Working Paper Series, N. 2, May 2016

Regular and promotional sales in new product life-cycle: A competitive approach

Mariangela Guidolin, Renato Guseo and Cinzia Mortarino
Department of Statistical Sciences
University of Padua
Italy

Abstract: In this paper, we consider the application of the Lotka–Volterra model with churn effects, LVch, (Guidolin and Guseo, 2015) to the case of a confectionary product produced in Italy and recently commercialized in a European country. Weekly time series, referring separately to quantities of regular and promotional sales, are available. Their joint inspection highlighted the presence of compensatory dynamics suggesting the study with the LVch to estimate whether competition between regular and promotional sales exists and how it affects product life-cycle. The study of sales under promotion with respect to regular ones represents a new way of dealing with promotional activities effects, whereas the innovation diffusion literature on new product growth has typically considered the effect of pricing and advertising through the generalized Bass model (Bass et al., 1994). In that model, the total amount of sales, regular plus promotional sales, is analyzed with a univariate approach, while price and advertising expenditures are used as exogenous inputs, without a feedback control. Conversely, exploiting the availability of two distinct time series and studying their interaction, our results show that competition has a symmetric character. Regular sales may access the residual market of those under promotion indicating the beneficial effect of promotional efforts, but the reverse effect is also present. Short-term forecasts on the evolution of the two series are then built with a two stage procedure based on an iterated SARMAX. The predicted values are further validated with observed real data. A comparison with Euler standard predictions is also performed.

Keywords: Competition modelling, Lotka-Volterra model with churn, Promotional sales, Regular sales, Short–term forecasts

Department of Statistical Sciences
University of Padua
Italy

UNIVERSITÀ DEGLI STUDI DI PADOVA
DIPARTIMENTO DI SCIENZE STATISTICHE

Contents

Department of Statistical Sciences
Via Cesare Battisti, 241
35121 Padova
Italy

tel: +39 049 8274168
fax: +39 049 8274170
http://www.stat.unipd.it

Corresponding author:
Cinzia Mortarino
tel: +39 049 827 4184
mortarino@stat.unipd.it
http://www.stat.unipd.it/~mortarino

Regular and promotional sales in new product life-cycle: A competitive approach

Mariangela Guidolin, Renato Guseo and Cinzia Mortarino
Department of Statistical Sciences
University of Padua
Italy

Abstract: In this paper, we consider the application of the Lotka–Volterra model with churn effects, LVch, (Guidolin and Guseo, 2015) to the case of a confectionary product produced in Italy and recently commercialized in a European country. Weekly time series, referring separately to quantities of regular and promotional sales, are available. Their joint inspection highlighted the presence of compensatory dynamics suggesting the study with the LVch to estimate whether competition between regular and promotional sales exists and how it affects product life-cycle. The study of sales under promotion with respect to regular ones represents a new way of dealing with promotional activities effects, whereas the innovation diffusion literature on new product growth has typically considered the effect of pricing and advertising through the generalized Bass model (Bass et al., 1994). In that model, the total amount of sales, regular plus promotional sales, is analyzed with a univariate approach, while price and advertising expenditures are used as exogenous inputs, without a feedback control. Conversely, exploiting the availability of two distinct time series and studying their interaction, our results show that competition has a symmetric character. Regular sales may access the residual market of those under promotion indicating the beneficial effect of promotional efforts, but the reverse effect is also present. Short-term forecasts on the evolution of the two series are then built with a two stage procedure based on an iterated SARMAX. The predicted values are further validated with observed real data. A comparison with Euler standard predictions is also performed.

Keywords: Competition modelling, Lotka-Volterra model with churn, Promotional sales, Regular sales, Short–term forecasts

1 Introduction

Product promotions in the form of price discounts are an effective marketing mix tool used by firms to increase short-term sales in accordance with short-term budget requirements (see Dawes, 2012). In fact, promotions may be used both for stimulating a new product's trial, but also in the case of mature markets, as observed by Raju (1995). Thus, a typical question arising from the use of promotions is to evaluate their effect on the dynamics of sales. Several marketing tools and theoretical approaches have been produced to answer this question and the literature dealing with the effect of promotions is very rich. In particular, a body of research deals with sales forecasting in the presence of promotional efforts. Some well defined streams of literature may be defined in this sense.

A first, widely accepted approach, described for instance in Trapero et al. (2013) and Trapero et al. (2014), is based on the combination between univariate statistical models and judgmental forecasting. As highlighted in Trapero et al. (2014), shorter product life cycles, increasing competition between products and brands, and an intense employment of marketing strategies, make sales forecasting an increasingly complex task and justify the use of such combination. Univariate forecasting methods are typically simple time series models such as exponential smoothing and ARIMA structures, that are used to produce baseline forecasts. These forecasts are then manually adjusted through managerial judgment, in order to take into account the presence of promotions or other marketing activities. For literature reviews on this methodology see for instance Ord and Fildes (2012), Trapero et al. (2013), Trapero et al. (2014), Huang et al. (2014), and Ma et al. (2016). In particular, Trapero et al. (2013) expressed some criticism on the accuracy of judgmental forecasts in the presence of promotions, finding that even a simple mathematical model may perform better than managerial judgement.

Another approach to forecasting promotional sales adopts a causal perspective by employing multiple regression models where promotional characteristics, such as price changes and advertising are used as input variables that may explain final sales. In particular, Cooper et al. (1999) and Divakar et al. (2005) produced well known methodologies in the business realm, namely Promocast and Chan4Cast. However, one of the recognized problems of these tools is the huge amount of data required to manage the models and the problem of variable selection, in order to eliminate potential problems of multicollinearity. To overcome this limitation various modeling solutions generally based on data mining techniques have been recently proposed. For instance Trapero et al. (2014) proposed a principal component regression in order to reduce the variable dimensionality, while Ma et al. (2016) and Ali et al. (2009) employed, respectively, Lasso regression and regression trees to achieve the purpose of variable selection.

A well established approach intended to evaluate the effect of marketing mix actions on sales dynamics by combining a regressive approach with time series data is the *intervention analysis*, which is essentially based on dynamic regression modeling, as described for instance in Makridakis et al. (1998) and applied in Trapero et al. (2014). A particular context where the effect of price variations has been modeled through an specific intervention function is that of new product growth, with the Generalized Bass Model (GBM), by Bass et al. (1994). In the GBM, marketing mix actions such as price reduction and advertising are considered as exogenous inputs, whose effect may be observed and estimated within a univariate model. In this domain, a specific effort was then dedicated to the analysis of optimal pricing strategies, see for instance Krishnan et al. (1999).

A common point to the above mentioned approaches, is the fact that promotions are considered as external actions whose impact is reflected on the pattern of final observed sales. A completely different perspective could be considering promotional sales as having their own trajectory, which dynamically interacts with that of sales with no price discounts, i.e. regular sales. This different perspective is made possible by the fact that often system databases separate between regular sales and promotional sales, yielding two different but related time series. The purpose of this paper is to study the dynamic interaction between promotional and regular sales and to investigate whether there may exist a compensatory behavior between the two series. In particular, we aim to study an intra-brand dynamics where the product under promotion may compete with the regular one. To our knowledge, such a *cannibalization* effect has not been studied in literature. Dawes (2012) analyzes the situation when price promotions for one pack size of a brand steal from the other pack-sizes of the same brand. Here we suggest that the same phenomenon may also occur between regular and promotional sales. To investigate our hypothesis we adopt a modeling approach which allows for the simultaneous treatment of two, possibly competing, time series, pertaining to the Lotka-Volterra family. Competition is an essential issue when studying the evolutionary dynamics of a product within a market. Despite the importance of the topic in product forecasting, the literature dealing with it has been developed just recently. This is probably due to complexity in managing systems of differential equations, describing the growth dynamics of each competitor and the corresponding inference. So far, competition modelling has essentially considered duopolistic markets, where products either have a simultaneous life-cycle or enter the market at different times. The latter has been termed diachronic competition, treated in literature by Krishnan et al. (2000), Savin and Terwiesch (2005), Guseo and Mortarino (2010; 2012; 2014; 2015), and recently with applications to energy context, by Guidolin and Guseo (2016). A characterizing aspect of the models proposed in these studies is the decomposition of word-of-mouth (WOM) into within-brand and cross-brand effects. A generalization of the models by Guseo and Mortarino (2012; 2014) has been proposed in Guidolin and Guseo (2015). This is a modified Lotka-Volterra system with churn effects (LVch), where competition exerts its effects both on the word-of-mouth components and on the size of each product residual market, allowing each product to access a portion of the residual market of its competitor.

In this paper, we consider the application of the LVch model to the case of a confectionary product produced in Italy and recently commercialized in a European country. Weekly time series, referring separately to quantities of regular and promotional sales, are available. Their joint inspection highlighted the presence of compensatory dynamics suggesting the study with the LVch to estimate whether competition between regular and promotional sales exists and how it affects product sales. Thus, we exploit the availability of two distinct time series and we study their interaction to describe their competition. Short-term forecasts on the evolution of the two series are then performed with a two stage procedure: once the mean behavior of each time series has been estimated with the LVch, the estimated trend is forecast with a SARMA and mean predictions are used as an external regressor in a SARMAX applied to the observed series.

The paper is organized as follows. Section 2 summarizes the main features of LVch model by emphasizing WOM effects decomposition and the local definition of residual markets that pertain to each competitor with possible churn effects. Section 3 discusses some inferential aspects of the multivariate approach. Section 4 suggests a simple way, based on a SARMA approximation, for the out-of-sample prediction of the local mean trajectories in a context which operates directly on the equations that do not present a closed–form solution. Section 5 describes an application to the diffusion in a European country of an Italian confectionary product with weekly data separating regular sales quantities and promotional sales quantities. Section 6 is devoted to the discussion of results and to the concluding remarks.

2 A Lotka-Volterra model with WOM decomposition and churn effects

The Lotka–Volterra model, LVch, proposed in Guidolin and Guseo (2015) combines two separate aspects that have relevance in describing competition in diachronic models.

The UCRCD models (restricted and unrestricted) proposed in Guseo and Mortarino (2014) emphasize the effect of communication in competition by separating within- and cross-product WOM effects and assuming a common residual market $[m_c - z_1(t) - z_2(t)]$ that is equally accessible to both competitors. Parameter m_c denotes the common fixed market potential and the quantities $z_i(t),\ i = 1, 2$, are the cumulative sales of two competing products entering the market diachronically. In Guseo and Mortarino (2015), the UCRCD is extended to deal with a *dynamic* market potential, which is however completely available to both competitors.

From a different point of view, the Lotka–Volterra family of models for competition, LV, does not consider cross effects in WOM dynamics and emphasizes a more flexible description of the residual market for each product by considering its own residual market with a possible gain derived from the competitor. A basic reference for the classical Lotka–Volterra family, LV, is Morris and Pratt (2003).

The duopolistic diachronic competition model proposed in Guidolin and Guseo (2015), LVch, is

$$\begin{aligned} z_1'(t) &= \left[p_{1a} + q_{1a}\frac{z_1(t)}{m_a}\right][m_a - z_1(t)], \quad t \leqslant c_2 \\ z_1'(t) &= \left[p_1 + \frac{a_1 z_1(t) + \alpha_2 b_1 z_2(t)}{m_1 + \alpha_2 m_2}\right]\{[m_1 - z_1(t)] + \alpha_2[m_2 - z_2(t)]\}, \ \ t > c_2 \\ z_2'(t) &= \left[p_2 + \frac{a_2 z_2(t) + \alpha_1 b_2 z_1(t)}{m_2 + \alpha_1 m_1}\right]\{[m_2 - z_2(t)] + \alpha_1[m_1 - z_1(t)]\}, \ \ t > c_2. \end{aligned} \tag{1}$$

The first equation describes the stand-alone monopolistic phase ($t \leq c_2$) with a standard Bass model (Bass, 1969), where m_a depicts the local market potential, $z_1(t)$ denotes the cumulative sales of the monopolist, and p_{1a}, q_{1a} represent the usual innovative (external) and imitative (internal) effects of the separate communication channels, respectively.

The second and third equations are defined during competition, $t > c_2$. In this couple of equations, each product's sales, $z_i'(t)$, $i = 1, 2$, are proportional to the specific residual market $\{[m_i - z_i(t)] + \alpha_j[m_j - z_j(t)]\}$, $i = 1, 2, j = 1, 2, i \neq j$, where m_i is the product's individual market potential under competition. Notice that m_1 and m_a may be different, to describe all situations when the beginning of competition corresponds also to a market expansion. The residual of each competitor is the sum of the product specific component, $m_i - z_i(t)$, and of a signed contribution proportional to the other's, $\alpha_j\,[m_j - z_j(t)]$. Parameter $\alpha_j, j = 1, 2$, modulates the size and the sign of this second element that measures the *churn* effect, which may be positive or negative depicting a product's partial access to the residual market potential of its competitor or a transfer of its own residual market potential. In particular, the case $\alpha_j < 0$ describes situations when product j reduces the perceived residual market of product i, thus reducing its competitor's instantaneous sales. Parameter $p_i, i = 1, 2$, defines innovative (or external) pressure towards adoption, while the WOM components have a complex structure characterized by a within-product component $[a_1 z_1(t)/(m_1 + \alpha_2 m_2)]$ and a cross-product one, $[\alpha_2 b_1 z_2(t)/(m_1 + \alpha_2 m_2)]$, for the first competitor and, similarly, $[a_2 z_2(t)/(m_2 + \alpha_1 m_1)]$ and $[\alpha_1 b_2 z_1(t)/(m_2 + \alpha_1 m_1)]$ for the second. Parameters α_1 and α_2, operating on the residual market potentials defining the churn effects, also affect the WOM level decomposition by controlling the relative size of the cross-product effects.

If the churn effects are symmetric, i.e., $\alpha_1 = \alpha_2 = 1$ (common residual market for both competitors), the LVch reduces to the unrestricted UCRCD model in Guseo and Mortarino (2014), where $m_1 + m_2 = m$, $a_1 = q_{1c} + \delta$ and $b_2 = q_2 - \gamma$.

In a different direction, the critical relevant differences between the LVch model and the traditional LV are initialization components and asymmetric WOM decompositions. Standard LV models assume $p_1 = p_2 = 0$, excluding an external initialization. Moreover, parameters b_1 and b_2 are forced to a zero level, $b_1 = b_2 = 0$, avoiding cross-product WOM effects. Conversely, products often generate either positive or negative WOM effects on their competitors due to the communication relationships among agents.

3 Statistical inference

3.1 Estimation

A reasonable and robust inferential methodology for estimating and testing the performance of the LVch model in specific applications may be implemented through the regression model

$$s(t) = f(t, \beta) + \varepsilon(t), \tag{2}$$

where $s(t) = [s_1(t), s_2(t)]$ denotes the instantaneous sales of both products. The transfer function $f(t, \beta)$ is described by Equations (1) and, for diachronic applications, it depends on the vector of parameters

$$\beta = \{m_a, p_{1a}, q_{1a}, m_1, p_1, a_1, \alpha_2, b_1, m_2, p_2, a_2, \alpha_1, b_2\}.$$

Details about an appropriate organization of the data and the corresponding model structure are given in Guidolin and Guseo (2015).

The error term $\varepsilon(t)$, with $E[\varepsilon(t)] = 0$, is usually assumed to be a white-noise (WN) or a more complex stationary process with local autoregressive-moving average components. Estimation of the parameters in Equation (2) may be performed through a two-phase procedure. In the first phase, we perform a nonlinear least squares (NLS) estimation and select the best nonlinear model to describe the *mean* trajectory of the series. This robust nonparametric method avoids further distributional assumptions on the error term. Following, for instance, Seber and Wild (1989), NLS is usually implemented through the Levenberg-Marquardt correction of the Gauss-Newton method. In the second phase, as will be detailed below, we use a suitable long-term prediction $f(t, \hat{\beta})$ based on the NLS solution, $\hat{\beta}$, as an input variable in an autoregressive–moving average process with a controlling covariate, namely an ARMAX model, to improve short-term predictions. This second part is necessary if a test (e.g., Durbin-Watson or Portmanteau tests) diagnoses a significant departure of residuals from a WN.

3.2 Short-term prediction of a nonlinear process

Stochastic nonlinear regressive models may have at least two different representations that depend upon the knowledge of a closed form solution regarding a specific equation:

$$y'(t) = G(y(t), \beta), \qquad \beta \in R^k,\, t \in R. \tag{3}$$

Suppose, as a first case, that $y(t) = F(t, \beta)$, with $\beta \in R^k$ is an explicit closed form solution to Equation (3) depending upon t and that the observable equation may be represented through a nonlinear regressive description, where $f(t, \beta) = F'(t, \beta)$ and $s(t)$, for $t = 1, 2, \ldots, T$ are instantaneous or rate data:

$$s(t) = f(t, \beta) + \varepsilon(t). \tag{4}$$

If the residual component $\varepsilon(t)$ may be defined through an ARMA process, we have

$$\Psi(B)[s(t) - f(t, \beta)] = \Theta(B)a(t) \tag{5}$$

with $\Psi(B)$ and $\Theta(B)$ backward polynomial operators and $a(t)$ a white noise, $a(t) \sim WN(0, \sigma^2)$.

An estimate for parameters $\beta \in R^k$, $\hat{\beta}$, may be obtained from Equation (4) with the nonlinear least squares method (NLS) omitting the parameters of the residual component $\varepsilon(t) = \Psi^{-1}(B)\Theta(B)a(t)$:

$$\hat{\beta} = \arg\min_{\beta} \sum_{t=1}^{T} [s(t) - f(t, \beta)]^2. \tag{6}$$

Using the explicit knowledge of function f, we may determine the mean predicted trajectory, $f(t, \hat{\beta})$, for $t = 1, 2, \ldots, T, T+1, \ldots, T+H$.

At this point, we introduce a modified ARMAX equation

$$\Psi(B)[s(t) - cf(t, \hat{\beta})] = \Theta(B)a(t), \quad t = 1, 2, \ldots T. \tag{7}$$

Parameter c allows for a test on the appropriateness of the global mean model $f(t, \beta)$ based on the NLS solution $\hat{\beta}$: its equilibrium level is 1 and an hypothesis test is based on a t-statistic. From equation (7) we are able to estimate $\hat{\Psi}(B)$ and $\hat{\Theta}(B)$, and use them to evaluate the predicted residuals

$$\hat{\varepsilon}(t) = \hat{\Psi}^{-1}(B)\hat{\Theta}(B)a(t), \quad t = T+1, T+2, \ldots T+H.$$

The sum $\hat{s}(t) = \hat{c}f(t, \hat{\beta}) + \hat{\varepsilon}(t)$ determines the predicted trajectory of the rate process for $t \in \{T+1, T+2, \ldots, T+H\}$.

Let us examine now the case when Equation (3) does not have an explicit closed form solution based on t so that the instantaneous observed process may be represented directly through

$$s(t) = G(S(t), \beta) + \varepsilon(t), \qquad t = 1, 2, \ldots, T, \tag{8}$$

with $S(t)$ the cumulative observations and $\varepsilon(t)$ is an ARMA process. Also in this case we may determine a NLS solution $\hat{\beta}$ for parameter β obtaining the mean trajectory through the model's fitted values:

$$\hat{G}(S(t), \hat{\beta}), \qquad t = 1, 2, \ldots, T. \tag{9}$$

The crucial point is that, outside the observations' range, for $t > T$, the lack of knowledge of transfer function does not provide a simple way to evaluate the predicted future mean trajectory.

A common approach to face this issue is the Euler's method for numerical solution of differential equations (see, e.g., Atkinson, 1989). This method relies on the approximation

$$y(t+1) - y(t) \simeq y'(t), \quad t \in R. \tag{10}$$

Substitution of (10) into Equation (3) leads to

$$y(t+1) \simeq y(t) + G(y(t), \beta), \quad t \in R. \tag{11}$$

The first mean predicted rate value outside the observations' window can thus be evaluated as

$$\hat{s}(T+1) = S(T+1) - S(T) \simeq \hat{G}(S(T), \hat{\beta}). \tag{12}$$

This formula allows subsequent computation of $\hat{S}(T+1)$ (as the sum of $S(T)$ and $\hat{s}(T+1)$) and iterating the process to obtain $\hat{s}(T+2)$ and following forecasts for the mean trajectory.

4 Our proposal

Instead of Euler's iterative method, which requires programmable software to be implemented, our proposal is to approximate the time series (9) through an ARMA, namely,

$$\Phi(B)\hat{G}(S(t), \hat{\beta}) = \Xi(B)a(t), \tag{13}$$

obtaining estimated coefficients for the backward polynomials $\Phi(B)$ and $\Xi(B)$. The forecasts of the series $\hat{G}(S(t), \hat{\beta})$ may be determined for $t \in \{T+1, T+2, \ldots, T+H\}$, namely,

$$\hat{G}(S(t), \hat{\beta}, \hat{\Phi}, \hat{\Xi}). \tag{14}$$

Similar results may be obtained if $\hat{G}(s(t), \hat{\beta})$ is a SARMA process with seasonal components, by adding appropriate polynomials of B^{ς}, where ς is the seasonal order parameter.

On the basis of Equation (13) we may represent forecasts (14) through

$$\hat{G}(S(t), \hat{\beta}, \hat{\Phi}, \hat{\Xi}) \simeq \hat{\Phi}^{-1}(B)\hat{\Xi}(B)a(t). \tag{15}$$

Similarly to (7), a short-term prediction of the process (15) may be rearranged through an ARMAX

$$\Psi(B)[s(t) - c\hat{\Phi}^{-1}(B)\hat{\Xi}(B)a(t)] = \Theta(B)a(t). \tag{16}$$

Alternatively for seasonal effects, a SARMAX approach may be more suitable.

Under a good approximation of the mean trajectory $\hat{G}(S(t), \hat{\beta})$ with an ARMA, we may determine a short-term prediction for the nonlinear mean trajectory $G(S(t), \beta)$ for $t \in \{T+1, T+2, \ldots, T+H\}$.

Notice that Equation (16) could also be compactly written as follows:

$$\Psi(B)s(t) = \Upsilon(B)a(t), \tag{17}$$

where $\Upsilon(B) = c\Psi(B)\hat{\Phi}^{-1}(B)\hat{\Xi}(B) + \Theta(B)$.

The methodology here proposed is quite simple to be implemented with standard time series software reducing computational effort. Nevertheless, not all systematic components $G(S(t), \beta)$ in Equation (8) have a well approximated linear representation. It is only an operational device for predictive purposes because the direct interpretation of parameters β is much more relevant for their original meaning within Equation (3).

5 Application of the LVch model to a confectionary product

Our data consist of weekly sales of an Italian confectionary product recently commercialized in a European country. Data, provided directly by the manufacturer, represent weight (in hundred kilograms) of product sold. However, due to confidentiality obligation, we scaled original values. Our observations report separately regular and promotional quantities. Data cover the period October 2012 – October 2015. Moreover, we also dispose of further 24 data (until March 2016) that we chose to use for forecasting validation purposes, as will be detailed below.

The inspection of Figure 1 suggested us that weeks with higher level of promotional sales often correspond to low regular sales. Conversely, we see peaks in regular sales when promotional quantities are small. This behavior led us modeling the data with a competition structure, as the LVch model described in Section 2. In our case, the two time series have a common origin and we use the simpler synchronic version of system (1), where $c_2 = 0$ and the first equation vanishes.

Table 1 shows the parameter estimates and the marginal confidence intervals (subscript 1 refers to the regular quantities, while subscript 2 denotes promotional ones). Given the wide local fluctuations, the R^2 value equal to 0.459091 is quite satisfactory.

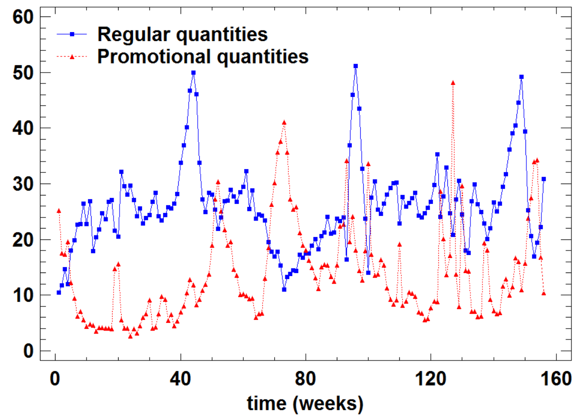

Figure 1: Weekly regular and promotional sold quantities of an Italian confectionary product in a European country (hundred kilograms, rescaled values). Data form October 2012 until October 2015.

Table 1: Parameter estimates of LVch model, ( ) marginal linearized asymptotic 95% confidence limits. Estimates performed on instantaneous data.

m_1	p_1	a_1	α_2	b_1	R^2
9644.77 (2900.96) (16388.6)	0.0122978 (-0.000950929) (0.0255466)	0.0324834 (-0.00327708) (0.0682438)	-1.59757 (-2.02226) (-1.17287)	0.032204 (-0.00383652) (0.0682446)	0.459091
m_2	p_2	a_2	α_1	b_2	D-W
4953.39 (1666.9) (8239.87)	0.0178985 (0.00383979) (0.0319571)	0.0467007 (0.0298265) (0.0635749)	-0.482776 (-0.510572) (-0.45498)	0.028165 (0.0122062) (0.0441238)	0.554084

If we substitute parameter estimates into the synchronic version of system (1), we obtain

$$z_1'(t) = [0.0123 + 0.000019\, z_1(t) - 0.000030\, z_2(t)]\,\{9644.77 - z_1(t) - 1.597565[4953.39 - z_2(t)]\}\,,$$
$$z_2'(t) = [0.0179 + 0.000157\, z_2(t) - 0.000046\, z_1(t)]\,\{4953.39 - z_2(t) - 0.482776[9644.77 - z_1(t)]\}\,.$$

For the regular sales, we estimate a market potential that is almost twice the promotional quantities market potential. Both series have a positive innovation coefficient. The WOM structure is typical of a true competition, since we observe positive within-product coefficients and negative cross-products ones. For both products, internal sales foster further adoptions and reduce the competitor's adoptions. When we turn to the analysis of the perceived residual market, this strong competition effect is confirmed. Both $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are negative. This means that the own specific residual market, $m_i - z_i(t)$, suffers a reduction due to the presence of its competitor.

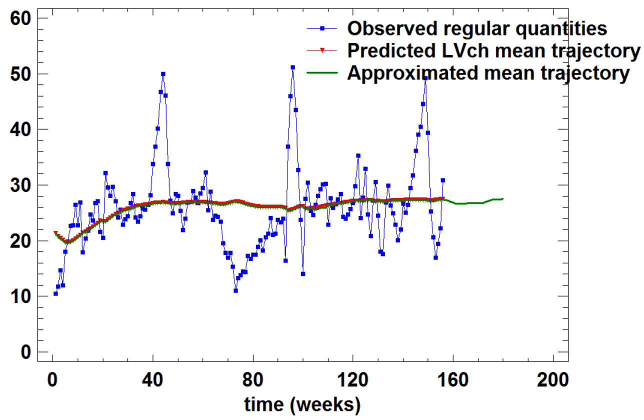

Figure 2: Weekly regular sold quantities. Observed data until October 2015, predicted LVch mean trajectory and its approximation, as in Equation (13).

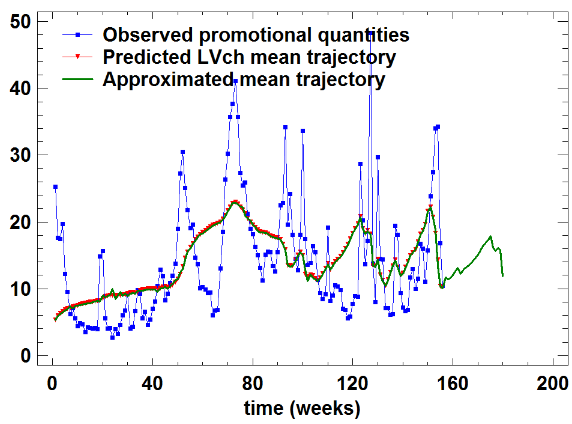

Figure 3: Weekly promotional sold quantities. Observed data until October 2015, LVch mean trajectory and its approximation, as in Equation (13).

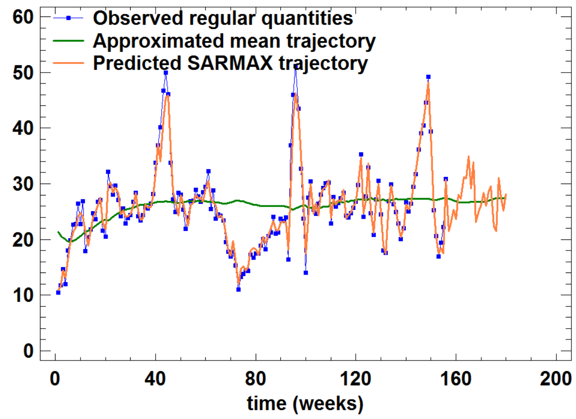

Figure 4: Weekly regular sold quantities. Approximated mean trajectory and SARMAX prediction of the original sale series.

The predicted LVch mean trajectory is shown in Figures 2 and 3 for regular and promotional quantities, respectively. The same figures show also the corresponding approximation with the method suggested in Section 4, Equation (13), and the related predictions 24 weeks ahead. In both cases, we can appreciate the high quality of the approximation which overlaps almost exactly the predicted LVch mean trajectory.

Figures 4 and 5 show the SARMAX refinement discussed in Section 4, Equation (16), for regular and promotional quantities, respectively. The agreement between observed quantities and predicted SARMAX trajectories is very good.

Figures 6 and 7 show in detail the forecasts obtained with the proposed method, their prediction limits and the true observed data from October 2015 until March 2016. The vertical red line separates the first observations, until October 2015 (t=156), from the new 24 values that were not used for estimation purposes. Predicted values show a pattern that mimics very well ex-post observed data, although the intensity of peaks—especially for promotional quantities—is underestimated.

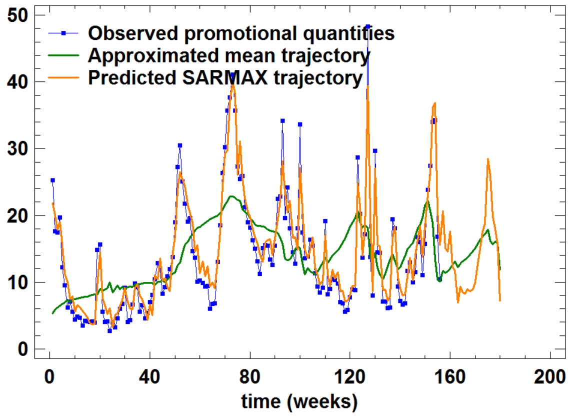

Figure 5: Weekly promotional sold quantities. Approximated mean trajectory and SARMAX prediction of the original sale series.

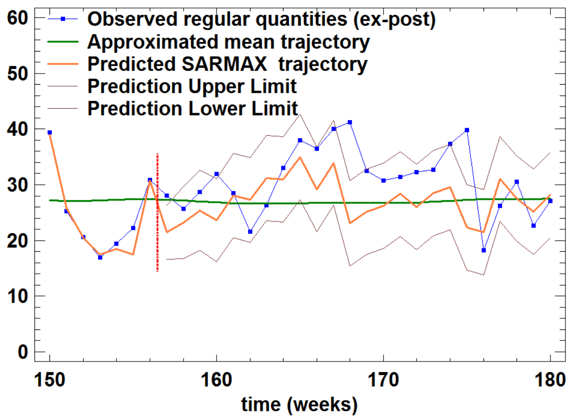

Figure 6: Weekly regular sold quantities. Observed data until March 2016, approximated mean trajectory, SARMAX predictions, prediction limits for SARMAX forecasts. The vertical red line separates the first observations, until October 2015 (t=156), from the new 24 values that were not used for estimation purposes.

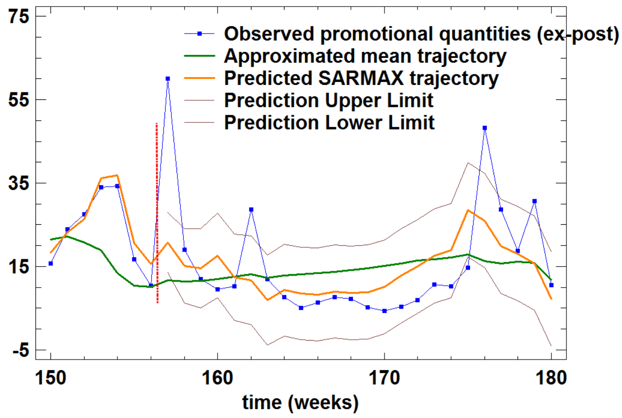

Figure 7: Weekly promotional sold quantities. Observed data until March 2016, approximated mean trajectory, SARMAX predictions, prediction limits for SARMAX forecasts. The vertical red line separates the first observations, until October 2015 (t=156), from the new 24 values that were not used for estimation purposes.

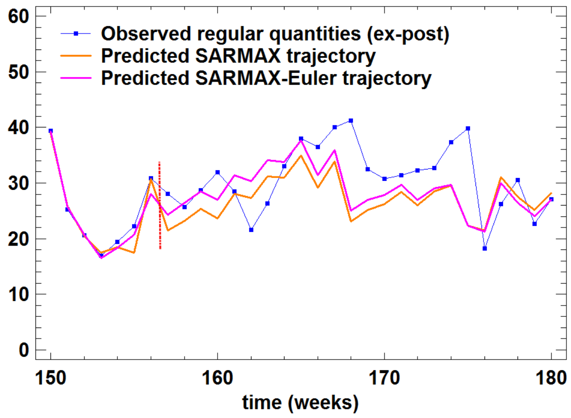

Figure 8: Weekly regular sold quantities. Observed data until March 2016, approximated mean trajectory, SARMAX predictions, SARMAX-Euler predictions. The vertical red line separates the first observations, until October 2015 (t=156), from the new 24 values that were not used for estimation purposes.

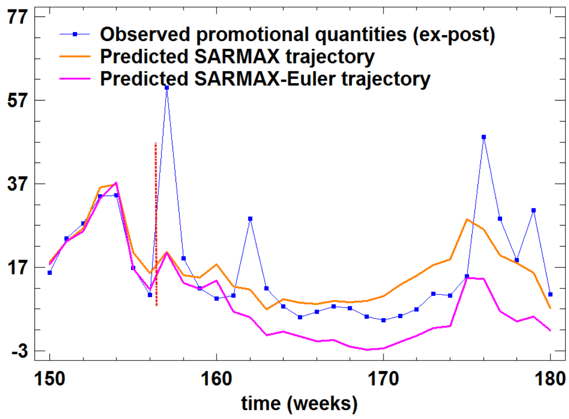

Figure 9: Weekly promotional sold quantities. Observed data until March 2016, approximated mean trajectory, SARMAX predictions, SARMAX-Euler predictions. The vertical red line separates the first observations, until October 2015 (t=156), from the new 24 values that were not used for estimation purposes.

As a final remark, we want to compare the suggested method with the traditional Euler approach (after a SARMAX refinement of the predicted Euler values, to obtain full comparability). Figures 8 and 9 show the predictions with both methods together with ex-post observed data. The red vertical line denotes again the start of the prediction period. For regular quantities the two methods give very similar results, maybe because the LVch mean predicted trajectory is quite stable. Conversely, SARMAX-Euler forecasts systematically underestimate promotional quantities and the proposed method performs better.

6 Discussion and concluding remarks

In this paper, we have faced the problem of evaluating the effect of promotions on regular sales of a product. Unlike previous literature on this topic, which essentially deals with univariate modeling, we exploited the possibility to study separately, but simultaneously, the series of regular and promotional sales of a product. Specifically, we analyzed these series with a modified Lotka-Volterra model with churn, LVch, and found that a real competition existed between the two, both in terms of cross-product WOM and residual market potential.

One could ask what the consequences of this result are from a product management point of view, since it may appear that promotional sales cannibalize regular ones and the advantage of promotional efforts may be questioned. To this extent, we may use the LVch model to assess the effect exerted on regular sales by a reduction or an increase in promotions. This could be achieved, for instance, by assuming

$z_2'(t) = 0$, for $t > T_o$ (an elimination of promotions since T_o), and examining the corresponding future values of regular instantaneous sales, $z_1'(t)$, in a neighborhood of T_o. Preliminary studies show that, given the LVch model, in our application, a decrease in promotional sales generates, after a very short period of increase in regular sales—due to elimination of the concurrent—an enduring progressive reduction in regular sales. This effect is obtained even if we assume that a fraction of the residual market of promotional sales is transferred to increase the residual market of regular sales, which confirms the essential role of promotions in sustaining a product's lifecycle. These preliminary results are in agreement with the conclusions sustained in Givon et al. (1995, 1997) with reference to piracy in software markets. The limited presence of illegal diffusion may sustain legal sales to some extent. In our context, we may use a more flexible model based on both regular and promotional sales avoiding to impose strong assumptions on a latent series as proposed in Givon et al. (1995).

Even though the LVCh is useful to model the mean trajectory of the series and allows a clear interpretation of parameters, nonetheless it is not immediate to perform short-term forecasts on sales evolution with it. To simplify this goal, we have proposed a new procedure based on a double ARMA modeling, which efficiently provides short-term forecasts by accounting for the high variability, in part due to seasonality, within the observed data. At this stage, our proposal does not consider other effects such as advertising actions: one possibility for future research is to extend the LVch model by miming the structure of the GBM, with intervention functions acting differently on regular and promotional sales.

References

Ali, O., Sayin, S., van Woensel, T., and Fransoo, J. (2009). "Sku demand forecasting in the presence of promotions". *Expert systems with Applications*, 36, pp. 12340–12348.

Atkinson, K. E. (1989). *An Introduction to Numerical Analysis, 2nd, ed.* Wiley: New York.

Bass, F. M. (1969). "A new product growth model for consumer durables". *Management Science*, 15, pp. 215–227.

Bass, F. M., Krishnan, T. V., and Jain, D. C. (1994). "Why the Bass model fits without decision variables". *Marketing Science*, 13, 3, pp. 203–223.

Cooper, L., Baron, P., Levy, W., Swisher, M., and Gogos, P. (1999). "Promocast trademark: A new forecasting method for promotion planning". *Marketing Science*, 18, 3, pp. 301–316.

Dawes, J. (2012). "Brand-pack size cannibalization arising from temporary price promotions". *Journal of Retailing*, 88, 3, pp. 343–355.

Divakar, S., Ratchford, B., and Shankar, V. (2005). "CHAN4CAST: A multichannel, multiregion sales forecasting model and decision up port system for consumer packaged goods". *Marketing Science*, 24, 3, pp. 334–350.

Givon, M., Mahajan, V., and Muller, E. (1995). "Software piracy: estimation of the lost sales and the impact on software diffusion". *Journal of Marketing*, 59, 1, pp. 29–37.

Givon, M., Mahajan, V., and Müller, E. (1997). "Assessing the relationship between the user-based market share and unit sales-based market share for pirated software brands in competitive markets". *Technological Forecasting and Social Change*, 55, pp. 131–144.

Guidolin, M. and Guseo, R. (2015). “Technological change in the U.S. music industry: within-product, cross-product and churn effects between competing blockbusters”. *Technological Forecasting and Social Change*, 99, pp. 35–46.

Guidolin, M. and Guseo, R. (2016). “The german energy transition: Modeling competition and substitution between nuclear power and renewable energy technologies”. *Renewable and Sustainable Energy Reviews*, 60, pp. 1498–1504.

Guseo, R. and Mortarino, C. (2010). “Correction to the paper “Optimal product launch times in a duopoly: Balancing life-cycle revenues with product cost””. *Operations Research*, 58, 5, pp. 1522–1523.

Guseo, R. and Mortarino, C. (2012). “Sequential market entries and competition modelling in multi-innovation diffusions”. *European Journal of Operational Research*, 216, 3, pp. 658–667.

Guseo, R. and Mortarino, C. (2014). “Within-brand and cross-brand word-of-mouth for sequential multi-innovations diffusions”. *IMA Journal of Management Mathematics*, 25, 3, pp. 287–311.

Guseo, R. and Mortarino, C. (2015). “Modeling competition between two pharmaceutical drugs using innovation diffusion models”. *Annals of Applied Statistics*, 9, 4, pp. 2073–2089.

Huang, T., Fildes, R., and Soopramanien, D. (2014). “The value of competitive information in forecasting FMCG retail product sales and the variable selection problem”. *European Journal of Operational Research*, 237, pp. 738–748.

Krishnan, T., Bass, F., and Jain, D. (1999). “Optimal pricing strategy for new products”. *Management Science*, 45, 12, pp. 1650–1663.

Krishnan, T. V., Bass, F. M., and Kumar, V. (2000). “Impact of a late entrant on the diffusion of a new product/service”. *Journal of Marketing Research*, XXXVII, pp. 269–278.

Ma, S., Fildes, R., and Huang, T. (2016). “Demand forecasting with high dimensional data: The case of SKU retail sales forecasting with intra- and inter-category promotional information”. *European Journal of Operational Research*, 249, pp. 245–257.

Makridakis, S., Wheelright, S., and Hyndman, R. (1998). *Forecasting: Methods and Applications*, *3rd ed.* Wiley: New York.

Morris, S. A. and Pratt, D. (2003). “Analysis of the Lotka-Volterra competition equations as a technological substitution model”. *Technological Forecasting and Social Change*, 70, 2, pp. 103–133.

Ord, J. and Fildes, R. (2012). *Principles of Business Forecasting*, *1st ed.* South Western Cengage Learning: Mason, Ohio.

Raju, J. (1995). “Theoretical models of sales promotions: Contributions, limitations, and a a future research agenda”. *European Journal of Operational Research*, 85, pp. 1–17.

Savin, S. and Terwiesch, C. (2005). “Optimal product launch times in a duopoly: Balancing life-cycle revenues with product cost”. *Operations Research*, 53, 1, pp. 26–47.

Seber, G. A. F. and Wild, C. J. (1989). *Nonlinear regression.* Wiley: New York.

Trapero, J., Kourentzes, and N., Fildes, R. (2014). “On the identification of sales forecasting models in the presence of promotions”. *Journal of the Operational Research Society*, 66, 2, pp. 299–307.

Trapero, J., Pedregal, D., Fildes, R., and Kourentzes, N. (2013). “Analysis of judgmental adjustments in the presence of promotions”. *International Journal of Forecasting*, 29, pp. 234–243.

Working Paper Series
Department of Statistical Sciences, University of Padua

You may order paper copies of the working papers by emailing wp@stat.unipd.it

Most of the working papers can also be found at the following url: http://wp.stat.unipd.it

UNIVERSITA DEGLI STUDI DI PADOVA DIPARTIMENTO DI SCIENZE STATISTICHE

Department of Statistical Sciences
University of Padua
Italy