Preisach Type Hysteresis Models with Everett Function in Closed Form

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The Preisach function is considered as a product of two special one dimensional functions, which allows the closed form evaluation of the Everett integral. The deduced closed form expression is included in static and rate dependent hysteresis models. The applicability and accuracy of the models are discussed and demonstrated fitting measured data. The developed hysteresis models, which are freely available for research and educational purpose, proved to be fast enough to be incorporated in electromagnetic software.

***Index Terms*—magnetic materials, hysteresis, Preisach model**

I. Introduction

The hysteresis of various magnetic materials has been widely investigated and several hysteresis models have been proposed [1]-[4]. In spite of this fact the application of the developed theory to actual engineering problems is challenging and most of the nowadays widespread commercial software cannot handle hysteresis. Electromagnetic simulations of devices with soft magnetic materials require the solution of nonlinear partial differential equations involving the update of many hysteresis models at each iteration step. Therefore computationally inexpensive hysteresis model is required to obtain solution within reasonable time [2].

In this paper Preisach type hysteresis models are described, with the Everett integral expressed in closed form [5], [6]. Therefore the magnetization can be expressed with a formula for arbitrary hysteresis loops in case of the classical Preisach model. The closed form Everett expression can be conveniently utilized in moving type [1] and rate dependent Preisach type hysteresis models [7]. The parameters of the models are identified fitting measured hysteresis loops. The applicability and accuracy of the models is discussed.

II. A Closed Form Expression of the Everett Function

The macroscopic hysteresis of magnetic materials can be modelled as a superposition of elementary rectangular hysteresis operators [1]. Integrating the contribution of all rectangular operators the classical Preisach model provides the magnetic flux density B as

$$B(t) = \iint_T \mu(h_1,h_2)\gamma\left(h_1,h_2,H(t)\right)dh_1dh_2 \tag{1}$$

where H is the magnetic field intensity at instant t, the hysteresis operator γ with h_2 switching up and h_1 switching down field can take the values $+1$ or -1, T denotes the Preisach triangle and $\mu(h_1,h_2)$ is the Preisach function. Introducing the Everett function

$$E(x,y) = \int_x^y \int_x^{h_2} \mu(h_1,h_2)dh_1dh_2\,, \tag{2}$$

where x, y is a turning point of the staircase line [1], instead of (1), the magnetic flux density can be calculated by addition of Everett functions. The Everett function can be measured [1, 7], however it is common to approximate the Preisach function analytically and perform the integration (2) numerically [8]. In this paper the following expression is introduced

$$\varphi_i(x) = \frac{a_i\, e^{-\frac{x-b_i}{c_i}}}{\left(1+e^{-\frac{x-b_i}{c_i}}\right)^2} = \frac{a_i/2}{1+\cosh\left(\frac{x-b_i}{c_i}\right)} = \frac{\alpha_i\, e^{-\beta_i x}}{\left(1+\gamma_i\, e^{-\beta_i x}\right)^2}\,.$$

With the assumption that he switching up and down fields are uncorrelated, the Preisach function can be approximated as

$$\mu(h_1,h_2) = \sum_{i=1}^{n} \varphi_i(h_1)\varphi_i(-h_2)\,, \tag{4}$$

which allows the evaluation of the integral (2) resulting in the closed form expression

$$E(x,y) = \sum_{i=1}^{n} \frac{\alpha_i^2}{\beta_i^2} \frac{\left(1-\gamma_i^2\right)\left(e^{\beta_i x}-e^{\beta_i y}\right)+\left(\gamma_i+e^{\beta_i y}\right)\left(1+\gamma_i e^{\beta_i x}\right)L_i}{\left(1-\gamma_i^2\right)^2\left(\gamma_i+e^{\beta_i y}\right)\left(1+\gamma_i e^{\beta_i x}\right)} \tag{5}$$

where $L_i = \log\frac{\left(1+\gamma_i e^{\beta_i y}\right)\left(\gamma_i+e^{\beta_i x}\right)}{\left(1+\gamma_i e^{\beta_i x}\right)\left(\gamma_i+e^{\beta_i y}\right)}$, $\alpha = ae^{b/c}$, $\beta = 1/c$ and $\gamma = e^{b/c}$. When $b=0$, then $\gamma=1$ and the Everett function (5) has undetermined form, which can be removed with the l'Hopital rule, resulting in

$$\lim_{\gamma\to 1} E(x,y) = \frac{1}{2\beta^2}\frac{\left(e^{\beta x}-e^{\beta y}\right)^2}{\left(1+e^{\beta x}\right)^2\left(1+e^{\beta y}\right)^2}\,. \tag{6}$$

The advantage of this formulation compared to other mathematical expressions derived for the Everett function [9] is that it results in closed form expression, which includes only the basic arithmetic operations (addition, subtraction, multiplication, and division), exponentiation to a real exponent and logarithms. Therefore the evaluation of (5) and (6) can be performed without the utilization of complex mathematical libraries and it is fast enough to allow the incorporation of this model in electromagnetic field calculations software.

III. STATIC, MOVING AND RATE DEPENDENT HYSTERESIS MODELS WITH CLOSED FORM EVERETT FUNCTION

In this section Preisach type hysteresis models, which utilize the closed form expression of the Everett function are presented and the parameters required to simulate the measured hysteresis loops [10] are identified. A reversible component in the form of

$$B_{rev}\left(H\right) = k_1 H + k_2 \tanh\left(\frac{H}{k_3}\right), \tag{7}$$

is added to the Preisach model to take into account the reversible part of the magnetization process. The parameters obtained for the classical Preisach model are summarized in Table I, where the fitting is performed only for the exterior hysteresis loop. Fig. 1 presents the measured and simulated hysteresis loops marked with blue and red. When all concentric loops are included in the optimization the fitting results in larger overall error. It can be observed that the outer loops fit well, however there is a considerable deviation in case of inner loops, which are closer to the demagnetized state.

TABLE I
THE PARAMETERS OF THE CLASSICAL PREISACH MODEL

a_i	b_i	c_i	k_i
0.113	56.468	6.035	9.521×10^{-3}
2.224×10^{-3}	30.197	284.54	0.241
1.812×10^{-2}	86.148	38.842	365.86

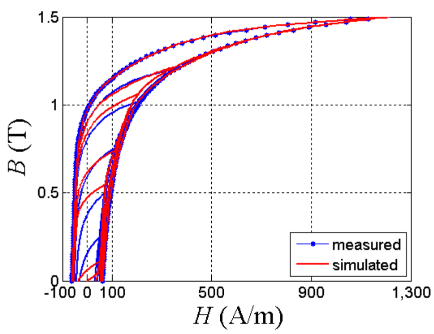

Fig. 1. The measured and simulated hysteresis loops with the Preisach model

The moving model improves the classical Preisach model through a feedback mechanism [1]. The effective magnetic field intensity [3] is defined with two additional parameters

$$H_m = H + m_1 B + m_2 B^3, \tag{8}$$

which is passed to the classical Preisach model as input. The moving model leads to a more accurate fitting of all measured concentric hysteresis loops as it is shown in Fig. 2. The identified parameters are summarized in Table II.

To take into account the frequency dependence of the hysteresis phenomena (e.g. the ”fattening” of the hysteresis loops with the increase of the frequency), a rate dependent hysteresis model with three additional parameters a_m, b_m and c_m is introduced [8], where the effective magnetic field intensity is defined with the following differential equation

$$\frac{dH_m}{dt} = a_m\left(H - H_m\right) - b_m\frac{dB}{dt} + c_m\frac{dH}{dt}. \tag{8}$$

Applying the chain rule of the differentiation, the rate of change of the magnetic flux density for increasing effective magnetic field intensities can be expressed as

$$\frac{dB}{dt} = \frac{dB}{dH_m}\frac{dH_m}{dt} = \left(2\int_{H_r}^{H_m}\mu\left(h_1, H_m\right)dh_1\right)\frac{dH_m}{dt}. \tag{9}$$

The dynamic permeability can be expressed in closed form resulting in

$$\frac{dB}{dH_m} = \sum_{i=1}^{n}\frac{\alpha_i}{1+\cosh\left(\dfrac{H_m-\beta_i}{\gamma_i}\right)}\left[\frac{1}{1+e^{\frac{H_r-\beta_i}{\gamma_i}}} - \frac{1}{1+e^{\frac{H_m-\beta_i}{\gamma_i}}}\right], \tag{10}$$

where $\kappa_i = \alpha_i^2/\left(\beta_i\gamma_i\right)$. Similar expression is obtained when the effective magnetic field intensities is decreased. The full paper will discuss more details of the presented hysteresis models, which are freely available online [11].

TABLE II
THE PARAMETERS OF THE MOVING MODEL

a_i	b_i	c_i	k_i	m_i
4.37×10^{-2}	46.78	12.141	0.12×10^{-3}	25.43
3.16×10^{-3}	0.11	236.5	0.24	11.8
1.86×10^{-2}	62.44	42.54	359.3	-

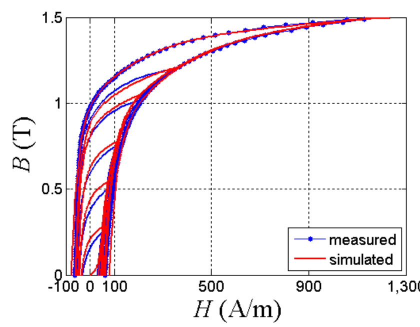

Fig. 2. The measured and simulated hysteresis loops with the Moving model

ACKNOWLEDGEMENTS

This work has been supported by the Bolyai János Fellowship of Hungarian Academy of Sciences, the PIAC-13-1-2013-0186 and KMR-12-1-2012-0008 projects of the National Development Agency Hungary and EUREKA project MetaFer.

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