Recursive Gauss-Newton based training algorithm for neural network modelling of an unmanned rotorcraft dynamics

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Abstract: The ability to model the time varying dynamics of an unmanned rotorcraft is an important aspect in the development of adaptive flight controller. This paper presents a recursive Gauss-Newton based training algorithm to model the attitude dynamics of a small scale rotorcraft based unmanned aerial system using the neural network (NN) modelling approach. It focuses on selection of optimised network for recursive algorithm that offers good generalisation performance with the aid of the cross validation method proposed. The recursive method is then compared with the off-line Levenberg-Marquardt (LM) training method to evaluate the generalisation performance and adaptability of the model. The results indicate that the recursive Gauss-Newton (rGN) method gives slightly lower generalisation performance compared with its off-line counterpart but adapts well to the dynamic changes that occur during flight. The proposed recursive algorithm was found effective in representing helicopter dynamics with acceptable accuracy within the available computational time constraint.

Keywords: Artificial neural network; system identification; rotorcraft dynamics; unmanned aerial system; recursive Gauss-Newton.

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1 Introduction

Rotorcraft based unmanned aerial systems (RUAS) are a commonly used platform of unmanned aerial vehicles (UAV) and this platform configuration have been widely used in numerous applications ranging from surveillance to safe and rescue operation. The rotorcraft or helicopter platform possesses the unique manoeuvring capabilities that can hover above or near targets, which make it more suitable than fixed wing aircraft for high building structural inspection. Research efforts had been directed in this field to enable the UAV to execute these application autonomously which subsequently could eliminate accident risks to human pilots and increase higher chances of successful mission implementation.

The miniature RUAS are regarded as an unstable non-linear system with fast responsive dynamics and this present challenges in designing the automatic flight control. The automatic flight control function is responsible for manipulating the inputs to a dynamical system to obtain a desired effect on its outputs without a human in the control loop. Most research platform of RUAS and commercial off the shelf (COTS) autopilot systems are based on standard linear controllers such as PID, LQR and H_∞ techniques (Mettler, 2003; Shim, 2000; Jiang et al., 2006). Even though the linear controllers are the preferred methods in controlling the RUAS, the linear controllers are well known to suffer from performance degradation when the RUAS operate beyond the linear operating conditions. Advanced non-linear controllers such as feedback linearisation, adaptive control and non-linear model based approaches have been suggested in the literature to overcome the limitation of linear

approaches with successful implementation in real flight tests (Kendoul, 2012). However, the capability of the RUAS automatic flight controller can be further improved with the development of self tunable and flexible controllers such as the NN based flight controller that can be deployed and integrated into different rotorcraft platforms in shorter development time. Another benefit of NN based controller includes the ability to adapt to platform changes such as payload, sensors or changes in the dynamic system.

In recent development of real-time adaptive control system, the NN approach has found growing success in the development of automatic flight control system due to their ability to learn complex mapping from the flight data. This would make the representation of the system dynamics much more straight forward, whereas the complex mathematical model is much more difficult to develop. The NN calculation is parallel in nature, which leads to faster calculation speed in an intensive computation problems (Balakrishnan and Weil, 1996). Furthermore, the NN has the ability to adapt well to a changing environment which makes it suitable for adaptive control application. Typical NN based control schemes can be categorised into six main classes as follows (Norgaard, 2000):

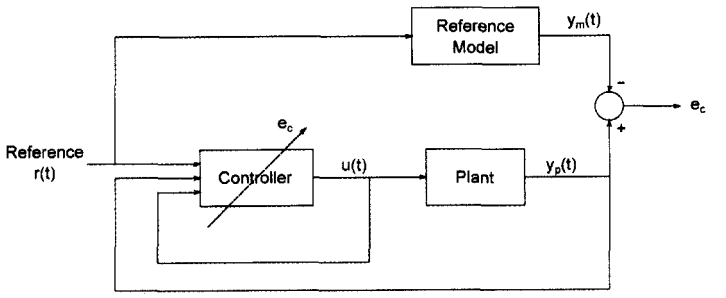
- direct inverse controller
- internal model controller (IMC)
- feed-forward controller with inverse model
- feedback linearisation controller
- optimal controller
- NN based model predictive controller (NNMPC).

Samal (2009), Norgaard (2000) and Agarwal (1997) suggested that the first five types of NN controller schemes fall under direct adaptive control class where the NN is used to update the controller parameters. Whereas, the NMPC is an indirect type of adaptive controller where the NN model is used to aid the existing conventional MPC controller to achieve the desired reference trajectory. Figure 1 shows the basic configuration structure of the direct and indirect adaptive control system for a dynamic system. In direct adaptive controllers, the control parameters are updated directly to minimise the tracking error. The calculation of the controller parameters or gains does not rely on the dynamic system to update the controller parameters. Whereas in the indirect adaptive control configuration, a dynamic model is used to predict the response of the dynamic plant. The prediction from the dynamic model is assumed to be identical to the actual response and this information is used to minimise the tracking error of the system.

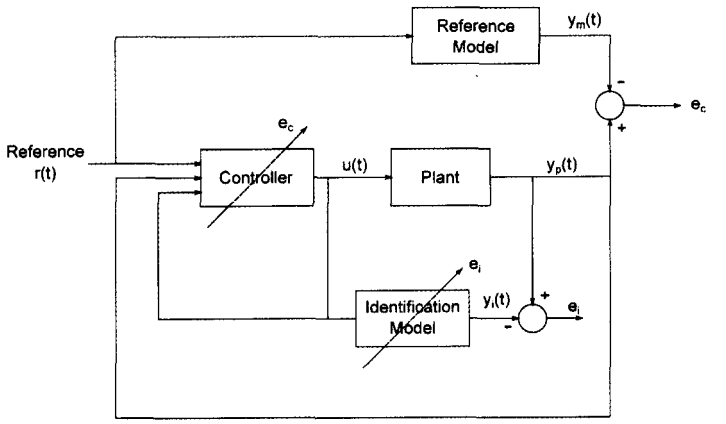
The non-linear and high order dynamics behaviour of a rotorcraft is typically hard to model using first principle modelling approach (direct physical understanding of forces and moments balance of the vehicle), and such approach can be inaccurate (Budiyono et al., 2009). Dynamic model obtained using the first principle approach depends on many parameters which needs to be carefully identified through direct measurements of geometrical or structural data. For aerodynamic parameters estimation, detailed experiment needs to be performed using wind tunnel facilities. The principle modelling approach requires considerable amount of theoretical knowledge and experiences about the rotorcraft flight and potentially would not produce a highly accurate results unless performed with extreme care. In certain cases, the agreement between the predicted and measured

dynamic behaviours is unsatisfactory because of the accumulated uncertainty and modelling simplification (Mettler, 2003; Kendoul, 2012).

Figure 1 The configuration of adaptive control system: (a) direct adaptive controller and (b) indirect adaptive controller



(a)



(b)

Since the helicopter dynamics is non-linear, the NN based system identification approach using the NNARX (Neural Network-Auto Regressive structure with eXtra inputs) model structure can be used to address such a problem. Here, the linear model structure such as ARX model structure was introduced as the basic NN model structure while static feed-forward multi-layered perceptron (MLP) network was used to introduce non-linearity into the model estimation. Similar to stability features of ARX model structure, the prediction from the NNARX model was considered stable since there was only pure algebraic relationship between prediction and past input and output measurements (Norgaard, 2000). This type of modelling technique demonstrates good general approximation capabilities for a reasonable non-linear system which make them ideal for the adaptive flight control application (Sjoberg et al., 1995; Calise and Rysdyk, 1998). Several research works on NN based approach using the popular feed-forward NNARX architecture can be found in Samal et al. (2008), Suresh et al. (2002) and Shamsudin

and Chen (2012) where the findings exhibit the capability of neural network approach and its effectiveness in modelling the dynamic response accurately using second order method such as the Levenberg-Marquardt (LM) method.

Although the NN is superior in terms of it prediction accuracy, the dynamic model identified from NN can be inaccurate due to many problems such as incorrect model structure selection, incorrect input vectors selection and over-fitting due to the excessive number of neurons. Previous attempts to model the non-linear unmanned helicopter dynamics using the off-line NN modelling has successfully modelled the dynamics of the helicopter, resulting in low mean or standard deviation of the residual values (Samal, 2009; Putro et al., 2009). However, past efforts in identifying the helicopter dynamics did not include the effect of embedded memory or model order of the NNARX networks on generalisation performance for the modelling problem considered. The validation methods introduced in this work can be used to ensure that the NNARX model fits well with observations and aid the neural network modeller to select the optimised network structure for prediction with an acceptable accuracy.

Furthermore, most NN based modelling techniques attempt to model the time varying dynamics of an UAS helicopter system using off-line modelling approach. The model which is generated and trained once from previously collected data is not able to represent the entire operating points of the flight envelope very well (Samal, 2009). Several attempts such as Samal (2009), Samal et al. (2008, 2009) were made to update the NN prediction model during flight using mini-batch LM training (LM training with small number of data samples). However due to a limited amount of processing power available in the real-time processor, such methods can only be employed to relatively small networks and they are limited to model uncoupled helicopter dynamics. In order to accommodate the time-varying properties of helicopter dynamics which changes frequently during flight, a recursive based learning algorithm is required to properly track the dynamics of the system under consideration. Furthermore, the usage of recursive algorithms such as recursive Gauss-Newton (rGN) or recursive Levenberg-Marquardt (rLM) reduces the computation complexity of the off-line training method without having to invert the full Hessian matrix in every iteration (Ngia and Sjoberg, 2000).

The present study is concerned with the modelling and identification of a helicopter based UAS. The multi-layer perceptron (MLP) network architecture is considered for this purpose and different network structures are compared and analysed to determine the optimised network structure, with good generalisation performance using the aid of the cross validation method proposed. Based on the network structure selection, a recursive prediction error algorithm such as the recursive Gauss-Newton method is proposed to train the neural network model. The generalisation and adaptability performance of the recursive algorithm is then compared with the off-line algorithm to verify if the prediction quality has improved over its off-line counterpart.

2 Platform description

The UAV platform which was used in this research is a conventional electric model helicopter known as TREX600, manufactured by ALIGN Co. Ltd. The helicopter model was selected due to its sufficient payload capacity, great manoeuvrability and low cost replacement parts. It was equipped with a standard Bell-Hiller stabiliser bar on the main rotor, which improves handling characteristics for human pilots by increasing the damping on the pitch and roll

responses. Furthermore, TREX600 was also equipped with a high efficiency high torque brushless motor that allows the helicopter to carry about 2 kg payloads with an operation time of about 15 m. The basic UAV platform shown in Figure 2 had been modified to make room for installing necessary electronic equipments which gathers flight data for dynamic modelling and control system design. Some key physical parameters of TREX600 RC helicopter are given in Table 1.

Figure 2 The TREX600 helicopter used in the system identification experiment with instrumentation equipment fitted between fuselage and landing gear (see online version for colours)

Table 1 The specification of the TREX600 ESP helicopter

Specifications	*TREX600 ESP*
Length	1.16 m
Height	0.4 m
Main Rotor Diameter	1.35 kg
Tail Rotor Diameter	0.24 m
Weight	3.3 kg
Endurance	15 min

3 Collection of flight data

The flight test was conducted on the UAV helicopter platform in calm weather conditions. Different flight manoeuvres were conducted to excite the desired dynamic of interest. For example, after the helicopter reached steady and level condition, the yaw dynamics was excited using only tail collective pitch command while other input commands were used to balance the helicopter in such a way to make the vehicle oscillate roughly around the operating point of interest.

Training and validation data were collected from specifically designed frequency swept excitation signal suggested in Tischler and Remple (2006). This type of signal is commonly used to collect experimental flight data in aircraft and rotorcraft system identification. The frequency swept excitation signal is not required to have constant amplitude.

It is recommended that the pilot executes two good low frequency cycle inputs (20 s) and then gradually increase the swept frequency to mid and higher frequencies before ending the manoeuvre in the trim position. Starting and ending the record in aircraft trim state enables concatenating flight data collected from several test runs while at the same time ensuring rich signal content.

All measurements of the helicopter's state variables were collected using an inertial measurement unit (IMU) where the data that were recorded during test were Euler angles: roll ϕ, pitch θ and yaw Ψ; angular rates in body coordinate frame: roll rate, p, pitch rate, q and yaw rate, r and body accelerations: a_x, a_y, a_z. The control inputs measured during the experiment were the stick deflection from the pilot's collective pitch δ_{col}, pedal δ_{ped}, longitudinal cyclic δ_{lon} and lateral cyclic δ_{lat}. The four servomotor signals

$$s = \left[s_{AILE}\ s_{AUX}\ s_{ELE}\ s_{RUD}\right]^T,$$

can be translated to pilot's stick positions (Input range = ± 1),

$$\delta = \left[\delta_{lon}\ \delta_{lat}\ \delta_{col}\ \delta_{ped}\right]^T,$$

by means of a linear transformation:

$$\delta = A^{-1}\left(s - s_{trim}\right) \tag{1}$$

where s_{trim} are the servo signals at trim values which indicate the necessary pulse width values to level the swash plate position. Matrix A (mixing gains) has to be determined through the measurement of servo signals for different stick positions to get the exact relationship between pulse width commands sent to the servos and the requested control inputs.

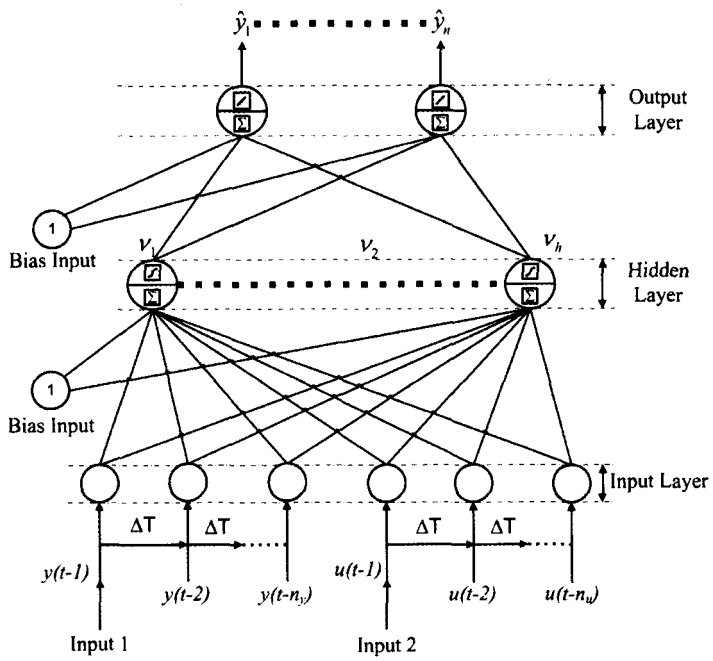
The common frequency range for the excitation signal used in rotorcraft system identification and control are between 0.3–20 rad/s. It is also recommended in Tischler and Remple (2006) that an identical filter should be used for all output and input signals with a cut-off frequency 5 times higher than the maximum excitation signal frequency. Hence to reduce the noise in sensors data, the cut-off frequency of the low pass filter used in this study was selected at 15 Hz. The sampling rate of the sensors was selected at 100 Hz which was at least 25 times higher than the maximum excitation frequency.

4 Neural network model structure

The NN based modelling approach was usually used to deduce the dynamic model of a system by taking into account the relationship between all inputs and outputs of the system. To simplify the modelling problem, the dynamic model of a helicopter was described and partition into smaller identification problems such as coupled roll-pitch equations, heave dynamics, yaw dynamics or coupling of heave and yaw dynamics or with some coupling combination among these coupling cases (Mettler, 2003). In our study, the attitude dynamics of the helicopter as coupled roll and pitch equations were considered in the system identification process. Although extra coupling from the effects of collective pitch and tail rotor collective were omitted, the current form of coupling pairings are still valid, particularly in low speed flight operation (Fan et al., 2009).

The NN input-output relationship of a dynamic system described in this research was adapted from standard ARX (Autoregressive structure with extra inputs) model structure as in Ljung (1999). In a NN based ARX (NNARX) model structure, the variable to be estimated and other influencing variables including their time lags are typically fed into a static feed-forward network such as multi-layer perceptron (MLP) network (Samarasinghe, 2007). The conceptual diagram of NNARX model structure used to identify the non-linear relationship of helicopter's attitude dynamics is shown in Figure 3. Note that the regression vector or inputs vector to the network is typically chosen to include n_y past output measurement data and n_u past input data. The number of past output and input data to be fed to the network is left for user choice.

Figure 3 The NNARX model structure with preselected regression vectors (see online version for colours)



The fully connected MLP network architecture containing only one hidden layer, was chosen to learn the non-linear relationship of the NNARX model. The output calculation from the MLP structure to represent the NNARX predictor is given as follows:

$$\hat{y}(t|\theta) = h_i(\varphi, \theta)$$
$$= F_i\left(\sum_{h=1}^{H} W_{ih} f_j\left(\sum_{j=1}^{m} w_{hj}\varphi_j + b1\right) + b2\right)$$

with,

$$h = 1, 2, 3 \cdots H$$
$$i = 1, 2, 3 \cdots n \tag{2}$$

and the parameter and regression vectors are given by:

$$\begin{aligned}\theta &= \left[w_{hj}\ W_{ih}\ b1\ b2\right] \\ \varphi(t) &= \left[\varphi_1\ \varphi_2\ \cdots\ \varphi_m\right] \\ &= \Big[y(t), y(t-1), \cdots, y(t-n_y), \quad u(t), u(t-1), \cdots, u(t-n_u)\Big]\end{aligned} \tag{3}$$

where w_{hj} is the weights matrix between the input layer and the hidden layer and W_{ih} is the weights matrix between the hidden layer and the output layer. The functions $f_j(*)$ and $F_i(*)$ are non-linear activation function for neurons in each hidden and output layers. The symbol H denotes the number of neurons in the hidden layer while $b1$ and $b2$ are the bias elements for the input layer and output layer. The number of inputs and outputs of the neural network are presented by m and n respectively.

5 Cross validation method

The choice of higher number of past output and input will results in a larger network architecture that will have a lower MSE but poor generalisation ability (Billings et al., 1992). This means that the network model predicts the estimation data set (training set) with great accuracy but fails to represent a new data that was not used in the training process. Large assignment of hidden neurons also contributes to poor generalisation performance (Wilamowski, 2009).

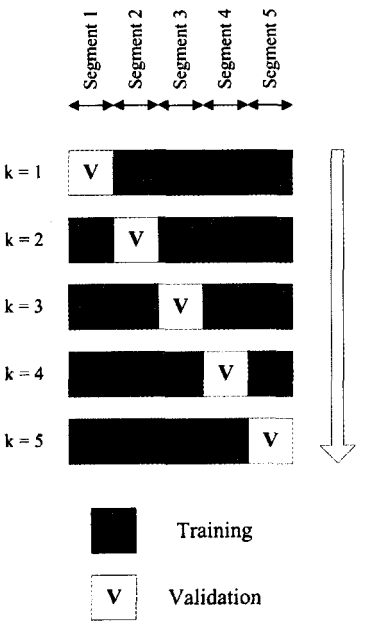
Cross validation is a statistical method that is normally used in data mining problems to determine the model structure selection and to compare generalisation performance of different learning methods. The simplest method to conduct validation analysis is to use the hold-out method where the measurement data is divided into training and test sets with user defined split ratio. Subsequently, the training set is used for model training and the test set data for error rate estimation of the trained model. However, the downside of this method is that the model evaluation can produces a high variance error. Depending on how the split ratio is defined, the prediction error evaluation may be inconsistent for different partitions of data that forms the training and test sets (Kohavi, 1995).

To overcome this problem and utilising the available overall data, the k-fold cross-validation method was used to reduce the variance by averaging error over k data segments. Kohavi (1995) suggested that cross validation with 10–20 folds would gives reasonable estimate with low bias and variance error. In this method, the measurement data N is split into k approximately equal M size data segments. Then, the training and validation are performed for k-iterations where within each iteration, a single portion of the data segment at a certain index location shown in Figure 4 will be used for validation after the training of the remaining $k-1$ data segments are completed. For each validation, the prediction MSE is calculated for the specific segment. The MSEs from each validation segment are averaged and combined together at the end of the iteration process using percentage root mean square error (% RMSE) as follows:

$$RMSE = \left[\frac{N}{kM}\frac{\sum_{i=1}^{k}\sum_{t=1}^{M}\left(\hat{y}_i(t) - y_i(t)\right)^2}{\sum_{t=1}^{N}\left(y(t) - \bar{y}(t)\right)^2}\right]^{1/2} \times 100 \tag{4}$$

where $\hat{y}_i(t)$ denoted the predicted NN model output from a specific k-validation data segment, $y_i(t)$ indicated the k-validation data segment and $\bar{y}(t)$ is the mean value of the measurement data.

Figure 4 The procedure of k-fold cross-validation for $k = 5$ (see online version for colours)



6 System identification methods

The system identification for inferring the helicopter dynamic model can be conducted using off-line (batch) and recursive based system identification methods. The estimation of a dynamic model in the off-line NN identification method involves the training process being carried out over some finite data gathered beforehand. Over the whole length of the data record, we determine the best weights (parameters vector θ) that give the best fit for the measurement data over repetitive iterations. Obviously, the off-line methods have a disadvantage such as it is unsuitable for tracking time varying dynamics, as the amount of computation time for the training phase in each iteration might exceed the available processing time (Norgaard, 2000). The adaptive control is an example where a model needs to be identified at the same time as a control law is calculated to compensate the time varying control variables. Even though the off-line training methods are not suitable for real-time implementation, several researchers have used a mini batch data size for the off-line training method implementation in real-time as proposed in (Samal, 2009; Puttige and Anavatti, 2006; Puttige, 2009). However, in these examples, the NN estimation and control were restricted only for the SISO case and smaller network due to the limited computation capabilities to invert big Hessian matrix at each iteration.

To overcome the disadvantages of the off-line training methods, the recursive based system identification methods can be used in tracking time varying dynamics. The recursive model estimation is a system identification technique that enables us to infer a model that adapts to time-varying dynamics based on real-time data coming from the system. In contrast to off-line training method, the recursive methods enforce update to NN parameter vector based only on a single data at current sample t. To achieve real-time implementation of the neural network based system identification, the estimation of neural network's parameter vector θ can be carried out using recursive algorithms as described in Billings et al. (1992), Norgaard (2000), Youmin and Li (1999), Ljung and Soderstrom (1983). The recursive identification algorithms have several advantages over the batch methods. The implementation of the method is simpler, less memory-consuming with faster convergence since the redundancy in the data set is effectively utilised (Norgaard, 2000).

Recursive algorithm can also be implemented similar to the off-line training where the recursive training is repeated several time on the finite training set Z_N, collected in advance (Norgaard, 2000; Billings et al., 1991, 1992). Figure 5 shows the difference between the batch algorithm, mini-batch algorithm, on-line recursive algorithm and repeated recursive algorithm. The parameter vector θ updating process usually start with initial random weights and it is carried out forward to the next iteration as computation progress. The implementation of batch and mini-batch algorithm is similar but differs in the number of data samples used for training. In the recursive algorithm methods, the parameter vector update is obtained in real-time as the measurement data become available from the instrumentation system. Figure 5(d) shows the implementation of recursive algorithm as an off-line method. The parameter vector θ is updated at each time sample t over a fixed data sample. At the end of first iteration, the last parameter vector θ is used as the initial update to the second iteration step. This iteration process will stopped if the mean square criterion converges to pre-defined threshold as in batch training algorithm implementation.

6.1 Off-line system identification with neural network

After selecting the model structure, the next step in the system identification process is to determine the best weights (vector parameters θ) that give the best fit between the NNARX model and measurement data. This is achieved by minimisation of error cost function. As mentioned in Norgaard (2000), the measurement of prediction's closeness to the true outputs of the system is given by mean square error (MSE) type criterion:

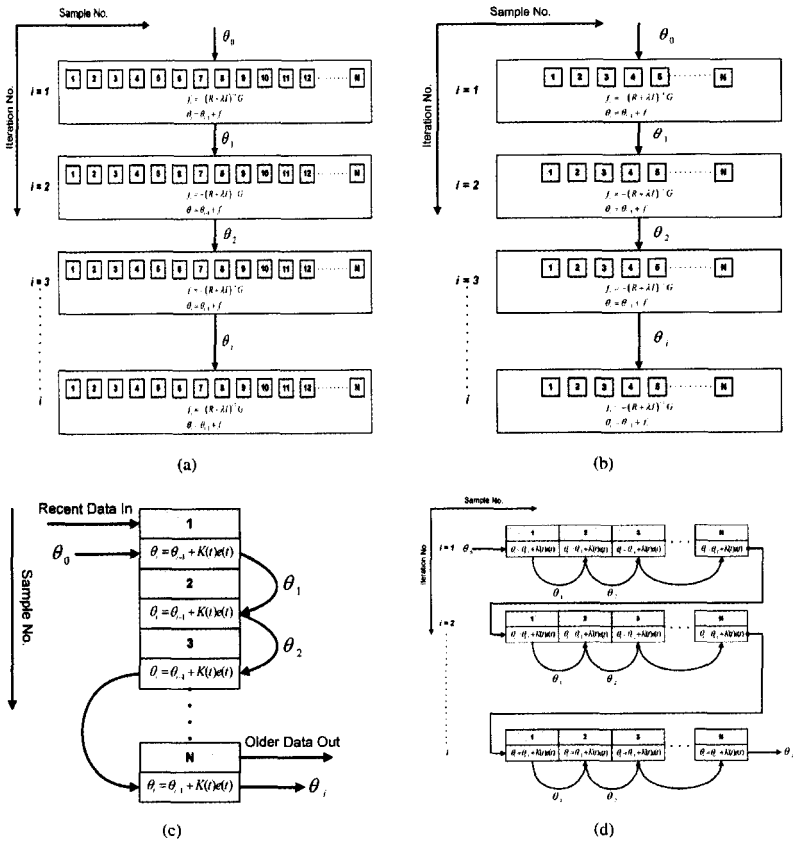
$$V_N(\theta, Z_N) = \frac{1}{2N}\sum_{t=1}^{N}[y(t) - \hat{y}(t|\theta)]^2 \tag{5}$$

with linear approximation of prediction error given by:

$$e(t, \theta) = y(t) - \hat{y}(t|\theta) \tag{6}$$

where N is the number of input-output pairs used as data test for training, $y(t)$ is the real measurement output of the system, $\hat{y}(t|\theta)$ is the predicted output vector and the training data set is given by $Z_N = [y(t), u(t)]$. For multiple input-output case (n outputs), the measurement output of the system $y(t)$ and predicted output $\hat{y}(t|\theta)$ will became a $n \times N$ matrix which would produced a vector of MSE criterion.

Figure 5 The different types of neural network model estimation methods: (a) batch algorithm; (b) mini-batch algorithm; (c) recursive algorithm and (d) repeated recursive algorithm (see online version for colours)



In order to minimise the cost function in equation (5), the Levenberg-Marquardt (LM) iterative search algorithm was used for the neural network training process. The optimisation process is carried out iteratively over a given data set to achieve the minimum error criterion. The LM optimisation algorithm uses the Gauss-Newton Gradient $G(\theta)$ and Hessian $R(\theta)$ matrices, which were derived specifically for the neural network model in Yu and Wilamowski (2011), Ngia and Sjoberg (2000) and Norgaard (2000). These important matrices are represented in the following equations:

$$G(\theta) = \frac{1}{N}\sum_{t=1}^{N} \psi(t|\theta)\left[e(t,\theta)\right] \tag{7}$$

$$R(\theta) = \frac{1}{N}\sum_{t=1}^{N} \psi(t|\theta)\left[\psi(t|\theta)\right]^{T} \tag{8}$$

where $\psi(t|\theta)$ is the Jacobian matrix that represent the first derivative of the one-step ahead prediction with respect to parameters vector θ. The procedure to calculate matrix $\psi(t|\theta)$ is

given in detailed in Yu and Wilamowski (2011) and Norgaard (2000). The Hessian matrix $R(\theta)$ has a dimension of $d \times d$ and Gradient vector $G(\theta)$ with dimension of $d \times 1$, where d is the total number of elements (weights + biases) in the parameter vector θ.

We could find the minimum of the error criterion by iteratively solving the following equations:

$$\theta^{(i+1)} = \theta^{(i)} + f^{(i)} \tag{9}$$

$$\left[R\left(\theta^{(i)}\right) + \lambda^{(i)} I\right] f^{(i)} = -G\left(\theta^{(i)}\right) \tag{10}$$

where $f^{(i)}$ is the search direction vector, I is the identity matrix and $\lambda^{(i)}$ is a damping factor used for deciding the step size. In order to determine λ, the indirect method used in Norgaard (2000) is adopted by calculating the following ratio to determine the accuracy of approximation:

$$r^{(i)} = \frac{2\left[V_N\left(\theta^{(i)}, Z_N\right) - V_N\left(\theta^{(i)} + f^{(i)}, Z_N\right)\right]}{\underbrace{\left(f^{(i)}\right)^T G\left(\theta^{(i)}\right) + \left(f^{(i)}\right)^T f^{(i)} \lambda^{(i)}}_{\text{reduction approximation}}} \tag{11}$$

The main purpose of introducing the ratio calculation is to measure how well the reduction of the criterion $V_N(\theta, Z_N)$ matches the reduction predicted by approximation terms in denominator of ratio calculation in equation (11). The damping factor λ is adjusted accordingly to the ratio $r^{(i)}$ by some factor (Norgaard, 2000). The procedure of the LM algorithm with indirect method to determine λ is given in Figure 6. The reduction approximation (denominator term in equation (11)) is most likely a close approximation to error criterion $V_N(\theta, Z_N)$, if the ratio $r^{(i)}$ value is close to one and parameter λ should be reduced by some factor. However, if the ratio $r^{(i)}$ is small or a negative value, parameter λ should be increased. Additional stopping criterions are normally introduced to this algorithm to prevent minimisation problems or to force early stopping such as stopping criteria based on maximum number of iterations, sum of square error that drops below a certain threshold, upper bound for gradient and maximum weight change, maximum value of parameter λ or early stopping criterion due to training time constraint.

6.2 *Recursive system identification with neural network*

Recursive model estimation is a system identification technique that enables us to infer a model that adapts to time-varying dynamics based on real-time data coming from the system. To achieve real-time implementation of NN based system identification, the estimation of neural network's parameter vector θ can be carried out using recursive algorithms such as described in Billings et al. (1992) and Youmin and Li (1999). Batch method described in the previous section is deemed unsuitable for tracking time varying dynamic as the amount of computation time for the training phase in each iteration might exceed the available processing time (Norgaard, 2000).

The recursive system identification method builds a model of the system at the same time as the measurement data is collected. The model is then updated at each time step, as new data become available. In our study, the weight updating procedure is calculated using

recursive Gauss-Newton (rGN) method. For every data sample, the parameter vector $\hat{\theta}(t)$ is updated by the recursive algorithm using the following equations:

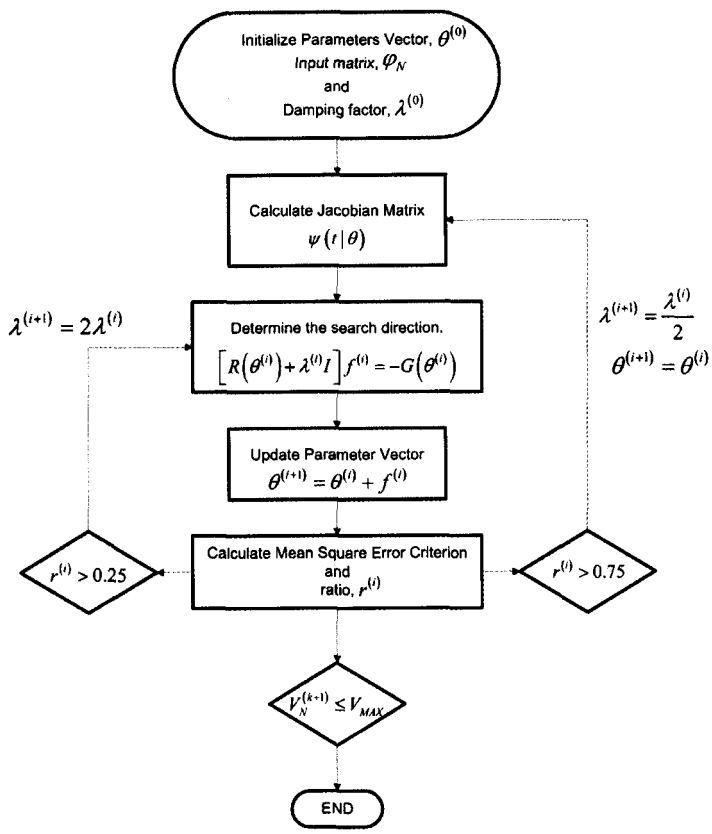
$$e(t) = y(t) - \hat{y}(t) \tag{12}$$

$$R(t) = R(t-1) + \gamma(t)\left[\psi(t)\hat{\Lambda}^{-1}(t)\psi^{T}(t) - R(t-1)\right] \tag{13}$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \gamma(t) R^{-1}(t)\psi(t)\hat{\Lambda}^{-1}(t)e(t) \tag{14}$$

where $R(t)$ is an approximation of Gauss-Newton Hessian matrix, $\hat{\theta}^{t}$ is the estimation of parameter vector of the NN model, $\hat{\Lambda}^{-1}(t)$ is the weighting matrix and $\gamma(t)$ is the gain sequence at the current time step t. The simplest choice of weighting matrix $\hat{\Lambda}^{-1}(t)$ is an identity matrix as suggested by Billings et al. (1992). The forgetting factor $\lambda(t)$ is defined as a constant scalar variable which accounts for the amount of past data information to be included in the error criterion function. If the forgetting factor is $\lambda(t) < 1$, the term would make the estimation more adaptable to changes and sensitive to noise. Whereas, if $\lambda(t) \to 1$ as time increases, more old data are included in the criterion and the adaptation would fluctuate less during the learning process (Youmin and Li, 1999).

Figure 6 The Levenberg-Marquardt (LM) algorithm with step involving λ determination



In practice, equations (12)–(14) are not calculated straightforward with inversion of matrix $R^{-1}(t)$ which requires computational complexity of $O(d^3)$ (Ngia and Sjoberg, 2000). Ljung and Soderstrom (1983) had shown that using matrix inversion theorem, the generalised rGN algorithm is rewritten to avoid full Hessian matrix inversion as follows:

$$P(t) = \left[P(t-1) - L(t) S^{-1}(t) L^T(t)\right] / \lambda(t) \tag{15}$$

$$S(t) = \psi^T(t) P(t-1) \psi(t) + \lambda(t) \hat{\Lambda}(t) \tag{16}$$

$$L(t) = P(t-1) \psi(t) S^{-1}(t) \tag{17}$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + L(t) e(t) \tag{18}$$

Note that the inversion of matrix $R^{-1}(t)$ had been reduced from full inversion of $d \times d$ matrix to $S^{-1}(t)$ with $n \times n$ dimension. Note that n denotes the number of outputs predicted in the model.

Equations (15)–(18) indicates that initial value of $P(0)$ ($d \times d$ matrix) and parameter vector $\hat{\theta}(0)$ need to be supplied by user at the beginning of the iteration. The initial parameter vector $\hat{\theta}(0)$ is usually selected as random values or pre-determined weights resulting from the off-line training. A common choice for $P(0)$ is $P(0) = \rho I$ with ρ being a large positive number (i.e., $10^2 \rightarrow 10^4$) indicating little confidence in $\hat{\theta}(0)$. This would cause the estimation to rapidly increase in the transient phase for a short period of time δt (Ljung and Soderstrom, 1983). As the estimation of parameters are quite poor at the beginning of the iteration, a lower forgetting factor should be selected at the initial stage for rapid adaptation and approaching unity as the time increases. The following strategies introduced by Ljung and Soderstrom (1983) is used for updating the forgetting factor term:

$$\lambda(t) = \lambda_o \lambda(t-1) + (1 - \lambda_o) \tag{19}$$

where λ_o and $\lambda(0)$ are design variables. The typical values of λ_o is 0.99 and $\lambda(0)$ is in the range of $0.95 < \lambda(0) < 1$.

According to Ljung and Soderstrom (1983), the recursion of equation (15) is numerically unstable due to round-off errors which build up and influence $P(t)$ to become indefinite. The numerical problem involving matrix $P(t)$ can also be corrected using several matrix factorisation techniques such as Potter's square root algorithm, Cholesky decomposition or UD factorisation which in turn gives better numerical properties compared with the straightforward calculation of $P(t)$ (Ljung and Soderstrom, 1983). The factorisation algorithms required roughly the same amount of computation to update $P(t)$ in equation (15) (Bierman, 1977). In this work, the Potter's square root algorithm is considered for the problem because of the algorithm's simple implementation.

The Potter's square root algorithm describes matrix $P(t)$ in terms of the following factorisation:

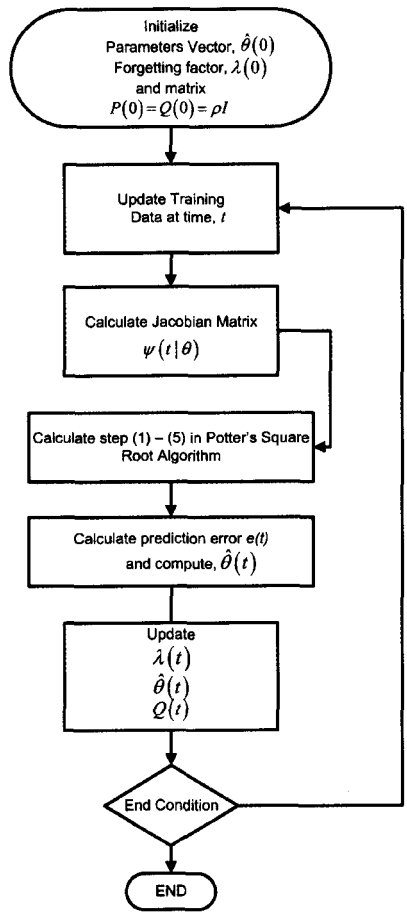
$$P(t) = Q(t) Q^T(t) \tag{20}$$

where $Q(t)$ is selected as a square non-singular matrix. The $Q(t)$ matrix is then calculated using the following algorithm in Table 2 at each time step. The implementation of recursive Gauss-Newton algorithm for NN based system identification using Potter's factorisation algorithm is given in Figure 7. Since the parameter vector $\hat{\theta}(t)$ in recursive algorithms is updated at the same time as the sensor data is collected, the update remains in indefinite loop as long as a stopping condition is supplied by the user.

Table 2 Potter's square algorithm

a) Initialise $P(0) = Q(0)Q^T(0)$ at time $t = 0$
b) For each time step t, update $Q(t-1)$ by performing step 1–6
1. $f(t) = Q^T(t-1)\psi(t)$
2. $\beta(t) = \lambda(t) + f^T(t) f(t)$
3. $\alpha(t) = 1/\left[\beta(t) + \sqrt{\beta(t)\lambda(t)}\right]$
4. $\bar{L}(t) = Q(t-1) f(t)$
5. $\bar{Q}(t) = \left[Q(t-1) - \alpha(t)\bar{L}(t) f^T(t)\right]/\sqrt{\lambda(t)}$
6. $Q(t) = \frac{\rho_{max}-\rho_{min}}{tr\{\bar{Q}(t)\}}\bar{Q}(t) + \rho_{min} I$
c) Compute parameter vector as:
$\hat{\theta}(t) = \hat{\theta}(t-1) + \bar{L}(t)\left[e(t)/\beta(t)\right]$

Figure 7 The recursive Gauss-Newton (rGN) algorithm with Potter's square root factorisation



Noted that the matrix $P(t)$ in equation (15) may happen to be singular or nearly singular if the model set contains too many parameters or if the input signal is not general enough (Ljung and Soderstrom, 1983). This problem can be overcome by introducing a lower and upper bounds on the eigenvalues of $P(t)$. Several variations of recursive Gauss-Newton algorithms such as Constant Trace (CT) and Exponential Forgetting and Resetting Algorithm (EFRA) have been proposed in various examples to overcome the unstable numerical $P(t)$ recursion (Norgaard, 2000; Salgado et al., 1988). By using the CT method, Step 6 in Table 2 is introduced to bound the eigenvalues of the $P(t)$. The ρ_{max} and ρ_{min} are the maximum and minimum eigenvalues respectively, and the values are selected so that $\rho_{max}/\rho_{min} \simeq 10^5$. The initial $Q(0)$ should be selected as a diagonal matrix, $\rho_{min} I \leq Q(0) \leq \rho_{max} I$.

7 Results and discussion

The experimental data from different flight manoeuvres was collected and concatenated into a single recording with the measurements of the servo PWM signals were rescaled to the appropriate pilot's input command range. The flight test data were divided into training, validation and test data sets. The training and validation data sets were used for purpose of NN training and model structure selection. The test data set was used for the final evaluation of the NN model prediction accuracy. The pilot's input command range is normalised between -1 to $+1$ for longitudinal cyclic, lateral cyclic and yaw pedal cyclic inputs, while the collective cyclic input is scaled between 0 to $+1$. Using the collected data, the suitable regression vector structure and hidden neurons size were determined using the k-fold cross validation technique previously discussed.

The lowest error resulting from the k-fold cross validation was used as the network structure for the recursive training algorithm. A fully connected MLP architecture was used for the NN training in cross validation with the number of hidden neurons gradually increased. The tangent hyperbolic and linear activation functions were used in the hidden and output layer of the NN model. In this study, the flight data obtained from the experiment was divided into 10 approximately equal segments. In the validation stage, the error calculation was then stored for every network structure and hidden neuron case. Subsequently, the stored error calculation was then retrieved at the end of the validation cycle for RMSE computation.

The results of k-fold cross validation for the off-line NN model is given in Figure 8. For each neuron size, different network structures were tested and compared with each other. From the plot, network structure with 3 past outputs and 1 past input (regression vector or input nodes with dimension size of 8) gives the lowest RMSE value for neurons size, $h = 4$. This network structure was then used as the basic architecture for comparing the generalisation performance of the recursive Gauss-Newton method with the off-line NN model. Note that the neuron size $h = 8$ gives a comparable low RMSE values. However, it does not indicates that the prediction displays good generalisation performance since neuron size $h = 5$, has a sudden increase in RMSE calculation. The effect of noise has affected the error calculation for the neuron size $h = 5 \rightarrow 8$. However, the validity test is still useful in aiding the selection of an appropriate network structure (Billings et al., 1992). Furthermore, it is not advisable to use an excessive number of neurons which may lead to an over-fitting problem. Finally, we arrive at the following network specifications (Table 3) that adequately represent the attitude dynamics of a model scaled helicopter.

Figure 8 The percentage of root mean square error (RMSE) for each network structure and number of neurons. The neural network training was carried out using off-line Levenberg-Marquardt (LM) (see online version for colours)

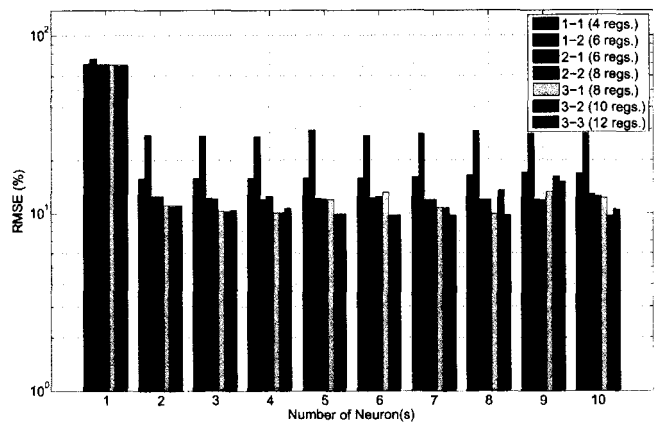


Table 3 The MLP neural networks model parameters

MLP network specifications for attitude dynamics	
Number of past outputs	3
Number of past inputs	1
Number of neurons in hidden layer	4
Activation function at hidden layer	Tanh
Activation function at output layer	Linear
Number of regressors	8
Total number of weights	46
Weight decay	0.0001

An example of the one-step ahead prediction of the angular rate responses that are estimated from the off-line neural network system identification is shown in Figures 9 and 10. The network is trained using the nearly optimal structure from Table 3. These predicted responses from neural network identification (NNID) are overlaid with the measured helicopter responses. The results indicate that one-step ahead NNID predictions overlap the test data almost perfectly as indicated by the magnitude order of the prediction error plot. This usually happens when the sampling frequency of the data collected is high compared with the frequency of the dynamic system as suggested in Norgaard (2000).

This work also utilised the k-fold cross validation method to identify the efficiency of the selected neural network training methods in estimating the attitude dynamics of the helicopter. In order to compare the generalisation performance of rGN method against the off-line LM method, the training of rGN is repeated a several of time on a finite data set instead of assuming that the data set increases with time as in the recursive training scheme. After reaching the maximum iterations or performance index threshold, the resulting parameter vector θ is then selected for cross validation. The rGN design parameters are initialised as $P(t) = 100I, \lambda_o = 0.99$ and $\lambda(0) = 0.995$. Figure 11 indicates the generalisation error plot for rGN and off-line LM methods for 10 runs. As seen in

this plot, we consider a network structure of 3 past output and 1 past input as a network structure for both training algorithms. The recursive training algorithm (rGN) exhibits a slightly higher generalisation error in cross validation compared with the off-line method. This indicates that training performed over a large data set would give better generalisation performance over the recursive method.

Even though the generalisation error of rGN is slightly higher than the off-line LM, the rGN is more adaptive to the changes in dynamic properties. As a comparative study of the adaptability between the off-line LM and rGN methods, the roll rate measurement from a new data set is considered. Figure 12 shows the prediction from a pre-trained model using off-line LM and rGN method. The corresponding error statistics for these prediction models is given in Table 4. In Figure 12(a) and (b), the off-line model (NN 1) is pre-trained using 1 past output and 2 past inputs with 4 hidden neurons (training with 746 samples) while the recursive model (NN 2) is also trained with the same model structure. The training of rGN is carried out using the sliding window method where older data is discarded from the window to allow present data to enter. Thus, it can be seen that the off-line model follows the output measurement accurately at the beginning of the data length and its prediction begins to deteriorate for the remaining data. Whereas, the prediction from the model trained using rGN algorithm adapts well to the dynamic changes that occur during flight even though it was not trained using the optimal structure. In Figure 12(c), the prediction model (NN 3) using the optimal model structure ($n_y = 3$, $n_u = 1$ with 4 hidden neurons) gives the best RMSE accuracy with 19.454% and 11.611% for roll rate and pitch rate respectively. Note that the RMSE values for recursive training (NN3) is slightly higher than results obtained in Figure 11 since the recursive training is done with a single pass to the data compared with the results obtained in repeated recursive training.

Figure 9 The prediction from MLP network model for roll dynamics. (a) The one-step ahead prediction and measurement data plot. (b) The error plot between one-step ahead prediction and the measurement data. The red dashed line indicates estimation from the neural network model while the solid blue line with 'x' marker represents the output measurement (see online version for colours)

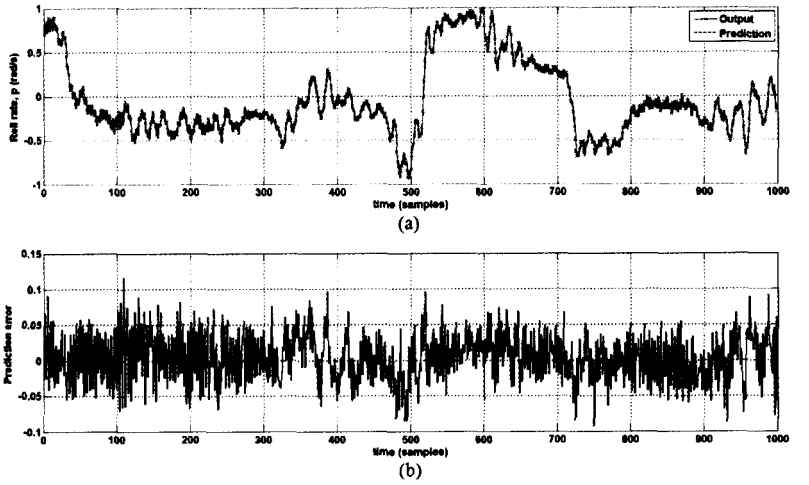


Figure 10 The prediction from MLP network model for pitch dynamics. (a) The one-step ahead prediction and measurement data plot. (b) The error plot between one-step ahead prediction and the measurement data. The red dashed line indicates estimation from the neural network model while the solid blue line with 'x' marker represents the output measurement (see online version for colours)

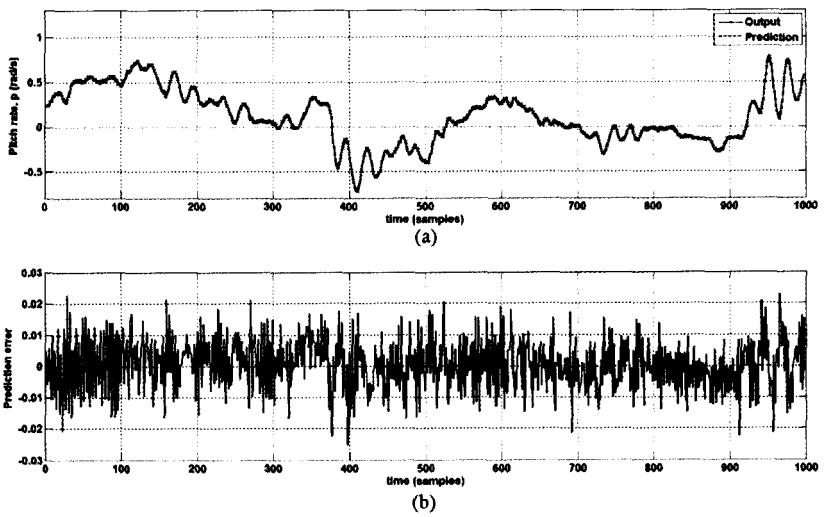


Figure 11 The percentage of root mean square error (RMSE) comparison plot for off-line Levenberg-Marquardt (LM) and recursive Gauss-Newton (rGN) training methods (see online version for colours)

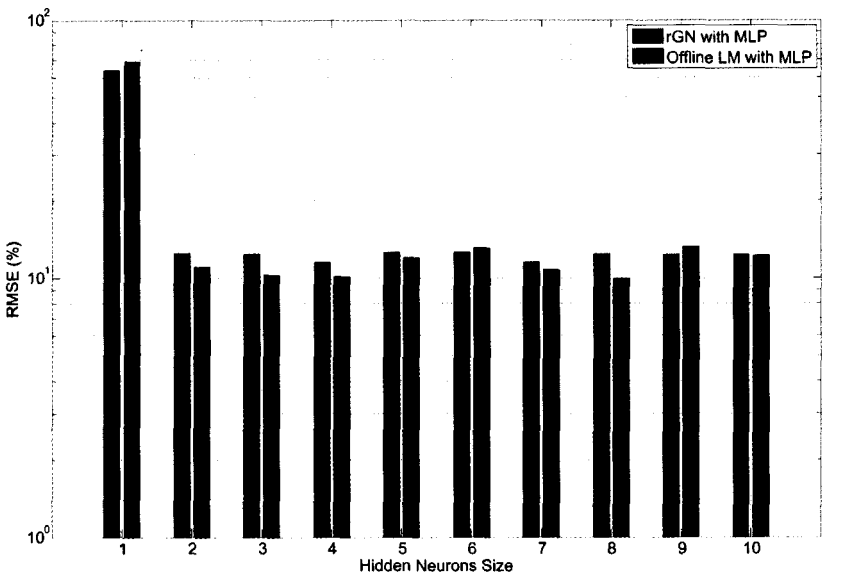
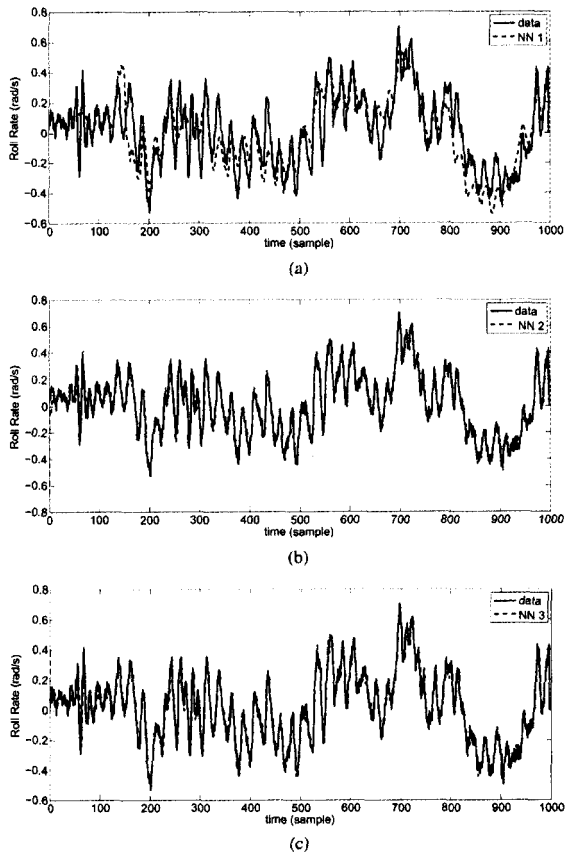


Figure 12 The prediction comparison of the off-line model trained by Levenberg-Marquardt (LM) method and recursive model trained using recursive Gauss-Newton (rGN) method against roll rate measurement. (a) NN model 1 was trained using the off-line LM algorithm. The NN model structure was set with $n_y = 1$ and $n_u = 2$ with 4 hidden neurons; (b) NN model 2 was trained using the recursive Gauss-Newton algorithm. The NN model structure was set with $n_y = 1$ and $n_u = 2$ with 4 hidden neurons and (c) NN model 3 was trained using the recursive Gauss-Newton algorithm. The NN model structure was set with optimised structure obtained from k-fold cross validation ($n_y = 3$ and $n_u = 1$ with 4 hidden neurons (see online version for colours)



In on-line system identification for adaptive control application, the training time for NN model needs to be less than the sampling time of the control loop. This is essential since the control decisions need to be updated at the specific timing requirement (less than 22 ms). There are two type of recursive algorithm method exists to approximate the non-linear dynamics in real-time:

- mini-batch methods (Samal, 2009; Puttige, 2009)
- recursive prediction methods as presented in previous section.

For mini-batch wise methods, the off-line training such as LM algorithm was used to train the NN in real-time by choosing smaller data length to achieve faster convergence time. Typically, a fixed amount of input-output data is collected and stored in a queue. Table 5 shows the average training time for mini-batch LM and rGN training algorithms for attitude dynamics identification using the optimal NN model structure ($n_y = 3, n_u = 1$ with 4 hidden neurons). The minimum criterion error (MSE) was selected at 0.001, 0.01 and 0.05 as stopping criteria for mini-batch LM training. The training time comparison test was conducted using a 400 MHz National Instrument's real-time embedded controller. Result from the comparison test shows that faster training convergence is achieved with smaller batch sizes. However, mini-batch method still requires a lot more computation resources and would not finish within targeted sampling period (22 ms). Attempts to reduce the training time of the NN training through manipulation of target MSE values could improve the algorithm training performance, but at the expense of poor training error. A recursive training algorithm such as rGN usually demonstrates faster prediction updates and offers rapid computation of weight adaptation with average training time of 3.88 ms. The average training time for rGN algorithm is well below the control loop sampling period (22 ms) and this indicates that such recursive training algorithms are well suited for real-time applications.

Table 4 Summaries of error statistics for training algorithms comparison

Error statistics				
Training	*System responses*	*RMSE*	*RMSE* (%)	R^2
NN 1	p	0.08891	42.376	0.8506
	q	0.03495	16.657	0.9647
NN 2	p	0.04907	23.366	0.9451
	q	0.03665	17.454	0.9691
NN 3	p	0.09289	19.454	0.9620
	q	0.04244	11.611	0.9863

Table 5 Summaries of error statistics for training algorithms comparison

		Average training time (ms)				
		Data sample				
	Target MSE	$N = 5$	$N = 10$	$N = 15$	$N = 20$	$N = 25$
Mini batch	0.001	38.29	44.98	46.373	50.84	54.78
LM 1		[5.02%]	[6.61%]	[7.13%]	[7.53%]	[7.89%]
Mini batch	0.01	24.48	25.11	24.89	26.38	27.53
LM 2		[13.19]	[15.72%]	[16.87%]	[18.00%]	[18.47]
Mini batch	0.05	23.22	23.13	26.38	27.37	28.86
LM 3		[22.07%]	[24.16%]	[26.83%]	[28.16]	[28.48]
				Data sample N=1		
rGN				3.88 [5.50%]		

8 Conclusion

The methods and results presented in this paper indicate the suitability and effectiveness of off-line and recursive based neural network modelling for representing coupled UAS helicopter dynamics with acceptable accuracy. Results indicate that although the generalisation error of rGN is slightly higher than off-line LM, the rGN is more adaptive to the changes in dynamic properties. The rGN method is also capable to produce a satisfactory prediction quality even-though the model structure was incorrectly selected. The generalisation and adaptability performance can be further improved by properly selecting the optimised network structure with the aid of k-fold cross validation method. It can also be concluded that the recursive method presented here is suitable to model the UAS helicopter in real-time within the computational resource constraints. These models can be further used for the design of adaptive flight controllers for autonomous flight.

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References

Agarwal, M. (1997) 'A systematic classification of neural-network-based control', *Control Systems, IEEE*,Vol. 17, No. 2, pp.75–93.

Balakrishnan, S.N. and Weil, R.D. (1996) 'Neurocontrol: a literature survey', *Mathematical and Computer Modelling*, Vol. 23, Nos. 1–2, pp.101–117, doi: 10.1016/0895-7177(95)00221-9.

Bierman, G.J. (1977) *Factorization Methods for Discrete Sequential Estimation*, Volume 128, Academic Press, New York.

Billings, S.A., Jamaluddin, H.B. and Chen, S. (1991) 'A comparison of the backpropagation and recursive prediction error algorithms for training neural networks', *Mechanical Systems and Signal Processing*, Vol 5, No. 3, pp.233–255.

Billings, S.A., Jamaluddin, H.B. and Chen, S. (1992) 'Properties of neural networks with applications to modelling non-linear dynamical systems', *International Journal of Control*, Vol. 55, No. 1, pp.193–224.

Budiyono, A., Joon, Y.K. and Daniel, F.D. (2009) 'Integrated identification modeling of rotorcraft-based unmanned aerial vehicle', *17th Mediterranean Conference on Control and Automation*, 24–26 June, IEEE, Thessaloniki, Greece, pp.898–903.

Calise, A.J. and Rysdyk, R.T. (1998) 'Nonlinear adaptive flight control using neural networks', *Control Systems, IEEE*, Vol. 18, No. 6, December, pp.14–25, ISSN 1066-033X, doi: 10.1109/37.736008.

Fan, C., Baoquan, S., Xuanping, C. and Yunhui, L. (2009) 'System identification and attitude control of a small scale unmanned helicopter', *IEEE International Conference on Robotics and Biomimetics, 2008. ROBIO 2008*, Bangkok, Thailand, pp.1342–1347.

Jiang, Z., Han, J.D., Wang, Y.C. and Song, Q. (2006) 'Enhanced LQR control for unmanned helicopter in hover', *1st International Symposium on Systems and Control in Aerospace and Astronautics, 2006. ISSCAA 2006*, January, Harbin, China, pp.1438–1443, doi: 10.1109/ISSCAA.2006.1627508.

Kendoul, F. (2012) 'Survey of advances in guidance, navigation, and control of unmanned rotorcraft systems', *Journal of Field Robotics*, Vol. 29, No. 2, pp.315–378, ISSN 1556-4967.

Kohavi, R. (1995) 'A study of cross-validation and bootstrap for accuracy estimation and model selection', *The 1995 International Joint Conference on Artificial Intelligence*, Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, pp.1137–1145.

Ljung, L. (1999) *System Identification: Theory for the User*, 2nd ed., Prentice Hall information and system sciences Series, Prentice Hall PTR, Upper Saddle River, NJ.

Ljung, L. and Soderstrom, T. (1983) *Theory and Practice of Recursive Identification*, The MIT Press series in signal processing, optimization, and control, MIT Press, Cambridge, Mass.

Mettler, B. (2003) *Identification Modeling and Characteristics of Miniature Rotorcraft*, 1st ed., Kluwer Academic Publishers, Boston.

Ngia, L.S.H. and Sjoberg, J. (2000) 'Efficient training of neural nets for nonlinear adaptive filtering using a recursive levenberg-marquardt algorithm', *IEEE Transactions on Signal Processing*, Vol. 48, No. 7, pp.1915–1927.

Norgaard, M. (2000) *Neural Networks for Modelling and Control of Dynamic Systems: A Practitioner's Handbook*, 2nd ed., Advanced textbooks in control and signal processing, Springer, Berlin, New York.

Putro, I.E., Budiyono, A., Yoon, K.J. and Kim, D.H. (2009) 'Modeling of unmanned small scale rotorcraft based on neural network identification', *2008 IEEE International Conference on Robotics and Biomimetics*, Bangkok, Thailand, pp.1938–1943.

Puttige, V.R. and Anavatti, S.G. (2006) 'Real-time neural network based online identification technique for a UAV platform', *International Conference on Computational Intelligence for Modelling, Control and Automation, 2006 and International Conference on Intelligent Agents, Web Technologies and Internet Commerce*, Sydney, NSW, p.92-92.

Puttige, V.R. (2009) *Neural Network based Adaptive Control for Autonomous Flight of Fixed Wing Unmanned Aerial Vehicles*, PhD Thesis, School of Aerospace, Civil and Mechanical Engineering, Australian Defence Force Academy, University of New South Wales, August 2009, URL: http://handle.unsw.edu.au/1959.4/43736.

Salgado, M.E., Goodwin, G.C. and Middleton, R.H. (1988) 'Modified least squares algorithm incorporating exponential resetting and forgetting', *International Journal of Control*, Vol. 47, No. 2, pp.477–491, URL: http://www.tandfonline.com/doi/abs/10.1080/00207178808906026.

Samal, M.K., Anavatti, S. and Garratt, M. (2009) 'Real-time neural network based identification of a rotary-wing UAV dynamics for autonomous flight', *IEEE International Conference on Industrial Technology, 2009. ICIT 2009*, Gippsland, VIC, pp.1–6.

Samal, M. (2009) *Neural Network based Identification and Control of an Unmanned Helicopter*, PhD Thesis, School of Engineering and Information Technology, University of New South Wales, Sydney, NSW, Australia.

Samal, M.K., Anavatti, S. and Garratt, M. (2008) 'Neural network based system identification for autonomous flight of an eagle helicopter', *17th World Congress The International Federation of Automatic Control*, Vol. 17, pp.7421–7426.

Samarasinghe, S. (2007) *Neural Networks for Applied Sciences and Engineering: From Fundamentals to Complex Pattern Recognition*, 1st ed., Auerbach Publications, Boca Raton, FL, ISBN 084933375X, 9780849333750.

Shamsudin, S.S. and Chen, X.Q. (2012) 'Identification of an unmanned helicopter system using optimized neural network structure', *International Journal of Modelling, Identification and Control*, Vol. 17, No. 3, pp.223–241.

Shim, D.H. (2000) *Hierarchical Flight Control System Synthesis for Rotorcraft based Unmanned Aerial Vehicles*, PhD Thesis, University of California, Berkeley.

Sjoberg, J., Zhang, Q.H., Ljung, L., Benveniste, A., Delyon, B., Glorennec, P-Y., Hjalmarsson, H. and Juditsky, A. (1995) 'Nonlinear black-box modeling in system identification: a unified overview', *Automatica* Vol. 31, No. 12, pp.1691–1724.

Suresh, S., Kumar, M.V., Omkar, S.N., Mani, V. and Sampath, P. (2002) 'Neural networks based identification of helicopter dynamics using flight data', *9th International Conference on Neural Information Processing ICONIP (2002)*, Vol. 1, pp.10–14.

Tischler, M.B. and Remple, R.K. (2006) *Aircraft and Rotorcraft System Identification : Engineering Methods with Flight-Test Examples*, AIAA education series. American Institute of Aeronautics and Astronautics, Reston, VA, 2006.

Wilamowski, B.M. (2009) 'Neural network architectures and learning algorithms', *Industrial Electronics Magazine, IEEE*, Vol. 3, No. x, pp.56–63.

Youmin, Z. and R. Li, X. (1999) 'A fast U-D factorization-based learning algorithm with applications to nonlinear system modeling and identification', *IEEE Transactions on Neural Networks*, Vol. 10, No. 4, pp.930–938.

Yu, H. and Wilamowski, B. (2011) *Levenberg-Marquardt Training*, chapter 12, p.1-1.