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MODELLING OF HEAT EXCHANGER USING RECURSIVE LEAST SQUARE (RLS)

N. A. Daud [1,a], S. S. Mohd Sukri [1,b], S. Md Salleh [1,c]

[1] Faculty of Mechanical and Manufacturing Engineering, Universiti Tun Hussein Onn Malaysia 86400, Parit Raja, Batu Pahat, Johor, Malaysia.
E-Mail: [a]nuratiqahdaud.atiq@gmail.com, [b]Sitisuhanah@gmail.com, [c]saliha@uthm.edu.my

ABSTRACT

Heat exchanger is a heat transfer device that is used to transfer thermal energy between two or more fluids available at different temperature. Modelling of heat exchanger helps to understand process that occurs during the operation. In order to understand it, a precise model definitely needed to be construct. The primary objective of this study is to develop a structural model of heat exchanger using Auto-regressive model with exogenous variables (ARX) model. In this study, data from heat exchanger experiment was used to determine the parameter of ARX equation. Using RLS algorithm, ARX parameters are optimized. Hence, the transfer function represents the plant for modelling. Validation test of autocorrelation and cross-correlation were used to validate between normalised data input and error. Based on the result on the second order model, A's values are -0.2397 and 0.0691 while B's values are -0.0381 and 0.0111 respectively. From validation test, all graphs are within the 95 percent of confident line.

Keywords: Heat exchanger・ARX modelling・RLS・Validation test・

INTRODUCTION

A heat exchanger may be defined as equipment that transfers the energy from a hot fluid to a cold fluid through a separating wall. The separating is between walls or into and out of a wall in a transient manner. A heat exchanger is widely used in the process industries particularly in chemical plants, power plants and so on [1,3]. In many industrial process and operations, heat exchanger is one of the simplest and important unit [4] for the transfer of thermal energy. The main purpose of exchanger is to maintain specific temperature conditions, which is achieved by controlling the exit temperature of one of the fluids (mainly hot fluid) in response to variations of the operating conditions [5].

For this research paper, the real system of heat exchanger that will be used is the shell and tube heat exchanger type since it offers a great flexibility to meet almost any service requirement. Shell and tube exchanger are designed with a bundle of round tubes mounted in a cylindrical shell with the tube axis parallel to that of the shell. One fluid flows across and between the tubes, the other flows in the inner tubes. The outlet temperature of the heat exchanger system has to be kept at a desired set point according to a process requirement.

System identification is the general process of developing a model for some particular system from given input-output data and the process of deriving a mathematical system model from observed data in accordance with some predetermined criteria [6,7]. The selected model structure that will be used was ARX model and estimation algorithm using Recursive Least Square (RLS). For model validation, the identified model will be tested using autocorrelation test and cross-correlation test. It is to determine how well the behaviour of the model corresponds to the data measured and the purpose for which the model used.

Based on in-depth study of adaptive filter, RLS had been applied to filtering noise adaptively. Through the simulation results prove that its performance is usually much better than using conventional designed to filter fixed [8]. This algorithm recursively find the filter coefficients that will minimize the weight linear that relating between the input signal and least square cost function. RLS is differ with Least Mean Squares (LMS) algorithms, where LMS is used to reduce the mean square error.

HEAT EXCHANGER MODELLING

Heat exchanger plant descriptions

The major purpose of the plant is to heat a liquid in a heat exchanger. The heat is supplied by hot water which is heated in the boiler. Plant is used to heat up liquid in a heat exchanger. The liquid is come from cold water that temperature based in room temperature. This project is mostly concerned with the modelling of the open loop hot temperature of heat exchanger. Preheated tank used in the plant is to warm the cold water. The cold water is flow from pipe to become hot water from the temperature transfer process. For the input liquid, the temperature is at room temperature. The temperature is around 25ºC and the preheated tank is maintaining the temperature at 60ºC. Hot water is heated in preheated tank (T12) with 60ºC. Figure 1 shows the plant descriptions of heat exchanger. With reference to the overall schematic diagram, it is observed that the system consist of a number of tank T11, T12, T13 and heat exchanger systems.

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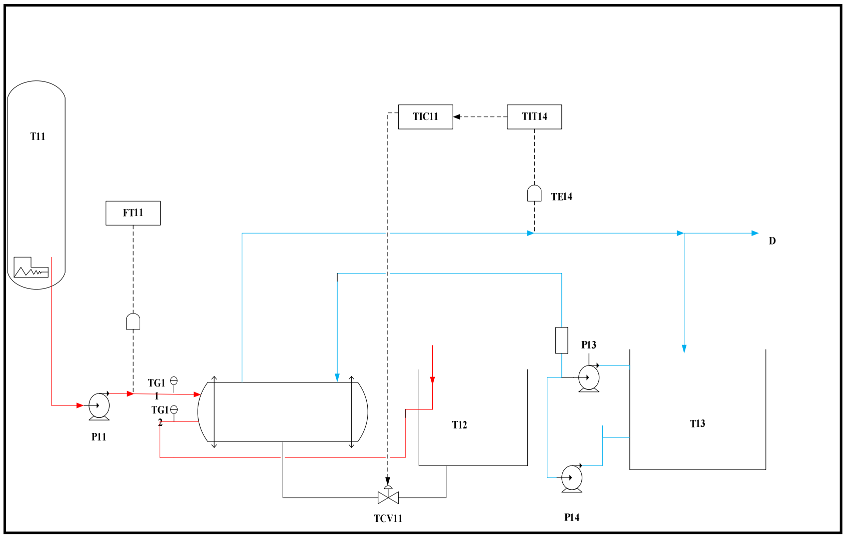

Figure 1: Plant Descriptions of the Heat Exchanger

System identification Autoregressive Exogenous (ARX) Model structure

System identification (SI) is a method to obtain a mathematical model of the dynamic system based on experimental data. There are four steps to determine model using SI. First, obtaining input and output data. Second, perform a model structure. Third, applied identification method. Forth, do a model validation. The model identifying will be tested by using autocorrelation and cross-correlation for model validation. This is to verify that the identified model fulfills the modeling requirement according to subjective and objective criteria of good model approximation.

In this paper, a discrete time ARX model is chosen for the model structure of the heat exchanger. This model is the simplest model includes the stimulus signal. The structure of an ARX model is show below in Figure 2.

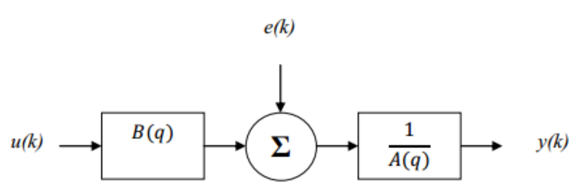

Figure 2: The ARX Model signal flow

The model structure of ARX relates the current output *y(t)* to a finite number of past outputs *y(t-k)* and inputs *u(t-k)*. The structure is entirely defined by three na, nb, and nk. na is represent to the number of poles and nb-1 is the number of zeros. Thus, nk is the pure time delay (dead-time) in system. For system under sampled-data control occasionally nk is equal to 1 if there is no dead-time. For multi input systems, nb and nb are row vectors. Which the i-th element gives the order delay associated with the ith input.

$$y(t) + a_1 y(t-1) + \cdots + a_{n_a} y(t-n_a) =$$

$$b_1 u(t-n_k) + \cdots + b_{n_b} u(t-n_k-n_b+1) \qquad (1)$$

The identification method for the ARX model is the least squares method. This is a special case of the prediction error method. The least square method is the most efficient polynomial estimation. This is because this method solves linear regression equations in analytical form [9].

Recursive Least Square (RLS) Algorithm

The concept of recursive least square was discovered and developed by Gauss but they leave unused and ignored. Generally any problem can be solved by adaptive filter in used of RLS. This algorithm recursively finds the filter coefficients that will minimize the weight linear that relating between the input signal and least square cost function. This are difference to other algorithm likewise the least mean squares (LMS). LMS aim is to reduce the mean square error. The algorithm can be described in the following step [13]:

The tap-input vector at time step *i* is define as;

$$U(i) = [u(i) \quad u(i-1) \quad \ldots \quad u(i-M+1)]^T \qquad (2)$$

The tap-weight coefficient vector equation at time step *n* is defines as:

$$w(n) = [w_0(n) \; w_1(n) \quad \ldots w_{M-1}(n)] \qquad (3)$$

The recursive parameter estimation algorithms are based on the data analysis. The data analysis is the input and output signal from the process to be identified. Many recursive identification algorithms were proposed.

In this part several wellknown recursive algorithms with forgetting factors implemented in recursive identification algorithm are summarized. This method can be used toward parameters estimate of ARX model. The algorithm can be written in following form [13]:

$$\hat{e}(k) = y(k) - \varphi^T(k)\,\hat{\vartheta}(k-1)$$
$$L(k) = \frac{C(k-1)\varphi(k)}{1+\varphi^T(k)C(k-1)\,\varphi(k-1)}$$
$$\hat{\vartheta}(k) = \hat{\vartheta}(k-1) + L(k)\hat{e}(k)$$
$$C(k) = C(k-1) - L(k)\varphi^T(k)C(k-1) \qquad (4)$$

Where L(k) denote gain matrix, C(k) equal to covariance matrix of the estimated parameters, ϑ(k) is the vector which contains the estimated parameters and φ(k) is the data or regression vector.

$$\hat{\vartheta}(k) = [a_1, \ldots, a_{na}, b_1, \ldots, b_{nb}]^T \qquad (5)$$

$$\varphi^T(k) = [y(k-1), \ldots, y(k-na), u(k-1), \ldots, u(k-nb)] \qquad (6)$$

This RLS algorithm assumes that the parameters of the model process are same. In many situations, nevertheless, the estimator will be required to track changes in a set of parameters. To manage with tracking the time-variant parameters some adjustment mechanism must be introduced in the previous basic equations.

Directional Forgetting Factor

It has been devised as one possible way to avoid the wind-up in Exponential Forgetting. The described version is from [10], and this forgetting method is used in the implementation in the following chapter. The information matrix evolves according to the incoming information. The directional forgetting algorithm is consider as discrete-time system with output y(t)∈R1, and the associated linear predictor ŷ(t)= φ(t)'ϑ, where φ(t)is the ndimensional observation vector. The estimation of the parameter vector can be recursively performed with the Directional Forgetting algorithm [11].

$$\epsilon(t) = y(t) - \varphi(t)'\hat{\vartheta}(t-1)$$
$$r(t) = \varphi(t)'P(t-1)\varphi(t) \qquad (8)$$
$$K(t) = \frac{P(t-1)\varphi(t)}{1+r(t)}$$
$$\hat{\vartheta}(t) = \hat{\vartheta}(t-1) + K(t) \in (t)$$

$$\beta(t) = \begin{cases} \mu - \dfrac{1-\mu}{r(t)}, \mu\epsilon(0,1) & if\ r(t) > 1 \\ 1 & if\ r(t) = 0 \end{cases}$$
$$P(t) = P(t-1) - \frac{P(t-1)\varphi(t)\varphi(t)'P(t-1)}{\beta(t)^{-1}+r(t)}$$

From the equation, the constant μ is the Forgetting Factor. If μ is close to one, the algorithm is slow to respond. More alertness is achieved by decreasing μ, at the price of the increased sensitivity to disturbance.

RLS algorithm is used to estimate the parameter of ARX model. The parameters of the model process are assumed as constant in this algorithm. The heat exchanger plant will be in transfer function state. The transfer function will be linked to MATLAB Simulink. The second order of transfer function had been chose to represent the plat of heat exchanger. To create the plant modelling in Simulink, standard signal was chose to be the input of model required. The type of standard signal used was step input standard signal. Figure 3 below show the picture of plant modelling with heat exchanger.

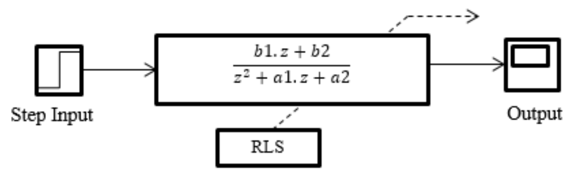

Figure 3: Plant modelling with RLS

In order to define the order of the plant, several analysis had been made. The analysis have gone through from first until fifth order and the results show that second order gave a better result in terms of means square error (MSE) value and time of simulation takes place. The second order of transfer function gave the least value of MSE and the fastest time of simulation.

Autocorrelation and cross-correlation

For validation test, autocorrelation and cross-correlation are used. Correlation is widely used in digital signal processing. This is because it is easy to understand and implement. It can be defined as degree of similarity between two signals. If the both signals are identical, then the correlation coefficient is equal to 1.

The autocorrelation function of a random signal described general dependence of the values of samples. The samples are at one time on the values of the samples at another time. Consider a random process x(t) in example is continuous-time, it is autocorrelation is written as:

$$R_{xx}(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau)dt$$

While T, is equal to period of observation. $R_{xx}(\tau)$ is a real-valued and an even function with a maximum at $\tau = 0$.

Cross-correlation is very useful to investigate the degree of association between two signals. The essential use of the cross-correlation function is providing measure of similarity. The similarities are between two signals which are a function of the delay between them. Both cross-correlation between x(t) and y(t) can be described by [12]:

$$R_{xy}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T}^{T} x(t)y(t+\tau)dt$$

Or

$$R_{yx}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T}^{T} y(t)x(t+\tau)dt \qquad (7)$$

Where T is the period of the observation and $R_{xy}(\tau)$ is a real valued.

Plant modelling using RLS

During in process of the project, the modelling of heat exchanger was done by using the RLS algorithm. By using RLS the second order steady state error is use to optimize the value of a and b. the flow chart of plant modelling using RLS shows in Figure 3.

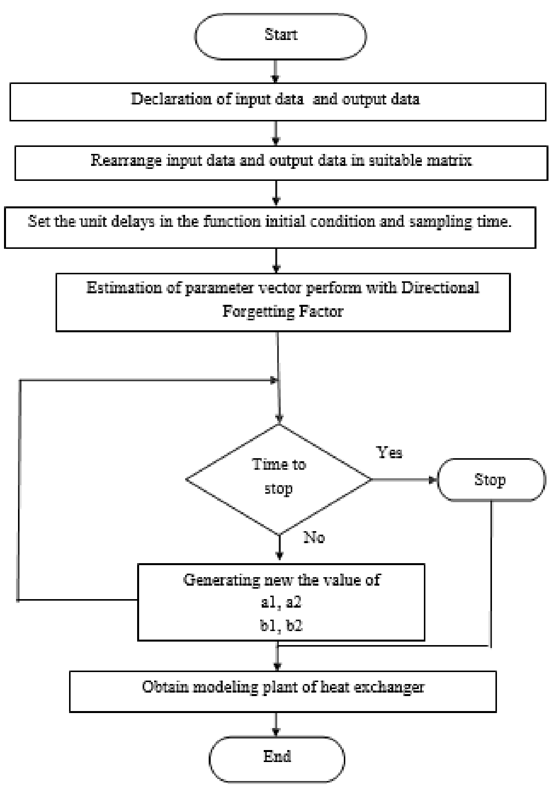

Figure 3: Flowchart of plant modelling using RLS

The input called FT11 data is the flow rate of heat exchanger. On the other hand, data of TIT14 is an output for temperature of heat exchanger. A plant of heat exchanger is use in term of transfer function to represent plant.

RESULTS AND DISCUSSIONS

Analysis has been done thoroughly based on the result obtained from heat exchanger experiment by using MATLAB programming. The result are presented the values of a's and b's which used to make a transfer function model. The ARX parameters were optimise using RLS algorithm using MATLAB Simulink. This giving the model parameters of a's and b's with second order. After getting the value of the parameters, the transfer function is built by using all of these parameters. All parameters knew as a1, a2 b1 and b2. The letter 'a' is representing the output and the letter 'b' is representing the input. The four variables are used because these parameters will fit the data obtained. The parameter model value are given in Table 1.

Table 1: The estimated value of ARX model using RLS method

Time (sec)	Parameter				MSE
	a1	a2	b1	b2	
1000	-0.0441	0.0545	-0.2754	0.3481	0.0054
2000	-0.0977	0.0873	-0.6127	0.5524	-0.0012
3000	-0.1044	0.0852	-0.6550	0.5372	-0.1115
4000	-0.0663	0.0425	-0.4163	0.2678	-0.1838
5000	-0.0381	0.0111	-0.2397	0.0691	-0.2773

The transfer function obtained in z-function. Compared to s-function, the z-function is for easier digitization process of realtime data. Paramaters value is based on 5000sec time respond.

$$tf = \frac{-0.2397z + 0.06911}{z^2 - 0.0381z + 0.01111}$$

Figure 4 shows a predicted output, 'Tout' and normalised ouput,'Tnout'. The graph of 'Tout' and 'Tnout' are quiet similar to each other except after 3000 second. There is some errors occur within significant value due to the unability of RLS algorithm optimizing the best parameters value for ARX model.

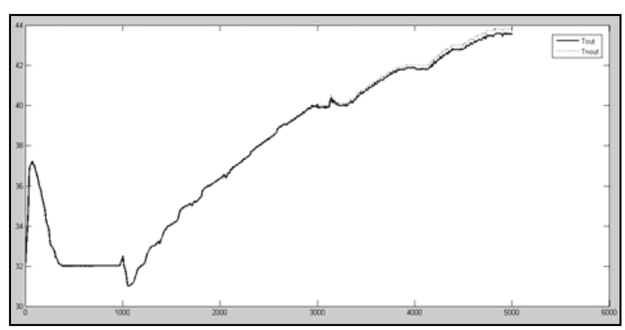

Figure 4: Graph of predicted output, 'Tout' and normalized output, 'Tnout'

The signal error show in Figure 5 are for all data produced between the predicted and real output data along the curve. The error is between +0.05 to -0.3. Note that RLS algorithm unable to find similarity betweent Tout and Tnout.

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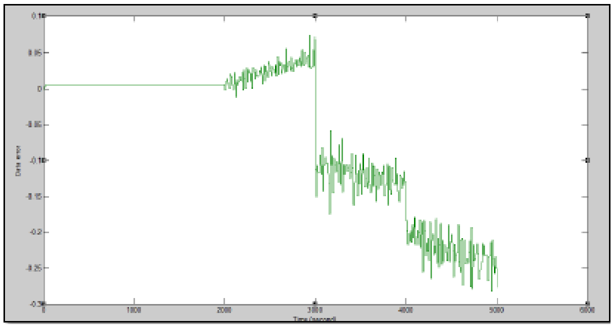
Figure 5: Signal error of ‘Tout’ and Output Data

Validation test includes autocorrelation, cross-correlation of input and residual, cross-correlation of input square and residual, cross-correlation of input square and residual square, and cross-correlation of residual and (input*residuals). Figure 6 and figure 7 to 10 show the result of autocorrelation and cross-correlation respectively. By correlating a signal with itself, repetitive patterns will stand out and make it easier to see. After the process runs for a few minutes, the result produce is a perfectly sharp spike and gives impact on the system’s latency. The result shows that the graph provides a smooth discrete time signal. For cross-correlation test, it is important to examine all the graph at once. To be known that the validation test is acceptable, all the cross-correlation graph should be produced within its confident line.

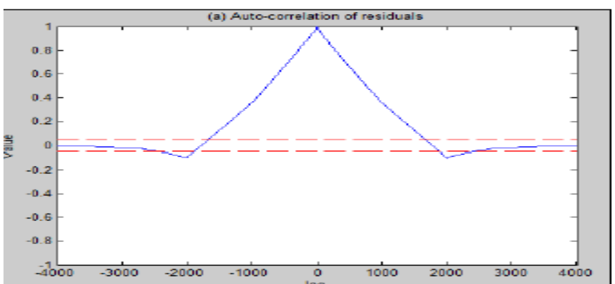

Figure 6: Autocorrelation test

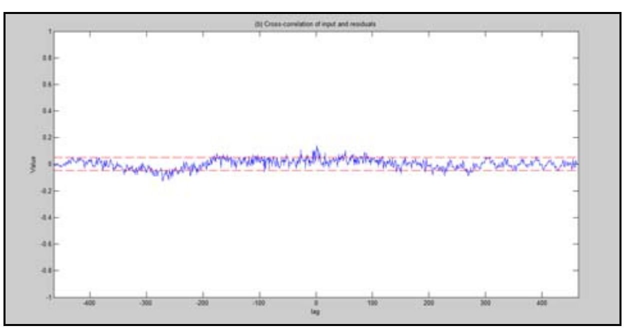
Figure 7: Cross-correlation of input and residual

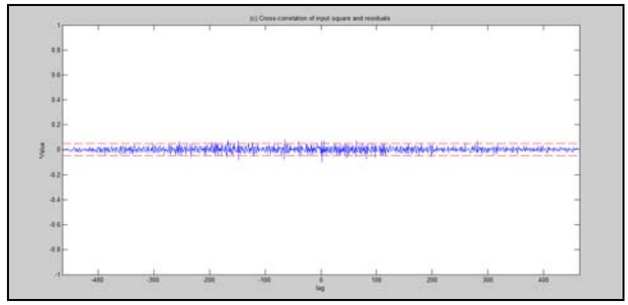
Figure 8: Cross-correlation of input square and residual

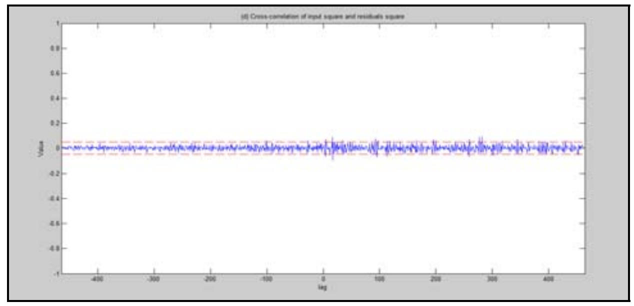
Figure 9: Cross-correlation of input square and residual square

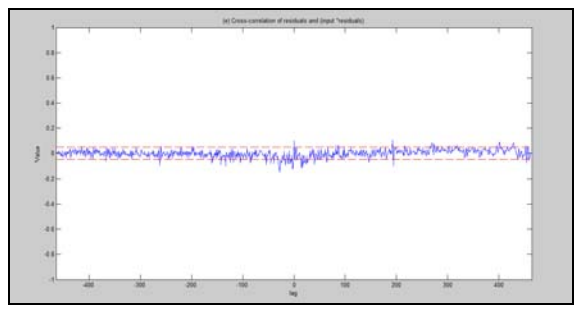
Figure 10: Cross-correlation of residual and (input*residuals)

CONCLUSIONS

The experiment of Heat Exchanger QAD model BDT921 was done in the Control Laboratory, FKEE UTHM. All of the 582 data from the graph of data recorder is used to find ARX parameters, transfer function of plant and compared the actual data with prediction model by recursive least square algorithm There are 4 parameters of ARX equation obtained in this project which are 2 outputs (a1 and a2) and 2 inputs (b1 and b2). These parameters is the simplest transfer function to describe the overall system of heat exchanger. As the small parameterscan represent the plant very well consideration of higher variables numbers are neglected. The parameters obtained will be used as the modelling plant for simulation process that represent the heat exchanger itself. It is important to understand that, minimum value of error shows the effectiveness of heat exchanger process.

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REFERENCE

[1] Weyer E., Szederkenyi G., Hangos K. (2000). Grey box fault detection of heat exchanger – Theory and application. Control Engineering Practice. Volume 8. Number 2. pp. 121-131(11). Elsevier.

[2] Raymond C., Shao F. (1994). Manual of QAD Model BDT 921/M3, “Boiler Drum and Heat Exchanger Process Control Training System. Unpublished.

[3] Belinda C. C. M. (2002). Modelling of a Hot Water System Drum and Heat Exchanger.

[4] Lennart L. (1999). System Identification: Theory for the User (2nd. Ed), Prentice-Hall.

[5] Mr.P.Sivakumar, Dr.D.Prabhakaran. Dr.T.Kannadasan. (2012). Temperature Control of Shell and Tube Heat Exchanger by Using Intelligent Controllers. International Journal of Computational Engineering Research, 285-291.

[6] Lennart L. (1988). System Identification Toolbox™: User’s Guide R2010b, the MathWorks, Inc., © COPYRIGHT 1988-2011.

[7] Komal R. Borisagar, G. R. Kulkarni (2010). Simulation and Comparative Analysis of LMS and RLS Algorithms Using Real Time Speech Input Signal. Global Journal of Researches in Engineering. 44-47.

[8] Noor Adzmira binti Mustapha (2011). General Polynomial Model to System Identification of Heat Exchanger Based on Hammerstein Warmer Model. Final Project UTHM.

[9] Navratil, P. and Ivanka, J. (2009). *Recursive Identification Algorithms Library*. Ministry of Educattion of Czech repucblic

[10] Kulhavý, R. (1987). *Restricted exponential forgetting in real time identification*. Automatica (Journal of IFAC)

[11] Bittanti, S., bolzern, P. and Campi, M. (1990) *Convergence and Exponential Of Identification Algorithm With Directional Forgetting Factor*. International Federation of Automatic Control. Great Britain

[12] Taghizadeh, S.R. (2000). *Digital signal Processing.* London: University of North London.

[13] Haykin, S. et. al. (1995) *Tracking of Linear Time-Variant Systems*. McMaster University, Canada.