THE

CONSOLIDATION BEHAVIOUR

OF

SOFT CLAYS

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ABSTRACT

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Increased understanding of engineering behaviour of soil requires sophisticated interplay between observation, theory and analysis.

Interactive Graphics programs are presented to handle numerical extensions to Terzaghi Consolidation Theory, permitting multi-layer analysis, time-dependant loading, and governing parameters varying with effective stress. The capabilities of such analysis are explored. Problems arise with non-linear stress-strain behaviour and there is considerable evidence that strain rate is also a significant variable for soft clays.

A model proposed by Bjerrum is investigated in detail. This suggests skeletal behaviour may be constituted of two relationships, termed Instantaneous Consolidation - comprised of stress-strain functions only, and Delayed Consolidation - relating only strain and time.

These processes may be considered independant, but progress simultaneously. The mathematical model of such consolidation, and its solution, has been developed by Garlanger. Improved numerical solution techniques are presented. Some minor theoretical inconsistencies are resolved, and some major ones identified. Later work establishes the theoretical validity of the scheme. After clarification and modification this is rather better than originally claimed.

The major modification is in treatment of the critical pressure, p_c. It is suggested that a discontinuity of stress-strain function is superfluous; the phenomenon is adequately defined by inclusion of a compatible strain-time function.

Experimental evidence for this and the general acceptability of Garlanger's scheme is based on some 30 consolidation tests of varying type, pressure, and prior delayed consolidation, on Grangemouth silty-clay. Other cases of laboratory and field behaviour are also considered.

It is concluded that the modified Garlanger theory is the most satisfactory approach currently available, and is potentially of considerable value for analysis of the Consolidation of Soft Clays.

DECLARATION

I hereby declare that this thesis has been composed by me and the material described herein is my own unaided work, apart from advice acknowledged in the text.

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PRINCIPAL NOTATION

a_v	Coefficient of Compressibility (Terzaghi)
a	Log e - log p constant below p_c (Garlanger)
b	Log e - log p constant above p_c (Garlanger)
c	Log e - log t constant (Garlanger)
e	Voids Ratio
k	Permeability
m_v	Cofficient of Volume Compressibility (Terzaghi)
p	Effective stress
p_c	Critical Effective Stress (see section 6.4)
q	Stress due to Time Dependant Loading
t	Time
t_i	Time Value of Instant Line (Garlanger)
t_L	Time Value of Limit Line
u	Excess Pore Pressure
z	Space Variable
Av	Compressibility Coefficient $\{= \Delta e/\Delta p\}$ (Garlanger)
[B]	Pore Pressure Coefficient Matrix for new time in Numerical Analysis
C_α	Coefficient of Secondary Consolidation (Terzaghi) {gradient e - log t}
Cv	Coefficient of Consolidation (Terzaghi) $\{= k(1+e_o)/av\ \gamma_w\}$
C_F	Coefficient of Consolidation (Gibson *et al.*) $\{ = \frac{k(e)\ (1+e_o)^2}{\gamma_w\ (1+e)} \frac{\partial p}{\partial e} \}$

%C Average degree of Consolidation

= Strain = $\{(e_o - e)/\Delta e\}_{average} \times 100\%$

[D] Pore Pressure Coefficient Matrix for old time in Numerical Analysis

H Total thickness of Soil Layer

NZ Number of Elements into which Layer is divided for Numerical Analysis

S Total Integrated Settlement

T Dimensionless Time $\{= Cv.t/H^2\}$

{U} Vector of Pore Pressures in Numerical Analysis

Z Dimensional space variable $\{= z/H\}$

β Strain $\{= (e_o - e)/\Delta e\}$ (Garlanger)

β^1 Stability Factor for Numerical Analysis (Terzaghi)

$\{= Cv \ \Delta t/\Delta z^2\}$

γ_w Density of Pore Water

Δ Incremental Value - used as Prefix

Δe Increment in Voids Ratio - must be defined corresponding to increment of time.

See specific instances in text. e.g.

Frequently used for 1 day $\{\Delta e = e_o - e_D\}$

Δp Increment of Applied Stress $\{= p_f - p_o\}$

Δt Time discretization in Numerical Analysis

Δz Space discretization in Numerical Analysis

μ Dimensionless Pore Pressure (Garlanger)

$\{= \mu/\Delta p\}$

SUFFICES (Below or Above)

a	Values of parameters reached in A-type test after 24 hours undrained under Δp.
f	Final value attained by variable
i	Numbering of Space Node in Numerical Analysis
ℓ	Layer Number (Multi-layer Analysis)
o	Initial Value of a Variable
r	First reading possible in test
+	Value of Variable at New Time {i.e. to be evaluated by subsequent iteration of Numerical Analysis}
D	Value of Variable after 1 day's Consolidation
W	Value of Variable after 1 week's Consolidation
M	Value of Variable after 1 month's Consolidation

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1.1 Introduction - General Remarks

Of major significance to the engineering behaviour of soils are the complex inter-relationships between stress, strain, and time. For clay soils, problems are compounded by the slow movements of pore fluid under the influence of hydraulic gradients resulting from conditions of applied stress. The term consolidation is used to describe the process of drainage of pore fluid with resulting transfer of internal stress from the fluid to the soil skeleton, as the soil system tends to a state of equilibrium under a given applied loading.

The present study seeks to extend understanding of soil behaviour in consolidation. To do so, a continual interplay is required between observation and experimentation, development of suitable theoretical frameworks, and satisfactory techniques of analysis and solution. The present range of work on this topic is, however, so large and involved that attention has had to be directed to one-dimensional behaviour, and in particular to the significance of time dependant behaviour of the soil skeleton.

Mathematical description of the consolidation processes rapidly becomes rather involved and the techniques of numerical analysis prove extremely valuable in enabling solution of models of soil behaviour of interest, but too complex for analytical techniques. Details of computer programs to handle a number of such schemes are to be presented herein. For these, advantage has been taken of the excellent facilities existing at Edinburgh University as a result of the current research project into Computer Aided Design in Soil Mechanics (see section 1.3 below). Indeed, the programs developed here are a contribution to this work.

The classic Terzaghi Theory of Consolidation is still by far the most widely used approach in engineering practice. Although not equipped to handle time dependant behaviour of the soil skeleton it can give a fair description of some aspects of consolidation and allow the significance of time effects to be assessed. A suite of programs for this analysis is presented (Chapter 2) in which a number of extensions to the theory are shown capable of numerical solution, relaxing somewhat the assumptions invoked by Terzaghi. Of particular significance here is the possibility of using a more realistic stress-strain relationship. It is demonstrated herein that time dependant skeletal behaviour may be considered to affect the stress-strain relationships. In particular, strain at constant effective stress, over a period of time, leads to a strengthening of the soil such that further loading generates relatively little strain, up to some critical level. Allowance for this in satisfactory stress-strain moduli enables Terzaghi theory to fit reasonably with observed soil behaviour (see below).

A considerable number of theories of consolidation have been proposed since Terzaghi's. It would appear that real behaviour is very complex, and most relationships non-linear. For a theory to have value for prediction purposes it must not only be capable of satisfactory description of observed behaviour, but it must also use parameters which may be readily obtained to acceptable accuracy from standard test procedures - preferably tests which are already familiar, or at least use generally available apparatus (see below). Non-linear theories of soil behaviour encounter a major problem here, for fitting non-linear curves to experimental data is not normally a practical proposition unless the law can be linearized by use of suitable functions of the relevant variables.

In the present study particular attention is directed to an approach which separates the stress-strain-time behaviour into independant but simultaneous stress-strain and strain-time functions whose non-linearity is handled by use of linear logarithmic relationships. The approach was developed by Bjerrum (1967) who suggested the terms instantaneous (for e-p) and delayed (for e-t) consolidation. Garlanger (1972) produced a full theory explicitly to handle this analysis. This has been examined in detail here (Chapter 4). The original numerical solution scheme proved rather inefficient and two preferable methods have been developed. Some minor corrections are made to the published text. Of greater consequence are a number of problems of interpretation of the scheme as it stands. A first approach is outlined, and its application to existing data appears to substantiate the impressive claims made for this scheme.

A detailed experimental investigation of time dependant skeletal behaviour in consolidation has therefore been undertaken. Particular attention was paid to examining the validity of Garlanger's method and the parameters used therein. This required the measurement of pore water pressures at the impermeable boundary. Tests were carried out on Grangemouth Silty Clay, using the Bishop and Skinner Hydraulic Oedometer. A matrix system was used for the tests to examine the variables pressure and prior delayed consolidation (samples were left to consolidate for up to 1 month before incrementing the load). Four types of test were used; i) conventional, ii) conventional, except the drain is left closed for 24 hours after increasing the load to ensure pore pressures reach equilibrium; iii) as no. ii) but using fairly high back-pressure to keep gases in solution, and iv) using small load increments to trace the stress-strain path. {Nos. i) - iii) use a load increment ratio ($\Delta p/p$) of 1.0, throughout.} Experimental work is subject to a variety of possible errors, but the elaborate programme adopted, and subsequent analysis, allows the validity of results to be established. It is then possible to examine the capabilities of

Terzaghi and Garlanger theories, not just for fitting, by judicious choice of parameters, but also for prediction purposes.

The experimental, numerical and theoretical evidence of the present work is drawn together with the various relevant literature in Chapter 6, and conclusions are drawn on the major aspects of one dimensional consolidation of soft clays. The acceptability of Garlanger's scheme (including several proposed modifications and clarifications) is finally reviewed in Chapter 7, in terms of a number of criteria of importance for a theory to have practical value. The limitations of the method are considered, as well as the support for the author's contention that this is the most satisfactory scheme currently available for soils where time-dependant skeletal behaviour is to be included in the consolidation theory.

1.2 Theories of Consolidation

Theoretical analysis of soil consolidation was first made possible by the work of Terzaghi {1923, 1925, see also 1960}, following his classic investigations into the engineering behaviour of soils. The derivation of the theory, and the underlying assumptions {succinctly stated by Taylor (1948), p.226}, are sufficiently well-known not to require presentation here. The governing equation for the process is written

$$Cv \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \tag{1.1}$$

$$\text{where } Cv = \frac{k}{\gamma_w mv} .$$

i.e. The coefficient of consolidation, Cv, depends upon the permeability, k; density of the pore water, γ_w; and the coefficient of volume compressibility, mv, which itself is based on a linear relationship between stress and strain.

$$mv = \frac{-1}{1 + e_o} \cdot \frac{de}{dp} = \frac{a_v}{1 + e_o} \tag{1.2}$$

The classical theory assumes Cv is constant over the whole of the consolidation process under consideration. In this respect,* it does not matter whether permeability and compressibility remain constant, or vary simultaneously, such that Cv remains constant. The pore pressure behaviour will be unaffected. The strains (and hence Average Degree of Consolidation - which to avoid confusion should always be defined as the integrated mean strain over the soil profile) will be affected by non-constant mv, however, as will permeability values, should these require to be determined from this theory.

Equation (1.1) being of the diffusion type, analytical solution

* But notice that varying k is not fully consistent with the theoretical derivation of equation (1.1).

proved possible for the simplest initial pore pressure distributions and boundary conditions. Later work has considerably extended the range of conditions amenable to analytical solution {see, in particular, Carslaw and Jaeger (1948) and the various references to Schiffman}. A serious problem here is the complexity of such solutions and as conditions become more involved, so the amount of tabulation or graphical presentation becomes increasingly unmanageable.

As will be seen in Chapter 2, conveniently programmed numerical methods allow rapid and quite cheap solutions of such cases, as well as treatment of more complex conditions not amenable to analytical solution. Of major significance here is the fact that soil behaviour in consolidation is not particularly well approximated by the simple linear relationships assumed by Terzaghi. Typical stress-strain behaviour is indicated on fig. 1.1. The various states of consolidation will be considered shortly. We begin, however, by noting that many workers prefer to use linear logarthimic stress-strain relationships. Such has been incorporated into an approach by Davis and Raymond (1965), who in fact obtain solutions by specifying an equivalent k-log p relationship, such that Cv remains constant, as noted above for the Terzaghi approach. This may be considered as a special case of Poskitt's (1969) theory, in which the e- and k-log p relationships may vary without having to satisfy constant Cv.

The numerical extensions to Terzaghi theory, to be presented in Chapter 2, allow coefficients Cv and mv to vary as functions of p. The input data for the programs includes arrays of values of the parameters at as many values of effective stress, p, as are required to satisfactorily approximate the known behaviour. A subroutine linearly interpolates between point values to obtain values of Cv and mv corresponding to effective stresses determined during the solution

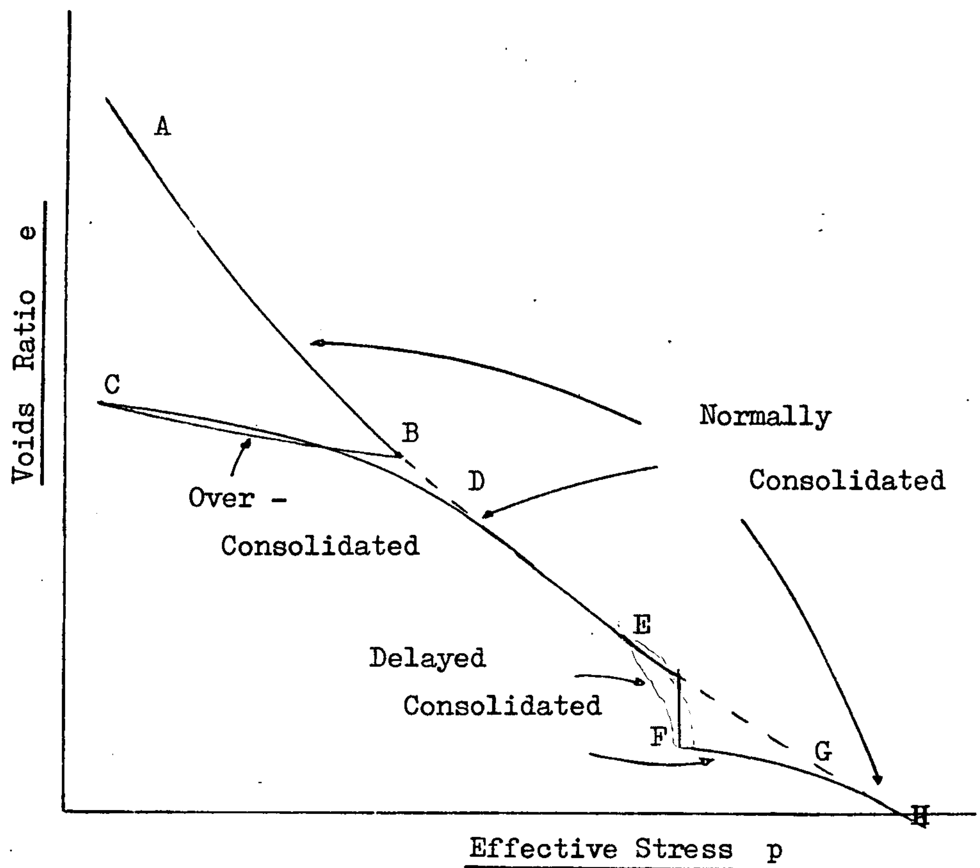


Fig 1.1 Typical Stress- Strain Behaviour of Soil Showing Definitions Used.

process. In principle, such parameters might result from a series of consolidation tests at varying p. In practice there are serious problems in obtaining even the standard parameters which have been in use for many years. Once non-linearity of soil behaviour is considered, interpretation of test data becomes more complex. Indeed the information available from conventional testing is rather limited, as noted in section 1.1, and current knowledge of real soil behaviour lags behind the potential of analysis. An instance high-lighted by the numerical scheme of chapter 2 is the difference in behaviour for "local" and "averaged" parameters. The former links Cv and mv values to p at each point in the soil profile; the latter links them to the effective stress averaged over the whole layer. During primary consolidation these will differ, whereas in the absence of excess pore pressures they may be taken to be the same. Specifying a given variation of Cv and mv with p, it may be seen numerically that significant differences in behaviour are predicted by use of "average" and "local" forms of analysis. Yet experimental techniques are insufficiently advanced to yield much data on this important point. This topic is considered in some detail in Section 6.

Further complication arises with real soil behaviour. It has long been known that all clays, to a greater or lesser extent, display continuing settlement after that predicted by Terzaghi theory has become negligible, and observed pore pressures "insignificant". In the present work the term secondary consolidation is used without precise definition to cover this process, which is usually attributed to the slow, viscous rearrangement of the soil particles at constant effective stress. A precise term will be introduced shortly, but first brief comment should be made on this process. Some workers use the term "creep" as apparently synonymous with secondary consolidation. The present author reserves 'creep' (by analogy with its use for single phase media) for multidimensional effects, which may take place at constant moisture content. It is clear that one-dimensional secondary consolidation settlement is due solely to expulsion of pore water from the voids. This may easily be demonstrated in the laboratory by preventing drainage from an oedometer sample, whereon settlement ceases. {This refers to a fully saturated soil where the pore fluid may be taken as incompressible. For an unsaturated soil gases may go into solution with consequent reduction of voids.} Slight build up of pore pressures in such cases has been reported on occasions. This would seem the likely behaviour, but such effects are small, especially after lengthy periods of drainage and the measuring equipment is too near the limits of its accuracy to yield much useful quantitative data here. However, such a secondary process of slow expulsion of pore water must be the result of a hydraulic gradient. Hence several workers have taken issue with definitions using zero or "insignificant" pore pressures, or effective stresses "constant".

Now consolidation is a complex phenomenon of flow of pore fluid through a medium displaying non-linear stress-strain-time behaviour. Since the different relationships cannot be isolated in practice, the author believes formal definitions of terms must refer to idealized behaviour in theoretical frameworks, after which real conditions may be identified where behaviour approximates to these definitions.

The most helpful approach in this case seems to be to consider two independant processes taking place simultaneously. The most useful terminology was felt to be that of Bjerrum (1967). Formalizing his definitions slightly:

Instantaneous Consolidation is that where strain in the soil skeleton results directly and instantaneously from the effective stress pertaining.

It may thus be described by e-p relationships, independant of time.

{It should be borne in mind that the slow drainage of pore fluid will cause effective stress to change with time. Hence strain occurs indirectly with time; the validity of the e-p relationships is not affected, however.}

{It may be useful to conceive Instantaneous Consolidation as that which would occur instantaneously were there no hydrodynamic behaviour to take place.}

Delayed Consolidation is that where strain in the soil progresses with time, independant of effective stress. It may thus be described by e-t relationships only. At large times, instantaneous consolidation becomes negligible, and a good approximation to this delayed consolidation may be observed.

The first reported method of handling secondary consolidation is due to Buisman (1936) who augmented Terzaghi theory with an empirically derived linear e-log t relationship fitted to the ultimate settlement predicted for the primary phase. Such an approach has been used here in the work on extended Terzaghi theory. By judicious fitting reasonable agreement with observed settlements may be obtained, though this gives too rapid settlement towards the end of the primary phase, changing rather abruptly to this secondary e-log t line. Observations indicate a more gradual transition.

Taylor and Merchant (1940) proposed a scheme in which the soil matrix displays the two processes defined above (i.e. an e-p relationship and an e-t relationship) occurring simultaneously. As would be expected, the "instantaneous consolidation" dominates at early stages, and the "delayed consolidation" at later times. Fig. 1.2 shows the soil behaviour postulated here. Christie (1964) later demonstrated that this method is mathematically identical to a technique presented by Gibson and Lo (1961); this latter will be discussed, since it is easier to visualise.

Gibson and Lo, then, consider the soil as the rheological model shown in fig. 1.3. The springs and dashpot are linear, whence instantaneous strain is a linear function of pressure. [illegible] linear function of time. Present evidence suggests that quite good prediction of settlement is now possible, but pore pressures tend to dissipate considerably less rapidly than has been observed, especially for tests after significant periods of delayed consolidation. {See e.g. Christie (op cit), Berre and Iversen (1972).} The author has programmed the analytical series solutions to facilitate application of this approach. A brief example of a typical case is given in section 4.6.2, figs. 4.10 and 4.11, below.

The author has also reformulated the governing equations to be capable of numerical solution. By this method it is now possible to solve cases where the springs, and/or dashpot, are non-linear. Work to be presented in this thesis suggests that the more important step would be to introduce a more realistic approximation to the stress-strain behaviour. The topic becomes very involved, and for present purposes it was preferred to direct attention to the actual behaviour of soil, and using this, to consider in detail an existing non-linear approach recently presented by Garlanger (see below). Modifications to Gibson and Lo's model have not, therefore, been pursued further.

A large number of models of one dimensional consolidation behaviour have been devised, and it is not possible to give detailed examination

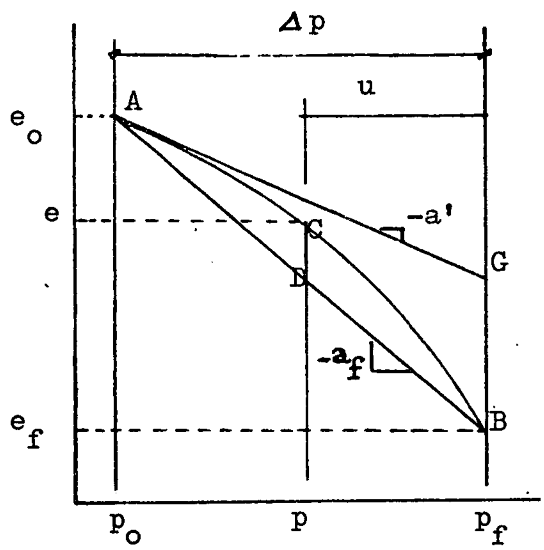


Fig 1.2 Taylor & Merchant Stress - Strain Model

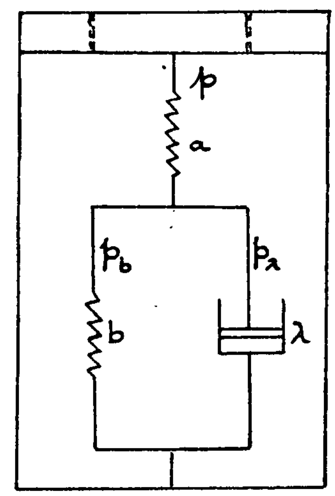


Fig 1.3 Gibson & Lo Rheological Model

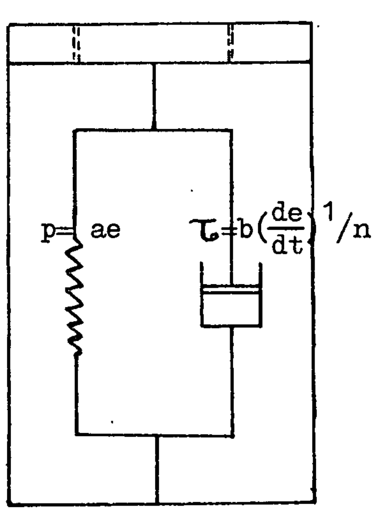


Fig 1.4 Barden Rheological Model

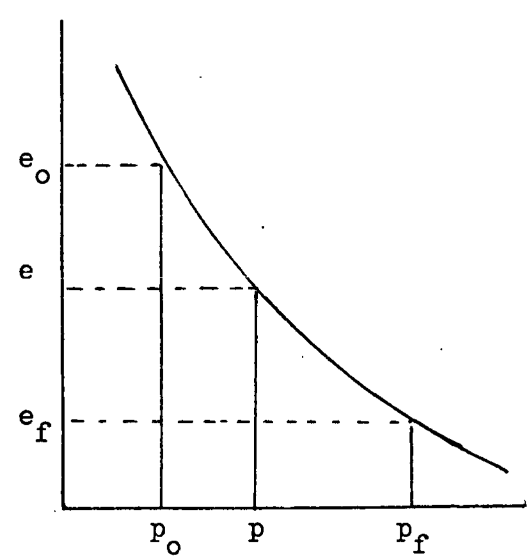


Fig 1.5 Barden Stress - Strain Model

of these here. Some points of significance to the present study drawn from various published theories, are, however, included in the detailed discussions of different aspects of consolidation behaviour presented in Chapter 6. Brief mention should perhaps be made of one further approach which has attracted some interest. Barden (1965) presented the rheological model shown in fig. 1.4, with the stress-strain-time relationship indicated in fig. 1.5. He suggested that both components of his rheological model (spring and dashpot, in parallel) were in fact non-linear, but chose to solve the case where only the dashpot did not behave linearly. Berre and Iversen (1972) found quite good agreement with their experimental data; they suggested this might have been better had a top-spring been included in the model. The present author believes suitable improvement might also have been obtained had the spring been non-linear to accommodate lower compressibility at early stages of consolidation following long periods of delayed consolidation. The major problem in this approach is again how to adequately describe the non-linearity.

Two states of consolidation in soils have traditionally been identified - namely normally-consolidated (NC) and over-consolidated (OC). As illustrated in fig. 1.1, the former term is applied to a soil under the highest state of effective stress it has experienced to date. On lowering the effective stress, from whatever cause, only a fraction of the previous strain is recovered, and the soil is described as over-consolidated. Re-application of load in such a case leads to fairly rapid consolidation with relatively little strain until the normally-consolidated state is reached at what is termed the preconsolidation stress. It appears that during delayed consolidation, an NC soil gains additional strength so that on further loading a "quasi - preconsolidation effect " is found at what is termed here the critical stress, p_c. The author has introduced the term Delayed Consolidated (DC) here (see fig. 1.1) to describe this soil state.

Definition of the critical pressure is particularly troublesome, since it cannot be directly observed but must be deduced from its effects. Detailed discussion of this can be found in Chapter 6. The observed change from the DC to NC state is affected by the strain rate (which will be affected by load increment and rate, and by thickness of the layer). It may, however, be possible to define a constant point, and develop a theoretical framework allowing for such an effect. The author believes this is what Bjerrum had in mind, but the examples given by Garlanger following his exposition of the theoretical treatment are not consistent with this.

These ideas on delayed consolidation were first presented by Bjerrum (1967), though they constitute a synthesis of several research approaches. In 1972, Garlanger published a consolidation theory based on this work, together with a numerical solution method for the rather complex governing equations. Since its publication it has been suggested that this is "the most powerful tool available at present for the analysis of one-dimensional consolidation where secondary behaviour is significant", {Clayton, 1973.} The approach does not, however, appear to have been subjected to detailed scrutiny. A major portion of the present project has therefore been devoted to theoretical and experimental investigations of Garlanger's theory. Improved numerical solution techniques are also suggested here. This approach has similarities with that of Gibson and Lo, but complex non-linear expressions are now used for instantaneous and delayed consolidation. Details are left to Chapter 4.

To summarize, then, a considerable number of approaches to one dimensional consolidation have been reported and only some of major importance can be investigated here. Terzaghi's approach is by far the most popular in current use, and extensions to conditions outside the simple linear case should prove valuable. The major advance on Terzaghi theory is inclusion of the time dependant behaviour of the soil matrix. Handling of this is still rather unsatisfactory, and a recent approach by Garlanger which appears promising is to be considered in detail.

1.3 Numerical Solution Methods

Before the advent of modern computing techniques the only theories of practical value were those capable of analytical solution - usually by reasonably straightforward series techniques. Even so the work involved in obtaining solutions could be considerable, and part of the attraction in the use of dimensionless variables lies in the fact that for commonly encountered problems the solutions may be plotted out or presented in tables which may then be converted to the real units under consideration. Numerical techniques of solution were known, though the subject has developed rapidly in recent years, but the work involved was normally prohibitive for hand calculation. A major aspect of Terzaghi's (1923) paper was that the governing equation was shown to be of the diffusion, or heat conduction, type, for which analytical solutions were available for the simpler boundary conditions.

Since Terzaghi's theory was first presented a number of theoretical developments have taken place, as discussed in the previous section. The availability of numerical methods which may be programmed and are thus relatively easy to use, has had a significant effect on these developments in recent years. Many equations and boundary conditions which were hitherto beyond consideration, now become amenable to analysis. Of particular interest here have been a number of techniques based on non-linear soil behaviour, and the comparison of such analyses with experimental data has contributed to knowledge of the phenomena involved.

In most cases the analytical solutions, where available, represent the easiest approach. These may be aided by computer if series solutions are lengthy. For example, Gibson and Lo's method requires fairly involved series calculations to obtain degree of consolidation at each time and pore pressures at each node. It is relatively simple to program these, giving results very easily. In a few cases analytical solutions are so involved that the numerical method is preferable. A case to be examined later (section 2.7) is for a two layer deposit under Terzaghi theory. The full analytical solution has been obtained (Gray 1945), but is, in fact, too lengthy for publication. It would seem to be completely superceded by numerical techniques which are relatively easy to handle. It is possible to check the accuracy of numerical schemes, or set the limits of discretization to ensure that a given accuracy criterion is satisfied. Such solutions may thus be obtained to the same degree of accuracy as from analytical methods. For this reason the author avoids the term "approximate solutions" which is sometimes applied to these.

The numerical methods used for the present problems are presented in the relevant sections. Some general comments may be made by way of introduction.

<u>A Finite Difference Expression</u> is one in which the continuous derivatives are replaced by step functions applying over small increments of the independant variables. There is, in fact, a complete calculus of finite differences. For present purposes it is sufficient to note that this results in various expressions for each derivative. The appropriate expression depends upon the solution scheme to be used. Here we are interested in functions in time and space. The continuous functions are replaced by expressions for the variables at a number of discrete points. Thus a grid of points is introduced and expressions used relating the values of variables at any point to those at adjacent points, from which new values of the variables may

be determined. For consolidation problems, which progress in time, two approaches may be identified.

Explicit Method

The expressions for variables at the new time are written only in terms of known values. It is thus relatively easy to advance the solution for all space nodes at each step. Unfortunately, the step size is very limited. Beyond a certain critical value the finite difference expressions are not valid and this is clearly demonstrated by results becoming ludicrous. For the more commonly encountered explicit schemes the conditions for stability of the equations are well-defined.

Implicit Method

New values at any point are given in terms of both known values and other new values. Given boundary conditions, the resulting equations at each space node at the new time may be solved simultaneously. Techniques are available for solution of large sets of simultaneous equations (usually most easily handled in matrix form) but this involves a fair amount of work at each step. The advantage is that such equations are always stable, so that larger time steps may be used. There is still some restriction on the time step, however, due to truncation errors. These occur because the functions should really be continuous. Exact representation of this in difference form would require lengthy series expressions. The difference expressions normally used employ only the major terms of such series, so that the accuracy attained depends on the size of discretization in time and space.

{For full details of Finite Differences the interested reader is referred to a standard text such as Crandall 1956, or Smith 1965.}

The inter-relation of step size and accuracy makes it difficult to compare efficiencies of different schemes. The situation is further confused by individual quirks of programming and machine types, efficiencies of the different compilers etc. At an early stage in the present work some general comparisons of explicit and implicit schemes were made. Both Schiffman (1972) and Desai and Johnson (1972) have compared computer (c.p.u.) times for a number of schemes to solve the basic Terzaghi equation. Their results suggest that with both schemes using the same size of step of the stability limit for explicit, an implicit scheme (Crank-Nicholson Method, as used in the programs of Chapter 2) takes some 3 - 4 times as long to run as the explicit.

No comments were made on accuracy, however, so the author carried out a detailed examination of two schemes. Both Explicit and Implicit (Crank-Nicholson) were written as simply as possible for the standard Terzaghi case. The time step for the implicit scheme was varied from $\eta = 1$ to 10 (when η = ratio of time step to explicit stability time step). As expected, the implicit scheme became more efficient on computer time as η increased, equalling the explicit method at $\eta \doteqdot 3.5$. At this value accuracy was very similar for the two schemes, being within ½% for pore pressure near the free draining boundaries at early times (the most critical case), using 10 space nodes per layer. It was also clearly shown that increasing the number of space nodes increases the accuracy. For example, $\eta = 10$ gave acceptable accuracy (~ ½%) with 50 nodes, moderate (~ 1%) for 20 nodes and poor for 10 nodes (~ 3% at worst). It was found that 5 nodes led to poor accuracy even at low η values.

Later work has established beyond doubt that truncation errors decrease as consolidation progresses and the rate of change of the variables decreases. An implicit scheme should use a small initial time step, increasing as consolidation takes place. Further comments are made in section 2.12. The above studies established the preferability of the implicit scheme which will be developed in chapter 2.

Later work on Garlanger's theory was somewhat different. An explicit scheme presented with the theory proved quite time-consuming (especially as large total times investigating the secondary consolidation process are frequently required). The equations involved are fairly complex (indeed, analytical solution is not possible here even for the simplest conditions). A full implicit difference method would therefore involve a very large amount of work in each step. Something of this nature has proved possible through a technique termed the method of lines. Some improvement in efficiency was found, but not a great amount. However, the scheme takes advantage of a library program which evaluates the accuracy after each step, and if unsatisfactory recalculates the whole step with a smaller time increment. Accuracy may thus be guaranteed to a criterion selected for the problem under investigation.

In investigating Garlanger's equations the author discovered that implicit expressions for some derivatives, but explicit expressions for the more complex ones, led to a scheme which was implicit in behaviour. This proved to be remarkably efficient and after detailed investigation of the effect of time step on accuracy, this was shown capable of quite acceptable results. Details of the programs for Garlanger's method are given in chapter 4.

1.4 Computer Techniques

The underlying theme of Edinburgh's research work in soil mechanics during recent years has been an investigation of the possibilities of computer-aided-design in this field, and the development of new computer methods. The author has therefore been able to utilise the excellent facilities existing here, and to take advantage of some previous work. Some investigations of the use of interactive graphics have already been done, and similar work has proved useful in the present study.

Computer-aided-design may range from the rapid performance of tedious and relatively trivial pieces of "number-grinding" through to a very sophisticated interaction with a designer who may be enabled to explore possibilities which could previously have proved far too time-consuming. An example of relevance to this project is the solution of complex differential equations by iterative numerical techniques. Such techniques were in existence when Taylor (1942) examined consolidation behaviour. However, the manual use of these was virtually unthinkable, so that he only presented such equations as might reasonably be solved. In this sense computer-aided-design may open up new possibilities, solving equations which could otherwise not be considered.

Interactive programming is the art of writing a program so that at certain points the user may be presented with a choice, and the program will continue accordingly. Traditional programming has been for batch jobs, where all the information is read in, and the program then proceeds to its predestined conclusion. It is generally rather more difficult to allow the user choice of different aspects of analysis, and the ability to alter determinants while the program is running. As this is relatively new to the civil engineering field some specific comments are made in section 1.4.3. It should perhaps be noted that the choice of program type depends on the job to be done. It is frequently very convenient for a designer to be able to examine a range of possibilities, and tell the program what to do next. But it does, of course, involve him being there, waiting while the next stage is carried out. For some "straight-through" analyses it may be more convenient to send off a complete job to run in "background mode" while the user gets on with another job on his terminal.

Which brings us to the choice of terminal and method of communication with the computer. Some design offices will have a small in-house computer dedicated to one task at any particular time. Nowadays it is frequently easier to have a terminal with remote access to a large machine. This allows use of facilities only available on large machines, and it is frequently more economical since a number of users can keep the machine working close to its full capacity. The usual procedure in such cases is time sharing. The computer spends a very short period of time with each user - but this is done so rapidly that each user has the impression that the machine is dedicated to his problem. The work reported here was done on a time sharing system on an S.R.C. research machine, shared by schools of research in computer-aided-design and artificial intelligence. A remote terminal in the Civil Engineering Department was employed. The simplest form of terminal is the "teletype" sort, which types commands and machine replies. The ordinary numerical work reported here would have been possible on such a terminal, which is the cheapest sort available.

However, this department has been investigating the application of graphics to engineering problems, and a visual display unit has, in fact, been used for most of the current work. Details of computer hard-ware may be found in section 1.4.2 below.

1.4.1 Summary of Computer Programs

Some 10 programs have been developed during the present project, dealing with various aspects of soil consolidation. They are:

A) Extended Terzaghi Theory

CONED 1	Single Layer Soil	Constant Parameters
CONED 2	Multi Layer Soil	Constant Parameters
CONED 3	Single Layer Soil)	Parameters varying
	)	with
CONED 4	Multi Layer Soil)	Effective Stress

B) Garlanger Theory

GARCON	Garlanger's Explicit Finite Difference Scheme
SECON	Semi-Implicit Finite Difference Scheme
CONGO	Fully Implicit, Method of Lines

C) Gibson and Lo's Theory

GIBLO	Series summations for Analytical Solutions

D) Analysis of Consolidation Test Results

CALCO	Simple calculation of 'e' values etc.
CALCOG	As above, with Graphics

The programs for A and B are quite involved and the necessary theories are developed in chapters 2 and 4 respectively, together with details of the numerical methods used. The programs for D are quite straightforward, but proved enormously useful in aiding analysis of experimental data obtained in the present study, especially for obtaining the parameters required for Garlanger's theory. Brief comments are made in section 5.4.

The Gibson and Lo program C) is relatively trivial and simply calculates the terms in the presented solutions to the required degree of accuracy at any points requested.

The programs accept data from an INPUT file, and write results to an OUTPUT file. The computer monitor's file-handling facilities are quite extensive, including editing, line-printer listing, copying, and moving files around the system.

Programs are written to be CONVERSATIONAL, i.e. during running a variety of messages are sent to the user who selects from the options presented, and the programs continue having responded to these choices. All programs begin by requesting input and output file names, followed by a variety of options depending on the available types of analysis. All programs, except GIBLO and CALCO, offer graphs of results obtained. Graphical sections are best written as continuous loops so that the various graphs may be examined in any order or replotted to different scales, without the programs having to be re-run. The user selects from a MENU of possible graphs, which is redisplayed after plotting. The option STOP is included here for selection when all desired graphs have been viewed.

The user thus gets the impression he is dealing with a reasonably intelligent machine speaking in English, or rather, perhaps, well understood engineering terminology, (i.e. he has no need of a programming language). Full user manuals have been prepared to enable the programs to be run by any casual user with a knowledge of consolidation theory, and the basics of operating a time-sharing terminal. Details of handling input data; running programs and the options available; graphs; handling of results; together with brief notes on theoretical implications of the various approaches where necessary; have been included. A worked example of the CONED programs is also presented. The manuals are not included with this thesis, but are available at the Department of Civil Engineering, Edinburgh University, as are full listings of all the programs described.

Flow diagrams are given in Appendix A.

1.4.2 Hardware

The present programs have been developed on a DEC/PDP.10 computer system. This is a research machine set up at Edinburgh under the auspices of Science Research Council for a number of projects on Computer-Aided-Design and Artificial Intelligence. The system is, in fact, quite large with many facilities beyond the scope of the present work. The machine operates a time-sharing approach through which up to some 30 users may be serviced. The main memory for processing is core space. Most memory, however, is stored on magnetic disk and only moved into main memory for execution. The time spent in execution in the core is termed "central processor unit" (c.p.u.) time, and is a useful measure of the work done, and hence, for a commercial system, cost. Sufficient space was available on disk for storage of all the FORTRAN files developed in the present work, though compiled files tended to require a considerable amount of space so that only a few of these could be kept on disk at any one time. Further storage was available on magnetic tape, and completed programs were archived here together with some old data files in case they might be further needed.

Access to the system was via a Tektronix 4010-1 terminal. This is a visual display unit (v.d.u.) of the storage tube variety. It is similar to the conventional teletype, except that characters are now displayed on a cathode ray tube screen. The main advantage is that pictures may now be drawn using specially designed graphics software (see 1.4.4).

Coupled to this was a Tektronix 4610-1 Hard Copy Unit. This is able to scan any required display on the video screen and produce a permanent copy on a light-sensitive paper. Examples are included in the results of later chapters.

Brief mention should be made of refresh display units. A file is created fully defining the required picture. This is then continuously re-drawn (frequency some 20 times per second) on a display screen, rather in the manner of a television picture. The picture may thus be altered as desired, and may give the appearance of moving. Programming becomes rather complex, since considerable care is needed in "buffering" the information for display, to enable this to be changed without affecting the picture until modifications are complete and the altered information quite ready for display. Refresh display units are quite expensive and require considerable computer time. The author was able to do some work on such a system at Edinburgh Regional Computing Centre, using the IGLU package (see section 1.4.4). The display screen was linked to a smallish computer, the PDP.15, operating as a satellite of the "EMAS" time sharing system. Fascinating as this work is, it was felt to be too sophisticated for the problems under consideration here. Because of its cost and complexity, its use should be confined to problems which absolutely require rapid alteration of pictures. One interesting facility is the "light-pen" which can identify a position on the screen by pointing at it. With suitable back-up programming this enables pictures to be moved, distorted, or even drawn, which seems to have possibilities in some engineering design work.

1.4.3 Interactive Programming

It should first be noted that all the programs developed in the present project were written for 'Version 1A of the FORTRAN 10 compiler'. A number of slightly different versions of FORTRAN exist on different machines; the above being the most recent version implemented on the PDP.10 since January 1974. In practice no difficulty should be experienced using a FORTRAN IV or other FORTRAN compiler for the programs developed here, although compatibility should be checked for any specific case.

Two sets of library facilities have been used. A package for solution of large sets of simultaneous O.D.E.'s has been used in the method of lines solution for Garlanger's equations (section 4.4.3). Graphics work is handled by calls to a package of subroutines GINO-F (section 1.4.4). These must be loaded with the relevant programs before these latter can be run.

Interactive programming allows a number of modes of analysis of any given data depending upon options to be selected as the program is running. The user is presented with a number of options which he selects as required. The program displays a certain "intelligence" in performing the necessary steps following any choice given.

The programmer must therefore consider the options which may efficiently be offered. These are presented by writing messages on the screen - ** MESSAGE ** (as indicated in the flow diagrams). The option selected is read and then used as necessary to control analysis in the rest of the program. As the number of options increases, the programming logic rapidly becomes more complex, especially where a number of options all bear on a certain topic. For well-structured programs the analyses should be split into a number of 'modules', each performing a well-defined task. Each of these may then be started, as appropriate, by conditions when the module is or is not to be used. This helps to clarify progress through a program. It is also useful should any section need to be altered, since this may be done without affecting the rest of the program. As far as possible, flow through the modules should be continuous, simply skipping any not required. Jumping from place to place always leads to problems and should be minimised.

The flow diagrams given (Appendix A) present a general impression of the functions performed by the various modules, and the way in which these are linked. Detail diagrams have been included for a couple of cases where a fairly complex procedure is entailed to achieve a given task.

Every effort has been made to present a clear and consistent logic to the user. Messages have a standard format. All interactions here require the user to type a single number. The message lists number codes corresponding to the various choices available. Other sorts of interaction were investigated including display of movable cursors to line up with given choices. It was found confusing to move from one technique and set of terminal controls to another. Much may be learnt of an interactive program during use and in trying to write a manual for an inexperienced user. The schemes presented here represent considerable work on improving the initial methods and logic of interaction.

All sorts of facilities could be written into interactive programs, including various degrees of sophistication in error recovery. Providing data is correct and the right options are selected, there is no problem. Should errors be made, however, it is possible to diagnose some, and to recover from some. The present study is primarily interested in the analyses, rather than this aspect of interactive work. However, in developing the programs, a few common errors were encountered which could easily be handled. Of the "recoverable" kind is selection from the menu of a non-existent graph, i.e. typing a wrong key. The menu is simply re-displayed. Of the "diagnosis" type is erroneous data for parameters varying with effective stress (in CONED 3 and 4). If the calculated effective stress falls outside the range of data given, a message is displayed indicating whether too high, or too low and the layer number in question, where applicable. The program pauses for the user to consider. If no further data is available, and the nearest values must suffice, the program may recover, and continue automatically selecting the relevant value. Some advice on errors is included in the user manual.

Interactive programming thus offers a large number of additional facilities for analysis. The main problems which arise are:

(1) To decide which facilities are important and to implement these.

(2) To present a consistent logic and well-understood messages to the user.

(3) To structure the program in an orderly fashion with a minimum of jumping around.

Obviously all options must be checked for satisfactory operation. It is not necessary (or indeed practicable) to check every permutation of selections but each individual mode of analysis must be examined.

Particular thought and detailed examination of a program is required to see where one mode of analysis is influenced by another and certain combinations of options may have to be tested. For the present programs, especially the CONEDs, this testing proved quite lengthy. Considerable use of the programs since then has indicated their general value and ease of operation, besides confirming that they are functioning correctly.

1.4.4. Graphics

Work on computer graphics has advanced considerably in recent years. Software enabling graphics display is now available for the non-specialist user in several sets of library subroutines or "Graphics Packages". These normally consist of a number of external functions which may be incorporated in a program to perform basic processes, such as drawing a line between two points. More sophisticated processes include transforming picture axes, and plotting graphs. In FORTRAN, a high-level graphics language usually consists of a number of calls to subroutines which perform given tasks, and require a number of parameters to define the process. To take an example (from GINO-F, see below) CALL LINETO (65.,60.) draws a straight line from the present point on the screen to the point 65., 60. (The position on the screen is expressed in x and y co-ordinates. For the Tektronix 4010-1 used in the present work the screen is divided into a grid of 1040. by 1040.)

The standard text on this topic is Newman and Sproull (1971). Principles of Interactive Graphics, though it is intended primarily for the specialist in this field.

For the present work three graphics packages have been considered. Various relevant features of these are compared in Table 1.1.

i) IGLU (Interactive Graphics for Lots of Users). Developed by Edinburgh Regional Computing Centre for refresh display work, this most entertaining system had to be rejected as too sophisticated and therefore too costly for the applications intended here. Its use seems likely to be confined to the E.R.C.C. network.

ii) STURGEON. Specifically developed for computer-aided-design in the Architectural Research Unit, this package has proved useful for previous work here in the Civil Engineering Department. It is quite efficient, but seems likely to be confined to the PDP-10 research machine at Edinburgh.

iii) GINO-F. In 1974, around the beginning of the present project, this general purpose graphics system was first being made available. It was included on the Edinburgh machines in early 1975, and as it was desired to develop programs for the present work capable of use on other machines it seemed the ideal answer. Since its introduction GINO-F has been greeted with enthusiasm throughout the U.K. and rapidly spread amongst the major computer installations. There can be little doubt that it is presently the most satisfactory package for work of the type developed here. Despite its general purpose nature it proved fairly easy to use, helped by good documentation (User Manual, CADC 1974).

Table 1.1 A comparison of Three Graphics Packages

Package	IGLU	STURGEON	GINO-F
Originator	Edinburgh Regional Computing Centre	Edinburgh University Architectural Research Unit	Computer-Aided-Design Centre, Cambridge.
Availability	Edinburgh - EMAS	Edinburgh - PDP.10	Generally, on many larger machines throughout U.K.
Display System	Refresh	Storage	Refresh or storage
Brief Description	Pictures defined individually and stored in Sub-Picture Table. Displayed by entering into Initialisation Blocks, which also govern position on screen, scale, brightness etc.	Pictures defined in a display file which is called as required to draw its contents on screen.	Subroutines called which draw directly on the screen. (Facilities available for segmenting or displaying separate pictures in refresh display.)
Recall/Picture Modification	Yes, by changing picture definition, and re-entering, or merely modifying initialization blocks.	Picture may be added to, but erasing or modification requires alteration of display file and re-drawing.	Picture may be added to. Otherwise, no.
Drawing facilities	Includes 3-dimensional drawing and transformations of axes.	Limited to basic drawing-lines.	Very comprehensive. 3-dimensional drawing etc. Includes facilities for graphs.
Interaction	Type-keys, Cursor, Light-Pen.	Type-keys, Cursor.	Type-keys, Cursor (and light-pen for refresh display).

1.5 Experimental Work

Theoretical and numerical considerations have raised a number of questions about real soil behaviour. Reported observations from both field and laboratory tend to fit with Bjerrum's model, although such investigations have not been directed at this framework. An experimental programme has therefore been carried out to examine delayed consolidation, and Garlanger's analysis of it, in some detail.

The most important consequence of Bjerrum's model is that, due to delayed consolidation, an 'aged' normally consolidated soil deposit will behave as if slightly over-consolidated, i.e. the compressibility is initially low, but changes at some higher stress to that usually associated with normal consolidation. For analysis purposes Garlanger has taken two linear log e - log p relationships, of gradients 'a' and 'b', below and above some critical stress level p_c, respectively. Such linearizations may over-simplify the real stress-strain behaviour. This point is taken up in Chapter 6.

Besides Bjerrum and his colleagues at Norwegian Geotechnical Institute (see e.g. Foss 1969, Bjerrum 1972 a, Engesgaar 1973, and, of particular importance to the whole problem of behaviour of soft clays, Bjerrum 1973), a number of workers have reported some form of increased strength which is believed to be a direct result of delayed consolidation. Leonards and Altschaeffl (1964) reported what they termed a "quasi-preconsolidation pressure" after secondary consolidation of some artificially re-sedimented clays. Below this pressure, strain was small and rapid; Cv values being some 20 to 40 times those above it. Barden (1969) noted that secondary consolidation will increase soil strength and thus alter subsequent consolidation behaviour, and some examples of this effect are given, showing the rapid initial dissipation of pore pressures resulting (see section 6.3).

There are also a number of reports of field deposits which according to stress history are normally consolidated, but which display some over-consolidation. Several of the authors have ascribed such behaviour to previous delayed consolidation. Some such cases are to be found in the contributions to the British Geotechnical Society Conference on Settlement of Structures - Session II. Simons (1974) in the general report on 'Normally Consolidated and Lightly Over-Consolidated Cohesive Materials' discusses delayed consolidation and critical pressure effects in some detail. Particular attention is paid to Bjerrum's work and Garlanger's analysis method is commended.

One paper from this conference is of particular interest, since it refers to the same soil as used for the present experimental work. Jarrett et al. (1974) identified a critical stress in the Grangemouth Silty-Clay, from records of settlements of a number of structures applying a variety of loads. This deposit has almost certainly never been subjected to greater overburden in its history, so the critical stress may reasonably be ascribed to delayed consolidation. (Other possibilities are leaching and chemical change, over a period of time, which may, in fact, be associated with delayed consolidation. The author has discussed this with geologists from Edinburgh University working on the Grangemouth area, and the consensus of opinion is that conditions here have not changed significantly since the start of formation of this deposit, shortly after the last glaciation, some 9 - 10,000 years ago.)

There is, then, a considerable amount of qualitative evidence from both laboratory and field, for delayed consolidation leading to a critical stress and slight over-consolidation effect. Quantitative data, however, is at present limited to some work at N.G.I. on a number of (mostly Norwegian) soft deposits for which the stress history is known, and the present degree of over-consolidation has been related to the delayed consolidation over the time since formation, or at least first attainment of current stress levels, of the deposits (see Berre and Bjerrum, 1973). The present project aims to provide some quantitative laboratory data on the influence of delayed consolidation.

Clearly there is scope for much more work on field behaviour. A number of problems arise here, however. Assuming a suitable building on a suitable deposit can be monitored, records of consolidation over a long period of time are required. For reasons which will be seen in Chapters 5 and 6, pore pressure data is important in the present study and this has generally only become available over recent years, and for major works. Equally important is accurate information on the soil, the applied loadings, and any factors of possible influence. The author has some detailed records of settlements of a number of structures in the Grangemouth area, over some 25 years. Unfortunately, this line of investigation has not proved very productive, due to the absence of further vital information on distributions of loading, and a number of influential factors which complicate the settlement curves. Nor can the influence of 2 and 3 dimensional field behaviour be adequately assessed.

For the present study experimental investigation has been confined to laboratory work under controllable conditions. This included the need to compare results from many samples which were initially as nearly identical as possible. Hence remoulded soil has been used throughout, although it may be noted that the influence of the delayed consolidation under examination is believed to account for some of the observed differences between "undisturbed" and remoulded soils. Much work remains to be done on undisturbed samples. Probably the most serious problem outstanding here is how to maintain the sample at field stresses during sampling, removal from the ground, and until fully prepared for testing. It seems likely that change in stress conditions will affect, if not totally destroy, the over-consolidation effect of interest here.

Before embarking on laboratory work, Garlanger's method was applied to some existing data for 3 remoulded soils (Christie, 1963). This gave surprisingly good agreements, tending to encourage further investigation of this model. A number of problems were encountered in obtaining suitable parameters and points raised here were borne in mind in the design of the subsequent experimental programme. Details of this analysis of existing data may be found in section 4.6.

One of the soils used for this initial examination, Grangemouth Silty-Clay, was chosen for the present experiments since it displayed significant delayed consolidation; was of considerable local interest; and was conveniently located for samples to be obtained. This experimental work consisted of a large number of consolidation tests with pore pressure measurements, conducted after 3 standard periods of delayed consolidation - 1 day, 1 week, and 1 month. The experiments were planned on a "factorial analysis" system, and sufficient tests carried out to enable the significance of the variables, and the random errors to be assessed. The influence of delayed consolidation on subsequent behaviour has thus been clearly established for this soil. A further objective of the work was to clarify the techniques for obtaining parameters for use in Garlanger's analysis method. Working from this, comparison of results with Garlanger's theory is considered in some detail. The results are also considered vis-a-vis Terzaghi theory, and the capabilities and limitations of this are discussed. Details of the experimental procedures, results and analyses, will be found in Chapter 5.

CHAPTER 2

Terzaghi Theory - Numerical Extensions

2.1 Introduction

The case has been presented (section 1.2 above) for extending the range of analysis of Terzaghi Theory somewhat using numerical techniques. Some discussion of use of this approach in engineering problems will be found in Chapter 6, viewed in the light of various aspects of soil behaviour examined in this thesis.

The model of soil behaviour proposed by Terzaghi has been extended to include the following conditions.

a) Single or Multi-layered soil profile.

b) Parameters Cv and mv made constant, or varying as functions of effective stress.

c) Loading Instantaneous or Time Dependant.

d) Drainage from Top, Bottom, or Both boundaries.

e) Any distribution of initial excess pore pressures.

It was envisaged that the program(s) would be written to allow the user to interact with the computer and select the modes of analysis required, during running. It became clear that problems with single layer only, or with constant parameters Cv and mv, could be programmed to run far more efficiently than possible as options in the full analysis. Hence, 4 programs have been developed.

CONED 1	Single Layer	Constant Parameters
CONED 2	Multi Layer	Constant Parameters
CONED 3	Single Layer	Varying Parameters
CONED 4	Multi Layer	Varying Parameters

The relevant program is easily selected and all have options c), d) and e) available. One further facility, of particular significance to later experimental and theoretical work, is the inclusion of a method of dealing with a critical stress, p_c. This may be handled by the programs 3 and 4, with suitable variations of parameters Cv and mv. A simple approach, available on all the programs, is to allow the parameters to take constant, but different, values depending whether or not p_c is exceeded. A stress history algorithm automatically keeps track of maximum effective stresses reached and selects rebound parameters should stresses fall.

In addition, several options are available to control the output.

f) Settlement or Pore Pressure information may be suppressed if not required (saving computer running time).

g) Results may be output Numerically, Graphically or Both.

h) Numerical results are given at each "output time" as selected by the user in the data file. Pore pressures are given at each node. The integrated total settlement at such times is also shown.

i) Where parameters vary with effective stress, additional results are available, as an option, showing average effective stresses in layers, and values of Cv and mv used, as interpolated by subroutine VARIED, from the data given.

j) Details of the numerical scheme are also available showing number of iterations and time steps used.

k) Graphs are available of:-

i) Pore Pressure profiles with depth at the specified output times.

ii) Settlement against Linear Time.

iii) Settlement against Logarithmic Time.

The user may specify maximum ordinates for these graphs as desired (he is advised of the maximum values attained).

The various sections of this chapter present the development of the methods used in the programs. The topics of interactive programming and graphics work are not dealt with here: some general comments were made in section 1.4. Considerable thought has been given, however, to ease of operation by an inexperienced user. A standard format is used for presentation of options, results etc. The programs display a little "intelligence" in diagnosing, and, where possible, being able to recover from, the more obvious errors. A full user manual has been prepared (though not included here) indicating how to set up data files and run the programs. Some knowledge of the theory and of simple computer terminal operation must be assumed, but the programs should be accessible to anyone with this basic information. A worked example, and details of the various modes of analysis are included.

The number of different options available has made it impossible to fully check every permutation possible through each program. However, every option has been checked for each of its possible settings. A number of specific checks on particular combinations of options have also been carried out. The programs have proved satisfactory when tested against published results (some of which are mentioned in Chapter 3) and in the course of routine use, including application to some experimental results to be considered in Chapter 5 below. The author is satisfied they are free of programming errors. Numerical accuracy is also believed to be satisfactory (within ½% accuracy in pore pressures at their most critical). This is discussed in section 2.12. There are some problems, however, concerning the model of soil behaviour used when Cv ceases to be constant and these are discussed in sections 2.9 and 2.11.

A flow diagram for CONED 4 is given in Appendix A. (The other 3 programs are simpler cases of this.)

2.2 Finite Difference Methods

The basis of finite difference techniques has already been mentioned in section 1.3, when it was noted that there are two basic types of schemes, namely, Explicit and Implicit. It was also noted that the latter scheme is more efficient for cases of the type considered here, and consequently this will be used in the programs to be developed below. For the sake of completeness, however, the equations of the Explicit method are first given.

Explicit Method

The derivatives are replaced by the following finite difference expressions

$$\frac{\partial u}{\partial t} = (u_i^+ - u_i)/\Delta t \qquad (2.1)$$

$$\frac{\partial^2 u}{\partial z^2} = (u_{i+1} - 2u_i + u_{i-1})/\Delta z^2 \qquad (2.2)$$

{where + indicates the value at the new time.}

{Fig. 2.1a shows this difference grid.}

Terzaghi's equation $$Cv\,\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \qquad (2.3)$$

becomes, after re-arranging,

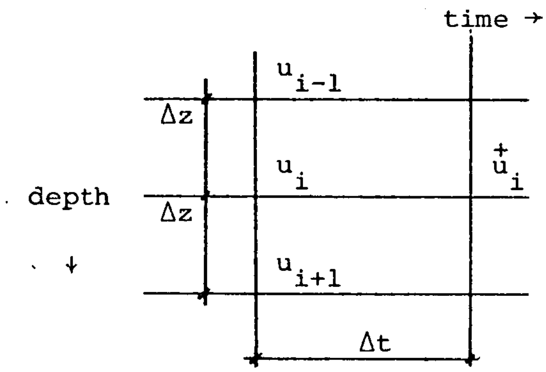
$$u_i^+ = u_i + \beta^1\{u_{i+1} - 2u_i + u_{i-1}\} \qquad (2.4)$$

where $\beta^1 = \frac{Cv\Delta t}{(\Delta z)^2}$

Finite Difference Grids in One Dimensional Consolidation

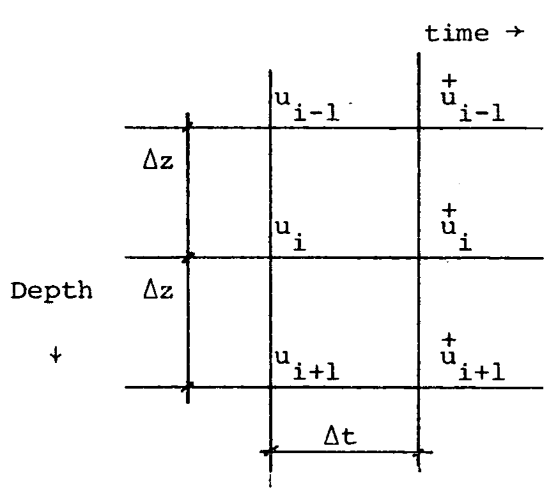
i) Explicit Method

Fig.2.1a



ii) Implicit Method

Fig.2.1b



Clearly the new pore pressure at any point may be determined directly from the old values at that and adjacent points.

The serious drawback is that the equations become unstable if $\beta^1 > \frac{1}{2}$, and this frequently imposes an unreasonably small limit on the time step. This is not to deny the attractiveness of the method for many problems, however. It may be very quickly and easily programmed, which is particularly advantageous for one-off jobs. Since it is reasonably well-known nowadays in the Soil Mechanics field, further discussion here is superfluous.

Implicit Method

The following finite difference expressions are used:-

$$\frac{\partial u}{\partial t} = (u_i^+ - u_i)/\Delta t \qquad (2.5)$$

and

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{(\Delta z)^2} \{\theta (u_{i+1}^+ - 2u_i^+ + u_{i-1}^+) + (1-\theta)(u_{i+1} - 2u_i + u_{i-1})\} \qquad (2.6)$$

where $\theta \geq 0.5$

{Notice that $\theta = 0$ gives the special case of eqn. (2.2); the explicit scheme.}

The most popular scheme, since it is believed to be the most efficient, is that due to Crank and Nicholson (1947), who use the midpoint, or central, differences at $\theta = 0.5$. The grid is shown in Fig. 2.1b.

After re-arrangment, Terzaghi's equation becomes

$$-\beta^1 u_{i-1}^+ + 2(1+\beta^1) u_i^+ - \beta^1 u_{i+1}^+ = \beta^1 u_{i-1} + 2(1-\beta^1) u_i + \beta^1 u_{i+1} \qquad (2.7)$$

$$\text{where } \beta^1 = \frac{Cv\Delta t}{(\Delta z)^2}$$

The equations must now be solved simultaneously to obtain new values, since each equation is in terms of other (unknown) new values as well as the old. This may be done provided initial and boundary conditions are specified.

The authors of the scheme (op.cit) have shown it to be stable for all β^1. Truncation errors, however, become of significance, and the time step must be selected to maintain acceptable accuracy. This is examined in section 2.12, below.

2.3 The Basic Equations in One Dimension

Applying the Crank-Nicholson Implicit scheme, then, the basic equation (2.7) may be written in matrix format -

$$[-\beta^1 \quad 2(1+\beta^1) \quad -\beta^1] \begin{Bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{Bmatrix}^+ = [\ \beta^1 \quad 2(1-\beta^1) \quad \beta^1] \begin{Bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{Bmatrix} \qquad (2.8)$$

This may be simplified by dividing through by β^1, and following the notation used by Toan (1973) we write

$$[-1 \quad C_L \quad -1] \begin{Bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{Bmatrix}^+ = [+1 \quad C_R \quad +1] \begin{Bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{Bmatrix} \qquad (2.9)$$

$$[B] \qquad \{U\}^+ \quad = \quad [\bar{D}] \qquad \{U\} \qquad (2.10)$$

where $C_L = 2(1 + \beta^1)/\beta^1$

$C_R = 2(1 - \beta^1)/\beta^1$

The final matrices are shown in Fig. (2.2) for all the space nodes. Boundary conditions are discussed below. It should be noted

Fig. 2.2 Matrices for Single Layer Soil

(Free drainage at top and bottom)

$$\begin{bmatrix} 0 & C_L & -1 \\ -1 & C_L & -1 \\ -1 & C_L & -1 \\ & \text{etc.} & \\ -1 & C_L & 0 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \\ \\ u_{n-1} \end{Bmatrix}^{+} = \begin{bmatrix} 0 & C_R & 1 \\ 1 & C_R & 1 \\ 1 & C_R & 1 \\ & \text{etc.} & \\ 1 & C_R & 0 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \\ \\ u_{n-1} \end{Bmatrix}$$

For an impermeable boundary, e.g. at bottom

$$\begin{bmatrix} -1 & C_L & -1 \\ -2 & C_L & 0 \end{bmatrix} \begin{Bmatrix} u_{n-1} \\ u_n \end{Bmatrix} \qquad \begin{bmatrix} 1 & C_R & 1 \\ 2 & C_R & 0 \end{bmatrix} \begin{Bmatrix} u_{n-1} \\ u_n \end{Bmatrix}$$

where

$$C_L = 2(1+\beta^1)/\beta^1$$

$$C_R = 2(1-\beta^1)/\beta^1$$

that [B] and [$\bar{D}$] are really square matrices of order $n \times n$. Only the diagonal and immediately adjacent positions have non-zero values, however, (termed tridiagonal matrix), so this is most efficiently handled as a matrix of order $3 \times n$. Considerable computer space is saved this way, and there is no loss of clarity provided it is kept in mind that the middle term of each row corresponds to the vector on that same row.

The right hand side of the equation in Fig. (2.2) contains only known values at the current point in time. [$\bar{D}$] and {U} may therefore be multiplied out to give a single vector {D}. This matrix equation could be solved by a number of techniques. The preferred method here is that of Gaussian Elimination. Subtracting a multiple of the previous row from the second and subsequent rows will eliminate one term of each row (for consistency, the first term each time). This leaves a set of equations in terms of u_i^+ and u_{i+1}^+ at each node. Since u_{i+1}^+ is known at the point adjacent to the lower boundary, all the values may now be determined by back-substitution, working systematically back up the matrix.

2.4 Boundary Conditions

Each boundary may be either free draining or impermeable. Mathematically, only 2 cases need be considered - one or both boundaries free draining. For convenience the scheme here allows the single free drain to be at either top or bottom, which more easily handles multi-layer soil profiles and asymmetrical initial pore pressure distribution (see below).

For a free draining boundary at the top

$$u_1 = 0 \qquad (2.11)$$

Hence, from equation 2.8, above, we write

$$[0 \quad c_L \quad -1] \begin{Bmatrix} - \\ u_2 \\ u_3 \end{Bmatrix}^+ = [0 \quad c_R \quad 1] \begin{Bmatrix} - \\ u_2 \\ u_3 \end{Bmatrix} \qquad (2.12)$$

A similar expression applies for a free draining lower boundary.

For an impermeable boundary, at the bottom, the standard technique is to note there can be no pressure gradient at such a point

$$\left(\frac{\partial u}{\partial z}\right)_n = 0 \qquad (2.13)$$

It then follows that the pore pressure at some imaginary point a short distance below n, must equal that an equal short distance above

$$u_{n+1} = u_{n-1} \qquad (2.14)$$

Equation (2.9) at node n may therefore be re-written omitting the imaginary point by use of (2.14)

$$[-2 \quad c_L \quad 0] \begin{Bmatrix} u_{n-1} \\ u_n \\ - \end{Bmatrix}^+ = [+2 \quad c_R \quad 0] \begin{Bmatrix} u_{n-1} \\ u_n \\ - \end{Bmatrix} (2.15)$$

Hence the free drainage boundary does not require an equation at that node, and the adjacent node is amended. For the impermeable boundary the equation at that node is amended. This means that the top and bottom of the matrix will depend on drainage conditions, to be specified by the user. The numbering of the points for the program had therefore to be treated as a variable which proved a little tedious.

2.5 Initial Conditions

Numerically, the computation may be started by any values of initial pore pressures, provided one or both boundaries are set to zero (i.e. drainage is permitted) to cause consolidation to occur. The values will clearly be chosen to represent actual conditions as closely as possible.

Case 1 : Initial Pore Pressures are constant with depth. They may either be analysed dimensionally, by giving the value as data, or non-dimensionally, calling the initial value 100% from which conversion to real units is relatively straightforward if desired.

Case 2 : Initial Pore Pressures varying with depth. The layer(s) of thickness Z, have been split into NZ elements (specified by the user) of thickness DZ each. The position of each node (numbered from the top, at the beginning of each element) is therefore known, so an array of pore pressure values is input corresponding to each of these nodes. Again, the values may be real or dimensionless.

It should be noted that if 1 dimensional theory is applicable, then Case 1 above should apply, except when analysis is begun in the middle of a consolidation process.

The programs automatically accept the user's choice of free drainage boundaries, and set u = 0 there for all times. Some confusion exists, however, as to when this should be done. In the continuous function u = 100% at t = 0

$$u = 0\% \quad \text{at } t = \lim_{\delta t \to 0} \delta t$$

and this is quite satisfactory.

It appears that when δt is finite an error is introduced by taking u = 100% at t = 0, and Toan (1973) compared solutions using a variety of u_1 values, with those for Fourier Series solutions.

He concluded an initial value between 33 - 50% gave best agreement. The author must take issue with this approach since (apart from it being unclear how exactly this conclusion is drawn from the data he presents) it seems clear that results must depend on size of space and time discretizations, and in the limit as δt becomes infinitesimal, u = 100% initially is satisfactory.

The present work, therefore, sets u to zero at t = δt and thereafter. The initial time step is made sufficiently small for this error to be negligible. Toan's work clearly shows (and the author has confirmed) that these errors rapidly disappear with time in any case. A small initial time step was necessary to keep truncation errors small (see 2.12) and the change of u, discussed here, did not impose any further restriction on this.

2.6 Time Dependant Loading

The analysis presented so far refers to the case of instantaneous loading, i.e. pore pressures are initially at some excess value over and above equilibrium conditions, and the ensuing consolidation is examined. It is reasonably straightforward to modify the analysis to the case of time dependant loading. Terzaghi's equation is modified to

$$Cv \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} - \frac{\partial q}{\partial t} \qquad (2.16)$$

The derivative $\frac{\partial q}{\partial t}$ is usually termed the rate of loading, but it includes the assumption (fundamental to Terzaghi theory) that any applied total stress is translated immediately into pore water pressure.

Consider the rate of loading to be linear over a small increment of time. Expressed in finite difference terms we have

$$\frac{\partial q}{\partial t} = \frac{q^{+} - q}{\Delta t} \qquad (2.17)$$

This term may be added into the finite difference scheme without complication since q may be determined explicitly at all times.

The scheme adopted here is for the user to input an array of "loading times" t_q, and an array of corresponding loads. The program assumes loading is linear between any two such points. Hence $\frac{\partial q}{\partial t}$ may be determined. The increment in stress is thus

$$\Delta q = \frac{\partial q}{\partial t} \cdot \Delta t \qquad (2.18)$$

over any time step Δt. This is added directly to the values contained in the {D} vector (see eqn. 2.10 above), bearing in mind that the original finite difference equations have been simplified by multiplying through by $\frac{2}{\beta^1}$. Hence the algebra leads to

$$\{D^1\} = \{D\} + \frac{2}{\beta^1} \times \Delta q \qquad (2.19)$$

Now any point at which the rate of loading changes must coincide with a time of calculation, otherwise errors result. Since such times will normally be of interest to the user, the program scans the array of loading times and adds to the output array any previously not given. Hence the user need not specify such times twice, any loading times will automatically be output.

Any initial loading is handled by setting the initial pore pressures accordingly. Hence time dependant loading always begins with q = O at t = O, so this point is specified by the program, and is not required in the data. It is quite possible for initial pore pressures and/or time dependant loading to be negative. This is examined in detail in Section 2.10.

2.7 Analysis of Multi-layer Soil Profile

Many soil profiles may best be analysed as a number of contiguous layers having different properties. Before discussing the numerical techniques for such analysis, existing solutions are considered.

Gray (1945) has presented the exact analytical solutions for the 2-layer case with both single and double drainage. More recently Schiffman and Stein (1970) have produced an analytical solution to the general layered consolidation problem. The solution to Gray's case is apparently too lengthy to be included in his paper; only final results are quoted, and the same is true of the general solution which must be even more daunting. It appears that in this latter case substantial computer work was entailed in evaluating the series solutions. Neither analytical solution is generally available and application in practice has not been reported.

Several approximate methods have been proposed. The first, by Terzaghi (1940) suggests averaging Cv values with weighting by thicknesses of the various layers. This simple technique takes no account of the positions of the various layers in relation to the permeable boundary. Glick (1945) in discussion of Gray's paper, suggested transforming the thickness of the 2nd layer to give the same Cv value in each. This is valid if

$$\frac{k_{\ell}\ mv_{\ell}}{k_{\ell+1}\ mv_{\ell+1}} = 1$$

and may be used for any number of layers where this is the case. Outside this case the scheme has no real value since errors can be large, and may only really be assessed by comparison with correct solutions. In the same discussion Barber (1945) suggested a scheme of what he termed "load transfer" to allow consolidation in each layer to be adjusted to correspond with final settlements in each, and Razouki (1974) suggests the best approximate method is a combination of these in the Glick-Barber Scheme - Transformation with load transfer. Even so, such a method is fairly tedious to employ with doubtful accuracy which cannot be assessed. The present author believes such schemes are rendered obsolete by the availability of cheap easily-used numerical techniques.

The first numerical scheme for analysis of a multi-layered profile was presented by Abbott (1960) and the example he presented has been re-examined in Section 3.5 below. Further such schemes have been reported by Jordan and Schiffman (1967), Schiffman and Stein (1969), Murray (1972) and Toan (1973), {the first 2 of these using the less efficient explicit approach}. Several features of the 2 last mentioned have been employed in the current programs.

Mention should also be made of an approximate technique for analysis of 2-layer profiles presented by Lee (1974). Since this has been used to examine some results of the programs, it is considered in some detail in Chapter 3.6.

Extension of the present numerical scheme to multi-layer analysis is based on consideration of behaviour at a layer interface. Consider a node i on the interface. We first assume this is a point within the top layer, ℓ, by extending this layer to an imaginary further node $i + 1$. We thus write the finite different approximation to Terzaghi's equation at node i (eqn.2. 9) including the term for time dependent loading (eqn. 2.19).

$$-\underline{u}^{+\ell}_{i-1} + C^{\ell}_{L}\, u^{+\ell}_{i} \quad \underline{-u^{+\ell}_{i+1}} = u^{\ell}_{i-1} + C^{\ell}_{R} u^{\ell}_{i} + \underline{u^{\ell}_{i+1}} + \frac{2}{\beta^{1\ell}}\,\Delta q \quad (2.20a)$$

Now, considering i to be a part of layer $\ell + 1$, with imaginary node i - 1

$$\underline{-u^{+\ell+1}_{i-1}} + C^{\ell+1}_{L}\, u^{+\ell+1}_{i} - u^{+\ell+1}_{i+1} = \underline{u^{\ell+1}_{i-1}} + C^{\ell+1}_{R}\, u^{\ell+1}_{i} + u^{\ell+1}_{i+1} + \frac{2}{\beta^{1\,\ell+1}}\,\Delta q \quad (2.20b)$$

where the imaginary terms are underlined.

Now it is also necessary to include the condition of continuity of flow at the interface.

$$k_{\ell}\left(\frac{\partial u}{\partial z}\right)_{\ell} = k_{\ell+1}\left(\frac{\partial u}{\partial z}\right)_{\ell+1}$$

This may be written in finite difference form at time t

$$\frac{k^{\ell}\,(\underline{u}^{\ell}_{i+1} - u^{\ell}_{i-1})}{2\;\Delta z^{\ell}} = \frac{k^{\ell+1}\,(u^{\ell+1}_{i+1} - \underline{u}^{\ell+1}_{i-1})}{2\;\Delta z^{\ell+1}} \qquad (2.21a)$$

and at $t + \delta t$

$$\frac{k^{\ell}\,(\underline{u}^{+\ell}_{i+1} - u^{+\ell}_{i-1})}{2\;\Delta z^{\ell}} = \frac{k^{\ell+1}\,(u^{+\ell+1}_{i+1} - \underline{u}^{+\ell+1}_{i-1})}{2\Delta z^{\ell+1}} \qquad (2.21b)$$

Combining .21a and b, we obtain

$$(u^{+\ell}_{i+1} + u^{\ell}_{i+1}) = K^{\ell}\,\{(u^{+\ell+1}_{i+1} + u^{\ell+1}_{i+1}) - (u^{+\ell+1}_{i-1} + u^{\ell+1}_{i-1})\} + (u^{+\ell}_{i-1} + u^{\ell}_{i-1})$$

$$\text{where } K^{\ell} = \frac{\{k^{\ell+1}}{\{k^{\ell}} \cdot \frac{\Delta z^{\ell}\;\}}{\Delta z^{\ell+1}\;\}} \qquad (2.22)$$

thus eliminating two of the imaginary points. The remaining two may now be eliminated between equations (2.20) and (2.22) resulting in the equation, when simplified and dropping the ℓ notation for u values,

$$-\overset{+}{u}_{i-1} + \Gamma_{L}\overset{+}{u}_{i} - K^{\ell+}u_{i+1} = u_{i-1} + \Gamma_{R} u_{i} + K^{\ell} u_{i+1} + \left(\frac{1}{\beta^{1\ell}} + \frac{K^{\ell}}{\beta^{1\,\ell+1}}\right)\Delta q$$

(2.23)

Fig. 2.3 Matrices for Multi-Layered Soil Profile

(Free drainage top and bottom)

$$
\underset{[B]}{\begin{bmatrix}
0 & C_L^1 & -1 \\
-1 & C_L^1 & -1 \\
 & \text{etc.} & \\
-1 & C_L^1 & -1 \\
-1 & \Gamma_L^1 & -K^1 \\
-1 & C_L^2 & -1 \\
 & \text{etc.} & \\
-1 & C_L^2 & -1 \\
-1 & \Gamma_L^2 & -K^2 \\
 & \text{etc.} & \\
-1 & C_L^L & -1 \\
 & \text{etc.} & \\
-1 & \Gamma_L^\ell & -K^\ell \\
 & \text{etc.} & \\
-1 & C_L^{N\ell} & -1 \\
-1 & C_L^{N\ell} & 0
\end{bmatrix}}
\underset{\{U\}^+}{\begin{Bmatrix}
u_2 \\ 3 \\ \\ n_1-1 \\ n_1 \\ n_1+1 \\ \\ \sum_1^2 n-1 \\ \sum_1^2 n \\ \\ \sum_1^2 n+4 \\ \\ \sum_1^\ell n \\ \\ \text{max}-2 \\ \text{max}-1
\end{Bmatrix}}
=
\underset{[D]}{\begin{bmatrix}
0 & C_R^1 & 1 \\
1 & C_R^1 & 1 \\
 & \text{etc.} & \\
1 & C_R^1 & 1 \\
1 & \Gamma_R^1 & K^1 \\
1 & C_R^2 & 1 \\
 & \text{etc.} & \\
1 & C_R^2 & 1 \\
1 & \Gamma_R^2 & K^2 \\
 & \text{etc.} & \\
1 & C_R^L & 1 \\
 & \text{etc.} & \\
1 & \Gamma_R^\ell & K^\ell \\
 & \text{etc.} & \\
1 & C_R^{N\ell} & 1 \\
1 & C_R^{N\ell} & 0
\end{bmatrix}}
\underset{\{U\}}{\begin{Bmatrix}
u_2 \\ 3 \\ \\ n_1-1 \\ n_1 \\ n_1+1 \\ \\ \sum_1^2 n-1 \\ \sum_1^2 n \\ \\ \sum_1^2 n+4 \\ \\ \sum_1^\ell n \\ \\ \text{max}-2 \\ \text{max}-1
\end{Bmatrix}}
$$

Layer 1 (rows u_2 to n_1)

Layer 2 (rows n_1 to $\sum_1^2 n$)

Layer ℓ (rows $\sum_1^2 n$ to $\sum_1^\ell n$)

Layer $N\ell$ (rows $\sum_1^\ell n$ to max−1)

For an impermeable boundary, e.g. at bottom

$$
\begin{bmatrix}
-1 & C_L^{N\ell} & -1 \\
-2 & C_L^{N\ell} & 0
\end{bmatrix}
\begin{Bmatrix}
\text{max}-1 \\ \text{max}
\end{Bmatrix}
\quad
\begin{bmatrix}
1 & C_R^{N\ell} & 1 \\
2 & C_R^{N\ell} & 0
\end{bmatrix}
\begin{Bmatrix}
\text{max}-1 \\ \text{max}
\end{Bmatrix}
$$

$$
\text{Where } K_L^\ell = \frac{Cv^{\ell+1} mv^{\ell+1}}{Cv^\ell \; mv^\ell} \cdot \frac{\Delta z^\ell}{\Delta z^{\ell+1}}
$$

$$
\Gamma_L = \tfrac{1}{2}\{C_L^\ell + K^\ell C_L^{\ell+1}\} \qquad \Gamma_R = \tfrac{1}{2}\{C_R^\ell + K^\ell C_R^{\ell+1}\}
$$

where $\Gamma_L = \frac{1}{2} \{C_L^{\ell} + K^{\ell} C_L^{\ell+1}\}$ (2.23a)

$\Gamma_R = \frac{1}{2} \{C_R^{\ell} + K^{\ell} C_R^{\ell+1}\}$ (2.23b)

Incorporation of this analysis into the CONED programs presented no major problems, although considerable care was required in the numbering systems required to position the various layers and corresponding parameters.

A set of the basic data is now required for each layer so the data file is somewhat lengthier.

It may be recalled that k is not included in the basic data since it may be determined from Cv and mv in the Terzaghi case

$$k = Cv\ mv\ \gamma_w \qquad (2.24)$$

Hence the present programs determine K^{ℓ} in equation 2.22 from

$$K^{\ell} = \frac{Cv^{\ell+1} mv^{\ell+1}}{Cv^{\ell}\ mv^{\ell}} \times \frac{\Delta z^{\ell}}{\Delta z^{\ell+1}} \qquad (2.25)$$

The final matrix equations, then, are as shown in Fig. 2.3.

2.8 Settlement Analysis

The present analysis applies, of course, only to settlements resulting from primary consolidation. The total settlement in any layer is simply the integration of the strains over that layer. For the present Extended Terzaghi theory simple linear stress-strain relations are used, though these may change for different ranges of effective stress to approximate non-linear relationships.

Now the settlement at any time t after the start of consolidation may be written for the Terzaghi case of mv constant as

$$S_t = \int_0^H mv\ .\Delta p\ .\ dz \qquad (2.27)$$

where the coefficient of compressibility $mv = \frac{1}{1+e_o} \cdot \frac{\Delta e}{\Delta p}$, and Δp is the change in effective stress at any point in the layer over time t.

Since mv may change as consolidation progresses, this should here be written incrementally over each time step

$$dS = \int_o^H (mv_p \quad \delta p).\, dz \qquad (2.28)$$

where δp is the change in effective stress over a finite time step of δt, and mv_p is the relevant value of this coefficient determined as a function of p.

The integration is performed numerically by application of the trapezoidal rule formula

$$dS = \frac{\Delta Z}{2} \{mv_1 \quad \delta p_1 + 2 \sum_{i=2}^{NZ-1} mv_i\, \delta p_i + mv_{NZ}\, \delta p_{NZ}\} \qquad (2.29)$$

where mv_i at each node is determined by reference to input data defining variation of mv with p. For the simple case this will be constant. More sophisticated analyses include different, but constant, m values above and below some critical pressure (see 2.9, below), and mv varying linearly with effective stress between point values given as data (see 2.11).

A subroutine, INTRAL, has been written to perform the necessary integration and this is called at each step of the program where settlement values are required. For a multi-layered soil it is called for each layer, and added to previous settlement to give a total value for each layer (which may be output as "Additional data" if desired by the user). The various layers are added to give total settlement at any desired time.

It may be mentioned here that the "Average Degree of Consolidation" must be defined in terms of the strain, by expressing the settlement as a percentage of the ultimate settlement.

The program writes calculated settlements to the output file for any output times specified by the user. Graphical presentation of these results is also available during running of the programs. This is written on a continuous loop so the user may first select the graph required (with either linear or logarithmic time scale) and then select the scale for settlements as desired (he is advised of the maximum settlement value attained). These graphs may be viewed as desired, replotting the scales, until the option STOP is selected. Normally the output times specified by the user, do not give sufficient values for a good plot of settlement, so these values (plotted *) are supplemented by further values (plotted Δ) automatically calculated and stored by the program during the time step iterations (these values may also be included in the numerical results if desired). Examples of graphical output are shown in Chapter 3.

2.9 The Critical Pressure

An important modification included in the present work is the ability to handle the effect of a critical pressure, p_c. It is well known that an overconsolidated soil may be reloaded up to a certain effective stress with relatively little accompanying strain, and such consolidation takes place quite rapidly. It now appears that a similar effect (though of less magnitude) is frequently encountered in deposits which, from their stress history, would be considered normally consolidated (see e.g. Bjerrum 1967, Simons 1974). This can be due to a variety of natural processes other than loading, including weathering and leaching and the changes in soil microstructure associated with secondary, or delayed, consolidation. This topic features considerably in later chapters of this thesis. It was thought useful to investigate the capabilities of extended Terzaghi theory in this problem. There are, however, many problems here which can only properly be discussed at a later stage. Only those which are of immediate interest are mentioned briefly here.

The scheme developed in CONED 1 and 2 maintains parameters Cv and mv constant, except that different constant values are used when the effective stress is below p_c. The linearizations of soil behaviour are shown in Fig. 2.4. These lead to the line CD, below p_c, being the same whether soil is unloaded or loaded, so that the parameters are termed "rebound" values RCv and Rmv.

The problem arises whether the parameters should be selected locally, i.e. depending on the effective stress at each node, or from the average stress in the layer. There are arguments for both approaches - and some further comments are made in Section 2.11, below, for the analyses where parameters are allowed to vary with effective stress. In the event it was decided to use the "average stresses" approach for several reasons. Firstly, it can be conveniently programmed into the present scheme. Secondly, since multi-layered analysis is possible, subdividing a layer should give a reasonable approximation to "local" behaviour. Thirdly, by adopting a similar stress-path to that undergone in the field, laboratory tests might give satisfactory parameters for such average stresses. The effect of subdividing the layer to examine local behaviour was tested in some detail and results of this work are considered in Section 3.4 below. It was concluded, however, that this depends on factors beyond the numerical work considered here, and full discussion of these is left to Chapter 6.

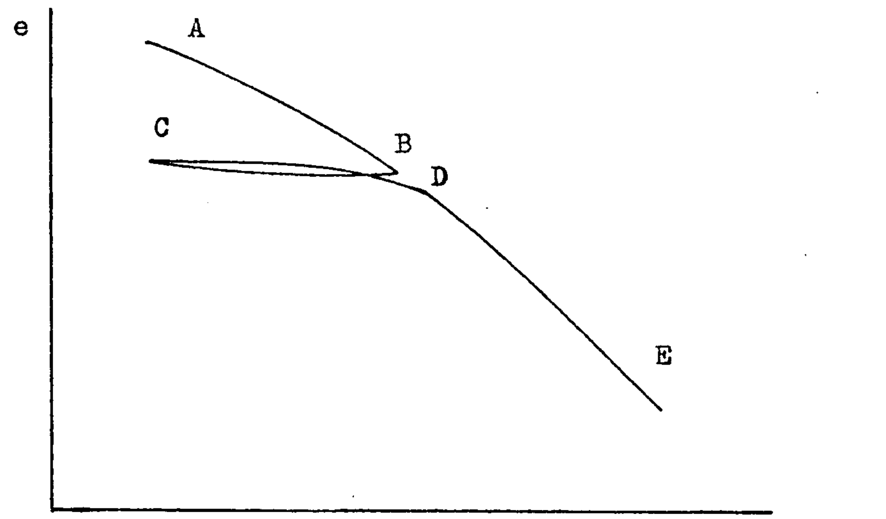


Fig 2.4a). Representation of Actual Non-linear Soil Behaviour.

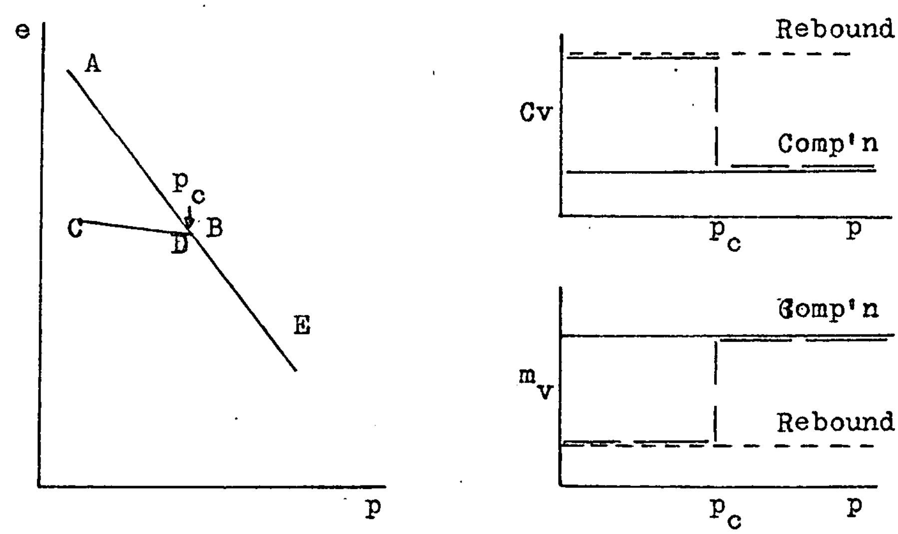


Fig 2.4b). CONED 1 & 2 Linearizations Using Constant Cv and mv values.

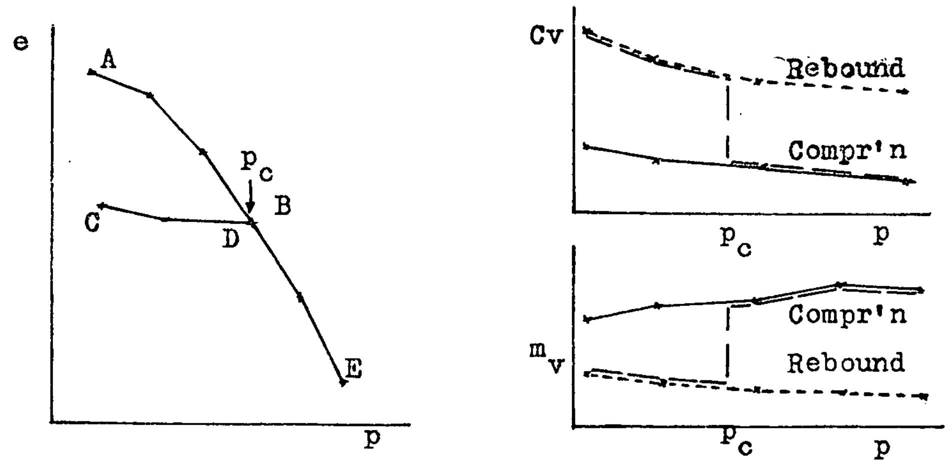


Fig 2.4c) CONED 3 & 4 Linearizations Using Cv and mv varying with Effective Stress. (Linear Interpolation between data values ×)

The scheme for the present programs is as follows. The data includes values of initial and critical stress, and rebound parameters RCv and Rmv. (It is recognized that for many consolidation problems such rebound data will not be available, or required, so these have been placed in the data files so that they may be omitted without causing any problems. The programs will simply register zero values here. Should they, in fact, be required they are automatically set equal to the compression values, and the user is notified accordingly in the results.) Before each iteration of the numerical solution the effective stress at each point is compared with the p_c value at that point. The compression or rebound parameters for the whole layer are then chosen depending whether the majority of points have passed p_c or not. This technique was found convenient because a stress history algorithm has been incorporated into the programs to allow for cases of unloading. As the effective stress increases, so the critical stress must be considered to increase, such that if, for any reason, it falls below the maximum level reached, rebound parameters should apply. This is handled here by setting the p_c value initially equal to that given in the data at all nodes. From there on if it is exceeded at any time it is set to this new value. Thus any state of time dependant loading is acceptable - some comments are made in Section 2.10.

Details of the points at which p_c is reached will frequently be of interest and, in any case, are valuable to keep track of the consolidation process. It will also be necessary to recalculate all the constants in the matrices. When the p_c condition is passed in any layer, then, the relevant time is considered as an output time. A message is included in the results that parameters change, giving the layer number for multi-layer analysis. Results are then output, the new parameters used as necessary in setting up the matrices, and the allowable time step recomputed for the existing conditions (see Section 2.12).

The technique of varying parameters with effective stress has still to be presented (Section 2.11). It is used in programs CONED 3 and 4. Here it need only be mentioned that two techniques are available for analysing the critical pressure. The first is similar to that already presented, except that parameters may now be varying above and below p_c. Rebound or compression parameters are again selected according to the average effective stress in a layer compared with p_c. The alternative method is to use compression values only and omit the p_c data, simply specifying the necessary variation of parameters with effective stress. This leaves the rebound parameters available for use should unloading occur.

2.10 Unloading - Negative Consolidation

On reduction of effective stresses, a soil will swell or heave. The CONED programs can quite satisfactorily deal with problems where the consolidation process takes place in the opposite sense from normal. A few minor modifications were required to enable satisfactory handling of the negative quantities. For instance, when the user selects "maximum" co-ordinates for the graphical outputs, these must be negative values. {The program automatically includes a note on such graphs to avoid errors of misinterpretation.} Detailed analyses of unloading problems were carried out for different loading conditions, with constant and varying parameters (not included in Chapter 3, since they merely confirm satisfactory behaviour of the programs, and are not of particular interest). It was also confirmed that the programs could handle problems of unloading, followed by loading (or vice versa).

It should be noted that the governing parameters will now be those for rebound, but the stress history technique will still function quite satisfactorily should the soil be loaded above its previous maximum stress level. As with all applications of these programs, the results are only as valid as is the theory used, and in this case attention should be drawn to the limitations of consolidation theory - particularly with regard to the assumption that the soil is fully saturated. Bishop et al. (1975) suggest that full saturation cannot be maintained if the pore pressure falls below some -80 kN/m^2. The topics of soil suction, and consolidation of partially saturated soils cannot be considered here, but clearly may control behaviour in real cases of unloading.

2.11 Parameters Varying with Effective Stress

For many soils the parameters are found to vary with the effective stress. A number of theoretical approaches have been developed to handle various types of non-linearity and some of these are discussed in Chapter 6 below. An approach which is easily adapted to the present computational method is to allow parameters to be determined from data which defines some curves of Cv and mv (or, in the present theory, k could replace one of these) as functions of the effective stress. To date, programs dealing with this have been reported by Murray (1972) and Toan (1973). Before indicating how this has been handled in programs CONED 3 and 4, some comments are required on the validity of the method and the assumptions made.

It is frequently reported that the e - p relationship for a soil is non-linear, and changing mv as a function of p is a quite acceptable method of expressing this (see Fig. 2.4). Once this is done, however, a basic assumption in the derivation of Terzaghi's Theory is affected and a problem arises in interpretation of the equation

$$Cv = \frac{k}{mv\ \gamma_w} \qquad (2.30)$$

The expression is still valid, but not constant. Cv varies with mv. One approach, presented by Abbott (1966) (see Section 3.5), has been to make mv and k vary as functions of the average degree of consolidation (which is a measure of average strain) in the same linear fashion, such that Cv remains constant. A preferable approach with analytical solutions, has been developed by Gibson et al. (1967) (see section 6.1).

Lowe (1974) has pointed out that Terzaghi theory may be applied incrementally. If a small increment of the consolidation process is considered over which Cv is constant, then the time factor solution is quite valid. By proceeding through small increments of the time factor solution, and determining t from the relevant Cv value at each step, using

$$T = \frac{Cv\ t}{H^2} \qquad (2.31)$$

it is possible to describe the entire consolidation process with varying Cv. The full method, which incorporates making Cv dependent upon strain rate, is presented as a hand calculation which seems extraordinarily tedious. Some further comments are made in Chapter 6, where strain rate effects are considered.

For the present work the Terzaghi equation will again be solved using the appropriate parameters at each increment, but this may easily be done in the programs without recourse to the dimensionless time. This is found to be confusing, since it must be defined with respect to a changing Cv value. (For multi-layer problems a further confusion is introduced with different Cvs in different layers.) All CONED programs have therefore been designed to present only real times to the user, although dimensionless time is used within the program to select the time step.

Accepting the technique of varying mv with p, there are two possible methods of continuing. The first is to specify the variation of permeability with p (from which Cv may be determined). Some success has been encountered here, notably at TRRL (Lewis et al. 1975), determining k from in-situ pumping tests. Problems remain in the interpretation of such results, though first indications suggest considerable improvements in prediction of actual consolidation behaviour. Such tests are expensive, however. From a theoretical point of view it appears that k should be a function of e. A number of attempts to liken soil permeability to flow in pipes have been made, using such parameters as roughness of the perimeter and shape factors. The most popular of these relationships is probably one due to Kozeny-Carmen. This has been used in a consolidation theory due to Hawley (1971). Two objections may be raised. Firstly, there is no direct way of assessing this, since e is a dependant variable in all testing, and can only be determined indirectly for different values of independent variable p, (and it may be mentioned, for many soils the independant time variable is also significant). Secondly, it seems most probable that permeability is dominated by structural effects such as channels and fissures, which may well be better handled as functions of p. This seems to be the argument of Rowe (1972) in his Rankine Lecture.

This leads into the second approach possible for the present work. This is to note that more frequently Cv values are what are actually determined, so it seems preferable to use these directly in the analysis.. It is therefore assumed here that the variation of Cv with effective stress is determined from a series of oedometer tests in the laboratory. The permeability, which is required for the continuity of flow condition at layer interfaces, is calculated by the program from Cv and mv (see Section 2.7), using appropriate values at each step.

Now the present numerical scheme requires Cv to be constant within any layer at any given time step. Hence Cv must be determined corresponding to the average effective stress over the layer. This is done by a Subroutine VARIED. The user gives an array of effective stresses and corresponding Cv and mv values. VARIED linearly interpolates between the data values to obtain parameters corresponding to the average p value given. The stress-strain model used is shown in Fig. 2.4c. Use of this average value is discussed further below.

The selection of rebound or compression parameters from the critical pressure, p_c, (discussed in Section 2.9) is still applicable. (VARIED is re-called to evaluate rebound parameters if required.) Normally for this analysis, however, it will be easier to set p_c to a small value, or simple omit it from the data, and show the effect in the way the compression values vary with effective stress. An example of this is included in Section 3.4.

It is clearly essential to include the initial effective stress, p_o, in the data for the varying parameters case, in order to correctly evaluate the average effective stress at each step.

The programs are designed to be as helpful to the user as possible.

Here, if the average p falls outside the range of values given as data an error message is output diagnosing the problem, and the program pauses. Normally all available data will have been given (but user should consider why this is insufficient), so the option CONTINUE allows the computation to proceed using the nearest value above or below, as the case may be. To do this every step might be very tedious, so if user CONTINUES twice, the program automatically does so in future. Effective stresses at each output time may be examined in the numerical results.

Since the solution technique is based on central differences the parameters Cv and mv should really correspond to the average p at the mid-point of the time step. This cannot be determined prior to the iteration since it depends upon the pore pressures to be calculated. Use of values corresponding to the start of each time step introduces only marginal errors here, however.

As the soil settles, layer thickness(es) will change. In CONED 3 and 4 this is included; the settlement is subtracted from the thickness and the layer re-divided into NZ elements. This is believed to be consistent with the data, since the determined values of Cv and mv will include this effect.

A number of tests were run to examine this method of analysis - some results are given in Section 3.3. Generally behaviour was very satisfactory. Only one point need be pursued here. The use of an average p value to determine Cv was mentioned above. Section 3.3 shows tests designed to assess the validity of this. A layer was re-analysed after being subdivided a number of times and results compared. This more closely approximates Cv being a function of p at each node. Further subdivision and re-analysis was carried out until reasonable agreement of results was obtained. It appears that some

8 sub-layers must frequently be used (but depending on the case) for acceptable accuracy for such local variation.

Murray (1975) drew the author's attention to an important physical process involved here. If the rapid consolidation near a free draining boundary significantly affects the governing parameters, the behaviour of the more central parts of the profile will now be controlled by the rate at which the pore water can seep through these boundaries. Barden (1965) has also hinted this effect may be significant. It is such a process that the technique of subdividing a layer gives a better approximation to. Such "local" methods should use parameters obtained on samples under reasonably constant effective stress with depth - i.e. using slow constant rate or small increment loading tests. Conventional tests ($\Delta p/p = 1$) will give parameters based on average p over the profile. Problems remain in successfully modelling the actual field stress path of interest.

In subsequent chapters it will be seen that strain rate has a fundamental effect on the stress-strain behaviour of clays near the normally consolidated state. A consistent theoretical approach will be examined. Here we simply note that the present method could prove valuable if satisfactory statements of varying parameters can be obtained; in practise this appears difficult.

2.12 The Time Step and Related Accuracy of Results

In Section 2.2 above, a factor β^1 was defined as

$$\beta^1 = \frac{Cv\ \Delta t}{\Delta z^2} \qquad \text{(2.4A bis)}$$

It was mentioned that for the explicit scheme presented $\beta^1 \leq \frac{1}{2}$ is necessary for the difference equations to be stable. Implicit schemes are stable for any time step. As mentioned in Section 1.4, truncation errors now become of significance, however. These are errors introduced by curtailing the numerical approximations to the functions after a few terms. (The Crank-Nicholson scheme is derived from writing the Taylor series expansions of the required derivatives. Only first order terms are used, so errors are of second order and above.) Use of the Crank-Nicholson scheme for Terzaghi's equation leads to truncation errors dependant on the space and time intervals used. They further depend upon the rate of change of the derivatives. This will vary with the case under examination. Since the programs here must be capable of handling any loading conditions, any realistic values of Cv, and indeed cases where Cv varies with effective stress (which itself is governed by the dependant variable u), a pragmatic approach must be adopted to the question of suitable increments of Δz and Δt.

Lengthy tests were carried out on the programs in an attempt to optimise both time and space intervals. This proved quite tedious, and only the main points emerging are summarized below. Errors were assessed by comparison with various published results and analytical solutions including several of the cases presented in Chapter 3. Now the errors in pore pressure, u, vary from point to point in time and space. It was soon established that critical errors occured at the space node adjacent to a free drainage boundary at times when u changed rapidly. Instantaneous loading was more critical than time-dependant.

It should be remembered that for such schemes additional accuracy is obtained only at the expense of further computer-time, so a realistic balance must be achieved, bearing in mind the accuracy to which parameters may be determined and the simplifying assumptions limiting the theory. Accuracy of ½% in pore pressures at their most critical was considered acceptable here; for the vast majority of cases much better than this was achieved.

Space Interval - most workers on these problems {e.g. Christie (1963), Murray (1972), Toan (1973), Razouki (1974)} have stressed the need for a sufficient number of elements in each layer - the consensus of opinion giving about 10 elements per layer. The author's results agree with this figure, although it appears that this may be reduced if a number of layers are used - the amount of the reduction increasing with the number of layers and as the differences in Cv values between layers decreases. In such cases the value of $\frac{\partial^2 u}{\partial z^2}$ will be lower. In any case not less than 5 nodes per layer should be used except in special circumstances. Although the space derivative will decrease with time the number of nodes must obviously be kept constant.

Time Interval - with Δz defined, the stability criterion β^1 may be used as a convenient control of the time step; it is, in fact, a dimensionless time value. Some confusion exists in the literature concerning the allowable values of β^1. It was mentioned in Section 1.4 that using a constant time step a value of β^1 around 3.5 - 4 was found necessary to give the same efficiency of running time as for the explicit method. Yet figures in the literative vary from < 1 to > 50! A few workers (notably Razouki 1974) have grasped that β^1 may increase dramatically as the values of the derivatives fall. Razouki made β^1 a function of time which he examined empirically, resulting in a small initial step increasing logarthimically with time. This cannot be quite correct; it should somehow be based on $\frac{\partial u}{\partial t}$, which does not decrease exactly logarithmically. No acceptable scheme has yet been devised, however, for predicting the time step necessary for acceptable accuracy at any stage of the consolidation process, so a similar logarithmic approach has been used here - taking care to err on the side of excessive accuracy (i.e. too small a step size).

Slight modifications have had to be made. The user of the CONED programs inputs an array of "output times" at which calculated results are to be printed and the pore pressure profile displayed graphically. For every change of Δt a substantial amount of recalculation had to be performed. It was therefore preferred to work using a constant time interval between any two output times. More steps were required but the calculation per step was significantly reduced. Following each output time the acceptable time step is again determined from the logarithmic criterion. This is adjusted down if necessary to give an integral number of steps to the next output, using this constant time step.

It should be noted that for a multi-layered soil profile the largest β^1 value must be kept within the required accuracy criterion, so the programs CONED 2 and 4 automatically scan the β^1 values and set the allowable time step from the largest of these, from which the others follow. A useful technique here is to keep β^1 values similar in the different layers which may be done by suitable numbers of elements, such that $Cv/\Delta Z^2$ is similar in each layer.

CHAPTER 3

Some Case Studies Using Extended Terzaghi Theory

Introduction

A large number of results have been obtained in the course of checking that the programs are functioning satisfactorily in the various modes of analysis of which they are capable. Most of these results are not of great interest in themselves, so are not included in this thesis. The cases below are presented as general illustrations of the analyses possible. Some points mentioned in Chapter 2 were investigated in detail and further discussion in the light of results obtained is included here. Some cases reported in the literature are considered, and a technique for analysis of layer pairs is described and compared with numerical results.

3.1 Single Layer, Constant Properties, Instantaneous Loading

This is the classic consolidation problem first solved by Terzaghi (1923). Carslaw and Jaeger (1947) list three possible analytical methods of solution; Laplace Transforms, Green's Functions, and Fourier Series.

Following Taylor (1948) the last of these techniques yields the well known solutions for the case of constant initial pore pressure with depth.

$$u = \sum_{m=0}^{\infty} \frac{2u_o}{M} \left(\sin \frac{Mz}{H}\right) \exp\left(-M^2T\right) \qquad (3.1)$$

where $T = \frac{Cv.t}{H^2}$ (Dimensionless Time Factor),

and $M = \frac{\pi}{2}(2m+1)$

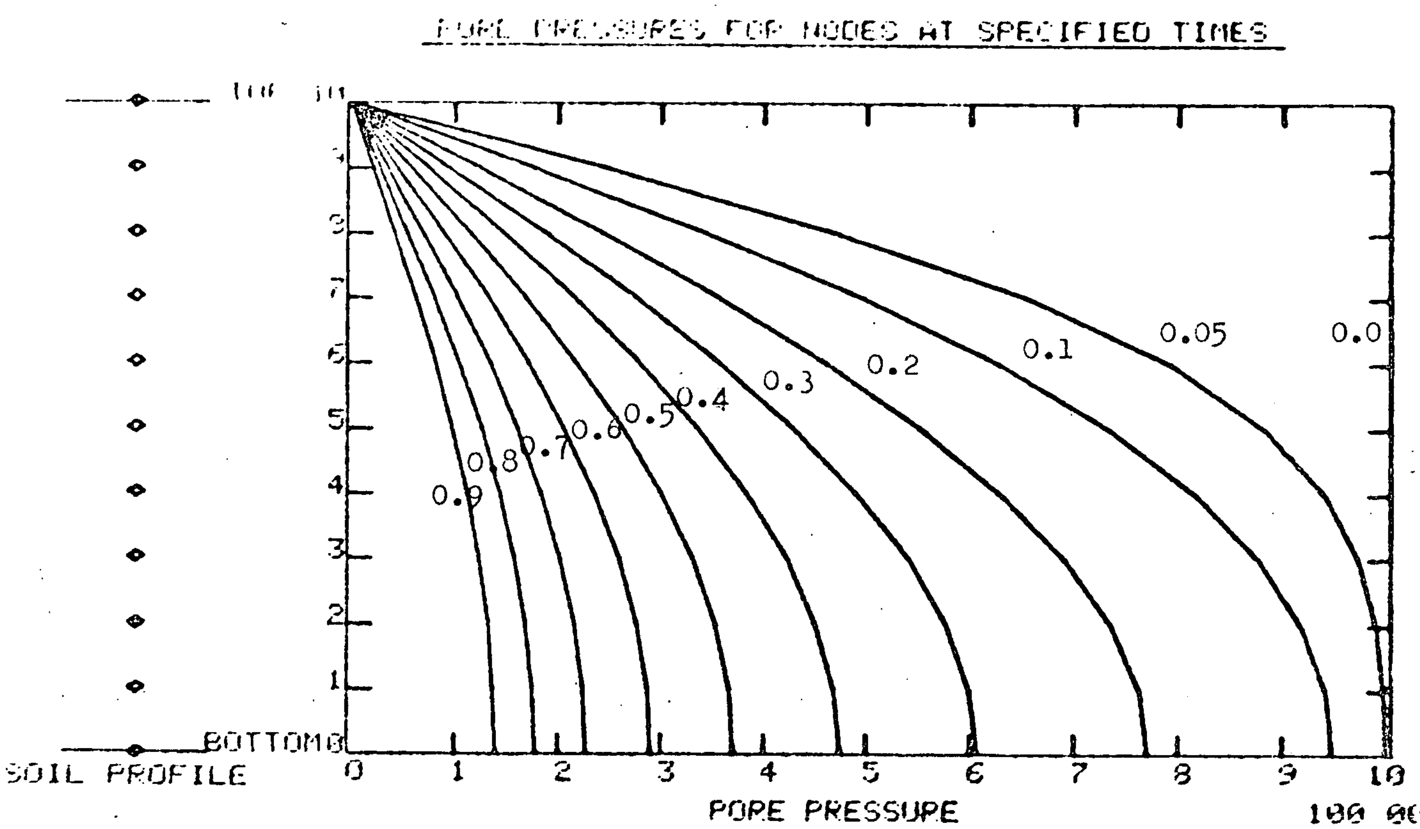


Fig 3.1 Dissipation of Pore Pressure for Different Values of Time Factor. T.

Single Layer, Constant Properties, Instantaneous Loading.

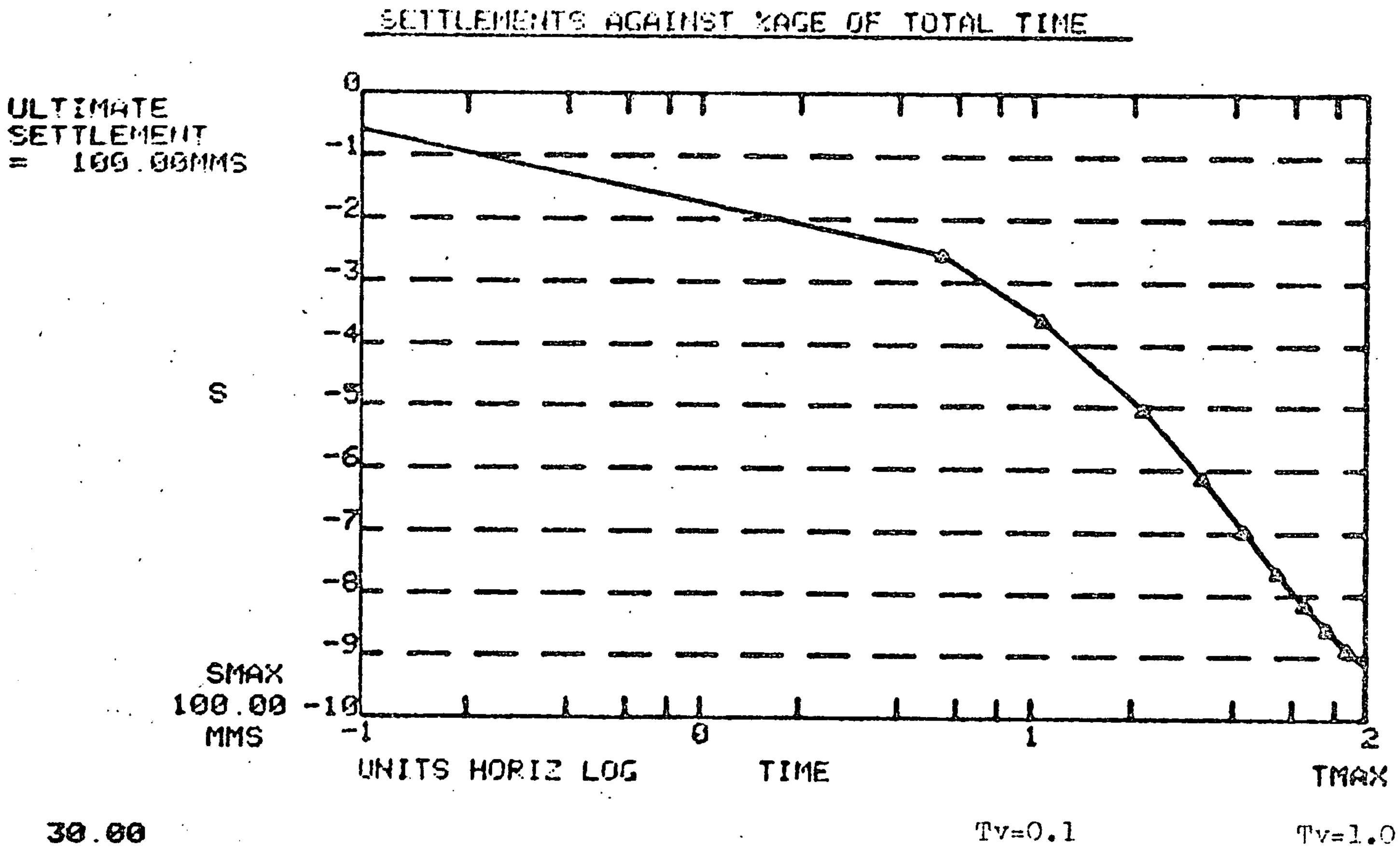


Fig 3.2 Progress of Degree of Consolidation with Time (Settlement = % Consolidation)

Single Layer with Constant Properties, Instantaneous Loading

The Degree of Consolidation U is given by

$$U = 1 - \sum_{m=O}^{\infty} \frac{2}{M^2} \exp(-M^2T) \tag{3.2}$$

Figs. 3.1 and 3.2 show the CONED solutions to this problem. A comparison of numerical results shows these are correct to within ½%.

Taylor (1948) (Fig. 10.10) gives solutions for cases where initial pore pressures change linearly with depth (IB) or are parabolic (II). Clearly the CONED programs can handle any initial distribution.

3.2 Single Layer, Constant Properties, Time Dependant Loading

Two cases were examined here in detail. Both gave excellent numerical agreement, but results are of relevance only to the specific schemes considered, so these are not presented herein.

Lumb (1963) has presented the rigorous analytical solution to the problem of a soil layer with no initial excess pressures, subjected to a load increasing directly with time to some maximum value (100%) at time t_o. This case was analysed where t_o corresponded to a dimensionless time of 0.5.

Gibson (1958) presented an analytical solution to the problem of a clay layer increasing in thickness with time, such as might occur during sedimentation, or in construction of an earth dam. He presents an example of the latter case. Now the CONED programs cannot handle the problem of changing layer thickness during the first season's construction - boundary conditions cannot be altered within the programs; a limitation here.

The problem of pore pressure dissipation between loading seasons is easily handled by CONED, giving excellent agreement with the results presented by Gibson on his hand-calculated explicit grid.

He uses the same process to explore behaviour on subsequent loading - linear with time - and though the newly added soil cannot be analysed here (change in thickness, as above), the previous season's fill is topped by a sand layer, so may be analysed as a free draining layer with linearly increasing load. Again excellent agreement was found.

CONEDs can, of course, handle far more complex loading conditions with ease (see 2.6 above).

3.3 Single and Multi-Layer, Cv varying with Effective Stress

A number of results have been obtained for cases where parameters vary with effective stress. Most of these are of no great interest in themselves, so comment is confined to some general remarks on the observed behaviour. One case is reported in detail to illustrate the main points arising.

Firstly, by way of an aside, the problem of constant Cv, but mv varying with p may be disposed of. This is handled quite satisfactorily by the programs, leading to variations in the settlement results reflecting the changes in mv as might be expected.

In practice we may encounter parameters Cv and mv both varying together. This will introduce uninterpretable differences in the numerical results, so for the rest of this discussion only cases with constant mv will be considered. It is then possible to compare progress towards a constant ultimate settlement.

The present example is taken from Murray (1972) who presented a graph illustrating that if a given soil layer is analysed using a constant value of 5 m^2/yr the predicted progress of consolidation is far more rapid than when analysed with Cv varying linearly with log effective stress from an initial value of 10 m^2/yr to a final figure of 1 m^2/yr. The case is for a layer of 5 m thickness with single drainage, under instantaneous loading of 100 kN/m^2.

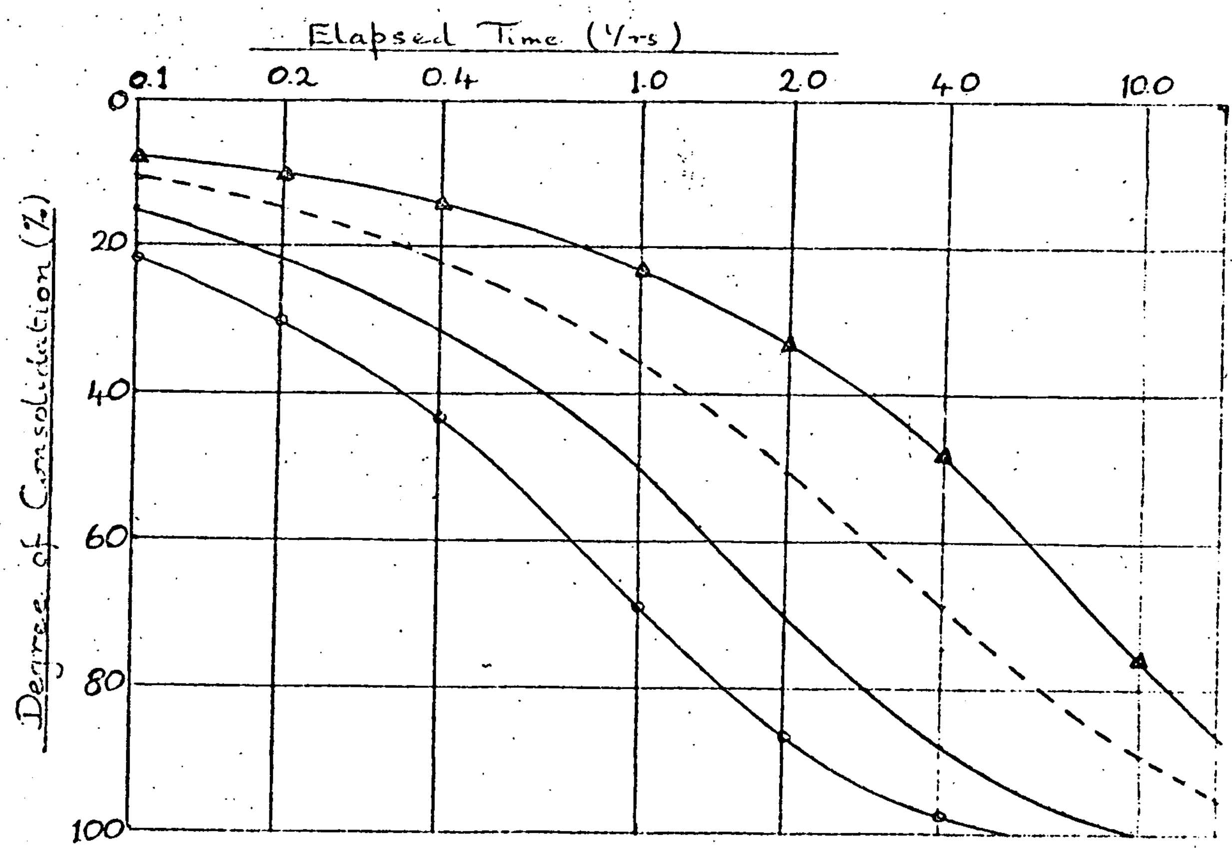
Fig. 3.3 indicates the final results obtained here. The plots using constant values of the maximum and minimum Cvs are also shown. It is intuitively obvious, but has been checked for all cases studied, that the varying parameter case will fall within these upper and lower bounds. The actual position will depend upon the manner in which Cv varies which will depend upon the manner of variation of effective stress. This in turn will be affected by the rate of loading in the time dependant loading case. It is frequently possible, however, to make an intelligent estimate of the variable parameters curve from the constant parameters solutions, and this gives a useful "first check" on solutions.

Reworking Murray's example, then, excellent agreement was obtained for the constant parameter solution, as might be expected since it is merely the simple analytically soluble case. For varying Cv, Murray does not fully define the problem. For linear variation with log p it is essential to state not just the increment used (100 kN/m^2) but also some fixed point, such as the initial effective stress which it is most convenient to use here. For present purposes a value of $p_o = 10\ kN/m^2$ has been used. This leads to $p_f = 110\ kN/m^2$, whence Cv changes by a factor of 10 over slightly more than 1 logarithmic cycle of stress. Comparison with the published results was poor, and it was concluded (by re-analysing with lower p_o values) that Murray had approximated zero initial effective stress by a value of $p_o = 0.001\ kN/m^2$, giving a stress range covering some 5 logarithmic cycles. In such a case, $p = 1\ kN/m^2$, after very little consolidation, already represents more than half the logarithmic interval, so the rate of consolidation may be expected to be close to the lower bound. This was, indeed, found to be the case for Murray's solution, which lies very close to the solution for Cv constant at 1 m^2/yr. Results of the analyses varying p_o are not included here since the case would not arise in practice, but the importance of correct specification of effective stresses is emphasised.

Results are given, however, to illustrate the effect of dividing the soil profile into a number of sublayers. It will be recalled that the CONED programs determine the variable Cv and mv values by reference to the average effective stress across any layer. However, increasing the number of sublayers in the analysis will tend to the solution for Cv varying locally with effective stress. Results for the present example (Fig. 3.4) are typical. Initially the stress distribution is uniform, so no difference between local and average stresses occurs. As soon as drainage is permitted the difference in effective stresses across the profile becomes considerable, and the resulting consolidation curves diverge, depending upon the extent to which Cvs differ. At large times the stress distribution again tends to be fairly uniform, so that the solutions tend towards the same final curve.

Near a free draining boundary p tends quite rapidly towards its final value p_f. If this implies a large decrease in Cv, the consolidation process may be considerably slowed down as water from the centre of the profile must flow out through this less permeable region. The local parameters solution describes such a model. The average parameter solution considers Cv to be determined from tests on samples which will model this stress distribution. In practice it might be expected that the variation of Cv with p would differ between the average and local approaches. Either of these has potential; preference should be given to the method where parameters are most acceptable. This involves not only ease of determination in the laboratory, but also knowledge of how such parameters might vary with layer thickness. It does not necessarily follow that variation in parameters with local effective stress in thick layers will be the same as that found in thin samples.

The present results show that a considerable number of sublayers are necessary for accuracy in the local parameters scheme. The "correct" solution for this case is not



Progress of Consolidation with Time

Fig 3.3

Effect of Using Varying Cv.

—o——o— Cv=10.0 m^2/year

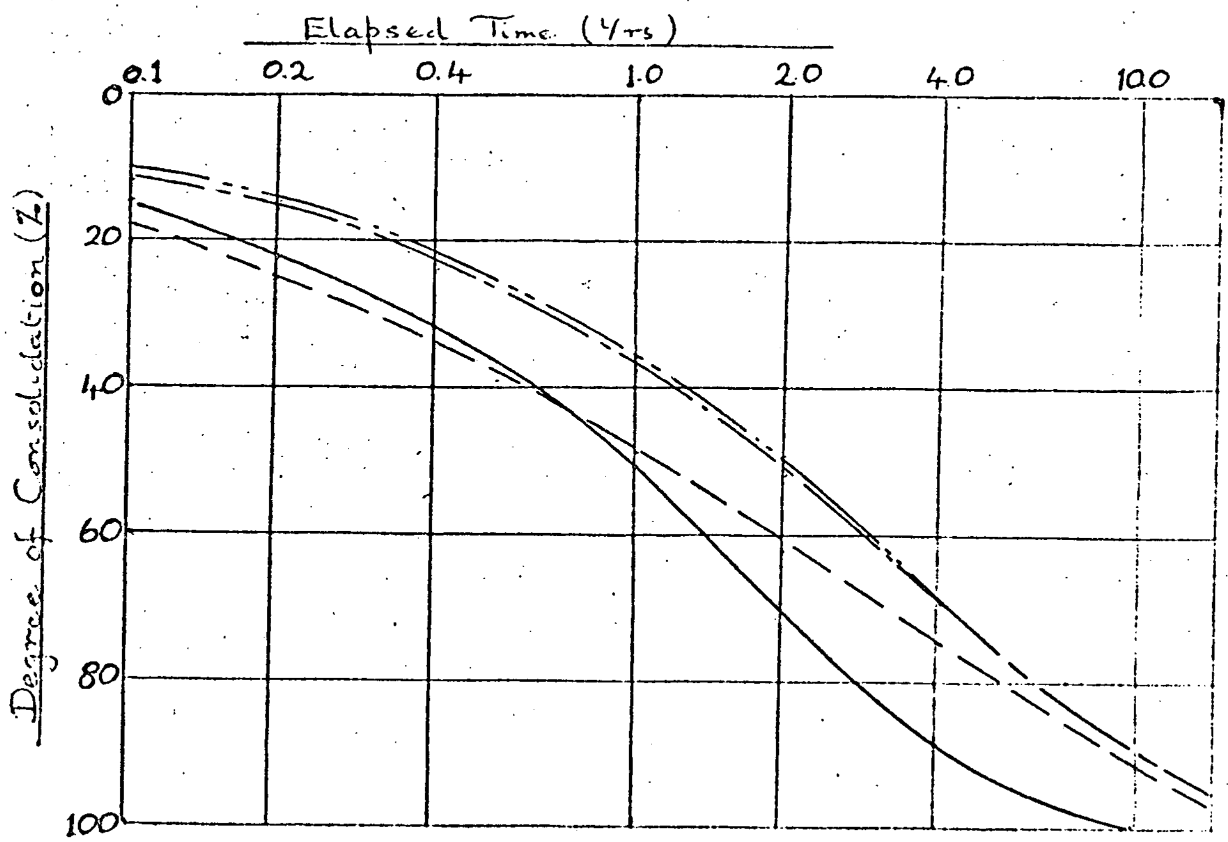
——— Cv= 5.0 m^2/year

—△——△— Cv= 1.0 m^2/year

- - - - - - - Cv varies linearly with log. effective stress (1 log cycle) from 10.0 to 1.0 m^2/year

(p_o=10kN/m^2 Δp=100kN/m^2)

(8 Sublayers used in Analysis)



Progress of Consolidation with Time

Fig. 3.4

Effect of Number of Sublayers used in Analysis

Cv varies linearly with log. effective stress (1 log cycle) from 10.0 to 1.0 m^2/year

(p_o=10kN/m^2 Δ p=100kN/m^2)

— — —	1 Sublayer
—- - —	4 Sublayers
—-- - —	8 Sublayers
————	Constant Cv=5.0 m^2/year

available, and the only method of examining accuracy is to increase the number of sublayers, re-analyse, and compare. Increasing sub-division markedly increases computer time necessary, so some sort of balance must be struck for practical purposes. For Murray's case, such balance is achieved at around 8 sublayers. A similar figure was obtained for most other cases considered so 8 sublayers may be suggested as a rough guide, but each case must be treated on its merits. So the local parameters case is somewhat less amenable to numerical solution, at least in terms of the schemes presented here.

On the other hand it is by no means certain that standard oedometer testing accurately models the stress distributions occurring in the field. This topic is really outside the range of present discussion. Some further comments are made in Chapter 6. Here it is sufficient to note that the programs can handle this analysis quite satisfactorily.

To summarise then, it is not clear whether Cv should vary with effective stress locally or as an average over any layer. The latter solution has been specifically programmed, but a good approximation to local behaviour can be obtained by splitting the profile into a considerable number of sublayers. The process is, however, tedious and time consuming. It is suspected that the average stresses method is of more practical value, but some experimental and theoretical problems remain to be solved here.

3.4 The Critical Pressure

The method of handling critical pressure is again based on the average effective stress in a layer. As mentioned in section 2.9, use of a large number of sublayers allows analysis of the case where parameters vary locally with effective stress at each node. A number of tests were run to ensure that the programs were operating satisfactorily, and the differences between "local" and "averaged" parameters were investigated.

The case illustrated here is for a 10 m layer displaying a critical pressure ratio (p_c/p_o) of 1.25, and a load increment ratio of ($\Delta p/p_o$) of 1.0, applied instantaneously. Parameters Cv and mv are multiplied and divided, respectively, by 4, below p_c, so that the permeability is constant. Full details are given on Fig. 3.5, which shows the results obtained.

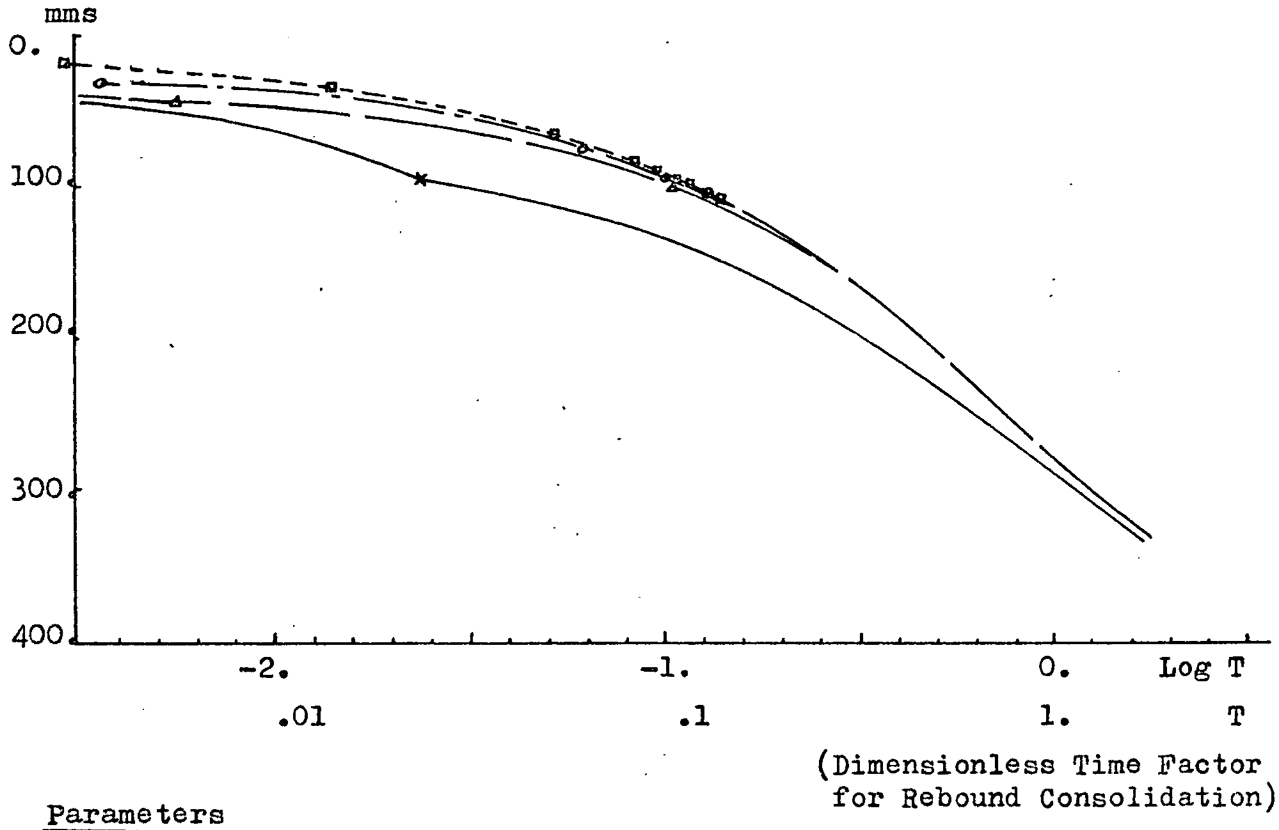
The results indicate the different solutions for the average and locally varying parameters. For the former, consolidation takes place quite rapidly up to p_c, and the effect of the change in Cv at this point is quite evident. The effect of doubling the number of sub-layers is to bring the analysis closer to the case of the change in parameters occurring locally whenever p exceeds p_c. Use of 2 sub-layers is not particularly helpful, but it is seen that further doubling to 4 and 8 quickly tends to acceptable agreement. The reason for this analysis indicating slower consolidation is that the effective stress near the drain soon exceeds the critical stress, so that the lower Cv value must be used which controls behaviour in the more central portions of the soil.

In checking the behaviour of the programs, the results shown, derived from CONED 1 and 2, were compared with those obtained from CONED 3 and 4 by the two techniques available for these latter (namely, setting parameters constant using p_c; and modelling the relationships by suitable variation of compression values, omitting p_c). In all such cases agreement was excellent.

Fig 3.5

Progress of Consolidation with Time

Effect of Critical Stress, p_c, and Number of Layers Used in Analysis



Parameters

$p_o = 100$ $p_c = 125$ $p_f = 200$ kN/m^2

	Rebound	Compression	
Cv=	0.4×10^{-2}	0.1×10^{-2}	m^2/s
mv=	0.1×10^{-3}	0.4×10^{-3}	m^2/kN

Analysis with p_c	No of Layers
————————	1
——— ———	2
———·———	4
– – – – – –	8

Symbols indicate points where average effective stress in any layer equal p_c .

3.5 Multi-Layer, Parameters Constant and Varying, Instantaneous Loading

Abbott (1960) was the first author of a finite difference scheme to solve the problem of consolidation of a multi-layer soil profile. Details of the scheme are not given here, since in principle it is identical to more recent techniques (such as presented in 2.7 above); but these latter are rather neater and easier to handle.

The case study presented, however, was re-analysed, firstly to check the validity of the CONED programs. The example is from Vreeswijk Lock, where pumping during construction drew the water level down some 6.5 m, and the 3 layers of clay, peat and clay settled accordingly. Fig. 3.6 gives a good example of the graphical output obtained direct from the computer, showing profiles of the dissipation of pore water pressure with time. (It has already been noted that times corresponding to each curve must be added by the user.) Abbott's results have been superimposed as dashed lines, where they can be separated. It will be seen that agreement is excellent. The discrepancies observable were also found by Murray (1972) and Christie (1963) (who used an analogue computer scheme) and the present results are indistinguishable from their values.

The settlement graph also agrees with Abbott's published results. He noted that the values used in this back-analysis were obtained after the scheme was completed, and speculated that the greater settlements observed in the field might have been due to higher original values of parameters. He later (1966) produced a scheme in which mv and k varied linearly with the total degree of consolidation in each layer (and hence Cv remained constant). The variable case, where initial mv values for each layer are approximately $2\frac{1}{3}$ × the final values (as used in constant case), produces a curve of similar shape to the observed settlements, beyond a point B. This has been demonstrated on Fig. 3.7 by transposing this curve down from B to B' on the actual settlement curve. The reason for significantly greater early settlements than calculated is not given. However, it seems most likely that it lies outside the factors being considered here (possibly due to settlements in the underlying sand layers) and it is reasonable to exclude it from this comparison.

Starting at B' then, it is demonstrated that settlement is slightly greater than predicted by the constant parameter case. Abbott's variable parameter solution gives good agreement, while the present author's attempt using both Cv and mvs varying by a factor of 2 leads to excessive settlements.

It had been hoped to compare CONED 4 results with Abbott's variable scheme. For the linear case degree of consolidation may be defined from the effective stress. However, once the parameters vary, the stress-strain relationship is non-linear, and degree of consolidation is usually expressed in terms of average strains.

Thus Abbott's technique makes k and mv dependant upon the average strain within a layer, and they are linear functions inter-related such that Cv always remains constant. This does not allow much versatility to deal with experimentally determined variations of the parameters. The two possible approaches of linking varying parameters to stress or strain were compared in section 2.11.

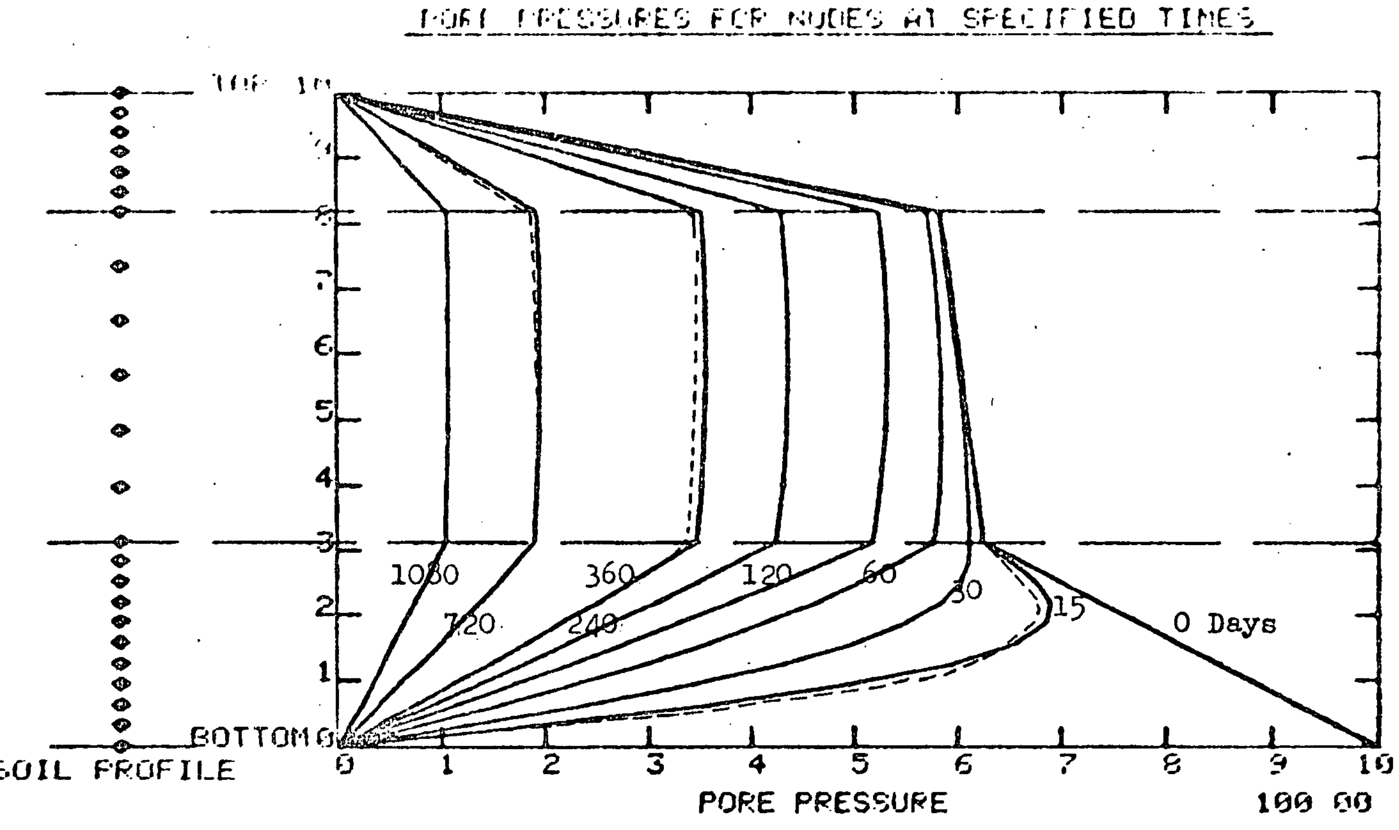


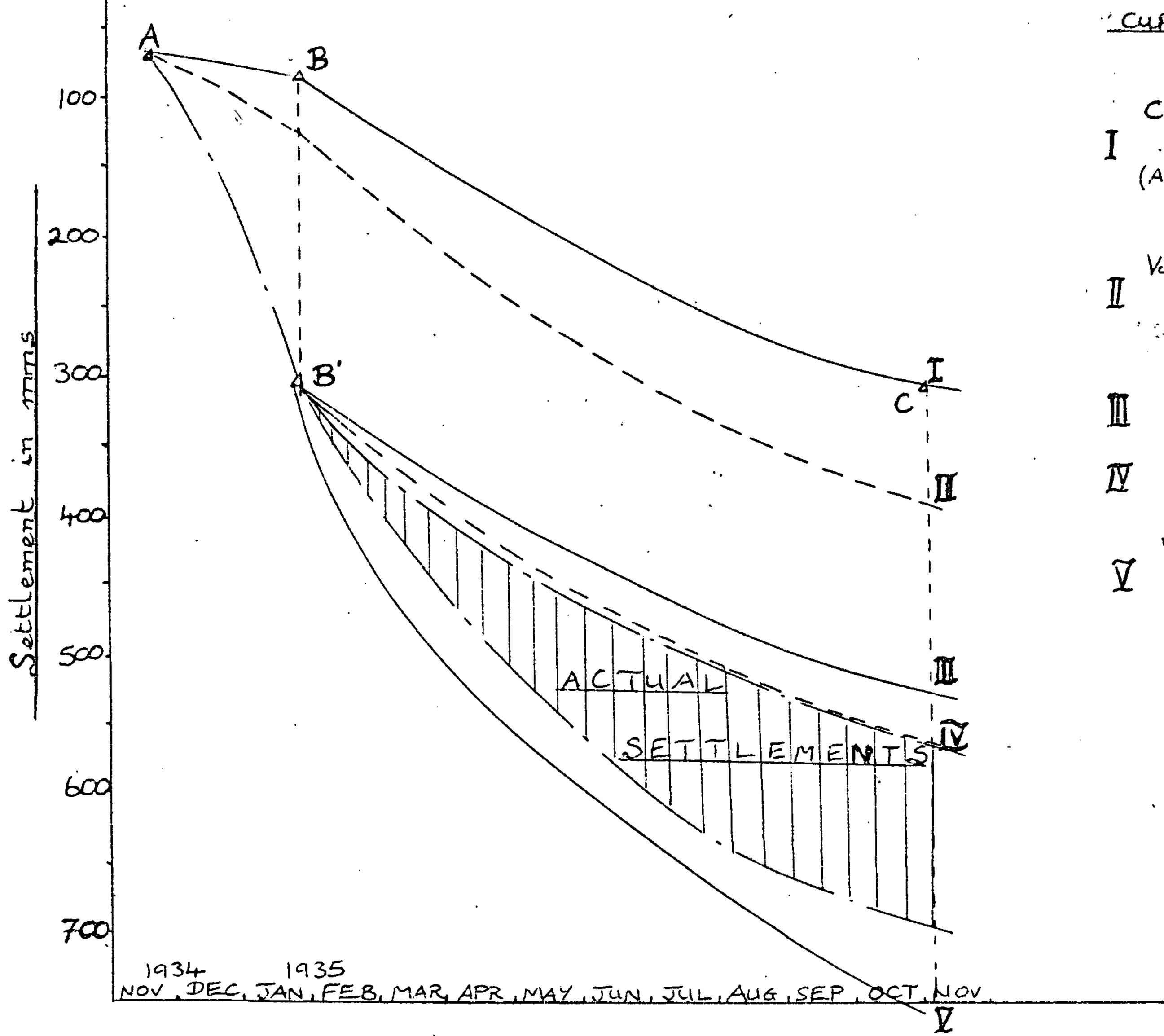
Fig 3.6 Abbott's Problem. 3 Layered Soil, Constant Properties, Instantaneous Loading. Dissipation of Pore Pressure with Time.

———— CONED 2 OUTPUT

-------- Abbott's Results (where distinguishable from above.)

Fig 3.7

ABBOTT VREESWIJK LOCK



CURVE		LAYER	m_v	k $\times 10^{-7}$	C_v $\times 10^{-6}$
I	Constant Parameters (Abbott & CONED)	1	.055	1.02	1.85
		2	.303	38.1	12.58
		3	.137	2.7	1.97
II	Varying Parameters (Abbott) Initial (Final - As Above I)	1	.128	2.37	As Above
		2	.705	88.5	
		3	.318	6.3	
III	As I. Transposed from B to B'				
IV	As II Transposed from B to B'				
V	Varying Parameters (CONED)	1	.128	-	3.70
		2	.705	-	25.16
		3	.318	-	3.94

3.6 Layer Pairs. Constant Parameters, Instantaneous Loading

Although in general the problems of multi-layer soil consolidation can be handled only by computer analysis of a numerical scheme, solutions are available for a two layer deposit and also for the important practical case of a deposit of multiple layer pairs (which is applicable to varved clays and clays with regular silt seams running horizontally). Lee (1974) presents a method making use of the geometric similarities of the consolidation curves. Briefly it is applied as follows.

The compressibility and permeability characteristics of the layers are defined by two parameters a and B (see below). Hence the time factor for 50% consolidation is read from a graph (Fig. 3.8) and a standard curve is then fitted through this point. This curve is not applicable for less than 20% consolidation for the case of "permeable top, permeable base" (henceforth PTPB) or for less than 40% consolidation for "permeable top, impermeable base" (PTIB).

Lee presents graphs of rate of settlement for some specified cases and it has been possible to test the program CONED 2 against some of these cases and then continue to consider the effects of varying some parameters. Firstly, though, it is necessary to present the theory used. Lee uses the following dimensionless parameters

$$a_{\ell,\ell+1} = \frac{m_\ell}{m_{\ell+1}} \; \frac{h_\ell}{h_{\ell+1}}$$

$$B_{\ell,\ell+1} = \frac{k_{\ell+1}}{k_\ell} \; \frac{h_\ell}{h_{\ell+1}}$$

{where m_ℓ is the coefficient of volume compressibility in layer ℓ; mv elsewhere in present text

For the case of a two layered soil, and bearing in mind we are using the parameter Cv $(= \frac{k}{\gamma_w m_v})$ we can write for the present work

$$a_{1,2} = \frac{m_1}{m_2} \cdot \frac{h_1}{h_2}$$

$$B_{1,2} = \frac{Cv_2\ m_2\ \gamma\omega_o\ h_1}{Cv_1\ m_1\ \gamma\omega_o\ h_2} = \frac{Cv_2\ m_2\ h_1}{Cv_1\ m_1\ h_2}$$

Now $\alpha = \frac{a}{B} = \frac{Cv_1}{Cv_2} \left(\frac{m_1}{m_2}\right)^2$

The time factor for this work is based on the total thickness of the deposit and the equivalent coefficient of consolidation for the whole deposit.

Hence $\bar{T} = \frac{\bar{c}\ t}{H^2}$

where $\bar{c} = \frac{H^2}{\left(\sum_1^n m_\ell h_\ell\right)\left(\sum_1^n \frac{h_\ell}{k_\ell}\right)}$

For our two layer deposit (or the deposit with multiple layer pairs) this results in

$$\bar{T} = \frac{T_1\ a\ B}{(1 + a)(1 + B)}$$

where $T_1 = \frac{c_1\ t}{p^2 h^2}$ p being the number of layer pairs and c_1 the coefficient for layer 1.

Results for degree of settlement against this time factor $\bar{T}$ were investigated for different values of the variables α (for both PTPB and PTIB), a, and p (for PTPB), and these are presented in Figs. 3.9 - 3.12. Lee's results for degree of consolidation against time were presented on fairly small scale graphs so it has not been possible to replot these with a high degree of accuracy. However, as far as may be determined these results are identical with these obtained in the present work for analogous cases, and such results have been indicated on the graphs with the symbol Ⓛ.

It will be noted immediately from the first graph (showing the effect of varying α) that there is no simple relationship between

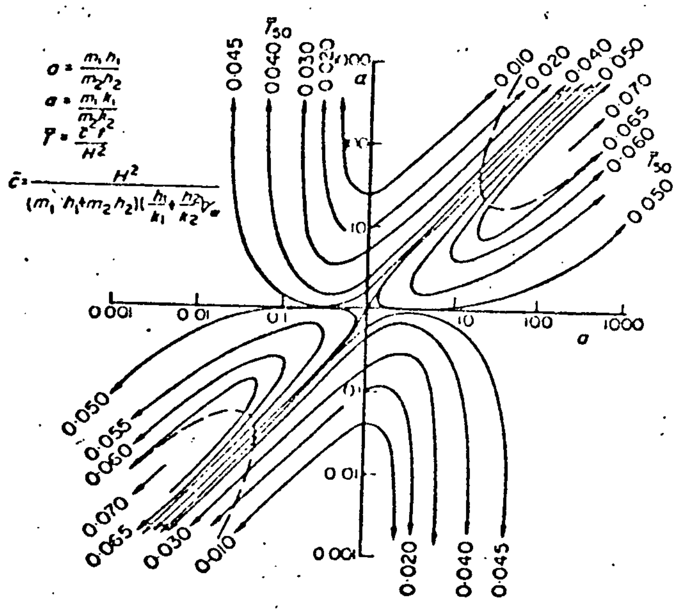


Fig 3.8 a) Contours of $\bar{T}_{50}$ for 2 layered soil (PTPB)

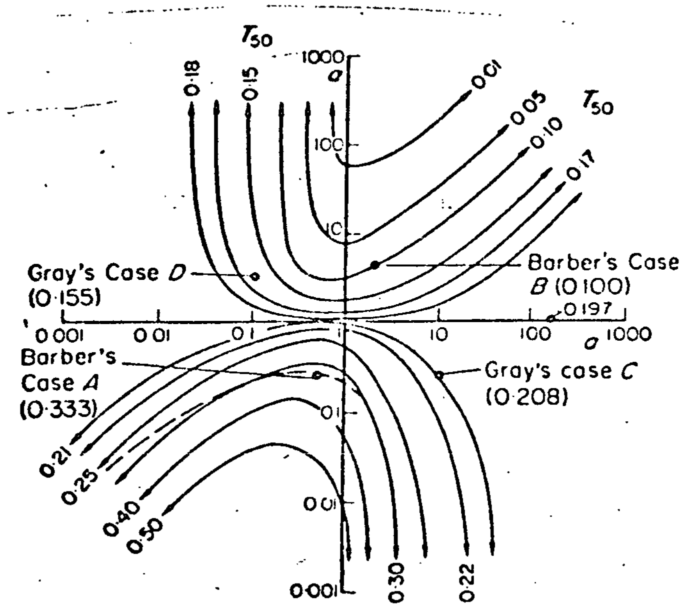
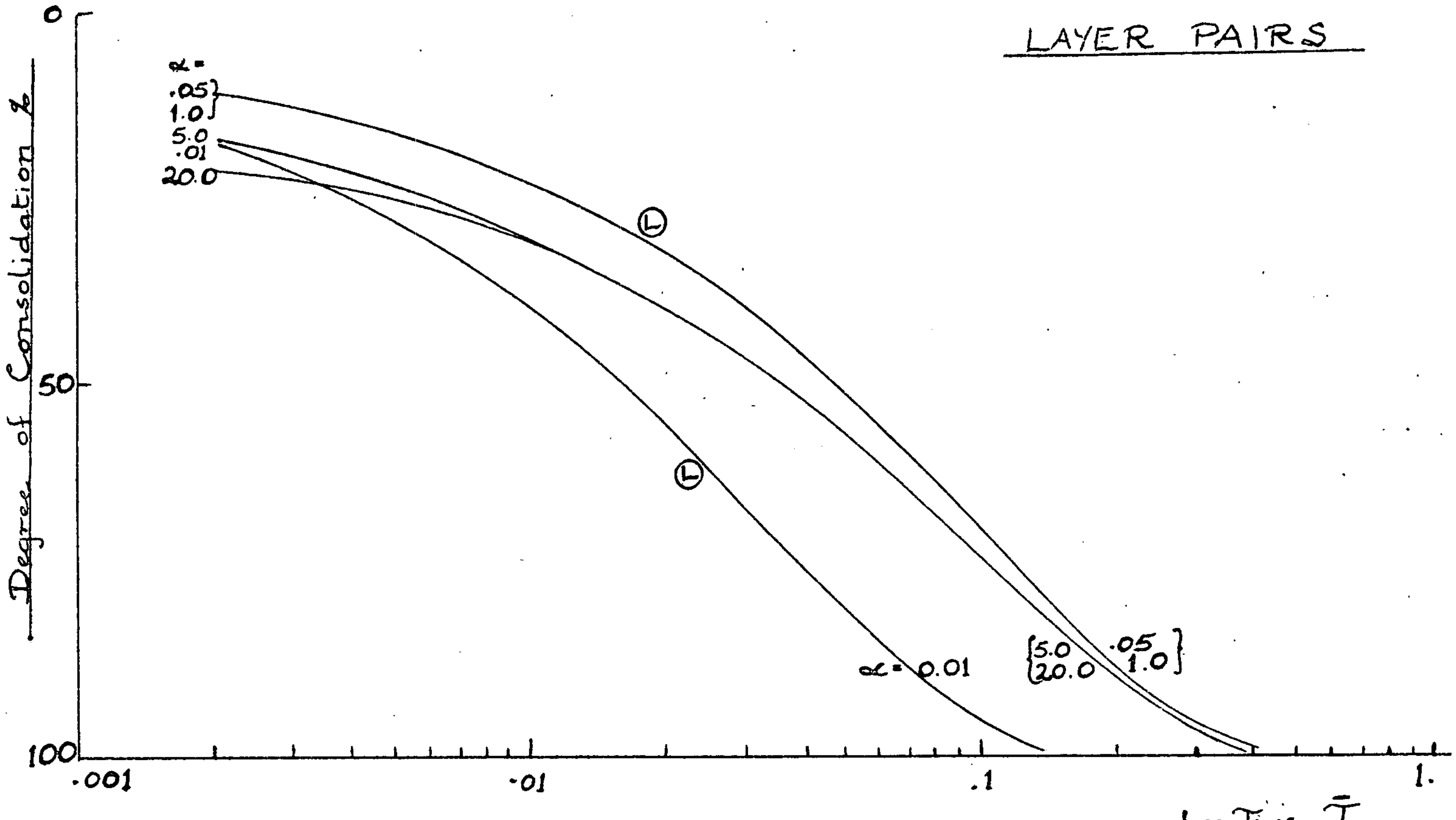


Fig 3.8 b) Contours of $\bar{T}_{50}$ for 2 layered soil (PTIB)

(After Lee(1974) figs 5.3 & 5.4)

Fig 3.9

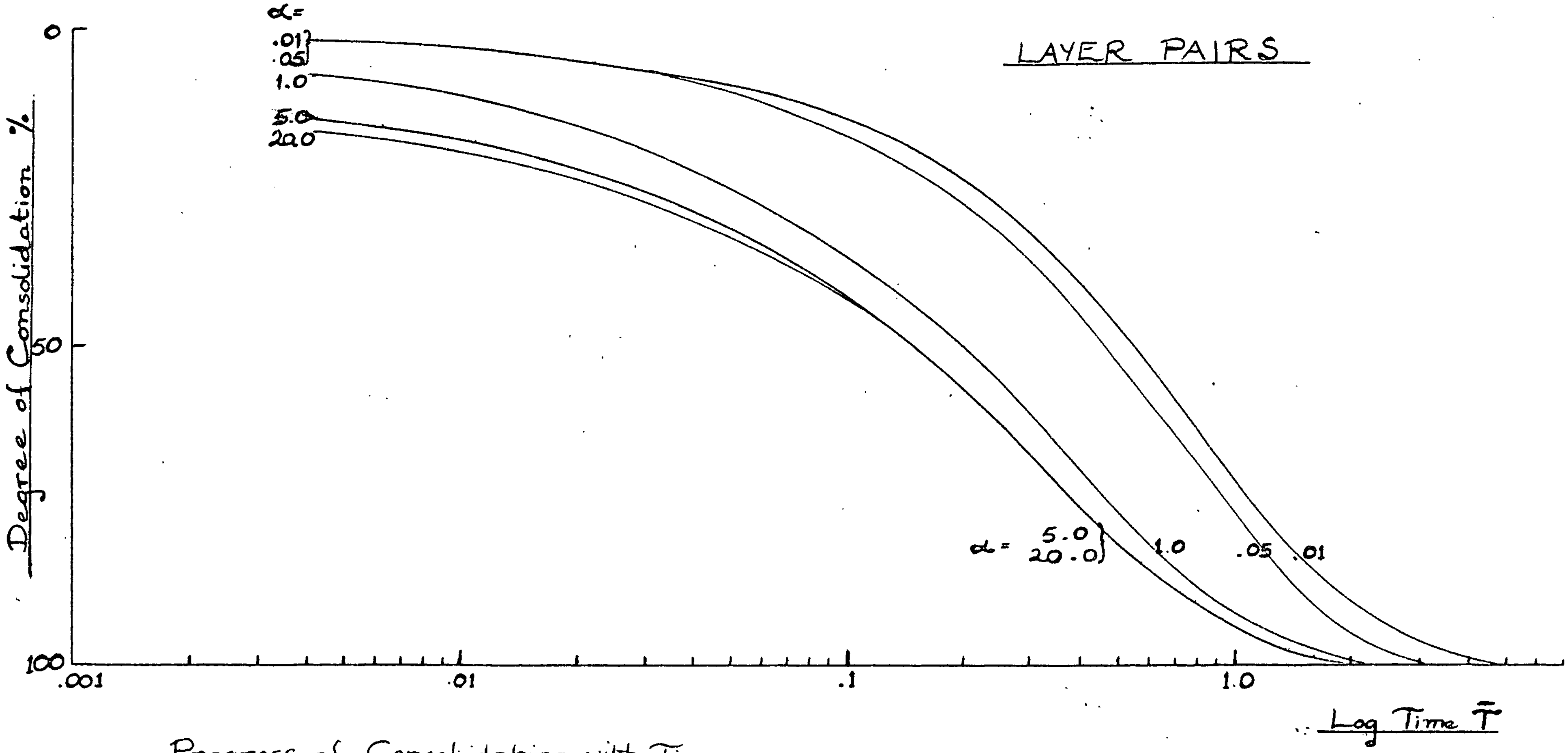
LAYER PAIRS



Progress of Consolidation with Time

Effect of α (a = 0.1 p = 1) (PTPB)

Fig 3.10



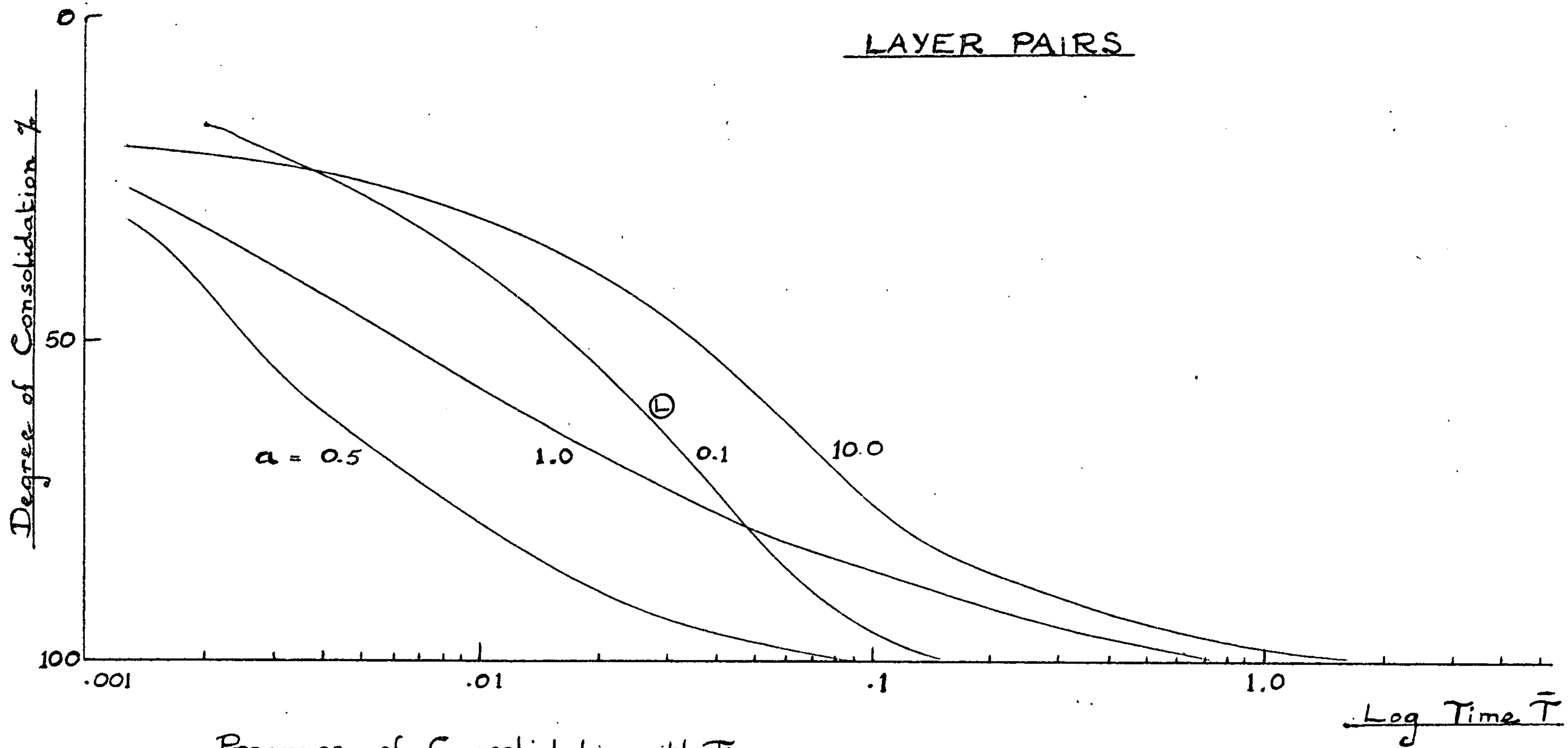
Progress of Consolidation with Time

Effect of α

(a=0.1 b=1) (PT1B)

Fig 3.11

LAYER PAIRS

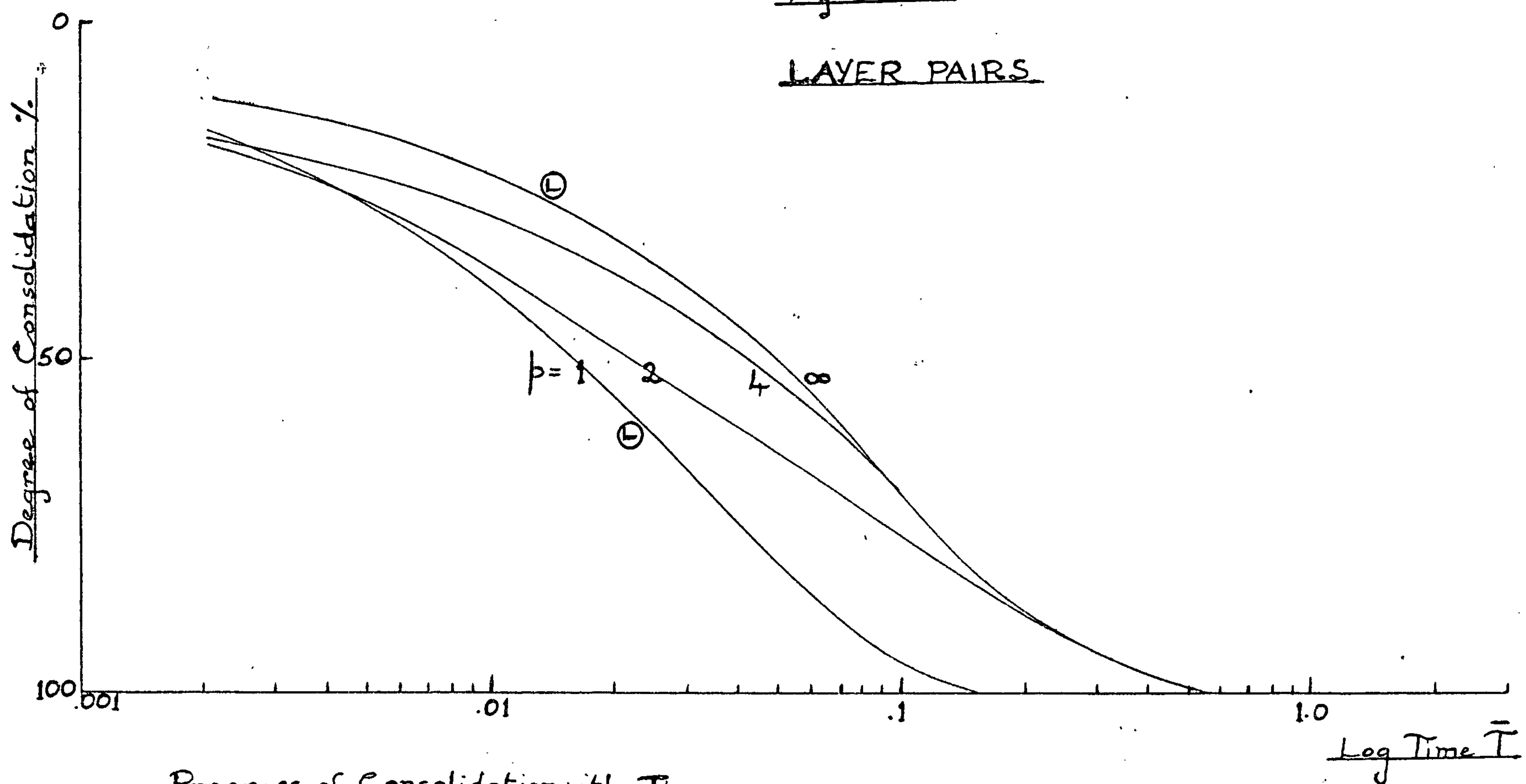


Progress of Consolidation with Time

Effect of a $(\alpha = 0{\cdot}01 \quad \beta = 1)$ (PTPB)

Fig 3.12

LAYER PAIRS



Progress of Consolidation with Time

Effect of N° of Pairs, p

(α = 0.01 a = 0.1) (PTPB)

α and rate of consolidation. Similarly graph 3.11 (although limited to 4 different values) indicates considerable complexity in any relationship. This is to be expected from the rather awkward definition of the time factor $\overline{T}$ which is required to apply to both layers, as opposed to 2 separate time factors for the different layers. Whilst this method may give useful results for practical purposes (see further discussion below) it is probably too complex a variable for much further theoretical consideration. The case of a two layer system with given properties where the only variable is the number of "layer pairs" seems rather more straightforward (see graph, Fig. 3.12). The thickness of the system is kept constant, and we consider it firstly as two layers A and B, then as two pairs of layers ABAB, identical but each layer being of ½ the previous thickness, and then as 4 layer pairs. Lee notes that the resulting rate of consolidation curve for an infinite number of layers is the same for any value of α, and furthermore is the same as for 1 layer pair in the case where $\alpha = 1$. To test this observation this curve ($p = 1$ at $\alpha = 1$) has been plotted on the graph of variation of number of pairs, p, as the result for $p = \infty$. It was found to give an excellent fit with the other results, becoming identical to the $p = 4$ case beyond about 70% consolidation. This gives us a useful assessment of the approximation involved in analysing a layered or varved clay as a finite number of pairs.

Returning now to consideration of use of the parameters a and α for the proposed approximate method, some comment must be made on Lee's contention that the shapes of the consolidation curves are sufficiently similar to justify fitting a standard curve through the point T_{50}. It is unfortunate that Lee does not include some graphs of variations in 'a' at a constant α, for the results of the present work (Fig. 3.11) show considerable variation in curve shapes. The results using constant 'a' and varying α, however, do all seem to have a fairly constant shape for the cases examined here. Obviously, though, the approximate method can lead to considerable error in the estimation of degree of consolidation and without more information on which cases it is reasonably applicable to, it cannot be considered altogether satisfactory.

Finally, the satisfactory behaviour of the program CONED 2 should be noted. Consolidation curves for the cases presented by Lee agreed well, and each test confirmed the value of $\overline{T}_{50}$ given for the appropriate values of a and α on Fig. 3.8.

CHAPTER 4

Garlanger's Theory of Consolidation

4.1 Introduction

This chapter contains details of the extension of Bjerrum's (1967) ideas on consolidation into a soluble theoretical framework as presented by Garlanger (1972). Development of the theory is followed by details of the original numerical solution technique, and two further numerical methods which are proposed here. All three of these have been programmed and shown capable of producing satisfactory results; the relative merits of the different schemes are compared.

The theory is applied to some existing long duration laboratory test results which show it to be a promising approach. The scheme clearly depends upon the validity of the parameters used, and the method of determining these is outlined. Although this seems to have been quite successful, further clarification of some points is required. There are some major problems concerning real soil behaviour and further examination of the effects of delayed consolidation would appear to be valuable. Some experimental work to these ends is presented in Chapter 5.

Some minor theoretical objections to Garlanger's theory are dealt with in the present chapter. There are also some major problems concerning assumptions made in deriving the theory, and in the meaning (and hence derivation) of several of the parameters used. It would be inappropriate to consider these here; they must be examined in the light of the available experimental evidence and of the many existing theories of soil behaviour in general, and consolidation in particular. This is done in Chapter 6.

4.2 Presentation of Theory

Bjerrum (1967) presented a graph showing the variation of e with log p at different periods of time after application of a consolidation load. This represented what he termed the "delayed consolidation" behaviour, i.e. the slow stress-strain-time behaviour of the soil which occurs in the absence of significant excess pore pressure effects. It was noted that for soils such as the alluvial deposits on many parts of the Norwegian coast the lines obtained from such analysis were reasonably straight and parallel. Garlanger (1972), following a suggestion by Hansen (1969), presented a similar graph, only assuming linear relationships between log e, log p and log t. Fig. 4.1 shows a typical presentation of consolidation behaviour using the "time-lines" approach.

Bjerrum first examined behaviour of the soil skeleton under effective stress, ignoring the time dependant drainage of water from the pores described by Terzaghi theory. Consolidation behaviour was then identified as:-

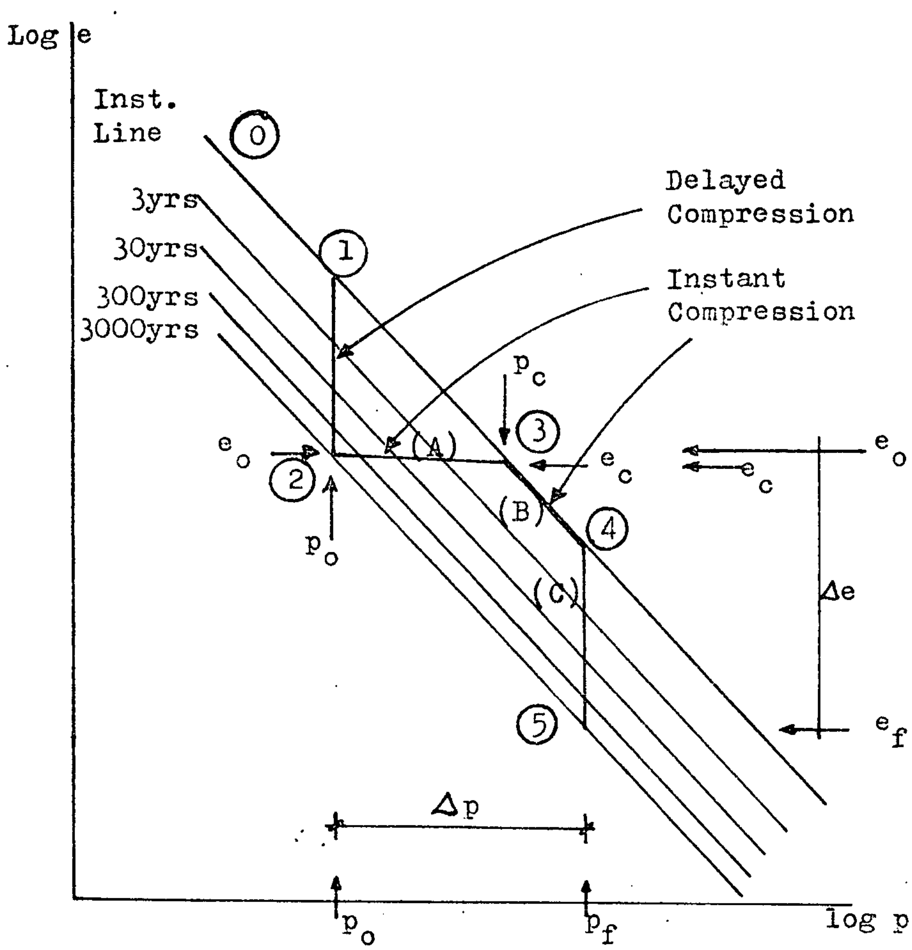
1. Instant compression - since in the absence of hydrodynamic lag this occurs instantaneously.
   a) With very little strain below some critical pressure, p_c.
   b) Parallel to the time lines above p_c.
2. Delayed consolidation - the gradual re-arrangement of the soil skeleton over a long period of time.

These types are termed A (for 1a), B (for 1b) and C (for 2), after the parameters a, b, and c respectively used to describe the stress, strain and time relationships.

From Fig. 4.1, it is clear how delayed (C) consolidation is responsible for a strengthening of the soil matrix, so that

Fig 4.1 Time Lines Diagram

(after Garlanger(1972) - Fig 1)



further load may be applied with little strain (A-consolidation) up to some critical value, p_c (which Bjerrum terms the apparent preconsolidation pressure by analogy with behaviour of an overconsolidated clay).

Garlanger has assumed that this model of the skeleton is quite valid for an element of soil in which the effective stress changes slowly over a period of time, i.e. this model may be used during a primary consolidation process, by solving the governing equations simultaneously with that for flow of the pore fluid, sometimes termed the continuity equation.

For A-type consolidation we write

$$\left(\frac{e}{e_o}\right) = \left(\frac{p}{p_o}\right)^{-a} \tag{4.1}$$

hence $$\frac{de}{dp} = -\frac{ae}{p} \qquad \left\{ = -\frac{ae}{p_o}\left(\frac{e}{e_o}\right)^{1/a} \right\} \tag{4.2}$$

For B-type consolidation

$$\left(\frac{e}{e_c}\right) = \left(\frac{p}{p_c}\right)^{-b} \tag{4.3}$$

hence $$\frac{de}{dp} = -\frac{be}{p} \qquad \left\{ = -\frac{be}{p_c}\left(\frac{e}{e_c}\right)^{1/b} \right\} \tag{4.4}$$

also $$e_c = e_o\left(\frac{p_c}{p_o}\right)^{-a} \tag{4.5}$$

Delayed, C-type consolidation is described by the equation

$$\frac{e}{e_c} = \left(\frac{p}{p_c}\right)^{-b} \left\{1 + \frac{t}{t_i}\right\}^{-c} \tag{4.6}$$

where c is the slope of the log e - log t plot during delayed consolidation, and t_i is time given to the instant line {this is considered in detail later - see, in particular, section 6.4 }.

The delayed consolidation was defined as occurring at constant effective stress, so Garlanger says the rate of delayed strain may be obtained by differentiating (4.6) with respect to time at constant pressure.

Hence
$$-\left(\frac{\partial e}{\partial t}\right)_c = c\cancel{\left(\frac{p'}{p_c}\right)^{b}} \frac{e}{(t+t_i)} \qquad (4.7A)$$

$$= \frac{ce}{t_i}\left(\frac{p}{p_c}\right)^{b/c}\left(\frac{e}{e_c}\right)^{1/c} \qquad (4.7B)$$

Note that in equation (4.7A), t is the time measured from when the soil is at (p_c, e_c). This cannot usually be determined with ease for a soil loaded from (p_o,e_o), and in any case will be different at different depths within the soil profile. Nevertheless, it is valid for all (positive and negative) values of t, and similarly (4.7B), which is used in the solution methods, is valid at all stages of the consolidation process, including pressures below p_c.

These relationships are now combined with the continuity expression

$$\frac{\partial e}{\partial t} = \frac{k\ (1+e_o)}{\gamma_w}\ \frac{\partial^2 u}{\partial z^2} \qquad (4.8)$$

In the solution of this Terzaghi used the principle of effective stress

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial t} \qquad (4.9)$$

which is, of course, still applicable here, and the stress-strain-time relationship

$$\frac{\partial e}{\partial t} = -a_v \frac{\partial p}{\partial t} \qquad (4.10)$$

where a_v is a constant value of $\frac{\partial e}{\partial p}$ termed the coefficient of compressibility. Following Taylor (1942), Garlanger considers the total strain to be comprised of two independant processes

$$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial p} \cdot \frac{\partial p}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_c \qquad (4.11)$$

This is now compatible with the previously presented stress-strain-time relationships. The first term of the R.H.S. is obtained from equation (4.2) or (4.4), depending whether p is below or above p_c, and the second term uses equation (4.7B).

The basic equations are now written in terms of dimensionless variables.

$$T = \frac{Cv\ t}{H^2} \cdot \qquad Z = \frac{z}{H} \qquad \mu = \frac{u}{\Delta p} = \{p_f - p\}/\Delta p$$

$$Cv = \frac{k\ (1+e_o)}{\gamma w\ A_v} \qquad A_v = \frac{\Delta e}{\Delta p} \qquad \beta = \frac{\{e_o - e\}}{\Delta e}$$

[See discussion of this definition of Cv in Section 6.1.]

Where Δp = the total change in effective stress

Δe = the total change in voids ratio.

The transforms used are

$$\frac{\partial\beta}{\partial T} = -\frac{\partial e}{\partial t} \cdot \frac{\partial t}{\partial T} /\Delta e = -\frac{H^2}{Cv\Delta e} \cdot \frac{\partial e}{\partial t}$$

$$\frac{\partial^2\mu}{\partial Z^2} = \frac{\partial^2 u}{\partial z^2} \cdot \left(\frac{\partial z}{\partial Z}\right)^2 /\Delta p = \frac{H^2}{\Delta p} \cdot \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial\mu}{\partial T} = \frac{\partial u}{\partial t} \cdot \left(\frac{\partial t}{\partial T}\right)/\Delta p = -\frac{H^2}{Cv\Delta p} \cdot \frac{\partial p}{\partial t}$$

$$\frac{\partial\beta}{\partial\mu} = -\frac{\partial e}{\partial u} \cdot \frac{\Delta p}{\Delta e} = \frac{\Delta p}{\Delta e} \cdot \frac{\partial e}{\partial p}$$

Now transforming equation (4.8)

$$\frac{\partial e}{\partial t} = \frac{k\ (1+e_o)}{\gamma w} \quad \frac{\partial^2 u}{\partial z^2}$$

$$-\frac{\partial\beta}{\partial T} = \left(\frac{H^2}{Cv\Delta e}\right) \quad (CvA_v) \left(\frac{\partial^2\mu}{\partial Z^2}\right) \left(\frac{\Delta p}{H^2}\right)$$

$$\underline{\underline{\frac{\partial\beta}{\partial T} = -\frac{\partial^2\mu}{\partial Z^2}}} \qquad (4.12)$$

{This is misprinted in Garlanger's text.}

From equation (4.11)

$$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial p} \cdot \frac{\partial p}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_c$$

$$-\frac{\partial\beta}{\partial T} = -\frac{H^2}{Cv\Delta e}\frac{\partial\beta}{\partial\mu}\cdot\frac{\Delta e}{\Delta p}\cdot\frac{\partial\mu}{\partial T}\cdot\frac{Cv}{H^2}\Delta p - \left(\frac{\partial\beta}{\partial T}\right)_c$$

$$\frac{\partial\beta}{\partial T} = \frac{\partial\beta}{\partial\mu}\frac{\partial\mu}{\partial T} + \left(\frac{\partial\beta}{\partial T}\right)_c$$

or, re-arranging

$$\frac{\partial\mu}{\partial T} = \left\{\frac{\partial\beta}{\partial T} - \left(\frac{\partial\beta}{\partial T}\right)_c\right\} / \frac{\partial\beta}{\partial\mu} \qquad (4.13)$$

From (4.7)

$$-\left(\frac{\partial e}{\partial t}\right)_c = \frac{ce}{t_i}\left(\frac{p}{p_c}\right)^{b/c}\left(\frac{e}{e_c}\right)^{1/c}$$

$$\left(\frac{\partial\beta}{\partial T}\right)_c = \frac{c(e_o-\beta\Delta e)}{T_i}\left\{\frac{p}{p_c}\right\}^{b/c}\left(\frac{e_o-\beta\Delta e}{e_c}\right)^{1/c} \qquad (4.14)$$

where $T_i = \frac{t_i\, Cv}{H^2}$ and $p = p_o + (1-\mu)\Delta p$

The normal boundary conditions are used - namely:

i) At a permeable, free draining boundary there is zero pore pressure

$$\mu = 0.0 \qquad (4.15)$$

It will be seen that this is insufficient for the numerical solution techniques proposed. Since the pore pressure here will clearly not vary with time, the author has included the condition

$$\frac{\partial\mu}{\partial T} = 0.0 \qquad (4.16)$$

here, which efficiently includes the effects of delayed consolidation at this boundary.

ii) At an impermeable boundary

$$\frac{\partial\mu}{\partial z} = 0.0 \qquad (4.17)$$

The average degree of consolidation is taken as the integration of the strain over the total soil depth, which may be written

$$U_j = \frac{\int_0^1 \beta_{i,j} dZ}{\int_0^1 dZ} \approx \frac{\Delta Z}{2} \{ \beta_{1,j} + 2 \sum_{i=2}^{n-1} \beta_{i,j} + \beta_{n,j} \} \qquad (4.18)$$

This, incidentally, corrects a misprint in Garlanger's paper.

4.3 Some Comments on the Theory

Garlanger's paper presents a mathematical treatment capable of solving problems with delayed consolidation. Unfortunately, however, no attempt is made to examine the theoretical implications of the equations used, or, indeed, the assumptions required to obtain these equations. A number of points here were found to be of considerable significance. In particular -

1. What exactly is the instant line? How is it determined? We note that the time lines are lines of constant secondary strain rate $(\frac{\partial e}{\partial t})_c$, not total strain {as is used by some rheologists - see e.g. Suklje (1969)}, and therefore may be obtained after primary consolidation is complete.
2. {Related to 1).} How may the critical stress, p_c, be determined? Is it considered constant, or can it be affected by such factors as sample thickness?
3. What takes place during "A"-type consolidation, and how is 'a' determined?
4. Is Bjerrum's representation of soil behaviour in terms of time lines valid? Can behaviour be uniquely described by such a diagram?

Such considerations have an important bearing on the value of the theory for prediction purposes. Some points could be made at this stage, but it was thought preferable to consider all aspects of these problems at a later stage, when further theoretical treatments and the results of experimental work could be taken into account. Section 4.6.1 (below) shows the techniques used initially to obtain the necessary parameters, and this also indicates the sections of Chapter 6 in which specific points are discussed.

Some points may be disposed of quite easily here. The change from e to log e linear relationships with log p, and log t, appears to be acceptable. Plotting out sets of data for a variety of soils over large ranges of e, p and t, of possible practical significance suggested either could be fitted with reasonable straight lines, and to within the accuracy available from current test methods, there were no grounds for preferring either e or log e plots.

Although e - log p and e - log t relationships have been more commonly used, several authors have used an exponential stress-strain relationship of the form

$$\Delta p = D \ \{\Delta e\}^n \qquad (4.19)$$

whence, taking logarithms

$$\log \Delta p = D + n \log \Delta e$$

which is of the same type as Garlanger's stress-strain relationships. The fundamental difference is that D is no longer considered a constant; it incorporates the effects of delayed consolidation.

The presentation of section 4.2 (above) is believed to be the simplest logical summary of Garlanger's ideas, and the equations resulting therefrom. However, it must be pointed out that Garlanger's text is not satisfactory as it stands. He attempts to present the several types of consolidation together in one equation

$$\left(\frac{e}{e_o}\right) = \left(\frac{p_c}{p_o}\right)^{-a} \left(\frac{p_f}{p_c}\right)^{-b} \left(\frac{t_i + t}{t_i}\right)^{-c} \qquad \text{(Gr.Eqn. 7)} \qquad (4.20)$$

It can only be assumed that p_f should read p for $p > p_c$; and for $p < p_c$

$$\left(\frac{e}{e_o}\right) = \left(\frac{p}{p_o}\right)^{-a} \left(\frac{t_i + t}{t_i}\right)^{-c} \qquad (4.21)$$

Now, these give e as a function of p and t, i.e. given p and t, e may be determined directly. This is clearly not what Garlanger intended, nor is it what he uses to derive solutions. Gibson and Andrews (1975) have noted that the above equations "are not proper constitutive relations for the clay structure" and, in fact, he has really used

$$de = - \langle a,b \rangle \; \frac{e}{p} \; dp - \frac{ce}{t_i} \left(\frac{e}{e_c}\right)^{1/c} \left(\frac{p}{p_c}\right)^{b/c} dt \qquad (4.22)$$

where $\langle\ \rangle$ is a singularity bracket; a applying below p_c, and b above.

It proved possible to develop the theory in section 4.2 without reference to the 'red herring' of equation (4.20), but it may be seen that the correct relationship (4.22) is the combination of equations (4.2), (4.4) and (4.7B).

4.4 Solutions of Garlanger's Equations

The equations were originally solved using an explicit finite difference scheme, basically as described by Garlanger. It became clear that for practical purposes such a scheme was rather heavy on computer time and consequently two further numerical techniques have been investigated. Computer flow diagrams for all three methods are presented in Appendix A.

It should be noted that the equations are quite complex, involving parameters raised to experimentally determined powers, and analytical solution is out of the question here.

4.4.1 The Explicit Method

The following finite difference expressions are used for the derivatives

$$\frac{\partial \beta}{\partial T} = \{\beta_i^+ - \beta_i\}/\Delta T \tag{4.23}$$

$$\frac{\partial^2 \mu}{\partial z^2} = \{\mu_{i-1} - 2\mu_i + \mu_{i+1}\}/\Delta z^2 \tag{4.24}$$

$$\frac{\partial \mu}{\partial T} = \{\mu_i^+ - \mu_i\}/\Delta T \tag{4.25}$$

The following steps summarise the procedure:

i) Initial Conditions are set, $\mu_i = 1.0$ $\beta_i = 0.0$

At the drainage boundary, $\mu_1 = 0.0$ and $\beta_1 = (e_o - e_1)/\Delta e$

where $e_1 = e_o \; (p_f/p_o)^{-a}$ if $p_f < p_c$

or $e_1 = e_c \; (p_f/p_c)^{-b}$ if $p_f \geq p_c$

ii) At each node, determine $(\frac{\partial \beta}{\partial T})_c$ from equation (4.14).

iii) Equation (4.12) gives $\frac{\partial \beta}{\partial T} = - \frac{\partial^2 \mu}{\partial z^2}$ which may be determined at each node from equation (4.24).

iv) $$\frac{\partial \beta}{\partial \mu_i} = \frac{1}{A_v} \frac{ae_i}{p_i} \quad \text{for } p_i < p_c \qquad (4.26A)$$

$$= \frac{1}{A_v} \frac{be_i}{p_i} \quad \text{for } p_i \geq p_c \qquad (4.26B)$$

where $e_i = (e_o^{\bullet} - \beta_i \Delta e)$

v) Knowing $\frac{\partial \beta}{\partial T}$, $(\frac{\partial \beta}{\partial T})_c$ and $(\frac{\partial \beta}{\partial \mu})$; $\frac{\partial \mu}{\partial t}$ is calculated using equation (4.13).

Garlanger here included the condition

$$\text{IF} \left(\frac{\partial \beta}{\partial T}\right)_c \geq \frac{\partial \beta}{\partial T} \text{ then } \frac{\partial \mu}{\partial T} = 0 \qquad (4.27)$$

which has, he says, been observed in both field and laboratory tests.

vi) Using equations (4.23) and (4.25), μ and β may now be determined at the new time {see comment below on size of step ΔT}.

vii) The average degree of consolidation is computed from equation (4.18).

viii) Repeat from step ii) until desired end condition reached. (Normally 100% consolidation reached, or some specified time reached.)

Now as for all explicit schemes of this type, there is a critical value of time step above which the equations will become unstable. The present study confirms Garlanger's solution by trial, that

$$\frac{\Delta T}{\Delta Z^2} \leq \frac{3}{4} \left(\frac{\partial \beta}{\partial \mu}\right)_{min} \qquad (4.28)$$

Garlanger omitted to mention that the minimum value of $\frac{\partial \beta}{\partial \mu}$ at any time step must be used, since this varies with depth.

More significantly, no mention was made of the order of errors produced by this scheme. It would appear that while equation (4.28) is sufficient for stability, excessive truncation errors can occur at early times. These errors depend upon the rate of change of the derivatives, so that at later stages of consolidation the limitation of stability on the time step ensures that these will not be significant (see method of dealing with truncation errors for the semi-implicit scheme (4.4.2), below).

A coefficient α was introduced into the time step criterion

$$\Delta T = \frac{3}{4} \times \left(\frac{\partial \beta}{\partial \mu}\right)_{min} \times \Delta Z^2 \times \alpha \qquad (4.29)$$

An investigation of the behaviour of the scheme then indicated that a value of α = 0.1 leads to sufficient accuracy for practical purposes (errors in μ and β are confined to the 4^{th} decimal place). This additional restriction on time step quickly becomes unnecessary, and α is set to 1 once 20% consolidation is reached.

The procedure as outlined above does not include the term for delayed consolidation at the free drainage boundary. The easiest way to include this, as mentioned in 4.2 above, is to include here the condition

$$\frac{\partial \mu}{\partial T_1} = 0 \qquad (4.16\ bis)$$

Substituting in 4.13, we may write

$$\frac{\partial \beta}{\partial T_1} = \left(\frac{\partial \beta}{\partial T}\right)_{c_1} \qquad (4.30)$$

which is included in the procedure at step vi).

The scheme was programmed in the standard manner of the present study, reading data from a file, and outputting numerical results to another file, specified by the user. Graphics facilities were added to give options of plots of %C against time, and %C against log time.

The allowable time step is very small, so that a large number of iterations are required, and it is normally only necessary to print out every 100^{th} iteration to obtain acceptable description of the consolidation process. Typical running time is in the order of

$2\frac{1}{2}$ minutes c.p.u. time - which on a time-sharing terminal can be around half an hour to obtain results. This is not too satisfactory from the point of view of convenience and also of cost.

Two implicit schemes are therefore examined below. The need to solve such complex equations simultaneously at many space nodes will clearly add significantly to the computation time per step. The efficiency of such schemes will therefore depend on the extent to which the time step may be increased over that required for stability in the explicit scheme. It was shown for the Terzaghi theory scheme (Chapter 2) that very large steps could be used at large times. Here we are interested in delayed consolidation continuing far beyond the end of the primary process, so implicit techniques may be expected to be of particular value for this.

4.4.2 The Semi-Implicit Scheme

A fully implicit finite difference scheme would require all the partial derivatives to be replaced by suitable expressions in terms of values of the variables at old and new times. This would clearly be quite involved, especially in the term for $(\frac{\partial \beta}{\partial T})_c$, leading to a very complicated matrix of simultaneous equations to be solved. It would therefore be expected to be rather time-consuming, if, indeed, suitable expressions could be found.

It is, however, possible to develop a scheme which is implicit in character, if the explicit expressions for $\frac{\partial \mu}{\partial \beta}$ and $(\frac{\partial \beta}{\partial T})_c$ are retained. The rest of the derivatives are expressed as central differences over the time step.

In equation (4.13) we may substitute for $\frac{\partial \beta}{\partial T}$ from (4.12) giving

$$\frac{\partial \mu}{\partial T} = \{- \frac{\partial^2 \mu}{\partial z^2} - (\frac{\partial \beta}{\partial T})_c \}/\frac{\partial \beta}{\partial \mu} \qquad (4.31)$$

or, rearranged for the present work,

$$\frac{\partial \mu}{\partial T} = \{- \frac{\partial^2 \mu}{\partial z^2} \cdot \frac{\partial \mu}{\partial \beta} - (\frac{\partial \beta}{\partial T})_c \cdot \frac{\partial \mu}{\partial \beta}\} \quad (4.32)$$

The well known Crank-Nicholson midpoint difference expression is written for $\frac{\partial^2 \mu}{\partial z^2}$

$$\frac{\partial^2 \mu}{\partial z^2} = \{\mu^+_{i-1} - 2\mu^+_i + \mu^+_{i+1} + \mu_{i-1} - 2\mu_i + \mu_{i+1}\}/2\Delta z^2 \quad (4.33)$$

Substitute this and equation (4.25) in (4.32) gives

$$\mu^+_i - \mu_i = - \frac{\Delta T}{2\Delta z^2} \frac{\partial \mu}{\partial \beta_i}\{ \mu^+_{i-1} - 2\mu^+_i + \mu^+_{i+1} + \mu_{i-1} - 2\mu_i + \mu_{i+1}\}$$

$$-\Delta T \frac{\partial \mu}{\partial \beta_i} \cdot (\frac{\partial \beta}{\partial T})_{c_i} \quad (4.34)$$

Since $(\frac{\partial \mu}{\partial \beta})_i$ may be determined from the explicit equation (4.26A) or (4.26B) depending whether p_i is below or above p_c, we may write

$$Ki = - \frac{\Delta T}{\Delta z^2} \quad (\frac{\partial \mu}{\partial \beta})_i \quad (4.35)$$

Writing this in (4.34) and rearranging, gives

$$-Ki\mu^+_{i-1} + 2(1+Ki)\mu^+_i - Ki\mu^+_{i+1} = Ki\mu_{i-1} + 2(1-Ki)\mu_i + Ki\mu_{i+1}$$

$$- 2\Delta T \frac{\partial \mu}{\partial \beta_i} (\frac{\partial \beta}{\partial T})_{c_i} \quad (4.36)$$

This is now in a standard tridiagonal form whereby the terms of the left hand side may be solved for simultaneously.

All the terms of the right hand side are known and may be used to calculate a single value d_i at each node in space. It has been found most efficient to obtain as many constant parameters as possible, and this is done by dividing through by Ki

$$-\mu^+_{i-1} + C_L\mu^+_i - \mu^+_{i+1} = \mu_{i-1} + C_R\mu_i + \mu_{i+1} + 2\Delta z^2 (\frac{\partial \beta}{\partial T})_{c_i} \quad (4.37)$$

where $C_L = \frac{2(1+Ki)}{Ki}$. $C_R = \frac{2(1-Ki)}{Ki}$ (4.38A & B)

Written in matrix form for all the nodes this gives

$$[B]\ \{\mu^{+}\} = \{D'\} \qquad (4.39)$$

The boundary conditions give $\mu = 0$ at the top

and $\frac{\partial \mu}{\partial z} = 0$ at the bottom,

this latter being dealt with by writing $\mu^{+}_{n+1} = \mu^{+}_{n-1}$ for a fictitious reflective point n + 1.

The final matrices are thus:

$$\begin{matrix} I = 1 \\ 2 \\ 3 \\ \\ \\ n-1 \\ n \end{matrix} \begin{bmatrix} 0 & C_L & -1 \\ -1 & C_L & -1 \\ -1 & C_L & -1 \\ & & \\ & & \\ -1 & C_L & -1 \\ -2 & C_L & 0 \end{bmatrix} \begin{Bmatrix} \mu^{+}_1 \\ \mu^{+}_2 \\ \mu^{+}_3 \\ \\ \\ \mu^{+}_{n-1} \\ \mu^{+}_n \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \\ \\ d_{n-1} \\ d_n \end{Bmatrix} \qquad (4.40)$$

where

$$d_i = \underset{[D]}{[1\ C_R\ 1]} \begin{Bmatrix} \mu_{i-1} \\ \mu_i \\ \mu_{i+1} \end{Bmatrix} + \left[2.\Delta z^2.\left(\frac{\partial \beta}{\partial T}\right)_{c_i}\right] \qquad (4.41)$$

The appropriate lines in [D] for permeable top and impermeable bottom boundaries respectively are

$[0 \quad C_R \quad 1]$ and $[2 \quad C_R \quad 0]$

These equations are now solved by the Gaussian Elimination technique.

With the new μ values determined we may now go back to equation (4.12) to obtain β values. Again replacing $\frac{\partial^2 \mu}{\partial z^2}$ and $\frac{\partial \beta}{\partial T}$ with the

Crank-Nicholson expressions, we obtain

$$\frac{\beta_i^+ - \beta_i}{\Delta T} = - \{\mu_{i-1}^+ - 2\mu_i^+ + \mu_{i+1}^+ + \mu_{i-1} - 2\mu_i + \mu_{i+1}\}/2\Delta Z^2 \quad (4.42)$$

Then, on rearranging

$$\beta_i^+ = \beta_i - \frac{\Delta T}{2\Delta Z^2} \{\mu_{i-1}^+ - 2\mu_i^+ + \mu_{i+1}^+ + \mu_{i-1} - 2\mu_i + \mu_{i+1}\} \quad (4.43)$$

which may readily be evaluated.

At the top boundary we use the further condition that $\frac{\partial\mu}{\partial T} = 0$, and simply write

$$\frac{\beta_i^+ - \beta_i}{\Delta T} = \left(\frac{\partial\beta}{\partial T}\right)_c \quad (4.44)$$

from equation (4.13), which leads to

$$\beta_1^+ = \beta_1 + \Delta T \left(\frac{\partial\beta}{\partial T}\right)_{c_1} \quad (4.45)$$

The average degree of consolidation is computed as before from equation (4.18).

This scheme was found to function well, without the limiting requirement of stability used in the explicit scheme. Truncation errors are the major problem and the scheme is rather sensitive to these. In the space dimension these were kept satisfactory by use of 10 elements; i.e. eleven nodes. The main problem is in the choice of time step. It has been mentioned that this may increase dramatically as the rate of change of the relevant derivatives (in this case $\frac{\partial\mu}{\partial T}$) falls. {See discussion of the similar problem in the implicit scheme for Terzaghi-type consolidation - section 2.12.} For equations of this type the best approach was felt to be a logarithmic increase in time step with time, using empirically determined constants to govern this.

The new time is calculated from the expression

$$T_v = 10^{tx^+} \quad (4.46)$$

where

$$tx^{+} = tx + \Delta tx \qquad (4.47)$$

It was found satisfactory to use an initial value (i.e. for the first time step) of

$$tx_{o} = - 4.0 \qquad (4.48)$$

and an increment of

$$\Delta tx = 0.025 \qquad (4.49)$$

i.e. 40 steps per log cycle of T .

The accuracy obtained from this time step criterion will clearly vary with the case to be solved, but it is believed to be always sufficient for practical purposes. A large number of widely varying cases have been run with errors in μ values confined to the 4^{th} decimal place. The constants in (4.47) have, in fact, been set well to the conservative side of what is deemed necessary, since the computer time is so low that efficiency here is not too important a factor.

The scheme has, in fact, proved remarkably efficient to run, typically taking under 10 seconds of cpu time.

Graphical output has been included in the program, and the user may view the average degree of consolidation plotted against linear or logarithmic real time.

A couple of minor modifications have been programmed to allow analysis of the special cases of a = 0, and c = 0, which the author wished to examine briefly. This simply involves the program in testing for such cases and skipping, or correctly evaluating, some expressions which would otherwise fail with zero values.

The scheme has been extended to allow for time dependant loading, and this is described in section 4.7 below.

4.4.3 A Fully Implicit Scheme   The Method of Lines

The development of a fully implicit and mathematically rigorous scheme for solution of equations of such complexity as those considered here, is a fairly sophisticated problem in Numerical Analysis, and the author was fortunate to receive invaluable assistance here from Dr. J.M. Aitchison of the Department of Applied Mathematics of this University.

At the present state of knowledge several suitable schemes are available for treatment of ordinary differential equations (O.D.E.s), but no satisfactory methods are available for partial differential equations (P.D.E.s) of the type considered here. It is, however, possible to reduce the P.D.E.s to O.D.E.s in a given solution plane by replacing other derivatives with finite difference expressions. This is the principle behind the Method of Lines. Once this is done a suitable library program may be used for the integration of complex O.D.E.s.

The library routine chosen solves large sparse matrices by a numerical technique due to Gear (1971), and goes by the name JMAG1O. The driver program, MAIN, reads in parameters, does all the necessary setting of initial data, and passes over all the information required (including an array of output times)when JMAG1O is called. From then on, control is in the hands of this library program which calls on two subroutines as and when needed.

The first subroutine, DIFFUN, defines the differential equations. The second, OUTPUT, controls the format of the output results.

DIFFUN considers two arrays: Y, which contains the dependant variable, and DY, which contains its derivatives. It is clearly most sensible here to consider the O.D.E.s in the time domain, so DY is made the derivative with respect to time. From the method of lines, the space derivatives are therefore replaced by finite difference terms. This will be done shortly. First, however, there is the complication that we are here dealing with two dependant variables, β and μ. Since these must be solved for simultaneously, both must be included in the Y matrix. This is handled by storing μ in locations 1 to NZ (here represented as Y(I)) and β in locations NZ + 1 to 2NZ (shown here as Y"(I)). Similarly for the time derivatives, DY(I) indicates $\frac{\partial\mu}{\partial T}$ in locations 1 to NZ, and DY"(I), $\frac{\partial\beta}{\partial T}$ in NZ + 1 to 2NZ.

Referring to equations (4.12) and (4.13) above, and noting that equations (4.14) and (4.26) already give satisfactory terms for $(\frac{\partial\beta}{\partial T})_c$ and $\frac{\partial\beta}{\partial\mu}$, there is just one derivative with respect to Z that requires a finite difference expression. The simplest second order term may be used -

$$\frac{d^2\mu}{dZ^2} = \{Y(I - 1) - 2Y(I) + Y(I + 1)\}/\Delta Z^2 \qquad (4.50)$$

(for I = 2 to NZ - 1).

Boundary conditions are handled in the normal manner - viz.

$$\left(\frac{d^2\mu}{dZ^2}\right)_1 = \{2Y(1) + Y(2)\}/\Delta Z^2 \qquad (4.51A)$$

and

$$\left(\frac{d^2\mu}{dZ^2}\right)_{NZ} = \{2Y(NZ - 1) - 2Y(NZ)\}/\Delta Z^2 \qquad (4.51B)$$

at the free drainage and impermeable boundary respectively.

We thus have the equations in O.D.E. form. Equations (4.12) and (4.13) may be written in the required derivative form. {(4.13), however, includes $\frac{d\beta}{dT}$ from the R.H.S. and since we are solving for this, it is replaced by $-\frac{d^2\mu}{dZ^2}$ from equation (4.12).}

Hence

$$DY = \frac{d\mu}{dT} = -\{\frac{d^2\mu}{dZ^2} + (\frac{d\beta}{dT})_c\}/\frac{d\beta}{d\mu} \qquad (4.52)$$

$$DY'' = \frac{d\beta}{dT} = -\frac{d^2\mu}{dZ^2} \qquad (4.53)$$

The library program uses a matrix manipulation technique to determine new values of the variable Y at each time step. An estimate of the truncation errors involved is also obtained here. This is compared with a specified accuracy criterion, and if excessive the time step is reduced and the step repeated. For present purposes an accuracy of 0.001 in Y was considered acceptable, so this has been incorporated in the program, but may easily be modified if desired.

Whenever a specified output time is passed back-interpolation is used to determine the required results. These are then output to a results file as specified by the user.

The scheme is again rather heavy on computer time. It typically takes around 2 minutes of c.p.u. time, which is little better than the Explicit method, although the results can now be guaranteed to the desired accuracy. The number of steps is considerably better than for the explicit scheme, but the amount of work in the simultaneous solution of each equation at each node is quite substantial.

4.5 Checking of Programs, and their Comparison

It is essential to ensure that the programs are functioning satisfactorily. First of all the three methods may be compared with each other. This has been done for many different problems, including checks for the experimental results described below and in Chapter 5. This has shown complete agreement, within the limits of numerical accuracy, between the different schemes.

It would have been desirable to check results against a published solution, but unfortunately Garlanger has not given sufficient data to reproduce any of his results. However, the author discovered that a program for the explicit method presented by Garlanger, had been written by Mr. K.R.F. Andrews, of King's College, London. He kindly agreed to run a sample set of data {for Grangemouth Clay - see Figs. 4.6&4.7 - pressure increments 8.75 to 17.5 psi}. Excellent agreement was again obtained.

It was anticipated that considerable improvements in running times could be achieved by use of implicit schemes, but these might be hampered if excessive calculation were required at each step. The efficiencies of the 3 schemes outlined above demonstrate this. The semi-implicit scheme, by keeping the simultaneous equations to simple tri-diagonal form, is by far the most efficient. On the other hand the accuracy is not proven. The author believes that used intelligently this is the most efficient and generally acceptable solution method for Garlanger's equations {it uses some $\frac{1}{25}$th to $\frac{1}{20}$th of the computer time required for the others}. Until its use is proven over a large number of cases, however, it should be checked occasionally against the Method of Lines solution. (During the present work it has also been checked against the Explicit scheme but this is clearly superfluous for practical purposes.)

4.6 Comparison of Theory with Some Previous Test Results

The first step in assessing the validity of this approach consisted of applying the theory to some experimental data due to Christie (1963) who had performed tests of up to 1 month duration on three soils which had displayed pronounced secondary consolidation behaviour. The existing analyses had shown that Gibson and Lo's theory led to rather better agreement than Terzaghi theory, but the treatment of pore water pressures was still far from satisfactory.

4.6.1 Method of Obtaining Parameters

The determination of the governing parameters is clearly crucial to the successful use of any theoretical approach. The full mechanics of these calculations are therefore presented for a specimen case, below. It must be stressed that these techniques are the best available treatment of the existing results, but better determination of some parameters may be obtained from other tests. Experimental investigation and full discussion of these topics follows in Chapter 5 and 6.

The available data consists of settlement and base pore pressure readings from a series of tests of 1 month duration. Deflexions are converted into e values by the "height of voids" technique. The time lines are now plotted (fig. 4.2) using the data from table 4.1. This data also gives the log e -log t relationship plotted on fig. (4.4).

The other information required is the log e - log p path followed by the soil over any increment of loading. This requires some assumption about the pressure distribution within the soil profile. It seems that the average stress at any instant must be used, and this is determined from the assumption that the pore pressure distribution is parabolic so that its average value is 2/3 of the value at the impermeable boundary. This approximate technique has been plotted on fig. (4.3) using the data of table 4.3. It was found to yield surprisingly good results. Nevertheless, the use of this is not altogether satisfactory. A preferable approach is given in section 6.4. Here we simply note that the technique of using parabolic isochrones is not valid at the earliest times, before pore pressure at the base has begun to fall.

The following list shows the parameters required and the method of obtaining these:

NZ Number of space elements for numerical work. Chosen by user to satisfy accuracy requirements. For most purposes NZ = 10 is used; this is found to give a good balance between accuracy and computer time.

Z Thickness of layer - at beginning of increment.

Cv The Coefficient of Consolidation describes the process of diffusion of pore fluid from the system. It should be determined by fitting strains, using Taylor or Casagrande method. (* 6.1 - see note p. 101).

Δe Change in voids ratio over increment. The results give degree of consolidation defined as a percentage of Δe. Since delayed consolidation is an ongoing process, some care is required in the choice of Δe which should be specified for a given period after loading commenced.

Δp Increment of Applied loading.

a Stress-strain constant below the critical pressure. Obtained by drawing tangent to the initial portion of the curve (4.3). Garlanger recommends determination from loading a sample in small increments up to p_c. (* 6.3).

b Stress-strain constant above the critical pressure. The slope of the time lines, obtained from fig. (4.2). (* 6.2).

c Strain-time constant for the soil independent of primary consolidation. Hence it is obtained from the log e - log t plot after the excess pore pressures have dissipated (fig. 4.4). (* 6.2).

t_i The time given to the instant line. This is best considered as the value required to correctly position the time-lines grid on the log e - log p plot. Garlanger does this by specifying the time line which passes through the point p_c. Its value is determined by drawing this time line, finding the e value where this intersects the line of constant final stress p_f, (from fig. 4.3) and hence determining the corresponding time on fig. (4.4). (* 6.4).

p_o Initial effective stress before loading applied.

p_c The critical pressure. The pressure at which the stress-strain constant changes from 'a' to 'b'. There is considerable difficulty about just what this means. Garlanger recommends loading a thin sample in small increments until the change occurs. For present purposes this data was not available, so the instant line was taken as a line of slope 'b' tangential to the log e - log p curve, and p_c is taken as the intersection of this with the line of slope a (see fig. 4.3). This important point needs far more clarification. (* 6.4).

e_o Initial voids ratio.

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* The techniques used here which are not altogether satisfactory, or need further discussion, are marked thus, *....., showing the section where more information may be found.

Table 4.1

Data for Construction of Time Lines Graph - Fig 4.2

Soil - Grangemouth (after Christie(1963))

Height of Soil Particles h_s = .3380 ins

Sample Thickness 2H = .9600 ins

Pressure p.s.i.	Time days	Defl^n d ins	Thickness 2H ins	Voids Ratio e	log e
0 –	–	.0512	.9088	1.69	0.228
8.75	1	.2036	.7564	1.28	0.107
	7	.2134	.7466	1.21	0.083
	14	.2175	.7425	1.20	0.079
	21	.2190	.7410	1.195	0.077
	24	.2202	.7398	1.190	0.0755
8.75 –	1	.2564	.7036	1.082	0.033
17.5	7	.2638	.6962	1.060	0.025
	14	.2662	.6938	1.053	0.0229
	21	.2670	.6930	1.050	0.0213
	23	.2673	.6928	1.049	0.0211
17.5 –	1	.3042	.6558	0.940	$\bar{1}.973$
35.0	7	.3093	.6507	0.925	$\bar{1}.966$
	14	.3111	.6489	0.920	$\bar{1}.964$
	21	.3123	.6477	0.916	$\bar{1}.962$
	28	.3134	.6466	0.913	$\bar{1}.960$
	33	.3144	.6456	0.910	$\bar{1}.959$

Table 4.2

Additional Data for log e - log t Graph - Fig 4.4

t(days)	1	7	14	21	28	33
t(mins)	1440	10100	20200	30200	40300	47500
log t	3.160	4.001	4.305	4.480	4.605	4.680

Table 4.3

Data for log p - log e plot fig 4.3

Load Increment 8.75 - 17.5 p.s.i.

Time	d	2H	e	log e	u	pav	log(pav)
0	.2202	.7398	1.190	0.0755			
$2\frac{1}{2}$	.2252	.7350	1.175	0.070	5.49	13.84	1.141
10	.2274	.7326	1.168	0.067	4.96	14.19	1.152
20	.2296	.7304	1.162	0.065	4.26	14.66	1.166
40	.2328	.7272	1.152	0.0615	3.60	15.10	1.179
65	.2358	.7242	1.144	0.0585	2.95	15.53	1.191
100	.2389	.7211	1.134	0.0545	2.30	15.92	1.203
180	.2432	.7168	1.122	0.0500	1.44	16.54	1.2185
270	.2464	.7136	1.111	0,045	0.97	16.85	1.227
440	.2511	.7089	1.099	0.041	0.55	17.14	1.234
755	.2544	.7056	1.084	0.036	0.10	17.43	1.241
						17.50	1.243

Table 4.4

Values Used in Data File

NZ=10 Z=0.7400 Cv=0.0020

Δe=0.140 Δp=8.75

a=0.015 b=0.20 c=0.0088 ti=13.8

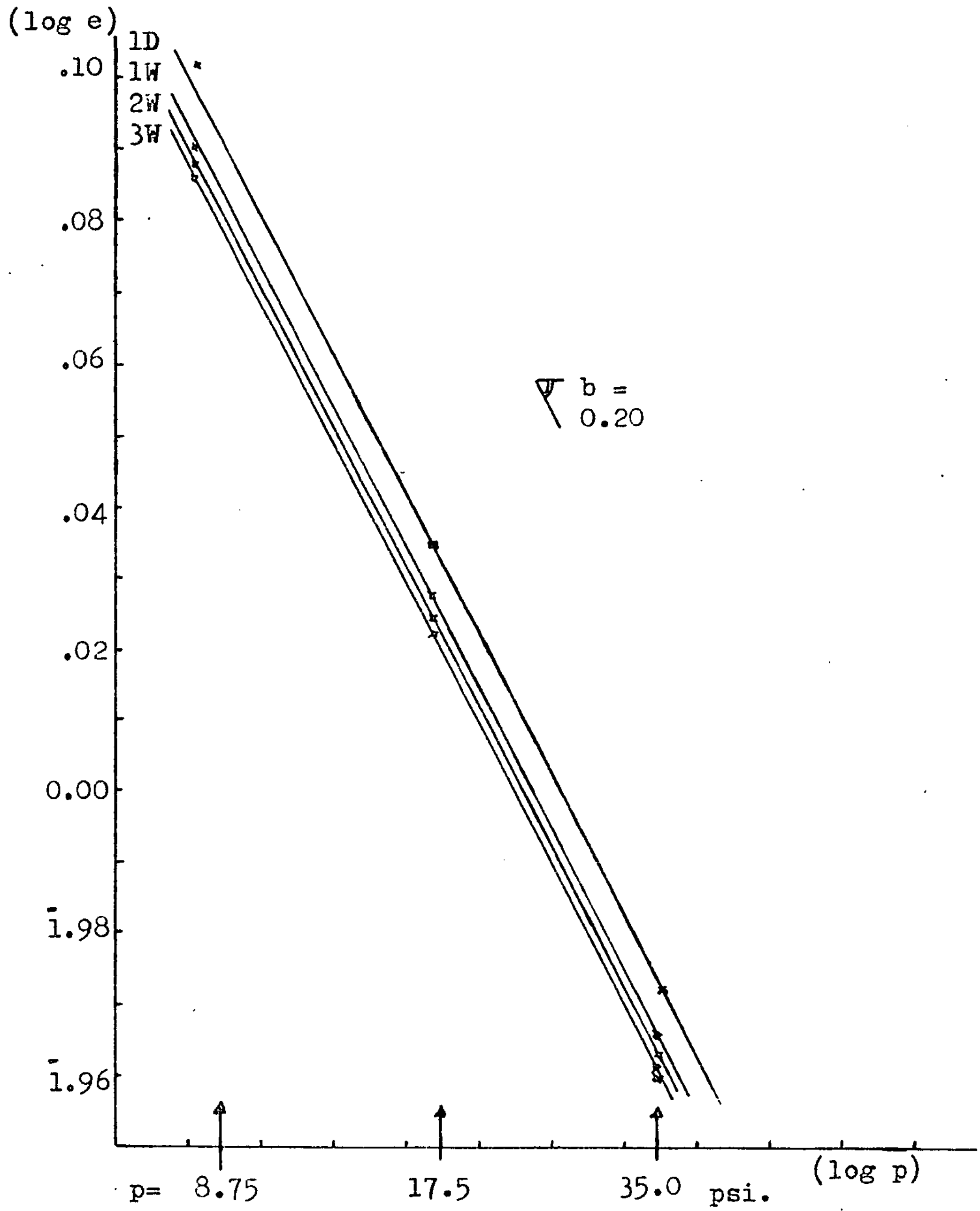
po=8.75 pc=13.40 eo=1.190

Notice that thickness is in inches, and time in minutes. Cv must therefore be given in units of ins^2/min

Fig 4.2 Garlanger Theory - Determination of Parameters

Grangemouth

Time Lines Diagram



Log e - Log p Plot for Series of Oedometer Tests

Fig 4.3 Garlanger Theory - Determination of Parameters

Log e- Log p Plot

Grangemouth Increment 8.75 - 17.5 p.s.i.

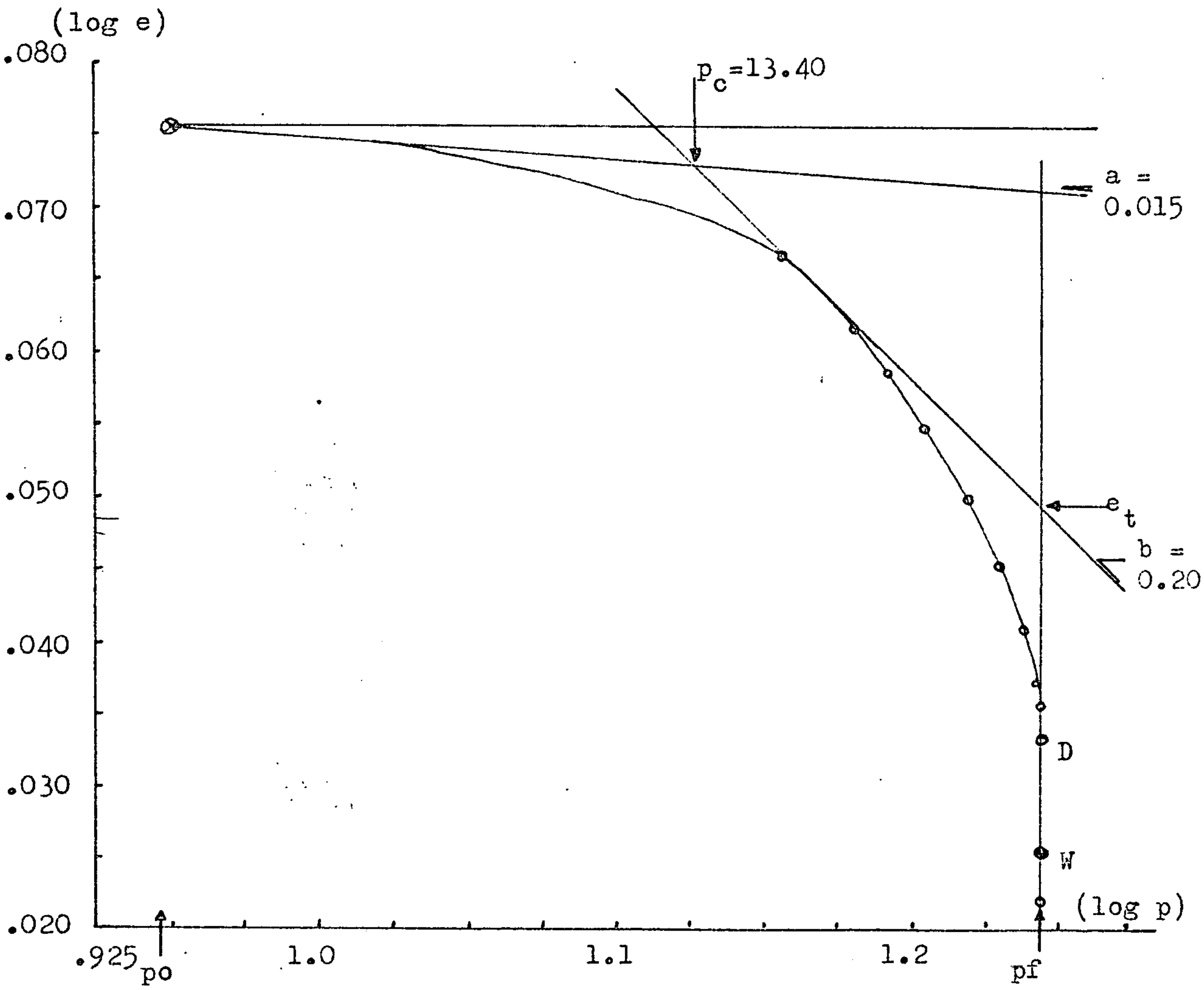


Fig 4.4 Garlanger Theory - Determination of Parameters

Grangemouth

Log e - log t Plot

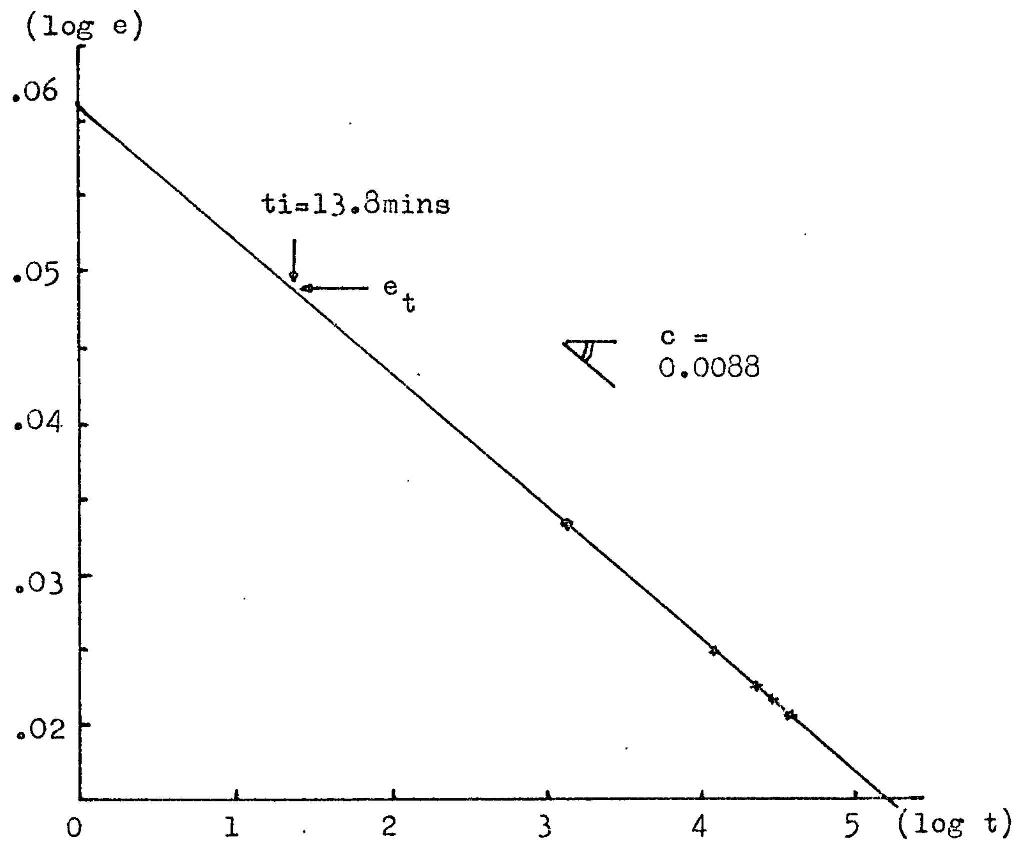


Table 4.4 shows the values of parameters required in the data file for numerical analysis. Real time is used here, and the most convenient units are minutes. Hence t_i is quoted in minutes. Length, z, is given in inches, so the Cv value must be in ^{2}/min. (The value shown here was obtained from Christie's results.) Pressures must be consistent (here in psi) but the units are immaterial since results are expressed as percentages of Δp. Parameters a, b, and c are dimensionless, and hence not affected by the pressure units chosen.

4.6.2 Grangemouth

The results for the log e - log p diagram (see fig. 4.2) do give reasonably parallel, straight time-lines. {The 1 day point for the first increment is rather high, which is because not all the primary consolidation was completed here.} The necessary parameters for Garlanger analysis were obtained without complication, leading to the results shown in figs. 4.5 - 4.9.

It appears that Terzaghi theory is fairly satisfactory for the initial increment of loading, but not so where the soil has undergone a significant amount of delayed consolidation prior to application of a load increment as in the other two cases here. The graph of dimensionless pore pressure at the impermeable boundary against dimensionless degree of consolidation is used extensively in the present study. For Terzaghi theory there is a unique relationship between these variables. Generally the parameters of a theory are fitted to give good agreement with strains, so this gives a useful indication of the accuracy of pore pressure agreement. The present results on this plot show that pore pressures fall rapidly after significant delayed consolidation. Garlanger theory (fig. 4.9) gives agreement which, in view of the uncertainties in obtaining parameters, is surprisingly good.

The author has also written a simple program to perform the series summations required for Gibson and Lo's theory (1961). Use of this for the increment of 8.75 - 17.5 psi gives the results shown in figs. (4.10-11). It is clear that this does not adequately handle the rapid fall in pore pressure at the early stages of consolidation, although there is some improvement on Terzaghi theory. Predicted settlements are rather less rapid than was observed experimentally. Christie (1963) considered this approach in detail and concluded that

Fig 4.5 GRANGEMOUTH

Pressure Increment 0. - 8.75 psi.

Time-Settlement Curves

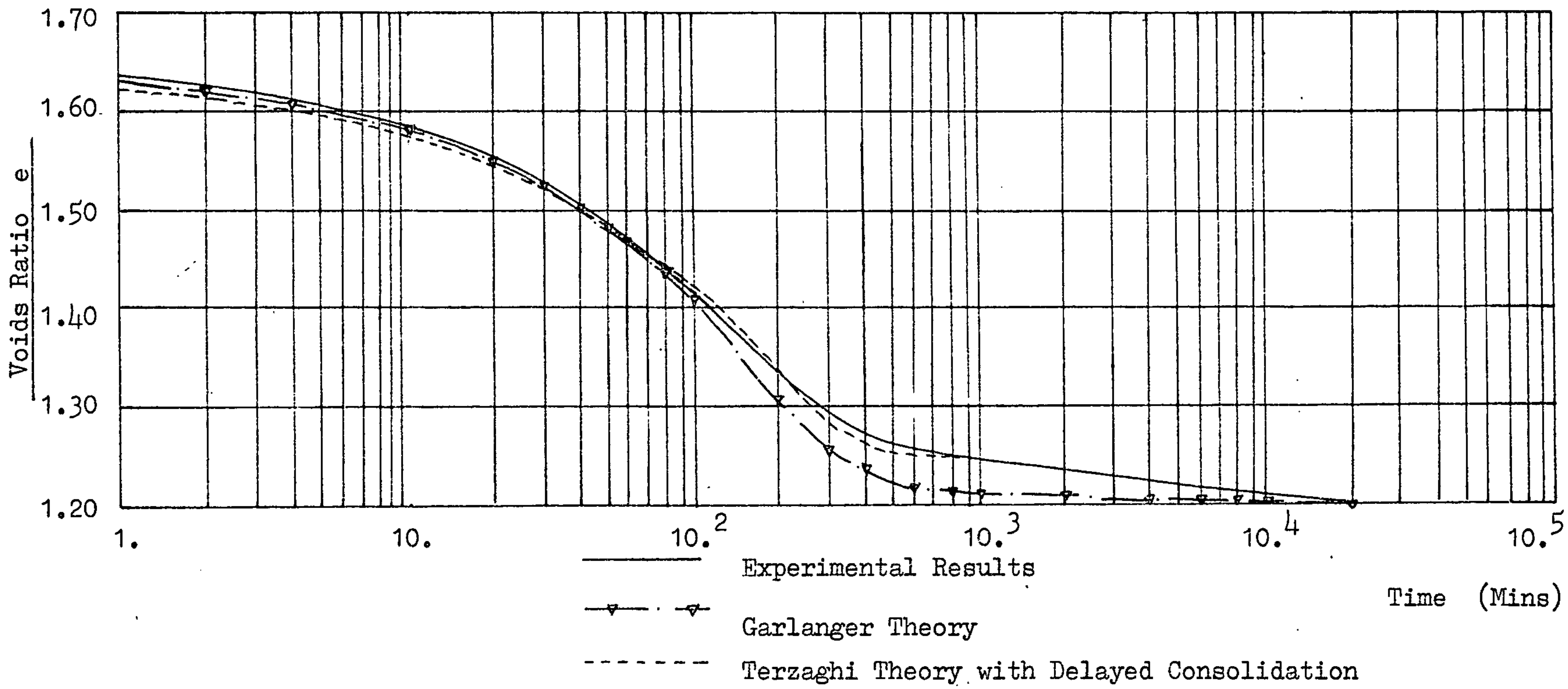


Fig 4.6 GRANGEMOUTH

Pressure Increment 8.75 - 17.5 psi.

Time-Settlement Curves

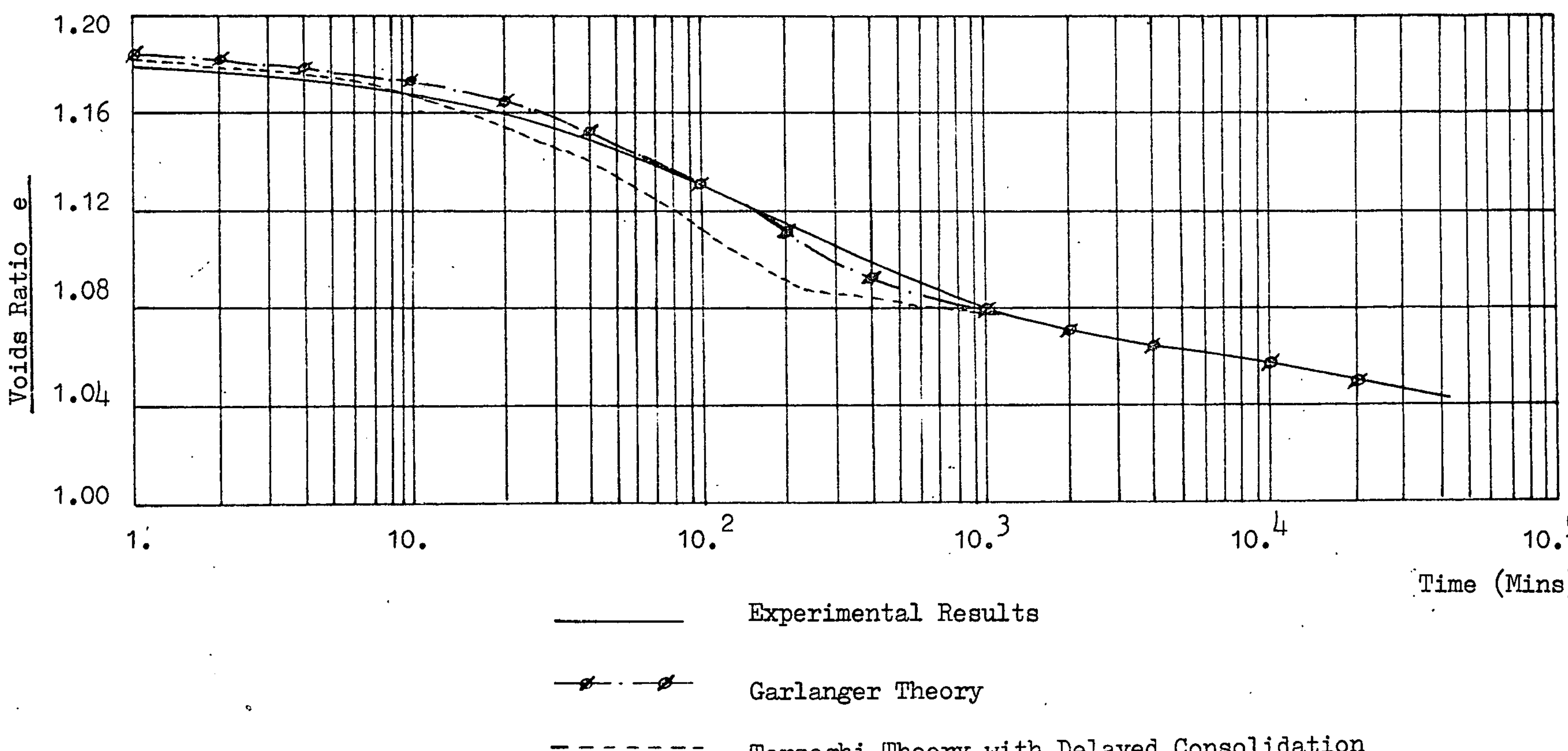


Fig 4.7 GRANGEMOUTH

Pressure Increment 8.75 - 17.5 psi.

Pore Pressure-Time Curves

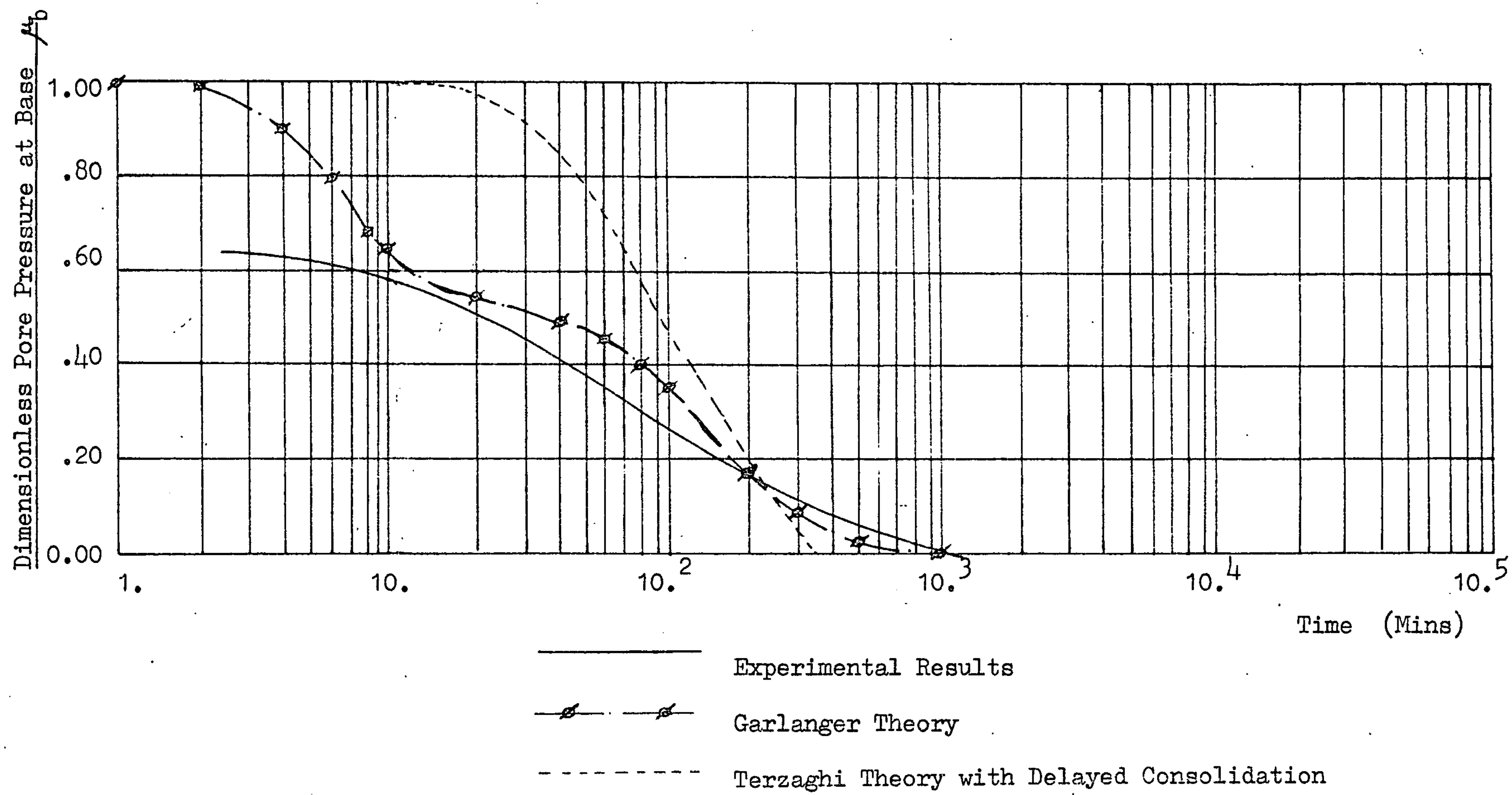


Fig 4.8 GRANGEMOUTH

Pressure Increment 17.5 - 35.0 psi.

Time-Settlement Curves

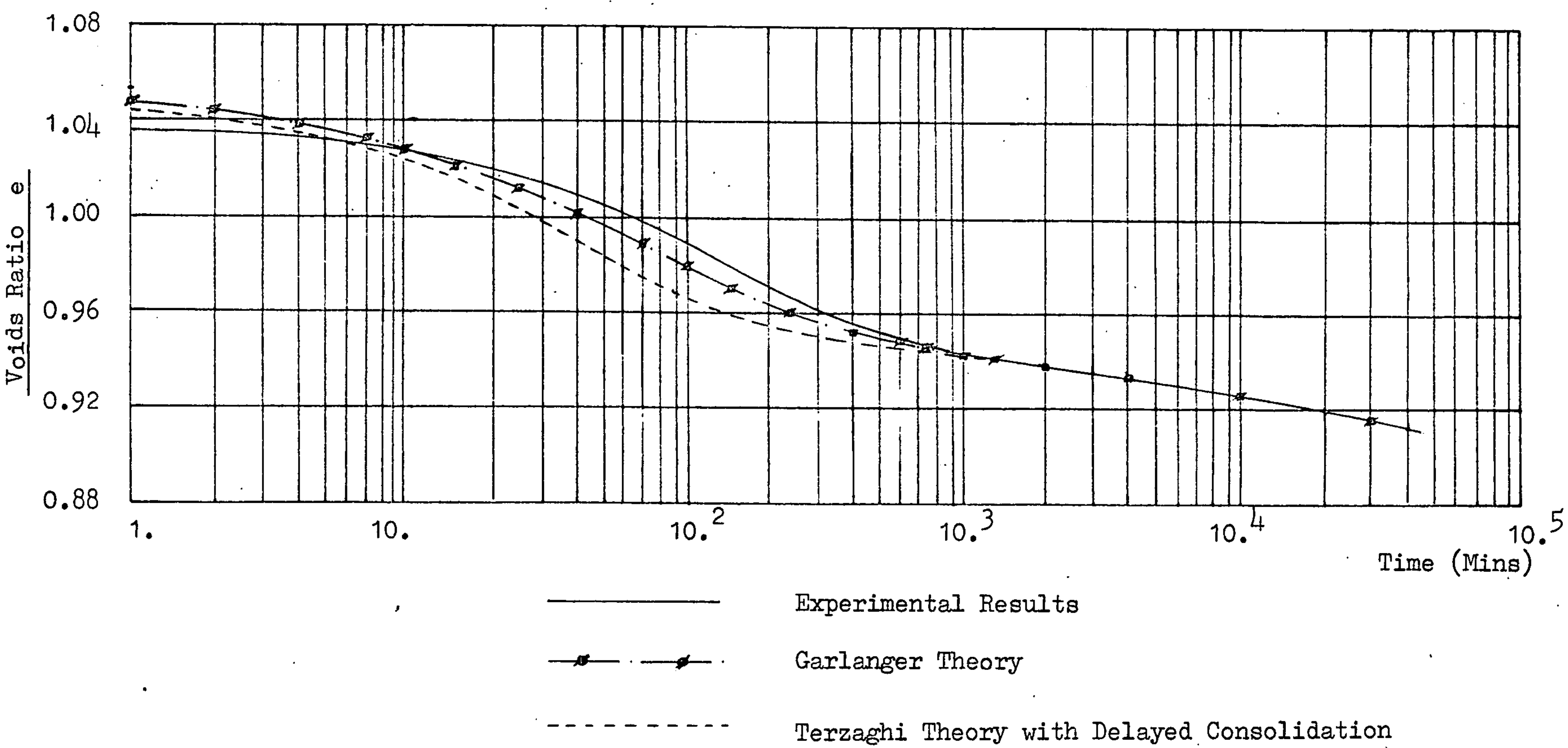
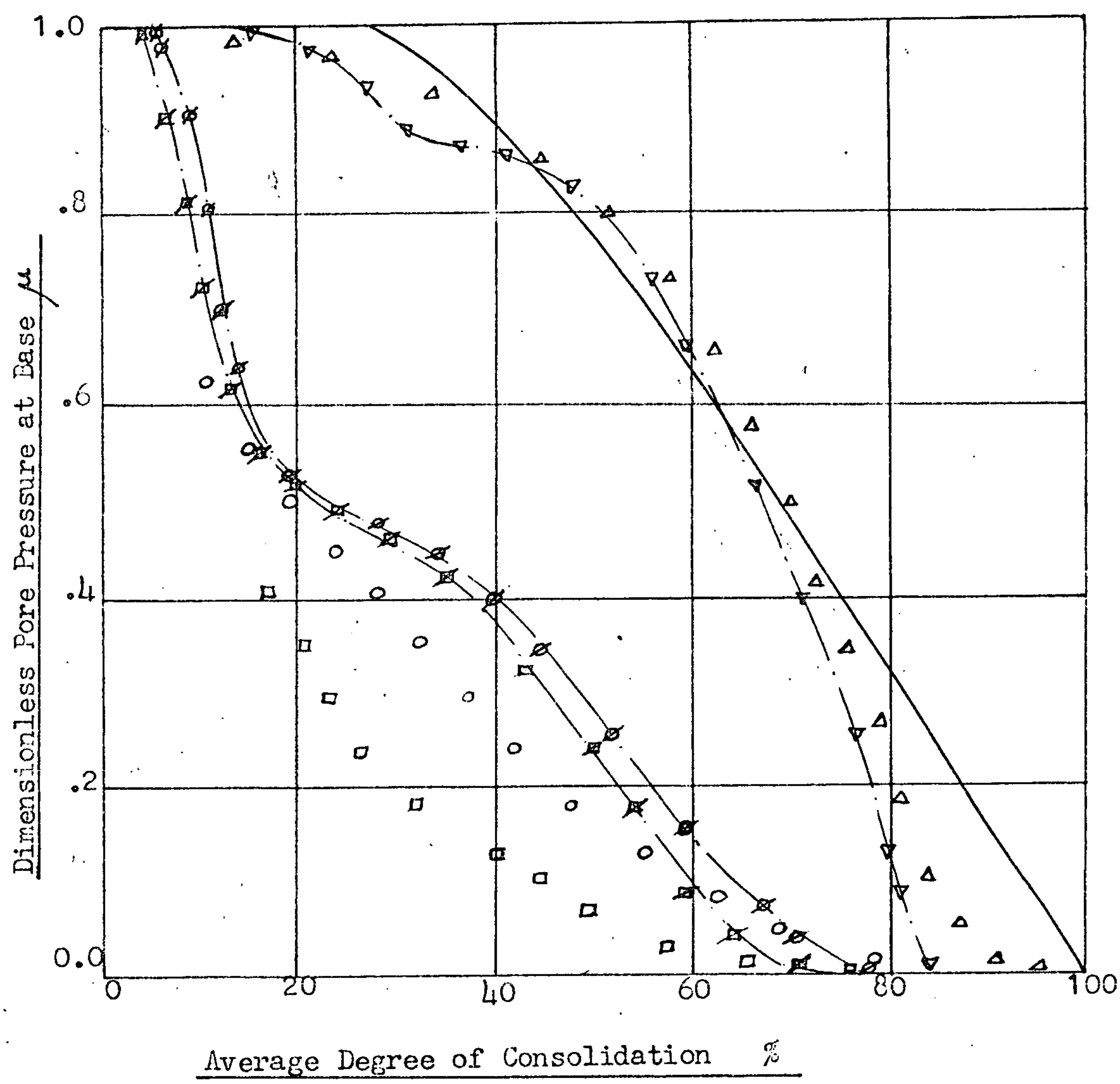


Fig 4.9

GRANGEMOUTH

Pore Pressure at base against Average Degree of Consolidation



Pressure Increment	Experiment	Garlanger	Terzaghi
0.0 - 8.75 psi	△ △ △ △	—▽— · —▽—	————
8.75-17.5 psi	○ ○ ○ ○	—ø— · —ø—	————
17.5 -35.0 psi	□ □ □ □	—ø— · —ø—	————

Fig 4.10 GRANGEMOUTH

Pressure Increment 8.75 - 17.5 psi.

Time-Settlement Curves

Showing Results for Gibson & Lo's Method

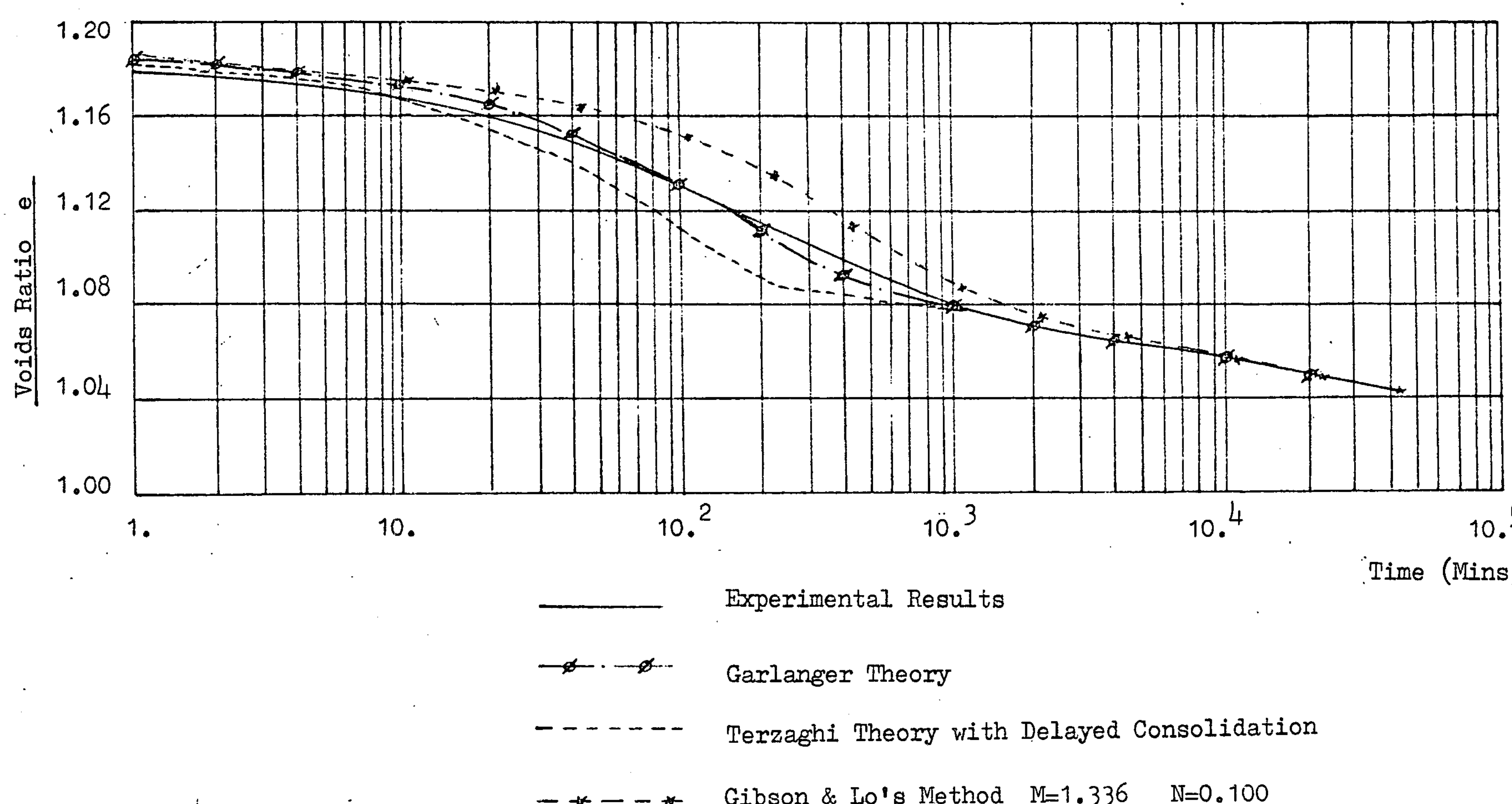
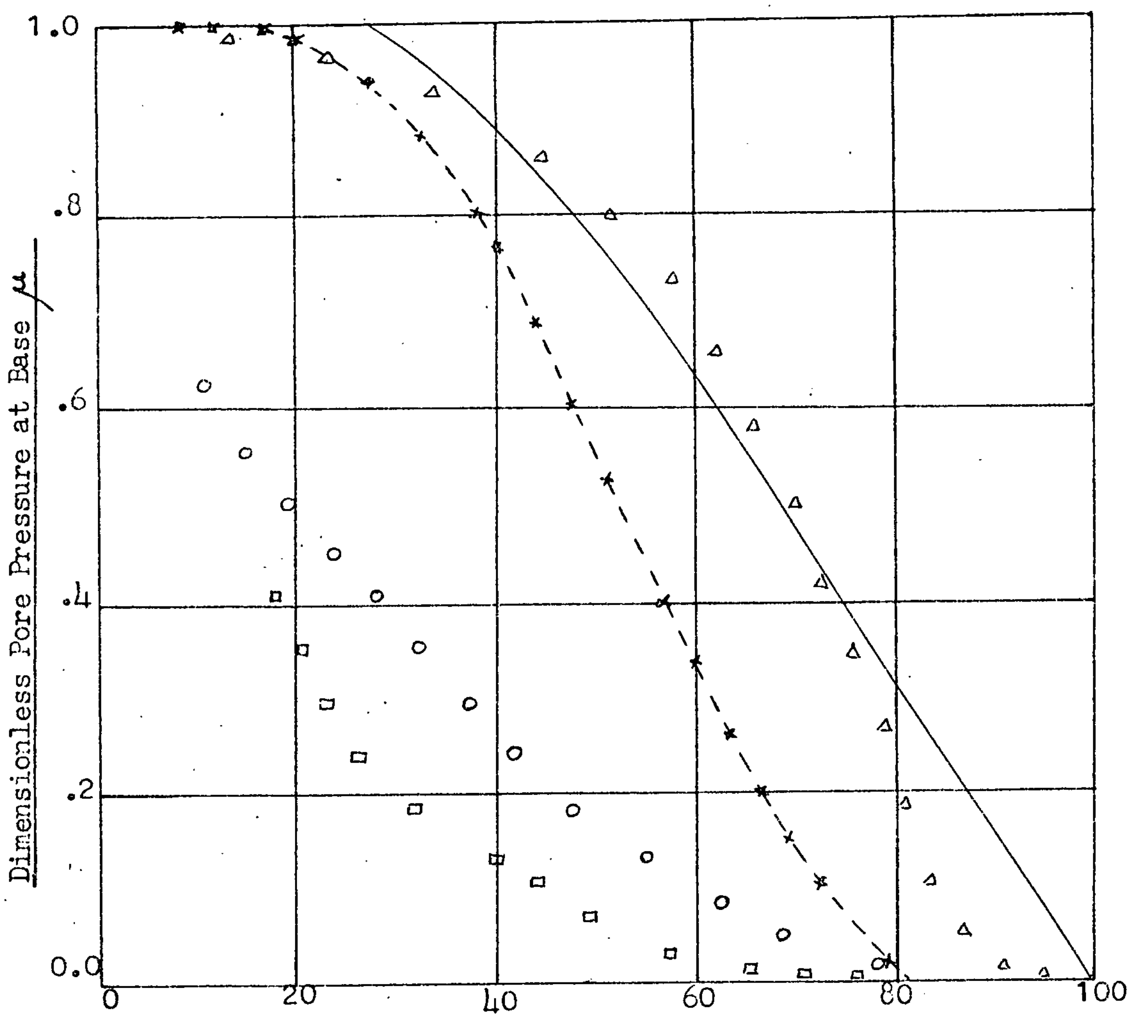


Fig 4.11

GRANGEMOUTH

Pore Pressure at base against Average Degree of Consolidation (Showing results from Gibson & Lo's Method)



———	Terzaghi Theory (All Increments)
△ △ △ △ △	Experiment - 0.0-8.75 psi.
o o o o o	Experiment - 8.75-17.5 psi.
□ □ □ □ □	Experiment - 17.5-35.0 psi.
--*-*	Gibson & Lo's Method for M=1.336 N=0.100 (Increment 8.75-17.5 psi.)

it goes some way towards describing the observed soil behaviour, but cannot satisfactorily handle the rapid early changes in pore pressure which occur in increments following significant delayed consolidation.

4.6.3 Leigh on Sea

The time lines here are parallel, but slightly curved. This has been handled by linearizing these over each time step. The technique appears satisfactory, as do the constructions for the other parameters, since excellent agreement is obtained with the experimental settlements (figs. 4.12 - 4.13).

The pore pressures (figs. 4.14 - 4.15) show the same effect as for Grangemouth that the dissipation takes place far more rapidly after a period of delayed consolidation. Again Garlanger theory can model this behaviour quite satisfactorily.

Fig 4.12 LEIGH-ON-SEA

Pressure Increment 2.19 - 4.38 psi

Time-Settlement Curves

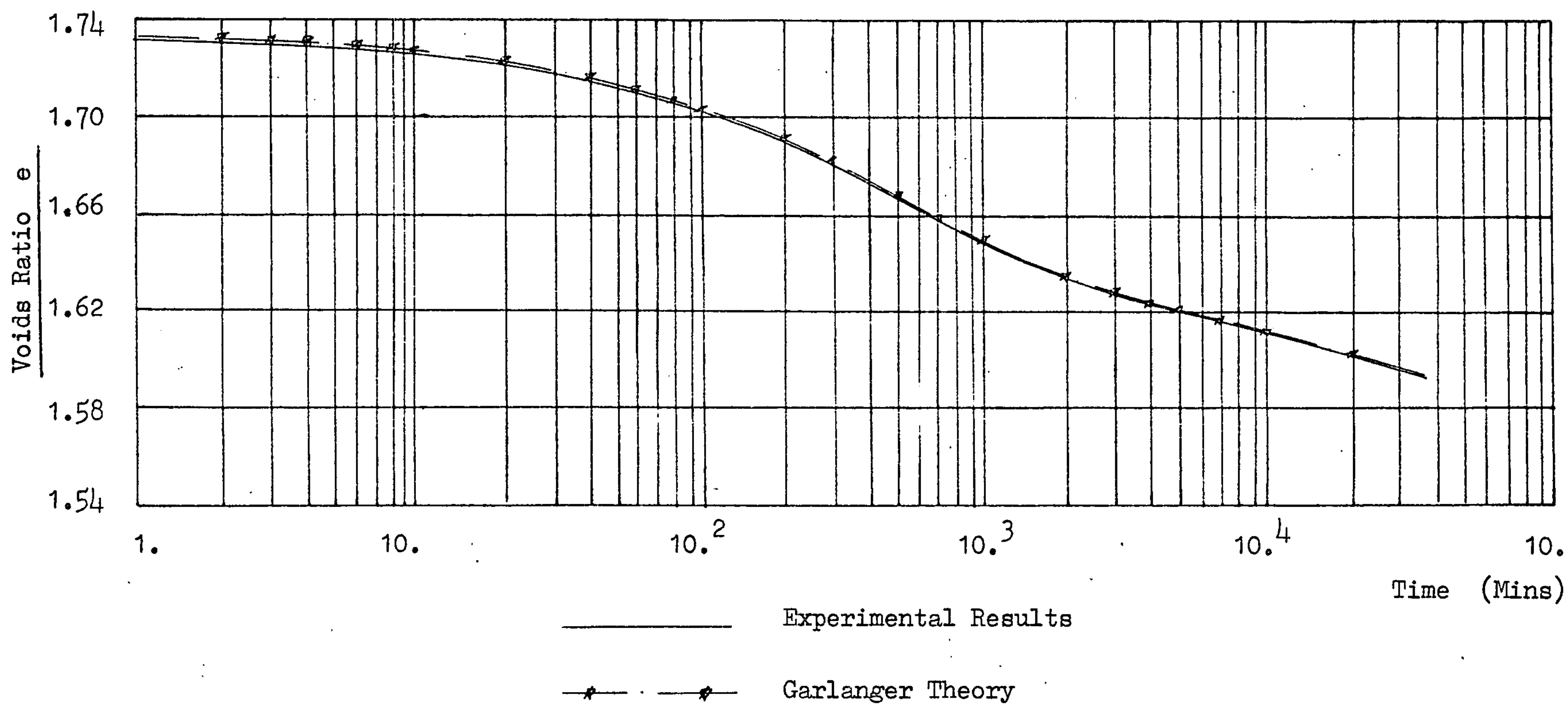


Fig 4.13 LEIGH-ON-SEA

Prssure Incremet 4.38 - 8.75 psi.

Time-Settlement Curves

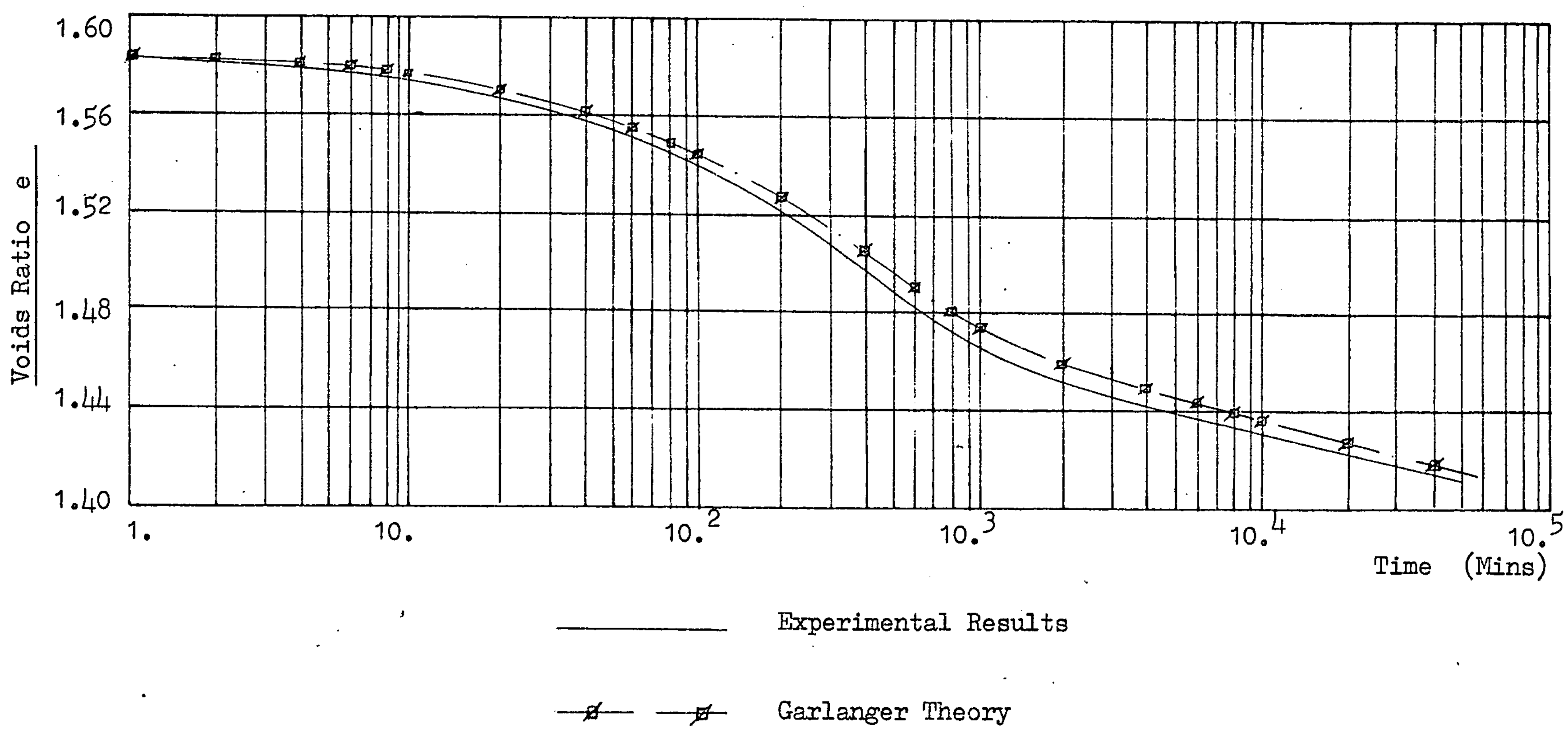


Fig 4.14 LEIGH-ON-SEA

Pressure Increment 4.38 - 8.75 psi.

Pore Pressure-Time Curves

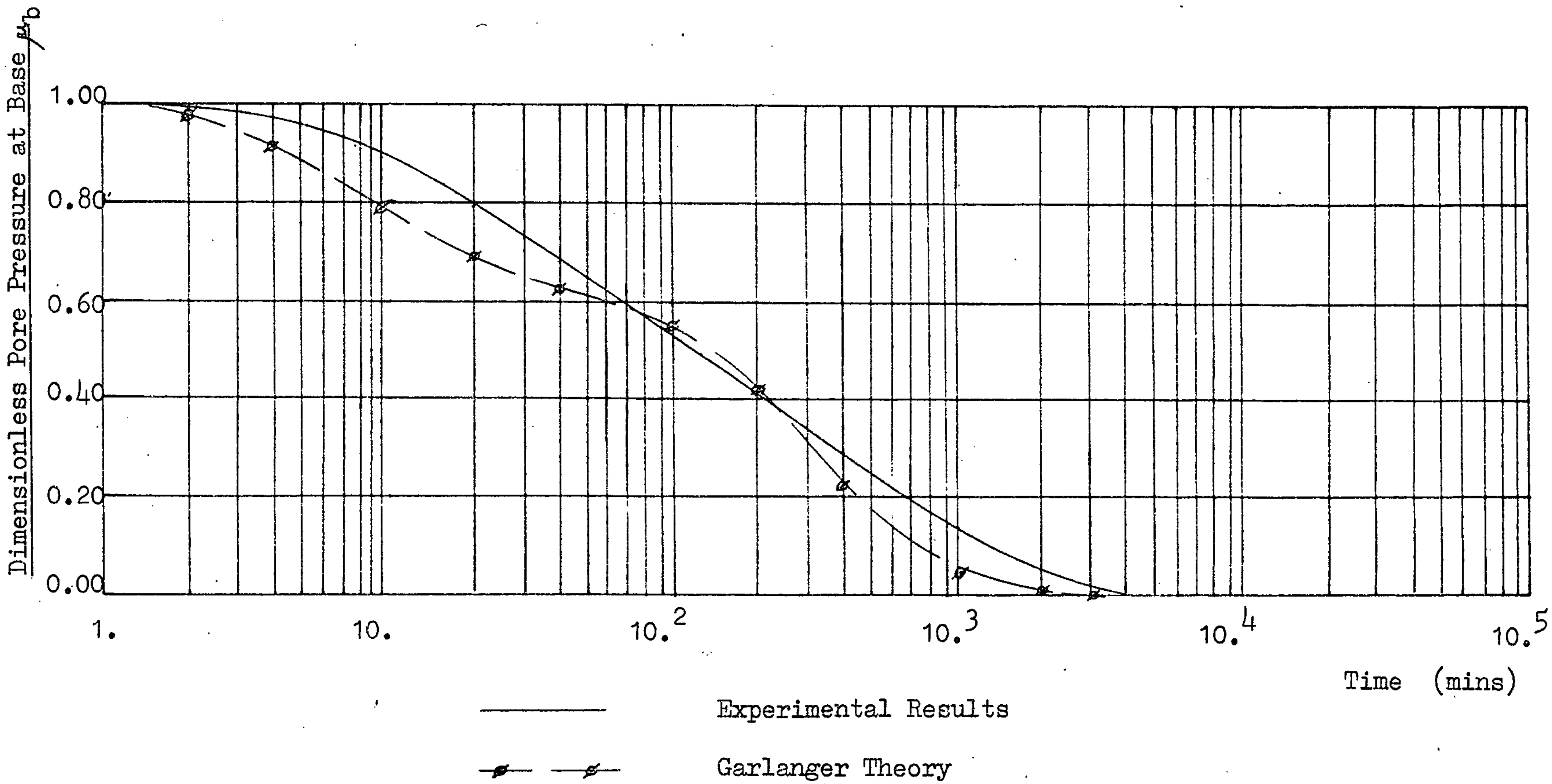
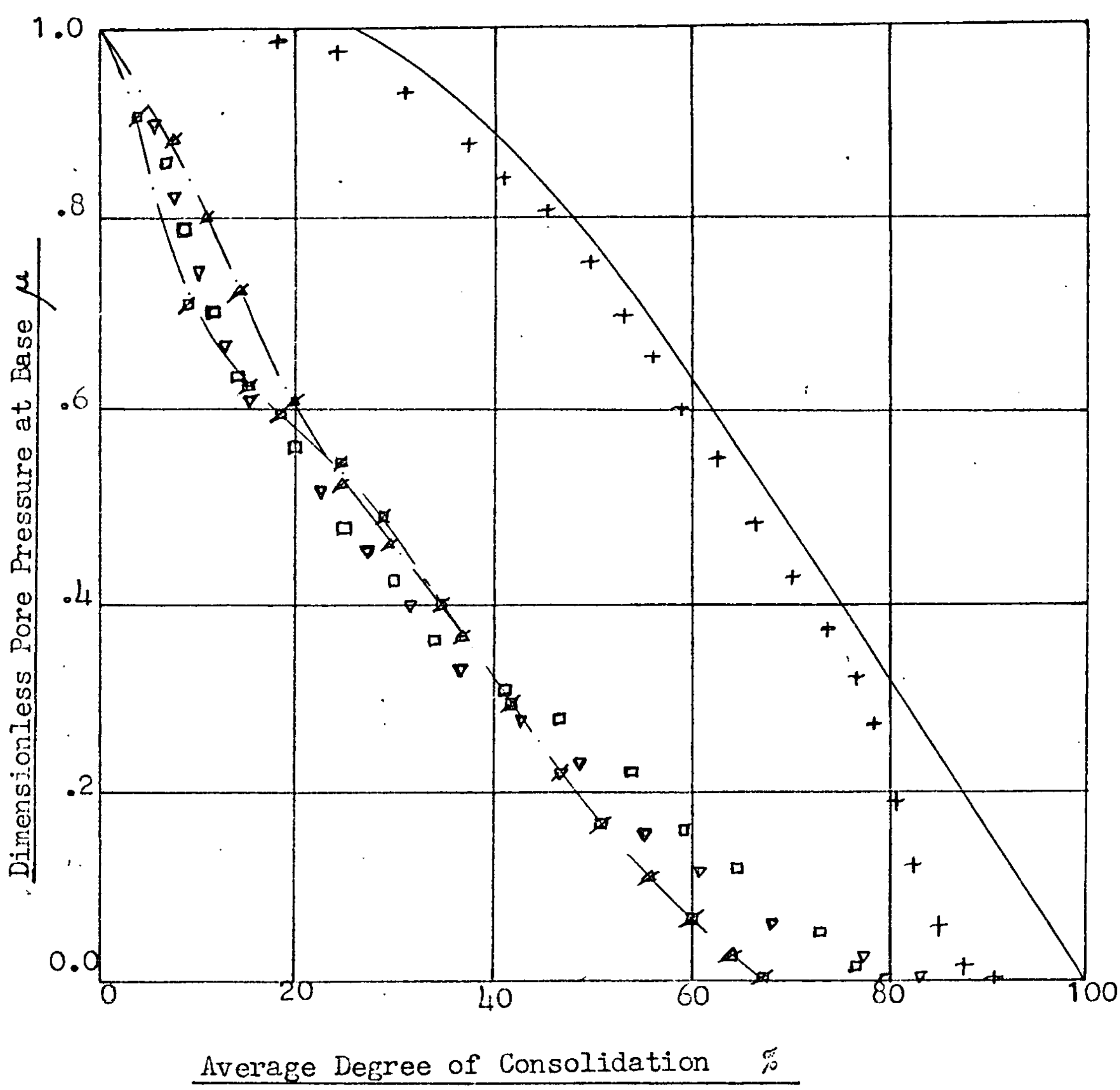


Fig 4.15

LEIGH-ON-SEA

Pore Pressure at base against Average Degree of Consolidation



Pressure Increment	Experiment	Garlanger	Terzaghi
0.0 - 2.19 psi	+ + + +		————
2.19- 4.38 psi.	▽ ▽ ▽ ▽	—⌀—·—⌀—	————
4.38-8.75 psi.	□ □ □ □	—⌀—·—⌀—	————

4.6.4 Bentonite

The time lines on the log e - log p diagram are once again slightly curved and are linearized over each load increment considered here. Primary consolidation is not complete after 1 day, so that this time line cannot be drawn.

Again the comparisons of experimental and theoretical results may be left to speak for themselves. Agreement for settlements is quite good (though less so than for the previous two soils examined). Pore pressure agreement is good for the lower increment (fig. 4.17) but less so for the higher increment (fig. 4.19).

Fig 4.16 BENTONITE

Pressure Increment 2.19 - 4.38 psi.

Time-Settlement Curves

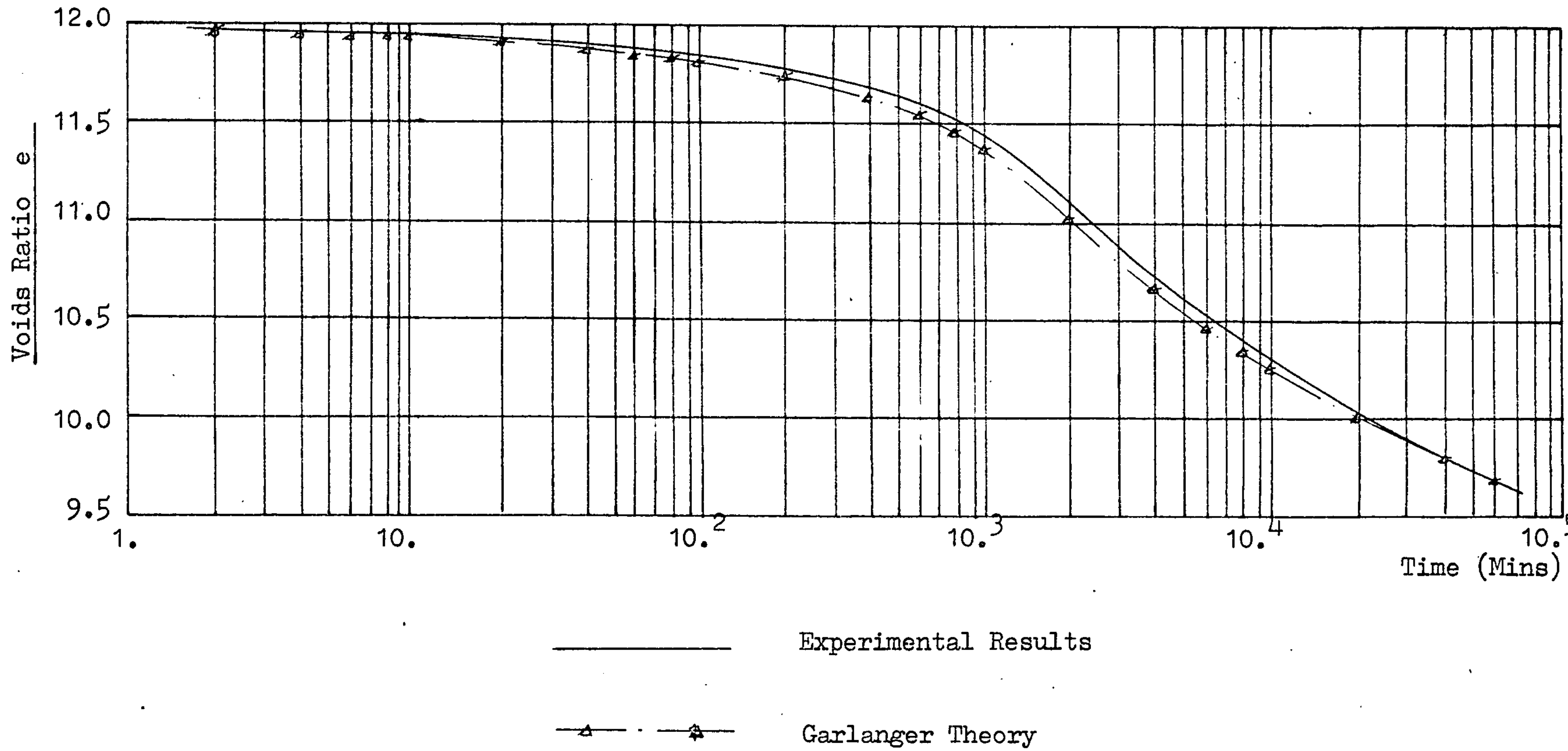


Fig 4.17

BENTONITE

Pore Pressure at base against Average Degree of Consolidation

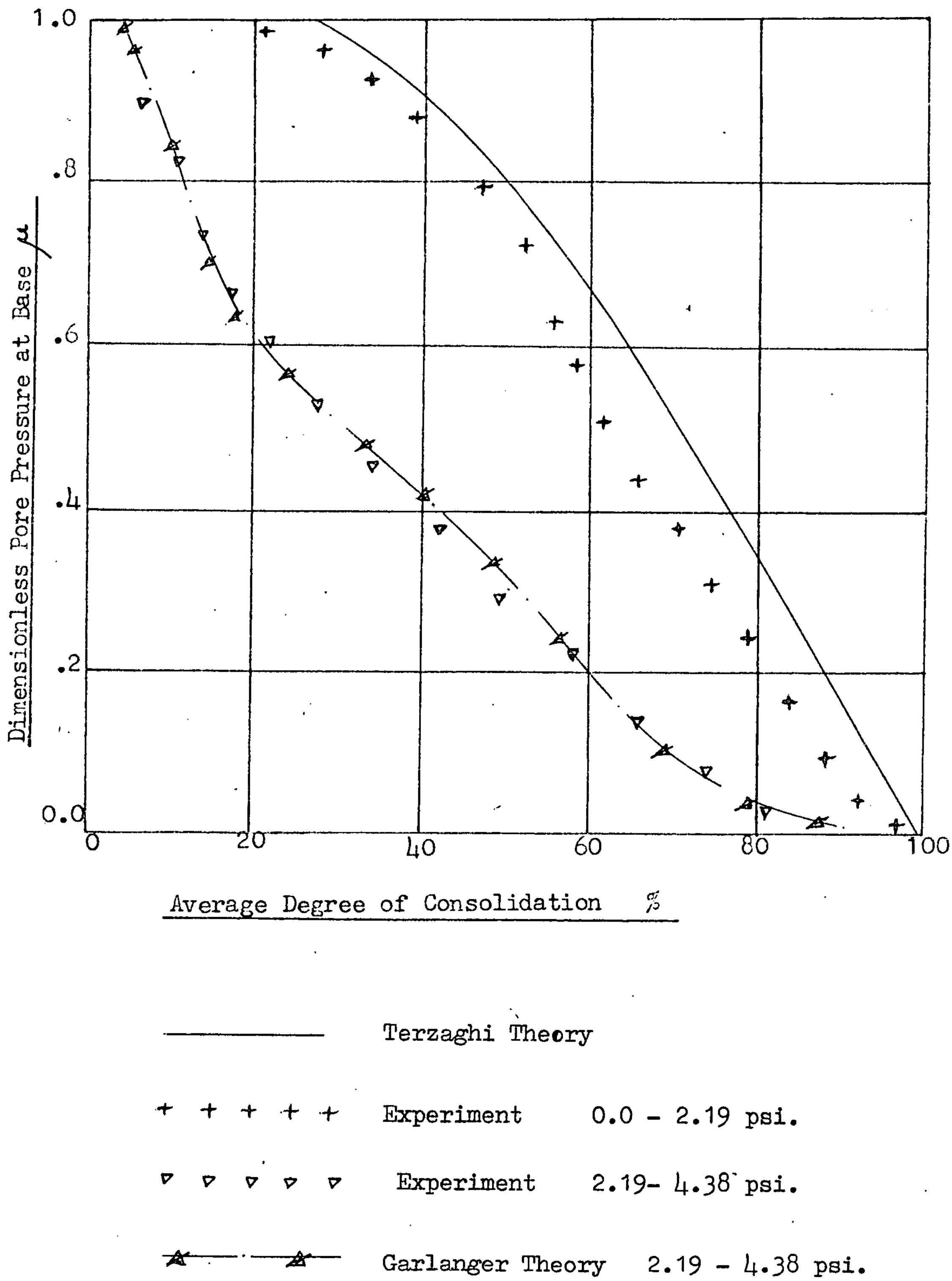


Fig 4.18 BENTONITE

Pressure Increment 4.38 - 8.75 psi.

Time-Settlement Curves

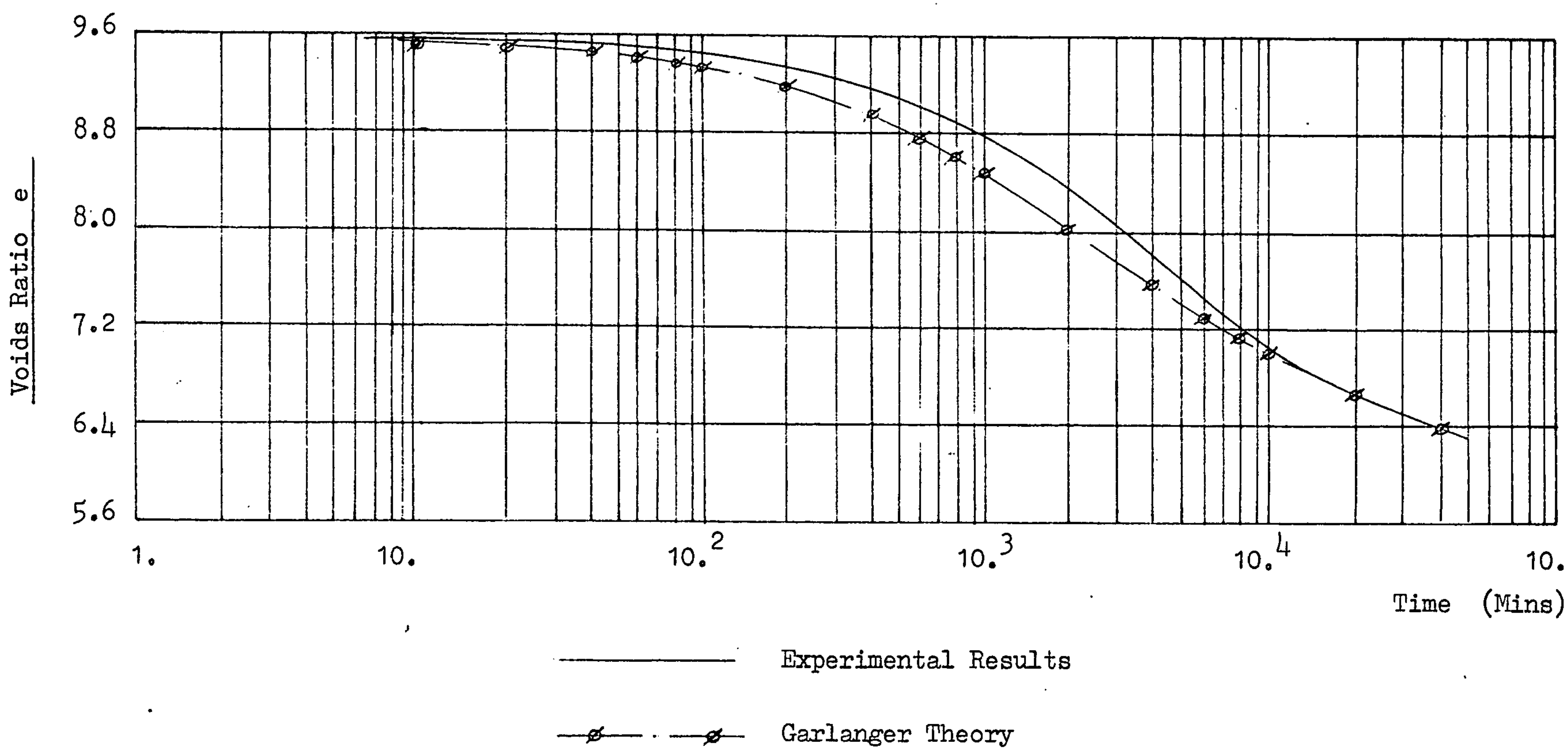
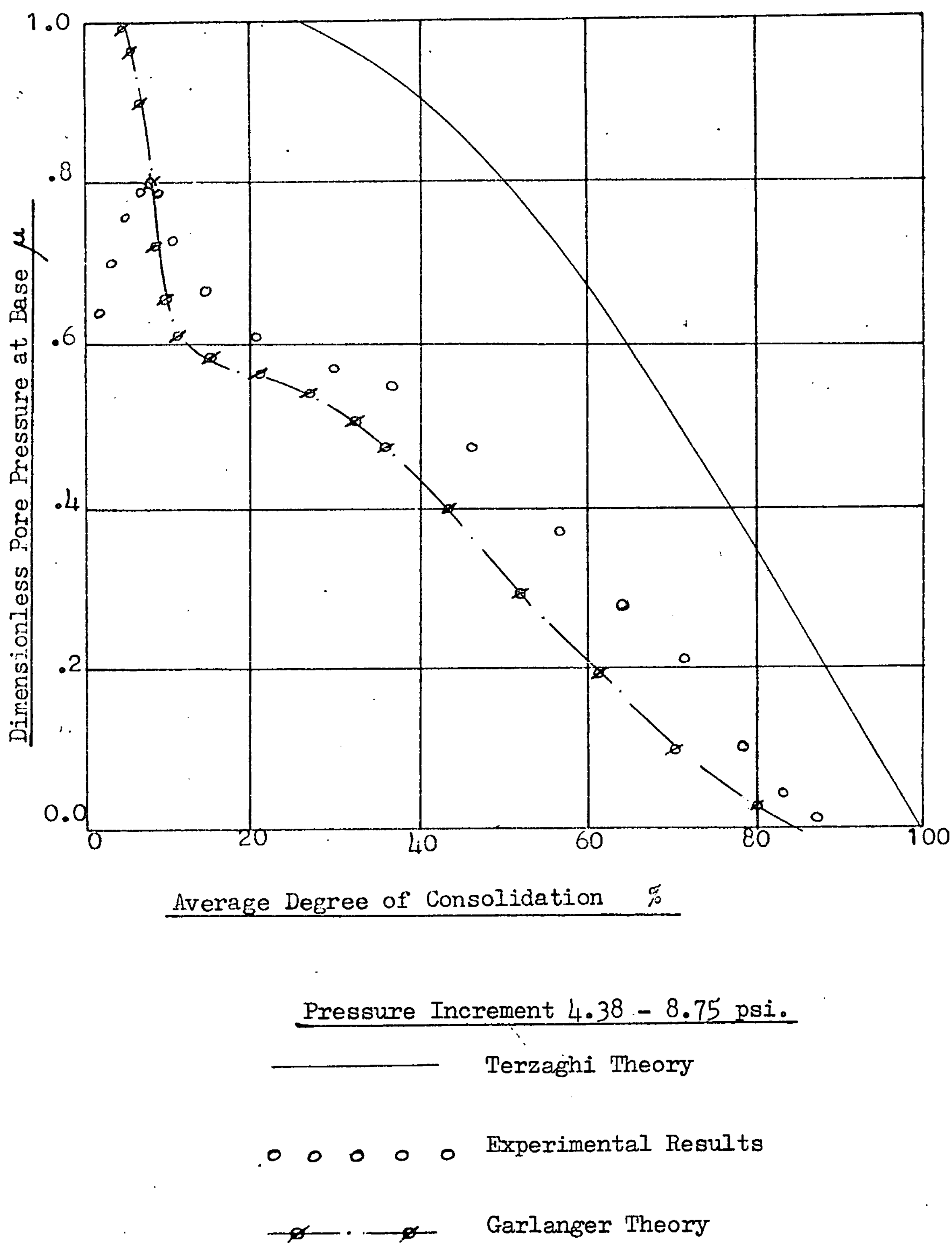


Fig 4.19

BENTONITE

Pore Pressure at base against Average Degree of Consolidation



4.6.5 Some General Remarks

While the soils considered above differ considerably, it must be noted that all have been selected for the pronounced secondary consolidation displayed. The graphs of pore pressure dissipation at the impermeable boundary against degree of consolidation (defined from the average strain), high-light the difference in observed behaviour caused by lengthy secondary consolidation prior to application of a load increment. It is clear that any theory postulating a simple linear relationship between effective stress and strain will not be able to handle such behaviour. However, the load increments applied immediately a sample is set up come close to such linear relationship as indicated by their closeness to the unique curve obtained from Terzaghi theory on such a plot.

The time-lines approach suggested by Bjerrum, and its subsequent implementation by Garlanger, represents the first consistent approach to take account of such effects as noted after significant delayed consolidation. The agreements obtained from this method for data collected several years previously are very encouraging. There are problems to be ironed out, but the graphs clearly move into the right general area.

The following Chapters try to assess how the problems may be tackled, and this requires investigations of the theoretical implications and of the actual soil behaviour.

4.7 Time Dependant Loading

In many practical situations the loading is better considered to be applied over a significant length of time. It has already been seen (section 2.6) how this may be handled for Terzaghi theory by adjustment of the numerical scheme. It was considered of value to attempt a similar approach here. Only the simplest case of loading proportional to time up to some ultimate load q at real time tq, has been considered here. By analogy with the method of section (2.6) this could be extended to more complex situations if required.

The rate of loading at any instant = $\frac{\partial q}{\partial t}$.

The effective stress relationship must now include this applied loading rate, and is now written

$$\frac{\partial p}{\partial t} = - \frac{\partial u}{\partial t} + \frac{\partial q}{\partial t} \qquad (4.71)$$

Substituting in the stress-strain-time relationship (4.11) this gives

$$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial p} \left\{- \frac{\partial u}{\partial t} + \frac{\partial q}{\partial t} \right\} + \left(\frac{\partial e}{\partial t}\right)_c \qquad (4.72)$$

To make this compatible with the solution technique, a dimensionless variable is defined

$$\theta = q/\Delta p \qquad (4.73)$$

then

$$\frac{\partial \theta}{\partial T} = \frac{\partial q}{\partial t} \times \frac{\partial t}{\partial T}/\Delta p = \frac{\partial q}{\partial t} \cdot \frac{H^2}{Cv\Delta p} \qquad (4.74)$$

Transforming (4.72) and rearranging into Garlanger's format gives

$$\frac{\partial \mu}{\partial T} = \frac{\partial \theta}{\partial T} + \left\{\frac{\partial \beta}{\partial T} - \left(\frac{\partial \beta}{\partial T}\right)_c \right\}/\frac{\partial \beta}{\partial \mu} \qquad (4.75)$$

For the present work the simplification is introduced that loading is linear up to t_q, after which it remains constant at $q = \Delta p$.

Then $\frac{\partial\theta}{\partial T} = \frac{1}{t_q} \times \frac{H^2}{Cv}$ $\quad t \leq t_q$ (4.76A)

$$\frac{\partial\theta}{\partial T} = 0 \qquad t > t_q \qquad (4.76B)$$

This method of analysis has been programmed into the semi-implicit scheme (section 4.4.2). This involves re-working a considerable amount of fairly tedious algebra. Only the results are quoted here.

The matrix equations (4.40 and 4.41) may be written

$$[B]\{\mu^+\} = [D]\{\mu\} + \{F\} \qquad (4.77)$$

where $F_i = 2 \times \Delta Z^2 + (\frac{\partial\beta}{\partial T})_{c_i}$ (4.78)

at each node, i.

It is found that the $\frac{\partial\theta}{\partial T}$ term may be most easily handled by the inclusion of a further term in the vector F, leading to (4.78) being replaced by

$$\underline{\underline{F_i = 2 \times \Delta Z^2 \times (\frac{\partial\beta}{\partial T})_{c_i} + 2 \times \frac{\partial\theta}{\partial T} \times \frac{\Delta T}{Ki}}} \qquad (4.79)$$

It should be noted that following the inclusion of time dependant loading in equation (4.71), the expression for p required in the program can no longer be determined from the dimensionless statement

$\mu = \{p_f - p\}/\Delta p$ (see section 4.2)

where $p_f = p_o + \Delta p$.

This was rearranged in the programs previously described to give

$$p = p_o + (1 - \mu)\Delta p$$

Inclusion of the effect of Time Dependant Loading results in this being modified to read

$p = p_o + (\gamma - \mu)\Delta p$ where $\gamma = t/t_q$ (4.80)

Relatively few modifications of the original scheme are required for this analysis, so it has been included as an option which may be selected if required by the user of the semi-implicit scheme program SECON. The only additional data required is the value of t_q.

The total primary strain at a free draining boundary takes place with the load application. Hence it can no longer be calculated once and for all before commencement of the time step loop, and the algorithm for calculation of strains had to be modified slightly at this boundary.

The logarithmic increase of time step described in section 4.4.2 was found to function quite satisfactorily using the same constants tx_o and Δtx (see equations 4.47 and 4.48). This is to be expected, since it was noted that the truncation errors depend upon rate of change of the relevant derivatives, and these will change less severely than in the instantaneous loading case.

Numerical Example

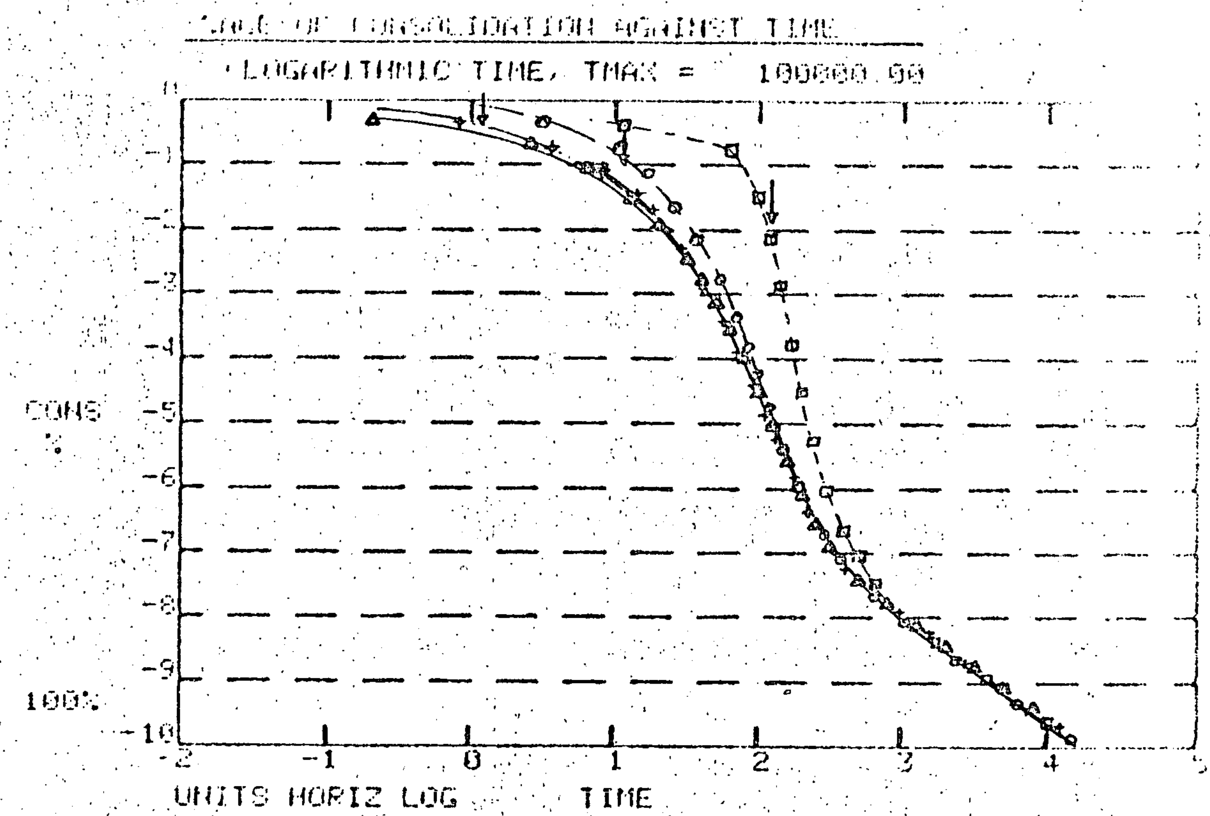
As an example of the effects predicted for time dependant loading the results for the 8.75 - 17.5 psi increments for Grangemouth are presented here. These are fairly typical of the various increments for the soils described in section 4.6. The full load increment is assumed to be reached (linearly with time) after various periods of time t_q. This is presented here as fractions of the Terzaghi time factor (i.e. TQ = 0.005 represents the fastest rate of loading, and TQ = 0.5, the slowest). These results are shown in figs. 4.20, 4.21 and for comparison the equivalent analysis using Terzaghi theory is shown in 4.22. (The relevant experimental results were shown in figs. 4.6, 4.7 and 4.9.)

Consolidation settlement obviously progresses less rapidly the slower the rate of loading. The main observation is that as the Garlanger theory indicates pore pressures falling much more rapidly at early stages of consolidation, so the maximum pore pressures reached on application of full time dependant loading are significantly

Fig 4.20

Effect of Loading Rate on Progress of Consolidation

(Showing Typical Computer Graphics Output for % C – log t results.)



TQ=

Instantaneous

0.005

0.05

0.5

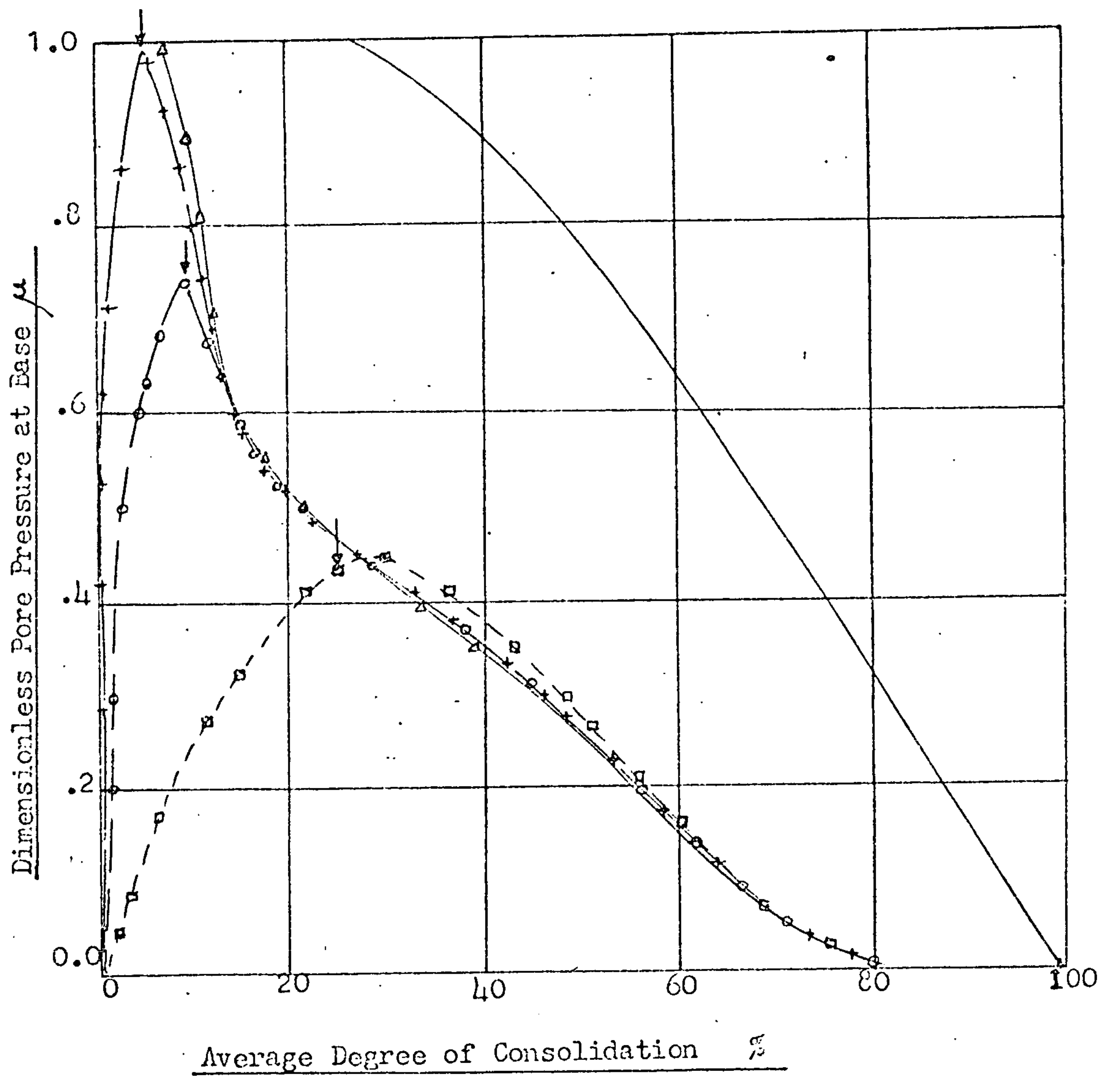
Load Increment q is applied linearly with time, reaching q = Δp at Terzaghi Dimensionless time of TQ, as shown. (Indicated by ↓)

Fig 4.21

GRANGEMOUTH Numerical Example

Effect of Loading Rate on Garlanger Analysis

Pore Pressure at Base against Degree of Consolidation



Terzaghi Analysis

Garlanger Analysis - TQ=

Instantaneous

0.005

0.05

0.5

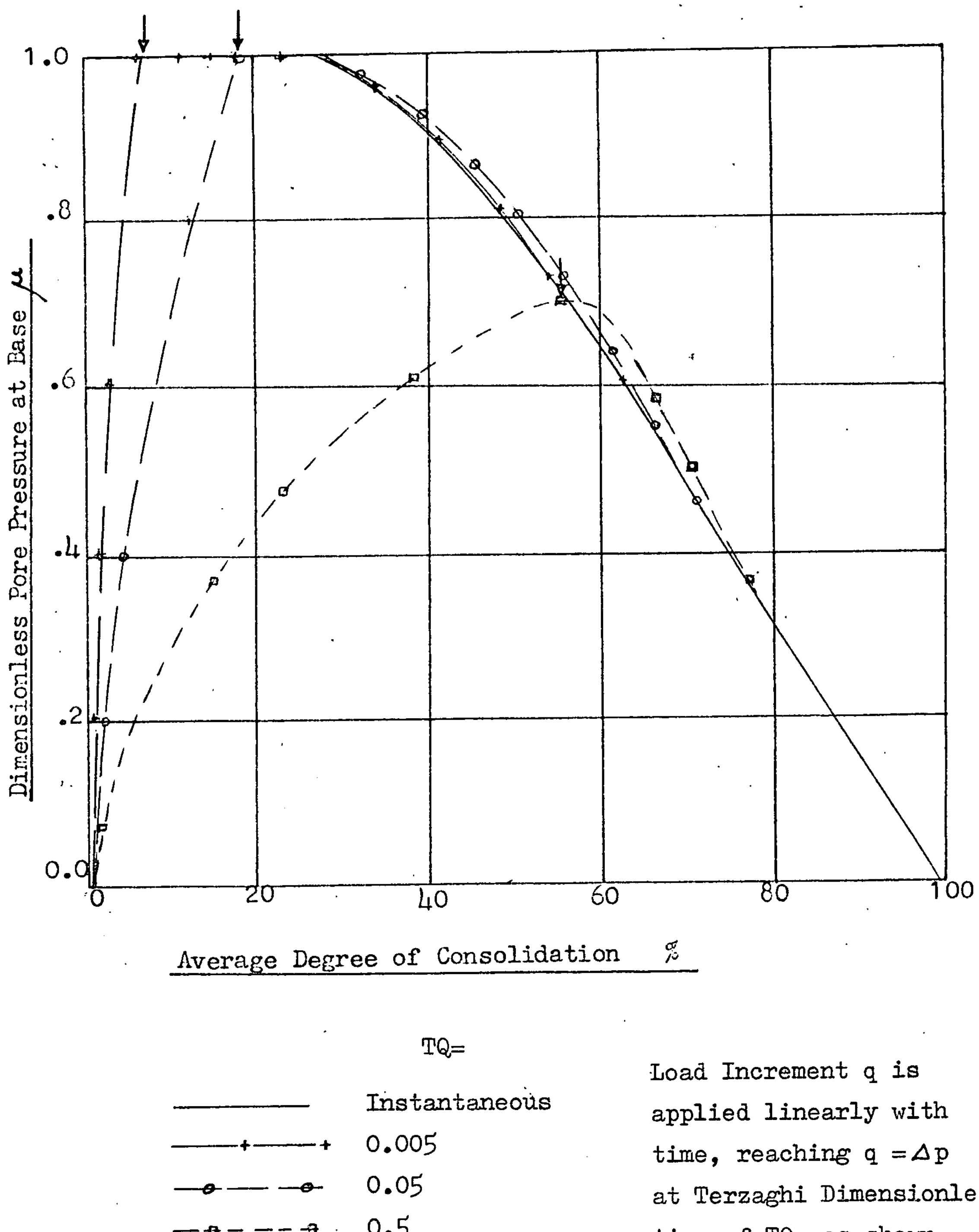
Load Increment q is applied linearly with time, reaching q = Δp at Terzaghi Dimensionless time of TQ, as shown. (Indicated by ↓)

Fig 4.22

GRANGEMOUTH Numerical Example

Effect of Loading Rate on Terzaghi Analysis

Pore Pressure at Base against Degree of Consolidation



Load Increment q is applied linearly with time, reaching q = Δp at Terzaghi Dimensionless time of TQ, as shown. (Indicated by ↓)

less, and dissipate again more rapidly than suggested by Terzaghi theory.

For both theories the graphs tend to converge towards the instantaneous loading solution. For most practical purposes the instantaneous solution may be sufficiently accurate for the base pore pressure - degree of consolidation plots from shortly after attainment of full loading (i.e. shortly after t_q). The consolidation settlement and pore pressure relationships with time, however, converge less rapidly, and although it is of some value to know that they tend to the instantaneous results at large times, for the progress of consolidation it will normally be necessary to analyse the particular case.

CHAPTER 5

Experimental Investigation

5.1 Introduction

Delayed Consolidation as described by Bjerrum (1967) is believed to be capable of accounting for some major deviations of real soil behaviour from that described by Terzaghi theory. An experimental program was carried out to give specific information about such delayed consolidation, and to examine the method of analysis proposed by Garlanger. Methods of obtaining the various governing parameters are shown, and the experimental evidence for the validity of these is considered.

The experimental work was performed on a soft silty-clay from Grangemouth, and general properties of this deposit are given in Section 5.2. A large number of consolidation tests were carried out after various periods of delayed consolidation, and the influence of this on behaviour is considered in detail. Four types of consolidation test have been used, and this allows the validity of the tests to be examined. Further to this, three load increments have been used for each test type, and since this was not found to significantly affect results, this allows the validity of tests of each type, for any given period of delayed consolidation, to be assessed. Essential details of the experimental procedures are given in Section 5.3. Much important information, however, is fairly tedious, so the full experimental method is relegated to Appendix B, along with drawings of the apparatus used.

A considerable volume of data has been collected, analysis of which was aided by a program CALCOG (Section 5.4) written to calculate stress and strain data from the observations and present this in useful tabular and graphical form. Some condensation of results has proved necessary for presentation purposes, but the major points, with appropriate experimental evidence, are considered in Section 5.5 under the heading of the various types of test, and the delayed consolidation process is examined specifically.

The capabilities of Terzaghi theory are considered and causes of discrepancies identified. It is shown that if proper allowance is made for the effects of delayed consolidation, reasonable description of behaviour is still possible. This involves non-linear stress-strain relationships, however, which are not easy to handle, although if successfully evaluated the extended Terzaghi theory programs CONED (Chapter 2) can solve resulting consolidation problems. More easily-handled approximate procedures may be of value here.

A more consistent approach to delayed consolidation is Garlanger's method, examined in Section 5.7. This is shown to lead to quite good description of the experimental results, demonstrating that these are in accordance with Bjerrum's time-lines model. This model and the results are compared in some detail for all aspects of consolidation behaviour and a number of points are raised from this. Some light is shed on the problems of critical pressure and instant line, which later leads to a satisfactory theoretical treatment (Section 6.4).

It is concluded that time dependant skeletal behaviour has a significant effect on the consolidation of this clay. The Garlanger-Bjerrum model provides a useful conceptual aid and practical technique for analysis of such behaviour.

5.2 Grangemouth Silty-Clay

The Grangemouth area, on the south bank of the river Forth, is well-known for its soft soil deposits and difficult foundation conditions. The material, termed locally Carse Clay, is a post-glacial estuarine mud. It has been widely reported in the literature (see, e.g. Skempton 1948, Pike and Saurin 1952), and the geological and geomorphological background to the area has been discussed by Sissons (1969, 1971). An investigation of the soil behaviour 'in-situ', and work on the undisturbed soil is currently under way at Glasgow University, under Professor H. Sutherland. Some of this work is reported by Jarrett et al. (1974), who suggest that the soil in the field displays a preconsolidation pressure of around 18 kN/m^2. This conclusion, from buildings in the area, is interesting since, as they note, no suggestion of a preconsolidation pressure has been found from laboratory tests on so-called undisturbed samples. For such a soft clay it appears that even the best techniques presently available cause undue disturbance. It is hoped that the present work on the remoulded clay may later be tied in with some of this work.

The soil used in the present experimental work was obtained from a borehole near that used by Christie (1963). The basic engineering properties are given in table 5.1, and these indicate that the soil is similar to that used in his experiments, so that direct comparison is possible. It was not considered worthwhile carrying out fairly expensive analyses of the mineralogical properties since accurate quantitative data is not possible in any case. The information given comes directly from Christie (op cit); obtained by differential thermal analysis and X-ray diffraction.

Table 5.1

Properties of Grangemouth Silty-Clay

Engineering Properties

Plastic Limit = 65%

Liquid Limit = 27%

Plasticity Index = 38%

pH = 7.85

Organic Matter = 1.75% by weight

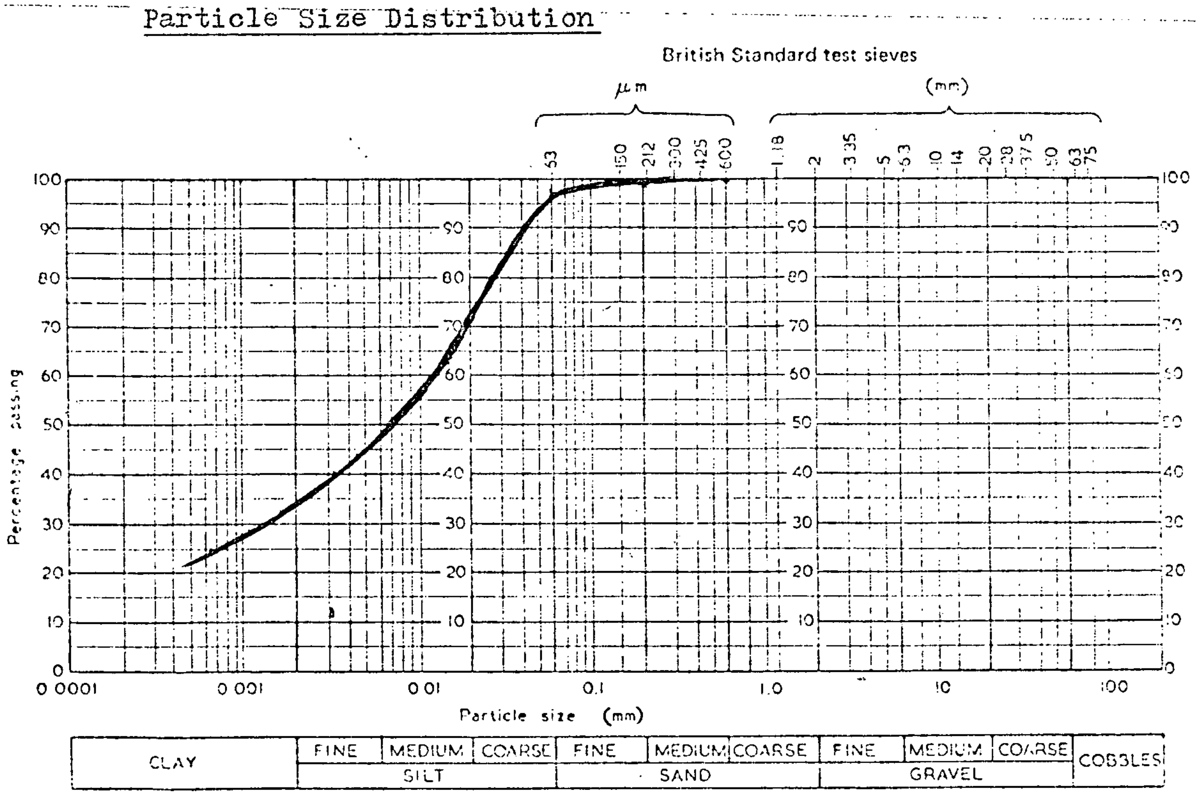
Tests from B.S.1377 1975

Specific Gravity of Soil Particles Gs = 2.69

Mineral Properties

Clay fraction composed mainly of Illite with some Kaolinite (Ratio roughly 4:1). Also contains a little vermiculite, quartz, and a trace of feldspar.

Particle Size Distribution



5.3 Experimental Procedures

The main experimental programme consisted of a series of consolidation tests designed to examine delayed consolidation and its influence upon later soil behaviour. Three types of test (A, B, and C - see below) were performed at each of three pressure increments following three periods of delayed consolidation {namely, 1 day (D), one week (W), and one month (M)}. The pressure increments were labelled R, S and T, corresponding to 60 - 120, 120 - 240 and 240-480 kN/m^2, respectively. A further set of tests were run (type E) to include back pressure on the sample - only 1 week of delayed consolidation has been used in these. Table 5.2 summarises the tests. The types of test were:

(A) The pressure increment is applied to the sample with the drain closed, and pore pressures are allowed to build up to their maximum values over a period of 24 hours. The drain is then opened and normal readings of pore pressure and deflection are taken as consolidation progresses.

(B) Loading is applied in increments with the drain open. This gives information concerning the early behaviour and the critical pressure. The total pressure change is divided into 10 equal increments which are applied at 10 minutes intervals, this being sufficient time for the rapid consolidation below p_c to be completed.

(C) The total load increment is applied with the drain open and normal deflection and pore pressure readings are taken.

(E) The procedure is as for (A), except that a back pressure of 240 kN/m^2 is used.

Details of the behaviour in these tests is considered under these headings in section 5.5.

A random programme was adopted for the tests to indicate whether the remoulded soil used changed between test series and also to allow comparison between the three cells used. In the event this had to be modified slightly to allow faulty tests to be repeated. The cell used was not found to affect results; nor did the soil properties change, as far as could be distinguished between the tests. Details of this random program have therefore been omitted here, and the various results may be analysed simply in terms of test type, amount of prior delayed consolidation, and pressure increment. Each series of tests was performed with progressively increasing pressure; this could not be randomised.

Tests were carried out in the Bishop and Skinner Hydraulic Oedometer, drawings of which are given in Appendix B1. Fig. 5.1 shows the general experimental layout. Briefly, the cell is designed as a flexible piston in a cylinder. This is placed in a Triaxial-Cell-type housing cylinder and the hydraulic pressure in this bears upon the piston. {Since the department was short of suitable triaxial cylinders, the author modified the design to be appropriate for the present work, and had 3 of these fabricated in the workshops.} From the bottom of the cell a tapping leads to one outlet valve, and from the centre "mushroom" on the top piston a fine-bore flexible tube leads to another outlet valve.

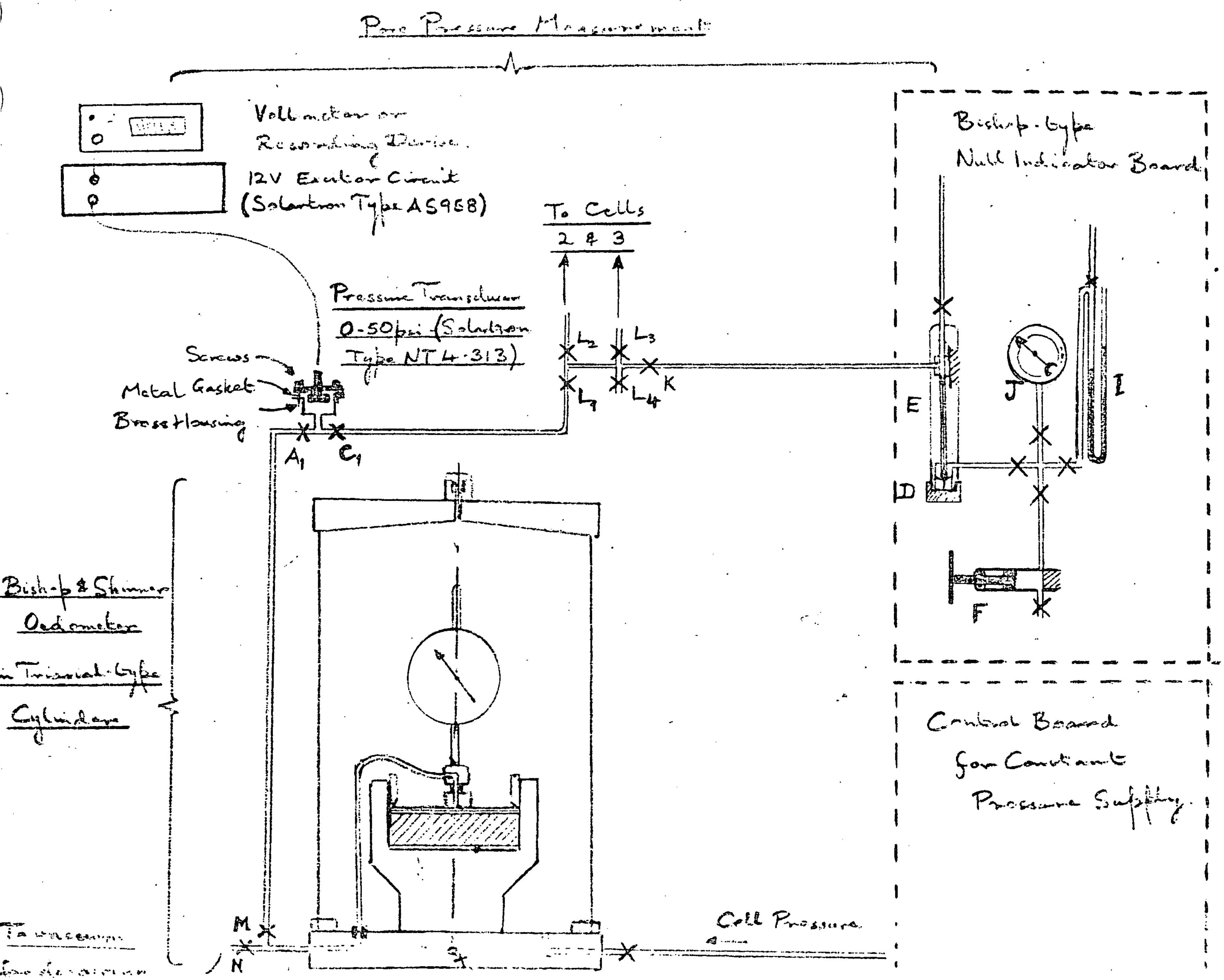
As use of this cell has not been widely reported to date, a number of preliminary tests were run to examine behaviour. The results of this work led to the procedures finally adopted for the experiments, which are detailed in Appendix B2. Details of this preliminary work are not included, but some general findings of significance should be mentioned.

Firstly, the flexible neoprene membrane was designed to allow the consolidation at the centre of the sample to be followed (by a dial gauge

TABLE 5.2

Matrix showing Experimental Programme

Pressure Increment \ Prior Delayed Consolidation	1D		1W		1M	
R 60-120 kN/m^2	A C	B	A C	B E	A C	B
S 120-240 kN/m^2	A C	B	A C	B E	A C	B
T 240-480 kN/m^2	A C	B	A C	B E	A C	B



Cell should be fully immersed in constant temperature water-bath. Transducer housing should be similarly immersed but not leads & connections which must be kept dry.

Drain from cell may be connected to burette (water level = midheight of cell) to measure volume changes.

SKETCH of EXPERIMENTAL SET-UP

BISHOP & SKINNER OEDOMETER

FIG 5.1

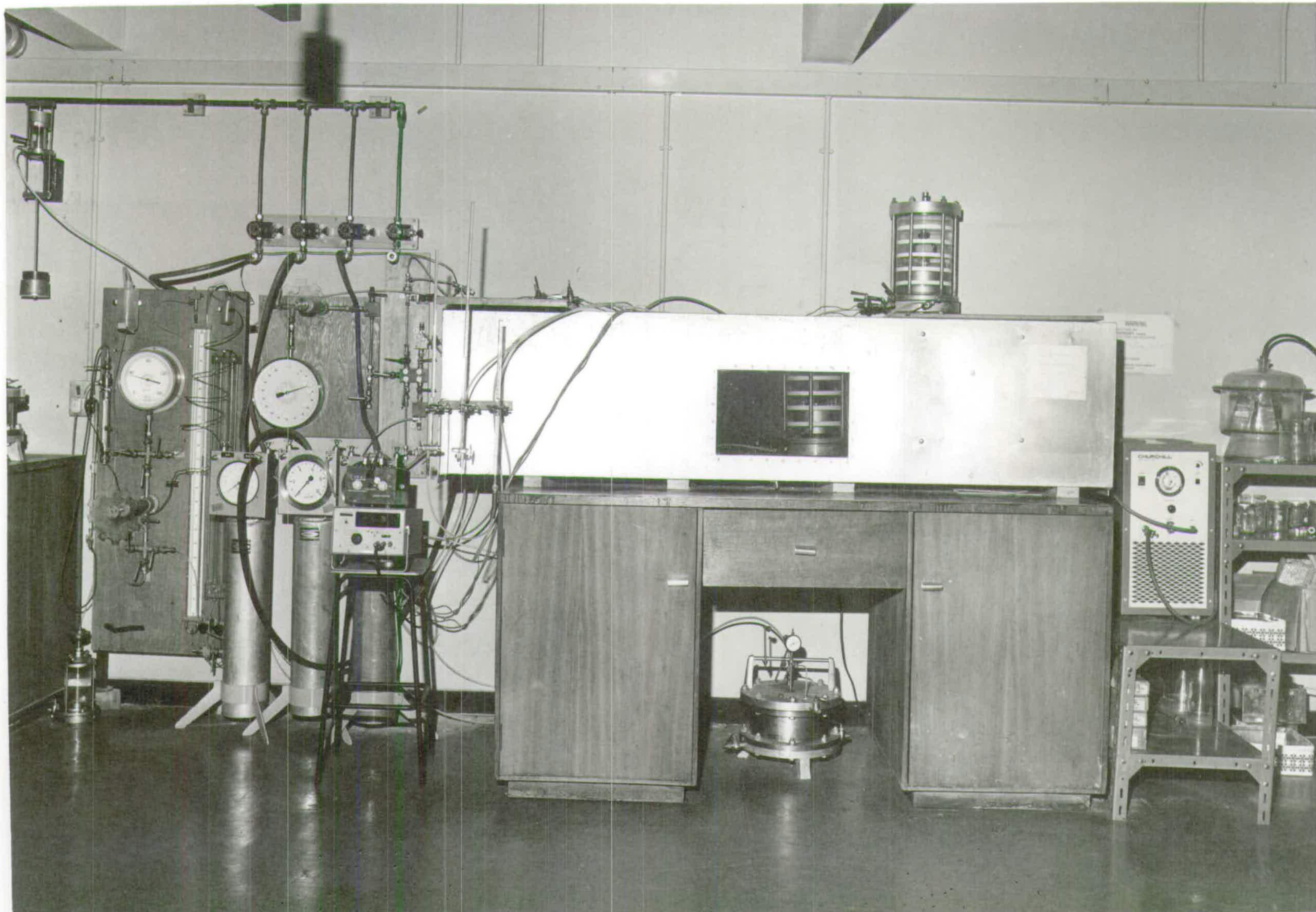


PLATE I EXPERIMENTAL APPARATUS

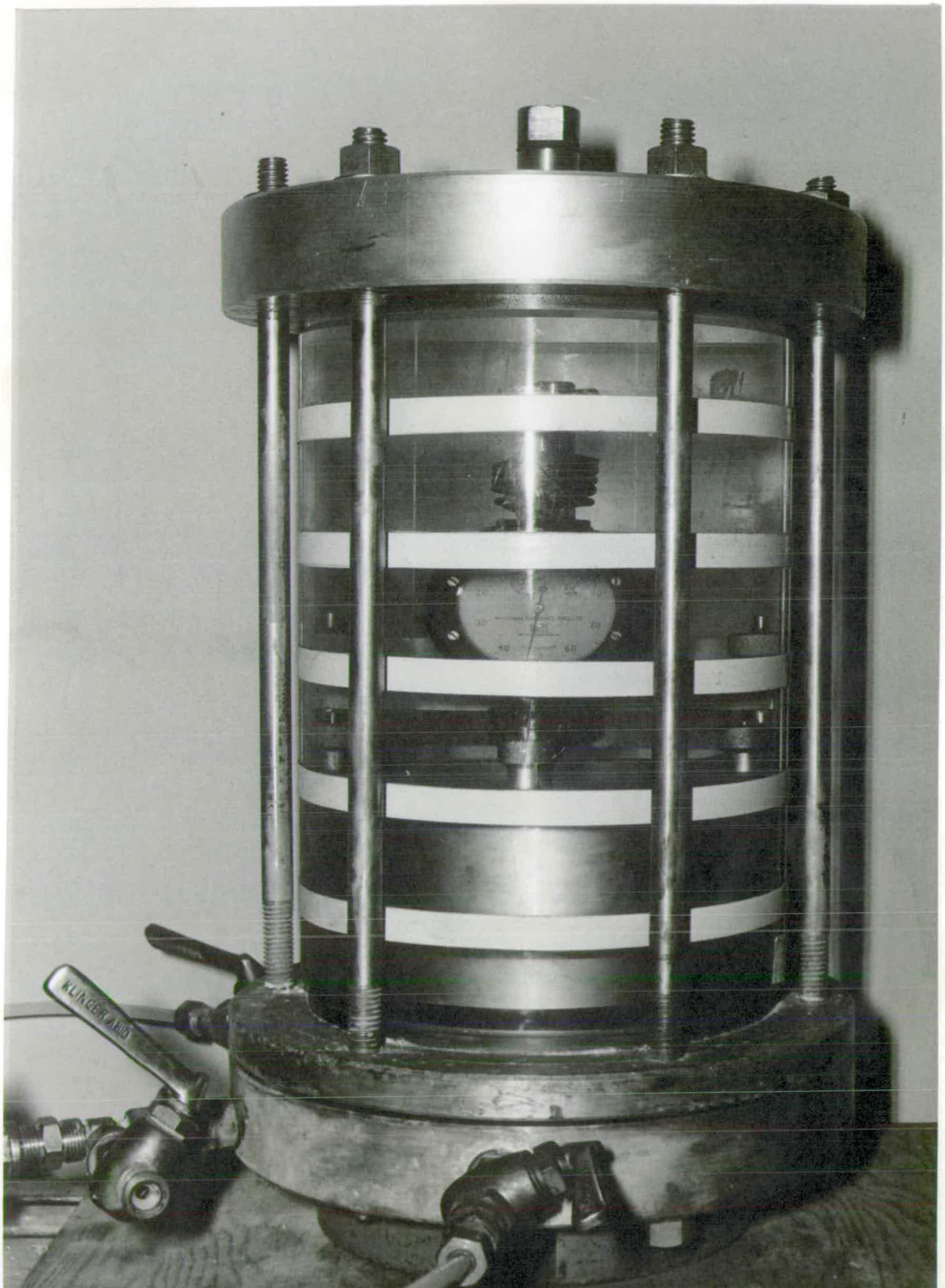


PLATE II THE BISHOP AND SKINNER OEDOMETER

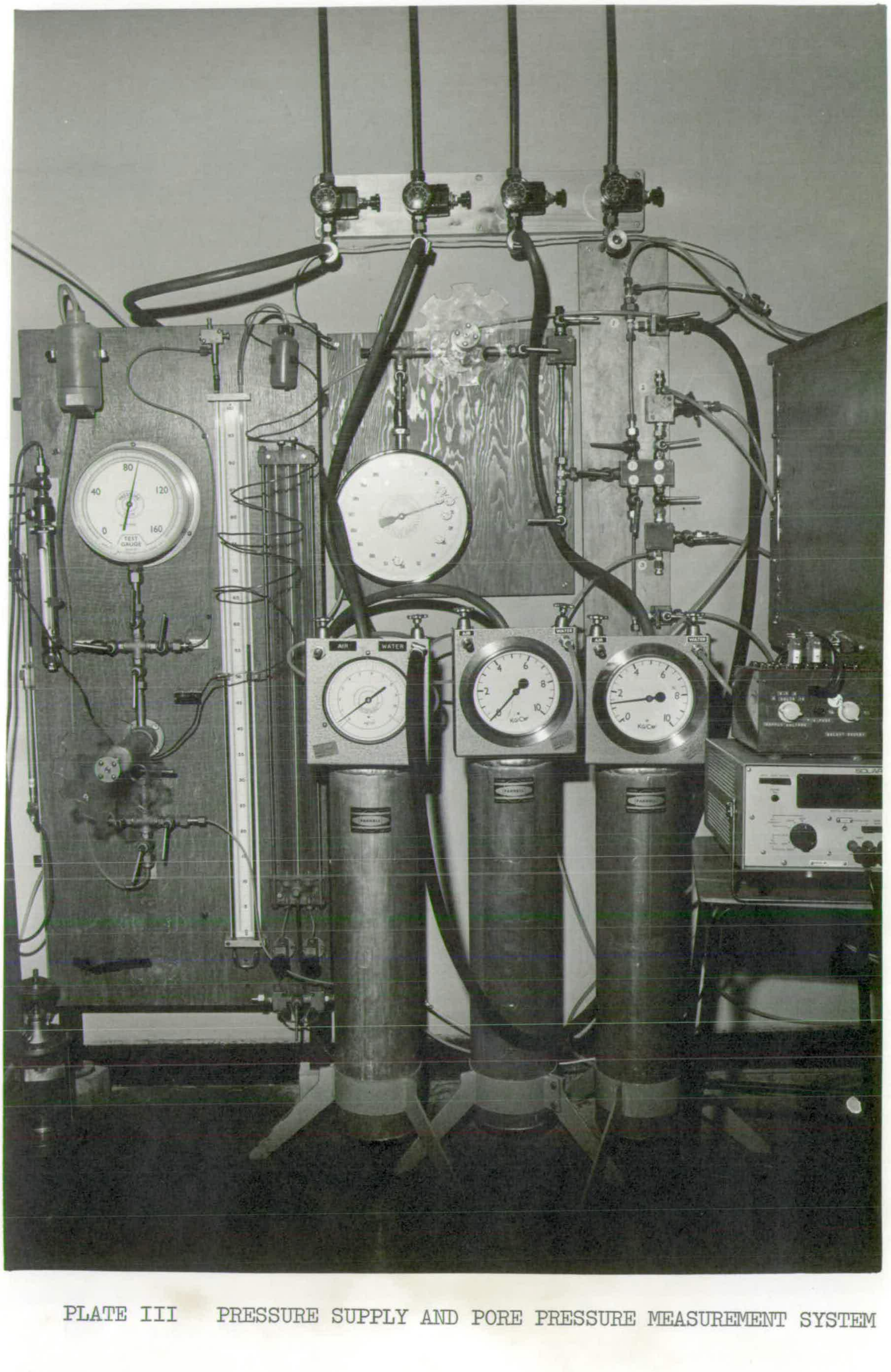


PLATE III PRESSURE SUPPLY AND PORE PRESSURE MEASUREMENT SYSTEM on the centre mushroom), thus avoiding the effects of side friction which occur when a rigid top platten is used. It was soon demonstrated that for such a case consolidation behaviour deviates significantly from one-dimensional, so that satisfactory analysis would not be possible for the present investigation. It therefore proved necessary to use a rigid top platten to overcome this. The resulting, indeterminable, problem of side friction has had to be accepted. The usual precaution of covering the cell sides with an inert silicon grease has been followed here.

Secondly, the top line was found to be unsuitable for drainage purposes, as the narrow bore imposes excessive friction losses on the flow. A special construction of the Terzaghi spring-in-pot rheological model gave "consolidation" governed only by the rate of flow of water from the cell. Using the fine-bore tube only for drainage, the model took up to 1 day to reach equilibrium. Pore water pressures, measured at the impermeable cell bottom, were found to fall in line with this.

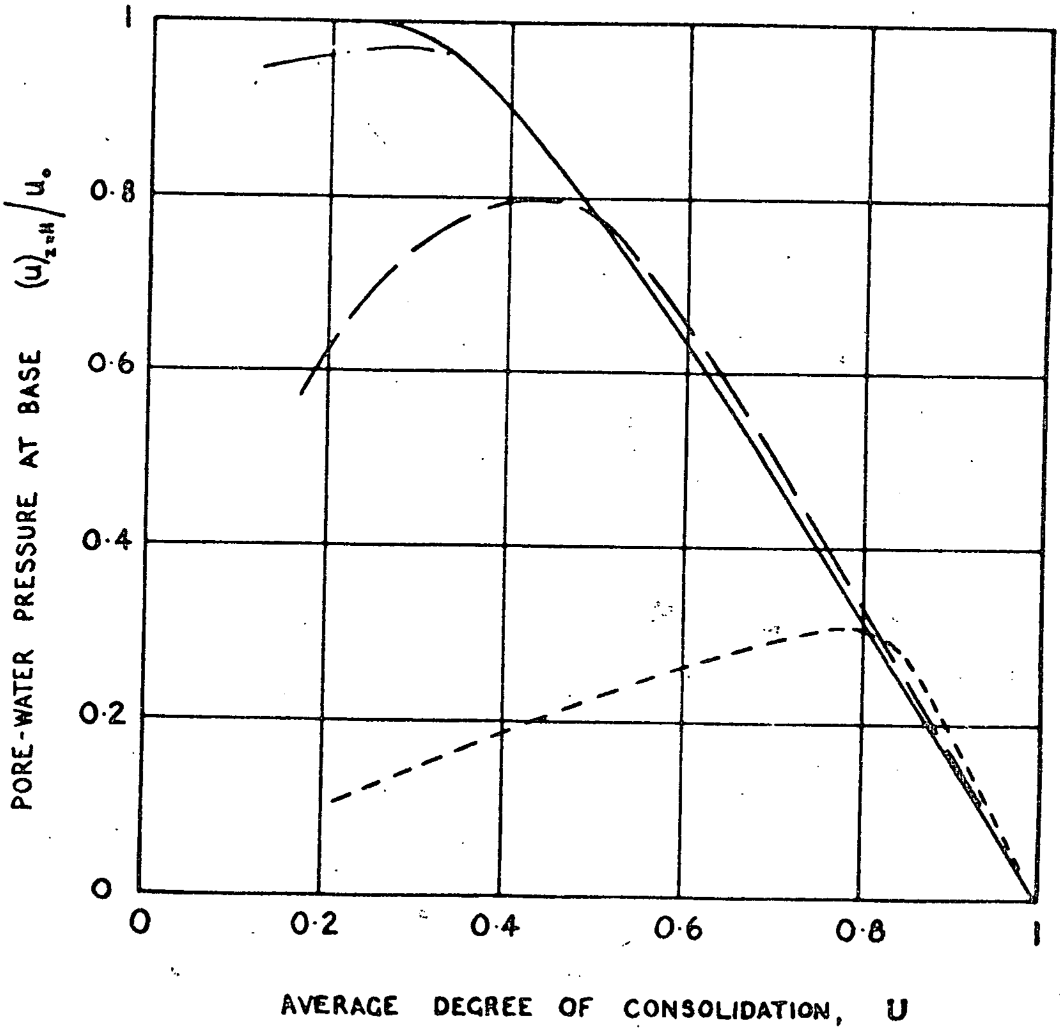
The same spring-in-pot model confirmed drainage from the bottom to be satisfactory, with immediate pore pressure response measured via the top line. However, the only way to fit the top piston in place is by gradually withdrawing first air and then de-aired water from above the sample, by applying a mild vacuum through the narrow-bore tube (see Appendix B1, fig. B1-2). This line must then become part of the pore pressure measurement system, in which no air can be tolerated (see below). Careful tests proved a 2 way valve system and suitable procedure (Appendix B2) to this end, but this has proved rather tedious and time consuming to use and cannot be recommended for routine tests. This the author considers to be a rather unsatisfactory feature of this oedometer.

Pore pressure measurements were made by transducer, with readings displayed on a digital voltmeter. As may be seen from fig. 5.1, the pressure line from cell to transducer may be opened back to the Bishop-type pore pressure board. This is useful both for calibration, and for forcing water through the system for initial de-airing and setting up.

Flexibility of the pore pressure measuring system is a problem. This is the volume change resulting from change of pressure within the system, and it leads to departures from the no-flow condition at the "impermeable" boundary, which can be of some significance. The apparatus does not allow this to be assessed during a test, but a figure of better than $15. \times 10^{-3}$ mm^3 per kN/m^2 was obtained for the apparatus used when fully de-aired. Hence the elaborate de-airing procedures used - any trapped air would give rise to significant flexibility. {The flexibility may be evaluated by pressurising the system against a sealed end or closed valve through the Bishop Null Indicator device. Movement of 1 mm of the mercury thread indicates a volume change of 0.787 mm^3.}

An analysis technique proposed by Christie (1965) proved useful here. Briefly, the Terzaghi equation is solved with an unusual boundary condition allowing for a flexibility X in the pore pressure measuring system. Graphs were produced showing the influence of flexibility in terms of a dimensionless parameter C = AH mv/X, where A and H are sample cross-sectional area and thickness, respectively, and mv is the conventional Terzaghi coefficient of volume compressibility. Details need not be reproduced here, but the graph for the case of free drainage, equivalent to the present C-type tests, is included to clarify this discussion, (Fig. 5.2). The equivalent graph for the A-type tests shows the pore pressure dissipation curves always to the right of the conventional Terzaghi curve, the discrepancies increasing

Fig 5.2 Analytical Solutions to the Terzaghi Equation with Unusual Boundary Condition.
(Flexibility X in Pore Pressure Measuring System)
(After Christie 1963, 1965)



PORE-WATER PRESSURE AT BASE OF SAMPLE PLOTTED AGAINST AVERAGE DEGREE OF CONSOLIDATION FOR VARIOUS VALUES OF C $(=AHm_v/X)$.

INITIAL PRESSURE IN MEASURING SYSTEM, $p_{mo} = 0$.

- – – – – – C = 1
- —— —— C = 10
- ——·—— C = 100
- ———— C = ∞

with decreasing C, and being greater at high pore pressure than at later stages of consolidation.

The critical case for the present study would be at early times, when according to the concepts outlined here, mv values would tend to be very low. Even in such cases, however, it could be shown that values of C in excess of 100 result for the fully saturated soil in the present experiments, from which the influence of such flexibility may be considered negligible. Comparison of fig. 5.2 with the results obtained here (figs. 5.5, 5.9 and 5.12) shows that the deviations from Terzaghi theory cannot be attributed to such flexibility.

Unfortunately it was found that the soil tended to become unsaturated over a period of time. This effect has been attributed to evolution of gases from the soil during the periods of delayed consolidation (discussed in sections 5.5.2-3). Some back-pressure tests (E-type) were therefore run, using a fairly high back-pressure of 240 kN/m^2, and this refinement proved very effective in maintaining full saturation. These tests considerably clarify behaviour, and incidentally demonstrate that the apparatus is functioning as intended. The apparatus and sample are found to remain suitably inflexible throughout these tests.

Temperature variations may lead to two undesirable effects.

i) The rate of secondary consolidation is affected. Lo (1961) reported a discernable change in rate for a variation of some $4^{\circ}C$, for a Grangemouth Clay similar to that used here. Changes of $\pm \frac{1}{2}^{\circ}C$ do not lead to detectable variations. (See Section 6.2.)

ii) Volume changes may be induced in the pore pressure measurement system, leading to problems similar to the pressure flexibility described above.

A constant temperature room was not readily available, so a neat solution was to keep the cells immersed in a constant temperature water bath. Transducer lines and housings (but not the electrical leads) were also immersed. This enabled temperatures to be maintained at $16 \pm \frac{1}{2}^{\circ}C$.

5.4 Computer Analysis

A large amount of repetative calculation was involved in the analysis of data from the experimental programme. It proved very useful here to write a computer program, CALCOG, capable of reducing deflection readings to void ratios, by the 'height of voids' method; reducing transducer readings to pore pressures; and determining the necessary functions of these for analysis. These are given as tabular numerical output.

It was relatively easy to extend this program to give graphical output. A MENU is presented allowing the following graphs to be selected as desired.

1. e - square root of time
2. e - log time
3. μ_b - square root of time
4. μ_b - log time
5. μ_b - degree of consolidation (average strain)
6. log e_{av} - log p_{av} (where p_{av} is the average effective stress over the layer, obtained by assuming a parabolic pore pressure distribution, whence $p_{av.} = p_f - \frac{2}{3}\mu_b \times \Delta p$.
   This assumption is of limited validity, and certainly not acceptable at early stages of consolidation - see note below.

   {μ_b is the dimensionless pore pressure at the impermeable boundary.}

These graphs have greatly facilitated comparison of results (see section 5.5) - graph 5, proving particularly useful here. They have also enabled the various parameters required for Terzaghi and Garlanger theories to be determined directly; further comments are made in sections 5.6 and 5.7.

A flow diagram for CALCOG may be found in Appendix A.

Note on p_{av}

The average log e - log p curve was used to determine a value for the instant line in Chapter 4. With only limited data available, t_i was defined as the time line tangential to this curve, and such a technique seemed to give quite good values. There are problems, both in plotting such a curve, and in the meaning of t_i. It will be seen that the incremental loading tests (B-type) have been introduced in order to obtain a more satisfactory indication of t_i. This avoids the need for average e - p curves in Garlanger analysis.

{It will further be seen, in section 6.4, that a satisfactory and theoretically consistent treatment of the t_i line and associated p_c value has been proposed, which completely eliminates the need for such attempts to identify an "instant" line - but this goes considerably beyond the present scheme.} The Garlanger analysis may be numerically integrated, to give average stress-voids ratio curves as consolidation progresses. Comparison with the experimental average curves (graph 6, above) shows generally good agreement, suggesting that the assumption of parabolic pore pressure distribution is reasonably acceptable. The exception to this is at early times, before the pore pressure is observed to dissipate. The "parabola" gradually spreads from the drain to the impermeable boundary, and it is only once the full profile of soil is affected that the expression for p_{av} from μ_b holds. However, knowing initial conditions the curve for this early behaviour may be drawn in by hand with acceptable accuracy.

5.5 Experimental Results

5.5.1 General Remarks

A substantial amount of experimental data has been obtained and some comments are made below on the rationalisations introduced to reduce this to a manageable form for analysis and for presentation purposes. The main aspects of the observed consolidation behaviour are considered under the headings of the four basic types of test. Details of the delayed consolidation are grouped together in a separate section. Analysis of the data by Terzaghi and Garlanger theories is left to sections 5.6 and 5.7 respectively.

Detailed comparison of the various analogous tests for the three pressure increments used indicates that pressure does not have a significant effect on the consolidation behaviour, at least over the range of pressures tested here. This is quite interesting because although in line with the Garlanger-Bjerrum model of soil behaviour, the author was quite prepared to find significant pressure effects. Some previous tests on 5 different remoulded soils using the Rowe oedometer (Tonks 1972) had suggested that Cv rose slightly with pressure. For the present data it appears that Cv in the equivalent tests (C-type) may, in fact, rise slightly with pressure. This is not observed for the other types of test, however. As will be seen in section 5.5.3 this effect on Cv is believed to be an aberration due to the decreased degree of saturation in the higher pressure tests which have necessarily been in the oedometer for greater periods of time. The only other observed behaviour which was found to be affected by pressure in the present tests was the amount of strain occurring in the A-tests when the pressure was increased with the drain closed.

This again suggests the degree of saturation decreases somewhat with pressure and, associated with this, the maximum pore pressure, u_a, obtained after 24 hours undrained, also decreases with pressure. It is misleading, however, to interpret these as direct pressure effects: they should really be related to the length of time the soil has been in the cell.

For the present purposes, then, the pressure variable may be dispensed with. All the test results have, of course, been analysed, but presentation here is considerably simplified by only having to display results for one pressure increment. In the subsequent sections all results are given where these may be clearly distinguished, (on at least one graph for each test type), but for graphs where results would tend to loose clarity only one increment is given.

For any given pressure increment, then, the amount of prior delayed consolidation has two major effects on behaviour. After increasing delayed consolidation

i) The voids ratio is lower. Hence on incrementing the pressure the initial strain is less, and the total change in voids ratio to reach the one-day line at the final effective stress is less (see figs. 5.6, 5.10).

ii) On incrementing the load the fall of pore pressure with time takes place more rapidly, especially at early times (see figs. 5.7 and 5.11).

Now the pore pressure behaviour at the impermeable boundary in the different tests may be easily compared by defining a dimensionless parameter $\mu_b = u_b/\Delta p$ where u_b is the observed real pore pressure at the impermeable boundary and Δp the increment of applied load. The change in voids ratio is a little more awkward to handle. A dimensionless parameter of strain may be introduced $\beta = (e_o - e)/\Delta e$ where e_o

is the initial voids ratio (after the appropriate delayed consolidation), e is the voids ratio at any particular point in time and space, and Δe is the total change in voids ratio. Since this last parameter depends on the period of time consolidation has undergone, time must also be defined. Δe is therefore defined for the period of 1 day's consolidation. By this time the influence of primary consolidation on the laboratory samples may be considered negligible, so that Δe is the same for all points in the soil profile. {During primary consolidation different depths in the profile will have undergone different amounts of settlement.} The parameter of major interest (and indeed the only deflection data capable of measurement) is the <u>average</u> deflection. Hence we work in terms of the average degree of consolidation $\%C = \{(e_o - e)/\Delta e\}av \times 100\%$.

This is the definition of strain frequently found in conventional consolidation analysis, although it was introduced to deal with the primary process in which some ultimate voids ratio e_f, is reached. The importance of relating e_f to some fixed point in time when the secondary process is included, has not always been appreciated. This terminology is employed in the graphs of μ_b - %C presented below. It is also consistent with Garlanger's analysis (to be presented later) provided e_o and Δe are correctly defined.

We notice, however, that e_o will vary with the amount of prior delayed consolidation. This definition therefore obscures a variable of major significance. It would also be useful to compare actual voids ratios. Unfortunately, the voids ratios observed for any given pressure and time are not identical for the different tests. A rationalisation of results has been introduced to enable direct comparisons here. We briefly recap to clarify this modification. A <u>test series</u> consisted of a standard method of setting up the sample and bringing it to a loading of 60 kN/m^2, from where the required period of delayed consolidation for the first test is allowed. The three tests in the series are then performed, each involving a doubling of applied load and delayed consolidation for the subsequent test. For each test series, the time lines diagram (i.e. the log e - log p plot corresponding to the available results for delayed consolidation) may thus be drawn. A typical example is given in fig. 5.3. From these, values of the gradient 'b' could be found. Excellent straight lines were obtained and values of 'b' were found to be satisfactorily constant, as shown in table 5.3. Slight variations were found, however, in the position of these lines between the different test series. This would not be expected, especially as the slurry used, and procedure for beginning the test series, did not vary. The differences may be ascribed solely to errors in determination of voids ratio which is very sensitive to the measurements of sample thickness, weight, and moisture content.

These measurements can only be made at the end of the test series and all values for the series follow from this. We note that errors here affect all e values linearly, so that the correction would involve only vertical movements of any plot of e against f(p) or f(t).

It was considered reasonable to assume that all the time lines grids from the various test series should lie in the same position on a log e - log p plot. A Master Time-Lines Diagram has therefore been drawn, using the averaged results from the various test series. Two methods of averaging the data were used.

1. The 1 day line of slope 'b_{av}' (table 5.3) was drawn through the mean value of e_{D-120} - the value of voids ratio after 1 day's consolidation at 120 kN/m^2. Knowing the value of c (the log e - log t relationship) the lines for any other times, again of slope b_{av}, may be added as desired.

2. The mean values of e were determined for 3 pressures (60, 120 and 240 kN/m^2) after 3 periods of consolidation, (1 day, 1 week and 1 month). This gave 3 points on each of the 3 time-lines, which were thus drawn.

Excellent agreement was obtained between these two averaging techniques. The final Master Diagram is shown in fig. 5.4.

Now this master diagram gives the best averaged data of the experimental observations of delayed consolidation. The time lines from any given test series are moved vertically slightly and rotated slightly. As noted above it is consistent with the assumption of experimental error causing the discrepancies in position to move an e - log p plot up or down. The error in moving a log e - log p plot in this manner is imperceptible for such small deviations as were found here (change in $e < 0.025$). Rotation, on the other hand, (i.e. assuming all values of b to be identical) is not strictly valid. For the analysis by Garlanger's method it is reasonable to assume the best available averaged parameters would be used, hence the data from the master diagram is used for analysis. It is not acceptable to rotate the individual test results in this manner, even if this could be accomplished with ease. (In practice such a "correction" would be rather involved.) The rationalisation here has been to consider each test separately (rather than in its test series) and move the calculated e values up or down slightly, so that the voids ratio after one day coincides with that for the master diagram for the given pressure. All experimentally observed voids ratios are then incremented by this difference. Hence the experimental differences in change of voids ratio will appear in the different values of e_o. This technique has been used for figs. 5.6, 5.10 and 5.13, showing voids ratio against logarithmic time.

TABLE 5.3

Coefficient 'b'

Test Series	b =
2	.220
3	.216
4	.210
5	.214
6	.212
7	.215
8	.224
9	.220
10	.216
11	.220
12	.220
14	.218
20	.210

Mean value of 'b' = 0.216

Fig 5.3

Typical Time Lines Diagram

(Test Series T5)

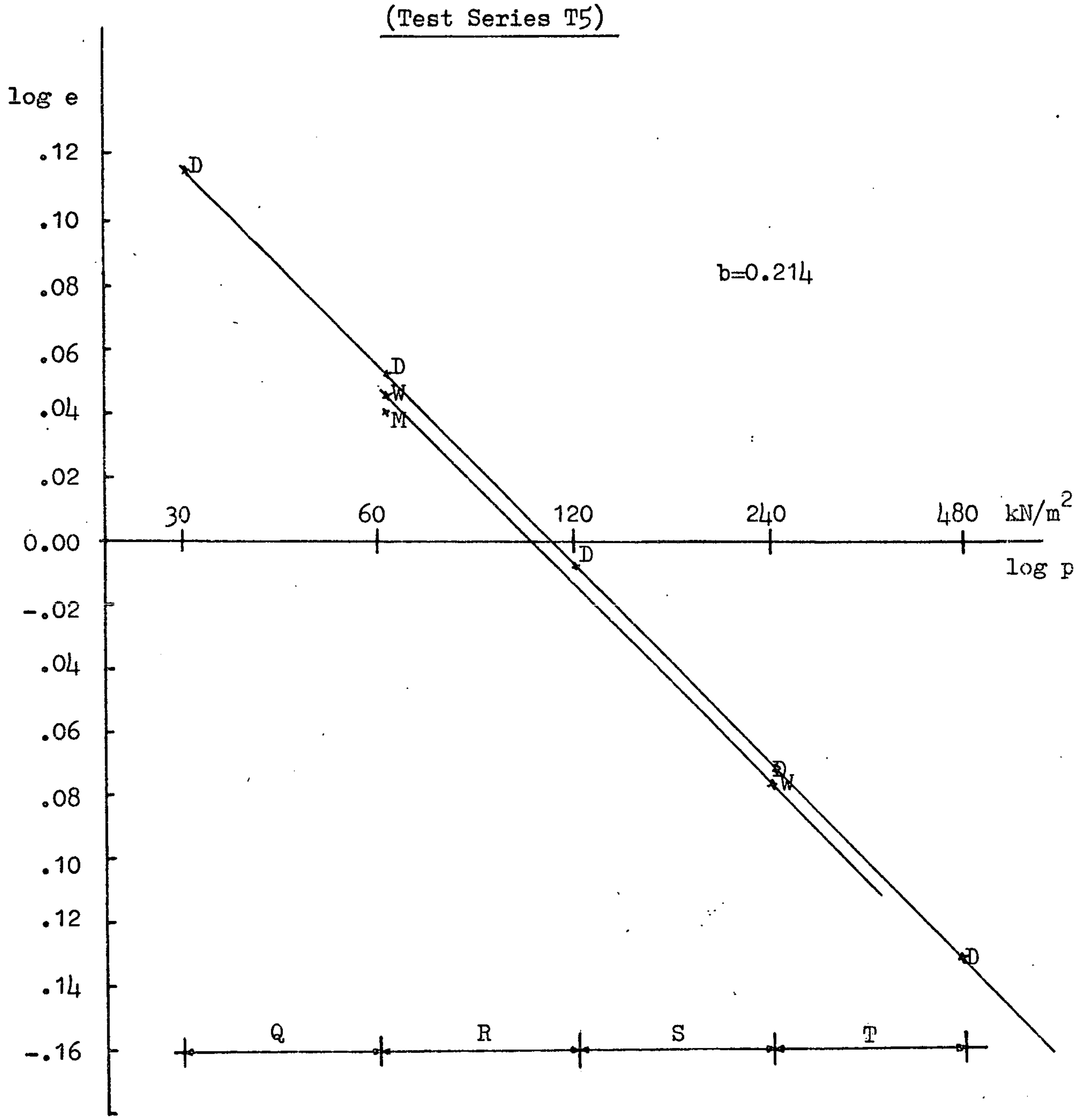
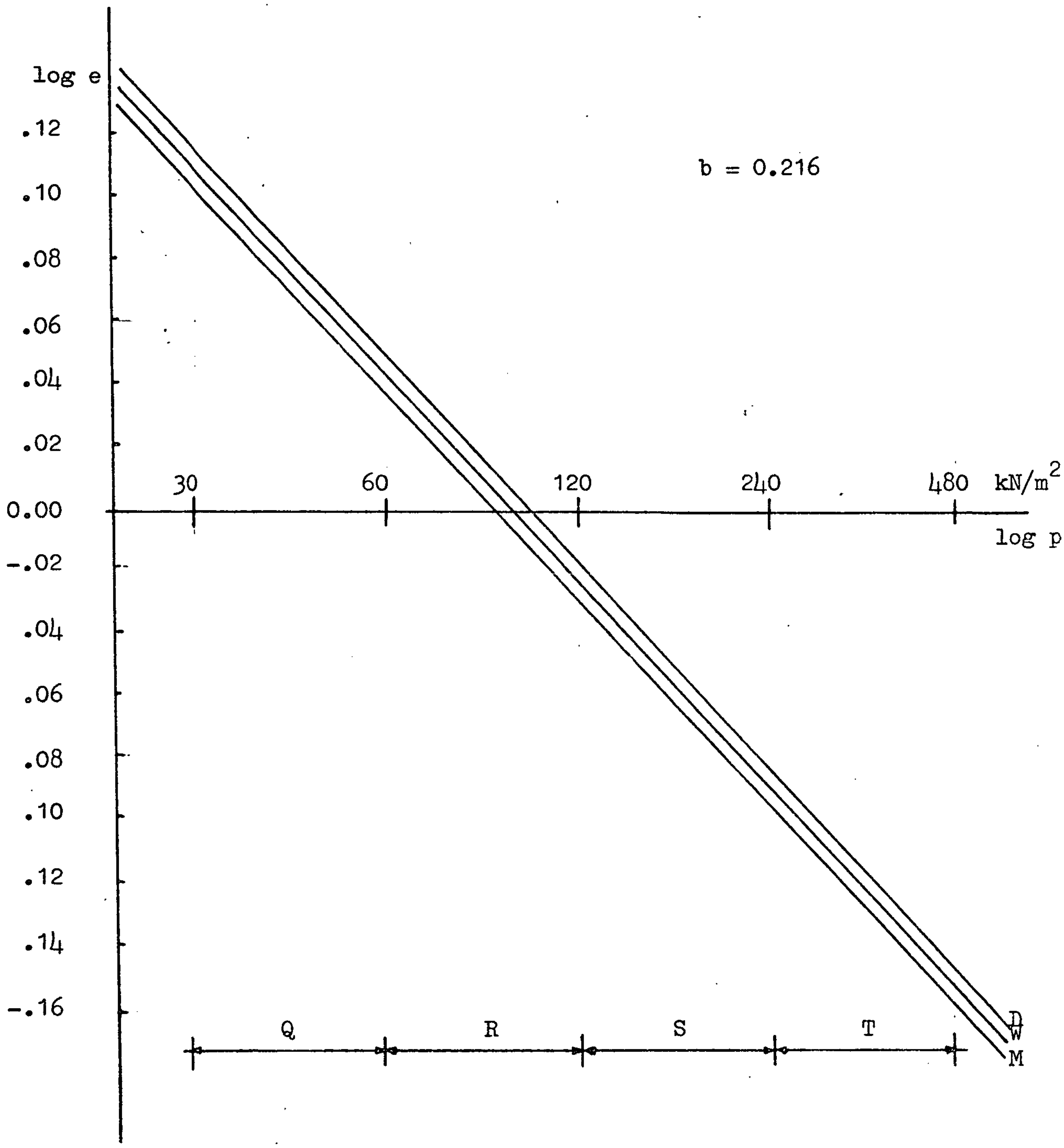


Fig 5.4

Grangemouth Silty Clay (Remoulded)

Time Lines - Master Diagram



Note Results are average of 13 test series

From these graphs it may be clearly seen that the major differences in e_o are due to the period of delayed consolidation. As mentioned before, the influence of the pressure is not significant. Comparing tests of the same type and same prior delayed consolidation for the three pressure increments, there is some scatter in values of e_o but this is random and small compared with that due to the influence of delayed consolidation.

The rationalisations introduced to enable comparison of e values for the test results do not affect these results in any way other than moving all values of e slightly up or down, and this is consistent with the assumption that all tests for any given final pressure should show the same voids ratio after 1 day (i.e. after primary consolidation is complete). Comparing results for the various periods of delayed consolidation shows the influence of this on behaviour and confirms that the soil behaves in line with the Bjerrum model. The pore pressure data shows clearly how the dissipation becomes more rapid following greater delayed consolidation.

Results are complicated somewhat by gas coming out of solution in long-term tests, so that the soil becomes only partially saturated. It is possible to allow for this in the A-tests. These may be compared with the C-tests to indicate the effects of partial saturation on the behaviour of these latter. The problem arises that some of the observations could be influenced by other effects and analysis might then not be possible. Some back-pressure tests were run (E-tests, section 5.5.4) which, it was anticipated, would keep all gases in solution, and hence allow any other departures of the experiments from assumed conditions to be assessed. From these results it may be demonstrated that behaviour was quite satisfactory - partial saturation is the only additional factor which need be considered in analysis of A, B and C tests.

5.5.2 A-Tests

Following the required period of delayed consolidation, the drain is closed and a load increment of ratio ($\Delta p/po$) 1.0 is applied. The build-up of pore pressure and any settlement is observed over a period of 24 hours. The drain is now opened and normal observations of consolidation are taken.

The main results obtained from the A-tests are shown in figs. 5.5 -7. From the plots of pore pressure against strain it may be seen that although there is some random scatter between the different pressure increments, a clear trend is established with increased delayed consolidation leading to lower pore pressures and greater deviation from predictions of Terzaghi theory. The graphs of voids ratios and pore pressure against time indicate lesser values of both these variables following increased delayed consolidation.

Conventional Terzaghi theory suggests that following an increase in applied load, the excess pore pressure jumps immediately to equal this load increment. The A-tests were carried out specifically to examine if this were the case and hence to enable the validity of the more conventional C-tests (section 5.5.3) to be assessed. It was found, as illustrated in fig. 5.8, that pore pressures build up gradually, and usually to rather less than 100% of the load increment. Accompanying this, significant strains took place. Such behaviour could not be attributed to flexibility of the pore pressure measurement system. It may be noted that for the first increment, Q, of each test series, applied immediately after setting up the sample, the pore pressure was always observed to rise to 100%, although the last few per cent took a significant time to be reached (see fig. 5.8). It could be demonstrated that leakage would not give rise to the effects observed. It was concluded that the soil becomes unsaturated during delayed compression. This occurred although the system was fully sealed so that air could not enter, and both system and sample had previously been fully de-aired. Gas held in solution in the soil-fluid system, even against a vacuum of some 760 mms Hg, appears to be released over a period of time and, indeed, would seem capable of exerting a small pressure such that water may be forced from the soil, and the gas occupy some space. {The "drain" led to a burette, the water level in which was kept very slightly above the top level of the sample, to ensure the soil was not subjected to suction. This may be considered as a negligibly small back pressure, never exceeding 0.1 kN/m^2.}

On removing samples from their cells at the end of testing it was noticed that a thin black layer (~ 0.5 mm) had formed on both top and bottom edges, and a strong odour of hydrogen sulphide was given off possibly with organic gases also present. Now the soil organic content is fairly low (see table 5.1) so that it would not normally be classified as "organic". The black colouration was not thought to be carbon. It is believed that this soil undergoes gleying (see, e.g. Bolt and Bruggenwert, 1976). Briefly, a soil normally exists in biological equilibrium, with aerobic bacteria living on dissolved oxygen in the pore fluid. On rigorous de-airing (as was the case here) anaerobic bacteria take over, reducing sulphates to sulphides and, in particular, converting iron salts to black iron oxide. Hydrogen sulphide is given off. Clearly the soil chemistry is complex and it is regretted that it was not possible to pursue this topic further. Whatever the precise mechanisms at work, the soil, even though well vacummed prior to testing, may become significantly unsaturated in anaerobic conditions.

Fig 5.5

GRANGEMOUTH A Tests

Pore Pressure at Base against Average Degree of Consolidation

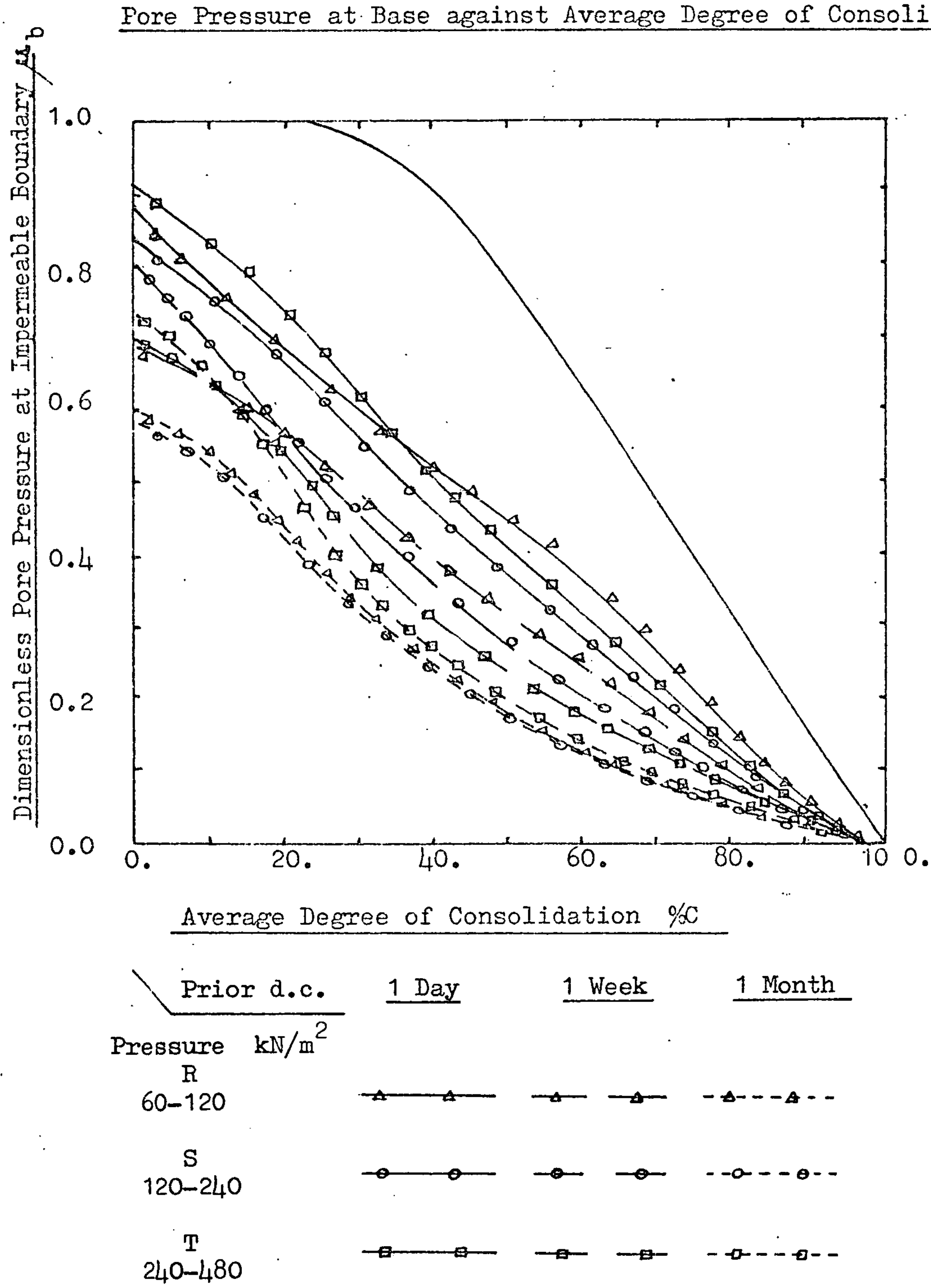
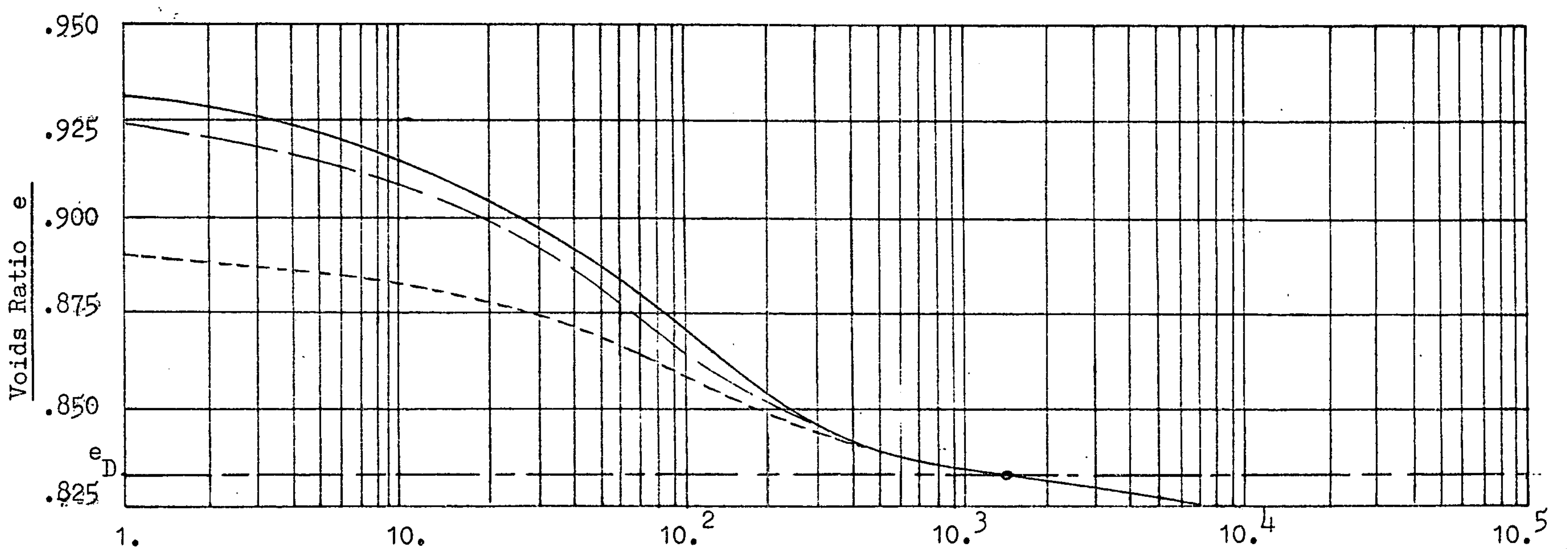


Fig 5.6 GRANGEMOUTH A-Test Settlement-Time Graph

Increment S 120-240 kN/m²



e_a =		Prior Delayed Consolidation
.939	———————	1 Day
.931	—— —— -	1 Week
.890	- - - - - - - -	1 Month

Fig 5.7 GRANGEMOUTH A-Test Pore Pressure-Time Graph

Increment S 120-240 kN/m²

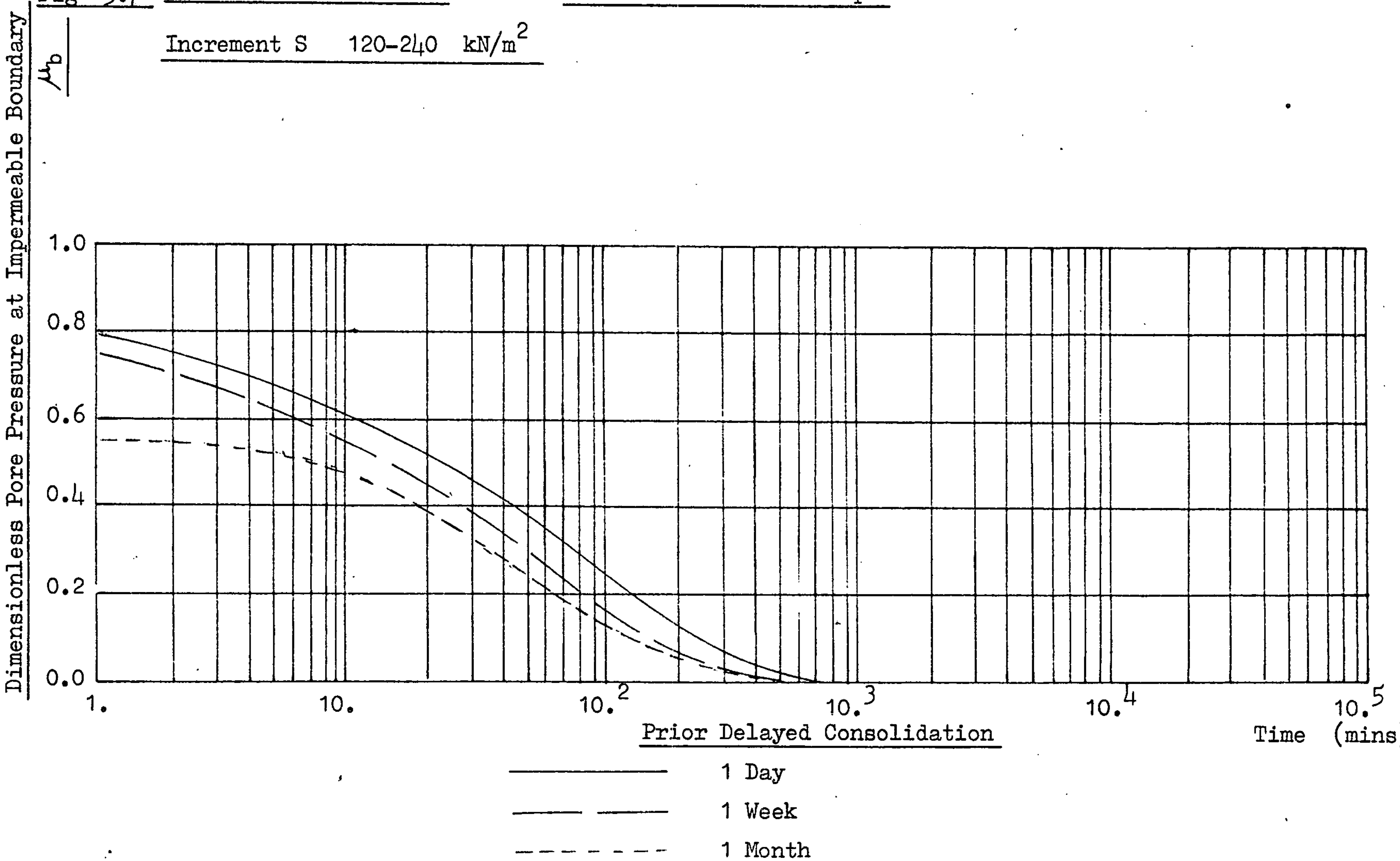
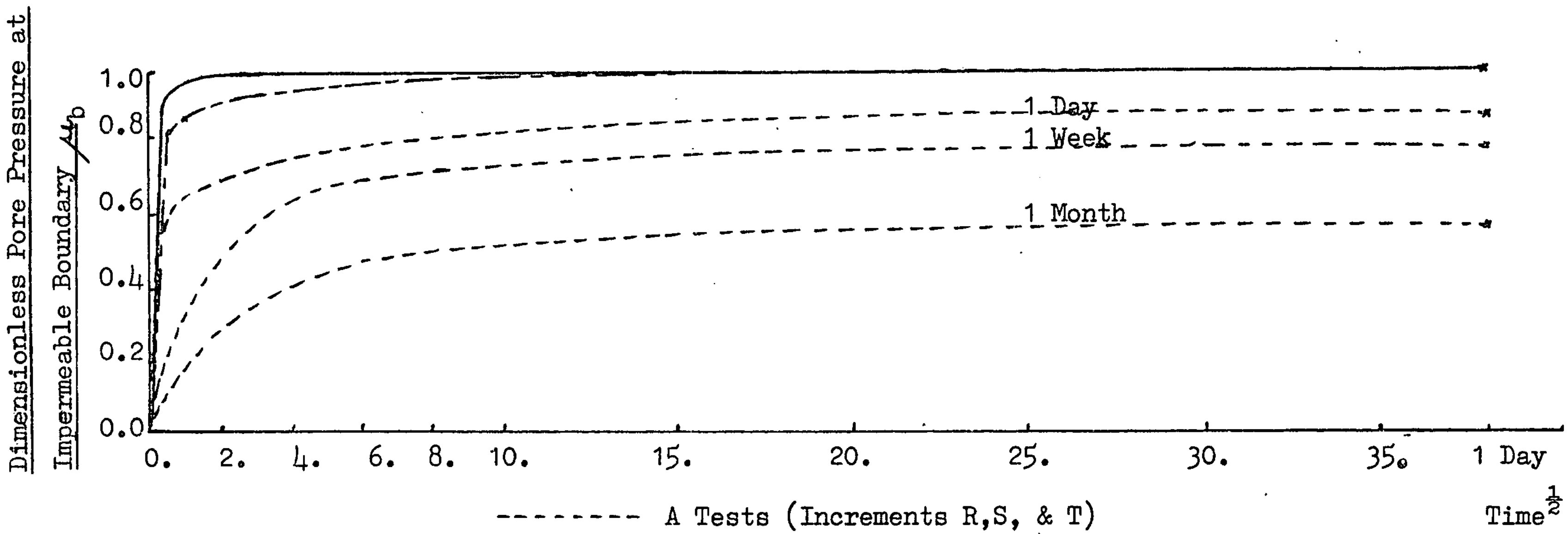


Fig 5.8 **Build-up of Pore Pressure on Application of Load Increment Ratio of 1.0 with Drain Closed, following Delayed Consolidation as Indicated**



- - - - - - - - A Tests (Increments R,S, & T)

—— – —— A Test (Increment Q see text)

———————— E Test (Increments R,S, & T after 1 week)

Although not 100% of the load increment, pore pressures reached a constant value after a period of time. It may be assumed that this corresponds to all the gas having been forced back into solution. It should be noted that the state of solution is a purely liquid phase, so that the soil system may now be considered fully saturated. The state of strain after 24 hours, just prior to opening the drain (denoted by suffix $_a$ - e.g. e_a, p_a), may be determined directly from the deflection gauge reading. It is reasonable to take the stress distribution constant with depth, whence p_a may be determined directly from the observed pore pressure. It was found that this new point (p_a, e_a) plotted on the time lines diagram approximately corresponding to the amount of prior delayed compression, i.e. the stress-strain state remained on the same time line as before the build-up. Agreement here was not exact, but then this test is fairly sensitive, and points were sufficiently close to their respective lines to suggest this is a useful technique for analysing unsaturated behaviour.

The various techniques available for analysis of results will be met in later sections. Here it should be noted that the critical effective pressure is affected by the fact that significant strain occurs in the build-up stage. Reference to the time lines diagram indicates that this would be expected to increase the values obtained for p_c. Because the soil state (p_a, e_a) is known, this may be allowed for. It can then be shown how delayed consolidation affects the value of p_c, having allowed for the complication of unsaturation.

Now it is important to note the difference between (p_a, e_a) and (p_o, e_o). The stress and voids ratio increments when the drain is opened after 24 hours at the new loading, will be rather less than those from the point (p_o, e_o). In the present graphs the definition of μ is kept as noted for section 5.5.1, i.e. the pore pressure is expressed as a fraction of total applied load increment. It will be seen later that better analysis is possible considering only the load increment from p_a to p_f. The average degree of consolidation in fig. 5.5 is defined now as $\{(e_a - e)/\Delta e\}_{av}$, i.e. the starting point for strain is taken as e_a. This is the voids ratio observed at the start of the test (i.e. when the drain is opened), but because of the "undrained" strain during the pore pressure build up stage, this definition of strain is not identical to that in the other tests, which begin at e_o.

5.5.3 C-Tests

Following the required period of delayed consolidation, a load increment of ratio 1.0 is applied with the drain open, and normal consolidation observations of strain and pore pressure are taken.

The major results are given in figs. 5.9 - 11. It will be noted immediately that pore pressures now increase with strain and time up to some maximum. During this stage, gas is being forced back into solution and strain is consequently rather rapid. The voids ratio-log time graph shows that significantly greater strain takes place in the first minute as the degree of delayed consolidation increases.

The influence of partial saturation makes the results of the C tests rather difficult to analyse. It is not possible to take the degree of saturation as a simple function of time. It appears that other factors are also of significance, hence the variations in the curves during the stage where pore pressures increase. At medium and later stages of consolidation, however, when the system is saturated, the effect of delayed consolidation is clearly shown, with results rather similar to those for the A-tests.

Values of the Terzaghi coefficients of consolidation are given in table 5.7, to be found in section 5.6. It appears that these increase

Fig 5.9

GRANGEMOUTH C Tests

Pore Pressure at Base against Average Degree of Consolidation

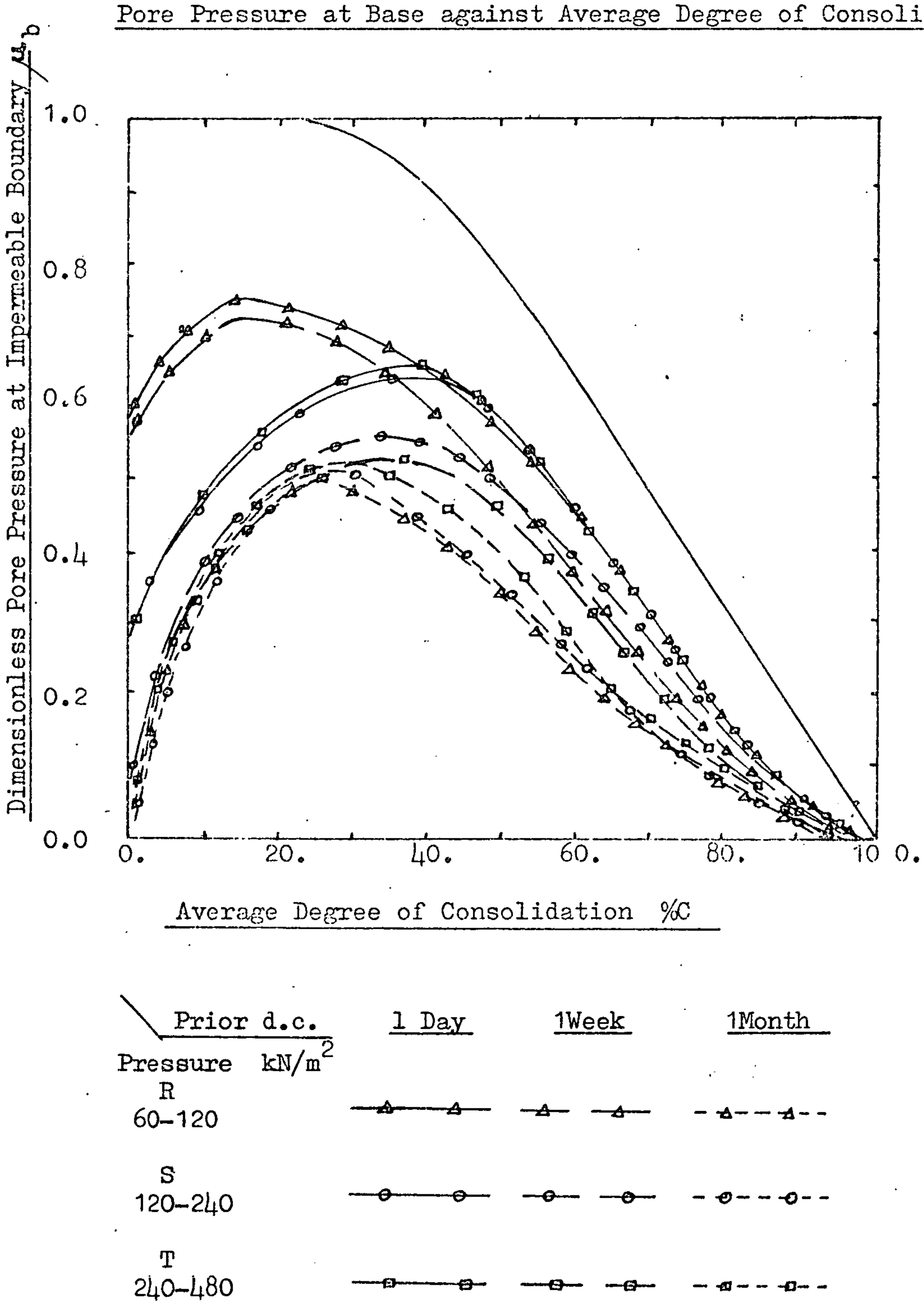


Fig 5.10 GRANGEMOUTH C-Test Settlement-Time Graph

Increment S 120-240 kN/m^2

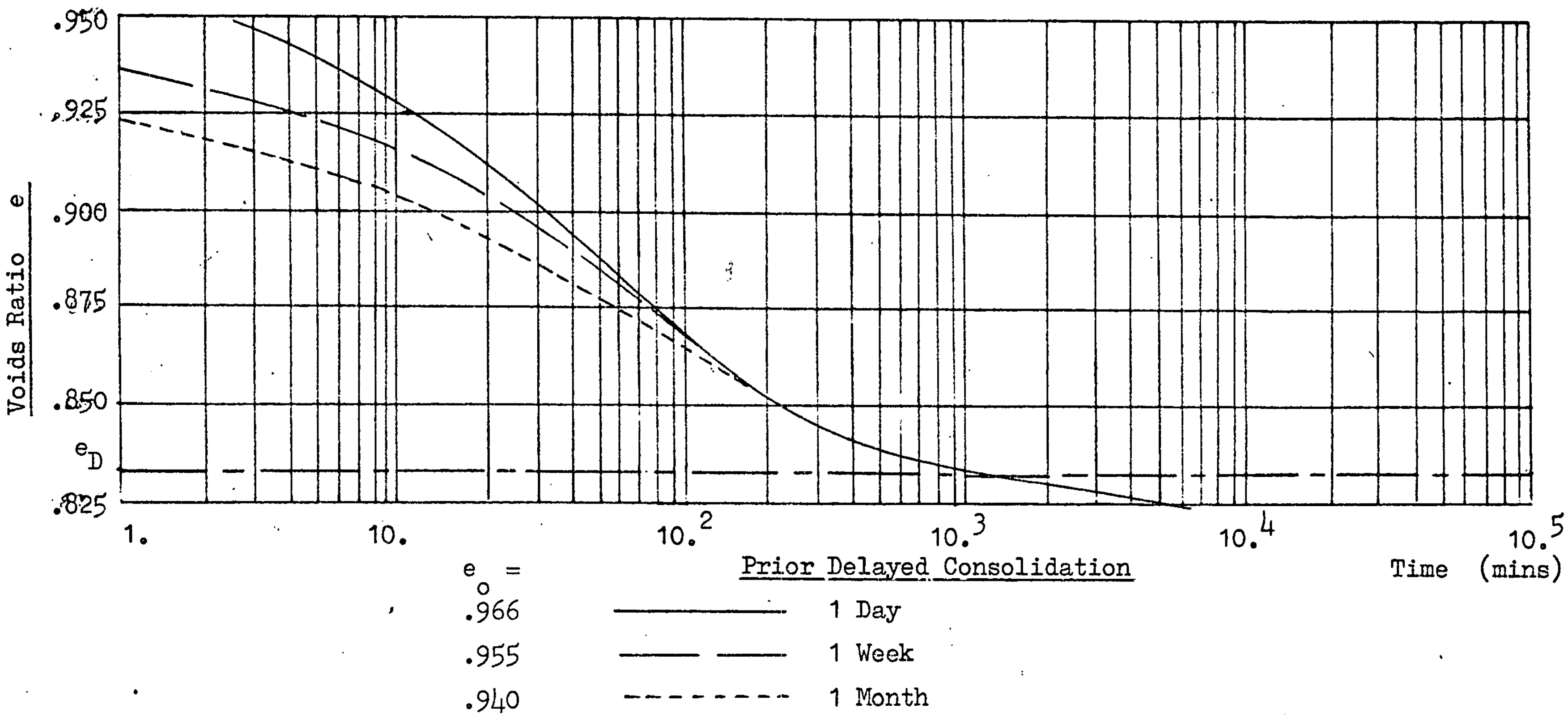
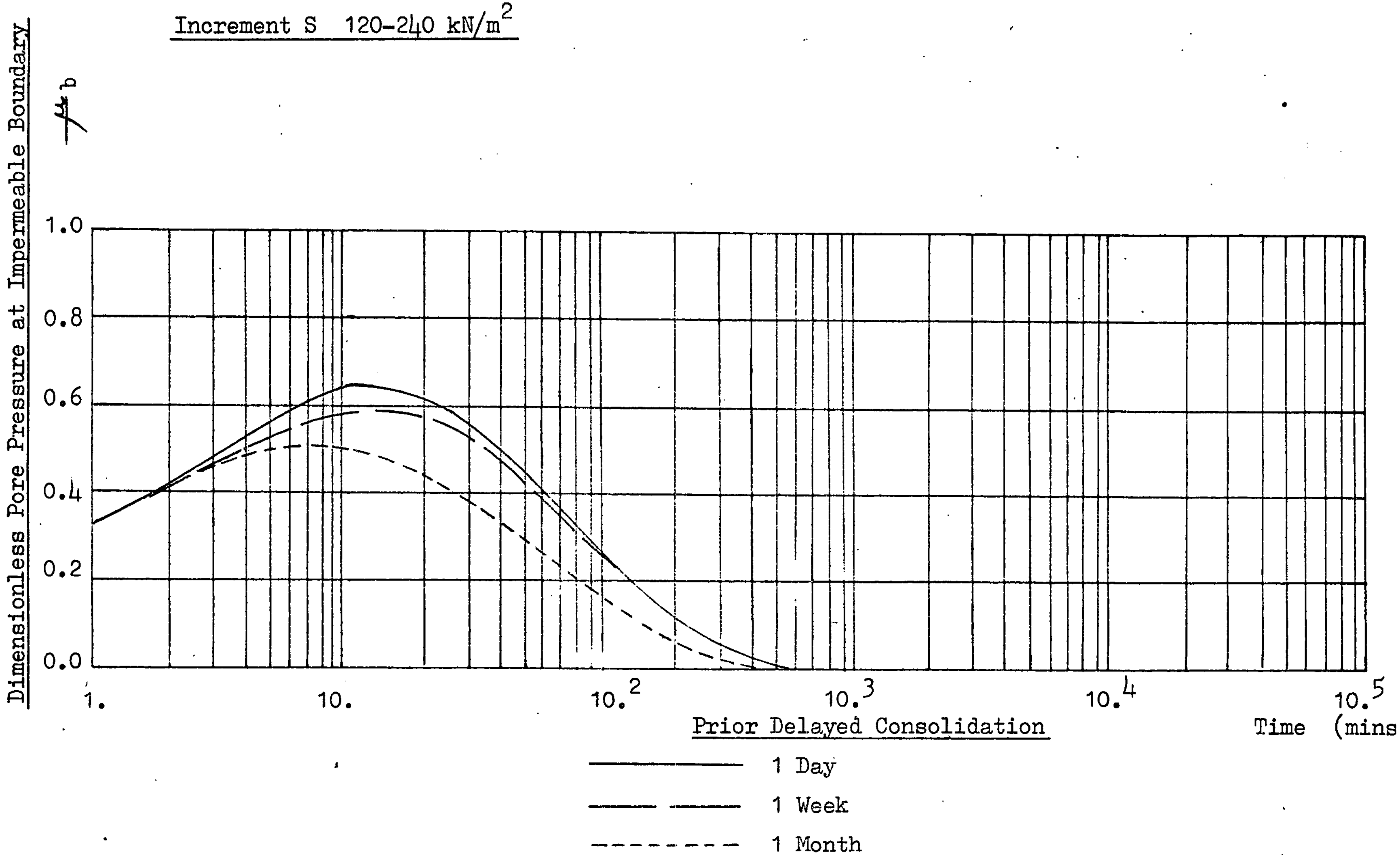


Fig 5.11 GRANGEMOUTH C-Test

Pore Pressure-Time Graph

Increment S 120-240 kN/m²



slightly with pressure. This is believed to be due to gases going more rapidly into solution, altering slightly the shape of the strain-time curve. Although a constant load increment ratio was used, the actual pressure increment increases. It is this pressure which forces the gases back into solution. As this pressure increases, so this effect proceeds more rapidly, resulting in curves of strain against square root of time, or logarithmic time, showing an initially high gradient gradually decreasing. These differ from the Terzaghi curves, and the fitting methods tend to give low values of t_{50} or t_{90}, i.e. the time for a given percentage of the Terzaghi process to have occurred is underestimated, leading to artificially high Cv values. As noted in section 5.5.1, the degree of saturation after a given period of delayed consolidation is found to decrease slightly with increasing pressure, as evidenced by the maximum pore pressure attained in the C-tests, and the build-up of pore pressure in the A-tests. It is believed that pressure is not really the significant variable here, though. The effect is due to the total length of time the samples have been in the oedometer cell {which, since the test program requires doubling the load for each test, must increase with pressure}. It may be noted that since a random test program was adopted the periods of delayed consolidation for increments prior to that under consideration exert some influence here. But such influence only affects the partial saturation phenomena not the basic skeletal behaviour as shown on the time lines.

5.5.4 E-Tests

The method of testing was identical to the A-type tests, except that a fairly high back-pressure was used (240 kN/m^2). The intention here was to keep all gas in solution and avoid the problems of unsaturation described above. It may be seen that this has been achieved.

The typical build-up of pore pressure against a closed drain was included in fig. 5.8, above. This reached almost 100% of the load increment, instantaneously. The remaining build-up (from around 97% to 99%) took place over a period of time. It may be seen that the system was very slightly flexible - a change in voids ratio of $\delta e \sim .001$ taking place over the 24 hours. This may be due to volume changes in the measuring apparatus and in the cell itself, under pressure. At any rate, this order of error is quite acceptable, and the E-tests data may be used as a standard against which to assess the A and C tests. Although equivalent C-type tests with Back Pressure have not been carried out, it seems reasonable to assume behaviour would be identical after the first 10 seconds or so, during which the pore pressure at the impermeable boundary would rise to some 95% of the applied load.

The acceptability of E-test results indicates that all the problems of the previously described A and C tests may be attributed to non-saturation effects. In particular we may note that no leakage was found in the E-tests, which might have been expected to be a worse case since pressures over-all were some 240 kN/m^2 higher.

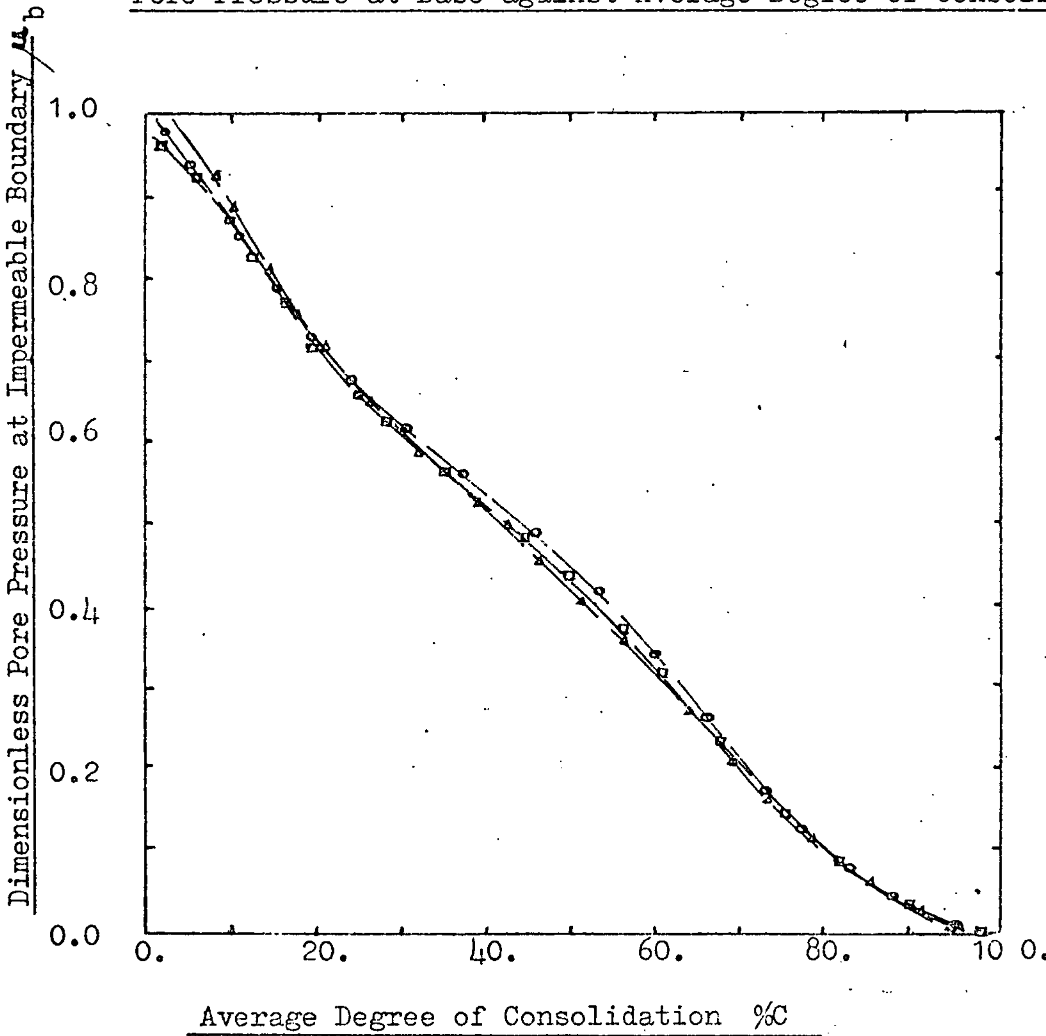
With the time and apparatus available it was only possible to carry out 3 E-tests. The same period of prior delayed consolidation was therefore employed (1 week) for each of 3 pressures. Since the pressure is found to have little effect on behaviour this should give fairly precise information on the acceptability of these results. (Use of the 3 periods of delayed consolidation would not have enabled experimental errors to be acceptably identified.)

Results are shown in figs. 5.12 - 14. It may be seen that excellent agreement is now obtained between the results at different

Fig 5.12

GRANGEMOUTH E Tests

Pore Pressure at Base against Average Degree of Consolidation



Prior d.c. / Pressure kN/m²	1 Week
R 60-120	—△— —△—
S 120-240	—○— —○—
T 240-480	—□— —□—

Fig 5.13 GRANGEMOUTH E-Test Settlement-Time Graph

Prior Delayed Consolidation 1 Week

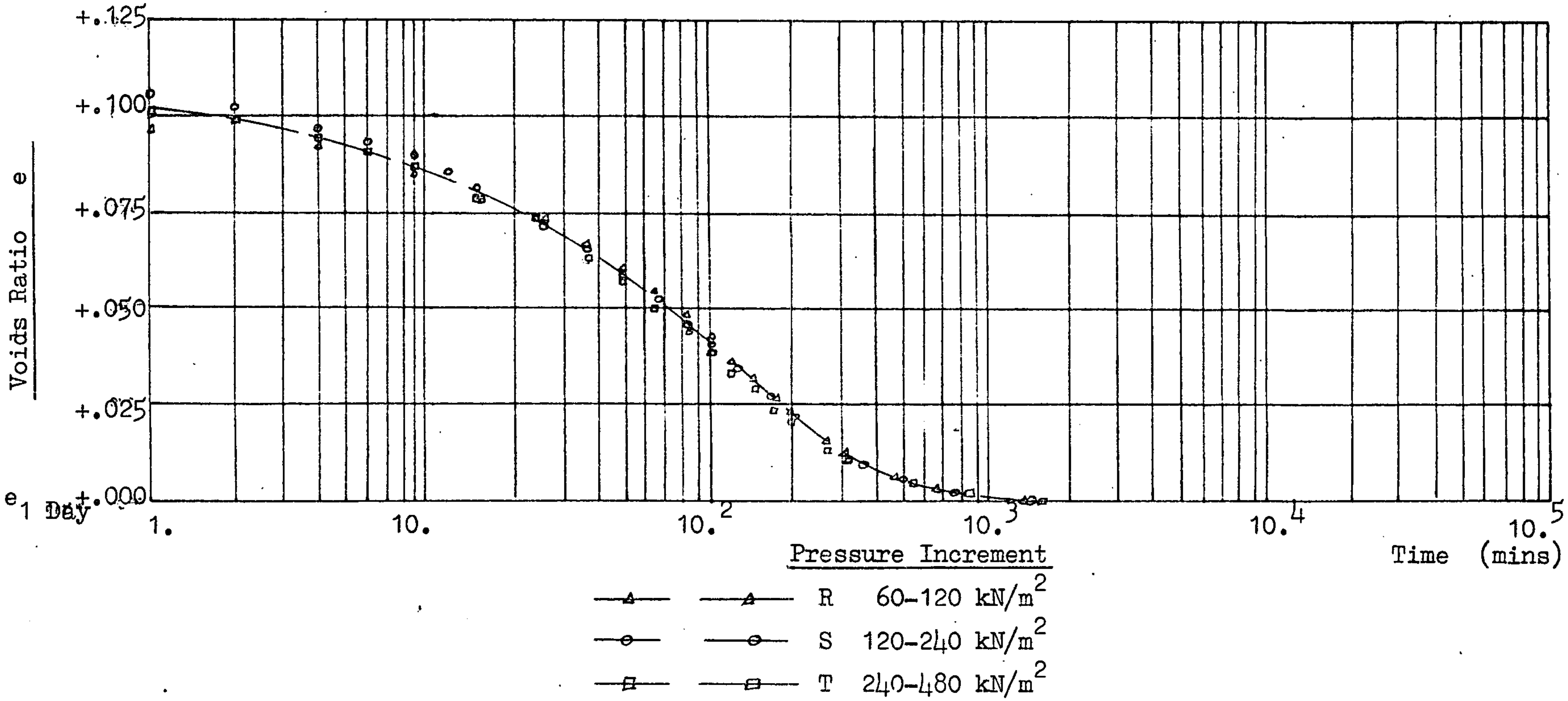
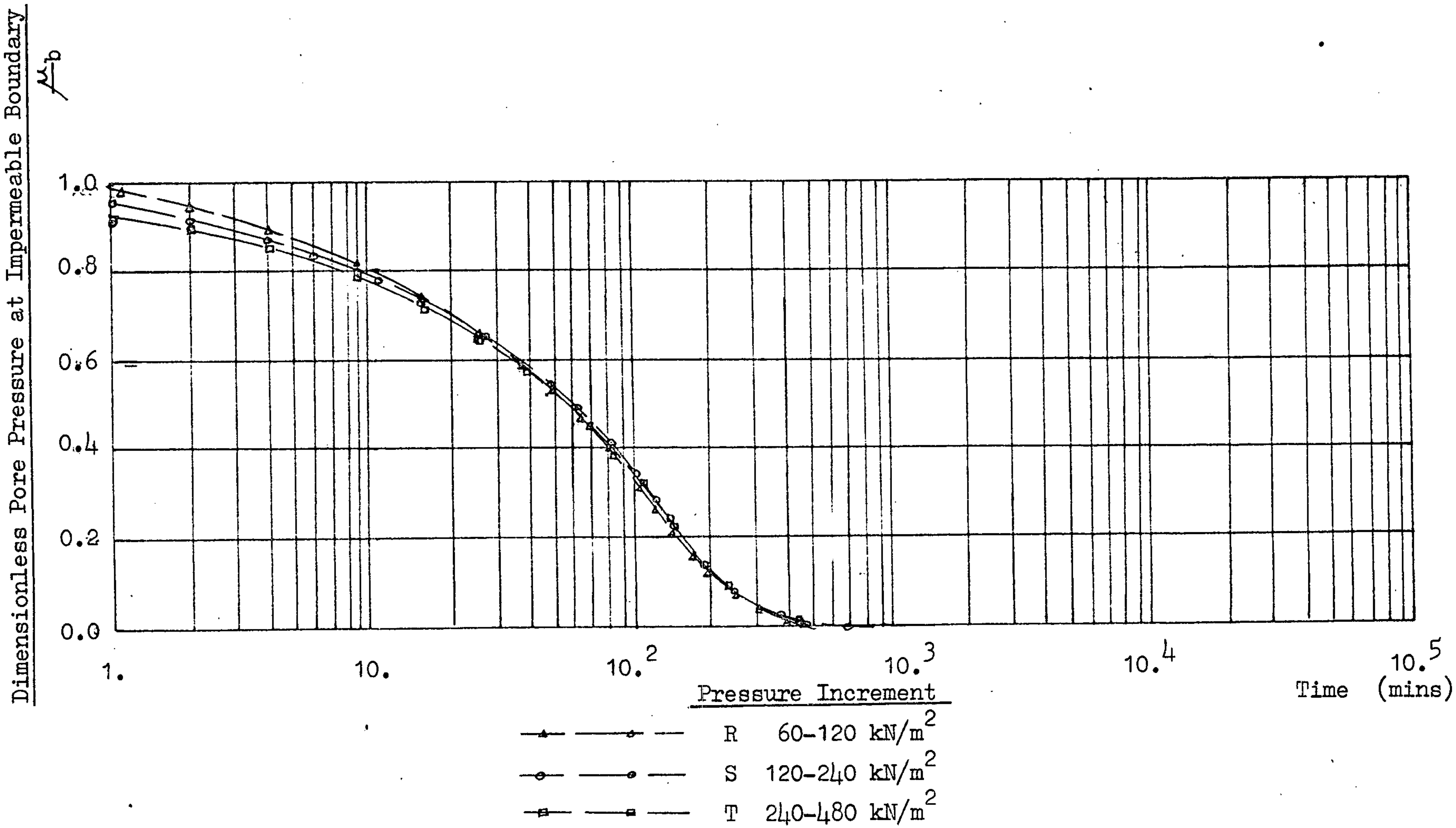


Fig 5.14 GRANGEMOUTH E-Test   Pore Pressure-Time Graph

Prior Delayed Consolidation 1 Week



pressures. The only slight discrepancy is in the initial pore pressures; for the second (S) test these fall slightly more rapidly from 100% than for the first increment, and for the T test the initial pore pressure reaches only 98.5%. It is possible that system flexibility has some influence here, but results are close enough to conclude that for a fully saturated soil the pore pressure does rise instantaneously to equal the applied load increment.

Referring to table 5.3 it may be seen that the value of b obtained for the test series T20 lies at the low end of the various values obtained. The e values at lower pressures were slightly low, and at high pressures about average. It is possible that the sample consolidates slightly more under the back-pressure when this is first applied, but there is not sufficient evidence to be significant. Similarly c values (table 5.5) are slightly on the low side of the mean, but well within the range of values obtained for the "free draining" tests. It is believed that such differences as may occur between back-pressure tests and the others are sufficiently small to be neglected.

The results graphs clearly show the settlement and pore pressure behaviour of the Grangemouth soil under a consolidating load. It will be seen in subsequent sections that such behaviour diverges significantly from that capable of analysis by standard Terzaghi theory and improved methods of analysis will be seen.

5.5.5 B-Tests

After the required period of delayed consolidation, the applied load is increased in 10 increments each of 10 minutes duration, and each equal to 1/10th of the initial load, whence the total load increment ratio of 1.0 is achieved after 100 minutes. The plots of results, fig. 5.15, show the values of applied loading for the increments labelled p_1 to p_{10}, where $p_{10} \equiv p_f$. Following Bjerrum (1973) these tests are intended to give specific information about sub-critical behaviour (A-type Consolidation) and the critical pressure.

Now before examining the results, a few points concerning behaviour in this test should be discussed. Ideally, we should like information on the stress-strain function only. Unfortunately the time dependant behaviour of the soil matrix is also present, and below p_c consolidation takes the stress-strain state of the soil across the time lines, such that the rate of delayed consolidation increases. To the author's knowledge there is no really satisfactory method of separating these effects. The present method does, however, allow some idea of the influence of delayed consolidation to be obtained. First, the possibility of $a = 0$ (i.e. strain below p_c may be attributed purely to delayed consolidation) may be discounted. It will be shown in section 6.3 that while this simplification gives an approximate method of some use for fitting later stages of consolidation, it implies that the soil goes instantaneously to the point (p_c, e_o), which experimental evidence shows is definitely not the case. 'a' must take some small but significant value. Now Bjerrum (op.cit) suggested incrementing the load every hour, while Garlanger hinted that this might be done as soon as 95% consolidation is attained. Some preliminary tests showed that below p_c primary consolidation was virtually complete after 10 minutes. This time interval was therefore chosen. The more rapid the possible loading, the less the influence of delayed consolidation.

It was found that for the first few increments a good straight line could be drawn through the 10 minute points. This line was taken to indicate the sub-critical stress-strain relationship. Values of the gradient 'a' are given in table 5.4. It may be seen that the values obtained are satisfactorily constant, so that it appears quite reasonable to use the mean value of a = 0.018 in subsequent analysis. The greater the prior delayed consolidation, the more reliable the value of a, since more increments give results not affected appreciably by delayed consolidation. As p_c is approached it was found that some settlement was still occurring after 10 minutes, and the e - p line started to curve downwards, away from the 'a' line. In terms of the present model, we say that as p_c is approached the influence of delayed consolidation increases significantly. This appears as a gradual curve from an 'a' line to a 'b' line, rather than the discontinuity of slope at p_c, suggested by the present model were time effects in the skeleton not present.

How, then, may p_c be determined? Full discussion may be found in section 6.4. The best definition consistent with Garlanger's model is to take the point of intersection of a line of slope 'a' from e_o, with the instant line. Now this instant line must be the minimum value of time line that the soil may reach. It was noted in Chapter 4 that although the tangent of slope b to the log e - log p curve was used as the best approach when that was the only available data, the instant line might be expected to actually take a rather lower value. According to the Garlanger-Bjerrum model, the actual soil log e - log p relationship will become increasingly distant from the instant line as thickness of the soil layer increases, since this increases the length of time required for hydrodynamic effects on the instantaneous consolidation, hence increasing the influence of delayed consolidation.

A further complication is that we have so far been discussing the <u>average</u> values of e and p in the layer. Now Garlanger's method relates behaviour to local parameters - i.e. the effective stress at each node is compared with p_c and 'a' or 'b' chosen accordingly. Clearly when a load increment crosses p_c, points near a drainage boundary will soon be above p_c, while points further away will remain in A-type consolidation for longer. This implies that a good value of p_c cannot be obtained from tests type A, C or E. At the end of an increment, however, when most primary consolidation is complete, the effective stress in a layer is fairly constant with depth. Hence, although there is the drawback of delayed consolidation, as noted above, the effective stress is kept rather more constant in the B tests. At the end of an increment in A-type consolidation the behaviour may therefore be related to average p, which is reasonably close to the local p value throughout the layer.

Now the observed p_c value will depend upon the strain rate, and it might therefore be expected that the present technique of loading every 10 minutes will impose, or at least tend to cause a given strain rate in the soil, which in turn will affect the observed value of p_c. This is of particular significance when we later wish to know the p_c value for prediction of behaviour in thick deposits. The limitations of the present technique are acknowledged. It will be seen in section 6.4 that a preferable approach to the problem of p_c can now be offered. Never-the-less, the B-tests represent an important step towards increased understanding of the phenonemon. They also enable us to directly assess the validity of the delayed consolidation model as interpreted by Garlanger.

The results obtained in the B-tests are shown in fig. 5.15. To facilitate comparison for tests at the different pressure increments, results are presented in terms of the logarithmic voids ratio relative to that after 1 day's consolidation. As noted in section 5.5.1, this rationalisation involves moving all the voids ratio values for a given test slightly up or down, so that the voids ratios after 1 day, e_D, coincide. It was also mentioned in 5.5.1 that this does not involve a "rotation" of the e values. Thus the actual differences between

TABLE 5.4

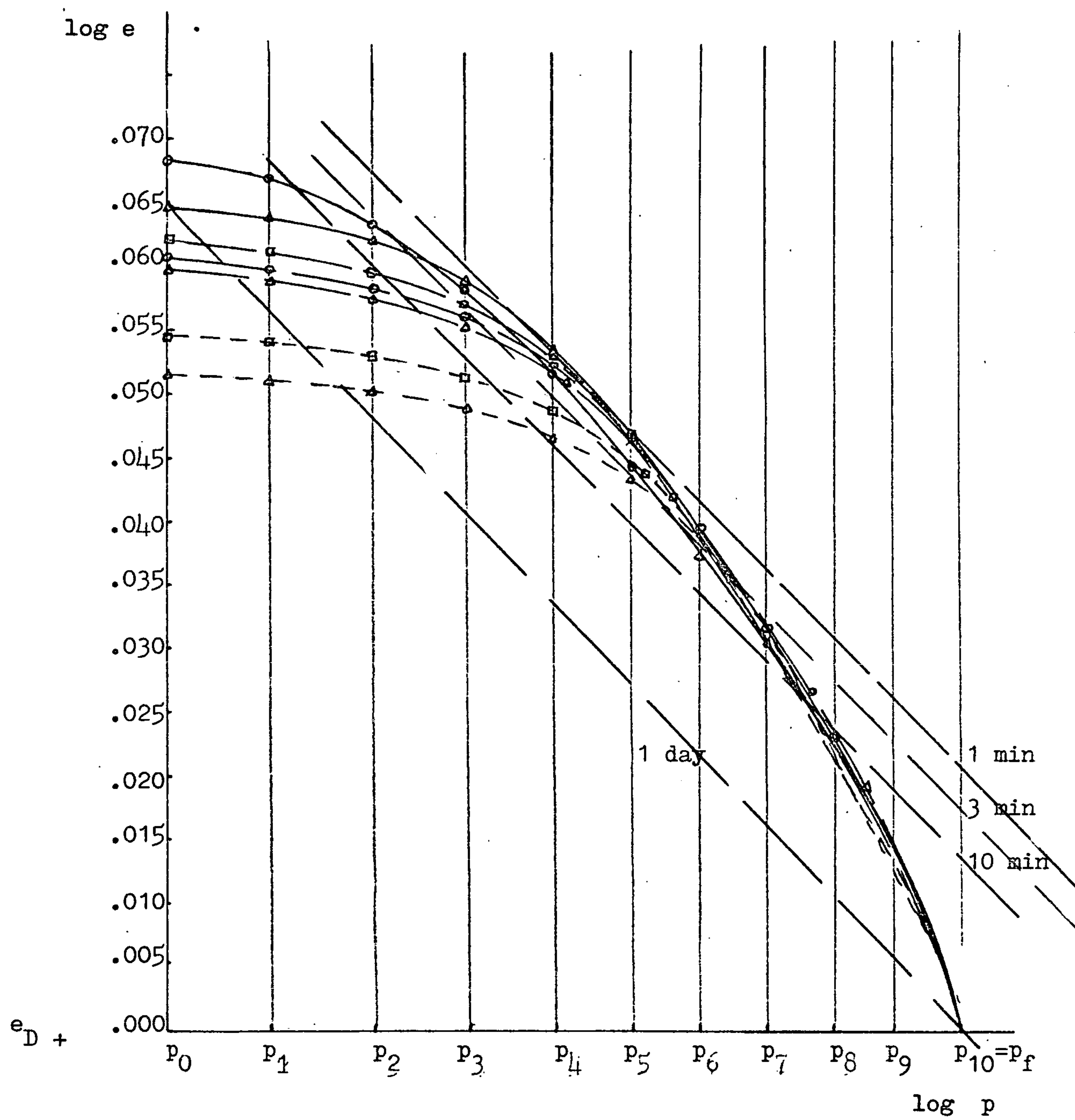
B - type Tests   Coefficient 'a'

Pressure Increment \ Prior Delayed Consolidation	1D	1W	1M
R 60 - 120 kN/m^2	.017	.018	.017
S 120 - 240 kN/m^2	.017	.021	.018
T 240 - 480 kN/m^2	.018	.017	.018

Coefficient 'a' may be taken as constant a = 0.018

Fig 5.15 B - Test Results

Log e - log p Plots



$\log(e_D)$		1 D	1 W	1 M
$\bar{1}.985$	R			
$\bar{1}.920$	S			
$\bar{1}.855$	T			

(Note Tests R-M and T-D not available)

values of Δe for the different tests are reflected in e_o values. For the mean data of the Master Time Lines Diagram (fig. 5.4) the logarithmic increments in Δe were found to be log (Δe) = 0.065, 0.059, 0.053 after delayed consolidation of 1 day, 1 week and 1 month, respectively. The individual results of fig. 5.15, for the various amounts of delayed consolidation, are grouped around these values although the scatter is rather greater than might have been hoped.

Two possible explanations may be presented for the scatter of results.

i) Variations in prior delayed consolidation. This is the assumption made in fig. 5.15, putting all values of e_D the same.

ii) Variations in instantaneous consolidation. If all scatter were due to this the 1D, 1W and 1M tests should each be started at their "correct" positions for e_o, and the variations in points for e_D compared.

A detailed examination of results in relation to the test series, and time lines diagrams, suggests both these effects play some part in the scatter, although i) is probably more significant. The results of fig. 5.15 suggest that the strain increment increases slightly with increasing effective stress. Accordingly to i) this would follow if the value of c decreased with p. The full analysis of delayed consolidation (section 5.5.6, below) suggests this is not the case. Nor is there any indication for ii) that the values of a or b increase with effective stress. It has been concluded that these variations are totally random.

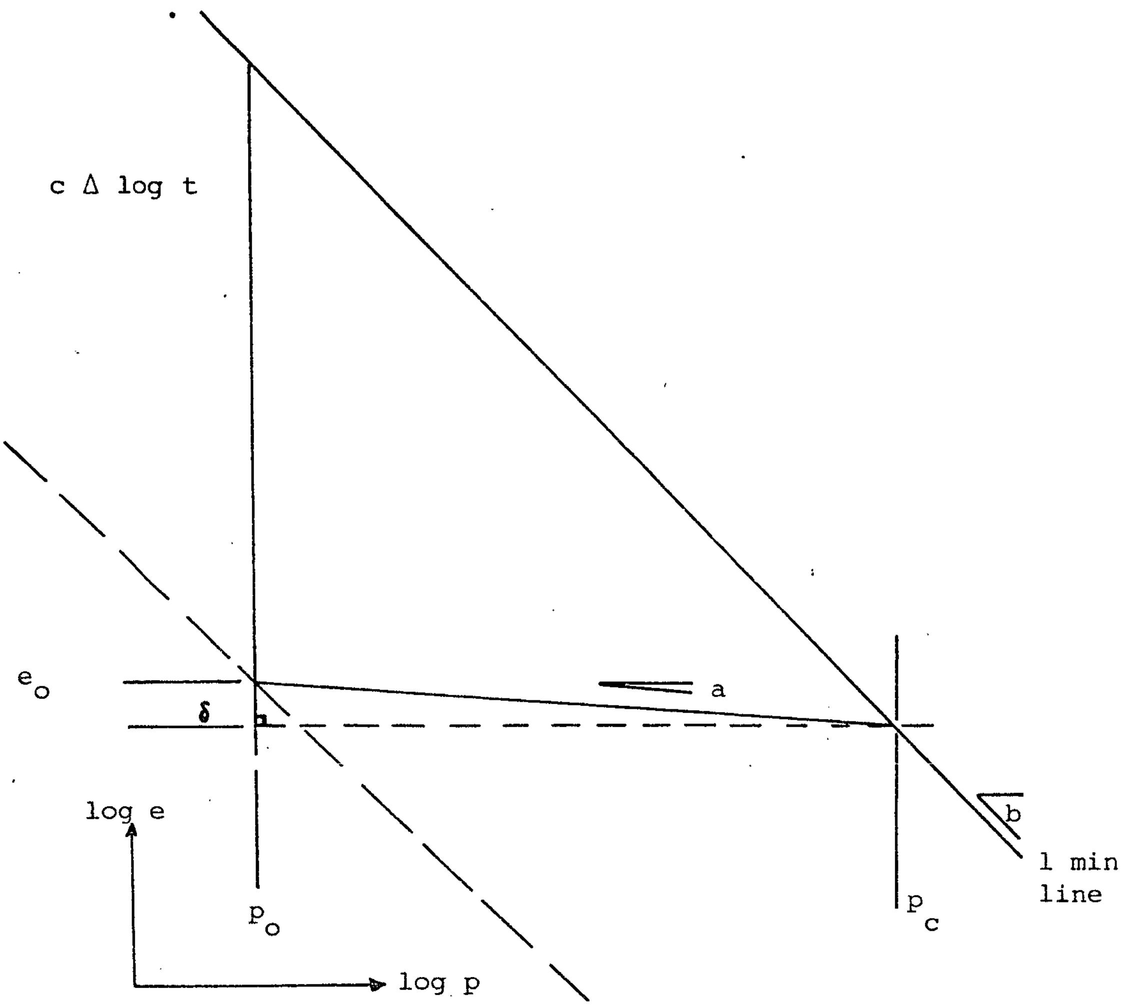
Now the influence of partial saturation should be mentioned. It is of importance to notice that the degree of saturation does not influence the stress-strain-time functions for the soil skeleton in the Garlanger model. All that is affected is the diffusion process of water through the soil in the primary consolidation. This in turn, influences both pore pressure behaviour, and the rate of instantaneous consolidation, but these are not fundamental to interpretation of the B-test.

For a fully saturated soil the excess pore pressures, theoretically, rise immediately to equal any increment of loading. Below p_c, they will fall again to zero quite rapidly. Above p_c, substantial pore pressures will remain after 10 minutes, when the load is next increased, and behaviour becomes complex. In the present experiments pore pressure remained at zero for the first few increments. It is thought that this applied stress was transferred almost immediately to the soil particles, since the soil was unsaturated, so that compression could take place rapidly, driving the gaseous phase back into solution. As the critical pressure was approached pore pressures started to build up, but as full saturation was not yet achieved this process took a little time. Initially such pore pressures built up to only a fraction of the load increment, but this increased until a couple of increments after passing p_c the rise was almost equal to the applied increment. Unsaturation just increases the complication of interpreting pore pressure behaviour, but, even for a saturated soil it is believed that pore pressures would show a gradual build-up as p_c was passed, due to the complications of delayed consolidation behaviour, rather than a sudden change from a to b type behaviour.

From the B-type tests, then, how is p_c best evaluated for use in Garlanger theory? We note that it is defined as the intersection of the 'a' line from (e_o, p_o) with the instant line. There appears to be no problem in drawing a line of gradient 'a', but the point (e_o, p_o) will depend on conditions prior to the increment of loading under consideration. The major problem is to identify the instant line. Drawing tangents of slope 'b' to the curves of fig. 5.15 gives 't_i' values between about 1 and 3 minutes. In spite of the scatter of Δe values, a fairly constant value for the instant line is thus obtained. To be consistent with Garlanger's theory, t_i should, indeed, be constant. It was decided to take the 1 minute line for the present analysis. This has the interesting property that $\log t_i = 0$ for the delayed consolidation function, and at one stage in the present investigation it was thought that this might indicate some limiting position for the instant line. It will be seen in section 6.4 that this is not the case. The observed instant line may be related simply to rate of strain. However, this goes some way beyond the scheme proposed by Garlanger, and for the present chapter it was thought better to keep to his model. It will also be seen in section 6.4 that differences between use of the 1 minute and 3 minute lines are negligible. The lower bound on t_i from the B-tests was chosen because the higher t_i values may then be interpreted as tending to this, but slightly influenced by delayed consolidation.

The method of determining p_c from this data is best explained by reference to fig. 5.16. After a given period of delayed consolidation at p_o the soil has a voids ratio of e_o, which lies a distance $c\,\Delta \log t$ vertically below the instant line. Drawing a line of gradient 'a' on the log e - log p plot, the point of intersection with the instant line may be computed, as shown. Because linear log - log relationships are used in the present model, a constant ratio of p_c/p_o is found corresponding to any given period of delayed consolidation.

Fig. 5.16 Determination of p_c



Now $\delta = a \log (\frac{p_c}{p_o})$

$$\delta + c\,\Delta \log t = b \log (\frac{p_c}{p_o})$$

$$c\,\Delta \log t = (b-a) \log (\frac{p_c}{p_o})$$

$$\text{or} \quad \log (\frac{p_c}{p_o}) = \frac{c}{(b-a)}\,\Delta \log t$$

From Master Diagram (fig.5.4) b-a = 0.216 - 0.018 = 0.198

for e_o	$c\,\Delta \log t$ =	∴ p_c/p_o =	$\Delta p_c/\Delta p$ =
after 1 day	.02116	1.279	0.279
1 week	.02682	1.366	0.366
1 month	.03326	1.472	0.472

5.5.6 Delayed Consolidation

In section 1.2 the delayed consolidation was defined to be a relationship between strain and time, independent of stress effects. At large times the influence of stress is sufficiently small for a good approximation to this delayed consolidation to be obtained. Plots of both linear and logarithmic strain against log time have been obtained for all the tests carried out (a typical example of the latter is given in fig. 5.17). The resulting values of C_α and c, respectively, are given in tables 5.5. and 5.6, these coefficients being the gradient of the best straight lines obtained. The two methods gave equally good fits to straight lines (i.e. e and log e plots appear equally acceptable here) and for the bulk of results points lay very accurately on such lines (i.e. the fit on fig. 5.17 is not unusual).

It is somewhat surprising to find that a change in gradient occurred quite consistently after around 7 days. {This was not anticipated and the 1 week test duration was in no way related to this.} Obviously this could only be observed in the 28 day tests, of which there are only 7, but this would seem sufficient number to prove the point. The reason for such behaviour is not known. Pore pressure readings (sensitive to 0.2 kN/m^2) were not affected. The process whereby gas is released could conceivably reach some critical stage after a given time at negligible pressure, but this seems somewhat abstruse. Certainly the soil is unsaturated after 1 day of delayed consolidation, so a fundamental change in behaviour seems unlikely.

It will be seen from the C_α and c values that although good straight lines may be fitted, the slopes obtained vary somewhat. Now for the tests of only 1 day's duration, the values are not particularly good as only the points for 12 and 24 hours could be used and minor pressure effects may still be present in the former. Values here do tend to be on the high side. Fitting lines using the average value from all tests suggested the curves could be tending to such a gradient. However, drawing such lines on plots from 7 and 28 day tests indicated that there were definite differences in slopes between tests. Such variations appear to be completely random. Determinations of mean values for pressure increment, type of test, and length of prior delayed consolidation (it would be surprising if this were significant) revealed no significant effects due to these. Constant temperature was maintained, so this cannot be significant. Nor could variations be attributed to the cell used or, indeed, to the test series. {This, incidentally, suggests side friction was not a contributing factor, since this effect might be expected to be constant within a test series, but vary between such series.} Another possible factor is vibration. The apparatus was kept well clear of this, the only possible source of vibration being when other cells were lifted from or placed in the water bath. Records were kept of this and no anomalies could be traced to such a cause.

The various models of delayed consolidation are examined in detail in section 6.2, but we note here that independence of c from pressure increment is in line with the Garlanger-Bjerrum model, giving parallel time lines. It seems reasonable here to proceed on the assumption that the variations observed are truly random and hence take an average value of rate of delayed consolidation over the total of 30 results.

It is interesting to compare the present results with those of Christie (1963). Analysis of his data gives values of c = 0.0088, 0.0081 and 0.0087 for pressure increments virtually identical to R, S and T here, respectively. These were 28 day tests (see data in table 4.1) and the values quoted are mean gradients over the whole of this time.

Examination of the curves suggests there may be a change in c at around 1 week, but data is not sufficiently precise here to be certain. The c values quoted above are in excellent agreement with the average delayed consolidation rate observed over 1 month in the present tests.

Lo (1961) also presented long term consolidation curves for Grangemouth Silty Clay, and also found a change in gradient after about 1 week. This will be discussed in detail in section 6.2.

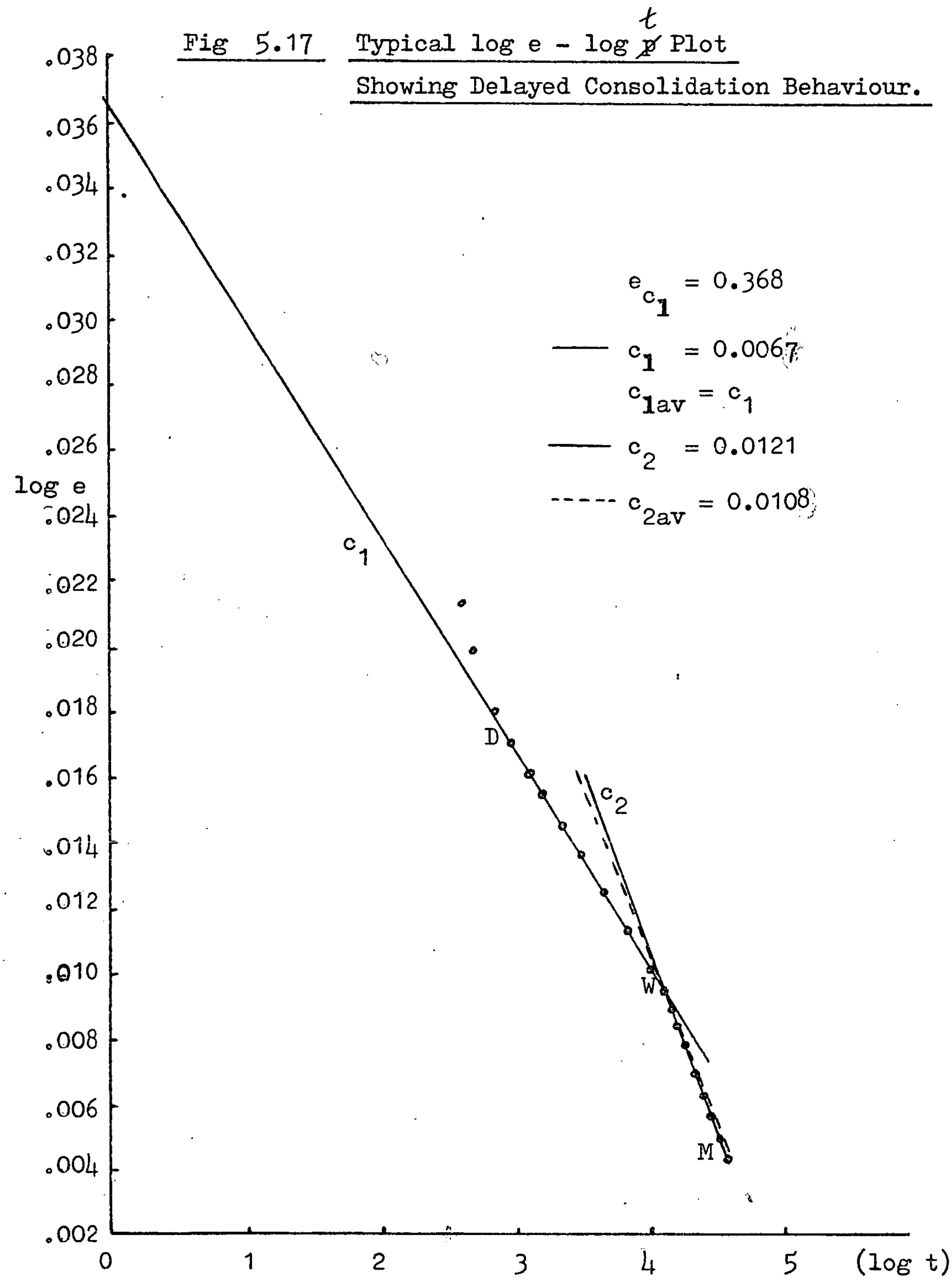


Fig 5.17 Typical log e - log t Plot
Showing Delayed Consolidation Behaviour.
e_{c_1} = 0.368
c_1 = 0.0067
$c_{1av} = c_1$
c_2 = 0.0121
c_{2av} = 0.0108
log e
.038
.036
.034
.032
.030
.028
.026
.024
.022
.020
.018
.016
.014
.012
.010
.008
.006
.004
.002
0
1
2
3
4
5
(log t)
c_1
c_2
D
W
M

TABLE 5.5

Delayed Consolidation Coefficient C_α

Mean Values $C_{\alpha_1} = 0.012$

$C_{\alpha_2} = 0.025$

Pressure Increment	Prior Delayed Consolidation / Test Type	1D	1W	1M
R 60-120 kN/m^2	A	{.010 {.026 (28)	.008 (7)	{.016 {.033 (28)
	B	.013 (7)	.021 (1)	.017 (1)
	C	.011 (7)	.010 (1)	{.008 {.020 (28)
	E	-	.010 (7)	-
S 120-240 kN/m^2	A	.008 (7)	{.014 {.030 (28)	.014 (7)
	B	{.010 {.029 (28)	.014 (7)	.017 (1)
	C	{.011 {.021 (28)	.013 (7)	.013 (1)
	E	-	.010 (7)	-
T 240-480 kN/m^2	A	.012 (1)	.008 (1)	{.009 {.013 (28)
	B	.014 (1)	.013 (7)	.014 (1)
	C	.010 (1)	.007 (7)	.006 (1)
	E	-	.012 (1)	-

Notes

(N) indicates number of days tests were run

{ for tests run 28 days, 2 straight lines were fitted to
{ the data (see text), given here as { 1 - 7 days
{ 7 -28 days

TABLE 5.6

Delayed Consolidation Coefficient $\bar{c}$ (log e_c/log t)

Mean Values c_1 = 0.0067

c_2 = 0.0108

Pressure Increment	Prior Delayed Consolidation / Test Type	1D	1W	1M
R 60-120 kN/m^2	A	{.0064 {.0121 (28)	.0056 (7)	{.0066 {.0121 (28)
	B	.0062 (7)	.0092 (1)	.0087 (1)
	C	.0076 (7)	.0073 (1)	{.0049 {.0086 (28)
	E	-	.0044 (7)	-
S 120-240 kN/m^2	A	.0055 (7)	{.0072 {.0110 (28)	.0041 (1)
	B	{.0048 {.0124 (28)	.0079 (7)	.0093 (1)
	C	{.0057 {.0084 (28)	.0071 (7)	.0081 (1)
	E	-	.0053 (7)	-
T 240-480 kN/m^2	A	.0091 (1)	.0083 (1)	{.0066 {.0106 (28)
	B	.0086 (1)	.0090 (7)	.0061 (1)
	C	.0057 (1)	.0059 (7)	.0049 (1)
	E	-	.0057 (1)	-

Notes

(N) indicates number of days tests were run
{ for tests run 28 days, 2 straight lines were
{ fitted to the data (see text), given here as
{1 - 7 days
{7 - 28 days

0a, Aver .0075

5.6 Terzaghi Analysis

The results of the present experimental work indicate major departures from Terzaghi theory are brought about by delayed consolidation. This theory remains, however, by far the most commonly used in practise. It is instructive to examine briefly its capabilities and limitations here.

First of all, the values of coefficient of consolidation, Cv, have been determined for the A, C, and E tests (tables 5.7 and 5.8). The necessary graphical constructions were drawn directly on the appropriate graphs given by the analysis program, CALCOG (see Section 5.4). Three fitting methods have been used.

i) Taylor's Method - Drawing strain against square root of time and fitting at 90% Consolidation (labelled Cv_{90}).

ii) Casagrande's Method - Drawing strain against logarithmic time and fitting at 50% Consolidation (Cv_{50}).

These two techniques are standard and well known; further discussion is superfluous (see e.g. Taylor (1948) p.238 ff).

iii) Fitting pore pressure data. A method is described by Bishop and Henkel (1963) (p.135 ff) whereby the experimental curve for pore pressure at the impermeable boundary is fitted to that predicted by Terzaghi theory. The dimensionless time for 50% dissipation may be determined as 0.380, whence

$$Cv_{pl} = \frac{0.380 \times H^2}{t_{50}}$$

where t_{50} is the experimentally determined time to reach dimensionless pore pressure $\mu = 0.50$. This was determined on the assumption that initial excess pore pressure equalled the applied load increment. It has been seen that this was not the case due to

TABLE 5.7

Cv Values A-Tests and E-Tests (mm^2/sec).

Pressure Increment \ Prior Delayed Consolidation		A-Tests 1D	A-Tests 1W	A-Tests 1M	E-Tests 1W
R	Cv_{90}	.024	.024	.014	.020
	Cv_{50}	.025	.022	.015	.022
60-120 kN/m^2	Cv_{p1}	.059	.072	.155	.051
	Cv_{p2}	.042	.032	.043	-
S	Cv_{90}	.012	.019	.018	.027
	Cv_{50}	.016	.017	.015	.032
120-240 kN/m^2	Cv_{p1}	.070	.099	.220	.052
	Cv_{p2}	.038	.046	.050	-
T	Cv_{90}	.026	.018	.025	.025
	Cv_{50}	.035	.012	.017	.034
240-480 kN/m^2	Cv_{p1}	.099	.176	.230	.053
	Cv_{p2}	.067	.066	.066	-

Notes Cv_{90} is determined from Taylor's square root fitting technique at 90% Consolidation. Cv_{50} is determined by Casagrande's method at T_{50}. Cv_{p1} and Cv_{p2} are determined from pore pressure data - see text.

TABLE 5.8

Cv Values C Tests mm^2/sec

Prior Delayed Consolidation / Pressure Increment		1D	1W	1M
R	Cv_{90}	.016	.027	.018
	Cv_{50}	.021	.028	.025
60-120 kN/m^2	Cv_{p1}	.096	.038	.103
S	Cv_{90}	.022	.038	.037
	Cv_{50}	.045	.038	.048
120-240 kN/m^2	Cv_{p1}	.043	.061	.238
T	Cv_{90}	.047	.032	.038
	Cv_{50}	.061	.032	.041
240-480 kN/m^2	Cv_{p1}	.070	.110	.127

Notes Cv_{90} is determined from Taylor's square root fitting technique at 90% Consolidation. Cv_{50} is determined by Casagrande's method at T_{50}. Cv_{p1} and Cv_{p2} are determined from pore pressure data - see text.

the influence of partial saturation. For C tests better information is not available. For A-tests, however, after 24 hours pore pressure build up against a closed drain, a maximum value μ_a corresponded to full saturation. A second attempt at fitting the pore pressure data was made, therefore, using t_{50} determined at $\mu = 0.5\ \mu_a$. This is termed Cv_{p2}.

The resulting coefficients show considerable random variation between tests. This is largely a result of departures in behaviour from Terzaghi theory making accurate fitting impossible. For methods i) and ii) the straight lines required for fitting were not usually well-defined leading to some inaccuracy in resulting coefficients. However, comparison of the various mean values against the variables, suggested for the A-tests:

a) Differences between methods i) and ii) are not significant;

b) Pressure increment is not significant;

c) Prior delayed consolidation is not significant.

A mean value of $Cv = 0.020\ mm^2/sec$ may be taken and departures from this are quite random.

For the C-tests similar analysis suggested:

a) Method ii) gives values of Cv some 30% higher than i);

b) Cv increases with increasing load;

c) Prior delayed consolidation is not significant.

A mean value of $Cv = 0.034\ mm^2/sec$ is obtained, but this is not very helpful in view of effects a) and b) here. The fact that Cv is rather higher in C-tests is believed to be due to inclusion of the strain resulting from closing of the gas voids which takes place fairly rapidly, so that a large proportion of total consolidation is soon achieved. This is consistent with Cv_{50} being larger than Cv_{90}. This is also believed to be the cause of the pressure effect b).

The higher the applied pressure (using a constant load increment ratio), the greater the actual excess pressure driving gases back into solution, hence the more rapidly this occurs.

There are not sufficient results of E-type tests for such analysis to be useful, though there is a hint that Cv here increases slightly with load, and Cv_{50} values are slightly higher.

Now methods i) and ii) fit the strain data. The consistency observed for A and E tests suggests that Terzaghi theory gives a reasonable fit with strains. (This is probably a result of the rapid rate of early consolidation compensating for the smallness of the strain.) Results for fitting pore pressures, on the other hand, are quite inconsistent, being much higher. This indicates that pore pressures dissipate far more rapidly than predicted by Terzaghi theory based on strains, as, of course, has already been clearly established. Values of Cv here rise with applied loading and with prior delayed consolidation, again in agreement with behaviour observed elsewhere. Although the discrepancy is reduced for the E-tests and for the A-tests when partial saturation is accounted for (Cv_{p2}), it is important to note significant discrepancy from the strain-fitting methods remains.

Some graphs of predicted against experimentally determined behaviour illustrate the problems of satisfactory analysis of pore pressures. A number of approaches have been examined as discussed below. Now the goodness of fit will depend firstly on the value of Cv, and we have seen that problems occur in determining this. Typical fitting to strain behaviour enables reasonably good agreement of strain against logarithmic time (fig. 5.18) for the A-tests. The delayed consolidation has been fitted with a linear strain-logarithmic time relationship of slope $C\alpha$ (see table 5,5) projected back to meet

Fig 5.18 Analysis of Experimental Results by Terzaghi Theory.

GRANGEMOUTH A-Test Settlement-Time Graph

Increment S 120-240 kN/m^2

Prior Delayed Consolidation - 1 Day

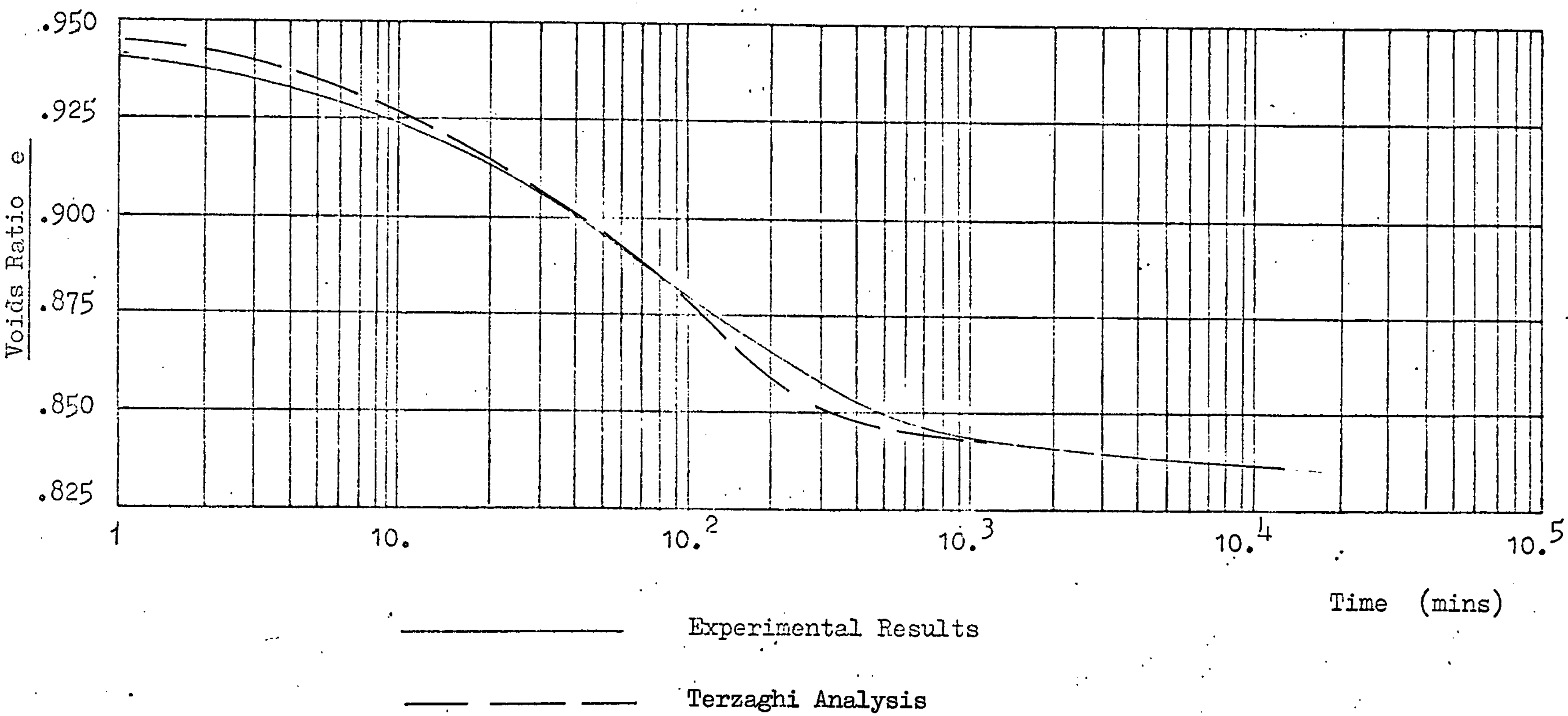


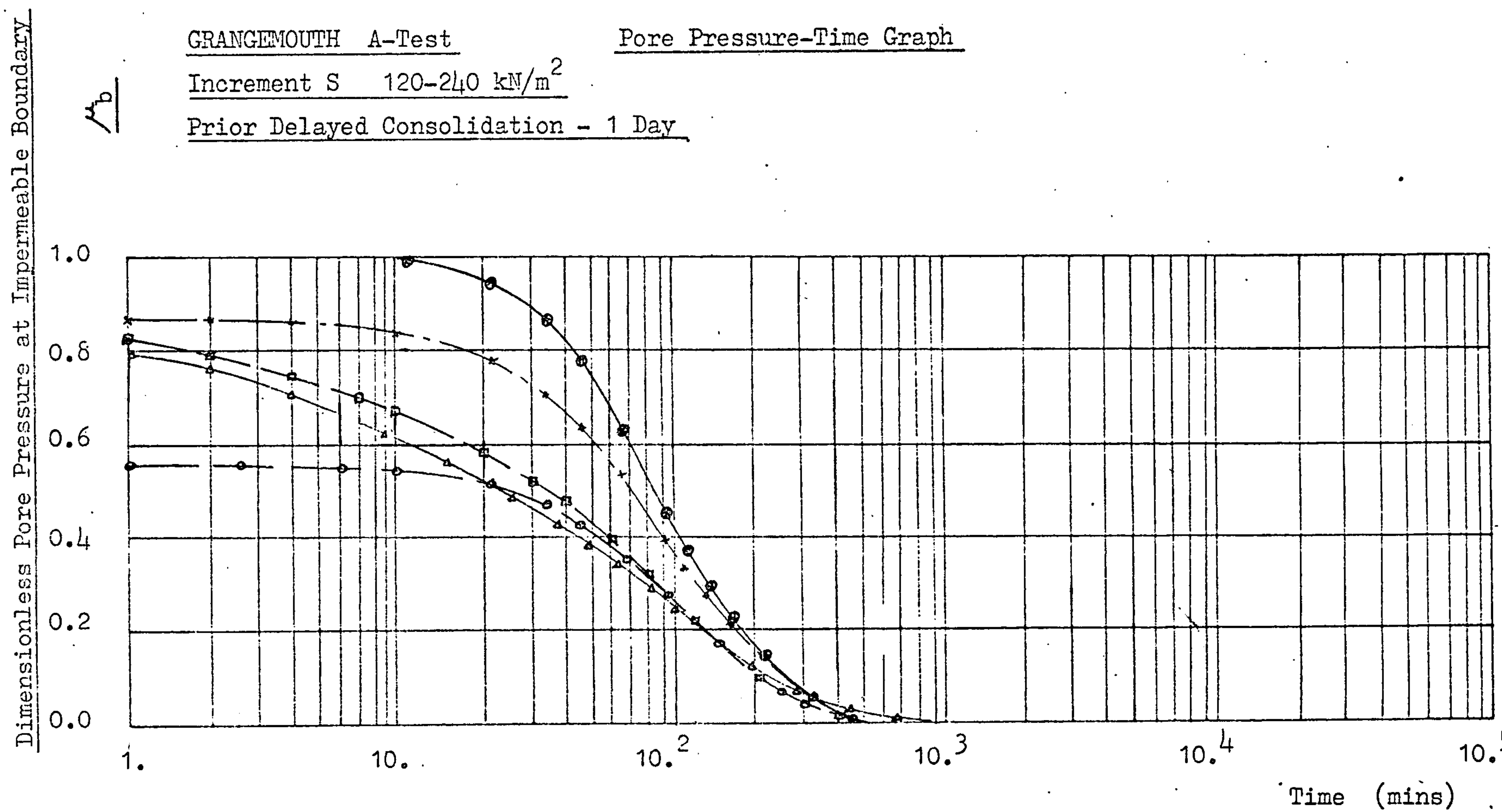
Fig 5.19 Analysis of Experimental Results by Terzaghi Theory.

GRANGEMOUTH A-Test

Pore Pressure-Time Graph

Increment S 120-240 kN/m^2

Prior Delayed Consolidation - 1 Day



- Experimental Results

Terzaghi Analysis

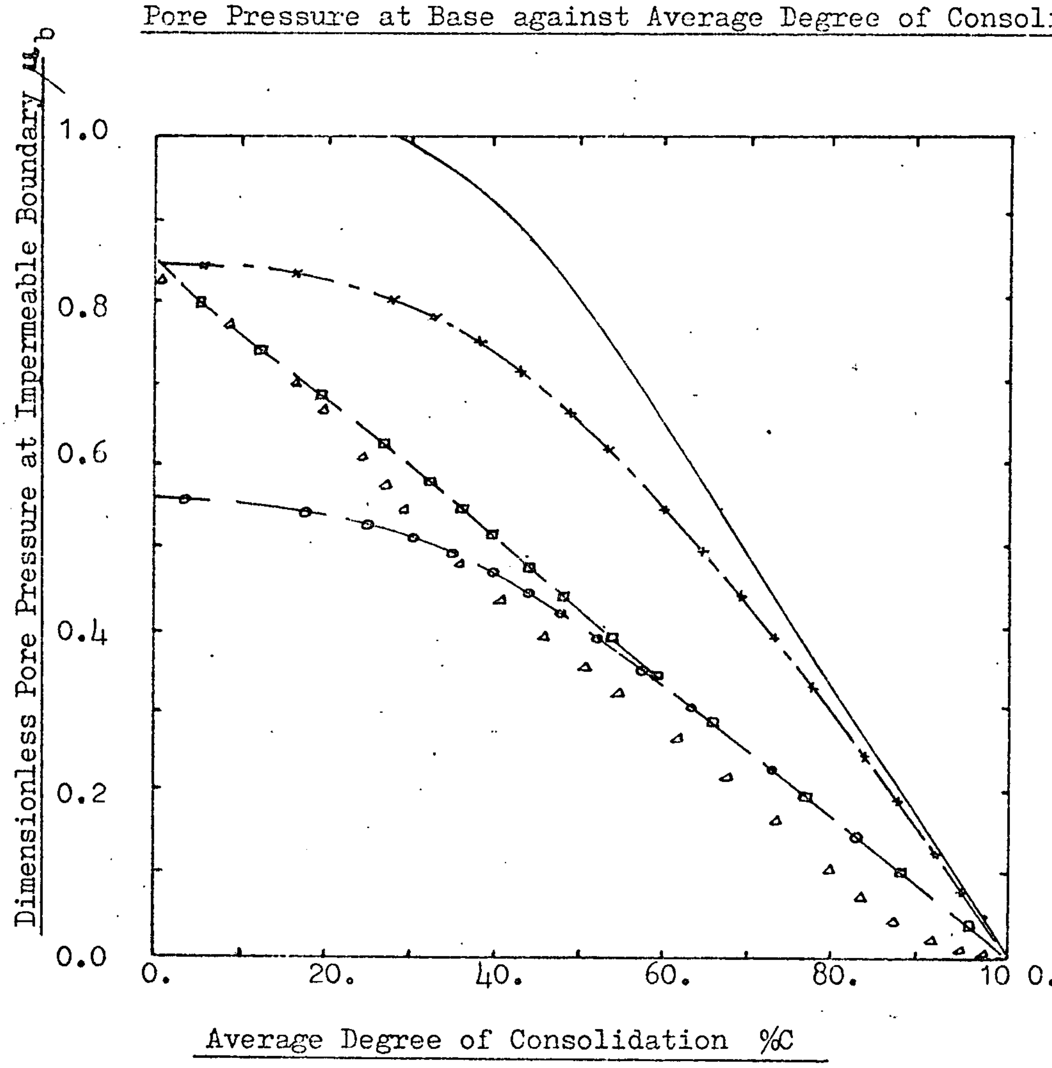
- Over Full Load Increment
- Starting at p_a
- Starting at p_c
- Using varying parameters with CONED Program (see text)

Fig 5.20

Analysis of Experimental Results by Terzaghi Theory

GRANGEMOUTH   A - Test   SAD.

Pore Pressure at Base against Average Degree of Consolidation



△ △ △ △ Experimental Results

Terzaghi Analysis

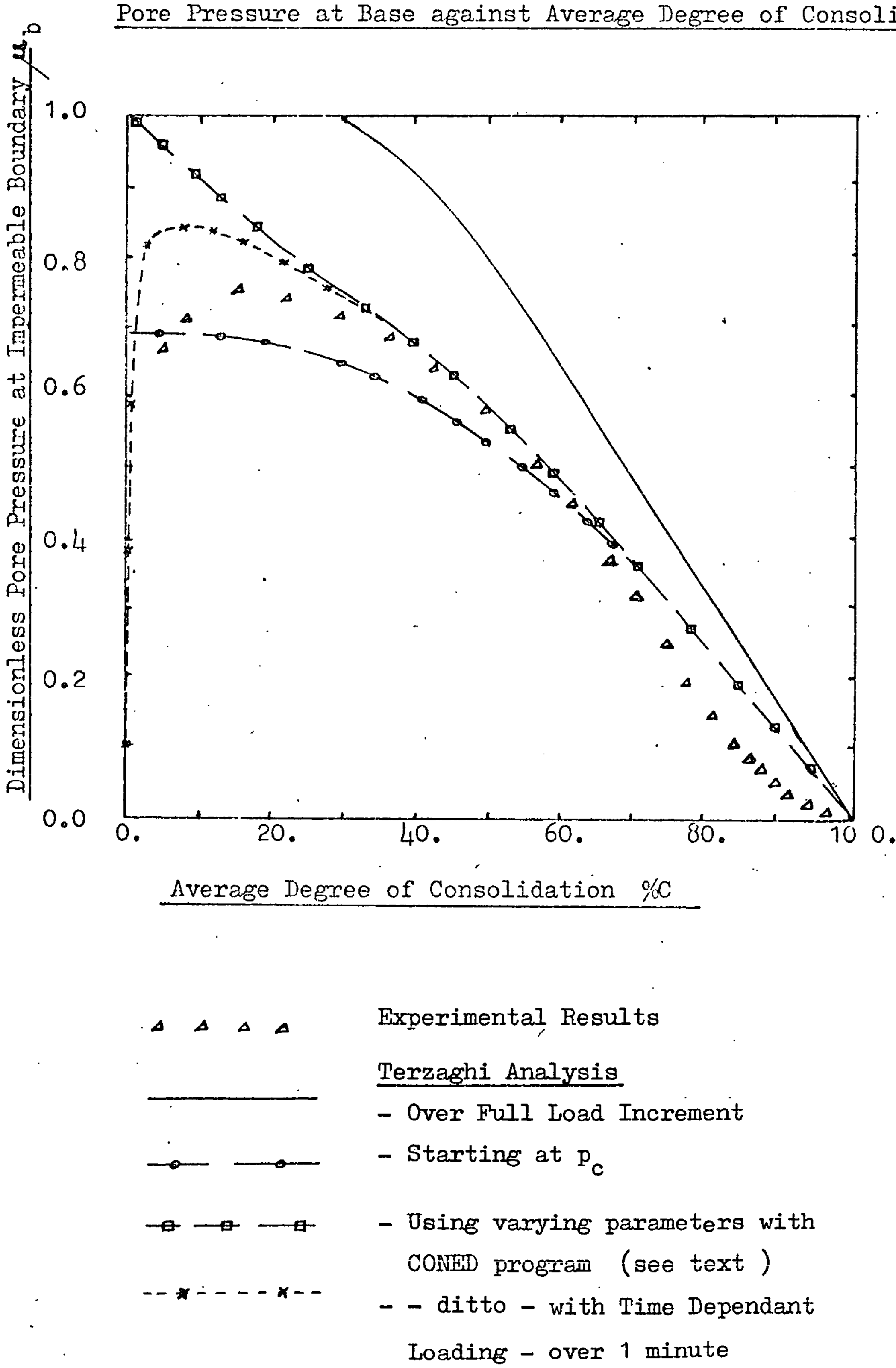
- Over Full Load Increment
- Starting at p_a
- Starting at p_c
- Using varying parameters with CONED program (see text)

Fig 5.21

Analysis of Experimental Results by Terzaghi Theory

GRANGEMOUTH   C - Test   RCD

Pore Pressure at Base against Average Degree of Consolidation



the predicted primary consolidation curve (after Buisman, 1936). Observed early strain proceeds rather more rapidly than predicted, and later strain rather less so. Fig. 5.19 shows pore pressures against logarithmic time when the best fit with strain has been obtained. The problem of best determination of Cv, however, may be avoided here by considering the relationship between dimensionless pore pressure at the impermeable boundary and average degree of consolidation (strain) (figs. 5.20 and 5.21 for A and C tests respectively). The relationship here is independant of Cv. Referring to these graphs several approaches may be considered.

Case 1

The conventional Terzaghi theory is applied over the full load increment. Predicted pore pressures are quite unsatisfactory.

Case 2

For the A-tests the increment from p_a to p_f may be considered. Hence the initial pore pressure is correct, but dissipation does not agree at all well.

Case 3

Consolidation below some critical pressure (p_c - as defined for Garlangér theory - see section 5.7) is assumed to take place instantaneously, whence consolidation is analysed over the stress increment p_c to p_f. Reasonably good agreement is now obtained for the majority of the process, but pore pressures observed at early stages are rather higher.

Case 4

From the above considerations it appears that consolidation between p_a and p_c takes place at a rapid, but not infinite, rate. Examination of the data, particularly from the B-tests, suggests that consolidation is initially very rapid (Cv high) with little strain (mv low), Cv falling to the values observed here as p_c is exceeded. As a first approximation a value of constant Cv some 10 times that applying ~~below~~ above p_c was used for this early behaviour (and a corresponding value of 1/10th mv was used to keep permeability constant). Such analysis was handled by the CONED programs described in Chapter 2. Agreement is now reasonably good at all stages of consolidation. (It should be noted that the difference this produced in strain-time behaviour was too small to show on fig. 5.18.)

The most important innovation to Terzaghi analysis, then, appears to be identification of some stress level below which the assumption of linear stress-strain behaviour is not even approximately valid. Now the theory is not designed for non-linear stress-strain, but numerical solution methods are available if the variation of this modulus, mv, with effective stress is known. Here the assumption of constant mv above and below p_c (with corresponding values of Cv) enables reasonably good agreement to be obtained. It seems probable that the stress-strain relationship is actually curved (whence mv varies gradually with effective stress) and while the CONED programs could handle such analysis, the problems of obtaining such parameters probably preclude use of such a method here.

A simple approximate approach may be of value here. The curve of case 3 above, fitting the increment p_c to p_f, may be determined from the tabulated series solutions for basic Terzaghi theory. This fits the majority of the consolidation process, including strain-time behaviour.

The pore-pressure behaviour may be estimated at early stages by a smooth curve from the correct initial pore pressure to become asymptotic to the maximum slope of the case 3 curve.

Hence the primary consolidation process may be reasonably well, if slightly awkwardly, described. Similarly the secondary consolidation may be fitted by a Buisman-type e - log t straight line. The author believes, however, that the absence of a consistent treatment of time-dependant behaviour of the soil skeleton leaves something to be desired. While suitable consolidation parameters may be fitted for laboratory rates of strain, no indication is given how these will vary for strain rates applicable for field cases. The problems of prediction and field behaviour are dealt with in Section 6.5.

5.7 Garlanger Analysis

In section 5.5.1 a master time-lines diagram (fig. 5.4) was presented as the best average of all the tests performed. Examination of Garlanger's method is best commenced by applying the analysis to this mean data. Results of the individual tests of the various types can then be considered in the light of this.

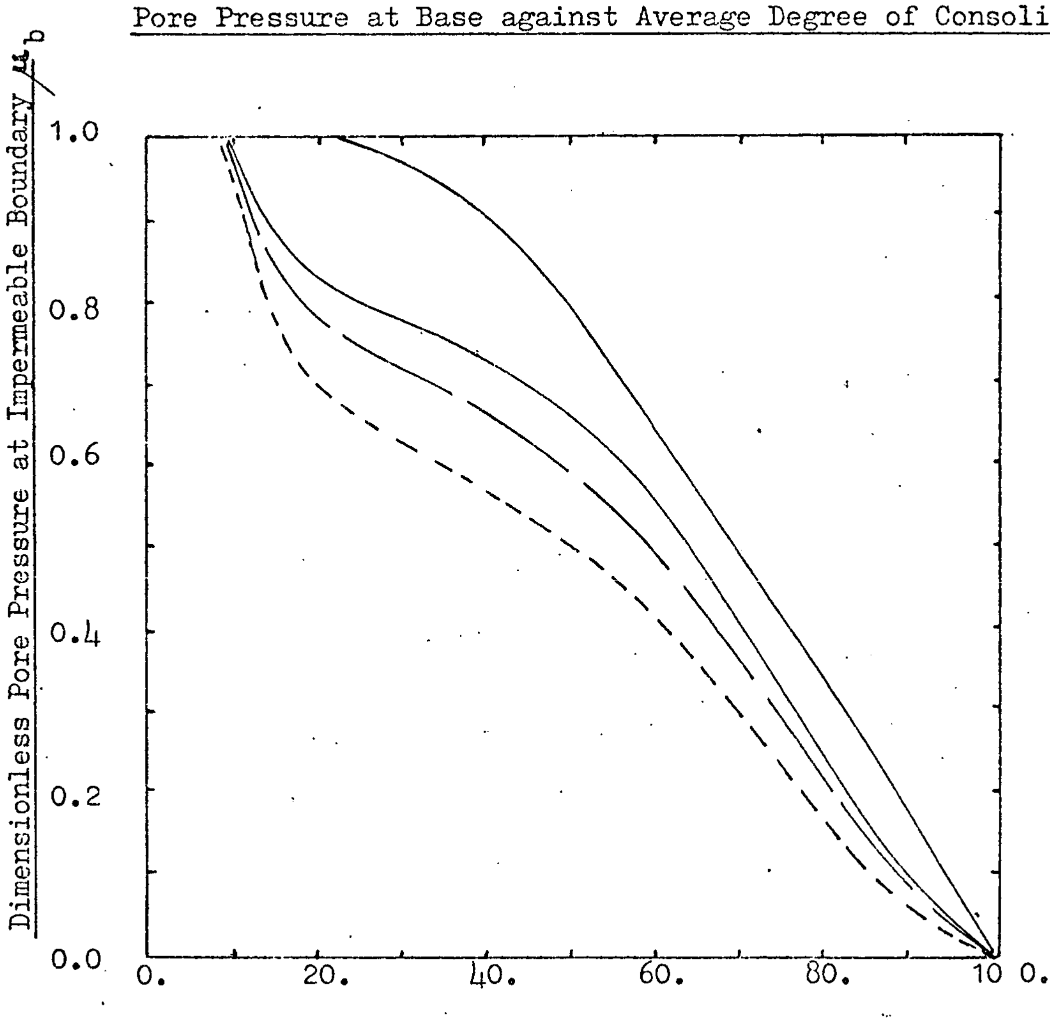
Now the time-lines were noticed to be independant of pressure increment, so only the influence of the period of prior delayed consolidation need be analysed. Results are shown on figs. 5.22 - 25 for the load increment S, 120 - 240 kN/m^2. All the parameters used are indicated on fig. 5.22. A few comments may be helpful. Fig. 5.25 shows details of the time-lines for this increment. It may be seen that e_o decreases with increasing prior delayed consolidation, and similarly Δe, the increment in voids ratio from e_o to e_D after 1 day's consolidation, decreases. From each value of e_o, a line of slope 'a' is drawn; the intercept of this with the "instant" time-line, t_i, giving the appropriate p_c value. From the B-tests (section 5.5.5), the instant line has been taken as the 1 minute line, and this is not affected by the prior delayed consolidation. Values of a, b, and c have been taken as constant throughout.

Results using this data, as indeed all results in this section, have been obtained using the program SECON (section 4.4.2) with occasional checks by the rigorous method, CONGO, showing these to be quite satisfactory. Strain-time behaviour (fig. 5.23) is very much as would be expected; the initial e value being lower for greater prior delayed consolidation, and all curves tending to converge as most of the instantaneous consolidation is completed. Pore pressure behaviour with time (fig. 5.24) shows the expected early fall in pore pressures

Fig 5.22 Garlanger Analysis using Mean Experimental Results

GRANGEMOUTH Increment S 120-240 kN/m^2

Pore Pressure at Base against Average Degree of Consolidation



a = .018 b = .216 c = .0067 ti = 1 min

	Prior dc.	e_o =	Δe =	p_c/p_o
————————	1 Day	.966	.134	1.279
———— ————	1 Week	.953	.121	1.366
- - - - - - - - - -	1 Month	.940	.108	1.472

Fig 5.23 Garlanger Analysis using Mean Experimental Results

GRANGEMOUTH

Increment S 120-240 kN/m^2

Time-Settlement Curves

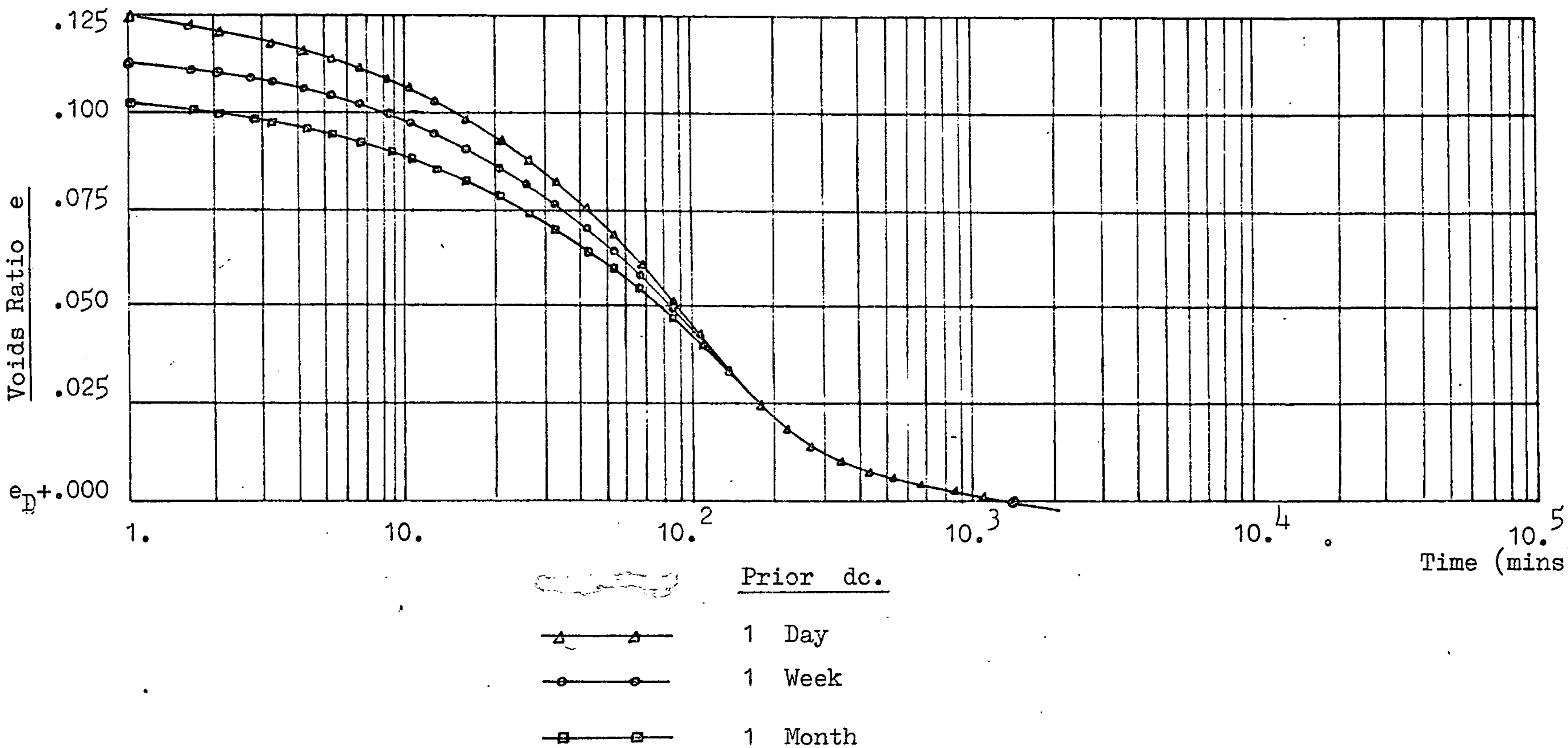


Fig 5.24 Garlanger Analysis using Mean Experimental Results

GRANGEMOUTH

Increment S 120-240 kN/m²

Pore Pressure - Time Curves

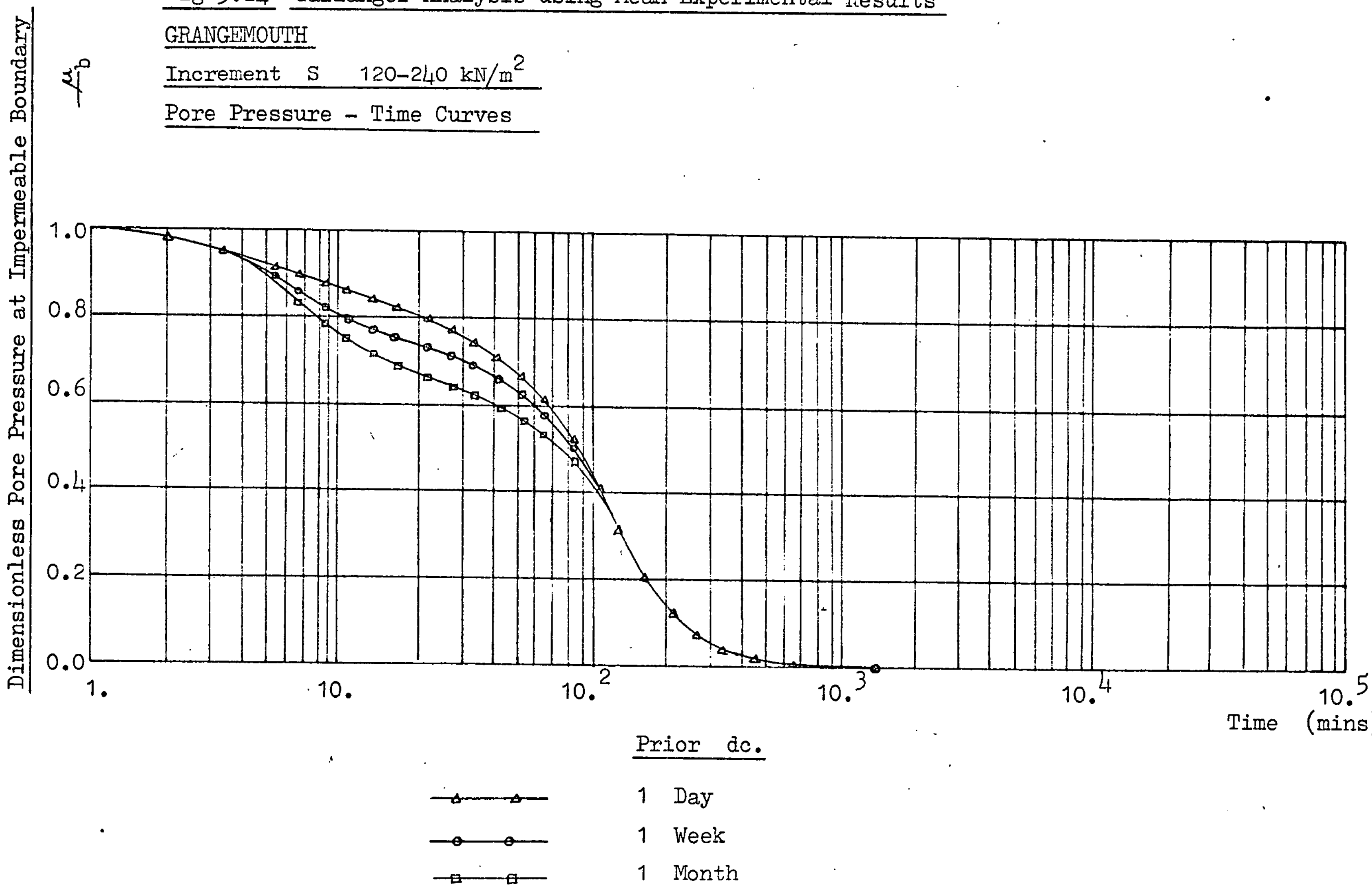
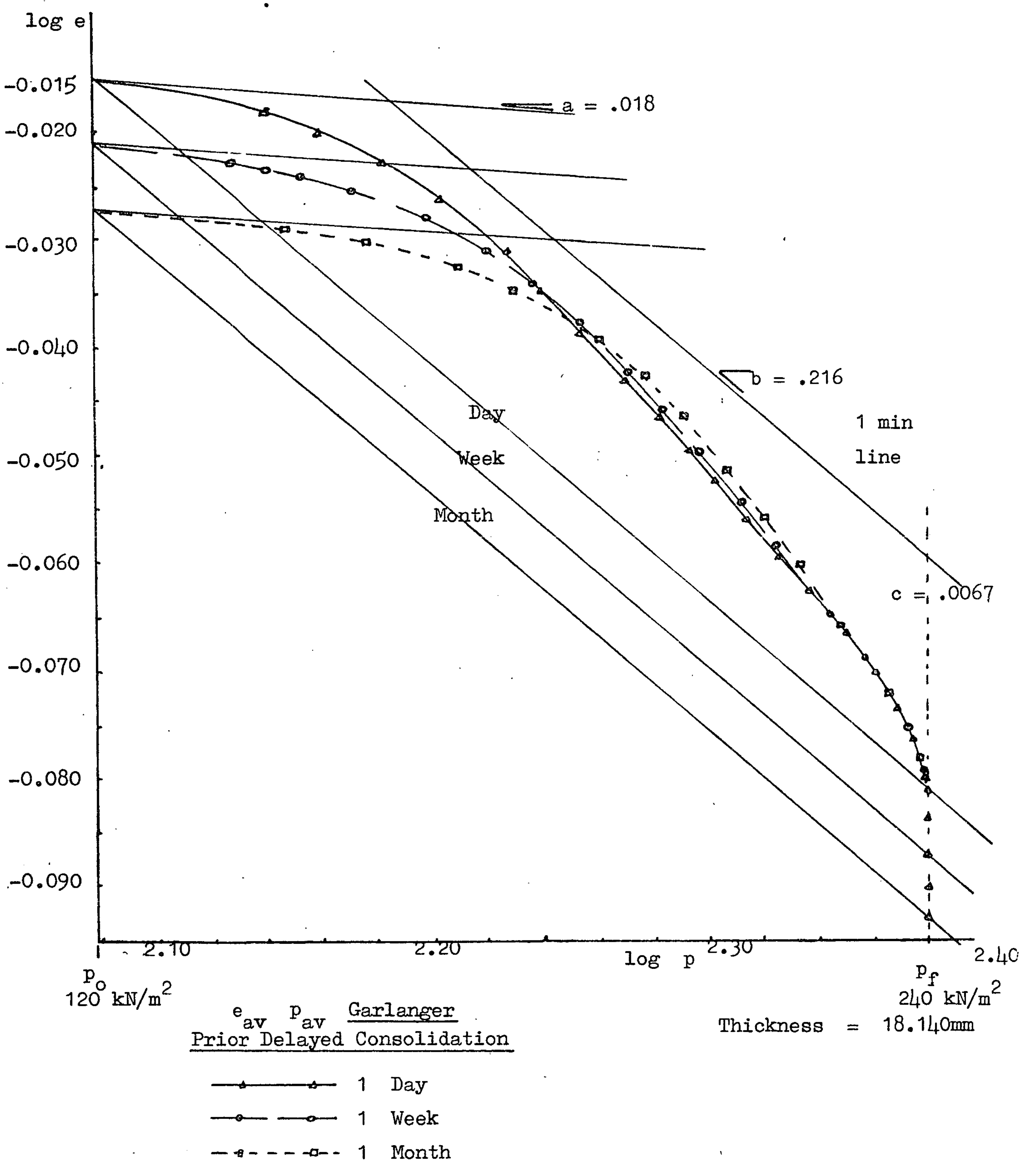


Fig 5.25 Time Lines Diagram for Mean Experimental Results

Increment S 120-240 kN/m^2



increasing as prior delayed consolidation is increased. It is interesting to note, however, that the three results follow a similar curve for the first couple of minutes. This is believed to be a result of behaviour being initially almost entirely governed by parameter 'a'. It is only when a significant proportion of the layer has reached a stress exceeding p_c, that the influence of 'b' starts to arrest this rapid dissipation. This, of course, occurs earlier, the lower p_c, i.e. the less prior delayed consolidation. The soil at the impermeable boundary will be the last to reach the critical stress. Once this has occurred, after some 10 - 20 minutes here, the curves tend to gradually converge again. Referring to fig. 5.22, the deviation of behaviour from Terzaghi theory increases with increased prior delayed consolidation. It is also noted that pore pressures at the impermeable boundary do not begin to fall until some 8% of the consolidation strain has occurred. This is due to the model assuming a diffusion process, so that dissipation only gradually spreads to the points distant from the drain.

It may be recalled that in Chapter 4, for want of a better method for the available data, t_i was determined from the time line tangential to the average stress-strain curve, stresses being determined on the assumption of parabolic pore pressure distribution (see below). The program SECON was modified slightly to give results of average stress and strain at each output time, by integrating the appropriate values at each space node. The resulting curves are plotted on figure 5.25. It is interesting that these tend to a similar curve at later stages of consolidation, and would tend to give similar values of t_i determined from the tangential time-line. This average plot is, incidentally, influenced by thickness of the soil layer (that corresponding to mean data for this increment being used). This is a further reason against use of the tangent time-line if this can be avoided.

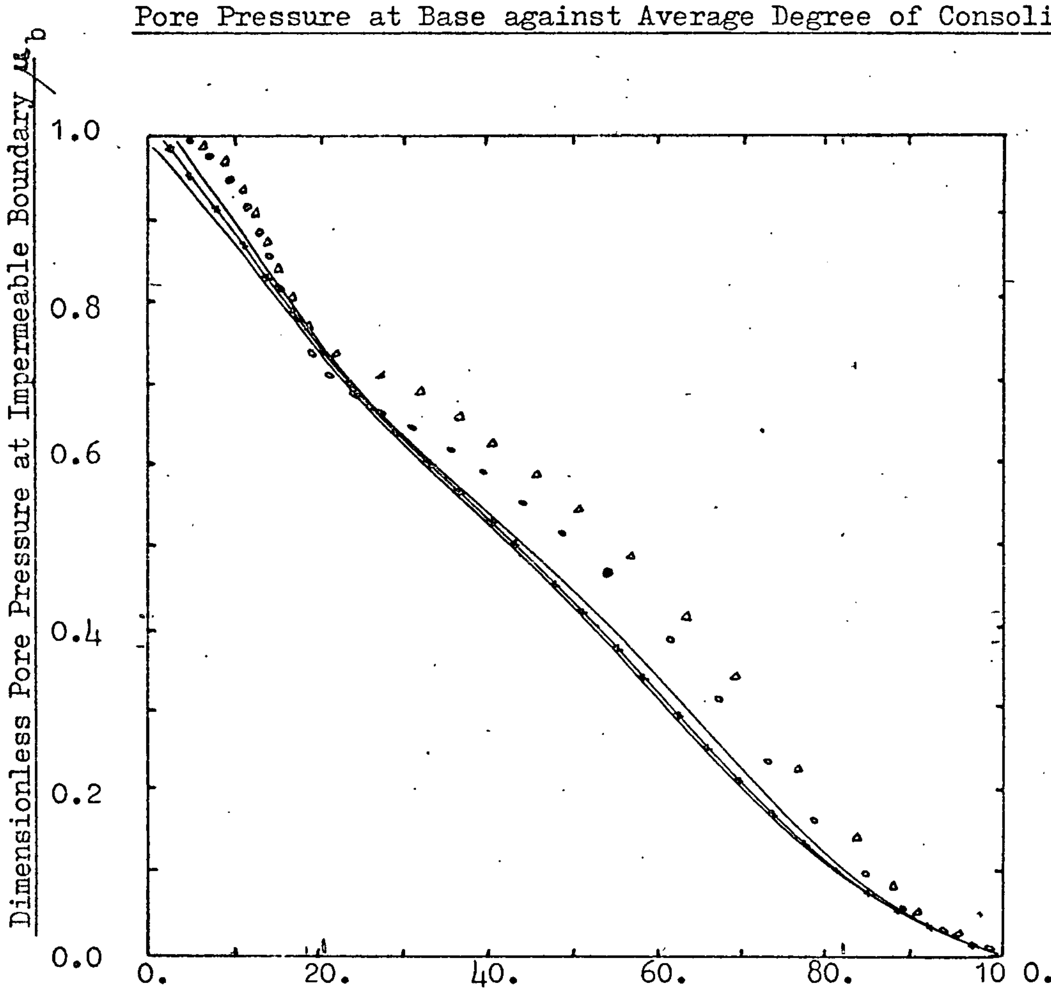
Now we begin comparison with the individual results by consideration of the E-type tests. Although only data after 1 week's delayed consolidation is available here, these tests, being fully saturated under back-pressure, should correspond directly to the results pertaining to the master diagram. Results are compared in figs. 5.26 - 28, (again, for simplicity, only increment S is shown for graphs involving time). Agreement of theory and experiment is quite good. The analysis does seem to give too slow a dissipation of pore pressure at very early stages, followed by too rapid a rate until p_c is reached. The subsequent consolidation behaviour seems to be handled fairly satisfactorily. The author believes this problem with early behaviour is a result of the simplification of the stress-strain relationship introduced here by Garlanger. A gradual curve to gradient 'b' seems more likely than the linear log e - log p relationship up to some point of discontinuity, p_c, as is used in the present model. This is discussed in section 6.4.

The parameters used to analyse E-tests, then, were the same as those of fig. 5.22 for the mean data. The variation of parameters in individual tests is responsible for some differences from this mean behaviour. It was noted that for the E-tests the total change in voids ratio for each increment was slightly lower than average, which accounts for the high predicted voids ratios at early times (fig. 5.27). Relating the value of e_o to the 1 minute line obtained using e_D and the average c value, as in fig. 5.25, a slightly higher value of p_c/p_o was obtained. The analysis was re-run using data identical to the mean values, except for these values of e_o, Δe, and p_c/p_o which were fitted to the specific test. As shown on the graphs, this gives rather better agreement of results. This, indeed, is to be expected, but as variations from mean

Fig 5.26 Garlanger Analysis of E Test Results

GRANGEMOUTH

Pore Pressure at Base against Average Degree of Consolidation



Average Degree of Consolidation %C

——————— Experimental Results × × × Increment S

Garlanger Analysis for Increment S:

△ △ △ Using Mean Data - see fig 5.22

○ ○ ○ Using Individual Test Data for e_o - see text

Fig 5.27 Garlanger Analysis of E Test Results

GRANGEMOUTH

Increment S 120-240 kN/m^2

Time-Settlement Curves

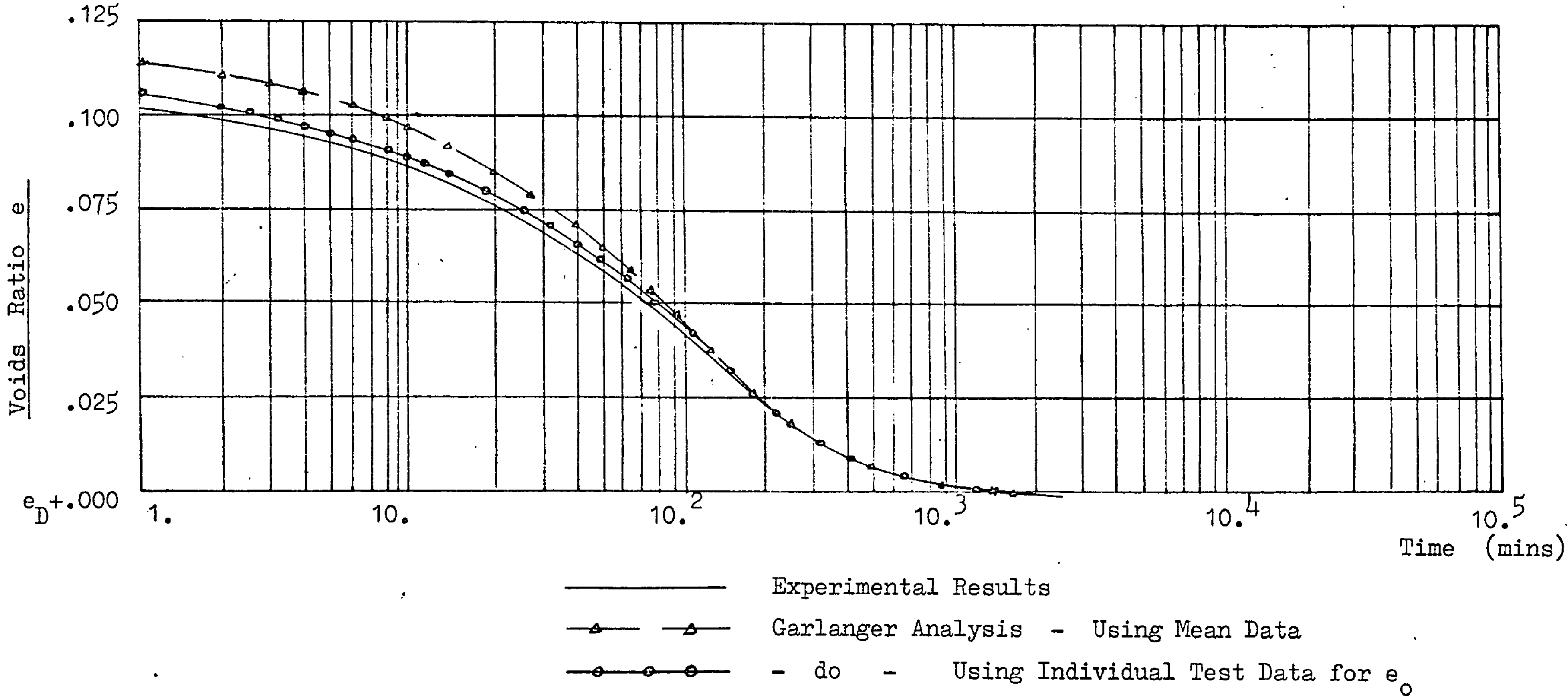
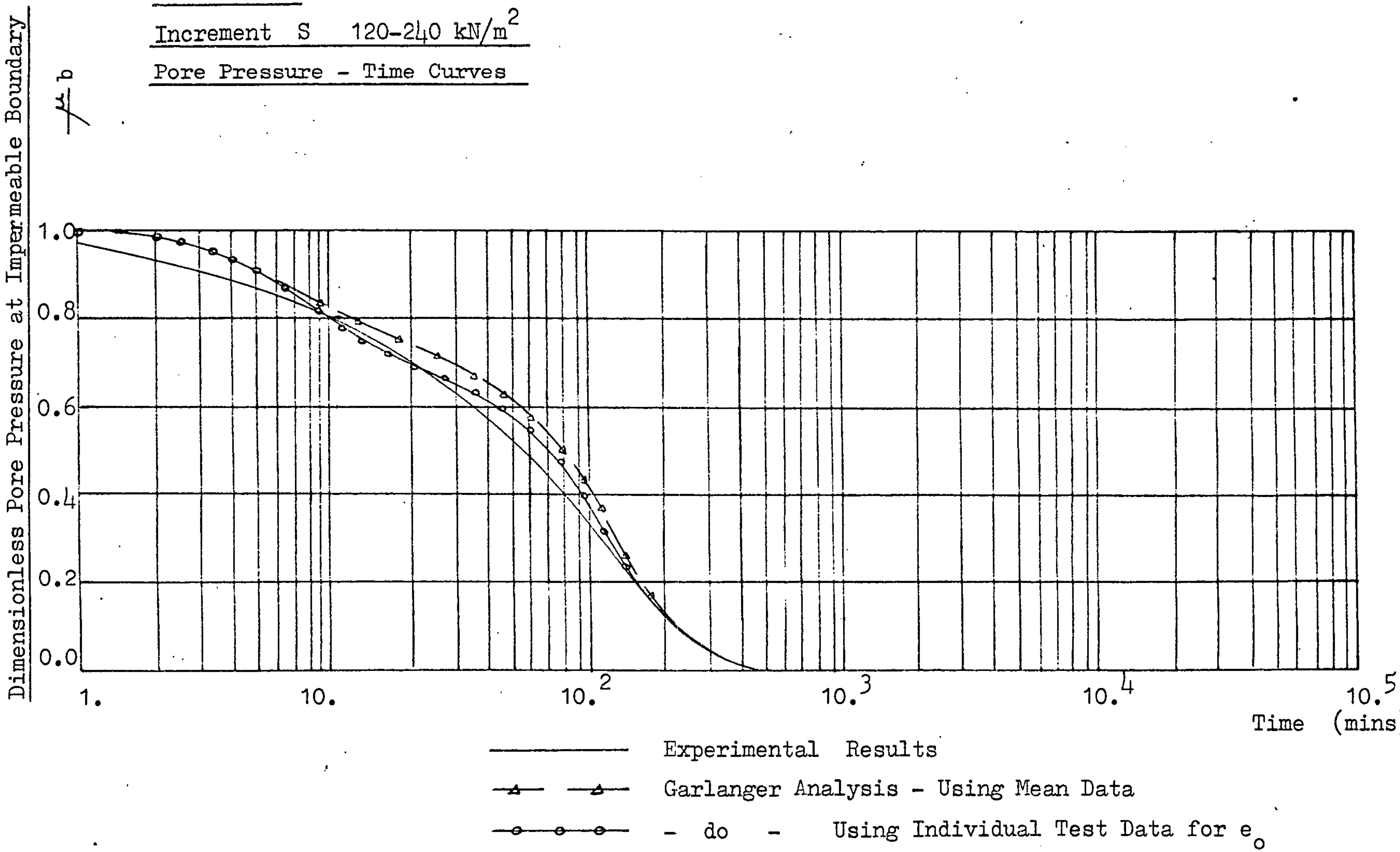


Fig 5.28 Garlanger Analysis of E Test Results

GRANGEMOUTH

Increment S 120-240 kN/m^2

Pore Pressure - Time Curves



behaviour are a result of random differences between tests, this best data for any specific test is of limited value for practical purposes. Further comments are made shortly.

Although the time-lines plots for these tests, equivalent to fig. 5.25, have not been included here, since they do not include further information of importance, it may be noted that the average log e - log p plots calculated from experimental results agreed well with those obtained from the Garlanger analysis for the correct e_o, Δe, and p_c/p_o values.

Analysis of the A-test results is complicated slightly by the influence of partial saturation. However, after the pore pressures have been allowed to build up for 24 hours against a closed drain, the sample may be taken as fully saturated; all the gaseous phase having been driven into solution. The value of voids ratio, e_a, is known, and the effective stress, p_a, constant with depth, may be determined from the constant pore pressure attained. As noted in the results section, this "undrained" process took the soil state to another point roughly on the time line it had reached due to prior delayed consolidation. Since a logarithmic scale is used for effective stress, the critical pressure may be determined using the ratios calculated for p_c/p_o as shown in fig. 5.22 for the requisite prior delayed consolidation, only p_c/p_a is now evaluated. Referring to fig. 5.32, $p_a = 146.0$ kN/m^2 and for 1 week's delayed consolidation $p_c/p_a = 1.366$, whence p_c is given. Alternatively, the instant line may be drawn through the point p_f, e_{c1}; determined from the voids ratio after 1 day and fitting the log e - log t time of slope c to this, from which the 1 minute point e_{c1} is obtained. The two methods of drawing the instant line will give slightly differing results. This is due to point (p_a, e_a) not falling exactly on the time line - the effect of this was usually found to be very small, as shown on fig. 5.32.

Figs. 5.29 - 5.31 show the comparison of Garlanger's method with experimental results for the A-tests. Again, for simplicity, only pressure increment S is presented here, but this is quite typical. Analysis is based on the mean data of fig. 5.22. The only differences in parameters are that consolidation is begun at (e_a, p_a), and the appropriate values of Δe and p_c must be used to be consistent. This data is given on fig. 5.29. From these graphs we see that consolidation following various periods of delayed consolidation can be well described by Garlanger's method. Progress of strain with time (fig. 5.30) gives particularly good agreement. Pore pressure behaviour still encounters some problems. The awkward early behaviour resulting from Garlanger's linearisation of the log e - log p relationship below p_c, was discussed above. This is very much in evidence here.

For the 1 month test for this increment the values of μ_a (dimensionless pore pressure after 24 hours undrained) and e_a were found to be rather low. The early dissipation of pore pressure during consolidation is correspondingly slow resulting in rather poor agreement with analysis results. Here the point (p_a, e_a) was found to plot around the 1 week line, rather than the 1 month line. The reason for this experimental anomaly is not known, but it may be seen that analysis correcting the initial soil state to be on the 1 month line (with μ_a = 0.69% corresponding to this) gives rather better agreement with the majority of the consolidation process.

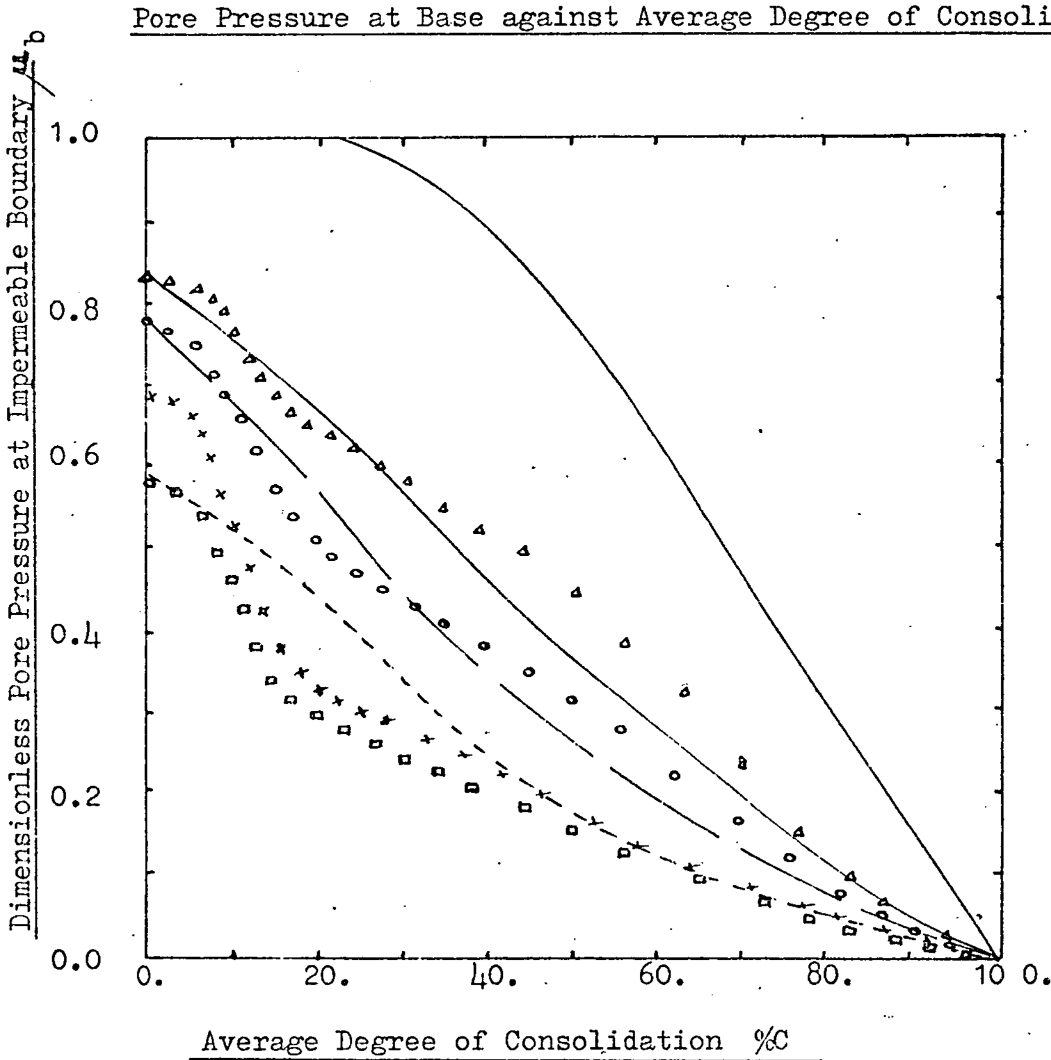
The average log e - log p plots were compared for experimental results and Garlanger analyses. As shown in fig. 5.32, this generally led to good agreement. For the case of increment S, 1 month test,

Fig 5.29 Garlanger Analysis of A Test Results

Increment S 120-240 kN/m^2

GRANGEMOUTH

Pore Pressure at Base against Average Degree of Consolidation



Garlanger Analysis	Experimental Results	Prior dc	Δe	p_a	p_c	$p_c/$
△ △ △	————	1 Day	.107	141.2	179.4	1.27
○ ○ ○	—— ——	1 Week	.098	146.0	199.0	1.36
□ □ □	- - - - - - - - -	1 Month	.058	175.3	229.6	1.31
* * *	Corrected (see text)	1 Month	.058	158.0	229.6	1.45

Fig 5.30 Garlanger Analysis of A Test Results

GRANGEMOUTH A-Test

Settlement-Time Graph

Increment S 120-240 kN/m^2

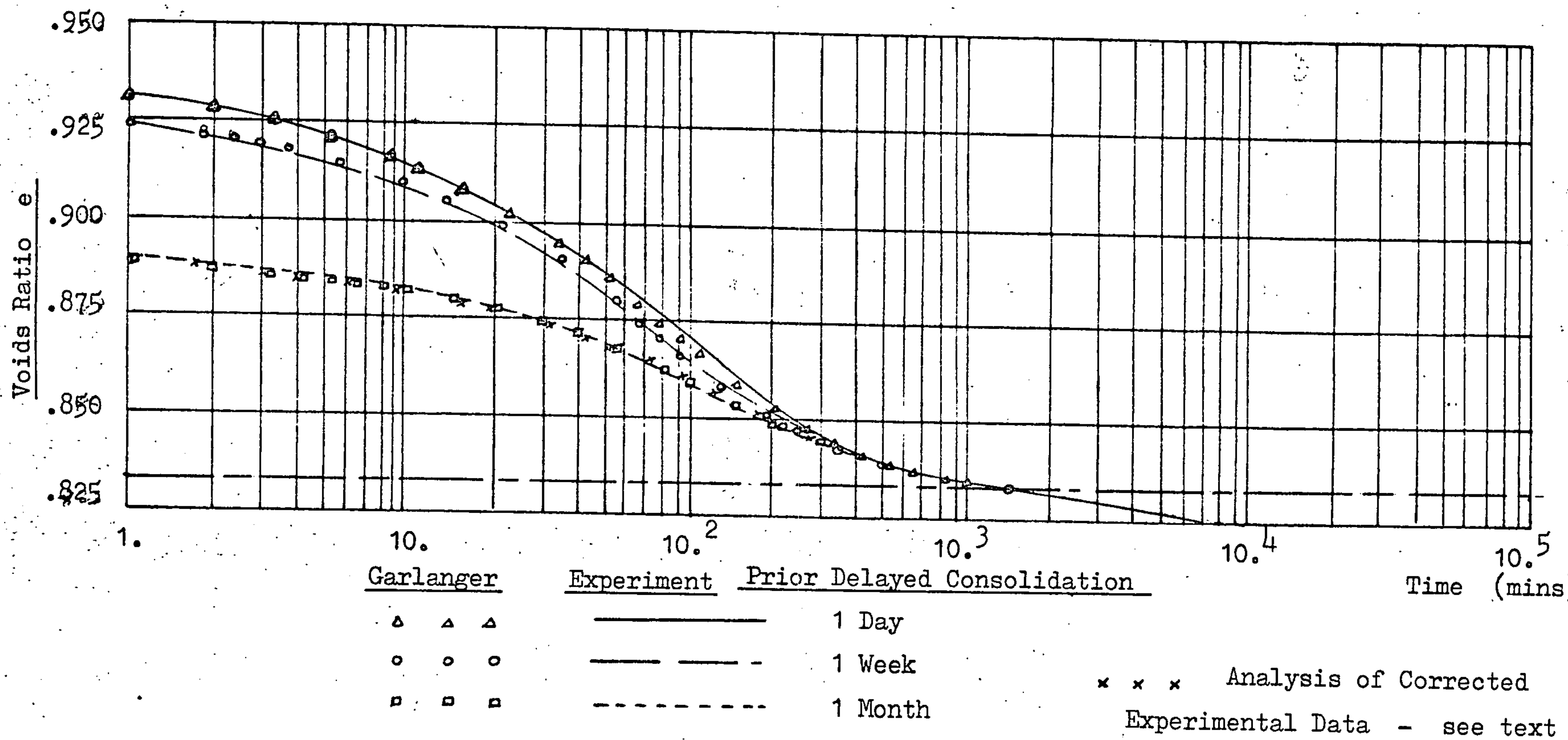


Fig 5.31 Garlanger Analysis of A Test Results

GRANGEMOUTH A-Test Pore Pressure-Time Graph

Increment S 120-240 kN/m^2

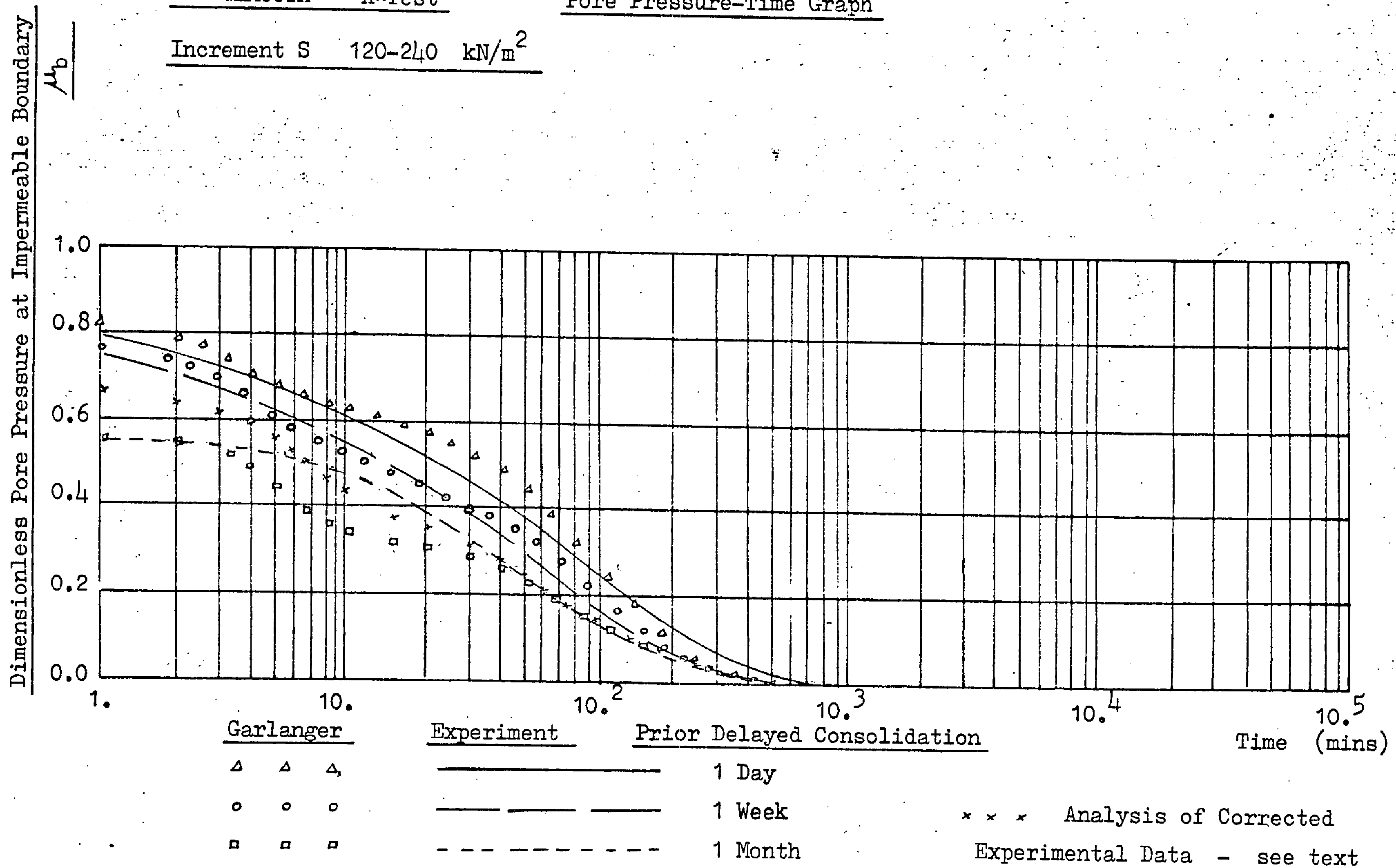
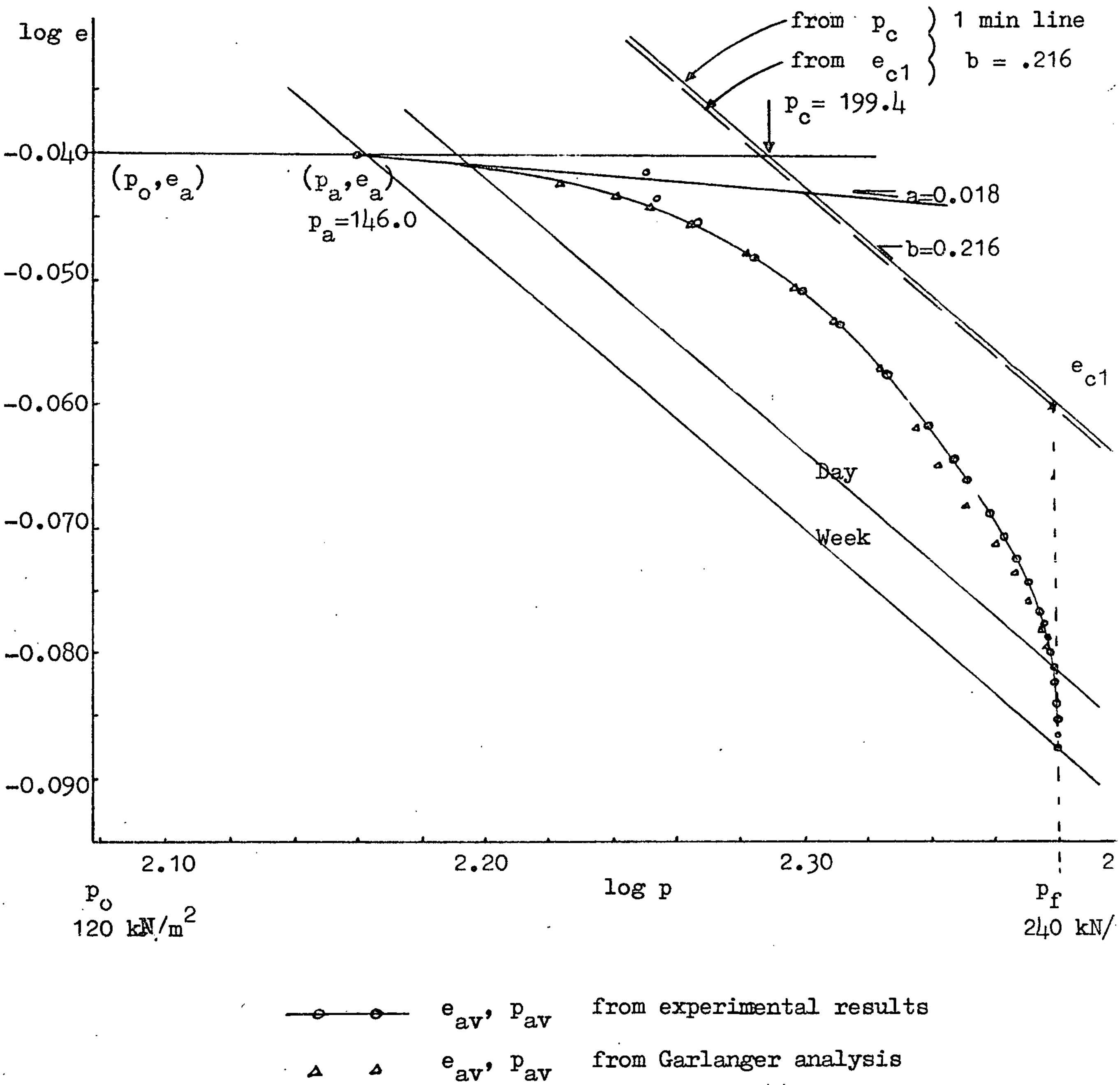


Fig 5.32 Time Lines Diagram for Test SAW



p_c/p_a = 1.366

discussed above, good agreement was again obtained once the value of p_a had been corrected to lie on the 1 month time-line.

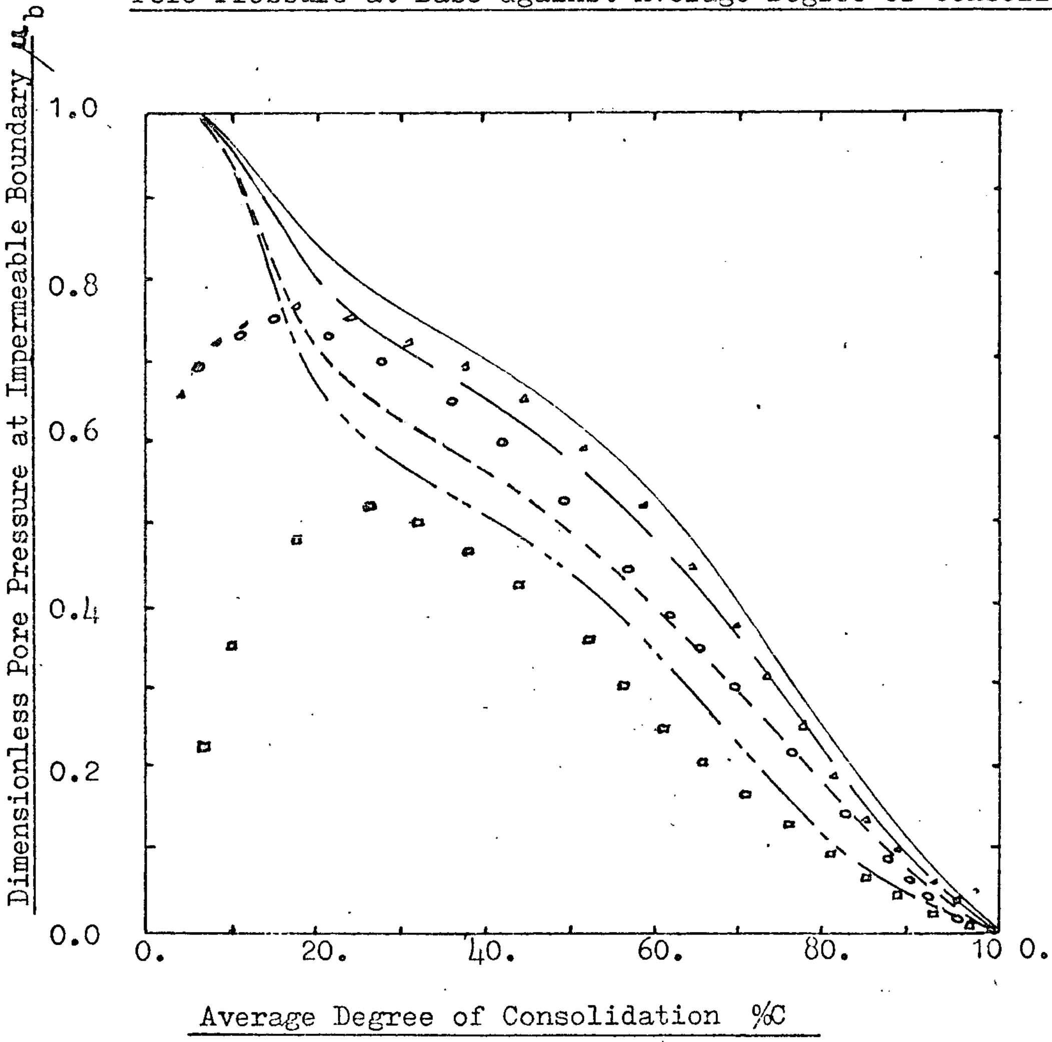
During some early analysis of experimental data the initial soil state was taken as (e_a, p_o), i.e. it was assumed that the initial pore pressure should equal the applied load increment. Referring to fig. 5.32, it may be seen that drawing a line of slope 'a' from this point to the instant line will give a value of p_c very close to the correct one (though, of course, the ratio p_c/p_o will not be of any value here, since it is affected by undrained strain as well as delayed consolidation). The good value of p_c here meant that although pore pressures began at 100%, they fell rapidly to join the correct curves shown in the present graphs. Hence Garlanger's analysis method depends primarily on the correct positioning of the point (p_c, e_c) and the time lines resulting from this. Even major variations in the assumed initial soil state, and the A-type consolidation, will result in variations only in that aspect of analysis, and this will not affect the satisfactory behaviour at later stages.

The effect of partial saturation on the C-type tests is rather more complex as the gaseous phase is being forced into solution while consolidation takes place. Hence the pore pressures are observed to build up to some maximum value while considerable strain occurs although this takes place fairly rapidly with time. It is again possible to compare this behaviour with that predicted by Garlanger analysis of the mean data. For simplicity only the pore pressure-degree of consolidation graph is included here, again for just one pressure increment, in fig. 5.33. It was, in fact, found that strain took place slightly faster than predicted at early times and gradually slowed up to join the predicted curves. The results shown compare experiments for the

Fig .5.33 Garlanger Analysis of C Test Results

GRANGEMOUTH

Pore Pressure at Base against Average Degree of Consolidation



Garlanger	Experimental	Prior dc
———————	△ △ △	1 Day
——— ———	o o o	1 Week
- - - - - - - - -	▫ ▫ ▫	1 Month
—— - - —— - -	Corrected (see text)	1 Month

three periods of delayed consolidation with theoretical results using the best averaged data for all the tests. It may be seen that the build up of pore pressure cannot be predicted. Once the maximum pore pressure has been attained, however, the analysis goes some way to fitting the experimental data. It seems probable that from some 20-30% degree of consolidation, the soil behaves as if fully saturated; the vast majority of the gaseous phase having been forced into solution. As may be seen from fig. 5.33, the actual pore pressures are rather lower than predicted. It has already been noted for E and A-tests, that Garlanger theory tends to slightly over-predict pore pressures, so this may account for some of the discrepancy. The rest is due to the influence of partial saturation, and unfortunately, for this type of test, it is not possible to assess the influence of this.

Agreement is worst, here, for the one month test. All the tests were also analysed using the values of parameters obtained by specifically fitting the results. For the 1 day and 1 week tests values were quite close to the mean data, and results of these analyses are very close to the mean data lines, so have not been included on fig. 5.33. For the 1 month test here, parameters were slightly different from the mean, in particular a higher value of p_c/p_o (= 1.50) (as compared with 1.47) was found, and analysis using this data gave rather better agreement. This simply shows that some discrepancy from Garlanger analysis is due to random variation in the experimental results which may be better fitted with specific parameters for each test.

In fact, all the experimental results, for A, C and E-tests, were also analysed using the best fit of each specific test to obtain parameters. Values of 'b' were derived from the relevant test series. 'a' was taken as 0.018, throughout, based on the B-tests. Drawing log e - log p curves for the tests considered here, and later comparison with such curves from Garlanger theory, suggests this value is as satisfactory as is possible for linearisation of this curve below p_c. The 1 minute line was again used as the instant line; the position of this was determined by extrapolating the log e - log t line for delayed consolidation, of slope c, back to 1 minute, and drawing the line of slope b through this point at the final effective stress. Two methods were possible here; the observed value of c could be used, or the averaged value (see section 5.5.6). The latter method was found to give slightly better results. Values of p_c could then be determined from the intersection of the line of slope 'a' with this instant line. Such values of p_c were found to be rather sensitive, but generally the ratios p_c/p_o for the three periods of delayed consolidation were around those predicted from the average results model as discussed in section 5.5.5 and shown in figs. 5.16 and 5.22.

Fitting the data for the individual tests gave better agreement with the experimental results than the Garlanger analyses using the mean data reported here. The current presentation, however, allows the effect of the period of delayed consolidation, and its analysis in the Garlanger-Bjerrum framework, to be examined in a more consistent manner. While the random variations between tests can be fitted, these tend to confuse understanding of the actual soil behaviour. If the theoretical method is to have any practical value, the parameters used should be constant and capable of being determined with reasonable accuracy. It has been shown in this section, that analysis using constant values of the best averaged parameters from the full test programme allows quite good agreement with the observed effects of delayed consolidation on subsequent soil consolidation behaviour for the Grangemouth Silty-Clay.

5.8 Concluding Remarks

A considerable body of experimental data on consolidation behaviour following various periods of delayed consolidation, has been obtained for a soft silty-clay known to exhibit significant time-dependant effects. It has been shown that an increase in strength of the soil skeleton occurs as delayed consolidation progresses, such that further loading results in a significant amount of fairly rapid consolidation with relatively little strain before the consolidation behaviour typical of a normally consolidated soil is observed. Such behaviour can be reasonably described by Terzaghi theory if the governing parameters, which now vary with effective stress, can be successfully obtained. A more generally satisfactory approach, however, is that proposed by Bjerrum, in which the influence of delayed consolidation on subsequent behaviour is automatically handled. It has been shown here that, using Garlanger's formalisation of this model, with linearised logarithmic stress-strain and strain-time functions, good description of the consolidation behaviour following various periods of delayed consolidation is obtained. Hence the experimental evidence supports the validity of this Garlanger-Bjerrum model.

From this work, however, it is suggested that the linear log e - log p relationship up to some critical effective stress, p_c, is probably an oversimplification of real soil behaviour at low stress increments above the level at which delayed consolidation has occurred. Comparing actual consolidation behaviour with that predicted by the linear model, and also considering tests using small load increments, a gradually curving stress-strain relationship reaching a gradient 'b' at around some stress value p_c, seems more likely. (i.e. p_c is an arbitrarily defined yield point representing a region of change, rather than a sharp physical point of discontinuity.)

It may be helpful to summarise the basic steps in application of Garlanger's method.

1. One or more series of at least three consolidation tests, of long duration must be performed over the stress range of interest.
2. From such results the best time-lines diagram may be constructed.
3. A series of incremental loading tests should be performed to identify the position of the instant line. This also gives a value for parameter 'a'.
4. Adding this instant line to the time-lines diagram, and drawing a line of slope 'a' from any value of e_o, the required value of p_c is obtained.

From this all the necessary parameters for use in the Garlanger programs may be evaluated.

CHAPTER 6

Aspects of Consolidation Behaviour

6.1 Overview of Consolidation Theory

It has been shown that the consolidation behaviour of Grangemouth silty-clay departs significantly from that suggested by simple Terzaghi theory. Strain rate affects the shape of the stress-strain relationship, resulting in pore pressure dissipation being influenced by prior delayed consolidation. Such behaviour is believed to be typical of soft clays and, indeed, of any clay brought close to the normally consolidated state, where effects of delayed consolidation are of importance. Some evidence will be examined in this Chapter.

Settlement behaviour was found to be surprisingly well fitted by conventional Terzaghi theory, but the pore pressures were quite inadequately handled. As demonstrated in section 5.6, acceptable fitting of the data has proved possible using suitable, non-linear stress-strain relationships, in an "extended Terzaghi" analysis. Besides being tedious to apply, such methods have no great value unless they can be applied to field thicknesses, which implies a knowledge of how the non-linear relationships are scaled up. It is believed that the Terzaghi stress-strain function is dependant on time and will change markedly with layer thickness. Predictions based on the behaviour of thin samples may be positively misleading.

The Garlanger-Bjerrum approach uses simultaneous linear log e - log p and log e - log t relationships which should be independent of layer thickness. There are still problems concerning the critical pressure, p_c, which appears to depend on thickness. Clarification is required before Garlanger's scheme has any real predictive value.

Satisfactory treatment of this will be presented in section 6.4.

In the present chapter the major aspects of consolidation are treated in detail. Further general discussion of Garlanger's theory is required before proceeding. The theory was developed by combining Bjerrum's (1967) ideas on consolidation with a model proposed by Hansen (1969), incorporating an additional stress-strain term for the rapid "sub-critical" behaviour. Now Bjerrum's time-lines diagram is a development of the work of Taylor (1942) (his fig. 4.2) whose model is shown in fig. 6.1. Taylor suggested two components of effective stress; the basic compression curve $\{f_b(e)\}$ and plastic resistance $\{p_p\}$

$$p = f_b(e) + p_p \qquad (6.1)$$

The plastic resistance, in turn, was split into two components; bond, p_b; and viscous resistance, p_v.

$$p_p = p_b + p_v \qquad (6.2)$$

Taylor has, in fact, introduced different theories to deal with these components, though he notes that "the combined use of ideas from two such theories must be the ultimate procedure......."

<u>Theory A</u> (also Taylor and Merchant (1940)), uses the expression

$$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial p} \cdot \frac{\partial p}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_c \qquad (6.3)$$

Constant linear relationships are used for

$$\frac{\partial e}{\partial p} = a_v \qquad (6.4)$$

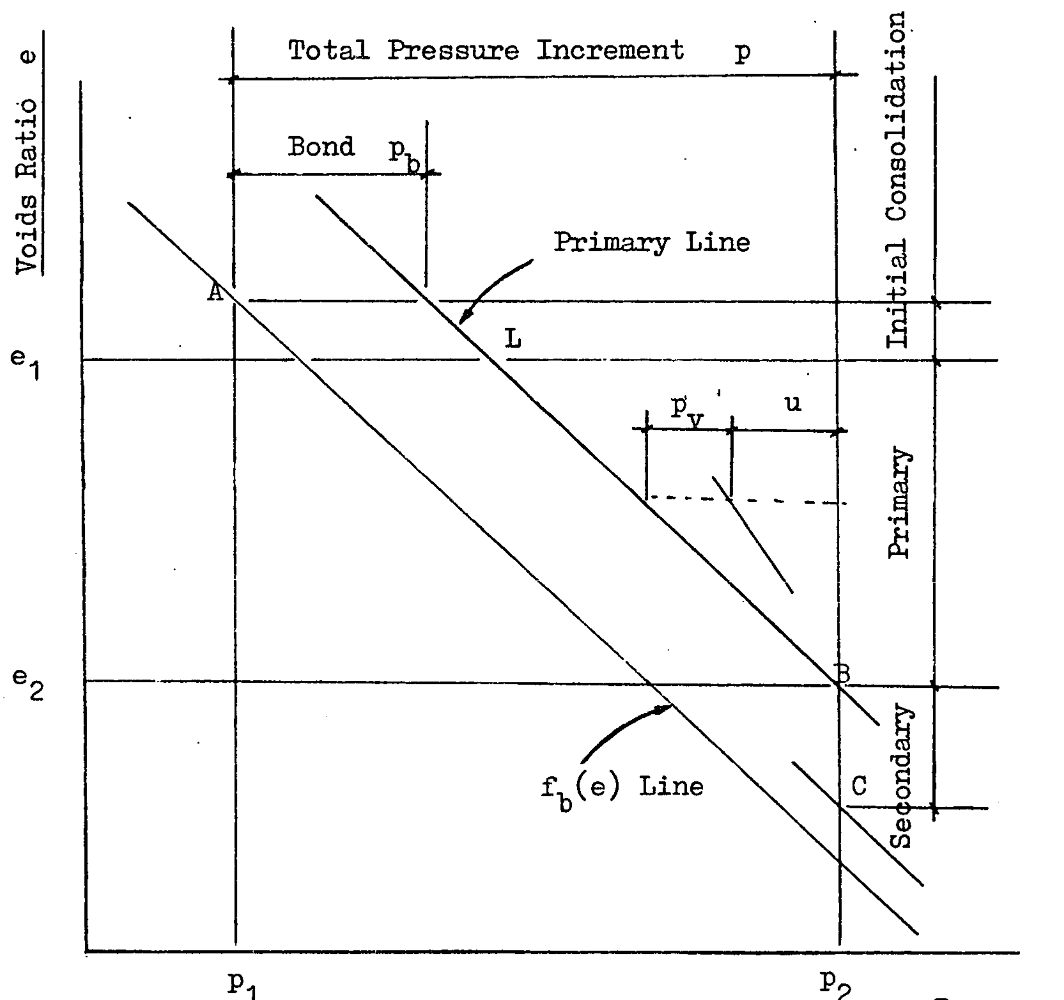
and

$$\left(\frac{\partial e}{\partial t}\right)_c = - \mu \{a_f(p-p_1) + (e-e_1)\} \qquad (6.5)$$

Brief comments were made in section 1.2 {together with a diagram of the stress-strain model (fig. 1.2)} and it was noted that this is mathematically identical to Gibson and Lo's method (fig. 1.3). It was mentioned that the major shortcoming is in treatment of pore

Fig 6.1 Consolidation Theory with Plasticity

AFTER D.W. Taylor (1942) (fig 42)



pressures at early times. The term $av(\partial p/\partial t)$ describes the primary line (L B on fig. 6.1), and $(\partial e/\partial t)$ is a simple expression of "secondary" consolidation; we can see how such secondary behaviour leads to an increase in bond. Inclusion of such bond in future consolidation is not possible, however.

Theory B considers viscous behaviour in "primary" consolidation, by making p_v a simple linear function of strain rate

$$p_v = \eta \frac{\partial e}{\partial t} \qquad (6.6)$$

Notice that this is total strain rate, rather than $(\frac{\partial e}{\partial t})_c$, hence the lines of constant strain rate will be of different slope from the primary line. Barden (1965) {again, see section 1.2 and figs. 1.4 and 1.5} has extended this approach to give a non-linear viscosity term. Progress with such schemes encounters three major problems.

i) It is difficult to observe the resistance-strain rate behaviour directly.

ii) It is not clear how such a relationship, established for laboratory samples, would alter for field thicknesses.

iii) Taylor does not allow for bond (previous delayed consolidation) in the scheme. Change of state from A to L on fig. 6.1 must be calculated before applying theory B, although it occurs simultaneously with primary consolidation. It is probably possible to include this in Barden's scheme, although there is no simple expression for the effect on viscous parameters.

Garlanger follows equation (6.3), but the terms are now complex and non-linear. The time-lines are lines of constant $(\frac{\partial e}{\partial t})_c$, so that when $\partial p/\partial t \rightarrow 0$ they are the same as Taylor's $f_b(e)$ lines, except that a linear log e - log p relationship is now proposed. Two steps are now introduced which allow a consistent treatment of viscous behaviour to be included.

i) We notice that the term for delayed consolidation

$$\left(\frac{\partial e}{\partial t}\right)_c = \frac{ce}{t_i}\left(\frac{e}{e_c}\right)^{1/c}\left(\frac{p}{p_c}\right)^{b/c} \qquad (6.7)$$

is valid for <u>any</u> values of e and p. The "primary" line is replaced by an "instant" line, t_i, through some point e_c, p_c. This point is, as yet, arbitrary: for any values of e_c and p_c a value of t_i may be determined, "positioning" this function correctly.

ii) "A-type" stress-strain behaviour is introduced.

$$\frac{\partial e}{\partial p} = a\,\frac{e}{p} \qquad (6.8)$$

This handles behaviour from A to L on fig. 6.1 (i.e. bond). It may, however, continue beyond L. Garlanger, in fact, continues it to meet the "instant" line at p_c, e_c. {It is important to note that this term can only be introduced once a consistent term for $\left(\frac{\partial e}{\partial t}\right)_c$ is available. Equation 6.7 is valid during A-type consolidation, indeed during this process $\left(\frac{\partial e}{\partial t}\right)_c$ increases greatly.}

Thus, if the instant line is defined as the greatest strain rate, $\left(\frac{\partial e}{\partial t}\right)_c$, undergone by the skeleton, we have a perfectly valid way of combining viscous and bond resistances. In place of those terms, though, it is easier to think of the components of structural resistance as a stress-strain relationship and a strain-time relationship.

While the model is useful as an attempt to reconcile the known factors influencing consolidation behaviour, the validity of the stress-strain and strain-time functions cannot be proven mathematically. Hence the experimental evidence of chapter 5; this and similar comparisons with real soil behaviour have, to date, shown the assumed relationships to be acceptable.

We mention briefly the "instant" line. It would appear that Taylor's "primary" line (which led to serious problems in its arbitrary position, causing a discontinuity between bond and viscosity) has been removed, only to be replaced by some equally imprecise "instant" line. Garlanger's published scheme had the stress-strain relationship change abruptly to

$$\frac{\partial e}{\partial p} = b\,\frac{e}{p} \tag{6.9}$$

at some point (p_c, e_c), where the 'a' line meets the instant line. His definitions of p_c and t_i appear to be circular. This topic will be discussed in detail in section 6.4, where the effect of the t_i and p_c values on predicted behaviour is compared with observed data. Suffice it here to note that

i) A satisfactory treatment, capable of predicting field behaviour as well as fitting known results, can now be offered.

ii) Mathematically the above approach holds good for any chosen p_c and t_i values.

It is well known that the load increment ratio ($\Delta p/p$) has an important effect on consolidation behaviour. The smaller $\Delta p/p$, the greater the departures from Terzaghi theory. It may now be seen that lower $\Delta p/p$ implies a larger proportion of A-type consolidation, so that the stress-strain relationship becomes more non-linear. It will be shown in section 6.3 that the Garlanger-Bjerrum approach provides a rational treatment of this effect, and of prior delayed consolidation, which is linked in that it affects the A-type behaviour.

Comment is also required on the equation

$$\frac{\partial e}{\partial t} = \frac{k(1+e_o)}{\gamma_w}\,\frac{\partial^2 u}{\partial z^2} \tag{6.10}$$

apparently used by Garlanger. This is based on the well-known assumptions of homogeneity, one-dimensional flow, complete saturation, incompressible fluid and soil particles, and the validity of Darcy's law. Gibson et al. (1967) have reworked the problem leading to the equation (in the present notation)

$$\pm \left(\frac{\gamma_s}{\gamma_f} - 1\right) \frac{d}{de}\left[\frac{k(e)}{1+e}\right] \frac{\partial e}{\partial z} + \frac{\partial}{\partial z}\left[\frac{k(e)(1+e_o)}{\gamma_w(1+e)} \frac{dp}{de} \cdot \frac{\partial e}{\partial z}\right] + \frac{1}{(1+e_o)} \cdot \frac{\partial e}{\partial t} = 0 \qquad (6.11)$$

The first term handles self-weight of soil and fluid. For the present this will be neglected (it is probably insignificant for thin samples, but may be important at field thicknesses - see section 6.5). This is most easily handled by assuming equal density of fluid and soil particles, $\gamma_s = \gamma_f$, whence this term disappears.

It should be noted that Darcy's law should be applied to the velocity of flow relative to the soil skeleton which is also moving. This accounts for the term (1+e). At the same time the moving skeleton introduces a correction to the length variable $(1 + e_o)$.

The first solution approach suggested by Gibson et al. involves writing

$$C_F \frac{\partial^2 e}{\partial z^2} = \frac{\partial e}{\partial t} \qquad (6.12)$$

where

$$C_F = \frac{-k(e)(1+e_o)^2}{\gamma_w(1+e)} \cdot \frac{\partial p}{\partial \bar{e}} = \text{constant} \qquad (6.13)$$

It is instructive to compare Garlanger's treatment with this consistent approach.

Let us remove the constant permeability restriction, and rewrite equation 6.10.

$$\frac{\partial e}{\partial t} = \frac{k(e)(1+e_o)}{\gamma_w} \cdot \frac{\partial p}{\partial e} \cdot \frac{\partial^2 e}{\partial z^2} = Cv \ \frac{\partial^2 e}{\partial z^2} \qquad (6.14)$$

where

$$Cv = \frac{k(e)(1+e_o)}{\gamma_w} \cdot \frac{\partial p}{\partial e} = \frac{k(e)(1+e_o)}{\gamma_w} \cdot \frac{\partial \mu}{\partial \beta} \cdot \frac{\Delta p}{\Delta e} \qquad (6.15)$$

the latter expression using Garlanger's transforms. Now this differs from Garlanger's published Cv by the variable permeability, and inclusion of $\partial\mu/\partial\beta$. It is believed that $\partial\mu/\partial\beta$ should have been included for consistent treatment. The impression is given, by its omission, that all parameters remain constant, whence Cv can be considered constant throughout consolidation. However, $\partial\mu/\partial\beta$ certainly varies according to the complex stress-strain-time behaviour of the skeleton. Allowing permeability to vary introduces the possibility of Cv remaining constant. It is well known in practise that k and $\partial e/\partial p$ tend to vary inversely, so that constant Cv may be a reasonable approximation. Apart from the "small strains" assumption (see below) this is the same as the approach first considered by Gibson et al. They also examined a non-linear theory, writing

$$C_F = C_o + \alpha(e - e_o).$$

It may well be that this, or some other non-linear scheme, will prove necessary for some soils. Before proceeding to this, however, full consideration should be given to the simpler approach. For the present study, then, it is assumed that Cv, as correctly defined by equation 6.15, remains constant. It is emphasised that this implies varying permeability.

The "small strains" assumption may be written

$$(1 + e_o) = (1 + e).$$

If this holds Cv is identical to C_F. It would be preferable to use C_F in the present scheme. Unfortunately, a variable 'e' in Cv cannot be reconciled with the transforms leading to Garlanger's dimensionless equations (see section 4.2). Further comment is made in section 6.5.

Finally, on this topic, it should be added that equations (6.14) and (6.15) hold without restriction on the stress-strain-time relationships for the skeleton, i.e. this is compatible with equation (6.3).

Brief mention should be made of two further approaches in which the behaviour of the soil skeleton is assumed to be uniquely described by stress, strain and rate of strain. Hawley (1971) produced a "Unified Theory of Consolidation". The physical and theoretical build up of this model is carried out with considerable care and Gibson et al.'s (op. cit) governing equation is recovered (6.11, above). To reach the final solution form, a complex theoretical expression links permeability to voids ratio and an unusual strain rate term is introduced. The theoretical basis of the latter seems questionable, and both steps require experimental justification. The approach holds some theoretical interest but its use in practice must await clarification of:

i) How governing parameters are obtained.

ii) The validity of these (and hence the theoretical expression used) in fitting laboratory data.

iii) The value of the scheme for predicting field behaviour.

Lowe (1974) presented a model in which the effect of strain rate is incorporated into Terzaghi's approach. A complex graphical and tabular procedure varies mv, and hence Cv, between small increments of time.

It is then assumed that Terzaghi's time factor solutions apply over small increments of constant Cv, so that the real times derived from this will reflect the influence of strain rate on Cv. It is not very profitable to go into a number of objections which may be raised, but the point should be made that average strain rate over the layer is used, {indeed, it is hard to see how the procedure could be extended to local strain}. This has already been discussed with reference to the numerical extensions to Terzaghi theory (Chapter 2). In fact, the CONED programs would appear to be a much neater treatment, and can also approximate local behaviour by use of a number of sublayers. The only modification required is suitable description of the variation in mv with strain rate. Lowe does present such a relationship, though its origin is somewhat obscure. It appears to be more of a fitting method than a prediction technique. Garlanger's method and the stress-strain-time approach was adopted just because such a method of varying mv seemed impracticable. The present author is of the opinion that

i) Lowe's extensions to Terzaghi theory introduce theoretical inconsistencies.

ii) The proposed variation in mv with strain rate is not a particularly helpful treatment of the stress-strain-time behaviour of the skeleton.

iii) The method is extraordinarily involved for hand computation, and a programmed scheme is to be preferred (the graphs and tables are not particularly conducive to such a scheme, however).

There are a number of outstanding questions concerning the real behaviour of the soil in both field and laboratory, and the acceptability of the present relationships for skeletal behaviour. These topics are taken up in the next few sections. To conclude the present discussion we note that despite a number of inconsistencies and confusing presentation, Garlanger has developed a scheme which enables theoretically acceptable treatment of consolidation where the skeletal behaviour depends upon strain rate as well as pressure and voids ratio. The main points of disagreement with the published scheme are:

i) Garlanger's equation 7, in which e is expressed as a function of p and t, is not valid. However, the incremental expressions for $\frac{\partial e}{\partial p}$ and $(\frac{\partial e}{\partial t})_c$, {6.8 and 6.7, above} are valid, and these are what have actually been used (discussed in section 4.3).

ii) Cv remains constant, as stated by Garlanger, but it is believed that his expression with constant compressibility and permeability is incorrect. The correct definition is given in expression 6.15, from which it may be seen that constant Cv would result from k and $\frac{\partial p}{\partial e}$ varying inversely.

It may be that in some cases the soil behaviour is more complex than described by the present model with constant parameters, but for this present study we shall confine investigation to the possibilities of this scheme. The indications so far are that this is satisfactory for at least some real soils.

6.2 Delayed Consolidation

Delayed consolidation has been defined in the present study as the relationship between strain of the soil skeleton, and time, independant of effective stress. At large times after application of a load increment the influence of the strain-effective stress relationship becomes negligible and a good approximation to the delayed consolidation function may be observed. Experimental results suggest that there are some random variations in the observed gradient of strain against logarithmic time and also that a significant change of slope takes place after some 7 days. Nevertheless, generally good agreement was found with use of Garlanger's method, based on a constant value of delayed consolidation coefficient.

An experimental investigation into the "Secondary Compression of Clays" was reported by Lo (1961). Several points are of interest to the present work. He begins by identifying three types of secondary consolidation curve. Type I quickly slows to negligible rate, type II is roughly linear with logarithmic time, and type III shows rate of settlement with logarithmic time increasing after some significant period. Lo performed tests on a remoulded Grangemouth clay believed to be similar to that used in the present experiments. He assigned this soil to type I where, he says, final settlement is reached after some three weeks. The author must take issue with this, since there is no doubt for the many tests reported in Chapter 5 that significant strain still occurred after 1 month, and, indeed, a longer duration test (not reported here) showed delayed consolidation continuing after 6 months. The only result of more than 1 week's duration published by Lo for Grangemouth shows the rate of delayed strain increasing with logarithmic time, so it is not clear why it was assigned to type I.

One possible reason may be that the increased rate was believed to be solely due to a rise in temperature. This result was used by Lo to illustrate how a rise of around 3°C caused a marked change in gradient of the e - log t curve. However, the effect occurred after some 7 days and such behaviour has been noted in the present experiments, where it is certainly not attributable to temperature change. Re-examining Lo's other published results it seems difficult to justify his contention that "variation in temperature has a predominant influence on both the rate and magnitude of secondary consolidation", although he may have had more data on this which was not published. In the present tests, all performed under constant temperature conditions, some variation in the delayed consolidation coefficient was found, and it seems that this is random, or at least due to unidentified causes. In view of this it would be hard to establish the effect of temperature unless this were fairly marked and Mesri (1973) suggests this is not the case. In quite a full review of secondary consolidation he comments that more importance is attached to temperature than deserved; it is probably insignificant.

The present author agrees with this view-point. Besides the reported test program, a number of other tests were run without the controlled temperature. Over a range of some 14 - 20°C the values of C_α and c fitted well within the range of results recorded for constant conditions. Similarly, Christie's data for this clay (see section 5.5.6) fits well although temperature was not controlled.

To what, then, may the change in gradient after some 7 days be attributed? It seems to correspond to Lo's type III curve. This, however, Lo only reported for undisturbed loosely deposited soils, and he ascribed the change in gradient to "breakage of bonds". The Grangemouth clay was remoulded, so this does not seem to be the same phenomenon. The change in gradient did not appear to be affected by pressure, so it is concluded that it must be linked to either strain rate or time. (Lo's sample was some 3/4" thick, so strain rate would have been similar after 7 days to that in the present tests.) The answer to this particular problem almost certainly requires a detailed examination of the chemistry and physics of the soil-fluid system going far beyond the scope of this project. Knowledge in this field is rapidly increasing (see, e.g. Mitchell 1975), but the author has not, to date, found an explanation which satisfies observed behaviour.

Davies (1975) has undertaken a lengthy experimental investigation of "creep" behaviour of Brown London Clay, Avonmouth Clay, and Upper Lias Clay. This was mostly stress-path tri-axial testing, unconfined horizontally and leading to eventual failure, but some constant rate of strain oedometer tests were also run. (The difference between creep, in the former, and delayed consolidation, in the laterally confined specimens, is not, perhaps, clearly enough brought out.) The deflection measuring apparatus was very sophisticated, and it is interesting to note that "strain instabilities" were observed, i.e. definite, and sometimes quite large changes of strain rate (in both creep and delayed consolidation) were observed for no identifiable reason. These appear to be quite random, although there were no repeated identical delayed consolidation tests to permit comparison. Such complex long-term behaviour of real soils goes well beyond treatment by present theoretical models.

The review of literature on long-term behaviour, by Davies, gives some interesting information on the influence of pressure on C_α (the delayed consolidation coefficient for e - log t) with respect to p_c. Of 21 references considered, all 12 which discuss p_c report

C_α increasing with pressure up to p_c. Seven of these suggest it increases linearly; the others are not committed on this point. The vast majority of the references quoted C_α constant with pressure above p_c. Mesri (1973), in his review, draws similar conclusions, as does Clayton (1973). We notice that this ties in well with the time-lines framework. The link between critical pressure and strain rate (see section 6.4) has not generally been appreciated, so that reports may be found of the delayed consolidation rate depending on pressure when, to the present author's mind, this is simply an expression of the soil state relative to the time lines. Consider a soil at the critical pressure p_c, with voids ratio e_c, and ignore primary consolidation effects (these do not affect behaviour of the soil skeleton in relation to delayed consolidation). The rate of strain reduces with lower e at constant p, or with lower p at constant e. If the time lines are constantly spaced logarithmically on a log e - log p plot (i.e. the log e - log t coefficient 'c' is constant), we find that for <u>any</u> straight line crossing the time lines parameter c will change linearly with log p and also with log e. Such a determination of c, however, really confuses understanding of behaviour by taking a wrong starting point. It is suggested that c or C_α may be taken as constant with pressure, providing it is remembered that the initial time corresponds to the soil's position relative to the time-lines, i.e. the e and p values, and <u>not</u> the absolute time from application of loading.Bjerrum's time-lines model gives a rational framework, which, being in line with the main stream of soils research, should help to clarify the picture.

It is possible that, having stated C_α correctly, it may be found, for some soils, to vary with pressure. The references mentioned above, and the present experiments suggest that for most soils and most pressure ranges of practical significance, this effect will probably be insignificant.

We are left, then, with an e - log t relationship which may reasonably be considered independant of pressure, but which may change in gradient with time or strain rate, and may be subject to "strain instabilities". At the present state of knowledge it seems that theoretical treatment must use some simple logarithmic relationship. It may be that this is all that is necessary for practical purposes, if the "roughness" observed on graphs of laboratory test resultscan be suitably smoothed out.

We note that Garlanger wishes to extrapolate the delayed consolidation behaviour observed in the later stages of a long term laboratory test, in both directions in time. Backwards (lower t values) gives the delayed consolidation rates during primary consolidation, i.e. this defines the time-lines grid from the instant-line until primary consolidation is complete. The assumption that such time-lines, governed by constant c_o, are valid during the primary stage, is made implicitly by Garlanger. As mentioned in section 6.1, by postulating a negligibly thin sample it seems reasonable to argue that such time-lines are valid, even at very small times. It appears debatable whether 'c' may be taken as constant here, but the acceptability of agreement between predicted behaviour and the results of chapter 5 is offered as evidence that this assumption holds.

Extrapolation forwards in time poses problems when we come to predict the behaviour of field deposits where, of course, delayed consolidation behaviour, both during and after the primary stages, is required for times far in excess of those observable in laboratory tests. For the present, as noted above, some simple logarithmic relationship must suffice. We may note, however, that should detailed information on delayed consolidation become available {perhaps through investigations of a number of foundations in a "troublesome" area}

Garlanger's method could easily be modified to handle this. The actual relationship may be approximated by a number of straight lines on a log e - log t plot of gradients c_n, applying between time-lines t_n and t_{n+1}. The program could quite simply select the appropriate t_n and hence c_n value for each point in the calculations.

To summarise, then:

1. Delayed Consolidation has been defined as a relationship between voids ratio and time at constant effective stress.
2. The influence of pressure {but see 4 below} and temperature on the rate of delayed consolidation are probably insignificant.
3. The rate of delayed consolidation may be uniquely determined by reference to the soil's e - p state in relation to a Bjerrum-type time-lines diagram.
4. Use of such a diagram clarifies the observation that rate of delayed consolidation rises with p, below p_c. This follows from the relation of e - p to the time-lines; it is misleading to consider it as a fundamental pressure effect.
5. Bjerrum's model uses parallel time lines, implying that the delayed consolidation coefficient is independant of pressure. This is believed to be quite satisfactory {see 2 above}. The model also takes such lines to be equally spaced with logarithm of time, which implies a linear e - log t relationship {Garlanger's log e - log t is equally acceptable}. Present evidence suggests some departure from linearity, although the significance of this may not be too serious.
6. Although current theoretical models must assume some simple delayed consolidation relationship, Garlanger's method could handle more complex relationships should such be established.

6.3 A-type Consolidation

The A-type consolidation is defined by a linear relationship, of gradient 'a', between log e and log p. Such behaviour may be considered elastic and reversible. The model, as originally presented, defines some critical stress level, p_c, above which this relationship ceases and B-type behaviour pertains. The present discussion treats p_c as an experimentally observed point of discontinuity of stress-strain relationship (obtained by linearising what is actually a curved plot). The problem of definition of p_c is satisfactorily resolved in section 6.4, but it would be confusing to introduce these ideas without detailed discussion here. We simply note that the present section is consistent with the developments to be presented in 6.4.

The validity of an elastic 'a' relationship should be easy to establish. Unfortunately, it is obscured by two effects which, together, prevent direct observation.

1) The diffusion process of drainage of pore fluid leads to effective stress varying with depth.

2) Together with the elastic behaviour, plastic deformation also takes place. This is described by the delayed consolidation function, c, which is linked to the rate of strain of the soil skeleton.

Since these processes cannot be satisfactorily isolated, it is necessary to compare actual behaviour with models in which the various components are solved simultaneously.

Now the experimental results reported in Chapter 5 tend to fit well with the Garlanger-Bjerrum model. The main discrepancies from Terzaghi theory were found to be in the early behaviour, and the results thereof. The changes in voids ratio with time, however, could be quite well fitted by Terzaghi's theory. As discussed in Section 6.1

this is believed to be because this voids ratio relationship may be established from first principles without the need for any assumption concerning the stress-strain-time behaviour of the skeleton. The only assumption required is that some coefficient, Cv, may be taken as constant throughout the consolidation process, and the arguments for this have been considered.

We thus find that the nature of the stress-strain-time relationship can only be established by reference to the available data on pore pressure at the impermeable boundary. The conclusions drawn from the present experimental work were that quite good agreement was obtained using Garlanger's scheme. It was suggested that it was probably an oversimplification to take the A-type log e - log p relationship as linear, and a curve from zero gradient to somewhat greater than the present 'a' value (which constitutes an average gradient) would give better agreement with observed behaviour. It should be noted that this suggested curve would still be an elastic e - p relationship; the delayed consolidation term $(\frac{\partial e}{\partial t})_c$ was properly included in the Garlanger analyses. Nevertheless, it would appear that for present purposes the simple linear A-type relationship should suffice. The improvement offered by a more complex relationship is fairly minor compared to the degree of improved agreement offered by the present scheme over predictions of the Terzaghi model.

To clarify discussion of pore pressure behaviour, 4 general types of curve are identified as shown in figs. 6.2. μ_b signifies the dimensionless excess pore pressure ($u/\Delta p$) at the impermeable boundary.

<u>Type I</u> has pore pressure initially equal to the load increment, where it remains some time before dissipating. Such behaviour is indicated by Terzaghi theory, which gives a unique curve on fig. 6.2a, but varies according to Cv on fig. 6.2b.

Fig 6.2 Types of Pore Pressure Behaviour

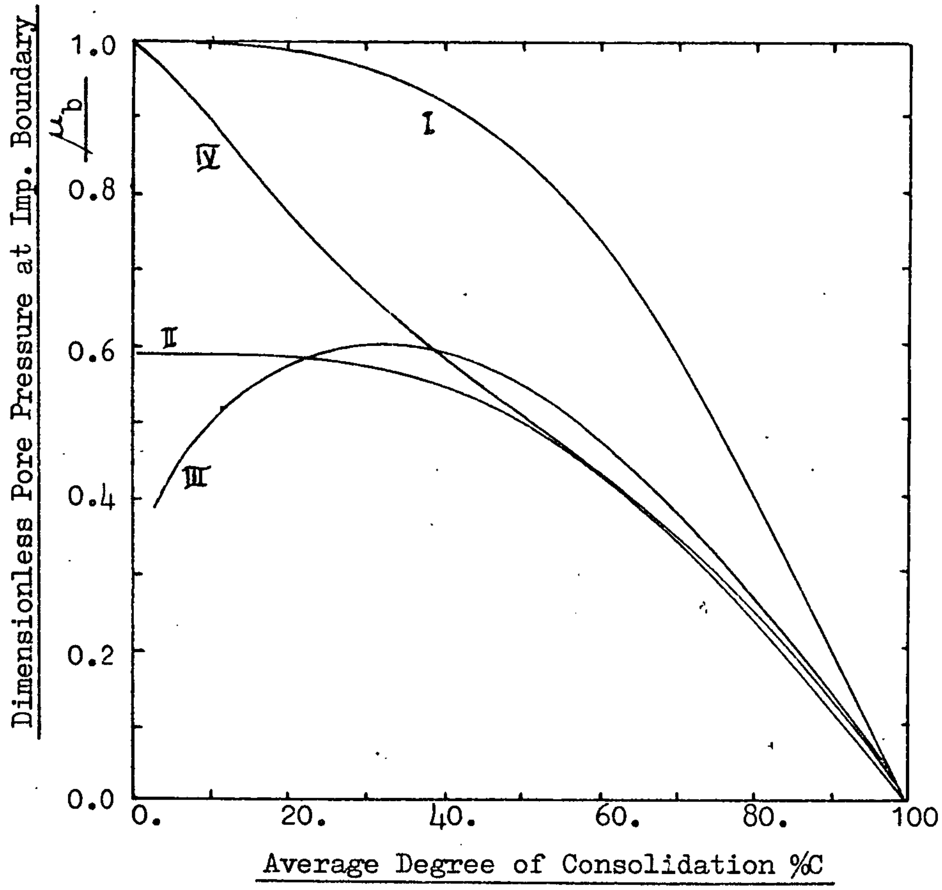


Fig 6.2a

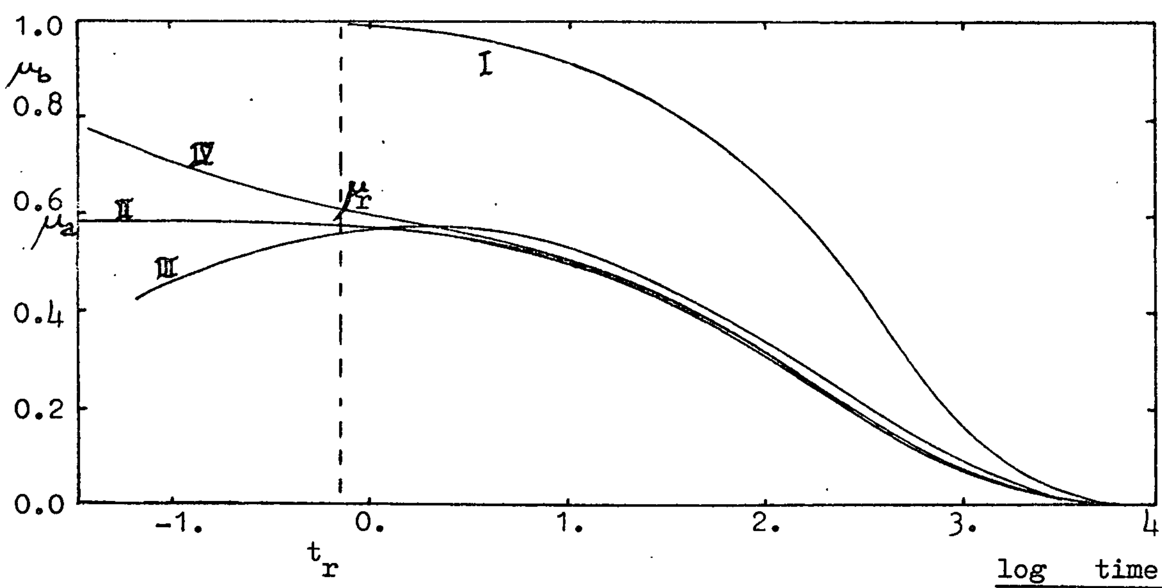


Fig 6.2b

<u>Type II</u> is similar, but initially only some fraction, μ_r of the applied load is observed, from which value the pore pressure falls.

<u>Type III</u> has pore pressure building up to some maximum value, μ_{max}, before dissipating.

<u>Type IV</u> has pore pressure initially equal to the applied load, but falling rapidly, especially at early times.

One complication should be mentioned. The first reading may only be obtained after some significant time t_r, and this frequently does not allow types II and IV. and sometimes, type III, to be distinguished. On a logarithmic time plot it is not clear what original value μ took, but fig. 6.2a allows behaviour to be clearly identified.

Garlanger theory predicts curves of type IV. The exact form of such curves will depend on the initial values (p_o, e_o), and hence the amount of A-type consolidation which takes place (see e.g. figs. 5.22, 6.7). Similarly the curves are affected by the gradient and spacing of the time-lines, and by the thickness of the soil layer (see section 6.5, and fig. 6.13).

As an approximation to this rapid early behaviour a type II curve might be considered. Such an approach, taking the initial pore pressure as some fraction μ_r of the total applied load, was reported for Terzaghi theory in section 5.6 above, although it was shown for the Grangemouth data that type IV curves were actually obtained, and for the purposes of the present investigation such an approximation was not acceptable. In cases where behaviour reasonably follows a type II curve, Terzaghi analysis setting $p_o = p_f - \mu_r \times \Delta p$ leads to a unique curve on the μ - % Consolidation plot fig. 6.2a, for any given μ_r value. This analysis may be useful, since it avoids the need to consider varying consolidation parameters in Terzaghi theory, as would be required for a type IV curve. As will be seen shortly, the author believes type IV is the more likely real behaviour, and there is little advantage in using Garlanger theory to approximate a type II curve, although this is quite possible, again setting $p_o = p_f - \mu_r . \Delta p$, if desired.

Two types of initial conditions may be identified (A-test and C-test of Chapter 5). In the first the drain is initially closed and pore pressure is allowed to reach equilibrium before consolidation commences. In such cases type III curves will not be found. The μ_a value reached, plus any strain noted here, is taken to indicate the extent of partial saturation. The present experimental evidence strongly supports μ reaching 1.0 for a fully saturated clay. It will, however, fall quite rapidly on opening the drain; increasingly so after greater delayed consolidation when the A-type component becomes greater.

For the conventional (C-test) consolidation, in which load is applied to a freely draining sample, curves of type II and III have been reported. For a fully saturated soil with incompressible grains, the pore pressure will rise rapidly to join a type IV curve. Since this falls rapidly, the meeting point, and maximum observed pore pressure, will only be a proportion of the applied load increment; the value being governed by the compressibility of the skeleton (affected, as mentioned, by delayed consolidation) and the time taken for the pore pressure to rise to this point. But why does this not take place instantaneously? It is the present author's belief that for a thin laboratory sample, times greater than about 1 minute signify partial saturation or flexibility effects. A short time is actually taken, however, and this is due to viscous behaviour of the soil fluid system. Even fully de-aired water is not totally rigid, and although such compressibility is negligible for strain behaviour it does lead to a very short, but observable, time being taken for "news" of the load increment to "filter through" the sample. Berry (1969) has developed his viscosity theory to allow for this, and, depending on the parameters used, predicts times in the order of 10 seconds for μ in typical laboratory samples to build up to 0.95 against a closed drain. He reports good agreement of predicted results with observed behaviour. {The E-tests reported in Chapter 5 show similar behaviour.} The time taken for such build up increases with thickness of soil. This topic has not been pursued for the present - there does not appear to be a straight-forward method of developing a similar approach for Garlanger theory.

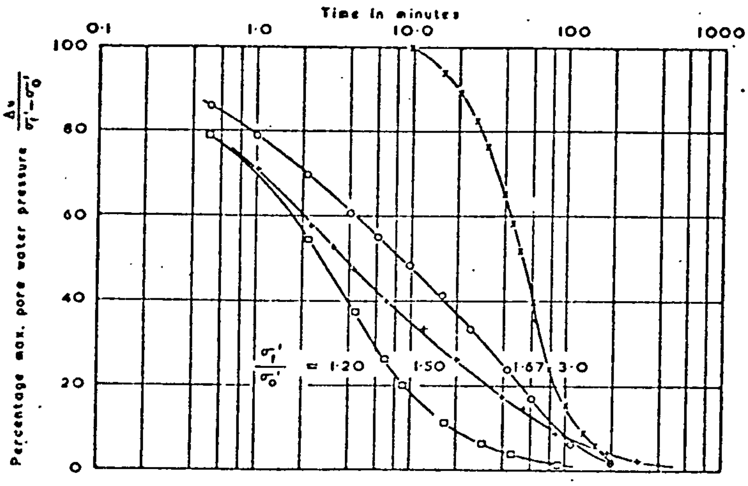
It is concluded that there will be significant differences in behaviour between A and C tests at very small times and strains, due to structural viscosity. For fully saturated soils (some back-pressure tests are virtually essential to confirm this), this is not believed to be of much practical importance provided the reasons are clearly understood, and qualitative allowance made.

Let us examine the literature in the light of these remarks. Berre and Iversen (1972), in a companion paper to Garlanger's, gave results of oedometer tests on an undisturbed Norwegian soft clay. Two load increments are reported for 4 thicknesses ranging from some 2 - 45 cms. All tests showed type II dissipation curves, except the thickest sample which peaked very slightly {this would agree with the viscous effect in pore pressure build up, mentioned above}. All were C-type tests with back-pressures of 45 p.s.i. (~ 310 kN/m^2), Garlanger showed graphs of excellent agreement between his theory and these results. Unfortunately, the parameters used are not indicated so it is not clear whether this refers merely to fitting methods, or whether results for, say, the thinnest samples could be used to satisfactorily predict behaviour of the thick samples. Analysis of the published results shows that $\mu_{max} \doteq 0.78$ for the first increment on the undisturbed soil. The second increment, applied one day later, gave $\mu_{max} \doteq 0.93$ ($\Delta p/p = 1.0$ in both cases). Sample thickness does not appear to influence these results. Now this is consistent with Bjerrum's time-lines model. The first increment includes considerable A-type, due to the extensive prior delayed consolidation. Hence the initial fall in pore pressures might be expected to be more rapid than for the second increment, so that the value of μ_{max} might be expected to be lower.

Taylor (1942) investigated the influence of load increment ratio ($\Delta p/p$) in some detail. He also mentioned the effect of delayed consolidation on subsequent behaviour. Although all the seeds of the present ideas are there, he does not appear to have directly linked these. Load Increment Ratio was mentioned briefly in section 6.1 and further comments can now be made. Consider any point (p_o, e_o) at which primary consolidation is sensibly complete. Apply a load increment Δp. As Δp increases, so the proportion of A-type consolidation, p_o to p_c, will decrease. Several workers have concluded that $\Delta p/p$ should be at least 1.0 to limit the influence of this phenomenon, and this is included in good practice for routine testing (see e.g. BS 1377). This may be seen as a useful "rule of thumb" but consideration of the stress-strain-time behaviour provides a more rational treatment. Now consider the above soil at effective stress, p_o, to which a constant value of Δp is applied. As delayed consolidation at p_o progresses, so e_o decreases and p_c increases (as discussed in sections 5.5.5 and 5.7). Hence the proportion subsequent A-type of delayed consolidation is found to be greater with resulting faster pore pressure dissipation. Thus $\Delta p/p$ and delayed consolidation effects are seen to be indirect results of the A-type consolidation.

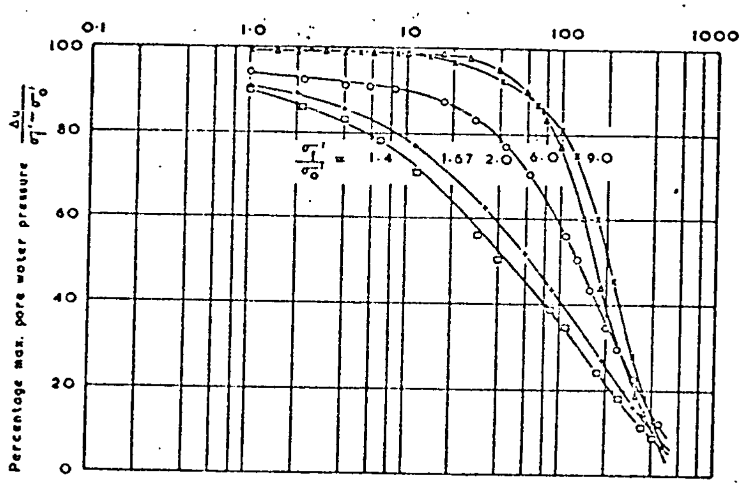
Simons and Beng (1969) report some investigations of load increment ratio, and of delayed consolidation (some results are reproduced in figs. 6.3 and 6.4). Two remoulded clays were tested (with back-pressures of some 40 p.s.i.) and the dissipation of pore pressure is clearly more rapid for the lower $\Delta p/p$. The first tests were performed immediately on setting up the samples, while 7 days were left before subsequent tests. The influence of this delayed consolidation will also be significant. The former tests are labelled A, and the latter B, on fig. 6.3, according to the present author's understanding of these results. It may be noted, in passing, that the tests were performed in Bishop and Skinner hydraulic oedometers similar to those used in the experiments of Chapter 5.

Barden and his colleagues at Manchester have reported a considerable amount of consolidation testing, done mainly in the Rowe cell. Pore pressures usually built up to equal the load increment against a closed drain. The influence of secondary consolidation has been considered, and some results from Barden (1969) have been included here as fig. 6.5 showing the effect of 20 days delayed consolidation on subsequent behaviour of kaolin and amorphous granular peat. The rapid initial fall in pore pressure is noted to be due to the increased resistance to compression of the soil skeleton. In terms of Barden's theories of consolidation (mentioned briefly in section 1.2) this would be handled by increased structural viscosity of the system. There would not appear, however, to be a readily applicable technique for predicting how the necessary parameters would be affected by this. It is interesting to note on fig. 6.5, that the influence of prior delayed consolidation becomes less for thicker samples (see below).



Symbol	σ_0' lb/sq.in	σ_1' lb/sq.in.	$\frac{\sigma_1'}{\sigma_0'}$	$\frac{\Delta u.max}{\sigma_1' - \sigma_0'}$	
□	75	90	1·2	97 %	B
+	30	45	1·5	98 %	B
o	45	75	1·67	90%	B
x	10	30	3·0	100%	A

Remoulded Tilbury Clay. Percentage maximum pore-water pressure plotted against time. 3 in. dia. hydraulic oedometer, back pressure 40 lb/sq. in.



Symbol	σ_0' lb/sq.in	σ_1' lb/sq.in	$\frac{\sigma_1'}{\sigma_0'}$	$\frac{\Delta u.max.}{\sigma_1' - \sigma_0'}$	
□	25	35	1·4	95 %	B
+	15	25	1·67	95 %	B ?
o	35	70	2·0	96%	B
▲	10	60	6·0	99%	A
x	10	90	9·0	99%	A

Remoulded Pisa Clay. Percentage maximum pore-water pressure plotted against time. 4 in. dia. hydraulic oedometer, back pressure 40 lb/sq. in.

Fig 6.3 Consolidation Test Results (After Simons & Beng, 1969) (below). Remoulded Tilbury Clay. Percentage maximum pore-water pressure plotted against average degree of consolidation

(below right). Remoulded Pisa Clay. Percentage maximum pore-water pressure plotted against average degree of consolidation

Classification tests

	Tilbury Clay	Pisa Clay
Liquid limit	91	72
Plastic limit	34	28
Plasticity index	57	44
Clay fraction	49	56
Activity	1·16	0·79
Organic content	6 to 10%	2·2%
Sulphate content	—	7·6%
Carbonate content	—	0·1%
Specific gravity	2·62	2·65

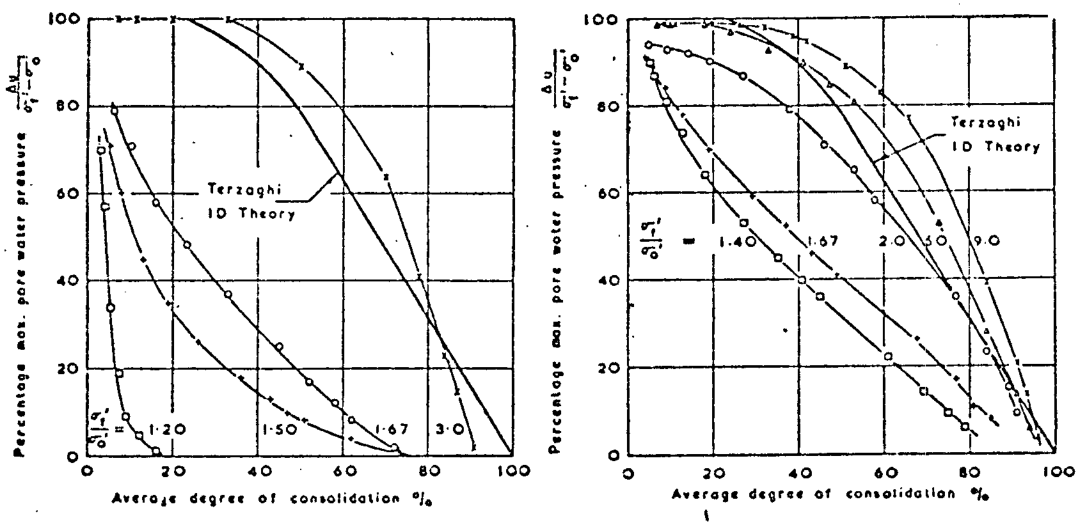
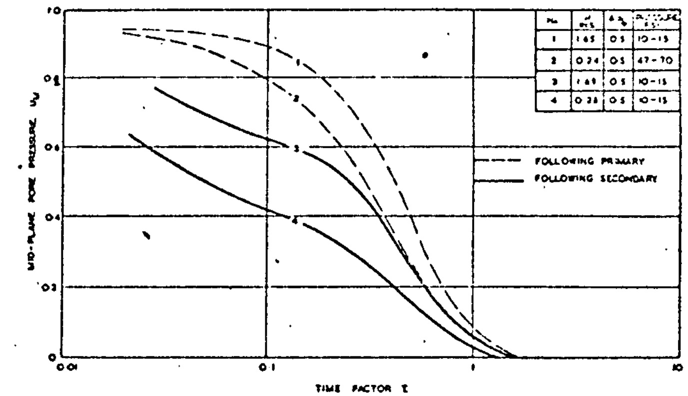
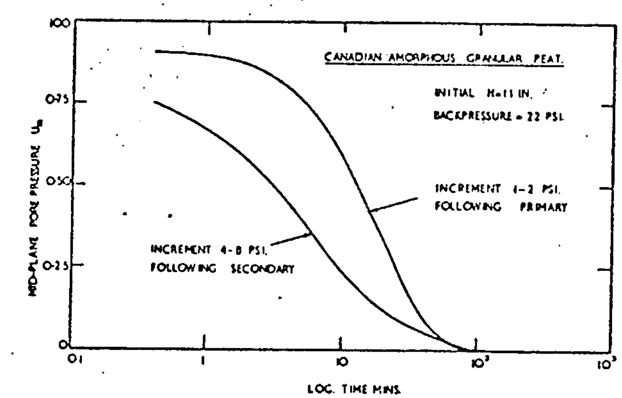


Fig 6.4 Consolidation test Results (After Simons & Beng 1969)



—MEASURED EFFECT OF PREVIOUS SECONDARY STAGE ON PORE PRESSURE DISSIPATION FOR KAOLIN



—MEASURED EFFECT OF PREVIOUS SECONDARY STAGE ON PORE PRESSURE DISSIPATION FOR AMORPHOUS GRANULAR PEAT

Fig 6.5 Consolidation Test Results (After Barden 1969)

Behaviour at field thicknesses is considerably less well-defined, since conditions usually depart from those carefully controlled in the laboratory. The author has examined a number of publications and particular mention should be made of work on embankments by the French Laboratoire Centrale des Ponts et Chaussées (1973), and the Transport and Road Research Laboratory, (Lewis et al., 1975). Field loading normally progresses over a significant period of time, but there is good evidence that the maximum pore pressures attained are significantly below those predicted by Terzaghi theory, suggesting a more rapid rate of early consolidation. Detailed analysis is not very profitable here, but it would seem that a suitable treatment of strain rate effects, such as that proposed by Garlanger, should lead to substantial improvements. The effects of time-dependant loading can be quite easily incorporated into the solution scheme, as reported in section 4.7 for the program SECON. The practical implications of such behaviour include the possibility of allowing substantially more rapid rates of construction of embankments where stability is a problem and pore pressures must be kept below certain bounds.

More will be said about the influence of thickness on behaviour in section 6.5, but it should be added here that the effect of A-type consolidation predicted by Garlanger's theory does become less for thicker deposits (though still significant for a 10 m layer - see fig. 6.13) and this is believed to be the case in practice.

The justification presented for A-type consolidation, then, is its ability to successfully handle observed behaviour. The best method of obtaining parameter 'a' at present, will be outlined in section 6.6. It is the author's belief that this approach may represent a useful step towards a link with the currently rather separated topic of shear strength. Similarities with elastic-plastic soil models begin to emerge (see section 6.4) and it may be that tests related to this will eventually realise the most satisfactory elastic function.

The following conclusions have been drawn.

1). For a fully saturated soil with incompressible particles, the pore pressure should rise to equal the applied load increment. Due to structural viscosity this takes a short time (~ 10 secs. for typical laboratory samples), but if the soil skeleton possesses considerable structural strength, with consequent rapid early consolidation, this short time may be sufficient for a maximum pore pressure substantially less than the applied load increment to be observed.

2). Partial saturation is more of a problem than is generally recognised because gases may come out of solution in an initially fully saturated soil,and some back-pressure testing is virtually essential before the influence of this may be discounted.

3). Observed behaviour is well in line with the Garlanger-Bjerrum model, which therefore seems to be a convenient technique for analysis. The linearised A-type stress-strain behaviour may be something of a simplification, but at the present state of knowledge refinement of this term is not justified.

4). The present model provides a valuable, rational framework for examination of the influence of prior delayed consolidation and load increment ratio on consolidation behaviour.

6.4 The Critical Pressure and t_L Method

The idea of some critical stress level, at which the gradient of the stress-strain relationship changes abruptly, has received considerable attention in recent years, particularly in relation to shear strength behaviour. Two of the most interesting approaches have been those at Norwegian Geotechnical Institute (of which the best summary is Berre and Bjerrum, 1973) and at Cambridge University (Burland, 1972, gives a good indication of the principles used). Although, to the author's knowledge, detailed comparison of these schemes has not been attempted, there are important similarities. Both envisage a soil matrix behaving approximately elastically up to some yield point, beyond which plastic behaviour obtains. The critical pressure discussed in the present work is simply the yield criterion under conditions of no lateral strain. Strength theory goes on to consider the yield locus in multi-dimensional stress space. This is outside the scope of the present study.

Before proceeding we must establish just what p_c represents. Bjerrum's last published ideas on this (1972 a, and b, 1973) were that an observed p_c would be the result of time-dependant skeletal behaviour. As sample thickness increased, so the retarding effect of the diffusion process would increase, average strain rate would be lower, and hence observed p_c lower. There is no suggestion here of a rigid bond independent of strain rate; p_c is purely an indirect result of what should more correctly be viewed as stress-strain-time behaviour. (The approach to be developed below is consistent with this, but it will be seen that Garlanger introduced a complication in his presentation). Lowe (1974), has suggested that "the entire conventional concept of preconsolidation should be abandoned", in favour of a strain rate approach. Taylor (1942), although presenting the concept of bond, seems to favour all plastic resistance related to strain rate (see section 6.1). This is certainly the interpretation taken by Barden (1968), who concludes that there is little evidence for a rigid bond which only breaks at some critical stress, although further information on this important point is required. Barden, though in a rather different theoretical model from Bjerrum's, is essentially in agreement on p_c being a result of the soil's viscosity. Burland (1972) shows a series of yield loci, which are affected by strain rate. He does, however, quote some evidence from Mitchell for an abrupt yield point which appears to be considered a fundamental soil property. This refers to an undisturbed Leda clay, which is a sensitive cemented deposit.

A number of experimental projects have been reported using constant rate of strain (C.R.S.) or constant rate of loading (C.R.L.) oedometer tests {see e.g. Crawford (1964), Jarrett (1967), Lowe et al. (1969), Davies (1975)}. The observed value of p_c again increases with rate of strain. Apart from some difficulties in interpretation of experimental data, the major problem with such tests is that they refer to constant average rate of strain, or loading, whereas the actual effective stress profile may vary significantly with depth. Such results, however, do tend to confirm the dependance of p_c on strain rate. It may be added that Davies (op. cit), for Undisturbed Avonmouth Clay, found evidence of structural breakdown associated with p_c.

The approach adopted in the present study, then, is to consider p_c as an indirect result of the stress-strain-time behaviour of the soil skeleton. It is recognised that there may be structural bonding, cementation and suchlike in some undisturbed clays, although present evidence suggests they are somewhat unusual. These must be excluded from present consideration.

Garlanger's interpretation of Bjerrum's model, defines an "instant line" through the point (p_c, e_c). As noted in Chapter 4, this serves two functions. The first is to position the grid of time-lines on the e-p plot. This could be done through any chosen point. The second function is to change the gradient of the stress-strain relationship, independant of time, from a, to b. The inclusion of this raises a number of problems in use of the model. How is p_c determined? How does it vary with sample thickness? It is now believed that such complications are an unnecessary result of the misapplication of stress-strain-time function in Garlanger's model. The time-lines of gradient b simply represent constant rates of skeletal strain. They should not, therefore, be governed by a stress-strain function, but by the consistent stress-strain-time behaviour, which in Garlanger's model requires correct formulation of independant but simultaneous stress-strain and strain-time functions. It is, therefore, proposed here that the idea of a change in gradient from 'a' to 'b' should be abandoned, and the skeletal behaviour may be adequately described by the two equations

$$\frac{\partial e}{\partial p} = \frac{ae}{p} \tag{6.16}$$

$$-\left(\frac{\partial e}{\partial t}\right)_c = c\left(\frac{p}{p_c}\right)^{-b}\left\{\frac{e}{t+t_i}\right\} = \frac{ce}{t_i}\left(\frac{p}{p_c}\right)^{b/c}\left(\frac{e}{e_c}\right)^{1/c} \tag{6.17}$$

We still have to position the time-lines grid correctly. The method is best illustrated by reference to the typical Master Diagram for Grangemouth clay (fig. 6.6). The grid of lines may be extended back as far as desired, assuming a linear log e - log t relationship at constant effective stress. It is thus possible to define a <u>limit time line</u>, t_L, through the point where the 'a' line intersects $p = p_f$. Garlanger's method may then be used for solution, setting

$p_c = p_f$ and giving t_L in place of t_i.

The variation of these results from Garlanger's published method will depend on the p_c value used by the latter. A soil element near the drainage boundary would initially experience a high rate of strain, tending to a stress-strain state above the "instant-line". Garlanger's B-type consolidation would prevent this, and force the soil along the instant line; the t_L approach would permit the appropriate strain rate to be attained. Thus the schemes only differ for elements wishing to cross the "instant line", so the variation will increase as more elements try to cross this, i.e. the larger the value taken for t_i, or the higher the average strain rate.

In Chapter 5 a value of t_i = 1 minute was used for Garlanger analysis. Examination of the average log e - log p plots for A, C, and E-tests shows they lay somewhat below this line. Thus only small sections of the profiles near the drain, would have been restrained by the b function. Differences from the t_L analyses should be small. Some typical comparisons of analysis are shown on fig. 6.7 - 6.9. Parameters used were identical, apart from p_c and t_i. For the t_L method, settlement progresses slightly less rapidly with time, while pore pressures dissipate a little faster. These combine to give a noticably more rapid fall in μ against strain (% C) (fig. 6.7). Many similar analyses have confirmed such behaviour.

Besides considering the previous analyses, the experimental results were also compared with the t_L method. Generally it was found that better agreement was obtained between observation and the new analysis, although this was not sufficiently dramatic to require much modification of the conclusions drawn in Chapter 5. It was suggested that the linear log e - log p gradient 'a' is probably an oversimplification. Removal of the p_c/b-type constraint leads to better agreement using this linear term, Fig 6.6 GRANGEMOUTH Master Time Lines Diagram

Showing t_L Line

Increment S after 1 Day Delayed Consolidation

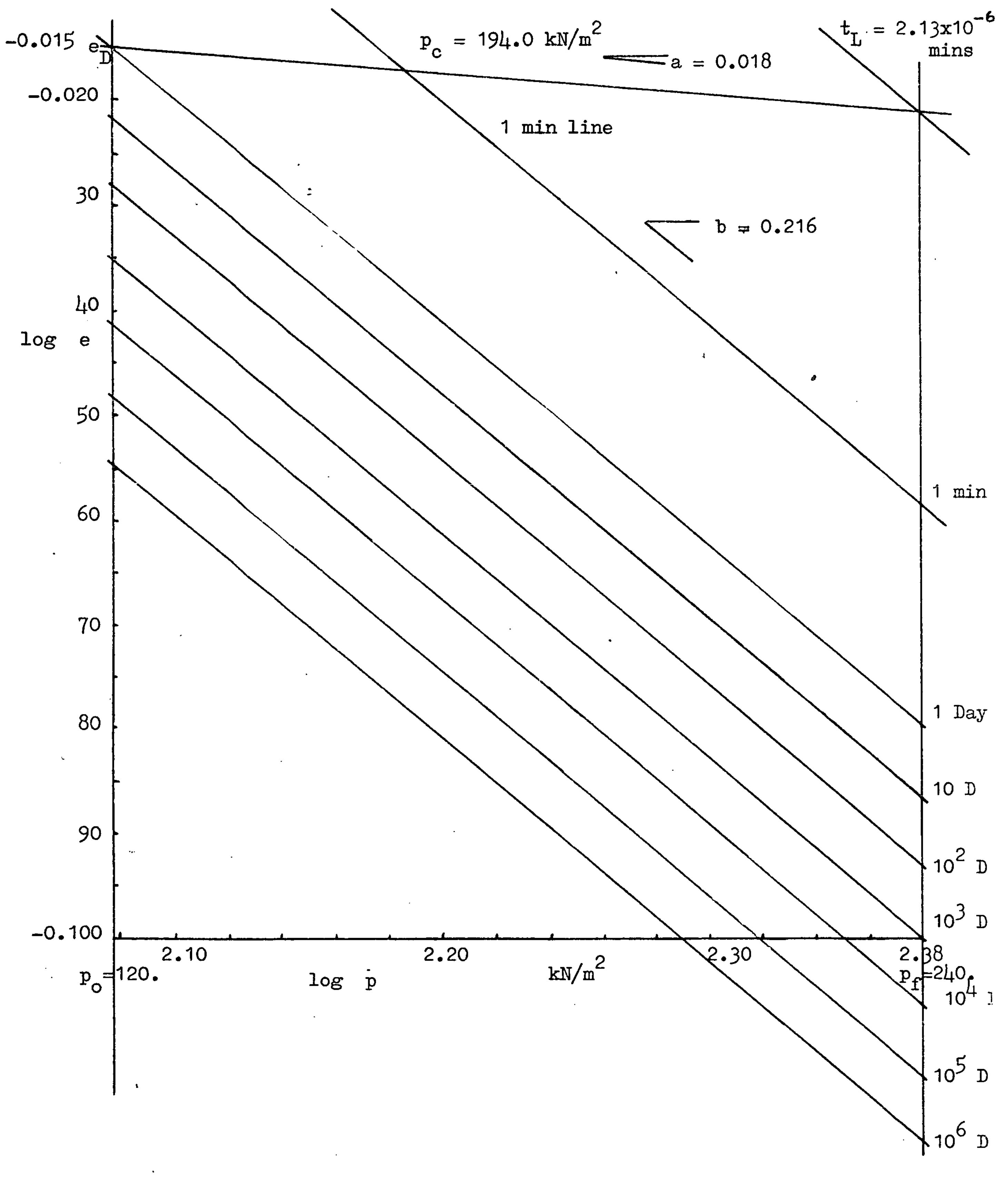
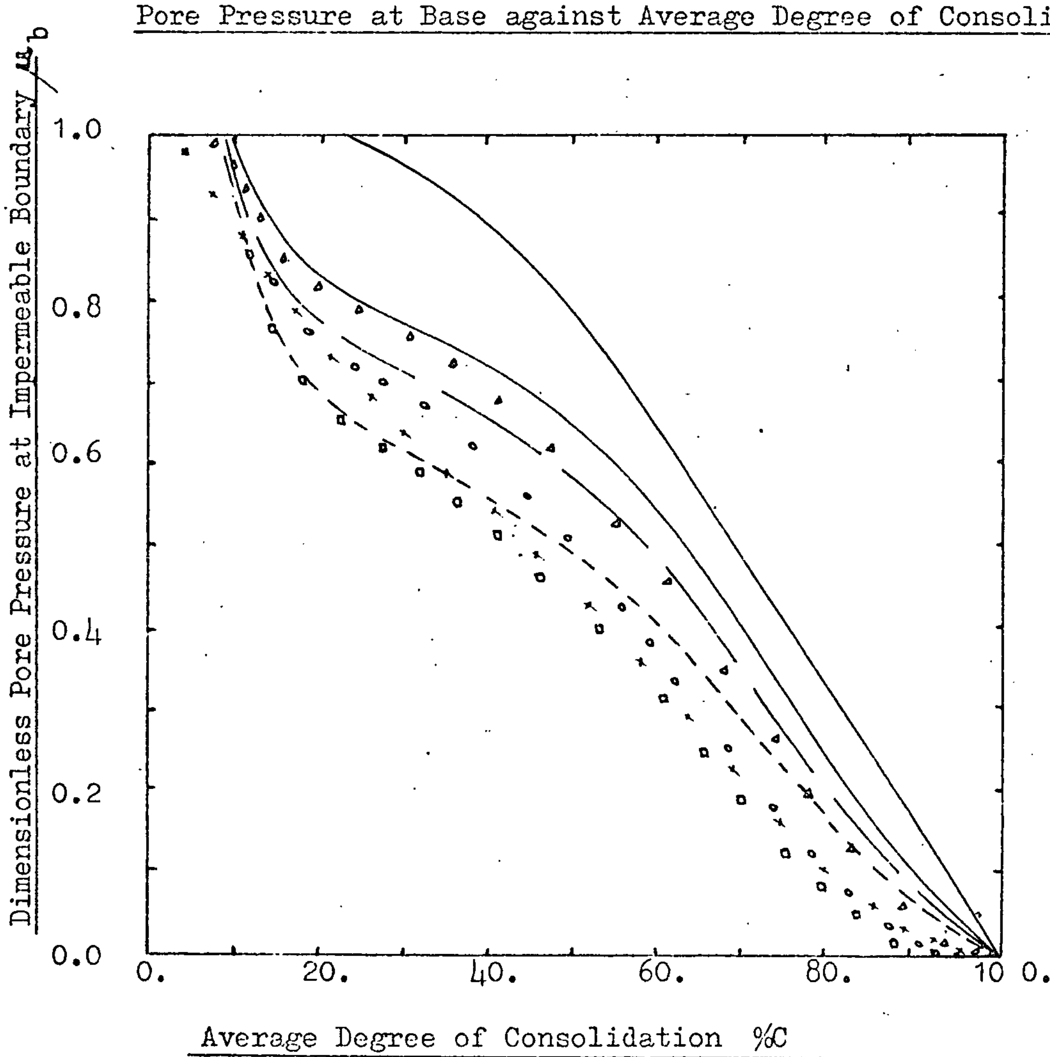


Fig 6.7 Comparison of Garlanger and t_L Analyses

GRANGEMOUTH Increment S 120.- 240. kN/m^2

Pore Pressure at Base against Average Degree of Consolidation



Garlanger ti = 1 min	Prior DC	t_L Analysis $p_c=p_f=240 kN/m^2$	t_L = (mins)
————	DAY	▲ ▲ ▲	2.13×10^{-6}
—— ——	WEEK	o o o	1.85×10^{-5}
- - - - - - -	MONTH	□ □ □	1.46×10^{-4}

(see fig 5.22 for parameters) (see fig 6.6)

× × × Experimental Result - E test after 1 week Delayed Consolidation.

Fig 6.8 Comparison of Garlanger and t_L Analyses

Settlement - Log time Graph

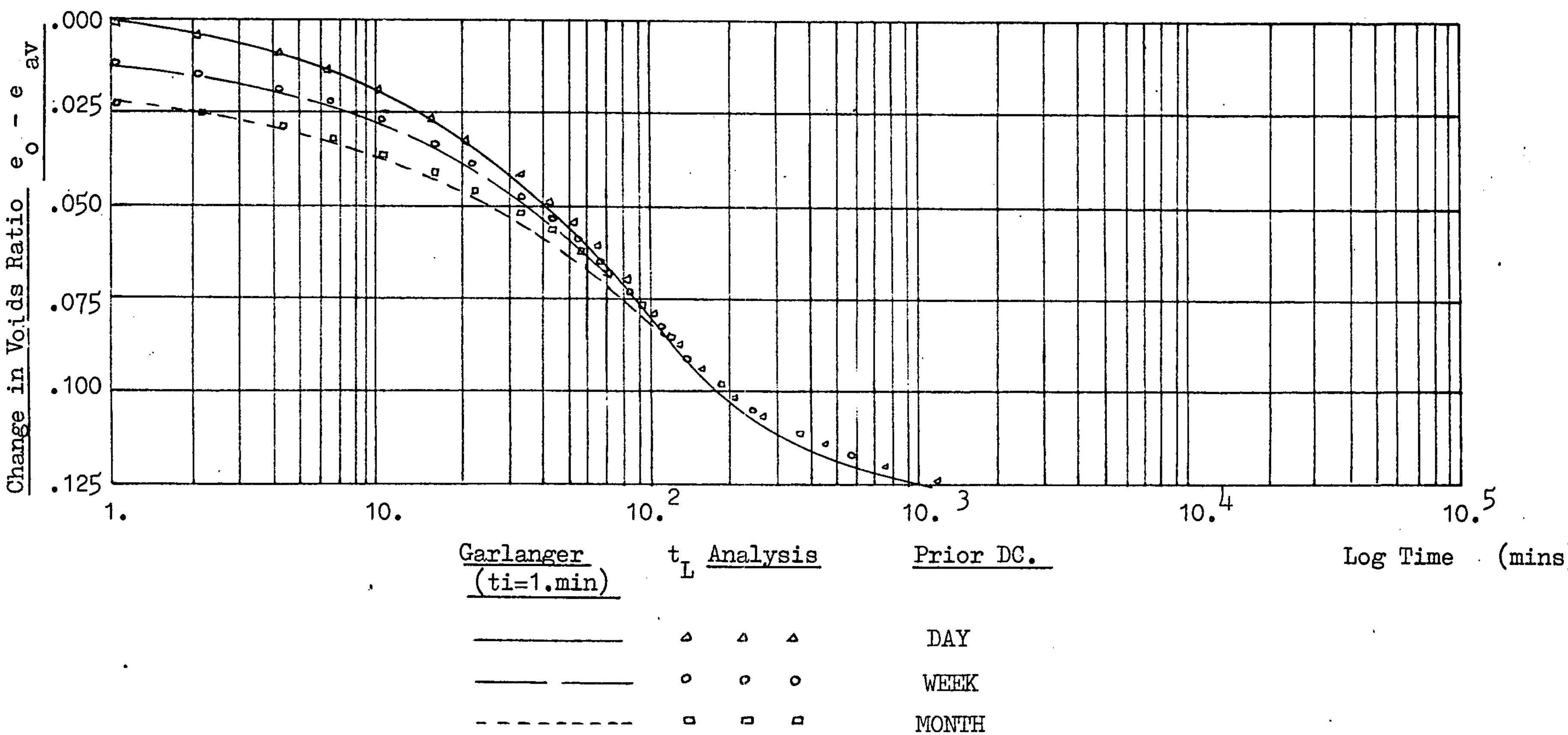
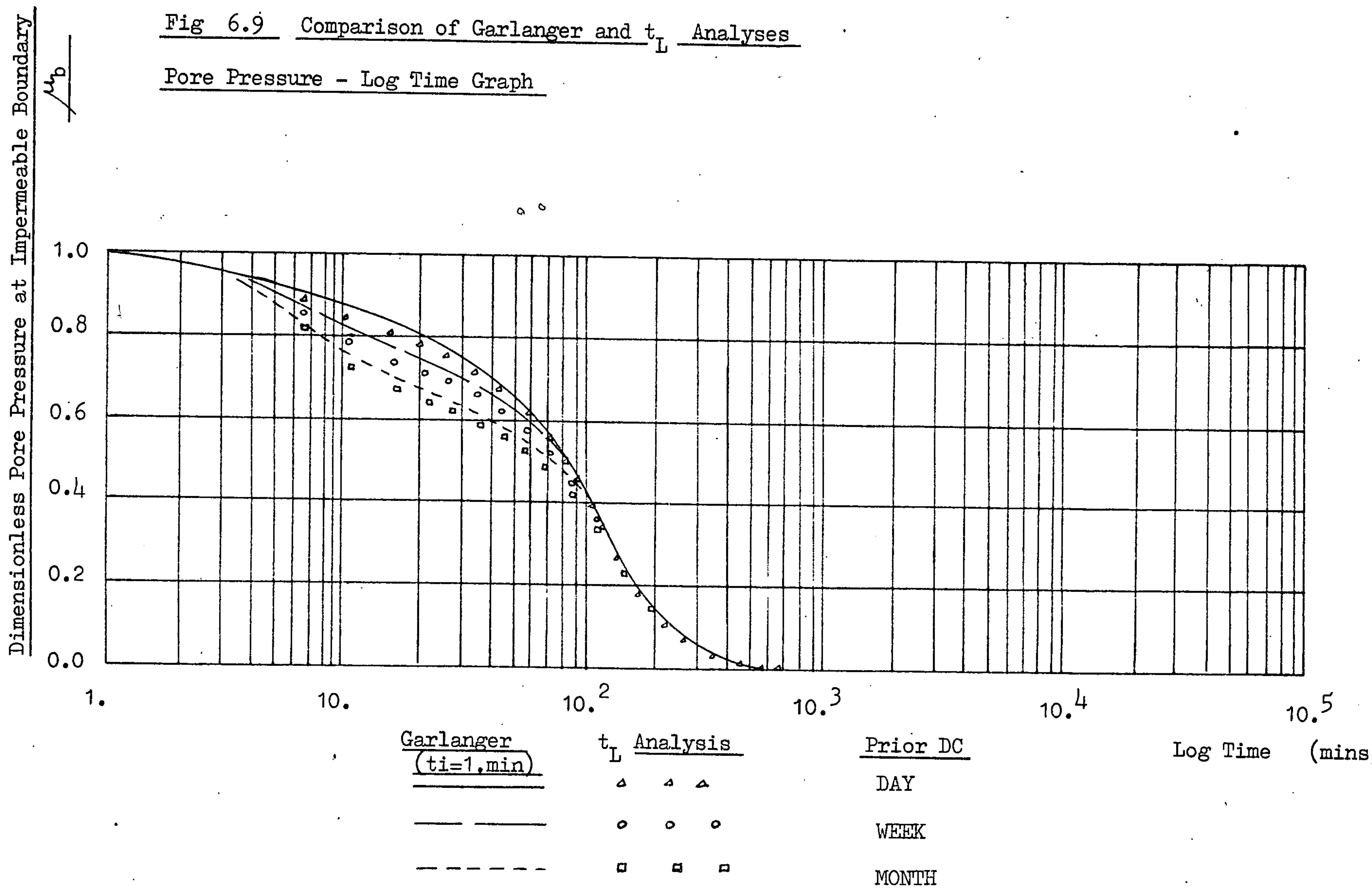


Fig 6.9 Comparison of Garlanger and t_L Analyses

Pore Pressure - Log Time Graph



but it is probably still the case that this represents something of an approximation. As will be discussed in Chapter 7, however, there would seem to be little justification in introducing further complexity into the model at present. This evidence substantiates the proposed t_L modification to Garlanger analysis.

The effect of sample thickness is to be examined in section 6.5. Here we simply note that t_L analyses were run for many test cases up to 10 m in thickness, and compared with Garlanger analysis using a variety of t_i values. Providing t_i was taken considerably lower than the lowest time-line reached by the average log e - log p curve, differences were slight. They increased with increasing t_i, becoming quite unacceptable once this average curve crossed t_i. The "allowable" t_i value thus varies with thickness. For the 1 metre layer of fig. 6.10 (section 6.5), 1 day was about the highest acceptable t_i. For the 10 metre layer t_i = 100 days was allowable, while for a thin (20 mm), laboratory sample, t_i = 1 min led to discernable differences from t_L analysis, as already noted.

The above comparisons serve only to illustrate the influence of t_L hypothesis on the established method. The author believes the t_L method is preferable, both theoretically and for use in practice, and there is no further need to consider a t_i value. We notice that the t_L method is consistent with Bjerrum's ideas, without requiring any assumption concerning p_c or t_i. We have thus dispensed with the major problem encountered in Chapter 5, where the B-tests were introduced in an attempt to evaluate p_c, and similarly the problem of interpretation of these results is removed. {It should be added that for the present the B-tests are still required to determine 'a' - see section 6.3.} Apart from simplifying laboratory testing, and the theoretical treatment, this offers a rational method of predicting behaviour of thick deposits - all the parameters may be determined for a thin sample, and remain constant.

The numerical solution of the t_L approach is virtually identical to Garlanger's method. Any of the programs GARCON, SECON or CONGO may be used (see Chapter 4). p_f should replace p_c in the data file, and t_L replaces t_i. A note on the t_L scheme is included in the User Manual. Some simplification results for the t_L scheme, but the effect of this on the programs is minimal, so that they have not been altered here. For thin samples (10 - 20 mms) strain rates near the drainage boundary are initially very high, and minor errors were encountered in SECON. Generally, however, the numerical techniques were more successful without the discontinuity at p_c, and the self-checking scheme CONGO was able to take rather larger steps, reducing c.p.u. time to around ½ minute (from 2 minutes). GARCON proved reasonably satisfactory, though as slow as ever. Numerical errors of up to 1.0% were found at very early times.

Concluding Remarks

1. The bulk of evidence suggests an apparent critical pressure, p_c, results from the stress-strain-time behaviour of the skeleton. The present treatment is consistent with this.

{1(a) A small but significant number of undisturbed clays exhibit "structural breakdown" around p_c. These are outside the scope of the present study.}

2. The problem of instant line and p_c, has been resolved by the hypothesis that such effects are already adequately treated by the simultaneous stress-strain and strain-time functions. B-type consolidation is omitted - it was thought to impose an unacceptable restriction on any element of soil having a strain rate greater than that corresponding to the instant line.

3. The time lines grid can be correctly positioned by a limit time line t_L, through the intersection of 'a' and p_f lines.
4. Comparison of this t_L approach with the experimental data of Chapter 5, and Garlanger analysis thereof, suggests the present hypothesis is well-justified, and theoretically it is more consistent with Bjerrum's ideas than was the t_i approach it supercedes.
5. The t_L method offers the significant practical advantage that all parameters may be determined from laboratory tests, and remain constant, so predictions may be made for thick field deposits.

6.5 Thickness of Soil Layer

Of major interest in the present study is the manner in which the thickness of soil layer affects consolidation. The behaviour predicted by Garlanger theory (using the t_L modification - see below) is considered and compared with known behaviour of field deposits and with the predictions of Terzaghi theory.

We begin by studying the Grangemouth soil used for the experimental work of Chapter 5. This was remoulded, and there may be important differences from the undisturbed soil. However, for the present we shall simply compare the known behaviour of a thin laboratory sample, with that predicted for a hypothetical deposit which is identical in every respect except thickness. This is directly analogous to predicting behaviour of a real field deposit from tests on representative undisturbed samples.

The stress-strain-time behaviour of the skeleton is defined by the master diagram of Chapter 5. An assumption must be made concerning the delayed consolidation function, and for the present demonstration a linear log e - log t relationship is used, i.e. c remains constant (see section 6.2). Hence the time lines may be drawn for large times, and the resulting grid is shown on fig. 6.10. Referring to this, we shall examine behaviour of a soil which has undergone 3,000 years delayed consolidation under constant effective stress of 120 kN/m^2, i.e. it is at the state (p_o, e_o) as shown. Three thicknesses are compared; a typical laboratory sample, 0.020 m (THIN); a medium field deposit, 1.0 m (MED); and a thick field deposit, 10.0 m (THICK). An incremental load of 120 kN/m^2 is applied instantaneously. Results are presented in a number of ways on figs. 6.10 - 6.16. The t_L method of analysis was used, as presented in section 6.4, where it was indicated that this is the most satisfactory approach for prediction for thick deposits, and comments were made on the comparison with other values of t_i in Garlanger's published scheme. The t_L method takes identical parameters for any value of layer thickness. Data is given on fig. 6.10. The value of the limit time line, t_L, was found to be 2.36 minutes.

The effects of sample thickness are best considered in terms of the various graphs. The plotsof average log e - log p are included in fig. 6.10. With increasing thickness, the rate of diffusion of pore fluid decreases, leading to a considerably less curved stress-strain plot. For very thick layers, the average strain rate would remain fairly constant. Agreement with Terzaghi theory might therefore be expected to improve for thicker layers, since the behaviour more closely approximates a linear stress-strain relationship.

It will be seen shortly that this is indeed the case. However, comparison with Terzaghi theory becomes a little confusing because of the signficance of definitions chosen. It is better to begin by establishing behaviour with real time. Fig. 6.11 shows settlement (as change in voids ratio) against real time. Fortuitously, the total change in voids ratio over 3,000 years = 0.125, so a strain scale (%C) has been added with 100% defined at 3,000 years - the loading age of the deposit.

Consolidation settlement progresses less rapidly with increasing thickness, and we may also note how according to conventional terminology, the proportion of "secondary" to "primary" consolidation decreases greatly for thicker samples. These predictions are well in line with what the author considers to be the best current knowledge of soil behaviour. Indeed, an e - log t graph of the form of fig. 6.11 was suggested by Taylor as long ago as 1942 (his fig. 64). He stated with considerable conviction that "for like increment durations there is more primary compression and less secondary compression in thick samples than there is in thin samples".

The actual stress-strain-time behaviour of the skeleton is obscured by the retarding effect of diffusion of pore fluid during the primary process. As this is completed the delayed consolidation relationship is displayed. Since the soil skeleton is identical for all thicknesses here, we should expect results to follow the same curve after primary consolidation. This is found to be the case. (It is actually a linear log e - log t relationship, so is not exactly linear on the e - log t plot, but, as previously noted, differences are imperceptible over the range of e of practical interest.)

Little need be said of pore pressure behaviour against real time (Fig. 6.12). Dissipation clearly proceeds far more rapidly for thin samples. The curves are of rather different shapes and differ from those predicted by Terzaghi theory. This is more clearly seen in figs. 6.13 and 6.15.

Comparing settlement predictions with those of Terzaghi theory, we note that the latter considers only primary consolidation. Defining Δe as 0.125 for 3,000 years, for all thicknesses, will clearly be nonsense. The only reasonable treatment requires <u>$\Delta e'$ to vary with sample thickness</u>. Average strain (%C) is then defined as $(e_o - e)_{av}/\Delta e'$. According to basic Terzaghi theory solutions, at dimensionless time T = 1.0, Fig 6.10 Effect of Sample Thickness on Behaviour of Thick Deposit after Significant Delayed Consolidation (Predictions from Garlanger Theory, using t_L Line)

Time Lines Diagram - Showing Average Stress - Strain Relationships

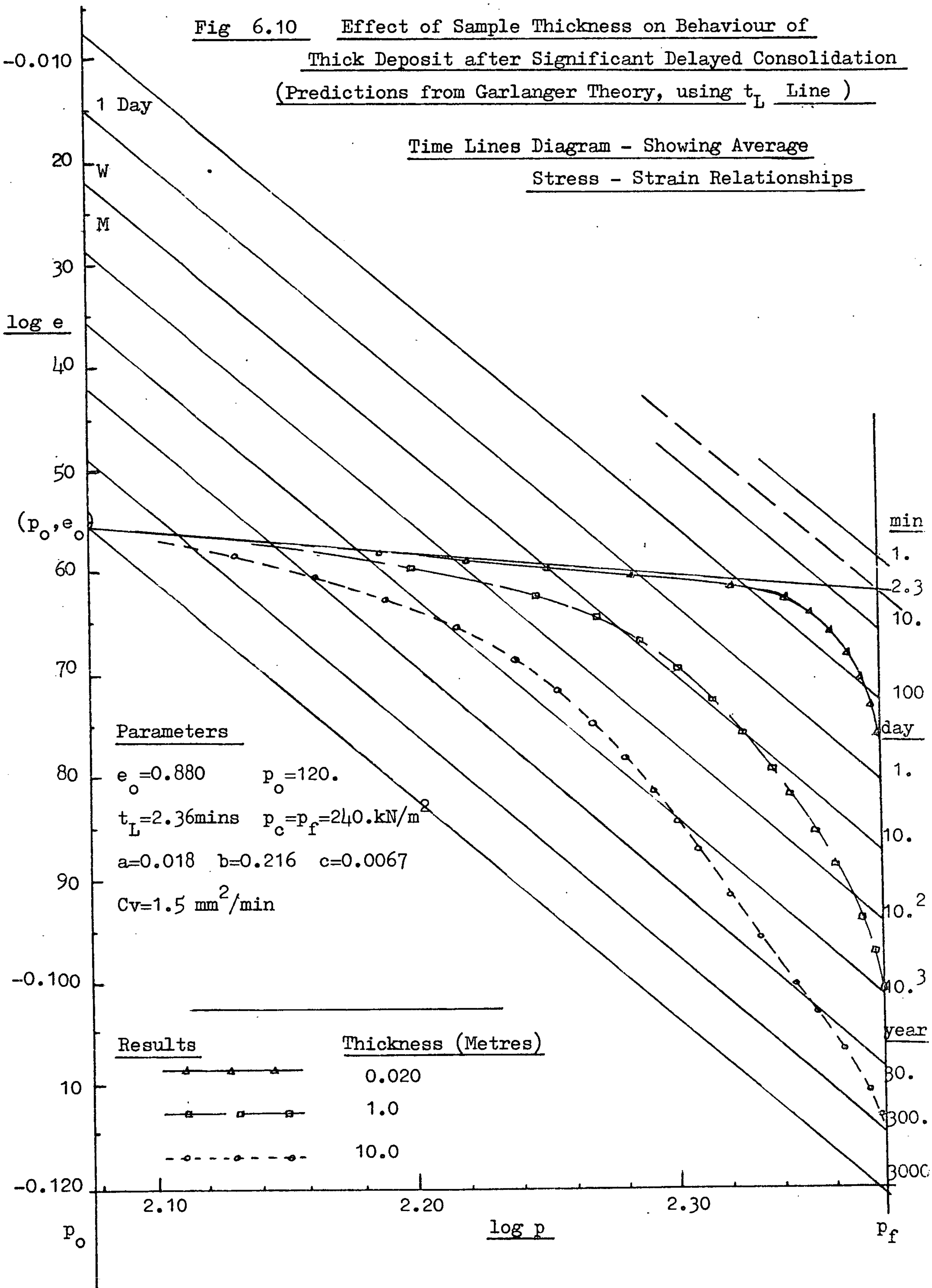


Fig 6.11 Effect of Thickness on Garlanger/t_L Analysis

Settlement - Log time Graph

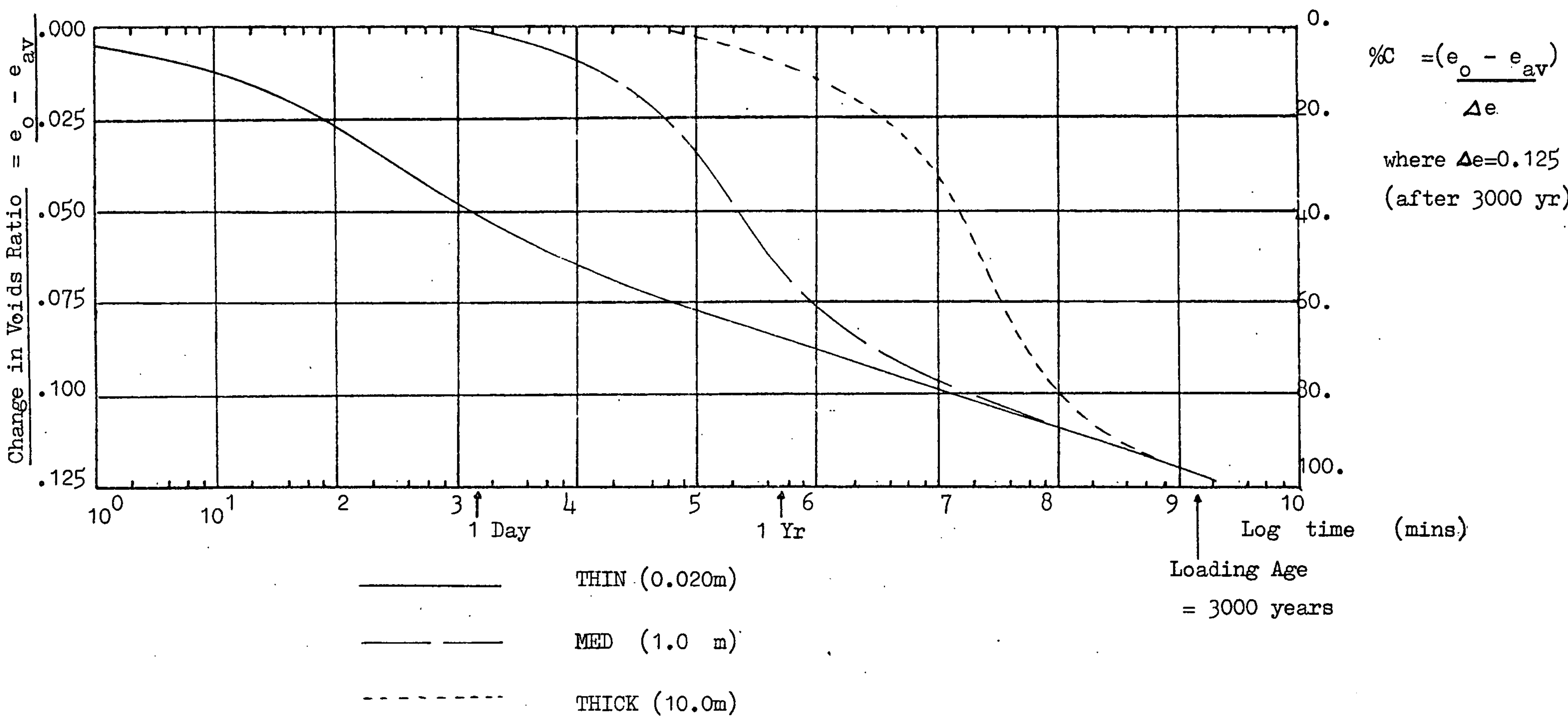


Fig 6.12 Effect of Thickness on Garlanger/t_L Analysis.

Pore Pressure - Log Time Graph

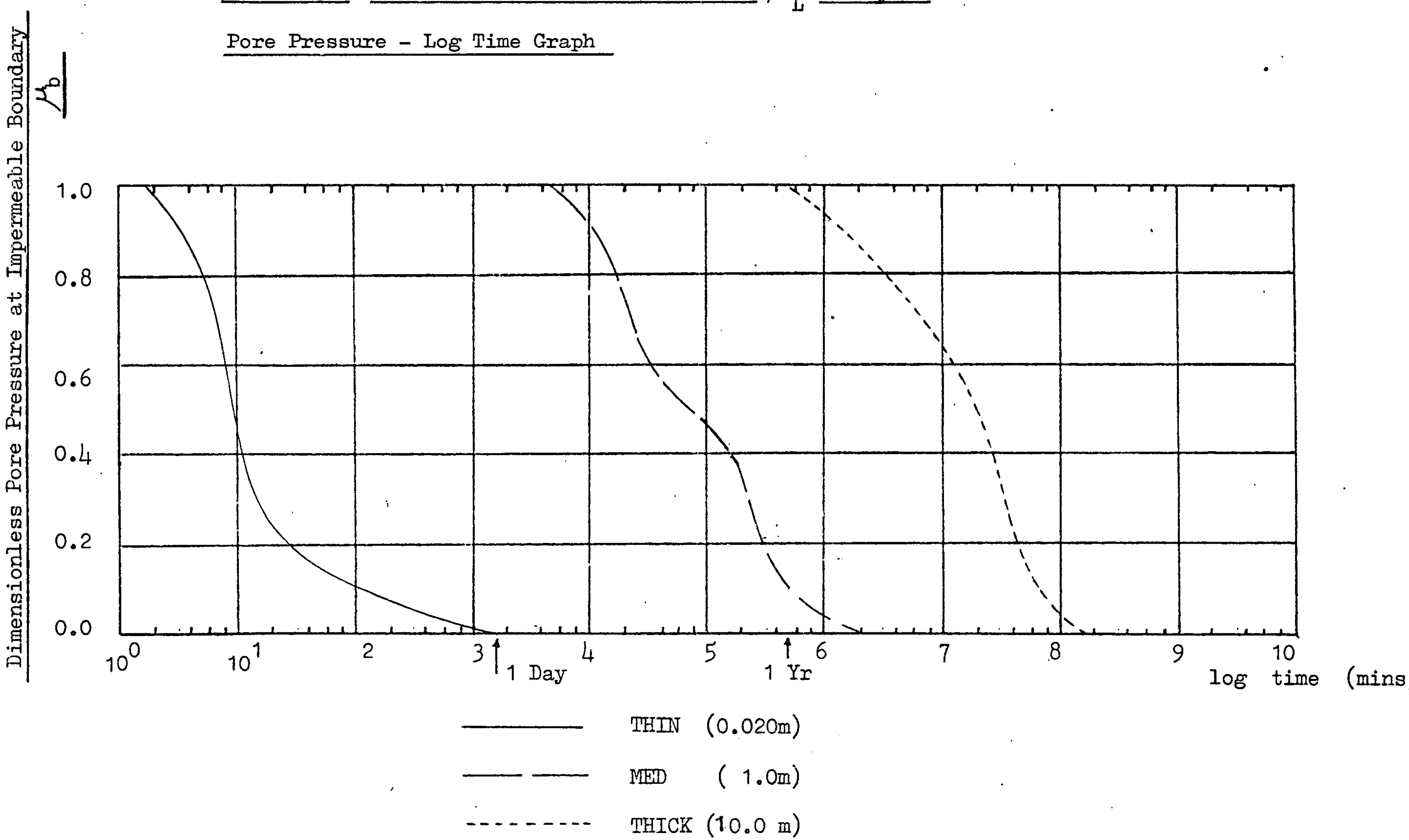
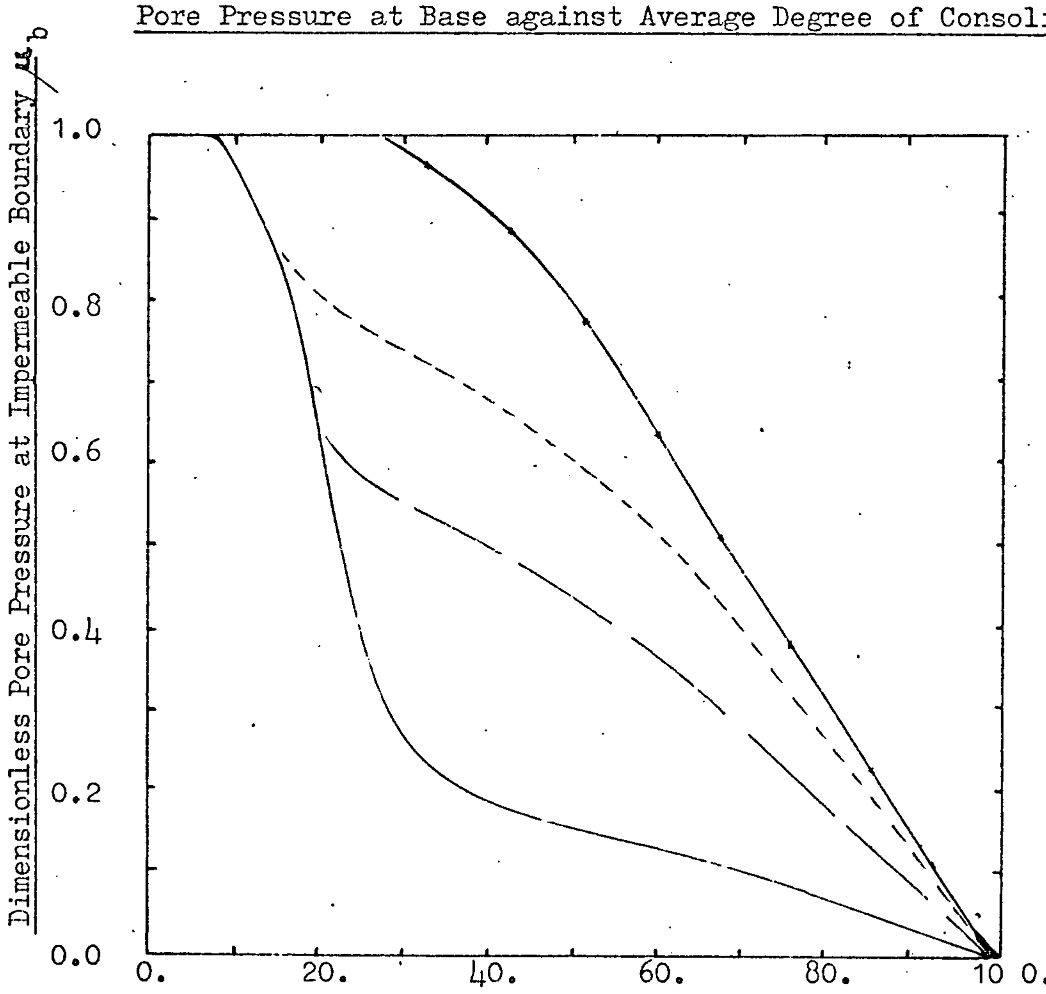


Fig 6.13 Effect of Thickness on Garlanger/t_L Analysis

GRANGEMOUTH

Pore Pressure at Base against Average Degree of Consolidation



Average Degree of Consolidation %C'

—•—•—	Terzaghi Theory
	Garlanger/t_L
————	THIN (0.020m)
—— ——	MED (1.0 m)
- - - - -	THICK (10.0m)

%C' defined as

$(e_o - e_{av}) / \Delta e'$

where $\Delta e'$ is total primary consolidation (see text)

Fig 6.14 Effect of Thickness of Garlanger/t_L Analysis

Settlement - Log Dimensionless Time

(See text for definition of %C')

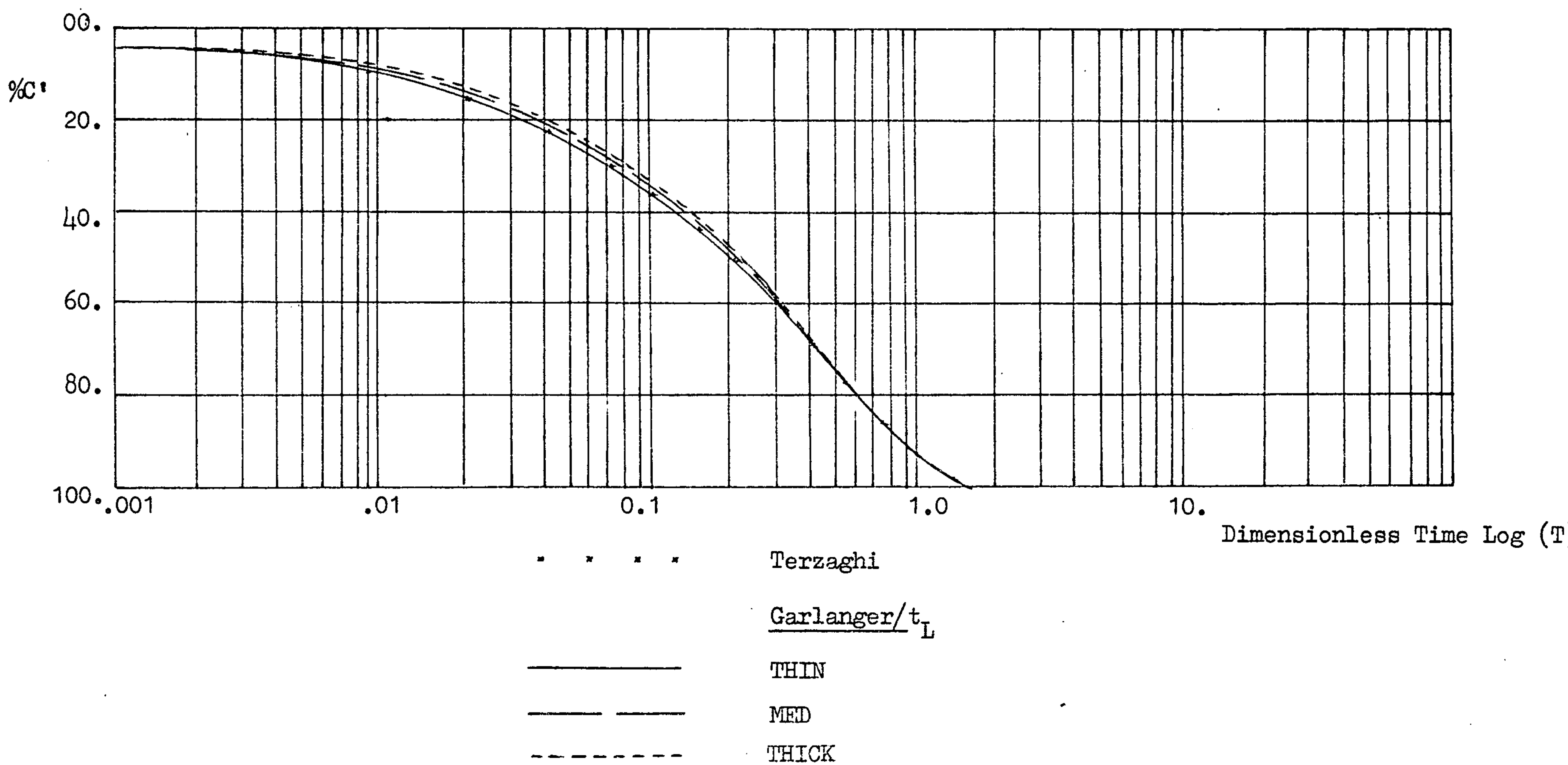


Fig 6.15 Effect of Thickness on Garlanger/t_L Analysis

Pore Pressure - Log Dimensionless Time

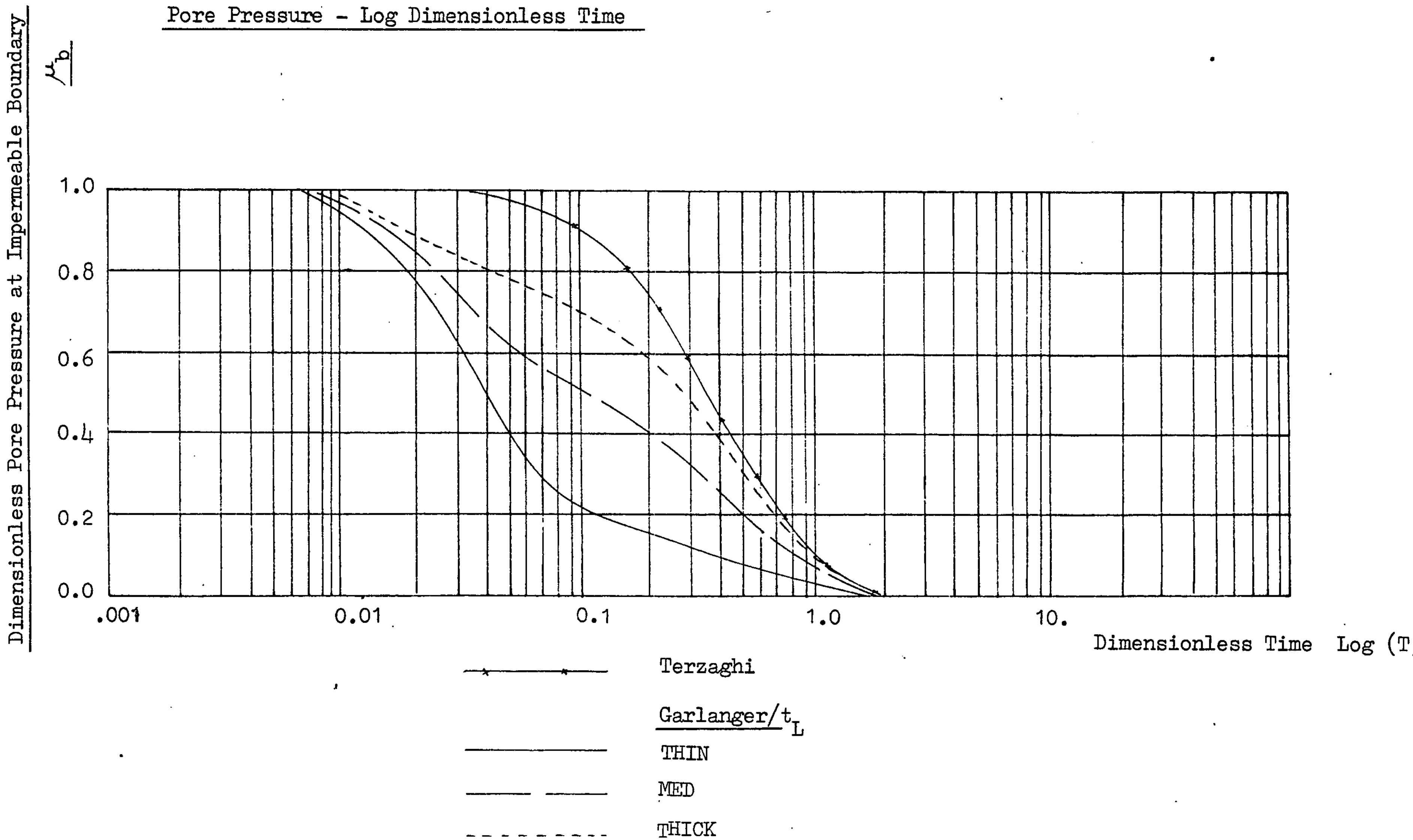
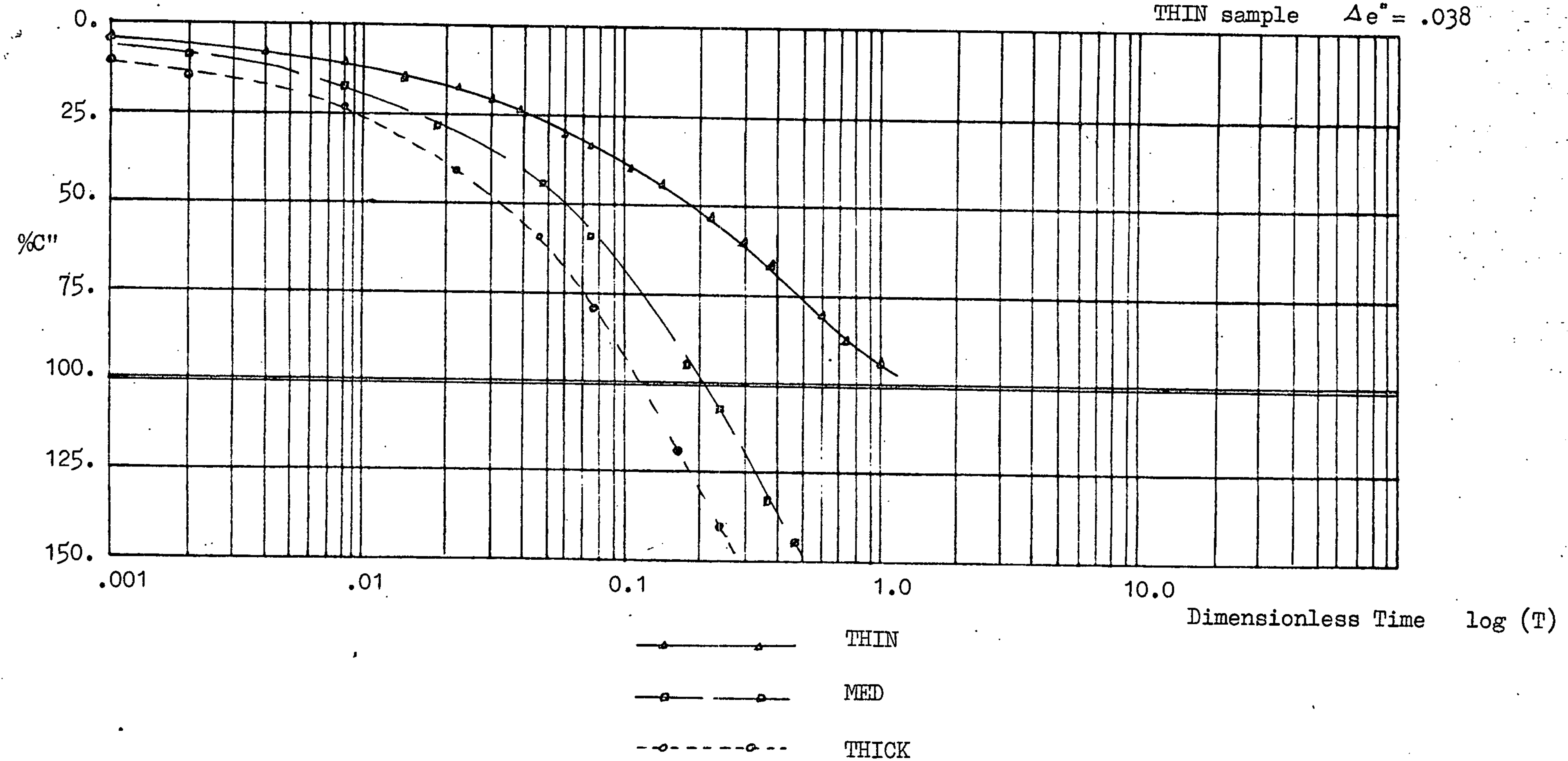


Fig 6.16 Effect of Thickness on Garlanger/t_L Analysis

Settlement - Log Dimensionless Time

%C" defined as constant from 100% primary Consolidation of THIN sample $\Delta e^{"} = .038$



the average degree of primary consolidation = 93%. The t_L analysis results have therefore been compared by defining %C' as 93% at T = 1.0, which involves multiplying all %C solutions by $93./C_1$ where C_1 is the original value of %C at T = 1.0. The converted results are compared on figs. 6.13 - 6.15.

Consider first dimensionless pore pressure against average strain (%C') (fig. 6.13). The departure from Terzaghi theory increases greatly for thin samples, due to the effect of strain rate. This raises the interesting paradox that better undisturbed samples, preserving the soil's natural state of slight overconsolidation, would give worse prediction of field behaviour, unless the significance of this strain rate effect is properly appreciated.

Pore pressure against dimensionless time (fig. 6.15) may be dealt with quite briefly. Garlanger/t_L analysis, handling the influence of strain rate on consolidation, predicts pore pressures dissipating more rapidly than indicated by Terzaghi theory; increasingly so for thinner samples (and also for increasing overconsolidation, as noted in Chapter 5).

It is difficult to be succinct about the strain - log T graph (fig. 6.14). The present definition of %C' has made the curves for Terzaghi theory and all thicknesses for Garlanger/t_L virtually coincidental. This would tie in with the fact that all present solutions to the equation

$$Cv \frac{\partial^2 e}{\partial z^2} = \frac{\partial e}{\partial t} \qquad (6.12 \text{ bis.})$$

which was discussed in section 6.1. Cv has been taken as constant and identical for all the solutions, and this equation holds for strain behaviour regardless of the stress-strain-time relationships for the skeleton.

We note that slight differences are found in fig. 6.14; strains at earlier times being a little less rapid for the thicker samples. The definitive numerical solutions by program CONGO, were used. These should be accurate to at least 3 significant figures, so numerical errors are imperceptible. It is believed that the present differences are due to departure from the "small strains" theory. The differences come to light because results have been compared after "correcting" for the different Δe values applying for the different thicknesses. Here we simply note that the effect is slight in comparison to the "unknowns" remaining in comparison of real behaviour and model predictions. The theoretical significance of the assumption of small strains was mentioned in section 6.1.

It would appear, then, that the strain predictions of Garlanger's theory, for the primary process, or up to around T = 1.0, can be adequately handled by Terzaghi theory, providing the correct value of Δe is used. It is frequently assumed, however, that Δe remains constant for any layer thickness. This is not the case where the soil skeleton behaves in time dependant manner. {The effect of defining Δe for a thin deposit and using this value for thicker layers is illustrated in fig. 6.16. Consolidation appears to proceed far more rapidly. Brief comments will be made shortly.}

Use of Terzaghi theory, when only strain predictions are required, may thus offer a simple solution technique without requiring use of a computer program. The following approach is suggested.

i) Cv is obtained from conventional laboratory tests, fitting <u>Settlement</u> data.

ii) Using the expression $Cv = H^2T/t$, the real time $t_{1.0}$ corresponding to T = 1.0, may be determined for the desired thickness H.

iii) From a time-lines diagram the value of $\Delta e_{1.0}$ corresponding to $t_{1.0}$ at $p = p_f$, may be determined.

iv) This, from Terzaghi theory, represents 93% primary consolidation. Hence the full Terzaghi solution may be evaluated for the correct primary settlement $\Delta e = \Delta e_{1.0}/0.93$.

It must be noted that this is an approximate procedure for primary consolidation strain only. For full analysis of the stress-strain-time behaviour of the soil numerical solution techniques are essential.

In the present analysis the values of p_o and e_o are taken as constant. While this is probably acceptable for a thin sample, the overburden in the field will vary with depth, and hence e_o will vary. Present remarks are confined to strict one dimensional theory, where any increment of loading at the surface, conveys a constant stress increment to all depths of the soil profile. In Terzaghi theory, taking a linear e - p relationship, it does not matter what absolute values p_o and e_o apply, providing the correct incremental values are used. Varying (p_o, e_o) with depth is adequately handled by choice of any arbitrary constant values , from which increments are calculated, for the whole layer. Treatment of e - log p, or log e - log p relationships is more difficult. Consider a soil element under 10 kN/m^2 overburden. A load increment of 90 kN/m^2 represents a doubling of log p. For a deeper element, under 100 kN/m^2 overburden, the same increment increases log p by only 0.28. This simple example illustrates that the greater the existing overburden, the less the influence of a given load increment on e (or log e), when this is a function of log p. An excellent example of the practical implications of this is given by Bjerrum (1967) comparing settlements of two buildings, one with soft clay near the surface, and the other at considerable depth. The former, of course, settled much more rapidly than the latter, other factors being roughly equal.

Coupled with varying p_o and e_o, the "self-weight" forces, due to the density differences between skeleton and pore fluid may be of significance in the governing equation (see Gibson et al.'s equation 6.11 in section 6.1). .

A number of approaches have been considered which might allow such effects to be treated, but major problems are soon encountered, both in analytical techniques, and in keeping the model comprehensible. The author is not aware of any quantitative data on these topics which might help in assessing their significance. It is believed that a suitable treatment of self-weight effects, and variation of stress and voids ratio with depth must eventually be tackled, but the time for this is probably after some degree of consensus is reached on the best treatment of the constant (p_o, e_o) problem.

As noted in section 6.3, there is evidence that pore pressures dissipate more rapidly in the field than predicted by Terzaghi theory. There is also evidence that settlements proceed more rapidly. Lewis et al (1975) suggest that Cv is actually higher in the field than in the laboratory and they prefer to use a Cv value based on laboratory mv and in-situ determination of permeability. They link this finding with the work of Rowe (1972), who has demonstrated the influence of such natural features as drainage channels, silt layers and soil structure generally on the field permeability. The importance of such factors and the possibility of higher field values of Cv must be noted; it is regretted that this goes beyond the scope of the present study. One factor which could be of significance here may be mentioned. The correct Cv value for Terzaghi theory should be based on the total primary strain, and where the skeleton behaves in time dependant fashion, this will increase for thicker samples. Fig. 6.16 has been drawn using the value of Δe for the primary consolidation of the thin laboratory sample to define strain (%C"). It may be seen that consolidation appears to progress far more rapidly for thick samples. If field behaviour is fitted from early settlements, without properly accounting for the delayed consolidation component occurring simultaneously, the impression might be given of Cv increasing in the field, when this is not actually the case.

To summarise the main points raised in this section -

1. For soft clays, time dependant behaviour of the skeleton must be included in the analysis, which considerably complicates the prediction of field consolidation.
2. Such analysis, using the Garlanger/t_L method shows pore pressure dissipating considerably more rapidly than predicted by Terzaghi theory; increasingly so for decreasing sample thickness.
3. This leads to the interesting paradox that improved sampling would give worse predictions, were strain rate effects not taken into account.
4. Settlement-time curves eventually converge to the same delayed consolidation curve. The thicker the sample, the less rapidly the diffusion of pore fluid proceeds, so that the delayed consolidation curve is joined after greater settlement. Hence in Terzaghi-type analysis, the amount of "primary settlement" increases with layer thickness.
5. An approximate method for such primary strain behaviour is proposed, which avoids the need to use the sophisticated numerical techniques of the Garlanger/t_L analysis.
6. Failure to account for this increasing total primary settlement may explain some reported instances of field Cv being considerably higher than laboratory values. However, natural soil features are also likely to be of significance and much remains to be discovered of the acceptability of laboratory Cv values.

7. The present Garlanger/t_L analysis, in common with other practicable schemes to date, does not include treatment of self-weight of the soil and pore fluid, or the value e_o varying with depth. These are probably insignficant in thin samples, but may be important for thick deposits: quantitative evidence on this is not yet available.

8. The t_L modification to Garlanger's method allows the scheme to directly predict consolidation of thick deposits. Results appear promising and in line with known behaviour.

6.6 Prediction of Field Consolidation

The findings of the present study may now be focussed on the main aim of the investigation: consideration of the best techniques currently available for the prediction of field consolidation from laboratory data. Discussion is restricted here to fully saturated soils with incompressible grains and pore fluid, behaving one-dimensionally.

Sampling The extent and quality of sampling will depend on the specific project. Obtaining good quality undisturbed samples in soft clay is something of an art. Considerable work has been done by a number of research organisations, and in keeping with the present approach, Bjerrum's (1973) discussion of the work at Norwegian Geotechnical Institute is recommended (a brief review with references leading more deeply into the topic). A number of undisturbed samples should be obtained. Supplementary site investigation data should include information on field stresses and shear strength. Larger projects may include field permeability tests, and in-situ investigations of consolidation; these will not be considered here.

Testing Ideally, the samples would be transferred from field to test apparatus under field stresses. In practise, reconsolidating samples to their field state is becoming increasingly widely adopted. It would seem best to set up the sample under effective stress p_o, and leave it overnight to ensure the starting point of tests is a well-identified state, (p_o, e_o), constant with depth. Several series of tests should be carried out over the stress range of practical importance. In general, conventional (C-type) oedometer tests are quite satisfactory, using a load increment ratio ($\Delta p/p$) of 1.0. From these the time-lines diagram may be drawn and parameters b and Cv determined. If the usual procedure of incrementing the load each day is adopted, only the one day line will be available here. A few long duration tests are required to establish the delayed consolidation function, c, from which the rest of the time-lines may be drawn. Long duration tests are not popular in commercial testing because they tie up apparatus and delay results. The extra work involved is, however, minimal, and for some soils the additional information can be extremely valuable.

Such tests will give sufficient data for application of Terzaghi-type prediction of strains (see below). For a fully consistent analysis of strain, or if pore pressure is to be considered, information on the stress-strain function 'a', is also required. At present the best method of obtaining this appears to consist of loading the sample in small increments from the field state of stress until strain rate effects are clearly becoming dominant. This is akin to the B-tests of Chapter 5, although the idea of identifying a p_c value from this has been largely superceded by the proposed consistent stress-strain-time approach (t_L method). As mentioned in section 6.4, there may also be a p_c effect due to structural breakdown in certain natural deposits, and such a test would allow this to be identified. Alternatively, constant rate of strain (or of loading) tests may be used here, but rates must be kept very low if the complexities of hydrodynamic lag are to be avoided.

The above tests suffice for Garlanger/t_L analysis parameters, and we note that pore pressure data is not required. This is a significant advantage since such data is difficult to obtain, and therefore costly. Remarks so far have taken real behaviour to conform closely with that of the Garlanger-Bjerrum model, so that we have only to establish parameter values. Every real soil deposit is a unique material and the validity of assumptions should be checked. The B-tests and long term tests should indicate the acceptability of linear log e - log p and log e - log t relationships. One or two conventional tests with pore pressure measurement would also be useful to check the validity of the analyses for both strain and pore pressure.

The significance of partial saturation was considered in Chapter 5, and a back-pressure test should be performed at an early stage in the investigation. If the soil does become unsaturated during testing, back-pressures are required throughout.

Analysis

For present purposes two cases are identified; where skeletal behaviour is (a) independant of strain rate, and (b) dependant on strain rate. For case (a), the situation where Cv remains constant, due to a linear stress-stress relationship of slope a_v, and permeability k is constant, is handled by basic Terzaghi theory. If the stress-strain behaviour is non-linear it is generally necessary to resort to numerical techniques of solution. In the programs developed in Chapter 2, the values of Cv, a_v and k are taken as constant over each discrete step of time, during which the Terzaghi equation holds. Results may thus be obtained for any desired variation of parameters, specified here as functions of the effective stress. This is very much a pragmatic approach and problems may be encountered in adequately defining how parameters vary; and how the variation in tests relates to variations for field deposits. Theoretically, the approach by Gibson et al. (1967) (see section 6.1) is more satisfactory, and this may readily be applied for the special case of constant Cv where a_v and k both vary in identical fashion, providing small strains may be assumed. The CONED programs can adequately handle problems of unlike contiguous layers consolidating together, and time dependant loading.

For case (b), including time dependant skeletal behaviour, only cases where Cv is assumed constant can be handled at present. Thus, if only strains are to be considered a modified version of the Terzaghi approach has been suggested (see section 6.5) which satisfactorily approximates behaviour without the need to use a computer program method. For the consistent treatment of strain, and to analyse pore pressure behaviour, it is necessary to solve the diffusion equation simultaneously with the stress-strain-time relationship for the soil skeleton. This involves the complex numerical techniques based on the scheme proposed by Garlanger (1972). The main steps involved may be summarised.

1. Parameters a, b, c, and Cv are obtained from the laboratory tests outlined above.
2. p_o and e_o are taken from field conditions. As discussed in section 6.5, present analysis requires constant values. This, and the omission of self-weight effects, introduces an approximation the significance of which has not been systematically investigated to date.

3. Δe should be defined corresponding to a specified time. The choice, however, is arbitrary bearing in mind that local strain, β, $\{=(e_o - e)/\Delta e\}$, and average strain (%C) in the numerical results will be defined by reference to Δe. For a consistent approach, Garlanger's definition is recommended, calling Δe the change in voids ratio for consolidation to reach the "loading age" of the deposit, i.e. $\Delta e = e_o - e_f$, where (p_o, e_o) and (p_f, e_f) lie on the same time line.

4. In place of Garlanger's t_i and p_c, which appear to vary with sample thickness, the t_L approach (section 6.4) is proposed. This considers the observed p_c to be a result of the soil's stress-strain-time behaviour, and adequately defined by such functions without the need for a discontinuity of stress-strain relationship. The time-lines grid is positioned by the line of gradient 'b' through the intersection of lines 'a' and p_f. This is termed the limit time-line, t_L.

All the parameters are thus available and remain constant for any thickness of soil layer, which is important for the method to have predictive value. Solution is handled quite easily by the programs developed in Chapter 4. The semi-implicit scheme, SECON, is recommended for routine work - it is very rapid. It has not, however, been proved free of numerical errors, so that intelligent checks against the rigorous method, CONGO, are recommended.

To date this scheme is confined to single soil layers, but time dependant loading can be successfully treated by SECON (see section 4.7).

CHAPTER 7

Concluding Remarks

The various facets of the present research have allowed a clearer picture of the behaviour of soft clays to be developed. Coupled with this, improved techniques of analysis and prediction have been proposed. In these concluding remarks the main findings of this thesis are reviewed, and the present status of the Garlanger/t_L approach is discussed.

This study began by examining Terzaghi's theory and the opportunities offered to extend the analysis capabilities by numerical techniques of solution. Multi-layer profiles, and time-dependant loading were shown capable of solution, as was any desired initial pore pressure distribution with depth. The interactive graphics facilities afford very convenient treatment of input and results. Problems with the consolidation parameters varying as functions of effective stress could also be quite conveniently handled. Theoretical complexities were encountered for such analysis and it was noted that the present numerical techniques relate parameters to the average effective stress over a layer. Localised parameters could be satisfactorily handled by use of a number of sub-layers, although this increases computer running time somewhat. Less satisfactory is the problem of defining the manner in which the consolidation parameters vary. Local parameters can probably be determined in laboratory tests only by applying very small load increments sufficiently slowly for effective stress to be sensibly constant with depth. This might be an acceptable procedure if behaviour were independant of time effects.

There is now a considerable body of evidence that this is not the case, at least for soft deposits and clays which are lightly over-consolidated. The influence of time upon the soil's stress-strain behaviour is believed to be of fundamental importance. Although such ideas date back to the work of Merchant (1939) and Taylor (1942), a method of including time dependant skeletal behaviour in consolidation analysis has not yet gained general acceptance. If the practising engineer is reluctant to engage in the complexities of such theories, the onus is perhaps on the researcher to

1) demonstrate the need for such analysis and the acceptability of the theoretical techniques.
2) Clarify and simplify where possible the analytical method and produce viable solution schemes.
3) Present clear statements of the capabilities and limitations of the scheme, including methods of determining parameters, and their validity for prediction purposes.

A major limitation of the schemes to date has been the demonstrable inability to account for the rapid pore pressure dissipation with relatively little strain observed during the early stages of consolidation in laboratory tests (i.e. the process now termed A-type Consolidation). Such behaviour can be shown to depend upon prior delayed consolidation. In the present study it has been shown that the time dependency of skeletal behaviour can account for this, if such analysis is coupled with a "time-lines" approach. To be fully consistent the time-lines should be considered rather as lines of constant rate of skeletal strain, from which it is evident that this increases rapidly during the early stages of consolidation. Although rather awkwardly presented, so that incorrect interpretations have been reported in the literature, it has now been established that Garlanger's consolidation theory does correctly incorporate an expression for delayed strain behaviour, valid at all stages of consolidation. A number of further clarifications and modifications of several theoretical inconsistencies have resulted in what the author believes to be the most satisfactory approach currently available. We may consider the scheme under a number of criteria of importance if it is to have real value in engineering practice.

a) Formulation of theoretically acceptable and compatible relationships

It has been established that Garlanger's method is theoretically sound and successfully describes consolidation as a process of diffusion of pore fluid under a potential gradient from a soil skeleton whose behaviour depends upon the effective stress, voids ratio, and rate of strain. The method of handling skeletal behaviour by two simultaneous, but independant functions, referring to stress-strain and strain-time processes appears to be a very useful concept. Details of theoretical treatment were given in section 6.1, but it must be added that some modification is required to the published scheme which was rather unsatisfactory as it stood. In particular, a discontinuity of the stress-strain relationship at some "critical stress, p_c", was included by Garlanger, and this is theoretically debatable, and introduced major problems in practical use of the method. A modification (termed the t_L method) has been proposed which is theoretically consistent, gives the scheme predictive value, and is believed to be more in line with real soil behaviour - see d) below.

b) Experimental validification of such relationships

The main experimental evidence in the present study is the lengthy series of oedometer tests carried out on remoulded Grangemouth Silty-Clay. It has been shown that Garlanger's scheme enables quite good fitting of these results and the influence of prior delayed consolidation can be successfully treated. Detailed consideration of the linear logarithmic functions used for A-type and delayed consolidation, suggests these may be slight oversimplifications, but the effects of such are small in comparison to the advances made over existing approaches to consolidation, and it has been concluded that at the present state of knowledge there is little justification for further complicating the model. Evidence from published consolidation data reinforces these findings and suggests the present model should be valuable for many soft clays.

c) Acceptability of Governing Parameters

The parameters employed should be capable of relatively straight forward and accurate determination by laboratory tests, preferably reasonably similar to conventional methods. The best approach for the present scheme has been outlined in section 6.6, and is along the lines of present techniques, although some supplementary long term and incremental loading tests are desirable (this might, in any case, be considered good practise for major projects). Rather more analysis of the data than usual is also required. The author found a simple computer program useful for the present large amount of data. Alternatively, data could be conveniently evaluated on a simple programmable calculator, or hand calculation is not unduly lengthy. In the author's opinion the additional work is well justified by the improved analysis available. Methods of determining parameters are then quite straightforward, and have been demonstrated in detail in the present text. The parameters remain constant with thickness of soil, and are therefore available for prediction of field behaviour.

d) Value for prediction of field consolidation

With parameters constant the application to thick deposits is evident. One problem should be mentioned. The concept of a critical stress is of considerable current interest. Garlanger's published scheme includes a stress level p_c at which the gradient of stress-strain function changes abruptly. It has been shown (section 6.4) that the non-linearity of stress-strain behaviour of interest in the present theory is adequately handled by the consistent expression for time dependant skeletal behaviour, and Garlanger's use of p_c introduces an unnecessary complication. Correcting this, by what is termed the limit time-line, t_L, method, enables the scheme to be used for any thickness of soil under completely constant governing parameters. {It is believed that certain clays, such as the highly sensitive, cemented Canadian Leda clay, may exhibit structural breakdown at some particular stress level. This is probably not dependant on strain rate and differs from the phenomenon under present consideration.}

e) Statement of Applicability and Validity

This is linked to b) above and there is always an element of subjectivity in what constitutes "sufficient" experimental evidence. The direct quantitative evidence to date is limited to Garlanger's work on deposits at Drammen, and the present detailed work on Grangemouth Clay together with limited investigation of Leigh-on-Sea clay and Bentonite. The time-lines diagrams on which the model is based, have been developed by Bjerrum and his associates for a considerable number of soft deposits - particularly around Scandinavia. Similar time-lines models have been developed by a number of workers since Taylor described such a scheme based on tests for the Boston Blue Clay. It appears that many soft clays do approximate quite closely to such simple elastic-plastic concepts. The importance of complex soil structural features should not be overlooked, but it would seem likely that these are of far less significance in soft deposits than in overconsolidated and stiff clays.

The present theoretical limitations are:

i) Cv is assumed constant

ii) self weight of soil grains and pore fluid is ignored

iii) initial stress p_o and voids ratio e_o are assumed constant with depth

iv) small strains are assumed.

The usual assumptions are made of homogeneity, incompressibility, one-dimensional behaviour, and validity of Darcy's Law.

f) Ease of Use

Although an approximate procedure has been suggested based on Terzaghi solutions for primary strain, the full solutions of Garlanger/t_L theory require programmed numerical techniques. Although universal access to a suitable program remains something of a problem, there is little doubt that the various computer bureax will make such available if the demand is there. The three programms developed at Edinburgh University enable solutions to be readily obtained from any desired input of parameter values. Besides numerical output of results, graphs of average strain against linear or logarithmic time are also available here.

GARCON - uses Garlanger's explicit solution scheme. This is slow and not particularly accurate. It is not recommended.

SECON - uses a semi-implicit numerical technique which is very fast. It is therefore recommended for general use, although accuracy is not fully established and should be checked occasionally.

CONGO - uses a fully implicit scheme (method of lines) of specified accuracy. It is roughly twice as fast as GARCON, and is recommended for rigorous solution.

The Consolidation Behaviour of Soft Clay is a complex problem, in soil mechanics. A consistent treatment in terms of fairly simple stress-strain and strain-time functions has been shown capable of quite accurate description of observed behaviour and the possibilities for prediction purposes in engineering practice appear encouraging. After the clarifications and modifications examined in the present thesis, it is concluded that the Garlanger/t_L method is currently the most satisfactory scheme available for analysis and prediction purposes.

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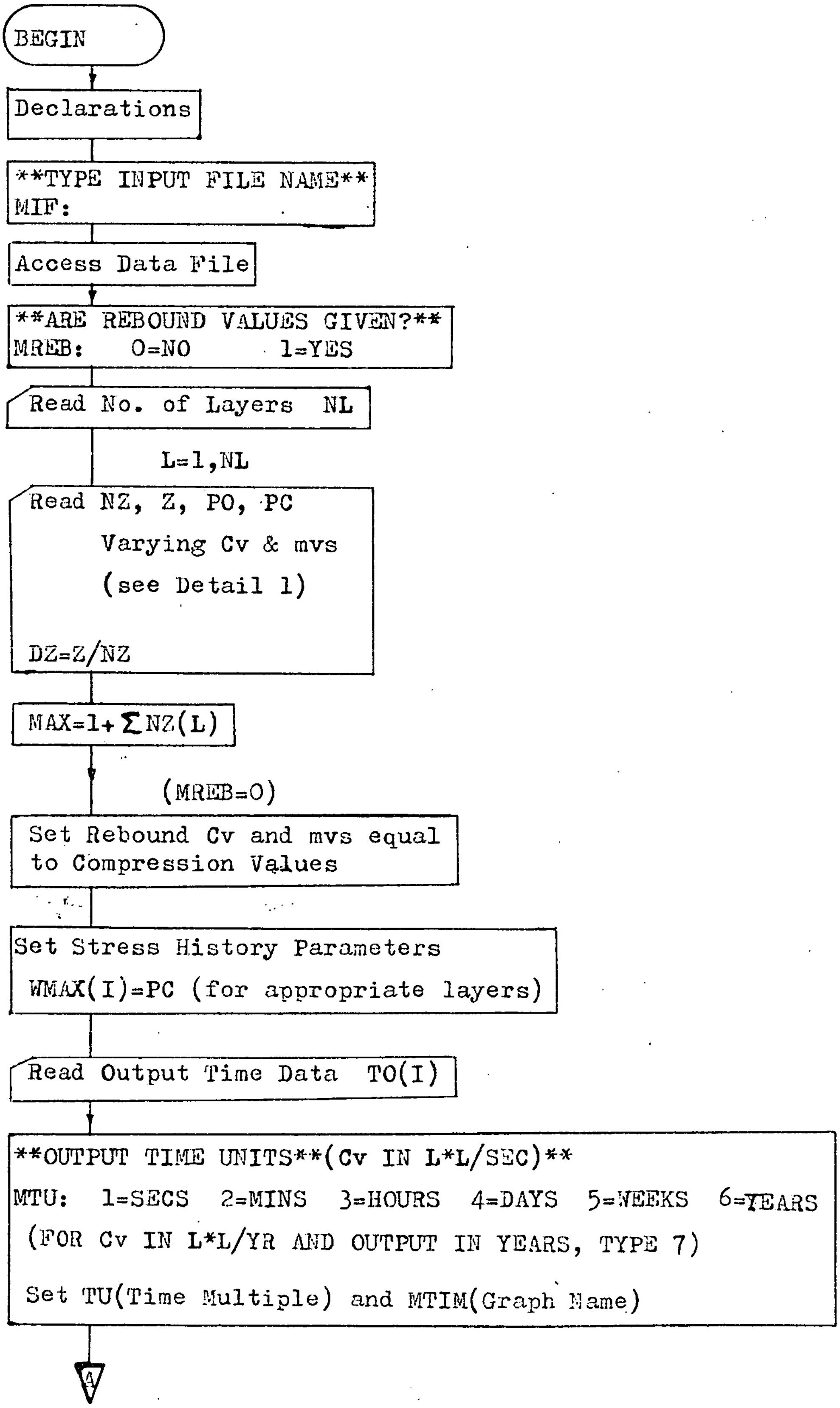
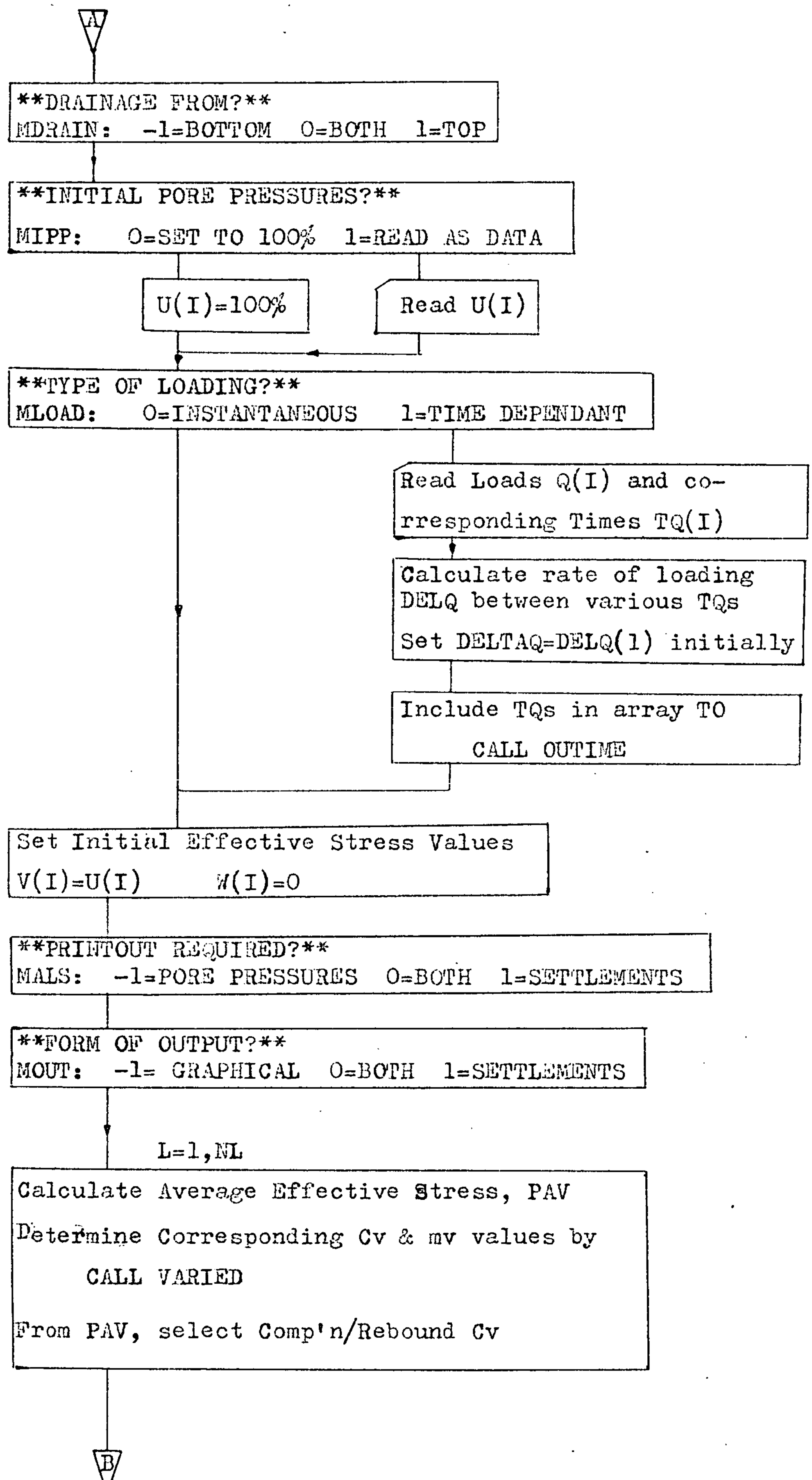
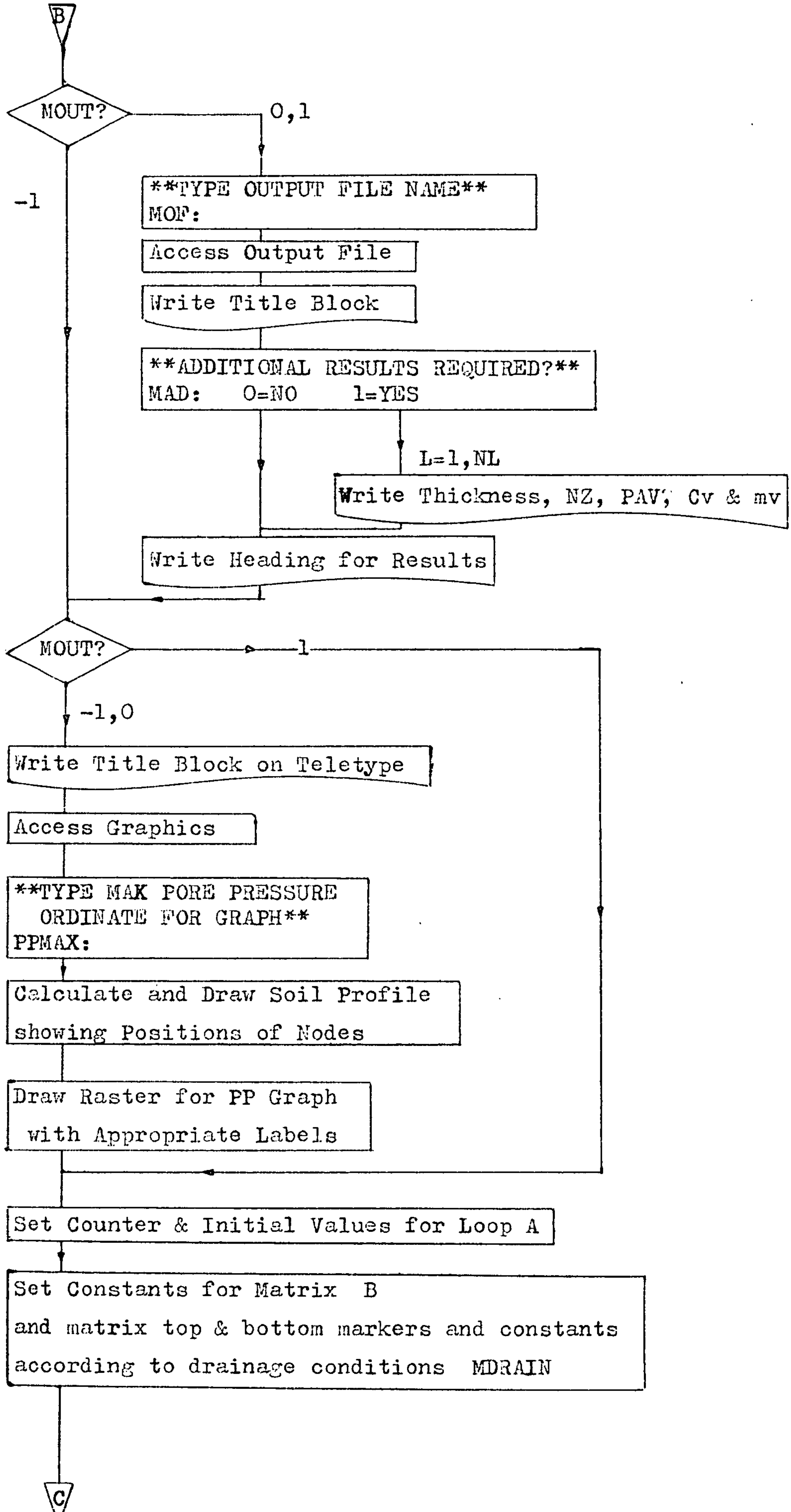
APPENDIX A

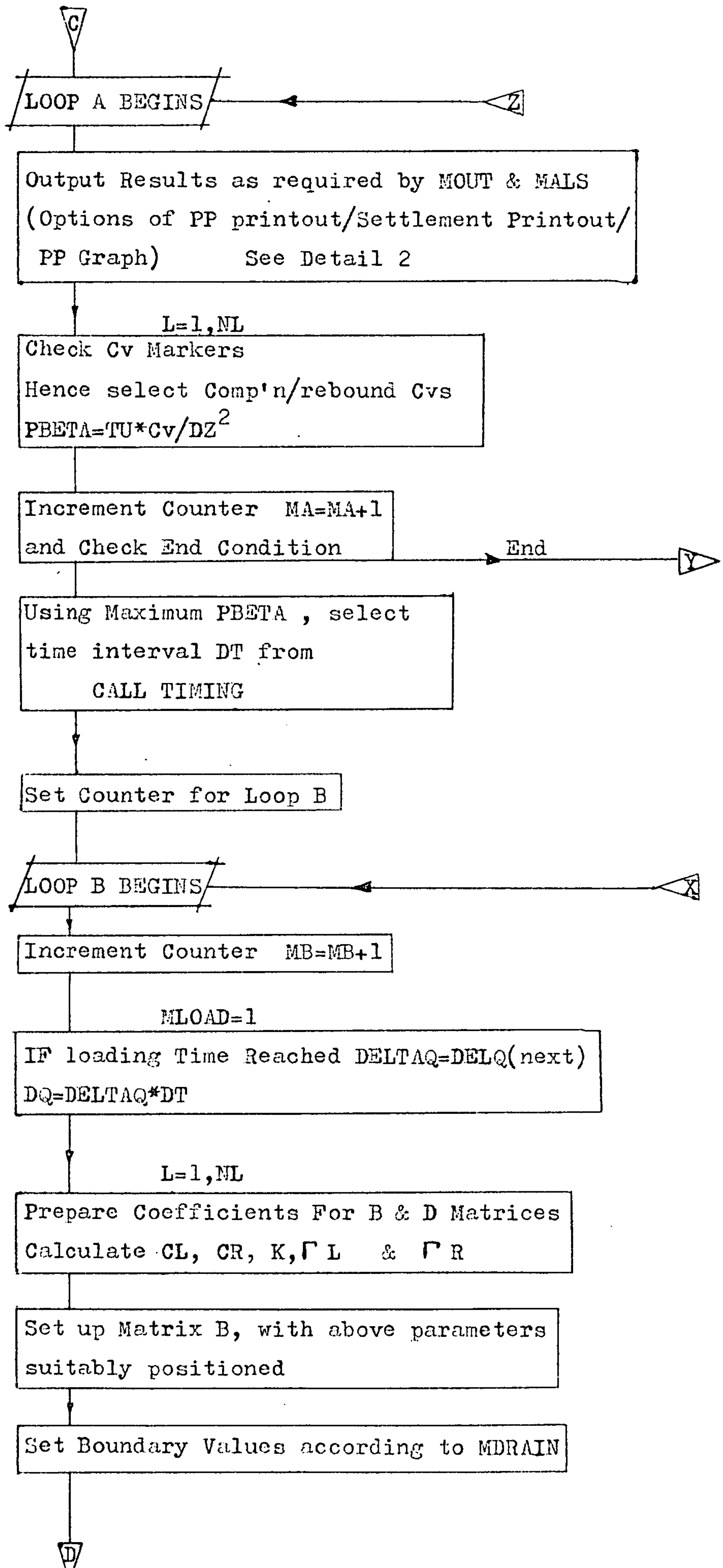
CONED4 FLOW DIAGRAM

MAIN




A
DRAINAGE FROM?
MDRAIN: -1=BOTTOM 0=BOTH 1=TOP
INITIAL PORE PRESSURES?
MIPP: 0=SET TO 100% 1=READ AS DATA
U(I)=100%
Read U(I)
TYPE OF LOADING?
MLOAD: 0=INSTANTANEOUS 1=TIME DEPENDANT
Read Loads Q(I) and co-rresponding Times TQ(I)
Calculate rate of loading DELQ between various TQs
Set DELTAQ=DELQ(1) initially
Include TQs in array TO
CALL OUTIME
Set Initial Effective Stress Values
V(I)=U(I) W(I)=0
PRINTOUT REQUIRED?
MALS: -1=PORE PRESSURES 0=BOTH 1=SETTLEMENTS
FORM OF OUTPUT?
MOUT: -1= GRAPHICAL 0=BOTH 1=SETTLEMENTS
L=1,NL
Calculate Average Effective Stress, PAV
Determine Corresponding Cv & mv values by
CALL VARIED
From PAV, select Comp'n/Rebound Cv
B


B
MOUT?
0,1
-1
TYPE OUTPUT FILE NAME
MOF:
Access Output File
Write Title Block
ADDITIONAL RESULTS REQUIRED?
MAD: 0=NO 1=YES
L=1,NL
Write Thickness, NZ, PAV, Cv & mv
Write Heading for Results
MOUT?
1
-1,0
Write Title Block on Teletype
Access Graphics
TYPE MAK PORE PRESSURE ORDINATE FOR GRAPH
PPMAX:
Calculate and Draw Soil Profile showing Positions of Nodes
Draw Raster for PP Graph with Appropriate Labels
Set Counter & Initial Values for Loop A
Set Constants for Matrix B and matrix top & bottom markers and constants according to drainage conditions MDRAIN
C


C
LOOP A BEGINS
Z
Output Results as required by MOUT & MALS
(Options of PP printout/Settlement Printout/
PP Graph) See Detail 2
L=1,NL
Check Cv Markers
Hence select Comp'n/rebound Cvs
PBETA=TU*Cv/DZ2
Increment Counter MA=MA+1
and Check End Condition
End
Y
Using Maximum PBETA , select
time interval DT from
CALL TIMING
Set Counter for Loop B
LOOP B BEGINS
X
Increment Counter MB=MB+1
MLOAD=1
IF loading Time Reached DELTAQ=DELQ(next)
DQ=DELTAQ*DT
L=1,NL
Prepare Coefficients For B & D Matrices
Calculate CL, CR, K, Γ L & Γ R
Set up Matrix B, with above parameters
suitably positioned
Set Boundary Values according to MDRAIN
D

D

Reduce B to Upper Triangular Matrix

Set up Vector D using appropriate parameters with current pore pressures

Hence Solve B * U^+ = D
to obtain new pore pressures, by Gaussian Elimination & Back Substitution

Hence Calculate New Effective Stresses
IF(MLOAD=1) V=V+DQ
W(I)=V(I)-U(I)

Compare W(I):WMAX(I) and set marker DH(I) for comp'n(1) or rebound(-1)
IF (W(I) > WMAX(I)) set WMAX(I)=W(I)

L=1,NL

Calculate av. eff. stress at midpoint of previous step. Hence determine corresponding Mv values by CALL VARIED
(Note - Calculation of Cvs suppressed here)

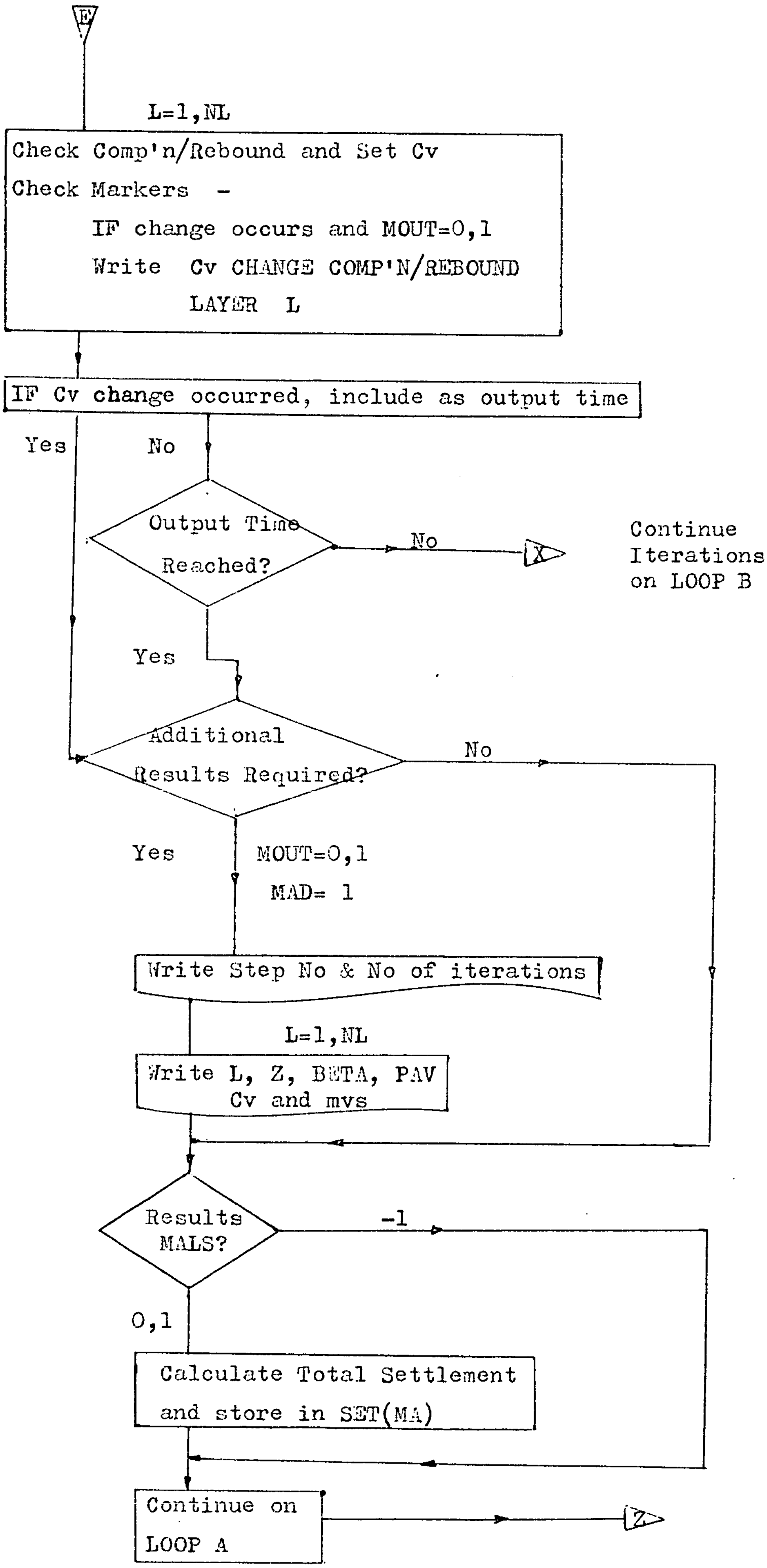
L=1,NL

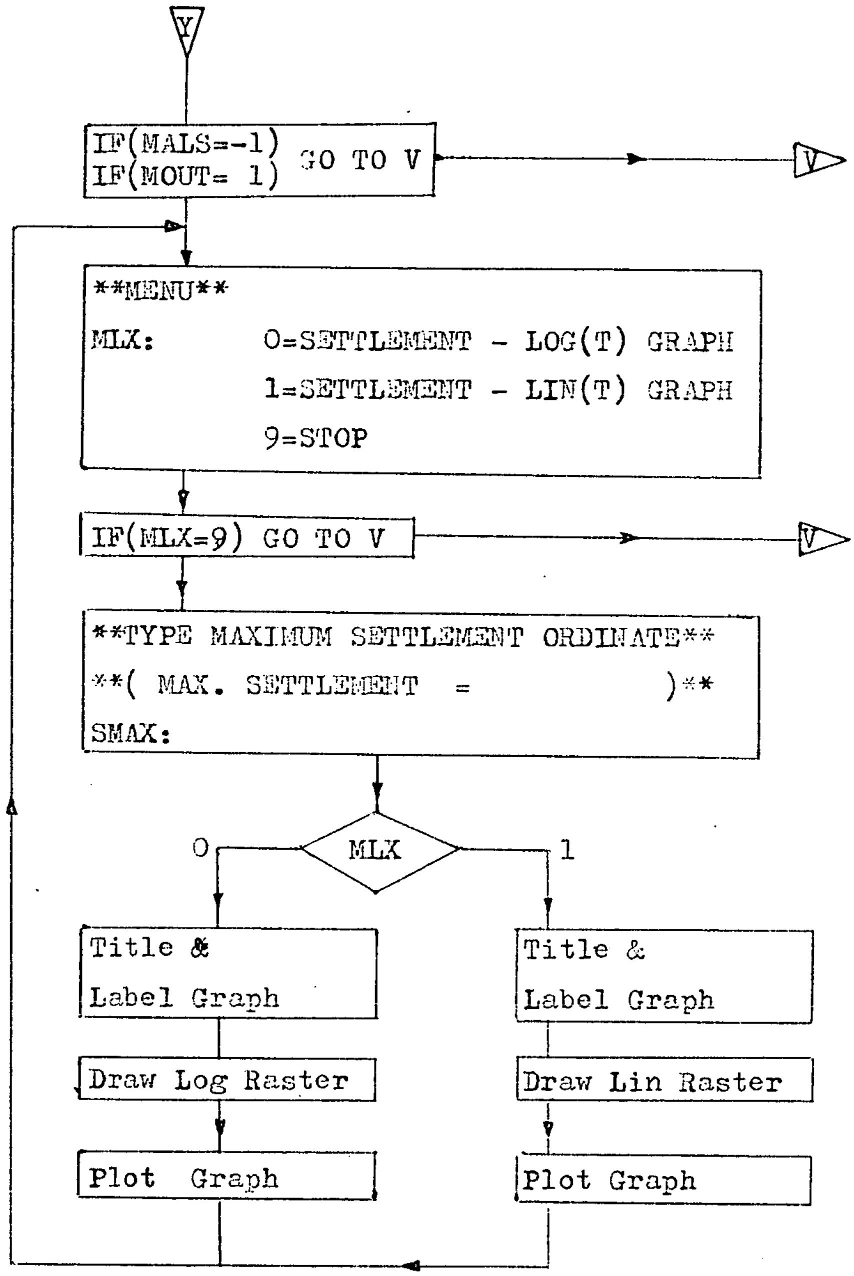
Calculate Settlement (with appropriate comp'n/rebound mv indicated by DH(I))
CALL INTRAL
Hence determine new thickness
Z=Z-s

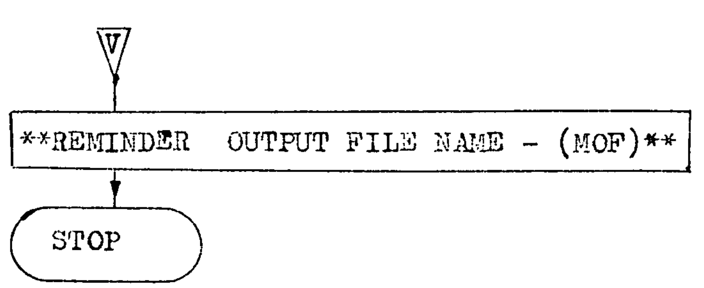
L=1,NL

Calculate current av. eff. stress
Hence Cv & mv values for next iteration of U(I) calculations
CALL VARIED

E

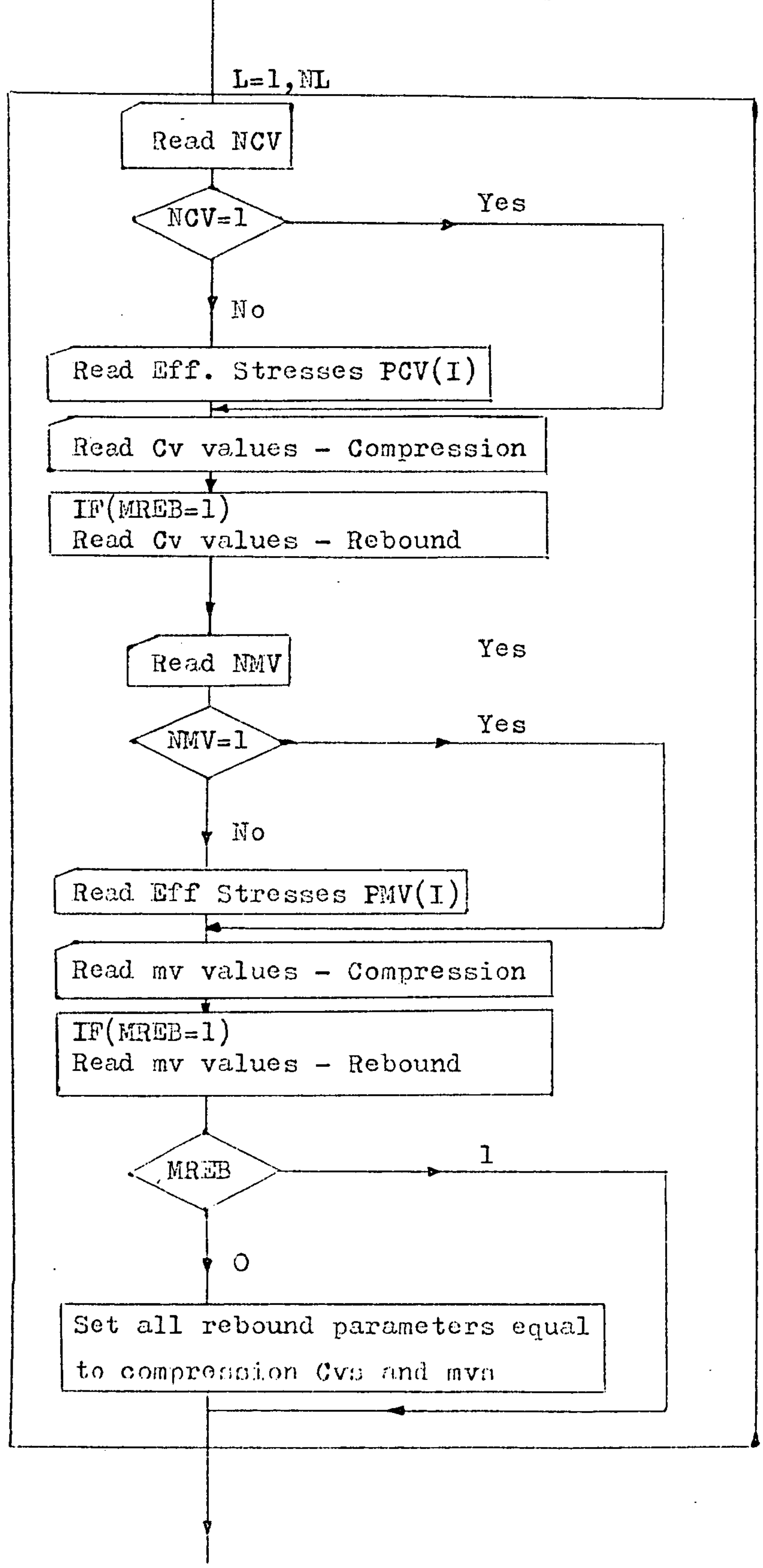

E
L=1,NL
Check Comp'n/Rebound and Set Cv
Check Markers -
IF change occurs and MOUT=0,1
Write Cv CHANGE COMP'N/REBOUND
LAYER L
IF Cv change occurred, include as output time
Yes
No
Output Time Reached?
No
X
Continue Iterations on LOOP B
Yes
Additional Results Required?
No
Yes
MOUT=0,1
MAD= 1
Write Step No & No of iterations
L=1,NL
Write L, Z, BETA, PAV Cv and mvs
Results MALS?
-1
0,1
Calculate Total Settlement and store in SET(MA)
Continue on LOOP A
Z


V
IF(MALS=-1)
IF(MOUT= 1)
GO TO V
V
MENU
MLX:
0=SETTLEMENT - LOG(T) GRAPH
1=SETTLEMENT - LIN(T) GRAPH
9=STOP
IF(MLX=9) GO TO V
V
TYPE MAXIMUM SETTLEMENT ORDINATE
(MAX. SETTLEMENT =)
SMAX:
MLX
0
1
Title & Label Graph
Draw Log Raster
Plot Graph
Title & Label Graph
Draw Lin Raster
Plot Graph

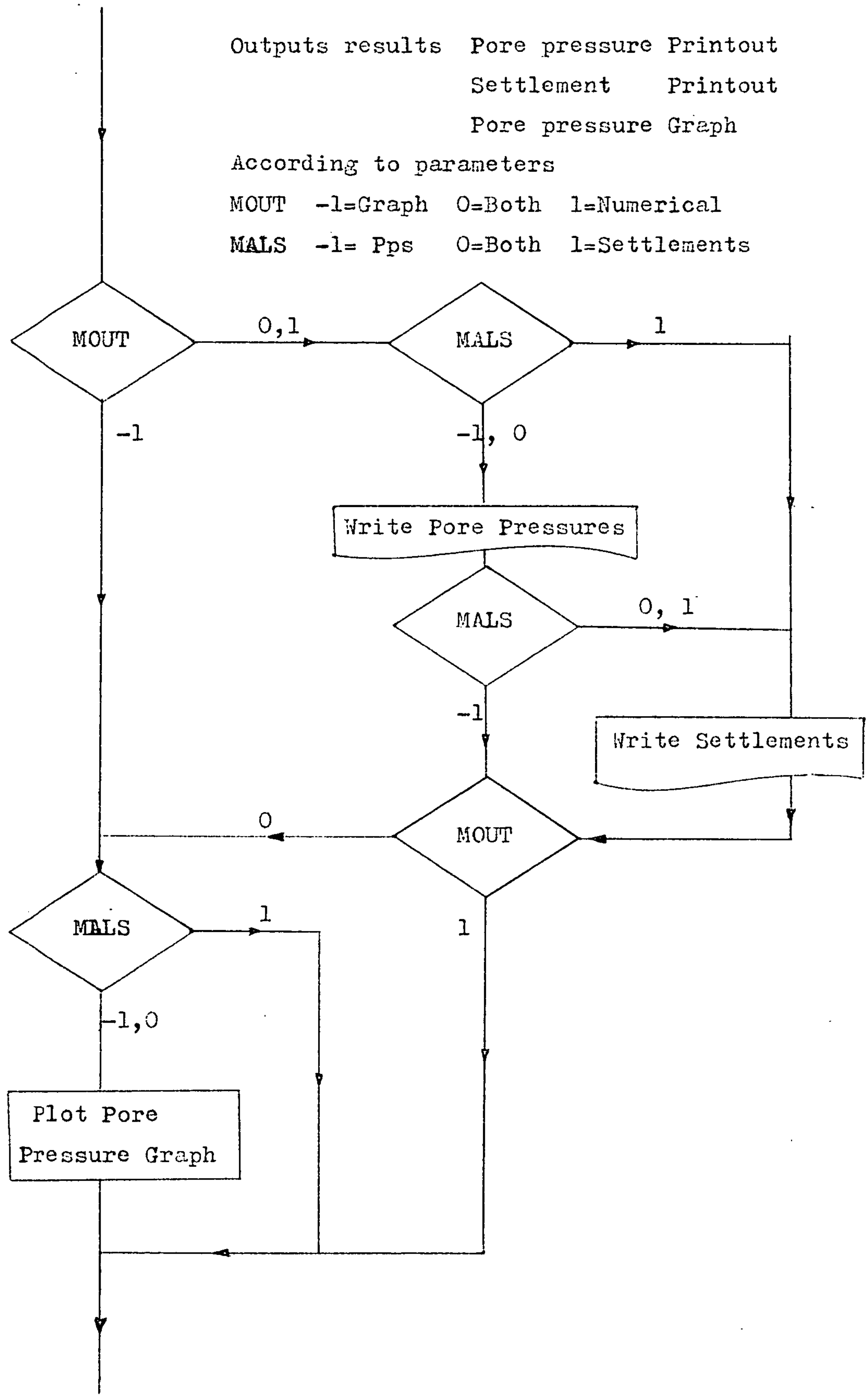

V
REMINDER OUTPUT FILE NAME - (MOF)
STOP

DETAIL 1

Reads in arrays of Cv & mv values for Corresponding arrays of effective stresses (PCV(I) & PMV(I), respectively).
Rebound values may either be read (MREB=1) or set to Compression values (MREB=0).



DETAIL 2

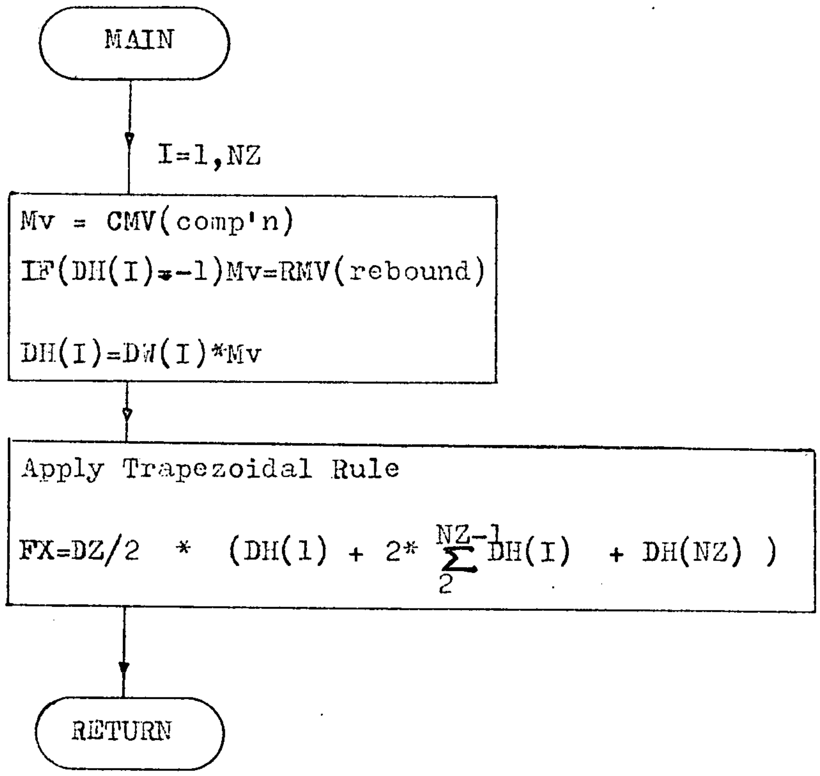


Outputs results Pore pressure Printout
Settlement Printout
Pore pressure Graph
According to parameters
MOUT -1=Graph 0=Both 1=Numerical
MALS -1= Pps 0=Both 1=Settlements
MOUT
0,1
MALS
1
-1
-1, 0
Write Pore Pressures
MALS
0, 1
-1
Write Settlements
0
MOUT
MALS
1
1
-1,0
Plot Pore Pressure Graph

SUBROUTINE INTRAL (NZ,DZ,CMV,RMV,DH,DW,FX)

Calculates Numerical Integration of

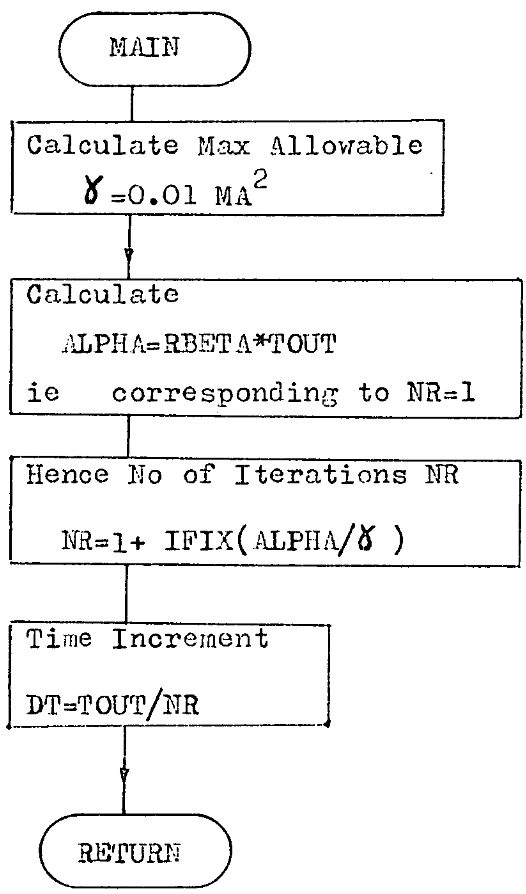
$\int_0^Z mv.\Delta p.dZ$ by Trapezoidal Rule



DH is given at each node as 1 for Compr'n, or -1 for Rebound
For CoNED 3&4, mvs vary with effective stress, but only
the relevant values are passed over here.

SUBROUTINE TIMING (RBETA,TOUT,MA,NR,DT)

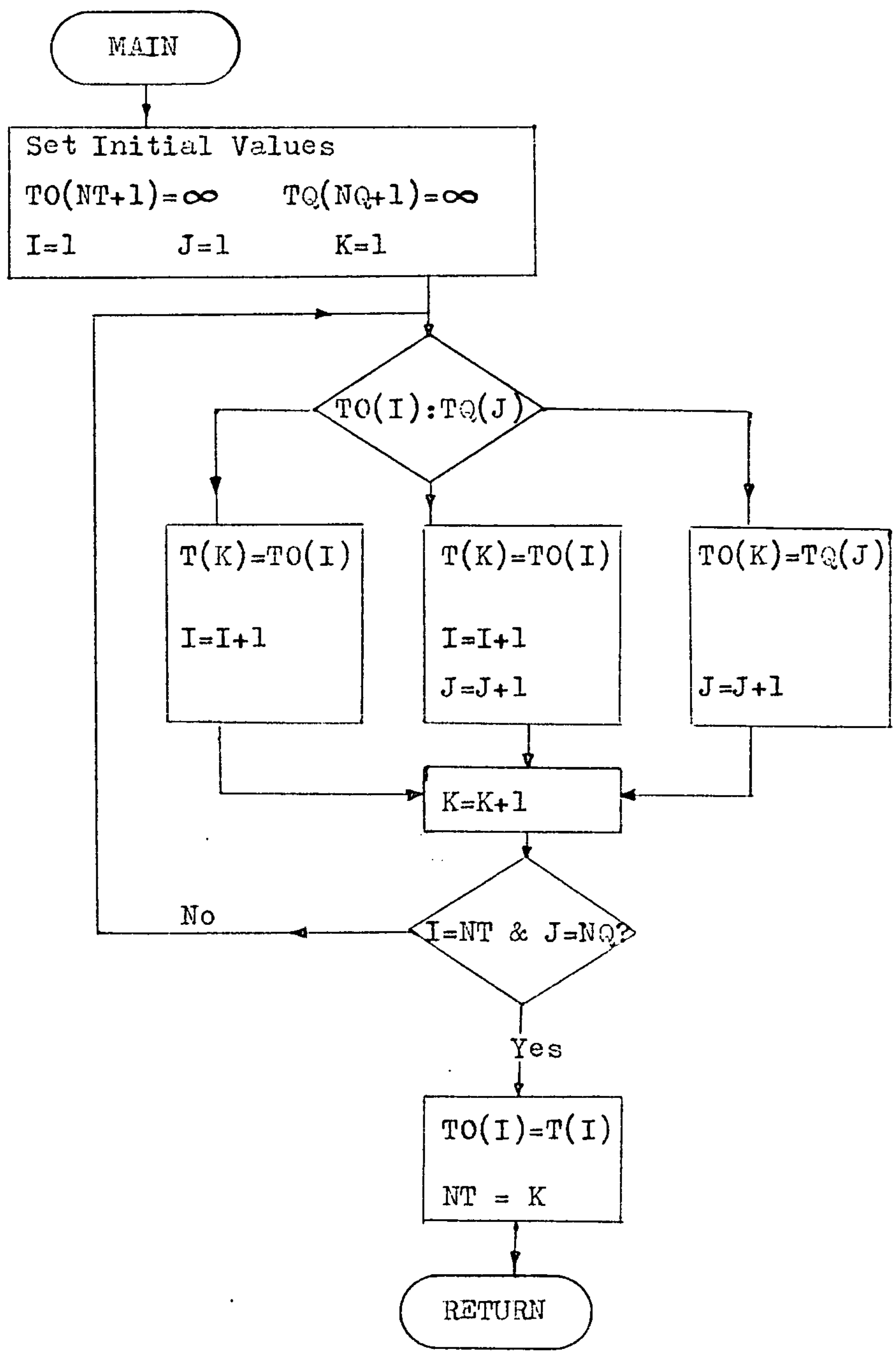
Calculates acceptable time increment,DT, to satisfy accuracy criterion limitting



Total Time Increment to next output time, TOUT? is split into NR equal increments of size DT, where DT is the largest permissible time step to keep β (or β max for a multi-layer profile) within the limits imposed by truncation error.

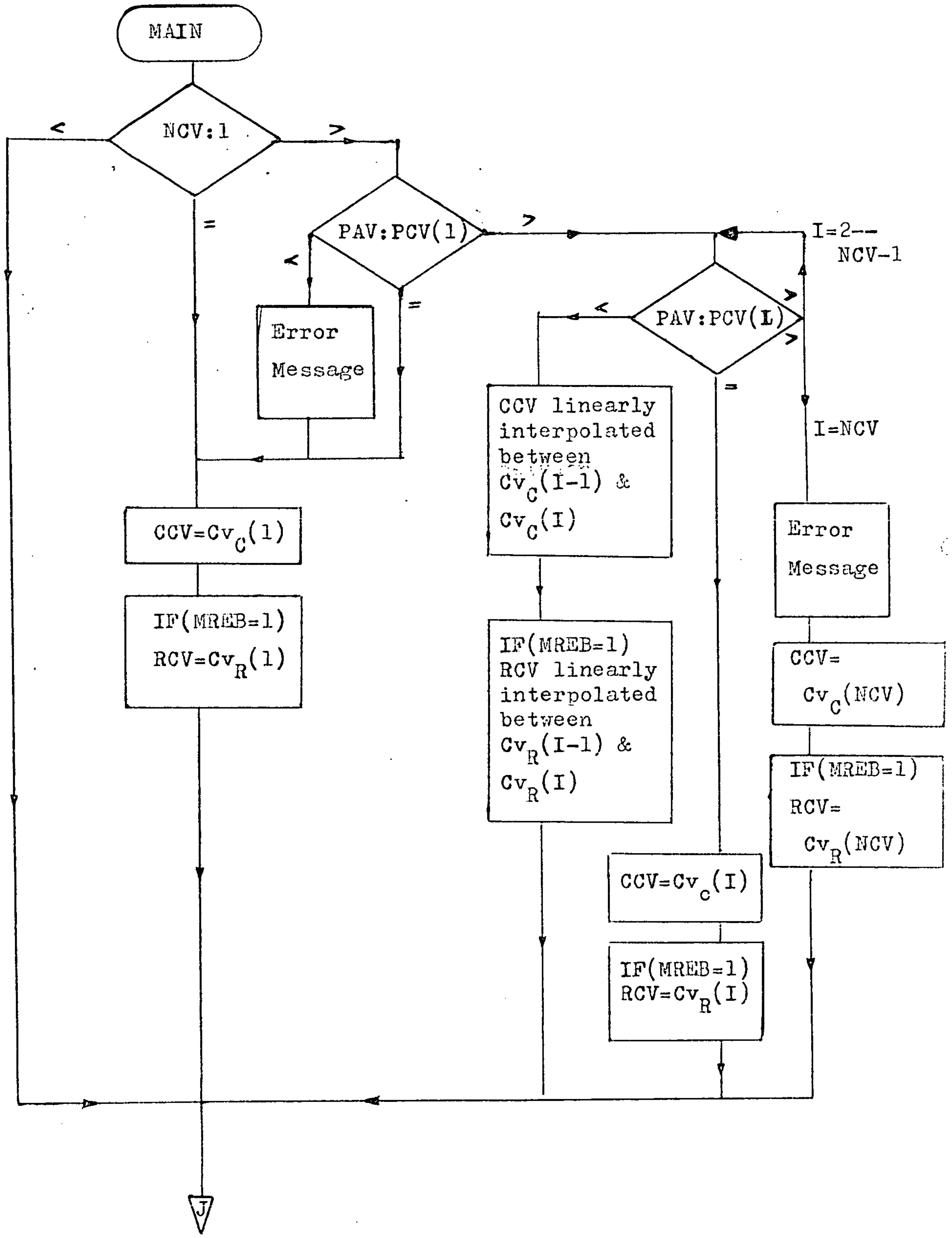
SUBROUTINE OUTIME (TO,TQ,NT,NQ)

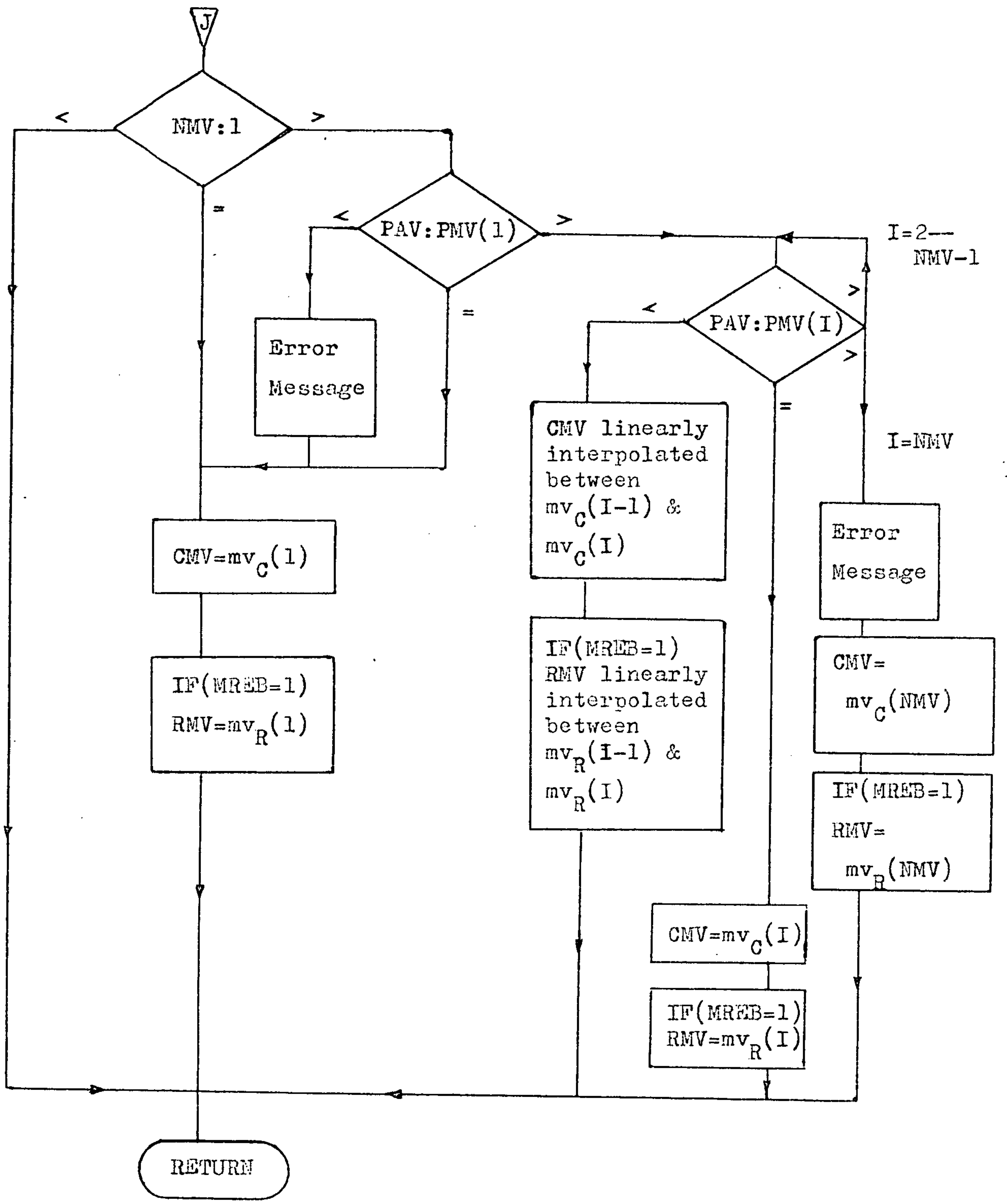
For Time Dependant Loading, this ensures all loading times are included in the Output Times Array, Hence points of Calculation.



SUBROUTINE VARIED (PAV,NCV,NMV,CCV,RCV,CMV,RMV,MREB)

Determines Cv & mv(Comp'n) (& rebound if MREB=1) corresponding to given av. eff. stress, PAV. Arrays of parameters & corresponding stresses (PCVs & PMVs) are supplied in a COMMON block, and VARIED linearly interpolates between relevent values.



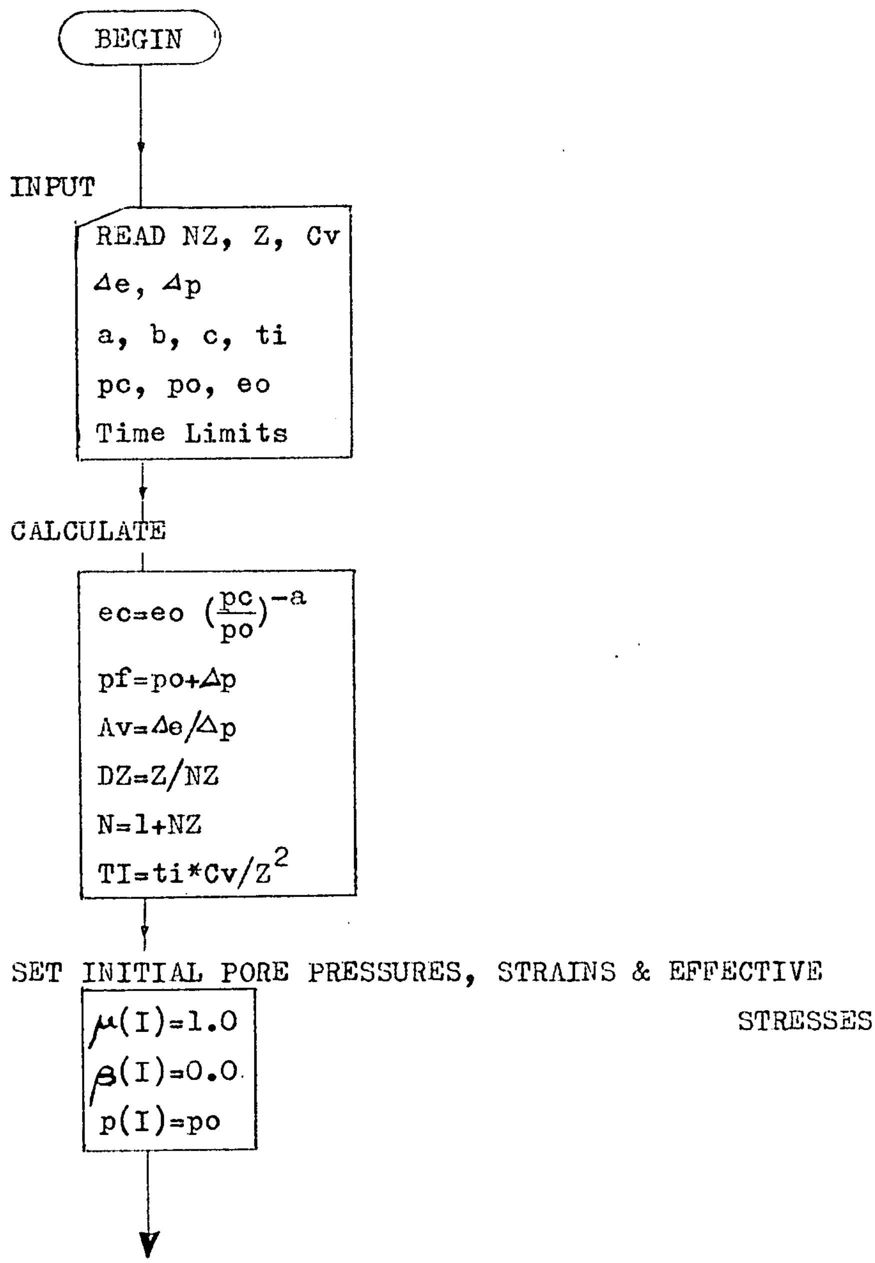


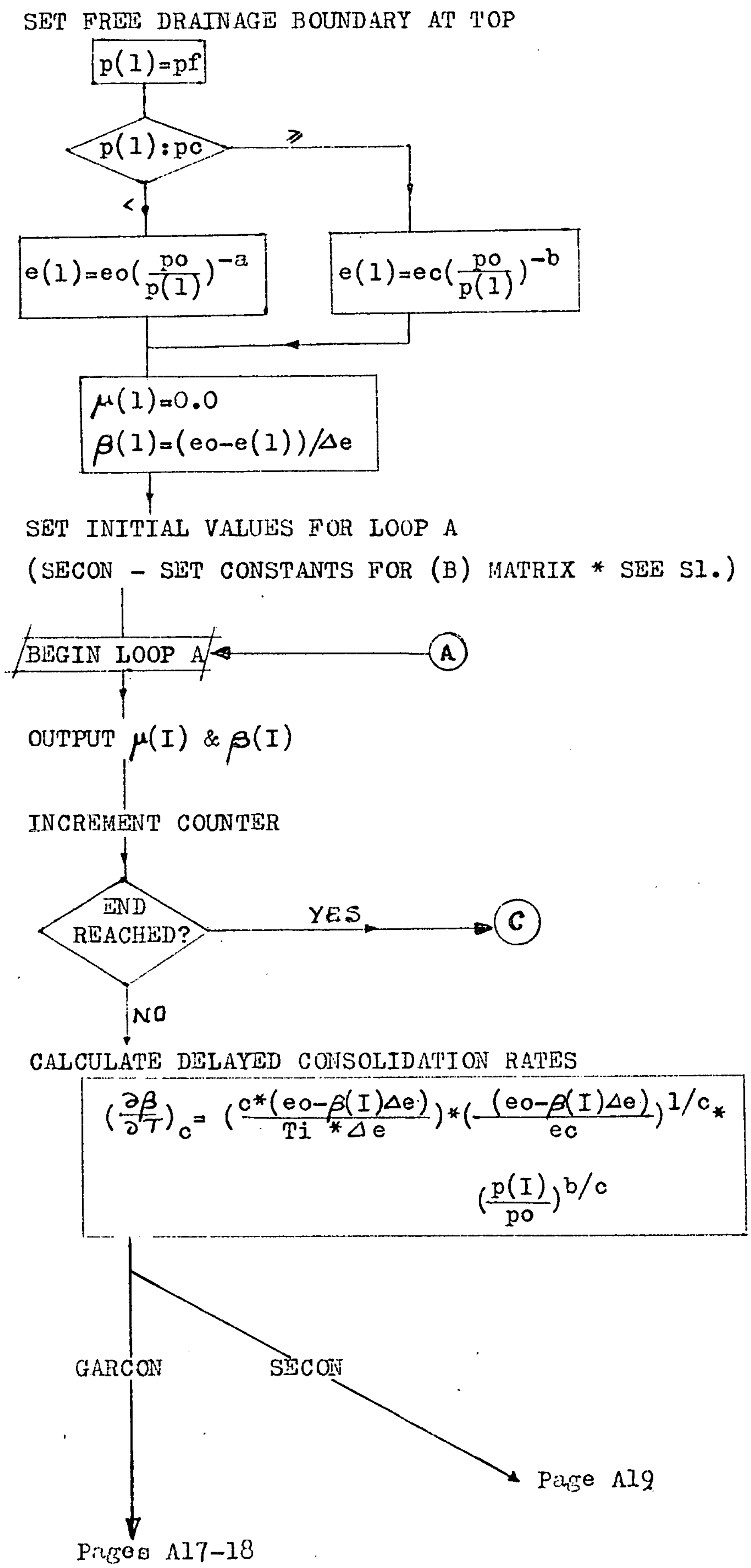
Notes: Error Message includes option to STOP or CONTINUE
A counter is also incremented each time this is displayed, so that after second warning this is suppressed (Details not shown).
Tests also included so that Cv or mv calculations may be by-passed if NCF or NMV = 0 respectively

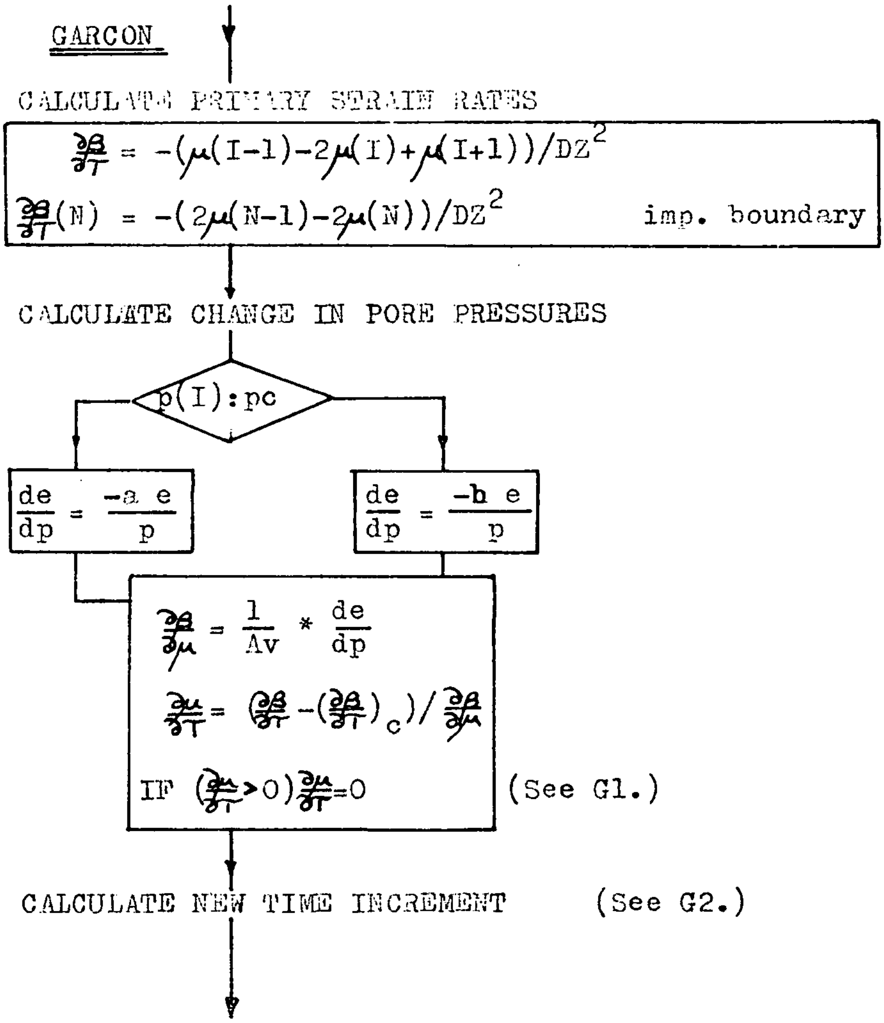
GARCOM & SECON

Note: Much of these programs is identical, so a single flow diagram is employed, with the differing central sections on separate pages, clearly headed.

For clarity, details of the considerable amount of interactive programming, graphics and input-output handling are omitted.




SET FREE DRAINAGE BOUNDARY AT TOP
p(1)=pf
p(1):pc
≥
<
e(1)=eo(po/p(1))^-a
e(1)=ec(po/p(1))^-b
μ(1)=0.0
β(1)=(eo-e(1))/Δe
SET INITIAL VALUES FOR LOOP A
(SECON - SET CONSTANTS FOR (B) MATRIX * SEE S1.)
BEGIN LOOP A
A
OUTPUT μ(I) & β(I)
INCREMENT COUNTER
END REACHED?
YES
C
NO
CALCULATE DELAYED CONSOLIDATION RATES
(∂β/∂T)c = (c*(eo-β(I)Δe)/Ti *Δe)*((eo-β(I)Δe)/ec)^1/c *
(p(I)/po)^b/c
GARCON
SECON
Page A19
Pages A17-18



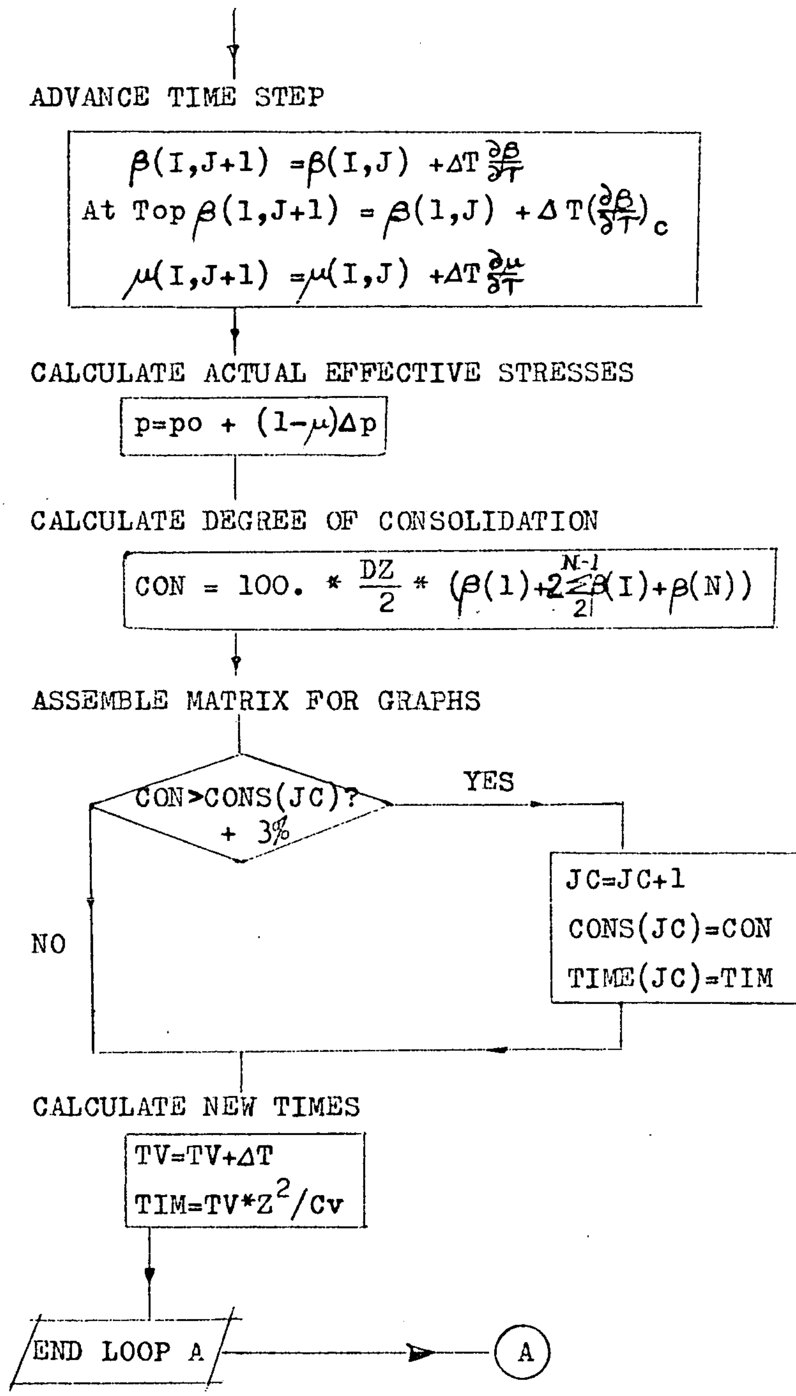
*****   *****   *****

Notes

G1. Garlanger includes this condition and reports laboratory tests have confirmed this behaviour

G2. For stability in GARCON $T = \frac{3}{4} * \left(\frac{\partial\beta}{\partial\mu}\right)_{min} * DZ^2$
This does not, however lead to acceptable truncation errors at early times, so 1/10th of this value has been used at less than 20% Consolidation.

GARCON


ADVANCE TIME STEP
β(I,J+1) =β(I,J) +ΔT ∂β/∂T
At Top β(1,J+1) = β(1,J) + ΔT(∂β/∂T)c
μ(I,J+1) =μ(I,J) +ΔT ∂μ/∂T
CALCULATE ACTUAL EFFECTIVE STRESSES
p=po + (1-μ)Δp
CALCULATE DEGREE OF CONSOLIDATION
CON = 100. * DZ/2 * (β(1)+2 Σ(2 to N-1) β(I)+β(N))
ASSEMBLE MATRIX FOR GRAPHS
CON>CONS(JC)? + 3%
YES
NO
JC=JC+1
CONS(JC)=CON
TIME(JC)=TIM
CALCULATE NEW TIMES
TV=TV+ΔT
TIM=TV*Z2/Cv
END LOOP A
A

SECON

CALCULATE NEW TIME INCREMENT

SET UP (B) & (D) MATRICES (SEE S2.)

REDUCE B TO UPPER TRIANGULAR MATRIX

SOLVE FOR μ(I) AT NEW TIME

Using Gaussian Elimination and Back Substitution

CALCULATE β(I) AT NEW TIME

$$\beta(I,J+1)=\beta(I,J) - \frac{\Delta T}{2*DZ^2} *(\mu(I+1,J+1)-2\mu(I,J+1)+\mu(I-1,J+1)+\mu(I+1,J)-2\mu(I,J)+\mu(I-1,J))$$

CALCULATE ACTUAL EFFECTIVE STRESSES

$$p=po+(1-\mu)\Delta p$$

CALCULATE DEGREE OF CONSOLIDATION

$$CON = 100.*\frac{DZ}{2} * (\beta(1)+2\sum_{2}^{N-1}\beta(I)+\beta(N))$$

ASSEMBLE MATRIX FOR GRAPHS

CON>CONS(JC)? + 3%

YES: JC=JC+1; CONS(JC)=CON; TIME(JC)=TIM

NO

CALCULATE NEW TIMES

TV=TV+ΔT

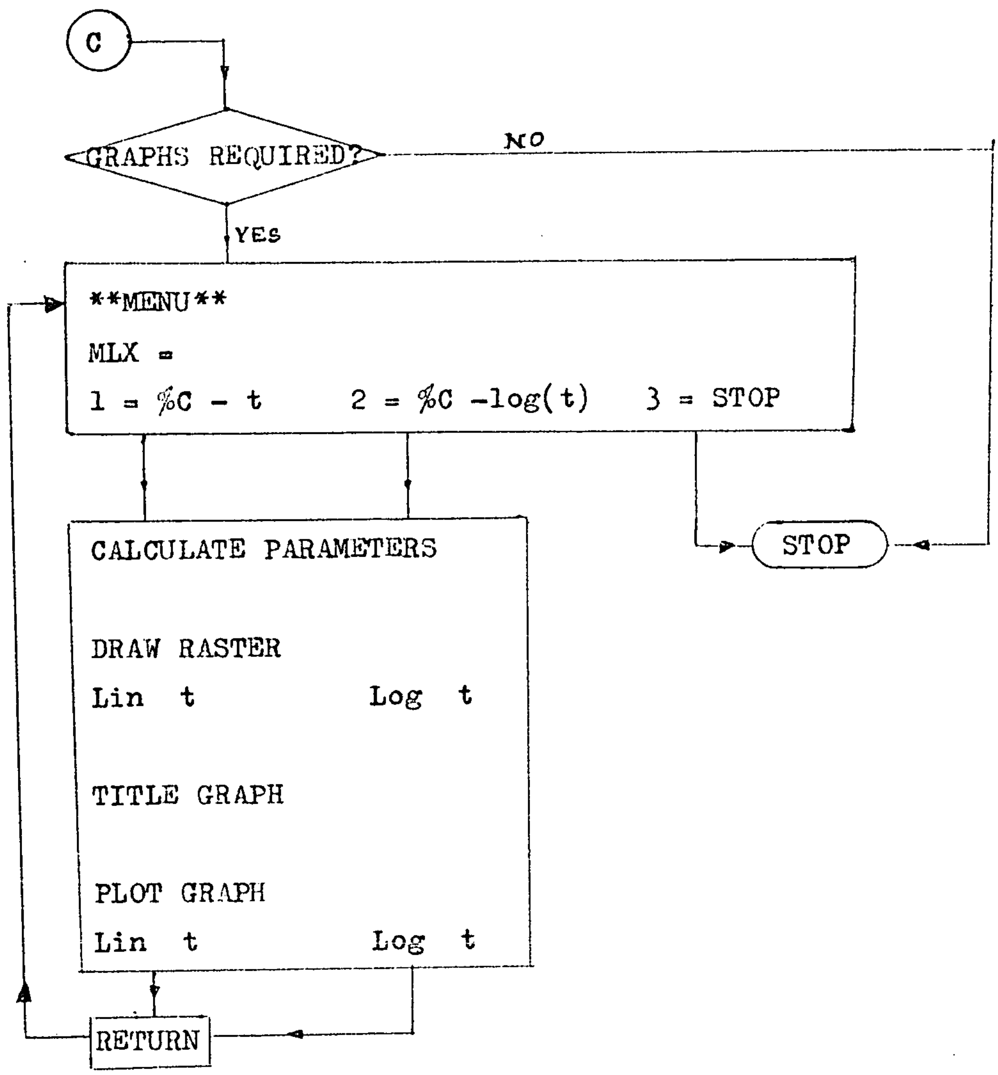
TIM=TV*Z^2/Cv

END LOOP A → (A)

***** ***** *****

S1. Computer time is saved by setting some coefficients of (B) outside the iterated loop.

S2. See text for handling of central difference equations


C
GRAPHS REQUIRED?
NO
YES
MENU
MLX =
1 = %C - t
2 = %C -log(t)
3 = STOP
STOP
CALCULATE PARAMETERS
DRAW RASTER
Lin t
Log t
TITLE GRAPH
PLOT GRAPH
Lin t
Log t
RETURN

CONGO

BEGIN

INPUT

READ NZ, Z, Cv
Δe, Δp
a, b, c, ti
po, pc, eo
NT
T(I) I=1,NT

CALCULATE

$ec=eo\left(\frac{pc}{po}\right)^{-a}$
pf=po+Δp
Av=Δe/Δp
DZ=Z/NZ
$Ti=ti*Cv/Z^2$

SET ACCURACY CRITERION

eps=0.001 (SEE C1.)

SET INITIAL PORE PRESSURES & STRAINS

Y(I)=1.0 ($=\mu$) (SEE C2.)
Y"(I)=0.0 ($=\beta$)

***** ***** ***** *****

Notes.

C1. Accuracy may easily be altered as required.

C2. JMAG10 solves one large matrix containing all variables. We use I=1 to NZ for μ, & L=NZ+1 to 2NZ for β. (Y"(I)).

SET FREE DRAINAGE BOUNDARY

$Y(1)=0.0$

$Y''(1)=(eo-ec*(\frac{pf}{pc})^{-b})/\Delta e$

IF $(pf<pc)$ $Y''(1)=(eo-ec*(\frac{pf}{pc})^{-a})/\Delta e$

WRITE HEADING FOR RESULTS

CALL JMAG10

STOP

***** ***** ***** *****

Notes.

JMAG10 is a library routine for solution of large sets of Ordinary Differential Equations. It is self-checking to the accuracy desired. If the accuracy criterion is not satisfied initially a smaller time step is selected and the calculations are repeated.

Two subroutines must be provided, to be called by JMAG10 - namely DIFFUN & OUTPUT shown below.

SUBROUTINE DIFFUN

(JMAG10)

INCREMENT COUNTER

CALCULATIONS TO DETERMINE DERIVATIVES

$p = pf - Y(I) * \Delta p$

$e = eo + Y''(I) * \Delta e$

$CREEP = \left(\frac{c * e}{Ti * \Delta e}\right) * \left(\frac{e}{ec}\right)^{1/c} * \left(\frac{p}{pc}\right)^{b/c}$

$\frac{\partial^2 \mu}{\partial Z^2} = (Y(I-1) - 2*Y(I) + Y(I+1))/DZ^2$

At impermeable bottom boundary

$\frac{\partial^2 \mu}{\partial Z^2} = (2*(Y(NZ-1) - Y(NZ)))/DZ^2$

$\frac{d\beta}{d\mu} = -a*e/(p*Av)$

IF(p>pc) $\frac{d\beta}{d\mu} = b*\frac{d\beta}{d\mu}/a$

$DY(I) = -\left(\frac{\partial^2 \mu}{\partial Z^2} + CREEP\right)/\frac{d\beta}{d\mu}$ $\quad \left(= \frac{d\mu}{dT}\right)$

$DY''(I) = -\frac{\partial^2 \mu}{\partial Z^2}$ $\quad \left(= \frac{d\beta}{dT}\right)$

(RETURN)

***** ***** ***** *****

Notes.

Called by JMAG10, this gives derivatives DY in terms of Y. Notice that here two formulae are required for DY and DY", the time derivatives of μ and β respectively. See also C2, above.

SUBROUTINE OUTPUT

JMAG10

↓

WRITE TIME

↓

WRITE Y(I)

↓

WRITE Y"(I)

↓

CALCULATE DEGREE OF CONSOLIDATION

$$SUM=100. *\frac{DZ}{2}*(Y''(1)+2\sum_{2}^{NZ-1} Y''(I)+Y''(NZ))$$

↓

WRITE SUM

↓

WRITE NO. OF ITNS, COMPUTER TIME ETC (OPTIONAL)

↓

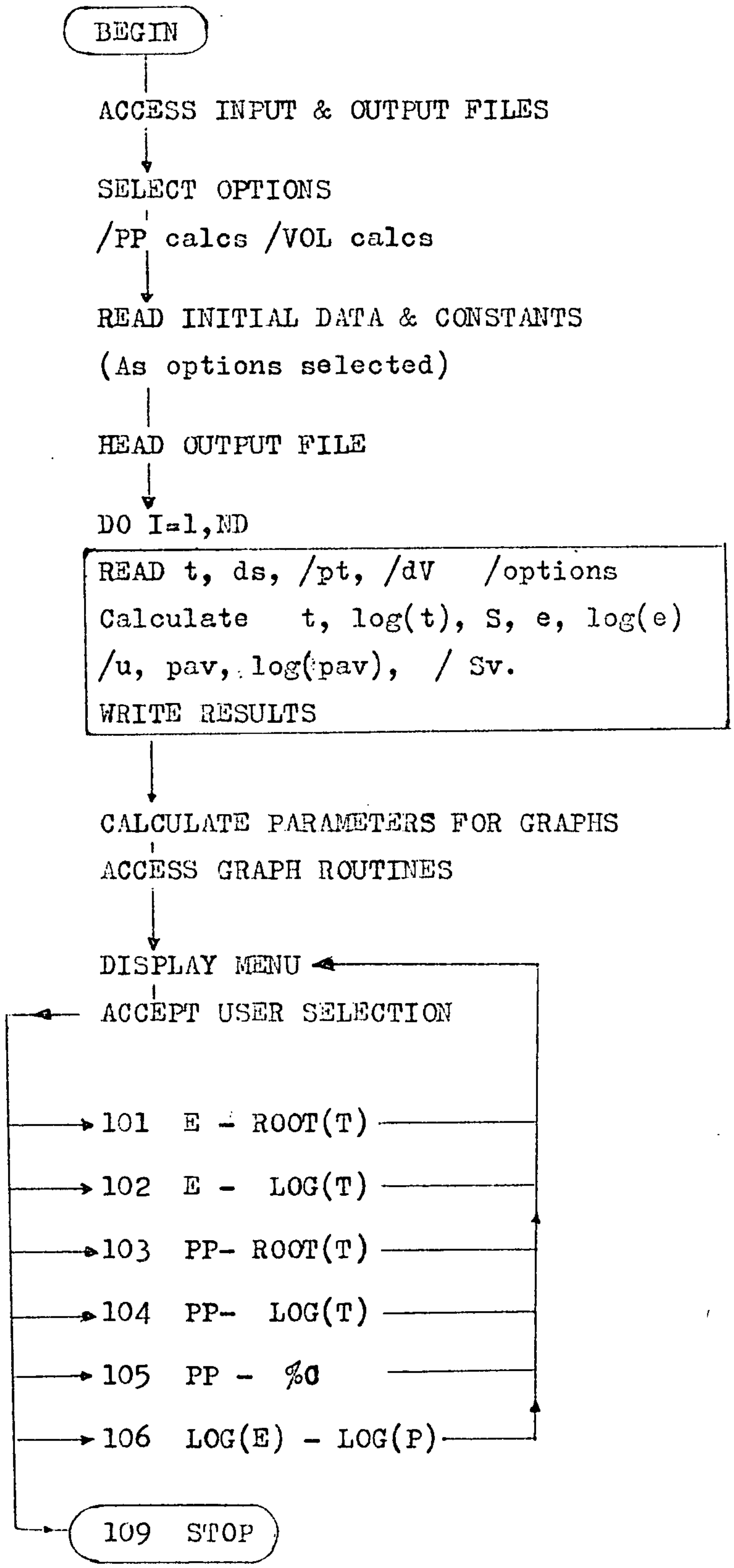
RETURN

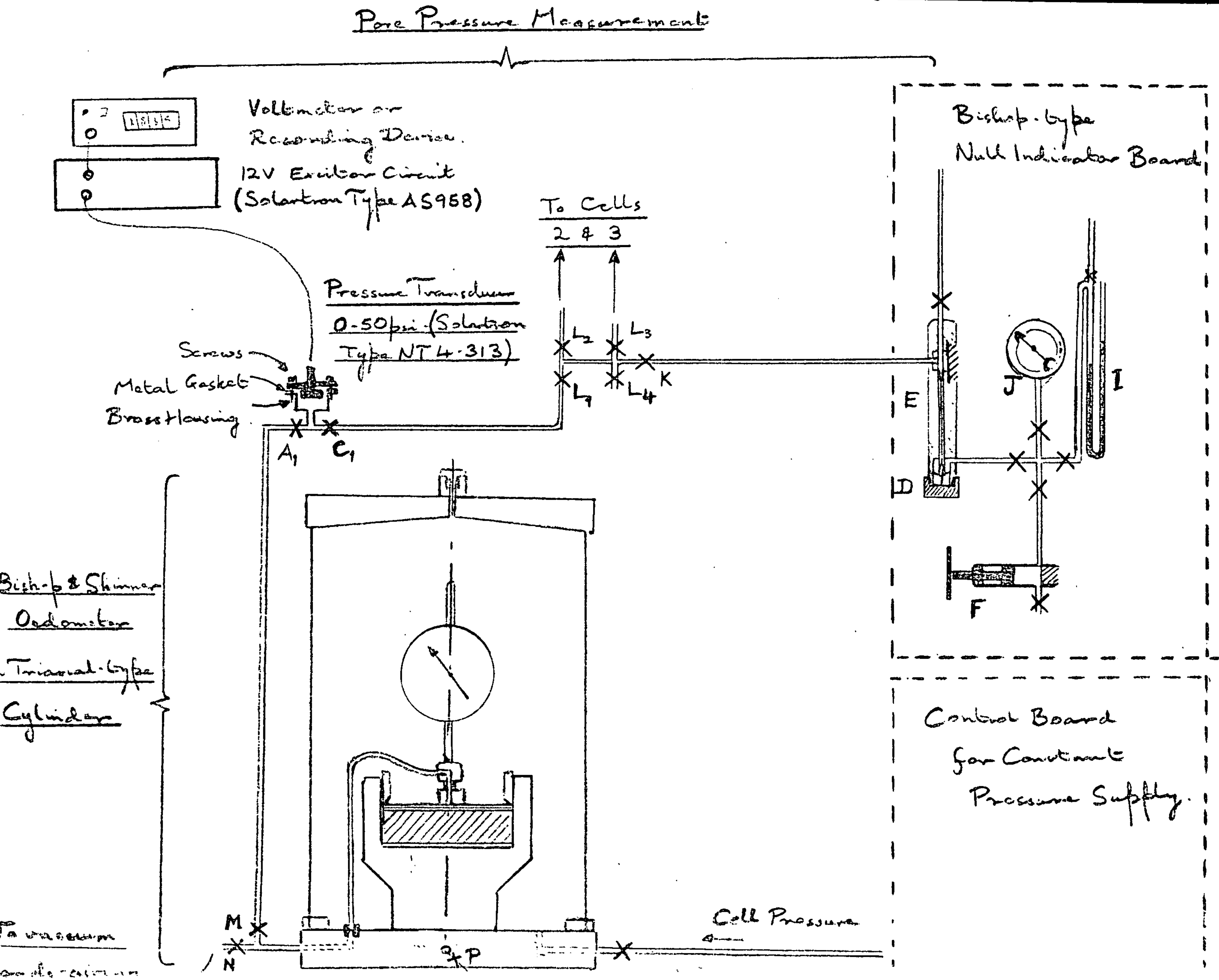
***** ***** ***** *****

Notes.

Called by JMAG10, this controls output, as defined by the array of output times

CALCOG


BEGIN
ACCESS INPUT & OUTPUT FILES
SELECT OPTIONS
/PP calcs /VOL calcs
READ INITIAL DATA & CONSTANTS
(As options selected)
HEAD OUTPUT FILE
DO I=1,ND
READ t, ds, /pt, /dV /options
Calculate t, log(t), S, e, log(e)
/u, pav, log(pav), / Sv.
WRITE RESULTS
CALCULATE PARAMETERS FOR GRAPHS
ACCESS GRAPH ROUTINES
DISPLAY MENU
ACCEPT USER SELECTION
101 E - ROOT(T)
102 E - LOG(T)
103 PP- ROOT(T)
104 PP- LOG(T)
105 PP - %C
106 LOG(E) - LOG(P)
109 STOP



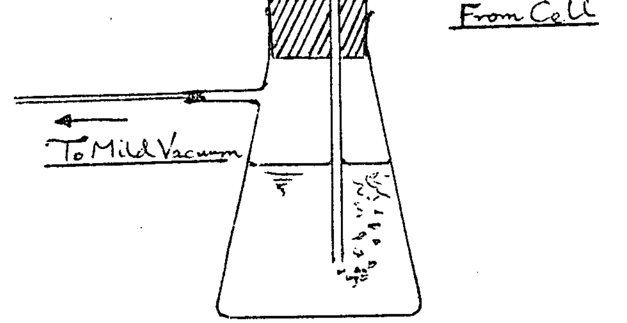
fully immersed in constant temperature water-bath. Transducer housing should be similarly immersed but not leads & connections which must be kept dry.

Drain from cell may be connected to burette (water level = midheight of cell) to measure volume changes.

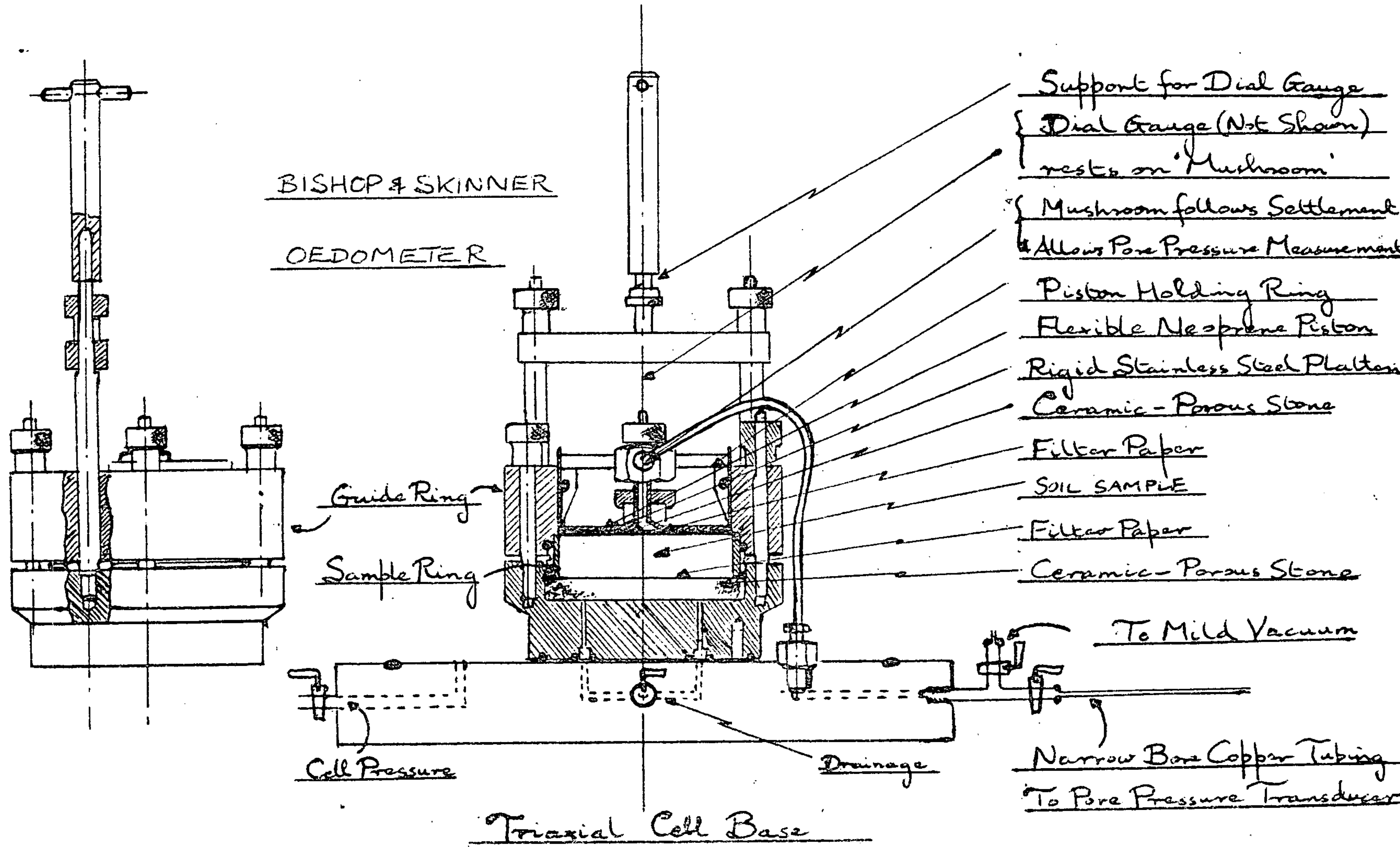
SKETCH of EXPERIMENTAL SET-UP

BISHOP & SKINNER OEDOMETER

Fig B1·1

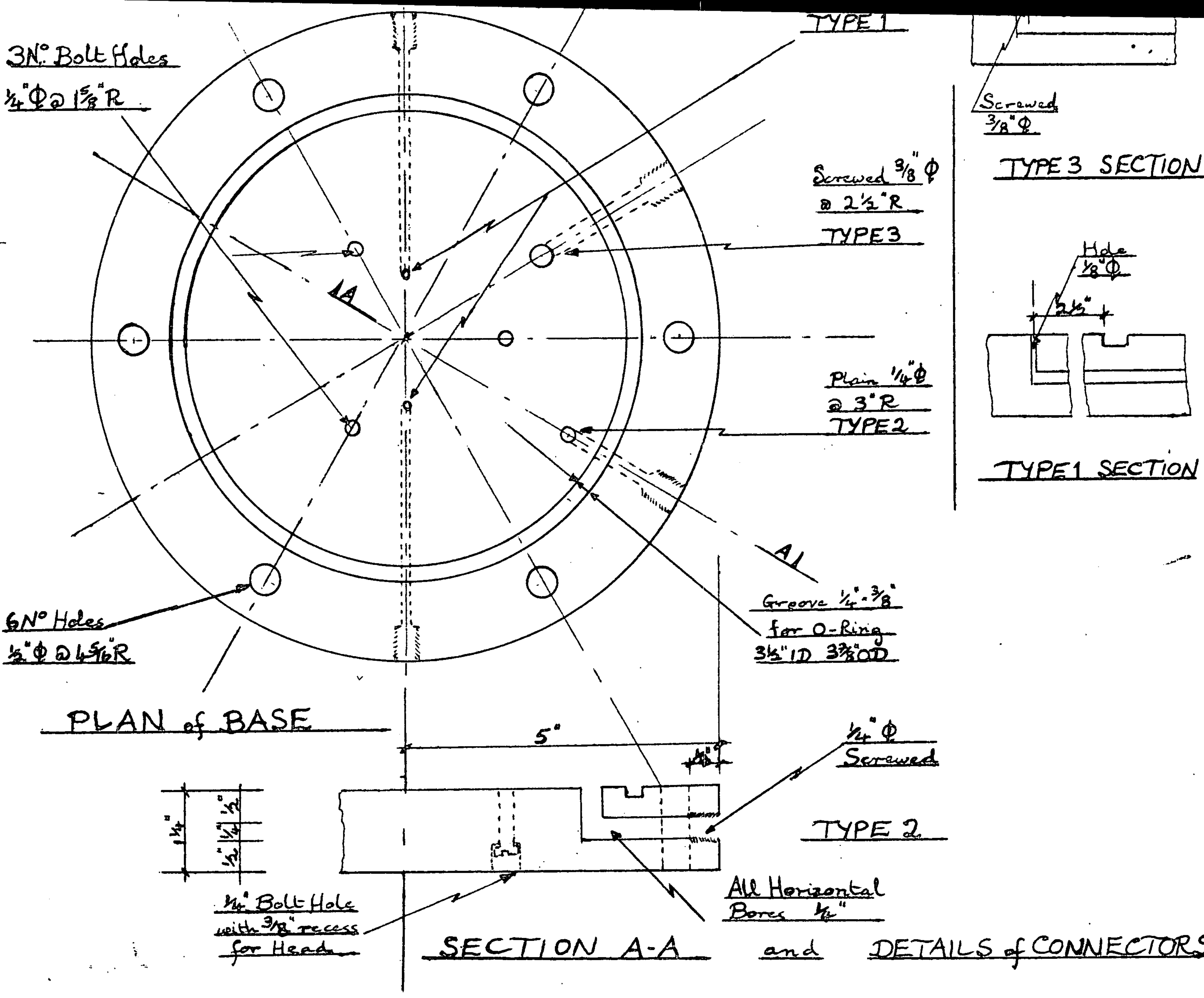


DE-AIRING BOTTLE



DETAILS of
BISHOP & SKINNER
OEDOMETER

Fig B1.2



Fabricated from 10"Φ, 1¼" thick Aluminium Plate

All Screw Threads for BSP. Gasfittings

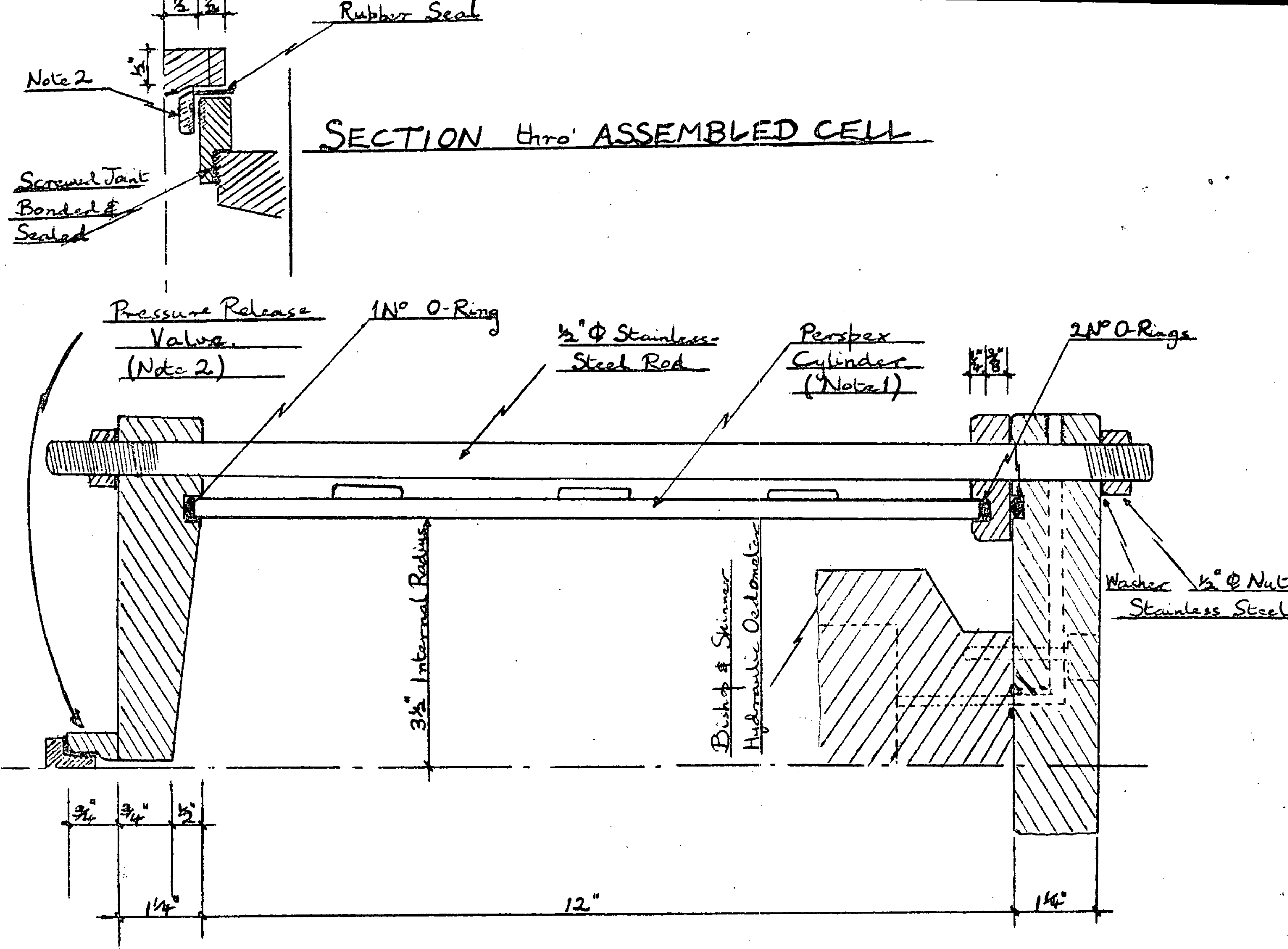
Horizontal Tappings take Klinger AB 10 valve.

DETAILS of CONSOLIDATION CELL

BASE PLATE

Fig B1·3

Scale ½":1" | July '76



ELE. N° EL25 414-10
Banded to take pressures up to 150psi.

2. Pressure Release
Tapping ¼" vertical, ⅛" inclined.
Nut & Sleeve fabricated from Brass

DETAILS of CONSOLIDATION CELL

CYLINDER & TOP-PLATE

Fig B1-4

Scale ½:1" | July '76

APPENDIX B.2

Standard procedure for consolidation testing

Equipment

The techniques described are for the modified version of the Bishop and Skinner Hydraulic Oedometer (Wykeham Farrance WF24501) - high quality consolidation testing with measurement of pore pressures. Constant temperature is maintained in the soil and pore pressure lines by immersing the apparatus (but not the transducers or their leads) in a constant temperature water bath.

Remoulding the soil

The aim is to compare a number of tests on a uniform material. The calculated amount of soil required plus ample for wastage is slurried to twice the liquid limit with demineralised, de-aired water in a plastic bucket with sealable top.

For each test a sample of approximately 300 mls is removed and vacuumed for at least 6 hours in the dessicator at approximately 30 inches of mercury vacuum. At the same time a porous plate should be vacuumed in a dish of de-aired water.

Preparation of cell

The main aim is to remove any trapped air. The system should be flushed through several times with demineralised, de-aired water - ensuring that the bottom porous plate is always covered. The burette (drainage) line should be flushed through and carefully checked for bubbles.

With about 1 cm of water in the cell, an inverted filter funnel connected to a vacuum line should be applied to the top of the cell for 1 hour (this may be reduced if the porous stone is still saturated from a previous test). Cardboard smeared with silicone vacuum grease has proved an effective gasket here. The drainage valve (P)* should now be kept closed.

The cell walls should now be liberally coated with silicone grease to minimise soil/wall friction. Greasing of the cell rings and O-rings prevents leakage (see Note on Leakage - below).

<u>Preparation of Pore Pressure System</u>

Several versions of top plate are available for this cell - see Note - below. The general procedure is similar for all, however. The problem is that pore pressure measurements must be made through the cell top, and placing this requires air to be drawn out through this line, so that full de-airing (which is essential for good pore pressures to be obtained) cannot be completed until the apparatus is set up.

The modified cell has two valves (M and N)* on a Tee so that the transducer and part of the line may be de-aired first. This should be done by flushing through de-aired water, holding the lines under pressure, further flushing etc.

Flexibility of the system may be determined by use of the Bishop Null Indicator device. A movement of 1 mm of the mercury thread $\equiv 0.787\ mm^3$. Movement of 6 mm over a pressure increment of 300 kN/m^2 was considered acceptable (giving flexibility $\doteqdot 15 \times 10^{-3}\ mm^3$ per kN/m^2). Note that the transducers must not be pressurised above 345 kN/m^2 (50 p.s.i.).

The pore pressure transducer should be calibrated against the gauge and manometer on the pore pressure board - or simply checked between a series of tests, the rigorous de-airing procedure should not be necessary.

By clamping a rigid seal over the tapping in the mushroom on the cell top, the flexibility of the whole system may be tested. The system should be left on 330 kN/m^2 for 1 hour or more to check for

* See fig. B1.1

leakage and appropriate action taken if necessary. (See also Note on Leakage below.)

In reporting the flexibility, the section up to the transducer (C) may be neglected since this is not employed in the actual tests.

Placing the sample

The sample ring is now filled very carefully with the de-aired slurry - taking care to entrap as little air as possible. A bottom filter paper should be used. The cell Guide Ring may be removed to give a flush top so that the sample may be scribed off to a thickness of 31.75 mm. The guide ring is now bolted in place and the slurry vacuumed using the inverted filter funnel device, for 1 hour. The slurry is then covered with a filter paper, and 50 mls of de-aired water pipetted on top of this. The porous stone, if required, is placed on top of the sample, floating it through the excess water on top. The vacuum is re-applied for 1 hour.

Meanwhile, a sample of slurry left over should be sent for determination of moisture content and any other tests desired.

Fitting Cell Top

The sides of the neoprene top piston should be well greased. Valve (M) to the transducer should remain closed. The open route via (N) connects the hole in the top membrane to a water bottle fitted to a mild vacuum device of around 100 mm Hg vacuum (see illustration). This allows air to be pulled from on top of the soil and observed bubbling in the water jar. Starting the vacuum, the top piece of the cell is now slid slowly down into place, expelling first the air and then the de-aired water. (When using the flexible top membrane, this should be coned upwards during placing to ensure all the air is pulled out. Great care is needed, however, not to overstress and rupture the rubber.)

Notice that this technique is not usually sufficient to remove all the air. Closing the vacuum line (N), more de-aired water should be flushed into the cell from the pore pressure board, by opening (M). This is then closed and further vacuum applied. The process should be repeated several times, gradually expending the water on top of the sample.

Standard Start of Tests

The dial gauge, ds, is now fitted and tightened down (it should, however, allow for slight swelling to occur). The presence of water above the sample complicates the initial setting. We assume the quantity is small and ds is set for a sample thickness of 31.75 mm. This must be checked, though, against the final thickness which can be determined with accuracy.

The pore pressure line should be closed from the null indicator set-up by closing valve (C). The vacuum line is disconnected, the cell housing cylinder bolted down, and the cell pressurised to a standard initial value of 30 kN/m^2. The transducer reading should climb quite rapidly to this reading. If this does not occur the system flexibility is unsatisfactory and the pressure should be removed and the cause investigated.

The cell should now be placed in the constant temperature water bath. The drain (P) may now be opened, noting the time, and the sample allowed to consolidate for 1 day. The zero reading on the transducer should be taken (notice that this is different from that set from the pore pressure board, due to differential head when the transducer is moved to a new position).

The Consolidation Tests

The actual consolidation tests are relatively simple to carry out.

Cell pressure (Q) and drainage (P) valves are closed. The cell pressure is now set to its new value[1] and sufficient time left for equilibrium to be sure in the pressure device.

The cell pressure valve (Q) is now opened (see note on Alternative Test Procedures - below) and the rise in pore pressure with time is noted. This should be allowed to build up for 24 hours, though most of the rise should occur quite rapidly. Any settlement should also be noted.

The drain (P) may now be opened, and the consolidation behaviour observed, for the desired length of time.

This procedure may be carried out for each standard test at a new pressure.

Unloading the Cell

Only after the cell has been unloaded can the analysis be carried out. Great care should be exercised to ensure the best possible results.

If a point on a Log(p) plot is required the p = O condition is useless, so the value of 30. kN/m^2 is again used. The sample, when removed, must be at an identifiable state of strain, i.e. on its secondary phase.

Following the final positive load increment, the cell pressure is reduced to 30 kN/m^2, with the drain open and the burette initially full. Again settlement and pore pressures are noted. This pressure must be maintained for at least 24 hours to accurately define the soil state.

* * *

1. The weight of the top platten must be taken into account. For the rigid stainless steel platten glued to the neoprene membrane this was found to exert a pressure of 0.7 kN/m^2 on the soil. This is most easily allowed for by reducing the applied cell pressures by this amount for each loading required.

The cell pressure is now again reduced to atmospheric, consolidation behaviour noted, and at least 24 hours allowed.

Before removing the sample the drain should be closed.

The sample and guide ring are removed together, once the apparatus has been dismantled. The thickness of the soil, filter papers and platten together should now be found with a micrometer. (The unloading procedure should ensure that soil swelling here is minimal.) The soil may now be removed from the ring, the platten and filter papers stripped, and the individual thicknesses determined.

The sample, in a suitable container, should quickly be weighed. It may now be sent off to the oven for drying and moisture content determination.

The pore pressure transducer calibration and flexibility should be checked.

Finally the cell should be thoroughly cleaned and left readyfor the next experiment.

Alternative Test Procedures

The above techniques describe the A-type test. The pore pressure is allowed to build up for 24 hours after incrementing the load, before the drain is opened. Classical Terzaghi theory predicts that this pore pressure will rise immediately to the value of the pressure increment. In fact, this rise will take place over a period of time, and does not always reach 100% of the applied pressure. Reasons for this are:-

1. deviations from classical theory
2. experimental errors.

Full analysis of this is complex; suffice it to note here that the pressure rise over the 24 hours should be recorded.

These results may be contrasted with those from the C-type test which is identical to the above, except that the drain is left open, so that consolidation begins immediately on incrementing the load. Since both these types of test have been recommended by different workers, it is useful to examine whether significant differences in results are obtained.

Recent work has highlighted the importance of the critical pressure, pc, on soil behaviour. One major factor here is thought to be the duration of previous secondary (or delayed) consolidation. This may be examined indirectly by A and C type tests, estimating the average effective stress in the soil from the pore pressure readings and hence plotting the stress strain relationship.

The B-type test has been devised to give detailed information about pc. With the drain (P) open, the load is incremented in small steps. Below pc the consolidation takes place rapidly so that each increment need last only 10 minutes. (Notice that delayed consolidation still occurs here, which is a complication, but over so short a time as 10 minutes this may reasonably be neglected.) Ten equal steps of 1/10th of the load increment each has been found to give satisfactory results. When p exceeds pc there will still be significant settlements after the 10 minutes, so that results cannot be directly interpreted. It may be preferred here to jump straight to the final load. Pore pressures should also be recorded during this test and should give further information about pc.

Some soils will evolve gases during consolidation, and tend to become unsaturated. A back pressure may be used to keep such gases in solution. The E-type test involves application of some 240 kN/m^2 pressure back through the drainage line, and the applied loading is in excess of this value. Otherwise the test is carried out exactly as for the A-type above.

Further details of Experimental Procedure

1. Leakage

Without great care this is a major problem for tests with this type of cell - even minor leakage completely ruins results. The following points should be observed.

The nuts holding down the guide ring should be fully tightened. The nuts on the pore pressure line between top and cell-base should be teflon-taped and well tightened, but care is needed not to damage the ends of the plastic tubing.

Grease should be applied liberally to all inside cell walls; the internal edges of the cell ring and 2 O-ring seals; and the outside walls of the neoprene piston.

Fitting the flexible piston presents problems as the rubber seems liable to rupture, which leads to leakage. (Such small holes may be temporarily repaired with a rubber patch 'araldited' in place, but this is now weak and liable to fail again.) The cell top must be placed with great caution.

Despite this, leakage past the top still presents something of a problem, and for this and other reasons a new rigid top piston has been designed and used - see below. Using this piston and painstaking care in setting up the cell, it has proved possible to obtain satisfactory results.

2. Top Pistons

The original design, as supplied by Wykeham Farrance, includes a flexible neoprene top piston to which the centre 'mushroom' is connected. The intention is that the consolidation behaviour is measured at the centre of the specimen so that it is largely unaffected by side friction. This admirable intention is only achieved at the price of significant departures from one dimensional behaviour. Preliminary investigations suggest that not only strain, but also pore water behaviour deviates unacceptably from conditions amenable to analysis.

The simplest solution to obtain one dimensional behaviour is to place a ceramic disc and rigid platten between the top of the soil and the piston. This system, using stainless steel plattens, was tried also, and found to suffer from three principal defects - viz.

a) The neoprene appears to be very weak and even with very careful handling is frequently damaged, in the form of pin-holes and small tears appearing. Temporary repair could not be relied on.

b) Pore pressure results suggested the system was too flexible, although the line from the transducer to the mushroom could be shown to be satisfactory. It is suspected that water could flow between the piston and the platten greatly increasing the flexibility.

c) Several workers have noted the problem of diffusion of water across rubber membranes. This could prove unsatisfactory in tests of 1 month's duration.

The best solution therefore appeared to be to bond the platten to the piston. This was done, Araldite proving a suitable adhesive. This form of top piston has proved satisfactory for all the oedometer tests reported here.

APPENDIX C

The Influence of Consolidation on Shear Strength

It is evident that the shear strength of a soil will be influenced by the state of consolidation. This topic is very involved and could not be considered in great detail in the present study. However, an interesting similarity was found between Bjerrum's model and the Critical State approach. This, in turn, leads to an examination of Hvorslev's theory of shear strength, from which it will be seen that the various types of behaviour in consolidation suggested by Bjerrum may be related to changes in cohesive and frictional strength parameters.

In Critical State Soil Mechanics (Schofield and Wroth 1968) the "Virgin Compression Curve" is given as a straight line relationship between water content, w, and logarithmic effective stress, σ', such that

$$w - w_o = - \ell \, \ell n \, \sigma' \qquad (C.1)$$

where w_o is the water content at $\ell n \, \sigma' = 0$. Now this Virgin Curve may be considered similar to Bjerrum's Instant Line, using an e - log σ' plot (i.e. reverting to Bjerrum's (1967) presentation, rather than Garlanger's log e - log σ' lines). The gradient of the instant line is defined as

$$C_c = \frac{\Delta e}{\log(\Delta\sigma')} = \frac{2.3 \, \Delta e}{\ell n \, (\Delta\sigma')} \qquad (C.2)$$

For a fully saturated clay

$$w = \frac{e}{G_s} \qquad (C.3)$$

whence, since both (C.1) and (C.2) are valid expressions of the virgin compression curve for a given clay,

$$\ell = \frac{C_c}{G_s \times 2.3} \qquad (C.4)$$

Now Hvorslev (1960) related the soil strength to an effective angle of friction ϕ'_e, which does not vary with water content, and a cohesion k, which is a function of water content. He also introduced the idea of an equivalent effective stress (σ'_e), which is the effective stress corresponding to any given water content on some normal consolidation curve. By relating shear strength to this equivalent stress, using a Coulomb type expression, he obtained

$$\frac{\tau}{\sigma'_e} = \frac{\sigma'}{\sigma'_e} \tan \phi'_e + k_o \qquad (C.5)$$

where k_o is the cohesive component on the equivalent stress line, which remains constant.

Hvorslev gives the equation of the equivalent stress line as

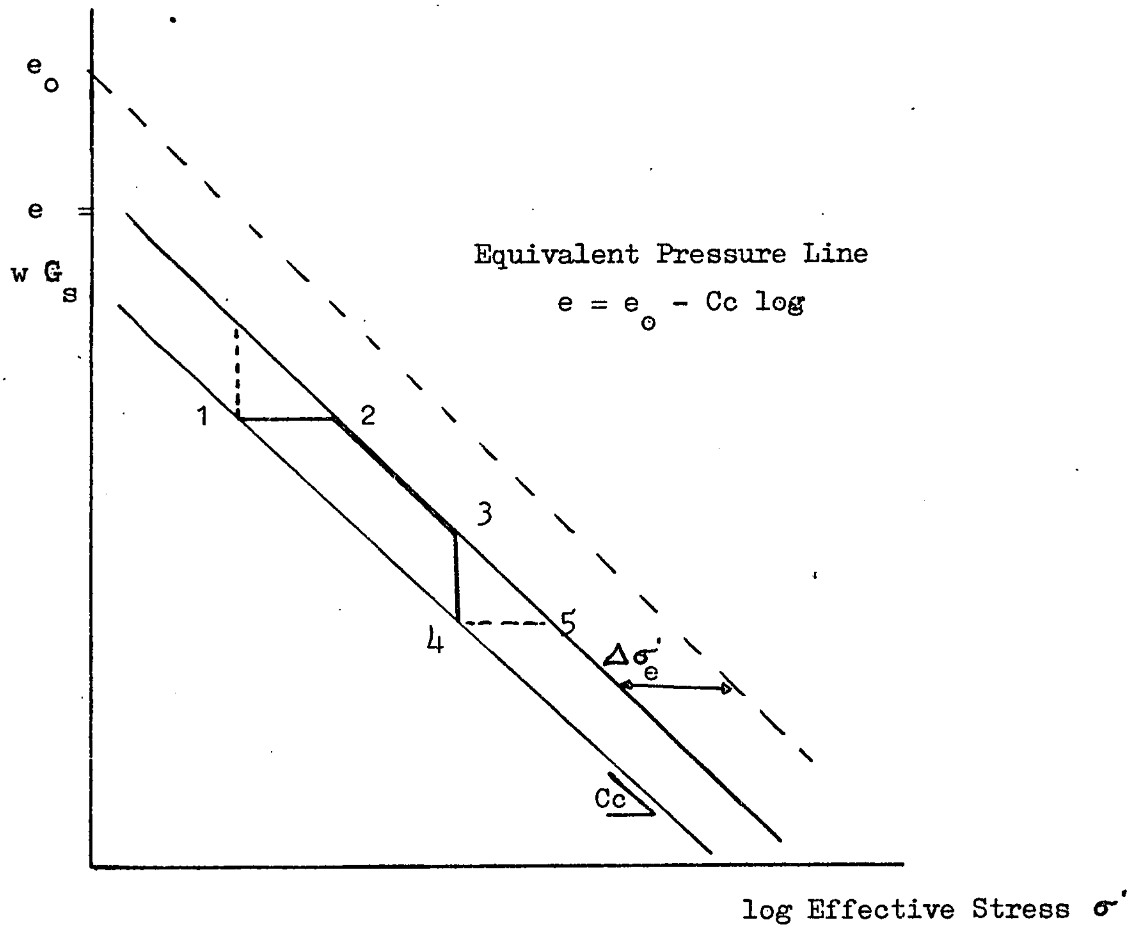
$$e = e_o - C_c \log (\sigma'_e) \qquad (C.6.)$$

which is the same gradient as the Virgin Curve and Instant Line already introduced. The question arises, however, as to the positions of all these in e - log σ space. Do they coincide, or are they different? In fact none of these lines are adequately defined for soft clays. All suffer from the problem discussed in previous sections of this thesis, that time or strain rate effects are of significance. Schofield and Wroth have taken Hvorslev's equivalent pressure line to be identical to their virgin curve. The present author believes Bjerrum's Instant Line is intended to describe the same behaviour as is the virgin curve, though for the present analysis these lines need not coincide, since behaviour will be examined relative to some arbitrary equivalent pressure line, the position of which is eliminated by considering only change in soil stress-strain states.

Multiplying through by σ'_e in equation (C.5) we may write

$$\tau = k + \sigma' \tan \phi'_e \qquad (C.7)$$

Fig C.1 Shear Strength Behaviour of Soft Clay during One Dimensional Consolidation.



(Note: definition of e_o & w_o, corresponding to $\log\sigma' = 0$. This differs from e_o used elsewhere in this thesis.)

where

$$k = k_o \sigma'_e = k_o \exp\{(w_o - w)/\ell\} \quad (C.8A)$$

$$= k_o \exp\{2.3 \Delta e/C_c\} \quad (C.8B)$$

The most convenient form of this equation for the present work relates the shear strength τ_n at any point n, to the effective stress at that point, and the equivalent stress parameters ϕ'_e and k_o, which are considered to be fundamental, and unaffected by the consolidation process.

$$\tau_n = k_o \sigma'_{e_n} + \sigma'_n \tan \phi'_{e_n} \quad (C.8C)$$

Referring to fig. C.1 we may now consider Bjerrum's types of consolidation behaviour.

For a-type consolidation very little strain occurs over the pressure increment up to p_c. We begin by taking the simplified case of no strain. Hence $e_2 = e_1$ and therefore $\sigma'_{e_2} = \sigma'_{e_1}$

$$\tau_2 - \tau_1 = k_o (\sigma'_{e_2} - \sigma'_{e_1}) + (\sigma'_2 - \sigma'_1) \tan \phi'_e \quad (C.8)$$

$$= (\sigma'_2 - \sigma'_1) \tan \phi'_e \quad (C.9)$$

i.e. the change in strength is directly proportional to the change in normal effective stress.

For the real case, allowing for a little strain, a small component due to $k_o (\sigma'_{e_2} - \sigma'_{e_1})$ will also contribute here.

For b-type consolidation

$$\tau_3 - \tau_2 = k_o (\sigma'_{e_3} - \sigma'_{e_2}) + (\sigma'_3 - \sigma'_2) \tan \phi'_e \quad (C.10)$$

Now we have seen that the line (2) - (3) is parallel to the equivalent stress line, but, depending which time line is considered, it will be positioned some distance away. We term this difference in stress $\Delta\sigma_e$ at constant voids ratio. Then

$$\sigma'_{e_3} - \sigma'_{e_2} = (\sigma'_3 + \Delta\sigma_e) - (\sigma'_2 + \Delta\sigma_e) = \sigma'_3 - \sigma'_2 \qquad (C.11)$$

$$\tau_3 - \tau_2 = (\sigma'_3 - \sigma'_2)\ (k_o + \tan\phi'_e) \qquad (C.12)$$

Thus we see that for consolidation progressing along a time line

$$\frac{\Delta\tau}{\Delta\sigma'} = k_o + \tan\phi'_e = \text{constant} \qquad (C.13)$$

The change in shear strength is again directly proportional to the change in effective stress.

For c-type consolidation the effective stress remains constant, $\sigma'_4 = \sigma'_3$. Hence

$$\tau_4 - \tau_3 = k_o\ (\sigma'_{e_4} - \sigma'_{e_3}) \qquad (C.14)$$

Now from (C.2)

$$\ell n\ (\Delta\sigma'_e) = \frac{2.3\ \Delta e}{C_c}$$

$$\tau_4 - \tau_3 = k_o\ \exp\{\frac{2.3(e_3 - e_4)}{C_c}\} \qquad (C.15)$$

This ties in well with the known behaviour. During delayed (c-type) consolidation the cohesive strength of the soil increases significantly - given here as exponentially with decrease in voids ratio.

We may further note that the change in voids ratio during delayed consolidation is frequently taken as constant with logarithmic time.

Then

$$\Delta e = C_\alpha \log \Delta t = \frac{C_\alpha}{2.3}\ \ell n\ \Delta t \qquad (C.16)$$

$$\text{From (C.2)} \quad \ell n(\Delta\sigma'_2) = \frac{C_\alpha}{2.3}\ \frac{2.3}{C_c}\ \ell n\ (\Delta t) \qquad (C.17)$$

$$\tau_4 - \tau_3 = k_o\{t_4 - t_3\}^{C_\alpha/C_c} \qquad (C.18)$$

Thus the cohesive component of shear strength of the soil increases with the increment in time raised to the power C_α/C_c, which is, of course, very much less than 1. It has not, as yet, proved possible to further examine this predicted relationship but it would appear to be promising, and qualitatively, at least, in keeping with known soil behaviour.

It must be emphasised that expression (C.18) refers to the delayed consolidation process so that time, t, refers to time line values which will not, in general, be the same as the absolute times from first loading the soil; they are rather an expression of the strain-rate dependency of the skeleton.

Combining Bjerrum's model of consolidation with Hvorslev's shear strength model, then, identifies a-type consolidation as that where the main change is in the frictional component; b-type as that where frictional and cohesive components are both directly proportional to the effective stress change, in constant ratio; and c-type as that where only the cohesive strength changes.