm would be worse off (relative to the full information refined equilibrium) in any equilibrium where consumers recognise low quality at the point of sale but do not approach firms endogenously as they are uninformed before approaching firms. Whilst in both cases the low firm is recognised as low quality, in cases where consumers approach firms randomly the low firm does not benefit from higher sale volumes. Furthermore greater equilibrium sales of the high firm means that they are selling to a wider range of high taste consumers. This results in a lower high firm price which means the low firm would lose more high taste consumers to the high firm. These forces mean that the low firm will strictly be worse off in signalling markets (with random market entry) than in the full information disclosure market (with endogenous market entry). Thus signalling could never occur even if allowed as the low quality firm would pre-emptively disclose their quality to induce the full information equilibrium presented in this paper.

On the other hand note that if the low firm were not able to disclose in this market<sup>3.21</sup> then the high quality firm would also not want to disclose even if it were possible for them to do so. Depending on the costs involved, the high firm may wish to signal at the point of the firm however so consumers recognise its high quality.<sup>3.22</sup> In this way the high firm benefits from a higher perceived value without losing customers through endogenous consumer market entry.

### 3.4.3 Multi-product Firms

The endogenous entry of consumers to the market has interesting implications in a setting of multi-product firms. Multi-product firms may also be thought to arise in part from high qualities seeking to thwart the unravelling mechanism of this paper by encouraging low taste consumers to approach them. In this extension we consider that there are two firms: one that sells both the high and low quality good (the “*both*” firm) and the other that sells only the low quality good (the “*low*” firm). We will consider only the full information subgame.

This is an interesting extension because a monopolistic screening problem appears for the both firm. The lower is the price they set for the low quality good, the more high taste consumers will opt for it cover the higher margin high quality good. In the case of endogenous market entry, the firm selling both goods will encounter fewer low taste consumers than the low quality firm that specialises in only selling this good. This ratio of high taste to low taste consumers amplifies the monopoly screening effect. The firm selling both goods will not want to price lower than the low

<sup>3.21</sup>For instance consider the case in the United States between 1907 and 1977 where bar association advertising rules prohibited lawyers from advertising.

<sup>3.22</sup>For instance while lawyer advertising was banned, large law firms invested heavily in opulent offices and amenities which may have been a mechanism to signal quality.

firm for fear of high taste consumers opting for it over the high quality good. The implication is that a firm with high quality goods trying to avert the unravelling result by also selling low quality goods will not be successful in this endeavour. Indeed in the full information subgame as  $\psi \rightarrow 0$ , the low firm still takes over the market:

**Proposition 6.** *In the unique refined equilibrium of the full information subgame, the both firm will price the low good at  $P_L + s$  where  $P_L$  is the low firm's price for this good. This means the both firm sells to all undirected consumers that visit it but low taste directed consumers still prefer to approach the low firm. As is the case in proposition 3 as  $\psi \rightarrow 0$  the ex ante indifferent consumer taste will approach 1,  $a_A \rightarrow 1$ . The both firm's profit will tend to zero and the low firm's profit to  $\frac{L}{4}$  in this refined equilibrium.*

*Proof.* The proof is included in appendix 3.D □

### 3.4.4 Many Firms

The full information equilibrium result where  $a_A \rightarrow 1$  as  $\psi \rightarrow 0$  extends to the setting of many firms. This can be seen in appendix 3.E. It also appears likely that this result would also extend to the whole game depending on how disclosure was modelled in a multifirm setting. This paper has so far modelled disclosure as one firm revealing both qualities. Whilst this is credible in the duopolistic setting as a firm can disclose the relative differences in the two goods, it does not extend easily to the multifirm setting.

A natural way in which disclosure could be added to the model would be to allow firms to choose to disclose their quality or to pool by choosing to stay silent.<sup>3.23</sup> Depending on what disclosures occur, there could be no disclosures with the asymmetric information subgame occurring, there could be all high quality or all low quality firms disclosing which would result in the full information subgame occurring, there could be some (but not all) of one group of firms disclosing resulting in a pooled quality level and a set of firms of known quality and finally there could be an information setting where three quality levels exist for low firms, high firms and an intermediate group of pooled firms.

With a sufficiently small search cost, the likely equilibrium would exhibit the low taste marginal consumer at a low quality firm indifferent to buying and leaving the market, while the low taste marginal consumer at a pooled firm is indifferent to buying and going to a low quality firm and the marginal consumer at the high quality firm is indifferent to buying and going to a pooled firm. In this setting the same unravelling result will occur as described in proposition 1. Consider a

<sup>3.23</sup>A second way that disclosure could be modelled is as firms exerting effort to shift the market's  $\psi$  value. The low firms may all choose to advertise more which leads to more informed consumers and more directed consumers. Depending on the costs of this advertising to lower  $\psi$  and other parameter values, this may be a profitable strategy for low quality firms.



split with consumers of taste  $[0, x)$  going to the low firms, consumers of taste  $[x, x')$  with  $x' > x$  going to the pooled firms and consumers of taste  $[x', 1]$  going to the disclosing high firms. In the absence of undirected consumers the high firm will want to price to make the consumer with taste  $x'$  indifferent between buying and leaving. This will lead to consumers of taste  $x'$  being better off going to a pooled firm ex ante which results in the same unravelling as discussed before. The same unravelling would also occur at the margin of the low firms and the pooled firms.<sup>3.24</sup> This will lead to low firms disclosing as pooled and high quality firms make no profits in the refined equilibria.

### 3.5 Conclusion

This paper has presented a consumer search model with two key features: consumers have heterogeneous marginal utility from quality and enter the market endogenously. It is shown that when consumers have full information in this simple setting the only equilibrium that survives refinement is one where only the low quality firms sell goods. This result comes about because the effect of quality on pricing decisions can be anticipated by consumers. The high firm will price to make one of the consumers approaching it indifferent between buying and searching further after this consumer has arrived at the firm. Anticipating this the consumer would be better off going to a low quality firm ex ante. Having lost such consumers the high quality firm will want to raise its price to make a higher taste consumer indifferent. And thus there is an unravelling result where all consumers end up shopping at the low quality firm.<sup>3.25</sup>

In an asymmetric information setting a pooling equilibrium may be sustainable if the quality of the high firm's goods are sufficiently higher than the quality of the low firm's goods. In such cases the low firm will prefer to pool with a higher perceived value of their good than disclose and get a higher quantity. On the other hand if the high firm started signalling or disclosing after consumers approach the firm then the low firm would not benefit from pooling and would be strictly better off disclosing. Thus we can offer an argument for why a high quality firm may choose costly signalling (at the point of the firm) over costless disclosure (before consumers approach firms) in search markets. The caveat is that where a high quality firm separates itself from a low quality firm through signalling there is no disincentive for the low quality firm to proactively disclose its quality. The low firm can disclose their quality and benefit from a greater number of consumers visiting the firm in equilibrium.

The analysis indicates that information provision is not necessarily harmful to a low quality

<sup>3.24</sup>The calculations for the case of unravelling with 3 levels of quality and many firms is shown in appendix 3.F.

<sup>3.25</sup>At a methodological level, consumer search models often have equilibria where no consumer searches beyond their initial firm. A further contribution of this paper is that it highlights that in such a market with heterogeneity, the assignment of consumers to their initial firm is important. In this paper's model the equilibrium induced by endogenous market entry differs markedly from the equilibrium induced by random assignment of consumers to firms (which can be seen by considering equations 3.9 to 3.14 when  $\psi = 1$ ).

firm in markets exhibiting search frictions. It may indeed be in the best interests of a low quality firm to disclose its low quality. Similarly it may not be in the interests of a high quality firm to disclose its high quality as that may deter consumers from approaching the firm for fear of high prices.

Whilst quality disclosure in this paper's model can boost a low firm's profits it is generally bad for welfare. Both consumer surplus and producer surplus are decreased by the disclosure of information to the market. In addition when quality is endogenised in the model it is shown that competition can lead to firms reducing the quality they offer consumers even when both firms can produce a good of any quality at the same cost. This implies that it may be welfare improving for regulators to impose minimal quality standards in the market. It may also be welfare improving for regulators to stop a low quality firm from voluntarily disclosing its quality to consumers until consumers are at the point of sale. This suggestion may be problematic in many cases as it could be argued firms and consumers have a moral right to voluntarily exchange truthful information before trading. On the other hand such disclosures adversely impact consumers and the high quality firm in the refined equilibrium presented in this paper. This is in addition to the issue in some markets (such as the United States legal services market) where the mechanisms for low quality disclosure produces undesirable externalities for other participants in the market.<sup>3.26</sup> Thus in certain cases it may on balance be better for a regulator or professional body to intervene to prevent low quality firm disclosures.

### 3.6 References

AboveTheLaw.com. Adventures in lawyer advertising: Battle of the hammers. <http://abovethelaw.com/2012/09/adventures-in-lawyer-advertising-battle-of-the-hammers/>, 2012.

George A. Akerlof. The market for lemons: Quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84(3):488–500, 1970.

American Bar Association. ABA aspirational goals for lawyer advertising. [http://www.americanbar.org/groups/professional\\_responsibility/resources/professionalism/professionalism\\_ethics\\_in\\_lawyer\\_advertising/abaaspirationalgoals.html](http://www.americanbar.org/groups/professional_responsibility/resources/professionalism/professionalism_ethics_in_lawyer_advertising/abaaspirationalgoals.html), 2016.

Joseph Bertrand. Book review of *theorie mathematique de la richesse sociale* and of *recherches sur les principes mathematiques de la theorie des richesses*. *Journal de Savants*, 67:499–508, 1883.

<sup>3.26</sup>For instance the American Bar Association regularly states that the antics of some advertising lawyers brings the legal profession into disrepute and it has been argued that such advertising has adversely impact the respect with which the public affords lawyers [Cebula, 1998].

- Oliver Board. Competition and disclosure. *Journal of Industrial Economics*, LVII(1):197–213, 2009.
- Fabio Caldieraro, Dongsoo Shin, and Andrew Stivers. Voluntary quality disclosure under price-signalling competition. *Managerial and decision economics*, 32(8):493–504, 2011.
- Richard J. Cebula. Does lawyer advertising adversely influence the image of lawyers in the United States? *The Journal of Legal Studies*, 27(2):503–516, 1998.
- Andrew Daughety and Jennifer Reinganum. Communicating quality: a unified model of disclosure & signalling. *RAND Journal of Economics*, 39(4):973–989, 2008.
- Peter A. Diamond. A model of price adjustment. *Journal of Economic Theory*, 3(2):156–168, 1971.
- David Dranove and Ginger Zhe Jin. Quality disclosure and certification: Theory and practice. *Journal of Economic Literature*, 48(4):935–963, 2010.
- Sanford J. Grossman. The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics*, 24(3):461–483, 1981.
- Maarten Jansson and Santanu Roy. Competition, disclosure and signalling. *The Economic Journal*, 125(582):86–114, 2015.
- Dan Levin, James Peck, and Lixin Ye. Quality disclosure and competition. *Journal of Industrial Economics*, LVII(1):167–196, 2009.
- Paul R. Milgrom. Good news and bad news: representation theorems and applications. *The Bell Journal of Economics*, 12(2):380–391, 1981.
- Reinhard Selten. A reexamination of the perfectness concept for equilibrium points in extensive games. *International Journal of Game Theory*, 4(1):25–55, 1975.
- The Economist. Why are no-frills airlines so cheap? <http://www.economist.com/blogs/economist-explains/2013/10/economist-explains-13>, 2013.
- The Telegraph. 48 reasons why you don't hear much from Michael O'Leary anymore. <http://www.telegraph.co.uk/travel/lists/Michael-OLearys-most-memorable-quotes/>, 2016.
- George Trebbi, Thomas Hayes, and Mary Walker. Characteristics of lawyers portrayed in television advertising: The effects of preference, client-role and advertisement type. *Journal of Professional Services Marketing*, 19(1):73–89, 1999.
- US Supreme Court. *Bates v. State Bar of Arizona*, 1977.
- Bryan Wilson. Texas law hawk commercials. <http://texaslawhawk.com/commercials/>, 2016.

## Appendices

### 3.A Proof of lemma 3

To prove lemma 3 we will show first that given sufficiently small search costs, no equilibrium can satisfy  $\frac{P_H}{H} = \frac{P_L}{L}$  before showing that given small search costs no equilibrium can exhibit  $\frac{P_H}{H} < \frac{P_L}{L}$ .

**Lemma 4.** *For a sufficiently low search cost, there are no equilibriums where  $\frac{P_H}{H} = \frac{P_L}{L}$ :*

*Proof.* In the first case note that in an equilibrium with  $\frac{P_H}{H} = \frac{P_L}{L} = 0$  then neither firm is making any profits. The high firm could earn positive profits by setting a small price and selling to high taste consumers.

In the second case we consider a putative equilibria where  $\frac{P_H}{H} = \frac{P_L}{L} > 0$ . We will use the notation  $x = \frac{P_H}{H} = \frac{P_L}{L}$  and note that  $a_A = x$  and the low firm gets all consumers with a lower taste and the high firm gets all consumers with a higher taste. At first we will assume  $a_T \leq 1$ . Using the demand function (3.4) we can write the profit for the low firm pricing at  $xL$ :

$$\begin{aligned}\pi_{\text{Low}}(xL) &= xL \frac{\psi}{2} \left[ x + \frac{s}{H-L} - x \right] \\ &= \frac{sxL\psi}{2(H-L)}\end{aligned}\tag{3.A.1}$$

where all sales are to undirected consumers that visit the low firm initially. Note that sales are entirely dependent on the extent of the search cost  $s$ . Now if the low firm instead reduced their price to  $xL - \epsilon$  where  $xL > \epsilon > 0$  then their profit would be:

$$\begin{aligned}\pi_{\text{Low}}(xL - \epsilon) &= [xL - \epsilon] \left[ \frac{\psi}{2} \left[ x + \frac{\epsilon + s}{H-L} - (x - \frac{\epsilon}{L}) \right] + (1 - \psi) \left[ x - (x - \frac{\epsilon}{L}) \right] \right] \\ &= xL \left[ \frac{\psi(\epsilon + s)}{2(H-L)} + \frac{\epsilon}{L} \left(1 - \frac{\psi}{2}\right) \right] - \epsilon \left[ \frac{\psi(\epsilon + s)}{2(H-L)} + \frac{\epsilon}{L} \left(1 - \frac{\psi}{2}\right) \right] \\ &= \pi_{\text{Low}}(xL) + xL \left[ \frac{\psi(\epsilon)}{2(H-L)} + \frac{\epsilon}{L} \left(1 - \frac{\psi}{2}\right) \right] - \epsilon \left[ \frac{\psi(\epsilon + s)}{2(H-L)} + \frac{\epsilon}{L} \left(1 - \frac{\psi}{2}\right) \right]\end{aligned}$$

Where sales are to undirected consumers who visit the firm initially as well as directed consumers with a taste less than  $a_A$ . As  $xL > \epsilon$  we will have  $xL \left[ \frac{\psi(\epsilon)}{2(H-L)} + \frac{\epsilon}{L} \left(1 - \frac{\psi}{2}\right) \right] - \epsilon \left[ \frac{\psi(\epsilon + s)}{2(H-L)} + \frac{\epsilon}{L} \left(1 - \frac{\psi}{2}\right) \right]$  positive for a sufficiently small  $s$ .

Now considering the case where  $a_T > 1$ . Using the demand function (3.4) we can write the profit for the low firm pricing at  $xL$ :

$$\begin{aligned}\pi_{\text{Low}}(xL) &= xL \frac{\psi}{2} [1 - x] \\ &= \frac{xL\psi}{2} (1 - x)\end{aligned}$$

where all sales are to undirected consumers that visit the low firm initially. Note that as  $a_T > 1$

then  $(1-x) < \frac{s}{H-L}$ . Now if the low firm instead reduced their price to  $xL - \epsilon$  where  $xL > \epsilon > 0$  and  $\epsilon$  is sufficiently small that  $a_T$  is still greater than one then their profit would be:

$$\begin{aligned}\pi_{\text{Low}}(xL - \epsilon) &= [xL - \epsilon] \left[ \frac{\psi}{2} \left[ 1 - \left(x - \frac{\epsilon}{L}\right) \right] + (1 - \psi) \left[ x - \left(x - \frac{\epsilon}{L}\right) \right] \right] \\ &= [xL - \epsilon] \left[ \frac{\psi}{2}(1-x) + \left(1 - \frac{\psi}{2}\right) \frac{\epsilon}{L} \right] \\ &= \pi_{\text{Low}}(xL) + xL \left[ \left(1 - \frac{\psi}{2}\right) \frac{\epsilon}{L} \right] - \epsilon \left[ \frac{\psi}{2}(1-x) + \left(1 - \frac{\psi}{2}\right) \frac{\epsilon}{L} \right]\end{aligned}$$

Where sales are to undirected consumers who visit the firm initially as well as directed consumers with a taste less than  $a_A$ . As  $(1-x)$  is small for a sufficiently small  $s$  and  $xL > \epsilon$  we will get  $xL \left[ \left(1 - \frac{\psi}{2}\right) \frac{\epsilon}{L} \right] - \epsilon \left[ \frac{\psi}{2}(1-x) + \left(1 - \frac{\psi}{2}\right) \frac{\epsilon}{L} \right]$  as positive and this is a profitable deviation.  $\square$

**Lemma 5.** *For a sufficiently low search cost, there are no equilibria where  $\frac{P_H}{H} < \frac{P_L}{L}$ :*

*Proof.* Suppose that  $\frac{P_H}{H} < \frac{P_L}{L}$ . We will denote  $x_L = \frac{P_L}{L}$  and  $x_H = \frac{P_H}{H}$ . In this case  $a_A = \frac{Hx_H - Lx_L}{H-L}$ .

We can note this implies we must have  $a_T < 1$  if the search cost is small. This is because  $a_T > 1$  would imply:

$$\begin{aligned}\frac{Hx_H - Lx_L + s}{H-L} &> 1 \\ L(1-x_L) + s &> H(1-x_H)\end{aligned}$$

Now as  $H > L$  and  $(1-x_H) > (1-x_L)$ , it is not possible for this to hold if the search cost is sufficiently small.

Now examining the case where  $a_T \leq 1$ , the case profit for the low firm is:

$$\begin{aligned}\pi_{\text{Low}}(x_L L) &= x_L L \left[ \frac{\psi}{2} \left[ \frac{Hx_H - Lx_L + s}{H-L} - x_L \right]_+ \right] \\ \pi_{\text{Low}}(x_L L) &= \frac{\psi x_L L}{2} \left[ \frac{H(x_H - x_L) + s}{H-L} \right]_+\end{aligned}$$

Where the firm only sells to undirected consumers that approach it initially if the search cost is sufficiently high (as  $x_H < x_L$ ). Note for a sufficiently small  $s$  this profit is zero. Now if the low firm lowers its price to  $x_L L - L\epsilon$ ,<sup>3.27</sup> it gets a profit of:

$$\begin{aligned}\pi_{\text{Low}}(x_L L - L\epsilon) &= (x_H L - \epsilon L) \frac{\psi}{2} \left[ \frac{x_H(H-L) + \epsilon L + s}{H-L} - (x_H - \epsilon L) \right] \\ \pi_{\text{Low}}(x_L L - L\epsilon) &= \frac{\psi(x_H L - \epsilon L)}{2} \left[ \frac{\epsilon L + s}{H-L} \right]\end{aligned}$$

Which is greater than  $\pi_{\text{Low}}(x_L L)$  for a sufficiently small value of  $s$ .  $\square$

<sup>3.27</sup>Note that this is always possible unless  $x_H = 0$  which is not possible from lemma 4.

### 3.B Equilibrium in a competitive market

In this appendix the benchmark model will be analysed in the absence of search frictions. All consumers buy one good knowing the two prices. There is no search cost and as a result the concept of directed/undirected consumers is not used here.

With similar logic as earlier in this paper it can be noted that the set of consumers who will buy from a given firm will be a convex set. It can also be noted that the high firm will not sell to all consumers as this would require a price of 0 which would result in no profits.

We will use the notation of  $[0, a_L]$  to describe the consumers who will not buy but will take the outside option;  $[a_L, a_A]$  is the set of consumers who will buy from the low firm and  $[a_A, 1]$  is the set of consumers who will buy from the high firm. Considering the condition for a consumer to be indifferent between the high and low firm's offering we get  $a_A = \frac{P_H - P_L}{H - L}$ . Considering a consumer indifferent between buying at the low firm and the outside option yields  $a_L = \frac{P_L}{L}$ . These can be used to write the firm profit functions:

$$\pi_H(P_L) = \max_{P_H} P_H \left[ 1 - \left[ \frac{P_H - P_L}{H - L} \right]_+ \right]_+ \quad (3.B.1)$$

$$\pi_L(P_L) = \max_{P_L} P_L \left[ \left[ \frac{P_H - P_L}{H - L} \right]_+ - \left[ \frac{P_L}{L} \right]_+ \right]_+ \quad (3.B.2)$$

Taking first order conditions of equations 3.B.1 and 3.B.2 yields the reaction functions  $P_H(P_L) = \frac{H - L + P_L}{2}$  and  $P_L(P_H) = \frac{LP_H}{2}$  which can be used to find the equilibrium price and profits:

$$P_L = \frac{L(H - L)}{4H - L} \quad P_H = \frac{2H(H - L)}{4H - L} \quad (3.B.3)$$

$$\pi_{L, \text{Comp}} = \frac{HL(H - L)}{(4H - L)^2} \quad \pi_{H, \text{Comp}} = \frac{4H^2(H - L)}{(4H - L)^2} \quad (3.B.4)$$

Examining the profit expressions (3.B.4), we can note in this setting the high firm's profit is always strictly higher than low firm's profit by a factor of  $\frac{4H}{L}$ .

Comparing the high firms profit in the search model (3.11) with no directed consumers ( $\psi = 1$ ) to the high firm profit in the competitive model (3.B.1) we find that in the search case profits are  $\frac{(3H-s)^2(4H-L)^2}{2H^2(9H-4L)^2}$  times higher. Taking the limit as  $s \rightarrow 0$  this ratio translates to  $\frac{9(4H-L)^2}{2(9H-4L)^2} = \left[ \frac{3(4H-L)}{\sqrt{2}(9H-4L)} \right]^2$ , If  $\frac{H}{L} \leq \frac{4\sqrt{2}-3}{9\sqrt{2}-12} \approx 3.65$  then this profit ratio is greater than one. Now examining the ratio of low firm profits it can be calculated that the low firms profits are  $\frac{(4L-3s)^2(4H-L)^2}{2L^2(9H-4L)^2}$ . In the limit as  $s \rightarrow 0$  this ratio approaches  $\left[ \frac{8\sqrt{2}H-2\sqrt{2}L}{9H-4L} \right]^2$ . This is more than one if  $\frac{H}{L} > \frac{2\sqrt{2}-4}{8\sqrt{2}-9} \approx -0.5$  which is satisfied.

### 3.C Derivation of Equilibrium when $a_T > 1$

We derive the equilibrium in the case when  $a_T > 1$  which is likely when  $\psi$  is low and  $a_A$  is close to 1. First rewriting  $Q_{\text{Low}}(P_L)$  (equation 3.7):

$$Q_{\text{Low}}(P_L) = \underbrace{\frac{\psi}{2} [1 - a_L(P_L)]}_{\text{Undirected - own}} + \underbrace{\frac{\psi}{2} \left[ a_A - \frac{s}{H-L} - a_{HLM} \right]}_{\text{Undirected Runoff from the } H \text{ firm}} + \underbrace{(1 - \psi) [a_A - a_L(P_L)]}_{\text{Directed}} \quad (3.C.1)$$

With the same solution steps as in section 3.3.2 we can derive the following expressions:

$$P_H = \frac{2H^2\psi - 8H^2 - 4HL\psi + 12HL + 2H\psi s + 2L^2\psi - 4L^2}{H\psi^2 - 2H\psi - 8H + 8L} \quad (3.C.2)$$

$$P_L = \frac{-HL\psi^2 - 4HL + H\psi^2 s + 2H\psi s + L^2\psi^2 + 4L^2}{H\psi^2 - 2H\psi - 8H + 8L} \quad (3.C.3)$$

$$\pi_H = \frac{2\psi(H^2\psi - 4H^2 - 2HL\psi + 6HL + H\psi s + L^2\psi - 2L^2)^2}{(H-L)(H\psi^2 - 2H\psi - 8H + 8L)^2} \quad (3.C.4)$$

$$\pi_L = \frac{(2-\psi)(-HL\psi^2 - 4HL + H\psi^2 s + 2H\psi s + L^2\psi^2 + 4L^2)^2}{2L(H\psi^2 - 2H\psi - 8H + 8L)^2} \quad (3.C.5)$$

$$a_A = \frac{8H^2 - 2H^2\psi - HL\psi^2 + 4HL\psi - 16HL + 2H\psi^2 s - 2H\psi s - 8Hs + L^2\psi^2 - 2L^2\psi + 8L^2 + 8Ls}{(H-L)(8(H-L) + 2H\psi - H\psi^2)} \quad (3.C.6)$$

$$a_T = \frac{8H^2 - 2H^2\psi - HL\psi^2 + 4HL\psi - 16HL + 2H\psi^2 s - 2H\psi s - 8Hs + L^2\psi^2 - 2L^2\psi + 8L^2 + 8Ls}{(H-L)(8(H-L) + 2H\psi - H\psi^2)} \quad (3.C.7)$$

### 3.D Multiproduct firms

Now consider that there is one firm that sells both high and low quality goods (the “both” firm) and one that only sells the low quality firm (the “low” firm).  $P_H$  is the price of the high good (sold by the both firm),  $P_B$  is the price of the low good from the both firm and  $P_L$  is the price of the low good from the low firm. The profits of the low and both firm are given by  $\pi_L$  and  $\pi_B$  respectively. We will consider only the full information subgame. We will redefine the ex ante indifferent consumer's taste of  $a_A$  to be:

$$a_A = \frac{P_H - \min(P_L, P_B)}{H - L} \quad (3.D.1)$$

We will use  $\lambda$  to denote the fraction of directed consumers with tastes lower than  $a_A$  that go to the low firm upon entering the market. Clearly  $\lambda = 1$  if in an equilibrium  $P_L < P_B$  and  $\lambda = 0$  if  $P_L > P_B$ . A consumer indifferent between buying and leaving a firm will choose to buy at the current firm.

We can note that no equilibrium can take place where  $P_B > P_L + s$ . In this case the both firm could reduce their price to  $P_L + s$  and sell to low taste undirected consumers at the both firm while the outside option for these consumers (of buying the low good at total cost  $P_L + s$ ) would stay the same. There can also be no equilibrium where  $P_L > P_B + s$ . In this case the low firm would sell

no quantity and make no profit. They could make strictly positive profits by pricing at another level (for instance this firm can guarantee a positive profit by selling to undirected consumers at a price of  $\frac{s}{2}$ ).

Now writing the firm profit functions for a putative equilibrium where  $|P_B - P_L| \leq s$ . In this case a consumer will never leave one firm to buy the low quality good from the other. The profit functions for each firm can be written as:

$$\pi_L = \max_{P_L} \overbrace{\frac{\psi}{2} P_L \left[ \min\left(1, \frac{P_H^E - P_L + s}{H - L}\right) - \frac{P_L}{L}\right]}^{\text{Undirected}} + \overbrace{(1 - \psi)\lambda P_L \left[ a_A - \frac{P_L}{L} \right]}^{\text{Directed}} \quad (3.D.2)$$

$$\begin{aligned} \pi_B = \max_{P_B, P_H} \overbrace{\frac{\psi}{2} \left[ P_B \left( \frac{P_H - P_B}{H - L} - \frac{P_B}{L} \right) + P_H \left( 1 - \frac{P_H - P_B}{H - L} \right) \right]}^{\text{Undirected}} \\ + \overbrace{(1 - \psi) \left[ (1 - \lambda) P_B \left( a_A - \frac{P_B}{L} \right) + P_H (1 - a_A) \right]}^{\text{Directed}} + \overbrace{\frac{\psi}{2} P_H \left[ 1 - \left( a_A + \frac{s}{H - L} \right) \right]}^{\text{High taste runoff from low firm}} \end{aligned} \quad (3.D.3)$$

At this point it can be seen that intuitively as the both firm lowers the price of the low quality good (within  $P - s \leq P_B \leq P_L + s$ ) it reduces the amount of the high quality good it can sell. The low firm faces no such problem. This leads to the following lemma:

**Lemma 6.** *For a small search cost and a nonzero fraction of undirected consumers, no equilibrium exists where  $P_B \leq P_L$ .*

*Proof.* We will show this by supposing the contrary such that  $P_B \leq P_L$  and hence  $a_A = \frac{P_H - P_B}{H - L}$ . We will first guess that in equilibrium  $\frac{P_H - P_L + s}{H - L} < 1$  Taking first order conditions of equations 3.D.2, 3.D.3 and rearranging

$$P_L = \frac{L(-2Ha_A\lambda\psi + 2Ha_A\lambda + 2La_A\lambda\psi - 2La_A\lambda + P_H^E\psi + \psi s)}{4H\lambda - 4H\lambda\psi + 2H\psi + 4L\lambda\psi - 4L\lambda} \quad (3.D.4)$$

$$P_H = \frac{H}{2} - \frac{L}{2} + p_B - \frac{1}{2\psi} (\psi(-H + L + a_A(H - L) + s) - 2(H - L)(a_A - 1)(\psi - 1)) \quad (3.D.5)$$

$$P_B = \frac{L(Ha_A\lambda\psi - Ha_A\lambda - Ha_A\psi + Ha_A - La_A\lambda\psi + La_A\lambda + La_A\psi - La_A + P_H^E\psi)}{2H\lambda\psi - 2H\lambda - H\psi + 2H - 2L\lambda\psi + 2L\lambda + 2L\psi - 2L} \quad (3.D.6)$$

Substituting in that in equilibrium  $P_H^E = P_H$  and  $a_A$  must satisfy  $a_A = \frac{P_H - P_B}{H - L}$  and solving these equations yields expressions for the prices of the putative equilibrium:<sup>3.28</sup>

$$P_H = \frac{(2H - 2L - \psi s)(2H\lambda\psi - 2H\lambda - H\psi + 2H - L\lambda\psi + L\lambda + L\psi - L)}{(H - L)(\psi + 2)(2\lambda\psi - 2\lambda - \psi + 2)} \quad (3.D.7)$$

$$P_B = \frac{L(2H - 2L - \psi s)(\lambda\psi - \lambda + 1)}{(H - L)(\psi + 2)(2\lambda\psi - 2\lambda - \psi + 2)} \quad (3.D.8)$$

$$a_A = \frac{2H - 2L - \psi s}{(H - L)(\psi + 2)} \quad (3.D.9)$$

<sup>3.28</sup>The expression for  $P_L$  is omitted as it is long but it can be found by substituting equations 3.D.9 and 3.D.7 into equation 3.D.4.



Now taking the difference between  $P_L$  and  $P_B$  and taking  $s$  to zero in the limit. This should be nonnegative by assumption.

$$P_L - P_B = \frac{L\psi(2H\lambda\psi - 2H\lambda - H\psi - 3L\lambda\psi + 3L\lambda + L\psi - L)}{(\psi + 2)(2\lambda\psi - 2\lambda - \psi + 2)(2H\lambda - 2H\lambda\psi + H\psi + 2L\lambda\psi - 2L\lambda)} \quad (3.D.10)$$

First considering the case where  $P_L > P_B$  and hence  $\lambda = 0$  this difference should be strictly positive. In this case it simplifies to:

$$P_L - P_B = \frac{L(-L - (H - L)\psi)}{H(2 - \psi)(\psi + 2)} \quad (3.D.11)$$

Which is strictly negative. Thus we cannot have an equilibrium where  $P_L > P_B$  for a small search cost. Now considering the other case where  $P_L = P_B$  and hence  $1 \geq \lambda \geq 0$ . The difference in equation 3.D.10 must be equal to zero. This is the case if either  $\psi = 0$ <sup>3.29</sup> or  $(2H\lambda\psi - 2H\lambda - H\psi - 3L\lambda\psi + 3L\lambda + L\psi - L) = 0$ . This second condition can only hold if  $\lambda = \frac{(H-L)\psi+L}{(3L-2H)(1-\psi)}$ . The numerator of this equation is always greater than  $L$  and the denominator is always less than  $L$ . As  $\lambda$  can only be in the range  $[0, 1]$  this indicates there is no equilibrium with equal low quality good prices.

Now we guess that  $\frac{P_H - P_L + s}{H - L} \geq 1$ . Taking first order conditions of equation 3.D.2 and rearranging:

$$P_L = \frac{L(2a_A\lambda\psi - 2a_A\lambda - \psi)}{4\lambda\psi - 4\lambda - 2\psi} \quad (3.D.12)$$

With the reaction functions for  $P_H$  and  $P_B$  being identical to equations 3.D.5 and 3.D.6. As  $P_L$  does not enter these other reaction functions the prices  $P_H, P_L$  and  $a_A$  in this equilibrium are identical to those given in equations 3.D.7, 3.D.8 and 3.D.9 respectively.  $P_L$  can be found by substituting these expressions into equation 3.D.12. Now taking the difference between  $P_L$  and  $P_B$  and taking  $s$  to zero in the limit. This should be nonnegative by assumption.

$$P_L - P_B = -\frac{L\psi(\psi^2 + 4\lambda - 2\lambda\psi^2 - 2\lambda\psi)}{2(\psi + 2)(\psi + 2\lambda(1 - \psi))(2 - 2\lambda(1 - \psi) - \psi)} \quad (3.D.13)$$

Which is negative for all  $\lambda$ . Hence we cannot have an equilibrium where  $\frac{P_H - P_L + s}{H - L} \geq 1$  and  $P_B \leq P_L$ .  $\square$

With  $\lambda = 0$  the first order condition for the both firm's low quality good pricing decision becomes:

$$\frac{\partial \pi_B}{\partial P_B} = \psi \left[ \frac{P_H - P_B}{H - L} - \frac{P_B}{L} \right] \quad (3.D.14)$$

<sup>3.29</sup>This can be seen intuitively by substituting  $\psi = 0$  into equations 3.D.2 and 3.D.3 and taking the first order conditions which are scalar multiples of each other.

Which will be positive for a small search cost and  $P_B < L$ .<sup>3.30</sup> This implies that the optimal  $P_B$  will be equal to  $P_L + s$  which is the top of the domain of possible prices satisfying  $|P_B - P_L| \leq s$ . Note that if the both firm posted a higher price they would experience a discontinuous in sales while the outside option of their consumers would stay the same.

Now using  $\lambda = 1$ ,  $a_A = \frac{P_H - P_L}{H - L}$  and  $P_B = P_L + s$  and using first order conditions to solve for equilibrium quantities for the case where  $\frac{P_H - P_L + s}{H - L} < 1$  as:

$$P_H = \frac{4H^2\psi - 8H^2 - 8HL\psi + 12HL + 2H\psi^2s - 4H\psi s + 4L^2\psi - 4L^2 - 3L\psi^2s}{(\psi + 2)(2H\psi - 4H - 3L\psi + 4L)} \quad (3.D.15)$$

$$P_L = \frac{2L(-H\psi + 2H + L\psi - 2L + 2\psi s)}{(\psi + 2)(4H - 2H\psi + 3L\psi - 4L)} \quad (3.D.16)$$

$$P_B = \frac{2HL\psi - 4HL + 2H\psi^2s - 8Hs - 2L^2\psi + 4L^2 - 3L\psi^2s - 6L\psi s + 8Ls}{(\psi + 2)(2H\psi - 4H - 3L\psi + 4L)} \quad (3.D.17)$$

$$a_A = \frac{2H - 2L + \psi s}{(H - L)(\psi + 2)} \quad (3.D.18)$$

Where  $a_A$  approaches 1 as  $\psi \rightarrow 0$ . This will imply that at some level of  $\psi$  we have  $\frac{P_H - P_L + s}{H - L} \geq 1$ . Now considering this case we can get the equilibrium quantities:

$$P_H = \frac{4H^2\psi - 8H^2 - HL\psi^2 - 6HL\psi + 12HL + 2H\psi^2s - 4H\psi s + L^2\psi^2 + 2L^2\psi - 4L^2 + 2L\psi s}{2(H - L)(\psi - 2)(\psi + 2)} \quad (3.D.19)$$

$$P_L = \frac{L(-H\psi^2 + 2H\psi - 4H + L\psi^2 - 2L\psi + 4L + 2\psi^2s - 2\psi s)}{2(H - L)(\psi - 2)(\psi + 2)} \quad (3.D.20)$$

$$P_B = \frac{(-HL\psi^2 + 2HL\psi - 4HL + 2H\psi^2s - 8Hs + L^2\psi^2 - 2L^2\psi + 4L^2 - 2L\psi s + 8Ls)}{2(H - L)(\psi - 2)(\psi + 2)} \quad (3.D.21)$$

$$a_A = \frac{2H - 2L + \psi s}{(H - L)(\psi + 2)} \quad (3.D.22)$$

Where again  $a_A$  approaches 1 as  $\psi \rightarrow 0$ .

### 3.E For Online Publication: Many Firms in a full information consumer search

There are  $h + l$  firms in a market,  $h$  of which sell a product of quality  $H$  (the high firms) and  $l$  of which sell goods with a lower quality of  $L$  (the low firms), where  $H > L$ . We assume  $h > 1$  and  $l > 1$  and that all firms produce their goods costlessly. We make the assumption that all firms of a particular quality level sell at the same price. This is done in order for this analysis to arrive at a similar equilibrium to that of section 3.3.2. With these assumption lemmas 1, corollary 2 and lemma 3 carry over with the only modification that directed consumers approach a quality

<sup>3.30</sup>This can be seen by substituting taking the first order condition for the optimal  $P_H$  (identical to equation 3.D.5) and substituting this into  $\frac{P_H - P_B}{H - L} - \frac{P_B}{L}$  which simplifies to  $\frac{1}{\psi} - \frac{P_B}{L} + \frac{a_A}{2} - \frac{s}{2(H - L)}$  which is positive as  $\frac{1}{\psi} > 1 > \frac{P_B}{L}$  and  $s$  is small.

level rather than a specific firm. As before we will denote the ex ante indifferent consumer  $a_A$  so consumers with a lower taste approach a low firm and higher taste consumers approach high firms.

$$Q_{\text{High}}(P_H) = \underbrace{\frac{\psi}{h+l} [1 - \max(a_{HL}(P_H), a_H(P_H))]_+}_{\text{Undirected - own}} + \underbrace{\frac{\psi l}{h(h+l)} \left[ 1 - \max\left(a_A + \frac{s}{H-L}, a_{HL}(P_H), a_H(P_H)\right) \right]_+}_{\text{Undirected runoff from the L firm}} + \underbrace{\frac{1-\psi}{h} [1 - \max(a_A, a_{HL}(P_H), a_H(P_H))]_+}_{\text{Directed}} \quad (3.E.1)$$

The demand function for the low firm can be written as:

$$Q_{\text{Low}}(P_L) = \underbrace{\frac{\psi}{h+l} [\min(1, a_T(P_L)) - a_L(P_L)]_+}_{\text{Undirected - own}} + \underbrace{\frac{\psi l}{h(h+l)} \left[ \min\left(a_A - \frac{s}{H-L}, a_T(P_L)\right) - \max(a_{HLM}, a_L(P_L)) \right]_+}_{\text{Undirected Runoff from the H firm}} + \underbrace{\frac{1-\psi}{h} [\min(a_A, a_T(P_L)) - a_L(P_L)]_+}_{\text{Directed}} \quad (3.E.2)$$

As before we examine the case when the search costs are sufficiently low that the high firm's marginal consumer is indifferent to the low firm.

$$Q_{\text{High}}(P_H) = \underbrace{\frac{\psi}{h+l} [1 - a_H(P_H)]_+}_{\text{Undirected - own}} + \underbrace{\frac{\psi l}{h(h+l)} \left[ 1 - \left(a_A + \frac{s}{H-L}\right) \right]_+}_{\text{Undirected runoff from the L firm}} + \underbrace{\frac{1-\psi}{h+l} [1 - a_A]}_{\text{Directed}} \quad (3.E.3)$$

### Case A: $a_T < 1$

Now assuming  $a_T < 1$  we can rewrite the low firm's demand function as:

$$Q_{\text{Low}}(P_L) = \frac{\psi}{2} [a_T(P_L) - a_L(P_L)] + \frac{\psi}{2} \left[ a_A - \frac{s}{H-L} - a_{HLM} \right] \quad (3.E.4)$$

And we can solve to get:

$$\begin{aligned} P_H &= \frac{2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 + HLh^2\psi - 3HLh^2 + HLhl\psi^2 - 4HLhl\psi - HLl^2\psi^2 + 2Hh^2\psi s + Hhl\psi^2s - 3Hhl\psi s - 2Hl^2\psi^2s - L^2h^2\psi}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2} \\ &\quad + \frac{L^2h^2 - L^2hl\psi^2 + L^2hl\psi + Lh^2\psi^2s + Lhl\psi s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2} \\ P_L &= \frac{HLh^2 + 2HLhl\psi + HLL^2\psi^2 - Hhl\psi^2s - Hhl\psi s - Hl^2\psi^2s - L^2h^2 - 2L^2hl\psi - L^2l^2\psi^2 + Lh^2\psi^2s + 2Lh^2\psi s + 2Lhl\psi^2s - Lhl\psi s - Ll^2\psi^2s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2} \\ \pi_H &= \frac{\psi}{(H-L)(h+l) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2 \right)^2} \left[ 2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 \right. \\ &\quad \left. + HLh^2\psi + HLhl\psi^2 - 3HLh^2 - 4HLhl\psi - HLl^2\psi^2 + 2Hh^2\psi s + Hhl\psi^2s - 3Hhl\psi s - 2Hl^2\psi^2s - L^2h^2\psi + L^2h^2 - L^2hl\psi^2 \right. \\ &\quad \left. + L^2hl\psi + Lh^2\psi^2s + Lhl\psi s \right]^2 \\ \pi_L &= \frac{(H+L\psi-L)}{L(H-L)(h+l) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2 \right)^2} \left[ HLh^2 \right. \\ &\quad \left. + 2HLhl\psi + HLL^2\psi^2 - Hhl\psi^2s - Hhl\psi s - Hl^2\psi^2s - L^2h^2 - 2L^2hl\psi - L^2l^2\psi^2Lh^2\psi^2s + 2Lh^2\psi s + 2Lhl\psi^2s - Lhl\psi s - Ll^2\psi^2s \right]^2 \\ a_A &= \frac{2Hh^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2 + 2h^2\psi s + 2hl\psi^2s - 2hl\psi s - l^2\psi^2s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2} \\ a_T &= \frac{H^2l\psi(3h+l\psi) - HLh^2(4-\psi) + 2H^2h^2 - HLhl\psi(6-\psi) - 2H(Ll^2\psi^2 + h^2s) + Hhl\psi s(\psi-5) - \psi(2Hl^2\psi s + L^2h^2)}{(H-L) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2 \right)} \\ &\quad + \frac{L^2h(2h-l\psi^2) + L^2l\psi(3h+l\psi) - Lh^2\psi s(\psi+2) - Lhs(3l\psi^2 - 2h) + Ll\psi s(5H+2l\psi)}{(H-L) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + Hl^2\psi^2 + Lh^2\psi^2 - 2Lh^2 + Lhl\psi^2 - 3Lhl\psi - Ll^2\psi^2 \right)} \end{aligned} \quad (3.E.5)$$

**Case B:**  $a_T \geq 1$

Now assuming  $a_T \geq 1$  we can rewrite the low firm's demand function as:

$$Q_{\text{Low}}(P_L) = \frac{\psi}{2} \overbrace{[1 - a_L(P_L)]}^{\text{Undirected - own}} + \frac{\psi}{2} \overbrace{\left[ a_A - \frac{s}{H-L} - a_{HLM} \right]}^{\text{Undirected Runoff from the } H \text{ firm}} \quad (3.E.6)$$

And we can solve to get:

$$\begin{aligned} P_H &= \frac{2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 - 3HLh^2 - 4HLhl\psi - HLI^2\psi^2 + 2Hh^2\psi_s + Hhl\psi^2_s - 3Hhl\psi_s - 2HI^2\psi^2_s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2} \\ &\quad + \frac{L^2h^2 + L^2hl\psi - Lh^2\psi^2_s - Lh^2\psi_s + Lhl\psi^2_s + Lhl\psi_s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2} \\ P_L &= \frac{HLh^2\psi^2 + HLh^2 + 2HLhl\psi + HLI^2\psi^2 - Hhl\psi^2_s - Hhl\psi_s - HI^2\psi^2_s - L^2h^2\psi^2 - L^2h^2}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2} \\ &\quad + \frac{Lh^2\psi_s - 2L^2hl\psi - L^2l^2\psi^2 - Lh^2\psi^2_s + 2Lhl\psi^2_s - Lhl\psi_s - LI^2\psi^2_s}{2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2} \\ \pi_H &= \frac{\psi}{(H-L)(h+l) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2 \right)^2} \left[ 2H^2h^2 + 3H^2hl\psi - 3HLh^2 \right. \\ &\quad \left. + H^2l^2\psi^2 - 4HLhl\psi - HLI^2\psi^2 + 2Hh^2\psi_s + Hhl\psi^2_s - 3Hhl\psi_s - 2HI^2\psi^2_s + L^2h^2 + L^2hl\psi - Lh^2\psi^2_s - Lh^2\psi_s + Lhl\psi^2_s + Lhl\psi_s \right]^2 \\ \pi_L &= \frac{1}{L(h+l) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2 \right)^2} \left[ Hhl\psi^2_s - HLh^2\psi^2 - HLh^2 \right. \\ &\quad \left. - 2HLhl\psi - HLI^2\psi^2 + Hhl\psi_s + HI^2\psi^2_s + L^2h^2\psi^2 + L^2h^2 + 2L^2hl\psi + L^2l^2\psi^2 + Lh^2\psi^2_s - Lh^2\psi_s - 2Lhl\psi^2_s + Lhl\psi_s + LI^2\psi^2_s \right]^2 \\ a_A &= \frac{2H^2h^2 + 3H^2hl\psi + H^2l^2\psi^2 - HLh^2\psi^2 - 4HLh^2 - 6HLhl\psi - 2HLI^2\psi^2 + 2Hh^2\psi_s + 2Hhl\psi^2_s - 2Hhl\psi_s - HI^2\psi^2_s}{(H-L) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2 \right)} \quad (3.E.7) \\ &\quad + \frac{L^2h^2\psi^2 + 2L^2h^2 + 3L^2hl\psi + L^2l^2\psi^2 - 2Lh^2\psi_s - Lhl\psi^2_s + 2Lhl\psi_s + LI^2\psi^2_s}{(H-L) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2 \right)} \\ a_T &= \frac{H\psi^2(HI^2 - Lh^2) - HLh(4h + 6l\psi) + H^2h(2h + 3l\psi) - 2H(LI^2\psi^2 + h^2s) - Hhl\psi_s(5 - \psi) - 2HI^2\psi^2_s + L^2h^2\psi^2 + 2L^2h^2 + 3L^2hl\psi}{(H-L) \left( 2Hh^2\psi + 2Hh^2 + Hhl\psi^2 + 3Hhl\psi + HI^2\psi^2 - Lh^2\psi^2 - Lh^2\psi - 2Lh^2 - 3Lhl\psi - LI^2\psi^2 \right)} \end{aligned}$$

**As**  $\psi \rightarrow 0$

As  $\psi \rightarrow 0$  we get  $a_A$  as defined by equation 3.E.5 approaching 1.<sup>3.31</sup> This indicates that as  $\psi \rightarrow 0$  we get  $a_T$  going above  $a_A$  and hence the second case will arise. As  $\psi \rightarrow 0$  we get  $a_A$  as defined by equation 3.E.7 approaching 1.

Thus in a similar result to proposition 3, the low quality firms take over the market in the full information as the number of directed consumers increases.

### 3.F For Online Publication: Three levels of quality in the market

The top firm has a quality of  $H$ . The middle firm has a quality of  $M$  and the lower firm has a quality of  $L$ . The ex ante indifferent consumers between the low and middle quality firms have a taste denoted  $a_A$  while the ex ante indifferent consumers between the high and middle quality firms have a taste denoted  $a_{A2}$ . There are  $\gamma$  firms in total,  $\beta$  lower quality firms and  $\alpha$  middle quality firms and so  $\gamma - \alpha - \beta$  high quality firms.

By setting up expressions similar to equation 3.1 and rearranging, we can derive expressions for the indifferent taste consumers at each firm. These can be seen in table 3.F.1:

<sup>3.31</sup>This can be seen by substituting  $\psi = 0$  into the equation and noting the feasible range of  $a_A$  is the closed interval between 0 and 1.

Notation	Location of consumer	Indifferent between	Formula
$a_A$	Ex Ante	Going to medium and low firm	$\frac{P_M^E - P_L^E}{M - L}$
$a_{A2}$	Ex Ante	Going to high and medium firm	$\frac{P_H^E - P_M^E}{H - M}$
$a_H$	High firm	Buying and going to medium firm	$\frac{P_H - P_M^E - s}{H - M}$
$a_{HLeave}$	High firm	Going to low firm and leaving market	$\frac{P_L^E + s}{L}$
$a_{MH}$	Medium firm	Buying and going to high firm	$\frac{P_H^E - P_M + s}{H - M}$
$a_M$	Medium firm	Buying and going to low firm	$\frac{P_M - P_L^E - s}{M - L}$
$a_{MLeave}$	Medium firm	Going to low firm and leaving market	$\frac{P_L^E + s}{L}$
$a_{LM}$	Low firm	Buying and going to medium firm	$\frac{P_M^E - P_L + s}{M - L}$
$a_L$	Low firm	Buying and leaving market	$\frac{P_L}{L}$

Table 3.F.1: Indifferent Consumers for model with three quality levels

Now we will examine the putative equilibria where  $s$  is small. We shall suppose that the search cost is sufficiently low that all of the following hold. The best outside for the marginal consumer at a high firm is to go to a medium firm, the best outside option for the highest taste marginal consumer at a medium firm is to go to a high firm. The best outside option for the bottom taste marginal consumer at a medium firm is to go to a low firm. The top and bottom marginal consumers at the low firm will go to a medium firm/ leave market respectively.

We write profit functions for each firm starting with the low firms:

$$\begin{aligned}
 \text{Quantity}_L(P_L) = & \overbrace{\frac{\psi}{\gamma} [\min(1, a_{LM}(P_L)) - a_L(P_L)]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi \alpha}{\beta} \left[ a_A - \frac{s}{M - L} - a_L(P_L) \right]_+}^{\text{Undirected Runoff from the middle firms}} \\
 & + \overbrace{\frac{\psi \gamma - \beta - \alpha}{\beta} [a_A - a_L(P_L)]_+}^{\text{Undirected Runoff from the top firms}} + \overbrace{\frac{(1 - \psi)}{\beta} [a_A - a_L(P_L)]_+}^{\text{Directed}}
 \end{aligned}$$

Now writing the demand function for the medium firms:

$$\begin{aligned}
 \text{Quantity}_M(P_M) = & \overbrace{\frac{\psi}{\gamma} [\min(1, a_{MH}(P_M)) - a_M(P_M)]_+}^{\text{Undirected - own}} + \overbrace{\frac{\psi \beta}{\alpha} \left[ a_{A2} - \max\left(a_A + \frac{s}{M - L}, a_M(P_M)\right) \right]_+}^{\text{Undirected Runoff from the bottom firms}} \\
 & + \overbrace{\frac{\psi (\gamma - \alpha - \beta)}{\alpha} \left[ a_{A2} - \frac{s}{H - M} - \max(a_A, a_M(P_M)) \right]_+}^{\text{Undirected Runoff from the top firms}} \\
 & + \overbrace{\frac{(1 - \psi)}{\alpha} [a_{A2} - \max(a_A, a_M(P_M))]_+}^{\text{Directed}}
 \end{aligned}$$

Now writing the demand function for the high firms:

$$\begin{aligned} \text{Quantity}_H(P_H) = & \underbrace{\frac{\psi}{\gamma} [1 - a_H(P_H)]_+}_{\text{Undirected - own}} + \underbrace{\frac{\psi \beta}{\gamma - \alpha - \beta} [1 - \max(a_{A2}, a_H(P_H))]_+}_{\text{Undirected Runoff from the bottom firms}} \\ & + \underbrace{\frac{\psi \alpha}{\gamma - \alpha - \beta} \left[ 1 - \max\left(a_{A2} + \frac{s}{H - M}, a_H(P_H)\right) \right]_+}_{\text{Undirected Runoff from the medium firms}} \\ & + \underbrace{\frac{(1 - \psi)}{\gamma - \alpha - \beta} [1 - \max(a_{A2}, a_H(P_H))]_+}_{\text{Directed}} \end{aligned}$$

## Algebra

The sympy package of python was used to solve for equilibrium values  $P_L, P_M, P_H, a_A, a_{A2}$ . The code to do this can be seen on the following pages. It can be found that in the case  $a_{MH} < 1$  &  $a_{LM} < 1$ , the case  $a_{MH} < 1$  &  $a_{LM} < 1$  and the case  $a_{MH} > 1$  &  $a_{LM} > 1$  we have  $a_A \rightarrow 1$  and  $a_{A2} \rightarrow 1$  as  $\psi \rightarrow 0$ .<sup>3.32</sup>

```
'''
Paper: It's Good to be Bad: A model of low quality dominance in a full information consumer search
market
Author: Stuart Baumann & Margaryta Klymak
Date: 6th of April 2017
Comments: This code solves the model where there are three quality levels as occurs when there are many
firms only some of which may disclose. Version 1.0 of sympy is used.
'''
# Preparing workspace and declaring variables
%reset -f
import sympy as sy
H, M, L, psi, a_A, a_A2, s, p_L, p_M, p_H, alpha, beta, gamma = sy.symbols('H M L psi a_A a_{A2} s p_L
p_M p_H alpha beta gamma')

# Writing Function to solve for equilibrium.
def GetEquilibrium(LowDemandExpr, MedDemandExpr, HighDemandExpr):
    global H, M, L, psi, a_A, a_A2, s, p_L, p_M, p_H, alpha, beta, gamma
    # Getting Reaction Functions
    LowPi = LowDemandExpr*p_L
    LowReactionFunction = next(iter(sy.solve(sy.diff(LowPi, p_L), p_L)))
    MedPi = MedDemandExpr*p_M
    MedReactionFunction = next(iter(sy.solve(sy.diff(MedPi, p_M), p_M)))
    HighPi = HighDemandExpr*p_H
    HighReactionFunction = next(iter(sy.solve(sy.diff(HighPi, p_H), p_H)))

    # Ex Ante Indifferent Consumers
    ExAnteIndifferent = {a_A: (p_M-p_L)/(M-L), a_A2: (p_H-p_M)/(H-M) }
    # Sub in a_A and a_A2
    LowFunction2 = LowReactionFunction.subs(ExAnteIndifferent)
    MedFunction2 = MedReactionFunction.subs(ExAnteIndifferent)
    HighFunction2 = HighReactionFunction.subs(ExAnteIndifferent)

    # Solving to get each as function of other two
    LowFunction3 = next(iter(sy.solve(LowFunction2 - p_L, p_L)))
    MedFunction3 = next(iter(sy.solve(MedFunction2 - p_M, p_M)))
    HighFunction3 = next(iter(sy.solve(HighFunction2 - p_H, p_H)))

    # Eliminate Low price from other two.
    MedFunction4 = next(iter(sy.solve(MedFunction3.subs(p_L, LowFunction3) - p_M, p_M)))
    HighFunction4 = next(iter(sy.solve(HighFunction3.subs(p_L, LowFunction3) - p_H, p_H)))

    # Sub MedFunction4 into HighFunction4 to get expression for high price. Then get medium and low
    prices
```

<sup>3.32</sup>Output of the code is not shown as the expressions are complicated and occupy many pages. Should you wish to replicate these results please find the python code on the author's (Baumann) website.

```

RawHighPrice = next(iter(sy.solve(HighFunction4.subs(p_M, MedFunction4) - p_H,
p_H)))
RawMedPrice = next(iter(sy.solve(MedFunction4.subs(p_H, RawHighPrice) - p_M,
p_M)))
RawLowPrice = next(iter(sy.solve(LowFunction3.subs({p_H: RawHighPrice, p_M: RawMedPrice}) - p_L,
p_L)))

# Simplifying prices
LowPrice = sy.factor(sy.simplify(RawLowPrice))
MedPrice = sy.factor(sy.simplify(RawMedPrice))
HighPrice = sy.factor(sy.simplify(RawHighPrice))

# Getting expressions for ex ante indifferent consumers
a_AExpr = sy.factor(sy.simplify((MedPrice - LowPrice)/(M-L)))
a_A2Expr = sy.factor(sy.simplify((HighPrice - MedPrice)/(H-M)))

# What do a_A and a_A2 tend to as $psi \rightarrow 0$. Note that direct substitution is ok
# as expressions will be continuous
a_ALowPsi = sy.factor(sy.simplify(a_AExpr.subs(psi,0)))
a_A2LowPsi = sy.factor(sy.simplify(a_A2Expr.subs(psi,0)))

# Dictionary To Return
EquilibriumValues = {'p_L': LowPrice, 'p_M': MedPrice, 'p_H': HighPrice, 'a_A': a_AExpr, 'a_A2':
a_A2Expr,
'a_ALimit': a_ALowPsi, 'a_A2Limit': a_A2LowPsi }

# Returning Equilibrium Values
return(EquilibriumValues)

# Case 1: This is guessing for the low and medium firms that the 1 never binds in the min function.
LowDemandExpr = (psi/gamma)*((p_M - p_L + s)/(M-L) - p_L/L) + ((psi*alpha)/(gamma*beta))*(a_A - s/(M-L)
- p_L/L) + ((psi*(gamma-beta-alpha))/(gamma*beta))*(a_A - p_L/L) + ((1-psi)/(beta))*(a_A - p_L/L)
MedDemandExpr = (psi/gamma)*((p_M - p_H - s)/(M-H) - (p_M - p_L - s)/(M-L)) + ((psi*beta)/(gamma*alpha)
)*(a_A2 - a_A - s/(M-L)) + ((psi*(gamma-beta-alpha))/(gamma*alpha))*(a_A2 - s/(H-M) - a_A) + ((1-
psi)/(alpha))*(a_A2 - a_A)
HighDemandExpr = (psi/gamma)*(1 - (p_H - p_M - s)/(H-M)) + ((psi*beta)/(gamma*(gamma-alpha-beta)))*(1 -
a_A2) + ((psi*alpha)/(gamma*(gamma-alpha-beta)))*(1 - a_A2 - s/(H-M)) + ((1-psi)/(gamma-alpha-beta)
)*(1 - a_A2)
EqCase1 = GetEquilibrium(LowDemandExpr, MedDemandExpr, HighDemandExpr)

# Case 2: We have a_{MH} > 1 but a_{LM} < 1
LowDemandExpr = (psi/gamma)*((p_M - p_L + s)/(M-L) - p_L/L) + ((psi*alpha)/(gamma*beta))*(a_A - s/(M-L)
- p_L/L) + ((psi*(gamma-beta-alpha))/(gamma*beta))*(a_A - p_L/L) + ((1-psi)/(beta))*(a_A - p_L/L)
MedDemandExpr = (psi/gamma)*(1 - (p_M - p_L - s)/(M-L)) + ((psi*beta)/(gamma*alpha))*(a_A2 - a_A - s/(M-
L)) + ((psi*(gamma-beta-alpha))/(gamma*alpha))*(a_A2 - s/(H-M) - a_A) + ((1-psi)/(alpha))*(a_A2 -
a_A)
HighDemandExpr = (psi/gamma)*(1 - (p_H - p_M - s)/(H-M)) + ((psi*beta)/(gamma*(gamma-alpha-beta)))*(1 -
a_A2) + ((psi*alpha)/(gamma*(gamma-alpha-beta)))*(1 - a_A2 - s/(H-M)) + ((1-psi)/(gamma-alpha-beta)
)*(1 - a_A2)
EqCase2 = GetEquilibrium(LowDemandExpr, MedDemandExpr, HighDemandExpr)

# Case 3: We have a_{MH} > 1 and a_{LM} > 1
LowDemandExpr = (psi/gamma)*(1 - p_L/L) + ((psi*alpha)/(gamma*beta))*(a_A - s/(M-L) - p_L/L) + ((psi*(
gamma-beta-alpha))/(gamma*beta))*(a_A - p_L/L) + ((1-psi)/(beta))*(a_A - p_L/L)
MedDemandExpr = (psi/gamma)*(1 - (p_M - p_L - s)/(M-L)) + ((psi*beta)/(gamma*alpha))*(a_A2 - a_A - s/(M-
L)) + ((psi*(gamma-beta-alpha))/(gamma*alpha))*(a_A2 - s/(H-M) - a_A) + ((1-psi)/(alpha))*(a_A2 -
a_A)
HighDemandExpr = (psi/gamma)*(1 - (p_H - p_M - s)/(H-M)) + ((psi*beta)/(gamma*(gamma-alpha-beta)))*(1 -
a_A2) + ((psi*alpha)/(gamma*(gamma-alpha-beta)))*(1 - a_A2 - s/(H-M)) + ((1-psi)/(gamma-alpha-beta)
)*(1 - a_A2)
EqCase3 = GetEquilibrium(LowDemandExpr, MedDemandExpr, HighDemandExpr)

```