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Computation of worst-case spacing errors of vehicle platoons based on a skew-μ method ⋆

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Abstract: A numerical method is presented for the computation of spacing error bounds in vehicle platoons. The resulted bounds can be applied to evaluate performance of specific platoon control algorithms. The upper bound calculation is based on the computation of worst-case induced $\mathcal{L}_2$-gains of systems defined between the disturbance inputs to the leader vehicle and the spacing errors of the follower vehicles. The worst-case is defined subject to heterogeneity in the vehicle dynamics. The method is applicable both in the frequency- and the time-domain. The latter enables the extension of robustness analysis to a wider class of uncertainties and communication topologies. The method is illustrated on a numerical example with leader and predecessor following control architecture.

1. INTRODUCTION

A challenge of our times is to handle the capacity overload of the road transportation infrastructure. One solution is to exploit the available capacities more efficiently. In order to increase road capacity and avoid congestions on highways the vehicles can be organized in automated platoons (Sheikholeslam and Desoer (1990),Bender (1991)). In this paper the focus is placed on the longitudinal control of vehicles where the control objective is to keep the distances between the vehicles small, while guaranteeing a high level of safety and performance. Performance and safety are closely related terms. Disturbances and change in the reference speed induce transients in the spacing which are propagated along the platoon. An important condition of scalability (the property that any number of vehicles may join the platoon) is *string stability*, Swaroop and Hedrick (1996), Shaw and Hedrick (2007), Xiao and Gao (2011) and Ploeg et al. (2014).

Whenever string stability is established, the performance of the automated platoon controllers can be characterized in terms of worst-case spacing error bounds. The worst-case is defined subject to all possible disturbances, uncertainties in the modeling and heterogeneity in the set of vehicles that constitute the platoon.

A possible choice for measuring errors is the $\mathcal{L}_\infty$ signal norm. A bound on the spacing error peaks may help determining the safe gaps between the vehicles. A numerical method to determine the induced $\mathcal{L}_\infty$ norm of vehicle platoons are presented in Rödönyi et al. (2014). They evaluated three different controllers subject to model uncertainty, platoon heterogeneity and communication delay, however the computational cost explodes exponentially as the number of vehicles grows.

In this paper induced $\mathcal{L}_2$ gains are computed for the systems defined between leader vehicle disturbances and spacing errors. The heterogeneity in vehicle dynamics is represented by a time-invariant uncertainty plus a common nominal vehicle model. Since the size of the uncertainty is fixed a priory a skew-μ analysis method is applied. The advantage of this approach as compared to previous results, Rödönyi et al. (2014), is that longer platoon can be examined. The prise to pay is that the calculated bounds are only upper-bounds (Meinsma et al. (1997)). The upper bounds are compared to lower-bounds. It is illustrated with the help of a numerical example that the upper-bounds computed based on the skew-μ method are tight.

The paper is organized as follows. In Section 2 the heterogeneous vehicle platoon model is described with a leader and predecessor control architecture. The skew-μ analysis and the lower bound of the induced $\mathcal{L}_2$ norm are discussed in Section 3 and tested in Section 4.

1.1 Notations

Let $\mathcal{L}_2^n$ denote the space of square integrable signals with norm defined by $\|x\|_2^2 = \int_0^\infty \|x(t)\|^2 dt$, where $\|x(t)\|$ denotes the Euclidean norm, i.e. $\mathcal{L}_2^n = \{x \in \mathcal{L}^n : \|x\|_2 < \infty\}$. The induced $\mathcal{L}_2$-gain of a system G is denoted by $\|G\|_\infty$. For linear time-invariant (LTI) stable systems $\|G\|_\infty = \sup_\omega \bar{\sigma}(G(j\omega))$, where $\bar{\sigma}(.)$ denotes the largest singular value of a matrix. I_n denotes the $n \times n$ identity matrix. The state-space representation of a system denoted by $M = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$, and the transfer function of M is $M(j\omega) := C(j\omega I - A)^{-1}B + D$. The upper linear

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fractional transformation is defined by $\mathcal{F}_U(M,\Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}$, where $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$. The ith element of vector x is denoted by x_i, the ith row (column) of matrix M is denoted by M_{i*} (M_{*i}), the ij element by M_{ij}. Matrix inequality $M > 0$ denotes that M is symmetric and positive definite. The transpose of a matrix M is denoted by M^T and the conjugate transpose by M^*.

2. HETEROGENEOUS VEHICLE PLATOON MODEL

2.1 Vehicle platoon model

The basic concept of the vehicle platoon in consideration is that the lead vehicle (indexed with $i = 0$) is driven by a human driver, and the motion of the follower vehicles ($i = 1, \dots, n$) are determined by on-board controllers.

The mathematical model of the ith vehicle longitudinal dynamics is the following continuous time state-space model (Rödönyi et al., 2014)

$$\dot{p}_i(t) = v_i(t), \tag{1a}$$
$$\dot{v}_i(t) = a_i(t), \tag{1b}$$
$$\dot{a}_i(t) = -\frac{1}{\tau_i}a_i(t) + \frac{g_i}{\tau_i}u_i(t), \tag{1c}$$

where p_i, v_i and a_i denote position, velocity and acceleration, respectively. u_0 denotes the acceleration demand computed from the pedal signals of the lead vehicle, u_i, $i = 1, 2, ..., n$ for the follower vehicles denote the acceleration demand generated by the controllers. The spacing errors between the vehicles are defined by

$$e_i(t) = p_i(t) - p_{i-1}(t) + L_i, \quad i = 1, \dots, n \tag{2}$$

where L_i denotes the prescribed constant gap between the vehicles. Without loss in generality L_i can be set to zero in the analysis.

Using the definition of spacing errors the vehicle dynamics (1) is reformulated using state variables $x_0 = a_0$ and $x_i = [e_i \ \delta_i \ a_i]^T$, $i = 1, \dots, n$, where $\delta_i = \dot{e}_i$,

$$\dot{x}_i(t) = A_i x_i(t) + A_{i,i-1}x_{i-1}(t) + B_i^u u_i(t), \ i = 0, 1, \dots, n, \tag{3}$$

where

$$A_0 = -\frac{1}{\tau_0}, \ B_0^u = \frac{g_0}{\tau_0},$$
$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, \quad A_{i,i-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix},$$
$$B_i^u = \begin{bmatrix} 0 \\ 0 \\ \frac{g_i}{\tau_i} \end{bmatrix}, \quad i = 1, \dots, n.$$

The open-loop platoon model with vehicle dynamics (3) reveals the form

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_d u_0(t), \tag{4}$$

where $x = [x_0, \ x_1^T, \dots, x_n^T]^T$ and $u = [u_1, \dots, u_n]^T$ and

$$A = \begin{bmatrix} A_0 & 0 & \dots & 0 \\ A_{1,0} & A_1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & A_{n,n-1} & A_n \end{bmatrix}$$
$$B_u = \mathrm{diag}\{0, B_1^u, \dots, B_n^u\}$$
$$B_d = [B_0^u \ 0 \ \dots \ 0]^T.$$

The closed-loop vehicle platoon with the following controller with constant spacing policy and leader and predecessor following architecture proposed by Swaroop (1994) is analyzed in the paper

$$u_1(t) = -k_1\delta_1(t) - k_2 e_1(t) + a_0(t) \tag{5a}$$
$$u_i(t) = -k_{1\beta}\delta_i(t) - k_{2\beta}e_i(t) + k_{a0}a_0(t) + k_{a1}a_{i-1}(t) - k_{1\alpha}\delta_i^0(t) - k_{2\alpha}e_i^0(t), \ i = 2, ..., n, \tag{5b}$$

where k_* are constant parameters and

$$e_i^0(t) := p_i(t) - p_0(t) = \sum_{j=0}^{i} e_j(t)$$
$$\delta_i^0(t) := v_i(t) - v_0(t) = \sum_{j=0}^{i} \delta_j(t).$$

The controller utilizes information from local radars and receives acceleration information through a V2V communication network, which assumed to be lossless (no delay, no pocket loss). Controllers (5) can be expressed as

$$u(t) = Kx(t) \tag{6}$$

where

$$K = \left[\begin{array}{c|ccc|ccc|c|ccc} 0 & -k_2 & -k_1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ k_{a0} & -k_{2\alpha} & -k_{1\alpha} & k_{a1} & k_{2\eta} & k_{1\eta} & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ k_{a0} & -k_{2\alpha} & -k_{1\alpha} & 0 & -k_{2\alpha} & -k_{1\alpha} & 0 & \dots & k_{2\eta} & k_{1\eta} & 0 \end{array}\right]$$
$$k_{1\eta} = -k_{1\alpha} - k_{1\beta}, \qquad k_{2\eta} = -k_{2\alpha} - k_{2\beta}.$$

Inserting controller (6) into the platoon model (4) the closed-loop platoon system, denoted by $\mathcal{G}_n$ can be derived in the form

$$\begin{aligned} \dot{x}(t) &= (A + B_u K)x(t) + B_d u_0(t) \\ e_n(t) &= Cx(t), \end{aligned} \tag{7}$$

where $C = [\,0|0 \ 0 \ 0|\dots|1 \ 0 \ 0\,]$.

2.2 Platoon heterogeneity

In a heterogeneous platoon the vehicle parameters τ_i, g_i may be different for every $i = 0, 1, \dots, n$. Shaw and Hedrick (2007) proposed an informal definition of heterogeneous string stability as "A heterogeneous vehicle string is string stable if the propagating errors stay uniformly bounded for all string lengths and vehicle type orderings." If the platoon is heterogeneous string stable, a vehicle can join the platoon at any location without the need for reconsidering the analysis for string stability. The number of the possible vehicle type orderings explodes as the number of vehicles in the platoon grows, therefore it can not be tested efficiently.

It is assumed that every parameter value $\hat{\tau}_i, \hat{g}_i$ is uncertain with constraint

$$\hat{\tau}_i \in [\underline{\tau}, \bar{\tau}], \quad \hat{g}_i \in [\underline{g}, \bar{g}], \tag{8}$$

where $\underline{\tau} = \min_i \tau_i$, $\bar{\tau} = \max_i \tau_i$ and $\underline{g}, \bar{g}$ are defined similarly. It was shown in Rödönyi et al. (2014) that the peak to peak gains of the platoon from u_0 to e_i $i > 0$ are convex functions over the reasonable range of parameters, thus it is sufficient to test the parameter vertices of the polyhedron.

The platoon model (7) is reformulated into linear fractional transformation (LFT) form for later $\mathcal{L}_2$-gain analysis. Let uncertainty set (8) be written as

$$\hat{\tau}_i = r_\tau + s_\tau \delta_{\tau_i} \tag{9a}$$
$$\hat{g}_i = r_g + s_g \delta_{g_i}, \quad i = 0, 1, \ldots, n, \tag{9b}$$

where $r_\tau = \frac{\bar{\tau}+\underline{\tau}}{2}$, $s_\tau = \frac{\bar{\tau}-\underline{\tau}}{2}$ and $\delta_{\tau_i} \in [-1, 1]$, r_g, s_g and δ_{g_i} defined similarly. Using these equations the uncertainties are equal to

$$\hat{g}_i = \mathcal{F}_U(M_g, \delta_{g_i}) \tag{10a}$$
$$\frac{1}{\hat{\tau}_i} = \mathcal{F}_U(M_\tau, \delta_{\tau_i}), \tag{10b}$$

where

$$M_g = \begin{bmatrix} 0 & s_g \\ 1 & r_g \end{bmatrix} \tag{11a}$$
$$M_\tau = \begin{bmatrix} -s_\tau & -s_\tau \\ 1 & 1 \end{bmatrix} \frac{1}{r_\tau}. \tag{11b}$$

In vehicle model (1) the uncertainties appear only in the derivative of a_i, which can be written using the uncertainty description (9) and (10) as

$$\dot{a}_i = -\mathcal{F}_U(M_\tau, \delta_{\tau_i}) a_i + \mathcal{F}_U(M_\tau, \delta_{\tau_i}) \mathcal{F}_U(M_g, \delta_{g_i}) u_i, \tag{12}$$

which equals to a single LFT defined by

$$\begin{aligned} \dot{a}_i &= -\frac{1}{p_\tau} a_i + \frac{p_g}{p_\tau} u_i + \frac{1}{p_\tau} w_{\tau_i} + \frac{1}{p_\tau} w_{g_i}, \\ w_{\tau_i} &= \delta_{\tau_i} z_{\tau_i}, \\ w_{g_i} &= \delta_{g_i} z_{g_i}, \\ z_{\tau_i} &= -\frac{s_\tau}{p_\tau} w_{\tau_i} + \frac{s_\tau}{p_\tau}(a_i - w_{g_i} - p_g u_i), \\ z_{g_i} &= s_g u_i. \end{aligned}$$

Using these equations the system description (7) leads to the following LFT form

$$e_n = \mathcal{F}_U\left(M_n, \Delta_n\right) u_0, \tag{13}$$

where $\Delta_n = \mathrm{diag}\{\delta_{\tau_0}, \ldots \delta_{\tau_n}, \delta_{g_0}, \ldots, \delta_{g_n}\}$ and

$$M_n = \left[\begin{array}{c|cc} A_M & B_\Delta & B_p \\ \hline C_\Delta & D_\Delta & D_{p\Delta} \\ C_p & D_{\Delta p} & D_p \end{array}\right] :=$$
$$\left[\begin{array}{c|ccc} A + (B_u + B_b)K & B_\tau & B_\tau & B_d + B_l \\ \hline C_\tau + D_{\tau u} K & D_\tau & D_\tau & D_{\tau u_0} \\ D_{gu} K & 0 & 0 & D_{gu_0} \\ C & 0 & 0 & 0 \end{array}\right],$$

$$B_b = \left[\begin{array}{cccc} 0 & 0 & \ldots & 0 \\ \hline 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & \frac{p_g}{p_\tau} & \ldots & 0 \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & \frac{p_g}{p_\tau} \end{array}\right], B_\tau = \left[\begin{array}{cccc} \frac{1}{p_\tau} & 0 & \ldots & 0 \\ \hline 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & \frac{1}{p_\tau} & \ldots & 0 \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & \frac{1}{p_\tau} \end{array}\right],$$

$$B_l = \left[\frac{p_g}{p_\tau} | \; 0 \; 0 \; 0 | \ldots | 0 \; 0 \; 0\right]^T,$$

$$C_\tau = \left[\begin{array}{c|ccc|c|ccc} \frac{s_\tau}{p_\tau} & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{s_\tau}{p_\tau} & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & \frac{s_\tau}{p_\tau} \end{array}\right],$$

$$D_\tau = \mathrm{diag}\{-\frac{s_\tau}{p_\tau}, -\frac{s_\tau}{p_\tau}, \ldots, -\frac{s_\tau}{p_\tau}\},$$
$$D_{\tau u} = \mathrm{diag}\{-\frac{s_\tau p_g}{p_\tau}, -\frac{s_\tau p_g}{p_\tau}, \ldots, -\frac{s_\tau p_g}{p_\tau}\},$$
$$D_{\tau u_0} = \left[-\frac{s_\tau p_g}{p_\tau} \; 0 \; \ldots \; 0\right]^T, \; D_{gu} = \mathrm{diag}\{s_g, s_g \ldots, s_g\},$$
$$D_{gu_0} = [s_g \; 0 \; \ldots \; 0]^T.$$

In matrices A and B_u, all τ_i and g_i, $i \geq 0$, are replaced respectively by nominal values p_τ and p_g.

3. $\mathcal{L}_2$-GAIN BOUNDS

We want to compute the $\mathcal{L}_2$-gain of the heterogeneous vehicle platoon: $\gamma_i := \|\mathcal{F}_U(M_i, \Delta_i)\|_\infty$. An upper bound of the induced $\mathcal{L}_2$ norm can be computed by the skew-μ analysis. As the number of uncertainties are increasing the conservatism of this method is increasing too (Meinsma et al., 1997). In order to evaluate the upper bound a method for lower bound computation is presented.

3.1 Polytopic method for computing a lower bound of the $\mathcal{L}_2$-gain

The vehicle parameter constraints (8) determine a polyhedron. A specific vehicle platoon j, which is a vertex of the polyhedron, characterized by defining the sequence of parameters $\lambda_j(i) = [\tau_{j_0}, \tau_{j_1}, \ldots, \tau_{j_i}, g_{j_0}, g_{j_1} \ldots, g_{j_i}]$. Let $\Lambda(i)$ denote the set of all possible sequences of parameters. Clearly the number of possible sequences is 4^{i+1}. Let the platoon system with parameters $\lambda_j(i)$ denoted by $\mathcal{G}_i(\lambda_j(i))$. A lower bound of the $\mathcal{L}_2$-gain can be computed as

$$\gamma_{PM_i} := \max_{\lambda_j(i)\in\Lambda(i)} \|\mathcal{G}_i(\lambda_j(i))\|_\infty. \quad (14)$$

3.2 Frequency-domain skew-μ analysis

For skew-μ computation the following lemma from Doyle et al. (1982) is applicable.

Lemma 1. For all $\|\Delta_i\|_\infty \leq 1$ the LFT $\mathcal{F}_U(M_i,\Delta_i)$ is well-posed, internally stable and for all $\omega \in \mathbb{R}$:

$$\bar{\sigma}(\mathcal{F}_U(M_i(j\omega),\Delta_i(j\omega))) \leq \gamma_i(j\omega) \quad (15)$$

if and only if

$$\mu_\Delta(M_i(j\omega)\,\mathrm{diag}\{\gamma_i(j\omega), I_{2i+2}\}) < \gamma_i(j\omega). \quad (16)$$

For structured singular value computation integral quadratic constraints (IQC) are used.

Lemma 2. (Scherer (2001)). Suppose there exist Hermitian Π^i and $\Pi^i_p = \begin{bmatrix} -\gamma_i^2 & 0 \\ 0 & 1 \end{bmatrix}$ for all $\|\Delta_i\|_\infty \leq 1$ such that

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{w}_i(j\omega) \\ \hat{z}_i(j\omega) \\ \hline \hat{u}_0(j\omega) \\ \hat{e}_i(j\omega) \end{bmatrix}^* \begin{bmatrix} \Pi^i(j\omega) & 0 \\ 0 & \Pi^i_p(j\omega) \end{bmatrix} \begin{bmatrix} \hat{w}_i(j\omega) \\ \hat{z}_i(j\omega) \\ \hline \hat{u}_0(j\omega) \\ \hat{e}_i(j\omega) \end{bmatrix} \geq 0 \quad (17)$$

holds, where $w_i = [w_{\tau_0}, w_{\tau_1},\ldots,w_{\tau_i}, w_{g_0}, w_{g_1},\ldots,w_{g_i}]^T$, $z_i = [z_{\tau_0}, z_{\tau_1},\ldots, z_{\tau_i}, z_{g_0}, z_{g_1},\ldots,z_{g_i}]^T$, $\hat{}$ denotes the Fourier-transform of the signal and

$$[\,*\,]^* \left[\begin{array}{cc|cc} \Pi^i_{11}(j\omega) & 0 & \Pi^i_{12}(j\omega) & 0 \\ 0 & -\gamma_i^2(j\omega) & 0 & 0 \\ \hline \Pi^i_{21}(j\omega) & 0 & \Pi^i_{22}(j\omega) & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} I \\ M_i(j\omega) \end{bmatrix} < 0 \quad (18)$$

for all $\omega \in \mathbb{R}$. Then $\mu_\Delta(M_i(j\omega)\,\mathrm{diag}\{\gamma_i(j\omega), I\})$ is less than γ_i.

In case of vehicle platoon the Δ_i contains real uncertainties, which can be described with the following IQC (Scherer and Weiland, 2000)

$$\Pi^i(j\omega) = \begin{pmatrix} -D_i(j\omega) & jG_i(j\omega) \\ -jG_i(j\omega) & D_i(j\omega) \end{pmatrix} \quad (19)$$

and $D_i = D_i^* > 0$, $G_i = -G_i^*$, both diagonal.

Inequality (18) is reformulated according to (19) as

$$[\,*\,]^* \left[\begin{array}{cc|cc} -D_i(j\omega) & 0 & jG_i(j\omega) & 0 \\ 0 & -\gamma_i^2(j\omega) & 0 & 0 \\ \hline -jG_i(j\omega) & 0 & D_i(j\omega) & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} I \\ M_i(j\omega) \end{bmatrix} < 0. \quad (20)$$

Then the induced $\mathcal{L}_2$ norm of system $\mathcal{F}_U(M_i\Delta_i)$ can be computed as $\gamma_{FSM_i} := \max_\omega\{\min \gamma_i(j\omega) \text{ subject to (20)}\}$.

3.3 Time-domain skew-μ analysis

In this subsection the index i, which denotes the number of the follower vehicles, is omitted for simplicity. The frequency-domain skew-μ analysis described in Section 3.2 can be reformulated in the time-domain with the Kalman-Yakubovich-Popov (KYP) lemma. First the dynamical multiplier $\Pi(j\omega)$ has to be factorized as

$$\Pi(j\omega) = \Phi(j\omega)^* P \Phi(j\omega), \quad (21)$$

where P is a constant real symmetric matrix and Φ is real rational proper stable function. The columns of Φ as $[\Phi_1\ \Phi_2]$ have minimal state-space realizations

$$\Phi_1 = \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right], \quad \Phi_2 = \left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right]. \quad (22)$$

With this multiplier factorization the following KYP lemma (Veenman and Scherer (2014)[Thm. 1]) is applicable.

Lemma 3. Suppose that the LFT representation (13) is well-posed. Then (13) is stable and the $\mathcal{L}_2$-gain is less than γ, if there exist X, P symmetric real matrices such that

$$[\,*\,]^T \begin{bmatrix} 0 & X & 0 & 0 \\ X & 0 & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & \Pi_p \end{bmatrix} \times \left[\begin{array}{ccc|cc} & I & & 0 & 0 \\ \hline A_1 & 0 & B_1 C_\Delta & B_1 D_\Delta & B_1 D_{p\Delta} \\ 0 & A_2 & 0 & B_2 & 0 \\ 0 & 0 & A_M & B_\Delta & B_p \\ \hline C_1 & C_2 & D_1 C_\Delta & D_1 D_\Delta + D_2 & D_1 D_{p\Delta} \\ \hline 0 & 0 & C_p & D_{\Delta p} & D_p \\ 0 & 0 & 0 & 0 & I \end{array}\right] < 0 \quad (23)$$

holds.

Veenman and Scherer (2014) suggested the following multiplier factorization in case of real uncertainty. Let the factorization matrices Φ and P have the following structure

$$\Phi(j\omega) = \begin{bmatrix} \phi_\nu(j\omega) & 0 \\ 0 & \phi_\nu(j\omega) \end{bmatrix}, \quad P = \begin{bmatrix} -P_1 & P_2 \\ P_2^T & P_1 \end{bmatrix}, \quad (24)$$

where $\phi_\nu = \mathrm{diag}\{\phi_{\nu_0},\ldots,\phi_{\nu_{2i+1}}\}$, $P_1 = \mathrm{diag}\{P_{1_0},\ldots,P_{1_{2i+1}}\}$, $P_2 = \mathrm{diag}\{P_{2_0},\ldots,P_{2_{2i+1}}\}$. For every uncertainty block search separately for an appropriate ϕ_{ν_k}, $k = 0,\ldots,2i+1$ as a basis function

$$\phi_{\nu_k}(j\omega) = \begin{bmatrix} 1 & \frac{1}{j\omega - \rho_k} & \ldots & \frac{1}{(j\omega-\rho_k)^{\nu_k}} \end{bmatrix}^T, \quad (25)$$

where $\rho_k < 0$, $\nu_k \in \mathbb{N}_0$ are the tuning parameters for a good approximation of the dynamic multiplier. With appropriate parameters the LMI (23) is solvable, and an upper bound of the $\mathcal{L}_2$-gain is computable in the time-domain.

To determine the parameters ν_k, ρ_k the resulting multipliers from the frequency-domain analysis is used. Let ω_m $m = 1,\ldots,N$ denote the grid points of the ω interval on which the inequality (20) is solved and $\Pi_k(j\omega_m)$ $k = 0,1,\ldots,2n+1$ the obtained multiplier at ω_m for the kth uncertainty block. To measure the goodness of the factorization the difference $F_k(m) = \Phi_k(j\omega_m)^* P_k \Phi_k(j\omega_m) - \Pi_k(j\omega_m)$ is introduced. A weighting function $W(j\omega_m)$ can be applied to penalize the error of the fit at the important frequencies where the gain, γ_m, is high. For a fixed ρ_k, ν_k the following optimization problem with variables λ_k, P_k is applicable to determine the goodness of the tuning parameters

$$\min \lambda_k \quad (26a)$$

$$\begin{bmatrix} \lambda_k I & F_k(m) W(j\omega_m) \\ W^*(j\omega_m) F_k(m) & I \end{bmatrix} > 0, \quad m = 1,\ldots,N. \quad (26b)$$

This inequality is equivalent to $\bar{\sigma}(F_k(m)W(j\omega_m)) < \sqrt{\lambda_k}$, then if λ_k sufficiently small an appropriate IQC factorization has been found. To determine IQC factorization for a fixed ν_k parameter the optimization problem (26) is solved, which is nonlinear in variable ρ_k.

4. NUMERICAL RESULTS

The vehicle platoon with $n + 1$ vehicles is tested. The controller parameters are $k_1 = 0.7$, $k_2 = 0.1127$, $k_{1\alpha} = 0.4642$, $k_{2\alpha} = 0.0564$, $k_{1\beta} = 0.2358$, $k_{2\beta} = 0.0564$, $k_{a1} = 0.0449$, $k_{a0} = 0.9551$ from Rödönyi et al. (2012). In the numerical analysis the effect of u_0 is considered separately for every spacing error e_i, $i = 1, \ldots, n$.

Three different case of vehicle heterogeneity will be examined:

(P1) only parameter τ_i may differ with nominal value $r_\tau = 0.7$ and $g_i = 1$, $i = 0, \ldots, n$;
(P2) only parameter g_i may differ with nominal value $r_g = 1$ and $\tau_i = 0.7$, $i = 0, \ldots, n$;
(P3) both parameters may differ with nominal values $r_\tau = 0.7$ and $r_g = 1$.

Three different deviation from the nominal value of the uncertain parameters is tested $s_\tau = s_g = 0.05$, 0.1 and 0.2 (on the figures with black, red and blue color, respectively).

The polytopic method (PM) in Section 3.1, the frequency-domain skew-μ analysis (FSM) in Section 3.2 and the time-domain skew-μ analysis (TSM) in Section 3.3 are tested in the three case of uncertainties. The LFT reformulation of the vehicle platoon system in Section 2.2 and the $\mathcal{L}_2$-gain bound computation in Section 3 were presented in case when both parameters are uncertain (P3), however the other two cases P1 and P2 can be derived from these easily.

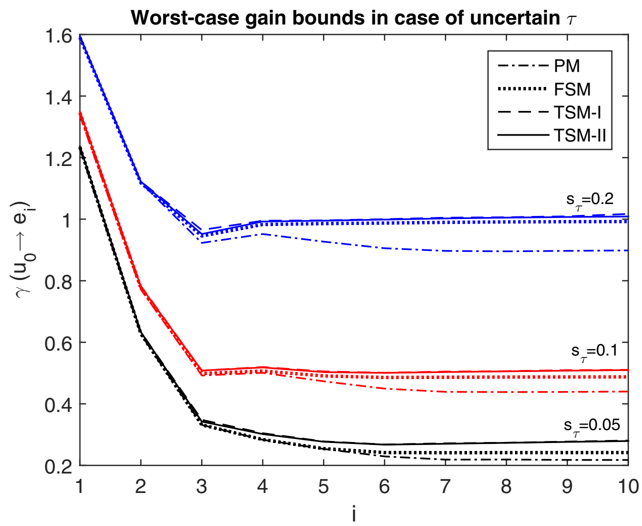

Fig. 1. $\mathcal{L}_2$-gain in case of uncertain τ_i parameters (P1) from input u_0 to the ith spacing error.

In the time-domain skew-μ analysis two different approach are considered: (i) The nonlinear optimization problems (26) are solved with weighting function $W(j\omega) = \frac{j\omega+1}{j\omega+0.1}$ and parameter $\nu_k = 5$ using Matlab function `patternsearch` (on the figures denoted by TSM-I). (ii)

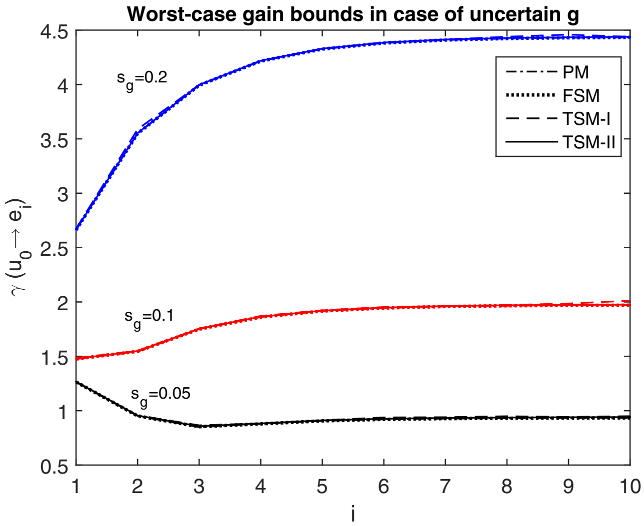

Fig. 2. $\mathcal{L}_2$-gain in case of uncertain g_i parameters (P2) from input u_0 to the ith spacing error. The four different method approximately coincide in this case.

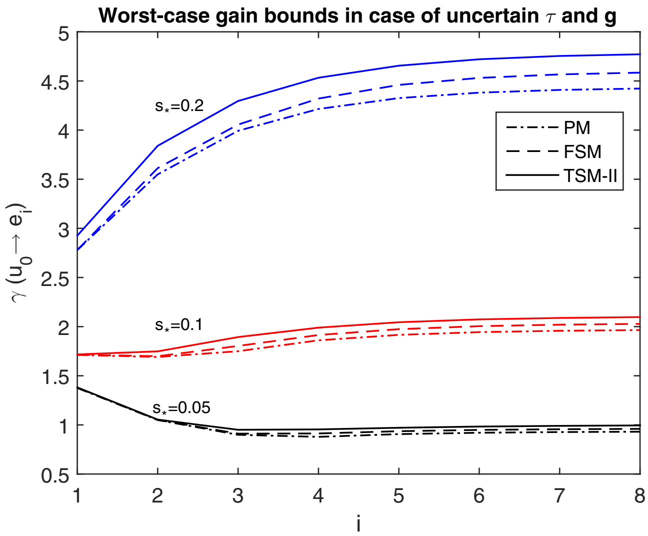

Fig. 3. $\mathcal{L}_2$-gain in case of uncertain τ_i, g_i parameters (P3) from input u_0 to the ith spacing error.

Using a fix ρ_k parameter the number of the basis functions ν_k has to be increased for a better approximation of the IQC (on the figures denoted by TSM-II). In the numerical analysis for $\rho_k = -1$ the number of the basis function is chosen as $\nu_k = 5$, which is still computationally tractable. Clearly, the first method is more time-consuming, because to find good tuning parameters a comprehensive search is necessary separately for every uncertainty parameter.

The $\mathcal{L}_2$-gain bounds from u_0 to spacing errors e_i are compared on Figure 1, 2, 3, in case of P1, P2 and P3, respectively. It can be concluded:

- The skew-μ analysis and the polytopic method is nearly the same in case of a few vehicle, and differ more at backward positions, $i > 4$. The number of uncertainty blocks is increasing with the number of vehicles, however, the upper bounds remain tight.

- The vehicle platoon is more sensitive to the parameter g_i than τ_i.
- In case of P3 the number of the uncertainty blocks are increasing with two by adding a new vehicle to the platoon, therefore the skew-μ analysis is more conservative compared to the case when only one parameter is uncertain (P1 and P2).
- The two different time-domain skew-μ approach is nearly coincide, however using fix tuning parameters the computation time is much shorter.
- Both of the time-domain methods provide similar results to the frequency-domain skew-μ method.
- The spacing errors stay bounded, therefore heterogeneous string stability can be deduced for this autonomous vehicle platoon.

5. CONCLUSIONS

For induced $\mathcal{L}_2$ norm computation of heterogeneous vehicle platoons a skew-μ analysis is presented in frequency- and time-domain. The skew-μ analysis with one full complex block (performance channel) and more than one real uncertainty block is not accurate. In order to evaluate the tightness of the upper bound, a polytopic method for lower bound computation of the induced $\mathcal{L}_2$ norm is presented.

On a vehicle platoon with leader and predecessor following control strategy the polytopic method, frequency and time domain skew-μ methods are compared. It is shown that the frequency-domain skew-μ analysis gives a good approximation of the $\mathcal{L}_2$-gain even with many uncertainty blocks, and that using fix tuning parameters with appropriate number of basis function the time-domain method provides nearly the same results as the frequency-domain method with tolerable computation time.

The time-domain skew-μ method can be extended to compute the worst-case gain of a time-varying delay system which occurs when the communication network is imperfect in the vehicle platoon.

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