

Review on Fluid Structure Interaction Solution Method for Biomechanical Application

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Abstract. Fluid-Structure Interaction engages with complex geometry especially in biomechanical problem. In order to solve critical case studies such as cardiovascular diseases, we need the structure to be flexible and interact with the surrounding fluids. Thus, to simulate such systems, we have to consider both fluid and structure two-way interactions. An extra attention is needed to develop FSI algorithm in biomechanic problem, namely the algorithm to solve the governing equations, the coupling between the fluid and structural parameter and finally the algorithm for solving the grid connectivity. In this article, we will review essential works that have been done in FSI for biomechanic. Works on Navier–Stokes equations as the basis of the fluid solver and the equation of motion together with the finite element methods for the structure solver are thoroughly discussed. Important issues on the interface between structure and fluid solvers, discretised via Arbitrary Lagrangian–Eulerian grid are also pointed out. The aim is to provide a crystal clear understanding on how to develop an efficient algorithm to solve biomechanical Fluid-Structure Interaction problems in a matrix based programming platform.

Introduction

Nowadays, fluid–structure interaction models are increasingly used in biomedical engineering applications. The one of challenging fluid–structure problems that can be found in the human body involves the cardiovascular system. However, the widely studied are cardiovascular and heart valve disease [1]. Solving and measuring these diseases in a direct way is impossible because they engages with complex geometry and flow thus to investigate problem involved in the complex interaction between fluids and solids, numerical techniques may be used to assess them [2,3].

This paper extensively reviewed FSI computational techniques based on numerical methods to solve FSI problems involving interaction between fluids and solids for biomechanical application. Basically, the numerical procedures to solve these FSI problems may be classified into two approaches: the monolithic approach and the partitioned approach. The monolithic approach treats the fluid and structure dynamics in the same mathematical structure to create a single system equation for the whole problem, which is solved simultaneously by a unified algorithm. The interfacial conditions are implicit in the solution procedure. On the other hand, the partitioned approach treats the fluid and the structure as two computational fields which can be solved separately with their respective numerical algorithm and mesh discretization. The interfacial conditions are used explicitly to communicate information between the fluid and structure solutions [4,5]. Therefore, the best approach that will be used in solving problem for biomechanics application is partitioned approach. This is due to the complex geometry and flow in the human body.

A further general classification of the FSI solution procedures is based upon the treatment of meshes: the conforming mesh methods and non-conforming mesh methods. The conforming mesh method consider the interface conditions as physical boundary conditions, which treat the interface location as part of the solution, and requires meshes that conform to the interface. Owing to the

movement and/or deformation of the solid structure, re-meshing (or mesh-updating) is needed as the solution. In contrast, the non-conforming mesh methods treat the boundary location and the related interface conditions as constraints imposed on the model equations so that non-conforming meshes can be utilized [2]. As a result for the problem involving the biomechanical application, the non-conforming mesh method is suitable to apply. This is because the fluid and solid equations for this method can be conveniently solved independently from each other with their respective grids, and re-meshing is not necessary.

An important consideration when simulating fluid flow problem by any numerical method is the choice of an appropriate kinematical description and this selection has three distinct types of description of motion: 1) the Lagrangian description, 2) the Eulerian description and 3) the ALE description. Lagrangian algorithms are mainly used in structural mechanics where each individual node of the computational mesh follows the associated material particle during the motion. The Lagrangian description allows an easy tracking of free surfaces and interfaces between different materials. Its weakness is its inability to follow large distortions of the computational domain without recourse to frequent remeshing operations. Eulerian algorithms are widely used in fluid dynamics problems. This is different with the Lagrangian method where it does not suffer from mesh distortion, because the mesh is spatially fixed during the simulation. However, it is more difficult to take into account the history of the material, as the element mesh is not connected to it. A second problem is that it is difficult to obtain an accurate description of the free surface and interfaces between different materials or different media (e.g., fluid-fluid and fluid-solid interfaces) and the free surface must be described in the fixed element mesh [6]. The Arbitrary Lagrangian Eulerian (ALE) method is a combination of the best features of both the Lagrangian and the Eulerian approaches. Originally, the method was introduced in the context of the finite difference method [7]. The ALE method is employed with the finite element method in fluid mechanics and fluid dynamics [8,9] and in fluid structure interaction [10]. ALE algorithms are particularly useful in flow problems involving large distortions in deforming boundaries.

Besides that, the numerical techniques for simulating flows with moving boundaries can be classified in two broad categories: (1) moving grid methods, mainly the arbitrary Lagrangian–Eulerian (ALE) approach [11]; and (2) fixed grid methods, such as the immersed boundary (IB) method developed by [12]. ALE methods employ a grid that is adapted to and moves and deforms with the moving boundary. However, a limitation of the ALE approach stems from the fact that the mesh conforms to the moving boundary and as such it needs to be constantly displaced and deformed following the motion of the boundary. The mesh moving step could be quite challenging and expensive for complicated 3D problems. This situation is further exacerbated in problems involving large structural displacements for which frequent remeshing at each time step of the simulation. Owing to this limitation, the ALE approach is only valid to FSI problems involving relatively small structural displacements. Otherwise, in fixed grid approaches, the entire computational domain (including both the fluid and structure domains) is discretized with a single, fixed, non-boundary conforming grid system (most commonly a Cartesian mesh is used as the fixed background mesh). The effect of a moving immersed boundary (IB) is accounted for by adding forcing terms to the governing equations of fluid motion so that the presence of a no-slip boundary at the interface can be felt by the surrounding flow. Because of the fixed grid arrangement, such methods are inherently applicable to FSI problems involving arbitrarily large structural displacements [13]. So, to solve problems involving biomechanical applications, both methods are suitable to use but the selection of the method depends on the complexity of cases and the deformations of the structural displacements.

Lastly to develop the FSI algorithm we must clearly know about the governing equation that will be used in biomechanical applications. Therefore, to select the equation, the properties of each domain must be identified. In the human body, the properties of the fluid are mostly in laminar regime only. Due to this reason, Navier–Stokes equations are an efficient equation that will be used as the governing equation for the fluid domain [14]. On the other hand, several approaches are commonly used to derive the governing equations for the structure domain. Among them are Newton's

second law of motion, D' Alembert's principle, principle of virtual displacements and the principle of conservation of energy [15]. Particularly, this paper will review the solution procedures to develop Fluid Structure Interaction (FSI) algorithm in biomechanics application. Nevertheless, this FSI algorithm will be consist of fluid, particle and structure dynamics solvers in a matrix based programming platform.

The review is organised as follows. First, the governing equations are given, treating the equations for fluid and structure solver. Next, this followed by the constraints concerning the fluid-structure coupling and the discretisation that will be used to form FSI algorithm. Lastly, the discussion and conclusion about this review on FSI solution method for biomechanics application will be explained.

Governing equations

Based on the solution of method and technique discussed earlier, biomechanics applications are suitable to apply these governing equations and the details are described as follows.

Fluid solver. First, the governing equations that describe the fluid domain are the 3D, unsteady incompressible continuity and Navier–Stokes equations, which in compact tensor notation read as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2.1)$$

$$\frac{\partial u_i}{\partial t} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (2.2)$$

where u_i are the Cartesian velocity components, p is the pressure divided by the density ρ , and Re is the Reynolds number of the flow based on a characteristic length and velocity scale in biomechanics application. d/dt is the material derivative defined as:

$$\frac{\partial}{\partial t} (.) = \frac{\partial}{\partial t} (.) + u_j \frac{\partial}{\partial x_j} (.) \quad (2.3)$$

Structure solver. In this biomechanics application, 3D motions of rigid bodies will be considered. Generally, the rigid bodies and the system are assumed elastically mounted and have damping respectively. The algorithm is general and can be applied to multiple rigid bodies. For the sake of convenience but without loss of generality, however, the FSI formulation for a single rigid body will be applied. Under these assumptions, the Newton's equation of motion that governs the motion of the structural components can be formulated in the inertial frame of reference in generalized form as follows:

$$M \frac{\partial^2 Q_i}{\partial t^2} + C \frac{\partial Q_i}{\partial t} + K Q_i = F_i + F_i^{ext} \quad (2.4)$$

where Q_i are the components of the unknown Lagrangian vector $\mathbf{Q}(t)$ describing the position of the structure. For pure translational motion: Q_i is the Cartesian position vector ($Q_i \equiv X_i$) of the rigid body, M is the mass of the object, C is the damping coefficient, K is the spring stiffness coefficient, F_i and F_i^{ext} are the components of the force F and F^{ext} acting on the rigid body from the surrounding fluid and external sources, respectively. The forces that will be exist in biomechanics application such as blood and muscle contraction. For pure rotational motion, Q is the vector with components the relative angles of the rigid body ($Q_i \equiv X_i$), M represents the moment of inertia, and F_i and F_i^{ext} are the moment vectors around the rotation axis arising from fluid-induced and external forces, respectively.

Fluid–structure coupling. The fluid and structural dynamics are coupled together at the fluid–structure interface T by the following boundary conditions:

$$u_i = U_i = \frac{\partial X_i}{\partial t} \text{ at } T \quad (2.5)$$

Eq. (2.4) comprises a system of second-order ordinary differential equations. Numerically these equations are typically solved by first transforming them into a system of first-order ordinary differential equations as follows:

$$\frac{\partial Q_i}{\partial t} = \Phi_i \quad (2.6)$$

$$\frac{\partial \Phi_i}{\partial t} + \frac{c}{M} \Phi_i + \frac{K}{M} Q_i = \frac{F_i + F_i^{ext}}{M} \quad (2.7)$$

Discretisation. In tackling the issue include the biomechanics application, the fluid and solid domain need to be discretised. Thus, finite-element once of the method that will be used for the discretisations for the fluid and solid domain corresponding to fluid–structure coupling and the contact law. However, discretisation for the fluid and solid domain in biomechanics application will be divided into two types; 1) time discretization 2) space discretization.

Discussion and Conclusion

Today have seen a countless number of numerical methods developed for the simulation of FSI problem in biomechanical application. These developments are due to the demand from a wide range of scientific and engineering disciplines. Meanwhile, the fast improvement of computational powers has made large-scale FSI simulations possible and has facilitated many realistic applications of these numerical techniques. Owing to the multidisciplinary nature of FSI problems especially in biomechanical application, in this review, we have emphasized the numerical procedures used by various methods to treat the interface conditions between fluids and structures.

Firstly, we reviewed is based on the basic numerical procedures to solve FSI problems in biomechanical application where monolithic and the partitioned approach must to consider. As mention earlier, the monolithic approach treats the fluid and structure dynamics in single numerical system for the whole problem in biomechanics application. This approach can potentially achieve better accuracy for a biomechanics problem, but it may require substantially more resources and expertise to develop and maintain such a specialized code. Conversely, the partitioned approach treats the fluid and the structure as two computational fields which can be solved separately with their respective mesh discretization and numerical algorithm. This approach is to combine available disciplinary (i.e., fluidic and structural) algorithms and cut the code development time by taking advantage of the numerical algorithms that have been validated and used for solving many complicated problems in biomechanics application. As a result, a successful partitioned method can be used to solve FSI problem in biomechanical application with sophisticated fluid and structural physics. The challenge of this approach is to arrange the disciplinary algorithms to attain correct and effective fluid–structure interaction solution with negligible code adjustment [2].

Beside that, to solve FSI problem in biomechanical application, we prefer to use non-conforming mesh methods rather than conforming mesh methods owing to their simplicity, efficiency and flexibility, as well as their capability to handle complex flows and large structural deformations. The Arbitrary Lagrangian Eulerian (ALE) method is a combination of the greatest features of the Lagrangian and the Eulerian approaches. Therefore, ALE method is the suitable methodologies that will be utilized to take care of FSI problem in biomechanics application. The key idea in the ALE formulation is the introduction of a computational mesh which can move with a velocity independent of the velocity of the materials particles. With this additional freedom with respect to the Eulerian and Lagrangian descriptions, the ALE methods succeeds to a certain extend in minimizing the problem encountered in the classical kinematical descriptions, while combining at the best their respective advantages [7].

As a conclusion, the solution method for FSI algorithm in biomechanical application must consist of three major numerical components: 1) the algorithm for solving the governing equations in a domain; 2) the iterative strategy for coupling together between the fluid and structural domains; and 3) the algorithm for effectively solving the grid connectivity for FSI problem in biomechanical application.

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