CONTRIBUTIONS TO THE EXPERIMENTAL STUDY OF THE GYROMAGNETIC ANOMALY OF FREE ELECTRONS

by

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CHAPTER 1

Introduction

The first suggestion that the electron might possess an intrinsic magnetic moment was made by A.L. Parson, in 1916. In an attempt to explain the mechanism of chemical binding (Parson, 1916), he postulated that the electron is itself magnetic and has, in addition to a negative charge, the properties of a circulating current. This was pictured as a negative charge distribution around a ring which rotated about an axis perpendicular to its plane, the peripheral velocity being comparable with that of light.

From this, it follows that the magnetic moment of the Parson magneton must be approximately $\frac{1}{2}rec$, where e is the electronic charge, c is the velocity of light, and r is the average radius of the circulating current distribution. Taking r to be about 1.5×10^{-9} cms., we find a value of 3.6×10^{-22} weber-metres for the magnetic moment of the Parson magneton; this is about fifty times greater than the moment of the Bohr magneton. It should be emphasised that, in Parson's view, the concept of the magneton is to be

substituted for that of the orbital electron, and not added to it.

Five years later, the magneton idea was developed further by Compton, in an effort to show that the ultimate unit of magnetism is the spinning electron (Compton, 1921). It had already been shown that this ultimate magnetic unit did not consist of any group of atoms, such as a chemical molecule (Compton and Trousdale, 1915), nor was it an atom as a whole (Compton and Rognley, 1920). By regarding the electron as being a rotating system with one degree of freedom and hence possessing an amount of energy $\frac{1}{2}h\omega$ at absolute zero (h is Planck's constant divided by 2π, and ω is the spin angular velocity), Compton showed that the spin angular momentum of the electron should be of the order of h. After citing evidence from an investigation of the rotation of the plane of polarisation of light by optically active substances (Allen, 1920), and from Wilson Cloud Chamber photographs showing helical tracks of electrons passing through paramagnetic substances such as air, Compton concludes: "The electron itself, spinning like a tiny gyroscope, is probably the ultimate magnetic particle."

Immediately afterwards, Crowther and

Schonland attempted to verify the Rutherford formula for the scattering of β-particles by nuclei (Crowther and Schonland, 1922); the formula had previously been verified by Chadwick for α-particles. Considerable disagreement between theory and their experimental results was found, and it was suggested that this might be due to the operation of some force between the scattering centre and the scattered particle which was not of the Coulomb type. In addition, they put forward the following explanation: "The theory that the electron is also a magneton, though not yet orthodox science, lends colour to the suggestion that, in a collision between two electrons, magnetic forces might be called into play."

These vague and tentative ideas about the properties of the electron were synthesised by Uhlenbeck and Goudsmit. They showed (Uhlenbeck and Goudsmit, 1925 and 1926) that many of the more puzzling features of atomic spectra, such as the doublet character of alkali atoms, and the anomalous Zeeman effect, could be understood immediately if it were assumed that the electron possessed an intrinsic angular momentum of $\frac{1}{2}h$, and an intrinsic magnetogyric ratio of e/m,

where m is the electron mass. Thus the intrinsic magnetogyric ratio is twice the orbital magnetogyric ratio; this is discussed in detail in Appendix I.

In the study of atomic spectra, it is customary to express the magnetic moment of an atom in any state in the form

$$\mu = gj\mu_o$$

where $\mu_o \equiv {}^{eh}/_{2m}$ is the Bohr magneton, j is the total angular momentum quantum number and measures the angular momentum in units of h, and g is the Landé splitting factor, or g-factor, and is a pure number. If we apply this notation to a system of only one electron, we may write by analogy

$$\mu = gs\mu_o \ .$$

From the postulates of Uhlenbeck and Goudsmit, we see immediately that $s = \frac{1}{2}$ and $\mu = {}^{e}/_{m} \times \frac{1}{2}h = \mu_o$. Thus, for an electron, we must have $g = 2$.

It was soon shown that electron spin could be included in a logical way in quantum mechanics, both within the framework of matrix mechanics (Heisenberg and Jordan, 1926), and using the formalism of wave mechanics (Pauli, 1927). It

turns out that the angular momentum $\underline{M}$ associated with a system having spin (quantum number) $\frac{1}{2}$ cannot be expressed in the form $\underline{M} = \underline{r} \wedge \underline{p}$, where $\underline{r}$ is the displacement of the system from the point about which $\underline{M}$ is calculated, and where $\underline{p}$ is the linear momentum. This emphasises the intrinsic nature of the electron's spin angular momentum.

The type of quantum mechanics just mentioned, where the spin of a particle was assumed from the start and merely added to the system as an additional degree of freedom, could not be used to describe a particle moving with a velocity approaching that of light; the relevant equations are not relativistically invariant. Dirac approached the problem of finding a relativistic wave equation by requiring, as a necessary condition for relativistic invariance, that the Hamiltonian for a free particle should be symmetric with respect to space and time derivatives, and hence linear in the space derivatives, (Dirac, 1928). In the presence of a magnetic field $\underline{B}$, it was shown that, in the non-relativistic limit, an additional term appears in the expression for the energy of the particle and is of magnitude $\frac{eh}{2m}B$. This is precisely the

energy possessed by a magnetic dipole of strength one Bohr magneton when aligned in a magnetic field. Further, it turns out that when such a particle moves in a central field of force, the orbital angular momentum $\underline{L}$ is not a constant of the motion, but that the vector sum of $\underline{L}$ and a second vector of magnitude $\frac{1}{2}h$ is a constant of the motion. Hence we conclude that an electron possesses an intrinsic or spin angular momentum of amount $\frac{1}{2}h$. These and related matters are considered in Appendix I.

Such was the success of Dirac's theory that no attempt was made to measure the magnetic moment of the electron in any direct fashion. It is certain that the much-quoted arguments of Bohr (for example, Mott, 1929), that an experiment of the Stern-Gerlach type could not demonstrate the existence of the magnetic moment of a free electron, did much to discourage any such investigation. Indeed, in their book on atomic collisions, Mott and Massey go so far as to say: "We must conclude that it is meaningless to assign to the free electron a magnetic moment." (Mott and Massey, 1949).

Interest in the subject was revived when various experimenters reported discrepancies

between Dirac theory and measured doublet separations in the spectra of hydrogen and deuterium (Houston, 1937; Williams, 1938). It was suggested that these were due to a perturbing interaction between the electron and the nucleus, though the mechanism of the perturbation was not specified (Pasternak, 1938). Investigators using optical methods were hampered in their efforts to check the Dirac theory by the Doppler broadening of the lines being large compared to the very small splittings which had to be observed.

In the immediate post-war years, refined optical beam methods and recently developed microwave techniques were applied to the problem, and a succession of papers, both theoretical and experimental, soon followed. Nafe, Nelson and Rabi used an atomic beam method to examine the hyperfine structure separation of atomic hydrogen and deuterium; they found a discrepancy between calculated and measured values which was about five times greater than their estimated experimental error (Nafe, Nelson and Rabi, 1947). Lamb and Retherford, using microwave techniques to examine the hyperfine structure of hydrogen, found a similar disagreement with theory (Lamb and Retherford, 1947); this latter was immediately

given a theoretical interpretation in terms of a shift of the energy levels caused by the interaction of the electron with the radiation field of the nucleus (Bethe, 1947). The results of Nafe et al. were confirmed independently, using a similar apparatus (Nagel, Julian and Zacharias, 1947), and an optical study of a line in the spectrum of ionised helium gave a result in fair agreement with Bethe's approximate calculation (Mack and Austern, 1947). Breit pointed out that, if the observed discrepancies really were due to an interaction between the electron and the nucleus, it would be difficult to understand why this interaction should be the same for the deuteron as for the proton. He claimed that the experimental evidence was not inconsistent with the electron's having a magnetic moment of the order of $\mu_o(1 + \alpha)$, where α is the fine structure constant (Breit, 1947). This implies that the g-factor of the electron should be approximately $2(1 + \alpha) = 2(1 + 0.0073)$.

Using an atomic beam apparatus to examine the Zeeman spectrum of gallium, Kusch and Foley concluded that their results could be explained by taking the electron g-factor to be $2(1 + 0.001145)$ (Kusch and Foley, 1947). The observations were

then extended to the spectra of sodium and indium, and it was found that theoretical and experimental results fitted best when the electron g-factor was taken to be $2(1 + 0.00119)$, with an uncertainty of 5 in the last place (Kusch and Foley, 1948a; 1948b).

Meanwhile, a new evaluation of radiative corrections to electron phenomena (avoiding the divergence difficulties which beset earlier calculations) had been accomplished by Schwinger (Schwinger, 1948; 1949). Using the methods of quantum electrodynamics, a radiative correction was derived, expressing the effect of the electromagnetic field of the electron upon itself. Working to second order in perturbation theory, Schwinger showed that when the theory is applied to an electron in a magnetic field, the correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of amount $\frac{1}{2}(\frac{\alpha}{\pi})\mu_o$. Thus the electron g-factor is $2(1 + \frac{1}{2}(\frac{\alpha}{\pi}))$, or $2(1 + 0.001162)$. A precision measurement of the hyperfine structure separation of the ground state of hydrogen and deuterium atoms, using an atomic beam method, confirmed Schwinger's calculation within the experimental error, and also agreed with the results of Kusch and Foley (Nafe and

Nelson, 1948).

A more precise determination of the g-factor for electrons bound in atoms could be achieved by combining the results of two independent experiments: in the first, the magnetic moment of the proton is determined in units of the Bohr magneton (μ_p/μ_o), while in the second, the ratio of the magnetic moment of the electron to that of the proton is found (μ/μ_p). The product of these two results gives $\mu/\mu_o \equiv gs = \frac{1}{2}g$, or one half of the g-factor for the bound electron.

The first determination of μ/μ_p was made by Taub and Kusch, using a molecular beam method (Taub and Kusch, 1949). More precise results have been obtained by observing the ratio of the electronic spin g-value of atomic hydrogen to the proton g-value in a sample of mineral oil in the same magnetic field (Koenig, Prodell and Kusch, 1952; Beringer and Heald, 1954; Geiger, Hughes and Radford, 1957). If the ratio μ/μ_p is required for the free electron, a relativistic correction is applied to account for the binding energy of the electron in the hydrogen atom. According to a recent review (Du Mond, 1958), the best value at present is (for free electrons)

$$\mu/\mu_p(\text{oil}) = 658.2288 \pm 0.0004 .$$

The first determination of $\mu_p(\text{oil})/\mu_o$ was made by Gardner and Purcell; the ratio of the nuclear magnetic resonance frequency of protons in a sample of mineral oil to the cyclotron frequency of free (low energy) electrons in the same magnetic field was measured (Gardner and Purcell, 1949). Their results, which gave a low value for the electron g-factor when combined with the above value of μ/μ_p, have recently been re-evaluated (Hardy and Purcell, 1960). A second measurement of $\mu_p(\text{oil})/\mu_o$, using the same technique, was carried out by Franken and Liebes (Franken and Liebes, 1956). Using the above best value of μ/μ_p, we find the following values for the g-factor for free electrons:

Gardner & Purcell (original): $g = 2\,(1 + 0.001146 \pm 0.000012)$

Gardner & Purcell (revised): $g = 2\,(1 + 0.001156 \pm 0.000002)$

Franken & Liebes : $g = 2\,(1 + 0.001165 \pm 0.000005)$

In step with the development of the more precise experimental methods outlined above, there have been successive refinements of the theory relating to the anomalous g-factor of the electron. The first calculation to fourth order was made by Karplus and Kroll, as far as terms in α^2 (Karplus

and Kroll, 1950). It yielded

$$g = 2\left\{1 + \tfrac{1}{2}\left(\frac{\alpha}{\pi}\right) - 2.973\left(\frac{\alpha}{\pi}\right)^2\right\}$$

$$\text{or} \quad g = 2\left\{1 + 0.0011454\right\}.$$

This lengthy calculation was considerably shortened by Sommerfield, using the formalism introduced by Schwinger (Sommerfield, 1957, 1958). The self-energy of the electron in a steady, externally applied magnetic field was calculated, and the magnetic moment of the electron identified as the coefficient of that term in the expression for the energy which was linear in the magnetic field. He found

$$g = 2\left\{1 + \tfrac{1}{2}\left(\frac{\alpha}{\pi}\right) - 0.328\left(\frac{\alpha}{\pi}\right)^2\right\}$$

$$\text{or} \quad g = 2\left\{1 + 0.0011596\right\}.$$

An identical result was obtained by Petermann in a quite independent calculation (Petermann, 1957).

As we have seen, it is possible to deduce the g-factor for free electrons if we know its value for electrons bound to atomic nuclei. Less indirect measurements would, however, be very desirable, and three experiments have been devised to measure directly the g-factor of free electrons. In a fourth experiment, the g-factor of the free electron has been compared with that of the valence electron in the sodium atom. These experiments will be discussed in the following chapters.

The author has been concerned (in conjunction with others) with the construction and testing of an apparatus designed by Farago (Farago, 1958). The success of this experiment depends largely on the formation of a suitable electron beam in crossed electromagnetic fields. In addition to constructing and developing some of the apparatus described in Chapter 3, notably the first proton resonance probe, the vacuum system and the special E.H.T. unit (no suitable commercial unit was available), the required type of electron beam has been produced and studied; this is described in Chapter 4. The theory relating to various aspects of the experiment is reviewed and presented in condensed form in Appendices I - IV. In Appendix V, the theory of focussing a beam of electrons in crossed fields is worked out, and a possible experimental arrangement is discussed.

CHAPTER TWO

Experiments Using Free Electrons

In this section we outline three g-factor experiments, all of which involve free electrons directly. Two of these experiments have already yielded results.

(a) Dehmelt's Experiment

The aim of this experiment (Dehmelt, 1958a, see also (1958b) was not to measure the g-factor of the free electron explicitly, but to compare it with that of the valence electron of the sodium atom.

Consider a volume filled with sodium vapour in which sodium atoms are oriented, that is to say, the spin of the valence electron has the same orientation for each atom. If free electrons are somehow produced in the same volume, having their spins oriented at random, collisions will occur between the sodium atoms and the electrons. As a result of these exchange collisions, free electrons will be oriented in the direction of the valence electron spins, and, since angular momentum must be conserved, partial disorientation of the sodium atoms will occur. Eventually an

equilibrium state will be reached, with partial orientation of both electrons and sodium atoms.

Suppose now that the equilibrium is upset by completely destroying the polarisation of the electrons. Equilibrium will be re-established by the sodium atoms giving up to the electrons a further degree of polarisation in a second series of exchange collisions. In this way, a change in the state of polarisation of the free electrons can produce a change in the state of polarisation of the sodium atoms.

It was shown by Kastler that partial orientation of alkali atoms may be produced and detected by optical means (Kastler, 1954). This technique, known as optical pumping, was applied by Dehmelt. Sodium vapour was contained in a vessel the walls of which were coated with caesium which, when irradiated with ultraviolet light, produces photo-electrons. To prevent the vapour atoms and the photo-electrons from making too many disorienting collisions with the walls, an inert buffer gas was added. Orientation of the sodium atoms, and hence of the electrons, was achieved by illuminating the bulb containing the vapour with circularly polarised sodium light. The intensity of light transmitted through the

vapour was detected by a photo-cell, and gave a measure of the polarisation of the sodium atoms: the greater the degree of orientation, the more intense will be the transmitted light.

A constant, uniform magnetic field, B_o, was applied to the sample, in the direction of the incident illumination, and a loop was arranged to produce a radio-frequency magnetic field in a direction perpendicular to B_o (this is equivalent to two oppositely-rotating fields with axes of rotation parallel to B_o). When the radio frequency became equal to the spin resonance frequency of electrons in the field B_o, energy was absorbed from the radio-frequency field by the partially oriented electrons, thereby destroying the polarisation of the electron gas. As a result, the degree of orientation of the sodium atoms was reduced, giving rise to a decrease in the current from the photo-cell. In practice, B_o was modulated at a low frequency to sweep through the field strength required for resonance. By applying an alternating voltage at the same low frequency to the x-plates of an oscilloscope, and by feeding to the y-plates the amplified photo-cell signal, the resonance curve could be observed.

To compute the g-factor in this experiment, both the applied resonance frequency and the magnetic field strength at resonance must be known; measurement of the former presents no problems but, rather than determine the magnetic field strength, an auxiliary experiment was performed in which transitions between hyperfine levels in the ground state of sodium were induced, in the same magnetic field, by an applied radio-frequency field. Combining the results of these two experiments with data acquired previously from atomic beam experiments, Dehmelt found that the g-factors for the free electron and for the valence electron of sodium were equal, to an accuracy of 3 parts in 100,000.

(b) Frisch's Experiment

The energy of a non-relativistic electron in a magnetic field B may be written as

$$E_{\ell,s} = \frac{1}{2m} p_z^2 + (2\ell + 1 + gs)\mu_o B$$

where p_z is the component of the electron's momentum which is parallel to B, ℓ is the orbital quantum number, and m, g, s and μ_o have their usual meanings. From this we have

$$E_{\ell,+\frac{1}{2}} - E_{\ell,-\frac{1}{2}} = g\mu_o B = h\omega_s ,$$

where $\omega_s = \frac{eg}{2m} B$ is the spin precession frequency. Also

$$E_{\ell+1,s} - E_{\ell,s} = 2\mu_o B = h\omega_c \ ,$$

where $\omega_c = \frac{e}{m} B$ is the cyclotron frequency.

Hence $g = 2^{\omega_s}/\omega_c$.

An experiment was suggested by Bloch to determine ω_s and ω_c for slow electrons (Bloch, 1953). The method makes use of the fact that, in a homogeneous magnetic field, the electron can exist only in certain stationary states, each with a well-defined energy. In the following paragraphs, we outline an experiment undertaken by Frisch which is based on Bloch's suggestion (Frisch , 1954; see also Tolhoek, 1956).

Thermal electrons are made to describe spiral orbits by entering a constant, homogeneous magnetic field B, applied in the z direction. The electron energy will then be $E_{\ell,s}$, as given above. When a potential well of depth ϕ is suddenly set up by the application of suitably chosen electrostatic fields, those electrons for which $E_{\ell,s} < e\phi$ will be trapped in the well, while those of greater energy will continue to spiral onwards. It may be shown that the result of establishing an additional, inhomogeneous, magnetic field with a gradient in the z direction

is to raise one side of the potential well and to lower the other. The effect of this will be to "blow" the more energetic electrons out of the trap. If the inhomogeneity is removed, and if the system is not disturbed in any way, a second application of the inhomogeneous magnetic field cannot eject any more electrons.

Suppose that, with the particular depth of well chosen, the establishment of the inhomogeneous field "blows out" all electrons having energy greater than $E_{\ell',-\frac{1}{2}}$, which is just greater than $E_{\ell'+1,-\frac{1}{2}}$ (it is easily seen that, if $g = 2$, these energies are equal, that is, there is a degeneracy). After the inhomogeneity is removed again, a radio-frequency magnetic field is applied to the trapped electrons, in a direction perpendicular to the z-axis. If the frequency of this field is $\omega = \omega_c$, absorption of radio-frequency energy will cause transitions in which ℓ increases by unity, and two such transitions will be $\ell' \rightarrow \ell' + 1$ and $\ell' + 1 \rightarrow \ell' + 2$. If, on the other hand, the applied frequency is $\omega = \omega_s$, absorption of radio-frequency energy will cause transitions ("spin flips") in which s goes from $-\frac{1}{2}$ to $+\frac{1}{2}$. In either case, subsequent application of the inhomogeneous magnetic field will eject

electrons and, after being accelerated, these ejected electrons may be detected. By determining the two frequencies at which energy is absorbed (the frequencies after the application of which electrons are "blown out"), the value of g may be determined.

If, in a practical arrangement, we take $B = 0.1$ webers/m^2 (= 1,000 gauss), the resonant frequencies will be about 1.8×10^{10} radians/sec., the corresponding wavelength being around 10 cms. Since $h\omega \doteq e\phi$, we find $\phi \doteq 10$ μ volts, so that the trapping voltage has to be very small. Because of space charge effects, only a very few (about ten) electrons can be trapped at a time, and the trapping time must be sufficient for the required transitions to occur. Hence, even when the cycle of operations is repeated many times per second, counting rates will be low and, while in the trap, electrons will move through large distances, so very low pressure (about 10^{-7} m.m. mercury) is essential. Another difficulty is that the ω_c resonance is much more intense than that for ω_s and, as the frequencies are very close together, the former may mask the latter. Further complications may arise on account of energy exchange with the surroundings through radiation.

As a result of these and other difficulties, this experiment has not yielded results so far.

(c) Crane's Experiment

It was recently reported (Schupp, Pidd and Crane, 1961) that the research programme started in 1952 by Crane had succeeded in measuring directly the g-factor of free electrons with an accuracy of about 2 parts in 1,000,000. An earlier version of this experiment had been described previously (Louisell, Pidd and Crane, 1954), but we shall consider here only the later and more successful version (first described by Louisell, Pidd and Crane, 1953).

The experimental arrangement is as follows: a pulsed (and unpolarised) beam of electrons from an electron gun, moving parallel to a constant and uniform magnetic field inside a 20 ft. long solenoid, is scattered through a right-angle by a thin gold foil. As shown by Mott scattering theory (see Appendix IV), the beam will then be partially polarised; it is allowed to drift forward in a helix.

In order that the electrons should perform a prescribed number of orbital revolutions in the constant magnetic field, the axial component of

their velocity is decreased by a pulse of about -40 volts applied to coaxial electrodes. At this point, an auxiliary magnetic field modifies the main field so that it decreases radially. The electrons in this "trap" oscillate parallel to the axis of the solenoid. A second voltage pulse is applied after they have described the required number of revolutions, and they spiral forward again. After a second Mott scatter, the electrons move parallel and antiparallel to the magnetic field and are detected.

As discussed in Appendix II the cyclotron and spin precession frequencies, ω_c and ω_s, of electrons in a pure magnetic field are related by the equation

$$(\omega_s - \omega_c)/\omega_c = \gamma(g/2 - 1).$$

Thus the relative orientation of the spin and momentum vectors changes by $\Delta \phi$ per revolution, where

$$\Delta \phi = 2\pi\gamma(g/2 - 1).$$

If the asymmetry in the counting rates in the two counters is measured as a function of

the number of orbital revolutions described in the magnetic field, a sinusoidal variation should be obtained. A whole period will be covered after n revolutions, where

$$2\pi = 2\pi\, n\, \gamma\left(\frac{g}{2} - 1\right)$$

so that $\frac{1}{n} = \gamma\left(\frac{g}{2} - 1\right)$.

Hence, a measurement of the number of orbital revolutions which correspond to a complete period in the variation of the scattering asymmetry gives directly the deviation of the g-factor from the value 2.

In the Crane experiment, the number of completed orbits is controlled by the time (up to 300 μsec.) spent by the electrons in the magnetic trap. Therefore the variation of the scattering asymmetry was measured as a function of the time spent in the magnetic trap, the frequency of the variation being $(\omega_s - \omega_c)/2\pi$. Instead of using the expression

$$\left(\frac{g}{2} - 1\right) = \frac{1}{\gamma} \frac{(\omega_s - \omega_c)}{\omega_c}$$

which involves a knowledge of the electron velocity (through the factor $\frac{v^2}{c^2}$), Crane uses

$$\left(\frac{g}{2} - 1\right) = \frac{m}{eB}(\omega_s - \omega_c).$$

It is therefore necessary to measure B and,

since the magnetic field in the trap is not homogeneous, this involves evaluating the time average of B as experienced by the electrons, which presents some difficulty.

Crane et al. determined the g-factor anomaly, $(\frac{g}{2} - 1)$, for electron energies between 50 and 100 kev. They found that the anomaly showed a slight dependence on energy, which is attributed (without complete conviction) to stray electrostatic charges in the region of the magnetic trap, but it is pointed out that this variation of the anomaly with energy might be real. After making certain assumptions, they arrive at their final result:

$$g = 2(1.0011609 \pm 0.0000024) \quad .$$

In an earlier report (Schupp, Pidd and Crane, 1959), it was suggested that the above-mentioned apparent variation of the g-factor anomaly with energy might be interpreted by assigning to the electron an electric dipole moment. By assuming the best theoretical value for g to be exact, they showed that such a dipole moment could not exceed $(1.8 \pm 1.4) \times 10^{-17} \times e$ coulomb-metres, and this point has been investigated further (Nelson, Schupp, Pidd and Crane, 1959). It is worth noting that the spin kinematics for an electron with an electric dipole moment has already been worked out

(Bargmann, Michel and Telegdi, 1959), and the effect of such a dipole moment on electron scattering has been calculated (Margolis, Rosendorff and Sirlin, 1959). The theoretical implications of the electron's having an electric dipole moment have been pointed out by Landau, and by Lee and Yang (Landau, 1957; Lee and Yang, 1957a).

CHAPTER THREE

The Present Experiment

An experiment similar to that of Crane et al. was suggested (Farago, 1958), and has been under development for the past three years. The variation of the angle between the spin and momentum vectors of a beam of polarised electrons, performing approximately circular orbits in a homogeneous electromagnetic field, is detected by Mott scattering. By measuring the variation of the state of polarisation of the beam as a function of the number of orbital revolutions described in the field, the quantity $(\omega_s - \omega_c)/\omega_c$ is obtained directly, where ω_s, ω_c are respectively the spin precession and cyclotron frequencies.

In two important respects, this experiment is much simpler than that of Crane: a polarised beam of electrons is obtained directly from a suitable beta-active isotope, and, by applying an electrostatic field in a direction perpendicular to the magnetic field, the number of orbital revolutions performed before detection may be controlled without the necessity of a magnetic trap of any kind.

Outline of the Experimental Method

The main features of the experimental arrangement are shown in Figure 3.i.

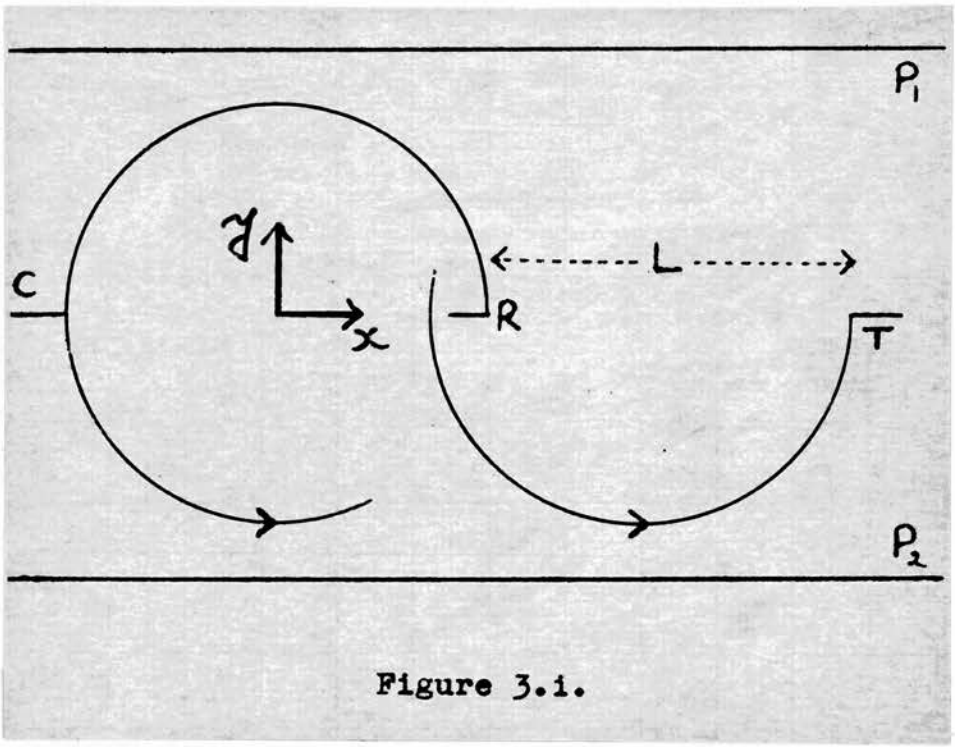


Figure 3.i.

The radioactive source, R, is placed in the crossed, homogeneous, electric and magnetic fields $\underline{E} = (0, E, 0)$, and $\underline{B} = (0, 0, B)$, and emits electrons into one hemisphere only. Let the magnitude of the velocity of these electrons be v; then, as explained in Appendix V, the electron orbits may be considered to be, to a good approximation, circular, of radius $\gamma mv/eB$, (where $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$), provided that $E/vB \ll 1$.

Because of the electric field, these "circular" orbits drift in the direction of the positive x-axis with a velocity E/B, the drift per revolution being $\delta = 2\pi\gamma mE/eB^2$. If the electrons drift a total distance, **L** (which is approximately equal to the diameter of the orbits), before striking the target, T, then the number of orbital revolutions described in the crossed fields will be $L/\delta = c\beta B/\pi E$, where $\beta = v/c$.

In the present experiment, the radius of the electron orbits is restricted to about 5 cms., so that $L \doteq 0.1$ m. For 128 kev electrons, $\beta = 0.6$, and so we require $B \doteq 0.025$ webers/m.^{2} (= 250 gauss). From Appendix II we find, if $E/cB \ll 1$,

$$(\omega_s - \omega_c)/\omega_c = \gamma(\tfrac{g}{2} - 1),$$

The relative orientation of the spin and momentum vectors therefore changes by an angle $\Delta\phi$ per orbital revolution, where

$$\Delta\phi = 2\pi\gamma(\tfrac{g}{2} - 1) \quad .$$

Hence, the relative orientation will change by 2π after n revolutions, where

$$2\pi = 2\pi n\gamma(\tfrac{g}{2} - 1) \quad .$$

Now $(\frac{g}{2} - 1) \doteq 0.001$ and, if $\beta = 0.6$, we find $\gamma = 1.25$. So the number of orbital revolutions required to give a complete period of relative spin precession is $n \doteq 800$ turns, which leads to $E \doteq 1,500$ volts/m. For a quarter of a period (which will transform a longitudinally polarised beam into a transversely polarised beam), 200 turns will be necessary, the required electric field being about 6,000 volts/m.

If the electric field is produced by establishing a potential difference between two parallel plates, P_1 and P_2, which are approximately 10 cms. apart, the necessary potential differences to give 800 and 200 turns will be about 150 volts and 600 volts respectively. Should we require that the electrons strike the target after only 10 turns, then $E = 120,000$ volts/m., the corresponding potential difference being 12,000 volts. We then have $E/cB = 0.016$, so that, for a number of orbital revolutions as low as 10, the approximation made above ceases to be valid.

When the target is a thin gold foil, the numbers of electrons scattered by the gold nuclei in directions parallel and anti-parallel to the magnetic field will not, in general, be equal. Hence, if detectors are placed above and below the foil,

the two counting rates will not normally be the same. Mott scattering theory, which is discussed in Appendix IV, shows that the counting rate asymmetry depends on the state of polarisation of the electron beam when it strikes the foil: the asymmetry is a maximum for a transversely polarised beam, and is zero for a longitudinally polarised beam.

It is clear from Figure 3.i that for a given magnetic field, the baffle C, together with the condenser plates, P_1 and P_2, limits the maximum particle energy (that is, the maximum diameter of orbit) which can reach the target T. Remembering that the orbits drift from left to right, we see that the position of the edge of the source-holder furthest from T will define the lowest particle energy which can reach the target; hence, the breadth of the source-holder is an important parameter. Likewise, the source must be mounted very close to the edge of the source-holder nearest to T, especially at low values of the electric field; otherwise, in spite of drifting towards the target, the electrons will strike the back of the source on completing the first orbit. It should be noticed that the angular aperture of the beam which, in practice, was taken to be about 30°, is limited by the separation of the condenser

plates. The actual dimensions of the apparatus used are given in the following section.

An important feature of the above electron-optical system is that, whatever their initial velocities, all electrons emitted from a point source with no z component of velocity are brought to a point again after an integral number of orbital revolutions. In other words, there is focussing in the plane of symmetry of the system. This point is discussed in Appendix V, where the possibility is considered of focussing in the z direction also.

Dimensions of the Electron-Optical System

In the course of the experiment, two arrangements have been used to produce the required electrostatic field distribution. The first, composed of brass and perspex components, was designed for the initial development work and has recently been used to obtain the preliminary data discussed in Chapter 4. The second is described in detail below.

The maximum size of the apparatus was controlled by the area of the magnetic field which was sufficiently homogeneous, and by the separation

of the magnet pole-faces. The minimum size was determined principally by considerations of machining tolerances and alignment difficulties. For these and for practical reasons, it was decided to use a condenser plate separation of 10.28 cms. By requiring that electrons be accepted from the source within an angle of 30° in the x-y plane, the mean electron orbit diameter was fixed at 8.166 cms. The two extremal orbits and the central orbit are shown, in Figure 3.ii, leaving the source R and subsequently striking the target T. From this, we find that the total distance drifted by the (almost) circular orbits before striking the source is 7.61 cms., the breadth of the source-holder is 0.278 cms., and the distance from the source at R to the baffle C is just one orbit diameter.

To correct for end effects and thus preserve the homogeneity of the electric field, equally spaced conducting grids of rectangular shape, each at a potential appropriate to its position, were placed between the condenser plates. These are shown in Figures 3.ii and 3.iii. To obtain the requisite potential on the grids, each was connected to its neighbour by a 4 MΩ precision resistor. The condenser plates P_1 and P_2 were

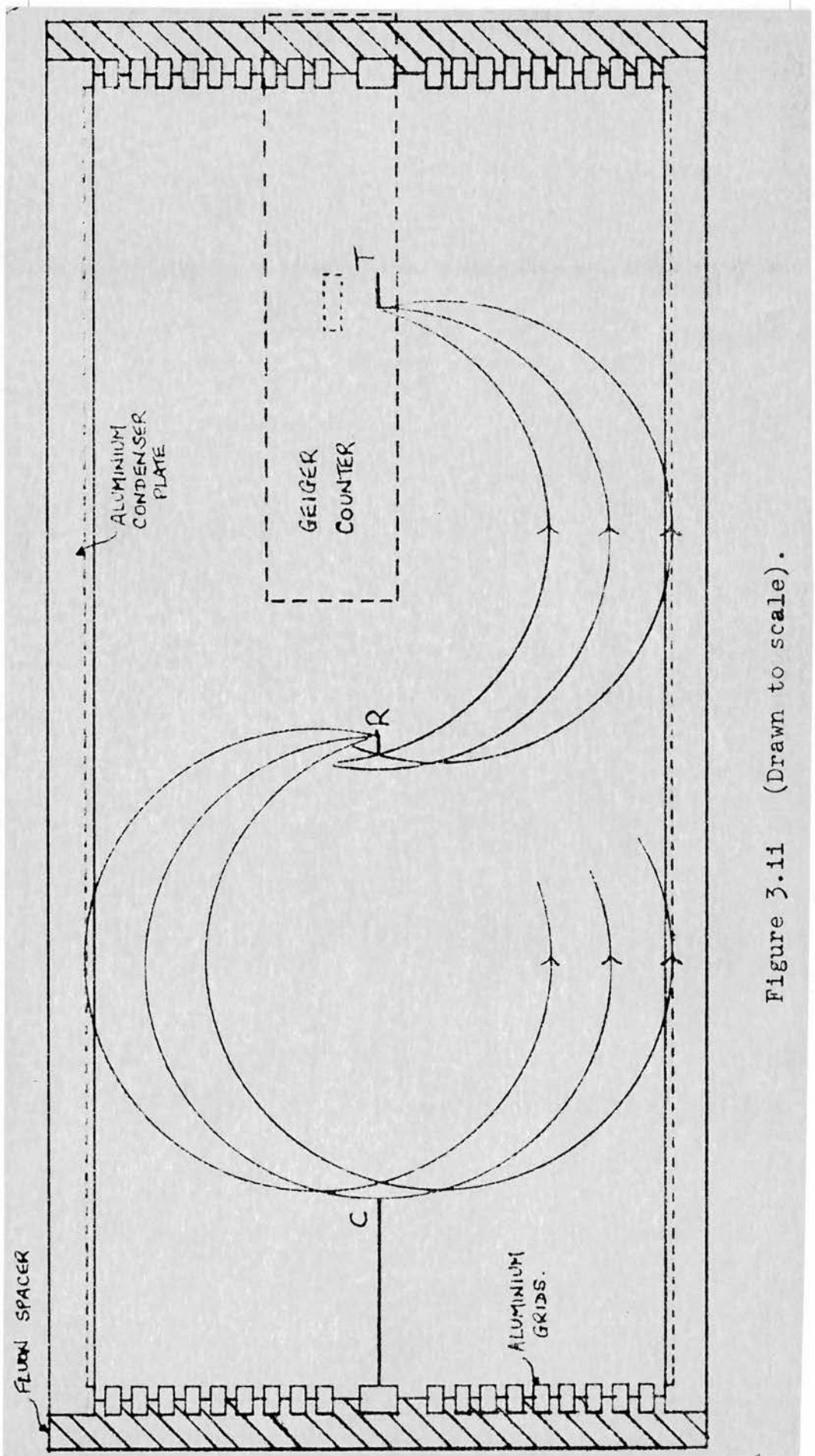


Figure 3.11 (Drawn to scale).

were raised to equal and opposite potentials, the centre grid then being at earth potential.

In an effort to reduce bremsstrahlung, condenser plates, grids, baffle, source-holder and target foil-holder were constructed of aluminium.

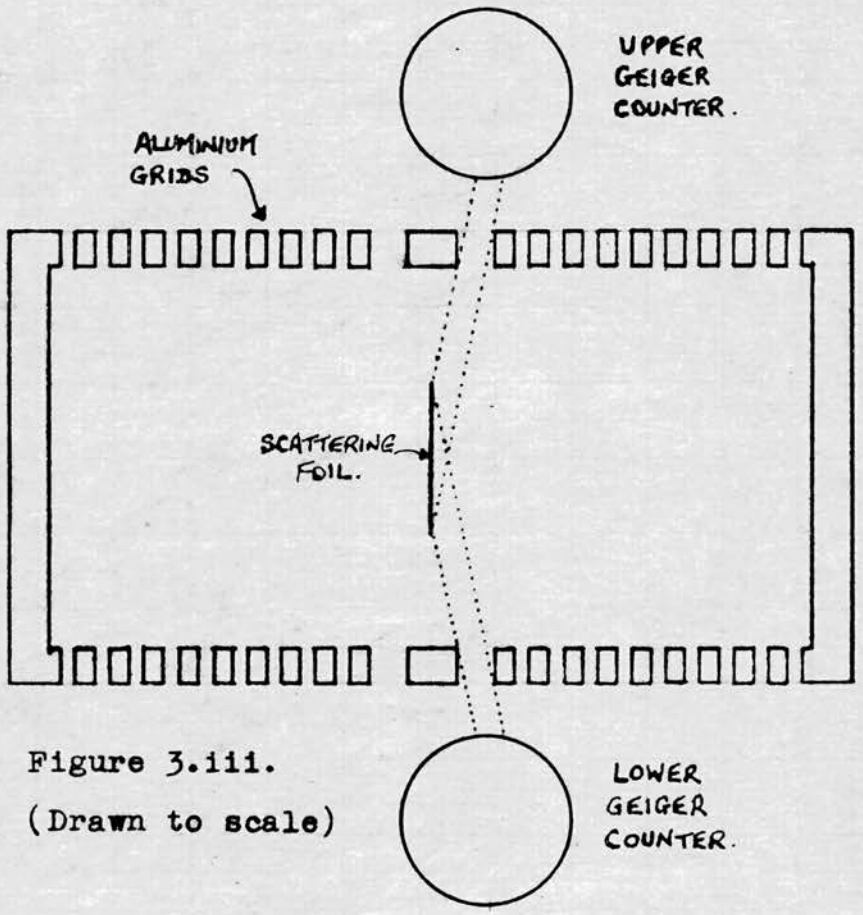


Figure 3.iii.
(Drawn to scale)

Because of its satisfactory mechanical and vacuum properties, end-pieces and grid spacers of accurately machined fluon were used, and the entire

electrostatic system was positioned in the plane of symmetry of the magnetic field by fluon spacers.

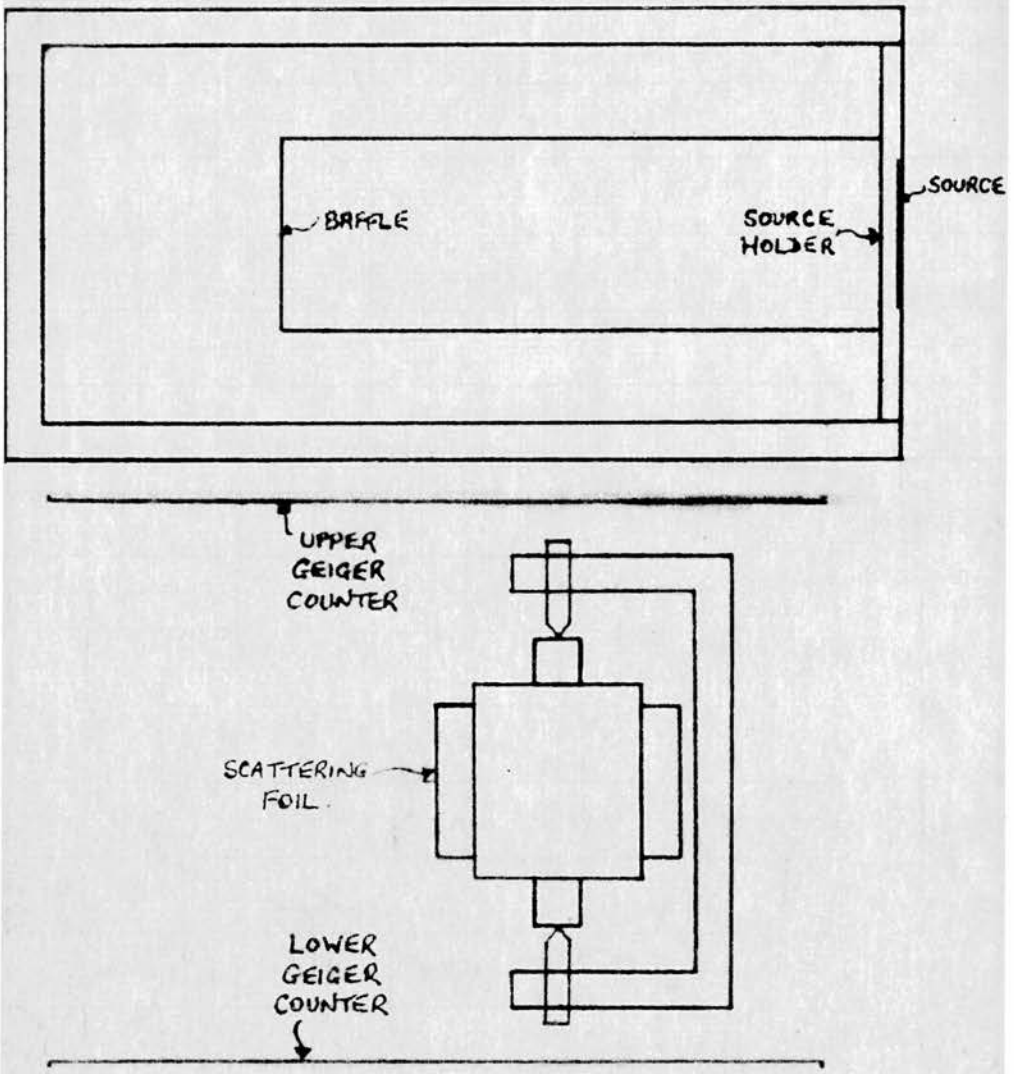


Figure 3.iv.
(Drawn to scale)

The central grid is shown in Figure 3.iv in side elevation. The target foil-holder could be rotated through 180° from outside the vacuum box so as to present to the beam either a thin gold

foil or a thicker aluminium foil (or, for some investigations, no foil at all). The source of S^{35} was deposited on a suitably shaped piece of aluminium rod, care being taken to ensure that most of the source was situated very close to the edge of the source-holder. Two small, thin-window Geiger counters were placed above and below the target foil, with their axes parallel to the grids, as shown in Figure 3.iii.

The Magnet

The soft-iron pole-faces were circular and were spaced 11.80 cms. apart, a shim of height and breadth 0.693 cms. being incorporated to improve the homogeneity of the magnetic field. The inside radius of the shim was 16.6 cms. Each pole was attached to a square, soft-iron plate, 1 in. thick and of side 27 1/4 ins., the length of each pole being 3.145 ins. The return path for the magnetic flux was through four triangular soft-iron corner pieces, each of area 24.6 sq. ins.

The magnet was energised by passing current through coils of copper strip, 400 turns of which were mounted on each pole piece. Current was supplied, from a mains stabiliser, through a rectifier capable of producing up to 10 amps.

Smoothing was provided by an LC-filter (1,000 μF, 0.5 henries), followed by two resistance-coupled banks of "floating" accumulators. This supply system will be described in detail elsewhere.

To measure the magnetic field in the gap between the pole-faces, a simple proton resonance device was constructed, and two small modulating coils were added to the pole pieces. The original circuit is shown in Figure 3.v, being based on a marginal oscillator arrangement (Watkins and Pound, 1951; Pound, 1952). By measuring the resonance frequency (around 1 Mc./s.) on a commercial detector, preliminary investigations showed that a current of about 3 amps produced a field of 0.025 webers/m^2. To measure magnetic field inhomogeneities, a more elaborate system (using simultaneously two proton resonance probes) was developed, and will be described elsewhere. After adding a small correcting coil to one pole piece, it was shown that, within the volume to be traversed by the electrons, the magnetic field was homogeneous to 4 parts in 10^4.

The Vacuum System

Electrons which describe 1,000 turns in the crossed fields before striking the foil will cover a path about 250 metres long. Clearly, a

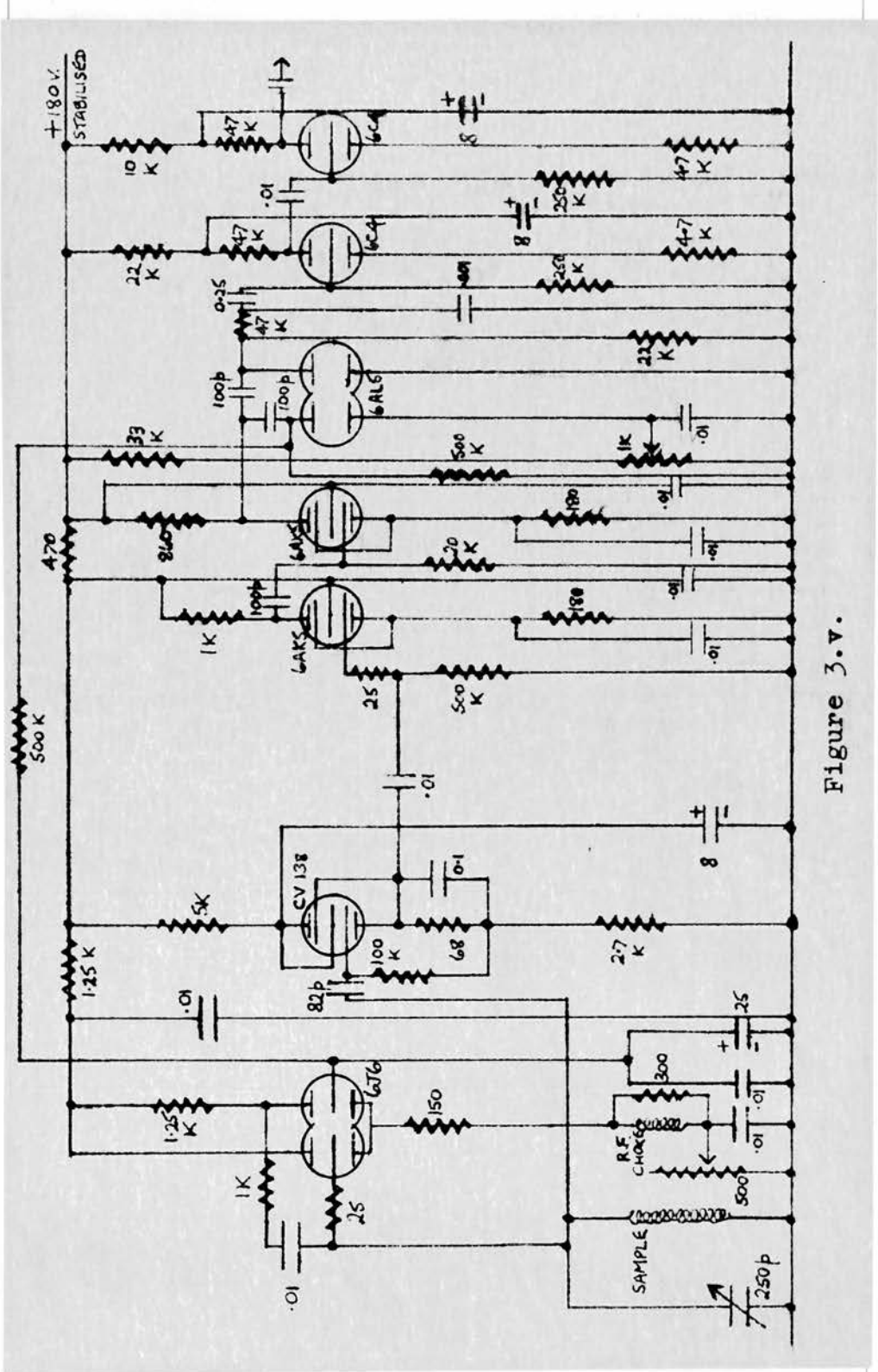

+180 V. STABILISED
10 K
47 K
22 K
0.25
33 K
840
470
1K
500 K
100p
6AK5
6AL6
6J6
CV 138
500 K
20 K
180
25
0.01
0.1
8
1.25 K
5K
100 K
68
2.7 K
82p
150
300
R.F. CHOKE
SAMPLE
250p

Figure 3.v.

good vacuum will be essential if the majority of electrons in the beam are not to be lost by gas scattering.

In order to estimate the required pressure, we may proceed as follows: each gas nucleus is surrounded by a "disc of influence" of radius about 5×10^{-9} cms., and hence of area, σ, about 8×10^{-17} cms.^{2} This is derived from the Rutherford scattering formula, assuming that a 128 kev. electron which is deviated by an angle of about 10^{-3} radians will be lost from the beam. Alternatively, the cross-section for 150 kev. electrons in hydrogen is about 6×10^{-19} cms.^{2} Since the cross-section is proportional to the square of the atomic number, this will be about 4×10^{-17} cms.^{2} for air, in fair agreement with the first result. If there are I electrons per square cm. passing through a gas containing n molecules per c.c., then the number lost from the beam in a distance dt will be

$$- dI = 2nI\sigma dt$$

since the chief constituents of air are diatomic. Hence

$$I(t) = I_o e^{-2n\sigma t}$$

where $I = I_o$ at $t = 0$.

Let us require that 90% of the initial beam

should be unscattered after travelling 250 metres. Then

$$\log_e(\tfrac{10}{9}) = 2n\sigma x\ 2.5 \times 10^4$$

$$\therefore\ {}^1/n \doteq 4 \times 10^{-11} \text{ ccs.}$$

Now the gas pressure, $P = nkT$, where k is Boltzmann's constant and T is the absolute temperature. Hence, taking $T = 300^{\circ}K$,

$$P = 10^{-3} \text{ dynes/cm}^2.$$

or $P = 10^{-6}$ mm. mercury.

The vacuum chamber consisted of a portion of circular sectioned brass tubing, of 11 ins. internal diameter and $\frac{1}{2}$ in. wall thickness. This was grooved top and bottom to accommodate rubber O-rings, and then fitted between the magnet pole faces. The chamber was evacuated through a 8 ins. x 3 ins. rectangular pipe, leading to a $5\frac{3}{4}$ ins. diameter and 9 ins. long circular sectioned pipe, to which was attached a Leybold Do501 oil diffusion pump. The pump oil used was Diffelen V, for which an ultimate pressure of less than 10^{-8} mm. mercury is claimed.

To prevent oil vapour from diffusing into the vacuum chamber, a water-cooled baffle consisting of three copper cups, each with just less than

half its base removed, was mounted immediately above the vapour pump. A Pollard-type liquid air trap (Pollard, 1959) was fitted: this consisted of a 9/16 ins. copper rod which penetrated into the system immediately above the water-cooled baffle, being insulated thermally from the pumping pipe by a stainless steel tube, 4 ins. long and of 0.01 ins. wall thickness. A brass reservoir for the liquid air was soldered to one end of the copper rod, and could be surrounded by a Dewar flask. A small, brass "fan" was attached to the inside end of the copper rod, to provide a large, cold surface area.

When the aluminium and fluon electrode system was placed in position inside the vacuum box, the pressure (as measured on an Edwards ionisation gauge) fell to about 10^{-5} mm. in four hours without liquid air, and to about - 2×10^{6} after a week, again without liquid air. The addition of liquid air reduced the pressure to around 10^{-6} mm. mercury. This was considered to be satisfactory, at least for a preliminary investigation. When the brass and perspex electrode system was used, pressures below about 6×10^{-6} mm. could not be achieved without the use of liquid air.

The E.H.T. Supply

We saw earlier that, to vary the number of orbital revolutions described by the electrons before being scattered by the foil from 800 to 10 turns requires an electric field which can be varied between about 1,500 volts/m. and 120,000 volts/m. Since the condenser plates are 10.14 cms. apart, the necessary potential difference between the plates must vary from approximately 150 volts to 12,000 volts.

Further, the potential difference, once established, must be stable over periods comparable with the necessary counting times; and it must be possible to measure the applied potential difference with sufficient accuracy. Now since

$$\frac{1}{n} = \gamma\left(\frac{g}{2} - 1\right) \doteq \gamma.10^{-3} \quad ,$$

where n is the number of complete orbits required to cause a relative spin precession of 2π, n must be determined with an accuracy of 1 part in 100 if g is to be measured with a precision of 1 part in 10^5. But n = "const." x B/E ; so we require to know B and E with a total accuracy better than 1 part in 100, and both fields must be stable to this degree. (The "constant" in the above equation is

v/π, where v is the magnitude of the velocity of the electrons striking the foil; clearly the energy selection of the electron-optical system must therefore be better than 2 parts in 100, since the energy is proportional to the square of v).

Bearing in mind the above considerations, a high voltage generator was constructed, the circuit diagram being shown in Figure 3.vi. The oscillator consists of two 6L6's in parallel, the level of oscillation being controlled by the screen potential of these valves. In turn, this is governed by the conductance of the ECC 33 which depends, through the d.c. amplifier consisting of two CV 358's and an ECC 35, on the potential applied to the right hand grid of the ECC 35. The control loop is completed by sampling the output of the rectifiers (on the negative side) and comparing this with a steady (positive) voltage from the helipotentiometer: the difference in potential is then the voltage applied to the right-hand grid of the ECC 35.

Six "Westalite" rectifiers and six H.T. condensers are used in the voltage tripling stage. The controlling voltage and two small voltages suitable for measurement by a precision

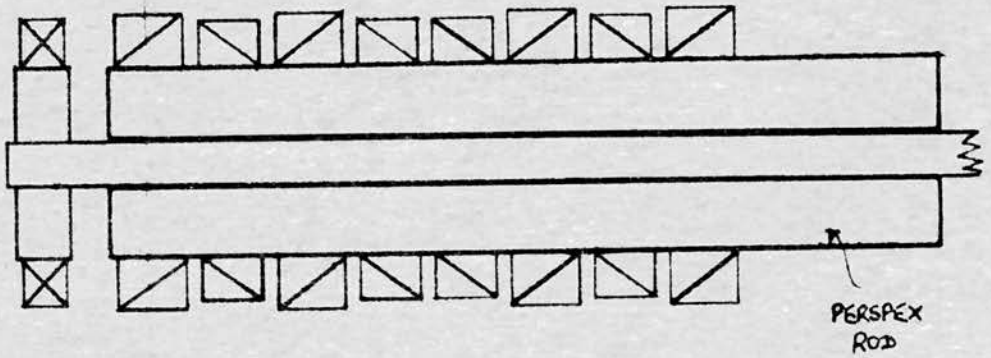


Figure 3.vii.
(Drawn to scale)

potentiometer are obtained by applying the output voltage of the tripler to a chain of high precision resistors. The radiofrequency transformer is shown in Figure 3.vii, the grid coil being coupled to the secondary of the transformer to an extent which may be varied manually.

The initial adjustment of the unit is as follows: close switch 1, thereby earthing the right-hand grid of the ECC 35. Set the 5 KΩ variable resistor to give + 350 volts on the stabilised H.T. line. Note the variation in the tripled voltage caused by moving the centre tap of the 50 KΩ variable resistor from one end of its range to the other, and adjust this resistor setting to give an output voltage approximately in the centre of the range available; the D.C. amplifier will then be at approximately the centre of its controlling range. Adjust the position of the grid coil to give maximum output consistent with no drop in the potential of the stabilised H.T. line. Open switch 1, and set the helipotentiometer to give the required output voltage.

Using the circuit as shown, the resonant frequency is around 77 kc./s., and the total maximum voltage is about 19 kilovolts (that is, + and - 8.5 kilovolts approximately). Lower

voltages may be obtained by decreasing the coupling between the grid coil and the secondary of the transformer. For output voltages below about 1 kilovolt, the oscillator became rather unstable, so a stack of dry batteries was used instead. Measurement of the voltage was made by applying the battery output to the precision resistor chain.

At a total output of 15 kilovolts, the 77 kc./s. ripple measured at the low voltage side of the 0.025 μF feedback condenser was less than 2 volts peak to peak; that is, the ripple is about 1 part in 10^4. At the same output the overall stability as measured across both the 2kΩ precision resistors in the output chain was better than 4 parts in 10^4 over a period of 24 hours.

Since the centre grid of the electrode system is not actually earthed, the exact equality of the positive and negative outputs is not essential. However, because of the proximity of various earthed components (vacuum box, Geiger counters), it is desirable that the potential of the x - z plane of the system should not be greatly different from zero. During a 24 hours test run, the "lopsidedness" of the output (15 kilovolts) varied between 56 and 24 parts in 10^5. This means that when an output of 15 kilovolts is applied to the electrode system, the potential of the mid-

plane differs from zero by about 4.25 volts.

From Figure 3.vi, it is clear that the output voltage may be calculated in terms of the potential differences across the two 2kΩ precision resistors. Strictly, this is possible only when the right-hand grid of the ECC 35 valve is at earth potential. In practice, if a high-precision measurement of the output voltage is required, it is always possible to alter the grid coil coupling so that, for any required voltage, the potential on the right-hand grid of the ECC 35 is actually zero. Then we find that:

Half total output = 6071 x P.D. across 2kΩ.

The error in the measurement of the output voltage arises from the tolerance in the values of the resistor chain components, and from the sensitivity of the precision potentiometer used to make the measurement. The chain consisted of 24 components, the stated tolerance being 0.1%. Temperature stability was achieved by immersing all the resistors in a large oil-bath. It is safe to say, therefore, that the error arising from the first source will be less than 1 part in 10^3. The lowest voltage to be measured on the potentiometer is about 0.025 volts; this arises when the total output is about 150 volts, which corresponds to around 800 orbital revolutions.

Changes in voltage of a few microvolts could be detected, but absolute measurement of voltage is limited by the stability of the standard cell. The error from the second source should therefore be less than 1 part in 10^5. From this, we see that the dominant error in the electric field intensity arises from instability in the output voltage. Over periods of several hours, this error will be less than 4 parts in 10^4.

The total resistance of the output precision resistor chain is just over 20 MΩ. With the aluminium and fluon electrode system, the total resistance of the potential divider is 88 MΩ. At maximum output, the total power consumption in the resistor chains is about 20 watts, of which less than 4 watts is dissipated inside the vacuum box.

The Source

For practical reasons, electrons with energies in the range 100 - 150 kev. are required. To reduce background to a minimum, a pure beta-emitter is desirable. Since several sources would be required in the development of the apparatus, an isotope was sought which was readily available, convenient to handle, and which could

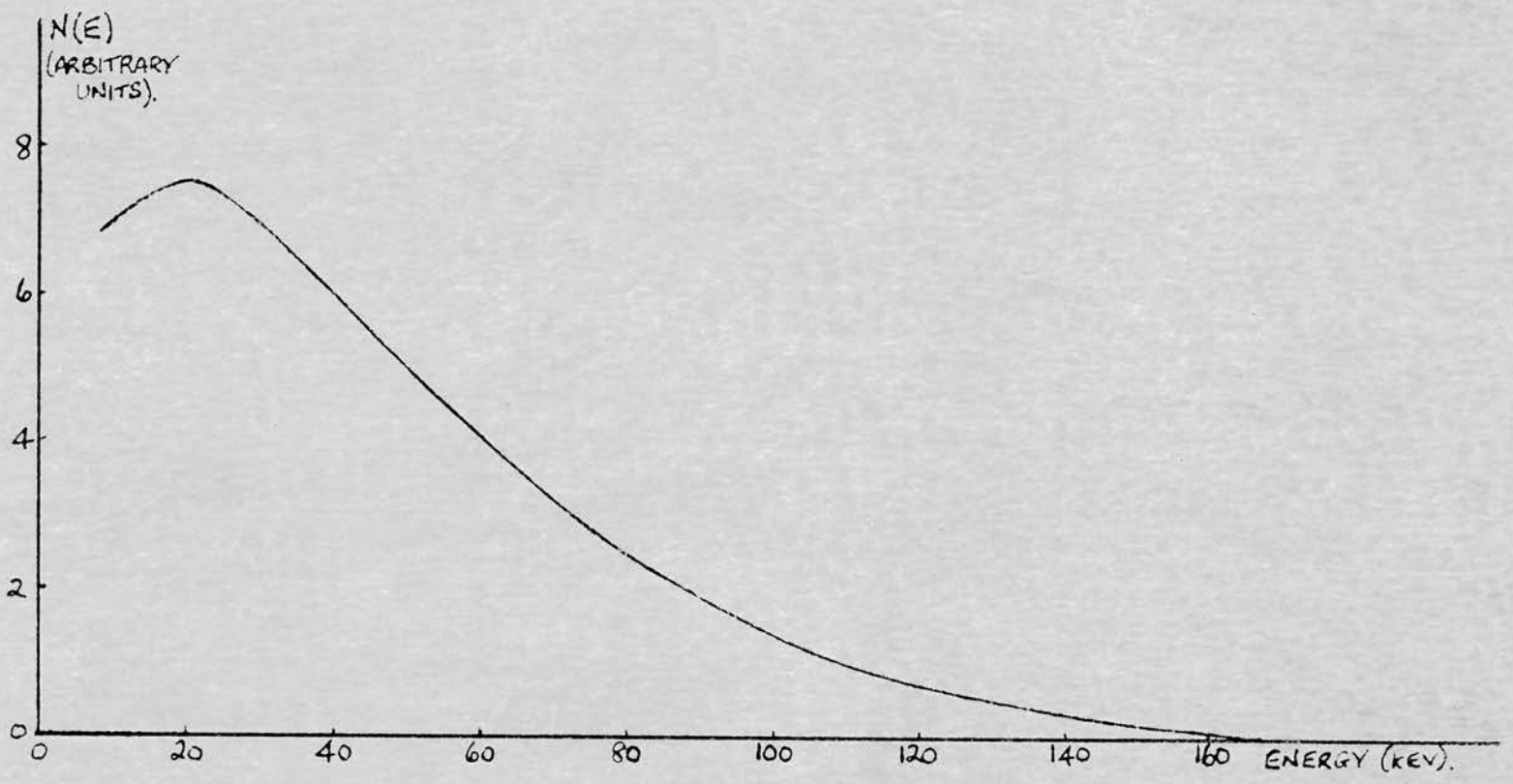


Figure 3.viii.

be deposited with reasonable precision on such backings as thin wires and small metal studs. Some of the mounts actually used are described in Chapter 4. The electrons emitted must be longitudinally polarised; as discussed in Appendix III, all known beta-emitters satisfy this condition to some extent. To minimise depolarisation in the source itself, and to give high counting rates without excessive source thickness, a carrier-free isotope of high specific activity is required.

The isotope S^{35} fulfills all the requirements listed above. The end-point energy is 167.4 kev. (Connor & Fairweather, 1957) and the half-life is 87 days (Hendricks, Bryner, Thomas & Ivie, 1943). Figure 3.viii shows the energy spectrum of S^{35}, as measured with a proportional counter (Cockroft & Insch, 1949; for further references, see Strominger, Hollander & Seaborg, 1958).

The Scatterer

Since the differential cross-section for the scattering of electrons by the Coulomb field of a nucleus is proportional to Z^2, where Z is the

atomic number of the scattering material, it is desirable to use scatterers of high atomic number, especially in cases such as the present experiment where counting rates are low. Further, the counting rate asymmetry in a single scattering experiment is proportional to Z. For these reasons, it is customary to use foils of gold (Z = 79) in Mott scattering experiments.

In experiments where absolute measurements are required, for example, where Mott cross-sections or where helicity of beta-particles have to be determined, it is necessary to use either very thin foils (of the order of 0.1 mg/cm^2.) or a series of foils of different thicknesses and to extrapolate results to zero foil thickness. If this is not done, then the effects of plural and multiple scattering in the foil will cause serious error (see, for example, Spiegel, Ruane, Anthony, Waldman & Miller, 1959).

While no such measurements were required in the present experiment, excessive foil thickness would mask any genuine spin-dependent counting rate asymmetry. It was convenient to use gold foil of 1 mg/cm^2 which, although very fragile, was found to possess adequate mechanical properties. The dimensions of the foil used with the aluminium

and fluon system are shown in Figure 3.iv.

The Detectors

Because of lack of space above and below the grid system, and because of the magnetic field, the only form of detector which could be used in the present experiment was a small, thin-window Geiger counter. The presence of the grids, and the effect of electrons spiralling in the magnetic field, restricted the scattering angle to around 90°.

From the table in Appendix IV, we see that, at energies around 100 kev, the product polarisation x asymmetry x cross-section is maximum at a scattering angle of 90°. Although this product increases rapidly as $\beta = v/c$ decreases, it was thought that, for electrons with β less than about 0.5, this advantage would be more than offset by increased absorption in the counter window.

In addition to being small, the counters used had to be non-magnetic and must have very thin windows. Side window counters supplied by Twentieth Century Electronics were used, the thickness of the mica windows being in the range 1-2 mg/cm^2. The metal-to-glass seals of these

counters are, in fact, slightly magnetic, but their effect on the magnetic field was found to be sufficiently small to be neglected. The positioning of the counters, which were in the form of circular cylinders $^{7}/_{8}$ ins. in diameter and about 4 ins. long, is shown in Figures 3.ii and 3.iii.

CHAPTER FOUR

Formation and Properties of an Electron Beam in Crossed Electromagnetic Fields

Using the aluminium and fluon electrostatic system described in the previous chapter, and a thick gold foil (1 mg./cm.^{2}), asymmetry determinations were made at three different voltages. The results obtained are tabulated below, where the suffix 1 refers to the upper Geiger counter, while 2 refers to the lower Geiger counter.

The number of turns, n, was calculated from the expression

$$n = \frac{LeB^2d}{2\pi\gamma mV} \quad ,$$

where L is the total distance drifted 2 7.61 cms.

B is the magnetic field strength = 2.74×10^{-2} webers/m^2.

d is the condenser plate separation = 10.28 cms.

$^e/_m$ is the electron charge-to-mass ratio = 1.76×10^{11} coulombs/kgm.

V is the total applied voltage.

Applied Voltage	No. of turns	Counting Rate/min. C_1	Counting Rate/min. C_2	Asymmetry $2(C_1-C_2)/(C_1+C_2)$
3.45 kv	40	268.4 ± 2.0	272.4 ± 1.8	0.165 ± 0.011
2.76 kv	50	272.8 ± 2.6	229.4 ± 2.4	0.173 ± 0.014
0.74 kv	186	260.7 ± 0.5	220.3 ± 0.5	0.168 ± 0.003

Now, for electrons with $\beta = 0.55$ (= 102 kev.), the number of turns required to produce a relative spin precession of 2π is approximately 720, if we take the g-factor anomaly to be 0.00116. Hence we should expect to find that the scattering asymmetry is a maximum after about 180 turns; we see from the above table that no such asymmetry was observed.

While this negative result was almost certainly due in part to multiple and plural scattering in the gold foil, it was felt that other factors, principally the electron optics of the system, were contributing as well. It was therefore decided that a more detailed study of the behaviour of electrons in the electromagnetic field should be undertaken.

For this investigation, which would be concerned with small numbers of orbital revolutions only, the brass and perspex system used in the initial stages of the experiment was adequate, and produced a more uniform electric field than the later system since it employed larger condenser plates.

A new S^{35} source, in the form of a circular disc about $^{3}/_{32}$ ins. in diameter, was prepared. To reduce the electron background, a small metal

cylinder, about 1 cm long, was fitted over the source, the axis of the cylinder being in the y direction, that is, in the direction of the required initial electron velocity.

The scattering foil and foil-holder were replaced by a piece of thin plastic scintillator (of type NE 103) mounted on the end of a perspex light-guide the axis of which coincided with the positive z-axis. The effective breadth of the crystal could be controlled by a moveable aluminium mask. Scintillations were detected by an E.M.I. type 6097B photomultiplier.

The spacing between the brass condenser plates was 12.70 cms., which corresponds to a 30° beam acceptance with mean orbit diameter 10.08 cms. Using the same magnetic field as before (2.74×10^{-2} webers/m.^{2}), we find $\beta = 0.63$ (150 kev.).

The effects of the magnetic field, B (2.74×10^{-2} webers/m.^{2}), and the electric field, E (Potential difference of 10.5 kv.), on the counting rate were studied with the crystal (1 mm. wide) placed 6.0 cms. away from the source. The following results were obtained:

B off, E off ; counting rate: 247 ± 12/min.

B on, E off ; counting rate: 285 ± 12/min.

B on, E on ; counting rate: 379 ± 12/min.

The small increase in counting rate produced by the magnetic field alone is due to scattering from the back of the source. In the following discussion, the term "background" means the counting rate with B on and E off, while "signal" means counting rate with B and E on minus background.

Because the background, as defined above, includes the true background of the photomultiplier, it is not constant even for periods of a few minutes. Therefore, background had to be sampled at regular intervals during the period of any count. To facilitate this, a two-channel counting system was set up: for the first half-minute, one scalar counted with E on, while for the next half-minute, the other scalar counted with E off. A device incorporating a synchronous motor which actuated microswitches was used so that the switching operation was fully automatic.

We shall now attempt to predict, from purely theoretical considerations, the manner in which counting rate should vary with applied voltage when the crystal detector is placed at a given constant distance from the source, the electron optical system having been set up in a given fashion. Later, we shall compare these predictions with experimental results.

Using those values of the various parameters which were actually employed in the experimental arrangement, we take:

Distance from source to crystal, $L = 7.0$ cms.

Separation of condenser plates, $d = 12.7$ cms.

Applied magnetic field, $B = 2.74 \times 10^{-2}$ webers/m.^{2}.

Now, if n is the number of turns described by the electrons before striking the crystal when the potential difference between the condenser plates is V volts, then

$$n = \frac{LeB^2d}{2\pi\gamma mV} \quad , \text{ where } \gamma = (1-\beta^2)^{-\frac{1}{2}} .$$

Figure 4.i shows how β and V are related for values of n between 8 and 20.

To investigate the electron optical properties of the system, we assume first that there is no baffle. Figure 4.ii then shows that, if θ is half the emission angle which is accepted by the system, then

$$r\,(1 + \sin\theta) = {}^{d}/_{2} \quad ,$$

where r is the radius of an electron orbit.

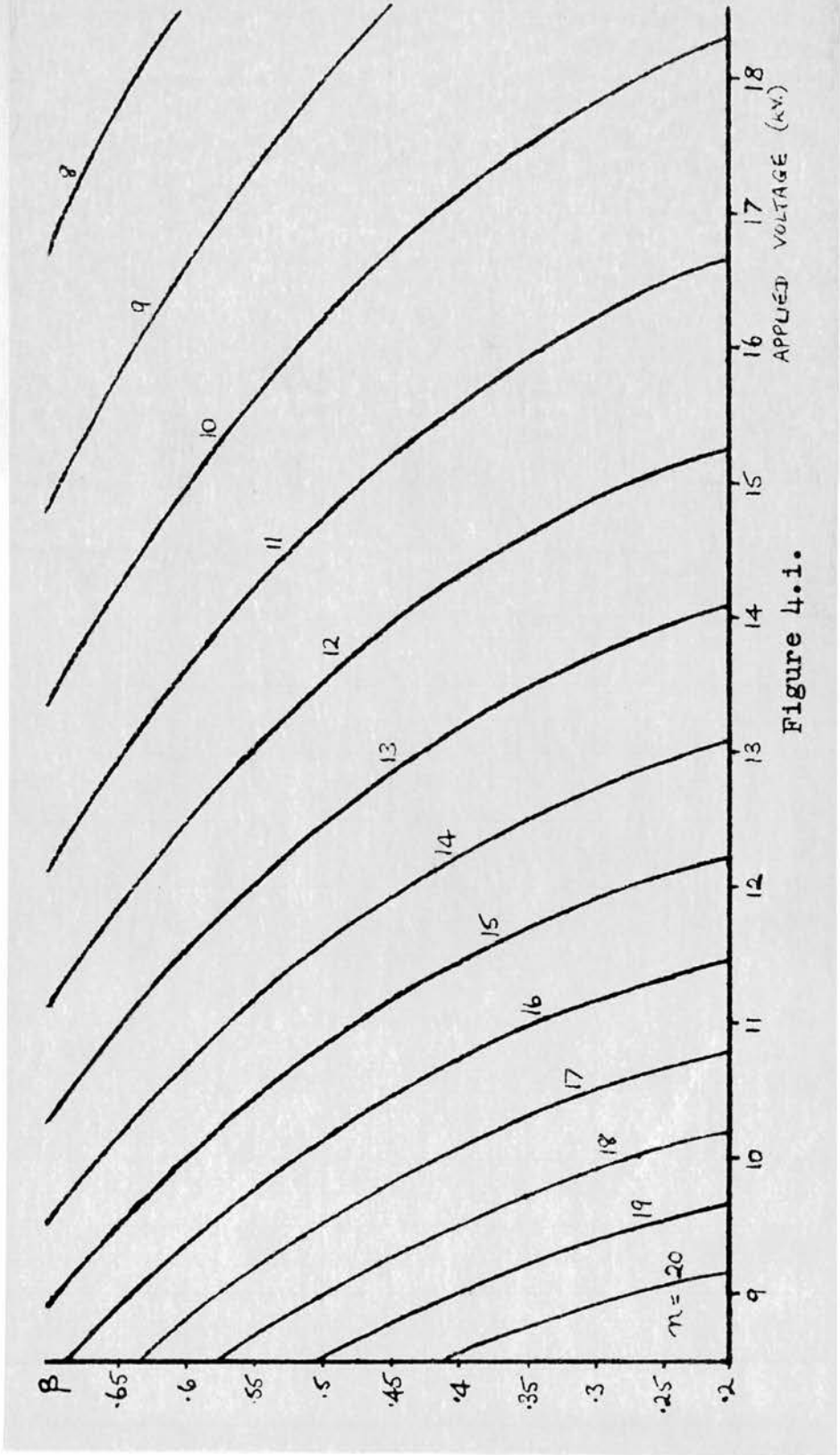
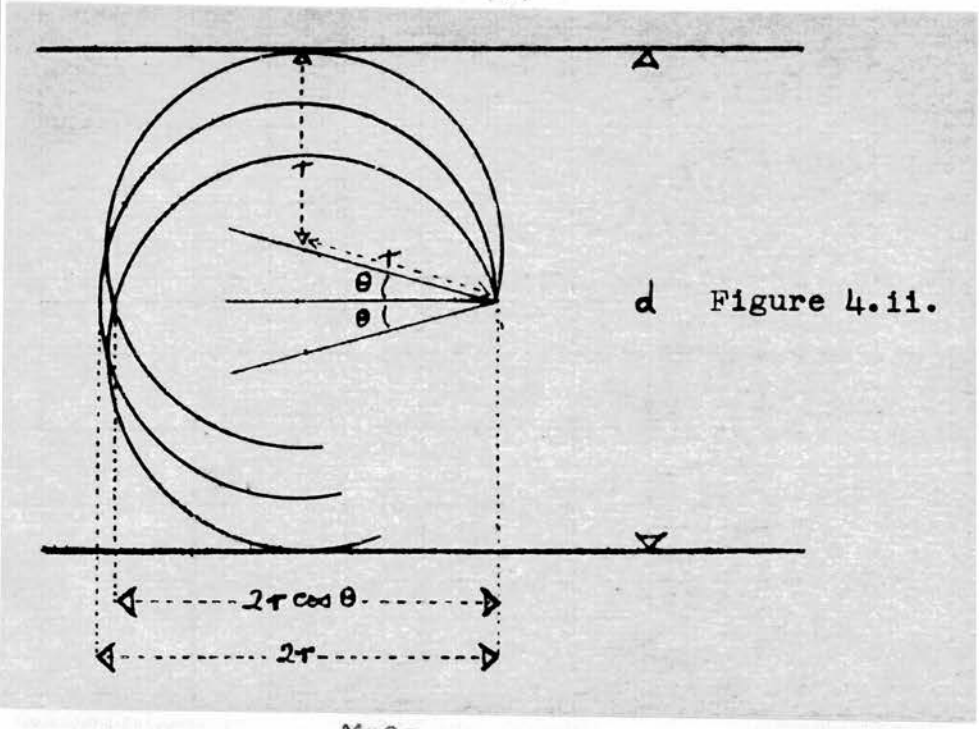

β
.65
.6
.55
.5
.45
.4
.35
.3
.25
.2
9
10
11
12
13
14
15
16
17
18
APPLIED VOLTAGE (K.V.)
n = 20
19
18
17
16
15
14
13
12
11
10
9
8

Figure 4.1.



d Figure 4.ii.

Now, $r = \frac{\gamma m \beta c}{eB}$, so that

$$1 + \sin\theta = \frac{1}{\gamma\beta} \cdot \frac{eBd}{2mc} \quad .$$

After half a revolution in the magnetic field, the breadth of the electron beam for a given energy will be $2r(1 - \cos\theta)$. The following table gives r, θ, and $2r(1 - \cos\theta)$ as functions of β.

β	r(cms.)	θ(deg.)	$2r(1-\cos\theta)$(cms.)
0.35	2.32	90	4.64
0.40	2.72	90	5.44
0.45	3.14	90	6.28
0.50	3.60	50	2.57
0.55	4.10	33.3	1.35
0.60	4.67	21.1	0.63
0.65	5.32	11.2	0.20

We now take account of the fact that the source, on a small brass stud, was mounted on a square rod of approximate breadth 0.5 cms.; and recall that a baffle was fitted, the distance from source to baffle being 8.0 cms.

It is readily seen that, if

$$2r \cos\theta > 7.5 \text{ cms.},$$

the source holder will not intercept the electron beam at all while, if

$$2r < 7.5 \text{ cms.},$$

the beam will be cut off completely by the source holder.

Similarly, if

$$2r \cos\theta > 8.0 \text{ cms.},$$

the entire beam will be cut off by the baffle while, if

$$2r < 8.0 \text{ cms.},$$

the baffle will not intercept the beam at all. This effect of the source holder and baffle restricting the transmitted beam is shown graphically in Fig. 4.iii.

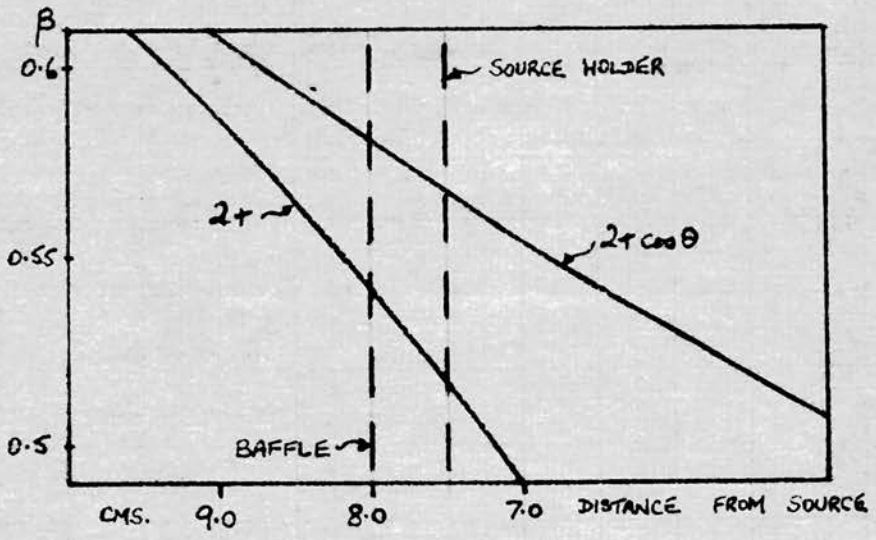


Figure 4.iii.

From this graph, we see that only those electrons for which β lies between 0.515 and 0.580 can be transmitted by this arrangement. Electrons of higher energy are cut off by the baffle, while those of lower energy are stopped by the source holder.

In order to predict counting rates, we assume that the number of electrons of a given energy which are transmitted is proportional to the "slit width" as determined from Figure 4.iii, taking the maximum width as unity. Further, we require to know how the number of electrons emitted by the source varies with β. This variation is shown in Figure 4.iv, which is derived from Figure 3.viii. The counting rate

scale has been normalised in an arbitrary fashion.

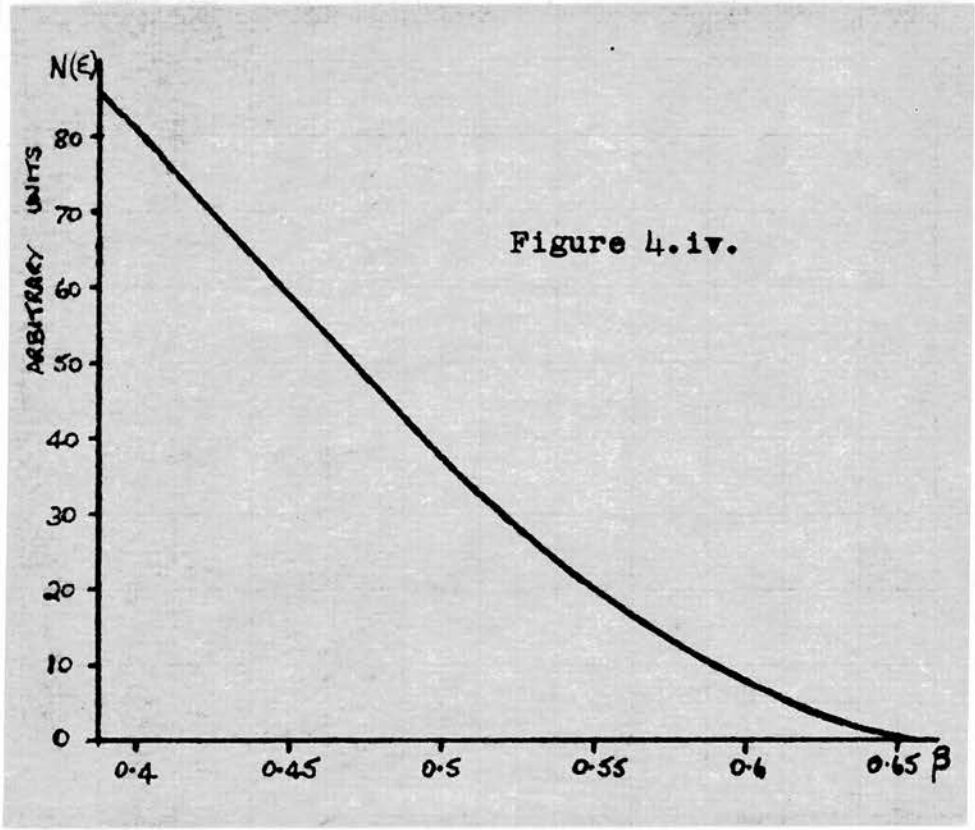


Figure 4.iv.

One further factor must be considered: if we assume perfect focussing in the x-y plane, with no focussing in the z-direction then the number of electrons which reach the detector will decrease linearly with the distance between source and detector; that is, with the number of turns described before the beam strikes the crystal.

From these considerations, the counting rate, c, should be given by

$$c = \frac{S(\beta)N(\beta)}{n}$$

where $S(\beta)$ is the "slit width" as given by Figure 4.iii, $N(\beta)$ is the number of electrons emitted by the source per unit range of β, as given by Figure 4.iv, and n is the number of turns. The quantities $S(\beta)$ and $N(\beta)$ are tabulated below:

β	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58
$S(\beta)$	0	0.15	0.55	0.95	1.00	1.00	0.75	0
$N(\beta)$		30.0	26.5	23.0	20.0	17.0	14.0	

Finally the counting rate is computed from the above, using Figure 4.i. This leads to the following table for the variation of counting rate with applied voltage.

V(kv)	18.0	17.8	17.6	17.5	17.4	17.2
β		0.52	0.53	0.54	0.54	0.56
n		9	9	9	9	9
c	0	0.50	1.62	2.42	2.42	1.89

V(kv)	17.0	16.8	16.6	16.5	16.4	16.2
β	0.57					
n	9					
c	1.17	0	0	0	0	0

V(kv)	16.0	15.8	15.6	15.5	15.4	15.2
β	0.52	0.535	0.55	0.56	0.565	
n	10	10	10	10	10	
c	0.45	1.80	2.00	1.70	1.40	0
V(kv)	14.8	14.6	14.5	14.4	14.2	14.0
β			0.52	0.53	0.55	0.57
n			11	11	11	11
c	0	0	0.41	1.33	1.82	0.95
V(kv)	13.8	13.6	13.5	13.4	13.2	13.0
β					0.53	0.55
n					12	12
c	0	0	0	0	1.22	1.67
V(kv)	12.8	12.6	12.5	12.4	12.2	12.0
β	0.57				0.53	0.55
n	12				13	13
c	0.88	0	0	0	1.12	1.54
V(kv)	11.8	11.6	11.5	11.4	11.2	11.0
β	0.57			0.52	0.545	0.57
n	13			14	14	14
c	0.81	0	0	0.32	1.50	0.75
V(kv)	10.8	10.6	10.5	10.4	10.2	10.0
β		0.525	0.53	0.55	0.57	0.52
n		15	15	15	15	16
c	0	0.60	0.99	1.33	0.70	0.28

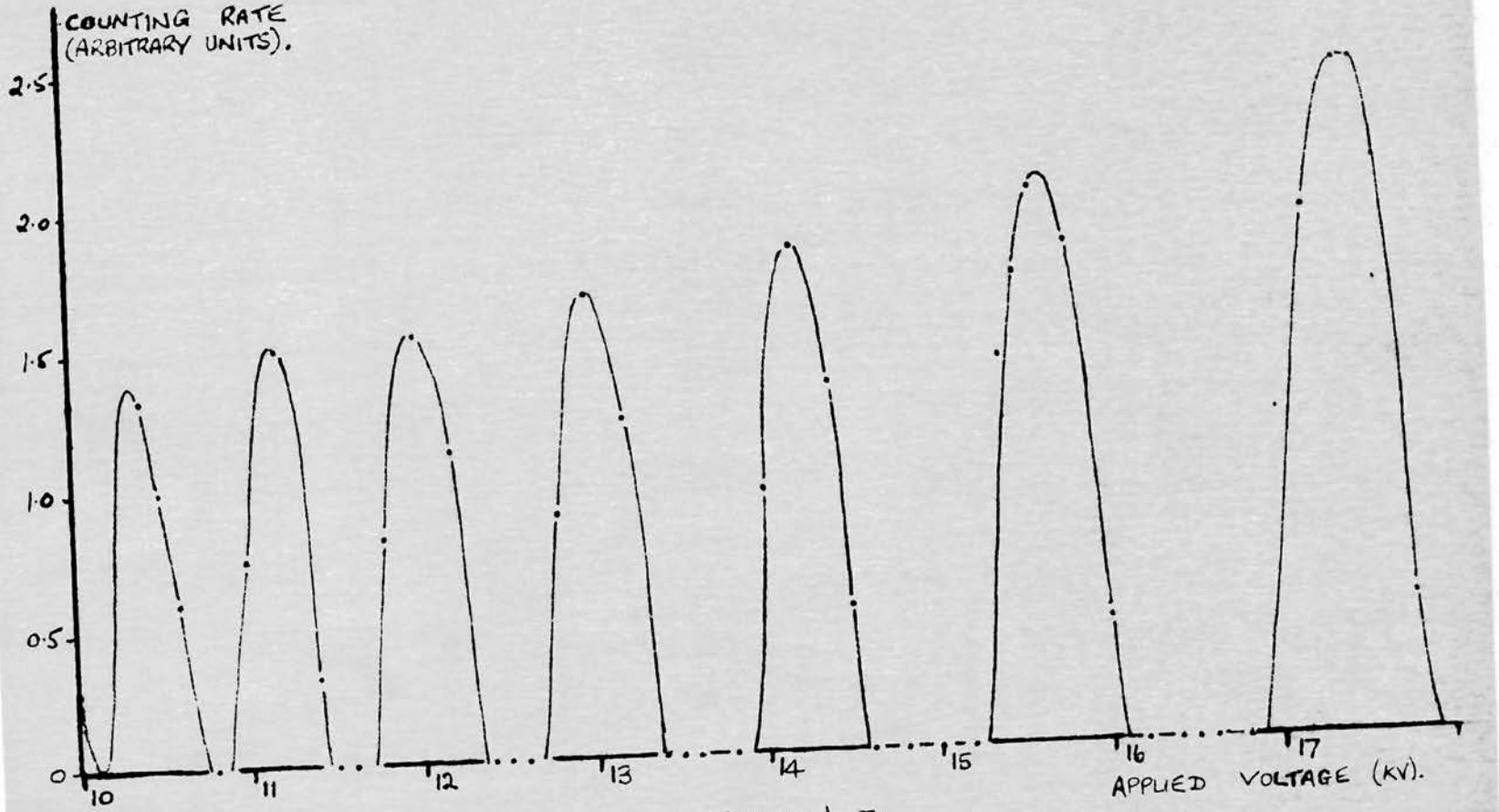

COUNTING RATE
(ARBITRARY UNITS).
2·5
2·0
1·5
1·0
0·5
0
10
11
12
13
14
15
16
17
APPLIED VOLTAGE (KV).

Figure 4.v.

Figure 4.v is a graph of this theoretical counting rate as a function of the total applied voltage.

This should be compared with the experimentally determined curve in Figure 4.vi. It is seen that, while there is poor agreement between the general shapes of the two curves, the maxima and minima occur at very nearly the same voltages in each. The fact that the observed counting rate between successive maxima does not drop to zero implies that there is some spreading of the beam in the x-direction; that is, focussing in the x-y plane is not perfect.

It is significant that, in the theoretical curve, the counting rate at successive maxima decreases more slowly than was found in practice as the voltage decreases. This implies that, in the above calculation, some attenuating factor has been overlooked.

When, as must be the case, the focussing in the x-y plane is <u>not</u> perfect, we may suppose that a fraction f of that beam is "lost" each turn. If $N(0)$ is the original number of electrons in the beam, the number surviving in the beam after n turns will be

$$N(n) = N(0)(1 - f)^n \ .$$

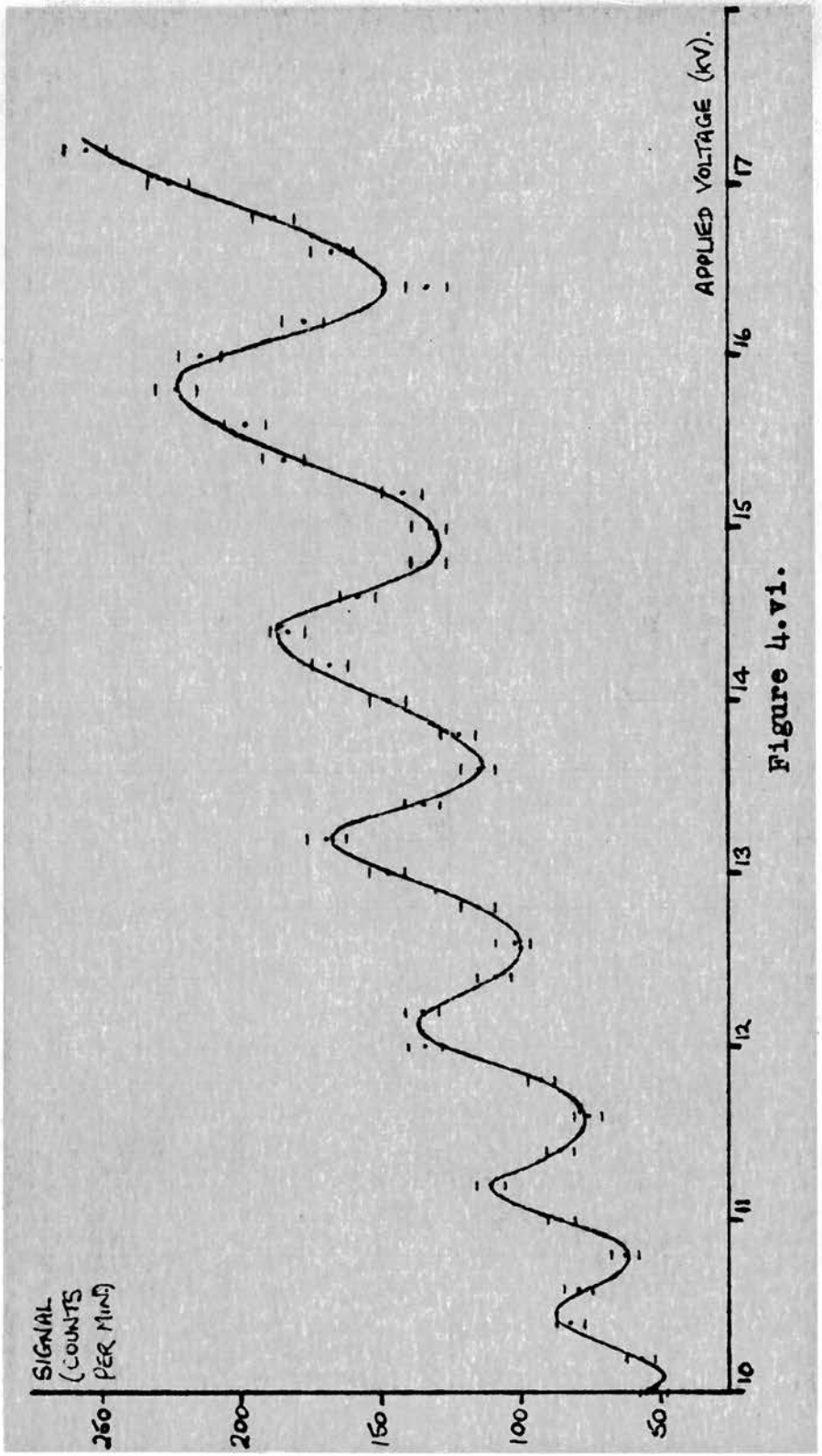

SIGNAL (COUNTS PER MIN.)
250
200
150
100
50
10
11
12
13
14
15
16
17
APPLIED VOLTAGE (KV).

Figure 4.vi.

If $f \ll 1$, then we may write

$$N(n) = N(0)(1 - nf),$$

so that the fractional decrease is

$$\frac{N(0) - N(n)}{N(0)} = nf.$$

Hence, provided the focussing is reasonably good ($f \ll 1$), the decrease in counting rate from this cause should be proportional to n. Combining this with the above-mentioned decrease arising from the absence of focussing in the z-direction, we might expect that the observed counting rate should be proportional to $1/n^2$.

If the theoretical counting rate is thus modified, the curve shown in Figure 4.vii is obtained. From this we see that, while the amplitudes of successive maxima approximate more closely to those found by experiment, the rate of decrease is still insufficient. The deficiency is doubtless due to gas scattering which, being an absorption type of effect, will increase exponentially with n :

$$C(n) = \text{const.} \times e^{-kn}.$$

At 10 kilovolts, the distance drifted per revolution is about 4.5 mm. Clearly, a detector

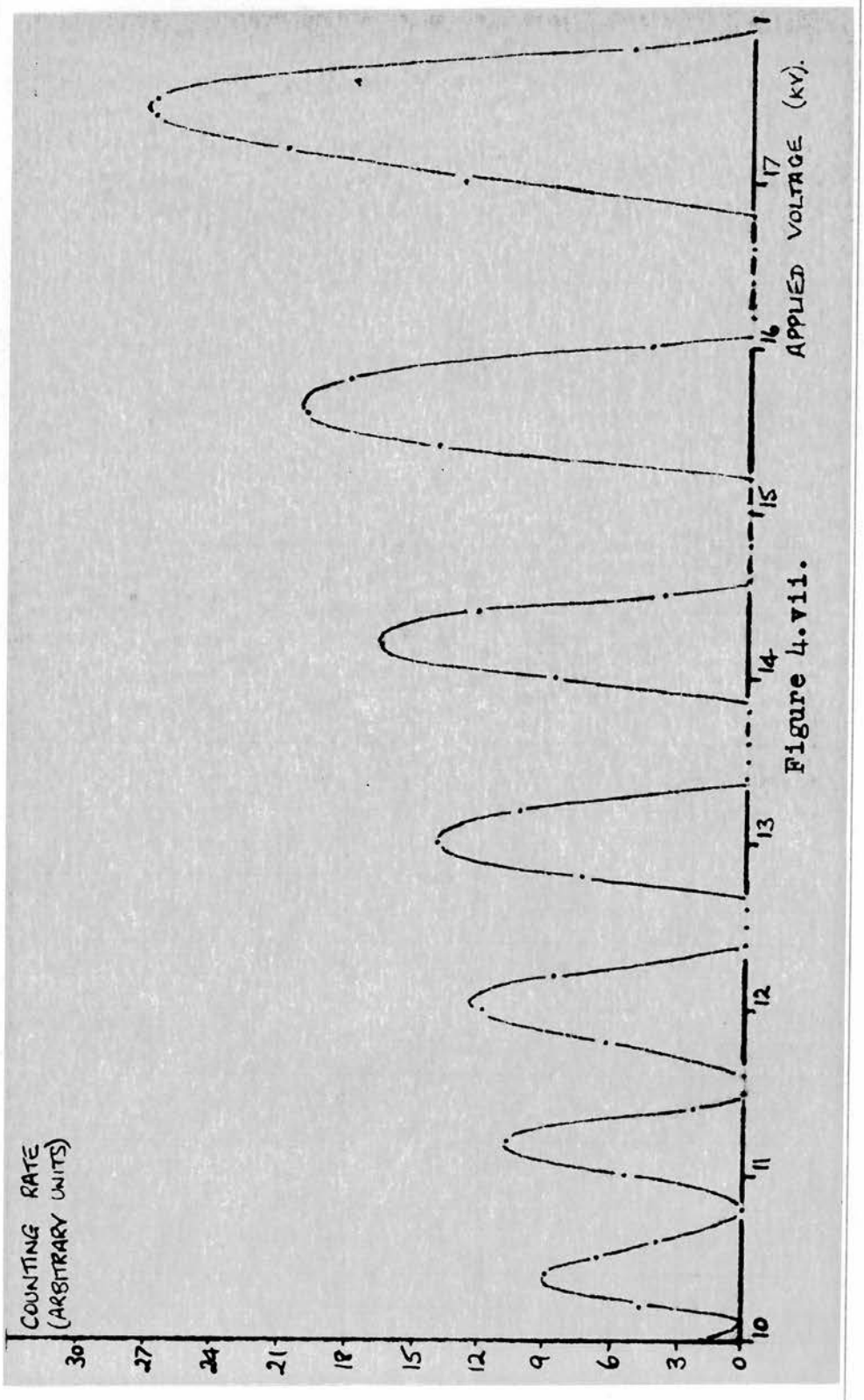

COUNTING RATE
(ARBITRARY UNITS)
30
27
24
21
18
15
12
9
6
3
0
10
11
12
13
14
15
16
17
APPLIED VOLTAGE (KV).

Figure 4.vii.

of width 1 mm. will be adequate to resolve maxima and minima in counting rate at such voltages. However, because of the finite size of the source and because of the cylinder fitted over the source, it must be expected that some electrons will fail to clear the source system at the end of the first turn when the voltage falls below about 10 kilovolts. This explains the decrease in counting rate at low voltages.

For the reasons discussed in Appendix V, two rectangular coils, each of three turns of wire, were placed one above and one below the electrode system, the longer sides of the coils being close to the edges of the condenser plates. At the expense of sharp focussing in the x-y plane, the magnetic field associated with a current passed through such an arrangement of coils should produce focussing parallel to the direction of the main magnetic field. The effect of the coils is to modify the otherwise uniform magnetic field distribution, causing the field lines to be convex outwards. Electrons with a component of velocity parallel to the z-axis are therefore affected by the component of the field parallel to the x-y plane; such electrons will experience a force tending to deflect them back towards the x-y plane.

Application of such focussing current should, therefore, increase signal strength, provided the electrons have spent a time in the crossed fields of the same order as their period of oscillation parallel to the z-axis. The increased signal will be achieved at the expense of the pronounced counting rate periodicity which was observed with no focussing field.

Before this was investigated experimentally the disc source was replaced by a deposit of S^{35} on a thin copper wire which was placed parallel to the magnetic field, and was surrounded on three sides by a screen of thin aluminium sheet. The active length of the wire was 11 mms., and the strength about 0.5 mC.

When focussing current was applied at a constant electric field setting, it was found that the counting rate increased rapidly as the focussing current increased. At 14.1 kilovolts, the counting rate of 3,000 per minute with no focussing was increased by almost 400% when the focussing current was 8 amps., and was still rising rapidly at 10 amps. However, the effect of an 8-amp. focussing current was to reduce the periodicity in the counting rate even at high values of the electric field intensity.

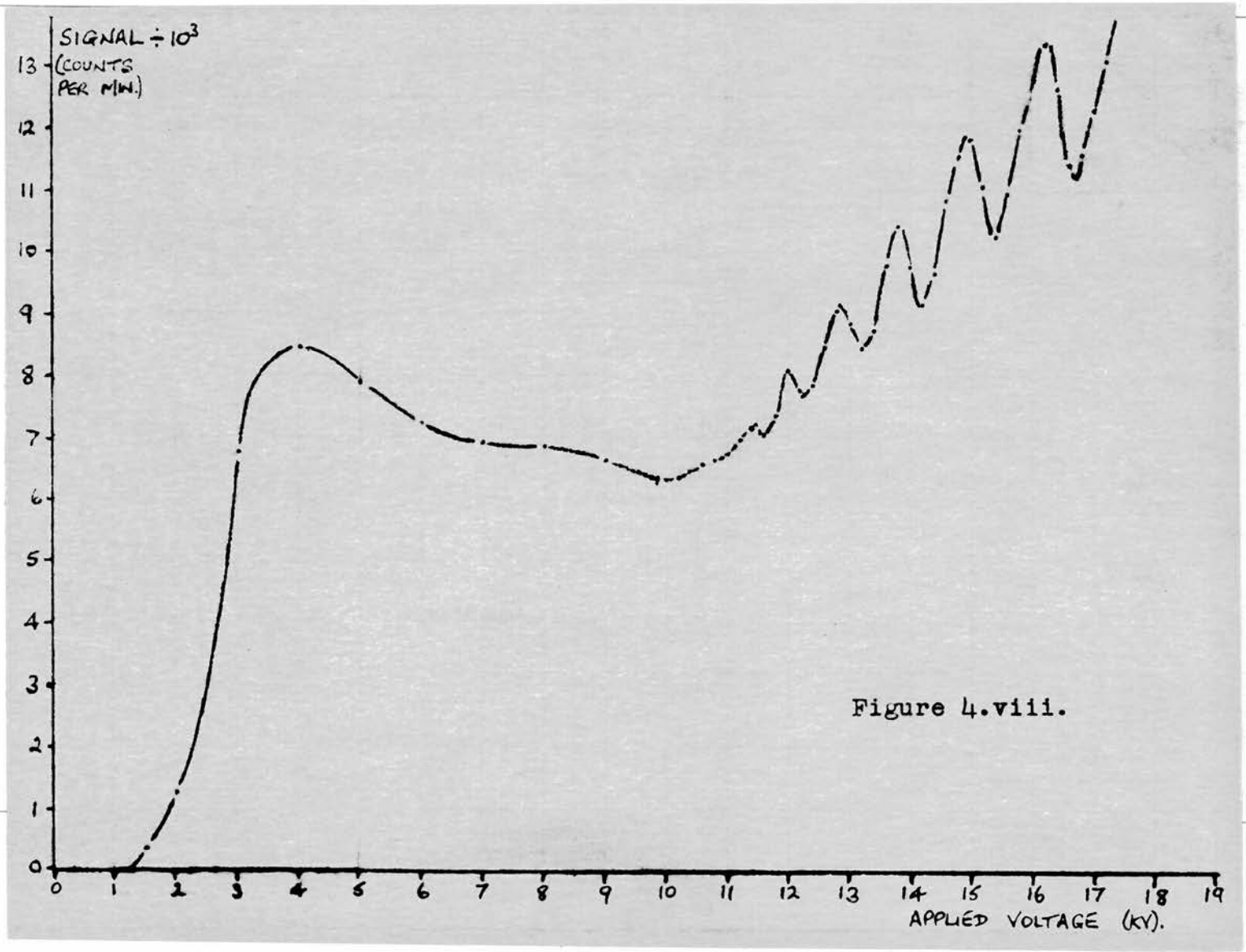


Figure 4.viii.

Figure 4.viii shows the variation of counting rate as a function of applied voltage when the focussing current was 8 amps., the drift distance being 7.8 cms. It is seen that the effect of focussing is to maintain the counting rate at an almost constant value over a wide range of electric field intensities. Since the crystal width used was 1 mm., any periodicity in the counting rate should show up even at the lowest field intensities. The sharp drop in counting rate below 3 kilovolts is caused by electrons failing to clear the aluminium screen round the source, at the end of the first orbit.

CHAPTER V

Conclusions

To sum up, it may be said that conclusive evidence for the existence of trochoidal orbits of the required type has been obtained. Further, it has been shown that, by the use of focussing in the direction of the magnetic field, it is possible to maintain the beam strength at the expense of sharp focussing in a plane perpendicular to the magnetic field.

The equation used to calculate the g-factor anomaly involves not only the number of orbital revolutions corresponding to a relative spin precession of 2π, but also the factor Υ. Hence, to compute accurately the anomaly, the electron energy must be sharply defined. This means that the mechanical adjustment of the baffle and the foil relative to the source, and the breadth of the source mounting, must be set very precisely. For this reason, it is not possible to overcome defocussing caused by field inhomogeneities simply by reducing the physical dimensions of the experiment.

A further difficulty is the following: to reduce the background of unwanted electrons to a

tolerable level, it was necessary to screen the source. As we have seen, the thickness of the screen between source and foil sets a lower limit on the drift per revolution of the electrons. Below this limit, electrons will strike the back of the screen and be lost. To achieve 800 orbital revolutions with the present arrangement required a minimum drift distance of about 0.1 m.m.

Even with a completely open source, the fact that the source has finite breadth will cause the beam strength to be severely attenuated as the drift distance per turn becomes very small. If an unscreened source has to be used, then it will be essential to use an energy-sensitive detector to cut out a substantial fraction of the background. Because of vacuum difficulties, perspex light pipes are undesirable. In any case, because of the upsetting influence of the magnetic field, and because of the restricted space, any light guide would have to be of such a length as to make the energy resolution of the system very poor. The development of a suitable solid state device would be most welcome.

Recently it has been shown that, while approximately 800 turns in the crossed fields are required to give a complete period of polarisation asymmetry, the g-factor anomaly may be computed

with the required precision (about 0.1%) even if only one quarter of the complete period is determined experimentally. This will be possible provided each point on the quarter-period curve is known with an accuracy of about 0.1%. Further, scattered electrons have been detected after performing about 150 turns in the crossed fields. It therefore seems likely that a measurement of the g-factor anomaly of free electrons may be possible in the near future.

APPENDIX I

Electron Spin

In this Appendix, we start by assuming that the electron does possess a spin, and then verify the non-quantum mechanical part of the Uhlenbeck and Goudsmit hypothesis: that the intrinsic magnetogyric ratio of the electron is $^{e}/_{m}$, and that this is twice the electron's orbital magnetogyric ratio. This is followed by a brief account of the Dirac relativistic wave equation, and some deductions from it are discussed.

We first deduce the orbital magnetogyric ratio for an electron. Without loss of generality, only the non-relativistic limit is considered. The magnetic vector potential $\underline{A}$ at position $\underline{r}$ due to current densities $\underline{j}'$ at positions $\underline{r}'$ which are confined to a volume V' is

$$\underline{A}(\underline{r}) = \frac{\mu_0'}{4\pi}\int_{V'} \frac{\underline{j}'(\underline{r}')\,dV'}{|\underline{r}-\underline{r}'|},$$

where $\mu_0' = 4\pi.10^{-7}$ henries/metre. For discrete, moving particles, each carrying a charge e and moving with a velocity $\underline{v}_i'$, this becomes

$$\underline{A}(\underline{r}) = \frac{\mu_0' e}{4\pi}\cdot\sum_i \frac{\underline{v}_i'}{|\underline{r}-\underline{r}'|},$$

so, for a single charged particle,

$$\underline{A}(\underline{r}) = \frac{\mu_0' e}{4\pi} \dot{\underline{r}}' \left(r^2 + r'^2 - 2\underline{r}\cdot\underline{r}'\right)^{-1/2}$$

$$\doteq \frac{\mu_0' e}{4\pi} \frac{\dot{\underline{r}}'}{r}\left(1 + \frac{\underline{r}\cdot\underline{r}'}{r^2}\right) \qquad (r' \ll r).$$

Using the relations

$$(\underline{r}' \wedge \dot{\underline{r}}') \wedge \underline{r} = (\underline{r}'\cdot\underline{r})\dot{\underline{r}}' - (\underline{r}'\cdot\dot{\underline{r}})\underline{r}'$$

$$\frac{d}{dt'}(\underline{r}\cdot\underline{r}')\underline{r}' = (\underline{r}\cdot\underline{r}')\dot{\underline{r}}' + (\underline{r}\cdot\dot{\underline{r}}')\underline{r}'$$

we find

$$\underline{A}(\underline{r}) = \frac{\mu_0' e}{4\pi}\left\{\frac{1}{r}\frac{d}{dt'}(\underline{r}') + \frac{1}{2r^3}\left[(\underline{r}'\wedge\dot{\underline{r}}')\wedge\underline{r} + \frac{d}{dt'}(\underline{r}\cdot\underline{r}')\underline{r}'\right]\right\};$$

since the moving charge is confined to a finite volume of space, and since we have taken the origin to be the centre of the system (of one particle), the time average of the $\frac{d}{dt'}$, terms must be zero. Hence

$$\underline{A}(\underline{r}) = \frac{\mu'_0 e}{4\pi} \frac{(\underline{r}'\wedge\dot{\underline{r}}')\wedge\underline{r}}{2r^3}$$

But the vector potential at $\underline{r}$ from a magnetic dipole $\underline{\mu}$ is

$$\underline{A}(\underline{r}) = \frac{\mu'_0}{4\pi} \frac{\underline{\mu}\wedge\underline{r}}{r^3}$$

so the orbiting electron is equivalent to a magnetic dipole

$$\underline{\mu} = \frac{e}{2} \cdot \underline{r}'\wedge\dot{\underline{r}}' \quad .$$

Now, the angular momentum of the particle about the origin is

$$\underline{L} = m\, \underline{r}' \wedge \dot{\underline{r}}$$

where m is the mass of the particle. So we have finally

$$\underline{\mu} = \frac{e}{2m} \underline{L}$$

whence the magnetogyric ratio for the orbital motion of the electron is

$$\mu/L = \frac{e}{2m} .$$

This result is readily derived in the special case when the electron orbit is assumed circular.

We shall now calculate the intrinsic magnetogyric ratio of the electron. Let the spin of the particle in the rest frame (R) be $\underline{s}$ and assume that there exists a four-vector $\underline{s}^4$ such that, in (R), it coincides with $\underline{s}$; thus:

$$\underline{s}^4 = (\underline{s}, s_4) \text{ ; in (R), } \underline{s}^4(R) = (\underline{s}, 0).$$

The four-velocity of the electron is denoted by

$$\underline{u}^4 = (\underline{u}, u_4) = \gamma(\underline{v}, ic)$$

where $\underline{v}$ is the ordinary velocity vector, $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$ and $\beta = v/c$, c being the velocity of light. In the rest frame of the

particle,

$$\underline{u}^4(R).\underline{s}^4(R) = (0, i\gamma c).(\underline{s}, 0) = 0 \ .$$

Since the scalar product of any two four-vectors is invariant under Lorentz transformation, we have, in all frames,

$$\underline{u}^4.\underline{s}^4 = 0 \qquad (I.1)$$

yielding

$$s_4 = \frac{i}{\gamma c}\,\underline{u}.\underline{s} \ .$$

We assume further that, in (R), $\underline{s}$ obeys the usual equation of motion

$$\dot{\underline{s}} = \alpha\, \underline{s} \wedge \underline{B} \qquad (R) \qquad (I.2)$$

where the dot indicates differentiation with respect to the proper time, and $\underline{B}$ is the magnetic field acting on the electron. We define α by

$$\underline{\mu} = \alpha\, \underline{M} \ ,$$

$\underline{\mu}$ being the magnetic moment and $\underline{M}$ the intrinsic angular momentum of the electron; α is thus the required intrinsic magnetogyric ratio.

If m is the mass of the electron, the equation of motion of the particle is

$$m\dot{\underline{u}}^4 = \underline{f}^4$$

$\underline{f}^4$ being the four-force acting.

For a homogeneous electromagnetic field, the

field tensor is (Stratton, 1941)

$$\underline{\underline{F}} = \begin{pmatrix} 0 & B_z & -B_y & -\frac{i}{c}E_x \\ -B_z & 0 & B_x & -\frac{1}{c}E_y \\ B_y & -B_x & 0 & -\frac{1}{c}E_z \\ \frac{1}{c}E_x & \frac{1}{c}E_y & \frac{1}{c}E_z & 0 \end{pmatrix}$$

We find $e\underline{\underline{F}}.\underline{u}^4 = \gamma e\left\{(\underline{E} + \underline{v}\wedge\underline{B}), \frac{1}{c}\underline{E}.\underline{v}\right\}$.
This is the usual Minkowski four-force acting on a particle of charge e in a homogeneous electromagnetic field. Hence the equation of motion of the electron is

$$\dot{\underline{u}}^4 = \frac{e}{m}\underline{\underline{F}}.\underline{u}^4 \qquad (I.3)$$

We now wish to generalise (to arbitrary frames of reference) equation (I.2) and its fourth component, which is

$$\dot{s}_4 = \frac{i}{\gamma c}\dot{\underline{u}}.\underline{s} \quad .$$

Such a generalisation must be bilinear in $\underline{\underline{F}}$ and $\underline{s}^4$ and must satisfy, from (I.1),

$$\underline{u}^4.\dot{\underline{s}}^4 + \dot{\underline{u}}^4.\underline{s} = 0 \qquad (I.4)$$

The most general bilinear equation (Michel, 1959) is of the form

$$\dot{\underline{s}}^4 = K_1\underline{\underline{F}}.\underline{s}^4 + K_2(\underline{u}^4.\underline{\underline{F}}.\underline{s}^4)\underline{u}^4 \qquad (I.5)$$

Using (I.3) and (I.4), and recalling that $\underline{\underline{F}}$ is

antisymmetric and that $(\underline{u}^4)^2 = -c^2$, we find

$$K_1 - K_2 c^2 = e/m \quad .$$

In the rest frame (R), (I.5) becomes

$$\dot{\underline{s}}^4 = K_1 \underline{\underline{F}} \cdot \underline{s}^4 \qquad (R)$$

and the spatial part of this is

$$\dot{\underline{s}} = K_1 \underline{s} \wedge \underline{B} \qquad (R)$$

Comparison with (I.2) gives immediately

$$K_1 = \alpha$$

and hence $$\dot{\underline{s}}^4 = \alpha \underline{\underline{F}} \cdot \underline{s}^4 \qquad (R) \qquad (I.6)$$

Comparing (I.3) and (I.6), we find

$$\alpha = e/m \qquad (I.7)$$

Thus the intrinsic magnetogyric ratio of the electron is

$$\mu/M = e/m \quad .$$

Using equations first given by Abraham (Abraham, 1903), Uhlenbeck and Goudsmit derived this result taking as a model a spherical, rotating electron, with a surface electrical charge (Uhlenbeck and Goudsmit, 1925).

The following outline of the Dirac wave equation for a relativistic electron is based on the treatment given by Schiff (Schiff, 1955).

If we take the positive square root of the usual relativistic expression for the energy E of a particle with momentum $\underline{p}$ and mass m, i.e.

$$E = + \left\{ c^2 \underline{p}^2 + m^2 c^4 \right\}^{\frac{1}{2}}$$

and make the usual substitution $\underline{p} \rightarrow - ih\nabla$, the resulting wave equation is not symmetrical with respect to space and time derivatives, and hence cannot be relativistic. Dirac modified the Hamiltonian for a free particle so as to make it linear in both space and time derivatives (Dirac, 1928). Thus, instead of a wave equation of the form

$$(E^2 - c^2 \underline{p}^2 - m^2 c^4)\psi = 0 \qquad (I.8)$$

he wrote

$$(E + c\underline{\alpha}.\underline{p} + \beta mc^2)\psi = 0 \qquad (I.9)$$

where $\underline{\alpha}$ and β are independent of E, $\underline{p}$, and of the space coordinates and the time.

To learn more about $\underline{\alpha}$ and β, we require that any solution of (I.9) should be a solution of (I.8) also. We find that the four quantities α_x, α_y, α_z, β anticommute in pairs, and so only one of the four can be diagonalised at a time; we seek a representation in which one of them, say β, is diagonal, and require that this representation should have as low a rank as possible.

It turns out that each may be represented by a 4 x 4 matrix which are abbreviated to

$$\underline{\alpha} = \begin{pmatrix} 0 & \underline{\sigma} \\ \underline{\sigma} & 0 \end{pmatrix} \quad , \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where each "element" is a 2 x 2 matrix, and where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

are the Pauli spin matrices. We now define three new 4 x 4 matrices, σ_x', σ_y', σ_z' by

$$\underline{\sigma}' = \begin{pmatrix} \underline{\sigma} & 0 \\ 0 & \underline{\sigma} \end{pmatrix} .$$

The reason for doing this will soon be clear.

If now the particle is in an electromagnetic field derived from scalar and vector potentials, ϕ and $\underline{A}$ respectively, the wave equation becomes

$$\left\{ E - e\phi + \underline{\alpha}.(c\underline{p} - ec\underline{A}) + \beta mc^2 \right\} \psi = 0 .$$

If this is multiplied from the left by

$$E - e\phi - \underline{\alpha}.(c\underline{p} - ec\underline{A}) - \beta mc^2 ,$$

we find, after much algebra

$$\left\{ (E-e\phi)^2 - (c\underline{p}-ec\underline{A})^2 - m^2c^4 + e\hbar c\underline{\sigma}'.\underline{B} + ie\hbar c\underline{\alpha}.\underline{E} \right\} \psi = 0 .$$

The first three terms are just those which occur in the relativistic Schrodinger equation (I.8) when extended to include the electromagnetic potentials. To find the physical significance of the last two terms, we proceed to the non-relativistic limit by writing $E = E' + mc^2$ and assuming that E' and $e\phi$ are small compared to mc^2. Then

$$(E - e\phi)^2 - m^2c^4 \doteq 2mc^2(E' - e\phi)$$

and so

$$E'\psi = \left\{\frac{1}{2m}(\underline{p} - e\underline{A})^2 + e\phi - \frac{eh}{2m}\underline{\sigma}.'\underline{B} - i\frac{eh}{2m}\underline{\alpha}.\underline{E}\right\}\psi .$$

It turns out that the last term, involving $\underline{E}$, is of order $(v/c)^2$ times $e\phi$, and hence is negligible; the second last term, involving $\underline{B}$, has the form associated with the energy of a magnetic dipole of moment

$$\underline{\mu} = \frac{eh}{2m}\underline{\sigma}' \qquad (I.10)$$

To investigate the orbital motion of an electron, we work with a <u>central</u> field of force, i.e. we take $\underline{A}$ to be zero and ϕ to be spherically symmetric. We find that the orbital angular momentum

$$\underline{L} = \underline{r} \wedge \underline{p}$$

does not commute with the Hamiltonian and therefore is not a constant of the motion. However, it turns

out that the quantity

$$\underline{L} + \tfrac{1}{2}h\underline{\sigma}'$$

does commute with the Hamiltonian, and hence is a constant of the motion. It therefore represents the <u>total</u> angular momentum of the system. The quantity $\frac{1}{2}h\underline{\sigma}'$ is called the spin angular momentum, $\underline{M}$, where

$$\underline{M} = \tfrac{1}{2}h\underline{\sigma}' \quad . \qquad (I.11)$$

It may be shown (Bohm, 1951) that for any two operators, A and B, the uncertainties in the measurement of these quantities, ΔA and ΔB, satisfy the relation

$$(\Delta A)^2(\Delta B)^2 \geq \overline{\left\{\tfrac{1}{2}(AB - BA)\right\}^2} \quad . \qquad (I.12)$$

This implies that, if two operators do not commute, they cannot be measured simultaneously with perfect precision.

It is readily seen that no one of σ_x', σ_y', σ_z' commutes with any other, and hence only one of the components of $\underline{\sigma}'$ may be measured exactly at any time. Since σ_z' is already diagonal, we choose σ_z' to be the component which is to be measured.

By inspection, the eigenvalues of σ_z' are +1 and -1. Hence, from (I.10) and (I.11),

spin magnetic moment of the electron, μ = $\frac{eh}{2m}$

spin angular momentum of the electron, M = $\frac{1}{2}h$.

It should be observed that the above results have not been added to the Schrodinger theory in any way; they have been, as it were, "built into" the Dirac equation.

It is usual to say that, in a magnetic field, the electron spin will be aligned either parallel or antiparallel to the direction of the field. If the field is in the z-direction, then we have $M_z = \pm \frac{1}{2}h$. Now

$$\underline{M}^2 = M_x^2 + M_y^2 + M_z^2 = \frac{1}{4}h^2(\sigma_x'^2 + \sigma_y'^2 + \sigma_z'^2)$$

$$\text{i.e.} \qquad \underline{M}^2 = \frac{3}{4}h^2$$

since the square of each component of $\underline{\sigma}'$ is unity. Hence

$$M_x^2 + M_y^2 = \underline{M}^2 - M_z^2$$
$$= \frac{3}{4}h^2 - \frac{1}{4}h^2$$
$$= \frac{1}{2}h^2 \neq 0 .$$

Thus, the angular momentum vector is not aligned along the direction of the magnetic field.

Without loss of generality, let us consider the case when $M_z = +\frac{1}{2}h$, that is, when σ_z' has

the eigenvalue +1. In the representation in which σ_z' is diagonal, with eigenvalue +1, what are the average values. $\bar{M}_x$ and $\bar{M}_y$, of M_x and M_y? If ψ_+ is an eigenfunction of σ_z' in this representation, then

$$\bar{M}_x = \tfrac{1}{2}h\, \psi_+^* \sigma_x' \psi_+$$

$$\bar{M}_y = \tfrac{1}{2}h\, \psi_+^* \sigma_y' \psi_+ \quad .$$

Possible eigenfunctions are easily seen to be

$$\psi_+^* = (1 \quad 0 \quad 0 \quad 0)$$

$$\text{or} \quad \psi_+^* = (0 \quad 0 \quad 1 \quad 0)$$

$$\text{or} \quad \psi_+^* = \frac{1}{\sqrt{2}}(1 \quad 0 \quad 1 \quad 0) \quad ,$$

and the same with negative signs. Using any of these, we find

$$\bar{M}_x = \bar{M}_y = 0 \, .$$

To confirm our earlier result that M_x and M_y are non-zero, we compute

$$\overline{M_x^2} = \tfrac{1}{4} h^2\, \psi_+^* (\sigma_x')^2 \psi_+$$

$$\overline{M_y^2} = \tfrac{1}{4} h^2\, \psi_+^* (\sigma_y')^2 \psi_+ \quad .$$

We find

$$\overline{M_x^2} = \overline{M_y^2} = \tfrac{1}{4} h^2, \quad \text{so that}$$

$$\overline{M_x^2} = \overline{M_y^2} = \tfrac{1}{2}h^2, \quad \text{as expected.}$$

From this, we conclude that even when one component

(the z-component here) of the angular momentum is precisely defined, the other two components are not zero; we must suppose that they cover simultaneously the entire range of values consistent with $\underline{M}^2 = \frac{3}{4}\hbar^2$ and $M_z = \frac{1}{2}\hbar$.

APPENDIX II

Spin Kinematics

Prior to the suggestion that electrons emitted in beta-decay should be longitudinally polarised, Mott scattering was the only experimentally realisable method of producing a beam of (partially) polarised electrons (see Appendix III). Because of this, and because early Mott scattering experiments had given negative results (Dymond, 1932 and 1934; Thomson, 1934; Richter, 1937), little interest was shown in either the theoretical or the experimental aspects of the motion of a spinning electron in an electromagnetic field.

A theoretical investigation of spin kinematics was undertaken by Tolhoek and de Groot (Tolhoek and de Groot, 1951a and 1951b) when it was realised that electrons emitted from aligned beta-active nuclei would be polarised (Tolhoek and de Groot, 1951c; for possible experimental method of alignment, see Gorter, 1948; Rose, 1948a and 1948b). Using Dirac theory, in which in electron g-factor is exactly 2, the effect of electromagnetic fields on the spin orientation of electrons in a beam was worked out. It was shown that, when a magnetic field is applied in a direction perpendicular to the beam, the spin precession frequency

and the cyclotron frequency are identical.

In a later paper (Tolhoek, 1956), the spin precession frequency was found to exceed the cyclotron frequency by a fractional amount equal to the g-factor anomaly, that is, by about one part in one thousand. This result was derived only for infinitesimal rotations, but was wrongly generalised to the case where the electrons perform large numbers of cyclotron revolutions in a transverse magnetic field. Hence it was concluded that, after a number of revolutions, $n = \frac{1}{4}(\frac{g}{2} - 1)^{-1} \doteq 250$, a longitudinally polarised beam would become transversely polarised, and vice versa. As we shall see, this expression is in error by a factor of $1/\gamma$ where $\gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$, v being the electron velocity; it is therefore accurate only in the limit of low electron energies.

The problem was studied in detail by Mendlowitz and Case, using Dirac theory (Case, 1954; Mendlowitz and Case, 1955; Case, 1957; Mendlowitz, 1958). They showed that, when a pure magnetic field was applied, perpendicular to the velocity of the electrons,

$$\frac{\omega_s - \omega_c}{\omega_c} = \gamma(\frac{g}{2} - 1),$$

where ω_s, ω_c are the spin precession and cyclotron frequencies respectively. This differs from

Tolhoek's (incorrect) result by the factor Υ.

An identical result was obtained independently by Carrassi who emphasised the importance of the factor Υ in the above equation, pointing out that rotation through a semi-circle would suffice to transform a longitudinally polarised beam of 250 Mev electrons into a transversely polarised beam. (Carrassi, 1958; see also Carrassi, 1957). He stressed that these results are obtained by assuming that the influence of a uniform magnetic field on the spin orientation of electrons may be described by a Dirac-Pauli equation, even when the electrons are required to have an anomalous magnetic moment. If, in fact, the anomaly is a purely radiative effect and not an intrinsic property of electrons, then Carrassi doubts whether a Dirac equation with an added Pauli-type term in the energy, proportional to $(\frac{g}{2} - 1)$, will suffice to describe the behaviour of an electron.

An entirely different approach to the subject of spin kinematics was adopted by Bargmann, Michel and Telegdi: to solve the problem for arbitrary spin in the relativistic case, they concluded that it was sufficient to produce a consistent set of covariant classical equations of motion (Bargmann, Michel and Telegdi, 1959). Because of the brevity of their paper, some steps in the argument are by

no means clear. Using the notation and some of the results of the last part of Appendix I, we shall consider their calculation in detail.

Because of the g-factor anomaly, equation (I.2), the equation of motion of the spin in the rest frame of the electron (R), has to be modified to

$$\dot{\underline{s}} = \frac{ge}{2m} \underline{s} \wedge \underline{B} \qquad \text{(II.1)}$$

The general equation of motion of the spin (I.5) then becomes

$$\dot{\underline{s}}^4 = \frac{e}{m} \left\{ \frac{g}{2} \underline{F}.\underline{s}^4 + \frac{1}{c^2} \left(\frac{g}{2} - 1\right)(\underline{u}^4.\underline{F}.\underline{s}^4)\underline{u}^4 \right\}. \qquad \text{(II.2)}$$

We shall use this equation to compute the rate at which a longitudinally polarised beam is transformed into a transversely polarised beam. To do this for motion in a plane, we express $\underline{s}^4$ in the laboratory frame, L, in terms of two unit four-vectors, $\underline{e}_\ell^4$ and $\underline{e}_t^4$, which describe the polarisation. If $s = (\underline{s}^4.\underline{s}^4)^{\frac{1}{2}}$ is the magnitude of $\underline{s}^4$, then

$$\underline{s}^4 = s \left\{ \underline{e}_\ell^4 \cos \phi + \underline{e}_t^4 \sin \phi \right\} ; \qquad \text{(II.3)}$$

we require

$$\underline{e}_\ell^4.\underline{e}_\ell^4 = \underline{e}_t^4.\underline{e}_t^4 = 1 \; ; \; \underline{e}_\ell^4.\underline{e}_t^4 = 0. \qquad \text{(II.4)}$$

Let $\hat{\underline{v}}$ be the unit vector in the direction of $\underline{v}$, and let $\hat{\underline{n}}$ be a unit vector in the plane of the motion and perpendicular to $\underline{v}$. The required unit vectors are then

$$\underline{e}_{\ell}^{4} = \gamma(\hat{\underline{v}}, i\frac{v}{c})$$
$$\underline{e}_{t}^{4} = (\hat{\underline{n}}, 0) \quad . \qquad \text{(II.5)}$$

From (II.4), we have

$$\dot{\underline{e}}_{\ell}^{4}\cdot\underline{e}_{t}^{4} + \underline{e}_{\ell}^{4}\cdot\dot{\underline{e}}_{t}^{4} = 0 \quad .$$

From (II.5), remembering that γ is not constant, we find

$$\dot{\underline{e}}_{\ell}^{4}\cdot\underline{e}_{t}^{4} = \gamma\frac{\hat{\underline{n}}\cdot\dot{\underline{v}}}{v} = \gamma\frac{\underline{e}_{t}^{4}\cdot\dot{\underline{u}}^{4}}{v}$$

and so

$$\underline{e}_{\ell}^{4}\cdot\dot{\underline{e}}_{t}^{4} = -\gamma\frac{\underline{e}_{t}^{4}\cdot\underline{u}^{4}}{v} \qquad \text{(II.6)}$$

Now $\underline{u}^{4}\cdot\underline{e}_{\ell}^{4} = \gamma(\underline{v}, ic)\cdot\gamma(\underline{v}, i\frac{v}{c}) = 0$;

so, from (II.2),

$$\dot{\underline{s}}^{4}\cdot\underline{e}_{\ell}^{4} = \frac{e}{m}\frac{g}{2}\underline{e}_{\ell}^{4}\cdot\underline{\underline{F}}\cdot\underline{s}^{4}$$
$$= s\frac{e}{m}\frac{g}{2}\underline{e}_{\ell}^{4}\cdot\underline{\underline{F}}\cdot(\underline{e}_{\ell}^{4}\cos\phi + \underline{e}_{t}^{4}\sin\phi),$$

so $$\dot{\underline{s}}^{4}\cdot\underline{e}_{\ell}^{4} = s\frac{e}{m}\frac{g}{2}\underline{e}_{\ell}^{4}\cdot\underline{\underline{F}}\cdot\underline{e}_{t}^{4}\sin\phi \ , \qquad \text{(II.7)}$$

since $\underline{a}^{4}\cdot\underline{\underline{F}}\cdot\underline{a}^{4} = 0$, where $\underline{a}^{4}$ is any four-vector.

Differentiating (II.3) with respect to the proper time, and recalling that, from (II.4), $\underline{e}_{\ell}^{4}$ and $\dot{\underline{e}}_{\ell}^{4}$ are orthogonal, we find, using (II.6),

$$\dot{\underline{s}}^{4}\cdot\underline{e}_{\ell}^{4} = -s\left\{\dot{\phi} + \frac{1}{v}\underline{e}_{t}^{4}\cdot\dot{\underline{u}}^{4}\right\}\sin\phi \ .$$

Using (I.3), this becomes

$$\dot{\underline{s}}^{4}\cdot\underline{e}_{\ell}^{4} = -s\left\{\dot{\phi} - \frac{e}{mv}\underline{u}^{4}\cdot\underline{\underline{F}}\cdot\underline{e}_{t}^{4}\right\}\sin\phi \ , \qquad \text{(II.8)}$$

since $\underline{\underline{F}}$ is antisymmetric. Equating (II.7) and (II.8) then gives

$$\dot{\phi} = \frac{e}{m}\left\{\frac{1}{v}\,u^4 - \frac{g}{2}\,e_\ell^4\right\}\cdot\underline{\underline{F}}\cdot e_t^4 \quad . \qquad \text{(II.9)}$$

From the definition of $\underline{\underline{F}}$ (see Appendix I), we find

$$\underline{\underline{F}}\cdot e_t^4 = (\hat{n}\wedge\underline{B},\ \frac{i}{c}\,\hat{n}\cdot\underline{E})$$

and substitution in (II.9) gives

$$\dot{\phi} = \frac{e\gamma}{2m}\left\{(g-2)\hat{v}\cdot\underline{B}\wedge\hat{n} + \frac{\hat{n}\cdot\underline{E}}{v}\,(g - \frac{g}{\gamma^2} - 2)\right\}. \qquad \text{(II.10)}$$

Since $\dot{\phi} \equiv \frac{d\phi}{d\tau} = \gamma\,\frac{d\phi}{dt}$, t being the time measured in L,

$$\frac{d\phi}{dt} = \frac{e}{2m}\left\{(g-2)\hat{v}\cdot\underline{B}\wedge n + \frac{\hat{n}\cdot\underline{E}}{v}\,(g - \frac{g}{\gamma^2} - 2)\right\}. \qquad \text{(II.10)}$$

Equation (II.10) gives the rate of change of the state of polarisation of a particle in a homogeneous electromagnetic field.

In the present experiment, since the motion is in a plane perpendicular to $\underline{B}$,

$$\hat{v}\cdot\underline{B}\wedge\hat{n} = \hat{n}\wedge\hat{v}\cdot\underline{B} = \hat{B}\cdot\underline{B} = B \quad . \qquad \text{(II.11)}$$

If now we introduce an angle θ defined by

$$\underline{E}\cdot\hat{v} = E\sin\theta\ ;\ \underline{E}\cdot\hat{n} = -E\cos\theta \qquad \text{(II.12)}$$

then $\frac{d\theta}{dt}$ is the angular velocity of the electron in L.

From the spatial part of (I.3)

$$\dot{\underline{v}} = \frac{e}{m} (\underline{E} + \underline{v} \wedge \underline{B})$$

from which we find, using (II.11) and (II.12)

$$\frac{d\theta}{dt} = \frac{e}{\gamma m} (B - \frac{E}{v} \cos \theta) \qquad \text{(II.13)}$$

Again using (II.11) and (II.12), equation (II.10) becomes

$$\frac{d\phi}{dt} = \frac{e}{2m} \left\{ (g-2)(B - \frac{E}{v}\cos \theta) + (\frac{g}{\gamma^2} \frac{E}{v} \cos \theta) \right\} . \qquad \text{(II.14)}$$

In order to calculate $\frac{d\phi}{d\theta}$ per revolution, we may eliminate the time by dividing (II.14) by (II.13). However, since γ, v and θ are all variables, the integration is very troublesome. We choose an alternative approach.

In the above, $\underline{E}$ and $\underline{B}$ are the field vectors in the laboratory system, L. Let $\underline{E}'$ and $\underline{B}'$ be the field vectors in a frame moving with velocity $\underline{V}$ relative to L. Lorentz transformation gives (Stratton, 1941), when $\underline{V}.\underline{B} = \underline{V}.\underline{E} = 0$,

$$\underline{E}' = \gamma_V (\underline{E} + \underline{V} \wedge \underline{B})$$

$$\underline{B}' = \gamma_V (\underline{B} - \frac{1}{c^2} \underline{V} \wedge \underline{E})$$

where $\gamma_V = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$. (II.15)

If we choose $\underline{V}$ such that $\underline{E}' = 0$, then

$$\underline{E} + \underline{V} \wedge \underline{B} = 0.$$

$$\therefore \underline{E} \wedge \underline{B} + (\underline{V}.\underline{B})\underline{B} - B^2\underline{V} = 0 \quad .$$

In addition, we use $\underline{V}.\underline{B} = 0$; then

$$\underline{V} = \frac{\underline{E} \wedge \underline{B}}{B^2}$$

If $\underline{E}$ and $\underline{B}$ are orthogonal, $|\underline{E} \wedge \underline{B}| = EB$, and so

$$|\underline{V}| = \frac{E}{B} \quad . \tag{II.16}$$

Hence we see that, if $\underline{E}$ and $\underline{B}$ are orthogonal, it is possible to define a frame moving at a speed $V = E/B$ in a direction perpendicular to both $\underline{E}$ and $\underline{B}$, in which a particle will be acted on by a magnetic field only. We denote variables in this system by primes.

In this moving system, equations (II.13) and (II.14) become

$$\frac{d\theta'}{dt'} = \frac{e}{\gamma' m} B'$$

$$\frac{d\phi'}{dt'} = \frac{e}{2m} (g - 2) B'$$

where $\gamma' = (1 - \frac{(v')^2}{c^2})^{-\frac{1}{2}}$. Division then gives

$$\frac{d\phi'}{d\theta'} = \frac{\gamma'}{2}(g - 2) \quad . \tag{II.17}$$

Now, since the motion of the electron is periodic, with period T, say (i.e. $\underline{s}^4$ returns to its original direction after a time T), we have from (II.3)

$$\begin{aligned} \underline{s}^4(t+T) &\equiv s \left\{ \underline{e}_l^4(t+T)\cos \phi(t+T) + \underline{e}_t^4(t+T)\sin\phi(t+T) \right\} \\ &= s \left\{ \underline{e}_l^4(t) \cos \phi(t+T) + \underline{e}_t^4(t)\sin\phi(t+T) \right\} \end{aligned}$$

$$\therefore \quad \underline{s}^4(t+T).\underline{s}^4(t) = s^2 \cos\left\{\phi(t+T) - \phi(t)\right\}$$

$$= s^2 \cos \Delta\phi, \quad \text{say,}$$

where $\Delta\phi$ is the change in the relative directions of the spin and momentum vectors after one complete revolution. Since the product of two four-vectors is Lorentz invariant, we have

$$\Delta\phi = \text{Lorentz invariant} . \qquad (II.18)$$

Returning to (II.17), if we consider one complete revolution in the primed system,

$$\frac{\Delta\phi'}{2\pi} = \frac{\gamma'}{2}(g - 2)$$

and so, from (II.18),

$$\frac{\Delta\phi}{2\pi} = \frac{\gamma'}{2}(g - 2) . \qquad (II.19)$$

All that remains is to express γ' in terms of known quantities. From the Lorentz transformation for velocity components, we have (taking $\underline{V}$ to be in the x direction),

$$v_x' = \frac{v_x - V}{1 - \frac{v_x V}{c^2}}$$

$$v_y' = \frac{v_y (1 - (\frac{V}{c})^2)^{\frac{1}{2}}}{1 - \frac{v_x V}{c^2}}$$

$$v_z' = v_z = 0.$$

Using the relations

$$\left(\frac{v'}{c}\right)^2 = \left(\frac{v_x'}{c}\right)^2 + \left(\frac{v_y'}{c}\right)^2$$

$$\left(\frac{v}{c}\right)^2 = \left(\frac{v_x}{c}\right)^2 + \left(\frac{v_y}{c}\right)^2$$

we find

$$\gamma' = \gamma_V \gamma \left(1 - \frac{v_x V}{c^2}\right) .$$

Substituting this in (II.19), and using (II.16), we find

$$\frac{\Delta\phi}{2\pi} = \gamma_V \gamma \left(1 - \frac{E}{cB} \cdot \frac{v_x}{c}\right)\left(\frac{g}{2} - 1\right)$$

$$\text{or } \frac{\Delta\phi}{2\pi} = \gamma\left(\frac{g}{2} - 1\right)\left(1 - \left(\frac{E}{cB}\right)^2\right)^{-\frac{1}{2}}\left(1 - \frac{E}{cB}\frac{v\cos\theta}{c}\right) .$$

In the present experiment $\frac{v}{c} \doteq 0.5$, $B \doteq 2 \times 10^{-2}$ webers/m^2 and $E \doteq 10^5$ volts/m. at most. So $\frac{E}{cB} \doteq \frac{1}{50}$; $\frac{v}{c}\frac{E}{cB} \doteq \frac{1}{100}$ and, to a good approximation, we find

$$\frac{\omega_s - \omega_c}{\omega_c} \equiv \frac{\Delta\phi}{2\pi} = \gamma\left(\frac{g}{2} - 1\right) . \qquad \text{(II.21)}$$

Thus, provided $\left(\frac{E}{cB}\right) \ll 1$, we have the same expression for $\frac{\omega_s - \omega_c}{\omega_c}$ as was deduced by Mendlowitz and Case, and by Carassi, for the special case when $E = 0$. In other words, provided $\left(\frac{E}{cB}\right) \ll 1$, the contribution of the electric field to the relative rate of spin precession is negligible.

Recently, Fradkin and Good have reviewed the mathematical theory of electron polarisation, and have shown the relations between the various approaches. (Fradkin and Good, 1961). The conditions under which the classical (non-quantum) limit may be applied are discussed; one such condition is that fields and potentials should exhibit negligible variations over the dimensions of the wave packet describing the particle.

APPENDIX III

Electron Polarisation

It might appear that the most likely way to produce a beam of polarised electrons would be by means of a Stern-Gerlach experiment. However, since the electron possesses an electric charge as well as a magnetic moment (neutral atoms are used in the normal Stern-Gerlach experiment), it is found that, unless the beam of electrons is confined to the symmetry plane of the inhomogeneous magnetic field by so fine a slit that diffraction occurs, a component of force due to the electron charge will so spread the beam that the desired splitting will be completely masked (Mott, 1929).

Another possibility might be to reflect an electron beam at an abrupt discontinuity of electric potential. By analogy with the behaviour of light on being reflected at a mirror (the Malus effect), it might be expected that the reflected beam should be partially polarised. It may be shown theoretically that no such polarisation will occur (Frenkel, 1929).

It is conceivable that, on interacting with a homogeneous or slowly varying electromagnetic

field, an unpolarised beam of electrons might become polarised. Tolhoek showed that, under these circumstances, an unpolarised beam would always remain unpolarised (Tolhoek, 1956).

Since electron spin is essentially a quantum mechanical phenomenon, it must not be expected that any of the above, purely classical, effects should be capable of changing the state of polarisation of an electron beam. Only a quantum mechanical process can produce a quantum mechanical effect.

Until recently, the only method whereby it could be demonstrated experimentally that polarised electrons were produced was Mott scattering - the scattering of electrons by the (screened) Coulomb field of heavy nuclei. This is discussed more fully in Appendix IV. Several other methods, all possible in principle, are listed by Tolhoek (Tolhoek, 1956), perhaps the most interesting being that proposed by Kastler and used later by Dehmelt in the experiment described in Chapter 2 (Kastler, 1954).

The first suggestion that electrons emitted in beta-decay should be polarised was made by Tolhoek and de Groot, five years before parity non-conservation in weak interactions was suspected

(Tolhoek and de Groot, 1951c). They found that for allowed transitions, if the decaying nuclei were aligned, then electrons emitted in a direction parallel or antiparallel to the axis of polarisation of the nuclei would be longitudinally polarised, while electrons emitted in a direction perpendicular to this axis would be transversely polarised. Further, they showed that the angular distribution of electrons emitted from the beta-decay of aligned nuclei would possess spherical symmetry if the corresponding beta-transitions were allowed, that is, if the emitted electrons carry off no orbital angular momentum. No attempt to verify this theory was made.

In 1956, Lee and Yang drew attention to the fact that, while there was experimental evidence that in strong and in electromagnetic interactions the parity of a system is conserved, parity conservation in weak interactions (Lee and Yang, 1956) was only an extrapolated hypothesis, altogether unsupported by experiment. They proposed a simple experiment to test parity conservation in beta-decay: if parity is not conserved, the angular distribution of electrons emitted from aligned beta-active nuclei should be asymmetric; in other words, if θ is the angle between the nuclear orientation axis and the direction of emission of

a particle, the numbers of electrons emitted in directions θ and $\pi - \theta$ should not be equal.

Such an experiment was immediately undertaken by Wu et al., who used a Co^{60} source, the nuclei of which were aligned (Wu, Ambler, Hayward, Hoppes and Hudson, 1957). A very large asymmetry of negative sign was found, showing that the emission of beta-particles is more favoured in the direction opposite to that of the nuclear spin. This was the first experimental evidence in favour of parity non-conservation in weak interactions, and contrasts strongly with the earlier predictions of Tolhoek and de Groot which were mentioned above.

Wu's experiment shows that, in beta-decay, there is a correlation between the nuclear spin (an axial vector) and the beta-particle momentum (a polar vector). In a second paper, Lee and Yang pointed out that such a correlation can be understood only in terms of a violation of the law of space inversion invariance in beta-decay (Lee and Yang, 1957b). They went on to propose a new theory of the neutrino, according to which the neutrino has only one spin state: the spin is always parallel to the momentum. This implies that the wave function describing the neutrino need have only two components instead of the usual four. One conclusion to be drawn from this is that

electrons emitted in beta-decay will be longitudinally polarised, whether or not the nuclear spins are aligned. Grodzins has restated their argument in a very simple form (Grodzins, 1959).

Lee and Yang's two component theory of the neutrino was immediately developed and the conclusion reached that, if the theory were correct, electrons emitted in beta-decay should be longitudinally polarised to an extent $\pm v/c$, where v is the magnitude of the velocity of the emitted particles; the positive sign applies to positrons, the negative sign to electrons (Jackson, Treiman and Wyld, 1957; Landau, 1957; Curtis and Lewis, 1957).

Several experiments devised to measure the longitudinal polarisation of beta-particles were carried out, the first reported being that of Frauenfelder et al. ; electrons from Co^{60} were sent through an electrostatic spin-twister (Tolhoek and de Groot, 1951b), the spin orientation being detected by Mott scattering. For $\beta = v/c = 0.49$, they found a polarisation $P = -0.40$ (Frauenfelder, Bobone, von Goeler, Levine, Lewis, Peacock, Rossi and De Pasquali, 1957). Using crossed electric and magnetic fields to rotate the electron spins, followed by Mott

scattering, Cavanagh et al. measured the longitudinal polarisation of beta-particles from two radioisotopes: for Co^{60} with $\beta = 0.6$, they found $P = - 0.65 \pm 0.13$, while for Au^{198}, with $\beta = 0.6$, the polarisation was $P = - 0.58 \pm 0.12$ (Cavanagh, Turner, Coleman, Gard and Ridley, 1957).

After a review of these and later experiments, Grodzins concludes that for all types of beta-decay transition (with one exception), the degree of longitudinal polarisation, or helicity, of the emitted beta-particles should be $\pm v/c$. Except for the isotope RaE, the experimental evidence is consistent with this (Grodzins, 1959).

Recently, two high-precision experiments to measure beta-particle polarisation have been reported. By the use of an electrostatic spin-twister followed by Mott scattering from gold foils of different thicknesses, Greenberg et al. measured the helicity of 194 kev electrons from Co^{60}. After making many corrections (but without correcting the Mott theory for screening of the nucleus, since no suitable calculation bas been performed), they find $P = -(0.994 \pm 0.057)\beta$ (Greenberg, Malone, Gluckstern and Hughes, 1960).

Ullman et al. investigated the helicity of beta-particles from four isotopes, using electron-electron (Møller or Bhabha) scattering: this has (Møller, 1932; Bhabha, 1936). This has

the advantage that the longitudinal polarisation of the particles is used directly, without the necessity of transforming to a state of transverse polarisation. Their results are summarised below

(1) P^{32}, electron emitter, $P = -v/c$ to within 2%.

(2) Au^{198}, electron emitter, $P = -v/c$ to within 3%

(3) Ga^{68}, positron emitter, $P = +v/c$ to within 10%

(4) RaE, electron emitter, $P = -0.75\, v/c$ to within 4% .

(Ullman, Frauenfelder, Lipkin and Rossi, 1961).

For reasons given in Chapter 3, it was decided to use a source of S^{35} in the present experiment. The first measurement of the helicity of beta-particles from S^{35} was made by Langevin-Joliot& Marty 1957, using an electrostatic spin-twister followed by Mott scattering. For 128 kev electrons ($\beta = 0.6$), they found $P = -(0.63 \pm 0.17)\beta$, the low value being attributed mainly to depolarisation in the rather thick source. A more recent determination by Murray, using a Cavanagh-type spin-twister followed by Mott scattering, yielded $P = -(1.14 \pm 0.03)\beta$, the beta-particle energy being 100 kev. (Murray, 1960). This high value was thought to be caused by errors in Sherman's computed values for the Mott asymmetry, which assume an unscreened nuclear field.

APPENDIX IV

Mott Scattering

It was first shown by Mott that the scattering of electrons by the Coulomb field of a nucleus involves the spin of the electron (Mott, 1929 and 1932). Thus, if an unpolarised beam of electrons is scattered, the scattered particles will be partially polarised, and the direction of polarisation will be perpendicular to the plane of scattering. Further, if a beam of electrons is partially polarised in a direction perpendicular to the plane of scattering, the numbers of particles scattered are not equal in the two directions making equal angles with the initial beam direction.

The underlying physical reason why such scattering is spin-dependent is that, on approaching the nucleus, the electron is moving through a highly inhomogeneous electric field which is varying on a microscopic scale. Because of its motion, the electron experiences not only an electric but also a magnetic field, and this latter interacts with the magnetic moment of the electron. This is known as spin orbit coupling. To investigate the effect quantitatively, Mott

considered the solutions which represent the scattering of an electron wave, according to the Dirac equation, with a spherically symmetric electrostatic potential. The calculation is outlined by Tolhoek, who also reviews some of the difficulties experienced in Mott scattering experiments (Tolhoek, 1956).

To understand more fully the effects of Mott scattering, suppose that an unpolarised beam of electrons, moving in the positive z direction, is scattered at T through a right angle (see sketch). We may imagine that the beam, of total intensity I, has half of its particles polarised in the positive y direction, the remainder being polarised in the negative y direction. Detectors are placed at X_+ and X_- on the positive and negative x-axes.

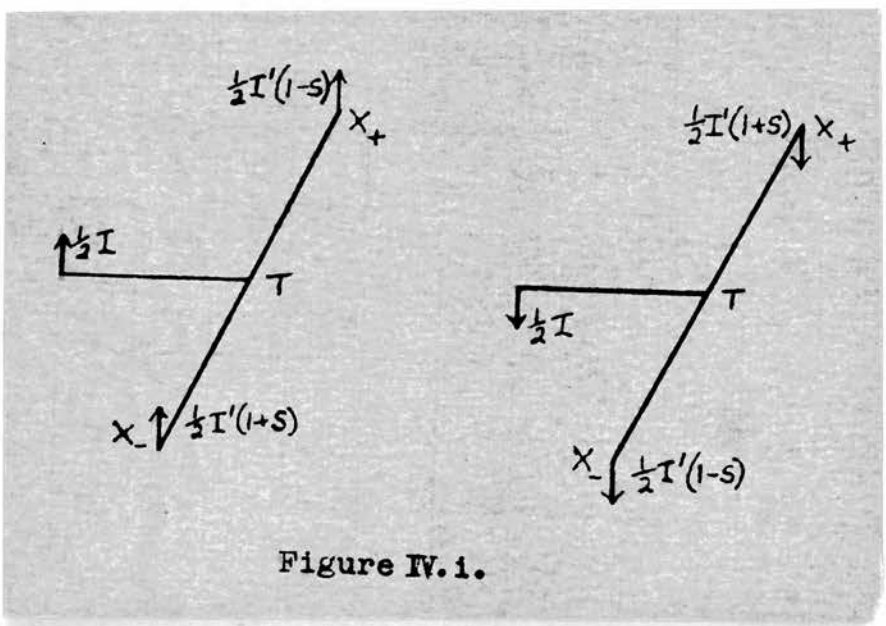


Figure IV.1.

The counting rates at X_+ and X_- will be:

$$C_+ = \tfrac{1}{2}I'(1 - S) + \tfrac{1}{2}I'(1 + S) = I'$$

$$C_- = \tfrac{1}{2}I'(1 + S) + \tfrac{1}{2}I'(1 - S) = I' \quad .$$

Hence the scattered intensities in the positive and negative x directions are equal, but in the positive-going beam, the ratio of the number, of electrons with spin "down" to the number with spin "up" is $1 + S/1 - S$, while in the negative-going beam, this ratio is $1 - S/1 + S$.

Consider now the scattering of an electron beam, of intensity I, in which a fraction P of the particles is polarised in the positive y direction. The fraction remaining, (1 - P), has randomly oriented spin (for an unpolarised beam, P = 0, while if the beam is completely transversely polarised, P = 1). As before, we have:

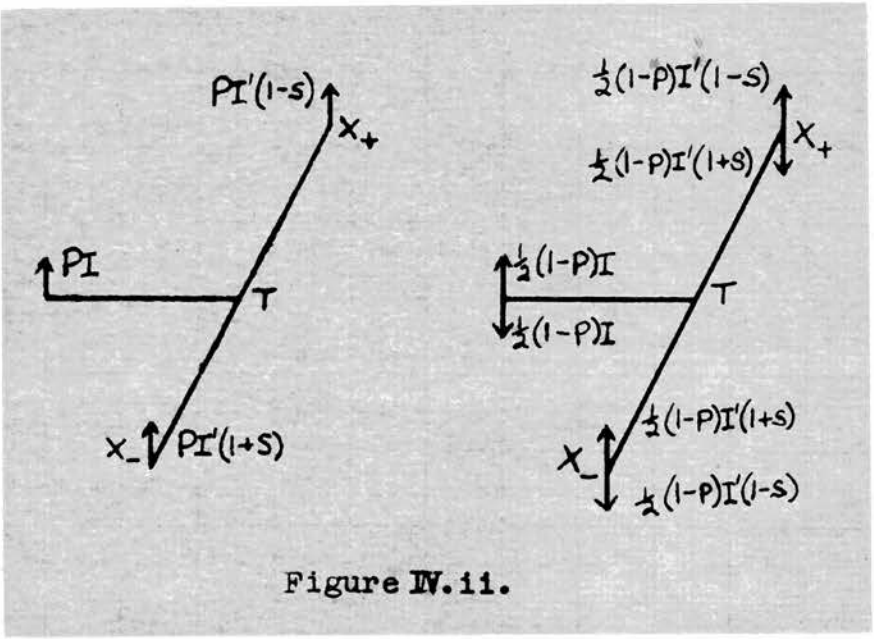


Figure IV.ii.

In this case, the counting rates at X_+ and X_- will be

$$C_+ = PI'(1 - S) + \tfrac{1}{2}(1 - P)I'\{(1 - S) + (1 + S)\}$$

$$= I'(1 - PS)$$

$$C_- = PI'(1 + S) + \tfrac{1}{2}(1 - P)I'\{(1 + S) + (1 - S)\}$$

$$= I'(1 + PS) .$$

$$\frac{C_+}{C_-} = \frac{(1 - PS)}{(1 + PS)} \neq 1 \quad \text{unless } P = 0, \text{ or } S = 0.$$

From this, we see that the scattered intensities in the positive and negative x directions are not, in general, equal. The theory shows that $S = S(\Theta)$, where Θ is the scattering angle (in the above, $\Theta = 90^{\circ}$), and that $S \propto Z$, the atomic number of the material of the scatterer. The quantity S is called the single scattering asymmetry.

In Appendix III, we saw that in many beta-decay processes, a fraction $v/c = \beta$ of electrons emitted with velocity v would be longitudinally polarised. After describing a sufficient number of turns in a homogeneous electromagnetic field, this longitudinal polarisation will be transformed into transverse polarisation, as explained in Appendix II. In this case, the degree of polarisation of the electron beam will be

$$P = \beta .$$

Thus, in choosing the most suitable electron energy for the present experiment, we should select the maximum value of the quantity

$$\beta S \, d\sigma/d\Omega$$

where $\beta = v/c = P$ is the degree of polarisation, S is the single scattering asymmetry, and $d\sigma/d\Omega$ is the differential scattering cross-section.

Recently, calculations of absolute cross-sections and asymmetries for Mott scattering have been greatly extended and improved, with the aid of electronic computers (Doggett and Spencer, 1956; Sherman, 1956). These calculations use potentials which do not take account of the screening of the nucleus by atomic electrons, and point nuclei are assumed. In the following table, which is based on Sherman's results, the differential cross sections, $d\sigma/d\Omega$, are in (barns per steradian) x 10^{-4}, S is the single scattering asymmetry, energies are in kev, θ is the scattering angle, and the scatterer is mercury (Z = 80). The bottom line of the table shows, for comparison, values of the single scattering asymmetries which have been derived directly from one of Mott's original papers (Mott, 1932).

θ	$\beta(= v/c)$	0.4	0.5	0.6	0.7	0.8
	Energy:	46	79	128	204	340
75°	$d\sigma/d\Omega$	3.81	1.59	0.725	0.337	0.148
	S	0.104	0.143	0.160	0.162	0.150
	$\beta.S\ d\sigma/d\Omega$	0.1585	0.1137	0.0696	0.0382	0.0178
90°	$d\sigma/d\Omega$	2.35	0.964	0.429	0.194	0.083
	S	0.234	0.261	0.271	0.265	0.242
	$\beta.S\ d\sigma/d\Omega$	0.2200	0.1258	0.0698	0.0360	0.0161
105°	$d\sigma/d\Omega$	1.66	0.656	0.281	0.122	0.050
	S	0.333	0.356	0.367	0.364	0.340
	$\beta.S\ d\sigma/d\Omega$	0.2211	0.1168	0.0619	0.0311	0.0136
90°	S (Mott)	0.123	0.240	0.278	0.265	0.224

A later calculation has been undertaken for gold (Z = 79), the electron energies being 121 kev (β = 0.59) and 75 kev (β = 0.49). (Sherman and Nelson, 1959). The cross-sections and asymmetries are only slightly different from those given above for mercury (Z = 80) at 128 kev and 79 kev respectively.

An experiment has recently been carried out by Spiegel et al. in which Mott scattering theory was checked (with a precision of about 2%) over a wide range of conditions. (Spiegel, Ruane, Anthony, Waldman and Miller, 1959). They used a monoenergetic electron beam and, to ensure that single scattering was being observed, foils of different thicknesses were employed, and the energy of the scattered electrons was analysed by a magnetic spectrometer. The ratio of the observed absolute cross-section to the calculated cross-section was computed for energies between 1.0 and 2.5 Mev and, in every case but one, this ratio was less than unity.

To account for this discrepancy, it was suggested that the theoretical Mott cross-sections should be corrected for: (a) the finite size of the nucleus; (b) the effect of screening of the nucleus by atomic electrons; (c) radiative

effects. All three effects tend to reduce the Mott cross-section but, even when these were allowed for, the discrepancy was still well outside the experimental error; it was felt that an accurate evaluation of the screening correction (the least certain of the three) might remove this.

It is worth noting that Tassie has worked out the single scattering asymmetry for electrons on gold (Tassie, 1957), using the screened field of gold previously calculated by Mohr and Tassie (Mohr and Tassie, 1954). He finds S = 0.28 for scattering of 121 kev electrons through ninety degrees. Comparing this with Sherman's unscreened field calculation for the scattering of 128 kev electrons on mercury (see above table) he concludes that, at least for these energies and large angles of scatter, the effect of screening of the nucleus by atomic electrons is altogether negligible. In view of this and of the results of Spiegel et al., it would appear that a thorough investigation of Mott scattering theory at low energies is very desirable, for it is surely reasonable to suppose that the effect of screening will decrease as the energy of the incident particle increases (since high energy electrons will approach closer to the nucleus, and hence will be less affected by the atomic electrons).

APPENDIX V

Electron Optics

We shall define an electric field, $\underline{E}$, and a magnetic field, $\underline{B}$, to be "crossed" if, at all points, they satisfy the relation

$$\underline{E}(\underline{r}).\underline{B}(\underline{r}) = 0 \quad . \qquad (V.1)$$

If $\underline{E}$ has no x component, we may write

$$E_x = 0 \; ; \quad E_y = -\frac{\partial \phi}{\partial y} \; ; \quad E_z = -\frac{\partial \phi}{\partial z} \quad ,$$

where ϕ is the scalar potential such that $\underline{E} = -\nabla\phi(y,z)$. Similarly, $\underline{B}$ is derived from a vector potential, $\underline{A}$: $\underline{B} = \nabla \wedge \underline{A}$; we shall consider only the special case where

$$A_x = A(y,z) \; ; \quad A_y = A_z = 0, \quad \text{so that}$$

$$B_x = 0 \qquad ; \quad B_y = \frac{\partial A}{\partial z} \quad ; \quad B_z = -\frac{\partial A}{\partial y} \quad .$$

Using (V.1), we find

$$\frac{\partial \phi}{\partial y} \cdot \frac{\partial A}{\partial z} = \frac{\partial \phi}{\partial z} \; \frac{\partial A}{\partial y}$$

which will be satisfied if

$$\phi(y,z) = f\{A(y,z)\} \; . \qquad (V.2)$$

Since $E^2 = E_y^{\,2} + E_z^{\,2}$; and $B^2 = B_y^{\,2} + B_z^{\,2}$,

$$\frac{E}{B} = \left\{ \frac{\left(\frac{\partial \phi}{\partial y}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2}{\left(\frac{\partial A}{\partial y}\right)^2 + \left(\frac{\partial A}{\partial z}\right)^2} \right\}^{\frac{1}{2}} .$$

Now $\frac{\partial \phi}{\partial y} = \frac{d\phi}{dA}\frac{\partial A}{\partial y}$; $\frac{\partial \phi}{\partial z} = \frac{d\phi}{dA}\frac{\partial A}{\partial z}$, from (V.2).

Squaring and adding leads to

$$\frac{E}{B} = \frac{d\phi}{dA} .$$

Hence, E/B will be <u>constant</u> if

$$\phi(y,z) = c_1 A(y,z) + c_2 \qquad (V.3)$$

where c_1, c_2 are constants. Then

$$\frac{E}{B} = \text{const.} = \frac{E_o}{B_o} \text{ , say,} \qquad (V.4)$$

where $E_o = E(0,0,0)$, $B_o = B(0,0,0)$.
In the following discussion, we consider only crossed fields which are derived from potentials satisfying the condition (V.3).

The analysis of the focussing properties of such fields is simplified when it is realised that there exists a coordinate system, moving with a unique velocity in the x direction, in which a moving particle experiences no electric field. Relative to this system, the motion of a particle is therefore determined by a pure magnetic field.

If two particles are emitted from the same point at the same instant, $t' = 0$, in this moving

frame, and if at some later time $t' = T'$, their positions in space again coincide, in the moving frame, then an observer in the fixed, laboratory, frame would detect that the two particles coincided in position at $t = 0$ and again at $t = T$. This is so since to every point in the moving frame there corresponds one, and only one, point in the laboratory frame, the two points being related by Lorentz transformation. From this we see that, if particles are emitted from a point and subsequently brought to a point in the moving system, that is, they are focussed, then there will be focussing in the laboratory system also.

As mentioned already, in Appendix II, the field vectors in the moving (primed) system are related to those in the laboratory (unprimed) system by the equations

$$\underline{B}' = \gamma_V(\underline{B} - \frac{1}{c^2}\,\underline{V}\wedge\underline{E})$$

$$\underline{E}' = \gamma_V(\underline{E} + \underline{V}\wedge\underline{B})$$

$$\gamma_V = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$$

where $\underline{V}$ is the relative velocity of the two systems. These equations hold only when $\underline{E}$ and $\underline{B}$ are perpendicular to $\underline{V}$, that is, when

$$\underline{V}.\underline{E} = \underline{V}.\underline{B} = 0.$$

Choosing $\underline{E}' = 0$, we find

$$\underline{V} = \frac{\underline{E} \wedge \underline{B}}{B^2}$$

and, using (V.1),

$$V = \frac{E}{B} = \frac{E_o}{B_o} = \text{const.},$$

from (V.4).

Hence, in a coordinate system moving in the x direction with a velocity $V = E_o/B_o$, a moving particle will be influenced by a pure magnetic field, $\underline{B}'$. Now

$$\underline{B}' = \gamma_V(\underline{B} - \frac{1}{c^2}\,\underline{V} \wedge \underline{E})$$

and $\underline{V} \wedge \underline{E} = VE = V^2 B$; so

$$B' = \gamma_V(B - \frac{V^2}{c^2}\,B)$$

$$\text{or} \quad B' = \frac{1}{\gamma_V}\,B \quad . \qquad \text{(V.5)}$$

Since $y' = y$ and $z' = z$, the field distribution will be the same in both frames, the relative velocity, V, appearing only in the scaling factor $1/\gamma_V$.

In the moving system, electrons with velocity $\underline{v}'$ will describe circular orbits of radius of curvature R', where

$$R' = \gamma' \frac{mv'}{eB'}$$

$$\text{with } \gamma' = \left\{1 - \left(\frac{v'}{c}\right)^2\right\}^{-\frac{1}{2}} \quad .$$

The angular frequency of the motion will be

$$\omega' = \frac{eB'}{\gamma' m} \quad .$$

If $\underline{v}$ is the velocity of an electron in the laboratory frame, then

$$v_x = (v_x' + V)(1 + \frac{v_x' V}{c^2})^{-1}$$

$$v_y = v_y'(1 - \frac{V^2}{c^2})^{\frac{1}{2}}(1 + \frac{v_x' V}{c^2})^{-1}$$

$$v_z = v_z'(1 - \frac{V^2}{c^2})^{\frac{1}{2}}(1 + \frac{v_x' V}{c^2})^{-1}$$

so the condition that the path of an electron, which is circular in the moving system, should approximate to a circle in the laboratory system is $V/v_x' << 1.$

Now $V = E/B$, and v_x' is of the order of magnitude of v'; so the above condition becomes

$$E/v'B << 1.$$

If this is so, then v' and v are of the same order, so we must have

$$E/vB << 1. \qquad (V.6)$$

This certainly implies

$$E/cB \equiv V/c << 1, \qquad (V.7)$$

so, in the limit of the orbits approximating closely to circles in the laboratory coordinate system, we have, to a good approximation $\gamma_V = 1.$

The radius and angular frequency of the (almost) circular orbits will then be

$$R = \gamma \frac{mv}{eB} \qquad (V.8)$$

$$\omega = \frac{eB}{\gamma m} \qquad (V.9)$$

with

$$\gamma = (1 - v^2/c^2)^{-\frac{1}{2}} \quad .$$

Suppose there exists a magnetic field which is everywhere perpendicular to a given plane. If there is a closed curve in this plane such that a particle of given momentum moves on this curve, the curve is called the <u>equilibrium orbit</u>. The problem which we wish to investigate is the following: if a particle, with momentum appropriate to the given equilibrium orbit, starts with a small initial displacement from, and at a small initial angle to the equilibrium orbit, under what conditions will the particle remain close to this orbit for all time?

It was suggested by Gabor that, a field of the following type might be suitable to producing focussing (Gabor, 1958):

$$\begin{aligned} E_x &= 0 \\ E_y &= E_o \cosh(kz)\cos(ky) \\ E_z &= E_o \sinh(kz)\sin(ky) \end{aligned} \qquad (V.10)$$

$$B_x = 0$$

$$B_y = -B_o \sinh(kz)\sin(ky) \qquad (V.11)$$

$$B_z = B_o \cosh(kz)\cos(ky) \quad .$$

The corresponding potentials are

$$\phi(y,z) = -\frac{E_o}{k} \cosh(kz)\sin(ky)$$

$$A_x(y,z) = -\frac{B_o}{k} \cosh(kz)\sin(ky)$$

$$A_y = A_z = 0$$

so that (V.3) is satisfied.

Since the relative velocity of the primed and unprimed systems is in the x direction,

$$y = y' \; ; \qquad z = z' \quad .$$

From (V.5),

$$B_o' = \frac{1}{\gamma_V} B_o \quad .$$

In the primed system, the equation of motion of an electron is

$$\gamma' m \ddot{\underline{r}}' = e \underline{v}' \wedge \underline{B}' \qquad (V.12)$$

Using the above transformation relations, (V.12) becomes, in components,

$$\ddot{x} = \omega \left\{ \dot{y} \cosh(kz)\cos(ky) + z \sinh(kz)\sin(ky) \right\}$$

$$\ddot{y} = -\omega \dot{x} \cosh(kz)\cos(ky) \qquad (V.13)$$

$$\ddot{z} = -\omega \dot{x} \sinh(kz)\sin(ky)$$

where, for convenience, the primes have been

dropped, and where we have written

$$\omega = \frac{1}{\gamma' \gamma_v} \frac{e}{m} B_o \quad . \qquad (V.14)$$

Note that B_o is the magnitude of the magnetic field as measured in the laboratory system.

In the plane $z = 0$, the equations (V.13) reduce to

$$\begin{aligned} \ddot{x} &= \omega \dot{y} \cos(ky) \\ \ddot{y} &= -\omega \dot{x} \cos(ky) \qquad (V.15) \\ \ddot{z} &= 0 \quad . \end{aligned}$$

The third equation gives $\dot{z} = \text{const.}$ If we consider an electron with initial velocity in the x - y plane, then $v^2 = v_o^2 = \text{const.} = \dot{x}^2 + \dot{y}^2$; so $\dot{z} = 0$, and the motion will be confined to the x - y plane for all time.

The first equation integrates immediately to

$$\dot{x} = \frac{\omega}{k} \sin(ky) + \text{const.}$$

If $\dot{x} = 0$ at the origin (that is, the source at the origin emits electrons with initial velocities perpendicular to the x-axis), then

$$\dot{x} = \frac{\omega}{k} \sin(ky) \quad . \qquad (V.16)$$

Then

$$\dot{y} = \left\{ v_o^2 - \left(\frac{\omega}{k}\right)^2 \sin^2(ky) \right\}^{\frac{1}{2}} \quad .$$

The differential equation of the orbit is then

$$\frac{dx}{dy} = \frac{\dot{x}}{\dot{y}} = \frac{(\frac{\omega}{k})\sin(ky)}{\left\{v_o^2 - (\frac{\omega}{k})^2\sin^2(ky)\right\}^{\frac{1}{2}}} \quad .$$

Writing $R = v_o/\omega$

where R has the dimension of length, we have

$$\frac{dx}{dy} = \frac{(1/Rk)\sin(ky)}{\left\{1 - (1/Rk)^2\sin^2(ky)\right\}^{\frac{1}{2}}} \quad .$$

If the electron path is to be a <u>closed</u> orbit, then at some point we must have $dy/dx = 0$, so a necessary condition for closed orbits is

$$\sin(ky) = \pm Rk$$

or $$|Rk| \leq 1 \quad . \qquad (V.17)$$

Substituting $(1/Rk)\sin(ky) = \sin\psi$,

$$x = R\int_o^{\psi} \frac{\sin\psi\, d\psi}{1 - (Rk)^2\sin^2\psi^{\frac{1}{2}}}$$

and hence

$$kx = \log\left\{\frac{\cos(ky) - \left\{(Rk)^2 - \sin^2(ky)\right\}^{\frac{1}{2}}}{1 - Rk}\right\} \qquad (V.18)$$

where condition (V.17) has been applied.

The "top" of the orbit is reached when $\sin(ky) = Rk$; let the corresponding value of x be X ; then

$$kX = \log\left\{\frac{1 + Rk}{1 - Rk}\right\}^{\frac{1}{2}} \quad .$$

Subtracting kX from (V.18) and rewriting the result in exponential form, we find

$$e^{-k(X-x)} = \frac{\cos(ky) - \left\{(Rk)^2 - \sin^2(ky)\right\}^{\frac{1}{2}}}{\left\{1 - (Rk)^2\right\}^{\frac{1}{2}}} \quad .$$

From the above, $Rk = \tanh(kX)$, and so we find

$$\cos(ky) = \frac{\cosh k(X - x)}{\cosh(kX)} \quad . \qquad (V.19)$$

Since cos is an even function, there are two values, $\pm y$, corresponding to a given value of x. In other words, the orbit is symmetrical about the x-axis. It is symmetrical, also, about an axis parallel to the y-axis through the point $x = X$, since cosh is an even function. The closed orbit thus found in the plane $z = 0$ is the required equilibrium orbit.

We may rewrite (V.19) as

$$\cos(ky) = \cosh(kx) - Rk \sinh(kx) \quad .$$

As k becomes vanishingly small, the fields become homogeneous, and the above reduces to

$$1 - \tfrac{1}{2}(ky)^2 = 1 + \tfrac{1}{2}(kx)^2 - (Rk)(kx)$$

leading to

$$(x - R)^2 + y^2 = R^2 \quad .$$

This is the equation of a circle of radius R, passing through the origin (that is, through the source), as might have been anticipated from the definition of R.

We now assume that a particle is displaced

from a point $(x,y,0)$ of the equilibrium orbit to a neighbouring point $(x + \alpha, y + \beta, \gamma)$. The components of the magnetic field at $x + \alpha$, $y + \beta$, γ are

$$B_x(y+\beta,\gamma) = 0$$

$$B_y(y+\beta,\gamma) = B_y(y,0) + \left(\frac{\partial B_y}{\partial y}\right)\beta + \left(\frac{\partial B_y}{\partial z}\right)\gamma$$

$$B_z(y+\beta,\gamma) = B_z(y,0) + \left(\frac{\partial B_z}{\partial y}\right)\beta + \left(\frac{\partial B_z}{\partial z}\right)\gamma$$

where, after the partial differentiation is carried out, we put $z = 0$. Using (V.11), (V.13) and (V.14), we find

$$\begin{aligned} \ddot{\alpha} &= \omega\left\{\dot{\beta}\cos(ky) - k\dot{y}\beta\sin(ky)\right\} \\ \ddot{\beta} &= -\omega\left\{\dot{\alpha}\cos(ky) - k\dot{x}\beta\sin(ky)\right\} \qquad \text{(V.20)} \\ \ddot{\gamma} &= -\omega k\dot{x}\gamma\sin(ky) \quad . \end{aligned}$$

The first equation integrates immediately to

$$\dot{\alpha} = \omega\beta\cos(ky) \quad . \qquad \text{(V.21)}$$

Using this and (V.16), the second equation gives

$$\ddot{\beta} = -\omega^2\beta\cos(2ky)$$

or

$$\ddot{\beta} = -\omega^2\beta\left\{1 - 2\sin^2(ky)\right\} \quad . \qquad \text{(V.22)}$$

The third equation gives, using (V.16),

$$\ddot{\gamma} = -\omega^2\gamma\sin^2(ky) \quad . \qquad \text{(V.23)}$$

If $k = 0$, that is, when the field is homogeneous, (V.22) becomes

$$\ddot{\beta} = -\omega^2\beta \quad ,$$

showing that, in the z = 0 plane, a small displacement in the y direction is restored by a quasi-elastic force, and the particle oscillates about the equilibrium orbit with angular frequency ω, which is the same as that of the orbital motion. This is simply an alternative description of the normal focussing effect in the symmetry plane. Notice that, when k = 0, there is no restoring force in the z direction.

To integrate (V.22) and (V.23), we shall replace the function $\sin^2(ky)$, which varies periodically over a small range, by its mean value. We have already seen that when y (and hence ky) has its maximum value, then

$$\sin(ky) = Rk$$

$$\text{so} \qquad (ky)_{max} = \sin^{-1}(Rk)$$

Writing $\sin\theta = Rk$, we have (V.24)

$$(ky)_{max} = \theta \ ,$$

and so

$$\overline{\sin^2(ky)} = \frac{1}{\theta}\int_0^{\theta} \sin^2(ky)\,d(ky)$$

$$\therefore \quad \overline{\sin^2(ky)} = \tfrac{1}{2}\left\{1 - \frac{\sin 2\theta}{2\theta}\right\} . \qquad (V.25)$$

Substituting this into (V.22) and (V.23) gives

$$\ddot{\beta} = -\omega^2\left(\frac{\sin 2\theta}{2\theta}\right)\beta \qquad (V.26)$$

$$\ddot{\gamma} = -\omega^2 \cdot \tfrac{1}{2}\left(\frac{\sin 2\theta}{2\theta}\right)\gamma \qquad (V.27)$$

From (V.27), we see that the electron will execute simple harmonic motion parallel to the z-axis with angular frequency

$$\omega_z = \omega \left\{ \tfrac{1}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right) \right\}^{\frac{1}{2}} . \qquad (V.28)$$

Equation (V.26) shows that, in the plane of the equilibrium orbit, the particle will execute simple harmonic motion parallel to the y-axis with angular frequency

$$\omega_y = \omega\left(\frac{\sin 2\theta}{2\theta}\right)^{\frac{1}{2}} \qquad (V.29)$$

The x component of the displacement may be found from (V.21):

$$\dot{\alpha} = \omega\beta \frac{1}{\theta} \int_0^{\theta} \cos(ky)\, d(ky)$$

$$= -\omega\beta\left(\frac{\sin\theta}{\theta}\right) .$$

If we write $\beta = \beta_o \sin(\omega_y t)$, then

$$\dot{\alpha} = -\omega\,\beta_o\left(\frac{\sin\theta}{\theta}\right)\sin(\omega_y t), \quad \text{and so}$$

$$\alpha = \beta_o \frac{\omega}{\omega_y}\left(\frac{\sin\theta}{\theta}\right)\cos(\omega_y t)$$

$$= \alpha_o \cos(\omega_y t), \quad \text{say.}$$

Hence, the displacement of the electron from the equilibrium orbit in a direction parallel to the x-axis also varies sinusoidally, and this variation has the same angular frequency as that in the y direction; but there is a phase difference of $\pi/2$ between the oscillations. The

ratio of the two amplitudes is

$$\frac{\alpha_o}{\beta_o} = \frac{\omega}{\omega_y}\left(\frac{\sin\theta}{\theta}\right) = \left(\frac{\sin\theta}{\theta}\right)\left(\frac{\sin 2\theta}{2\theta}\right)^{-\frac{1}{2}} ,$$

from (V.29); so $\alpha_o/\beta_o \longrightarrow 1$ as $k \longrightarrow 0$.

If we consider orbits which approximate closely to circles, then the period of one orbital revolution will be

$$T \doteq \frac{2\pi}{\omega} .$$

Since we are considering k to be small,

$$\frac{\sin 2\theta}{2\theta} \doteq 1 - \frac{1}{6}(2\theta)^2$$

$$= 1 - \frac{2}{3}(Rk)^2 .$$

So, to this degree of approximation, (V.28) gives

$$\omega_z = \omega\left\{\frac{1}{2}\cdot\frac{2}{3}(Rk)^2\right\}^{\frac{1}{2}} = \omega\frac{Rk}{\sqrt{3}}$$

$$\therefore \quad T_z = \frac{2\pi}{\omega_z} = \frac{2\pi}{\omega}\frac{\sqrt{3}}{Rk} = T\frac{\sqrt{3}}{(Rk)} \qquad (V.30)$$

If, for example, $Rk = 0.1$, then

$$T_z \doteq 17T$$

so that a complete cycle of oscillation in the z direction would require about 17 orbital revolutions.

From (V.29) we see that, when Rk is small, $\omega_y \equiv \omega_x \doteq \omega$, so that the focussing properties

of the homogeneous field ($k = 0$) are scarcely affected. In one complete orbital revolution, the oscillating electron will cross the equilibrium orbit (that is, will be brought to a focus) twice.

Even when Rk is small, the foci produced in the $x - y$ plane will not occur after exactly integral numbers of orbital revolutions. Further, (V.30) shows that, when Rk is small, focussing in the z direction requires $\sqrt{3}/2Rk$ orbital revolutions and this will not, in general, be integral. Therefore a field distribution of the type considered cannot produce precise focussing on a target after a large integral number of orbital revolutions. However, even this somewhat imperfect focussing will confine the spread of an electron beam both in the plane of the equilibrium orbit and in a direction perpendicular to this plane.

If α_o and γ_o are the amplitudes of the oscillations parallel to the x- and z-axes respectively, then

$$\alpha = \alpha_o \sin \omega_x t \quad ; \quad \gamma = \gamma_o \sin \omega_z t$$

and so, at $t = 0$,

$$\dot{\alpha}_o = \alpha_o \omega_x \quad ; \quad \dot{\gamma}_o = \gamma_o \omega_z \quad .$$

Let the angle between the direction of the initial velocity, v_o, and the z-axis be $(\pi/2 - \phi_2)$, and between the projection of v_o on the x-y plane and the y axis be ϕ_1.

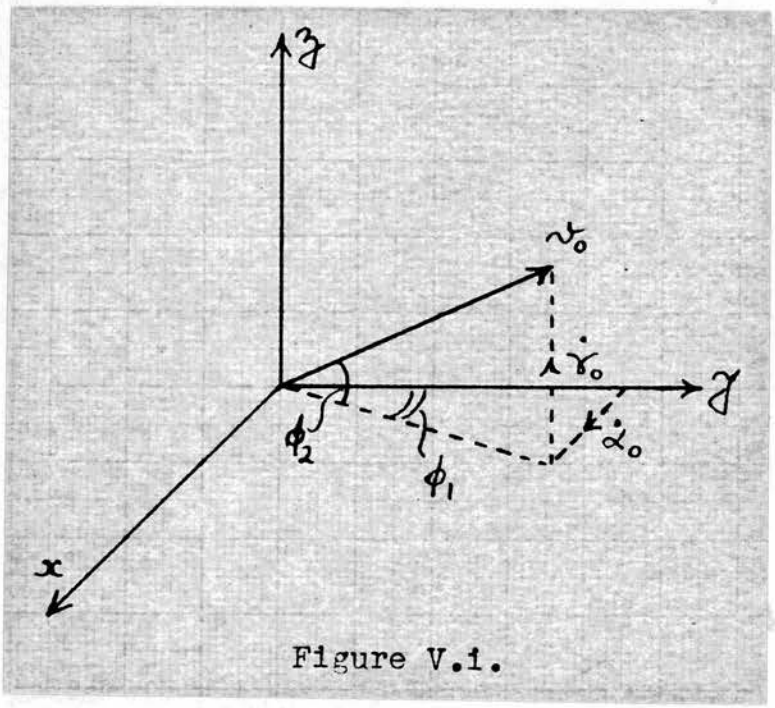


Figure V.i.

Then, from Figure V.i,

$$\phi_1 = \frac{\dot{\alpha}_o}{v_o} = \frac{\alpha_o \omega_x}{v_o}$$

$$\phi_2 = \frac{\dot{\gamma}_o}{v_o} = \frac{\gamma_o \omega_z}{v_o}$$

Using (V.29) and (V.28), and recalling that $v_o = R\omega$, we find, when Rk is small,

$$\phi_1 = \frac{a_o}{R}\left(\frac{\sin 2\theta}{2\theta}\right)^{\frac{1}{2}} = \frac{a_o}{R}$$

$$\phi_2 = \frac{\gamma_o}{R}\left\{\tfrac{1}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)\right\}^{\frac{1}{2}} = \frac{\gamma_o}{R} \cdot \frac{Rk}{\sqrt{3}} \quad .$$

We shall now consider one possible arrangement whereby fields approximating to those of (V.10) and (V.11) might be realised in practice.

Suppose there are four long conductors running parallel to the x-axis having (y,z) coordinates (a,b), (-a,b), (-a,-b) and (a,-b); let these at (a,b), (a,-b) carry a current -I while those at (-a,b), (-a,-b) carry a current +I. The resulting vector potential is

$$A_x(y,z) = -\frac{I\mu'_o}{4\pi} \log \frac{\left\{(y-a)^2+z^2+b^2\right\}^2 - (2zb)^2}{\left\{(y+a)^2+z^2+b^2\right\}^2 - (2zb)^2}$$

where $\mu'_o = 4\pi \times 10^{-7}$ henries/metre,

or $A_x(y,z) = C \log F(y,z)$, say.

Then

$$B_x = 0$$

$$B_y = \frac{\partial A_x}{\partial z} = \frac{C}{F}\frac{\partial F}{\partial z} = CG_1(y,z)$$

$$B_z = \frac{\partial A_x}{\partial y} = -\frac{C}{F}\frac{\partial F}{\partial y} = -CG_2(y,z) \quad .$$

Carrying out the differentiations, and confining our investigation to the x - y plane, we find

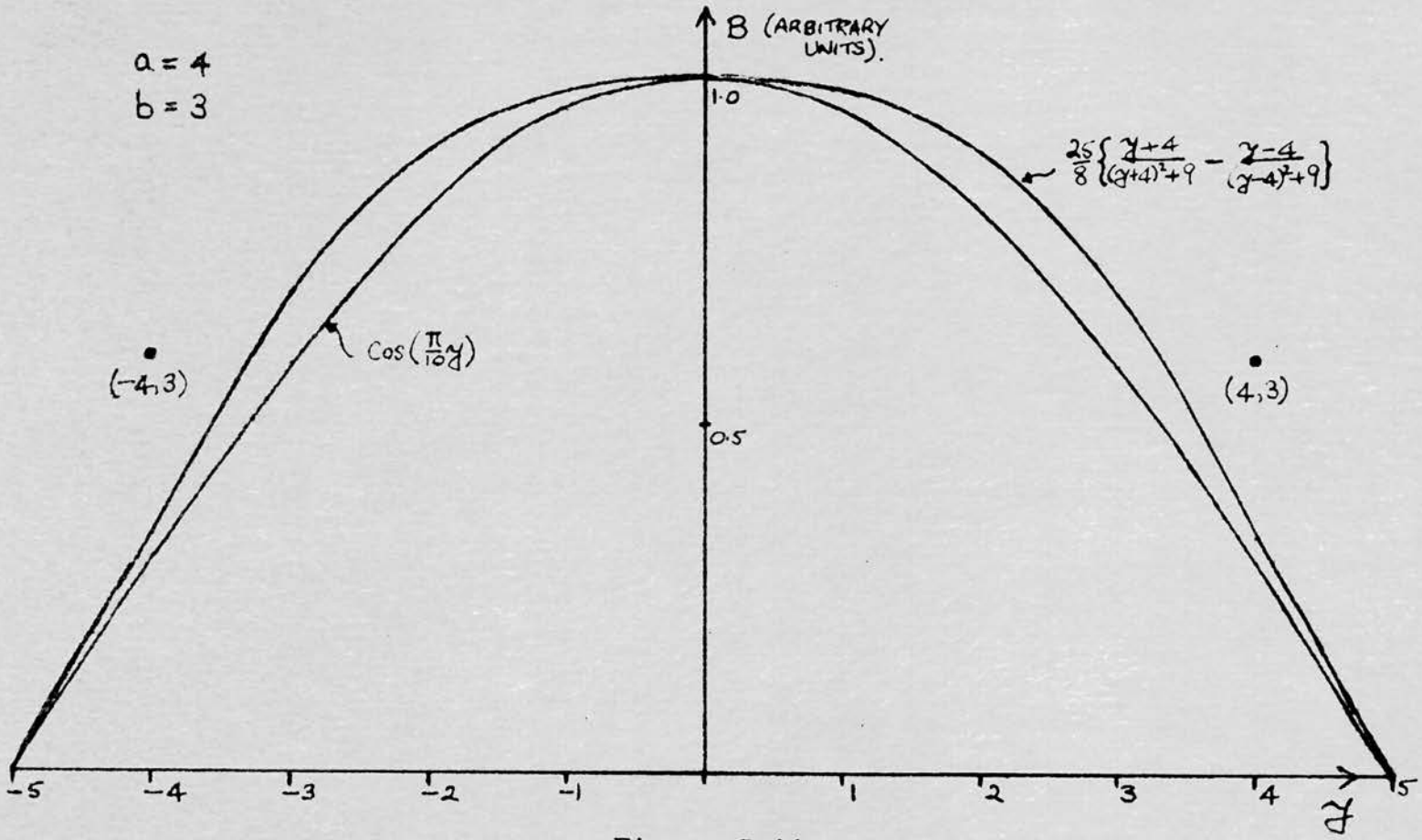

a = 4
b = 3
B (ARBITRARY UNITS).
1·0
0·5
$\frac{25}{8}\left\{\frac{y+4}{(y+4)^2+9} - \frac{y-4}{(y-4)^2+9}\right\}$
$\cos\left(\frac{\pi}{10}y\right)$
(-4,3)
(4,3)
-5
-4
-3
-2
-1
1
2
3
4
5
y

Figure V.ii.

$$B_{xo} = 0$$

$$B_{yo} = CG_1(y,0) = 0$$

$$B_{zo} = -CG_2(y,0)$$

$$= 4C \left[\frac{y+a}{(y+a)^2+b^2} - \frac{y-a}{(y-a)^2+b^2} \right] .$$

The distribution of B_{zo} with y is shown in Figure V.ii; it has a maximum value of

$$\frac{I\mu'_o}{4\pi} \cdot \frac{8a}{(a^2+b^2)}$$

and falls to zero at $y = \pm (a^2 + b^2)^{\frac{1}{2}}$.

The third of equations (V.11) gives, with $z = 0$,

$$B_{zo} = B_o \cos (ky).$$

Considering the expression

$$\frac{I\mu'_o}{4\pi} \frac{8a}{(a^2+b^2)} \cos \tfrac{\pi}{2}(a^2+b^2)^{-\frac{1}{2}} y$$

which vanishes at $y = \pm (a^2+b^2)^{\frac{1}{2}}$ and has a maximum value of $\frac{I\mu'_o}{4\pi} \cdot \frac{8a}{(a^2+b^2)}$, we see that the above system of four parallel conductors will approximate to the required focussing field provided

$$B_o = \frac{I\mu'_o}{4\pi} \cdot \frac{8a}{(a^2+b^2)} \qquad (V.31)$$

and $$k = \tfrac{\pi}{2}(a^2+b^2)^{-\frac{1}{2}} . \qquad (V.32)$$

As before, let us require $Rk = 0.1$. In the present experiment, $R = 5 \times 10^{-2}$ m. and $B_o = 2 \times 10^{-2}$ webers/m.^{2}. Then $k = 2$ m.$^{-1}$ and so (V.32) gives

$$(a^2 + b^2) = \frac{10}{16} \text{ m.}^2$$

If we take $a = b$. we find $a = 0.5$ m. and so (V.31) gives

$$I \doteqdot 30{,}000 \text{ amps.}$$

If a 50 kilowatt power supply were available, a <u>possible</u> arrangement could consist of two rectangular coils, each $1\frac{1}{2}$m. x 1 m. and spaced 1 m. apart. Each coil would consist of 300 turns of 10 gauge copper wire, through each turn of which was passed a current of 100 amps. The total coil wire diameter would be about 6 cms. and the total resistance about 5 ohms. Current would be supplied at about 500 volts.

To produce an electric field orthogonal to the magnetic field described above, we take four conductors in the form of cylinders charged with $+Q$ and $-Q$ coulombs per unit length, corresponding to and co-axial with the currents $+ I$ and $-I$. The scalar potential $\phi(y,z)$ has the same form as A_x, the constant C being replaced by $-Q/4\pi\varepsilon_o$, where $\varepsilon_o = (36\pi \times 10^9)^{-1}$ farads/metre. We see then that

$$\phi(y,z) = \text{const. } A_x(y,z),$$

so that condition (V.3) is satisfied.

We find, putting $z = 0$,

$$E_{xo} = 0$$

$$E_{yo} = \frac{Q}{4\pi\varepsilon_o} \cdot 4 \left\{ \frac{y+a}{(y+a)^2+b^2} - \frac{y-a}{(y-a)^2+b^2} \right\}$$

$$E_{zo} = 0 \ .$$

The maximum value of E_{yo} is $\frac{Q}{4\pi\varepsilon_o} \frac{8a}{(a^2+b^2)}$.

We saw, in Chapter 3, that the maximum value of E that might be required in the present experiment is about 10^5 volts/m., which corresponds to about 12 turns in the field. Taking $a = b = 0.5$ m., we then find

$$Q \doteqdot \tfrac{4}{3} \times 10^{-6} \text{ coulombs.}$$

To find the required potential of the conductors, we use the general expression for the potential at any point (y,z):

$$\phi(y,z) = -\frac{Q}{4\pi\varepsilon_o} \log \frac{\left\{(y-a)^2+z^2+b^2\right\}^2 - (2zb)^2}{\left\{(y+a)^2+z^2+b^2\right\}^2 - (2zb)^2} \ .$$

The potential at a point on a cylinder centred at (a,b) and of radius **r** may be found by substituting $y = a + r$; $z = b$ in the above. Putting $a = b = 0.5$ metres and $r = 0.05$ metres, we find

$$\phi(a + r, a) = 10^5 \text{ volts,}$$

where we have used $Q = \frac{4}{3} \times 10^{-6}$ coulombs.

Since we are interested mainly in the case when a hundred or more turns are performed in the crossed fields before scattering, potentials of the order of 10^4 volts would suffice, and these may be obtained without difficulty.

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