FUZZY RANDOM REGRESSION TO IMPROVE COEFFICIENT ESTIMATION FOR MALAYSIAN AGRICULTURAL INDUSTRY

HAMIJAH BINTI MOHD RAHMAN

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Faculty of Computer Science and Information Technology
Universiti Tun Hussein Onn Malaysia

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ABSTRACT

Conventional model setting of production planning is developed with numerical crisp values. Additionally coefficient values must be determined before the model is set. It is however troublesome and complex for decision maker to provide rigid values and determining the coefficient values for the model. Building the production planning model with precise values sometimes generates improper solution. Hence, this study proposes a fuzzy random regression method to estimate the coefficient values for which statistical data contains simultaneous fuzzy random information. A numerical example illustrates the proposed solution approach whereby coefficient values are successfully deduce from the statistical data and the fuzziness and randomness were treated based on the property of fuzzy random regression. The implementation of the fuzzy random regression method shows the significant capabilities to estimate the coefficient value to further improve the model setting of production planning problem which retain the simultaneous uncertainties.

ABSTRAK

Menentukan nilai pemalar kadang-kala sukar dan kompleks untuk pembuat keputusan. Walau bagaimanapun nilai pemalar adalah penting untuk pelbagai tujuan, khususnya untuk mengukur hubungan dalam ungkapan algebra dan membina model matematik. Selain itu, memberi nilai yang tepat bagi pemalar adalah sukar apabila ia melibatkan maklumat kabur, dan kewujudan maklumat rawak meningkatkan lagi kesukaran untuk menentukan nilai pemalar. Oleh itu, kajian ini mencadangkan satu kaedah regresi rawak kabur untuk menganggarkan nilai pemalar di mana data statistik yang mengandungi maklumat kabur dan rawak digunakan. Contoh nilai numerik menunjukkan cadangan pendekatan penyelesaian di mana nilai-nilai pemalar berjaya disimpulkan dari data statistik, dan kekaburan serta kerawakan dirawat berdasarkan regresi rawak kabur. Pelaksanaan kaedah regresi rawak kabur menunjukkan kebolehan untuk menganggar nilai pemalar untuk memperbaiki masalah dalam model perancangan pengeluaran di mana ketidaktentuan wujud.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

FRRM	Fuzzy Random Regression Model
FRM	Fuzzy Regression Model
MRB	Malaysian Rubber Board
MREPC	Malaysian Rubber Export Promotion Council
MIDA	Malaysia Investment Development Authority
DOA	Department of Agriculture
MAE	Mean Absolute Error
MSE	Mean Square Error
RMSE	Root Mean Square Error

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

This chapter explains the background of the study which involves the estimating coefficient values from historical data whereby fuzzy random information coexist. A fuzzy random regression analysis is introduced to estimate coefficient of a linear programming from a statistical data which contain fuzzy random data. The problem statement, objectives, scope and significance of the study are also presented in this chapter.

1.1 Background study

Coefficient or weight value is a constant that represents the rate of change of one variable as a function of changes in the others. The coefficient value is used to explain the relation between dependent and independent variables. Equation 1.1 (Montgomery *et al*., 2012) shows the example of coefficient used in linear regression analysis.

$$y = ax + b \tag{1.1}$$

Given in the Equation (1.1), a indicates the coefficient that represent the rate of change of variable y as a function of change x. This coefficient value plays an important role in the development of a linear programming. However, developing this linear programming model requires coefficient values for each of the attribute that included in the model. Commonly, coefficient values in the model are provided by the decision maker. However, determining coefficient sometimes is difficult in the absence of relevant data. In fact, the decision maker might provide imprecise value; and may vary from one decision maker to another where many uncertainties are occurring at the same time. Therefore, an appropriate method is necessary to estimate the coefficient value when the historical data available. A method to solve the problem of model's coefficient estimation is by using regression analysis. This regression analysis is a statistical technique for modeling the relationship between variables (Montgomery *et al.*, 2012). This technique is supported by effective statistical analysis dealing with numeric precise data. In other words, the obtained statistical model is not considering the imprecision or uncertainty in the data. However, in real life application, there exists uncertainty fuzzy data in which subjective human estimation play a main role. As a result, a fuzzy regression models was introduced to handle with the fuzzy uncertainty input-output data (Tanaka *et al*., 1982).

On the other hand, it is a common situation in real life problem where the uncertainty is occurring simultaneously fuzzy with random data. Fuzziness is comes from the vague or imprecise information when mistakes can come from man, machine or the expert himself. Meanwhile, the randomness occurs when the observed data is collected randomly.The fuzzy random situation is explained as follows. Assume that an expert is responsible to evaluate the production of two sample products. Randomness occurs because it is not known which product may produce higher quantity, and vice versa. Meanwhile, fuzziness originates from the lingual notion such as ‘sufficiently larger’, ‘is about’ etc. used by the decision maker to express the preference instead of the precise figures.

When such simultaneous uncertainties occur in the data, an appropriate mathematical programming model is necessary to handle these uncertainties. Hence, the fuzzy random regression is introduced to deal with the hybrid uncertainty where the fuzziness and randomness are the central roles (Watada *et al*., 2009). The fuzzy random regression uses confidential interval to include more possibility in its prediction strategies.

1.2 Problem statement

Real life application of production planning problem requires mathematical programming structure such as linear programming to model the relationship between variables. The coefficient information is necessary as it is important to model the relationship between the input-output variables. However, the required coefficient value is not always available on time, yet it raises the complexities to model the production planning mathematically.

The difficulties occurred as the decision maker difficult to decide the exact coefficient values. The decision also, might vary from one decision maker to another. The differences among decision maker can be treated statistically, but each fuzzy term should be treated by considering the formalism of fuzzy set. Therefore, it is important to ensure that the tool to estimate the coefficient value is sufficiently can handle the explained situation.

Additionally, the historical data which is captured from the real situation may contain simultaneous fuzzy random information whereby the collective data which is collected randomly contain imprecision. Moreover, the real life application is commonly include the state of randomness and fuzziness. The classical regression analysis is unable to deal with the uncertainties. Besides, the existence of random data is beyond the scope of fuzzy regression approach. Motivated by the above-mentioned problems, a fuzzy random regression method is proposed to be applied to estimate the model's coefficient value in the fuzzy random environment.

1.3 Objectives

The main objective of this study is to implement the fuzzy random regression approach to determine the fuzzy random based coefficient value under fuzzy random uncertainties. Specifically, the objectives of this study are:

1. to estimate the coefficient value from statistical data which contain fuzzy random information
2. to develop linear programming model for selected parameter in agricultural industrial application
3. to compare the fuzzy random regression with the fuzzy regression approach.

1.4 Scope

The scope of this study encompasses of several items. It is the aim of this study to estimate the coefficient value using fuzzy random regression approach for developing a mathematical programming model. The application of this research is implemented in Malaysian Agricultural Industry i.e., rubber and paddy. As for Malaysian rubber industry, two types of natural rubber production is selected which is dry and latex. The selected rubber historical data is for 13 year interval beginning 2000 until 2012. Meanwhile, two kinds of products of Malaysian paddy industry is selected which are paddy and rice. Six years of paddy historical data is been selected beginning 2006 until 2011.

1.5 Significance of the study

In real practice, a decision maker provides the coefficient information. However, the coefficient value sometime is not easy to obtain or determine beforehand which makes the relevant coefficient value is unavailable on the time it is required. Therefore, the fuzzy random regression method is beneficial to estimate the coefficient value especially when the information is not available.

Conventionally, in mathematical programming, the coefficient is assumed as a constant and precise value. However, in real practical application, the presence of uncertainties (e.g. fuzzy and random information) in the real data could not be ignored. It is important to remain the uncertainties in the modeling of real situation to avoid misleading result.

The methodology presented in this study is able to assist coefficient values extraction from statistical data with the treatment of fuzziness and randomness. Hence, at one particular point of view, the fuzzy random regression method is important to facilitate the decision maker to decide the coefficient value and further modeling the real situation problem mathematically. Thus, the difficulties of decision maker to decide the coefficient value can be reduced and uncertainties situation can be treated sufficiently.

1.6 Dissertation overview

This report is presented in five chapters. The first chapter explains the background study, problem statement, objective, scope and significant of the study. Chapter two discusses a related literature review to this study. Chapter three describes the research methodology while results and discussions are explained in Chapter four. The last chapter conclude the report and provides some suggestion for future work.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter is focuses in explaining a review on prediction model, classical regression approach; fuzzy regression approach fuzzy random regression approach and also production planning. The review on the Malaysian rubber industry and paddy industry is also presented as a background.

2.2 Uncertainties

Uncertainty theory is a branch of mathematics where it is used to model the human uncertainty (Liu, 2012). Generally, the uncertainty can be defined as that which is precisely known (Booker and Ross, 2011). The phenomenon of uncertainty is a common feature in real world whereby it happened when some situation can be quantified by uncertain measure. The concept of uncertainty was introduced by Zadeh where he defined the fuzzy set concept as a class of objects encountered in the real physical world do not have precisely defined criteria of membership (Zadeh, 1965). Uncertainty in the data is not only occurring in fuzzy situation, as the data may exist randomly. Moreover, uncertainty may exist simultaneously fuzzy with random data. In this section, the fuzzy theory, random and also concept of fuzzy random variable are explained.

2.2.1 Fuzzy theory

The term “Fuzzy Logic” originates from the word fuzzy which is deal with fuzziness. Fuzziness means a situation or problem that has imprecision, uncertainty or incomplete of information. Fuzzy Logic is a small part of the fuzzy set theory that is been introduced to solve real world problems. It is different from Crisp Logic in the conventional mathematical operation. Crisp Logic is a classical logic which is valid for ideal cases with more precision.

More specifically, fuzzy logic may be viewed as an attempt at formalization of two remarkable human capabilities to converse, reason and make rational decision in an environment of imprecision, uncertainty, incompleteness of information, conflicting information and partiality of truth (Zadeh, 2008). The main motivation to apply fuzzy logic is because the algorithm can manipulate vague, imprecise and uncertainty concepts. It also has the capability to operate on perception-based information in which come from human natural language.

2.2.2 Random/ probability theory

Random theory or probability is a quantity that is measured in connection with random experiment. The probability theory comprises mathematically based theories and methods for investigating random phenomena (Beichelt, 2006). Commonly, random phenomena occur in connection with random experiment in which outcomes of the random experiment could not be predicted with certainty. An example of random experiment is like counting the number of vehicle arriving at a filling station a day (Beichelt, 2006).

The probability theory is different from fuzzy theory where this theory is lacking in capability to operate perception-based information (Zadeh, 2002). The perception-based information is based on the human natural language-proposition which describes one or more perception. However, this probability theory has limitation whereby it unable to operate on perception-based information (Zadeh, 2002).

2.2.3 Fuzzy random variable

The concept of fuzzy random variable was introduced by Kwarkenaak, who defined these variables as a measurable function that linked the probability with the fuzzy numbers. Randomness is caused by some chance mechanism whereas fuzziness is brought about by dimness of perception (Kwarkenaak, 1978). Fuzzy random variables may be used to describe and characterize situations where the statistical data has some element of uncertainties (Kwarkenaak, 1979). Since then, various extensions of these variables had been studied. Puri and Ralescu (1986) define a concept of fuzzy random variables and the expectation of these variables by investigating some of their properties.

Moreover, Lopez-Diaz and Gil (1997) discussed the constructive definition of fuzzy random variables that would play an essential role in the constructive definition of their integrals, which will be especially valuable to perform practical computations and to develop further results concerning the integration of these variables. Furthermore, Gil *et al*., (2006), analysed the interpretation, modelling and impact of the fuzzy random variables. Yang and Gao (2010) investigate the fuzzy random variables by their own characteristic and uniqueness. In all of these works, a fuzzy random variable is defined as a function that assigned a fuzzy subset in order to find the possible output of a random experiment. The different definitions in the literature disagree on the measurability condition imposed to the mapping, and in the properties of the output space, but all of them intend to model situations that incorporates fuzziness and randomness (Cousco *et al.,* 2007).

Based on the explanation, it is apparently realized that the uncertain information are originated from many resources. Even though the solution which uses precise value is satisfactorily good, retaining the uncertainties in the developed model is necessarily important to remain the underlying intention and to address some limitation.

2.3 Prediction model

Predicting the future is a basic problem in which people have to solve every day. It is a component of planning, decision-making, memory and causal reasoning (Griffiths and Tenenbaum, 2011). The ability to accurately predict the future outcomes of complex systems has been the goal of forecasters for decades (Cave, 2011). Prediction is widely been used in real life application such as in weather forecasts, medical studies, quality testing and many more. For example, statistical data from the weather department is used as to compare prior weather conditions with current weather to predict future weather. Prediction is also applied in medical studies where it uses statistic data to validate the effectiveness of any drug before the drug can be prescribed. Besides that, the quality testing also used data in order to identify whether any product that the company produced will pass the quality test.

In order to accomplish a prediction, an appropriate method is necessary as a tool in assisting to predict accurately. Several methods for prediction have been suggested and implemented such as Kalman filters, Artificial Neural Network and also regression analysis. The Kalman filter is taken from control theory and is used to resolve the statistical error. However, this method has done little to replace forecast using human judgement by people knowledgeable in their filed (Cave, 2011). Meanwhile, the Artificial Neural Network is suitable been applied when there is huge data provided. Besides that, a regression analysis is a well-known method applied in prediction problem when the statistical historical data still intact.

2.4 Regression analysis

Regression analysis is one of the techniques uses in statistical data for coefficient estimation. This technique is supported by effective statistical analysis dealing with precise crisp data. The importance of this regression approach is to investigate and model the relationship between variables. Regression analysis is used in evaluating the functional relationship between the dependent and independent variables and also in determining the best-fit model for describing the relationship (Yang and Ko, 1997).

This technique is widely used in forecasting and prediction and had been used over a decades. There also a study of regression where the researchers identified the algebraic relationship between classical regression and total least square estimation (Huffle and Vandewall, 1987). Furthermore, regression also had been used in medical problem where the equation was used to estimate the total body water for diabetes's patient (Chertow *et al*., 1997). The applications of regression are numerous and occur in almost every field such as engineering, chemical science, economic and others (Montgomery *et al*., 2012). Thus, the regression analysis have a significant role in finding relationship between variables and apparently useful for various application.

However, the classical regression has limitation in treating uncertainty that commonly exist in real life application. Therefore, improvement on fuzzy regression is useful to make the method capable to solve the limitation in the traditional regression method.

2.4.1 Fuzzy regression

Fuzzy regression is a regression analysis on fuzzy data in dealing with the fuzziness environment (Yang and Ko, 1997). The first linear of regression analysis with fuzzy model was proposed by Tanaka *et al*., (1982). This method was introduced as to cope with the imprecise data coming from fuzzy environments where human subjective estimates play a main role. Fuzzy regression has been studied and applied widely in the fuzzy environment. The concept of fuzzy regression related to Tanaka approach has been discussed including a consideration of fuzzy least- squares regression models (Shapiro, 2005).

The study of fuzzy regression technique is continues by improve the fuzzy regression proposed by Tanaka *et al*., (1982) with the introducing fuzzy input-output data using shape-preserving fuzzy arithmetic operation the fuzzy partition (Song *et al*., 2005). Moreover, new method for computation of fuzzy regression was proposed where the approach is depends on the entropy's properties to rectify previous problems in fuzzy linear regression with crisp input and fuzzy output (Pasha *et al*., 2007). The fuzzy regression also applied as to solve problem in agricultural industry where it used to decide the fuzzy weight based on the criteria of oil palm fruit (Nureize and Watada , 2010). Meanwhile, different approach has also been proposed for computation of fuzzy linear regression based on the goal programming technique to estimate upper and lower fuzzy band with fuzzy output and crisp inputs (Abdollahzadeh *et al*., 2010).

Classical fuzzy regression is written as follows:

$$\begin{aligned} &\mathrm{Y} = [Y_j] = [A_1 x_{j1} + A_2 x_{j2} + \cdots + A_n x_{jn}] = \mathbf{A}\mathbf{x}_j^{\,t} \\ &x_{j1} = 1;\ j = 1,2,\cdots n \end{aligned} \tag{2.1}$$

where regression coefficient A_i is a triangular-shaped fuzzy number $A_i = \langle a_i, h_i \rangle$ with centre a_i and spread h_i. In Equation (2.1), $\mathbf{x}_j$ is a value vector of all criteria for the j^{th} sample.

Equation (2.1) can be written as Equation 2.2 which referring to the extension principle:

$$Y_j = \mathbf{A}\mathbf{x}_j^{\ t} = \left\langle \mathbf{a}\mathbf{x}_j^{\ t}, \mathbf{h}\,|\,\mathbf{x}_j\,|^t \right\rangle \tag{2.2}$$

where $|\mathbf{x}_j| = (|x_{j1}|, |x_{j2}|, \cdots, |x_{jK}|)$. The output of the fuzzy regression (2.1) results in fuzzy number, whose coefficients are fuzzy numbers.

The regression model with fuzzy coefficients can be described using the lower boundary $\mathbf{a}\mathbf{x}_j^{\ t} - \mathbf{h}\,|\,\mathbf{x}_j\,|^t$, centre $\mathbf{a}\mathbf{x}_j^{\ t}$ and upper boundary $\mathbf{a}\mathbf{x}_j^{\ t} + \mathbf{h}\,|\,\mathbf{x}_j\,|^t$. A sample $(y_j, \mathbf{x}_j)$, $j = 1,2,\cdots,n$ is defined for the total evaluation with centre y_j, spread d_j as a fuzzy number $y_j = \left\langle y_j, d_j \right\rangle$. The inclusion relation between the model and the samples should be written as Equation 2.3:

$$\begin{aligned} y_j + L^{-1}(\alpha) d_j &\le \mathbf{a}\mathbf{x}_j^{\ t} + L^{-1}(\alpha)\mathbf{h}\,|\,\mathbf{x}_j\,|^t \\ y_j - L^{-1}(\alpha) d_j &\ge \mathbf{a}\mathbf{x}_j^{\ t} - L^{-1}(\alpha)\mathbf{h}\,|\,\mathbf{x}_j\,|^t \end{aligned} \tag{2.3}$$

In other words, the fuzzy regression model is built to contain all samples in the model. This problem results in a linear program.

Using the notations of observed data $(y_j, \mathbf{x}_j)$, $y_j = \left\langle y_j, d_j \right\rangle$, $\mathbf{x}_j = [x_{j1}, x_{j2}, \cdots, x_{jK}]$ for $j = 1,2,\cdots,n$ and fuzzy coefficients $\mathbf{A}_i = \left\langle \mathbf{a}_i, \mathbf{h}_i \right\rangle$ for $i = 1,2,\cdots,K,$ the regression model can be mathematically written as the following linear programming problem:

$$\begin{aligned} \min_{\mathbf{a},\mathbf{h}} \quad & \sum_{j=1}^{n} \mathbf{h}\,|\,\mathbf{x}_j\,|^t \\ \text{subject to} \quad & y_j + d_j \le \mathbf{a}\mathbf{x}_j^{\ t} + \mathbf{h}\,|\,\mathbf{x}_j\,|^t,\ j = 1,2,\cdots,n, \\ & y_j - d_j \ge \mathbf{a}\mathbf{x}_j^{\ t} - \mathbf{h}\,|\,\mathbf{x}_j\,|^t,\ j = 1,2,\cdots,n, \\ & \mathbf{h} \ge 0. \end{aligned} \tag{2.4}$$

A fuzzy regression is obtained by solving the linear programming problem in Equation (2.4). The fuzzy regression model is also capable to treat non-fuzzy data by setting the spread to 0 in the above equations. The formulation of regression model (2.4) is then used to estimate the weight or coefficient. From the explanation of fuzzy regression, it shows that this technique is suitable to be applied in the situation where fuzzy data is exist.

2.4.2 Fuzzy random regression

Fuzzy random regression is a regression technique that was proposed in coping with the fuzzy random data. In real world regression analysis, statistical data may be linguistically imprecise or vague where the real data cannot be characterized by using only the formalism of random variables as there is existence of stochastic and fuzzy uncertainty (Watada *et al*., 2009). Statistical inference with fuzzy random data transfers the fuzziness into parameter estimators and it may be necessary to defuzzify the vague parameter at level decision making (Nather, 2006).

Regression based on fuzzy random data is widely been used and many researcher study this technique to solve problem with these fuzzy random data (Rodriguez *et al*., 2009). The study of this fuzzy regression analysis is based on fuzzy random variables with confidence intervals in the fuzzy multi attribute decision making was been proposed, as to enable decision makers to evaluate and find the importance weight (Watada, 2011). Determining weight in multi-attribute decision making is important, so a fuzzy random regression to estimate attribute importance in total evaluation within the bound of hybrid uncertainty is proposed (Nureize and Watada, 2011). Moreover, the fuzzy random regression method also has been proposed as an integral component of regression models in handling the existence of fuzzy random information (Nureize and Watada, 2011).

The expected value of a fuzzy variable is presented as follows:

Definition 2.1: Let Y be a fuzzy variable. Under the assumption that the two integrals are finite, the expected value of Y is defined as follows in (Watada *et al*., 2009):

$$E[Y] = \int_0^{\infty}\left(\frac{1}{2}\left[1+\sup_{t\geq r}\mu_Y(t)-\sup_{t<r}\mu_Y(t)\right]\right)dr-\int_{-\infty}^{0}\left(\frac{1}{2}\left[1+\sup_{t\leq r}\mu_Y(t)-\sup_{t>r}\mu_Y(t)\right]\right)dr \qquad (2.5)$$

Following from Equation (3.1), the expected value of Y is defined as $E[Y]=\dfrac{a^l+2c+a^r}{4}$ where *(a^l, c, d^r)* is a triangular fuzzy number and c is a center value.

Definition 2.2: Let us assume that (Ω, Σ, Pr) is a probability space and F_v is a collection of fuzzy variables defined on possibility space $(\Gamma, P(\Gamma), \mathrm{Pos})$. A fuzzy random variable is a map $X : \Omega \to F_v$ such that for any Borel subset B of $\Re$, $Pos\{X(\omega) \in B\}$ is a measureable function of ω.

Let V be a random variable on probability space $(\Omega, \Sigma, \mathrm{Pr})$. It makes $X(\omega) = (V(\omega) - 2, V(\omega) + 2, V(\omega) + 6)_\Delta$ a triangular fuzzy variable for every $\omega \in \Omega$ on some possibility space $(\Gamma, P(\Gamma), \mathrm{Pos})$. As a result, X is a triangular fuzzy random variable.

The expected value of the fuzzy variable $\mathrm{X}(\omega)$ is denoted by $E[X(\omega)]$ for any fuzzy random variable X on Ω, (Liu and Liu, 2003). Thus, the expected value of the fuzzy random variable X is defined as the mathematical expectation of the random variable $E[X(\omega)]$.

Definition 2.3: Let X be a fuzzy random variable defined on a probability space $(\Omega, \Sigma, \mathrm{Pr})$. The expected value of X is defined as follows in (Watada *et al*., 2009):

$$E[X] = \int_\Omega \left[\int_0^\infty \left(\frac{1}{2} \left[1 + \sup_{t \geq r} \mu_{Z(\omega)}(t) - \sup_{t < r} \mu_{Z(\omega)}(t) \right] \right) dr - \int_{-\infty}^0 \left(\frac{1}{2} \left[1 + \sup_{t \leq r} \mu_{Z(\omega)}(t) - \sup_{t > r} \mu_{Z(\omega)}(t) \right] \right) dr \right] \mathrm{Pr}(d\omega) \tag{2.6}$$

Definition 2.4: Let X be a fuzzy random variable defined on a probability space $(\Omega, \Sigma, \mathrm{Pr})$ with expected value e. The variance of X (Watada *et al*., 2009) is defined as Equation 2.7.

$$\mathrm{var}[X] = E\left[(X\text{-}e)^2\right] \tag{2.7}$$

where $e = E[X]$ given by Definition 2.3.

The preliminary of fuzzy random regression is explained through fuzzy random variable whereby two main components which develop the model were expected value and variance.

In fuzzy random regression method, the confidence intervals are expressed through the expectations and variances of fuzzy random variables. Fuzzy random data Y_j (output) and X_{jk} (input) for all $j = 1, \ldots, N$ and $k = 1, \ldots, K$ are defined as follows:

$$Y_j = \bigcup_{t=1}^{M_{Y_j}} \left\{ \left(Y_j^t, Y_j^{t,l}, Y_j^{t,r} \right)_{\Delta}, p_j^t \right\}, \tag{2.8}$$

$$X_{jk} = \bigcup_{t=1}^{M_{X_{jk}}} \left\{ \left(X_j^t, X_j^{t,l}, X_j^{t,r} \right)_{\Delta}, q_{jk}^t \right\}, \tag{2.9}$$

That means all values are given fuzzy variables with probabilities, where fuzzy variables $\left(Y_j^t, Y_j^{t,l}, Y_j^{t,r}\right)_\Delta$ and $\left(X_j^t, X_j^{t,l}, X_j^{t,r}\right)_\Delta$ are obtained with probability p_j^t and q_{jk}^t for $j = 1,\ldots,n$, $k = 1,\ldots,K$ and $t = 1,\ldots,M$ or $t = 1,\ldots,M_{X_{jk}}$, respectively. The fuzzy random data is shown in **Appendix A.**

Let us denote a fuzzy linear model with fuzzy coefficients $A_k^*,\ldots,A_K^*$ as follows:

$$\mathbf{Y}_j^* = \mathbf{A}_k^* X_{j1} + \cdots + \mathbf{A}_K^* X_{jK}, \tag{2.10}$$

where each Y_j^* denotes an estimate of the output and $\mathbf{A}_k^* = \left(\left[\frac{A_k^{*l} + A_k^{*r}}{2} \right], A_k^{*l}, A_k^{*r} \right)_\Delta$ are symmetric triangular fuzzy coefficients when triangular fuzzy random data X_{ik} are given for *j = 1,… n, k = 1,…, K* as shown in the table of fuzzy random data.

Based on the expected value and variance of fuzzy random variable, the one-sigma confidence $(1 \times \sigma)$ interval is shown in the following:

$$I[e_X, \sigma_X] \underline{\underline{\Delta}} \left[E(X) - \sqrt{var(X)}, E(X) + \sqrt{var(X)} \right] \tag{2.11}$$

Hence, the fuzzy random regression model with $\sigma -$ confidence intervals is described as follows:

$$\begin{aligned} &\min_{\mathrm{A}} \quad J(A) = \sum_{k=1}^{K} \left(A_k^r - A_k^l \right) \\ &A_k^r \geq A_k^l, \\ &\mathrm{Y}_{\mathrm{j}}^* = \mathrm{A}_{\mathrm{j}}^* I\left[e_{X_{j1}}, \sigma_{X_{j1}}\right] + \ldots + \mathrm{A}_{\mathrm{K}}^* I\left[e_{X_{jK}}, \sigma_{X_{jK}}\right] \supset_{\tilde{h}} I\left[e_{Y_j}, \sigma_{Y_j}\right] \\ &j = 1,\ldots,n; k = 1,\ldots,K, \end{aligned} \tag{2.12}$$

The solution of the fuzzy random regression model with confidence interval can be rewritten as a problem of n samples with one output and K input interval values (Watada *et al*., 2009). In other words, it means that when a larger number of *n* and *K* involves, the complexity of the model is increases.

2.5 Production planning

By definition, production-planning models include decisions on production and inventory quantities; however in addition, there might be resource acquisition and allocation decision, such as adding to the work force and upgrading the training of the current work force (Graves, 1999). In the other words, production planning is a procedure to fix the production goals and to estimate the resources which are required to achieve the goals. The production planning and scheduling involves different complicated task dealing with a hierarchy of decision making problem in the manufacturing environment which require cooperation among multiple function unit such as production, accounting and marketing in an organization (Torabi *et al*., 2010).

Furthermore, this planning is an important method to improve and achieve the goals of any production process and also help the decision maker to deal with forecasting, planning, scheduling and executing the required production work (Messac *et al*., 2002). Today, production planning has become very important part of production management because companies have to react to dynamic market conditions and rising customer requirements for shorter delivery times, lower prices and better quality of services (Denisa, 2012). One of the main objectives in production planning is to maximize the production rate of the produced products by choosing appropriate methods and raw materials (Messac *et al*., 2002).

The importance of production planning influenced many studies on this planning and a lot of methods are applied as to make this planning process become more effective. Messac *et al*., (2002) proposed an optimization-based model for production planning, and physical programming as an effective method to optimize the production planning process where the model seek to minimize cost and manufacturing time, while maximizing production rate. Pastor *et al*., 2008 proposed mixed-integer linear programming in a case of production planning in a woodturning company as to achieve the goal to meet the demand at minimum cost, which include the cost of any overtime plus that of any subcontracting.

The application of production planning is widely been applied in many sector including agricultural industry. For example, Itoh *et al*., (2003) proposed a model of crop planning which contain uncertain values which may support decision making of agricultural farms. This crop planning is proposed as to maximize the minimum value of all total gains according to the probabilities. Besides that, Haneveld and Stegeman (2005) proposed a method to write crop succession requirements as constraint in a linear programming model for agricultural production planning. This crop succession requirement has important implication for production planning as to maintain the high yielding of crop production. From the previous studies, it is shown that the production planning has its important role especially in decision making whereby it can be used to improve many parameters such as production rate, profit, etc.

2.6 Application to industrial sector

This study is applied in the industrial sector in Malaysia which is the agricultural industry. Two kinds of Malaysian agricultural industries were selected which are the rubber sector and paddy sector. In this section, these two kinds of industries were explained.

2.6.1 Rubber industry

In the last seventeen years (1990 – 2007), the rubber consumption increased from 187,592 tonnes to 579 248 tonnes where the natural rubber was the main material used (Malaysian Rubber Board). This industry has been evolved through the years and transformed into a more integrated industry and also have gained a multi-billion ringgit. The natural rubber industry is one of Malaysia's economic contributors where this industry contributed almost RM36.4 billion in export earnings in 2012 (Market Watch 2012).

The rubber products accounted for 3.9% of Malaysia's total exports for manufacturing products. There are more than 500 manufacturers included in the Malaysian rubber product industry which produces latex products, tyres and tyre-related products and also industrial and general rubber products (Malaysian Investment Development Authority). In the latex products sub sector, 125 manufacturers are involving which produce gloves, condom, catheters and others. In fact, Malaysia is the world's leading producer and exporter of catheters, latex thread and natural rubber medical gloves (Malaysian Rubber Export Promotion Council).

Meanwhile, currently there are 120 companies in the tyre and tyre-related products sub sector. There are nine companies as tyre producer, while the other companies produce retread, tyre treads for retreading, valves and other accessories. In addition, the industrial and general rubber product sub sector comprises 185 companies producing various types of rubber products including hoses, tubing, electrical and electronic, machinery and equipment and many more. The rapid growth of the industry would enable Malaysia to become the world’s largest consumer of natural rubber latex and also the other rubber products.

Nowadays, Malaysia rubber industry is in the recovery stage due to the slow production of the commodity. Malaysia is currently the fourth largest consumer of natural rubber in the world after Thailand, Indonesia and Vietnam (Malaysian Rubber Export Promotion Council). Malaysia aims to become the world's largest producer of natural rubber compete with Thailand, Indonesia and Vietnam. Therefore, a strategic planning should be applied in order to maintain the rubber industry competitiveness.

In order to achieve the national goal, a production planning model is necessary whereby a fuzzy random tool is utilized to find coefficient values for the development of the production planning. The production planning is then used to assist decision making on determining the factor that would give higher impact to the production and further realize the industry aspiration to contribute RM52.9 billion to the Gross National Income by 2020.

2.6.2 Paddy industry

The Malaysian paddy and rice industry is highly protected crop and always been given special treatment from the government regarding on the importance of rice as a staple food commodity. The industry is heavily regulated because of its social, political and economic importance (Nadia *et al*., 2012). Malaysia produce 70% of its rice consumption and remaining 30% is imported from the other nations like Thailand and Vietnam. In 2011, the paddy production is 2,578.519 tonnes while the rice production is 1,661,280 tonnes and the average national paddy production is 3,748 tonnes per hectare (Department of Agriculture). Meanwhile, the total apparent consumption of rice is 2, 692,000 tonnes.

The importance of rice industry is not only as the country's staple food, but the industry also provides the main livelihood to about 300 000 farmers who rely on paddy farming as their main source of income. The paddy in Malaysia is mainly grown by smallholders with farm size of about 2.0 hectares. Land utilization for paddy production is currently at 674,928 hectare which is 76% in Peninsular Malaysia (515,657 ha) while Sarawak and Sabah accounted for 18% (118,919 ha) and 6% (40,352 ha) of the total hectarage respectively (Nadia *et al*., 2012).

Now, rice remains as the country's most important crop in terms of cultivation, being the principle staple food for most of the populace (Dano and Samonte, 2005). In fact, for the past fifty years, the Malaysian government has allocated billions of Ringgit in order to increase rice production including R & D, credit facilities, subsidized retail price, guaranteed minimum price, extension support, fertilizer subsidies and irrigation investment (Fahmi *et al*., 2013). Thus, a good strategy is necessary as to remain the production of rice in Malaysia. One of the strategy that could be applied to improve the existing strategies is production planning. This production planning can be used in decision making level whereby the decision maker will examine the factors that would be effect the rice production. Exploring such option is beneficial as to maintain the higher production of rice in Malaysia.

2.7 Chapter summary

This chapter presents a review on the several interrelated topic which is useful in this study. The output from the review is beneficial to obtain better understanding on the selected method to address the issues of predicting the model's coefficient; discussed in this study.

CHAPTER 3

METHODOLOGY

3.1 Overview

In this chapter, the research methodology is explained. The explanation is including the steps involved in the research activities such as collection data, preparation data, coefficient estimation and comparative study of two methods that applied in this study. Furthermore, the explanations on the steps that included in the methods which are the fuzzy regression and fuzzy random regression is provided.

3.2 Terminology and notations

In this study, the following notation is used to define the models:

x	: decision variable
$A*$	: attribute weight vector or coefficient
ex_{ik}	: expected value of attribute j for sample i
σx_{ik}	: variance of attribute j for sample i
$[\underline{a}_k, \overline{a}_k]$	: interval number

3.3 Research activity

The research activity of this study is including the collection and preparation data, estimation model's coefficient, application of the methods which are fuzzy regression and fuzzy random regression and also comparative study between these methods. The research activity can be represented by a flow chart as shown in Figure 3.1.

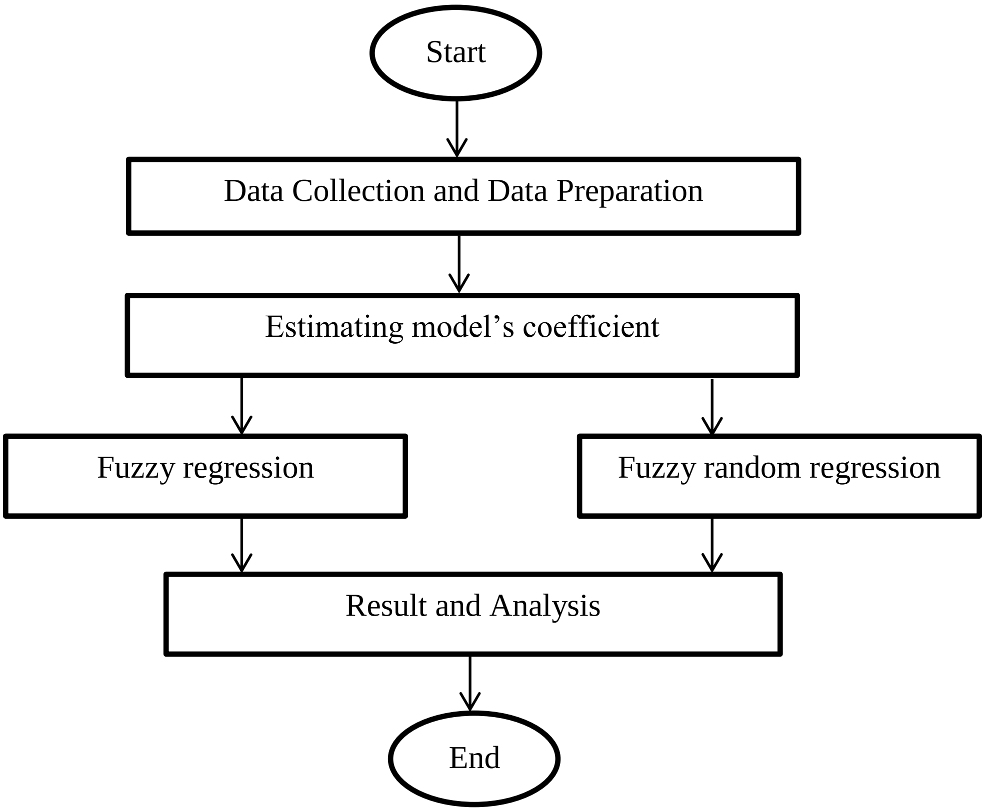

Figure 3.1.Flowchart of the research methodology

3.3.1 Data collection and data preparation

This study used a real statistical data of Malaysian rubber industry which has been collected from the Malaysian Rubber Board. The Malaysian Rubber Board was established as to assist in the development and modernisation of the Malaysian rubber industry in all aspect. It also provides Malaysian rubber historical data which is published to public. As for this study, 13 years of statistical rubber data is been selected from the year 2000 until 2012.

Other than rubber data, this study also uses Malaysian paddy data as second dataset. The statistical data Malaysian paddy industry is acquired from the Malaysian Department of Agriculture. The data that is been selected is from the year 2006 until 2011. Table 3.1 shows the selected parameter for the rubber dataset and paddy dataset. The two datasets were selected to demonstrate the applicability of fuzzy random regression approach to simultaneously improve coefficient estimation and treat fuzzy random uncertainties. The sample of rubber and paddy dataset is shown in Table 3.2 and Table 3.3, respectively.

Table 3.1: Selected parameter for the rubber dataset and paddy dataset

Datasets	**Industry**	
	Rubber	**Paddy**
Parameter	Natural rubber production Planted area Number of companies provided Employment in the rubber sector Natural rubber consumption.	Paddy production Rice production Planted Area Average yield of paddy Total apparent consumption

Table 3.2: Sample of rubber dataset (Malaysian Rubber Board, www.lgm.gov.my)

Year	**Production**		
	Dry	**Latex**	**Total**
2000	774,248.00	153,360	927,608
2001	761,594.00	120,473	882,067
2002	775,334.00	114,498	889,832
2003	854,619.00	131,028	985,647
2004	960,841.00	207,894	1,168,735
2005	935,529.00	190,494	1,126,023
2006	1,073,698.00	209,934	1,283,632
2007	1,023,190.00	176,363	1,199,553
2008	918,656.00	153,709	1,072,365
2009	746,106.00	110,913	857,019
2010	846,813.00	92,428	939,241
2011	916,270.00	79,940	996,210
2012	846,813.00	75,985	922,798

Table 3.3: Sample of paddy dataset (Department of Agriculture, www.doa.gov.my)

Year	**Production**		
	Paddy	**Rice**	**Total**
2006	2,187,519	1,407,220	3,594,739
2007	2,375,604	1,530,971	3,906,575
2008	2,353,032	1,516,470	3,869,502
2009	2,511,043	1,620,259	4,131,302
2010	2,464,831	1,588,457	4,053,288
2011	2,578,519	1,661,260	4,239,779

3.3.2 Estimating model coefficient

The main purpose in this dissertation methodology is to estimate the coefficient values for the selected statistical data. The dataset for the respective parameters were listed in Table 3.1. For the rubber industry, data were collected for rubber production, number of companies, planted area, number of employment and natural rubber consumption. Meanwhile, the data collected in paddy industry are paddy and rice production, planted area, average yield and number of consumption.

Even though the data containing in the table (i.e. Table 3.2 and Table 3.3) is presented in crisp and precise value, however it may inherently contain fuzziness and randomness. Fuzziness in the data is captured whenever the crisp values may represent some interval; shows flexibility. The data entry errors may also contributes to the fuzzy information, whereby the entered data has some imprecision. On the other hand, the statistical industrial data is presenting cumulative value of national level yield. The data is actually a collection from a city, district, state and so on. It makes the distribution of the data is randomly obtained from several sources to form the collective figures. The above mentioned situation explains that the statistical data in contains simultaneous fuzzy random information. A treatment to this situation of data is necessary to retain the actual meaning of those imprecision and the treatment is sufficiently performed.

Additionally, the statistical data as shown in Table 3.2 and Table 3.3 for example, doesn't show the relationship among others. This relationship is important to determine which dependent variable is significant to the independent variable. Let us assume that two products of A and B are used to determine the total production of the respective industry. By knowing the coefficient of the predictor variable, it is useful to determine which product is significant to the total production. Classical regression is a tool which is usually used to estimate coefficient values. However classical regression is unable to deal with fuzzy information. After then, fuzzy regression is introduced to address the limitation of fuzzy or imprecise information in the data. In this rubber statistical data, there exist the hybrid uncertainties which are fuzzy and random data. As for that problem, the classical regression technique and fuzzy regression is incapable to deal with the data. Therefore, with the existence of these fuzzy and random uncertainties, fuzzy random regression is proposed to estimate the coefficient based on the statistical data.

Determining coefficient or weight value is important to set up the programming model for production planning and to ensure the developed model will not produce misleading result. However, the difficulties in obtaining the model attribute's coefficient may cause problem and the decision of coefficient or weight value is crucial to the model's result. Furthermore, the existence of fuzzy random variable in the statistical data also should be considered due to the incapability of conventional method to handle these uncertainties. Therefore, the fuzzy random regression is proposed in this study as to estimate the model's coefficient under a hybrid fuzzy random environment. The calculation of fuzzy random regression as to estimate the coefficient is based on the model (3.1).

3.3.3 Result and analysis

The proposed method fuzzy random regression in this study is demonstrated to determine the coefficient values from the statistical data. The process of analysis is been done to analyse the outcome from the fuzzy random regression model. The coefficient values deduced from fuzzy random regression is a confidential interval which is in the form of close interval $[\underline{a}_k, \overline{a}_k]$ where a is the weight attribute. Meanwhile, the coefficient value deduced from fuzzy regression is in the form of $[a_i, h_i]$ where a_i is the central value and h_i is the spread of the weight attribute.

The outcomes from the demonstration was now analysed in order to determine the decision factor or attribute which has higher coefficient; shows the significant. The larger value of coefficient shows the more significant of the factor towards the evaluation. On the other hand, the estimated coefficient also is useful to develop production planning model and to predict the future outcome of the rubber and paddy production.

3.4 Application of methods

This section is spent to explain two methods of estimating the coefficient values which is fuzzy random regression and fuzzy regression.

3.4.1 Fuzzy regression

Fuzzy regression algorithm to estimate the coefficient is as follows:

Step 1: Developing fuzzy regression model for estimation

1.1 Eliciting the lower boundary and upper boundary

Determine the lower boundary $\mathbf{ax}_j^{\,t} - \mathbf{h}\,|\,\mathbf{x}_j\,|^t$, centre $\mathbf{ax}_j^{\,t}$ and upper boundary $\mathbf{ax}_j^{\,t} + \mathbf{h}\,|\,\mathbf{x}_j\,|^t$. A sample $(y_j, \mathbf{x}_j)$, $j = 1,2,\cdots,n$ is defined for the total evaluation with centre y_j, spread d_j as a fuzzy number $y_j = \langle y_j, d_j \rangle$ based on Equation (2.2).

1.2 Estimating the attribute’s coefficient

Set regression model as the Equation (2.4) to estimate coefficient.

Step 2: Decision-making and analysis

The estimated coefficient will be deduced by fuzzy regression. The coefficients is in the form of $[a_i, h_i]$ are obtained (2.4) where *a* is the centre and *h* is the spread. Whenever the spread is 0, it is understood that the non-fuzzy value is concerned. The larger value of coefficient shows the more significant of the factor towards the total evaluation.

3.4.2 Fuzzy random regression

Fuzzy random regression algorithm to estimate the coefficient is as follows:

Step 1: Eliciting the confidence interval

Compute the confidence interval of each fuzzy random by inducing the expected value and variance of fuzzy random variable to construct the one-sigma confidence interval, $I[e_X, \sigma_X]$.

1.1 Calculate the expected value $E[X]$ of triangular fuzzy random variable X according to the Definition 2.3 and Equation (2.6).

1.2 Calculate the variance of X based on Equation (2.7).

Step 2: Estimate the coefficient

Construct fuzzy random regression analysis to estimate the coefficient. Let A_k denote an attribute for $k = 1,\ldots,K$ and Y_j^* is the total evaluation for $j = 1,\ldots,n$, where n is the number of candidate alternatives to be evaluated. A fuzzy random regression model is described as follows:

$$
\begin{aligned}
\min_{\mathrm{A}} \quad & J(A) = \sum_{k=1}^{K} \left(\bar{a}_k - \underline{a}_k \right) \\
\text{subject to:} \quad & \bar{a}_k \geq \underline{a}_k, \\
& y_j^r + (e_{Y_j} + \sigma_{Y_j}) \leq \sum_{k=1}^{K} \bar{a}_k \left(e_{X_{j1}} + \sigma_{X_{j1}} \right), \\
& y_j^l - (e_{Y_j} - \sigma_{Y_j}) \geq \sum_{k=1}^{K} \underline{a}_k \left(e_{X_{j1}} - \sigma_{X_{j1}} \right), \\
& j = 1,\ldots,n; \quad k = 1,\ldots,K
\end{aligned}
\tag{3.1}
$$

Step 3: Decision-making and analysis

The coefficients $[\underline{a}_k, \bar{a}_k]$ are obtained from fuzzy random regression model (3.1). The lower boundary $\underline{a}_k$ and upper boundary $\bar{a}_k$ shows the flexibility of the coefficient values which is included during estimation.

From the stated algorithm, regression approach is structured in two common steps which is data model preparation and result analysis for both fuzzy and fuzzy random situation. The weight estimated by both model was then useful to develop forecasting model for fuzzy and fuzzy random circumstances.

3.5 Implementation of the proposed method

The use of the proposed method is demonstrated in this study on a two sets of models of rubber and paddy production in Malaysia. The increase in both rubber and paddy consumption has escalated the demand for their production respectively. In order to sustain its competitive edge, the industry needs to improve its productivity and this goal should be achieved with a plan based on the available resource. Based on the situation, a mathematical model namely linear programming can be developed to assist resource planning and manage the target efficiently. To address this industrial

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