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Natural Logic, Medieval Logic and Formal Semantics

Abstract. This paper provides a comparative analysis of the issue of natural logic: the "formalizational approach", prevalent in contemporary logic, and the "regimentational approach", prevalent in medieval logic, as exemplified by the 14th-century nominalist philosopher, John Buridan. The differences between the two are not as great as they may first appear: a little tweaking of standard quantification theory can take us surprisingly close to Buridan's logic. However, as the conclusion of the paper points out, there still are some fundamental differences between the resulting "reconstructed Buridanian logic" and Buridan's logic itself, discussed in detail in the author's recent monograph.

NATURAL LANGUAGE AND THE IDEA OF A "FORMAL SYNTAX" IN BURIDAN

The idea of the universality of logic is based on the conviction that despite the immense diversity of human languages, there are certain invariant features of human reasoning, carried out in any natural language whatsoever, that allow the formulation of universal logical laws, applicable to any language. It is precisely for expressing these universal, invariant aspects of human reasoning that in modern logic we construct an artificial language, which is then conceived to serve as a more direct linguistic expression of those invariant conceptual structures that are variously expressed by various natural languages.

But this is not the only possible way to achieve the desired transparency of conceptual structure through the transparency of syntax. The way the 14th-century nominalist philosopher, John Buridan (and medieval logicians in general) achieved this was by using, *not* a full-fledged artificial language, but an artificially "regimented" Latin. We can get a nice, yet relatively simple illustration of what this "regimentation" of Latin consists in if we take a closer look at how Buridan introduces the idea that every simple categorical proposition of Latin can be reduced to the "canonical" subject-copula-predicate form. After briefly stating the division of propositions into categorical and hypothetical, and the description of categorical propositions as those that consist of subject and predicate as their principal parts, Buridan remarks:

> … a verb has to be analyzed into the verb 'is' as third adjacent, provided that the proposition is assertoric [*de inesse*] and in the present tense [*de praesenti*], and into the participle of that verb, as for example, 'A man runs' is to be analyzed into 'A man is running', and similarly, 'A man is' into 'A man is a being'.[1]

English speakers might at once notice that the proposed transformation does not always yield equivalent sentences, given the tendency in English to use the simple present tense to signify habitual action as opposed to the continuous present tense, consisting of the copula and the appropriate participle, which is used to express present action. For instance, if I say 'I smoke', I may simply want to express that I am a smoker, a person who has the habit of smoking, but this does not mean that I am actually smoking, which would properly be expressed by the sentence 'I am smoking'. In fact, in accordance with Buridan's theory of predication, according to which the affirmative copula expresses the identity of the *supposita*, that is, the referents of the terms flanking it, a more appropriate rendering of his proposed transformation would be 'I am [identical with] someone smoking'.

But Buridan might answer that this is merely a difference in the different syntactical "clues" a different language uses to indicate a different sort of underlying conceptual construction. The simple present tense of English, when it is used to signify habitual action, should then not be analyzed into a participle and a simple assertoric copula, but *perhaps* (somewhat unidiomatically) into a participle and an adverbially modified copula, as in 'I am usually smoking',[2] where we just express in the surface syntax of this sentence an adverbial modifier that is unmarked in the simple tense (as is the implicit copula), but which is nevertheless present in the corresponding mental proposition. In any case, it is in this spirit that Buridan answers four questions he raises in connection with the issue of the "canonical form" of categorical propositions:

[1] SD 1.3.2.

[2] Alternatively, one might say that the best explication of 'I smoke' expressing the habit is 'I am a smoker', where the nominal definition of 'smoker' may explicate the habit, as in 'x is a smoker iff x has the habit of smoking'. But as Buridan often remarks, "examples are not to be verified", i.e., it does not matter whether we provide "the right analysis" here, as long as it serves to illustrate the point.

> But then some questions arise. The first concerns what such a copula signifies. The second is whether that copula is a principal part of a categorical proposition. The third question is whether the proposition 'The one lecturing and disputing is a master or a bachelor' is categorical or hypothetical; for it seems that it is hypothetical, since it has two subjects and two predicates. The fourth question is the same concerning the proposition 'A man who is white is colored'; for it seems that it is hypothetical, since here we have two subjects, two predicates and two copulas; and also because it seems to be equivalent to 'A man is colored, who is white' which is apparently hypothetical.[3]

In his reply, Buridan provides the rationale for the canonical subject-copula-predicate structure in terms of what modern linguists would certainly recognize as "deep structure", and what for Buridan is the conceptual structure of the corresponding mental proposition:

> To the first question we should reply that a spoken proposition has to signify a mental proposition [..]. A mental proposition, however, involves a combination of concepts [*complexio conceptuum*], and so it presupposes in the mind some simple concepts, to which it adds a complexive concept, by means of which the intellect affirms or denies one of those [presupposed simple] concepts of the other. Thus, those presupposed concepts are the subject and the predicate in a mental proposition, and they are called the matter of the mental proposition, for they are presupposed by the common form of a proposition, just as matter is presupposed by the substantial form in [the process of] generation. And then it is clear that the subject and the predicate of the spoken proposition signify in the mind the subject and the predicate of the mental proposition. The copula 'is' signifies an affirmative complexive concept, whereas the copula 'is not' signifies a negative complexive concept; and the intellect is unable to form that complexive concept except when it has formed those which are the subject and the predicate, for it is impossible to have the combination [*complexio*] of the predicate with the subject without the predicate and the subject. And this is what Aristotle meant[4] when he said that 'is' signifies a certain composition which cannot be understood without the components.[5]

What fundamentally justifies sticking to the idea of this "canonical form" according to Buridan is that no matter how a mental proposition is expressed

[3] SD 1.3.2.
[4] Aristotle, *On Interpretation,* 1, 16b24.
[5] SD 1.3.2.

in the ("surface") syntax of a spoken language, the concept of the copula is there in the mental proposition. Therefore, indicating it in the syntax of the spoken proposition merely explicates the presence of the complexive concept of the copula in the corresponding mental proposition. Indeed, this explication is *always* justified because, as Buridan now explains in his answer to the second question, that complexive concept *has to* be present in *any* mental proposition:

> To the second question we should reply that the copula is truly a principal part of the proposition, because there could not be a categorical proposition without it; and also because it can be compared to a form of the subject and the predicate, and the form is a principal part of a composite.[6]

Thus, given that the copula is the "formal", principal part of a categorical proposition, i.e., it is that complexive concept (proposition-forming functor) in the mind without which the concepts corresponding to the terms would not constitute a proposition, it is obvious that no matter how complex those terms and the corresponding concepts are, if they are joined by one copula, then they form one proposition. This is precisely the basis of Buridan's answer to the third question:

> To the third question we should reply that that proposition is categorical; for it does not contain two categoricals, as there is only one copula here; neither are there several subjects, nor several predicates here, for the whole phrase 'the one lecturing and disputing' is a single subject […], namely, a conjunctive subject, and the whole phrase 'master or bachelor' is likewise a single predicate, although disjunctive.[7]

As this remark clearly illustrates, Buridan would allow complex terms in either the subject or the predicate positions of otherwise simple, categorical propositions. In fact, given the potentially unlimited complexity of the terms of categorical propositions, these propositions may exhibit a *very* complex structure *within* their terms, despite the simplicity of the general subject-copula-predicate scheme. For it is not just the (iterable) "Boolean" operations of disjunction, conjunction and negation that can yield potentially infinite complexity in these terms, but also the fact that any proposition can be turned into a term (by forming a "that-clause") or into a determination of a determinable term (in the form of a relative clause). For example, if we take the proposition 'A man is running', it can easily be transformed into the term 'That a man is running', which can then

[6] SD 1.3.2.

[7] Ibid. Note that in Buridan's usage, 'hypothetical' in this context simply means 'complex', as opposed to the widespread modern usage that makes it equivalent to 'conditional'.

be the subject of another proposition, e.g., 'That a man is running is possible' or a part of another more complex term in another proposition, as in 'That a man is running is believed by Socrates'. Again, taking the proposition 'A man is white', and inserting a relative pronoun after its subject, we get another complex term 'A man who is white', which can then be the subject in the proposition 'A man who is white is colored'.

Now if we look at this proposition in this way, namely, as having a complex subject term built up from a head noun as the *determinable* determined by a relative clause, then it should be obvious why Buridan gives the following answer to the problem raised in connection with this proposition:

> To the fourth question we should reply that there is one predicate here, namely, 'colored', which by the mediation of the copula is predicated of the whole of the rest as of its subject, namely, of the whole phrase: 'man who is white'; for the whole phrase: 'who is white' functions as a determination of the subject 'man'. And the case is not similar to 'A man is colored, who is white', for there are two separate predicates here, which are predicated separately of their two subjects, and there is not a predicate here which would be predicated by the mediation of one copula of the whole of the rest. And although these [propositions] are equivalent, they are not equivalent if we add a universal sign. For positing the case that every white man runs and there are many others who do not run, the proposition 'Every man who is white runs' is true, and is equivalent to: 'Every white man runs'; but the proposition 'Every man, who is white, runs' is false, for it is equivalent to: 'Every man runs and he is white'.[8]

Buridan's response to the objection in terms of distinguishing two interpretations of the relative clause indicated by different word order is particularly revealing of his practice of using a "regimented Latin" to make logical distinctions. Indeed, the difference between the syntactical devices used in English and Latin to make the same distinction is also very instructive concerning the advantages and disadvantages of developing logical theory in a "regimented" natural language, as opposed to doing the same using an artificial language, as we usually do nowadays.

Let us take a closer look at the syntax and the semantics of the propositions distinguished here, both in English and in Latin:

(1) Homo qui est albus est coloratus
(2) A man who is white is colored
(3) Homo est coloratus qui est albus
(4) A man, who is white, is colored

[8] SD 1.3.2.

(5) Omnis homo qui est albus currit ↔ (5') Omnis homo albus currit
(6) Every man who is white runs ↔ (6') Every white man runs
(7) Omnis homo currit qui est albus ↔ (7') Omnis homo currit et ille est albus
(8) Every man, who is white, runs ↔ (8') Every man runs, and he is white

Every other line here is the English translation of the Latin of the preceding line. Yet, the syntactical devices by which the Latin and the English sentences bring out the intended conceptual distinction are obviously different (word order vs. punctuation). Nevertheless, the important thing from our present point of view is that these different devices can (and do) bring out the *same* conceptual distinction.

As should be clear, the fundamental difference in all the contrasted cases is whether the relative clause is construed as a *restrictive relative clause*, forming part of the complex subject term, or it is construed as a *non-restrictive relative clause*, making a separate claim referring back to the simple subject of the main clause.

The "regimentation" of the syntax of a natural language, therefore, is the *explication*, and occasionally even the *stipulation*, of *which* syntactical structures of the given language are supposed to convey *which* conceptual constructions. The governing principle of Buridan's regimentation of his technical Latin seems to be what may be called the *principle of scope-based ordering*. This principle is most clearly at work in the "Polish notation" in modern formal logic (where the order of application of logical connectives is indicated by their left-to-right ordering), but something similar is quite clearly noticeable in Buridan's rules of logical syntax in general.

To be sure, Buridan never goes as far as to organize Latin according to the rules of a formal syntax in the way a modern artificial language is constructed.[9] And for all his views about the conventionality of language, even he would shy away from re-rewriting the rules of Latin grammar to fit the requirements of the syntax of an artificial language. Rather, he uses the existing grammatical, structural features of Latin (sometimes stretching, and sometimes bending them a little) to make conceptual distinctions. However, once such a distinction is somehow made, using some such existing syntactical device, Buridan's regimentation of Latin consists in his insistence on the point that this syntactical device should be consistently regarded as expressing this conceptual distinction, at least when we use language for the purposes of logic (as opposed to, for example, using it to do poetry).

[9] I tried to do this once for a *tiny* fragment of Latin with an explicitly listed finite vocabulary for the purposes of illustration, and even that resulted in an extremely complex, unwieldy system. See (Klima 1991).

REGIMENTATION VS. FORMALIZATION

Thus, even if doing logic by means of a full-fledged artificial, formal language did not even emerge as a theoretical alternative for Buridan, given the fact that in our time this is the dominant approach to logic, we should pause here a little to reflect on the theoretical as well as the practical advantages and disadvantages of these two different approaches.

One apparent disadvantage of Buridan's "regimentational" approach in comparison to the modern "formalizational" approach is that an informal system can never be as exact as a formal one, given all the possible ambiguities and arbitrariness of an informal approach. By contrast, in the formal approach, the rules of interpretation in a formal semantics and the manipulations with formulae in a formal syntax are fixed by the highest standards of mathematical exactitude, which can never be matched by any sort of informal approach. Therefore, it seems that Buridan's approach suffers from an inherent *inexactitude* that can be overcome only by the formalizational approach.

Again, Buridan's approach renders the construction of logical theory in a fundamental sense *unfinishable*. Given the immense variety and variability of the syntactical forms of a natural language, a logical theory based on its regimentation will never cover *all* syntactically possible constructions in a natural language. By contrast, since in an artificial language we have an explicit and effective set of construction rules, we can formulate logical laws that apply to all possible well-formed formulae of that language without having to worry about possible formulae that may not be covered by these laws.

Furthermore, Buridan's approach seems to be plagued by what may be termed its *linguistic provincialism*. If logical rules and distinctions are formulated in terms of the regimentation of the existing syntactical devices of a particular natural language, then, given the obvious syntactical diversity of natural languages, this approach seems to threaten the universality of logical theory. Indeed, following the lead of the syntax of a particular natural language may even provide "false clues" concerning what we may mistakenly believe to be the universal conceptual structure of Mentalese. By contrast, the formalizational approach provides equal access for speakers of all languages to *the same* "conceptual notation", which directly reflects the structure of the common mental language of all human beings endorsed by Buridan. So, apparently, even Buridan's logic would be much better off if it were also couched in an artificial, formal language.

Finally, if we use the natural language embodying our logic in our *reflections* on *the same* natural language, then we are obviously running the risk of Liar-type paradoxes, which are bound to emerge under the resulting conditions of *semantic closure*, first diagnosed as such by Alfred Tarski.[10] By contrast, an artificial language embodying our logical theory can serve as the *object language* of the considerations concerning the syntax and semantics of this language which are to be carried out in a distinct *meta-language.* In this way we avoid the risk of paradox, since keeping the object language apart from the meta-language eliminates semantic closure.

Perhaps, these would be the most obvious reactions against Buridan's "regimentational" approach coming from someone comparing it to the modern "formalizational" approach. Nevertheless, these considerations may not be sufficient to establish once and for all the "absolute superiority" of the modern approach over Buridan's. For if we take a closer look at the modern practice, we can see that it is not much better off concerning these issues.

It must be conceded at the beginning that the mathematical exactitude of a formal logical system is unmatched by any "natural" logic (i.e. a logical system based on a certain regimentation of reasoning in some natural language). But the exactitude in question concerns only the formal system in and of itself. Concerning the formal system, we may have absolutely rigorous proofs of consistency or inconsistency, completeness or incompleteness, etc., which we may never have concerning an "unfinishable" system of "natural" logic. However, as soon as we use a formal logical system to represent and evaluate natural language reasoning, the uncertainties and ambiguities of interpretation return with a vengeance, as anyone who has ever tried to impart "formalization skills" in a symbolic logic class can testify. "Formalization" is the largely intuitive process of translating natural language sentences to formulae of a formal language based on the linguistic competence of the speakers of the natural language in question and their understanding of the import of the symbols of the formal language. Therefore, this process involves just as much inexactitude, uncertainty and ambiguity as does working with "unregimented" natural language expressions in general.

This difficulty can be overcome by constructing a formal syntax for an interesting portion of a natural language, in the vein of the approach of Richard Montague and Imre Ruzsa,[11] which can then serve as the basis for an exact and effective translation procedure into the artificial language of a formal logical system. In this way, having a formally constructed (not to say, "regimented") part of a natural language at our disposal, the problem of the inexactitude of

[10] Cf. (Tarski 1944). The gist of the idea of semantic closure is that a language that contains its own truth-predicates and has the means of referring to its own sentences is semantically closed, which is quite obviously the case with natural languages. According to Tarski, in a semantically closed language, Liar-type paradoxes ('This sentence is false' – is this true or false?) are bound to arise. For a more recent, generalized version of Tarski's argument, see (Priest 1984).

[11] Cf. (Montague 1973), (Montague 1974), (Ruzsa 1989).

the otherwise merely intuitive formalization process can certainly be avoided. However, given that the formal syntax can only cover a sufficiently interesting, yet manageable, part of a natural language, this approach brings out most clearly the inherently "unfinished" character of the enterprise as far as the representation of *all possible forms of natural language reasoning* is concerned. Thus, the formalizational approach can overcome the problems of inexactitude only by carving out a manageable part of natural language reasoning, thereby making explicit the "unfinished" character of the enterprise. Buridan's regimentational approach, in comparison, simply acknowledges from the start that it can only explicate and regulate certain manageable types of natural language reasoning, and it does this with the requisite degree of exactitude, yet without introducing the explicit, full-fledged formal syntax of an artificial language that would allegedly universally reflect the structure of Mentalese underlying all natural linguistic structures.

Since the process of formalization as it is commonly practiced is based on the linguistic competence of the speakers of particular natural languages, it involves just as much "linguistic provincialism" as does the regimentational approach. Actually, it is quite instructive to observe the differences between different Montague-style approaches to formalization motivated by different languages, especially if they are also motivated by certain logical considerations that are "most natural" in those languages.

But we can also say that the syntax of standard predicate logic as we know it was also motivated by some fairly "provincial" linguistic considerations, namely, considerations concerning the language of mathematics, rather than any actual natural language. This is probably the historical reason for the notorious "mismatch" between the syntax of predicate logic on one hand, and the syntax of various natural languages on the other, which otherwise agree among themselves in those of their syntactic features that predicate logic systematically fails to match. Consider again sentences (5)-(8) listed above:

(5) Omnis homo qui est albus currit ↔ (5') Omnis homo albus currit
(6) Every man who is white runs ↔ (6') Every white man runs
(7) Omnis homo currit qui est albus ↔ (7') Omnis homo currit et ille est albus
(8) Every man, who is white, runs ↔ (8') Every man runs, and he is white

In modern predicate logic, these sentences have to be represented in terms of the basic vocabulary of the formal language of this logic. In that language, besides the logical constants (which Buridan would recognize as syncategorematic terms, such as negation, conjunction, conditional, etc.), we have primitive symbols referring to individuals, namely, individual names (intuitively corresponding to proper nouns) as well as variables (roughly corresponding to pronouns),[12] and predicates (corresponding to common terms). All complex expressions are built up from these primitive symbols by means of an explicit set of rules that effectively determine which strings of these symbols are to be regarded as well-formed. Frege's original rationale for this type of construction was that he regarded all common terms as functional expressions: on this conception, a common term, such as 'man', denotes a function from individuals to the two truth-values, the True and the False. Thus, the term itself is essentially predicative; it needs to be completed with a referring expression picking out an individual to yield a complete sentence that denotes one of these truth values. Therefore, since for Frege *all* common terms denote functions of this sort, *all common terms are essentially predicative*. Accordingly, in the sentences above, even their grammatical subject terms need to be construed as predicates of individuals, which are picked out by variables bound by the quantifier word 'every' or 'omnis'. It is for this reason that universal sentences in this logic are to be represented as universally quantified conditionals. Since the subject terms of these sentences are not regarded as having the function of restricting the range of individuals to be considered in determining whether the sentence is true, these sentences will have to be interpreted as concerning all individuals in the universe, stating of them all conditionally that *if* they fall under the subject, *then* they also fall under the predicate.

Thus, (6) and (8), and the corresponding Latin sentences as well, would on this approach be parsed as expressing the same as

(6'') For everything (it holds that) if it is a man and it is white, then it is running

(8'') For everything (it holds that) if it is a man, then it is white and it is running

In other words, using the variable x in place of the pronoun,

(6''') For every x, if x is a man and x is white, then x is running

(8''') For every x, if x is a man, then x is white and x is running

And these, using the symbols of predicate logic, directly yield the matching formulae:

(6'''') $(\forall x)[(Mx \,\&\, Wx) \supset Rx]$

(8'''') $(\forall x)[Mx \supset (Wx \,\&\, Rx)]$

[12] The problems of representing anaphoric pronouns with bound variables of quantification theory generated a whole new field of research in the eighties, primarily inspired by Peter Geach's reflections on "donkey-sentences", coming from medieval logic, and especially from Buridan. For a summary account of those developments and their comparison to Buridan's ideas, see (Klima 1988).

However, given Buridan's radically different conception of the semantic function of common terms, he would provide a very different parsing for (6) and (8) (or rather for (5) and (7)). For on his conception, common terms have the function of signifying several individuals indifferently (as opposed to singular terms that would signify one individual as distinct from any other), and correspondingly they *supposit*, i.e., stand for (some of) these individuals in the context of a proposition in which the term is actually used for this purpose. Therefore, on Buridan's reading, (6) and (8) (or rather (5) and (7)) do not make a conditional claim about all individuals in the universe, but rather a categorical claim about a restricted range of individuals, namely, those marked out by the subject term, i.e., the *supposita* of the subject.

In fact, as anyone checking her own linguistic intuitions in English can testify, Buridan's analysis, coming from a "provincial" natural language, namely, Latin, matches much better the intuitions of speakers of another "provincial" natural language, namely, English. For English speakers would also find it "more natural" to understand the corresponding sentences as being categorical claims about a restricted range of individuals, rather than conditional claims about absolutely everything. To be sure, further reflection on the implications of this sort of analysis may further influence one's judgment on what "the correct" analysis of these sentences ought to be, but at least it should be clear that the Fregean analysis is definitely *not the only possible* or even the "most natural" one.

Thus, the Fregean analysis, being only one possible theoretical option, turns out to be just as provincial as Buridan's approach based on a particular natural language. Nevertheless, one may still object that at least for the Fregean analysis we have a working formal system with all the advantages of mathematical exactitude going for it, whereas we have nothing comparable for Buridan's approach. But this is simply not true.

BURIDAN'S LOGIC AS A LOGIC OF RESTRICTED QUANTIFICATION

As I have argued in several earlier papers, a simple, conservative extension of predicate logic can go a long way toward capturing in an exact form much of medieval logic in general and Buridan's logic in particular. Once we enhance the language of standard predicate logic with *restricted variables*, and provide the appropriate formal interpretation for their semantic evaluation in a formal semantic system, the resulting system at once becomes capable of capturing an enormous amount of traditional logic, and especially Buridan's version of it. We do not have to go into the technical details of constructing that formal system[13]

[13] For the technically-minded reader, a semantic system of this sort is available in (Klima 2001).

to explain its basic intuitive idea and its important philosophical implications concerning the relationships between this "enhanced predicate logic", classical predicate logic, and Buridan's informal logic.

The "basic intuitive idea" can be articulated in the following principles of construction:

(1) Restricted variables function as variables in classical predicate logic, i.e., they are quantifiable terms that fill in the argument places of predicate letters.
(2) Restricted variables have the general form of 'v.Av', where v is what is referred to as the operator variable of the restricted variable, and 'Av' as the matrix of the restricted variable, which is a well-formed formula open in v (i.e., having at least one occurrence of v that is not bound by a quantifier). The operator variable may itself be a restricted variable, in which case we can refer to it as a "nested" restricted variable (a restricted variable "nested" in another); other restricted variables occurring in the matrix of a restricted variable are spoken of as "embedded" in that restricted variable.
(3) Restricted variables pick their values in a value-assignment *not* from the entire domain of interpretation ("universe of discourse"), but from the extension of their matrix, i.e., from the set of individuals of which the matrix is true (under a certain value-assignment of variables).
(4) If the extension of the matrix of a restricted variable is empty, then the restricted variable has no value (which in the formal system can be represented by assigning an arbitrary value to it, outside the domain of interpretation, a so-called "zero-entity", a technical device I owe to Imre Ruzsa). When a restricted variable has no value (i.e., technically, its value is outside the domain of discourse), then its value cannot fall within the extension of any predicate, i.e., all simple affirmative predications containing this variable in the argument of a predicate letter will come out as false.

Having these "principles of construction" in place, we can obtain a system that (i) reflects more faithfully the syntax and semantics of natural languages than standard predicate logic,[14] (ii) naturally extends itself to a generalized quantification theory, (iii) it not only matches, but surpasses standard predicate logic in expressive power, and (iv) provides an analysis of categorical propositions perfectly in tune with Aristotelian logic, validating all relations of the traditional *Square of Opposition* and the traditionally valid syllogistic forms.

Let us now take these four points in turn, and see exactly how the system constructed in accordance with (1)-(4) can obtain these results.

(i) Predicate logic formulae using unrestricted quantification exhibit a compositional structure involving propositional connectives that are nowhere

[14] For a precise characterization of the notion "faithfulness" involved in this intuitive claim, see (Klima 1988).

to be found in the corresponding natural language sentences (be they English, Latin or even Hungarian, etc.). If we take a look, e.g., at (6’’’’)-(6) above, the structural mismatch is obvious. But the same sort of mismatch becomes even more striking if we change the quantifier from universal to particular (or “existential”), which requires that the main conditional be replaced with a conjunction in the resulting formula, whereas no such change is apparent in the syntax of the corresponding natural language sentence. Indeed, the variation of the natural language determiner does not require any change at all in the rest of the sentence, whereas changing the corresponding quantifier always requires a change in the propositional connectives of the formula following it, if a corresponding formula can be produced at all.

Therefore, there is no single propositional connective that could fill the place of the question mark in the following semi-formal schemata:

For every *x* }
For some *x* }
For the *x* } *Fx ? Gx*
For most *x* }
For five *x* }

so that we would get correct representations of the following sentence-schemata, which obviously exhibit a uniform structure (just as would the corresponding Latin, etc.):

(1) Every }
(2) Some }
(3) The } *F*(‘s) is/are *G*(‘s)
(4) Most }
(5) Five }

Among these schemata, (1) and (2) can be represented in predicate logic only with formulae involving different propositional connectives, (3) and (5) demand complex formulae to provide their correct truth-conditions (such as the Russellian formula: ‘$(\exists y)\{[Fy \mathbin{\&} (\forall x)(Fx \rightarrow x = y)] \mathbin{\&} Gy\}$’ for (3)), and for (4) there is demonstrably no quantificational formula that would provide its correct truth-conditions.[15]

(ii) By contrast, in the system of predicate logic enhanced with restricted variables (as well as with the requisite set of quantifiers), the following formula schema provides an intuitive formalization of (1)-(5): ‘(Q*x*.*Fx*)(*Gx*.)’. This states that Q *x* that is an *F* is a *G*, or in the plural form, that Q *x*’s that are *F*’s are *G*’s, where Q stands for any of the appropriate determiners or “quantifier words”

[15] For the proof, if “most” is understood as “more than half of the”, see (Barwise and Cooper 1981, 214-215.)

of English (and *mutatis mutandis* the same goes for any other natural language). This immediately establishes the claim that this system naturally extends itself to a generalized quantification theory.[16]

(iii) People who argue for the superiority of modern predicate logic over "traditional", Aristotelian logic often refer to (various versions of) De Morgan's famous example as proof that the Aristotelian analysis of categorical propositions, and correspondingly Aristotelian syllogistic, is incapable of handling reasoning involving relational terms. Intuitively, the following looks like a valid inference: 'Every man is an animal; therefore, every man's head is an animal's head'. However, there is no way of parsing this inference along traditional lines so it would fit into a valid Aristotelian syllogistic form.

Medieval logicians, taking their cue from Aristotle's *Prior Analytics*, treated such inferences under the heading *de syllogismis ex obliquis*, i.e., "on syllogisms involving oblique terms", which is to say, terms in cases other than the nominative case, such as the genitive "man's" in the conclusion of De Morgan's example.[17] To be sure, "standard syllogistic" treats the terms of a syllogism as unbreakable units (just as propositional logic treats atomic sentences as such units), although it allows complex terms as substituends of such units. Therefore, when the validity of an inference turns on the conceptual connections between parts of such complex terms, "standard syllogistic" is indeed inapplicable (just as uniform quantification theory, involving only monadic predicates, is unable to handle inferences with multiply quantified sentences.) So, to account for such inferences, Buridan and others distinguished between the *terms of the syllogism* and the *terms of the propositions*, where the *terms of the syllogism* (in particular, the middle term) can be parts of the *terms of the propositions*, and provided further syllogistic rules in terms of this distinction, referring to the intrinsic complexity of the terms of the propositions involved.

Correspondingly, the predicate logic with restricted variables inspired by Buridan provides a compositional semantics for formulae that represent the internal structure of propositions with complex terms. Therefore, this logic has no more difficulty in handling such inferences than standard predicate logic does. There are, however, some important and instructive differences between the two.

[16] For good surveys of the booming research on generalized quantifiers in the mid-eighties, see (Van Benthem and Ter Meulen 1985), and (van Benthem 1986). For a recent survey of later developments see (Westerståhl 2005).

[17] For Buridan's treatment, see SD 5. 8.

In standard predicate logic, the De Morgan-example can be reconstructed as follows:

$(\forall x)(Mx \rightarrow Ax)$
For every *x*, if *x* is a man, then *x* is an animal

$(\forall x)(\forall y)[(Mx \,\&\, Dxy) \rightarrow (Ax \,\&\, Dxy)]$[18]
For every *x* and every *y*, if x is a man and *y* is the head of *x*, then *x* is an animal and *y* is the head of *x*.

Using restricted variables, the same example can be reconstructed in the following way:

$(\forall x\ Mx)(\exists y\ Ay)(x. = y.)$[19]
Every (*x* that is a) man is (identical with) some (y that is an) animal

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$(\forall x.(\exists y.My)(Dxy.))(\exists u.(\exists v.Av)(Duv.))(x. = u.)$
Every (*x* that is a) head of some (*y* that is a) man is (identical with) a (*u* that is a) head of some (*v* that is an) animal[20]

One important difference between these two reconstructions is that if we drop the parenthetical phrases in the semi-formal sentences that are simply transcribed into the formulae with restricted variables, then we get perfectly good English sentences, which cannot be done with the semi-formal sentences transcribed into the standard formulae. This quite clearly indicates the close match between the syntax of the natural language sentences and the formulae with restricted variables.

Another important difference is that while the standard formulae are true if there are no men or they have no heads, those with restricted variables in those circumstances would be false. Therefore, according to the formalization with restricted variables, the inference is not formally valid, unless there is a further premise to guarantee that if there are men, then there are men's heads. Actually, this is how it should be. After all, even if it is actually true, it is not a *logical truth* (i.e., a truth based on the meaning of logical connectives) that if there are men, then they have heads. Therefore, the formulation with restricted variables provides an even better analysis of the natural language sentences, in the sense that it better reflects our semantic intuitions as to what is and what is not implied by the sentences in question.

[18] I am providing here the "stronger", but "more intuitive" formalization of this sentence. Cf. (Merrill 1977).

[19] To simplify formulae with restricted variables, the matrix of a restricted variable may be omitted after its first occurrence.

[20] For a similar analysis with the same results, see (Orenstein 2000). For a detailed discussion of the neat syntactical match between restricted quantification and natural language sentences, see (Klima 1988).

Thus, we have to conclude that the "Buridan-inspired" predicate logic with restricted variables, besides covering more than standard predicate logic does as far as non-standard quantifiers are concerned, can handle what standard predicate logic can, indeed, while sticking more faithfully to the syntactic construction of natural languages and reflecting better our semantic intuitions concerning reasoning in natural languages.

(iv) What accounts for the difference between the judgments of the two different formalizations concerning the validity of De Morgan's example is their difference in attributing vs. denying existential import to universal affirmative propositions. The reason why De Morgan's example at first appears to be intuitively valid is that we tend to tacitly assume the non-logical truth that if there are men, then they have heads too. However, a formally valid inference has to yield truth from truth with any terms, which is actually not obvious with De Morgan's example. Consider the following, analogous example: 'Every man is an animal; therefore, every man's hat is an animal's hat'. Suppose there are men, but no man has a hat, which is certainly possible. In that case it is obviously true that every man is an animal, but is it true that every man's hat is an animal's hat? Or take the following, perhaps even more obvious example: 'Every horse is an animal; therefore, every horse's wing is an animal's wing'. Knowing that there are no winged horses, and hence no horse's wings, we would naturally tend to reject the conclusion. To be sure, one may still understand this conclusion conditionally, as saying that *if* something is a horse's wing, *then* it is an animal's wing, but that conditional reading would lose precisely the matter-of-fact character of the original categorical claim.

Indeed, other examples can bolster our intuition that even if universal affirmatives may occasionally have the force of a conditional, hypothetical claim, especially when they are supposed to express a law-like statement; nevertheless, it is simply wrong to assume that they *always* have to be interpreted this way. Consider for example the case of Mary boasting to her friends that every boy kissed her at the party yesterday. If her friends later find out that there were no boys at the party, then they will certainly take her for a liar, rather than accept her claim as being "vacuously" true on account of her universal claim expressing a universally quantified conditional with a false antecedent. Such and similar examples could be multiplied *ad nauseam*. What is important, though, is the fact that we do have the intuitive distinction between the categorical and hypothetical readings of universal affirmatives; therefore, a logic that can acknowledge both of these readings is certainly preferable to one that can only handle one of them. Since predicate logic with restricted variables is a conservative extension of standard predicate logic in the sense that all formulae of the standard logic are formulae of the logic enhanced with restricted variables, the latter is of course capable of representing whatever the former can, but not *vice versa*.

CONCLUSION: BURIDAN'S "NATURAL LOGIC" VS. ITS RECONSTRUCTION

Nevertheless, although this has to be the end of this lecture, this is far from being the end of the story of comparing medieval and modern logic. The foregoing could serve merely to illustrate that heeding medieval logicians' *regimentation* of *natural language*, we may be able to come up with some *more natural formalization* in a *formal language*. But, as I argue in detail in my monograph on John Buridan (Klima 2009), where this lecture comes from, Buridan would still not be quite happy with this formal reconstruction of his logic. And the reason would not be its formalism (after all, Buridan also uses some symbolism time and again), but rather its restricted applicability in other areas, where Buridan's logic still has important lessons to teach us. In particular, even if quantification theory with restricted variables can easily be extended to cover a great deal of Buridan's modal and temporal logic, it cannot quite properly handle Buridan's treatment of inten/ional contexts generated by words signifying our mental acts. Moreover, Buridan would not be quite happy with restricted variables representing his common terms, since for him there are *simple* common terms, say, *F*, the semantic properties of which are different from a *complex term*, such as 'an x that is an *F*'. In fact, in various contexts, Buridan would sharply distinguish between the logical import of the two. Finally, and even more importantly, Buridan would reject both the Quinean idea of ontological commitment usually associated with quantification theory and the global distinction between object language and meta-language, built into the very construction of this theory. As I argue in my book, this double rejection allows Buridan to work out a third alternative "between" a Quinean and a Meinongian approach to ontological commitment, as well as a viable logical theory for semantically closed natural languages, avoiding Liar-type paradoxes. But this much may be just enough by way of "a shameless plug" to finish this lecture.

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