

## Mathematical Modeling of Industrial Heat Exchanger System

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**Abstract.** In manufacturing and industrial fields used heat exchanger to control of temperature weather as a boiler or cooling system. This system is not stable as the temperature output can easily disturb by noise and other disturbance such as surrounding temperature. To improve the heat exchanger system performance, the mathematical model's needed. The heat exchanger mathematical model in this case is constructed using dynamic modelling based on real parameters of the heat exchanger. The simulation result shows almost similar trend of responses with the experiment result, it means they are can used as a model of the heat exchanger.

### Introduction

Process control is a branch of control engineering that deals with the operation of plants in industries such as petrochemicals, steel, glass, and energy. this research focus on the heat exchanger mathematical model constructed using dynamic modelling based on real parameters and simulate it using Matlab software to define the response and achieve it to compare it with real response from real plant. Afterwards, simulations are realized using these parameters. Finally, real time temperature experiment set is implemented using the same parameters with simulation. And the results are discussed.

### Mathematical Model of product liquid temperature in the Heat Exchanger

By using the energy balance equation [4], the energy supplied to the exchanger must equal the energy removed. For precise analysis, the heat loss to the environment must be determined. Here, however, to simplify the analysis well insulated for heat exchanger is assumed. When the heat exchanger is well insulated from the environment then all the energy that is lost by the primary medium must leave the exchanger with the secondary medium. Then

Rate of energy accumulation

= rate of energy in by flow or convection

– rate of energy out by flow or convection

+ net rate of heat addition to system from surroundings

Flow of heat into the tube side depends on the hot water flow rate and its temperature:

$$\therefore q_h = w_h \cdot C_{ph}(T_{hi} - T_{ho}) + q_{hi} \quad (1)$$

The heat flow into the tube is the difference between the heat from the hot liquid incoming and the heat flowing out to the product liquid. This net flow determines the rate of temperature change of the tube side as:

$$T_{ho}(t) = \frac{w_h}{\rho_h V_h} (T_{hi}(t) - T_{ho}(t)) + \frac{U_h A_h}{\rho_h V_h C_{ph}} (T_{co}(t) - T_{ho}(t)) \quad (2)$$

The differential equation describing the product liquid temperature in the shell side is:

$$T_{co}(t) = \frac{w_c}{\rho_c V_c} (T_{ci}(t) - T_{co}(t)) + \frac{U_c A_c}{\rho_c V_c C_{pc}} (T_{ho}(t) - T_{co}(t)) \quad (3)$$

To complete the dynamics. The time delay between the measurement and the exit flow is described by the relation:

$$\tilde{T}_{Co}(t) = T_{Co}(t - t_d) \tag{4}$$

Where  $\tilde{T}_{Co}$ , is the measured temperature of the product water, and  $t_d$  is the time delay. There may also be a delay in the measurement of the hot liquid temperature  $T_{ho}(t)$ , which would be modeled in the same manner. Equation (2) is nonlinear because the state variable  $T_{ho}(t)$ , is multiplied by the control input  $w_c$ . The equation can be linearized about  $\tilde{T}_{ho}(t)$  (a specific value of  $T_{ho}(t)$ ), so that  $T_{hi}(t) - T_{ho}(t)$  is assumed constant for purposes of approximating the nonlinear term, which we will define as  $\Delta T_h$ . In order to eliminate the  $T_{ci}(t)$  term in Eq. (3), it is convenient to measure all temperatures in terms of deviation in degrees from  $T_{ci}(t)$ . [4] The resulting equations are:

$$T_{ho}(t) = -\frac{U_h A_h}{\rho_h V_h C_{ph}} T_{ho}(t) + \frac{U_h A_h}{\rho_h V_h C_{ph}} T_{Co}(t) + \frac{w_h}{\rho_h V_h} \Delta T_h \tag{5}$$

$$T_{Co}(t) = \frac{U_c A_c}{\rho_c V_c C_{pc}} T_{ho}(t) - \left( \frac{w_c}{\rho_c V_c} + \frac{U_c A_c}{\rho_c V_c C_{pc}} \right) T_{Co}(t) \tag{6}$$

where:

$q_h$  → heat energy flow for tube side J/sec

$w_h, w_c$  → mass flow rate of the tube and shell side kg/sec

$C_{ph}, C_{pc}$  → heat capacity of the tube and shell side J/kg °C

$q_{hi}$  → heat energy addition to tube side from surrounding J/sec

$T_{hi}, T_{ho}$  → inlet and outlet temperature of the tube °C

$T_{hi}(t)$  → inlet temperature of tube side °C

$T_{ho}(t)$  → outlet temperature of the tube side °C

$T_{Co}(t)$  → outlet temperature of the tube shell side °C

$\rho_h, \rho_c$  → density of the hot and cold liquid, in kg/cm<sup>3</sup>

$V_h, V_c$  → volume of the tube and shell side cm<sup>3</sup>

### Mathematical Model Based on Real Parameters Heat Exchanger Temperature

The block diagram as shown in figure 1 represents the product liquid temperature open loop for the heat exchanger in the plant:



Fig. 1. The open loop heat exchanger system.

- Gain for conversion between the temperature to voltage  $G_{TV} : \frac{\text{output range"voltage"}}{\text{input range"temperature"}} = 0.093 \text{ V/C}^\circ$
- Gain for conversion between the voltage to current  $G_{VI} : \frac{\text{output range"current"}}{\text{input range"voltage"}} = \frac{4 \text{ mA}}{V}$
- Gain current to pressure converter  $G_{I/P} : 0.74 \text{ psi/mA} = 0.0527 \text{ kg/cm}^2 \cdot \text{mA}$
- Valve transfer function  $G_{VT} : \frac{1.48}{0.067s+1}$

### Result and Conclusion

The heat exchanger mathematical model in this paper constructed using dynamic modeling based on real parameters of the heat exchanger. The temperature setpoint is 40°C but the output is around 44.5°C. The open loop system shows that response below for both cases. The simulation result shows almost similar trend of responses with the experiment result, it means they are can use as a model of the complex heat exchanger which clear at table 1 and figure 2. That is achieving by the mathematical model for use it for many ways of control design.

Table 1. Response parameters comparison

Response Type	Ts(sec)	Tr(sec)	OV%	Ess%
Experiment response	185	79	1.2	12.5
Simulation response	149	59	0	14.13

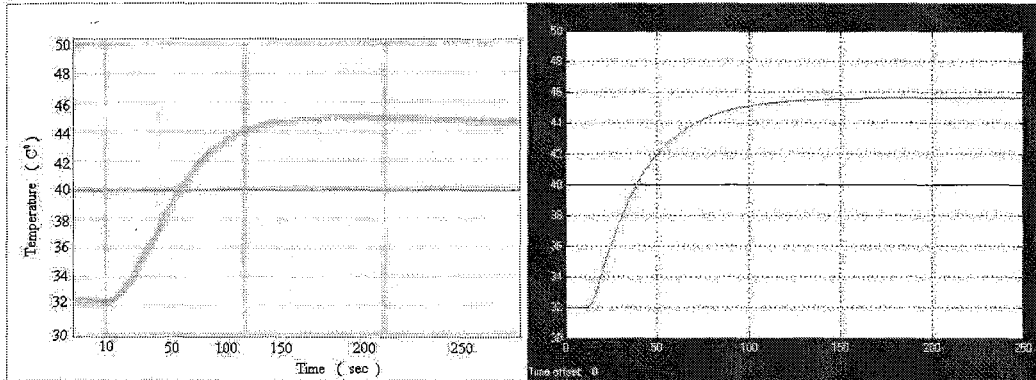


Fig.2. Comparison between experimental and simulation open loop response for temperature of the heat exchanger.

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