

論文

Quantification of sustainability as a welfare limit in social-ecological systems

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Abstract

The increasing burden exerted by human activities on natural capitals is expected to seriously jeopardize their stable functioning in the future. The situation urgently requires an operational measure of social-ecological resilience, in light of which the root cause of the instability of the institutional structure of our society could be quantitatively reexamined. By developing a version of Bayesian hierarchical modeling, this paper presents a dynamic and stochastic framework in which a stratified social structure basically determines the sustainability of ecological systems. Specifically, the framework is applied to coastal ecosystems that are trapped in their barren states, i.e., urchin barrens. In the application, using a hypothetical land-use model, the paper regards the social structure as a mathematical operator acting on probability distributions of slow parameters, and thus illustrates how the institutional dimension of society reveals itself in ecological systems.

Keywords : Bayesian hierarchical modeling; social structure; social-ecological resilience; sustainability; urchin barrens

1 Introduction

The ever-increasing burden of human activities on the environment is compromising nature's ability to produce stable ecosystem services. This concern is closely related to the concept of safe operating spaces in the Anthropocene, which requires environmental models to evaluate such spaces both qualitatively and quantitatively, as the growing literature on resilience considers operationalization of the concept as an important characteristic of theoretical development (Leslie et al. [2015], Allen et al. [2016], Quinlan et al. [2016], Verburg et al. [2016]). Regardless of the quantitative analysis, the model should be based on a dynamic framework because the primary

issue is where the system will converge in the long term.

This paper attempts to contribute to the above-mentioned theme by presenting a quantifiable dynamic framework and its application to a coastal ecosystem of seaweed beds, with its hypothetical model simulating a quantitative valuation. The framework is constructed to satisfy three requirements, as suggested by Verburg et al. (Verburg et al. [2016]): (1) to consider all possible social-ecological development paths rather than some selected scenarios; (2) to model interactions of society and nature; and (3) to facilitate extraction of information or implementation of environmental policies by explicitly expressing the relationship between social structure and its effects on sustainability.

The notion of resilience has a wide spectrum of meanings (Angeler and Allen [2016], Gunderson et al. [2010]). For example, engineering resilience applies only to “behavior of a linear system, or behavior of a non-linear system in the immediate vicinity of a stable equilibrium where a linear approximation is valid” (Folke [2006]), because it focuses on the time for a system to return to the previous equilibrium; hence, returning to the original stability is a precondition of the notion (Holling [1996]). On the other hand, the ability to absorb perturbations without shifting to an alternative basin of attraction is called ecological resilience (Scheffer [2009]).

By contrast, this paper deals with the concept of social-ecological resilience, i.e., robustness of ecological systems in terms of persistent supply of ecosystem services so as to meet the current and future needs of both humans and nature; by emphasizing the social aspect of the definition, we interchangeably use the term “sustainability” (Marchese et al. [2018]). As such, social-ecological resilience focuses on “the underlying rules and structures such as values, social norms, laws and policies that govern everyday choices” (WWF [2016]). This viewpoint has been emphasized in the literature (Ostrom [2009], Hinkel et al. [2014], Poe [2014]), but it remains difficult to examine how ecological regimes could shift with social structural dynamics, such as demographic transitions and shifts in economic policies. In other words, incorporating such social hierarchical components into an ecological model to enable ecological regime shifts to be driven by the fundamental structure of society is not straightforward; it requires a rather systematic framework. As one such theoretical framework, a version of Bayesian hierarchical modeling is developed in this study in order to obtain a metric of sustainability. The metric, in turn, facilitates the derivation of a quantitative boundary for any society to remain sustainable, which is subsequently shown by an

application.

1.1 Institutions, rapid reversible changes, and long-term slow shifts

Empirical studies have shown that institutional structure in society has significant effects on the sustainability of natural capitals. For example, in their study of an agropastoral system in Madagascar, von Heland and Folke observed a close relationship between social-ecological resilience and local culture, emphasizing that the persistent supply of ecosystem services is deeply rooted in their social imaginary of clan and moral order (von Heland and Folke [2014]). Similarly, socio-cultural institutions, such as customary tenure and taboos, prevented coral reefs from being exploited by outsiders (Cinner et al. [2016]). In a slightly different context, Arrow et al. attempted to grasp the dynamic effects of the more visible impacts of social structure on ecosystem services, and they proposed the notion of comprehensive wealth to evaluate social sustainability (Arrow et al. [2012]). They noted the temporal effects of technology, population, and institutional quality on wealth.

The shrinking area of arable land in Japan is an example of population dynamics influencing natural capitals and their ecosystem services (MAFF [2016]); the post-war industrialization in Japan was accompanied by population growth and urbanization, resulting in fragmentation and shrinkage of habitats for wildlife. Now, conversely, rapid aging of the population is occurring, leading to shrinkage of the young labor force, especially in rural areas. This has left once-cultivated lands untended, increasing the number of fields unsuitable for agriculture and possibly turning them into recovered habitats for insects such as wild bees. It is not clear what such changes in an exogenous parameter such as demographic aging could mean for the long-term quality and quantity of ecosystem services. As most parameters, including population growth, are not fixed, failure to recognize how the shift in slow variables affects the dynamics of natural stocks could lead to a miscalculation of the system resilience (Carpenter et al. [2001]). This issue is related to the long-term effects of parameter shifts on social sustainability.

At the same time, short-term revisions of social or economic policies could trigger a minor regime shift in the environment. An example of this is the marine ecosystems in the neighborhood of nuclear power plants in the aftermath of the Great East Japan Earthquake and subsequent nuclear accident at Fukushima in 2011 (Sato [2013], Mizuguchi [2015]); after the accident, the

Japanese government ordered full-scale safety inspections of all nuclear facilities across the country, resulting in temporary discontinuation of heated effluents, i.e., sea water used as “once-through” coolant of reactors, being discharged into the sea. The lack of waste heat discharge into the sea for at least a few years caused a noticeable regime shift in the surrounding coastal seaweed beds. This happened because the local sea temperature decreased, for instance, up to 2 °C at the Takahama nuclear plant; previously, such a remarkable fall was possible partly because the total volume of effluents from all the nuclear plants is estimated to be as much as one quarter of the total river flow into the sea across Japan each year (Masuda [2012]). At the same time, the temperature dip brought coastal seaweed beds and their grazers that are adaptable to lower temperatures back to life, causing a minor regime shift in the coastal areas. This is an example of short-term, local consequences of parameter change where ecological resilience often appears intact; in this case, discontinuing the effluent discharge brought the coastal ecosystem back to its old state that existed before the power plants were built.

Conversely, local losses of ecological resilience could be masked by long-term, seemingly unaffected, global sustainability. Large-scale natural disasters often reveal the gap between them; for example, in his study on flora and fauna in the coastline hit by the tsunami due to the Great East Japan Earthquake in 2011 (Nagahata [2012]), the author refers to his findings as numerous local “minor extinctions” of insects, although they are short of major “extinction of species”. Such insects could not survive the natural disturbances because their innate adaptability to the stochastic events had been weakened by human alterations of the environment, such as habitat fragmentation; “rather than the tsunami itself, greater impacts on them were caused by the isolation and segmentation of sandy and marshy areas which had been accelerated by land-use changes.”

These examples require us to explain a few theoretical and empirical issues: What governs the slow parameter shift and how? What should we expect of the quality of ecosystem services at the end of the repeated parametric changes (long-term effects on sustainability)? In other words, how can we predict social-ecological resilience at the start of the parametric shift?

To answer these questions, a stratification of social structure is useful: the basic layer of social structure relevant to parametric transition, such as long-term demographic movements, fundamental technological innovations such as artificial intelligence, and social norms of value, is

assumed to be the fundamental driver of the parametric shift. Over this basic layer is the second layer of political, economic, or environmental policies, such as a specific energy policy in a wide spectrum of policy alternatives, which are visible reflections of the first layer. In this two-layer structure, the former is the more persistent societal structure from which the latter materializes as its characteristics, valid only over a short period of time. Relating the former to the latter requires a Bayesian hierarchical modeling framework.

2 Method and Model

2.1 Framework for quantifying sustainability

A theoretical framework for the quantification of social-ecological resilience can be explained using the terminology of Bayesian hierarchical modeling (Cressie and Wikle [2011]).

The approach divides a stochastic data generation model into three sub-levels to grasp the entire process in light of the conditional dependency. At the top level is a data model that describes the distribution of data given by a true hidden process. Then, directly below it is a process model that deals with the hidden process. At the bottom level lies a parameter model that governs the dynamics of the hidden process.

When applied to our analysis of social-ecological resilience, the data model V corresponds to the valuation of ecosystem services generated by a vector of natural capital stocks, \mathbf{Z} . The establishment of a functional relationship between the two is a focus of empirical studies on ecosystem services (Kumar [2010], Karevia et al. [2011]). Rather than delving into the issue further, we simply postulate the general abstract relationship between the two quantities as $V = V(\mathbf{Z})$ by following the production function approach that connects “the environment as input” with the production of social welfare from ecosystem services (Barbier et al. [2009]). This expression implies that natural capitals generate a measurable form of social welfare.

Next, the process model is presented along with its stochastic dynamics of natural capitals, denoted by $d\mathbf{Z}(t)$, where t denotes the time and d is the differential operator. The dynamics are conditioned by a given parameter vector $\boldsymbol{\theta}$, which is a slow variable. The vector consists of various indicators governing the dynamics of natural capitals, such as endogenous growth rates of certain organisms, carrying capacity of the habitat, and diffusion coefficients of stochastic disturbances.

Considering these effects on natural capitals, we have $d\mathbf{Z}(t) = d\mathbf{Z}(t | \theta)$. Owing to the stochastic nature of the growth of natural capitals, the social benefits from their ecosystem services become a stochastic process.

Finally, the parameter model describes how institutional and cultural backgrounds of society govern the temporal changes of a parameter. For instance, as in the previous example, a policy could bring about a minor regime shift of marine ecosystems by creating a new environment through a fall in sea temperature. As these events occur only probabilistically, a mechanism connecting a parametric change (sea temperature fall) to its driver (energy policy) should be of a probabilistic form; this is where we introduce the parameter model, which describes the temporal transition of a parameter as a function of social structure. More specifically, we regard the first layer of social structure as a mathematical operator in function spaces to which parameter distributions belong, and the second layer of social structure is reflected in how the operator behaves in replacing a current parameter value with a new one. Some technicalities involved in the treatment are elucidated in the following example.

In summary, the framework for analyzing the social-ecological resilience comprises the trinity of data, process, and parameter models, wherein the data model corresponds to the utilitarian evaluation of ecosystem services; the process model, to the ecological dynamics of natural capitals; and the parameter model, to the institutional and cultural structure of society.

From the standpoint of cause and effect, the parameter model is the most basic of the three models, followed by the process model and the data model; this is because social structure ultimately regulates the final persistent state of ecosystem services through parametric updates. However, the opposite is true for the sequence of the causation; social structure itself certainly shifts in response to the extent to which society has benefited from ecosystem services, which is apparent in the two-way interactions of culture and ecosystems, e.g., in the agropastoral system of Tandroy. In the following model, the entire social structure is assessed using a metric of social-ecological resilience. In this sense, the framework of the data, process, and parameter models could be a feedback system (see Figure 1).

2.2 Model and simulations

A mathematical model of marine ecosystems facilitates understanding of the three-step approach

for operationalizing social-ecological resilience. For ease of comprehension, a process model is presented first.

The resilience of marine ecosystems has long been studied in the context of the regime shift in coral reef ecosystems (Nystr and Folke [2001], Scheffer et al. [2001], Hughes et al. [2003], Bellwood et al. [2004], Adger et al. [2005], Folke [2006]). Similarly, the regime-shift dynamics of seaweed beds, particularly urchin barrens, has attracted considerable attention (Conversi et al. [2015], Dakos et al. [2015], Ling et al. [2015], Möllman et al. [2015], Rocha et al. [2015]).

Such deterministic models of coastal ecosystems (Carpenter et al. [1999], May [1977]) are rewritten to consider a stochastic growth model of two interacting natural stocks. The following model serves as a numerical and visual simulation of regime shifts, and it clarifies the connections between model parameters and properties of regime-shift dynamics.

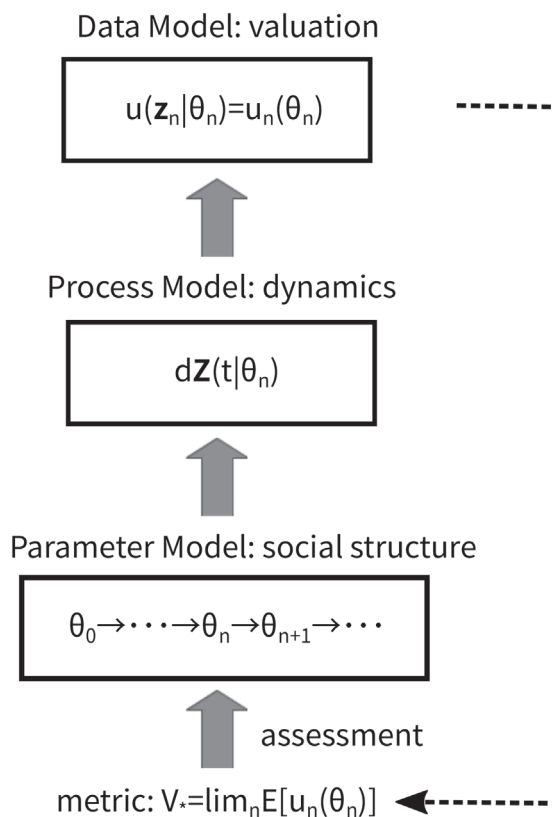


Fig. 1 : Framework of the data, process, and parameter models

Let $X(t)$ be the biomass of seaweed (or coral) at time t , and let $H(t)$ be the population of sea urchins (or crown-of-thorns starfish) feeding on it at the same time. Suppose that the growth rates of both $X(t)$ and $H(t)$ are intertwined quadratically, as described in the following system of stochastic differential equations :

$$\frac{dX(t)}{X(t)} = \left[rX(t) \left(1 - \frac{X(t)}{\kappa} \right) - H(t) \frac{\beta X(t)^2}{\beta_0^2 + X(t)^2} \right] dt + \sigma_1 dW_1(t), \quad (1)$$

$$\frac{dH(t)}{H(t)} = \left[\alpha (X(t) - \bar{x})^2 - \alpha_0 H(t) \right] dt + \sigma_2 dW_2(t), \quad (2)$$

where $r > 0$, $\kappa > 0$, $\beta > 0$, $\sigma_1 > 0$ in (1) and $\alpha > 0$, $0 < \alpha_0 < 1$, $\bar{x} \geq 0$, $\sigma_2 > 0$ in (2) are all temporarily fixed parameters with $W_i(t)$ ($i=1, 2$) independent standard Brownian motions ($E[W_i(t)] = 0$, $Var[W_i(t)] = t$).

The left-hand side of (1) is the ratio of the increase in seaweed biomass to the original biomass, i.e., the growth rate of seaweed. The right-hand side consists of a familiar logistic growth expression of the biomass (where r is the endogenous growth rate and κ the carrying capacity) minus an S-shaped function of the seaweed consumption by urchins, plus a term of stochastic disturbances; if $\beta_0 = X(0)$, the deterministic part is similar to that in equation (3) in a previous article (May [1977]), in which H is a constant density of herbivores. As opposed to the case of the classic model, H is not a constant but a random variable; its growth rate, or the right-hand side of (2), changes stochastically, depending both quadratically on the biomass of algae and linearly on its own decrease, where $\alpha_0 > 0$ is the death rate of the urchins. For example, the rate is relatively high if certain infectious diseases occur or if conservationists remove urchins to prevent the barren state. Both \bar{x} (growth threshold of seaweed biomass) and α (adjustment speed) are equally critical in determining the position and number of attractors of the dynamical system. The stochastic part of the model is a white noise of Black-Scholes type in financial markets (Stojanovic [2002]); equations (1) and (2) are the simplest two-dimensional Black-Scholes equation with their deterministic parts replaced with a typical biological growth model.

In general, the bottom-up (environmental) drivers of algae growth are basically embodied in the parameter vector

$$\theta = (r, \kappa, \beta, \beta_0, \sigma_1, \alpha_0, \bar{x}, \alpha, \sigma_2),$$

whereas the top-down (predator) pressures occur mainly in (2), although there need not be a clear dichotomy between the two sources of controls (Conversi et al. [2015]). Obviously, the vector makes the dynamic paths of two natural stocks differ. The difference is illustrated by some examples of deterministic and stochastic simulations of model (1) and (2). Besides the trajectories of both stocks, we also consider social welfare on the paths that are relevant to sustainability of society.

3 Simulations: dynamic paths of natural stocks and welfare

3.1 Deterministic paths

First, we investigate dynamic paths in deterministic cases : $\sigma_1 = \sigma_2 = 0$. For a given parameter vector, letting $dX(t) = dH(t) = 0 (\forall t \geq 0)$ in (1) and (2) yields the loci of the stationary state of both stocks. For example, the top-left panel in Figure 2 shows these loci for $\theta_1 = (r, \kappa, \beta, \beta_0, \sigma_1, \alpha_0, \bar{x}, \alpha, \sigma_2) = (3, 12, 2, 2, 0, \frac{1}{5}, 0, \frac{1}{15}, 0)$, where the unique equilibrium point occurs at the intersection of the two stationary loci. The adjustment speed is relatively low ($\alpha \approx 0.067$); urchins multiply slowly relative to the algal growth. The dynamic path from the initial value converges to the equilibrium point, suggesting local stability of the point.

To evaluate temporal change in social benefits of the natural stocks, suppose that the social welfare can be measured by a simple function such as

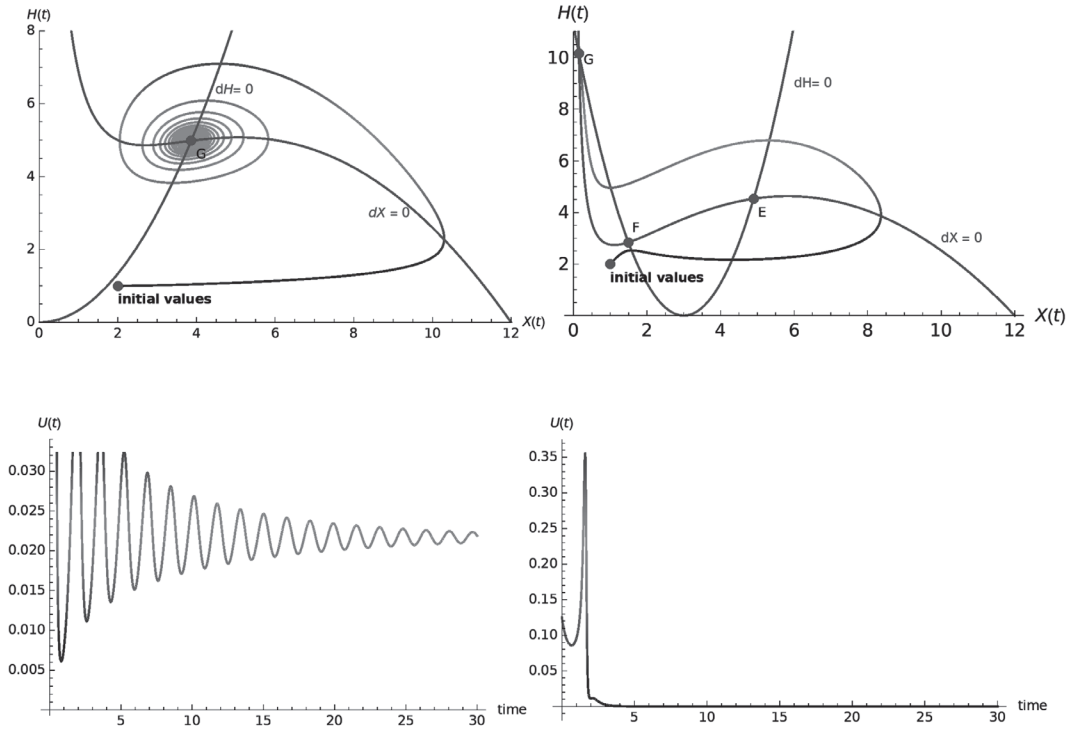


Fig. 2 : Deterministic loci of $(X(t), H(t))$ and their welfare paths (1). The top two panels show deterministic loci with parameter θ_1 (top-left) and θ_2 (top-right), while the bottom two panels show their corresponding temporal $U(t)$ paths; the initial values are $(x, h) = (2, 1)$ (top-left) and $(x, h) = (1, 2)$ (top-right)

$$U(t) \equiv u(X(t), H(t)) = X(t)^\theta H(t)^\eta, \quad (3)$$

where $\theta > 0$ and $\eta < 0$ represent the degrees of social desirability of each stock. Let $\theta = 0.8$ and $\eta = -3$; for simplicity, we ignore the utility of sea urchins as a delicacy. The welfare path on the stock trajectories is an indicator of sustainability.

An example of the welfare along the path in Figure 2(top-left) is plotted below it (bottom-left). The welfare path shows a damping oscillation; seaweeds grow with the population of sea urchins remaining low, whence it slowly increases as the path tends to equilibrium point G.

In contrast to the unique equilibrium in the above-mentioned example, another path in Figure 2 (top-right) shows a case with multiple equilibria for parameter $\theta_2 = (r, \kappa, \beta, \beta_0, \sigma_1, \alpha_0, \bar{x}, \alpha, \sigma_2) = (3, 12, 2, 1, 0, \frac{1}{5}, 3, \frac{1}{4}, 0)$; three equilibria, namely E, F, and G, exhibit different stability

characteristics. In the figure, the locus initiating from a point in the neighborhood of F is repelled by F and then goes around E before finally being absorbed in the basin of attraction of G. The ultimate situation represents an ecologically barren state, as point G corresponds to the state in which numerous crown-of-thorns starfish consume the majority of the coral cover. The bottom-right panel shows the welfare path that flattens once trapped in the basin of attraction.

In such a state near G, the coastal ecosystem would produce limited ecosystem services; getting out of the barren state requires a change in some parameter values because we assume no stochastic disturbances, such as hurricanes. For example, controlled removal of urchins causes α_0 , i.e. the mortality of urchins, to increase rapidly. Let θ_3 be a new parameter with the original value of $\alpha_0 = 0.2$ in θ_2 replaced with $\alpha_0 = 0.9$: $\theta_3 = (r, \kappa, \beta, \beta_0, \sigma_1, \alpha_0, \bar{x}, \alpha, \sigma_2) = (3, 12, 2, 1, 0, \frac{9}{10}, 3, \frac{1}{4}, 0)$. Then, the parameter shift from θ_2 to θ_3 makes point G in Figure 2 disappear; thus, no attractor is generated in the neighborhood of G. Once most urchins have disappeared, the system under the new parameter vector converges to the stable point E (top-left in Figure 3); the figure below it (bottom-left) shows the corresponding welfare path along the locus.

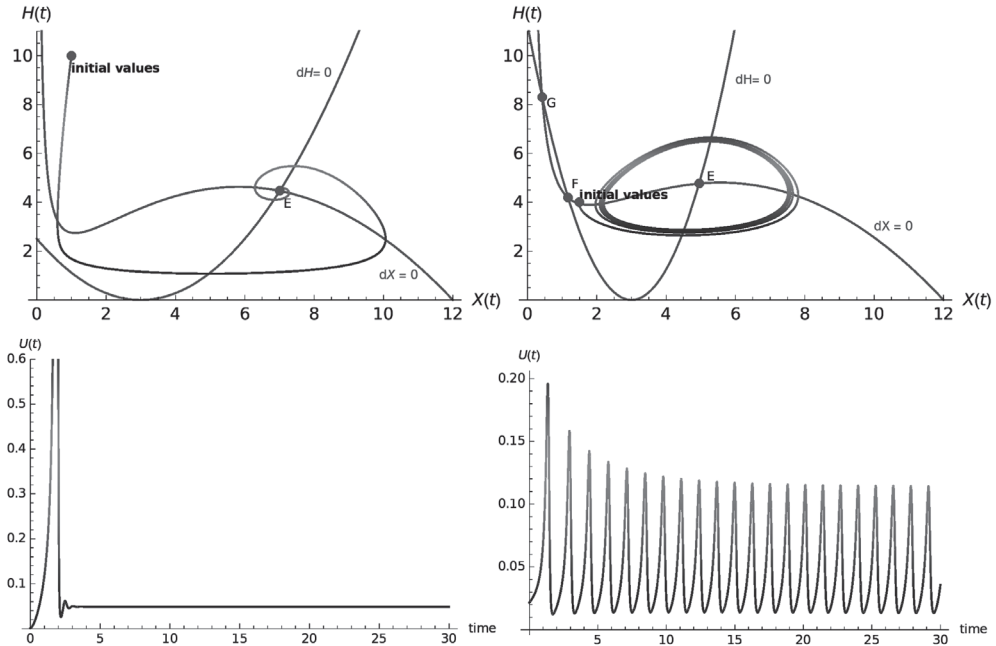


Fig. 3 : Deterministic loci of $(X(t), H(t))$ and their welfare paths (2). The top two panels show deterministic loci with parameter θ_3 (top-left) and θ_4 (top-right) while the bottom two panels show their corresponding temporal $U(t)$ paths; the initial values are $(x, h) = (1, 10)$ (top-left) and $(x, h) = (1.5, 4)$ (top-right), respectively.

Returning to a case of multiple generated equilibria, the top-right figure in Figure 3 shows a path orbiting around the equilibrium point E for parameter $\theta_4 = (r, \kappa, \beta, \beta_0, \sigma_1, \alpha_0, \bar{x}, \alpha, \sigma_2) = (3, 12, 2, \frac{15}{10}, 0, \frac{1}{5}, 3, \frac{1}{4}, 0)$; the bottom-right figure in Figure 3 is the corresponding welfare path. Next, we consider how these two deterministic paths (two panels on the right in Figure 3) are influenced by stochastic disturbances.

3.2 Stochastic paths

Next, letting $\sigma_i > 0 (i = 1, 2)$, we examine the effects of stochasticity using Monte Carlo simulations (Stojanovic [2002]). Let $\sigma_1 = 0.3$ and $\sigma_2 = 0.2$, leaving the other parameters in θ_4 unaltered. With this introduction of stochasticity, θ_4 changes to θ_5 :

$$\theta_5 = (r, \kappa, \beta, \beta_0, \sigma_1, \alpha_0, \bar{x}, \alpha, \sigma_2) = (3, 12, 2, \frac{15}{10}, \frac{3}{10}, \frac{1}{5}, 3, \frac{1}{4}, \frac{2}{10}).$$

Figure 4 (left) shows an example of stochastic trajectories that deviate from the deterministic orbit toward point G, the urchin barren attractor. The path nearly follows the orbiting trajectory in Figure 3 (top-right) partly because both of them have the same values of parameters except for stochastic components. This implies that stochastic disturbances could cause a regime shift (Reed et al. [2011]). Reflecting the figure on the left in Figure 4 about the H-axis and then rotating it 90° clockwise gives another figure on the right, which shows the phase shift in a more conventional manner (cf. Ling et al. [2015], Filbee-Dexter [2014]).

Next, we examine welfare properties with the natural capitals subject to the stochastic dynamics as in Figure 3. How the social welfare evolves has been discussed in the context of sustainability. For example, given a discounted rate $\delta > 0$, the discounted sum of welfare up to time t , $V(t) = \int_0^t e^{-\delta s} u(X(s), H(s)) ds$, was considered in certain contexts of sustainable development (Arrow et al. [2012], Mäler et al. [2009]). As a very simple measure using the sum, $V'(t) > 0 (\forall t > 0)$ could represent a sustainable path of natural capitals. For comparison with the criterion, we simulate the time path of $V(t)$ subject to the stochastic dynamics in (1) and (2).

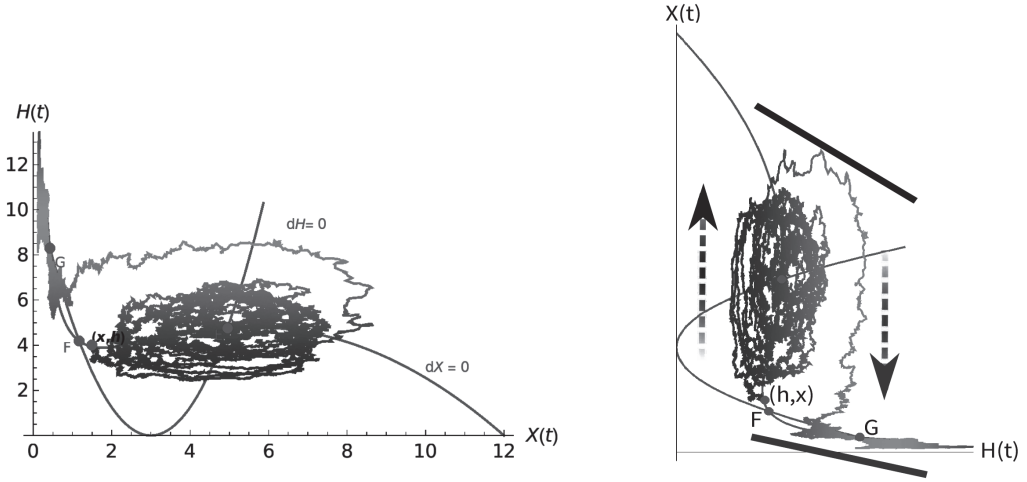


Fig. 4 : Stochastic path of $(X(t), H(t))$ with θ_5 and $(x, h) = (1.5, 4)$

Simulating $V(t)$ is not as straightforward as in deterministic cases. For the calculation, let $\mathbf{Z}(t) = (X(t), H(t))$ and note that the social welfare function $u(\mathbf{Z}(t))$ is of class C^2 , the set of twice continuously differentiable functions. Then, applying Ito's formula to $u(\mathbf{Z}(t))$ yields

$$u(\mathbf{Z}(t)) = u(\mathbf{Z}(0)) + M(t) + \int_0^t \mathcal{L}u(\mathbf{Z}(s)) ds, \quad (4)$$

where $M(t)$ is a local martingale such that

$$M(t) = \int_0^t u_x(\mathbf{Z}(s)) \sigma_1 X(s) dW_1(s) + \int_0^t u_H(\mathbf{Z}(s)) \sigma_2 H(s) dW_2(s),$$

and \mathcal{L} is an operator on functions such that

$$\begin{aligned} \mathcal{L}u(\mathbf{Z}(s)) &= \frac{1}{2} (\sigma_1^2 X^2(s) \cdot u_{XX}(\mathbf{Z}(s)) + \sigma_2^2 H^2(s) \cdot u_{HH}(\mathbf{Z}(s))) \\ &+ \left[rX(s) \left(1 - \frac{X(s)}{\kappa} \right) - H(s) \frac{\beta X(s)^2}{X(0)^2 + X(s)^2} \right] X(s) \cdot u_X(\mathbf{Z}(s)) \\ &+ \left[\alpha (X(s) - \bar{x}) - \alpha_0 H(s) \right] H(s) \cdot u_H(\mathbf{Z}(s)). \end{aligned}$$

In the definitions stated above, u_x and u_H are the partial derivatives of u with respect to X and H ,

respectively. Similarly, u_{XX} and u_{HH} are the second partial derivatives with respect to X and H , respectively (for example, see Bass [2011], Theorem 39.3). Then, the integration by parts formula yields

$$d(e^{-\delta t} u(\mathbf{Z}(t))) = e^{-\delta t} du(\mathbf{Z}(t)) - \delta e^{-\delta t} u(\mathbf{Z}(t)) dt,$$

which implies that

$$e^{-\delta t} u(\mathbf{Z}(t)) = u(\mathbf{Z}(0)) - \delta \int_0^t e^{-\delta s} u(\mathbf{Z}(s)) ds + \int_0^t e^{-\delta s} dM(s) + \int_0^t e^{-\delta s} \mathcal{L}u(\mathbf{Z}(s)) ds.$$

Based on the calculation, Figure 5 shows a result of simulations. On the left is another dynamic path of natural capitals with parameter θ_5 and initial condition $(x, h) = (\frac{15}{10}, 4)$ (the same conditions as in Figure 4). This stochastic path can be viewed from the 3D perspective with a horizontal time axis added to it, which is shown on the left in Figure 6. For $\delta = 0.02$, another figure (right) in Figure 5 shows the corresponding time derivative of discounted welfare, or $V'(t) = e^{-\delta t} u(\mathbf{Z}(t))$, in which the spikes correspond to orbiting trajectories around point E, shooting up as the volume of seaweeds increases and then falling as the sea urchins become more populous and eat them away. This path clearly shows that the increments of welfare are negligible, or $V'(t) \approx 0$, around time 10 onward, when the trajectory of natural capitals is trapped in the neighborhood of point G, i.e., the basin of attraction of urchin barrens.

This result supports the idea of considering urchin barrens as unsustainable states of ecosystem services. Nonetheless, this does not mean that urchin barrens are irreversible; as sustainability is path-dependent, we could have a stock path as in Figure 6 (the one on the right), in which the stock dynamics gets out of the urchin barren around time 20 once trapped around time 4. Such reversible urchin barrens could occur, especially under larger diffusion coefficients (for example, $(\sigma_1, \sigma_2) = (0.5, 1)$ in this case).

Given the welfare criteria based on (3), it is the shift of parameters that caused the path characteristics to differ; hence, it is necessary to consider how parameter changes affect welfare more systematically. Moreover, if we are to quantify the notion of sustainability in terms of a certain welfare function, the point of time at which the sustainability of the stock dynamics is evaluated is equally important. For example, in the simulation of Figure 6, the end point of time

is 30, when the system (the figure on the left) is trapped in the neighborhood of point G; at that time, it does not seem sustainable whereas the other one (on the right) does even though it was once trapped in the same region itself. These two issues are addressed using the two remaining components of the Bayesian hierarchical model, namely the data model and the parameter model.

4 Data model and parameter model

4.1 Data model

The preceding examples illustrate that a parameter shift could cause well-known stability and instability results of equilibrium in ecological systems. Since identifying the parameters that contribute the most to such characteristics becomes even more difficult as the parameter dimension increases, theoretical considerations are required regarding the basic mechanism that drives parameter shifts.

It is the social structure that lies behind parameter shifts. How this structure stipulates the dynamical characteristics of ecological systems has been largely explained in theory, but not in modeling; given a certain structure, capturing the crucial shifts of ecological systems in terms of certain indicators has been a main interest, which is an engineering approach to social-ecological

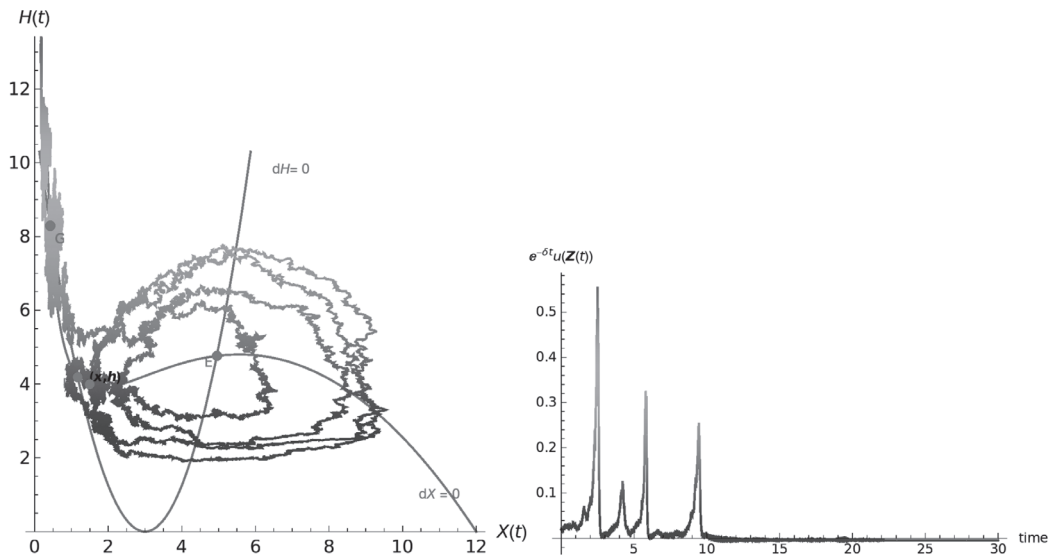


Fig. 5: Another stochastic path of $(X(t), H(t))$ with θ_5 and $(x, h) = (1.5, 4)$ on the left; shown on the right is the corresponding value of $e^{-\delta t} u(Z(t))$ with $\delta = 0.02$, *i.e.*, $V'(t)$, as a function of time.

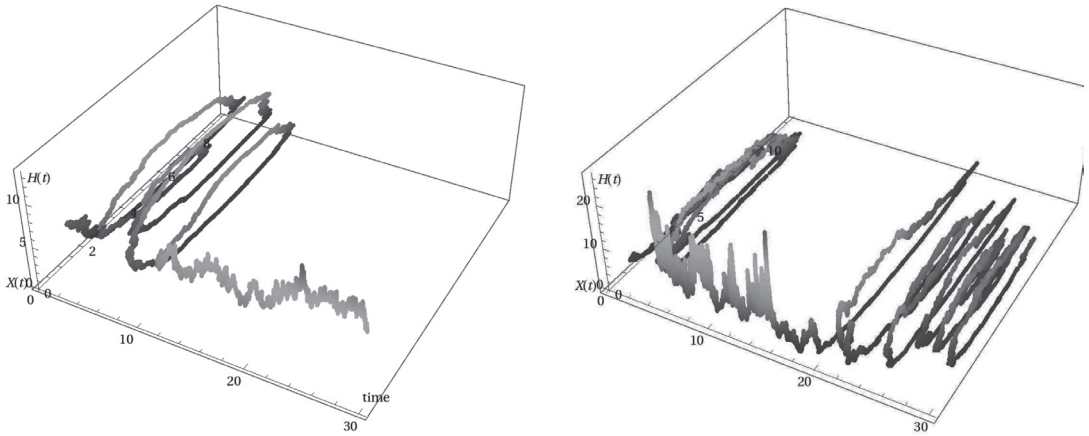


Fig. 6 : Stochastic paths of $(X(t), H(t))$ from the 3D perspective ; on the left is the 3D time-path corresponding to the left figure in Figure 5 ; on the right is a 3D path of natural stocks with the diffusion coefficients in θ_3 replaced with $(\sigma_1, \sigma_2) = (0.5, 1)$.

systems (Scheffer et al. [2015]). Instead, stressing the social respect, we present a model in which the social structure governs the direction of long-term parameter shifts. Furthermore, irrespective of the valuation method applied, different results could be obtained, depending on characteristics of structure itself. We illustrate this point in the framework of the data and parameter models.

The data model connects the dynamic flows of ecosystem services from natural capitals with their social valuations. Suppose that the parameter is fixed, $\theta = \theta_0$, during time interval $[0, t_1]$. As in the simulations, the value affects the dynamical paths of natural stocks. Assume that the path, $\mathbf{Z}(t | \theta_0)$, has the Markov property, which implies that knowledge of the entire history of the path up to any given time provides no more useful information than knowledge of the path at that time. Let U be a continuous function for evaluating ecosystem services. Then, the discounted total value of the services generated along the path in the first period is:

$$u(\mathbf{z} | \theta_0) = E_{\mathbf{z}} \left[\int_0^{t_1} e^{-\delta t} U(\mathbf{Z}(t | \theta_0)) dt \right], \quad (5)$$

where \mathbf{z} is the initial stock of natural capitals at $t=0$, $\delta > 0$ a discount rate, and $E_{\mathbf{z}}$ the integral with respect to the conditional distribution of the path starting from \mathbf{z} .

After having fluctuated subject to ecological dynamics with $\theta = \theta_0$, the natural stocks reach to a new level, \mathbf{z}_1 , at the end of the first period. Observing the new state of natural stocks, the society

adjusts its attitudes, such as environmental policies about CO₂ emission, to the new stock level, which causes the old parameter θ_0 to shift to θ_1 at the start of the second period $[t_1, t_2)$. This process of updating the parameter is represented by a response function K :

$$\theta_1 = K(z_1, \theta_0) \equiv K_1(\theta_0),$$

where the response ignores the initial stock level, z ; the society is assumed to have short memories in that it regards only the current state of stocks as relevant to the environment. Then, the new value of the parameter remains fixed during the second period.

In the same manner, a sequence of slow parameters, $\theta_0 \rightarrow \theta_1 \rightarrow \dots$, is generated by each social response function $\theta_n = K_n(\theta_{n-1})$. The parameter shift affects the phase of ecological dynamics, as seen in Figure 2 and Figure 3, where the removal of sea urchins allows an attractor to disappear. In parallel with the parameter shift, different flows of ecosystem services will be generated, which are valued at each stock level and parameter :

$$u(z_n | \theta_n) = E_n \left[\int_{t_n}^{t_{n+1}} e^{-\delta t} U(Z(t | \theta_n)) dt \right], n=0, 1, \dots$$

where z_n is the stock level at the start of period n , and E_n is the expectation operator conditional on z_n . Let $u(z_n | \theta_n) \equiv u_n(\theta_n)$, and $\{u_n\}$ be referred to as the data model.

4.2 Social process of parameter revisions

This section explains how parameter shifts can be described more systematically in relation to the social structure behind them. First, as opposed to the assumption of a fixed parameter in each period, suppose that each θ_n has its own distribution. Social structure is then defined as the driving force of these distributions.

Let $p_0(\theta_0)$ be the density function of θ_0 , a prior. Using the probability distribution, the values of ecosystem services in the data model are given weights such that :

$$V(z) = \int_Q u(z | \theta_0) dp_0(\theta_0) \equiv \int_Q u_0(\theta_0) p(\theta_0) d\theta_0,$$

where Q denotes a compact support in the n -dimensional space, i.e., the parameter domain.

The point-to-point transition $\theta_0 \rightarrow \theta_1$ is replaced by a distributional shift $p_0 \rightarrow p_1$ that is defined

by a social response function, $K_1(\theta_1, \theta_0) \geq 0$, such that

$$p_1(\theta_1) = \int_Q K_1(\theta_1, \theta_0) dp_0(\theta_0), \int_Q K_1(\theta_1, \theta_0) d\theta_1 = 1,$$

where the integral kernel K_1 represents the probabilistic response to the new stock level, z_1 . Repeating the definition for $n = 2, 3, \dots$, we have a sequence of parameter distributions, $\{p_n\}$, on Q . In general, let G_n be the operator mapping p_n to p_{n+1} : $G_n p_n = p_{n+1}$, or

$$p_{n+1}(\theta) = (G_n p_n)(\theta) = \int_Q K_n(\theta, \theta') p_n(\theta') d\theta'. \quad (6)$$

Since Q is assumed to be a compact set, G_n is a positive compact operator (Lax [2002], Chapter 23).

We refer to G_n as the social structure in period n .

Let $\mathcal{G}_n = G_n G_{n-1} \cdots G_0$. Since $p_n = \mathcal{G}_n p_0$, we have

$$V(z_n) = \int_Q u_n(\theta_n) dp_n(\theta_n) = \int_Q u_n(\theta_n) d(\mathcal{G}_n p_0)(\theta_n).$$

Set $V(z_n) = V_n$. Then, if there exists the limit of V_n as $n \rightarrow \infty$, it could be a surrogate measure of sustainability (Bennett et al. [2009]) because it integrates (1) valuation of ecological services and (2) social effects on ecological dynamics into a combined frame.

5 Norm of sustainability and rigid social structure

5.1 Dichotomy in sustainability evaluation

In comparison with other indicators of social-ecological resilience, what theoretical characteristics does the surrogate measure have?

First, it is comprised of two possibly separable parts of u_n and G_n . The former is concerned with the valuation of ecological services from natural capitals. Since satisfying the current and future demands for the services is the essence of sustainable development, this is the norm, by which “good” or “bad” states of natural stocks are assessed in the context of welfare attained. By contrast, the latter, independent of any normative criteria, focuses on the effects of the social structure on the trajectories of natural capitals. In that sense, it is a neutral description of

resilience projected onto the societal backgrounds. With the two components connected, it becomes clear whether or not society regards its trajectories of natural stocks as sustainable. The notion of surrogate measure, therefore, is included in the category of social sustainability, but is not a resilience indicator; this especially holds true in that a positive assessment of sustainability based on u_n could be wrongly associated with a loss of resilience of ecosystems, an important point of the study, which is elaborated below.

Looking into u_n more closely, it clearly depends on function U that evaluates the flows of ecosystem services. When considering provisional services such as fisheries, the function often takes the form of producer surplus or income (GDP). Even in certain models that discuss the safe operating space, market valuation is applied to evaluating sustainability (Hossain et al. [2017]). In our simulations, however, U does not include any prices, but depends only on fluctuations of the stock level.

There are two reasons for not introducing the market mechanism into the valuation of stock trajectories. One of them is obviously the market failure: the “invisible hand” of the market mechanism with prices as signals for adjustment fails to attain efficient allocations of natural resources although the mechanism itself is supported by the fundamental theorems of welfare economics; exploitation and pollution of natural stocks ensue because the principle cannot be applied to ecosystem services that are open-access or have no property rights clearly defined. It is such areas of ecosystem services that have been deteriorated in both quality and quantity.

In an attempt to recover the reliability of the market principle, several mechanisms for internalization of externalities have been proposed, such as the Pigouvian taxes or emission trading system. In tandem with these, the pricing of non-market services of natural capitals has been eagerly considered, including studies on imputing a shadow price to resilience regarded as an asset (Walker et al. [2009]). These approaches are eventually reduced to the design and implementation of a reliable mechanism for managing natural assets, whether as a complement or as a substitute for the market price mechanism, and are yet to be studied. Nevertheless, we do not recommend any valuation scheme here, but rather focus on another direction of the relationship between valuation and institution.

Managing natural resources sustainably is concerned not only with mechanism design, but also with flexibility or its loss in social structure; institutions are not flexible enough to respond to

changing norms of sustainability: “the crux of the matter is not only to create functional institutions but also, as known from institutional theory, that inefficient or ineffective norms, rules, and values often persist because institutions are ‘sticky’ and not easily replaced nor designed, developed, or changed” (Olsson et al. [2015]). Institutional loss of flexibility has its root in the fundamental social structure, which prevents the feedback between society and ecological systems from working together successfully for sustainability. Some authors refer to the failure as gilded traps: “reinforcing feedbacks between social and ecological systems in which social drivers (e.g., population growth, globalization, and market demand) increase the value of natural resources as the ecological state moves closer to a tipping point” (Steneck et al. [2011]). In the context of \mathcal{G}_n , we can think of such gilded traps as the stationary repetition of a fixed social structure, which is the next focus of our arguments.

5.2 Inflexible social structure and convergence to stationary distribution

An example of a natural environment in the process of losing resilience is the coastal area in eastern Japan before the great tsunami in 2011. The land use there brought about the fragmentation of habitat for living creatures, causing minor extinctions. Social responses against such land use had been weak, and seemed basically unchanged. Expressed mathematically, the response functions, or integral kernels of parameter shift, have changed only negligibly (with the Lebesgue measure zero) from a certain period, despite the sequence of natural capitals $\mathbf{z} \rightarrow \mathbf{z}_1 \rightarrow \dots$ tending toward less resilience.

Consider a mathematical representation. Let K be the kernel for which $K_n = K$ for period $n = 1, 2, \dots$ and \mathbf{G} be the positive compact operator defined by the same kernel. For any initial parameter density p_0 , the revision of social policies leads to the n -th density of the parameter such that $p_n = \mathcal{G}_n p_0 = (\mathbf{G} \cdot \dots \cdot \mathbf{G}) p_0 = \mathbf{G}^n p_0$. Letting $n \rightarrow \infty$, we have $p_n = \mathcal{G}_n p_0 \rightarrow p_*$, where p_* is the eigenfunction relative to the maximum eigenvalue 1 of the operator $\mathbf{G} : \mathbf{G} p_* = p_*$ (Lax [2002], Theorem 2, pp.256-258). Since the multiplicity of the eigenvalue 1 is one, the limit density of the parameter is uniquely determined for any prior probability density p_0 .

Suppose that $u_n \rightarrow u$ in L^2 -norm $\| \cdot \|$, where u is a certain norm for evaluating social sustainability. Then,

$$V_* \equiv \int_Q u(\theta) dp_*(\theta) = \int_Q u(\theta) p_*(\theta) d\theta \equiv (u, p_*),$$

where (\cdot, \cdot) denotes the inner product in the function space. Similarly, set $V_n = (u_n, p_n)$, $V_{n*} = (u_n, p_*)$. Then, we have

$$\begin{aligned} |V_n - V_*| &= |(V_n - V_{n*}) + (V_{n*} - V_*)| \\ &\leq |(u_n, p_n - p_*)| + |(u_n - u, p_*)| \\ &\leq \|u_n\| \|p_n - p_*\| + \|u_n - u\| \|p_*\|. \end{aligned}$$

Since the sequence $\|u_n\|$ is bounded by the convergence assumption, $u_n \rightarrow u$, the inequality implies:

$$\int_Q u_n(\theta) p_n(\theta) d\theta = V_n \rightarrow V_* = \int_Q u(\theta) p_*(\theta) d\theta.$$

Thus, the stationary structure of parameter revisions yields a measure of social-ecological sustainability, V_* .

Making our theory more concrete, we define \mathbf{G} as a description of land use structure in society. If any society maintains a static structure of land use for a long time, the distribution of land use will ultimately be lead to p_* , which in turn yields the measure of social-ecological sustainability V_* . It is naturally expected that a higher value of V_* indicates a wider and safer space left for human activities, since it is an integrated criterion of dynamic paths of natural stocks and society. This, however, is not necessarily true because higher values of V_* could mean deterioration and loss of resilience of the natural environment, which is illustrated using examples in the bubble periods of Japan.

6 Application

6.1 Hypothetical model of land use

This section presents an example of the surrogate measure for a hypothetical social structure. The idea of surrogate measure is based on the limit distribution of parameters governed by a given social structure. Maintaining the idea of valuation in the context of social structure, we slightly modify the treatment of parameter shifts to apply it to ecosystem services of coastal seaweed beds

in Japan.

According to a 1989-91 survey, the total area of seaweed beds across Japan is estimated to be approximately 200,000 ha, having shrunk by 6,400 ha over 13 years (Fisheries Agency [2015], Fujita [2010], MERI [2012]); the decreasing tendency continues to be observed at present : “The approximately 1000 km of coastline across Japan has turned into a barren desert with no seaweeds growing.” (Mastunaga [2010]) The losses are mainly attributed to sea urchins and herbivorous fishes flourishing under the rising sea temperature. In addition, there exist some cultural and social retardants of seaweed growth, such as replacement of natural shorelines with man-made seashores, hindering of the cycle of nitrogen and phosphorus nutrients, pesticides, or domestic and industrial pollutants found in sediment particles, changing dietary culture, heated effluents from power plants, and insufficient management of shrinking forests that provide coastal ecosystems with various nutrients found in humus via rivers.

In the definition of surrogate measure, it is in the dynamics of natural stocks that temporal parameter shifts are supposed to occur, which in turn induce the sequence of social welfare. Instead, assume that they occur not in the dynamics of natural capitals, but outside of it, having direct effects on a social welfare function such that

$$V(X, H | \theta) = X^\theta \exp\left(-\frac{1}{2}(H - \theta)^2\right), \quad (7)$$

where the parameter $\theta > 0$ is one-dimensional and stochastic, shifting with land-use changes. Given the parameter value, the social well-being increases with increasing seaweed biomass as well as increasing sea-urchin population at low levels owing to the utility of sea urchins as a delicacy; at higher levels, their utility decreases¹.

For instance, we categorize four types of land use that affect the eutrophication level in coastal areas: (1) agricultural use (pollution and/or eutrophication by agricultural runoffs, denoted by A), (2) domestic use (pollution and/or eutrophication by domestic drainage, denoted by D), (3) industrial use (pollution and/or eutrophication by industrial use, including heated effluents from power plants, denoted by I), and (4) other uses (mainly preserved forest areas, denoted by F).

¹ The different approach to parameter shifts will help to avoid technical complexities involved in treating initial values simultaneously shifting with parameters in the stock dynamics in the original definition.

Thus, the parameter is a one-dimensional discrete random variable with four possible values, θ_A , θ_D , θ_I , and θ_F , whose subscripts indicate that the values depend on each land-use change upstream. Further, suppose that $\theta_I \leq \theta_D \leq \theta_A \leq \theta_F$; specifically, for calculating the welfare values, let $\theta_I=0.2$, $\theta_D=0.5$, $\theta_A=2$, $\theta_F=3$.

Let $\mathbf{p}_0 = (p_0^D, p_0^I, p_0^A, p_0^F)$ be the initial distribution of θ , where

$$p_0^k = \text{Probability}[\theta = \theta_k], \quad k=D, I, A, F.$$

We assume that $p_0^k > 0 (k=D, I, A, F)$. The stationary distribution is uniquely determined, regardless of the initial distribution, by $\mathbf{p}_* = \lim_{n \rightarrow \infty} \mathbf{G}^n \mathbf{p}_0$, where \mathbf{G} is the given social structure.

6.2 Calibrations of social structural operator

In the finite dimensional setting, the operator \mathbf{G} becomes a stochastic matrix that represents a transition of land use under the social structure. Let θ_{ij} denote the transition probability of land use from i to j , where $i, j=A, D, I$, and F . For example, θ_{AD} denotes the transition probability of land use from agriculture to domestic use, whereas θ_{AA} denotes the probability of no change from agricultural use.

First, assume that the stochastic matrix is such that

$$\mathbf{G} = \begin{bmatrix} \theta_{DD} & \frac{3}{4}(1 - \theta_{II}) & \frac{3}{4}(1 - \theta_{AA}) & \frac{1}{20}(1 - \lambda) \\ \frac{1}{3}(1 - \theta_{DD}) & \theta_{II} & \frac{1}{8}(1 - \theta_{AA}) & \frac{1}{20}(1 - \lambda) \\ \frac{1}{3}(1 - \theta_{DD}) & \frac{3}{16}(1 - \theta_{II}) & \theta_{AA} & \frac{9}{10}(1 - \lambda) \\ \frac{1}{3}(1 - \theta_{DD}) & \frac{1}{16}(1 - \theta_{II}) & \frac{1}{8}(1 - \theta_{AA}) & \lambda \end{bmatrix}.$$

As noted above, the diagonal elements of the matrix denote no change probabilities. For example, the (4,4) entry of the matrix, $0 < \lambda < 1$, is the forest preservation rate, while the other elements of the fourth column describe land-use changes of forest; 90%, $0.9(1 - \lambda)$, is converted into agricultural use, and the remaining $0.1(1 - \lambda)$ is equally divided between industrial and domestic uses. They sum up to unity. On the other hand, summing up each element across columns indicates the total land use for each purpose; for example, the sum of the first row indicates the total domestic land use.

Second, we rewrite θ_{DD} , θ_H , θ_{AA} in terms of the forest preservation rate λ . This requires us to clarify to what social structures \mathbf{G} is calibrated. For this purpose, postulate two land-use structures: (1) the bubble structure of the economy, typically observed from 1985 to 1990 in Japan, and (2) the non-bubble structure from 1991 to 2010. Owing to the availability of data, we replace land-use statistics with water use data for each purpose (see Table 1, MLIT [2014])².

In the data, which reflects land-use changes to certain degrees, we note the shift in average water use between two periods: (1) 1975 to 1990 and (2) 1991 to 2010. We regard this as a social structural transition of land use, and the model \mathbf{G} is based on it. For example, the domestic water-use average changed from 137 million tons to 161 million tons between the two periods; the ratio of 161/137 is interpreted as the no change probability of domestic use plus the conversions to domestic use from the other uses:

$$\theta_{DD} + \frac{3}{4}(1 - \theta_H) + \frac{3}{4}(1 - \theta_{AA}) + \frac{1}{20}(1 - \lambda) = \frac{161}{137}. \quad (8)$$

Similarly, applying the same interpretation to the industrial and agricultural uses yields two other equations:

$$\frac{1}{3}(1 - \theta_{DD}) + \theta_H + \frac{1}{8}(1 - \theta_{AA}) + \frac{1}{20}(1 - \lambda) = \frac{132}{150}, \quad (9)$$

$$\frac{1}{3}(1 - \theta_{DD}) + \frac{3}{16}(1 - \theta_H) + \theta_{AA} + \frac{9}{10}(1 - \lambda) = \frac{568}{580}, \quad (10)$$

2 Domestic use includes water consumption at restaurants, hotels, schools, hospitals, and business offices; industrial water implies water used for washing raw materials and products, water used in boilers, and water used for temperature control. Agricultural uses are mainly water for irrigation, including water for livestock.

	Year	Domestic	Industrial	Agricultural
Stable growth period	1975	114	166	570
	1976	123	165	570
	1977	126	162	570
	1978	128	156	570
	1979	130	153	570
	1980	128	152	580
	1981	133	148	580
	1982	134	145	580
	1983	140	145	585
	1984	142	144	585
Bubble period	1985	143	144	585
	1986	144	141	585
	1987	146	142	585
	1988	149	142	585
	1989	153	144	586
	1990	158	145	586
Post-bubble period	1991	159	148	586
	1992	161	147	586
	1993	161	144	586
	1994	163	141	587
	1995	163	140	585
	1996	164	138	590
	1997	165	138	589
	1998	164	137	586
	1999	164	135	579
	2000	164	134	572
	2001	163	129	564
	2002	163	123	560
	2003	161	121	557
	2004	162	121	552
2005	159	126	549	
2006	157	126	547	
2007	157	128	546	
2008	155	123	546	
2009	154	116	544	
2010	154	117	544	
Average	1975 to 1990	137	150	580
	1991 to 2010	161	132	568
	1991 to 2000	163	140	585
	2011 to 2010	159	123	551
	1975 to 1984	130	154	576

Table. 1 : Water Use in Japan: 1975 to 2010 (intake base; Unit: million tons)

The three independent equations (8), (9), and (10) are solved for three variables θ_{DD} , θ_{II} , and θ_{AA} , given the forest preservation rate λ :

$$\theta_{DD} = \frac{43365295 \lambda - 20725819}{20262300} \equiv \theta_{DD}(\lambda),$$

$$\theta_{II} = \frac{1668660 \lambda - 116023}{1688525} \equiv \theta_{II}(\lambda),$$

$$\theta_{AA} = \frac{27334240 \lambda - 12086947}{15196725} \equiv \theta_{AA}(\lambda),$$

Putting them back into \mathbf{G} yields

$$\mathbf{G}_\lambda = \begin{bmatrix} \theta_{DD}(\lambda) & \frac{3}{4}(1 - \theta_{II}(\lambda)) & \frac{3}{4}(1 - \theta_{AA}(\lambda)) & \frac{1}{20}(1 - \lambda) \\ \frac{1}{3}(1 - \theta_{DD}(\lambda)) & \theta_{II}(\lambda) & \frac{1}{8}(1 - \theta_{AA}(\lambda)) & \frac{1}{20}(1 - \lambda) \\ \frac{1}{3}(1 - \theta_{DD}(\lambda)) & \frac{3}{16}(1 - \theta_{II}(\lambda)) & \theta_{AA}(\lambda) & \frac{9}{10}(1 - \lambda) \\ \frac{1}{3}(1 - \theta_{DD}(\lambda)) & \frac{1}{16}(1 - \theta_{II}(\lambda)) & \frac{1}{8}(1 - \theta_{AA}(\lambda)) & \lambda \end{bmatrix},$$

where the requirement of every element in \mathbf{G}_λ lying between 0 and 1 is satisfied if

$$\frac{20725819}{43365295} < \lambda < \frac{1107787}{1172035},$$

or, approximately, $0.478 < \lambda < 0.945$. We refer to \mathbf{G}_λ as the bubble structure of society.

In comparison with the bubble structure, another structure is introduced as a benchmark. This structure $\tilde{\mathbf{G}}_\lambda$ is generated by replacing the right-hand side in (8) with its reciprocal $137/161$; this is a time reversal occurring only with domestic water use while the other ratios remain unchanged. With this alteration, the no change probabilities are recalculated as in (8), (9), and (10), leading to their new values:

$$\tilde{\theta}_{DD}(\lambda) = \frac{50962135 \lambda - 43280707}{23811900},$$

$$\tilde{\theta}_{II}(\lambda) = \frac{1960980 \lambda - 741919}{1984325},$$

$$\tilde{\theta}_{AA}(\lambda) = \frac{32122720 \lambda - 19957291}{17858925}.$$

Putting these values back again into \mathbf{G} yields the definition of $\tilde{\mathbf{G}}_\lambda$, which is referred to as the non-

bubble structure of society. The range of the forest preservation rate that guarantees that every element in $\tilde{\mathbf{G}}_\lambda$ is positive and less than 1 is $\frac{43280707}{50962135} < \lambda < 1$, i.e., approximately $0.849 \leq \lambda < 1$.

The non-bubble structure of society has some relevance to the land-use reality in Japan. This is explained in terms of the stationary distribution, or the limit distribution, of land use for $\lambda = 0.94$; it tends approximately to

$$\tilde{\mathbf{p}}_{94} = (\tilde{p}_{94}^D, \tilde{p}_{94}^I, \tilde{p}_{94}^A, \tilde{p}_{94}^F) \approx (0.103, 0.086, 0.164, 0.647)$$

where \tilde{p}_{94}^k is the stationary distribution for $k = D, I, A$, and F . In other words, if the forest preservation rate is 94% under the non-bubble structure, the ratios of stationary land use are 64.7% and 16.4% for forest and agricultural land, respectively; with $\lambda = 0.95$, the stationary values are 68% and 15%, respectively. Table 2 shows a recent time series of land uses in Japan that are close to these stationary values.

Year	Forest (%)	Cultivated areas (%)
1990	68.43	14.38
1991	68.41*	14.27
1992	68.39*	14.17
1993	68.37*	14.05
1994	68.35*	13.94
1995	68.33*	13.82
1996	68.33*	13.70
1997	68.31*	13.58
1998	68.29*	13.46
1999	68.27*	13.35
2000	68.25	13.25
2001	68.28*	13.15
2002	68.31*	13.07
2003	68.34*	12.99
2004	68.38*	12.93
2005	68.41	12.87
2006	68.43*	12.81
2007	68.44*	12.76
2008	68.46*	12.70
2009	68.48*	12.64
2010	68.48	12.60
2011	68.48*	12.51
2012	68.47*	12.48
2013	68.47*	12.45
2014	68.46*	12.40
2015	68.46	12.33**

(*): FAO estimates
(**): Official statistics in Japan

Table. 2 : Forest and Cultivated Land Ratios to Total Land Areas in Japan;
(source: Food and Agriculture Organization)

6.3 Influences of social structure on sustainability

Given a social structure \mathbf{G}_λ with the forest preservation rate λ , let $d\mathbf{p}_\lambda(\theta)$ denote the stationary distribution (density) of θ derived as the limit $\mathbf{p}_\lambda \equiv (\mathbf{p}_\lambda^D, \mathbf{p}_\lambda^I, \mathbf{p}_\lambda^A, \mathbf{p}_\lambda^F) = \lim_{n \rightarrow \infty} \mathbf{G}_\lambda^n \mathbf{p}_0$. Using this distribution, the expected social welfare based on (7) is as follows :

$$V_\lambda(X, H) \equiv \int V(X, H | \theta) d\mathbf{p}_\lambda(\theta) = \sum_{k: D, I, A, F} X^{\theta_k} \exp\left(-\frac{1}{2}(H - \theta_k)^2\right) \mathbf{p}_\lambda^k. \quad (11)$$

This function evaluates the sustainability of society in that it quantifies social welfare in terms of the stationary distribution to which the given social structure ultimately leads. If natural stocks are fixed, such as $(X, H) = (x, h)$ in (11), the value of the function is determined for the initial value, which is the case with the original definition of surrogate measure. However, in this application, rather than fixing the size of natural capitals at some point of time, we allow it to change; as the stocks fluctuate subject to the dynamics in (1) and (2) with their parameters fixed, $V_\lambda(X(t), Y(t))$ yields a corresponding time series of stationary values of ecosystems services. Then, the focus is to clarify how the time path of welfare is related to different types of social structures, especially to each rate of forest preservation. We illustrate this using the same simulation method as that used in previous sections.

First, as in (4), the welfare process is defined by

$$V_\lambda(\mathbf{Z}(t)) = V_\lambda(\mathbf{Z}(0)) + M_\lambda(t) + \int_0^t \mathcal{L}V_\lambda(\mathbf{Z}(s)) ds,$$

where M_λ and $\mathcal{L}V_\lambda$ are similarly defined as in (4) with $u(\cdot)$ replaced with $V_\lambda(\cdot)$. Second, fix the parameters in the stock dynamics as θ_5 ; then, compare the effects of two rates of forest preservation, $\lambda = 0.94$ and $\lambda = 0.93$, on the stationary values of well-being under two different social structures.

Figure 7 shows a few results of simulations. The top-left figure in Figure 7 shows a dynamic path of natural stocks starting at $\mathbf{z} = (2, 2)$. Superimposed in the background are contours of $V_{94}(X, H) - V_{93}(X, H)$ under the non-bubble structure with $\lambda = 0.94$ and $\lambda = 0.93$, respectively; this is calculated from (11). The blue background represents the region of $V_{94} - V_{93} < 0$; the no color region represents the opposite. Starting from the point \mathbf{z} , the stock path moves horizontally to the

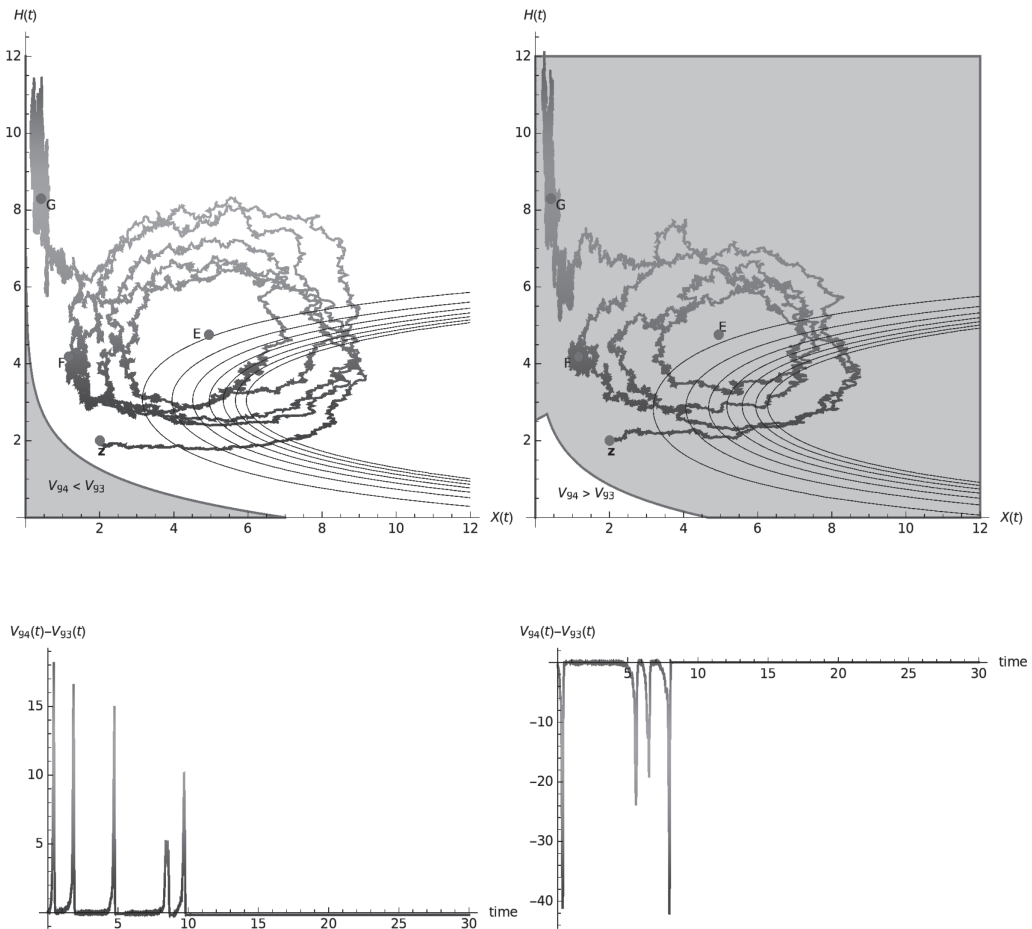


Fig. 7 : Comparison of welfare relative to different rates of forest reservation; the two figures on the top show the loci of $(X(t), H(t))$ against a colored background, i.e., contours of difference, $V_{94}(X, H) - V_{93}(X, H)$, for the two rates of forest reservation $\lambda = 0.94$ and $\lambda = 0.93$. The two figures on the bottom show the corresponding welfare $V_{94}(X(t), H(t)) - V_{93}(X(t), H(t))$ as a function of time.

right, where the contours gain altitude, and then goes around point E back down toward point F; the up and down process in terms of welfare is shown in the figure below it (bottom-left). Before being absorbed into the basin of attraction of point G, the path stays in the white region, implying that $V_{94} \geq V_{93}$; in principle, the more forests that are preserved, the better is the welfare. However, after around time 10, the welfare effect of forest preservation becomes negligible, as observed from the flat line nearly overlapping with the time axis subsequently. This is because in the neighborhood of G along the vertical H -axis runs a narrow region of $V_{94} = V_{93}$ (not clearly seen in Figure 7); this suggests that forest preservation efforts might not contribute to the production

of coastal ecosystem services as much as they would unless the stock path was trapped in the basin.

In distinct contrast to the non-bubble structure, the bubble structure of society produces different results on sustainability, which are seen in the two panels on the right in Figure 7. The path (top-right) shows a stock path similar to the path in the non-bubble structure (top-left), except that the blue-painted region covers most of the background, approximately meaning that $V_{94} \leq V_{93}$ in the quadrant. This is clearly seen from the corresponding welfare path (bottom-right). When the stock path is absorbed in the neighborhood of point G around time 8, the social welfare flattens again, suggesting a similar implication for the effect of forest preservation on the sustainability of welfare. Hence, except for the state of urchin barrens, for greater welfare under the bubble structure of society, cutting down more trees is preferable to sticking to the principle of environmental preservation.

7 Discussion and conclusion

For various values of λ , $\{G_\lambda\}$ represents a mathematical family of social structures in terms of different land uses as forests. As λ tends to zero, the corresponding society uses less land as forests and more land as factories, commercial facilities, or residential areas. Under these structural perturbations, coastal seaweed beds are supposed to produce limited ecosystem services. However, diminishing forest areas are not necessarily linked with decreasing coastal ecosystem services, which is the case with the bubble structure of society, shown in Figure 7. If a societal progression in that direction remains unrestricted, the accumulated burden on the environment may reveal itself in the stochastic development of events such as natural disasters. Only then, a structural shift of society itself, such as $G_\lambda \rightarrow \tilde{G}_\lambda$, could be initiated as reorganizations of political processes, cultural values, and economic policies. This is the feedback process shown in Figure 1 by the arrow from the bottom up to the parameter model.

In fact, in the latter half of the 1980s in Japan, such a structural change rapidly occurred, causing the economic bubble to burst. Looking back over the period, various deregulation policies of financial markets, such as liberalization of interest rates and elimination of barriers between commercial banks and investment banks, had been adopted (Ishii [2011]). With the help of the

policy shift and active stock markets, financing instruments for companies had pushed them towards equity financing, even strengthening in-house fund management sectors. In an attempt to make up for the lost borrowers, banks competed for larger shares of real-estate loans, which accelerated land speculations by bank-affiliated non-banks. For example, the now-defunct Long-Term Credit Bank (LTCB) of Japan, which was once reluctant to partake in real-estate lending, withdrew its self-imposed ban and increased the loan balances two-fold to ¥17.555 billion in five years. Back then, the bank projected that the boom in property investments was solid, backed up by long-term demands for urban office buildings for financial and information services; the view seems at least partly vindicated as shown by constant increases in domestic water use in Table 1, which includes water for offices. With expenditure frenzies on luxurious items, such as imported cars and arts, the land-loan bubble was extended to less populated local areas, encouraged by a new law on resort development in 1987, resulting in a controversy related to environmental protection. As its demographic background, there exists rapid aging in the local communities, which means decreasing areas of cultivated lands. These social trends are partly captured in G_λ . In the long-term assessment of sustainability, it is the entire stratified structure of society, such as demography, economic and social policies, and cultural trends, or, “the way consumerist societies are organized” (WWF [2016]) that matters.

A few aspects of the three-step approach to the resilience metric remain to be addressed. First, the data model is not built on the basis of real data; instead, it is a general theoretical description for evaluating ecosystem services. In proper Bayesian hierarchical modeling, the data model is constructed from data on ecosystem services of seaweed beds, such as the time series of the market size of processed marine products. Although rough estimates of marketable services are available (Fishery Agency [2015]), versatile ecosystem services in coastal ecosystems, including health-related regulating services such as the removal of microbiological contamination, should be taken into account (Lamb et al. [2017]). The method of extending the current model to a real valuation model translates directly into the degree of reliability of the surrogate measure as an actual indicator of the sustainability of society.

Second, equation (1) and (2) may not always reflect the dynamics of actual biological environments. Especially, recent studies in the telegraph noise suggest that such perturbations should replace its stochastic part (Liu and Zhu [2018]).

Third, a more fundamental objection of the metric may be raised in the instrumental, utilitarian view of ecosystem services. As is often pointed out, “trust in neighbors, empathy, mindfulness, and purpose, rather than an accumulation of things” are the foundation of well-being indispensable to social sustainability (Chan et al. [2016]). Such social capitals are nurtured and maintained by constant contact with nature, or human interactions with nature, and not just by the mere existence of natural capitals as a precondition. In this sense, connecting the volume of natural capital directly to social well-being by an exact functional relationship such as in equation (3) would pose a theoretical challenge. The method of estimation thereof will be highly significant because the human-nature relationship yields irreplaceable core values of natural capitals, on which every culture or civilization is founded.

Acknowledgments The author sincerely thanks several anonymous reviewers of early versions of the paper for their valuable comments.

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