CONTOUR MATCHING USING ANT COLONY OPTIMIZATION AND CURVE EVOLUTION

RATHIAH BINTI HASHIM (KETUA)
ROSMILA ABDUL KAHAR
ROSDINAR ABDUL KAHAR
SAADI YOUNES (GRA – SARJANA)

GERAN FRGS
VOT 0740

Faculty of Computer Science And Information Technology
Universiti Tun Hussein Onn Malaysia

ABSTRACT

Shape retrieval is a very important topic in computer vision. Image retrieval consists of selecting images that fulfil specific criteria from a collection of images. This thesis concentrates on contour-based image retrieval, in which we only explore the information located on the shape contour. There are many different kinds of shape retrieval methods. Most of the research in this field has till now concentrated on matching methods and how to achieve a meaningful correspondence. The matching process consist of finding correspondence between the points located on the designed contours. However, the huge number of incorporated points in the correspondence makes the matching process more complex. Furthermore, this scheme does not support computation of the correspondence intuitively without considering noise effect and distortions. Hence, heuristics methods are convoked to find acceptable solution. Moreover, some researches focus on improving polygonal modelling methods of a contour in such a way that the resulted contour is a good approximation of the original contour, which can be used to reduce the number of incorporated points in the matching. In this thesis, a novel approach for Ant Colony Optimization (ACO) contour matching that can be used to find an acceptable matching between contour shapes is developed. A polygonal evolution method proposed previously is selected to simplify the extracted contour. The main reason behind selecting this method is due to the use of a stopping criterion which must be predetermined. The match process is formulated as a Quadratic Assignment Problem (QAP) and resolved by using ACO. An approximated similarity is computed using original shape context descriptor and the Euclidean metric. The experimental results justify that the proposed approach is invariant to noise and distortions, and it is more robust to noise and distortion compared to the previously introduced Dominant Point (DP) Approach. This work serves as the fundamental study for assessing the Bender Test to diagnose dyslexic and non-dyslexic symptom in children.

ABSTRAK

Dapatan semula bentuk merupakan topik yang amat penting dalam penglihatan komputer. Dapatan semula imej melibatkan pemilihan dari koleksi-koleksi imej yang memenuhi kriteria tertentu. Tesis ini menjurus kepada dapatan semula imej berasaskan kontur, di mana kami meneroka hanya maklumat yang terletak pada kontur bentuk. Terdapat pelbagai jenis kaedah dalam dapatan semula bentuk. Sehingga kini, kebanyakan penyelidikan di dalam bidang ini tertumpu kepada kaedah-kaedah pemadanan dan cara untuk mencapai kesepadanan hubungan yang lebih baik. Proses pemadanan ini terdiri daripada mendapatkan kesepadanan hubungan antara titik-titik yang terletak pada kontur-kontur yang direka. Walau bagaimanapun, kewujudan jumlah yang besar bagi titik-titik pemadanan di dalam kesepadanan hubungan tersebut sehingga pemadanan menjadi lebih kompleks. Selain itu, kaedah ini tidak mengambilkira kesan hingar dan gangguan-gangguan lain. Oleh itu, kaedah heuristik digunakan untuk mencari penyelesaian yang lebih bermakna. Tambahan pula beberapa penyelidikan memfokuskan kepada penambahbaikan kaedah-kaedah pemodelan poligon kontur supaya kontur yang dihasilkan mempunyai penghampiran yang baik berbanding kontur asal, di mana ia boleh digunakan untuk mengurangkan bilangan titik-titik pemadanan. Dalam tesis ini, satu pendekatan baru untuk pemadanan kontur bagi Pengoptimuman Koloni Semut (PKS), boleh digunakan dalam mencari pemadanan antara bentuk-bentuk kontur. Satu kaedah evolusi poligon yang dicadangkan sebelum ini telah dipilih bagi memudahkan kontur yang telah diekstrak. Tujuan utama di sebalik pemilihan kaedah ini ialah disebabkan penggunaan kriteria pemberhentian yang perlu ditentukan pada peringkat awal. Proses pemadanan yang diformulasikan sebagai Masalah Tugasan Kuadratik (MTK) telah diselesaikan dengan menggunakan PKS. Satu penghampiran keserupaan dikira menggunakan bentuk penghurai konteks asal dan metrik Euclidean. Sebagai kesimpulan dari keputusan eksperimen, pendekatan ini tidak varian kepada bunyi bising dan gangguan-gangguan, dan lebih teguh kepada bunyi bising dan gangguan-gangguan berbanding dengan Pendekatan Titik Dominan (TD) sebelumnya. Projek ini merupakan asas kepada penilaian di dalam Bender Test yang digunakan untuk mengesan simptom dyslexic dalam kalangan kanak-kanak.

PUBLICATIONS

A fair amount of the material presented in this thesis has been published in various refereed conference proceeding and journal as stated below:

1. **Younes Saadi**, Rathiah Binti Hashim and Rosmila Abdul-Kahar, Contour Matching: A heuristic Approach. International Journal of Intelligent Information Processing. (2012) (Indexed by **DBLP**)
2. **Younes saadi**, Rathiah Binti Hashim, Rosmila Abdul-Kahar. Ant Colony Matching: A Curve Evolution Approach. International conference on Communication and Informations Science **IEEE** (ICCIS2012). Gyeongju. Fouth Korea.
3. **Younes Saadi**, Rathiah Binti Hashim and Rosmila Abdul-Kahar, A New Approach for Shape Dissimilarity Retrieval Based on Curve Evolution and Ant Colony Optimization, The Third International Conference on Recent Trends in Information Processing & Computing- IPC 2012. (Accepted and will be Published by **Springer Verlag**).
4. **Younes Saadi**, Rathiah Binti Hashim and Rosmila Abdul-Kahar, Shape Recognition Using Heuristic Algorithm, International Journal of Signal & Image Processing IJSIP. (Accepted, indexed by **DBLP**).
5. **Younes Saadi**, Rathiah Binti Hashim, Rosmila Abdul-Kahar. Using Video Games to Detect Dyslexia Symptoms Malaysian Technical Universities International Conference on Engineering & Technology (MUiCET 2011).

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CHAPTER 1

INTRODUCTION

1.1 Background study

Shape retrieval is a very important topic in computer vision. It is an act of selecting images that fulfil a specific criterion from a collection of images (Nacereddine *et al.*, 2007). It is based on shape matching, which is one of the fundamental techniques in verifying those criteria. Shape matching is classified into two main categories, region based matching and contour based matching. This research focused only on contour based matching.

In contour based matching, only the information located on the shape contour is explored. The contour consists of a collection of points located along the edges, formulating a bipartite graph that can be structured as a Quadratic Assignment (QA) (Kaick *et al.*, 2007). Most of the research in this field has till now concentrated on how to achieve a meaningful correspondence between contour points (Kaick *et al.*, 2007). Based on the distribution of the contour points, a bipartite graph can be formulated, and the problem is structured as Quadratic Assignment Problem (QAP). This is considered as one of the NP-Hard problems. Finding an optimal solution for such problems is so difficult and in some cases, impossible (Ruberto and Morgera, 2011). For this complexity reason, heuristics techniques are convoked to find acceptable solutions instead of using exact methods.

On the other hand, some researches focus on improving polygonal approximation methods of a contour in such a way that the resulted contour contains only the most dominant points that can be used to extract meaningful information (Parvez and Sabri, 2010) which can reduce the correspondence complexity.

1.2 Motivation

Shape matching has been deeply studied by many researchers (Veltkamp *et al.*, 2001). However, challenges still remain. The first challenge is the invariance. Since shape in many applications is often discussed based on the property of invariance, shape matching is expected to be invariant to transformations such as example translation, scale and orientation. The second challenge is tolerance as noise, blur, crack and deformation are usually introduced when the shape of an object is extracted from an image. In this case, a shape matching is required to be robust to these imperfections.

Moreover, in contour based matching, the points along the contours as a bipartite graph can be figured out and formulated as a QA. This is considered as an NP-hard problem (Ruberto and Morgera, 2011). In order to find an acceptable solution, heuristics techniques are often used. The main idea is to compute the mapping between two contours by minimizing the global dissimilarity. Many research studies have been proposed. For example, Hungarian method uses the simple greedy matching (Papadimitriou and Stieglitz, 1982), and COPAP (Scot and Nowak, 2006) takes into account the order preserving. However, the main drawback of these techniques is the omission of proximity information measurement between feature points on the same shape contour. For this reason, Kaick *et al.* (2007) proposed an Ant Colony Optimization (ACO) approach based on incorporating proximity into an optimization framework. However, the huge number of points incorporated in the correspondence makes the matching more complex and less accurate (Ruberto and Morgera, 2011). A modified ACO matching approach has been proposed by Ruberto and Morgera (2011) based on genetic algorithm; only dominant points are used instead of the sampling distribution of contour points, which improves the correspondence accuracy and reduces the complexity.

In this research, a new ACO matching approach based on Discrete Curve Evolution (DCE) was proposed. In order to reduce the number of contour points incorporated in the correspondence, a polygonal approximation proposed previously (Latecki and Lakamper, 2000) was selected to simplify the extracted contour. Mainly, it simplifies the contour by neglecting distortions while at the same time preserving the perceptual appearance at a level sufficient for object recognition. To test the effectiveness of our approach, an MPEG-7 subset described by Ruberto and Morgera (2011) was used to test shape retrieval considering noise and distortions effect. The results were also compared with the previous work of ACO matching by Ruberto and Morgera (2011).

1.3 Objectives

The objectives of this research are:

(i) To design a shape contour matching algorithm based on Ant Colony Optimization and DCE techniques.

(ii) To develop a prototype for shape matching which will be known as `myMatch`

(iii) To test the prototype and evaluate its efficiency by comparing it with an existing approach, namely Dominant Point (DP).

1.4 Scope of study

This research focused only on Ant Colony Optimization approach as a model of 2D contour matching. We used the original shape context which is not invariant to rotation. `Not invariant to rotation` means that the accuracy of the matching process is affected by rotation. Only the invariance to distortion and noise was investigated in this approach. This approach was tested using MPEG-7 dataset following the work of Ruberto and Morgera (2011) which was the only research on matching algorithm based on ACO and approximation method.

1.5 Significance of study

Shape matching has numerous applications in both computer vision and graphics. The significance of this study is to provide a robust matching invariant to noise and distortion, which can be used in many different potential applications:

(i) This research can be used in registration as a matching model. The registration can be used for various applications such as assistance in surgery and also for reconstruction of data from different images.

(ii) It could be used in the field of medical image analysis such as matching two images to identify diseased tissue or to detect tumours.

(iii) It could be used in similarity measure for 2D shape based object retrieval and recognition.

(iv) This technique can be a basis for modelling new computer aided detection software for automated detection of abnormalities.

1.6 Thesis outline

This thesis is organized as a progressive study of our approach using ACO matching and DCE. Each chapter shows a stage in the research and is based on the foundation provided by previous chapters.

In Chapter 2, the basic concepts used in contour matching are initially followed by a review on Ant Colony Optimization (ACO). A review has been done on previous work, focusing on contour correspondence based heuristics techniques and polygonal approximation techniques. Some concepts on polygonal approximation are also presented. This survey is not exhaustive but serves to set the context for the contribution of this thesis and also summarizes the problems and issues of the topic.

The focus in Chapter 3 is on establishing the direction of research that takes into account the issues decorticated in Chapter 2. In this chapter, the proposed approach consists of ACO matching based on DCE is introduced briefly by using original shape context as a descriptor. The main idea is to reduce the number of the contour points along the contour in such a way that the simplified contour preserves the original frame of the shape. This simplification provides a solution for the problem related to the complexity of correspondence and also improves the accuracy of correspondence by reducing noise and distortion effect. Finally, the flowchart of the proposed approach is presented.

Chapter 4 describes the implementation part of this research, which is part of the research contribution. It consists of a detailed explanation of the DCE algorithm used with the ACO matching. The mathematical model of DCE is described and the different functions used in the algorithm are detailed. Furthermore, the original ACO algorithm is explained with its mathematical model. Finally, some problems encountered during the implementation of the approach are discussed.

In Chapter 5, the obtained results are discussed. The experimental results are justified for each class which provide a good analysis for the efficiency of the proposed approach. Moreover, a comparison between the proposed approach and Dominant Point approach (Ruberto and Morgera, 2011) is carried out in order to prove the robustness of the proposed approach.

Finally, Chapter 6 concludes the thesis with a summary of the contributions. Some limitations of the proposed approach are highlighted, and possible directions and future research are presented.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Nowadays, shape analysis has become a hot topic in computer vision. It has been widely adopted in many applications of computer graphics. In particular, shape matching—one of the fundamental techniques of shape analysis—plays a primordial role in shape retrieval, recognition and classification, and medical registration (Tsapanos *et al.*, 2011). Shape matching has reached a state of maturity in which many real products based on shape matching are commercialized in different areas (Aaron *et al.*, 2011). In the commercial domain, shape matching methods are being used to retrieve and classify images, for personal and institutional needs like security and military. In the medical domain, shape matching is used in radiology to diagnose and to assess medical images to determine the progress and the suitable treatment options.

Shape matching is application dependent. Different applications may have different requirements on invariance and tolerance to noise, distortion, blur, transformation, scale and orientation. Thus, it is difficult to design a universal method which is suitable for all applications. Nowadays, many techniques have been proposed but most of them only focus on the applications where shape is invariant to transformation.

Based on the representation techniques, shape matching techniques can be classified into two categories: contour based matching and region based matching (Luciano and Roberto, 2009). This research is focusing on contour based matching techniques, in which only the information located on the shape contour is explored. Most of the past research concentrated on how to achieve a meaningful correspondence (Kaick *et al.*, 2011). On the other hand, some research focused on improving representation methods of a contour in such a way that the resulted contour could be used to extract meaningful information (Luciano and Roberto, 2009).

Over the last years, several correspondence based matching methods have been introduced; some of them are heuristic-based. They represent an acceptable solution for the cases where the matching is formulated as an NP-hard problem (Kaick *et al.*, 2007). Furthermore, many methods for representing shape contours by approximation have been proposed. The rest of this chapter will discuss on correspondence based contour matching, polygonal approximation, Ant Colony Optimization (ACO) and some issues related to the topic.

2.2 Fundamentals of shape representations

Shape plays a crucial role in human object classification and identification (Beeck *et al.*, 2001). It is also regarded as being one of the predominant features determining the perceived similarity of images (Lu *et al.*, 2007). The term shape, however, is used in a variety of meanings. The task of defining shape is complex (Luciano and Roberto, 2009). The point is that if shapes are to be successfully analysed, we must precisely know what they are and what properties they exhibit, and often take into account the way they are perceived by humans. Therefore, this research adopts a definition of shape which tries to take into account what the object is and to decide on shape equivalence under a generic class of transformations were adopted. Regarding this aspect, a shape can be considered as any connected set of points. Many fundamental concepts are described in order to put the reader in the context of this research.

As shown in Figure 2.1, shape representation can be classified into three categories: representation based contour, representation based region and representation based transformations. According to the type application, a representation model is selected. The most useful model is region based representation because using the whole surface gives more efficiency than using contour. However in some particular application, using the whole surface is not significant. Contour based representation is useful in many types of applications especially in medical image processing and image detection.

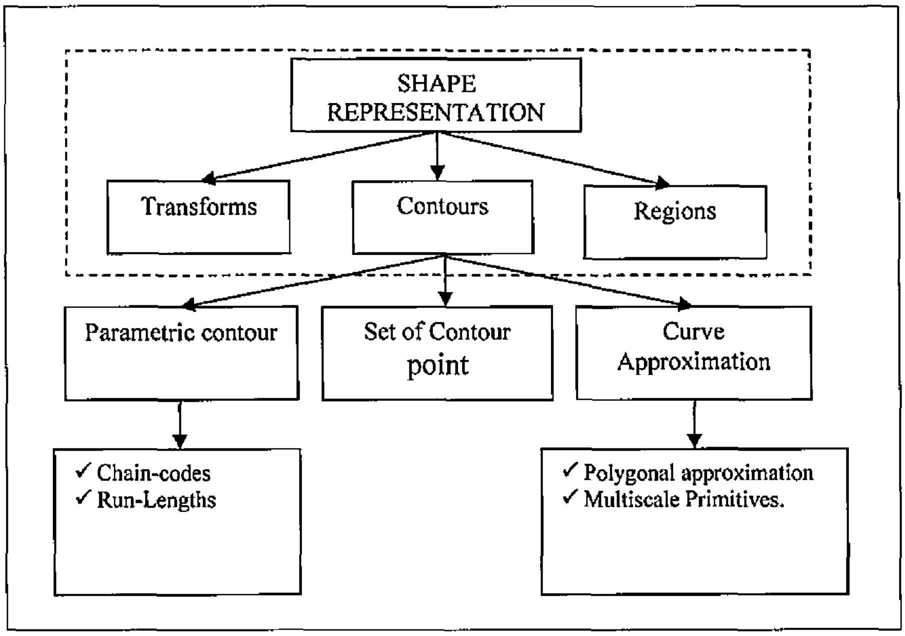

Figure 2.1: A taxonomy of contour based shape representation techniques (Luciano and Roberto, 2009)

2.2.1 Shape representation by contour based method

Shape representation generally consists of finding effective and perceptually important shape features based on either shape boundary information or boundary plus interior content (Zheng and Doerman, 2006). Shape representation and description techniques can be generally classified into three classes of methods (Luciano and Roberto, 2009). The focus in this research is on contour based representation. Figure 2.2 shows fundamental contour points; it comprises set of consecutive points forming the general frame of the shape. It is very important to mention that all the contour tracing techniques used to obtain contour points are for the same results.

This discussion focuses only on contour based methods, which is the focus of this research. According to Luciano and Roberto (2009), contour shape methods only exploit shape boundary information.

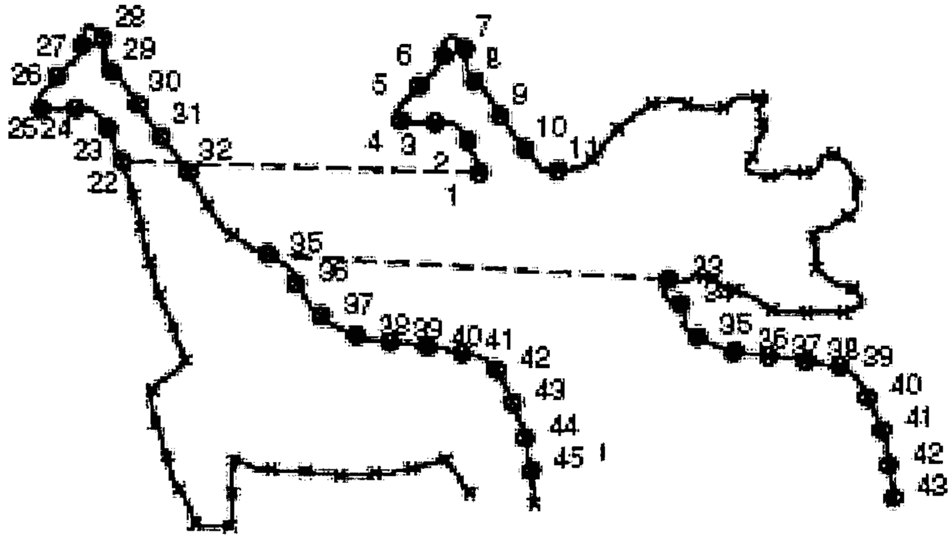

Figure 2.2: Contour based representation (Kaick *et al.*, 2007)

(1) Parametric Contour

Representing shapes directly by binary images implies some drawbacks. For example, it demands large storage space and does not explicitly identify the shape elements. Hence, an alternative approach has been introduced based on parametric representation. This alternative approach involves representing the contour as a parametric curve thus implying a sequential order along it as shown in Figure 2.3.

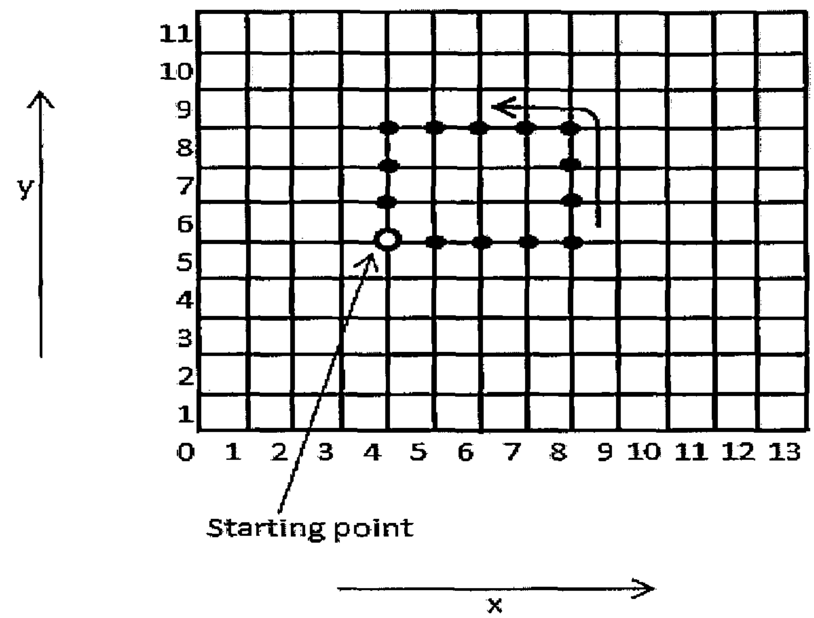

Figure 2.3: Parametric contour representation on a discrete grid.

(2) Set of Contour Points

Several approaches in representing the shape as a parametric contour are based on tracking the shape boundary in a given order. Assuming that $g(p,q)$ is a binary image where $g(p,q)=1$ for shape pixels and $g(p,q)=0$ for background pixels creates a set of contour points stored in the variable cs, which is of the above defined contour set type.

Square tracing algorithm (see Figure 2.4) is adopted to trace the contour. Given a digital pattern and a group of black pixels on a background of white pixels (a grid), a black pixel is located and declared as a "start" pixel. (Locating a "start" pixel can be done in a number of ways. For instance, at the bottom left corner of the grid, each column of pixels is scanned from the bottom going upwards starting from the leftmost column and proceeded to the right until a black pixel encountered. Then, that pixel is declared as a "start" pixel).

In order to extract the contour of the pattern, every time a black pixel is found, the direction is turned to the left, and every time a white pixel is found, the direction is turned to the right, until the start pixel is encountered again. The black pixels found during the loop constitute the contour of the pattern.

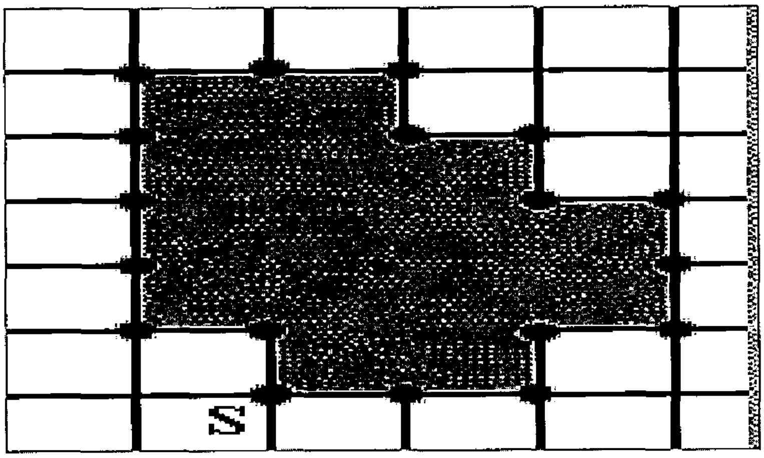

Figure 2.4: Square tracing.

(3) Curve Approximations

The third approach is the most relevant to this research. Representing shape boundaries is based on approximating or interpolating them. Among many techniques for curve approximation, vision researchers have recognized that rather representing whole contours by a single function or series, it is generally more interesting to derive piecewise representations that approximate each contour portion by a geometric primitive such as a straight line segment (Pavilidis and Horowitz, 1974). The contour points that define a contour segmentation are called dominant points, and can be defined using different ways. One of the methods incorporated with Ant Colony matching was proposed by Luciano and Roberto (2009). More details concerning this method and the previous work on polygonal approximation methods are described in Section 2.7.

2.2.2 Similarity measures

Shape similarity measure consists of computing the extent to which one shape differs from another. Hence, the way how the similarity is computed becomes an essential ingredient in shape matching (Veltcamp *et al.*, 2001). Under pattern recognition perception, a similarity measure is a function which is defined on pairs of patterns to indicate the degree of resemblance of the patterns (Veltkamp *et al.*, 2006). In most of related literatures, it is agreeable that a desirable similarity measure is a metric or semi-metric.

Various typical similarity measures have been reviewed adequately by Veltkamp *et al.* (2001, 2006) and Singh *et al.* (2011) involving many areas such as discrete metric, Minskowski distance, bottleneck distance, Hausdorff distance, turning function distance, signature function distance, Fréchet distance, nonlinear elastic matching distance, reflection distance, area of overlap, area of symmetric difference, template metric, Banach-Mazur distance, and Earth Mover's distance and fuzzy theory. An interesting research of similarity transformation between two 2D point sets was introduced by Werman *et al.* (1995). In these research studies, the image normalization must be applied to the images before comparison to guarantee that the computed distance is symmetric with respect to the two images. Another condition of this method is that the correspondence between the point sets must be known. They confirmed that their method can be used to compute the distance between images under different conditions, including cases where the images are treated asymmetrically. Generally, shape feature can be used as similarity measurement between two images with clutter environment, complex objects and transformation. For each way to construct the feature, the similarity measurement and the tool for performing are also adjusted to gain the highest result relying on subjectivity of authors. Therefore, there is no standard for constructing feature and similarity measurement.

2.3 Correspondence-based contour matching

Correspondence-based contour matching works with a point-to-point matching between a model contour and a target contour to evaluate similarity measures (Figure 2.5). The correspondence between each pairwise point is calculated before proceeding with the matching. A huge number of research were done on contour correspondence. The existing techniques can be classified into two categories, conventional and heuristics. Many metrics have been developed. Hausdorff distance assigns to each point of one set the distance to its closest point in the other set and takes the maximum over all these values (Kirchberg *et al.*, 2006). It is used as common metric even if it is sensitive to noise and outliers and is not invariant to translation, scale and rotation. Considering this last fact, the matching may be computationally difficult and the shapes need to be compared with different rotation, scale and position.

Another approach based on point-to-point correspondence has been presented by Belongie *et al.* (2006) known as shape context (Figure 2.6). For each contour point, a histogram map representing the quantized length and orientation of the vector departing from that point to the others is created. Such histograms are then put into log polar space and joined to form the context of the shape (Figure 2.7). The matching between shapes is realized by matching the context maps. This can be formulated as a Quadratic Assignment (QA). It is one of the NP-hard problems, thus heuristic techniques are convoked to find optimal solutions for the correspondence.

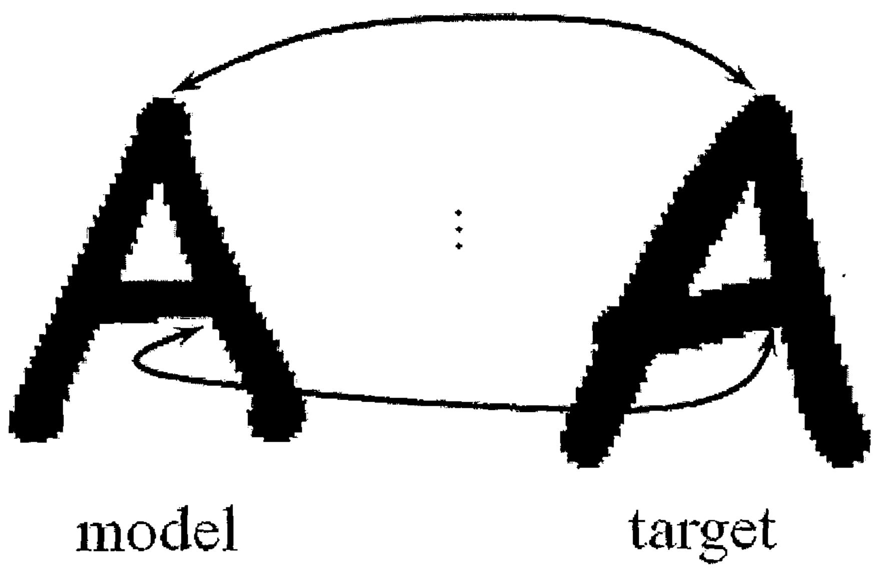

Figure 2.5: Finding correspondence between points on shape (Kaick *et al.*, 2007)

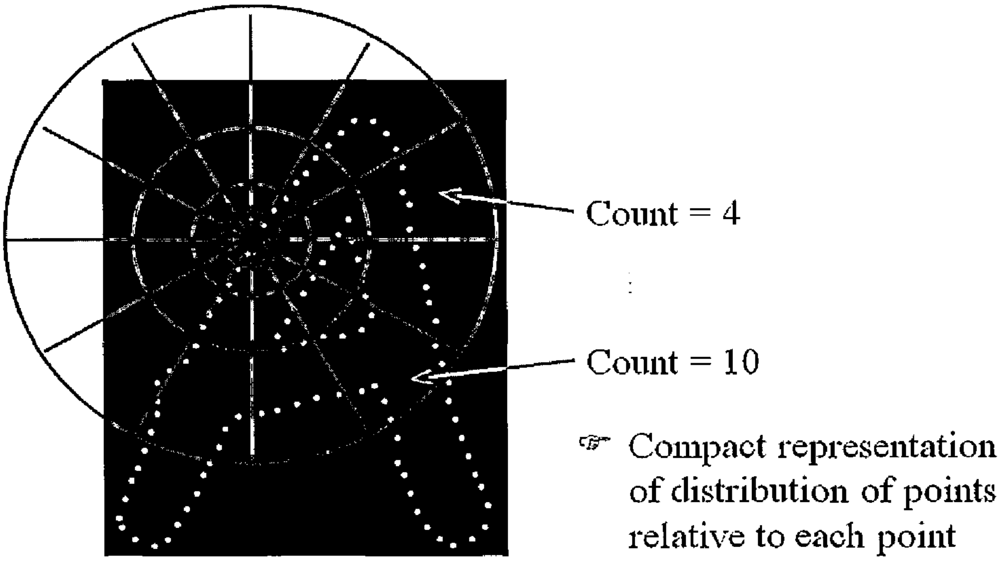

Figure 2.6: Designing shape context descriptor along the contour (Kaick *et al.*, 2007)

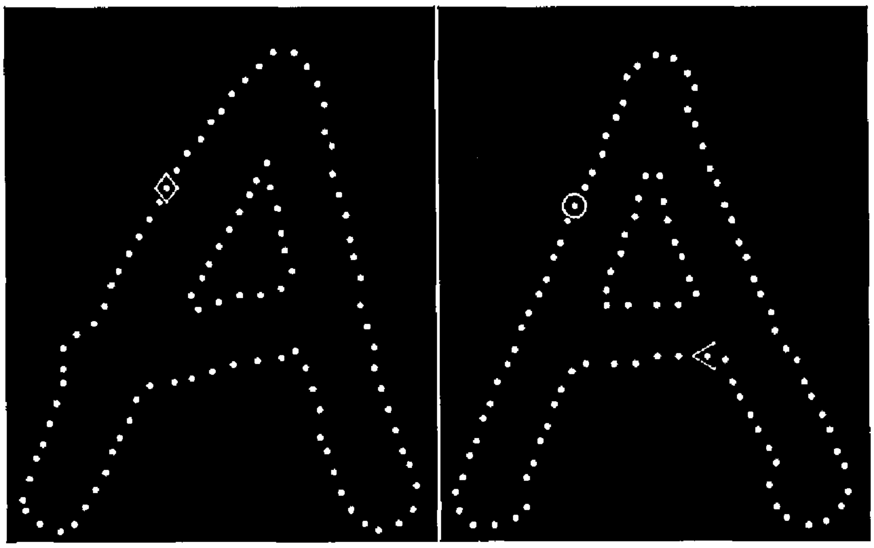

Figure 2.7: Comparison of point sets (Kaick *et al.*, 2007)

A classification based on methods with aligning transformation and methods with direct transformation without aligning has been introduced by Kaick *et al.* (2011). It consists of 5 classes which are discussed below.

2.3.1 Transformation and alignment search

The correspondence method in this class is based on two phases. In the first phase, a transformation for aligning the shape is proceeded, and then the correspondence is applied. In this approach the correspondence is derived from the proximity of the aligned points. There are three categories of alignment, i.e., rigid alignment, piecewise rigid alignment and non rigid alignment.

The rigid alignment relies on the fact that the transformations used for alignment can be derived from a small set of sample points. For example, if we consider a rigid transformation between two point sets in 3D, its parameters (given by a rotation matrix and a translation vector) can be derived from an initial configuration of three points and their transformed positions. After sampling a transformation, the algorithm can either verify the quality of the alignment or vote on the transformation.

For the piecewise rigid alignment, majority of the methods consist of using one global transformation to match two shapes. A different class of methods generalizes this idea by applying transformations to local portions of the shapes (Kaick *et al.*, 2011). Chang and Zwicker (2008) introduced a technique where the transformations are applied in a piecewise rigid manner to establish a correspondence between articulated shapes.

Non-rigid alignment has been described by Allen *et al.* (2003), Sumner *et al.* (2004) and Pauly *et al.* (2005). Different transformations are assigned to each vertex on the shape. The problem is formulated as finding the best transformation that brings each vertex in a reference shape close to its counterpart in the target shape, and a regularization term is added to enforce the similarity of transformations across neighbouring vertices. The difficulty in this setting is avoiding solutions that are local minima. This is achieved by initializing the methods with a set of corresponding marker points and solving the optimization in a multi-level fashion. The optimization can be posed as a nonlinear least squares problem and solved with a Newton-based method (Styner *et al.*, 2003).

2.3.2 Correspondence search

The characteristic of the methods of this class is that they work primarily with the pairwise assignments between feature points without searching for transformations that align the shapes (Kaick *et al.*, 2011). The correspondence problem is typically posed as optimizing an objective function. The objective is based on the quality of pairwise assignments, and the compatibility between pairs of such assignments. The solution is found by using well-known discrete or continuous optimization methods. A special group of methods in discrete optimization utilize a tree-based search to explore the solution space (Paulsen *et al.*, 2003).

If the objective being optimized is only composed of a similarity term, then the formulation becomes a Linear Assignment Problem (LAP). This simplified objective can be solved by the simplex algorithm, since it is a special case of a linear program (Papadimitriou *et al.*, 1982). However, if the correspondence is constrained to a one-to-one mapping, the problem becomes that of finding an optimal matching in a weighted bipartite graph, which can be solved more efficiently by the Hungarian algorithm (Kaick *et al.*, 2007).

On the other hand, if the objective comprises both the linear and quadratic terms, we arrive at a Quadratic Assignment Problem (QAP), which is known to be NP-hard (Kaick *et al.*, 2011). Several techniques have been proposed to compute approximate solutions to this problem. One group of methods poses the problem as an integer optimization, which is relaxed to the continuous setting and solved with a continuous optimization technique. Examples include the soft-assign technique (Gold *et al.*, 1995), concave programming (Maciel *et al.*, 2003), approximations based on linear programming (Berg *et al.*, 2005), and spectral clustering (Leordeanau *et al.*, 2005). It can also be formulated in probabilistic terms and solved as a convex optimization problem (Zass *et al.*, 2008).

Another group of methods solves the problem by computing an optimal labeling of a graph. For example, the problem can be posed in terms of a Markov network where the set of labels corresponds to matching points on the target shape (Anguelov *et al.*, 2004). Other methods make use of metaheuristics for combinatorial optimization such as Ant Colony Optimization (Kaick *et al.*, 2007). One more option is to sample the space of correspondences in search of a solution, guided by geodesic distances and importance sampling (Tevs *et al.*, 2009).

2.3.3 ICP and variants

This class is characterised mainly by the Iterative Closest Point (ICP) method, which iteratively computes a correspondence by alternating between two steps. In the first step, the method searches for an alignment between the shapes. In the second step, a correspondence is derived from the alignment. Finally, this procedure is reiterated by using the correspondence to estimate a new aligning transformation. Thus, we call it a hybrid search method because it searches for both alignment and correspondence solutions, which in turn, affect each other. The different variants of the ICP algorithm are obtained when the two steps are solved in different manners (Rusinkiewicz *et al.*, 2001).

2.3.4 Use of embeddings

The non-rigid alignment of shapes, especially that of articulated shapes, can also be accomplished by first embedding the shapes in a space where the configuration of the rigid parts is normalized, and then treating the problem simply as a case of rigid alignment in this embedding space (Elad *et al.*, 2003). The rigid alignment can then be obtained by any of the methods discussed previously. The key to create such embedding is to obtain an intrinsic representation of the shape which is invariant to bending, and then utilize this representation to embed the shape in a new ambient space, so that the intrinsic geometry of the shape is translated into its extrinsic geometry in this new space. This embedding can be obtained with techniques such as Multi-Dimensional Scaling (MDS) (Bronstein *et al.*, 2006, 2008), or the spectral transform (Jain *et al.*, 2007; Mateus *et al.*, 2008).

2.3.5 Partial correspondence

The main problem in this class is defined as finding a subset of shape elements for which a meaningful correspondence can be computed, as opposed to finding a full correspondence that could include additional parts or features which do not exist in both shapes. This task composed of two sub problems: searching for an optimal subset of k feature points that match consistently, and finding the correspondence between these k elements (Zass *et al.*, 2008). One approach to determine the subset is to examine the objective function in search of sharp increases in the alignment error, which appear when an outlier point is added to the set of matched points (Zheng *et al.*, 2006). Alternatively, an estimate on the number of outlier features can be provided to the optimization, which limits the number of points that appear in the computed correspondence (Maciel *et al.*, 2003).

2.4 Significance of correspondence search based on heuristic approaches

Generally speaking, a heuristic is a method that achieves good (but not necessarily optimal) results at low expense. However, which results are to be considered good heavily depends on the application at hand.

Analyzing the contour shape (see Figure 2.8), we can figure out the points along the contour as a bipartite graph, which can be formulated as a Quadratic Assignment. This is one of the NP-hard problems. Finding an optimal solution for such problem seems difficult by using conventional methods. Heuristic methods are used to find an acceptable solution for such cases.

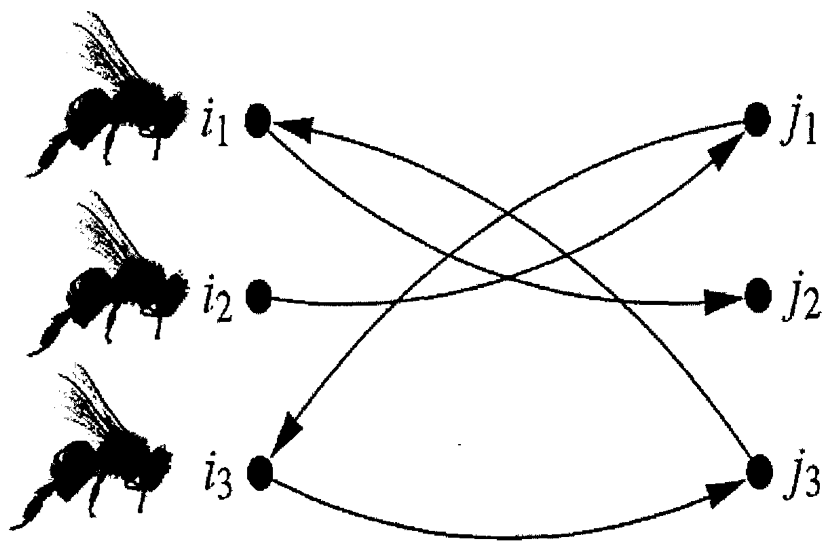

Figure 2.8: Heuristic matching model based Bee Colony.

It might seem surprising to mention the Hungarian method (Papadimitriou *et al.*, 1982), as one of the well studied methods in this context. In fact it is based on solving the complexity which is a part of the combinatorial optimization solved by using ICP scheme (Ehrgott and Gandibleux, 2004). Following the requirements of the applications, many version of heuristic approaches have been proposed (Lakamper and Sobel, 2008). A review about correspondence introduced by Kaick *et al.* (2011) clearly shows the importance of heuristic methods in solving complexity related to shape matching issues.

This does not mean that heuristic methods might be useful for all matching applications, but it means: firstly, the applicability of such methods is only related to the domain of application, for example, sometimes in image registration it is enough to get an approximated matching to obtain the abnormalities; secondly, the applicability depends on the requirements of the matching, for example, in some cases considering proximity between the points along the contour is compulsory which allow the usage of heuristic methods as a key for the solution.

2.5 Ant Colony Optimization (ACO)

Ant Colony Optimization is a heuristic approach based ants (see Figure 2.9). It is used to find approximate solutions to difficult optimizations. The inspiring source of ACO is the pheromone trail laying behaviour of real ants, which use pheromone as a communication medium (Dorigo and Thoms, 2004). By analyzing the biological prototype example, ACO is modelled by using indirect communication of a colony of simple agents, called artificial ants, mediated by artificial pheromone trails. The trail values of these pheromone are modified at runtime based on a problem-dependent heuristic function and the amount of pheromone deposited by the ants while they traverse between their colony and a food source (Dorigo and Thomas, 2004).

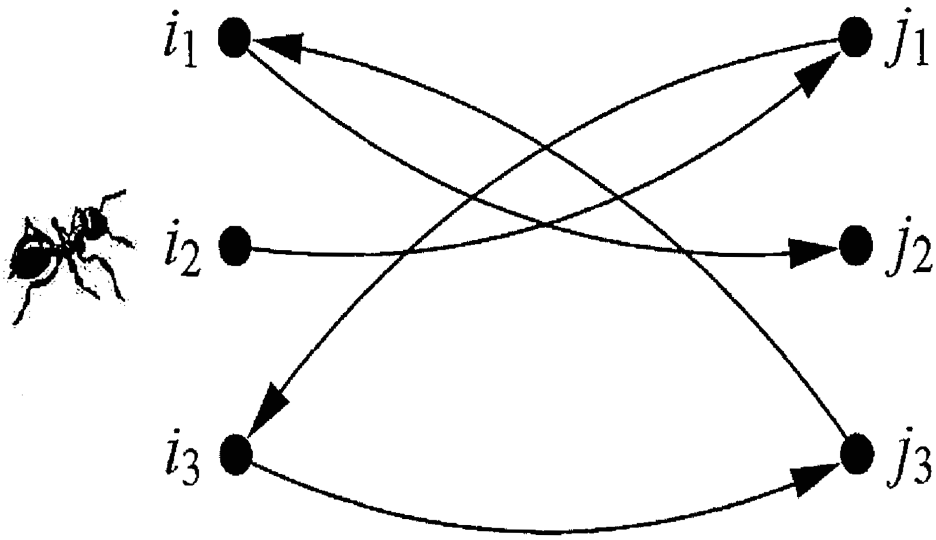

Figure 2.9: Ant Colony Optimization model.

2.6 Ant Colony Optimization (ACO) matching

Kaick *et al.* (2007) described a matching algorithm based on Ant Colony Optimization; it consists of considering the proximity measured between feature points on the same shape. The matching is formulated as two sets of points (see Figure 2.10) I and J where the ants cross these two sets doing a complete tour. All the movements produce a collection of possible paths between the two sets. During the building of these paths, ants release the pheromone with different amounts for each possible path. A bigger amount of pheromone on a path means that it is more eligible in terms of cost of correspondence. The traversing from a vertex $i \in I$ to a vertex $j \in J$ is given by the following equation:

$$p_{ij}^{k} = \frac{\alpha\tau_{ij}+(1-\alpha)\eta_{ij}}{\sum_{l\in N_i}[\alpha\tau_{il}+(1-\alpha)\eta_{il}]} \qquad (2.1)$$

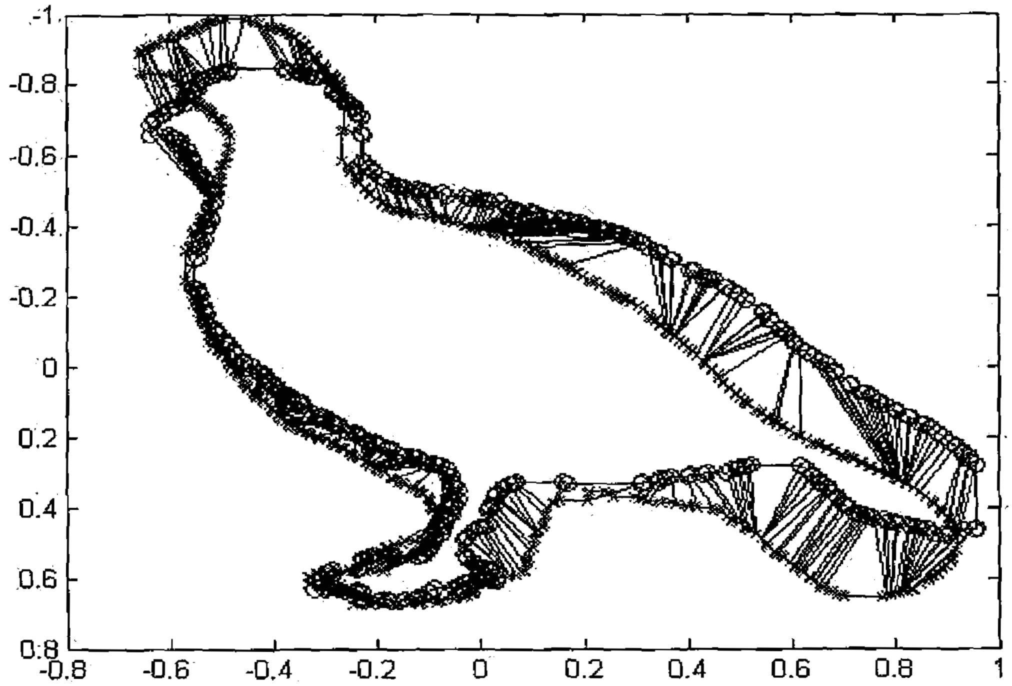

Figure 2.10: A matching computed by ACO.

The pheromone accumulated on the edge (i, j) is quantified by τ_{ij}, η_{ij}, indicating the desirability (or probability) of traversing (i, j) based on heuristic information, $N_i = \{l \in J : (i, l) \in E\}$ is the immediate neighbourhood of vertex i. The parameter $0 \leq \alpha \leq 1$ regulates the influence of pheromones over heuristic information. After a complete tour of the ants, an ACO iteration, the cost of solutions is computed as defined in sec. Pheromones are updated at the end of ACO iteration. First, pheromones are evaporated at n constant pheromone rate (Kaick *et al.*, 2007) $\rho, 0 \leq \rho \leq 1$.

Pheromone evaporation:

$$\tau_{ij} \leftarrow (1 - \rho)\tau_{ij} \tag{2.2}$$

where ρ is the pheromone evaporation rate. The new pheromone deposition on the edges that were traversed by the ants is regulated by pheromone deposition:

Pheromone deposition:

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \quad (2.3)$$

where $\Delta\tau_{ij}^{k}$ is the amount of pheromone that an ant k has deposited on the edge(i, j).

Cost function:

The formulation of the problem is done by a QAP. When augmenting the shape descriptor R with proximity information, the general objective function is in the form:

$$\mathrm{QAP}(\pi, R, I, J) = (1 - \nu)S(\pi, R, I, J) + \nu\chi(\pi, I, J) \quad (2.4)$$

where $0 \leq \nu \leq 1$ is used to control parameter between S and the proximity χ, and the arguments I, J represents the two points sets, π is a mapping for every point, I corresponds a point in J, and R is the set of shape descriptors. The forms S and χ were detailed by Kaick *et al.* (2007).

Figure 2.7 shows a matching computed by ACO. We can see the huge number of points along the contours incorporated in the correspondence, which makes the matching more complex. Furthermore, this scheme cannot allow us to compute the correspondence intuitively without considering noise effect and distortions.

A solution has been proposed previously (Ruberto and Morgera, 2011) based on dominant point approach. However, this technique does not take into consideration the human judgment. It only consists of controlling the number of dominant points on the resulted contour by using Genetic Algorithms.

2.7 Previous work on correspondence

A detailed survey of shape correspondence was proposed by Kaick *et al.*, (2011). As explained previously in Chapter 1, finding a meaningful matching between shapes requires a meaningful correspondence (Hong and Soatto, 2005). There are many different approaches to build a contour shape matching. Most of them consist of identifying the points along the contour shape by using specific descriptors and then computing the contour correspondence. In some cases, a form of transformation is required (Ruberto and Morgera, 2011) and to calculate the degree of similarity between shapes, a suitable metric is selected.

Motivated by problems from image analysis, many shape matching algorithms have been proposed (Shwartz *et al.*, 1987; Leitao *et al.*, 2002; Chui *et al.*, 2003; Yanjuan *et al.*, 2004; Rongjiang *et al.*, 2005; Bronstein *et al.*, 2006). The majority of these techniques can often be directly applied to contour correspondence without considering the order between contour points, which may affect the matching performance (Kaick *et al.*, 2007).

One of the fundamental ways to find correspondence contours is to compute the similarity between the elements of each contour (Liu *et al.*, 2004). Gold *et al.* (1998) provided an algorithm that works on noisy 2D and 3D shapes using a combination of optimization techniques to find a good suboptimal solution. This technique is fast and robust to noise and thus became a reference to many optimization methods for a long time.

Maciel and Costeria (2003) proposed a method consists of formulating the selected features as an integer optimization, and then a concave build function is used to find global optimal solution to the formulated problem. The main feature of this method is that it can use any criterion provided and it can be translated into functions with continuous second derivatives.

Zheng and Doermann (2006) proposed a technique based on shape context consists of maximizing the preservation of binary neighbourhood. For information during matching, each point is considered as a node in a simple graph, and two nodes are connected if they are neighbours. The optimal match is the one with the maximum number of matches.

Berg *et al.* (2005) proposed an algorithm based on QAP formulation, it is solved by using two-step method where the cost function has terms based on similarity of corresponding geometric blur point descriptors as well as the geometric distortion between pairs of corresponding feature points.

Scott and Nowak (2006) proposed a method based on enforcing point ordering. The cost of each possible pairing of points is computed and then the assignment problem is solved using dynamic programming.

Lakaemper *et al.* (2008) proposed a method based on particle filter. It is used to establish correspondence between 2D point sets representing shapes. The advantage of this method is that the global constraints can be learned.

Using the dynamic programming, Liu *et al.* (2004) reported an excellent matching based on discarding and skipping small features introduced by noise which give more credibility to the correspondence. The method is fast, invariant to the geometric transformations and feature preserving.

The only similar work to the proposed approach in this study was done by Ruberto and Morgera (2011). A dominant point scheme based on genetic algorithm is applied on the shape contour before proceeding with the matching using ACO. Furthermore, Ruberto and Morgera have modified shape context to be invariant to rotation. The experimental results demonstrate the effectiveness of the framework.

To enhance the context, it is very important to highlight some popular methods based on transformation techniques. Sclaroff and Pentland (1995) introduced a modal matching based on the idea of describing objects in terms of generalized symmetries. In contrast to previous methods, which are based on the correspondence of shapes elements, modal matching utilizes a finite element formulation that allows for an object's eigenmodes to be computed directly from available image information.

Jain *et al.* (2007) proposed a hybrid algorithm for finding a meaningful correspondence between triangle meshes. Based on the geodesic affinities, the meshes are transformed into the spectral domain and then the spectral embeddings are matched after verifying suitable steps to ensure a coherent ordering. Many conventional methods can be applied for the resulting spectral domain to compute the correspondence.

Finally, another approach consists of finding correspondence without defining local shape descriptors is used in many methods. Siddiqi *et al.* (1999) proposed a method based on shock graphs. It consists of derivation of structural structures from the shocks. A tree matching algorithm is constructed in order to find the best set of corresponding nodes between shock's trees in polynomial time.