NON-DARCY MIXED CONVECTION FROM A HORIZONTAL PLATE EMBEDDED IN A NANOFLUID SATURATED POROUS MEDIUM IN THE PRESENCE OF HEAT SOURCE/SINK

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ABSTRACT

Non-Darcy mixed convection in porous media plays an important role in several applications such as in geothermal operations, petroleum industries and thermal insulation. The problem of Non-Darcy mixed convection from horizontal plate embedded in a nanofluid saturated porous media in the presence of heat source/sink is numerically studied using different types of nanofluid as Copper (Cu), Alumunium (Al_2O_3) and Titanium (TiO_2). The model used for the nanofluid incorporates the effect of the volume fraction parameter and thermal conductivity in the presence of heat source/sink along the porous horizontal plate in case of non-Darcy. The mathematical model formulation is obtained by reducing non-linear partial differential equation governing equations to first order ordinary differential equations. The obtained governing equations have been solved numerically by using fourth-fifth Runge-Kutta Fehlberg method with shooting technique. Dimensionless velocity and temperature profiles for different values of parameters are presented in graph and tabular. Salient features of the results are analyzed and discussed.

ABSTRAK

Olakan haba campuran bukan Darcy dalam media yang berliang memainkan peranan yang penting dalam beberapa aplikasi seperti dalam pemanasan bumi, industri petroleum dan penebat haba. Permasalahan olakan campuran bukan Darcy dari plat mendatar yang terbenam dalam bendalir nano tepu berliang dengan kehadiran sumber haba/singki haba dikaji secara berangka menggunakan pelbagai jenis bendalir nano seperti kuprum (Cu), alumunium (Al_2O_3) dan titanium (TiO_2). Model yang digunakan adalah gabungan bendalir nano dengan kesan jumlah pecahan parameter dan kekonduksian haba dengan kehadiran sumber haba/singki haba bersama-sama plat mendatar berliang dalam kes bukan Darcy. Formulasi model matematik diperolehi dengan menurunkan persamaan bukan linear kepada persamaan pembezaan biasa. Persamaan pembezaan yang diperolehi diselesaikan secara berangka dengan kaedah Runge-Kutta Fehlberg peringkat keempat-kelima dengan teknik meluru. Kelajuan tanpa dimensi dan profile suhu diperolehi dari nilai-nilai parameter yang berbeza ditunjukkan dalam bentuk graf dan jadual. Ciri-ciri utama keputusan dianalisis dan dibincangkan.

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CHAPTER 1

INTRODUCTION

1.1 Research Background

Nanotechnology is one of widely technology rapidly progress in various fields. Chemistry, physics, materials science and biotechnology are some nanotechnology applications. It is due to their structures that are determined on the nanometre scale. Nanofluid is a fluid containing nanometre sized particles known as nanoparticles. It is typically made of metals, oxides, carbines or carbon nanotubes. The fluids are engineered colloidal suspension of nanoparticles in a base fluid. Water, ethylene glycol and oil are commonly example base fluid. Nanofluid has novel properties that make them potentially useful widely in heat transfer. It includes engine cooling or vehicle thermal management, domestic refrigerator and so on. This is because nanofluid exhibit enhanced thermal conductivity and the convective heat transfer coefficient compared to the based fluid.

Non-Darcy mixed convection in porous medium also has various applications such as geothermal operations, petroleum industries, thermal insulation, design of solid matrix heat exchangers, chemical catalytic reactors and many others. Non-Darcy mixed convection from a plate embedded in a porous medium has been studied in many literatures. Abdelgaied and Mohamed (2013) had studied mixed convection flow of nanofluid over a vertical surfaces embedded in a porous medium with temperature dependent viscosity. Rosca et al. (2012) discussed non-darcy mixed convection from a horizontal plate embedded in a nanofluid saturated porous media.

Ghalambaz and Noghrehabadi (2014) examine the effect of heat generation/absorption on natural convection of nanofluids over the vertical plate embedded in a porous medium using drift-flux model. Rabeti et al. (2013) analysed forced convection heat transfer over a horizontal plate embedded in a porous medium saturated with a nanofluid in the presence of heat sources.

Non-Darcy mixed convection from a horizontal plate embedded in a nanofluid saturated porous media in the presence of heat source/sink has not been analysed yet. The present study aims to analyse the steady mixed convection boundary layer flow past a horizontal impermeable surface embedded in a porous medium filled by a nanofluid in the presence of heat source/sink. The governing partial differential equation is transformed into an ordinary differential equation. The set of nonlinear ordinary differential equations with boundary conditions are reduced to the first order differential equations and have been solved by using fourth-fifth order Runge-Kutta Fehlberg method in conjunction with shooting technique.

1.2 Problem Statement

Fluid heating and cooling are important in many industrial and engineering applications. Aerodynamics extrusion of plastic sheet and the cooling of metallic plate in a cooling techniques are needed for cooling any sort of high energy device.

Conventional heat transfer fluids have limited heat transfer capabilities due to their low thermal conductivity in enhancing the performance and compactness of many engineering electronic devices. Meanwhile, metal have thermal conductivity three times higher than fluids. Thus it is natural to combine the two substance to produce a medium for heat transfer that would behave like fluid and have thermal conductivity same with metal.

The presence of nanoparticle and heat source/sink in the fluids may increases appreciably the effect of thermal conductivity and viscosity of the base fluid and consequently enhances the heat transfer characteristic. Therefore, there is strong needed to develop advanced heat transfer fluids with high conductivities to enhance thermal characteristics. Based on these, there are few questions arise:

1. How to generate the mathematical model?

2. What are the effects the presence of heat source/sink toward the temperature and velocity profiles for Cu, Al_2O_3 and TiO_2?
3. What are the effects without the presence of heat source/sink toward the temperature and velocity profiles for Cu, Al_2O_3 and TiO_2?
4. What are the comparison toward the temperature and velocity profile for Cu, Al_2O_3 and TiO_2 in the presence of heat source/sink and without the presence of heat source/sink?

1.3 Objectives

The objectives of this thesis are :

1. To determine the effects of several pertinent parameters for copper (Cu), alumina (Al_2O_3) and titanium (TiO_2) without of presence of heat source/sink.
2. To determine the effect of heat source and heat sink on steady mixed convection boundary layer flow in the case Cu, Al_2O_3 and TiO_2.
3. To analyze the steady mixed convection boundary layer flow past a horizontal impermeable surface embedded in a porous medium filled by a nanofluid in the presence of heat source and heat sink.

1.4 Significance of the study

Fluid heating and cooling are important in industry and engineering area. So the effective cooling techniques are needed for cooling any sort of high energy device. Unfortunately, fluids have limited heat transfer capabilities to act as a medium for heat transfer and it is recommend to find an advanced heat transfer fluids with substantially higher conductivities that can enhance thermal characteristics.

Many engineering discipline can be applied the fluid heating and cooling such as transpiration cooling, thin film solar energy collector device and so on. In this research the presence of heat source/sink in the nanofluids can enhance the thermal conductivities of heat transfer fluids.

1.5 Scope of study

In order to achieve all the objectives, it is important to outline scope of study to avoid any diverted. Three scopes has been defined and listed as follow:

1. This study will focus on nanofluid incorporates the effect of the volume fraction parameter and thermal conductivity in the presence and without heat source/sink along the horizontal plate on non-Darcy for Cu, Al_2O_3 and TiO_2.
2. The mathematical model is solved by using fourth-fifth order Runge-Kutta Fehlberg method.
3. The results from previous research are compared to the current achieved result.

1.6 Flow Chart and Gantt Charts

Figure 1.1 shows a summary flow of this project. In Chapter 1, contains research of background, problem statement, objectives, important of study, scope of study, flow chart and Gantt chart. Chapter 2, diverse of literature review that are used as guidance to conduct the study. Methodology of this research include process involve and software use have been explained in Chapter 3. Then, the results are discussed in Chapter 4. Finally, conclusion and recommendation for further study are stated in Chapter 5.

Table 1.1 shows the Gantt chart for Project 1 and that gives the outline and durations to complete the task. Master Project 1 cover on Chapter 1 until Chapter 3.

Meanwhile, Table 1.2 is the Gantt chart for Master Project 2 and this cover the whole project but more focus on the results that have been obtained.

1.7 Flow chart of research

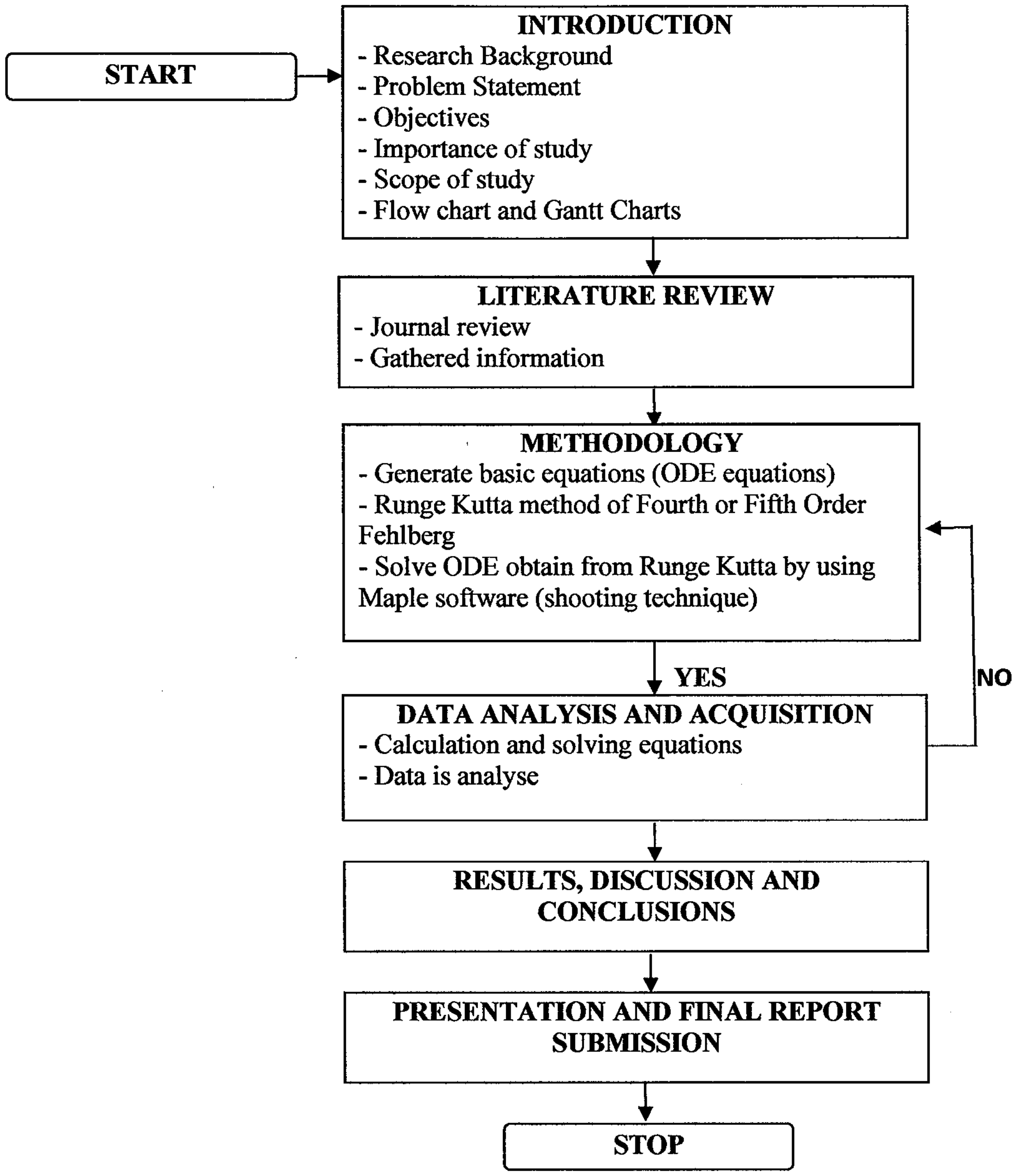

Figure 1.1 Project research flow chart

1.8 Gantt chart of research

Project 1 Activities	Weeks											
	3	4	5	6	7	8	9	10	11	12	13	14
1. Confirmation of thesis title and supervisor.	■											
2. Final project briefing by coordinator and proposal submission.		■										
3. Chapter 1 (Introduction) - Research background - Problem statement - Objective - Scope of study - Significance of study - Expected results - Flow charts and Gantt charts			■	■	■							
4. Mid semester break (1 week)						■						
5. Chapter 2 (Literature Review) - Non-Darcy - Reynolds number - Forchheimer law - Porous medium - Nanofluid - Mixed Convection - Heat source/sink						■	■					
6. Chapter 3 (Methodology) - Formulation of the problem - Solution of the problem - Numerical solutions								■	■			
7. Submission of report to supervisor & correction										■		
8. Submission of report to FSTPi											■	
9. Presentation of final project 1												■

Table 1.1: Gantt chart for Master Project 1

Project 2 Activities	Weeks														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Registration of new semester.	■														
2. Updating previous chapters 1, 2 and 3 with new information.		■	■	■											
3. Chapter 4 (Results & Discussion) - Introduction - Velocity and temperature profile on Cu, TiO_2 and Al_2O_3 - Table sleep and velocity heat transfer on Cu, TiO_2 and Al_2O_3 - Comparison of results for $f''(0)$ and $-\theta'(0)$ with previous published work					■	■	■	■	■	■					
4. Chapter 5 (Conclusion) - Conclusion - Recommendation											■	■			
5. Submission of reports to supervisor.													■		
6. Submission of reports to FSTPi														■	
7. Final presentation of the whole project.															■

Table 1.2: Gantt chart for Master Project 2

CHAPTER 2

LITERATURE RIVIEW

Non-Darcy flow is fluid flow that deviates from Darcy's law, which assumes laminar flow in the formation. It observed in high-rate fluid wells when the flow converging to the wellbore reaches flow velocities exceeding the Reynolds number.

2.1 Non-Darcy Law

Darcy law of flow can be described as a linear relationship between volumetric flow rate (Darcy velocity) and pressure (head or potential) gradient. It has been a fundamental principle in analyzing flow and transport processes in porous and fractured media. Experimentally, fluid flow deviation from this linear relation is defined as non-Darcy flow. Various terms are used to described this behaviour such as turbulent flow, initial flow, high velocity flow and other (Firoozabadi and Katz,1979).

Although, Darcy law has been used widely in the study of porous medium phenomena, but there are ample evidence stated that high-velocity non-Darcy flow may occur in many subsurface systems like in the flow near the well of oil and gas production, water pumping and liquid waste injection.

In order to identify the behaviour of non-Darcy flow, a criteria must be determined. Chilton and Colburn (1931) had published their work on the criterion for non-Darcy flow behaviour in porous media. There are two types of criteria, the Reynolds number and Fochheimer number. These criteria have been used in the past for identifying non-Darcy law in the beginning.

2.2 Reynolds number

In fluid mechanics and heat transfer, the Reynolds number (Re) is a dimensionless quantity that gives a measure of ratio of inertial force (V_ρ) to forces (μ/L). It is used to predict similar flow patterns in different fluid flow situations.

Reynold numbers arise when performing dimensional analysis of fluid dynamics and heat transfer problems. It can be used to determine dynamics similitude between different experimental cases and characterize different flow regimes such as laminar or tubulent flow. Laminar flow occurs at low Reynolds numbers where viscous forces are dominant. It is characterized by smooth and constant fluid motion. Meanwhile turbulent flow occurs at high Reynolds numbers. It is dominated by inertial forces which has potential to produce random eddies, vortices and other flow fluctuations.

Reynolds number can be defined as

$$Re = \frac{\text{Inertial force}}{\text{Viscous forces}} = \frac{\rho v L}{\mu} = \frac{vL}{v}$$

where,

- v is velocity of object relative to the fluid
- L is a characteristic linear dimension
- μ is dynamic viscosity of the fluid (kg/(m.s) or N.s/m^2)
- v is the kinematic viscosity ($v = \mu/ p$) (m^2/s)
- ρ is the density of the fluid (kg/m^3)

In circular tubes, when transition from laminar to turbulent flow occurs over a range of Reynolds numbers about 2300 to 4000, inspite the nature of the fluid or the dimensions of the pipe or the average velocity. Reynolds number can be indicate in the form of a range. If Reynolds number is below than 2300 so the flow is considered laminar. When it is above 4000 the flow will be classified turbulent. If it is in between these two limits, the flow is known as transition flow.

2.3 Forchheimer law

Theng and Zhao (2000) in his thesis entitle "An extension of Darcy's law to non-Stokes in porous media'' stated that Philip Forchheimer (1991) was investigated the fluid flow through porous media in the high velocity regime. He observed that as the flow velocity increases, the inertial effects start dominating the flow. In order to solve these high velocity inertial effects, he suggested the inclusion of an inertial term representing the kinetic energy of the fluid to the Darcy equation.

Thus, Forchheimer law is recommended in order to avoid future confusion with Reynolds number. Forchheimer number, F_0 defined as

$$F_0 = \frac{k\beta\rho v}{\mu}$$

where,

- k is permeability, (10^{-15} m^2)
- β is non-Darcy coefficient, (10^8 m^{-1})
- ρ is fluid density, (g/cm^3)
- v is superficial velocity of the fluid, (cm/s)
- μ is fluid viscosity, (Pa-s)

Forchheimer number is more clear compare to Reynolds number. Based on Forchheimer formula, all the involved parameters are clearly defined and can be determined. It is clear that this definition can be applied to all types of porous materials, as long as the permeability and non-Darcy coefficient can be determined experimentally or empirically when no experimental data is available (Li and Engler, 2001).

2.4 Porous medium

In general, porous medium can be defined as a material containing pores. 'Matrix' or frame is skeletal portion of the material. Generally, fluid (gas or liquid) will fill the pores. Porous medium is characterised by its porosity. Besides that, other properties of the medium are permeability, tensile strength, electrical conductivity and so on.

Generally, the dynamics of fluids in porous medium is dominated by the porous matrix which pervades the entire volume and leads to an efficient dissipation of the fluid's kinetic energy. In return, the internal dynamics becomes rather simple, essentially described by Stokes' law. In typical experiments the quantities of interest are measured by areas that cross many pores, and such space averaged (macroscopic) quantities change in a regular manner with respect to space and time. Hence it is accommodating to theoretical treatment.

According to the book by Neild and Bejan (2006), the observer sees only one or two channels or one or two closed cavities when the distance is short. It is practicable to use convectional fluid mechanics and convective heat transfer to describe what happen at each point of the fluid and solid filled spaces in this case. If the distance is large, there are many channels and cavities in the problem solver's field of vision, the complications of the flow paths rule out convectional approach. In this limit, volume-averaging and global measurement (permeable and conductivity) are useful in describing the flow and in simplifying the description.

Refer to the book Dynamics of fluid in porous media written by Bear (2013), he summarized that porous medium can be defined as

- Heterogeneous or multiphase matter occupied a portion of space. At least one of the phases comprising this matter is not solid. It may be gaseous or liquid phases. Solid matrix is solid phase, meanwhile void space or pore space is space within the porous medium domain that is not part of the solid matrix.

- The solid phase should be distributed throughout the porous medium within the domain occupied by porous medium. Solid must be present inside each representative elementary volume. An essential characteristic of a porous medium is that specific surface of the solid matrix is relatively high. In many respects, this characteristic dictates the behaviour of fluids in porous media. Another basic feature of a porous medium is that the various openings comprising the void space are relatively narrow.

- At least some of the pores comprising the void space should be interconnected. The interconnected pore space is sometimes known as effective pore space. As far as flow though porous media is concern, unconnected pore space may be considered as part of solid matrix. Certain portion of interconnected pore space may, in fact also ineffective as far as flow through the medium is concerned.

2.5 Nanofluid

'Nanofluid' is referred to a liquid containing a dispersion of submicronic solid particles (nanoparticles). The term was created by Choi (1995). Meanwhile, Masuda et al (1993), observed the characteristic feature by nanofluid that is thermal conductivity enhancement. Based on this phenomenon, Buongiorno and Hu (2005) suggested the possibility of using the nanofluid in advanced nuclear systems. The types of particles used in nanofluid might be categorized as follows (Michaelides 2014) :

- Metals, such as Au, Ag, Cu and Fe.
- Oxides, such as CuO, Cu_2O, Fe_2O_3, Al_2O_3, SiO_2 and TiO_2.
- Carbon nanotubes (CNTs), which have shown the highest conductivity enhancement: Single-walled (DWCNT), and multi-walled (MWCNT) are three types CNTs commonly used.
- Other particles such as Si compounds

Water is the most commonly used fluid followed by refrigerants and organic fluid (primarily oils). It is due to the fact that fluid is made of asymmetrical molecules with electric dipole or tripole characteristic. Besides that, nanoparticles are subjected to electric potentials that cause their aggregation or clustering.

2.5.1 Properties of nanofluid

Refer to article by Wang, X.Q and Mujumdar (2008), nanofluids can be considered to be the next generation heat transfer fluids as they offer exciting new possibilities to enhance heat transfer performance compared to pure liquid. The fluids contain micro sized metallic particles, so they are expected to have superior properties compared to conventional heat transfer fluids. The larger relative area of nanoparticles compared to those of conventional particles, should not only significantly improve heat transfer capabilities but also should increase the stability of suspensions.

Heat transfer fluids provide an environment for adding or removing energy system.Their efficacies depend on their physical properties like thermal conductivity, viscosity, density and heat capacity. However it has a weakness where low thermal conductivity is frequently the primary limitation for heat transfer fluid. Thus, to improve this weakness Lee and Choi (1996) use nanoparticles as additives to modify heat transfer fluids to improve their performance. In addition, a method to improve the thermal conductivity of the mixtures is dispersion or suspension of high thermal conductivities in heat transfer fluid where it is called as 'nanofluid' (Wang et. al 1999).

2.5.2 Nanoparticles

Nanoparticles are particles between 1 and 100 nanometres in size. Meanwhile in nanotechnology, it can be defined a small object that behaves as a whole unit with respect to its transport and properties known as particle. Recently, nanoparticles research in area intense scientific interest due to its wide variety of potential applications such as in biomedical, chemistry, optical, electronic field and so on.

Applications of nanoparticles provides an effectives way of improving heat transfer characteristic of fluids (Eastman et al., 1997). It is because the diameter is less than 100nm and this condition exhibit properties different from those of conventional solids. Nanophase powders have larger relative surface areas and great potential for heat transfer enhancement compare to micron-sized particles. Some initially results (Eastman et al., 1997) showed that thermal conductivity is increase approximately 60% and can be obtain for the nanofluid consisting of water and 5% vol CuO nanoparticles.

Heat transfer performance of the fluid might be significantly improved by depending nanophase particles in heating or cooling fluid. There are main reason listed as follows by Xuan and Li (2000):

- The suspended nanoparticles increase the surface area and heat capacity of the fluid.
- The suspended nanoparticles increased the effective (or apparent) thermal conductivity of the fluid.
- The interaction and collision among particles, fluid and the flow passage surface are intensified.
- The mixing fluctuation and turbulence of the fluid are intensified.
- The dispersion of nanoparticles flattens the tranverse temperature gradient of the fluid.

2.6 Mixed Convection

Combined forced convection and natural convection or well known as mixed convection happen when natural convection or forced convection mechanism react together to transfer heat. Natural convection in saturated porous medium has recently received considerable attention due its considerable of applications in geophysics and energy related engineering problems. Natural circulation in geothermal reservoirs, aquifers, porous insulation, packed bed reactors; sensible heat storage beds and beds of fossil fuels such as oil shale and coal which have been fragmented for in situ energy extraction are some example of application of mixed convection (Plumb and Huenefeld, 1981).

2.6.1 Effect of nanoparticles diameter on mixed convection heat transfer

In general, nanoparticles with small diameter will give more benefits to nanofluid due to a large surface area so it may changes their thermal conductivity remarkably. Thermal conductivity of nanofluid has been measured with several nanoparticles volume fraction, material and dimensions in several base fluids and all finding show that thermal conductivity of nanofluid is higher than the base fluids (Mirmasoumi and Behzadmehr, 2008).

There are several evidence to show that smaller diameter of nanoparticles will give higher thermal conductivity. Oxide ceramic nanofluid consisting of CuO or Al_2O_3 nanoparticles in water or ethylene glycol exhibit enhanced thermal conductivity had been demonstrate by Lee et al. (1999). Besides that, Masuda et al. (1993) noted that using Al_2O_3 particles having mean diameter of 13nm at 4.3% volume fraction increase the thermal conductivity of water under stationary conditions by 30%.

In addition, larger particles with an average diameter of 40nm led an increase of less than 10% (Lee et al., 1999). Moreover, Eastman et al. 2001, in his article stated that the effective thermal conductivity of metallic nanofluid increased by up to 40% for the nanofluid consisting of ethylene glycol containing approximately 0.3% volume Cu nanoparticles of mean diameter less than 10nm.

2.6.2 Cases of mixed convection

According to Yunus et al. (2007), combined forced and natural convection generally described by three cases:

- First case

  The first case occurs when the buoyant motion is in the same direction as the force motion, thus enhancing the heat transfer. For example, a fan blowing upward on a hot plate. The air being forced upward over the plate adds to the heat transfer since heats naturally rise up.

- Second case

  The second case is when natural convection acts in the opposite way of the force convection. An example, a fan forcing air upward over a cold plate. In this case the buoyancy force of the cold air naturally causes it to fall, but the air being forced upward opposes this natural motion, keeping the cool air hovering around the cold plate. This in turn diminishes the amount of heat transfer.

- Third case

  The third case is transverse flow. This happen when the buoyant motion acts perpendicular to the force motion. These encourage fluid mixing and enhance heat transfer. An ample example is air flowing horizontally over a hot or cold pipe. This situation can encourage phase changes which often creates a very high heat transfer coefficient.

2.7 Heat

Heat can be defined as transfer of kinetic energy from one medium or object to another or from an energy source to a medium or object. Energy transfer can occur in three ways radiation, conduction and convection. An example of heat radiation is warming of the earth by sun where radiation does not need intervening medium. Heat by conduction happen when two material objects are in direct contact and the temperature of one is higher than the temperature of the other.The temperature is tend to be equalize. In addition, heat by convection take place when the motion of a liquid or gas carries energy from a warmer region to a cooler reagion.

2.7.1 Heat source/sink

Heat source is an object or mechanism that creates heat and the term is associated with thermodynamics cycles. There are various of heat sources that exist in nature, from the sun to the earth and some are completely manmade. The most common use for a heat source is the commercial production of electricity.

In thermodynamics a heat sink is a heat reservoir that can absorb an arbitrary amount of heat without significantly changing temperature. Practical heat sinks for electronic devices must have temperature higher than surroundings to transfer heat by convection, radiation and conduction.

Fluid is undergoing exothermic or endothermic chemical reaction is an example study of heat generation or absorption in moving fluids. According to Rana and Bhargava (2011), exact modelling of the internal heat generation or absorption is quite difficult, some simple mathematical models can express its average behaviour for most physical situation. Heat generation or absorption has been assumed to be constant, space depandent or temperature dependent.

Heat source/sink had been studied in many literatures. Sparrow and Cess (1961) in his research considered temperature dependent heat absorption in their work on steady stagnation point flow and heat transfer. Meanwhile, Watanebe (1991) investigated the forced and free mixed convection boundary layer flow with uniform suction or injection on a vertical flat plate.

CHAPTER 3

METHODOLOGY

In this chapter, the governing equations and the method that been used in this research will be explained in detail. Methodology is an important part to determine a guideline, direction and the process of the project working so that the objectives can be achieved. Section 3.1 described the physical model and formulation of the model. Meanwhile the solution of the problem and the governing equation are explained in section 3.2. Section 3.3 presented the overview of numerical solution of the problem. In numerical solution we explained the set of nonlinear ordinary differential equation with the boundary conditions that are reduced to first order differential equation by using fourth-fifth order Runge-Kutta Fehlberg method.

3.1 Formulation of the problem

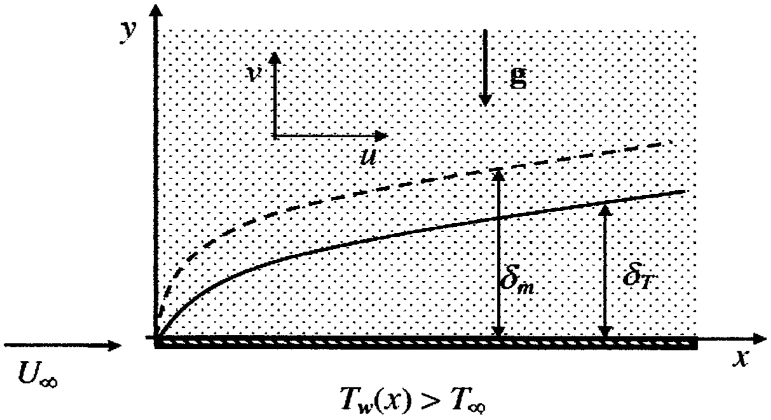

Figure 3.1: Physical model and coordinate system

Consider the problems of steady mixed convection flow in a saturated porous medium above a heated horizontal impermeable surface and a temperature-dependent heat source or sink is assumed to be presented in the flow. Figure 3.1 shows physical model for the case of free convection flow. y is the coordinate measured normal to the surface where x is axis pointing toward the porous medium. The dashed lines refer to the momentum boundary layer and the full lines refer to the thermal boundary layer respectively. In this problem, U_∞ is assumed as constant velocity outside the boundary layer. T_∞ is the uniform temperature of the ambient nanofluid and temperature of the plate is $T_w(x) > T_\infty$.

We apply the Boussinesq approximation and the nanoparticles are assumed to be suspended in the nanofluid using either surfactant or surface charge technology. It is further assumed that local thermal equilibrium in the porous medium and the homogeneity. The physical properties of the nanofluid are given in table 3.1 (Oztop and Abu-Nada, 2008).

Property	Water	Cu	Al_2O_3	TiO_2
c_p (J/ kg K)	4179	385	765	686.2
ρ (kg/m^3)	997.1	8933	3970	4250
k (W/m K)	0.613	400	40	8.9538
α x 10^7 (m^2/s)	1.47	1163.1	131.7	30.7
β x 10^{-5} (1/K)	21	1.67	0.85	0.9

Table 3.1: Physical properties of fluid and nanoparticles

Under the above assumption and adopting the nanofluid model proposed by Tiwari and Das (2007), along the non-Darcy law Ergun (1952), in the presence of heat source/sink, the basic equations can be written in the Cartesian coordinates x and y in form as (Nield and Bejan, 2006).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0, \qquad (3.1)$$

$$u+\rho_{nf,\infty}\frac{K'}{\mu_{nf}}u^2=-\frac{K}{\mu_{nf}}\frac{\partial p}{\partial x}, \qquad (3.2)$$

$$v+\rho_{nf,\infty}\frac{K'}{\mu_{nf}}v^{2}=-\frac{K}{\mu_{nf}}\left(\frac{\partial p}{\partial y}+\rho_{nf}g\right), \tag{3.3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^{2}T}{\partial x^{2}}+\frac{\partial^{2}T}{\partial y^{2}}\right)-\frac{Q_{0}(T-T_{\infty})}{(\rho C_{p})_{nf}}, \tag{3.4}$$

where

$$\rho_{nf}=\rho_{nf,\infty}\left[1-\beta_{nf}(T-T_{\infty})\right]. \tag{3.5}$$

Equation (3.1) to (3.5) are Boussinesq approximation. From equations (3.2) and (3.3), we obtained

$$\frac{\partial u}{\partial v}-\frac{\partial v}{\partial x}+\frac{\rho_{nf}}{\mu_{nf}}K'\left[\frac{\partial(u^{2})}{\partial y}-\frac{\partial(v^{2})}{\partial x}\right]=-\frac{g\rho_{nf,\infty}K\beta_{nf}}{\mu_{nf}}\frac{\partial T}{\partial x}. \tag{3.6}$$

3.2 Solution of the problem

It is convenient to reduce governing equations (3.1) to (3.5) to ordinary differential equation by introducing the stream function ψ defined as $u=\frac{\partial\psi}{\partial y}$ and $v=\frac{\partial\psi}{\partial y}$, equation (3.6) and (3.4) become

$$\frac{\partial^{2}\psi}{\partial y^{2}}+\frac{\rho_{nf,\infty}}{\mu_{nf}}K'\frac{\partial}{\partial y}\left[\left(\frac{\partial\psi}{\partial y}\right)^{2}\right]=-\frac{g\rho_{nf,\infty}K\beta_{nf}}{\mu_{nf}}\cdot\frac{\partial T}{\partial x}, \tag{3.7}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^{2}T}{\partial y^{2}}, \tag{3.8}$$

with boundary conditions

$$\begin{aligned}&v=0,\quad T_{w}=T_{\infty}+Ax^{\frac{1}{2}}\quad at\quad y=0,\\&u\to U_{\infty},\quad T\to T_{\infty}\quad as\quad y\to\infty\end{aligned} \tag{3.9}$$

For the sake of simplicity, $\rho_{nf,\infty}$ will replace with ρ_{nf} and other physical characteristics are denote below

$$
\begin{aligned}
&\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \\
&\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \\
&\mu_{nf} = \frac{\mu_{nf}}{(1-\phi)^{2.5}}, \\
&(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \\
&\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \\
&\beta_{nf} = (1-\phi)\beta_{nf} + \phi\beta_s
\end{aligned}
\tag{3.10}
$$

Following Lai and Kulacki (1987), the similarity variables are

$$
\begin{aligned}
&\psi = (\alpha_f u_\infty x)^{\frac{1}{2}} f(\eta), \\
&\eta = (U_\infty x/\alpha_f)^{\frac{1}{2}} \left({}^{y}\!/_{x} \right), \\
&\theta(\eta) = (T - T_\infty)/(T_w - T_\infty).
\end{aligned}
\tag{3.11}
$$

Therefore, the partial differential equations (3.7) and (3.8) are transformed to the following ordinary (similarity) equations, respectively

$$
f'' + (1-\theta)^{2.5}\left(1-\theta+\frac{\phi\rho_s}{\rho_f}\right)G\left[(f')^2\right]' = \frac{1}{2}\left(1-\phi+\frac{\phi(\rho_s\beta_s)}{(\rho_f\beta_f)}\right)\lambda(\eta\theta' - \theta),
\tag{3.12}
$$

$$
\frac{k_{nf}/k_f}{1-\phi+\phi(\rho C_p)_s/(\rho C_p)_f}\theta'' = \frac{1}{2}(\theta f' - f\theta') - \delta,
\tag{3.13}
$$

with the corresponding boundary conditions

$$\begin{aligned} &f(0)=0, \qquad &&\theta(0)=1 \;\; at \;\; \eta=0, \\ &f'(\eta)\rightarrow 1 \qquad &&\theta(\eta)\rightarrow 0 \;\; as \;\; \eta\rightarrow\infty, \end{aligned} \tag{3.14}$$

Here G is the non-Darcy or inertial parameter and λ is the mixed convection parameter, where it can be defined as

$$G=\frac{K'U_\infty}{\nu_f}=\frac{K'U_\infty\rho_f}{\mu_f}, \qquad \text{and} \qquad \lambda=\frac{Ra_x}{Pe_x^{\frac{3}{2}}}, \tag{3.15}$$

where $Ra_x = g\rho_f K\beta_f(T_w - T_\infty)x/(\mu_f\alpha_f)$ is the local Rayleigh number and $Pe_x = U_\infty x/\alpha_f$ is local Péclet number for the porous medium.

The term $Q_0(T - T_\infty)$ is assumed to be the amount of heat generated/absorbed per unit volume. Q_0 is a constant, which may take a positive or negative value. When the wall temperature T_w exceeds the free stream temperature T_∞, the source term represent the heat source, $Q_0 > 0$ and heat sink, $Q_0 < 0$ whereas $T_w < T_\infty$, the opposite relationship is true, (Rana and Bhargava, 2011).

3.3 Numerical solutions

The set of nonlinear ordinary differential equations (3.12) and (3.13) with boundary conditions (3.14) are reduced to the first order differential equations and have been solved by using fourth-fifth order Runge-Kutta Fehlberg method in conjunction with shooting technique with Υ, λ, δ, q and P_r as prescribed parameters. The terms in equations (3.12) and (3.13) are define as

$$\frac{d^2 f}{d\eta^2}=F(f,f',\theta,\eta), \tag{3.16}$$

$$\frac{d^2\theta}{d^2\eta^2}=G(f,f',\theta,\theta',\eta), \tag{3.17}$$

$$f' = \frac{df}{d\eta} = u; \qquad f'' = \frac{d^2 f}{d\eta^2} = u' = F(f,u,u',\theta,\eta), \tag{3.18}$$

$$\theta' = \frac{d\theta}{d\eta} = w; \qquad \theta'' = \frac{d^2\theta}{d\eta^2} = w'' = G(f,u,\theta,w,\eta), \tag{3.19}$$

$$f' = u, \tag{3.20}$$

$$u' = F(f,u,u',\theta,\eta), \tag{3.21}$$

$$\theta' = w, \tag{3.22}$$

$$w' = G(f,u,\theta,w,\eta), \tag{3.23}$$

subject to conditions from equation (3.14)

$$f(0) = 0, \qquad \theta(0) = 1,$$

$$f'(\infty) = 1, \qquad \theta(\infty) = 0. \tag{3.24}$$

Thus by using equations (3.16) to (3.23), the governing equations (3.12) and (3.13) to be solved by MAPLE 18 programming are the system of first order differential equations as follows:

$$f = f(\eta), \tag{3.25}$$

$$f' = \frac{d}{d\eta} f(\eta) = u(\eta), \tag{3.26}$$

$$f'' = \frac{d^2}{d\eta^2} f(\eta) = \frac{d}{d\eta} u(\eta), \tag{3.27}$$

$$\theta = h(\eta), \tag{3.28}$$

$$\theta' = \frac{d}{d\eta} h(\eta) = w(\eta), \tag{3.29}$$

$$\theta'' = \frac{d}{d\eta} w(\eta), \tag{3.30}$$

$$\frac{d}{d\eta} u(\eta) - \left(\frac{1}{(2 + 4.(G.B.q.u(\eta)))} \right).G.(B2).\lambda.(\eta.w(\eta) - \theta(\eta)) = 0, \tag{3.31}$$

$$\frac{d}{d\eta}w(\eta)-\left(\frac{E}{C}\right).\left(\left(\frac{1}{2}\right).(\theta(\eta).u(\eta)-f(\eta).w(\eta))-\delta.\theta(\eta)\right)=0, \tag{3.32}$$

where

$$K_{nf}=(ks)+2.(kf)-2.\gamma.((kf)-(ks)), \tag{3.33}$$

$$B=\left(1-\gamma+\gamma.\frac{(ps)}{(pf)}\right), \tag{3.34}$$

$$E=\left(1-\gamma+\gamma.\frac{(ps).(cps)}{(pf).(cpf)}\right), \tag{3.35}$$

$$G=(1-\gamma)^{2.5}, \tag{3.36}$$

$$K_f=(ks)+2.(kf)+\gamma.((kf)-(ks)), \tag{3.37}$$

$$C=\frac{K_{nf}}{K_f}, \tag{3.38}$$

$$B2=\left(1-\gamma+\gamma.\frac{(ps).(Bs)}{(pf).(Bf)}\right), \tag{3.39}$$

The asymptotic boundary conditions ∞ given in (3.24) were replaced by a finite value of 10 for similarity variable η max as follows:

$$u(10)=1, \qquad \theta(10)=0$$

In the Appendix A section, MAPLE 18 programming for fourth-fifth order Runge-Kutta Fehlberg method is given for finding the values of heat transfer and velocity. ($f'(0)$ and $-\theta'(0)$) respectively is given in detail. In the following section, the results are discussed in detail.

CHAPTER 4

RESULT AND DISCUSSION

4.1 Introduction

Chapter 4 presents the results of ordinary differential equation (3.12) and (3.13) with boundary conditions (3.14) that are solved numerically by applying a shooting technique using fourth-fifth order Runge Kutta Fehlberg method. In the following section, the results are discussed in detail. Non-Darcy mixed convection from a horizontal plate embedded in a nanofluid saturated porous media in the presence of heat source/sink are studied for difference values of parameters.

4.2 Velocity and temperature profile on Cu, TiO_2 and Al_2O_3

In this research, copper (Cu), aluminum oxide (Al_2O_3) and titanium oxide (TiO_2) nanoparticles and water as a base fluid have been used. The relative tolerance was set to 10^{-10}. The obtained results are presented for several values of parameters in some figures and tables. For the sake of simplicity, we will denote non-Darcy or inertial parameter (G) by q and nanoparticle volume fraction (ϕ) by Υ. In order to validate our method, we have compared the results of $f'(0)$ and $-\theta'(0)$ Cu nanoparticles with Rosca (2012).

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