

# Equivalent Electrical Lumped Component Modeling of E-shaped Patch Flat Lens Antenna Unit Cell

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**Abstract**—An equivalent electrical lumped component modeling of E-shaped patch flat lens antenna unit cell was designed and analyzed in this paper. The proposed element model consists of two identical circuits connected by a coupling transformer. The lumped component parameters were determined analytically considering the physical structure of the unit cell at 4.2 GHz. The circuit was implemented and simulated using MULTISIM<sup>V10</sup> software. The transmission loss and reflection loss performance of the unit cell can be controlled by adjusting the values of inductance ( $L_p$ ) and capacitance ( $C_p$ ) at the resonance frequency. A low transmission loss of 2.8 dB and a large phase shift range of 320° were achieved. The theoretical results were compared with the CST Microwave Studio simulations and a good agreement was obtained.

**Keywords**—flat lens antenna; equivalent circuit model; aperture coupled; transmission loss; phase shift;

## I. INTRODUCTION

The development of compact, light weight and a high directivity flat lens antenna is an attractive choice for space applications. The ongoing development of wireless communication systems and digital radar for remote sensing creates a demand of developing high performance flat lens antennas. In spite of the several existing design approaches, a typical flat lens antenna configuration consists of an array of microstrip patches which are coupled or joined together by phase shifters [1].

The basic theory of operation of flat lens antenna unit cell is to collimate the feed spherical electromagnetic incident wave into planar wave front at the back of the aperture [2]. Therefore, the unit cell element must be designed to establish the needed phase adjustment. The required phase compensation value of the unit cell depends on the incident wave angle and its location on the surface of the array. The significant point of designing a unit cell for flat lens antenna is to obtain the essential requirement phase shift range of up to 360°.

The aim of this paper is to design an equivalent electrical lumped component model of flat lens antenna unit cell. This is to obtain the physical insight of the structure and to analytically determine the lumped element parameters while considering the element's physical characteristics. In the related literature, an equivalent electrical model of several

rectangular patch antennas loaded by different slot configurations are proposed [3-5]. An equivalent circuit model of discrete lens antenna unit cell patches connected by metalized via is also presented [6].

The main contributions of this paper are to design and simulate an equivalent circuit model for E-shaped flat lens antenna unit cell based on aperture-coupled patches. The organization of the rest of this paper is as follows: In section II, The electrical modeling of the unit cell is presented, followed by the unit design configuration in section III. The simulation results and comparisons of this work are discussed in section IV. Lastly, the conclusion of the work is given in section V.

## II. ELECTRICAL MODELING OF THE UNIT CELL

To achieve the required phase compensation of flat lens antenna unit cell, E-shaped patches are proposed and designed as sketched in Fig.1. The adjustable slot length on the patch is the key parameter that controls the phase of the unit cell. To obtain a physical insight of the unit cell, an equivalent lumped-element electrical model of the E-shaped patches for the unit cell was designed and simulated. The proposed model consists of two resonators connected by a coupling transformer model.

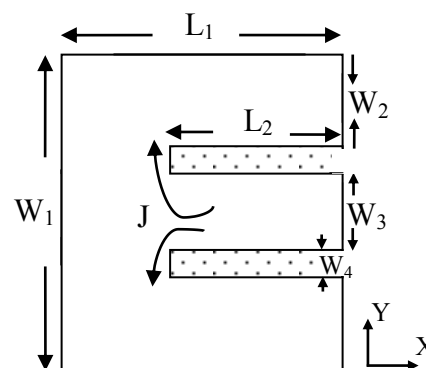


Fig. 1. E-shaped patch for flat lens antenna unit cell

The equivalent electrical modeling of ordinary rectangular patch antenna can be designed as parallel RLC components, which are based on cavity model as shown in Fig. 2(a) [6, 7]. It is shown in Fig. 2(b) that the current distribution is at its

maximum in the center of the patch and evenly spread on the corners as the electric field is excited in the Y-direction.

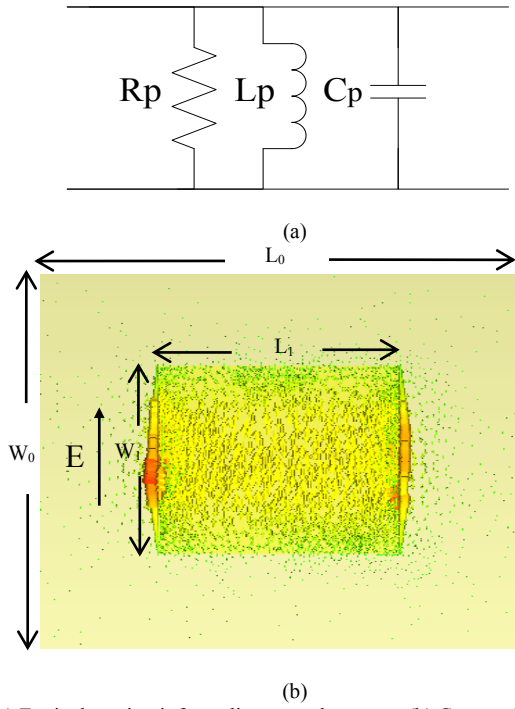
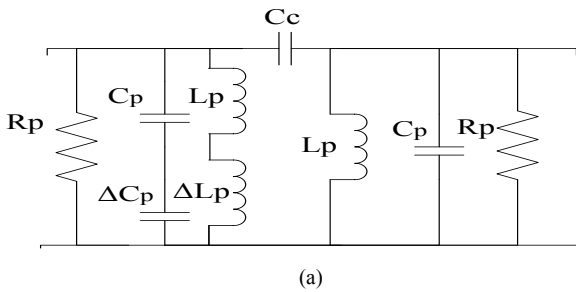
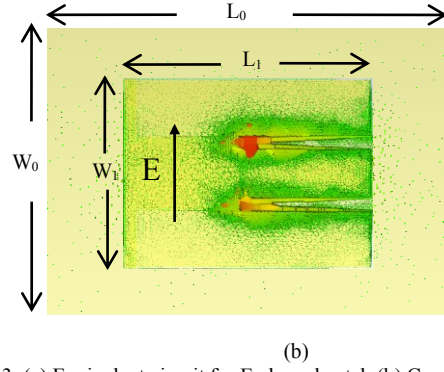


Fig.2. (a) Equivalent circuit for ordinary patch antenna (b) Current distribution of patch antenna

However, when two parallel slots are embedded into the patch, the current distribution changes. Hence, the resonant feature of the patch is also altered. As shown in Fig.3 (b) the E-shaped patch has three arms which affect the current distribution on the surface of the patch. In the center arm the current flows normally and represents the ordinary RLC equivalent circuit but for the two side arms the current has to bend around the slot and this creates longer current path as illustrated in Fig 1. This increment of current path results an additional series inductance ( $\Delta L_p$ ) [4]. The slot also contributes to additional series capacitance ( $\Delta C_p$ ) as shown in Fig 3(a). Therefore, the equivalent circuit model of the E-shaped patch unit cell has two resonators which are connected by a coupling capacitor ( $C_c$ ).



(a)



(b)

Fig.3. (a) Equivalent circuit for E-shaped patch (b) Current distribution of E-shaped patch

As the proposed flat lens antenna unit cell consists of two back-to-back E-shaped patches, the equivalent circuit of the unit cell is also comprises of two identical circuits linked by a coupling transformer as sketched in Fig 4.

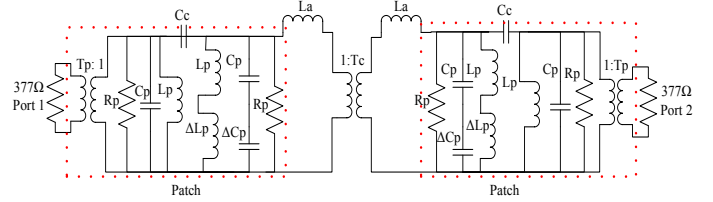


Fig.4. Equivalent circuit model of E-shaped patch unit cell

The equivalent circuit parameters are analytically determined as follows:

The value of  $L_p$  and  $C_p$  can be defined as [4, 8].

$$C_p = \frac{\epsilon_e \epsilon_0 L_1 W_1}{2h} \text{ F} \quad (1)$$

$$L_p = \frac{1}{(2\pi f_r)^2 C} \quad (2)$$

Where  $L_1$  and  $W_1$  represents the length and width of the patch respectively,  $f_r$  is the operating frequency of the unit cell,  $\epsilon_e$  is the effective dielectric constant,  $\epsilon_0$  is free space permittivity,  $h$  is the thickness of the substrate and  $y_0$  is the distance from the source to the patch and  $F = \cos^{-2}\left(\frac{\pi y_0}{L_1}\right)$ .

The slot inductance can be calculated by [9].

$$\Delta L = \frac{Z_1 + Z_2}{16\pi f_r F} \tan\left(\frac{\pi f_r L_2}{c}\right) \quad (3)$$

In this equation,  $Z_1$  and  $Z_2$  are the characteristic impedances of the E-shaped patch wings, which represents as microstrip lines of widths  $W_2$  and  $W_3$  respectively.

Here,  $Z_1$  and  $Z_2$  can be determined by the following equations:

$$Z_1 = \frac{120\pi}{\left(\frac{W_2}{h} + 1.393 + 0.667 \ln\left(\frac{W_2}{h} + 1.444\right)\right)} \quad (4)$$

$$Z_2 = \frac{120\pi}{\left(\frac{W_3}{h} + 1.393 + 0.667 \ln\left(\frac{W_3}{h} + 1.444\right)\right)} \quad (5)$$

Where the value of  $W_2 = \frac{1}{2}(W_1 - W_3) - W_4$  and  $W_3 = W_1 - 2W_2 - 2$ .

The slot capacitance ( $\Delta C_p$ ) can be calculated as a gap capacitance between the center arms and side arm of the E-shaped patch with length of  $L_2$  [10]

$$\Delta C_p = 2L_2 \frac{\epsilon_0}{\pi} \left[ \ln \left( 2 \frac{1+\sqrt{k'}}{1-\sqrt{k'}} \right) + \ln \coth \left( \frac{\pi W_4}{4h} \right) + 0.013 C_f \frac{h}{W_4} \right] F \quad (6)$$

$$\text{Where } k' = \sqrt{1 - k^2} \quad \text{and } k^2 = \frac{1 + \frac{W_2}{W_4} + \frac{W_3}{W_4}}{\left(1 + \frac{W_2}{W_4}\right) \left(1 + \frac{W_3}{W_4}\right)}$$

The resistance ( $R_p$ ) of the circuit can be determined as [11]

$$R_p = \frac{1}{2(G_1 + G_{12})} \cos^2 \left( \frac{\pi y_0}{L_1} \right) \quad (7)$$

$$G_1 = \frac{1}{120\pi^2} \int_0^\pi \left[ \frac{\sin(k_{1,2})(w_1 \cos(\frac{\theta}{2}))}{\cos\theta} \right]^2 (\sin\theta)^3 d\theta$$

$$G_{12} = \frac{1}{120\pi^2} \int_0^\pi \left[ \frac{\sin(k_{1,2})(w_1 \cos(\frac{\theta}{2}))}{\cos\theta} \right]^2 J_0(k_0 l \sin\theta) (\sin\theta)^3 d\theta$$

In these equations,  $k_{1,2}$  represents number of waves at the center arm and side arms with resonance frequencies of  $f_r$  and  $f'_r$  respectively, and can be calculated as:

$$f_r = \frac{1}{\sqrt{L_p C_p}}, \quad f'_r = \frac{1}{\sqrt{L' C'}}$$

$$\text{Where } L' = L_p + \Delta L_p \quad \text{and } C' = \frac{C_p \Delta C_p}{C_p + \Delta C_p}$$

$$\text{While } k_1 = \frac{2\pi f_r}{c_0} \quad \text{and } k_2 = \frac{2\pi f'_r}{c_0}$$

The coupling capacitance ( $C_c$ ) value of the circuit can be determined as [12]

$$C_c = \frac{-(C_p + C') + \sqrt{((C_p + C')^2 - 4C_p C' \left(1 - \frac{1}{C_{pf}^2}\right))}}{2} \quad (8)$$

The radiation resistances of the three coupling transformers are considered and calculated in the transforming ratios ( $T_p$  and  $T_c$ ). The equation for the turn ratio of aperture coupled transformer ( $T_c$ ) has been confirmed from the investigation carried out in [13] and can be calculated as  $T_c = \frac{L_2}{2W_1}$  (9)

where  $L_2$  is the slot length and  $W_1$  is patch width.

For the transforming ratio ( $T_p$ ) of both coupling transformers in port 1 and port 2 of the circuit can be determined as [3, 14],

$$T_p = \frac{R_r}{\eta_0} \quad (10)$$

All the calculation values of the equivalent circuit parameters are summarized in Table I.

Table I. Equivalent circuit component values at 4.2 GHz

Parameter	Value
$C_p$ (pF)	3.1
$L_p$ (nH)	0.45
$R_p$ ( $\Omega$ )	80
$\Delta L_p$ (nH)	0.2
$\Delta C_p$ (pF)	0.92
$C_c$ (pF)	2.2
$T_p$	0.2
$T_c$	0.5
$R_r$ ( $\Omega$ )	77
$y_0$ (mm)	7.2
$L_a$ (pH)	0.002

### III. UNIT CELL DESIGN CONFIGURATION

In this work the unit cell dimensions are organized in a square shape with the side length of  $L_0 = 36$  mm which is equivalent to  $0.5\lambda_0$  at 4.2 GHz. The rectangular patch dimensions ( $15.5 \times 16.75$  mm<sup>2</sup>) are optimized for a better performance. The unit cells were designed on a standard FR4 epoxy substrate having relative permittivity ( $\epsilon_r$ ) of 4.3, dielectric loss tangent ( $\tan \delta$ ) of 0.02 and physical thickness of 1.6 mm. The unit cell physical parameters and main features are summarized in Table II.

Table II. Important parameters of the unit cell

Parameter	Value
Design frequency	4.2 GHz
Unit cell size	36mm x 36 mm
Patch size	$L_1=16.75$ mm, $W_1=15.5$ mm
Thickness of the cell	$t=3.305$ mm
Substrate	FR4: $h=1.6$ mm, $\epsilon_r=4.3$
Slot dimensions	Adjustable

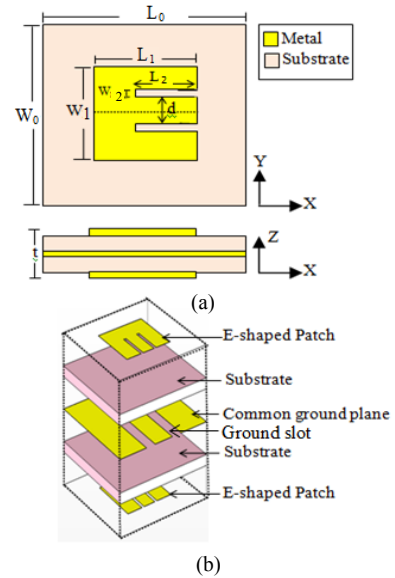


Fig.5. Unit cell design configurations (a) E-shaped patch unit cell. (b) Exploded diagram of the element.

#### IV. DISCUSSION OF SIMULATION AND COMPARISONS

The equivalent circuit was implemented and simulated using MULTISIM<sup>V10</sup> software. A network analyzer was connected to the circuit to collect the scattering parameters of the unit cell. To validate the equivalent circuit simulation results, the flat lens antenna unit cell has been simulated and analysed by using commercially available CST microwave studio. An infinite periodic boundary condition (PEC and PMC) are employed.

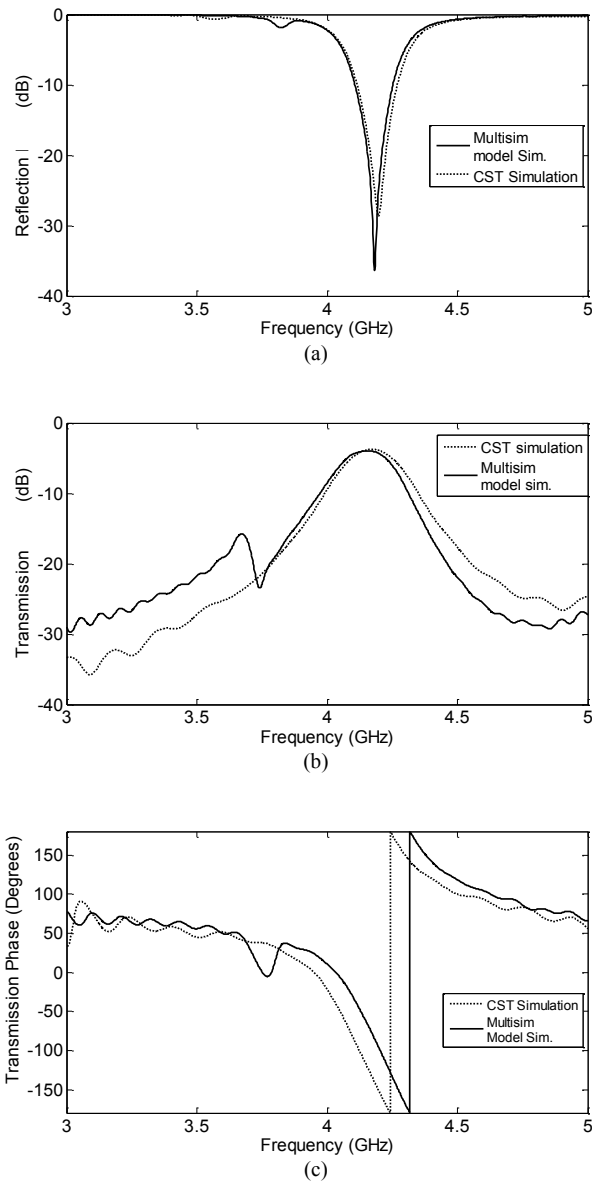


Fig. 6. S-parameters of the unit cell (a) Reflection Loss (dB), (b) Transmission Loss (dB), and (c) Transmission Phase (deg).

#### V. CONCLUSIONS

An equivalent circuit modeling of E-shaped flat lens antenna unit cell was designed and simulated. The lumped component values were determined analytically considering the physical structure of the unit cell at 4.2 GHz. The theoretical results were compared with the CST Microwave Studio simulations and a good agreement was obtained. More detailed results will be discussed in the presentation.

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