

DITHER EFFECT OF DRUM BRAKE SQUEAL

by

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DECLARATION

I hereby declare that the work reported in this thesis is the result of my own investigation and that no part of the thesis has been plagiarized from external sources. Materials taken from other sources are duly acknowledged by given explicit references.

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ABBREVIATIONS

AC	alternating current
CEA	complex eigenvalue analysis
dB (A)	unit of A-weighting sound pressure level
DOE	design of experiment
DOF	degree of freedom
FEM	finite element method
FFT	Fast Fourier transform
FRF	frequency response function
LMS	an innovative engineering company that provides service focused on attributes such as system dynamics, structural integrity and sound quality.
NVH	noise, vibration and harshness
PI	Physik Instrumente, the world's leading provider of nanopositioning products and systems. They develop and manufacture piezo components, actuators and motors.
PolyMAX	a new procedure for Modal Parameter Estimation from LMS
PZT	Lead Zirconate Titanate (Material of Piezo actuator)
R _{90%}	the maximum efficiency of dither force where the ratio of vibration amplitude reduction to the applied dither amplitude is highest and the suppression is at least 90% of the initial squeal amplitude.
RMS	root mean square
RPM	revolutions per minute
SAE	Society of Automotive Engineers, globally active professional association and standards organization for engineering professionals in various industries

SCLM	spectral criterion based linearisation method
SPL	sound pressure level
UTM	universal testing machine
WHO	world health organization

NOMENCLATURE

Symbol	Description	Unit
a	acceleration	m/s^2
C	capacitance of piezoceramic stack	F
c_1	damping coefficient of damper 1	(N.s)/m
c_2	damping coefficient of damper 2	(N.s)/m
c_c	contact damping coefficient	(N.s)/m
c_x	effective damping coefficient in normal direction	(N.s)/m
c_y	effective damping coefficient in tangential direction	(N.s)/m
D	Rayleigh dissipation function	J
$d(t)$	dither force (Transfer function)	N
f	frequency	Hz
F	force	N
$F_{dither}(t)$	dither force	N
F_f	friction force	N
F_N	normal force	N
$f_{NL}(\cdot)$	nonlinear component in transfer function	-
$h(t)$	impulse response (Transfer function)	-
$H(j\omega)$	control system (Transfer function)	-
k_1	internal spring stiffness 1	N/m
k_2	internal spring stiffness 2	N/m
k_c	contact stiffness	N/m
k_x	effective dynamic stiffness in normal direction	N/m
k_y	effective dynamic stiffness in tangential direction	N/m

M	mobility	(m/s)/N
m	dynamic mass	Kg
M_c	mobility element of damping	(m/s)/N
M_k	mobility element of stiffness	(m/s)/N
M_m	mobility element of mass	(m/s)/N
m_x	effective dynamic mass in normal direction	Kg
m_y	effective dynamic mass in tangential direction	Kg
N	mode number	-
P	follower force	N
$P_{actuator}$	power supplied to piezoceramic actuator	W
$r(t)$	force applied on the system (Transfer function)	N
T	kinetic energy	J
t	time	s
U	potential energy	J
V_B	sliding velocity	m/s
$V_{relative}$	relative velocity between sliding and shoe vibration	m/s
V_{rms}	root mean square voltage	V
V_{pp}	peak to peak voltage	V
$\ddot{x}(t)$	acceleration in normal direction	m/s ²
$\dot{x}(t)$	velocity in normal direction	m/s
$x(t)$	displacement in normal direction	m
$\ddot{y}(t)$	acceleration in tangential direction	m/s ²
$\dot{y}(t)$	velocity in tangential direction	m/s
$y(t)$	displacement in tangential direction	m
α	gradient of the friction-velocity graph	-

γ	dither direction	°
δ_1	elongation of spring k_1	m
δ_2	elongation of spring k_2	m
δ_c	elongation of spring k_3	m
ζ	damping ratio	%
η	hysteretic loss factor	-
θ_1	spring 1 location from y-axis	°
θ_2	spring 2 location from y-axis	°
θ_{drum}	Rotation of drum	°
λ	complex eigenvalue	-
λ_x	complex eigenvalue in normal direction	-
λ_y	complex eigenvalue in tangential direction	-
$\mu(t)$	friction coefficient	-
μ_s	static friction coefficient	-
τ	braking torque	Nm
ψ_n	amplitude of the mode	-
ω_n	natural frequency	Hz

KESAN GENTAR TERHADAP BUNYI KIUT BREK GELENDONG

ABSTRAK

Bunyi kiut brek kenderaan merupakan pencemaran bunyi jalanraya yang disebabkan oleh getaran yang teraruh daripada geseran. Paras tekanan bunyi dari bunyi kiut adalah lebih tinggi daripada 70 dB dan ia boleh menyebabkan kerosakan pendengaran jikalau pendedahan jangka panjang. Tesis ini menghuraikan penggunaan daya berfrekuensi tinggi, biasanya dirujuk sebagai sistem gantaran untuk mengurangkan bunyi kiut brek gelendong. Penggerak piezoseramik digunakan sebagai punca gantaran. Penggunaan sistem gantaran ini telah berjaya mengurangkan paras tekanan bunyi kiut brek gendang dari 85 dB ke tahap bunyi latar belakang sebanyak 38 dB. Model matematik dengan dua darjah kebebasan telah dibina untuk menyiasat ciri-ciri bunyi kiut brek gelendong dan keberkesanan sistem gantaran pada keadaan yang berlainan. Parameter model matematik ini ditakrif berdasarkan nilai yang diperolehi dari analisis kebolehergerakan yang diukur pada arah normal dan tangen. Model ini kemudiannya disahkan dengan keputusan eksperimen semasa kiut. Keberkesanan sistem gantaran dalam pengurangan bunyi kiut brek gelendong dikaji pada empat frekuensi gantaran yang berlainan pada arah tangen dan juga arah normal. Berdasarkan keputusan berangka, sistem pengantaran adalah lebih berkesan semasa gantaran dikenakan pada arah tangen dan juga semasa frekuensi gantaran yang lebih rendah digunakan. Selain itu, sistem kawalan ini memerlukan daya gantaran yang lebih rendah pada kelajuan yang rendah bagi mengurangkan bunyi kiut. Keputusan ramalan berdasarkan model ini menunjukkan sekaitan yang tinggi dengan keputusan eksperimen. Sistem gantaran juga menghasilkan jalur sisi di kiri dan kanan frekuensi kiut pada sebelum bunyi kiut dipadamkan sepenuhnya.

DITHER EFFECT OF DRUM BRAKE SQUEAL

ABSTRACT

Vehicle brake squeal is a typical traffic noise pollution which is caused by friction induced vibration. The sound pressure level of the brake squeal is higher than 70 dB and it can cause hearing damage if long-term exposure. This thesis describes the use of high frequency excitation force for suppressing drum brake squeal, commonly referred to as dither control. A piezoceramic actuator is used to generate the dither force. The application of dither has successfully quenched the drum brake squeal from 85 dB to background noise level of 38 dB. A bi-axial two DOF mathematical model is developed to investigate the characteristic of drum brake squeal and the effect of dither on drum brake squeal at various operating parameters. The model parameters are defined based on the mobility measurement in both normal and tangential directions. This model is then validated with the measured results during squeal. The effectiveness of dither is investigated in four excitation frequencies in both tangential and radial directions. The numerical results show that dither control is more efficient in tangential direction and during low dither excitation frequency. Besides, at low sliding speed, lower dither force is needed to suppress the brake squeal. The predicted results based on the developed model shows high correlation with the measured results. The existences of dither excite the sidebands of the squeal peak with equal frequency spacing at both sides before complete suppression of the brake squeal.

CHAPTER ONE

INTRODUCTION

1.1 Overview

This chapter provides the basic information of drum brake system, brake squeal and brake squeal elimination methods. Besides, the problem statement and contribution of this research work are discussed.

1.2 Background

Brake squeal is one of the common sources of traffic noise pollution. The sound pressure level of the brake squeal can be up to 90 dB with the frequency range of 1 kHz to 16 kHz (Liu and Pfeifer, 2003). Based on the statement from World Health Organization (WHO), long term exposure to noise above 60 dB increases the risk of myocardial infarction (Kim, 2007). Brake squeal also contributes to the risk in high blood pressure, hypertension and sleep disturbances. Although the impact of noise pollution to human health is relatively low compared to air pollution, but the annoyance caused cannot be ignored. Due to the impact on health, vehicles quietness is becoming one of the major considerations for the end users when choosing a vehicle. Although brake squeal has been proved to have no impact on braking performance, the brake noise, vibration and harshness (NVH) issues affect greatly customer satisfaction and the repair on NVH has always dominated the warranty claimed among auto parts (Glišović and Miloradović, 2010).

Dither control is a type of active control system which is an open loop that can be used to suppress brake squeal. The advantage of dither control is no feedback is needed and no sensor is used. The establishment of dither control system is simpler

compared to active feedback control. Dither control is defined as the quenching of friction induced vibration by using a high frequency, low amplitude periodic excitation force without any feedback from the system (Morgül, 1999). The periodic force tends to smoothen the unstable sliding motion of the vibration such as surface sticking, locking and spragging by providing them enough energy to slide freely. In more scientific way, the excitation force tends to change the Coulomb type friction into viscous like-damping which is more stable (Thomsen, 1999). Dither excitation is proved to be useful in quenching a few type of friction induced vibration including binary flutter (Hoffmann et al., 2005) and negative damping mechanism (Thomsen, 1999). Cunefare and Graf (2002a) investigated experimentally the application of dither control on disc brake squeal. The harmonic force is generated from a piezoceramic actuator installed on the calliper piston. They successfully suppressed disc brake squeal from 80 dB to 30 dB (background noise level) with a 20 kHz dither signal at actuation level of 153 V_{rms}. Dither control has been used to suppress the automotive wiper squeal (Stallaert et al., 2006). Dither can be applied in tangential or normal direction to the sliding surface and applying dither in tangential direction is proved to be more effective compared to dither applied in normal direction (Michaux, 2005). However, all of the published experimental work used normal dither in controlling of unstable vibration since applying tangential dither is difficult. For the case of drum brake squeal, applying tangential dither is possible due to its assembly.

1.3 Motivation of work

This research investigates the characteristic and mechanism of brake squeal excitation using the mathematical models. The application of dither control on drum brake squeal has not been done before and the investigation of the effectiveness and

behaviour of dither applied is important. Besides, the suppression mechanism of the dither control has to be determined for better understanding of dither control system on friction induced vibration.

1.4 Problem statement

Application of dither control on drum brake system has not been reported due to its difficulty in the installation of piezoceramic actuator in the limited space and only a small actuator can be placed in the system. The amplitude of the excitation is one of the most important parameter in the dither control system. A small actuator typically will have smaller excitation amplitude for a given amount of voltage supplied. This limitation causes the dither control to have insufficient dither amplitude to quench the drum brake squeal.

Dither control has always been used to suppress friction excited instability; however the dither is applied on the flat surface, where the direction of dither control can be properly defined. In the literature, the dithers are applied in normal direction to the sliding surface, since the installation is much easier. For a drum brake system which has curve sliding surface, the identification of location of centre of contact is difficult. Application of dither control on specific direction is difficult. Besides, the suppression mechanism of dither control has not been reported in the literature.

1.5 Objectives

This research is purposely done to study the mechanism of drum brake squeal excitations and effectiveness of dither control on it. The main objectives of the research are to;

- Develop a mathematical model to investigate the characteristic of drum brake squeal.
- Validate the developed model experimentally based on modal analysis.
- Apply dither control on the model of drum brake squeal.
- Suppress the drum brake squeal experimentally using dither.

1.6 Contribution

Minimal model is commonly used to investigate the characteristic and response of the friction induced vibration; however all these models have not been validated with experiment and the characteristics of the model obtained has not been compared to experiment. In this research, a two DOF model of drum brake squeal is constructed based on the mobility of the drum brake system measured in both directions of the friction and normal force. The development of the model has been described in good detail, and the resulting frequency response function of the model matches the experimental data.

Application of dither control on suppression of drum brake squeal has not been reported in the literature. The development of the experimental technique of dither control system applied in tangential direction to suppress drum brake squeal is novel and has never been reported before. This has greatly reduced the dither actuation level, V_{rms} compared to the normal direction as applied in disc brake squeal suppression by previous researchers.

From the available literature review, dither is proven to be effective in quenching of friction induced vibration; however the literature did not cover the mechanism of how the dither affects the system stability. This research has shown that the dither effect can be successfully modelled using the bi-axial two DOF model

which is capable of showing the excited sidebands of the squeal frequency which has been detected in the experiment.

1.7 Scope and limitation

This work was done based on laboratory setup drum brake system which was installed on the UTM machine. Thus, a small passenger car drum brake system was used. In order to produce squeal consistently, the brake lining of the brake shoe was removed and replaced with a rivet. The rivet was mounted on the location of centre of contact which was determined based on the wear condition of the lining. The use of rivet caused the distribution force in the brake to change to a concentrated force on the rivet. Since the rivet was mounted on the location of centre of pressure, the response of the brake shoe subjected to the concentrated force is assumed to be the same as the distribution force.

1.8 Thesis overview

This thesis is focused on the suppression of drum brake squeal using dither control. Before the dither control is applied on the experimental setup of a drum brake system, the characteristic of the drum brake squeal is studied using a mathematical model. This model is verified experimentally where the modal properties of the drum brake shoe are properly defined. The characterization of dither control is also done numerically before it is applied in the experimental setup of a drum brake system. The dither control is applied at two different speeds. The dither amplitude and frequency are tuned to achieve the optimum dither frequency and amplitude for suppression of squeal. This thesis is divided into five chapters.

Chapter One introduces the drum brake squeal with the discussion of squeal suppression methods. The general problem of the brake squeal is stated and briefly discussed. The objectives, contribution and motivation of the research are stated.

Chapter Two reviews the literature of brake squeal. Firstly, the proposed mechanisms of drum brake squeal by previous researchers are discussed together with the parametric studies. The methods used to analyse the brake squeal are also discussed with the advantages and limitations of each method. The squeal prevention and suppression methods proposed by previous researchers are also presented.

Chapter Three describes the methodology of how the model is developed and how experiment is done. The equations of motion of the mathematical model are derived. The steps of verification of the model using the experimental data are discussed. The experimental setup for modal analysis and squeal analysis are described and the devices and equipment used are stated. This chapter also describes the installation of piezoceramic stack actuator on the drum brake system and also the electric circuit of the dither signal generation.

Chapter Four presented both the experimental and simulation results of the dither control system. The results are discussed and compared with work done by other researchers on similar case.

Chapter Five summarises and concludes all the findings of the research and the recommendations for future work are presented.

CHAPTER TWO

LITERATURE REVIEW

2.1 Overview

This chapter covers the literature on the excitation mechanisms of brake squeal followed by the analytical method of the system stability. The analytical models from previous researchers and the characteristics of the system stability are discussed. The application of dither control is also included.

2.2 Drum brake system

Drum brake was first used in 1902 as a stopping device for vehicle and it is still widely used presently on heavy duty trucks, buses and also the rear wheel of some passenger cars due to its advantages over other braking systems. The schematic diagram of drum brake system is shown in Figure 2.1. The main components of drum brake system included the drum, a pair of shoes, backing plate and wheel hydraulic cylinder.

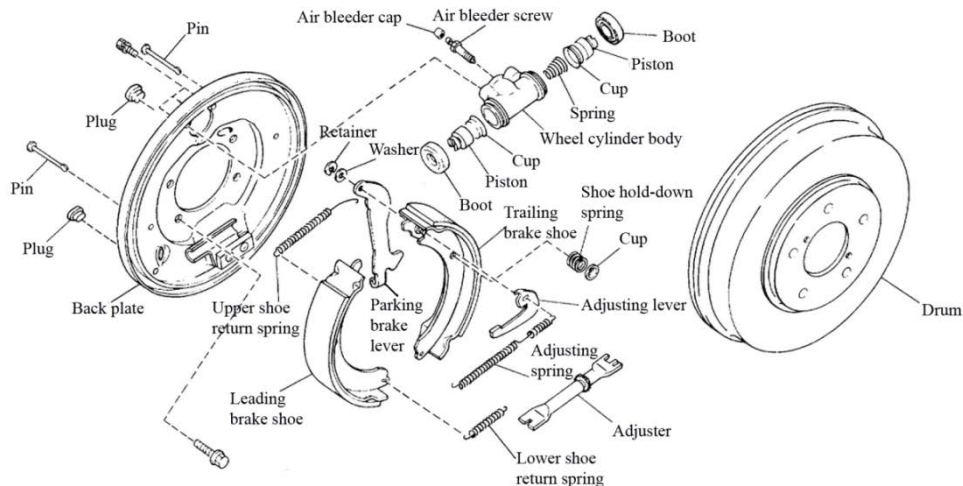


Figure 2.1 Schematic diagram of drum brake system (AutoZone, 2014).

During braking, the hydraulic fluid in the wheel cylinder transfers the braking pressure from the brake lever to the pistons which push the brake shoes against the rotating drum to stop the vehicle. There are several return springs which allow the brake shoes to return to their rest position after the brake pressure is released (preventing shoe-drum contact when driving). The adjuster is used to adjust the rest position of the brake shoes in order to maintain the optimum distance from the drum since the shoes must travel a greater distance to reach the drum as the shoe linings get thinner due to wear. The parking brake lever uses the cable actuated force instead of hydraulic force to press on the drum to stop the vehicle if there is failure on the hydraulic system. It is also used during parking since the hydraulic pressure may drop with time.

Drum brake is more easily incorporated with emergency brake due to its self-applying characteristic which increases the braking force with the same amount of braking pressure applied. The rotating drum will drag the leading shoe to press harder on the drum and increase the braking force. Due to the large contact area during braking, the drum brake shoe lasts longer (lower wearing rate) and has lower heat generation per unit area than the disc brake pad. The manufacturing cost of drum brake is relatively lower, but it has more parts and more difficult to service. Although the response time of drum brake is slower compared to disc brake, it is still widely used due to its advantages.

2.3 Brake squeal

Brake squeal is a phenomenon of dynamic instability which is caused by self-excited friction induced vibration. In drum brake squeal, the friction force is generated when the shoe lining is sliding on the drum inner surface. The friction

force causes the drum brake assembly to vibrate and generate squeal when some of the frequency modes are excited at particular condition. Brake squeals are mostly generated from the heavy vehicle such as buses and trucks which are using drum brake. Due to the large momentum of the heavy vehicles, higher friction coefficient material lining is typically used since larger friction force is needed to stop the vehicle. The use of high friction lining is more likely to excite brake squeal (Liu and Vyletel, 2012). Furthermore, in order to create higher braking torque, larger diameter drums are typically used. Larger diameter drum has less resistance to squeal due to its modal properties.

2.4 Mechanisms of brake squeal

Brake squeal is a type of self-excited friction induced vibration which remained unsolved although there have been a lot of improvements. There are several excitation mechanisms that have been used to describe the brake squeal namely the binary flutter instability (Hoffmann et al., 2002, Hoffmann and Gaul, 2003, Kang, 2008), negative damping (Ouyang et al., 1998, Shin et al., 2002, Kang, 2008), sprag-slip (Sinou et al., 2003, Hoffmann and Gaul, 2004, Keitzel and Hoffmann, 2006) and follower force nature of the friction force (Chan et al., 1995, Mottershead and Chan, 1995). Among these mechanisms, binary flutter instability is the most widely discussed in literature.

In the case of mechanical sliding where stick-slip phenomenon exists, the motion can turn out to be intermittent. This intermittent motion may lead to large amplitude of vibration or system limit cycle. The stick-slip phenomenon has high tendency to occur when the static friction coefficient is larger than dynamic friction coefficient (Spurr, 1961, Feeny et al., 1998, Giannini and Sestieri, 2006). The stick-

slip motion can be divided into two phases, stick and slip which occur alternatively during sliding. In stick phase, the surface is sticking together with zero relative motion. The sliding surface is deformed and the system accumulates potential energy. Once the accumulated potential energy is greater than the surface sticking, surface sliding will occur where the system dissipates the potential energy accumulated in the stick phase as kinetic energy. Shin et al. (2002) investigated the stick-slip phenomenon in the disc brake system using a two DOF model and showed that the system damping is the main key suppression of the model instability and the instability can be eliminated or reduced by increasing the system damping. Yang et al. (2009) included the non-smooth bifurcations and stick-slip transition in the analysis of brake noise and observed that grazing-sliding bifurcation and stick-slip chaos can also be found during the sliding motion. Nakano and Maegawa (2010) studied the stick-slip vibration using a one DOF sliding model and classified three situations for sliding system with regard to stick-slip vibration which are unstable system, stable system, and robust-stable system. A robust-stable system is always stable no matter how much vibration is induced where the energy of vibration is dissipated perfectly. They found that stick-slip motion with saw-tooth like motion is easily detected at low sliding speed, high contact load and large variation between static friction coefficient and sliding friction coefficient.

Binary flutter mechanism, sometimes referred to as mode coupling mechanism is an excitation mechanism that causes two or more nearby natural modes of the system (in term of frequency) to couple and produce a single mode with large amplitude (Hoffmann et al., 2004). The coupling of these modes is caused by the friction force which is a function of contact stiffness. Thus, the value of friction coefficient plays a key role in the binary flutter mechanism. Many parametric studies

have been made on the mode coupling mechanisms of brake squeal (Hoffmann and Gaul, 2003, Huang et al., 2006, Lignon et al., 2006, Huang et al., 2009). Sinou and Jezequel (2007) studied the influence of damping on the mode coupling mechanism and observed that the tendency of squeal can be reduced by optimizing the structural damping ratio. Huang et al. (2006) showed the coupling of shoe and drum modes by the stiffness of the lining material causes the brake squeal. They observed that not all the modes with similar frequency can be merged where the mode shapes play an important role in mode coupling. Other than small frequency separation, the modes must have strong interaction and compatible mode shapes in order to excite mode-coupling mechanism. Hultén (1993) developed a mathematical model to study the flutter instability of drum brake vibration which is excited by mode-coupling mechanism as shown in Figure 2.2. The coupling of the imaginary part (mode) can be achieved by increasing the friction coefficient, μ as shown in Figure 2.3. The black surface represents stable condition meanwhile the white surface represents unstable condition. The results showed that the imaginary part of the modes came closer with the increases of the value of friction coefficient until the modes coalesced at the critical friction coefficient $\mu=0.35$.

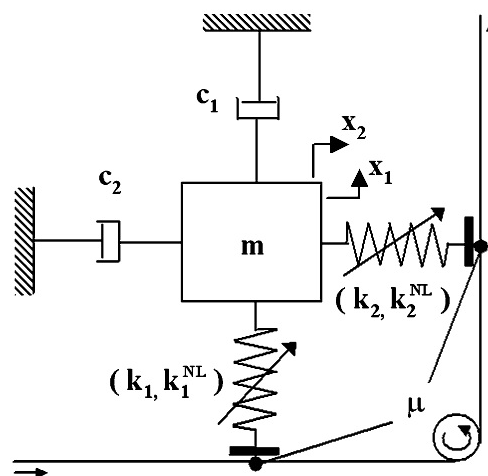


Figure 2.2 Mode-coupling model by Hultén (1993).

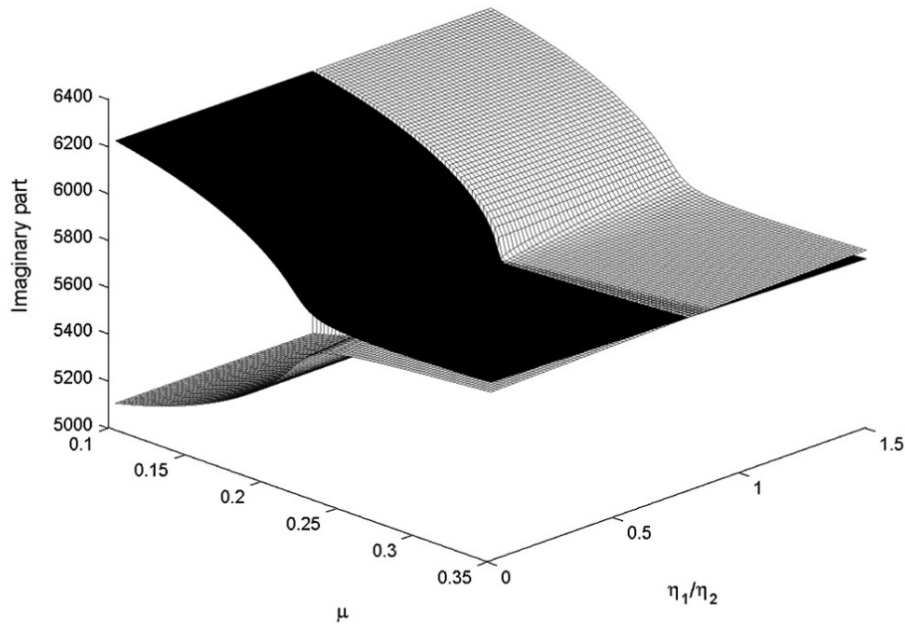


Figure 2.3 The change of imaginary part with friction coefficient, μ and damping ratio η_1/η_2 (Sinou and Jezequel, 2007).

Another well-known brake squeal mechanism is negative damping mechanism which is based on the phenomena of the decreases of friction coefficient with the sliding speed which can result in the instability (Shin et al., 2004, Kang et al., 2009b). The model of Shin et al. (2002) and the friction-velocity relationship are shown in Figure 2.4. This model included two masses m_1 and m_2 sliding on each other where a negative friction velocity slope (α) is included. However, the contact stiffness and damping of the system interface is excluded in this model. Experiment by Chen and Zhou (2003) proved that the negative friction-velocity slope is not a necessary condition since squeal also occurs in cases with positive friction-velocity slope. Although the influence of the negative friction-velocity slope to the squeal occurrence is less, compare to binary flutter instability, the non-linearity effect on the system stability cannot be neglected since it will change the stability region of the system and previous researchers have included the negative friction-velocity

characteristic as the secondary excitation mechanism of brake squeal (Ouyang et al., 1998, Fritz et al., 2007, Kang et al., 2009b).

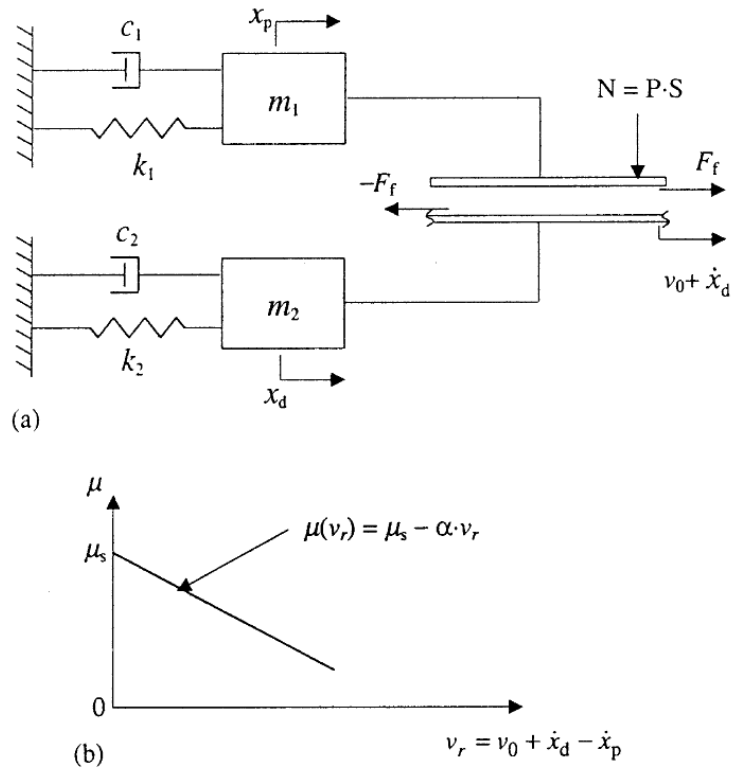


Figure 2.4 (a) Model by Shin et al. (2002) which emphasizes on the effect of (b) negative friction-velocity characteristic.

Sprag-slip is an excitation mechanism where the geometry of the parts is taken into account. There are two types of actions for this mechanism, which are locking and sliding. The sprag-slip mechanism can be explained with an elastic beam in contact with a sliding surface as shown in Figure 2.5. In this model, an elastic beam with stiffness \tilde{K} and inclination angle γ is having contact with a sliding surface with the contact stiffness K . When the beam inclination angle to the sliding surface is unfavourable, vigorous vibration can be observed (Sinou et al., 2003, Hoffmann and Gaul, 2004, Keitzel and Hoffmann, 2006). During the locking phase, the beam bent and gained potential energy. Once the friction force is insufficient to lock the contact

point, the beam will slide and all the potential energy stored is converted into kinetic energy. Sinou et al. (2003) analysed the non-linear sprag-slip model using centre manifold theory to simplify the equations by reducing the order of the dynamic system and preserving the dynamic behaviour near the Hopf bifurcation point. They concluded that sprag-slip is only noticeable in non-steady sliding state. The effective way to suppress sprag-slip mechanism is by finding the acceptable dynamic behaviour. Elimination of the dynamic behaviour of sprag-slip is difficult since there is no static state in sprag-slip mechanism, and simple addition of system damping will not effectively suppress the sprag-slip mechanism. The excitation of sprag-slip mechanism is strongly influenced by the friction coefficient and contact angle (Keitzel and Hoffmann, 2006).

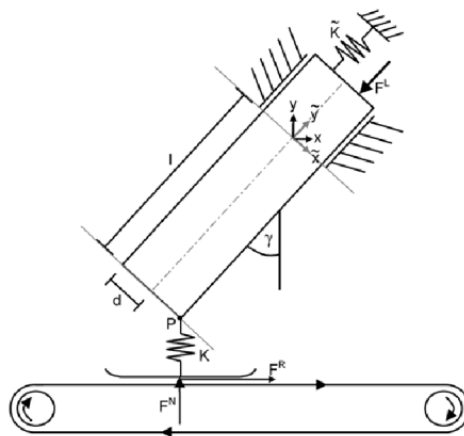


Figure 2.5 Sprag-slip model (Hoffmann and Gaul, 2004).

Frictional follower force is a non-conservative force which depends on the displacement of the system (Kinkaid et al., 2003). Ziegler pendulum has been used to study the frictional follower force (Kessler et al., 2007) where the follower force P acts on the mass m_2 at the angle of $\alpha\phi_2$ as shown in Figure 2.6. In the case of drum brake squeal, the frictional follower force is generated by the direction change of friction force due to the rotational and deformation of the geometry. Mothershead is

one of the well-known researchers who studied the influence of follower load in brake squeal in the early years (Mottershead and Chan, 1995, Chan et al., 1995, Mottershead, 1998). However, lately several researchers concluded that the influence of frictional follower force to the system stability is insignificant in the case of brake squeal (Heilig and Wauer, 2003, Kang et al., 2009b).

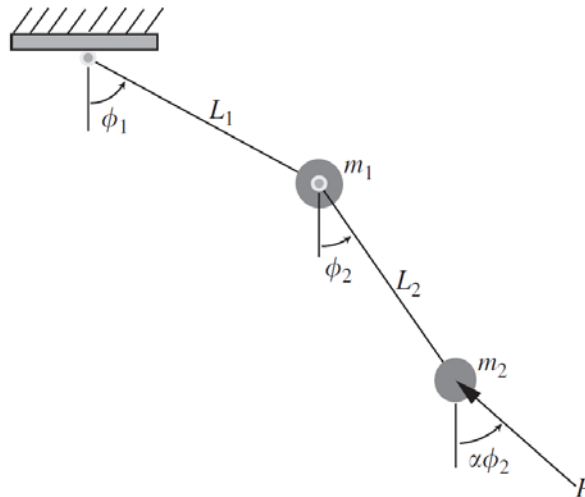


Figure 2.6 Ziegler pendulum model to describe frictional follower force (Kessler et al., 2007).

2.5 Stability analysis of brake squeal

There are several methods to analyse the stability of the brake system including complex eigenvalues, analytical modelling and transient analysis. Each has particular advantages and limitations, the choice of the proper method is important to ensure the analysis carried out is correct and effective.

Among these methods, complex eigenvalue analysis using the finite element method (FEM) is the most common method in the analysis of brake squeal (Huang et al., 2006, Hassan et al., 2009, Kang, 2009, Coudeyras et al., 2009, Grange et al., 2009, Ouyang et al., 2009, Sinou, 2010). FEM is a numerical method which can be used to analyse the vibration system by minimizing the variational calculus to obtain

approximate response of the system. FEM is powerful in analysing complex geometry which is difficult to do using other analytical methods. Sinou (2010) studied the transient and stationary non-linear behaviours which included the contact loss at the sliding interface. He found that additional mode can be excited which is different from the excited mode calculated from the linear system. In the study of disc brake squeal, Ouyang et al. (2009) included the heat conduction analysis and thermal deformation in the transient analysis. They found that contact pressure distribution is affected by thermal deformation subsequently affecting the vibration level which lead to the lowering of vibration amplitude. Kang et al. (2009b) developed a gyroscopic non-conservative brake system to predict the squeal propensity. They emphasized that negative slope of the friction coefficient – velocity curve is important in generation of squeal. Although FEM is powerful, it is time consuming especially when analysing high frequency drum brake squeal or large number of degree of freedoms.

Complex eigenvalue analysis can be used to measure the instability of a dynamic vibration system indicated by the positive real part of the complex eigenvalue (Liles, 1989, Brooks et al., 1993, Ripin, 1995, Sinou et al., 2009). However, this analysis is limited to steady sliding state since it linearised all the nonlinearities in the system. In the case of drum brake squeal, complex eigenvalue analysis is valid only if the friction force is constant. Nevertheless, in real situation of drum brake squeal, the friction force is dependent on the sliding velocity and also the system vibration. Both will influence the magnitude of the friction force and also allow a negative relative sliding velocity (particularly at very low speed when stopping where friction force changes direction). During the low velocity motion, the tangential motion of the sliding pairs may overcome the gross sliding speed of the

interface. Thus, in order to include nonlinearity of the friction force into the system, transient analysis is more suitable for such low speed or stopping condition. Kang (2008) investigated the dynamic instability of a sliding oscillator using complex eigenvalue analysis including both mode coupling and also negative slope instability.

Grange et al. (2009) proposed a new method which is spectral criterion based linearisation method (SCLM) to analyse brake squeal behaviour. This method is based on linearisation of nonlinear dynamic response of brake with unilateral contact and friction conditions. The basic of this method is in finding an equivalent linear system to replace the nonlinear dynamic system of brake squeal based on the brake response in squealing state obtained experimentally. The results obtained are comparable to the complex eigenvalue analysis and also experimental results. Moreover, SCLM is able to identify separation areas in the contact surface and other structural modes which contributed to the unstable mode.

Hoffmann et al. (2004) used the harmonic balance approach to calculate the limit cycle of the vibration in steady state response of non-linear differential equation. By assuming the system time history response can be expressed in the form of Fourier series (frequency domain), the limit cycle of the response can be estimated.

2.6 Modelling of brake squeal

Friction is a complex phenomenon and various models have been developed to explain the various parameters affecting friction. Stefanski et al. (2006) provided an excellent review of the friction model which in general can be divided into static and dynamic model with the inclusion of the acceleration that determines the level of friction in addition to the velocity. This single DOF dynamic friction model is able to show the chaotic response of the system (Stefański et al., 2003). The friction

characteristics independent of the system dynamics, which include the Coulomb friction model, are termed as insensitive friction model. Friction characteristics which are sensitive to the system dynamics are termed as sensitive friction model. The complex sensitive friction model allows for large relative velocity and took into consideration the zero crossing of the relative velocity (Wojewoda et al., 2009). Application of sensitive friction model requires robust numerical algorithm. In this thesis, classical Coulomb approach is adopted since this model was shown to be sufficient to approximate the real friction force due to the dominant role of mean level of frictional resistance in real systems (Wojewoda et al., 2009). Mathematical models have been developed to study dynamic instability problems especially in disc brake squeal (Crolla and Lang, 1991, Shin et al., 2002, Hoffmann et al., 2002, Sinou et al., 2003, Wagner et al., 2007, Kang, 2008) and machine chattering (Chandiramani and Pothala, 2006).

Shin et al. (2002) developed a model of disc brake squeal where the analysis concentrated on the negative damping excitation mechanism. They found that the damping of the disc and pad is important key for quenching the brake squeal; however the addition of damping on the disc or the pad independently may also make the system more unstable. In the Shin's model, the contact stiffness and damping between the disc-pad contacts are omitted. Hoffmann and Gaul (2003) developed a model of friction induced vibration to clarify the mode-coupling instability of self-excited friction induced vibration. They also emphasised that adding the structural damping can suppress the mode coupling friction induced vibration. Wagner et al. (2007) modelled the disc brake system as a wobbling disc which represents the orthogonal modes of the elastic disc. In addition, drum brake squeal has also been studied using Acoustic Quality Control to achieve the optimised

vibrational properties to avoid squeal (Haverkamp and Koopmann, 2004). Acoustic Quality Control is a low cost measurement technique that can be done very quickly and suitable for squeal tendency testing for commercial use. They proposed that the torsional modes of the drum brake shoe play the important role in squeal generation.

A non-linear two DOF model based on (Hultén, 1993, Hultén, 1997) was developed by Sinou and Jezequel (2007) using two cubic stiffnesses to represent the contact condition. The results in terms of mode coupling in friction induced vibrations are important since the effect of damping is shown to strongly influence the response of the system with the existence of optimal value of ratio of damping factors of the two modes which pushes the Hopf bifurcation point which in this case is the critical friction coefficient μ_o to a higher value pointing to a more stable system.

Pei and Tan (2009) proposed that modal interactions of the disc brake are important on the natural frequency and the deflection of the disc in disc brake system. The modal interaction is considered by the inclusion of a non-diagonal element in stiffness matrix. Some significant errors are found in the analysis of natural frequency and modes deflection at high speed if the modal interaction is neglected. The modal interaction causes the deflection of a given mode affects the deflection of other modes and the effect becomes stronger with the increase of rotating speed.

2.7 Characteristics of brake squeal

Kirilov (2009) investigated axisymmetric flexible rotor perturbed by dissipative and non-conservative force originated at the contact with anisotropic stator which is important for the excitation of the sub critical flutter as generally occur in brake squeal at low speed. The self-excited vibration is due to the unstable

modes in the sub critical speed range which can be related to the exceptional points at the corners of the singular eigenvalue surfaces.

An exhaustive review by Kirillov and Verhulst (2010) on the destabilization of non-conservative system has shown that in particular the dissipation-induced instabilities are related to singularities on the stability boundary. Ziegler's paradox was demonstrated using a two DOF system and the critical follower load was shown to be significantly lower for the case when damping is not zero. In the case of circulatory system of rotor dynamics, both the drum and disc brakes were reviewed and the equation of motion were proved to be exactly in the form presented by Bottema (1955) where interestingly the relative damping coefficients are denoted by the damping ratio and natural pulsations (i.e. the natural frequency of the individual sub-system). The solution of which showed that there are selected distribution of damping that can increase the critical load which can be used to attain a more stable system. Other than the said combination of damping parameters, destabilisation can occur if the damping is increased.

Cantone and Massi (2011) studied the effect of the structural damping to the squeal propensity where a disc-beam system is used to study the disc brake squeal for both experiment and finite element analysis. They found that homogenous distribution of the added damping is able to reduce squeal propensity, meanwhile the inhomogeneous distribution of the added damping can contribute to the mode coupling which increases the squeal propensity. Nouby et al. (2010) investigated the factors influencing squeal propensity by integrating finite element analysis with statistical regression techniques in disc brake system where the combined approach of complex eigenvalue analysis (CEA) and design of experiments (DOE) techniques are used. They found that the increase of the Young's Modulus of the back plate can

significantly reduce the squeal propensity. Besides, the modification of the friction material of brake lining by adding chamfer at both sides and by adding slot configurations are also able to reduce squeal propensity.

Based on the literature reviewed, most of the publications are based on the hypothesis that the mode coupling mechanism of the brake squeal is between the diametral modes of the drum itself (Lee et al., 2001, Wagner et al., 2007) or between the modes of brake components (Massi et al., 2006, Chevillot et al., 2008, Kang et al., 2009a). However, the experimental data showed that the contact of the brake shoe on the drum was able to generate modes other than the pure diametral modes. Kung and Saligrama (2000) developed an approach based on the modal participation factor to identify the interaction of such modes using FEM.

2.8 Squeal prevention and suppression methods

Brake squeal is the resulting noise from the self-excited friction induced vibration. Lowering the friction coefficient is the most effective way to suppress squeal, however the braking performance will be reduced. An alternative solution is needed which can suppress or prevent brake squeal, and still preserve the braking performance. The most direct solution is geometry modifications which changes the dynamic behaviour of brake system and prevents excitation of unstable vibration. Hamid et al. (2013) modified the geometry of drum brake shoes by adding shims which increases the stiffness of the brake shoes and they have successfully reduced the shoe vibration by 80%.

Bergman et al. (2000) investigated the effect of brake pad surface geometry on occurrence of brake squeal. They found that there is reduction in squeal when the pad material is removed at certain location. This is due to the change of centre of contact

which changes the dynamic behaviour of the system. Brake squeal can also be reduced by increasing the system damping. The constrained layer damping is added on the brake pad to reduce the squeal noise (Triches et al., 2004). With the layer damping, the squeal has been successfully reduced about 20 dB at particular frequencies. Previous researchers (Ikeuchi, 1988, Niwa et al., 1998) carried out geometry modification and the improvements are significant, however full suppression of the brake squeal has not been reported.

Active feedback control has also been applied to suppression brake squeal by using piezoceramic actuator (Wagner et al., 2004). This is done by cancellation of squeal wave by an identical squeal wave with opposite phase generated from the piezoceramic actuator. The piezoceramic is used as actuator and also as a sensor in the system. The application of active feedback control is effective, but it requires complex sensing devices and expensive. Another alternative solution is by using piezoelectric shunt damping which require only single actuator which is shunted to an electric branch (Neubauer and Oleskiewicz, 2008). By using the negative capacitance shunt, they have successfully increased the frequency range for the stabilized brake system.

Structural modification is one of the common methods to reduce brake squeal propensity in the cost effective way. It is a passive counter system that prevent squeal from occurring. Massi et al. (2009) proposed structural modification of the disc rotor for a disc brake system to avoid squeal. It was done by adding mass at certain location on the rotor which causes the natural frequencies of the rotor to shift apart and avoid mode-coupling mechanism. They have proved experimentally that the squeal can be totally eliminated if the distribution of added mass is at the right place. They have also showed that the brake squeal is always noticeable in low rotational

speed since the quick variations of the system dynamics at high speed will prevent the growth of the vibration due to the non-constant dynamic with the rotor rotation.

Giannini (2009) proposed the used of tuned fuzzy damper to suppress brake squeal generated from beam-disc system. Fuzzy damper is a device with specific distribution of mass which is characterised by several oscillators. The results showed that the device was able to suppress brake squeal from occurring if the fuzzy damper is attached on the beam, meanwhile the installation of fuzzy damper on the disc could reduce the amplitude of the squeal.

2.9 Open loop active vibration control

In vibrational system, dither is defined as a disturbance high frequency vibrational signal applied to affect the low-frequency behaviour of the system. Typically, dither signal is applied on a friction induced vibrational system to suppress the vibration. The dither signal forces the contact point to sweep quickly back and forth at a certain range around its nominal position with frequency higher than system vibrational frequency which tends to linearize the nonlinearity of the system. Dither tends to smoothen the nonlinearity of the system which turns the Coulomb type friction into viscous-like damping (Thomsen, 1999, Cunefare and Graf, 2002b). Dither is an open loop active vibration control that can be applied without any transducer. The transfer function of the dither control system is given as shown in Figure 2.7. The input $r(t)$ represents the force applied on the system including normal and friction forces. The dither force is represented as $d(t)$ which is presented as another input force. The output of the nonlinear system $f_{NL}(\cdot)$ is written as Eq. 2.1 where $h(t)$ is the impulse response of the filter $H(j\omega)$.

$$y(t) = h(t) * f_{NL}(x(t)) = h(t) * f_{NL}(r(t) + d(t)) \quad (2.1)$$

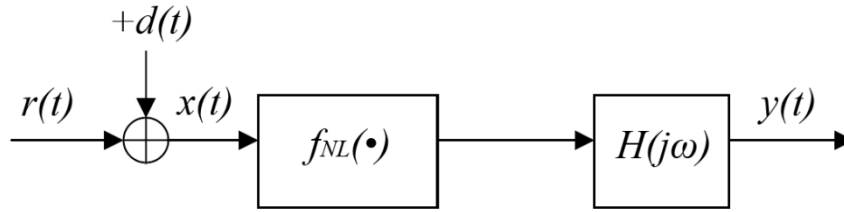


Figure 2.7 Transfer function of the dither control system.

Dither acts to smoothen the friction sliding by overcoming the surface locking between the rough sliding surfaces. It helps the sliding surface to 'jump over' the surface locking or provide instantaneous high impact force to the sliding in order to overcome the surface locking. Thus the effectiveness of dither is frequency and amplitude dependent. Only at certain frequency and amplitude, the effectiveness of dither is excellent in quenching of unstable vibration. It is totally dependent on the sliding condition and surface roughness. In the case of suppression of brake squeal, dither force can be applied in different direction relative to the sliding direction suppresses vibration with different levels of effectiveness. In some cases, the dither force applied in normal direction to the sliding surface is more effective in quenching than in the friction force direction and vice versa.

Although dither can be used to suppress the large amplitude of friction induced vibration, it can sometime destabilize or magnify system vibration under certain conditions (Michaux, 2005). The stabilizing effect of dither depends on dither frequency, dither amplitude, and sliding velocity; a slight change in these parameters can affect system stability. The advantage of dither control system in drum brake squeal is the working condition of the dither which does not rely on the braking condition and feedback signal is unnecessary. Since dithering is an open loop active