Acta Geod Geophys (2015) 50:461–477
DOI 10.1007/s40328-015-0105-9

Evaluation of hydraulic conductivity in shallow groundwater formations: a comparative study of the Csókás' and Kozeny–Carman model

N. P. Szabó · K. Kormos · M. Dobróka

Received: 6 December 2014 / Accepted: 21 February 2015 / Published online: 4 March 2015
© Akadémiai Kiadó 2015

Abstract The Kozeny–Carman equation has achieved widespread use as a standard model for estimating hydraulic conductivity of aquifers. An empirically modified form applicable in shallow formations called Csókás' formula is discussed, which is based on the relation between the effective grain-size and formation factor of freshwater-bearing unconsolidated sediments. The method gives a continuous estimate of hydraulic conductivity along a borehole by using electric and nuclear logging measurements without the need of grain-size data. In the first step, synthetic well-logging data sets of different noise levels are generated from an exactly known petrophysical model to test the noise sensitivity of the Csókás' method and to assess the degree of correlation between the results of Csókás' and Kozeny–Carman model. In the next step, borehole logs acquired from Hungarian sites are processed to make a comparison between the Csókás' formula and the Kozeny–Carman equation including grain-size data measured on rock samples. The hydraulic conductivity logs derived separately from the Csókás' and Kozeny–Carman formulae show reliable interpretation results, which are also validated by the Hazen's formula and statistical factor analysis. The fundamental goal of Professor Csókás' research was to derive some useful hydraulic parameters solely from well-logging observations. This idea may be of importance today since the input parameters can be determined more accurately by advanced measurement techniques. Hence, the Csókás' formula may inspire the hydrogeophysicists to make further developments for a more efficient exploration of groundwater resources.

Keywords Hydrogeophysics · Hydraulic conductivity · Csókás' formula · Kozeny–Carman equation · Factor analysis

N. P. Szabó (✉) · K. Kormos · M. Dobróka
Department of Geophysics, University of Miskolc, Miskolc-Egyetemváros 3515, Hungary
e-mail: norbert.szabo.phd@gmail.com

1 Introduction

Hydraulic conductivity is one of the most important hydraulic rock properties in solving hydrogeophysical problems. By definition, it is the rate of flow under unit hydraulic gradient through unit cross-sectional area of aquifer, which depends not only on the type and structure of rock matrix but also the properties of fluid that fills the pore space. In hydrogeology the hydraulic conductivity is normally determined from laboratory measurements or well-site aquifer tests. However, these information are quite local which should be extended to the whole length of the borehole or as good to a bigger measurement area. Ground geophysical surveying methods are used to investigate some related quantities from which the lateral variation of hydraulic conductivity can be detected acceptably. Several types of well-logging probes are available also in boreholes to measure electrical, nuclear and acoustic properties of the surrounding rocks that are useful to estimate the vertical distribution of hydraulic conductivity. The hydraulic information acquired from several wells can be correlated by means of surface geophysical measurements. The similarity and expandability of hydraulic conductivity data obtained from different sources were discussed by Zilahi-Sebess et al. (2007). During the geological site characterization of low and intermediate radioactive waste deposit in Bátaapáti (South-West Hungary) it was experienced that the shape of permeability curves obtained from well-logging measurements correlates properly with that of hydrogeological pumping tests. The well-log derived hydraulic conductivity standardized to hydrogeological information can be related to some physical quantities measured by surface geophysical methods. However, the hydraulic conductivity estimated from ground geophysical observations is valid only for the investigated block, which depends on the spatial distribution of porosity, too. The suitable hydrogeophysical methods and several applications in estimating hydraulic conductivity can be found in Rubin and Hubbard (2005) and Kirsch (2009).

In shallow clastic sediments the evaluation of hydraulic conductivity requires the preliminary knowledge of porosity and grain-sizes. In the absence of direct geophysical measurements one is confined to measure some related physical parameters or to take rock samples from the borehole to extrapolate hydrogeological information for a local area. Several regression relationships are available in the literature, e.g. between porosity and bound-water saturation and permeability or between Stoneley wave slowness and permeability, to make satisfactory predictions, but the values of regression coefficients must usually be set to each area particularly. In order to avoid core sampling, Professor Csókás (1995) worked out a comprehensive interpretation method to give an estimate to permeability and hydraulic conductivity of unconsolidated freshwater-bearing formations based purely on well-logging data. By incorporating the field experiments of Alger (1971), the Hazen's effective grain-size is possible to be substituted by the Archie's formation factor which can be measured directly from well logs. The derived formula including porosity and true resistivity of aquifers gives a continuous estimate to hydraulic conductivity for the entire length of the borehole. Additional to it, the Csókás' method comprises the determination of critical filtration velocity, which can be used to estimate the highest value of sand-free yield in the knowledge of filter radius and effective layer-thickness. The hydrogeophysical parameters estimated by the Csókás' method provide with valuable information for groundwater exploration and help to establish a suitable technique for exploiting underground water resources.

As a continuation of Professor Csókás' research, in this paper, a comparative study is made between the Csókás' and the grain-size based Kozeny–Carman model. The Csókás' method is first applied to synthetic well-logging data sets to test the noise sensitivity of parameter estimation and to quantify the accuracy of interpretation results. Then, the re-processing of a Hungarian data set published previously by Professor Csókás is made to compare the Csókás' and Kozeny–Carman methods. At last, a new Hungarian case study is shown to demonstrate that hydraulic conductivities estimated by the Csókás' method are in close agreement both with laboratory measurements and independent results of multi-variate statistical (factor) analysis of borehole logs. The authors recommend the Csókás' method primarily for hydrogeophysicists to expand hydraulic test data for the whole length of a borehole or correlate them between several wells drilled in the groundwater investigation site.

2 The Kozeny–Carman equation

The Darcy's equation is one of the basic equations in hydrogeology which describes the flow of a fluid through a porous formation

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{\kappa}{\Phi \mu} \nabla p, \tag{1}$$

where κ (m^2) denotes permeability, Φ (v/v) is porosity, μ (Ns/m^2) is dynamic viscosity, $\mathbf{u}$ (m) is the relative displacement vector of the fluid and p (N/m^2) is the pore pressure. Hydraulic conductivity $K = \kappa\rho g/\mu$ (m/s) as a related quantity expresses the ease with which the fluid flows through the pore system, which is influenced by several properties of the rock matrix and pore fluid such as density (ρ) and viscosity of pore-filling fluid, distribution of grain-sizes, amount of porosity and degree of water saturation in primary porosity rocks (g (cm/s^2) is the normal acceleration of gravity). In the first approximation, the connection between hydraulic conductivity and grain-size is represented by a simple empirical formula (Hazen 1892)

$$K = C_H d_{10}^2, \tag{2}$$

where C_H is the Hazen's empirical coefficient (in the interval of 0.4 and 10), d_{10} is the representative grain diameter at 10 % cumulative frequency relative to which the one-tenth part of the sample is finer by weight. A more reliable approach was suggested by Kozeny (1927), which was later modified by Carman (1937). According to the Kozeny–Carman (hereafter KC) model the rock with intergranular porosity is considered to be an assembly of capillary channels in which the Navier–Stokes equation satisfies. The following form of the KC equation is one of the most widely used formulas for the estimation of hydraulic conductivity (Bear 1972)

$$K = \frac{\rho_w g}{\mu} \frac{d^2}{180} \frac{\Phi^3}{(1 - \Phi)^2}, \tag{3}$$

where d (cm) is the dominant grain diameter, Φ (v/v) is the porosity of formation and ρ_w (g/cm^3) is the density of pore-fluid (K is given in units of cm/s). In Eq. (3) the term of $\Phi^3/(1 - \Phi)^2$ characterizes the compactness of the rock. The dominant grain diameter can be derived from those representative values of the grain-size distribution curve for which the 10 % and 60 % parts of the sample are finer by weight, respectively (Juhász 2002)

$$d = \frac{d_{10} + d_{60}}{2} \sqrt{\frac{d_{10}}{d_{60}}}, \tag{4}$$

where d_{60} is the grain diameter at 60 % cumulative frequency. Rock samples can be taken from boreholes to measure the grain-size and the amount of porosity can be estimated by well log analysis. The correlation between grain-size and porosity should be found before well logs are applied to calculate the continuous curve of hydraulic conductivity by Eqs. (3)–(4) along the borehole axis. The knowledge of the hydraulic conductivity is of great importance in the determination of water reserves as well as in the management and protection of groundwater supply. Equation (3) is considered as a standard calculation method to estimate hydraulic conductivity. The accuracy of the estimation is highly limited of course by the uncertainty of the related parameters. The estimation error of hydraulic conductivity normally is one (or one and a half) order of magnitude, thus it is the change of hydraulic conductivity that can be preferably evaluated by geophysical methods than its absolute value.

3 The Csókás' method

In the absence of core samples one should rely on borehole logging data for the estimation of petrophysical properties of rocks. Csókas (1995) worked out a deterministic procedure

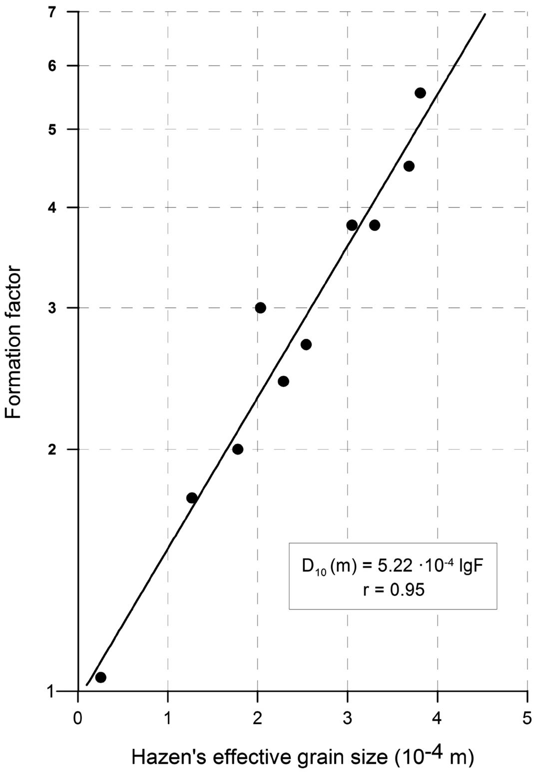

Fig. 1 Alger's empirical relation between formation factor and dominant grain-size

to extract hydraulic conductivity of freshwater formations only from well logs. The starting-point in formulating the Csókás' (hereafter CS) model is the assumption of the KC equation. On the other hand, it is also assumed that there is an empirical connection between some types of well logs and hydraulic conductivity. A constraint relation suggested by Alger (1971) connecting the dominant grain-size to the Archie's formation factor forms the bridge between well-logging measurements and hydraulic conductivity of aquifers.

In rocks saturated fully with water the formation factor (F) is defined as the ratio of the resistivity of rock (R_0) to that of the pore-water (R_w)

$$F = \frac{R_0}{R_w}. \tag{5}$$

Alger (1971) found a direct proportionality between the formation factor and grain-size of freshwater saturated sediments in the laboratory, which is opposite of that experienced in the oilfields (i.e. in brine saturated rocks). The Hazen's effective grain diameter (d_{10}) determined from sieve analysis was empirically related to the formation factor of unconsolidated sediments

$$d_{10} = C_d \lg F, \tag{6}$$

where C_d is a site-specific constant. The value of $C_d = 5.22 \times 10^{-4}$ in Eq. (6) was proposed for not too poorly sorted sediments with a formation factor less than 10. This condition is normally fulfilled in shallow clastic aquifers. Alger (1971) suggested an interpretation methodology based on Eq. (6) by using different types of well-logging suits adapted from the oilfield to evaluate freshwater-bearing shaly sands. The regression Eq. (6) is illustrated in Fig. 1, where the Pearson's correlation coefficient $r = \mathrm{cov}(d_{10}, F)/(\sigma_{d_{10}}\sigma_F)$ indicates a strong relation between the variables.

The dominant grain-size in Eq. (3) can be defined as the diameter of a homo-disperse conglomerate of grains the surface of which equals to that of the real sample with actual grain-size distribution and same density. The value of dominant grain-size can be given from the grain-size distribution curve. Additional to Eq. (4), there exist several formulae for the estimation of dominant grain diameter. The uniformity coefficient $U = d_{60}/d_{10}$ as a shape parameter describing the form of the grain-size distribution curve quantifies the degree of uniformity in a granular material. Kovács (1972) connected the uniformity coefficient of sands to dominant grain-size (d) as $d/d_{10} = 1.919 \cdot \lg U + 1$. For poorly sorted sediments ($U > 5$), quantity U is inversely proportional to the logarithm of hydraulic conductivity. For not so badly sorted sands ($2.0 \le U \le 2.5$) the previous equation takes the form as

$$d = 1.671 \cdot d_{10}. \tag{7}$$

By combining Eqs. (5)–(7) one can obtain

$$d = 1.671 \cdot C_d \lg \frac{R_0}{R_w}. \tag{8}$$

The determination of hydraulic conductivity requires the preliminary knowledge of effective porosity and shale volume of groundwater formations. Archie (1942) suggested the following empirical formula developed from laboratory measurements made on numerous samples

$$F = \frac{a}{\Phi^m}, \tag{9}$$

where m is the cementation exponent (in poorly compacted sediments $m \sim 1.5$–1.7) and a is the tortuosity coefficient ($a \sim 1$). The study of Alger (1971) showed that the formation factor depends not only on the porosity, but also on the resistivity of pore-water and grain-size in freshwater saturated sediments with primary porosity. Ogbe and Bassiouni (1978) coupled the tortuosity factor with porosity and formation factor

$$a^2 = \left(\frac{R_0}{R_w}\Phi\right)^{1.2}. \tag{10}$$

The constants a and m in Eq. (9) represents the textural properties of the rock. They are unvarying or only slowly varying with depth and are treated as zone parameters in well log interpretation. The effective porosity in shaly sands younger than Tertiary can be calculated as $\Phi_e = \Phi\ (1 - V_{sh})$ where the porosity can be extracted from gamma–gamma or neutron–neutron measurements and the shale volume (V_{sh}) can be estimated from the natural gamma-ray intensity log (Larionov 1969)

$$V_{sh} = 0.083\left(2^{3.7\, i_{GR}} - 1\right), \tag{11}$$

where i_{GR} is the natural gamma-ray index. Equation (11) does not depend on the water type, but cautions should be made in rocks including radioactive non-clay minerals or fractures filled with uranium- or thorium rich water. Dispersed clay particles also can considerably modify the resistivity of pore-water, which then affects the formation factor appearing in Eq. (6).

The hydraulic conductivity appears in Eq. (1) which was properly modified by Kovács (1972)

$$K = \frac{1}{5}\frac{g}{\upsilon}\frac{\Phi^3}{(1-\Phi)^2}\left(\frac{d}{\alpha}\right)^2, \tag{12}$$

where α is the average shape factor of sample particles in the range of 7 and 11 for sands (the average is 10). The kinematic viscosity of water υ (m^2/s) can be expressed in the function of formation temperature, thus the ratio of gravity acceleration and kinematic viscosity for the water in Eq. (12) is $g/\upsilon = 5.517 \times 10^4 \cdot C_t$ (m/s), where C_t is a temperature dependent constant calculated as $C_t = 1 + 3.37 \times 10^{-2}\mathrm{T} + 2.21 \times 10^{-4}\mathrm{T}^2$ (where T is given in units of $°C$). Pirson (1963) published another form of the Kozeny equation used to predict permeability

$$\kappa = \frac{1}{5}\frac{\Phi^3}{(1-\Phi)^2}\left(\frac{1}{a \cdot S}\right)^2, \tag{13}$$

where S (m^2/m^3) denotes the specific surface of the rock. By comparing Eqs. (12) and (13) the following identical equation can be derived

$$\left(\frac{d}{\alpha}\right)^2 = \left(\frac{1}{a \cdot S}\right)^2. \tag{14}$$

Gálfi and Liebe (1981) summarized several empirical relations between the specific electric resistance and hydraulic conductivity for sands and gravels. In freshwater-bearing sediments the electric current is hardly conducted through the spaces between the grains, but mainly on the surfaces of particles. Thus, the resistivity is inversely proportional to the specific surface. By assuming that the sedimentary rock is composed of spherical particles the specific surface can be calculated as

$$S = 6\frac{(1-\Phi)}{d}, \tag{15}$$

The combination of Eqs. (8) and (15) gives

$$S^2 = \frac{36(1-\Phi)^2}{\left(1.671 \cdot C_d \cdot \lg\frac{R_0}{R_w}\right)^2}. \tag{16}$$

The hydraulic conductivity can be calculated by the CS model on the basis of Eqs. (10), (12), (14) and (16) in units of m/s

$$K = C_k \frac{\Phi^3}{(1-\Phi)^4} \frac{\left(\lg\frac{R_0}{R_w}\right)^2}{\left(\frac{R_0}{R_w}\Phi\right)^{1.2}}, \tag{17}$$

where $C_k = 855.7 \cdot C_t C_d^2$ is a proportionality constant. According to Csókás (1995) good aquifers are characterized by hydraulic conductivities K (m/s) $> 10^{-6}$, while aquitards are indicated by K (m/s) $< 3 \times 10^{-8}$. The uniqueness of the CS formula resides in the fact that all parameters in Eq. (17) can be derived from borehole geophysical logs therewith a continuous (in situ) estimate can be given for hydraulic conductivity along a borehole.

4 Validation by factor analysis

The CS method-based hydraulic conductivity can be validated by laboratory measurements or aquifer tests. Its feasibility can also be demonstrated by making a comparison with independent well-logging data processing methods. The highest accuracy and reliability can be normally achieved when all suitable logs are processed simultaneously. Multivariate statistical methods usually employ several well logs and large statistical samples to provide an optimal solution. In this study, the shale volume is estimated from the simultaneous processing of nuclear and resistivity logs. Then an empirical connection between shale volume and hydraulic conductivity is used to calculate the vertical distribution of the hydraulic conductivity of groundwater formations. The advantage of the factor analysis approach is that it utilizes all types of well logs sensitive to the presence of shale in one interpretation procedure instead of using a single log, e.g. natural gamma-ray intensity data in Eq. (11), which normally gives less reliable results.

Factor analysis is traditionally used to reduce the dimensionality of multivariate statistical problems (Lawley and Maxwell 1962). Additionally, it can be used to enhance information on latent variables hidden in the data set. Factor analysis has been used both in hydrocarbon (Szabó 2011; Szabó and Dobróka 2013; Puskarczyk et al. 2014) and groundwater exploration (Szabó et al. 2014). The statistical procedure is applicable to transform numerous geophysical data types into smaller number of variables called factors. As a result, a few factors explain the determinant amount of total variance of measurement data, which can be connected to petrophysical properties of the observed geological structure. Consider the decomposition of the N-by-K standardized data matrix $\mathbf{D}$ including different types of well-logging data

$$\mathbf{D} = \mathbf{F}\mathbf{L}^{\mathrm{T}} + \mathbf{E}, \tag{18}$$

where $\mathbf{F}$ denotes the N-by-M matrix of factor scores, $\mathbf{L}$ is the K-by-M matrix of factor loadings, $\mathbf{E}$ is the N-by-K matrix of residuals, M is the number of extracted factors (T indicates the operator of matrix transpose). The factor scores as elements aligned in the first column of matrix $\mathbf{F}$ represent the well log of the first factor, which explains the largest part of variance of the well-logging data. The individual weights of each data type associated with the factors are given in the matrix of factor loadings, which practically measure the degree of correlation between the factors and original data. Since the factors are assumed to be uncorrelated, the covariance matrix of observed data can be written in the following form

$$\mathbf{R} = \frac{1}{N}\mathbf{D}^{\mathrm{T}}\mathbf{D} = \mathbf{L}\mathbf{L}^{\mathrm{T}} + \boldsymbol{\Psi}, \tag{19}$$

where $\boldsymbol{\Psi} = \mathbf{E}^{\mathrm{T}}\mathbf{E}/N$ is the diagonal matrix of specific variances. In case of $\boldsymbol{\Psi} = \mathbf{0}$ the problem reduces to the solution of an eigenvalue problem, which is equivalent to principal component analysis. Otherwise matrices $\mathbf{L}$ and $\boldsymbol{\Psi}$ are estimated jointly by optimizing the following objective function (Jöreskog 2007)

$$\Omega(\mathbf{L}, \Psi) = \mathrm{tr}(\mathbf{R}\text{-}\mathbf{L}\mathbf{L}^{\mathrm{T}} - \boldsymbol{\Psi})^2 = \min. \tag{20}$$

After solving Eq. (20) the factor scores can be calculated by the hypothesis of linearity (Bartlett 1953)

$$\mathbf{F}^{\mathrm{T}} = \left(\mathbf{L}^{\mathrm{T}}\boldsymbol{\Psi}^{-1}\mathbf{L}\right)^{-1}\mathbf{L}^{\mathrm{T}}\boldsymbol{\Psi}^{-1}\mathbf{D}^{\mathrm{T}}. \tag{21}$$

After the rotation of factor loadings the factor logs can be compared to parameters of the petrophysical model. Regression tests performed after factor analysis may reveal the relationships between the factors and petrophysical properties of rocks.

An empirical formula suggested by Szabó et al. (2014) provides an estimate of shale volume in North-East Hungarian shallow sediments

$$V_{sh} = 27.4e^{0.015F_1'} - 26.5, \tag{22}$$

where F_1' is the first factor scaled into the interval of 0 and 100. Sallam (2006) found a nonlinear regression relation between shale volume and hydraulic conductivity derived from hydraulic pumping tests. In this paper, shale volume calculated from Eq. (22) is substituted into Sallam's empirical formula to get hydraulic conductivity in units of m/day

$$K = 49.79e^{-12.51 \cdot V_{sh}}. \tag{23}$$

The above procedure is tested in a North Hungarian data set (Sect. 7), which provides as reference for comparing it to the CS method. It can be mentioned that other type of statistical approaches can also be used to derive unmeasurable parameters of the petrophysical model. In iterative inversion procedures the model parameters (i.e. porosity, water saturation, shale content, matrix volume etc.) are estimated simultaneously. The advantage of inverse modeling is that the estimation errors of petrophysical parameters are also given, which characterize quantitatively the accuracy and reliability of inversion results. The hydraulic conductivity can be derived from the results of inverse modeling by using proper empirical equations.

5 Synthetic modeling experiment

The CS method is first tested in a simulated geological environment. Synthetic well logs are calculated on a petrophysical model that has exactly known parameters. Two groups of model parameters are distinguished, i.e. layer- and zone parameters. The former vary layer-by-layer, while the latter are practically unvarying in the processing interval (groundwater zone). The layer parameters are porosity (Φ), shale volume (V_{sh}), sand volume (V_{sd}), grain size at 10 % relative frequency (d_{10}) and dominant grain size (d). The porous formations are fully saturated with freshwater, thus the water saturations of the flushed (S_{x0}) and uninvaded zones (S_w) are set to unity (100 %), respectively. The model describes an unconsolidated sedimentary formation made up of five homogeneous beds (tracks 6 and 8 in Fig. 2). The second and fourth layers represent coarse-grained sandy aquifers and the other ones show silts with low permeability. Hydraulic conductivity is calculated from well logs such as natural gamma-ray intensity (GR), spontaneous potential (SP), neutron-neutron intensity (NN), bulk density (ρ_b), shallow and deep resistivity (R_s, R_d).

The layer parameters are related to well-logging data by means of empirical modeling equations. The following set of probe response equations modified to fully saturated shaly sands can be used to calculate the theoretical values of borehole logging data (Alberty and Hashmy 1984)

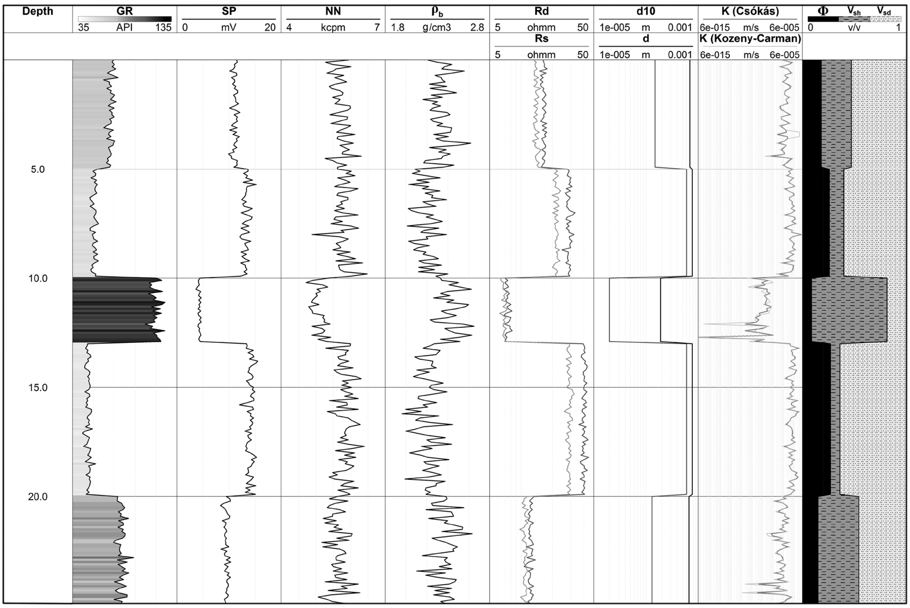

Fig. 2 Synthetic well logs contaminated with 5 % Gaussian distributed noise and results of hydraulic conductivity estimation

Table 1 Parameters of the petrophysical model

Zone parameter	Definition	Symbol	Value (unit)
Gamma-ray intensity	Shale	GR_{sh}	160 API
	Sand	GR_{sd}	25 API
Spontaneous potential	Shale	SP_{sh}	0 mV
	Sand	SP_{sd}	15.53 mV
	Temperature constant	C	70
Neutron–neutron intensity	Shale	NN_{sh}	4.8 kcpm
	Sand	NN_{sd}	7.2 kcpm
	Pore-fluid	NN_f	3.1 kcpm
Bulk density	Mud-filtrate	ρ_{mf}	1.0 g/cm^3
	Shale	ρ_{sh}	2.55 g/cm^3
	Sand	ρ_{sd}	2.65 g/cm^3
Electric resistivity	Mud-filtrate	R_{mf}	9 ohm-m
	Pore-water	R_w	15 ohm-m
	Shale	R_{sh}	2 ohm-m
Textural parameters	Cementation exponent	m	1.5
	Saturation exponent	n	1.9
	Tortuosity factor	a	1.0
Hydraulic parameters	Dynamic viscosity	μ	0.019 Pa s
	Acceleration of gravity	g	981 cm/s^2

$$SP = SP_{sh}V_{sh} - C \cdot \lg\frac{R_{mf}}{R_w}(1 - V_{sh}), \tag{24}$$

$$GR = GR_{sd} + \frac{1}{\rho_b}(V_{sh}GR_{sh}\rho_{sh} + V_{sd}GR_{sd}\rho_{sd}), \tag{25}$$

$$\rho_b = \Phi\rho_{mf} + V_{sh}\rho_{sh} + V_{sd}\rho_{sd}, \tag{26}$$

$$NN = \Phi NN_f + V_{sh}NN_{sh} + V_{sd}NN_{sd}, \tag{27}$$

$$R_s = \left[\left(\frac{V_{sh}^{(1-0.5\ V_{sh})}}{R_{sh}^{1/2}} + \frac{\Phi^{m/2}}{(aR_{mf})^{1/2}}\right)\right]^{-2}, \tag{28}$$

$$R_d = \left[\left(\frac{V_{sh}^{(1-0.5\ V_{sh})}}{R_{sh}^{1/2}} + \frac{\Phi^{m/2}}{(aR_w)^{1/2}}\right)\right]^{-2}, \tag{29}$$

$$\Phi + V_{sh} + V_{sd} = 1. \tag{30}$$

The zone parameters with their chosen values appearing in Eqs. (24)–(29) are specified in Table 1. Equation (30) is a constraint relation called the material balance equation, which can be used to specify the relative fractions of rock constituents per unit volume of rock and to reduce the number of unknowns by deriving one parameter (normally V_{sd}) from the other ones. In the next step, quasi measured well logs are generated by adding different amount of noise to the (noiseless) synthetic data. The hydraulic conductivity is calculated first by the KC model based on Eq. (3), then it is estimated separately by the CS formula using Eq. (17). In the latter case, constant C_k is chosen as 3.2587×10^{-4}. An example result of hydraulic conductivity estimation is shown in Fig. 2. In the first five tracks the input well logs are plotted, which are contaminated with 5 % Gaussian distributed noise. In the following track the vertical distributions of grain-sizes and in track 8 the compositions of rocks are represented. The estimated hydraulic conductivity curves can be compared in track 7, which show a close agreement between the results of the CS and KC methods.

The discrepancy between the estimation results of the CS and KC procedures is measured by three different quantities. The data distance quantifies the deviation between the noiseless and noisy (quasi measured) well-logging data sets. Since the measurement data have different magnitudes and measurement units the relative data distance is introduced as

$$D_d = \left(\frac{1}{P \cdot N} \sum_{p=1}^{P} \sum_{k=1}^{N} \left(\frac{d_{pk}^{(0)} - d_{pk}^{(noisy)}}{d_{pk}^{(noisy)}} \right)^2 \right)^{1/2} \times 100\,(\%), \tag{31}$$

where $d_{pk}^{(0)}$ and $d_{pk}^{(noisy)}$ are the k-th noiseless and noisy data in the p-th depth, respectively. The model distance measures the overall misfit between the estimated hydraulic conductivity logs

$$D_m = \left(\frac{1}{P} \sum_{p=1}^{P} \left(\frac{\lg K_p^{(KC)} - \lg K_p^{(CS)}}{\lg K_p^{(CS)}} \right)^2 \right)^{1/2} \times 100\,(\%), \tag{32}$$

where $K_p^{(KC)}$ and $K_p^{(CS)}$ denote the hydraulic conductivity in the p-th depth estimated by the KC and CS methods, respectively. If well-logging or other derived data sets are considered as samples of random variables, the tools of classical statistics can be applied to quality control purposes. In this study, the Pearson's correlation coefficient is used to determine the strength of relationship between the hydraulic conductivity logs

$$r = \frac{\mathrm{cov}\left(\mathrm{K}^{(\mathrm{KC})}, \mathrm{K}^{(\mathrm{CS})}\right)}{\left[\mathrm{cov}(K^{(KC)}, K^{(KC)})\mathrm{cov}(K^{(CS)}, K^{(CS)})\right]^{1/2}}, \tag{33}$$

where cov denotes the sample covariance operator. If r is close to unity, the correlation between the two logs is strong. It is concluded from the demonstrated example that the result is as noisy as input data. The accuracy of hydraulic conductivity depends on the uncertainty of well-logging data.

The accuracy of well logs is usually different depending on the probe type, technical data of well-logging operation, drilling environment and actual geological setting. To simulate the conditions of real measurements, the synthetic (noiseless) data are contaminated by different amount of noise. These data sets are separately processed to test the noise sensitivity of the interpretation procedure. In this particular case, the experiment may give valuable information on the accuracy of permeability estimation. The k-th synthetic datum is contaminated with random noise by

$$d_k^{(noisy)} = d_k^{(0)}(1 + \mathcal{N}(\mu, \sigma)), \tag{34}$$

where $\mathcal{N}$ is a Gaussian distributed random number, μ is the expected value chosen as 0, σ is the standard deviation proportional to the level of data noise. The scale parameter of the probability distribution function was chosen as 1/100 part of noise level (%). Each computer run produces a noisy data set the noise level of which deviates around an overall error calculated by Eq. (31). In this experiment, the synthetic data generated by Eqs. (24)–(29) are contaminated with 1–10 % Gaussian distributed noise. For simulating non-Gaussian data distributions, two additional data sets are involved that contain outliers (three times higher amount of noise is added randomly to the 1/6 part of the Gaussian distributed data). The numerical results of hydraulic conductivity estimation can be found in Table 2. Relative data distances calculated by Eq. (31) are listed in the first column. Model distance defined in Eq. (32) increases proportionally with the data distance. The correlation coefficient based on Eq. (33) shows only a slight decay with the increase of noise level. It is also observable that the correlation between the CS and KC model-based hydraulic conductivities is still strong with extreme noises and non-Gaussian distributed data. It is shown that the CS method gives consistent results, but the accuracy of hydraulic conductivity largely depends on the amount of data noise. The final results given by the CS method are close to those of the KC method, thus, it is rather the data noise that affects the accuracy of the solution than modeling errors the influence of which is relatively smaller. The synthetic experiments confirm the reliability of the CS procedure.

6 Case study I

The CS method has been tested earlier in Well K-564 drilled for Jászberény City's Waterworks in Hungary (Csókás 1995). Natural gamma-ray intensity (*GR*), spontaneous potential (*SP*), short normal and lateral apparent resistivity logs (R_a) were recorded with analogue logging equipment in the 320 mm diameter borehole (at a scale of 1:200). The measured signals were squared originally by the author by performing electrofacies analysis. The zone parameters of probe response functions can be found in Table 3. The *SP* log is applied to estimate the resistivity of pore-water ($R_w = 1.75 \cdot R_{we}$), where the equivalent water resistivity (R_{we}) calculated throughout the logging interval is 6.9–9.3 ohm-m. The R_a log is corrected for the effect of mud-filtrate resistivity and borehole diameter to give an estimate to formation resistivity R_0, formation factor F and porosity Φ. The input well logs (tracks 1–3) and necessary derived parameters (tracks 4–5), grain-sizes (track 6) and volumetric rock composition (track 8) are plotted in Fig. 3. The CS formula-derived hydraulic conductivity log can be found in track 7. The interpretation result is

Table 2 Results of noise sensitivity tests

Noise level	Data distance (%)	Model distance (%)	Correlation coefficient
0 % Gaussian (noiseless)	0	0.04	1.00
1 % Gaussian	1.01	0.85	1.00
2 % Gaussian	1.99	1.47	0.99
3 % Gaussian	3.05	2.06	0.99
4 % Gaussian	4.01	2.78	0.99
5 % Gaussian	5.02	3.58	0.99
10 % Gaussian	10.06	6.31	0.98
5 % Gaussian (incl. outliers)	8.97	4.31	0.98
10 % Gaussian (incl. outliers)	20.83	7.31	0.96

Table 3 Zone parameters applied in borehole K-564

Zone parameter	Definition	Symbol	Value (unit)
Gamma-ray deflection	Shale	GR_{sh}	47 mm
	Sand	GR_{sd}	0 mm
Spontaneous potential	Constant	C	1.5
Temperature	Formation	T_f	15 °C
Density	Mud	ρ_m	1.1 g/cm^3
Electric resistivity	Mud	R_m	9.31 ohm-m
	Mud-filtrate	R_{mf}	10.31 ohm-m
Hydraulic parameters	Dynamic viscosity	μ	0.019 Pa s
	Acceleration of gravity	g	981 cm/s^2

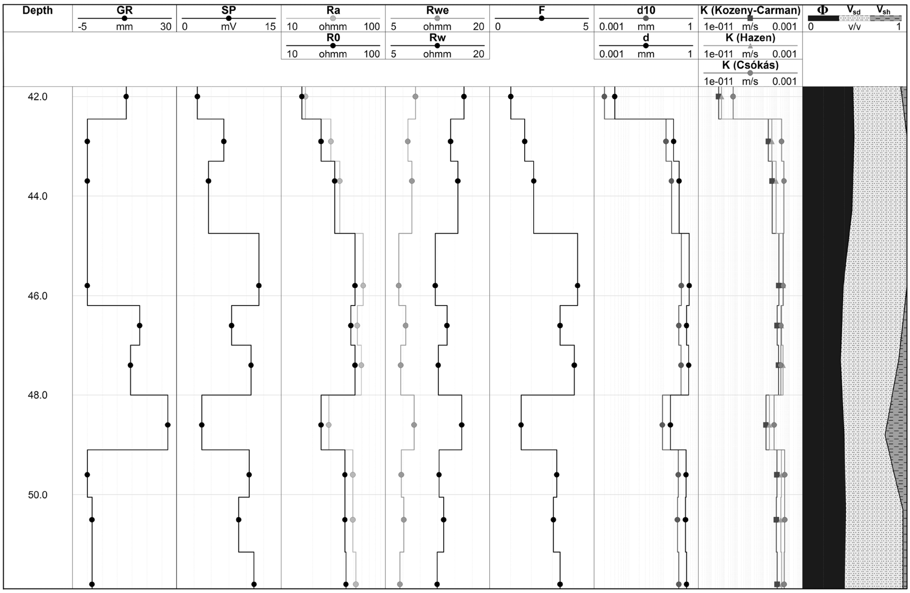

Fig. 3 Well logs of measured quantities and hydraulic conductivity estimated in borehole K-564

confirmed by hydraulic conductivity estimates calculated by Eqs. (2)–(3). The Hazen's equation is used in the form as $K = 116 \cdot d^2$, where the dominant grain-size d is given in the units of cm (the same quantity is plotted in the units of mm in track 6 of Fig. 3). The result of CS method correlates acceptably with those of the Hazen's and KC formulae.

The deviation between the hydraulic conductivity curves is maximum one-half order of magnitude. The model distance between the $K^{(CS)}$ and $K^{(KC)}$ logs is $D_m = 13\ \%$, while it is 9.8 % between the results of the CS and Hazen's methods. The depth intervals of 41.0–47.5 m and 48.0–52.0 m were designated for water production. As a result of well log analysis, the flow rate in permeable intervals was estimated as 632 l/min according to the CS procedure, which was confirmed by hydraulic pumping test data suggesting an optimal discharge rate of 550 l/min (Csókás 1995).

7 Case study II

The second test well named Baktalórántháza-1 is located in Szabolcs-Szatmár-Bereg County, North-East Hungary. The primary aim of the geophysical survey was the investigation of the geological structure for the purpose of hydrocarbon exploration. Although neither oil nor gas was found, the borehole was still remained remarkable in the aspect of prospecting thermal water resources. Lower Pannonian deposits are varying in the logged interval such as gravel, clayey sand, clayey silt, clayey marl and bituminous clay. The depth level of 240 m is the boundary of Pleistocene and Pannonian periods.

Natural gamma-ray intensity (*GR*), spontaneous potential (*SP*), shallow resistivity (*RS*), gamma-gamma (*GG*) and neutron-neutron (*NN*) logs are available in the interval of 240–460 m as they are illustrated in tracks 1–4 of Fig. 4. Grain-size data are also provided from laboratory measurements made on 118 rock samples. The significant values of the grain-size distribution curves (d_{10}, d_{60}, d) are plotted by different colors in track 6. The zone parameters are listed in Table 4. The data set was previously processed by factor analysis to give an estimate to shale content, which was also confirmed by laboratory data (Szabó et al. 2014). The shale volume is extracted from well logs by using the local relationship between the shale volume (track 8) and first factor (track 5) based on Eq. (22). Then, an approximate solution for the distribution of hydraulic conductivity is given by Eq. (23). For the application of the CS and KC methods a porosity is calculated from Eq. (27), where the values of zone parameters can be read from the *NN* versus *GG* crossplot (Table 4). In the knowledge of porosity and shale volume, the sand volume (V_{sd}) can be calculated from Eq. (30). The volumetric rock composition is illustrated in the last track. The CS method requires the prior knowledge of formation factor, which is normally

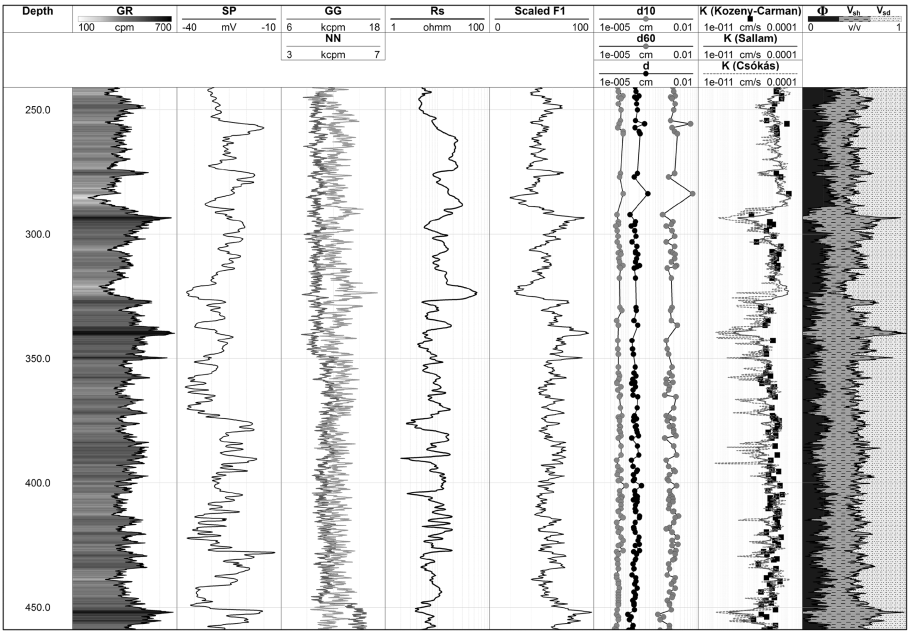

Fig. 4 Well logs of observed quantities and hydraulic conductivity estimated in borehole Baktalórántháza-1

Table 4 Zone parameters applied in borehole Baktalórántháza-1

Zone parameter	Definition	Symbol	Value (unit)
Gamma-ray deflection	Shale	GR_{sh}	685 cpm
	Sand	GR_{sd}	188 cpm
Neutron–neutron intensity	Shale	NN_{sh}	4 kcpm
	Sand	NN_{sd}	7.5 kcpm
	Pore-fluid	NN_{f}	1 kcpm
Textural parameters	Cementation exponent	m	2.15
	Tortuosity factor	a	0.62
Hydraulic parameters	Dynamic viscosity	μ	0.019 Pa s
	Acceleration of gravity	g	981 cm/s^2

derived from SP and R_0 logs. In the lack of some related quantities, the Humble-formula is used to calculate formation factor ($F = 0.62/\Phi^{2.15}$). The KC method cannot discard the measurement of grain sizes. Thus, the hydraulic conductivity is estimated only to that places where the rock samples have been previously taken from the borehole. On the contrary, the CS method gives an estimate to hydraulic conductivity without grain-size data along the entire length of the borehole. The results of hydraulic conductivity estimation can be seen in track 7. The CS method-based hydraulic conductivity log shows the highest degree of variation, because the porosity is calculated from a rather noisy neutron log. For seeking a smoother solution, it is recommended to calculate porosity from another source, e.g. by the inversion of all suitable well logs.

The numerical results show that the result of CS method is in close agreement with that of the KC procedure. The model distance is $D_m = 3.4\ \%$ averaged for the places of recovered rock samples. The Pearson's correlation coefficient between the CS and KC model-based hydraulic conductivities is $r = 0.96$. The hydraulic conductivity estimated by the combination of multivariate factor analysis and Sallam's method also fits acceptably to that of the CS procedure, where the correlation coefficient is $r = 0.72$ and the model distance is $D_m = 8.8\ \%$ for the logged interval. The feasibility of the CS method have been demonstrated in shallow hydrogeological environments.

8 Conclusions

It is concluded that the Csókás' method gives a satisfactory estimate to the vertical distribution of hydraulic conductivity in groundwater wells. In this study, it is confirmed by the results of the Kozeny–Carman procedure. Synthetic modeling studies show consistent solutions since the hydraulic conductivities based on the two methods keeps strong correlation with increasing level of data noise. In the well-site, not only other empirical formula or multivariate factor analysis based procedures, but also laboratory measurements show a close agreement with the results of Csókás' method. According to field experiences, an optimal solution can be obtained in medium or coarse grained (well-sorted) unconsolidated sediments with formation factor less than 10. In case of highly cemented aquifers the estimation results show considerable deviations from the Kozeny–Carman model. Also in very fine grained rocks, e.g. in loess, the hydraulic conductivities show sometimes more than one order of magnitude difference, which may require the revision of the Alger's formula. The empirical relationship between the grain-sizes and formation factor is advised to be specify in the given area. It requires the use of grain-size distributions and properly corrected resistivity curves. Once the Csókás' formula is validated by this empirical equation, any interval along the borehole can be properly evaluated. The Csókás' method gives a continuous in situ estimate to hydraulic conductivity in typical unconsolidated aquifers, which is of high importance in the interpolation of aquifer test data. This hydrogeophysical information can also be extended to larger areas by means of well-to-well correlation techniques, which may also improve the efficiency of complex geophysical surveys.

A tribute is paid to Professor János Csókás (1918–2000), whose borehole geophysical methodology, presented in this paper, is hereby recommended to the community of hydrogeophysicists, which can be further developed by advanced measurement techniques applicable to a more accurate observation of input parameters. Some new results have been grown in the geophysical practice may seem to be integrated fruitfully, for instance, the pore-space can be imaged by high-resolution micro-tomography or nuclear magnetic resonance measurements to improve the accuracy of effective porosity (Jarzyna et al. 2012). According to the theory of electrokinetic phenomena in porous media the grain diameters can be replaced by the effective pore radius in permeability estimation, which shows a close empirical relation with porosity. The effective pore radius was validated in the laboratory over a wide range of pore sizes by optical image analysis and the theory was tested using North Sea core samples by Glover and Walker (2009). The resistivity of pore-water and tortuosity factor included in the derivation of Csókás' formula are key-parameters, which can be estimated with their estimation errors by the interval inversion of well-logging data (Dobróka and Szabó 2011). Factor analysis of the same logs can give a reliable estimate to the amount of shaliness (Szabó et al. 2014). Moreover, one of our ongoing research studies shows that a certain statistical factor directly correlates with the hydraulic conductivity of aquifers. The above reasons may inspire the hydrogeophysicists to make further developments in the improvement of the Csókás' method to increase the efficiency of the hydrogeophysical exploration of groundwater resources.

Acknowledgments The first author as the leading researcher of OTKA project no. PD-109408 thanks to the support of the Hungarian Scientific Research Fund as well as to the support of the János Bolyai Research Fellowship of the Hungarian Academy of Sciences. The third author as the leading researcher of OTKA project no. K-109441 thanks to the support of the Hungarian Scientific Research Fund. The investigation was partly based on the previous research work of the TÁMOP (project No. 4.2.1.B-10/2/KONV-2010-0001) Center of Excellence. The described work was carried out as part of the TÁMOP-4.2.2.A-11/1/KONV-2012-0049 project in the framework of the New Hungarian Development Plan. The realization of this project is supported by the European Union, co-financed by the European Social Fund (as a support for the second author). The authors thanks to László Hursán titular associate professor of the Department of Geophysics in University of Miskolc for the professional consultation on well-site applications.

References

Alberty M, Hashmy K (1984) Application of ULTRA to log analysis. In: SPWLA symposium transactions, Paper Z, pp 1–17

Alger RP (1971) Interpretation of electric logs in fresh water wells in unconsolidated formation. SPE Reprint Series vol 1, p. 255

Archie GE (1942) The electrical resistivity log as an aid in determining some reservoir characteristics. SPE, Trans AIME 146(1):54–62

Bartlett MS (1953) Factor analysis in psychology as a statistician sees it. In: Nordisk Psykologi's 456 Monograph 3. Almqvist and Wiksell, Uppsala, pp 23–34

Bear J (1972) Dynamics of fluids in porous media. Dover Publications Inc., New York

Carman PC (1937) Fluid flow through granular beds. Trans Inst Chem Eng 15:150–166

Csókás J (1995) Determination of yield and water quality of aquifers based on geophysical well logs (in Hungarian). Magy Geofiz 35(4):176–203

Dobróka M, Szabó NP (2011) Interval inversion of well-logging data for objective determination of textural parameters. Acta Geophys 59:907–934

Gálfi J, Liebe P (1981) The permeability coefficient in clastic water bearing rocks (in Hungarian). Vízü Közl 63(3):437–448

Glover PWJ, Walker E (2009) Grain-size to effective pore-size transformation derived from electrokinetic theory. Geophysics 74(1):E17–E29

Hazen A (1892) Some physical properties of sands and gravels. Annual Report. Massachusetts State Board of Health, Boston, pp 539–556

Jarzyna JA, Krakowska PI, Puskarczyk E, Wawrzyniak-Guz K, Bielecki J, Kwiatek WM, Gruszczyk M (2012) Spatial distribution of petrophysical properties on the basis of laboratory results, well logging and seismic data. Geosci Eng 1(2):87–92

Jöreskog KG (2007) Factor analysis and its extensions. In: Cudeck R, MacCallum RC (eds) Factor analysis at 100, Historical developments and future directions. Lawrence Erlbaum Associates, New Jersey

Juhász J (2002) Hidrogeology (in Hungarian). Akadémiai Kiadó, Budapest

Kirsch R (ed) (2009) Groundwater geophysics. A tool for hydrogeology, 2nd edn. Springer, Berlin

Kovács G (1972) Hydraulics of filtration (in Hungarian). Akadémiai Kiadó, Budapest

Kozeny J (1927) Ueber kapillare Leitung des Wassers im Boden. Sitzungsber Akad Wiss 136(2a):271–306

Larionov VV (1969) Radiometry of boreholes (in Russian). Nedra, Moscow

Lawley DN, Maxwell AE (1962) Factor analysis as a statistical method. Statistician 12:209–229

Ogbe D, Bassiouni Z (1978) Estimation of aquifer permeabilities from electric well logs. Log Anal 19(5):21–27

Pirson SJ (1963) Handbook of well log analysis. Prentice-Hall Inc., New York

Puskarczyk E, Jarzyna J, Porebski SJ (2014) Application of multivariate statistical methods for characterizing heterolithic reservoirs based on wireline logs—example from the Carpathian Foreland Basin (Middle Miocene, SE Poland). Geological Quarterly, in press

Rubin Y, Hubbard SS (2005) Hydrogeophysics. Water Science and Technology Library Series, vol 50. Springer, Dordrecht

Sallam O. M. 2006: Aquifers parameters estimation using well log and pumping test data, in arid regions—step in sustainable development. In: The 2nd international conference on water resources and arid environment, 26–29 Nov, Riyadh, pp 1–12

Szabó NP (2011) Shale volume estimation based on the factor analysis of well-logging data. Acta Geophys 59(5):935–953

Szabó NP, Dobróka M (2013) Extending the application of a shale volume estimation formula derived from factor analysis of wireline logging data. Math Geosci 45(7):837–850

Szabó NP, Dobróka M, Turai E, Szűcs P (2014) Factor analysis of borehole logs for evaluating formation shaliness: a hydrogeophysical application for groundwater studies. Hydrogeol J 22:511–526

Zilahi-Sebess L, Fancsik T, Török I, Kovács AC (2007) The possibilities of estimating hydraulic conductivity based on geophysical measurements (in Hungarian). Magy Geofiz 48(3):99–111