# Projectile coherence: The Van Cittert - Zernike theorem revisited



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# Projectile coherence: The Van Cittert - Zernike theorem revisited

**I. Fabre*[1], F. Navarrete*[2], L. Sarkadi†[3] and R. O. Barrachina*[4]**

* Centro Atómico Bariloche and Instituto Balseiro, Comisión Nacional de Energía Atómica and Universidad Nacional de Cuyo, Av. Bustillo 9500, 8400 Bariloche, Argentina.

† Institute for Nuclear Research of the Hungarian Academy of Sciences (ATOMKI), H-4001 Debrecen, Pf. 51, Hungary

**Synopsis** We study the time evolution of an incoherent mixture of identical particles by solving the Liouville - von Neumann equation for the corresponding density matrix. We demonstrate the quantum mechanical equivalent of van Cittert - Zernike theorem, and apply these results to the quantitative analysis of the coherence of a beam of particles in atomic collisions.

Direct sunlight exhibits spatial coherence over a length of tens of $\mu$m at the Earth surface [1], even though it is produced by a conspicuously incoherent source, the Sun. This effect, where the light emitted from an incoherent source becomes approximately coherent at large distances, was demonstrated by Pieter H. van Cittert [2] in 1934 (see also [3]).

On the other hand, the study of the equivalent coherence properties of particle beams has largely relied on this and other results borrowed from light optics [4]. Even though this strategy is certainly sound for a qualitative analysis of particle coherence, it should be validated by means of a Quantum Mechanical approach in order to be applied to quantitative studies.

In this communication we analyze the time evolution of an incoherent mixture of particles of mass $m$, described by identical wave packets located at different positions upon a much larger region of dimension $D$. We evaluate the time evolution of corresponding density matrix $\langle \mathbf{r}|\rho|\mathbf{r}'\rangle$ by solving the Liouville - von Neumann equation. We exemplify these results by showing in figure 1 how the distribution of wavefronts for an incoherent mixture of particles traversing a slit smooths out at large distances.

Actually, it can be shown that a coherence length $\sigma$ can be defined, such that for $|\mathbf{r}-\mathbf{r}'| < \sigma$ the system behaves coherently (i.e. $\rho$ can be approximated by the product of pure states). In particular, $\sigma$ coincides with the coherence length of each individual wave packet in the Fresnel limit, as expected; while in the Fraunhofer limit the standard expression of van Cittert - Zernike theorem is recovered, $\sigma \approx \gamma t/mD$. Here the factor $\gamma$ depends on the shape of the incoherent distribution, and on how its width is defined. Finally, we show how these and other results of the present theoretical description can be of importance for the study of the coherence properties of particle beams in atomic collisions [5].

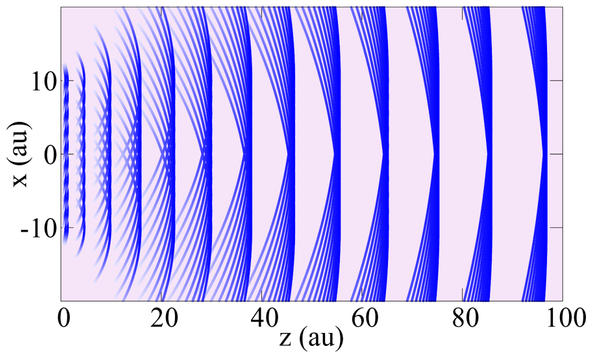


**Figure 1**. Wavefronts for an incoherent mixture of particles traversing a slit of width $D = 20$ au with velocity $v = 1$ au along the $z$ direction. For the sake of simplicity, the individual wave packets are assumed to be of gaussian shape.

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[1]E-mail: ignacio.fabre@ib.edu.ar
[2]E-mail: navarrete@cab.cnea.gov.ar
[3]E-mail: sarkadil@atomki.mta.hu
[4]E-mail: barra@cab.cnea.gov.ar