MATHEMATICAL PROGRAMMING MODELS FOR LIVESTOCK PRODUCTION

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Ph D

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1983

DECLARATION

I declare that this thesis has been composed by me and that the work is my own.

Three papers, Glen (1980a, 1980b, 1983), arising from this work have been published before submission of this thesis. This was done with the approval of the supervisors. The published papers are contained in Appendix A.

ABSTRACT

Agricultural production in the developed world is becoming concentrated in large, specialised units largely as a consequence of the development and adoption of new technologies. The increase in the scale and complexity of agricultural operations has contributed to the problems involved in agricultural planning, and as a result formal planning techniques, based on the construction and analysis of mathematical models, have been developed. The use of mathematical models in agricultural planning is reviewed but although many different models have been proposed, their practical application has been limited because in many cases their development has been technique rather than problem orientated.

The objective of this study is the development of models for use in planning livestock production, particularly in relation to intensive livestock operations in which least cost rations should be used throughout the production process. The formulation of least cost rations is complicated by the way in which the nutrient requirements of livestock are often specified. Since ration formulation is more complicated for ruminant species, the main emphasis of this study is on beef cattle.

In the system recommended by the Ministry of Agriculture, Fisheries and Food in the UK, difficulties arise in formulating rations for beef cattle because the energy requirements of the animal depend on the energy concentration of the ration. A parametric programming method for formulating least cost rations to meet these nutrient standards is developed. The economic performance of a beef production unit also depends on the feeding policy, i.e. the daily sequence of rations used. A dynamic programming (DP) model is developed to determine the feeding policy to produce animals of specified liveweight at minimum cost using least cost rations. This DP model can be used with other systems for specifying the nutrient requirements of cattle, and a method for calculating the required least cost rations is developed for the system recommended by the US National Research Council.

The market value of beef cattle depends on the the grade of beef produced. Since the grade of beef is affected by feeding policy, the body composition of the animal should be taken into account in the DP model. Although the model could be extended in this way, nutrient standards which consider the composition of liveweight gain are not available for beef cattle. A published pig growth model, in which liveweight gain is separated into fat free and fatty tissue components, is used to illustrate the use of the extended DP model.

Intensive livestock production is often combined with crop production, with some of the crops being used for livestock feeding. In planning operations of this type, the interaction between the crop and livestock production must be considered. A linear programming (LP) model of an integrated crop and intensive beef production enterprise is developed, using the DP model to evaluate the coefficients of the cattle feeding activities in the LP model.

By using mathematical programming models in series in this way the limitations of particular techniques can be overcome. The computational experience with each model is presented and it is suggested that, with the exception of the enterprise LP model, these models could be developed further for use by individual farmers.

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank all those, including Robin Day, Roy Fawcett, Sandy Gibson and Mitch Lewis, who have contributed to the completion of this thesis, but in particular I would like to acknowledge the special contribution made by the following:

Paul Williams for his guidance and advice.

Jim Harkins for his advice and for sharing his practical experience, and particularly for agreeing to continue to act as a supervisor following his retirement.

Margaret for her support and encouragement.

Ewan and Kirsty for the bed-time stories they missed.

CONTENTS

CHAPTER 1

AGRICULTURAL PRODUCTION

1.1 INTRODUCTION

In the agriculturally advanced nations, agriculture is becoming concentrated in larger and more specialised units than are found in traditional agricultural systems. Modern agricultural systems are characterised by a substantial investment in capital equipment and the use of intensive methods of production to obtain higher crop and livestock yields than can be achieved using traditional methods. As a result of the adoption of modern agricultural systems the agricultural production of the developed world has increased, although the number of people directly employed in farming has decreased.

The principal factor influencing the evolution of modern agriculture has been technology. Technological progress has been responsible for the development of agricultural equipment, fertilisers, pesticides, new plant varieties, drugs for the control of livestock diseases and improvements in the genetic potential of livestock. Many of the technological developments, e.g. agricultural machinery and environmentally controlled livestock housing, are capital intensive. The adoption of these technologies has involved the substitution of capital for labour, and historically this process has been facilitated by the availability of alternative employment opportunities for farm workers. In some cases the cost of the new technologies is such that their adoption can only be justified if the size of the operation is increased to achieve economies of scale in crop and livestock production, and this has been an important factor in concentrating agriculture in large specialised units. The adoption of capital intensive technologies clearly depends on the availability of finance, but the effective use of any technology requires appropriate technical knowledge and management skills. Thus although some technological developments, e.g. new plant varieties and drugs for the control of livestock diseases, are not dependent on the scale of operations, the widespread adoption of these technologies in the developing world has been restricted by the lack of adequate technical and managerial expertise.

Technological progress has improved the agricultural production potential of both land and labour. The adoption of production systems based on technological developments, together with associated changes in the organisation and management of agriculture in the developed world, have enabled much of this potential to be realised, resulting in a reduction in the unit cost of production and an increase in the productivity of land and labour in agricultural production.

1.2 CROP PRODUCTION

In the past crop production throughout the world was based on cultivation systems which involved a sequence of cropping, the crops in the sequence being chosen in relation to the climate and soil conditions. By adopting a sequence of cropping the fertility of the soil could be maintained, and weeds and plant pests could be controlled. In addition, the sequence of cropping could be chosen in relation to the availability of labour and the distribution of the requirement for labour throughout the year. However, technological progress has produced dramatic changes in the cultivation systems used in the agriculturally advanced world. As a result of the development and use of agricultural machinery, the labour requirements for crop production have been reduced and the speed at which cultivation and harvesting operations can be performed has increased. Chemical herbicides and pesticides can provide an effective means of controlling weeds and pests, and by using fertilisers, soil fertility can be maintained without the need for periods of fallow.

There has been a rapid and significant increase in crop production in the agriculturally advanced world as a result of the adoption of modern systems of crop production. The main factors influencing the level of crop production are the area of land under cultivation and the crop yields. The reserves of land in the world are finite and although climate imposes limits on the area of land which can be cultivated, capital investment in, for example, drainage and irrigation schemes can be used to overcome some climatic effects and increase the area of land under cultivation. Although technological progress has enabled some previously uncultivated land to be used for crop production, there has also been a substantial loss of land to non-agricultural uses in many countries. Consequently, most of the increase in crop production in the agriculturally advanced world has been achieved by increases in crop yields.

A number of factors have contributed to the increase in crop yields. One of the most important of these factors has been the development of new plant varieties. Some of these new varieties can be grown successfully using traditional methods of production, but with other varieties intensive cultivation techniques involving, for example, high rates of fertilser application, are required if these varieties are to realise their potential yields. The development and use of chemical fertilisers has also had an important effect on increasing crop yields, and indeed many of the high yielding plant varieties depend on heavy application of fertilisers. Fertilisers are used to supply the nutrients required for plant growth, but to obtain the best results the use of fertilisers must be properly managed.

Traditionally, damage to crops and reduction in yields caused by pests, disease and weeds has had a serious effect on crop production. The Food and Agriculture Organization of the United Nations (1970) has estimated that throughout the world lossses in the field and in storage due to pests, disease and weeds amounted to about one third of total production. Modern chemical insecticides have had a significant effect in reducing losses due to insect pests, although most pests develop resistance after prolonged exposure to a pesticide. Losses due to disease can also be reduced by modern chemical techniques, either as seed treatments or by foliar application. In addition to these chemical methods of controlling pests and diseases, new plant varieties resistant to common pests and diseases have been developed, although sometimes these new varieties may be attacked by new strains of pest or disease.

If weeds are present in a crop, the crop yield will be reduced because of competition for the available nutrients, light and water. In most traditional cultivation systems weed control was achieved by crop rotation, ploughing and labour intensive methods such as hoeing. The development of chemical herbicides, particularly selective herbicides, i.e. herbicides which can be used to destroy or check the growth of weeds without affecting the crop in which the weeds are growing, has provided an effective method for the control of most types of weed. As a result of the development of chemical herbicides, new cultivation systems involving, for example, sowing seeds directly into uncultivated soil, have been developed.

The use of chemicals to control pests, diseases and weeds may, however, have a harmful effect on the environment due to the build up of chemicals in soil and water, and this has brought about some reconsideration of non-chemical control strategies. For example, in pest control the environmental impact of chemical pesticides, together with increasing pest resistance to these pesticides, have resulted in the development of integrated approaches to pest management which involve the use of a combination of biological controls (e.g. predators and parasites), genetic improvement (e.g. plant breeding for pest resistance) and cultural practices (e.g. crop rotation and timing of harvesting) along with the use of pesticides, in order to reduce the quantity of toxic chemicals used in controlling pests.

The overall effect of these technological developments has resulted in an improvement in the crop production performance of agriculturally advanced countries. These developments have also reduced the variability in crop production and enabled better prediction of crop yields. The increase in crop yields in the UK as a result of changes in agriculture is illustrated in Table 1. However, even in the agriculturally advanced countries there are significant variations in crop yields. Examples of the trends in wheat yields in some developed countries are given in Table 2. Some of these variations can be explained in terms of factors such as climate and soil type. For example, wheat in North America has traditionally been produced in the prairie regions where rainfall is low, and consequently wheat yields are low by international standards. Another factor which affects crop yields is fertiliser usage. For example, the Organisation for Economic Co-operation and Development (OECD) (1976) has estimated that nutrients from fertilisers accounted for more than one third of the yield of winter wheat in the UK.

Technological developments have been responsible for much of the progress in crop production. In the future it can be expected that technological progress will continue, with the development of new plant varieties and new cultivation systems producing increases in crop yields. Other developments may also have a significant effect on crop production. For example, advances in biotechnology may help to speed up the process of developing new plant varieties selected on the basis of particular characteristics, such as resistance to drought. On a worldwide basis, crop production could be increased if existing technologies and cultivation systems were more widely adopted, but this can only be achieved through education in the techniques of modern crop production and the development of the necessary technical and managerial skills.

1.3 LIVESTOCK PRODUCTION

Livestock production outside the extensive grazing regions of the world, such as those in North America, has traditionally been associated with mixed farming units. However, as a result of technological progress, livestock production in the agriculturally advanced world is becoming increasingly specialised and capital intensive. Although much of traditional livestock production is land extensive, being based on the grazing of pasture, technological progress has improved crop production performance to such an extent that in many areas the economic value of land is greater when used for the production of crops rather than forage for livestock. In addition, technological progress in animal nutrition and disease control, together with the development of environmentally controlled housing for livestock, have enabled livestock to be fed in confinement, thus reducing the need for a large land base as capital is substituted for land. As a result of these developments, systems based on intensive feeding in indoor units dominate poultry and pig production in the agriculturally advanced world. Technological progress in nutrition, disease control and pasture management have also improved performance in the livestock production which remains land based. Although all livestock production in the agriculturally advanced world has benefitted from these developments, the advances in the poultry, pig, beef and dairy sectors have been more significant than in sheep farming where the influence of these developments has largely been limited to the control of disease and parasites.

The first major large scale application of the technological advances in livestock production came through the development of poultry and egg production systems based on feeding precisely formulated rations to poultry housed in environmentally controlled units. The types of bird used in these systems have been developed through selective breeding for economically desirable traits such as feed conversion performance, growth rate and egg production rate. The use of environmentally controlled units to house the birds enables the effects of temperature stress to be overcome. In addition, birds can be more easily managed in indoor units and the handling of feed, waste and eggs can be mechanised, thus increasing the productivity of labour. However, since birds maintained in confinement in indoor units are more susceptible to disease, the development of this type of production system would not have been possible without the availability of effective disease and parasite control techniques.

The adoption of these production systems has been accompanied by structural changes in the organisation of poultry and egg production, with poultry and egg producers combining with feed suppliers, packers and distributors to create vertically integrated production and marketing systems in which effective planning and control is of vital importance. As a result of these changes, poultry and egg production in the agriculturally advanced countries have become concentrated in large indoor units producing poultry and eggs of consistent quality.

Pig production in the agriculturally advanced world has experienced changes similar to those in poultry production. Developments in nutrition have enabled balanced, largely cereal based, rations to be used to meet the nutrient requirements of pigs at all stages in their life cycle, while progress in disease and parasite control has removed the need for a large land base. These technological advances, together with the availability of environmentally controlled housing and labour saving machinery, have led to the development of systems for pig production based on intensive feeding. The types of pig used in these systems have been developed through selective breeding in respect of factors such as feed conversion efficiency, growth rate and carcase composition. Selective breeding programmes have also been used to improve reproductive performance in areas such as breeding interval, conception rate and the number of piglets successfully weaned per pregnancy. These improvements in the genetic potential have improved the economic efficiency of pig production. As with poultry, pig production in the agriculturally advanced world is becoming concentrated in large indoor units with a degree of horizontal and vertical integration of the industry in many countries.

The development and adoption of intensive beef production systems has been less dramatic than is the case with poultry and pigs, mainly because the conversion of feedstuffs into meat in ruminants is relatively inefficient in comparison with monogastric species. Intensive systems for the production of beef have been developed as a result of advances similar to those with poultry and pigs, but these systems are only of major importance in North America where, for example, in the US between 25 and 30 percent of beef production is associated with intensive feeding (Martin (1979)). In the US system, cattle are raised on pasture and then transported to feedlots where they are fattened on cereal based diets until they reach a liveweight of 500 to 600 kg at an age of about 14 to 16 months. Cattle feeding operations of this type in the US have become concentrated in large units with 422 feedlots accounting for 52 percent of the 25 million fed cattle marketed in the US in 1977, the remaining 48 percent coming from 130,000 feedlots (Martin (1979)). Although intensive beef production systems have been developed outside North America, these systems are not widely used. For example, in the UK the barley beef system, which involves feeding barley and protein supplement to calves to produce finished beef at a liveweight of approximately 400 kg in about 12 months, is little used.

The economic efficiency of intensive beef production is highly dependent on the relative prices of grain and beef. For this reason beef production based on extensive grazing is more efficient, in economic terms, in many parts of the world, and the efficiency of these extensive operations can often be improved with better pasture management. Semi-intensive production systems, involving both concentrate feeding and grazing of well managed pasture, have also been developed, e.g. the 18 month beef system used in the UK. There are thus a large number of different types of beef production systems, most of which involve the application of aspects of technological progress in areas such as nutrition, disease control and improvements in genetic potential.

Although in many countries there are close links between dairy farming and beef production, with the dairy herd being a source of both beef and calves for beef production, dairy farming is becoming a specialised activity in the agriculturally advanced world. Technological progress in, for example, the mechanisation of milking and the bulk handling of milk, has resulted in the substitution of capital for labour and improved the productivity of labour in milk production. Milk yields per cow have increased as a result of advances in nutrition, involving, for example, the use of concentrates, and the development of better grazing and fodder production methods. Improvements in the genetic potential, resulting from selective breeding programmes and the use of artificial insemination, have also contributed to the increase in milk yields per cow. In order to take advantage of these developments, milk production is becoming concentrated in larger more specialised units, and as in the rest of the livestock sector, a high level of managerial and technical skill is required to ensure that resources are used in an efficient manner.

Technological progress in the crop and livestock sectors has been responsible for the improvement of traditional extensive grazing systems and the development of new livestock production systems based on intensive feeding. The most dramatic changes have resulted from the development of intensive feeding systems and the introduction of environmentally controlled housing. These intensive production systems have some associated problems, such as waste disposal, especially when concentrated in large units, but the adoption of these systems has resulted in improvements in livestock production performance as can be seen from improvements in indices such as milk yields per cow and eggs laid per bird per year, and examples of trends in these indices for the UK are given in Table 3. Feed conversion efficiency also appears to have improved (OECD (1976)) but this is not always apparent from indices such feed intake per unit of output, since feed conversion efficiency is affected by factors such as the liveweight and carcase composition of the animal and by the type of feedstuff used. For example, feed conversion efficiency tends to decrease as liveweight increases and to increase as more concentrates are used. Thus changes in the market weight, carcase composition and feeding system will affect overall feed conversion performance.

Intensive feeding systems are generally cereal based and livestock production systems based on intensive feeding are economically viable only if large quantities of feedstuffs are available at reasonable prices. As a result of the adoption of these systems, a significant proportion of the cereal consumption of the developed world is used for animal feeding. For example, in OECD countries in 1971-1973, animal feeding accounted for almost 70 percent of total cereal consumption, while in the developing countries it accounted for less than 10 percent of cereal consumption (OECD (1976)). Since feedstuffs, especially cereals, are such an important input to intensive livestock production systems, the introduction of these systems in developing countries may be restricted by the demend for cereals for direct human consumption. In the developed world, an increase in the scale of intensive livestock production is likely to be constrained not only by the prices and availability of feedstuffs, but also by problems associated with the environmental impact of these operations.

CHAPTER 2

PLANNING IN AGRICULTURE

2.1 INTRODUCTION

Planning in the agricultural sector is carried out at levels ranging from the individual farm to the regional or national level. For an individual farm, planning may be concerned with daily operations, such as animal feeding, or the planning horizon may extend from one year, as in planning crop and livestock operations within a given availability of resources, to the long term as is the case in investment planning. At the regional or national level the degree and nature of planning in the agricultural sector will depend on the ideology of the government and on the stage of development of the country involved.

The first stage in the planning process should involve identifying the planning objectives, defining the range of acceptable solutions and specifying the criterion for evaluating possible solutions. The planning objective will depend on the nature of the investigation, but at the farm level it is often convenient to consider planning in terms of financial objectives such as profit maximisation, although other factors are also relevant. Farmers value their way of life and they are therefore strongly motivated to survive as farmers and maintain their independence. For this reason farmers may be prepared, as noted by Barnard and Nix (1975), to sacrifice profit in return for a relatively stable level of income or, because peer group standing is an important motivation for farmers (e.g. Gasson (1973)), they may accept a reduction in income in return for the intrinsic satisfaction gained from obtaining high crop or livestock yields. Even when farmers are strongly profit motivated it will be necessary, as noted by Arnold and Bennet (1975), to take account of personal idiosyncracies, such as a preference for crop production, in planning farm operations. However, although non-financial objectives are important in planning at farm level, financial objectives, especially profit maximisation, are commonly used in farm planning, with the range of possible solutions being limited by operational considerations which reflect non-financial objectives through, for example, a limitation on the crops which can be grown.

Planning in the agricultural sector of the developed world has increased in complexity as a result of the changes in agricultural systems. Technological progress has been one of the principal factors influencing the development of these systems and in addition, the adoption of new technology directly affects planning. For example, at farm level the adoption of a capital intensive new technology must be viewed in the context of the long term future development of the enterprise, while the effective use of any technology requires careful planning at the operational level. Technological progress has also been a major factor in increasing both the scale and degree of specialisation of the agriculture of the developed world and these changes have implications for planning. Agricultural planning is characterised by uncertainty in prices and variability in crop and livestock yields due to uncertainty in weather conditions and the incidence of disease and pest attack. Although technological progress has helped to reduce the variability in crop and livestock yields, the increasing specialisation in modern agriculture means that risks cannot be spread as widely among a number of operations as is the case with traditional mixed farming systems. For this reason uncertainty should be taken into account in the planning process. In addition, the international nature of agricultural trade means that production decisions at farm level may be influenced by global conditions, and thus international environmental and economic factors should be considered in planning, particularly at national level.

Traditionally judgement based on experience has been the basis for planning in agriculture and although this approach has been successful, the increase in specialisation in agricultural production and the adoption of capital intensive production systems has led to the development of more formal planning methods. These formal planning techniques are based on the construction and analysis of mathematical models, i.e. sets of equations which describe the operation of a system. Once a solution to the model has been derived and tested, the solution can be implemented and then, by monitoring actual performance and comparing this with predicted performance, the solution can be controlled. This model building based approach has been characterised by, for example, Ackoff and Sasieni (1968) as the Operational Research approach:-

1. Formulating the problem
2. Constructing the model
3. Deriving a solution
4. Testing the model and evaluating the solution
5. Implementing and maintaining the solution.

Although the phases in this approach are listed serially, some back-tracking will generally be required depending on the outcome of particular phases.

The important feature of this approach is the emphasis on formal model building. A number of different types of model have been developed for planning in the agricultural sector, with the choice of model being influenced by the nature of the problem. It is therefore convenient to illustrate the range of models which has been developed for use in the agricultural sector by considering the different types of planning problems which arise at farm and regional or national level. Since many of the problems at farm level relate to either crop production or livestock production, these two aspects are considered separately, for although some models have attempted to integrate both crop and livestock production at farm level, these models have tended to concentrate on one aspect.

2.2 CROP PRODUCTION MODELS

Crop production involves a number of operations which must be performed at certain times of the year, and in planning crop production the seasonal nature of the operations and any seasonal constraints on the availability of resources, such as labour and machinery, must be taken into account. Another characteristic of crop production is the influence of uncertainty on both crop yields and the timing of operations. Some of the variation in crop yields can be reduced by the use of fertilisers, and since crop nutrient requirements can generally be specified in advance, a policy for fertiliser application can therefore be determined at the planning stage. Pest and disease attacks are sources of uncertainty in crop production and although some protection can be obtained through measures such as seed treatment, the effect of a pest or disease attack depends on uncertain factors. Thus a strategy for controlling pests or diseases should be influenced by the way in which an attack progresses, taking account of factors such as the intensity of the attack, the stage of development of the crop and the response to the control measure. Weather conditions, however, constitute the major source of uncertainty in crop production and although in some crops it may be possible to take some action to reduce the influence of weather conditions, e.g. by use of irrigation or by rescheduling harvesting operations, there is generally only limited scope for actions of this type and therefore the importance of uncertainty in weather conditions should be recognised at the planning stage.

Since the determination of cropping policy is of fundamental importance in crop production, this aspect of planning crop production is considered first. Methods used for planning harvesting operations are then discussed and this is followed by an examination of techniques used to evaluate investment decisions associated with crop production, particularly with respect to equipment. Finally, models used to evaluate strategies for controlling pests and diseases in crops are considered.

2.2.1 Cropping Policy

Planning crop production involves determining the crops to be grown, the area to be used for each crop, the fertiliser application policy and the crop rotation policy. A number of techniques have been used for planning crop production taking account of known operational constraints. Mathematical programming models have been widely used in this area and indeed Nix (1979) identified the first published example of the use of linear programming (LP) in agriculture being that of Heady (1954) who illustrated the development of a simple LP model to determine the allocation of arable land to two crops. More realistic LP models for planning crop production were subsequently developed, and in a number of cases planning systems based on this type of LP model were designed to simplify the use of LP models in this area. For example MASCOT (Bond et al. (1970)), an LP based planning system operated by ICI in the UK, incorporated a matrix generator to set up the LP model and a report writer to produce results in a form which is meaningful to the farmer. Similar features were included in the LP based planning package developed by McCarl et al. (1978) for large grain farms in the US, while the Agricultural Development and Advisory Service in England and Wales developed an LP based planning system (James (1972)) which used standard matrices, typical of farms in a region, to reduce the amount of data collection required.

Most LP based crop planning models involve maximising a profit function subject to limitations on resources and other requirements such as crop rotation. Even in cases where the normal LP assumptions of linearity and divisibility are valid, an LP model may not reflect the objectives of farmers who are motivated by other factors, e.g. a desire to maintain independence or peer group standing (e.g. Gasson (1973)). Wheeler and Russell (1977) suggested that the different goals of farmers could be taken into account in a goal programming model, i.e. an LP model in which the objective function represents the overall weighted level of over or under achievement of goals. A disadvantage of the goal programming approach is that the post optimal analysis which can be performed is of limited value, but it was argued that goal programming could be used to generate a range of plans for investigation in greater depth using LP.

Because of the seasonal nature of cropping operations it is important to incorporate the timing of the various operations in an LP model. Although the timing of crop production operations and the associated labour and equipment requirements are included in most crop planning LP models, some models include more detail of the operations involved. For example, Audsley et al. (1978) developed an LP model to compare different cultivation systems in arable farming, and included the starting times of harvesting operations as variables in the model, but a weakness of this model is that weather conditions were assumed to be deterministic.

A number of mathematical programming methods which incorporate uncertainty have been developed and have been suggested for use in planning crop production. Chance-constrained LP, in which restrictions on probabilities are represented by linear inequalities, was developed by Charnes and Cooper (1959) but although the chance-constrained model has the same size and structure as the deterministic version of the model, the economic consequences of violating a constraint cannot be evaluated directly. Cocks (1968) developed a stochastic LP approach in which all possible outcomes were assumed to be represented by their probabilities of occurrence and illustrated the use of this method with a crop planning example. This discrete stochastic programming model was extended by Rae (1971a) to accommodate forecasts and different types of utility function. The use of this extended model was illustrated by Rae (1971b) but the major limitation of this type of approach is that the LP model is very large in comparison with the deterministic version.

Game theory models solved by LP have also been suggested (e.g. McInerney (1967)) for dealing with uncertainty in the planning of farm operations, and McInerney (1969) and Hazell (1970) have illustrated the use of this approach to investigate the trade-off between expected and worst possible functional value in crop production. This trade-off was also investigated by Maruyama (1972) using an LP model which incorporated uncertainty in the objective function, the restraints and the input-output coefficients. This model was also used to generate associated information, such as the probability of ruin, which it was argued, could be more easily understood by farmers than conventional measures of uncertainty such as the standard deviation or the variance of income. However, this approach is likely to be of little practical value because of the difficulty of obtaining estimates of the relevant coefficients and their associated probabilities.

Portfolio theory (Markowitz (1959)) has also been used to provide a framework for analysing agricultural decisions under uncertainty. In this approach both expected return and risk, measured in terms of the variance of the return, are considered and the decision which maximises the expected return for a specified level of variance in the return is determined. This decision is said to be E-V efficient and quadratic programming can be used to determine the E-V efficient frontier, i.e. the set of E-V efficient decisions. Although the quadratic programming approach is theoretically appealing because it incorporates income variance and the covariance of the possible activities, there has been little practical application of this approach because of the difficulty of assessing the E-V utility function of an individual farmer. Extensions of this approach involving, for example, the calculation of associated lower income bounds (Scott and Baker (1972)) also suffer from the difficulty of relating the results to the decision making behaviour of individuals. Hazell (1971) suggested a linear alternative to E-V efficiecy which involved minimising mean absolute income deviation for a specified expected income using parametric LP, but although solution methods are more widely available than for the quadratic programming problem, it is again difficult to relate the results to practical decision making. Boussard and Petit (1967) assumed that farmers attempted to maximise expected income subject to a specified probability of obtaining some minimum level of income. This behaviour was represented in an LP model by using a 'focus loss' constraint for each activity with an uncertain income, but covariance was not taken into account.

There are therefore considerable computational and practical disadvantages associated with mathematical programming models incorporating uncertainty. However, although uncertainty is not incorporated into the formulation of a deterministic crop planning LP model, the results from the model can be of considerable value to the farmer if the known sources of uncertainty are taken into account in using the model to plan crop production and to evaluate contingency plans. Brink and McCarl (1978) carried out experiments to investigate the decision making behaviour of farmers using an LP model which incorporated uncertainty. The results from these experiments indicated that risk aversion was not, in general, an important factor in the choice of farm plan and it was suggested that the benefits from measuring and incorporating risk and risk aversion in the planning model were small. For these reasons deterministic LP models are more widely used than models which incorporate uncertainty and there is evidence to suggest that the use of these models has an effect on decision making. For example, Debertin et al. (1981) used a modified version of the LP model of McCarl et al. (1978) to evalute the influence of LP models on the decision making of farmers. It was found that farmers were prepared to make changes in decisions relating to the areas under different crops as a consequence of being exposed to the results of the model, but that they were less likely to make changes relating to the purchase of equipment or the use of labour, probably because these changes were more difficult to implement. In most cases the main benefits from using deterministic crop planning LP models arise, as noted by Debertin et al. (1981), from improved understanding of the interactions within a farm business.

Dynamic programming (DP) models have also been used in crop planning. For example, Burt and Allison (1963) used a stochastic DP model to determine the crop rotation system to maximise present value over a specified planning period in the dry farming regions of the US. In this problem decisions on fallowing land or planting wheat were evaluated using soil moisture content as the state variable. Whan et al. (1976) used a deterministic Markov model to determine crop rotation policy with particular reference to sugar cane production. In a development of this model, Whan et al. (1978) introduced crop quality as a stochastic element in the Markov model and determined the long run steady state solution by solving an LP model. However, since the crop production decisions of individual producers are likely to be influenced by market price expectations, this long run solution is likely to be of limited value, especially in dealing with crops subject to wide price fluctuations.

The policy for fertiliser application should also be considered in planning crop production. Many of the crop planning LP models assume that chemical fertilisers are used at some specified rate in crop production. Animal wastes are also widely used as fertilisers in crop production and the possibility of using animal wastes for this purpose is included in some models, particularly where an enterprise is involved in both crop and livestock production (e.g. Ashour and Anderson (1975), Weinsink and Miner (1977)). However, a limitation of these models is that fertiliser policy must be specified and thus crop yield was not considered as a function of fertiliser applied, and in addition the effect of previous applications of fertiliser was not included in these models.

A DP model was used by Kennedy et al. (1973) to evaluate fertiliser application policy taking account of residual fertiliser available from previous applications of fertiliser, particularly as it affects year-round cropping in many tropical environments. In this model it was assumed that the carryover of fertiliser between periods was a proportion of the fertiliser available in the earlier period, and a solution procedure based on an inductive approach was suggested. Although, as noted by Godden and Helyar (1980), the simple solution to this DP model is dependent on the form of the equations for fertiliser carryover, the DP approach could be applied using other forms of the fertiliser carryover function but it would be preferable to use a backward DP formulation (White (1969)), in which the initial fertiliser available is specified, since the effect of varying the initial conditions could be investigated more easily. Godden and Helyar (1980) developed an alternative method for determining fertiliser rates assuming that fertiliser carryover from each previous application was a simple function of the time since each application. A heuristic solution procedure was suggested but, as noted by Kennedy (1981b), the solution obtained using this approach will generally not be optimal. In these models fertiliser carryover was assumed to be deterministic, but in the DP model developed by Stauber et al. (1975) for evaluating fertiliser application policy the fertiliser carryover process was assumed to be stochastic and the cost of fertiliser application was incorporated.

The control of weeds is another aspect of crop planning which has been investigated through the development and analysis of models. For example, Menz et al. (1980) used a simulation model to evaluate methods for controlling perennial weeds in a defined area. A simple spatial model was used with the geographical area being divided into a number of equal squares which were assumed to be identical, and although this assumption is unlikely to be of general validity, the approach could form the basis for further research. A similar approach has been used by Auld et al. (1979) for controlling weeds on a single farm.

Weed and disease infestation can be controlled by crop rotation, and Fisher and Lee (1981) have developed a deterministic DP model to determine the crop rotation policy for control of the weed, wild oats, and the disease, crown rot, in a wheat crop in New South Wales. It was assumed that the farmer could either grow wheat with or without a post-emergence herbicide to control wild oats, or grow a summer crop (sorghum), or maintain a weed free fallow. The state variables in the DP model were the soil moisture content, the level of crown rot infestation and the wild oat population. Crop yield was assumed to be a linear function of soil moisture and the model was solved for the expected value of rainfall. Although the stochastic nature of commodity prices was not taken into account in this model, experimentation with a range of prices suggested that the solution was not particularly sensitive to changes in these prices. Some of the agronomic data used in this model was approximate and it was noted that one of the most significant outcomes of the development of this model was the initiation of research to provide the information required for the construction of this type of model.

2.2.2 Harvesting Operations

The scheduling of harvesting operations is an important aspect of planning crop production but although most crop planning models attempt to incorporate harvesting, it tends to be dealt with in a very general way which ignores the complexities associated with the scheduling of harvesting operations. For most crops the value of the crop depends on the timing of harvesting and since the harvesting capacity of a farm is limited, the whole crop cannot be harvested when it ripens. Losses occur if harvesting is delayed, the losses increasing with the time between ripening and harvesting. Therefore in scheduling harvesting operations the harvesting capacity and the relationship between time of harvesting and the value of the crop should be taken into account. In addition, since weather conditions affect both the ripening of the crop and the ability to perform harvesting operations, the uncertainty in weather conditions should also be taken into account in harvest scheduling.

Attempts have been made to include the time of harvesting in crop planning LP models, but since it is difficult to incorporate the relationship between crop value and time of harvesting in the LP framework, it is generally necessary to assume that weather conditions are deterministic (e.g. Audsley et al. (1978)). For these reasons a number of different models have been developed for use in scheduling harvesting operations, with simulation models being widely used. For example, a stochastic simulation model of grain harvesting, in which weather and grain moisture content were the stochastic elements, was developed by Donaldson (1968) in order to assess the influence of harvesting capacity and work rate. Simulation models using historical weather data have been developed by Ryan (1973) and by Philips and O'Callaghan (1974) to evaluate machine use policies and equipment capacities in harvesting. Simulation models have also been used to determine the time to start harvesting operations (e.g. Chen and Chi-Chen Yang (1980)) and heuristic methods have been suggested by Singh and Holtman (1979) for harvest scheduling.

Mathematical programming models have also been developed for scheduling harvesting operations. An LP model, in which both forecast and historical weather data were used to estimate factors such as available work hours, was developed by Fokkens and Puylaert (1981) for planning daily harvesting operations in a large Dutch grain farm formed as a result of the reclamation of land in the Zuyderzee. In this case the harvesting capacity which minimised costs, including the cost of field losses, was first determined by simulation. Van Elderen (1980) has compared the use of simulation and LP models in scheduling harvesting operations using historical weather data. The schedule produced by LP had significantly lower costs largely because weather data for an entire harvest season was available with the LP model, whereas in planning harvesting operations it would be necessary to use forecast weather data. A number of DP models have also been developed for scheduling harvesting operations. For example, Morey et al. (1972) developed a stochastic DP model for scheduling harvesting operations for corn and soybeans with weather conditions, defined in terms of the probability of working on any day, and harvesting rate being used as the stochastic elements. However, a weakness of this model is that harvesting rate was assumed to be independent of the probability of working on any day. A stochastic DP model has also been developed by Miyake et al. (1979) for scheduling the harvesting of tobacco. Since the value of the tobacco crop depends on the time of harvesting, emphasis was placed on determining the optimal starting date for harvesting and on deriving decision rules for hiring additional labour to supplement the full-time work force. In this model the probability of a day being suitable for harvesting was derived from historical data, but in implementing this type of model it would be desirable to incorporate short-term weather forecast data.

There are a number of crops where individual elements (e.g. fruit, vegetables) mature over an extended period. These crops are generally harvested by a succession of selective harvests, and in planning this type of harvesting operation the timing of each selective harvest and the size or grade of individual elements to be harvested should be considered as decision variables. Cauliflower is an example of a crop which matures over an extended period and Corrie and Boyce (1972) have developed a deterministic DP model for planning the harvesting of this crop, but a major weakness of this model is that the influence of weather on maturation and harvesting was not considered. Cucumber is another crop which reaches maturity over an extended period with fruit of different sizes being present on the plants. Chen et al. (1976) have developed a stochastic simulation model to evaluate multi-stage policies for harvesting cucumber in the south east of the US, but although the effect of weather on harvesting was included in this model the effect of weather on crop growth was not considered. Tobacco is also harvested in a number of passes as the crop matures. Yang and Sowell (1981) have developed a mixed integer programming model for scheduling the harvesting of flue-cured tobacco assuming that the tobacco must be placed in barns for curing immediately after harvesting. Integer variables were used for the number of barns filled on any day, but the value of the crop was assumed to be independent of timeliness in harvesting.

The wide range of models which have been developed for scheduling harvesting operations is partly a reflection of the fact that these operations vary from crop to crop. Although harvesting operations are generally highly seasonal, there are some crops for which harvesting can be undertaken throughout the year. For example, sugar cane grown for biomass rather than sugar production can be harvested throughout the year and Mishoe et al. (1979) have developed an LP model for scheduling the harvesting of this crop. However, for most crops harvesting must take place during a relatively short period of intensive activity and techniques which aid the planning of these operations can be of substantial economic benefit.

2.2.3 Capital Investment for Crop Production

Crop production in the agriculturally developed world is dependent on capital investment, particularly in equipment. Although most cropping operations have an associated equipment requirement, the major investment in equipment tends to be connected with harvesting operations. The equipment available should be taken into account in planning crop production but the determination of equipment requirements should form part of the longer term planning. The economic value of the equipment should be considered in evaluating capital investment in equipment and since most types of equipment have a limited life, policies for replacement of equipment must also be investigated.

Since many operations in crop production are machine dependent, models developed for cropping operations which involve the use of equipment can often be used to evaluate both operating policy and equipment requirements. For example, simulation models such as those of Donaldson (1968), Dalton (1971), Ryan (1973) and Philips and O'Callaghan (1974) can, by changing the equipment availability, be used to evaluate the equipment requirements or they can be used to evaluate operating policy for a specified set of equipment. However, a weakness of these simulation approaches is that only specified sets of equipment can be evaluated, and although the existing complement of equipment on a farm can be taken into account (e.g. Dalton (1971)), neither the method of achieving a desired set of equipment nor the possibility of hiring equipment or subcontracting some operations were considered.

Crop planning LP models can also be used to evaluate equipment requirements by running the model with different constraints according to the assumed availability of equipment (e.g. Debertin et al. (1981)). Some crop planning LP models include equipment levels as variables but the solution must be modified to allow for the discrete nature of equipment. For example, Audsley et al. (1978) round the solution to the nearest integer and re-solve the LP with the variable constrained to the integer value. However, scale economies were not considered and the solution procedure will not guarantee an optimal solution. These limitations can only be overcome by adopting an integer programming formulation.

Mathematical programming models which recognise the discrete nature of equipment have also been developed for use in evaluating investment decisions. A mixed integer programming model was developed by Danok et al. (1978) to determine the machinery requirement and the associated cropping plan and labour requirements for a state farm in Iraq. Integer variables were used for each type of machine and the model was solved using the decomposition method developed by Benders (1962). A mixed integer programming model was also used by Amir et al. (1978) to evaluate equipment investment, but in this case integer variables were used to define possible combinations of equipment and constraints specified incompatible systems from these combinations of equipment. Although constraints on capital, energy use and working hours were also specified in this model, the timeliness costs in harvesting were not considered in the objective function. Danok et al. (1980) also used integer variables to represent sets of equipment in a mixed integer programming model and ran the model under specified weather conditions which defined the availability of field operating days during critical seasons such as ploughing and harvesting. The choice of probability level for good field days was arbitrary but the results of these runs were used to determine if there existed a set of machinery which performed well over a wide range of weather conditions. A robust solution of this type was found although the associated set of equipment was not optimal under any particular weather conditions. It was suggested that this indicated that the results from mixed integer programming models of this type could be strongly biased towards the expected level of good field days and that sensitivity analysis should therefore be performed on mixed integer programming models of this type.

Although a mixed integer programming formulation of the equipment selection problem provides a better representation of reality than an LP formulation, mixed integer programming models are more costly to construct and a greater computational effort is required to solve these models. Hence, although mixed integer programming models may be valuable as research tools, this type of model is not suitable for use by individual farmers. Simulation models are also costly to build and Dalton (1971) argued that unless representative models of different types of farm were constructed, then simulation models would only be applicable in large farms as tools for evaluating equipment investment decisions.

A number of other methods have been suggested for evaluating equipment investment decisions. For example, Boyce and Rutherford (1971) developed a simple cost based model for evaluating combine harvesters. Audsley and Boyce (1974) extended this model to include a wet grain store and a high temperature drier and, by evaluating the cost for a number of different combinations of equipment, determined the minimum cost system. The main limitations of these models are that average figures were used for the daily availability of equipment and, since linear relationships were assumed between the cost and size of equipment, economies of scale were not considered. The failure to take account of scale economies is also a weakness of the iterative method suggested by Singh and Gupta (1980) to evalute equipment. A stochastic inventory model has been developed by Sanders and Lalor (1972) to determine the requirement for harvesting equipment. In this model the harvesting capacity was considered to constitute an inventory for which weather conditions indirectly created a demand, the demand for equipment being higher in a poor weather season. However, neither the discrete nature of harvesting equipment nor economies of scale were taken into account in this model.

The performance of agricultural equipment deteriorates with age and although this deterioration in performance can be reduced by maintenance, the strategy for equipment replacement should also be considered. The effect of tax allowances should, as noted by Chisholm (1974), be taken into account in evaluating equipment replacement policies. Kay and Rister (1976) developed a model to determine the optimal replacement policy for farm equipment over an infinite time horizon taking account of tax allowances. Inflation also influences replacement decisions since it affects the resale value of equipment while tax allowances for depreciation are typically based on historic costs, and Bates et al. (1979) extended the model of Kay and Rister (1976) to incorporate inflation. Although in both these cases the importance of equipment reliability was recognised, both models failed to take account of technological progress. Methods for incorporating technological change in replacement decisions in general have been suggested (e.g. White (1969)) but the major difficulty lies in forecasting technological progress. Crabtree (1981) took account of inflation, tax allowances and interest rates in the evaluation of agricultural machinery investment decisions, and although technological progress was not incorporated explicitly, the need to take account of savings in labour requirements and improvements in performance was recognised. However, a limitation of this approach is that it was assumed that machinery must be replaced after a specified period.

Models have also been developed to evaluate capital investment in facilities other than equipment. For example, Barry (1972) developed a multi-period LP model for planning the expansion of a farm through the purchase of additional land which was assumed to become available in multiples of units of fixed area. The effects of asset indivisibility were dealt with by running the model with different levels of land investment during the planning period. This type of approach would be inappropriate if a large number of investments over time were to be investigated. Another unsatisfactory feature of this approach is that it was assumed that land became available when required, whereas in reality it would generally be necessary to wait for suitable land to become available. Taxation factors should also be taken into account in planning the long term development of a farm. Reid et al. (1980) outlined a method for incorporating US investment tax credit in a multi-period mathematical programming model of a farm firm. This problem should be formulated using integer variables but there is no evidence that this was done. In this model projects appear to be selected on the basis of internal rate of return, a method which is generally unsatisfactory where projects are dependent on other investments or where investments are alternatives.

2.2.4 Pest and Disease Control

Crops are subject to attack by pests and diseases and this can cause a substantial reduction in both the yield and the value of the crop. The impact of a pest or disease attack can be reduced by control measures such as the application of chemical pesticides or fungicides, and these measures are often used by farmers as an insurance against crop damage, partly because control strategies are difficult to evaluate. Complex biological processes are involved when a crop is affected by pests or disease and although there has been considerable research effort in applied ecology, it is only in the recent past that, as noted by Conway (1977), attempts have been made to link ecological and economic factors through the use of mathematical models.

The main biological factors which should be considered in analysing crop-pest systems are the size and age structure of the pest population, the presence of predators and parasites, and the stage of development of the crop. The effect of a pest on a crop depends on the dynamic interaction between these biological factors and other factors, such as weather conditions and the effectiveness of any control agents. A number of models have been developed for evaluating policies for pesticide application but the earliest models of this type concentrated on simple crop-pest systems and ignored the uncertainty in weather conditions and the influence of predators and parasites. Headley (1972) developed a model which related the cost of crop damage to pest density and time, and used this model to investigate the policy for controlling the pest through the single application of a pesticide, with marginal analysis being used to determine the level to which the pest should be reduced at an entomologically specified time. In investigating the policy for pesticide application it is more useful to consider the pest population level at which a control action should be initiated and this level is generally defined as the economic threshold. Hall and Norgaard (1973) investigated the timing and application policy for pesticides using a model which contained functions for the growth of the pest population, the damage caused by the pest, the crop yield, the pesticide efficiency and the pesticide cost. The quantity and timing of pesticide application were determined by marginal analysis but only a single application of pesticide was considered. This model is not suitable for practical application and in a reply to a comment by Borosh and Talpaz (1974), Hall and Norgaard (1974) stated that the model should only be regarded as a basic exploration of the definition of the economic threshold.

Multiple applications of pesticide are frequently used to control pests and this should be taken into account in evaluating strategies for pesticide use. Chatterjee (1973) developed a model which incorporated multiple applications of pesticide, but the functions used in this model were not specified explicitly. In this model the pest survival rate was assumed to be independent of the cost of a pesticide application, although this may not be a major limitation since the rate of application is generally specified by the manufacturer, and in some cases it may be illegal to depart from this rate. Another model which allowed for multiple pesticide applications was developed by Talpaz and Borosh (1974), but since the unit of time was defined in terms of degree days, factors which depend on calendar time could not be incorporated directly. Further limitations of this model arise from the assumption that pesticide applications must be equally spaced and the assumption that each pesticide application reduces the pest population to the same level.

Models for evaluating pest control through the use of pesticides should take account of pesticide effectiveness and the longer term effects of a pest control strategy. Most pests exposed to intensive use of pesticides over an extended period will develop some resistance to the pesticide and thus the effectiveness of the pesticide will decrease with use. Hueth and Regev (1974) investigated the effect of increasing resistance to pesticide by constructing a model of a pest attack on a crop and studying the resulting optimality conditions under simplifying assumptions regarding, in particular, the nature of the susceptibility of the pest to the pesticide.

No pesticide is completely effective in controlling a pest and further generations of the pests which survive can cause damage not only to the farmer who carried out the control action but also to farmers in neighbouring areas. Regev et al. (1976) used a simple model of a crop-pest system to investigate pesticide application policies from the point of view of both the farmer applying the pesticide and society. Although the influence of crop quality on the value of the crop was considered in this model, the cost of pesticide application was not. Further simplifying assumptions were required because of limitations in the solution method and only the steady state version of the problem was considered, thus ignoring the dynamic nature of the system.

The major limitations of these early models of crop-pest systems arise from the failure to take account of the uncertainty in weather conditions and the influence of the presence of predators, parasites and other pests. In some cases these simplifications were adopted because the chosen solution method was unable to cope with the complexities of the biological processes involved. Simulation does not suffer these limitations and this approach has been used in a number of studies. For example, Reichelderfer and Bender (1979) developed a deterministic simulation model to evaluate chemical and biological methods to control the Mexican bean beatle in a soybean crop. In this model the development of the pest population and the growth of the crop were simulated, with the size and age structure of the pest population determining leaf consumption. The loss in crop yield was assumed to be a function of defoliation. The model was used to evaluate a large number of strategies and it was found that biological control, through the use of a parasitic wasp, was more cost effective than chemical control. However, a limitation of this model is that the development of both pest and crop were assumed to be independent of weather conditions. In a simulation of the European corn borer in corn crops in the US, Loewer (1976) used weather conditions as the main factor influencing the development of the pest population but the development of the crop and the influence of the pest on the crop were not considered. Pesticide application was not considered in this model and the only control strategy investigated was the use of sprinkler irrigation at certain times, although it was noted that such strategies were likely to be impractical.

Simulation models have also been used to investigate the boll weevil in cotton crops. Talpaz et al. (1978) developed a simulation model of a boll weevil attack on a cotton crop using temperature, humidity and solar radiation data, but the presence of other pests was not considered in this model. Brown et al. (1979) noted that although applications of insecticide early in the season can control the boll weevil, the insecticide also destroys insects which help to regulate other pests, principally the cotton bollworm and the tobacco budworm. The effect of both the boll weevil and these other two pests was taken in account in a simulation model developed by Brown et al. (1979) by interfacing a cotton growth model with models of the insect pests. A modification of the boll weevil simulation model of Jones et al. (1977) was used to simulate both the population dynamics of the boll weevil and the damage to the crop resulting from the feeding and oviposition behaviour of the boll weevil as functions of weather, crop development, insecticide application, predation and parasitism. In the model of the other two pests, damage to the crop was a function of larval numbers and age. The overall model was used to evaluate the economic consequences of different insecticide application schedules and it was noted that this model could provide a useful tool for studying the principles of pest management, particularly integrated pest management programmes which involve the use of a combination of chemical, cultural and biological control methods to reduce damage to the environment.

One of the benefits of using simulation models to evaluate pest control policies is that alternative policies can be evaluated on a comparable basis. However, since a set of simulation runs will be required for each policy, simulation methods may be computationally inefficient, particularly in cases where a large number of policies are to be evaluated. This difficulty can be overcome by developing optimisation models for pest control. The early optimisation models in this area used techniques such as marginal analysis which were unable to deal with the complex dynamic nature of the biological processes involved, but more recent models have attempted to incorporate these features.

An optimisation model was constructed by Conway et al. (1975) to investigate insecticide spraying strategies for controlling the sugar cane froghopper, a pest which can cause serious damage to sugar cane. The population dynamics of the pest were modelled using Leslie matrices (Leslie (1945)), i.e. essentially a Markov model. Pest damage was a function of pest population and the solution to the model was derived using a DP approach. The effect of weather conditions was not included in this model and difficulties were experienced in collecting data. Shoemaker (1979) developed a deterministic DP model to determine the best combination of chemical, biological and cultural methods for controlling the alfalfa weevil in an alfalfa crop under a specified weather pattern. In a development of work in this area, Shoemaker (1982) constructed a stochastic DP model to determine the control strategy for a univoltine pest, i.e. a pest with one generation per year, and demonstrated the use of this model in determining the policy for controlling the alfalfa weevil. The main stochastic element in this four-dimensional DP model was weather, with three weather patterns being considered, and a method to reduce the computational requirements for solving this model was suggested. Feldman and Curry (1982) noted that DP is an appropriate technique for developing optimisation models of pest-crop systems because stochastic features can be incorporated in the model, but argued that there is still a need to develop an overall theoretical framework to enable integrated pest management strategies to be evaluated for realistic pest and crop systems.

In spite of the development of disease resistant plant varieties, disease is still a major source of crop damage, and unlike the situation with respect to pests, there has been little progress in developing techniques for evaluating disease management schemes. Much of the work in modelling crop disease has been concerned with estimating crop yield losses due to disease, with most of the models using regression to relate yield losses to the observed level of the disease at one or more stages in the devolopment of the crop (e.g. King (1976), Melville et al. (1976), Mundy (1973)). The main limitation of these models is that causal relationships, e.g. relating disease loss to the presence of a pathogen, were not considered. Simulation models have been proposed for use in evaluating crop disease control policies, although these models can be difficult to validate. For example, Teng et al. (1977) developed a simulation model of a barley leaf rust epidemic and Teng et al. (1980) noted that this model was difficult to validate completely because of the lack of suitable data. Teng and Gaunt (1980), in a review of the literature on modelling disease and yield loss due to disease in cereal crops, argued that progress in developing methods for evaluating crop disease management schemes would depend on closer cooperation between specialists in different areas and the linking of models for disease development and crop growth. However, as noted by Anderson (1971), there is also a need to ensure that agronomic experiments are designed to provide the type of information required for the development of models for use in planning.

Although the purpose of modelling crop-pest and crop-disease systems is generally to investigate strategies for pest or disease control, models have also been used to obtain a better understanding of the systems involved. For example, a simulation model of cereal aphids in a cereal crop was developed by Carter et al. (1982) to explain the population dynamics of cereal aphids. In this model the development of the aphid population was influenced by temperature and the growth stage of the crop, but the effect of natural enemies was ignored. Crop growth was assumed to be a function of accumulated temperature, but the effect of the aphid population on the development of the crop was not considered. It was intended to use this model to help establish simple rules for forecasting cereal aphid outbreaks, but the rules investigated proved unsatisfactory, partly because of the limitations of the model.

Most of the models for evaluating pest and disease control strategies have been developed for use as research tools but there is also a need for techniques which can be used by individual farmers. Some of the techniques which have been developed for use in research could form the basis of methods for use at farm level, especially with the availability of cheap computing facilities, but other approaches for evaluating crop protection strategies may be more suitable for use at farm level. For example, Norton (1979) illustrated the use of a pay-off matrix to evaluate policies for controlling potato blight using historical data. Only a limited number of outcomes, e.g. levels of attack, can be considered using this approach and the decision problem is often more complex than the decision to spray or not spray which was considered in this case. Carlson (1970) demonstrated how subjective probabilities could be used in a Bayesian decision theoretic approach to determine disease control policies. A decision theoretic approach was also used by Webster (1977) to determine the utility maximising spraying strategy against a fungal parasite of wheat, and a tree diagram was used to show the possible combinations of four factors which affect susceptibility to attack. However, the disadvantage of this approach is that the utility function of the individual farmer must first be established. A simple decision tree approach was used by Mumford (1981) to illustrate the problem of controlling pests in sugar beet but although, as noted by Cox (1981), it was assumed that the decision variables could take only discrete values, e.g. the application rate for the pesticide was fixed, this may not be a serious limitation in many cases. The approach could, however, be improved by allowing for the revision of probabilities as a result of the actions taken.

The ultimate aim of model building effort in this area is the development of effective systems for controlling pests and diseases in crops. An example of the type of system which could be produced is EPIPRE, a computer based pest and disease management system for wheat, developed initially for the Netherlands (Rijsdijk (1982)). In this system the farmer makes regular observations of the crop. Data from these observations are input to the model of the crop system, and recommendations on, for example, the use of pesticide and fungicide, are returned to the farmer. Only very limited information on the model was given, but it was reported that the results were encouraging. This system is at present mounted on a mainframe computer, but it is hoped to produce a microcomputer based system which can be used by individual farmers.

2.3 LIVESTOCK PRODUCTION MODELS

Methods of livestock production range from traditional methods based on foraging to intensive methods in which livestock are supplied with food. Intensive methods of production have increased in importance in the livestock sector of the agriculture of the developed world as these methods have been found to be more efficient in economic terms than traditional methods, but the performance of traditional livestock production systems has also improved as a result of advances in, for example, pasture management. The operations involved in livestock production depend on the type of livestock, the production system used and the nature of the product, e.g. meat, eggs or milk. The provision of an adequate source of food, in the form of either foraging or feedstuffs supplied directly to the livestock, is of fundamental importance in livestock production, but other operations such as breeding, replacement of livestock with unsatisfactory performance and waste disposal may also be involved, depending on the nature of the livestock unit.

In intensive operations food must be supplied to the livestock at all times, but even in traditional pasture based systems it may be necessary to supply food to the livestock at certain times, e.g. during the winter or in drought conditions. The formulation of feeds to meet specified nutrient requirements may therefore be an aspect of planning operations in both intensive and traditional production systems, and methods for feed formulation are considered first. The formulation of feeds is, however, only one aspect of livestock feeding. In intensive operations feeding policy, i.e. the sequence of rations to feed, must be determined while in pasture based systems other factors, such as pasture growth, must be considered, and methods for planning production policy in intensive and pasture based systems are considered separately. Methods used for planning breeding and replacement policies for livestock are then discussed, and this is followed by an examination of methods for evaluating waste disposal policies in intensive livestock operations. Finally, methods for planning livestock production operations taking account of the interactions with other activities on the farm, e.g. crop production, are considered.

2.3.1 Diet Formulation

In the livestock feeding context a diet is defined in terms of the proportions of constituent foodstuffs while a ration is defined in terms of the quantities of constituent foodstuffs. The formulation of least cost diets of specified nutrient content by LP (e.g. Dent and Casey (1967), Beneke and Winterboer (1973)) was one of the earliest examples of the use of mathematical programming in agriculture, and LP continues to be widely used in this area, particularly by feed mix companies.

In the LP formulation of the least cost feed mix problem it is assumed that the coefficients specifying the nutrient contents of the ingredients are constants. The estimates of these coefficients are mean values derived from analysis of a large number of samples of the feedstuffs concerned and clearly there will be some variability in these results. If this variability is ignored, as is generally the case, the solution to the least cost diet LP model will on average meet the specified requirements for a particular nutrient on only 50 per cent of occasions, assuming that the distributions involved are normal. Diets which meet the nutrient requirements at a higher confidence level can only be obtained by taking this variability into account. Van de Panne and Popp (1963) have suggested a quadratic programming formulation of the feed mix problem to take the variability in protein content into account, where the variability is measured by variance. Although protein is not the only stochastic constraint in the formulation of feed mixes it is frequently the limiting factor in livestock feeding and in addition, protein supplements are generally expensive. Chen (1973) used this quadratic programming model to determine the relationship between the cost and the probability of meeting a feed specification, while Rahman and Bender (1971) used a linear approximation of variance in the stochastic diet formulation model to enable the model to be solved using LP. Although approaches which take the variability in nutrient content into account are theoretically superior to the deterministic approach, the post optimal analysis which can be performed with these models is limited. This variability could be taken into account in a deterministic model by including a safety allowance in the specification of the mix, and the shadow prices from this model could be interpreted directly. However, the models which take the variability in nutrient content into account could be used to set safety allowances in a more rational way.

2.3.2 Ration Formulation

The major limitation of using least cost diets for livestock feeding is that livestock performance, e.g. liveweight gain or milk yield, is not taken into account. In order to take livestock performance into account in livestock feeding, the nutrient requirements, i.e. the energy, protein, minerals, vitamins, etc., required to produce a specified level of performance must be known in order to calculate the quantities of feedstuffs, i.e. the ration, required to produce this level of performance at least cost. Systems for specifying the nutrient requirements of different livestock are recommended by bodies such as the Agricultural Research Council (ARC) in the UK and the National Research Council (NRC) in the US, although these bodies generally adopt different systems for specifying the nutrient requirements of a particular type of livestock and these differences affect the method for determining the least cost ration to achieve a specified level of livestock performance.

The influence of the system used to specify nutrient requirements can be illustrated by considering the formulation of least cost rations for beef cattle. Different systems for specifying the nutrient requirements of beef cattle have been adopted by the Ministry of Agriculture, Fisheries and Food (MAFF) in the UK (MAFF (1975)) and the NRC in the US (NRC (1976)). In these systems the nutrient requirements of beef cattle are specified in terms of the energy, protein, minerals, etc. required to produce a specified daily liveweight gain in an animal of given liveweight. Although the recommended allowances for protein, minerals, etc. in these two systems are different, the main difference between the two systems lies in the method adopted to specify the energy requirements.

The system adopted by the MAFF (MAFF (1975)) is based on the system developed by the ARC (ARC (1965)). In this system the food energy requirements of the animal and the energy content of feedstuffs are expressed in terms of metabolisable energy, i.e. the food energy which can be used by the animal after faecal, urinary and methane losses have been deducted, and it is assumed that the metabolisable energy requirements of the animal depend on the metabolisable energy concentration of the ration. An approach developed by Kennedy (1972) can be used to formulate least cost rations satisfying the nutrient requirements recommended by the MAFF (1975). This method involves using LP to determine the least cost ration, of specified metabolisable energy concentration, which supplies the energy, protein, etc. required by the animal. The overall least cost ration can then be determined by repeated solution of the LP problem for different values of the metabolisable energy concentration of the ration. However, this approach is time consuming and in practice only a limited number of values of the metabolisable energy concentration of the ration can be considered. To overcome the difficulties associated with the ARC based system for beef cattle, Harkins et al. (1974) developed an alternative system based on net energy, where net energy is metabolisable energy less the heat production of the animal. This variable net energy system allows rations to be formulated in an additive manner, and hence least cost rations can be formulated by LP. This system is recommended by the MAFF (MAFF (1975)), but it is based on approximations which can cause significant errors in ration formulation.

A revised system for specifying the nutrient requirements of cattle has been developed by the ARC (1980). This system is similar to the ARC (1965) system in that the metabolisable energy requirement is a function of the metabolisable energy concentration of the ration, but rumen degradable protein and undegradable protein are also considered, and both these factors are functions of the metabolisable energy intake of the animal. Crabtree (1982) has developed an iterative LP based system for formulating least cost rations to satisfy the nutrient standards recommended by the ARC (1980). This system, like that of Kennedy (1972), involves searching over all possible combinations of the metabolisable energy concentration of the ration, and consequently a large number of LP problems must be solved.

The NRC in the US (NRC (1976)) recommend the use of the net energy system developed by Lofgreen and Garrett (1968) for determining the energy requirements of beef cattle. In this system the net energy requirement, i.e. the metabolisable energy less heat production, of the animal is separated into a component for maintenance of body functions and a component for the production of liveweight gain, and it is assumed that food energy is first used for maintenance with any remaining food energy being used to produce liveweight gain. The efficiencies of utilisation of food energy for maintenance and gain are different and must therefore be specified separately for all feedstuffs. In this system the ration required to produce a specified liveweight gain in an animal of given liveweight depends on the composition of the ration and Brokken (1971a) has suggested a separable programming model to take these interdependencies into account in the formulation of least cost rations for beef cattle. Brokken (1971a) also suggested a method to determine the maximum profit rate of liveweight gain, but in this approach it was assumed that the rate of gain remained constant throughout the fattening period and a complex solution procedure was required.

There are limitations in both the MAFF (1975) and NRC (1976) systems for specifying the nutrient requirements of beef cattle. For example, the differences between breeds of cattle and between individual animals are not considered, although it has been suggested by the NRC (1976) that allowances can be made for these factors. Another limitation of these systems is that the effects of environmental stress in extremes of temperature are not taken into account. In chill stress some energy, which would otherwise be available for liveweight gain, is diverted to heat production for the maintenance of body temperature, while in heat stress feed intake is likely to be reduced and food energy will be diverted to dispersing burdensome heat production. Brokken (1971b) has extended the ration formulation model of Brokken (1971a) to allow for the effects of heat or chill stress by considering the heat increment, i.e. the difference between metabolisable energy and net energy. Economic analysis by Brokken (1971b) suggested that the increased ration costs of taking heat increment into account in ration formulation could be more than offset by the benefits from the resulting improvements in performance, although this clearly depends on the degree of stress, ingredient prices and the way the animal adjusts its feed intake when stress conditions occur. A further limitation of these systems is that the composition of the carcase, for example in terms of its fat free and fatty tissue content, is not taken into account. The NRC (1976) has assumed that the feeding system used will not have a major effect on carcase composition when economic feeding policies are adopted but this assumption has been questioned (e.g. Moe and Tyrrell (1973)).

In livestock production the rations are generally formulated to produce liveweight gain. However, at certain times, e.g. in drought conditions or when livestock are over-wintered under cover, it may be desirable to feed maintenance rations. The maintenance requirements of livestock of different liveweights are specified by bodies such as the ARC in the UK and the NRC in the US. The formulation of least cost maintenance rations is usually simpler than the formulation of least cost rations for the production of liveweight gain. For example, simple LP models can be used to determine the least cost rations to satisfy the maintenance requirements recommended by the MAFF (1975) or the NRC (1976) for cattle, and Vere (1972) used LP to formulate least cost maintenance rations for sheep in drought conditions in Australia.

The importance of incorporating growth response in the formulation of rations for pigs was recognised by Dent (1965) who proposed a method for determining least cost rations for pigs using data from feeding trials to estimate the nutrient requirements of the animal.

An LP model was used to determine the least cost ration of given energy and protein content, and then the overall least cost ration for a specified level of animal performance was determined by systematically changing the energy and protein content of the ration. A disadvantage of this approach is that a large number of LP problems must be solved to determine the overall least cost ration. A further limitation of the approach is that the composition of the liveweight gain in terms of, for example, fat free and fatty tissue is not considered.

It is very important to take carcase composition into account in feeding pigs since pig carcases are graded in terms of their composition, with higher prices being obtained for lean meat. Nutrient allowances for growing pigs have been recommended by both the ARC (1967) and the NRC (1979), but although in both cases it is recognised that feed intake affects both liveweight gain and the composition of this gain, in neither case are allowances expressed in terms of the nutrients required to produce liveweight gains of specified composition. A pig growth model in which daily liveweight gain is separated into fat free and fatty tissue components has been developed by Whittemore and Fawcett (1976), and a development of this model has been incorporated by Fawcett et al. (1978a, 1978b) in an LP model for formulating least cost rations to produce daily liveweight gains of specified composition in pigs of known liveweight.

In dairy farming the need to take milk yield into account in feeding dairy cattle has long been recognised (e.g. Redman (1952)) but many of the applications of LP in formulating rations for dairy cattle, e.g. Howard et al. (1968), failed to take milk yield into account. In the US, Dean et al. (1969) developed a profit maximising LP model for dairy cattle ration formulation by using a piecewise linear function to approximate the non-linear response of milk production to net energy intake, the milk production response being estimated from data from a limited series of feeding trials. Brown and Chandler (1978) in the US suggested an LP model for determining profit maximising rations for dairy cattle using data from more widely based feeding trials and used four sets of linear equations to approximate the non-linear milk production response to energy, dry matter, crude protein and crude fibre intake. A stepwise solution procedure was used to determine the optimal ration but it is not clear that the solution obtained will be globally optimum and it is likely that a simpler approach, e.g. running a least cost ration LP model, such as the LP based dairy feed formulation system of Jones et al. (1980), for a number of different milk production levels, would yield more acceptable results. In the UK the MAFF (1975) have adopted a metabolisable energy system to specify the energy requirements of dairy cattle. Since this system is additive, an LP model can be used for least cost ration formulation. The profit maximising ration can then be determined by running this LP model with different levels of milk production.

A further complication in formulating rations for dairy cows arises because the milk production from an individual cow varies during a lactation, with peak production occurring in about the fifth week after calving. Group feeding is often practised for dairy cows maintained in confinement, and under these conditions there is no control over the feed intake of an individual animal. For this reason herds of milking cows are often divided into groups depending on their stage in the lactation cycle and Spahr (1977) suggested that several rations should be used depending on the stage in the lactation cycle, with cows moving from one group to another as required. Smith (1976) reported that the net income in a two group system for dairy cows was greater than in a one group system even though the total milk production was lower, but the problem of determining the appropriate level of milk production was not considered by Smith (1976) or Spahr (1977).

Ration formulation is an important aspect of livestock feeding, especially in intensive livestock production and LP is a powerful and widely used technique for formulating least cost rations. Although some difficulties arise in formulating rations to supply the nutrients required for specified levels of livestock performance, these difficulties can generally be overcome by using more complex LP based approaches.

2.3.3 Feeding Policy for Intensive Livestock Production

The use of least cost rations in intensive livestock production systems which are concerned with increasing the liveweight of livestock, will ensure that specified liveweight gains can be achieved at minimum cost. However, in this type of operation the economic efficiency of the operation will also depend on the feeding policy, i.e. the sequence of rations, used to produce livestock of some desired final liveweight. In livestock production operations in which the weight at which the livestock are sold can be varied, the effect of marketing policy should also be considered in evaluating feeding policy, since the selling price of the livestock depends on the liveweight. In evaluating feeding policy the ration used should, in theory, change on a daily basis to reflect the increase in weight of the livestock, but for practical administrative reasons daily changes in rations may be undesirable. In addition, a substantial computational load may be involved in evaluating feeding policy if daily increments in liveweight are considered, and for these reasons it may be necessary to assume that the daily ration remains constant for a number of days. However, in evaluating feeding policy it is desirable to keep the length of this feeding period as short as possible so that the effects of changes in the nutrient requirements of the growing livestock can be taken into account.

Barnard (1969) noted that although theoretically marginal analysis could be used to determine the feeding policy which produced the greatest margin of income over feed costs, the ability to apply this theory in practice was limited because the required input-output relationships are constantly changing due to the dynamic nature of livestock growth. Traditional economic analysis of livestock feeding based on the analysis of production functions is thus inadequate because, as noted by Brokken et al. (1976), it is difficult to incorporate factors such as rate of gain, fattening period, individual differences between animals and differences in initial and final weights in the traditional production function framework.

Since livestock feeding is a sequential decision process, with each decision affecting subsequent decisions, dynamic programming (DP) would appear to be a valid approach to determine livestock feeding policy. Meyer and Newett (1970) used a DP model to determine the optimal operating policy, in terms of initial liveweight, final liveweight, length of feeding period and feeding policy, for a beef feedlot operation, i.e. a beef production system in which cattle are purchased at some initial liveweight and transported to the feedlot where they are fed on high energy rations until some specified liveweight is attained. The state variables in the DP model were animal liveweight and the time to reach this liveweight, and it was assumed that a number of least cost diets were available for feeding to the animals. The decision variables in this DP model were the diet used and the number of periods for which this diet was fed, and although the diet remained constant throughout a feeding period the quantity of diet fed, i.e. the ration, increased with animal liveweight. The approach was illustrated using the net energy system of Lofgreen and Garret (1968) and it was assumed that a limited number of least cost diets of specified net energy concentrations for maintenance and gain were available for feeding, where these least cost diets had previously been determined by LP. However, because of the form of this net energy system the use of least cost diets of this form will not, in general, ensure that a specified liveweight gain is achieved at least cost, even if diets of all possible combinations of net energy concentrations for maintenance and gain are considered. For this reason the operating policy obtained using this approach is unlikely to be optimal.

Dynamic programming is, however, potentially a powerful technique for determining livestock feeding policy. Kennedy (1972) developed a DP model to determine the optimal feeding and marketing policies for beef cattle which were fed intensively for part of the year, with grazing being allowed during the pasture growing season. It was assumed that cattle were to be fed according to the nutrient standards recommended by the ARC (1965) and least cost rations to produce specified gains in animals of known liveweight were first determined using an iterative LP approach. In the DP model decisions were assumed to be made every 28 days and thus the daily liveweight gain remained constant during periods of 28 days, the cost of producing this gain being determined from the least cost ration in the middle of the period. In order to incorporate the possibility of pasture grazing in the model a number of simplifying assumptions were made regarding, for example, the nutritional value of grass and the effect of stocking rate on grass consumption. The difficulty in determining the value of grass as a feedstuff was also noted although the method adopted, namely an annual charge based on the opportunity cost of land plus a direct pasture management charge, is probably an over-simplification. Examination of the results obtained using the model over 26 periods each of 28 days suggests that the method for dealing with conditions at the end of the planning period may need improvement since, for example, cattle were sold at or near the end of the planning period without replacement. In spite of these limitations the approach does attempt to incorporate the important decisions associated with this type of beef cattle operation and it could provide the basis for further development work.

A similar DP approach was also used by Kennedy et al. (1976) to determine the feeding and selling strategy in broiler production. The decision variable used in this DP model was the energy density of the feed intake, and LP was used outside the DP model to determine the least cost diets of specified energy density. A simple DP model was used by Clark and Kumar (1978) to determine the feeding and marketing strategies for beef production. Only an extremely narrow range of policies was considered in this model and since the imputed value of liveweight gain was evaluated at the end of each stage in the solution procedure, the influence of cash flow was not considered. A DP model has also been used by Topham (1979) to determine calf feeding policies taking account of the milk from the mother and using least cost feeds determined by LP. In this approach only a narrow range of feeding policies was evaluated and although the use of enterprise grown feedstuffs was considered, the possibility of purchasing a feed which could be grown on the farm was not taken into account.

Although LP has been widely used in diet and ration formulation, the use of LP in evaluating feeding policy is more limited. For example, Wilton et al. (1974) and Ashour and Anderson (1975) included animal feeding in models which integrated aspects of crop and beef production. However, in both these cases predetermined rates of liveweight gain were used throughout the fattening period, and the inclusion of livestock feeding was limited to determining the rations to produce these liveweight gains. If these models were modified to increase the number of liveweight intervals during which the rate of liveweight gain was fixed, then, by systematically changing the rate of liveweight gain in each liveweight interval, these models could be used to determine optimal feeding and marketing strategies, although the computational effort would be substantial. Klein et al. (1979) used an LP model to evaluate feeding policies for turkey production, particularly in relation to the use of rapeseed meal produced from recently developed rapeseed varieties having lower levels of growth and reproduction inhibitors than traditional rapeseed varieties. However, a limitation of this approach is that the diets which can be used must be specified in advance in terms of their rapeseed meal and energy content.

Enumerative techniques were used by Battese et al. (1968) to determine the rations for pigs fed separated milk and grain. Feeding trial results were used to derive expressions for the time to reach final liveweight, the quantity of milk, the quantity of grain and the carcase composition as functions of the milk and grain content of the diet. These expressions, together with cost and revenue data, were used to derive expressions for profit per unit time and profit per pig and then the optimal rations were found by enumeration. Errors in the feed consumption model have been noted by Townsley (1969) and a further disadvantage of this approach is that it is necessary to assume that the diet remains constant throughout the fattening period.

Heady et al. (1976) used weight gain isoquants estimated from feeding trials to derive an expression for the profit from pig production, and then used an enumerative approach to determine the pig feeding policy which maximised profit per unit time. In this approach it was assumed that initial and final liveweights were fixed and that the ration remained constant over specified liveweight ranges. A similar approach was also used by Melton et al. (1978) to evaluate the impact of different criteria on beef fattening operations, but clearly only a limited number of policies can be evaluated in this way. Bhide et al. (1980) used a similar approach to determine the feeding policy for beef cattle using criteria such as the cost of feedstuffs and the time to achieve total weight gain, although in this case the model was solved analytically where possible. Kennedy (1981) has noted that the formulation of problems using this approach was probably constrained by the solution technique and argued that DP would have been a more satisfactory method for tackling problems of this type.

Econometric analysis was used by Crabtree (1977) to evaluate feeding policies in bacon pig production and although carcase composition was included in the analysis, the approach is of limited use since only one diet was used over the entire growth period. Moreover, to perform the marginal analysis to determine the feeding policy to maximise profit per unit time, it was necessary to assume that the final liveweight was fixed and that both feed intake and dietary protein content could be considered separately in analysing growth and carcase quality response.

Brokken et al. (1976) have noted the difficulties of incorporating factors such as the rate of gain, the fattening period and the initial and final weights of livestock in a production function and have suggested a framework for the economic analysis of feeding policies for feedlot cattle. In this approach it was assumed that both the appetite and the voluntary energy intake of an animal, and hence its performance, depend on the nutrient concentration of the ration, and a system which related animal performance to the energy concentration of the ration was developed. The energy allowances recommended by the NRC (1976) were used and experimental results were used to establish relationships for both appetite and voluntary energy intake as functions of the energy concentration of the ration. The system was used to evaluate feeding policies under criteria such as profit per animal and profit per animal per unit time. However, the approach depends on the functions used being differentiable and on the assumption that both the diet and the rate of gain remain constant throughout the fattening period.

Simultation models have also been used to evaluate feeding and marketing policy in intensive livestock operations. For example, simulation models have been used by Dent (1971) to evaluate pig production policies, by Ryan (1974) to evaluate beef production policies and by Greig et al. (1977) to evaluate poultry production policies. In using a simulation model to investigate livestock production strategy, each policy to be evaluated must be specified at the start of a run of the model, although a ration formulation LP model may be associated with the simulation model to ensure that least cost rations are used throughout the the production process. In the pig production simulation model of Dent (1971) least cost rations were calculated outside the simulation model and the development of groups of pigs was simulated. The only stochastic element in this model was the market price of pigs and it was assumed that all pigs in a group were sold at the same time. In the beef feedlot simulation model of Ryan (1974) the development of individual animals was modelled on a daily basis, with least cost rations, which were calculated outside the simulation model, being used to produce constant daily liveweight gains between specified liveweights. This model attempted to incorporate the stochastic nature of the feeding response by random sampling of the growth rate to be achieved by an individual animal. This stochastic element was introduced to evaluate the effect of selling animals individually once a particular weight was attained, rather than selling a group of animals when the average liveweight reached a specified level. Although the importance of individual differences in the feeding response of livestock has been recognised by others (e.g. Barnard (1969), Dent (1971), Fawcett (1973)), additional administrative work will be involved in selling individual animals when a specified liveweight is reached and, as noted by Dent (1971), the opportunity cost of having a portion of livestock housing underused may be high. The practical significance of attempting to model individual differences in feeding response may therefore be limited.

The major disadvantage of simulation methods in evaluating feeding and marketing policies for livestock is that these methods are inefficient for evaluating a large number of possible policies since a separate run of the simulation model is required for each policy which is to be investigated. However, simulation models are generally easier to explain than other types of model and may therefore be more readily accepted by decision makers.

2.3.4 Livestock Production on Pasture

In spite of the development of intensive livestock production systems, the grazing of pasture still plays an important part in livestock production especially in the beef, dairy and sheep sectors. Even in North America where intensive beef production systems are widely used, the grazing of pasture makes an important contribution to beef production since in most cases cattle are reared on pasture before being transferred to a feedlot. The economic efficiency of extensive livestock production has benefitted from the adoption of improved methods of pasture management and, because of the seasonal nature of pasture growth, the use of concentrates to supplement feeding during part of the year has also improved the economic performance. In extensive livestock production the feed intake of the livestock cannot be controlled as in intensive operations, although it can be regulated to some extent by grazing strategy. The evaluation of extensive livestock production systems is therefore more complex than the evaluation of intensive systems since not only livestock feed requirements but also the seasonal variation in both pasture growth and digestibility should be taken into account. Moreover, the growth and digestibility of pasture depends not only on uncertain weather factors but also on controllable variables such as the stocking rate, the application of fertilisers and the use of irrigation.

There have been a number of studies which have examined intensive feeding of livestock in combination with extensive grazing. Halter and Dean (1965) constructed a simulation model of a combined cattle ranch and feedlot operation in which cattle were reared on the ranch and transferred to the feedlot in the spring at a rate determined mainly by the feed supply on the range. The stochastic elements in this model were weather conditions and the prices of both products and feedstuffs. Although the influence of both weather and grazing on feed supply was considered in this model, the feeding of cattle in the feedlot was based on an average cost per head per day. Kennedy (1972) developed a DP model for beef production where intensive feeding could be combined with pasture grazing. In this model the pasture growth was deterministic and the model was run with different stocking rates during the pasture growing season to evaluate the effect of stocking rate on operating policy. In the LP model of an integrated beef and crop production enterprise developed by Wilton et al. (1974), the pasture growth was also deterministic and only cows from the breeding herd and replacement heifers were assumed to have access to pasture in summer.

Modelling livestock production based on the grazing of pasture should involve the development and integration of livestock and pasture growth models. Simulation models are well suited to this approach, but although this approach has been used for investigating extensive cattle production (e.g. Morley et al. (1978), Loewer et al. (1981)), much of the work has been concerned with sheep. Arnold et al. (1977) developed a simulation model for food intake, liveweight change and wool production in growing sheep and this model was combined with a simulation model of the growth of subterranean clover pasture in South Australia (Galbraith et al. (1980)) to investigate grazing and stocking policy. Both the sheep and pasture models relied heavily on subjective assessments of some of the relationships involved and the models were therefore difficult to validate. Sibbald et al. (1979) argued that in modelling hill sheep farming operations the wide variation in pasture quality, both within and between seasons, and the preferences of sheep for certain components of the pasture should be taken into account, but because of the lack of adequate data, the model outlined by Sibbald et al. (1979) also relied heavily on subjective assessment of certain relationships. White et al. (1983) developed a simulation model of a breeding ewe flock, but this models also included a number of relationships containing subjective elements. The need for further research in modelling herbage growth was also noted by Edelsten and Newton (1977) who used a simulation model to investigate stocking and grazing policy for a flock of lowland sheep.

In extensive grazing the food intake of the livestock can be regulated by confining the livestock to certain areas. Geisler et al. (1979) used a simple model of lamb production to evaluate the use of an autumn catch crop for fattening lambs. The feed intake of the lambs was controlled by folding, but it was assumed that the rate of liveweight gain remained constant during the whole, or major parts of, the fattening period. In sheep farming the stocking rate affects not only the wool weight per animal but also the wool fibre diameter, with both wool weight per animal and fibre diameter decreasing with increasing stocking rate. Since finer fibre generally attracts a higher price, the effect of stocking rate on economic performance is complex. White and Morley (1977) developed a simulation model taking account of the influence of stocking rate on both wool production and wool fibre diameter, but it was difficult to establish the relationships between these factors.

A further factor which can be controlled in a pasture based livestock production system is the use of fertiliser to promote pasture growth. Richards and Hobson (1979) considered the problem of determining the nitrogenous feriliser rate for a dairy farm in which part of the grassland was used for grazing and the remainder was used for silage production. In this model the food requirements of the cows were expressed in terms of the dry matter intake, while the dry matter contents of the grass and silage were derived from total nitrogen supply, i.e. fertiliser plus soil nitrogen. The fertiliser rate and the division of the grassland between grazing and silage were determined by numerical solution of equations for dry matter requirements, but neither the effect of weather nor the dynamic nature of the feed requirements were considered.

The major limitation to the development of models for evaluating extensive livestock production systems is the lack of data on some of the relationships involved in the operation of these systems. Many of the models which have been developed are based on subjective assessment of some of the relationships and further research is required to enable the modelling effort in this area to proceed. The major requirement is for research on pasture growth and digestibility, and particularly on the influence of grazing on these factors.

2.3.5 Livestock Breeding and Replacement

The types of livestock used in agriculturally advanced countries, particularly in intensive livestock operations, have been developed through selective breeding programmes for traits which are economically desirable. Since experimental evaluation of breeding programmes is both costly and time consuming, model building methods have been used to evaluate breeding systems. For example, Smith (1964) used a simple index to estimate the genetic improvement obtained by different breeding schemes and Moave (1966) evaluated crossbreeding systems assuming that the profit in commercial livestock production could be expressed as a function of the reproductive performance of the parents and the productive efficiency and quality of their offspring. However, these methods for evaluating breeding systems do not take account of other factors, such as feeding and marketing policy, which influence the economic efficiency of livestock production.

Breeding systems have been examined in the context of the operating environment by incorporating breeding activities in a model of a livestock production system. For example, Long et al. (1975) used an LP model of a beef production enterprise to evaluate straightbred breeding systems which differed in the mature weight of the breeding females. The same model was used by Fitzhugh et al. (1975) to evaluate crossbreeding systems from mating straightbred sire and dam lines, and by Cartwright et al. (1975) to evaluate more complex breeding plans. Morris and Wilton (1975) investigated the influence of mature cow weight on the economic efficiency of beef production using the LP model of Wilton et al. (1974) which included calf breeding and rearing activities in a model of an integrated beef and crop production enterprise. However, with this type of approach a large number of runs of the LP model would be required to evaluate the complete range of operating policies since factors such as breeding scheme and feeding policy must be specified at the start of a run. Congleton and Goodwill (1980) noted that in the models of Wilton et al. (1974) and Morris and Wilton (1975) the age structure of the herd was independent of the mating policy, and developed a simulation model to evaluate breeding programmes for a beef cattle herd, but feeding policy was not considered in this model.

Livestock breeding can only be included in a rather broad and general way in models of livestock production systems, but methods have been suggested to deal with livestock breeding in a more detailed way. For example, Schneeberer et al. (1982) applied portfolio theory to the selection of dairy sires for use in artificial insemination, but in this approach the utility function of the decision maker must be known and therefore this method is not likely to be of much practical value. Reproductive performance is determined by intrinsic biological factors and by management policies, and simulation methods have also been used to evaluate management policies for improving reproductive performance. For example, Oltenacu et al. (1981) used a simulation model (Oltenacu et al. (1980)) to evaluate management policies, particularly with respect to heat detection and the use of artificial insemination, for improving reproductive performance in a dairy herd, but the parameters of this model had to be derived indirectly. Optimisation models have also been used to improve reproductive performance in livestock production, an example being the LP model of Johns and Pearse (1970) which was used to evaluate mating policies for lamb production in New South Wales. Mating policy was defined in terms of the time of mating, and the model was used to determine the optimal mating policy under different market and weather conditions.

In the examples which have been used to illustrate methods for evaluating livestock breeding policies it was assumed that the livestock were bred either to satisfy some specified requirement or because of the nature of the operations, e.g. to replace livestock which were sold on reaching a certain liveweight. In livestock operations where the performance of the livestock deteriorates with age, e.g. in milk or egg production, decisions must also be taken on the replacement of livestock with unsatisfactory performance. Pioneering work in this area was carried out by White (1959) who developed a deterministic DP model to derive the optimal replacement policy for laying hens and suggested that this model could form the basis for a stochastic model in which other factors which influence the decision to replace birds could be included, e.g. variation in the rate of laying, the time to reach maturity and mortality. It would also be desirable to include feed costs in a stochastic model since these vary with the rate of egg production. Throsby (1965) has noted that DP is an appropriate technique for livestock replacement problems and a deterministic DP model has also been used by Low and Brookhouse (1967) to determine the replacement policy for birds in a commercial egg production unit, with the replacement policy being defined in terms of housing dates and laying season lengths.

Cows in dairy herds are replaced to cover natural losses and the disposal of cows with unsatisfactory milking performance. Smith (1977) used a DP model containing stochastic elements to determine the replacement policy for dairy cows, with the states in the model being defined in terms of the lactation number and the milk production in a lactation. Improvements in genetic capacity were incorporated by assuming that the milk production of the replacement cow was one per cent greater than that of the cow it replaced, but since the milk production states were separated by 250 pounds of milk and the range of possible milk production during a lactation was 5000 - 12000 pounds, these improvements in genetic potential may not manifest themselves. The influence of cow size on milk yield should have been included in this model but it was noted that this was not done because of lack of data. Stewart et al. (1977) developed a deterministic DP model for replacing cows in a dairy herd. The state of a cow in this model was defined in terms of lactation number, body weight, 305-day milk yield and milk fat percentage, while the measure of performance used was the expected present value of milk and beef sales, less feeding and replacement costs. The model was used to examine the sensitivity of the solution to changes in prices and milk yields over a ten year planning period, but difficulties were encountered in obtaining reliable data on many of the biological and economic components of the system.

Simulation models have also been used to evaluate livestock replacement policies. For example, Gartner and Herbert (1979) used a simulation approach to evaluate dairy cow replacement policies, taking account of the genetic improvement resulting from the use of artificial insemination with performance tested sires. However, a time interval of one year with a fixed feeding policy was used in this model and a further weakness is that models of this type are very difficult to validate. Simulation methods have also been used to investigate replacement strategy for other types of livestock. For example, Walsingham et al. (1977) used a simulation model of a commercial rabbit production operation to evaluate replacement policies for breeding stock, although the cost of these policies was not considered.

Livestock breeding and replacement policy forms only part of the management policy of a livestock production unit and ideally breeding and replacement decisions should be made in the context of the operation of the entire system. Although some methods for investigating breeding and replacement policy, e.g. Wilton et al. (1974), Long et al. (1975), have incorporated aspects of other activities, these approaches have concentrated on the primary products of the system. For example, in beef cattle operations much of the effort has focussed on the primary product while secondary products, such as cull cows from the breeding herd, have received little attention although the feeding and marketing of cull cows affects both replacement policy and economic performance. Yager et al. (1980)

noted that in many regions in the US a spring calving policy is adopted and that under this policy operators are more likely to cull in autumn and spring, with old cows being culled in the autumn after calves have been weaned and cows which have failed to calve successfully being culled in the spring. Since cows removed from the breeding herd tend to be marketed immediately, the spring calving policy results in a distinct seasonal pattern in both the supply and market price of cows. A stochastic DP model was developed by Yager et al. (1980) to evaluate feeding and marketing policies for cows removed from breeding herds in the US. In this model the market price of cows was the only stochastic variable and least cost rations, determined by LP, were used for animal feeding. The results from this model suggested that significant benefits could be obtained from alternative cow disposal strategies.

2.3.6 Waste Disposal

The handling and disposal of livestock waste is an important problem in livestock production, particularly in intensive operations. Since livestock waste has considerable value as a fertiliser, land spreading is often the preferred method for waste disposal, especially when some of the feeds used in livestock production are grown on land which is owned or leased by the producer. However, to reduce the risk of envoronmental pollution there will generally be restrictions on the amount of waste which can be disposed of in this way. Equipment has also been developed for treating animal waste in a way which reduces the pollution potential and facilitates its economic utilisation. Mathematical models have been used to evaluate the operation of equipment of this type, an example being the model of a pig slurry treatment system developed by Audsley et al. (1977).

Land spreading of livestock waste is generally the most economic method of waste disposal in intensive livestock production operations where suitable land is available. O'Callaghan et al. (1971) used a simulation model to evaluate policies for collecting, storing and land spreading of the waste from a pig fattening unit. Empirical relationships were used to estimate the amount of waste produced by pigs fed according to a specified feeding system, and the model was used to determine the rate at which this waste could be spread without producing water pollution or soil and crop damage. Rainfall and other climatic factors were incorporated in the simulation model developed by Wensink and Miner (1977) for investigating the sizing and operating policy of waste retention ponds in cattle feedlots. Ashour and Anderson (1975) used an LP model to determine the limit to the development of beef cattle feedlots in an area where land was used for both feed production and waste disposal subject to environmental constraints. Although ration formulation and waste disposal costs were included in the model, the same ration was used for all animals in the feedlot and it was assumed that the waste produced per animal was independent of the ration used.

In less intensive operations the waste produced may not be sufficient to satisfy the fertiliser requirements for crop production and additional nutrients may be required from other sources, e.g. chemical fertilisers. Dodd et al. (1975) developed an LP model of a dairy or beef grassland farm in which some of the land was used to produce silage for winter feeding. The model was used to determine the optimal policy for spreading manure on the land which was used for silage production, allowing for the purchase of additional fertiliser from chemical or animal sources. An LP model was also used by Coote et al. (1976) to evaluate waste disposal policies on a dairy farm, and Safley et al. (1979) used an LP approach to evaluate waste disposal policies on a dairy farm, taking account of pollutant levels and the nitrogen requirements of the crop.

The methods of waste disposal which can be adopted by a livestock producer are generally affected by statutory regulations, and models have been developed to investigate the effect of regulations of this type in the US. For example, Forster (1975) used a deterministic simulation model of a typical US beef feedlot to evaluate the impact of possible water pollution control policies. The model included equations to provide estimates of both the price expectations of producers and the prices realised. However, since there was no stochastic element in the model the results were dominated by the cyclic nature of the historic price movements in the US beef sector, and therefore the model would be of little value to individual producers. Ashraf and Christensen (1974) developed an LP model to investigate the impact of different water pollution standards on dairy farms with crop production activities. Variables for fertiliser use and different methods of waste disposal were included in the LP model, and because competition for labour was intensified during spring and autumn as a result of using the new waste disposal systems, particular attention was paid to the seasonal labour requirements for livestock, crop and waste disposal activities. Since the waste disposal methods were mutually exclusive or involved discrete levels of investment, a mixed integer programming formulation would have been more appropriate, and because of these difficulties the model was solved case by case for each waste disposal system. However, if factors which affect waste disposal on specific farms were included, e.g. slope of land, rainfall, soil type and level of water pollution, this model could be used to evaluate waste disposal policies for individual operators.

2.3.7 Planning in a Livestock Production Unit

The operation of a livestock production unit involves dynamic interactions between a number of factors. In an intensive livestock production unit only livestock related factors may be involved, e.g. livestock feeding, livestock replacement and waste disposal, but where livestock production forms only part of the operations of an enterprise, the interactions with the other activities, e.g. crop production, should be taken into account in planning the operations of the livestock production unit. Ideally the models used for planning livestock operations should incorporate these interactions, but because of the complexity of the systems involved many of the models are restricted to particular aspects of livestock production.

The complex nature of biological processes should also be taken into account in planning livestock operations. For example, in dairy farming the milk production of an individual cow varies during a lactation and the quantity of milk produced depends on the feeding policy and the milk production potential of the animal. Other factors, such as the replacement policy for cows, should also be considered in evaluating operating policy for dairy farms but most of the models of dairy production consider only a limited number of aspects of dairy operations. Indeed, many of the models of dairy production have been restricted to ration formulation, but although the variation in milk yield during a lactation was taken into account by Spahr (1977), many dairy cattle ration formulation models have considered only the overall milk production level (e.g. Brown and

Chandler (1978), Jones et al. (1980)). An LP model was used by Reyes et al. (1981) to determine the optimal level of milk production during a lactation, but although the influence of different weight gain and loss strategies was considered, the variation in milk production during a lactation was not included. The effect of differences in milk yield between cows was taken into account in a two stage approach developed by Amir et al. (1980) to determine the herd composition which maximised long term net income from milk production in a dairy farm in Israel. Cows were classified in terms of their milk production potential, and enumeration was used to determine the optimal number of lactations for cows of each production class. The net incomes from cows in each production class were then used as the objective function coefficients in an LP model, but the feeding policy during the calving cycle and the age structure of the herd were not considered.

A number of different mathematical programming models have been developed for use in planning the operations of livestock production units. Mathematical programming models of intensive livestock production generally have concentrated on feeding and marketing policy (e.g. Meyer and Newett (1970), Kennedy et al. (1976)) and although some models of this type have incorporated other activities, such as crop production or waste disposal, the use of these models has been restricted by the nature of the simplifying assumptions. For example, although calf rearing, cattle feeding and crop production activities were included in the LP model of Wilton et al. (1974), cattle growth rates and marketing policy were fixed, and since it was assumed that the age distribution and attrition rates of the animals were constant, this model could not be used to examine the transient behaviour resulting from, for example, a change in the size of the herd. LP models incorporating livestock and crop production activities have also been developed by Ashour and Anderson (1975) and by Miller et al. (1978), but although only a limited number of activities were included in these models, useful information for planning purposes could be obtained from these models by carrying out sensitivity analysis. LP models have also been developed for use in planning the long term development of a livestock enterprise. For example, an LP model was used by Swart et al. (1975) to plan the development of a large US dairy farm. The model integrated livestock and cropping activities and was used to determine herd size, animal selling policy and cropping policy and to evaluate opportunities for renting additional land. Cattle were classified in terms of age and sex but it was necessary to specify the ration for animals in each class.

A major limitation of traditional LP models in farm planning is that the risk resulting from variation in, for example, market prices and crop yields, is not taken into account. Although LP has been used with game theory (e.g. McInerney (1969), Hazell (1970)) and in extensions of portfolio theory (e.g. Hazell (1971)) to take account of risk in agricultural planning, these methods have generally been illustrated by crop production examples. However, LP has also been used in combination with other techniques to deal with some aspects of risk in livestock production. For example, Gebremeskel and Shumway (1979) illustrated the development of a risk-constrained LP model of a cow-calf operation based on the grazing of pasture. This model was used to produce farm plans using decision theoretic concepts which assumed a knowledge of the utility function of the farmer. However, this type of approach may only add complexity without providing the insight which it is generally recognised (e.g. Balm (1980)) can be obtained by analysing the results of a traditional LP model in the light of the risks which are known to exist. LP has also been used in combination with simulation to take account of some of the risks in livestock production. For example, Trebeck and Hardaker (1972) proposed a combined simulation and LP approach for evaluating cattle production policy in farms in New South Wales where stocking rate was determined at the start of the production cycle and subsequent feeding policy was influenced by weather conditions. A stochastic LP model was used to determine feeding policy under different rainfall conditions, and the rainfall encountered was then simulated. This model was used to determine both the expected value and variance of profit under different stocking rates, and although the influence of other stochastic elements was ignored, it was argued that the results gave some indication of the risks involved.

Simulation models have also been used for planning both the operations and investment policy of livestock production units. Dent (1971) used a simulation model to evaluate livestock performance and capital investment in a pig production unit consisting of a breeding unit and two fattening units, where pigs were transferred between fattening units when they reached a certain weight. In this model the pigs in a group grew at a specified average rate, but the market price of pigs was stochastic. Nye et al. (1980) also used a simulation model to evaluate the facilities required for pig production, but in this case the model contained no stochastic elements.

A simulation model must contain sufficient detail of the operation of the real system to be of practical value for planning, but the cost of building a model of an individual unit may not be justified, particularly for small enterprises. The cost to the individual farm can be reduced by using representative farm models, but Blackie and Dent (1974) argued that the management information potential of simulation models could be improved by consructing 'skeleton' models representing the logical structure of livestock enterprises, and then applying the appropriate model to an individual enterprise using data for that enterprise. Leung et al. (1979) advocated a similar approach and developed a general framework for simulating livestock production systems. The stochastic nature of livestock production was incorporated in this generalised model and the model was used to evaluate pig production systems in Hawaii. Simulation models of intensive livestock production have also been used to provide guidelines for individual producers. For example, Ryan (1974) developed a beef feedlot simulation model in which the growth of individual animals was stochastic and this model was used to compare the effect of selling animals on either a group or individual basis, while Blackie and Dent (1976) used a simulation model of a pig production unit to evaluate the commercial viability of new production systems.

Simulation models have also been developed to evaluate policy in farms in which livestock production is combined with crop production. For example, Lovering and McIsaac (1981) have used a deterministic simulation model of a dairy farm to evaluate methods of forage conservation, harvesting policy, feeding policy and waste disposal systems, but the use of pasture and the raising of replacement cows were not considered in this model. A simulation model has also been used by Klein and Sonntag (1982) to examine management strategies in a combined beef, forage and grain farm in Canada. A number of other simulation models have, as noted in Section 2.3.4, been developed for livestock production based on the grazing of pasture, but the major limitation in the development of these models has been the lack of data on the interactions involved in these systems, particularly with respect to pasture growth and livestock grazing.

Most of the simulation models of livestock operations are either deterministic for a given set of input data, or incorporate a very small number of stochastic elements. The absence of a stochastic element in livestock growth is not likely to be of major importance in models of intensive livestock production units in which the feed inputs and the environment can be controlled, since the results can be taken to relate to average livestock performance, particularly in operations where group feeding is practised. In models of operations of this type it may be sufficient to consider market prices as the only stochastic variable. Where livestock production is combined with crop production, either for grazing or production of winter feeding, weather is a major source of uncertainty. In simulation models of operations of this type it would therefore be desirable to incorporate the relationship between weather conditions and crop growth so that the effect of different weather conditions could be investigated by running the model with different sets of weather data, but the lack of adequate data on these relationships has generally prevented the adoption of this approach. The weather data used in this type of approach could be either historical or simulated. Phillips (1971) has criticised the use of historical weather data as being an unnecessary restriction on the generality of a model but Ryan (1973) has argued that historical weather data can be regarded as a sample provided by nature, and that the use of historical data overcomes the validation problems associated with simulated weather data.

In addition to mathematical programming and simulation models, other less complex methods have also been proposed for planning livestock operations. For example, Audsley et al. (1976) used a simple model of the total feed requirements of dairy cows over the whole of the winter feeding period to evaluate different feeding systems and methods for storing and handling winter forage. It was assumed that winter rations were composed of two components, namely bulk forage and concentrate, and that least cost rations must satisfy only two constraints, namely energy and dry matter appetite. This approach simplified the formulation of rations since it was implicitly assumed that protein requirements would be satisfied automatically, but protein has been found to be a critical requirement (e.g. Van de Panne and Popp (1963)) and protein supplements can be costly. Simple cost based models, such as that used by Bryden (1978) to compare the benefits of sheep and deer farming, may also be useful for the initial analysis of investment decisions, while simple cash flow models, such as the dairy herd cash flow model of France et al. (1982) which was adapted for use on a programmable calculator, may be of some value in comparing operating policies, particularly when access to computing facilities is limited.

It has been seen that a large number of models have been developed for use in farm planning, but it has been noted (e.g. Kennedy (1973), Nix (1979), Bywater (1981)) that few of these models have been used in practice and that many are primarily research or teaching aids. The adoption of a farm planning technique depends not only on the development of an adequate model but also requires the development of associated information and control systems. A system for the regular collection and recording of relevant data, such as the management information system for dairy producers outlined by Bywater (1981), must be available if a model is to be used on a regular basis, but the limited use of many of these models may be partly because they are too complex for routine application.

Kennedy (1973) argued that an important reason for the limited use of many of these farm planning techniques was that the development of the control systems, which should be asociated with the planning techniques, was largely ignored. Planning in agriculture takes place in an uncertain environment and therefore the plans for attaining some specified objective should be revised regularly using the most recent data available, and in order to do this a control system is required to monitor discrepancies between the outcomes predicted by the model and the actual results. Many of the parameters in models of agricultural systems are estimates and these should be revised on the basis of new information. For example, Bayes' theorem is a convenient method for revising probability distributions and has been used by Bullock and Logan (1970) for forecasting livestock prices and these forecasts were then used for marketing and production decisions.

The application of an integrated planning and control system was illustrated by Kennedy (1973) using a simulation model of a beef feedlot in which the animals were classified by their gain potential on the basis of their weight gain over a number of periods. Three gain potential classes, i.e. average, above average and below average, were considered and the DP model of Kennedy (1972) was used to evaluate the effect on gross margins of not using the optimal ration for animals of each gain potential class. The actual weight gain of each animal was then simulated and the gain potential class of each animal was revised at the end of each period using Bayes' theorem. The effect of varying the length of a feeding period, i.e. the time between successive simulated weighings of the animals, was also investigated. Although a control system in which the performance of animals is continually monitored is costly to set up and operate, Kennedy (1973) argued that this type of system would be appropriate where the cost of errors is high and where actual performance can be monitored frequently at relatively low cost.

2.4 REGIONAL AND NATIONAL LEVEL MODELS

In most countries some planning is undertaken in the agricultural sector at regional or national level, although the extent of this planning depends on the philosophy of the government and on the stage of development of the country. The objectives of this type of planning also vary from country to country but it is likely to be concerned with factors such as the level of domestic food production, the provision of employment, agricultural production for export markets or regional policy. The extent of planning in the agricultural sector is greatest in centrally planned economies and in developing countries where plans can be implemented directly by government. In other countries indirect methods, such as grants and price supports, may be used to achieve desired goals in the agricultural sector, and in these countries mathematical models may be used to evaluate these policies or to investigate the effect of proposed legislation, e.g. environmental pollution control legislation, in the agricultural sector. Even in countries where plans cannot be implemented directly by government, the construction and analysis of models of the whole or part of the agricultural sector may improve understanding of this sector of a national economy.

2.4.1 Agricultural Sector Models

A number of models of the agricultural sectors of different countries have been developed either for use in planning agricultural development or as research tools for policy evaluation. The principal models of this type are based on mathematical programming methods, but econometric and simulation methods have also been used. The major limitation of econometric models of the agricultural sector at national level, e.g. the Wharton Agricultural Model of the US (Chen (1977)), is that these models are highly aggregated and it is difficult to relate changes at the aggregate level to changes at the regional or farm level. Chen (1977) has noted that econometric analysis has failed to make a substantial contribution to policy formulation in agiculture.

Simulation methods have also been used to model the agricultural sector at national level. For example, the Michigan State University Agricultural Sector Simulation Team (1973) developed a model of the Nigerian economy, with particular emphasis being placed on the agricultural sector because of the importance of agriculture to the economy of a developing country. This model has been used to evaluate strategies in a number of areas, e.g. policies for tetse fly eradication and policies for stimulating output of various products, while components of this model have been adapted for other countries. For example, Miller and Halter (1973) modified the cattle component of this model to evaluate policies to shift the Venezuelian cattle industry from traditional to modern production methods. The major difficulty with simulation models of the agricultual sector is that they are very difficult to validate, but models of this type may be useful, as suggested by the Michigan State University Agricultural Sector Simulation Team (1973), in helping decision makers in developing countries to identify new and economically feasible policy options.

Many agricultural sector models are LP based in spite of the limitations of LP, particularly the linearity assumption which implies constant returns to scale. Walker and Monypenny (1976) noted that since these LP based models were generally related to particular economic and environmental conditions, it is difficult to make comparisons between models. These LP based models are frequently referred to as aggregative programming models since they use data which has been aggregated on either a farm or regional basis, with this data being obtained from farm operations over large sections of the country. The data requirements of models of this type are substantial, and Sluczanowski (1976) noted that these large scale LP based models could be characterised by the extent of their data bases.

Regional production and demand patterns were used in the LP model developed by Heady and Egbert (1964) to analyse the efficient allocation of crop production in the US. In this model the US was divided into 122 regions but only a limited number of cropping activities were considered and no livestock activities were included. The demand for each commodity was estimated for an assumed set of commodity prices using a national demand model, and the LP model was used to determine the crop production pattern which minimised supply cost. Although there were no theoretical developments associated with this approach, a significant effort was involved in assembling the basic data and in setting up the model.

A representative farm approach (see, for example, Sharples (1969)) was used in the multi-year national model of the US Department of Agriculture (Sharples and Schaller (1968)), with separate LP sub-models being used for each of a number of representative farms defined by factors such as farm size and soil type. Only crop production activities were included in this model, and although it had been intended to extend the model to include livestock activities, Walker and Monypenny (1976) reported that the development of this model was discontinued in 1973 because of data collection difficulties and changes in the structure of US agriculture. The model incorporated flexibility constraints which imposed upper and lower bounds on allowable changes from one year to another and helped to overcome some of the difficulties associated with inadequate understanding of the influence of both the non-monetary objectives and the attitudes to risk of farmers. These flexibility coefficients were assumed to be independent of year-to-year changes in economic and environmental conditions. Sahi and Craddock (1974) have suggested an approach for estimating these coefficients taking account of changes in factors which might influence their values, but the variables used in this linear estimation procedure must also be included in the multi-year model, thus increasing the size of the model and compounding the difficulties of data collection.

An aggregative programming model of Australian agriculture (Walker and Dillon (1976)) has been developed based on LP models of 500 representative farms, each assumed to represent the structure of a group of 300 to 400 farms in 56 regions. Although inter-farm trade was not included in this model, an attempt was made to take account of the influence of uncertainty on decision making through the use of focus loss constraints (Boussard and Petit (1967)), i.e. it was assumed that farmers attempted to maximise expected income subject to a specified probability of obtaining some minimum level of income. The consequences of implementing the solutions from the model were then evaluated under simulated weather conditions. The advantage of adopting a highly disaggregated approach, by using a large number of representative farms, is that errors resulting from aggregation bias can be reduced. Some degree of bias is inevitable in large scale LP models of this form, although Buckwell and Hazell (1972) have proposed a classification system, based on cluster analysis, to reduce this bias. Walker and Monypenny (1976) considered that this LP model of Australian agriculture was over ambitious in attempting to incorporate farmer behaviour and weather uncertainty in a highly disaggregated multi-period model.

An aggregative programming model has also been developed for the agricultural sector in Britain (Buckwell and Thomson (1978), Thomson and Buckwell (1979)). This model was based on LP models of over 40 representative farms defined in terms of activities, resources and technology by aggregating across farm sizes and regions. Since the factors causing structural change, e.g. amalgamation and fragmentation of farms, in British agriculture were assumed to be distinct from those associated with resource allocation between and within farms, a Markov model was used in an independent exercise to estimate the future structure of the agricultural sector. The models for each representative farm were run separately, and therefore inter-farm trade in intermediate goods such as store livestock could not be included in the aggregative programming model. A more aggregated model based on six farm types has been used to examine store livestock trading, but the aggregation errors increased and the computational experience with this model suggested that an iterative approach, involving repeated solution of the models for each representative farm, was a more cost effective method of dealing with inter-farm trading. This model was developed as a research tool to assess policy options and to evaluate the long term impact of technological change. Longmire (1980) adapted the model to incorporate livestock feed requirements and links with feed compounders, and then used this revised model to investigate the demand for livestock feedstuffs in

Britain. In this analysis feed prices were varied by up to 50 per cent, and the effect on demand was determined assuming that other factors remained constant. However, in reality changes of this magnitude would influence livestock prices through the production decisions of individual producers, and thus the results from this part of the analysis are of limited value.

In the centrally planned European countries a number of LP based agricultural sector models have been developed, and Csaki (1978) has noted that in most cases these models are associated with five-year and longer term plans. Csaki (1978) has described the structure of a model of the Hungarian agricultural sector which was developed at the International Institute for Applied Systems Analysis for use in a study of world food and agriculture. This model consisted of a number of interconnected models, with LP being used for the socialist agriculture and food processing sectors, and non-linear optimisation models being used for household expenditure and private agriculture. The main reason for developing this model was to provide a tool to analyse the Hungarian agricultural system and improve understanding of the interaction of the elements of the system. It was also argued that since the model reflected the operational and decision making practices of Hungarian agriculture and food processing, the model could be used for forecasting in these sectors. Multi-period LP models have also been developed for forecasting in the agriculture sector of market based economies. For example, Andersen and Stryg (1976) used a multi-period regionally based LP model for forecasting agricultural development in Denmark. To use an LP model for forecasting purposes the parameters in the model must have a purely descriptive foundation. In this case this was achieved either by parametric variation or by changing the coefficients in successive runs to obtain the best agreement with actual performance, but the coefficients were assumed to be independent of both economic and environmental conditions.

Large scale LP models have also been used for planning the development of agriculture in developing countries, although data collection is a major problem in these countries. Randhawa and Heady (1964) constructed a regionally based LP model of Indian agriculture, the objective function being the maximisation of the value of the production of 16 major crops in 17 regions. This model contained constraints to limit the degree of specialisation in each region but production costs were not taken into account. A similar model, with 25 field crops in 17 regions, was used by Sherbiny and Zaki (1974) to examine the development of Egyptian agriculture. The major limitations of the model of Randhawa and Heady (1964) were overcome by using net revenue from agricultural production as the objective function and by incorporating additional agronomic constraints, e.g. for crop rotation, with particular attention being paid to irrigated land since this land can be used to produce more than one crop per year. It was argued that programming models of this type could play a useful role in the development of agriculture in developing countries because the gains identified by the model were obtained by looking at the entire agricultural sector. It was also argued that these gains could be achieved by offering price guarantees derived from the model solution, rather than by dictating what should be produced by farmers. Farm based LP models of the agricultural sector of developing countries have also been proposed. For example, Odero-Ogwell and Clayton (1973) constructed a farm based aggregative programming model of a region of Kenya, taking account of land, labour and capital in the definition of production units.

The major limitation of all these agricultural sector LP models is that in all cases the relationship between market prices and agricultural production was ignored. A methodology was proposed by Samuelson (1952) to model prices and output as endogenous variables by using linear supply and demand functions in a model in which the objective function was the maximisation of social welfare, defined as the surplus to consumers and producers, i.e. the area between the demand and supply curves to the left of their intersection. Takayama and Judge (1964) demonstrated how this spatial equilibrium model could be converted to a quadratic programming model with the same objective function, and extended the approach to deal with the case in which a number of products and regions were involved. Although the objective function used in these models is artificial, it has been argued (e.g. McCarl and Spreen (1980)) that the solution to the quadratic programming model is a reasonable representation of industry behaviour under competition.

The multi-product spatial equilibrium model of Takayama and Judge (1964) was adapted by Duloy and Norton (1973) in developing CHAC, a mathematical programming model of Mexican agriculture named after a Mayan rain god. This model was developed as a tool for examining pricing policies, employment programmes and certain investment decisions in a country where significant government intervention is possible, and because of the importance of agriculture to the Mexican economy, links between the agricultural sector and the rest of the economy were incorporated. A total of 33 short cycle crops in 20 regions were considered in the model but livestock and perennial crop production were not included. Demand functions were generally specified on a national basis, and spatial price differentials were used to reflect the transportation costs associated with each region.

Hazell and Scandizzo (1974) argued that both the supply response and return on investment could be overstated by CHAC because the attitude of individual farmers to risk was not incorporated in the model, and suggested a modification to allow for this risk. However, an additional set of assumptions was required to ensure that the resulting model produced economically meaningful results, and although a linearisation procedure was suggested to allow the quadratic programming model to be solved using LP codes, it was necessary to have estimates of the utility functions of individual farmers. For these reasons this modified model is unlikely to be of much practical value in agricultural planning.

Demand and supply relationships as functions of price were incorporated in a mathematical programming model of the agricultural sector of Portugal which was developed by Egbert and Kim (1975a), based on the major crop and livestock production activities in eleven regions. The objective function of this model involved the maximisation of the sum of social welfare and the value of exports, less all relevant costs. A linear approximation of this non-linear objective function was used to enable the model to be solved using LP. Egbert and Estacio (1975) considered that the major defects of this model were related to the labour supply functions, the limited range of investment opportunities and the static nature of the model which considered only a single year. Although each of these defects could be overcome, the resultant model would be very large, and it was reported that experience with a similar model for Brazil had indicated that the computational costs would be much greater. Aggregation bias is also a potential source of error in the results from this model, and Egbert and Kim (1975b) examined the effects of different levels of aggregation in this model. It was found that aggregation bias could have had serious implications if the results had been implemented, but that these errors could be minimised by careful attention to model formulation, particularly with respect to the activities in each region.

2.4.2 Agricultural Sub-Sector Models

The agricultural sector models considered in the previous section were concerned with the whole or major parts of the agricultural sector at regional or national level. Some of these models have also been used to analyse particular aspects of agricultural production on a regional or national basis. For example, as noted in the previous section, Longmire (1980) used a modified version of the aggregative programming model of British agriculture (Buckwell and Thomson (1978)) to investigate the demand for concentrate feeds for livestock in Britain, while Miller and Halter (1973) used a modification of the cattle component of a model of the Nigerian economy (Michigan State University Agricultural Sector Simulation Team (1973)) to evaluate cattle production policies for Venezuela. However, it is often necessary to consider a sub-sector in more detail than can be incorporated into agricultural sector models, and a number of different types of model have been developed for various applications at sub-sector level.

Modern methods of agricultural production have been the subject of legislation in many countries, and in some cases mathematical models have been used to investigate the impact of proposed legislation. Since legislation in areas such as waste disposal from intensive livestock production units has a direct impact on individual producers, models of individual agricultural units can be used to investigate the effect of legislation in these areas. For example, the LP model of a dairy farm developed by Ashraf and Christensen (1974) and the deterministic simulation model of a beef feedlot developed by Forster (1975) have, as discussed in Section 2.3.6, been used to examine the impact of waste disposal regulations on producers in the US.

Models of the operations of individual agricultural units can also be used to evaluate agricultural development policy. For example, a model of a pig production unit was used by Singh et al. (1980) to evaluate research programmes for pig production in Hawaii, but only a limited number of feeding and marketing policies could be investigated because of the form of the model. Beck et al. (1982) used a simulation model of beef production in agriculturally underdeveloped regions to assess the financial returns and risks associated with pasture improvement schemes. Beef prices and seasonal conditions were stochastic elements and the model incorporated a simple breeding herd model similar to that described by Granger and Walsh (1959). The model was used to compare improved and non-improved pasture under the same price and seasonal conditions, and it was argued that the use of this model helped to bridge the gap between commercial production and experimental field trials based on small numbers of non-breeding stock.

Less detailed models of farm level operations have also been used for studies at national level. A simple model incorporating an LP ration formulation model was used by Matulich (1978) to examine the influence of technology on the efficiency of different sized dairy units, and to investigate the effect of technological change on the future structure of the US dairy industry. An LP model of calving and milk production activities was developed by Killen and Keane (1978) to determine the seasonal prices to be paid to milk producers in Ireland to compensate for the higher production costs at certain times of the year. The milk production from an individual cow varies during a lactation and since grass is an important feed for dairy cows, the seasonal growth pattern of grass influences calving decisions, and hence milk production, during the year. Although some surplus milk can be used for the manufacture of storeable milk products, milk production at national level should be organised to meet demand for liquid milk and fresh milk products. The solution to the LP of Killen and Keane (1978) gave the calving pattern which minimised production costs subject to demand constraints, while the solution to the dual problem gave the seasonal prices to pay producers to reflect production costs. Although the seasonal growth pattern of grass was taken into account in this model, neither the effects of stocking rate on grass consumption nor the age distribution of the herd were considered, and hence the model is not adequate for application by individual producers.

Although models of operations at farm level can be used to evaluate certain aspects of agricultural policy at regional or national level it is often necessary to construct a model of the entire sub-sector. A number of techniques have been used in developing sub-sector models, but mathematical programming models appear to have been used most widely, although simulation has also been used. For example, Sere and Doppler (1981) used a simulation model to evaluate beef production systems in Togo, but this model did not include factors, such as seasonal variation in the availability of forage, which are important features of the West African savanna belt. A simulation model was also used by Sullivan et al. (1981) to evaluate cattle production in East Africa.

In the US, LP has been used in a number of studies of the beef production sector. For example, Byrkett et al. (1976) examined the location of the cattle feeding industry in the US using an LP model which incorporated constraints to limit the expansion of cattle feeding in areas in which there was strong competition for land. The model was used to determine factors which had a significant impact on the location of this industry by running the model with and without the constraints relating to these factors and comparing the results. However, a weakness of this approach is that many of the constraints were established in a rather subjective way.

A multi-period LP model of the US beef production industry was used by Miller et al. (1980) to determine the policy to maximise the production of the two top grades of beef, subject to cost and feedstuff availability constraints. The US was divided into five beef production regions, and cattle entering the finishing phase could be transported between regions. The grade of beef was determined by the feeding regime and the age of the animal, but only a limited number of feeding regimes were considered in each region. An extension of this LP model was used by Yorks et al. (1980) to examine the effects on beef production of minimising production cost or energy use, subject to constraints on the quantity and quality of beef produced. In both these studies it would have been more appropriate to use net income from beef production as the objective function since this would more accurately reflect the operational decisions of individual producers. The influence of quality, cost and energy use could then have been investigated by parameterisation of constraints. In addition it would have been desirable to incorporate supply and demand relationships in the model, although this would have introduced non-linearities. Supply and demand relationships for pork in North America were established by Martin and Zwart (1975) using econometric methods, and these relationships were then used to develop a quadratic programming model of the North American pork sector. This model was used to attempt to explain spatial and temporal variation in this sector, and also to evaluate policy options such as changes in tarriffs between the US and Canada. However, the same basic data was used to estimate coefficients in the econometric relationships and to validate the quadratic programming model.

A major limitation of LP in modelling agricultural sub-sectors is that constant returns to scale are assumed, and thus economies of scale which can be achieved through planning on a regional or national basis cannot be incorporated in the standard LP framework. To take account of scale economies in milk production and processing, Kloth and Blakley (1971) constructed a separable programming model of milk production and processing in the US. The objective function used was minimisation of assembly, processing and distribution costs, and the model was used to determine the size and location of milk processing plants in the US.

Mathematical programming sub-sector models have also been used to examine issues in crop production at regional and national level. For example, Shaw (1970) used a simple LP model to examine the distribution of maincrop potato production in Britain. This model was basically a transportation model, and although a number of simplifying assumptions were made, e.g. the price premium for certain varieties was ignored, the model could form the basis for further work in, for example, the evaluation of policies for controlling potato production. Aggregative programming techniques have also been applied to parts of the crop production sector, an example being the multi-period LP model developed by Abalu (1975) of perennial crop production in Cameroon. A total of 45 different crop combinations and five different land classes were considered in this model, and the objective function was maximisation of the present value of a 20 year stream of benefits. However, crop yields were not regarded as functions of crop age and thus it would be possible to have the full yield in the first year after planting. Nieuwoudt et al. (1976) used the formulation proposed by Hazell and Scandizzo (1974) to model risk aversion and supply and demand relationships in developing a model of peanut production in the US. This model was used to evaluate the effect of price support policies on peanut farming in the US, but as noted in the previous section, the methodology used in constructing this model has limitations.

2.5 MODELS IN AGRICULTURAL PLANNING

Agricultural systems are characterised by their complex nature, involving the dynamic interaction of a number of biological components, and the influence of uncertainty in, for example, weather and market conditions. It has been shown that a wide range of mathematical models of agricultural systems have been developed, and that because of the complexity of the systems involved, these models concentrate on particular aspects of agricultural production.

In many cases the use of models of agricultural systems has been restricted to research or teaching aids. A number of reasons have been advanced for the limited practical application of many models. For example, Kennedy (1973) has suggested that this situation has arisen because the development of agricultural planning techniques has lacked balance, with most of the effort being concerned with the methodolgy of planning, and insufficient attention being paid to the development of the control systems which are essential for the effective implementation of planning techniques. Nix (1979) has argued that much of the development of planning techniques has been of a theoretical nature and has had little impact on practical decision making because of the complexity of the techniques involved.

Even in cases where attempts have been made to simplify the use of agricultural planning techniques so that they can be used by individual farmers, as is the case with LP based planning systems such as MASCOT (Bond et al. (1970)), Nix (1979) noted that this effort had not been particularly successful. It is, however, worth noting that in the case of the LP based planning systems considered by Nix (1979), the farmers did not have direct control over the use of the planning system, and the involvement of the farmer was limited to supplying data and then examining the results some time later. The experience of Debertin et al. (1981) has suggested that in cases where computer based models can be run interactively or where the results can be made available quickly, then the use of techniques such as LP can have a significant impact on the decision making behaviour of farmers. Although the scope of the evaluation of Debertin et al. (1981) was limited, it was concluded that the experience gained from using the model improved understanding of the problems involved in planning farm operations. Benefits of this kind are difficult to evaluate, but can only be achieved if the decision maker has some understanding of the nature of the model, and therefore decision makers must be properly educated in the use of models if these planning techniques are to be accepted.

The technique orientation of some of the work in developing models of agricultural systems has involved simplifications which may limit the use of particular models. For example, LP models have been developed for use in evaluating agricultural equipment requirements, although these models cannot cope with the discrete nature of capital equipment or the economies of scale which can be achieved in this area. Agricultural systems are complex and therefore simplifications and approximations must be made in the model building process, but this should not be done simply because of the limitations of the type of model employed.

The emphasis on techniques rather than problems can also result in model building effort being focussed on problems which can be modelled accurately, although these may not be the most important problems facing the decision makers. For example, the formulation of least cost diets is a problem which can be tackled using LP, but the farmer is still faced with the problem of deciding how much of a particular diet should be used for livestock feeding. Ration formulation is much more complex than diet formulation, especially for ruminant species, because of the way in which the nutrient requirements are specified. In intensive operations in particular, ration formulation is much more important than diet formulation and there is the additional problem of deciding the sequence of rations to use.

If mathematical models are to be more widely used in agricultural planning, then the development of models of agricultural systems must be problem orientated. In some cases this may involve developing models as research tools, initially to improve understanding of certain agricultural systems, and eventually the results from this research effort may be used by decision makers in the agricultural sector. To be of value, a model must be an adequate representation of reality. Consequently, any simplifying assumptions and approximations should not be over restrictive, and these should be made in relation to the problem under investigation, rather than the proposed solution method. It is also desirable that the model should be in a form that can be used by individual decision makers.

2.6 OBJECTIVE OF THIS STUDY

The complexity of agricultural systems is such that mathematical models must be concerned with particular aspects of these systems. The objective of this study is the development of models for use in planning livestock production. The main aspects of livestock production which are considered are feeding to produce liveweight gain, particularly in intensive production systems, and the interaction of this activity with crop production. Other related activities, such as the breeding of livestock, are not considered although it would be possible to incorporate additional activities where appropriate. Since the formulation of feeds which supply specified nutrient requirements is more difficult for ruminant species, the main emphasis of this study is on beef cattle. However, the basic approach could be applied to other species, and this is demonstrated with respect to pig production. In all cases published data are used in the development of the models, but the underlying principles could be applied using suitable data from other sources.

Ration formulation is an important aspect of beef production. The method for formulating rations for beef cattle depends on the system used to specify the nutrient requirements of the animals. With the system which is recommended by the MAFF (1975) in the UK, difficulties arise in formulating rations for beef cattle because the metabolisable energy requirements of the animal depend on the metabolisable energy concentration of the ration. The variable net energy system was developed by Harkins et al. (1974) to overcome these difficulties and allow rations to be formulated in an additive manner. However, in Chapter 3 it is shown that this system is based on approximations which can cause significant errors in ration formulation. An iterative approach for formulating rations has been suggested by the MAFF (1975), but this approach is of little use in formulating least cost rations. Although the method proposed by Kennedy (1972) can be used to formulate least cost rations satisfying the nutrient standards recommended by the MAFF (1975), the method involves using an iterative approach and can be time consuming. In Chapter 3 a method for formulating least cost rations to satisfy these nutrient standards is developed. The approach involves using parametric LP to obtain a piecewise representation of the cost of the ration as a function of its energy concentration, and then using differential calculus to determine the least cost ration. This approach overcomes the limitations associated with other methods proposed for formulating rations to meet the nutrient standards recommended by the MAFF (1975) and the approach could be extended for use with the ARC (1980) system.

Although least cost rations should be used at each stage in livestock production, the economic performance of a livestock production unit will also depend on the feeding policy, i.e. the daily sequence of rations, used in the production process. A number of approaches have been proposed for evaluating feeding policies in intensive beef units, but all have limitations. For example, in the approaches of Meyer and Newett (1970), Kennedy (1972) and Ryan (1974) only a limited number of rations of the available feedstuffs were considered for feeding to the animals at any stage in the production process, while in the approach of Brokken et al. (1976), as in that of

Ryan (1974), the rate of liveweight gain of the animals must be specified. In Chapter 4 a DP model is developed to determine the feeding policy to produce beef cattle of specified liveweight from animals of known initial liveweight at minimum cost. This feeding policy involves using least cost rations formulated using the parametric LP model. This approach overcomes the limitations of previous methods since least cost rations are used throughout the production process and it is not necessary to make any assumptions about the rate of liveweight gain during any part of this process. The DP model can also be used to evaluate the feeding and marketing strategy for an intensive beef production unit. Any appropriate system can be used to specify the nutrient requirements of the animals, provided that it is possible to determine the least cost rations to produce specified daily liveweight gains in animals of known liveweight. It is demonstrated how the approach can be used with the system used by the NRC (1976) in the US to specify the nutrient requirements of beef cattle, and an improved method for calculating the required least cost rations is developed.

This DP approach could be used to evaluate the feeding policy for any livestock species, provided the required least cost rations can be determined. Only the liveweight of the animals is considered in developing the DP model of beef cattle feeding, but since the grade of the beef produced also affects the economic efficiency of beef production, it would be desirable to include the influence of feeding policy on carcase composition. The nutrient standards used for beef cattle do not take account of the effect of feeding policy on the composition of the liveweight gain, but the DP model could be extended to incorporate the body composition of the animals if suitable nutrient standards were available. In Chapter 5 the pig growth model of Fawcett et al. (1978a, 1978b), in which the composition of liveweight gain is separated into fat free and fatty tissue components, is used to extend the DP feeding policy model to take account of both liveweight and body protein content in assessing the feeding policy for pig production. The approach could be used for any species in which the development of the animal is described in terms of two variables, such as liveweight and body protein content, provided the least cost rations to produce specified daily changes in these variables can be determined.

Intensive livestock production is often combined with crop production, with some of the crop production being used for livestock feeding and other feedstuffs being purchased. In planning operations of this type the interactions between crop and livestock production must be taken into account. In previous attempts to model operations of this type, e.g. Wilton et al. (1974), Ashour and Anderson (1975), the possibility of purchasing feedstuffs which could be grown by the enterprise was ignored, and it was necessary to assume that the rate of liveweight gain remained constant during major parts of the production process. In Chapter 6 an LP model is developed for an integrated crop and intensive beef production enterprise in which some of the feedstuffs may be enterprise grown. The limitations of previous models of this type of enterprise are overcome by using the DP model of beef cattle feeding to derive the values of some of the coefficients in this LP model.

It can be seen that a set of mathematical programming models is to be developed for use in planning livestock production. It will be shown that each of the models used is of a form which is most appropriate to a particular type of problem, and that by using a series of different models, the major limitations associated with a particular type of model can be overcome. Most of the models developed in this study could be developed further for use by individual farmers, although it would also be necessary to develop associated information and control systems to use the models in this manner.

CHAPTER 3

RATION FORMULATION FOR BEEF CATTLE

3.1 INTRODUCTION

Ration formulation is an important aspect of the operation of many livestock enterprises, particularly in intensive operations in which the livestock are fed entirely on purchased feedstuffs. It involves determining the quantities of individual feedstuffs to feed to an animal in order to supply the nutrients, i.e. the energy, protein, minerals etc., required by the animal. The formulation of a ration is thus different from the formulation of a diet which in this context is defined in terms of the proportions of its constituent feedstuffs and is formulated to have specified concentrations of individual nutrients. Although LP has been widely used in the formulation of least cost diets (e.g. Dent and Casey (1967), Beneke and Winterboer (1973)), the formulation of least cost rations is more complex, especially for ruminants, because of the nature of the systems used to specify the nutrient requirements of livestock.

In intensive livestock operations rations should be formulated to supply sufficient nutrients to produce specified levels of performance in terms of, for example, liveweight gain or milk production. Different systems have been proposed for specifying the nutrient requirements of livestock by bodies such as the ARC in the UK and the NRC in the US. The method of ration formulation depends on the type of livestock and the system used to specify the nutrient requirements.

Nutrient allowances for beef cattle are recommended by the MAFF (1975) in the UK, but difficulties arise in formulating least cost rations to satisfy these recommendations because the energy requirement of the animal depends on the energy concentration of the ration. In this chapter a method is developed for formulating least cost rations for beef cattle to meet the nutrient standards recommended by the MAFF (1975) in the UK.

3.2 THE NUTRIENT REQUIREMENTS OF BEEF CATTLE

The nutrient requirements of beef cattle are specified in terms of the energy, protein, minerals etc., required to produce a specified liveweight gain in an animal of known liveweight. The two principal nutrients in formulating rations for beef cattle are energy and protein, and in order to illustrate the proposed method of least cost ration formulation only these nutrients will be considered, since other nutrients are easily incorporated into the ration formulation model.

3.2.1 Energy Requirements

The energy required by an animal is supplied by the food consumed. The portion of the food energy which can be used by the animal after faecal, urinary and methane losses have been deducted, is called metabolisable energy (ME). Part of the ME is used in heat production and since this heat is of no use to the animal except in a cold environment, it is considered as a loss from the food energy. Net energy (NE) is the ME less the heat production, and represents the part of the food energy which is available for maintenance of body functions and for production of, for example, liveweight gain or milk.

The system used by the MAFF (1975) to specify the energy requirements of cattle is based on the system recommended by the ARC (1965). In this system food energy is expressed in ME terms. The ME concentrations of feedstuffs commonly used in feeding beef cattle are listed (MAFF (1975)) and the ME concentration of a ration or diet is obtained by summing the ME contents of the constituent feedstuffs. Thus for a diet consisting of a proportion x_i, $x_i \geq 0$, of feedstuff i, i=1,2,3,....k, of ME concentration r_i (in MJ/kg dry matter), the ME concentration, R, of this diet is given by

$$R = \sum r_i x_i \tag{3.1a}$$

where
$$\sum x_i = 1 \tag{3.1b}$$

In the system used by the MAFF (1975) to specify the nutrient requirements of beef cattle, the energy requirements are specified in NE terms and the total NE requirement is separated into a requirement for maintenance of body functions and a requirement for liveweight gain. For an animal of liveweight W (in kg) which is required to produce a liveweight gain G (in kg) the NE requirements (in MJ) recommended by the MAFF for maintenance, E_m, and gain, E_g, are given by

$$E_m = 5.67 + 0.061W \tag{3.2a}$$

$$E_g = G(6.28 + 0.0188W)/(1.0 - 0.3G) \tag{3.2b}$$

The efficiency of utilisation of dietary ME in supplying the NE requirements of the animal for maintenance and gain depends on the ME concentration of the ration. For a feedstuff or ration with ME concentration R (in MJ/kg dry matter), the efficiencies of utilisation of ME for maintenance, k_m, and for gain, k_g, are given by

$$k_m = 0.55 + 0.016R \qquad (3.3a)$$

$$k_g = 0.0435R \qquad (3.3b)$$

However, for simplicity the MAFF recommend the adoption of a single value of 0.72 for k_m.

The ME requirements (in MJ) for maintenance, M_m, and gain, M_g, are obtained by dividing the corresponding NE requirements from (3.2) by the appropriate efficiency from (3.3). Thus, using $k_m = 0.72$,

$$M_m = 1.39E_m \qquad (3.4a)$$

$$M_g = 23.0E_g/R \qquad (3.4b)$$

where E_m and E_g are given by (3.2). These expressions do not include the 5% safety margin recommended by the MAFF (1975).

The total ME requirement of the animal is thus a function M(R) of the ME concentration, R, of the ration and is given by

$$M(R) = M_m + M_g \qquad (3.5)$$

where M_m and M_g are given by (3.4). The quantity, Q(R), (in kg) of this ration required to satify the ME requirements of the animal is given by

$$Q(R) = M(R)/R \qquad (3.6)$$

where Q(R) must not exceed the dry matter (DM) appetite of the animal.

3.2.2 Protein Requirements

Protein standards for beef cattle are not as well established as those for energy. In the US, the NRC (1976) has proposed protein standards which depend on animal liveweight and liveweight gain, while in the UK the ARC (1965) suggests that protein requirements also depend on the ME concentration of the ration. However, the protein standards recommended by the ARC (1965) are widely regarded as understating the requirements (e.g. Preston and Willis (1974)). The

MAFF (1975) consider only the energy requirements of beef cattle, but the Agricultural Development and Advisory Service (ADAS) of the MAFF has published protein requirements (ADAS (1976)) which are independent of the ME concentration of the ration. The protein requirements are listed for a number of values of animal liveweight and liveweight gain, with intermediate values being obtained by interpolation. In a revised system produced by the ARC (1980), both the rumen degradable protein and the undegradable protein are considered as functions of the ME intake of the animal in specifying the nutrient requirements of cattle. However, in illustrating the proposed method of ration formulation it will be assumed that protein requirements depend only on animal liveweight and liveweight gain. It would, however, be possible to modify the approach to allow for protein requirements which depend on the ME concentration of the ration.

3.3 THE VARIABLE NET ENERGY SYSTEM

Although the system used by the MAFF (1975) to specify the nutrient requirements of beef cattle can be easily used to predict the performance of growing animals, it is not well suited for the formulation of rations in general and least cost rations in particular. Ration formulation involves using an iterative procedure, such as that outlined by the MAFF (1975), because of the interaction between the ME requirements of the animal and the ME concentration of the ration, and thus the MAFF (1975) system is not suitable for the formulation of least cost rations. For this reason the MAFF (1975) also recommend the variable net energy system developed by Harkins et al. (1974) from principles suggested by MacHardy (1965). This variable NE system is designed to allow rations to be formulated in an additive manner, thus making it suitable for the formulation of least cost rations by LP.

3.3.1 Principles of the Variable Net Energy System

The variable net energy system was developed from the NE system recommended by the ARC (1965). In the variable NE system two further concepts are introduced, namely animal production level, and the NE of a feedstuff or ration for maintenance and production at a specified animal production level.

The animal production level, A, of an animal of liveweight, W, which is required to achieve a daily liveweight gain G is defined as

$$A = (E_m + E_g)/E_m \qquad (3.7)$$

where E_m and E_g are given by (3.2). For the feedstuff or ration used to produce the required liveweight gain in this animal, the overall efficiency of utilisation of ME for maintenance and production, k_{mp}, is given by

$$k_{mp} = (E_m + E_g)/(M_m + M_g) \qquad (3.8)$$

where E_m and E_g are given (3.2) and M_m and M_g are given by (3.4). By using expressions (3.4), (3.7) and (3.8) it can be shown that, when a feedstuff or ration of ME concentration, R, is used to produce an animal production level A, the efficiency of utilisation of ME for maintenance and production can be expressed in the form

$$k_{mp} = AR/(1.39R + 23.0(A - 1)) \qquad (3.9)$$

Hence, the NE concentration for maintenance and production, n_{mp}, of a feedstuff or ration of ME concentration, R, when used to produce animal production level A, is given by

$$n_{mp} = AR^2/(1.39R + 23.0(A - 1)) \qquad (3.10)$$

3.3.2 Use of the Variable Net Energy System for Ration Formulation

Suppose that a ration has to be formulated to produce a liveweight gain G in an animal of liveweight W and that k feedstuffs, with feedstuff i having ME concentration r_i, i=1,2,3,....,k, are available for ration formulation. Then the procedure for ration formulation using the variable NE system consists of the following steps :-

(1) Determine the total NE requirement, E, of the animal from

$$E = E_m + E_g \qquad (3.11)$$

where E_m and E_g are given by (3.2). If the NE allowances recommended by the MAFF (1975) are to be used, then a 5% safety margin should be included.

(2) Determine the animal production level, A, from (3.7).

(3) For each available feedstuff, calculate the NE concentration for maintenance and production, n_i, at this animal production level by substituting in (3.10)

i.e. $$n_i = Ar_i^2/(1.39r_i + 23.0(A - 1)) \qquad (3.12)$$

(4) Choose quantities q_i (in kg dry matter) of feedstuff i such that

$$\sum n_i q_i = E \qquad (3.13)$$

and such that the quantity, q, fed, $q = \sum q_i$, does not exceed the dry matter appetite of the animal. In addition, the ration must satisfy the protein requirements etc. of the animal.

By using the variable NE system least cost rations can be formulated by LP.

3.3.3 Errors in Using the Variable Net Energy System

The variable NE system uses approximations which can lead to significant errors in ration formulation. Although MacHardy (1965) and Harkins et al. (1974) recognised that the variable NE system could produce errors when used in ration formulation, the extent of these errors has never been investigated fully. The source of these errors can be demonstrated by comparing the NE requirement of the animal with the NE which would be supplied by the ration formulated using the variable NE procedure.

It can be seen by substituting for n_i, given by (3.12), in (3.13), that the NE requirement of the animal can be expressed in the form

$$E = A\sum [q_i r_i^2/(1.39r_i + 23.0(A - 1)] \qquad (3.14)$$

For the ration formulated by the variable NE procedure, the quantity, q_i, of feedstuff i has been set to satisfy (3.13) and thus the ME concentration, r, of this ration is given by

$$r = \sum r_i q_i/q \qquad (3.15)$$

For this ration the NE concentration for maintenance and production, n, at the required animal production level is given, from (3.10), by

$$n = Ar^2/(1.39r + 23.0(A - 1))$$

and substituting for r from (3.15) yields

$$n = A(\sum r_i q_i)^2/q(1.39\sum r_i q_i + 23.0q(A - 1)) \qquad (3.16)$$

Thus, the NE which would be supplied, E', by this ration at the required animal production level, is given by

$$E' = nq$$

and, by substituting for n from (3.16), E' can be expressed as

$$E' = A(\sum r_i q_i)^2/(1.39\sum r_i q_i + 23.0q(A - 1)) \qquad (3.17)$$

Clearly, at the required animal production level, the NE supplied, given by (3.17), differs from the NE requirement, given by (3.14), unless A=1. The error could be quantified by expressing the difference between the NE supplied and the NE requirement as a percentage of the NE requirement. However, this method of expressing the error would be rather misleading since, if the ration formulated by the variable NE procedure were fed to the animal, the animal production level obtained would differ from the required value, and therefore the NE supplied to the animal would differ from that given by (3.17).

In order to overcome this difficulty, consider the diet, defined in terms of the proportions of constituent feedstuffs, with the same composition as the ration formulated by the variable NE procedure. The proportion by weight of feedstuff i in this diet is q_i/q, and the ME concentration, r, of this diet is given by (3.15). With the variable NE ration formulation procedure, the quantity of this diet which is fed to the animal is q. However, in order to satisfy the NE requirements of the animal, the quantity, Q, of this diet which is required must satisfy

$$E = Qn$$

where E is given by (3.11) or (3.14) and n is given by (3.16),

i.e. $$Q = E/n \qquad (3.18)$$

Thus the error, ξ, arising from the use of the variable NE system can be quantified by determining the difference between the quantity of the diet supplied and the quantity of the diet required, and expressing this difference as a percentage of the quantity required

i.e. $$\xi = 100(q - Q)/Q \qquad (3.19)$$

The magnitude of this error depends on the animal production level and on the composition of the ration, i.e. ξ depends on A, r_i, and q_i, $i=1,2,3,....,k$. To get some indication of the importance of this error, the errors occurring in two component rations were investigated. Since a large number of cases, with different values of

A, r_i and q_i, must be considered, a FORTRAN program was written to perform the calculations.

A value of the animal production level was chosen and the program was used to calculate the error in each of the two component rations which could be formed from all possible combinations of feedstuffs of ME concentrations (in MJ/kg DM) r_i=6.0,6.5,7.0,7.5,....,14.5,15.0, in quantities (in kg DM) q_i=0,0.5,1.0,1.5,....,11.5,12.0. In these calculations, the only rations which were considered were those in which the total quantity of the feedstuffs was consistent with the specified animal production level and the NE supplied by the ration. A summary of the results is given in Table 4. In this table the largest percentage errors, as defined by (3.19), found for the specified animal production levels are listed together with the compositions of the associated rations.

From Table 4 it can be seen that even for two component rations, the errors arising from using the variable NE system can be significant. The largest errors found for two component rations would result in an under-supply of food energy to the animal, but small errors which would result in an over-supply of food energy were also found in some rations. For two component rations the greatest errors occur when feedstuffs with extreme ME concentrations are used. It is also likely that larger errors could arise at intermediate values of the variables used to define the rations, and in rations composed of more than two feedstuffs.

The variable NE system was developed to allow rations to be formulated in an additive manner, but this has been achieved by using approximations which cause errors in ration formulation. These errors are small for low values of the animal production level but can be significant at higher values. Although in many applications the simplicity of use of the variable NE system has much to recommend it, the existence of these errors of varying magnitude may warrant the use of the more complex basic ME system in ration formulation.

3.4 A PARAMETRIC PROGRAMMING METHOD FOR BEEF CATTLE RATION FORMULATION

In formulating rations to supply the nutrient requirements recommended by the MAFF (1975), difficulties arise because of the interaction between the ME concentration of the ration and the ME requirements of the animal. The iterative approach to ration formulation outlined by the MAFF (1975) is of little use in least cost ration formulation, while the variable NE system, which is also recommended by the MAFF (1975), is based on approximations and it has been shown that the use of this system can cause significant errors in ration formulation.

An approach developed by Kennedy (1972) can be used to formulate least cost rations which supply the nutrient requirements recommended by the MAFF (1975). This method involves first using LP to determine the least cost ration of specified ME concentration which supplies the ME, protein etc. required to produce a specified daily liveweight gain. The ME concentration of the ration is then changed and the ME requirement is changed to the appropriate value. The LP is then solved to determine the least cost ration of this ME concentration. By repeated solution of the LP for different values of the ME concentration, the overall least cost ration for the required liveweight and daily liveweight gain can be found. This procedure is time consuming and in practice only a limited number of values of the ration ME concentration can be considered.

3.4.1 The Ration Formulation Model

Assume that k feedstuffs are available for ration formulation and define

r_i = ME concentration of feedstuff i (in MJ/kg DM)

p_i = protein concentration of feedstuff i (in g/kg DM)

c_i = cost per kg DM of feedstuff i.

Suppose that a diet of ME concentration R has been formulated with the proportion, by weight, of feedstuff i being x_i, $x_i \geqslant 0$. Then the ME concentration, R, of this diet is given by (3.1), and the cost, c(R), per kg DM of this diet is

$$c(R) = \sum c_i x_i \tag{3.20}$$

If this diet is fed to an animal of liveweight W which is required to achieve a daily liveweight gain G, then defining

M(R) = ME requirement of animal when fed ration of ME concentration R, given by (3.5)

P = daily protein requirement of animal (in g),

the quantity Q(R) of this diet is given by (3.6), i.e. $Q(R)=M(R)/R$. Now Q(R) must not exceed the DM appetite of the animal and it will be assumed that M(R), and hence Q(R), is not defined if Q(R) would exceed the DM appetite of the animal. In addition, this ration must supply the protein requirements of the animal,

i.e. $Q(R)\sum p_i x_i \geqslant P$

or $$\sum p_i x_i \geqslant PR/M(R) \tag{3.21}$$

The cost of this ration, K(R), is given by

$$K(R) = Q(R)c(R)$$

or $$K(R) = M(R)c(R)/R \tag{3.22}$$

The least cost ration for this animal is that which minimises K(R), given by (3.22), and satisfies the protein requirements etc. of the animal. Now c(R) is not, in general, uniquely defined since diets of ME concentration R can be formulated in a number of ways. However, if C(R) is defined to be the cost of the least cost diet of ME concentration R which will satisfy the protein requirements of the animal, the following procedure can be used to determine the least cost ration to feed to the animal:-

(1) Use LP to determine the least cost diet of ME concentration R, which will satisfy the protein requirements of the animal:

$$C(R) = \min \sum c_i x_i \qquad (3.23a)$$

subject to

$$\sum x_i = 1 \qquad (3.23b)$$

$$\sum r_i x_i = R \qquad (3.23c)$$

$$\sum p_i x_i \geqslant PR/M(R) \qquad (3.23d)$$

$$x_i \geqslant 0$$

Note that other nutrients could be incorporated into this LP model.

(2) Determine the cost, K(R), of the least cost ration of ME concentration R for this animal :

$$K(R) = M(R)C(R)/R \qquad (3.24)$$

(3) Repeat steps (1) and (2) for a number values of the ME concentration to determine the overall least cost ration for the animal.

This procedure will produce the optimal ration if sufficient values of R are considered. However, this procedure can be improved by using parametric programming to obtain a piecewise linear representation of C(R) over the range of R, and then using differential calculus to determine the minimum overall cost. Such an approach will produce the optimal ration with negligible error.

3.4.2 The Parameterisation Procedure

By parameterising the right hand side of (3.23) a piecewise linear representation of C(R) over the range of R, $R_L \leqslant R \leqslant R_U$, can be obtained. Since the right hand side of (3.23d) is not in a form suitable for direct parameterisation, it is first necessary to approximate PR/M(R) by piecewise linearisation over the range of R. This can be done by determining the values of PR/M(R) for $R=R_j=R_L+j\Delta$, j=0,1,2,......,n, where Δ is the linearisation interval, $\Delta>0$, and n is integral such that $(R_U-R_L)/\Delta \leqslant n < (R_U+\Delta-R_L)/\Delta$ with intermediate values being obtained by linear interpolation, i.e. for $R_j \leqslant R < R_{j+1}$, j=0,1,2,3,......,n-1:

$$PR/M(R) = u_j + (R - R_j)v_j \tag{3.25a}$$

where
$$u_j = PR_j/M(R_j) \tag{3.25b}$$

and
$$v_j = (u_{j+1} - u_j)/(R_{j+1} - R_j) \tag{3.25c}$$

The LP of equations (3.23) can, by introducing a surplus variable x_{k+1} in (3.23d), be expressed in the form

$$C(R) = \min Z = \mathbf{CX} \tag{3.26a}$$

$$\mathbf{AX} = \mathbf{D} \tag{3.26b}$$

$$\mathbf{X} \geqslant \mathbf{0}$$

where $\mathbf{A}=\{a_{ij}\}$, $\mathbf{C}=\{c_j\}$, $\mathbf{D}=\{d_i\}$ and $\mathbf{X}=\{x_i\}$ are appropriately defined. Suppose that the LP is solved for some value, r, of the ME concentration of the ration, $R_j \leqslant r < R_{j+1}$, j=0,1,2,3,......,n-1, and that an optimal solution exists with optimal basis matrix **B**. Then if the columns of **A** are permuted so that **A**=(**B**,**N**), (3.26b) can be expressed in the form (see, for example, Hadley (1962))

$$\mathbf{BX}_B + \mathbf{NX}_N = \mathbf{D} \tag{3.27}$$

where $\mathbf{X}_B=\{x_{Bi}\}$ is the vector of optimal basic variables and $\mathbf{X}_N$ is the vector of non basic variables in the optimal solution. Premultiplying (3.27) by $\mathbf{B}^{-1}=\{\beta_{ij}\}$,

$$\mathbf{X}_B + \mathbf{B}^{-1}\mathbf{N}\mathbf{X}_N = \mathbf{B}^{-1}\mathbf{D} \tag{3.28}$$

yields the solution

$$\mathbf{X}_B = \mathbf{B}^{-1}\mathbf{D} \tag{3.29a}$$

$$\mathbf{X}_N = \mathbf{0} \tag{3.29b}$$

By letting $\mathbf{C}=(\mathbf{C}_B,\mathbf{C}_N)$ and substituting for $\mathbf{X}_B$ from (3.28), the optimal value of the objective function of (3.26), C(r), can be expressed as

$$C(r) = \mathbf{C}_B\mathbf{B}^{-1}\mathbf{D} - (\mathbf{C}_B\mathbf{B}^{-1}\mathbf{N} - \mathbf{C}_N)\mathbf{X}_N \tag{3.30}$$

Substituting $\mathbf{X}_N=\mathbf{0}$ in (3.30) yields

$$C(r) = \mathbf{C}_B\mathbf{B}^{-1}\mathbf{D}$$

or

$$C(r) = \boldsymbol{\lambda}\mathbf{D} \tag{3.31}$$

where $\boldsymbol{\lambda}=\{\lambda_j\}=\mathbf{C}_B\mathbf{B}^{-1}$ is the optimal basis of the dual of (3.26).

If r is increased to $r+\delta$ where $0\leqslant\delta\leqslant R_{j+1}-r$, then $\mathbf{D}$ will change to $\tilde{\mathbf{D}}=\{\tilde{d}_i\}$ with

$$\tilde{d}_1 = 1 = d_1$$

$$\tilde{d}_2 = r + \delta = d_2 + \delta$$

$$\tilde{d}_3 = u_j + (r - R_j)v_j + v_j\delta = d_3 + v_j\delta$$

This change in the right hand side of (3.26b) will change the solution from $\mathbf{X}_B$ to $\tilde{\mathbf{X}}_B=\{\tilde{x}_{Bi}\}$ where

$$\tilde{\mathbf{X}}_B = \mathbf{B}^{-1}\tilde{\mathbf{D}} \tag{3.32a}$$

and from the definition of $\mathbf{D}$ and $\tilde{\mathbf{D}}$, $\tilde{x}_{Bi}$, i=1,2,3, can be expressed as

$$\tilde{x}_{Bi} = x_{Bi} + (\beta_{i2} + v_j\beta_{i3})\delta \tag{3.32b}$$

This solution will remain optimal provided $\tilde{x}_{Bi}\geqslant 0$, i=1,2,3, and since $\delta\geqslant 0$, it can be seen that the solution will remain optimal provided,

$$\delta \leqslant -x_{Bi}/(\beta_{i2} + v_j\beta_{i3}) \qquad \text{if} \qquad \beta_{i2} + v_j\beta_{i3} < 0,\ i=1,2,3.$$

It is also required that $\delta\leqslant R_{j+1}-r$, and hence the range of δ, $0\leqslant\delta\leqslant\delta_u$, over which the solution remains optimal in this interval, or part interval, can be established. Over this range of δ the cost of the least cost diet with ME concentration $r+\delta$ which will satisfy the protein requirements of the animal is given by

$$C(r+\delta) = \boldsymbol{\lambda}\mathbf{D}$$

i.e. $$C(r+\delta) = \lambda_1 + (r+\delta)\lambda_2 + (\tilde{u}_j + (r+\delta)v_j)\lambda_3 \qquad (3.33a)$$

where $$\tilde{u}_j = u_j - v_jR_j \qquad (3.33b)$$

Parametric LP can thus be used to obtain a piecewise linear representation of C(R) over the range of R. Suppose that LP (3.23) has been solved for some value, r, of R, $R_j \leqslant r < R_{j+1}$. If this solution ceases to be optimal within the appropriate interval, or part interval, i.e. $\delta_u < R_{j+1}-r$, then the LP must be resolved at the value of R at which the solution changes. If the solution remains optimal to the end of the interval, i.e. $\delta_u = R_{j+1}-r$, the solution at the start of the next interval can be obtained from (3.32) and (3.33) with $\delta = R_{j+1}-r$, and the range over which this solution remains optimal can be determined as before. In order to extend the procedure to cover the entire range of R it is necessary to allow for ranges where no feasible solution exists. Parameterisation of the right hand side of LP (3.23) can easily be extended to determine the range over which no feasible solution exists. For example, when solving the LP by the two phase simplex method this involves determining the range of phase I optimality and overall infeasibility in any interval or part interval.

By using this procedure C(R) can be approximated by a piecewise linear function of m segments with boundaries at $R=\rho_0,\rho_1,\rho_2,\ldots\ldots,\rho_m$. If an optimal feasible solution to LP (3.23) exists in the k^{th} segment, i.e. for $\rho_{k-1} \leqslant R < \rho_k$, and if $R_j \leqslant \rho_{k-1} < R_{j+1}$, $j=0,1,2,3,\ldots\ldots,n-1$, then from (3.29a),

$$C(R) = \lambda_{k1} + R\lambda_{k2} + (\tilde{u}_j + Rv_j)\lambda_{k3} \qquad \rho_{k-1} \leqslant R < \rho_k \qquad (3.34)$$

where $\tilde{u}_j$ and v_j are given by (3.33b) and (3.25c) respectively and $\boldsymbol{\lambda}_k = \{\lambda_{ki}\}$ is the optimal dual basis for LP (3.23) in the k^{th} segment.

3.4.3 Determination of the Least Cost Ration

The parameterisation procedure is used to derive the cost, C(R), of the least cost diet of ME concentration, R, over the range of R. The quantity of diet, i.e. the ration, of ME concentration R required by the animal is obtained by substituting in (3.6) and can be expressed in the form

$$Q(R) = (1.39E_mR + 23.0E_g)/R^2 \qquad (3.35)$$

where E_m and E_g are given by (3.2), and where it has been assumed that k_m=0.72.

Suppose that a piecewise linear representation of C(R) has been derived and that an optimal feasible solution to LP (3.23) exists in the k^{th} segment of this piecewise linearisation, i.e. for $\rho_{k-1} \leqslant R < \rho_k$. In this segment the cost K(R) of feeding the least cost ration of ME concentration R is given by

$$K(R) = Q(R)C(R)$$

and by substituting for Q(R) from (3.35) and C(R) from (3.34), K(R) can be expressed in the form

$$K(R) = \alpha_1 + \alpha_2/R + \alpha_3/R^2 \qquad \rho_{k-1} \leqslant R < \rho_k \qquad (3.36)$$

where

$$\alpha_1 = 1.39(\lambda_{k2} + v_j\lambda_{k3})E_m$$

$$\alpha_2 = 1.39(\lambda_{k1} + \tilde{u}_j\lambda_{k3})E_m + 23.0(\lambda_{k2} + v_j\lambda_{k3})E_g$$

$$\alpha_3 = 23.0(\lambda_{k1} + \tilde{u}_j\lambda_{k3})E_g$$

Any turning points within this segment can be determined by differentiating K(R) with respect to R and equating to zero, and since R≠0 the turning point is given by

$$R = -2\alpha_3/\alpha_2 \qquad \text{provided} \qquad \rho_{k-1} \leqslant R < \rho_k.$$

This will be a minimum turning point if $\alpha_3 > 0$. The minimum cost ration in this segment can then be determined by evaluating the cost of the rations with $R=\rho_{k-1}$ and $R=\rho_k$, and at the minimum turning point if it exists in this segment.

The method for determinung the overall least cost ration involves performing the same calculations for each segment in which an optimal feasible solution to the LP exists and finding the ration of least cost overall. Although the approach has been described for the case where a single value of 0.72 has been used for the efficiency of utilisation of ME for maintenance, k_m, the approach can be extended to allow for k_m being dependent on the ME concentration of the ration as specified in equation (3.3a). The single value of k_m=0.72 recommended by the MAFF has been adopted to simplify the illustration of the approach.

3.5 USE OF THE RATION FORMULATION MODEL

The beef cattle ration formulation model is used to formulate least cost rations which will supply the energy and protein required to produce specified liveweight gains in animals of known liveweight, using the energy requirements specified by MAFF (1975) and the protein requirements specified by ADAS (1976). Although it is generally recognised (e.g. ARC (1965)) that the appetite of ruminants depends on the digestibility of the feed intake, the DM appetite values used in illustrating the use of this ration formulation model are those specified by ADAS (1976) which, like those specified by MAFF (1975), depend only on animal liveweight. An alternative approach which attempts to take account of the influence of digestibility on feed intake has been suggested by ADAS (1976). This approach involves considering dietary indigestible organic matter, but this method was not used in this study because of its tentative nature. In the modified ARC (1980) system DM intake is a function of dietary ME concentration, and the ration formulation model could be extended to take account of this.

The formulation of least cost rations can be of particular importance in intensive beef cattle operations because the economic efficiency of these operations can be improved by ensuring that the animals are fed on optimal rations at every stage in the production process. In order to ensure that optimal rations are used at all times, the ration formulation model can be used to determine the least cost rations for producing specified liveweight gains in animals of known liveweight for a large number of values of animal liveweight and liveweight gain.

The ration formulation model has been tested by using it to formulate least cost rations satisfying the energy and protein requirements of animals of liveweight 100,101,102,103,.....,500 kg which are required to produce daily liveweight gains of 0,0.25,0.50,0.75,.....,1.50 kg, provided such liveweight gains are possible for particular liveweights. Nine feedstuffs were available for ration formulation and details of these are given in Table 5. A FORTRAN program (see Appendix B1) was written to perform the calculations and the program was run on an IBM 3081. The computational time to calculate all the least cost rations, approximately 2800 in this case, depends on the linearisation interval, Δ, used. For a linearisation interval of 1.00, the central processor time was approximately 22 seconds, and when the linearisation interval was reduced to 0.10, the central processor time was approximately 27 seconds. Examples of the output from the program are presented in Table 6 in which, for each liveweight and liveweight gain, a block of entries gives details of the composition and cost of the least cost ration. Blocks in which all the entries are zero indicate that these liveweight gains cannot be achieved at the specified liveweights. Although the accuracy of the ration formulation procedure should improve with decreasing linearisation interval, only minor differences in ration composition and cost were found when the linearisation interval was reduced from 1.00 to 0.10.

3.5.1 Discussion of Results

It can be seen from the results of Table 6 that if an animal is to achieve a constant daily liveweight gain then the ration should change from day to day as the liveweight increases. In practice, however, the ration would not be changed as frequently as this. The results obtained using the data of Table 5 indicate that for a given daily liveweight gain there will be few, if any, sudden changes in the composition of the rations as the liveweight increases from 100 kg to 500 kg. Thus if a constant daily liveweight gain is required, fixed rations can be used over periods in which the change in the composition of least cost rations falls within prescribed limits. For the feedstuff data and liveweight gain values considered in this study there is only one example of the ration changing suddenly as the liveweight is increased. This occurs for a liveweight gain of 0.50 kg per day between liveweights 170 kg and 171 kg (see Table 6), when oat straw enters the ration at a high level. Sudden changes of this type in the ration may not produce the desired daily liveweight gain because the functioning of the rumen would take time to adjust to the new dietary regime, and therefore in practice it would be desirable to introduce changes in the ration gradually over a longer period.

From the results of Table 6 it can be seen that for a given liveweight the rations become more concentrated, with proportionately less roughage in the form of oat straw, as the daily liveweight gain increases. For daily liveweight gains of 0.75 kg or more (0.50 kg or more for animals of 170 kg or less) all concentrate rations of barley and soya bean meal are used. The change to an all concentrate ration as the daily liveweight gain increases from 0.50 kg to 0.75 kg (0.25 kg to 0.50 kg for animals of 170 kg or less) accounts for the drop in dry matter intake between these liveweight gain values.

The rations produced by this model have been examined by the Animal Nutrition Department of the East of Scotland College of Agriculture (ESCA). It was noted (Lewis (1983), personal communication) that although the rations of Table 6 satisfy the nutrient and appetite standards used in developing the model, the high roughage rations produced for daily liveweight gains of 0.25 kg or less (0.50 kg or less for liveweights greater than 170 kg) would not be acceptable in practice because the total quantity of feed required would exceed the DM appetite of cattle on low digestibility diets. The DM appetite values specified by ADAS (1976) apply to balanced diets of reasonable digestibility. The influence of digestibility on DM appetite was recognised by ADAS (1976), but the method suggested to take this dependence into account was not used in this study because of its tentative nature.

The Animal Nutrition Department of ESCA use ESCALF, a modified version of the Beef Calculating Model of ESCA (Rehman et al. (1978)) in advisory work. ESCALF (ESCA (1981)) is used to formulate rations satisfying the energy and protein standards of the ARC (1980) system, but although appetite is considered as a function of dietary digestibility, the method used differs from that of the ARC (1980) system. Rations can be formulated for the case where a fixed quantity of forage is available on a daily basis, or the case where forage is offered ad libitum, taking account of the appetite of the animal as a function of the digestibility of its feed intake. The model first calculates the forage intake and then determines the minimum quantity and composition of concentrate required to supplement the forage to produce a specified level of animal performance. The model does not attempt to calculate least cost rations, although an assessment can be obtained of the financial consequences of using the rations formulated in this way.

The rations produced by ESCALF are therefore based on different nutrient standards and formulated on a different basis from the rations produced by the ration formulation model developed in this study. In addition the Animal Nutrition Department of ESCA do not use the feedstuff nutritive values recommended by MAFF (1975) but use values obtained from feedstuff evaluation studies carried out by ESCA (ESCA (1982)). The rations produced by the two methods are therefore not directly comparable. However, to illustrate the differences in the rations produced by the two methods, ESCALF has been used to formulate rations required by cattle of 100 kg liveweight to produce daily liveweight gains of 0.25, 0.50, 0.75 and 1.0 kg. To allow some comparison with the rations produced by the beef cattle ration formulation model, the forage available per day as straw was set at the quantity required in the rations of Table 6, i.e. the straw available was set at 2.56 kg DM for a daily liveweight gain of 0.25 kg and no straw was allowed in the rations for daily liveweight gains of 0.50 kg and above. The only other feedstuffs considered in formulating rations by ESCALF were barley and soya bean meal, i.e. the feedstuffs used in the rations of Table 6. The rations formulated by

ESCALF are shown in Table 7 along with the corresponding rations from Table 6. Ration costs are not given since these are not produced by ESCALF. The major difference in the rations produced by the two methods occurs in the case of a daily liveweight gain of 0.25 kg. The maximum amount of straw used by ESCALF, taking account of feedstuff digestibility, is 1.1 kg compared with 2.56 kg of straw in the ration produced by the ration formulation program. For daily liveweight gains of 0.50 kg or more, the rations produced by the two methods are different, but are of a similar nature in terms of their barley and soya content. However, for the feedstuff data of Table 5, it was found that the rations produced by ESCALF would not supply the basic energy requirements recommended by the MAFF (1975) for daily liveweight gains of 0.75 kg and 1.0 kg in animals of 100 kg liveweight. Similar results were obtained for other liveweights, suggesting that the all concentrate rations produced by the ration formulation model would be suitable for use in intensive beef production systems where it is generally not economic to feed animals at or near maintenance levels.

The major limitation of the beef cattle ration formulation model is that it does not take account of the influence of dietary digestibility on feed intake. This limitation could be overcome by expressing the DM appetite of an animal as a function of dietary ME concentration, as in the ARC (1980) system, and because of the nature of the ration formulation model this change could be incorporated easily. It would also be desirable to extend the model to include rumen degradable protein and undegradable protein as functions of the ME concentration of the ration, as in the ARC (1980) system.

3.6 CONCLUSIONS

It has been demonstrated that although the variable net energy system, proposed by Harkins et al. (1974) and recommended by the MAFF (1975), is of a form which enables least cost rations to be formulated by LP, the system is based on approximations which can cause errors in ration formulation. The magnitude of these errors depends on the composition of the ration, and can be significant.

A parametric LP method has been developed which allows least cost rations, satisfying the nutrient requirements recommended by the MAFF (1975), to be formulated with negligible error. This approach overcomes the difficulties which arise in ration formulation because of the interaction between the energy requirements of the animal and the energy concentration of the ration in the MAFF system for specifying the energy requirements of beef cattle. The computational experience with the approach suggests that it would be practical for use in formulating least cost rations for beef cattle in both advisory work and in commercial operations, but it would be necessary to modify the model to consider DM appetite as a function of dietary digestibility, as in the ARC (1980) system. It would also be desirable to extend the model to include rumen degradable protein and undegradable protein as function of dietary ME concentration, as in the ARC (1980) system. The basic approach used in the ration formulation model is such that these changes could be incorporated easily.

CHAPTER 4

FEEDING POLICY FOR BEEF CATTLE

4.1 INTRODUCTION

In intensive beef cattle production systems, least cost rations should be used at every stage in the production process. However, the formulation of least cost rations is only one aspect of planning the operations of an intensive beef production unit since the economic efficiency of these operations is determined by the feeding policy, i.e. the daily sequence of rations used in the production process, and by the purchase and selling prices of the animals. The market value of beef cattle is governed by the weight of the animal and the grade of beef. The grade of beef depends on the composition of the carcase in terms of, for example, its fat free and fatty tissue content, and is influenced by the feeding policy of the production unit. Feeding policy is therefore of fundamental importance to the economic performance of an intensive beef production operation.

Although it has been recognised that feeding policy affects the carcase composition of beef cattle, neither the MAFF (1975) in the UK nor the NRC (1976) in the US take account of the influence of feeding policy on carcase composition in specifying the nutrient requirements of beef cattle. The NRC (1976) assumes that feeding policy will not have a major effect on the carcase composition when economic feeding practices are adopted, but this assumption has been questioned (e.g. Moe and Tyrrell (1973)). The nutrient requirements and the carcase composition of beef cattle are also influenced by genetic factors, but differences between breeds of cattle are not taken into account in either of these systems for specifying the nutrient requirements of beef cattle. A further limitation of these systems is that the effect of environmental stress resulting from, for example, extremes of temperature is not taken into account in assessing the nutrient requirements of beef cattle, although the NRC (1976) considers that allowances can be made for these factors. However, in spite of these limitations both the MAFF (1975) and NRC (1976) systems are widely used in formulating rations for beef cattle, although because of the differences in the methods used to specify the energy requirements, different approaches are required to formulate least cost rations to meet these different standards.

In this chapter a method is developed for determining the feeding policy of an intensive beef production unit in which animals are fed according to the nutrient standards recommended by the MAFF (1975). The method is developed using the least cost ration formulation model developed in Chapter 3, and assuming that carcase composition is not affected by feeding policy. It is shown that this feeding policy model can be used to investigate the marketing policy of an intensive beef production unit. This approach can also be used when other systems are used to specify the nutrient requirements of beef cattle, provided that it is possible to determine the least cost rations to produce specified liveweight gains in animals of known liveweight. It is shown how the approach can be used when cattle are fed according to the nutrient standards recommended by the NRC (1976) and a new method is developed to reduce the computational load in formulating least cost rations which satisfy the NRC (1976) standards.

4.2 FEEDING POLICY MODEL

The criterion which is used to evaluate possible feeding policies will depend on the nature of the operations of the beef production unit. For example, if the unit has a contract to supply animals of specified liveweight then, in the short term at least, a suitable criterion might be the cost of producing animals of this liveweight from animals of known initial liveweight, where the cost will include the feeding costs associated with feeding least cost rations and overhead costs such as rent, handling costs and veterinary costs. In cases where the initial purchase weight and final selling weight can be varied, a criterion such as the return per animal per period might be appropriate to evalutate feeding policy.

4.2.1 Cost Minimisation Model

In many cases part of the problem of evaluating feeding policies will be concerned with determining the feeding policy to produce animals of specified liveweight from animals of known initial liveweight at minimum cost, and possibly within some specified time period. A DP approach can be used to tackle this problem. In order to use DP it is first necessary to define the state of an animal in suitable terms. Although it would be desirable to describe the state of an animal in terms of its liveweight and carcase composition, both the MAFF (1975) and the NRC (1976) express the nutrient requirements of beef cattle in terms of only liveweight and liveweight gain. For this reason it is assumed that when cattle are fed according to the nutrient standards recommended by the MAFF (1975) or the NRC (1976), then the state of an animal can be described solely in terms of its liveweight. Initially the model is developed for the case where cattle are fed according to MAFF (1975) standards.

In order to examine the effect of varying the final liveweight of the animals produced, a backward DP formulation (White (1969)) is adopted. Define $F_n(W)$ to be the cost of producing an animal of liveweight W from an animal of liveweight W_0 in n feeding periods, each of d days, using an optimal feeding policy. Then

$$n \geq 1 \qquad F_n(W) = \min_{h>0} [F_{n-1}(W-h) + K + C(W-h,h)] \tag{4.1a}$$

$$F_0(W_0) = 0 \tag{4.1b}$$

where K is the overhead cost per feeding period and C(W-h,h) is the cost of producing a liveweight gain h in d equal daily increments, starting with an animal of liveweight (W-h) and using least cost rations. Clearly the costs in this model could be discounted and the model could be extended to deal with time dependent feeding and overhead costs.

The data required for this model depend on the feeding period length, d days, the interstate interval, w kg, the initial liveweight, W_0 kg, and the final liveweight, W_F kg. The DP recurrence relations (4.1) should be solved for $W=W_0,W_0+w,W_0+2w,\ldots,W_F$. For simplicity W_0 and W_F should be chosen such that $(W_F-W_0)/w$ is integral, and in practice it will generally be sufficient to take $w \geq 1$. It can be seen from the definition of C(W-h,h) that least cost rations should be calculated for liveweights $W=W_0,W_0+w/d,W_0+2w/d,\ldots,W_F$, and daily liveweight gains iw/d, $i=1,2,\ldots,[dG(W)/w]$, where G(W) is the maximum possible daily liveweight gain of an animal of liveweight W and [X] denotes the integral part of X. In practice however, it will generally be sufficient to calculate least cost rations for a more limited number of liveweights, e.g. $W=W_0,W_0+w,W_0+2w,\ldots,W_F$, and liveweight gains iw/d, i=1,2,....,[dG(W)/w], and then obtain intermediate values by interpolation. When the costs of the required least cost rations have been determined, C(W-h,h) can be evaluated by summing the costs of the appropriate rations. If feedstuff costs change from period to period, then least cost rations should be calculated for each time period.

Although the DP model of (4.1) can be used to determine the overall minimum cost of producing animals of liveweight W from animals of liveweight W_0, a more direct approach is possible if $F_n(W)$ is redefined to relate to production in at most n feeding periods. Then

$$n \geqslant 1 \quad F_n(W) = \min\,[F_{n-1}(W),\ \min_{h \geqslant 0}\,[F_{n-1}(W-h) + K + C(W-h,h)]] \qquad (4.2a)$$

$$F_0(W_0) = 0 \qquad (4.2b)$$

By using this formulation the number of feeding periods required to produce animals of liveweight W from animals of liveweight W_0 at minimum cost can be found directly. It should also be noted that once the optimal feeding schedule from liveweight W_0 to liveweight W_F has been found, the optimal feeding schedules from any initial liveweight W_p, $W_0 \leqslant W_p < W_F$, to any final liveweight W_s, $W_0 < W_s \leqslant W_F$, are embedded in the results.

This DP approach could be used with other systems for specifying the nutrient requirements of beef cattle, provided that the necessary least cost rations can be obtained. This approach could also be used with animals of any species to determine the optimal feeding policy to increase the liveweight of an animal by a specified amount, provided again that the required least cost rations can be derived. However for most other species, e.g. pigs, it would be important to take carcase composition into account, and hence this DP model may not be so useful for other species.

4.2.2 Return Maximisation Model

In the case where both the purchase weight and the selling weight of the animals can be varied, then since the length of a production cycle will typically be less than a year and since costs and revenues may vary from cycle to cycle, a suitable criterion for evaluating feeding policies might be the return per animal per period. Thus if

$F(W_p, W_s, n)$ = cost of the optimal feeding schedule from liveweight W_p to liveweight W_s in n feeding periods (from (4.1))

$S(W_s)$ = selling price of an animal of liveweight W_s

$B(W_p)$ = purchase price of an animal of liveweight W_p

then the optimal return per period, $R(W_p, W_s)$, for purchasing animals at liveweight W_p and selling at liveweight W_s is given by

$$R(W_p, W_s) = \max_{n>0} \left[(S(W_s) - B(W_p) - F(W_p, W_s, n))/n \right] \qquad (4.3)$$

The optimal combination of purchase weight and selling weight can then be found by repeated solution of (4.3). The approach involves first specifying the range of possible values of purchase weight and selling weight. In practice it will generally be sufficient to consider values within these ranges at intervals of 10 kg or more. The purchase and selling prices of animals of the weights to be considered within these ranges must also be specified. A pair of values of purchase weight, W_p, and selling weight, W_s, are then chosen and the value of n which maximises $R(W_p, W_s)$ found. The value of either W_p or W_s is then changed and the optimal return for this new pair of values found. By repeating this process until all commercially acceptable combinations of purchase weight and selling weight have been covered, the operating policy which maximises return per animal per period can be determined.

4.2.3 Use of the Model

The use of the feeding policy model has been tested by using it to determine the feeding policy for an intensive beef production unit in which the objective is to minimise the cost of producing animals of specified liveweight from animals of known initial liveweight. For this case the approach involves first using the ration formulation model developed in Chapter 3 to determine least cost rations, and then using DP to determine the optimal feeding policy by solution of the recurrence relations (4.2). A FORTRAN program (see Appendix B2) has been developed to perform the calculations for solution of the DP recurrence relations. To use this program it is first necessary to determine least cost rations for appropriate values of animal liveweight and liveweight gain. This involves using the least cost ration formulation program developed in Chapter 3 to produce a data file which is used as an input data file for the DP model solution program.

In the development of the DP model (4.2) it was assumed that the rate of liveweight gain remained constant throughout a feeding period of length d days. In the DP model solution program the liveweight gains which can be achieved during a feeding period are limited to integral multiples of 1 kg, and thus the interval between successive states in the DP model is 1 kg. For this reason the daily liveweight gains which must be considered are restricted to i/d, $i=1,2,3,\ldots,[dG(W)]$, and to ensure that a reasonable range of daily liveweight gains is considered, it is therefore desirable to use a feeding period of at least 3 days. However, since the rate of liveweight gain is assumed to remain constant during a feeding period, it is possible that the solution to the model will involve a substantial increase in the rate of liveweight gain between successive periods. Such an increase would be undesirable from a practical animal feeding viewpoint, and to reduce the possibility of sudden and substantial increases in the rate of liveweight gain the length of a feeding period should not exceed 10 days.

The approach has been tested using representative data for an intensive beef production unit in which animals of 500 kg liveweight are to be produced from animals of 100 kg liveweight. The length of a feeding period was set at 4 days, and the ration formulation model of Chapter 3 was used to determine least cost rations for animals of liveweight 100,101,102,.....,500 kg which were required to produce daily liveweight gains of 0.25,0.50,0.75,.....,1.50 kg, provided that these liveweight gains were possible for particular liveweights. The least cost rations for the intermediate liveweights which were required for the DP model were obtained by interpolation. The programs were run on an IBM 3081 and the central processor time to solve the DP recurrence relations was approximately 1 second. An example of the output from the DP model solution program, using the feedstuffs specified in Table 5 for ration formulation, is shown in Table 8.

4.2.4 Discussion of Results

Once the DP model (4.2) has been used to determine the feeding policy to produce animals of specified final liveweight from animals of specified initial liveweight at minimum cost, the results can be used to determine the optimal feeding policy for all intermediate combinations of initial and final liveweight. However, care must be taken in attempting to generalise from a limited set of results, since feeding policy clearly depends on the nutrient standards used for animal feeding and on the prices and nutrient compositions of the feedstuffs available for ration formulation. In addition, the results from this model are not directly comparable with the results from other feeding policy evaluation models (e.g. Meyer and Newett (1970), Kennedy (1972), ESCA (1981)) because these models use different nutrient standards and are based on more restrictive assumptions.

The results from using this DP model will also be affected by the assumptions made in developing the model. In reality daily liveweight gain is a continuous variable, but only a set of discrete values can be considered in the model, these values depending on the choice of feeding period and interstate interval. The maximum daily liveweight gain of an animal should also be a continuous function of liveweight, but because the permitted values of daily liveweight gain form a discrete set, there will be discrete changes in the maximum daily liveweight gain as the liveweight changes. In the method developed in Chapter 3 for formulating least cost rations to satisfy the basic nutrient standards of the MAFF (1975), least cost rations are only defined if the ration does not violate the dry matter appetite of an animal of particular liveweight, and the maximum daily liveweight gains can be obtained from the output to the ration formulation program. For the daily liveweight gain values used, i.e. 0.25,0.50,0.75,.....,1.50 kg, the development of an animal growing at the maximum rate is shown in Figure 1.

The approach has been applied using a number of different sets of feedstuff data and nutrient standards, and four cases are considered to illustrate the influence of feedstuff prices, feedstuff nutritive values and nutrient standards on feeding policy. In case (a) the feedstuff data of Table 5 is used and rations are formulated to satisfy the basic energy allowances of the MAFF (1975). In this case the rations are found to include soya as a protein supplement, and in case (b) the price of soya is increased and the effect on feeding policy is examined. In case (c) the rations are formulated to satisfy the increased energy allowances (i.e. including the 5% safety margin) recommended by the MAFF (1975), using the feedstuff data of Table 5. In each of these three cases the rations in the optimal feeding policy are composed of barley plus a protein supplement, and in case (d) the effect of substituting barley by a hypothetical feedstuff with a higher digestible crude protein content is investigated. The results for these four cases are discussed below.

(a) The feedstuffs of Table 5 were used to formulate least cost rations using the basic energy allowances given by expressions (3.4), i.e. the 5% safety margin adopted by the MAFF (1975) was not included. The feeding policy for this case is presented in Table 8, and involves feeding the animals to grow at a rate corresponding closely to the maximum rate of gain of the animal (see case (a) of Figure 1), with the final liveweight being reached after 284 days, i.e. 71 feeding periods. It can be seen from Table 8 that generally the rations are composed of feedstuffs 1 and 5, i.e. barley and soya bean meal. The composition of these rations, i.e. the diet, remains reasonably constant over periods in which the rate of liveweight gain is constant. Changes in the diet occur when there is a change in the rate of liveweight gain, but in each case the change in the composition of the diet is relatively small although the change in the quantity of diet required, i.e. the ration, is more pronounced. Since sudden changes in the ration may not produce the desired daily liveweight gain, it would be desirable in practice to introduce these changes more gradually, so that the change in the growth rate is more gradual. It is not possible to constrain the results from the DP model to prevent sudden changes in the daily sequence of rations, but some of the problems could be overcome by reducing the interval between successive liveweight values in the DP model solution program.

The feeding policy produced by the DP model has been examined by the Animal Nutrition Department of the ESCA and the rations were judged (Lewis (1983), personal communication) to be satisfactory for use in intensive beef production systems, such as the barley beef system. However, in practice it would be necessary to examine the sequence of rations produced by the model and, where there are sudden changes in the ration, it would be necessary to modify the rations so that changes are introduced gradually.

The feeding policy produced by the DP model has been compared with the feeding policy produced by ESCALF (ESCA (1981)), although since the rations produced by ESCALF are formulated on a different basis the results, as noted in Chapter 3, are not directly comparable. ESCALF can be used to determine the feeding policy in a period of specified length (maximum 300 days) which is divided into three component periods of equal length. The user must specify the initial liveweight and the daily liveweight gain in each of the three component periods, and the ration for the middle day of each of these periods is calculated. To allow some comparison with the results from the DP model, the initial liveweight, the length of the total feeding period and the daily liveweight gain in each component period were chosen using the DP model results for this case.

It can be seen from Table 8 that at the start of feeding period number 8 the liveweight is 128 kg and that the daily liveweight gain is 1.25 kg till the end of period 19, i.e. for 48 days, with the daily liveweight gain in subsequent periods being 1.5 kg. For this reason ESCALF was run for an animal of liveweight 128 kg which was to be fed for 144 days, with a daily liveweight gain of 1.25 kg in the first component period of 48 days and a daily liveweight gain of 1.5 kg in the two subsequent component periods. The rations produced by ESCALF for each component period are shown in Table 9, together with the corresponding rations from the DP model. Although differences in the two sets of rations are to be expected because of differences in the nutrient standards and the basis of ration formulation, it can be seen that the two sets of rations are similar in composition, particularly in the last two component periods, i.e. liveweights of 188 kg and more. The rations produced by ESCALF are of lower weight and, for the feedstuff data of Table 5, these rations would not satisfy the basic energy requirements recommended by MAFF (1975). However, the major difference between the DP model and ESCALF is that the DP model can be used to determine the optimal feeding policy without the need to specify the rate of liveweight gain, whereas in using ESCALF the rate of liveweight gain must be specified and thus repeated use of ESCALF in a systematic manner would be required to determine the optimal feeding policy.

(b) In case (a) soya was included in the rations as a protein supplement. The effect of increasing the price of soya by 300% (i.e. to 68.8 p/kg DM) was investigated, with other feedstuff data remaining as in Table 5, and with rations being formulated to satisfy the basic energy allowances as before. The development of the animal for this case is shown in Figure 1, case (b). It can be seen that the animal grows more slowly than in case (a), and that the rate of liveweight gain is less than the maximum rate for the first 120 days, with the animal reaching the final liveweight after 300 days, i.e. 75 feeding periods. In this case the rations are generally composed of feedstuffs 1 and 6, i.e. barley and sugar beet pulp, the proportion of barley in the ration being lower than for the corresponding situation in case (a), although soya is present at low levels following an increase in the rate of liveweight gain.

The results clearly depend on the cost and nutrient composition of the available feedstuffs and on the associated overhead cost per animal per unit time. However, the results for this case indicate that it is not always optimal to assume that animals should be fed to achieve maximum daily liveweight gain.

(c) The MAFF (1975) recommend the inclusion of a 5% safety margin over the basic energy allowances given by (3.4), although the justification for adopting this safety margin is not given. The inclusion of this safety margin affects both ration formulation and feeding policy. The computer program developed for formulating least cost rations satisfying the basic energy allowances was modified to include this safety margin. The development of an animal growing at the maximum rate is shown in

Figure 2, the final liveweight being reached in 283 days. The optimal feeding policy, using the feedstuffs of Table 5 to formulate least cost rations satisfying the increased energy allowances, involves feeding the animals to grow at a rate which is close to the maximum growth rate of the animal (see case (c) of Figure 2), with the final liveweight being reached after 288 days, i.e. 72 feeding periods. The rations used are similar to those of case (a), being composed mainly of barley and soya, although the proportion of barley in the ration is higher than the corresponding ration for case (a) and the weight of the ration is greater.

(d) In cases (a) and (c) soya was included in the rations to provide a protein supplement in barley based rations, and for this reason the effect of substituting a hypothetical feedstuff with a higher digestible crude protein (DCP) content was investigated. For example, one case involved substituting barley by a feedstuff with ME concentration 13.2 MJ/kg DM, DCP content 118 g/kg DM and costing 8.7 p/kg DM, and substituting barley straw by a feedstuff with ME concentration 6.9 MJ/kg DM, DCP content 12 g/kg DM and costing 2.1 p/kg DM. The other feedstuff data remained as in Table 5 and the ration formulation program was used to formulate least cost rations satisfying the increased energy allowances recommended by the MAFF (1975). The development of an animal for this case is shown in Figure 2, case (d). The feeding policy involves using rations composed entirely of the hypothetical substitutes for barley and barley straw. It can be seen from Figure 2 that the animal grows much more slowly in this case, taking 332 days, i.e. 83 periods, to reach the final liveweight, and that after 36 days the growth rate is always less than the maximum growth rate. Similar results were found with other hypothetical substitutions, indicating again, as in case (b), that beef cattle should not always be fed to achieve the maximum daily liveweight gain.

Although only a small number of cases has been considered, the results demonstrate that in an optimal beef cattle feeding policy, the daily liveweight gain depends on the animal liveweight, the nutrient standards used and the prices and nutrient composition of the feedstuffs available for ration formulation. For this reason it is not possible to derive rules for optimal feeding policies which will be valid under all conditions. However, it may be possible to use the approach to determine optimal feeding rules which apply to a specified set of feedstuffs and a limited range of feedstuff prices.

4.3 BEEF CATTLE FEEDING USING UNITED STATES NUTRIENT STANDARDS

A number of different standards for specifying the nutrient requirements of beef cattle have been proposed by bodies such as the ARC (1965) in the UK and the NRC (1976) in the US. Although there are differences in the allowances for protein, minerals etc. recommended in these systems, the main difference between the systems lies in the way in which energy requirements are specified and this affects the method of ration formulation. In order to use the methods developed in Section 4.2 for determining the feeding policy for beef cattle, it is first necessary to determine least cost rations for appropriate values of animal liveweight and daily liveweight gain. Therefore in order to demonstrate the use of the feeding policy model when cattle are fed according to the nutrient standards recommended by the NRC (1976), the system used to specify the energy requirements and the method of ration formulation to meet these standards must first be considered.

4.3.1 Energy Requirements

The NRC recommend the use of the net energy system developed by Lofgreen and Garrett (1968) for determining the energy requirements of beef cattle. In this system it is assumed that the total NE requirement of the animal can be separated into a component for maintenance and a component for the production of liveweight gain. The daily NE requirement for maintenance, E_m (in Mcal), of an animal of liveweight, W (in kg), is assumed to depend only on liveweight, with

$$E_m = 0.077W^{0.75} \tag{4.4}$$

The NE requirement for liveweight gain, E_g (in Mcal), of an animal of liveweight, W (in kg), which is required to produce a daily liveweight gain, G (in kg), is given by separate expressions for steers and heifers and is assumed to depend only on liveweight and gain.

Steers : $$E_g = (0.05272G + 0.00684G^2)W^{0.75} \tag{4.5a}$$

Heifers : $$E_g = (0.05603G + 0.01265G^2)W^{0.75} \tag{4.5b}$$

It is assumed that food energy will first be used to meet, or attempt to meet, the NE requirement for maintenance, and any remaining food energy will be used to produce liveweight gain. Since the efficiencies of utilisation of food energy for maintenance and gain differ, the food energy available for these two functions must be considered separately. In the NRC system it is assumed that the food energy available for maintenance and the food energy available for gain, after the maintenance requirements have been met, can be specified in Mcal/kg DM for all feedstuffs. These NE concentrations for maintenance and gain are tabulated (NRC (1976)) for feedstuffs commonly used for feeding beef cattle. The NE concentrations for maintenance and gain of diets and rations composed of a number of these feedstuffs can be calculated from these tables.

Suppose that a diet consists of a proportion, by weight, y_i of feedstuff i, i=1,2,....,k, of NE concentration for maintenance m_i, and NE concentration for gain g_i. Then the NE concentrations of the diet for maintenance, m, and gain, g, are given by

$$m = \sum m_i y_i \qquad (4.6a)$$

$$g = \sum g_i y_i \qquad (4.6b)$$

where $$\sum y_i = 1 \qquad (4.6c)$$

If this diet is fed to a steer of liveweight W, which is required to achieve a daily liveweight gain G, then the NE requirements of the steer for maintenance, E_m, and gain, E_g, are given by expressions (4.4) and (4.5a) respectively. The quantity, in kg, of this diet required for maintenance is E_m/m, and the quantity required for gain is E_g/g. Thus the quantity, Q (in kg), of this diet required, i.e. the ration, is

$$Q = E_m/m + E_g/g \qquad (4.7)$$

A portion of the ration is therefore used to satisfy the NE requirement for maintenance and the remainder is used to satisfy the NE requirement for growth. In addition to satisfying the energy requirements, the ration must also satisfy all the other nutrient requirements of the animal, in particular the protein requirements, and must not violate the appetite constraints of the animal.

4.3.2 Ration Formulation Model

With the NE system recommended by the NRC, LP can be used to determine the least cost diet with specified NE concentrations, m, for maintenance and, g, for gain (see equations (4.6)). However, as can be seen from (4.7), the quantity of this diet, i.e. the ration, required to produce a specified liveweight gain in an animal of known liveweight depends on the values of m and g, and it would therefore be necessary to search all values of m and g to determine the least cost ration.

Brokken (1971a) has suggested an LP model for ration formulation which takes account of the interdependences in the NE system recommended by the NRC. Suppose it is required to determine the minimum cost ration to feed to an animal of liveweight W to achieve a daily liveweight gain G. The NE requirements of the animal for maintenance, E_m, and gain, E_g, are given by (4.4) and (4.5) respectively, and let

P = protein requirement of the animal (in grams)

U = maximum DM intake of the animal (in kg)

L = minimum DM intake of the animal (in kg)

where P is a function of liveweight and liveweight gain, and U and L are functions of liveweight.

Assume that k feedstuffs are available for ration formulation and let

c_i= cost per kg DM of feedstuff i

m_i= NE concentration for maintenance of feedstuff i (Mcal/kg DM)

g_i= NE concentration for gain of feedstuff i (Mcal/kg DM)

p_i= protein concentration of feedstuff i (g/kg DM)

If a ration consisting of quantity x_i (kg DM), $x_i \geq 0$, of feedstuff i,

$i=1,2,\ldots,k$, is fed to the animal, then from (4.6), the NE concentrations for maintenance, m, and gain, g, are given by

$$m = \sum m_i x_i / Q \tag{4.8a}$$

$$g = \sum g_i x_i / Q \tag{4.8b}$$

where
$$Q = \sum x_i \tag{4.8c}$$

i.e. Q is the quantity fed.

Suppose that when this ration is fed to the animal, a fraction f, by dry matter weight, of the ration is used to meet the NE requirement for maintenance. Then, an amount, Qf (in kg DM), of this ration is used for maintenance, i.e. $mQf=E_m$, and substituting for m from (4.8a),

$$Qf\sum m_i x_i / Q = E_m$$

i.e.
$$\sum m_i x_i = E_m / f \tag{4.9a}$$

and since a fraction (1-f) is used to meet the NE requirement for gain,

$$\sum g_i x_i = E_g / (1-f) \tag{4.9b}$$

A ration which supplies the NE requirements of the animal by using a fraction f by DM weight of the ration to supply the NE requirement of the animal must satisfy (4.9). Since the ration must also satisfy the DM intake constraints and protein requirements of the animal, the least cost ration of this form is given by the solution of the LP:

$$\min Z = \sum c_i x_i \tag{4.10a}$$

subject to

$$\sum m_i x_i = E_m / f \tag{4.10b}$$

$$\sum g_i x_i = E_g / (1-f) \tag{4.10c}$$

$$\sum x_i \leqslant U \tag{4.10d}$$

$$\sum x_i \geqslant L \tag{4.10e}$$

$$\sum p_i x_i \geqslant P \tag{4.10f}$$

$$x_i \geqslant 0, \quad i=1,2,3,\ldots\ldots,k.$$

This LP formulation can be extended to incorporate other nutrients.

The overall least cost ration can be found by solving LP (4.10) for all values of f, although obviously in practice only a limited number of cases can be evaluated. However, Brokken (1971a) has shown that, by piecewise linearisation of E_m/f and $E_g/(1-f)$, a separable programming formulation will yield the overall optimal solution with negligible error.

In general, piecewise linearisation of E_m/f and $E_g/(1-f)$ over the range of f would not be entirely satisfactory because of the asymptotic behaviour as f tends to 0 and 1 respectively. In this problem however, it is never possible to have f=0 and in intensive production systems it will seldom, if ever, be economic to feed animals at or near maintenance levels, corresponding to $f \to 1$. The range of possible values of f is thus reduced and upper and lower bounds for f can be established. For example, for a ration with NE concentrations m and g for maintenance and gain respectively,

$$f = E_m/m/(E_m/m + E_g/g) = 1/(1 + mE_g/gE_m)$$

and since for all rations of available feedstuffs i, i=1,2,3,....,k,

$$\rho_L = \min_i[m_i/g_i] \leqslant m/g \leqslant \max_i[m_i/g_i] = \rho_U$$

then

$$f_L = 1/(1 + \rho_U E_g/E_m) \leqslant f \leqslant 1/(1 + \rho_L E_g/E_m) = f_U \qquad (4.11)$$

It may be possible to establish narrower limits for f by taking the maximum dry matter intake of the animal into account as suggested by Brokken (1971a).

A piecewise linear approximation of E_m/f and $E_g/(1-f)$ over the range of f defined by (4.11) can be obtained by determining the values of E_m/f and $E_g/(1-f)$ for $f=f_j=f_L+j\Delta$, j=0,1,2,....,n, where Δ is the linearisation interval, $\Delta>0$, and n is the integer satisfying $(f_U-f_L)/\Delta \leqslant n < (f_U+\Delta-f_L)/\Delta$, with intermediate values being determined by linear interpolation. The smaller the linearisation interval, the smaller the error due to interpolation, although in practice a value of Δ in the range 0.005 - 0.01 should be adequate. A separable programming formulation for the least cost ration formulation problem is then:

$$\text{Minimise} \qquad Z = \sum_{i=1}^{k} c_i x_i \tag{4.12a}$$

$$\text{subject to} \qquad \sum_{i=1}^{k} m_i x_i - \sum_{j=0}^{n} x_{k+1+j} E_m / f_j = 0 \tag{4.12b}$$

$$\sum_{i=1}^{k} g_i x_i - \sum_{j=0}^{n} x_{k+1+j} E_g / (1-f_j) = 0 \tag{4.12c}$$

$$\sum_{j=0}^{n} x_{k+1+j} = 1 \tag{4.12d}$$

$$\sum_{i=1}^{k} x_i \leqslant U \tag{4.12e}$$

$$\sum_{i=1}^{k} x_i \geqslant L \tag{4.12f}$$

$$\sum_{i=1}^{k} p_i x_i \geqslant P \tag{4.12g}$$

$$x_i \geqslant 0, \quad i=1,2,3,\ldots,k+n+1$$

with at most two adjacent x_{k+1+j}, $j=0,1,2,\ldots,n$, non zero.

The formulation of (4.12) follows that of Brokken (1971a) except that different limits for f are used. Since the LP model (4.12) is convex it can be solved by the simplex method.

4.3.3 An Improved Method for Solving the Ration Formulation Model

It has been shown that a large number of least cost rations must be calculated in order to use the DP model (4.1) or (4.2). In formulating rations to satisfy the nutrient standards recommended by the NRC (1976), the computational load involved in determining the least cost rations for the required values of liveweight and liveweight gain could be reduced if parametric programming methods could be used, but since both the left and right hand sides of (4.12) change when either the liveweight or the liveweight gain is changed, standard parametric programming techniques are not applicable. However, the nature of this ration formulation problem is such that the solution to the LP model (4.12) for a specified liveweight and liveweight gain can be used to obtain a basic solution to the LP model for a different liveweight but the same liveweight gain.

Consider the ration formulation problem for liveweight W_1 and daily liveweight gain G. By introducing a slack variable x_{k+n+2} in (4.12e) and surplus variables x_{k+n+3} and x_{k+n+4} in (4.12f) and (4.12g) respectively, the LP model (4.12) can be expressed in the form

$$\min Z = \mathbf{CX} \tag{4.13a}$$

$$\mathbf{AX} = \mathbf{D} \tag{4.13b}$$

$$\mathbf{X} \geqslant \mathbf{0}$$

where $\mathbf{A}=\{a_{ij}\}$, $\mathbf{C}=\{c_j\}$, $\mathbf{D}=\{d_i\}$ and $\mathbf{X}=\{x_i\}$ are appropriately defined. Suppose that an optimal feasible solution to (4.13) exists and that $\mathbf{B}=\{\beta_{ij}\}$ is the optimal basis matrix, then if the columns of $\mathbf{A}$ are permuted so that $\mathbf{A}=(\mathbf{B},\mathbf{N})$, (4.13b) can be expressed in the form (see, for example, Hadley (1962)):

$$\mathbf{BX_B} + \mathbf{NX_N} = \mathbf{D} \tag{4.14}$$

Premultiplying (4.14) by $\mathbf{B}^{-1}$,

$$\mathbf{X_B} + \mathbf{B}^{-1}\mathbf{NX_N} = \mathbf{B}^{-1}\mathbf{D} \tag{4.15}$$

yields the solution

$$\mathbf{X_B} = \mathbf{B}^{-1}\mathbf{D} \tag{4.16a}$$

$$\mathbf{X_N} = \mathbf{0} \tag{4.16b}$$

By letting $\mathbf{C}=(\mathbf{C_B},\mathbf{C_N})$, the optimal value Z_0 of the objective function of (4.13) can be expressed as

$$Z_0 = \mathbf{C_B B}^{-1}\mathbf{D} - (\mathbf{C_B B}^{-1}\mathbf{N} - \mathbf{C_N})\mathbf{X_N}$$

If the liveweight is now changed to W_2 and the daily liveweight gain remains unchanged, then it can be seen from (4.4) and (4.5) that the new coefficients of x_{k+1+j}, $j=0,1,2,\ldots,n$, in (4.12b) and (4.12c) are obtained by multiplying the corresponding coefficients in the original model, i.e. for the ration formulation problem with liveweight W_1, by $(W_2/W_1)^{0.75}$, provided the range of f defined for the original problem is adequate. If (4.12d) is multiplied throughout by $(W_2/W_1)^{0.75}$, then it can be seen from (4.12) that effectively all the original coefficients of variables x_{k+1+j}, $j=0,1,2,\ldots,n$, have been multiplied by the factor $(W_2/W_1)^{0.75}$ while the coefficients of all the other variables remain unchanged. As a result of the change in the liveweight, the right hand side of (4.12) will change from $\mathbf{D}$ to $\tilde{\mathbf{D}}=\{\tilde{d}_i\}$, where

$$\tilde{d}_1 = 0,\ \tilde{d}_2 = 0,\ \tilde{d}_3 = (W_2/W_1)^{0.75},\ \tilde{d}_4 = \tilde{U},\ \tilde{d}_5 = \tilde{L},\ \tilde{d}_6 = \tilde{P},$$

where $\tilde{U}$, $\tilde{L}$ and $\tilde{P}$ denote the maximum DM intake, minimum DM intake and protein requirement respectively of the animal of liveweight W_2. Hence, the least cost ration formulation LP model for liveweight W_2 and liveweight gain G can be written

$$\min Z = \mathbf{CX} \qquad (4.17a)$$

$$\tilde{\mathbf{B}}\mathbf{X}_B + \tilde{\mathbf{N}}\mathbf{X}_N = \tilde{\mathbf{D}} \qquad (4.17b)$$

where $\mathbf{X}_B$ and $\mathbf{X}_N$ are the vectors for the basic and non-basic variables in the solution to the original problem (4.13), given by (4.16), and

$$\tilde{\mathbf{B}} = \{\gamma_j \beta_{ij}\},$$

$$\text{with} \quad \gamma_j = \begin{cases} (W_2/W_1)^{0.75} & \text{if } j^{th} \text{ element of } \mathbf{X}_B \text{ is } x_{k+1+i},\ i=0,1,2,\ldots,n \\ 1 & \text{otherwise} \end{cases}$$

and where $\tilde{\mathbf{N}}$ is defined similarly in terms of the elements of $\mathbf{N}$. Premultiplying (4.17b) by $\mathbf{B}^{-1}$ yields

$$\mathbf{G}\mathbf{X}_B + \mathbf{B}^{-1}\tilde{\mathbf{N}}\mathbf{X}_N = \mathbf{B}^{-1}\tilde{\mathbf{D}} \qquad (4.18)$$

where $\mathbf{G}=\mathbf{B}^{-1}\tilde{\mathbf{B}}$ is a diagonal matrix with principal diagonal elements γ_i. Premultiplying (4.18) by $\mathbf{G}^{-1}$ yields

$$\mathbf{X}_B + \mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{N}}\mathbf{X}_N = \mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{D}} \qquad (4.19)$$

where $\mathbf{G}^{-1}$ is a diagonal matrix with principal diagonal elements $1/\gamma_i$. Clearly (4.19) yields a basic solution to the new ration formulation problem,

$$\mathbf{X}_B = \mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{D}} \qquad (4.20a)$$

$$\mathbf{X}_N = \mathbf{0} \qquad (4.20b)$$

although this solution may not be feasible. The objective function corresponding to the basic solution (4.20) is given by

$$Z = \mathbf{C}_B\mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{D}} - (\mathbf{C}_B\mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{N}} - \mathbf{C}_N)\mathbf{X}_N$$

The basic solution given by (4.19) can be used to find the optimal solution to the new ration formulation problem by first evaluating $\mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{D}}$ and $\mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{N}}$. This can be done by performing simple operations on the optimal solution to the original problem, given by (4.15):-

(a) Evaluation of $\mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{D}}$: first use right hand side parameterisation methods (e.g. Hadley (1962)) to determine $\mathbf{B}^{-1}\tilde{\mathbf{D}}$ from $\mathbf{B}^{-1}\mathbf{D}$ and then multiply the i^{th} element by $1/\gamma_i$.

(b) Evaluation of $\mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{N}}$: by noting that if $\mathbf{B}^{-1}\mathbf{N}=\{e_{ij}\}$, it can be seen from the definition of $\tilde{\mathbf{N}}$ that

$$\mathbf{B}^{-1}\tilde{\mathbf{N}} = \{\delta_j e_{ij}\}$$

$$\text{with } \delta_j = \begin{cases} (W_2/W_1)^{0.75} & \text{if } j^{th} \text{ element of } \mathbf{X}_N \text{ is } x_{k+1+i},\ i=0,1,\ldots,n \\ 1 & \text{otherwise} \end{cases}$$

and hence

$$\mathbf{G}^{-1}\mathbf{B}^{-1}\tilde{\mathbf{N}} = \{\delta_j e_{ij}/\gamma_i\}$$

Having performed operations (a) and (b), the objective function corresponding to this basic solution can be obtained. The optimal solution to the new ration formulation problem can then be found by performing simplex iterations if this basic solution is feasible, or by first performing dual simplex iterations if this basic solution is optimal but not feasible.

The use of this approach for obtaining basic solutions to the ration formulation problems depends on the initial range of f for piecewise linearisation of E_m/f and $E_g/(1-f)$ being sufficient to be applicable in the initial problem and in all subsequent problems with the same daily liveweight gain but different liveweights. From (4.4) and (4.5) it can be seen that for a specified daily liveweight gain G, the ratio E_g/E_m is independent of liveweight. Hence if the range of f defined by (4.11) is used, this range will be adequate for all liveweights for the specified daily liveweight gain. For this reason the range defined by (4.11) was used rather than the limits used by Brokken (1971a), which take account of the maximum DM intake.

To use this approach the values of liveweight and daily liveweight gain for which least cost rations are required must first be established. The procedure for determining least cost rations for the required values of liveweight and daily liveweight gain involves setting the gain to one of the required values and then solving the LP (4.12) for the highest value of liveweight for the specified gain. The liveweight is then reduced to the next highest value and the above approach is used to obtain a basic solution to this new problem from the previous optimal solution. If this solution is feasible or non-feasible but optimal, simplex or dual simplex iterations are performed to obtain the optimal solution to the new problem, otherwise the new LP is solved from scratch. The procedure is repeated, i.e. the liveweight is reduced and the new ration formulation problem is solved by first obtaining a basic solution to the new problem from the optimal solution to the previous problem, until all the liveweight values for the specified daily liveweight gain have been covered. The procedure is then repeated for all the required values of daily liveweight gain.

4.3.4 Computational Experience

The overall approach to determining the feeding policy for an intensive beef production unit in which animals are fed according to the nutrient standards recommended by the NRC (1976) has been applied to a unit in which the objective is to minimise the cost of producing animals of specified liveweight from animals of known initial liveweight. In this case the approach involves first using the LP model (4.12) to determine least cost rations and then using DP to determine the optimal feeding schedule by solution of the recurrence relations (4.2). In order to perform the calculations two FORTRAN programs were developed. The first program (see Appendix B3) is used to perform the calculations to determine least cost rations for appropriate values of animal liveweight and liveweight gain by solving the ration formulation LP model (4.12) by the improved solution procedure. The output from this program is then used as the input for the second program (see Appendix B4) which is used to perform the calculations for the solution of the DP recurrence relations.

The approach has been tested using representative cost data for an intensive beef production unit in which animals of liveweight 500 kg are to be produced from animals of liveweight 100 kg at minimum cost, with 13 feedstuffs available for ration formulation. The length of a feeding period was 4 days, and the LP model (4.12) was used to determine least cost rations for animals of liveweight 100,101,102,.....,500 kg which are required to produce daily liveweight gains of 0.25,0.50,0.75,.....,1.50 kg, provided such liveweight gains are possible for particular liveweights. The remaining least cost rations required for the DP model were obtained by interpolation.

The programs were run on an IBM 3081. The central processor time to calculate the least cost rations (approximately 2400) using a linearisation interval, Δ, of 0.01 was approximately 7 seconds. When the linearisation interval was reduced to 0.005, the central processor time was increased by approximately 75% and no significant differences were found in the compositions or costs of rations. The central processor time to solve the DP recurrence relations was approximately 1 second. An example of part of the output from the program for solving the DP model is shown in Table 10. The form of the output is similar to the output from the program used for the MAFF nutrient standards, but because of the form of the LP model (4.12), the rations satisfying the NRC nutrient standards may be composed of up to five feedstuffs.

The improvement in computational efficiency achieved by using the improved solution procedure rather than solving each ration formulation LP from scratch was investigated. For a specified liveweight gain the saving in computational time will depend on the number of values of liveweight to be considered and on the interval between successive liveweights. For the case considered above, where approximately 2400 rations were evaluated using a linearisation interval of 0.01, the central processor time when each ration formulation LP was solved from scratch was approximately five times longer than the corresponding case using the improved solution procedure, i.e. the improved solution procedure yields a significant reduction in the computational time. The approach has also been tested using a Cromemco System 3 microcomputer, but the computational times were very long, partly due to the lack of floating point hardware and the speed of accessing diskette files. However, with future developments in hardware it should be practical to use this approach on microcomputer systems.

4.4 CONCLUSIONS

The proposed approach for determining the feeding policy for an intensive beef production unit involves first determining least cost rations, then using a DP model to determine the daily sequence of these least cost rations. The approach can be used with any system for specifying the nutrient requirements of beef cattle, provided that the required least cost rations can be determined. This approach offers two significant advantages over previous methods. Firstly it ensures that optimal rations are used at all stages in the production process, and secondly it is not necessary to assume that the rate of liveweight gain must remain constant throughout the whole, or part, of this process. In addition, through the development of a parameterisation procedure which exploits the structure of the associated ration formulation model, the computational efficiency of the approach has been improved for the case where rations are formulated to meet the nutrient standards recommended by the NRC (1976).

The results from using the approach have demonstrated that the nature of an optimal feeding policy will depend on the nutrient standards, and the prices and nutrient composition of the feedstuffs available for ration formulation. The computational experience with realistically sized problems suggests that the approach would be practical for use in advisory work and that, with improvements in microcomputer systems, it could be applied in commercial intensive beef production units.

CHAPTER 5

FEEDING POLICY FOR PIG PRODUCTION

5.1 INTRODUCTION

Pig production in the agriculturally advanced world is becoming concentrated in large scale indoor units in which pigs are bred and fattened. The types of pig used in these intensive production systems have been developed through selective breeding for economically desirable traits, and for a given type of pig the economic performance of a fattening unit depends on the market value of the animals produced and the feeding policy of the unit. The market value of a pig is determined by the weight of the animal and the quality of the carcase, and since premium prices can be obtained for lean meat, pig carcases are frequently graded by weight and quality. Carcase quality is governed mainly by the genetic potential of the pig and the food it has consumed. Therefore for a given type of pig, the feeding policy of a fattening unit affects both the costs of production and the market value of the pigs produced, and since carcase quality is a major determinant of market value it should be taken into account in evaluating pig production policy.

In order to evaluate possible feeding policies for pig production the nutrients, i.e. the energy, protein, minerals, vitamins and water, required to produce liveweight gains of specified body composition must be known. The ARC (1967) in the UK and the NRC (1979) in the US both publish recommended nutrient allowances for growing pigs but, although both bodies recognise that feed intake affects both liveweight gain and the composition of this gain, neither body specifies the nutrient requirements in terms of the nutrients required to produce daily liveweight gains of specified composition. The composition of liveweight gain and the associated nutrient requirements were incorporated in the model of pig growth developed by Whittemore and Fawcett (1976). In this model daily liveweight gain was separated into fat free and fatty tissue components, and a development of this pig growth model has been used by Fawcett et al. (1978a, 1978b) in constructing an LP model to determine the least cost rations to produce daily liveweight gains of specified composition in terms of the fat free and fatty tissue content. However, although it is generally recognised that least cost rations should be used throughout the fattening period, the overall efficiency of the operation is also affected by the sequence of rations, i.e. the feeding policy, used to produce pigs of the required weight and carcase composition, and thus the model of Fawcett et al. (1978a, 1978b) has limitations for the evaluation of pig production policy.

In this chapter the approach developed for determining the feeding policy of an intensive beef production unit is extended for use in pig production by taking account of body composition in the feeding policy model. The approach involves first determining the least cost rations to produce daily liveweight gains of specified composition, and then using DP to determine the sequence of these least cost rations to produce pigs of the required liveweight and carcase composition at minimum cost. The method can be used with any pig growth model in which the development of the animal can be expressed in terms of two variables, such as liveweight and body protein content, provided that the least cost rations required to produce specified changes in these variables can be determined. The development of the method is illustrated using the pig growth model of Fawcett et al. (1978a) and a modified version of the ration formulation model of Fawcett et al. (1978b).

5.2 PIG GROWTH AND RATION FORMULATION

The growth of a pig of a specified type is controlled by the nutrient content, i.e. the energy, protein, mineral, vitamin and water content, of the food it consumes and by the ambient temperature. Although each nutrient has specific functions, the principal nutrients in pig growth are energy and protein. Energy is required to maintain body functions and for processes, such as new tissue synthesis, connected with growth. The weight gain in growth is accompanied by changes in the fatty and fat free body tissue together with associated development of bone, skin etc. The fatty tissue is composed of lipid and water while the fat free tissue is composed of protein and water, this protein being produced by conversion of dietary protein.

5.2.1 The Pig Growth Model

In the pig growth model of Fawcett et al. (1978a) the daily liveweight gain of the growing pig is separated into fat free and fatty tissue components expressed in terms of protein retention, P_r, and lipid retention, L_r, respectively. It is assumed that the daily liveweight gain, ΔW, is given by

$$\Delta W = \alpha P_r + 1.1 L_r \qquad (5.1)$$

where α represents the ratio of fat free body gain to protein retention. Genetic potential affects the maximum rate of protein deposition, and for a specified type of pig it is assumed that the daily protein retention must not exceed a finite maximum, $\bar{P}_r$, i.e.

$$P_r \leqslant \bar{P}_r \tag{5.2}$$

It is also assumed that the ratio of lipid retention to protein retention must exceed a minimum value, γ_L, i.e.

$$L_r \geqslant \gamma_L P_r \tag{5.3}$$

Consider a pig of liveweight W (in kg) and total protein mass P_T (in kg) and let E_m (in MJ) denote the daily maintenance energy requirement, excluding the energy required for protein synthesis. For example (Whittemore and Fawcett (1976)), E_m may be expressed as

$$E_m = 0.475W^{0.75} - 0.365P_T \tag{5.4}$$

If the required daily liveweight gain ΔW (in g) is to be composed of protein content P_r (in g) and lipid content L_r (in g) and the pig is fed a ration of digestible crude protein content P (in g/kg dry matter), then in the model of Fawcett et al. (1978a), the food energy requirement, E (in MJ), is given by

$$E = 0.0121P + (0.0115 + 0.0073/Z)P_r + 0.0535L_r + E_m \tag{5.5}$$

where Z is the ratio of protein retained, P_r, to total protein synthesised, P_t, and depends on the maturity of the animal.

Total daily protein synthesis is composed of new protein synthesis plus resynthesis of part of the protein which has been broken down. The daily protein synthesis depends on the quantity and quality of the dietary crude protein intake. In the approach of Fawcett et al. (1978a, 1978b), chemical value is used as a measure of protein quality and is determined by the essential amino acid in the feed which is limiting when the amino acid profile of the feed is compared with the preferred amino acid profile of the animal. The amino acid profile of a feed is obtained by expressing the content of each amino acid in the feed as a percentage of protein mass. The chemical value of a feed is given by the minimum value obtained when the concentration of each essential amino acid in the feed is divided by the corresponding value in the preferred profile. If a pig is fed a ration of digestible crude protein content, P, and chemical value V, $0<V<1$, then the daily new protein synthesis, P_n, must satisfy

$$P_n \leqslant VP$$

If the ratio of new protein synthesis to total protein synthesis is Y, where Y depends on the maturity of the animal, then from the definition of Y and Z,

$$P_r \leqslant PZV/Y \tag{5.6}$$

Although in general all essential amino acids should be considered in determining chemical value, only histidine, lysine and methionine + cystine will be considered in the subsequent discussion as Fawcett et al. (1978b) found that these were the only limiting groups in fifteen commonly used feedstuffs.

5.2.2 Limitations of the Pig Growth Model

The major factors influencing the response to food intake in a healthy pig are the genetic strain of the animal, sex and ambient temperature. The genetic potential will determine the maximum rate of deposition of fat free tissue, the ratio of fat free to fatty tissue in liveweight gain and the efficiency of conversion of food energy. In the model of Fawcett et al. (1978a) the maximum rate of protein retention and the minimum value of the ratio of lipid to protein retention must be specified and will depend on the type of pig. The efficiency of food conversion affects the coefficients in equations (5.4) and (5.5) and in developing these relationships average values have been used although ideally relationships for a specified breed and sex of pig should be determined.

The growth of the pig is also influenced by ambient temperature. Heat production is a natural part of the pig's body processes and this heat must be dispersed to maintain body temperature at the normal level. If the ambient temperature is too high then heat stress will cause the pig to eat less and grow more slowly. In the UK heat stress can generally be dealt with by increasing ventilation at relatively low cost, and therefore the ommission of heat stress from the model is unlikely to be a major limitation for pig production in the UK.

At low temperatures food energy is diverted from productive processes in order to increase heat production so that body temperature can be maintained. Cold animals will therefore grow more slowly and convert food less efficiently. If the food supply is not sufficient to provide the energy required to maintain body temperature, body fat stores, if available, will be used for heat production and the animal will lose weight. The effect of cold stress is not considered in the model of Fawcett et al. (1978a), although the model could be extended to incorporate this factor. For example, in the model of Whittemore and Fawcett (1976) it is assumed that the pig cannot maintain body temperature without diverting food energy from productive processes when the temperature falls below a critical temperature T_c (in degrees C) which depends on the liveweight W (in kg) of the pig, where

$$T_c = 23.8 - 0.15W \qquad (5.7)$$

At temperatures below this critical temperature food energy is used to provide heat to maintain body temperature, the heat required, Q (in MJ), depending on liveweight, W, critical temperature, T_c, (from (5.7)) and ambient temperature T (in degrees C), where

$$Q = 0.016W^{0.75}(T_c - T) \quad (5.8)$$

Thus when ambient temperature is less than the critical temperature for a pig of liveweight W, the food energy required to produce a specified liveweight gain as determined by (5.5) must be increased by Q, given by (5.8), to allow for cold stress. In the subsequent discussion, however, the effect of ambient temperature on pig growth will not be considered.

5.2.3 The Ration Formulation Model

Consider a pig of liveweight W (in kg) which is required to achieve a daily liveweight gain ΔW (in g) involving protein retention P_r (in g) and lipid retention L_r (in g) then, using the pig growth model of Fawcett et al. (1978a), ΔW, P_r, and L_r must satisfy (5.1). Suppose that k feedstuffs are available for ration formulation and for feedstuff i let

c_i = cost per kg DM

e_i = digestible energy content in MJ/kg DM

p_i = digestible crude protein content in g/kg DM

h_i = histidine concentration

l_i = lysine concentration

m_i = methionine + cystine concentration

where the histidine, lysine and methionine + cystine concentrations are expressed as percentages of protein mass, and let H, L and M denote the concentrations, as percentages of protein mass, of histidine, lysine and methionine + cystine respectively in the preferred amino acid profile of the animal.

Suppose that the pig is fed a ration containing quantity x_i, in kg DM, $x_i \geq 0$, $i=1,2,3,\ldots,k$, of feedstuff i. The histidine concentration of the ration as a percentage of protein mass is

$$(\sum h_i p_i x_i)/\sum p_i x_i = \sum h_i p_i x_i / P$$

where $P=\sum p_i x_i$ is the protein content of the ration, and hence the chemical value, V, of the ration is given by

$$V = \min [\sum h_i p_i x_i/HP, \sum l_i p_i x_i/LP, \sum m_i p_i x_i/MP] \tag{5.9}$$

The protein retention, P_r, must satisfy (5.6) and hence, substituting for V from (5.9),

$$P_r \leqslant \min [Z\sum h_i p_i x_i/HY, Z\sum l_i p_i x_i/LY, Z\sum m_i p_i x_i/MY] \tag{5.10}$$

and (5.10) can be expressed in a form suitable for LP as

$$(Y/Z)P_r - \sum h_i p_i x_i/H \leqslant 0 \tag{5.11a}$$

$$(Y/Z)P_r - \sum l_i p_i x_i/L \leqslant 0 \tag{5.11b}$$

$$(Y/Z)P_r - \sum m_i p_i x_i/M \leqslant 0 \tag{5.11c}$$

Clearly other amino acids could be taken into account in the same way.

Suppose that for this pig $L_r=\gamma P_r$ where $\gamma \geqslant \gamma_L$, then from (5.1),

$$P_r = \Delta W/(\alpha + 1.1\gamma) \tag{5.12}$$

with $P_r \leqslant \bar{P}_r$. Since the ration fed to the animal must satisfy (5.5) and (5.9), the least cost ration to achieve a liveweight gain ΔW with protein retention P_r and lipid retention $L_r=\gamma P_r$ is given by the solution of the LP :

$$\text{Minimise } C = \sum c_i x_i \tag{5.13a}$$

$$\text{subject to } \sum (e_i - 0.0121 p_i) x_i = E_m + bP_r \tag{5.13b}$$

$$\sum (h_i p_i/H) x_i \geqslant (Y/Z)P_r \tag{5.13c}$$

$$\sum (l_i p_i/L) x_i \geqslant (Y/Z)P_r \tag{5.13d}$$

$$\sum (m_i p_i/M) x_i \geqslant (Y/Z)P_r \tag{5.13e}$$

$$x_i \geqslant 0 \qquad i=1,2,3,\ldots,k,$$

where P_r is given by (5.12), E_m is given by, for example, (5.4), and b is given by

$$b = 0.0115 + 0.0073/Z + 0.0535\gamma$$

and where Y and Z are functions of the maturity of the animal.

5.3 FEEDING POLICY MODEL

The feeding policy of a pig production unit is defined in terms of the daily sequence of rations used in the fattening process. Fawcett et al. (1978a, 1978b) used the pig growth model to consider the characteristics of an optimal pig feeding policy and concluded that in the early stages of the production process high protein diets should be used to produce the maximum daily protein deposition. Although, as noted by Fawcett et al. (1978b), this conclusion is in accordance with feeding practice, it is based on implicit assumptions about the relative costs of supplying dietary protein and energy. In the model of Fawcett et al. (1978a, 1978b) the development of the animal must be specified in terms of the rate of liveweight gain and the composition of this gain, and therefore a large number of runs of this model would be required to determine the optimal feeding policy to produce pigs of specified liveweight and body composition.

The feeding policy of a pig production unit affects the cost of production and the weight and carcase composition of the pigs produced. In a unit producing pigs of specified weight and carcase composition, an optimal feeding policy will involve feeding least cost rations throughout the fattening period in such a way that the total production cost is minimised. If the least cost rations to produce daily liveweight gains of specified body composition can be determined, then by extending the beef cattle feeding DP model (4.1) to include both the liveweight and body composition of the animal, DP can be used to determine the optimal feeding policy for pig production, without the need to make assumptions about any of the characteristics of an optimal feeding policy. With the pig growth model of Fawcett et al. (1978a), these least cost rations can be determined by the solution of (5.13). By changing the final weight and carcase composition, this extended DP model can then be used to examine alternative production and marketing strategies for a pig production unit.

5.3.1 The Dynamic Programming Model

In the DP model for beef cattle feeding (4.1), the state of an animal was defined solely in terms of its liveweight. However, in order to take account of the influence of both liveweight and carcase composition in pig production, the state of a pig in the DP model must be defined in terms of liveweight and body composition. With the pig growth model of Fawcett et al. (1978a), body composition is expressed in terms of the protein and lipid content, and since liveweight, protein content and lipid content are related by (5.1), the state of the animal can be defined completely in terms of liveweight and body protein content. As in the beef cattle feeding model (4.1), a backward DP formulation (White (1969)) is used in order to be able to examine the effect of varying the final liveweight and carcase composition. Define $F_n(W,P_T)$ to be the cost of producing a pig of liveweight W with total protein content P_T from a pig initially of liveweight W_I and protein content P_I in n feeding periods, each of d days, using an optimal policy. Then

$$n \geq 1 \quad F_n(W,P_T) = \min_{y,z \geq 0} [F_{n-1}(W-y,P_T-z) + C(W-y,P_T-z,y,z) + K] \quad (5.14a)$$

$$F_0(W_I,P_I) = 0 \quad (5.14b)$$

where K is the overhead cost per feeding period and $C(W-y,P_T-z,y,z)$ is the cost of producing a liveweight gain y with protein content z in a pig of liveweight W-y and protein content P_T-z in d equal daily increments using least cost rations.

The DP model can be used to determine the overall minimum cost of producing pigs of specified liveweight and protein content and could be extended to accommodate time dependent feeding and overhead costs.

5.3.2 Solving the Dynamic Programming Model

For numerical work the state of the animal, expressed in terms of liveweight and body protein content, must be defined in discrete units. If the liveweight interval is w (in kg) and the protein content interval is p_T (in kg), the DP recurrence relations (5.14) must be solved for all possible combinations of liveweight, $W=W_I, W_I+w, W_I+2w, \ldots, W_F$, and protein content, $P_T=P_I, P_I+p_T, P_I+2p_T, \ldots, P_F$, by considering all possible liveweight gains, y, and protein increments, z, from each of these states at every stage in the solution procedure. The computational load and the fast access storage required for computer solution must also be taken into account in defining the states and possible transitions from these states at any stage in the solution procedure.

For computational convenience, the values of liveweight gain and protein increment over feeding periods of d days, i.e. the values of y and z, should be chosen to be integral multiples of w and p_T respectively. For example, if w=1 kg then $y=1,2,3,\ldots,[dG(W)]$ kg where G(W) is the maximum daily liveweight gain of a pig of liveweight W, and consequently the values of daily liveweight gain, ΔW, which should be considered in determining least cost rations would be $\Delta W=1/d, 2/d, 3/d, \ldots, [dG(W)]/d$ (in kg). Similar considerations apply in the choice of values for daily protein retention, P_r, and this can be dealt with by choosing suitable values of the ratio γ, $\gamma \geqslant \gamma_L$, and obtaining P_r from (5.12) for each of the required values of ΔW. For example, γ can be chosen so that the daily fat free gains take the values $0.5\Delta W$, $0.6\Delta W$, $0.7\Delta W$, $0.8\Delta W$.

From the definition of $C(W-y, P_T-z, y, z)$, it can be seen that the LP model (5.13) should be used to determine all the least cost rations involved in increasing the liveweight from (W−y) to W and the protein content from (P_T-z) to P_T in d equal daily increments, and that these calculations must be performed for each of the possible combinations of the state, defined in terms of liveweight and body protein content, at the start and end of a period of d days. In practice, however, it will generally be sufficient to calculate least cost rations for the middle day of a period of d days. For example, if w=1 kg and d=5 days and the number of possible values of daily liveweight gain is limited to five and the number of values of γ is limited to four, then if W_I=20 kg and W_F=100 kg, approximately 2×10^5 least cost rations must be calculated. However, since a change in the liveweight, protein content or protein increment, or any combination of these factors, affects only the right hand side of (5.13), the calculation procedure can be improved by using right hand side parameterisation methods (see, for example, Hadley (1962)) rather than solving each LP problem from scratch.

5.3.3 Return Maximisation Model

In pig production units which do not have contracts to produce pigs of specified market grade, the weight and carcase composition of the pigs produced can be changed to suit the market conditions. Under these conditions a suitable criterion for evaluating possible operating policies might be the return per animal per period, and an extension of the return maximisation model (4.3) developed for intensive beef production could be applied. Assume that pigs are introduced into the fattening system at a fixed liveweight and body composition and that final liveweight and body composition can be varied to some extent. Thus if

$F_n(W_S,P_S)$ = cost of producing a pig of liveweight W_S and body protein content P_S in n feeding periods using the optimal feeding policy derived from (5.14)

$S(W_S,P_S)$ = selling price of a pig of liveweight W_S and body protein content P_S

C_I = cost of producing a pig for the start of the fattening process

then the optimal return per animal per feeding period, $R(W_S,P_S)$, from producing pigs of liveweight W_S and body protein content P_S is given by

$$R(W_S,P_S) = \max_{n>0} [(S(W_S,P_S) - C_I - F_n(W_S,P_S))/n] \qquad (5.15)$$

The optimal combination of selling weight and carcase composition can be found by numerical solution of (5.15) for all commercially acceptable combinations of W_S and P_S. In addition, if the liveweight W_I and protein content P_I at the start of the fattening process can be varied, the optimal combination of starting and selling conditions can be obtained in a similar way by allowing C_I to be a function of W_I and P_I and redefining R and F_n to be functions of W_I, P_I, W_S and P_S.

5.4 USE OF THE PIG PRODUCTION MODEL

A suite of three FORTRAN programs has been developed to solve the DP cost minimisation model and thus determine the optimal feeding policy to produce pigs of specified liveweight and body protein content from pigs of given initial liveweight and body protein content. The first program (see Appendix B5) solves the DP recurrence relations (5.14), with least cost rations being calculated as they are required. This approach was adopted to reduce the fast access storage requirement, although at the expense of increased computational load. However, a right hand side parameterisation routine was incorporated in this program to reduce the computational load associated with the solution of the ration formulation LP model (5.13). Although this program uses the pig growth model of Fawcett et al. (1978a), it could be modified to use any suitable pig growth model. The second program (see Appendix B6) uses the output from the first program to determine the development of a pig, in terms of its liveweight and body protein content at the end of each feeding period, in the optimal feeding policy for the production of an animal of specified liveweight and body protein content. The third program (see Appendix B7) calculates the least cost rations for the optimal policy determined by the second program and prints the results. The second and third programs could be combined to enable the optimal feeding policy to be determined in a single step.

5.4.1 Data Requirements

To run the first program the operating data for the pig fattening unit must be supplied. This dataset specifies the nutrient composition and cost of each of the feedstuffs available for ration formulation, the weight and body composition of pigs entering the fattening unit and the maximum final weight of the pigs. In addition, the feeding period length, i.e. the value of d, used in the DP model (5.14) must be specified, and computational considerations must be taken into account in choosing the value of this parameter.

In the development of the DP model (5.14) it was assumed that the rate of liveweight gain remained constant throughout a feeding period of length d days. In the DP model solution program the liveweight gains, in kg, which can be achieved during a feeding period are restricted to integral values, i.e. the liveweight interval, w, between successive liveweight states in the DP model (5.14) is 1 kg, and thus the daily liveweight gains which are considered are restricted to $1/d, 2/d, 3/d, \ldots, [dG(W)]/d$ (in kg), where $G(W)$ is the maximum daily liveweight gain of a pig of liveweight W.

The choice of feeding period length also determines the possible daily protein increments associated with the daily liveweight gains. With the pig growth model of Fawcett et al. (1978a), daily protein increments can be related to the increase in fat free tissue by (5.1). In the DP model solution program the body protein content is expressed in terms of the fat free tissue weight, and the increase in fat free tissue weight during a feeding period is restricted to integral multiples of 100 g. For each liveweight and daily liveweight gain, ΔW, up to four values of protein increment can be considered, with the values of the lipid to protein retention ratio, γ, being determined from (5.12) such that the corresponding daily fat free gains are $0.5\Delta W$, $0.6\Delta W$, $0.7\Delta W$ and $0.8\Delta W$. Hence the maximum daily liveweight gain determines the maximum rate of protein deposition.

In order to ensure that a reasonable range of daily liveweight gains is considered with this version of the DP model solution program it is desirable to use a feeding period of at least 3 days. Since the daily liveweight gain remains constant during a feeding period there is a possibility that the solution will involve large increases in the rate of daily liveweight gain between successive feeding periods, and to reduce the possibility of this occurring the feeding period length should not exceed 10 days.

The values of the parameters of the pig growth model are also required to run the DP model solution program. The values of all the parameters except α, i.e. the ratio of fat free body gain to protein gain (see equation (5.1)), are given by Fawcett et al. (1978a). However, the value of α can be determined from the cases for which least cost rations were evaluated by Fawcett et al. (1978b). For example, for the case of a 20 kg pig required to produce a daily liveweight gain of 600.6 g with a protein content of 100 g and a lipid content of 100 g, it can be seen from (5.1) that $\alpha=4.906$. The same value of α was obtained for the other cases considered by Fawcett et al. (1978b) and this value was used in testing the DP model. From an analysis of tables given by Fawcett et al. (1978b), Z, the ratio of protein retention to protein synthesis, and Y, the ratio of new protein synthesis to total protein synthesis, were expressed as functions of liveweight.

The form of the least cost ration formulation LP model (5.13) is different from that used by Fawcett et al. (1978b), but the results produced by the two models should be the same. However, when the same feedstuff data was used to formulate least cost rations for the cases considered by Fawcett et al. (1978b), substantial differences were found in the results. In the LP model used by Fawcett et al. (1978b) a coefficient x_3 is used for the energy cost of protein deposition where, from Fawcett et al. (1978a), x_3 is given by (after correction of a typographical error)

$$x_3 = 0.0115 + 0.0073/Z$$

For a 20 kg pig, Z=0.2116 (in agreement with figures used by Fawcett et al. (1978b)), and thus x_3=0.046, but it was found, by substituting in the energy balance constraint of the LP tableau of Fawcett et al. (1978b) after correcting a typographical error, that a different value of x_3 had been used in the examples of least cost rations given by Fawcett et al. (1978b). For example, for a 20 kg pig required to produce a daily liveweight gain of 600.6 g with a protein content of 100 g and a lipid content of 100 g, it was found that x_3=0.086. Similar values for x_3 were obtained for other least cost rations considered by Fawcett et al. (1978b). These results indicate that there is an error in the data used by Fawcett et al. (1978b) and this has been confirmed (Fawcett (1983), personal communication). Because of this error the results from the pig production DP model cannot be compared with the published results of Fawcett et al. (1978b).

5.4.2 Computational Experience

The programs for determining the optimal feeding policy to produce pigs of specified liveweight and carcase composition have been tested using representative cost data for an enterprise in which pigs of 20 kg and protein content 3.0 kg (Whittemore and Fawcett (1976)) can be fed to a maximum liveweight of 100 kg, with four feedstuffs available for ration formulation. The feeding period length, d, was 5 days and thus the daily liveweight gain values considered were 200 g, 400 g, 600 g, 800 g and 1000 g, provided that such gains were possible for particular liveweights. The feedstuff data used to test the model are presented in Table 11 and were taken from Fawcett et al. (1978b). The programs were run on an IBM 3081. The central processor time to solve the DP recurrence relations was approximately 140 seconds. The central processor time to trace the development of a pig of specified final liveweight and body protein content was approximately 1 second, and the central processor time to calculate the associated least cost rations and print the results was approximately 3 seconds. The form of the output from the third program is shown in Table 12.

5.4.3 Discussion of Results

The results in Table 12 illustrate the form of the output from the programs but, since the results are dependent on both the pig growth model and on the feedstuffs and feedstuff prices used, general conclusions regarding feeding policy cannot be derived from these results. However, the results can be used to illustrate some aspects of the use of the DP model.

The results in Table 12 relate to the production of a pig of 100 kg liveweight with a body protein content of 13.29 kg, using a pig type for which the maximum rate of liveweight gain is 800 g per day and the maximum rate of fat free gain is 640 g per day, corresponding to a maximum rate of protein deposition of 130.5 g per day. It can be seen from Table 12 that the least cost rations are composed of only two feedstuffs. This is a direct consequence of the least cost ration formulation model used since, in general, only one amino acid will be limiting, i.e. only one of the constraints (5.13c), (5.13d), (5.13e) in the LP model (5.13) will be binding. It may be desirable to include additional dietary constraints in the ration formulation model, but this will not affect the DP model.

In solving the DP recurrence relations (5.14), only the ration for the middle day of each feeding period was considered in order to reduce the computational load associated with the DP model solution program. In theory the ration should change on a daily basis to reflect the increase in weight and change in body composition of the growing pig, but for practical administrative reasons daily changes in the ration are undesirable. If the feeding period used in the DP model is short then the effect of keeping the ration constant during a feeding period will not be significant, and in this case the most appropriate ration to use is the ration determined for the middle day of the feeding period. This ration can then be fed by dividing it into the number of feeds required during a day according to the practice of the pig production unit (e.g. Thornton (1973)).

If it is desired to maintain constant diets, defined in terms of the proportions of constituent feedstuffs, over longer periods, then diets can be averaged over feeding periods in which the compositions of the rations in the feeding policy determined by the DP model remain reasonably constant, with the total daily intake of this diet, i.e. the ration, increasing as the pig grows. For example, in Table 12 the composition of the ration remains reasonably constant during feeding periods 3 to 10 inclusive and also during feeding periods 11 to 21 inclusive. An average diet could be determined for each of these extended periods and this could be used as the constant diet in the appropriate extended period, with the daily quantity of this diet increasing with time. If sudden changes in diet are found it may be necessary to change diets gradually since the pig may react adversely to sudden changes in diet. The effect of using constant diets over a number of feeding periods could be examined by using the pig growth model to investigate the effect of this policy on the final weight and carcase composition of the pigs produced.

The feeding period length used in producing the results of Table 12 was 5 days and from these results it can be seen that after feeding period 2 the liveweight of the pig increases at a rate of 800 g per day. This corresponds to the assumed maximum daily liveweight gain of pigs of 24 kg or more. During feeding periods 3 to 10 inclusive the rate of protein deposition is 130.5 g per day, corresponding to the deposition of 640 g of lean tissue per day, i.e. the assumed maximum rate for a daily liveweight gain of 800 g. The composition of the ration remains reasonably constant during these feeding periods, the ration on the middle day being composed of 81.5% barley and 18.5% soya. During feeding periods 11 to 21 inclusive the rate of protein deposition is 81.5 g per day, corresponding to the deposition of 400 g of lean tissue per day, i.e. the assumed minimum rate for a daily liveweight gain of 800 g. The composition of the ration is also reasonably constant during these feeding periods, the ration on the middle day being composed of 98.9% barley and 1.1% soya, i.e. the energy and protein concentrations of the rations in these feeding periods are lower than those of the rations in the earlier feeding periods.

Similar results were obtained for the production of pigs of the same liveweight, i.e. 100 kg, but different body protein content. For example, the feeding policy to produce of a pig of 100 kg liveweight with a body protein content of 14.03 kg is shown in Table 13. It can be seen that after feeding period 2 the liveweight increases at a rate of 800 g per day, with the rate of protein deposition equal to the assumed maximum rate, i.e. 130.5 g per day, during feeding periods 3 to 13 inclusive, and falling to 81.5 g per day, i.e. the assumed minimum rate for a daily liveweight gain of 800 g, during subsequent feeding periods. It can also be seen that the cost of producing a pig with a body protein content of 14.03 kg is slightly lower than the cost of producing a pig of the same liveweight but with a body protein content of 13.29 kg. This result is due to the type of pig involved and the feedstuff data used.

In the results for the cases considered the rate of protein deposition in the final stages of growth is always equal to the assumed minimum rate for a daily liveweight gain of 800 g. This result suggests that it would be desirable to consider more than four values of protein increment for each liveweight gain value considered in the DP model solution program so that daily fat free gains of less than $0.5\Delta W$, where ΔW is the daily liveweight gain, could be considered. However, since the objective of this part of the study is to demonstrate that a DP model can be used to evaluate feeding policy when the state of an animal is defined in terms of two variables, it was felt that the additional effort involved in modifying the program was not justified.

In spite of the difficulties in establishing the values of some of the parameters in the DP model, the results in Table 12 and 13 are in agreement with the conclusions of Fawcett et al. (1978b) that pigs should be fed to produce maximum daily protein deposition in the early stages of growth. In developing the DP model it was not necessary to make any assumptions about the characteristics of an optimal feeding policy, and although the above result is in accordance with feeding practice, this result should not be regarded as being of general validity since it is likely to depend on the costs and nutrient compositions of the available feedstuffs. However, the DP model could be used to investigate the nature of optimal feeding policies for different feedstuff data.

In the DP (5.14) model the state of a pig was defined in terms of liveweight and body protein content. However, body protein content is not immediately meaningful to pig producers as a measure of carcase quality since pig carcases are graded in terms of the lean meat content rather than the protein content. The lean meat content of the carcase depends on the body protein content of the pig and can be estimated from equation (5.1) by making allowances for the ratio of carcase weight to animal liveweight. For example, for a pig of liveweight W and body protein content P_T, the percentage lean content, L_c, of the carcase was estimated by Whittemore and Fawcett (1975) to be given by

$$L_c = 426P_T/W \qquad (5.16)$$

Since this measure of carcase quality is more meaningful to a pig producer than body protein content, the output from the program could be modified to relate to producing a pig of given liveweight with a carcase of specified lean content. In the UK the lean meat content of a pig carcase is often estimated from measurement of backfat thickness. Since backfat thickness is related to body protein content, the pig growth model could, as suggested by Whittemore and Fawcett (1975), be used to obtain an estimate of backfat thickness from body protein content.

5.4.4 Validation of the Pig Production Model

The suitability of the feeding policies produced by the DP model has been investigated using the computer based Edinburgh Model Pig (e.g. Whittemore and Fawcett (1975)) of the ESCA. To use the Edinburgh Model Pig the initial liveweight, the initial body composition, the required final liveweight and the maximum rate of lean tissue deposition must be specified together with the diet and the quantity of diet to be fed on a weekly basis, with at most three different diet types being permitted. The feeding policy produced by the DP model was used as input data for the Edinburgh Model Pig and the development of the pig was examined. Since the time intervals at which the rations change in the DP model and the Edinburgh Model Pig are different, the results from the DP model which were used as input data for the Edinburgh Model Pig were chosen so that changes in diet occurred after an integral number of weeks.

In the results of Table 12, where the length of a feeding period is 5 days, the liveweight at the start of feeding period 4 is 28 kg and the composition of the ration remains reasonably constant for 5 weeks, i.e. until the end of feeding period 10, with the rate of lean tissue deposition in this time interval being 640 g per day. For the Edinburgh Model Pig the initial liveweight was therefore set at 28 kg, the initial body protein content was set at 4.2 kg and the maximum rate of lean tissue deposition was set at 640 g per day. The diet specified in the first 5 weeks for the Edinburgh Model Pig was the diet produced by the DP model for the middle of the 5 week period from the start of feeding period 4 till the end of feeding period 10, i.e. composed of 81.8% barley and 18.2% soya and having a digestible energy (DE) concentration of 13.1 MJ/kg DM, a digestible crude protein (DCP) concentration of 137.6 g/kg DM and a chemical value of 0.69, with the quantity of this diet increasing from 1.63 kg in week 1 to 1.81 kg in week 5. The diet in subsequent weeks for the Edinburgh Model Pig was the diet in feeding period 16 of Table 12, i.e. composed of 98.9% barley and 1.1% soya and having a DE concentration of 12.7 MJ/kg DM, a DCP concentration of 80.7 g/kg DM and a chemical value of 0.48, with the quantity of this diet increasing from 2.6 kg in week 6 to 2.95 kg in the thirteenth and subsequent weeks. Using this dataset for the Edinburgh Model Pig it was found (Fawcett and Gibson (1983), personal communication) that the time to produce a pig of 100 kg liveweight was 91 days and the protein content of this pig was 12.94 kg. These results correspond closely to the results of Table 12 where, starting with a pig of 28 kg liveweight and a body protein content of 4.24 kg, a time of 90 days is required to produce a pig of 100 kg liveweight and a body protein content of 13.29 kg.

A similar analysis was performed using the feeding policy of Table 13 as input to the Edinburgh Model Pig. In this case it was found that starting with a pig of 29 kg liveweight and a body protein content of 4.4 kg, then the time to produce a pig of 100 kg liveweight was 91 days and the protein content of this pig was 13.79 kg. These results correspond closely to the results of Table 13 where, starting with a pig of 29 kg liveweight and a body protein content of 4.37 kg (by interpolation), a time of 89 days is required to produce a pig of 100 kg liveweight and a body protein content of 14.03 kg.

These results from the Edinburgh Model Pig suggest that, in spite of the difficulties encountered in establishing parameter values from published sources, the pig growth component of the DP model corresponds closely to the pig growth component in the Edinburgh Model Pig. The differences in the results may be due to differences in the parameter values, e.g. the ratio of fat free body gain to protein gain, or to differences in particular components of the pig growth model, e.g. the expression used to specify the daily maintenance energy requirement. Differences in the results also arise because the composition of the ration changes from feeding period to feeding period in the DP model, while the diet remains constant for a specified number of weeks in the Edinburgh Model Pig.

5.5 CONCLUSIONS

The approach which was developed in previous chapters for determining the feeding policy of an intensive beef production unit has been extended for use in pig production where both animal liveweight and body composition must be taken into account in evaluating feeding policy. The optimal feeding policy involves feeding least cost rations throughout the fattening period and in order to use the method it is necessary to be able to determine the least cost rations to produce liveweight gains of specified body composition in pigs of known liveweight. A DP model is used to determine the sequence of these least cost rations to produce pigs of specified liveweight and carcase composition at least cost. The DP model can also be used to evaluate possible operating policies under different feedstuff and market conditions. The main advantages of this approach are that it is an optimisation approach and it is not necessary, as in the model of Fawcett et al. (1978a, 1978b), to make any assumptions about the rate of liveweight gain or the rate of change of body composition during the fattening period.

A suite of computer programs has been developed to solve the DP model. The results from using these programs suggest that it would be desirable to consider more than four values of daily protein increment for each value of daily liveweight gain. The present versions of the programs have been developed as a research tool for use on a mainframe computer. For this reason the programs are not suitable for use by individual producers although they could be used in advisory work. However, by using techniques such as path restriction (Norman (1972)) to reduce the computational load in solving the DP model, it would be possible to modify the programs to run on the more powerful of the present generation of microcomputers, and in this form the programs could be used by individual producers. It would also be desirable to extend the model to include the influence of ambient temperature, particularly with respect to cold stress. This could be done by incorporating in the pig growth model the energy requirement, given by (5.8), for maintenance of body temperature when the ambient temperature falls below the critical temperature, given by (5.7). Although it would not be desirable to include ambient temperature as a state variable in the DP model, since this would dramatically increase the computational load, it would be possible to use the extended model to investigate the effect of different house temperatures on pig production policy.

Although the method has been developed using a particular pig growth model, the approach can be used with any pig growth model provided the least cost rations to produce daily liveweight gains of specified body composition can be determined. The programs could be enhanced for use in research and in advisory work, and with further development the method could be made suitable for use by individual producers. The approach could also be used for any species in which development must be expressed in terms of two variables, such as liveweight and body protein content, provided that it is possible to calculate the least cost rations required to produce specified changes in these variables.

CHAPTER 6

INTEGRATED CROP AND INTENSIVE LIVESTOCK PRODUCTION

6.1 INTRODUCTION

Livestock production is often combined with crop production, with some of the crops being used for livestock feeding. This type of combined operation is more complex than the intensive livestock systems considered in the previous chapters since the interactions between crop and livestock production must be taken into account in the planning process. The methods of livestock production in these combined operations range from extensive land based systems to systems based on intensive feeding. In the extensive production systems the feed intake of the livestock cannot be controlled directly, but in cases where crop production is combined with intensive livestock production, the methods which were developed in the previous chapters can be used for evaluating livestock production policy. Since the method for evaluating livestock production policy is simplified if the body composition of the growing livestock can be ignored, as has been assumed for beef cattle, the subsequent discussion will be restricted to the case where crop production is combined with intensive beef production.

In an integrated crop and intensive beef cattle enterprise some, or all, of the crop production is used for animal feeding. The cattle for the intensive production unit may be obtained from a calf breeding and rearing part of the enterprise or they may be purchased and transported to the unit. Crops produced by the enterprise can, where appropriate, be used for feeding cattle in the intensive production unit or sold at market prices. Other feedstuffs, including supplies required because of insufficient enterprise grown production, can be purchased. The main factors which must be taken into account in evaluating the performance of this type of enterprise are the costs and liveweights of animals entering the intensive production unit, the selling prices and liveweights of animals leaving the unit, the feeding system used in the production process, the costs of purchased feedstuffs, the costs of crop production and the market value of crops. The market value of beef cattle is, as noted in Chapter 4, determined partly by the grade of beef, but since neither the MAFF in the UK nor the NRC in the US take account of the influence of feeding policy on body composition, it is assumed that the feeding policy does not affect the grade of beef when economic feeding practices are adopted. This is the assumption made by the NRC (1976).

The methods which have been developed for evaluating operating policy in integrated crop and intensive beef production enterprises, e.g. Clark (1973), Wilton et al. (1974), Ashour and Anderson (1975), make a number of simplifying assumptions, the most restrictive being that the rate of liveweight gain must remain constant throughout the whole, or major parts of, the fattening period. This limitation is overcome in the mathematical programming approach which was developed in Chapter 4 for evaluating production policy in an intensive beef production unit. However, this approach cannot be used where there are restrictions on the availability of some feedstuffs or where one or more of the feedstuffs can be obtained from different sources at different costs, as is the case in an integrated crop and intensive beef enterprise when additional supplies can be purchased if the enterprise grown production of particular feedstuffs is not sufficient for animal feeding. Previous attempts to model the operations of an integrated crop and intensive beef enterprise have also failed to deal with the possibility of purchasing feedstuffs which can be grown by the enterprise.

In this chapter an LP model is developed for an integrated crop and intensive beef enterprise. The model overcomes the limitations of previous models of this type of enterprise by using the beef cattle feeding DP model which was developed in Chapter 4 to obtain the values of some of the coefficients in the LP model. In order to simplify the analysis, only the crop production and intensive beef production activities are considered in developing this LP model, and it is therefore assumed that any calf breeding and rearing activities form a separate part of the enterprise, with appropriate transfer payments being made when weaned calves are moved to the intensive production unit. The model could, however, be extended to include calf breeding and rearing activities.

6.2 MODEL DEVELOPMENT

In an integrated crop and intensive beef production enterprise in which some of the crop production is used for cattle feeding, the feeding policy in the intensive production unit is influenced by the crop production policy of the enterprise. Since the land in an enterprise of this type can be used to produce crops for sale at market prices, the opportunity costs involved must be taken into account in evaluating the use of enterprise grown feedstuffs.

It is assumed that the animals in the intensive production unit are fed according to the nutrient standards recommended by a body such as the MAFF (1975) or the NRC (1976), and that it is possible to determine least cost rations for the production of specified daily liveweight gains in animals of known liveweight. Least cost rations should be used at all stages in the production process, but the rations used will depend on the feedstuffs available for ration formulation. If there are no restrictions on the supply of feedstuffs then the DP model (4.1) of beef cattle feeding can be used to determine the daily sequence of rations to use in the production unit. However, if the supply of some of the feedstuffs is limited, as is the case with enterprise grown feedstuffs, this DP model is not directly applicable.

6.2.1 Ration Types

The formulation of least cost rations for cattle feeding depends on the feedstuffs available for ration formulation. In integrated crop and intensive beef production operations in which the supply of some feedstuffs is limited, the feedstuffs available for ration formulation can be taken into account by introducing the concept of ration type.

Consider an enterprise which can grow n crop products on its land where m, $m \leqslant n$, of these crop products can be used as feedstuffs in formulating rations for cattle in the intensive production unit. In specifying the feedstuffs which can be produced by the enterprise, care must be taken in dealing with complementary crop products, such as barley grain and barley straw, which can be used for animal feeding. Each of these complementary crop products should be defined separately, although clearly the supplies of each are related. Other feedstuffs can be purchased for ration formulation, and it is assumed that the supplies of these feedstuffs are not restricted and that the unit cost of these feedstuffs is independent of the quantity purchased. The approach could, however, be extended to accommodate quantity discounts and restrictions on the supply of some purchased feedstuffs. The supply of the m feedstuffs which can be grown by the enterprise is limited and, by defining a ration type in terms of the enterprise grown feedstuffs designated as being available for ration formulation, then 2^m different ration types can be identified. For convenience, ration type 1 is defined as being that where no enterprise grown feedstuffs are designated as being available for ration formulation, i.e. rations of this type are formulated entirely from purchased feedstuffs, and this is the only ration type for which there are no limitations on the availability of feedstuffs.

It is assumed that when a set of enterprise grown feedstuffs are designated as being available for ration formulation, then least cost rations of the corresponding type can be formulated, using purchased feedstuffs as required. These least cost rations should be formulated to satisfy appropriate nutrient standards using methods such as those considered in Chapters 3 and 4. It should, however, be noted that even though a particular enterprise grown feedstuff is designated as being available for ration formulation, the least cost rations of this type, for some values of animal liveweight and daily liveweight gain, may not include this enterprise grown feedstuff, i.e. a ration type is defined in terms of the enterprise grown feedstuffs available for ration formulation even though some of these feedstuffs may not be included in some least cost rations of this ration type. For ration formulation the enterprise grown feedstuffs should be priced at the variable cost of production. In planning the operations of this type of enterprise, these costs must be estimated on the basis of expected yield. It should be noted that opportunity costs should not be included in pricing enterprise grown feedstuffs, since these costs are automatically taken into account in the LP model of the enterprise. This approach therefore overcomes the difficulties which arise with methods, such as marginal analysis (e.g. Barnard (1969)), in which it is necessary to determine the opportunity costs of enterprise grown feedstuffs.

6.2.2 Cattle Feeding

In the intensive production unit cattle are fed to produce liveweight gain. The liveweight gain which an animal can achieve in any day is a continuous variable with an upper limit depending on the type of animal, its liveweight and the feedstuffs available. In the LP model of the enterprise the development of animals of a specified type is considered at discrete time intervals, and the possible values of animal liveweight at these time intervals are limited to specified discrete values. Consequently the possible values of daily liveweight gain are limited to discrete values which depend on the time interval and the discrete set of liveweight values used in the LP model, with the influence of feedstuff availabilities on liveweight gain being taken into account in the LP model.

Suppose it is required to increase the liveweight of an animal from W_i to W_j, in kg, $W_j \geqslant W_i$, in a time interval composed of N feeding periods each of d days, where the set of possible liveweight values at the start and end of the time interval is defined so that successive liveweights are separated by ω kg. If the maximum liveweight gain in any time interval of Nd days is $G\omega$ (in kg), where G is integral, then the maximum daily liveweight gain is $G\omega/Nd$. The set of discrete values of daily liveweight gain is then $\{0,\omega/Nd,2\omega/Nd,3\omega/Nd,\ldots,G\omega/Nd\}$, and the possible values of liveweight, W_j, at the end of the time interval in which animals are initially of liveweight W_i is $W_j=W_i,W_i+\omega,W_i+2\omega,\ldots,W_i+G\omega$, provided these daily liveweight gains can be achieved by an animal of liveweight W_i.

Suppose that the feedstuffs required for the formulation of least cost rations of type r are available. Then if the limitations on the supply of the appropriate enterprise grown feedstuffs are ignored, a simple extension of the DP model (4.1) can be used to determine the feeding policy to produce a specified liveweight gain at least cost using rations of this type. Define $F_n^r(W)$ to be the cost of producing an animal of liveweight W from an animal of liveweight W_i in n feeding periods, each of d days, using an optimal feeding policy consisting of rations of type r. Then

$$n \geqslant 1 \qquad F_n^r(W) = \min_{h \geqslant 0} [F_{n-1}^r(W-h) + K + C_r(W-h,h)] \qquad (6.1a)$$

$$F_0^r(W_i) = 0 \qquad (6.1b)$$

where K is the overhead cost per feeding period and $C_r(W-h,h)$ is the cost of producing a liveweight gain h in d equal daily increments starting with an animal of liveweight (W-h) and using least cost rations of type r.

For a given set of available feedstuffs, the DP model (6.1) can be used to determine the optimal sequence of daily liveweight gains to increase the liveweight from W_i to W_j, $W_j=W_i,W_i+\omega,W_i+2\omega,\ldots,W_i+G\omega$, at minimum cost in a time interval composed of N feeding periods each of d days. Although limitations on the supply of enterprise grown feedstuffs are not taken into account in the DP model (6.1), these restrictions are incorporated in the LP model of the enterprise. Clearly if a liveweight gain $G\omega$ is to be achieved in N periods each of d days, the animal must grow at a constant rate $G\omega/Nd$. However, for intermediate values of the liveweight at the end of a time interval, the optimal sequence of daily liveweight gains is obtained from the solution of the DP model (6.1) and consequently the rate of liveweight gain throughout the time interval may not be constant.

The choice of the separation, ω, of successive liveweights at the start or end of a time interval will depend on the length of the time interval, i.e. on the values of N and d, and on computational considerations. For example, if a time interval is composed of 10 feeding periods each of 4 days, i.e. the time interval is of length 40 days, then a suitable separation of successive liveweights at the start or end of a time interval would be 10 kg. Using these values, i.e. $\omega=10$, $N=10$, $d=4$, the daily liveweight gains to be considered are 0,0.25,0.50,.....,1.50 kg, provided these daily liveweight gains are possible for animals of specified liveweight. If the minimum liveweight at which animals can enter the intensive production unit is W_0 and the maximum liveweight of animals in the unit is W_F, then for a given set of available feedstuffs least cost rations should be calculated for liveweights $W_0, W_0+\omega/Nd, W_0+2\omega/Nd, W_0+3\omega/Nd, \ldots, W_F$ and daily liveweight gains $i\omega/Nd$, $i=0,1,2,3,\ldots,G$, provided that these liveweight gains are possible for animals of any particular liveweight. However, as noted in Chapter 4, it will generally be sufficient to calculate least cost rations for these daily liveweight gains for a more limited number of values of liveweight and then obtain intermediate rations by interpolation.

6.2.3 The Linear Programming Model

The variables in the LP model of the enterprise relate to the activities of animal feeding, animal buying, animal selling, crop production and crop selling. The animal feeding activities are defined in terms of the number of animals fed to increase their liveweight from W_i to W_j, $W_j \geqslant W_i$, in time interval t, composed of N feeding periods each of d days, using rations of type r, i.e. the only enterprise grown feedstuffs which can be used for ration formulation are those defined in this type of ration. The associated cost of feeding the animal is obtained from the solution of the DP model (6.1). For any designated set of available feedstuffs, the total quantities of each of the corresponding enterprise grown feedstuffs in the rations required to increase the liveweight of an animal by a specified amount in a time interval are determined by tracing the sequence of daily liveweight gains from the solution of the DP model, and summing the quantities of each enterprise grown feedstuff contained in the daily rations required to produce this sequence of daily liveweight gains.

The variables and coefficients used in the LP model of this integrated crop and intensive beef production enterprise, for a planning period of T time intervals, are defined as follows:-

Variables:

x_{ijrt} - number of animals fed ration type r to increase liveweight from W_i to W_j, $W_j \geqslant W_i$, in time interval t

y_{it} - number of animals of liveweight W_i bought at end of time interval t

z_{jt} - number of animals of liveweight W_j sold at end of time interval t

z_{jT+1} - number of animals of liveweight W_j kept at end of the planning period

u_k - hectares used for production of crop k, k=1,2,3,....,n

v_k - tonnes of crop k produced for sale, k=1,2,3,....,n

Coefficients:

a_{ijrk} - quantity, in kg DM, of enterprise grown feedstuff k, k=1,2,....,m, in ration type r, $r=2,3,....,2^m$ to increase liveweight from W_i to W_j in any time interval (from solution of (6.1))

c_{ijr} - cost of using ration type r, $r=1,2,....,2^m$, to increase the liveweight of an animal from W_i to W_j in any time interval (from solution of (6.1))

s_{jt} - selling price of animal of liveweight W_j in time interval t

b_{it} - buying price of animal of liveweight W_i in time interval t

f_i - terminal valuation of an animal of liveweight W_i

d_k - DM content, in g/kg, of feedstuff k, k=1,2,....,m

g_k - quantity, in kg DM, of feedstuff k produced per hectare, k=1,2,....,n

p_k - profit contribution per tonne of crop k, k=1,2,....,n

In addition, the following terms are defined for the right hand side of the LP model:

N_i - number of animals of liveweight W_i initially

M_t - maximum number of animals in the intensive production unit in time interval t

A - hectares for crop production

The terminal valuation, f_i, of an animal of liveweight, W_i, is required to overcome the problems associated with having animals of below the normal selling weight at the far time horizon. The size of the model will clearly depend on the extent of the planning period. For simplicity it will be assumed that crop production is to be planned over a period of one year while animal production is to be planned over a period of one year from the time at which all crops have been harvested. The crop and animal production planning periods can be made to coincide by specifying the quantities of enterprise grown feedstuffs available initially, introducing additional variables and constraints for the enterprise grown feedstuffs stored at the start of each time interval, and considering the time at which each crop is harvested.

Taking profit contribution as the criterion for evaluating operating polices, the LP model of the enterprise is:-

$$\text{Max } P = \sum s_{jt} z_{jt} - \sum b_{it} y_{it} - \sum c_{ijr} x_{ijrt} + \sum p_k v_k + \sum f_i z_{iT+1} \qquad (6.2a)$$

subject to:

Feedstuff availability -

$$\sum a_{ijrk} x_{ijrt} + d_k v_k - g_k u_k = 0 \quad k=1,2,\ldots,m \qquad (6.2b)$$

Animal sales -

$$\sum x_{ijrt} - z_{jt} \geqslant 0 \quad \forall\ j,t \qquad (6.2c)$$

Continuity -

$$\sum x_{j1rt+1} - \sum x_{ijrt} + z_{jt} - y_{jt} = 0 \quad \forall\ j,t \qquad (6.2d)$$

Animal numbers -

$$\sum x_{ijrt} \leqslant M_t \quad \forall\ t \qquad (6.2e)$$

Initial conditions -

$$\sum x_{ijr1} = N_i \quad \forall\ i \qquad (6.2f)$$

Terminal conditions -

$$z_{jT+1} - \sum x_{ijrT} + z_{jT} - y_{jT} = 0 \quad \forall\ j \qquad (6.2g)$$

Farm size -

$$\sum u_k \leqslant A \qquad (6.2h)$$

$$x_{ijrt},\ y_{it},\ z_{jt},\ u_k,\ v_k \geqslant 0$$

Although variables x_{ijrt}, y_{it} and z_{jt} should be strictly integral, this requirement can be neglected as the errors introduced by rounding the solution to the LP problem will be small, since in commercial operations the numbers of animals at each stage in the production process will be large. Other constraints could clearly be incorporated in this LP model. To use the model it necessary to forecast crop yields and market prices of crops and livestock, and if required the model could be used to investigate the sensitivity of the results to forecasting errors in these areas.

6.3 USE OF THE MODEL

From the description of the LP model (6.2), it can be seen that before the model can be solved the coefficients c_{ijr} and a_{ijrk} must be determined for each possible ration type, r, and for all the possible values of animal liveweight at the start and end of time intervals of Nd days. The procedure for determining these coefficients involves first determining the least cost rations to produce specified daily liveweight gains in animals of known liveweight, using a specified set of available feedstuffs, i.e. a particular ration type. Methods for calculating least cost rations for beef cattle have been described in Chapters 3 and 4. These least cost rations are then used to determine the cost c_{ijr} of the optimal feeding schedule to increase the liveweight from W_i to W_j in a time interval, composed of N feeding periods each of d days, by solving the DP model (6.1) for all possible liveweights at the start and end of a time interval, using only rations of type r, $r=1,2,\ldots,2^m$. The quantities, a_{ijrk}, of each enterprise grown feedstuff k, $k=1,2,3,\ldots,m$, to increase the liveweight from W_i to W_j using rations of type r, $r=2,3,\ldots,2^m$, are determined from the solution to the DP model.

This process of determining c_{ijr} and a_{ijrk} is repeated for each ration type. Two FORTRAN programs were developed to calculate these coefficients. The first program (see Appendix B8) evaluates least cost rations for specified liveweights and liveweight gains, and is an extension of the least cost ration formulation program developed in Chapter 3 for the nutrient standards recommended by the MAFF (1975). The second program (see Appendix B9) uses the output from the first program to solve the DP model (6.1) and calculate the coefficients c_{ijr} and a_{ijrk}.

Because of the size of the LP model (6.2) a FORTRAN matrix generator program (see Appendix B10) was developed to set up the model in IBM MPS format (IBM (1972)). The procedure for setting up the LP model (6.2) thus uses a suite of three FORTRAN programs, i.e. a program for determining least cost rations, a program to calculate the coefficients c_{ijr} and a_{ijrk}, and the matrix generator program. The LP model can then be solved using the MPSX program product (IBM (1972)). The optimum operating policy for the enterprise can then be determined by inspecting the solution of the LP model. This last part of the procedure could be speeded up by use of a report writer.

6.3.1 Computational Experience

The approach has been tested using representative data (e.g. from Rix (1980)) for an integrated crop and intensive beef production enterprise which can produce four crop products, two of which can be used for animal feeding, i.e. a total of four ration types were available for animal feeding. Cattle were introduced into the intensive production unit at a liveweight of 100 kg and could be fed to a maximum liveweight of 420 kg. The length of the time interval for the LP model was 40 days, composed of 10 feeding periods each of 4 days, and the separation of successive liveweight values at the start or end of a time interval in the LP model was 10 kg. The crops which could be grown by the enterprise were barley, wheat and potatoes. The enterprise grown crop products which could be used for cattle feeding were barley grain and barley straw, and since these are complementary crop products, this had to be taken into account in constraints (6.2b) and (6.2h). Cattle were fed according to the standards recommended by the MAFF (1975), and least cost rations were calculated for animals of liveweight 100,101,102,.....,420 kg which were required to achieve daily liveweight gains 0,0.25,0.50,.....,1.50 kg, provided that these liveweight gains were possible for particular liveweights.

The programs were run on an IBM 3081. For each ration type the central processor time to calculate the least cost rations was approximately 13 seconds and the central processor time to solve the DP model and evaluate the coefficients c_{ijr} and a_{ijrk} for each ration type, r, was approximately 3 seconds. For the case where the planning period was 9 intervals of 40 days, i.e. approximately 1 year, the central processor time for the matrix generator was approximately 14 seconds.

The size of the LP model also depends on the feedstuff data used since, for a particular liveweight and liveweight gain, each of the ration types involving enterprise grown feedstuffs need only be considered if the corresponding least cost rations include some of these fedstuffs. For example, in case (A) considered in the next Section, the LP model involves 642 rows and 8405 variables (including one slack variable for each row) with a non-zero element density of

0.78%, and the central processor time to solve this LP model was 34 seconds. For case (B), which is also considered in the next Section, the LP model involved 642 rows and 8153 variables (including one slack variable for each row) with a non-zero element density of 0.81%, and the central processor time to solve this LP model was 62 seconds. The results produced by the MPSX program product (IBM (1972)) for these cases are too lengthy to reproduce, but some aspects of the operating policies derived by the model can be considered.

6.3.2 Discussion of Results

The results from this LP model of an integrated crop and intensive beef production enterprise will depend on the size of the enterprise and the operating policy, particularly in relation to the nutrient standards used for animal feeding, the crop production policy, the product prices and the input costs. For this reason the results from a limited number of runs of the model cannot be used to produce general conclusions regarding the operating policy of this type of enterprise.

It would be interesting to compare the results from this LP model with the published results from other models of enterprises of this type. To perform direct comparisons with other models it would be necessary to run this model with the same data, i.e. the same nutrient standards and the same crop and livestock data. In cases where nutrient standards other than those of the MAFF (1975) or the NRC (1976) have been used it would be necessary to develop new computer programs to calculate least cost rations. For example, Clark (1973) used the variable net energy system of McHardy (1965) in an LP model of a combined crop and beef production enterprise. Even in cases where MAFF (1975) or NRC (1976) standards were used it would be necessary to extend the LP model to include additional features, such as the breeding and rearing activities considered by Wilton et al. (1974). A considerable amount of work would therefore be involved in modifying and developing computer programs to perform direct comparisons with the published results from other models, and since the comparisons would only relate to particular sets of operating data, they would be of limited value. For these reasons direct comparisons with the published results from other models were not undertaken.

This LP model of a combined crop and intensive beef production enterprise differs from other models of this type of enterprise in two major respects. Firstly it is not necessary to specify the rate of liveweight gain during any part of the production process, and secondly the source of feedstuffs used for animal feeding is determined by the model, since some feedstuffs which could be grown by the enterprise can be purchased if required. By examining the results from a number of runs of the model the effect of including these factors can be evaluated, and this may give some indication of the importance of including these factors in models of operations of this type.

It has been shown in Chapter 4 that, in the absence of restrictions on the availabilities of feedstuffs, the rate of liveweight gain in an optimal feeding policy depends on the feedstuffs available for ration formulation and on the nutrient standards used. The LP model was therefore run using some of the combinations of feedstuff data and nutrient standards which were used to demonstrate the use of the DP model of beef cattle feeding in Chapter 4. In the cases considered in Chapter 4 it was found that when the basic energy allowances recommended by the MAFF (1975), i.e. excluding the 5% safety margin, and the feedstuff data of Table 5 were used (i.e. case (a) of Section 4.2.4), then in the optimal feeding policy the animals grew at a rate which corresponded closely to the maximum rate of gain. However, when the 5% safety margin recommended by the MAFF (1975) was included and the hypothetical feedstuffs defined in case (d) of Section 4.2.4 were used, the growth rate was generally less than the maximum growth rate. Since these two cases represent extremes in terms of the times to reach the maximum liveweight for the cases considered in Chapter 4, these two combinations of nutrient standards and feedstuff data were used to examine operating policy in an integrated crop and intensive beef production unit, and to investigate the importance of the distinguishing features of this model. The results for these two cases are discussed in cases (A) and (B) below.

(A) In this case the basic energy allowances recommended by the MAFF (1975) for beef cattle were used, i.e. the 5% safety margin was not included. In Chapters 3 and 4 when these energy allowances were used to formulate least cost rations using the feedstuff data of Table 5 it was found that the only feedstuffs used in the rations were barley, soya bean meal and oat straw. For this reason, and to allow some comparison with the results of case (a) of Section 4.2.4, it was assumed that only these feedstuffs, but with barley straw substituted for oat straw, could be purchased by the enterprise. The crops which could be grown by the enterprise were barley, wheat and potatoes, and the only enterprise grown feedstuffs which could be used for cattle feeding were barley grain and barley straw. The total area available for crop production was 138 ha, with at most 80 ha for barley, at most 80 ha for wheat and at most 30 ha for potatoes. The intensive production unit could accommodate at most 200 cattle, and for simplicity it was assumed that initially the unit contained 200 animals of 100 kg liveweight. These animals could be sold at a liveweight of 400 to 420 kg. The results for this case, in terms of the cattle feeding policy and the crop production policy, are summarised in Tables 14 and 15 respectively.

From Table 14 it can be seen that the animals are sold at a liveweight of 420 kg after six time intervals of 40 days, i.e. after 240 days. These animals are then replaced by a group of 200 animals of 100 kg liveweight, and in the next two time intervals the feeding policy for this second group is the same as that for the first group. The difference in feeding policy in the final time interval may be due to the figures used as terminal valuations of animals of less than the normal selling weight. The development of an animal in the first six time intervals is shown in Figure 3, with the development of an animal growing at the maximum rate and the development of an animal in the optimal policy of case (a) of Section 4.2.4 shown for comparison. It can be seen that, except during the third time interval, i.e. days 81 to 120, the animal grows at a rate corresponding closely to the maximum rate, the lower rate in the third time interval being necessary to allow a liveweight in the range 400 to 420 kg to be achieved in an integral number of time intervals. The development of an animal in this case is thus similar to the development in the case where only animal feeding activities were considered.

It can also be seen from Table 14 that, except in the sixth time interval, the same ration is fed to all the animals in the group. Only in the sixth time interval was it necessary to round the solution for variables which should be integral. In the sixth time interval 80 animals are fed rations of type 1, i.e. with no enterprise grown feedstuffs available, while the remaining 120 animals are fed rations of type 4, i.e. with enterprise grown barley and barley straw available, although in this case no enterprise grown barley straw is used and ration type 2, i.e. with only enterprise grown barley available, is an alternative solution to the LP model. However, the liveweight gain for all the animals in the group is the same. Two different ration types are required in this time interval because there is not sufficient enterprise grown barley available to feed rations of type 4 to the whole group, but the rations were found to be identical in terms of their barley and soya content. It can be seen from Table 14 that only ration of types 1 and 4 are used throughout, and further examination of rations of these types revealed that for daily liveweight gains of 0.50 kg or more, the compositions of least cost rations of types 1 and 4 are identical for the feedstuff data used in this case. For this reason there is not a dramatic change in the rations between time intervals 3 and 4, although the ration type changes from 1 to 4. Clearly in practice it would not be necessary to distinguish between purchased and enterprise grown barley in ration formulation.

From the crop production policy shown in Table 15 it can be seen that the maximum areas are used for the production of wheat and potatoes, with the remaining 28 ha being used for barley. Even though all of this barley is used for cattle feeding it is more economic in this case to use the available land to produce other crops and to purchase additional barley for cattle feeding. In this case none of the enterprise grown barley straw is used for cattle feeding. The results for this case indicate that the source of feedstuffs (i.e. purchased or enterprise grown) should be considered in LP models of this type of enterprise.

(B) In this case the energy allowances were increased by the 5% safety margin recommended by the MAFF (1975). The livestock, feedstuff and crop data was the same as in case (A), but with the hypothetical cereal and cereal straw defined in case (d) of Section 4.2.4, substituted for barley and barley straw. The results for this case, in terms of the cattle feeding policy and the crop production policy, are summarised in Tables 16 and 17 respectively.

From Table 16 it can be seen that the animals are sold at a liveweight of 410 kg at the end of the sixth time interval, i.e. after 240 days. These animals are replaced by a group of 200 animals of 100 kg liveweight, and the feeding policy for this second group of animals in the three remaining time intervals is the same as that for the first group. The development of an animal in the first six time intervals is shown in Figure 4, with the development of an animal growing at the maximum rate and the development of an animal in the optimal policy of case (d) of Section 4.2.4 shown for comparison. It can be seen that, except in the second and third time intervals, i.e. days 41 to 120, the animal grows at the maximum rate, and that the overall development is more rapid than in case (d) where only animal feeding activities were considered. However, the results from this case indicate that when the other activities of the enterprise are considered, it is not always optimal to assume that animals should be fed to achieve maximum daily liveweight gain.

It can also be seen from Table 16 that, except in the fifth time interval, the same ration is fed to all the animals in the group. In the fifth time interval, 7 animals are fed rations of type 3, i.e. with enterprise grown cereal straw available, while the remaining 193 animals are fed rations of type 4, i.e. with enterprise grown cereal and cereal straw available. Although different ration types are used in this time interval, the rations were found to be identical in composition. However, unlike the situation in case (A), it was found that corresponding least cost rations of the different types generally exhibit small differences in composition.

From the crop production policy shown in Table 17 it can be seen that the maximum areas are used for the production of both wheat and potatoes, with the remaining 28 ha being used for the production of the hypothetical cereal. Even though all this enterprise grown cereal is used for cattle feeding it is more economic in this case, as in case (A), to produce other crops and to purchase additional supplies of the hypothetical cereal for cattle feeding. However in this case, unlike case (A), some of the hypothetical cereal straw is used for cattle feeding.

The results from cases (A) and (B) demonstrate that in modelling the operations of an integrated crop and intensive beef production enterprise it may not always be optimal to assume that the animals should be fed to achieve the maximum daily liveweight gain. The results also indicate that the source of feedstuffs used in the intensive production unit should be considered in the model, particularly for feedstuffs which can either be purchased or grown by the enterprise. The inclusion of these factors increases the size of the LP model. In the results for the two cases considered, the optimal policy involved feeding for maximum liveweight gain over much of the production process, and feeding for liveweight gains close to the maximum value over the remaining parts of this process. It may therefore be possible to reduce the size of the LP model by considering only liveweight gain values close to the maximum. However, further research would be required to determine the validity of this approach.

6.4 CONCLUSIONS

A linear programming model has been developed for an integrated crop and intensive beef enterprise in which some of the feedstuffs which can be used for cattle feeding are grown on land which is owned or leased by the enterprise. The approach involves first determining the least cost rations to produce specified daily liveweight gains in animals of known liveweight. The method used to determine these least cost rations will depend on the system used to specify the nutrient requirements of the animals. A DP model is then used to determine the optimal feeding policy to increase the liveweight of an animal of known initial liveweight by a specified amount in a given time interval, using the least cost rations determined in the first stage of the approach. The results from this DP model are used to determine the coefficients for the animal feeding activities in the LP model of the enterprise. The approach therefore involves using a number of optimisation models in series.

By using this approach, the production and marketing policies for both the crop and livestock parts of the enterprise can be evaluated, taking account of the interactions between these two parts of the enterprise. The advantages of this approach are firstly that it is not necessary to assume that the rate of liveweight gain is constant during major parts of the production process and secondly the source of feedstuffs contained in the rations used at each stage in the production process is determined by the model. This approach also overcomes the difficulties associated with pricing enterprise grown feedstuffs, since the opportunity costs involved in using enterprise grown feedstuffs for livestock feeding are automatically taken into account. The model could be extended to include other activities, such as cattle breeding. The size of the model is such that it is necessary to use commercial LP codes, and for this reason it is not suitable for use by individual farmers, although it could be used in research and advisory work.

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 MODELS OF AGRICULTURAL SYSTEMS

Modern agricultural systems have been developed largely as a result of technological progress. Since many of these new technologies are capital intensive, agricultural production in the developed world is becoming concentrated in larger and more specialised units than are found in traditional agricultural systems. The adoption of production systems based on these new technologies has resulted in improvements in yields in both the crop and livestock sectors, but although technological progress has helped to reduce the variability in crop and livestock yields, the greater degree of specialisation and the need to obtain adequate returns from capital intensive production systems have added to the problems involved in planning in the agricultural sector.

The increased complexity of modern agricultural systems has led to the development of formal planning techniques based on the construction and analysis of mathematical models. A wide range of mathematical models of agricultural systems or sub-systems has been developed for use at levels ranging from the individual farm to the regional or national level, but with most of the published work relating to farm level operations. The ultimate objective of model building effort in this area should be concerned with improving decision making in agriculture, but it has been noted that many of the models have been used only as research tools or teaching aids. However, the use of models in research and education can help to improve understanding of the operation of agricultural systems, particularly those which involve the dynamic interaction of a number of biological processes. In some cases, especially where model building effort has been in progress for a limited time, e.g. with respect to crop-pest systems, there is a need for further research to establish the relationships between the various components of the systems involved. The experience and knowledge gained in the model building process can be valuable in directing research programmes to provide the basic data required, and ultimately models which can be used by decision makers may be produced. In certain areas there is therefore a need to develop models as research tools as a first step in the process of producing practical aids to decision making in agriculture.

Although some models of agricultural systems have been developed specifically as research tools, many models which have been proposed for use in planning aspects of agricultural production have had only limited application. Kennedy (1973) has suggested that, in farm planning in particular, the limited use of these models has arisen because methodological advances have not been accompanied by the development of the associated information and control systems which are required for implementation. This may be partly a reflection of the relative importance attached to the development of methodology, in comparison with the importance attached to the solution of practical problems, in academic circles. The emphasis on methodology has tended to result in the development of complex models which are of little practical relevance. Attempts have, however, been made to incorporate model building methodology in planning systems in such a way that the systems are suitable for use by individual farmers, but Nix (1979), citing the experience with computerised LP based planning systems which were designed for ease of use, noted that even systems designed in this way may not receive widespread acceptance by farmers. It has been argued in this study that the limited application of these LP based planning systems may be partly due to the fact that farmers were not directly involved in the use of these planning systems, since there is some evidence to suggest that farmers are more likely to make use of a planning system in their decision making if they are more directly involved in its use.

Part of the problem associated with using some of the models which have been developed for use in agricultural planning is that the use of a computer is required for the model solution phase. In the past this has generally involved using remote computing facilities and often the postal service, with its associated delays, has been used for transmitting both input data and results. The development of microcomputers has brought about a dramatic reduction in the cost of computing equipment, so that powerful computing facilities can now be purchased by individual farmers at relatively low cost. Some of the planning systems developed on mainframe computers for advisory work could be developed for use on microcomputers, but it will be necessary to convince farmers of the benefits of investing in microcomputers and associated software. It is likely that microcomputers will be purchased by farmers initially to perform simple but tedious tasks such as budget preparation and cash flow analysis, and Nix (1979) has noted that simple systems are being developed for use in these areas. As farmers become familiar with the use of microcomputers they will become more receptive to the idea of using microcomputers to perform more sophisticated analyses, based on the use of mathematical models, but it will be necessary to ensure that the software is both robust and simple to use. It will also be necessary to ensure that farmers have the necessary knowledge and skills to use these models effectively.

7.2 THE LIVESTOCK PRODUCTION MODELS DEVELOPED IN THIS STUDY

This study has been concerned with developing models of livestock production. The main emphasis of the model building has been in relation to intensive livestock systems and the interaction of livestock and crop production activities in integrated crop and intensive livestock operations. Mathematical models have been developed in the past for many aspects of livestock production, but frequently major simplifications have been required, either because of the complexities of the systems involved or because of the limitations of the techniques used. In particular, in evaluating livestock feeding policies it has been necessary to assume that the rate of liveweight gain remained fixed during major parts of the production process, and in models of combined crop and intensive livestock production operations the possibility of purchasing feedstuffs which could be grown by the enterprise was not considered.

Historically the use of mathematical models in livestock production has been associated with the application of LP in the formulation of least cost feeds of specified nutrient composition. However, this application of LP is of limited value since livestock performance should be considered in evaluating livestock feeding policy, especially in intensive operations. It is widely recognised that least cost rations should be used throughout the livestock production process, but the formulation of least cost rations is complicated by the way in which the nutrient requirements of livestock are often specified. This is particularly important in the case of ruminant species. A further complication arises because the method of ration formulation depends on the type of livestock and on the system used to specify the nutrient requirements, with different systems being recommended by bodies such as the ARC in the UK and the NRC in the US.

The system recommended for beef cattle by the MAFF (1975) in the UK is based on the system developed by the ARC (1965). In this system the metabolisable energy requirement of the animal depends on the metabolisable energy concentration of the ration, and this interdependency causes difficulties in ration formulation. The variable net energy system was developed by Harkins et al. (1974) from the ARC (1965) system to overcome these difficulties and to allow rations to be formulated in an additive manner. This variable net energy system is suitable for least cost ration formulation by LP but it is based on approximations which can cause significant errors in ration formulation. The magnitude of these errors has been investigated and it has been demonstrated that the magnitude of the errors associated with using this system depends on the composition of the ration. Hence, although the variable net energy system simplifies the formulation of least cost rations, it is unreliable in that it can give rise to errors of varying magnitude.

An iterative approach has been suggested by Kennedy (1972) for formulating least cost rations for beef cattle to satisfy the nutrient requirements recommended by the ARC (1965). This approach involves determining least cost rations for a number of different values of the metabolisable energy concentration of the ration and searching for the overall least cost ration, but this procedure is time consuming. In this study a parametric programming method has been developed for formulating least cost beef cattle rations satisfying the nutrient requirements of the MAFF (1975). This approach involves using parametric programming to derive a piecewise relationship for the cost of the ration as a function of its metabolisable energy concentration, and then using differential calculus to determine the least cost ration. This approach enables least cost rations to be formulated with negligible error, and the method could be extended to incorporate other factors, such as rumen degradable protein and undegradable protein, as functions of the metabolisable energy concentration of the ration, as in the system proposed by the ARC (1980). Although this parametric programming method for beef cattle ration formulation has been developed on a mainframe computer, it could be developed for use by individual farmers on microcomputer systems.

In livestock production not only is it necessary to determine the least cost rations to produce specified daily liveweight gains in livestock of given liveweight, but it is also necessary to determine the sequence of these rations to use in the production process. A DP model has been developed to determine the feeding policy to produce beef cattle of specified liveweight from animals of known initial liveweight at minimum cost using least cost rations formulated by means of the parametric programming method. By using this approach it is ensured that least cost rations will be used at all stages in the production process and it is not necessary to make any assumptions about the rate of liveweight gain during this process. Beef cattle feeding policy has been investigated using this approach and it has been shown that the rate of liveweight gain associated with an optimal feeding policy depends on the feedstuffs available for ration formulation and on the nutrient standards used. This DP model can also be used to investigate marketing strategy for an intensive beef production unit.

The beef cattle feeding DP model can be used in conjunction with any system for specifying the nutrient requirements of beef cattle, provided that the least cost rations to produce specified daily liveweight gains in animals of known liveweight can be determined. The use of this DP model has also been demonstrated for the case where beef cattle are fed to meet the nutrient standards recommended by the NRC (1976), and an improved method for calculating the required least cost rations has been developed. The overall approach for evaluating beef cattle feeding policy could be developed for use on microcomputers, using the nutrient standards recommended by either the MAFF (1975) or the NRC (1976).

A limitation of the approach developed for evaluating beef cattle feeding policy is that the grade of beef produced is not considered as a function of feeding policy. If there existed beef cattle nutrient standards which specified the nutrients required to produce daily liveweight gains of specified composition in terms of, for example, the fat free and fatty tissue content, then the DP model could be extended to take account of both liveweight and body composition in evaluating beef cattle feeding policy, provided that it was possible to determine the least cost rations to produce daily liveweight gains of specified body composition. Nutrient standards of this form are not available for beef cattle but Fawcett et al. (1978a, 1978b) have developed a model of pig growth in which daily liveweight gain is separated into its fat free and fatty tissue content. This pig growth model has been used to demonstrate how the beef cattle feeding DP model can be extended to take account of both liveweight and body protein content in evaluating feeding policy for pig production. The approach could be used with any type of livestock and any model of livestock growth which considered the development of the livestock in terms of two components, such as liveweight gain and body protein increment, provided that the least cost rations to produce specified changes in these variables can be derived.

The computational load associated with solving the DP model in which two variables are used to specify the state of an animal is much greater than the case where only liveweight is considered, but the computational experience with solving the pig production DP model has demonstrated the feasibility of the approach. With improvements in the computational procedure it should be possible to use this model as the basis of a planning tool which could be mounted on the more powerful of the present generation of microcomputers. The DP model could be extended to include additional decision variables, such as the temperature in the production unit, but an extended model of this type is likely to be of little practical significance since the associated computational load would be very large.

Intensive livestock production is often coupled with crop production in such a way that some of the crop production is used for livestock feeding. The major limitations of previous models of this type of integrated operation are that it was necessary to specify the rate of liveweight gain during major parts of the production process and the possibility of purchasing feedstuffs which could be grown by the enterprise, either as the sole source or to supplement enterprise production, was ignored. These limitations have been overcome by using the DP model to determine the minimum cost method of producing specified liveweight gains over a given period, using a set of defined feedstuffs for livestock feeding. The results from the DP model are used to obtain the values of coefficients for livestock feeding activities in an LP model which also incorporates the livestock buying, livestock selling, crop production and crop selling activities of the enterprise. This approach also overcomes the difficulties associated with pricing enterprise grown feedstuffs. Although other related activities, such as livestock breeding and rearing, are not incorporated in this model, it would be possible to extend the model to include these activities. However, it is only possible to consider livestock growth in terms of liveweight since the size of the model would increase dramatically if more than one variable was used to describe livestock development. The operating policy of an integrated crop and intensive beef production enterprise has been investigated using this LP model and it has been shown that in operations of this type it is not always optimal to assume that animals should be fed to achieve maximum daily liveweight gain. The results also indicate that the source of feedstuffs used in the intensive production unit should be considered in evaluating operating policy. Because of the size of the model and the need to use a commercial LP code, the model is not suitable for use by individual farmers, although it may be useful in research and advisory work.

7.3 CONCLUSIONS

In this study mathematical programming models have been developed for different aspects of livestock production. By using the appropriate models in series, the operating policy for an integrated crop and intensive beef production operation can be evaluated without, as in previous methods, the need to specify the nature of the livestock feeding policy. When the beef cattle nutrient standards of the MAFF (1975) are used the approach involves first determining the least cost rations to produce specified daily liveweight gains in animals of known liveweight, using a parametric programming method which involves the use of the differential calculus. The next stage involves using a DP model to determine the minimum cost method of producing specified liveweight gains over a given period using a defined set of feedstuffs for ration formulation. Finally the results from this DP model are used to establish the values of some of the coefficients in an LP model which incorporates activities relating to both crop and livestock production. The overall approach in this case thus involves using a series of optimisation models, namely an LP model which incorporates the differential calculus in a parameterisation procedure, a DP model and finally an LP model, to evaluate the operating policy for the enterprise.

By using the optimisation methods in series in this way the limitations of particular techniques can be overcome. It is ensured that least cost rations will be used at each stage in the production process and it is not necessary to assume that the rate of liveweight gain is fixed during any part of this process, although it is necessary to specify a discrete set of possible values of daily liveweight gain. The approach also overcomes the difficulties associated with pricing feedstuffs which are grown for livestock feeding since the opportunity costs involved are automatically taken into account.

This type of mathematical programming approach can be adopted for any livestock species, provided that it is possible to determine the least cost rations to produce specified daily liveweight gains in livestock of known liveweight. Although in theory this approach could be used where the development of the livestock is described in terms of two variables, such as liveweight and body protein content, the size of the resultant LP model of the enterprise would make the approach impractical. The DP model can, however, be used independently for any livestock species and it has been demonstrated, using the example of pig production, that the use of this DP model is feasible in the case where livestock development is defined in terms of two variables.

The use of optimisation models in series in this way has not been suggested previously for use in agricultural planning, and in addition techniques have been developed to improve the solution procedures for some of the optimisation models developed in this study. In particular it has been demonstrated that by using the differential calculus in association with LP parameterisation it is possible to derive least cost rations satisfying the nutrient standards recommended by the MAFF (1975) for beef cattle. This approach could be applied more generally to obtain the solution to other optimisation problems where LP parameterisation can be used to obtain a piecewise representation of the objective function, with differential calculus being used to obtain the overall optimal solution. A new LP parameterisation procedure has also been developed to reduce the computational load in calculating a large number of least cost rations satisfying the nutrient standards recommended by the NRC (1976). In calculating these least cost rations the coefficients of some of the variables in the ration formulation LP model change by the same factor, while the coefficients of all the other variables remain unchanged. LP parameterisation methods have generally concentrated on the right hand side and the objective function, with relatively little work being concerned with the interior of the LP matrix. The parameterisation method developed for calculating these least cost rations relates to the interior of the LP matrix and could be used for other LP problems in which the coefficients of the variables change in the same way as in this ration formulation problem.

This study has been concerned with developing models for evaluating livestock production policy, particularly in intensive operations. Since livestock production policy depends on the type of livestock, the feedstuffs available for ration formulation and the market prices of both feedstuffs and livestock, it is not possible to derive general results which will be valid under all conditions. However, models such as the DP model of beef cattle feeding could be used to attempt to obtain optimal rules for beef cattle feeding which would be valid for a specified set of feedstuffs and the range of feedstuff prices which are likely to arise in the short to medium term. The approach advocated in this study is therfore different from that which traditionally has been associated with the economic analysis of livestock production, with its emphasis on the investigation of the characteristics of optimal operating policies.

Large scale intensive livestock production is an important part of the agriculture of the developed world. Although intensive livestock production systems are more efficient in economic terms than traditional systems, these intensive systems require large energy inputs and are biologically inefficient in terms of the use of resources, particularly cereals, which could be used for direct human consumption. With increasing pressure on food supplies and non-renewable energy resources it is important to improve the economic efficiency of livestock production through the use of models such as those developed in this study.

The models developed in this study could be used in research and advisory work. The LP model of an integrated crop and intensive beef production enterprise is, by its nature, not suitable for use by individual farmers, but the other models developed in this study could be developed further for use at farm level, although it may be necessary to adapt the approach for specific applications. In some cases it may be sufficient simply to modify the models to include other factors, such as additional nutrients, while in other cases it may be necessary to extend the approach to deal with different types of livestock or different nutrient standards. Published data on livestock performance have been used throughout this study and therefore the problems of implementation have not been considered. In practice it would also be necessary to develop systems for monitoring actual performance so that model parameters could be revised in the light of discrepancies between predicted and actual performance. The next stage of work in this area should be concerned with developing systems which are suitable for use on microcomputers by individual farmers. This will involve improving the computational efficiency of some of the solution methods and developing software which is both reliable and simple to use.

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TABLE 1

EXAMPLES OF TRENDS IN CROP YIELDS IN THE UK

Crop	Average Annual Crop Yields (Tonnes/ha) 1948-1952	1958-1962	1968-1972	1978-1981
Wheat	2.72	3.40	4.04	5.51
Barley	2.52	3.09	3.63	4.27
Oats	2.28	2.54	3.52	4.08
Potatoes	19.0	20.0	26.7	33.1

(Source : Food and Agriculture Organization (1949-))

TABLE 2

EXAMPLES OF TRENDS IN WHEAT YIELDS IN DIFFERENT COUNTRIES

Country	Average Annual Wheat Yields (Tonnes/ha) 1948-1952	1958-1962	1968-1972	1978-1981
Australia	1.12	1.25	1.15	1.39
Canada	1.28	1.16	1.72	1.84
France	1.83	2.39	3.83	4.94
Italy	1.52	1.80	2.38	2.68
UK	2.72	3.40	4.04	5.51
USA	1.12	1.63	2.11	2.25

(Source : Food and Agriculture Organization (1949-))

TABLE 3

EXAMPLES OF TRENDS IN AVERAGE ANNUAL MILK AND EGG YIELDS IN THE UK

Year	1964/65	1969/70	1974/75	1979/80
Average milk yield per cow (litres)	3537	3738	3989	4668
Average egg yield per laying hen	204	218	231	247

(Source : Ministry of Agriculture, Fisheries and Food (1972-))

TABLE 4

LARGEST ERRORS FOUND IN TWO COMPONENT RATIONS

FORMULATED USING THE VARIABLE NET ENERGY SYSTEM

Animal Production Level	Largest Error (%)	Composition of Ration with Largest Error			
		Feedstuff 1		Feedstuff 2	
		ME Conc (MJ/kg DM)	Quantity (kg DM)	ME Conc (MJ/kg DM)	Quantity (kg DM)
1.1	-0.48	6.0	5.0	15.0	2.0
1.2	-1.43	6.0	5.0	15.0	2.0
1.3	-2.47	6.0	5.0	15.0	2.0
1.4	-3.49	6.0	5.0	15.0	2.0
1.5	-4.42	6.0	4.5	15.0	2.0
1.6	-5.16	6.0	4.5	15.0	2.5
1.7	-5.54	6.0	4.0	15.0	3.0
1.8	-5.93	6.0	3.5	15.0	3.0
1.9	-5.82	6.0	4.0	15.0	4.5
2.0	-5.79	6.0	3.0	15.0	4.0
2.1	-5.81	6.0	3.0	15.0	4.5
2.2	-5.19	6.0	2.5	15.0	5.0
2.3	-5.12	6.0	2.5	15.0	5.5

TABLE 5

FEEDSTUFF DATA FOR BEEF CATTLE RATION FORMULATION

Food Number	Food Name	ME MJ/kg DM	DCP g/kg DM	Cost p/kg DM
1	Barley	13.7	82	10.6
2	Maize	14.2	78	13.7
3	Oats	11.5	84	10.3
4	Sugar beet pulp	12.2	61	11.1
5	Soya bean meal	12.3	453	17.2
6	Dried grass	10.6	136	13.2
7	Barley straw	5.8	8	3.1
8	Oat straw	6.8	9	3.5
9	Hay	8.4	39	8.2

Abbreviations : ME - metabolisable energy
DCP - digestible crude protein
DM - dry matter

Source : ME and DCP values from MAFF (1975).

TABLE 6

EXAMPLES OF OUTPUT FROM BEEF CATTLE RATION FORMULATION PROGRAM

LIVE WEIGHT (KG)	RATION SPEC	DAILY LIVEWEIGHT GAIN (KG) 0.00	0.25	0.50	0.75	1.00	1.25	1.50
100	FOOD NO,%	8 92.1	8 85.4	5 14.9	5 14.5	5 14.6	0 0.0	0 0.0
	FOOD NO,%	5 7.9	5 10.3	1 85.1	1 85.5	1 85.5	0 0.0	0 0.0
	FOOD NO,%	0 0.0	1 4.2	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	2.26	3.00	1.82	2.21	2.68	0.00	0.00
	COST (P)	10.37	15.65	21.08	25.53	31.03	0.00	0.00
101	FOOD NO,%	8 92.1	8 85.6	5 15.0	5 14.5	5 14.6	0 0.0	0 0.0
	FOOD NO,%	5 7.9	5 10.4	1 85.0	1 85.5	1 85.4	0 0.0	0 0.0
	FOOD NO,%	0 0.0	1 4.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	2.27	3.02	1.83	2.22	2.69	0.00	0.00
	COST (P)	10.43	15.72	21.17	25.63	31.14	0.00	0.00
	.							
	.							
	.							
	.							
170	FOOD NO,%	8 91.3	8 88.2	5 15.7	5 15.0	5 14.2	5 12.6	0 0.0
	FOOD NO,%	5 8.7	5 11.8	1 84.3	1 85.0	1 85.8	1 87.4	0 0.0
	FOOD NO,%	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	3.06	4.05	2.36	2.81	3.36	4.03	0.00
	COST (P)	14.37	20.75	27.46	32.58	38.77	46.09	0.00
171	FOOD NO,%	8 91.3	8 88.2	8 77.0	5 15.0	5 14.2	5 12.6	0 0.0
	FOOD NO,%	5 8.7	5 11.8	5 12.4	1 85.0	1 85.8	1 87.4	0 0.0
	FOOD NO,%	0 0.0	0 0.0	1 10.6	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	3.07	4.07	4.63	2.82	3.37	4.04	0.00
	COST (P)	14.44	20.81	27.54	32.68	38.87	46.20	0.00
	.							
	.							
	.							
	.							
499	FOOD NO,%	8 90.9	8 92.2	8 90.0	5 4.6	5 3.6	5 2.6	5 1.0
	FOOD NO,%	5 9.1	5 7.8	5 7.5	1 95.4	1 96.4	1 97.4	1 99.0
	FOOD NO,%	0 0.0	0 0.0	1 2.5	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	6.87	8.81	10.68	5.56	6.44	7.53	8.91
	COST (P)	32.63	40.21	50.26	60.55	69.79	81.11	95.07
500	FOOD NO,%	8 90.9	8 92.2	8 90.0	5 4.5	5 3.6	5 2.6	5 1.0
	FOOD NO,%	5 9.1	5 7.8	5 7.5	1 95.5	1 96.4	1 97.4	1 99.0
	FOOD NO,%	0 0.0	0 0.0	1 2.5	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	6.89	8.82	10.70	5.56	6.45	7.54	8.92
	COST (P)	32.68	40.27	50.34	60.64	69.89	81.22	95.19

TABLE 7

RATIONS PRODUCED BY ESCALF AND RATION FORMULATION PROGRAM

Liveweight/ Liveweight Gain	Ration Composition (DM Basis)	ESCALF	RFP
100 kg	Straw (%)	50.0	85.4
0.25 kg/day	Barley (%)	38.5	4.2
	Soya (%)	11.5	10.3
	Quantity (kg)	2.2	3.0
100 kg	Barley (%)	83.0	85.1
0.50 kg/day	Soya (%)	17.0	14.9
	Quantity (kg)	1.8	1.8
100 kg	Barley (%)	74.0	85.5
0.75 kg/day	Soya (%)	26.0	14.5
	Quantity (kg)	2.1	2.2
100 kg	Barley (%)	71.0	85.5
1.00 kg/day	Soya (%)	29.0	14.5
	Quantity (kg)	2.5	2.7

TABLE 8

OUTPUT FROM BEEF CATTLE DP MODEL SOLUTION PROGRAM - MAFF STANDARDS

PERIOD NUMBER	LIVEWEIGHT (KG) START	END	CUMULATIVE COST (P)	RATION REQUIRED ON FIRST DAY OF PERIOD COMPOSITION (DM BASIS) FOOD	%	FOOD	%	FOOD	%	QUANTITY (KG)
1	100	104	124.8	5	14.6	1	85.5	0	0.0	2.68
2	104	108	251.4	5	14.6	1	85.4	0	0.0	2.72
3	108	112	379.9	5	14.6	1	85.4	0	0.0	2.76
4	112	116	510.1	5	14.6	1	85.4	0	0.0	2.80
5	116	120	642.2	5	14.7	1	85.3	0	0.0	2.84
6	120	124	776.1	5	14.7	1	85.3	0	0.0	2.88
7	124	128	911.8	5	14.7	1	85.3	0	0.0	2.92
8	128	133	1077.2	5	13.2	2	6.4	1	80.4	3.56
9	133	138	1244.5	5	13.0	1	87.0	0	0.0	3.63
10	138	143	1414.3	5	13.1	1	86.9	0	0.0	3.68
11	143	148	1586.7	5	13.1	1	86.9	0	0.0	3.74
12	148	153	1761.6	5	13.1	1	86.9	0	0.0	3.79
13	153	158	1938.8	5	13.0	1	87.0	0	0.0	3.85
14	158	163	2118.5	5	12.9	1	87.1	0	0.0	3.90
15	163	168	2300.4	5	12.8	1	87.2	0	0.0	3.96
16	168	173	2484.7	5	12.6	1	87.4	0	0.0	4.01
17	173	178	2671.3	5	12.5	1	87.5	0	0.0	4.06
18	178	183	2860.3	5	12.4	1	87.6	0	0.0	4.12
19	183	188	3051.5	5	12.3	1	87.8	0	0.0	4.17
20	188	194	3282.2	5	9.8	1	90.3	0	0.0	5.10
21	194	200	3516.0	5	9.6	1	90.4	0	0.0	5.18
22	200	206	3753.0	5	9.5	1	90.5	0	0.0	5.25
23	206	212	3992.9	5	9.2	1	90.8	0	0.0	5.33
24	212	218	4235.8	5	9.0	1	91.0	0	0.0	5.40
25	218	224	4481.7	5	8.8	1	91.2	0	0.0	5.47
26	224	230	4730.6	5	8.6	1	91.4	0	0.0	5.55
27	230	236	4982.6	5	8.4	1	91.6	0	0.0	5.62
28	236	242	5237.5	5	8.2	1	91.8	0	0.0	5.70
29	242	248	5495.4	5	8.0	1	92.0	0	0.0	5.77
30	248	254	5756.3	5	7.8	1	92.2	0	0.0	5.84
31	254	260	6020.2	5	7.6	1	92.4	0	0.0	5.92
32	260	266	6286.9	5	7.4	1	92.6	0	0.0	5.99
33	266	272	6556.5	5	7.1	1	92.9	0	0.0	6.06
34	272	278	6829.0	5	6.9	1	93.1	0	0.0	6.14
35	278	284	7104.4	5	6.7	1	93.3	0	0.0	6.21
36	284	290	7382.7	5	6.5	1	93.5	0	0.0	6.28
37	290	296	7664.0	5	6.3	1	93.7	0	0.0	6.36
38	296	302	7948.1	5	6.1	1	93.9	0	0.0	6.43
39	302	308	8235.1	5	5.9	1	94.1	0	0.0	6.50
40	308	314	8525.0	5	5.7	1	94.3	0	0.0	6.58

TABLE 8 (Continued)

PERIOD NUMBER	LIVEWEIGHT (KG) START	END	CUMULATIVE COST (P)	RATION REQUIRED ON FIRST DAY OF PERIOD COMPOSITION (DM BASIS) FOOD	%	FOOD	%	FOOD	%	QUANTITY (KG)
41	314	320	8817.9	5	5.6	1	94.4	0	0.0	6.65
42	320	326	9113.7	5	5.4	1	94.6	0	0.0	6.72
43	326	332	9412.4	5	5.2	1	94.8	0	0.0	6.80
44	332	338	9714.1	5	5.1	1	94.9	0	0.0	6.87
45	338	344	10018.7	5	4.9	1	95.1	0	0.0	6.95
46	344	350	10326.2	5	4.7	1	95.3	0	0.0	7.02
47	350	356	10636.5	5	4.6	1	95.4	0	0.0	7.09
48	356	362	10949.6	5	4.4	1	95.6	0	0.0	7.17
49	362	368	11265.3	5	4.1	1	95.9	0	0.0	7.24
50	368	374	11583.8	5	3.9	1	96.1	0	0.0	7.31
51	374	380	11905.1	5	3.7	1	96.3	0	0.0	7.38
52	380	386	12229.0	5	3.5	1	96.5	0	0.0	7.46
53	386	392	12555.6	5	3.2	1	96.8	0	0.0	7.53
54	392	398	12885.0	5	3.0	1	97.0	0	0.0	7.60
55	398	404	13217.1	5	2.8	1	97.2	0	0.0	7.67
56	404	410	13552.1	5	2.7	1	97.3	0	0.0	7.75
57	410	416	13890.1	5	2.6	1	97.4	0	0.0	7.82
58	416	422	14231.0	5	2.4	1	97.6	0	0.0	7.89
59	422	428	14574.8	5	2.3	1	97.7	0	0.0	7.97
60	428	434	14921.5	5	2.2	1	97.8	0	0.0	8.04
61	434	440	15271.1	5	2.1	1	97.9	0	0.0	8.11
62	440	446	15623.7	5	2.0	1	98.0	0	0.0	8.19
63	446	452	15979.2	5	1.9	1	98.1	0	0.0	8.26
64	452	458	16337.6	5	1.8	1	98.2	0	0.0	8.34
65	458	464	16699.0	5	1.7	1	98.3	0	0.0	8.41
66	464	470	17063.3	5	1.6	1	98.4	0	0.0	8.48
67	470	476	17430.5	5	1.5	1	98.5	0	0.0	8.56
68	476	482	17800.6	5	1.4	1	98.6	0	0.0	8.63
69	482	488	18173.7	5	1.3	1	98.7	0	0.0	8.70
70	488	494	18549.7	5	1.2	1	98.8	0	0.0	8.78
71	494	500	18928.6	5	1.1	1	98.9	0	0.0	8.85

Note: Period length is 4 days

TABLE 9

FEEDING POLICIES PRODUCED BY ESCALF AND DP MODEL

Weight Range/ Weight Gain	Ration Composition (DM Basis)	ESCALF	DP Model
128 - 188 kg	Barley (%)	75.0	87.0
1.25 kg/day	Soya (%)	25.0	13.0
	Quantity (kg)	3.4	3.9
188 - 260 kg	Barley (%)	94.0	91.3
1.50 kg/day	Soya (%)	6.0	8.7
	Quantity (kg)	4.8	5.5
260 - 332 kg	Barley (%)	94.0	93.6
1.50 kg/day	Soya (%)	6.0	6.4
	Quantity (kg)	5.8	6.3

TABLE 10

EXAMPLE OF OUTPUT FROM DP MODEL SOLUTION PROGRAM - NRC STANDARDS

PERIOD NUMBER	LIVEWEIGHT (KG) START	LIVEWEIGHT (KG) END	CUMULATIVE COST ($)	RATION REQUIRED ON FIRST DAY OF PERIOD COMPOSITION (DM BASIS) FOOD	%	FOOD	%	FOOD	%	FOOD	%	FOOD	%	QUANTITY (KG)
1	200	205	1.557	12	5.0	4	77.4	2	17.7	0	0.0	0	0.0	5.48
2	205	210	3.136	12	4.7	4	77.4	2	17.9	0	0.0	0	0.0	5.59
3	210	216	5.006	12	3.5	6	4.3	4	92.2	0	0.0	0	0.0	5.70
4	216	222	6.879	12	2.8	6	2.0	4	95.1	0	0.0	0	0.0	5.84
5	222	228	8.767	12	2.2	4	97.7	0	0.0	0	0.0	0	0.0	5.97
6	228	234	10.683	12	1.8	4	97.7	2	0.4	0	0.0	0	0.0	6.11
7	234	240	12.628	12	1.5	4	97.7	2	0.8	0	0.0	0	0.0	6.24
8	240	246	14.602	12	1.1	4	97.7	2	1.2	0	0.0	0	0.0	6.38
9	246	252	16.603	12	0.8	4	97.6	2	1.6	0	0.0	0	0.0	6.51
10	252	258	18.632	12	0.5	4	97.6	2	1.9	0	0.0	0	0.0	6.64
11	258	264	20.689	12	0.1	4	97.8	2	2.0	0	0.0	0	0.0	6.76
12	264	270	22.780	4	97.8	2	2.2	0	0.0	0	0.0	0	0.0	6.89
13	270	276	24.905	4	97.7	2	2.3	0	0.0	0	0.0	0	0.0	7.01
14	276	282	27.066	4	97.6	2	2.4	0	0.0	0	0.0	0	0.0	7.13
15	282	288	29.261	4	97.5	2	2.5	0	0.0	0	0.0	0	0.0	7.25
16	288	294	31.491	4	97.3	2	2.7	0	0.0	0	0.0	0	0.0	7.38
17	294	300	33.755	4	97.2	2	2.8	0	0.0	0	0.0	0	0.0	7.50

Note: Period length is 4 days

TABLE 11

FEEDSTUFF DATA FOR PIG PRODUCTION DP MODEL

Food Number	Food Name	DE MJ/kg DM	DCP g/kg DM	Amino Acid as % of Protein Mass			Cost p/kg DM
				Lysine	Histidine	Methionine	
1	Barley	12.7	77	3.2	2.1	3.6	8.4
2	Soya	15.0	410	6.2	2.6	2.8	18.0
3	Fish meal	15.1	580	7.4	2.3	4.0	33.0
4	Tallow	29.0	-	-	-	-	31.5

Abbreviations: DE - digestible energy
DCP - digestible crude protein
DM - dry matter

(Source: Fawcett et al. (1978b))

TABLE 12

OUTPUT FROM PIG PRODUCTION DP MODEL SOLUTION PROGRAM

CASE 1 – FEEDING POLICY TO PRODUCE 100 KG PIG WITH BODY PROTEIN CONTENT OF 13.29 KG

PERIOD NUMBER	LIVEWEIGHT (KG) START	END	PROTEIN (KG) START	END	TOTAL COST (£)	RATION REQUIRED ON MIDDLE DAY OF PERIOD COMPOSITION (DM BASIS) FOOD	%	FOOD	%	FOOD	%	FOOD	%	PERIOD QUANTITY (KG)
1	20	21	3.00	3.16	0.33	2	9.7	1	90.3	0	0.0	0	0.0	0.59
2	21	24	3.16	3.59	1.05	2	11.8	1	88.2	0	0.0	0	0.0	1.42
3	24	28	3.59	4.24	1.92	2	20.4	1	79.6	0	0.0	0	0.0	1.57
4	28	32	4.24	4.90	2.80	2	19.8	1	80.2	0	0.0	0	0.0	1.61
5	32	36	4.90	5.55	3.69	2	19.2	1	80.8	0	0.0	0	0.0	1.65
6	36	40	5.55	6.20	4.60	2	18.7	1	81.3	0	0.0	0	0.0	1.69
7	40	44	6.20	6.85	5.52	2	18.2	1	81.8	0	0.0	0	0.0	1.72
8	44	48	6.85	7.50	6.46	2	17.8	1	82.2	0	0.0	0	0.0	1.76
9	48	52	7.50	8.16	7.41	2	17.4	1	82.6	0	0.0	0	0.0	1.79
10	52	56	8.16	8.81	8.38	2	17.0	1	83.0	0	0.0	0	0.0	1.82
11	56	60	8.81	9.22	9.54	2	1.5	1	98.5	0	0.0	0	0.0	2.59
12	60	64	9.22	9.62	10.71	2	1.4	1	98.6	0	0.0	0	0.0	2.63
13	64	68	9.62	10.03	11.90	2	1.3	1	98.7	0	0.0	0	0.0	2.67
14	68	72	10.03	10.44	13.10	2	1.2	1	98.8	0	0.0	0	0.0	2.70
15	72	76	10.44	10.85	14.31	2	1.2	1	98.8	0	0.0	0	0.0	2.74
16	76	80	10.85	11.26	15.54	2	1.1	1	98.9	0	0.0	0	0.0	2.77
17	80	84	11.26	11.66	16.79	2	1.0	1	99.0	0	0.0	0	0.0	2.81
18	84	88	11.66	12.07	18.04	2	1.0	1	99.0	0	0.0	0	0.0	2.84
19	88	92	12.07	12.48	19.31	2	0.9	1	99.1	0	0.0	0	0.0	2.88
20	92	96	12.48	12.89	20.60	2	0.9	1	99.1	0	0.0	0	0.0	2.91
21	96	100	12.89	13.29	21.90	2	0.8	1	99.2	0	0.0	0	0.0	2.95

Notes: (1) Period length is 5 days
(2) Total cost refers to the cumulative total cost

TABLE 13

OUTPUT FROM PIG PRODUCTION DP MODEL SOLUTION PROGRAM

CASE 2 – FEEDING POLICY TO PRODUCE 100 KG PIG WITH BODY PROTEIN CONTENT OF 14.03 KG

PERIOD NUMBER	LIVEWEIGHT (KG) START	LIVEWEIGHT (KG) END	PROTEIN (KG) START	PROTEIN (KG) END	TOTAL COST (£)	RATION REQUIRED ON MIDDLE DAY OF PERIOD COMPOSITION (DM BASIS) FOOD	%	FOOD	%	FOOD	%	FOOD	%	QUANTITY (KG)
1	20	21	3.00	3.16	0.33	2	9.7	1	90.3	0	0.0	0	0.0	0.59
2	21	24	3.16	3.59	1.05	2	11.8	1	88.2	0	0.0	0	0.0	1.42
3	24	28	3.59	4.24	1.92	2	20.4	1	79.6	0	0.0	0	0.0	1.57
4	28	32	4.24	4.90	2.80	2	19.8	1	80.2	0	0.0	0	0.0	1.61
5	32	36	4.90	5.55	3.69	2	19.2	1	80.8	0	0.0	0	0.0	1.65
6	36	40	5.55	6.20	4.60	2	18.7	1	81.3	0	0.0	0	0.0	1.69
7	40	44	6.20	6.85	5.52	2	18.2	1	81.8	0	0.0	0	0.0	1.72
8	44	48	6.85	7.50	6.46	2	17.8	1	82.2	0	0.0	0	0.0	1.76
9	48	52	7.50	8.16	7.41	2	17.4	1	82.6	0	0.0	0	0.0	1.79
10	52	56	8.16	8.81	8.38	2	17.0	1	83.0	0	0.0	0	0.0	1.82
11	56	60	8.81	9.46	9.36	2	16.7	1	83.3	0	0.0	0	0.0	1.86
12	60	64	9.46	10.11	10.35	2	16.3	1	83.7	0	0.0	0	0.0	1.89
13	64	68	10.11	10.77	11.36	2	16.0	1	84.0	0	0.0	0	0.0	1.92
14	68	72	10.77	11.17	12.55	2	1.3	1	98.7	0	0.0	0	0.0	2.68
15	72	76	11.17	11.58	13.76	2	1.3	1	98.7	0	0.0	0	0.0	2.72
16	76	80	11.58	11.99	14.98	2	1.2	1	98.8	0	0.0	0	0.0	2.75
17	80	84	11.99	12.40	16.21	2	1.1	1	98.9	0	0.0	0	0.0	2.79
18	84	88	12.40	12.80	17.46	2	1.1	1	98.9	0	0.0	0	0.0	2.82
19	88	92	12.80	13.21	18.72	2	1.0	1	99.0	0	0.0	0	0.0	2.86
20	92	96	13.21	13.62	20.00	2	0.9	1	99.1	0	0.0	0	0.0	2.89
21	96	100	13.62	14.03	21.29	2	0.9	1	99.1	0	0.0	0	0.0	2.93

Notes: (1) Period length is 5 days
(2) Total cost refers to the cumulative total cost

TABLE 14

ENTERPRISE MODEL - CASE (A) : CATTLE FEEDING POLICY

Time Period	Ration Type	Number of Animals	Weight at Start (kg)	Weight at End (kg)
1	1	200	100	140
2	1	200	140	190
3	1	200	190	240
4	4	200	240	300
5	1	200	300	360
6	1	80	360	420
	4	120	360	420
7	1	200	100	140
8	1	200	140	190
9	4	200	190	250

Notes:

(1) Ration Type 1 - no enterprise grown feedstuffs available
Ration Type 2 - enterprise grown barley available
Ration Type 3 - enterprise grown barley straw available
Ration Type 4 - enterprise grown barley and barley straw available

(2) At end of sixth time interval 200 animals of 420 kg liveweight are sold and 200 animals of 100 kg are bought.

TABLE 15

ENTERPRISE MODEL - CASE (A) : CROP PRODUCTION POLICY

Product	Area (ha)	Production (t)	Quantity Fed (t)	Quantity Sold (t)
Barley	28.0	140.0	140.0	-
Barley Straw	28.0	70.0	-	70.0
Wheat	80.0	560.0	-	560.0
Potatoes	30.0	1050.0	-	1050.0

TABLE 16

ENTERPRISE MODEL - CASE (B) : CATTLE FEEDING POLICY

Time Period	Ration Type	Number of Animals	Weight at Start (kg)	Weight at End (kg)
1	1	200	100	140
2	3	200	140	180
3	3	200	180	230
4	3	200	230	290
5	3	7	290	350
	4	193	290	350
6	4	200	350	410
7	1	200	100	140
8	3	200	140	180
9	3	200	180	230

Notes:

(1) Ration Type 1 - no enterprise grown feedstuffs available
Ration Type 2 - enterprise grown cereal available
Ration Type 3 - enterprise grown cereal straw available
Ration Type 4 - enterprise grown cereal and cereal straw available

(2) At end of sixth time interval 200 animals of 410 kg liveweight are sold and 200 animals of 100 kg are bought.

TABLE 17

ENTERPRISE MODEL - CASE (B) : CROP PRODUCTION POLICY

Product	Area (ha)	Production (t)	Quantity Fed (t)	Quantity Sold (t)
Cereal	28.0	140.0	140.0	-
Cereal Straw	28.0	70.0	57.5	12.5
Wheat	80.0	560.0	-	560.0
Potatoes	30.0	1050.0	-	1050.0

FIGURE 1

CATTLE GROWTH USING BASIC ENERGY ALLOWANCES

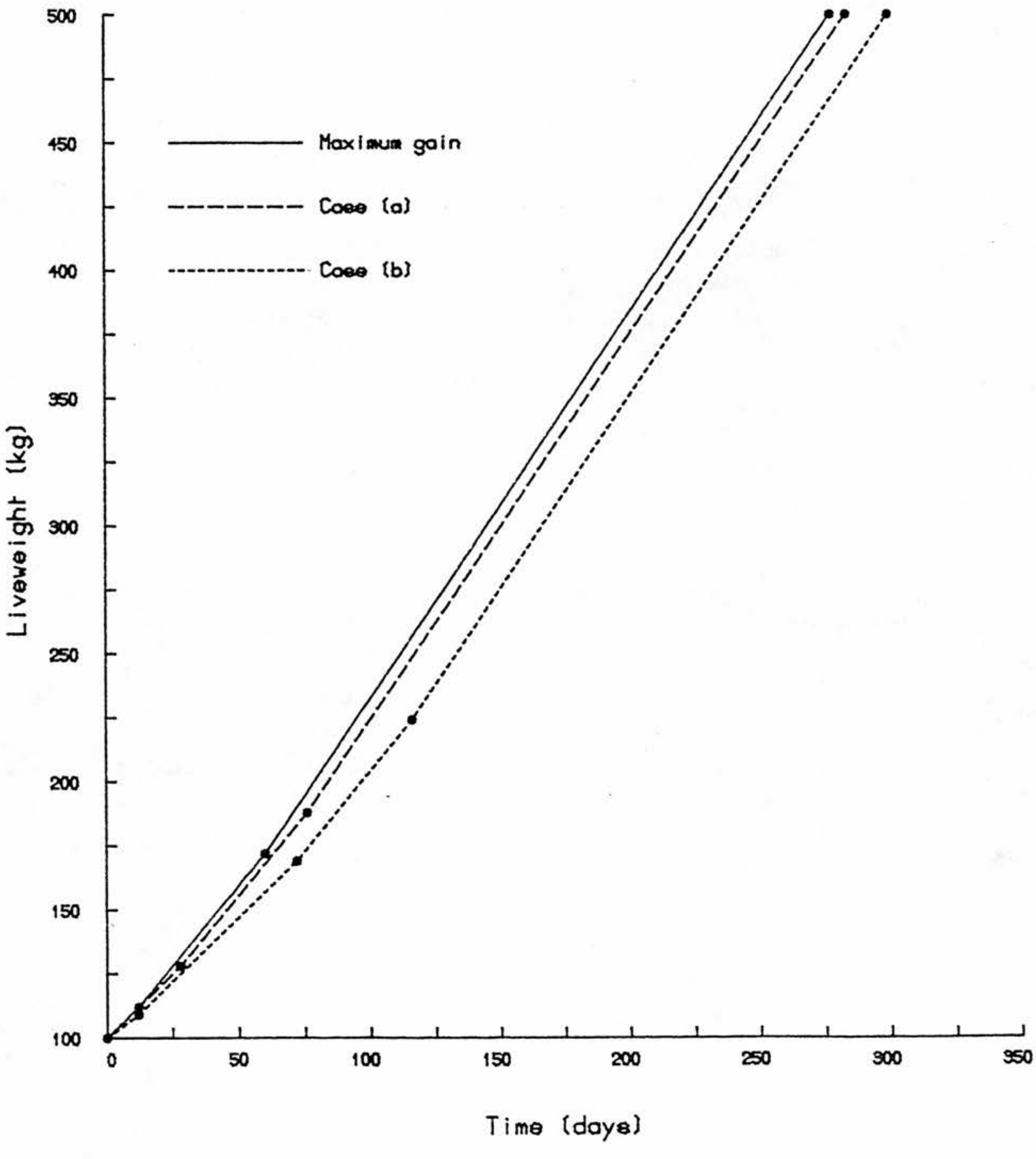


FIGURE 2

CATTLE GROWTH USING INCREASED ENERGY ALLOWANCES

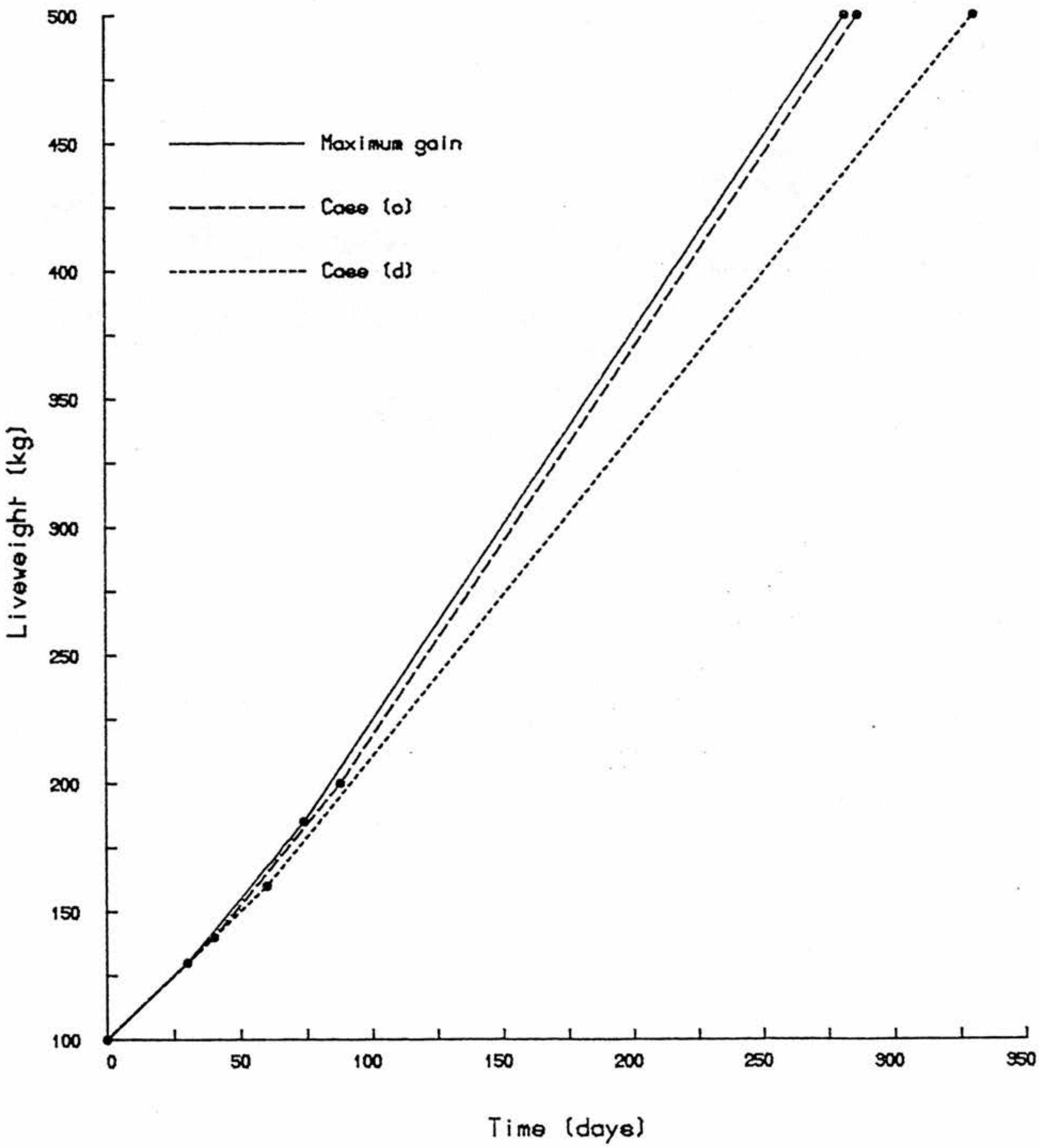


FIGURE 3

CATTLE PRODUCTION POLICIES USING BASIC ENERGY ALLOWANCES

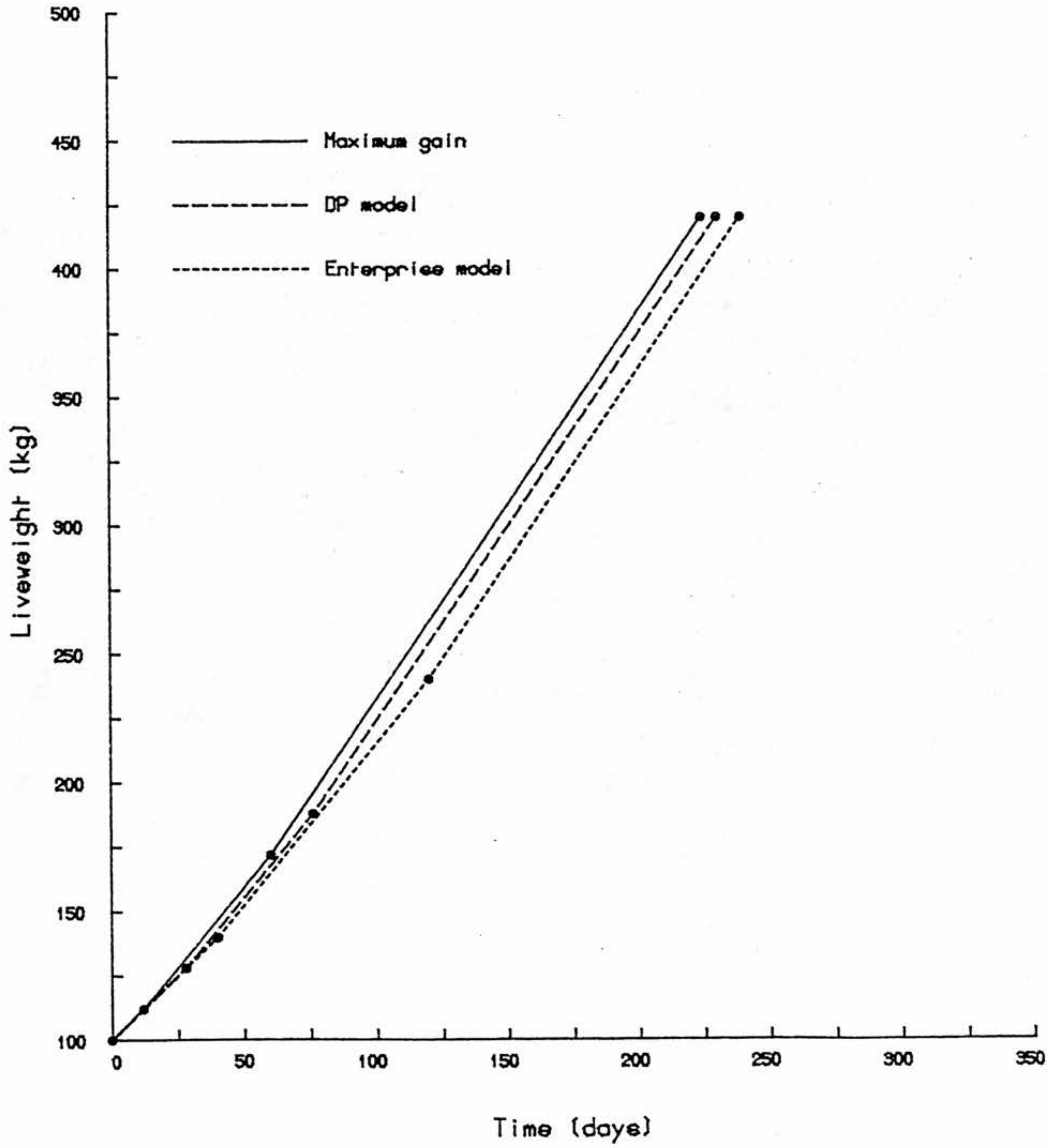
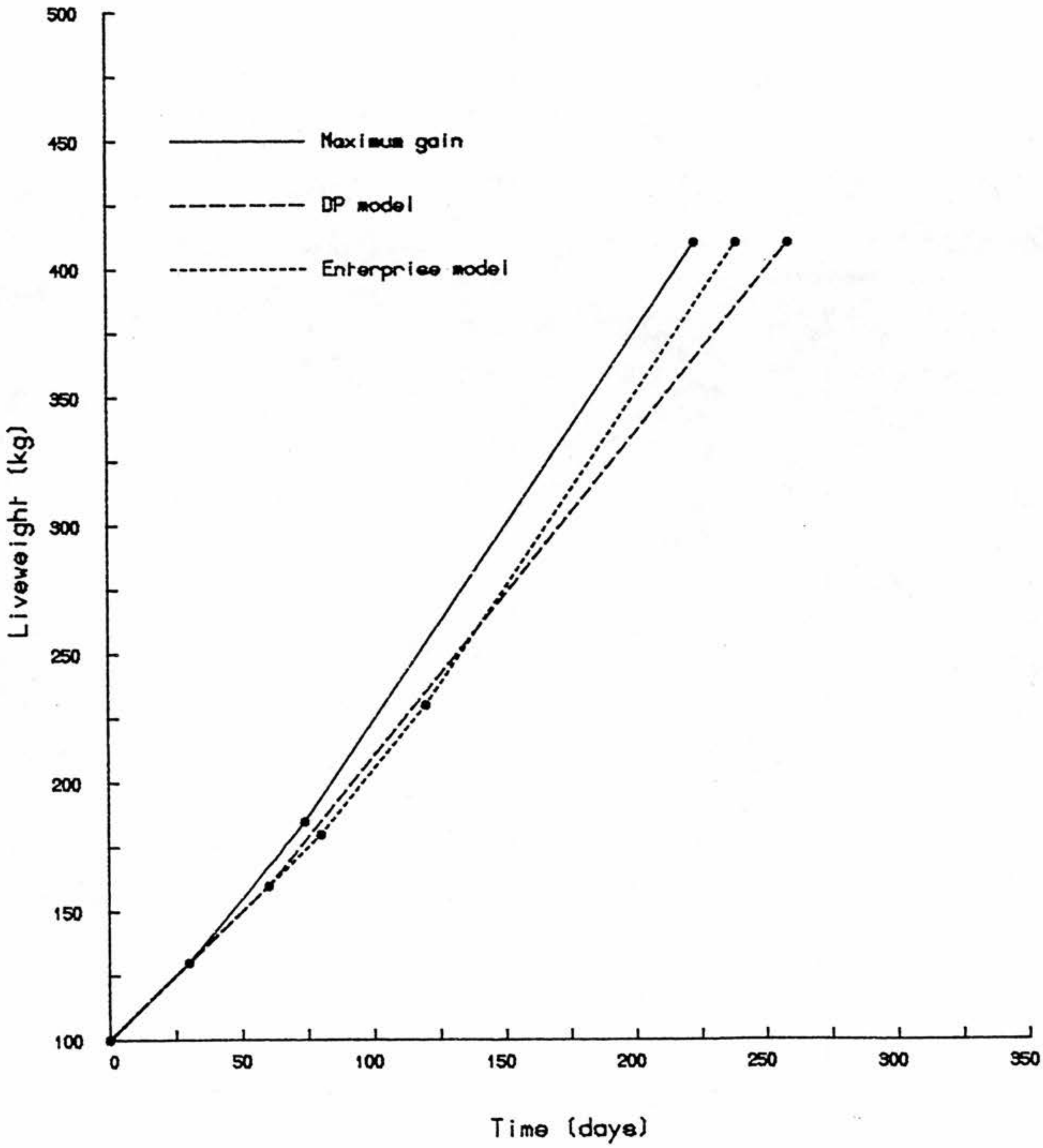


FIGURE 4

CATTLE PRODUCTION POLICIES USING INCREASED ENERGY ALLOWANCES



APPENDIX A1

A MATHEMATICAL PROGRAMMING APPROACH TO BEEF FEEDLOT OPTIMIZATION

Published in Management Science, Vol. 26, pp 524-535, 1980

MANAGEMENT SCIENCE
Vol. 26, No. 5, May 1980
Printed in U.S.A.

A MATHEMATICAL PROGRAMMING APPROACH TO BEEF FEEDLOT OPTIMIZATION*

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The efficient operation of a beef cattle feedlot is governed by the purchase and selling weights and prices of the animals and by the feeding system used. The optimal feeding system will involve feeding least cost rations to animals at every stage in the production process. A method is described for determining optimal feeding systems which meet the nutrient standards recommended by the US National Research Council. The approach involves first using linear programming (LP) to determine least cost rations to produce specified liveweight gains in animals of known liveweight. Dynamic programming (DP) is then used to determine the optimal sequence of rations to feed in order to produce animals of specified liveweight from animals of known initial weight at minimum cost, using the least cost rations determined from the LP model. The results from this new DP model can then be used to determine the optimal combination of purchase weight, selling weight and feeding system. It is shown that in order to use the DP model, the LP model must be solved a large number of times and a new method is developed to produce these solutions more efficiently. The approach is applied to a representative problem and the computational experience is presented.
(PROGRAMMING—LINEAR, APPLICATIONS; DYNAMIC PROGRAMMING—APPLICATIONS; INDUSTRIES—ACRICULTURE/FOOD)

1. Introduction

In recent years intensive methods of beef production have become increasingly important in agriculturally advanced nations as these methods have frequently been found to be more efficient, in economic terms, than traditional methods of beef production based on the grazing of pasture. These intensive methods of beef production are based on the fattening of cattle in feedlots. In a typical feedlot operation, cattle are purchased at some initial liveweight and transported to the feedlot where they are fed on high energy diets until they reach some specified selling weight.

The economic efficiency of a feedlot is influenced by the purchase and selling weights and prices of the animals and by the feeding system used. A number of approaches for evaluating feedlot operating policies have been proposed but all have limitations. For example, in the approaches of Kennedy [6], Meyer and Newett [8] and Ryan [11], only a limited number of the possible rations of the available feedstuffs are considered for feeding to the animals in the feedlot at any stage in the production process, while in the approaches of Brokken [3] and Brokken et al. [4], as in that of Ryan [11], the rate of liveweight gain remains constant throughout the feeding period. In this paper an approach is developed which ensures that the complete range of feeding systems is evaluated. The approach involves first using the separable linear programming (LP) model developed by Brokken [3] to determine the optimal ration from the available feedstuffs to produce a specified liveweight gain in an animal of known liveweight. Dynamic programming (DP) is then used to deter-

* Accepted by David G. Dannenbring; received November 5, 1978. This paper has been with the author 2 months for 2 revisions.

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0025-1909/80/2605/0524$01.25

mine the feeding schedule to produce animals of specified liveweight from animals of known initial weight at minimum cost, using the optimal rations determined from the LP model. The results from this new DP model can then be used to determined the optimal policy for the feedlot, e.g. in terms of the optimal combination of purchase weight, selling weight and feeding system. It is shown that in order to use the DP model, the LP model must be solved a large number of times and a new method is developed to produce these solutions more efficiently.

2. Nutrient Requirements of Beef Cattle

Several different systems for specifying the nutrient requirements of beef cattle have been proposed by bodies such as the National Research Council (NRC) in the US [10] and the Agricultural Research Council in the UK [1]. In the systems currently in use, the nutrient requirements are specified in terms of the energy, protein, minerals, roughage levels etc. required to produce a specified liveweight gain in an animal of given liveweight. Although the recommended allowances for protein, minerals, etc. may differ, the main difference between the systems lies in the method adopted for specifying energy requirements and since this affects the method of ration formulation, it is necessary to give a brief outline of the energy system used in this study, namely the net energy system recommended by the NRC [10].

Energy Requirements

The NRC recommend the use of the net energy system developed by Lofgreen and Garrett [7] for determining the energy requirements of beef cattle. Net energy (NE) is the energy which is available for maintenance of body functions and production of liveweight gain after faecal and urinary losses and losses due to methane and heat production have been deducted.

In the NE system recommended by the NRC for beef cattle it is assumed that the total NE requirement can be separated into a component for maintenance and a component for the production of liveweight gain. The daily NE requirement for maintenance, E_m (in Mcal), of an animal of liveweight, W (in kg), is assumed to depend only on liveweight, with

$$E_m = 0.077\,W^{0.75}. \tag{1}$$

The NE requirement for liveweight gain, E_g (in Mcal), of an animal of liveweight, W (in kg), which is required to produce a daily liveweight gain, G (in kg), is given by separate expressions for steers and heifers and is assumed to depend only on liveweight and gain.

$$\text{Steers:}\quad E_g = (0.05272\,G + 0.00684\,G^2)\,W^{0.75}, \tag{2a}$$

$$\text{Heifers:}\quad E_g = (0.05603\,G + 0.01265\,G^2)\,W^{0.75}. \tag{2b}$$

It is assumed that food energy will first be used to meet, or attempt to meet, the NE requirement for maintenance, and any remaining food energy will be used to produce liveweight gain. Since the efficiencies of utilization of food energy for maintenance and gain differ, the food energy available for these two functions must be considered separately. In the NRC system it is assumed that the food energy available for maintenance and the food energy available for gain, after the maintenance requirements have been met, can be specified in Mcal/kg dry matter (DM) for all feedstuffs.

These NE concentrations for maintenance and gain are tabulated [10] for feedstuffs commonly used for feeding beef cattle. The NE concentrations for maintenance and gain of diets and rations composed of a number of these feedstuffs can be calculated from these tables. In this paper a diet is defined in terms of the proportions of its constituent feedstuffs, while a ration refers to the quantity of a diet required.

Suppose that a diet consists of a proportion y_i of feedstuff i, $i = 1, 2, \ldots, k$, of NE concentration for maintenance m_i, and NE concentration for gain g_i. Then the NE concentrations of the diet for maintenance, m, and gain, g, are given by

$$m = \sum m_i y_i, \tag{3a}$$

$$g = \sum g_i y_i; \tag{3b}$$

where

$$\sum y_i = 1. \tag{3c}$$

If this diet is fed to a steer of liveweight W, which is required to achieve a daily liveweight gain G, then the NE requirements of the steer for maintenance, E_m, and gain, E_g, are given by expressions (1) and (2a) respectively. The quantity, in kg, of this diet required for maintenance is E_m/m, and the quantity required for gain is E_g/g. Thus the quantity, Q (in kg), of this diet required, i.e. the ration, is

$$Q = E_m/m + E_g/g. \tag{4}$$

A portion of the ration is therefore used to satisfy the NE requirement for maintenance and the remainder is used to satisfy the NE requirement for growth. In addition to satisfying the energy requirements the ration must also satisfy all the other nutrient requirements of the animal, in particular the protein requirements, and must satisfy the appetite constraints of the animal.

Although this NE system is widely used a number of limitations in the system have been noted. For example the effects of environmental stresses, e.g. extremes of temperature, on feed requirements and the differences between breeds of cattle are not taken into account in the system, although the NRC [10] considers that allowances can be made for these factors. A further limitation of the system is that it does not take account of the composition of the carcass in terms of its fat and protein content, and although the NRC [10] assume that the feeding system used will not have a major effect on carcass composition when economic feeding policies are adopted, this assumption has been questioned (e.g. Moe and Tyrrell [9]).

3. Ration Formulation

Linear programming has been widely used in the formulation of diets for animal feeding (e.g. Beneke and Winterboer [2]). For cattle, LP can be used with the NE system recommended by the NRC to determine the least cost diet with specified NE concentrations, m, for maintenance and, g, for gain (see equations (3)). However, as can be seen from (4), the quantity of this diet, i.e. the ration, required to produce a specified liveweight gain in an animal of known liveweight depends on the values of m and g, and it would therefore be necessary to search all values of m and g to determine the least cost ration.

Ration Formulation Model

Brokken [3] has suggested an LP model for ration formulation which takes account of the interdependences in the NE system recommended by the NRC. Suppose it is required to determine the minimum cost ration to feed to an animal of liveweight W to achieve a daily liveweight gain G. The NE requirements of the animal for maintenance, E_m, and gain, E_g, are given by (1) and (2) respectively, and let

P = protein requirement of the animal (in grams),

U = maximum DM intake of the animal (in kg),

L = minimum DM intake of the animal (in kg),

where P is a function of liveweight and liveweight gain, and U and L are functions of liveweight.

Assume that k feedstuffs are available for ration formulation and let

c_i = cost per kg DM of feedstuff i,

m_i = NE concentration for maintenance of feedstuff i (Mcal/kg DM),

g_i = NE concentration for gain of feedstuff i (Mcal/kg DM),

p_i = protein concentration of feedstuff i (g/kg DM).

If a ration consisting of quantity x_i (in kg DM), $x_i \geqslant 0$, of feedstuff i, $i = 1, 2, \ldots, k$, is fed to the animal, then from (3), the NE concentrations for maintenance, m, and gain, g, are given by

$$m = \sum m_i x_i / Q,$$

$$g = \sum g_i x_i / Q$$

where

$$Q = \sum x_i \quad \text{is the quantity fed.}$$

Suppose that when this ration is fed to the animal, a fraction f, by dry matter weight, of the ration is used to meet the NE requirement for maintenance. Then, substituting for m from (5a),

$$Qf \sum m_i x_i / Q = E_m$$

i.e.

$$\sum m_i x_i = E_m / f \tag{6a}$$

and since a fraction $(1 - f)$ is used to meet the NE requirement for gain,

$$\sum g_i x_i = E_g / (1 - f). \tag{6b}$$

A ration which supplies the NE requirements of the animal by using a fraction f by DM weight of the ration to supply the NE requirement of the animal must satisfy (6). Since the ration must also satisfy the DM intake constraints and protein requirements of the animal, the least cost ration of this form is given by the solution of the LP:

$$\min Z = \sum c_i \; x_i, \tag{7a}$$

$$\text{subject to} \quad \sum m_i \; x_i = E_m / f, \tag{7b}$$

$$\sum g_i \; x_i = E_g / (1 - f), \tag{7c}$$

$$\sum \; x_i \leqslant u, \tag{7d}$$

$$\sum \; x_i \geqslant L, \tag{7e}$$

$$\sum p_i \; x_i \geqslant P, \tag{7f}$$

$$x_i \geqslant 0, \qquad i = 1, 2, 3, \ldots, k.$$

This LP formulation can be extended to incorporate other nutrients.

The overall least cost ration can be found by solving LP (7) for all values of f, although obviously in practice only a limited number of cases can be evaluated. However, Brokken [3] shows that by piecewise linearization of E_m/f and $E_g/(1 - f)$ a separable programming formulation will yield the overall optimal solution with negligible error.

In general, piecewise linearizaion of E_m/f and $E_g/(1-f)$ over the range of f would not be entirely satisfactory because of the asymptotic behavior as f tends to 0 and 1 respectively. In this problem however, it is never possible to have $f = 0$ and in feedlot operations it will seldom, if ever, be economic to feed animals at or near maintenance levels, corresponding to $f \to 1$. The range of possible values of f is thus reduced and upper and lower bounds for f can be established. For example, for a ration with NE concentrations m and g for maintenance and gain respectively,

$$f = E_m/m \Big/ (E_m/m + E_g/g) = 1/(1 + mE_g/gE_m)$$

and since for all rations of available feedstuffs i, $i = 1, 2, 3, \ldots, k$,

$$\rho_L = \min_i [m_i/g_i] \leqslant m/g \leqslant \max_i [m_i/g_i] = \rho_U$$

then

$$f_L = (1 + \rho_U E_g/E_m) \leqslant f \leqslant 1/(1 + \rho_L E_g/E_m) = f_U. \tag{8}$$

Narrower limits for f may be established by taking the maximum dry matter intake of the animal into account [3].

A piecewise linear approximation of E_m/f and $E_g/(1-f)$ over the range of f defined by (8) can be obtained by determining the values of E_m/f and $E_g/(1-f)$ for $f = f_j = f_L + j\Delta$, $j = 0, 1, 2, \ldots, l$, where Δ is the linearization interval, $\Delta > 0$, and l is the integer satisfying $(f_U - f_L)/\Delta \leqslant l < (f_U + \Delta - f_L)/\Delta$, with intermediate values being determined by linear interpolation. The smaller the linearization interval, the smaller the error due to interpolation, although in practice a value of Δ in the range $0.005 - 0.01$ should be adequate. A separable programming formulation for the least cost ration formulation problem is then

$$\min Z = \sum_{i=1}^{k} c_i x_i \tag{9a}$$

$$\text{subject to} \quad \sum_{i=1}^{k} m_i x_i - \sum_{j=0}^{l} x_{k+1+j} E_m/f_j = 0, \tag{9b}$$

$$\sum_{i=1}^{k} g_i x_i - \sum_{j=0}^{l} x_{k+1+j} E_g/(1-f_j) = 0, \tag{9c}$$

$$\sum_{j=0}^{l} x_{k+1+j} = 1, \tag{9d}$$

$$\sum_{i=1}^{k} x_i \leqslant U, \tag{9e}$$

$$\sum_{i=1}^{k} x_i \geqslant L, \tag{9f}$$

$$\sum_{i=1}^{k} p_i x_i \geqslant P, \tag{9g}$$

$$x_i \geqslant 0, \quad i = 1, 2, 3, \ldots, k + l + 1.$$

with at most two adjacent x_{k+1+j}, $j = 0, 1, 2, \ldots, l$, nonzero.

The formulation of (9) follows that of Brokken [3] except that different limits for f are used. Since the LP model (9) is convex it can be solved by the simplex method.

4. The Feeding Schedule

Although the use of the ration formulation LP model is feedlot operations will ensure that least cost rations are used at every stage in the production process, the economic efficiency of the feedlot will also depend on the feeding schedule, i.e. the daily sequence of rations, used to produce animals of some desired final liveweight. The criterion which is used to evaluate possible feeding schedules will depend on the nature of the feedlot operation. For example, if the feedlot has a contract to supply animals of specified liveweight then, in the short term at least, a suitable criterion might be the cost of producing animals of this weight from animals of known initial weight, where the cost will include the feeding costs associated with feeding least cost rations and feedlot overhead costs such as rent, handling costs and veterinary costs. In cases where the initial purchase weight and final selling weight can be varied, a criterion such as the return per animal per period might be appropriate.

Cost Minimization Model

In many cases, part of the problem of evaluating feeding schedules will be concerned with determining the feeding schedule to produce animals of specified liveweight from animals of known initial liveweight at minimum cost and possibly within some specified time period. A DP approach can be used to tackle this problem. In order to use DP it is first necessary to define the state of an animal in suitable terms. Although it might be desirable to describe the state of an animal in terms of its liveweight and carcass composition, the NRC [10] assume that when economic feeding practices are adopted, the feeding system used will not have a major effect on carcass composition. In this case, since rations are formulated to meet NRC standards [10], it will be assumed that the state of an animal can be described solely in terms of its liveweight.

Define $F_n(W)$ to be the cost of producing an animal of liveweight W from an animal of liveweight W_0 in n periods, each of t days, using an optimal feeding policy. Then

$$n \geq 1, \qquad F_n(W) = \min_{h > 0}\left[F_{n-1}(W-h) + K + C(W-h,h)\right], \tag{10a}$$

$$F_0(W_0) = 0 \tag{10b}$$

where $C(W-h,h)$ is the cost of producing a liveweight gain h in t equal daily increments, starting with an animal of liveweight $(W-h)$ and using least cost rations and K is the overhead cost per period.

Clearly the model could be extended to deal with time dependent feeding and overhead costs and costs could be discounted. Constraints on the rations to be fed could also be incorporated. For example, a constraint could be imposed to ensure that the energy content of the rations used in any period is greater than that of the rations used in the previous period.

The data required for this model depend on the period length, t days, the interstate interval, w kg, the initial liveweight, W_0 kg, and the final liveweight, W_F kg. The DP recurrence relations (10) should be solved for $W = W_0, W_0 + w, W_0 + 2w, \ldots, W_F$. For simplicity W_0 and W_F should be chosen such that $(W_F - W_0)/w$ is integral, and in practice it will generally be sufficient to take $w > 1$. It can be seen from the definition of $C(W-h,h)$ that the LP model should be used to determine least cost rations for liveweights $W = W_0, W_0 + w/t, W_0 + 2w/t, \ldots, W_F$, and liveweight gains iw/t, $i = 1, 2, \ldots, [tG(W)/w]$, where $G(W)$ is the maximum possible daily liveweight gain of an animal of liveweight W. In practice however, it will generally be sufficient to calculate least cost rations for a more limited number of liveweights, e.g. $W = W_0, W_0 + w, W_0 + 2w, \ldots, W_F$, and liveweight gains iw/t, $i = 1, 2, \ldots, [tG(W)/w]$, and then obtain intermediate values by interpolation. When the costs of the required least cost rations have been determined, $C(W-h,h)$ can be evaluated by summing the costs of the appropriate rations. If feedstuff costs change from period to period, the LP model should be used for each time period.

Although the formulation of (10) can be used to determine the overall minimum cost of producing animals of liveweight W from animals of liveweight W_0, a more direct approach is possible if $F_n(W)$ is redefined to relate to production in at most n periods. Then

$$n \geqslant 1, \qquad F_n(W) = \min\Big[F_{n-1}(W), \min_{h>0}\big[F_{n-1}(W-h) + K + C(W-h,h)\big]\Big], \tag{11a}$$

$$F_0(W_0) = 0. \tag{11b}$$

By using this formulation the number of periods required to produce animals of liveweight W from animals of liveweight W_0 at minimum cost can be found directly. It should also be noted that once the optimal feeding schedule from liveweight W_0 to liveweight W_F has been found, the optimal feeding schedules from any initial liveweight W_p, $W_0 \leqslant W_p < W_F$, to any final liveweight W_s, $W_0 < W_s \leqslant W_F$, are embedded in the results.

This DP approach could be used for animals of any species to find the optimal feeding schedule between any two liveweights, provided the required least cost rations could be derived. However, for most other species, e.g. pigs, it would be important to take carcass composition into account, and hence this DP model is of limited application.

Return Maximization Model

In the case where both the purchase weight and the selling weight of the animals can be varied, then since the length of a production cycle will typically be less than a year and since costs and revenues may vary from cycle to cycle, a suitable criterion for evaluating feeding schedules might be the return per animal per period. Thus if

$F(W_p, W_s, n)$ = the cost of the optimal feeding schedule from liveweight W_p to liveweight W_s in n periods (from (10)),

$S(W_s)$ = the selling price of an animal of liveweight W_s,

$B(W_p)$ = the purchase price of an animal of liveweight W_p,

then the optimal return per period, $R(W_p, W_s)$ for purchasing animals at liveweight W_p and selling at liveweight W_s is given by

$$R(W_p, W_s) = \max_{n>0}\big[(S(W_s) - B(W_p) - F(W_p, W_s, n))/n\big]. \tag{12}$$

The optimal combination of purchase weight and selling weight can then be found by repeated solution of (12). The approach involves first specifying the range of possible values of purchase weight and selling weight. In practice it will generally be sufficient to consider values within these ranges at intervals of 10 kg or more. The purchase and selling prices of animals of the weights to be considered within these ranges must also be specified. A pair of values of purchase weight, W_p, and selling weight, W_s, are then chosen and the value of n which maximizes $R(W_p, W_s)$ found. The value of either W_p or W_s is then changed and the optimal return for this new pair of values found. This process is repeated until all combinations of purchase weight and selling weight have been covered.

5. An Improved Method for Solving the Ration Formulation Model

It has been shown that in order to use the DP model, the ration formulation LP model must be used to determine a large number of least cost rations. The computational load involved in determining the least cost rations for the required values of liveweight and liveweight gain could be reduced if parametric programming methods could be used, but since both the left and right hand sides of (9) change when either the liveweight or the liveweight gain is changed, standard parametric programming techniques are not applicable. However, the nature of this ration formulation problem is such that the solution to the LP model (9) for a specified liveweight and liveweight gain can be used to obtain a basic solution to the LP model for a different liveweight but the same liveweight gain.

Consider the ration formulation problem for liveweight W_1 and liveweight gain G. By introducing a slack variable x_{k+l+2} and surplus variables x_{k+l+3} and x_{k+l+4} in (9e), (9f) and (9g) respectively, LP (9) can be expressed in the form

$$\min \quad Z = CX, \tag{13a}$$

$$AX = D, \tag{13b}$$

$$X > 0,$$

where $A = [a_{ij}]$, $C = [c_j]$, $D = [d_i]$ and $X = [x_i]$ are appropriately defined. Suppose that an optimal feasible solution to (13) exists and that $B = [b_{ij}]$ is the optimal basis matrix, then if the columns of A are permuted so that $A = (B, N)$, (13b) can be expressed in the form (see, for example, Hadley [5])

$$BX_B + NX_N = D. \tag{14}$$

Premultiplying (14) by B^{-1},

$$X_B + B^{-1}NX_N = B^{-1}D \tag{15}$$

yields the solution

$$X_B = B^{-1}D; \qquad X_N = 0. \tag{16}$$

By letting $C = (C_B, C_N)$, the optimal value Z_0 of the objective function can be expressed as

$$Z_0 = C_B B^{-1} D - (C_B B^{-1} N - C_N) X_N.$$

If the liveweight is now changed to W_2 and the liveweight gain remains unchanged, then it can be seen from (1) and (2) that the new coefficients of x_{k+1+j}, $j = 0, 1, 2, \ldots, l$, in (9b) and (9c) are obtained by multiplying the corresponding coefficients in the original model, i.e. for the ration formulation problem with liveweight W_1, by

$(W_2/W_1)^{0.75}$, provided the range of f defined for the original problem is adequate. If (9d) is multiplied throughout by $(W_2/W_1)^{0.75}$, then it can be seen from (9) that effectively all the original coefficients of variables $x_{k+1+j}, j = 0, 1, 2, \ldots, l$, have been multiplied by the factor $(W_2/W_1)^{0.75}$ while the coefficients of all the other variables remain unchanged. As a result of the change in the liveweight, the right hand side of (9) will change from D to $\tilde{D} = [\tilde{d}_i]$, where

$$\tilde{d}_1 = 0, \tilde{d}_2 = 0, \tilde{d}_3 = (W_2/W_1)^{0.75}, \tilde{d}_4 = \tilde{U}, \tilde{d}_5 = \tilde{L}, \tilde{d}_6 = \tilde{P},$$

where $\tilde{U}$, $\tilde{L}$ and $\tilde{P}$ denote the maximum DM intake, minimum DM intake and protein requirement respectively of the animal of liveweight W_2. Hence, the least cost ration formulation LP model for liveweight W_2 and liveweight gain G can be written

$$\min Z = CX, \tag{17a}$$

$$\tilde{B}X_B + \tilde{N}X_N = \tilde{D} \tag{17b}$$

where X_B and X_N are the vectors for the basic and nonbasic variables in the solution to the orginal problem (13), given by (16),

$$\tilde{B} = [\gamma_j b_{ij}],$$

with

$$\gamma_j = \begin{cases} (W_2/W_1)^{0.75} & \text{if } j\text{th element of } X_B \text{ is } x_{k+1+i},\ i = 0, 1, 2, \ldots, l, \\ 1 & \text{otherwise,} \end{cases}$$

and where $\tilde{N}$ is defined similarly in terms of the elements of N. Premultiplying (17b) by B^{-1} yields

$$GX_B + B^{-1}\tilde{N}X_N = B^{-1}\tilde{D} \tag{18}$$

where $G = B^{-1}\tilde{B}$ is a diagonal matrix with principal diagonal elements γ_i. Premultiplying [18] by G^{-1} yields

$$X_B + G^{-1}B^{-1}\tilde{N}X_N = G^{-1}B^{-1}\tilde{D} \tag{19}$$

where G^{-1} is a diagonal matrix with principal diagonal elements $1/\gamma_i$. Clearly (19) yields a basic solution to the new ration formulation problem,

$$X_B = G^{-1}B^{-1}\tilde{D}; \qquad X_N = 0$$

although this solution may not be feasbile. The objective function corresponding to this basic solution is given by

$$\tilde{Z} = C_B G^{-1}B^{-1}\tilde{D} - (C_B G^{-1}B^{-1}\tilde{N} - C_N)X_N.$$

The basic solution given by (19) can be used to find the optimal solution to the new ration formulation problem by first evaluating $G^{-1}B^{-1}\tilde{D}$ and $G^{-1}B^{-1}\tilde{N}$. This can be done by performing simple operations on the optimal solution to the original problem, given by (15):

(a) Evaluation of $G^{-1}B^{-1}\tilde{D}$: first use right hand side parameterization methods [5] to determine $B^{-1}\tilde{D}$ from $B^{-1}D$ and then multiply the ith element by $1/\gamma_i$.

(b) Evaluation of $G^{-1}B^{-1}\tilde{N}$: by noting that if $B^{-1}N = [e_{ij}]$, it can be seen from the definition of $\tilde{N}$ that

$$B^{-1}\tilde{N} = [\,\delta_j e_{ij}\,]$$

where

$$\delta_j = \begin{cases} (W_2/W_1)^{0.75} & \text{if } j\text{th element of } X_N \text{ is } x_{k+1+i},\ i = 0, 1, \ldots, l, \\ 1 & \text{otherwise,} \end{cases}$$

and hence

$$G^{-1}B^{-1}\tilde{N} = [\,\delta_j e_{ij}/\gamma_i\,].$$

Having performed operations (a) and (b), the objective function corresponding to this basic solution can be obtained. The optimal solution to the new ration formulation problem can then be found by performing simplex iterations if this basic solution is feasible, or by first performing dual simplex iterations if this basic solution is optimal but not feasible.

The use of this approach for obtaining basic solutions to the ration formulation problems depends on the initial range of f for piecewise linearization of E_m/f and $E_g/(1-f)$ being sufficient to be applicable in the initial problem and in all subsequent problems with the same liveweight gain but different liveweights. From (1) and (2) it can be seen that for a specified gain G, the ratio E_g/E_m is independent of liveweight. Hence if the range of f defined by (8) is used, this range will be adequate for all liveweights for the specified liveweight gain. For this reason the range defined by (8) was used rather than the limits used by Brokken [3], which take account of the maximum DM intake.

To use this approach the values of liveweight and liveweight gain for which least cost rations are required must first be established. The procedure for determining least cost rations for the required values of liveweight and liveweight gain involves setting the gain to one of the required values and then solving the LP for the highest value of liveweight for the specified gain. The liveweight is then reduced to the next highest value and the above approach is used to obtain a basic solution to this new problem from the previous optimal solution. If this solution is feasible or nonfeasible but optimal, simplex or dual simplex iterations are performed to obtain the optimal solution to the new problem, otherwise the new LP is solved from scratch. The procedure is repeated, i.e. the liveweight is reduced and the new ration formulation problem is solved by first obtaining a basic solution to the new problem from the optimal solution to the previous problem, until all the liveweight values for the specified liveweight gain have been covered. The procedure is then repeated for all the required values of liveweight gain.

6. Computational Experience

The overall approach to optimizing feedlot operations has been applied to a feedlot in which the objective is to minimize the cost of producing animals of specified liveweight from animals of known initial liveweight. In this case the approach involves first using the LP model to deterimine least cost rations and then using DP to determine the optimal feeding schedule by solution of the recurrence relations (11). In order to perform the calculations two FORTRAN programs were developed. The first program is used to perform the calculations to determine least cost rations for appropriate values of animal liveweight and liveweight gain by solving the ration formulation LP model by the improved solution procedure. The output from this program is then used as the input for the second program which is used to perform the calculations for the solution of the DP recurrence relations.

The approach has been tested using representative cost data for a feedlot in which animals of liveweight 500 kg are to be produced from animals of liveweight 100 kg at minimum cost, with 13 feedstuffs available for ration formulation. For the DP model the interstate interval, w, was 1 kg and the period length, t, was 4 days. The LP model was used to determine least cost rations for animals of liveweight 100, 101, 102, . . . , 500 kg which are required to produce daily liveweight gains of 0.25, 0.50, 0.75, . . . , 1.50 kg, provided such liveweight gains are possible for particular liveweights. The remaining least cost rations required for the DP model were obtained by interpolation.

The programs were run on an IBM 370/168. The central processor time to calculate the least cost rations (approximately 2400) using a linearization interval, Δ, of 0.01 was approximately 4 seconds. When the linearization interval was reduced to 0.005 no significant differences were found in the compositions or costs of rations and the central processor time was approximately 7 seconds. The central processor time to solve the DP recurrence relations was approximately 1.5 seconds. An example of the output from the program for solving the DP model is shown in Table 1. Note that because of the form of the LP model (9), rations will be composed of at most five feedstuffs.

The improvement in computational efficiency achieved by using the suggested method for solving the ration formulation LP by first trying to obtain an improved initial basic solution to the LP problem rather than solving each ration formulation LP from scratch was investigated. For a specified liveweight gain the saving in computational time will depend on the number of values of liveweight to be consid-

TABLE 1

Example of Output from DP *Model Solution Program*

Period Number	Liveweight (KG) Start	Liveweight (KG) End	Cumulative Cost ($)	Ration Required on First Day of Period Composition (DM Basis) Food	%	Food	%	Food	%	Food	%	Food	%	Quantity (KG)
1	200	205	1.557	12	5.0	4	77.4	2	17.7	0	0.0	0	0.0	5.48
2	205	210	3.136	12	4.7	4	77.4	2	17.9	0	0.0	0	0.0	5.59
3	210	216	5.006	12	3.5	6	4.3	4	92.2	0	0.0	0	0.0	5.70
4	216	222	6.879	12	2.8	6	2.0	4	95.1	0	0.0	0	0.0	5.84
5	222	228	8.767	12	2.2	4	97.7	0	0.0	0	0.0	0	0.0	5.97
6	228	234	10.683	12	1.8	4	97.7	2	0.4	0	0.0	0	0.0	6.11
7	234	240	12.628	12	1.5	4	97.7	2	0.8	0	0.0	0	0.0	6.24
8	240	246	14.602	12	1.1	4	97.7	2	1.2	0	0.0	0	0.0	6.38
9	246	252	16.603	12	0.8	4	97.6	2	1.6	0	0.0	0	0.0	6.51
10	252	258	18.632	12	0.5	4	97.6	2	1.9	0	0.0	0	0.0	6.64
11	258	264	20.689	12	0.1	4	97.8	2	2.0	0	0.0	0	0.0	6.76
12	264	270	22.780	4	97.8	2	2.2	0	0.0	0	0.0	0	0.0	6.89
13	270	276	24.905	4	97.7	2	2.3	0	0.0	0	0.0	0	0.0	7.01
14	276	282	27.066	4	97.6	2	2.4	0	0.0	0	0.0	0	0.0	7.13
15	282	288	29.261	4	97.5	2	2.5	0	0.0	0	0.0	0	0.0	7.25
16	288	294	31.491	4	97.3	2	2.7	0	0.0	0	0.0	0	0.0	7.38
17	294	300	33.755	4	97.2	2	2.8	0	0.0	0	0.0	0	0.0	7.50

ered and on the interval between successive liveweights. For the case considered above where approximately 2400 rations were evaluated using a linearization interval of 0.01, the central processor time when each ration formulation LP was solved from scratch was approximately 20 seconds.

7. Conclusions

The suggested approach to optimizing feedlot operations involves first using an LP model to determine least cost rations, then using a DP model to determine optimal feeding schedules, and finally analyzing the results of the DP model. This approach offers two significant advantages over previous methods. Firstly it ensures that optimal rations are used at all stages in the production process, and secondly it is not necessary to assume that the rate of liveweight gain must remain constant throughout the whole, or part, of this process. In addition, improvements have been made in the method for solving the ration formulation LP model, and this has enhanced the computational efficiency of the approach. The computational experience with a realistically sized problem suggests that the approach would be practical in commercial feedlot operations.

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APPENDIX A2

A PARAMETRIC PROGRAMMING METHOD FOR BEEF CATTLE RATION FORMULATION

Published in Journal of the Operational Research Society,

Vol. 31, pp 689-698, 1980

J. Opl Res. Soc. Vol. 31, pp. 689 to 698
Pergamon Press Ltd 1980. Printed in Great Britain

0160-5682/80/0801-0689$02.00/0

A Parametric Programming Method for Beef Cattle Ration Formulation

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Ration formulation is an important aspect of the operation of many beef cattle enterprises. The method for beef cattle ration formulation depends on the system used to specify the nutrient requirements of the animals. With the system which is recommended by the Ministry of Agriculture, Fisheries and Food in the U.K., difficulties arise in ration formulation because the energy requirements of the animal depend on the energy concentration of the ration. A method for formulating rations to satisfy the recommended nutrient standards is developed. The approach involves using parametric linear programming to obtain a piecewise representation of the cost of the ration as a function of the energy concentration of the ration, and then using differential calculus to determine the least cost ration.

INTRODUCTION

RATION formulation is an important aspect of the operation of many livestock enterprises, particularly in intensive operations in which animals are fed entirely on purchased feedstuffs. It involves determining the quantities of individual feedstuffs to feed to an animal in order to supply the nutrients, i.e. the energy, protein, minerals etc. required by the animal. The formulation of a ration is thus different from the formulation of a diet which in this context is defined in terms of the proportions of its constituent feedstuffs and is formulated to have specified concentrations of individual nutrients. Much work has been carried out on the formulation of least cost diets, with linear programming (LP) being widely used in this work (e.g. Beneke and Winterboer,[1] Dent and Casey[2]). However, for ruminants, ration formulation is more complex than diet formulation and, especially in intensive livestock enterprises, it is more important.

In these operations rations should be formulated to supply sufficient nutrients to produce a specified liveweight gain in an animal of known liveweight. Several different systems have been proposed for specifying the nutrient requirements of animals by, for example, the Agricultural Research Council[3] in the U.K. and the National Research Council[4] in the U.S. The method of ration formulation depends on the system used for specifying the nutrient requirements. In this paper a method is developed for formulating least cost rations for beef cattle to meet the nutrient standards recommended by the Ministry of Agriculture, Fisheries and Food (MAFF) in the U.K. The MAFF recommendations are based partly on the Agricultural Research Council (ARC) system.[3]

NUTRIENT REQUIREMENTS OF BEEF CATTLE

The nutrient requirements of beef cattle are specified in terms of the energy, protein, minerals, etc. required to produce a specified liveweight gain in an animal of known liveweight. In order to illustrate the proposed method of ration formulation only the two principal nutrients, namely energy and protein, will be considered since other nutrients are easily incorporated into the ration formulation model.

Energy requirements

The energy required by an animal is supplied by the food consumed. The portion of the food energy which can be used by the animal after faecal, urinary and methane losses have been deducted, is called metabolisable energy (ME). Part of the ME is used in heat production and since this heat is of no use to the animal except in a cold environment, it is considered as a loss from the food energy. Net energy (NE) is the ME less the heat production and represents the part of the food energy which is available for maintenance of body functions and for production, e.g. liveweight gain, milk production.

In the system recommended by the MAFF[5] for specifying the nutrient requirements of ruminants, food energy is expressed in ME terms. The ME concentrations of feedstuffs commonly used in feeding beef cattle are listed[5] and the ME concentration of a ration or diet is obtained by summing the ME contents of the constituent feedstuffs. Thus for a diet consisting of a proportion x_i, $x_i \geq 0$, of feedstuff i, $i = 1, 2, 3, \ldots, I$, of ME concentration r_i (in MJ/kg dry matter), the ME concentration, R, of this diet is given by

$$R = \Sigma\, r_i x_i \tag{1a}$$

where

$$\Sigma\, x_i = 1. \tag{1b}$$

The beef cattle energy requirements recommended by the MAFF[5] are based on the NE system recommended by the ARC.[3] For an animal of liveweight W (in kg) which is required to produce a liveweight gain G (in kg) the NE requirements (in MJ) recommended by the MAFF for maintenance, E_m, and gain, E_g, are given by

$$E_m = 5.67 + 0.061W \tag{2a}$$

$$E_g = G(6.28 + 0.0188W)/(1.0 - 0.3G) \tag{2b}$$

The efficiency of utilisation of dietary ME in supplying the NE requirements of the animal for maintenance and gain depends on the ME concentration of the ration. For a feedstuff or ration with ME concentration R (in MJ/kg dry matter), the efficiencies of utilisation of ME for maintenance, k_m, and for gain, k_g, are given by

$$k_m = 0.55 + 0.016R \tag{3a}$$

$$k_g = 0.0435R. \tag{3b}$$

However, for simplicity the MAFF recommend the adoption of a single value of 0.72 for k_m.

The ME requirements (in MJ) for maintenance, M_m, and gain, M_g, are obtained by dividing the corresponding NE requirements from (2) by the appropriate efficiency from (3). Thus, using $k_m = 0.72$,

$$M_m = 1.39E_m \tag{4a}$$

$$M_g = 23.0E_g/R \tag{4b}$$

where E_m and E_g are given by (2). These expressions do not include the 5% safety margin recommended by the MAFF.[5]

The total ME requirement of the animal is thus a function $M(R)$ of the ME concentration, R, of the ration and is given by

$$M(R) = M_m + M_g \tag{5}$$

where M_m and M_g are given by (4). The quantity $Q(R)$ (in kg) of this ration required to satisfy the ME requirements of the animal is given by

$$Q(R) = M(R)/R \tag{6}$$

where $Q(R)$ must not exceed the dry matter (DM) appetite of the animal.

Protein requirements

Protein standards for beef cattle are not as well established as those for energy. In the U.S., the National Research Council[4] has proposed protein standards which depend on animal liveweight and liveweight gain, but in the U.K. the ARC[3] suggests that protein requirements also depend on the ME concentration of the ration. However, the protein standards recommended by the ARC are widely regarded as understating the requirements (e.g. Preston and Willis[6]). The protein requirements recommended by the MAFF[7] are independent of the ME concentration of the ration. Protein requirements are listed[7] for a number of values of animal liveweight and liveweight gain, with intermediate values being obtained by interpolation. In illustrating the proposed method of ration formulation, it will be assumed that protein requirements depend only on animal liveweight and liveweight gain. It would, however, be possible to modify the approach to allow for protein requirements which depend on the ME concentration of the ration.

RATION FORMULATION

In formulating rations to supply the nutrient requirements recommended by the MAFF, difficulties arise because of the interaction between the ME concentration of the ration and the ME requirements of the animal. The MAFF[5] recommend an iterative approach to ration formulation, but even for two component rations this can be a time consuming process and would be of little use in least cost ration formulation. In order to overcome these difficulties in ration formulation, the variable NE system was developed[8] from the ARC based NE system. With the variable NE system rations can be formulated in an additive manner, thus making it suitable for the formulation of rations by LP. This system is also recommended by the MAFF[5] but it is based on approximations which can cause significant errors in ration formulation.

A method for formulating least cost rations which supply the nutrient requirements recommended by the MAFF has been developed by Kennedy.[9] This method involves first using LP to determine the least cost ration of specified ME concentration which supplies the ME, protein etc. required by an animal of known liveweight which is required to achieve a specified liveweight gain. The ME concentration of the ration is then changed and the ME requirement is changed to the appropriate value. The LP is then solved to determine the least cost ration of this ME concentration. By repeated solution of the LP for different values of the ME concentration, the overall least cost ration for the required liveweight and liveweight gain can be found. This procedure is time consuming and in practice only a limited number of values of the ration ME concentration can be considered.

A method for least cost ration formulation

Assume that l feedstuffs are available for ration formulation and define

r_i = ME concentration of feedstuff i (in MJ/kg DM),
p_i = protein concentration of feedstuff i (in g/kg DM),
c_i = cost per kg DM of feedstuff i.

Suppose that a diet of ME concentration R has been formulated with the proportion of feedstuff i being x_i, $x_i \geqslant 0$. Then the ME concentration, R, of this diet is given by (1) and the cost $c(R)$ per kg DM of this diet is

$$c(R) = \Sigma\, c_i x_i. \tag{7}$$

If this diet is fed to an animal of liveweight W which is required to achieve a daily liveweight gain G, then defining

$M(R)$ = ME requirement of animal when fed ration of ME concentration R, given by (5)

and

P = daily protein requirement of animal (in g),

the quantity $Q(R)$ of this diet is given by (6), i.e. $Q(R) = M(R)/R$. Now $Q(R)$ must not exceed the DM appetite of the animal and it will be assumed that $M(R)$, and hence $Q(R)$, is not defined if $Q(R)$ would exceed the DM appetite of the animal. In addition, this ration must supply the protein requirements of the animal,

i.e.

$$Q(R)\,\Sigma\, p_i x_i \geqslant P$$

or

$$\Sigma\, p_i x_i \geqslant PR/M(R). \tag{8}$$

The cost of this ration, $K(R)$, is given by

$$K(R) = Q(R)c(R)$$

or

$$K(R) = M(R)c(R)/R. \tag{9}$$

The least cost ration for this animal is that which minimises $K(R)$, given by (9), and satisfies the protein requirements, etc. of the animal. Now $c(R)$ is not, in general, uniquely defined since diets of ME concentration R can be formulated in a number of ways. However, if $C(R)$ is defined to be the cost of the least cost diet of ME concentration R which will satisfy the protein requirements of the animal, the following procedure can be used to determine the least cost ration to feed to the animal:

(1) Use LP to determine the least cost diet of ME concentration R which will satisfy the protein requirements of the animal:

$$C(R) = \min \Sigma\, c_i x_i \tag{10a}$$

$$\text{subject to} \quad \Sigma\, x_i = 1 \tag{10b}$$

$$\Sigma\, r_i x_i = R \tag{10c}$$

$$\Sigma\, p_i x_i \geqslant PR/M(R) \tag{10d}$$

$$x_i \geqslant 0.$$

Note that other nutrients could be incorporated into this LP model.

(2) Determine the cost, $K(R)$, of the least cost ration of ME concentration R for this animal:

$$K(R) = M(R)C(R)/R. \tag{11}$$

(3) Repeat steps (1) and (2) for a number values of the ME concentration to determine the overall least cost ration for the animal.

This procedure will produce the optimal ration if sufficient values of R are considered. However, this procedure can be improved by using parametric programming to obtain a piecewise linear representation of $C(R)$ over the range of R and then using differential calculus to determine the minimum overall cost. Such an approach will produce the optimal ration with negligible error.

The parameterisation procedure

By parameterising the right hand side of (10) a piecewise linear representation of $C(R)$ over the range of R, $R_L \leqslant R \leqslant R_U$, can be obtained. Since the right hand side of (10d) is not in a form suitable for direct parameterisation, it is first necessary to approximate $PR/M(R)$ by piecewise linearisation over the range of R. This can be done by determining the values of $PR/M(R)$ for $R = R_j = R_L + j\Delta$, $j = 0, 1, 2, \ldots, n$, where Δ is the linearisation interval, $\Delta > 0$, and n is integral such that $(R_U - R_L)/\Delta \leqslant n < (R_U + \Delta - R_L)/\Delta$ with intermediate values being obtained by linear interpolation, i.e. for $R_j \leqslant R < R_{j+1}$, $j = 0, 1, 2, 3, \ldots, n-1$,

$$PR/M(R) = u_j + (R - R_j)v_j \tag{12a}$$

where

$$u_j = PR_j/M(R_j) \tag{12b}$$

$$v_j = (u_{j+1} - u_j)/(R_{j+1} - R_j). \tag{12c}$$

The LP of equations (10) can, by introducing a surplus variable x_{l+1} in (10d), be expressed in the form

$$C(R) = \min Z = \mathbf{CX} \tag{13a}$$

$$\mathbf{AX} = \mathbf{D} \tag{13b}$$

$$\mathbf{X} \geqslant \mathbf{0}$$

where $\mathbf{A} = [a_{ij}]$, $\mathbf{C} = [c_j]$, $\mathbf{D} = [d_i]$ and $\mathbf{X} = [x_j]$ are appropriately defined. Suppose that the LP is solved for some value, r, of the ME concentration of the ration, $R_j \leqslant r < R_{j+1}, j = 0, 1, 2, 3, \ldots, n-1$, and that an optimal solution exists with optimal basis matrix $\mathbf{B}$. Then if the columns of $\mathbf{A}$ are permuted so that $\mathbf{A} = (\mathbf{B},\mathbf{N})$, (13b) can be expressed in the form (see, for example, Hadley[10])

$$\mathbf{BX}_B + \mathbf{NX}_N = \mathbf{D} \tag{14}$$

where $\mathbf{X}_B = [x_{Bi}]$ is the vector of optimal basic variables and $\mathbf{X}_N$ is the vector of non basic variables in the optimal solution. Premultiplying (14) by $\mathbf{B}^{-1} = [\beta_{ij}]$,

$$\mathbf{X}_B + \mathbf{B}^{-1}\mathbf{NX}_N = \mathbf{B}^{-1}\mathbf{D} \tag{15}$$

yields the solution

$$\mathbf{X}_B = \mathbf{B}^{-1}\mathbf{D}; \qquad \mathbf{X}_N = \mathbf{0}. \tag{16}$$

By letting $\mathbf{C} = (\mathbf{C}_B, \mathbf{C}_N)$ and substituting for $\mathbf{X}_B$ from (15), the optimal value of the objective function of (13), $C(r)$, can be expressed as

$$C(r) = \mathbf{C}_B\mathbf{B}^{-1}\mathbf{D} - (\mathbf{C}_B\mathbf{B}^{-1}\mathbf{N} - \mathbf{C}_N)\mathbf{X}_N \tag{17}$$

Substituting $\mathbf{X}_N = \mathbf{0}$ in (17) yields

$$C(r) = \mathbf{C}_B\mathbf{B}^{-1}\,\mathbf{D}$$

or

$$C(r) = \Lambda\mathbf{D} \tag{18}$$

where $\Lambda = [\lambda_j] = \mathbf{C}_B\mathbf{B}^{-1}$ is the optimal basis of the dual of (13).

If r is increased to $r + \delta$ where $0 \leqslant \delta \leqslant R_{j+1} - r$, then $\mathbf{D}$ will change to $\bar{\mathbf{D}}\,[\bar{d}_i]$ with

$$\begin{aligned} \bar{d}_1 &= 1 &&= d_1 \\ \bar{d}_2 &= r + \delta &&= d_2 + \delta \\ \bar{d}_3 &= u_j + (r - R_j)v_j + v_j\delta &&= d_3 + v_j\delta. \end{aligned}$$

This change in the right hand side of (13b) will change the solution from $\mathbf{X}_B$ to $\bar{\mathbf{X}}_B = [\bar{x}_{Bi}]$ where

$$\bar{\mathbf{X}}_B = \mathbf{B}^{-1}\bar{\mathbf{D}} \tag{19a}$$

and from the definition of $\mathbf{D}$ and $\bar{\mathbf{D}}$, $\bar{x}_{Bi}$, $i = 1, 2, 3$, can be expressed as

$$\bar{x}_{Bi} = x_{Bi} + (\beta_{i2} + v_j\beta_{i3})\delta \tag{19b}$$

This solution will remain optimal provided $\bar{x}_{Bi} \geqslant 0$, $i = 1, 2, 3$, and since $\delta \geqslant 0$, it can be seen that the solution will remain optimal provided,

$$\delta \leqslant -x_{Bi}/(\beta_{i2} + v_j\beta_{i3}) \quad \text{if} \quad \beta_{i2} + v_j\beta_{i3} < 0, \quad i = 1, 2, 3.$$

It is also required that $\delta \leqslant R_{j+1} - r$, and hence the range of δ, $0 \leqslant \delta \leqslant \delta_u$, over which the solution remains optimal in this interval, or part interval, can be established. Over this range of δ the cost of the least cost diet with ME concentration $r + \delta$ which will satisfy the protein requirements of the animal is given by

$$C(r + \delta) = \Lambda\overline{\mathbf{D}}$$

i.e.

$$C(r + \delta) = \lambda_1 + (r + \delta)\lambda_2 + (\tilde{u}_j + (r + \delta)v_j)\lambda_3 \quad (20a)$$

where

$$\tilde{u}_j = u_j - v_j R_j. \quad (20b)$$

Parametric LP can thus be used to obtain a piecewise linear representation of $C(R)$ over the range of R. Suppose that LP (10) has been solved for some value, r, of R, $R_j \leqslant r < R_{j+1}$. If this solution ceases to be optimal within the appropriate interval, or part interval, i.e. $\delta_u < R_{j+1} - r$, then the LP must be resolved at the value of R at which the solution changes. If the solution remains optimal to the end of the interval, i.e. $\delta_u = R_{j+1} - r$, the solution at the start of the next interval can be obtained from (19) and (20) with $\delta = R_{j+1} - r$, and the range over which this solution remains optimal can be determined as before. In order to extend the procedure to cover the entire range of R it is necessary to allow for ranges where no feasible solution exists. Parameterisation of the right hand side of LP (10) can easily be extended to determine the range over which no feasible solution exists. For example, when solving the LP by the two phase simplex method this involves determining the range of phase I optimality and overall infeasibility in any interval or part interval.

By using this procedure $C(R)$ can be approximated by a piecewise linear function of m segments with boundaries at $R = \rho_0, \rho_1, \rho_2, \ldots, \rho_m$. If an optimal feasible solution to LP (10) exists in the kth segment, i.e. for $\rho_{k-1} \leqslant R < \rho_k$, and if $R_j \leqslant \rho_{k-1} < R_{j+1}$, $j = 0, 1, 2, 3, \ldots, n-1$, then from (16a),

$$C(R) = \lambda_{k1} + R\lambda_{k2} + (\tilde{u}_j + Rv_j)\lambda_{k3} \qquad \rho_{k-1} \leqslant R < \rho_k \quad (21)$$

where $\tilde{u}_j$ and v_j are given by (20b) and (12c) respectively and $\Lambda_k = [\lambda_{ki}]$ is the optimal dual basis for LP (10) in the kth segment.

Determination of the least cost ration

The parameterisation procedure is used to derive the cost, $C(R)$, of the least cost diet of ME concentration R over the range of R. The quantity of diet, i.e. the ration, of ME concentration R required by the animal is obtained by substituting in (6) and can be expressed in the form

$$Q(R) = (YR + Z)/R^2 \quad (22)$$

where

$$Y = 1.39E_m$$

and

$$Z = 23.0E_g$$

where E_m and E_g are given by (2) and it has been assumed that $k_m = 0.72$.

Suppose that a piecewise linear representation of $C(R)$ has been derived and that an optimal feasible solution to LP (10) exists in the kth segment of this piecewise linearisation, i.e. for $\rho_{k-1} \leqslant R < \rho_k$. In this segment the cost $K(R)$ of feeding the least cost ration of ME concentration R is given by

$$K(R) = Q(R)C(R)$$

and by substituting for $Q(R)$ from (22) and $C(R)$ from (21), $K(R)$ can be expressed in the form

$$K(R) = \alpha_1 + \alpha_2/R + \alpha_3/R^2 \qquad \rho_{k-1} \leq R < \rho_k \qquad (23)$$

where

$$\alpha_1 = (\lambda_{\cdot k2} + v_j\lambda_{\cdot k3})Y$$

$$\alpha_2 = (\lambda_{\cdot k1} + \tilde{u}_j\lambda_{\cdot k3})Y + (\lambda_{\cdot k2} + v_j\lambda_{\cdot k3})Z$$

$$\alpha_3 = (\lambda_{\cdot k1} + \tilde{u}_j\lambda_{\cdot k3})Z.$$

Any turning points within this segment can be determined by differentiating $K(R)$ with respect to R and equating to zero, and since $R \neq 0$ the turning point is given by

$$R = -2\alpha_3/\alpha_2 \quad \text{provided} \quad \rho_{k-1} \leq R < \rho_k.$$

This will be a minimum turning point if $\alpha_3 > 0$. The minimum cost ration in this segment can then be determined by evaluating the cost of the rations with $R = \rho_{k-1}$ and $R = \rho_k$ and at the minimum turning point if it exists in this segment.

The method for determining the overall least cost ration involves performing the same calculations for each segment in which an optimal feasible solution to the LP exists and finding the ration of least cost overall. Although the approach has been described for the case where a single value of 0.72 has been used for the efficiency of utilisation of ME for maintenance, k_m, the approach can be extended to allow for k_m being dependent on the ME concentration of the ration as specified in equation (3a). The single value of $k_m = 0.72$ recommended by the MAFF has been adopted to simplify the illustration of the approach.

USE OF THE RATION FORMULATION MODEL

The beef cattle ration formulation model is used to formulate least cost rations which will supply the energy and protein requirements recommended by the MAFF for producing specified liveweight gains in animals of known liveweight. This approach to beef cattle ration formulation can be of particular importance in intensive beef cattle operations because the economic efficiency of these operations can be improved by ensuring that the animals are fed on optimal rations at every stage in the production process. In order to ensure that optimal rations are used at all times, the ration formulation model can be used to determine the least cost rations for producing specified liveweight gains in animals of known liveweight for a large number of values of animal liveweight and liveweight gain. These rations could then be used to determine the feeding schedule, i.e. the daily sequence of rations to feed to the animals, to optimise the operation of an intensive beef cattle enterprise.[11]

The ration formulation model has been tested by using it to formulate least cost rations satisfying the energy and protein requirements of animals of liveweight 100, 101, 102, 103, ..., 500 kg which are required to produce daily liveweight gains of 0, 0.25, 0.50, 0.75, ..., 1.50 kg, provided such liveweight gains are possible for particular liveweights. Nine feedstuffs were available for ration formulation and details of these are given in Table 1. A FORTRAN program was written to perform the calculations and the program was run on an ICL 2980. Examples of the output from the program are presented in Table 2 in which, for each liveweight and liveweight gain, a block of entries gives details of the composition and cost of the least cost ration. Blocks in which all the entries are zero indicate that these liveweight gains cannot be achieved at the specified liveweights. The computational time to calculate all the least cost rations, approximately 2800 in this case, depends on the linearisation interval, Δ, used. The central processor times for the calculations are listed in Table 3 for a number of values of the linearisation interval. Although the accuracy of the ration formulation procedure should improve with

TABLE 1. FEEDSTUFF DATA

Food number	Food name	ME MJ/kg DM	DCP g/kg DM	Cost p/kg DM
1	Barley	13.7	82	10.6
2	Maize	14.2	78	13.7
3	Oats	11.5	84	10.3
4	Sugar beet pulp	12.2	61	11.1
5	Soya bean meal	12.3	453	17.2
6	Dried grass	10.6	136	13.2
7	Barley straw	5.8	8	3.1
8	Oat straw	6.8	9	3.5
9	Hay	8.4	39	8.2

Abbreviations: ME—metabolisable energy; DCP—digestible crude protein; DM—dry matter.
Source: ME and DCP values from MAFF.[5]

TABLE 2. EXAMPLES OF OUTPUT FROM PROGRAM

LIVE WEIGHT (KG)	RATION SPEC	DAILY LIVEWEIGHT GAIN (KG) 0.00	0.25	0.50	0.75	1.00	1.25	1.50
100	FOOD NO,%	8 92.1	8 85.4	5 14.9	5 14.5	5 14.6	0 0.0	0 0.0
	FOOD NO,%	5 7.9	5 10.3	1 85.1	1 85.5	1 85.5	0 0.0	0 0.0
	FOOD NO,%	0 0.0	1 4.2	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	2.26	3.00	1.82	2.21	2.68	0.00	0.00
	COST (P)	10.37	15.65	21.08	25.53	31.03	0.00	0.00
101	FOOD NO,%	8 92.1	8 85.6	5 15.0	5 14.5	5 14.6	0 0.0	0 0.0
	FOOD NO,%	5 7.9	5 10.4	1 85.0	1 85.5	1 85.4	0 0.0	0 0.0
	FOOD NO,%	0 0.0	1 4.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	2.27	3.02	1.83	2.22	2.69	0.00	0.00
	COST (P)	10.43	15.72	21.17	25.63	31.14	0.00	0.00
	⋮							
170	FOOD NO,%	8 91.3	8 88.2	5 15.7	5 15.0	5 14.2	5 12.6	0 0.0
	FOOD NO,%	5 8.7	5 11.8	1 84.3	1 85.0	1 85.8	1 87.4	0 0.0
	FOOD NO,%	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	3.06	4.05	2.36	2.81	3.36	4.03	0.00
	COST (P)	14.37	20.75	27.46	32.58	38.77	46.09	0.00
171	FOOD NO,%	8 91.3	8 88.2	8 77.0	5 15.0	5 14.2	5 12.6	0 0.0
	FOOD NO,%	5 8.7	5 11.8	5 12.4	1 85.0	1 85.8	1 87.4	0 0.0
	FOOD NO,%	0 0.0	0 0.0	1 10.6	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	3.07	4.07	4.63	2.82	3.37	4.04	0.00
	COST (P)	14.44	20.81	27.54	32.68	38.87	46.20	0.00
	⋮							
499	FOOD NO,%	8 90.9	8 92.2	8 90.0	5 4.6	5 3.6	5 2.6	5 1.0
	FOOD NO,%	5 9.1	5 7.8	5 7.5	1 95.4	1 96.4	1 97.4	1 99.0
	FOOD NO,%	0 0.0	0 0.0	1 2.5	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	6.87	8.81	10.68	5.56	6.44	7.53	8.91
	COST (P)	32.63	40.21	50.26	60.55	69.79	81.11	95.07
500	FOOD NO,%	8 90.9	8 92.2	8 90.0	5 4.5	5 3.6	5 2.6	5 1.0
	FOOD NO,%	5 9.1	5 7.8	5 7.5	1 95.5	1 96.4	1 97.4	1 99.0
	FOOD NO,%	0 0.0	0 0.0	1 2.5	0 0.0	0 0.0	0 0.0	0 0.0
	QTY (KG)	6.89	8.82	10.70	5.56	6.45	7.54	8.92
	COST (P)	32.68	40.27	50.34	60.64	69.89	81.22	95.19

TABLE 3. COMPUTATIONAL TIMES ON AN ICL 2980

Linearisation interval	1.00	0.50	0.25	0.10
Central processor time (sec)	51	51	59	72

decreasing linearisation interval, only minor differences in ration composition and cost were detected on decreasing the linearisation interval from 1.00 to 0.10.

Discussion of Results

It can be seen from the results of Table 2 that if an animal is to achieve a constant daily liveweight gain then the ration should change from day to day as the liveweight increases. In practice, however, the ration would not be changed as frequently as this. The results obtained using the data of Table 1 indicate that for a given daily liveweight gain there will be few, if any, sudden changes in the composition of the rations as the liveweight increases from 100 kg to 500 kg. Thus if a constant daily liveweight gain is required, fixed rations can be used over periods in which the change in the composition of least cost rations falls within prescribed limits. For the liveweight gain values considered in this study there is only one example of the ration changing suddenly as the liveweight is increased. This occurs for a liveweight gain of 0.50 kg per day between liveweights 170 kg and 171 kg (see Table 2), when oat straw enters the ration at a high level.

From the results of Table 2 it can be seen that for a given liveweight the rations become more concentrated, with proportionately less roughage in the form of oat straw, as the daily liveweight gain increases. For daily liveweight gains of 0.75 kg or more (0.50 kg or more for animals of 170 kg or less) all concentrate rations of barley and soya bean meal are used. The change to an all concentrate ration as the daily liveweight gain increases from 0.50 kg to 0.75 kg (0.25 kg to 0.50 kg for animals of 170 kg or less) accounts for the drop in dry matter intake between these liveweight gain values. Although these all concentrate rations satisfy the energy and protein requirements of the animal it is possible that, in cases where the dry matter intake is significantly less than the maximum dry matter intake, these rations may not satisfy the appetite of the animal. This difficulty could be overcome by specifying the minimum dry matter intake of the animal or the minimum roughage content of the ration for each liveweight and liveweight gain. However, the MAFF do not specify such figures and this may be an area where further research is required. The ration formulation model could be extended to deal with constraints on the minimum dry matter intake of the animal or the minimum roughage content of the ration.

CONCLUSIONS

The proposed method for beef cattle ration formulation allows least cost rations to be formulated to meet the beef cattle nutrient requirements recommended by the MAFF. The approach overcomes the difficulties which arise in ration formulation because of the interaction between the energy requirements of the animal and the energy concentration of the ration in the MAFF system for specifying the energy requirements of beef cattle. The computational experience with the approach suggests that it would be practical for use in beef cattle ration formulation in commercial operations and in advisory work.

ACKNOWLEDGEMENT

The author wishes to thank Mr J. Harkins of the Edinburgh School of Agriculture for his advice.

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APPENDIX A3

A DYNAMIC PROGRAMMING MODEL FOR PIG PRODUCTION

Published in Journal of the Operational Research Society,

Vol. 34, pp 511-519, 1983

J. Opl Res. Soc. Vol. 34, No. 6, pp. 511-519, 1983
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0160-5682/83/060511-09$03.00/0

A Dynamic Programming Model for Pig Production

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The feeding policy of a pig production unit affects both the cost of production and the weight and carcase composition of the pigs produced. Since the market value of the pigs produced is determined by the weight and composition of the carcase, feeding policy has a major influence on the economic performance of the unit. In order to evaluate possible feeding policies, the effect of feed intake on both the weight and the body composition of the growing pig must be known, and since an optimal policy will involve using least cost rations, it must be possible to determine the least cost rations to produce liveweight gains of specified body composition. A dynamic programming model to determine the optimal feeding policy to produce pigs of specified weight and carcase composition is developed using a published pig growth model which allows the formulation of the required least cost rations, and the use of this dynamic programming model is illustrated.

INTRODUCTION

PIG PRODUCTION in the U.K. is becoming concentrated in large-scale indoor units in which pigs are bred and fattened. The market for pigs in the U.K. is graded in terms of weight and carcase quality, with premium prices being obtained for lean meat. Carcase quality is determined mainly by the genetic potential of the pig and the food it has consumed. The genetic potential has been improved by selective breeding programmes which have produced pig types with economically desirable characteristics in terms of factors such as feed conversion performance and carcase quality.

The economic performance of a pig production unit depends to a large extent on the feeding policy, i.e. the daily sequence of rations, in the unit since, for a given type of pig, the feeding policy affects both the cost of production and the market value of the pigs produced. In order to evaluate possible feeding policies, the nutrients, i.e. the energy, protein, minerals, vitamins and water, required to produce liveweight gains of specified body composition must be known. The Agricultural Research Council[1] in the U.K. and the National Research Council[2] in the U.S. both publish recommended nutrient allowances for growing pigs but, although both bodies recognise that feed intake affects both liveweight gain and the composition of this gain, neither body specifies the nutrient requirements in terms of the nutrients required to produce daily liveweight gains of specified composition. A model for pig growth has been developed by Whittemore and Fawcett[3] in which daily liveweight gain is separated into fat free and fatty tissue components, and a development of this model has been used by Fawcett *et al.*[4,5] to determine the least cost rations to produce daily liveweight gains of specified composition using linear programming (L.P.). However, although it is generally recognised that least cost rations should be used throughout the fattening period, the overall efficiency of the operation is also affected by the sequence of rations used to produce pigs of the required weight and carcase composition.

In this paper a method is developed for determining the optimal feeding policy to produce pigs of specified weight and carcase composition. The approach involves using dynamic programming (D.P.) to determine the sequence of least cost rations to produce pigs of the required liveweight and carcase composition at minimum cost and is similar to the method developed by Glen[6] for beef feedlot operations, although in the case of beef cattle, the carcase composition was not taken into account. The method can be used with any pig growth model in which the development of the animal can be expressed in terms of two variables, such as liveweight and body protein content, provided that the least cost rations required to produce specified changes in these variables can be determined. In order to illustrate the development of the method, the pig growth model of Fawcett et al.[4] and a modified version of the ration formulation model of Fawcett et al.[5] are used.

PIG GROWTH

The growth of a pig of a specified type is controlled by the nutrient content, i.e. the energy, protein, mineral, vitamin and water content, of the food it consumes and by the ambient temperature. Although each nutrient has specific functions, the principal nutrients in pig growth are energy and protein. Energy is required to maintain body functions and for processes, such as new tissue synthesis, connected with growth. The weight gain in growth is accompanied by changes in the fatty and fat free body tissue together with associated development of bone, skin etc. The fatty tissue is composed of lipid and water, while the fat free tissue is composed of protein and water, this protein being produced by conversion of dietary protein.

The pig growth model

In the model of Fawcett *et al.*,[4] the daily liveweight gain of the growing pig is separated into fat free and fatty tissue components expressed in terms of protein retention, P_r, and lipid retention, L_r, respectively. It is assumed that the daily liveweight gain, ΔW, is given by

$$\Delta W = \alpha P_r + 1.1\, L_r, \tag{1}$$

where α represents the ratio of fat free body gain to protein retention. Genetic potential affects the maximum rate of protein deposition, and for a specified type of pig it is assumed that the daily protein retention must not exceed a finite maximum, $\bar{P}_r$, i.e.

$$P_r \leqslant \bar{P}_r. \tag{2}$$

It is also assumed that the ratio of lipid retention to protein retention must exceed a minimum value, γ_L, i.e.

$$L_r \geqslant \gamma_L P_r. \tag{3}$$

Consider a pig of liveweight W (in kg) and total protein mass P_T (in kg) and let E_m (in MJ) denote the daily maintenance energy requirement, excluding the energy required for protein synthesis. For example (Whittemore and Fawcett[3]), E_m may be expressed as

$$E_m = 0.475\, W^{0.75} - 0.365\, P_T. \tag{4}$$

If the required daily liveweight gain ΔW (in g) is to be composed of protein content P_r (in g) and lipid content L_r (in g) and the pig is fed a ration of digestible crude protein content P (in g/kg dry matter), then in the model of Fawcett *et al.*[4] the food energy requirement, E (in MJ), is given by

$$E = 0.0121\, P + (0.0115 + 0.0073/Z)P_r + 0.0535\, L_r + E_m, \tag{5}$$

where Z is the ratio of protein retained, P_r, to total protein synthesised, P_t, and depends on the maturity of the animal.

Total daily protein synthesis is composed of new protein synthesis plus resynthesis of part of the protein which has been broken down. The daily protein synthesis depends on the quantity and quality of the dietary crude protein intake. In the approach of Fawcett *et al.*,[4,5] chemical value is used as a measure of protein quality and is determined by the essential amino acid in the feed, which is limiting when the amino acid profile of the feed is compared with the preferred amino acid profile of the animal. The amino acid profile of a feed is obtained by expressing the content of each amino acid in the feed as a percentage of protein mass. The chemical value of a feed is given by the minimum value obtained when the concentration of each essential amino acid in the feed is divided by the corresponding value in the preferred profile. If a pig is fed a ration of digestible crude protein content, P, and chemical value V, $0 < V < 1$, then the daily new protein synthesis, P_n, must satisfy

$$P_n \leq VP.$$

If the ratio of new protein synthesis to total protein synthesis is K, where K depends on the maturity of the animal, then from the definition of K and Z,

$$P_r \leqslant PZV/K. \tag{6}$$

Although in general all essential amino acids should be considered in determining chemical value, only histidine, lysine and methionine + cystine will be considered in the subsequent discussion, as these were found to be the only limiting groups by Fawcett *et al.*[5] in 15 commonly used feedstuffs.

Limitations of the pig growth model

The major factors influencing the response to food intake in a healthy pig are the genetic strain of the animal, sex and ambient temperature. The genetic potential will determine the maximum rate of deposition of fat free tissue, the ratio of fat free to fatty tissue in liveweight gain and the efficiency of conversion of food energy. In the model of Fawcett *et al.*,[4] the maximum rate of protein retention and the minimum value of the ratio of protein to lipid retention must be specified and will depend on the type of pig. The efficiency of food conversion affects the coefficients in equations (4) and (5), and in developing these relationships average values have been used, although ideally relationships for a specified breed and sex of pig should be determined.

The growth of the pig is also influenced by ambient temperature. Heat production is a natural part of the pig's body processes, and this heat must be dispersed to maintain body temperature at the normal level. If the ambient temperature is too high, then heat stress will cause the pig to eat less and grow more slowly. In the U.K., heat stress can generally be dealt with by increasing ventilation at relatively low cost.

At low temperatures, food energy is diverted from productive processes in order to increase heat production so that body temperature can be maintained. Cold animals will therefore grow more slowly and convert food less efficiently. If the food supply is not sufficient to provide the energy required to maintain body temperature, body fat stores, if available, will be used for heat production, and the animal will lose weight. The effect of cold stress is not considered in the model of Fawcett *et al.*,[4] although the model could be extended to incorporate this factor. For example, in the model of Whittemore and Fawcett,[3] it is assumed that the pig cannot maintain body temperature without diverting food energy from productive processes when the temperature falls below a critical temperature T_c (in °C) which depends on the liveweight W (in kg) of the pig, where

$$T_c = 23.8 - 0.15\,W. \tag{7}$$

At temperatures below this critical temperature food energy is used to provide heat to maintain body temperature, the heat required H (in MJ) depending on liveweight, W, critical temperature T_c [from (7)] and ambient temperature T (in °C), where

$$H = 0.016\,W^{0.75}\,(T_c - T). \tag{8}$$

Thus when ambient temperature is less than the critical temperature for a pig of liveweight W, the food energy required to produce a specified liveweight grain as determined by (5) must be increased by H to allow for cold stress. In the subsequent discussion, however, the effect of ambient temperature on pig growth will not be considered.

The ration formulation model

Consider a pig of liveweight W (in kg) which is required to achieve a daily liveweight gain ΔW (in g) involving protein retention P_r (in g) and lipid retention L_r (in g); then, using the pig growth model of Fawcett *et al.*,[4] ΔW, P_r and L_r must satisfy (1). Suppose that k feedstuffs are available for ration formulation and for feedstuff i let

c_i = cost per kg dry matter (DM),
e_i = digestible energy content in MJ/kg DM,
p_i = digestible crude protein content in g/kg DM,
h_i = histidine concentration,
l_i = lysine concentration,
m_i = methionine + cystine concentration,

where the histidine, lysine and methionine + cystine concentrations are expressed as percentages of protein mass, and let H, L and M denote the concentrations, as percentages of protein mass, of histidine, lysine and methionine + cystine respectively in the preferred amino acid profile of the animal.

Suppose that the pig is fed a ration containing quantity x_i, in kg DM, $x_i \geqslant 0$, $i = 1,2,3, \ldots, k$, of feedstuff i. The histidine concentration of the ration as a percentage of protein mass is

$$(\Sigma h_i p_i x_i)/\Sigma p_i x_i = h_i p_i x_i/P,$$

where

$$P = \Sigma p_i x_i$$

is the protein content of the ration, and hence the chemical value, V, of the ration is given by

$$V = \min[\Sigma h_i p_i x_i/HP, \Sigma l_i p_i x_i/LP, \Sigma m_i p_i x_i/MP]. \tag{9}$$

The protein retention, P_r, must satisfy (6) and hence, substituting for V from (9),

$$P_r \leqslant \min[Z\,\Sigma h_i p_i x_i/HK, Z\,\Sigma l_i p_i x_i/LK, Z\,\Sigma m_i p_i x_i/MK] \tag{10}$$

and (10) can be expressed in a form suitable for LP as

$$(K/Z)P_r - \Sigma h_i p_i x_i/H \leqslant 0 \tag{11a}$$

$$(K/Z)P_r - \Sigma l_i p_i x_i/L \leqslant 0 \tag{11b}$$

$$(K/Z)P_r - \Sigma m_i p_i x_i/M \leqslant 0. \tag{11c}$$

Clearly other amino acids could be taken into account in the same way.

Suppose that for this pig $L_r = \gamma P_r$ where $\gamma \geqslant \gamma_L$; then from (1),

$$P_r = \Delta W/(\alpha + 1.1\gamma) \tag{12}$$

with $P_r \leqslant \bar{P}_r$. Since the ration fed to the animal must satisfy (5) and (9), the least cost ration to achieve liveweight gain ΔW with protein retention P_r and lipid retention $L_r = \gamma P_r$ is given by the solution of the LP:

$$\text{Minimise } C = \Sigma c_i x_i \tag{13a}$$

$$\text{subject to } \Sigma(e_i - 0.0121 p_i)x_i = E_m + bP_r \tag{13b}$$

$$\Sigma(h_i p_i/H)x_i \geqslant (K/Z)P_r \tag{13c}$$

$$\Sigma(l_i p_i/L)x_i \geqslant (K/Z)P_r \tag{13d}$$

$$\Sigma(m_i p_i/M)x_i \geqslant (K/Z)P_r \tag{13e}$$

$$x_i \geqslant 0 \quad i = 1,2,3, \ldots, k,$$

where P_r is given by (12), E_m is given by, for example, (4), and b is given by

$$b = 0.0115 + 0.0073/Z + 0.0535\gamma,$$

and where K and Z are functions of the maturity of the animal.

THE FEEDING POLICY

The feeding policy of a pig production unit affects the cost of production and the weight and carcase composition of the pigs produced. In a unit producing pigs of specified weight and carcase composition, an optimal feeding policy will involve feeding least cost rations throughout the fattening period in such a way that the total production cost is minimised. If the least cost rations to produce liveweight gains of specified body composition can be determined, then dynamic programming (D.P.) can be used to determine the optimal feeding policy for the unit and, by changing the final weight and carcase composition, this D.P. model can be used to examine alternative production and marketing strategies. With the pig growth model of Fawcett *et al.*,[4] these least cost rations can be determined by the solution of (13).

The dynamic programming model

In the D.P. model, the state of a pig is defined by its liveweight and carcase composition. With the pig growth model of Fawcett *et al.*,[4] the carcase composition is defined in terms of the protein and lipid content, and since liveweight, protein content and lipid content are related by (1), the state can be defined completely in terms of liveweight and protein content. In order to examine the effect of varying the final liveweight and carcase composition, a backward D.P. formulation is used (White[7]). Define $F_n(W, P_T)$ to be the cost of producing a pig of liveweight W with total protein content P_T from a pig initially of liveweight W_1 and protein content P_1 in n periods, each of t days, using an optimal policy. Then

$$n \geqslant 1 \qquad F_n(W, P_T) = \min_{y,z \geqslant 0} [F_{n-1}(W-y, P_T - z) + C(W-y, P_T - z, y, z) + K] \tag{14a}$$

$$F_0(W_1, P_1) = 0, \tag{14b}$$

where $C(W-y, P_T - z, y, z)$ is the cost of producing a liveweight gain y with protein content z in a pig of liveweight $W-y$ and protein content $P_T - z$ in t equal daily increments using least cost rations and where K is the overhead cost per period.

The D.P. model can be used to determine the overall minimum cost of producing pigs of specified liveweight and protein content and could be extended to accommodate time dependent feeding and overhead costs.

Solving the dynamic programming model

For numerical work the state of the animal, expressed in terms of liveweight and protein content, must be defined in discrete units. If the liveweight interval is w (in kg) and the protein content interval is p_T (in kg), the D.P. recurrence relations (14) must be solved for all possible combinations of liveweight, $W = W_1, W_1 + w, W_1 + 2w, \ldots, W_F$, and protein content, $P_T = P_1, P_1 + p_T, P_1 + 2p_T, \ldots, P_F$, by considering all possible liveweight gains, y, and protein increments, z, from each of these states at every stage in the solution procedure. In addition, in defining the states and possible transitions from these states at any stage in the solution procedure, the computational load and the fast access storage required for computer solution must be taken into account. For computational convenience, the values of liveweight gain and protein increment over periods of t days, i.e. the values of y and z, should be chosen to be integral multiples of w and p_T, respectively. For example, if $w = 1$ kg then $y = 1, 2, 3, \ldots, [tG(W)]$ kg, where $G(W)$ is the maximum daily liveweight gain of a pig of liveweight W, and $[X]$ denotes the integral part of X, and consequently the values of daily liveweight gain ΔW which should be considered in determining least cost rations would be $\Delta W = 1/t, 2/t, 3/t, \ldots, [tG(W)]/t$ (in kg). Similar considerations apply in the choice of values for daily protein retention, P_r, and this can be dealt with by choosing suitable values of the ratio γ, $\gamma \geqslant \gamma_L$, and obtaining P_r from (12) for each of the required values of ΔW. For example, γ can be chosen so that the daily fat free gains take the values $0.5\Delta W$, $0.6\Delta W$, $0.7\Delta W$, $0.8\Delta W$.

From the definition of $C(W - y, P_1 - z, y, z)$, it can be seen that the L.P. model (13) should be used to determine all the least cost rations involved in increasing the liveweight from $(W - y)$ to W and the protein content from $(P_1 - z)$ to P_1 in t equal daily increments and that these calculations must be performed for each of the possible combinations of the state, defined in terms of liveweight and protein content, at the start and end of a t day period. In practice, however, it will generally be sufficient to calculate least cost rations for the middle day of a period of t days. For example, if $w = 1$ kg and $t = 5$ days and the number of possible values of daily liveweight gain is limited to five and the number of values of γ is limited to four, then if $W_1 = 20$ kg and $W_F = 100$ kg, approximately 2×10^5 least cost rations must be calculated. However, since a change in the liveweight, protein content or protein increment, or any combination of these factors, affects only the right hand side of (13), the calculation procedure can be improved by using right hand side parameterisation methods (see, for example, Hadley[8]) rather than solving each L.P. problem from scratch.

Extensions of the dynamic programming model

In pig production units which do not have contracts to produce pigs of specified market grade, the weight and carcase composition of the pigs produced can be changed to suit the market conditions, and in this case a suitable criterion for evaluating possible operating policies would be the return per animal per period. Assume that pigs are introduced into the fattening system at a fixed liveweight and carcase composition and that final liveweight and carcase composition can be varied to some extent. Thus if

$F_n(W_S, P_S)$ = cost of producing a pig of liveweight W_S and protein content P_S in n periods using the optimal policy from (14),

$S(W_S, P_S)$ = selling price of a pig of liveweight W_S and protein content P_S,

C_1 = cost of producing a pig for start of the fattening process,

then the optimal return per animal per period, $R(W_S, P_S)$, from producing pigs of liveweight W_S and protein content P_S is

$$R(W_S, P_S) = \max_{n>0} [(S(W_S, P_S) - C_1 - F_n(W_S, P_S))/n]. \tag{15}$$

The optimal combination of selling weight and carcase composition can be found by numerical solution of (15) for all commercially acceptable combinations of W_S and P_S. In addition, if the liveweight W_1 and protein content P_1 at the start of the fattening process can be varied, the optimal combination of starting and selling conditions can be obtained in a similar way by allowing C_1 to be a function of W_1 and P_1 and redefining R and F_n to be functions of W_1, P_1, W_S and P_S.

USE OF THE MODEL

A suite of two FORTRAN programs has been developed to solve the D.P. cost minimisation model and thus determine the optimal feeding policy to produce pigs of specified liveweight and protein content from pigs of given initial liveweight and protein content. The first program solves the D.P. recurrence relations and incorporates a right hand side parameterisation routine for solution of the ration formulation L.P. model (13). Although this program uses the pig growth model of Fawcett *et al.*,[4] it can be modified to use any suitable pig growth model. The second program uses the output from the first program to determine the optimal feeding policy for the specified final liveweight and carcase composition and prints the results.

Data requirements

To run the first program the operating data for the pig fattening unit must be supplied. This dataset specifies the nutrient composition and cost of each of the feedstuffs available for ration formulation, the weight and body composition of pigs entering the fattening unit and the maximum final weight of the pigs. In addition, the length of the feeding period used in the D.P. model (14) must be specified, and in choosing the value of this parameter computational considerations must be taken into account.

In the development of the D.P. model (14), it was assumed that the rate of liveweight gain remained constant throughout a feeding period of length t days. In the present version of the D.P. model solution program, the liveweight gains, in kg, which can be achieved during a feeding period are restricted to integral values, i.e. the liveweight interval, w, between successive states in the D.P. model (14) is 1 kg, and thus the daily liveweight gains which are considered are restricted to $1/t, 2/t, 3/t, \ldots, [tG(W)]/t$ (in kg), where $G(W)$ is the maximum daily liveweight gain of a pig of liveweight W. The choice of feeding period length also determines the possible daily protein increments associated with these daily liveweight gains. In the present version of the program, the body protein content is expressed in terms of the fat free tissue weight, and the increase in fat free tissue weight during a feeding period is restricted to integral multiples of 100 g. For each liveweight and liveweight gain, ΔW, up to four values of protein increment can be considered, with the values of the lipid to protein retention ratio, γ, being determined from (12) such that the corresponding daily fat free gains are $0.5\Delta W$, $0.6\Delta W$, $0.7\Delta W$ and $0.8\Delta W$.

In order to ensure that a reasonable range of daily liveweight gains is considered with this version of the D.P. model solution program, it is desirable to use a feeding period of at least 3 days. Since the daily liveweight gain remains constant during a feeding period, there is a possibility that the solution will involve large increases in the rate of daily liveweight gain between successive feeding periods, and to reduce the possibility of this occurring the feeding period length should not exceed 10 days.

Computational experience

The programs for determining the optimal feeding policy to produce pigs of specified liveweight and carcase composition have been tested using representative cost data for an enterprise in which pigs of 20 kg and protein content 3.0 kg (Whittemore and Fawcett[3]) can be fed to a maximum liveweight of 100 kg, with four feedstuffs available for ration formulation. The feeding period length, t, was 5 days, and thus the daily liveweight gain values considered were 200, 400, 600, 800 and 1000 g, provided that such gains were possible for particular liveweights. Feedstuff data and the values of the parameters of the pig growth model were taken, or derived, from Fawcett *et al.*[5] The programs were run on an IBM 370/165. The central processor time to solve the D.P. recurrence relations was 368 seconds, and the central processor time to trace the optimal solution and print the results for a specified liveweight and protein content was 7 seconds.

Discussion of results

From the description of the D.P. model it can be seen that the quality of the carcase is expressed in terms of the protein content. In the market for pigs, carcases are graded in terms of their lean meat content, which can be estimated from measurement of backfat thickness. Although protein content is related to the lean meat content of the carcase, protein content is not immediately meaningful to pig producers as a measure of carcase quality. However, protein content can be related to the lean meat percentage in the carcase using equation (1) and making allowances for the ratio of carcase weight to animal liveweight. An example of the output from the second program is given in Table 1, which shows the optimal feeding policy to produce a pig of 100 kg liveweight with a carcase of 55% lean meat. If required, the development of the protein content of the animal can also be printed in the output.

The results in Table 1 demonstrate the form of the output from the programs but, since the results are dependent on both the pig growth model and the feedstuff data used, general conclusions regarding feeding policy cannot be derived from these results. However, the results can be used to illustrate some practical problems associated with using the D.P. model.

TABLE 1. EXAMPLE OF OUTPUT FROM PROGRAM
OPTIMAL FEEDING PLAN TO PRODUCE PIG OF LIVEWEIGHT 100 KG WITH CARCASE OF 55% LEAN MEAT

Period No.	Liveweight (kg) Start	Liveweight (kg) End	Total cost (£)	Ration required on middle day of period — Composition (DM basis) Food	%	Food	%	Food	%	Food	%	Quantity (kg)
1	20	21	0.33	Soya	9.7	Barley	90.3	—	—	—	—	0.59
2	21	25	1.24	Soya	13.1	Barley	86.9	—	—	—	—	1.79
3	25	30	2.29	Soya	21.6	Barley	78.4	—	—	—	—	1.90
4	30	35	3.35	Soya	20.9	Barley	79.1	—	—	—	—	1.95
5	35	40	4.43	Soya	20.3	Barley	79.7	—	—	—	—	2.00
6	40	45	5.53	Soya	19.8	Barley	80.2	—	—	—	—	2.04
7	45	50	6.66	Soya	19.3	Barley	80.7	—	—	—	—	2.09
8	50	55	8.02	Soya	2.2	Barley	97.8	—	—	—	—	3.06
9	55	60	9.41	Soya	2.0	Barley	98.0	—	—	—	—	3.11
10	60	65	10.81	Soya	1.9	Barley	98.1	—	—	—	—	3.16
11	65	70	12.24	Soya	1.8	Barley	98.2	—	—	—	—	3.20
12	70	75	13.68	Soya	1.8	Barley	98.2	—	—	—	—	3.25
13	75	80	15.14	Soya	1.7	Barley	98.3	—	—	—	—	3.30
14	80	85	16.62	Soya	1.6	Barley	98.4	—	—	—	—	3.34
15	85	90	18.12	Soya	1.6	Barley	98.4	—	—	—	—	3.39
16	90	95	19.64	Soya	1.5	Barley	98.5	—	—	—	—	3.44
17	95	100	21.18	Soya	1.5	Barley	98.5	—	—	—	—	3.48

It can be seen from Table 1 that the least cost rations are composed of only two feedstuffs. This is a direct consequence of the least cost ration formulation model used since, in general, only one amino acid will be limiting, i.e. only one of the constraints (13c), (13d), (13e) in the L.P. model (13) will be binding. It may be desirable to include additional dietary constraints in the ration formulation model, but this will not affect the D.P. model.

In the output from the program shown in Table 1, only the ration for the middle day of each feeding period is printed. In theory the ration should change on a daily basis to reflect the increase in weight and change in body composition of the growing pig, but for practical administrative reasons daily changes in the ration are undesirable. If the feeding period used in the D.P. model is short, then the effect of keeping the ration constant during a feeding period will not be significant, and in this case the most appropriate ration to use is the ration determined for the middle day of the feeding period. This ration can then be fed by dividing it into the number of feeds required during a day according to the practice of the unit (e.g. Thornton[9]).

If it is desired to maintain constant diets, defined in terms of the proportions of constituent feedstuffs, over longer periods, then diets can be averaged over feeding periods in which the compositions of the rations determined by the D.P. model remain reasonably constant, with the total daily intake of this diet, i.e. the ration, increasing as the pig grows. For example, from Table 1 the composition of the ration remains reasonably constant during feeding periods 3–7 inclusive and also during feeding periods 8–17 inclusive. An average diet could be determined for each of these extended periods, and this could be used as the constant diet in the appropriate extended period, with the daily quantity of this diet increasing with time. If sudden changes in diet are found, it may be necessary to change diets gradually since the pig may react adversely to sudden changes in diet. The effects of using constant diets over a number of feeding periods could be examined by using the pig growth model to investigate the effect of this policy on the final weight and carcase composition of the pigs produced.

The present versions of the programs have been developed as a research tool for use on a mainframe computer. For this reason the programs are not suitable for use by individual producers, although they could be used in advisory work. However, by incorporating techniques, such as path restriction,[10] to reduce the computational load, it would be possible to modify the programs to run on the more powerful of the present generation of microcomputers, and in this form the programs could be used by individual producers. The next stage of this work will involve developing the model for use on a 16 bit microcomputer and modifying the model to allow for changes in ambient temperature.

CONCLUSIONS

A method has been developed for determining the optimal feeding policy for a pig production unit. The optimal policy involves feeding least cost rations throughout the fattening period, and in order to use the method it is necessary to be able to determine the least cost rations to produce liveweight gains of specified body composition in pigs of known liveweight. A D.P. model is used to determine the sequence of these least cost rations to produce pigs of specified final weight and carcase composition at least cost. The D.P. model can also be used to evaluate possible operating policies under different feedstuff and market conditions. Although the method has been developed using a particular pig growth model, the approach can be used with any pig growth model provided the least cost rations to produce liveweight gains of specified body composition can be determined. The method can be used as a research tool and in advisory work for planning the operations of pig producers, but with further development the method could be made suitable for use by individual producers.

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APPENDIX B1

BEEF CATTLE RATION FORMULATION PROGRAM - MAFF STANDARDS

```
      DEFINE FILE 7(502,352,L,IAV7)
      DIMENSION AA(3,50),OBF(50),RVAL(161),PCVAL(161),LWVAL(20),
     1GVAL(11),DMAVAL(20),DCPVAL(20,11),COSMIN(11),LCINGR(11,3),
     2DCPINT(11),DCP(11),INGS(3),COMP(11,3),XB(3),QFED(11)
      COMMON/LPCOM/NCONS,NVARS,NGTS,NG,NGL,NLE,NL1,IC,NC
      COMMON/FECOM/AA,OBF,NRVAL,RMAX,NC1,EPSI
      IAV7=1
      NVMAX=50
      NGMAX=11
      NRMAX=161
      NLWMAX=20
      NCONS=3
      NLTS=0
      NEQS=2
      NGTS=1
      NC=NCONS+1
      NC1=NC+1
      NL1=NLTS+1
      NLE=NLTS+NEQS
      EPSI=0.0005
      READ (5,1010) NVARS,LWMIN,LWMAX,LWINCR
 1010 FORMAT (4I4)
      IC=NVARS+NC+NGTS
      NG=NVARS+NGTS
      NGL=NG+NLTS
      IF (LWMAX.GT.LWMIN) GO TO 5
      WRITE (6,4010)
 4010 FORMAT (33H MAX LIVEWEIGHT LE MIN LIVEWEIGHT)
      GO TO 999
    5 I=LWMAX-LWMIN
      J=I/LWINCR
      K=J*LWINCR
      IF (I-K.EQ.0) GO TO 7
      WRITE (6,4015)
 4015 FORMAT (68H DIFFERENCE BETWEEN MAX LW AND MIN LW NOT A MULTIPLE OF
     1 LW INCREMENT)
      GO TO 999
    7 DO 10 J=1,NVMAX
      OBF(J)=0.0
      DO 10 I=1,NCONS
   10 AA(I,J)=0.0
      DO 20 I=1,NCONS
   20 READ (5,1020) (AA(I,J),J=1,NVARS)
 1020 FORMAT (12F6.2)
      READ (5,1020) (OBF(I),I=1,NVARS)
      DO 22 I=1,NRMAX
```

```
      PCVAL(I)=0.0
   22 RVAL(I)=0.0
      READ (5,1030) RMIN,RMAX,RINT
 1030 FORMAT (2F6.2,F6.3)
      IF (RMIN.LT.RMAX) GO TO 25
      WRITE (6,4020)
 4020 FORMAT (23H RMIN GREATER THAN RMAX)
      GO TO 999
   25 IF (RMIN.GT.5.999.AND.RMAX.LT.15.001) GO TO 27
      WRITE (6,4030)
 4030 FORMAT (50H RMIN OR RMAX OUTSIDE PERMITTED RANGE : 6.00-15.00)
      GO TO 999
   27 NRVAL=0
      R=RMIN-RINT
   30 NRVAL=NRVAL+1
      IF (NRVAL.LE.NRMAX) GO TO 32
      WRITE (6,4040) NRMAX
 4040 FORMAT (37H TOO MANY VALUES OF R IMPLIED,MAXIMUM,I4)
      GO TO 999
   32 R=R+RINT
      RVAL(NRVAL)=R
      IF (RMAX-R.GT.0.0001) GO TO 30
      RVAL(NRVAL)=RMAX
      READ (5,1040) NLWVAL
 1040 FORMAT (I3)
      IF (NLWVAL.LE.NLWMAX.AND.NLWVAL.GT.1) GO TO 35
      WRITE (6,4050) NLWMAX
 4050 FORMAT (58H NO. OF VALUES OF LIVEWEIGHT OUTSIDE PERMITTED RANGE :
     1 2-,I3)
      GO TO 999
   35 DO 40 I=1,NLWMAX
   40 LWVAL(I)=0
      READ (5,1050) (LWVAL(I),I=1,NLWVAL)
 1050 FORMAT (18I4)
      READ (5,1040) NGVAL
      IF (NGVAL.LE.NGMAX.AND.NGVAL.GT.1) GO TO 45
      WRITE (6,4060) NGMAX
 4060 FORMAT (56H NO. OF VALUES OF LW. GAIN OUTSIDE PERMITTED RANGE :  2
     1-,I3)
      GO TO 999
   45 DO 50 I=1,NGMAX
      DCP(I)=0.0
      DCPINT(I)=0.0
   50 GVAL(I)=0.0
      READ (5,1060) (GVAL(I),I=1,NGVAL)
 1060 FORMAT (11F5.2)
      DO 60 I=1,NLWMAX
      DMAVAL(I)=0.0
      DO 60 J=1,NGMAX
   60 DCPVAL(I,J)=0.0
      READ (5,1020) (DMAVAL(I),I=1,NLWVAL)
      DO 70 I=1,NLWVAL
   70 READ (5,1020) (DCPVAL(I,J),J=1,NGVAL)
      READ (5,1070) PERCOS
 1070 FORMAT (F7.2)
      IF (LWMIN.GE.LWVAL(1).AND.LWMIN.LT.LWVAL(NLWVAL)) GO TO 72
      WRITE (6,3010) LWVAL(1),LWVAL(NLWVAL)
 3010 FORMAT (45H MINIMUM LIVEWEIGHT OUTSIDE PERMITTED RANGE :,
```

```
     1I5,2H -,I5)
      GO TO 999
   72 IF (LWMAX.LE.LWVAL(NLWVAL)) GO TO 74
      WRITE (6,3020) LWVAL(1),LWVAL(NLWVAL)
 3020 FORMAT (45H MAXIMUM LIVEWEIGHT OUTSIDE PERMITTED RANGE :,
     1I5,2H -,I5)
      GO TO 999
   74 DO 80 I=1,NGMAX
      COSMIN(I)=0.0
      QFED(I)=0.0
      DO 80 J=1,NCONS
      COMP(I,J)=0.0
   80 LCINGR(I,J)=0
      NDAYS=1.0/GVAL(2)+0.1
      WRITE (7'1) LWMIN,LWMAX,LWINCR,NGVAL,NDAYS,PERCOS
      WRITE (6,2110) (GVAL(I),I=1,NGVAL)
 2110 FORMAT (1H1,5H LIVE,2X,7H RATION,17X,24H DAILY  LIVEWEIGHT  GAIN/
     17H WEIGHT,2X,5H SPEC,27X,5H (KG)/6H  (KG),9X,11F8.2)
C
C  SET INITIAL VALUES FOR DMA,DMAINT,DCP,DCPINT AND YME
C
      IF (LWVAL(1).EQ.LWMIN) GO TO 250
      DO 210 I=2,NLWVAL
      IF (LWVAL(I).LT.LWMIN) GO TO 210
      K1=I-1
      GO TO 230
  210 CONTINUE
  230 K2=K1+1
      LWUP=LWVAL(K2)
      LWDIF=LWUP-LWVAL(K1)
      WINCS=LWMIN-LWVAL(K1)
      WINCS=WINCS/LWINCR
      DMAINT=((DMAVAL(K2)-DMAVAL(K1))/LWDIF)*LWINCR
      DMA=DMAVAL(K1)+DMAINT*(WINCS-1.0)
      DO 240 I=1,NGVAL
      DCPINT(I)=((DCPVAL(K2,I)-DCPVAL(K1,I))/LWDIF)*LWINCR
  240 DCP(I)=DCPVAL(K1,I)+DCPINT(I)*(WINCS-1.0)
      GO TO 270
  250 DMAINT=0.0
      DMA=DMAVAL(1)
      DO 260 I=1,NGVAL
      DCPINT(I)=0.0
  260 DCP(I)=DCPVAL(1,I)
      K2=1
      LWUP=LWMIN
  270 YINC=1.39*0.061*LWINCR
      YME=1.39*(5.67+0.061*LWMIN)-YINC
      ZINC=23.0*0.0188*LWINCR
      ZMENUM=23.0*(6.28+0.0188*LWMIN)-ZINC
      IPOS=1
C
C  ITERATE FOR EACH LIVEWEIGHT
C
      DO 600 LW=LWMIN,LWMAX,LWINCR
      IF (LW.LE.LWUP) GO TO 330
      K1=K2
      K2=K1+1
      LWUP=LWVAL(K2)
```

```
      LWDIF=LWUP-LWVAL(K1)
      WINCS=LW-LWVAL(K1)
      WINCS=WINCS/LWINCR
      DMAINT=((DMAVAL(K2)-DMAVAL(K1))/LWDIF)*LWINCR
      DMA=DMAVAL(K1)+DMAINT*WINCS
      DO 320 I=1,NGVAL
      DCPINT(I)=((DCPVAL(K2,I)-DCPVAL(K1,I))/LWDIF)*LWINCR
  320 DCP(I)=DCPVAL(K1,I)+DCPINT(I)*WINCS
      GO TO 350
  330 DMA=DMA+DMAINT
      DO 340 I=1,NGVAL
  340 DCP(I)=DCP(I)+DCPINT(I)
  350 YME=YME+YINC
      ZMENUM=ZMENUM+ZINC
C
C   ITERATE FOR EACH LIVEWEIGHT GAIN
C
      DO 500 IG=1,NGVAL
      ZME=(ZMENUM*GVAL(IG))/(1.0-0.3*GVAL(IG))
      LR=0
      DO 420 IR=1,NRVAL
      R=RVAL(IR)
      TME=YME+ZME/R
      DMI=TME/R
      IF (DMI.GT.DMA) GO TO 420
      PCVAL(IR)=DCP(IG)/DMI
      IF (LR.GT.0) GO TO 420
      LR=IR-1
      IF (LR.GT.0) GO TO 410
      LR=1
      RMIN=RVAL(1)
      GO TO 420
  410 RMIN=(YME+SQRT(YME*YME+4*DMA*ZME))/(2.0*DMA)
      PCVAL(LR)=(DCP(IG)*RVAL(LR))/(YME+ZME/RVAL(LR))
  420 CONTINUE
      IF (LR.GT.0) GO TO 440
      COSMIN(IG)=99999.0
      QFED(IG)=0.0
      DO 430 I=1,NCONS
      LCINGR(IG,I)=0
  430 COMP(IG,I)=0.0
      GO TO 500
  440 CALL LCFEED(LR,RMIN,RVAL,PCVAL,YME,ZME,COSTL,Q,INGS,XB)
      COSMIN(IG)=COSTL
      QFED(IG)=Q
      DO 450 I=1,NCONS
      COMP(IG,I)=XB(I)
  450 LCINGR(IG,I)=INGS(I)
  500 CONTINUE
      IPOS=IPOS+1
      WRITE (7'IPOS) COSMIN,QFED,LCINGR,COMP
      DO 510 I=1,NGVAL
  510 IF (COSMIN(I).GT.99990.0) COSMIN(I)=0.0
      WRITE (6,2120) LW,(LCINGR(I,1),COMP(I,1),I=1,NGVAL)
 2120 FORMAT (/I5,11H  FOOD NO,%,11(I3,F5.1))
      DO 520 J=2,NCONS
  520 WRITE (6,2130) (LCINGR(I,J),COMP(I,J),I=1,NGVAL)
 2130 FORMAT (6X,10H FOOD NO,%,11(I3,F5.1))
```

```
      WRITE (6,2140) (QFED(I),I=1,NGVAL)
 2140 FORMAT (6X,10H QTY  (KG),11(1X,F7.2))
      WRITE (6,2150) (COSMIN(I),I=1,NGVAL)
 2150 FORMAT (6X,10H COST  (P),11(1X,F7.2))
  600 CONTINUE
  999 STOP
      END
C
C
C
C
C
      SUBROUTINE LCFEED(LR,RMIN,RVAL,PCVAL,YME,ZME,COSTL,Q,INGS,XB)
      DIMENSION AA(3,50),OBF(50),RHS(3),XB(3),DV(3),BETA(3,3),
     1RANVAL(200,7),RVAL(161),PCVAL(161),A(5,55),ID(3),INGRED(200,3),
     2INGS(3),BASINV(200,3,3),XBNIT(3),INGNIT(3)
      COMMON/LPCOM/NCONS,NVARS,NGTS,NG,NGL,NLE,NL1,IC,NC
      COMMON/FECOM/AA,OBF,NRVAL,RMAX,NC1,EPSI
      MAXNIT=200
      NITS=0
      COSNIT=99999.0
      QNIT=0.0
      NITIND=0
      DO 80 I=1,NCONS
      XBNIT(I)=0.0
   80 INGNIT(I)=0
      DO 90 I=1,MAXNIT
      DO 90 J=1,7
   90 RANVAL(I,J)=0.0
      DO 95 I=1,MAXNIT
      DO 95 J=1,NCONS
      INGRED(I,J)=0
      DO 95 K=1,NCONS
   95 BASINV(I,J,K)=0.0
      RHS(1)=1.0
      ICRN=1
      NFSNO=0
      INTNO=LR
      INTNO1=INTNO+1
      R=RMIN
      GO TO 105
  100 R=RVAL(INTNO)
  105 RU=RVAL(INTNO1)
      PCONC=PCVAL(INTNO)
      PCONCU=PCVAL(INTNO1)
      DU=RU-R
      PCGRAD=(PCONCU-PCONC)/(RU-RVAL(INTNO))
      IF (INTNO.EQ.LR) PCONC=PCONCU-PCGRAD*DU
      DELTAU=DU
      DELTAL=0.0
C
C  SET UP AND SOLVE L.P.
C
  110 RHS(2)=R
      RHS(3)=PCONC
      CALL LPSUB(AA,OBF,RHS,A,ID,NFSIND)
      DO 120 I=1,NCONS
  120 XB(I)=A(I,IC)
```

```
      DO 125 I=1,NCONS
      BETA(I,1)=A(I,NG+1)
      BETA(I,2)=A(I,NG+2)
  125 BETA(I,3)=-A(I,NG)
      IF (NFSIND.EQ.1) GO TO 200
      DV(1)=-A(NC,NG+1)
      DV(2)=-A(NC,NG+2)
      DV(3)=A(NC,NG)
C
C  DETERMINE RANGE OF SOLUTION IN CURRENT INTERVAL OR PART INTERVAL
C
  130 DO 140 I=1,NCONS
      WV1=BETA(I,2)+PCGRAD*BETA(I,3)
      IF (WV1.GT.-1.0E-8) GO TO 135
      DELTA=-XB(I)/WV1
      IF (DELTA-DELTAU.LT.-1.0E-8) DELTAU=DELTA
      GO TO 140
  135 IF (WV1.LT.1.0E-8) GO TO 140
      DELTA=-XB(I)/WV1
      IF (DELTA-DELTAL.GT.1.0E-8) DELTAL=DELTA
  140 CONTINUE
      DO 150 I=1,NCONS
      DO 145 J=1,NCONS
  145 BASINV(ICRN,I,J)=BETA(I,J)
      INGRED(ICRN,I)=ID(I)
      IF (ID(I).GT.NVARS) INGRED(ICRN,I)=0
  150 CONTINUE
      DO 155 I=1,NCONS
  155 RANVAL(ICRN,I)=DV(I)
      RANVAL(ICRN,4)=R+DELTAL
      RANVAL(ICRN,5)=PCONC-PCGRAD*R
      RANVAL(ICRN,6)=PCGRAD
      RANVAL(ICRN,7)=R+DELTAU
      ICRN=ICRN+1
      IF (ICRN.LE.MAXNIT) GO TO 165
      NITS=NITS+1
      NITIND=1
      IF (DU-DELTAU.LT.0.0001) GO TO 160
      RNIT=R+DELTAU+EPSI
      IF (RMAX-RNIT.LT.0.0001) NITIND=0
      R=RNIT
      DELTAL=-EPSI
      GO TO 300
  160 INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) NITIND=0
      RNIT=RVAL(INTNO)
      R=RNIT
      DELTAL=0.0
      GO TO 300
  165 IF (DU-DELTAU.LT.0.0001) GO TO 180
      R=R+DELTAU+EPSI
      IF (RMAX-R.LT.0.0001) GO TO 300
      DU=RU-R
      IF (DU.GT.0.0001) GO TO 170
      INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) GO TO 300
      INTNO1=INTNO+1
      GO TO 100
```

```
  170 DELTAU=DU
      DELTAL=-EPSI
      PCONC=PCONCU-PCGRAD*DU
      GO TO 110
C
C  NEW RANGE FOR R. CALCULATE NEW SOLUTION
C
  180 INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) GO TO 300
      INTNO1=INTNO+1
      WV1=DELTAU
      WV2=PCGRAD
      R=RVAL(INTNO)
      RU=RVAL(INTNO1)
      PCONC=PCVAL(INTNO)
      PCONCU=PCVAL(INTNO1)
      DU=RU-R
      PCGRAD=(PCONCU-PCONC)/DU
      DELTAU=DU
      DELTAL=0.0
      DO 190 I=1,NCONS
      XB(I)=XB(I)+(BETA(I,2)+WV2*BETA(I,3))*WV1
      IF (XB(I).LT.-1.0E-8) GO TO 110
      IF (ABS(XB(I)).LT.1.0E-8) XB(I)=0.0
  190 CONTINUE
      GO TO 130
C
C  DETERMINE RANGE OF INFEASIBILITY
C
  200 NFSNO=0
  210 NFSNO=NFSNO+1
      ITYPE=0
      DO 220 I=1,NCONS
      WV1=BETA(I,2)+PCGRAD*BETA(I,3)
      IF (WV1.GT.-1.0E-8) GO TO 215
      DELTA=-XB(I)/WV1
      IF (DELTA-DELTAU.LT.-1.0E-8) DELTAU=DELTA
      GO TO 220
  215 IF (WV1.LT.1.0E-8) GO TO 220
      DELTA=-XB(I)/WV1
      IF (DELTA-DELTAU.GT.1.0E-8) DELTAL=DELTA
  220 CONTINUE
      IF (NFSNO.GT.1) GO TO 250
      DO 230 I=1,6
  230 RANVAL(ICRN,I)=-10000.0
      RANVAL(ICRN,4)=R+DELTAL
      DO 240 I=1,NCONS
  240 INGRED(ICRN,I)=0
  250 R=R+DELTAU+EPSI
      IF (DU-DELTAU.GT.0.0001) GO TO 265
      ITYPE=1
      INTNO=INTNO+1
      R=RVAL(INTNO)
      IF (INTNO.LT.NRVAL) GO TO 255
      RANVAL(ICRN,7)=R
      ICRN=ICRN+1
      GO TO 300
  255 INTNO1=INTNO+1
```

```
      WV1=DELTAU
      WV2=PCGRAD
      RU=RVAL(INTNO1)
      PCONC=PCVAL(INTNO)
      PCONCU=PCVAL(INTNO1)
      DU=RU-R
      PCGRAD=(PCONCU-PCONC)/DU
      DELTAU=DU
      DELTAL=0.0
      DO 260 I=1,NCONS
      XB(I)=XB(I)+(BETA(I,2)+WV2*BETA(I,3))*WV1
      IF (XB(I).LT.-1.0E-8) GO TO 270
      IF (ABS(XB(I)).LT.1.0E-8) XB(I)=0.0
  260 CONTINUE
      GO TO 210
  265 RANVAL(ICRN,7)=R-EPSI
      GO TO 275
  270 RANVAL(ICRN,7)=R
  275 ICRN=ICRN+1
      IF (ICRN.GT.MAXNIT) GO TO 290
      IF (RMAX-R.LT.0.0001) GO TO 300
      DU=RU-R
      IF (DU.LT.0.0001) GO TO 280
      DELTAU=DU
      DELTAL=-EPSI
      PCONC=PCONCU-PCGRAD*DU
      GO TO 110
  280 IF (ITYPE.EQ.0) INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) GO TO 300
      INTNO1=INTNO+1
      IF (ITYPE.EQ.0) GO TO 100
      DELTAL=0.0
      GO TO 110
  290 NITS=NITS+1
      NITIND=1
      IF (DU-DELTAU.LT.0.0001) GO TO 295
      RNIT=R+DELTAU+EPSI
      IF (RMAX-RNIT.LT.0.0001) NITIND=0
      R=RNIT
      DELTAL=-EPSI
      GO TO 300
  295 IF (ITYPE.EQ.0) INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) NITIND=0
      INTNO1=INTNO+1
      RNIT=RVAL(INTNO)
      R=RNIT
      DELTAL=0.0
      GO TO 300
C
C  *****  DETERMINATION OF LEAST COST FEED  *****
C
  300 NCRN=ICRN-1
      RANVAL(ICRN,4)=R
      DO 350 ICRN=1,NCRN
      RL=RANVAL(ICRN,4)
      RU=RANVAL(ICRN,7)
      IF (RANVAL(ICRN,1).LT.-9999.0) GO TO 330
      DVAR1=RANVAL(ICRN,1)
```

```
      DVAR2=RANVAL(ICRN,2)
      DVAR3=RANVAL(ICRN,3)
      WV1=DVAR1+DVAR3*RANVAL(ICRN,5)
      WV2=DVAR2+DVAR3*RANVAL(ICRN,6)
      A1=WV2*YME
      A2=WV1*YME+WV2*ZME
      A3=WV1*ZME
C
C  DETERMINE TURNING POINT OF COST FUNCTION
C
      RTP=-1.0
      IF (A3.LT.1.0E-8) GO TO 310
      IF (A2.GT.-1.0E-8) GO TO 310
      RTP=-2*A3/A2
C
C  DETERMINATION OF MINIMUM COST FOR RANGE
C
  310 COSTL=A1+(A2+A3/RL)/RL
      RV=RL
      COST=A1+(A2+A3/RU)/RU
      IF (COST-COSTL.GT.-1.0E-8) GO TO 320
      COSTL=COST
      RV=RU
  320 IF (RTP.LT.RL.OR.RTP.GT.RU) GO TO 340
      COST=A1+(A2+A3/RTP)/RTP
      IF (COST-COSTL.GT.-1.0E-8) GO TO 340
      COSTL=COST
      RV=RTP
      GO TO 340
  330 COSTL=99999.0
      RV=99999.0
  340 RANVAL(ICRN,1)=COSTL
  350 RANVAL(ICRN,2)=RV
C
C  DETERMINATION OF OVERALL MINIMUM COST
C
      COSTL=RANVAL(1,1)
      RV=RANVAL(1,2)
      K=1
      IF (NCRN.EQ.1) GO TO 410
      DO 400 ICRN=2,NCRN
      IF (RANVAL(ICRN,1)-COSTL.GT.-1.0E-8) GO TO 400
      COSTL=RANVAL(ICRN,1)
      RV=RANVAL(ICRN,2)
      K=ICRN
  400 CONTINUE
  410 Q=0
      DO 420 I=1,NCONS
      XB(I)=0.0
  420 INGS(I)=0
      IF (COSTL.GT.99990.0) GO TO 500
      Q=(YME+ZME/RV)/RV
      RHS(2)=RV
      RHS(3)=RANVAL(K,5)+RV*RANVAL(K,6)
      DO 440 I=1,NCONS
      INGS(I)=INGRED(K,I)
      IF (INGS(I).EQ.0) GO TO 440
      DO 430 J=1,NCONS
```

```
  430 XB(I)=XB(I)+RHS(J)*BASINV(K,I,J)
      XB(I)=XB(I)*100.0
      IF (XB(I).GT.0.1) GO TO 440
      XB(I)=0.0
      INGS(I)=0
  440 CONTINUE
      DO 450 I=1,NCONS
      DV(I)=XB(I)
  450 INGRED(1,I)=INGS(I)
      L=3
      DO 480 I=1,NCONS
      IF (INGS(I).GT.0) GO TO 460
      ID(I)=L
      L=L-1
      GO TO 480
  460 K=1
      DO 470 J=1,NCONS
      IF (J.EQ.I) GO TO 470
      IF (INGS(J).GT.INGS(I)) K=K+1
  470 CONTINUE
      ID(I)=K
  480 CONTINUE
      DO 490 I=1,NCONS
      XB(ID(I))=DV(I)
  490 INGS(ID(I))=INGRED(1,I)
C
C  CHECK WHETHER ENTIRE E.C. RANGE COVERED IN ONE ITERATION
C
  500 IF (NITS.EQ.0) GO TO 600
      IF (COSTL.GT.COSNIT) GO TO 520
      COSNIT=COSTL
      QNIT=Q
      DO 510 I=1,NCONS
      XBNIT(I)=XB(I)
  510 INGNIT(I)=INGS(I)
  520 IF (NCRN.EQ.MAXNIT.AND.NITIND.EQ.1) GO TO 540
  525 DO 530 I=1,NCONS
      XB(I)=XBNIT(I)
  530 INGS(I)=INGNIT(I)
      COSTL=COSNIT
      Q=QNIT
      GO TO 600
  540 ICRN=1
      NFSNO=0
      DO 550 I=1,MAXNIT
      DO 550 J=1,7
  550 RANVAL(I,J)=0.0
      DO 560 I=1,MAXNIT
      DO 560 J=1,NCONS
      INGRED(I,J)=0
      DO 560 K=1,NCONS
  560 BASINV(I,J,K)=0.0
      R=RNIT
      RU=RVAL(INTNO1)
      DU=RU-R
      IF (DU.GT.0.0001) GO TO 570
      INTNO=INTNO+1
      IF (INTNO.NE.NRVAL) GO TO 100
```

```
      GO TO 525
  570 PCONC=PCONCU-PCGRAD*DU
      DELTAU=DU
      GO TO 110
  600 RETURN
      END
C
C
C
C
C
      SUBROUTINE LPSUB(AA,C,RHS,A,ID,NFSIND)
      COMMON/LPCOM/M,N,NGTS,NG,NGL,NLE,L1,IC,M1
      DIMENSION AA(3,50),C(50),A(5,55),RHS(3),ID(3)
      IW=M+2
      NCMAX=55
C
C  SET UP LP MATRIX
C
      DO 5 I=1,IW
      DO 5 J=1,NCMAX
    5 A(I,J)=0.0
      DO 10 J=1,N
      A(M1,J)=C(J)
      DO 10 I=1,M
   10 A(I,J)=AA(I,J)
      DO 15 I=1,M
      A(I,IC)=RHS(I)
   15 A(I,NG+I)=1.0
      DO 20 I=1,NGTS
   20 A(NLE+I,N+I)=-1.0
      DO 25 I=1,M
   25 ID(I)=NG+I
C
C  SET UP PHASE I AND FIND PIVOTAL COLUMN
C
      KC=0
      Q=0.0
      DO 40 J=1,NG
      S=0.0
      DO 30 I=L1,M
   30 S=S-A(I,J)
      A(IW,J)=S
      IF(S.GE.Q) GO TO 40
      Q=S
      KC=J
   40 CONTINUE
      S=0.0
      DO 50 I=L1,M
   50 S=S-A(I,IC)
      A(IW,IC)=S
      IF(KC.GT.0) GO TO 120
      IW=IW-1
C
C  SIMPLEX ITERATION
C
  100 KC=0
      Q=0.0
```

```
      DO 110 J=1,NGL
      IF (A(IW,J).GE.Q) GO TO 110
      Q=A(IW,J)
      KC=J
  110 CONTINUE
      IF (KC.EQ.0) GO TO 200
  120 KR=0
      Q=1.0E70
      DO 130 I=1,M
      IF(A(I,KC).LE.1.0E-8) GO TO 130
      P=A(I,IC)/A(I,KC)
      IF(P.GT.Q) GO TO 130
      Q=P
      KR=I
  130 CONTINUE
      IF(KR.EQ.0) GO TO 250
      P=A(KR,KC)
      ID(KR)=KC
      DO 140 J=1,IC
  140 A(KR,J)=A(KR,J)/P
      DO 160 I=1,IW
      IF(I.EQ.KR) GO TO 160
      DO 150 J=1,IC
      IF(J.EQ.KC) GO TO 150
      A(I,J)=A(I,J)-A(KR,J)*A(I,KC)
      Q=ABS(A(I,J))
      IF(Q.LT.1.0E-8) A(I,J)=0.0
  150 CONTINUE
  160 CONTINUE
      DO 170 I=1,IW
  170 A(I,KC)=0.0
      A(KR,KC)=1.0
      GO TO 100
C
C  CHECK FOR PHASE II COMPLETION AND FEASIBILITY
C
  200 IF (IW.EQ.M+1) GO TO 210
      IW=IW-1
      GO TO 100
  210 DO 220 I=1,M
      IF (ID(I).GT.NGL) GO TO 230
  220 CONTINUE
      NFSIND=0
      GO TO 999
  230 NFSIND=1
      GO TO 999
  250 WRITE (6,2000)
 2000 FORMAT (19H SOLUTION UNBOUNBED)
      STOP
  999 RETURN
      END
```

APPENDIX B2

BEEF CATTLE DP MODEL SOLUTION PROGRAM - MAFF STANDARDS

```
      DEFINE FILE 7(502,352,L,IAV7),11(64,4008,L,IAV11),
     112(64,4008,L,IAV12)
      DIMENSION FEEDC(501,11),GVAL(11),COSINC(11),COST1(501),COST2(501),
     1IPOL(501),IGROW(501),COSMIN(11),QFED(11),LCINGR(11,3),COMP(11,3)
      NRECS=64
      IAV7=1
      IAV11=1
      IAV12=1
      MAXNLW=501
      NGMAX=11
      READ (7'1) LWMIN,LWMAX,LWINCR,NGVAL,NDAYS,PERCOS
      DO 50 I=1,MAXNLW
      IGROW(I)=0
      DO 50 J=1,10
   50 FEEDC(I,J)=-9999.9
      NLW=LWMAX-LWMIN+1
      NDL1=NDAYS-1
      IPOS=1
      DO 60 LW=1,NLW,LWINCR
      IPOS=IPOS+1
      READ (7'IPOS) COSMIN,QFED,LCINGR,COMP
      DO 60 IG=1,NGMAX
   60 FEEDC(LW,IG)=COSMIN(IG)
      IF (LWINCR.EQ.1) GO TO 100
      N=NLW-LWINCR
      DO 90 LW=1,N,LWINCR
      L2=LW+LWINCR
      DO 70 IG=1,NGVAL
      WV1=FEEDC(LW,IG)
      IF (WV1.GT.99990.0) FEEDC(LW,IG)=-9999.9
      WV2=FEEDC(L2,IG)
      IF (WV2.GT.99990.0) FEEDC(L2,IG)=-9999.9
      IF (FEEDC(LW,IG).LT.-9990.0) GO TO 65
      IF (FEEDC(L2,IG).LT.-9990.0) GO TO 65
      COSINC(IG)=(WV2-WV1)/LWINCR
      GO TO 70
   65 COSINC(IG)=-9999.9
   70 CONTINUE
      L1=LW+1
      DO 80 L=L1,L2
      I=L-1
      DO 75 IG=1,NGVAL
      IF (COSINC(IG).LT.-9990.0) GO TO 75
      FEEDC(L,IG)=FEEDC(I,IG)+COSINC(IG)
   75 CONTINUE
   80 CONTINUE
```

```
   90 CONTINUE
  100 DO 250 LW=1,NLW
      LWG=0
      FEEDC(LW,1)=NDAYS*FEEDC(LW,1)
      DO 200 IG=2,NGVAL
      IF (FEEDC(LW,IG).LT.0.0) GO TO 200
      LWG=LWG+1
      IF (LW+LWG.LE.NLW) GO TO 110
      FEEDC(LW,IG)=-9999.9
      GO TO 200
  110 L1=LW
      L2=L1+1
      IF (FEEDC(L2,IG).GT.0.0) GO TO 120
      FEEDC(LW,IG)=-9999.9
      GO TO 200
  120 NINC=NDL1*LWG
      L=1
      CINCR=FEEDC(L2,IG)-FEEDC(L1,IG)
      COST=NDAYS*FEEDC(L1,IG)
      DO 150 J=LWG,NINC,LWG
      K=J/NDAYS
      IF (K.LT.L) GO TO 140
      L=K+1
      L1=LW+K
      L2=L1+1
      IF (FEEDC(L2,IG).GT.0.0) GO TO 130
      FEEDC(LW,IG)=-9999.9
      GO TO 200
  130 CINCR=FEEDC(L2,IG)-FEEDC(L1,IG)
  140 K=J-K*NDAYS
      COST=COST+NDAYS*FEEDC(L1,IG)+K*CINCR
  150 CONTINUE
      FEEDC(LW,IG)=COST/NDAYS
  200 CONTINUE
  250 CONTINUE
      DO 310 I=1,MAXNLW
      IPOL(I)=0
  310 COST2(I)=9999999.0
      COST2(1)=0.0
      NPERS=LWMAX-LWMIN
      L1MIN=1
      L1MAX=1
      ICHAN=11
      IPOS=0
      DO 500 N=1,NPERS
      NPREQ=N
      DO 320 I=1,MAXNLW
      IPOL(I)=0
  320 COST1(I)=COST2(I)
      DO 400 L1=L1MIN,L1MAX
      COSTA=COST1(L1)
      IF (COSTA.GT.9999990.0) GO TO 400
      L2=L1
      DO 350 IG=2,NGVAL
      L2=L2+1
      IF (FEEDC(L1,IG).LT.0.0) GO TO 350
      COST=COSTA+PERCOS+FEEDC(L1,IG)
      IF (COST-COST2(L2).GT.-0.0001) GO TO 350
```

```
      COST2(L2)=COST
      IPOL(L2)=L1
  350 CONTINUE
  400 CONTINUE
      L1MIN=N
      L1MAX=L1MAX+NGVAL-1
      IF (L1MAX.GT.NLW) L1MAX=NLW
      DO 410 I=1,NLW
      IF (IPOL(I).EQ.0) GO TO 410
      IPOL(I)=IPOL(I)
  410 CONTINUE
      IF (IPOS.LT.NRECS) GO TO 420
      ICHAN=ICHAN+1
      IPOS=0
  420 IPOS=IPOS+1
      WRITE (ICHAN'IPOS) COST2,IPOL
      DO 460 I=2,NLW
      IF (IPOL(I).GT.0) GO TO 500
  460 CONTINUE
      GO TO 510
  500 CONTINUE
  510 IF (NPREQ.NE.NPERS) GO TO 530
      NPREQ=NPREQ+1
      IPOS=IPOS+1
      IF (IPOS.LE.NRECS) GO TO 520
      ICHAN=ICHAN+1
      IPOS=0
  520 IPOS=IPOS+1
  530 IGROW(NPREQ)=LWMAX
      INEXT=NLW
      NPREQ=NPREQ-1
      DO 600 N=1,NPREQ
      IPER=NPREQ-N+1
      IPOS=IPOS-1
      IF (IPOS.GT.0) GO TO 550
      ICHAN=ICHAN-1
      IPOS=NRECS
  550 READ (ICHAN'IPOS) COST2,IPOL
      COST1(IPER)=COST2(INEXT)
      INEXT=IPOL(INEXT)
      IGROW(IPER)=INEXT+LWMIN-1
      IF (INEXT.EQ.0) GO TO 610
  600 CONTINUE
  610 IF (IPER.EQ.1) GO TO 630
      NPREQ=NPREQ-IPER+1
      DO 620 N=1,NPREQ
      IPER1=N+IPER-1
      COST1(N)=COST1(IPER1)
  620 IGROW(N)=IGROW(IPER1)
      IGROW(NPREQ+1)=LWMAX
  630 IF (LWINCR.EQ.1) GO TO 650
      WRITE (6,2110)
 2110 FORMAT (1H1,5X,21H OPTIMAL FEEDING PLAN///31H PERIOD  LIVEWEIGHT
     1CUMULATIVE/7H NUMBER,4X,5H (KG),7X,5H COST/8X,11H START  END,4X,4H
     2 (P))
      DO 640 N=1,NPREQ
  640 WRITE (6,2120) N,IGROW(N),IGROW(N+1),COST1(N)
 2120 FORMAT (I5,2X,2I6,F10.1)
```

```
      GO TO 999
  650 WRITE (6,2210)
 2210 FORMAT (1H1,25X,21H OPTIMAL FEEDING PLAN///71H PERIOD  LIVEWEIGHT
     1 CUMULATIVE  RATION REQUIRED ON FIRST DAY OF PERIOD/7H NUMBER,4X,
     25H (KG),7X,5H COST,7X,23H COMPOSITION (DM BASIS),4X,9H QUANTITY/
     38X,11H START  END,4X,4H (P),5X,3(10H FOOD   % ),2X,5H (KG)/)
      DO 660 N=1,NPREQ
      K=IGROW(N)
      L=IGROW(N+1)
      IPOS=K-LWMIN+2
      IG=L-K+1
      READ (7'IPOS) COSMIN,QFED,LCINGR,COMP
  660 WRITE (6,2220) N,K,L,COST1(N),(LCINGR(IG,I),COMP(IG,I),I=1,3)
     1,QFED(IG)
 2220 FORMAT (I5,2X,2I6,F10.1,3X,3(I4,F6.1),F7.2)
  999 STOP
      END
```

APPENDIX B3

BEEF CATTLE RATION FORMULATION PROGRAM - NRC STANDARDS

```
      DEFINE FILE 7(502,484,L,IAV7),8(502,484,L,IAV8)
      DIMENSION AA(7,20),A(8,130),ID(6),LWVAL(20),DMAVAL(20),
     1DMCVAL(20),GVAL(11),DCPVAL(20,11),DCP(11),DCPINT(11),COSMIN(11),
     2QFED(11),SOL(6),INGS(6),RHS(6),LCINGR(11,6),COMP(11,6),
     3BETA(11,6,6),FVAL(101),FINVAL(101),GINVAL(101),NIC(11),IDB(11,6),
     4B(11,7,130),C(130),RH4(11),NFL(11),NFU(11)
      INTEGER*2 LCINGR
      COMMON/LPCOM/NCONS,NL1
      IAV7=1
      IAV8=1
      NGMAX=11
      NVMAX=20
      NLWMAX=20
      NFMAX=101
      NCONS=6
      NLTS=1
      NEQS=3
      NGTS=2
      NC=NCONS+1
      NC1=NC+1
      NL1=NLTS+1
      NLE=NLTS+NEQS
      READ (5,1010) NVARS,LWMIN,LWMAX,LWINCR
 1010 FORMAT (4I4)
      IF (NVARS.LT.NVMAX) GO TO 3
      WRITE (6,4005) NVMAX
 4005 FORMAT (25H TOO MANY FEEDSTUFFS.MAX ,I4)
      GO TO 999
    3 IF (LWMAX.GT.LWMIN) GO TO 5
      WRITE (6,4010)
 4010 FORMAT (33H MAX LIVEWEIGHT LE MIN LIVEWEIGHT)
      GO TO 999
    5 I=LWMAX-LWMIN
      J=I/LWINCR
      K=J*LWINCR
      IF (I-K.EQ.0) GO TO 10
      WRITE (6,4015)
 4015 FORMAT (68H DIFFERENCE BETWEEN MAX LW AND MIN LW NOT A MULTIPLE OF
     1 LW INCREMENT)
      GO TO 999
   10 NTOT=NVMAX+NFMAX
      NTOT1=NTOT+1
      NVARS1=NVARS+1
      NCOLS=NTOT+NC+NGTS
      DO 15 I=1,NC
      DO 15 J=1,NVMAX
```

```
   15 AA(I,J)=0.0
      DO 20 I=2,3
   20 READ (5,1020) (AA(I,J),J=1,NVARS)
 1020 FORMAT (12F6.2)
      READ (5,1020) (AA(5,J),J=1,NVARS)
      DO 25 J=1,NVARS
      AA(1,J)=1.0
   25 AA(6,J)=1.0
      DO 30 J=1,NCOLS
   30 C(J)=0.0
      READ (5,1020) (C(J),J=1,NVARS)
      READ (5,1030) NLWVAL
 1030 FORMAT (I3)
      IF (NLWVAL.LE.NLWMAX.AND.NLWVAL.GT.1) GO TO 35
      WRITE (6,4030) NLWMAX
 4030 FORMAT (58H NO. OF VALUES OF LIVEWEIGHT OUTSIDE PERMITTED RANGE :
     1 2-,I3)
      GO TO 999
   35 DO 40 I=1,NLWMAX
   40 LWVAL(I)=0
      READ (5,1040) (LWVAL(I),I=1,NLWVAL)
 1040 FORMAT (18I4)
      READ (5,1030) NGVAL
      IF (NGVAL.LE.NGMAX.AND.NGVAL.GT.1) GO TO 45
      WRITE (6,4040) NGMAX
 4040 FORMAT (56H NO. OF VALUES OF LW. GAIN OUTSIDE PERMITTED RANGE :  2
     1-,I3)
      GO TO 999
   45 DO 50 I=1,NGMAX
      DCP(I)=0.0
      DCPINT(I)=0.0
   50 GVAL(I)=0.0
      READ (5,1050) (GVAL(I),I=1,NGVAL)
 1050 FORMAT (10F5.2)
      DO 60 I=1,NLWMAX
      DMAVAL(I)=0.0
      DMCVAL(I)=0.0
      DO 60 J=1,NGMAX
   60 DCPVAL(I,J)=0.0
      READ (5,1020) (DMAVAL(I),I=1,NLWVAL)
      READ (5,1020) (DMCVAL(I),I=1,NLWVAL)
      DO 70 I=1,NLWVAL
   70 READ (5,1020) (DCPVAL(I,J),J=1,NGVAL)
      READ (5,1060) PERCOS
 1060 FORMAT (F7.2)
      READ (5,1080) FMIN,FMAX,FINT
 1080 FORMAT (3F6.3)
      IF (LWMIN.GE.LWVAL(1).AND.LWMIN.LT.LWVAL(NLWVAL)) GO TO 72
      WRITE (6,3010) LWVAL(1),LWVAL(NLWVAL)
 3010 FORMAT (45H MINIMUM LIVEWEIGHT OUTSIDE PERMITTED RANGE :,
     1I5,2H -,I5)
      GO TO 999
   72 IF (LWMAX.LE.LWVAL(NLWVAL)) GO TO 74
      WRITE (6,3020) LWVAL(1),LWVAL(NLWVAL)
 3020 FORMAT (45H MAXIMUM LIVEWEIGHT OUTSIDE PERMITTED RANGE :,
     1I5,2H -,I5)
      GO TO 999
   74 DO 80 I=1,NGMAX
```

```
      COSMIN(I)=0.0
      QFED(I)=0.0
      NFL(I)=0
      NFU(I)=0
      DO 80 J=1,NCONS
      COMP(I,J)=0.0
   80 LCINGR(I,J)=0
      DO 85 I=1,NGMAX
      DO 85 J=1,NC
      DO 85 K=1,NCOLS
   85 B(I,J,K)=0.0
      NDAYS=1.0/GVAL(2)+0.1
      WRITE (8'1) LWMIN,LWMAX,LWINCR,NGVAL,NDAYS,PERCOS
C      WRITE (6,2110) LWMIN,LWMAX,LWINCR,NGVAL,NDAYS,PERCOS
C 2110 FORMAT (5I4,F5.2)
      DO 90 I=1,NFMAX
      FVAL(I)=0.0
      FINVAL(I)=0.0
   90 GINVAL(I)=0.0
      F=FMIN-FINT
      NF=0
  100 NF=NF+1
      IF (NF.LE.NFMAX) GO TO 110
      WRITE (6,4050) NFMAX
 4050 FORMAT (38H TOO MANY VALUES OF F IMPLIED.MAX NO :,I4)
      GO TO 999
  110 F=F+FINT
      FVAL(NF)=F
      IF (F.LT.FMAX) GO TO 100
      FVAL(NF)=FMAX
      NFMAX=NF-1
      DO 120 NF=1,NFMAX
      F=FVAL(NF)
      FINVAL(NF)=1.0/F
  120 GINVAL(NF)=1.0/(1.0-F)
      RHOLO=AA(2,1)/AA(3,1)
      RHOUP=RHOLO
      DO 140 I=2,NVARS
      WV=AA(2,I)/AA(3,I)
      IF (WV.LT.RHOLO) RHOLO=WV
      IF (WV.GT.RHOUP) RHOUP=WV
  140 CONTINUE
C
C  FIND RANGE OF F FOR PIECEWISE LINEARISATION FOR EACH LWG
C
      DO 180 IG=2,NGVAL
      R=((0.05272+0.00684*GVAL(IG))*GVAL(IG))/0.077
      NFLO=0
      NFUP=0
      FLO=1.0/(1.0+RHOUP*R)
      FUP=1.0/(1.0+RHOLO*R)
      IF (FLO.GT.FMIN.AND.FUP.LT.FMAX) GO TO 155
      IF (FLO.GT.FMIN) GO TO 150
      FLO=FMIN
      NFLO=1
  150 IF (FUP.LT.FMAX) GO TO 155
      FUP=FMAX
      NFUP=NFMAX
```

```
  155 IF (NFLO.EQ.1) GO TO 165
      DO 160 I=2,NFMAX
      IF (FLO.GE.FVAL(I)) GO TO 160
      NFLO=I-1
      GO TO 165
  160 CONTINUE
  165 IF (NFUP.EQ.NFMAX) GO TO 175
      DO 170 I=2,NFMAX
      IF (FUP.GT.FVAL(I)) GO TO 170
      NFUP=I
      GO TO 175
  170 CONTINUE
      NFUP=NFMAX
  175 NFL(IG)=NFLO
  180 NFU(IG)=NFUP
C
C  SET INITIAL VALUES FOR DMA,DMAINT,DMC,DMCINT,DCP,DCPINT AND YME
C
      IF (LWVAL(NLWVAL).EQ.LWMAX) GO TO 250
      DO 210 I=2,NLWVAL
      IF (LWVAL(I).LT.LWMAX) GO TO 210
      K2=I
      GO TO 230
  210 CONTINUE
  230 K1=K2-1
      LWUP=LWVAL(K2)
      LWLO=LWVAL(K1)
      LWDIF=LWUP-LWLO
      WINCS=LWUP-LWMAX
      WINCS=WINCS/LWINCR
      DMAINT=((DMAVAL(K2)-DMAVAL(K1))/LWDIF)*LWINCR
      DMA=DMAVAL(K2)-DMAINT*(WINCS-1.0)
      DMCINT=((DMCVAL(K2)-DMCVAL(K1))/LWDIF)*LWINCR
      DMC=DMCVAL(K2)-DMCINT*(WINCS-1.0)
      DO 240 I=1,NGVAL
      DCPINT(I)=((DCPVAL(K2,I)-DCPVAL(K1,I))/LWDIF)*LWINCR
  240 DCP(I)=DCPVAL(K2,I)-DCPINT(I)*(WINCS-1.0)
      GO TO 270
  250 DMAINT=0.0
      DMA=DMAVAL(NLWVAL)
      DMCINT=0.0
      DMC=DMCVAL(NLWVAL)
      DO 260 I=1,NGVAL
      DCPINT(I)=0.0
  260 DCP(I)=DCPVAL(NLWVAL,I)
      K1=NLWVAL
      LWLO=LWMAX
  270 IPOS=1
      RHS(2)=0.0
      RHS(3)=0.0
      RHS(4)=1.0
      DO 275 IG=1,NGMAX
      NIC(IG)=0
      RH4(IG)=1.0
      DO 275 I=1,NCONS
      IDB(IG,I)=0
      DO 275 J=1,NCONS
  275 BETA(IG,I,J)=0.0
```

```
C
C   ITERATE FOR EACH LIVEWEIGHT
C
      LW=LWMAX+LWINCR
      DO 900 LWC=LWMIN,LWMAX,LWINCR
      LW=LW-LWINCR
      IF (LW.GE.LWLO) GO TO 330
      K2=K1
      K1=K2-1
      LWUP=LWVAL(K2)
      LWLO=LWVAL(K1)
      LWDIF=LWUP-LWLO
      WINCS=LWUP-LW
      WINCS=WINCS/LWINCR
      DMAINT=((DMAVAL(K2)-DMAVAL(K1))/LWDIF)*LWINCR
      DMA=DMAVAL(K2)-DMAINT*WINCS
      DMCINT=((DMCVAL(K2)-DMCVAL(K1))/LWDIF)*LWINCR
      DMC=DMCVAL(K2)-DMCINT*WINCS
      DO 320 I=1,NGVAL
      DCPINT(I)=((DCPVAL(K2,I)-DCPVAL(K1,I))/LWDIF)*LWINCR
  320 DCP(I)=DCPVAL(K2,I)-DCPINT(I)*WINCS
      GO TO 350
  330 DMA=DMA-DMAINT
      DMC=DMC-DMCINT
      DO 340 I=1,NGVAL
  340 DCP(I)=DCP(I)-DCPINT(I)
  350 WP=LW
      WP=WP**0.75
      YME=0.077*WP
      DO 360 IG=1,NGVAL
      COSMIN(IG)=99999.0
      QFED(IG)=0.0
      DO 360 I=1,NCONS
      LCINGR(IG,I)=0
  360 COMP(IG,I)=0.0
      RHS(1)=DMA
      RHS(6)=DMC
C
C  ITERATE FOR EACH LIVEWEIGHT GAIN
C
      DO 800 IG=2,NGVAL
      DO 400 I=1,NCONS
      SOL(I)=0.0
  400 INGS(I)=0
      RHS(5)=DCP(IG)
      ZME=WP*(0.05272+0.00684*GVAL(IG))*GVAL(IG)
      IF (NIC(IG).GT.0) GO TO 500
  405 DO 410 I=1,NC1
      DO 410 J=1,NCOLS
  410 A(I,J)=0.0
      DO 420 J=1,NVARS
      A(NC,J)=C(J)
      DO 420 I=1,NCONS
  420 A(I,J)=AA(I,J)
      RH4(IG)=1.0
      RHS(4)=1.0
      NFLO=NFL(IG)
      NFUP=NFU(IG)
```

```
      J=NVARS
      DO 450 NF=NFLO,NFUP
      J=J+1
      A(2,J)=-YME*FINVAL(NF)
      A(3,J)=-ZME*GINVAL(NF)
  450 A(4,J)=1.0
      NV=J
      IC=NCONS+NV+NGTS+1
      NG=NV+NGTS
      NGL=NG+NLTS
      DO 460 I=1,NGTS
  460 A(NLE+I,NV+I)=-1.0
      DO 470 I=1,NCONS
  470 A(I,NG+I)=1.0
      DO 480 I=1,NCONS
  480 A(I,IC)=RHS(I)
      ITYPE=0
      CALL LPDS(NG,NGL,IC,ITYPE,A,INGS,NFSIND)
      IF (NFSIND.EQ.0) GO TO 485
      NIC(IG)=0
      GO TO 800
  485 COSMIN(IG)=-A(NC,IC)
      DO 490 I=1,NCONS
      IDB(IG,I)=INGS(I)
  490 SOL(I)=A(I,IC)
      DO 495 I=1,NCONS
      DO 495 J=1,NCONS
  495 BETA(IG,I,J)=A(I,NG+J)
      NIC(IG)=IC
      DO 497 I=1,NC
      DO 497 J=1,NCOLS
  497 B(IG,I,J)=A(I,J)
      GO TO 700
C
C  DERIVE BASIC SOLUTION FROM OPTIMAL SOLUTION FOR PREVIOUS LIVEWEIGHT
C
  500 IC=NIC(IG)
      ICL1=IC-1
      NV=IC-NC-NGTS
      NV1=NV+1
      DO 505 I=1,NCONS
  505 INGS(I)=IDB(IG,I)
      DO 510 I=1,NCONS
      DO 510 J=1,ICL1
  510 A(I,J)=B(IG,I,J)
      DO 515 I=1,NCONS
      DO 515 J=IC,NCOLS
  515 A(I,J)=0.0
      WV=YME/AYME
      RH4(IG)=RH4(IG)*WV
      RHS(4)=RH4(IG)
      DO 530 J=1,NVARS
      A(NC,J)=0.0
      DO 520 I=1,NCONS
      IF (INGS(I).EQ.J) GO TO 530
  520 CONTINUE
      S=C(J)
      DO 525 I=1,NCONS
```

```
      K=INGS(I)
      IF (K.LE.NVARS.OR.K.GT.NV) GO TO 525
      A(I,J)=A(I,J)/WV
  525 S=S-C(K)*A(I,J)
      A(NC,J)=S
  530 CONTINUE
      DO 545 J=NVARS1,NV
      A(NC,J)=0.0
      DO 535 I=1,NCONS
      IF (INGS(I).EQ.J) GO TO 545
  535 CONTINUE
      S=0.0
      DO 540 I=1,NCONS
      K=INGS(I)
      IF (K.GT.NVARS.AND.K.LE.NV) GO TO 540
      A(I,J)=A(I,J)*WV
  540 S=S-C(K)*A(I,J)
      A(NC,J)=S
  545 CONTINUE
      DO 560 J=NV1,ICL1
      A(NC,J)=0.0
      DO 550 I=1,NCONS
      IF (INGS(I).EQ.J) GO TO 560
  550 CONTINUE
      S=0.0
      DO 555 I=1,NCONS
      K=INGS(I)
      IF (K.LE.NVARS.OR.K.GT.NV) GO TO 555
      A(I,J)=A(I,J)/WV
  555 S=S-C(K)*A(I,J)
      A(NC,J)=S
  560 CONTINUE
      DO 565 I=1,NCONS
      S=0.0
      DO 562 J=1,NCONS
  562 S=S+BETA(IG,I,J)*RHS(J)
  565 A(I,IC)=S
      S=0.0
      DO 570 I=1,NCONS
      K=INGS(I)
      IF (K.GT.NVARS) GO TO 570
      S=S-C(K)*A(I,IC)
  570 CONTINUE
      A(NC,IC)=S
      IOPT=1
      NG=NV+NGTS
      NGL=NG+NLTS
      DO 575 I=1,NGL
      IF (A(NC,I).GT.-1.0E-8) GO TO 575
      IOPT=0
      GO TO 580
  575 CONTINUE
  580 IFEA=1
      DO 585 I=1,NCONS
      IF (A(I,IC).GT.-1.0E-8) GO TO 585
      IFEA=0
      GO TO 590
  585 CONTINUE
```

```
  590 IF (IOPT.EQ.1.AND.IFEA.EQ.1) GO TO 485
      IF (IOPT.EQ.0.AND.IFEA.EQ.0) GO TO 405
      ITYPE=1
      IF (IOPT.EQ.1.AND.IFEA.EQ.0) ITYPE=-1
      CALL LPDS(NG,NGL,IC,ITYPE,A,INGS,NFSIND)
      IF (NFSIND.EQ.0) GO TO 485
      NIC(IG)=0
      DO 595 I=1,NCONS
      IDB(IG,I)=0
      DO 595 J=1,NCONS
  595 BETA(IG,I,J)=0.0
      IF (NFSIND.EQ.-1) GO TO 405
      GO TO 800
C
C  SORT INGREDIENTS IN DESCENDING FOOD NO. ORDER
C
  700 DO 710 I=1,NCONS
      IF (INGS(I).LE.NVARS) GO TO 705
      INGS(I)=0
      SOL(I)=0.0
      GO TO 710
  705 QFED(IG)=QFED(IG)+SOL(I)
  710 CONTINUE
      DO 720 I=1,NCONS
      IF (INGS(I).EQ.0) GO TO 720
      SOL(I)=(SOL(I)/QFED(IG))*100.0
      IF (SOL(I).GT.0.05) GO TO 720
      SOL(I)=0.0
      INGS(I)=0
  720 CONTINUE
      L=NCONS
      DO 750 I=1,NCONS
      IF (INGS(I).GT.0) GO TO 730
      ID(I)=L
      L=L-1
      GO TO 750
  730 K=1
      DO 740 J=1,NCONS
      IF (J.EQ.I) GO TO 740
      IF (INGS(J).GT.INGS(I)) K=K+1
  740 CONTINUE
      ID(I)=K
  750 CONTINUE
      DO 760 I=1,NCONS
      J=ID(I)
      COMP(IG,J)=SOL(I)
  760 LCINGR(IG,J)=INGS(I)
  800 CONTINUE
      AYME=YME
      IPOS=IPOS+1
      WRITE (8'IPOS) COSMIN,QFED,LCINGR,COMP
C       WRITE (6,2120) LW,(COSMIN(I),I=1,NGMAX),(QFED(I),I=1,NGMAX)
C 2120 FORMAT (I4,11F10.3/4X,11F10.3)
C       DO 810 I=1,NCONS
C  810 WRITE (6,2130) (LCINGR(IG,I),COMP(IG,I),IG=1,NGMAX)
C 2130 FORMAT (4X,11(I3,F7.1))
  900 CONTINUE
C
```

```
C  SORT RESULTS IN INCREASING LIVEWEIGHT ORDER
C
      JPOS=1
      WRITE (7'JPOS) LWMIN,LWMAX,LWINCR,NGVAL,NDAYS,PERCOS
      K=IPOS-1
      IPOS=IPOS+1
      DO 910 I=1,K
      IPOS=IPOS-1
      JPOS=JPOS+1
      READ (8'IPOS) COSMIN,QFED,LCINGR,COMP
  910 WRITE (7'JPOS) COSMIN,QFED,LCINGR,COMP
  999 STOP
      END
C
C
C
C
C
      SUBROUTINE LPDS(NG,NGL,IC,IT,A,ID,NFSIND)
      COMMON/LPCOM/M,L1
      DIMENSION A(8,129),ID(6)
      IW=M+1
      IF (IT.EQ.0) GO TO 10
      IF (IT.EQ.1) GO TO 100
      GO TO 300
   10 DO 20 I=1,M
   20 ID(I)=I+NG
      IW=IW+1
      DO 25 J=1,IC
   25 A(IW,J)=0.0
C
C  SET UP PHASE I AND FIND PIVOTAL COLUMN
C
      KC=0
      Q=0.0
      DO 40 J=1,NG
      S=0.0
      DO 30 I=L1,M
   30 S=S-A(I,J)
      A(IW,J)=S
      IF(S.GE.Q) GO TO 40
      Q=S
      KC=J
   40 CONTINUE
      S=0.0
      DO 50 I=L1,M
   50 S=S-A(I,IC)
      A(IW,IC)=S
      IF(KC.GT.0) GO TO 120
      IW=IW-1
C
C  SIMPLEX ITERATION
C
  100 KC=0
      Q=0.0
      DO 110 J=1,NGL
      IF (A(IW,J).GE.Q) GO TO 110
      Q=A(IW,J)
```

```
      KC=J
  110 CONTINUE
      IF (KC.EQ.0) GO TO 200
  120 KR=0
      Q=1.0E70
      DO 130 I=1,M
      IF(A(I,KC).LE.1.0E-8) GO TO 130
      P=A(I,IC)/A(I,KC)
      IF(P.GT.Q) GO TO 130
      Q=P
      KR=I
  130 CONTINUE
      IF(KR.EQ.0) GO TO 250
      P=A(KR,KC)
      ID(KR)=KC
      DO 140 J=1,IC
  140 A(KR,J)=A(KR,J)/P
      DO 160 I=1,IW
      IF(I.EQ.KR) GO TO 160
      DO 150 J=1,IC
      IF(J.EQ.KC) GO TO 150
      A(I,J)=A(I,J)-A(KR,J)*A(I,KC)
      Q=ABS(A(I,J))
      IF(Q.LT.1.0E-8) A(I,J)=0.0
  150 CONTINUE
  160 CONTINUE
      DO 170 I=1,IW
  170 A(I,KC)=0.0
      A(KR,KC)=1.0
      GO TO 100
C
C  CHECK FOR PHASE II COMPLETION AND FEASIBILITY
C
  200 IF (IW.EQ.M+1) GO TO 210
      IW=IW-1
      GO TO 100
  210 DO 220 I=1,M
      IF (ID(I).GT.NGL) GO TO 230
  220 CONTINUE
      NFSIND=0
      GO TO 999
  230 NFSIND=1
      GO TO 999
  250 WRITE (6,2000)
 2000 FORMAT (19H SOLUTION UNBOUNBED)
      STOP
C
C  DUAL SIMPLEX ITERATION
C
  300 KR=0
      Q=0.0
      DO 310 I=1,M
      IF (A(I,IC).GE.Q) GO TO 310
      Q=A(I,IC)
      KR=I
  310 CONTINUE
      IF (KR.EQ.0) GO TO 100
      KC=0
```

```
      Q=-1.0E70
      DO 320 J=1,NGL
      IF (A(KR,J).GT.-1.0E-8) GO TO 320
      P=A(IW,J)/A(KR,J)
      IF (P.GT.Q) GO TO 320
      Q=P
      KC=J
  320 CONTINUE
      IF (KC.EQ.0) GO TO 370
      P=A(KR,KC)
      ID(KR)=KC
      DO 330 J=1,IC
  330 A(KR,J)=A(KR,J)/P
      DO 350 I=1,IW
      IF (I.EQ.KR) GO TO 350
      DO 340 J=1,IC
      IF (J.EQ.KC) GO TO 340
      A(I,J)=A(I,J)-A(KR,J)*A(I,KC)
      Q=ABS(A(I,J))
      IF (Q.LT.1.0E-8) A(I,J)=0.0
  340 CONTINUE
  350 CONTINUE
      DO 360 I=1,IW
  360 A(I,KC)=0.0
      A(KR,KC)=1.0
      GO TO 300
  370 NFSIND=-1
  999 RETURN
      END
```

APPENDIX B4

BEEF CATTLE DP MODEL SOLUTION PROGRAM - NRC STANDARDS

```
      DEFINE FILE 7(502,484,L,IAV7),11(64,4008,L,IAV11),
     112(64,4008,L,IAV12)
      DIMENSION FEEDC(501,11),GVAL(11),COSINC(11),COST1(501),COST2(501),
     1IPOL(501),IGROW(501),COSMIN(11),QFED(11),LCINGR(11,6),COMP(11,6)
      INTEGER*2 LCINGR
      NRECS=64
      IAV7=1
      IAV11=1
      IAV12=1
      MAXNLW=501
      NGMAX=11
      READ (7'1) LWMIN,LWMAX,LWINCR,NGVAL,NDAYS,PERCOS
      DO 50 I=1,MAXNLW
      IGROW(I)=0
      DO 50 J=1,10
   50 FEEDC(I,J)=-9999.9
      NLW=LWMAX-LWMIN+1
      NDL1=NDAYS-1
      IPOS=1
      DO 60 LW=1,NLW,LWINCR
      IPOS=IPOS+1
      READ (7'IPOS) COSMIN,QFED,LCINGR,COMP
      DO 60 IG=1,NGMAX
   60 FEEDC(LW,IG)=COSMIN(IG)
      IF (LWINCR.EQ.1) GO TO 100
      N=NLW-LWINCR
      DO 90 LW=1,N,LWINCR
      L2=LW+LWINCR
      DO 70 IG=1,NGVAL
      WV1=FEEDC(LW,IG)
      IF (WV1.GT.99990.0) FEEDC(LW,IG)=-9999.9
      WV2=FEEDC(L2,IG)
      IF (WV2.GT.99990.0) FEEDC(L2,IG)=-9999.9
      IF (FEEDC(LW,IG).LT.-9990.0) GO TO 65
      IF (FEEDC(L2,IG).LT.-9990.0) GO TO 65
      COSINC(IG)=(WV2-WV1)/LWINCR
      GO TO 70
   65 COSINC(IG)=-9999.9
   70 CONTINUE
      L1=LW+1
      DO 80 L=L1,L2
      I=L-1
      DO 75 IG=1,NGVAL
      IF (COSINC(IG).LT.-9990.0) GO TO 75
      FEEDC(L,IG)=FEEDC(I,IG)+COSINC(IG)
   75 CONTINUE
```

```
 80 CONTINUE
 90 CONTINUE
100 DO 250 LW=1,NLW
    LWG=0
    FEEDC(LW,1)=NDAYS*FEEDC(LW,1)
    DO 200 IG=2,NGVAL
    IF (FEEDC(LW,IG).LT.0.0) GO TO 200
    LWG=LWG+1
    IF (LW+LWG.LE.NLW) GO TO 110
    FEEDC(LW,IG)=-9999.9
    GO TO 200
110 L1=LW
    L2=L1+1
    IF (FEEDC(L2,IG).GT.0.0) GO TO 120
    FEEDC(LW,IG)=-9999.9
    GO TO 200
120 NINC=NDL1*LWG
    L=1
    CINCR=FEEDC(L2,IG)-FEEDC(L1,IG)
    COST=NDAYS*FEEDC(L1,IG)
    DO 150 J=LWG,NINC,LWG
    K=J/NDAYS
    IF (K.LT.L) GO TO 140
    L=K+1
    L1=LW+K
    L2=L1+1
    IF (FEEDC(L2,IG).GT.0.0) GO TO 130
    FEEDC(LW,IG)=-9999.9
    GO TO 200
130 CINCR=FEEDC(L2,IG)-FEEDC(L1,IG)
140 K=J-K*NDAYS
    COST=COST+NDAYS*FEEDC(L1,IG)+K*CINCR
150 CONTINUE
    FEEDC(LW,IG)=COST/NDAYS
200 CONTINUE
250 CONTINUE
    DO 310 I=1,MAXNLW
    IPOL(I)=0
310 COST2(I)=9999999.0
    COST2(1)=0.0
    NPERS=LWMAX-LWMIN
    L1MIN=1
    L1MAX=1
    ICHAN=11
    IPOS=0
    DO 500 N=1,NPERS
    NPREQ=N
    DO 320 I=1,MAXNLW
    IPOL(I)=0
320 COST1(I)=COST2(I)
    DO 400 L1=L1MIN,L1MAX
    COSTA=COST1(L1)
    IF (COSTA.GT.9999990.0) GO TO 400
    L2=L1
    DO 350 IG=2,NGVAL
    L2=L2+1
    IF (FEEDC(L1,IG).LT.0.0) GO TO 350
    COST=COSTA+PERCOS+FEEDC(L1,IG)
```

```
      IF (COST-COST2(L2).GT.-0.0001) GO TO 350
      COST2(L2)=COST
      IPOL(L2)=L1
  350 CONTINUE
  400 CONTINUE
      L1MIN=N
      L1MAX=L1MAX+NGVAL-1
      IF (L1MAX.GT.NLW) L1MAX=NLW
      DO 410 I=1,NLW
      IF (IPOL(I).EQ.0) GO TO 410
      IPOL(I)=IPOL(I)
  410 CONTINUE
      IF (IPOS.LT.NRECS) GO TO 420
      ICHAN=ICHAN+1
      IPOS=0
  420 IPOS=IPOS+1
      WRITE (ICHAN'IPOS) COST2,IPOL
      DO 460 I=2,NLW
      IF (IPOL(I).GT.0) GO TO 500
  460 CONTINUE
      GO TO 510
  500 CONTINUE
  510 IF (NPREQ.NE.NPERS) GO TO 530
      NPREQ=NPREQ+1
      IPOS=IPOS+1
      IF (IPOS.LE.NRECS) GO TO 520
      ICHAN=ICHAN+1
      IPOS=0
  520 IPOS=IPOS+1
  530 IGROW(NPREQ)=LWMAX
      INEXT=NLW
      NPREQ=NPREQ-1
      DO 600 N=1,NPREQ
      IPER=NPREQ-N+1
      IPOS=IPOS-1
      IF (IPOS.GT.0) GO TO 550
      ICHAN=ICHAN-1
      IPOS=NRECS
  550 READ (ICHAN'IPOS) COST2,IPOL
      COST1(IPER)=COST2(INEXT)
      INEXT=IPOL(INEXT)
      IGROW(IPER)=INEXT+LWMIN-1
      IF (INEXT.EQ.0) GO TO 610
  600 CONTINUE
  610 IF (IPER.EQ.1) GO TO 630
      NPREQ=NPREQ-IPER+1
      DO 620 N=1,NPREQ
      IPER1=N+IPER-1
      COST1(N)=COST1(IPER1)
  620 IGROW(N)=IGROW(IPER1)
      IGROW(NPREQ+1)=LWMAX
  630 IF (LWINCR.EQ.1) GO TO 650
      WRITE (6,2110)
 2110 FORMAT (1H1,5X,21H OPTIMAL FEEDING PLAN///31H PERIOD  LIVEWEIGHT
     1CUMULATIVE/7H NUMBER,4X,5H (KG),7X,5H COST/8X,11H START  END,4X,4H
     2 (P))
      DO 640 N=1,NPREQ
  640 WRITE (6,2120) N,IGROW(N),IGROW(N+1),COST1(N)
```

```
2120 FORMAT (I5,2X,2I6,F10.2)
     GO TO 999
 650 WRITE (6,2210)
2210 FORMAT (1H1,35X,21H OPTIMAL FEEDING PLAN///31H PERIOD  LIVEWEIGHT
    1 CUMULATIVE,17X,37H FEED REQUIRED ON FIRST DAY OF PERIOD/7H NUMBER
    2,4X,5H (KG),7X,5H COST,24X,19H RATION COMPOSITION,21X,9H QUANTITY
    3/8X,11H START  END,4X,4H (P),5X,6(10H FOOD   % ),2X,5H (KG))
     DO 660 N=1,NPREQ
     K=IGROW(N)
     L=IGROW(N+1)
     IPOS=K-LWMIN+2
     IG=L-K+1
     READ (7'IPOS) COSMIN,QFED,LCINGR,COMP
 660 WRITE (6,2220) N,K,L,COST1(N),(LCINGR(IG,I),COMP(IG,I),I=1,6)
    1,QFED(IG)
2220 FORMAT (I5,2X,2I6,F10.2,3X,6(I4,F6.1),F8.2)
 999 STOP
     END
```

APPENDIX B5

PIG PRODUCTION DP MODEL SOLUTION PROGRAM

```
      DIMENSION COST1(80,241),COST2(80,241),IPOL(80,241),EW(80,2),
     1PRCONT(80,2),R(80,2),F(80,2),C(10),E(10),P(10),ACID(10,3),REF(3),
     2AA(5,17),A(6,18),B(4,5,18),RHS1(4),RHS2(4),NFS(4),IBV(4),IDB(4,4),
     3G(4),PRV(5,4),PRD(5,4),IFAULT(8)
      COMMON/COM1/NCONS,NC,IC,NG,NL1,NGL,ICL1,NGP1,NGP2
      COMMON/COM2/B,IDB,RHS1,RHS2,NFS
      COMMON/COM3/A,IBV
      COMMON/COM4/AA
      MAXNLW=80
      MAXNPT=241
      MAXNV=10
      MAXLWG=5
      MAXNG=4
      MINLW=20
      MAXLW=200
      NDAYS=5
      X1=4.906
      NCONS=4
      NLTS=0
      NEQS=1
      NGTS=3
      NC=NCONS+1
      NC1=NC+1
      NL1=NLTS+1
      NLE=NLTS+NEQS
      ICHAN=11
      DO 10 I=1,8
   10 IFAULT(I)=0
      DO 20 I=1,MAXNLW
      DO 20 J=1,MAXNPT
      COST1(I,J)=999999.9
      COST2(I,J)=999999.9
   20 IPOL(I,J)=0
      DO 30 I=1,MAXNLW
      DO 30 J=1,2
      EW(I,J)=0.0
      PRCONT(I,J)=0.0
      F(I,J)=0.0
   30 R(I,J)=0.0
      DO 40 I=1,MAXNV
      C(I)=0.0
      E(I)=0.0
      P(I)=0.0
      DO 40 J=1,3
   40 ACID(I,J)=0.0
      IC=MAXNV+NCONS+NGTS
```

```
      DO 50 I=1,NC
      DO 50 J=1,IC
   50 AA(I,J)=0.0
      IC=IC+1
      DO 60 I=1,NC1
      DO 60 J=1,IC
   60 A(I,J)=0.0
      DO 70 I=1,4
      RHS1(I)=0.0
      RHS2(I)=0.0
      NFS(I)=0
      DO 70 J=1,NC
      DO 70 K=1,IC
   70 B(I,J,K)=0.0
      DO 80 I=1,NCONS
      IBV(I)=0
      DO 80 J=1,4
   80 IDB(J,I)=0
      DO 90 I=1,MAXNG
      G(I)=0.0
      DO 90 J=1,MAXLWG
      PRD(J,I)=0.0
   90 PRV(J,I)=0.0
C
C  READ IN DATA AND CHECK FOR ERRORS
C
      READ (5,1010) NVARS,LWMIN,LWMAX,LWGMAX,IGAMAX
 1010 FORMAT (5I4)
      NLWMAX=LWMAX-LWMIN
      NPTMAX=3*NLWMAX+1
      IF (NVARS.GT.MAXNV) IFAULT(1)=1
      IF (LWMIN.LT.MINLW) IFAULT(2)=1
      IF (LWMAX.GT.MAXLW) IFAULT(3)=1
      IF (LWGMAX.GT.MAXLWG) IFAULT(4)=1
      IF (IGAMAX.GT.MAXNG) IFAULT(5)=1
      IF (NLWMAX.GT.MAXNLW) IFAULT(6)=1
      IF (NLWMAX.LT.1) IFAULT(7)=1
      IF (NPTMAX.GT.MAXNPT) IFAULT(8)=1
      K=0
      DO 100 I=1,8
      IF (IFAULT(I).EQ.0) GO TO 100
      K=K+1
  100 CONTINUE
      IF (K.EQ.0) GO TO 120
      WRITE (6,3000) K
 3000 FORMAT (I4,26H INPUT ERROR(S) OF TYPE :-)
      DO 110 I=1,8
      IF (IFAULT(I).EQ.0) GO TO 110
      WRITE (6,3010) I
 3010 FORMAT (30X,I2)
  110 CONTINUE
      GO TO 999
  120 DO 130 I=1,NVARS
  130 READ (5,1020) C(I),E(I),P(I),(ACID(I,J),J=1,3)
 1020 FORMAT (6F6.1)
      READ (5,1030) (REF(I),I=1,3)
 1030 FORMAT (3F6.1)
      READ (5,1040) PERCOS
```

```
 1040 FORMAT (F6.1)
      READ (5,1050) PRTMIN,PTINC
 1050 FORMAT (2F6.3)
      PRTMIN=PRTMIN/X1
      PTINC=PTINC/X1
      DO 140 I=1,LWGMAX
      READ (5,1060) (PRV(I,J),J=1,IGAMAX)
 1060 FORMAT (4F6.2)
      DO 140 J=1,IGAMAX
      IF (I.NE.LWGMAX) GO TO 140
      G(J)=(X1/PRV(I,J)-X1)/1.1
  140 PRV(I,J)=PRV(I,J)/X1
      READ (5,1070) RZ,DZ,RK,DK
 1070 FORMAT (4F10.6)
C
C  CALCULATE MODEL PARAMETERS
C
      K=LWMIN-MINLW
      RZ=RZ+K*DZ
      RK=RK+K*DK
      DZ=0.5*DZ
      DK=0.5*DK
      RZ=RZ+DZ
      RK=RK+DK
      RO=(RK/RZ)*1000.0
      WAV=LWMIN+0.5
      EW0=0.475*WAV**0.75
      DP=0.5*NDAYS*PTINC
      DP=0.365*DP
      PRT0=0.365*PRTMIN+DP
      A2=(0.0115+0.0073/RZ)*1000.0
      WV=PRTO
      DO 150 I=1,NLWMAX
      DO 150 J=1,2
      RZ=RZ+DZ
      RK=RK+DK
      R(I,J)=(RK/RZ)*1000.0
      F(I,J)=(0.0115+0.0073/RZ)*1000.0
      WAV=WAV+0.5
      EW(I,J)=0.475*WAV**0.75
      WV=WV+DP
  150 PRCONT(I,J)=WV
      WV=0.5*0.365*NDAYS
      DO 160 I=1,IGAMAX
      G(I)=0.0535*G(I)*1000.0
      DO 160 J=1,LWGMAX
  160 PRD(J,I)=WV*(PRV(J,I)-PRV(J,1))
      DP=0.365*PTINC
C
C  SET UP LP MATRIX
C
      NG=NVARS+NGTS
      NGL=NG+NLTS
      NGP1=NG+1
      NGP2=NG+2
      ICL1=NG+NCONS
      IC=ICL1+1
      DO 170 I=1,NVARS
```

```
      AA(NC,I)=NDAYS*C(I)
      AA(1,I)=E(I)-0.0121*P(I)
      DO 170 J=1,3
  170 AA(J+1,I)=(P(I)*ACID(I,J))/REF(J)
      DO 180 I=1,NGTS
  180 AA(NLE+I,NVARS+I)=-1.0
      DO 190 I=1,NCONS
      IBV(I)=NG+I
  190 AA(I,NG+I)=1.0
      DO 200 I=1,NC
      DO 200 J=1,ICL1
  200 A(I,J)=AA(I,J)
C
C  ***** CALCULATE MEASURE OF PERFORMANCE AT END OF PERIOD 1 *****
C
      PRTOT=PRT0
      EM=EW0-PRTOT
      PR=PRV(1,1)
      RHS1(1)=EM+PR*(A2+G(1))
      RHS2(1)=PR*R0
      A(1,IC)=RHS1(1)
      DO 210 I=2,NCONS
  210 A(I,IC)=RHS2(1)
      ITYPE=0
      CALL LPDS(ITYPE,A,IBV,NFSIND)
      NFS(1)=NFSIND
      CALL STORLP(1,ITYPE)
      CALL TRANLP(1)
      LWG=0
      LWV=0
      LWH=2
      LW2LO=1
      LW2UP=LWGMAX
      IF (LW2UP.GT.NLWMAX) LW2UP=NLWMAX
C
C  ITERATE FOR EACH LWT AT END OF PERIOD 1
C
      DO 290 LW2=LW2LO,LW2UP
      LWG=LWG+1
      IF (LWG.EQ.1) GO TO 250
      LWH=LWH+1
      IF (LWH.EQ.2) GO TO 220
      LWV=LWV+1
      LWH=1
  220 PR=PRV(LWG,1)
      PRTOT=PRCONT(LWV,LWH)
      R0=R(LWV,LWH)
      EM=EW(LWV,LWH)-PRTOT
      A2=F(LWV,LWH)
      RH1=EM+PR*(A2+G(1))
      RH2=PR*R0
      ITYPE=0
      CALL RESET(2,RH1,RH2,D1,D2)
      IF (NFS(2).EQ.1) GO TO 230
      NFSIND=0
      CALL PARARH(D1,D2,A,ITYPE)
      IF (ITYPE.EQ.1) GO TO 240
  230 CALL LPDS(ITYPE,A,IBV,NFSIND)
```

```
      NFS(2)=NFSIND
  240 CALL STORLP(2,ITYPE)
  250 CALL TRANLP(2)
C
C  ITERATE FOR EACH VALUE OF GAMMA
C
      DO 290 IGAMV=1,IGAMAX
      IF (IGAMV.EQ.1) GO TO 280
      PR=PRV(LWG,IGAMV)
      RH1=EM-PRD(LWG,IGAMV)+PR*(A2+G(IGAMV))
      RH2=PR*R0
      ITYPE=0
      CALL RESET(3,RH1,RH2,D1,D2)
      IF (NFS(3).EQ.1) GO TO 260
      NFSIND=0
      CALL PARARH(D1,D2,A,ITYPE)
      IF (ITYPE.EQ.1) GO TO 270
  260 CALL LPDS(ITYPE,A,IBV,NFSIND)
      NFS(3)=NFSIND
  270 CALL STORLP(3,ITYPE)
  280 IF (NFSIND.NE.0) GO TO 290
      IPT2=(IGAMV-1)*LWG+1
      COST2(LW2,IPT2)=-A(NC,IC)+PERCOS
  290 CONTINUE
      WRITE (ICHAN) LWMIN,LWMAX,LWGMAX
      NRECS=1
      DO 300 LW=1,LW2UP
      NRECS=NRECS+3
  300 WRITE (ICHAN) LW,(IPOL(LW,I),I=1,NRECS)
C
C  ***** ITERATE FOR EACH STAGE *****
C
      IF (NLWMAX.EQ.1) GO TO 610
      NLWL1=NLWMAX-1
      LW1LO=0
      LW1UP=0
      NRMIN=1
      DO 600 IPER=2,NLWMAX
      NRECS=NRMIN
      DO 310 I=LW2LO,LW2UP
      NRECS=NRECS+3
      DO 310 J=1,NRECS
      COST1(I,J)=COST2(I,J)
  310 IPOL(I,J)=0
C
C  SET UP INITIAL CONDITIONS FOR THIS STAGE
C
  315 LW1LO=LW1LO+1
      LW1UP=LW2UP
      IF (LW1UP.GT.NLWL1) LW1UP=NLWL1
      LWV=LW1LO
      LWH=2
      PR=PRV(1,1)
      PRTOT=PRCONT(LWV,LWH)
      R0=R(LWV,LWH)
      EM=EW(LWV,LWH)-PRTOT
      A2=F(LWV,LWH)
      RH1=EM+PR*(A2+G(1))
```

```
      RH2=PR*R0
      ITYPE=0
      CALL RESET(1,RH1,RH2,D1,D2)
      IF (NFS(1).EQ.1) GO TO 320
      NFSIND=0
      CALL PARARH(D1,D2,A,ITYPE)
      IF (ITYPE.EQ.1) GO TO 330
  320 CALL LPDS(ITYPE,A,IBV,NFSIND)
      NFS(1)=NFSIND
  330 CALL STORLP(1,ITYPE)
      CALL TRANLP(1)
C
C  ITERATE FOR EACH LWT AT START OF PERIOD
C
      IPT1UP=3*(LW1LO-1)+1
      LW1000=1000*(LW1LO-1)
      DO 550 LW1=LW1LO,LW1UP
      IPT1UP=IPT1UP+3
      LW1000=LW1000+1000
      IF (LW1.EQ.LW1LO) GO TO 360
      LWV=LW1
      LWH=2
      PR=PRV(1,1)
      PRTOT=PRCONT(LWV,LWH)
      R0=R(LWV,LWH)
      EM=EW(LWV,LWH)-PRTOT
      A2=F(LWV,LWH)
      RH1=EM+PR*(A2+G(1))
      RH2=PR*R0
      ITYPE=0
      CALL RESET(2,RH1,RH2,D1,D2)
      IF (NFS(2).EQ.1) GO TO 340
      NFSIND=0
      CALL PARARH(D1,D2,A,ITYPE)
      IF (ITYPE.EQ.1) GO TO 350
  340 CALL LPDS(ITYPE,A,IBV,NFSIND)
      NFS(2)=NFSIND
  350 CALL STORLP(2,ITYPE)
  360 CALL TRANLP(2)
C
C  ITERATE FOR EACH LWT AT END OF PERIOD
C
      LWG=0
      LW2LO=LW1+1
      LW2UP=LW1+LWGMAX
      IF (LW2UP.GT.NLWMAX) LW2UP=NLWMAX
      DO 550 LW2=LW2LO,LW2UP
      LWG=LWG+1
      IF (LWG.EQ.1) GO TO 400
      LWH=LWH+1
      IF (LWH.EQ.2) GO TO 370
      LWH=1
      LWV=LWV+1
  370 PR=PRV(LWG,1)
      PRTOT=PRCONT(LWV,LWH)
      R0=R(LWV,LWH)
      EM=EW(LWV,LWH)-PRTOT
      A2=F(LWV,LWH)
```

```
      RH1=EM+PR*(A2+G(1))
      RH2=PR*R0
      ITYPE=0
      CALL RESET(3,RH1,RH2,D1,D2)
      IF (NFS(3).EQ.1) GO TO 380
      NFSIND=0
      CALL PARARH(D1,D2,A,ITYPE)
      IF (ITYPE.EQ.1) GO TO 390
  380 CALL LPDS(ITYPE,A,IBV,NFSIND)
      NFS(3)=NFSIND
  390 CALL STORLP(3,ITYPE)
  400 CALL TRANLP(3)
C
C  ITERATE FOR EACH VALUE OF GAMMA
C
      DO 550 IGAMV=1,IGAMAX
      IF (IGAMV.EQ.1) GO TO 430
      PR=PRV(LWG,IGAMV)
      RH1=EM-PRD(LWG,IGAMV)+PR*(A2+G(IGAMV))
      RH2=PR*R0
      ITYPE=0
      CALL RESET(4,RH1,RH2,D1,D2)
      IF (NFS(4).EQ.1) GO TO 410
      NFSIND=0
      CALL PARARH(D1,D2,A,ITYPE)
      IF (ITYPE.EQ.1) GO TO 420
  410 CALL LPDS(ITYPE,A,IBV,NFSIND)
      NFS(4)=NFSIND
  420 CALL STORLP(4,ITYPE)
C
C  ITERATE FOR EACH VALUE OF PROTEIN CONTENT AT LWT LW1
C
  430 PT=PRTOT
      IPT2=LWG*(IGAMV-1)
      D1=0.0
      DO 550 IPT1=1,IPT1UP
      IPT2=IPT2+1
      IF (COST1(LW1,IPT1).LT.999990.0) GO TO 440
      D1=D1-DP
      GO TO 550
  440 IF (IPT1.EQ.1) GO TO 510
      D1=D1-DP
      RH1=RH1+D1
      ITYPE=0
      IF (NFSIND.EQ.0) GO TO 470
      DO 450 I=1,NC
      DO 450 J=1,ICL1
  450 A(I,J)=AA(I,J)
      A(1,IC)=RH1
      DO 460 I=2,NCONS
  460 A(I,IC)=RH2
      A(NC,IC)=0.0
      D1=0.0
      GO TO 500
  470 ITYPE=1
      DO 490 I=1,NCONS
      XB=A(I,IC)+A(I,NGP1)*D1
      IF (XB.GT.-1.0E-8) GO TO 480
```

```
      ITYPE=-1
      GO TO 490
  480 IF (XB.LT.1.0E-8) XB=0.0
  490 A(I,IC)=XB
      A(NC,IC)=A(NC,IC)+A(NC,NGP1)*D1
      D1=0.0
      IF (ITYPE.EQ.1) GO TO 520
  500 CALL LPDS(ITYPE,A,IBV,NFSIND)
  510 IF (NFSIND.NE.0) GO TO 550
  520 COST=-A(NC,IC)+COST1(LW1,IPT1)+PERCOS
      IF (COST-COST2(LW2,IPT2).GT.-0.001) GO TO 550
      COST2(LW2,IPT2)=COST
      IPOL(LW2,IPT2)=LW1000+IPT1
  550 CONTINUE
      LW2LO=LW1LO+1
      NRMIN=3*LW1LO+1
      NRECS=NRMIN
      DO 560 LW=LW2LO,LW2UP
      NRECS=NRECS+3
  560 WRITE (ICHAN) LW,(IPOL(LW,I),I=1,NRECS)
  600 CONTINUE
  610 NRECS=1
      DO 620 LW=1,NLWMAX
      NRECS=NRECS+3
      WRITE (6,2010) LW
 2010 FORMAT (I4)
  620 WRITE (6,2020) (COST2(LW,I),I=1,NRECS)
 2020 FORMAT (4X,10F12.1)
  999 STOP
      END
C
C
C
C
C
      SUBROUTINE STORLP(K,ITYPE)
      DIMENSION AA(5,17),A(6,18),B(4,5,18),IBV(4),IDB(4,4),RHS1(4),
     1RHS2(4),NFS(4)
      COMMON/COM1/NCONS,NC,IC,NG,NL1,NGL,ICL1,NGP1,NGP2
      COMMON/COM2/B,IDB,RHS1,RHS2,NFS
      COMMON/COM3/A,IBV
      COMMON/COM4/AA
      IF (NFS(K).NE.0) GO TO 30
      IF (ITYPE.EQ.1) GO TO 70
      DO 10 I=1,NC
      DO 10 J=1,IC
   10 B(K,I,J)=A(I,J)
      DO 20 I=1,NCONS
   20 IDB(K,I)=IBV(I)
      RETURN
   30 DO 40 I=1,NC
      DO 40 J=1,ICL1
   40 B(K,I,J)=AA(I,J)
      B(K,1,IC)=RHS1(K)
      DO 50 I=2,NCONS
   50 B(K,I,IC)=RHS2(K)
      B(K,NC,IC)=0.0
      DO 60 I=1,NCONS
```

```
   60 IDB(K,I)=NG+I
      RETURN
   70 DO 80 I=1,NC
   80 B(K,I,IC)=A(I,IC)
      RETURN
      END
C
C
C
C
C
      SUBROUTINE TRANLP(K)
      DIMENSION B(4,5,18),IDB(4,4),RHS1(4),RHS2(4),NFS(4)
      COMMON/COM1/NCONS,NC,IC,NG,NL1,NGL,ICL1,NGP1,NGP2
      COMMON/COM2/B,IDB,RHS1,RHS2,NFS
      KP1=K+1
      RHS1(KP1)=RHS1(K)
      RHS2(KP1)=RHS2(K)
      DO 10 I=1,NC
      DO 10 J=1,IC
   10 B(KP1,I,J)=B(K,I,J)
      DO 20 I=1,NCONS
   20 IDB(KP1,I)=IDB(K,I)
      NFS(KP1)=NFS(K)
      RETURN
      END
C
C
C
C
C
      SUBROUTINE RESET(K,RH1,RH2,D1,D2)
      DIMENSION A(6,18),B(4,5,18),IBV(4),IDB(4,4),RHS1(4),RHS2(4),NFS(4)
      COMMON/COM1/NCONS,NC,IC,NG,NL1,NGL,ICL1,NGP1,NGP2
      COMMON/COM2/B,IDB,RHS1,RHS2,NFS
      COMMON/COM3/A,IBV
      D1=RHS1(K)
      D2=RHS2(K)
      RHS1(K)=RH1
      RHS2(K)=RH2
      D1=RH1-D1
      D2=RH2-D2
      DO 10 I=1,NC
      DO 10 J=1,IC
   10 A(I,J)=B(K,I,J)
      DO 20 I=1,NCONS
   20 IBV(I)=IDB(K,I)
      IF (NFS(K).EQ.1) GO TO 30
      RETURN
   30 A(1,IC)=RH1
      DO 40 I=2,NCONS
   40 A(I,IC)=RH2
      RETURN
      END
C
C
C
C
```

```
C
      SUBROUTINE PARARH(D1,D2,A,ITYPE)
      DIMENSION A(6,18)
      COMMON/COM1/NCONS,NC,IC,NG,NL1,NGL,ICL1,NGP1,NGP2
      ITYPE=1
      DO 30 I=1,NCONS
      WV=0.0
      DO 10 J=NGP2,ICL1
   10 WV=WV+A(I,J)
      WV=WV*D2
      XB=A(I,IC)+A(I,NGP1)*D1+WV
      IF (XB.GT.-1.0E-8) GO TO 20
      ITYPE=-1
      GO TO 30
   20 IF (XB.LT.1.0E-8) XB=0.0
   30 A(I,IC)=XB
      WV=0.0
      DO 40 I=NGP2,ICL1
   40 WV=WV+A(NC,I)
      WV=WV*D2
      A(NC,IC)=A(NC,IC)+A(NC,NGP1)*D1+WV
      RETURN
      END
C
C
C
C
C
      SUBROUTINE LPDS(IT,A,ID,NFSIND)
      DIMENSION A(6,18),ID(4)
      COMMON/COM1/M,NC,IC,NG,L1,NGL,ICL1,NGP1,NGP2
      IW=NC
      IF (IT.EQ.-1) GO TO 300
      IW=IW+1
      DO 20 J=1,IC
   20 A(IW,J)=0.0
C
C  SET UP PHASE I AND FIND PIVOTAL COLUMN
C
      KC=0
      Q=0.0
      DO 40 J=1,NG
      S=0.0
      DO 30 I=L1,M
   30 S=S-A(I,J)
      A(IW,J)=S
      IF(S.GE.Q) GO TO 40
      Q=S
      KC=J
   40 CONTINUE
      S=0.0
      DO 50 I=L1,M
   50 S=S-A(I,IC)
      A(IW,IC)=S
      IF(KC.GT.0) GO TO 120
      IW=IW-1
C
C  SIMPLEX ITERATION
```

```
C
  100 KC=0
      Q=0.0
      DO 110 J=1,NGL
      IF (A(IW,J).GE.Q) GO TO 110
      Q=A(IW,J)
      KC=J
  110 CONTINUE
      IF (KC.EQ.0) GO TO 200
  120 KR=0
      Q=1.0E70
      DO 130 I=1,M
      IF(A(I,KC).LE.1.0E-8) GO TO 130
      P=A(I,IC)/A(I,KC)
      IF(P.GT.Q) GO TO 130
      Q=P
      KR=I
  130 CONTINUE
      IF(KR.EQ.0) GO TO 250
      P=A(KR,KC)
      ID(KR)=KC
      DO 140 J=1,IC
  140 A(KR,J)=A(KR,J)/P
      DO 160 I=1,IW
      IF(I.EQ.KR) GO TO 160
      DO 150 J=1,IC
      IF(J.EQ.KC) GO TO 150
      A(I,J)=A(I,J)-A(KR,J)*A(I,KC)
      Q=ABS(A(I,J))
      IF(Q.LT.1.0E-8) A(I,J)=0.0
  150 CONTINUE
  160 CONTINUE
      DO 170 I=1,IW
  170 A(I,KC)=0.0
      A(KR,KC)=1.0
      GO TO 100
C
C  CHECK FOR PHASE II COMPLETION AND FEASIBILITY
C
  200 IF (IW.EQ.M+1) GO TO 210
      IW=IW-1
      GO TO 100
  210 DO 220 I=1,M
      IF (ID(I).GT.NGL) GO TO 230
  220 CONTINUE
      NFSIND=0
      GO TO 999
  230 NFSIND=1
      GO TO 999
  250 WRITE (6,2000)
 2000 FORMAT (19H SOLUTION UNBOUNBED)
      STOP
C
C  DUAL SIMPLEX ITERATION
C
  300 KR=0
      Q=0.0
      DO 310 I=1,M
```

```
      IF (A(I,IC).GE.Q) GO TO 310
      Q=A(I,IC)
      KR=I
  310 CONTINUE
      IF (KR.EQ.0) GO TO 100
      KC=0
      Q=-1.0E70
      DO 320 J=1,NGL
      IF (A(KR,J).GT.-1.0E-8) GO TO 320
      P=A(IW,J)/A(KR,J)
      IF (P.LT.Q) GO TO 320
      Q=P
      KC=J
  320 CONTINUE
      IF (KC.EQ.0) GO TO 370
      P=A(KR,KC)
      ID(KR)=KC
      DO 330 J=1,IC
  330 A(KR,J)=A(KR,J)/P
      DO 350 I=1,IW
      IF (I.EQ.KR) GO TO 350
      DO 340 J=1,IC
      IF (J.EQ.KC) GO TO 340
      A(I,J)=A(I,J)-A(KR,J)*A(I,KC)
      Q=ABS(A(I,J))
      IF (Q.LT.1.0E-8) A(I,J)=0.0
  340 CONTINUE
  350 CONTINUE
      DO 360 I=1,IW
  360 A(I,KC)=0.0
      A(KR,KC)=1.0
      GO TO 300
  370 NFSIND=1
  999 RETURN
      END
```

APPENDIX B6

PIG PRODUCTION POLICY PROGRAM

```
      DIMENSION IPOL(80,241),LWTOPT(80),IPTOPT(80),IFAULT(8)
      MAXNLW=80
      MAXNPT=241
      MINLW=20
      MAXLW=200
      MAXLWG=5
      DO 10 I=1,8
   10 IFAULT(I)=0
C
C  READ IN DATA AND CHECK FOR ERRORS
C
      READ (11) LWMIN,LWMAX,LWGMAX
      NLWMAX=LWMAX-LWMIN
      NPTMAX=3*NLWMAX+1
      IF (LWMIN.LT.MINLW) IFAULT(1)=1
      IF (LWMAX.GT.MAXLW) IFAULT(2)=1
      IF (LWGMAX.GT.MAXLWG) IFAULT(3)=1
      IF (NLWMAX.GT.MAXNLW) IFAULT(4)=1
      IF (NLWMAX.LT.1) IFAULT(5)=1
      IF (NPTMAX.GT.MAXNPT) IFAULT(6)=1
      READ (5,1010) LWT,IPT
 1010 FORMAT (2I4)
      LWV=LWT-LWMIN
      IF (LWV.LT.1.OR.LWV.GT.NLWMAX) IFAULT(7)=1
      IF (IPT.LT.1.OR.IPT.GT.MAXNPT) IFAULT(8)=1
      K=0
      DO 20 I=1,8
      IF (IFAULT(I).EQ.0) GO TO 20
      K=K+1
   20 CONTINUE
      IF (K.EQ.0) GO TO 40
      WRITE (6,3000) K
 3000 FORMAT (I4,26H INPUT ERROR(S) OF TYPE :-)
      DO 30 I=1,8
      IF (IFAULT(I).EQ.0) GO TO 30
      WRITE (6,3010) I
 3010 FORMAT (30X,I2)
   30 CONTINUE
      GO TO 999
   40 DO 45 I=1,MAXNLW
      LWTOPT(I)=0
   45 IPTOPT(I)=0
      IF (NLWMAX.GT.1) GO TO 50
      LWTOPT(1)=LWT
      IPTOPT(1)=IPT
      NPER=1
```

```
      GO TO 210
   50 MAXPER=NLWMAX
      NLWL1=NLWMAX-1
C
C  DETERMINE NUMBER OF PERIODS REQUIRED
C
      NPER=0
      LW2UP=LWGMAX
      IF (LW2UP.GT.NLWMAX) LW2UP=NLWMAX
      NRECS=1
      DO 60 LW=1,LW2UP
      NRECS=NRECS+3
   60 READ (11) L,(IPOL(LW,I),I=1,NRECS)
      LW1LO=0
      LW1UP=0
      DO 100 IPER=2,MAXPER
      DO 70 I=1,NLWMAX
      DO 70 J=1,NPTMAX
   70 IPOL(I,J)=0
      LW1LO=LW1LO+1
      LW1UP=LW2UP
      IF (LW1UP.GT.NLWL1) LW1UP=NLWL1
      LW2LO=LW1LO+1
      LW2UP=LW1UP+LWGMAX
      IF (LW2UP.GT.NLWMAX) LW2UP=NLWMAX
      NRECS=3*LW1LO+1
      DO 80 LW=LW2LO,LW2UP
      NRECS=NRECS+3
   80 READ (11) L,(IPOL(LW,I),I=1,NRECS)
      K=IPOL(LWV,IPT)
      IF (K.EQ.0) GO TO 90
      NPER=IPER
      IP=K
      GO TO 100
   90 IF (NPER.EQ.0) GO TO 100
      GO TO 110
  100 CONTINUE
  110 IF (NPER.GT.0) GO TO 120
      IF (LWV.GT.LWGMAX) GO TO 115
      LWTOPT(1)=LWT
      IPTOPT(1)=IPT
      NPER=1
      GO TO 210
  115 K=1
      WRITE (6,3020) K
 3020 FORMAT (35H ERROR - NO SOLUTION FOUND AT STAGE,I2)
      GO TO 999
  120 LWTOPT(NPER)=LWT
      IPTOPT(NPER)=IPT
      LWV=IP/1000
      IPT=IP-LWV*1000
      LWT=LWV+LWMIN
      NP=NPER-1
      LWTOPT(NP)=LWT
      IPTOPT(NP)=IPT
      MAXREC=NPER
      NPERL2=NPER-2
C
```

```
C   ***** DETERMINE OPTIMAL POLICY AT EACH STAGE *****
C
      DO 200 ICOUNT=1,NPERL2
      MAXREC=MAXREC-1
      REWIND 11
C
C  READ POLICY MATRIX AT END OF PERIOD 1
C
      LW2UP=LWGMAX
      IF (LW2UP.GT.NLWMAX) LW2UP=NLWMAX
      READ (11) LWMIN,LWMAX,LWGMAX
      NRECS=1
      DO 130 LW=1,LW2UP
      NRECS=NRECS+3
  130 READ (11) L,(IPOL(LW,I),I=1,NRECS)
C
C  READ POLICY MATRIX AT END OF EACH SUBSEQUENT PERIOD
C
      LW1LO=0
      LW1UP=0
      DO 170 IPER=2,MAXREC
      IF (IPER.NE.MAXREC) GO TO 150
      DO 140 I=1,NLWMAX
      DO 140 J=1,NPTMAX
  140 IPOL(I,J)=0
  150 LW1LO=LW1LO+1
      LW1UP=LW2UP
      IF (LW1UP.GT.NLWL1) LW1UP=NLWL1
      LW2LO=LW1LO+1
      LW2UP=LW1UP+LWGMAX
      IF (LW2UP.GT.NLWMAX) LW2UP=NLWMAX
      NRECS=3*LW1LO+1
      DO 160 LW=LW2LO,LW2UP
      NRECS=NRECS+3
  160 READ (11) L,(IPOL(LW,I),I=1,NRECS)
  170 CONTINUE
      IP=IPOL(LWV,IPT)
      IF (IP.GT.0) GO TO 180
      K=2
      WRITE (6,3020) K
      WRITE (6,3021) NPER,ICOUNT,IPER
 3021 FORMAT (3I6)
      GO TO 999
  180 LWV=IP/1000
      IPT=IP-LWV*1000
      LWT=LWV+LWMIN
      NP=NP-1
      LWTOPT(NP)=LWT
  200 IPTOPT(NP)=IPT
  210 K=0
      IPT=1
      WRITE (6,2000) K,LWMIN,IPT
 2000 FORMAT (3X,23H OPTIMAL FEEDING POLICY//29H  PERIOD  LIVEWEIGHT  PR
     1OTEIN//I6,2I10)
      DO 220 I=1,NPER
  220 WRITE (6,2010) I,LWTOPT(I),IPTOPT(I)
 2010 FORMAT (I6,2I10)
      WRITE (10) LWMIN,LWMAX,LWGMAX,NPER
```

```
      WRITE (10) (LWTOPT(I),I=1,NPER)
      WRITE (10) (IPTOPT(I),I=1,NPER)
  999 STOP
      END
```

APPENDIX B7

PIG PRODUCTION FEEDING POLICY PROGRAM

```
      DIMENSION EW(80,2),PRCONT(80,2),R(80,2),F(80,2),C(10),E(10),P(10),
     1ACID(10,3),REF(3),AA(5,17),A(6,18),IBV(4),G(4),PRV(5,4),PRD(5,4),
     2ING(4),SOL(4),LCING(4),COMP(4),ID(4),LWTOPT(80),IPTOPT(80),
     3IFAULT(8)
      COMMON/COM1/NCONS,NC,IC,NG,NL1,NGL,ICL1,NGP1,NGP2
      MAXNLW=80
      MAXNPT=241
      MAXNV=10
      MAXLWG=5
      MAXNG=4
      MINLW=20
      MAXLW=200
      MAXITS=200
      NDAYS=5
      X1=4.906
      NCONS=4
      NLTS=0
      NEQS=1
      NGTS=3
      NC=NCONS+1
      NC1=NC+1
      NL1=NLTS+1
      NLE=NLTS+NEQS
      DO 10 I=1,8
   10 IFAULT(I)=0
      DO 30 I=1,MAXNLW
      LWTOPT(I)=0
      IPTOPT(I)=0
      DO 30 J=1,2
      EW(I,J)=0.0
      PRCONT(I,J)=0.0
      F(I,J)=0.0
   30 R(I,J)=0.0
      DO 40 I=1,MAXNV
      C(I)=0.0
      E(I)=0.0
      P(I)=0.0
      DO 40 J=1,3
   40 ACID(I,J)=0.0
      IC=MAXNV+NCONS+NGTS
      DO 50 I=1,NC
      DO 50 J=1,IC
   50 AA(I,J)=0.0
      IC=IC+1
      DO 60 I=1,NC1
      DO 60 J=1,IC
```

```
   60 A(I,J)=0.0
      DO 80 I=1,NCONS
   80 IBV(I)=0
      DO 90 I=1,MAXNG
      G(I)=0.0
      DO 90 J=1,MAXLWG
      PRD(J,I)=0.0
   90 PRV(J,I)=0.0
C
C  READ IN DATA AND CHECK FOR ERRORS
C
      READ (5,1010) NVARS,LWMIN,LWMAX,LWGMAX,IGAMAX
 1010 FORMAT (5I4)
      NLWMAX=LWMAX-LWMIN
      NPTMAX=3*NLWMAX+1
      IF (NVARS.GT.MAXNV) IFAULT(1)=1
      IF (LWMIN.LT.MINLW) IFAULT(2)=1
      IF (LWMAX.GT.MAXLW) IFAULT(3)=1
      IF (LWGMAX.GT.MAXLWG) IFAULT(4)=1
      IF (IGAMAX.GT.MAXNG) IFAULT(5)=1
      IF (NLWMAX.GT.MAXNLW) IFAULT(6)=1
      IF (NLWMAX.LT.1) IFAULT(7)=1
      IF (NPTMAX.GT.MAXNPT) IFAULT(8)=1
      K=0
      DO 100 I=1,8
      IF (IFAULT(I).EQ.0) GO TO 100
      K=K+1
  100 CONTINUE
      IF (K.EQ.0) GO TO 120
      WRITE (6,3000) K
 3000 FORMAT (I4,26H INPUT ERROR(S) OF TYPE :-)
      DO 110 I=1,8
      IF (IFAULT(I).EQ.0) GO TO 110
      WRITE (6,3010) I
 3010 FORMAT (30X,I2)
  110 CONTINUE
      GO TO 999
  120 DO 130 I=1,NVARS
  130 READ (5,1020) C(I),E(I),P(I),(ACID(I,J),J=1,3)
 1020 FORMAT (6F6.1)
      READ (5,1030) (REF(I),I=1,3)
 1030 FORMAT (3F6.1)
      READ (5,1040) PERCOS
 1040 FORMAT (F6.1)
      READ (5,1050) PRTMIN,PTINC
 1050 FORMAT (2F6.3)
      PRTMIN=PRTMIN/X1
      PTINC=PTINC/X1
      DO 140 I=1,LWGMAX
      READ (5,1060) (PRV(I,J),J=1,IGAMAX)
 1060 FORMAT (4F6.2)
      DO 140 J=1,IGAMAX
      IF (I.NE.LWGMAX) GO TO 140
      G(J)=(X1/PRV(I,J)-X1)/1.1
  140 PRV(I,J)=PRV(I,J)/X1
      READ (5,1070) RZ,DZ,RK,DK
 1070 FORMAT (4F10.6)
      READ (10) I,J,K,NPER
```

```
      IF (I.EQ.LWMIN.AND.J.EQ.LWMAX.AND.K.EQ.LWGMAX) GO TO 145
      WRITE (6,3020)
 3020 FORMAT (26H ERROR - INCONSISTENT DATA)
      GO TO 999
  145 READ (10) (LWTOPT(I),I=1,NPER)
      READ (10) (IPTOPT(I),I=1,NPER)
C
C  CALCULATE MODEL PARAMETERS
C
      K=LWMIN-MINLW
      RZ=RZ+K*DZ
      RK=RK+K*DK
      DZ=0.5*DZ
      DK=0.5*DK
      RZ=RZ+DZ
      RK=RK+DK
      RO=(RK/RZ)*1000.0
      WAV=LWMIN+0.5
      EW0=0.475*WAV**0.75
      DP=0.5*NDAYS*PTINC
      DP=0.365*DP
      PRT0=0.365*PRTMIN+DP
      A2=(0.0115+0.0073/RZ)*1000.0
      WV=PRT0
      DO 150 I=1,NLWMAX
      DO 150 J=1,2
      RZ=RZ+DZ
      RK=RK+DK
      R(I,J)=(RK/RZ)*1000.0
      F(I,J)=(0.0115+0.0073/RZ)*1000.0
      WAV=WAV+0.5
      EW(I,J)=0.475*WAV**0.75
      WV=WV+DP
  150 PRCONT(I,J)=WV
      WV=0.5*0.365*NDAYS
      DO 160 I=1,IGAMAX
      G(I)=0.0535*G(I)*1000.0
      DO 160 J=1,LWGMAX
  160 PRD(J,I)=WV*(PRV(J,I)-PRV(J,1))
      DP=0.365*PTINC
C
C  SET UP LP MATRIX
C
      NG=NVARS+NGTS
      NGL=NG+NLTS
      NGP1=NG+1
      NGP2=NG+2
      ICL1=NG+NCONS
      IC=ICL1+1
      DO 170 I=1,NVARS
      AA(NC,I)=NDAYS*C(I)
      AA(1,I)=E(I)-0.0121*P(I)
      DO 170 J=1,3
  170 AA(J+1,I)=(P(I)*ACID(I,J))/REF(J)
      DO 180 I=1,NGTS
  180 AA(NLE+I,NVARS+I)=-1.0
      DO 190 I=1,NCONS
      IBV(I)=NG+I
```

```
  190 AA(I,NG+I)=1.0
      DO 200 I=1,NC
      DO 200 J=1,ICL1
  200 A(I,J)=AA(I,J)
C
C  CALCULATE LEAST COST RATION FOR PERIOD 1
C
      PTD=NDAYS*PTINC
      CUCOST =0.0
      PRTOT=PRTO
      EM=EW0-PRTOT
      LW1=0
      LW2=LWTOPT(1)-LWMIN
      LWG=LW2
      IF (LWG.EQ.1) GO TO 210
      LWH=1
      LWV=LWG/2
      K=LWV*2
      IF (K.NE.LWG) LWH=2
      PRTOT=PRCONT(LWV,LWH)
      RO=R(LWV,LWH)
      EM=EW(LWV,LWH)-PRTOT
      A2=F(LWV,LWH)
  210 IPT1=1
      IPT2=IPTOPT(1)
      PT1=PRTMIN
      PT2=PRTMIN+LW2*PTD+(IPT2-1)*PTINC
      WV1=IPT2-IPT1
      WV2=LWG
      WV1=(WV1/WV2)+1.05
      IGAMV=WV1
      PR=PRV(LWG,IGAMV)
      RHS1=EM-PRD(LWG,IGAMV)+PR*(A2+G(IGAMV))
      RHS2=PR*RO
      A(1,IC)=RHS1
      DO 220 I=2,NCONS
  220 A(I,IC)=RHS2
      ITYPE=0
      CALL LPDSRAT(ITYPE,A,IBV,NFSIND)
      IF (NFSIND.EQ.0) GO TO 230
      WRITE (6,3100)
 3100 FORMAT (35H ERROR - NO FEASIBLE SOLUTION FOUND)
      GO TO 999
  230 WRITE (6,2000)
 2000 FORMAT (1H1,32X,23H OPTIMAL  FEEDING  PLAN///87H  PERIOD  LIVEWEIG
     1HT    PROTEIN     TOTAL   RATION  REQUIRED  ON  MIDDLE  DAY  OF  PER
     2IOD/8H  NUMBER,4X,5H (KG),6X,5H (KG),6X,5H COST,9X,23H COMPOSITION
     3 (DM BASIS),9X,9H QUANTITY/9X,11H START END ,11H START  END,3X,4H
     4(#),2X,4(10H FOOD   % ),2X,5H (KG)/)
C
C  CALCULATE LEAST COST RATION FOR EACH PERIOD
C
      DO 400 IPER=1,NPER
      IF (IPER.EQ.1) GO TO 280
      LW1=LW2
      LW2=LWTOPT(IPER)-LWMIN
      IPT1=IPT2
      IPT2=IPTOPT(IPER)
```

```
      PT1=PT2
      PT2=PRTMIN+LW2*PTD+(IPT2-1)*PTINC
      LWG=LW2-LW1
      LWH=1
      LWV=LWG/2
      K=LWV*2
      IF (K.NE.LWG) LWH=2
      LWV=LW1+LWV
      R0=R(LWV,LWH)
      EM=EW(LWV,LWH)-PRCONT(LWV,LWH)
      A2=F(LWV,LWH)
      WV1=IPT2-IPT1
      WV2=LWG
      WV1=(WV1/WV2)+1.05
      IGAMV=WV1
      PR=PRV(LWG,IGAMV)
      PRD1=(IPT1-1.0)*DP
      RH1=EM-PRD1-PRD(LWG,IGAMV)+PR*(A2+G(IGAMV))
      RH2=PR*R0
      D1=RH1-RHS1
      D2=RH2-RHS2
      RHS1=RH1
      RHS2=RH2
      ITYPE=1
      DO 260 I=1,NCONS
      WV=0.0
      DO 240 J=NGP2,ICL1
  240 WV=WV+A(I,J)
      WV=WV*D2
      XB=A(I,IC)+A(I,NGP1)*D1+WV
      IF (XB.GT.-1.0E-8) GO TO 250
      ITYPE=-1
      GO TO 260
  250 IF (XB.LT.1.0E-8) XB=0.0
  260 A(I,IC)=XB
      WV=0.0
      DO 270 I=NGP2,ICL1
  270 WV=WV+A(NC,I)
      WV=WV*D2
      A(NC,IC)=A(NC,IC)+A(NC,NGP1)*D1+WV
      IF (ITYPE.EQ.0) GO TO 280
      CALL LPDSRAT(ITYPE,A,IBV,NFSIND)
      IF (NFSIND.EQ.0) GO TO 280
      WRITE (6,3100)
      GO TO 999
  280 CUCOST=CUCOST+(PERCOS-A(NC,IC))/100.0
C
C  SORT INGREDIENTS IN DESCENDING FOOD NO ORDER AND PRINT RESULTS
C
      QFED=0.0
      DO 320 I=1,NCONS
      ING(I)=IBV(I)
      SOL(I)=A(I,IC)
      IF (ING(I).LE.NVARS) GO TO 310
      ING(I)=0
      SOL(I)=0.0
      GO TO 320
  310 QFED=QFED+SOL(I)
```

```
  320 CONTINUE
      DO 330 I=1,NCONS
      IF (ING(I).EQ.0) GO TO 330
      SOL(I)=(SOL(I)/QFED)*100.0
      IF (SOL(I).GT.0.05) GO TO 330
      SOL(I)=0.0
      ING(I)=0
  330 CONTINUE
      L=NCONS
      DO 360 I=1,NCONS
      IF (ING(I).GT.0) GO TO 340
      ID(I)=L
      L=L-1
      GO TO 360
  340 K=1
      DO 350 J=1,NCONS
      IF (J.EQ.I) GO TO 350
      IF (ING(J).GT.ING(I)) K=K+1
  350 CONTINUE
      ID(I)=K
  360 CONTINUE
      DO 370 I=1,NCONS
      J=ID(I)
      COMP(J)=SOL(I)
  370 LCING(J)=ING(I)
      LWT1=LW1+LWMIN
      LWT2=LW2+LWMIN
      WRITE (6,2010) IPER,LWT1,LWT2,PT1,PT2,CUCOST,
     1(LCING(I),COMP(I),I=1,4),QFED
 2010 FORMAT (I6,3X,2I5,1X,2F6.2,F7.2,1X,4(I4,F6.1),F7.2)
  400 CONTINUE
  999 STOP
      END
C
C
C
C
C
      SUBROUTINE LPDSRAT(IT,A,ID,NFSIND)
      DIMENSION A(6,18),ID(4)
      COMMON/COM1/M,NC,IC,NG,L1,NGL,ICL1,NGP1,NGP2
      IW=NC
      IF (IT.EQ.-1) GO TO 300
      IW=IW+1
      DO 20 J=1,IC
   20 A(IW,J)=0.0
C
C  SET UP PHASE I AND FIND PIVOTAL COLUMN
C
      KC=0
      Q=0.0
      DO 40 J=1,NG
      S=0.0
      DO 30 I=L1,M
   30 S=S-A(I,J)
      A(IW,J)=S
      IF(S.GE.Q) GO TO 40
      Q=S
```

```
      KC=J
   40 CONTINUE
      S=0.0
      DO 50 I=L1,M
   50 S=S-A(I,IC)
      A(IW,IC)=S
      IF(KC.GT.0) GO TO 120
      IW=IW-1
C
C  SIMPLEX ITERATION
C
  100 KC=0
      Q=0.0
      DO 110 J=1,NGL
      IF (A(IW,J).GE.Q) GO TO 110
      Q=A(IW,J)
      KC=J
  110 CONTINUE
      IF (KC.EQ.0) GO TO 200
  120 KR=0
      Q=1.0E70
      DO 130 I=1,M
      IF(A(I,KC).LE.1.0E-8) GO TO 130
      P=A(I,IC)/A(I,KC)
      IF(P.GT.Q) GO TO 130
      Q=P
      KR=I
  130 CONTINUE
      IF(KR.EQ.0) GO TO 250
      P=A(KR,KC)
      ID(KR)=KC
      DO 140 J=1,IC
  140 A(KR,J)=A(KR,J)/P
      DO 160 I=1,IW
      IF(I.EQ.KR) GO TO 160
      DO 150 J=1,IC
      IF(J.EQ.KC) GO TO 150
      A(I,J)=A(I,J)-A(KR,J)*A(I,KC)
      Q=ABS(A(I,J))
      IF(Q.LT.1.0E-8) A(I,J)=0.0
  150 CONTINUE
  160 CONTINUE
      DO 170 I=1,IW
  170 A(I,KC)=0.0
      A(KR,KC)=1.0
      GO TO 100
C
C  CHECK FOR PHASE II COMPLETION AND FEASIBILITY
C
  200 IF (IW.EQ.M+1) GO TO 210
      IW=IW-1
      GO TO 100
  210 DO 220 I=1,M
      IF (ID(I).GT.NGL) GO TO 230
  220 CONTINUE
      NFSIND=0
      GO TO 999
  230 NFSIND=1
```

```
      GO TO 999
  250 WRITE (6,2000)
 2000 FORMAT (19H SOLUTION UNBOUNBED)
      STOP
C
C  DUAL SIMPLEX ITERATION
C
  300 KR=0
      Q=0.0
      DO 310 I=1,M
      IF (A(I,IC).GE.Q) GO TO 310
      Q=A(I,IC)
      KR=I
  310 CONTINUE
      IF (KR.EQ.0) GO TO 100
      KC=0
      Q=-1.0E70
      DO 320 J=1,NGL
      IF (A(KR,J).GT.-1.0E-8) GO TO 320
      P=A(IW,J)/A(KR,J)
      IF (P.LT.Q) GO TO 320
      Q=P
      KC=J
  320 CONTINUE
      IF (KC.EQ.0) GO TO 370
      P=A(KR,KC)
      ID(KR)=KC
      DO 330 J=1,IC
  330 A(KR,J)=A(KR,J)/P
      DO 350 I=1,IW
      IF (I.EQ.KR) GO TO 350
      DO 340 J=1,IC
      IF (J.EQ.KC) GO TO 340
      A(I,J)=A(I,J)-A(KR,J)*A(I,KC)
      Q=ABS(A(I,J))
      IF (Q.LT.1.0E-8) A(I,J)=0.0
  340 CONTINUE
  350 CONTINUE
      DO 360 I=1,IW
  360 A(I,KC)=0.0
      A(KR,KC)=1.0
      GO TO 300
  370 NFSIND=1
  999 RETURN
      END
```

APPENDIX B8

ENTERPRISE MODEL - RATION FORMULATION PROGRAM

```
      DEFINE FILE 9(352,352,L,IAV9)
      DIMENSION AA(3,50),OBF(50),RVAL(161),PCVAL(161),LWVAL(20),
     1GVAL(11),DMAVAL(20),DCPVAL(20,11),COSMIN(11),LCINGR(11,3),
     2DCPINT(11),DCP(11),INGS(3),COMP(11,3),XB(3),QFED(11)
      COMMON/LPCOM/NCONS,NVARS,NGTS,NG,NGL,NLE,NL1,IC,NC
      COMMON/FECOM/AA,OBF,NRVAL,RMAX,NC1,EPSI
      IAV9=1
      NVMAX=50
      NGMAX=11
      NRMAX=161
      MAXNLW=351
      NLWMAX=20
      NCONS=3
      NLTS=0
      NEQS=2
      NGTS=1
      NC=NCONS+1
      NC1=NC+1
      NL1=NLTS+1
      NLE=NLTS+NEQS
      EPSI=0.0005
      READ (5,1010) NVARS,LWMIN,LWMAX,LWINCR
 1010 FORMAT (4I4)
      IC=NVARS+NC+NGTS
      NG=NVARS+NGTS
      NGL=NG+NLTS
      IF (LWMAX.GT.LWMIN) GO TO 5
      WRITE (6,4010)
 4010 FORMAT (33H MAX LIVEWEIGHT LE MIN LIVEWEIGHT)
      GO TO 999
    5 I=LWMAX-LWMIN
      J=I/LWINCR
      K=J*LWINCR
      IF (I-K.EQ.0) GO TO 7
      WRITE (6,4015)
 4015 FORMAT (68H DIFFERENCE BETWEEN MAX LW AND MIN LW NOT A MULTIPLE OF
     1 LW INCREMENT)
      GO TO 999
    7 I=LWMAX-LWMIN+1
      IF (I.LE.MAXNLW) GO TO 8
      WRITE (6,4017)
 4017 FORMAT (36H TOO MANY LIVEWEIGHT VALUES REQUIRED)
      GO TO 999
    8 DO 10 J=1,NVMAX
      OBF(J)=0.0
      DO 10 I=1,NCONS
```

```
   10 AA(I,J)=0.0
      DO 20 I=1,NCONS
   20 READ (5,1020) (AA(I,J),J=1,NVARS)
 1020 FORMAT (12F6.2)
      READ (5,1020) (OBF(I),I=1,NVARS)
      DO 22 I=1,NRMAX
      PCVAL(I)=0.0
   22 RVAL(I)=0.0
      READ (5,1030) RMIN,RMAX,RINT
 1030 FORMAT (2F6.2,F6.3)
      IF (RMIN.LT.RMAX) GO TO 25
      WRITE (6,4020)
 4020 FORMAT (23H RMIN GREATER THAN RMAX)
      GO TO 999
   25 IF (RMIN.GT.5.999.AND.RMAX.LT.15.001) GO TO 27
      WRITE (6,4030)
 4030 FORMAT (50H RMIN OR RMAX OUTSIDE PERMITTED RANGE : 6.00-15.00)
      GO TO 999
   27 NRVAL=0
      R=RMIN-RINT
   30 NRVAL=NRVAL+1
      IF (NRVAL.LE.NRMAX) GO TO 32
      WRITE (6,4040) NRMAX
 4040 FORMAT (37H TOO MANY VALUES OF R IMPLIED,MAXIMUM,I4)
      GO TO 999
   32 R=R+RINT
      RVAL(NRVAL)=R
      IF (RMAX-R.GT.0.0001) GO TO 30
      RVAL(NRVAL)=RMAX
      READ (5,1040) NLWVAL
 1040 FORMAT (I3)
      IF (NLWVAL.LE.NLWMAX.AND.NLWVAL.GT.1) GO TO 35
      WRITE (6,4050) NLWMAX
 4050 FORMAT (58H NO. OF VALUES OF LIVEWEIGHT OUTSIDE PERMITTED RANGE :
     1 2-,I3)
      GO TO 999
   35 DO 40 I=1,NLWMAX
   40 LWVAL(I)=0
      READ (5,1050) (LWVAL(I),I=1,NLWVAL)
 1050 FORMAT (18I4)
      READ (5,1040) NGVAL
      IF (NGVAL.LE.NGMAX.AND.NGVAL.GT.1) GO TO 45
      WRITE (6,4060) NGMAX
 4060 FORMAT (56H NO. OF VALUES OF LW. GAIN OUTSIDE PERMITTED RANGE :  2
     1-,I3)
      GO TO 999
   45 DO 50 I=1,NGMAX
      DCP(I)=0.0
      DCPINT(I)=0.0
   50 GVAL(I)=0.0
      READ (5,1060) (GVAL(I),I=1,NGVAL)
 1060 FORMAT (11F7.4)
      DO 60 I=1,NLWMAX
      DMAVAL(I)=0.0
      DO 60 J=1,NGMAX
   60 DCPVAL(I,J)=0.0
      READ (5,1020) (DMAVAL(I),I=1,NLWVAL)
      DO 70 I=1,NLWVAL
```

```
   70 READ (5,1020) (DCPVAL(I,J),J=1,NGVAL)
      READ (5,1070) PERCOS
 1070 FORMAT (F7.2)
      IF (LWMIN.GE.LWVAL(1).AND.LWMIN.LT.LWVAL(NLWVAL)) GO TO 72
      WRITE (6,3010) LWVAL(1),LWVAL(NLWVAL)
 3010 FORMAT (45H MINIMUM LIVEWEIGHT OUTSIDE PERMITTED RANGE :,
     1I5,2H -,I5)
      GO TO 999
   72 IF (LWMAX.LE.LWVAL(NLWVAL)) GO TO 74
      WRITE (6,3020) LWVAL(1),LWVAL(NLWVAL)
 3020 FORMAT (45H MAXIMUM LIVEWEIGHT OUTSIDE PERMITTED RANGE :,
     1I5,2H -,I5)
      GO TO 999
   74 DO 80 I=1,NGMAX
      COSMIN(I)=0.0
      QFED(I)=0.0
      DO 80 J=1,NCONS
      COMP(I,J)=0.0
   80 LCINGR(I,J)=0
      NDAYS=1.0/GVAL(2)+0.1
      WRITE (9'1) LWMIN,LWMAX,LWINCR,NGVAL,NDAYS,PERCOS,NCONS,NVARS,
     1GVAL
      WRITE (6,2110) (GVAL(I),I=1,NGVAL)
 2110 FORMAT (1H1,5H LIVE,2X,7H RATION,17X,24H DAILY  LIVEWEIGHT  GAIN/
     17H WEIGHT,2X,5H SPEC,27X,5H (KG)/6H  (KG),9X,11F8.2)
C
C  SET INITIAL VALUES FOR DMA,DMAINT,DCP,DCPINT AND YME
C
      IF (LWVAL(1).EQ.LWMIN) GO TO 250
      DO 210 I=2,NLWVAL
      IF (LWVAL(I).LT.LWMIN) GO TO 210
      K1=I-1
      GO TO 230
  210 CONTINUE
  230 K2=K1+1
      LWUP=LWVAL(K2)
      LWDIF=LWUP-LWVAL(K1)
      WINCS=LWMIN-LWVAL(K1)
      WINCS=WINCS/LWINCR
      DMAINT=((DMAVAL(K2)-DMAVAL(K1))/LWDIF)*LWINCR
      DMA=DMAVAL(K1)+DMAINT*(WINCS-1.0)
      DO 240 I=1,NGVAL
      DCPINT(I)=((DCPVAL(K2,I)-DCPVAL(K1,I))/LWDIF)*LWINCR
  240 DCP(I)=DCPVAL(K1,I)+DCPINT(I)*(WINCS-1.0)
      GO TO 270
  250 DMAINT=0.0
      DMA=DMAVAL(1)
      DO 260 I=1,NGVAL
      DCPINT(I)=0.0
  260 DCP(I)=DCPVAL(1,I)
      K2=1
      LWUP=LWMIN
  270 YINC=1.39*0.061*LWINCR
      YME=1.39*(5.67+0.061*LWMIN)-YINC
      ZINC=23.0*0.0188*LWINCR
      ZMENUM=23.0*(6.28+0.0188*LWMIN)-ZINC
      IPOS=1
C
```

```
C  ITERATE FOR EACH LIVEWEIGHT
C
      DO 600 LW=LWMIN,LWMAX,LWINCR
      IF (LW.LE.LWUP) GO TO 330
      K1=K2
      K2=K1+1
      LWUP=LWVAL(K2)
      LWDIF=LWUP-LWVAL(K1)
      WINCS=LW-LWVAL(K1)
      WINCS=WINCS/LWINCR
      DMAINT=((DMAVAL(K2)-DMAVAL(K1))/LWDIF)*LWINCR
      DMA=DMAVAL(K1)+DMAINT*WINCS
      DO 320 I=1,NGVAL
      DCPINT(I)=((DCPVAL(K2,I)-DCPVAL(K1,I))/LWDIF)*LWINCR
  320 DCP(I)=DCPVAL(K1,I)+DCPINT(I)*WINCS
      GO TO 350
  330 DMA=DMA+DMAINT
      DO 340 I=1,NGVAL
  340 DCP(I)=DCP(I)+DCPINT(I)
  350 YME=YME+YINC
      ZMENUM=ZMENUM+ZINC
C
C   ITERATE FOR EACH LIVEWEIGHT GAIN
C
      DO 500 IG=1,NGVAL
      ZME=(ZMENUM*GVAL(IG))/(1.0-0.3*GVAL(IG))
      LR=0
      DO 420 IR=1,NRVAL
      R=RVAL(IR)
      TME=YME+ZME/R
      DMI=TME/R
      IF (DMI.GT.DMA) GO TO 420
      PCVAL(IR)=DCP(IG)/DMI
      IF (LR.GT.0) GO TO 420
      LR=IR-1
      IF (LR.GT.0) GO TO 410
      LR=1
      RMIN=RVAL(1)
      GO TO 420
  410 RMIN=(YME+SQRT(YME*YME+4*DMA*ZME))/(2.0*DMA)
      PCVAL(LR)=(DCP(IG)*RVAL(LR))/(YME+ZME/RVAL(LR))
  420 CONTINUE
      IF (LR.GT.0) GO TO 440
      COSMIN(IG)=99999.0
      QFED(IG)=0.0
      DO 430 I=1,NCONS
      LCINGR(IG,I)=0
  430 COMP(IG,I)=0.0
      GO TO 500
  440 CALL LCFEED(LR,RMIN,RVAL,PCVAL,YME,ZME,COSTL,Q,INGS,XB)
      COSMIN(IG)=COSTL
      QFED(IG)=Q
      DO 450 I=1,NCONS
      COMP(IG,I)=XB(I)
  450 LCINGR(IG,I)=INGS(I)
  500 CONTINUE
      IPOS=IPOS+1
      WRITE (9'IPOS) COSMIN,QFED,LCINGR,COMP
```

```
      DO 510 I=1,NGVAL
  510 IF (COSMIN(I).GT.99990.0) COSMIN(I)=0.0
      WRITE (6,2120) LW,(LCINGR(I,1),COMP(I,1),I=1,NGVAL)
 2120 FORMAT (/I5,11H  FOOD NO,%,11(I3,F5.1))
      DO 520 J=2,NCONS
  520 WRITE (6,2130) (LCINGR(I,J),COMP(I,J),I=1,NGVAL)
 2130 FORMAT (6X,10H FOOD NO,%,11(I3,F5.1))
      WRITE (6,2140) (QFED(I),I=1,NGVAL)
 2140 FORMAT (6X,10H QTY  (KG),11(1X,F7.2))
      WRITE (6,2150) (COSMIN(I),I=1,NGVAL)
 2150 FORMAT (6X,10H COST  (P),11(1X,F7.2))
  600 CONTINUE
  999 STOP
      END
C
C
C
C
C
      SUBROUTINE LCFEED(LR,RMIN,RVAL,PCVAL,YME,ZME,COSTL,Q,INGS,XB)
      DIMENSION AA(3,50),OBF(50),RHS(3),XB(3),DV(3),BETA(3,3),
     1RANVAL(200,7),RVAL(161),PCVAL(161),A(5,55),ID(3),INGRED(200,3),
     2INGS(3),BASINV(200,3,3),XBNIT(3),INGNIT(3)
      COMMON/LPCOM/NCONS,NVARS,NGTS,NG,NGL,NLE,NL1,IC,NC
      COMMON/FECOM/AA,OBF,NRVAL,RMAX,NC1,EPSI
      MAXNIT=200
      NITS=0
      COSNIT=99999.0
      QNIT=0.0
      NITIND=0
      DO 80 I=1,NCONS
      XBNIT(I)=0.0
   80 INGNIT(I)=0
      DO 90 I=1,MAXNIT
      DO 90 J=1,7
   90 RANVAL(I,J)=0.0
      DO 95 I=1,MAXNIT
      DO 95 J=1,NCONS
      INGRED(I,J)=0
      DO 95 K=1,NCONS
   95 BASINV(I,J,K)=0.0
      RHS(1)=1.0
      ICRN=1
      NFSNO=0
      INTNO=LR
      INTNO1=INTNO+1
      R=RMIN
      GO TO 105
  100 R=RVAL(INTNO)
  105 RU=RVAL(INTNO1)
      PCONC=PCVAL(INTNO)
      PCONCU=PCVAL(INTNO1)
      DU=RU-R
      PCGRAD=(PCONCU-PCONC)/(RU-RVAL(INTNO))
      IF (INTNO.EQ.LR) PCONC=PCONCU-PCGRAD*DU
      DELTAU=DU
      DELTAL=0.0
C
```

```
C  SET UP AND SOLVE L.P.
C
  110 RHS(2)=R
      RHS(3)=PCONC
      CALL LPSUB(AA,OBF,RHS,A,ID,NFSIND)
      DO 120 I=1,NCONS
  120 XB(I)=A(I,IC)
      DO 125 I=1,NCONS
      BETA(I,1)=A(I,NG+1)
      BETA(I,2)=A(I,NG+2)
  125 BETA(I,3)=-A(I,NG)
      IF (NFSIND.EQ.1) GO TO 200
      DV(1)=-A(NC,NG+1)
      DV(2)=-A(NC,NG+2)
      DV(3)=A(NC,NG)
C
C  DETERMINE RANGE OF SOLUTION IN CURRENT INTERVAL OR PART INTERVAL
C
  130 DO 140 I=1,NCONS
      WV1=BETA(I,2)+PCGRAD*BETA(I,3)
      IF (WV1.GT.-1.0E-8) GO TO 135
      DELTA=-XB(I)/WV1
      IF (DELTA-DELTAU.LT.-1.0E-8) DELTAU=DELTA
      GO TO 140
  135 IF (WV1.LT.1.0E-8) GO TO 140
      DELTA=-XB(I)/WV1
      IF (DELTA-DELTAL.GT.1.0E-8) DELTAL=DELTA
  140 CONTINUE
      DO 150 I=1,NCONS
      DO 145 J=1,NCONS
  145 BASINV(ICRN,I,J)=BETA(I,J)
      INGRED(ICRN,I)=ID(I)
      IF (ID(I).GT.NVARS) INGRED(ICRN,I)=0
  150 CONTINUE
      DO 155 I=1,NCONS
  155 RANVAL(ICRN,I)=DV(I)
      RANVAL(ICRN,4)=R+DELTAL
      RANVAL(ICRN,5)=PCONC-PCGRAD*R
      RANVAL(ICRN,6)=PCGRAD
      RANVAL(ICRN,7)=R+DELTAU
      ICRN=ICRN+1
      IF (ICRN.LE.MAXNIT) GO TO 165
      NITS=NITS+1
      NITIND=1
      IF (DU-DELTAU.LT.0.0001) GO TO 160
      RNIT=R+DELTAU+EPSI
      IF (RMAX-RNIT.LT.0.0001) NITIND=0
      R=RNIT
      DELTAL=-EPSI
      GO TO 300
  160 INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) NITIND=0
      RNIT=RVAL(INTNO)
      R=RNIT
      DELTAL=0.0
      GO TO 300
  165 IF (DU-DELTAU.LT.0.0001) GO TO 180
      R=R+DELTAU+EPSI
```

```
      IF (RMAX-R.LT.0.0001) GO TO 300
      DU=RU-R
      IF (DU.GT.0.0001) GO TO 170
      INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) GO TO 300
      INTNO1=INTNO+1
      GO TO 100
  170 DELTAU=DU
      DELTAL=-EPSI
      PCONC=PCONCU-PCGRAD*DU
      GO TO 110
C
C  NEW RANGE FOR R. CALCULATE NEW SOLUTION
C
  180 INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) GO TO 300
      INTNO1=INTNO+1
      WV1=DELTAU
      WV2=PCGRAD
      R=RVAL(INTNO)
      RU=RVAL(INTNO1)
      PCONC=PCVAL(INTNO)
      PCONCU=PCVAL(INTNO1)
      DU=RU-R
      PCGRAD=(PCONCU-PCONC)/DU
      DELTAU=DU
      DELTAL=0.0
      DO 190 I=1,NCONS
      XB(I)=XB(I)+(BETA(I,2)+WV2*BETA(I,3))*WV1
      IF (XB(I).LT.-1.0E-8) GO TO 110
      IF (ABS(XB(I)).LT.1.0E-8) XB(I)=0.0
  190 CONTINUE
      GO TO 130
C
C  DETERMINE RANGE OF INFEASIBILITY
C
  200 NFSNO=0
  210 NFSNO=NFSNO+1
      ITYPE=0
      DO 220 I=1,NCONS
      WV1=BETA(I,2)+PCGRAD*BETA(I,3)
      IF (WV1.GT.-1.0E-8) GO TO 215
      DELTA=-XB(I)/WV1
      IF (DELTA-DELTAU.LT.-1.0E-8) DELTAU=DELTA
      GO TO 220
  215 IF (WV1.LT.1.0E-8) GO TO 220
      DELTA=-XB(I)/WV1
      IF (DELTA-DELTAU.GT.1.0E-8) DELTAL=DELTA
  220 CONTINUE
      IF (NFSNO.GT.1) GO TO 250
      DO 230 I=1,6
  230 RANVAL(ICRN,I)=-10000.0
      RANVAL(ICRN,4)=R+DELTAL
      DO 240 I=1,NCONS
  240 INGRED(ICRN,I)=0
  250 R=R+DELTAU+EPSI
      IF (DU-DELTAU.GT.0.0001) GO TO 265
      ITYPE=1
```

```
      INTNO=INTNO+1
      R=RVAL(INTNO)
      IF (INTNO.LT.NRVAL) GO TO 255
      RANVAL(ICRN,7)=R
      ICRN=ICRN+1
      GO TO 300
  255 INTNO1=INTNO+1
      WV1=DELTAU
      WV2=PCGRAD
      RU=RVAL(INTNO1)
      PCONC=PCVAL(INTNO)
      PCONCU=PCVAL(INTNO1)
      DU=RU-R
      PCGRAD=(PCONCU-PCONC)/DU
      DELTAU=DU
      DELTAL=0.0
      DO 260 I=1,NCONS
      XB(I)=XB(I)+(BETA(I,2)+WV2*BETA(I,3))*WV1
      IF (XB(I).LT.-1.0E-8) GO TO 270
      IF (ABS(XB(I)).LT.1.0E-8) XB(I)=0.0
  260 CONTINUE
      GO TO 210
  265 RANVAL(ICRN,7)=R-EPSI
      GO TO 275
  270 RANVAL(ICRN,7)=R
  275 ICRN=ICRN+1
      IF (ICRN.GT.MAXNIT) GO TO 290
      IF (RMAX-R.LT.0.0001) GO TO 300
      DU=RU-R
      IF (DU.LT.0.0001) GO TO 280
      DELTAU=DU
      DELTAL=-EPSI
      PCONC=PCONCU-PCGRAD*DU
      GO TO 110
  280 IF (ITYPE.EQ.0) INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) GO TO 300
      INTNO1=INTNO+1
      IF (ITYPE.EQ.0) GO TO 100
      DELTAL=0.0
      GO TO 110
  290 NITS=NITS+1
      NITIND=1
      IF (DU-DELTAU.LT.0.0001) GO TO 295
      RNIT=R+DELTAU+EPSI
      IF (RMAX-RNIT.LT.0.0001) NITIND=0
      R=RNIT
      DELTAL=-EPSI
      GO TO 300
  295 IF (ITYPE.EQ.0) INTNO=INTNO+1
      IF (INTNO.EQ.NRVAL) NITIND=0
      INTNO1=INTNO+1
      RNIT=RVAL(INTNO)
      R=RNIT
      DELTAL=0.0
      GO TO 300
C
C  *****  DETERMINATION OF LEAST COST FEED  *****
C
```

```
  300 NCRN=ICRN-1
      RANVAL(ICRN,4)=R
      DO 350 ICRN=1,NCRN
      RL=RANVAL(ICRN,4)
      RU=RANVAL(ICRN,7)
      IF (RANVAL(ICRN,1).LT.-9999.0) GO TO 330
      DVAR1=RANVAL(ICRN,1)
      DVAR2=RANVAL(ICRN,2)
      DVAR3=RANVAL(ICRN,3)
      WV1=DVAR1+DVAR3*RANVAL(ICRN,5)
      WV2=DVAR2+DVAR3*RANVAL(ICRN,6)
      A1=WV2*YME
      A2=WV1*YME+WV2*ZME
      A3=WV1*ZME
C
C  DETERMINE TURNING POINT OF COST FUNCTION
C
      RTP=-1.0
      IF (A3.LT.1.0E-8) GO TO 310
      IF (A2.GT.-1.0E-8) GO TO 310
      RTP=-2*A3/A2
C
C  DETERMINATION OF MINIMUM COST FOR RANGE
C
  310 COSTL=A1+(A2+A3/RL)/RL
      RV=RL
      COST=A1+(A2+A3/RU)/RU
      IF (COST-COSTL.GT.-1.0E-8) GO TO 320
      COSTL=COST
      RV=RU
  320 IF (RTP.LT.RL.OR.RTP.GT.RU) GO TO 340
      COST=A1+(A2+A3/RTP)/RTP
      IF (COST-COSTL.GT.-1.0E-8) GO TO 340
      COSTL=COST
      RV=RTP
      GO TO 340
  330 COSTL=99999.0
      RV=99999.0
  340 RANVAL(ICRN,1)=COSTL
  350 RANVAL(ICRN,2)=RV
C
C  DETERMINATION OF OVERALL MINIMUM COST
C
      COSTL=RANVAL(1,1)
      RV=RANVAL(1,2)
      K=1
      IF (NCRN.EQ.1) GO TO 410
      DO 400 ICRN=2,NCRN
      IF (RANVAL(ICRN,1)-COSTL.GT.-1.0E-8) GO TO 400
      COSTL=RANVAL(ICRN,1)
      RV=RANVAL(ICRN,2)
      K=ICRN
  400 CONTINUE
  410 Q=0
      DO 420 I=1,NCONS
      XB(I)=0.0
  420 INGS(I)=0
      IF (COSTL.GT.99990.0) GO TO 500
```

```
      Q=(YME+ZME/RV)/RV
      RHS(2)=RV
      RHS(3)=RANVAL(K,5)+RV*RANVAL(K,6)
      DO 440 I=1,NCONS
      INGS(I)=INGRED(K,I)
      IF (INGS(I).EQ.0) GO TO 440
      DO 430 J=1,NCONS
  430 XB(I)=XB(I)+RHS(J)*BASINV(K,I,J)
      XB(I)=XB(I)*100.0
      IF (XB(I).GT.0.1) GO TO 440
      XB(I)=0.0
      INGS(I)=0
  440 CONTINUE
      DO 450 I=1,NCONS
      DV(I)=XB(I)
  450 INGRED(1,I)=INGS(I)
      L=3
      DO 480 I=1,NCONS
      IF (INGS(I).GT.0) GO TO 460
      ID(I)=L
      L=L-1
      GO TO 480
  460 K=1
      DO 470 J=1,NCONS
      IF (J.EQ.I) GO TO 470
      IF (INGS(J).GT.INGS(I)) K=K+1
  470 CONTINUE
      ID(I)=K
  480 CONTINUE
      DO 490 I=1,NCONS
      XB(ID(I))=DV(I)
  490 INGS(ID(I))=INGRED(1,I)
C
C  CHECK WHETHER ENTIRE E.C. RANGE COVERED IN ONE ITERATION
C
  500 IF (NITS.EQ.0) GO TO 600
      IF (COSTL.GT.COSNIT) GO TO 520
      COSNIT=COSTL
      QNIT=Q
      DO 510 I=1,NCONS
      XBNIT(I)=XB(I)
  510 INGNIT(I)=INGS(I)
  520 IF (NCRN.EQ.MAXNIT.AND.NITIND.EQ.1) GO TO 540
  525 DO 530 I=1,NCONS
      XB(I)=XBNIT(I)
  530 INGS(I)=INGNIT(I)
      COSTL=COSNIT
      Q=QNIT
      GO TO 600
  540 ICRN=1
      NFSNO=0
      DO 550 I=1,MAXNIT
      DO 550 J=1,7
  550 RANVAL(I,J)=0.0
      DO 560 I=1,MAXNIT
      DO 560 J=1,NCONS
      INGRED(I,J)=0
      DO 560 K=1,NCONS
```

```
  560 BASINV(I,J,K)=0.0
      R=RNIT
      RU=RVAL(INTNO1)
      DU=RU-R
      IF (DU.GT.0.0001) GO TO 570
      INTNO=INTNO+1
      IF (INTNO.NE.NRVAL) GO TO 100
      GO TO 525
  570 PCONC=PCONCU-PCGRAD*DU
      DELTAU=DU
      GO TO 110
  600 RETURN
      END
C
C
C
C
C
      SUBROUTINE LPSUB(AA,C,RHS,A,ID,NFSIND)
      COMMON/LPCOM/M,N,NGTS,NG,NGL,NLE,L1,IC,M1
      DIMENSION AA(3,50),C(50),A(5,55),RHS(3),ID(3)
      IW=M+2
      NCMAX=55
C
C  SET UP LP MATRIX
C
      DO 5 I=1,IW
      DO 5 J=1,NCMAX
    5 A(I,J)=0.0
      DO 10 J=1,N
      A(M1,J)=C(J)
      DO 10 I=1,M
   10 A(I,J)=AA(I,J)
      DO 15 I=1,M
      A(I,IC)=RHS(I)
   15 A(I,NG+I)=1.0
      DO 20 I=1,NGTS
   20 A(NLE+I,N+I)=-1.0
      DO 25 I=1,M
   25 ID(I)=NG+I
C
C  SET UP PHASE I AND FIND PIVOTAL COLUMN
C
      KC=0
      Q=0.0
      DO 40 J=1,NG
      S=0.0
      DO 30 I=L1,M
   30 S=S-A(I,J)
      A(IW,J)=S
      IF(S.GE.Q) GO TO 40
      Q=S
      KC=J
   40 CONTINUE
      S=0.0
      DO 50 I=L1,M
   50 S=S-A(I,IC)
      A(IW,IC)=S
```

```
      IF(KC.GT.0) GO TO 120
      IW=IW-1
C
C  SIMPLEX ITERATION
C
  100 KC=0
      Q=0.0
      DO 110 J=1,NGL
      IF (A(IW,J).GE.Q) GO TO 110
      Q=A(IW,J)
      KC=J
  110 CONTINUE
      IF (KC.EQ.0) GO TO 200
  120 KR=0
      Q=1.0E70
      DO 130 I=1,M
      IF(A(I,KC).LE.1.0E-8) GO TO 130
      P=A(I,IC)/A(I,KC)
      IF(P.GT.Q) GO TO 130
      Q=P
      KR=I
  130 CONTINUE
      IF(KR.EQ.0) GO TO 250
      P=A(KR,KC)
      ID(KR)=KC
      DO 140 J=1,IC
  140 A(KR,J)=A(KR,J)/P
      DO 160 I=1,IW
      IF(I.EQ.KR) GO TO 160
      DO 150 J=1,IC
      IF(J.EQ.KC) GO TO 150
      A(I,J)=A(I,J)-A(KR,J)*A(I,KC)
      Q=ABS(A(I,J))
      IF(Q.LT.1.0E-8) A(I,J)=0.0
  150 CONTINUE
  160 CONTINUE
      DO 170 I=1,IW
  170 A(I,KC)=0.0
      A(KR,KC)=1.0
      GO TO 100
C
C  CHECK FOR PHASE II COMPLETION AND FEASIBILITY
C
  200 IF (IW.EQ.M+1) GO TO 210
      IW=IW-1
      GO TO 100
  210 DO 220 I=1,M
      IF (ID(I).GT.NGL) GO TO 230
  220 CONTINUE
      NFSIND=0
      GO TO 999
  230 NFSIND=1
      GO TO 999
  250 WRITE (6,2000)
 2000 FORMAT (19H SOLUTION UNBOUNBED)
      STOP
  999 RETURN
      END
```

APPENDIX B9

ENTERPRISE MODEL - COEFFICIENT EVALUATION PROGRAM

```
      DEFINE FILE 9(352,352,L,IAV7),10(351,136,L,IAV10),
     111(397,64,L,IAV11)
      DIMENSION FEEDC(351,11),GVAL(11),COSMIN(11),QFED(11),CMINA(11),
     1QFEDA(11),LCINGR(11,3),COMP(11,3),LCINGA(11,3),COMPA(11,3),
     2COMP1(3),COMP2(3),LCING1(3),LCING2(3),QTYFED(11,3),Q1(3),QDIF(3),
     3IGROW(10),COST(2,61),IPOL(9,61),IFAULT(12)
      IAV9=1
      IAV10=1
      IAV11=1
      MAXNLW=351
      NGMAX=11
      MAXNFP=10
      MAXNLV=61
      MAXNCF=3
      MAXLWT=600
      MAXNC=3
      LWDIF=10
      READ (5,1010) NDPTP,NCF,LWINT
 1010 FORMAT (3I3)
      IPOS=1
      READ (9'IPOS) LWMIN,LWMAX,LWINCR,NGVAL,NDAYS,PERCOS,NCONS,NVARS,
     1GVAL
      DO 10 I=1,12
   10 IFAULT(I)=0
      IF (LWMAX.LE.LWMIN) IFAULT(1)=1
      IF (NGVAL.GT.NGMAX) IFAULT(2)=1
      IF (LWINCR.NE.1) IFAULT(3)=1
      IF (NCF.GT.MAXNCF) IFAULT(4)=1
      NFPPTP=NDPTP/NDAYS
      K=NFPPTP*NDAYS
      IF (NDPTP-K.NE.0) IFAULT(5)=1
      MAXLW=(LWMAX/LWINT)*LWINT
      MINLW=(LWMIN/LWINT)*LWINT
      IF (MINLW.EQ.MAXLW) IFAULT(6)=1
      IF (MAXLW.GT.MAXLWT) IFAULT(7)=1
      IF (NFPPTP.GT.MAXNFP) IFAULT(8)=1
      NLVMAX=GVAL(NGVAL)*NDPTP
      IF (NLVMAX.GT.MAXNLV) IFAULT(9)=1
      NLW=MAXLW-MINLW+1
      IF (NLW.GT.MAXNLW) IFAULT(10)=1
      IF (NCONS.GT.MAXNC) IFAULT(11)=1
      IF (LWINT.NE.LWDIF) IFAULT(12)=1
      K=0
      DO 20 I=1,12
      IF (IFAULT(I).EQ.0) GO TO 20
      K=K+1
```

```
   20 CONTINUE
      IF (K.EQ.0) GO TO 40
      WRITE (6,4010) K
 4010 FORMAT (1H1,I3,25 H ERROR(S) IN INPUT DATA :)
      DO 30 I=1,12
      IF (IFAULT(I).EQ.0) GO TO 30
      WRITE (6,4010) I
 4020 FORMAT (6X,19H INPUT ERROR NUMBER,I3)
   30 CONTINUE
      GO TO 999
   40 DO 50 I=1,MAXNLW
      DO 50 J=1,NGMAX
   50 FEEDC(I,J)=-9999.9
      DO 60 J=1,MAXNCF
      Q1(J)=0.0
      QDIF(J)=0.0
      DO 60 I=1,NGMAX
   60 QTYFED(I,J)=0.0
      NDL1=NDAYS-1
      FACT=NDAYS/100.0
      NUCF=NVARS-NCF
      IPOS=MINLW-LWMIN+1
      DO 70 LW=1,NLW
      IPOS=IPOS+1
      READ (9'IPOS) COSMIN,QFED,LCINGR,COMP
      DO 70 I=1,NGVAL
      WV=COSMIN(I)
      IF (WV.GT.99990.0) WV=-9999.9
   70 FEEDC(LW,I)=WV
      PERCOS=NFPPTP*PERCOS
      IPOSB=0
      LWT=MINLW-1
      IPOS=MINLW-LWMIN+1
C
C  CALCULATE FEED COST PER PERIOD AND QUANTITY OF FEEDSTUFFS REQUIRED
C
      DO 400 LW=1,NLW
      LWT=LWT+1
      IPOS=IPOS+1
      READ (9'IPOS) COSMIN,QFED,LCINGR,COMP
      LWG=0
      FEEDC(LW,1)=NDAYS*FEEDC(LW,1)
      QFED1=QFED(1)*FACT
      IF (NCF.EQ.0) GO TO 120
      DO 110 I=1,NGVAL
      DO 110 J=1,NCF
  110 QTYFED(I,J)=0.0
      DO 115 I=1,NCONS
      IF (LCINGR(1,I).LE.NUCF) GO TO 120
      N=LCINGR(1,I)-NUCF
  115 QTYFED(1,N)=QFED1*COMP(1,I)
  120 DO 350 IG=2,NGVAL
      LWG=LWG+1
      IF (FEEDC(LW,IG).LT.0.0) GO TO 350
      IF (LW+LWG.LE.NLW) GO TO 130
      FEEDC(LW,IG)=-9999.9
      GO TO 350
  130 IPOSA=IPOS
```

```
      L1=LW
      L2=L1+1
      IF (FEEDC(L2,IG).GT.0.0) GO TO 140
      FEEDC(LW,IG)=-9999.9
      GO TO 350
C
C     FIRST DAY OF FEEDING PERIOD
C
  140 NINC=NDL1*LWG
      L=1
      FCOST=FEEDC(L1,IG)
      QFED1=QFED(IG)*0.01
      DO 150 I=1,NCONS
      COMP1(I)=COMP(IG,I)
  150 LCING1(I)=LCINGR(IG,I)
      DO 160 I=1,NCONS
      IF (LCING1(I).LE.NUCF) GO TO 165
      N=LCING1(I)-NUCF
  160 QTYFED(IG,N)=QFED1*COMP1(I)
  165 GPD=(1.0*LWG)/(1.0*NDAYS)
      IF (GPD.GT.1.001) GO TO 200
      IPOSA=IPOSA+1
      CINCR=FEEDC(L2,IG)-FEEDC(L1,IG)
      READ (9'IPOSA) CMINA,QFEDA,LCINGA,COMPA
      QFED2=QFEDA(IG)*0.01
      DO 170 I=1,NCONS
      COMP2(I)=COMPA(IG,I)
  170 LCING2(I)=LCINGA(IG,I)
      IF (GPD.GT.0.999) GO TO 200
      IF (NCF.EQ.0) GO TO 200
      DO 175 I=1,NCF
      Q1(I)=0.0
  175 QDIF(I)=0.0
      DO 180 I=1,NCONS
      IF (LCING2(I).LE.NUCF) GO TO 185
      N=LCING2(I)-NUCF
  180 QDIF(N)=QFED2*COMP2(I)
  185 DO 190 I=1,NCONS
      IF (LCING1(I).LE.NUCF) GO TO 200
      N=LCING1(I)-NUCF
      Q1(N)=QTYFED(IG,N)
  190 QDIF(N)=QDIF(N)-Q1(N)
C
C     SUBSEQUENT DAYS OF FEEDING PERIOD
C
  200 DO 300 J=LWG,NINC,LWG
      K=J/NDAYS
      IF (K.LT.L) GO TO 280
      L=L+1
      L1=LW+K
      L2=L1+1
      IF (FEEDC(L2,IG).GT.0.0) GO TO 220
      FEEDC(LW,IG)=-9999.9
      IF (NCF.EQ.0) GO TO 350
      DO 210 I=1,NCF
  210 QTYFED(IG,I)=0.0
      GO TO 350
  220 IF (K.EQ.L) GO TO 240
```

```
      L=K+1
      IPOSA=IPOS+K
      READ (9'IPOSA) CMINA,QFEDA,LCINGA,COMPA
      QFED1=QFEDA(IG)*0.01
      DO 230 I=1,NCONS
      COMP1(I)=COMPA(IG,I)
  230 LCING1(I)=LCINGA(IG,I)
      GO TO 250
  240 QFED1=QFED2
      DO 245 I=1,NCONS
      COMP1(I)=COMP2(I)
  245 LCING1(I)=LCING2(I)
  250 CINCR=FEEDC(L2,IG)-FEEDC(L1,IG)
      IPOSA=IPOSA+1
      READ (9'IPOSA) CMINA,QFEDA,LCINGA,COMPA
      QFED2=QFEDA(IG)*0.01
      DO 255 I=1,NCONS
      COMP2(I)=COMPA(IG,I)
  255 LCING2(I)=LCINGA(IG,I)
      IF (NCF.EQ.0) GO TO 280
      DO 260 I=1,NCF
      Q1(I)=0.0
  260 QDIF(I)=0.0
      DO 265 I=1,NCONS
      IF (LCING2(I).LE.NUCF) GO TO 270
      N=LCING2(I)-NUCF
  265 QDIF(N)=QFED2*COMP2(I)
  270 DO 275 I=1,NCONS
      IF (LCING1(I).LE.NUCF) GO TO 280
      N=LCING1(I)-NUCF
      Q1(N)=QFED1*COMP1(I)
  275 QDIF(N)=QDIF(N)-Q1(N)
  280 F=J-K*NDAYS
      F=F/NDAYS
      FCOST=FCOST+FEEDC(L1,IG)+F*CINCR
      IF (NCF.EQ.0) GO TO 300
      DO 290 I=1,NCF
  290 QTYFED(IG,I)=QTYFED(IG,I)+Q1(I)+F*QDIF(I)
  300 CONTINUE
      FEEDC(LW,IG)=FCOST
  350 CONTINUE
      IPOSB=IPOSB+1
  400 WRITE (10'IPOSB) LWT,QTYFED
C
C  ***** SOLVE DP MODEL FOR EACH LW AND LWG OVER TIME PERIOD *****
C
      N1=MAXNFP-1
      DO 410 I=1,N1
      IGROW(I)=0
      DO 410 J=1,MAXNLV
  410 IPOL(I,J)=0
      IGROW(MAXNFP)=0
      NFPL1=NFPPTP-1
      IPOS=1-LWINT
      IPOSA=1
      WRITE (11'1) MINLW,MAXLW,LWINT,NGVAL,NCF
      DO 700 LW=1,NLW,LWINT
      IPOS=IPOS+LWINT
```

```
      READ (10'IPOS) LWT,QTYFED
      LWTL1=LWT-1
      L2MAX=1
C
C  NO LIVEWEIGHT GAIN OVER TIME PERIOD
C
      DO 420 I=1,NFPPTP
  420 IGROW(I)=LWT
      FCOST=NFPPTP*FEEDC(LW,1)+PERCOS
      IF (NCF.EQ.0) GO TO 435
      DO 430 I=1,NCF
  430 Q1(I)=NFPPTP*QTYFED(1,I)
  435 IRG=1
      IPOSA=IPOSA+1
      WRITE (11'IPOSA) LWT,IRG,FCOST,Q1,IGROW
      WRITE (6,2001) LWT,IRG,FCOST,Q1,IGROW
      IF (NFPPTP.GT.1) GO TO 500
C
C  ONE FEEDING PERIOD PER TIME PERIOD
C
      DO 480 IRG=2,NGVAL
      L2MAX=L2MAX+LWINT
      FCOST=FEEDC(LW,IRG)
      IF (FCOST.LT.0.0) GO TO 450
      FCOST=FCOST+PERCOS
      IGROW(1)=L2MAX+LWTL1
      IF (NCF.EQ.0) GO TO 470
      DO 440 I=1,NCF
  440 Q1(I)=QTYFED(IRG,I)
      GO TO 470
  450 FCOST=9999999.0
      IGROW(1)=0
      IF (NCF.EQ.0) GO TO 470
      DO 460 I=1,NCF
  460 Q1(I)=0.0
  470 IPOSA=IPOSA+1
  480 WRITE (11'IPOSA) LWT,IRG,FCOST,Q1,IGROW
      WRITE (6,2001) LWT,IRG,FCOST,Q1,IGROW
      GO TO 700
C
C  SOLVE RECURRENCE RELATIONS FOR EACH LIVEWEIGHT GAIN OVER TIME PERIOD
C
  500 DO 690 IRG=2,NGVAL
      DO 510 I=1,NFPL1
      IGROW(I)=0
      DO 510 J=1,MAXNLV
  510 IPOL(I,J)=0
      IGROW(NFPPTP)=0
      COSTL2=9999999.0
      IPOLL2=0
      L2MAX=L2MAX+LWINT
      DO 520 I=1,2
      DO 520 J=1,MAXNLV
  520 COST(I,J)=9999999.0
      COSTA=0.0
      DO 530 IG=1,NGVAL
      FCOST=FEEDC(LW,IG)
      IF (FCOST.LT.0.0) GO TO 530
```

```
      FCOST=FCOST+COSTA
      COST(2,IG)=FCOST
      IPOL(1,IG)=1
  530 CONTINUE
      IF (NFPL1.EQ.1) GO TO 580
      DO 570 N=2,NFPL1
      L=LW-1
      DO 540 I=1,L2MAX
      COST(1,I)=COST(2,I)
  540 COST(2,I)=9999999.0
      DO 560 L1=1,L2MAX
      L=L+1
      COSTA=COST(1,L1)
      IF (COSTA.GT.9999990.0) GO TO 560
      IG=0
      DO 550 L2=L1,L2MAX
      IG=IG+1
      IF (IG.GT.NGVAL) GO TO 560
      FCOST=FEEDC(L,IG)
      IF (FCOST.LT.0.0) GO TO 550
      FCOST=FCOST+COSTA
      IF (FCOST-COST(2,L2).GT.-0.001) GO TO 550
      COST(2,L2)=FCOST
      IPOL(N,L2)=L1
  550 CONTINUE
  560 CONTINUE
  570 CONTINUE
  580 IG=NGVAL
      N=L2MAX-IG+1
      IF (N.GE.1) GO TO 590
      N=1
      IG=L2MAX
  590 L=LW+N-2
      IG=IG+1
      DO 600 L1=N,L2MAX
      L=L+1
      IG=IG-1
      COSTA=COST(2,L1)
      IF (COSTA.GT.9999990.0) GO TO 600
      FCOST=FEEDC(L,IG)
      IF (FCOST.LT.0.0) GO TO 600
      FCOST=FCOST+COSTA
      IF (FCOST-COSTL2.GT.-0.001) GO TO 600
      COSTL2=FCOST
      IPOLL2=L1
  600 CONTINUE
C
C  TRACE OPTIMAL PATH FOR THIS PROBLEM
C
      IF (COSTL2.LT.9999990.0) GO TO 620
      FCOST=9999999.0
      IF (NCF.EQ.0) GO TO 670
      DO 610 I=1,NCF
  610 Q1(I)=0.0
      GO TO 670
  620 IGROW(NFPPTP)=L2MAX+LWTL1
      IPER=NFPPTP
      INEXT=IPOLL2
```

```
      DO 630 N=1,NFPL1
      IPER=IPER-1
      IGROW(IPER)=INEXT+LWTL1
  630 INEXT=IPOL(IPER,INEXT)
      FCOST=COSTL2+PERCOS
      IF (NCF.EQ.0) GO TO 670
      IPOSB=LW
      IGL1=IGROW(1)-LWT
      IG=IGL1+1
      READ (10'IPOSB) L,QTYFED
      DO 640 I=1,NCF
  640 Q1(I)=QTYFED(IG,I)
      IF (NFPPTP.EQ.1) GO TO 670
      DO 660 N=2,NFPPTP
      IPOSB=IPOSB+IGL1
      N1=N-1
      IGL1=IGROW(N)-IGROW(N1)
      IG=IGL1+1
      READ (10'IPOSB) L,QTYFED
      DO 650 I=1,NCF
  650 Q1(I)=Q1(I)+QTYFED(IG,I)
  660 CONTINUE
  670 IPOSA=IPOSA+1
      WRITE (11'IPOSA) LWT,IRG,FCOST,Q1,IGROW
      WRITE (6,2001) LWT,IRG,FCOST,Q1,IGROW
 2001 FORMAT (2I4,F12.2,3F7.2,10I4)
  690 CONTINUE
  700 CONTINUE
  999 STOP
      END
```

APPENDIX B10

ENTERPRISE MODEL - MATRIX GENERATOR PROGRAM

```
      DEFINEFILE 11(397,64,L,IAV11),12(397,64,L,IAV12),
     113(397,64,L,IAV13),14(397,64,L,IAV14)
      DIMENSION VNAME(6),RNAME(9),Q(3),IGROW(10),SP(60),BP(60),
     1DATNAM(2),IFAULT(10),NANS(60),YIELD(9),CROPPC(9),VALUE(60),
     2MAXNAN(15),CROPA(9),CROPDM(9)
      DATA VNAME/'X   ','SEL ','BUY ','KEEP','CSEL','AREA'/,
     1RNAME/'OBJ ','FEED','SALE','CONT','NUMB','INIT','TERM','STRA',
     2'FARM'/
      IAV11=1
      IAV12=1
      IAV13=1
      IAV14=1
      LWDIF=10
      MAXVIT=15
      MAXVNR=8
      MAXBSV=60
      MAXCF=3
      MNCROP=9
C
C  READ DATA AND CHECK FOR ERRORS
C
      DO 10 I=1,10
   10 IFAULT(I)=0
      READ (5,1010) DATNAM
 1010 FORMAT (2A4)
      READ (5,1020) LWLO,LWUP,NGV,ITMAX,NRMAX,MAXNCF,NCROP
 1020 FORMAT (2I4,5I3)
      IF (ITMAX.GT.MAXVIT) IFAULT(1)=1
      IF (NRMAX.GT.MAXVNR) IFAULT(2)=1
      DO 20 I=1,NRMAX
      ICHAN=10+I
      READ (ICHAN'1) LWMIN,LWMAX,LWINT,NGVAL,NCF
      IF (LWMIN.NE.LWLO) IFAULT(5)=IFAULT(5)+1
      IF (LWMAX.NE.LWUP) IFAULT(6)=IFAULT(6)+1
      IF (LWINT.NE.LWDIF) IFAULT(7)=IFAULT(7)+1
   20 IF (NGVAL.NE.NGV) IFAULT(8)=IFAULT(8)+1
      IF (MAXNCF.GT.MAXCF) IFAULT(3)=1
      IF (NCF.NE.MAXNCF) IFAULT(4)=1
      IF (NCROP.GT.MNCROP) IFAULT(9)=1
      IF (NCROP.LT.MAXNCF) IFAULT(10)=1
      K=0
      DO 30 I=1,10
      IF (IFAULT(I).EQ.0) GO TO 30
      K=K+1
   30 CONTINUE
      IF (K.EQ.0) GO TO 50
```

```
      WRITE (6,2010) K
 2010 FORMAT (I3,25H TYPES OF ERROR IN DATA :)
      DO 40 I=1,10
      IF (IFAULT(I).EQ.0) GO TO 40
      WRITE (6,2020) IFAULT(I),I
 2020 FORMAT (6X,I3,17H ERROR(S) OF TYPE,I3)
   40 CONTINUE
      GO TO 999
   50 DO 60 I=1,MAXBSV
      NANS(I)=0
      VALUE(I)=0.0
      SP(I)=0.0
   60 BP(I)=-9999999.0
      DO 70 I=1,MAXVIT
   70 MAXNAN(I)=0
      DO 75 I=1,MNCROP
      YIELD(I)=0.0
      CROPA(I)=0.0
      CROPDM(I)=0.0
   75 CROPPC(I)=0.0
      LMIN=LWMIN/LWINT
      LMAX=LWMAX/LWINT
      READ (5,1030) (SP(I),I=LMIN,LMAX)
 1030 FORMAT (10F7.2)
      READ (5,1030) (BP(I),I=LMIN,LMAX)
      DO 80 I=LMIN,LMAX
   80 BP(I)=-BP(I)
      READ (5,1030) (VALUE(I),I=LMIN,LMAX)
      READ (5,1040) (NANS(I),I=LMIN,LMAX)
 1040 FORMAT (10I7)
      READ (5,1040) (MAXNAN(I),I=1,ITMAX)
      IF (NCROP.EQ.0) GO TO 95
      READ (5,1050) FAREA
 1050 FORMAT (F7.2)
      READ (5,1030) (CROPA(I),I=1,NCROP)
      READ (5,1030) (YIELD(I),I=1,NCROP)
      READ (5,1030) (CROPDM(I),I=1,NCROP)
      READ (5,1030) (CROPPC(I),I=1,NCROP)
      DO 90 I=1,NCROP
   90 YIELD(I)=-YIELD(I)
   95 ITML1=ITMAX-1
      NGVL1=NGVAL-1
      IITIN=(LMIN-1)*100
      I0=0
      FP1=1.0
      FM1=-1.0
C
C  ***** ROWS SECTION *****
C
      WRITE (10,3000) DATNAM
 3000 FORMAT (4HNAME,10X,2A4/4HROWS)
      WRITE (10,3010) RNAME(1)
 3010 FORMAT (2H N,2X,A3)
      IF (NCROP.EQ.0) GO TO 115
      DO 110 K=1,NCROP
  110 WRITE (10,3020) RNAME(2),I0,K
 3020 FORMAT (2H E,2X,A4,2I1)
  115 DO 120 IT=1,ITMAX
```

```
      IIT=IITIN+IT
      DO 120 I=LMIN,LMAX
      IIT=IIT+100
  120 WRITE (10,3030) RNAME(3),IIT
 3030 FORMAT (2H G,2X,A4,I4)
      DO 130 IT=1,ITML1
      IIT=IITIN+IT
      DO 130 I=LMIN,LMAX
      IIT=IIT+100
  130 WRITE (10,3040) RNAME(4),IIT
 3040 FORMAT (2H E,2X,A4,I4)
      IF (ITMAX.EQ.1) GO TO 160
      DO 150 I=2,ITMAX
      IF (I.GT.9) GO TO 140
      WRITE (10,3050) RNAME(5),I0,I
 3050 FORMAT (2H L,2X,A4,2I1)
      GO TO 150
  140 WRITE (10,3060) RNAME(5),I
 3060 FORMAT (2H L,2X,A4,I2)
  150 CONTINUE
  160 DO 170 I=LMIN,LMAX
  170 WRITE (10,3070) RNAME(6),I
 3070 FORMAT (2H E,2X,A4,I2)
      DO 180 I=LMIN,LMAX
  180 WRITE (10,3070) RNAME(7),I
      IF (NCROP.EQ.0) GO TO 200
      WRITE (10,3080) RNAME(8)
 3080 FORMAT (2H E,2X,A4)
      WRITE (10,3090) RNAME(9)
 3090 FORMAT (2H L,2X,A4)
C
C  ***** COLUMNS SECTION  *****
C
  200 WRITE (10,3100)
 3100 FORMAT (7HCOLUMNS)
C
C  VARIABLE X I,J,R,T
C
      DO 300 IT=1,ITMAX
      NRIT=IT
      DO 260 NR=1,NRMAX
      NRIT=NRIT+100
      ICHAN=NR+10
      IPOS=1
      READ (ICHAN'1) LWMIN,LWMAX,LWINT,NGVAL,NCF
      IIT=IITIN+IT
      DO 250 I=LMIN,LMAX
      JIT=IIT
      IIT=IIT+100
      JMAX=I+NGVL1
      DO 240 J=I,JMAX
      JIT=JIT+100
      IPOS=IPOS+1
      READ (ICHAN'IPOS) LWT,IRG,FCOST,Q,IGROW
      IF (FCOST.GT.9999990.0) GO TO 240
      FCOST=-FCOST/100.0
      WRITE (10,3110) VNAME(1),I,J,NRIT,RNAME(1),FCOST
 3110 FORMAT (4X,A1,2I2,I3,2X,A3,7X,F12.2)
```

```
      IF (NCF.EQ.0) GO TO 215
      DO 210 K=1,NCF
      IF (Q(K).LT.0.005) GO TO 210
      WRITE (10,3120) VNAME(1),I,J,NRIT,RNAME(2),I0,K,Q(K)
 3120 FORMAT (4X,A1,2I2,I3,2X,A4,2I1,4X,F12.2)
  210 CONTINUE
  215 WRITE (10,3130) VNAME(1),I,J,NRIT,RNAME(3),JIT,FP1
 3130 FORMAT (4X,A1,2I2,I3,2X,A4,I4,2X,F12.2)
      IF (IT.EQ.ITMAX) GO TO 220
      WRITE (10,3130) VNAME(1),I,J,NRIT,RNAME(4),JIT,FM1
  220 IF (IT.EQ.1) GO TO 230
      IITL1=IIT-1
      WRITE (10,3130) VNAME(1),I,J,NRIT,RNAME(4),IITL1,FP1
      IF (IT.GT.9) GO TO 225
      WRITE (10,3135) VNAME(1),I,J,NRIT,RNAME(5),I0,IT,FP1
 3135 FORMAT (4X,A1,2I2,I3,2X,A4,2I1,4X,F12.2)
      GO TO 235
  225 WRITE (10,3140) VNAME(1),I,J,NRIT,RNAME(5),IT,FP1
 3140 FORMAT (4X,A1,2I2,I3,2X,A4,I2,4X,F12.2)
      GO TO 235
  230 WRITE (10,3140) VNAME(1),I,J,NRIT,RNAME(6),I,FP1
  235 IF (IT.NE.ITMAX) GO TO 240
      WRITE (10,3140) VNAME(1),I,J,NRIT,RNAME(7),J,FM1
  240 CONTINUE
  250 CONTINUE
  260 CONTINUE
C
C VARIABLE SEL I,T
C
      IIT=IITIN+IT
      DO 270 I=LMIN,LMAX
      IIT=IIT+100
      WRITE (10,3150) VNAME(2),IIT,RNAME(1),SP(I)
 3150 FORMAT (4X,A3,I4,3X,A3,7X,F12.2)
      WRITE (10,3160) VNAME(2),IIT,RNAME(3),IIT,FM1
 3160 FORMAT (4X,A3,I4,3X,A4,I4,2X,F12.2)
      IF (IT.EQ.ITMAX) GO TO 265
      WRITE (10,3160) VNAME(2),IIT,RNAME(4),IIT,FP1
      GO TO 270
  265 WRITE (10,3170) VNAME(2),IIT,RNAME(7),I,FP1
 3170 FORMAT (4X,A3,I4,3X,A4,I2,4X,F12.2)
  270 CONTINUE
C
C  VARIABLE BUY I,T
C
      IIT=IITIN+IT
      DO 280 I=LMIN,LMAX
      IIT=IIT+100
      WRITE (10,3150) VNAME(3),IIT,RNAME(1),BP(I)
      IF (IT.EQ.ITMAX) GO TO 275
      WRITE (10,3160) VNAME(3),IIT,RNAME(4),IIT,FM1
      GO TO 280
  275 WRITE (10,3170) VNAME(3),IIT,RNAME(7),I,FM1
  280 CONTINUE
  300 CONTINUE
C
C  VARIABLE KEEP I
C
```

```
      DO 310 I=LMIN,LMAX
      WRITE (10,3180) VNAME(4),I,RNAME(1),VALUE(I)
 3180 FORMAT (4X,A4,I2,4X,A3,7X,F12.2)
  310 WRITE (10,3190) VNAME(4),I,RNAME(7),I,FP1
 3190 FORMAT (4X,A4,I2,4X,A4,I2,4X,F12.2)
C
C  VARIABLE CSEL K
C
      IF (NCROP.EQ.0) GO TO 400
      DO 320 K=1,NCROP
      WRITE (10,3200) VNAME(5),I0,K,RNAME(1),CROPPC(K)
 3200 FORMAT (4X,A4,2I1,4X,A3,7X,F12.2)
  320 WRITE (10,3210) VNAME(5),I0,K,RNAME(2),I0,K,CROPDM(K)
 3210 FORMAT (4X,A4,2I1,4X,A4,2I1,4X,F12.2)
C
C  VARIABLE AREA K
C
      DO 350 K=1,NCROP
      WV=YIELD(K)*CROPDM(K)
      WRITE (10,3210) VNAME(6),I0,K,RNAME(2),I0,K,WV
      IF (K.NE.1) GO TO 330
      WRITE (10,3220) VNAME(6),I0,K,RNAME(8),FP1
 3220 FORMAT (4X,A4,2I1,4X,A4,6X,F12.2)
      GO TO 340
  330 IF (K.NE.2) GO TO 340
      WRITE (10,3220) VNAME(6),I0,K,RNAME(8),FM1
      GO TO 350
  340 WRITE (10,3220) VNAME(6),I0,K,RNAME(9),FP1
  350 CONTINUE
C
C  ***** RHS SECTION *****
C
  400 WRITE (10,3300)
 3300 FORMAT (3HRHS)
      IF (NCROP.EQ.0) GO TO 415
      DO 410 K=1,NCROP
  410 WRITE (10,3310) RNAME(2),I0,K,I0
 3310 FORMAT (4X,4HRHS1,6X,A4,2I1,4X,I9)
  415 DO 420 IT=1,ITMAX
      IIT=IITIN+IT
      DO 420 I=LMIN,LMAX
      IIT=IIT+100
  420 WRITE (10,3320) RNAME(3),IIT,I0
 3320 FORMAT (4X,4HRHS1,6X,A4,I4,2X,I9)
      DO 430 IT=1,ITML1
      IIT=IITIN+IT
      DO 430 I=LMIN,LMAX
      IIT=IIT+100
  430 WRITE (10,3320) RNAME(4),IIT,I0
      IF (ITMAX.EQ.1) GO TO 460
      DO 450 IT=2,ITMAX
      IF (IT.GT.9) GO TO 440
      WRITE (10,3310) RNAME(5),I0,IT,MAXNAN(IT)
      GO TO 450
  440 WRITE (10,3330) RNAME(5),IT,MAXNAN(IT)
 3330 FORMAT (4X,4HRHS1,6X,A4,I2,4X,I9)
  450 CONTINUE
  460 DO 470 I=LMIN,LMAX
```

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  470 WRITE (10,3330) RNAME(6),I,NANS(I)
      DO 480 I=LMIN,LMAX
  480 WRITE (10,3330) RNAME(7),I,I0
      IF (NCROP.EQ.0) GO TO 600
      WRITE (10,3340) RNAME(8),I0
 3340 FORMAT (4X,4HRHS1,6X,A4,6X,I9)
      WRITE (10,3350) RNAME(9),FAREA
 3350 FORMAT (4X,4HRHS1,6X,A4,6X,F12.2)
C
C  ***** BOUNDS SECTION *****
C
      WRITE (10,3400)
 3400 FORMAT (6HBOUNDS)
      DO 510 K=1,NCROP
  510 WRITE (10,3410) VNAME(6),I0,K,CROPA(K)
 3410 FORMAT (10H UP BOUND1,4X,A4,2I1,4X,F12.2)
  600 WRITE (10,3500)
 3500 FORMAT (6HENDATA)
  999 STOP
      END
```