Sensory dead zones and time delays in neural feedback control

John Milton, Gabor Stepan and Tamas Insperger

***Abstract*— Sensory dead zones are intrinsic components of the neural control of human balancing. Numerical and analytical studies of the resulting time-delayed switching models for balance control suggest that transient stabilizations of an inverted pendulum are possible. In other words, falls can be an intrinsic property of the same mechanisms designed to prevent them! These observations raise the possibility that the increased risks of falling in the elderly may be a consequence of age-dependent changes in the size of sensory dead zones.**

I. Introduction

Falls are leading causes of mortality and morbidity in the elderly. However, little is known about the mechanisms that cause falls. Recently, video capture studies have shown that only $\approx 24\%$ of falls in elderly subjects who live in an extended care facility can be attributed to "slips and trips" [1]. The majority of falls were associated with incorrect weight transfer and, in particular, 13% of falls occured during quiet standing. These observations support the utility of investigations into human balance control based on the study of fluctuations in the vertical displacement angle, θ, or alternatively in the center of pressure (COP) while subjects are standing. Here we suggest that the statistical properties of the fluctuations in θ as well as the occurence of falls can be attributed, at least in part, to the presence of a sensory dead zone for the detection of θ. By the term "sensory dead zone" we mean the presence of a threshold below which changes in sensory input are not reflected by changes in motor output.

II. Balance control

Current control theoretic investigations into human balance are motivated by considerations of the stabilization of an inverted pendulum by time-delayed feedback [2-4]. The equations of motion take the form of second-order delay differential equations (DDE), for example

$$\ddot{\theta}(t) - \omega_{\mathrm{n}}^2 \sin\theta(t) = f(\theta_\tau, \dot{\theta}_\tau)\,, \tag{1}$$

where τ is the time delay and ω_{n} is the natural angular frequency of the pendulum hung downward [4]. Postural balance during quiet standing is typically modeled as a planar pendulum and hence $\omega_{\mathrm{n}} = \sqrt{3g/2\ell}$ where g is the acceleration due to gravity and ℓ is the length of the pendulum. Stick balancing at the fingertip is modeled as a pendulum on a moving cart and hence $\omega_{\mathrm{n}} = \sqrt{6g/\ell}$. Plausible choices of the feedback, f, can be obtained from a knowledge of the shortest pendulum that can be stabilized for a given τ [2]. In writing (1) we have introduced the notations $\theta_\tau := \theta(t-\tau)$, $\dot{\theta}_\tau := \dot{\theta}(t-\tau)$; $\dot{\theta} := \mathrm{d}\theta/\mathrm{d}t$; and $\ddot{\theta} := \mathrm{d}^2\theta/\mathrm{d}t^2$. In order to obtain a solution it is necessary to define appropriate initial functions, $\Phi(s)$, where $s \in [t_0-\tau, t_0]$ for the initial time t_0.

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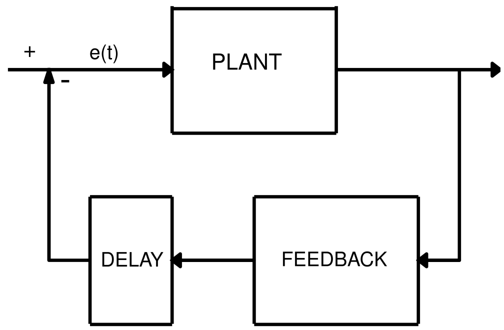

Fig. 1. Block diagram of a time-delayed negative feedback system. The '-' sign indicates that the feedback signal acts to decrease the value of the controlled variable and $e(t)$ is the error signal.

Feedback control corresponds to a looped structure in which the output is fed back, after a time delay τ, to influence future outputs (Figure 1). The plant corresponds to the left-hand side of (1) and the error signal, $e(t)$, is equal to the difference between $\theta(t)$ and the desired outcome, θ_{des}. For balance control, $\theta_{\mathrm{des}} = 0$ and is equal to the fixed point of (1) determined by setting $\ddot{\theta}(t) = 0$, $\dot{\theta}(t) = 0$ and $\theta(t) = \theta_{\mathrm{des}}$. The determination of the stability of the feedback control corresponds to the linear stability analysis of the fixed point. The linearized equation for the error is

$$\ddot{e}(t) - \omega_{\mathrm{n}}^2 e(t) = -k_{\mathrm{p}} e_\tau - k_{\mathrm{d}} \dot{e}_\tau$$

where $e_\tau := e(t-\tau)$, $\dot{e}_\tau := \dot{e}(t-\tau)$, $\ddot{e}(t) := \mathrm{d}^2 e/\mathrm{d}t^2$, and $k_{\mathrm{p}}, k_{\mathrm{d}}$ are, respectively, the proportional and derivative gains. The goal of the feedback is to make $e(t)$ as small as possible and hence the negative signs in the right-hand side. This corresponds to negative feedback.

III. Sensory uncertainty

A critical question for feedback control concerns the sensitivity of the sensor for measuring changes in $e(t)$. In mathematical models, such as (1), it is implicitly assumed that infinitesimally small changes in $e(t)$ can be accurately measured. However, all real sensors, including those in the peripheral nervous system, have limited sensitivity, namely

$$e(t) = \begin{cases} 0 & \text{if } |e(t)| < \Pi\,, \\ e(t) & \text{otherwise}\,, \end{cases} \tag{2}$$

where Π is a sensory threshold. Consequently when $|e(t)| < \Pi$ changes in sensory input do not result in changes in controlling forces. Hence there is a *sensory dead zone*. When Π is very small compared to the fluctuations in the controlled variable, it would be expected that the effects of a sensory dead zone on dynamics would be buried within the intrinsic noisy perturbations. However, in the case of human balance control it is possible that Π is large enough to influence the observed dynamics. A well known example of threshold crossing in human balance control is the "safety net" characteristics of the ankle-hip-step strategies used by humans to maintain balance in the face of increasingly large perturbations [18].

During quiet standing the fluctuations in θ are of the order of tenths of a degree. The vertical displacements of the center of pressure (COP) are of the order of 0.004-0.006m. Hence, if we assume an inverted pendulum of length 1m, the fluctuations in θ are of the order of $0.2 - 0.4^o$. The magnitude of these fluctuations are too small for the detection of movements by both visual and vestibular sensors [5-6]. Consequently the primary sensors for estimating $e(t)$ are proprioceptive, namely muscle spindles, Golgi tendon organs, and cutaneous mechanoreceptors. Estimates of the threshold for the detection of ankle movements suggest that $\Pi \approx 0.05 - 0.08^o$ for unmodulated muscle activity in the ankle joint [5,7]. The threshold increases ten-fold to $\approx 0.5^o$ when agonist muscles are actively modulated [8]. Taken together these observations suggest that muscle contraction is not the only force available for balance control during quiet standing, but that the biomechanical properties of the hip, knee and ankle joints make important contributions.

Dynamic evidence in support of these observations comes from the analysis of the fluctuations in the COP during quiet standing in terms of a correlated random walk [9]. In this approach, the two-point correlation function, $K(\Delta t)$, is interpreted as

$$K(\Delta t) \approx \Delta t^{2H}\,, \tag{3}$$

where $H = 0.5$ for a simple random walk. Experimental observations indicate that for small Δt the random walk exhibits persistence ($H < 0.5$), namely movements in one direction are followed by movements in the same direction. Persistence can be interpreted as open-loop control for small Δt, an observation that is consistent with the presence of a sensory dead zone [7]. Figure 2 shows that when $H = 0.5$ for postural sway, $\Delta t \approx 0.4$s. The average velocity for human postural sway is $0.2 - 0.3^o/s$ [10-11] implying that $\Pi \approx 0.08^o$.

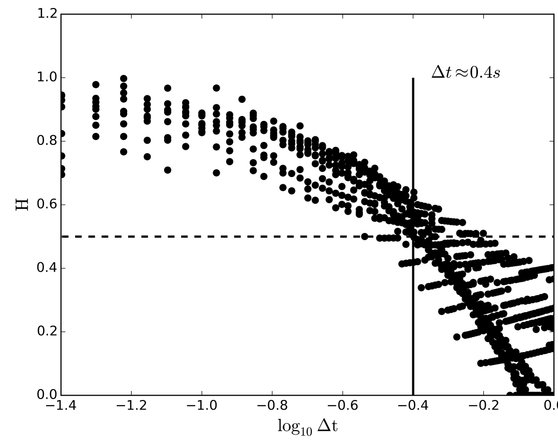

Fig. 2. Scaling exponent, H, for 9 healthy subjects, ages 18-25 years. The fluctuations in the COP were measured using a force platform while subjects stood quietly with eyes closed. The horizontal dashed line corresponds to $H = 0.5$.

In the case of stick balancing the sensory dead zone is related to the difficulty that the visual system experiences in estimating θ in the anterior-posterior (AP) plane [12-13]. Three observations suggest that this is a major control problem for stick balancing: 1) highly skilled stick balancers are unable to maintain stick balancing for longer than 5s when one eye is covered; 2) the fluctuations in θ are larger in the AP plane than in the medial-lateral (ML) plane; and 3) stick falls primarily occur in the AP plane. Figure 3 shows the AP and ML error when a person attempts to quickly align their fingertip under a hanging target under conditions when they cannot viewed both the hanging target and their fingertip at the same time. The alignment error is much greater in the AP ($\approx 3^o$ for 4 subjects; 120 trials) than in the ML ($\approx 0.3^o$) direction.

IV. Switching models

The presence of a sensory dead zone suggests that (1) becomes of the form [7,14-17], for example,

$$\ddot{\theta}(t) - \omega_{\mathrm{n}}^2\theta(t) = \begin{cases} 0 & \text{if } |\theta_\tau| < \Pi\,, \\ -k_{\mathrm{p}}\theta_\tau - k_{\mathrm{d}}\dot{\theta}_\tau & \text{otherwise}\,. \end{cases} \tag{4}$$

The dynamics of (4) are complex [7,16,19-21]. Briefly, the sensory dead zone is a strong small scale nonlinearity since the fixed point is destroyed. The presence of the dead zone has no effect on large-scale stabilization. In other words if the linear system is asymptotically stable when $\Pi = 0$, then it will be stable when $\Pi \neq 0$. However, the presence of this nonlinearity can lead to to small amplitude chaotic oscillations, referred to as *micro-chaos*. From this point of view it is important to note that it was suggested previously by Yamada [22] that the fluctuations in COP during quiet standing are chaotic.

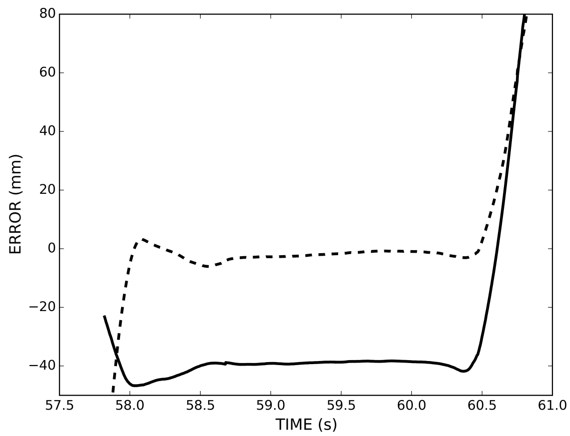

Fig. 3. The error between the position of the fingertip and hanging spherical reflective marker (9mm diameter) in the AP (solid line) and ML (dashed line) direction. The negative sign means that the subject under-estimates the position of the suspended marker. A reflective marker was attached to a thimble placed on the finger and the errors were measured using three motion capture cameras (Qualisys Oqus 300, 500 Hz). Time pressure was enforced by having the subject quickly reposition their fingertip under a different marker every 3s. The vertical distance between the fingertip and hanging marker was maintained at 0.56m. Under these conditions the subject could not simultaneously see both markers.

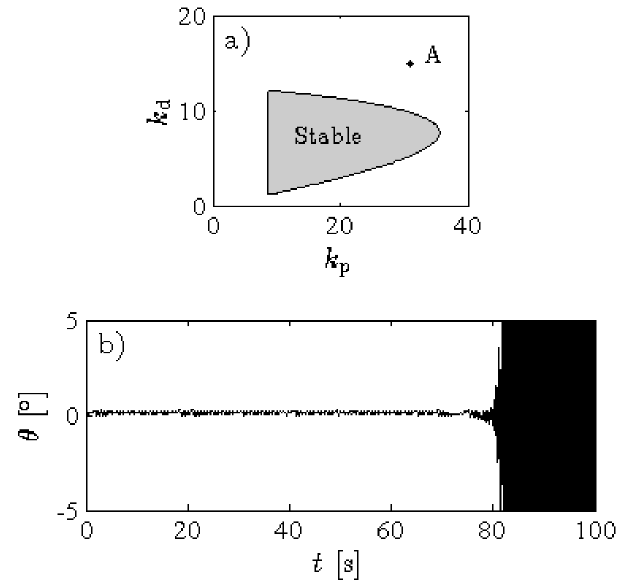

Fig. 4. a) Stability diagram and b) time series (bottom) for (4). The stable domain is indicated by the gray shading. The time series was obtained by setting the control gains (A • in top figure) in the unstable region, i.e., both the open-loop and closed-loop systems are unstable. However, the switching due to the dead zone creates a transient chaos. $\ell = 1.7$m and initial conditions: $\theta(0) = 0^o$, $\dot{\theta}(0) = 0.4^o s^{-1}$, $\Phi(s) = 0^o$ with $s \in [-\tau, 0)$.

Figure 4 draws attention to a counter-intuitive property of (4), namely the inverted pendulum can be transiently stabilized for up to 1-2 minutes [2,7,16]. In this computer simulation the two gains k_p and k_d are chosen such that the fixed point is asymptotically unstable.

V. Example

In order to explore the relationship between a sensory dead zone, time-delayed feedback and transient stabilization we consider the Eurich-Milton model for postural sway during quiet standing [14]. In dimensionless form this model becomes

$$\dot{x}(t) = \begin{cases} x(t) + C & \text{if } x_\tau < -1\,, \\ x(t) & \text{if } -1 \le x_\tau \le 1\,, \\ x(t) - C & \text{if } x_\tau > 1\,, \end{cases} \tag{5}$$

where $x_\tau := x(t-\tau)$. This model incorporates three properties of human balance control: 1) an unstable upright position in the absence of feedback, 2) stabilizing time-delayed feedback, and 3) a sensory dead zone. The fixed point is unstable. Whenever x exceeds a threshold a constant corrective force C is applied after a time delay τ. The solutions of (5) depend only on two parameters, τ and C. It can be readily shown that three types of limit cycle oscillations are possible [14].

Now, assume that the threshold condition is checked only at certain discrete time instants $t_j = j\Delta t$ where $\tau = r\Delta t$ with r an integer [16,23]. This assumption is justified since neural feedback is not likely to be a continuous function of time, but presumably has a digital quality reflecting the observation that spatially separated neurons communicate by discrete action potentials. Thus (5) becomes

$$\dot{x}(t) = \begin{cases} x(t) + C & \text{if } x(t_j - r\Delta t)) < -1\,, \\ x(t) & \text{if } -1 \le x(t_j - r\Delta t) \le 1\,, \\ x(t) - C & \text{if } x(t_j - r\Delta t) > 1 \end{cases} \tag{6}$$

where $t \in [t_j, t_{j+1})$.

If $r = 0$, then the solution of (6) gives the scalar map

$$x(t_{j+1}) = \begin{cases} ax(t_j) + b & \text{if } x(t_j) < -1\,, \\ ax(t_j) & \text{if } -1 \le x(t_j) \le 1\,, \\ ax(t_j) - b & \text{if } x(t_j) > 1\,, \end{cases} \tag{7}$$

where $a = \exp(\Delta t)$ and $b = C(\exp(\Delta t) - 1)$. In the interval $x_j \in [-2, 2]$, the map shown in Figure 5 is identical to the micro-chaos map [16,21]. In this case micro-chaos results from a discretely sampled time delayed system with a dead zone, which is a kind of quantization of the input signal around the origin [7,16]. For different values of a and b, the system experiences different behaviors. If $b < a - 1$ then the system is unstable. If $b < a(a-1)$, then the solution is transiently bounded for a period of time, then exponentially grows. This is the case of transient micro-chaos. Finally when $b > a(a-1)$ there is micro-chaos around the origin.

Figure 6 summarizes the behaviors of (7) in terms of C and Δt. For the same parameters that (7) exhibits micro-chaos (regions labelled MC1, MC2 and MC3), (5) exhibits stable limit cycle oscillations [7,16]. Each of the parameter spaces for which the micro-chaotic solutions exist are extended by a region of transient microchaos (regions labelled TC1, TC2

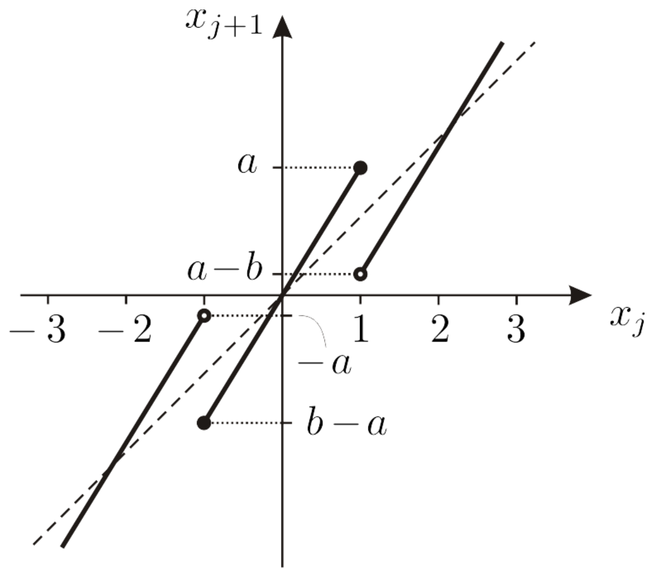

Fig. 5. The map given by (7). Over the interval $x_j \in [-2, 2]$ this map is identical to the micro-chaos map.

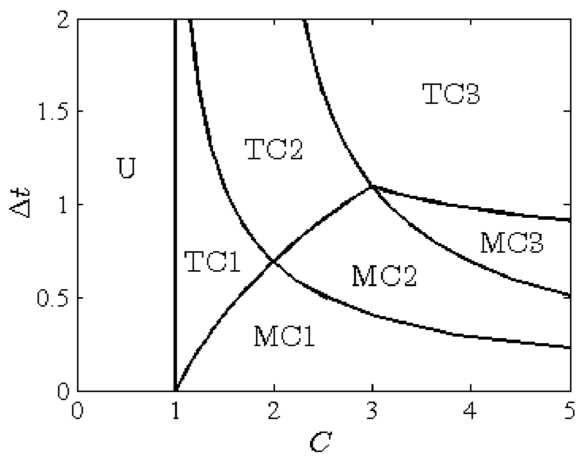

Fig. 6. The steady state behavior of (7) as a function of C and Δt.

and TC3). In other words, the effect of the transient micro-chaos is to extend the parameter range for which balance can be maintained temporarily.

VI. Discussion

Our observations suggest that falls can be a manisfestation of the same control mechanisms designed to prevent them. In particular an increased risk of falling may be related to age-dependent changes in sensory dead zones which result in the control system being tuned to transient micro-chaos. This mechanism may provide an explanation as to why some falls occur in active elderly subjects during quiet standing after a certain time in the absence of cardiac arrhythmias or epileptic seizures [1].

The role of a sensory dead zone in the control of balance during quiet standing remains an open question. There are certainly a number of advantages of switching-type feedback for balance control. During quiet standing part of balance control can be attributed to the biomechanical properties of the ankle, knee and hip joints. Thus it would be anticipated that neural control strategies which act "only when needed" would be energetically favored [24-25]. Moreover in the presence of noisy perturbations the addition of a threshold minimizes the destabilizing effects of over control.

Many investigators have favored the use of continuous types of feedback control for balance [26-28]. However, it has proven to be surprisingly difficult to distinguish a nested control strategy that contains both open and closed-loop control from a strategy that relies on continuous feedback using systems identification techniques [16,29]. The importance of the possibility that sensory dead zones are involved in balance control is that it suggests that the increased risk of falling in the elderly may be related to age-dependent changes in Π. If so, then our observations would support the utility of techniques based on chaos control and stochastic resonance for lowering the risk of falling. An obvious advantage of these approaches is that they can safely and readily implemented.

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