

Social Cost of Carbon: What Do the Numbers Really Mean?

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Abstract: The Social Cost of Carbon (SCC) is estimated by integrated assessment models and is widely used by government agencies to value the climate impacts of rulemakings, however, the core discussion around SCC so far was focused on validity of obtained numerical estimates and related uncertainties while largely neglecting a deeper discussion of the SCC applicability limits stemming from the calculation method. This work provides a conceptual mathematical background and the economic interpretation that is behind the SCC calculation in the three widely used integrated assessment models. Policy makers need to be aware of the difference between the commonly implied and the actual meaning of SCC that substantially limits its applicability in practice. The presented results call for a critical revision of the SCC concept and the SCC calculation methods in integrated assessment models.

One Sentence Summary: The Social Cost of Carbon as estimated by the three widely used integrated assessment models and as used in policymaking has a different meaning than commonly implied, which makes it largely irrelevant to the currently applied policy context.

Main Text:

The concept of the Social Cost of Carbon (SCC) appeared in the early publications of Nordhaus (1) and dates back to the first works on the Dynamic Integrated Climate-Economy (DICE) model (2). SCC gained momentum for policy making in 2000's (3) and since then was widely used by a large number of organizations in policy making e.g. Worldbank (4), US EPA (5), UK (3). While according to more recent publications by Nordhaus (1) the SCC did not play a decisive role in the evaluation of the US climate related policies, an earlier publication by Nordhaus (16) reported “regulations with more than \$1 trillion of benefits have been written for the United States that use the SCC in their economic analysis”. The SCC concept is well integrated in the current policy context and therefore plays an important role in assessments of climate related action. The United States Government’s Interagency Working Group on Social Cost of Carbon is using the SCC according to respective regulation (5) relying for the purposes of the SCC estimation on the FUND¹ (6,7) and PAGE² (8-11) models along with the DICE model.

¹ Climate Framework for Uncertainty, Negotiation and Distribution (FUND) Model

² Policy Analysis of the Greenhouse Effect (PAGE) Model

There are a few definitions of SCC e.g. "The social cost of carbon refers to the estimate of the monetary value of world-wide damage done by anthropogenic CO₂ emissions" (3), "social cost of carbon' is defined as the monetary value of the damage done by emitting one more tonne of carbon at some point of time" (3), "it is the change in the discounted value of economic welfare from an additional unit of CO₂-equivalent emissions" (1), or "it is the change in the discounted value of the utility of consumption per unit of additional emissions, denominated in terms of current consumption" (12), or "SCC estimates the dollar value of reduced climate change damages associated with a one-metric-ton reduction in carbon dioxide (CO₂) emissions" (13) to name a few. In DICE, SCC is calculated as a ratio of the marginal value for the emissions equation to the marginal value for the consumption equation on optimal trajectories - those that deliver the maximum to the objective function, which represents societal utility. "The ratio calculates the economic impact of a unit of emissions in terms of t-period consumption as a numéraire" (12). The DICE model (1) estimates the optimal Carbon Price (CP) that is a marginal abatement cost - a known function depending on time and the amount of abated carbon, which is a decision variable. A standard DICE 2016 model³ run produces vividly different SCC and CP for the tail of the trajectory, see **Fig. 1**. The difference between SCC and CP, generally speaking, is not confined to the tail of the optimal trajectory, so the same model, with the exception that the utility function is replaced with the one that is not weighted by population (14), produces visibly different SCC and CP also at the head of the optimal trajectory, see **Fig. 2**. The difference between SCC and CP can be positive and negative as evidenced by these figures, and the moment in time when this difference becomes well visible can be rather close to the start of the modeling time interval as evidenced by results obtained from a modified version of the DICE 2016 model where only an additional temperature constraint $T < 2.4$ °C is added and the rest of the model is left unchanged⁴. The CP is a direct result of the optimization, and SCC is a co-product obtained via marginals on that same optimal trajectories of the model. As both represent the price/cost of the carbon by their names and are expressed in the same units, the puzzle is why there is a difference between the two and what implications does it have for applications.

³ Source: <http://www.econ.yale.edu/~nordhaus/homepage/homepage/DICE2016R-091916ap.gms> (accessed on October 23, 2019).

⁴ The application of a direct temperature constraint is justified by a deficiency in the damage function that is unable to capture (or just translate to a monetary value) all potential damage stemming from increased GHG concentrations in the atmosphere. Such constraint was applied and reported in DICE-2013 (Introduction and User Manual) and later in DICE-2016 (<https://data.nber.org/reporter/2017number3/nordhaus.html>).

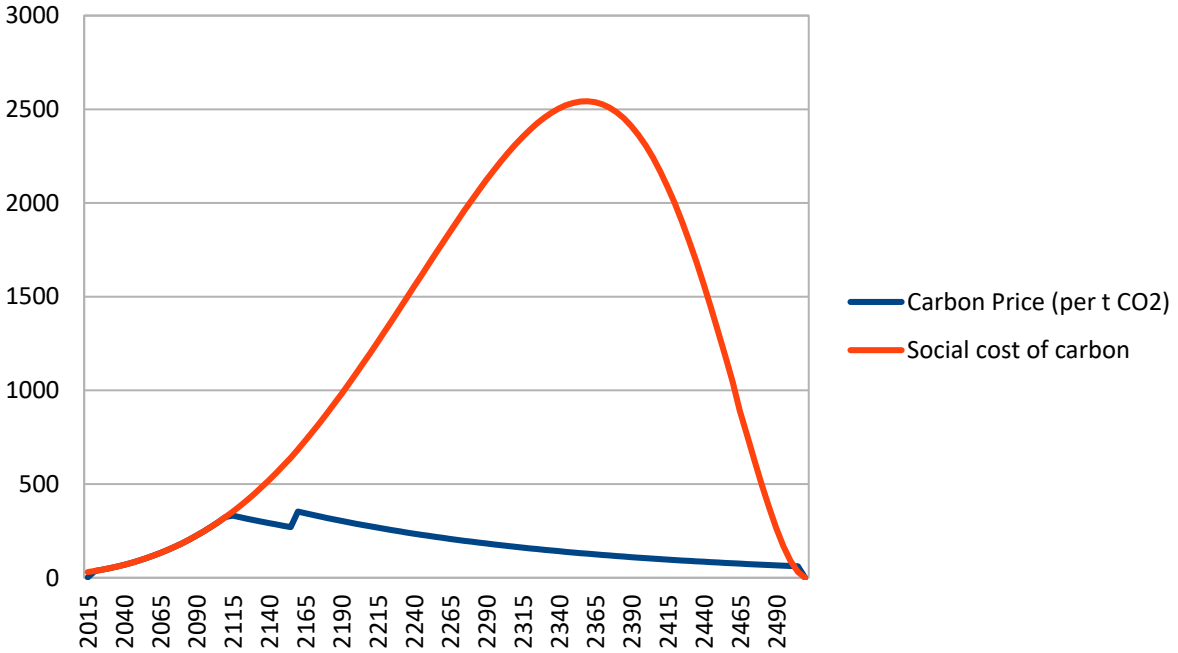


Fig. 1. Social cost of carbon and the carbon price as estimated by the unmodified DICE 2016 model.

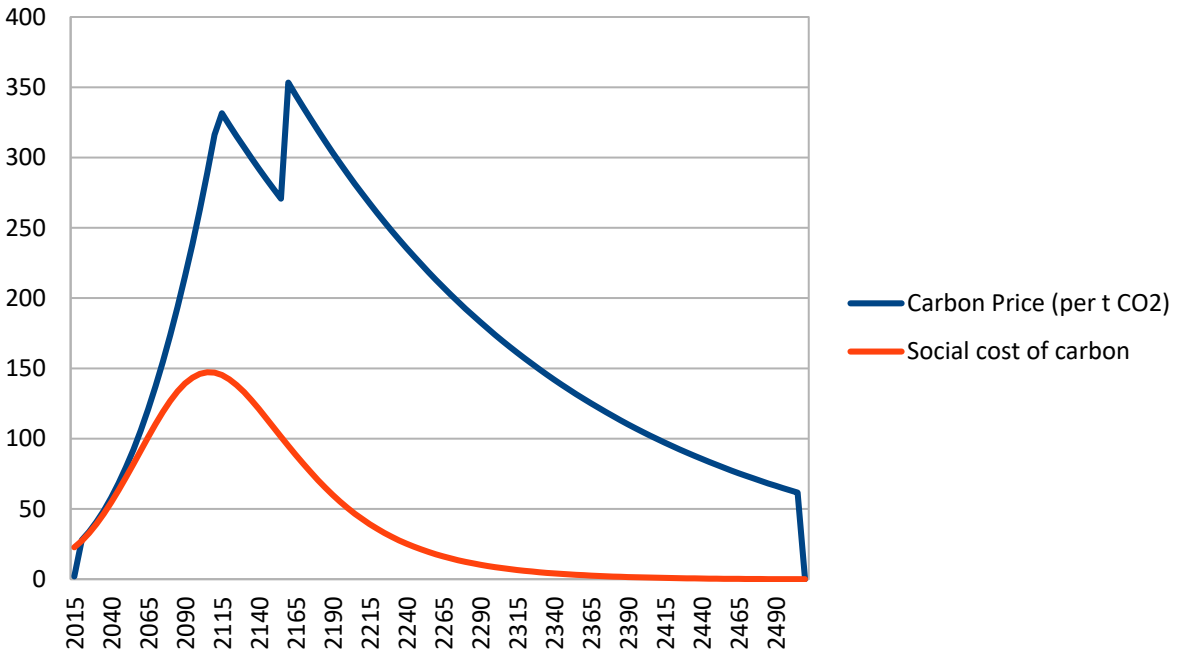


Fig. 2. Social cost of carbon and the carbon price as estimated by a modified version of the DICE 2016 model where only the utility function is replaced with utility that is not weighted by population size.

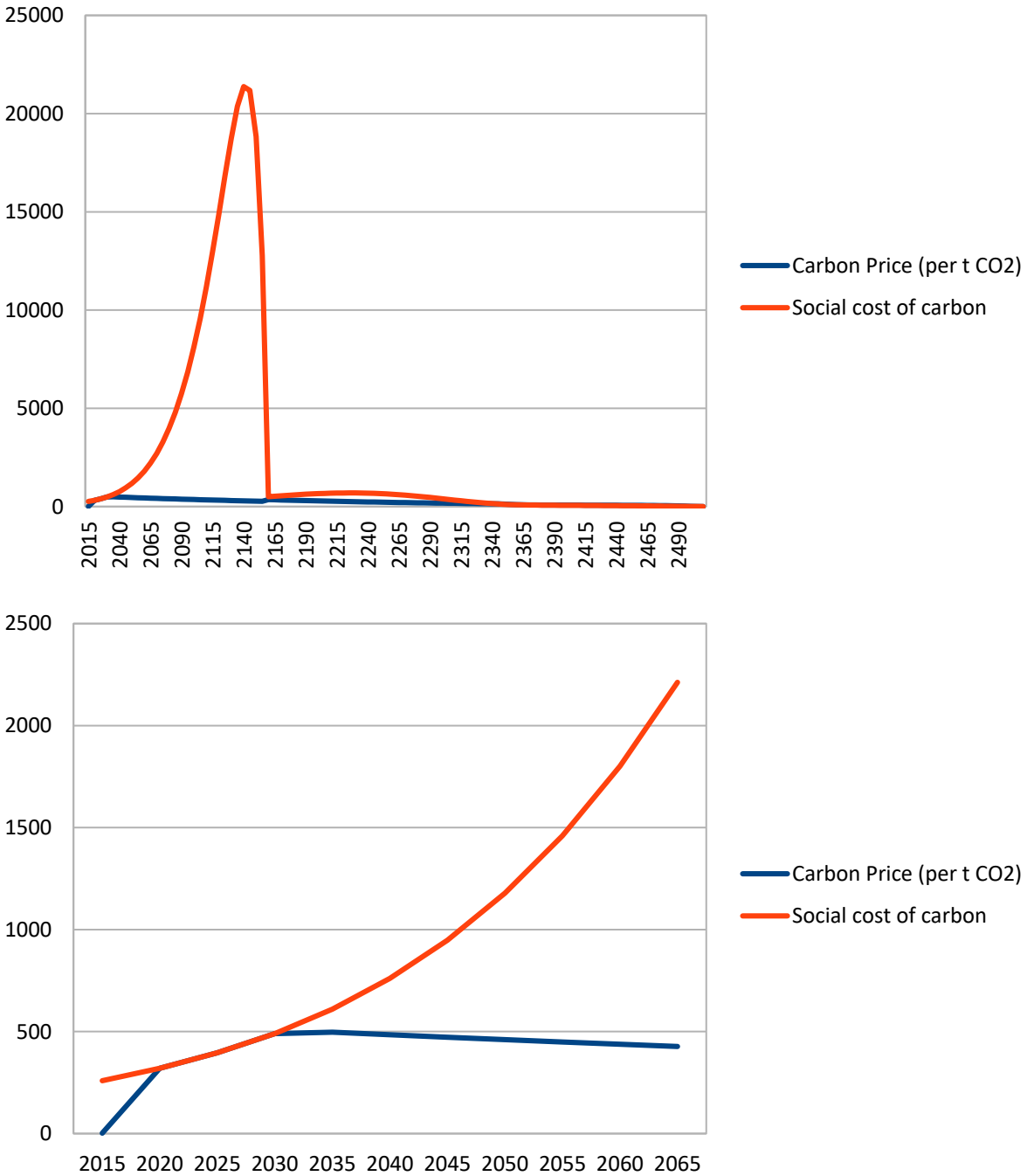


Fig. 3. Social cost of carbon and the carbon price as estimated by a modified version of the DICE 2016 model where only an additional temperature constraint $T < 2.4^{\circ}\text{C}$ is added. (Upper panel: full time span of 2015-2500, lower panel: zoom-in to a shorter 50-year time period 2015-2065.)

Unfortunately, the literature does not say anything clear on that and therefore cannot help, e.g.: "With an optimized climate policy (abstracting away from complications due to tax or regulatory distortions or inconsistent treatment in different sectors), the *SCC will equal the carbon price*; this in turn is equal to the marginal cost of emissions reduction" (12), which contradicts the fact of the difference between SCC and CP shown in **Fig. 1 - Fig. 3**, or, (in a cost-benefit analysis

(CBA) model) "the marginal social cost of carbon is the marginal damage done at the optimal level of abatement" (3), etc. Despite the lack of clarity, policy makers are keen on employing SCC for various reasons.

Definition and Calculation

To some degree the optimal carbon price is shadowed by the SCC - a combination of the two indirect products of the model – the marginal of the emissions equation and the marginal consumption. As these two are just the Lagrangian multipliers (15) in a constrained utility maximization problem, their meaning is respectively: (a) by how much the optimal utility would change if one ton of CO₂ would be added to an emission balance equation and a new problem would be solved, and (b) by how much the utility would change if one dollar would be added to the consumption equation and a new problem would be solved. These values are expressed respectively in (a) units of utility per one ton of CO₂ and (b) units of utility per one dollar of additional consumption. Both values being Lagrangian multipliers assume that the corresponding balance equation is perturbed (modified) by a "sufficiently small" quantity (we assume for simplification that one ton of CO₂ and one dollar in our example satisfy that requirement).

The equation

$$E.m + x * C.m = 0 \tag{1}$$

as implemented in DICE 2016 and 2013 (where *E.m* and *C.m* are marginal values for emission and consumption equations respectively) describes that the effects of adding one ton of CO₂ to the emissions equation and simultaneous adding of *x* dollars lead to a new optimization problem that has the same optimal value of social utility (objective function) as the original problem, i.e. one ton of CO₂ is being compensated by *x* dollars in consumption. This allows one to call *x* an "exchange rate" between one ton of CO₂ of emissions added to the equation and dollars that have to be added to the consumption to keep the "status quo" in terms of utility assuming the optimal solution in a new problem. This new problem is obtained from the original problem by perturbing its emissions and consumption equations and is further referred to as "perturbed problem". The "exchange rate" can be seen as a price compensating extra one ton of emissions to keep the societal "status quo", which justifies the name SCC.

A fully compatible alternative way of looking at SCC should be noted for further explanation. If *x* dollars are not added while one ton of CO₂ is added to the emissions equation, the situation (in the sense of the optimal value of the utility function) is equal to the case when the emission equation is kept untouched, but *x* dollars are subtracted from the consumption equation. Both these perturbed problems would have, generally speaking, different optimal trajectories, yet the same optimal value of the objective function.

Interpretation

Here we provide an interpretation of a perturbed problem - the one with a perturbed equation(s). We start with a question what is the meaning of the correction of the emissions equation by that one tCO₂ in a particular year? The industry may *decide* for whatever reason to emit just a bit more than planned (whether the plan is optimal or not), however, that would imply that the

abated quantity⁵ and/or the capital investment⁶ (both are the only *decision* variables in DICE) should change so that the total production and associated emissions go up. So, in case of a change in industrial emissions, the correction of the emission balance equation is not justified. Such correction of the equation is justified if the *uncontrolled* emissions (in DICE 2016 commonly referred to as land emissions) need to be corrected.

For the ease of storytelling, we will consider adding emissions and refer to the equivalent monetary damage (decreased consumption) that it creates. A similar consideration of reducing emissions and its equivalent in terms of increased consumption is also valid.

According to a note made earlier in the text, subtracting x dollars from the consumption equation (while keeping emission balance equation intact) would achieve exactly that same effect on the optimal value of utility as adding one ton to emission balance equation (while keeping the consumption intact). This newly subtracted consumption x has to have the same "external" nature i.e. being something that is not controlled by the model similar to the discussed case of the nature of the emissions perturbation. Such an external source for consumption decrease could be a disaster that is not taken into account in the damage function of the model⁷. However, that is a rather virtual disaster, as it implies a pulse that only affects consumption, but does not affect the capital.

As we have noted earlier, this interpretation of SCC is fully compatible with the interpretation comparing an unperturbed problem with a perturbed one where a perturbation in emissions (plus or minus one ton CO₂) is compensated by the perturbation in consumption (minus or plus x dollars respectively), so that the optimal social utility remains the same.

Discussion and Implications for Policy Context

The SCC equates emissions' and consumption perturbations in a perturbed problem in such a way that the maximum societal utility on an optimal trajectory of the new perturbed problem remains the same as in the unperturbed problem. This, however, has nothing to do with any deviation of actual emissions under control from the estimated optimal plan. For example, both over-emitting and under-emitting as compared to the optimum would lead to losses in utility and by that create a social cost in a wider sense. From this perspective, the SCC as calculated in (Eq. 1), seems to be an irrelevant concept to justify or enforce keeping emissions on an optimal trajectory in whatever form including its application as a carbon tax.

SCC only comes handy in case if, because of the reasons beyond the controls embedded into the model i.e. unforeseen event, the emission equation gets disturbed. In this case it only can estimate the monetary damage of such an event in the sense that if there were an "external" source for increasing consumption by that amount, that event would not have had any impact on the utility. In no case SCC can provide a guidance on how to re-distribute consumption and investment after such event has been discovered - to answer these questions one has to carry out an optimization of the new (perturbed) problem.

⁵ The related decision variable is denoted as the emission control rate, MIU(t) in the DICE 2016 GAMS source code.

⁶ The related decision variable is denoted as the gross savings rate as fraction of gross world product, S(t) in the DICE 2016 GAMS source code.

⁷ For a symmetric consideration (emission reduction) positive non-zero x is harder to justify within the existing model, this would be an increase in consumption that is not supported by production as conceptualized in the model.

It is needless to say that SCC and carbon price are not comparable despite they are expressed in the same units. As the DICE model is run and an optimal solution is found, the CP is the only optimal carbon price (cost of carbon) in the societal context as reflected by the models' utility function. This social optimality is unconditional on the value of SCC.

While there are cases where the numerical value of SCC in DICE happens to be close to CP for a relatively long time of 30-100 years after the start of the modeling period (see *Fig. 1* and *Fig. 2*), for other model setups, CP can be overestimated by SCC by the factor of four already in 50 years after the start of the modeling period (see *Fig. 3*). This overestimation is not conditional on any model parameters' uncertainty and stems only from the used calculation method.

The findings obtained from the analysis of the SCC calculation in the DICE model are relevant beyond the scope of DICE itself. The discovered semantic issue is in the attempt to apply the SCC value (obtained via perturbed problem employing non-human made emissions that are different from those in the original problem) to shape human-made emissions through policies like a carbon tax. The FUND (6,7) and PAGE (8-11) models that also estimate SCC, while being structurally different from DICE (12), both employ the same idea of an "emission pulse" that increases total emissions by a pre-defined amount over certain period of time to generate a new "marginal" model (that is otherwise equal to the original), which then provides a trajectory to derive the SCC value. These newly added emissions are not caused by any change in the abatement and are the basis for the SCC estimation. The DICE model vividly demonstrates inconsistencies resulting from the application of such constructed SCC to enforce desired abatement.

The presented research results call for a clear specification of the meaning of SCC and revising the methods for its estimation within an integrated modeling approach.

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Acknowledgments: The authors acknowledge the early discussions around the DICE model with their IIASA colleagues Elena Rovenskaya, Artem Baklanov, Fabian Wagner, Thomas Gasser, Petr Havlik, and others. These discussions spurred the interest in a deeper exploration of DICE that ultimately resulted in the presented analysis. The authors are grateful to IIASA Deputy Director General for Science Leena Srivastava for her attention and feedback. The authors acknowledge Dr. Linus Mattauch from the University of Oxford for sharing the view on an early draft of this manuscript. The authors acknowledge William Nordhaus for making the DICE source code and documentation openly available as well as the clean and transparent model structure that all made the present research possible.

Funding: Austrian Science Fund (FWF): P31796-N29/"Medium Complexity Earth System Risk Management" (ERM). European Research Council Synergy Grant number 610028 Imbalance-P:

Effects of phosphorus limitations on Life, Earth system and Society (Seventh Framework Programme of the European Union).

Author contributions: NK has conceptualized the problem; NK and AS have carried out the investigation; MO has contributed to funding acquisition; NK, AS, and MO have discussed the results in the process of investigation; NK has drafted the paper; all co-authors have contributed to writing the manuscript.

Competing interests: Authors declare no competing interests.

Data and materials availability: The DICE, FUND, and PAGE models are freely available online.

Supplementary Materials: None.