PÉTER MEKIS

Atomic Descriptions in Dynamic Predicate Logic

Abstract. We introduce a version of dynamic predicate logic (*DPL*, see (Groenendijk &al. 1991) as a framework to model in a compositional way the dynamics of definite descriptions put forward in (Lewis 1979). The resulting system, *dynamic predicate logic with descriptions* (*DPLD*) borrows the concept of a referent system from an upgraded version of *DPL* introduced in (Groenendijk &al. 1996). It is an interesting feature of *DPLD* that not only formulas but also individual terms are capable of updating discourse information.

1 INTRODUCTION

David Lewis, in his seminal paper (Lewis 1979) that served as a philosophical basis for the project of dynamic semantics, put forward a cluster of linguistic phenomena that deserve dynamic treatment. One of them is the anaphoric use of definite descriptions. Strangely enough, in more than thirty years since Lewis's paper this phenomenon has apparently not been studied within the framework of dynamic semantics.[1]

According to Lewis, examples like "The dog got in a fight with another dog" show that "[i]t is not true that a definite description 'the F' denotes x if and only if x is the one and only F in existence. Neither is it true that 'the F' denotes x if and only if x is the one and only F in some contextually determined domain of discourse." (348.) Instead, "[t]he proper treatment of descriptions must be more like this: 'the F' denotes x if and only if x is the most salient F in the domain of discourse, according to some contextually determined salience ranking." (ibid.)

---

[1]On the other hand, it has been studied to a certain extent within the *DRT* framework; see (Eijck &al. 1997, 189ff.) The *DRT* treatment is completely different from ours.

This view contradicts the classical view on descriptions dominant in logic ever since Frege and Russell at crucial points.

Salience may be determined by the context of utterance or antecedent expressions. Both can be found in Lewis's following example:

(1) The cat is in the carton. The cat will never meet our other cat, because our other cat lives in New Zealand. Our New Zealand cat lives with the Cresswells. And there he'll stay, because Miriam would be sad if the cat went away.

In Lewis's analysis, the referent of the first occurrence of "the cat" is determined by external conditions, and the second one refers back to the first. Then, the term "our other cat" introduces another discourse referent, and "our New Zealand cat" refers back to "our other cat", which has meanwhile been associated with the term "New Zealand".

All of these four description occurrences deserve special attention. The semantic phenomena involved in this sample discourse are very complex and have far-reaching implications, only a small portion of which can be discussed in this paper. In particular,

(a) we are going to focus on atomic descriptions like "the cat" or "the man", since they seem to be more capable of linking anaphorically, while complex descriptions tend to behave in the Frege–Russell way;

(b) we attempt to model the anaphoric use of atomic descriptions, like "the cat" referring back to "a cat" and "the one that lives with us" referring back to "the cat" in the discourse "We have a cat that lives with us, and another one that lives in new Zealand. The one that lives with us is old";

(c) we also attempt to model the way atomic descriptions take discourse referents from external sources (or even introduce new discourse referents) when they have no salient antecedent, like "the cat" in the opening sentence of Lewis's example "the cat is in the carton":

The technical problem to be solved is that of representing the way salience information is gathered, updated and put to use in a discourse. In our approach, salience is a relation between discourse referents and predicates. Since the former are semantic entities, while the latter syntactic ones, this is a mixed type of information. Clearly, a predicate itself is a better choice as an object of salience information than its extension. But we cannot rule out the possibility that intension plays a role in salience ranking, as the following example may suggest:

(2) A bachelor has a date with an old maid. $^{??}$The man is nervous. The woman is bored.

Whether and how anaphoric linking in this example works is beyond the scope of this paper.

We take Lewis's statement "[i]t is not true that a definite description 'the
F' denotes x if and only if x is the one and only F in existence" as an
exaggeration. An account of definite descriptions that takes the uniqueness and
existence conditions attached to "the smallest prime number" or "the author
of *Counterfactuals*" as merely extreme cases of salience ranking would be highly
counterintuitive. The use of descriptions that involves uniqueness and existence
(henceforth, we refer to a description interpreted this way as a *classical description*)
is just as legitimate as the anaphoric one, and it may even have priority over
that. But descriptions, like so many other natural language expressions, are tools
that—in spite of being of the same form—have different uses. In this paper we
attempt to model one of these uses.

2   THE DYNAMICS OF ATOMIC DESCRIPTIONS

Cross-sentential anaphora is one of the two semantic challenges that *DPL*
took on (the other being the anaphoric relations in donkey-sentences, which we
will not discuss here). Let us see an example of it and the way *DPL* treats it.

(3) A man walks in the park. He meets a woman.

Standard first-order logic (henceforth, *PL*) translates this discourse as

(3a) $\exists x \left(\text{man}(x) \ \& \ \texttt{walk\_in\_the\_park}(x) \right.$
$\left. \& \ \exists y \left(\text{woman}(y) \ \& \ \texttt{meet}(x, y) \right) \right)$[2]

This translation reflects the anaphoric link between "he" in the second sentence
and "a man" in the first by placing all the occurrences of x within the scope
of $\exists x$. The price is that the translation is not compositional; no syntactic
component of the formula translates the first sentence of (3). On the other hand,

(3b) $\exists x \left(\text{man}(x) \ \& \ \texttt{walk\_in\_the\_park}(x) \right)$
$\& \ \exists y \left(\text{woman}(y) \ \& \ \texttt{meet}(x, y) \right)$

is not a correct translation, since the last occurrence of x is not bound by the
quantifier $\exists x$. In *PL*, (3a) is not equivalent with (3b).

DPL offers a framework in which the syntactic scope of a quantifier occur-
rence (that is, the formula to which the quantifier is prefixed in the syntactic
construction) separates from its semantic scope (that is, the part of the discourse
in which it is able to bind variable occurrences). In *DPL*'s semantics, due to
the dynamic rules governing existential quantification and conjunction, (3a) is
equivalent with (3b).

Moreover, in *DPL* anaphoric linking is possible even between the conclusion
of a consequence and one of its premises. Consider the following example and
its attempted translations:

---

[2]As a tribute to professor Ruzsa, we are going to use his versions of the logical constants,
wherever it is possible, including $\&$ as conjunction, $\sim$ as negation, $\supset$ as implication, $\equiv$ as
the biconditional and I as the descriptor. See, for example, (Ruzsa 1981, 16ff.)

(4) *Witness*: A man entered the house and switched the light on. He had a knife. *Inspector*: So, he switched the light on.

(4a) $\{\exists x\,(\texttt{man}(x)\ \&\ \texttt{enter\_the\_house}(x)\ \&\ \texttt{switch\_the\_light\_on}(x),$
$\texttt{have\_a\_knife}(x))\} \models \texttt{switch\_the\_light\_on}(x)$

(4b) $\forall x\,(\texttt{man}(x)\ \&\ \texttt{enter\_the\_house}(x)\ \&\ \texttt{switch\_the\_light\_on}(x)$
$\&\ \texttt{have\_a\_knife}(x)) \supset \texttt{switch\_the\_light\_on}(x))$

This is an example that cannot be properly translated into *PL* even if we give up compositionality. In standard semantics, (4a) is not a valid consequence, and (4b) is not even an consequence. On the other hand, *DPL* has a dynamic consequence relation that makes "cross-inferential" anaphora possible, and makes (4b) valid.

In *DPLD* we want to deal with similar examples involving atomic definite descriptions. The first one and its compositional translation are as follows:

(5) A man walks in the park. He meets a woman. The man hugs her. A man watches from a distance. He walks a dog. The dog is bored. The man is jealous.

(5a) $\exists x\,\texttt{man}(x)\ \&\ \texttt{walk\_in\_the\_park}(x)\ \&\ \exists y\,\texttt{woman}(y)\ \&\ \texttt{meet}(x,y)\ \&$
$\texttt{hug}(\mathrm{I}w\,\texttt{man}(w),y)\ \&\ \exists z\,\texttt{man}(z)\ \&\ \texttt{watch\_from\_a\_distance}(z)$
$\&\ \exists v\,\texttt{dog}(v)\ \&\ \texttt{walk}(z,w)\ \&\ \texttt{bored}(\mathrm{I}w\,\texttt{dog}(w))\ \&$
$\texttt{jealous}(\mathrm{I}w\,\texttt{man}(w))$

In this discourse, all occurrences of descriptions refer back anaphorically to indefinite ones. One particularly interesting point is that the two occurrences of "the man" have two different antecedents, both of which are occurrences of the same expression "a man". Let us call d_1 and d_2 the discourse referents introduced by the first and the second occurrences of $\exists x$, respectively; and let us call s_1 the salience information that the predicate $\texttt{man}$ is associated with d_1, and s_2 the salience information that the predicate $\texttt{man}$ is associated with d_2. Now, *DPLD* has to interpret this formula in such a way that s_1 is passed to the first occurrence of $\mathrm{I}w\,\texttt{man}(w)$, but it is replaced with s_2 in the course of the updating process before the second occurrence of $\mathrm{I}w\,\texttt{man}(w)$.

DPLD also has to treat descriptions that lack possible antecedents, like Lewis's

(6) The cat is in the carton. She is asleep.

(6a) $\texttt{in\_the\_carton}(\mathrm{I}x\,\texttt{cat}(x))\ \&\ \texttt{asleep}(x)$

If we meet an atomic description at the beginning of a discourse, there are two possible scenarios for evaluation. One is that—like Lewis suggests—we have salience information from some external sources. This means that the evaluation of a discourse may not start with an empty discourse information state. The other possibility is that, for some reason or another, we lack the salience information needed; for example, this is the opening sentence of a novel. In this case, the description is most likely to introduce a new discourse referent, behaving the same way as existential quantification. In either of the scenarios, the occurrence of x in the second formula is bound by the description operator in the first.

We have to deal with "cross-inferential" anaphora, too:

(7) *Witness*: A woman entered the house. She switched the light on. A man waited outside.
Inspector: So, the one that switched the light on was a woman.

(7a) $\langle \exists x \, \texttt{woman}(x) \, \& \, \texttt{enter\_the\_house}(x), \texttt{switch\_the\_light\_on}(x),$
$\exists y \, \texttt{man}(y) \, \& \, \texttt{wait\_outside}(x) \rangle \models$
$\texttt{woman}(\mathrm{I}z \, \texttt{switch\_the\_light\_on}(z))$

As the example suggests, *DPLD*'s consequence relation has to make it possible for a description in its conclusion to be bound by an existential quantifier in one of its premises. The example has another peculiarity that deserves attention. The description $\mathrm{I}z \, \texttt{switch\_the\_light\_on}(z)$ uses a predicate that was associated with the discourse referent in one sentence later than it was introduced. This is another phenomenon in the dynamics of salience ranking to be taken account of.

Finally, let us consider the behavior of a description with respect to negated sentences.

(8) John doesn't own either a car or a motorcycle. [??]The car is too expensive for him.

(8a) $\sim \big((\exists x \, (\texttt{car} \, \& \, \texttt{own}(\texttt{John}, x)) \vee \exists y \, (\texttt{motorcycle} \, \& \, \texttt{own}(\texttt{John}, x)) \big)$
$\& \, \texttt{too\_expensive}(\mathrm{I}x \, \texttt{car}(x), \texttt{John})$

Clearly, the description "the car" in the second sentence cannot refer back to the indefinite expression "a car" in the first. Instead, if the definite description is acceptable here at all, it introduces a new discourse referent. On the other hand, an atomic description in a negated sentence seems to be able to bind later variable occurrences:

(9) It is not the case that the cat is in the carton. She is in the garden.

(9a) $\sim \texttt{in\_the\_carton}(\mathrm{I}x \, \texttt{cat}(x)) \, \& \, \texttt{in\_the\_garden}(x)$

Even if the referent of "the cat" is determined by external conditions, the pronoun "she" in the second sentence refers back to the description. Thus, negation seems to be externally dynamic with respect to atomic descriptions. This is essentially different from the analogous case with existential quantification:

(9′) It is not the case that there is a cat in the carton. [??]She is in the garden.

(9a′) $\sim (\exists x \, \texttt{cat}(x) \, \& \, \texttt{in\_the\_carton}(x)) \, \& \, \texttt{in\_the\_garden}(x)$

Here, the pronoun "she" appears to behave deictically, and thus the corresponding occurrence of x in the second formula is best interpreted as free.

We choose not to take this last phenomenon into account in *DPLD*. The reason is simple; no proper dynamic version of negation is known in a first-order framework. This is a fundamental problem of dynamic semantics, already observed in (Groenendijk *& al.* 1991, 99ff.), and to all appearances, it has not been solved yet.

3   INTERLUDE: THE DYNAMICS OF CLASSICAL DESCRIPTIONS

After making clear what we are interested in, a quick word on what we are
not interested in. There is another way of using dynamic semantics in the
analysis of definite descriptions; it concerns classical descriptions instead of
Lewis's anaphoric ones. It is well-known that standard first-order logic cannot
deal with Russell's analysis of descriptions in a compositional way.[3] It is less
well-known that dynamic predicate logic solves this problem. This is an obvious
consequence of its refined manner of handling anaphoric relations.

(10) The present king of France is bald. He shaves himself.

(10a) $\exists x \left(\forall y \left(\texttt{present\_king\_of\_France}(y) \equiv y = x \right) \right.$
        $\left. \& \texttt{bald}(y) \;\&\; \texttt{shave}(x, x) \right)$

(10b) $\exists x \, \forall y \left(\texttt{present\_king\_of\_France}(y) \equiv y = x \right)$
        $\& \; \texttt{bald}(y) \;\&\; \texttt{shave}(x, x)$

While "the present king of France" is a syntactic component in (10), its
first-order translation, $\exists x \, \forall y \left(\texttt{present\_king\_of\_France}(y) \equiv y = x \right)$ is not a
component in (10a). Nevertheless, it is present in the formula (10b), and in *DPL*
(10a) is equivalent with (10b). Similarly, the translation of the first sentence
is a component of (10b), but not of (10a). However, all occurences of x in
$\texttt{bald}(y)$ and $\texttt{shave}(x, x)$ are anaphorically linked to $\exists x$. Thus, unlike *PL*, *DPL*
offers a proper treatment of Russellian definite descriptions. Compositionality
also makes the introduction of a classical description operator straightforward,
either with a Russellian or with a Strawsonian semantics, and thus makes the
semantic modelling of the dynamics of existence and uniqueness conditions
straightforward. This idea is developed in (Eijck 1993), but it has little in
common with our project.

4   THE SYNTAX AND SEMANTICS OF *DPL* AND *DPLR*

We are going to present dynamic predicate logic in two versions, which have
the same syntax, but differ in their semantics. One of them is the well-known
DPL presented in (Groenendijk & *al.* 1991). The second version, which we
refer to as *DPLR*, differs from the original one at one important point: the
use of referent systems. Referent systems were introduced in the semantics in
(Groenendijk & *al.* 1996), along with the concept of a *peg*. (In fact, *DPLR* is the
extensional part of the system presented in (Groenendijk & *al.* 1996).) Pegs are
the technical equivalents of discourse referents; that is, abstract entities which
are introduced by indefinite descriptions like "a man", and to which discourse
information is attributed instead of being attributed to the variables themselves,

[3]Cf. e.g. (Gamut 1991, 164.))

or the elements of the domain of discourse. We are going to refer to them as discourse referents instead of pegs.[4]

Syntax

The syntax of *DPL* and *DPLR* is identical to that of standard predicate logic with identity. For simplicity, the only function symbols allowed are individual constants. Individual terms (henceforth, terms) are either variables or constants. Besides these, we have n-ary predicates in the non-logical vocabulary. Logical primitives are $\sim$, $\&$, $\exists$, $=$, $($, $)$; te rest of the logical symbols are defined in terms of these.

   i $\varphi \vee \psi \iff_{\mathrm{d}} \sim (\sim \varphi \,\&\, \sim \psi)$
  ii $\varphi \supset \psi \iff_{\mathrm{d}} \sim (\varphi \,\&\, \sim \psi)$
 iii $\varphi \equiv \psi \iff_{\mathrm{d}} (\varphi \supset \psi) \,\&\, (\psi \supset \varphi)$
 iv $\forall x\, \varphi(x) \iff_{\mathrm{d}} \sim \exists x \sim \varphi$

Now, we define formulas as usual:

(1) If P is an n-place predicate and $t_1, \ldots, t_n$ are terms, then $P(t_1, \ldots, t_n)$ is an atomic formula.
(2) If t_1 and t_2 are terms, then $t_1 = t_2$ is an atomic formula.
(3) Nothing else is an atomic formula. Atomic formulas are formulas.
(4) If φ and ψ are formulas, then $(\varphi \,\&\, \psi)$ is a formula.
(5) If φ is a formula, then $\sim \varphi$ is a formula.
(6) If φ is a formula and x is a variable, then $\exists x\, \varphi$ is a formula.
(7) Nothing else is a formula.

The semantics of DPL

Although the orignal version of *DPL* is well-known, we reintroduce it in a nutshell. Dynamic semantics is semantics of information state updating. Discourse information is based on an ordinary first-order structure $\mathcal{M} = \langle U, \varrho \rangle$, U being the universe of discourse and ϱ being the interpretation function. If a is an individual constant and P is an n-place predicate, then $\varrho(a) \in U$ and $\varrho(P) \subseteq U^n$. This structure is not updated in the process of evaluating a discourse; it is not part of the discourse information.

We will use cylindrification as a basic operation on assignment sets, defined as

$$V[x] =_{\mathrm{d}} \{v : \text{ for some } v', v' \in V \text{ and } v[x]v'\},$$

[4]The term *discourse referent* was coined in (Karttunen 1975), along with the idea that discourse referents should be identified with natural numbers. We find this expression more expressive and less idiosyncratic than *peg*.

where

$$v[x]v' \iff_d \text{ for all variables } y \text{ different from } x, v'(y) = v(y).$$

The value $|t|^{DPL}_{\mathcal{M},V}$ of a term t is $\varrho(t)$ if it is a constant, and $v(t)$ if it is a variable. Thus, the evaluation function $[\![\varphi]\!]^{DPL}_{\mathcal{M}}$ of formulas updates information states as follows (we omit the upper index wherever it is not confusing):[5]

(1) $[\![P(t_1, \ldots, t_n)]\!]_{\mathcal{M}}(V) =_d \left\{ v \in V : \langle |t_1|_{\mathcal{M},V}, \ldots, |t_n|_{\mathcal{M},V} \rangle \in \varrho(P) \right\}$;

(2) $[\![t_1 = t_2)]\!]_{\mathcal{M}}(V) =_d \left\{ v \in V : \langle |t_1|_{\mathcal{M},V} = |t_2|_{\mathcal{M},V} \rangle \right\}$;

(3) $[\![(\varphi \ \& \ \psi)]\!]_{\mathcal{M}}(V) =_d [\![(\psi)]\!]_{\mathcal{M}} \circ [\![(\varphi)]\!]_{\mathcal{M}}(V)$;

(4) $[\![\sim \varphi]\!]_{\mathcal{M}}(V) =_d \left\{ v \in V : [\![\varphi]\!]_{\mathcal{M}}(\{v\}) = \emptyset \right\}$;

(5) $[\![\exists x \, \varphi]\!]_{\mathcal{M}}(V) =_d [\![\varphi]\!]_{\mathcal{M}}(V[x])$.

Finally, a dynamic consequence relation $\langle \varphi_1, \ldots, \varphi_n \rangle \models_{DPL} \psi$ is defined as

$$\langle \varphi_1, \ldots, \varphi_n \rangle \models_{DPL} \psi \iff_d \text{ for all } \mathcal{M} \text{ and } V,$$

$$\text{if } [\![(\varphi_n)]\!]_{\mathcal{M}} \circ \cdots \circ [\![(\varphi_1)]\!]_{\mathcal{M}}(V) \neq \emptyset,$$

$$\text{then } [\![(\varphi_n)]\!]_{\mathcal{M}} \circ [\![(\varphi_n)]\!]_{\mathcal{M}} \circ \cdots \circ [\![(\varphi_1)]\!]_{\mathcal{M}}(V) \neq \emptyset.$$

That is, $\langle \varphi_1, \ldots, \varphi_n \rangle \models_{DPL} \psi$ is valid iff whenever the evaluation process of the discourse $\langle \varphi_1, \ldots, \varphi_n \rangle$ does not result in an empty set of assignments, neither does the evaluation of $\langle \varphi_1, \ldots, \varphi_n, \psi \rangle$. Note that, unlike in *PL*, the premises of a consequence relation form an ordered sequence.

The semantics of DPLR

DPLR is a variant of *DPL* that applies a more detailed model of discourse information. In *DPL*, an information state is a set of assignments, and assignments are total functions from the set of variables to the domain of discourse. In this semantic framework discourse referents do not play an explicit role. *DPLR* differs from *DPL* in making the use of discourse referents transparent, by means of referent systems.

Discourse information is represented in *DPLR* as a triple $\mathcal{I} = \langle d, r, V \rangle$. d is a natural number, and serves as the set of the discourse referents that are in use at a certain point of a discourse. As it is standard in set theory, numbers are identified with the sets of their predecessors, that is, $0 = \emptyset$ and $n = \{0, \ldots, n-1\}$. The elements of d serve as discourse referents in a given information state. r is a partial function from the set of variables to n. The pair $\langle d, r \rangle$ is called a referent

---

[5]Instead of updating a set of assignments as a whole, (Groenendijk & *al.* 1991) updates each assignment separately. Our version is standard in dynamic systems that give an explicit definition of information state. We first saw *DPL* presented this way in (Kálmán & *al.* 2001, 62ff.)

system. V is a set of evaluation functions from d to U. Thus, if a variable x has a value, then its value is given by $v(r(x))$ for each $v \in V$. But it is not the case that every variable has a value; in fact, the evaluation of any discourse only necessitates that a finite number of them has. A variable that does not occur in a discourse does not have a value. Discourse information is treated in a dynamic way; in the the course of evaluating a discourse, each occurrence of a term updates the actual referent system, the same way as each occurrence of a formula does.

Some technical concepts will be useful in the definition of semantics. The first is that of updating of a referent system with a new variable. This will be used in the evaluation of quantified formulas.

$$\langle d, r \rangle [x] =_{\mathrm{d}} \langle d + 1, (r \setminus \{\langle x, i \rangle : i < d\}) \cup \{\langle x, d \rangle\} \rangle$$

That is, a new discourse referent is introduced into the referent system as the value of the variable x. Meanwhile, if x already had a value, the old value is deleted. A set of assignments V is updated with a new discourse referent in a similar way:

$$V[d] =_{\mathrm{d}} \{v : (\exists v' \in V)\, (\exists u \in U)\; v = v' \cup \{\langle d, u \rangle\}\}$$

(Note that unlike variables, discourse referents are not reused in the evaluation process; and although we don't rule out the possibility that different variables are associated with the same discourse referent, nothing in our semantics enforces such a situation.)

To make the following definitions more transparent, we will follow a simple notational rule for information states and first-order structures: $I = \langle d, r, V \rangle$, $I' = \langle d', r', V' \rangle$, $I'' = \langle d'', r'', V'' \rangle$ etc; and $\mathcal{M} = \langle U, \varrho \rangle$, $\mathcal{M}' = \langle U', \varrho' \rangle$ etc. This way it will be easy to identify the semantic components.

A discourse information state $\mathcal{I}$ is updated by updating both $\langle d, r \rangle$ and V:

$$\mathcal{I}[x] =_{\mathrm{d}} \langle \langle d, r \rangle [x], V[d] \rangle^6$$

We call a discourse information state *empty* iff it has an empty set of assignments. An empty information state may or may not have an empty referent system.

The value $|t|_{\mathcal{M}, \mathcal{I}}^{DPLR}$ of a term t is $\varrho(t)$ if it is a constant, and $v(r(t))$ if it is a variable with a previous occurrence. Variables with no previous occurrences have no value. Thus, the evaluation function $[\![\varphi]\!]_{\mathcal{M}}^{DPLR}$ of formulas is essentially partial. (We omit the upper indices wherever it is not confusing.) Formulas with semantically free variables have no semantic value. The semantic value of a formula, if it exists, is a function from discourse information states to discourse information states, defined in the following clauses:

---

[6]As is customary, for convenience, we identify $\langle \langle a, b \rangle, c \rangle$ with $\langle a, \langle b, c \rangle \rangle$. Also, we identify $\langle a \rangle$ with a.

(1) $[\![P(t_1,\ldots,t_n)]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}}$
$\qquad \langle d, r, \{v \in V : \langle |t_1|_{\mathcal{M},\mathcal{I}},\ldots,|t_n|_{\mathcal{M},\mathcal{I}} \rangle \in \varrho(P)\} \rangle$;

(2) $[\![t_1 = t_2)]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} \langle d, r, \{v \in V : \langle |t_1|_{\mathcal{M},\mathcal{I}} = |t_2|_{\mathcal{M},\mathcal{I}} \rangle \} \rangle$;

(3) $[\![(\varphi \ \& \ \psi)]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} [\![(\psi)]\!]_{\mathcal{M}} \circ [\![(\varphi)]\!]_{\mathcal{M}}(I)$;

(4) $[\![\sim \varphi]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} \left\langle d, r, \left\{v \in V : [\![\varphi]\!]_{\mathcal{M}}(\langle d, r, \{v\}\rangle) \text{ is empty}\right\} \right\rangle$;

(5) $[\![\exists x \, \varphi]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} [\![\varphi]\!]_{\mathcal{M}}(\mathcal{I}[x])$.

Finally, the definition of dynamic consequence is as follows:

$\qquad \langle \varphi_1,\ldots,\varphi_n \rangle \models \psi$ iff for all $\mathcal{M}$ and $\mathcal{I}$, if

$$[\![\psi]\!]_{\mathcal{M}} \circ [\![\varphi_n]\!]_{\mathcal{M}} \circ \cdots \circ [\![\varphi_n]\!]_{\mathcal{M}}(\mathcal{I})$$

exists and

$$[\![\varphi_n]\!]_{\mathcal{M}} \circ \cdots \circ [\![\varphi_n]\!]_{\mathcal{M}}(\mathcal{I})$$

is nonempty, then

$$[\![\psi]\!]_{\mathcal{M}} \circ [\![\varphi_n]\!]_{\mathcal{M}} \circ \cdots \circ [\![\varphi_n]\!]_{\mathcal{M}}(\mathcal{I})$$

is nonempty.

The problem of partiality

We have seen how *DPLR* makes the semantic role of a discourse referent explicit. The price of this is partiality. In *DPLR*, a semantically free variable occurrence—that is, an occurrence of a variable that has not been associated with a discourse referent—results in a semantic value gap. This is not necessarily a problem. As it turns out from the above definitions, the evaluation process of a discourse $\langle \varphi_1,\ldots,\varphi_n \rangle$ does not necessarily starts with an empty referent system. If a variable x occurs in a discourse without an antecedent $\exists x$, it may still have a value, provided by the initial information state. This way *DPLR* gives an account of the deictic use of pronouns of natural language.

However, a natural language discourse does not always come to a halt when a new pronoun is introduced without salient discourse referent. It is more likely that a new discourse referent is tacitly introduced. This resembles the case of using an atomic description without any salient referent, like in example (6) above. For this reason, in *DPLD* we do not follow (Groenendijk & *al.* 1996) in making semantic rules partial.

5   THE SYNTAX AND SEMANTICS OF *DPLD*

In this section we introduce *DPLD*, a system of dynamic predicate logic with atomic descriptions, as a modified version of *DPLR*. The main differences are the use of the descriptor in the syntax, and salience ranking as a component of discourse information in semantics.

Syntax

In all our examples, we used one-place predicates in the descriptions. We restrict ourselves to them in the syntax, because we do not intend to model the dynamics of embedded and open descriptions, like $Ix\ R(x, Iy\ P(y))$ ("the man's friend") and $Ix\ R(x, y)$ ("his friend"). Adapting our semantic rules to atomic descriptions with many-place predicates is straightforward but slightly complicated.

We define terms by the following clauses:

  (1) Variables and individual constants are terms.
  (2) If P is a one-place predicate and x is a variable, then $Ix\ Px$ is a term.

The rest of the syntax remains intact.

Semantics

As in *DPLR*, the definition of semantics begins with the concept of a discourse information state. We use referent systems and assignment sets as in *DPLR* (but as we will see, they are updated differently). We enrich discourse information states with a new component, salience ranking; $\mathcal{I} = \langle d, r, S, V \rangle$, where S is a salience ranking.

A salience ranking is an ordered tuple of salience information bits. Salience bits, on their turn, are ordered pairs. If P is a one-place predicate and d is a discourse referent, then $\langle P, d \rangle$ is a salience bit. (Note that salience bits have both syntactic and semantic elements.) Let us explain its use through simple examples.

Let P be a one-place predicate, d a discourse referent, and let the salience information bit $s = \langle P, d \rangle$ be at the head of our actual salience ranking. Now, if a description $Ix\ Px$ occurs in the course of evaluation, then our actual referent system $\langle d', r \rangle$ is updated with the pair $\langle x, d \rangle$. On the other hand, if we find a number of bits of the form $s = \langle P, d \rangle$ in our actual salience ranking, then the referent system is updated with the discourse referent in the leftmost bit. And finally, if there is no bit of the form $\langle P, d \rangle$ on the list, then x is associated with a new discourse referent, just like in the case of *DPL*'s existential quantification.

The formal definition of updating a referent system is

$$\langle d, r \rangle [x/d'] =_d \langle \mathsf{max}(d, d'+1), r \setminus \{\langle x, i \rangle : i < d\}) \cup \{\langle x, d' \rangle\}\rangle$$

The difference between this update definition and the one given in the last section is that x is not necessarily associated with a new discourse referent. As a consequence, different variables can be associated with the same discourse referent. An occurrence of $Ix\ P(x)$ can refer back to $\exists y\ P(y)$ in this way; although x is not bound by $\exists y$, x and y are associated with the same discourse referent, and hence they have the same value.

Let, again, P be a one-place predicate, and t a term associated with the discourse referent d; that is, $r(t) = d$. Let the atomic formula $P(t)$ occur at a certain point of the discourse. Then the salience ranking S is updated with a new bit $\langle P, d\rangle$; that is, this pair is attached to the head of the ranking:

$$S[P(t)] =_\mathrm{d} \big\langle \langle P, r(t)\rangle, S\big\rangle$$

Since the last information bit is always the leftmost one, the discourse referent this bit offers is more salient than its rivals that are offered further down in the ranking. In simple terms, the last thing mentioned is the most salient.

It will be useful to define a function $S(P)$ that gives back the most salient discourse referent associated with a predicate P in a given salience ranking S. If there is no salient discourse referent, then the value of $S(P)$ is P. We define $S(P)$ recursively with respect to the length of S.

(1) If $S = \emptyset$, then $S(P) = P$;

(2) if $S = \big\langle \langle P, d\rangle, S'\big\rangle$ for some d and S', then $S(P) = d$;

(3) if $S = \big\langle \langle Q, d\rangle, S'\big\rangle$ for some Q that is different from P, d and S', then $S(P) = S'(P)$.

The last component of a discourse information state is a set V of assignments. It is updated with a discourse referent the same way as in *DPL*. There are two ways of updating the discourse information state with a new variable x. In the first case, x is associated with an existing discourse referent:

$$\mathcal{I}[x/d'] =_\mathrm{d} \big\langle \langle d, r\rangle[x/d'], S, V\big\rangle$$

In the second case, a new discourse referent is introduced along with a new variable:

$$\mathcal{I}[x] =_\mathrm{d} \big\langle \langle d, r\rangle[x/d], S, V[d]\big\rangle$$

We have seen that terms are capable of updating discourse information. Now we define this update function $[\![t]\!]_\mathcal{M}^{DPLD}$.

(1) $[\![x]\!]_\mathcal{M}(\mathcal{I}) =_\mathrm{d} \begin{cases} \mathcal{I}[x] & \text{if } x \notin \mathsf{dom}(r); \\ \mathcal{I} & \text{if } x \in \mathsf{dom}(r); \end{cases}$

(2) $[\![a]\!]_\mathcal{M}(\mathcal{I}) =_\mathrm{d} \mathcal{I}$;

(3) $[\![\mathrm{I}x\ P(x)]\!]_\mathcal{M}(\mathcal{I}) =_\mathrm{d} \begin{cases} \mathcal{I}[x] & \text{if } S(P) = P; \\ \mathcal{I}[x/d'] & \text{if } S(P) = d'. \end{cases}$

That is, an occurrence of a variable x that has not yet been associated with a discourse referent behaves like an existential quantifier in the sense that it introduces a new discourse referent. Occurrences of x that are already associated with a discourse referent leave discourse information unchanged. Individual constants do not change discourse information either. An occurrence of an atomic description $\mathrm{I}x\ P(x)$ for which there is no salient referent introduces

a new one. Otherwise the x of $\mathrm{I}x\ P(x)$ is associated with the most salient discourse referent.

The value $|t|_{\mathcal{M},\mathcal{I}}^{DPLD}$ of a term t is defined as follows:

(1) $|a|_{\mathcal{M},\mathcal{I}} =_{\mathrm{d}} \varrho(t)$;

(2) $|x|_{\mathcal{M},\mathcal{I}} =_{\mathrm{d}} v(r(x))$;

(3) $|ixP(x)|_{\mathcal{M},\mathcal{I}} =_{\mathrm{d}} v(r(x))$.

Thus every occurrence of any term has a value in *DPLD*. Unlike *DPLR*, the semantics of *DPLD* is not a partial one.

Now we are ready to evaluate formulas. The information update function $[\![\varphi]\!]_{\mathcal{M}}^{DPLD}$ is defined in the following clauses:

(1) $[\![P(t)]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} \langle d', r', S'[P(t)], \{v \in V' : |t|_{\mathcal{M},\mathcal{I}} \in \varrho(P)\}\rangle$,
where $\langle d', r', S', V'\rangle = [\![t]\!]_{\mathcal{M}}(\mathcal{I})$;

(2) $[\![P(t_1, \ldots, t_n)]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}}$
$\langle d', r', S', \{v \in V' : \langle|t_1|_{\mathcal{M},\mathcal{I}}, \ldots, |t_n|_{\mathcal{M},\mathcal{I}}\rangle \in \varrho(P)\}\rangle$,
where $n > 1$ and $\langle d', r', S', V'\rangle = [\![t_n]\!]_{\mathcal{M}} \circ \cdots \circ [\![t_1]\!]_{\mathcal{M}}(\mathcal{I})$;

(3) $[\![t_1 = t_2)]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} \langle d', r', S', \{v \in V' : \langle|t_1|_{\mathcal{M},\mathcal{I}} = |t_2|_{\mathcal{M},\mathcal{I}}\rangle\}\rangle$,
where $\langle d', r', S', V'\rangle = [\![t_2]\!]_{\mathcal{M}} \circ [\![t_1]\!]_{\mathcal{M}}(\mathcal{I})$;

(4) $[\![(\varphi\ \&\ \psi)]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} [\![(\psi)]\!]_{\mathcal{M}} \circ [\![(\varphi)]\!]_{\mathcal{M}}(I)$;

(5) $[\![\sim \varphi]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} \langle d, r, S, \{v \in V : [\![\varphi]\!]_{\mathcal{M}}(\langle d, r, S, \{v\}\rangle)\ \text{is empty}\}\rangle$;

(6) $[\![\exists x\ \varphi]\!]_{\mathcal{M}}(\mathcal{I}) =_{\mathrm{d}} [\![\varphi]\!]_{\mathcal{M}}(\mathcal{I}[x])$.

Thus, the definition deviates from the one of *DPL* only in the evaluation of atomic formulas. Dynamic inference is defined the same way as in *DPL*.

To see how discourse information updating works, it will be instructive to see the evaluation of a particular formula. Let us consider the first three sentences of example (5). To make the formula shorter, we abbreviate the predicates `man`, `walk_in_the_park`, `woman`, `meet` and `hug` as P, Q, T, R and S, respectively.

(5′) A man walks in the park. He meets a woman. The man hugs her.

(5a′) $\exists x\ P(x)\ \&\ Q(x)\ \&\ \exists y\ T(y)\ \&\ R(x, y)\ \&\ S(\mathrm{I}w\ P(w), y)$

We present the evaluation process of the formula in an informal fashion.

(1) We start with an empty referent system, an empty salience ranking and an empty assignment set. That is, we do not make use of an external context.

(2) $\exists x\ P(x)$ introduces a new discourse referent, and associates x with it. Salience information associates the new discourse referent with the predicate P. $d_1 = 1$; $r_1 = \{\langle x, 0\rangle\}$; $S_1 = \langle P, 0\rangle$; $V_1 = \{\{\langle 0, u\rangle\} : u \in \varrho(P)\}$.

(3) $Q(x)$ updates salience information. $d_1 = d_1$; $r_2 = r_1$; $S_2 = \langle\langle Q, 0\rangle, < P, 0\rangle\rangle$; $V_1 = \{\{\langle 0, u\rangle\} : u \in \varrho(P), u \in \varrho(Q)\}$.

(4) $\exists y\ T(y)$ introduces a new discourse referent, associates y with it, and updates salience information. $d_3 = 2$; $r_3 = \{\langle x, 0\rangle, \langle y, 1\rangle\}$;

$S_3 = \langle\langle T, 1\rangle, \langle Q, 0\rangle, \langle P, 0\rangle\rangle$; $V_3 = \{\{\langle 0, u\rangle, \langle 1, u'\rangle\} : u \in \varrho(P), u \in \varrho(Q), u' \in \varrho(T)\}$.

(5) $R(x, y)$ does not change either the referent system or the salience ranking. $d_4 = d_3$; $r_4 = r_3$; $S_4 = S_3$; $V_4 = \{\{\langle 0, u\rangle, \langle 1, u'\rangle\} : u \in \varrho(P), u \in \varrho(Q), u' \in \varrho(T), \langle u, u'\rangle \in \varrho(R)\}$.

(6) $\mathrm{I}w\ P(w)$ introduces a new variable in the referent system. Since P occurs only once in S_4, this occurrence determines the discourse referent associated with w: $r_5(w) = S_4(P) = 0$. Thus, $d_5 = d_4$; $r_5 = \{\langle x, 0\rangle, \langle y, 1\rangle, \langle w, 0\rangle\}$; $S_5 = S_4$; $V_5 = V_4$.

(7) Finally, $S(\mathrm{I}w\ P(w), y)$ does not change either the referent system or the salience ranking. $d_6 = d_5$; $r_6 = r_5$; $S_6 = S_5$; $V_6 = \{\{\langle 0, u\rangle, \langle 1, u'\rangle\} : u \in \varrho(P), u \in \varrho(Q), u' \in \varrho(T), \langle u, u'\rangle \in \varrho(R), \langle u, u'\rangle \in \varrho(S)\}$.

REFERENCES

Eijck, J. van, 1993, The dynamics of description. *Journal of Semantics* 10, 223–267.

Eijck, J. van and Kamp, H., 1997, Representing discourse in context. In J. van Benthem and A. ter Meulen (eds.), *Handbook of Logic and Language*. Elsevier, Amsterdam & MIT Press, Cambridge (MA), 179–237.

Gamut, L. T. F., 1991, *Logic, Language, and Meaning*. University of Chicago Press, Chicago. (L. T. F. Gamut is a collective pseudonym for J. van Benthem, J. Groenendijk, D. de Jongh, M. Stokhof and H. Verkuyl.)

Groenendijk, J. and Stokhof, M., 1991, Dynamic predicate logic. *Linguistics and Philosophy* 14, 39–100.

Groenendijk, J., Stokhof, M. and Veltman, F., 1996, Coreference and modality. In S. Lappin (ed.), *The Handbook of Contemporary Semantic Theory*, Blackwell, 179–214.

Kálmán, L. and Rádai, G., 2001, *Dinamikus szemantika* (*Dynamic Semantics*; in Hungarian). Budapest, Osiris.

Karttunen, L., 1975, Discourse referents. In J. McCawley (ed.), *Syntax and Semantics 7: Notes from the Linguistic Underground*. Academic Press, New York, 363–385.

Lewis, D., 1979, Scorekeeping in a language game. *Journal of Philosophical Logic* 8, 339–359.

Ruzsa, I., 1981, *Modal Logic with Descriptions*. Martinus Nijhoff, De Hague.