ARX Model of Four Types Heat Exchanger Identification

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Abstract - Heat exchangers are devices built for efficient heat transfer from one fluid to another and widely used in engineering process. The four heat exchangers consist of shell and tube, spiral, concentric (double pipe), and plate heat exchangers available in UTHM Laboratory. This paper is aim to identify model nonlinear of the four heat exchanger types. To identify this model, need to use mathematical models of dynamic system using measure the input and output signal of the real system. The process identification requires select model structure, parameter estimation and model validation. ARX (autoregressive with exogenous input) model structure is chosen to fitting criterion and parameter estimation determined and the data measured from the real system will be used model validation. The M-File MATLAB program has generated to simulate this model to analyze the heat exchangers performance and the best model of these heat exchangers is obtained that based on the fitting which more than 50% and was close to 100%.

Keyword: Identification, ARX model, four types heat exchanger, M-File MATLAB Program.

I. INTRODUCTION

Heat exchanger which is a facility that was installed in UTHM is called four types heat exchanger model HE 158C. There are also four different designs of heat exchangers: Shell and tube, spiral heat exchanger, concentric heat exchanger and plate heat exchanger (cocurrent and counter current) [1]. To design a controller of HE158C requires an identification system to find a mathematical equation. There are many methods identification system are linear models (ARX, ARMX, OE, GPM, Steady State) are convenient from a computational perspective, and furthermore, their analysis and use for control design is much simpler than in the case of nonlinear models (NLARX, NLOE, NNARX, NNARMAX) [2]. The mathematical models equations of ARX model are the foundation of most scientific and engineering method data can be obtained by either a theoretical approach based on physical laws, or an experimental approach based on obtained measurements from the system. There are cases when linear models not well enough which is when the system is highly nonlinear and when the system is nonlinear and it operates over a wide range of operation Figure 1 will show the real system of four types heat exchanger model HE158C. The main objective of this paper is to find the mathematical model for four types heat exchanger using ARX model. For all types heat exchanger HE158C have different on the variable input and output. The number input and output have a five data in co current and counter current with the input is a temperature and the output is a water flowrate of each type heat exchanger. The estimation algorithm use linear least square method to fit linear model to data. A linear model is defined as an equation that is linear in the coefficients, efficiency, simplicity of operation and robustness.

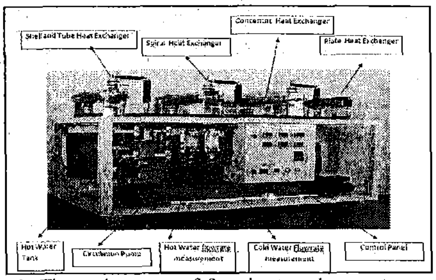

Figure 1: The real system of four heat exchanger types model HE158C.

The system identification is an offline system. The first selection of the experiment about nonlinear ARX models conducting by Qinghua Zhang and Lennart Ljung [5] is about NLARX (Nonlinear Autoregressive with exogenous inputs) models which are frequently used in black-box nonlinear system identification. The main difficulty is that in general there is no easy way to compute the mathematical expectation of an output conditioned by past measurements. An optimal solution would require intensive numerical computations related to nonlinear filtering. The purpose of this paper is to investigate simple non-optimal prediction methods. It is shown that caution must be paid when using such methods, since the prediction behaviours may be radically different, depending on some detailed choices. In this paper, the authors present dynamic models for identification of linear time invariant system in process industries are ARX (Autoregressive with exogenous input) and FIR (Finite Impulse Response) models. However, need very large amount of data to reduce variance error, in addition ordinary ARX model structures lead to inconsistent model parameters. The model parameters developed using linear least square method. The second selection by Tatang Mulyana [7] presented a simple method to determine a discrete time in heat exchanger model HE158C. The all discrete time model resulted is a first order system. The model can be designed with four different types of heat exchangers and two stainless steel sump tanks for hot and cold water sources. The four heat exchangers consist of shell and tube, spiral, concentric (double pipe), and plate heat exchangers. This paper covers a discrete time model of the four heat exchanger types. The modelling process includes the following steps, i.e. experiment design, selection of model structure, choice of criterion to fit, parameter estimation, and model validation. The experiment designs are to study the reliability and accuracy of the existing equipment available and to collect the real data set input-output and data pre-processing. The parameters of interest are estimated using the least squares method. The results are calculated using MATLAB program.

II. SYSTEM IDENTIFICATION

System identification is the process of developing or improving a mathematical model of a physical system using experimental data. System identification is especially useful for modelling systems and cannot easily represent in terms of first principles. This is accomplished by adjusting parameters within a given model until the result matches relatively well with the measured data. Model validation can be carried out by using a separate set of data, which differs from estimating data, to evaluate the model's properties and for validation purpose. Figure 2 show flowchart of procedure system identification.

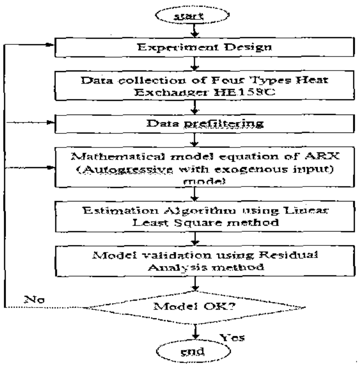

Figure 2: Flowchart of procedure system identification

Using experimental data obtained from input-output relations to model dynamic systems then Figure 3 will show a block diagram of input and output relations to model dynamics systems.

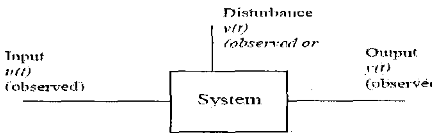

Figure 3: An input and output relations to model dynamic system

III. METHODOLOGY

In this methodology part, there are two parts which system identification process and software. The system identification is the experimental approach to process modelling. System identification includes the following steps: experimental design, model structure, and parameter estimation-linear least square, model validation-residual anlysis. M-File MATLAB program is used for doing the system identification process which is to analysis the graph of the system. The coefficient of an optimisation problem will be solved from obtained the numerical values of the model parameters. Experimental design is purpose to obtain good experimental data and includes the choice of the measured variables and of the character of the input and output signal. The data co current and counter current of four types heat exchanger HE158C is getting when step system identification is running. A suitable model structure is chosen using prior knowledge and trial and error and ARX model is obtaining a simple equation model. Figure 4 will show ARX model structures.

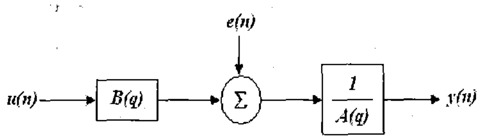

Figure 4: ARX model structures

The input and output relations with dynamic model describe as linear difference are obtaining by equation (1):

$$y(t) + a_1 y(t-1) + \dots + a_{n_a} y(t-n_a) = b_1 u(t-1) + \dots + b_{n_b} u(t-n_b) \quad (1)$$

Linear difference equation (1) is corresponding to the model by equation (2):

$$y(t) = G(q,\theta)\, u(t) \quad \text{where } G(q,\theta) = \frac{B(q)}{A(q)} \quad (2)$$

So, an ARX model equation by equation (3):

$$y(k) = \frac{B(q)}{A(q)}\, u(k) \quad (3)$$

where

$$A(q) = 1 + a_1 q^{-1} + \dots + a_{n_a} q^{-n_a}$$
$$B(q) = b_1 q^{-1} + \dots + b_{n_b} q^{-n_b} \quad (4)$$

The linear least square is a method to fit linear data. A linear model is defined as equation that is linear in coefficients. For example, polynomials are linear but Gaussian is not. To illustrate the linear least square fitting process, suppose that have n data points that can be modelled by a first degree polynomial. Equation (5) will show the equation linear least square.

$$\hat{\theta}_N^{LS} = \left[\frac{1}{N}\sum_{t=1}^{N} \varphi(t)\varphi^T(t)\right]^{-1} \frac{1}{N}\sum_{t=1}^{N} \varphi(t) y(t) \quad (5)$$

The ARX model is tested in order to reveal any inadequacies and verify the identified model according to objectives of good model approximation. The example of model validation is residual analysis test.

IV. RESULT AND ANALYSIS

The result and analysis of ARX model four types heat exchanger HE158C consist of experimental data, convert data into transfer function, convert transfer function to state-space, running m-file program and analysis the result. The data co current and counter current of four types heat exchanger is simulating using M-File MATLAB program. The different of flow is based on the experiment data. First experiment is when the cold water flow rate (FT2) will be varied in the range of 2, 4, 6, 8, 10 Litre/min. The hot water flow rate (FT1) and temperature will stay approximately constant at about 10 Litre/min and 70°C respectively. Second experiment is when the hot water flow rate (FT1) is varied and cold water flow rate (FT2) will be constant. Table 1 - Table 4 show the experimental data. These both of table show that at first five data taken of TT1 with TT4 and TT3 with TT2. The data reading for the experiment based on flow rate and temperature. The data will be taken every minute 1 until minute 5. Even though the flows of water are different way, the experimental data between both of them are slightly different due to the temperature of both hot water and cold water.

TABLE 1: DATA EXPERIMENTAL FOR SHELL & TUBE HEAT EXCHANGER

No. Exp.	Types	co current				counter current			
		Min	FT1	FT2	TT2	Min	FT1	FT2	TT3
1	Shell and Tube	1	10	2	34.4	1	10	2	48.1
		2	10	4	33.6	2	10	4	48.1
		3	10	6	34.7	3	10	6	46.2
		4	10	8	44.2	4	10	8	43.3
		5	10	10	22.8	5	10	10	43.2
2		Min	FT1	FT2	TT4	Min	FT1	FT2	TT4
		1	2	10	43.7	1	2	10	42.7
		2	4	10	43.4	2	4	10	44.7
		3	6	10	43.4	3	6	10	45.1
		4	8	10	43.8	4	8	10	45.3

TABLE 2: DATA EXPERIMENTAL FOR SPIRAL HEATEXCHANGER

No. Exp.	Types	co current				counter current			
		Min	FT1	FT2	TT2	Min	FT1	FT2	TT3
1	Spiral	1	10	2	34.4	1	10	2	49.1
		2	10	4	34.2	2	10	4	48.1
		3	10	6	33.1	3	10	6	40.2
		4	10	8	33.0	4	10	8	43.1
		5	10	10	32.6	5	10	10	43.2
2		Min	FT1	FT2	TT4	Min	FT1	FT2	TT4
		1	2	10	42.9	1	2	15	45.0
		2	4	10	45.5	2	4	15	44.5
		3	6	10	47.0	3	6	15	46.6
		4	8	10	47.2	4	8	15	47.4
		5	10	10	47.8	5	10	15	48.2

TABLE 3: DATA EXPERIMENTAL FOR COCENTRIC HEAT EXCHANGER

No. Exp.	Types	co current				counter current			
		Min	FT1	FT2	TT2	Min	FT1	FT2	TT3
1	Cocentric	1	10	2	30.3	1	10	2	34.5
		2	10	4	31.6	2	10	4	33.3
		3	10	6	30.7	3	10	6	32.8
		4	10	8	30.6	4	10	8	32.5
		5	10	10	30.6	5	10	10	32.4
2		Min	FT1	FT2	TT4	Min	FT1	FT2	TT4
		1	2	10	47.2	1	2	15	49.5
		2	4	10	51.2	2	4	15	49.1
		3	6	10	48.7	3	6	15	49.1

TABLE 4: DATA EXPERIMENTAL FOR PLATE HEAT EXCHANGER

No. Exp.	Types	co current				counter current			
		Min	FT1	FT2	TT2	Min	FT1	FT2	TT3
1	Plate	1	10	2	31.5	1	10	2	49.6
		2	10	4	31.4	2	10	4	48.1
		3	10	6	31.3	3	10	6	46.2
		4	10	8	31.2	4	10	8	43.1
		5	10	10	31.2	5	10	10	43.2
2		Min	FT1	FT2	TT4	Min	FT1	FT2	TT4
		1	2	10	31.9	1	2	15	37.2
		2	4	10	29.5	2	4	15	39.3
		3	6	10	34.7	3	6	15	38.7
		4	8	10	38.1	4	8	15	38.1
		5	10	10	37.9	5	10	15	37.2

Table 5 and Table 6, will show the all data for co current and counter current training apparatus model HE158C.

TABLE 5: CONVERSION DATA INTO TRANSFER FUNCTION FOR CO CURRENT

Type	Transfer Function (z-domain)	Transfer Function
Shell & Tube	[illegible]	[illegible]
Spiral	[illegible]	[illegible]
Concentric	[illegible]	[illegible]
Plate	[illegible]	[illegible]

Table 5 and Table 6 will show the all result of four types heat exchanger for co current and counter current from Table 1, Table 2, Table 3 and Table 4 conversion into z-transform and after that convert into transfer function in equation (6):

$$tf = \frac{y(z)}{u(z)} \qquad (6)$$

TABLE 6: CONVERSION DATA INTO TRANSFER FUNCTION FOR COUNTER CURRENT

Temperature	Transfer Function (z-domain)	Transfer Function
Shell & Tube	$Y(Z) = \frac{0.1939 - 0.2243z}{z^3 - 1.0265z^2 - 1.9807z + 1.0042}$	$TF = \frac{0.19392 - 0.22432z^2}{z^4 - 2.93652z^3 - 0.04425z^2 + 2.90492z - 1.0042}$
Spiral	$Y(Z) = \frac{0.1223 - 0.0158z}{z^3 - 1.09194z^2 - 0.1360z + 1.0562}$	$TF = \frac{0.1223 - 0.0158z^2}{z^4 - 2.09194z^3 + 0.95514z^2 + 1.1932z - 1.0562}$
Concentric	$Y(Z) = \frac{0.1627 - 0.4580z}{z^3 - 1.9597z^2 - 0.1967z + 1.1564}$	$TF = \frac{0.1627 - 0.4580z^2}{z^4 - 2.9597z^3 + 1.7637z^2 + 1.3531z - 1.1564}$
Plate	$Y(Z) = \frac{-0.9801 - 0.1075z}{z^2 - 1.9801z^2 - 0.9801}$	$TF = \frac{-0.9801 - 0.1075z^2}{z^4 - 2.9801z^3 + 1.9801z^2 + 0.9801z - 0.9801}$

TABLE 7: CONVERSION TRANSFER FUNCTION TO STATE-SPACE FOR CO CURRENT

Type	Transfer Function	State-space
Shell & Tube	[illegible]	[illegible]
Spiral	[illegible]	[illegible]
Concentric	[illegible]	[illegible]
Plate	[illegible]	[illegible]

TABLE 8: CONVERSION TRANSFER FUNCTION TO STATE-SPACE FOR CO CURRENT

Type	Transfer Function	State-space
Shell & Tube	[illegible]	[illegible]
Spiral	[illegible]	[illegible]
Concentric	[illegible]	[illegible]
Plate	[illegible]	[illegible]

Table 7 and Table 8 shows the conversion from the transfer function to state-space for co current and counter current. Table 9 shows data input output QAD BDT921.

TABLE 9: DATA INPUT OUTPUT QAD BDT921

No	Input	Output
1	25.1	22.2
2	25.1	22.2
3	25.1	22.2
4	25.1	22.2
5	25.1	22.2
6	25.1	22.2
.	.	.
.	.	.
.	.	.
500	25.1	42.5

Figure 5 will show the command of input and output data that obtained using M-File MATLAB program.

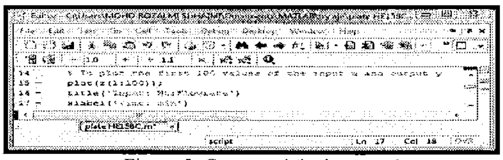

Figure 5: Command for input and output

In the system identification, the data will be set up by using the command for IDDATA object with sampling interval. Figure 6 will show the command for create IDDATA object.

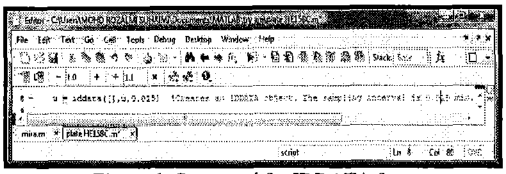

Figure 6: Command for IDDATA for co current

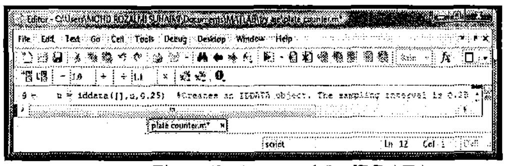

Figure 7: Command for IDDATA

From Figure 6 and Figure 7, will show the command for create IDDATA object to identification routines. The sampling time for co current that will be used is 0.25 min while for counter current; the model structure that will be used is ARX model. To computes the linear least square to executed in M-File MATLAB program, command like Figure 8 will be used.

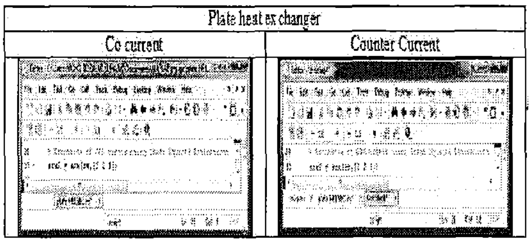

Figure 8: Command for linear least square

Figure 8 will show the command for linear least square of co current and counter current plate heat exchanger and for co current the orders that will be used is [3 1 1] while for counter current, the order is [2 2 2]. The value for the orders is choosing randomly like Figure 9.

No	ORDER na	nb	nk
1	1	2	2
2	2	2	1
3	2	2	2
4	3	1	1
5	3	1	2
6	3	2	2

Figure 9: Orders of HE158C

After having settled on the preceding three choices, have at least implicitly, arrived at particular model the one in the set that describes the data according to the chosen criterion. Figure 10 will show the command comparing between validation data and measured model.

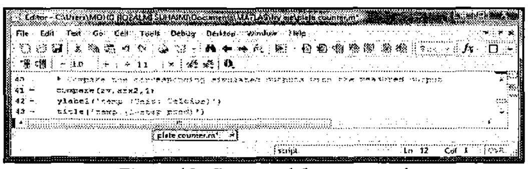

Figure 10: Command for compared

Figure 10 show the validation model is in range from 201 until 350. The basic program is COMPARE(DATA, SYS,M). DATA refer to IDDATA, SYS is referring to IDARX and M is referring to prediction horizon and in this project M is equal 1. For residual analysis, use autocorrelation and cross-correlation and Figure 11 will show the command for residual analysis.

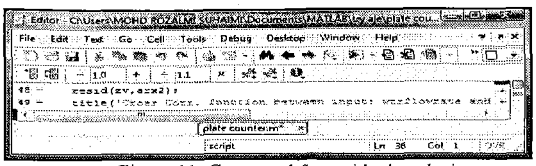

Figure 11: Command for residual analysis

Figure 12 will show the input and output data for co current and counter current HE158C.

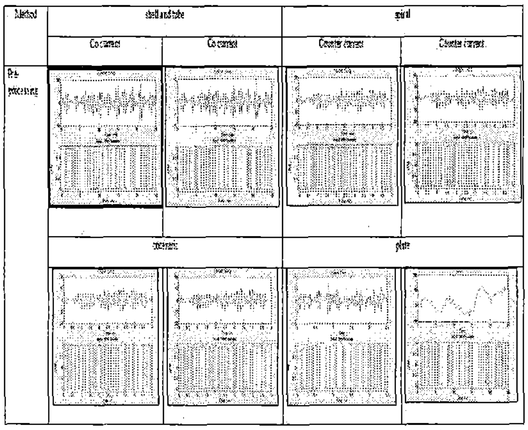

Figure 12 Input and output data for co current and counter current HE158C

Figure 13 will show the % fitting for co current and counter current HE158C that counter current shows the best fitting which is 75.91% compared with co current only 65.74% but both of result shows that the types of co current and counter current is a good model because it is was close to 100%. For residual analysis auto-correlation and cross correlation, also shows that the model is good as like in Figure 14. Figure 15 will shows the Bode plot diagram for co current and counter current HE158C. Figure 16 will show the estimated using ARX model for co current and counter current.

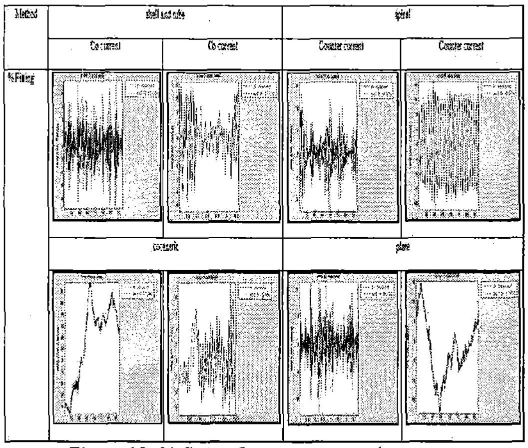

Figure 13: % fitting for co current and counter current HE158C

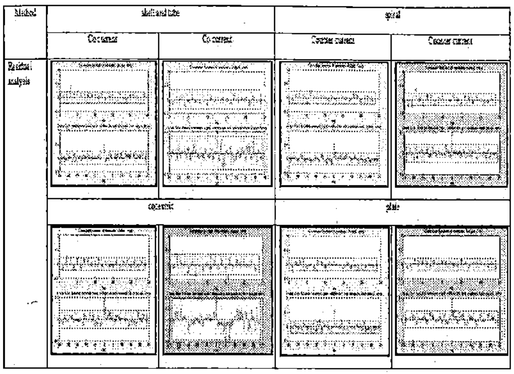

Figure 14: Autocorrelation and cross correlation for concurrent and counter current HE158C

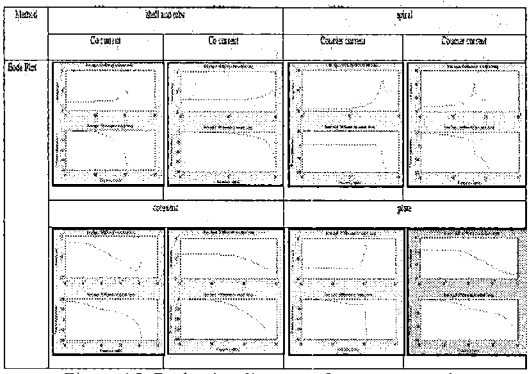

Figure 15: Bode plot diagram of co current and counter current HE158C

Method	shell and tube		spiral		concentric		plate	
	Co current	Counter current	Co current	Counter current	Co current	Counter current	Co current	Counter current
Estimated	LF :3.3206 FPE: 3.4511	LF :2.832584 FPE: 2.900944	LF :4.0713 FPE: 4.2226	LF :4.11456 FPE: 4.2775	LF :3.9615 FPE: 4.1121	LF :3.3883 FPE: 3.5216	LF :3.3904 FPE: 3.5223	LF :3.1420 FPE: 3.2865

Figure 16: Estimated using ARX model for co current and counter current HE158C

V. CONCLUSION

As a conclusion, after all of the process in the system identification of four types heat exchanger HE158C will be done, the mathematical models are found from using experimental data both of the experiment. Every types of heat exchanger show the fitting more than 50% and were close to 100%. It means that each types both of model have high quality to be training. Co current concentric shows the best fitting from model HE158C which is 89.32%. From the analysis, the mathematical modeling of ARX model for model HE158C. For co current concentric HE158C:

$A(q) = 1 - 0.7021 q^{-1} - 0.4988 q^{-2} + 0.206 q^{-3}$
$B(q) = 0.4123 q^{-2} + 0.1807 q^{-3}$

The order of [na nb nk] that will be used is [3 2 2] which is the loss function and FPE is 3.09365 and 3.24833.

In this paper, m-File MATLAB program is running with the experimental data of input and output. Based on input and output, the system identification can be done by using other methods either linear or nonlinear such as ARMAX, Output error model, General polynomial model, neural network and compare each models to get the best result.

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