

NOVEL SCHEME FOR ANALYSING THE NOISE LIMITATIONS OF
AVERAGE AND GUIDED SOLITON COMMUNICATION SYSTEMS

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ABSTRACT

This thesis presents analytical and numerical investigations of optical soliton transmission in optical fibre communication systems. The basic principles of nonlinear pulse propagation in optical fibre are discussed followed by discussion of the main limitations for an amplified soliton based system. The goal of this work is to study in-depth the soliton propagation and to analyse the bit pattern noise analysis and multi-slot analysis of system performance. First, a performance comparison of mqocss, VPI and OptSim for single and WDM transmission systems is presented. The purpose of this comparison is to assess the degree of variability between independently written numerical simulation codes and the commercial software available in the market. Secondly, a study of average soliton systems with 35, 50 and 70 km amplifier spacing is presented both analytically and numerically. The effect of modifying the existing rule of thumb formulae is presented in terms of a design diagram. This is then compared to the numerical simulations. The introduction of optimum jitter parameter, $\alpha_{sq} = 0.05$ in the formulation of GH limitation and optimum ratio of B_{elec} to B_{opt} , $BRAT = 0.4$, of the ASE limitation improve the better comparison between analytical and numerical results. A series of design diagram with Gordon-Haus (GH) jitter, signal to noise ratio (SNR) and soliton collapse limited transmission is also introduced. The same study has been repeated for guided soliton systems. Finally a significant portion of this work is devoted to the noise analysis of GH and SNR limited system transmission. A novel technique is introduced which is designed to distinguish between the effects of dispersion and jitter in analysing the limitations of system performance. These are represented in the form of probability distribution function on amplitude and timing jitter, by introducing the two methods (A and B), referring to the eye diagrams of the system limitations. Another technique analysis is presenting the energy plots of E_{n+1} and E_{n-1} , the energy adjacent to bit under consideration. The plot also explains the limitation due to amplified soliton, ASL.

Key words: Optical amplification, Guiding Centre Soliton, Guided Soliton, GH and SNR limited transmission, Noise analysis.

ABSTRAK

Tesis ini membentangkan penyelidikan secara analitik dan numerik bagi penghantaran 'soliton' optik di dalam sistem komunikasi gentian optik. Prinsip asas perambatan denyut tak linear dalam gentian optik akan dibincangkan diikuti dengan perbincangan had-had perambatan bagi sistem soliton yg diamplifikasikan. Objektif kajian ini adalah untuk mendalami perambatan soliton seterusnya menganalisis prestasi sistem pada corak hingar untuk setiap 'bit' dan 'multi bit'. Perkara pertama yang dibincangkan ialah perbandingan dari segi prestasi di antara mqcoss, VPI and OptSim bagi sistem penghantaran tunggal dan sistem penghantaran WDM. Tujuan perbandingan ini ialah untuk menilai darjah kebolehubahan di antara kod pengaturcaraan dan perisian komersil. Kedua, kajian tentang sistem "average soliton". Reka bentuk sistem dan hasil dapatan pada jarak penguat 35, 50 dan 70 km (yang membentuk keseluruhan jarak 3500 km) akan di bincangkan secara analitik dan numerik termasuk kesan mengubah parameter-parameter kepada formula sedia ada. Pengenalan kepada nilai optimum "jitter" , $\alpha_{sq} = 0.05$ dalam formulasi had GH dan nilai optimum nisbah B_{elec} to B_{opt} , $BRAT = 0.4$ pada had ASE menambah baik perbandingan antara output analitik dan numerik. Di bahagian ini juga diperkenalkan gambarajah rekabentuk sistem penghantaran yang terhad kepada Gordon Haus (GH), nisbah isyarat-hingar (SNR) dan 'keruntuhan' soliton. Kajian serupa juga dibuat untuk "guided soliton" dan analisis yang sama diulang untuk sistem ini. Akhirnya bahagian terpenting ditumpukan kepada analisis hingar sistem penghantaran yang dihadkan oleh GH and SNR. Satu teknik baru diperkenalkan untuk membezakan di antara kesan-kesan "dispersion" dan "jitter" dalam menganalisis prestasi bagi sistem tersebut. Analisis yang dilakukan adalah dalam bentuk fungsi taburan kebarangkalian pada amplitud dan masa, dengan memperkenalkan dua cara (A dan B) merujuk kepada bentuk "eye diagram" bagi setiap had system. Teknik lain ialah mempamerkan plot tenaga di E_{n+1} and E_{n-1} , iaitu tenaga bersebelahan "bit" yang dikaji. Plot tenaga ini juga boleh menerangkan had yang disebabkan oleh soliton yang dikuatkan, ASL.

Perkataan kunci: Penguatan optik, Guiding Centre Soliton, Guided Soliton, Penghantaran terhad GH dan SNR, analisis hingar.

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LIST OF ABBREVIATIONS

ASE	Amplified Spontaneous Emission
ASL	Average Soliton Limit
BER	Bit Error Rate
BRAT	Bandwidth Ratio
BW	Bandwidth
CW	Continuous Wave
DCF	Dispersion Compensating Fibre
DSF	Dispersion Shifted Fibre
EDFA	Erbium Doped Fibre Amplifier
FBG	Fibre Bragg Grating
FWHM	Full Width Half Maximum
FWM	Four Wave Mixing
GH	Gordon-Haus
GVD	Group Velocity Delay
NF	Noise Figure
NLSE	Non-Linear Schrödinger Equation
NRZ	Non Return to Zero
OSNR	Optical Signal to Noise Ratio
PDF	Probability Distribution Function
RZ	Return to Zero
SBS	Stimulated/Spontaneous Brillouin Scattering
SC	Soliton Collapse
SMF	Single Mode Fibre
SNR	Signal to Noise Ratio
SPM	Self Phase Modulation
SRS	Stimulated/Spontaneous Raman Scattering
WDM	Wavelength Division Multiplexing/Multiplexer
XPM	Cross Phase Modulation

CHAPTER 1

INTRODUCTION TO FIBRE-OPTIC TRANSMISSION

1.1 Historical perspective of fibre optic communication system

The rapid progress of optical communications is basically stimulated by the continuous increase in the demand for telecommunication services. Researchers and designers of optical networks live in a permanent quest for new techniques to augment the capacity and flexibility of the existing communications systems/networks, and also for the development of new concepts to meet the requirements of long distance and/or high capacity transmission systems. Optical communication has been proven to be the only reasonable choice to meet such a demanding requirement (Senior J.M., 1992). Optical fibre can transmit ultra high speed information with extremely low loss over a wide range of wavelengths. By virtue of this outstanding property, fibre-optic communication technologies have been applied to a variety of transmission systems throughout the world, such as international undersea networks and terrestrial links (Agrawal G.P., 2002).

In the early 1960s the idea of optical waves for communications was faced with two main problems; those were the availability of a suitable source of such waves and the need for a suitable medium of transmission delayed the progress. Both problems were solved by the invention of lasers in 1960 and the availability of GaAs semiconductor lasers in 1970, the first proposal of glass fibre and the subsequent development of low loss glass fibre that can guide the optical waves over long distances (Senior J.M., 1992). An optical communication system based on a single mode fiber transmitted 2 Gb/s over 40 km in 1981 (Yamada J. I., Kimura T., 1981). The communication technology has shown rapid progress since then. In just 20 years the bit rates have increased to 40 Gb/s and more. Below is the summary of time line

on progress and development of fibre optic communication systems (Mollenauer L.F., 1986, Mollenauer L.F., 1988, Mollenauer L.F, S.G.E, Haus H.A., 1991, Hasegawa A, Kodama Y., 1990, Hasegawa A., Kodama Y., 1991).

- 1960: Invention of the laser
- 1970: The first optical fibre was fabricated, with losses of 20 dB/km, and operating in the region of 1 μ m
- 1978: Commercial operation of the first generation of optical communication systems
 - multimode fibres, near 0.8 μ m
 - up to 10 km spacing between repeaters
 - bit rates around 45 Mb/s
- Beginning of the 80's: Second generation of optical communication systems
 - multimode fibres, near 1.3 μ m
 - up to 20 km spacing between repeaters
 - bit rates around 100 Mb/s
- 1987: Commercial operation of the second generation of optical communication systems
 - monomode fibres, near 1.3 μ m
 - up to 50 km spacing between repeaters
 - bit rates around 1.7 Gb/s
- Second of the 80's: third generation of optical communication systems
 - monomode fibres, near 1.55 μ m
 - 60-70 km spacing between repeaters
 - no improvement on bit rate
- End of the 80's: development of EDFA's
 - reduced coupling losses
 - amplification process: stimulated emission
 - 30 dB gain
 - low pump powers: 30 mW
- Beginning of the 90's: commercial operation of the third generation of optical communication systems, with bit rates around 2.5 Gb/s, and deploying EDFA's
- Mid 90's: modern optical communication systems of the fourth generation
 - EDFA's
 - coherent detection
- WDM technology
 - terrestrial and submarine transmissions
- Suggestion and development of dispersion compensation techniques
 - motivation for the fifth generation of optical communication systems, based on optical solitons

1.2 Solitons in optical communication

Solitons have been studied in many fields of science; however, the most promising applications of soliton theory are in the field of optical communications. In such systems information is encoded into light pulses and transmitted through optical fibers over large distances. Optical communication systems are no longer the pipedreams of the future. Commercial systems have been in operation since 1977. The transatlantic undersea optical cable has been developed, which is expected to transmit around 40,000 telephone conversations simultaneously. The development of optical fibres, which are the basis of such systems, has led to a revolution in communications technology.

In 1973 Hasegawa and Tappert proposed that soliton pulses could be used in optical communications through the balance of nonlinearity and dispersion. They showed that these solitons would propagate according to the nonlinear Schrodinger equation (NLSE), which had been solved by the inverse scattering method a year earlier by Zakharov and Shabat. At that time there was no capability to produce the fibres with the proper characteristics for doing this and the dispersive properties of optical fibres were not known. Also, the system required a laser which could produce very short pulse widths, which also was unavailable. Mollenauer, Stollen and Gordon (1985, 1986, 1988, 1991) at AT&T Bell Laboratories then experimentally demonstrated the propagation of solitons in optical fibers.

The original communications systems employed pulse trains with pulse widths of about one nanosecond. However, there was still some distortion due to fibre loss. This was corrected by placing electronic repeaters every several tens of kilometres. In the mid 1980's it was proposed that by sending in an additional pump wave along the fibre, the fibre loss could be compensated through a process known as Raman scattering. In 1988, Mollenauer and his group showed that this could indeed be done by propagating a soliton over 4000 km without the need for electronic repeaters. This was one of the first demonstrations of an all-optical transmission system. Such systems became even more common with the development of the Erbium amplifier.

Below is a summary of the time line on significant progress and development of solitons in optical communication systems.

- 1838: observation of solitary waves in water
- 1895: mathematical description – KdV equation
- 1972: optical solitons as solutions to the NLSE
- 1973: dispersion and nonlinear phenomena
- 1980: experimental demonstration on propagation of soliton in optical fibres by Mollenauer, Stolen and Gordon. This is followed by extensive experiments conducted by the group at Bell Laboratories and confirmed the predictions of nonlinear Schrödinger equation.
- 1986: The analysis of nonlinear effect (*Gordon-Haus* effect) by Haus H. A and Gordon J. P when ASE noise from amplifiers is considered in the system.
- 1990: EDFAs –pioneering work in the development of diode-pumped Erbium-doped amplifiers by Desurvive, Mears, Tachibana, Zynskind (worldwide)
- 90's: soliton control techniques for example sliding guiding filter concept introduced by Gordon et al 1992, Mollenauer et al. 1993, in an attempt to push the transmission capacity to a higher limits.
- 1998: 40 channel WDM, combining optical solitons of different wavelengths, demonstrated a data transmission of 1 terabit per second (1,000,000,000,000 units of information per second).
- 2001: the practical use of solitons deployed submarine telecommunications equipment in Europe carrying real traffic using John Scott Russell's solitary wave.

1.3 Objectives of Study

Analytical and numerical investigations of an optical soliton transmission in optical fibre communication systems are studied. Specifically the main aim of this work is to analyse the noise resulted from this study with the following objectives:

1. To investigate the soliton propagation in fibre optics.

2. To model the propagation of soliton in fibre optics with the use of non-linear Schrödinger equation.
3. To numerically simulate the systems.
4. To compare and contrast the output between the analytical and the numerical simulations.
5. To perform analysis on the noise in the system used.

In general, these objectives have been achieved using two soliton systems namely the guiding centre soliton and the guided soliton which will be elaborated in chapter 5 and 6, respectively.

1.4 Problem Statement

In designing a soliton transmission system, there are several effects which must be taken into consideration. The design of the system depends not only on the system length but also on the data rate and amplifier span which are required. As shall be seen there must be compromises made in the system design brought about by the requirements of low timing jitter on arrival at the detector, high signal to noise ratio and an acceptable average power.

This study shows two system designs. First is the transmission of soliton in fibre with amplifier which is named as guiding centre soliton system and second is the guided soliton system where filter is included in the transmission line after every amplifier. With a transmission distance is limited to 3500 km, one of the most serious problems which must be overcome if these systems are to become viable is a consequence of the amplification of the signal along the transmission line. Amplifiers must be included in order to compensate for the loss of the optical fibre and any other components which are included in the system. However the inclusion of amplifier introduce some noise into the system as well as amplifying the signal. Noise degrades the signal to noise ratio. In a nonlinear soliton system the effect is more drastic and the ability of the soliton to re-adjust to accommodate small changes lead to a random timing jitter, known as Gordon-Haus jitter, as the soliton propagates (Gordon G. P., Haus H. A, 1986). Including a filter in a transmission line can reduce the rate of Gordon-Haus jitter build up in

a soliton system but introduces an additional loss. This can be improved by a small increase in gain in the amplifier.

What is needed in this study is to look into the limitations imposed on these systems, particularly limitations due to timing jitter (GH jitter) and amplitude jitter (SNR). How these limitations differ from each other by studying the noise produced at the end of the propagation. Keeping this in mind, the model based on the receiver design for systems impaired by pulse jitter is proposed. This will give the analysis on the system performance in terms of the probability distribution function in the form of amplitude and of time. Analysis on the energy in time slots adjacent to the bit under consideration can also be done.

1.5 Scope and Delimitation

The study focuses on the limitations of the soliton propagation in an optical fibre giving emphasize on the amplified soliton emission (ASE) and on the Gordon Haus (GH) limitations. The study is delimited by a distance of propagation up to 3500 km only. The systems used are also delimited to guiding centre soliton (GCS) and guided soliton (GS), and only on a single-channel soliton transmission.

1.6 Chapter Overview

An overview of the chapter contents is as follows:

- Chapter 1. A brief history of optical fibre and soliton in optical communication systems development.
- Chapter 2. An introduction to pulse propagation in optical fibres, which includes the basic propagation equation, and condition and characteristic of propagation of optical solitons.
- Chapter 3. Introduces the amplified soliton; this includes the limiting factors imposed by optical fibres and amplifiers (EDFA), the soliton control technique and an introduction to the concept of design diagram.
- Chapter 4. A review of commercial simulation software, VPI and Optsim, in the study of optical transmission systems. These simulators are compared with a simulator written at Aston.

- Chapter 5. Studies the average soliton system by comparing the analytical model with the numerical simulations.
- Chapter 6. Studies the guided soliton system; look at the effect of putting the filter in the system. Again the numerical and analytical simulations are compared.
- Chapter 7. Studies the bit pattern noise analysis in terms of their probability distribution functions for GH and ASE limitation, both in average soliton and guided soliton systems.
- Chapter 8. Studies the multi-slot analysis of system performances which introduces a new analysis of system performance based on considering the energy in time slots adjacent to the bit under consideration.
- Chapter 9. Finally conclusions for the thesis are presented along with suggestions for future work.

CHAPTER 2

PULSE PROPAGATION IN OPTICAL FIBRES

The telecommunications industry discovered the optical fibre as a medium for efficient information transfer between locations being several kilometers apart from each other, with the invention of low-loss optical fibres (Horiguchi M, H.O 1976). A multitude of different fibre types are commercially available, which offer different signal propagation characteristics. For details, see (<http://www.corningfiber.com>, <http://www.lucnet>, <http://www.furukawa.jp>).

In the following section, the basic propagation equation for a single-mode optical fibre is presented. Following the introduction of the nonlinear Schrödinger equation, NLSE, a discussion on pulse propagation governed by group velocity dispersion GVD, self phase modulation SPM and a combination of GVD and SPM is presented.

2.1 Basic Propagation Equation

In this section, the propagation equation for the slowly varying amplitude of the electric field in optical single-mode fibres is presented. Details of the derivation can be found in Marcuse D (1991) and Agrawal G.P (2007).

Starting from Maxwell's equations, the optical field evolution in a dielectric medium can be described by the wave equation as follows

$$\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \frac{\partial^2}{\partial t^2} \vec{P} = 0 \quad (2.1)$$

where $\Delta \vec{E} = \nabla \times \nabla \times E$ is the curl of the electric field vector,

$\vec{P} = \vec{P}_L + \vec{P}_{NL}$ is the electric polarisation vector, with \vec{P}_L is the linear

polarisation and \vec{P}_{NL} is the non linear polarisation

c is the speed of light in vacuum,

μ_0 is the vacuum permeability

For silica-based optical fibres, \vec{P} can be described via \vec{E} as follows

$$\vec{P} = \frac{1}{\mu_0 c^2} \left\{ \chi^{(1)} \vec{E} + \chi^{(3)} \vec{E}^3 \right\} \quad (2.2)$$

where $\chi^{(1)}$ is the first order susceptibility, defining the linear evolution behaviour,

$\chi^{(3)}$ is the third order susceptibility, responsible for the nonlinear

propagation characteristics. Equation 2.2 is related to the ratio of the

nonlinear, n_2 and n_0 linear refractive index via $\frac{n_2}{n_0} = \frac{3}{8} \text{Re}\{\chi^{(3)} \vec{E}^3\}$.

Assuming that the fundamental mode of the electric field is linearly polarized in the x or y direction (with z being the propagation direction), its value can be approximately described using the method of separation of variables by

$$E(x, y, z, t) = \text{Re}\{F(x, y)A(z, t) \exp[j(\omega_0 t - \beta_0 z)]\} \quad (2.3)$$

where $F(x, y)$ is the transversal field distribution,

$A(z, t)$ is the complex field envelope (or slowly varying amplitude)

describing electric field evolution in the propagation direction z and time t ,

with $|A|^2$ corresponding to the optical power.

For single-mode fibre, SMF, the fundamental transverse mode, HE_{11} , is approximately given by a Gaussian distribution over the fibre radius. $A(z, t)$ is determined as a solution of the generalized nonlinear Schrödinger equation, which is given by

$$\frac{\partial}{\partial z} A + \beta_1 \frac{\partial}{\partial t} A + \frac{i}{2} \beta_2 \frac{\partial^2}{\partial t^2} A = i\gamma |A|^2 A - \frac{\alpha}{2} A \quad (2.4)$$

where $A(z, t)$ is the slowly varying amplitude of the electric field

β_1 and β_2 are chromatic dispersion where the pulse envelope moves with

group velocity $v_g = 1/\beta_1$ and the effects of group-velocity dispersion

GVD are governed by β_2

γ is the fibre nonlinear coefficient

α is the fibre losses coefficient

Equation (2.4) describes the evolution of the slowly varying field amplitude over a nonlinear, dispersive fiber. It describes the most important propagation effects for pulses of widths larger than 5 ps. It can, however, be extended to include higher order GVD and other nonlinear effects (such as stimulated Raman scattering and pulse self-steepening) which might be of importance for ultra-short pulse propagation or wide bandwidth applications.

In order to further simplify equation (2.4) it is helpful to use a frame of reference moving with the pulse at the group velocity $v_g = 1/\beta_1$ by making the transform

$$T = t - \frac{z}{v_g} = t - \beta_1 z. \quad (2.5)$$

Equation (2.4) then becomes

$$\frac{\partial}{\partial z} A + \frac{i}{2} \beta_2 \frac{\partial^2}{\partial T^2} A = i\gamma |A|^2 A - \frac{\alpha}{2} A \quad (2.6)$$

where it includes the effects of group velocity dispersion, nonlinearity and loss. In the case where fibre loss is neglected ($\alpha = 0$), equation (2.6) is known as the nonlinear Schrödinger equation (NLSE)

To discuss the solutions of equation 2.6 in a normalized form (Agrawal G.P 2007), we introduce,

$$\tau = \frac{T}{T_0}, \quad U = \frac{A}{\sqrt{P_0}} \quad (2.7)$$

where T_0 is the rms pulse width and P_0 is the peak power of the pulse.

Equation (2.6) becomes $\frac{\partial A}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - i\gamma |A|^2 A = 0$ and from equation 2.7,

$$\left(\frac{\partial \tau}{\partial T} \right)^2 = \frac{1}{T_0^2} \text{ and } \frac{\partial U}{\partial A} = \frac{1}{\sqrt{P_0}} \quad (2.8)$$

Now, $\frac{\partial A}{\partial U} \frac{\partial U}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial \tau^2} \left(\frac{\partial \tau}{\partial t} \right)^2 - i\gamma |A|^2 A = 0$

$$\Rightarrow \sqrt{P_0} \frac{\partial U}{\partial z} + \frac{i\beta_2}{2T_0^2} \frac{\partial^2 A}{\partial \tau^2} - i\gamma |A|^2 A = 0 \quad (2.9)$$

Multiplying i to equation (2.9),

$$i\sqrt{P_0} \frac{\partial U}{\partial z} - \frac{1}{2} \frac{\beta_2}{T_0^2} \frac{\partial^2 A}{\partial \tau^2} + \gamma |A|^2 A = 0 \quad (2.10)$$

but dispersion length, $L_D = \frac{T_0^2}{|\beta|}$, then equation (2.10) becomes

$$i\sqrt{P_0} \frac{\partial U}{\partial z} - \frac{1}{2L_D} \frac{\partial^2 A}{\partial \tau^2} + \gamma|A|^2 A = 0 \quad (2.11)$$

Dividing equation (2.11) by $\sqrt{P_0}$ becomes

$$i \frac{\partial U}{\partial z} - \frac{1}{2L_D \sqrt{P_0}} \frac{\partial^2 A}{\partial \tau^2} + \frac{\gamma|A|^2 A}{\sqrt{P_0}} = 0 \quad (2.12)$$

Using equation (2.7),

$$U = \frac{A}{\sqrt{P_0}} \quad \text{then} \quad \frac{\partial U}{\partial A} = \frac{1}{\sqrt{P_0}} \quad (2.13)$$

$$\text{Then} \quad \frac{\partial U}{\partial z} = \frac{\partial A}{\partial z} \frac{\partial U}{\partial A} = \frac{1}{\sqrt{P_0}} \frac{\partial A}{\partial z} \quad \text{and} \quad \frac{\partial^2 U}{\partial \tau^2} = \frac{\partial^2 A}{\partial \tau^2} \frac{\partial^2 U}{\partial A^2}$$

$$\Rightarrow \frac{\partial^2 A}{\partial \tau^2} = \frac{\partial^2 U}{\partial \tau^2} \left(\frac{\partial A}{\partial U} \right)^2 \quad (2.14)$$

Substitute equations (2.13),(2.14) in (2.12),

$$i \frac{\partial U}{\partial z} - \frac{1}{2L_D \sqrt{P_0}} \frac{\partial^2 U}{\partial \tau^2} \left(\frac{\partial A}{\partial U} \right)^2 + \frac{\gamma|A|^2 A}{\sqrt{P_0}} = 0$$

$$\Rightarrow i \frac{\partial U}{\partial z} - \frac{1}{2L_D \sqrt{P_0}} \left(\sqrt{P_0} \right)^2 \frac{\partial^2 U}{\partial \tau^2} + \frac{\gamma|A|^2 A}{\sqrt{P_0}} = 0 \quad (2.15)$$

$$\text{or rewrite equation (2.15),} \quad i \frac{\partial U}{\partial z} - \frac{s\sqrt{P_0}}{2L_D} \frac{\partial^2 U}{\partial \tau^2} + \frac{\gamma|A|^2 A}{\sqrt{P_0}} = 0 \quad (2.16)$$

$$\text{Where } s = \text{sgn}(\beta_2) = \begin{cases} +1; \beta_2 > 0 \\ -1; \beta_2 < 0 \end{cases}$$

$$\text{and again, from equation (2.7), } U = \frac{A}{\sqrt{P_0}} \Rightarrow A = U\sqrt{P_0} \Rightarrow |A|^2 = |U|^2 P_0$$

the last term in equation (2.15) becomes.

$$\frac{\gamma|A|^2 A}{\sqrt{P_0}} = \frac{\gamma|U|^2 P_0 U \sqrt{P_0}}{\sqrt{P_0}} = \gamma P_0 |U|^2 U = \frac{|U|^2 U}{L_{NL}} \quad (2.17)$$

Therefore the propagation equation in (2.16) is conveniently rewritten as,

$$i \frac{\partial U}{\partial z} = \frac{\sqrt{P_0}}{L_D} s \frac{\partial^2 U}{\partial \tau^2} - \frac{1}{L_{NL}} |U|^2 U \quad (2.18)$$

where $s = \text{sgn}(\beta_2)$. L_D and $L_{NL} = 1/\gamma P_0$ are the dispersion length and nonlinear length respectively.

The dispersion length L_D and the nonlinear length L_{NL} provide the length scales over which the dispersive or nonlinear effects become important for pulse evolution along the fibre length L . When the fibre length L is such that $L \ll L_{NL}$ and $L \ll L_D$, neither dispersive nor nonlinear effects play a significant role during pulse propagation. When the fibre length L is such that $L \ll L_{NL}$ and $L > L_D$, the pulse evolution is governed by GVD and the nonlinear effects play a minor part; the width of the pulse increases. When the fibre length L is such that $L \ll L_D$ and $L > L_{NL}$, the pulse evolution is governed by SPM and dispersive effects play a minor part; the pulse suffers spectral broadening. This can happen for relatively wide pulses with a peak power $P_0 > 1W$. When the fibre length L is such that $L > L_D$ and $L > L_{NL}$ or L is longer or comparable to both L_D and L_{NL} , both dispersion and nonlinearity have a significant effect on the pulse propagation leading to a qualitatively different behaviour compared with that expected from GVD and SPM alone.

2.1.1 Pulse propagation governed by Group-velocity Dispersion (GVD)

In this section, the linear regime where the effect of dispersion dominates over nonlinearity is considered, when $L \ll L_{NL}$ and $L > L_D$ or equivalently,

$$\frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \ll 1. \text{ GVD involves the temporal broadening of a pulse as it propagates}$$

through an optical fiber. It determines how much an optical pulse would broaden on propagation inside the fibre. To study the effect of GVD alone, γ is set to zero in equation (2.6) and can be rewritten as,

$$\frac{\partial}{\partial z} A = -\frac{i}{2} \beta_2 \frac{\partial^2}{\partial T^2} A - \frac{\alpha}{2} A \quad (2.19)$$

The loss has no effect on the dispersive behaviour as it can be eliminated by using the normalised amplitude $U(z, T)$ defined by

$$A(z, T) = \sqrt{P_0} \exp(-\alpha z/2) U(z, T) \quad (2.20)$$

then $\frac{\partial U}{\partial A} = \frac{1}{\sqrt{P_0} \exp(-\alpha z/2)}$.

The normalized amplitude $U(z, T)$ satisfies the partial differential equation

$$i \frac{\partial U}{\partial z} = \frac{1}{2} \beta_2 \frac{\partial^2 U}{\partial T^2} \quad (2.21)$$

A solution to this equation can be found when operating in the Fourier domain to transform $U(z, T)$ to produce $\tilde{U} = (z, \omega)$ whose relationship is given by

$$U(z, T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{U}(z, \omega) \exp(-i\omega T) d\omega \quad (2.22)$$

This satisfies the ordinary differential equation (ODE)

$$i \frac{\partial \tilde{U}}{\partial z} = -\frac{1}{2} \beta_2 \omega^2 \tilde{U} \quad (2.23)$$

The homogeneous solution of this equation is

$$\tilde{U}(z, \omega) = \tilde{U}(0, \omega) \exp\left(\frac{i}{2} \beta_2 \omega^2 z\right) \quad (2.24)$$

where $\tilde{U}(0, \omega)$, the Fourier transform of the pulse at $z = 0$, is given by

$$\tilde{U}(0, \omega) = \int_{-\infty}^{\infty} U(0, T) \exp(i\omega T) dT \quad (2.25)$$

Equation 2.24 shows the effect of GVD alone, which causes a phase change of all the frequency components of the initial signal $\tilde{U}(0, \omega)$ proportional to the propagation distance z . The phase changes given by $\phi = \frac{\beta_2 \omega^2 z}{2}$ can lead to an alteration of the pulse shape as the pulse propagates. This solution shows that these phase changes vary as the square of the frequency through the ω^2 term. The phase change is also proportional to β_2 which indicates the phase behaviour is different in the anomalous and normal dispersion regions depending on the sign of β_2 .

The general time domain solution can be found by substituting equation 2.24 in equation 2.22 to give

$$U(z, T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{U}(0, \omega) \exp\left(\frac{i}{2} \beta_2 \omega^2 z - i\omega T\right) d\omega \quad (2.26)$$

In order to study the effect of GVD on a pulse after transmission, it is useful to consider the case of a Gaussian pulse as its mathematical form lends itself to this mathematical analysis because it is easily integrable. The Gaussian input pulse is given by (Marcuse D, 1980).

$$U(0, T) = \exp\left(-\frac{T^2}{2\tau_0^2}\right) \quad (2.27)$$

where τ_0 is the half width $1/e$ intensity point of the initial pulse. For a Gaussian pulse, this can also be expressed in terms of the full width at half maximum through

$$T_{FWHM} = 2(\ln 2)^{1/2} \tau_0 \approx 1.665\tau_0 \quad (2.28)$$

Using this input pulse and carrying the substitution and integrations in equations 2.25-2.27 gives the amplitude at any point z along the fibre (Agrawal G.P 2001).

$$U(z, T) = \frac{\tau_0}{(\tau_0^2 - i\beta_2 z)^{1/2}} \exp\left(-\frac{T^2}{2(\tau_0^2 - i\beta_2 z)}\right) \quad (2.29)$$

The solution shows the pulse maintains a Gaussian shape as it propagates but the pulse broadens temporally with transmission that is the width increases with z as

$$\tau_z = \tau_0 \sqrt{1 + \left(\frac{z}{L_D}\right)^2} \quad (2.30)$$

where $L_D = T_0^2 / |\beta_2|$. This equation shows that the shorter the pulse the more quickly the dispersive broadening as the rate of broadening is inversely proportional to L_D .

Figure 2.1 is a straightforward plot from equation 2.30 showing the shortest pulse broadens more quickly than other pulses. The pulse width of the shortest pulse broadens significantly more with distance than other pulses. The initial widths, τ_0 1-5 were chosen to make a comparison on how quickly the broadening effect with pulsewidth.

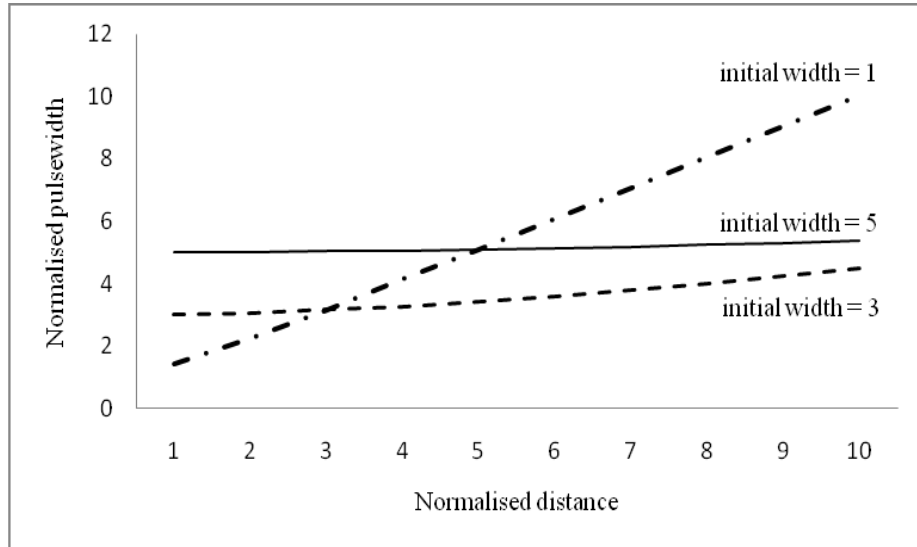


Figure 2.1: A plot of equation 2.30 for normalized pulsewidth versus normalized distance for different Gaussian pulses with initial width=1,3 and 5 (for $\beta_2 = 1$).

Comparing equation 2.7 with 2.9, it can be seen that the pulse frequency components have been subjected to a phase change. It can be seen from Marcuse D (1980), by rewriting equation 2.9 in the form,

$$U(z, T) = |U(z, T)| \exp[i\phi(z, T)] \quad (2.31)$$

$$\text{where } \phi(z, T) = -\frac{\text{sgn}(\beta_2)(z/L_D) T^2}{1 + (z/L_D)^2} + \frac{1}{2} \tan^{-1}\left(\frac{z}{L_D}\right) \quad (2.32)$$

The instantaneous frequency change across the pulse is given by the derivative of the time dependence of the phase $\phi(z, T)$ that is

$$\delta\omega(T) = -\frac{\partial\phi}{\partial T} = \frac{\text{sgn}(\beta_2)(2z/L_D) T}{1 + (z/L_D)^2} \frac{1}{T_0^2} \quad (2.33)$$

Equation 2.33 shows that the frequency changes linearly across the pulse and that is referred to as the linear frequency chirp. It depends on the sign of β_2 . In the normal dispersion regime ($\beta_2 > 0$) $\delta\omega$ is negative when $T < 0$ at the leading edge of the pulse but increases across the temporal profile of the pulse and vice versa with the anomalous dispersion regime. Comparing with equation 2.30 for an unchirped pulse, the amount of spectral broadening is independent of the sign of β_2 and the pulse broadens in the same amount for normal and anomalous dispersion regimes for any given value of L_D . The effect is somewhat different when launching a Gaussian pulse that is linearly chirped and whose function is

$$U(0, T) = \exp\left(-\frac{(1+iC) T^2}{2 T_0^2}\right) \quad (2.34)$$

where C is a chirp parameter. The function in terms of z is derived as in (Agrawal G.P. 2001).

$$U(z, T) = \frac{T_0}{\left[[T_0^2 - i\beta_2 z(1+iC)]\right]^{1/2}} \exp\left(-\frac{(1+iC) T^2}{2[T_0^2 - i\beta_2 z(1+iC)]}\right) \quad (2.35)$$

from which the broadening factor with distance is given by the relation (Marcuse D, 1981)

$$\frac{T}{T_0} = \left[\left(1 + \frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2 \right] \quad (2.36)$$

GVD parameter β_2 and chirp parameter C determine the broadening factor in this equation. In the normal dispersion region ($\beta_2 > 0$), for the case when the pulse is positively chirped ($\beta_2 C > 0$), broadening occurs immediately after transmission but it undergoes a narrowing process when the pulse is negatively chirped ($\beta_2 C < 0$) as in Figure 2.2.

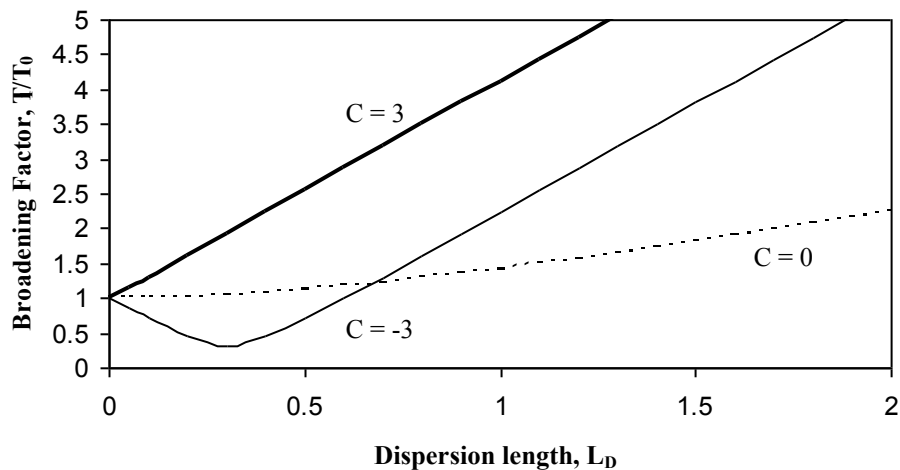


Figure 2.2: A plot of equation 2.36. Broadening factor for a chirped Gaussian pulse as a function distance for unchirped Gaussian pulse, positive and negative chirp. The pulse with positive chirp undergoes compression before broadening

In this section, the GVD parameter β_2 has been used throughout. It is commonly more interesting to determine the dependence of the inverse of the group velocity on wavelength rather than on frequency. This dependence is described by the

dispersion parameter D and its slope with respect to wavelength, λ . The following relationships hold

$$D = \frac{d}{d\lambda} \left(\frac{1}{v_g} \right) = -\frac{2\pi c}{\lambda^2} \beta_2 \quad \text{or} \quad \beta_2 = \frac{d}{d\omega} \left(\frac{1}{v_g} \right) = -\frac{\lambda^2}{2\pi c} D \quad (2.37)$$

D is typically measured in units ps/nm-km. It determines the broadening for a pulse of bandwidth $\Delta\lambda$ after propagating over a distance z , or equivalently, the time offset of two pulses after a distance z , which are separated in the spectral domain by $\Delta\lambda$. The relationship between dispersion and wavelength is described in Figure 2.3 and between normal and anomalous dispersion and wavelength, in Figure 2.4.

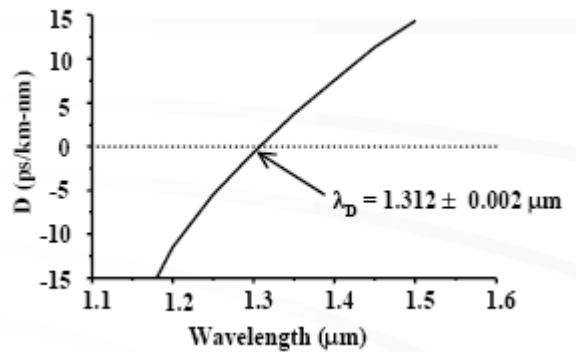
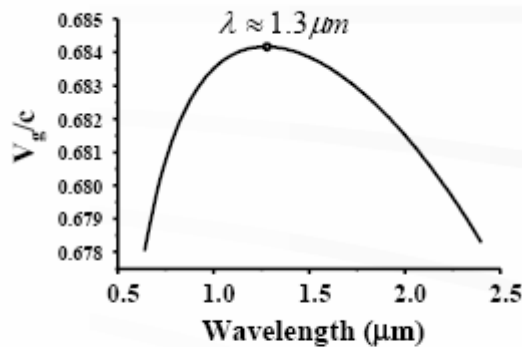


Figure 2.3: Variation of the dispersion parameter D with wavelength for a standard monomode fibre showing that zero dispersion occurs at $\lambda \approx 1.3 \mu\text{m}$ (Li T., 1985).



Normal dispersion	Anomalous dispersion
$D < 0, \text{ or } \beta_2 > 0 : \lambda < 1.3 \mu\text{m}$	$D > 0, \text{ or } \beta_2 < 0 : \lambda > 1.3 \mu\text{m}$

Figure 2.4: Anomalous and normal dispersion regimes (Li T., 1985).

2.1.2 Pulse propagation governed by SPM

In this section, the effect of dispersion term in equation 2.18 is considered to be negligible compared to the nonlinear term. In that case the pulse evolution is governed by nonlinear effect that leads to spectral broadening of the pulse (Agrawal G.P. 2001). This happens when the fibre length L is such that $L_D \gg L > L_{NL}$ or equivalently.

$$\frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \gg 1 \quad (2.38)$$

In standard single mode fibres, this condition holds for relative long pulses ($\tau_0 > 100ps$) and peak power $P_0 \sim 1W$.

To study the effect of SPM mathematically (where the effect of dispersion $\beta_2 = 0$ and taking into account the effect of loss, α in equation 2.6 and using the same process in section 2.1.1, we have

$$\frac{\partial U}{\partial z} = i \frac{\exp(-\alpha z)}{L_{NL}} |U|^2 U \quad (2.39)$$

where α accounts for fibre loss, which plays an important part in the pulse propagation and $L_{NL} (= 1/\gamma P_0)$ is the nonlinear length. The nonlinear parameter, γ is related to the nonlinear index coefficient n_2 by $\gamma = n_2 \omega_0 / c A_{eff}$. Equation 2.39 can be solved to give the solution

$$U(z, T) = U(0, T) \exp(i \phi_{NL}(z, T)) \quad (2.40)$$

where $U(0, T)$ is the input pulse amplitude at $z = 0$ and ϕ_{NL} is the nonlinear phase shift which increases with fibre length, L given by

$$\phi_{NL}(z, T) = |U(0, T)|^2 \frac{z_{eff}}{L_{NL}}. \quad (2.41)$$

with the effective transmission distance z_{eff} (which is smaller than z due to fibre losses) given by

$$z_{eff} = [1 - \exp(-\alpha z)] / \alpha \quad (2.42)$$

The maximum pulse shift occurs at the centre of the pulse $T = 0$ with magnitude of

$$\phi_{max} = z_{eff} / L_{NL} = \gamma P_0 z_{eff} \quad (2.43)$$

where $L_{NL} = (\gamma P_0)^{-1}$.

This shows the physical significance of L_{NL} that is the effective propagation distance at which $\phi_{\max} = 1$.

To impose a frequency chirp on the pulse, take the partial derivative of the phase shift in equation 2.41 with respect to T ,

$$\delta\omega = -\frac{\partial\phi_{NL}}{\partial T} = \frac{z_{eff}}{L_{NL}} \frac{\partial}{\partial T} |U(0,T)|^2 \quad (2.44)$$

This frequency chirp is now time dependent and increases with magnitude as the propagation distance increases. New frequencies are self-generated leading to spectral broadening. To see the SPM effect in spectral broadening consider a super- Gaussian pulse with an initial field given by

$$U(0,T) = \exp\left[-\frac{1}{2}\left(\frac{T}{\tau_0}\right)^{2m}\right] \quad (2.45)$$

The SPM-induced chirp $\delta\omega(T)$ for this pulse is

$$\delta\omega(T) = \frac{2m}{T_0} \frac{z_{eff}}{L_{NL}} \left(\frac{T}{\tau_0}\right)^{2m-1} \exp\left[-\left(\frac{T}{\tau_0}\right)^{2m}\right] \quad (2.46)$$

where m is the order of super Gaussian pulse ($m=1$ is for Gaussian).

From equation 2.45 and 2.46 the relationship between phase and the corresponding time is plotted using $m = 1$ and $m = 4$ and is presented in figures 2.5 and 2.6.

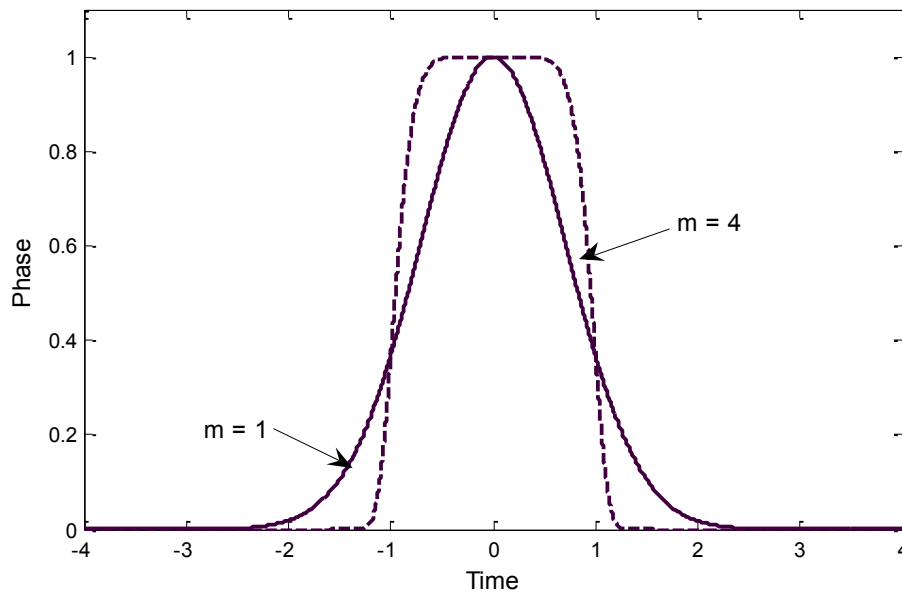


Figure 2.5: A plot of equation 2.46. The effect of transmission of a pulse with a unit width T_0 and effective transmission distance equal to the non linear length L_{NL} . Amplitude of 1st and 4th order Gaussian pulses with time.

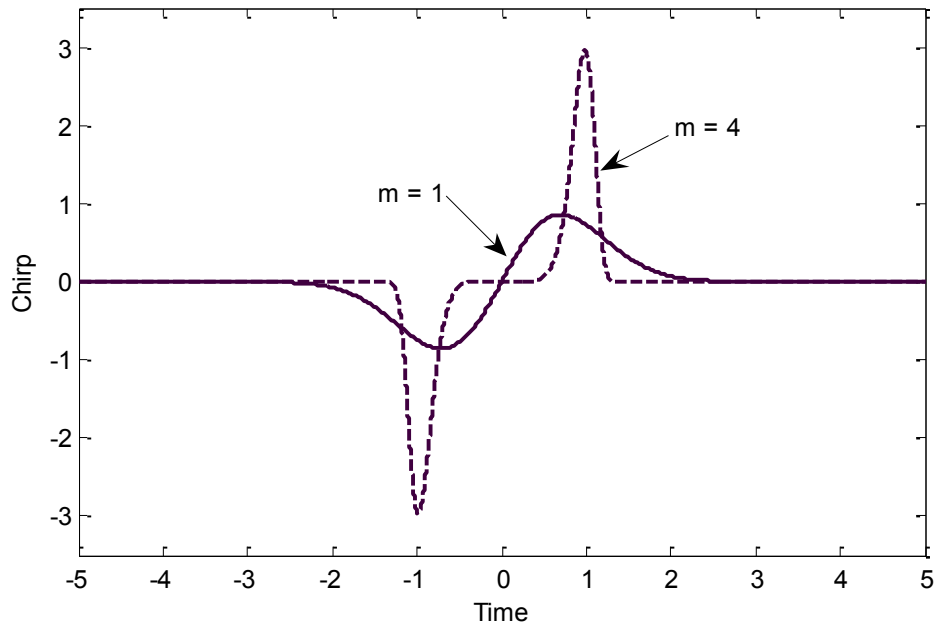


Figure 2.6: A plot of equation 2.46. The effect of transmission of a pulse with a unit width T_0 and effective transmission distance equal to the non linear length L_{NL} . Induced chirp caused by phase changes across 1st and 4th order Gaussian pulses.

2.1.3 Pulse Propagation Governed by GVD and SPM

Previously GVD and SPM were treated separately. In this section the case is considered when the fibre length $L \geq L_D$ and $L \geq L_{NL}$ where dispersion and nonlinearity act together as the pulse propagates along the fibre. Temporal and spectral changes that occur when the effect of GVD and SPM are combined, are considered in this section.

To start with, consider the lossless case of equation 2.6. The NLSE equation can then be written as

$$i \frac{\partial A}{\partial z} = \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} - \gamma |A|^2 A \quad (2.50)$$

This equation can be normalized using

$$U = \frac{A}{\sqrt{P_0}} \Rightarrow u = NU = N \frac{A}{\sqrt{P_0}}, \quad Z = \frac{z}{L_D}, \quad \tau = \frac{T}{T_0} \quad (2.51)$$

to give

$$i \frac{\partial U}{\partial Z} = \text{sgn}(\beta_2) \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} - N^2 |U|^2 U \quad (2.52)$$

where N is defined as

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}. \quad (2.53)$$

Eliminating N from equation 2.52 by introducing $u = NU = \sqrt{\gamma L_D} A$ to take the standard form of NLSE:

$$i \frac{\partial u}{\partial Z} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0 \quad (2.54)$$

where $\text{sgn}(\beta_2) = -1$ is taken for anomalous GVD; for normal dispersion regime, the dispersive term (2nd term in equation 2.54) should be negative.

The parameter N governs the relative importance of the effects of GVD and SPM on the pulse propagation:

$N \ll 1$: dispersive effects are dominant

$N \gg 1$: nonlinear effects are dominant

$N \approx 1$: both dispersive and nonlinear effects are important

$N = 1$: SPM and GVD are in the anomalous and normal dispersion regimes

Consider again equations 2.33 and 2.46. The evolution of the shape of an initially unchirped Gaussian pulse in a normal-dispersion regime of a lossless fibre is shown in figure 2.7. The pulse broadens more rapidly with the presence of SPM ($N=1$). This can be understood by recalling that new frequency components that are red-shifted near the leading edge and blue-shifted near the trailing edge of the pulse, are generated continuously as it propagates down the fibre. “In the normal-dispersion regime, the red components are traveling faster than the blue components, SPM leads to an enhanced rate of pulse broadening compared with the expected from GVD alone” (Agrawal G.P, 2001). But the situation is different when considering pulses propagating in the anomalous-dispersion regime as in figure 2.8. This is due to a negative sign of β_2 in equation 2.33. “The pulse broadens initially at a rate much lower than that expected in the absence of SPM and then appears to reach a steady state for $z > 4L_D$ ” (Agrawal G.P 2001).

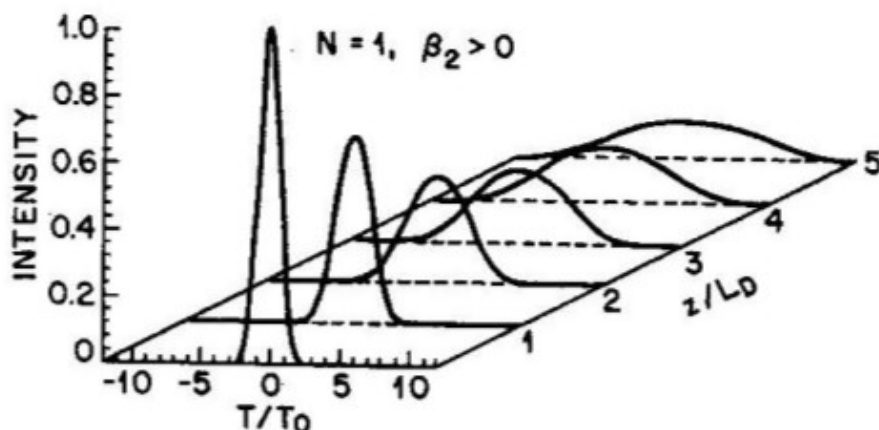


Figure 2.7: Evolution of pulse shapes over a distance z/L_D for an initially unchirped Gaussian pulse propagating in the normal dispersion regime of the fibre (Agrawal G.P,2001).

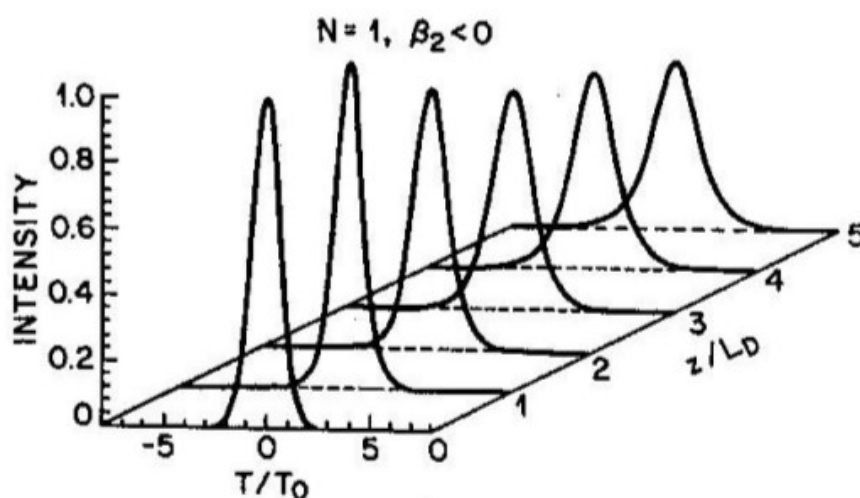


Figure 2.8: Evolution of pulse shapes over a distance z/L_D for an initially unchirped Gaussian pulse propagating in the anomalous dispersion regime of the fibre. (Agrawal G.P, 2001)

2.2 Optical solitons: characteristics and conditions for propagation

The existence of solitons in optical fibre is a result of interplay between the dispersive (GVD) and nonlinearity (SPM) effects. Optical solitons are pulses that exhibit particular shape and intensity, so there is a perfect balance between the frequency chirps produced by the GVD in the anomalous dispersion regime and the SPM. This section discusses the properties of solitons in lossless media. The fundamental soliton

has a shape which remains unchanged during propagation while higher order solitons have a shape which evolves periodically. This section also looks at the interaction between solitons, where two solitons that collide in a fibre recover their shape and initial intensity after the collision.

2.2.1 Fundamental and higher order solitons

The mathematical description of solitons employs the NLSE. Consider again equation 2.54. The solution to this equation was first solved using the inverse scattering method (Zakharov V.E. & Shabat A.B. 1972, Miwa T, 1999). Consider the first order (N=1) solution of NLSE which has the general form of solution

$$u(z, \tau) = 2\zeta \operatorname{sech}(2\zeta\tau) \exp(2i\zeta^2 z) \quad (2.55)$$

where ζ is the soliton amplitude. Normalising $u(0,0)$ by setting $2\zeta = 1$ gives the fundamental (N=1) soliton solution in the form of hyperbolic-secant,

$$u(z, \tau) = \operatorname{sech}(\tau) \exp(iz/2) \quad (2.56)$$

This equation indicates that if the hyperbolic-secant pulse whose width T_0 and the peak power P_0 are chosen such that $N = 1$ in equation 2.53 then

$$P_0 = \frac{|\beta_2|}{\gamma T_0^2} \quad (2.57)$$

is launched into an ideal lossless optical fibre, thus it will propagate undistorted indefinitely with no change in shape. Equation 2.57 shows that the peak power is inversely proportional to the square of the pulsewidth. The further solution to the NLSE is

$$u(0, \tau) = N \operatorname{sech}(\tau) \quad (2.58)$$

where N is an integer related to the soliton order. From equation 2.53, the required peak power is N^2 times that of a fundamental soliton for the same pulsewidth. Higher order solitons, corresponding to $N > 1$, do not maintain their shape during propagation along the fibre. This is due to the unusual evolution process where pulse splitting occurs during transmission only for the pulses to reform back into its original form at $z = m\pi/2$, where m is an integer. Thus the distance or the soliton period at which a higher order soliton recovers its original shape is given by

$$z_0 = \frac{\pi}{2} L_D = \frac{\pi}{2} \frac{T_0^2}{|\beta_2|} \quad (2.59)$$

To understand the reason for the stable solution it is useful to consider the sign of the chirps induced on the pulse by GVD and SPM. The sign of the GVD induced chirp depends on the sign of the β_2 (positive or negative sign in normal- or anomalous-dispersion regime respectively) whilst that of the SPM induced chirp is always of the same sign since the frequency shift is opposite to the gradient of the pulse power profile. In the anomalous-dispersion regime, the two chirps are opposed and can cancel out to form a stable soliton whilst in the normal-dispersion regime they have the same sign and do not cancel out, thus accumulate with transmission to produce unstable pulses.

2.2.2 Interaction between solitons

In order to increase the information carried in the transmission system, it is desirable to launch the pulses close together. Unfortunately the overlap of the closely spaced solitons leads to mutual interaction and therefore to serious performance degradation of the soliton transmission system. This is as a result of the small finite tails of the soliton that extend into neighboring solitons which in turn form a superposition that causes them to propagate at different velocities (Gordon G.P 1983). Another study showed that the inclusion of fibre loss also leads to dramatic increase in soliton interactions (Blow K.J, Doran N.J 1983). Several schemes for the reduction of the pulse interaction have been proposed; the use of Gaussian-shaped pulses (Chu P.L, Desem C. 1983), introduction of phase difference between neighboring solitons (Anderson D., Lisak M. 1986), the use of third order dispersion (Chu P.L, Desem C. 1985, 1987) are among others.

Consider a pair of solitons at the input of transmission line which can be described by

$$u(0, \tau) = \text{sech}\{\tau - q_0\} + r \text{sech}\{r(r + q_0)\} \exp(i\theta) \quad (2.60)$$

Where $2q_0$ is the initial (normalised) separation ($2q_0 = T_B/T_0$ and B is the bit rate), r is the relative amplitude and θ is the relative phase of the two input pulses. The interaction of the pulses depends on their relative amplitude, r and relative phase, θ . Numerical simulations and calculations have shown that for equal phase ($\theta = 0$) and

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