AI-aided, incremental numerical approach for finite strain poroelasticity: On the brain tissue deformation

H. Dehghani and A. Zilian

Institute of Computational Engineering and Sciences, Department of Engineering, Faculty of Science, Technology and Medicine, University of Luxembourg

May 21, 2021

UNIVERSITÉ DU LUXEMBOURG

Poroelasticity

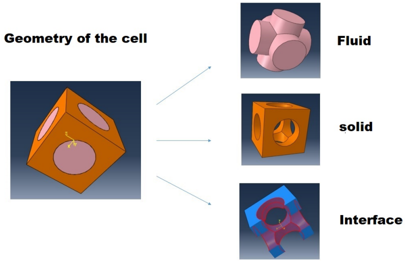


Figure: Phase Decoupling.

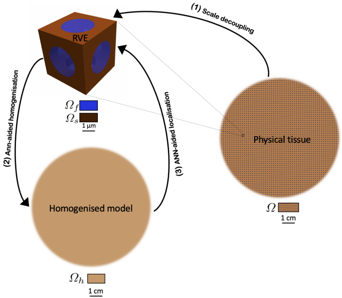
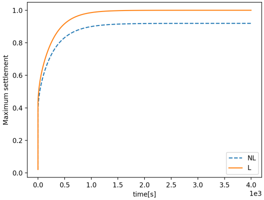


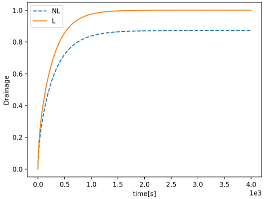
Figure: Phases and Scales.

Poroelastic response

why incremental analysis?

$$\boldsymbol{U}_{t+\Delta t} = \boldsymbol{U}_t + \Delta \boldsymbol{U}$$





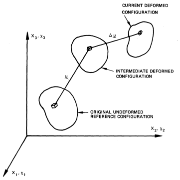
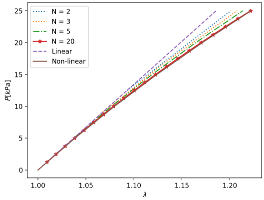


Figure 2.1 -- Original and Deformed Configurations



F. Rembert and Jr. Jones. INCREMENTAL ANALYSIS OF NONLINEAR STRUCTURAL MECHANICS PROBLEMS WITH APPLICATIONS TO GENERAL SHELL STRUCTURES. *U.S. NAVAL ACADEMY*

Poroelasticity

$$\begin{aligned} \nabla \cdot \sigma &= 0 && \text{in } \Omega_f \\ \sigma &= -p\mathrm{I} + \mu\left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^{\mathrm{T}}\right) && \text{in } \Omega_f \\ \nabla \cdot \boldsymbol{v} &= 0 && \text{in } \Omega_f \end{aligned}$$

$$\begin{aligned} \nabla \cdot \tau &= 0 && \text{in } \Omega_s \\ \tau &= \mathbb{C}\mathrm{e} && \text{in } \Omega_s \end{aligned}$$

$$\begin{aligned} \tau \boldsymbol{n} &= \sigma \boldsymbol{n} && \text{on } \Gamma \\ \dot{\boldsymbol{u}} &= \boldsymbol{v} && \text{on } \Gamma \end{aligned}$$

$$\nabla_{\boldsymbol{x}} \cdot \tau_E = 0,$$

$$\begin{aligned} \tau_E &:= \tilde{\mathbb{C}} \nabla_{\boldsymbol{x}} \boldsymbol{u}^{(0)} - \tilde{\alpha} p^{(0)} \\ \dot{p}^{(0)} &= -M[\,\tilde{\alpha} : \nabla_{\boldsymbol{x}} \dot{\boldsymbol{u}}^{(0)} + \nabla_{\boldsymbol{x}} \cdot \langle \boldsymbol{w} \rangle_f] \\ \langle \boldsymbol{w} \rangle_f &= -\mathrm{K} \nabla_{\boldsymbol{x}} p^{(0)} \end{aligned}$$

$$\begin{aligned} \langle \boldsymbol{w} \rangle_f &= \left\langle \boldsymbol{v}^{(0)} \right\rangle_f - \phi \dot{\boldsymbol{u}}^{(0)} \\ \mathrm{K} &:= \langle W \rangle_f \,, \quad \tilde{\alpha} := \phi \mathrm{I} - \mathrm{Tr}\, \langle \mathbb{M} \rangle_s \\ \tilde{\mathbb{C}} &:= \langle \mathbb{C} + \mathbb{C}\mathbb{M} \rangle_s \,, \quad M := -\frac{1}{\langle \mathrm{Tr}\, \mathrm{Q} \rangle_s} \\ \langle \psi \rangle_k &= \frac{1}{|\Omega|} \int_{\Omega_k} \psi(\boldsymbol{x}, \boldsymbol{y})\, \mathrm{d}\boldsymbol{y} \quad k = f, s, \end{aligned}$$

Implicit interdependency
between effective coefficients

Cell/RVE Problems

$$
\begin{aligned}
\nabla_{\boldsymbol{y}}^2 \mathsf{W}^\mathsf{T} - \nabla_{\boldsymbol{y}} \boldsymbol{P} + \mathsf{I} &= 0 && \text{in } \Omega_f \\
\nabla_{\boldsymbol{y}} \cdot \mathsf{W}^\mathsf{T} &= 0 && \text{in } \Omega_f \\
\mathsf{W} &= 0 && \text{on } \Gamma \\
\langle \boldsymbol{P} \rangle_f &= 0
\end{aligned}
$$

$$
\begin{aligned}
\nabla_{\boldsymbol{y}} \cdot (\mathbb{C}\mathrm{Q}) &= 0 && \text{in } \Omega_s \\
(\mathbb{C}\mathrm{Q})\boldsymbol{n} + \boldsymbol{n} &= 0 && \text{on } \Gamma
\end{aligned}
$$

$$
\begin{aligned}
\nabla_{\boldsymbol{y}} \cdot (\mathbb{C}\mathbb{M}) &= 0 && \text{in } \Omega_s \\
(\mathbb{C}\mathbb{M})\boldsymbol{n} + \mathbb{C}\boldsymbol{n} &= 0 && \text{on } \Gamma \\
\langle \mathcal{A} \rangle_s &= 0
\end{aligned}
$$

Efficient?

Localisation and Microscopic properties update

$$\langle \varepsilon^{(1)} \rangle = \langle \mathbb{M} \rangle \varepsilon^{(0)} + \langle \mathbf{Q} \rangle p^{(0)}$$

Back to the incremental formulation

$$\langle \varepsilon^{(1)}_{t+\Delta t} \rangle = \langle \varepsilon^{(1)}_{t} \rangle + \langle \Delta \varepsilon^{(1)} \rangle$$
$$\mathbf{F} = \langle \varepsilon^{(1)}_{t+\Delta t} \rangle / (1 - \phi) + \mathbf{I}$$
$$\mathbf{E} = \frac{1}{2} \left(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I} \right)$$
$$\phi = 1 - (1 - \phi_i) J$$

where,

$$J := \det(\mathbf{F})$$

$$W(I_1, J) = C_{10}(I_1 - 3) - 2 C_{10} \ln(J) + \frac{1}{D_1} \ln(J)^2$$
$$C_{10} = \frac{E_i}{4(1 + \nu_i)}$$
$$D_1 = \frac{6(1 - 2\nu_i)}{E_i}$$
$$C^t_{ijkl} = \frac{\partial W}{\partial E_{ij} \partial E_{kl}}$$

Sufficiently small increment

$$C^t_{ijkl} \approx C_{ijkl}$$
$$E = \frac{C_{11}(C_{11} + C_{12}) - 2 C_{12}^2}{C_{11} + C_{12}}$$
$$\nu = \frac{C_{12}}{C_{11} + C_{12}}$$

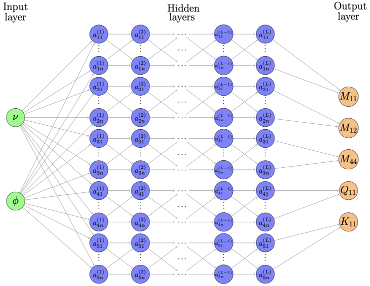
Application of ANN

$$(\mathbb{M}, \mathbf{Q}, \mathbf{K}) = ANN(\mathbb{C}, \mu, \phi)$$

$$(M_{11}, M_{12}, M_{44}, Q_{11}, K_{11}) = ANN(E, \nu, \mu, \phi)$$

$$(M_{11}, M_{12}, M_{44}, Q_{11}, K_{11}) = ANN(\nu, \phi)$$

No extrapolation risk!



$$z_k^{(i)} = w_{kj}^{(i)} a_j^{(i-1)} + b_k^{(i)}$$

$$a_k^{(i)} = ReLU(z_k^{(i)})$$

$n = 50$ $L = 3$

$n = 50$ $L = 3$

$n = 20$ $L = 3$

$n = 50$ $L = 3$

$n = 10$ $L = 3$

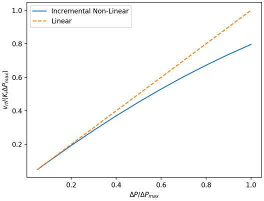
Incremental nonlinear algorithm

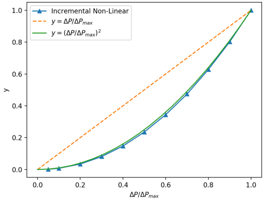
$$\boldsymbol{u}^{(0)}_{t+\Delta t} = \boldsymbol{u}^{(0)}_{t} + \Delta \boldsymbol{u}^{(0)}$$
$$p^{(0)}_{t+\Delta t} = p^{(0)}_{t} + \Delta p^{(0)}$$

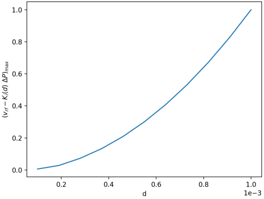
$$\begin{aligned} 0 = &\int_{\partial\mathcal{B}} (\boldsymbol{t}^t_s + \Delta \boldsymbol{t}_s) \cdot \delta \boldsymbol{u}^{(0)} \, \mathrm{d}S - \int_{\mathcal{B}} \left(\boldsymbol{\tau}^t_E + (\tilde{\mathbb{C}}_t : \Delta \boldsymbol{\varepsilon}^{(0)} - \tilde{\boldsymbol{\alpha}}_t \Delta p^{(0)}) \right) : \nabla_{\boldsymbol{x}} \, \delta \boldsymbol{u}^{(0)} \, \mathrm{d}V \\ &+ \int_{\mathcal{B}} \left(\frac{1}{M_{t-\Delta t}} \dot{p}^{(0)}_t + \frac{1}{M_t} \Delta \dot{p}^{(0)} \right) \delta p^{(0)} \, \mathrm{d}V + \int_{\partial\mathcal{B}} (\boldsymbol{v}^t_{rf} - \mathbf{K}_t \Delta \nabla_{\boldsymbol{x}} p^{(0)}) \cdot \boldsymbol{n} \, \delta p^{(0)} \, \mathrm{d}S \\ &- \int_{\mathcal{B}} (\boldsymbol{v}^t_{rf} - \mathbf{K}_t \Delta \nabla_{\boldsymbol{x}} p^{(0)}) \cdot \nabla_{\boldsymbol{x}} \, \delta p^{(0)} \, \mathrm{d}V + \int_{\mathcal{B}} (\tilde{\boldsymbol{\alpha}}_{t-\Delta t} : \dot{\boldsymbol{\varepsilon}}_t + \tilde{\boldsymbol{\alpha}}_t : \Delta \dot{\boldsymbol{\varepsilon}}) \, \delta p^{(0)} \, \mathrm{d}V \\ &\forall \, \delta \boldsymbol{u}^{(0)}, \, \delta p^{(0)}. \end{aligned}$$

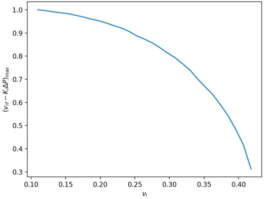
Deviation from Darcy's law

$$y = \frac{v_{rf} - (K_i \Delta P)}{v_{rf,max} - (K_i \Delta P_{max})} \qquad \Delta P_{max} = E_i/4$$





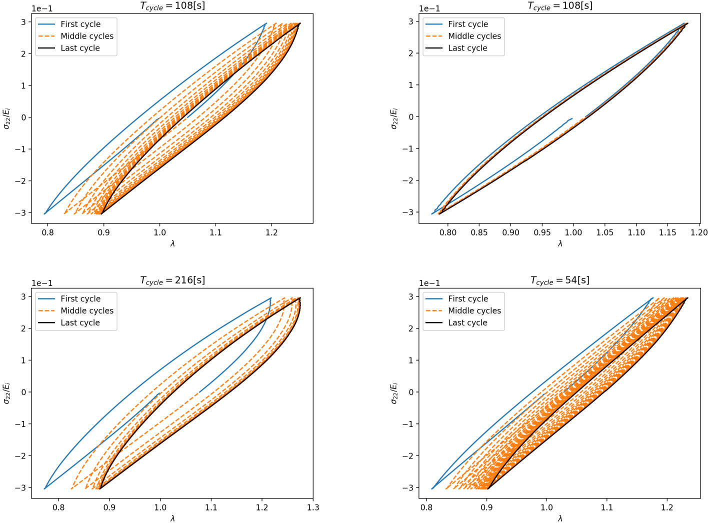




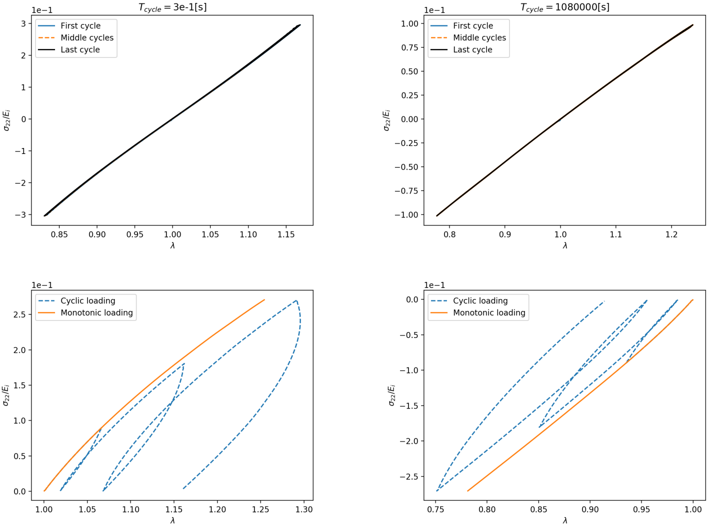
Uniaxial cyclic test simulating brain tissue

$L_c = 1[mm]$ $\mu_c = 10^{-3}[Pa.s]$ $d = 20^{-6}[m]$ $f_c = 10^{-3}[N]$

$\nu_i = 0.3[-]$ $E_i = 13.5 * 10^3[Pa]$ $T_{cycle} = 108[s]$ $width = 9.5L_c$ $length = 14L_c$



Uniaxial cyclic test simulating brain tissue



Thank you for your attention