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New Results in Modeling the Phenomenon of Acoustic Hysteresis

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SUMMARY

Interpretation of seismic data is significantly constrained by the extrapolation of measured acoustic properties - in laboratory - of rocks in a given physical (pressure) environment. To reasonably interpret laboratory measurements, a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. It is well known that the change of acoustic wave velocity propagating in rocks is nonlinear with respect to pressure and the quasistatic elastic properties of rocks are hysteretic. In this paper a petrophysical model is presented which provides the connection between the propagation velocity of acoustic wave and rock pressures both in case of pressurization and depressurization cycles. The developed model also describes well and explains the mechanism of acoustic hysteresis. The model is based on the idea that the microcracks in rocks close during pressurization and reopen during depressurization. The model was applied to acoustic P wave velocity data sets. Measurements were carried out at various incremental pressures and the parameters of the petrophysical model were determined by a linearized inversion method. The calculated data matched accurately with measured data proving that the new rock physical model describing acoustic hysteresis applies well in practice.

Introduction

It is well known that the change of acoustic wave velocity propagating in rocks under pressure is highly nonlinear and the quasistatic elastic properties of rocks are hysteretic (Ji et al. 2007). The observable non-elastic response to pressure (acoustic hysteresis) may be caused by the processes: irreversible closure of microcracks, irreversible compaction of pore spaces as well as improvement of contact conditions. According to the theory of irreversible closure of microcracks, the microcracks closed during pressurization do not reopen during subsequent depressurization (Birch 1960). After the conception of irreversible compaction of pore spaces, the pores which collapsed at higher pressures do not recover their original shapes or dimensions at lower pressures (Jones and Wang 1981). By idea of the improvement of contact conditions, the contact conditions are modified by local ductile cushions of weak, alteration materials (e.g., chlorite, sericite or serpentine) along grain boundaries and microcracks (Hashin and Shtrikman 1963). Namely in a rock, grains themselves act as perfectly elastic units, while the contacts between these grains often display non-linear elastic behaviour. As a result, the rock will show an overall elastically non-linear behaviour characterized by hysteresis.

The idea that the pressure-acoustic velocity connection can be characterized by exponential function is well-known but the developed empirical models are based on purely on mathematical curve fitting, however the physical meaning is unclear (Ji et al. 2007). To reasonably interpret laboratory measurements, a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper we present a quantitative petrophysical model, which explains the mechanism of pressure dependence of acoustic velocity and describes well the acoustic hysteresis.

The pressure dependent velocity model in case of pressurization and depressurization cycles

The phenomenon is well-known that wave velocity is increasing with pressure directly and was explained on various rock mechanical studies (Birch 1960). One of the most frequently used mechanisms for explaining the phenomenon is based on the closure of microcracks in rocks under pressure (Brace and Walsh 1964). For this reason we introduce parameter N as a specific number of open microcracks. At the development of the petrophysical model (Dobróka and Somogyi Molnár 2012) we restricted ourselves to uniaxial stress state and longitudinal acoustic waves.

If we create a stress increase $d\sigma$ in the rock (pressurization cycle), we find that dN (the change of the number of open microcracks) is directly proportional to the applied stress increase $d\sigma$. At the same time dN is directly proportional to N. We can unify both assumptions in the following differential equation

$$\mathrm{d}N = -\lambda N \,\mathrm{d}\sigma \;\rightarrow\; N = N_0 \exp(-\lambda\sigma)\,, \tag{1}$$

where λ is new (positive) material quality dependent petrophysical constant (Dobróka and Somogyi Molnár 2012) and N_0 is the number of open microcracks at stress-free state ($\sigma = 0$). The negative sign represents that at increasing stress - with closing microcracks - the number of the open microcracks decreases. We assume also a linear relationship between the infinitesimal change of the propagation velocity dv - due to stress increase - and dN

$$\mathrm{d}v = -\alpha\,\mathrm{d}N\,, \tag{2}$$

where proportionality factor α is a material characteristic. The negative sign represents that the velocity increases with decreasing number of cracks. Solving Eqs. (1-2) jointly, we obtain the following velocity model which provides a theoretical connection between the propagation velocity and rock pressure during pressurization

$$v = v_0 + \Delta v_0 \left(1 - \exp(-\lambda\sigma)\right)\,, \tag{3}$$

where v_0 is the propagation velocity at stress-free state ($\sigma=0$) and $\Delta v_0=\alpha N_0$ is a new (stress-independent) petrophysical constant (Dobróka and Somogyi Molnár 2012).

To characterize depressurization cycle, $n=N_0-N$ as the number of closed microcracks is required to be introduced. If we decrease the pressure (from a maximum pressure value σ_m) the closed microcracks start to open again, so decreasing velocity can be measured. Therefore we assume dn (the change of the number of closed microcracks) being proportional with the number of closed microcracks and the stress decrease $d\sigma$

$$dn = \lambda' n\, d\sigma \;\rightarrow\; n = n_m \exp\left(-\lambda'(\sigma_m - \sigma)\right), \tag{4}$$

where λ' is another new material characteristic constant (which differs from the previously introduced parameter λ) and n_m is the number of closed microcracks at maximum pressure value σ_m. Combining Eq. (2) and Eq. (4) by using the formulas $dN = -dn$ and $\alpha n_m = \Delta v_m$ one can find

$$v = v_m - \Delta v_m \left(1 - \exp\left(-\lambda'(\sigma_m - \sigma)\right)\right). \tag{5}$$

Eq. (5) shows the propagation velocity – pressure function of depressurization cycle. In the two limiting cases (at pressure value $\sigma=\sigma_m$ and $\sigma=0$) Eq. (5) gives v_m and $v_1 = v_m - \alpha n_m \left(1 - \exp\left(-\lambda'\sigma_m\right)\right)$ respectively, (here the notation $v(0) = v_1$ was used). This gives the formula (similar to Eq. (3))

$$v = v_1 + \Delta v_1 (1 - \exp(-\lambda'\sigma)), \tag{6}$$

with the notation $\Delta v_1 = -\alpha n_m \exp\left(-\lambda'\sigma_m\right)$.

Note that in the range of high pressures, reaching a critical pressure the reversible range is exceeded and new microcracks open due to destruction of the sample, hence increasing velocity is observed. This effect is outside of our present investigations. To avoid the creation of new microcracks - not to exceed the reversible range -, we increased the pressure only up to one third of the critical uniaxial strength.

Case study

The pressure dependent velocity model was tested on longitudinal wave velocity data sets. The pulse transmission technique was used for P wave velocity measurements (Dobróka and Somogyi Molnár 2012). We performed wave velocity measurements on many different sandstone samples originated from oil-drilling wells. Specimens were subjected to uniaxial stresses of up to 20 MPa by an electromechanical pressing device and wave velocities - as a function of pressure - were measured at adjoining pressures during pressurization and depressurization cycles. Two typical test results (Sample 1 and Sample 2) are presented in the paper. Sample 1 was a fine-, medium-grained, while Sample 2 a coarse-grained sandstone. Our measurements showed that the longitudinal velocity is directly proportional to pressure. Moreover a slight difference between the characteristics of the pressurization and depressurization curves was found that can be explained by the phenomenon of acoustic hysteresis. The explanation of acoustic hysteresis by Birch (1960) was followed: the microcracks closed during pressurization do not reopen completely during depressurization; there is always a certain amount of irreversibility. This irreversibility in our model is denoted by two different parameters λ and λ' characterizing the pressurization and the depressurization cycles, respectively.

Proving the validity and applicability of the introduced velocity model, we present the interpretation of measurement data of the described samples. The parameters (v_0, Δv_0, λ, v_1, Δv_1, λ') appearing in the model equations (in case of pressurization and depressurization) can be determined by processing measurement data based on joint inversion method (Dobróka et al. 1991). Since the relevant data sets contained relatively low amount of noise and the problem was overdetermined, the Gaussian Least Squares Method was used. The inversion results (Dobróka et al. 1991) for each sample can be seen in

Table 1. For the characterization of the accuracy of inversion estimates, the estimation errors of individual inversion parameters are also provided within the brackets.

Table 1 *Model parameters with estimation errors of pressurization and depressurization cycles estimated by linearized inversion using the developed velocity model.*

Sample	Pressurization			Depressurization		
	v_0 (km/s)	Δv_0 (km/s)	λ (1/MPa)	v_1 (km/s)	Δv_1 (km/s)	λ' (1/MPa)
Sample 1	2,69 (±0,0187)	0,96 (±0,0051)	0,1094 (±0,0028)	2,69 (±0,0070)	0,89 (±0,0083)	0,1889 (±0,0078)
Sample 2	2,60 (±0,0295)	0,86 (±0,0114)	0,1334 (±0,0012)	2,56 (±0,0112)	0,81 (±0,0134)	0,2988 (±0,0069)

With the estimated parameters the velocities can be calculated (separately at pressurization and depressurization) at any pressure by substituting them into Eq. (3) or Eq. (6). The results are shown in Fig. 1-2, where the solid line shows the calculated velocity-pressure function produced by the velocity model, while asterisks represent the measured data.

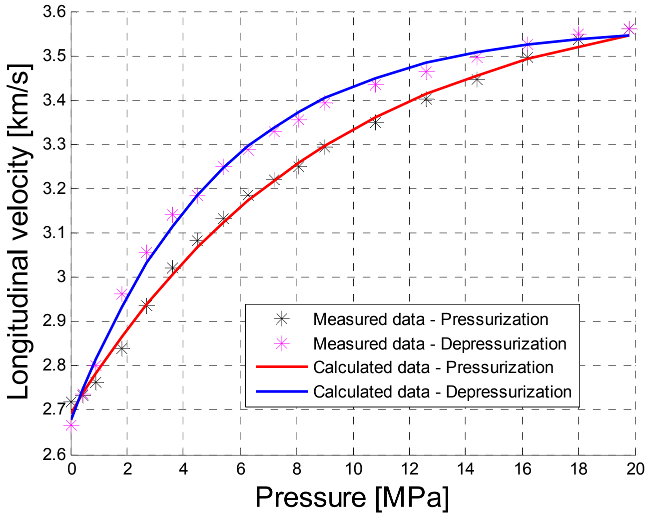

Figure 1 *Velocity vs. pressure curves for pressurization-depressurization cycles of Sample 1.*

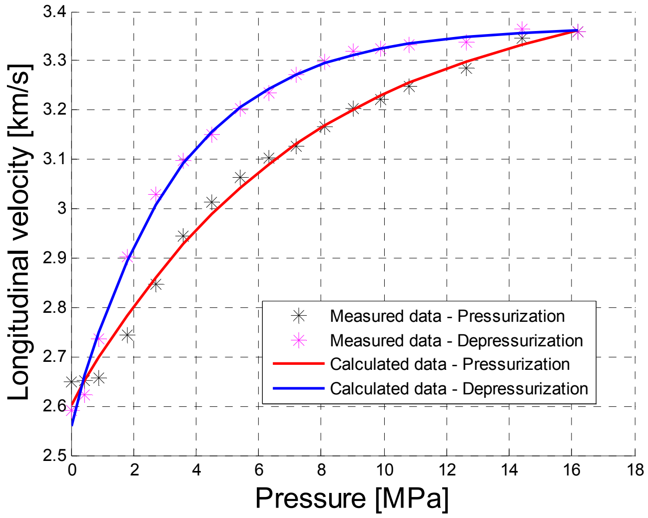

Figure 2 *Velocity vs. pressure curves for pressurization-depressurization cycles of Sample 2.*

The figures show that the calculated curves are in good accordance with the measured data proving that the petrophysical model (Eq. (3) and Eq. (6)) describing the acoustic hysteresis applies well in practice. It can be also seen that the model characterizes well both pressurization and depressurization cycles. Based on Birch's theory at $\sigma=0$, higher P velocity (v_0) value is expected during depressurization cycle than at pressurization period, because the microcracks closed during pressurization do not reopen entirely during decreasing pressure. As a result, there would be less open microcracks at the end of depressurization stage than at pressurization stage at the same pressure, i.e. the velocity would be higher. In contrast, we can see that the depressurization curve at low pressure range drops below the pressurization curve (in different degrees at each sample). The reason is that the new microcracks were created during pressurization. As it was mentioned before in order to avoid creating new microcracks in samples, they were loaded during our measurements only up to one third of the critical uniaxial strength. However the hysteresis of the pressure-velocity curves indicates that the specimens were exceeded their elastic limits and suffered quasi-plastic deformation under uniaxial stress. Describing this phenomenon requires further investigations.

The measure of fitting in data space (difference between measured and calculated data) was calculated according to the root mean square error. The value of data misfit was obtained 0,48% for Sample 1 and 0,66% for Sample 2 at the end of the inversion procedure. The application of the proposed model resulted in approximately the same data misfit on several sandstone samples.

Conclusions

We presented a new petrophysical model describing the acoustic hysteresis which provides the connection between the propagation velocity of acoustic wave and rock pressure, both in case of pressurization and depressurization periods. The model (valid only in reversible/elastic range) is based on the idea that microcracks are opened and closed under the change of pressure. Based on the model the acoustic hysteresis can be expressed by two different parameters λ and λ' because the closed microcracks do not reopen entirely during depressurization. The suggested model was applied to acoustic velocity data measured on core samples. By means of inversion-based data processing, the model parameters were determined from measurement data, thus, calculated data could be produced by the implementation of the petrophysical model in the forward problem. The inverse problem was highly overdetermined; hence the inversion procedure was numerically stable and could be handled by a linear inversion technique. The calculated data match accurately with measured data proving that the petrophysical model describes well both pressurization and depressurization cycles. As it was shown the data misfit was small (less than 1%), which supports the reliability of the inversion results and the accuracy, feasibility of the developed petrophysical model.

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References

Birch, F. [1960] The velocity of compression waves in rocks to 10 kilobars, Part 1. *Journal of Geophysics Research*, **65**, 1083-1102.

Dobróka, M., Gyulai, Á., Ormos, T., Csókás, J. and Dresen, L. [1991] Joint inversion of seismic and geoelectric data recorded in an underground coal-mine. *Geophysical Prospecting*, **39**, 643-665.

Dobróka, M. and Somogyi Molnár, J. [2012] New petrophysical model describing the pressure dependence of seismic velocity. *Acta Geophysica*, **60**, 371-383.

Hashin, Z. and Shtrikman, S. [1963] A variation approach to the theory of the elastic behaviour of multiphase materials. *Journal of the Mechanics and Physics of Solids*, **11**, 127-140.

Ji, S., Wang, Q., Marcotte, D., Salisbury, M.H. and Xu, Z. [2007] P wave velocities, anisotropy and hysteresis in ultrahigh-pressure metamorphic rocks as a function of confining pressure. *Journal of Geophysical Research*, **112**, B09204.

Jones, L.A. and Wang, H.F. [1981] Ultrasonic velocities in Cretaceous shales from the Williston basin. *Geophysics*, **46**, 288-297.

Walsh, J.B. and Brace, W.F. [1964] A fracture criterion for brittle anisotropic rock. *Journal of Geophysics Research*, **69**, 3449-3456.