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Mathematical Models to support territorial reorganization decisions in the public sector

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Introduction

Facility location decisions represent a critical element in strategic planning in both private and public sectors, as they can have a strong and lasting impact on operational and logistic performance. For example, in the public sector, the position of a set of structures providing a service in a region may strongly affect the accessibility of users and, hence, the quality of the service itself; as in the case of hospitals, schools or any type of emergency service. On the other hand, in the private sector, the position of facilities in the study region may affect the profit gained by the firm; for example, in the case of a new retail outlet, the selected site may influence the market share captured from the competitors, while, in the context of a supply chain, the position of a new warehouse, and then its distances from producing plants, may strongly influence the distribution costs incurred by the decision maker.

In order to determine suitable sites for new facilities, different approaches can be employed. In the OR literature, the two classes of problems mainly used to address these kind of decisions are *facility location* and *districting problems*.

A *Facility Location Problem* (FLP) is aimed at finding the best position for a set of facilities in order to optimize a specific objective function. In presence of more facilities to be located, also an *allocation problem* has to be solved, consisting in the assignment of customers to the patronized facility. In the context of FLPs several formulations have been proposed in literature in terms of objective functions, features of facilities to be located, demand to be served and location space, interaction rules between facilities and demand, and many applications have been analyzed.

On the other hand, a *Districting Problem* (DP) consists of partitioning a given region, divided into elementary units, in a fixed number of areas, named *districts* in such a way some requirements (i.e. topology and dimension) are satisfied. In the service-oriented application, given a set of facilities (already located or to be located) in a region, districting problems are aimed at designing the *territories* in which each facility has to provide its service(s).

These class of models have represented and still represent a viable decision support tool for institutions and firms that plan to open new facilities or to expand their capacity in a given region and/or market.

While the mentioned models are mainly oriented to locate new facilities, in a given supply system, some occurring circumstances could require strategies oriented to reduce costs and/or improve the system performance; therefore, actions related to the re-organization of the current facilities network may be undertaken. For example, in the private sector, many factors may quickly change during the facilities lifetime, such as market structure and conditions, distribution of demand (and its uncertainty), presence of new competitors and financial needs. On the other hand, in the public sector, economic conditions may impose constraints on public expenditure that could result in policies oriented to the rationalization of service supply systems in crucial areas, such as healthcare, education, public transport. These changes could make the existing supply system inefficient and/or unsustainable, and require reorganization processes, in order to cut costs and increase the efficiency of the offered services.

In a general economic and political context characterized by growing cuts to public expenditure and a review process of the welfare state, public services have and are still undergoing significant transformations, generally oriented to reduce administrative, managerial and operational burden and costs. Therefore, central and local authorities are more interested in the rationalization of the current systems of facilities, through downsizing and merging processes, rather than to the expansion or opening issues. In this sense, it could be useful and interesting to develop appropriate tools to support this kind of decisions.

In order to re-organize an existing supply system, composed by a set of facilities already sited in a given location space, each providing several types of services, different strategies could be adopted, such as the closure of some active facilities, their repositioning in different points of the location space, the downsizing of the capacities of the available services and so on. Each re-organization action perturb the interaction between the facilities and the demand, and could produce some side effects that should be carefully evaluated.

In this context, decisions may depend on various factors such as the nature of services and the characteristics of the market (competitive or non-competitive), the objectives to be achieved and the constraints to be satisfied. Therefore, apart from the evident practical interest, the problem appear of interest also from a methodological point of view.

In this work, after a review of the existing body of literature, we present some mathematical models for exploring re-organization decisions about facilities in a non-competitive context.
In particular, the thesis can be divided into three parts. The first part, comprising Chapter 1 and 2, is devoted to the analysis of the literature background.
In Chapter 1 the two above classes of problems, i.e. facility location and districting problems, are separately introduced first and, then, the linkage between them is highlighted, showing how is it possible to formulate a districting problem in terms of location-allocation problem.
In Chapter 2, the attention is focused on that contributions in literature that, through adaptations of traditional location models, have dealt more specifically with the re-organization problem, i.e with those actions aimed at modifying an existing set of facilities. In most of the analyzed papers, the problem arises when some occurred (or forecasted) changes in the distribution of the demand have made (or could make) the system obsolete or inefficient. Therefore, the re-organization is mainly intended as a possibility to adapt the overall organization of the system to such changes.
From the analysis of the literature, it is possible to notice that few attention has been paid to the situation in which the re-organization process is motivated by some budget constraints that make unsustainable the system and aims at reducing managerial and operational costs. In this case, the re-organization represents an opportunity for the planner to reduce costs but it may produce a perturbation of the previous demand allocation with a subsequent potential detriment of the service quality offered to the users. Therefore, to effectively solve these kinds of problems, decision support models should be able to find a trade-off solution between two inherently conflicting goals: the maximization of the benefit associated to the reorganization process (taking into account the planner perspective) and the minimization of the damage due to reallocation of the demand (taking into account the user perspective).
In the second part some re-organization models have been proposed.
In particular, in Chapter 3 a theoretical framework for the formulation of rationalization models has been defined, in which different strategies and models have been introduced and proposed. One specific version of the model, concerning the shrinking of an existing service, has been deeply analyzed. The model has been tested on a set of randomly generated instances in order to show that a good range of problems can be solved to optimality through the use of a commercially available solver (CPLEX).
In Chapter 4, two applications related to different real-world problems are illustrated, in order to show how rationalization models can be effectively exploited. The first application

concerns the problem of the re-organization of a public university system on a regional scale, while the second deal with the re-organization of a school system.

In the third part, consisting of Chapter 5, some strategies to address redistricting decisions have been defined and some mathematical models have been formulated. These models have been applied on a real-world problem related to the re-organization of administrative division of Italian local authorities; in particular, the re-order of the current partition of Italian regions into provinces (which correspond to the Level 3 of the classification of territorial administrative units provided by the NUTS Eurostat System). In order to build some instances for testing the performances of the models and their capability of solving realistic problems, some benchmark instances, built on the real data of five Italian regions, have been considered and results provided by the different models are analyzed and compared

Finally, some conclusions and directions for further research are drawn.

Chapter 1

Facility Location and Districting Problems

1.1 Introduction

Location decisions represent a critical element in the strategic planning of services, in both private and public sectors, as they can have a strong and lasting impact on their operational and logistic performance. In the OR literature, the two classes of problems mostly used to address decisions related to the spatial organization of services are: *Facility Location* and *Districting Problems*.

A *Facility Location Problem* (FLP) is aimed at identifying the optimal position to assign to one or more structures (*facilities*), in a given space, in order to satisfy a demand (actual or potential) coming from a set of customers. Examples of facility location problems are: determining the most efficient position for a distribution center and/or a warehouse in the context of a supply chain, selecting the site of a public service among several candidate points, positioning infrastructures in order to improve the performance of a transportation network. In these problems, the solution is strongly influenced by the objective that should guide the decision making process (maximization of the service accessibility, minimization of the management costs) and by the set of constraints that should be satisfied (geographical, environmental, economic, logistic and/or technological). The use of mathematical models has been demonstrated particularly suitable to tackle them; therefore, investigators have focused on formulations and algorithms able to describe and answer to several different questions which usually arise in practical applications. Exhaustive reviews of models and methods have been provided by Drezner and Hamacher (2002) and Eiselt and Marianov (2011).

On the other hand, a *Districting Problem* (DP) consists of partitioning a given region in a fixed number of areas, named *districts*. In this context, the region is currently divided into elementary units, each of them associated with a set of parameters (e.g., population, area), that have to be then grouped in districts in such a way that constraints on topology and dimensions are respected. In the applications related to the public sector, given a set of facilities (already located or to be located) in a region, districting problems are aimed at designing the *territories* in which each facility has to provide its service(s).

In this chapter we describe separately the two classes of problems; in particular we introduce their basic features, the fundamental mathematical models used to describe most of the problems of practical interest and their possible applications. Then, we focus on the linkage between the two class of problems, showing how it is possible to formulate a districting problem in terms of location-allocation problem.

1.2 Elements of Facility Location Problems

In general, a *FLP* aims at identifying the best position to assign to a set of facilities, in a given location space, in order to satisfy a demand (actual or potential), according to a certain objective to be optimized and a set of constraints to be satisfied.

From this definition, it is clear that the fundamental elements of a FLP are (see also Eiselt and Laporte, 1995; ReVelle and Eiselt, 2005):

- *Location Space;*
- *Facilities*;
- *Demand*;
- *Interaction*;
- *Objectives,* to optimize;
- *Constraints,* to be satisfied.

The ***location space*** generally corresponds to the space where customers are present and facilities are to be located. It can be a geographical area (e.g. a region or a city) or not; for example, this latter is the case of positioning a company in a market described as a space in a set of economic variables.

It is possible to distinguish between *continuous*, *network*, and *discrete* problems. In the first case, facilities may be positioned everywhere in the location space; at most there could be some *forbidden zones* where locations are not allowed due to geographical obstacles or technical constraints. In the second case, the location of a facility is restricted to the nodes and/or the edges of a *network*; while in the last one, it has to be chosen within a set of candidate sites.

The characteristics of the location space and the considered application generally drive the adoption of the *metric,* i.e. the function used to measure distances between pairs of elements of the space (facilities and/or demand points). Distance may refer to a physical length, a period of time, or it can be estimated on the basis of other criteria.

The ***facilities*** are the objects (to be located) that will provide services and/or goods in order to satisfy the demand. Classical examples are: industrial or commercial structures (e.g., retail outlets, plants, warehouses, bank branches), public services sites (e.g., schools, hospitals, fire stations), transportation and logistics infrastructures (e.g., bus-terminals, cross-dockings, metro stations). Facilities are usually characterized by attributes, such as the number and the types of provided services, their capacity, their attractiveness, the costs associated with their establishment and operation.

In general, the size of the facilities is so small as compared to the space they are located in, that they can be considered dimensionless *(puntual problems)*. Examples of these problems are: finding the location of an assembly plant within a country *(continuous),* or selecting the site of a school in one of several candidate points in a city *(discrete)* or identifying the position of a new bus stop on a road network *(network).*

The problems in which the size of the facilities is significant in comparison to the space they are to be located in, are referred to as *layout problems*. Examples in this class include the siting of a drilling machine in a workshop.

A fundamental characteristic of a FLP is the number of new facilities to be located, that can be either pre-specified or a decision variable. The simplest case is the *single-facility problem*, in which the position of only one facility has to be determined. When more facilities have to be located (*multi-facility problem)*, it is necessary to face the additional issue of determining from which facility the demand of the customers has to be satisfied; this is usually referred to as the *allocation problem*. In this context it is possible that customers are free to patronize their own facilities (i.e. customers of supermarkets or trade centers) or they are obliged to follow predetermined criteria (i.e. students assigned to schools on the basis of their residences).

The ***demand*** is represented by the actors (already located) in the location space that require the goods and/or services. Depending on the specific application, they can be defined as customers, users, residents, population centers and so on.

As well as the facilities, also the demand can be continuously distributed in the location space or can be concentrated in a discrete set of points. When the demand is continuously distributed in the location space, it is possible to discretize it through appropriate procedures that consist in partitioning, according to some criteria, the study region into a finite number of sub-areas. To each sub-area is then associated a value of demand, equal to the sum of the demands coming from all the points falling inside it, and a point, usually the centroid, in which it is considered concentrated. During these operations particular attention should be paid to approximations and errors introduced in the model.

Depending on the application, demand can be deterministic or stochastic. In both these cases, it can be estimated either by combining current data and/or attributes or by using appropriate forecasting tools.

Another important element of FLP is the ***interaction*** between the objects involved, the existing and the new ones to be located. In particular, two kinds of interactions have to be taken into account: *customer-facility* and *facility-facility* interactions.

Customer-facility interactions concern all the effects that the presence of new facilities in the location space produces on the customers. In this context, a particularly important issue is related to the assignment of customers to the facilities. Drezner and Eiselt (2002) differentiated between *location-allocation* and *location-choice models*. Within location-allocation models the demand is allocated to the facilities compulsory (e.g. schools in the United States); while in location-choice models customers choose among the available facilities according to a utility function, which, in general, combines attributes of facilities and distances between customers-facilities (e.g., retail outlets).

Facility-facility interactions take into account all the effects that the presence of single new facilities produce on each other (existing or to be located). In some cases there is competition in order to capture as much of the demand as possible (i.e. commercial stores of different companies); while in other applications facilities cooperate in order to assure a certain level of accessibility to the users (i.e. bank offices, public service sites, franchising stores).

Location decisions can be made according to different criteria or ***objective functions,*** whose choice may depend on the nature of the service and the specific application. Being proximity (distance or travel time) one of the fundamental aspect of location analysis, many models seek to minimize a function of the distances or travel times between the customers and the facilities at which they receive the service.

Finally, the problem can be characterized by many ***constraints***. Typical examples are topological constraints (i.e. minimum and/or maximum distances between facilities, zoning laws), capacity constraints (i.e. maximum demand that each facility can serve), technical and/or technological restrictions, economic and budget constraints.

Depending on the combinations of the above elements, a wide range of mathematical models can be defined. This variety has suggested proposals of classification on the basis of different schemes, as discussed by Francis et al. (1983), Brandeau and Chiu (1989), Eiselt and Laporte (1995), Hamacher and Nickel (1998), ReVelle and Eiselt (2005) and ReVelle et al. (2008).

1.3 Location Problems in the private and public sector

There are several ways of classifying location models and problems; one of the most adopted is based on the dichotomy between public versus private sector.

The *private sector* is the part of a country's economic system run by individuals or groups, usually by means of firms, with the aim of making profit. It is legally regulated by the state, in the sense that businesses within one country are required to comply with the laws in that country, but not controlled by it.

The segment of the economy under control of the government is known as the *public sector.* The composition of the public sector varies by country, but in most cases it includes such services as: the police, military, public roads, public transit, education and healthcare. The organizations of the public sector (public ownership) can take several forms, ranging from direct administration to partial outsourcing, but in any case the government plays a key role in management terms as it is responsible of providing the performances with reference to the users (*accountability*).

The main difference between the location of public and private facilities lies in the nature of the criteria adopted in the decion-making process.

In the ***private sector***, the objective is mainly the profit maximization (or cost minimization) and the capture of largest market shares from competitors. Generally, in such problems a trade-off solution between *fixed location costs,* incurred for establishing and operating new facilities at some points, and *variable costs*, incurred for production and transportation, is searched. High transportation costs and low facilities costs imply decentralization, the reverse implies a few large central facilities. Even if the location of a private facility may have some *external effects* on the environment and the population, only the private benefits of the decision maker are taken into account. In order to control such effects, central governments have two possibilities of influencing private location decisions and drive them towards more socially acceptable solutions: by additional constraints and by additional costs. In the first case the alternatives open to the private firms are limited, for example by means of law; while in the second case an additional cost (a sorty of penalty) is introduced in the objective function, for example by means of taxes and subsidies. The latter method appears to be favored currently in the thinking of many governmental policymakers; a difficult and challenging areas of research in public policy concerns the methods to optimally determine the order of magnitude of such measures so as achieve desired goals.

In the ***public sector***, the objectives of the decision-making process tipically consist in the social cost minimization or in the universality, efficiency and equity of the services.

The main issue in this class of problems is that objectives and constraints are no easily definable and/or quantifiable. In order to overcome this aspect, the most adopted methodology is to identify some *proxy measures* to quantify the utility of users and/or their *accessibility* to services. One of the most common *proxy* is the distance between users and their assigned facilities. As most of public services are *desirable*, the smaller this quantity more accessible the system is to the users; therefore, the decision maker will tend to position facilities as close as possible to them (*pull objective*). On the contrary, when facilities are considered undesirable, customers aim at avoiding their presence and try to stay far away from them (*push objectives*).

According with this assumption, objectives are in most cases expressed as functions of distances between customers and facilities and they are optimized subject to constraints on investment, which can be in turn expressed as an explicit limitation on the budget available or on the maximum number of facilities to be located.

The objective functions can be classified in two classes:

- ***efficiency measures***, in which the average accessibility of users to the service is taken into account, without any consideration about the fairness in the access to the facilities. According to this kind of measures, an efficient location pattern is presumably one in which the average distance of users from facilities is minimized within a given budget (*mini-sum* problems); or, alternatively, some predetermined level of service, in terms of maximum distance between each user and its assigned facility, is met at minimum total cost.
- ***equity measures,*** in which the decision maker is interested in finding solutions that assure a certain fairness in the access to facilities. Models with such objectives attempt to locate facilities, so that the distances may be as similar to each other as possible. Various expressions have been proposed, based on the minimization of measures related to the

distribution of distances between customers and facilities; examples include the variance, the mean absolute deviation or the Gini coefficient. For more details, see Marsh and Schilling (1994) and Eiselt and Laporte (1995).

Finally, it should be underlined that locational decision problems in practice can involve multiple, conflicting and incommensurate evaluation criteria and, in this sense, they are multiobjective in nature. In order to tackle FLPs formulated using multiple conflicting objectives, appropriate multiobjective techniques are needed, some of which are reviewed by Current et al. (1990) and Farahani et al. (2010).

1.4 Discrete Location Problems in the Public Sector

In this section, the attention has been focused on three classes of facility location models frequently used for the design of public services:

- *Median Models,* in which the efficiency is measured in terms of average distance between customers and facilities (*mini-sum objectives*);
- *Covering Models,* in which the quality of the solution is related to the ability of facilities to cover demand within a maximum prespecified value, named *coverage radius*;
- *Center Models*, in which the maximum distance between customers and their patronized facilities is minimized, in order to protect the customers in the worst condition (*minimax objectives*).

As we will focus on the discrete versions of the above problems, we will refer to a finite set of demand points and to a finite set of potential facility sites. In particular, we will adopt the common following notation:

I	set of demand nodes, indexed by i ($\|I\| = n$);
J	set of potential facility sites, indexed by j ($\|J\| = m$);
d_{ij}	distance between nodes $i \in I$ and $j \in J$;
a_i	population at the demand node i;
y_j	binary variable equal to one if and only if a new facility is located in j;
x_{ij}	allocation binary variable equal to 1 if and only if customer at node i is assigned to facility j.

1.4.1 Median Models

The *p-median problem* find the optimal location of exactly p facilities, so that the weighted sum of the distances between customers and their assigned facilities is minimized. Since the number n of customers is known, by dividing the objective by n, the minimum average weighted distance between customers and facilities is obtained.

The first explicit formulation of the p-median problem is attributed to Hakimi (1964). Even though his application was in the field of telecommunications, more precisely in the location of switching centers on a graph, the *p-median model*[1] has been since then extensively used as a basis to build problems related to the location of public services. From the point of view of public decision-making, the p-median objective maximizes the accessibility, in terms of average proximity of customers to a facility. For example, if a region is represented by a network whose nodes are patient locations, and whose edges are roads, locating p hospitals according to the solution of a p-median will minimize the average travel distance for patients attending those hospitals.

The formulation of *p-median problem* is now well known and used profusely, in the following form:

$$min\, z = \sum_{i \in I} \sum_{j \in J} a_i d_{ij} x_{ij} \tag{1.1}$$

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I \tag{1.2}$$

$$x_{ij} \leq y_j \qquad \forall i \in I, \forall j \in J \tag{1.3}$$

$$\sum_{j \in J} y_j = p \tag{1.4}$$

$$x_{ij}, y_j \in \{0,1\} \qquad \forall i \in I, \forall j \in J \tag{1.5}$$

Constraints (1.2) force each demand point to be assigned to only one facility. Constraints (1.3) allow demand point i to be assigned to a point j only if there is an open facility in that location. Finally, the last constraint (1.4) sets the number of facilities to be located.

The set of constraints (1.3), also known as Balinski conditions, can be replaced by the following more condensed set:

$$\sum_{i \in I} x_{ij} \leq M y_j \qquad \forall j \in J \tag{1.6}$$

where M represents an arbitrary large quantity. Also in this case, any allocation to node j is avoided, unless there is a facility open in that node.

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[1] The name *p-median* derives from the concept of a median vertex, which is the vertex of a graph for which the sum of the lengths of the shortest paths to all other vertices is the smallest.

Since its first formulation, the p-median problem has been modified to be adapted to specific problems or to allow a better *real world* implementation in the public sector.

One of the main rigidity of the p −median problem is that it presents a complete inelastic demand with respect to distance. People travel to the closest facility regardless of the distance they need to cover. In 1972, Holmes et al. presented a formulation that considered that people would not travel beyond a given threshold distance. In essence the p-median objective was replaced by the following one:

$$max \quad z = \sum_{i \in I} \sum_{j \in J} a_i (S - d_{ij}) x_{ij} \tag{1.7}$$

and the group of constraints (2) was reformulated as follows:

$$\sum_{j \in J} x_{ij} \leq 1 \qquad \forall i \in I \tag{1.8}$$

Since it is not guaranteed that every node will be assigned to a facility (1.8), the model will tend to leave uncovered those demand nodes that are farther than S from any located facility $(S - d_{ij} \leq 0)$, as their assignments would introduce negative addends in the objective function (1.7) to be maximized. The model was applied to locate public day care facilities in Columbus, Ohio.

In the same work, Holmes et al. also introduced the so called *capacitated p-median problem*. In this model, facilities have a limited capacity and therefore the following constraint needs to be added:

$$\sum_{i \in I} x_{ij} \leq C_j y_j \qquad \forall j \in J \tag{1.9}$$

where C_j is the maximum capacity level for facility sited in j.

Sometimes it may be necessary not only to constrain the maximum demand to be allocated to each facility, but also a minimum threshold service level. In the p-median formulation, this is achieved by adding the following set of constraint:

$$\sum_{i \in I} x_{ij} \geq c_j y_j \qquad \forall j \in J \tag{1.10}$$

where c_j is the minimum capacity level for facility sited in j. In the public sector, the central governments use to introduce this kind of minimum requirements or to avoid strong inefficiencies or to obtain a more balanced distribution of customers between facilities (*balancing requirements*). Carreras and Serra (1999) were the first to consider this aspect in the analysis of the effect of the deregulation of pharmacies in a region of Spain.

Another problem when implementing the *p-median problem* is related to the distance parameter. The model supposes that distances (or travel times), such as demands, are fixed and may not change. This assumption sometime may be too restrictive; if we want to locate fire stations in a city, for example, we know that travel times may change during a single day and therefore an optimal location during traffic peak hours may be inefficient in valley hours. On the other hand, the demand may also change during the day. Some areas may be crowded during daytime and empty during night time. Serra and Marianov (1998) introduced the concept of *regret* and *minmax objectives* when locating fire stations in Barcelona (Spain) taking into account what they called *changing networks*. Basically, uncertainty was treated using the classic scenario approach, in which different patterns of demand or travel times are represented in different scenarios. The formulation in both cases is straightforward, as it can be obtained from the classic one through the introduction of a third index k, associated to the single scenario. This way, a_i^k is the demand coming from node i in scenario k and d_{ij}^k the distance or travel time bertween nodes i and j in scenario k.

In the *min-max approach*, over a range of possible demand scenarios, facilities are located in such a way to minimize the maximum average travel time when evaluated for all scenarios. *Minmax* p *−median problem* is then formulated as follows:

$$min\, M \qquad (1.11)$$

$$\sum_{i\in I}\sum_{j\in J}\frac{a_i^k d_{ij}^k}{W_k}x_{ij}^k \le M \qquad \forall k \in K \qquad (1.12)$$

$$\sum_{j\in J} x_{ij}^k = 1 \qquad \forall i \in I, \forall k \in K \qquad (1.13)$$

$$x_{ij}^k \le y_j \qquad \forall i \in I, \forall j \in J, \forall k \in K \qquad (1.14)$$

$$\sum_{j\in J} y_j = p \qquad (1.15)$$

$$x_{ij}^k, y_j \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall k \in K \qquad (1.16)$$

Constraints (1.12) are directly related to the objective. Having M to be greater than the weighted average travel time evaluated for each scenario k (l.h.s.), it represents the maximum average travel time across scenarios, to be minimized (1.11). The interpretation of the other groups of constraints (1.13-1.16) is straightforward, as they are the same constraints introduced in the classical formulation of p-median problem (1.2-1.5), considered for each scenario k.

In the regret approach, facilities are positioned so as to minimize the *maximum regret*, i.e. the difference between *(a)* the optimal average travel time that would be obtained if the decision maker had planned its sites for the scenario that actually occurs; and *(b)* the value of average travel time that was actually obtained

If the regret objective is used, constraints (1.12) have to be replaced by the following:

$$\sum_{i\in I}\sum_{j\in J}\frac{a_i^k d_{ij}^k}{W_k}x_{ij}^k - Z_k \leq M \qquad \forall k \in K \quad (1.17)$$

where Z_k is the optimal value of the objective function, found by applying the original p-median formulation to each scenario individually. The unknown variable M represents the largest regret evaluated over all scenarios.

Although the p-median problem is of interest, its emphasis on the average accessibility to the services is not sufficient in the context of emergency services because it makes no provision for the extremes values and it is possible that a solution to this problem leaves some demand points too far from the nearest facility. In order to overcome this aspect, other classes of objectives need to be considered, as shown in the next sections.

1.4.2 Covering Models

Covering models are based on the concept of *coverage radius*, i.e. a maximum pre-specified value for either distance or travel time. A facility is said to cover a demand point if their mutual distance does not exceed this maximum value. An example is the case in which it is desired that the population in a given area have access to a health care center within a given distance, say 2 miles. It is said that a customer in this area is covered if he has a health care center within 2 miles of her/his home.

In this case, the service is considered equally good if provided by facilities at different distances, as long as both distances are smaller than a maximum value, that represent an *acceptable proximity*.

There are two basic formulations of covering models:

- *Set Covering Location Problem (SCLP)*, seeking to minimize the number of facilities needed for full coverage of the population in a given location space;
- *Maximum Covering Location Problem (MCLP)*, aiming at maximizing covered population, given a limited number of facilities or budget.

The SCLP seeks to locate the minimum number of facilities needed to obtain mandatory coverage of all demands (Toregas et al., 1971; Toregas and ReVelle, 1973). In other words, each demand point need to have at least one facility located within some distance or time standard S. The first application of this model was in the area of emergency services (ReVelle et al. 1976).

Assuming the following notation:

S	target distance for coverage;
$N_i = \{j \in J: d_{ij} \leq S\}$	set of all those sites that are within distance S from the demand node i.

the SCLP can be formulated as follows:

$$min\, z = \sum_{j \in J} y_j \tag{1.18}$$

$$\sum_{j \in N_i} y_j \geq 1 \qquad \forall i \in I \tag{1.19}$$

$$y_j \in \{0,1\} \qquad \forall j \in J \tag{1.20}$$

The objective (1.18) minimizes the number of required facilities to cover all the demand. Constraints (1.19) state that the demand at each node i must be covered at least by one facility located within the distance S.

Although public services should be available to everybody, as modeled by the LSCP, the MCLP recognizes that mandatory coverage of all people in all occasions and no matter how far they live, could require excessive resources. Thus, MCLP does not force coverage of all demand but, instead, seeks the location of a fixed number of facilities, most probably insufficient to cover all demand within the standards, in such a way that covered demand is maximized. The fixed number of facilities is a proxy for a limited budget.

Church e ReVelle (1974) and White and Case (1974) formulated the MCLP as follows:

$$min\, z = \sum_{i \in I} a_i x_i \tag{1.21}$$

$$\sum_{j \in N_i} y_j \geq x_i \qquad \forall i \in I \tag{1.22}$$

$$\sum_{j \in J} y_j = p \tag{1.23}$$

$$x_i, y_j \in \{0,1\} \qquad \forall i \in I, \forall j \in J \tag{1.24}$$

where the binary variable x_i indicates if a demand node i is covered or not.

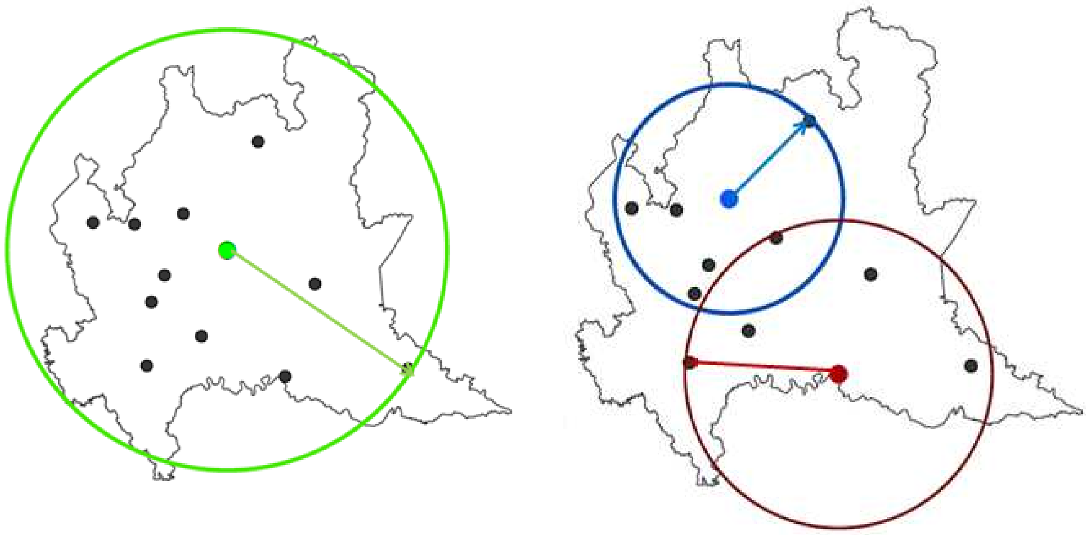
The objective function (1.21) maximizes the weighted sum of covered demand nodes. Constraints (1.22) state that the demand at node i is covered $(x_i = 1)$ whenever at least one

facility is located within the time or distance standard S. Constraints (1.23) give the total number of facilities that can be sited.
It has to be noticed that, while the LSCP ignores the magnitude of the demand coming from each node, in the MCLP this aspect is strongly considered as it aims at covering as much demand as possible rather than as much nodes as possible.
Application of these models in the public sector range from emergency services to location of bus stops (Gleason, 1975), health clinics (Eaton et al., 1981), hierarchical health services (Moore and ReVelle, 1982), and many other applications.
An interesting generalization of the MCLP considers the simultaneous location of several types of facilities, each characterized from its own standard distance (Schilling et al., 1979). The model was formulated for the location of fire-fighting services in the city of Baltimore; in particular, depots for two types of servers (pump and ladder companies). The objective of the model was the coverage of the maximum number of people by both companies, each one sited within its own standard distance. Marianov and ReVelle, (1991, 1992) extended the model to the case in which two or more servers of each type are needed, because the attendance of only one of each is not enough (as in police or fire emergencies).
In most cases, a server cannot attend more than one call at a time; consequently, the system can become congested if one or more calls arrive when a server is busy. In this case, different approaches can be used. When congestion is not expected to be severe, the approach (Hogan and ReVelle, 1986) consists in allocating more than one server to each demand node within the standard distance (*redundant coverage*). On the contrary, when congestion is expected to be more severe, a probabilistic approach is more appropriate. Stochastic models consist of maximizing expected coverage of each demand node (Daskin, 1983; Daskin, 1995) or of constraining the probability, that at least one server (for each demand node) is available, to be greater than or equal to a specified value α.

1.4.3 Center Models

This class of problems involves locating one or more facilities in such a way that every demand receives its service from the closest facility and the maximum distance between each demand node and its facility is as small as possible. While the *mini-sum* problems optimize an aggregated measures of access to the facilities, these problems focus on the customers in the worst condition and seek to minimize the distance that separates him from its assigned facility.

Suppose that only one facility has to be located in a given location space where a set of demand node exists; using the Euclidean distance, the problem of minimizing the maximum distance is equivalent to finding the center of the smallest circle enclosing all points (Figure 1.1,a). In the same way, the problem of locating a predetermined number $\boldsymbol{p}$ of facilities is equivalent to cover every point through $\boldsymbol{p}$ circles with the smallest possible radius (Figure 1.1,b). For this reason this class of problems is also known as *center problems.*



(a) $p = 1$ **(b)** $p = 2$

Fig.1.1- $\boldsymbol{p}$ **−center solutions**

The problem was first introduced in the literature by Hakimi (1964) for the location of a single facility on a network (*Absolute Center Problem*); its formulation is now well known in the following form:

$$min\, z = D \quad (1.24)$$

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I \quad (1.25)$$

$$x_{ij} \leq y_j \qquad \forall i \in I, \forall j \in J \quad (1.26)$$

$$\sum_{j \in J} y_j = p \quad (1.27)$$

$$\sum_{j \in J} d_{ij} x_{ij} \leq D \qquad \forall i \in I \quad (1.28)$$

$$x_{ij}, y_j \in \{0,1\} \qquad \forall i \in I, \forall j \in J \quad (1.29)$$

Constraints (1.25-1.27) and (1.29) are identical to (1.2-1.5) of the $p-$median problem. Constraints (1.28) are directly related to the objective. Having D to be greater than the distance of each demand node from its assigned, it represents the maximum distance (1.28), to be minimized (1.24).

In some cases, it could be required to assign different values of importance to the single demand nodes, based, for example, on population densities. A node representing a densely populated area may require an higher protection against emergency than a node representing a rather sparsely populated area. Such differences may be reflected into the model by assigning to each node a weight and minimizing the maximum weighted distance. In this case, constraints (28) should be modified as follows:

$$a_i \sum_{j \in J} d_{ij} x_{ij} \leq D \qquad \forall i \in I \quad (1.30)$$

Another extension is related to the introduction of capacity restrictions on the facilities (Pinar, 2006).

Center location problems commonly arise in emergency service location, where the goal of quick response times is significantly more important than any efficiency consideration, related to the cost of delivering that service. In the case of a fire station, for example, when a fire breaks out, it is crucial to arrive at the fire as quickly as possible. Similarly, for ambulances the human loss is strictly related to the response time. Consequently, in all these circumstances it makes sense to adopt p-center objectives.

1.5 Districting Problems

Districting Problems are aimed at grouping small geographic areas, called *basic areas* or *territorial units* (counties, zip code or company trading areas), generally associated with a set of attributes (i.e. demand, population), into a given number of larger geographic clusters, called *territories* or *districts*, in a way that the latter are acceptable according to some planning criteria. The most relevant adopted criteria are:

- *Integrity*: each territorial unit cannot be split between two or more districts;
- *Population equality (*or *population balance)*: districts should have approximatively the same size, in terms of population;
- *Compactness*: the shape of districts has to be almost compact, without elongated parts. Thus a round-shaped district is deemed to be acceptable, while an octopus- or an eel-like one is not;
- *Contiguity*: in each district, it should be possible to walk from any point to any other point of the district, without ever leaving it; in other words, no holes or isolated parts should be present;
- *Respect of existing boundaries (administrative or geographic)*: districts should be coherent with existing boundaries. In the case of geographic borders, this is desirable in order to avoid discontinuities in the solution. For example, it could be difficult to travel between two points sited in the opposite parts of a river; in this sense, they should not be included in the same district. In the same way, also mountains or lakes can represent obstacles for the districts' contiguity. In the case of administrative boundaries, the conformity to existing divisions is desirable in order to avoid to split already existing official or normative regions.

This problem has been largely applied in the context of political districts, i.e. territories within elections have to be performed (Hess et al., 1965; Williams, 1995; Hojati, 1996; George et al., 1997; Mehrotra et al., 1998; Bozkaya et al., 2003). The design of political district is particularly important in democracies where each territory elects a single member to a parliamentary assembly (*majority system*). This is, for example, the case of Canada, New Zealand, and most countries in the United States and Germany. As the territorial subdivision of an area in districts may significantly influence election results, in these problems the goal is to design *fair* districts, so as no political party should be able to take advantage from territorial subdivision in order to gain seats. With this aim, the main planning criteria to be taken into account are the *size* and *shape* of districts. As concern the first aspect, districts should have approximately the same number of voters (*population balance criteria*) in order to respect the principle of *one man-one vote*; while as concerns the second aspect, districts are required to be to be compact and non distorted. The bad malpractice of manipulating district boundaries in order to create a political advantage for a particular party or group is known as *gerrymandering*. This word was used for the first time in 1812 by the Boston Gazette, in reaction to a particular event. Indeed, in 1812 the then Governor Gerry signed a bill that redistricted the Massachusetts state in order to benefit his Democratic-Republican Party and to be elected again. When mapped, one of the district in the Boston area was said to resemble the shape of a salamander (Figure 2). Then, the term gerrymander comes form the combination of the governor's last name (Gerry) and the word salamander.

Fig.1.2- Gerrymandering

Districting problems have been largely applied also to other contexts, ranging from the design of territories for public services, such as schools (Ferland and Guénette, 1990; Schoepfle and Church, 1991), waste collection facilities (Hanafi et al., 1999), emergency services (Baker et al., 1989; D'Amico et al., 2002), to the design of sales territories (Hess and Samuels, 1971; Shanker et al., 1975; Zoltners and Sinha, 1983; Fleischmann and Paraschis, 1988; Drexl and Haase, 1999). Only few papers in literature consider districting problems independently from a practical background (Kalcsis et al., 2005). For an exhaustive review on political districting, see Ricca et al. (2011).

1.6 Service- Oriented Districting Problems

In the public sector, given a set of facilities (already located or to be located) in a region, districting problems are aimed at designing the *territories* in which each facility has to provide its service(s). Unlike political districting, in this kind of applications units have to be grouped with reference to the facilities and their characteristics (i.e. position, capacity). Therefore, the problem can be viewed as a location-allocation problem in which facilities have been located and units have to be assigned to them; units assigned to the same facility represent a territory.

Palermo et at. (1977) and Fertand and Gudnette (1990) deal with the problem of school districting, i.e. the partition of all basic area in the region under consideration into a number territories, one for each school. In these problems territorial units can be represented by census tracks, streets or streets segments; while criteria generally taken into account in the planning process are capacity limitations and equal utilization of the schools, maximal or average travel distances for students, good accessibility and racial balance.

Hanafi et al. (1999) address the districting problem for the organization of the solid waste disposal service. In a first step, the region under consideration is partitioned into sectors, where each sectors consists of a set of streets or street segments in which waste has to be collected on a certain day. Afterwards, routes for the garbage trucks within the sectors are computed separately. According to Hanafi et al. (1999), the overall time for collecting garbage should be minimized (compactness), the time for collecting garbage should be approximately the same for all sectors (balance) and the sectors should be contiguous. Moreover, territories should allow the planning of *good* or *efficient* routes.

D'Amico et al. (2002) report on a case study for police district design, where police departments have to partition their jurisdiction into so-called command districts. A closely related problem is described by Baker et al. (1989), which face the task of designing so-called primary response areas for county ambulances. As reported, the main design criteria for the territories are workload balance, geographical compactness and contiguity.

Apart from the specific application, there are several elements shared by most of the models proposed in literature. In the following we illustrate the fundamental elements of typical districting problems.

1) Basic Units

A districting problem involves a set I of *territorial units*. These units are geographical objects in the plane: points (e.g. geo-coded addresses), lines (e.g. street segments) or geographical areas (e.g. zip code areas, counties), generally represented by polygons (Figure 1.3).

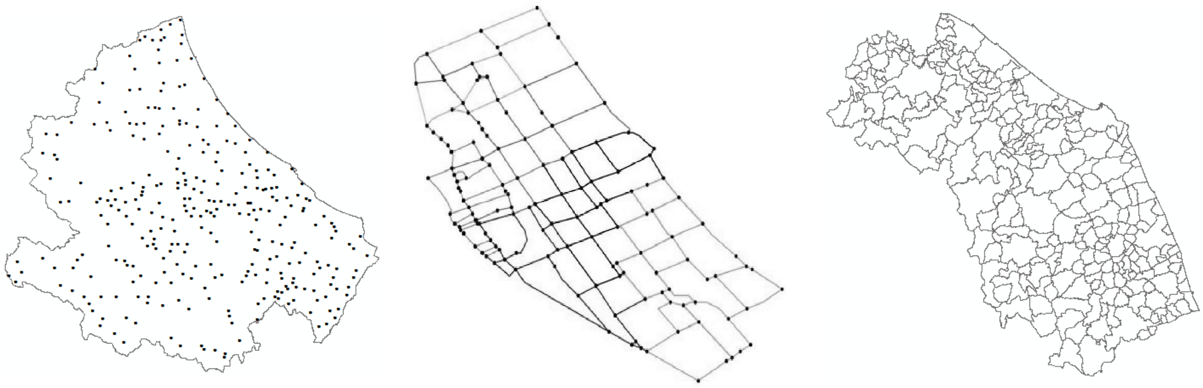


Fig.1.3- Examples of basic units (points, lines, areas/polygons)

In case of non-punctual objects, to each unit $i \in I$ is associated a point c_i, called *center*, whose coordinates (x_i, y_i) correspond to an its representative point (i.e. the centroid).
In Figure 1.4 it is shown a representation of the centers of a set of territorial units within a region. The example deals with an italian region (Marche) and the centers correspond to the positions of the related city halls.
These points are generally used as basis for the computation of the distances among basic units.

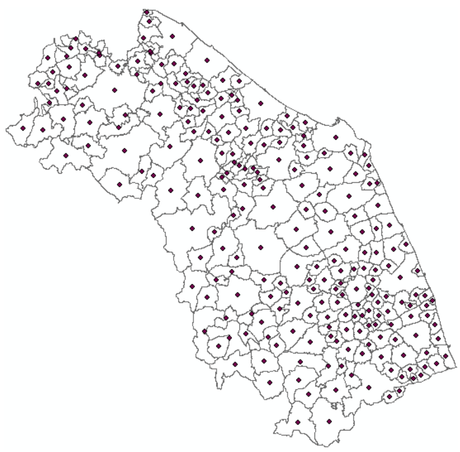


Fig.1.4- Centres of basic units

Usually one or more quantifiable attributes are associated with each basic unit. Typical examples are demand, number of inhabitants, extension, etc. We will assume here, that for each basic unit $i \in I$ just a single attribute a_i is given.

2) District Centers

In general, districts are obtained by assigning basic units to a set J of *centers,* which often correspond to the points where facilities are located or are to be located (e.g. fire stations, hospitals, schools). Usually the location of the *district centers* is part of the planning process and their positions are selected among the centers of the basic units ($J \subset I$). Another important aspect is related to the number p of districts to design; in most cases it is pre-specified but it may be also a decision variable of the problem.

3) Integrality constraints

In most application, it is required that every basic unit is contained in exactly one district; hence, that districts define a partition of the set I.

Let $I_k \subset I$ denote the k-th district, then:

$$\bigcup_{k=1}^{p} I_k = I \text{ and } I_k \cap I_j = \emptyset \quad \forall k \neq j$$

This requirement is motivated by several factors. First of all, unique allocations result in transparent responsibilities among single facilities; moreover, they allow to define in a stable way the boundaries of territories. See Figure 1.5 for possible partitions for the problems shown in Figure 1.3.

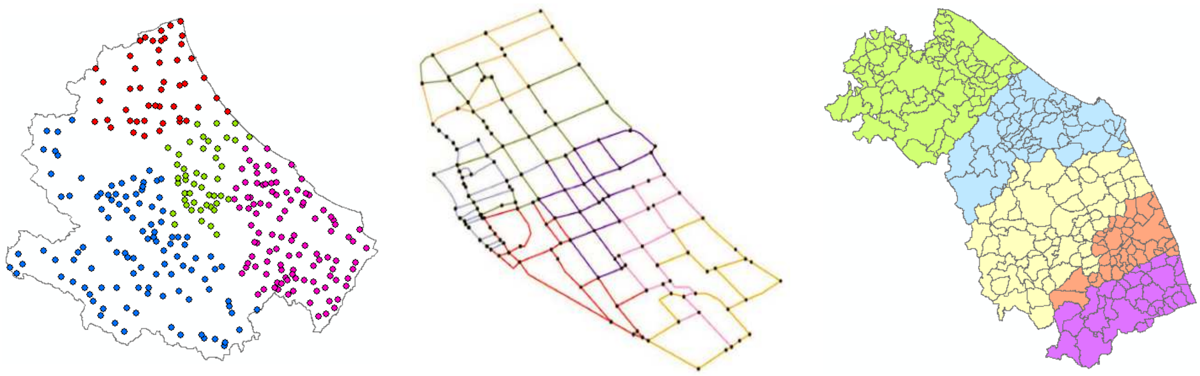


Fig.1.5- Examples of Territories (points, lines, areas/polygons)

4) Balance

Usually districts are required to be balanced, i.e of equal size, with reference to one or more attributes. Starting from the attribute values related to the the single units, it is possible to obtain the size of the whole districts. In general, the aggregation scheme is additive, then the attribute or the *size* of territory k is given by the sum of the values of the contained basic units. Formally:

$$A(I_k) = \sum_{i \in I_k} a_i$$

Due to the discrete structure of the problem and the integrality constraints, perfectly balanced territories can generally not be accomplished. Different indicators can be defined in order to measure balance; a common way consists in the computation of the relative percentage deviation of the district size from their average size $\bar{A} = \frac{\sum_{i \in I} a_i}{p}$. The larger this deviation is, the worse is the balance of the territory.

5) *Contiguity*

Districts are said to be contiguous when it is possible to travel from one point to any another point of the same district without leaving it; or, in other words, when it is constituted of a single part.

Ensure as well as check the contiguity of a provided solution is a difficult task. When basic units are points, a district is said to be contiguous if the *convex hull*[2] of the points incuded in the district does not intersect the convex hull of the basic units of any other district. If the basic units are non-punctual, i.e. lines or polygons, in order to check the contiguity of districts, it is required to collect some input information about the adjacency between pairs of units. Usually two units are said to be adjacent if their geographical representations have a non-empty intersection; in particular, polygons need to share a common border, segments have to meet in a crossroad. This information can be represented through a matrix, known as *adiacency matrix*, or a *contiguity graph*, in which vertices correspond to the units in I and two nodes are connected by an edge if and only if they are adjacent.

6) *Compactness*

A territory is said to be geographically compact if it is somewhat round-shaped and undistorted. Although being a very intuitive concept, a rigorous definition of compactness does not exist. Young (1988), Niemi et al. (1990) and Horn et al. (1993) propose several measures to asses the compactness of a district, none of which is comprehensive. Some measures fail to detect districts that are obviously noncompact, while others assign a low rating to visibly compact districts (Williams,1995). Spatial or visual compactness can be evaluated using relative or absolute measures. While the former determine compactness using shape only, the latter measure compactness taking into account shape as well as area. Absolute measures are, however, biased against large areas in the sense that a large circle is

[2] Smallest convex polygon enclosing all points

less absolutely compact than a small circle. Common relative measures are the Roeck test and Schwartzberg test. The former calculates the ratio of the territory area to the area of the smallest enclosing circle, while the latter determines the ratio of the districts perimeter length to the circumference of a circle with equal area. Hess et al. (1965) propose an absolute measure, called the moment of inertia, which calculates the dispersion of the district area about its center. One way to measure the moment of inertia is to compute the sum of the squared Euclidean distances from the center of the district to the centers of the basic areas.

The smaller the moment of inertia is, the more compact the district is. Note that, despite its deficiencies, this measure is often used in the literature since it can be calculated easily and, moreover, incorporated into linear programs without effort. Apart from geographical compactness, there also exists the concept of population or demographic compactness. Here, a district is said to be compact if its population is concentrated in a relatively small area. Hess et al. (1965), for example, propose the population weighted moment of inertia as a measure.

$$\sum_{k=1}^{p} \sum_{i \in I_k} a_i d_{ik}$$

1.7 Combining Districting and Facility Location Problems

The analogy between the two classes of above problems is first introduced in 1965 by Hess et al., that formulate a districting problem in terms of location-allocation problem, in which customers correspond to the single territorial units and the p facilities to be located to the district centers. The idea is to identify the location for the p centers and assign each territorial unit to exactly one of them; hence, all the basic units allocated to the same center constitute a district. Associating to each unit an attribute corresponding to its demand, balance of the territories is achieved by imposing capacity restrictions on the facilities.

The model introduced by Hess et al. can be formulated as follows:

$$min\, z = \sum_{i \in I} \sum_{j \in J} p_i d_{ij}^2 x_{ij} \qquad (1.31)$$

$$\sum_{j \in I} x_{ij} = 1 \qquad \forall i \in I \quad (1.32)$$

$$\sum_{j \in I} x_{jj} = p \qquad (1.33)$$

$$(1-\tau)\bar{P}x_{jj} \leq \sum_{i \in I} p_i x_{ij} \leq (1+\tau)\bar{P}x_{jj} \qquad \forall j \in I \quad (1.34)$$

$$x_{ij} \in \{0,1\} \qquad \forall i,j \in I \quad (1.35)$$

The objective function (1.31) is a *compactness measure*, defined as the sum of the weighted distances between the single territorial units and the related district centers. Constraints (1.32) ensure that each territorial unit is assigned to exactly one district center (*integrality*). Constraints (1.33) impose that the number of district (centers) is equal to p. The two groups of constraints (1.34) represent the conditions on the maximum and minimum allowable population for each district (*population balance*); in particular, they define the maximum tolerance τ from the ideal size $\bar{P}\left(\bar{P} = \frac{\sum_{i \in I} p_i}{n}\right)$. Finally constraints (1.35) define the nature of the decision variables.

The parameter τ strongly affect the solution provided by the model and the time for solving it. The smaller τ is, the better will be the balance of the districts; but, on the other hand, if τ is too small, i.e. the time for optimally solving the model generally increases and territories tend to be no longer compact and connected.

As this mixed integer linear program is NP-hard, its practical use is fairly limited. To this end, Hess et al. used a location-allocation heuristic to solve the problem. In this heuristic, the simultaneous location-allocation decisions of the underlying facility location problem are decomposed into two independent phases, a location and an allocation phase, which are iteratively performed until a satisfactory result is obtained.

1) *Initialization*: select (randomly) p units as district centers;
2) *Allocation Phase:* assign the units to the selected centers so as to obtain perfectly balanced districts at the minimum cost. In this step, a *transportation problem*, in which the set of origins corresponds to the set of current centers, all with supplies equal to $\bar{P}$, and the set of destinations corresponds to the set of territorial units, each with demand equal to the related population p_i, is solved. Given that the unit transportation cost from i to j is equal to d_{ij}^2, the problem consists of determining the quantity to be supplied from each origin towards each destination so as to minimize the total transportation costs and satisfy the constraints on the total demands and supplies. In this problem the assignment variables are not binary $\left(x_{ij} \in [0,1]\right)$; indeed, x_{ij} represent the fraction of demand coming from unit i allocated to center j.
3) *Split Resolution*: the solution of the transportation problem is perfectly balanced but usually violates the integrality property, as portions of the same territorial unit may be allocated to different districts. In this phase, the solution of the transportation problem is adjusted so that each territorial unit is assigned to exactly one district, i.e. so as each

fractional variables is rounded to one or zero. Since there are many possibilities for the rounding, it is necessary to find the one that results in as much as possible balanced territories *(Split Resolution Problem - SRP)*. Hess and Samuels (1971) propose a simple rule, which exclusively assigns each *split unit* to the district (center) which owns the largest share.

4) *Location Phase:* once the districts are obtained, it is necessary to update the set of district centers. There exist several approaches for determining a new configuration of district centers. A fairly simple and commonly used method is to solve in each district, resulting from the last phase, a *single facility median problem* and find the so-called *centroids* (Fleischmann and Paraschis, 1988; George et al., 1997).
5) *Stopping Criteria:* Steps 2-4 are repeated until the procedure converges, i.e. the centers do not change in two successive iterations.

Hess et al. (1965) provide an application related to the design of Delaware legislative districts in 1964, where $n = 650$ and $p = 35$.

Starting from this contribution, other authors develop political districting methods based on a location approach. Hojati (1996) suggests a three-phases procedure, in which instead of adopting an iterative strategy based on successive adjustments of the centers, they are located only once at the beginning of the procedure and this choice is permanent. This method is applied to the territory of the city of Saskatoon (Canada) for the provincial elections in 1993, and the obtained district map is compared with the institutional one showing good results for compactness.

The procedure proposed in George et al. (1997) follows the iterative location/allocation approach pioneered by Hess et al. (1965), but with the main difference that a new method for assigning territorial units to districts is adopted. For this step, the authors introduce a minimum cost network flow problem. The proposed iterative procedure alternates the location of the p centers with the assignment of territorial units according to the solution of the above minimum cost network flow problem. It stops when the difference between the value of two successive optimal solutions of this problem is sufficiently small.

The authors apply their algorithm to the design of parliamentary districts in New Zeland in 1993 and study the performance of four versions of their PD procedure; they also compare the results obtained with their automated algorithm with the New Zealand institutional districts obtained by a manual procedure.

Although the transportation problem and the split resolution can be done rather efficiently using specialized methods, computational experiments show that running times of location-allocation algorithms are still too high for large scale problems with several thousand basic units and hundreds of districts (Kalcsics et al., 2005).

1.8 Conclusions

Facility location decisions represent a critical element in strategic planning in both private and public sectors, as they can have a strong and lasting impact on operational and logistic performance.

A facility location problem is aimed at finding the best position for a set of facilities within a given region in order to optimise a specific objective function. Starting from this general framework, several formulations may be defined in terms of objective function, features of facilities to be located, demand to be served, and location space (ReVelle et al., 2007). In the last decades, there have been many applications concerning the location of both public facilities (i.e. schools or post offices, emergency services, fire stations, hospitals, ambulances) and private (i.e. plants, warehouses, industrial sites) facilities. These models represent a viable decision support tool for institutions and firms that are planning to open new facilities in a given region and/or market. In recent years, however, due to the general interest to reduce costs and to improve efficiencies, companies and institutions have been also interested in the re-configuration of existing supply systems. In the next chapter models addressing re-organizational decisions are introduced and analyzed.

Chapter 2

Re-organization Problems

2.1 Introduction

Historically, facility location models have been a viable decision support tool for institutions and firms that are planning to open new facilities in a given region (Drezner and Hamacher, 2002). Indeed, most of the proposed methods address problems in which services have to be organized ex-novo.

Sometimes it could be necessary to reorganize an existing facility system. Motivations for such decisions can be various; the most discussed in the literature is related to the variability of parameters during the facilities lifetime, such as demand, costs, market structure, competition. Due to this variability, decisions taken on the basis of certain conditions could reveal inefficient later and require some changing in the organization of the system. The *territorial re-organization* is here intended as a process aimed at modifying the set of facilities operating in a given region, in terms of their number, positions, capacities of the offered services, in order to optimize some objectives and satisfy a set of constraints. In the literature, several approaches have been proposed to describe and solve similar problems in a variety of situations.

In this chapter we review the extant literature related to the problem of re-organization of an existing supply system. A classification of the main contributions is proposed and within each class the main models are analyzed in detail and described. Finally, the main gaps are highlighted and, with reference to the latters, the goal of this work is described.

2.2 Re-organization Problems

Facility location decisions are often made at a strategic level as they require large investments. Accordingly, facilities are expected to operate for a considerable time span, during which several parameters and conditions may change; tipically, demand distribution in the location space, costs, competition. Therefore, decisions taken on the basis of the current situation could become inefficient in the future.

Let us assume, for example, that we optimally located a set of facilities in a location space to serve a given demand (Figure 2.1) and that ten years later the demand has increased and its distribution has changed making some of the existing facilities closer to areas with low demand and away from some high demand areas. Therefore, existing facilities may no longer be able to provide adequate service, which yields to an intolerable increase in total weighted distance traveled by the customers (Figure 2.2).

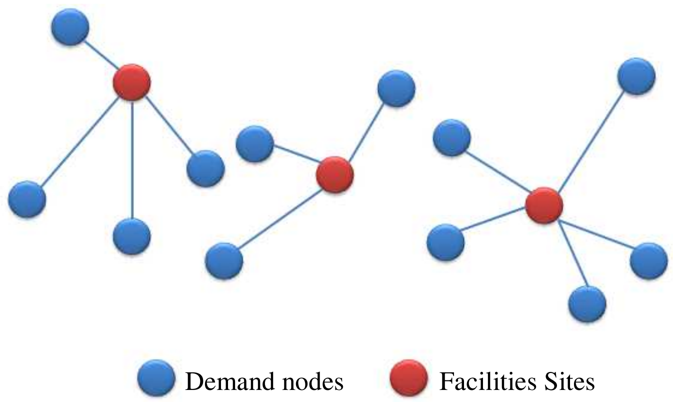


Fig. 2.1 - Current Situation

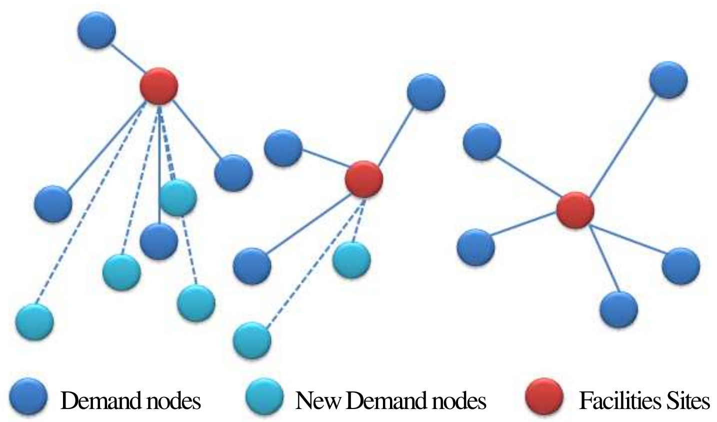


Fig. 2.2 - New Demand Distribution

In this situation the decision maker could decide to reorganize the service, by expanding the service, i.e. opening new facilities (Figure 2.3), or repositioning some facilities, i.e. closing some existing facilities and opening new ones in different point of the location space (Figure 2.4).

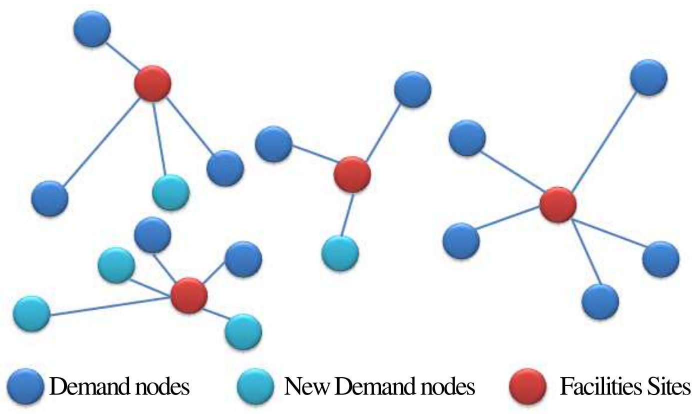


Fig. 2.3 - Expanded Service

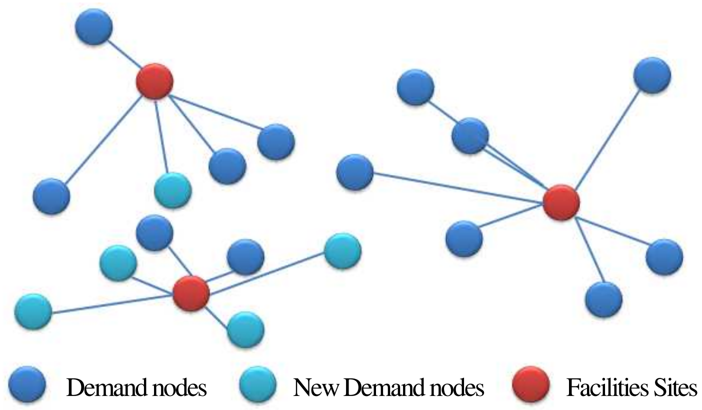


Fig. 2.4 - Repositioned Facilities

Also other motivations for this kind of problems may arise. In the private, sector many factors may quickly change during the lifetime of the facilities, such as market structure and conditions, distribution of demand, presence of new competitors, and financial needs. In the public sector, economic conditions may impose constraints on public expenditure that could result in policies oriented to the rationalization of service supply systems in crucial areas, such as healthcare, education, public transport. These changes could make the existing supply system inefficient and/or unsustainable, and require decisions aimed at increasing the efficiency of the offered services.

In literature there are many papers that address the problem of modifying the system of available facilities in a given location space. We classified them, according to the adopted approach, in:

- ***Ex-ante Re-organization***, in which the decisions are taken before the changes have occured; not only on the basis of the current situation but also taking into account in advance any predictable change (forecasting tools);
- ***Ex-post Re-organization***, in which the decisions are taken once changes have already occurred.

In the first class, we may distinguish between two classes of problems:

- *Multiperiod Models:* decisions are made period per period, over a time horizon, on the basis of future deterministic data (estimated demand, costs);
- *Stochastic Models:* decisions are made in a single period (*now and here*), on the basis of estimated probabilities about the occurrence of future scenarios.

In the second class, there are models that, starting from a given configuration of the system of facilities, aim at modifying it in order to optimize a given objective and satisfy a set of constraints.

In Figure 2.5 we summarize the classification scheme introduced above and in the next sections we are going to analyze the main contributions in the single classes of problems identified.

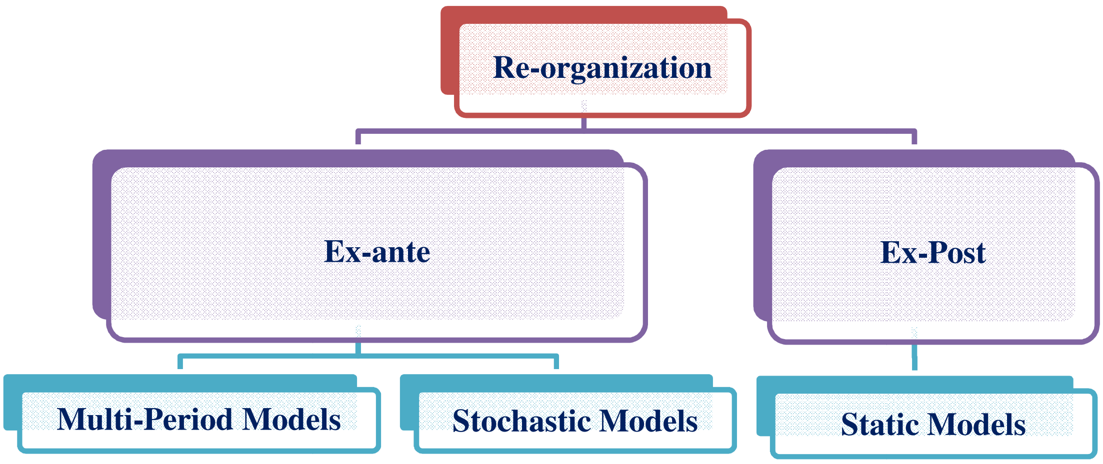


Fig. 2.5 - Re-organization actions

2.3 Multi-period Models

In the literature, many models explore the possibility to change the current organization of the facilities in the location space, by closing some of them and opening others in different points. This approach has been mainly adopted in multi-period models.

One of the first discrete facility location problem to be extended to a multi-period setting has been the $p-$median problem.

Suppose we have a set I of n nodes that originate demand in every period t of a finite planning horizon (a_{it}) and assume that the set J of nodes for potential facilities location is included in I $(J \subseteq I)$. The problem aims at finding the best location of p facilities in each time period. Introducing a third index t, related to the generic time period, the formulation of the classic $p-$median problem can be easily adapted to the multi-period setting, as follows:

$$min\, z = \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} a_{it} d_{ij}\, x_{ijt} \qquad (2.1)$$

$$\sum_{j \in J} x_{ijt} = 1 \qquad \forall i \in I, \forall t \in T \qquad (2.2)$$

$$\sum_{i \in I} x_{ijt} \leq M y_{jt} \qquad \forall j \in J, \forall t \in T \qquad (2.3)$$

$$\sum_{j \in J} y_{jt} = p \qquad \forall t \in T \qquad (2.4)$$

$$x_{ijt} \geq 0, y_{jt} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall t \in T \qquad (2.5)$$

where the variables y_{jt} indicate if a facility is sited (or not) in $j \in J$ in period $t \in T$ and x_{ijt} if demand point $i \in I$ is allocated (or not) to a facility $j \in J$ in period $t \in T$. The optimal configuration of located facilities may change period per period according to the changes in the demand patterns. In this case, there is no limitation about the number of facility location changes or relocations between periods, therefore the optimal solution in each $t \in T$ can be found by solving the related single period $p-$median problem. Obviously this assumption may be too restrictive in real-life applications; indeed, it is much more reasonable to include some constraints in order to reflect tolerable levels of disruption organization and to consider costs associated with the relocation of facilities.

In order to overcome this problem, Wesolowsky and Truscott (1975) include opening and closing costs for the facilities.

The problem proposed can be formulated as follows:

$$min\, z = \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} a_{it} d_{ij}\, x_{ijt} + \sum_{t \in T} \sum_{j \in J} g_{jt}\, z'_{jt} + \sum_{t \in T} \sum_{j \in J} h_{jt}\, z''_{jt} \qquad (2.6)$$

(2.2)- (2.5)

$$\sum_{j \in J} z'_{jt} \le m_t \qquad \forall t \in T \qquad (2.7)$$

$$y_{jt} - y_{jt-1} + z''_{jt} - z'_{jt} = 0 \qquad \forall j \in J, \forall t \in T \qquad (2.8)$$

$$z''_{jt}, z'_{jt} \in \{0,1\} \qquad \forall j \in J, \forall t \in T \qquad (2.9)$$

where m_t is the maximum number of facilities which can be opened in each period $t \in T$, whereas the binary variables z'_{jt} (z''_{jt}) are equal to 1 if a facility is opened (closed) at $j \in J$ in $t \in T$ and 0 otherwise. The cost g_{jt} and h_{jt} ($j \in J, \forall t \in T$) are the opening and closing costs respectively. Given that in each time period the number of available facilities has to be equal to p (2.4) and the maximum number of new facilities that can be opened is equal to m_t (2.7), the model aims at finding a trade-off solution between static distribution costs and expenditures for relocating facilities (2.6).

Galvão and Santibañez-Gonzalez (1992) assumed that the number of operating facilities does not need to be equal in all periods; in each time period $t \in T$, a prespecified number of facilities p_t must be operating.

Dias et al. (2007) point out that these models ignore the fact that reopening a facility has in general a smaller cost that opening it for the first time (for instance, land acquisition costs are incurred only once). Accordingly, they proposed a model which takes this aspect into account. Decisions variables are now required to distinguish wheter a facility is being opened for the first time or re-opened. A primal-dual heuristic is proposed for obtaining lower and upper bounds on the optimal value. The gap is closed using a branch-and-bound approach.

Moreover, in the above multi-period facility location problems, facilities can be opened and closed more than once during the planning horizon. In many situations, it is not reasonable to instal a facility and remove it, for instance, in the following period. This may make sense for seasonal facilities such as wharehouses which can be rented for short time interval. However, this cannot be assumed in general. Early, researchers have noticed this important aspect and have considered structural constraints which limit the number of openings and closing in the same locations during the planning horizon. Such constraints often state that once established, a facility should be kept opened until the end of the planning horizon.

The above models extend, in different way, the classic $p-$median problem to a multi-period setting; therefore, the number of desired facilities in each time period is pre-specified and the positions are adjusted period per period in order to minimize the total cost, defined as sum of total afference costs and relocating costs.
An interesting version of the multi-period $p-$median problem, is the one in which exactly p facilities have to be located over the planning horizon and the removal of facilities is not allowed once they are established. In this case, the number of facilities in each period is not given but, on the contrary, it is a decision variable and the model considers the speed at which the number of facilities increases in the location space over the planning horizon. The formulation of such problem is the following:

$$min\ z = \sum_{t\in T}\sum_{i\in I}\sum_{j\in J} a_{it}\, d_{ij} x_{ijt} \tag{2.10}$$

$$\sum_{j\in J} x_{ijt} = 1 \qquad \forall i \in I, \forall t \in T \tag{2.11}$$

$$\sum_{i\in I} x_{ijt} \le M y_{jt} \qquad \forall j \in J, \forall t \in T \tag{2.12}$$

$$\sum_{j\in J} y_{jt} = p_t \qquad \forall t \in T \tag{2.13}$$

$$y_{jt} \ge y_{j,t-1} \qquad \forall j \in J, \forall t = 2 \dots T \tag{2.14}$$

$$x_{ijt}, y_{jt} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{2.15}$$

where $1 \le p_1 \le p_2 \le \cdots \le p_t = p$.
Another class of interesting multi-period models are the ones that extend the uncapacitated facility location problems (UFLP) to a multi-period setting.
Denoting with f_{jt} the fixed cost for operating a facility at $j \in J$ in period $t \in T$ and c_{ijt} the cost for satisfying all the demand of customer $i \in I$ in period $t \in T$ from facility $j \in J$, the *multi-period uncapacitated facility location problem* can be formulated as follows:

$$min\ z = \sum_{t\in T}\sum_{j\in J} f_{jt}\, y_{jt} + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J} c_{ijt}\, x_{ijt} \tag{2.16}$$

(2.2)- (2.3), (2.5)

Despite the $p-$median models, in this case the number of facilities is not pre-specified but it is determined in each time period so as to minimize total costs, given by costs incurred for establishment and operation of facilities over time at several possible sites$\left(\sum_{t\in T}\sum_{j\in J} f_{jt}\, y_{jt}\right)$ and variable costs, for serving demand in each time period t from the available facilities $\left(\sum_{t\in T}\sum_{i\in I}\sum_{j\in J} c_{ijt}\, x_{ijt}\right)$.

Again, this problem has little multi-period flavor as it can be decomposed into $|T|$ single-period problems.

A more relevant extension of the model was proposed by Warszawski (1973) who included opening costs for facilities. These costs are incurred whenever a facility is opened (even if the same facility has operated in some past period). Denoting by g_{jt} the cost for opening a facility at $j \in J$ in period $t \in T$, the model proposed by Warszawski (1973) differs from the basic version of multi-period UFLP by considering the following quadratic objective function:

$$min\, z = \sum_{t \in T} \sum_{j \in J} f_{jt}\, y_{jt} + \sum_{t \in T} \sum_{j \in J} g_{jt}\, y_{jt}(1 - y_{j,t-1}) + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} c_{ijt}\, x_{ijt} \tag{2.17}$$

with $y_{j0} = 0, \forall j \in J$.

Another extension of the multi-period UFLP was proposed by Canel and Khumawala (1997) who introduced a new group of variables z_{jt} explicitly indicating whether or not a facility is opened at $j \in J$ in period $t \in T$. They propose a profit maximization problem as follows:

$$max \quad z = \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} r_{ijt}\, x_{ijt} - \sum_{t \in T} \sum_{j \in J} f_{jt}\, y_{jt} - \sum_{t \in T} \sum_{j \in J} g_{jt}\, z_{jt} \tag{2.18}$$

(2.2)- (2.3), (2.5)

$$z_{jt} \geq y_{jt} - y_{jt-1} \qquad \forall j \in J, \forall t \in T \tag{2.19}$$

$$z_{jt} \in \{0,1\} \qquad \forall j \in J, \forall t \in T \tag{2.20}$$

with $y_{j0} = 0, \forall j \in J$. In this model r_{ijt} represents the revenue for supplying all the demand of customer $i \in I$ in period $t \in T$ from a facility operating in $j \in J$. Furthermore, M in constraints (2.3) is replaced by a pre-specified maximum number of customers that each facility $j \in J$ can supply (*Capacitated version*).

Roodman and Schwarz (1977) considered for the first time the situation in which a set of facilities was already operating before the beginning of the planning horizon and could be then removed. Accordingly, the set of locations J is partitioned into two sets: J^c, i.e. the set of facilities which are operating before the beginning of the planning horizon, and J^o, i.e. the set of potential locations for new facilities.

A more comprehensive model is then the following:

$$min\ z = \sum_{t \in T} \sum_{j \in J} f_{jt}\, y_{jt} + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} c_{ijt}\, x_{ijt} \tag{2.21}$$

(2.2)- (2.3), (2.5)

$$y_{jt} \leq y_{j,t-1} \qquad \forall j \in J^c, t = 2, \dots, |T| \tag{2.22}$$

$$y_{jt} \geq y_{j,t-1} \qquad \forall j \in J^o, t = 2, \dots, |T| \tag{2.23}$$

Conditions (2.22) impose that once an existing facility $\mathrm{j} \in \mathrm{J}^{\mathrm{c}}$ become not operating ($\mathrm{y}_{\mathrm{jt}} = 0$), it remains not operating until the end of the planning horizon; while conditions (2.22) impose that, once opened a facility in $j \in J^o$ $(y_{jt} = 0)$, it remains open. Therefore, even if the meaning of variables y_{jt} does not change (they continue to indicate if a facility is operating or not at site j in period t), the different monotonicity conditions on the sets J^o and J^c make possible only the closure action for the existing facilities and only the opening action at the new sites, in both cases one time during the planning horizon.

Roodman and Schwarz (1977) consider a pre-specified maximum number of customers that can be served by each facility in each period. Furthermore, not all facilities can serve all customers. These aspects are accomodated by replacing (2.3) with:

$$\sum_{i \in P_{jt}} x_{ijt} \leq n_{jt} y_{jt} \qquad \forall j \in J, \forall t \in T \tag{2.24}$$

where P_{jt} and n_{jt} are respectively the set and maximum number of customers that can be served by a facility located in $j \in J$ in period $t \in T$.

A reformulation of this model was proposed by Van Roy and Erlenkotter (1982). The idea is to consider binary decision variables representing the status change of a location instead of the traditional location variables. Accordingly, for an existing facility $j \in J^c$, z_{jt} is equal to 1 if the facility is removed at the end of period t (thus operating in periods $1 \dots t$) and 0 otherwise. For an existing facility $j \in J^o$, z_{jt} is equal to 1 if the facility starts operating at the beginning of period t (thus, operating in periods $t, \dots, |T|$) and 0 otherwise.

Using the new decision variabes, the following model can be considered:

$$min\ z = \sum_{t \in T} \sum_{j \in J} F_{jt}\, z_{jt} + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} c_{ijt}\, x_{ij}^{t} \tag{2.25}$$

$$\sum_{j \in J} x_{ij}^{t} = 1 \qquad \forall i \in I, \forall t \in T \tag{2.26}$$

$$\sum_{t \in T} z_{j}^{t} \leq 1 \qquad \forall\, j \in J \tag{2.27}$$

$$x_{ij}^{t} \leq \sum_{\tau \in \bar{T}_{jt}} z_{j\tau} \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{2.28}$$

$$x_{ij}^{t} \geq 0; z_{j}^{t} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{2.29}$$

The objective function (2.25) is the traditional sum of fixed and variable costs. The only difference is in the parameters F_{jt}. Being F_{jt} multiplied to the status change variable z_{jt}, when this latter is equal to 1, it has to take into account the costs associated to all those periods in which facility has been operating. In particular:

$$F_{jt} = \begin{cases} f_{j1} + \cdots + f_{jt} & j \in J^{c}, t \in T \\ f_{jt} + \cdots + f_{j|T|} & j \in J^{o}, t \in T \end{cases}$$

Constraints (2.26) guarantee the assignment of the assignment of all the demand in each time period. Constraints (2.27) impose that each facility may change its status at most once during the whole planning horizon. In the conditions (2.28), it has been denoted with $\bar{T}_{j\tau}$ the set of possible periods for changing the status of a facility $j \in J$ so that it can be operating in period t:

$$\bar{T}_{j\tau} = \begin{cases} \{t, \ldots, |T|\} & j \in J^{c}, \tau \in T \\ \{1, \ldots, t\} & j \in J^{o}, \tau \in T \end{cases}$$

Then, the demand coming from node i can be assigned to a facility $j \in J^{o}$ in t, if it has been opened before t or in t $\left(\sum_{\tau=1}^{t} z_{j\tau} = 1\right)$; while it can be assigned to a facility $j \in J^{c}$, if it will get close after t $\left(\sum_{\tau=t}^{T} z_{j\tau} = 1\right)$. In this sense an exisiting facility has to change formally its status one time during the planning horizon; this does not mean that it has necessarly to be closed; infact, $z_{jT} = 1$ indicates that facility has been operating all over the planning horizon. Finally constraints (2.29) define the nature of decision variables.

The applicability of the above formulation is restricted significantly by one shortcoming, the absence of capacity constraints. In the static UFLP, facility size decisions are determined simultaneously with location decisions and the capacities established are fully utilized. In dynamic problems, capacities established in earlier periods become constraints for the subsequent periods: full capacity utilization in every period is unlikely. By ignoring these capacity decisions, the DUFLP assumes that capacity adjustment in each period is perfectly flexible.

Denoting with C_j the capacity for a facility operating in $j \in J$ and a_{it} the demand coming from node i in period t, a capacitated version of the above problem can be obtained by introducing in the (2.25-2.29) the following group of constriants:

$$\sum_{i \in I} a_{jt} x_{ijt} \leq C_j \sum_{\tau \in \bar{T}_{jt}} z_{j\tau} \qquad \forall j \in J, \forall t \in T \qquad (2.30)$$

The model (2.25-2.30) was addressed by Saldanha-da-Gama (2002).

The capacity constraints (2.30) may be quite restrictive when it comes to practical applications. Indeed, by considering fixed values no adjustment of the capacities is possible during the planning horizon according to the needs. One attempt is to consider different values of capacities C_{jt}, one for each time period. Nevethless, this is still short from a practical point of view as no relation exist among these values. Taking into account these observations, Shulman (1991) introduced capacity constraints in the model formulation, allowing the expansion of existing facilities to cope with evolving demand conditions. Extending and adapting this idea to a broader supply chain scenario, Melo et al. (2006) introduced mixed integer programming models to minimise the cost for a multi-commodity, multi-echelon, dynamic network by means of step-wise reallocation of capacities under the assumptions that all existing facilities are operating at the start of the planning horizon; if an existing facility is closed, it cannot be reopened; and when a new facility is opened, it will remain in operation.

Hinojosa et al. (2000) extend the models proposed by Roodman and Schwarz (1977) and Van Roy and Erlenkotter (1982) by considering more than one facility echelon and multiple commodities. This problem is later extended by Hinojosa et al. (2008) in order to include inventory decisions.

Albareda-Sambola et al. (2009) extend the model proposed by Roodman and Schwarz (1977) to handle the so-called multi-period incremental service facility location problem. The problem is motivated by some practical problems which require a multi-period plan for progressively covering the demand in some area. Accordingly, the coverage level progressively increases over time until all customers/demand points are served.

Wilhelm et al. (2013) formulated the strategic dynamic supply chain reconfiguration problem, to prescribe the location and capacity of each facility in such a way to minimize total cost by allowing the dynamic reconfiguration of the network (i.e., by opening facilities, expanding, downsizing and/or contracting their capacities, and closing facilities) over time to accommodate changing trends in demand and/or costs

2.4 Stochastic Models

Another possible approach to tackle with the uncertainty related with the parameters of the problem is to take decision at present time by considering ex-ante the probability associated to some uncertain future conditions (stochastic approach).

Berman and Drezner (2008) develop a first formulation for the *p-median problem under uncertainty*. The proposed model aimed at locating p facilities at the present time given that, with known probabilities, up to q additional facilities in the future may be located.

There are practical situations that should be modeled by the $p-$median model under uncertainty. For example, suppose that there is a limited budget that allows the location of p facilities and there may be additional budget available in the future for expansion. Another example is the location of a chain of stores, specialty restaurants, coffee houses, or others, planned for a market area. The planner is not sure about the success of the chain. If the chain is successful, more facilities will be built in the future. The model can also be used in a non-probabilistic context. Suppose that we know that in five years an additional facility will be built and suppose that the planning horizon is for 20 years. The first p facilities contribute to the profit for 20 years and the additional facility contributes to it for only 15 years. Present value considerations can also be incorporated into the calculation. Instead of viewing α as a probability, it can be viewed as a function of the additional facility's relative contribution to the profit. A combination of these approaches can also be useful. There is a probability that in a few years demand will increase uniformly at all demand points. Such an increase will require the establishment of more facilities. Several scenarios for adding one, two, or more facilities can be incorporated into the model. The probability of each scenario can be multiplied by the extent of the increase in demand. The shorter service time for the extra facilities can also be incorporated into the calculations.

Therefore, the initial locations of the facilities should be determined considering the probability of change in the number of facilities in the future as well

Consider a graph $G(N, L)$ with a set N of n nodes, and a set L of m links. The shortest distance between node i and node j is d_{ij}. Demand at node $i \in N$ is w_i. When several facilities are available on the network, each customer selects the closest facility. The p −median model seeks the locations for p facilities such that the total weighted distance is minimized. Suppose now that in the future some new facilities will be added to the system. Suppose that up to q new facilities can be added, and the probability that r facilities are added $(0 \leq r \leq q)$ is given and equal to α_r. By definition $\sum_{r=0}^{q} \alpha_r = 1$; for example, if the probability of adding a facility is α and these events are independent,

then α_r follows a binomial distribution. The p −*median model under uncertainty* seeks the location for p facilities that minimize the expected weighted distance when additional facilities are added in the future.

In the following, it is possible to prove that the p −*median model under uncertainty* satisfies the Hakimi property, i.e. an optimal solution exist with all the facilities located on nodes. Consequently, it suffices to consider only location on nodes of the network.

Let P be the set of existing facilities, $K_r(P)$ a set of r nodes in the set $(N - P)$ and let $F_r(P)$ be the best value of the objective function when r facilities are added in the future to P. The value of $F_r(P)$ is:

$$F_r(P) = \min_{K_r(P) \in (N-P)} \left\{ \sum_{i \in N} w_i \min_{j \in P \cup K_r(P)} \{d_{ij}\} \right\} \tag{2.31}$$

The p −*median model under uncertainty* is:

$$\min_{P} \left\{ F(P) = \sum_{r=0}^{q} \alpha_r F_r(P) \right\} \tag{2.32}$$

Note that when α_r and all the other α’s are equal to zero, the p −*median model under uncertainty* is equivalent to the $p + r$ −*median problem.*

Introducing the following groups of decision variables:

y_j^p	binary variable equal to 1 if and only if one of the p facilities is located at j;
y_j^{p+r}	binary decision variable equal to 1 if and only if a facility is located at node j when r new facilities should be located on the network;
x_{ij}^{p+r}	binary decision variable equal to 1 if and only if demand node i is assigned to a facility located at node j when $(p + r)$ facilities are located on the network;

the problem can now be formulated as follows:

$$min\ z = \sum_{r=0}^{q} \alpha_r \sum_{i=1}^{n} w_i \sum_{j=1}^{n} d_{ij}\, x_{ij}^{p+r} \tag{2.33}$$

$$\sum_{j=1}^{n} y_j^p = p \tag{2.34}$$

$$\sum_{j=1}^{n} y_j^{p+r} = r \qquad r = 1 \dots q \tag{2.35}$$

$$x_{ij}^p \leq y_j^p \qquad i,j = 1 \dots n \tag{2.36}$$

$$x_{ij}^{p+r} \leq y_j^p + y_j^{p+r} \qquad i,j = 1 \dots n;\ r = 1 \dots q \tag{2.37}$$

$$\sum_{j=1}^{n} x_{ij}^{p+r} = 1 \qquad i = 1 \dots n;\ r = 0 \dots q \tag{2.38}$$

$$x_{ij}^{p+r} \in \{0,1\} \qquad i,j = 1 \dots n;\ r = 0, 1 \dots q \tag{2.39}$$

$$y_j^p \in \{0,1\} \qquad j = 1 \dots n \tag{2.40}$$

$$y_j^{p+r} \in \{0,1\} \qquad j = 1 \dots n;\ r = 1 \dots q \tag{2.41}$$

The objective function (2.33) is to minimize the expected cost of serving all the demand nodes, taking into account that r new facilities may be located in the future, $r = 0, 1 \dots q$. In Constraints (2.34) (constraints (2.35)) we make sure that p (r new) facilities should be located. Constraints (2.36) (Constraints (2.37)) ensure that node i can be asigned to facility located at node j when p ($p + r$) facilities are located. Constraints (2.38) guarantee that each demand node is assigned to a facility for all possible numbers of facilities to be located. Constraints (2.39), (2.40), (2.41) define the binary nature of decision variables.

It has to be noticed that the proposed approach did not allow the closure of the facilities that have been located during the first round, neither the modification of their capacity.

Sonmez and Lim (2012) proposed a solution approach that can determine the initial locations and the future relocations of facilities in the case that demand is subject to change and also the number of future facilities is uncertain. The aim was to minimise the initial and expected future weighted distances without exceeding the given budget for opening and closing facilities. Also in this case, however, there is no possibility of altering the capacity of the facilities.

2.5 Ex-post Reorganization Models

In this class we introduce those models aimed at modifying the current spatial organization of a given service, in order to react to some occurred circumstances (i.e. changes in the distribution of the demand, financial need, etc.) and improve the efficiency of the system.

Wang et al. (2003) introduce a model addressing the situation in which, due to some occurred changes in the distribution of users demand, the existing facility system no longer provides adequate service and the relocation of the existing facilities in the location space is required in order to improve the accessibility of users to the service. Indicating with J_1 the set of nodes where existing facilities are located and with J_2 the set of nodes at which new facilities may be located, the facility relocation model aims at finding the sites of the existing facilities $j \in J_1$ to close and the sites for new facilities $j \in J_2$ to open so that the total weighted travel distance is minimized. As, whenever a facility is opened or closed, the decision maker incurr a cost, indicated with c_j, a constraint on the budget available for facility relocation is considered.

The model can be formulated as follows:

$$min\ z = \sum_{i \in I} a_i \sum_{j \in J} d_{ij} x_{ij} \tag{2.42}$$

$$x_{ij} \leq y_j \qquad \forall i \in, I, \forall j \in J \tag{2.43}$$

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I \tag{2.44}$$

$$\sum_{j \in J} y_j = p \tag{2.45}$$

$$\sum_{j \in J_1} c_j (1 - y_j) + \sum_{j \in J_2} c_j y_j \leq b \tag{2.46}$$

$$x_i \geq 0, y_j \in \{0,1\} \qquad \forall i \in I, j \in J \tag{2.47}$$

where $\{x_{ij}\}$ are the classical assignment variables while $\{y_j\}$ represent a new group of binary variables equal to 1 if and only if in j a facility is open (note that if $j \in J_1$, $y_j = 1$ means that facility has been kept open or that it has not been closed; while if $j \in J_2$, $y_j = 1$ means that a new facility has been opened in j).

The objective function (2.42) is expressed as total weighted distance between users and their assigned facilities. Constraints (2.43-2.44) guarantee the assignment of all the demand coming from each demand node i towards open facilities ($y_j = 1$). Constraint (2.45) define the number of desired active facilities while contraint (2.46) express the budget limitation for the relocation of facilities. In particular, the total cost incurred (l.h.s.), defined as sum of the closing costs $\left(\sum_{j \in J_1} c_j \left(1 - y_j\right)\right)$ and the opening costs $\left(\sum_{j \in J_2} c_j y_j\right)$, has to be smaller than the maximum budget b available (r.h.s). Finally, constraints (2.47) define the nature of the introduced decision variables.

Wang et al. (2003) applied the model to tackle the real problem of locating/relocating bank branches in Amherst, New York.

In 2007, ReVelle et al. address the re-organization problem from another perspective. In this case, the motivations for modifying the current facilities configuration don't lie in the change of the distribution of the demand and, therefore, in the worsening of the service accessibility, but in some occurred financial circumstances that impose to contain costs and, then, to reduce the number of operating facilities. Consequently, any action is not aimed at improving the accessibility of users to the service, but, on the contrary, the perturbation of the previous demand allocation generally causes the wosening of the service quality offered to the users.

Suppose to have a set of facilities $j \in J$ already located in a region ($|J| = m$), that provide a given service to users $i \in I$. The decision-maker has to shrink the service by closing a pre-specified number p of facilities ($p < m$). These actions will produce some side effects on the users. In particular, assuming that in the current configuration each user is allocated to its closest facility, the closure of the single facility imposes the re-allocation of its assigned demand to a farthest faciliity, with a consequent increase of costs and the worsening of the quality of the offered service. In order to take into account such damage on the users, ReVelle et al. introduced two parameters: a tolerance level β, expressed as maximum percentage of increasing of the current distance $\bar{d}_i$ of each user i from its assigned facility, over which user percieves the worsening of its condition and he can be considered damaged, and a standard distance S within which each user has necessarly to be covered.

Therefore, given that in the final configuration each user has to be covered within a standard distance S, the model aims at identifying the p facilities to be closed in order to mnimize the number of damaged users, i.e. for which the distance from its closest facility increases more than β times from its current value $\bar{\bar{d}}_i$.

Indicating with:

$M_i = \{j \in J : d_{ij} \leq (1 + \beta)\bar{\bar{d}}_i\}$ the set of facilities j that are distant from i within β times the current distance $\bar{\bar{d}}_i$ to its closest facility;

$N_i = \{j \in J: d_{ij} \leq S\}$ — the set of facilities j that are able to cover user i within the standard distance S;

x_i — binary variable equal to 1 if and only if user i is in worse condition after the closure of the facilities;

y_j — binary variable equal to 1 if and only if facility j is kept opened;

the *Planned Shrinkage Model* can be formulated as follows:

$$min\, z = \sum_{i \in I} a_i x_i \tag{2.48}$$

$$\sum_{j \in N_i} y_j \geq 1 \qquad \forall i \in I \tag{2.49}$$

$$x_i \geq 1 - \sum_{j \in M_i} y_j \qquad \forall i \in I \tag{2.50}$$

$$\sum_{j \in J} y_j = p \tag{2.51}$$

$$x_i, y_j \in \{0,1\} \qquad \forall i \in I, j \in J \tag{2.52}$$

The objective (2.48) minimizes the population that will be uncovered within a threshold level after the closing of p facilities. Thus, the first group of constraints (2.49) guarantees that all the population will be covered within the distance standard S. The second group (2.50) accounts for population loss at node i if there is no facility within$(1+\beta)\bar{d}_i$. The use of β allows for a range of *nearest* facilities to be modeled. If $\beta = 0\%$, then the set M_i will consist of the nearest facility j to node i or the nearest facilities if there is a tie. Alternatively, if $\beta = 25\%$,, as an example, then the nearest set will consist of those facilities at most 25% farther than the closest facility. Finally, constraint (2.51) sets the number of facilities to be kept opened.

Apart from the above formulation, suitable for services in the public sector, ReVelle et al. (2007) formulated another version of the *Planned Shrinkage Model*, for dealing with facilities closure problems in the private sector. In particular, the model consider firms under situations of pressing financial needs that need to cede market share to competitors.

Recently models dealing with re-organization of existing facility systems have began to appear also in the field of districting problems. In this case, the problem consists of modifying the current partition of territorial units in districts, in order to satisfy some planning criteria (*re-districting problem*). In the final configuration the number of the districts, and hence the operating facilities, may be smaller, higher or equal to the current one. The extant literature offers a good number of example of redistricting problems, most of which addressing electoral and political versions of the problem (for a complete survey, see Duque et al., 2007). In the context of service-oriented application, motivations for re-districting lie in the changes in the distribution of the units attributes (i.e. service demand, population) that may result in a non balanced district map. The expansion of cities, people migration and uneven changes of service demand in somea areas are examples of forces that pressure the redefinition of districts Therefore, they generally have the aim of improving the quality of solution with reference to some planning criteria. For instance, Eagleson et al. (2002) develop a method for school redistricting; D'Amico et al. (2002) suggest an approach for dealing with police districts redesign.

Silva de Assis et al. (2014) were the first to propose a mathematical model to address the problem of reshaping districts for energy metering.

Suppose to have a set I of territorial units already grouped in p districts, where the binary label l_{ik} indicates if the unit i is currently assigned to district k ($k = 1..p$). Each territorial unit is associated with a set of attributes A and w_i^a is the value of attribute $a \in A$ associated to the generic unit $i \in I$. The target value for each district, with reference to the generic attribute a is equal to $\mu_a = \frac{\sum_{i \in I} w_i^a}{p}$. Introducing the binary variables x_{ik}, equal to 1 if and only if unit i is assigned to district k, and y_{ijk}, equal to 1 if and only if both units i and j are assigned to district k, the model has been formulated as follows:

$$min\, z_1 = \sum_{k=1}^{p} \max_{i,j \in I}\{d_{ij} y_{ijk}\} \tag{2.53}$$

$$min\, z_2 = \sum_{k=1}^{p} \sum_{a \in A} \sum_{i \in I} |w_i^a x_{ik} - \mu_a| \tag{2.54}$$

$$y_{ijk} \leq x_{ik} \qquad \forall i,j \in I, \forall k = 1..p \tag{2.55}$$

$$y_{ijk} \leq x_{jk} \qquad \forall i,j \in I, \forall k = 1..p \tag{2.56}$$

$$y_{ijk} \geq x_{ik} + x_{jk} - 1 \qquad \forall i,j \in I, \forall k = 1..p \tag{2.57}$$

$$\sum_{k=1}^{p} x_{ik} = 1 \qquad \forall i \in I \tag{2.58}$$

$$\sum_{k=1}^{p} \sum_{i \in I} l_{ik}(1 - x_{ik}) \leq L \tag{2.59}$$

$$\sum_{i \in \cup_{v \in D} N^v / D} x_{ik} - \sum_{i \in D} x_{ik} \geq 1 - |D| \qquad \forall D \subset I, \forall k = 1..p \tag{2.60}$$

$$x_{ik}, y_{ijk} \in \{0,1\} \qquad \forall i,j \in I, \forall k = 1..p \tag{2.61}$$

The first objective function (2.53) minimizes the sum of the greatest distances between pairs of units within the same district (compactness). The second objective function (2.54) minimizes the sum of deviations of district attributes values from the related target (balance). Constraints (2.55-2.57) ensure that, for each pair of unit $(i,j) \in IxI$ and each $k = 1..p$, variable y_{ijk} is equal to 1 if and only if both units are assigned to the same district k. Constraint (2.58) assure that each unit i is assigned to a single district. Constraint (2.59) limits to L the number of units that can change district. Constraints (2.60) guarantee that all the units assigned to a district are connectd to each other. Indicated with N_i the set of territorial units adjacent to i, the contiguity property is verified as the constraints impose that any subset of units $D \subset I$ can be entirely assigned to to the same district k ($\sum_{i \in D} x_{ik} = |D|$) if and only if in its neighborhood ($\bigcup_{v \in D} N^v / D$) there is at least another unit belonging to district k ($\sum_{i \in \bigcup_{v \in D} N^v / D} x_{ik}$). Finally, constraints (2.61) define the nature of the above introduced decisions variables.

In order to solve the model, Silva de Assis et al. (2014) proposed a solution framework based on a greedy randomized adaptive search procedure and multicriteria scalarization techniques to approximate the Pareto frontier. The computational experiments show the effectiveness of the method for a set of randomly generated networks and for a real-world network extracted from the city of São Paulo.

2.6 Gaps in the extant literature

The *territorial re-organization* of an existing supply system consists of modifying the set of facilities that currently provide the service(s) in the study region, or in terms of their number and/or position, or in terms of capacities and/or offered services. For example, the action aimed at reducing (increasing) the number of available facilities is known as shrinking (expansion) action while the one aimed at changing the position of available facilities, without changing their number, is known as relocation action.

Motivations for reorganization decisions can be various. The most discussed in the literature is related to the variability of parameters during the facilities lifetime, such as demand, costs, market structure, competition. Due to this variability, decisions taken on the basis of certain conditions could reveal inefficient later and require some changing in the organization. In literature, particular attention is paid to possible occuring changes in the demand distribution within the location space.

In the above sections we classified the proposed approaches into two groups:

- *Ex-ante Re-organization models*, in which the decisions are taken before the changes have occured, on the basis of forecasting about the future conditions;
- *Ex-post Re-organization,* in which the decisions are taken once changes have already occurred.

Within the firs class, a distinction has been introduced between *Deterministic Multi-Period Models* and *Stochastic Models.* In the former the trend of parameters over time is known (deterministic) and on the basis of such conditions decisions are made period per period over an extended planning horizon; while in the last case, decisions are made in a single period on the basis of limited knowledge (probability) about the future trend of input parameters.

In the class of *ex-post re-organization decisions*, models start from an existing configuration of the facilities and aim at modifying it according to some objectives to optimize and some constraints to be satisfied.

The main gap is that most of analysed contributions deal with situations in which the re-organization problem arises because of a changing in the distribution of the demand that has made obsolete the current organization (Wang et al., 2003; Silva de Assis et al. 2014). In the context of static re-organization decisions, only the contribution by ReVelle et al. (2007) analyses the problem from a diferent perspective. The authors analyze the situation in which the current organization is the best one in terms of accessibility of users (average distance, maximum distance) and the re-organization is motivated by economic conditions that make the current system unsustainable and impose to shrink the service. If we assume that the current configuration is the best one from the users perspective, any re-organization action will damage them as it perturbs the current optimal allocation.

Therefore, in this case, the problem is completely different as, apart from potential efficiency gains for the decision maker, any re-organization action will produce some side effects, such as the increase of costs faced by users (in terms of accessibility to the service) and, potentially, the worsening of the quality of the offered service (measurable in terms of drop of coverage, worsening of users' satisfaction and congestion of the remaining facilities). Hence, to effectively solve these kinds of problems, decision support models should be able to find a trade-off solution between two inherently conflicting goals: the maximization of the benefit from the planner perspective and the minimization of the damage from the users perspective.

The model proposed by ReVelle et al. ams at identify a prespecified number p of facilities to close in order to contain as much as possible the degradation of the service level provided to the users. It has to be highlighted that the benefits for the planner is modeled in a very simple way, i.e. in terms of total number of facilities to be closed. Moreover, the demand re-allocation does not consider constraints on the capacity of the remaining facilities; the damage on the users is measured as number of users that after the re-allocation have to cover a distance (travel time) higher than a given threshold.

In order toaddress this kind of probems from a more general perspective a theoretical framework should be defined in which different rationalization actions are defined and modeled, different objective for both the different actors involved are formalized, and different models are formulated.

Another gap is that in most of the considered models, the re-organization actions implemented are the modification of the number of available facilities (service expansion vs service shrinking) and the relocation of existing faciliites in new points of the location space. Few contributions, only in multi period setting, deal with the possibility of expanding or reducing the capacity of single facilities.

Furthermore, most of the surveyed models represent facilities that are able to provide only one type of service. In various applications this assumption is not adequate. For example, in the case of public facilities (e.g., hospitals), sites often host complex structures capable of providing multiple services to users (e.g., different wards). In these cases, it may not be necessary to close the whole facility but just downsizing it by reducing the range of offered services. This also applies to production sites within a supply chain, where plants may be both entirely closed down or downsized by dismantling some existing manufacturing lines.

Finally, in the context of districting problems few contributions deal with the problem of the re-organization. Silva de Assis et al. (2014) address the problem of reshaping the boundaries of existing districts in a region in order to achieve a better balance. New versions of the problem can be defined.

2.7 Conclusions

Historically, facility location models have been a viable decision support tool for institutions and firms that plan to open new facilities in a given region. In recent years, due to the general interest to reduce costs and improve efficiency, companies and institutions have been more interested in the *re-organization* of existing supply systems, by implementing various rationalization actions. In this context, decisions may depend on various factors, such as the nature of the service, the characteristics of the market (competitive or non-competitive) and so on. In this sense, it could be useful and interesting to develop appropriate tools to support this kind of decisions.

Chapter 3

Mathematical Models
for the territorial reorganization
of a facility network in the public sector

3.1 Introduction

As introduced in the above chapters, in the field of location analysis, various models have been formulated to address decisions related to the territorial re-organization of existing services. In most cases, the decision maker is motivated by the obsolescence of the facility network with reference to a new distribution of the demand in the location space; therefore, any action is aimed at making it more efficient, i.e. as closer as possible to the new positions of users. The goal of such models generally consists of changing, within a given budget, the position and/or the number of located facilities in order to adapt the system to the occurred changes and better serve users.

We want to analyze those situations in which the re-organization is motivated by some occurred financial needs that make the system unsustainable and require the reduction of costs. Unlike the above situation, the current demand allocation could be here optimal from the users perspective; in this sense, any modification may produce a worsening of the quality of the offered service. Therefore, the problem is inherently multi-objective; hence, the planner should aim at combining conflicting goals: the need for achieving economic efficiencies and the need to minimize the discomfort or the damage caused to the users. We will refer to such re-organizational problem as *rationalization problem.*

In this chapter, we present a mathematical model to support shrinking decisions about facilities in a non-competitive context. The model considers a set of facilities already sited in a given location space, each providing different types of services, and aims at identifying the set of services and/or facilities to be closed in order to optimize the benefit (from the planner perspective) and limit the damage deriving from the shrinking process (from the users perspective). In such model two different rules have been considered in order to re-allocate the demand after the re-organization of the system. The model has been tested on a set of randomly generated instances in order to show that a good range of problems can be solved to optimality through the use of a commercial solver (CPLEX). Finally, further rationalization strategies are introduced and, accordingly, a new version of the model is proposed.

3.2 Rationalization problems in the public sector

Historically, facility location models have been a viable decision support tool for institutions and firms that plan to open new facilities or expand their capacity in a region. However, in a given supply system, some occurring circumstances could require strategies oriented to reduce costs and/or improve the system performance; therefore, actions related to the rationalization of the current facilities network may be undertaken.

In the private sector many factors may quickly change during the facilities lifetime, such as market structure, distribution of demand (and its uncertainty), presence of new competitors and financial needs. In the public sector, economic conditions may impose constraints on public expenditure that could result in policies oriented to the rationalization of services in crucial areas, such as healthcare, education, public transport. These changes could make the existing system inefficient and/or unsustainable, and require its re-organization, through shrinking and/or merging processes, in order to cut costs and increase the overall efficiency.

The rationalization actions constitute strategic decisions that have to be planned by taking into account different perspectives. Indeed, while the planner would be interested in the improvement of the performance in terms of efficiency, by identifying the set of facilities that, if closed or relocated, would optimize a certain benefit index (including, for example, cost measures for facilities operations), users will be damaged by the loss of one or more facilities, as this will increase the accessibility cost. Indeed, apart from potential efficiency gains, such processes will produce some side effects, such as the increase of costs faced by users (in terms of accessibility to the service) and, potentially, the worsening of the quality of the offered service (measurable in terms of drop of coverage, worsening of users' satisfaction and congestion of the remaining facilities).

Therefore, to effectively solve *rationalization problems*, decision support models should be able to find a trade-off solution between two inherently conflicting goals: the maximization of the benefit produced by the re-organizational process (taking into account the planner

perspective) and the minimization of the damage deriving from it (taking into account the user perspective).
In literature many models have been formulated to address similar problems; but, as underlined in Chapter 2, most of them deal with the re-organization of existing services in response to an occurred (or forecasted) change in the distribution of the demand; therefore the goal of the decision maker is to improve the quality of the service from users' perspective (single-objective problem). On the contrary, only few contributions analyze the situations in which the re-organization process is motivated by the unsustainability of the current configuration and aims at reducing costs, even by worsening the service provided to users. Hence, starting from the theoretical framework of the location analysis, this gap suggests the possibility to develop a new class of models to support decisions in these contexts, by experimenting different approaches to the problem and formulating different versions of the models, in terms of rationalization strategies to be implemented, objectives to be optimized and constraints to be satisfied. Moreover, such problems may inspire a huge number of practical applications with interesting characteristics, especially in the public sector.
Among the contributions analyzed in the above chapter, the only one that could be classified as a rationalization problem, is the *Planned Shrinkage Model*, by ReVelle et al. (2007). It considers a firm operating in a non-competitive market that has to shrink its services for economic reasons. The objective is to identify the p facilities to be closed, among the existing ones, so as to contain as much as possible the degradation of the provided service level. One of the main limits of this model consists in the indicator defined for measuring the benefit for the planner; indeed, they assume that the closure of each facility provides the same benefit and, consequently, they simply impose a constraint on the total number of facilities to be closed. Moreover, the demand re-allocation does not consider constraints on the capacity of the remaining facilities and it is performed according the nearest-reallocation rule. Finally, the damage on the users is measured as amount of demand re-allocated to a facility farther than a given threshold distance, considered as maximum acceptable worsening level.
In order to overcome these aspects, in the next section a new mathematical model for the shrinking of an existing service is introduced. The model considers a set of facilities that provide several services to users (multi-type facilities) and explores, together with the closure of whole facilities, also the possibility of closing single services. Moreover the services have limited capacities; therefore, in the re-allocation of the uncovered demand, this aspect is taken into account.

3.3 A new Mathematical Model to Shrink an existing service

3.3.1 Problem description

Suppose the presence of a given number of facilities in a location space, providing different types of services. As an example, in figure 3.1 a system in which three facilities (A,B,C), offer five type of services (1,2,3,4,5) is represented. Each facility is characterized by its own portfolio of offered services; for example, it is possible to notice that facility B offers all the services except the type 1, while facility A provides only services 1,2,5. This assumption is quite realistic; indeed, in the case of public facilities, sites often host complex structures capable of providing multiple services to users (i.e. wards of hospitals, degree programs of universities). Each service may be characterized by different capacities at the different facilities (i.e. number of beds associated to a ward of a specific hospital, maximum number of students that can enroll to a degree program of a given university).

Moreover, it is assumed that each demand node in the location space requires all the provided services. In Figure 3.1, users are represented by nodes a,b,c,d.

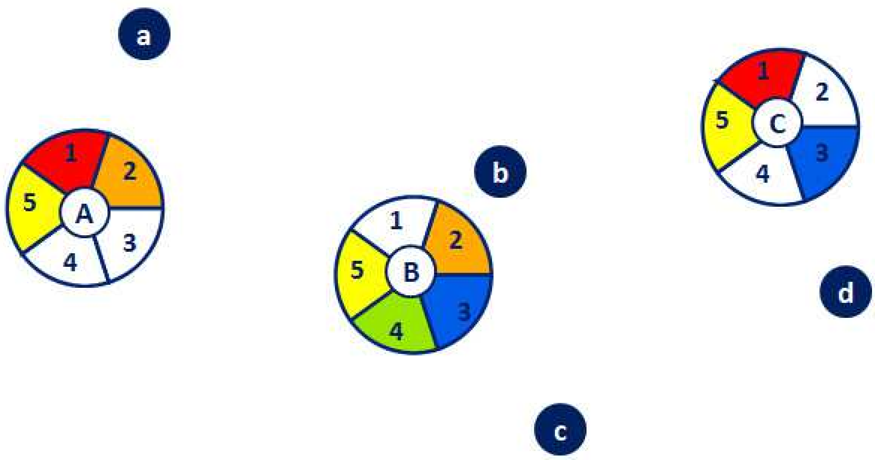
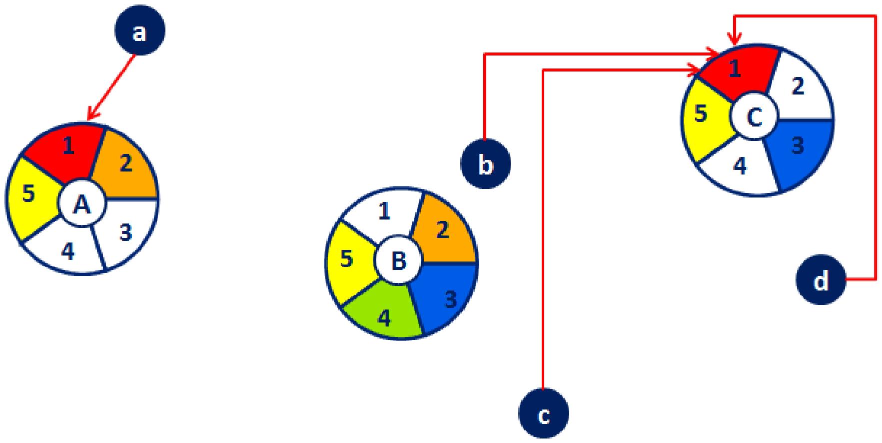
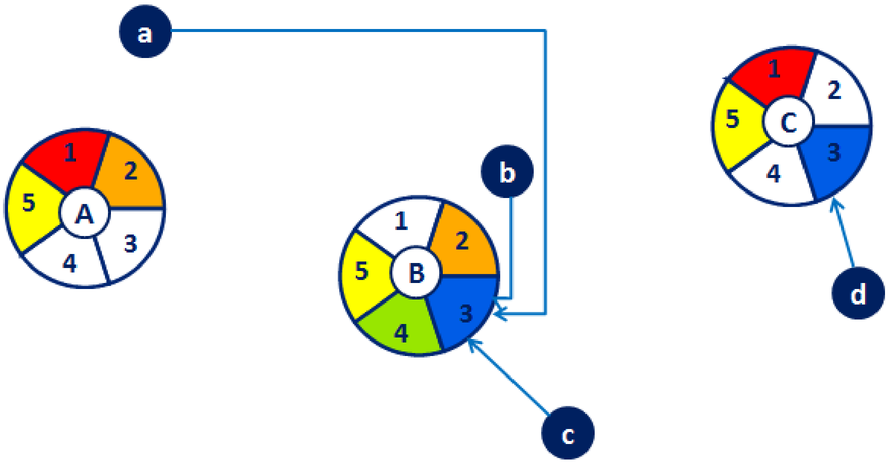


Fig. 3.1- Multi-type facility network

Now, assume that, in order to reduce costs and improve the efficiency of the whole system, the planner aims at closing some of the existing facilities or shrinking the portfolio of services offered by each of them. This rationalization process is not trivial; indeed, the closure of a service involves users that were previously assigned to it and that could then decide either to patronize another available facility offering the same service or even to renounce to the service itself. Therefore, a crucial aspect is represented by the interaction between users and facilities. The rule that every user follows to select the facility to patronize strongly depend on the type of considered service. The most widely adopted assumes that users choose the closest facility; this assumption implicitly considers all facilities being equally attractive with reference to the single service. In Figure 3.2 the case of closest assignment is shown; in particular, the distribution of users among facilities is represented for services 1 (a) and 3 (b). Note that users are assigned to the closest facility providing the required service; then, for each demand node, if the closest facility does not offer certain services, it will be assigned to a further facility to receive it. For example, in the case of demand node 'a', its closest facility 'A' does not provide services 3 and 4; therefore it patronizes 'B' to receive them.



(a)-Distribution of users requiring service 1



(b)-Distribution of users requiring service 3

Fig. 3.2- Closest allocation model

This allocation model is particular suitable, for example, in the case of emergency services. In many other real applications, it can be reasonably supposed that facilities are characterized by different attractiveness and, hence, the distance is not the only factor to be considered in the choice, as empirically proved by Bucklin (1971), Hodgson (1981), McLafferty (1988), Lowe and Sen (1996), Bruno and Improta (2008). In these cases it is possible to define (on the basis of a variety of factors and a given interaction model) the utility of a facility j for a user at node i, that can be also seen as the probability that i will select j. Coherently, the distribution of the demand among available facilities occurs according to these probabilities. This is typical, for instance, of gravity models, in which the probability is assumed proportional to the attractiveness of the facility and to a decreasing function of the distance from it (Joseph and Kuby, 2011). In Figure 3.3, an example in which users, requiring service 5, distribute among available facilities in a probabilistic fashion is shown. It is possible to notice, for example, that a significant percentage of users from node 'a' select facility B, even if it is farther than A. This probabilistic behavior characterizes, for example, the choice of students about the university to attend for a specific degree program; as proved by Bruno and Genovese (2012), they do not select only on the basis of the distance, but also on the basis of other factors, such as the prestige of the university site and the life quality of the city hosting the facility.

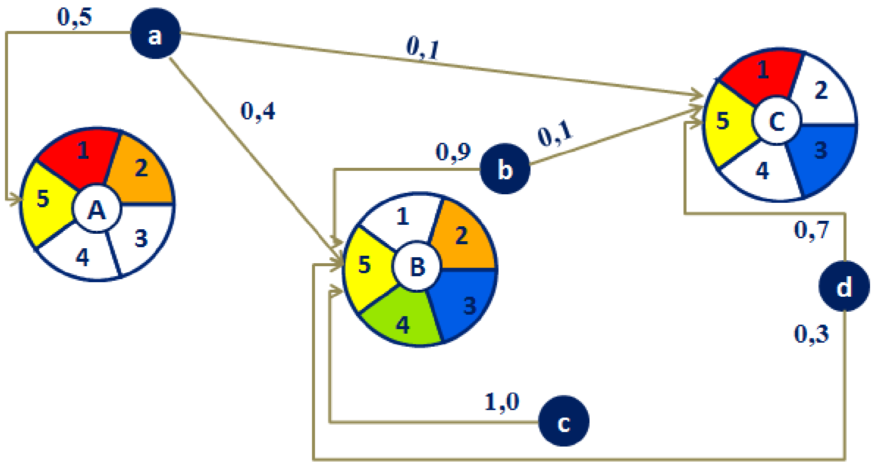


Fig. 3.3- Probabilistic allocation model

In a multi-type facility network, obviously there could be the coexistence of services with different allocation rules; in the case of hospitals, for example, the choice of patients for emergency services will be based only on the distance-factor (closest-allocation rule), while the choice for more specialized services will be also affected by other factors.

For each single service, the afference matrices associated to the two above situations, i.e. the matrices of the fractions of demand coming from each node and assigned to each facility, will be very different; in the case of the nearest-allocation rule, it will be binary (all or nothing allocation rule), while in the second case, it will be composed by elements ranging in the interval $[0,1]$.

In the problem under consideration, the analysis of the current allocation matrix is particularly important, as it could provide, for each provided service, useful information about the interaction between users and facilities and, consequently, it could suggest the mechanism to be adopted for the re-allocation of the demand after the closure of some services. Indeed, it would be coherent to assume that users continue to choose among the remaining facilities according to the same current rules.

In the proposed model, whenever a service gets closed, the assigned demand is re-allocated, on the basis of a mechanism which is coherent with the current behavior, among the facilities still providing the same service. As consequence, the planner could pay an extra-cost to expand the capacities related to the still active services, in order to satisfy the re-allocated demand after the shrinking process. Therefore, under the hypothesis that the planner wishes to achieve a certain level of benefit, a possible objective could be represented by the minimization of the total extra-capacity cost.

3.3.2 The model

With reference to a set of facilities already positioned in a location space, each providing a given set of capacitated services, the proposed model aims at identifying the set of services to be closed in order to achieve a minimum benefit for the planner and minimize the extra-capacity cost due to the reallocation of the demand among the active facilities.

In order to formulate such model, the following parameters have to be introduced:

I	the set of demand nodes, indexed by i ($\|I\| = n$);
J	the set of existing facilities, indexed by j ($\| J \| = m$);
K	the set of different types of services to be provided, indexed by k ($\|K\| = q$);
l_{kj}	the binary label equal to 1 if and only if facility j currently provides service k;
L_j	the portfolio of services provided by facility j $\left(L_j = \{k \in K : l_{kj} = 1\}\right)$;
N_j	the number of services provided by facility j $\left(\|L_j\| = N_j\right)$;
U_k	the set of facilities providing service k $\left(U_k = \{j \in J : l_{kj} = 1\}\right)$;
C_{kj}	the capacity of service k at facility j;
a_{ik}	the total demand coming from node i for service k;
d_{ij}	distance between the demand node i and the facility j;
α_j^{ik}	the fraction of demand a_{ik} currently assigned to j ($0 \leq \alpha_j^{ik} \leq 1$);
c_{kj}	the unit cost to expand the capacity of service k at facility j;

f_{kj} the benefit deriving from the closure of service k at facility j;

f_j the additional benefit deriving from the closure of the whole facility j;

B the minimum benefit to be obtained;

and the following groups of decision variables have to be defined:

s_{kj} the binary decision variable equal to 1 if and only if service k, currently provided by the facility j, gets closed;

y_j the binary decision variable equal to 1 if and only the whole facility j gets closed;

x_j^{ik} the non-negative decision variable representing the fraction of demand a_{ik} assigned to facility j after the rationalization process;

Δ_{kj} the non-negative decision variable denoting the extra-capacity needed for service k at facility j to satisfy the re-allocated demand;

According to the above notation, the model can be formulated as follows:

$$min\ z = \sum_{k \in K} \sum_{j \in J} c_{kj}\, \Delta_{kj} \tag{3.1}$$

$$s_{kj} \leq l_{kj} \qquad \forall j \in J, \forall k \in K \tag{3.2}$$

$$x_j^{ik} + s_{kj} \leq l_{kj} \qquad \forall i \in I, \forall j \in J, \forall k \in K \tag{3.3}$$

$$\sum_{j \in J} x_j^{ik} = 1 \qquad \forall i \in I, \forall k \in K \tag{3.4}$$

$$x_j^{ik} = f(\alpha_j^{ik}, d_{ij}, s_{kj}) \qquad \forall i \in I, \forall j \in J, \forall k \in K \tag{3.5}$$

$$\sum_{i \in I} a_{ik} x_j^{ik} - \Delta_{kj} \leq C_{kj} \qquad \forall j \in J, \forall k \in K \tag{3.6}$$

$$y_j - \frac{1}{N_j} \sum_{k \in K} s_{kj} \leq 0 \qquad \forall j \in J \tag{3.7}$$

$$y_j + \left(N_j - \sum_{k \in K} s_{kj} \right) \geq 1 \qquad \forall j \in J \tag{3.8}$$

$$\sum_{j \in J} \sum_{k \in L_j} f_{kj} s_{kj} + \sum_{\boldsymbol{j \in J}} f_j y_j \geq B \tag{3.9}$$

$$s_{kj} \in \{0/1\}, y_j \in \{0/1\}, x_j^{ik} \geq 0, \Delta_{kj} \geq 0 \qquad \forall i \in I, \forall j \in J, \forall k \in K \tag{3.10}$$

The objective function (3.1) represents the minimization of the total extra-capacity cost to be incurred to satisfy demand.

Constraints (3.2) ensure that a service k at facility j may be closed ($s_{kj} = 1$) if and only if it is currently offered by that facility ($l_{kj} = 1$). Constraints (3.3) impose that, for each node i, the demand for service k can be assigned only to facilities j that offered ($l_{kj} = 1$) and still offer it ($s_{kj} = 0$). Conditions (3.4) guarantee that, for each node i and service k, all the demand is covered, thanks to the contribution of facilities still providing that service. It can be noticed that this set of constraints also assures that, for each service k, there will always exist at least one facility providing it.

Conditions (3.5) rule the allocation of the demand after the closure of some existing services. The explicit expression of these conditions depends on the assumptions on the interaction model between users and facilities, which are related to the specific considered application. In the following section some formulations of this group of constraints will be proposed and described.

Constraints (3.6) indicate that, for each service k, the total demand assigned to j have to not exceed the total capacity of facility j, including the extra-capacity Δ_{kj}.

Constraints (3.7) and (3.8) define the relations between the variables s_{kj}, associated to the closure of the single services k at facilities j, and the variables y_{kj}, associated to the closure of the whole facilities. In particular, conditions (3.7) impose that, if the number of closed services at a given facility j $\left(\sum_{k\in K} s_{kj}\right)$ is lower than the total number of the provided services $\left(N_j\right)$, the facility is still open $\left(y_j = 0\right)$; while conditions (3.8) assure that, if all the services provided by a given facility j have been closed $\left(\sum_{k\in K} s_{kj} = N_j\right)$, facility j has to be closed itself $\left(y_j \geq 1\right)$.

Constraint (3.9) expresses the need for the planner to obtain a minimum benefit value B. The total benefit is calculated as the sum of the benefits related to the closure of the single services $\left(\sum_{j\in J} \sum_{k\in L_j} f_{kj} s_{kj}\right)$ and of an additional benefit achieved closing facilities as a whole $\left(\sum_{\boldsymbol{j}\in \boldsymbol{J}} f_j y_j\right)$.

Finally, constraints (3.10) define the nature of decision variables.

3.4 Rules for the Re-allocation of the demand

A special discussion is required for constraints (3.5), which drive the re-allocation of the demand after the closure of some existing services. As introduced above, the re-allocation of the demand has to be performed according rules that depend on the underlying spatial interaction model, i.e. the process by means of which users select the facility to patronize.

The current allocation matrix $\{\alpha_j^{ik}\}$ could provide very useful information about the choice model adopted by users. Therefore, starting from it, a coherent re-allocation mechanism has to be defined. In the following, we distinguish between two different situations: the first in which users select the closest facility and the second in which they distribute among available facilities according to probabilities, depending on a given utility function.

Accordingly, the following re-allocation rules have been defined:

- *Closest re-assignment rule*: if each user selects, for the generic service k, the closest facility providing it, he will continue to choose on the basis of the distance and he will be re-allocated to the closest facility still providing the required service;
- *Probabilistic re-assignment rule*: if each user chooses among the available facilities in a probabilistic fashion, on the basis of a given measure of the relative perceived utility, he will continue to choose among the remaining facilities according to the same probabilities.

3.4.1 Closest re-assignment

In literature, many formulations of Closest Assignment Constraints (CAC) have been proposed. A good review is provided by Espejo et al (2012). The need for CAC arises when the assignment of users to closest facilities is not guaranteed by the objective function, as in the case of equitable location models (Marin, 2011) and interdiction models (Church et al., 2004; Liberatore, 2011).

Supposing that in the current configuration each user patronizes the closest facility offering the required service ($\alpha_j^{ik} = 0{,}1,\ \forall i, j, k$), it is necessary to ensure that, after the closure of a service k, each uncovered user i is reallocated to the closest facility still offering k. It is possible to guarantee this mechanism by restricting the assignment variables to be integer in the group (3.10) ($x_j^{ik} \in \{0,1\}$ instead of $x_j^{ik} \geq 0$) and replacing in the model (3.1-3.10) constraints (3.5) with the following ones, obtained by reformulating the CAC proposed by Berman et al.(2009):

$$\sum_{t \in J} d_{it} x_t^{ik} + (F - d_{ij})(l_{kj} - s_{kj}) \leq F \qquad \forall i \in I, \forall j \in J, \forall k \in K \qquad (3.11)$$

where F is a very large positive number (for example, $F = max_i\{\sum_{j\in J} d_{ij}\}$).

For each service k, constraints (3.11) are trivially satisfied for all those facilities j that didn't offer k ($l_{kj} = 0 \Rightarrow s_{kj} = 0$) or don't offer it anymore ($l_{kj} = 1 \wedge s_{kj} = 1$). In all the other cases, i.e. for all those facilities still providing service k ($l_{kj} = 1 \wedge s_{kj} = 0$), they impose:

$$\sum_{t\in J} d_{it} x_t^{ik} \le d_{ij} \qquad \forall i \in I, \forall j \in J, \forall k \in K \qquad (3.12)$$

Being the assignment variables binary, for each demand node i and service k, only one term of the sum at the l.h.s. of (3.12) will be greater than zero. The inequalities hold for each facility j, if and only if i is assigned, among all those facilities still providing k, to the facility t positioned at the minimum distance $\left(t: d_{it} = min_{j\in J: l_{kj}-s_{kj}=1}\{d_{ij}\}, \forall i \in I, k \in K\right)$.

3.4.2 Probabilistic re-assignment

If currently the demand is distributed among active services in a probabilistic fashion, it is reasonable that the re-allocation mechanism will be based on the same probabilities.

In particular, it can be supposed that, for each service k, the current fraction of demand coming from i and being satisfied in j (α_j^{ik}), represents a good estimation of the probability that users from i select facility j to receive service k. After the closure of a given number of services, it would be coherent to assume that users continue to choose among the remaining facilities in a probabilistic fashion; but in this case, the initial probabilities values α_j^{ik} should be updated by taking into account that closed services cannot be patronized anymore.

According to this assumption, a first formulation of (3.5) may be represented by the not linear expression:

$$x_j^{ik} = \frac{\alpha_j^{ik}(1 - s_{kj})}{\sum_{t\in J} \alpha_t^{ik}(1 - s_{kt})} \qquad \forall i \in I, \forall k \in K, \forall j \in J \qquad (3.13)$$

First of all, these constraints impose that, for each node i, the demand for service k can be assigned only to facilities j that offered ($l_{kj} = 1$) and still offer it ($s_{kj} = 0$). Indeed, if a facility j did not offer service k ($l_{kj} = 0$) or it has been closed, no fraction of demand will be assigned to it, being respectively $\alpha_j^{ik} = 0$ and $s_{kj} = 1$. On the other hand, if facility j still offers service k ($s_{kj} = 0$), the fraction of demand from i assigned to j $\left(x_j^{ik}\right)$ is calculated by normalizing the current fraction α_j^{ik} over the sum of the fractions assigned to the other facilities still providing k. More precisely, if service k remains active at every facility $j \in U_k$, the allocation does not change $\left(x_j^{ik} = \alpha_j^{ik}\right)$, being the denominator equal to 1; on the contrary, if service k has been closed at some facilities, the denominator is lower than 1 $\left(\sum_{t\in J} \alpha_t^{ik}(1 - s_{kt}) < 1\right)$ and then the fraction allocated to each active facility is higher than the current value α_t^{ik}.

It is possible to demonstrate that conditions (3.13) are equivalent to the following groups of constraints:

$$x_j^{ik} + s_{kj} \leq l_{kj} \qquad \forall i \in I, \forall k \in K, \forall j \in J \qquad (3.3)$$

$$\sum_{j\in J} x_j^{ik} = 1 \qquad \forall i \in I, \forall k \in K \qquad (3.4)$$

$$x_j^{ik} \leq \frac{\alpha_j^{ik}}{\alpha_j^{ik}} x_t^{ik} + s_{kt} \qquad \forall i \in I, \forall k \in K, \forall j,t \in U_k: j \neq t \qquad (3.14)$$

This linearization has been obtained by adapting the procedure proposed by Aros-Vera et al. (2013) with reference to a Logit model (see Appendix A).

Then, as constraints (3.3) and (3.4) are already included in the proposed formulation, the final form of the proposed model is given by (3.1-3.10) replacing (3.5) with (3.14).

It must be highlighted that a necessary condition for the consistency of this linearization is that α_j^{ik} values have to be strictly positive $\forall i \in I, \forall k \in K, \forall j \in U_k$. This may be not considered as a restrictive assumption because it is reasonable to assume that, for each service k, the probability that users from i select a facility $j \in U_k$ is higher than zero. If the initial allocation does not satisfy this requirement, i.e. there exists at least a demand node i from which users do not select a facility $j \in U_k$, it will be sufficient to assign an arbitrary low value ε to the corresponding α_j^{ik} and to modify the other ones consequently.

3.5 Computational results

The above model, in the version with probabilistic re-allocation rule, was tested on randomly generated instances, that were obtained according to the following procedure.

Step 1: The cardinality of sets I, J, K has been assigned; in particular $|I| = 100, 200$; $|J| = 8, 10, 12$; $|K| = 5, 10, 15$. For each triplet $(|I|, |J|, |K|)$, 5 different instances have been generated and solved.

Step 2: $|I| + |J|$ points have been randomly positioned in a 100x100 square, with a uniform distribution, and the Euclidean distances d_{ij} between them are computed.

Step 3: the matrices $\{a_{ik}\}$, $\{l_{kj}\}$ and $\{\alpha_j^{ik}\}$ have been randomly generated. In particular:

- a population value p_i, generated from a Gamma distribution with parameters a=5 and b=80, has been associated to each point $i \in I$. With this setting, the probability function provides values in the range [0,1000], with a shape that reproduces a typical distribution of the population in a given area. For each node i we assumed the demand for each service k to be proportional to the population value p_i, through a factor β_k $(a_{ik} = \beta_k\, p_i)$ uniformly distributed in the range $[0.0, 0.2]$. This mechanism allowed avoiding that demands coming from the same node for the different services are very unbalanced.
- l_{kj} values have been generated according to a Bernoulli probability distribution with parameter equal to 0.3;
- In order to allocate the demand to the existing facilities, we considered a gravity model, as suggested by Joseph and Kuby (2011). In order to do this, we associated with each pair (k, j), such that $l_{kj} = 1$, an attractiveness value M_{kj}, randomly generated in the range $[1,20]$. Then, on the basis of these values and of the Euclidean distances d_{ij} between each demand node i and each facility j, we calculated the probabilities α_j^{ik}, according to the following formula:

$$\alpha_j^{ik} = \frac{u_j^{ik}\left(M_{kj}, d_{ij}\right)}{\sum_{j \in U_k} u_j^{ik}\left(M_{kj}, d_{ij}\right)}$$

where:

$$u_j^{ik}\left(M_{kj}, d_{ij}\right) = \frac{M_{kj}}{d_{ij}^{1.5}}$$

Step 4: On the basis of a_{ik} and α_j^{ik} values, the total demand A_{kj} for the specific service k at each facility j has been evaluated $\left(A_{kj} = \sum_i \alpha_j^{ik} a_{ik}\right)$. Then, for each service k, the capacities of the facilities $j \in U_k$ have been calculated on the basis of the value $A_k^* = \max_{j \in U_k}\{A_{kj}\}$. In particular, for each facility j offering k, we fixed $C_{kj} = \gamma_{kj}\, A_k^*$, assuming:

$$\gamma_{kj} = \begin{cases} 0.25 & 0.00 < \dfrac{A_{kj}}{A_k^*} \leq 0.25 \\ 0.50 & 0.25 < \dfrac{A_{kj}}{A_k^*} \leq 0.50 \\ 0.75 & 0.50 < \dfrac{A_{kj}}{A_k^*} \leq 0.75 \\ 1.00 & 0.75 < \dfrac{A_{kj}}{A_k^*} \leq 1.00 \end{cases}$$

Step 5: c_{kj} and f_{kj} values have been fixed equal to 1 for each pair (k, j), while f_j values equal to 0 for each facility j. This way B represents the minimum number of services to be closed and it is fixed as a percentage of 20% of the total number of active services $\left(\sum_{k,j} l_{kj}\right)$.

The test problems have been solved using Cplex 12.2 on an Intel Core i7 with 1.86 GHz and 4 GB of RAM. In Table 3.1 the running times (minimum, maximum, average, expressed in seconds) needed to obtain the optimal solutions are indicated. The corresponding average number of variables and constraints are also specified.

Problem Parameters			**Problem Size**		**CPU Time (Seconds)**		
$\lvert J \rvert$	$\lvert I \rvert$	$\lvert K \rvert$	**# Variables**	**# Constraints**	**Minimum**	**Average**	**Maximum**
8	100	5	4081	7981	4.67	12.58	28.32
		10	8161	17041	13.00	64.95	157.23
		15	12241	23021	48.38	122.87	237.97
8	200	5	8081	18201	21.71	51.64	93.96
		10	16161	29681	27.39	90.27	179.00
		15	24241	47081	94.33	627.61	1592.13
10	100	5	5101	10841	11.37	23.26	37.40
		10	10201	21241	18.08	597.97	2507.82
		15	15301	31641	210.18	535.48	1052.71
10	200	5	10101	22301	50.96	108.94	183.09
		10	20201	38521	52.61	488.68	1254.12
		15	30301	60101	495.44	1329.25	3251.64
12	100	5	6121	13221	17.64	40.03	67.63
		10	12241	28601	168.01	591.21	1515.92
		15	18361	40611	273.19	2147.29	4124.01
12	200	5	12121	33281	130.06	259.92	766.69
		10	24241	49761	220.55	1604.62	4672.19
		15	36361	69601	1517.38	3960.73	7399.34

Table 3.1 – Computational Results

Results show that the CPU time generally depends on the combination of the cardinality of the sets I, J, K; however, the most critical parameter appears to be $|K|$. Problems with lower values of $|K|$ (for example, 5 and 10) can be optimally solved within reasonable times while higher values of $|K|$ (for instance, 15) determine significant increases. Even more critical running times occur for the combination $(|J| = 12,\ |I| = 200,\ |K| = 15)$. Further tests performed on instances with $|J| = 12,\ |I| = 200,\ |K| = 20$ show that the solver is not always capable to obtain the optimal solution within the time limit of 3h. The dimension of the problems optimally solved appears, however, compatible with many cases of real applications.

3.6 The evolution of the proposed model: some preliminary results

In the above model, we explored the possibility to shrink a system composed by a set of facilities providing different types of services in a given region, or by closing whole facilities or by downsizing them by the closure of single services.

In the following, we analyze the possibility to transfer operating capacity among two services at the same facility. The service t that will cede part of its operating capacity will be referred as *shrinking service*, while the service k that will get adding capacity will be referred as *expanding service*. Obviously, a cost will be associated to this operation, depending on the pair of services between which the transfer occurs and on the facility that host them. It is assumed that the expansion of a service ex-novo is more expensive than any capacity transfer between services located at the same facility. This means that the *unit extracapacity cost*, i.e. the unit cost for expanding ex-novo the capacity of a service k at a facility j (c_{kj}), is higher than any *unit capacity transfer cost*, i.e. the cost for reallocating one unit of capacity from a service t provided by j toward k.

Considering this possibility, the re-allocated demand due to the shrinking process may be covered also by using the capacities available at the other services.

With this aim we introduce the following further groups of decision variables:

r_{kj-}	the binary decision variable equal to 1 if and only if service k at facility j cedes part of its capacity to another service of the same facility (*shrinking service*);
r_{kj+}	the binary decision variable equal to 1 if and only if service k at facility j gets adding capacity, or ex-novo or from another service of the same facility (*expanding service*);
$q_{t\,k}^{j}$	the non-negative decision variable representing the amount of capacity transferred from service t to service k of facility j. For each service k, the capacity re-conversion variable of one service toward itself ($t = k$) is used as artificial variable for the formulation of some logical constraints.

With this notation, the shrinking model can be modified as follows in order to take into account capacity re-allocation:

$$min\, z = \sum_{j\in J}\sum_{k\in K} c_{kj}\Delta_{kj} + \sum_{t\in K:t\neq k} g^j_{tk}\, q^j_{t\,k} \qquad (3.15)$$

$$s_{kj} + r_{kj+} + r_{kj-} \leq l_{kj} \qquad \forall j\in J, \forall k \in K \qquad (3.16)$$

$$q^j_{kk} \leq M s_{kj} \qquad \forall\, j\in J; \forall\, k\in K \qquad (3.17)$$

$$r_{kj-} \leq \sum_{t\in K:t\neq k} q^j_{k\,t} \leq C_{kj} r_{kj-} \qquad \forall\, j\ \in J\,;\ \forall\, k\in K \qquad (3.18)$$

$$q^j_{kk} + \sum_{t\in K:t\neq k} q^j_{k\,t} = C_{kj} s_{kj} \qquad \forall\, j\ \in J\,;\ \forall\, k\in K \qquad (3.19)$$

$$r_{kj+} \leq \Delta_{kj} + \sum_{t\in K:t\neq k} q^j_{tk} \leq r_{kj+} M \qquad \forall\, j\in J\,; \forall\, k,t\in L_j : k\neq t \qquad (3.20)$$

$$\sum_{i\in I} A_{ik}\, x^{ik}_j \leq C_{kj} + \left(\sum_{t\in K} q^j_{t\,k} + \Delta_{kj}\right) - \sum_{t\in K} q^j_{k\,t} \qquad \forall\, j\ \in J, \forall\, k\in K \qquad (3.21)$$

(3.3-3.5)-(3.7-3-9)

$$x^{ik}_j,\ s_{kj},\ , r_{kj+}, r_{kj-}, y_j \in \{0,1\} \qquad \forall i\in I, \forall k\in K, \forall j\in J \qquad (3.22)$$

$$q^j_{kt} \geq 0, \Delta_{kj} \geq 0, \qquad \forall\, i\ \in I\,;\ \forall\, j\in J;\ \forall\, k,t\in K \qquad (3.23)$$

The objective function (3.15) represents the total cost incurred to cover the reallocated demand, given by the total extracapacity cost ($\sum_{j\in J}\sum_{k\in K} c_{kj}\Delta_{kj}$) and the total capacity transfer cost ($\sum_{t\in K:t\neq k} g^j_{tk}\, q^j_{t\,k}$).

Constraints (3.16) assure that if a facility j provides service k, it may be closed ($s_{kj} = 1$), expanded ($r_{kj+} = 1$) or shrinked ($r_{kj-} = 1$).

Constraints (3.17-3.19) define the possible origins for the capacities re-allocations. In particular, they differentiate between closing and shrinking services; indeed, while the shrinking is here a strategy exclusively aimed at transfering capacity, the closure is not. Therefore, a service may be closed without ceding any amount of capacity while it cannot be shrinked without ceding anything. In order to guarantee this, constraints (3.17) impose that artificial variables q^j_{kk} may assume positive values only when services k gets closed ($s_{kj} = 1$). This means that, after the closure of the service k, its capacity may be transferred towards any other service t $\left(\sum_{t\in K:t\neq k} q^j_{k\,t}\right)$ provided by j or toward itself (q^j_{kk}); while in the case of shrinking service only re-allocations toward different services are possible ($q^j_{kk} = 0$).

Constraints (3.18) assure, on one hand, that if a service k at a facility j is selected as a shrinking service ($r_{kj-} = 1$), it has to cede at least one unit of capacity to any other service provided by j; moreover, the total ceding amount cannot exceed its available capacity C_{kj}. Constraints (3.19) impose that when a service k gets closed, all its capacity has to be reallocated, or toward anyother service t provided by j $\left(\sum_{t\in K:t\neq k} q^j_{k\,t}\right)$ or toward itself (q^j_{kk}). The self-re-allocation does not impose any cost; therefore, when the model will close a facility, it will tend to re-allocate all the related capacity toward itself, unless the transfer toward any service in the same facility is needed.

Constraints (3.20) assure that if a service k is an expanding facility ($r_{kj+} = 1$), it has to get at least one unit of additional capacity, either ex novo (Δ_{kj}) or from the other services $\left(\sum_{t\in K:t\neq k} q^{j}_{tk}\right)$. No limit on the maximum amount of capacity that it may accept is imposed, being M a very large number.

Constraints (3.21) impose that the total demand allocated to a service k at a facility j hasn't to exceed its total capacity C_{kj}, including any additional capacity, either introduced ex novo (Δ_{kj}) or deriving from the other services $\left(\sum_{t\in K:t\neq k} q^{j}_{tk}\right)$, and excluding any amount of capacity ceded toward the other services of the facility $\left(\sum_{t\in K} q^{j}_{k\,t}\right)$. Note that the total additional ($\Delta_{kj} + \sum_{t\in K:t\neq k} q^{j}_{tk}$) and ceded capacity $\left(\sum_{t\in K} q^{j}_{k\,t}\right)$ cannot be simultaneously positive quantities.

Constraints (3.3-3.5, 3.7-3.9) are the same introduced in the first version of the model. In particular, groups (3.3-3.5) rule the reallocation of the demand, while groups (3.7-3.9) define the possibility of closing whole facilities and impose the achievement of a minimum benefit from the rationalization process.

Finally, groups (3.22-3.23) define the nature of decision variables.

In order to understand the behavior of the model, a simple example is discussed below. In Figure 3.4 it is represented a system, composed by 3 facilities (A,B,C), providing three types of services (1,2,3) to the users present in the location space (a,b,c,d,e,f,g,h,i,l). Currently facilities B and C offer all the services, while facility A only services 1 and 3. The size of the generic node, representing facility j, is proportional to the related total capacity C_j $\left(C_j = \sum_k C_{kj}\right)$; while sectors inside it are associated to the services provided by j and the size of each single sector is proportional to the capacity of the related service k $\left(C_{kj}\right)$. In the current configuration, users are assigned to the closest facility. It could be possible that the same user is allocated to different facilities for the various services; for example, node 'a' is allocated to facility A for services 1 and 3 and to C for service 2, as A does not offer this latter. In the figure, for each user i, next to the edge pointing to the patronized facility, an indication of the service that user recieves from that facility and the amount of the related demand is provided. According to this allocation, each service k at each facility j will be characterized by a total captured demand $\left(A_{ik} = \sum_i a_{ik}\alpha^{k}_{ij}\right)$ represented in the figure as saturation level of the related sector.

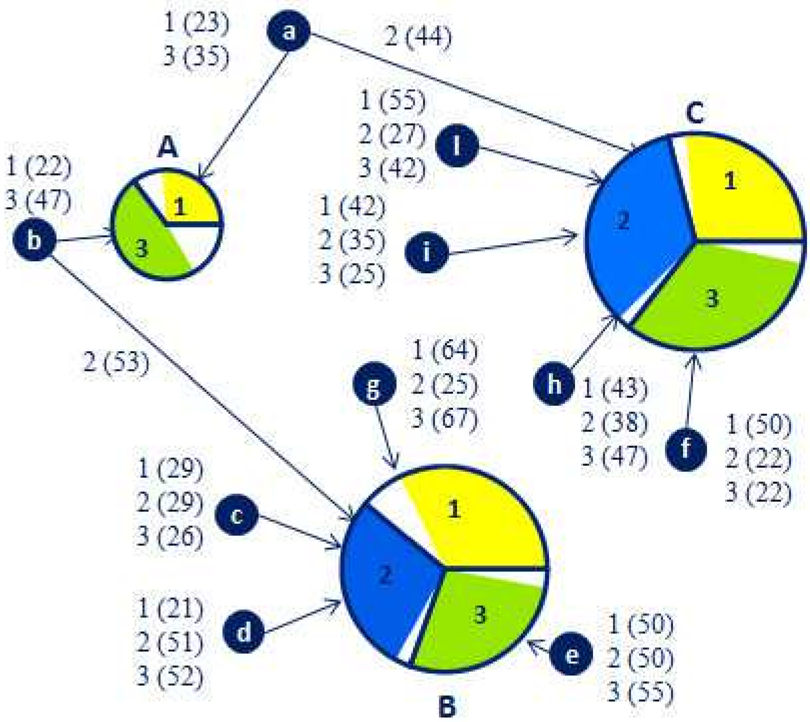


Fig.3.4 Example 1-Current situation

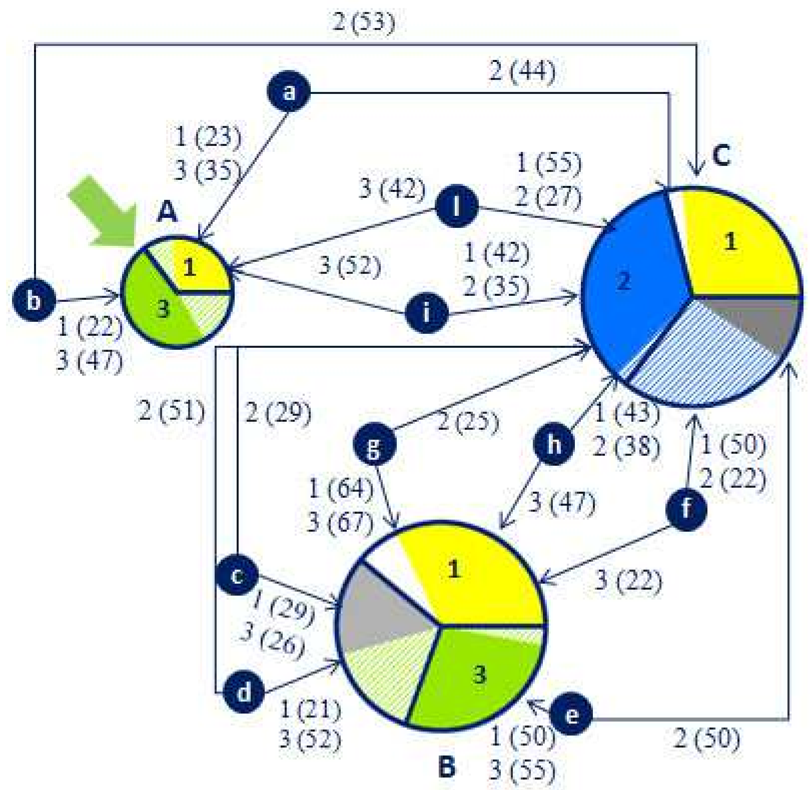


Fig.3.5 Example 1-Solution p=2

In this model, the planner has three alternatives to cover the demand re-allocated, after the closure process, toward a service k at a facility j:

- use the residual capacity of service k at facility j $\left(C_{kj} - \sum_i A_{ik}\alpha_{ij}^k\right)$;
- re-allocate capacity not used from the other services t provided by the same facility $\left(\sum_{t\in K:t\neq k} q_{tk}^j\right)$;
- expand ex-novo the capacity of service k at facility j $\left(\Delta_{kj}\right)$.

With reference to the introduced example, by imposing the closure of two services ($p = 2$), the model provides the solution represented in Figure 3.5. It closes service 2 at B and service 3 at C; accordingly, users $\{f, h, i, l\}$, that were previously allocated to facility C for all the services, now are re-allocated toward facility A e B to receive service 3. Moreover, after the closure of service 2 at facility B, C remains the only facility offering service 2; therefore all users are assigned to it to receive that service.

The services that need to be expanded to cover the re-allocated demand are service 3 at A, to cover additional demand coming from 'i' and 'l', service 3 at B, to cover re-allocated demand coming from 'h' and 'f' and, finally, service 2 at C, to cover demand coming from all users in the location space. In Figure 3.4 it is possible to notice that, service 2 at C and service 3 at B use their own residual capacities and part of tha capacities coming from the closed service at their facility. While in the case of service 3 at facility A, no closure occur; therefore, in addition to its own residual capacity, the needed demand is partly coming from service 1 (that is shrinked) and partly added ex-novo (green arrow in the figure).

In order to test this new version of the model, a new set of istances has been generated according with the procedure introduced in section 3.5. In particular, as the new set of variables and contraints introduced in the model depend on the number of services and facilities, we fixed the number of users ($n = 100$). The number of facilities and services have varied in the set $\{8,10,12\}$; for each pair $(|J|, |K|)$, 5 different instances have been generated and solved. The test problems have been solved for two different values of parameter p, equal to 10% and 20% the total number of available services.

In Table 3.2 the computational times to optimally solve the model are reported.

			CPU Time (Seconds)					
			Minimum	Maximum	Average	Minimum	Maximum	Average
$\lvert J \rvert$	$\lvert I \rvert$	$\lvert K \rvert$	p=10%*$\sum_{k,j} l_{kj}$			p=20%*$\sum_{k,j} l_{kj}$		
8	100	8	8.56	19.43	12.40	22.01	166.17	66.22
		10	9.09	87.69	32.90	27.36	147.36	78.93
		12	5.65	122.41	39.18	11.52	251.59	136.176
10	100	8	6.36	37.19	20.46	24.53	205.68	122.00
		10	6.83	55.47	21.98	23.78	457.43	337.58
		12	35.25	179.38	92.15	240.20	3766.17	1470.39
12	100	8	6.07	25.13	11.54	23.33	282.45	126.936
		10	10.82	74.05	39.586	12.91	286.80	166.532
		12	173.83	397.07	260.78	508.49	2762.67	1854.00

Table 3.2 – Computational Results

Results show that the CPU time generally depends on the combination of the cardinality of the sets J and K. In particular, the number of offered services seems to affect more than the number of facilities the computational times. Probably this is due to the fact that the capacity re-allocation options do not depend on the number of existing facilities but on the number of offered services at single facilities. Another parameter that may affect the number of re-allocation options is the number p of closed services; as higher it is, more demand will be re-allocated and more extracapacity at the remaining services will be needed. Note that by fixing the cardinalities of sets J and K, CPU times increase a lot by varying the percentage of closed services from 10% to 20%. This preliminary results suggests that the it is not possible to optimally solve by commercial solver great instances of the problem; therefore, it could be useful to develop some algorithmic approaches to the problem in order to cope with larger instances.

Finally, as concerns the model, new versions may be formulated by allowing the shrinking of the services without re-allocate any demand, the introduction of constraints about the maximum expansion allowed for existing facilities, equity constraints for the final distribution of the demand, and so on.

3.7 Conclusions

In this chapter we have proposed a new mathematical model to support territorial re-organization decisions in non-competitive contexts. The model assumes the presence of a set of facilities offering different types of services to users (multi-type facilities) and explores, together with the closure of whole facilities, also the possibility of closing single services. The re-allocation of the demand after the shrinking process may be performed according different rules (closest or probabilistic re-assignment), by taking into account that limited capacities are available at the remaining services. The objective function is represented by the extra-cost to be paid in order to satisfy the reallocated demand, while constraints expressing the need of obtaining a target benefit from the shrinkage process are included.

The model is quite general and can be adapted to a wide range of real applications, especially in the public context. Computational results show that optimal solutions can be obtained using commercial solvers for instances whose characteristic and sizes can be also representative of real problems.

Further rationalization strategies are introduced and, accordingly, a new version of the model has been proposed. With reference to this latter, some preliminary results are presented and directions for future research are drawn.

In the next chapter, two applications related to different real-world problems are discussed.

Chapter 4

Two real-world applications of territorial reorganization problems

4.1 Introduction

In a general economic and political context characterized by growing cuts to public expenditure and an overall review process of the welfare state, public services have and are still undergoing significant transformations, generally oriented to reduce administrative, managerial and operational burden and costs. In such processes, also the territorial re-organization of existing service facilities may be required, through shrinking and/or merging actions. In Italy, for example, in the last years there has been a huge number of reform proposals aimed at regulating the re-organization of services in crucial sectors, such as healthcare, education, justice and so. In order to take such decisions, central and local authorities are interested in finding solutions which provide a good balance between efficiency purposes and "public interest" (service accessibility).

The need to produce and compare solutions characterized by measurable indicators, along with the complexity of the problem, in terms of different objectives to be taken into account and constraints to be satisfied, suggest the need for developing appropriate methodologies for decision support.

With this aim, in this chapter we want to show how rationalization models could provide useful information to support the decision making processes in these contexts..

In particular, two applications related to different real-world problems are illustrated and solved. The first concerns the shrinking of a public university system on a regional scale and the second is related to the re-organization of a school system located in a given study region.

In both cases, the models have been solved considering real case studies and the results are shown and discussed.

4.2 The rationalization of a University system on a regional scale

In the last decades a general growth of higher education demand occurred in industrialized countries (Craig 1981; Robinson and Ralph 1984; Garnier and Hage 1991), as a consequence of the increase in perceived value of education (Ramirez and Boli 1987) and of a wide expansion of jobs requiring higher skills (Walters 1984). In this context, European authorities promoted a dynamic process which aimed at the definition of a certain number of policy decisions in order to establish a European Higher Education Area by 2010. The most important result of this process was the definition of a common framework for degree qualifications (Bachelor and Master Level), in order to make the different higher educational systems more comparable.

In Italy, the main consequence of this reorganizational process was the strong increase in the number of academic sites and degree courses. From 1995 to 2009 the total number of institutions rose from 60 to 86; in the same period, while the number of cities hosting main academic sites increased from 45 to 57, the number of cities hosting detached university sites (off-main campus) doubled, increasing from 93 to 185. In Figures 4.1, the old and the new distributions within the country are shown.

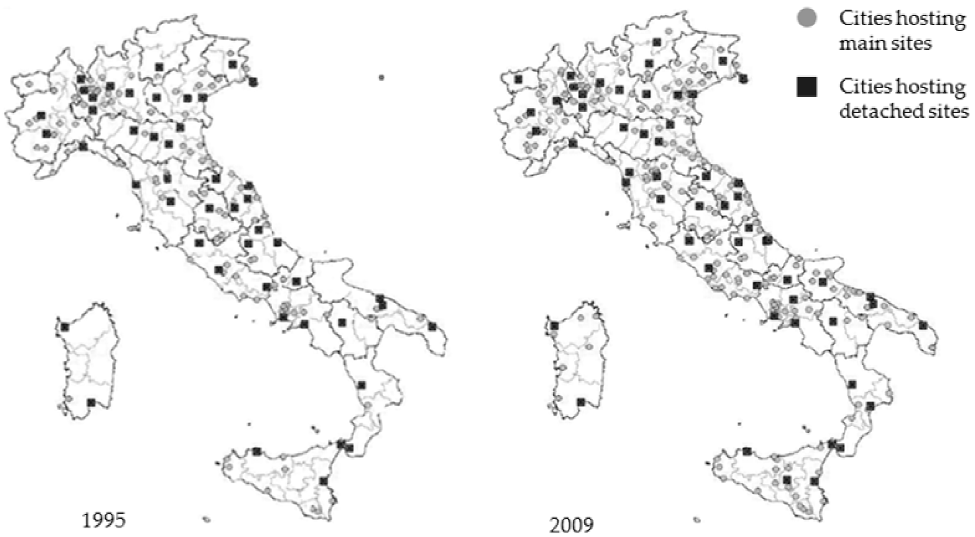


Fig.4.1 – Cities with main campus and detached sites.
A comparison between 1995 and 2009

However, some recent analyses (performed by the Italian Ministry of University; see, for instance, CNVSU, 2011) reveale that the growth of the supply have produced an inefficient system characterized by a high percentage of degree programs attracting demand levels much lower than target values fixed by central government and a considerable number of detached academic sites, hosting a low number of degree courses (Figure 4.2).

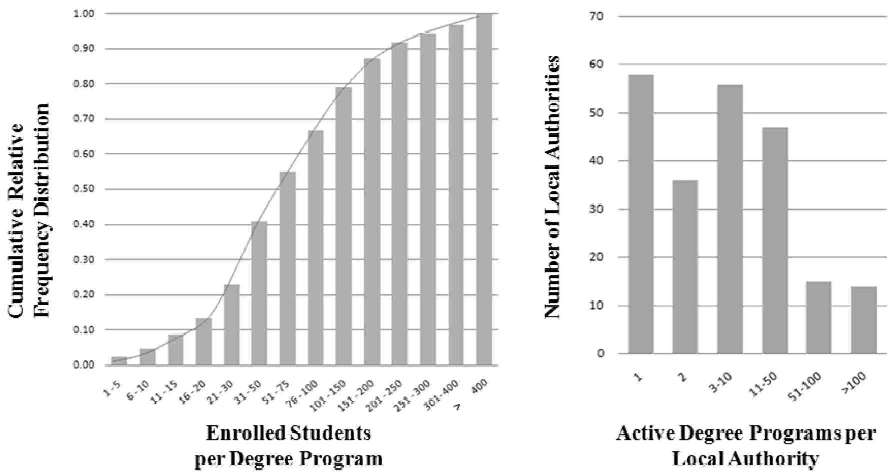


Fig.4.2- Italian Degree Programs

For these reasons, in a general context characterized by policies oriented to reduce public expenditure, the current system has been considered unsustainable. Therefore, re-organization strategies with the objective of rationalizing the supply system have to be implemented. Considering that the mobility of students across Italian regions is quite low (Bruno and Genovese, 2012), rationalization strategies may be defined at a regional level.

In this context, we apply the proposed shrinking model to analyze an Italian regional university system. We consider the second most populated region in Italy (Campania), hosting 7 public universities, with 5 Engineering Faculties. These faculties offer a wide range of degree programs, which can be classified into 9 different groups, and attract a very large number of students.

In order to evaluate the potentialities of the proposed models to provide decision support in such context, it has been adapted to address the specific real-world problem under analysis. In the next section the characteristics of such real instance are shown and the results provided by the model are shown.

4.3 The case of the Engineering Faculties of Campania Region

With reference to an existing universitary system, composed by a set of faculties (the facilities) offering various degree programs (the different services) to the students of the region (users), we supposed the presence of a decision-maker (for instance the Regional Government) interested in rationalizing the whole system by shutting down some services in order to find solutions which provide a good balance between efficiency purposes and "public interest" (service accessibility).

Therefore, the problem can be effectively described by the general shrinking model presented in chapter 3, which may be adapted as illustrated in the following.

With reference to the current organization of the system, the following parameters are considered:

J	the set of the five nodes where the 5 faculties are located;
K	the set of different types of degree programs offered by the existing faculties;
l_{kj}	the binary label equal to 1 if and only if faculty j currently provides degree program k;
L_j	the set of degree programs provided by faculty j;
U_k	the set of faculties providing degree program k;
C_{kj}	the maximum capacity of degree program k at faculty j;

In Figure 4.3 the position of the five faculties in the location space is represented (NA, PT, SUN, BN, SA), while in Table 4.1 the 9 classes of provided degree programs are listed and the indication of the presence of the single program at each faculty is reported. Note that, reading the table by rows, it is possible to obtain, for each single service k, the set of faculties providing it (U_k); while reading it by coloumns, it is possible to desume, for each single faculty j, the set of provided programs (L_j).

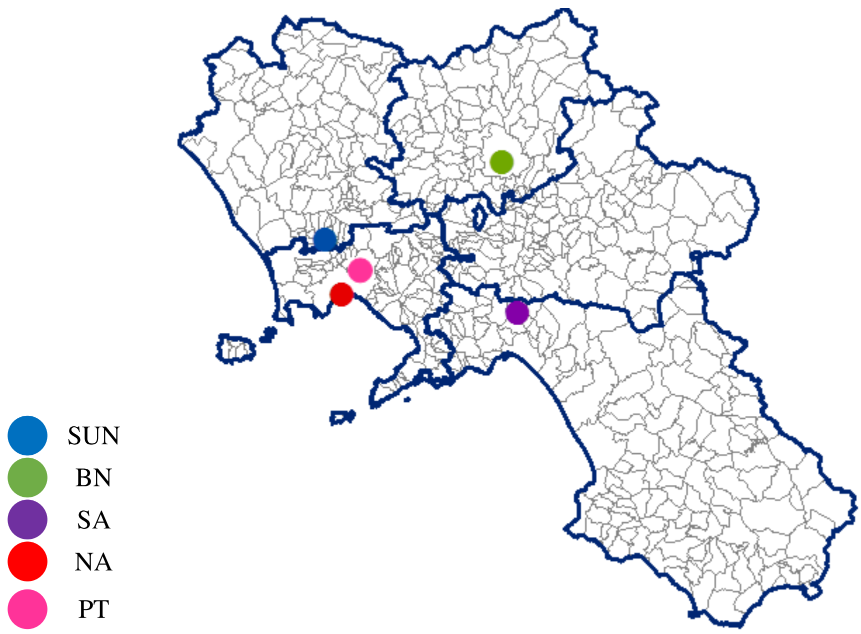


Fig.4.3- Engineering Faculties within Campania Region

		NA	*SUN*	*PT*	*SA*	*BN*	*Total*
1	***Civil***	1	1	1	1	1	**5**
2	***Environmental***	1	1	1	1	-	**4**
3	***IT***	1	1	-	1	1	**4**
4	***Electronic***	1	1	-	1	-	**3**
5	***Telecommunication***	1	-	1	-	1	**3**
6	***Aerospace***	1	1	-	-	-	**2**
7	***Mechanical***	1	1	-	1	-	**3**
8	***Chemical***	1	-	-	1	-	**2**
9	***Management***	1	-	1	1	-	**3**
	Total	**9**	**6**	**4**	**7**	**3**	**29**

Table 4.1- Degree Programs offered by Engineering Faculties within Campania Region

The demands for the 9 different classes of services, i.e. students that require to enroll to the single degree programs, are considered according to their residence address. The distribution of the demand in the location space is continue, as theoretically students may come from any point of the region. In order to aggregate demand data in a manageable way, a discretization of the location space is adopted, as introduced by Bruno and Improta (2008). This way, Campania Region has been divided into 58 internal zone and a further zone (59), representing the rest of the world outside the region, is considered (Figure 4.4).

Accordingly, in the following we will denote with:

I — the set of demand nodes coinciding with the 59 zones;

a_{ik} — the total number of enrollments coming from zone i for the degree program k;

α_j^{ik} — the fraction of students (a_{ik}) coming from zone i requiring the degree program k and enrolled at faculty j ($0 \leq \alpha_j^{ik} \leq 1$).

Fig. 4.4 – Zoning System of the Campania Region

As concern demand data, values for a_{ik} and α_j^{ik} were derived from official data about enrollments in the academic year 2008-2009; in Table 4.2, for each degree program k and each faculty j, the total number of enrollments $\left(A_{kj} = \sum_i \alpha_j^{ik} a_{ik}\right)$ and the related capacity $\left(C_{kj}\right)$ (in terms of maximum number of students) are shown. In particular, capacities are defined in multiples of 150, according to the requirements of the Italian Ministry of University. There is also the possibility of considering this capacity by merging similar degree programs, as shown in Table 4.2.

		NA		*SUN*		*PT*		*SA*		*BN*		*Total*	
		Enr.	*Cap.*	*Enr.*	*Cap.*	*Enr.*	*Cap.*	*Enr.*	*Cap.*	*Enr.*	*Cap.*	*Enr.*	*Cap.*
1	*Civil*	156	300	147	200	89	100[3]	154	300	77	150	**623**	**1050**
2	*Environmental*	49	150	32	100	20	50[3]	62	150	-	-	**163**	**450**
3	*IT*	318	450	80	100[1]	-	-	143	300	97	150	**638**	**1000**
4	*Electronic*	107	300	35	50[1]	-	-	82	150	-	-	**224**	**500**
5	*Telecommunication*	77	150	-	-	35	150	-	-	16	150	**128**	**450**
6	*Aerospace*	221	450	30	50[2]	-	-	-	-	-	-	**251**	**500**
7	*Mechanical*	442	600	66	100[2]	-	-	131	150	-	-	**639**	**850**
8	*Chemical*	135	300	-	N/A	-	-	80	150	-	-	**215**	**450**
9	*Management*	401	450	-	N/A	29	150	159	300	-	-	**589**	**900**
	Total	**1906**	**3150**	**390**	**650**	**173**	**450**	**811**	**1500**	**190**	**450**	**3423**	**6150**

Table 4.2: Enrollments to Engineering Degree Programs offered by Campania Universities

For each degree program k, the distribution of students, among the faculties providing it, is represented through five maps, one for each faculty. From the map associated to the generic faculty j, it is possible to desume the percentage of students attracted from each zone. In Figure 4.5, it is shown such representation for the specific degree program of 'Civil Engineering'. It is possible to notice that the distribution of students is not regulated by a closest assignment rule. Indeed, as proved by Bruno et al. (2012), who formulated a model to represent the spatial distribution of university students over the Italian territory, their choice is based not only on the basis of the distance but also of other attractiveness factors such as the prestige of the university sites and the socio-economic status of the city hosting it.

Therefore, on the basis of this evidence, we decided to include in the model a group of constraints that impose a probabilistic re-assignment, i.e. proportional to the initial allocation probabilities.

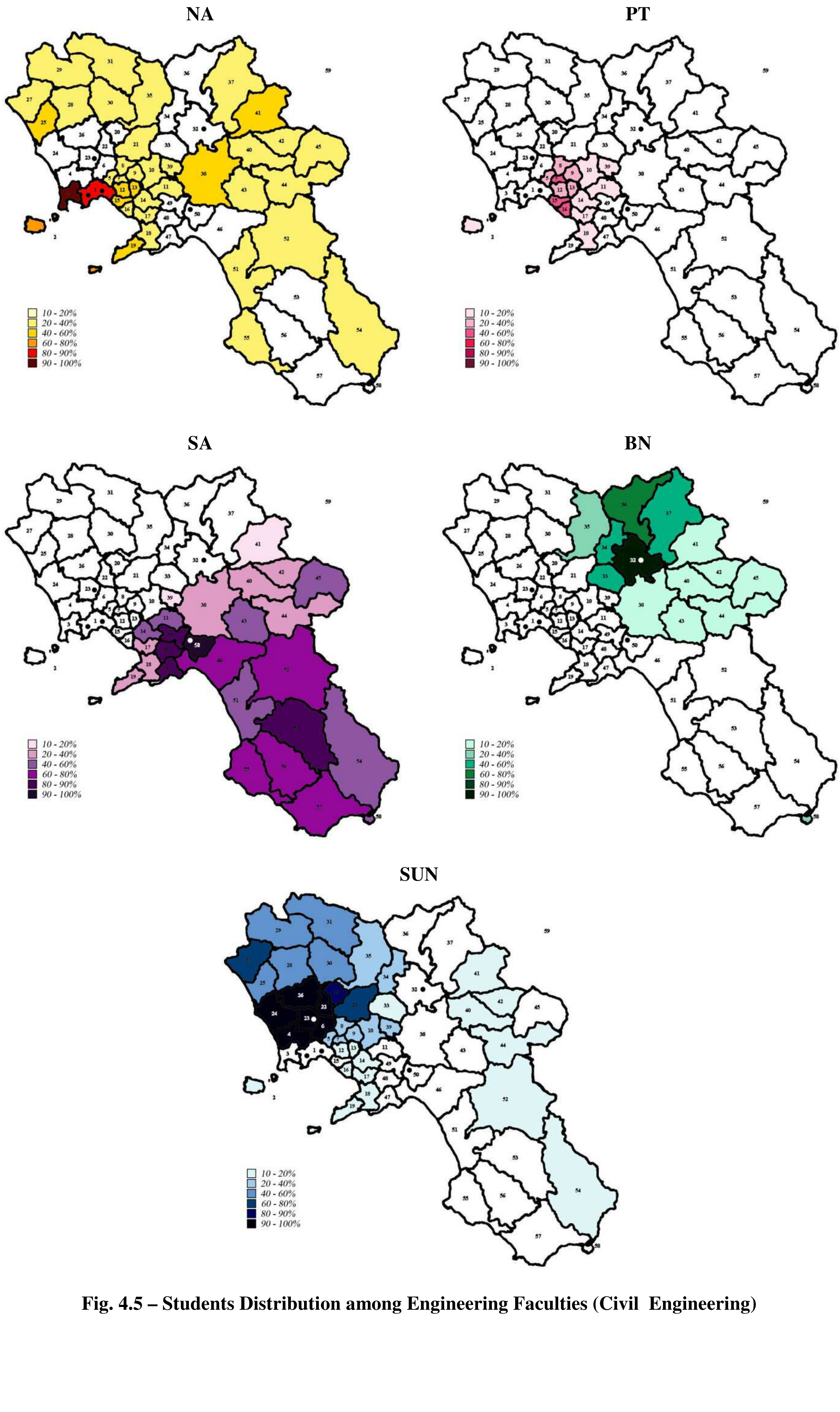


Fig. 4.5 – Students Distribution among Engineering Faculties (Civil Engineering)

Finally, as concerns the cost parameter, i.e.:

c_{kj} the unit cost to expand the capacity of degree program k at faculty j;

f_{kj} the benefit deriving from the closure of degree program k at faculty j;

f_j the additional benefit deriving from the closure of the whole faculty j;

B the minimum benefit to be obtained;

we assumed f_{kj} equal to 1 for any pair (k,j) and $f_j = 0$ for each faculty j, as a thorough estimation of these benefit parameters would require much deeper analysis and significant efforts. Then we also assumed c_{kj} equal to 1 for any pair (k,j).

With this adaptation, the model that has been applied to the problem under investigation has taken the following form:

$$min\, z = \sum_{k \in K} \sum_{j \in J} \Delta_{kj} \tag{4.1}$$

$$s_{kj} \leq l_{kj} \qquad \forall j \in J, \forall k \in K \tag{4.2}$$

$$x_j^{ik} + s_{kj} \leq l_{kj} \qquad \forall i \in I, \forall j \in J, \forall k \in K \tag{4.3}$$

$$\sum_{j \in J} x_j^{ik} = 1 \qquad \forall i \in I, \forall k \in K \tag{4.4}$$

$$x_j^{ik} \leq \frac{\alpha_j^{ik}}{\alpha_j^{ik}} x_t^{ik} + s_{kt} \qquad \forall i \in I, \forall k \in K, \forall j,t \in U_k : j \neq t \tag{4.5}$$

$$\sum_{i \in I} a_{ik} x_j^{ik} - \Delta_{kj} \leq C_{kj} \qquad \forall j \in J, \forall k \in K \tag{4.6}$$

$$\sum_{j \in J} \sum_{k \in L_j} f_{kj} s_{kj} \geq B \tag{4.7}$$

$$s_{kj} \in \{0/1\},\ x_j^{ik} \geq 0, \Delta_{kj} \geq 0 \qquad \forall i \in I, \forall j \in J, \forall k \in K \tag{4.8}$$

The main modifications concern: the objective function, that here indicates the total extra-capacity units needed to satisfy the re-allocated demand; and the group of constraints (4.7) as, on the basis of the choice of benefit parameters, the l.h.s. of the inequalities here represent the total number of closed services, instead of the total benefit achieved by the planner.

In order to evaluate the efficiency of the system over this number, we considered (4.7) as an equality constraint and we fixed the number of services to be closed (B) in a range from 1 to the maximum feasible value, equal to 20. In particular, in order to satisfy the group of constraints (4.4) which ensure the availability of at least one service of each type, the maximum number of services that could be closed is given by the difference between the total number of active services $\left(\sum_{k,j} l_{kj} = 29\right)$ and the number of different service types provided to users ($|K| = 9$).

Figure 4.6a illustrates the variation of the objective function, representing the required extra-capacity, while Figure 4.6b shows the overall degree of saturation, over the value of B. This indicator has been calculated as the ratio between the total demand $\left(\sum_{i,k} a_{ik}\right)$ and the total available capacity, i.e. the total capacity of active services plus the activated extra-capacities $\left(\sum_{j,k} C_{kj}\left(l_{kj} - s_{kj}\right) + \Delta_{kj}\right)$. Comparing these figures, it emerges that the closure of a significant number of services (B ≤11) can be performed with no investment in terms of extra-capacity. This aspect can be interpreted as a signal of the current inefficiency of the overall system, that seems to be presenting a significant level of redundancy in terms of the offered services. This fact is confirmed by the current low value of the capacity saturation level (equal to 3423/6150=0.54, Figure 4.6b).

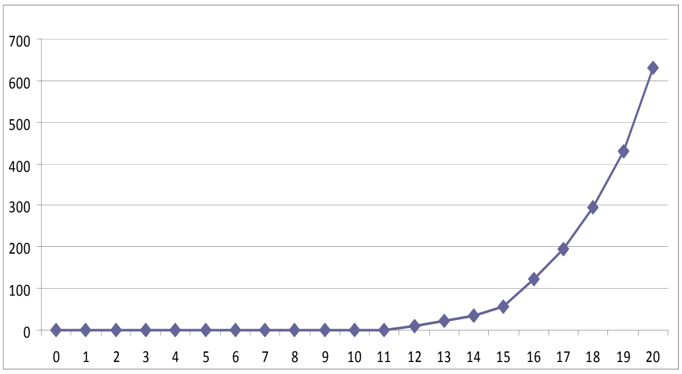


Fig. 4.6 (a) – Objective Function by varying *B* value

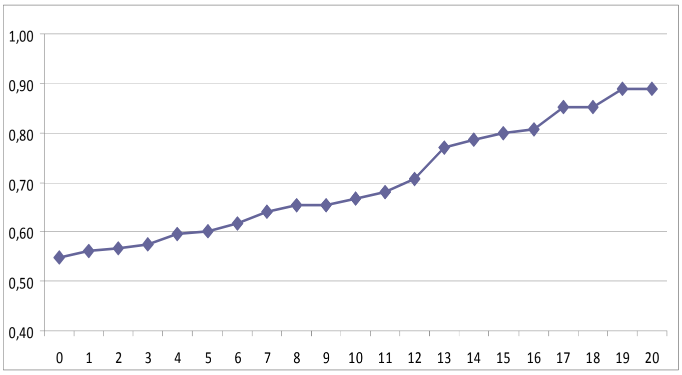


Fig. 4.6 (b) – Overall degree of saturation by varying *B* value

The reduction of the number of active services (with no activation of extra-capacity) allows the improvement of the efficiency of the system, obtaining a maximum degree of capacity saturation of about 0.70. In order to reach better efficiency levels it is necessary to invest in extra-capacity for $B \geq 12$. However, with higher numbers of closed services, as the system gets more and more congested, even large increases in extra-capacity produce very limited improvements in the saturation degree (for instance, $B \geq 16$). The maximum feasible value for B is 20 due to the fact that the model imposes that there must be active at least one service of each type.

As an illustrative example, in Table 4.3 we show the results obtained for $B = 11$. In particular the table provides, for each degree program at each faculty, the degree of capacity saturation (i.e. the ratio between enrollments and capacity) before and after the closure of the 11 services (marked, in the table, with the N/A value for the degree of capacity saturation meaning that the value cannot be calculated as the service is now closed).

		NA		*SUN*		*PT*		*SA*		*BN*	
		Before	**After**	**Before**	**After**	**Before**	**After**	**Before**	**After**	**Before**	**After**
1	***Civil***	0.77	0.88	089	0.97	0.61	N/A	0.42	0.45	0.34	0.35
2	***Environmental***	0.64	0.77	0.52	0.57	0.57	N/A	0.35	0.37	-	-
3	***IT***	0.74	0.95	0.99	N/A	-	-	0.29	0.30	0.28	0.31
4	***Electronic***	0.33	0.59	0.83	N/A	-	-	0.23	N/A	-	-
5	***Telecommunication***	0.61	N/A	-	-	0.15	N/A	-	-	0.08	0.84
6	***Aerospace***	0.43	0.53	0.93	N/A	-	-	-	-	-	-
7	***Mechanical***	0.73	0.88	0.91	N/A	-	-	0.60	0.61	-	-
8	***Chemical***	0.66	0.82	-	-	-	-	0.31	N/A	-	-
9	***Management***	0.93	0.99	-	-	0.21	N/A	0.25	0.25	-	-
	Total	0.67	0.77	0.85	0.42	0.32	N/A	0.34	0.30	0.23	0.50

Table 4.3 – Degree of capacity saturation before and after the closure of *B* = 11 services

It can be noticed that the model tends to close services with lower enrollment rates, as re-allocating these demand portions is the least costly option. In particular this solution is characterized by the closure of all the degree programs offered by the Engineering Faculty at Parthenope University.

The information provided by the model are very interesting as they allow to evaluate the performance of the system according to a set of possible choices, in terms of closed services, and compare all the produced scenarios in terms of measurable indicators. Therefore, this evidence suggests the adoption of such approaches to support decisions in similar contexts.

4.4 The problem of the reorganization of a school system

In the current economic climate, characterized by growing cuts to public expenditure, public services (e.g., healthcare, education, policing) have undergone significant transformations (Sancton, 2000). In this context, Italy has been interested by a progressive merging process of educational institutions. Indeed, while the highly centralized system originally allowed the functioning of diversified institutions on the basis of the offered educational level (for instance, kindergartens, primary schools, junior secondary schools), recently a new regulation has been adopted, oriented at promoting a higher degree of autonomy of educational institutions. However, in order to benefit from autonomy, schools have to comply with a series of requirements. In particular they must have a students' population between 500 and 900 units that has to be demonstrated stable in this range for the last 5 years. If these requirements are not satisfied, schools must merge themselves with other institutions (belonging to any of the three categories) in order to form clusters that should have a minimum students' population of 1000 units and include institutions from each educational level. This merging strategy allows schools rationalizing administrative and management offices, coherently to the cited policies of reduction of public expenditure. In practice, this process should be implemented by grouping schools in clusters, and letting each cluster being managed (in a centralized way) through the definition of a single cluster centre, providing shared administrative and managerial services. In this context, the availability of tools and models to assess and implement clustering (for grouping schools together) and locational choices (for locating clusters centers) could be useful for Local Authorities engaged in decision making activities that may provide cost savings and minimize, at the same time, the worsening of the service level for users.

In the operational research literature, several authors studied problems related to the organization of school systems; a typical problem concerns the so-called school districting, i.e. the partitioning of the demand coming from a given region in groups of students attending each school. In this problems school and class capacity constraints must be satisfied, various social objectives have to be achieved (for instance, racial balance) and some territorial aspects related to the contiguity of districts have to be considered to allow students from the same neighborhood to be assigned to the same school. The problem also occurs when reorganization actions have to be planned, such as the opening or the closing of a school, the modifying of the capacities of existing schools. In these cases, a perturbation of the previous demand allocation occurs, therefore the districts have to be redesigned considering a potential

worsening of the accessibility of users to the service. In this context many models and methods have been defined and applied (see, for instance: Ploughman et al., 1968; Holloway et al., 1975; Brown, 1987; Lemberg and Church, 2000; Caro et al., 2004).
However, compared to classical school districting cases, the reorganizational problem faced by Italian institutions can be considered a more strategic problem, as it involves the organizational structure of the school system. Indeed, while the first class of problems concerns the assignment of students to schools (existing or to be located) in order to minimize a certain cost or distance function, in the second problem existing schools, with the related students' population, have to be grouped together under a shared management centre, in order to improve the efficiency in terms of operating costs. These problems can be addressed by using adaptation of Facility Location models. Linking to this body of literature, we present a location model aimed at describing and solving the school clustering problem as defined in the Italian case.

The model is oriented to identify, within a given location space, the set of school facilities that, if merged together in a cluster (and, therefore, sharing management and administration service), could improve the efficiency of the system; moreover, the model should also identify the best location for the cluster centers. While the reorganization of the system represents an opportunity for the planner to reduce costs, it may generally produce a detriment of the service quality offered to the users. For this reason, through the imposition of appropriate constraints, the model must provide solutions that represent a good trade-off between the goal of the decision maker and the need of the users. The proposed model has been tested on a real-world case and the obtained results have been shown and commented.

4.5 A Mathematical Model for the School Clustering problem

As mentioned above, the reorganizational process of the school system (i.e. the grouping of schools in clusters) is aimed at reducing management costs, as each cluster will be managed in a centralized way through the definition of a single cluster centre. In this case, the current position of schools is assumed to be fixed and no demand re-allocation will occur. Therefore the reorganization will not have a direct effect on users' accessibility to the service. However, if we consider only efficiency aspects, the solution provided by the model could be considered not desirable (or equitable) from users perspective (Marsh and Shilling, 1994; Eiselt and Laporte, 1995). For example, the dimension of clusters is an important factor to be taken into account, as over-dimensioned clusters could have some side effects on the complexity of the managerial structure and, therefore, on the service level offered to users. For this reason, the planner should find a trade-off solution between the need to minimize costs and the need of keeping the discomfort caused to users below a given threshold. The problem is inherently multi-objective; however, it can be also modeled by means of a single-objective mathematical programming formulation, in which one of the objectives is included in the model as constraint. In order to avoid mentioned organizational inefficiency, aspects related to the location, composition and dimension of each cluster have to be considered. In particular:

- *location,* concerns the position of the cluster's centre, to be chosen among schools assigned to the same cluster;
- *composition*, concerns the type of schools to be included in each cluster;
- *dimension*, is related to the students' population of each cluster. In addition to the minimum threshold required by governmental regulations (1000 students), a limit on the maximum dimension could be taken into account, in order to obtain more balanced solutions.

If we assume to define the number p of clusters to be created, the problem consists of identifying the best position to assign to the clusters' centres and in the allocation of schools to each cluster.

Denoting with:

I	the set of nodes corresponding to the positions of each school;
J	the set of potential locations for clusters' centres ($J \subseteq I$);
K	the set of school types or levels ($K = \{1,2,3\}$, with $k = 1$ identifying the pre-primary level, $k = 2$ the primary level and $k = 3$ the lower secondary level);
l_{ik}	a binary label equal to 1 if and only if node i hosts schools of type k;
a_{ik}	the number of students of school type k at the school in i;
d_{ij}	the distance between nodes i and j;
y_j	a binary variable equal to 1 if and only if node j is a cluster's centre;
x_{ij}	a binary variable equal to 1 if and only if node i is assigned to the cluster with centre in j;

we can formulate the following mathematical model (sizing model):

$$min \;\; z = \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \tag{4.9}$$

$$x_{ij} \leq y_j \qquad \forall i \in I, \forall j \in J \tag{4.10}$$

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I \tag{4.11}$$

$$\sum_{j \in J} y_j = p \tag{4.12}$$

$$\sum_{i \in I} l_{ik} x_{ij} \geq y_j \qquad \forall\, j \in J, \forall k \in K \tag{4.13}$$

$$\sum_{i \in I} \sum_{k \in K} a_{ik} x_{ij} \geq N_{min} y_j \qquad \forall j \in J \tag{4.14}$$

$$y_j \in \{0/1\}; x_{ij} \in \{0/1\} \qquad \forall i \in I, \forall j \in J \tag{4.15}$$

The objective function (4.9) represents the average distance between schools and their assigned cluster's centre, to be minimized. This objective represents one of the classical compactness measure used in the literature related to districting and clustering models. Constraints (4.10), (4.11), (4.12) are classical $p-$median constraints, ensuring that: (4.10) node i is assigned to j only if node j is a cluster's centre; (4.11) each node i is assigned to only one cluster; (4.12) the number of clusters is equal to p. Conditions (4.13) impose that in each cluster j there is at least one school of each level k. Conditions (4.14) ensure that each cluster j has a students' population higher than N_{min}. Constraints (4.15) define the nature of decision variables.

It should be highlight that the presence of constraints (4.14) could lead to solutions characterized by a very skewed distribution of students' population among the produced clusters. In order to take into account balancing objectives, an additional set of constraints on the maximum population for each cluster has to be considered as follows:

$$\sum_{i \in I} \sum_{k \in K} a_{ik} x_{ij} \leq N_{max} \qquad \forall j \in J \tag{4.16}$$

The introduction of constraints (4.16) would limit the maximum dimension of a cluster and produce more balanced solutions with higher values of objective function. In order to support the decision maker in the choice of trade-off solutions between the objective function and balancing aspects, it could be possible to perform a sensitivity analysis in terms of N_{max}, by varying this parameter between a lower bound $LB_{N_{max}}$ and an upper bound $UB_{N_{max}}$. Indeed, decreasing N_{max}, it is possible to evaluate the trade-off between the objective function (expressing a rationalization need) and the balancing constraint.

As regards the upper bound $UB_{N_{max}}$, it is represented, for each value p, by the maximum dimension of clusters obtained solving the model (4.9-4.15); while an ideal lower bound can be assumed equal to the average students population for each group ($\bar{a} = \frac{1}{p}\sum_i \sum_k a_{ik}$). However, this value does not take into account the integrity constraints imposing that the population of each school has to be entirely assigned to the same cluster and the constraints on the composition of the groups. In order to calculate a more reliable lower bound value, a partitioning model was devised. Using the notation introduced above, the model can be formulated as follows:

$$min \quad z = N_{max} \tag{4.17}$$

$$\sum_{j=1}^{p} x_{ij} = 1 \qquad \forall i \in I \tag{4.18}$$

$$\sum_{i \in I} l_{ik} x_{ij} \geq 1 \qquad \forall\, j \in \{1, \dots, p\}, \forall k \in K \tag{4.19}$$

$$\sum_{i \in I} \sum_{k \in K} a_{ik} x_{ij} \leq N_{max} \qquad \forall\, j \in \{1, \dots, p\} \tag{4.20}$$

$$x_{ij} \in \{0/1\} \qquad \forall i \in I, \forall\, j \in \{1, \dots, p\} \tag{4.21}$$

The objective (4.17) consists in the minimization of the maximum cluster size N_{max}. Constraints (4.18) indicate that each school i can be assigned to exactly one of the p clusters. Constraints (4.19) assure that, for each cluster j, there is at least a school i of level k, while conditions (4.20) state that the dimension of each cluster cannot exceed the value N_{max}. Constraints (4.21) concern the binary nature of the decision variables.

For each value p, the solution provided by this model represents the most balanced partition of the set of the schools in p groups; for $N_{max} \leq LB_{N_{max}}$, it is not possible to have feasible clusters.

The proposed model (and its variant including constraints (4.16)) will be tested on a real-world case study in the next section. This model can be solved by varying the parameter N_{max} between a lower bound $LB_{N_{max}}$and an upper bound $UB_{N_{max}}$. In particular, $LB_{N_{max}}$ can be obtained solving the model (4.17-4.21); as regards the upper bound $UB_{N_{max}}$, it is represented, for each value p, by the maximum dimension of clusters obtained solving the model (4.17-4.21).

4.6 The case study

The case study is focused on the aggregation of school institutions related to an urban district in the Municipality of Naples (about 12 Kmq) with more than 100.000 inhabitants. In this area there are 29 schools of different levels with a total number of 9077 students. The distribution of students is characterized by significant differences across the schools (ranging from a minimum of 40 students to a maximum of 798). The current arrangement is based on 11 clusters grouping all the schools. Figure 4.7 shows the position of each school and their aggregation in clusters. Table 4.4 indicates the composition of each cluster with the indication of students' population of each school. It has to be highlighted that the current organization does not satisfy governmental requirements both on the minimum students' population of clusters and on their composition, as most of the clusters do not include schools of each level (as prescribed).

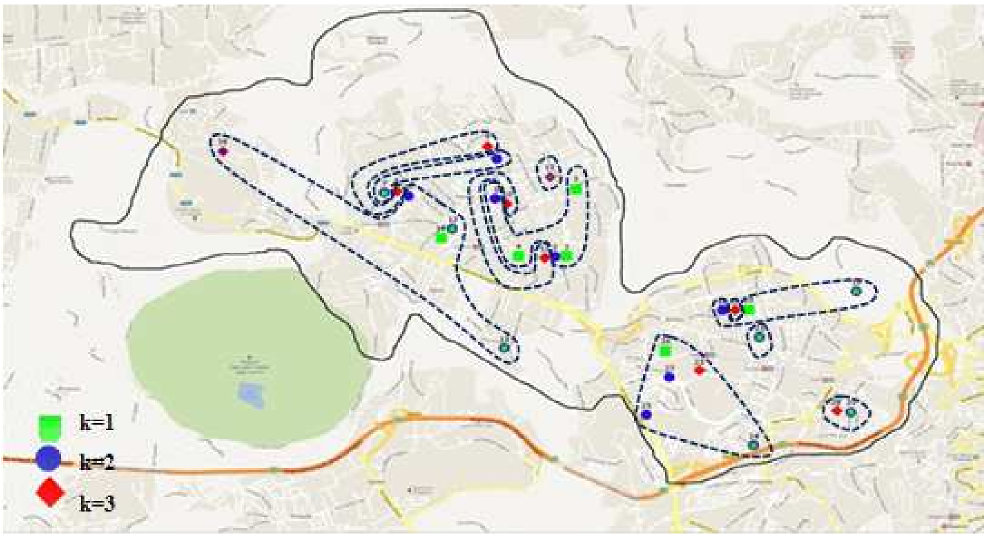


Fig.4.7- Location of schools and current organization in clusters

Clusters	Schools i	Levels k 1	2	3	Total Population
I	1	-	433	-	**968**
	2	-	239	-	
	3	88			
	4	168			
	5	40	-	-	
II	6	-	601	-	**1202**
	7	277	324		
III	8	-	-	730	**880**
	9	-	-	150	
IV	10	-	-	770	**990**
	11	-	-	220	
V	12	-	474	-	**1025**
	13	100	66	-	
	14	155	-	-	
	15	20	75	-	
	16	-	96	39	
VI	17	76	80	100	**256**
VII	18	-	424	-	**749**
	19	55	138	-	
	20	132	-	-	
VIII	21	229	569	-	**798**
IX	22	-	-	700	**700**
X	23	-	-	203	**647**
	24	64	110	-	
	25	-	79	-	
	26	43	-	-	
	27	-	148	-	
XI	28	-	-	358	**862**
	29	160	344	-	

Table 4.4. Students' population data

The proposed model has been applied to the case study and optimally solved using CPLEX 12.2 on an Intel Core i7 with 1.86 GigaHertz and 4.00 GigaBytes of RAM. Running times to obtain solutions are very limited (few seconds).

In the following, results are illustrated and discussed. In particular, the basic version of the model (including constraints 4.10-4.15 and fixing $N_{min} = 1000$ and $I = J$) was considered first; then, the balanced one was evaluated, with the addition of constraints (4.16). Table 4.5 indicates the results obtained by varying the number p of clusters to be created from 2 to 7. For each solution, the number of students for each school (in decreasing order), the average and maximum distances between schools and the related cluster centers are reported.

As expected, in the case of the sizing model, the objective function decreases over p. The range of the size of each cluster, in general, also tends to decrease, even if this condition is not assured. For example, considering the passage from $p = 5$ to $p = 6$, even if the average distance decreases of 17.18% (from 0.75 to 0.64 km), the solution for $p = 6$ is less balanced as the minimum dimension decreases from 1371 to 1055.

Number of clusters p	**2**	**3**	**4**	**5**	**6**	**7**
Average distance	1.28	1.02	0.87	0.75	0.64	0.60
Maximum distance	3.05	3.05	3.05	2.50	2.50	2.50
Students for each cluster	5321	3877	3877	2411	2411	1466
	3756	3756	2247	2247	1539	1445
		1444	1509	1539	1445	1444
			1444	1509	1371	1371
				1371	1256	1256
					1055	1055
						1040

Table 4.5- Sizing model, *Nmin* = 1000

As above mentioned, the second version of the model has been implemented by varying the parameter N_{max} between a lower bound $LB_{N_{max}}$ and an upper bound $UB_{N_{max}}$. In the Table 4.6 the bounds of the feasible range $[LB_{N_{max}}, UB_{N_{max}}]$, for each value of p, are reported.

	p=2	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**
$LB_{N_{\max}}$	4539	3026	2270	1816	1513	1297
$UB_{N_{\max}}$	5321	3877	3877	2411	2411	1466

Table 4.6- Feasible ranges for Nmax

The model including constraints (4.17-4.21) has been implemented, for each value of p, by varying N_{max} in the range $[LB_{N_{max}}, UB_{N_{max}}]$, with the following discrete step:

$$\Delta = \frac{UB_{N_{max}} - LB_{N_{max}}}{10} \tag{4.22}$$

such that:

$$N_{max} = LB_{N_{max}} + k_1 * \Delta, \; with\; k_1 = 0, \dots, 10 \tag{4.23}$$

The comparison of the solutions provided by the two models is summarized in Figure 4.8, where, for each p, the value of the average distance between schools and their respective cluster by varying the value of N_{max} is reported. In order to interpret the results, a single curve (for a given value of $p = \bar{p}$) can be considered. It is possible to notice that for $N_{max} = LB_{N_{max}}(\bar{p})$ we obtain a perfectly balanced solution with the maximum value of objective function. Increasing N_{max}, so gradually relaxing the balancing condition, the model provides better results in terms of objective function. In particular the minimum value is obtained when $N_{max} = UB_{N_{max}}(\bar{p})$. Of course for $N_{max} > UB_{N_{max}}(\bar{p})$, this does not vary anymore, as constraints (8) are not active. Coherently in Figure 4.8, for a given value of $p = \bar{p}$, the shape is monotonically decreasing until the value of the upper bound is reached. Of course this last conditions is obtained for lower values of N_{max} as p increases.

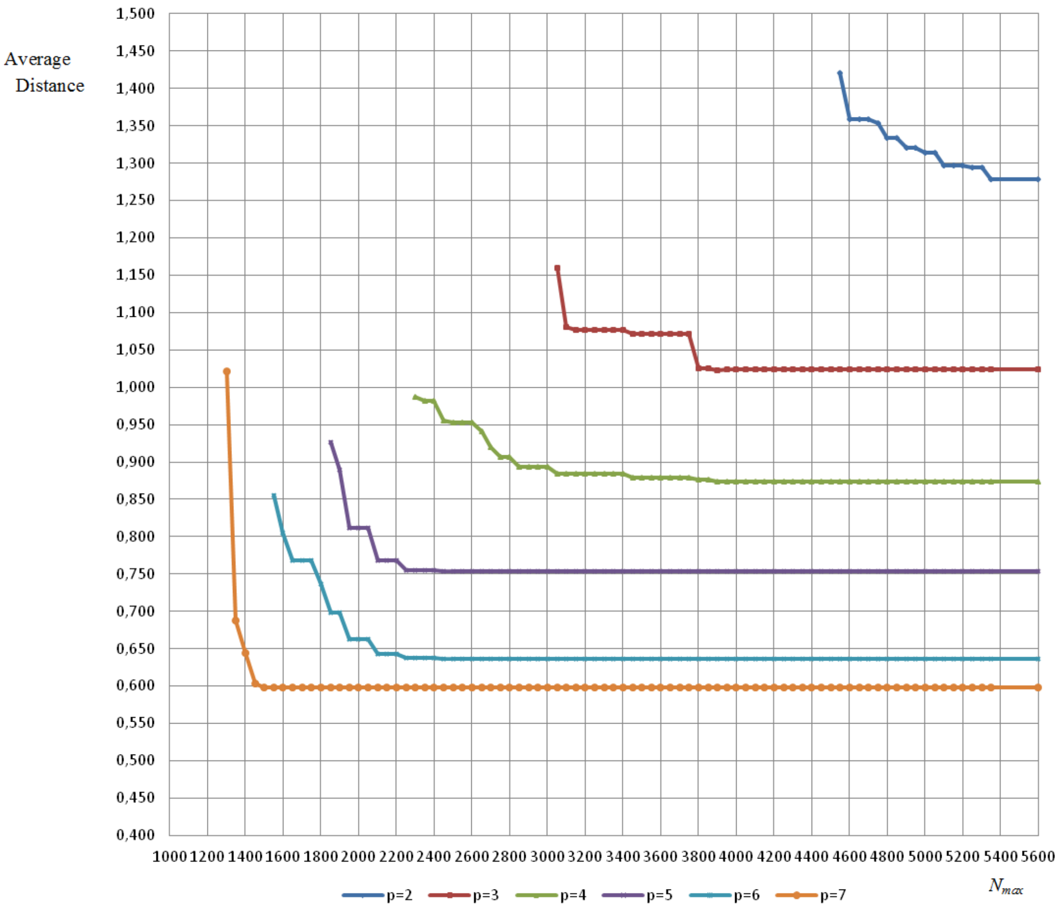


Fig. 4.9- Average distance in function of N_{max}

Furthermore, Figure 4.9 can have interesting managerial implications; in fact it can support the choice of the triplet (p, N_{max}, z). In particular, the graph can be interpreted in two ways: by fixing a value of objective function or, alternatively, by fixing a specific maximum value of clusters.

The first interpretation consists in drawing a horizontal straight line ($z = f_0$) that allows the identification of:

- the values of p that ensure the achievement of that value of objective function;
- the corresponding values of maximum number of clusters.

In this case, it could be possible to identify the most preferable combination (p, N_{max}) for achieving the defined objective function value.

The second interpretation consists in drawing a vertical straight line ($N = N_{max}$) that allows identifying:

- the values of p that allow satisfying the constraint on N_{max};
- the corresponding objective function values.

In this case, it could be possible to compare the possible combinations (p, z) for forming clusters of the given maximum size. For example, considering $N_{max} = 2400$, it can be understood that at least 4 clusters have to be formed. From Table 4.7, it can be seen that the objective function value improves in a very significant way increasing p from 4 to 5.

	p=4	**p=5**	**p=6**	**p=7**
z	**28.48**	**21.90**	**18.51**	**17.35**

Table 4.7- Objective function variation for Nmax = 2400

It can be then derived that further improvements (obtainable by considering an increased number of clusters) follow a diminishing returns law; therefore, this would suggest that opening more than 5 school clusters might not produce improvements that can justify the increase in costs, as $p = 6$ and $p = 7$ return very small improvements in the objective function.

4.7 Conclusions

In this chapter, two applications related to two different real-world territorial re-organizational problems have been introduced and analyzed.

The first application concerns the problem of the rationalization of a public university system on a regional scale. In a given system, in which existing Faculties may be interpreted as facilities offering a set of services (different degree programs) to users (students), the problem consists of shrinking the supply, in order to find solutions which provide a good balance between efficiency purposes (reduction of operating costs) and “public interest” (service accessibility). The model introduced in the Chapter 3 has been adapted to solve this problem with reference to the case of Engineering Faculties in the Campania Region.

The second application concerns the reorganization of a school system located in a given region. In particular, with reference to the requirements indicated for the Italian case, the problem consists of merging pre-primary, primary and secondary schools in integrated institutions managed in a centralized way. In this case a location problem has been formulated to find the optimal merging option, according to a set of objectives related to the efficiency of the solution and the consequences of such process from users perspective.

In both cases, the models have been solved considering real case studies and the presentation of the results show how they can be effectively used to produce useful information to support the decision making process.

Further development of the research may include the application to different cases and the extension of the model to address the reorganization of different categories of public services (for instance, healthcare and administrative systems).

Chapter 5

Redistricting Decisions: Models and Applications

5.1 Introduction

In many cases the public services are organized in districts, i.e. in non-overlapping areas covering all the study region, within which each facility provides the considered service(s). This kind of organization is particularly suitable when the assignment of demand nodes to the available facilities is compulsory. It is the case, for example, of the school districts, the health districts, the territories for the organization of mobile services, such as the waste collection service. Also in these cases, due to some occurring circumstances, it could be necessary to re-organize the service by closing some of the existing facilities and, hence, by modifying the current partition in districts. In order to do this, it is possible to adopt different strategies: for instance, merging existing districts or closing down some of them and reassigning their territorial units to active districts. In the final configuration, the active facilities will be responsible of wider areas, with some potential side effects that should be carefully assessed and evaluated.

This practical situation determines a need for mathematical programming models aimed at providing decision support in these contexts, in order to plan evidence-based redistricting actions.

In this chapter, we propose some novel mathematical programming formulations aimed at supporting redistricting decisions. Furthermore, the real-world specific problem that will be addressed is presented in details and then, with reference to some real instances, results provided by the different models are analyzed and compared. Finally, some conclusions are drawn.

5.2 Redistricting Problems for the reorganization of public services

Suppose to have a certain number of territorial units within a region, already grouped in a given number of districts. In each district, a facility, generally located in correspondence of a specific unit, is responsible for providing some services to the population of the district itself and, for this reason, it will be named *chief unit*.

Consider that the decision maker is interested to the shrinking of the service, and therefore, to the closure of some active facilities. In order to do this, he may adopt different strategies: for instance, merging existing districts or closing down some of them and reassigning their territorial units to active districts. This process will result in less facilities covering wider areas and serving higher amounts of demand; therefore it generally produces some side effects on the users that should be carefully evaluate.

Such models should be defined in such a way that both planner and user perspectives are considered. Indeed, while the planner would be interested in performance improvements in terms of efficiency and cost-saving, by identifying the set of facilities that, if suppressed, maximize a certain benefit index (including, for example, cost saving measures related to facilities and services operations), users will be damaged by the loss of these centers that are responsible for providing many essential services. In order to limit the worsening of the service level provided to users, the re-assignment of single territorial units to the still active facilities could be performed by taking into account appropriate criteria; for example, the size of resulting districts, that should be more balanced as possible in order to avoid congestion situations, their extension, in order to avoid high distances between users and facility within single districts, and so on.

Considering the huge number of feasible options and the multiple planning criteria that could be taken into account, the availability of appropriate decision support methodologies, in order to determine possible configurations, could be beneficial.

It is clear that redistricting problems involve aspects from both the re-organizational and districting problems. Indeed, similarly to the re-organizational problems, they involve decisions about facilities to be closed or relocated; once this decision has been made, a new districting problem has to be solved. However, this districting problem has to take into account the preexisting territorial configuration (namely, districts that have not been closed). This kind of problems arises quite frequently, as, normally, local government reforms produce modifications in existing configurations rather than producing them *ex-novo*.

In the following, some mathematical models are introduced and tested on some real-world cases, showing their potential for providing insights and support to stakeholders and policy makers.

5.3 Redistricting models

Suppose to have a certain number of territorial units in a region, already grouped in districts; in each district, a specific unit, named *chief unit*, is devoted to provide a given set of services to the population. In order to reduce the total management costs, it could be required to reorganize the system by reducing the number of chief units and then the related districts. Generally, in this process the planner may define some efficiency requirements that all districts in the final configuration have to meet, for example in terms of minimum population or area. In the following, we indicate as *feasible* a district meeting the defined requirements, *unfeasible* otherwise. As result of the reorganization process, services will be concentrated in a subset of chief units that generally will need to cover wider areas, with some potential side effects that should be carefully assessed and evaluated.
In such a process, the following decisions have to be performed:

- *Closure decision*, i.e. the identification of the subset of chief units to be shut down;
- *Reallocation decision*, i.e. the reassignment of territorial units to active chief units.

Concerning closure decisions, defined requirements can drive the decision making process in different ways. In particular, we can distinguish between the following approaches:

- *prescriptive*, if all districts not meeting the requirements in the current configuration get closed;
- *optimal*, if it is possible to decide *which* facilities have to be closed, provided that in the final configuration all districts have to meet the given requirements. In this case, the choice is made on the basis of an objective function and a set of considered constraints.

In practice, the number of districts in the solution provided by the prescriptive approach is given *a priori*, by how many feasible districts are in the current configuration and, then, by the definition of the requirements. On the other hand, the optimal approach, allowing a wider choice, may produce better solutions in terms of objective function and/or performance indicators.
Considering the reallocation decision, it is possible to identify the following main strategies:

- *Merging existing district*s, i.e. reallocating closed districts as whole to one of the active chief units;
- *Reassigning territorial units*, i.e. reallocating the single territorial units among active chief units. In this case, it could be decided to reassign only the territorial units related to the closed districts or, within certain limits, also the others.

It is obvious that reallocating single territorial units generally provides better solutions; however it has to be highlighted that this may require significant reorganization efforts as, in this case, territorial units belonging to the same district can be generally reassigned to different chief units.

On the basis of the combination of these two aspects (Closure and Reallocation decisions), we defined the four classes of redistricting models reported in Table 5.1.

		Closure Decision	
		Prescriptive	***Optimal***
Reallocation Decision	***Single Territorial Units***	*Prescriptive Reassigning Model*	*Optimal Reassigning Model*
	Entire Existing Districts	*Prescriptive Merging Model*	*Optimal Merging Model*

Table 5.1 – Classes of redistricting problems

In the following we illustrate the mathematical programming formulations of these problems. In particular, reassigning and merging models are separately described by comparing, for each of them, the prescriptive versions to the optimal ones. In the description of each formulation, the following elements can be distinguished:

- *objective function*: as usual in districting models, a measure of compactness of the resulting districts has been assumed as objective function, also representing a proxy to take into account the accessibility of users to provided services in the new configuration;
- *physical requirements constraints*: they take into account the conditions imposed by the decision maker (for instance, on minimum population, area, number of territorial units of districts);
- *reassignment constraints*: they rule the reassignment of territorial units to active districts and chief units;
- *other constraints*: they can be introduced to consider further aspects of the problem. Typical additional constraints could concern the contiguity of resulting districts, the respect of existing boundaries, the presence of special districts.

In order to formulate such models, the following common notation is introduced:

I	set of territorial unit ($\|I\| = m$);
$J \subset I$	set of chief units, i.e. units in which the facilities providing the considered services are located, also corresponding to the set of existing district ($\|J\| = m$);
$J^* \subset J$	set of special districts or chief units ($\|J^*\| = q$);
$\bar{c}_j \in I$	centroid of district j, corresponding to the $1-median$ solution in the district j;
r_{ij}	binary label equal to 1 if unit i is initially allocated to district j, 0 otherwise;
d_{il}	distance between units i and l;

Furthermore, the following sets of decision variables have to be defined:

y_j	binary variable equal to 1 if and only if the chief unit of district j gets closed;
x_{ij}	binary variable equal to 1 if and only if unit i is assigned to the chief unit of district j.

Formulations are presented assuming that requirements are defined in terms of minimum population and area per each district; therefore, the further notation below has to be considered:

p_i	population of unit i;
s_i	area of unit i;
$P_j = \sum_{i \in I} p_i r_{ij}$	total population of district j;
$S_j = \sum_{i \in I} s_i r_{ij}$	total area of district j;
P_{min}	minimum required amount of population per district;
S_{min}	minimum required extension per district.

5.3.1 Reassigning Models (Prescriptive vs Optimal)

The *Prescriptive Reassigning Model (PRM)* considers the shut-down of the chief units that do not meet the requirement and reassigns the related territorial units (previously part of its district) to the ones that have been kept active.

The *PRM* can be formulated as follows:

$$\min z = \sum_{i \in I} \sum_{j \in J} p_i d_{ij}^2 x_{ij} \tag{5.1}$$

$$P_{min}(1 - y_j) \leq P_j \qquad \forall j \in J - J^* \tag{5.2}$$

$$S_{min}(1 - y_j) \leq S_j \qquad \forall j \in J - J^* \tag{5.3}$$

$$(1 - y_j)\, r_{ij} \leq x_{ij} \leq 1 - y_j \qquad \forall i \in I, \forall j \in J \tag{5.4}$$

$$\sum_{j \in I} x_{ij} = 1 \qquad \forall i \in I \tag{5.5}$$

$$y_j = 0 \qquad \forall j \in J^* \tag{5.6}$$

$$y_j \in \{0,1\}; x_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{5.7}$$

The objective function (5.1) is one of the classical measures of compactness, defined as sum of the weighted square distances among each unit i and the chief unit of its assigned district j. Constraints (5.2-5.3) represent the requirements constraints. In particular, they impose that only districts having an extension larger than S_{min} and a population larger than P_{min} can be kept open. Expressions (5.4-5.5) are the reassignment constraints, which rule the reallocation mechanism of territorial reference units to districts that have been kept active. In particular, constraints (5.4) impose that only units belonging to closed districts can be affected by reallocation decisions, being redistributed across active districts, while constraints (5.5) ensure the allocation of each territorial unit to one (and only one) district. Equations (5.6) represent examples of other constraints, indicating the presence of a set of special districts that cannot be closed. Finally, constraints (5.7) define the nature of the decision variables being introduced in the model.

The *Optimal Reassigning Model (ORM)* differs from the *Prescriptive* one for the criterion used to select districts to be closed. In this case, every district, apart the special ones, represents a good candidate for the closure. Then, the model is aimed at determining *how many* and *which* chief units have to be closed, in such a way that the reassignment process will produce new feasible districts. Among all the solutions, the model selects the most efficient one in terms of objective function, minimizing the average distance between each territorial unit and its own chief unit (5.1).

As concerns the formulation, it is sufficient to replace the groups of constraints (5.2-5.3) with the following ones:

$$\sum_{i \in I} p_i x_{ij} \geq P_{min}(1 - y_j) \qquad \forall j \in J - J^* \qquad (5.8)$$

$$\sum_{i \in I} s_i x_{ij} \geq S_{min}(1 - y_j) \qquad \forall j \in J - J^* \qquad (5.9)$$

Constraints (5.8) assure that the population of a district which is kept active $(y_j = 0)$ is at least equal to P_{min}; while constraints (5.9) impose similar conditions on the area.

The *ORM* identifies the minimum number k^* of chief towns to be closed in order to produce feasible districts. However, it is also possible to include in the model an additional constraint about the minimum total number k of chief-towns to be closed, as shown in the following equation (5.10):

$$\sum_{j \in J} y_j = k \qquad (5.10)$$

Of course, in order to find feasible solutions, k must be at least higher than k^*.

Both the versions of the model may also include an explicit formulation of the contiguity condition.

5.3.2 Merging Models (Prescriptive vs Optimal)

In this class of models, the strategy consists of aggregating entire existing districts. With this aim, each *current district* j can be considered as a single *territorial unit* $(I = J)$, with the total population P_j and extension S_j concentrated in correspondence of its *centroid* $\bar{c}_j$. Then, here, the terms *territorial unit*, *existing district* and *centroid* can be considered equivalent. In the current configuration, each centroid $\bar{c}_j$ is assigned to the related chief town c_j; therefore, $\{r_{ij}\}$ is an identity matrix of order m. When a certain chief unit c_j gets closed, the related district, as a whole, has to be assigned to another active chief unit and, hence, to be merged with another district.

In particular, the *Prescriptive Merging Model (PMM)* closes the chief units that do not meet the requirements (5.2-5.3) and reassigns the related entire districts to the ones that have been kept active.

On the other hand, the *Optimal Merging Model (OMM)* determines the chief units to be closed in such a way that the reassignment of the related districts will produce new feasible districts.

Compared with the mathematical formulations of the *PRM* and *ORM*, the corresponding merging models require the following modifications:

- the objective function (1) has to be modified in order to consider the distance among the centroid of each district $t \in J$ and its assigned chief unit, weighted for the total population of the district itself:

$$z = \sum_{t,j \in J} P_t d^2_{\bar{c}_t j} x_{tj} \tag{5.11}$$

- the reassignment constraints (5.4-5.5) have to be adapted by considering that each district represents a single territorial unit $(I = J)$:

$$\left(1 - y_j\right) r_{tj} \leq x_{tj} \leq 1 - y_j \qquad \forall t, j \in J \tag{5.12}$$

$$\sum_{j \in J} x_{tj} = 1 \qquad \forall t \in J \tag{5.13}$$

Also in this case, both the versions of the model may also include an explicit formulation of the contiguity condition, that should be here referred to entire districts and not to single territorial units.

Table 5.2 summarizes the characteristics of the introduced formulations, in terms of elements of the model and represents a possible framework for the proposed redistricting models. Furthermore Table 5.3 reports, for each of the models introduced above, the number of variables and constraints, expressed as a function of the number of territorial units, *n,* the total number of existing districts, *m*, and the number of special ones, *q*. It is worth to notice that the structure of all the four models is very similar. However, it is has to be remarked that the *Merging* models, not involving any sort of considerations on the territorial units, present a much lower complexity in terms of number of variables and constraints, since, in this case, *n*=*m*. Therefore, the expressions for the number of variables and constraints can be obtained accordingly starting from the ones of the reassigning models.

			Closure Decision	
			Prescriptive	***Optimal***
Reallocation Decision	***Single Territorial Units***		***PRM***	***ORM***
		Objective Function	(5.1)	(5.1)
		Physical Requirements constraints	$(5.2-5.3)$	$(5.8-5.9)$
		Reassignment Constraints	$(5.4-5.5)$	$(5.4-5.5)$
		Other Constraints	(5.6)	$(5.6-5.10)$
	Entire Existing Districts		***PMM***	***OMM***
		Objective Function	(5.11)	(5.11)
		Physical Requirements constraints	$(5.2-5.3)$	$(5.8-5.9)$
		Reassignment Constraints	$(5.12-5.13)$	$(5.12-5.13)$
		Other Constraints	(5.6)	$(5.6-5.10)$

Table 5.2 – Characteristics of formulations of the described redistricting models

Models	**# Variables**	**# Constraints**
Prescriptive Reassigning (PRM)	$m*(n+1)$	$(2m+1)*(n+1)-(q+1)$
Optimal Reassigning (ORM)		
Prescriptive Merging (PMM)	$m*(m+1)$	$(2m+1)*(m+1)-(q+1)$
Optimal Merging (OMM)		

Table 5.3 – Structural properties of introduced models

5.4 The problem of the reorder of Italian provinces

A recent trend observed in western economies has concerned the rationalization of the administrative structures of local authorities, including a reduction in the number of administrative levels and units and the consequent rearrangement of their boundaries (Warner, 2010). For instance, in the last two decades, in the United Kingdom, several structural changes to local government were implemented, whereby a number of new unitary authorities were created in parts of the country which previously operated a *two-tier* system of counties and districts (Martin, 2002); furthermore, the number of districts was significantly reduced (Leach, 2009).

At the same time, in Italy there has been a heated and ongoing debate considering the opportunity of merging and rearranging territorial administrative units of the same level, such as provinces. This is testified by a series of reform proposals aimed at regulating this aspect by modifying the current configuration (ANSA, 2013). However, these proposals have typically found huge difficulties in reaching a consensus both in political contexts and in the public opinion, also due to the difficulty faced by governments in implementing solutions capable of combining the need for more efficient territorial configurations and the safeguard of the accessibility of public services in local communities (ANSA, 2013).

The problem of reorganizing the administrative divisions of italian provinces is very similar to the general problem introduced above.

Indeed, we have a set of regions in the country, each composed by a given number of municipalities and/or local communities (territorial units), already grouped in provinces (districts). One of the last governmental proposals (ANSA, 2013) imposed constraints about the minimum population (P_{min}=300.000 inhabitants) and the minimum extension (S_{min}=2.500 km^2) that all provinces in the final configuration had to meet. Accordingly, the idea was to reorganize, within each region, the current partition of municipalities in provinces, so as to meet the fixed targets.

In order to test the formulated models, they have been applied to this particular problem and the above values for the efficiency requirements have been assumed to drive the decision making process.

In particular, five benchmark problems, built on the real data of five Italian regions, have been considered. In the following section, the characteristics of the considered instances are analyzed in detail.

5.5 Five Real-world Instances

Each Italian region is divided into n territorial units, corresponding to the single municipalities (*territorial units*), already grouped in m provinces (*districts*), each one associated with a specific unit, named *chief unit*, devoted to provide some services to the population of the district itself. Among these m chief units, there exists a particular one, named *regional chief unit*, that provides also other kind of services to the whole region and, for this reason, cannot be closed; therefore, the province related to this unit has been considered as special district $(q = 1)$.

In Figure 5.1 the geographic position of the selected regions within the country is highlighted; while in the Table 5.4 the indication of the number n of municipalities and the number m of provinces, for each region, are provided. It is possible to notice that the selected Regions are the most significant in terms of number of districts and municipalities, apart from Sicilia and Sardinia Regions, that have not been considered as, being islands, they are often subject to different regulations.

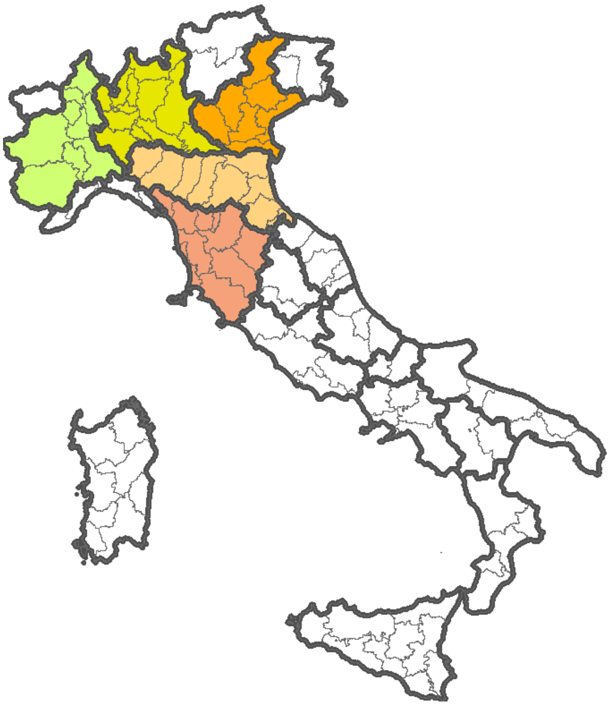


Fig.5.1- Position of the Selected Regions

Region	m	n
Lombardia	12	1544
Toscana	10	287
Piemonte	9	1206
Sicilia	9	390
Emilia Romagna	9	348
Sardinia	8	377
Veneto	7	581
Puglia	6	258
Campania	5	551
Calabria	5	409
Lazio	5	378
Marche	5	239
Abruzzo	4	305
Liguria	4	235
Friuli V. Giulia	4	218
Trentino Alto Adige	2	333
Molise	2	136
Basilicata	2	131
Umbria	2	92
Valle d'Aosta	1	74

Table 5.4-Number of municipalities (n) and provinces (m) per region

For each considered region, population and extension attributes associated to each unit have been obtained by using the most recent figures provided by the National Office of Statistics data (ISTAT, 2011).

The distances among municipalities have been calculated as shortest paths (in km) among them on the road network (considering motorways, national and regional roads). For the computation of such distances, each unit i has been approximated with a representative point, corresponding to the position of the related city hall, whose coordinates are provided by the National Office of Statistics data. This alternative has been preferred to the choice of the geometric centroids, as in this way it could have been possible to obtain points not linkable to the road network (within a given tolerance), for example in the middle of a river or of a lake, and, hence, not consistence for the calculation of the distances.

We use the ArcGIS 10.2 environment as a platform for geographic data storage as well as for the analysis and visualization of solutions provided by the models. The geographic data related to Italian municipalities are provided in shapefile format by the National Office of Statistics data (ISTAT).

In the following we report, for each of the five considered regions, two maps; the former related to its subdivision in municipalities, with the indication of their centroids (Fig.5.2,5.4,5.6,5.8,5.10) and the latter representative of the current partition of these municipalities in provinces (Fig.5.3, 5.5, 5.7, 5.9, 5.11). Moreover, a table (Table 5.5, 5.6, 5.7, 5.8, 5.9) provides a description of the current configuration reporting, for each province j, the number of territorial units, the total population P_j, the area S_j in km^2 and the radius R_{mj}, i.e. the distance between the province chief town and the farthest municipality assigned to it.

5.5.1 Lombardia Region

Lombardia Region is the largest region in Italy in terms of number of inhabitants (almost 10 million). Most of the population is concetrated in the provinces of Milano (almost 3 million), Brescia and Bergamo (almost 1 million, each). While these latter are feasible also in terms of extension, Milano is infeasible, having an area smaller than the minimum threshold of 2.500 kmq; nevertheless, being Milano the regional chief town, the related district cannot be closed by the models. Among the other provinces, only Pavia satisfy both requirements. As concerns the provincial radius, apart the case of Brescia district, whose radius value is significantly high (114 Km), the other values are quite reasonable.

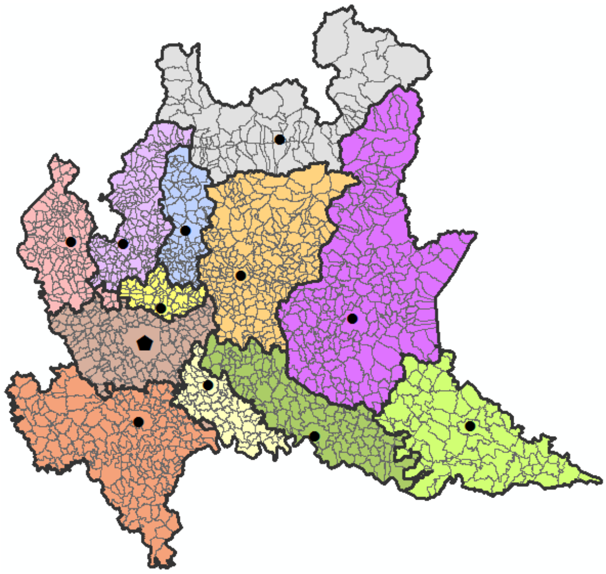


Fig.5.2-Municipalities of Lombardia Region **Fig.5.3-provinces of Lombardia Region**

	Districts (Provinces)	Territorial units (Municipalities)	Population	Area (kmq)	R_{mj} (km)
	Bergamo	244	1.086.277	2.745,94	65,63
	Brescia	206	1.238.044	4.785,62	114,68
	Como	160	586.735	1.279,04	63,12
	Cremona	115	357.623	1.770,46	65,13
	Lecco	90	336.310	814,58	41,84
	Lodi	61	223.755	782,99	43,33
	Mantova	70	408.336	2.341,44	59,64
	Milano*	134	3.038.420	1.575,65	59,23
	Monza	55	840.129	405,41	24,88
	Pavia	190	535.822	2.968,64	71,23
	Sondrio	78	180.814	3.195,76	79,62
	Varese	141	871.886	1.198,11	46,00
	Total	**1.544**	**9.704.151**	**23.864**	
	Average	**129**	**808.679**	**1.989,64**	**61,19**

Table 5.5 - Characteristics of the case study of Lombardia Region

5.5.2 Piemonte Region

Piemonte Region is characterized by an area of 25,402 kmq and a population of about 4,6 million. The district related to Torino, the regional chief town, is the widest, with an extension of 6.817,28 square kilometers, and the most populated, with a number of inhabitants of 2,2 million. Among the others, only Alessandria and Cuneo provinces meet both the requirements. As concerns the provincial radius, the highest values correspond to the provinces of Cuneo (114,41 km) and Vercelli (105,53Km). The other values are quite reasonable.

Fig.5.4-Municipalities of Piemonte Region

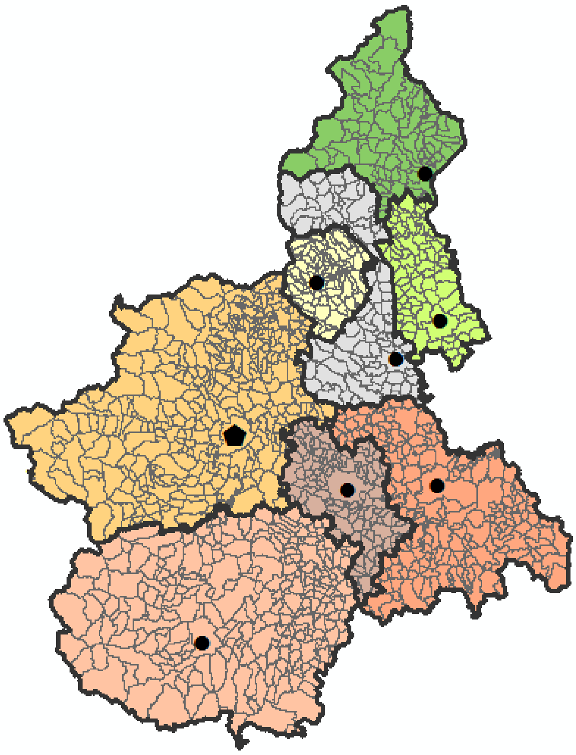


Fig.5.5-Current provinces of Piemonte Region

	Districts (Provinces)	Territorial units (Municipalities)	Population	Area (kmq)	Rmj (km)
	Alessandria	190	427.229	3.558,83	70,65
	Asti	118	217.573	1.510,19	70,76
	Biella	82	182.192	913,28	36,77
	Cuneo	250	586.378	6.894,94	114,41
	Novara	88	365.559	1.340,28	67,81
	Torino*	315	2.240.768	6.817,28	99,61
	Verbania	77	160.264	2.260,91	81,46
	Vercelli	86	176.941	2.081,64	105,53
	Total	**1.206**	**4.356.904**	**25.377,35**	
	Average	**151**	**544.613**	**3.172,17**	**80,88**

Table 5.6 - Characteristics of the case study of Piemonte Region

5.5.3 *Veneto Region*

Veneto, with its 4,8 million inhabitants, is the fifth most populated region in Italy. Five out of its seven provinces are characterized by population values around 900 thousands of inhabitants (Padova, Treviso, Venezia, Verona, Vicenza); but, among the latter, only two satisfy the requirement on the minimum extension. As concerns the other two provinces (Belluno and Rovigo), the population values are very low, around 200 thousand of inhabitants. A particular case is represented by the province of Belluno, characterized by the widest area in the region and the lowest number of inhabitants. The highest value of provincial radius correspond to Venezia (93,56 km) and it is due to the elongated shape of the district.

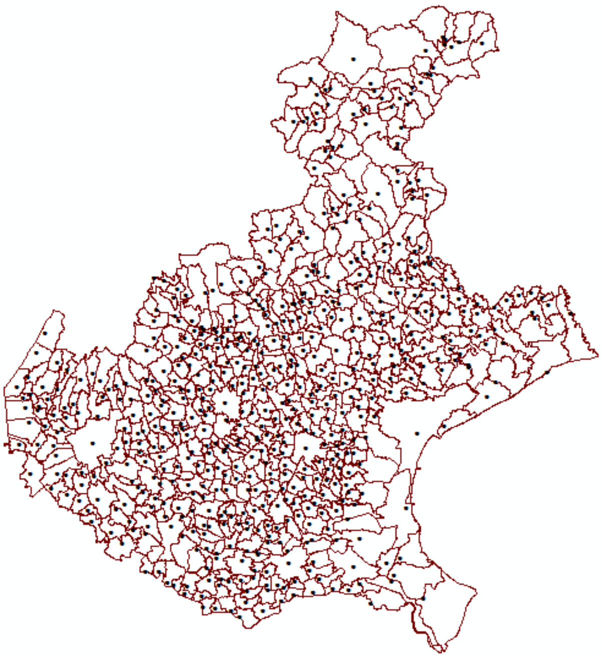


Fig.5.6-Municipalities of Veneto Region

Fig.5.7-Current provinces of Veneto Region

	Districts (Provinces)	Territorial units (Municipalities)	Population	Area (kmq)	Rmj (km)
	Belluno	69	210.001	3.672,26	76,03
	Padova	104	921.361	2.144,15	67,43
	Rovigo	50	242.349	1.819,35	64,63
	Treviso	95	876.790	2.479,83	53,07
	Venezia*	44	846.962	2.472,91	93,56
	Verona	98	900.542	3.096,39	65,44
	Vicenza	121	859.205	2.722,53	75,85
	Total	**581**	**4.857.210**	**18.407,42**	
	Average	**83**	**693.887**	**2.629,63**	**70,86**

Table 5.7 - Characteristics of the case study of Veneto Region

5.5.4 *Emilia Romagna Region*

Emilia Romagna Region is characterized by an area of about 22.452 kmq and a population of about 4,3 million of inhabitants (Table 5.8). All the provinces, apart Piacenza and Rimini, satisfy the requirement on the minimum population; while only five out of nine (Bologna, Ferrara, Modena, Parma and Piacenza) meet the one on the extension. The higher values of provincial radius correspond to the provinces of Ferrara (81,58 km), Forlì (81,60 km) and Parma (90,11km), in which the chief towns are located in decentralized positions.

Fig.5.8-Municipalities of Emilia Romagna Region

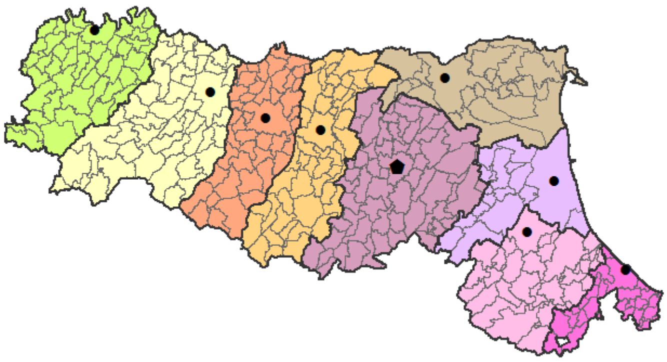


Fig.5.9-Current provinces of Emilia Romagna Region

	Districts (Provinces)	Territorial units (Municipalities)	Population	Area (kmq)	R_{mj} (km)
	Bologna*	60	976.243	3.702,32	68,22
	Ferrara	26	353.481	2.635,12	81,58
	Forlì	30	390.738	2.378,40	81,65
	Modena	47	685.777	2.688,02	79,92
	Parma	47	427.434	3.447,48	90,11
	Piacenza	48	284.616	2.585,86	77,65
	Ravenna	18	384.761	1.859,44	66,57
	Reggio nell'Emilia	45	517.316	2.291,26	64,23
	Rimini	27	321.769	864,88	55,28
	Total	**348**	**4.342.135**	**22.452,78**	
	Average	**39**	**482.459**	**2.494,75**	**73,91**

Table 5.8 - Characteristics of the case study of Emilia Romagna Region

5.5.5 *Toscana Region*

Toscana Region is characterized by an area of about 23.000 square kilometers and a population of about 3,7 million inhabitants (Table 5.5). Among the ten provinces in which it is subdivided, only the district of Firenze, satisfy both requirements. Indeed, the wider provinces, like Arezzo, Siena, Grosseto, do not meet the requirement about the total population; while, the most populated provinces, like Pisa and Lecco, have an area below the minimum feasible value. As concern the provincial radius, notice that the highest values correspond to the provinces whose chief units are located in a very decentralized position, such as Livorno, Siena and Pisa (Fig.5.3).

Fig.5.10-Municipalities of Toscana Region

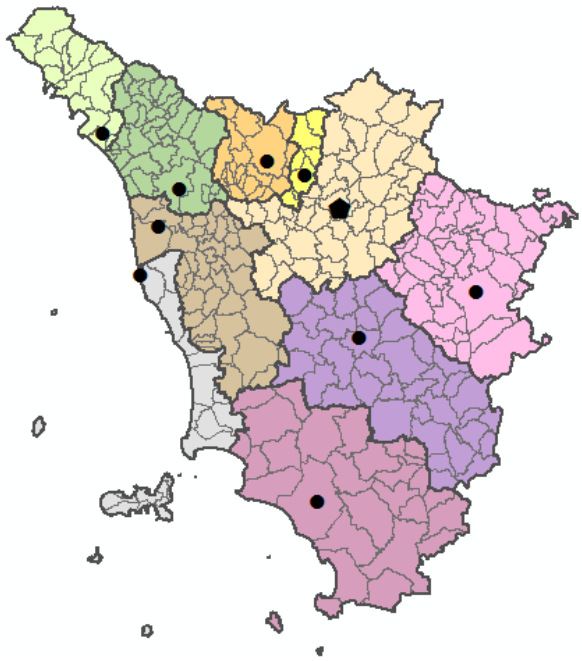


Fig.5.11-Current provinces of Toscana Region

	Districts (Provinces)	Territorial units (Municipalities)	Population	Area (kmq)	R_{mj} (km)
	Arezzo	39	343.676	3.233,08	85,13
	Firenze*	44	973.145	3.513,69	74,17
	Grosseto	28	220.564	4.503,12	90,99
	Livorno	20	335.247	1.213,71	148,21
	Lucca	35	388.327	1.773,22	73,19
	Massa	17	199.650	1.154,68	72,54
	Pisa	39	411.190	2.444,72	112,38
	Pistoia	22	287.866	964,12	49,68
	Prato	7	245.916	365,72	29,90
	Siena	36	266.621	3.820,98	105,31
	Total	**287**	**3.672.202**	**22.987,04**	
	Average	**29**	**367.220**	**2.298,70**	**84,15**

Table 5.9 - Characteristics of the case study of Toscana Region

5.6 Model testing

The test problems described above have been solved using Cplex 12.2 on an Intel Core i7 with 1.86 GHz and 4 GB of RAM. Table 5.10 reports, for every instance (region), the size of each solved model, in terms of number of variable and constraints, and the computational times (in seconds) for running it. Even if Merging models, as we expected, require shorter computational times, also reassigning models can be solved with very limited running times in the case of larger instances (Piemonte and Lombardia), characterized by a significant number of districts and territorial units.

	Parameters			Merging Models				Reassigning Models			
				Istances'size		Run Times (sec)		Istances'size		Run Times (sec)	
Region	m	n	q	**#var**	**#con**	**PMM**	**OMM**	**#var**	**#con**	**PRM**	**ORM**
Lombardia	12	1544	1	156	323	1.80	1.86	18540	38623	21.10	28.22
Piemonte	8	1206	1	72	151	2.03	1.96	9656	20517	14.65	19.10
Veneto	7	581	1	56	118	1.88	1.85	4074	8728	5.32	6.94
Emilia Romagna	9	348	1	90	188	1.97	1.87	3141	6631	3.58	3.77
Toscana	10	287	1	110	229	2.67	2.03	2880	6048	3.00	3.62

Table 5.10 – Computational times for solving the considered instances

The *optimal models* have been solved first assuming k as decision variable, so as to find the minimum number of districts (k_{min}) to be closed in order to have feasible districts, and then by considering it as a parameter and varying it for values higher than k_{min}. Tha maximum value k_{max} that it may assume is equal to $m - q$, i.e. the current number of districts minus the number of special ones. As in each considered instance, the number of special districts is equal to one, the solution provided for $k = k_{max}$ is trivial as it corresponds to only one district coincinding with the whole region and chief town located in the one of the special district.

Obviously, by increasing the number of closed districts the constraints related with minimum requirements will tend to become redundant as the models will provide few larger districts. In any case it could be interesting to compare the solutions provided by the optimal models, by fixing k and varying the rationalization strategy (merging/reassigning) or by fixing the strategy and varying parameter k.

In order to do that, we introduce the following key performance indicators:

- the average, minimum and maximum values for area, population and Province Radius;
- *Population Variance Index* (VAR_{pop}): the mean square deviation of the population of the new provinces from the average population value. This index can be assumed as a measure of the uniformity of the population distribution across the resulting provinces.
- *Area Variance Index* (VAR_{sup}): the mean square deviation of the areas of the new provinces from the average area value. The index can be considered as a measure of the uniformity of the area distribution across the resulting provinces.
- *Hoover Index* (I_H), defined as half of the sum of the differences between the percentages of population and area of each province compared to the regional total values:

$$\mathrm{I_H} = \frac{1}{2} \cdot \sum_{j \in J} \left| \frac{\mathrm{P_j}}{P} - \frac{\mathrm{S_j}}{\mathrm{S}} \right| * 100 \tag{5.14}$$

where P and S are the total population and area of the region. The index is utilised to measure the distribution of population across the resulting provinces (Long and Nucci, 1997). According to the index, the population is fairly distributed if a province accounting for 10% of the regional population also accounts for the 10% of the area. This way, $\mathrm{I_H} = 0$ if each province accounts for the same share of area and population; $\mathrm{I_H}$ gets closer to 100 as the unbalances in population distribution grow. With reference to this specific application, this index will keep track of potential improvements provided by the models in terms of population distribution (to be obtained, for example, by merging densely populated provinces to less crowded ones).

For each region, the maps provided by the Prescriptive models and Optimal models, solved for each value of parameter $k_{min} \leq k \leq k_{max} - 1$, are reported in the Appendix B.

It has to be underlined that all the models have been applied not considering explicitly contiguity constraints. Contiguity conditions have been assessed *a posteriori*, heuristically modifying solutions to enforce them in presence of non-contiguities

5.6.1 Reorder of Lombardia Region

In the case of Lombardia Region, prescriptive models close the eight infeasible districts (Como, Cremona, Lecco, Lodi, Mantova, Monza, Sondrio Varese) and reassign the related territorial units to the remaining four (Milano, Bergamo, Brescia, Pavia). While in the PMM, each closed districts is entirely re-assigned to the same chief-town (Fig.5.13), in the PRM municipalities belonging to the same closed district may be split among different chief towns (Fig.5.12). The two solutions are characterized by a very limited number of provinces covering a much wider area. In particular, the resulting Bergamo and Brescia districts account for very large areas with a very high radius.

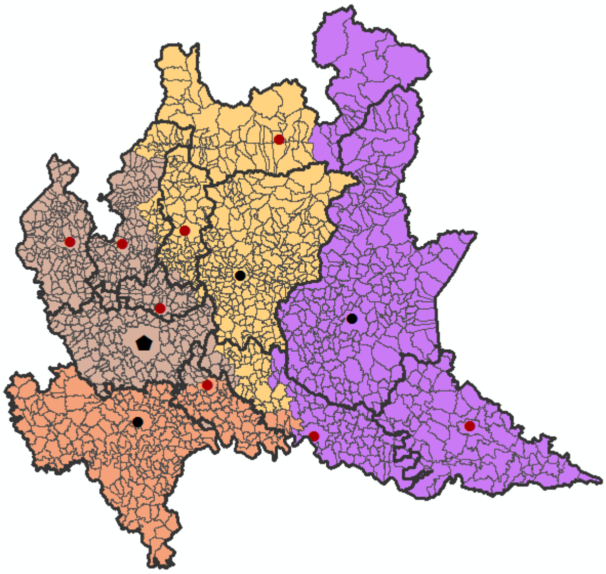


Fig.5.12- Map of PRM Solution

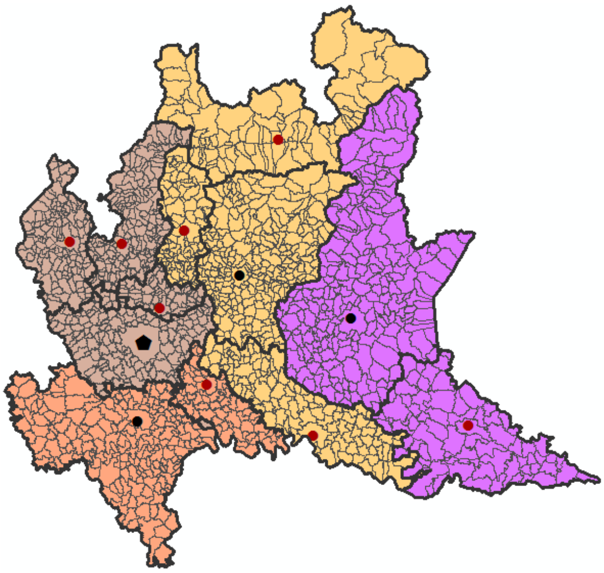


Fig.5.13- Map of PMM Solution

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Milano	482	5.389.647	4.371,51	119,7
	Bergamo	463	1.703.223	6.187,80	127,7
	Brescia	356	1.879.153	9.575,82	175,8
	Pavia	243	732.128	3.728,52	76,4

Table 5.10- Characteristics of PRM Solution

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Milano	490	5.337.170	4.458,22	119,7
	Bergamo	527	1.961.024	8.526,74	179,6
	Brescia	276	1.646.380	7.127,07	137,2
	Pavia	251	759.577	3.751,63	76,4

Table 5.11- Characteristics of PMM Solution

In order to compare solutions with the same number of active provinces, we show in the following the maps provided by the optimal models for $k = 6$, even if the ORM is able to provide a feasible map also by closing a lower number of districts, equal to five ($k = k_{min} = 5$). It is possible to notice, from Figures 5.14 and 5.15, that the two models close different chief towns; but in both the solutions, Brescia, Bergamo and Milano remain active. Apart from Milan, that presents in the two solutions a huge number of inhabitants, the distribution of the total population across the remaining provinces appears to be more balanced in the case of OMM than in the case of ORM, due to the presence of Sondrio, with a value almost equal to the minimum requirement, and Mantova, with almost 460 thousands of inhabitants. In terms of radius, on the contrary, the solution provided by the ORM appear to be better; indeed, in the OMM solution the provinces of Lecco and Cremona are characterized by an high value of provincial radius.

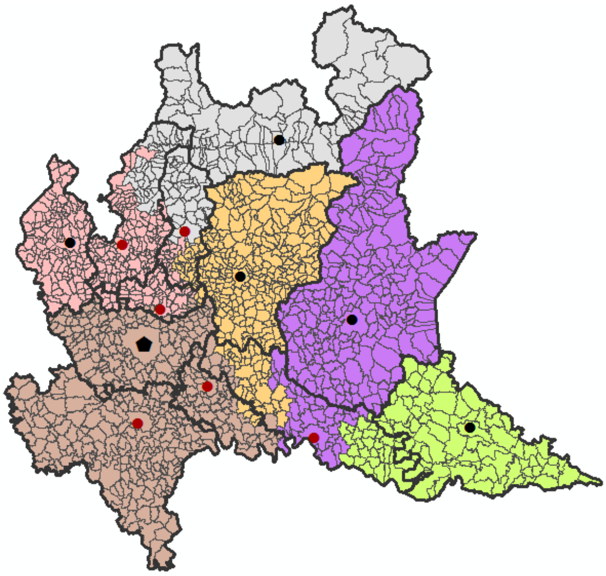


Fig.5.14- Map of ORM solution (k_{min}<k=6)

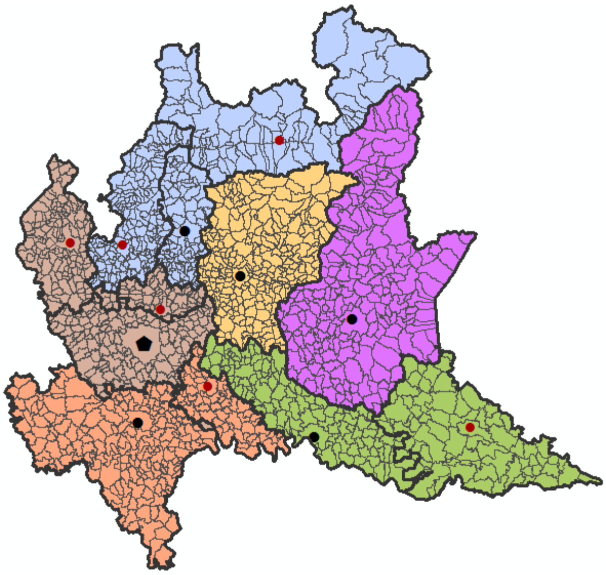


Fig.5.15- Map of OMM solution (k=k_{min}=6)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Milano	402	3.981.914	5.490,05	109,6
	Bergamo	319	1.349.125	3.424,24	65,9
	Brescia	242	1.372.875	5.447,30	114,7
	Varese	333	2.190.396	2.504,57	83,2
	Sondrio	148	350.182	4.147,30	96,6
	Mantova	100	459.659	2.850,18	64,5

Table 5.12- Characteristics of ORM solution (k_{min}<k=6)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Milano	330	4.750.435	3.179,2	119,7
	Bergamo	244	1.086.277	2.745,9	65,6
	Brescia	206	1.238.044	4.785,6	114,7
	Lecco	328	1.103.859	5.289,4	156,5
	Pavia	251	759.577	3.751,6	76,4
	Cremona	185	765.959	4.111,9	123,8

Table 5.13- Characteristics of OMM solution (k=k_{min}=6)

In order to compare the solutions provided by the models, by varying the parameter k, in the following the values of the above indicators for the single solutions are reported. The results are shown in Tables 5.14 (a,b) considering the decreasing number of districts in the optimal solution. By fixing k, the average values of the number of territorial units, area and population per district, are the same for all the solutions, being obtainable by dividing the total regional values for the number of active districts. Therefore, it is more interesting compare the solutions by the values of variance, range (min-max) and Provincial Radius. The minimum number of districts to be closed in order to obtain a feasible configuration is equal to five; only the ORM is able to provide a solution for this value of parameter k. In particular, the solution appears more balanced in terms of distribution of population and area; indeed, even if the maximum values of two attributes increase of the +25% and +12% respectively, the minimum values increase much more, of the +154% and +428,00% respectively. It has to be noticed that the minimum extension related to this solution is lower than the minimum requirement of 2500 kmq; this is due to the fact that it corresponds to the special district of Milano. As concerns the provincial radius, the average value increases a lot passing from 61,2 km to 104,9 km. With reference to this indicator, the solution provided for $k = 6$ results better, providing an average value of 89,1 km.

	m-k	**Population**			***VAR$_{pop}$* (10^5)**	**Area (km^2)**			***VAR$_{sup}$* (10^3)**
		Avg	**Min**	**Max**		**Avg**	**Min**	**Max**	
Current Configuration	12	808.679	180.814	3.038.420	7,80	1.988,6	405,4	4.785,6	1,27
ORM (k=5)	7	1.386.307	459.659	3.824.491	11,25	3.409,1	2.140,5	5.378,3	1,09
ORM (k=6)	6	1.617.359	350.182	3.981.914	13,41	3.977,3	2.504,6	5.490,1	1,28
OMM (k=6)	6	1.617.359	759.577	4.750.435	15,47	3.977,3	2.745,9	5.289,4	0,96
ORM (k=7)	5	1.940.830	459.659	4.810.285	16,70	4.772,7	2.501,8	6.418,5	1,94
OMM (k=7)	5	1.940.830	765.959	5.510.012	20,03	4.772,7	2.745,9	6.930,8	1,54
ORM (k=8)	4	2.426.038	459.659	5.548.540	21,88	5.965,9	2.850,2	7.356,1	2,12
OMM (k=8)	4	2.426.038	1.103.859	5.510.012	20,68	5.965,9	4.516,4	7.127,1	1,27
PRM (k=8)	4	2.426.038	732.128	5.389.647	20,39	5.965,9	3.728,5	9.575,8	2,62
PMM (k=8)	4	2.426.038	759.577	5.337.170	20,06	5.965,9	3.751,6	8.526,7	2,24
ORM (k=9)	3	3.234.717	2.054.681	5.546.107	20,02	7.954,6	7.229,7	9.268,7	1,14
OMM (k=9)	3	3.234.717	1.646.380	5.867.635	22,96	7.954,6	7.127,1	8.701,3	0,79
ORM (k=10)	2	4.852.076	2.328.674	7.375.477	35,69	11.931,8	11.388,7	12.475,0	0,77
OMM (k=10)	2	4.852.076	1.646.380	8.057.771	45,34	11.931,8	7.127,1	16.736,6	6,79

	m-k	**Number of territorial Units**				**Radius (km)**			***Hoover Index***
		Avg	**Min**	**Max**		**Avg**	**Min**	**Max**	
Current Configuration	12	129	55	244		61,2	24,9	114,7	36,37
ORM (k=5)	7	221	100	287		104,9	59,2	165,0	32,16
ORM (k=6)	6	257	100	402		89,1	64,5	114,7	30,10
OMM (k=6)	6	257	185	330		109,4	65,6	156,5	35,63
ORM (k=7)	5	309	100	488		122,0	64,5	165,0	25,39
OMM (k=7)	5	309	185	581		116,1	65,6	156,5	27,74
ORM (k=8)	4	386	100	597		111,3	64,5	156,5	26,84
OMM (k=8)	4	386	276	581		139,0	119,7	156,5	27,74
PRM (k=8)	4	386	243	482		124,9	76,4	175,8	37,22
PMM (k=8)	4	386	251	527		128,2	76,4	179,6	36,32
ORM (k=9)	3	515	429	596		134,4	109,6	156,5	26,86
OMM (k=9)	3	515	276	696		147,9	137,2	156,5	24,00
ORM (k=10)	2	772	489	1.055		183,6	175,8	191,4	23,73
OMM (k=10)	2	772	276	1.268		175,5	137,2	213,9	12,90

Tables 5.14a (top) and 5.14b (bottom)–
Comparison of the solutions provided by the models for the case of Lombardia Region

For values higher than five, both optimal models provide feasible solutions. In most cases ORM provides more balanced solutions in terms of population; however it has to be underlined that these benefits are counterbalanced by more organizational efforts due to the fact that, as it can be observed by the maps, the new districts are significantly different compared to the current ones. Prescriptive models produce solutions with the minimum number of provinces (4). This is due to the fact that they are obliged to close all the unfeasible districts in the current configuration. Even if the objective function of the PRM is, of course, better, the solutions are quite similar considering all the introduced indicators. The limited number of districts determines a significant worsening of all the indicators compared to the current configuration. In particular the average value of Province Radius almost doubles, passing from 61,2 km to over 120,0 km with the worst case of more than 170,14 km compared to the current 112 km.

Looking at the Hoover index, that is considered to be the premier measure for keeping track of equitable districting configurations (Long and Nucci, 1997), it can be seen that solutions are characterized by comparable values, even improving the current situation. The best result is given by the OMM for $k = 10$. The relatively high value of this index for all the solutions, depends on the peculiar configuration of Lombardia region, in which metropolitan areas (very densely populated) and Alpine sub-regions (much less crowded) are very close to each other.

5.6.2 *Reorder of Piemonte Region*

In the case of Piemonte Region, the prescriptive models close the five districts that do not satisfy the requirements (Verbania, Novara, Vercelli, Biella, Asti). The two solutions are characterized by a very limited number of provinces, equal to three. From the maps (Figures 5.16, 5.17), it is possible to notice that in both the solutions the boundaries of province of Cuneo do not change, as opposite to the other two. In particular, the province of Torino is partially modified, while the province of Alessandria changes a lot, covering a much wider area with a very high radius (206,3 Km).

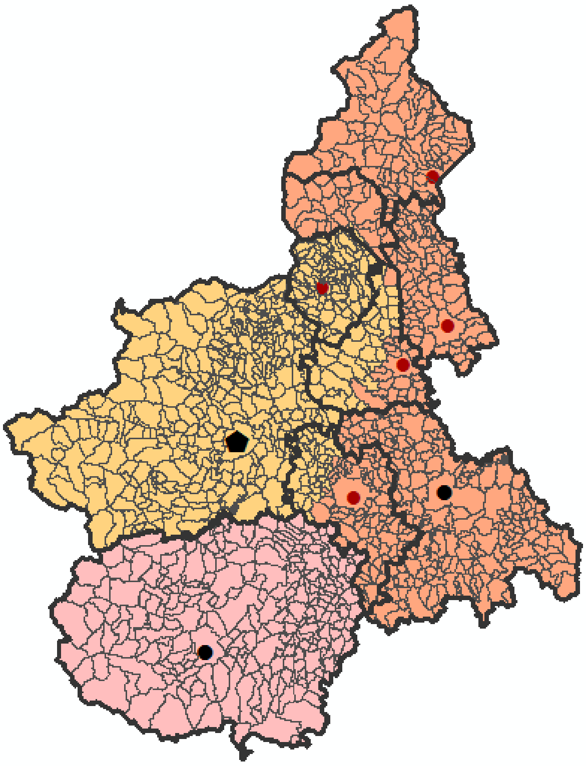


Fig.5.16- Map of PRM Solution

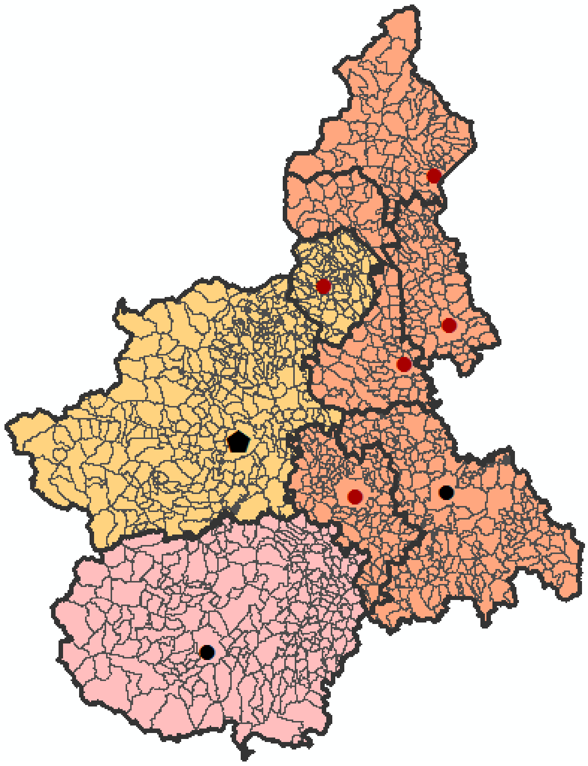


Fig.5.17- Map of PMM Solution

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Torino	467	2.522.388	8.965,8	102,9
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	489	1.248.138	9.516,6	206,3

Table 5.14- Characteristics of PRM Solution

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Torino	397	2.422.960	7.730,6	101,4
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	559	1.347.566	10.751,8	206,3

Table 5.15- Characteristics of PMM Solution

Both the optimal models are able to provide solutions by closing a minimum number of districts equal to three ($k = k_{min} = 3$). The ORM closes Biella, Novara and Asti provinces; while the OMM closes Asti, Vercelli and Verbania. In this case the solutions are quite similar (Figures 5.18, 5.19). The only thing that has to be noticed is that, even if the Vercelli and Biella provinces are in both cases merged, in the ORM also togheter with some other municipalities belonging to Novara, the value of provincial radius is quite different, higher in the ORM (105,5 Km) than in the OMM (88,8 km). This is due to the fact that in the first case the chief town is Vercelli, that is in a decentralized position with the reference to the new resulting province; while in the last case the chief town is Biella, that is better positioned.

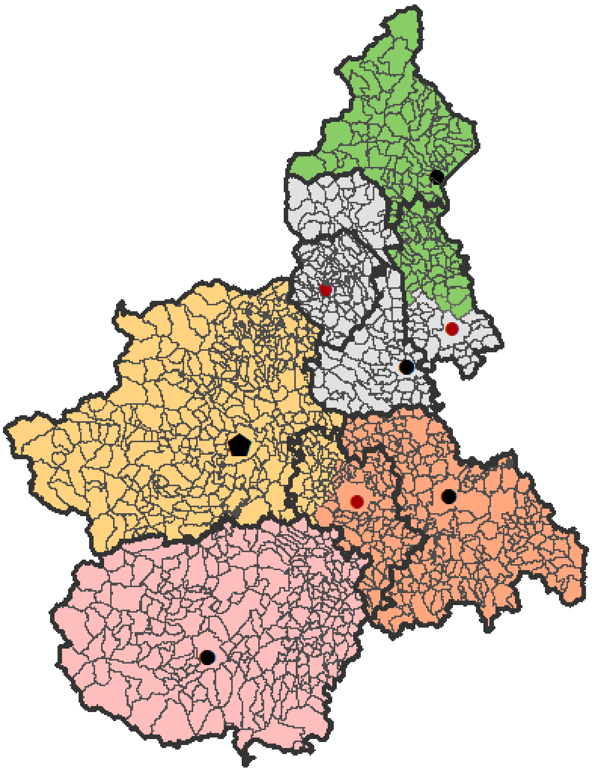


Fig.5.18- Map of ORM Solution ($k=k_{min}=3$)

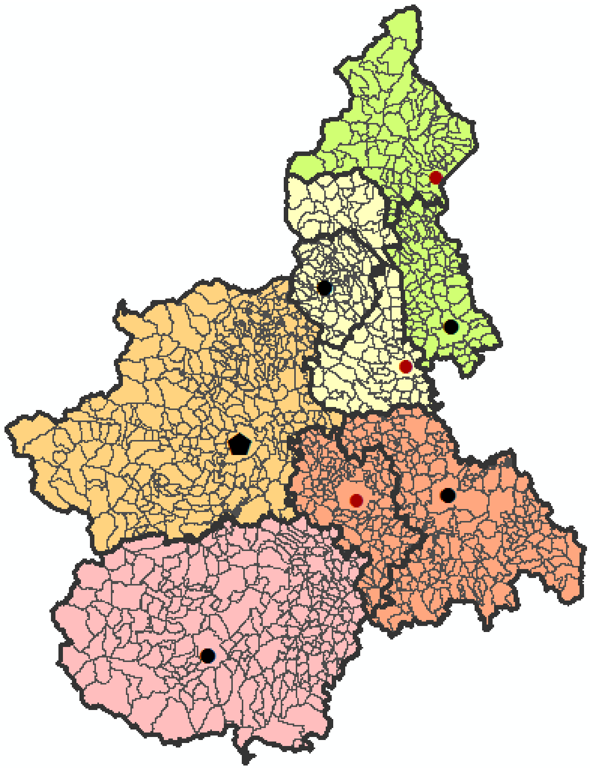


Fig.5.19- Map of OMM Solution ($k=k_{min}=3$)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Verbania	141	355.109	3.085,0	154,8
	Torino	347	2.275.353	7.167,4	99,6
	Vercelli	197	534.011	3.577,7	105,5
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	271	606.053	4.652,4	70,7

Table 5.16- Characteristics of ORM Solution ($k=k_{min}=3$)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Novara	165	525.823	3.601,2	155,4
	Torino	315	2.240.768	6.817,3	99,6
	Biella	168	359.133	2.994,9	88,8
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	308	644.802	5.069,0	83,4

Table 5.17- Characteristics of OMM Solution ($k=k_{min}=3$)

In Tables 5.18 (a,b) the above indicators for the solutions provided by the models, by varying parameter k, are reported. The results are shown considering the decreasing number of districts in the optimal solution. It is possible to notice that in most case the solutions provided by the optimal models are quite similar. For $k < 6$, the solutions of the OMM appear to be a little bit more balanced in terms of distribution of population and area across active provinces. Moreover for $k = 5$ the solution provided by the OMM is the best in terms of average provincial radius

	m-k	Population			VAR_{pop} (10^5)	Area (km^2)			VAR_{sup} (10^3)
		Avg	Min	Max		Avg	Min	Max	
Current Configuration	8	544.613	160.264	2.240.768	7,02	3.172,2	913,3	6.894,9	2,41
ORM (k=3)	5	871.381	355.109	2.275.353	7,91	5.075,5	3.085,0	7.167,4	1,88
OMM (k=3)	5	871.381	359.133	2.240.768	7,73	5.075,5	2.994,9	6.894,9	1,79
ORM (k=4)	4	1.089.226	586.378	2.315.474	8,26	6.344,3	4.664,1	7.619,1	1,26
OMM (k=4)	4	1.089.226	586.378	2.240.768	7,78	6.344,3	5.069,0	6.894,9	0,86
ORM (k=5)	3	1.452.301	676.186	2.832.523	11,98	8.459,1	5.220,4	13.957,6	4,79
OMM (k=5)	3	1.452.301	586.378	2.458.341	9,44	8.459,1	6.894,9	10.154,9	1,63
PRM (k=5)	3	1.452.301	586.378	2.522.388	9,84	8.459,1	6.894,9	9.516,6	1,38
PMM (k=5)	3	1.452.301	586.378	2.422.960	9,23	8.459,1	6.894,9	10.751,8	2,03
ORM (k=6)	2	2.178.452	1.339.438	3.017.466	11,87	12.688,7	10.429,0	14.948,4	3,20
OMM (k=6)	2	2.178.452	1.312.185	3.044.719	12,25	12.688,7	10.154,9	15.222,4	3,58

	m-k	Number of territorial Units				Radius (km)			*Hoover Index*
		Avg	Min	Max		Avg	Min	Max	
Current Configuration	8	151	77	315		80,9	36,8	114,4	28,26
ORM (k=3)	5	241	141	347		109,0	70,7	154,8	23,98
OMM (k=3)	5	241	165	315		108,3	83,4	155,4	24,57
ORM (k=4)	4	302	250	368		110,0	70,7	155,4	23,12
OMM (k=4)	4	302	250	333		114,0	83,4	158,5	24,57
ORM (k=5)	3	402	316	574		140,0	107,7	157,0	10,01
OMM (k=5)	3	402	250	523		133,2	114,4	158,5	23,61
PRM (k=5)	3	402	250	489		141,2	102,9	206,3	22,56
PMM (k=5)	3	402	250	559		140,7	101,4	206,3	25,15
ORM (k=6)	2	603	552	654		157,7	157,0	158,5	10,35
OMM (k=6)	2	603	523	683		157,7	157,0	158,5	9,90

Tables 5.18a (top) and 5.20b (bottom)–
Comparison of the solutions provided by the models for the case of Piemonte Region

5.6.3 Reorder of Veneto Region

In the case of Veneto Region, the solutions provided by the prescriptive models close the four infeasible districts (Treviso, Belluno, Padova, Rovigo) and reassign the related territorial units to the chief towns of feasible provinces (Verona, Vicenza, Venezia). In both solutions, the province of Verona is quite similar to the current configuration (in particular in the PMM, it remains unchanged), while resulting Venezia and Vicenza provinces result substantially modified, accounting for very large areas and population (Figures 5.20,5.21). Among them, the highest provincial radius corresponds to the province of Venezia that has a more elongated shape.

With reference to the optimal models, both the versions provide feasible solutions by closing only 2 districts ($k = k_{min} = 2$). The corresponding maps are shown in Figures 5.22 and 5.23.

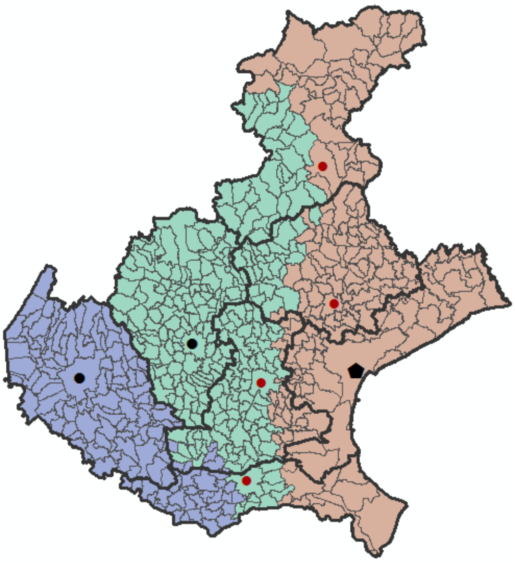


Fig.5.20- Map of PRM Solution

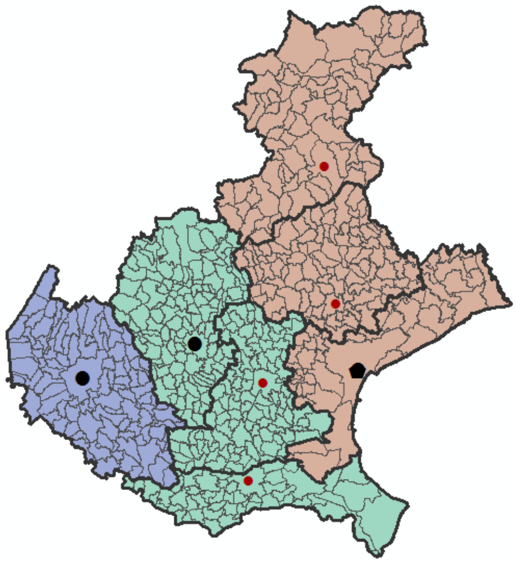


Fig.5.21- Map of PMM Solution

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Verona	134	1.003.549	3.862,1	90,5
	Vicenza	263	1.972.077	6.627,1	147,5
	Venezia	184	1.881.584	7.918,3	165,1

Table 5.19- Characteristics of PRM Solution

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Verona	98	900.542	3.096,4	65,4
	Vicenza	275	2.022.915	6.686,0	124,3
	Venezia	208	1.933.753	8.625,0	165,1

Table 5.20- Characteristics of PRM Solution

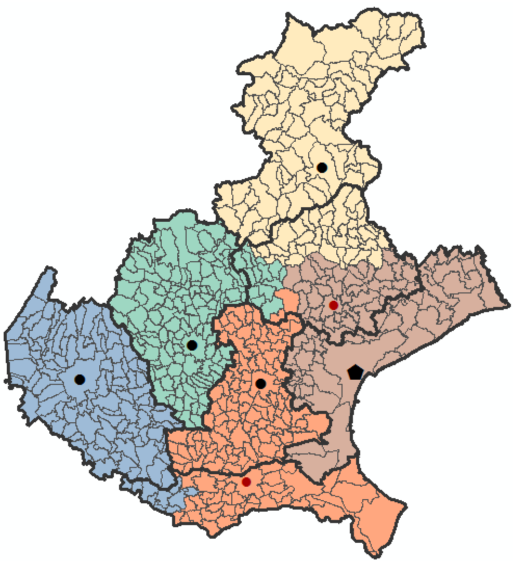


Fig.5.22- Map of ORM Solution ($k=k_{min}=2$)

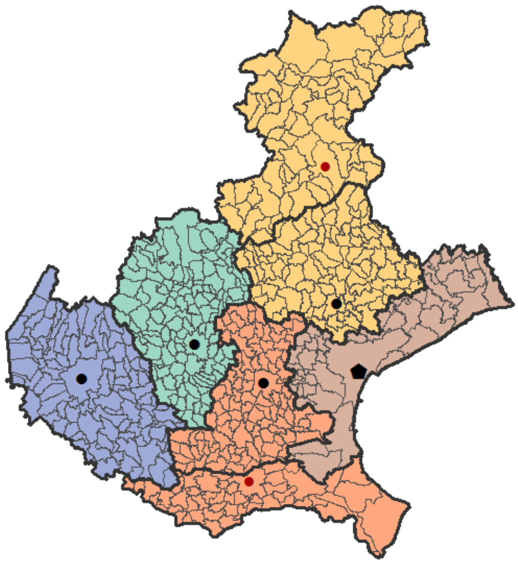


Fig.5.23- Map of OMM Solution ($k=k_{min}=2$)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Verona	108	922.251	3.303,2	78,0
	Vicenza	135	976.036	3.045,3	75,9
	Belluno	107	475.074	4.621,7	76,0
	Venezia	85	1.316.124	3.593,8	93,6
	Padova	146	1.167.725	3.843,4	79,5

Table 5.21- Characteristics of ORM Solution ($k=k_{min}=2$)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Verona	98	900.542	3.096,4	65,4
	Vicenza	121	859.205	2.722,5	75,9
	Treviso	164	1.086.791	6.152,1	134,1
	Venezia	44	846.962	2.472,9	93,6
	Padova	154	1.163.710	3.963,5	100,9

Table 5.22- Characteristics of OMM Solution ($k=k_{min}=2$)

In both the solutions Verona, Vicenza and Venezia provinces remain active with a very similar configuration to the current one (in particular, the OMM let them unchanged while the ORM assign them few adding municipalities). Padova province is merged with most of the municipalities of Rovigo, while Treviso is merged quite completely merged with Belluno (mergings are complete in the case of OMM). Despite the similarity of the two maps, the maximal provincial radius in the case of the OMM solution is much more higher, as within the province deriving from the merging of Belluno and Treviso, the chief town is kept open in Treviso that is located in a very decentralized position. In Tables 5.23 (a,b) the above indicators for the solutions provided by the models, by varying parameter k, are reported. The results are shown considering the decreasing number of districts in the optimal solution.

	m-k	**Population**			***VAR**$_{pop}$* **(10^5)**	**Area (km^2)**			***VAR**$_{sup}$* **(10^3)**
		Avg	**Min**	**Max**		**Avg**	**Min**	**Max**	
Current Configuration	7	693.887	210.001	921.361	3,21	2.629,6	1.819,4	3.672,3	0,61
ORM (k=2)	5	971.442	475.074	1.316.124	3,19	3.681,5	3.045,3	4.621,7	0,61
OMM (k=2)	5	971.442	846.962	1.163.710	1,44	3.681,5	2.472,9	6.152,1	1,49
ORM (k=3)	4	1.214.303	475.074	1.819.893	5,99	4.601,9	3.862,1	4.984,6	0,52
OMM (k=3)	4	1.214.303	846.962	2.022.915	5,49	4.601,9	2.472,9	6.686,0	2,12
ORM (k=4)	3	1.619.070	1.003.549	1.972.077	5,35	6.135,8	3.862,1	7.918,3	2,07
OMM (k=4)	3	1.619.070	846.962	2.923.457	11,36	6.135,8	2.472,9	9.782,4	3,65
PRM (k=4)	3	406.335	1.003.549	1.972.077	5,35	6.135,8	3.862,1	7.918,3	2,07
PMM (k=4)	3	1.619.070	900.542	2.022.915	6,24	6.135,8	3.096,4	8.625,0	2,81
ORM (k=5)	2	2.428.605	1.881.584	2.975.626	7,74	9.203,7	7.918,3	10.489,2	1,82
OMM (k=5)	2	2.428.605	1.933.753	2.923.457	7,00	9.203,7	8.625,0	9.782,4	0,82

	m-k	**Number of territorial Units**				**Radius (km)**			***Hoover Index***
		Avg	**Min**	**Max**		**Avg**	**Min**	**Max**	
Current Configuration	7	83	44	121		70,9	53,1	93,6	19,07
ORM (k=2)	5	116	85	146		80,6	75,9	93,6	15,33
OMM (k=2)	5	116	44	164		94,0	65,4	134,1	11,05
ORM (k=3)	4	145	107	221		88,6	76,0	94,4	15,65
OMM (k=3)	4	145	44	275		98,5	65,4	134,1	11,05
ORM (k=4)	3	194	134	263		134,4	90,5	165,1	4,60
OMM (k=4)	3	194	44	373		117,3	93,6	134,1	11,05
PRM (k=4)	3	194	134	263		134,4	90,5	165,1	4,60
PMM (k=4)	3	194	98	275		118,3	65,4	165,1	7,04
ORM (k=5)	2	291	184	397		156,3	147,5	165,1	4,28
OMM (k=5)	2	291	208	373		144,7	124,3	165,1	7,04

Tables 5.23a (top) and 5.23b (bottom)–
Comparison of the solutions provided by the models for the case of Veneto Region

5.6.4 Reorder of Emilia Romagna Region

In the Emilia Romagna region, five districts out of nine (Forlì, Piacenza, Reggio Emilia, Ravenna, Rimini), do not satisfy the specified requirements; therefore prescriptive models gets closed such districts and mantain active the others. The maps corresponding to the solutions provided by the PRM and PMM are shown in Figure 5.24 and 5.25, respectively. It is possible to notice that they are quite similar; the only difference is that in the ORM the provinces of Reggio Emilia and Ravenna are split between two active provinces, while they are entirely assigned to only one province by the OMM.

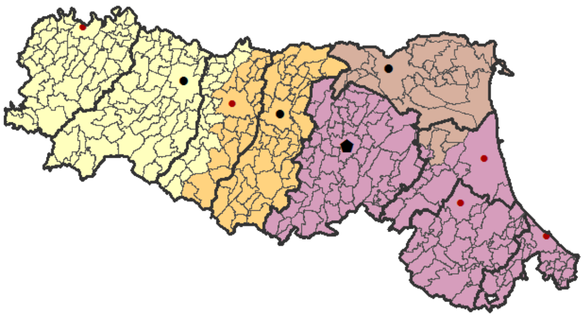


Fig.5.24- Map of PRM Solution

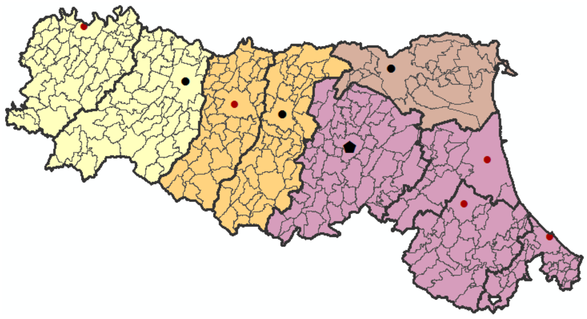


Fig.5.25- Map of PMM Solution

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Parma	117	864.068	6.977,8	138,9
	Modena	70	1.051.075	4.034,8	79,9
	Bologna	131	2.011.108	8.496,4	148,4
	Ferrara	30	415.884	2.943,7	81,6

Table 5.24- Characteristics of PRM Solution

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Parma	95	712.050	6.033,3	138,9
	Modena	92	1.203.093	4.979,3	89,2
	Bologna	135	2.073.511	8.805,0	148,4
	Ferrara	26	353.481	2.635,1	81,6

Table 5.25- Characteristics of PMM Solution

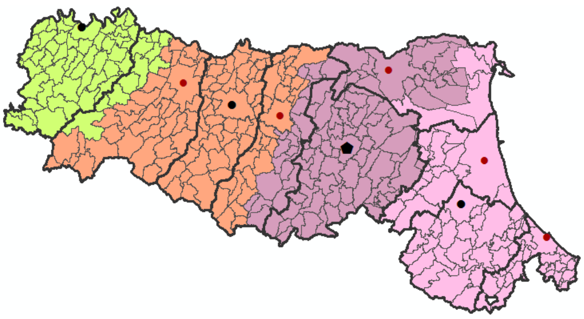


Fig.5.26- Map of ORM Solution (k_{min} <k=5)

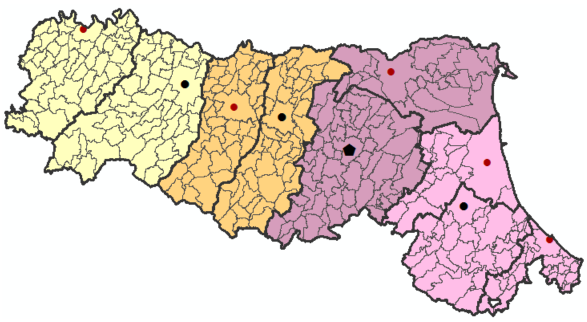


Fig.5.27- Map of OMM Solution (k_{min} <k=5)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Piacenza	59	359.139	3.342,4	77,6
	Reggio E.	107	1.348.923	6.538,2	117,1
	Bologna	101	1.463.648	6.551,8	87,8
	Forlì	81	1.170.425	6.020,4	95,4

Table 5.26- Characteristics of ORM Solution (k_{min} <k=5)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Parma	95	712.050	6.033,3	138,9
	Modena	92	1.203.093	4.979,3	89,2
	Bologna	86	1.329.724	6.337,4	122,1
	Forlì	75	1.097.268	5.102,7	84,7

Table 5.27- Characteristics of OMM Solution (k_{min} <k=5)

In order to compare solutions with the same number of districts, also those provided by the optimal models for k = 5 are shown (Figure 5.26, 5.27). Optimal models are not constrained to close the infeasible districts; indeed, in the case of ORM, together with Ravenna and Rimini, also Ferrara, Modena and Parma provinces gets closed, even if they satisfy both the requirements. In the case of OMM, with reference to the prescriptive models, the only difference consists in the closure of Forlì instead of Ferrara province. Comparing the values of the values of the provincial indicators associated to the four solutions (Tables 5.24,5.25,5.26,5.27), it is possible to notice that the ones provided by the optimal models are well homogeneus in terms of distribution of population and extension. In particular, the one provided by the ORM is the best in terms of provincial radius.

	m-k	Population			*VAR_{pop}* (10^5)	Area (km^2)			*VAR_{sup}* (10^3)
		Avg	Min	Max		Avg	Min	Max	
Current Configuration	9	482.459	284.616	976.243	2,20	2.494,8	864,9	3.702,3	0,83
ORM (k=2)	7	620.305	353.481	976.243	2,39	3.207,5	2.543,1	4.982,2	0,90
ORM (k=3)	6	723.689	359.139	1.194.264	2,87	3.742,1	2.559,7	4.982,2	0,97
ORM (k=4)	5	868.427	359.139	1.419.964	4,85	4.490,6	2.559,7	6.538,2	1,69
OMM (k=4)	5	868.427	353.481	1.203.093	3,41	4.490,6	2.635,1	6.033,3	1,33
ORM (k=5)	4	1.085.534	359.139	1.463.648	4,99	5.613,2	3.342,4	6.551,8	1,53
OMM (k=5)	4	1.085.534	712.050	1.329.724	2,67	5.613,2	4.979,3	6.337,4	0,67
PRM (k=5)	4	1.085.534	415.884	2.011.108	6,72	5.613,2	2.943,7	8.496,4	2,57
PMM (k=5)	4	1.085.534	353.481	2.073.511	7,45	5.613,2	2.635,1	8.805,0	2,56
ORM (k=6)	3	1.447.378	1.170.425	1.945.870	4,33	7.484,3	6.020,4	8.489,0	1,30
OMM (k=6)	3	1.447.378	1.097.268	2.015.501	4,96	7.484,3	5.102,7	9.025,5	2,09
ORM (k=7)	2	2.171.068	1.225.840	3.116.295	13,37	11.226,4	8.489,0	13.963,8	3,87
OMM (k=7)	2	2.171.068	1.229.366	3.112.769	13,32	11.226,4	8.324,6	14.128,2	4,10

	m-k	Number of territorial Units				Radius (km)			*Hoover Index*
		Avg	Min	Max		Avg	Min	Max	
Current Configuration	9	39	18	60		73,9	55,3	90,1	15,66
ORM (k=2)	7	50	26	81		85,4	66,6	117,1	12,36
ORM (k=3)	6	58	41	81		90,3	77,6	117,1	11,53
ORM (k=4)	5	70	41	107		93,1	77,6	117,1	9,38
OMM (k=4)	5	70	26	95		92,5	68,2	138,9	14,07
ORM (k=5)	4	87	59	107		94,5	77,6	117,1	6,62
OMM (k=5)	4	87	75	95		108,7	84,7	138,9	10,47
PRM (k=5)	4	87	30	131		112,2	79,9	148,4	14,71
PMM (k=5)	4	87	26	135		114,5	81,6	148,4	14,07
ORM (k=6)	3	116	81	141		111,9	95,4	138,9	9,58
OMM (k=6)	3	116	75	140		115,2	84,7	138,9	8,76
ORM (k=7)	2	174	141	207		143,7	138,9	148,4	9,58
OMM (k=7)	2	174	140	208		143,7	138,9	148,4	8,76

Tables 5.28a (top) and 5.28b (bottom)–
Comparison of the solutions provided by the models for the case of Emilia Romagna Region

The optimal models provide solutions also for lower values of parameter k. The values of the above indicators associated to the all the solutions provided by the optimal models by varying parameter k are reported in the Tables 5.28 (a,b).

5.6.5 Reorder of Toscana Region

In the case of Toscana Region, only one of the current districts meet both the requirements; therefore, the Prescriptive models (PRM, PMM) provide the same trivial solution with only one district, corresponding to the whole region, having its center in Firenze. In this case the population and the extension of the district correspond obviously to the total poplation and area of the region, while the maximum Radius is equal to 214,92 km.

As concerns the Optimal Models, the minimum number of districts to be closed in order to have feasible districts is equal to 6 for the OMM and to 5 for the ORM. In order to compare solutions with the same number of active districts, in Fig.5.12 and Fig.5.13 the solutions provided by the ORR and OMM for $k = 6$ are shown, with the indication, in Tables 5.11 and 5.12 respectively, of attributes values per district.

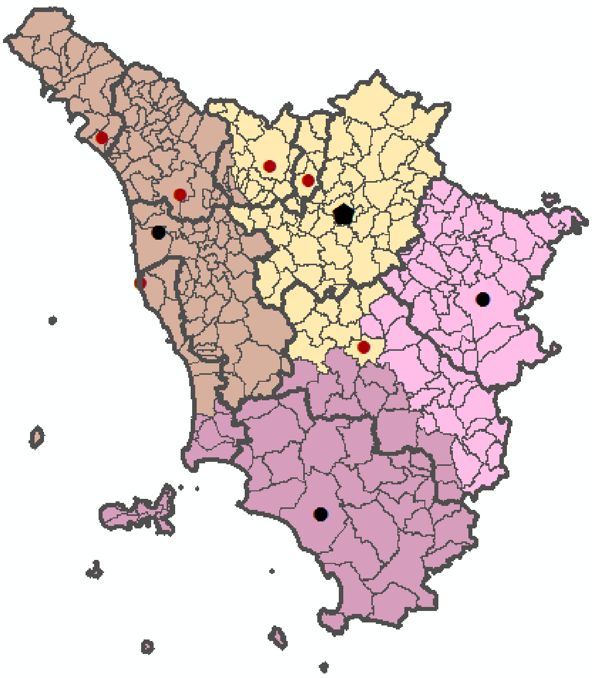


Fig.5.28- Map of ORM Solution (k=k_{min}=6)

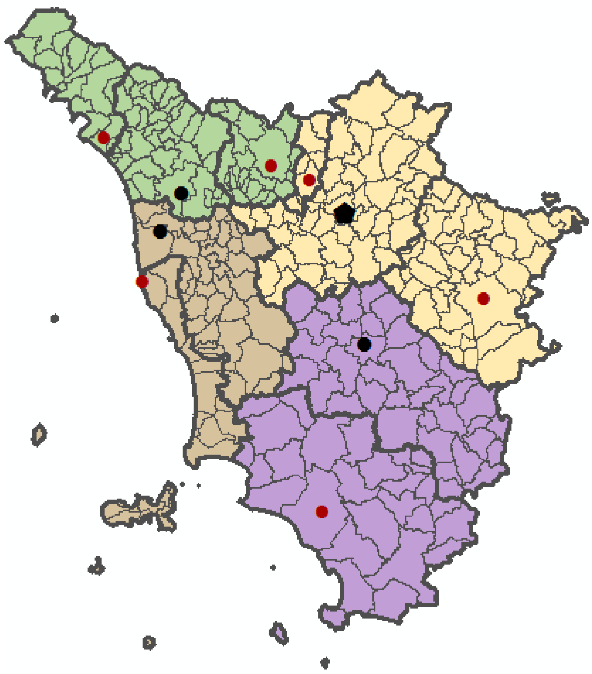


Fig.5.29- Map of OMM Solution (k_{min} <k=6)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Arezzo	56	434.974	4.878,8	89,4
	Firenze	76	1.588.247	5.552,2	88,9
	Pisa	103	1.291.845	6.115,5	117,8
	Grosseto	52	357.136	6.440,5	122,6

Table 5.29- Characteristics of ORM Solution (k_{min} <k=6)

	Districts	TUs	Pop.	Area (km^2)	R_{mj} (km)
	Lucca	74	875.843	3.892,0	113,9
	Firenze	90	1.562.737	7.112,5	159,6
	Pisa	59	746.437	3.658,4	155,1
	Siena	64	487.185	8.324,1	137,9

Table 5.30- Characteristics of OMM Solution (k_{min} <k=6)

The two models select different district to be closed. It is clear that the solution provided by the ORM is much better in terms of provincial radius. Indeed, while in the ORM solution the chief town are in a quite central position within the districts, in the OMM solution the chief towns are in all the cases quite decentralized. As concerns the distribution of the extension across provinces, it results more balanced in the case of ORM; on the contrary, the distribution of population is more balanced in the OMM.

In the following, the optimal solutions provided by the various models are compared on the basis of the above indicators (Table 5.31 a,b). The results are shown considering the decreasing number of districts in the optimal solution.

	m-k	Population (10^3)			*VAR_{pop}* (10^5)	Area (km^2)			*VAR_{sup}* (10^3)
		Avg	**Min**	**Max**		**Avg**	**Min**	**Max**	
Current Configuration	10	378,4	199,7	973,1	2,24	2.130	365,7	4.503,1	1,41
ORM (k=5)	5	734,4	357,1	1.586,0	5,31	4.597	2.585,7	6.440,5	1,52
ORM (k=6)	4	918,1	357,1	1.588,2	6,16	4.879	4.878,8	6.440,5	0,69
OMM (k=6)	4	918,1	487,2	1.562,7	4,59	5.746,8	3.658,4	8.324,1	2,33
ORM (k=7)	3	1.224,1	398,6	1.981,8	7,94	7.662,3	6.115,5	9.392,1	1,65
OMM (k=7)	3	1.224,1	487,2	1.850,6	6,88	7.662,3	6.586,3	8.324,1	0,94
ORM (k=8)	2	1.836,1	1.422,4	2.249,8	5,85	11.493,5	7.243,7	15.743,4	6,01
OMM (k=8)	2	1.836,1	1.334,4	2.337,8	7,09	11.493,5	6.586,3	16.400,7	6,94
PMM=PRM (k=9)	1	3.672,2	3.672,2	3.672,2	-	22.987,0	22.987,0	22.987,0	-

	m-k	Number of territorial Units				Radius (km)			*Hoover Index*
		Avg	**Min**	**Max**		**Avg**	**Min**	**Max**	
Current Configuration	10	29	7	44		81,8	29,9	148,2	27,65
ORM (k=5)	5	57	49	74		102,4	75,2	122,6	57,77
ORM (k=6)	4	72	52	103		104,7	88,9	122,6	55,34
OMM (k=6)	4	72	59	90		141,6	113,9	159,6	22,95
ORM (k=7)	3	96	62	122		133,3	117,8	159,6	43,37
OMM (k=7)	3	96	64	112		150,8	137,9	159,6	22,95
ORM (k=8)	2	144	120	167		185,0	155,1	214,9	14,45
OMM (k=8)	2	144	111	176		185,0	155,1	214,9	7,69
PMM=PRM (k=9)	1	287	287	287		214,9	214,9	214,9	1,00

Tables 5.31a (top) and 5.31b (bottom)–
Comparison of the solutions provided by the models for the case of Toscana Region

5.7 Conclusions

In this chapter, we introduced different approaches that can be used to perform redistricting decisions and we proposed accordingly four class of mathematical models. In particular, the models have been used to address the real problem of the reorder of Italian provinces and tested on some benchmark problems, build on real-world instances derived from Italian regions territorial configurations.

The results obtained on the case study show how models provide solutions with different characteristics and performances, in dependence on the criteria used to tackle closure decisions and reassigning process. Computational results highlight that models can be solved in very limited running times, even for instances of significant size.

Further researches will be addressed at enhancing the models formulations in order to take into account further practical operational aspects. In particular, the issue of the contiguity constraints will be explored, looking for ways of directly incorporating these into the models.

Conclusions and Outlooks

In this work we analyzed the problem of the spatial re-organization of an existing service and formulated some mathematical models in order to support such decisions in the context of a public service. In the OR literature, the two classes of problems mainly used to address location decisions are *facility location* and *districting problems.*

While the mentioned models are mainly oriented to locate new facilities, some occurring circumstances could require strategies oriented to reduce costs and/or improve the system performance; therefore, actions related to the re-organization of the current facilities network may be undertaken. For example, in the public sector, in a general economic and political context characterized by growing cuts to public expenditure and a review process of the welfare state, public services have and are still undergoing significant transformations, generally oriented to reduce administrative, managerial and operational burden and costs.

In order to re-organize an existing supply system different strategies could be adopted, such as the closure of some active facilities, their repositioning in different points of the location space, the downsizing of the capacities of the available services and so on. Each re-organization action perturb the interaction between the facilities and the demand, and could produce some effects that should be carefully evaluated.

The contents have been sub-divided according to three Sections: the first Section was focused on the analysis of the context (Chapter 1) and of the state of the art (Chapter 2); the second Section was devoted to the illustration of new facility location models for the spatial re-organization of a given service network (Chapter 3) and their application to real case studies (Chapter 4); then, in the last Section, we provided specific versions of districting models that can be used as support in decisions about spatial re-organization.

Various contributions have been proposed concerning the problem of the *spatial re-organization of a facility system*, i.e. the problem of modifying a set of pre-existing facilities, either in terms of their number (shrinking/expansion), or their position (relocation), or in terms of the capacities of the single offered services (downsizing). The analysis of the literature (Chapter 2) revealed that the re-organization has been traditionally intended as an opportunity for the decision maker to adapt the system to some changed (or changing) conditions, such as demand distribution. Therefore, the aim of such problems is to re-organize the system so as to improve as more as possible its efficiency, usually looking for a trade-off solution between the cost due to the modification of the current configuration (opening, closing, downsizing, relocating facilities) and the saving associated to such process. However it emerged that little attention was paid to the fact that the spatial re-organization of an existing supply system may be motivated also by other circumstances. In the public sector, for example, reductions to public expenditure may impose strategies oriented to the rationalization of public services, such as education, health, administration. Therefore, in these cases, the problem is completely different as any re-organization action produces some side effects, such as the increase of costs faced by users (in terms of accessibility to the service) and, potentially, the worsening of the quality of the offered service (measurable in terms of drop of coverage, worsening of users' satisfaction and congestion of the remaining facilities). Hence, to effectively solve these kinds of problems, decision support models should be able to find a trade-off solution between two inherently conflicting goals: the maximization of the benefit from the planner perspective and the minimization of the damage from the users perspective.

In this context, decisions may depend on various factors such as the nature of the service and the characteristics of the market (competitive or non-competitive), the objectives to be achieved and the constraints to be satisfied. In this sense, it could be useful and interesting to develop appropriate tools to support this kind of decisions.

In this context the objective of the second Session was the proposal of a general theoretical framework including facility location models for exploring re-organization decisions about facilities in a non-competitive context (Chapter 3). The models assumed the presence of a set of facilities offering different types of services and gave the possibility to implement different actions regarding the single service at a given facility (i.e. *closure*, *reduction, expansion* or *transformation*) or a whole facility (*closure, merging*). The implementation of any action produces some side effects on the users, that have to carefully evaluated; in particular, whenever a service is closed and/or downsized the interaction between users and facilities is perturbed as the demand needs to be re-allocated. For this reason different demand re-allocation rules were considered either deterministic (re-allocation to the closest available facility) or probabilistic (users are attracted by the remaining facilities according their preferences). The objective function was represented by the extra-cost to be paid in order to host the re-allocated demand while constraints expressing the need of obtaining a target benefit from the supply reduction were included. Within the various versions, a specific one was deeply analyzed and tested on a set of randomly generated instances. The obtained results show that it is possible to optimally solve a significant ranges of instances using a commercial solver (CPLEX). The proposed models were then applied to analyze two real-world problems. (Chapter 4). In particular they were applied to solve problems concerning the re-organization of a public university system and of a school system located in a given region, considering specific requirements indicated by the Italian government for the rationalization of these kinds of contexts.

In the third Section, we explored the re-organization issues in the context of districting problems (Chapter 5). A re-districting problem aims at redefining the current partition of elementary units into districts in a given study region, according to a set of planning criteria. Such problems may arise when the current organization does not satisfy some efficiency requirements or when there is the exigency to modify the actual number of districts. With reference to such situations, we proposed a general framework of mathematical programming models based on different approaches. In particular, the models were applied to the real case of the reorder of Italian provinces and tested on some benchmark problems, build on real-world instances derived from Italian regions territorial configurations. The obtained results on the case study show how models provide solutions with different characteristics and performances, in dependence on the criteria used to tackle closure decisions and reassigning process. Computational tests on the benchmark problems highlight that models can be solved in very limited running times, even for instances of significant size.

The obtained results provided either by facility location and by districting based models show that the use of mathematical models can actually represent a suitable and reliable support for these kinds of problems.

Further researches may be follow different research paths. First of all further efforts may be paid to enhance models' formulations in order to take into account other practical operational aspects or, in the case of districting models, some theoretical requirements (i.e. contiguity constraints).

Another interesting perspective may be represented by the analysis of further application fields in the public sector (health care, justice) in which these kinds of models can be fruitfully used.
Furthermore, multi-period version of the problem can be explored in order to implement progressively the modifications to the existing system. Indeed, the main shortcoming of a static approach, in which decisions are taken in a single-period and implemented in the same moment, is the sudden change of the district map, that results in a *non-stability* from both planner and user perspectives. Actually, we are working on a multi-period version of re-districting models in which decisions are made over time, so as to modify the current partition in an incremental way and make the changes gradual.

Appendix A

In the following we prove that the combination of the three sets of constraints (3.3), (3.4), (3.14) guarantee that fractions of demand assume the same values defined by the not linear expressions (3.13).

For each service k, consider the following subsets of J:

A_k subset of facilities that did not provide service k $\left(A_k = \{j \in J: l_{kj} = 0\}\right)$;

B_k subset of facilities at which service k has been closed$\left(B_k = \{j \in J: l_{kj} = 1, s_{kj} = 1\}\right)$;

C_k subset of facilities that still provide service k $\left(C_k = \{j \in J: l_{kj} = 1, s_{kj} = 0\}\right)$.

Note that the above introduced subsets form a partition of J; indeed, B_k and C_k form a partition of the set of facilities providing k ($B_k \cup C_k = U_k, B_k \cap C_k = \emptyset$) and A_k is the complementary set of U_k with reference to J ($A_k = J - U_k$).

The equivalence between (3.3,3.4,3.14) and (3.13) is trivially proved for the facilities $j \in A_k \cup B_k$. Indeed, conditions (3.3) impose:

$$x_j^{ik} = 0 \qquad \forall i \in I, \forall k \in K, \forall j \in A_k \cup B_k$$

equivalently to (3.12), being respectively in A_k and B_k $\alpha_j^{ik} = 0$ and $s_{kj} = 1$.

Then, it has to be proved only for facilities $j \in C_k$.

Conditions (3.14), for each service k, define a proportional relationship between the fractions of demand assigned to each pair of facilities j and t belonging to $U_k = B_k \cup C_k$.

For each pair $(j, t) \in U_k \times U_k$, one of the following conditions can occur:

a. $j \in C_k, t \in B_k$: facility j provides service k ($s_{kj} = 0$); while t not anymore ($s_{kt} = 1$);

b. $j \in B_k, t \in C_k$:facility t provides service k ($s_{kt} = 0$), but j not anymore ($s_{kj} = 1$);

c. $j, t \in B_k$: service k has been closed at both facilities j and t ($s_{kt} = s_{kj} = 1$);

d. $j, t \in C_k$: : service k is still provided by both the facilities j and t ($s_{kt} = s_{kj} = 1$).

It is possible to demonstrate that conditions (3.14) become active only in the last case.

With this aim, consider the paired conditions, associated to (j, t):

$$\begin{cases} x_j^{ik} \le \frac{\alpha_j^{ik}}{\alpha_t^{ik}} x_t^{ik} + s_{kt} \\ x_t^{ik} \le \frac{\alpha_t^{ik}}{\alpha_j^{ik}} x_j^{ik} + s_{kj} \end{cases} \qquad \forall i \in I$$

From Table A.1, in which the expressions of the above conditions in the single cases are reported, it is easy to understand that in the first three cases the constraints are trivially satisfied $\forall i \in I$.

a	*b*	*c*	*d*
$\begin{cases} x_j^{ik} \le 1 \\ 0 \le \frac{\alpha_t^{ik}}{\alpha_j^{ik}} x_j^{ik} \end{cases}$	$\begin{cases} 0 \le \frac{\alpha_j^{ik}}{\alpha_t^{ik}} x_t^{ik} \\ x_t^{ik} \le 1 \end{cases}$	$\begin{cases} 0 \le 1 \\ 0 \le 1 \end{cases}$	$\begin{cases} x_j^{ik} \le \frac{\alpha_j^{ik}}{\alpha_t^{ik}} x_t^{ik} \\ x_t^{ik} \le \frac{\alpha_t^{ik}}{\alpha_j^{ik}} x_j^{ik} \end{cases}$

Table A.1 – Possible expressions of conditions (2.13) for a generic pair $(j, t) \in U_k \times U_k$

In the case *d* the two conditions become equivalent to the following one:

$$x_t^{ik} = \frac{\alpha_t^{ik}}{\alpha_j^{ik}} x_j^{ik} \qquad \forall i \in I$$

Then, for a particular user i and service k, it is possible to express all the fractions of the demand assigned to the facilities in $C_k = \{t_1, \dots, t_c\}$ as a function of the same fraction x_j^{ik} $(j \in C_k)$. Therefore, replacing in the (3.4), we have:

$$\sum_{j \in J} x_j^{ik} = \sum_{j \in A_k} x_j^{ik} + \sum_{j \in B_k} x_j^{ik} + \sum_{j \in C_k} x_j^{ik} = \sum_{j \in C_k} x_j^{ik} = x_{t_1}^{ik} + \dots + x_{t_c}^{ik}$$

$$= \left(\frac{\alpha_{t_1}^{ik}}{\alpha_j^{ik}} + \dots + \frac{\alpha_{t_c}^{ik}}{\alpha_j^{ik}} \right) x_j^{ik} = 1$$

hence:

$$x_j^{ik} = \frac{\alpha_j^{ik}}{\alpha_{t_1}^{ik} + \dots + \alpha_{t_c}^{ik}} = \frac{\alpha_j^{ik}}{\sum_{t \in C_k} \alpha_t^{ik}} \tag{A.1}$$

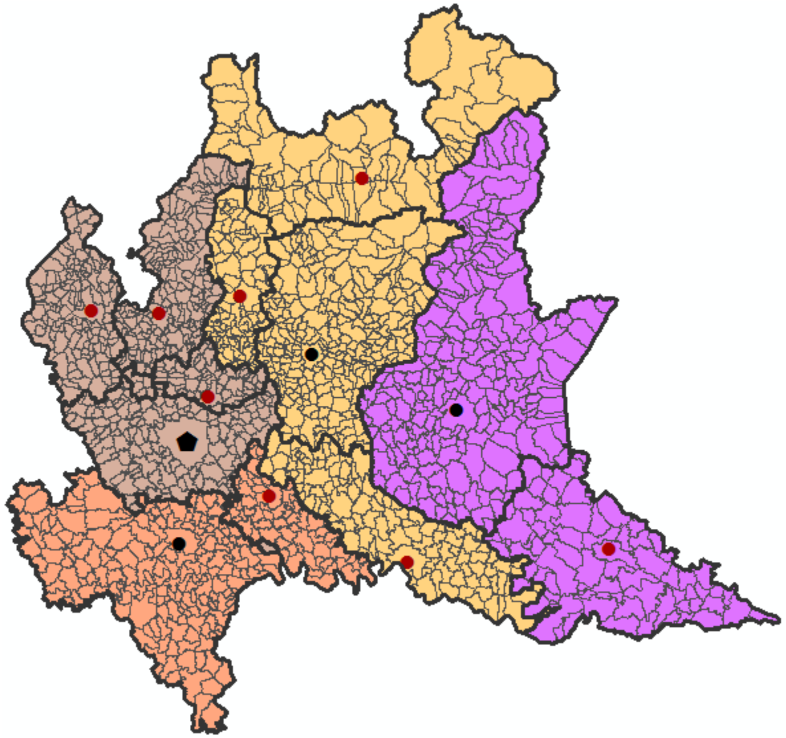
For a given service k, equation (A.1) holds for all the facilities in the set C_k and each user i; then we can generally write:

$$x_j^{ik} = \frac{\alpha_j^{ik}}{\sum_{t \in C_k} \alpha_t^{ik}} \qquad \forall i \in I, \forall k \in K, \forall j \in C_k \tag{A.2}$$

which is equivalent to (3.13) $\forall j \in C_k$.

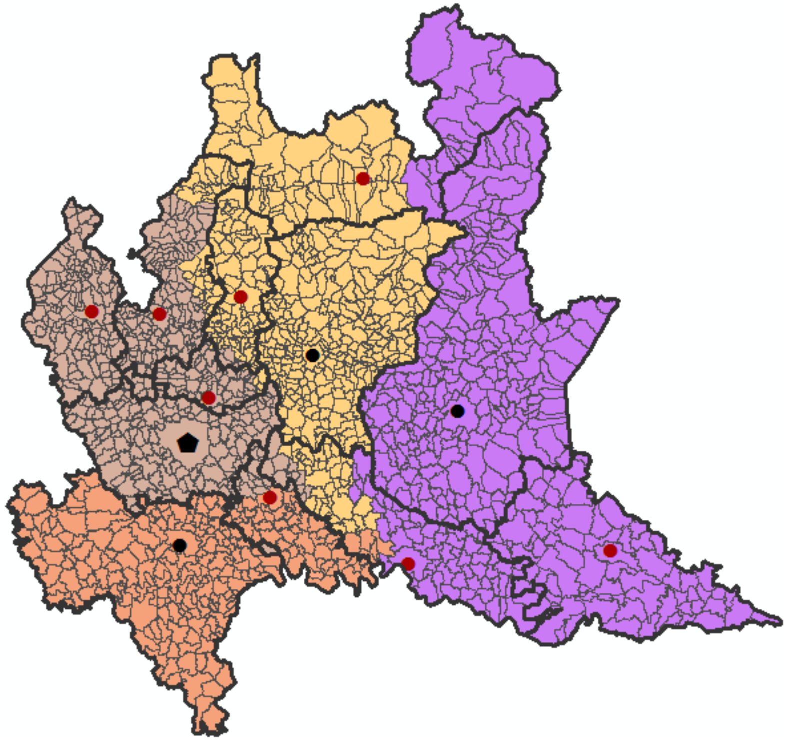
Appendix B

B.1 Lombardia Region



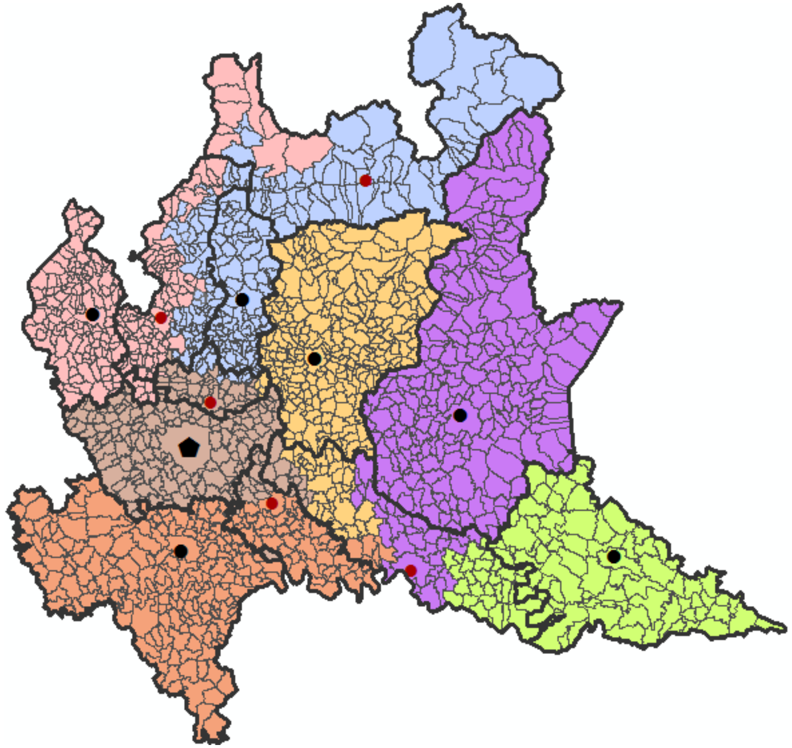
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	490	5.337.170	4.458,22	119,7
	Bergamo	527	1.961.024	8.526,74	179,6
	Brescia	276	1.646.380	7.127,07	137,2
	Pavia	251	759.577	3.751,63	76,4

PMM



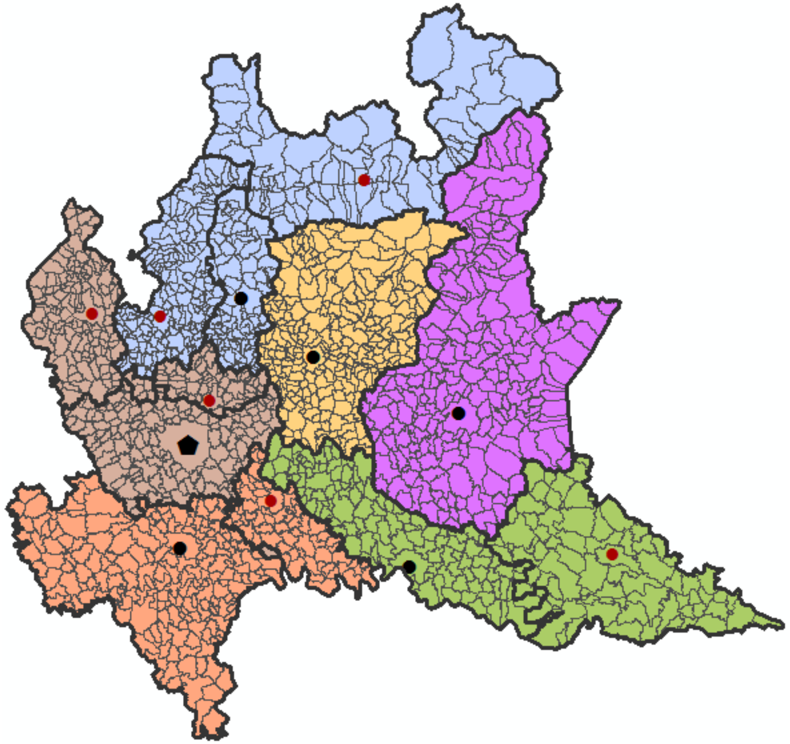
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	482	5.389.647	4.371,51	119,67
	Bergamo	463	1.703.223	6.187,80	127,70
	Brescia	356	1.879.153	9.575,82	175,77
	Pavia	243	732.128	3.728,52	76,39

PRM



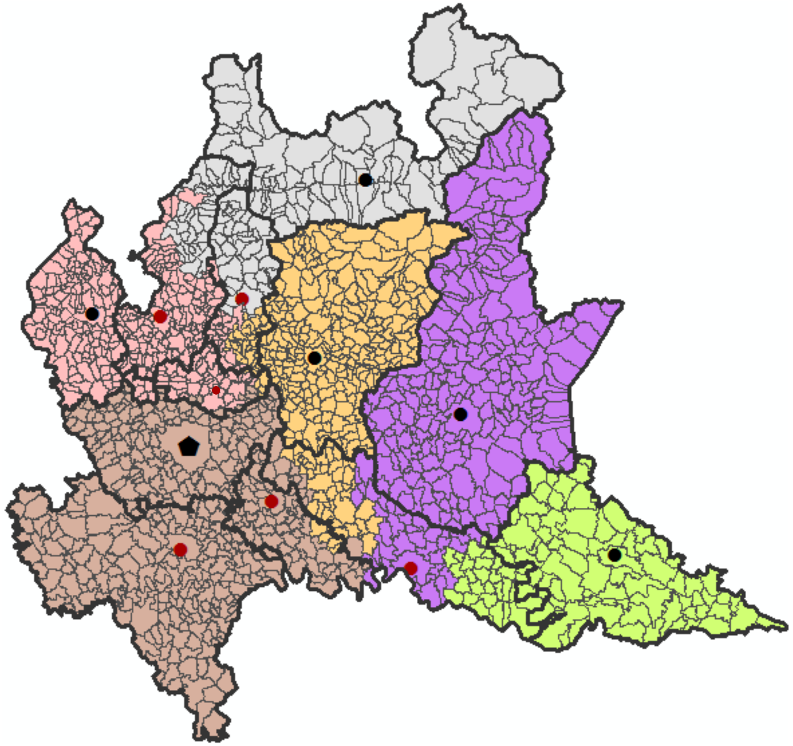
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Varese	242	1.184.360	2.503,11	165,0
	Milano	196	3.824.491	2.140,51	59,2
	Bergamo	287	1.311.593	3.237,54	97,9
	Brescia	238	1.366.144	5.378,33	114,7
	Pavia	243	732.128	3.728,52	76,4
	Mantova	100	459.659	2.850,18	64,5
	Lecco	238	825.776	4.025,45	156,5

ORM- $k=k_{min}=5$



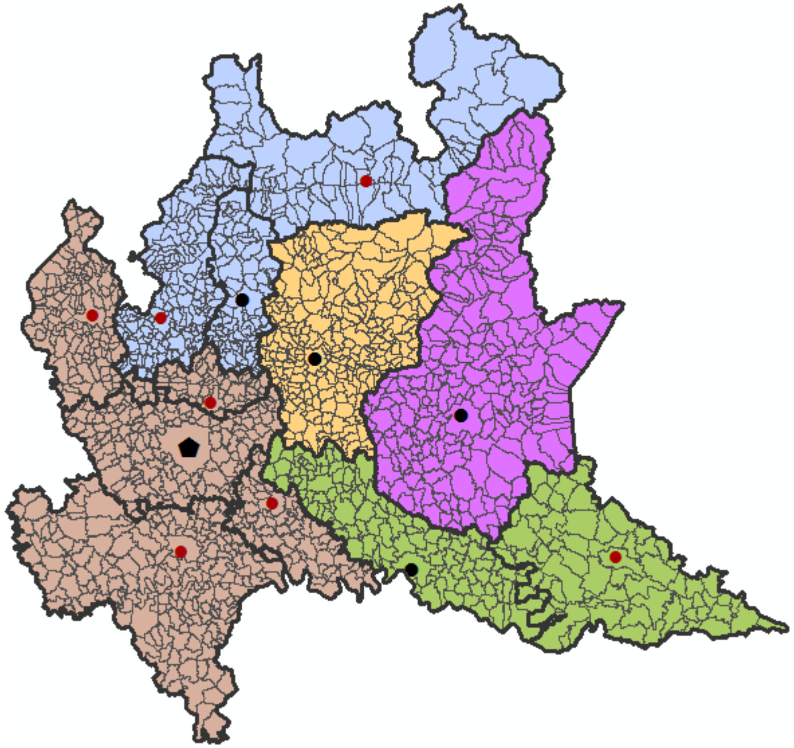
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	330	4.750.435	3.179,2	119,7
	Bergamo	244	1.086.277	2.745,9	65,6
	Brescia	206	1.238.044	4.785,6	114,7
	Pavia	251	759.577	3.751,6	76,4
	Cremona	185	765.959	4.111,9	123,8
	Lecco	328	1.103.859	5.289,4	156,5

OMM-k=k_{min}=6



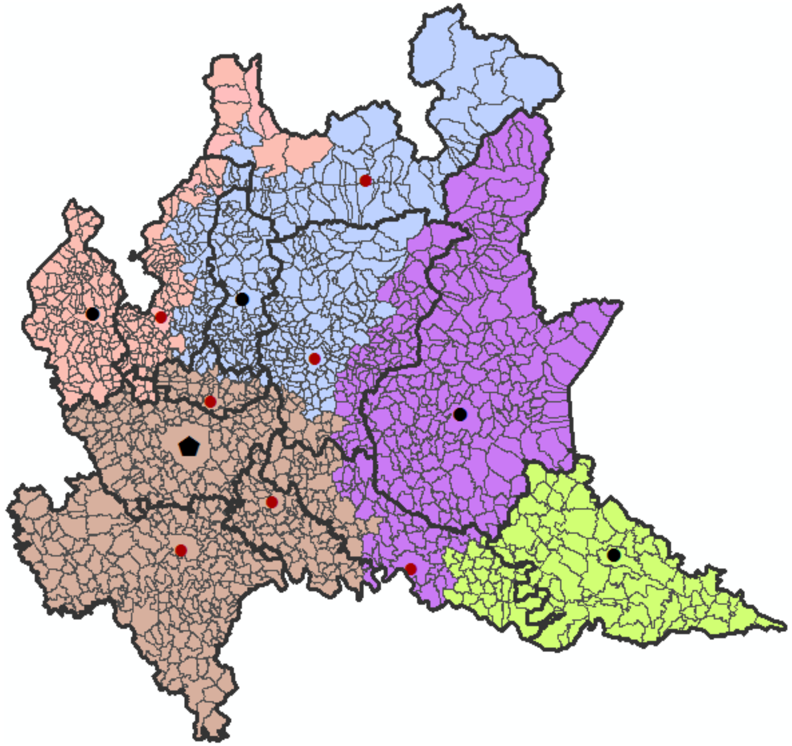
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Varese	333	2.190.396	2.504,57	83,2
	Sondrio	148	350.182	4.147,30	96,6
	Milano	402	3.981.914	5.490,05	109,6
	Bergamo	319	1.349.125	3.424,24	65,9
	Brescia	242	1.372.875	5.447,30	114,7
	Mantova	100	459.659	2.850,18	64,5

ORM-k =6



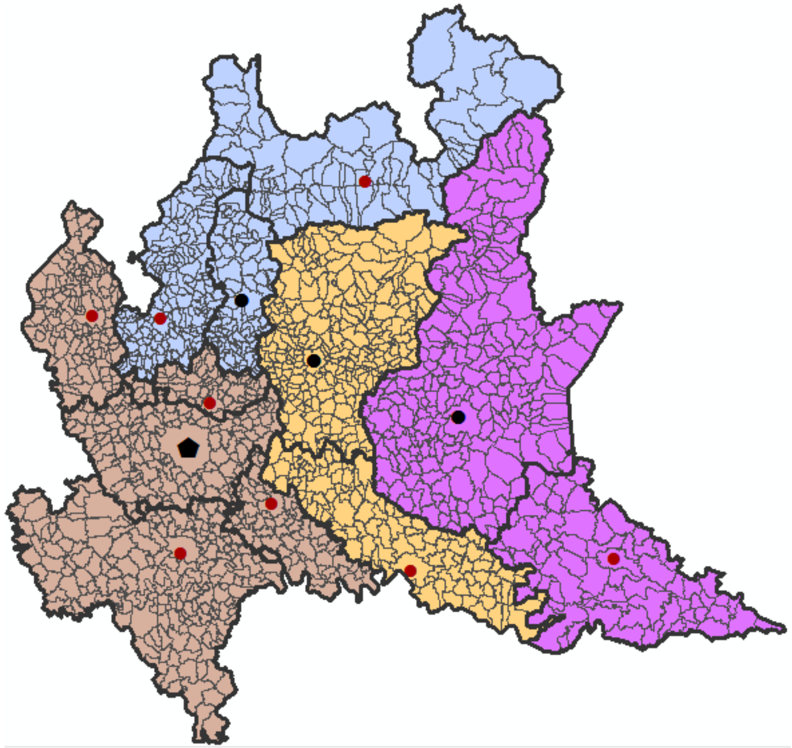
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	581	5.510.012	6.930,81	119,7
	Bergamo	244	1.086.277	2.745,94	65,6
	Brescia	206	1.238.044	4.785,62	114,7
	Cremona	185	765.959	4.111,90	123,8
	Lecco	328	1.103.859	5.289,38	156,5

OMM-k =7



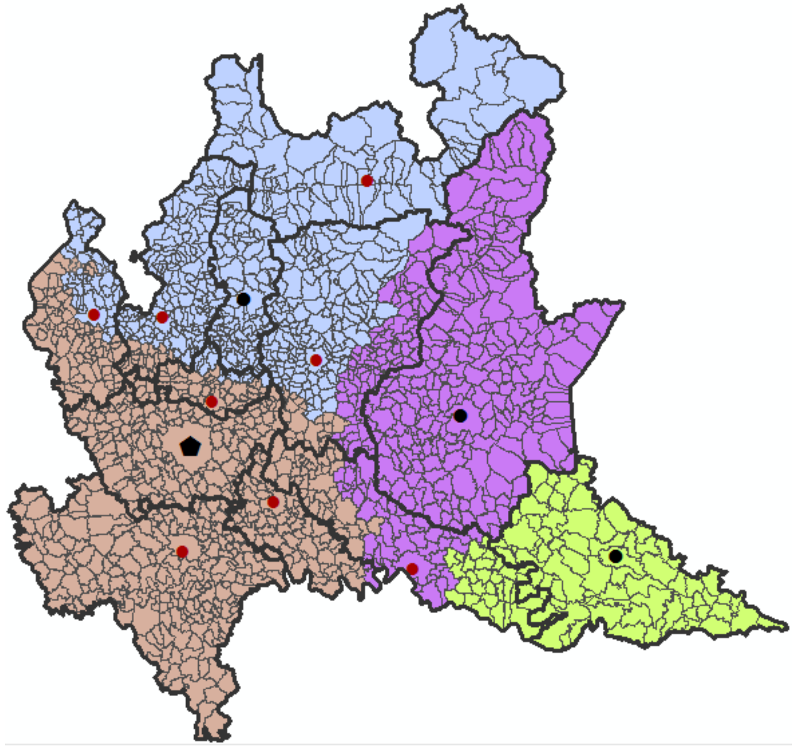
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Varese	238	1.248.231	2.501,8	165,0
	Milano	488	4.810.285	6.338,5	109,6
	Brescia	329	1.643.704	6.418,5	114,7
	Mantova	100	459.659	2.850,2	64,5
	Lecco	389	1.542.272	5.754,7	156,5

ORM-k =7



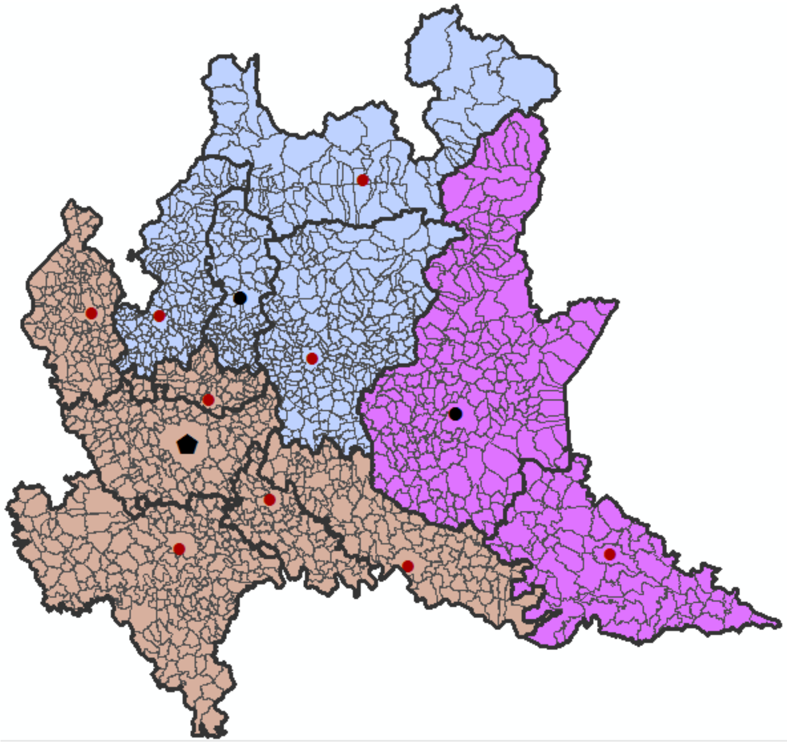
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	581	5.510.012	6.930,81	119,7
	Bergamo	359	1.443.900	4.516,40	142,7
	Brescia	276	1.646.380	7.127,07	137,2
	Lecco	328	1.103.859	5.289,38	156,5

OMM-k =8



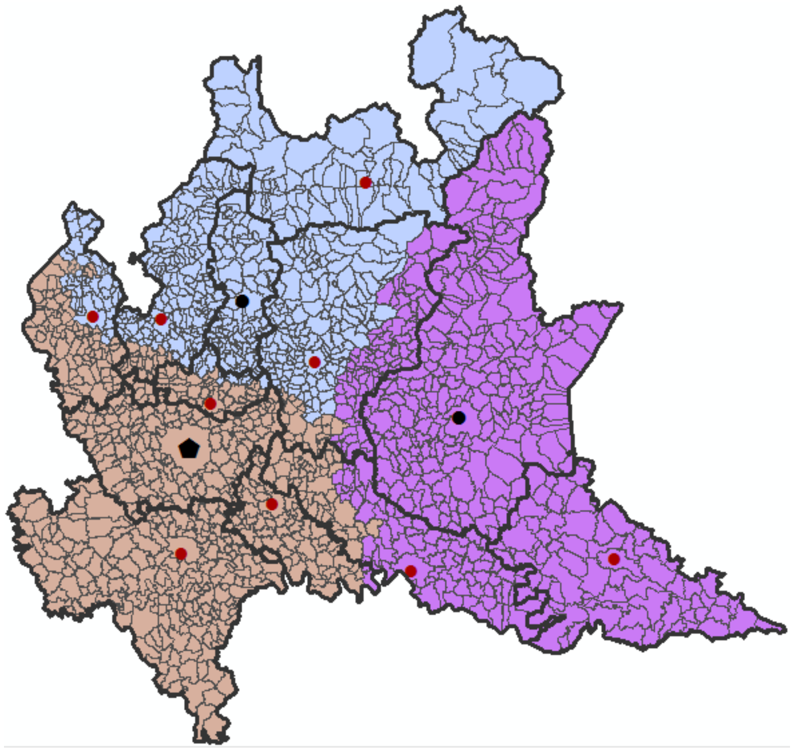
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	597	5.548.540	7.238,8	109,6
	Brescia	329	1.643.704	6.418,5	114,7
	Mantova	100	459.659	2.850,2	64,5
	Lecco	518	2.052.248	7.356,1	156,5

ORM-k =8



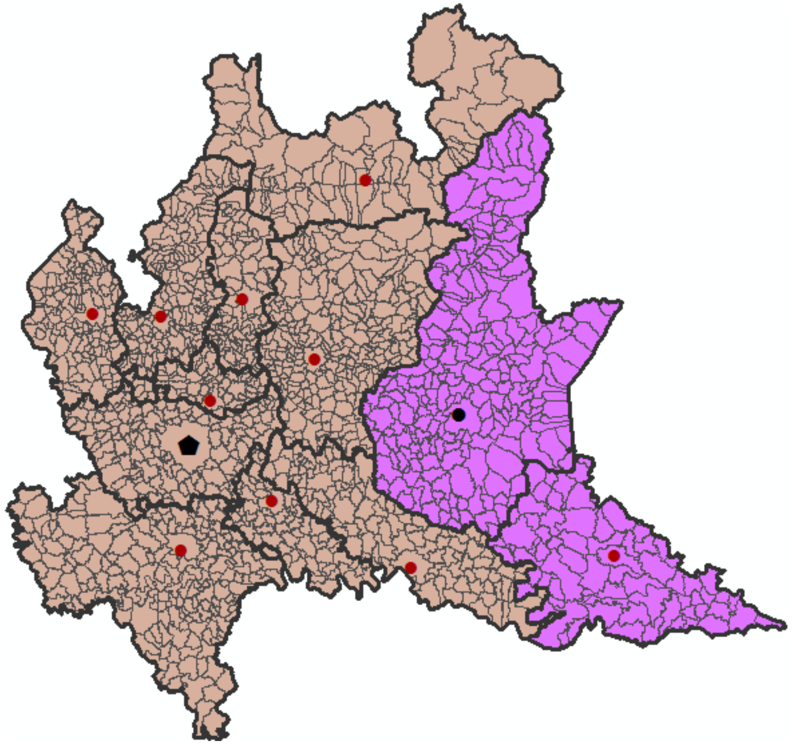
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	696	5.867.635	8.701,27	149,9
	Brescia	276	1.646.380	7.127,07	137,2
	Lecco	572	2.190.136	8.035,31	156,5

OMM-k =9



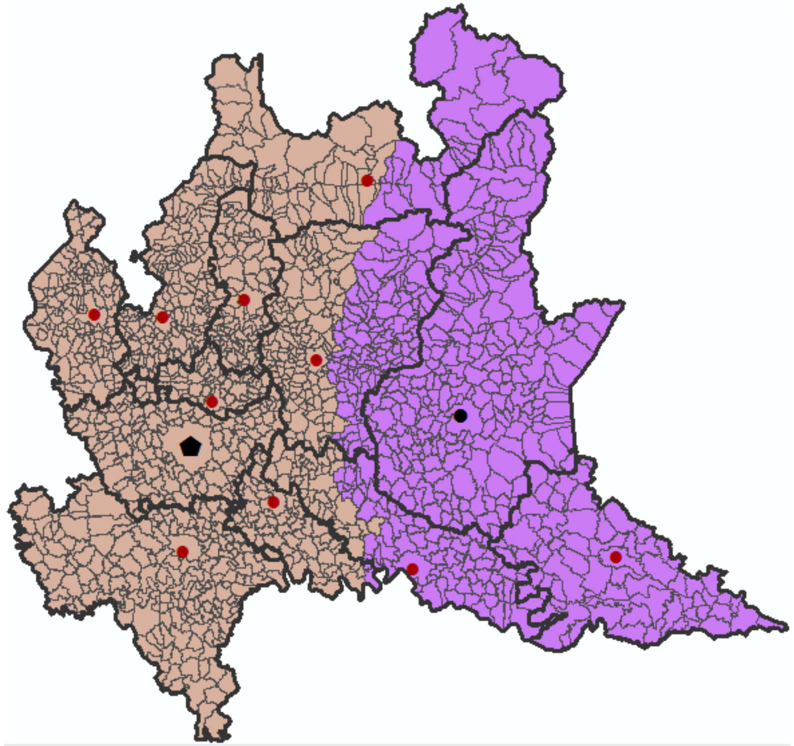
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	596	5.546.107	7.229,7	109,6
	Brescia	429	2.103.363	9.268,7	137,2
	Lecco	519	2.054.681	7.365,3	156,5

ORM-k =9



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	1.268	8.057.771	16.736,59	213,9
	Brescia	276	1.646.380	7.127,07	137,2

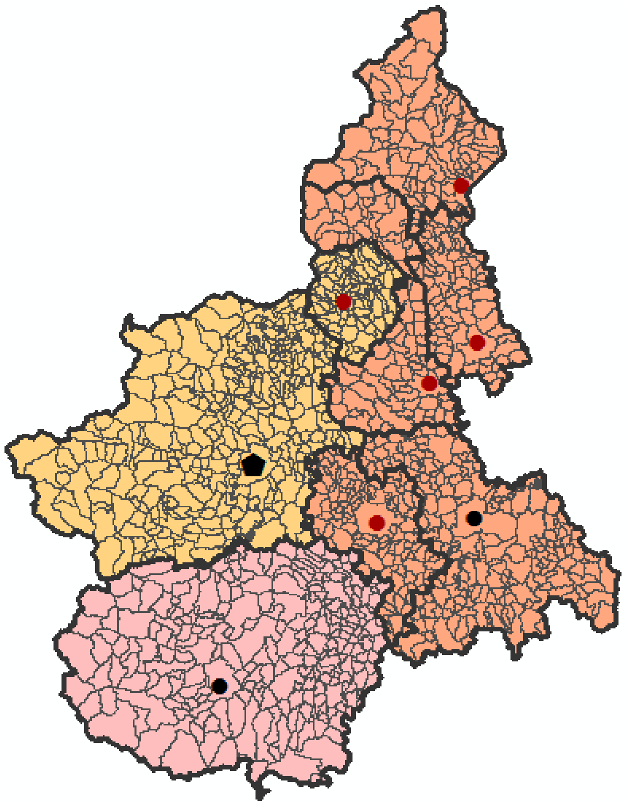
OMM-k =10



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Milano	1.055	7.375.477	12.475,0	191,4
	Brescia	489	2.328.674	11.388,7	175,8

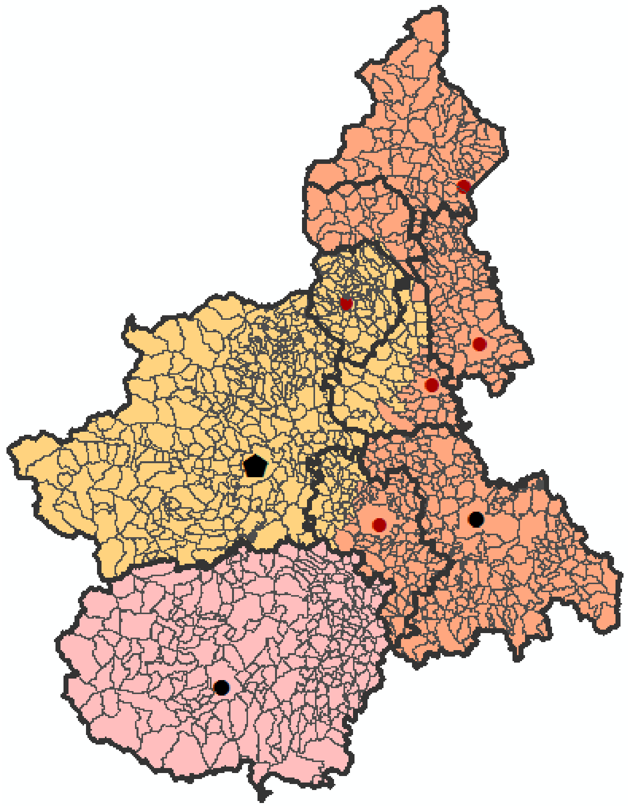
ORM-k =10

B.2 Piemonte Region



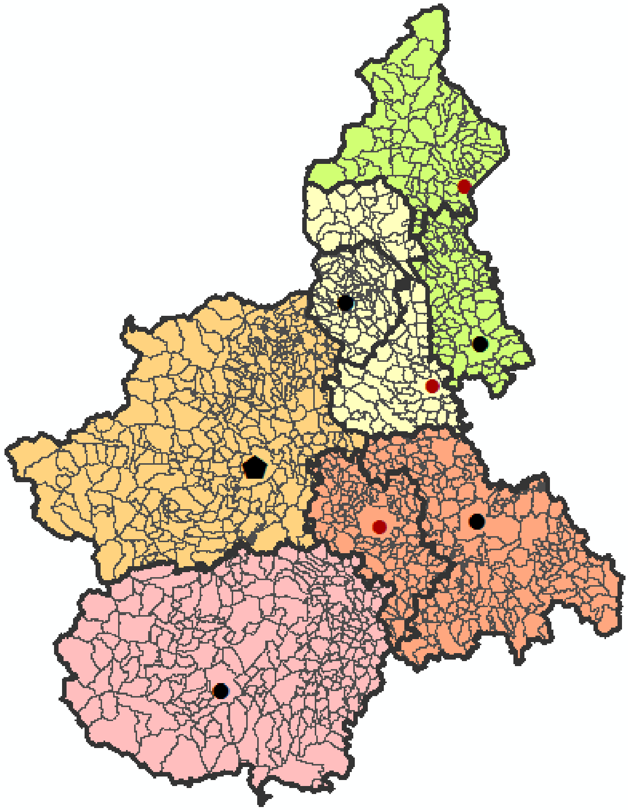
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	397	2.422.960	7.730,6	101,4
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	559	1.347.566	10.751,8	206,3

PMM



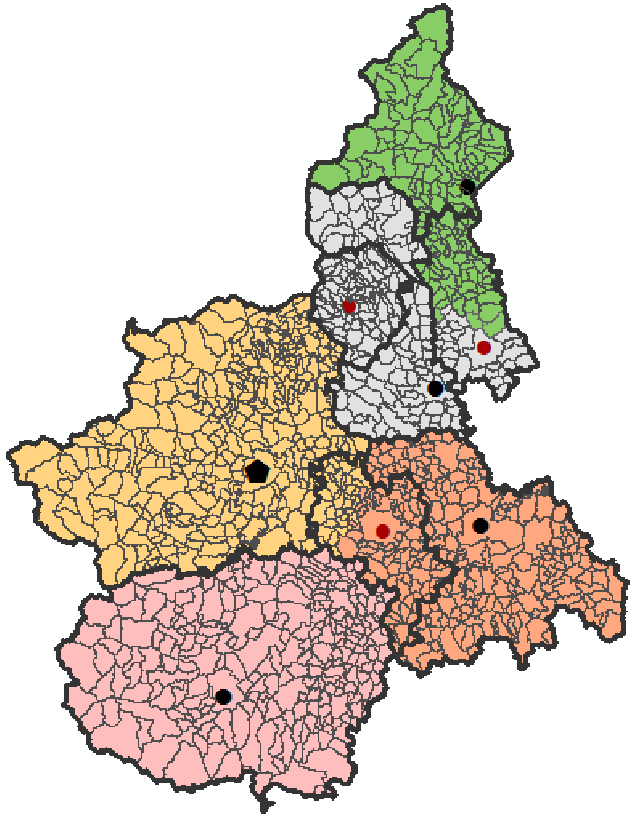
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	467	2.522.388	8.965,8	102,9
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	489	1.248.138	9.516,6	206,3

PRM



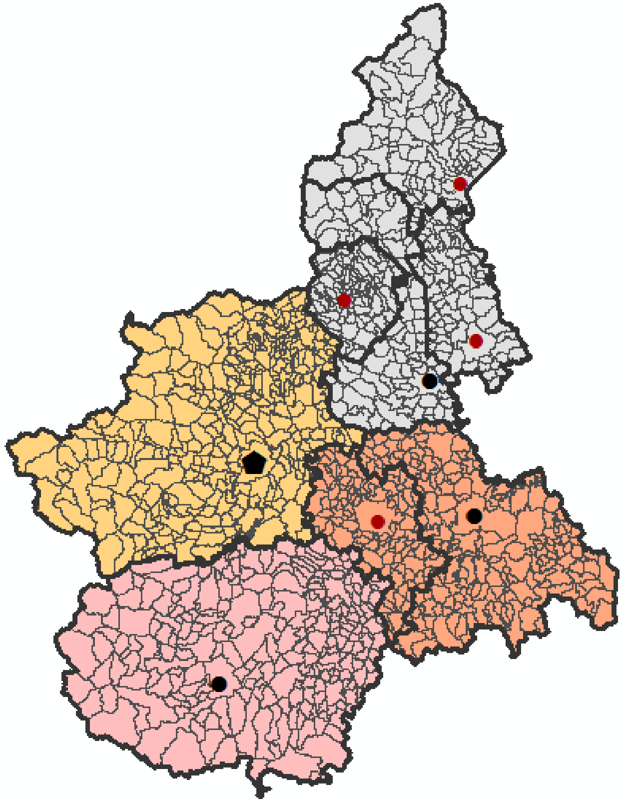
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	315	2.240.768	6.817,3	99,6
	Novara	165	525.823	3.601,2	155,4
	Biella	168	359.133	2.994,9	88,8
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	308	644.802	5.069,0	83,4

OMM- $k=k_{min}=3$



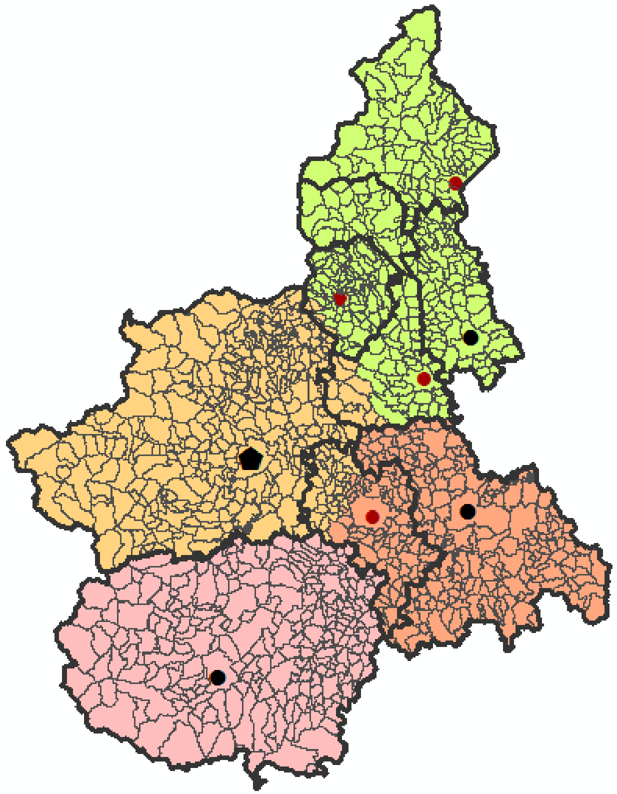
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Verbania	141	355.109	3.085,0	154,8
	Torino	347	2.275.353	7.167,4	99,6
	Vercelli	197	534.011	3.577,7	105,5
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	271	606.053	4.652,4	70,7

ORM- $k=k_{min}=3$



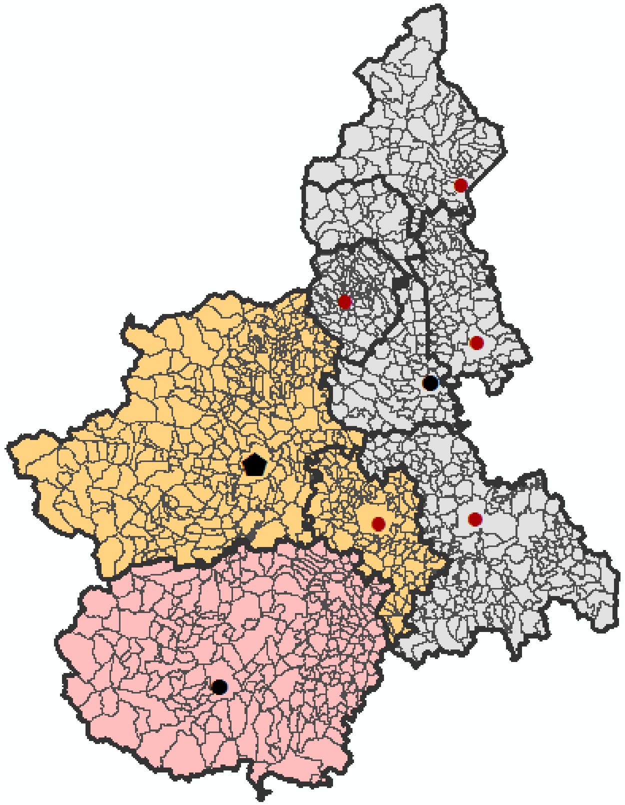
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	315	2.240.768	6.817,3	99,6
	Vercelli	333	884.956	6.596,1	158,5
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	308	644.802	5.069,0	83,4

OMM-k=4



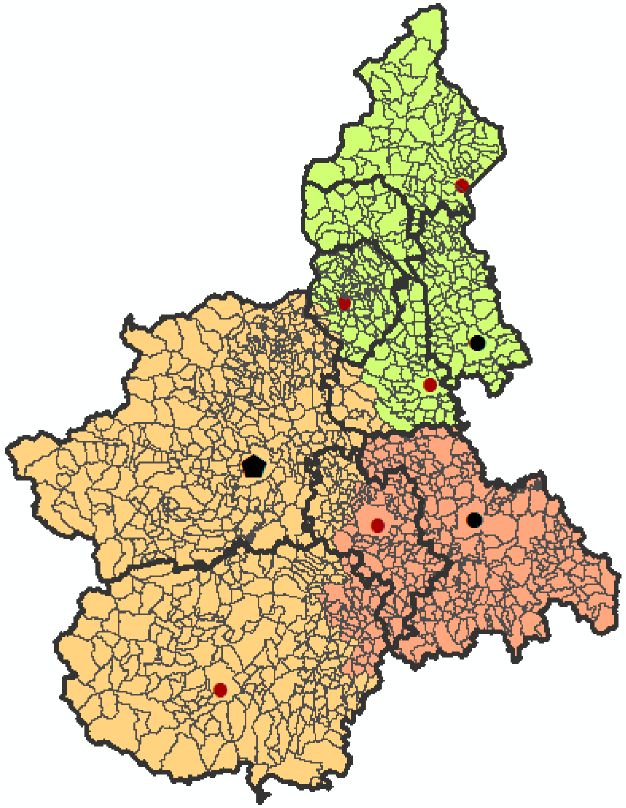
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	368	2.315.474	7.619,1	99,6
	Novara	316	848.195	6.199,3	155,4
	Cuneo	250	586.378	6.894,9	114,4
	Alessandria	272	606.857	4.664,1	70,7

ORM-k =4



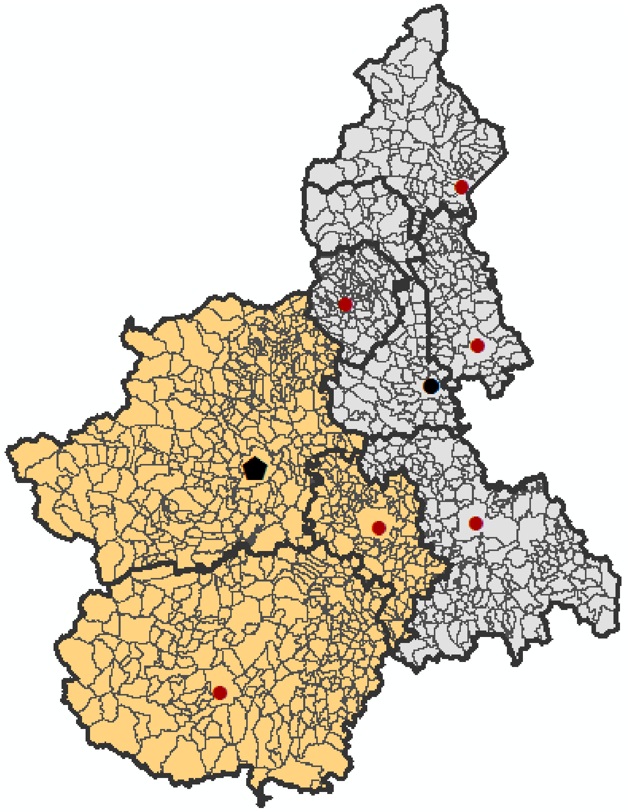
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	433	2.458.341	8.327,5	126,7
	Vercelli	523	1.312.185	10.154,9	158,5
	Cuneo	250	586.378	6.894,9	114,4

OMM-k =5



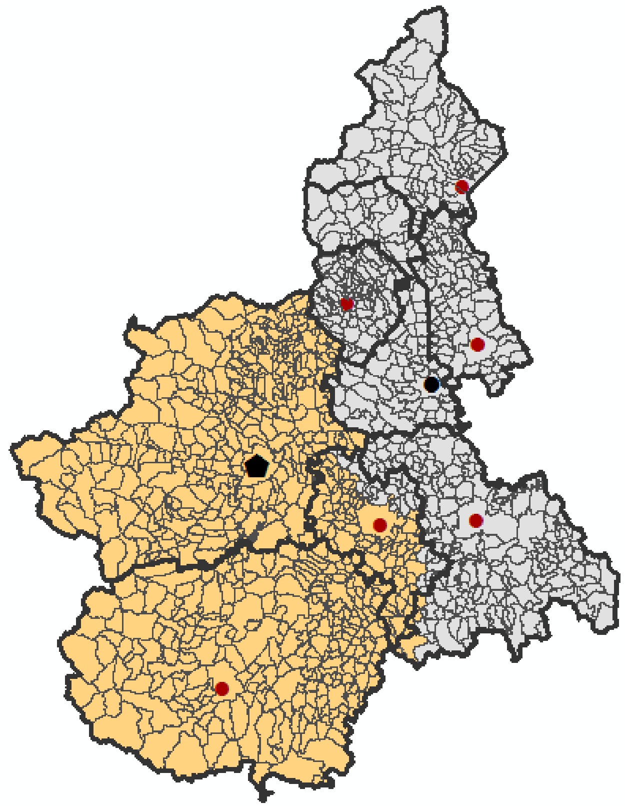
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	574	2.832.523	13.957,6	157,0
	Novara	316	848.195	6.199,3	155,4
	Alessandria	316	676.186	5.220,4	107,7

ORM-k =5



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	683	3.044.719	15.222,4	157,0
	Vercelli	523	1.312.185	10.154,9	158,5

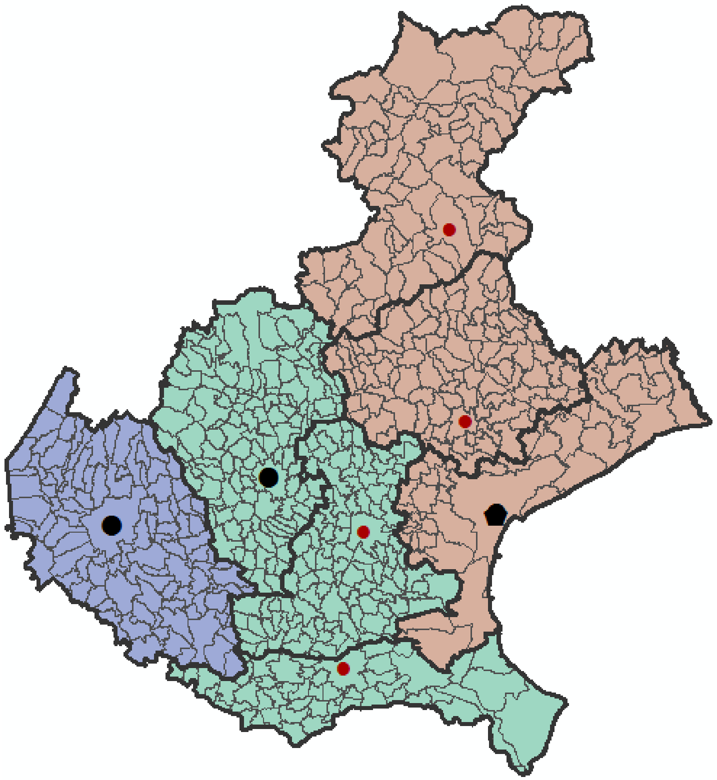
OMM-k =6



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Torino	654	3.017.466	14.948,4	157,0
	Vercelli	552	1.339.438	10.429,0	158,5

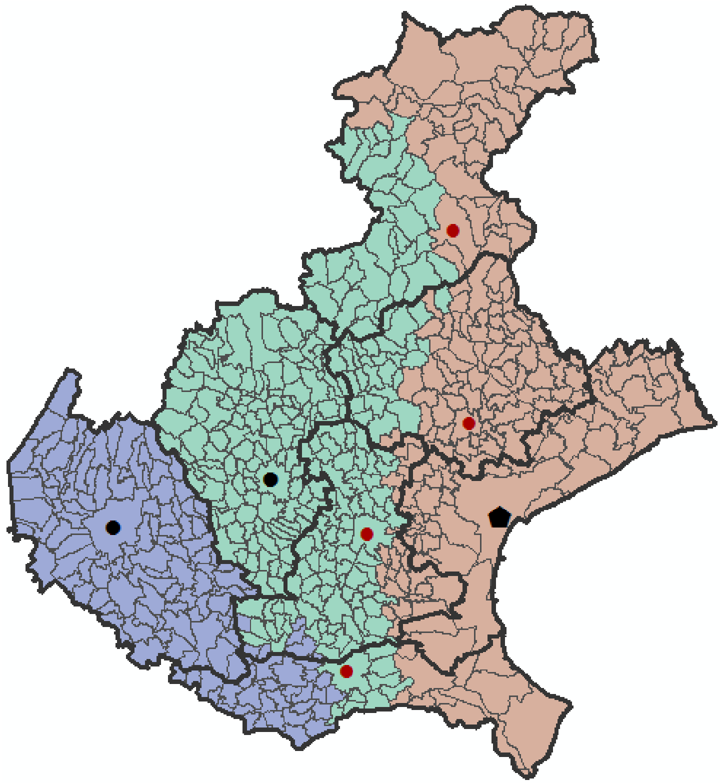
ORM-k =6

B.3 Veneto Region



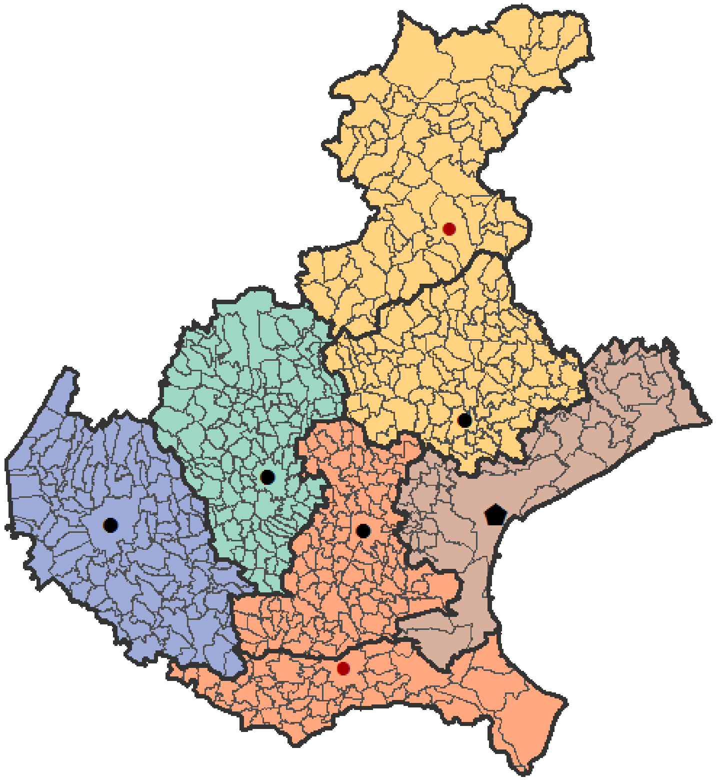
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Verona	98	900.542	3.096,4	65,4
	Vicenza	275	2.022.915	6.686,0	124,3
	Venezia	208	1.933.753	8.625,0	165,1

PMM



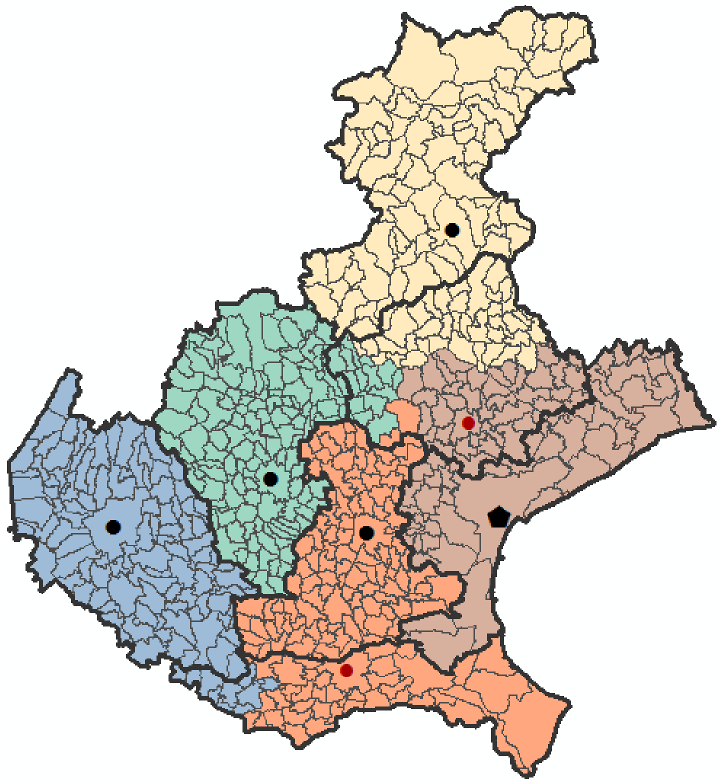
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Verona	134	1.003.549	3.862,1	90,5
	Vicenza	263	1.972.077	6.627,1	147,5
	Venezia	184	1.881.584	7.918,3	165,1

PRM



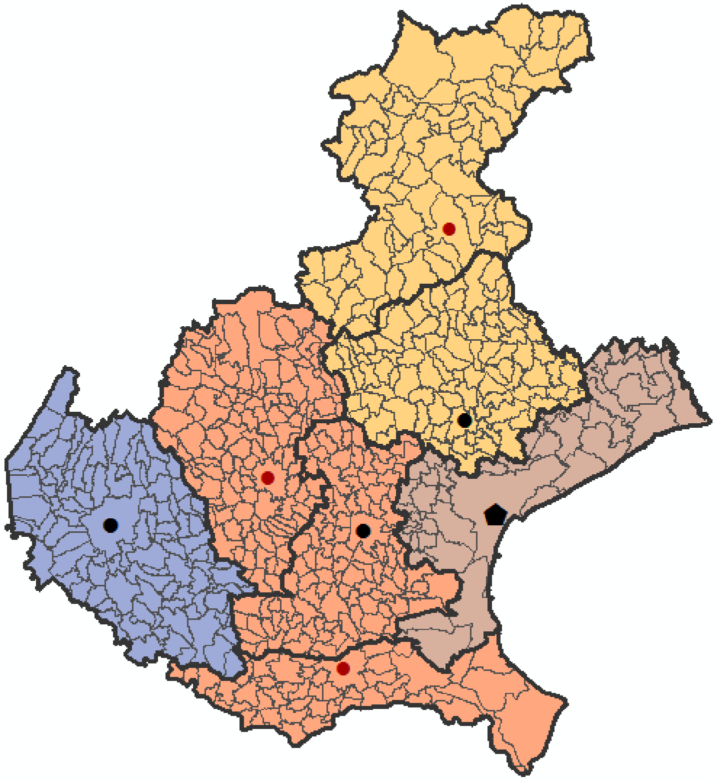
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Verona	98	900.542	3.096,4	65,4
	Vicenza	121	859.205	2.722,5	75,9
	Treviso	164	1.086.791	6.152,1	134,1
	Venezia	44	846.962	2.472,9	93,6
	Padova	154	1.163.710	3.963,5	100,9

OMM- $k=k_{min}=2$



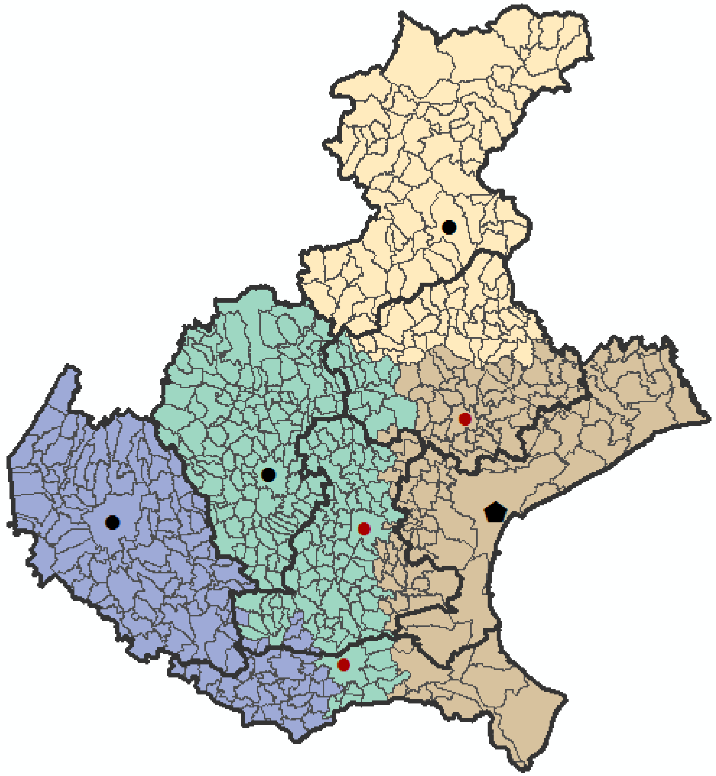
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Verona	108	922.251	3.303,2	78,0
	Vicenza	135	976.036	3.045,3	75,9
	Belluno	107	475.074	4.621,7	76,0
	Venezia	85	1.316.124	3.593,8	93,6
	Padova	146	1.167.725	3.843,4	79,5

ORM- $k=k_{min}=2$



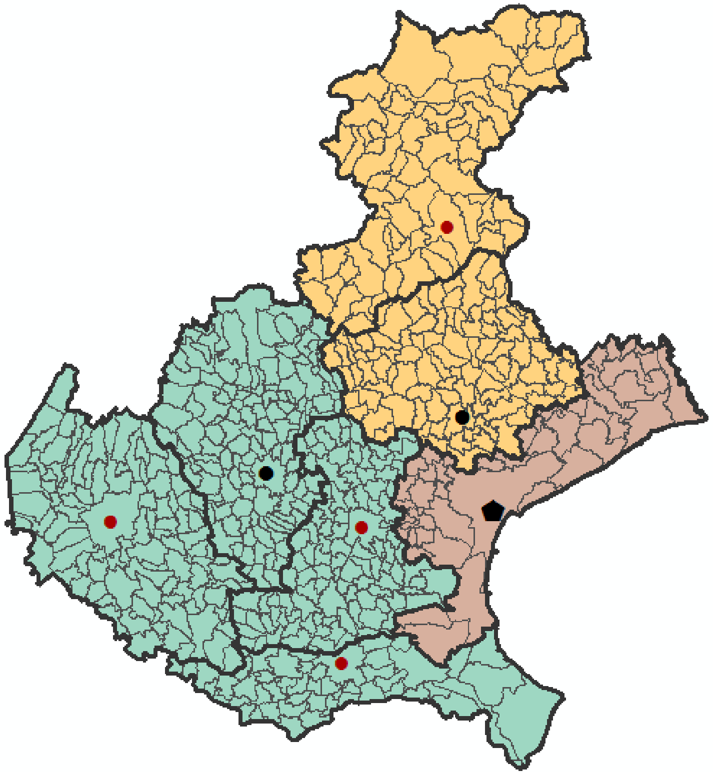
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Verona	98	900.542	3.096,4	65,4
	Treviso	164	1.086.791	6.152,1	134,1
	Padova	275	2.022.915	6.686,0	100,9
	Venezia	44	846.962	2.472,9	93,6

OMM-k=3



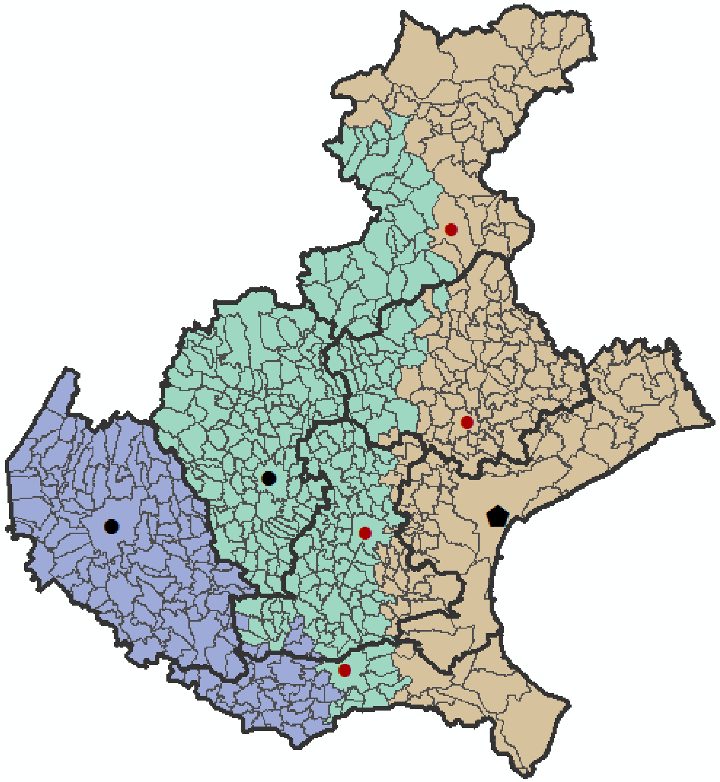
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Verona	134	1.003.549	3.862,1	90,5
	Vicenza	221	1.819.893	4.984,6	94,4
	Belluno	107	475.074	4.621,7	76,0
	Venezia	119	1.558.694	4.939,0	93,6

ORM-k =3



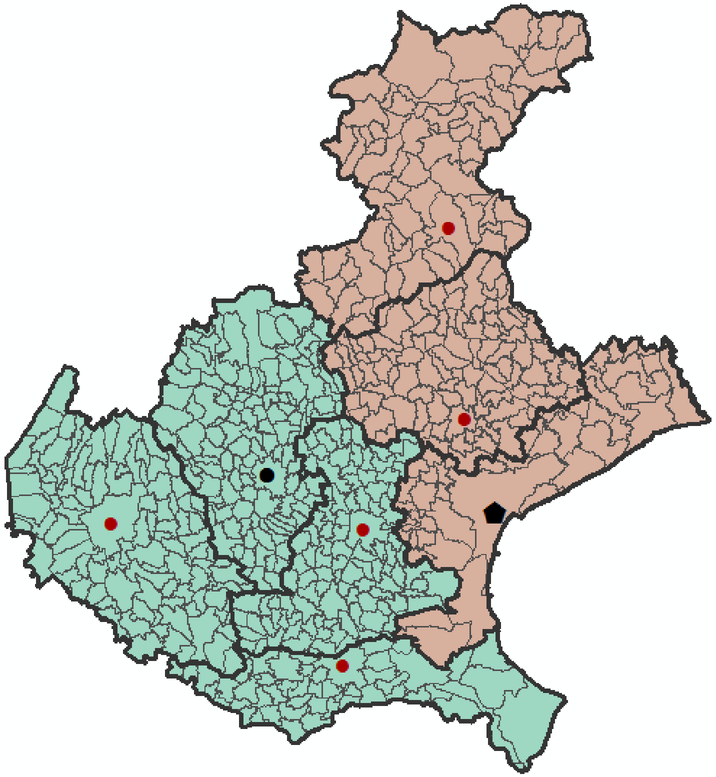
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Treviso	164	1.086.791	6.152,1	134,1
	Vicenza	373	2.923.457	9.782,4	124,3
	Venezia	44	846.962	2.472,9	93,6

OMM-k =4



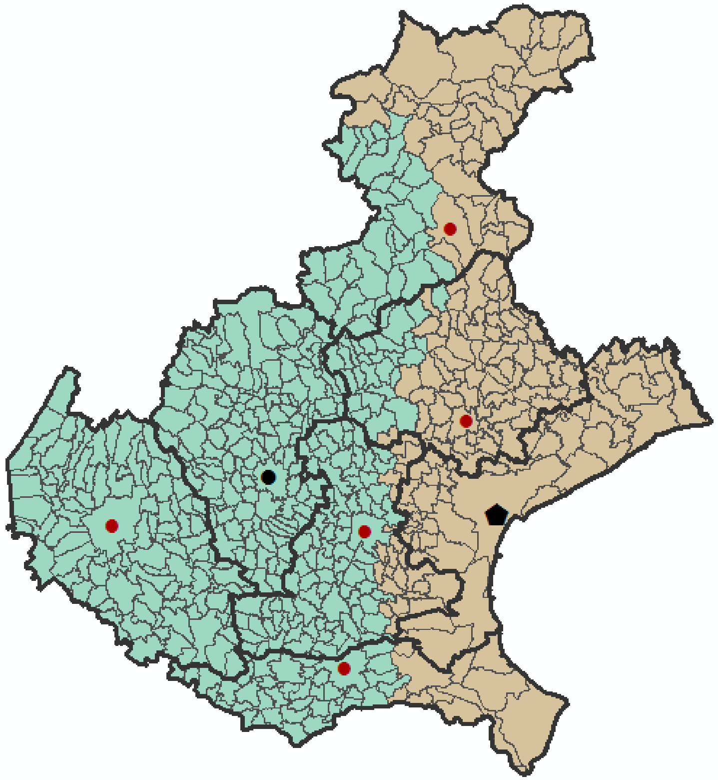
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Verona	134	1.003.549	3.862,1	90,5
	Vicenza	263	1.972.077	6.627,1	147,5
	Venezia	184	1.881.584	7.918,3	165,1

ORM-k =4



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Vicenza	373	2.923.457	9.782,4	124,3
	Venezia	208	1.933.753	8.625,0	165,1

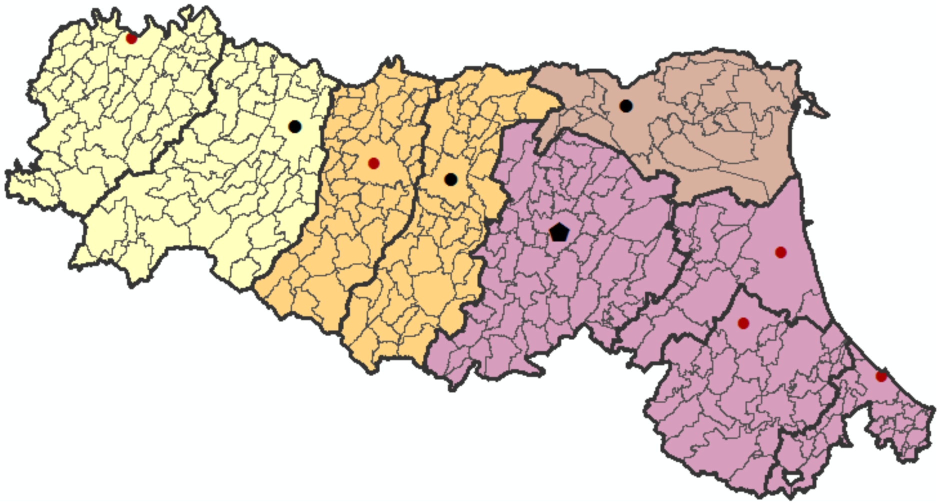
OMM-k =5

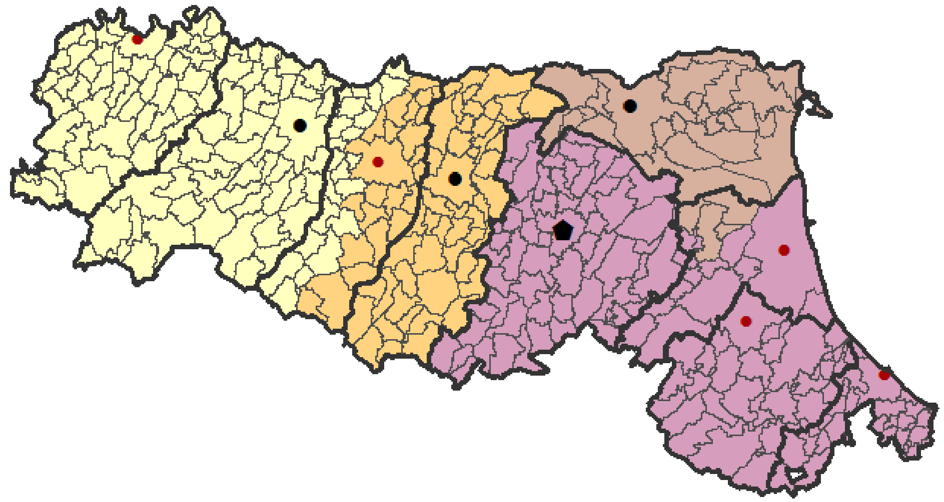


	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Vicenza	397	2.975.626	10.489,2	147,5
	Venezia	184	1.881.584	7.918,3	165,1

ORM-k =5

B.4 Emilia Romagna Region



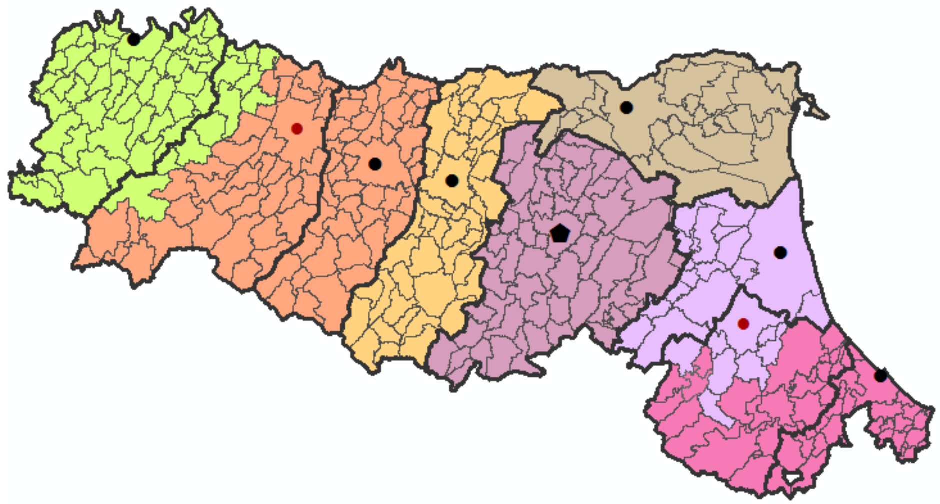


	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Parma	95	712.050	6.033,3	138,9
	Modena	92	1.203.093	4.979,3	89,2
	Bologna	135	2.073.511	8.805,0	148,4
	Ferrara	26	353.481	2.635,1	81,6

PMM

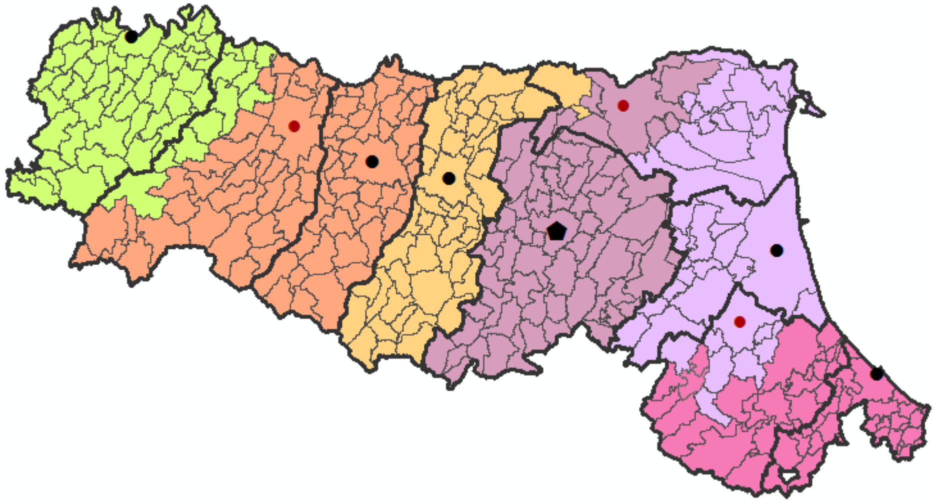
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Parma	117	864.068	6.977,8	138,9
	Modena	70	1.051.075	4.034,8	79,9
	Bologna	131	2.011.108	8.496,4	148,4
	Ferrara	30	415.884	2.943,7	81,6

PRM



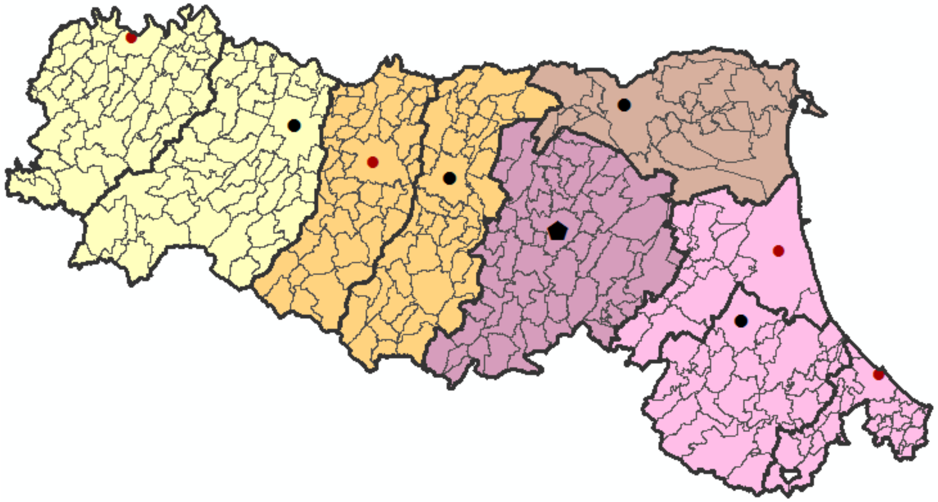
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Piacenza	59	359.139	3.342,4	77,6
	Reggio Emilia	81	870.227	4.982,2	117,1
	Modena	47	685.777	2.688,0	79,9
	Bologna	60	976.243	3.702,3	68,2
	Ferrara	26	353.481	2.635,1	81,6
	Ravenna	26	555.248	2.543,1	66,6
	Rimini	49	542.020	2.559,7	106,5

ORM- $k=k_{min}=2$



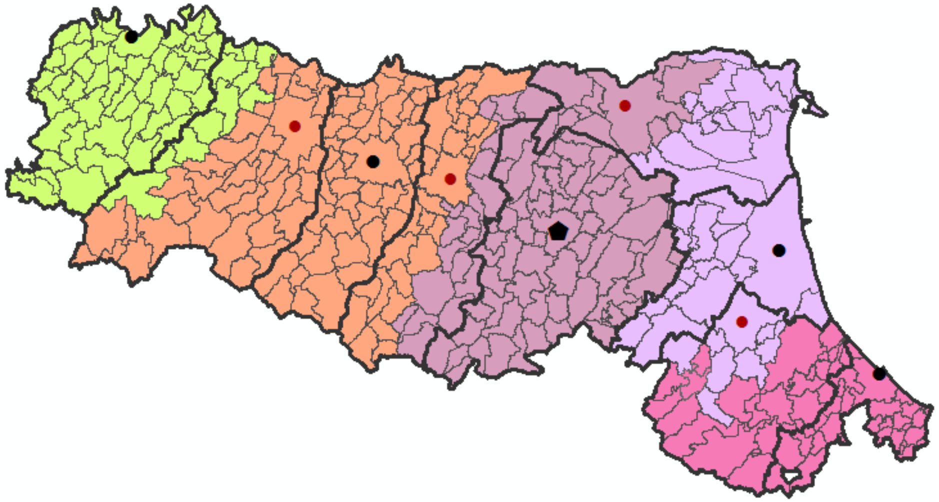
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Piacenza	59	359.139	3.342,4	77,6
	Reggio Emilia	81	870.227	4.982,2	117,1
	Modena	49	704.396	2.879,0	79,9
	Bologna	69	1.194.264	4.592,5	78,7
	Ravenna	41	672.089	4.097,0	81,9
	Rimini	49	542.020	2.559,7	106,5

ORM-k =3



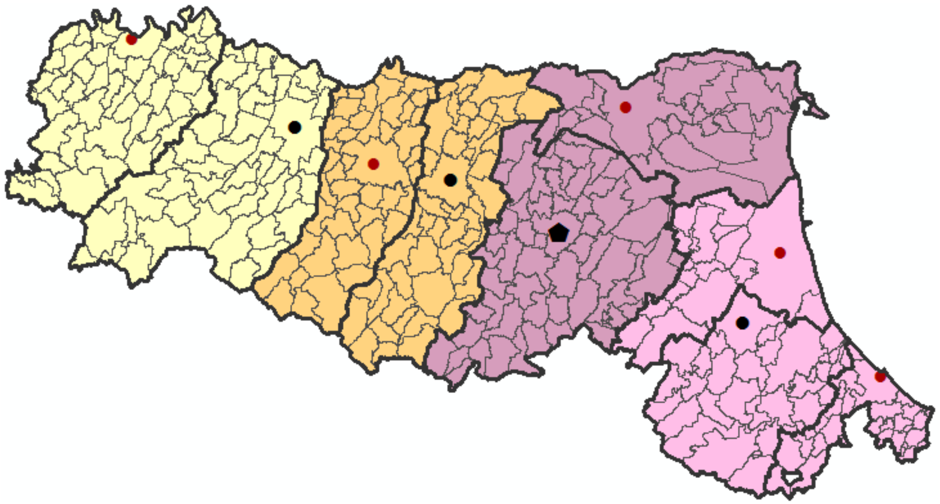
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Parma	95	712.050	6.033,3	138,9
	Modena	92	1.203.093	4.979,3	89,2
	Bologna	60	976.243	3.702,3	68,2
	Ferrara	26	353.481	2.635,1	81,6
	Forlì	75	1.097.268	5.102,7	84,7

OMM-k =k_{min}=4



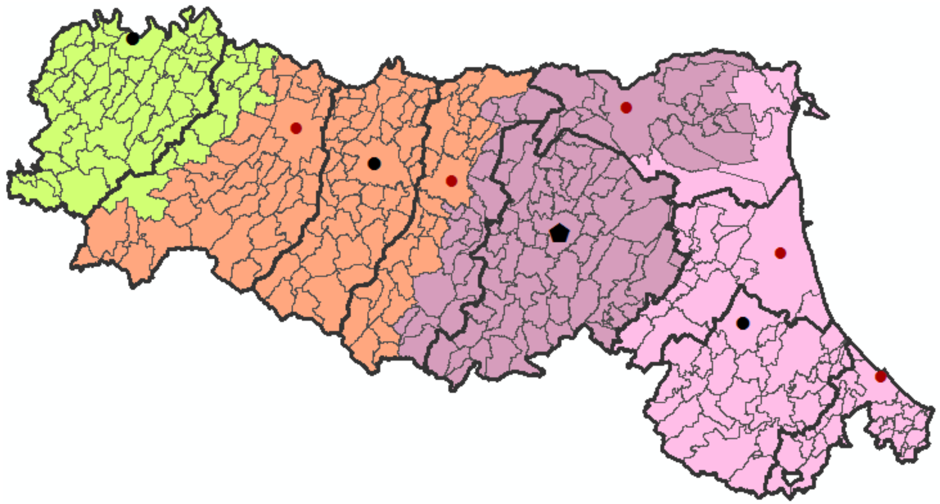
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Piacenza	59	359.139	3.342,4	77,6
	Reggio Emilia	107	1.348.923	6.538,2	117,1
	Bologna	92	1.419.964	5.915,6	82,3
	Ravenna	41	672.089	4.097,0	81,9
	Rimini	49	542.020	2.559,7	106,5

ORM-k =4



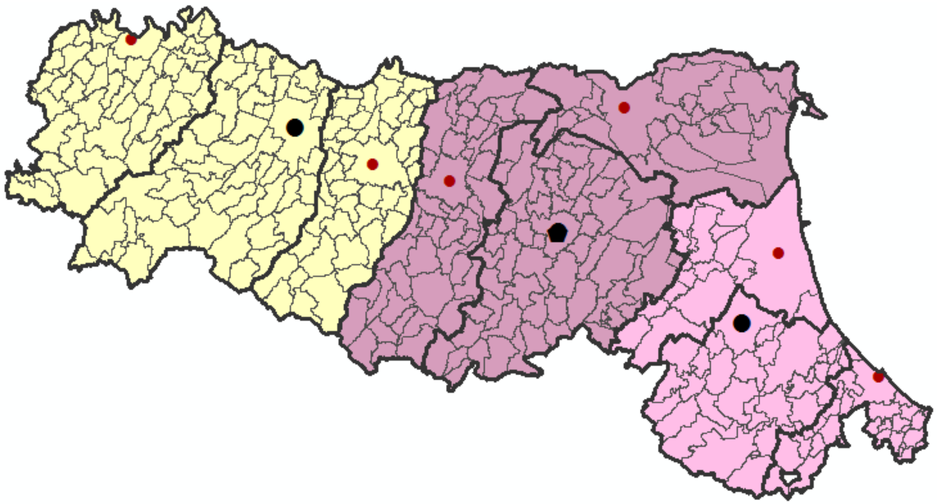
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Parma	95	712.050	6.033,3	138,9
	Modena	92	1.203.093	4.979,3	89,2
	Bologna	86	1.329.724	6.337,4	122,1
	Forlì	75	1.097.268	5.102,7	84,7

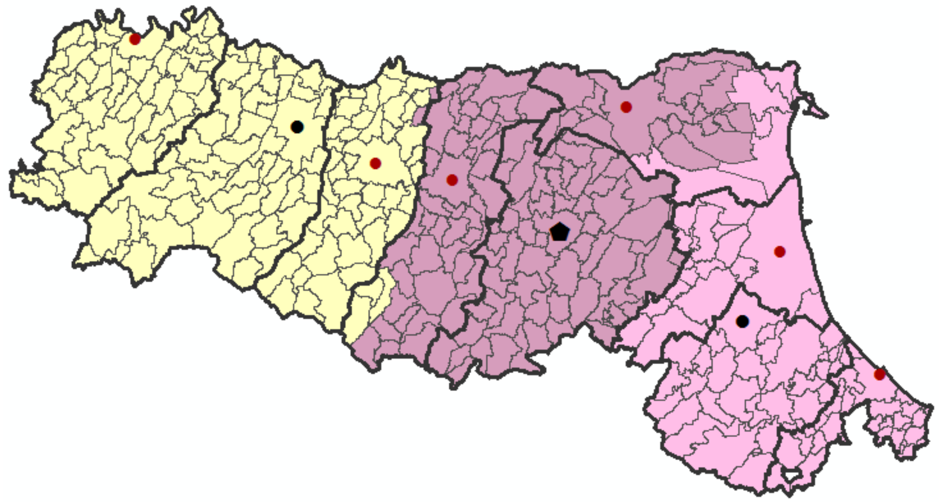
OMM-k =5



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Piacenza	59	359.139	3.342,4	77,6
	Reggio Emilia	107	1.348.923	6.538,2	117,1
	Bologna	101	1.463.648	6.551,8	87,8
	Forlì	81	1.170.425	6.020,4	95,4

ORM-k =5



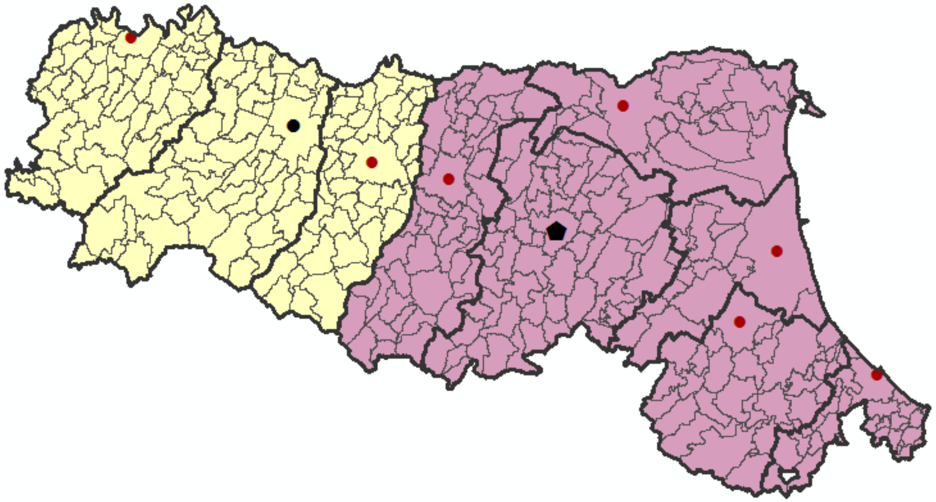


	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Parma	140	1.229.366	8.324,6	138,9
	Bologna	133	2.015.501	9.025,5	122,1
	Forlì	75	1.097.268	5.102,7	84,7

OMM-k =6

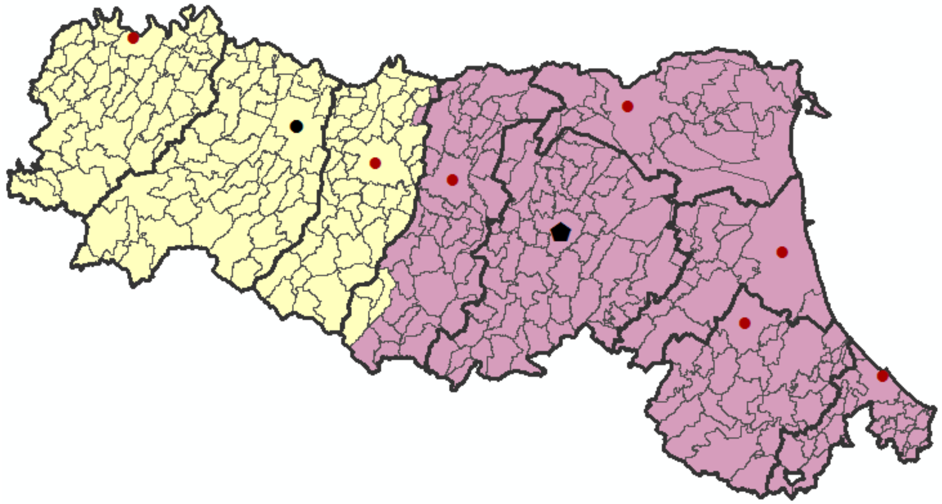
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Parma	141	1.225.840	8.489,0	138,9
	Bologna	126	1.945.870	7.943,3	101,4
	Forlì	81	1.170.425	6.020,4	95,4

ORM-k =6



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Parma	140	1.229.366	8.324,6	138,9
	Bologna	208	3.112.769	14.128,2	148,4

OMM-k =7



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Parma	141	1.225.840	8.489,0	138,9
	Bologna	207	3.116.295	13.963,8	148,4

ORM-k =7

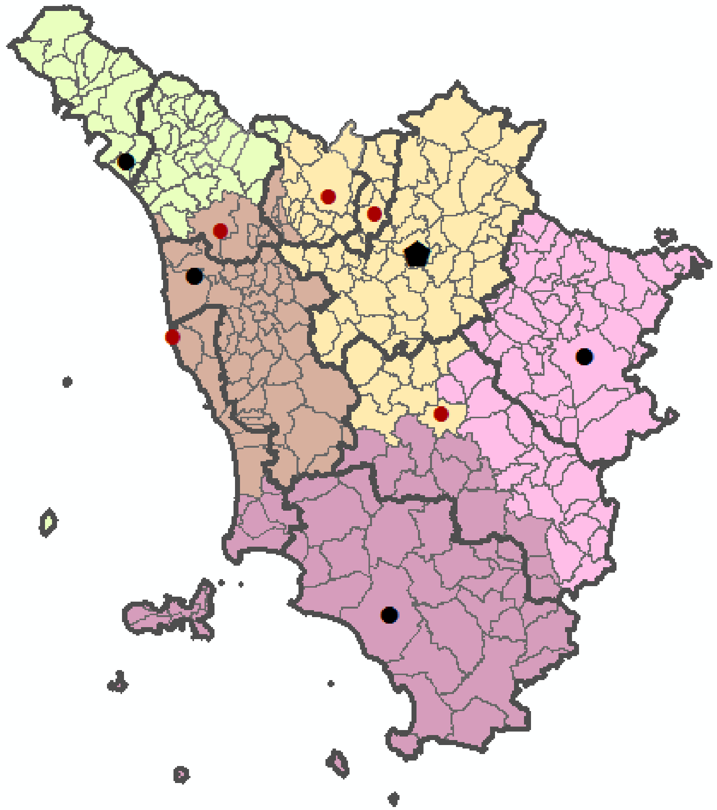
B.5 Toscana Region

	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Firenze	287	3.672.202	22.987,0	214,9

PMM

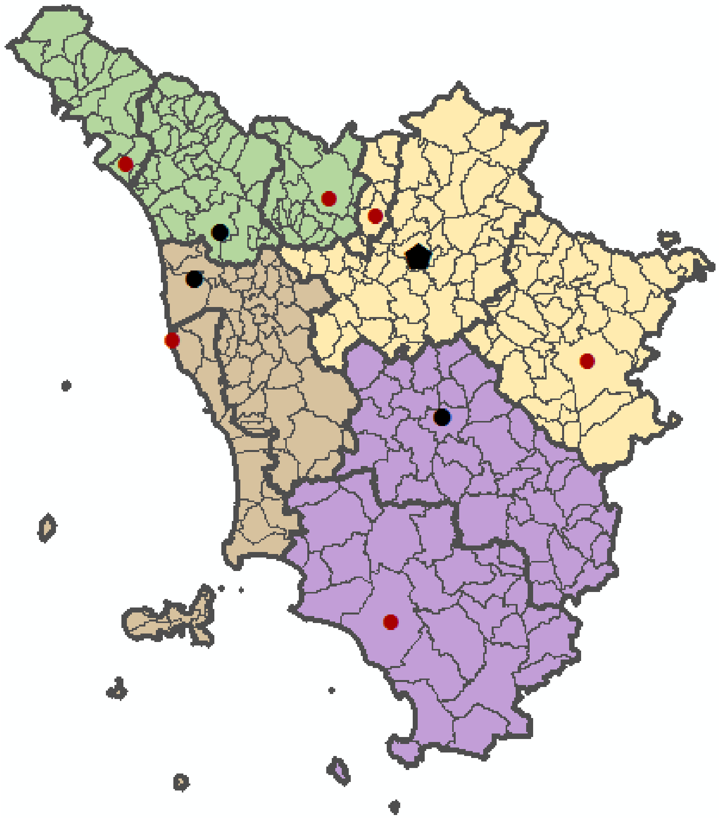
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Firenze	287	3.672.202	22.987,0	214,9

PRM



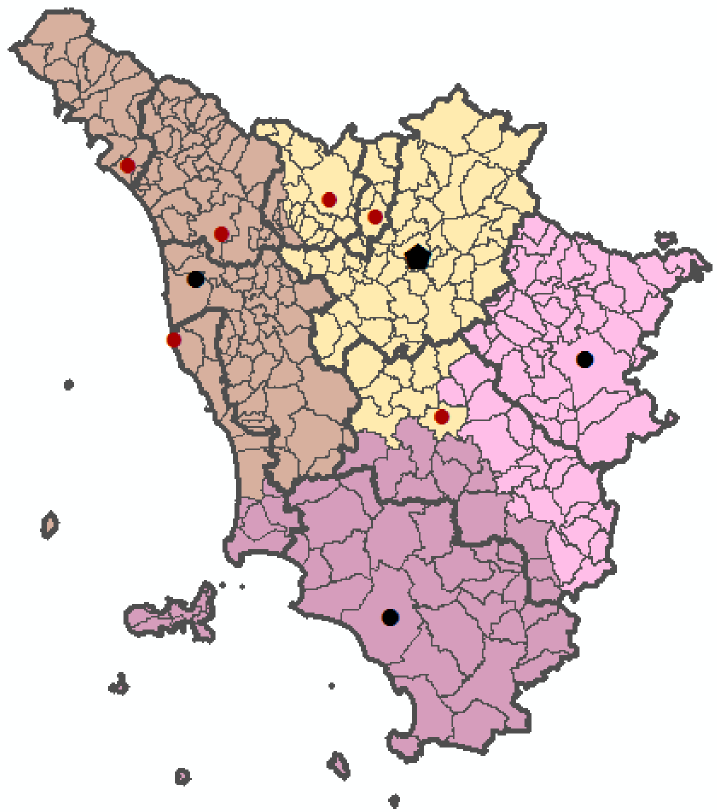
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Massa	49	368.534	2.585,7	112,4
	Firenze	74	1.585.999	5.477,3	75,2
	Pisa	56	925.559	3.604,8	112,4
	Arezzo	56	434.974	4.878,8	89,4
	Grosseto	52	357.136	6.440,5	122,6

ORM- $k=k_{min}=5$



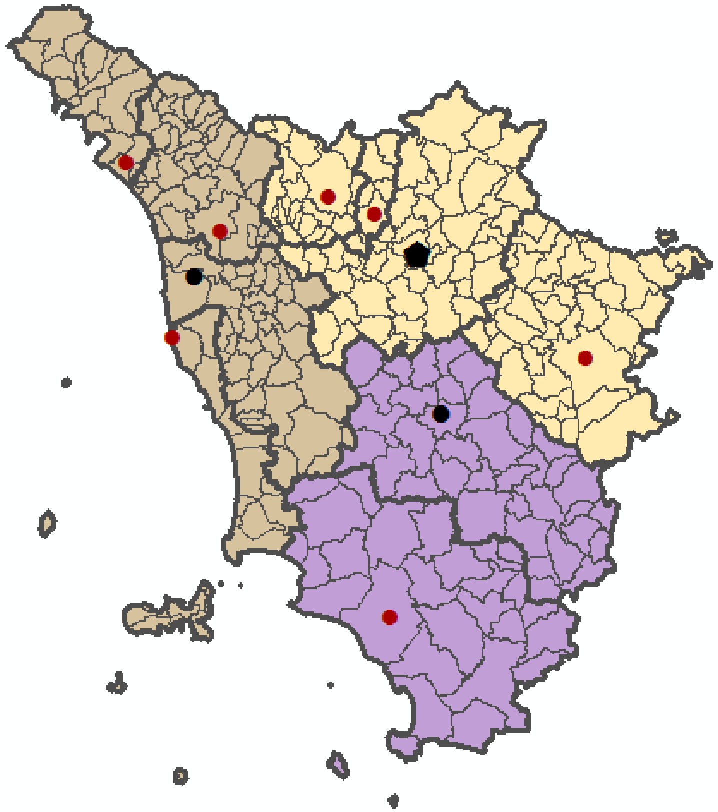
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Lucca	74	875.843	3.892,0	113,9
	Firenze	90	1.562.737	7.112,5	159,6
	Pisa	59	746.437	3.658,4	155,1
	Siena	64	487.185	8.324,1	137,9

OMM-k=k_{min}=6



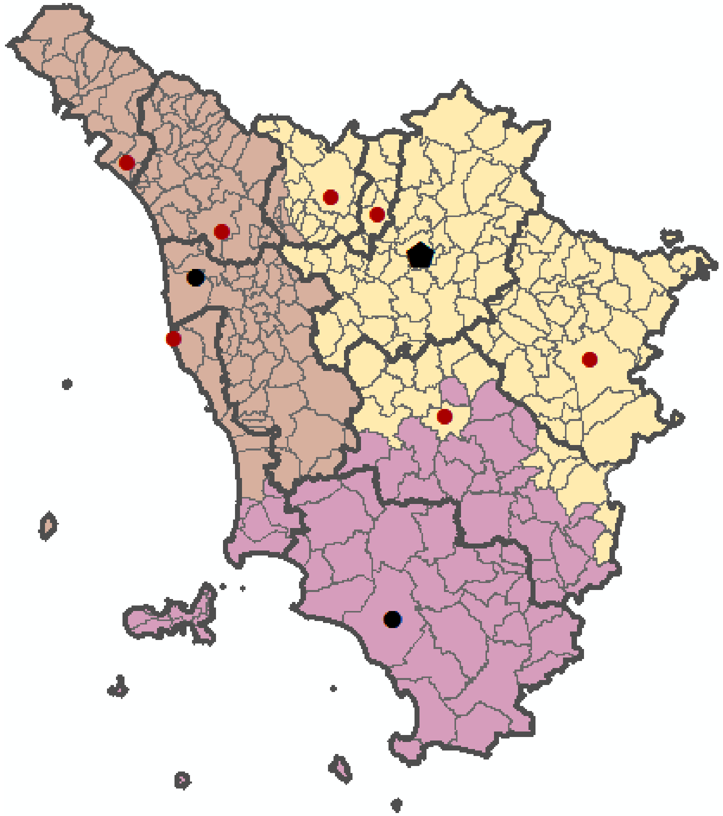
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Firenze	76	1.588.247	5.552,2	88,9
	Pisa	103	1.291.845	6.115,5	117,8
	Arezzo	56	434.974	4.878,8	89,4
	Grosseto	52	357.136	6.440,5	122,6

ORM-k =6



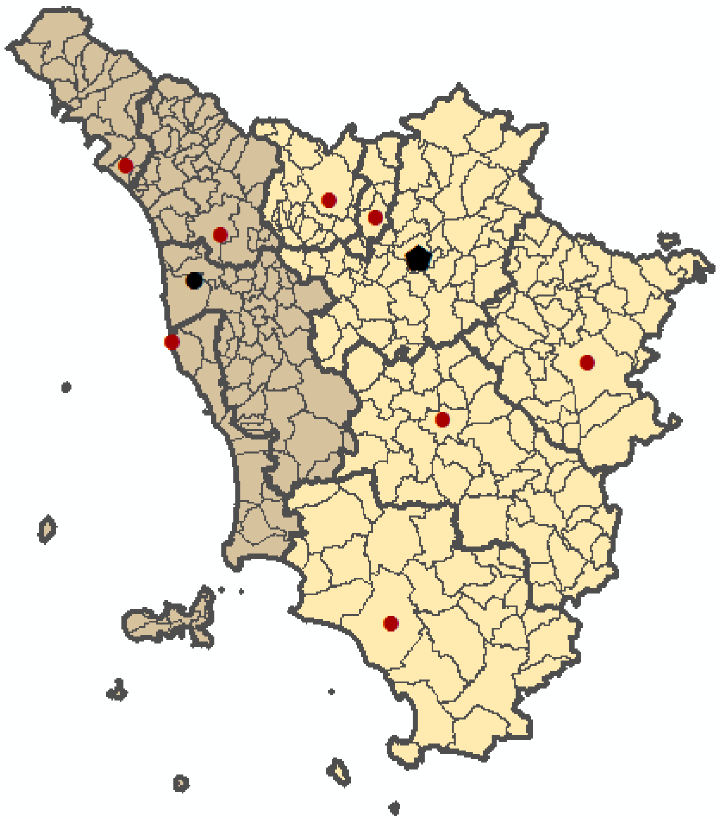
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Firenze	112	1.850.603	8.076,6	159,6
	Pisa	111	1.334.414	6.586,3	155,1
	Siena	64	487.185	8.324,1	137,9

OMM-k =7



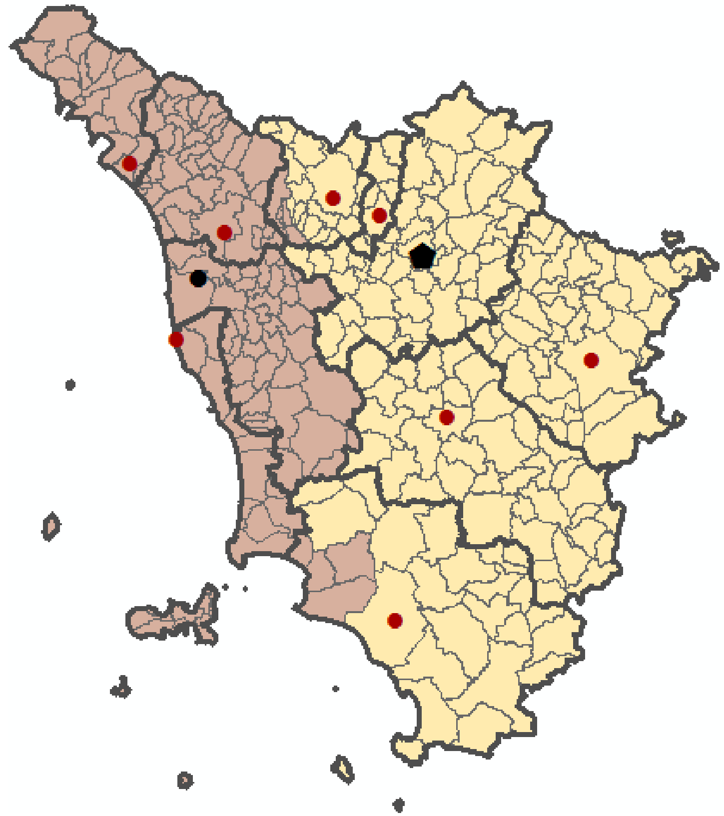
	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Firenze	122	1.981.771	9.392,1	159,6
	Pisa	103	1.291.845	6.115,5	117,8
	Grosseto	62	398.586	7.479,5	122,6

ORM-k =7



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Firenze	176	2.337.788	16.400,7	214,9
	Pisa	111	1.334.414	6.586,3	155,1

OMM-k =8



	Districts	TUs	Pop.	Area (kmq)	R_{mj} (km)
	Firenze	167	2.249.764	15.743,4	214,9
	Pisa	120	1.422.438	7.243,7	155,1

ORM-k =8

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