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Morgan- Voyce Approach for Solution Bratu Problems

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نهج مورغان فويس لحل مسائل براتو

ملخص

معادلات براتو من المعادلات المهمه في مسائل الكهرباء والبلازما .الهدف من هذا البحث هو تصميم تقريب لمورغان فويس لحل مسألة براتو قدمنا متعددة الحدود مورغان فويس مع مجموعة من اهم الخصائص فعالية الخوارزميه المقترحه تم توضيحها من خلال ثلاثه امثله عدديه.

Abstract

Bratu equations are substantial in electrostatic and plasma problem. The aim of this paper is to design morgan-voyce approaches for solving bratu problem; Morgan- voyce polynomials and some of its important properties were introduced; the effectiveness of the proposed approach is demonstrated by considering three numerical examples.

1. Introduction

The bratu problem is more used in applied mathematics, physical and chemical phenomena [1-4].Many of numerical and approximate methods have been studied to solve bratu equation, for example, Bernoulli polynomial [5], Finite difference [6], Bernstion polynomial [7] and B-spline collocation method [8].

Consider the Bratu boundary value problem of the form:

$$\ddot{u}(t) + \lambda\, e^{u(t)} = 0 \qquad 0 < x < 1$$

$$u(0) = u(1) = 0 \qquad for\ \lambda > 0 \qquad \ldots(1)$$

The outlined of the paper is as follows:- section 2, we explained and construct the Morgan-Voyce polynomial, some properties of this polynomial discussed in section 3,we analyzed the Morgan-Voyce approach for the solution of bratu problem in section 4, in section 5, numerical examples are presented, finally, the main conclusion are given in section 6.

2- Morgan-Voyce Polynomial:-

Morgan-Voyce polynomial introduced as recently as 1959 in the analysis of ladder network [9], many authors used this polynomial for solving problem, such as [10- 12].

For n ≥ 2 , the Morgan-Voyce polynomial $M_n(x)$ defined by :-

$$M_n(x) = (x+2)M_{n-1}(x) - M_{n-2}(x) \ ,$$

$$M_0(x) = 1\ ,\ \ M_1(x) = x + 1 \ \ \ldots(2)$$

While the explicit form is

$$M_n(x) = \sum_{k=0}^{n-1} \binom{n+k-1}{2k}\ x^k \ ,n \geq 1 \qquad \ldots(3)$$

using.(2) & (3), the first six Morgan-Voyce polynomial are

$M_0(x) = 1$,

$M_1(x) = x + 1$,

$M_2(x) = x^2 + 3x + 1,$

$M_3(x) = x^3 + 5x^2 + 6x + 1,$

$M_4(x) = x^4 + 7x^3 + 15x^2 + 10x + 1,$

$M_5(x) = x^5 + 9x^4 + 28x^3 + 35x^2 + 15x + 1$,

$M_6(x) = x^6 + 11x^5 + 45x^4 + 84x^3 + 70x^2 + 21x + 1,$

3- Important Properties of Morgan-Voyce polynomial: - [13]

In this section, some important properties of M.V were discussed

1. Generating function

$M(x,t) = \sum_0^\infty M_n(x)t^n = (1-t)\,[1-(xt+2t-t^2)]^{-1}$

2. Zeros

$$M_n(x): x_r = -4\,sin^2\left[\frac{2r-1}{2n+1} * \frac{\pi}{2}\right],$$
$$r = 1,2,\dots,n$$

3. Orthogonaltiy:-

$M_n(x)$ is orthogonal polynomial over (-4,0) with to weight function $\sqrt{\frac{-x}{x+4}}$

4. Differential Equations:-

$M_n(x) = x(x+4)\ddot{u} + 2(x+1)\dot{u} - n(n+1)u = 0$

5. Simson Formula:-

$$M_{n+1}(x)M_{n-1}(x) - M_n{}^2(x) = x$$

4. Approach of Morgan-Voyce:-

In this section, Morgan-Voyce polynomial are used to obtain the approximate solution of Bratu problem which is given in eq(1).

we first approximate u(x) as:-

$$u(x) \cong C_i M_i(x) \qquad \dots(4)$$

where C_i^s are unknown coefficients and $M_i(x)$ are Morgan-Voyce polynomial, let $x_0, x_1, \dots, x_n$ be n+1 grid points in the interval [a,b] so that , $x_i = a + ih$. Then by substituting the Eq(4) in eq(1) we get:-

$$\sum_{i=0}^n C_i \ddot{M_i}(x) + \lambda \exp(\sum_{i=0}^n C_i M_i(x)) = 0,$$
$$i = 0,1,\dots,n \qquad \dots(5)$$

and by substitut the boundary conditions unknown coefficients C_i will be reduced, finally by using discretization collocation point, a nonlinear system equation will be obtained that will be solved by Newton iteration method using Matlab program.

5. Numerical Examples:-

In this section, we explain the applicability and accuracy of the proposed method in three numerical examples.

Example 1:-

Consider the general form of Bratu equation

$$\ddot{u} + e^u = 0\,, \quad 0 < x < 1,$$
$$u(0) = u(1) = 0 \qquad \dots(6)$$

Exact solution $u(x) = -2\ln\left[\frac{cosh\left(x-\frac{1}{2}\right)\frac{\theta}{2}}{cosh\frac{\theta}{4}}\right],$

θ *solution of* $\theta - 2\sqrt{\lambda}cosh\frac{\theta}{4}$

first we assume

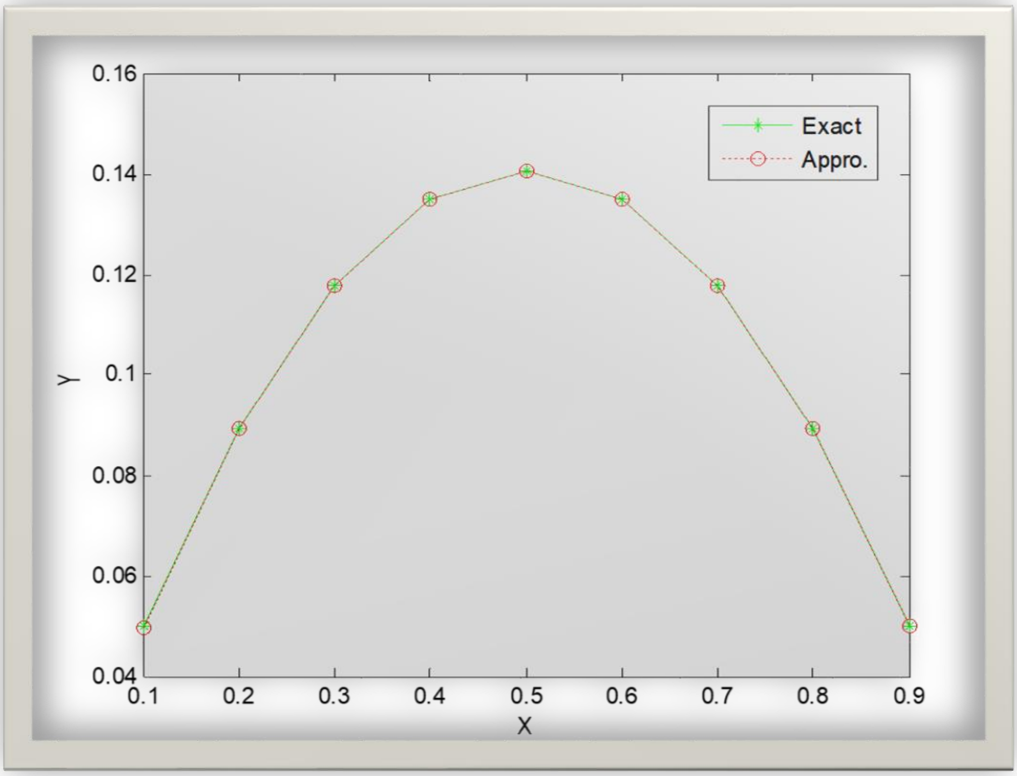
$$u(x) = \sum_{i=0}^5 C_i M_i(x) \qquad \dots(7)$$

then by deriving Eq(7) twice with respect to x and substituting the initial conditions ,we obtain $\ddot{u}$ and by substituting eq(6) and using collocation point we get system with 4 unknown coefficients. After solving this system by Newton iteration method the approximate solution of Eq.(6) will be found. Table.1 and fig.1 explain the comparison between the exact and approximate solutions.

Table 1: Exact and Approximate results of Example 1

x	Exact	Approximate	Absolute Error
0.1	0.049847	0.049765	0.000092
0.2	0.089189	0.089199	0.00001
0.3	0.117609	0.117600	0.000009
0.4	0.134790	0.134889	0.000099
0.5	0.140539	0.140545	0.000006
0.6	0.134790	0.134779	0.000011
0.7	0.117609	0.117607	0.000002
0.8	0.089189	0.089189	0.0000000
0.9	0.0498468	0.049837	0.000011

Fig.1



Example 2:-

Consider the other Bratu problem $\lambda = 2$

$\ddot{u} + 2e^{u} = 0$, $u(0) = u(1) = 0$

with the exact solution $u(x) = -2\ln(\cos(x))$

the comparison between the exact and approximate solution for N=8 presented by table 2 and fig. 2.

Example 3:-[14]

Finally consider the general form of Bratu problem with $\lambda = \pi^2$

$\ddot{u} - \pi^2 e^{u} = 0$, $0 < x < 1, u(0) = u(1) = 0..(6)$

The exact solution $u(x) = -\ln\left[1 + \cos(\frac{1}{2} + x)\pi\right]$

The absolute error between the exact and approximate solution with N=6 is shown in Table 3 and Fiq.3.

Table 2 :Computed absolute errors for example 2

x	Exact	Approximate	Absolute Error
0.1	0.114411	0.114352	0.000059
0.2	0.206419	0.206418	0.000001
0.3	0.273879	0.273886	0.000007
0.4	0.315089	0.315087	0.000002
0.5	0.328952	0.328951	0.000001
0.6	0.315089	0.315088	0.000001
0.7	0.273879	0.273877	0.000011
0.8	0.206419	0.206418	0.000001
0.9	0.114411	0.114421	0.00001

Fig.2

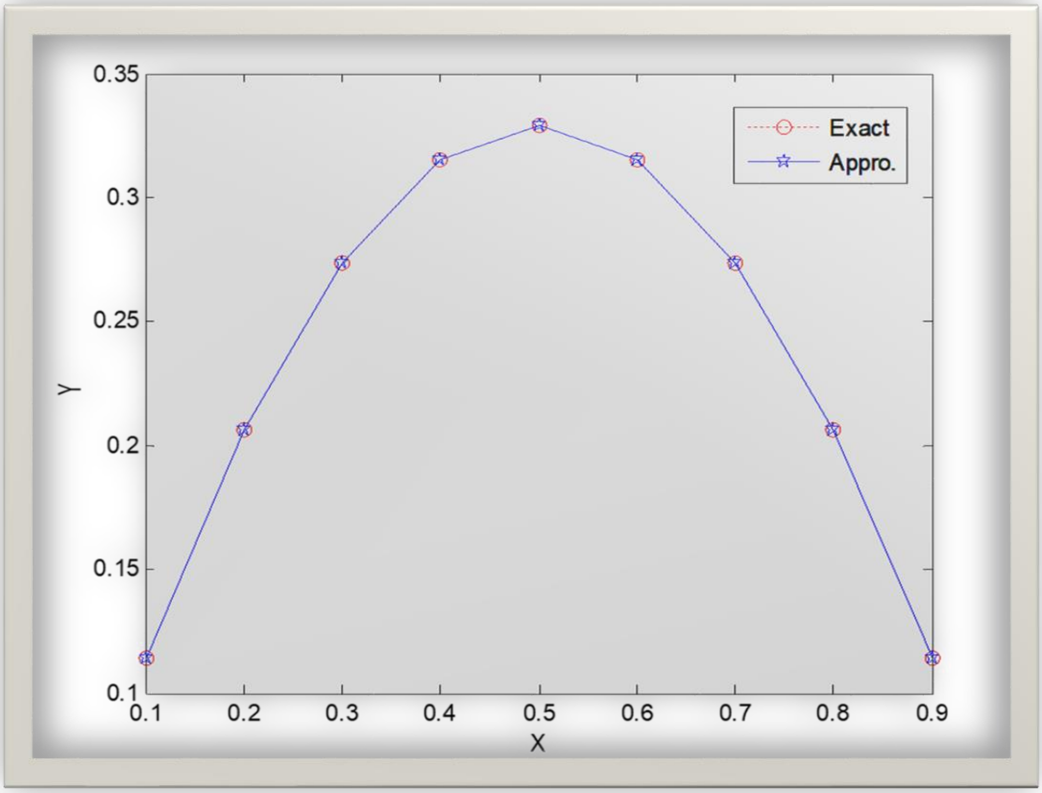
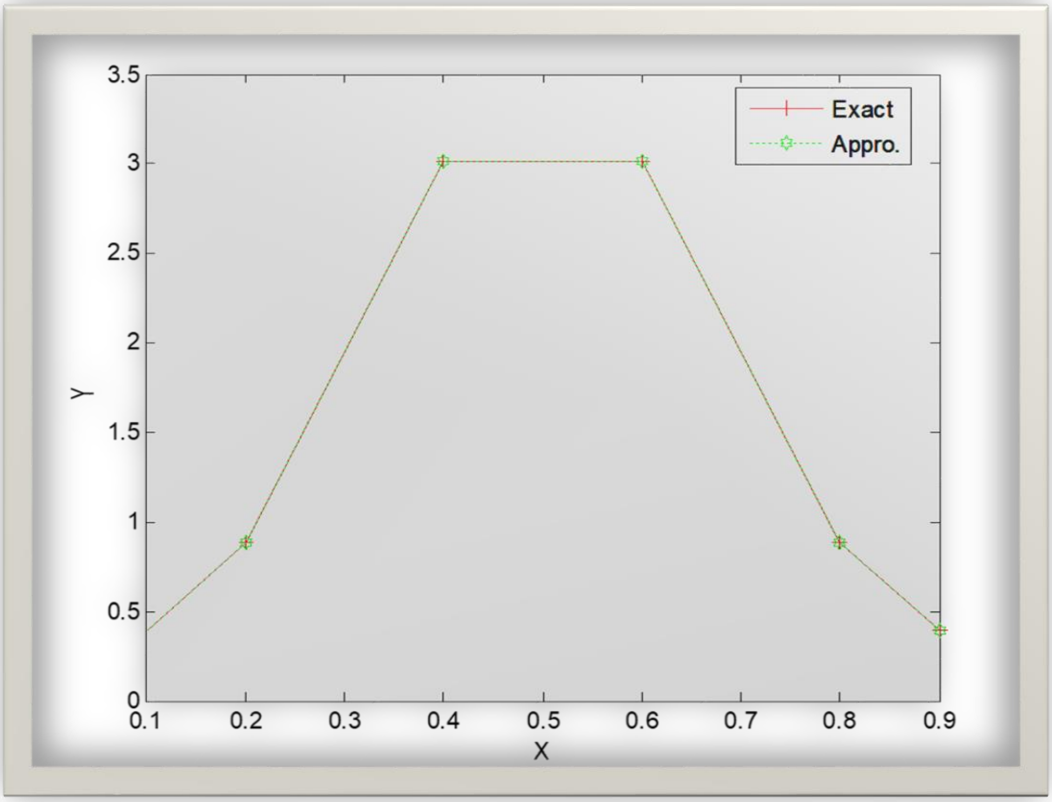


Table 3 : absolute errors for example 3

x	Exact	Approximate	Absolute Error
0.1	0.396640	0.396632	0.000008
0.2	0.886211	0.886222	0.000011
0.4	3.017089	3.017088	0.000001
0.6	3.017089	3.017087	0.000001
0.8	0.886211	0.886161	0.00005
0.9	0.396640	0.396592	0.000048

Fig.3



6. Conclusion:-

Approximate solution for Bratu problem by using Morgan- Voyce approach has been obtained. This method decreased into a system of nonlinear algebraic equations which solved by Newton iteration method. Numerical examples have been offered applicability and validity of the technique.

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