

A STUDY OF MULTIPLE CRACKS ON DYNAMIC CHARACTERISTIC AND STRESS INTENSITY FACTOR IN CANTILEVER BEAMS

MOHD AZIM BIN MD JANIS

A project report submitted in partial
fulfillment of the requirement for the award of the
Degree of Master of Mechanical Engineering

Faculty of Mechanical and Manufacturing Engineering
Universiti Tun Hussein Onn Malaysia

JULY 2015

ABSTRACT

Two dimensional finite element models of multiple cracks cantilever beam were established and the dynamic characteristic of the model were studied. The model was developed by using ANSYS and the finite element mesh model was divided into six nodes or eight nodes plane element by using PLANE183. The dynamic characteristic of the cantilever beam were simulated and discussed under the difference crack size and crack location. The mode shape behaviour showed increased trend when the cracks exists in cantilever beam. The natural frequency of cantilever beam were dropped and the shape behavior were changed when the cracks exists. The natural frequency was decreased when the crack size increased while the natural frequency increased as the crack location far away from the fixed end cantilever beam. On the basis, using the fracture mechanics and finite element method, the stress intensity factors were analyzed when the cracks exists in the cantilever beam. The stress intensity factors were studied under changes of cracks size and cracks location. The model was simulated by finite element software and the results were collected and analyzed. These studies establish good information for multiple cracks identification and fracture analysis of cantilever beam.

ABSTRAK

Model dua dimensi unsur terhingga bagi rasuk julur yang mempunyai beberapa retak dibangunkan dan ciri-ciri dinamik bagi model tersebut dikaji. Model dibangunkan dengan menggunakan *ANSYS* dan model unsur terhingga dibahagi kepada enam nod atau lapan nod unsur satah dengan menggunakan *PLANE183*. Simulasi bagi ciri-ciri dinamik rasuk jalur itu dijalankan dan keputusannya dibincangkan mengikut situasi perubahan saiz retak dan kedudukan retak. Ciri-ciri bentuk menunjukkan perubahan apabila terdapat retak pada rasuk jalur. Frekuensi tabii bagi rasuk jalur akan berkurang dan tingkah laku bentuk berubah apabila keretakan wujud. Frekuensi tabii akan berkurang apabila saiz retak pada model bertambah manakala frekuensi tabii meningkat apabila kedudukan retak menjauhi dari hujung tetap rasuk jalur. Secara asas, dengan menggunakan mekanik patah dan kaedah unsur terhingga, faktor tegasan dianalisis apabila terdapat retak yang wujud pada rasuk jalur. Faktor tegasan dikaji di bawah faktor perubahan saiz retak dan perubahan kedudukan retak. Simulasi model dijalankan dan data dikumpul dan dianalisis. Kajian ini dapat memberi maklumat yang baik berkenaan pengesanan beberapa retak dan analisis mekanik patah bagi rasuk jalur.

CONTENTS

TITLE	i
DECLARATION	ii
ACKNOWLEDGEMENT	iii
ABSTRACT	iv
ABSTRAK	v
CONTENTS	vi
LIST OF TABLES	ix
LIST OF FIGURES	x
LIST OF SYMBOLS AND ABBREVIATIONS	xiii
 CHAPTER 1 INTRODUCTION	 1
1.1 Beam structure	1
1.2 Vibration analysis non-destructive testing method on damage structure	3
1.3 Problem statement	4
1.4 Objective of the study	4
1.5 Scope of the study	5

CHAPTER 2 LITERATURE REVIEW 6

2.1	Overview	6
2.2	Non-destructive testing	6
2.3	Fracture mechanics and stress intensity factor (SIF)	7
2.4	Mathematical modelling of vibration of cracked beam	10
2.5	Review of previous studies	12

CHAPTER 3 METHODOLOGY 19

3.1	Overview of the finite element method	19
3.2	ANSYS program	20
3.3	Overview of the study	21
3.3.1	Verification of the natural frequency	22
3.3.2	The mode shape of the model	26
3.3.3	Study the natural frequency of cracked cantilever beam under different crack size and crack location	27
3.3.4	Stress Intensity Factor	27
3.3.5	Study the SIF of cracks in cantilever beam under different cracks size and cracks location	30
3.3.6	Study the relationship of SIF and natural frequency	30

CHAPTER 4 RESULTS & DISCUSSION 31

4.1	Overview	31
4.2	Verification	32
4.3	Mode shape of the vibration	33

4.4	Study the natural frequency of cracked cantilever beam under different crack size and crack location	37
4.4.1	Influence of crack size on natural frequency	37
4.4.2	Influence of crack location on natural frequency	39
4.5	Study the SIF of cracks in cantilever beam under different cracks size and cracks location	42
4.5.1	Influence of crack size on SIF	42
4.5.2	Influence of crack location on SIF	44
4.6	Study the relationship of SIF and natural frequency	47
CHAPTER 5 CONCLUSION		50
5.1	Overview	50
5.2	Conclusion	50
5.3	Recommendation	52
REFERENCES		53

LIST OF TABLES

3.1	The natural frequency from experimental test and simulation	22
3.2	The dimension of cantilever beam used for verification	24
4.1	Result of verification of the model	32
4.2	Percentage error between previous experimental results and present study	33
4.3	Percentage error of previous simulation results and present study	33
4.4	Result of natural frequencies under different set of crack size	37
4.5	Result of natural frequencies under different set of crack location	40
4.6	Result of K_I of first crack a_1 under different crack size	42
4.7	Result of K_I of second crack a_2 under different crack size	43
4.8	Result of K_I under different crack location for position load a	45
4.9	Result of K_I under different crack location for position load b	45
4.10	Data of natural frequencies and K_I for first crack a_1	47
4.11	Data of natural frequencies and K_I for second crack a_2	48

LIST OF FIGURES

1.1	Application of cantilever beam in aircraft wing	2
1.2	A cantilever beam	2
1.3	The cantilever beam having two cracks	3
2.1	Modes of crack a) mode I – opening, b) mode II – shearing and c) mode III – tearing	8
2.2	Model 1 of crack	9
2.3	Multiple cracks on beam (a) represented by massless rotational spring model (b)	10
2.4	Crack modeled as spring, a) bars in longitudinal vibration, b) beams in pure bending vibration	13
2.5	Geometry of the cracked cantilever beam	14
2.6	Component of whole structure and dividing into the finite number of element	14
2.7	The geometry of crack gear tooth modeled as cantilever beam	16
2.8	The structure model of cracked gear	16
2.9	The model of cracked gear for SIF analysis	17
2.10	Schematic diagram of cracked beam model and moving mass	18
2.11	The reduction of SIF with increasing of % SMA wire in composite	18
3.1	Steps performed in ANSYS	20
3.2	Flowchart of the study process	21
3.3	Geometry of cantilever beam having two cracks	23

3.4	The geometry of PLANE183 of a) 8-node quadrilateral element, b) 8-node triangular element and c) 6-node triangular element	24
3.5	Two dimensional cantilever beam having two cracks after meshing in ANSYS	25
3.6	Flowchart of modal analysis by ANSYS	26
3.7	The shape of crack tip	28
3.8	Flowchart of SIF analysis by ANSYS	29
3.9	The position of applied load ' a ' and ' b '	30
4.1	The shape behaviour of undamaged cantilever at mode 1 of vibration	34
4.2	The shape behaviour of undamaged cantilever beam at mode 2 of vibration	34
4.3	The shape behaviour of undamaged cantilever beam at mode 3 of vibration	35
4.4	The shape behaviour of damaged cantilever beam at mode 1 of vibration	35
4.5	The shape behaviour of damaged cantilever beam at mode 2 of vibration	36
4.6	The shape behaviour of damaged cantilever beam at mode 3 of vibration	36
4.7	Graph of natural frequencies in different dimensionless crack size for a) mode one, b) mode two and c) mode three	38
4.8	Graph of natural frequencies in different dimensionless crack location for a) mode one, b) mode two and c) mode three	41
4.9	K_I versus dimensionless crack size at first crack a_1 under two position applied load	43
4.10	K_I versus dimensionless crack size at first crack a_2 under two position applied load	44
4.11	K_I versus dimensionless crack location under different crack location for position load a	46

4.12	K_I versus dimensionless crack location under different crack location for position load b	46
4.13	Natural Frequency vs K_I for first crack a_1	48
4.14	Natural Frequency vs K_I for second crack a_2	49

LIST OF SYMBOLS AND ABBREVIATIONS

K_I	-	Stress intensity factor
K_{IC}	-	Critical stress intensity factor
σ	-	Tensile stress
π	-	Pi
F	-	Correction factor
E	-	Modulus young
I	-	Moment of area
a	-	Crack size
s	-	Crack location
ν	-	Poisson ratio
L	-	Length of cantilever beam
w	-	Width
h	-	Thickness
x,y	-	Cartesian coordinate
α	-	Dimensionless crack size
β	-	Dimensionless crack location
$[M]^e$	-	Element mass matrix
$[K]^e$	-	Element stiffness matrix
$\{W\}^e$	-	Element displacement vector
$\{F\}^e$	-	Element load vector
ρ	-	Density
A	-	Cross sectional area
k	-	Torsional stiffness at open crack
I	-	Moment of inertia

$[K]_c$	-	Crack element stiffness matrix
f	-	Natural frequency
NDT	-	Non-destructive testing
SIF	-	Stress intensity factor
FEM	-	Finite element method

CHAPTER 1

INTRODUCTION

1.1 Beam structure

Beam are one the important structural element that already extensively been used in many engineering application. It is considered as a horizontal engineering structure used to support horizontal load and have capability to withstand load by resisting bending. It is usually long, straight and prismatic members that support loads which act perpendicular to the longitudinal axis of the member. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment. There are many engineering structure that been develop based on beam element such as bridge, skyscrapers, robot arm and aircraft wing.

Currently, there are many types of beams exists and it is classified based on which they are supported. The simply supported beam is beams that have support on both end and it is free to rotate. The fixed beams have support and fixed on both of their end. An over-hanging beam extends beyond its support on one end while the continuous beams have more than two supports thoroughly the beam. The cantilever beam has fixed support only at one end and free at another end. The trussed beam is added a cable or rod to form a truss in order to strengthen the beam (Philpot, 2011).



Figure 1.1: Application of cantilever beam in aircraft wing

The cantilever beam is a one type of beam that extensively used in engineering structure and one of the applications of cantilever beam is shown in Figure 1.1. The cantilever beam is fixed at only one end and carries the load to the support where it is resisted by moment and shear stress. The construction of cantilever beam allows for overhanging structure without external structure. It also can be constructed with trusses or slabs. The diagram of the cantilever beam is shown in Figure 1.2.

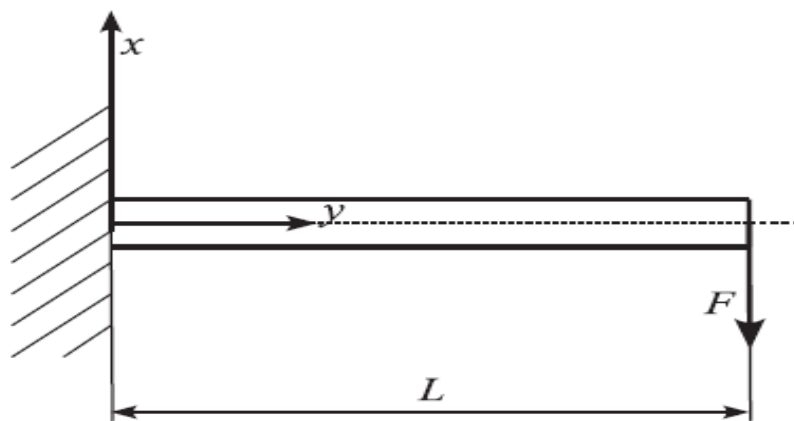


Figure 1.2: A cantilever beam (Carrera *et al.*, 2011)

However, the cantilever beam also cannot escape from structural failure due to the presence of cracks that develop mainly from fatigue loading. So it is useful for detection of the crack earlier before it become fracture.

1.2 Vibration analysis non-destructive testing method on damage structure

The safety of structures such as buildings and bridges and also their structural components such as beams, columns and canopies already become a public concern in all developed countries. The crack is developed due to fatigue loading. A crack occurs in a structural element causes some local variation in stiffness which affects the dynamic behavior of the element and the whole structure. The natural frequencies, amplitude of forced vibration and area of dynamic stability shall change due to the existence of the crack. An analysis on these changes is used to identify the location and the depth of the crack. This information of that dynamic behavior change will enable us to determine the degree of sustainability of the structural element and the whole body structure.

Vibration analysis which can be used to detect structural defects such as cracks can offers an effective and inexpensive non-destructive testing for structure. Recently, vibration analysis of cracked beam is one of the subjects which have attracted the attention of many researchers in this area. Thus, for this research, the rectangular cross-sectional cantilever beam having multiple cracks was used for analysis of vibration and the model was developed using ANSYS.

The schematic diagram of the cantilever beam used for this research is shown in Figure 1.3.

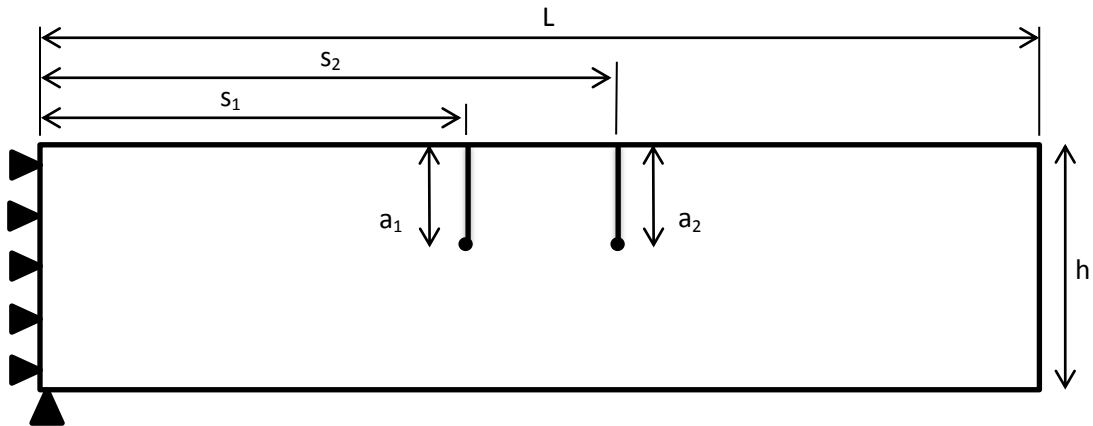


Figure 1.3: The cantilever beam having two cracks

1.3 Problem statement

The vibration analysis can be used as non-destructive testing method to identify and detect the damage beam. From the vibration analysis, the dynamic characteristic such as natural frequencies and their mode shape can be analyzed for detection of the cracks. The failure of the crack beam is governed by the stresses in the vicinity of the crack tip. The singular stress contribution is characterized by the stress intensity factor (SIF). The SIF play an important role in determining the fracture of the beam. Both vibration analysis and SIF study can give good information for crack identification and fracture analysis of the engineering structure.

Previously, there was not much study done by the researcher that focus on finding both dynamic characteristic and SIF in one research project. Therefore, in this study, both the natural frequency of the model and SIF of multiple cracks in cantilever beams were identified simultaneously and discussed. The relationship between natural frequency and the SIF also identified.

1.4 Objective of the study

Based on the problem statement, the objectives of this research are

- i. To analyze the natural frequency of the multiple cracks in cantilever beam model and verified the results with previous experiment and simulation to ensure the model developed was correct for further study.
- ii. To analyze the mode shape behavior of the multiple cracks in cantilever beam.
- iii. To find out the natural frequency and stress intensity factor (SIF) of multiple cracks in cantilever beam under different crack size and crack location.
- iv. To find out the relationship between the natural frequency and the SIF of cracks in cantilever beam.

1.5 Scope of the study

The study covers modelling and simulation of double crack in cantilever beam using ANSYS for finite element analysis. The scope of study was to investigate the effect of crack size and crack location to the natural frequencies and the SIF. The mode shape of the model also studied to see the behavior of cantilever beam having double cracks. The cantilever beam was made of C30 steel with dimensions of $0.02 \times 0.02 \times 0.8 \text{ m}^3$. The cantilever beam had rectangular cross sectional and double cracks. It assumed to be uniform and the boundary condition was fixed-free end.

CHAPTER 2

LITERATURE RIVIEW

2.1 Overview

In this chapter, a comprehensive review on the area topic involve in this study are provided and explained. It is reviewed about the concept of non-destructive testing, the stress intensity factor (SIF) together with the growth of the crack and the previous studies done by researcher around the world. The mathematical modelling of vibration of cracked beam also included.

2.2 Non-destructive testing

An engineering structure like cantilever beam is built to perform such a certain function and the user expect it will perform their function well during a certain period of time. However, during the operation of structure, they will expose under static, cyclic loading and environmental effects that may cause structural defects such as crack. If this defect did not take any prevention, it may lead to catastrophic failure of the structure. Thus, for the safety reason, it is importance for inspection on the structure to maintain the quality of structure and also to diagnose and detect the failure earlier before it can harm human. There are many procedures and testing that can be used to detect the crack on the engineering structure like cantilever beam and the user now likely to use the technique that can be examined the structure without affecting their performance.

Non-destructive testing (NDT) is an analysis technique used in science and engineering based on the application of physical principles employed for the purpose of determining the characteristic of materials, component or system and for detecting and assessing defects on the structure without affecting in any way their future usefulness and serviceability. These techniques play an important role in quality control during various stage of manufacturing, finishing structure and also during operation of the structure while retaining its structural integrity (Raj *et al.*, 2007).

There are many different NDT techniques that already proposed for different materials and components. The choice of specific NDT techniques are depends on many factors including availability, accessibility and suitability based on analysis and past experience. Some of the proposed NDT techniques that already used for identification of fracture materials are visual inspection, liquid penetrant testing, eddy current testing, magnetic particle testing, ultrasonic testing, radiographic testing and thermographic (Raj *et al.*, 2007).

The new reliable and inexpensive methods are now become the subject of attention to the researchers around the world. One of the NDT technique use vibration analysis on the damage structure offer the user an inexpensive method to detect the structural defect such as crack. This high capability technique will give us valuable information of the defect structure contain about what type of changes occur in the vibration characteristic, how these change can be detected and how the condition of the structure (Kisa, 2004).

2.3 Fracture mechanics and stress intensity factor (SIF)

Fracture mechanics is a field of solid mechanics that study about the mechanical behavior of cracked bodies subjected to an applied load. It deals with irreversible process of rupture due to nucleation and cracks growth. The formation of the cracks is a complex fracture process in which it depends on the microstructure of a particular crystalline or amorphous solid, applied loading and environment (Perez, 2004).

Crack on the structure initially occur when the energy available for crack growth is sufficient to overcome the resistance of the material. Material resistance against the crack growth includes surface energy and plastic work. Crack on structure

growth because of influenced from external forces. The crack form can be divided into three type of mode as shown in Figure 2.1 (Shao *et al.*, 2013):

- Mode I (opening) – the crack is opened and extended under tensile stress and it is perpendicular to direction of propagation.
- Mode II (shear/sliding) – the crack is slide and extended under shear stress and it is parallel to direction of propagation.
- Mode III (tearing) – the crack is torn and extended under shear stress and both parallel to crack surface and crack front.

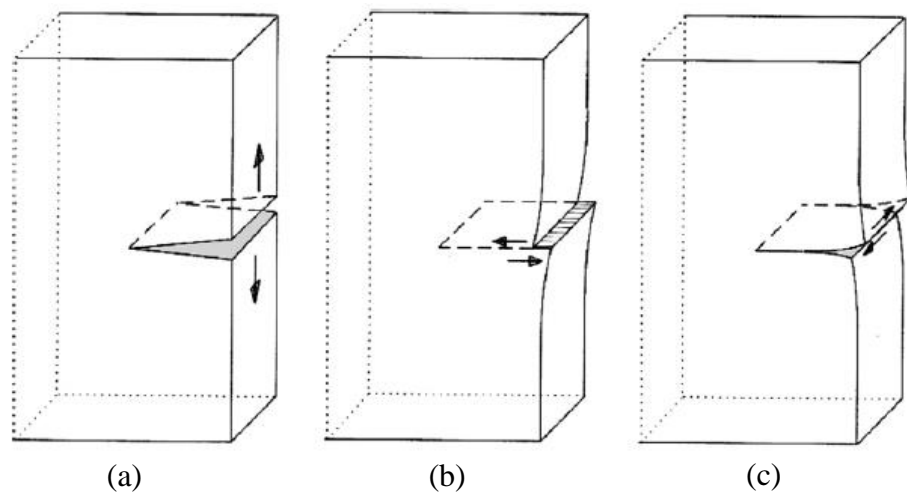


Figure 2.1: Modes of crack a) mode I – opening, b) mode II – shearing and c) mode III – tearing (Shao *et al.*, 2013)

In the field of fracture mechanics, there is some important parameter measured that used to determine either the crack will occur on structure or not. The critical stress intensity factor, K_{IC} or called also as fracture toughness is the measurement of the material to prevent unstable crack growth capacity and it is the material property. The capacity for material to resists the structure to initiate crack is depends directly with the value of K_{IC} for any material. There is possibility to test the K_{IC} values of various materials by experimental methods as shown in Figure 2.2. The penetrating crack with length $2a$ is in the center of the sample and the following stress distribution near the crack tip can be proved (Shao *et al.*, 2013).

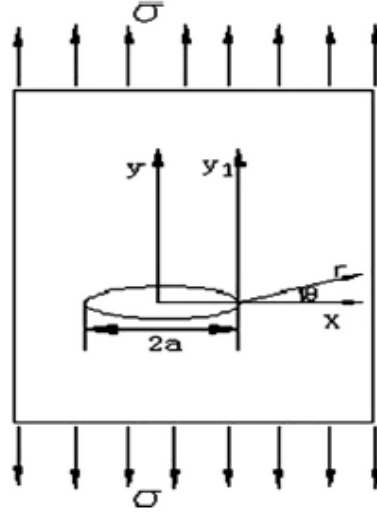


Figure 2.2: Model 1 of crack
(Shao *et al.*, 2013)

$$\sigma_x = \frac{K_1}{\sqrt{2\pi r}} \left\{ \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \right\} \quad (2.1)$$

$$\sigma_y = \frac{K_1}{\sqrt{2\pi r}} \left\{ \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \right\} \quad (2.2)$$

$$\tau_{xy} = \frac{K_1}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (2.3)$$

From equation 2.1, 2.2 and 2.3, it proved that the stress component all have K_1 which is relevant to crack size, shape and stress. K_1 is called stress intensity factor (SIF) in fracture mechanics. The formulation of K_1 is given by (Shao *et al.*, 2013):

$$K_1 = F\sigma\sqrt{\pi a} \quad (2.4)$$

where F is correction factor (based on the geometry of crack), σ is an applied stress and a is the length of crack.

In fracture mechanics, K_1 play an important role in determines whether any structure start to broken or not. If the applied stress on the crack tips increase, the K_1 value also increases. When the value of K_1 reaches a limit in which the value is

equal or higher than the fracture toughness, K_{IC} of any material, the structure start broken.

2.4 Mathematical modelling of vibration of cracked beam

For the investigation of the vibration of multiple cracks in cantilever beam, the geometry used is shown in Figure 2.3. The cracks is modelled as a massless rotational springs. $\alpha_i = \frac{a_i}{h}$ and $\beta_i = \frac{s_i}{L}$ are the dimensionless crack size and dimensionless crack location ($i=1,2,\dots$). h is a thickness of beam while L is a length of beam.

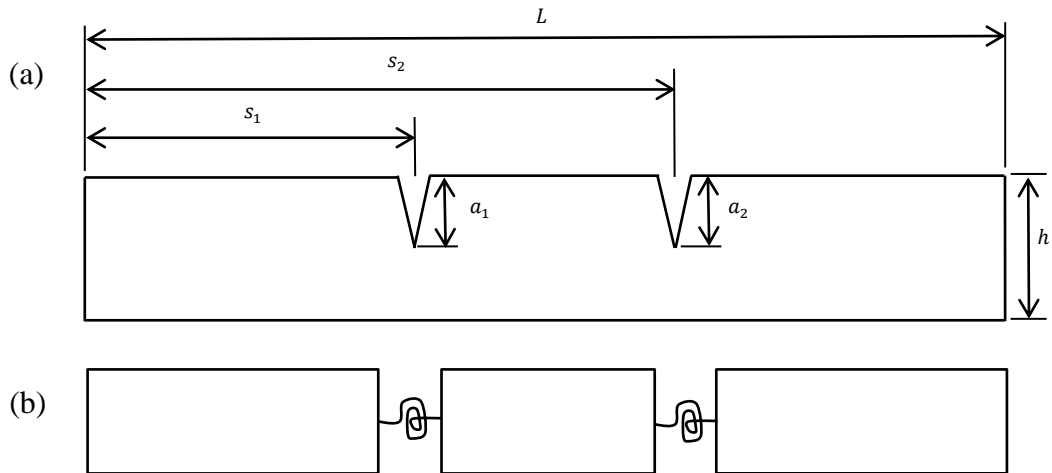


Figure 2.3: Multiple cracks on beam (a) represented by massless rotational spring model (b) (Lee, 2009b)

The finite element equation of beam element having length ΔL based on Euler-Bernoulli theory is given as:

$$[M]^e \{\ddot{W}\}^e + [K]^e \{W\}^e = \{F\}^e \quad (2.5)$$

where $[M]^e$ is the element mass matrix, $[K]^e$ is the element stiffness matrix, $\{W\}^e$ is the element displacement vector $\{w_i \quad \theta_i \quad w_{i+1} \quad \theta_{i+1}\}^T$ and $\{F\}^e$ is the element load vector. The element mass matrix, $[M]^e$ is defined as:

$$[M]^e = \frac{\rho A \Delta L}{420} \begin{bmatrix} 156 & 22\Delta L & 54 & -13\Delta L \\ 22\Delta L & 4(\Delta L)^2 & 13\Delta L & -3(\Delta L)^2 \\ 54 & 13\Delta L & 156 & -22\Delta L \\ -13\Delta L & -3(\Delta L)^2 & -22\Delta L & 4(\Delta L)^2 \end{bmatrix} \quad (2.6)$$

where ρ is a density and A is a cross sectional area. The element stiffness matrix is defined as:

$$[K]^e = \frac{EI}{(\Delta L)^3} \begin{bmatrix} 12 & 6\Delta L & -12 & 6\Delta L \\ 6\Delta L & 4(\Delta L)^2 & -6\Delta L & 2(\Delta L)^2 \\ -12 & -6\Delta L & 12 & -6\Delta L \\ 6\Delta L & 2(\Delta L)^2 & -6\Delta L & 4(\Delta L)^2 \end{bmatrix} \quad (2.7)$$

where E is a Young's modulus and I is a moment of area. The massless rotational spring model is connected with node j and $j+1$ and the deflection of node j and $j+1$ are same ($w = w_{j+1}$). So, the rotations θ_j and θ_{j+1} are coupled through crack stiffness matrix as shown below:

$$[K]_c = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \quad (2.8)$$

where k is a torsional stiffness at open crack. Ostachowicz & Krawczuk (1991) proposed k in their paper as:

$$k_1 = \frac{bh^2E}{72\pi\alpha^2 f_1(\alpha)} \quad (2.9)$$

where $f_1(\alpha) = 0.6384 - 1.035\alpha + 3.7201\alpha^2 - 5.1773\alpha^3 + 7.553\alpha^4 + 7.332\alpha^5 + 2.4909\alpha^6$ and b is a width of beam. Another k equation is proposed by Dimarogonas & Paipetis (1983) as:

$$k_2 = \frac{EI}{5.346hf_2(\alpha)} \quad (2.10)$$

where $f_2(\alpha) = 1.8624\alpha^2 - 3.95\alpha^3 + 16.375\alpha^4 - 37.226\alpha^5 + 76.81\alpha^6 - 126.9\alpha^7 + 172\alpha^8 - 143.97\alpha^9 + 66.56\alpha^{10}$. The element mass matrix, $[M]^e$, element stiffness matrix, $[K]^e$ and crack stiffness matrix, $[K]_c$ are assemble together to form the global mass and stiffness matrices, $[M]$ and $[K]$. The equation of motion contains $[M]$ and $[K]$ are shown below:

$$[M]\{\ddot{W}\} + [K]\{W\} = \{F(t)\} \quad (2.11)$$

When the beam is excited by a sinusoidal force at frequency ω_i rad/s,

$$\{F(t)\} = \{F^*\} \sin \omega_i t \quad (2.12)$$

The global generalized displacement vector $\{W\}$ is also in a sinusoidal motion as shown below:

$$\{W\} = \{W^*\} \sin \omega_i t \quad (2.13)$$

Finally, the equation of motion (2.11) becomes:

$$([K] - \omega_i^2 [M])\{W^*\} = \{F^*\} \quad (2.14)$$

2.5 Review of previous studies

Ruotolo & Surace (1997) proposed the use of modal analysis of the lower modes for non-destructive detection and sizing of cracks in beams. By using the finite element model of the structure, the dynamic behaviour could be calculated analytically and possible to formulate the inverse problem in optimization terms and then utilize solution procedures employing genetic algorithms. The damage assessment technique that used to simulated and experimental data related to different damage of cantilever steel beam could detect the presence of damage and also could estimate both the crack positions and sizes with satisfactory precision.

Hu & Liang (1993) proposed two damage modelling technique to nondestructively identify cracks in beam structures. In the first model, the crack structure in which the local flexibility due to a crack was modelled as a massless spring with infinitesimal length as shown in Figure 2.4. It was used Castigliano's theorem and the perturbation technique to derive a theoretical relationship between the eigenfrequency change, the location and the extent of discrete cracks. In the continuum damage model, it used the stress concept combine with Hamilton's principle to derive similar relationship that is cast in a continuum form. By using this proposed model, the continuum damage model could be used first to identify the discretizing elements of a structure that contain cracks while a spring damage model was used to quantify the location and depth of the crack in each damage element.

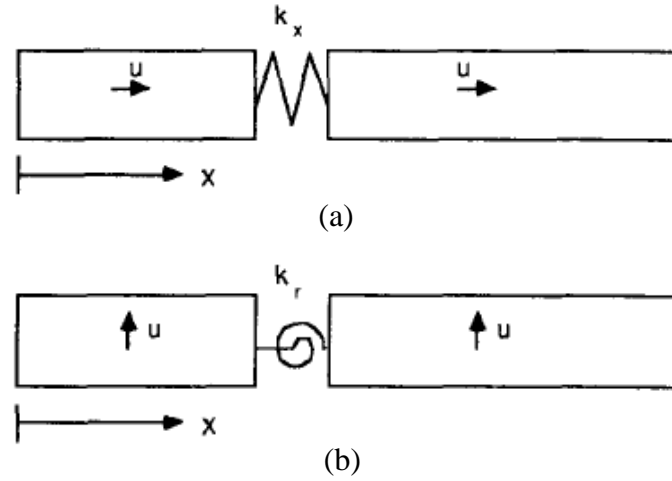


Figure 2.4: Crack modeled as spring, a) bars in longitudinal vibration, b) beams in pure bending vibration (Hu & Liang, 1993)

Lee (2009a) studied a simple method based on the massless rotational spring model for the crack, the finite element method and the Newton-Raphson method in order to identify multiple cracks in beams. The forward problem was solved using finite element method based on Euler-Bernoulli beam theory where the node that represents the crack had three degrees of freedom while each of other nodes had two degree of freedom. The rotations of the node that represents the crack were connected through the cracked stiffness matrix. The inverse problem was solved iteratively for the crack locations and sizes by the Newton-Raphson method. The $2n$ natural frequency measurements were required to identify n cracks in a beam. Result from the studied proved that the detected crack locations and sizes were similar with the actual problems.

Kisa, *et al.* (1998) used the analytical method to investigate the vibration mode shape and the natural frequencies of a cracked cantilever beam. The model used in the study was a cantilever beam having a uniform cross section, A and a transverse edge crack of depth ' a ' at a variable position ξ as shown in Figure 2.5. This study integrates the finite element method and the component mode synthesis by divide the beam into two components related by a flexibility matrix which incorporate the interaction forces. The cantilever beam was divided into two components at the crack section leading to a substructure approach and each component was divided into finite elements with two nodes and three degree of freedom at each node as shown in Figure 2.6.

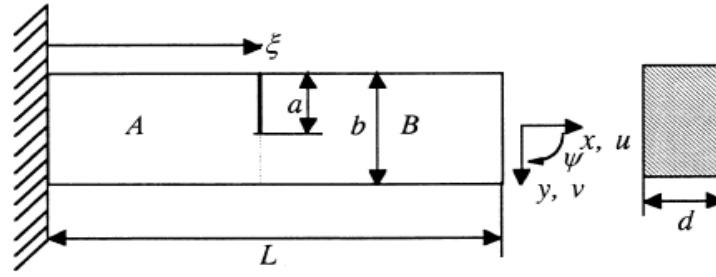


Figure 2.5: Geometry of the cracked cantilever beam
(Kisa *et al.*, 1998)

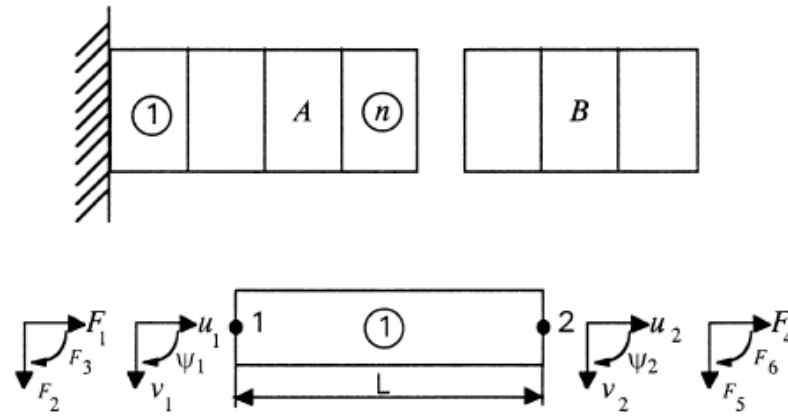


Figure 2.6: Component of whole structure and dividing into the
finite number of element (Kisa *et al.*, 1998)

A systematic finite element approach was proposed by the authors to determine the relationship between damage locations, damage size and corresponding changes in the Eigen parameters of a cantilever beam and suggested a finite element model for dynamic analysis of an edge-cracked beam. The crack was modelled as a massless rotational spring where stiffness was calculated by using fracture mechanics in order to determine the crack characteristic from vibration measurements. The results showed that the ratios of the natural frequencies of the cracked to the intact beam decrease with the increase of the crack to beam ratio.

Kisa & Brandon (2000) again in his paper studied about the effect of cracks on the dynamical characteristics of a cantilever beam that have multiple open-edge transverse cracks. The finite element method and the component mode synthesis methods were used by the authors to identify the flexibilities due to cracks for several crack depths and locations. The component was modelled by cantilever beam finite elements that have two nodes and three degree of freedom at each node. By

using the reduction of natural frequencies and change in the mode shape, the presence and the nature of the cracks in a structure could be detected.

Ostachowicz & Krawczuk (1991) proposed a numerical method of comparing the natural frequencies of cracked cantilever beam with single and multiple cracks at different locations. From the calculation showed that the position and the magnitude of cracks in cantilever beam definitely will affect the natural frequencies.

Yoon *et al.* (2009) proposed the mode shape curvature method to identify the crack in free-free beam. The differences between the curvatures of mode shapes of free-free beam was used by the authors in order to detect the locations and sizes of narrow and wide notches. The mode shape baseline data was generated using the gapped middle point by smoothing the curvature mode shapes near the damaged area.

Patil & Maiti (2005) in their study estimate the locations and sizes of multiple cracks in cantilever beams based on measurement of natural frequencies. The crack locations were predicted by developing a damage index algorithm by using the strain energy of different segments of the beam. Verification of the theory was done by performing an experiment from which they measured the natural frequencies and calculated the strain energy of 10 segments on a cantilever beam.

Kam & Lee (1994) proposed the strain energy method to identify the size of crack in free-free beams. The identification of crack size was based on natural frequencies and mode shape of the beam in order to evaluate the additional strain energy of the beam due to crack. Verification of proposed method was done by applying it to the experimental results from five cracked and healthy beam.

Some previous research in determining stress intensity factor (SIF) for crack structure was conducted by using Finite Element Method (FEM). Chan, *et al.* (1970) stated in their study that the FEM with special computational procedure can be used to find SIF of crack tip in various shape under different types of loading condition. The program based on displacement method, written by Fortran IV language and operated on a CDC6600 computer was used by the author for FEM analysis. The results were compared to other analytical solution and it showed the accuracy of the prediction was satisfactory without the use of excessive computer time and could be improved within the limitations of the computer.

Shao *et al.* (2013) in their research studied the influence of cracks on two dynamic characteristic and SIF of gears simultaneously. The dynamic characteristic

such as natural frequencies and vibration shape of the gears were investigated when tooth cracks in the gear appear while the influences of crack position and length on the dynamic characteristic of the gear structure were also discussed. By using theory of fracture mechanics the FEM by ANSYS, the SIF were analyzed when the cracks on gear appeared.

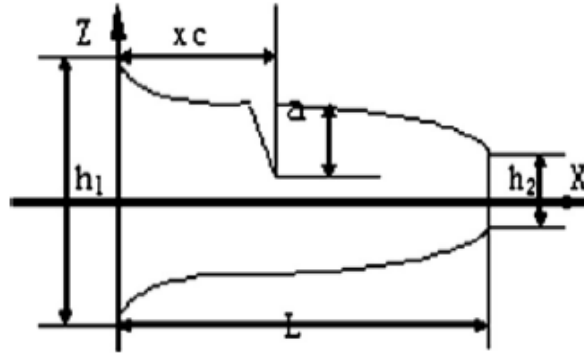


Figure 2.7: The geometry of crack gear tooth modeled as cantilever beam (Shao *et al.*, 2013)

For the simulation of gear tooth, a two dimensional model of gear tooth was established based on cantilever beam model as shown in Figure 2.7 while three dimensional gear structure was built for simulation of gear structure as shown in Figure 2.8.

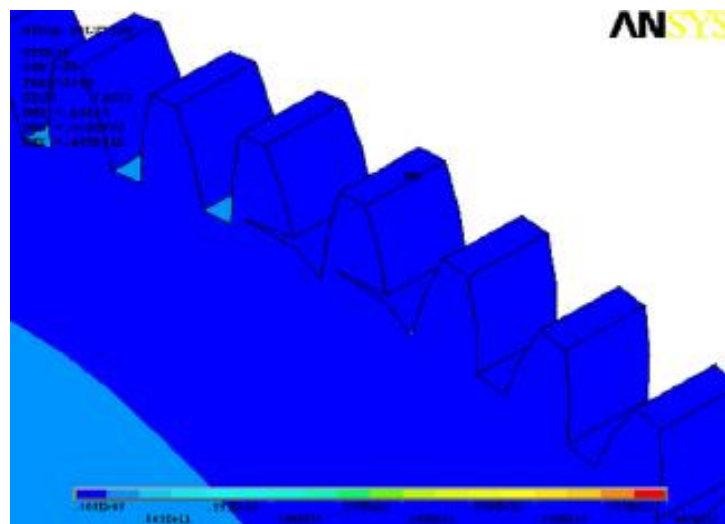


Figure 2.8: The structure model of cracked gear (Shao *et al.*, 2013)

For analysis of SIF, the crack locations were set at the pitch circle and tooth root and the model was shown in Figure 2.9.

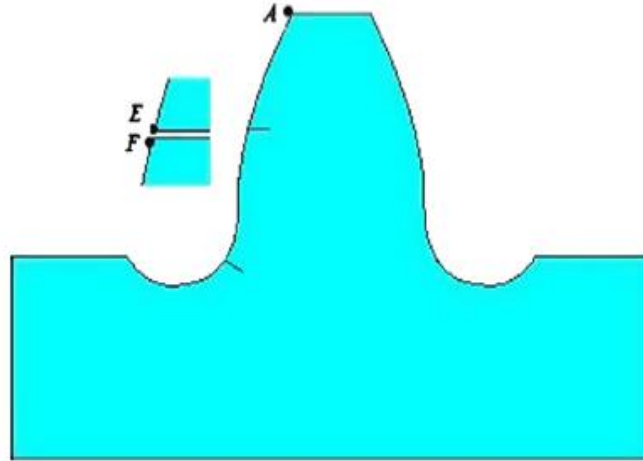


Figure 2.9: The model of cracked gear for SIF analysis (Shao *et al.*, 2013)

The dynamic characteristic was discussed and the result showed that when the cracks appear on the gear tooth, the natural frequencies of the gear drop and the vibration shape change. The natural frequencies decreases were bigger when the cracks occur at the tooth rather than at the top of the tooth and the vibration shapes of the gear body are different from without the crack. The SIF analysis was also discussed after the simulation done and the result showed that with the increase of gear module, the SIF was decreased and furthermore the random fracture of the gear was reduced.

Another previous research about SIF was conducted by Alebrahim *et al.* (2015). In this research, the vibration analysis of hybrid epoxy composite beam under moving mass was investigated and a schematic diagram of cracked beam model and moving mass was shown in Figure 2.10. The composite beam was made up from carbon fibre epoxy and shape memory alloy wire epoxy. The open crack was located in the middle of the beam and the SIF at the crack tip was studied.

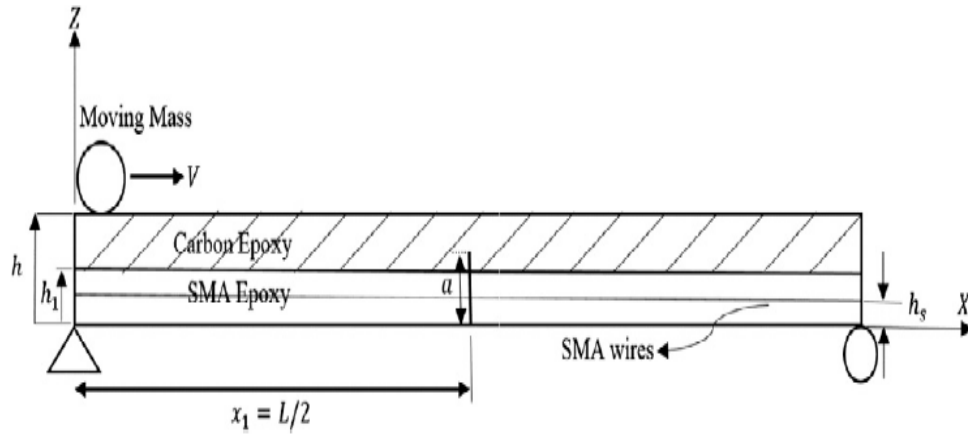


Figure 2.10: Schematic diagram of cracked beam model and moving mass (Alebrahim *et al.*, 2015)

The cracked on the composite beam was simulated by using FEM and the two dimensional model was programmed by using Matlab software. The effect of percentage of SMA wire in composite on the SIF on the crack tip was discussed. Based on the graph in Figure 2.11, the result showed that the SIF was decreased when the percentage of SMA wires in the structure of the beam was increased.

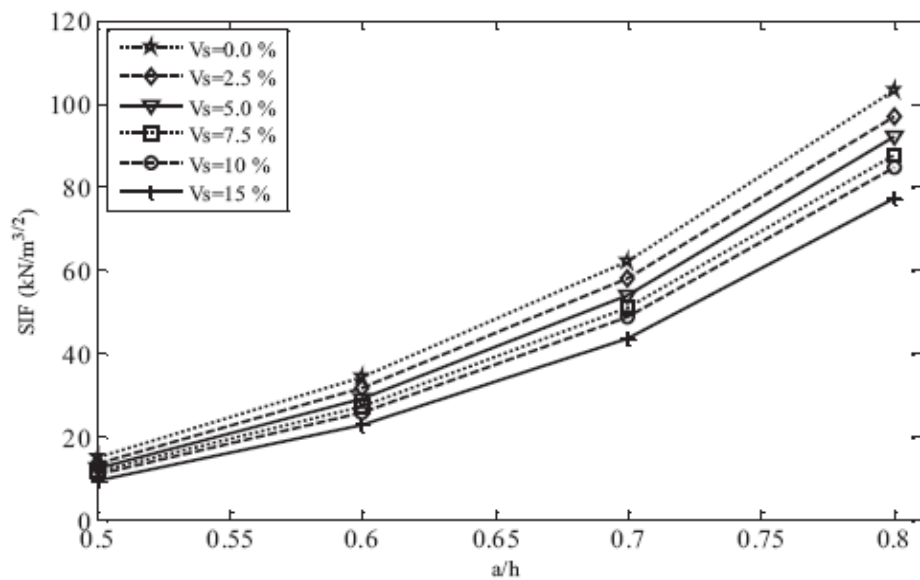


Figure 2.11: The reduction of SIF with increasing of % SMA wire in composite (Alebrahim *et al.*, 2015)

CHAPTER 3

METHODOLOGY

3.1 Overview of the finite element method

The finite element method (FEM) is a numerical methods and procedure that can be applied to obtain solution to a variety of problems in engineering. The problems in engineering that can be applied using FEM are steady, transient, linear or nonlinear problems in stress analysis, heat transfer, fluid flow and electromagnetism problems. The history of modern FEM was started as early as 1900s when some researchers approximated and modeled elastic continua using discrete equivalent elastic bars. However, the person being recorded as first person to develop FEM was Courant in his paper published in the early 1940s. Courant used piecewise polynomial interpolation over triangular sub-regions to investigate torsion problems (Moaveni, 2003).

The use of FEM in the field of vibration based damage detection has been significantly increased with the introduction of more accurate and faster computer simulation software in recent years. The finite element software allow for modelling and simulation of proposed structures that generate results with high accuracy and almost same as the real life cases. This allows the scientist and engineers to evaluate the design requirements and specifications before manufacturing and saved the cost and time during mass production.

For this research for identification of multiple cracks in cantilever beam, the beam was modeled and simulated by using one of the most advanced and comprehensive finite element program called as ANSYS. This study was performed

in three steps, which is pre-processing step, solution step and post processing step by ANSYS. The steps were shown in Figure 3.1.

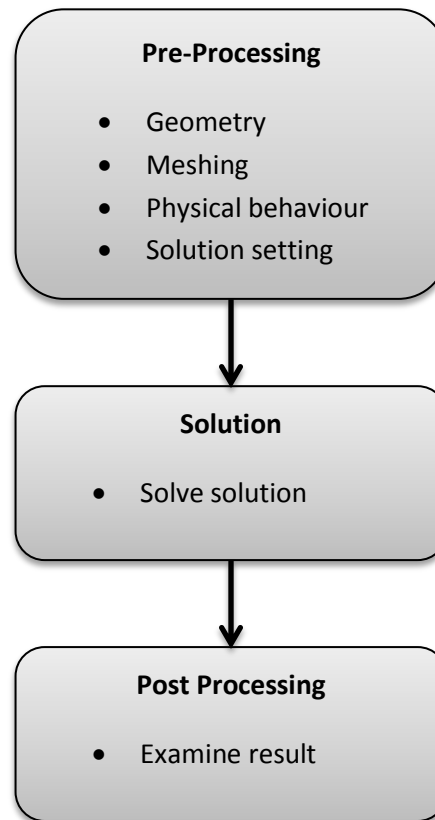


Figure 3.1: Steps performed in ANSYS

3.2 ANSYS program

The finite element program ANSYS was released for the first time in 1971. ANSYS is a comprehensive general purpose finite element computer program that contains over 100,000 lines of code. It has capability to perform static, dynamic, heat transfer, fluid flow and electromagnetic analysis and already leading the finite element analysis program for over 20 years. The application of ANSYS program are worldwide and used in many engineering fields such as aerospace, automotive, electronics and nuclear (Moaveni, 2003).

3.3 Overview of the study

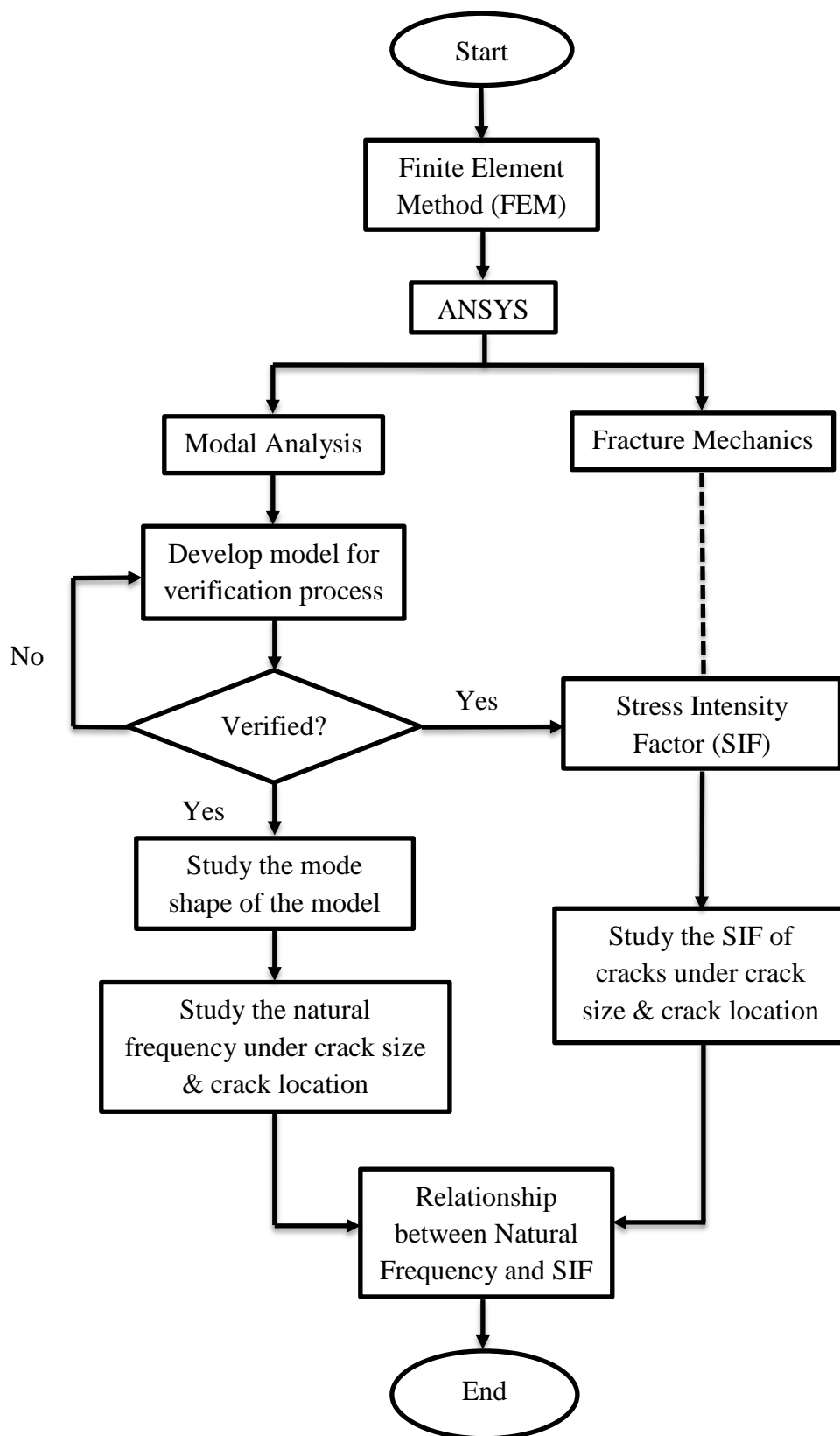


Figure 3.2: Flowchart of the study process

In this study, the modal analysis (natural frequency and mode shape) and the stress intensity factor (SIF) of multiple cracks in cantilever beam were simulated by using computation FEM. The summary of the study process was shown in Figure 3.2.

3.3.1 Verification of the natural frequency

Verification process is a compulsory step to recognize the output data is a correct value. For this research, the results of the simulation using ANSYS were compared with the previous experiment and simulation data.

For modal analysis of multiple cracks in cantilever beam, two dynamic characteristic were identified, the natural frequencies and the mode shape of the beam. Previous experiment test was conducted by Ruotolo & Surace (1997) on cantilever beams with and without cracks to measure the natural frequencies. The cantilever beam was made of C30 steel and had dimensions of $0.02 \times 0.02 \times 0.8 \text{ m}^3$. The cantilever beams had two cracks and the crack parameters were $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\beta_1 = 0.3182$ and $\beta_2 = 0.6812$ where α was a dimensionless crack size and β was a dimensionless crack location. The crack was obtained by wire erosion with a 0.1 mm diameter wire to produce notches of 0.13 mm wide. The later research was conducted by Lee (2009b) to simulate the experimental results of Ruotolo & Surace (1997).

Table 3.1: The natural frequency from experimental test and simulation

	Condition of Beam	1 st mode	2 nd mode	3 rd mode
		natural frequency f_1 (Hz)	natural frequency f_2 (Hz)	natural frequency f_3 (Hz)
Experimental Measurements (Ruotolo & Surace, 1997)	Undamaged	24.175	152.103	424.455
	Cracked	24.044	149.268	409.287
Simulation Measurements 2D model Ansys (Lee, 2009b)	Cracked	24.108	149.09	408.73

Based on the research conducted by Ruotolo & Surace (1997) and Lee (2009b), another modelling and simulation were conducted to compare and verified the result with the previous studies. A two dimensional of cantilever beam having two cracks with length L and thickness h were established as shown in Figure 3.3. The cantilever beam was modelled and simulated using ANSYS. The first crack had the location at s_1 and size of a_1 while the second crack located at s_2 and had crack size a_2 . The dimension of the cantilever beam was kept similar as the previous one and shown in Table 3.2. The crack size was set in dimensionless unit, α ($\alpha = \frac{a}{h}$) while dimensionless crack location, β is $\frac{s}{L}$.

As the one end of the cantilever beam was fixed, the boundary condition of the cantilever beam was set to allow the one end was fixed and the other end was free. The finite element mesh model was divided into six nodes or eight nodes plane element by using PLANE183 as shown in Figure 3.4. The 2D model after meshing by using ANSYS was shown in Figure 3.5. The boundary condition and loads were applied to analyze the dynamic characteristics of the undamaged and double cracks cantilever beam in order to verify with previous studies. The material properties $E = 181 \text{ GPa}$, $\rho = 7860 \text{ kg/m}^3$ and poisson's ratio $\nu = 0.29$ are used for this study.

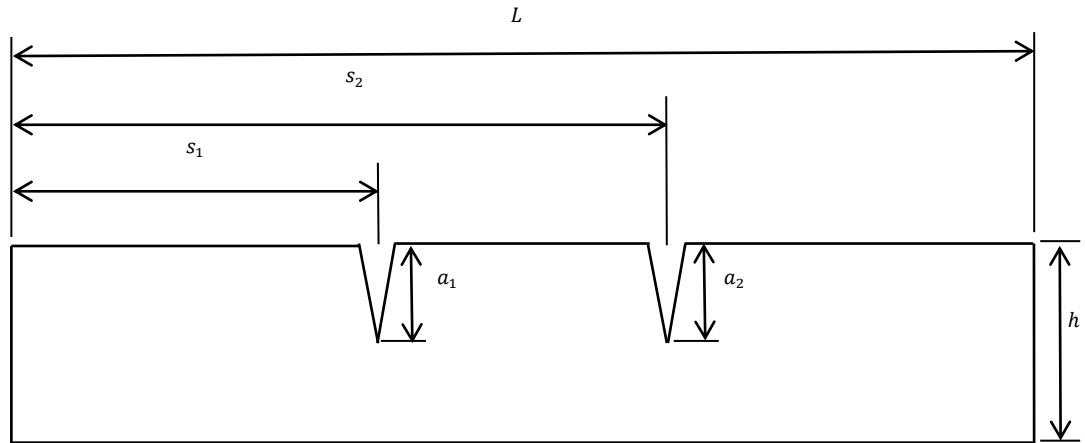


Figure 3.3: Geometry of cantilever beam having two cracks

Table 3.2: The dimension of cantilever beam used for verification

Item	Dimension (m)
L	0.8
s_1	0.25456
s_2	0.54496
h	0.02
a_1	0.004
a_2	0.006

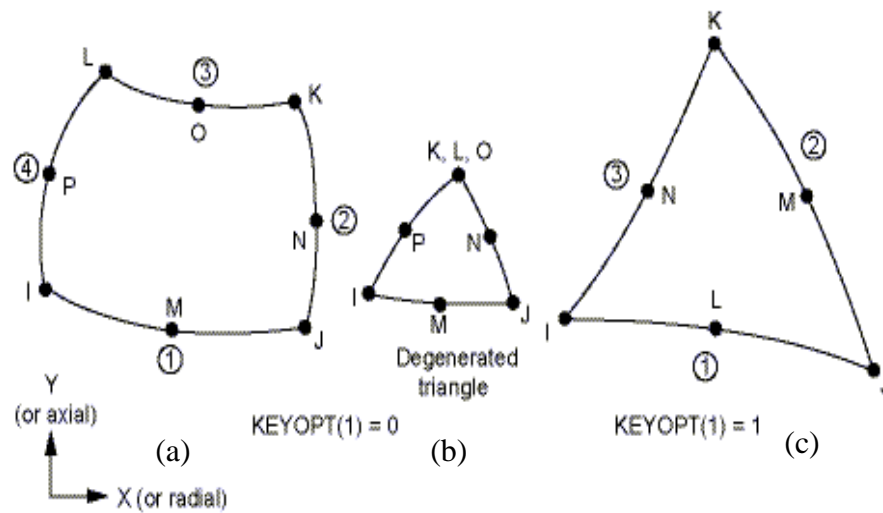


Figure 3.4: The geometry of PLANE183 of a) 8-node quadrilateral element, b) 8-node triangular element and c) 6-node triangular element

REFERENCES

- Alebrahim, R., Haris, S. M., Nik Mohamed, N. A. & Abdullah, S. (2015). Vibration analysis of self-healing hybrid composite beam under moving mass, *Composite Structure*, 119, pp. 463-476.
- Carrera, E., Giunta, G. & Petrolo, M (2011). Beam Structures: Classical & Advanced Theories. West Sussex: John Wiley & Sons, Ltd.
- Chan, S. K., Tuba, I. S. & Wilson, W. K. (1970). On the finite element method in linear fracture mechanics. *Engineering Fracture Mechanics*, 2, pp. 1-17.
- Dimarogonas, A. D. & Paipetis, S. A. (1983). *Analytical methods in rotor dynamic*. London: Elsevier Applied Science.
- Hu, J & Liang, R. Y. (1993). An integrated approach to detection of cracks using vibration characteristics. *Journal of Franklin Institute*, 330, pp. 841-853.
- Kam, T. Y. & Lee, T. Y. (1994). Identification of crack size via an energy approach. *Journal of Nondestructive Evaluation*, 13, pp. 1-11.
- Kisa, M. (2004). Free vibration analysis of a cantilever composite beam with multiple cracks. *Composites Science and Technology*, 64, pp. 1391-1402.
- Kisa, M., Brandon, J. & Topcu, M. (1998). Free vibration analysis of cracked beams by a combination of finite elements and component mode synthesis methods. *Computers and Structures*, 67, pp. 215-223.
- Kisa, M., Brandon, J. (2000). Free vibration analysis of multiple open-edge cracked beams by component mode synthesis. *Structural Engineering and Mechanical*, 10(1), pp. 81-92.
- Lee, J. (2009a). Identification of multiple cracks in a beam using natural frequencies, *Journal of Sound and Vibration*, 320, pp. 482-490.

- Lee, J. (2009b). Identification of multiple cracks in a beam using vibration amplitudes. *Journal of Sound and Vibration*, 326, pp. 205-212.
- Moaveni, S. (2003). *Finite Element Analysis – Theory and Application with Ansys*, 2nd ed. Pearson Education, Inc.
- Ostachowicz, W. M. & Krawczuk, M. (1991). Analysis of the effect of cracks on the natural frequencies of a cantilever beam, *Journal of Sound and Vibration*, 150, pp. 191-201.
- Patil, D. P. & Maiti, S. K. (2005). Experimental verification of a method of detection of multiple cracks in beams based on frequencies measurements. *Journal of Sound and Vibration*, 281, pp. 439-451.
- Perez, N. (2004). *Fracture Mechanics*. Boston: Kluwer Academic Publishers.
- Philpot, T. A. (2011). *Mechanics of Materials: An Integrated Learning System*. 2nd ed. Hoboken: John Wiley & Sons, Inc.
- Raj, B., Jayakumar, T. & Thavasimuthu, M. (2007). *Practical Non-Destructive Testing*, 3rd ed. Oxford: Alpha Science International Ltd.
- Ruotolo, R. & Surace, C. (1997). Damage assessment of multiple cracked beams: numerical results and experimental validation. *Journal of Sound and Vibration*, 206, pp. 567-588.
- Shao, R., Dong, F., Wang, W. & Jia, P. (2013). Influence of cracks on dynamic characteristics and stress intensity factor of gears. *Engineering Failure Analysis*, 32, pp. 63-80.
- Yoon, M. K., Heider, D., Gillespie Jr., J. W., Ratcliffe, C. P. & Crane, R. M. (2009). Local damage detection with the global fitting method using mode shape data in notched beams, *Journal of Nondestructive Evaluation*, 28, pp. 63-74.