Mode I Stress Intensity Factors for Slanted Cracks in Round Bars

A.E Ismail

Abstract – *This paper presents numerical study on the stress intensity factors (SIF) for slanted surface crack subjected to mode I loading. Tremendous amount of works can be found in discussing the normal crack under various types of loadings. According to literature survey, there is no significant information on the SIFs for slanted cracks available. Therefore, the purpose of this paper is to develop the slanted numerical crack model using ANSYS finite element program and to analyze the behavior of various type of geometries of slanted cracks. Since no SIFs for slanted cracks are available, then the present model is compared with the previous normal crack for the validation purposes. It is found that the present model is well agreed with the existing model. Several important parameters are used; crack aspect ratio, a/b, relative crack depth, a/D and slanted angle, θ. It is realized that such parameters played important roles in determining the SIFs where if a/b and a/D are increased, the SIFs are also increased. When the slanted angles are introduced, the SIFs decreased when compared with the normal cracks. Instead of mode I SIF, mode II are also induced and therefore affecting the structural reliability. However, mode II SIF is relatively insignificant compared with mode I SIF.*

Keywords: *Slanted Cracks, Round Bar, Mode I, Stress Intensity Factors, Surface Cracks*

Nomenclature

F_I	Mode I dimensionless stress intensity factor
F_{II}	Mode II dimensionless stress intensity factor
K_I	Mode I stress intensity factor
K_{II}	Mode II stress intensity factor
D	Circular bar diameter
R	Radius of the bar
E	Modulus Young
v	Poisson's ratio
a	Crack depth/Minor axis of semi-ellipse
b	Major axis of semi-ellipse
h	Crack width
x	Arbitrary point of the crack front
a/b	Crack aspect ratio
a/D	Relative crack depth
x/h	Normalized coordinate
σ_a	Axial stress
q_{ij}	Stress tensor
u_j	Displacement vector
w	Strain energy density
δ_{ij}	Kronecker delta
x_i	Coordinate axis
q	Crack extension vector
$\sigma_{k,j}^{aux}$	Stress auxiliary field
$\varepsilon_{k,l}^{aux}$	Strain auxiliary field
$u_{k,i}^{aux}$	Displacement auxiliary field

I. Introduction

The present of cracks in any mechanical components can have detrimental effects on the reliability and integrity during services. Crack normally formed due to several factors such as metallurgical or mechanical defects [1]. The formation of cracks significantly affected the structural performances. Linear elastic fracture mechanics is generally used to analyze the behavior of these cracks and it is has a great interest in the last several decades [2-5]. All works reported in [2-12] considered mainly on the normal or transverse cracks subjected to mode I loading. The solution of stress intensity factor (SIF) for various crack geometries are summarized by Murakami & Tsuru [4]. However, lacks of solutions of SIFs especially for slanted cracks are available. Carpinteri [5], Shin & Cai [6], Fonte & Freitas [7] and Ismail *et al.* [8] studied the transverse surface cracks in round bars and the behavior of such cracks are almost established. In other papers, transverse cracks are also subjected to combined loadings [9, 10]. In this paper, the solution of SIFs for slanted surface cracks is analyzed and discussed. The slanted surface crack is modeled using ANSYS finite element analysis program. There are four important parameters are considered such as crack aspect ratio, *a/b*, relative crack depth, *a/D*, slanted angle, θ and normalized coordinate, *x/h*. Then, the SIFs along the crack front are related with such parameters.

Manuscript received January 2007, revised January 2007

A.E Ismail

II. Calculating Stress Intensity Factors

The evaluation of J-integral around the crack tip is based on the domain integral method initially introduced by Shih *et al.* [11]. This integral formulation uses area integration for 2D and volume integration for 3D problems which offer much better accuracy than contour integral and it is also much easier to implement numerically. Eq. (1) represents the 2D domain J-integral taking accounts the absence of thermal strain, path dependent plastic strain, body forces occur within the integration area [12]

$$J = \int A\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i}\right]\frac{\partial q}{\partial x_i}dA \qquad (1)$$

where q_{ij} is the stress tensor, u_j is the displacement vector, w is the strain energy density, δ_{ij} is the Kronecker delta, x_i is the coordinate axis and q is referred to as the crack extension vector. The direction of q is similar with x-axis of the local coordinate specified at the crack tip and it is normally chosen as zero at nodes along the contour Γ. It is also a unit vector for all nodes inside Γ except the midside nodes and known as virtual crack extension nodes. For 3D problems, the principal is similar to the 2D problems. However, domain integral representation of the J-integral becomes volume integration [13].

Two approaches for calculating stress intensity factor (SIF) are available in ANSYS software, interaction integral method and displacement extrapolation method. The first method is used because much easier to implement numerically and it is also offers better accuracy and fewer mesh requirement. This method is similar to the domain integral method for J-integral evaluation describe previously. The discussion on the domain integral methods can be found elsewhere [13]. The interaction integral is defined as in Eq. (2).

$$I = -\int_V q_{ij}\left[\sigma_{k,l}\varepsilon_{k,l}^{aux}\delta_{i,j} - \sigma_{k,j}^{aux}u_{k,i} - \sigma_{k,j}u_{k,i}^{aux}\right]dv \Big/ \int_s \delta q_n ds \qquad (2)$$

where σ_{ij}, ε_{ij} and u_i are the stress, strain and displacement and $\sigma_{k,j}^{aux}$, $\varepsilon_{k,l}^{aux}$ and $u_{k,i}^{aux}$ are the stress, strain and displacement of the auxiliary field and q_i is the crack extension vector.

$$\sigma_{ij}^{aux} = \frac{K_I^{aux}}{\sqrt{2\pi r}}f_{ij}^{I}(\theta) + \frac{K_{II}^{aux}}{\sqrt{2\pi r}}f_{ij}^{II}(\theta) + \frac{K_{III}^{aux}}{\sqrt{2\pi r}}f_{ij}^{III}(\theta) \qquad (3)$$

$$u_{ij}^{aux} = \frac{K_I^{aux}}{2\mu}\sqrt{\frac{r}{2\pi}}g_j^{I}(\theta,v) + \frac{K_{II}^{aux}}{2\mu}\sqrt{\frac{r}{2\pi}}g_j^{II}(\theta,v) + \frac{2K_{III}^{aux}}{\mu}\sqrt{\frac{r}{2\pi}}g_j^{III}(\theta,v) \qquad (4)$$

$$\varepsilon_{i,j}^{aux} = \frac{1}{2}\left(u_{i,j}^{aux} + u_{j,i}^{aux}\right) \qquad (5)$$

An expression for the energy release rate in terms of mixed-mode SIFs is defined as for plane strain condition as in Eqs. (6)-(8)

$$J = \frac{\left(K_I^2 + K_{II}^2\right)\left(1-\upsilon^2\right)}{E} + \frac{K_{III}^2\left(1+\upsilon\right)}{E} \qquad (6)$$

$$J = \frac{\left[\left(K_I + K_I^{aux}\right)^2 + \left(K_{II} + K_{II}^{aux}\right)^2\right]\left(1-\upsilon^2\right)}{E} + \frac{\left(K_{III}^2 + K_{III}^{aux}\right)\left(1+\upsilon\right)}{E} \qquad (7)$$

$$J = J + J^{aux} + I \qquad (8)$$

The interaction integral can be associated with the SIFs as Eq. (9)

$$I = \frac{2(1-\upsilon^2)}{E}\left(K_I K_I^{aux} + K_{II}K_{II}^{aux}\right) + \frac{1}{\mu}K_{III}K_{III}^{aux} \qquad (9)$$

By setting $K_I^{aux} = 1$ and $K_{II}^{aux} = K_{III}^{aux} = 0$,

$$K_I = \frac{E}{2(1-\upsilon^2)}I \qquad (10)$$

By setting $K_{II}^{aux} = 1$ and $K_I^{aux} = K_{III}^{aux} = 0$ and $K_{III}^{aux} = 1$ and $K_I^{aux} = K_{II}^{aux} =$ 0 leads to the relationship between modes II and III SIFs with I, respectively.

$$K_{II} = \frac{E}{2(1-\upsilon^2)}I \qquad (11)$$

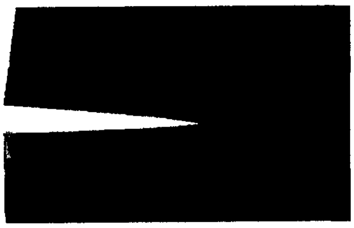
$$K_{III} = \mu I \qquad (12)$$

where, K_i is a stress intensity factor with i is a loading mode, i = 1, 2 and 3. J-integral can be represented as J and I is a interaction domain integral. While, E and μ is a modulus of elasticity and modulus of rigidity, respectively.

III. Methodology

The calculation of stress intensity factor (SIF) for the surface crack geometry has received much attention due to its frequent use in analyzing linear elastic fracture mechanics problems [4-9]. ANSYS finite element analysis is used to numerically model the cracks shown in Figure 1 where there are two cracks to be considered, normal and slanted cracks. Figures 1(a) and 1(b) depicted the nomenclature of surface cracks used in this work. A special attention is given to the crack tip by employing 20-node iso-parametric quadratic brick elements. The square-root singularity of stresses and strains is modeled by shifting the mid-point nodes to the quarter-point locations around the crack-tip region. The finite element model used in this work is revealed in Figure 2.

There are four important parameters used in this work, crack aspect ratio, a/b, relative crack depth, a/D, slanted angle, θ and normalized coordinate, x/h, where, a and b are the minor and major ellipse, respectively. The around the crack front. The calculations of SIFs are based on the energy method (J-integral) around the crack front. Similar patterns of crack deformations can be observed for a/b = 0.8 crack aspect ratio as in Figure 7. It is also depicted that the crack face distances are wider than when compared with the slanted cracks leading to decrease the SIFs. Even though the SIFs for slanted cracks are lower than normal cracks but there are another type of failure mode existed which is mode II failure mechanisms. Therefore, the formation of slanted cracks should not be underestimated in determining the structural reliability.



(a)

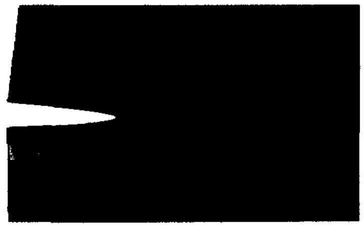
(b)

(c)

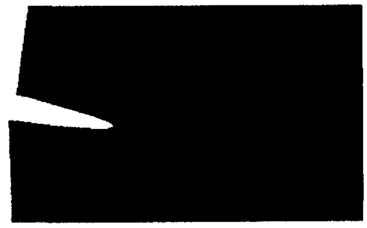
Fig. 6. Surface crack behavior (a/b = 0.2) of different slanted angle, θ (a) 0^0, (b) 15^0 and (c) 30^0.

While, Figure 8 indicated the SIFs behavior obtained at the deepest point along the crack front or at x/h = 0. Generally, the SIFs patterns are almost similar with the normal cracks except the SIFs are slightly lowered when slanted angles are introduced. Instead of mode I SIF reduction, mode II SIFs increased on the other hand. However, the increments of mode II SIFs are relatively insignificant compared with mode I SIFs.

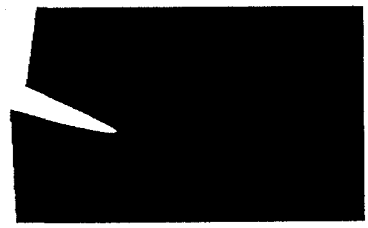
Another interesting behavior of cracks considered in this work is when higher a/b ratio is used (a/b > 1), cracks become deeper but crack mouth or length shorter. Therefore, it is then reduced the capability of cracks to open effectively. This phenomenon strongly reduced the mode I SIFs.



(a)



(b)



(c)

Fig. 7. Surface crack behavior (a/b = 0.0) of different slanted angle, θ (a) 0^0, (b) 15^0 and (c) 30^0.

V. Conclusion

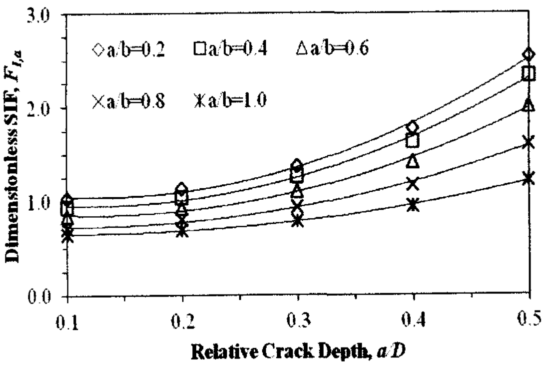
This paper discussed numerically the stress intensity factor (SIF) for normal and slanted surface cracks in round bars. ANSYS finite element program is used to model the crack. The present model is first validated with the previous model and it is found that the present model is well agreed with the existing model. There are three slanted angles are used 0^0, 15^0 and 30^0 where the relative crack depth are varies in the range of 0.1 to 0.5. Other type of cracks are not considered. It is found that:

I. The introductions of slanted surface cracks have reduced the SIFs along the crack front.

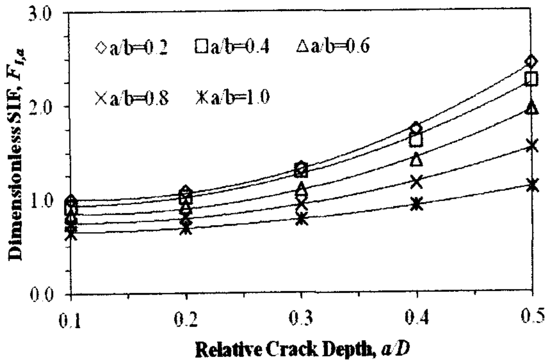
II. Even though lower values of SIFs are obtained for slanted cracks compared with normal cracks, but the present of mode II SIFs should not be underestimated the structural reliability.

Acknowledgements

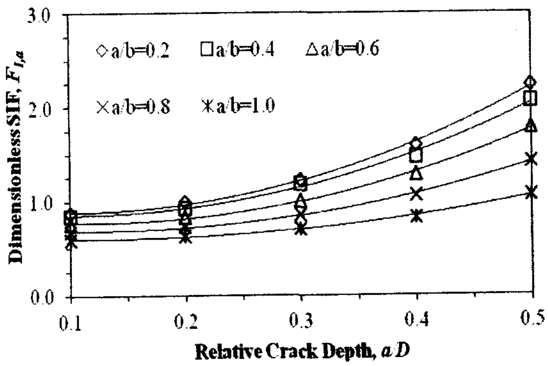
Author acknowledged Universiti Tun Hussein Onn Malaysia (UTHM) and Office of Research, Innovation, Commercialization and Consultancy (ORICC) for sponsoring this paper using Vot. 1001.



(a)



(b)



(c)

Fig. 8. SIFs along the crack front, x/h for a/b = 0.8 of different slanted angle, θ, (a) 0, (b) 15^0 and (c) 30^0.

References

[1] A. E. Ismail, A. K. Ariffin, S. Abdullah and M. J. Ghazali, Off-set Crack Propagation Analysis under Mixed Mode Loadings, *International Journal of Automotive Technology*, Vol. 12, n. 2, pp. 225-232, 2011.

[2] S. Raju and J. C. Newman, *Fracture Mechanics*, ASTM Special Technical Publication, Vol. 905, n. 17, pp. 789–805, 1986.

[3] J. Toribio, J. C. Matos and B. Gonzalez, Numerical Modelling of Crack Shape Evolution for Surface Flaws in Round Bars under Tension Loadings, *Engineering Failure Analysis*, Vol. 16, n. 2, pp. 618–630, 2009.

[4] Y. Murakami and H. Tsuru, *Stress intensity factor handbook*. Pergamon, New York, 1986.

[5] A. Carpinteri, Elliptical-arc Surface Cracks in Round Bars, *Fracture of Engineering Materials and Structures*, Vol. 15, n. 11, pp. 1141–1153, 1999.

[6] C. S. Shin and C. Q. Cai, Experimental and Finite Element Analyses on Stress Intensity Factors of an Elliptical Surface Crack in a Circular Shaft under Tension and Bending, *International Journal of Fatigue*, Vol. 129, n. 3, pp. 239–264, 2004.

[7] M. D. Fonte and M. D. Freitas, Stress Intensity Factors for Semi-elliptical Surface Cracks in Round Bars under Bending and Torsion, *International Journal of Fatigue*, 21 (5), 457-463, 1999.

[8] A. E. Ismail, A. K. Ariffin, S. Abdullah and M. J. Ghazali, Stress Intensity Factors for Surface Cracks in Round Bar unde rSIngle and Combined Loadings, *Meccanica*, 47, 1141-1156, 2012.

[9] A. E. Ismail, A. K. Ariffin, S. Abdullah and M. J Ghazali, Stress Intensity Factors under Mode III Loadings. *International Review of Mechanical Engineering*, Vol. 7, n. 3, pp. 578-582.

[10] A. E. Ismail, A. K. Ariffin, S. Abdullah and M. J Ghazali, Effect of Crack Faces Opening Mechanisms on the Combined Stress Intensity Factors under Bending and Torsion Moments, *International Review on Modeling and Simulation*, Vol. 4, n. 6, pp. 3232-3238.

[11] C. F. Shih, B. Moran and T. Nakamura, Energy Release Rate along a Three-Dimensional Crack Front in a Thermally Stressed Body, *Journal of Fracture*, 30 (2), 79-102, 1986.

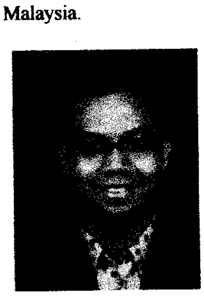
[12] S. Courtin, C. Gardin, G. Be'zine and H. Ben Hadj Hamouda, Advantages of the J-integral Approach for Calculating Stress Intensity Factors when Using the Commercial Finite Element Software ABAQUS, *Engineering Fracture Mechanics*, 72, 2174–2185, 2005.

[13] M.C Walters, G.H Pauline and R.H Dodds Jr, Interaction Integral Procedures for 3D Curved Cracks Including Surface Tractions, *Engineering Fracture Mechanics*, Vol. 72, pp. 1635-1663, 2005.

[14] ANSYS Manual User Guide, 2013.

Authors' information

Universiti Tun Hussein Onn Malaysia, Batu Pahat, 86400 Johor, Malaysia.



Al Emran Ismail was born in 1977 and obtained his first and second degrees from Universiti Putra Malaysia (UPM) in 2000 and Universiti Teknologi MARA (UiTM) in 2003, respectively. After 5 years working in Universiti Tun Hussein Onn Malaysia (UTHM), he continued his studies in Universiti Kebangsaan Malaysia (UKM) for PhD degree and graduated in 2012. His expertises in the area of fracture mechanics. He is also interested in natural fiber reinforced composites for crashworthiness applications.