

# From Floquet to Dicke: quantum spin-Hall insulator interacting with quantum light

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Time-periodic perturbations due to classical electromagnetic fields are useful to engineer the topological properties of matter using the Floquet theory. Here we investigate the effect of *quantized* electromagnetic fields by focusing on the quantized light-matter interaction on the edge state of a quantum spin-Hall insulator. A Dicke-type superradiant phase transition occurs at arbitrary weak coupling, the electronic spectrum acquires a finite gap and the resulting ground state manifold is topological with Chern number  $\pm 1$ . When the total number of excitations is conserved, a photocurrent is generated along the edge, being pseudo-quantized as  $\omega \ln(1/\omega)$  in the low frequency limit, and decaying as  $1/\omega$  for high frequencies with  $\omega$  the photon frequency. The photon spectral function exhibits a clean Goldstone mode, a Higgs like collective mode at the optical gap and the polariton continuum.

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*Introduction.* The conventional band theory of solids has attracted renewed attention after the theoretical prediction and experimental discovery of topological insulators (TIs)[1–3]. The topological protection of their edge/surface states, together with the spin-momentum locking due to the strong spin orbit coupling, yields a variety of novel phenomena, such as magnetoelectric effect, axion electrodynamics, surface Hall effect, not to mention topological superconductivity and Majorana fermions. In addition to their theoretical appeal, they also hold the promise to realizing practical quantum computers and spintronic devices.

In spite of the growing interest, materials with non-trivial topological properties are still scarce. Achieving a topological band structure often requires engineering the Bloch band structure. Recently, time periodic perturbations, mostly due to electromagnetic fields, have been proposed and used[4–8] to manipulate the band structure using the Floquet theory, the temporal analogue of Bloch states. Although the steady state of the resulting Floquet topological insulators often possesses a topology different from that of their static parents, the actual occupation of these states is still far from being understood[9] albeit it is an essential ingredient for determining the physical response of the system. The occupation depends on the sources of relaxation, e.g. coupling to heat baths, momentum scattering, interaction etc., and the conventional ground-state picture with the minimal energy configuration cannot be used for these driven models. To circumvent this problem, a classical driving field can be replaced by its quantum counterparts having its own quantum dynamics. This idea has been pursued for the creation of Majorana fermions using not only classical driven systems[10] but rather their quantum counterparts using high-Q electromagnetic cavity[11].

The properties of classical and quantized electromagnetic fields have long been investigated. In connec-

tion with their interaction with a single atom[12, 13], e.g., many similarities have been demonstrated between the corresponding Rabi and Jaynes-Cummings models. Besides the similarities, quantum fields also provide novel effects and collective phenomena such as spontaneous symmetry breaking and the superradiant phase transition[12, 14, 15] due to light-matter interaction in the Dicke model[16, 17].

In this work, we combine TIs and quantized electromagnetic fields to explore the quantum mechanical version of the model of Ref. [18], shown to display a Floquet topological phase transition. We demonstrate that a superradiant quantum phase transition exists in the quantum context, and is accompanied by a transition to a topologically non-trivial ground state manifold and by an induced photocurrent.

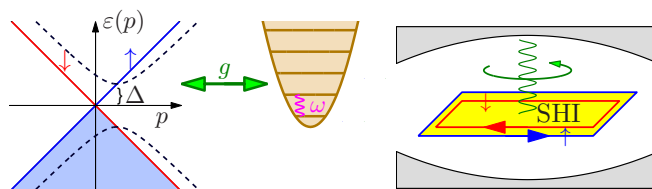


FIG. 1. (Color online) Left: a single  $\omega$  frequency boson mode is coupled to a continuum of fermionic degrees of freedom, which is a quantum spin-Hall insulator in the present case. This is the essential difference between our setup and conventional Dicke/Tavis-Cummings models, where the fermions possess a discrete spectrum. The new fermionic spectrum becomes gapped and shifted (dashed line). Right: cartoon of the experimental setup corresponding to Eq. (1) of SHI with spin filtered edge states placed into an optical cavity.

*The model.* We consider the edge dynamics of a spin-Hall insulator[19] (SHI), interacting with circularly polarized quantized electromagnetic field through the Zeeman

term (see Fig. 1). The Hamiltonian is

$$H = \omega a^\dagger a + v \sum_p p S_p^z + \frac{g}{\sqrt{L}} \sum_p (a^\dagger S_p^- + a S_p^+), \quad (1)$$

Here  $v$  is the Fermi velocity of the edge state,  $\omega$  and  $a$  denote the energy and annihilation operator of the cavity mode, and  $c_{p,\sigma}$  annihilates an electron with momentum  $p$  and spin  $\sigma$ . The last term describes the Zeeman interaction with the spin operators expressed as  $S_p^+ = c_{p,+}^\dagger c_{p,-}$  and  $S_p^z = \sum_{\sigma=\pm} \sigma c_{p,\sigma}^\dagger c_{p,\sigma}$ . The Zeeman coupling depends on the photon's frequency [12] as  $g = \sqrt{\tilde{g}|\omega|}$ , and  $L$  is the dimensionless length of the edge [20].

Let us first highlight the connection of Eq. (1) to Floquet topological insulators using the one body picture of the polaritons. Considering just a single momentum  $p$ , the Hamiltonian reduces to the Jaynes-Cummings model[12]. The spectrum consist of energy pairs  $\epsilon_\pm(p, m) = \omega(m - \frac{1}{2}) \pm \sqrt{(vp - \frac{\omega}{2})^2 + \frac{g^2}{L}m}$  with  $m$  being a positive integer, plus a single mode  $\epsilon_-(p, 0) = -vp$  [21]. These polariton energies are closely related to the Floquet spectrum of the same problem in the presence of classical electromagnetic field [5], when the photon state is prepared in a coherent state with  $m \gg 1$ . Importantly, similar to the classical case, the spectrum becomes gapped due to interaction with the quantized light field for all  $m > 0$  [22], and for each fixed  $m > 0$ , the contribution of all  $\epsilon_-(p, m)$  states amounts in an electric current identical to that in the classical case, quantized as  $j = \frac{e\omega}{2\pi}$  in agreement with Floquet adiabatic charge pumping.

This one body description has limited meaning, since all spins interact simultaneously with the same bosonic mode, and the many-body spectrum involves all of them. The Hamiltonian Eq. (1) is, however, integrable[23] for any number of spins, and coincides with the inhomogeneous Dicke model [24]. In addition to the energy, the total number of excitations,  $\mathcal{Q} = a^\dagger a + \frac{1}{2} \sum_p S_p^z$  is a conserved quantity, which generates a  $U(1)$  symmetry. Without  $g$ , the photon state is unpopulated and the total spin is unpolarized, therefore  $\langle \mathcal{Q} \rangle = 0$ . Note that this applies to positive helicity ( $\Lambda$ ) of light,  $\Lambda = 1$ , for negative helicity ( $\Lambda = -1$ ) the replacement  $a \leftrightarrow a^\dagger$  should be made in the Zeeman term, and  $a^\dagger a - \frac{1}{2} \sum_p S_p^z$  is conserved.

*Mean field theory.* In the thermodynamic limit  $L \rightarrow \infty$  the bosonic state becomes macroscopically populated, and the mean field description  $\langle a \rangle \approx \sqrt{nL} \exp(i\phi)$  becomes exact [14, 15]. The  $U(1)$  symmetry of the normal phase is spontaneously broken and the bosonic field acquires a nonzero, macroscopic mean value, implying superradiance. Note that integrating out the photon field in Eq. (1) leads to an effective long range electron-electron interaction of the form  $-\tilde{g} \sum_{p,k} S_p^+ S_k^-$ . In contrast to the short range interactions in low dimensional models,

this long-ranged interaction facilitates the  $U(1)$  symmetry breaking even at finite temperatures.

The ground state is determined by either choosing the absolute minimum energy configuration[14, 15] from all possible  $\mathcal{Q}$  sectors, referred to as *unconstrained* solution, or by fixing the total number of excitations[25–27] in a *constrained* solution (i.e. grand canonical ensemble). The former situation is achievable in the presence of coupling to external noise or dissipation, inducing transition between various  $\mathcal{Q}$  sectors, while the constrained solution corresponds to a situation in which polariton decay, arising from coupling the cavity mode or the electronic excitations to other modes, is slow compared with the time required to reach thermal equilibrium at a fixed polariton number. The  $\mathcal{Q} = 0$  constraint is taken into account by adding  $-\lambda\mathcal{Q}$  to the Hamiltonian with  $\lambda$  the Lagrange multiplier[25, 26].

Within mean field theory, the fermionic excitation spectrum reads as

$$E_\alpha(p) = \alpha \sqrt{\left(vp - \frac{\lambda}{2}\right)^2 + \Delta^2}, \quad (2)$$

where  $\alpha = \pm$  and, similarly to the case of classical light[5] and to the one body picture, a gap  $\Delta = g\sqrt{n}$  opens in the spectrum due to the the quantized light – matter interaction. The mean field parameters,  $n$ ,  $\phi$  and  $\lambda$  are determined by minimizing the ground states energy

$$\frac{E_0}{L} = (\omega - \lambda)n - \frac{\rho}{2} \int_{-W}^W d\varepsilon \sqrt{\left(\varepsilon - \frac{\lambda}{2}\right)^2 + g^2 n}. \quad (3)$$

where  $\rho = 1/\pi v$  is the 1D density of states and  $W$  is a high energy cutoff. The mean field equations,  $\partial E_0/\partial n = 0$  and  $\partial E_0/\partial \lambda = 0$  yield in the weak coupling limit

$$\omega - \lambda = \frac{\rho g^2}{2} \ln\left(\frac{2W}{g\sqrt{n}}\right), \quad \lambda = -\frac{4n}{\rho}, \quad (4)$$

respectively. Both equations are to be solved in the constrained case, while the unconstrained case requires only the first one in Eq. (4) with  $\lambda = 0$  and the second equation should be ignored.

The phase of the order parameter,  $\phi$  remains undetermined in both cases since the ground state energy shows a Mexican hat in the space of  $\text{Re}\langle a \rangle$  and  $\text{Im}\langle a \rangle$ . This degree of freedom corresponds to a gapless Goldstone mode from the broken  $U(1)$  symmetry in the superradiant phase of Eq. (1), similar to the Tavis-Cummings model[28]. By tuning the phase  $\phi$  adiabatically from 0 to  $2\pi$ , one sweeps through the degenerate ground state manifold with no energy cost. The  $(p, \phi)$  space can be mapped to the unit sphere with  $\mathbf{d}_\alpha = (\langle S_p^x \rangle, \langle S_p^y \rangle, \langle S_p^z \rangle) = (\Delta \cos \phi, \Delta \sin \phi, vp - \lambda/2)/E_\alpha(p)$ . The winding number of the mapping defines the Chern

number, which reads for band  $\alpha$  and given helicity of light  $\Lambda = \pm 1$  as

$$\mathcal{C}_\alpha = \int_{-\infty}^{\infty} dp \int_0^{2\pi} \frac{d\phi}{4\pi} \mathbf{d}_\alpha \cdot (\partial_p \mathbf{d}_\alpha \times \partial_\phi \mathbf{d}_\alpha) = -\alpha \Lambda. \quad (5)$$

The  $\mathcal{C}_\alpha$  is analogous to the Chern number of the Floquet case[18]. The superradiant ground state manifold is thus topologically non-trivial, irrespective of the value of  $\lambda$ .

We emphasize again that the phase degree of freedom from  $U(1)$  symmetry breaking in a 1D fermionic model arises due to the photon mediated infinitely long range interaction between electrons, while artificially created phases define similar Chern numbers in XY spin chain[29]. A photocurrent can also generated along the edge as  $\langle j \rangle = -e\lambda/2\pi$ , depending on the value of  $\lambda$ .

Not only the bosonic field develops a macroscopic population, but the time reversal symmetry is also broken, which is indicated by the finite electronic spin density perpendicular to the  $z$  direction as

$$\langle S^\pm \rangle = -\exp(\mp i\phi) \frac{\rho\Delta}{2} \ln\left(\frac{2W}{\Delta}\right). \quad (6)$$

*Unconstrained solution.* The ground state is the absolute minimum energy configuration over all possible  $\mathcal{Q}$  sectors as in Refs. [14, 15], since the total excitation number is not fixed. The order parameter follows a typical weak-coupling, BCS-like form as

$$n = \left(\frac{2W}{g}\right)^2 \exp\left(-\frac{4\omega}{\rho\tilde{g}}\right) = \frac{4W^2}{\tilde{g}\omega} \exp\left(-\frac{4}{\rho\tilde{g}}\right), \quad (7)$$

where the  $\omega$  dependence is made explicit, and the gap,  $\Delta = 2W \exp(-2/\rho\tilde{g})$  is independent of  $\omega$ . Since the electrons feel the photon mode through the opening of the gap, which is frequency independent, the physical response of the spin-Hall edge becomes also completely  $\omega$  independent. In contrast to the conventional Dicke model, a superradiant quantum phase transition occurs for arbitrarily small values of the coupling,  $g$ , while a critical coupling was required for the conventional, homogeneous Dicke case. Eq. (7) indicates an infinite order quantum phase transition as a function of  $\tilde{g}$  as opposed to the second order transition of the conventional Dicke model[15]. Since  $\lambda = 0$  due to the lack of constraint, the photocurrent along the edge is zero.

*Constrained solution.* Now we turn to the more relevant and interesting case with a fixed total number of excitations[25–27] using both expression from Eq. (4). In the low frequency limit, the solution to Eq. (4) is obtained approximately as

$$n \approx \frac{(\rho g)^2}{8} \ln\left(\frac{4\sqrt{2}W}{\rho g^2}\right) = \frac{\rho^2 \tilde{g} \omega}{8} \ln\left(\frac{4\sqrt{2}W}{\rho \tilde{g} \omega}\right) \quad (8)$$

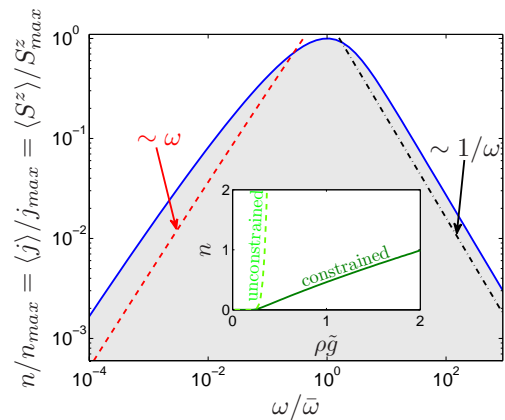


FIG. 2. (Color online) The weak-coupling solution of the mean field equations, Eq. (4) is plotted for the constrained case, the low and high frequency asymptotes are indicated by dashed and dash-dotted lines, agreeing with the Floquet solution of the corresponding classical model[18]. The unconstrained solution in Eq. (7) coincides with the constrained one in the high frequency limit. The inset shows the evolution of  $n$  for  $\omega\rho = 1$  as a function of the Zeeman coupling for the constrained and unconstrained cases (solid and dashed line, respectively).

to logarithmic accuracy. In the opposite, high frequency limit the solution coincides with Eq. (7). Due to the conserved number of total excitations, the superradiance of the cavity mode implies a finite photocurrent, which is also indicated by the finite value of  $\lambda$ . The relevant energy scale of the problem separates the low and high frequency limits, which is signaled by a maximum in the photon occupation number, occurring at

$$\bar{\omega} = \frac{8W}{\rho\tilde{g}} \exp\left(-\frac{2}{\rho\tilde{g}} - \frac{1}{2}\right), \quad (9)$$

where the maximum of the photon number and current are  $n_{max} = \rho^2 \tilde{g} \bar{\omega} / 16$  and  $j_{max} = -2en_{max} / \pi\rho$ , as shown in Fig. 2. While the  $\omega$  dependence features a maximum for fixed  $\tilde{g}$ , the  $\tilde{g}$  dependence of these quantities is monotonic for fixed  $\omega$ , shown in the inset of Fig. 2. The photocurrent behaves as

$$\langle j \rangle = \begin{cases} -\frac{e\omega}{2\pi} \frac{\rho\tilde{g}}{2} \ln\left(\frac{4\sqrt{2}W}{\rho\tilde{g}\omega}\right) & \text{for } \omega \ll \bar{\omega}, \\ -\frac{8eW^2}{\pi\rho\tilde{g}\omega} \exp\left(-\frac{4}{\rho\tilde{g}}\right) & \text{for } \omega \gg \bar{\omega}. \end{cases} \quad (10)$$

The photocurrent arises from the constraint, which represents an effective magnetic field along the edge, inducing a photocurrent through the magnetoelectric effect[1]. In the low frequency limit, the current is pseudo-quantized as it grows proportional to  $\omega$  (with  $\log(1/\omega)$  corrections). This is similar to the Floquet case, where a perfectly quantized current without log-corrections follows from the adiabatic charge

pumping[30]. The difference between the quantum and classical description arises because the condition of charge pumping that the Hamiltonian should change slowly and periodically in time is not satisfied here by the quantized photon field and quantum fluctuations do not allow for purely periodic time evolution and destroy quantization. This parallels to the breakdown of the classical equipartition theorem, which predicts vanishing energy per degree of freedom at absolute zero, which is violated by quantum fluctuations, which produce the zero-point energy of a quantum harmonic oscillator.

In the high frequency limit, the current decays slowly as  $1/\omega$  with increasing frequency, which again agrees with the Floquet case. The ground state energy in this case is always above the absolute minimum energy state of the unconstrained case, although these approach each other fast in the  $\omega \gg \bar{\omega}$  limit similarly to other quantities, because  $\lambda$  vanishes as  $\sim -1/\omega$  with increasing frequency and  $\lambda = 0$  in the unconstrained case. By allowing for  $Q \neq 0$  in the constrained case, the superradiant phase behaves similarly to  $Q = 0$  and is accompanied by a finite photocurrent along the edge. For some  $\omega$  and  $\tilde{g}$  pairs, the current may vanish, which indicates that the constrained solution in a given  $Q \neq 0$  sector coincides with the absolute ground state.

Due to the magnetoelectric phenomenon[1], a finite magnetization develops in the  $z$  direction along the edge as  $\langle S^z \rangle = \langle j \rangle / ev$ , unlike in the unconstrained case.

*Photon spectral function.* Having discussed the electronic properties exhaustively, now we turn to the properties of the photon mode in the superradiant phase, which contains the polariton spectrum and reveals the stability of the mean field solution. Its Green's function is obtained by evaluating the Gaussian fluctuation correction of the order parameter around the mean-field value[25]. At  $T = 0$ , it reveals a zero energy Goldstone mode for any finite  $g$  due to the breaking of the  $U(1)$  symmetry[31]. In the  $(a, a^+)$  Nambu space, the Green's function close to zero frequency becomes singular as

$$\mathcal{G}(\Omega \approx 0)^{-1} = \begin{bmatrix} \Omega + \frac{g^2 \rho}{4} & \frac{g^2 \rho}{4} \\ \frac{g^2 \rho}{4} & -\Omega + \frac{g^2 \rho}{4} \end{bmatrix}, \quad (11)$$

and  $\Omega$  is understood as  $\Omega + i0^+$ . The spectral function  $A(\Omega) = \text{Im Tr } G(\Omega)/\pi$ , measurable by the absorption coefficient of the cavity, features the Goldstone mode as  $A(\Omega \rightarrow 0) = \frac{g^2 \rho}{2} \delta(\Omega)$  from the Gaussian fluctuations of the order parameter. The spectral weight of the Goldstone mode vanishes at  $g = 0$  due to the absence of phase transition there, and the spectral function assumes its non-interacting value as  $A(\Omega \geq 0) = \delta(\Omega - \omega)$ . The structure of the spectral function for vanishing  $\Omega$  agrees with that of the finite-spin Tavis Cummings model[31]. In addition, a high energy mode is expected as a remnant of the bare cavity mode  $\omega$ . This mode gets damped as  $\sim \pi g^2 \rho$  for small  $\Delta$  and is renormalized towards smaller

frequencies before it hits the gap edge at  $\Omega = 2\Delta$  and merges with it. This optical gap is the amplitude mode (or Higgs like mode) where the spectral function displays a square-root singularity, as shown in Fig. 3, and this sets the threshold energy for continuum polariton excitations for  $\Omega > 2\Delta$ .

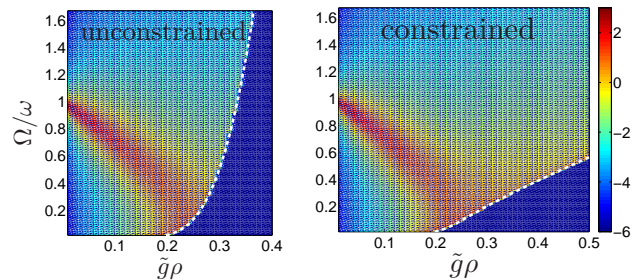


FIG. 3. (Color online) The contour plot of  $\ln[\omega A(\Omega)]$  reveals the polariton excitation energies and their spectral weights for  $W/\omega = 100$ ,  $\omega\rho = 1$ . The white dashed line denotes the minimal excitation energy  $2\Delta$ , above which a continuum is formed. There is always a Goldstone mode at  $\Omega = 0$ , omitted from these plots.

*Discussion.* A single photon mode in Eq. (1) is realized in a quantum LC circuit[32] or is selected from a ladder of cavity modes by placing a dispersive element into the cavity such as prism or nonlinear dielectric material (with wavevector dependent refractive index). The latter results in a non-equidistant ladder of excitations, which are eliminated by using dielectric mirrors (with high reflectivity for the basic cavity mode, and high transmission for inharmonic frequencies). Tuning  $\omega$  and  $g$  is possible by changing the dielectric constant or the position of mirrors in the cavity, thus exploring various regions of the phase diagram. Multimode photon fields yield similar results as was demonstrated in Refs. [14, 26, 33]. This is most directly seen by integrating out the photon modes, which further enhances the effective electron-electron interaction.

The SHI is realized using either condensed matter[1] or cold atomic[10, 34, 35] settings. The electromagnetic field appears not only through the Zeeman term but via a vector potential. Neutral cold atoms in optical traps or chiral edge states (with spin parallel to momentum) [35] do not couple to it, though. Otherwise, it can be incorporated into our calculations, and its main effect is the suppression of  $\omega$  in Eq. (4), which is negligible for  $\omega \gg v_{ec} \sqrt{\rho \tilde{g}} / g_{\text{eff}} \mu_B$ , where  $c$  is the speed of light,  $g_{\text{eff}}$  is the effective  $g$ -factor of the edge states and  $\mu_B$  is the Bohr magneton. When a weak external field is coupled to the photon field, the phase of the order parameter can be tuned slowly and the quantized Chern number can be probed through a Thouless type[30] adiabatic charge pumping.

The superradiant phase transition occurs for arbitrary small coupling between the photon field and the spin-



Hall edge, and is characterized by a Chern number  $\pm 1$  in the space of the 1D Brillouin zone and the phase of the order parameter. The unconstrained solution, corresponding to the absolute ground state of the system, does not carry a net electric current. By keeping the total number of excitations fixed, a finite electric current carrying state develops, which behaves analogously to its Floquet counterpart. This remotely parallels to the idea of quantum time crystals[36], which do not occupy the absolute ground state of a system but appear in an excited sector[37]. The conventional Dicke model exhibits quantum chaos[15] away from the thermodynamic limit as a precursor to quantum phase transition at  $L \rightarrow \infty$ . It would be interesting to investigate whether and how quantum chaotic behaviour sets in in the present model for finite  $L$ . Extending our work to higher dimensional systems, which are suspected to become Floquet topological insulators such as graphene[38] or HgTe/CdTe quantum wells[4] or their cold atomic realizations, promises to be a fruitful enterprise.

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