

## RUZSA'S WORK

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### Imre Ruzsa—A Man of Consequence

**Abstract.** The singular aim and task of this paper is to present an overview of the life and work of Imre Ruzsa.

#### 1 AN UNUSUAL ROUTE TO PHILOSOPHY

Ruzsa's life was rather different from a typical academic career. He was born on the 12<sup>th</sup> of May in 1921 in Budapest but grew up in a little town in the south-eastern part of Hungary as the son of a tailor. His family couldn't send him to high school, so after finishing elementary school, he worked as an assistant to his father. At the age of seventeen, he left his father's house and worked as a tailor's assistant first in Debrecen, a city in Eastern Hungary, then in Budapest. He joined to the Social Democratic Party once in Budapest, and was admitted to the illegal Communist Party. He worked as the printer for the party newspaper and for this activity, he was convicted to eleven years in prison in 1942. The first time he heard about mathematics beyond the common arithmetical operations was from a Communist economist in the prison courtyard. Like other political prisoners, he was sent to the front lines in a forced labour company in 1944. He escaped and survived the rest of the war with false documents in Budapest.

After the war, he finished high school on an accelerated track, then in 1947, he began his university studies at the Faculty of the Humanities of Budapest University. He attended diverse lectures on philosophy and Hungarian linguistics. After the reorganization of university programs in 1950, he became a student of mathematics, physics and descriptive geometry at the new Faculty of Science. By taking a look at his university records, we can note the intellectual level of the Mathematical Institute of Budapest University in those years: in fact, to this day, all professors listed there are regarded as prominent figures of the history of modern mathematics. At the very least, let me mention the name of Lipót Fejér, Rózsa Péter and Alfréd Rényi. Ruzsa's academic records indicate that he fulfilled the requirements of this institute with flying colors. In personal

conversations, he himself remarked that his mathematical studies allowed him to keep a bit more of a distance from politics at that point. But politics and history didn't release him: in the year 1953, a few weeks before Stalin's death, he was arrested again and sentenced to five years for "war crimes and crimes against the people". He was set free in the next year and rehabilitated in 1957.

In 1956, he finished his university studies and began teaching mathematics in a geological polytechnic. In 1960, he was invited to the University as a lecturer of mathematical calculus. From 1962 on, he taught mathematics for philosophy students. In these years, he began his research into modal and deontic logic. Earlier, while still a student, he had studied mathematical logic with Rózsa Péter, by this time, they were at the same department—Rózsa Péter read Ruzsa's writings in the sixties and gave him extensive advice and comments, especially on the philosophy of mathematics and on the mathematical aspects of logic. Ruzsa was in contact with the other great master of mathematical logic in Hungary, László Kalmár, too, but in philosophical logic, Ruzsa had no ancestors and mentors in Hungary at all, nor did he have any opportunities to study abroad either. He simply used the library and began corresponding with Arthur N. Prior, whose ideas had the greatest influence on him.

## 2 EARLY WORK IN PHILOSOPHY

In the year 1965, the Philosophy Institute of the Hungarian Academy of Sciences offered Ruzsa a job as a research fellow. He accepted it but kept his position at the university as a part-time assistant professor. During the sixties, he wrote several pieces about the philosophy of mathematics: a book for teachers (1967), an article in the *Hungarian Philosophical Review*, a series of articles for scientists and science teachers, remarkable lecture notes on mathematics for students of philosophy (1964), as well as a book for the broader public (1968). Ruzsa had many popular writings on mathematics and often connected the popularization of mathematics with philosophy. He was awarded the Manó Beke prize for popularizing mathematics in 1971.

A second group of his early papers consists of eight survey papers about contemporary research in philosophical logic for the *Hungarian Philosophical Review*. It was not merely academic reasons that led him to write the last three of these about research on symbolic logic in the Soviet Union. Ruzsa needed to prove that modern logic in philosophy didn't threaten the ideological foundations of the Communist regime. He presented what was effectively an argument from authority, showing that symbolic logic was an accepted research area in the Soviet Union. Investigations by Soviet logicians at that time, like Vladimir Smirnov or Aleksandr Zinoviev, who was later to become a political dissident, and some others, were carried out in accordance with logical research at leading

Western universities. Within Hungarian philosophy, during the sixties, Ruzsa stood almost alone with his research program. It would be unjust to deny that there were some philosophers who tried to integrate some tools and ideas of modern logic into university education and research; but mostly these efforts led to no more than a confused mixture of modern and obsolete ideas. In the realm of education, it was Sándor Szalai who did the best work in terms of integrating some modern logic into the logic curriculum at Budapest University during the forties and fifties; but the effects of his work were limited because Szalai was not a logician, not even a philosopher, but a sociologist who knew a fair amount about logic. The situation was paradoxical because in mathematical logic, Hungary had an abundance of great scholars, such as László Kalmár, Rózsa Péter and their numerous students. It was Ruzsa's mission to convey their knowledge to philosophy.

The third group of papers includes the first results of his own research in logic. He focused on two topics: deontic logic and the connection between logic and probability theory. His doctoral thesis at the Hungarian Academy of Sciences (the degree was called "candidate of mathematical science") was based on the latter topic. The title was "Random models of logical systems"; it was prepared without an official supervisor and Ruzsa mentions in the documents no mentor or advisor in Hungary. Ruzsa didn't subsequently return to this topic. Another branch of his early work was, however, the very beginning of a continuous line of research for the decades to come. Deontic systems are in fact special cases of modal logic; Ruzsa's survey papers from the same time display his interest into general modal logic and Kripke semantics. The idea of semantic value gaps which turned out to be the central thought of his logical work emerged during this time from his study of Arthur Prior's work and from correspondence with him.

### 3 AT THE DEPARTMENT OF LOGIC

In 1970, both the structure of the departments and the curriculum for philosophy students was reorganised at Eötvös University in Budapest. Mathematics was banned from the curriculum, but Ruzsa received a new task: he joined in the teaching of logic. The newly founded Department of Logic was in charge of the course of study in logic, which consisted of two main components up until the transition period in 1989-90: two or three semesters of formal logic and two semesters of dialectical logic. The basic principle was that the true logic of Marxist-Leninist philosophy was dialectical logic and formal logic was just a subordinated preliminary study to it. Dialectical logic meant, according to the head of department, a sort of materialistically transformed Hegelian logic; it in fact required no more formal logic than a minimal knowledge of Aristotelian syllogistic.

However, this situation made it possible to teach some real logic to young philosophers as long as one resigned oneself to steering clear of questioning the superiority of dialectical logic. Ruzsa published the first version of his lecture notes in logic in 1969, even though he taught the lectures on logic only the next year, and accepted the invitation to the Department of Logic as an associated professor in 1971. This was a great turn both in Ruzsa's life and in logic education.

There was a threefold difference between earlier "formal" logicians at Budapest university and Ruzsa. Firstly, he had the requisite mathematical background to follow contemporary research and contribute to it. Secondly, he didn't bother with improving and modernising old teaching materials and curricula but wrote a completely new one built on modern logic (and improved it over the next thirty years).<sup>1</sup> Thirdly, he didn't go into discussions about what real dialectical logic was supposed to be. Other people in Hungary, as well as in other Eastern-block countries, tried to sell under the name "dialectical logic" some more or less modern methodology of science and were therefore drawn into conflicts with Hegelian dialectical logicians. Ruzsa didn't interfere with the affairs of dialectical logicians. Instead, he responded in sarcastical short articles when mathematical logic was attacked for sneaking antidialectic, metaphysical, neopositivistic etc. ways of thinking into Marxist philosophy, charges brought on by people who had no real knowledge about the subject. As he was a fellow at a department led by the most militant dialectical logician, he didn't expect anything more in those years than that they leave him to work and teach.

In spite of the often astonishing circumstances, the seventies were fruitful years for Ruzsa both in terms of research and teaching. In modal logic, he generalized Prior's idea of truth value gaps to semantic value gaps and on this basis, he elaborated a Kripke-style semantics for various systems of first-order modal logic. His first paper about these systems was his (1973b). His dissertation based on this research, entitled *Individuals in modal logic*, earned him the degree "Doctor of Philosophical Science" at the Hungarian Academy of Sciences. The expanded English version of the dissertation was published by Martinus Nijhoff Publishers (1981). Ruzsa was appointed full professor in 1978.

In terms of teaching logic, beyond the lectures for philosophy students, Ruzsa was given the task of teaching logic and mathematics for students in theoretical linguistics. There was a lucky coincidence between this task and his new interest in the logical modeling of natural languages. Ruzsa became acquainted with Montague semantics in mid-seventies and immediately began to investigate how the idea of semantic value gaps might be implemented into Montague grammar. This idea led to more substantial changes in Montague semantics than

<sup>1</sup>Let us emphasize among the different versions the legendary "three-volume one" (1973).

in Kripke semantics but also proved to be even more fruitful.<sup>2</sup> For these investigations and for his teaching activity, he received recognition within a circle of younger linguists and some of them joined him as personal students, participants and guest speakers at his seminars in the seventies and eighties.

In 1977, an opportunity arose to expand the group of “formal logicians” within the Department of Logic with two new lecturer appointments. Ruzsa planned to orchestrate the celebration of the 100th birthday of symbolic logic (the centenary of Frege’s *Begriffsschrift*), and as a first task, he assigned to one of the new lecturers (namely, me) the translation of a selection from Frege’s writings. This volume was published with a slight delay, in 1980, accompanied by an issue of the Hungarian Philosophical Review which contained numerous papers on Frege and modern logic and translations of Frege’s articles “The Thought” and “Negation”. It was only Ruzsa’s own extensive programmatic article on Frege and the importance of modern logic for philosophy (1979) that was published exactly for the centenary in the Hungarian Philosophical Review. That is, the Review did not undertake to publish a special Frege-issue, as Ruzsa’s original intention had been. Nevertheless, the centenary of the *Begriffsschrift* was an important step towards the formation of Ruzsa’s school. The work of Frege offered a common starting point for the areas where Ruzsa’s writings and educational activity gained influence over the previous years: logic, philosophy of mathematics, linguistics, philosophy of language. The works published for this occasion therefore reached all the actual and potential students of Ruzsa and sympathizers of his work.

Moreover, this time, Ruzsa could appear in public together with some of his students and members of his circle. In Hungarian philosophy, symbolic logic had often been associated with logical positivism (not only in the era of Marxism-Leninism, but earlier, too). When Ruzsa, as a modern logician, was accused of smuggling neopositivistic influences into Marxist philosophy, he found this charge was awkward and at the same time, also amusing. For he had no special sympathy for logical positivism at all. Carnap belonged, of course, to the most widely cited authors in his monographs, but mostly it was not out of Ruzsa’s agreement with Carnap’s claims. Ruzsa was much more inclined towards realism; he was not a Frege-type Platonist, but his position was closer to Frege than to Carnap. This way,, the centenary celebration also offered an opportunity to present as the founding father of symbolic logic a thinker as far from neo-positivism or any sort of positivism as Frege was.

<sup>2</sup> His first publication on this area was (1980).

## 4 THE NEW DEPARTMENT

Through the eighties, the Ruzsa school flourished. There was a favourable turn of circumstances: in 1982, the unwanted marriage with dialectical logic could be broken off and a new “Group for symbolic logic and the methodology of science” was founded, headed by Ruzsa. It was an odd, unconventional unit that was subordinated to no departments but to the institute of philosophy (“Marxism-Leninism”) only; yet it didn’t have the rank and rights of a department until 1984. The new department began to publish a yearbook called *Tertium non datur*. Besides the members of the department and Ruzsa’s PhD students, several linguists, philosophers and other scholars wrote articles and reviews for the *Tertium*; its table of contents showed that Ruzsa and his circle were now gaining considerable influence in the humanities as well, among people interested in modern methodology.<sup>3</sup> In the yearbook we could now break with the earlier strategy of keeping distance from debates; by this time, it contained several sharply critical papers. In the opening volume, Ruzsa and five of his younger colleagues published a paper that dissected a logic textbook that was in use at teacher-training colleges and unified obsolete ideas of traditional logic with dialectic materialistic slogans. At times, the analysis would change into satire. We had considerable fun putting together this critique, but didn’t foresee the consequences of it: during the next academic year, the textbook was withdrawn by the ministry of education. But to tell the truth, we didn’t yet gather up the confidence to criticize dialectical logic.

Perhaps this is the time to say something about Imre himself as a man. It is not easy because his personality was rather hidden. Autonomy and steadfastness were among his major traits. He had chosen a path for himself and nobody could divert him from it. He wasn’t interested in success, praise or money; he did what he thought was the right thing to do and that was all there was to it. He was very helpful. I think most of the colleagues who knew him are indebted to him, but only few of us can claim to have had a truly personal conversation with him. He endured the humiliating situations that occurred at the Department of Logic with calm irony and on rare occasions, with sarcastic remarks—it was only by the end of the nineties that I understood how deeply he was insulted by them, when we compiled a repertory volume from the volumes of *Tertium non datur* and he wanted to devote two pages of a four-page foreword to this topic. I needed hours to convince him that Comrade Erdei (the head of that department) didn’t deserve so much attention any more. Well, his good sense of humour and irony

<sup>3</sup> Let me illustrate this influence by quoting the names of the Hungarian authors of *Tertium*: András Bárd, Katalin Bimbó, István Bodnár M., Balázs Dajka, Katalin É. Kiss, Özséb Horányi, Márta Fehér, László Kálmán, Ferenc Kiefer, Gyula Klíma, Imre Komlósi L., András Kornai, Judit Maár, Anna Madarász Zsigmond, Márta Maleczki, András Máté, Tamás Mihálydeák, Sándor Önódy, Kornél Solt, Anna Szabolcsi, Zoltán Szabó [Gendler], Tibor Szécsényi.

often helped him in difficult situations. He was a quiet person, never a loud word even if he was angered. On the other side, during the occasional relaxed moment, he liked to make jokes. In the eighties, he wrote a “Dictionary for patho-logicians”. Its entries contain “explanations” of logical notions that often mix wordplay with pin-pricks at colleagues and profound remarks. Unfortunately, most of them are basically untranslatable wordplays in Hungarian, but let me quote one that (hopefully) works in English as well:

*Inconsistency*: a heavy and contagious disease. Especially widespread among philologists. The reduction of texts is the only cure.

In the eighties, Ruzsa published two large monographs. The first was *Classical, modal and intensional logic* (1984). It contained less technical details but a thorough analysis of the philosophical literature on logic, especially on the logic of modalities. In this book, Ruzsa explored the philosophical motivations behind his logic with semantic value gaps and gave in-depth arguments about its advantages. The second book is the two-volume *Logical Syntax and Semantics*—volume I: (1988), volume II: (1989), in which the author gives a self-contained, comprehensive introduction to logical theory, together with the foundations of logical syntax and completing his survey with a description of a formalized fragment of Hungarian. This is the main work of Ruzsa; I shall say more in a bit about its first, metalogical chapter.

## 5 THE LAST YEARS

Ruzsa retired from professorship and from chairing the department in 1990. This unavoidable step together with the fact that some members of the department along with other colleagues from the Ruzsa-circle went on to pursue their careers abroad, at outstanding universities—which was otherwise very much a welcome fact—made the activities of the department somewhat more difficult and less effective. On the other side, at the end of the eighties began our cooperation with the Algebraic Logic department of the Alfréd Rényi Mathematical Institute of the Academy, which made it possible to start the Logic Graduate School, one of the very first graduate programs in Hungary. Although formally Ruzsa was not the leader of this graduate school, he did play a definitive role in its first years, up until the end of the 1990s. He published improved English versions of the two most important chapters of *Logical syntax and semantics: Intensional logic revisited* (1991) and *Introduction to metalogic* (1997). In 1991 he was awarded the Széchenyi Prize, the highest state honour for achievements in science. In 1998 he was appointed professor emeritus. His last larger work was a new textbook of logic (1998), even more comprehensive than the three-volume one.

Ruzsa's advanced age and failing health gradually decreased his involvement in logic and the department. But his former students who visited him over the last years had the chance to witness his spirit remaining the same throughout.

## 6 PHILOSOPHY OF MATHEMATICS

After this biographical outline, let me speak in some detail about Ruzsa's work in two closely connected areas that received less attention in the conference program: philosophy of mathematics and metalogic. Through the sixties, his writings on the philosophy of mathematics emerged not so much from his research interests, but mostly as responses to the interests of his readers. As he writes in the foreword of *Between Mathematics and Philosophy*, he observed that many of his students couldn't buy the lecture notes (1964) to his mathematics lectures for philosophy students (in which he discussed foundations and philosophy of mathematics in detail) because the copies were bought off by interested outsiders. He had written a book presupposing some mathematical knowledge, mainly for teachers of mathematics, but it was, again, not enough. So he published *Between Mathematics and Philosophy* (1968) for the larger public, setting forth in a popularizing style the mathematical background needed. But he didn't regard this area his field of research; his goal was merely to summarize the basics of various trends in the philosophy of mathematics from his own perspective, and convey them to the Hungarian public. So we can't speak about his philosophy of mathematics in the proper sense, I will therefore content myself with characterizing Ruzsa's point of view.

Ruzsa focuses on introducing the three classical schools in the philosophy of mathematics: logicism, intuitionism and the Hilbert-school or formalism. His main stress is on the contents of and mathematical motivations behind these trends, their connection with research in the foundations of mathematics, but he also sets forth some critical remarks. His general opinion is that all the three schools capture something from the real nature of mathematics, but each of them is one-sided; that is, he argues for some sort of eclecticism. He has the most sympathy for Hilbert's program which he demarcates from the formalist philosophy of mathematics. He agrees with this program in that foundational problems should be solved by mathematical tools, while staying away from destroying what was constructed in mathematics. He argues that Gödel's second incompleteness theorem has serious consequences for Hilbert's program but he does not consider them fatal. But he does criticize formalists for rejecting the importance of content in mathematics. Theorems of mathematics have their content, they are true propositions and it happens only for the sake of metamathematical investigations that we abstract from their content and regard them just as syntactical strings. This moderate, realistic understanding of Hilbert's



program and the sympathy for it is characteristic of the philosophical writings of Péter and Kalmár, too. But logicism is evaluated by Ruzsa in a more favorable way than by his predecessors, as he lays more stress on the philosophical, realist side of Frege's and Russell's logicism. On the other side, he criticizes intuitionists rather sharply.

Why were these writings of Ruzsa that contain little by way of original insights so popular during the sixties? Today, readers may be astonished by the occasional Marxist detours and quotations from Engels or Lenin in these works. The official prescriptions of that era were such that it was allowed to expose non-Marxist philosophical views but only when the exposition was accompanied by a thorough Marxist criticism of them. Beyond the fact that Ruzsa surveyed an area that was virtually unknown in Hungary, the novelty of his writings was that he devoted far more space to the exposition and objective analysis of the various philosophies of mathematics than to their criticism. Actually, this was a similar approach as the one found in the works of the circle of George Lukács. For example, in the book *Trends in contemporary bourgeois philosophy* by György Márkus and Zádor Tordai from 1964, we find similar efforts: the authors present the different philosophical schools and thinkers from a Marxist perspective, but the primary stress is on the exposition and analysis of the views. There was a rather sizeable distance between this attitude and the practice of Soviet Marxism, whose main concern was to classify non-Marxist thinkers as mechanical materialists, objective and subjective idealists and to discover traces of Marxist truth in their writings. It must be remarked that the Marxist detours were sincere—both from the side of Ruzsa and from the Lukácsists. They all had some rather abstract commitment to Marxism and socialism—not the actual positions of party ideologists, of course. They tried to preserve as much from Marxism as they found acceptable—there was, of course, no place for criticizing Marxism where it was not acceptable.

## 7 METALOGIC AND THE PHILOSOPHY OF LOGIC

After 1970, Ruzsa stopped publishing on the philosophy of mathematics. However, in his main work *Logical Syntax and Semantics* (1988, 1989) he made an important contribution to the circularity problem in foundations, that is, to the problem that logic has its semantical foundations in set theory but on the other hand, set theory is a theory which can prove its theorems within a logical framework. Ruzsa had always taught that a logical theory is useless if it has no intuitively acceptable semantical foundations. He criticized relevant logics of Zinoviev and systems of entailment for lacking such foundations and regarded Kripke semantics not only as a technical tool but as a way to make explicit the real content of modal logic. In his textbooks and lecture notes, the language

of logic is introduced in a purely semantical way. Inference rules are just mentioned but hardly anything further than that; on the elementary level, there are no formal deductions at all. Students should learn how to check the validity of a given inference by the methods of truth-tables, Venn-diagrams or semantic tableaux; they are not expected to find out consequences of a given set of premises. The methods are semantical and their correctness is likewise confirmed by informal semantical considerations. (There is, of course, no formal semantics at this level.) This is in accord with Ruzsa's rather strong realist commitment that is present in his writings about the philosophy of mathematics and in the paragraphs and chapters concerning the philosophy of logic in his logical writings. The method to begin logic with semantically defined logical constants is present in *Logical Syntax and Semantics*, too; but quite surprising, that actually shows why Ruzsa was not a Platonist, in spite of all of his realist commitments.

The logical theory constructed there starts with introducing symbols of first-order logic—logical constants and variables—into the language of communication (metalanguage). The single difference between their introduction and the usual way is that every variable is declared, that is, it is specified what values they are allowed to take. In this way, the extended metalanguage preserves the property presupposed about the language of communication, namely, that every proposition has one and only one truth-value. Only this much is needed by way of informal semantical considerations behind the metalanguage logic. In order to prove that the axioms of this theory are true, the metalanguage is extended with class abstractions that are constructed from monadic open sentences and it is enough to introduce some minimal class theory which needs no axioms but just definitions of the empty set, the subset relation and the usual binary operations.

The concepts and assumptions needed for the theory of canonical calculi concern language as the class of expressions, that is, finite strings over a finite but nonempty alphabet (the class of letters). Using the operation of concatenation, the class of expressions can be described in an axiomatic way. Canonical calculi define inductive classes within the class of all expressions as strings deducible by a given finite set of rules. (Rules contain mostly a distinguished letter not contained in the alphabet: the arrow, and we are also allowed to use other auxiliary letters.) This very simple machinery suffices for the following:

- to represent calculi by strings of the original alphabet;
- to produce hypercalculi that define the class of all calculi;
- to introduce Gödel numbering, using as “numbers” the strings formed solely from an arbitrary element of the alphabet;
- to prove that there are certain subclasses of the language that can be defined in the metalanguage but are not inductive (although their complements with respect to the set of all expressions are).

This last claim is in fact a Gödel-type theorem now. The following step is the introduction of Markov-algorithms that is natural and easy in this language because the formalisms of canonical calculi and Markov-algorithms are very similar. Roughly speaking, the single difference between the two is that in executing an algorithm, the next step is always determined; in executing a deduction within a canonical calculus, we are free to choose the next step among the allowed ones. Enumerability and decidability by algorithms are defined as usual, and it is easy to show that enumerable sets of expressions are the same as the inductively definable ones. A set is decidable iff both the set itself and its complement is enumerable. With respect to these facts, the theorem mentioned above has as a simple corollary a Church-type theorem: there are enumerable but undecidable sets of expressions.

Real first-order logic follows only after this theory of canonical calculi and algorithms. We can inductively define the language of first-order logic and the set of provable formulas. Within this first-order logic, the theory of canonical calculi (CC) can be formalized and we can prove via metalanguage argumentation that all the theorems of CC are true. In fact, this is the only statement for which we need to use metalanguage logic and set theory. In other words, metalanguage logic has to be accepted on the basis of intuitive semantical background considerations only as far as it is applied to classes of expressions, that is, strictly finite objects. The only place for infinity is that we need a weak form of the induction principle in our metalanguage argumentation. Metalanguage set theory is basically no more than an inventory of abbreviations; its theorems are in fact truths of metalanguage logic.

Everything else turns out surprisingly simple: it follows from the previous theorems that CC is not decidable and every theory which is an inductive class of theorems containing CC is negation-incomplete. Real set theory is a first-order theory defined inductively, and we can use set theoretical propositions in constructing semantics for first-order logic only if we can prove them within this first-order set theory. This whole construction is the answer to the question of how the priority of semantics should be understood: we should accept some semantical considerations before we can construct the syntax of our logic, but these considerations are reduced to a minimum that fulfils the Hilbertian requirement of finiteness. In the formal construction, the priority belongs to syntax and deducibility; there is no Platonic heaven of mathematical objects that we know about without knowing an axiomatic theory of them. Most of the details of Ruzsa's construction of the foundations of logic are not his own inventions; but the construction as a whole is both well-considered and well-founded on the philosophical side and elegant on the mathematical side.<sup>4</sup>

<sup>4</sup> This paper was supported by the Hungarian National Scientific Research Foundation OTKA, project No. 68043.

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