

LINEAR QUADRATIC GAUSSIAN (LQG) CONTROLLER DESIGN FOR
SERVO MOTOR

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ABSTRACT

This paper presents the development of Linear Quadratic Gaussian controller design for DC servo motor. The accuracy of the control system is directly proportional to servo motor precision. However, the precision of DC servo motor system difficult to achieved due to the ignorance of the sensor noise and plant disturbance. Therefore LQG controller has been design to control speed and position of DC servo motor. In LQG controller designed, the noise sensor and plant disturbance has been considered as white noise Gaussian. This controller has be designed based on combination of Steady-state Linear Quadratic (LQ) Optimal Control and Steady-state Kalman Filter State Estimation by solving matrix Riccati equation in order to determine steady-state feedback gain, K and steady-state Kalman gain, G . MATLAB Simulink and M-File approached have been done to simulate the design. Besides that, Arduino microcontroller board also has been explored in order to do real-time Simulation between LQG controller and DC servo motor. The step response of closed-loop digital control system using LQG controller and opened-loop control system have been compared to verified that the performance of closed-loop control system better than opened-loop system

ABSTRAK

Kertas kerja ini membentangkan perkembangan rekabentuk pengawal Linear Kuadratik Gaussian (LQG) bagi motor servo arus terus. Ketepatan system kawalan adalah berkadar langsung dengan ketepatan motor servo. Walaubagaimanapun, ketepatan servo motor arus terus sukar untuk dicapai disebabkan oleh pengabaian bunyi sensor dan gangguan sistem. Oleh sebab itu, pengawal LQG telah dicipta untuk mengawal kelajuan dan kedudukan motor servo arus terus. Dalam perekaan pengawal LQG, bunyi sensor dan gangguan system telah dianggap sebagai putih hingar Gaussian. Pengawal ini telah dibina berdasarkan gabungan keadaan mantap Linear Kuadratik (LQ) Kawalan Optimum dan keadaan mantap anggaran penapis Kalman dengan menyelesaikan persamaan matrik Riccati bagi menentukan gandaan maklumbalas keadaan mantap, K dan gandaan Kalman keadaan mantap, G . Pendekatan menggunakan perisian MATLAB Simulink dan M-File telah dilakukan bagi mensimulasikan rekabentuk pengawal. Selain itu, papan Arduino mikropengawal juga telah dikaji untuk melakukan simulasi masa nyata antara pengawal LQG dan motor servo arus terus. Sambutan langkah gelung tertutup system kawalan digital menggunakan pengawal LQG dan system kawalan gelung terbuka telah dibandingkan untuk mengesahkan bahawa prestasi system kawalan gelung tertutup adalah lebih baik berbanding system gelung terbuka.

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LIST OF SYMBOLS AND ABBREVIATION

LQG	-	Linear Quadratic Gaussian
LQR	-	Linear quadratic Regulator
LQ	-	Linear Quadratic
MATLAB	-	Matrix Laboratory
GUI	-	Graphical Interface User
LEQ	-	Linear Estimation Quadratic
LTR	-	Loop Transfer Recovery
EMF	-	Electromotive Force
DARE	-	Discrete Algebraic Riccati Equation
DC	-	Direct Current
BDC	-	Brush Direct Current
IR LED	-	Infrared Light emitting diode
GA	-	Genetic Algorithm
IC	-	Integrated Circuit
PWM	-	Pulse Width Modulation
t_d	-	Delay Time
t_r	-	Rise Time
t_p	-	Peak Time
M_p	-	Maximum Overshoot
t_s	-	Settling Time
R_a	-	Armature Resistance
L_a	-	Armature Inductance
V_a	-	Armature Voltage
i_a	-	Armature Current
T_m	-	Torque
e_b	-	Back Electromotive Force (Back EMF)

J_m	-	Motor Inertia
B_m	-	Viscous Damping Coefficient
ω_m	-	Angular Velocity
θ_m	-	Angular Displacement
	-	Angular Acceleration
K_b	-	Back-EMF Constant
K_t	-	Torque Constant
J_G	-	Mass moment inertia
J_L	-	Moment of inertia of the load
J_N	-	Performance Index
σ^2	-	Variance
K_e	-	Voltage Constant
G_s	-	Continuous Transfer Function
G_z	-	Discrete Transfer Function
P	-	Steady-state Riccati Equation
K	-	Optimal Feedback Gain Matrix
G	-	Steady-state Kalman Gain
τ	-	Duration of DC servo motor ON
T	-	Period of the rotation of DC servo motor in one cycle

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CHAPTER 1

INTRODUCTION

1.1 Project Background

Servo motor is a motor that is used to control position or speed in closed-loop control systems. It is generally designed for the computer, numeric control machines, industrial equipment, weapon industry, speed control of alternators and to control mechanism of full automatic regulators as the first starter, and also to quickly and correctly starting a system. In the field of control mechanical linkages and robots, research works are mostly found on DC motors. Therefore the requirement of a servo motor is to turn over a wide range of speeds and also to perform position and speed based on instruction given has been developed.

Properties of DC motor such like inertia, physical structure, shaft resonance and shaft characteristics, electrical and physical constant are variable. Besides that, the velocity and position tolerance of servo motor which are used at the control systems are nearly identical. Therefore, servo motor must be controlled to ensure the stability of the close-loop control system. Thus, Linear Quadratic Gaussian (LQG) controller has been proposed to control the speed and position of servo motor.

Linear Quadratic Gaussian control is one of the most fundamental optimal control problems that concerns uncertain linear systems disturbed by additive white Gaussian noise. LQG controller is simply the combination of a Linear Quadratic (LQ) Optimal Control and Kalman Filter State Estimation.

1.2 Problem Statement

Servo motors have received great attention in recent decades. Compare to other motor types, servo motors can deliver more torque and power for the same size. Besides that, it also provides safe and reliable operation and requires less maintenance which make the used of servo motor received a great attention. Specifically, with latest development in microprocessor and power electronics, very accurate and highly efficient servo motor control systems have been developed. The need for a feedback element was required in order to make control system stable. However, the feedback not only increases the cost but also makes the control of position and speed of servo motor becomes complex. Further, the control accuracy is directly proportional the precision of servo motor control. However, the precision of servo motor control has been difficult to achieve due to the ignorance of the plant disturbance and inaccuracies in feedback measurement cause by sensor noise. Therefore, Linear Quadratic Gaussian (LQG) controller has been proposed to overcome the sensor less position and speed control due to this problem. The main proposed alternative of this project were to control and estimate the motor position and speed besides to overcome the weakness of other digital controller.

1.3 Objectives of Project

The main objective of this project was to design and develop controller for servo motor. Besides that, the goals that want to be achieves are:

- i. To design Linear Quadratic Gaussian (LQG) controller for servo motor.
- ii. To control the position and speed of servo motor with Linear Quadratic Gaussian using MATLAB software.
- iii. To analyse the simulation result of Linear Quadratic Gaussian (LQG) based on specification of servo motor.

1.4 Scope of Project

This project is primarily concerned to build control system for control servo motor. The scopes of this project are:

- i. Design and produce the simulation of the LQG controller
- ii. Implement LQG controller to DC servo motor
- iii. Compare the step response of closed-loop control system using LQG controller with open-loop system.

1.5 Thesis Outline

This thesis consists of five chapters that will explain more details about this project. The first chapter will put in plain words about the project overview which are about project background, problem statements, objectives and scopes.

The second chapter will review about the transient specification of control system. Besides that, various LQG controller designed and hardware component for DC servo motor also have been reviewed. From this literature review, different approaches of controller design for servo motor can be studied. However, since LQG controller for DC servo will be design in this project, therefore the comparison has been done at the end of the chapter to select the best method in designed controller.

The third chapter is about the development flow chart to design control system for DC servo motor and LQG controller and mathematical modelling of the DC servo motor. This chapter will discuss the details on the methodology for this project.

The fourth chapter will discuss about the results for open-loop and closed-loop of DC servo motor system. In addition, the results of open-loop and closed-loop system will be compared to choose the better system.

The fifth chapter which also as the last chapter will be discuss about the conclusion and recommendation of the project. The recommendations suggested are proposed with the intention of improving the project in future.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter informs us all about the research before the controller has been set up. To build this project, it requires the knowledge that not readily offhand. There are three main parts need to be investigated in order this project will be successful, namely mathematical modelling design, controller design and hardware implementation. Before LQG controller of servo motor has been designed, the specification of the control system has been researched in order to meet transient response specification. Since LQG controller will be design to control servo motor, therefore servo motor will be selected as mathematical modelling system in this project. The controllability and observability of the system were also has been studied to determine either the system can be observe or not. Besides that, the method to design LQG controller was based on a general methodology that is called Linear Quadratic (LQ) Optimal Control. Besides that, since LQG controller is simply the combination of a Kalman Filter with a LQ Optimal control, therefore the approach that will be used to design controller is determines state-feedback gains matrix K for LQ Optimal Control and Kalman Filter gain matrix (G). After that, method to combine LQ optimal control and Kalman filter estimation has been studied in order to implement Linear Quadratic Gaussian (LQG) controller. Optical encoder and Arduino software also have been explored to ensure the hardware can be implemented smoothly.

2.2 Transient Response Specification of Control System

Transient response specifications describe desired performance characteristics of a control system, whether they are continuous or discrete time which are specified in terms of time-domain quantities. The performance characteristics of a control system frequently are specified in terms of the transient response to a unit step input since unit step input is easy to generate and it is sufficiently drastic to provide useful information on both the transient response and the steady-state response characteristic of the system.

Transient response of a system to a unit step-input depends on initial conditions which generally the standard initial condition will be used: the system is at rest initially and all its time-derivatives are zero [2]. Besides that, transient response of a control system often exhibits damped oscillation before reaching steady state.

The transient response of a control system can be specified using 5 characteristic specifications which are delay time t_d , rise time t_r , peak time t_p , maximum overshoot M_p and settling time t_s as shown in Figure 2.1.

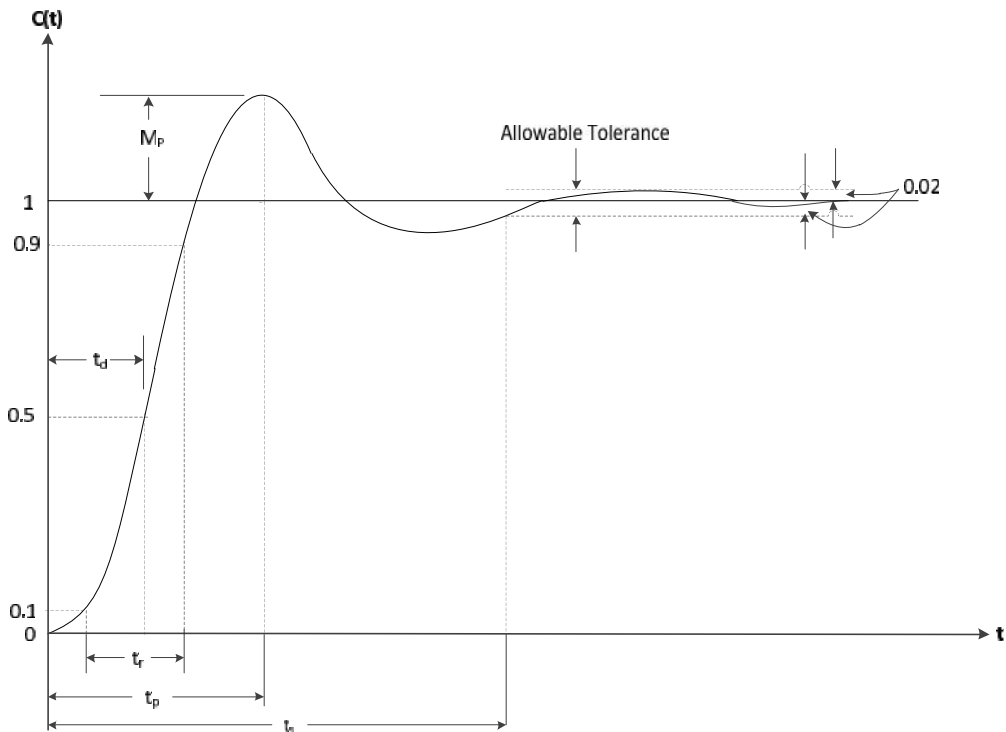


Figure 2.1: Unit-step response curve of transient response

Figure 2.1 show unit step response curve of transient response specification t_d , t_r , t_p , M_p and t_s . The delay time, t_d can be defined as the time required for the response to reach half the final value the very first time while the peak time, t_p is the time required for the response to reach the first peak of the overshoot. The rise time, t_r is the time required for the response to rise from 10% to 90%, or 5% to 95% or 0% to 100%, depending on the situation of the control system. Besides that, the maximum overshoot, M_p describe the maximum peak value of the response curve which measured from unity. However, if the final steady-state value of the response differs from unity, thus it is common to use the maximum percent overshoot which defined by the relation [2, 26]

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

The maximum percentage of overshoot directly indicates the relative stability of the system. The settling time, t_s is the time required for the response curve to reach steady state and stay within a range about the final value of a size specified as an absolute percentage of the final value, usually 2%.

The transient response specifications of time-domain are quite important since most of control systems are time domain system. Therefore, the control system being designed must be modified until the transient response specifications has been satisfied [2].

2.3 DC Servo motor Modelling

The DC servomotor considered in the project was a permanent magnet DC motor as shown in Figure 2.2.

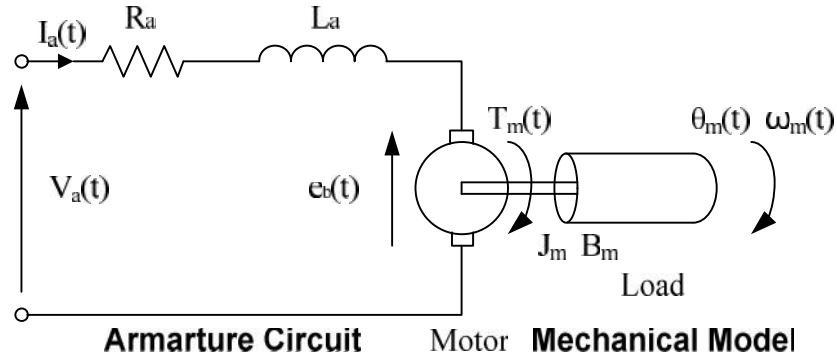


Figure 2.2: DC Servo Motor Modelling

By applying Kirchhoff's Voltage law to the armature circuit, the voltage and electromagnetic torque equation can be expressed as [2]

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + e_b \quad (2.1)$$

Where i_a is the armature current, R_a is the armature resistance, L_a is the armature inductance, and e_b represent the back electromotive force (back EMF). Most permanent magnet DC motor can also operate generators. In other words, if motor shaft rotating, a voltmeter connected across the motor terminal will indicate a voltage that is proportional to the angular velocity of the motor shaft, ω_m .

According to Lenz's law, the polarity of the induced back EMF voltage, e_b always opposed the applied voltage, V_a . Back EMF can be expressed as a function of the angular velocity, ω_m , of the motor as follows

$$e_b = K_b \omega_m \quad (2.2)$$

With K_b is the back-EMF constant [15].

$$T_m = K_t i_a(t) \quad (2.3)$$

For a mechanical system undergoing pure rotational motion, Newton's second law states that the sum of applied torques is equal to the product of the mass moment inertia, J_G and angular acceleration of the body,

$$\sum_{i=1}^n T_i = J_G \alpha \quad (2.4)$$

In the case of the DC servomotor, J_G is equal to the sum of the mass moment of inertia of the motor armature, J_m and the load, J_L with J_L usually negligible. The net torque generated by the motor is equal to motor torque, T_m minus the rotational viscous friction. The rotational viscous friction associated with the motor is proportional to the motor angular velocity, ω_m .

$$T_m - B_m \omega_m = (J_m + J_L) \alpha_m \quad (2.5)$$

$$T_m - B_m \omega_m = J_m \alpha_m \quad (2.6)$$

where B_m is the viscous damping coefficient. Replacing the angular acceleration of the motor with rate-of change of angular velocity yield:

$$T_m - B_m \omega_m = J_m \frac{d\omega_m}{dt} \quad (2.7)$$

The torque, velocity and position can be related by:

$$T_m = J_m \frac{d\omega_m}{dt} + B_m \omega_m \quad (2.8)$$

$$\omega_m = \frac{d\theta_m}{dt} \quad (2.9)$$

$$T_L = B_m \omega_m \quad (2.10)$$

with T_L is the load torque and θ_m is the motor displacement [1, 2, 5, 15, 20].

2.4 Controllability and Observability

The concept of controllability and observability has been introduced by Kalman Theorem in 1960. The definitions of controllability and observability can be described as:

- (i) A system is said to be state controllable if all the modes are affected by the input vector, otherwise it is not state controllable.
- (ii) A system is said to be state observable if all its modes appear at the output.

Based on Kalman's Theorem, concept of controllability and observability can be simplified as [1, 2, 25, 31]

Theorem 1: A continuous or discrete system is completely state controllable if and only if the rank of the composite $n \times nr$ matrix was

$$\text{rank}[B : AB : A^2B : \dots : A^{n-1}B] = n \quad (2.11)$$

Theorem 2: A continuous or discrete system is completely state observable if and only if the rank of the composite $n \times n$ matrix was

$$\text{rank}[C^T : A^T C^T : (A^T)^2 C^T : \dots : (A^T)^{n-1} C^T] = n \quad (2.12)$$

2.5 Optimal Control

Optimal control is one particular branch of modern control that sets out to provide analytical designs system that sets out to provide analytical designs of the system that is the end result of an optimal design which is not supposed merely to be stable, but have a certain bandwidth, or satisfy any one of the desirable constraints associated with classical control. However, it is supposed to be the best possible system of a particular type which represents the word of optimal. If it is both optimal and possesses a number of the properties that classical controls suggest are desirable, hence the controller is much the better [4, 20].

Linear optimal control is a special sort of optimal control where the plant that is controlled is assumed linear and the controller, the device that generates the optimal control is also constrained to be linear. Linear controllers are achieved by following the quadratic performance indices by using linear optimal control which is termed as Linear Quadratic (LQ) method. The term 'linear' specified that the systems considered were assumed linear and the term "quadratic" describe the use of

performance measures that involve the square of an error signal. In discrete system systems, the cost function which is also called the performance index is generally the form

$$J_N = \sum_{k=0}^N L[y(k), r(k), u(k)] \quad (2.13)$$

In equation (2.13), k known as the sample instant and N is the terminal sample instant. The control system outputs are $y(k)$, the inputs are $r(k)$ while $u(k)$ are the control inputs to the plant. The control inputs are always constrained in a physical system.

Since the quadratic cost function is simpler and the cost function is logical, thus the design technique for optimal linear regulator control systems based on quadratic cost functions is developed. This design results in a control law of the form

$$u(k) = -K(k)x(k) \quad (2.14)$$

where K denoted by full-state feedback gain [1, 2, 3, 4, 5].

2.6 The Quadratic Cost Function

The development of optimal control design begin by considering the case of a quadratic cost function of the states and the control signals, that is

$$J_N = \frac{1}{2} \sum_{k=0}^N [x^T(k)Q(k)x(k) + u^T(k)R(k)u(k)] \quad (2.15)$$

where N is finite and $Q(k)$ and $R(k)$ is symmetric. The linear plant is described by [1, 2]

$$x(k+1) = A(k)x(k) + B(k)u(k) \quad (2.16)$$

$$y(k) = C(k)x(k) + D(k)u(k) \quad (2.17)$$

where

$x(k)$ = state vector (n -vector)

$u(k)$ = control vector (r-vector)

A = $n \times n$ system matrix

B = $n \times r$ input matrix

C = $m \times n$ output matrix

D = $m \times r$ connection matrix

2.7 The Principle of Optimality

For a linear discrete plant which has been described in equation (2.16) and (2.17), the optimal control can be determined by

$$u^o(k) = f[x(k)] \quad (2.18)$$

that minimizes the quadratic cost function of equation (2.15)

$$J_N = \frac{1}{2} \sum_{k=0}^N [x^T(k)Q(k)x(k) + u^T(k)R(k)u(k)]$$

where N is finite and $Q(k)$ is positive semi-definite and $R(k)$ is positive definite. In equation (2.18), the superscript o denotes that the control is optimal. The principle of optimality may be stated as follows:

If a closed-loop control $u^o(k) = f[x(k)]$ is optimal over the interval $0 \leq k \leq N$, it is also optimal over any subinterval $m \leq k \leq N$, where $0 \leq m \leq N$. [1]

The principle of optimality can be defined using scalar F_k as

$$F_k = x^T(k)Q(k)x(k) + u^T(k)R(k)u(k) \quad (2.19)$$

Then, equation (2.13) can be expressed as

$$J_N = F_0 + F_1 + \cdots + F_{N-1} + F_N$$

Now, let S_m be the cost from $k = m$ to $k = N$; that is,

$$S_m = J_N - J_{N-m} = F_{N-m+1} + F_{N-m+2} + \dots + F_{N-1} + F_N$$

The principle of optimality states that if J_N is the optimal cost, then it also applied to S_m , for $m = 1, 2, \dots, (N+1)$. The principle of optimality can be apply by minimizing $S_1 = F_N$, after that F_{N-1} was choosing to minimize

$$S_2 = F_{N-1} + F_N = S_1^\circ + F_{N-1}$$

This step was continuing until $S_{N+1} = J_N$ is minimized. The controller that has been design using this procedure is known as dynamic programming. The general design procedure will be developed in next section [1, 2, 4].

2.8 Linear Quadratic Optimal Control

Linear Quadratic Optimal Control can be developed based on Quadratic Cost Function and the principle of optimality. Quadratic optimal control problems can be solved by many different approaches such as conventional minimization method using Lagrange multipliers.

A linear discrete plant for linear quadratic optimal was described in equation (2.16) and (2.17) as

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) \\ y(k) &= C(k)x(k) + D(k)u(k) \end{aligned}$$

while the optimal control determine by using equation (2.18)

$$u^\circ(k) = f[x(k)]$$

that minimizes the quadratic function

$$J_N = \frac{1}{2}x^T(N)Sx(N) + \frac{1}{2}\sum_{k=0}^N [x^T(k)Q(k)x(k) + u^T(k)R(k)u(k)] \quad (2.20)$$

where

$Q(k) = n \times n$ positive definite or positive semidefinite Hermitian matrix (or real symmetric matrix)

$R(k) = r \times r$ positive definite Hermitian matrix (or real symmetric matrix)

$S(N) = n \times n$ positive definite or positive semidefinite Hermitian matrix (or real symmetric matrix)

Matrices Q and R are selected weight the relative importance of the performance measures caused by state vector $x(k)$ ($k = 0, 1, 2, \dots, N-1$), the control vector $u(k)$ ($k = 0, 1, 2, \dots, N-1$) and the final state $x(N)$, respectively. Besides that, N is finite which make the feedback gain matrix $K(k)$ becomes time-varying matrix [1, 2, 4, 6]

The initial state of this at the system is at some arbitrary state $x(0) = c$. The final state $x(N)$ may be fixed, in which case the term $\frac{1}{2}x^T(N)Sx(N)$ is removed from the performance index of Equation (2.20) and instead the terminal condition $x(N) = x_f$ is imposed where x_f is the fixed terminal state. If the final state $x(N)$ is not fixed, then the first term in Equation (2.20) represent the weight of the performance measure due to the final state. In the minimization problem, the inclusion of the term $\frac{1}{2}x^T(N)Sx(N)$ in the performance index J implies that the final state $x(N)$ desire to be as close to the origin as possible [1, 2, 3, 4].

2.9 Steady-State Quadratic Optimal Control

When the control process is finite (when N is finite), the feedback gain matrix $K(k)$ becomes a time-varying matrix. However, for the quadratic optimal control of an infinite stage process (where N is infinity), as N approaches infinity the optimal control solution becomes a steady-state solution and the time-varying gain matrix $K(k)$ becomes a constant gain matrix. A constant gain matrix $K(k)$ is called a steady-state gain matrix and can be written as K [1, 2, 3].

In order to solve steady-state quadratic optimal control problem, steady-state quadratic optimal control of a regulator system has been considered. The plant equation is given by equation (2.16)

$$x(k + 1) = A(k)x(k) + B(k)u(k)$$

For $N = \infty$, the performance index may be modified to

$$J_N = \frac{1}{2} \sum_{k=0}^N [x^T(k)Q(k)x(k) + u^T(k)R(k)u(k)]$$

The term $\frac{1}{2}x^T(N)Sx(N)$ which appeared in equation (2.20) is not included because the optimal regulator system is stable so that the value of J_N converges to a constant, thus $x(k)$ becomes zero and $\frac{1}{2}x^T(\infty)Sx(\infty) = 0$.

Steady-state matrix $P(k)$ was defined as P with $P(k)$ is called as Riccati Equation

$$P(k) = Q + A'P(k+1)[I + BR^{-1}B'P(k+1)]^{-1}G \quad (2.21)$$

An alternative expression for P can be derived from equation (2.21):

$$P = Q + A'PA - A'PB(R + B'PB)^{-1}B'PA \quad (2.22)$$

Equation (2.22) was defined as Steady-State Riccati Equation. The steady gain matrix can be obtained in terms of P as

$$K = (R + B'PB)^{-1}B'PA \quad (2.23)$$

The optimal control law for steady-state operation is given by

$$u(k) = -Kx(k) \quad (2.24)$$

The performance index associated with the steady-state optimal control law can be defined as

$$J_{\min} = \frac{1}{2}x'(0)Px(0) \quad (2.25)$$

In many practical systems, instead of using a time-varying gain matrix $K(k)$, a gain matrix will be approximate by constant gain matrix K [1, 2, 3]

2.10 State Estimation

State estimation is a technique to estimates the state of a plant from the information that is available concerning the plant. The system that estimates the state of another system is generally called observer or state estimator. The plant is described by equation (2.16) and (2.17) with $D(k) = 0$

$$\begin{aligned}x(k + 1) &= A(k)x(k) + B(k)u(k) \\y(k) &= C(k)x(k)\end{aligned}$$

where $y(k)$ are the plant signals that will be measured. The state of the system to be observed are $x(k)$ and the state of the observer, $q(k)$. Besides that, the desire $q(k)$ should be approximately equal to $x(k)$. Since the observer will be implemented on a computer, the signals $q(k)$ are then available for feedback calculations [1, 2]

2.10.1 Observer Model

The observer design criterion to be used is that transfer function matrix from the input $u(k)$ to the observer state $q_i(k)$ be equal to that from the input $u(k)$ to the system $x_i(k)$, for $i = 1, 2, \dots, n$. Equations describing the observer of Figure 2.3 will be developed based on this criterion [1, 27].

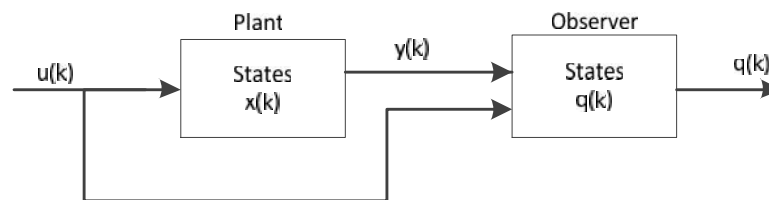


Figure 2.3: System illustrating state estimation

The z-transform of equation (2.16) and (2.17) can be described as

$$zX(z) = AX(z) + BU(z) \quad (2.26)$$

$$Y(z) = CX(z) \quad (2.27)$$

Thus it can be described that

$$X(z) = (zI - A)^{-1}BU(z) \quad (2.28)$$

Since the observer has two inputs, $y(k)$ and $u(k)$, the observer state equation can be written as

$$q(k + 1) = Fq(k) + Gy(k) + Hu(k) \quad (2.29)$$

where the matrices F , G and H are unknown. Taking the z -transform of equation (2.31) and solving $Q(z)$ yields

$$Q(z) = (zI - F)^{-1}[GY(z) + HU(z)] \quad (2.30)$$

From Equation (2.27)

$$Y(z) = CX(z) \quad (2.31)$$

Substitution of equation (2.32) and (2.30) into Equation (2.32) yields:

$$\begin{aligned} Q(z) &= (zI - F)^{-1}[GCX(z) + HU(z)] \\ &= (zI - F)^{-1}[GC(zI - A)^{-1}B + H]U(z) \end{aligned} \quad (2.32)$$

Recall that based on design criterion, the transfer function matrix from $U(z)$ to $Q(z)$ same as from $U(z)$ to $X(z)$. Therefore equation (2.30) become

$$Q(z) = (zI - A)^{-1}BU(z) \quad (2.33)$$

Then, from equation (2.33) and (2.34)

$$\begin{aligned} (zI - A)^{-1}B &= (zI - F)^{-1}[GC(zI - A)^{-1}B + H] \\ (zI - A)^{-1}B &= (zI - F)^{-1}GC(zI - A)^{-1}B + (zI - F)^{-1}H \end{aligned}$$

or

$$[I - (zI - F)^{-1}GC](zI - A)^{-1}B = (zI - F)^{-1}H$$

This equation can be expressed as

$$(zI - F)^{-1}[zI - (F + GC)](zI - A)^{-1}B = (zI - F)^{-1}H$$

Equating the post multiplying matrices of $(zI - F)^{-1}$ yields

$$(zI - A)^{-1}B = [zI - (F + GC)]^{-1}H \quad (2.34)$$

This equation is satisfied when H equal to B and

$$A = F + GC \quad (2.35)$$

Therefore, from equation (2.35), the observer state equation can be written as

$$q(k + 1) = (A - GC)q(k) + Gy(k) + Hu(k) \quad (2.36)$$

and the design criterion of equation (2.34) is satisfied. Since G is unspecified, hence the observer is a dynamic system described by Equation (2.36) with $y(k)$ and $u(k)$ as inputs and with the characteristic equation

$$|zI - A + GC| = 0 \quad (2.37)$$

The observer state equation is known as prediction observer, since the estimate at time $(k+1)T$ is based on the measurement at time kT [1, 2, 4, 6].

2.10.2 Errors in Estimation

The error vector, $e(k)$ in the state-estimation process is define as

$$e(k) = x(k) - q(k) \quad (2.38)$$

Then, from equation (2.16), (2.17) and (2.37), equation (2.38) can also be describe as

$$\begin{aligned} e(k+1) &= x(k+1) - q(k+1) \\ &= Ax(k) + Bu(k) - (A - GC)q(k) - GCx(k) - Bu(k) \end{aligned} \quad (2.39)$$

or

$$e(k+1) = (A - GC)[x(k) - q(k)] = (A - GC)e(k) \quad (2.40)$$

Hence, the error dynamics have a characteristic equation given by

$$|zI - (A - GC)| = 0 \quad (2.41)$$

which is similar to equation (2.38).

The errors in state estimation may occur due to the choice of initial conditions for the observer. Since the initial states of the plant will not be known, thus the initial error $e(0)$ will not be zero. However, since the provided observer is stable, this error will tend to zero with increasing time.

Besides that, the errors in state estimation can be also due to the plant disturbances and the sensor errors. The complete state equation of the plant in equation (2.16) and (2.17) become

$$x(k+1) = A(k)x(k) + B(k)u(k) + B_1w(k) \quad (2.42) \quad y(k) =$$

$$C(k)x(k) + D(k)u(k) + v(k) \quad (2.43)$$

When the plant disturbance vector $w(k)$ and the measurement inaccuracies $v(k)$ are considered. Hence, if equation (2.42) and (2.43) is used in deriving the error state equations, Equation (2.40) can be describe as

$$e(k+1) = (A - GC)e(k) + B_1w(k) - Gv(k) \quad (2.44)$$

For this case, the errors in the state estimation will tend to zero if the estimator is stable. However, the excitation terms in Equation (2.44) will prevent the errors from reaching a value of zero. Therefore, optimal state estimation by using Kalman filtering will be considered to ensure the errors in state estimation become zero [1,2].

2.11 Optimal State Estimation – Kalman Filters.

The plant is assumed to be described as

$$\begin{aligned}x(k+1) &= A(k)x(k) + B(k)u(k) + B_1w(k) \\y(k) &= C(k)x(k) + D(k)u(k) + v(k)\end{aligned}$$

where $x(k)$ [$n \times 1$] are the states, $u(k)$ [$r \times 1$] are the known inputs, $w(k)$ [$s \times 1$] are the random plant disturbances, $y(k)$ [$p \times 1$] are the measurement and $v(k)$ [$p \times 1$] are the random inaccuracies in the measurement [1,2]

The random inputs $w(k)$ and $v(k)$ in equation (2.42) and (2.43) are assumed to be uncorrelated and to have Gaussian distributions with the properties:

$$\begin{aligned}E[w(k)] &= 0, & E[v(k)] &= 0 \\cov[w(j), w(k)] &= E[w(j)w^T(k)] = R_w\delta_{jk} & (2.45) \\cov[v(j), v(k)] &= E[v(j)v^T(k)] = R_v\delta_{jk}\end{aligned}$$

with $E[.]$ is the mathematical expectation operation and $cov[.]$ denotes the covariance [16, 22, 23]. Besides that, δ_{jk} is the Kronecker delta function, defined by

$$\delta_{jk} = \begin{cases} 0, & j \neq k \\ 1, & j = k \end{cases}$$

Hence $w(k)$ and $v(k)$ are discrete white-noise sequence with Gaussian distributions.

The error estimation has been described by equation (2.38)

$$e(k) = x(k) - q(k)$$

The covariance of the vector is denoted as $P(k)$ [$n \times n$], with

$$P(k) = E[e(k)e^T(k)] \quad (2.46)$$

where the diagonal elements of $P(k)$ are the average square errors of the estimation. The cost function for the minimization process is chosen as the trace of $P(k)$ which is the sum of the diagonal elements or the sum of the average squared errors of estimation.

$$\begin{aligned}
J(k) &= \text{tr}P(k) = E[e_1^2(k)] + E[e_2^2(k)] + \dots + E[e_n^2(k)] \\
&= \sigma_{e_1}^2(k) + \sigma_{e_2}^2(k) + \dots + \sigma_{e_n}^2(k)
\end{aligned} \tag{2.47}$$

Where $\sigma_{e_1}^2(k)$ is the variance of $e_1(k)$. This cost function can also be expressed as

$$J(k) = E[e(k)e^T(k)]$$

Since the system of equation (2.42) and (2.43) has random inputs and the measurements have random error, so the actual estimation errors can never be determined [24]. Hence, only the statistical characteristics of the estimation errors will be considered, and minimize a function of the expected values of these errors. Hence the Kalman filter is optimal only on the average [1, 24].

Besides that, an additional property of the Kalman filter is the cost function

$$J'(k) = \text{tr}E[e(k)Qe^T(k)] \tag{2.48}$$

with $J'(k)$ is minimized for any Q positive semidefinite matrix. Hence the sum of cost function can be describe as

$$J_N = \sum_{k=0}^N E[e_1^2(k)] + E[e_2^2(k)] = \sum_{k=0}^N E[e(k)e^T(k)]$$

will is also minimized. The mathematical development requirement to minimize the cost function resulting Kalman filter equation as

$$q(k) = \bar{q}(k) + G(k)[y(k) - C\bar{q}(k)] \tag{2.49}$$

$$\bar{q}(k+1) = Aq(k) + Bu(k) \tag{2.50}$$

where $\bar{q}(k)$ is the predicted state estimate at the sampling instant k , and $q(k)$ is the actual state estimate at k . The gain matrix $G(k)$ is the Kalman gain, and is calculated from the covariance equations

$$G(k) = M(k)C^T[CM(k)C^T + R_v]^{-1}$$

$$\begin{aligned}
 P(k) &= M(k) - G(k)CM(k) \\
 M(k+1) &= AP(k)A^T + B_1R_wB_1^T
 \end{aligned}
 \tag{2.51}$$

with $M(k)$ is known as covariance of the prediction errors:

$$M(k) = E\{[x(k) - \bar{q}(k)][x(k) - \bar{q}(k)]^T\} \tag{2.51}$$

while $P(k)$ is defined in equation (2.46). The second equation in (2.51) can be substituted into the third equation of equation (2.51) to expressed as a single difference equation in $M(k)$.

$$M(k+1) = A[I - G(k)C]M(k)A^T + B_1R_wB_1^T \tag{2.52}$$

In steady-state Kalman filter case, $G(k)$ becomes constant and the filter equation (2.48) has the characteristic equation [1, 2]

$$|zI - (A - GC)| = 0$$

2.11.1 The Discrete Kalman Filter Algorithm

In the discrete Kalman filter algorithm, Kalman filter estimates the process by using a form of feedback control. The filter estimates the process state at some time and then obtains the feedback in the form of noise measurement. Kalman filter can be divided into two groups which are time update equations and measurement update equations.

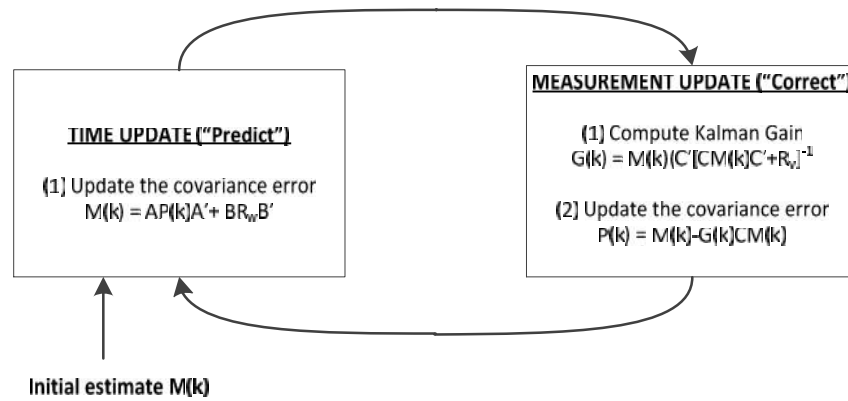


Figure 2.4: Discrete Kalman Filter process

Figure 2.4 show the operation of Kalman filter. The operation can be described in two terms which are time update equations and measurement update equations. The time update equations are to update the current error covariance estimation to obtain the estimation of a *priori* for the next time step. The measurement update equations are responsible for the feedback such as for incorporating a new measurement into the *priori* estimate to obtain an improved *posteriori* estimate. Time update can also be thought of as a predictor equation while the measurement update can be thought as corrector equation [1, 16].

2.12 Linear Quadratic Gaussian (LQG)

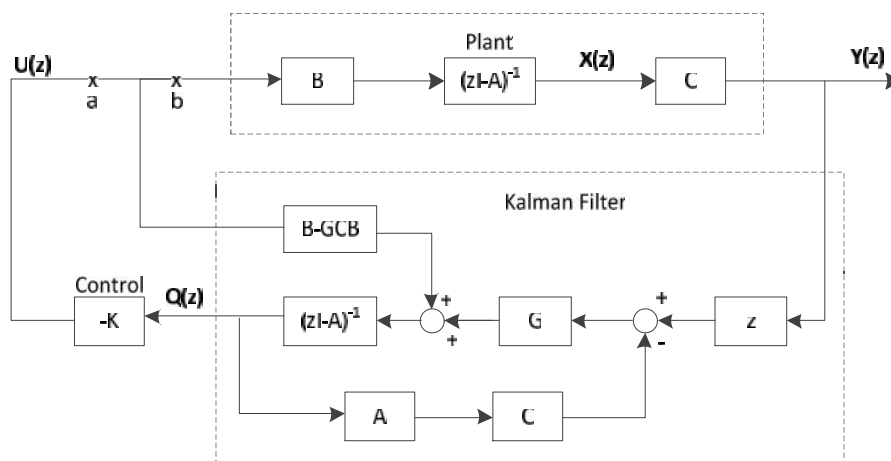


Figure 2.5: LQG Control System

Figure 2.5 show the block diagram of LQG control system which has been derived from equation (2.49) and (2.50). Figure 2.5 describe LQG control system can be implement by using Linear Quadratic (LQ) optimal control and Kalman filter

which Kalman filter will be act as estimator. Besides that, the implementation of full-state feedback control systems using Kalman filters as state estimators can also result in systems that are not robust. Hence the control-estimator transfer function, $D_{ce}(z)$ will be determined by simplify the block diagram of Figure 2.5 which can be performed with DARE [1]. Combination of the control-estimator transfer function also known as LQG digital controller transfer function, $G_D(z)$.

Since LQG is implemented based on LQ optimal control and Kalman filter, hence Separation Principle will be used. The Separation Principle is utilized in the LQG design as a two-step process

Step 1. Obtain the optimal state-feedback matrix gain (K) based on Steady-State Quadratic Optimal Control (Section 2.9).

$$K = (R + B'PB)^{-1}B'PA$$

Step 2. Obtain the Kalman Filter gain matrix (G) by using Optimal State Estimation method (Section 2.11)

$$G(k) = M(k)C^T[CM(k)C^T + R_v]^{-1}$$

2.13 Matrix Laboratory (MATLAB)

MATLAB is a high-level language and interactive environment that has been developed by MathWorks for numerical computation, visualization and programming. MATLAB allows matrix manipulations, plotting of functions and data implementation of algorithm, creation of user interfaces and interfacing with programs written in other language including C, C++, Java and FORTRAN.

The MATLAB application is built around the MATLAB language and most use of MATLAB involves typing MATLAB code into the Command Window or executing text file containing MATLAB code and functions. However as the number of command increases or trial and error is done by changing certain variable, M-files will be helpful to run the command efficiently. An m-file or scrip file is a simple text file to place MATLAB commands

MATLAB also supports developing application with graphical interface features. MATLAB includes GUIDE (GUI development environment) for graphical

designing GUIs. Besides that, MATLAB also supports Simulation and Model-Based Design (Simulink). Simulink is a block diagram environment for multidomain simulation and Model-Based Design. It provides a graphical editor, customizable block libraries and solvers for modeling and simulating dynamic system.

2.14 Arduino Mega 2560

Arduino is a microcontroller platform that has captured the imagination of electronic enthusiasts. The Arduino Mega is a microcontroller board based on the ATmega1280. It has 54 digital input/output pin, 16 analog inputs, 4 UART which known as hardware serial ports. It also contains 16 MHz crystal oscillator, a USB connection, a power jack, an ICSP header and a reset button.

The Arduino Mega can be power via the USB connection or with external power supply. The board can operate on an external supply 6 to 20 volts. If external supply was less than 7V, 5V pin may supply less than 5V and the board may unstable. However, if external supply more than 12V, the voltage regulator may overheat and damage the board. Therefore, the recommended supply range is 7 to 12V.

Each of the 54 digital pins on the Mega can be used as an input or output by define `pinMode()`, `digitalWrite()`, and `digitalRead()` functions. Each pin on Arduino board can provide or receive a maximum of 40 mA and has an internal pull-up resistor of 20-50Kohm. Pin assignment on Arduino board was shown in Figure 2.6.

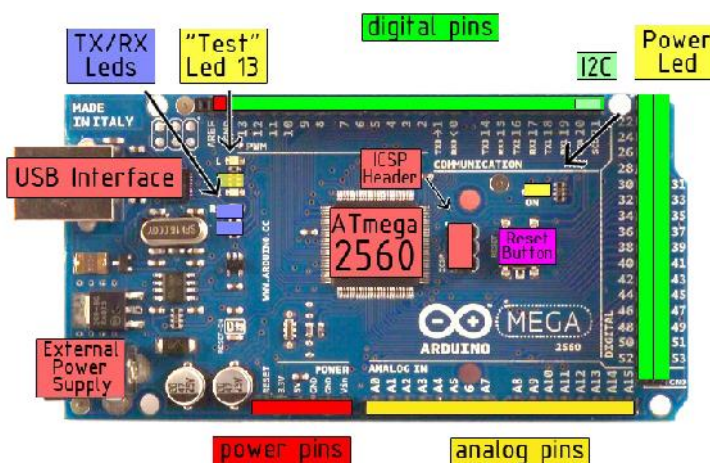


Figure 2.6: Arduino Mega 2560

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