Apparatus for Measuring Strain and Applying Stress, with an account of some Experiments on the behaviour of Iron and Steel under Stress,

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CONTENTS.

Sections

Introduction.

The behaviour of metals under stress has long been the subject of investigation both by mathematicians and physicists, so that the laws of strength are tolerably complete. Owing to the importance of iron and steel in construction, these materials have been subjected to very extensive tests, particularly in simple tension and compression.

Numerous tests of cylindrical iron and steel bars in torsion are also available; the bulk of these being tests to destruction of samples of material used in actual machines and structures designed by engineers. In such tests scientific accuracy is not of much importance; the chief consideration being the obtaining of sufficient data for use in design. The most accurate torsional work upon iron and steel has been the work of physicists, and nearly all their investigations have been conducted upon specimens of very small sectional area; the reasons for this no doubt being that such specimens in the form of wires are easily obtainable of great uniformity in size and quality, while large test pieces are costly to prepare, and moreover cause considerable difficulty in testing, because of the magnitude of the forces involved. Owing to the mode of manufacture, the physical properties of wire often differ to a considerable extent from turned specimens of iron and steel. These differences may be caused by the hardening effect of the drawing, minute cracks in the wires, want of roundness, and the like; It therefore appeared probable that experiments on the lines indicated by physicists would be of some service, and it was with this idea that the investigation was commenced.

The chief difficulty in the accurate investigation of the torsional properties of metal bars lies in the lack of suitable apparatus for the work; and after reviewing the chief machines available for measuring strain and applying torque - to all of which there seemed some objection - it was resolved to design and construct special appliances for the work.

Attention was first directed to the design and construction of a self-contained instrument for measuring strain, which should be sufficiently accurate to measure strains of one second of arc; and after some experiments an instrument was constructed which satisfied these conditions.* A modification of this was used in the work of this paper, and is described in Section II.

In most machines for applying torque the construction is such that the weigh lever can only be used for torsion in one direction, and the ends of the specimen are fixed, so that it is impossible, for instance, to apply a bending moment and torque, or a tension and torque, together.

A machine was therefore constructed to allow of torque in either direction, and also permit of the application of a uniform bending moment and a pure torque to give a combined stress. A separate device was constructed for giving the combined stresses of tension and torsion.

*Coker "On Instruments for measuring Small Torsional Strains;" Phil. Mag., 1899. December

Description of the apparatus.

Instrument for measuring strains.

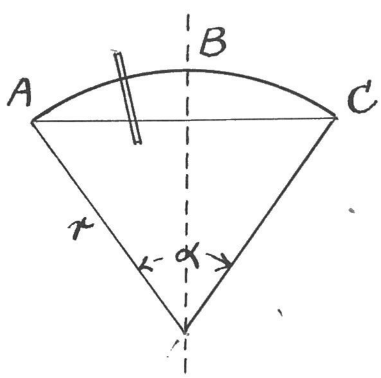
In making measurements of small strains it is a great advantage to use an instrument which will read directly, and which is self-contained and wholly supported on the specimen under test, thereby avoiding external scales whose positions with respect to the specimen may be changed by a disturbing element, such as slipping of the grips, applied bending moment, and the like. In order to meet these conditions an instrument was designed for the purpose of these experiments, and is shown in sectional elevation by Fig.1 and in side elevation by Fig.2. It consists of a graduated circle A mounted upon a chuck plate B, provided with three centering screws adjustable by hand. Upon the Vernier plate J an arm O carries an extension K, upon which is secured a frame X carrying a thick wire P. The movement of the wire is observed by a reading microscope carried in the sleeve R of an arm S mounted upon the short cylinder C, which latter is gripped upon the test bar by screws L.

The reading microscope has an eye-piece T provided with a glass scale, and a right-angled prism is interposed between this and the objective W, so that readings can be easily taken. The tube Q is free to slide or rotate in its guide R, but, in order to readily focus the wire, this latter is carried in a frame X pivoted upon the Vernier plate J, and adjusted by a screw V.

The microscope arm S is secured to the cylinder C by a divided collar, the two halves of which are pivoted on one side, and the free ends are clamped by screws.

If it is desirable to turn the telescope round or to release it altogether, the screw may be thrown out of engagement. Readings are taken from one edge of the thick wire, and as this edge is very distinct, it fatigues the eye much less than a spider line or scratch upon glass, which latter have the further disadvantage of being of appreciable thickness. Fig.5 shows the appearance of the field of view of the reading microscope and the wire P.

No appreciable error is caused by the fact that the divisions upon the glass scale of the microscope are linear measurements, while the movement of the wire is in a circle. For if ABC be the path of the wire and AC the chord, then the error is the difference between the arc ABC and its chord AC when the angle is a small quantity of the first order.



i.e. $\Delta = r\alpha - 2r\sin\frac{\alpha}{2}$

$$= r\alpha - 2r\left(\frac{\alpha}{2} - \frac{\alpha^3}{2^3\,\underline{|3}} + \frac{\alpha^6}{2^6\,\underline{|6}} - \&c\right)$$

$$= r\left(\frac{\alpha^3}{2^3\,\underline{|3}} - \frac{\alpha^6}{2^5\,\underline{|6}} + \&c\right)$$

a small quantity of the third order, and therefore negligible. In practice this is shown to be the case, as no difference can be observed between the parallelism of the wire P and scale for any position of the former. The reading of the microscope scale may therefore be taken as directly proportional to the angular displacement, and the calibration is effected by moving the wire through a definite angle and noting the equivalent reading of the micrometer eye-piece.

It is essential that the graduated circle be set accurately upon the bar, with its plane perpendicular thereto and its centre coinciding with the longitudinal axis of the bar. An arrangement was devised to effect this, consisting of a pair of divided collars a, the halves pivoted together at b, and secured by nuts c. The collars are wedge-shaped, in radial section to engage with correspondingly wide-angled grooves; upon the chuck plate and cylinder only the angled sides being in contact, so that the collars are readily fixed or freed when required. The lower halves d of the divided collars are connected by one or more distance-pieces e, so that when the former grip their respective grooves each piece has one degree of freedom with respect to the clamp, and this is sufficiently suppressed by the frictional grip of the collars, thereby causing the parts to act as one rigid whole for setting the instrument on the bar.

Both main pieces are chucked by set-screws, and with a little experience this can be effected as accurately as by self centering chucks.*

* Coker "On a new Instrument, etc." Phil. Mag. December 1899

In nearly all machines for applying torque some bending is also present, and it is therefore necessary to eliminate any possible errors due to this cause.

If the bar is bent in the plane containing its centre line and the observation wire, it has the effect of causing new parts of the wire to come opposite the scale, but no error in reading is caused thereby. If, however, the bar is bent in a plane at right angles to this, the effect of the bending will be read as an addition to or subtraction from the twist. Bending in any other plane may be resolved into components in these two planes, and it is therefore only necessary to eliminate the error due to bending in a plane perpendicular to the plane of the paper.

The error may be got rid of by using two reading microscopes set opposite to one another, and a mean reading taken, but as this doubles the labour of observation, it is inconvenient. Another plan is to arrange the wire mid-way between the sections gripped by the set-screws: then if

$2a$ be the length under measurement,

θ = angle of bending at first section

φ = angle of bending at second section

error in reading becomes $a(\sin\theta + \sin\varphi)$

and if θ and φ are equal and opposite the error vanishes. A specimen stressed by two equal and opposite couples applied at its ends bends into the arc of a circle, and fulfils the necessary condition for the equality of θ and φ, and in the application of torque this condition has been fulfilled. As a matter of precaution the observation wire is always set in the plane of bending.

In order to test the accuracy of the instrument torsion tests were made, (I) with no bending, (II) with bending moment of known amount. As an example the following may be quoted.

Turned bar of rivet steel — Torsion arm = 15·00

Diameter 0.662 — Calibration

Length under test — 1 min. = 54·4 divns

Table I

Torque in inch lbs	No Bending Moment		Bending Mt 480 inch lbs		Bending Mt 800 inch lbs		Bending Mt 1120 inch lbs	
	Readg	Δ	Readg	Δ	Readg	Δ	Readg	Δ
0	0		0		0		0	
7·5	51	−51	51	−51	50	−50	50	−50
15·0	102	−51	101	−50	101	−50	100	−50
22·5	153	−51	152	−51	150	−49	150	−50
30·0	204	−51	203	−51	200	−50	201	−51
37·5	256	−52	254	−51	251	−51	252	−51
45·0	307	−51	306	−52	301	−50	303	−51
52·5	357	−50	356	−50	352	−51	354	−51
60·0	408	−51	406	−50	404	−52	404	−50
67·5	459	−51	457	−51	455	−51	453	−49
75·0	509	−50	507	−50	505	−50	503	−50
0	2		1		1		2	

The differences caused by bending will be discussed in Section X

Machine for applying Torque and Bending.

The apparatus used for applying twist in either direction, and for the combined stresses of twist and bending, is shown in side elevation by Fig.6, in plan by Fig.7, and in sectional and elevation by Fig.8. The machine was specially constructed for the work of this paper, and consists essentially of two similar and equal castings A, bored axially to receive double coned spindles B, the outer ends of which project through the castings and are secured by nuts C. At the weigh-lever end the cone is secured to the casting by studs D, but at the other end, in order to take up the twist upon the specimen, the cone is gripped partly by the back-nut C, and partly by a plate E pressed against its face by studs.

Each casting is bored at right angles to the axis to receive arms F, G, of which the former are used for hanging weights therefrom to give the torque, while the latter G one carries a link H having an adjusting screw I and nut J whereby the weigh levers F can always be brought to the horizontal position; the final adjustment being made with a sensitive level K, while the other carries a balance weight L. The ends of the specimen M under test are secured in grips N N upon the projecting ends of the cones B.

In order to obtain a pure torque and a pure bending moment, both acting at the same time, each casting is supported by a ring (Fig.9) encircling the spindles B, and furnished with friction rollers P running in grooves in the spindles B. The rings have bearings Q turning in stirrups R, these latter being hung from an horizontal bar S by adjustable vertical hangers T, and we get a pure torque of a known amount throughout the specimen.

Bending Moment. Into the outer ends of the nuts C are screwed projecting arms V, of known length and carrying weights at their ends. These put a bending couple upon the specimen without shear, the arm of the couple being the distance of the weight from the hanger I. With this arrangement simple twist and simple bending or any combination thereof can be applied to a specimen with ease. The specimen is free to take up its own position of equilibrium, since it is imperfectly constrained - (the specimen can be easily rocked about even when fully loaded, but always comes back to the first position after a few oscillations)-and the condition of stress is accurately known.

Corrections - Twist. The results of tests on the friction of the roller bearing show that the friction is so small a quantity as not to introduce any sensible error.

Bending. The friction error is that due to the stirrups embracing the bearings Q, which latter were made large purposely. No experiments were made in which the bending moment varied during the experiment; consequently it was sufficient in each case to calculate the error due to friction for the particular load applied, and make the small corrections necessary. This has been done.

Apparatus for applying Torque and Tension.

The apparatus for applying torque and tension is shown in general elevation by Fig. 11, and in plan by Fig.12. A detailed section of some of the parts with the measuring instrument fixed to the specimen is shown by Fig.13.

The specimen A was screwed into a turned piece B, having a slotted hole above, and tapped to receive a screw C centred in a corresponding depression in a piece of tool steel D resting upon a plate E, which latter was carried by four bolts F depending from a cast iron beam G mounted upon two pillars H. Below the specimen screwed into a turned piece J, carrying a sleeve and pulley L, the lower end of the piece being fitted to receive a nut M and hanger N for weights. The torque was applied by weights attached to fine steel bands made of clock spring, which latter were attached to the pulley at convenient points, and passed over guide-pulleys O mounted on ball-bearings. The applied torque was balanced above by a double-ended lever P, keyed to the piece B so that its axis passed through the point of suspension, and furnished with screws at its outer ends, so that the ends pressed equally against the pillars H. The weights used for applying the torque were made by Oertling or were copies therefrom, and the twelve 200 pound weights for applying the tension were standard weights forming part of the equipment of the 100 ton Buckton testing machine in the laboratory. The method of suspension ensures that all the tension load is evenly distributed in the section of the specimen, and there is no correction for friction, as the load is a dead-weight one.

In applying the torque a small correction must be made for the friction of the pulleys. This was determined as follows:

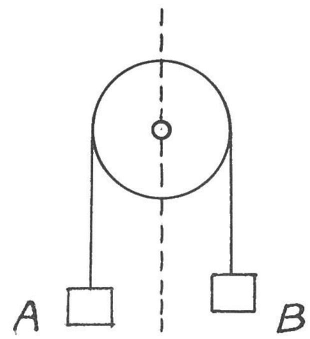


Fig 14

The pulley was first balanced by winding lead strips round its arms until it would stand in any position, or when rotated by a smart pull, continue to revolve several minutes. Next, equal weights (A, B) were attached to the spring steel tape passing over the pulley with the ends vertical, and the additional mass required to just start the pulley in one direction was determined. The weights were then reversed, and the additional mass required was again determined; the mean value of the two being taken.

Let T_1 be the tension on one side:

T_2 " " " " the other side

T_3 = tension horizontally

We have

$$\frac{T_2}{T_1} = \varepsilon^{\mu\pi}$$

$$\frac{T_3}{T_1} = \varepsilon^{\mu\frac{\pi}{2}}$$

very approximately;

from which we get by a simple transformation

$$T_3 = \sqrt{T_1 T_2}$$

This value was calculated throughout the range. As an example the following numbers may be quoted for the left-hand pulley:

Table II

T_1	$T_3 = \sqrt{T_1 T_2}$
2	2.0078
4	4.008
6	6.0088
8	8.0095
10	10.0113
12	12.0118
14	14.012
16	16.013
18	18.014
20	20.015

The correction for the other pulley was slightly less.

III. Method of Experimenting.

The diameter of the bar to be tested was first ascertained by a micrometer caliper, the mean of several readings being taken. The measuring instrument was then applied, and the calibration value of the readings ascertained. The bar was then placed in the testing machine, and the balance weights adjusted to give zero torque. If bending moment had to be applied, this was effected before the final adjustment of the reading microscope, care being taken to bring the wire midway between the two sets of clamping screws, and also to set the wire in the most favourable position for taking accurate readings viz the plane in which bending takes place. Unless readings are taken at equal intervals of time, the time effect of a stress will show itself, and it is therefore very necessary that the separate loadings be at equal intervals. It was found that the most convenient interval was one and a half minutes, this being necessary to bring the weigh-beam back to its zero position, and all readings were taken with this interval, except where otherwise stated.

The form of the stress strain-curve.

Before taking up the detailed examination of the relation of stress to strain, it is of interest to consider the stress strain curve as a whole.

Fig. 15 shows the general nature of the stress strain curve for a wrought iron or steel bar of circular section, when subjected to a gradually increasing torque. Starting from no torque, and gradually increasing the load upon the bar, the relation between stress to strain is found to be a linear one until near the point a, when the defect from linearity is first noticed in the gradual creeping up of the readings; the whole twist upon the bar being at the rate of from 1° to 2° per inch of length.

At a the yield point occurs, and there is a large increase in the strain, with no increase in the loading. The material has also changed from a nearly perfectly elastic to a semi-plastic condition, and the bar when released from load will no longer go back to zero, but shows a very considerable set. The material has also hardened by the process, and the curve rises at first quickly to a point c, and then more slowly until fracture occurs at d; the strain then being generally considerably more than one hundred-fold the strain within the perfectly elastic condition.

The form of the curve at the yield point.

The first sign of deviation from the linear law indicates the failure of the elastic state at the fibres most severely strained - viz. the outer ones - and a semi-plastic condition is entered upon, which as the loading proceeds extends inwardly until a more or less uniform shear stress is established throughout the section. The passage from the one state to the other state in a solid specimen requires a certain range of stress so that the diagram at the point a exhibits a well marked rounding.

If we call q_0 the maximum shear at the outer surface

r_0 = radius of the specimen

r_p = radius to which plasticity extends.

Then the resistance of the bar to torque is the sum of the resistances due to (1) the still elastic core; (2) the semi-plastic shell; and may be represented by

$$T = 2\pi q_0 \frac{1}{r_0} \int_0^{r_p} r^3 dr + 2\pi q_0 \int_{r_p}^{r_0} r^2 dr$$

where r = any radius

$$= \frac{\pi}{2} q_0 \frac{r_p^4}{r_0} + 2\pi q_0 \left(\frac{r_0^3}{3} - \frac{r_p^3}{3} \right)$$

up to the point where perfect elasticity prevails $r_0 = r_p$ and

$$T = \frac{\pi}{2} q_0 r_0^3$$

but when the specimen is wholly plastic

$$r_p = 0$$

and we get

$$T = \frac{2}{3}\pi q_0 r_0^3$$

which is $\frac{4}{3}$ of the value at the elastic limit.

It would therefore appear that if the bar changes [redacted] from the elastic to the plastic condition at the yield point, the maximum torque will be four-thirds of that value at which the first marked deviation from perfect elasticity occurs.

[redacted]

The result of experiment shows a fair agreement with this conclusion. In the example of a Wrought Iron specimen quoted in the next section, Table III Col I, the first marked deviation occurs below 375 inch pounds, while the material failed at 525 inch pounds, giving a ratio of 1·4.
Taking another case for the steel specimen quoted in the same section Table IV Col I; the first deviation occurred below 675 inch lbs, and failure took place at 870 inch pounds, corresponding roughly to a ratio of 1·29, which is very close to $\frac{4}{3}$
Having regard to the difficulty of observing exactly the first sign of failure, it seems probable that the conditions assumed are not far from the truth.

Recovery of elasticity with time.

If a bar of iron or steel be subjected to a torque causing a permanent strain in it, the condition of the bar becomes quite different; it no longer obeys Hooke's law, and the strain for a given stress is now greater than before the increase, being more marked at the higher loads. As an example we may take that of a turned wrought iron bar of length between centres of 4.00 inches; diameter 0.472; calibration value of readings 1 min. = 12.85 divisions. The following results were obtained:

Column I

Table III

Torque inch pounds	Read'g	Δ
0	0	
75	243	243
150	486	243
225	730	244
300	972	242
375	1226	254
450	1484	268
480	1624	140
510	1784	160
525	went off scale	

The load was now removed, and immediately afterwards re-applied, with the following result:

[redacted].

Table III contd.

Column II

Torque inch lbs	Reading	Δ
0	0	
75	245	245
150	500	255
225	762	262
300	1031	269
375	1320	289
300	1071	249
225	822	249
150	563	259
75	298	265
0	26	272

It will be seen at once that the bar exhibits quite different qualities from that shown before. The stress is now no longer proportional to the strain, and the curve showing the relation between the two no longer returns upon itself, but forms a looped figure.

Similar results are obtainable from bars subjected to tensional stress.*†

* Ewing "On The measurement of small strains in the Testing of Materials and Structures." Proc. Royal Society, May, 1898.

† Muir "on the Recovery of Iron from overstrain." Phil Trans 1899

If the bar be tested again after a short interval of time the recovery will be found to be very marked. At the end of one hour a test of the bar gave the results shown by Column III; there being a marked falling off of the increments at the higher loads. Thus the strain caused by increasing the torque from 300 inch pounds to 375 inch pounds now caused only 272 units of strain, instead of 289; and similarly at the end of three hours we find a further decrease to 268 units. The recovery of the bar was tested at suitable intervals of time, as shown in the annexed table; the effect becoming less apparent as the time increased; but practically perfect recovery was reached at the end of two days, and very little change was noticeable after this time.

Table III

Col III
One hour afterwards

Torque inch lbs	Read^g	Δ
0	0	
75	246	246
150	498	252
225	753	255
300	1019	266
375	1291	272
300	1043	248
225	793	250
150	536	257
75	276	260
0	7	269

Table III cont^{d}

Torque inch lbs	Col IV: 3 hours after, Read^{g}	Δ	Col V: 6 hours after, Read^{g}	Δ	Col VI: 12 hours after, Read^{g}	Δ	Col VII: 1 day after, Read^{g}	Δ	Col VIII: 2 days after, Read^{g}	Δ
0	0		0		0		0		0	
		245		246		245		245		243
75	245		246		245		245		243	
		246		244		243		243		245
150	491		490		488		488		488	
		252		249		245		246		247
225	743		739		733		734		735	
		256		256		247		244		245
300	999		995		980		978		980	
		265		260		253		249		246
375	1264		1255		1233		1227		1226	
		245		245		242		244		247
300	1019		1010		991		983		979	
		249		248		246		243		243
225	770		762		745		740		736	
		254		250		247		243		247
150	516		512		498		497		489	
		254		255		249		246		245
75	252		257		249		251		244	
		259		256		248		246		245
0	-7		1		1		0		-1	

Table III Cont^d

Torque inch lbs	Col IX: 3 days after — Read^g	Col IX: Δ	Col X: 5 days after — Read^g	Col X: Δ	Col XI: 6 days after — Read^g	Col XI: Δ
0	0		0		0	
75	245	245	245	245	244	244
150	492	247	489	244	489	245
225	738	246	734	245	783	244
300	983	245	979	246	979	246
375	1127	244	1225	246	1225	246
300	972	245	979	246	978	247
225	728	244	734	245	738	245
150	481	247	489	245	488	245
75	234	247	243	246	243	245
0	-10	244	0	243	0	243

The results may be shown graphically by direct plotting, but it is more convenient to adopt the plan of subtracting from each reading a number proportional to the torque, and plot the new set of readings thus obtained. The method is due to Prof. Ewing, and is used in Fig. 16 ; the diminution in the case being 200 units of scale reading for a torque of 75 inch pounds. A time recovery curve, Fig. 17 has been plotted; the ordinates of which correspond to the reading under a torque of 375 inch pounds, and the abscissae arc times; this curve shows in a marked manner how rapid is the recovery at first .

As a means of comparison with the last bar, a steel bar was now tested, the specimen being classed as machinery steel; i.e. semi-mild. Length under test 4.00; Diameter .425; Calibration 1 min. = 1285 divisions.

The following results were obtained:

~~Torque In.-Lbs.~~

Col I

Torque inch lbs ~~Reading~~	Readg.	Difference.
0	0	
75	385	385
150	770	385
225	1157	387
300	1543	386
375	1930	387
450	2323	393
525	2710	387
600	3103	393
675	3503	400
750	3920	420
780	4231	311
810	4675	444
825	4929	254
840	5309	380
855	6009	700
870	went off scale	

The load was removed and again applied by increments of 75 inch-pounds, until a limit of 750 inch-pounds was reached; the load being afterwards reduced by 75 inch-pounds to nothing. Tests were made at intervals of time, as recorded in Table IV, which latter shows that the recovery is much slower in the former case; and even at the end of 19 days the specimen showed signs of the initial overstrain.

The curves (Fig 18 showing the results of the experiments were) plotted by the indirect method previously described; 350 units being subtracted for an increment of 75 inch-pounds of torque. The rate of recovery is (also) shown by Fig. 19, corresponding to Fig. 17 of the former case. The readings at 750 inch-pounds are plotted as ordinates, and the times as abscissae. The difference between ~~the~~ (these latter) curves is very apparent. From the diagrams it is apparent that the bar recovers very rapidly at first like the ~~W I~~ Wrought Iron but this rate of recovery soon slackens and becomes less & less apparent as the time increases and unless a very considerable time is given the recovery does not become complete (cp Table XV Col X)

Table IV

Columns	I		II		III		IV		V	
Torque inch lbs	Readg.	Δ	Readg.	Δ	Readg.	Δ	Readg.	Δ	Readg.	Δ
			Immediately after I		1 hour after I		3 hours after I		6 hours after I	
0	0		0		0		0		0	
		385		395		395		392		390
75	385		395		395		392		390	
		385		397		395		392		393
150	770		792		790		784		783	
		387		410		405		403		401
225	1157		1202		1195		1187		1184	
		386		410		416		406		406
300	1543		1612		1611		1593		1590	
		387		414		412		414		407
375	1930		2026		2023		2007		1997	
		390		422		418		422		415
450	2320		2448		2441		2429		2412	
		387		412		418		415		417
525	2707		2860		2859		2844		2829	
		393		442		425		424		415
600	3100		3302		3284		3268		3244	
		400		444		422		431		430
675	3500		3746		3706		3609		3674	
		420		460		460		438		434
750	3920		4206		4166		4137		4108	
				384		398		399		393
675			3822		3768		3738		3715	
				406		398		399		400
600			3416		3370		3339		3315	
				407		406		403		404
525			3009		2964		2936		2911	
				410		416		408		408
450			2509		2548		2528		2503	
				407		413		411		411
375			2192		2135		2117		2092	
				415		411		415		413
300			1777		1724		1702		1679	
				418		419		416		413
225			1359		1305		1286		1266	
				424		416		422		426
150			935		889		864		840	
				438				431		426
75			497				433		414	
				445				430		428
0			52				3		-14	

Table III cont^d

Columns	VI		VII		VIII		IX	
Torque inch lbs	1 day after I		3 days after I		6 days after I		19 days after I	
	Read^g	Δ	Read^g	Δ	Read^g	Δ	Read^g	Δ
0	0		0		0		0	
75	390	390	390	390	389	389	389	389
150	778	388	777	387	780	391	778	389
225	1175	397	1170	393	1168	388	1167	389
300	1579	404	1564	394	1557	389	1555	388
375	1988	409	1968	404	1957	400	1944	389
450	2407	419	2380	412	2360	403	2337	393
525	2822	415	2795	415	2770	410	2728	391
600	3242	420	3216	421	3189	419	3128	400
675	3669	427	3643	427	3616	427	3533	405
750	4103	434	4077	424	4038	422	3940	407
675	3706	397	3694	383	3643	395	3549	391
600	3308	398	3292	402	3245	398	3159	390
525	2907	401	2891	401	2845	400	2766	393
450	2497	410	2485	406	2446	399	2374	392
375	2087	410	2080	405	2044	402	1981	393
300	1675	412	1672	408	1638	406	1587	394
225	1261	414	1260	412	1229	409	1191	396
150	837	424	839	421	814	415	793	398
75	415	422	416	423	407	417	393	400
0	-9	424	-1		-11	418	-4	397

The position of the yield-point as affected by previous stress.

The effect of a previous stress upon the properties of a bar have been explained in Section VI, and it remains to point out that overstrain in one direction has a very considerable influence upon the yield-point or curve, separating the elastic from the plastic stage; in fact it disappears but gradually reappears again - the recovery in the case of iron being practically complete in two days, while for steel 19 days effects partial recovery only. In both cases, if sufficient time be given, the yield-curve will assume a definite position above the last position, and this rise is augmented by every overstrain. As an example we may take that of a wrought-iron bar, having a length under test of 4.00 inches; diameter 0.420 inches; calibration 1 min. = 12.8 divisions.

The bar was subjected to stress extending beyond the yield-point, and afterwards left to rest for a minimum period, of ~~two~~ 1½ days, when the load was repeated. Eight tests were made, and each time there was a perceptible rise in the yield-curve. The observations were plotted with each curve, ~~and~~ spaced 1000 divisions from its neighbour. The curves are given in Fig. 20, and require no further explanation.

Twist in alternately opposite directions.

It has long been a common assumption that the limits of elasticity for a bar subjected to torsion lie equally distant from the position of no torque, and this is no doubt true for a specimen not previously strained.

Apparently the first theoretical discussion of the problem is that by James Thomson,* and in his original paper he makes the further assumption "that the limits of elasticity in a substance which has already been strained beyond its limits of elasticity, are equal on the two sides of the shape which it has when in equilibrium without disturbing force." This note, added in October, 1877, goes on to say: "A supposition which may be true or may not be true. Experiment is urgently needed to test it; for its truth or falseness is a matter of much importance in the theory of elasticity."

The paper further points out that these assumptions lead to the important result that if a wire be overstrained, its strength to resist torsion in the original direction is twice that in the other direction.

From the mathematical point of view, Thomson's conclusions may be arrived at as follows:

If a specimen be subjected to stress sufficient to cause a uniform shear throughout, and then be released, we have a

* Cambridge and Dublin Mathematical Journal, 1848, and Article "Elasticity", Enc.Brit.

new distribution of shear throughout the section, which may be expressed by

$$\text{shear} = q_0 - ar$$

where q_0 = original shear at external radius

r = any radius

a = a constant

Since the bar is in equilibrium, we must have

$$\int_0^{r_0} (q_0 - ar)\, 2\pi r^2\, dr = 0$$

giving $\quad a = \frac{4}{3}\frac{q_0}{r_0}$

Thus the shear in the bar is given by the expression

$$q = q_0\left(1 - \frac{4}{3}\frac{r}{r_0}\right)$$

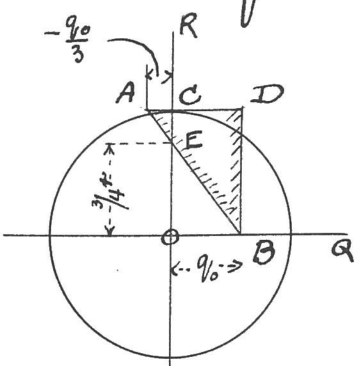


Fig 21

The distribution may be shown graphically as in Fig. 21 by a line AB where $OB = q_0$ and $AC = -\frac{q_0}{3}$

This line evidently crosses the axis OR at the distance OE $= \frac{3}{4}r$ and gives a point on the circle of no stress.

Clearly if no change has taken place in the limits of elasticity the maximum shear is q_0 at the centre, and evidently the bar will now stand a torque given by the expression

$$T = \int_0^{r_0}\left[\frac{4}{3}\frac{q_0}{r_0}r + q_0\left(1 - \frac{4}{3}\frac{r}{r_0}\right)\right] 2\pi r^2\, dr = \frac{2}{3}\pi q_0 r_0^3$$

[redacted] while in the opposite direction the torque will be given by the expression

$$T = \int_0^{r_0} \left[\frac{2}{3}\frac{q_0 r}{r_0} - q_0\left(1 - \frac{4}{3}\frac{r}{r_0}\right)\right] 2\pi r^2 dr = \frac{1}{3}\pi q_0 r_0^3$$

[redacted]

It remains to be seen if the assumptions are justified.

In order to examine this point, a wrought iron specimen was taken, having a length under test of 4.00 inches; diameter 0·634 ; calibration value 1 min. = 12.76 divisions of the scale.

[redacted]

The specimen was set in the machine so that torque could be applied in either direction, and observations were made of the strains for loads which in turn caused permanent set in both the positive and negative directions

These readings are plotted in Fig. 22, and from an inspection of this and the table it is apparent that the stress-strain curve was approximately linear before the yield-point was reached. The return curve was less so, but as soon as torsion was applied in the negative direction the linearity disappeared, and the strains though irregular became greater and greater as the torque increased. The material finally gave way under a torque of about -1100 inch-pounds. The torque was now reversed, and the stress-strain curve became approximately linear until the zero torque was reached, from which point the curve began to bend over to the left, until a torque of 1175 inch-pounds was reached.

In order to roughly test the behavior of the specimen still further, the applied torque was continued, but no strain measurements were taken.

Each complete cycle produced a hardening effect on the bar; widening its limits of endurance each time until a final limit of 1750 inch-pounds of torque was reached after 14 reversals of the stress. The bar was now cracked in several places along the minute seams of impurities, and further experiment seemed useless.

This experiment demonstrates that the limits of elasticity do not remain in their original positions, and further it is shown that stress carried beyond the elastic limit in one direction reduces the other limit to zero. The conclusion derived from the theory above that the bar is twice as strong to resist torsion in the original direction as in the other is also not borne out by the experiment.

A second bar of wrought iron was next examined in the same manner, only four cycles being performed, of which the first two are shown in Fig. 23 , and the last two in Fig. 24 . These curves exhibit the same general properties as the one described above. It is evident from Figs 23 & 24 that after the first reversal of stress there is no perceivable yield-point; all such critical points being absent. The commonly received idea that raising the elastic limit in one direction lowers it in the contrary direction does not hold good here, since all critical points vanish.

The further development of the idea that the distance apart of the limits is a constant, appears to have no physical basis for the torsion, of iron.

The influence of Tension on Torsion.

I. Tension within the elastic limit, and torsion within the elastic limit.

Among the notable experiments made upon the influence of tension upon torsion are those of McFarlane upon steel pianoforte wire.*

From the article it does not appear that any experiments were made to ascertain whether tension within the elastic limit has any influence upon torsion within the elastic limit, but in any case it was thought worth while to make the experiments, as specimens of much larger diameter could be dealt with.

The first specimen tried had the comparatively large diameter of $\frac{3}{4}$ inches, and the maximum tension load which could be applied was 3000 pounds. Repeated experiments failed to show any difference in the torsional properties of the bar, whether loaded or unloaded.

A second bar was then prepared, having a diameter of $\frac{1}{2}$ an inch, and the experiment was repeated with tension loads varying from 200 to 3000 pounds; the latter corresponding to a stress of 15300 pounds per square inch. The diameter of the pulley was 41.62 inches, so that a weight of one pound in each pan corresponded to a torque of 41.62 inch pounds.

*Enc.Brit. Art. Elasticity.

The length of the specimen was 8 inches, and the calibration value gave 1 division of the scale = 16.85 seconds. [redacted].

The following table gives a summary of the results obtained:

Table V

Tension load on specimen. pounds	Mean value of reading corresponding to 1 pound in each pair = 41.62 in.Lbs.		Mean of columns 2 and 3.
	Torque increasing	Torque diminishing	
200	59.00	58.98	58.99
600	59.00	58.74	58.87
1000	58.56	58.70	58.63
2000	58.59	58.70	58.65
3000	58.75	58.65	58.70

As will be seen from the last column, the values obtained differ very little; in no case varying more than one-half of one per cent.

The above experiments were carried out for me during the latter part of 1898, and the beginning of 1899, by Mr. Colpitts - then a student in the Civil Engineering Department of the University. As a test of the accuracy of these results, a third bar was prepared, having a diameter of 0.375 inches; length 8.00. A new objective was fitted to the measuring instrument to render it much more sensitive. The calibration value gave 1 minute of arc = 62.4 divisions of the scale. This necessitated low torques, in order to prevent the observation wire from passing out of the field of view. A new pulley was used, having a diameter of 20.82 inches, and weighing with hangers, etc., 120 pounds.

A test was first made with no tension load beyond that of the pulley, and immediately afterwards a load of 2400 pounds was applied; corresponding to an increased load of 21700 pounds per square inch. The readings obtained are shown in Table VI.

Table VI

Load in each pan in pounds	No Tension except weight of pulley		Tension 2400 pounds. + weight of pulley.	
	Read^g.	Diff^{ce}.	Read^g.	Diff^{ce}.
0.4	0		0	
		196		194
0.5	196		194	
		196		195
0.6	391		389	
		195		193
0.7	586		582	
		197		195
0.8	783		777	
		196		196
0.9	979		973	
		195		194
1.0	1174		1167	

affects the angular distortion in the same manner.

Note

Considerable difficulty was experienced in making accurate readings when the tensional load was applied as the time of vibration of the heavy weights with respect to the specimen was so large that any accidental motion due to the putting on or taking off the weights was very difficult to damp out

A deep four-armed vane was attached to the hanger and dipped into a water-trough on the floor; this effected a great improvement but did not wholly counteract the vibration

The readings obtained in Table VI show a slight diminution when a tensional load is applied but owing to the difficulties of observation mentioned above the author feels he cannot lay much stress upon them. The observations were repeated with very nearly the same result

As will be seen in the next section bending affects the angular distortion in the same manner.

Effect of tensional stress on the yield-point.

The only experiments upon the yield-point appear to be those of McFarlane.* These showed that a tension lowered the yield-point. Reasoning from this result, Lord Kelvin concludes that a compression stress would raise it, but no experiments appear to have been made to verify this conclusion.

In order to examine the effect of tension at or about the yield-point, a bar of wrought iron was taken and cut into two parts; one specimen was turned truly parallel to a convenient diameter (0.424 inches), and the second was made exactly the same size. Both specimens were tested and found to be perfectly cylindrical, as far as could be ascertained by a micrometer gauge.

The first specimen was then tested in the ordinary way, with the result shown in Column I, Table VII. The first noticeable deviation occurred when each pan was loaded with a weight of 16 pounds - corresponding to a torque of 333 inch-pounds; the maximum torque being 385 inch-pounds.

The second bar was then stressed; but before the tension load was applied a preliminary reading was taken to see whether the readings agreed with those from the first specimen; and as will be seen, (Col.II) the agreement is very close.

The specimen was now loaded with an additional 2400 pounds - corresponding to an increase of stress of 17.900 pounds per square inch - and a torque applied by increments, As shown by Column III, a slight deviation was noticed at 333 inch-pounds, and failure was accomplished by a torque of 360 inch-pounds.

* Loc.cit.

This result shows in a marked way the lowering of the yield-point by tension, and confirms McFarlane's experiments.

Table VII

Load in each pan lbs	Col I: No Tension Readg	Col I: Δ	Col II: No Tension Readg	Col II: Δ
2	0		0	
3	157	157	155	155
4	312	155	312	157
5	469	157	468	156
6	625	156	625	157
7	781	156	781	156
8	935	154	937	156
9	1090	155	1094	157
10	1247	157	1250	156
11	1403	156	1407	157
12	1559	156	1564	157
13	1716	157		
14	1872	156		
15	2029	157		
16	2188	159		
17	2356	168		
17·5	2476	120		
18	3426	950		
18·2	3826	400		
18·5	went off scale			

Load in each pan lbs	Col III: Tension 2400 lbs Readg	Col III: Δ
2	0	
3	156	156
4	311	155
5	467	156
6	624	157
7	782	158
8	938	156
9	1094	156
10	1251	157
11	1406	155
12	1564	158
13	1722	158
14	1878	156
15	2034	156
15·5	2115	81
16·0	2221	106
16·5	2341	120
17·0	2813	472
17·3	went off scale	

The results of Table are plotted & are shown on Fig 25

Tension beyond the elastic limit.

A machinery steel bar turned to a mean diameter of 0.537 with a length of 8.00 under test was chosem. The bar was first placed in the torsion machine, and gave the results under test shown by Col.I, Table VIII. The mean twist for 75 inch-pounds was 278 divisions. The specimen was then set in the tension grips of the Buckton Testing Machine, and a gradually increased load applied, until a permanent set of 0.18 inches was produced. Immediately after, a fresh test in torsion was made, the results obtained being shown by Col.II. The results are also plotted in Fig. 26, from which it will be seen that the effect of the tensional over-strain has entirely altered the properties of the material under torsion. The strain is no longer proportional to the stress; the deviation being even more marked than in the case of specimens upset by a previous overstrain by torsion.

Table VIII

Steel Specimen

Diam^r 0·537 Length 8·00

12·75 divisions of scale = 1 min of arc

Column I No Tension			Column II Specimen permanently lengthened 0·18"		
Torque inch lbs	Read^g	Δ	Torque inch lbs	Read^g	Δ
0	0		0	0	
75	278	278	75	299	299
150	557	279	150	600	301
225	835	278	225	921	321
300	1112	277	300	1260	339
375	1390	277	375	1624	364
			450	2030	406
			525	2514	484
			600	3126	612
			525	2834	292
			450	2536	298
			375	2236	300
			300	1936	300
			225	1631	305
			150	1321	310
			75	1010	311
			0	700	310

Effect of Torsion on Tensing.

The [redacted] effect of torsion upon the properties of a bar subjected to tensional stress was only examined below the elastic limit.

The measuring instrument used was of the Ewing type,* each unit of extension representing $\frac{1}{50000}$ inches upon an 8 inch length.

A series of tests were made, beginning with no tension and increasing by equal increments until the yield-point of the material was reached. The results are recorded in Table IX, and it will be seen that no difference was observable, whether the bar was twisted or not, provided the elasticity of the bar remained unimpaired.

* On Measurements of Small Strains in the Testing of Materials and Structures. By J.A.Ewing, F.R.S., Proc. Royal Society, May, 1895.

Table IX

Loads pounds	No Torsion		Torque of 141 inch lbs		Torque of 282 inch lbs		Torque of 423 inch lbs	
	Read^g^	Δ	Read^g^	Δ	Read^g^	Δ	Read^g^	Δ
200	0		0		0		0	
		15		15		16		15
400	15		15		16		15	
		14		15		15		16
600	29		30		31		31	
		14		14		15		14
800	43		44		46		45	
		16		15		15		15
1000	59		59		61		60	
		14		15		14		15
1200	73		74		75		75	
		14		15		14		15
1400	87		89		89		90	
		15		14		14		14
1600	102		103		103		104	
		14		15		14		14
1800	116		118		117		118	
		15		15		14		16
2000	131		133		131		134	
		15		14		15		13
2200	146		147		146		147	
		15		14		14		15
2400	161		161		160		162	

Effect of bending on Torsion.

One of the most interesting cases of stress which occurs in practice is that of torsion combined with bending; a subject which has received little or no attention from the experimental side. The apparatus desrribed in section II enables uniform twist and uniform bending to be applied to a bar in any proportion, and the torsional strain to be accurately measured without reference to any external body; so that the bar can assume its own position of equilibrium without affecting the readings of the measuring instrument.

Attention has been [redacted] directed to the influence of bending on the torsional properties of a bar.

Bending within the elastic limit, and torsion within the elastic limit.

In a previous section it has been shown that the effect of a tension produced little or no effect upon the torsional properties of a bar while in the elastic state. It might be expected, therefore, that a bending action which results in a varying tension and compression upon the longitudinal fibres would have but little effect upon the strain. As an example we may take the case quoted in section II of a rivet steel bar, in which an increase in the bending moment caused a slight diminution of the strain per unit torque.

Similar decrements were found in every case of the same type; as for example in the case of a semi-mild steel bar of diameter 0.869 inches, and of length 8.00 inches, under test.

The unit reading corresponds to $\frac{1}{54}$ minutes of arc.

Table X

Torque in lbs.	No Bending Moment		640 inch-pounds.	
	Reading	Difference	Reading	Difference
0	0		0	
		166		163
75	166		163	
		166		164
150	332		327	
		166		166
225	498		493	
		165		166
300	663		659	
		167		165
375	830		824	

Bending beyond the elastic limit.

A more interesting case was that of a steel bar in which the bending moment was gradually increased until a permanent set was given to the bar in the plane of bending. The [redacted] readings obtained, are shown in Table XI, and a summary of the results in Table XII. It will be noticed that when the bar is bent the true value of the torque is given by its apparent value multiplied by $\cos\theta$ where θ = angle of bending of the weigh-lever about its axis.

The results are also plotted upon the diagram, Fig. 27, from which it will be seen that the slight diminution in the readings within the elastic limit is followed by a much greater rise when the yield point is reached.

Table ~~XII~~ XI

Torque inch pounds	No Bending Moment Read.g	Δ	Bending Mt. 224 inch lbs Read.g	Δ	Bending Mt. 377 inch lbs Read.g	Δ	Bending Mt. 529 inch lbs Read.g	Δ	Bending Mt. 752 inch lbs Read.g	Δ
0	0		0		0		0		0	
		137		137		136		135		135
7.5	137		137		136		135		135	
		137		136		135		136		136
15.0	274		273		271		271		271	
		136		136		137		136		136
22.5	410		409		408		407		407	
		136		135		136		135		135
30.0	546		544		544		542		542	
		136		136		135		135		134
37.5	682		680		679		677		676	
		136		137		135		135		135
45.0	818		817		814		812		811	
		136		135		136		135		137
52.5	954		952		950		947		948	
		137		136		136		136		136
60.0	1091		1088		1086		1083		1084	

Table XI cont^d

Torque inch pounds	Bending M^t 922 inch lbs Read^g	Bending M^t 922 inch lbs Δ	Bending M^t 1092 inch lbs Read^g	Bending M^t 1092 inch lbs Δ	Bending M^t 1432 inch lbs Read^g	Bending M^t 1432 inch lbs Δ	Bending M^t 1772 inch lbs Read^g	Bending M^t 1772 inch lbs Δ
0	0		0		0		0	
		135		136		138		139
7·5	135		136		138		139	
		136		137		139		140
15·0	271		273		277		279	
		137		137		138		140
22·5	408		410		415		419	
		135		136		137		141
30·0	543		546		552		560	
		137		137		138		140
37·5	680		683		690		702	
		137		137		138		140
45·0	817		820		828		842	
		137		138		139		141
52·5	954		958		967		983	
		136		137		138		140
60·0	1090		1095		1105		1123	

Table XII

Bending Moment	Readings for Torque of 60 in lbs	∠ of bend ^{g} θ at weigh-lever	cos θ	Corrected Reading = Reading × cos θ
0	1091	0°	1.000	1091
224	1088	30′	·999	1088
337	1086	48′	·999	1086
529	1083	1° " 54′	·999	1083
752	1084	2° " 24′	·999	1084
922	1090	3° " 10′	·998	1088
1092	1095	3° " 20′	·998	1092
1432	1105	4° " 30	·997	1102
1772	1123	7° " 0′	·993	1115

Effect of bending upon the yield point.

The experiment was performed in a similar manner to that in Section VIII. Two bars cut from the same rod were turned up to exactly the same size. One of them was tested in the ordinary manner, and the other was subjected to bending moment, and then twisted beyond the yield-point.

The readings are recorded in Table XIII, from which it appears that there was quite a remarkable lowering of the yield-point; the reason for which is not at first apparent, until it is noticed that the specimen took a permanent set; the ends being bent to a considerable degree.

At first sight this might appear to be a mere time effect; but in the author's opinion the probable cause was the increase of stress, due to the torque applied later. Apparently the maximum stress due to bending was of itself insufficient to cause yield, but the application of a torque caused the principal stresses to assume the values

$$p_1 = \frac{p_n}{2} + \sqrt{\frac{p_n^2}{4} + q^2}$$

$$p_2 = \frac{p_n}{2} - \sqrt{\frac{p_n^2}{4} + q^2}$$

where p_n = normal stress due to bending

q = shear stress due to applied torque.

If we adopt Rankine's Theory of Maximum Stress, then p_1 in this case passed the working limit of the material, and a set resulted.

On the maximum strain theory of St.Venant

if e_1 = principal strain

m = Poisson's ratio

Then

$$Ee_1 = p_1 - \frac{p_2}{m}$$

$$= \quad = \frac{m-1}{2m}p_n + \frac{m+1}{2m}\sqrt{p_n^2+4q^2}$$

and since m has a value between 3 and 4 for steel, it is clear that the addition of a shear stress q would cause an increase in the value of e_1, which if [redacted] below the limit before might increase sufficiently to cause failure.

[redacted].

The relation of stress to strain after the permanent set is clearly shown by a further test indicated in the Tables and Figure 28 . There is now considerable hysteresis in the relation of stress to strain.

The author has not been able to find any other experiments bearing upon the position of the yield-point as affected by bending. The yield point is however known to be lowered by tension as mentioned previously.

A case which bears considerable resemblance to the case of permanent set last quoted, is one by M'Farlane,* who has shown that if a wire is twisted to nearly its limit of torsional elasticity, an increase in pull will cause the torque to give the wire a permanent set. This latter case can be easily explained in the same manner as the one described above.

* Art. "Elasticity", Enc.Brit., Par.21.

Table XIV

C Combined Bending and Twist

Torque inch pounds	No Bendg Mt Readg	Δ	No Bendg Mt Readg	Δ	Torque inch lbs	Bending Mt 668 inch lbs Readg	Δ	Torque inch lbs	3 hours after last Test Readg	Δ
0	0		0		0	0		0	0	
75	195	195	196	196	75	196	196	75	201	201
150	390	195	392	196	150	392	196	150	403	202
225	587	197	585	193	225	590	198	225	610	207
300	784	197	782	197	300	805	215	300	818	208
375	980	196			375	1570	765	375	1084	266
420	1105	125						300	890	194
435	1149	44						225	689	201
450	1191	42						150	486	203
465	1233	42						75	284	202
480	1276	43						0	76	208
495	1322	46								
510	1368	46								
525	1418	50								
540	1468	50								
555	1516	48								
570	1578	62								
585	1676	98								
605	2376	700								
615	3476	1100								

(Bending column, after 1570:) after 2 minutes went off scale entirely

Effect of annealing.

It has long been known that iron and mild steel stressed beyond the limit of elasticity regain their elastic properties when heated to a red heat and allowed to cool slowly. The process may be repeated many times without apparently changing the elastic properties of the material. The yield-point, however, is found to alter in position as the annealings proceed. In a particular case* a mild steel bar which in an ordinary test would give an extension of 25 per cent upon a ten-inch length, and a yield-point of about 18 tons per square inch, was stretched approximately $\frac{1}{4}$ inches, and annealed in the ordinary manner after each operation. Throughout the experiment the bar appeared to recover its elastic properties after each annealing, and finally broke with a total extension of approximately 100%. The yield-point remained fairly constant, except at the end, when it experienced a rise.

Copper treated in the same manner has been drawn out by the author to considerably more than double its length in this way without causing fracture.

Remarkable advances in our knowledge of annealing have been recently obtained by Muir,+ acting upon a suggestion of Prof. Ewing.

Muir has shown that comparatively low temperatures, such as boiling water, will restore a strained bar to its elastic condition. The yield-point, however, alters during the process, and is always higher than in the original condition.

* Note on the Endurance of Steel Bars subjected to repetitions of Tensional Stress; by E.G.Coker, B.Sc.

\+ The Recovery of Iron from overstrain; by James Muir,- Phil.Trans, 1899.

After a few applications of stress, followed by heating, in boiling water, or even water at 50° C, the bar fractures with a total extension not very different from a bar stressed to breaking without special treatment. The annealing at low temperatures, therefore, appears to be less complete than that at a high temperature.

In order to discover what effect a temperature of 100°C would exert on a bar overstrained by a torque, the steel bar III, which had been [redacted] used, (for experiments on the recovery of elasticity with time) (See Section VI Table IV) was selected, and after boiling for 15 minutes was overstrained [redacted], giving results (Col. X , Table XV) practically identical with those of Col. I , Table IV , for the first part of the curve. [redacted] As in practice it is very troublesome to get exactly the same calibration value for each setting of the instrument this latter (stripped of the reading microscope and wire holder) remained on the bar during the heating and the labour of comparing readings of whose unit values differ by a small amount was thereby avoided. Each stress operation causing overstrain was succeeded by a heating in water at 100° C for 15 minutes and in all the bar was stressed eight times. The readings obtained are given in Table XV and are plotted in the ordinary manner, Fig 29, the curves being spaced 100 units apart for convenience. As might be expected the curves show a general agreement with

Table XV

Columns X: Torque inch lbs	X: Read^g	X: Δ	XI: Torque inch lbs	XI: Read^g	XI: Δ	XII: Torque inch lbs	XII: Read^g	XII: Δ
0	0		0	0		0	0	
75	385	385	75	390	390	75	389	389
150	770	385	150	780	390	150	776	387
225	1158	388	225	1169	389	225	1165	389
300	1545	387	300	1558	389	300	1554	389
375	1934	389	375	1946	388	375	1941	387
450	2326	392	450	2337	391	450	2330	389
525	2715	389	525	2727	390	525	2718	388
600	3110	395	600	3115	388	600	3107	389
675	3510	400	675	3507	392	675	3496	389
750	3914	404	750	3897	390	750	3882	386
825	4340	426	825	4291	394	825	4273	391
900	4803	463	900	4701	410	900	4671	398
930	went off scale		975	5318	617	975	5090	419
			990	5578	260	1025	5393	303
			1015	went off scale		1050	went off scale	

Table XV cont^d.

Columns XIII Torque inch lbs	Read^g	Δ	XIV Torque inch lbs	Read^g	Δ	XV Torque inch lbs	Read^g	Δ
0	0		0	0		0	0	
75	386	386	75	386	386	75	387	387
150	775	389	150	773	387	150	774	387
225	1164	389	225	1157	384	225	1160	386
300	1552	388	300	1543	386	300	1547	387
375	1941	389	375	1931	388	375	1933	386
450	2330	389	450	2318	387	450	2320	387
525	2722	392	525	2706	388	525	2708	388
600	3110	388	600	3092	386	600	3099	391
675	3499	389	675	3481	389	675	3496	397
750	3893	394	750	3869	388	750	3891	395
825	4295	402	825	4261	392	825	4293	402
900	4697	402	900	4653	392	900	4705	412
975	5111	414	975	5056	403	975	5130	425
1050	5586	475	1050	5486	430	1050	5575	445
1080	went off scale		1080	5683	197	1125	6097	522
			1110	went off scale		1155	went off scale	
			~~1125~~	~~went~~				

Table XV cont^d.

Columns XVI and XVII

XVI Torque inch lbs	XVI Read^g	XVI Δ	XVII Torque inch lbs.	XVII Read^g	XVII Δ
0	0		0	0	
75	387	387	75	388	388
150	773	387	150	777	389
225	1159	387	225	1168	391
300	1544	385	300	1557	389
375	1930	386	375	1946	389
450	2317	387	450	2335	389
525	2704	387	525	2725	390
600	3093	389	600	3118	393
675	3482	389	675	3513	395
750	3874	392	750	3909	396
825	4268	394	825	4310	401
900	4666	398	900	4719	409
975	5076	410	975	5129	410
1050	5513	437	1050	5540	411
1105	went off scale		1125	5998	458
			1200	went off scale	

those obtained by Muir having regard to the fact that the stress is non-uniform

In conclusion the Author desires to express his thanks to Prof.r Bovey, Dean of the Faculty of Applied Science, McGill University, who placed the resources of the Testing Laboratory of the Civil Engineering Department at his disposal, and also to Mr Withycombe, Mechanical Superintendent, who gave much help in the preparation of the apparatus.

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