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Magnetohydrodynamic stagnation-point flow towards a stretching/shrinking sheet with slip effects☆

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ABSTRACT

The steady two-dimensional stagnation-point flow over a linearly stretching/shrinking sheet in a viscous and incompressible fluid in the presence of a magnetic field is studied. The governing partial differential equations are reduced to nonlinear ordinary differential equations by a similarity transformation, before being solved numerically by a shooting method. Results show that the skin friction coefficient decreases, but the heat transfer rate at the surface increases when the effect of slip at the boundary is taken into consideration. Dual solutions are found to exist for the shrinking sheet, while for the stretching sheet, the solution is unique.

1. Introduction

The flow and heat transfer of a viscous and incompressible fluid over a stretching sheet has attracted the interest of many researchers in view of its applications in many industrial manufacturing processes. Some examples are in the glass blowing, the cooling and/or drying of papers and textiles, the extrusion of a polymer in a melt-spinning process, metals and plastics, continuous casting and spinning of fibers, etc. Crane [1] was the first who studied the two-dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate and obtained an exact solution in closed analytical form. Since then, many authors have studied various aspects of this problem, such as Chiam [2], Mahapatra and Gupta [3], Ishak et al. [4,5], etc., who have studied the flow behaviors due to a stretching sheet in the presence of magnetic field, considering some other physical features such as power-law velocity and buoyancy effect, with various surface heating conditions.

Compared to a stretching sheet, less work has been done on the flow over a shrinking sheet. It seems that the steady hydrodynamic flow due to a shrinking sheet for a specific range of the suction parameter was first studied by Miklavčič and Wang [6], where the existence of the exact solution to the Navier–Stokes equations was reported. They also found that the flow is unlikely to exist unless adequate suction on the boundary is imposed since the vorticity of the shrinking sheet is not confined within a boundary layer. Later, Wang [7] showed that with an added stagnation flow to contain the vorticity, similarity solutions may exist. Recently, Faraz et al. [8] studied the two-dimensional viscous flow over a shrinking sheet by employing an analytical approach. On the other hand, the problem of stagnation flow over a shrinking sheet immersed in a micropolar fluid was considered by Ishak et al. [9] and Lok et al. [10], while Bhattacharyya and Layek [11] considered the boundary layer stagnation-point flow towards a shrinking sheet with thermal radiation and suction/blowing effects. They found that dual solutions exist for a certain range of the shrinking parameter.

All of the above-mentioned studies considered no slip condition at the boundaries. Andersson [12] noticed that in certain situations, the assumption of the flow field obeys the conventional no-slip condition at the boundary does no longer apply and should be replaced by partial slip boundary condition. Later, Wang [13] gave an exact solution of the Navier–Stokes equations for the flow due to a stretching boundary with slip. Wang [14] has also considered the effect of stagnation slip flow on the heat transfer from a moving plate. The effects of slip on the flow of an elastic-viscous fluid with some other physical features have been studied by Ariel et al. [15] and Ariel [16]. The heat transfer problem in a viscous fluid over an oscillatory stretching surface with slip has been investigated by Abbas et al. [17], while Fang et al. [18] studied the MHD slip flow and found only one physical solution for any combination of the slip, magnetic and mass transfer parameters. Later, Fang et al. [19] solved the problem of viscous fluid over a shrinking sheet with a second order slip flow model, without considering the heat transfer aspects, and reported an exact solution of the governing Navier–Stokes equations. The thermal boundary layers over a shrinking sheet with mass transfer but without slip has been also studied by Fang and Zhang [20]. In another paper, Fang et al. [21] solved analytically the magnetohydrodynamic flow under slip conditions over a shrinking sheet and reported the existence of multiple solution branches for certain parameter domain.

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Nomenclature

a, c	constants
B_0	uniform magnetic field
C_f	skin friction coefficient
f	dimensionless stream function
k	thermal conductivity
L	slip length
M	magnetic parameter
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	surface heat flux
Re_x	local Reynolds number
S	proportionality constant
T	fluid temperature
T_w	surface temperature
T_∞	ambient temperature
u, v	velocity components along the x and y directions, respectively
U	external flow velocity
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

α	thermal diffusivity
γ	thermal slip parameter
δ	velocity slip parameter
ε	stretching/shrinking parameter
η	similarity variable
θ	dimensionless temperature
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
σ	electrical conductivity
τ_w	surface shear stress
ψ	stream function

Subscripts

w	condition at the surface
∞	ambient condition

Superscript

$'$	differentiation with respect to η

The effect of slip on the stagnation-point flow towards an impermeable shrinking sheet has been studied by Bhattacharyya et al. [22], while Aman and Ishak [23] studied the flow and heat transfer over a permeable shrinking sheet.

In this paper, we study the behaviour of the hydromagnetic stagnation-point flow towards a stretching/shrinking sheet with slip effect on the boundary. The effect of magnetic, slip and stretching/shrinking parameters on the skin friction coefficient and the heat transfer rate at the surface is analyzed and discussed. To the best of our knowledge, this problem has not been studied before.

2. Mathematical formulation

Consider a two-dimensional stagnation-point flow towards a linearly stretching/shrinking sheet of constant temperature T_w immersed in an incompressible viscous fluid. It is assumed that the external flow velocity varies linearly along the x-axis, i.e., $U(x) = ax$, where a is a positive constant. A uniform magnetic field of strength B_0 is assumed to be applied in the positive y-direction normal to the plate. The induced magnetic field is assumed to be small compared to the applied magnetic field and is neglected. Under these assumptions along with the boundary layer approximations, the system of equations, which model the boundary layer flow, are given by (see Ishak et al. [5] and Bhattacharyya et al. [22])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho}(U-u) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where (u,v) are the fluid velocities in the (x,y) directions, T is the temperature in the boundary layer, ν is the kinematic viscosity, α is the thermal diffusivity, ρ is the fluid density and σ is the electrical conductivity. The appropriate boundary conditions for the velocity components with slip condition at the surface and the temperature are given by (see Bhattacharyya et al. [22])

$$\begin{aligned} &u=cx+L(\partial u/\partial y),\quad v=0,\quad T=T_w+S\frac{\partial T}{\partial y}\quad \text{at}\quad y=0\\ &u\to U(x),\quad T\to T_\infty\quad \text{as}\quad y\to\infty \end{aligned} \tag{4}$$

where c is the stretching/shrinking rate of the sheet with $c>0$ for stretching and $c<0$ for shrinking, L denotes the slip length, S is a proportionality constant and T_∞ is the ambient temperature.

We introduce now the following similarity transformation:

$$\eta=\left(\frac{U}{\nu x}\right)^{1/2}y,\quad \psi=(\nu xU)^{1/2}f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

where η is the independent similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which identically satisfies Eq. (1). Using Eq. (5), we obtain

$$u=axf'(\eta)\ \text{and}\ v=-(\nu a)^{1/2}f(\eta) \tag{6}$$

where primes denote differentiation with respect to η.

Substituting Eqs. (5) and (6) into Eqs. (2) and (3), we obtain the following nonlinear ordinary differential equations:

$$f'''+ff''+1-f'^2+M\left(1-f'\right)=0 \tag{7}$$

$$\theta''+\Pr f\theta'=0 \tag{8}$$

where $M=\sigma B_0^2/(\rho a)$ is the magnetic parameter and $\Pr=\nu/\alpha$ is the Prandtl number. The boundary conditions (4) now become

$$\begin{aligned} &f(0)=0,\quad f'(0)=\varepsilon+\delta f''(0),\quad \theta(0)=1+\gamma\theta'(0)\\ &f'(\eta)\to 1,\quad \theta(\eta)\to 0\quad \text{as}\quad \eta\to\infty \end{aligned} \tag{9}$$

where $\varepsilon=c/a$ is the stretching/shrinking parameter, with $\varepsilon>0$ for stretching and $\varepsilon<0$ for shrinking, $\delta=L(a/\nu)^{1/2}$ is the velocity slip parameter and $\gamma=S(a/\nu)^{1/2}$ is the thermal slip parameter.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f=\frac{\tau_w}{\rho U^2/2},\quad Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \tag{10}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{11}$$

with μ and k being the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables (5), we obtain

$$\frac{1}{2}C_f \mathrm{Re}_x^{1/2} = f''(0), \quad Nu_x/\mathrm{Re}_x^{1/2} = -\theta'(0) \tag{12}$$

where $\mathrm{Re}_x = Ux/\nu$ is the local Reynolds number.

Following Magyari [24], it can be shown that Eq. (8) with the boundary conditions (9) for $\theta(\eta)$ has the following integral form analytical solution

$$\theta(\eta) = \frac{J(\infty;\mathrm{Pr}) - J(\eta;\mathrm{Pr})}{\gamma + J(\infty;\mathrm{Pr})} \tag{13}$$

where

$$J(\eta;\mathrm{Pr}) = \int_0^\eta \left[\exp\left(-\mathrm{Pr}\int_0^t f(z)dz\right)\right]dt, \quad J(\infty;\mathrm{Pr}) = \int_0^\infty \left[\exp\left(-\mathrm{Pr}\int_0^t f(z)dz\right)\right]dt \tag{14}$$

3. Results and discussion

The transformed Eqs. (7) and (8) subjected to the boundary conditions (9) were solved numerically using the Runge–Kutta–Fehlberg method with shooting technique for some values of the governing parameters, namely, the magnetic parameter M, the velocity slip parameter δ, the thermal slip parameter γ and the stretching/shrinking parameter ε when the Prandtl number Pr is fixed to unity. It is worth mentioning that the computation is made until the solution exists up to the smallest value of ε where the results for the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ are converge and both velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles satisfy the far field boundary conditions asymptotically. Comparisons of the values of $f''(0)$ for the shrinking case ($\varepsilon < 0$) with those obtained by Wang [7] and Bhattacharyya et al. [22] for several values of ε are listed in Table 1. It is observed that the results show a very good agreement.

The variation of the skin friction coefficient $C_f\mathrm{Re}_x^{1/2}$ and the local Nusselt number $\mathrm{Nu}_x/\mathrm{Re}_x^{1/2}$ (representing heat transfer rate at the surface) for selected values of the velocity slip parameter δ are shown in

Table 1
Comparison with previously published data for the values of $f''(0)$, when $M = 0$ and $\delta = 0$ (no slip) for shrinking case ($\varepsilon < 0$).

ε	Present results		Wang [7]		Bhattacharyya et al. [22]	
	Upper branch	Lower branch	Upper branch	Lower branch	Upper branch	Lower branch
−0.25	1.4022		1.40224		1.40224051	
−0.3	1.4276					
−0.4	1.4686					
−0.5	1.4957		1.49567		1.49566948	
−0.615	1.5072				1.50724089	
−0.75	1.4893		1.48930		1.48929834	
−1	1.3288	0	1.32882	0	1.32881689	0
−1.15	1.0822	0.1167	1.08223	0.116702	1.08223164	0.11667340
−1.18	1.0004	0.1784				
−1.2465	0.5543	0.5543	0.55430		0.55428565	0.55428565

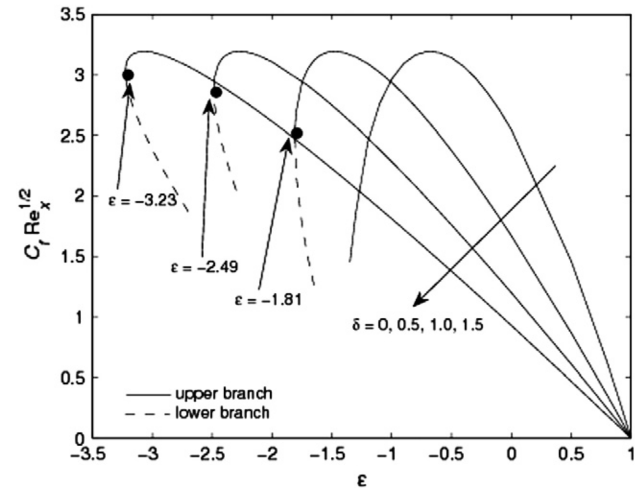

Fig. 1. Variation of the skin friction coefficient $C_f\mathrm{Re}_x^{1/2}$ with stretching/shrinking parameter ε for some values of velocity slip parameter δ when $M = 0.1$.

Figs. 1 and 2, respectively. It was found that dual solutions exist for the shrinking case ($\varepsilon < 0$), while for the stretching case ($\varepsilon > 0$), the solution is unique. However, based on our computations, no dual solutions could be found when the velocity slip parameter δ is absent ($\delta = 0$). In Figs. 1 and 2, the solid lines denote the upper branch solution, while the dash lines denote the lower branch solution. The value of $C_f\mathrm{Re}_x^{1/2}$ is zero when $\varepsilon = 1$. This is due to the fact that, for this case, the fluid and the solid boundary move with the same velocity, and thus there is no friction at the fluid–solid interface. However, there is heat transfer at the boundary for this case, see Fig. 2, even though there is no friction, since the fluid and the solid surface are at different temperatures.

Since Eqs. (7) and (8) are uncoupled, the flow field is not affected by the thermal field. Thus, the thermal slip parameter γ and the Prandtl number Pr have no effect on the flow field. For this reason, for each values of γ and Pr, the function f and its derivatives are identical. Fig. 3 presents the temperature profiles $\theta(\eta)$ for the selected values of the thermal slip parameter γ when $\mathrm{Pr} = 1$, $M = 0.1$, $\varepsilon = -2.48$ and $\delta = 1$. The effect of thermal slip is seen to reduce the fluid temperature in the boundary layer, which in turn decreases

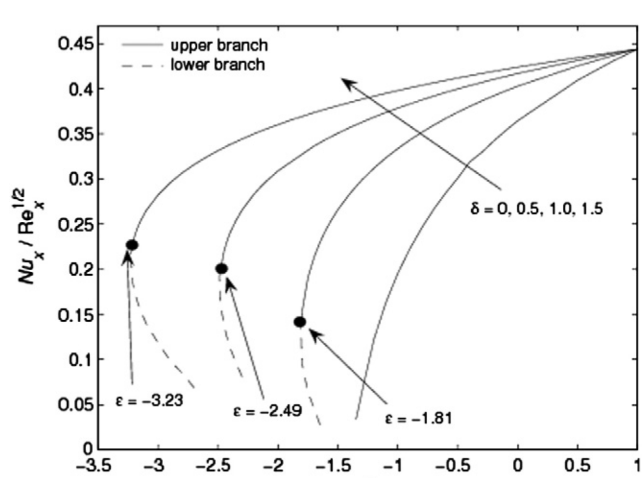

Fig. 2. Variation of the local Nusselt number $\mathrm{Nu}_x/\mathrm{Re}_x^{1/2}$ with stretching/shrinking parameter ε for some values of velocity slip parameter δ when $M = 0.1$, $\mathrm{Pr} = 1$ and $\gamma = 1$.

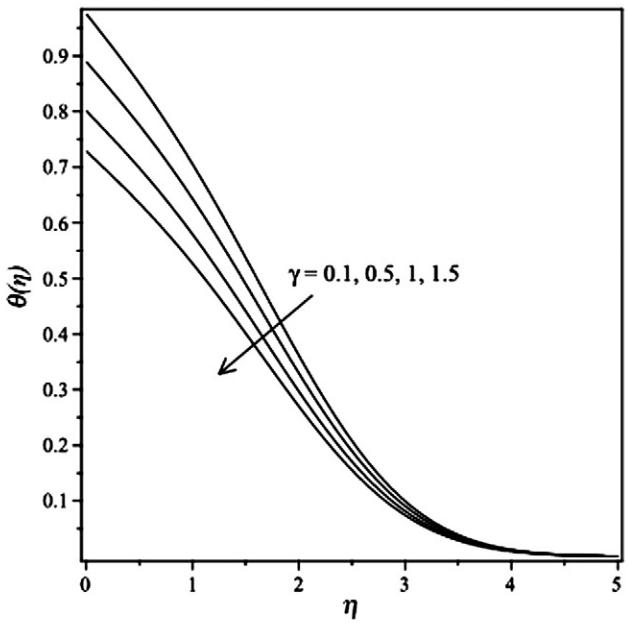

Fig. 3. Temperature profiles $\theta(\eta)$ for some values of γ when $\Pr = 1, M = 0.1, \varepsilon = -2.48$ and $\delta = 1$.

the temperature gradient at the surface. Thus, the heat transfer rate at the surface $Nu_x/Re_x^{1/2}$ increases as γ increases. Between the two solutions as presented in Figs. 1 and 2, we expect that the first solution (upper branch solution) is stable and most physically relevance, while the second solution (lower branch solution) is not, since the second solution exists only for certain range of the shrinking/stretching parameter ε.

Figs. 4 and 5 present the streamlines sketched by the stream function ψ from Eq. (5) for $M = 1, \delta = 1$ when $\varepsilon = 0.5$ (stretching sheet) and $\varepsilon = -3.5$ (shrinking sheet), respectively. It can be seen that the streamlines for $\varepsilon = 0.5$ are quite simple and symmetric about the vertical axis due to the equal force (in opposite directions) that exerts to stretch the sheet, and the pattern is almost similar to the normal stagnation-point flow without slip. On the other hand, the streamlines for $\varepsilon = -3.5$ are more complicated with a horizontal line separate the flows into two regions.

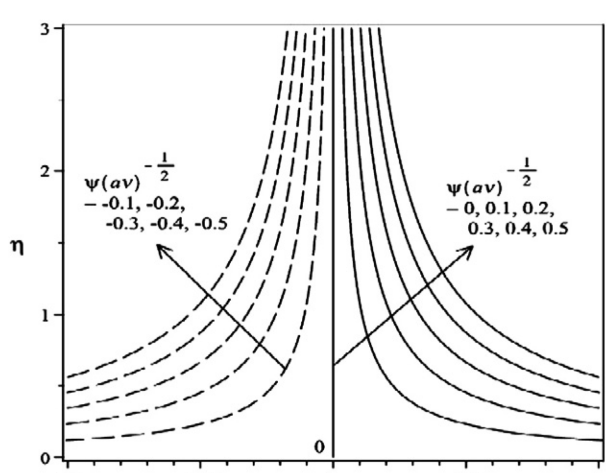

Fig. 4. Streamlines for two-dimensional flow when $M = 1$, $\delta = 1$ and $\varepsilon = 0.5$ (stretching sheet).

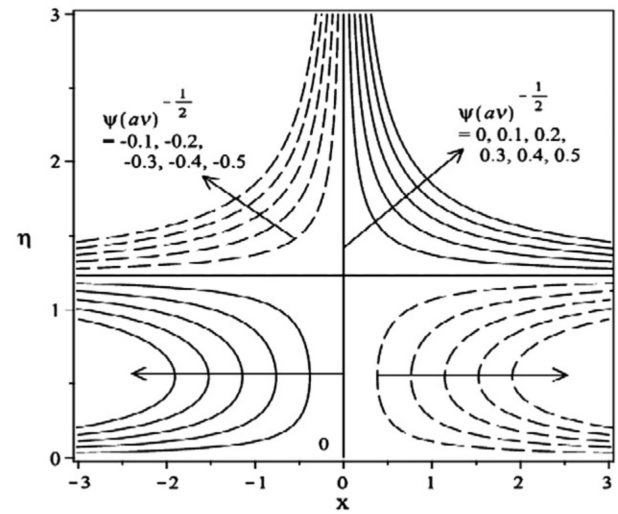

Fig. 5. Streamlines for two-dimensional flow when $M = 1$, $\delta = 1$ and $\varepsilon = -3.5$ (shrinking sheet).

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